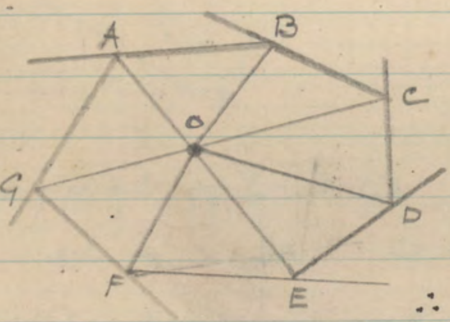
Regular Figure	No of equal Sides. n.	Angle at Centre $\frac{360}{n}$	1 Interior Angle	$2n-4$	Exterior Angle	Sum of Ext ∠s
Triangle	3	120°	60°	2	120°	360°
Square	4	90°	90°	4	90°	360°
Pentagon	5	72°	108°	6	72°	360°
Hexagon	6	60°	120°	8	60°	360°
Octagon	8	45°	135°	12	45°	360°
Decagon	10	36°	144°	16	36°	360°

Corollary 4 If the sides of a polygon are produced in order, the sum of the exterior ∠s so formed is equal to 4 rt ∠s.

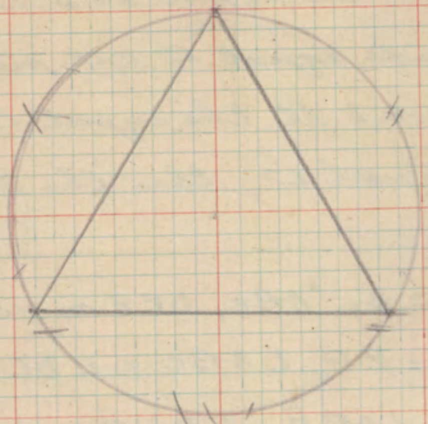


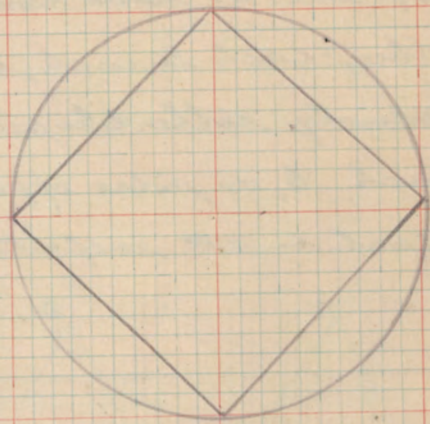
Consider the 7 sided figure ABCDEFG with it's sides produced in order, at every vertex, the sum of the exterior ∠ and the interior angle is two rt ∠s ∴ the sum of all the ∠s = 7 × 2 = 14 rt ∠s, but by the previous corollary the sum of the interior angles of a 7 sided figure is $2n-4$ rt ∠s where n = no. of sides, i.e in this fig 10 rt ∠s ∴ the sum of the exterior ∠s = 14 − 10 = 4 rt ∠s

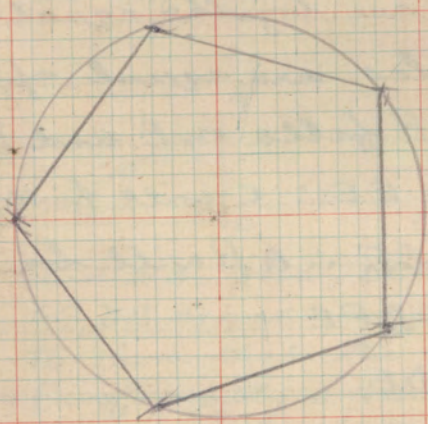
Alternate proof = 7 × 2 rt ∠s = 14 rt ∠s
∠s at Centre = 4 rt ∠s ∴ interior ∠s of figure = 10 rt ∠s.

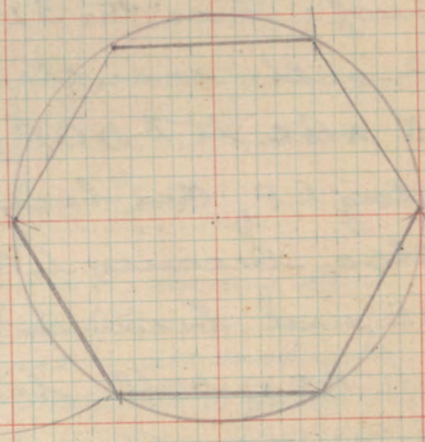
$2n - 4$

$6 - 4 = 2$ rt Ls



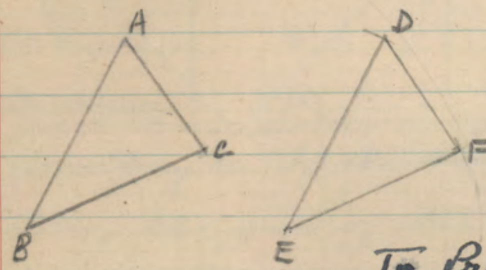






Proposition 16. Theorem

If two Δs have two $\angle$s of the one equal to 2 $\angle$s of the other, each to each, and one side equal to one side, namely, either the sides adjacent to the equal $\angle$s in each Δ, or sides opposite to equal $\angle$s; then the Δs are equal in all respects.



Given 2 Δs ABC, DEF in which BC = EF, $\angle ABC = \angle DEF$ and either $\angle ACB = \angle DFE$ or $\angle BAC = \angle EDF$.

To prove Δs are equal, in all respects

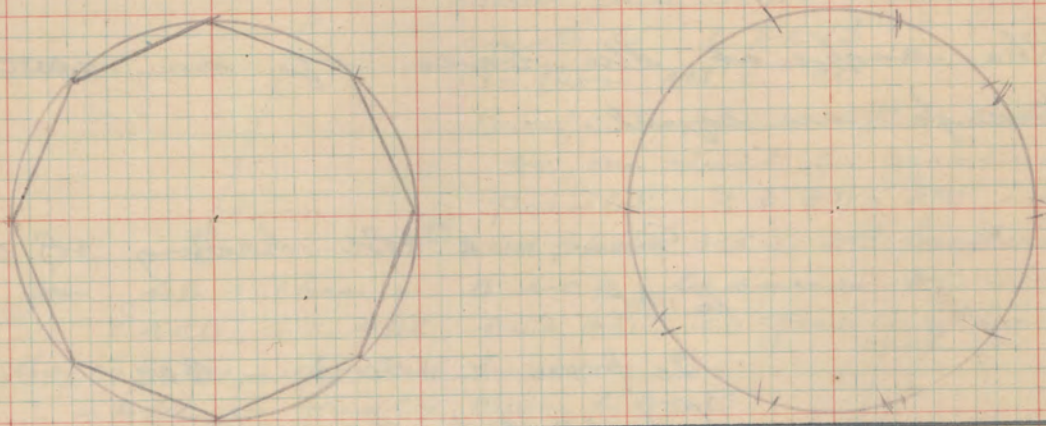
Proof: Since the 3 $\angle$s of a Δ are together equal to 2 rt $\angle$s in either case the 3rd angles are equal apply Δ ABC to Δ DEF so that B falls on E and BC lies along EF, then because BC = EF $\therefore$ C will fall on F also because $\angle ABC = \angle DEF$ $\therefore$ AB will be along DE, and because $\angle ACB = \angle DFE$

$\therefore$ AC will be along DF

The A the pt of intersection of AB + AC will coincide with D the pt of intersection of DE + DF

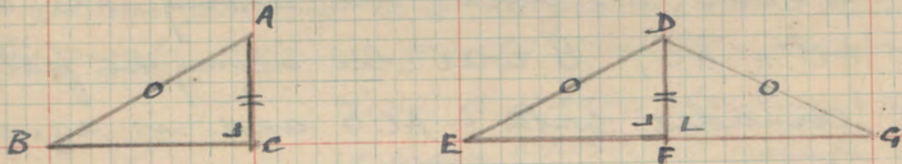
$\therefore$ the Δs coincide + are equal in all respects.

Q.E.D.



Proposition 17. Theorem

If two rt ∠d △s have their hypotenuses equal and one side of the one equal to one side of the other, the △s are equal in all respects.

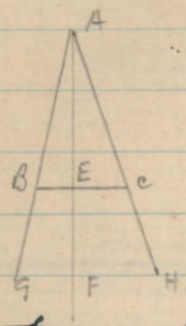


Given 2 △s ABC, DEF rt ∠d at C + F, having their hypotenuses AB, DE equal and AC equal to DF.
To Prove that the △s are equal in all respects.
Proof: apply △ ABC to △DEF so that A falls on D and AC lies along DF. Then because AC = DF ∴ C will fall on F. Let B fall on G on the side of DF opposite E. Now ∠DFE and ∠DFG are rt ∠s
∴ EFG is a st line and DEG is a △ but DE = DG
∴ ∠DEF = ∠DGF.
Then in △s DEF, DGF we have ∠DFE = ∠DFG (rt ∠s)
∠DEF = ∠DGF (proved) + DF common
∴ △s are equal in all respects
∴ △s ABC, DEF are equal in all respects.

Q.E.D

Proposition 9. Theorem.

The angles at the base of an isosceles triangle are equal.



Given a $\triangle$ ABC having AB equal to AC.

To prove $\angle$ ABC = $\angle$ BAC

~~Let AF cut BC in E~~

~~Proof~~ Cons. Draw AF bisecting $\angle$ BAC

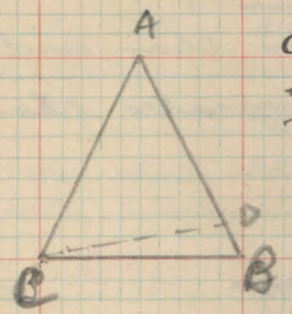
let AF cut BC in E

Proof: Fold $\triangle$ ABC about AF then because $\angle$ BAE = $\angle$ CAE, AB will lie along AC, and because AB = A'C, B will ~~lie along AC~~ fall on C $\therefore$ $\angle$ ABC coincides with $\angle$ ACB and is equal to it. Q.E.D.

Suppose AB be produced to G and AC to H, then $\angle$ CBG = $\angle$ BCH for these are supplements of equal angles ABC, ACB.

Proposition 10 Theorem.

If two ∠s of a triangle are equal, the sides opposite to them are equal. [33]



Given a Δ ABC having ∠ABC equal to ∠ACB.

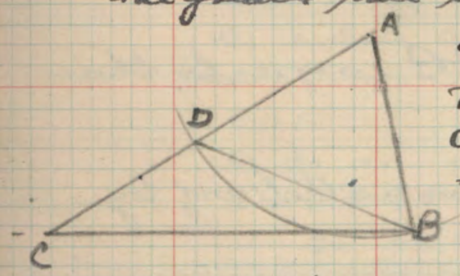
To prove AB = AC.

Proof: Suppose AB is not equal to AC, one must be greater. If possible let AD = AC, where D is a pt in AB or AB produced

Join CD. Then in Δ ACD because AC = AD
∴ ∠ACD = ∠ADC, but ADC is greater than ∠ABC the interior opposite ∠
∴ ∠ADC is greater than ∠ACB
∴ ∠ACD is greater than ∠ACB, the part greater than the whole, which is impossible.
∴ AB cannot be unequal to AC
∴ AB = AC QED

Prop 11. Theorem

If one side of a Δ is greater than another the ∠ opposite the greater side is greater than the ∠ opposite the less.



Given a Δ ABC having AC greater than AB

To prove ∠ABC is greater than ∠ACB

Cons: with centre A & radius AB describe an arc cutting AC in D. Join BD

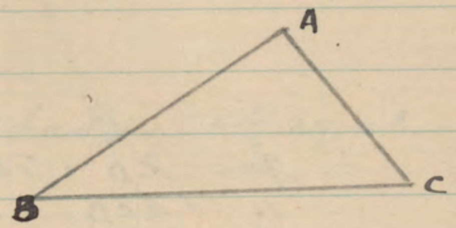
Proof In Δ ABD, because AB = AD
∴ ∠ ABD = ∠ADB

but the exterior ∠ ADB is greater than interior opposite ∠ACB ∴ ∠ABD is greater than ∠ACB
again the whole ∠ABC is greater than its part ∠ABD
still more, then, is ∠ABC greater than ∠ACB

Q.E.D.

Prop 12. Theorem

If one ∠ of a Δ is greater than another, the sides opposite the greater ∠ is greater than the side opposite the less



Given a Δ ABC having ∠ABC less than ∠ ACB.

To prove AB is greater than AC.

Proof: AC may be either greater, than, equal to, or less than AB. If AC is greater than AB ∴ ∠ABC is greater than ∠ACB but by hypothesis this is not so.

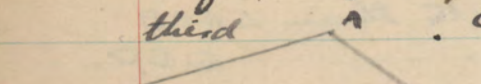
again, if AC = AB, ∠ABC = ∠ACB, but by hypothesis, this is not so

∴ AC is less than AB ie AB is greater than AC. Q.E.D.

Note. This method of proof is known as the Proof by Exhaustion.

Prop 13 Theorem

Any two sides of a Δ ABC are together greater than the third



Given Δ ABC. To Prove that BA + AC together greater than BC.

Proof: Because BC is a st line

∴ It is the shortest distance between 2 pts B + C

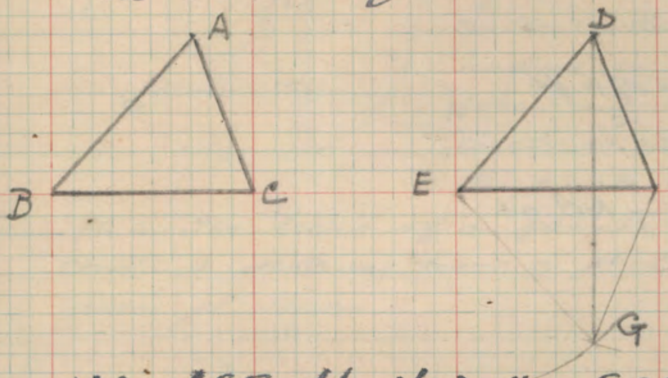
∴ any other way from B to C such as along BA + AC is greater than BC. Q.E.D.

Prop 14. Theorem

If any 2 ∠s have 2 sides of one equal to 2 sides of the other respectively and the ~~other~~ included ∠s equal, the Δs are equal in all respects.

Prop 15 Theorem.

If two Δs have the 3 sides of one equal to the 3 sides of the other respectively & the included ∠s equal the Δs are equal in all respects.



Given 2 Δs ABC, EDF, in which AB = DE, BC = EF, AC = DF

To Prove Δs are equal in all respects.

Proof apply Δ ABC to Δ DEF so that B falls on E and BC on EF, Then because BC = EF, C will fall on F. Let A fall at G on the side of EF opposite D then EGF is the new position of Δ ABC Join DG. In Δ EDG because ED = EG

∴ ∠EDG = ∠EGD

also in Δ FDG, because FD = FG ∴ ∠FDG = ∠FGD

∴ the whole ∠EDF = ∠EGF. Then in Δs EDF, EGF we have ED = EG, FD = FG, and included ∠EDF = included ∠EGF

∴ Δs are equal in all respects

But Δ EGF is Δ ABC in a new position

∴ Δs ABC, DEF, are equal in all respects. Q.E.D.

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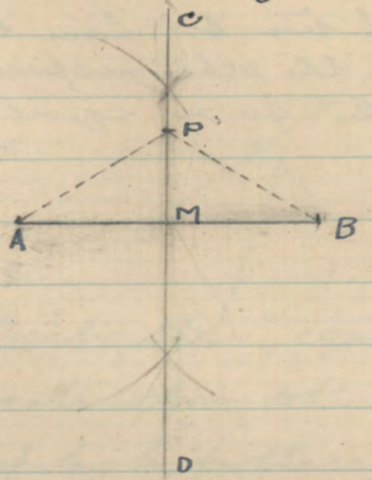
Proposition 19. Theorem.

The locus of a pt equidistant from two fixed pts is a st line bisecting at rt angles the line joining the two fixed points.



Given: two pts A and B.

To prove: that the locus of a pt equidistant from A and B is the perp. bisector of AB.

Cons: Draw CD bisecting AB at rt ∠s at the pt M.

Take any pt P in CD.

Join PA, PB.

Proof. In Δ PMA, PMB we have

AM = BM (Cons.) PM common, & included ∠s PMA, PMB equal being rt ∠s (Cons)

∴ Δs are equal in all respects ∴

∴ PA = PB

i.e., P is equidistant from A and B.

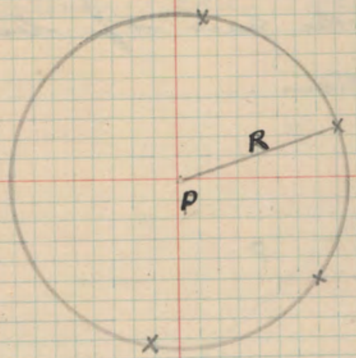
In the same way it may be shown that any other pt in CD is equidistant from A & B also that any pt outside CD is not equidistant from A & B.

∴ CD which bisects AB ~~and~~ at rt ∠s is the locus of a pt equidistant from A and B.

Q.E.D

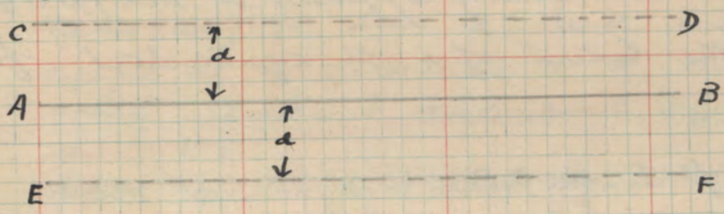
16/3/25 ~~Proposition 19.~~ Loci

Definition. If a point may move but is subject to certain restrictions, the path which it traces out is called locus.



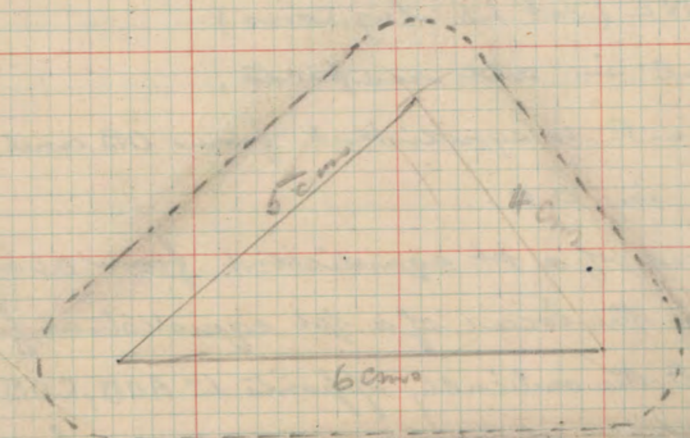
Example. a ~~position~~ point moves but keeps at a constant distance R from a fixed point P. The path it traces out is a circle, in other words the locus of the moving point is a circle whose centre is the fixed pt P. and whose radius is the constant distance R. When we speak of a locus being a circle, it is the O^ce (circumference) of the circle that is meant.

Example 2. The locus of a point which moves keeping at a constant distance from a st line is a st line // to the given st line.



AB is the given st line
d ——— distance from it
CD and EF are the loci

Exercise Draw a Δ 4, 5 & 6, cms long. Draw the locus of a pt which moves, keeping 1 cm from the sides.



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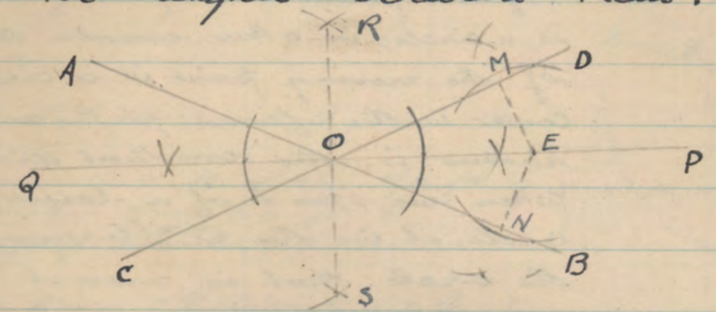
Proposition 20. Theorem

The locus of a point equidistant from two given intersecting straight lines is the pair of bisectors of the angles between them.



Given two st lines AB, CD intersecting at O and PQ bisecting the vertically opposite $\angle^s$ AOC, BOD

To prove PQ is the locus of a pt equidistant from AB and CD.

Cons: Take any pt E in OP.

Draw EM perp to OD and EN perp to OB

Proof. In Δs OME, ONE we have

$\angle$ MOE = $\angle$NOE (cons), OE Common, and

$\angle$ OME = $\angle$ONE (rt $\angle$s by cons)

$\therefore$ Δs are equal in all respects,

$\therefore$ EM = EN i.e. E is equidistant from OB and OD and E is any pt in OP.

$\therefore$ OP is the locus of a pt equidistant from OD and OB

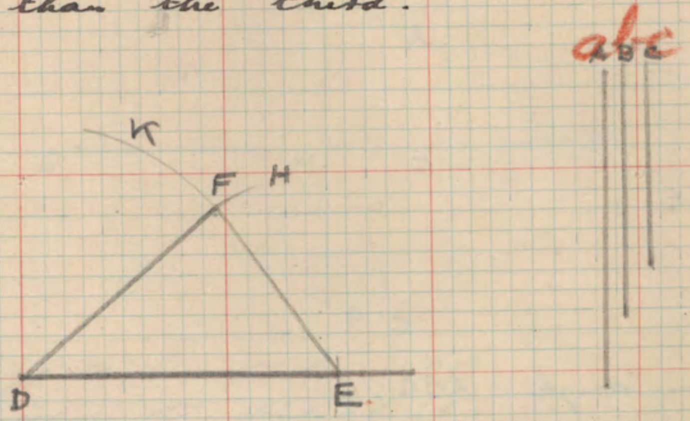
Similarly OQ is the locus of a pt equidistant from OA + OC

Also if RS bisects the vertically opposite $\angle^s$ AOD, COB it may be shown to be the locus of a pt equidistant from AO and OD, OC and OB. Q.E.D.

Note PQ and RS are at rt $\angle^s$.

Proposition 21 Problem

To describe a Δ having its sides equal to three given st lines, any of which are together greater than the third.



Let A, B, C be the 3 given st lines, any two being together greater than the third

Draw ~~Take~~ any st line DE equal to a and with centre D and radius equal to A describe a ⊙ cutting DE ~~at~~ at E
~~with Centre D and rad. equal to B describe a ⊙ KF~~
with centre E + rad. equal to C describe a ⊙ HF Cutting KF at F join DF, EF

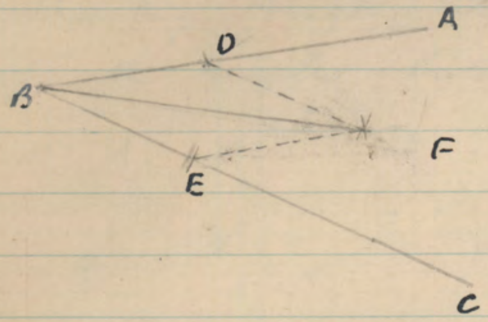
then DEF is the req. Δ

DE = A, DF = B and EF = C. Q.E.F

16th Feb 1935. Form C.

Proposition 22. Problem

To bisect a given angle



Given an angle ABC.

It is required to bisect it.

Cons: With centre B and any convenient radius describe an arc cutting AB in D and BC in E with centres D and E and the same radius describe arcs meeting in F.

Join BF.

Then BF bisects the angle ABC. Join DF, EF.

Proof: In Δs BDF, BEF we have

BD = BE (radii), BF common,

and DF = EF (radii of equal $\odot$s)

$\therefore$ Δs are equal in all respects

$\therefore$ $\angle$ DBF = $\angle$ EBF

Q.E.F.

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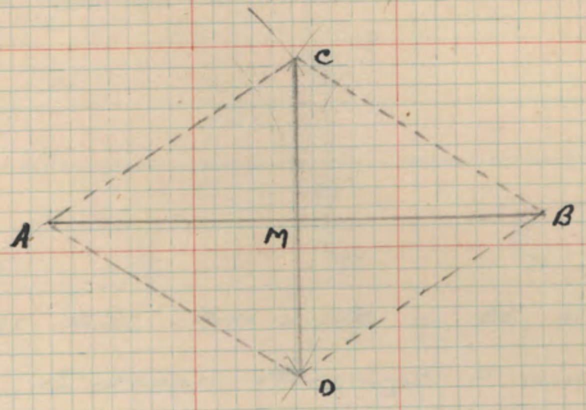
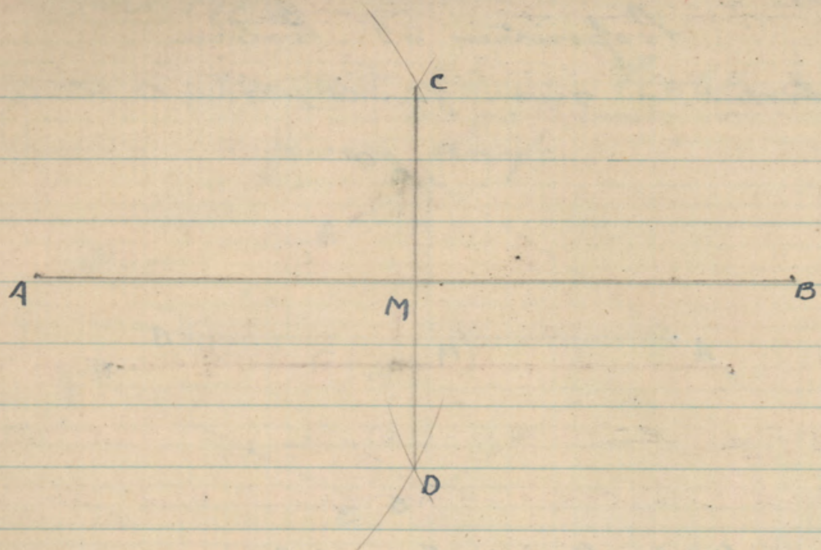
Proposition 23 Problem.

To bisect a given finite straight line

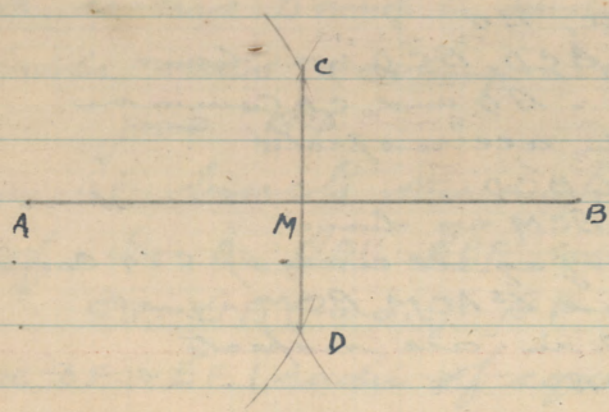


Given a finite straight line AB.
It is required to bisect it.
Cons: With centre A and any convenient radius draw arcs.
With centre B and ~~any~~ same radius draw arcs cutting
the first ~~and~~ at C and D.
Join CD and let CD cut AB in M
Then AB is bisected at M as required.
Join AC, AD, BC, BD
Proof. In the Δs ACD, BCD we have
AC = BC, AD = BD and CD common
$\therefore$ Δs are equal in all respects
$\therefore$ $\angle$ ACD = $\angle$ BCD.
Then in Δs ACM, BCM we have
AC = BC (radii of equal circles) CM common
and the included $\angle^s$ ACM, BCM equal
$\therefore$ Δs are equal in all respects
$\therefore$ AM = MB
i.e AB is bisected at M.

Q.E.F.



Exercise 1. AB = 3.7"



Ex: 2. AB = 6.9 cm

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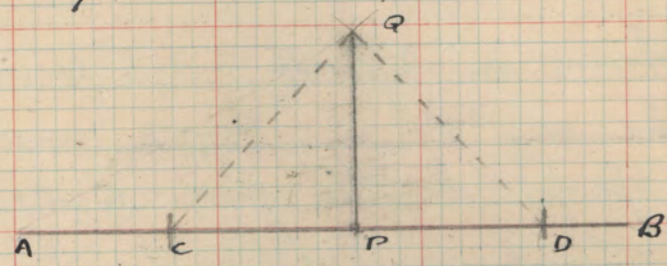
Proposition 24. Problem

To draw a perpendicular to a given straight line from a given point in it.



Given a straight line AB with a pt P in it

It is required to draw through P a str line perp to AB.

Cons With centre P and any convenient radius describe a ⊙ cutting AB in C and D

With centres C and D and any convenient radius describe arcs intersecting in Q.

Join PQ.

Then PQ is the perp required

Join QC, QD

Proof In Δs CPQ, DPQ we have

PC = PD (radii of same ⊙)

QC = QD (radii of equal ⊙s) and PQ common

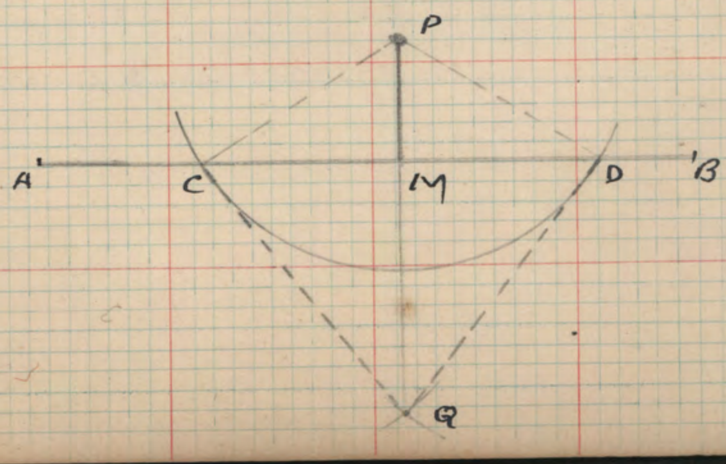
∴ Δs are equal in all respects

∴ ∠ QPC = ∠ QPD, and these are adjacent angles

∴ PQ is perp to AB. Q.E.F.

Proposition 25 Problem

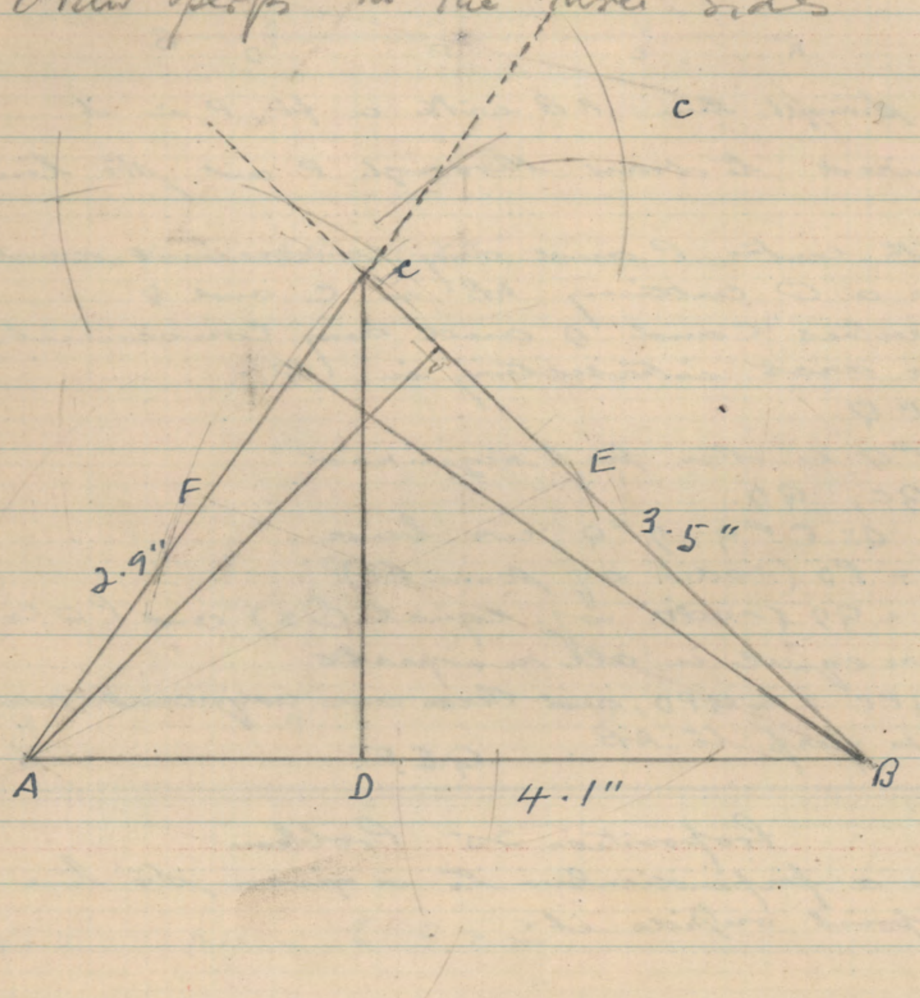
To draw a perpendicular to a given str line from a given point outside it.



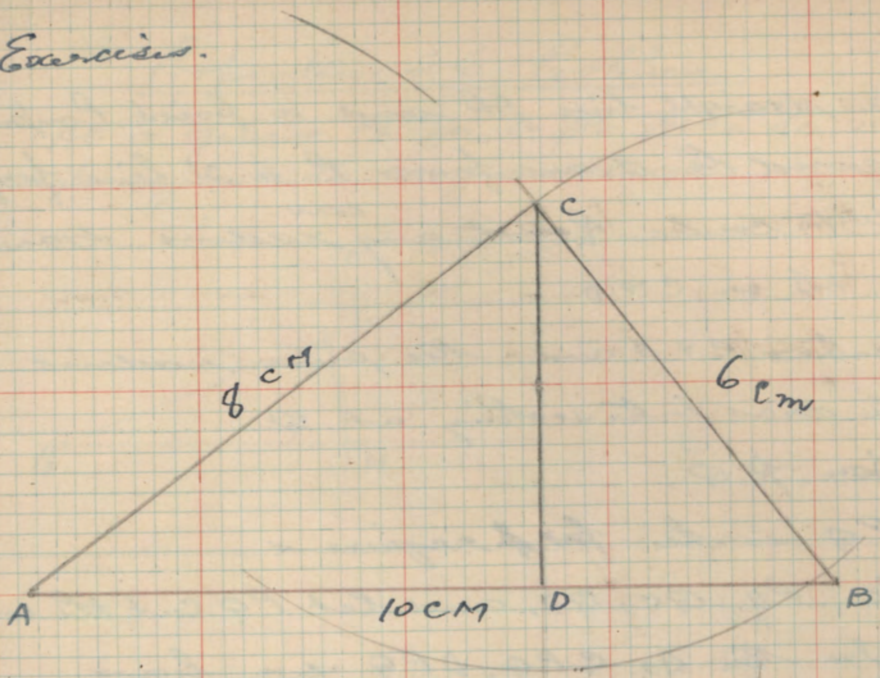
Exercises.

1\. Draw a Δ with sides 4.1", 3.5", 2.9"

B. Take any Pt in the Δ and from it draw perps to the three sides



Exercises.



The length of CD = 4.8

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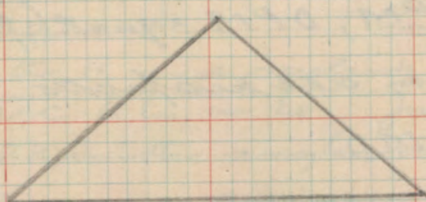
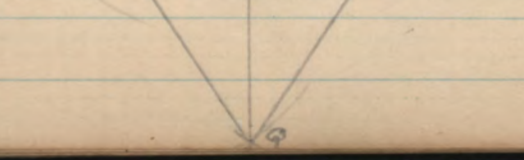


Given a straight line AB and a point P outside it.
It is required to draw from P a st line perp to AB
Cons. With centre P and any conv radius draw a ⊙
cutting AB in C & D.
With centres C & D and the same radius
describe arcs intersecting at Q.
join PQ.
Then PQ is the ~~perp~~ required
~~join~~ PC, PD, QC, QD, and let PQ cut AB in M.
Proof. In the $\triangle$s CPQ, DPQ we have
CP = DP (radii of the same ⊙)
CQ = DQ (radii of equal ⊙s) and PQ common
∴ $\triangle$s are equal in all respects
∴ $\angle CPQ = \angle DPQ$
then in $\triangle$s CPM, DP~~M~~ we have
CP = DP, PM common
& included $\angle$s CPM, DPM equal (proved)
∴ $\triangle$s are equal in all respects
∴ $\angle PMC = \angle PMD$
and these are adjacent $\angle^s$
∴ PM is perp to AB. Q.E.F.



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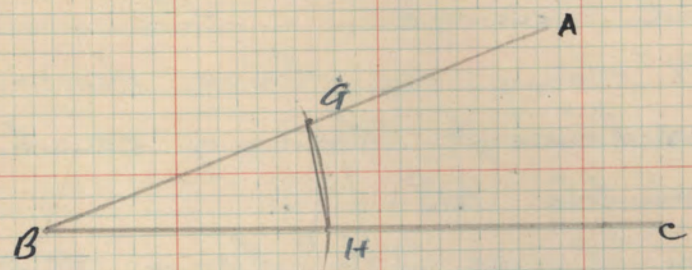
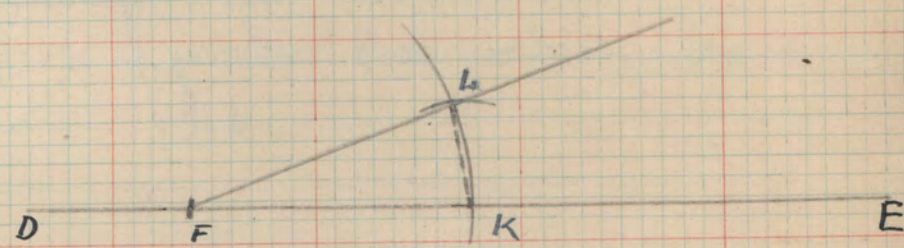
Proposition 26. Problem.

At a given pt in a st line to make an angle equal to a given angle.



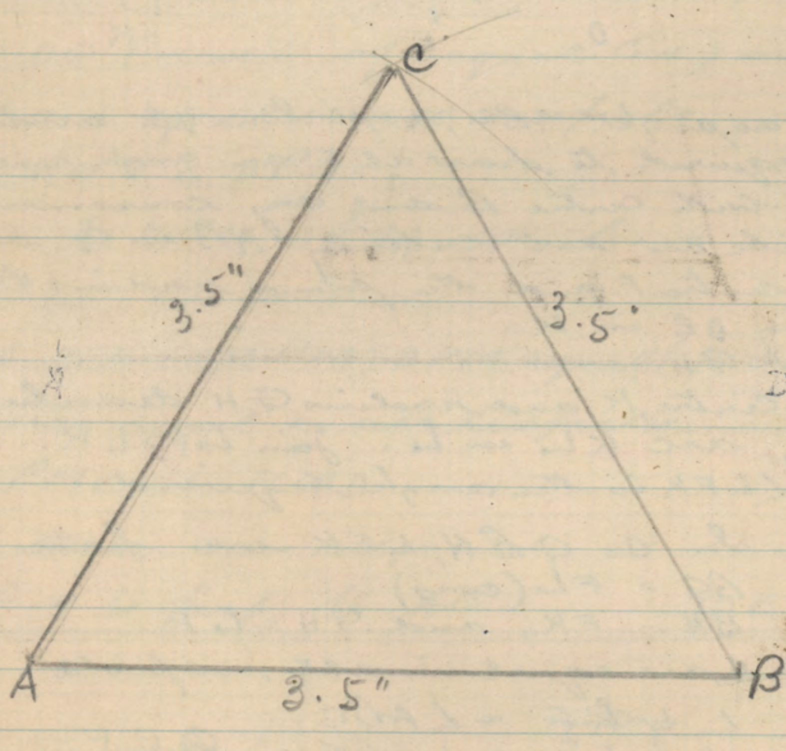


Given an angle ABC, a st line DE and a pt F in it.
It is required to draw at F an angle equal to the $\angle$ ABC.
Cons. With centre B and any convenient radius describe an arc cutting AB in G and BC in H
With centre F and the same radius draw an arc cutting DE in K.
Join GH.
With centre K and radius GH describe an arc cutting arc KL in L. Join LF, LK.
Then $\angle$LFK is the angle required.
Proof. In Δs GBH, LFK we have
BG = FL (cons)
BH = FK, and GH = LK
$\therefore$ Δs are equal in all respects
$\therefore$ $\angle$ L ~~KF~~ = $\angle$ABC.
$\angle$LFE = $\angle$ABC Q.E.F.

Exercises.

1. On a given st line AB 3.5" long describe an equilateral triangle.
2. On AB 1.7" long as base describe an isosceles triangle having each of its sides double AB.
3. On a given base 2.6" long describe an isosceles Δ having each of its sides 1.9" long.

1.



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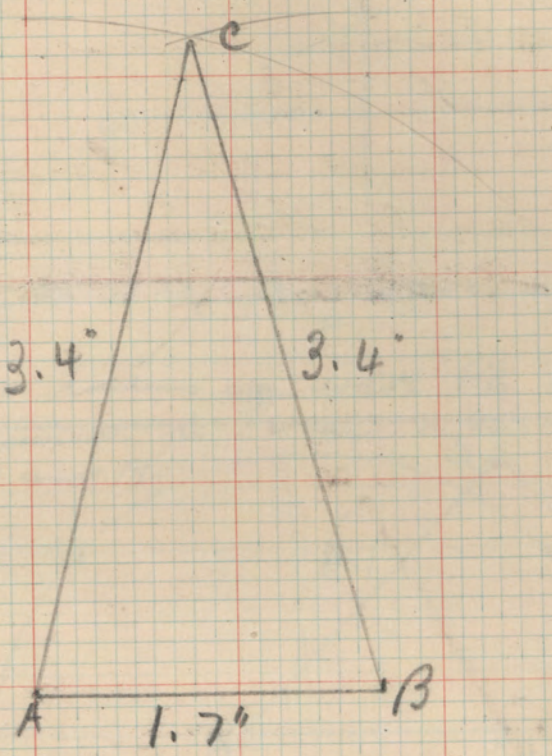
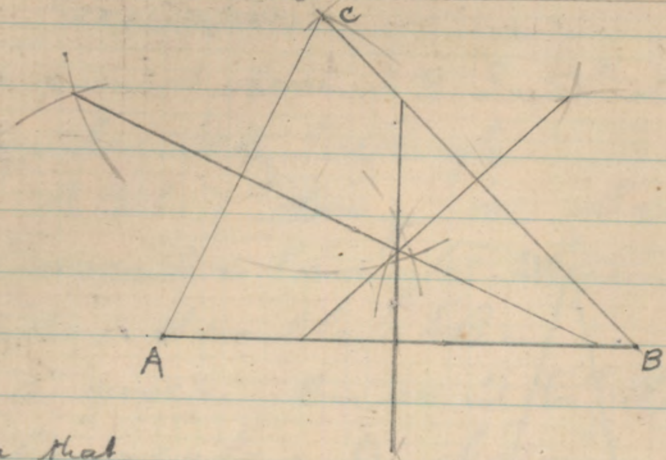


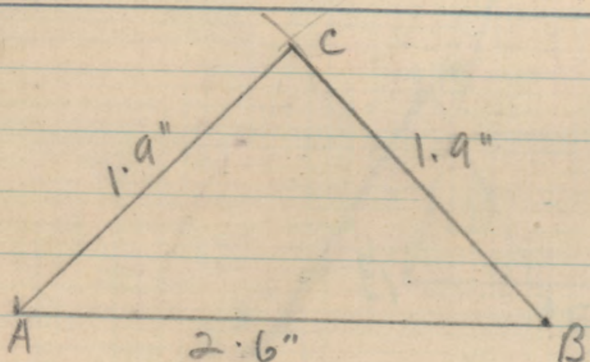
Prop 19 Exercise Draw a scalene Δ and draw the perpendicular bisectors of the three sides. What do you notice about these three new lines?

[50]
17/3/20



We notice that
the perpendiculars meet at a certain pt in the Δ ABC

3

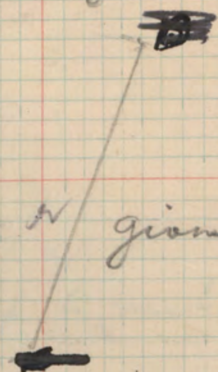


4. Describe a $\odot$ of given radius which shall pass through two given points.
When is this problem incapable of solution? (write out this in full.)

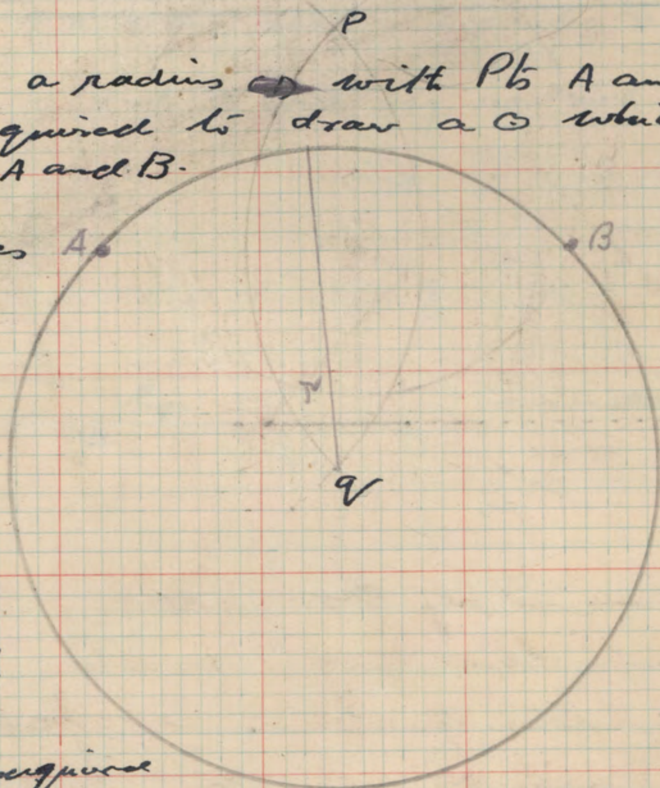
5. Find a pt equidistant from two given pts A & B.

6. With ruler & compasses make a right angle
State your construction and give a proof

4/ This problem is incapable of solution when the radius given is shorter than half the distance between the given pts. [51]



Given a radius with Pts A and B
It is required to draw a ⊙ which shall pass through A and B.



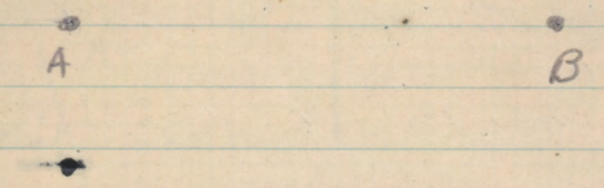
Cons: with centres A + B and the given radius describe arcs intersecting in q.
with pt q and the same radius describe a ⊙ cutting through A + B.

∴ this is the ⊙ required

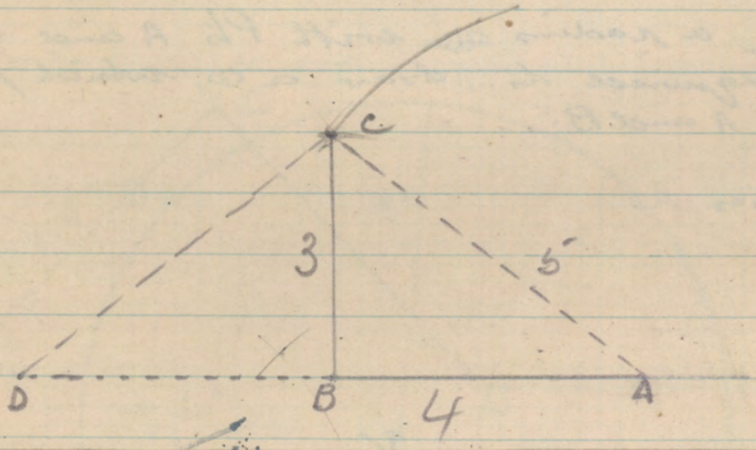
Q. E. F.

[52]

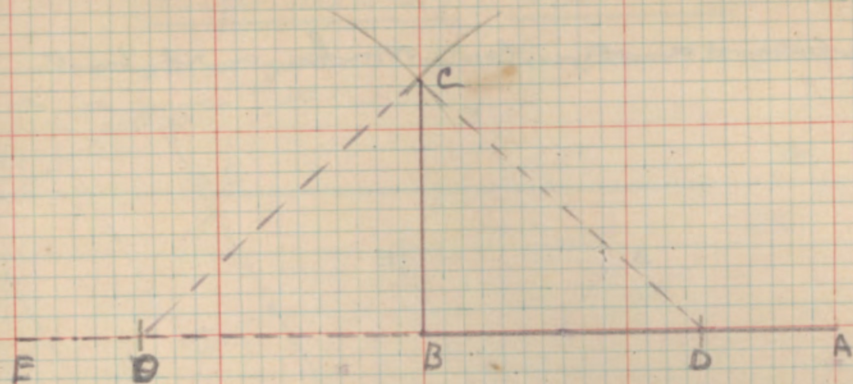
5



6.



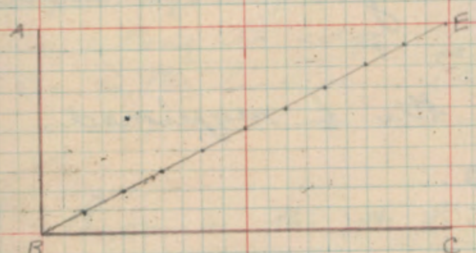
6.



Given a st line AB
It is required to draw through B a perp to make the angle a rt L.
Cons. From B produce a line towards E.
With Centre B and any radius describe a ⊙ cutting AE in D + O.
With Centres O and D and any radius describe arcs

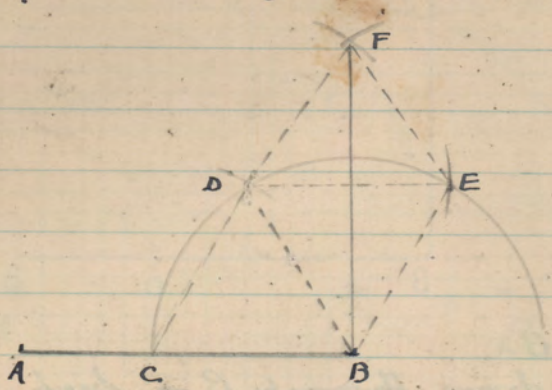
Page 32. Exercises on Loci

No 3



[54]

6. With compasses and ruler make a rt ∠.



Given a st line AB

It is required to draw a st line from B at rt ∠s to AB.

Cons: With centre B and any convenient radius draw a ⊙ cutting AB in C.

With centre C and the same radius describe an arc cutting the ⊙ in D.

With centre D and the same radius describe another arc cutting the ⊙ in E.

With centres D and E and any convenient radius describe arcs intersecting in F

Join BF. Then ABF is the ∠ required.

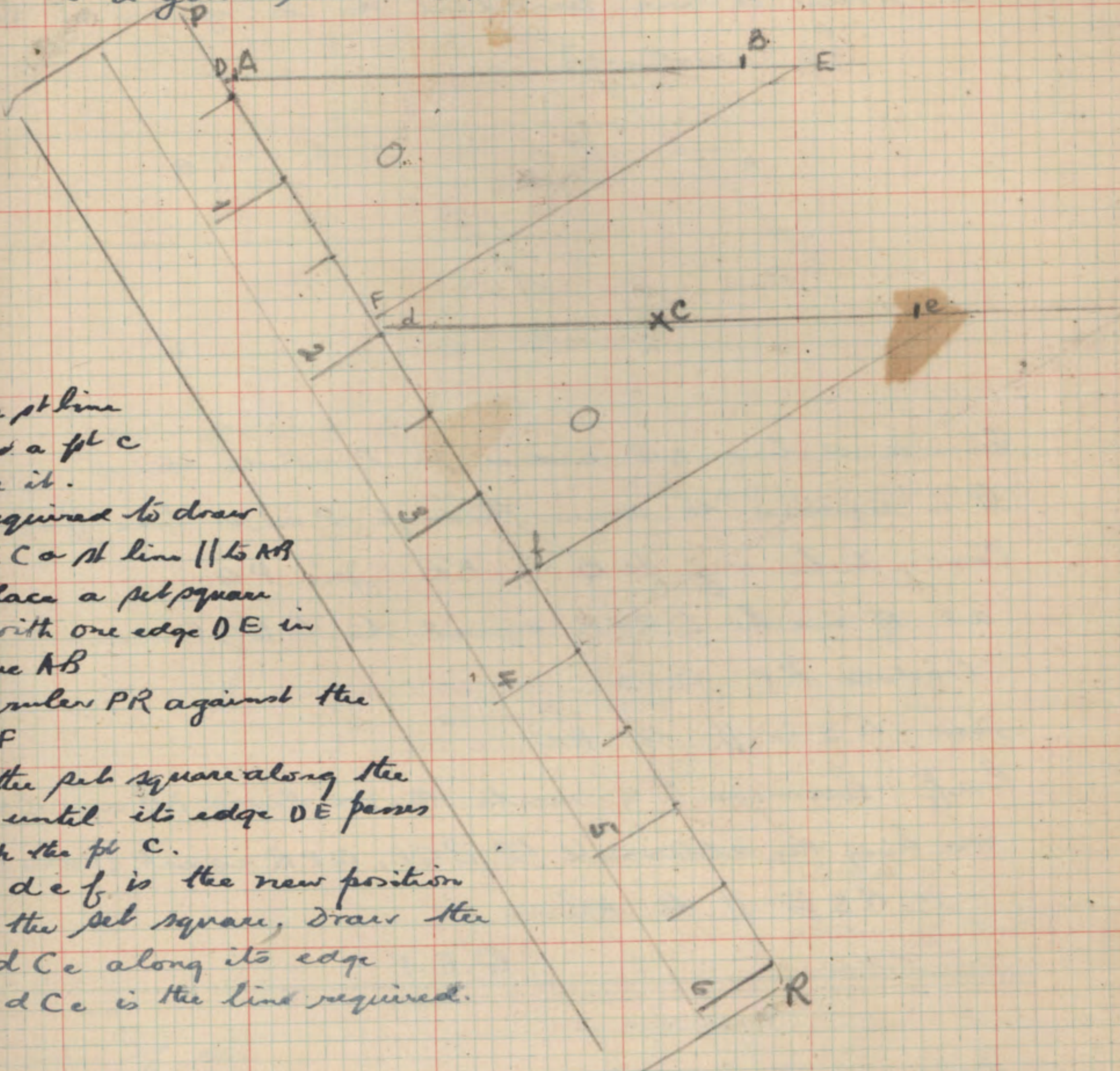
~~Proof~~ Join DE, DF, EF, EB, BD.

Proof

9/3/35

Proposition 27. Theorem

Through a given pt to draw a st line parallel to a given st line.

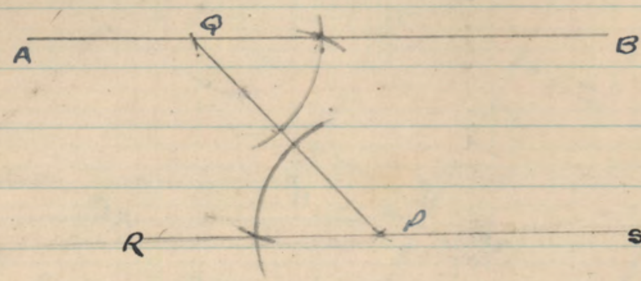


Given a st line AB and a pt C outside it.
It is required to draw through C a st line || to AB
Con. Place a set square DEF with one edge DE in the line AB
Lay a ruler PR against the edge DF
Slide the set square along the ruler until its edge DE passes through the pt C.
When d e f is the new position ~~of~~ of the set square, draw the line d C e along its edge
Then d C e is the line required.

Proof. Because the st line PR on the ruler meets the st line DE and de making the ∠ EDF equal to the ∠ edf (each being the same ∠ of the set square)
∴ de is || to AB. Q.E.F.

Proposition 27. Problem

To draw a line parallel to a given st line
[2nd method]

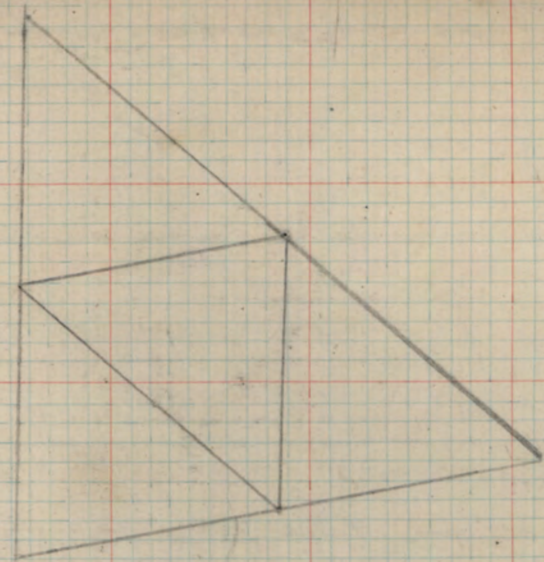


Given a st line AB and a pt P outside it.
It is required to draw through P a st line || to AB
Cons. Take any pt Q in AB. Join PQ.
at P make ∠ QPR equal to ∠ BQP
Produce RP to S
Then RS is || to AB
Proof Because PQ meets the 2 st lines AB and RS
making the alternate ∠s BQP, QPR equal (Cons)
∴ RS is || to AB.

Q.E.F

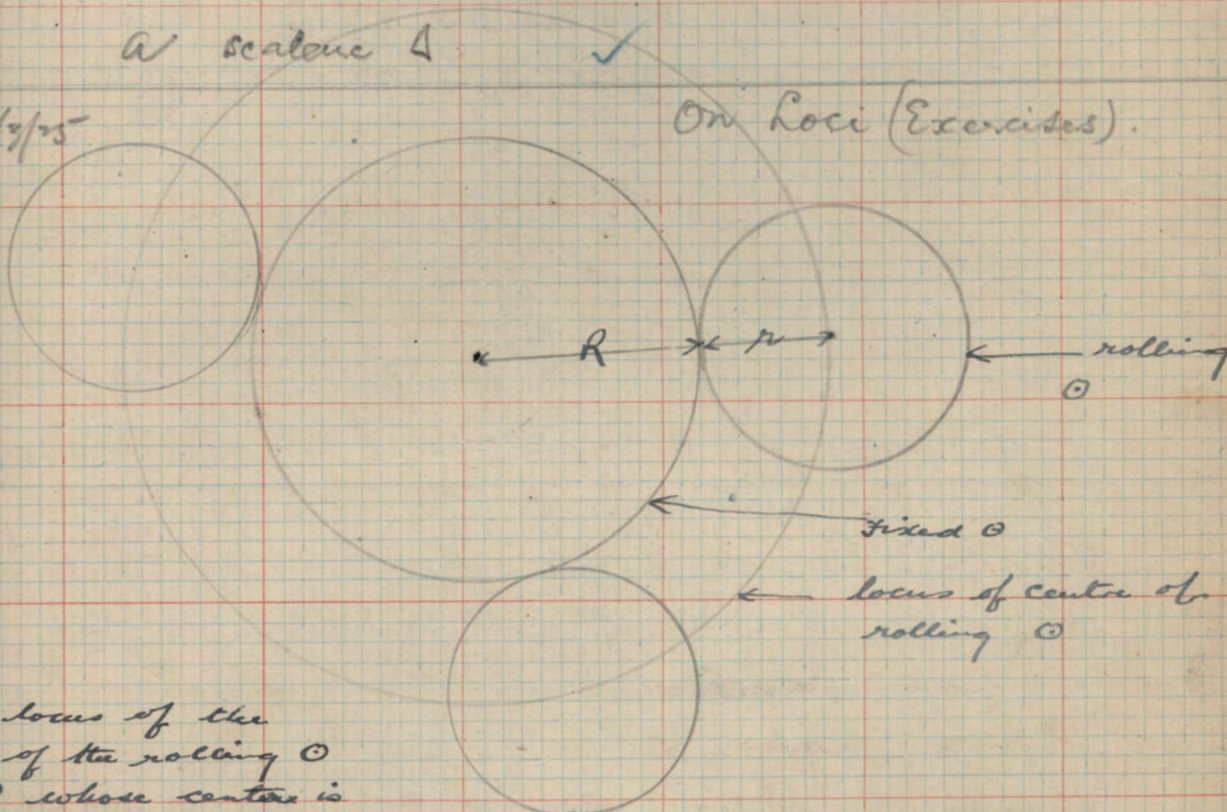
Exercise

Draw a scalene △.
Through its vertices draw st lines || to the opposite sides
so as to form a new △



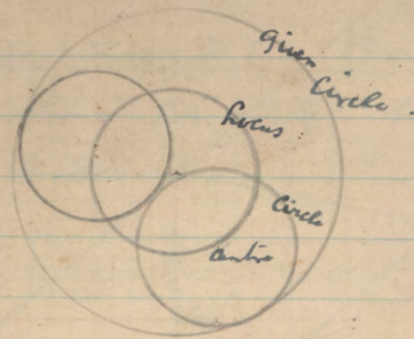
a scalene Δ ✓

24/3/25 On Loci (Exercises).



1 The locus of the centre of the rolling ⊙ is a ⊙ whose centre is the centre of the fixed ⊙, and whose radius is the sum of the 2 ⊙s.

No 2



The locus of the centre of a rolling ⊙ is the ⊙ ~~of who the ⊙~~ whose centre is the centre of the fixed ⊙ and whose radius is from the centre of the ~~given~~ fixed ⊙ to the centre of the rolling ⊙

No 4.

The locus of the middle pt of a st line is a st line draw ‖ to the given st line of unlimited length