INVESTIGATING OPPORTUNITIES TO LEARN GRADE TEN ALGEBRA: CASE STUDIES OF THREE CATHOLIC SECONDARY SCHOOLS.

BY

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I declare that Investigating Opportunities To Learn Grade Ten Algebra : Case Studies Of Three Catholic Secondary Schools is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references

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SIGNATURE   DATE

(Sr)

This thesis is dedicated to Mother Agnes SJI for believing in me and inspiring me to work hard, all those years ago.

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List of acronyms

C2005   Curriculum 2005

FET   Further Education and Training

NCLB   No Child Left Behind

NCS   National Curriculum Statement

NCTM   The National Council of Teachers of Mathematics

PROM/SE   Promoting Rigorous Outcomes in Mathematics and Science Education

OBE   Outcomes Based Education

OTL   Opportunities to Learn

SACMED   Southern African Consortium for Monitoring Educational Quality

TIMSS   Third International mathematics and Science Study

USAID   United States Agency for International Development

RESEARCH SUMMARY

The purpose of this thesis is to investigate opportunities to learn (OTL) algebra by grade ten learners at three Catholic secondary schools in South Africa. Performance in mathematics is poor and is a great cause for concern. Despite the government's effort to make education open and available to all, underperformance has continued among the black majority who were previously marginalised in the former regime. This thesis focuses on the OTL which are afforded learners who are given the chance to attend classes.

This thesis met its aims through an extensive review of related literature and the implementation of practical research. The latter was carried out through case studies conducted in three schools where lessons were observed and interviews conducted with the respective teachers. Literature on how OTL mathematics are created is lacking in South Africa. Real OTL still needs to be created if the expected level of performance is to be achieved.

The research produced a number of key findings: the learners were given the right to attend class but were subjected to different OTL, learning to convert within and between the different registers of representation of algebraic concepts is necessary to provide learners with OTL, it is not enough for learners to master certain facts and procedures, and learning is enhanced if the means to make the conversion necessary for concept building is developed and the OTL provided. The teacher's approach influences the way OTL are realised and utilised by learners.

The main conclusion drawn from this research is that the OTL afforded the grade ten learners were not the same and that different chances to make conversion within and between registers of representation of algebra concepts were given. Giving the teachers guidelines without expounding the meaning of specific terms such as 'convert' leaves gaps in their practices and results in some learners receiving adequate OTL and others not.

This research argues for a more involved capacity building programme for in-service teachers to acquaint them with the expected learner-centred approaches to lesson delivery as well as familiarise them with the terminology used in defining terms in the syllabus.

Key terms:

Opportunities to learn, Algebra, Teaching and Learning, Intended and enacted curriculum, Procedural, Constructivism, Registers of Representation, Mathematical Knowledge.

CHAPTER ONE

STUDY BACKGROUND

1.1   Introduction

> *I loved grade ten algebra.*
> *I've never been more sure*
> *About the world and my place in the world*
> *Than I was in grade ten math class.*
> *I teased out intricate equations*
> *With unknowns of x and y,*
> *And glowed with the confident knowledge*
> *I could always find the correct answers*
> *In the back of the book.*
>
> (Poem by Leggo, 2008:187, in response to Skovsmose, (2008:168))

The learner in the above poem loved grade ten algebra because he could "tease out intricate equations with unknowns of x and y." Is this all there is to grade ten algebra? The grade ten math class described sounds like a mysterious place where one performs tricks of seemingly no value as the answers were printed at the back of the book. The learner's confidence depends on this knowledge. The learner appreciates the opportunity for self-evaluation because he can find the answers in the back of the book. It seems as though the learner has little understanding about what is actually taking place and is consequently denied the opportunity to learn algebra meaningfully. The educator is absent from the poem and the textbook is the only authority recognized, especially as it confirms his learning. It is possible that this is the kind of learner who is independent and frequently works out problems on his own during the mathematics lessons. The learner's situation must be understood in the context of the view that education is a multilevel, complex and highly contextualized system (Gau, 1997). The question to ask is to what extent is this learner's experience and practice typical of most grade ten learners?

1.2   Motivation

My interest is in studying curriculum in general and the learning and teaching of secondary school mathematics in particular. Within the subject of mathematics, algebra is of special concern. Early on in my teaching career, I realized that algebra plays an important role in the general performance of learners of mathematics. Consequently, I often used an algebra test to select students for any advanced level mathematics course. These days I teach in an education system where the screening of learners on the grounds of performance in algebra is neither desirable nor permitted. I am motivated to provide opportunities to learn (OTL) mathematics to all our learners.

Though the term OTL will be explained in greater detail in the chapters that follow, it is appropriate to refer to it as an ideal for the learning situation. As a concept in education OTL can help in the pursuit of good practice in the learning and teaching situation because it takes into consideration all stakeholders, from those who make decisions about what is to be taught, those who implement it, to the learner and the supporting materials available to all. For this reason, I wished to better understand how groups of grade ten learners comprehend and learn algebra, an important section of mathematics. Three secondary Catholic schools in South Africa were identified in which this study could take place. Many questions were posed, for example, what opportunities does the system provide to actually develop competence in this area of mathematics?

Algebra is one of the main branches of mathematics to which learners are gradually exposed throughout high school. It is described as a generalized form of arithmetic where symbols, letters and signs are used in place of or together with numbers. These symbols have different meanings and interpretations in different situations. Many students seem to have different perceptions about these symbols, letters and signs and this affects their understanding of the mathematics involved. In South Africa for example, within the Further Education and Training (FET) band (Grades 10-12), as part of Learning Outcome 2 (Functions and Algebra), learners are expected to multiply, factorize, and simplify different algebraic expressions up to and including trinomials, solve linear equations, quadratic equations by factorisation,

exponential equations of the form $ka^x+p = m$, linear inequalities in one variable and illustrate the solution graphically, and linear equations in two variables simultaneously (numerically, algebraically and graphically) (Department of Education, 2005). This suggests a skills-based perspective on algebra.

From the above, it is clear that knowledge of algebra is a critical part of mathematical achievement. Furthermore, the skills required are needed when handling other topics such as geometry and trigonometry because at some stage these problems demand algebraic manipulation. It follows then that a weak mastery of algebra disadvantages learners in the other topics and indeed in all areas that require the problem solving skills that are developed during algebra classes. Although not every learner is expected to reach a high level of proficiency in algebra, it is important that they be afforded the opportunity to learn it. If learners acquire knowledge of algebra they can apply or use it in other areas where such knowledge is a prerequisite or an advantage.

The research challenge therefore is to examine how algebra can be learnt and understood so that there can be a general improvement in performance in mathematics. This study explores what opportunities are provided for grade ten learners in selected schools to learn algebra. Once again, algebra is important because it is useful in linking different areas of mathematics and in preparing students for college, the workforce, and citizenship.

In this study, the officially prescribed algebra content is compared with what actually takes place in the classrooms. Is what is expected reflected in the learners' workbooks as well in the educator's schedules? It is important to find out what intellectual sense the FET learners make of the algebra they are expected to learn. What do they actually learn in class? How do they learn this algebra? This study explores the learning of algebra, analyses the results and formulates a picture of the opportunities that students in some classrooms in South Africa have.

I have chosen Catholic secondary schools because the Catholic Church has long been involved in schooling in South Africa: their first school opened in 1949 (Grace, 2010). Potterton and Johnstone (2007) note that Catholic schools form a small

fraction of the total number of schools in South Africa, yet their influence has been both remarkable and enduring. Catholic schools in South Africa are distinctive because of the philosophy underpinning their approach to education. The dignity of the child is seen as central to process of education.

Worldwide, the Catholic Church encourages co-operation and co-existence of diverse educational institutions in order to safeguard her objectives in the face of cultural pluralism (The Catholic School, 1997). Thus, while policies and opportunities differ from place to place, a Catholic school has a place in any national school system (Vatican II, 1966).The Catholic church, particularly through her schools, is interested in promoting the common good (Grace, 2010) and this is in line with the overall goals in teaching mathematics in the sense that proficiency in the subject opens doors to many opportunities in life. It is from among the Catholic secondary schools in South Africa that the three schools for inclusion in the study were chosen.

Another motivation in undertaking this research is that the national government in South Africa has identified the learning and teaching of mathematics and science as a priority in the educational needs of the country (Teachers Without Borders, 2010). The Government is faced with the challenge to encourage capability in mathematics and science. This starts at school and must be firmly established before it can translate into a useful skill in the work place. Learners need be given the opportunity to learn effectively. At present harsh disparities are still observable in the conditions under which education takes place in the country. Such disparities can no longer be blamed on apartheid (Do Soweto kids need Blackwash Black Consciousness? Tuesday, January 12, 2010) For example, it has been noted that the teachers at former Model C schools work for seven to eight hours a day while at many schools in the townships, teachers teach for half that duration per day. Against this background it is worthwhile to study how the opportunities to learn vary at the selected schools. These findings provide useful information which contributes towards the understanding of OTL.

On a more personal level, as a mathematics educator who has taught in three different countries over the last twenty five years, I am keen to make a contribution in this area by drawing on my experience, observations and practices of teaching

mathematics in varied contexts. I seek to engage with how OTL mathematics are created and can be improved in different conditions.

Evidence from the current study will help develop new knowledge from what is observed, heard and learnt from the encounters with the respondents. I hope to make a contribution to scholarship in the field of mathematics education, and in particular to the teaching and learning of algebra in ways that seek to embrace the experiences of all learners, especially those previously marginalized in the mathematics classrooms.

1.3 Background

South Africa has a history that is strongly influenced by racial prejudice and discrimination (Mazibuko, 2000). Education in South Africa is very closely linked to the political history of South Africa. By 1912 when the Union of South Africa was established, South Africa had evolved politically as a racially segregated and discriminatory country. When the 1912 constitution came into effect, black people were not given any political rights. Education was also already segregated. The government provided education for white people in state run schools and no provision was made for the indigenous people. A few were tolerated in the existing schools, but the majority had to be catered for by various religious organizations, that is, non-governmental organizations. Until 1953, almost all education for the African population of South Africa was provided by non-governmental organizations. The Roman Catholic Church and the Anglican Church were the main providers of education for Africans in South Africa. Other churches also made important contributions.

In 1953 apartheid's architect, Hendrik Verwoerd said it was no use teaching the Bantu mathematics when they could not use it in practice (Vithal, Alger and Keitel, 2005). As a result of this pronouncement two differently resourced education systems evolved in South Africa: a well-resourced education system for whites and a comparatively poor one for the majority. Mathematics was not seen as part of the curriculum for the black majority. This legacy of inferior education lasted until the introduction of a single education system in 1994 but the effects appear to have lived

on for much longer. For this reason, South Africa's education system is said to be at a critical juncture in its history (Teachers Without Borders, undated). Though the past cannot be undone, some of the experience gained can provide insight on how to handle the situation today.

According to Moloi (2005), the shortage both in intake and success in school mathematics in South Africa can largely be blamed on a curriculum that was patently skewed in favour of a small minority of learners who would proceed to university training in areas such as engineering, manufacturing, medicine and other so called 'hard skills.' It provided little to no opportunity to learners who only needed to apply mathematical skills in ordinary life situations. The curriculum was heavily content-laden, encouraged rote learning of mathematical techniques and algorithms and lent itself to very little application in the everyday experiences of learners.

Moloi (2005) observes that besides the universally known cognitive challenges that learners have to contend with in learning mathematics, in South Africa the then apartheid regime made access to this learning area particularly difficult on three fronts. First, discriminatory provision for education on the basis of race limited severely the availability of adequate and appropriate resources for Black learners who constituted the majority of the learner population in the country. Second, whatever learning support materials (LSM), particularly textbooks, were available were based on western philosophies and were found not adaptable to local indigenous knowledge systems. The locus of the underlying pedagogy was on teaching rather than learning. Consequently, the curriculum was packaged into time-bound subject syllabi which required highly contrived and theoretical contexts in order to be accessed cognitively. Third, the use of imposed foreign languages for instruction affected the acceptability of the curriculum, made learning in general very difficult and the learning of mathematics in particular virtually impossible, (Moloi, 2005).

In South Africa broad participation (access) and quality achievement in mathematics have been prioritized for equity and general redress of historical inequalities (Department of Education, 2004). But the fact that the learners have access to mathematics education does not necessarily translate into an opportunity to learn.

According to Moloi (2005), it appears that alongside the challenges of social transformation, South Africa faces a challenge of improving the quality of learning outcomes, particularly in mathematics. Moloi further claims that, although there is overall unsatisfactory achievement of outcomes by learners of mathematics, the problem affects certain sectors of the population more than others. He mentions that learners in rural settings and learners from low socio-economic status seem to be the most vulnerable.

There were attempts during the apartheid era to introduce an alternative mathematics curriculum to replace the one prescribed for the black majority. For example, *People's Education for People's Power* became one of the key action fighting plans, (Bopane, undated). *People's Mathematics for People's Power* was one of the products of this strategy. People's Mathematics was part of People's Education, a counter-hegemonic movement to remedy the crisis in education in South Africa then. The significance of mathematics stems from its role as a gatekeeper (Bopane, undated) and it is still one today.

In an attempt to redress the long standing inequalities, the democratic government has developed a new curriculum (White paper, 1995). Access to mathematics is explicitly defined as a human right in itself (Vithal et al, 2005). The Constitution of South Africa is very clear about the intention of creating a new frame of reference in all spheres of life and in mathematics education in particular.

To ensure that every learner has access to mathematics learning, a new subject called 'mathematical literacy' was introduced into the curriculum to cater for those who do not have the need to pursue mathematics to an advanced level (Sidiropoulos, 2008). It is my experience that those who are challenged by mathematics tend to opt for the seemingly 'user friendly' mathematical literacy course. Sidiropoulos (2008) investigated the Government's fundamental commitment to the provision of mathematics and mathematical literacy for every learner in the South African system. She remarks that the opportunity for all learners to become mathematically literate requires an understanding of how teachers understand and implement the curriculum within the unequal and demanding contexts of schools after apartheid. This study examines to what extent this commitment by the

government translates into meaningful OTL mathematics in the grade 10 classroom in three Catholic secondary schools in South Africa.

From my experience as a learner and teacher of mathematics, learning with understanding involves many different aspects that help to explain it. No single definition can capture it completely for understanding comprises many aspects such as comprehension of concepts, skills in carrying out procedures efficiently, problem solving, capacity for logical thought, and a disposition to consider mathematics as a worthwhile endeavour. The National Council of Teachers of Mathematics (NCTM) (June, 2009) observes that mathematics consists of different topical strands which are interconnected, such as geometry and algebra. They suggest that in order to enable students to learn with understanding, it is advisable to have a coherent curriculum that effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on or connect with other ideas, thus enabling the development of skill proficiency and problem solving abilities. This advice is tantamount to saying that learners should be helped to learn algebra in ways that help them to build the skills that are demanded of them in life.

All the aspects from the NCTM mentioned above lead to the development of confidence in the process of learning. In most classrooms learners often lack the opportunity to engage in learning with understanding. Their learning of mathematics is imposed rather than acquired through their own experience (Ojose, 2008). As a result, the learners do not own their mathematics as it is like a foreign language to them. It appears that the majority of students in secondary schools are not able to connect by themselves the knowledge domains that constitute manipulative algebra on the one hand and instrumental algebra for problem-solving on the other.

Students must learn mathematics with understanding and so actively build new knowledge from experience and prior knowledge. Learning mathematics with understanding is essential (NCTM, 2009). According to Kim and Kasmer (2006), reasoning mathematically is fundamental to learning mathematics with understanding. When reasoning is effectively promoted and fostered through predicting and justifying results and making sense of observed phenomena, students develop a deeper understanding and connection of mathematical ideas. How many

learners are given the opportunity to develop efficient reasoning in the classroom? Malloy (2002:18) notes that though learning is supposed to be a pleasurable experience it is rarely so for many children when it comes to mathematics: "learning mathematics for intellectual pleasure is not wide spread; it tends not to happen for most children." According to her, children often like mathematics in the lower grades and they gradually willingly or are forced to withdraw from mathematics by the time they leave middle school. The fact that this does not happen with other subject areas means that there is something about mathematics that discourages or scares learners off.

Moloi (2005) reports that when in 2000 South Africa participated in the second study conducted by the Southern Africa Consortium for Monitoring Educational Quality (SACMEQ), a project known as SACMEQ II, in which 15 countries from southern and eastern Africa participated, the learners performed particularly poorly in mathematics. Although only Grade six learners were involved in the SACMEQ II study, the pattern has not been different from that shown by other international comparative studies of mathematical achievement. The lack of achievement in mathematics by learners leaves important gaps in their education and tends to narrow the range of career options open to them as more and more careers have mathematics as one of their entry requirements. According to a PROM/SE (Promoting Rigorous Outcomes in Mathematics and Science Education) report (2009) 'opportunity to learn' is one of the most important factors influencing student achievement and, if students are provided with an opportunity to learn, they will do so. It is therefore important to understand how opportunities to learn mathematics are distributed in the classroom and how the learner's experience them during algebra lessons.

It has already been said that algebra is often cited as one of the most difficult topics, yet it is an important area within the subject as it affects the learning of other areas within mathematics. This raises important issues with respect to student's access to mathematics and mathematical ideas generally. As Jonassen and Land (2000) asserts, it is an undisputed fact that many students who are capable of learning algebra are not doing so. This is of particular concern with the historically

marginalized students who in the context of South Africa are mostly black and female.

Reeves and Muller (2005) cite recent research which contains evidence that in South Africa there is a high level of under-performance, particularly amongst learners at schools in high poverty areas. But as McDonnell (1995) says, students can only be held accountable for their academic performance to the extent that the community, broadly defined, has offered them the tools to master the content expected of them. Similarly the PROM/SE report stresses that it is equally important that the opportunity to learn curriculum content is available to all students in all schools. The present study seeks to explore in the context of South African grade ten classrooms how OTL is characterised in the selected Catholic schools.

Vithal and Volmink (2005) trace the development of the mathematics curriculum in South Africa from early apartheid years to the recent Revised National Curriculum Statement. During the apartheid era access to mathematics education was restricted for the Black majority. Now, in the new Official Curriculum access to mathematics is explicitly defined as a human right in itself and linked to a definition of mathematics as a human activity and a product of investigation of different cultures. Because of this awareness, every learner in South Africa has to engage in mathematics education. In grade ten, a learner must take either pure mathematics or mathematical literacy.

1.4 Statement of the problem

Fifteen years after the end of apartheid, the majority of learners in secondary schools still have problems learning mathematics and tend to underperform. Sidiropoulos (2008) notes that most studies point to the fact that the country's students do not measure up globally in mathematics and mathematical literacy competencies. While facilities have 'improved' and access to education is more open, learners still appear to lack the opportunity to learn and achieve the expected standards. In spite of the fact that access is now relatively open, the majority of learners still do not perform well, and especially not at the end of the secondary school when they write public examinations. The course that is examined at the end of the secondary schooling

actually begins in grade ten, which is why the present study focuses on the grade ten learning of mathematics and in particular the learning of algebra.

Given that access to learn mathematics is relatively open, the problem of underperformance can perhaps be traced to the actual "opportunity to learn" in the classroom itself. In other words, what is it that learners do in the name of learning mathematics when they are in the classroom?

In general many students in secondary school find mathematics uninteresting and difficult to learn and comprehend. Conceptual understanding is of central concern for mathematics education world-wide (Mwakapenda, 2005). He argues that while attention has been paid to exploring the understanding in mathematics, only a few studies have researched student understanding of specific concepts. This study is interested in the specific topic of algebra and how it is taught to and learnt by grade ten learners at three Catholic secondary schools in South Africa. For Lew (1999), algebra seems to be far more difficult for students than is expected. Research shows that the concepts of variables and functions make serious epistemological obstacles for many students and even teachers. Could these issues be the real obstacles to providing the OTL mathematics for the South African learners as well?

Having reviewed more than a decade of Conference Proceedings of the Southern African Association for Research in Mathematics and Science Education (SAARMSE), Vithal and Volmink (2005) found no research that spoke directly to mathematics curriculum reforms at a systemic level, with a few exceptions such as the research related to the Third International Mathematics and Science Study (TIMSS). They observed that while the South African curriculum reforms have been shaped and changed by both international and national shifts and developments in mathematical education, theory and practice, very little evidence exists that research has played any significant role in the direction or form taken by curriculum over time. National Studies specific to South Africa, with indigenous research designs developed to recognize and respond to the vast inequality and diversity of teaching and learning conditions in the country, which could help to theoretically explain the situation that exists, have not been conducted (Vithal et al, 2005). Such research

would need to be a combined effort of seasoned and experienced researchers, not the work of a single researcher. They also concluded that many of the present and past curriculum reforms were driven largely by conjecture, stereotypes, intuition, assertion and a host of untested assumptions rather than by research. This research is an attempt to contribute to the call to have research that can help explain the situation that prevails in the chosen Catholic schools with regards to the opportunities to learn grade ten algebra.

Vithal and Volmink (2005) examine theories and practices that have informed particular reforms in South Africa. These they called 'curriculum roots.' They identify historical events such as colonialism, apartheid and democracy as having influenced mathematics education respectively. Curriculum developments in the Western countries have played a part in influencing curriculum development, sometimes without scrutiny as to their applicability to the South African context. People's Mathematics was also identified as a root because it left its marks on post-apartheid mathematics reform. Problem solving approaches introduced in the late 1980s and early 1990s also left their mark though they did not reach all classrooms partly because of the differential allocation of resources and large classes. They argue that constructivism which took root as a strong epistemology but with weak pedagogy, could not be applied in the mathematics classrooms due to its diverse and unequal conditions. This failed because of its weak social construction. As a curriculum root constructivism led to a theoretically driven mathematics curriculum reform but had limited impact because it failed to develop a praxis that factored in the socio-economic and political dimensions of the apartheid education as a whole. Taking cognizance of the curriculum roots is important for the current study because a better understanding of the system can be obtained by considering the historical experiences that have shaped it over the years. Mathematics education has passed through a number of stages of development which have all left their mark and which cannot be ignored when opportunities to learn are to be considered at any level of education. Their influence continues to this day.

Existing research has dealt with different factors that influence the teaching and learning of mathematics, such as, political and economic factors (Vithal, Alder and Keitel, 2005), mathematics anxiety (Chinn, 2008), coherence and focus (Watanabe,

2007) and teacher beliefs (Jenkins, 2007). According to Lins and Kaput (2004), much research has been conducted in the past concerning students' error patterns and misconceptions concerning levels of development. Few questions regarding what sense they make of the condition, however, seem to have been posed to the students themselves, especially those who are failing. Research on students' beliefs has usually been directed to general beliefs about mathematics, not algebra. This means it is worthwhile to look at what students are actually thinking about and with, rather than what they are failing to do and checking this against what they are expected to do. Capturing and making sense of the experiences of learners inside the mathematics, and, in particular, the algebra classroom is an important goal of the present research. Consequently, further study is needed to clarify the relationship between OTL and algebra achievement for grade ten learners in South Africa.

1.5   Research Questions

In more specific terms, the focus of the present study is in answering the following research questions:

1. What is the content and context of the official mathematics curriculum in grade ten?
2. How is the curriculum enacted in the selected grade ten classrooms?
3. How can the Opportunities to Learn (OTL) grade ten mathematics be characterized and explained from the official and enacted curricula?

In the first question, I explore the following sub-questions:

a) What are the key content themes that are prescribed for the grade ten learners?
b) What is the rationale for including the themes in the grade ten curriculum?
c) How are the themes to be taught in the classroom? That is, what guidelines and/or suggestions are provided to the teachers in terms of approaches, resources, and assessments?

In the second question, I explore the following sub-questions:

a) How do teachers translate the prescribed work into the enacted mathematics curriculum in their classrooms?

b) What do grade ten learners learn from the prescribed official mathematics curriculum in South Africa?

c) What are the relations between what the standards require, what teachers teach and what is tested?

In the third question, I explore the following sub-question:

How can the concept of OTL be used to understand and explain the teaching and learning of mathematics in the grade ten classrooms?

1.6 Objectives of the study

The objectives of this study are:

1. to gain insight into how the concept of opportunities to learn can be used to explain the teaching and learning of grade ten algebra;

2. to explore the relationship between the official and the enacted curricula in South Africa; and

3. to understand the structure and practice of mathematics teaching and learning in selected classrooms in South Africa with regard to the chances given to learners to learn the subject.

1.7 Delimitations of the study

As discussed earlier, the study focuses on Catholic schools in South Africa. The researcher has extensive experience of working within this sector of schools both in this country and abroad. In some ways the study seeks to explore the question of how successful these schools are in providing real access to mathematics by examining what OTL are provided in the grade ten mathematics classrooms.

The study will cover three Catholic Secondary schools in South Africa. One school is in the township area of Pretoria. Learners who attend these schools have to pay fees for their tuition because the schools are classified as private. One of the schools enrols about four hundred learners and it is co-educational. It has three grade ten mathematics classes: two for mathematics and one for mathematical literacy. The

learners can choose which subject they wish to take. The other two are convent schools located in the city. The convent school teaches learners from grade R to grade twelve on the same premises.

The study is concerned primarily with the opportunity for grade ten learners to learn mathematics and in particular algebra in different contexts and environments. A curriculum analysis and an empirical investigation with teachers, learners and mathematics subject advisors were done. Class observations and interviews have constituted an integral part of the investigation. They have provided insight into better understanding the challenges of teaching and learning mathematics. The methodology employed will be discussed in Chapter three.

1.8 Limitations of the study

The study has been undertaken during a relatively unstable period in the history of the education system in South Africa. Many changes are taking place. It is possible that recommendations that emanate from the study might be overtaken by events because major movements in the practice of education are anticipated and the direction at this time is not very clear. To minimize these possible limitations, the researcher has been careful to explore OTL as a key concept in the study and only use the context of South Africa as a case in point. The key questions around access to powerful mathematical ideas for learners (Ball, 2008) are perennial questions in mathematics education both in South Africa and elsewhere on the globe. Vithal and Volmink (2005) have warned that there are severe limitations on the extent to which educational research can inform policy and implementation. The reasons for this are many and include the fact that research does not extend much beyond academic circles, that change is affected by other factors besides education and that some research findings are overtaken by events.

In this particular study, it is also possible that data might be influenced by the fact that the researcher is a member of the Catholic Church, presently teaches at one of the selected schools and has in the past taught at one of the other schools for two and a half years. However, through careful planning by the researcher and professional guidance by the research supervisor, it is hoped that these limitations

will not handicap the success of the study. Member checking, validation and reliability techniques that are explored in Chapter Three have helped to minimize these potential liabilities.

1.9 Feasibility of the Study

Access to respondents who provided the data essential to the study was easily obtainable. I was ready to undertake the study because of previous experience in research. The skill of critically evaluating research gained on a previous course of study was useful. Networking with other students of research boosted and refined the skills essential for the success of the study.

1.10 Organization of the study

The following organizational structure was used to present the study and its findings:

Chapter 1 describes the problem and places it in perspective by giving an outline of the context, background and key questions. It also provides the rationale for the study as well as its objectives, justification, scope and limitations.

Chapter 2 consists of a review of related literature pertaining to the concept of OTL as applied to the teaching and learning of mathematics. The importance of a review of literature is to help develop a conceptual framework for the study. The literature has helped the researcher to gain insight into the problem and become acquainted with existing research so as to avoid duplication.

The research design and methodology is explained in Chapter Three. The way the research was carried out and how the results were analysed is clarified.

Data collected is presented and analysed in Chapter Four.

The findings of the research are discussed in Chapter Five.

Finally, a summary, conclusions and recommendations are presented in Chapter Six. A list of references and appendices are provided thereafter.

1.11 Conclusion

In this chapter I introduced the problem and posed questions which this study seeks to answer. I now move on to review literature on the various aspects of topic in the next chapter.

CHAPTER TWO

LITERATURE REVIEW

2.1   Introduction

Curriculum is understood in many ways and has been the subject of study from a number of perspectives (Murphy and Hall, 2008). Curriculum is a broad field and covers a number of distinct bodies of scholarship. In this review, I examine three that have influenced curriculum thinking in terms of curriculum design, implementation and development in schools with reference to South Africa. First, I describe the Curriculum in South Africa; its history, development and the structure of the education system. Then the concept of opportunity to learn is reviewed in as far as it relates to mathematics teaching and learning. This is followed by a section exploring the teaching and learning of mathematics in general and of algebra in particular. Finally the conceptual and theoretical frameworks guiding the study are presented and a conclusion drawn.

Lewy (1991) describes curriculum as a massive, comprehensive and ill-defined field and says that any effort to conceptualize it is necessarily arbitrary. There is no single definition and so no single line of inquiry on curriculum matters. Apparently there is much more to curriculum as a field of study than the products of the process that produces curriculum materials. There are competing ideologies, legal considerations, processes, bureaucratic regulations, financial restraints and other factors that impinge on and to a considerable degree determine what is taught. Still, curriculum remains one of the most important means to meet the needs of any country or nation because it grooms its future citizens. This makes the study of curriculum a worthwhile endeavour.

Connelly et al. (2008) regard curriculum and instruction as multi-dimensional and engage in a dynamic interplay between practice, context and theory. It follows that this dialogue shapes and is shaped by experiences of curriculum stakeholders such as students, parents, teachers, educators, curriculum policy makers and administrators. In this study attention is paid particularly to the classroom where learners interact with the

prescribed content together with their teachers in order to develop the desired competences for their benefit and that of the country at large. What happens in the classroom is of interest to all stakeholders.

2.2   The Curriculum in South Africa

The curriculum in South Africa is shaped by politics. South Africa has gone through three different forms of government: colonialism, apartheid and democracy. After the end of the apartheid era the new democratic government committed itself to the transformation of education. Key policy documents and legislation stress the principle of education as a basic human right enshrined in the Constitution. According to the Bill of Rights contained in the Constitution of the Republic of South Africa, 1996 (Act 108 of 1996), everyone has the right to basic education. This includes adult basic education and further education which the State, through reasonable measures, must make progressively available and accessible.

The Department of Education (2003a:2) holds that The Constitution of the Republic of South Africa forms the basis for social transformation in our post-apartheid society. Education influences social transformation by ensuring that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of the population. If social transformation is to be achieved, all South Africans have to be educationally affirmed through the recognition of their potential and the removal of artificial barriers (OTL) to the attainment of qualifications (The New Constitution, 1996). What the Constitution is calling for can only be achieved through appropriate curriculum, one which addresses the needs of society and is accessible to everybody.

2.3   Education Structures in South Africa

This section offers an overview of the education system in South Africa and provides the context under which algebra is taught and learnt by grade ten learners.

South Africa has a single national education system, organized and managed largely

by nine provincial subsystems. The Ministry of Education was established in May 1994 to handle education and training at national level. It is assisted by the Department of Education.

In 1995, the South African government through the Department of Education revamped the Education System emerging from the apartheid era by shifting from content-based to outcomes-based education (OBE) (Vithal, et al, 2005). The Department of Education determines the curriculum which is issued in National Curriculum Statements (NCS) together with the corresponding subject assessment guidelines. The Department of Education provides the schools and therefore the teachers with specifications that should be met and also gives a broad, flexible time frame. For example, commencing from January 2009 schools in the Gauteng Province are issued with work schedules for the mathematics learning area, for grades 10-12, complete with expected dates of completion.

The NCS grades 10-12 (general) adopt an inclusive approach. Although it specifies the minimum requirements for all learners it aims to develop a high level of knowledge and skills. To help the learners achieve the expected level of competence, the school should provide the necessary conditions for this to take place. It is acknowledged that all learners should develop to their full potential (Prinsloo, 2001). This takes place if learners receive the necessary support in terms of quality instruction and appropriate resources used in a supportive environment.

Mathematics is described in the NCS as a key subject and it is compulsory up to Grade 9; there after learners choose either Maths or Maths Literacy. Either way mathematics provides access to a wide variety of learning. Moreover, being literate in Mathematics is an essential requirement for the development of the responsible citizen, the contributing worker and the self-managing person. To meet this requirement all learners in grade ten attend mathematics classes. In this way learners are given the opportunity to develop the required mathematics competences by the school they attend. However, this is not always accompanied by the corresponding opportunity to learn the designated material in an effective manner.

Vithal and Volmink (2005:5) observe that in South Africa, "successive curriculum

reforms may be characterized as waves of change, each bringing in a tide of new ideas and practices, taking some away, leaving some behind, and changing some." They aptly describe how and why the teaching and learning of mathematics has changed over the years. They coined the term 'curriculum roots' which refer to different theoretical and philosophical underpinnings brought about by each wave of change. The curriculum as they see it was influenced by developments in Western countries imported into the South African system at different times. However, the implementation of these imported theories was also strongly influenced by internal forces such as colonialism, apartheid and more recently, democracy. The theories were implemented where people were divided on racial grounds and three separate educational systems existed in which schools did not enjoy an equal distribution of resources. The intended curriculum was the same, but the implemented curriculum was not the same, for the Black majority, mathematical education was largely inferior.

The late 1980s and early 1990s saw a concerted effort on the part of all mathematics educators to transform the teaching and learning of mathematics (Vithal et al. 2005). There was an indigenous response in the form of people's mathematics, developed as an opportunity for the black majority to study relevant mathematical skills. People's Mathematics (PM) was an independent development in South Africa and differed from other varieties in that it not only adopted the stance of critique but also emphasized action against those practices which inhibit human possibility (Julie, 2004). The broad umbrella goals of People's Mathematics were political, intellectual and mathematical empowerment.

The end of apartheid also saw the introduction of yet another western theory: the principle of constructivism (Vithal and Volmink, 2005). Constructivism is internationally recognized as a theory with much to offer mathematics education (Jarworski, 1994 and Matthews, 1998). The constructivist view recognizes the dependence of what is learned on previous knowledge and the experience of the learner. Learners do not passively receive knowledge but actively construct new knowledge based on prior knowledge and meaningful learning requires active involvement (Cobb and Steffe, 1983; Fennema and Romberg, 1999).

In South Africa constructivism is not mentioned directly as the adopted theory but is implied in statements such as this one: "Knowledge in mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships" (National Curriculum Statement, NCS Grades 10-12, (General), and 2003: 2 and Mwakapenda, 2005). Constructivism seems to fit with another statement in the NCS (2003) which says: "OBE encourages a learner-centred and activity-based approach to education." In a learner-centred approach the role of the teacher changes from one of transmitter to one of facilitator, coach, mediator, prompter and someone that helps students develop and assess their understanding. The learner thus plays a more active part in the process of building new knowledge. According to Fennema and Romberg (1999), it is important that learners build new mathematical knowledge by reflecting on their thinking and actions while they solve problems because they construct meaning for a new idea or process by relating it to ideas or processes that they already understand. They add that for an idea to be understood it must be related to other ideas.

2.4   Constructivism

Psychology helps us to understand how people learn and therefore it is of vital importance that the classroom practitioner heeds this knowledge. However, many theories have been put forward by different psychologists. This study adopts the constructivist approach to teaching and learning. Constructivism began with Vico (1668-1744) who pointed that "the only possible knowledge we can have is about what we construe" (Constructivist Psychology, undated). There are three major constructivist traditions: educational constructivism, philosophical constructivism and sociological constructivism. Educational constructivism is relevant to this study. Educational constructivism includes personal constructivism and has its origin with Piaget (1970, 1980). In the present day it is most clearly enunciated by Glaserfeld (1995) and social constructivism which has its origins with Vygotsky (Matthews, 1998).

According to Piaget, knowledge is built during the process of disequilibrium where individuals internally experience cognitive conflict when confronted with new information. During disequilibrium, prior knowledge cannot explain new experiences.

Therefore, through accommodating new knowledge and assimilating it with prior knowledge, individuals build internal structures of knowledge unique to them (Piaget, 1970). Piaget (1970) claims that interactions in the classroom can facilitate knowledge development because interaction creates cognitive conflict which can change thinking. Piaget claimed that peer interactions stimulate student reflections about ideas that other students present.

Piaget (1971) claims that human beings normally pass through four stages of cognitive development roughly defined according to age. The fourth and final stage which he termed the 'formal operational stage' (age 11-16 and onwards), is when human beings begin to think abstractly, reason logically and draw conclusions from the information available, as well as apply all processes to hypothetical situations. Children develop abstract thought and can easily conserve and think logically in their mind. This is the stage where most grade ten learners are expected to be by virtue of their age. It is appropriate that they learn algebra because it requires that learners think abstractly and logically so they are prepared for what follows in their life. Piaget's theory of constructive learning has a wide ranging impact on teaching and learning methods in education and is an underlying theme of many educational reform movements.

Vygotsky (1896-1934) says that individuals construct knowledge in the zone of proximal development through social interaction with more knowledgeable others, peers, teachers or acquaintances (1935). The zone of proximal development is the difference between what the learner can do without help and what he/she can do with help. When interacting with others, individuals learn as they communicate their thinking. For Vygotsky the interaction between pupils in the social context of the classroom is important for knowledge development. Learning happens when individuals construct their own interpretations through language, and to gain knowledge one needs an encounter with another person (not always a teacher), book or event. Vygotsky (1978) noted that social interaction not only initiates changes in thinking but also alters current thinking. Individuals gradually internalize the talk that occurs during interactions. For Vygotsky one expands knowledge and for Piaget one builds knowledge. For Vygotsky, further development is cultural but for Piaget it is biological. This issue is further elaborated below.

In one of his lectures Vygotsky explained:

> The environment is the source of development of these specifically human traits and attributes, most importantly because these historically evolved traits of human personality, which are latent in every human being due to the organic makeup of heredity, exist in the environment, but the only way they can be found in each individual human being is on the strength of his being a member of a certain social group… and during the course of their development children acquire, as their personal property, that which originally represented only a form of their external interaction with the environment.
>
> (The Vygotsky Reader, 1935:338-354)

Vygotsky believed educators' role was to give children experiences that were within their zone of proximal development, thereby encouraging and advancing their individual learning (Wikipedia.org/wiki/zone_of_proximal_development).

Learning is a social and cognitive process in which children share their thinking. Both perspectives, the cognitive and the socio-cultural, are essential in the learning process and complement one another (Cobb, 1994). This line of thinking is called social constructivism and puts great emphasis on communicating and negotiating as a process of constructing knowledge (Cobb, Yackel and Wood, 1990). In the mathematics classroom, teachers and students continually use each other's contributions to resolve disequilibrium and develop individual knowledge. However, during the process of negotiating and sharing with a knowledgeable teacher, students come to understand the mathematical meanings of the wider society, taken-as-shared-meanings (Cobb, Yackel and Wood, 1991). Students adapt to the actions of others in the course of on-going interactions. When children are constructing their knowledge, teachers must be able to pose tasks that help children construct meaningful conceptual knowledge that builds on their prior understandings.

2.5   Opportunity to learn

"A major goal of school mathematics programs is to create autonomous learners" (Posametier and Jaye, 2006:14). Creation of autonomous learners is gradual and needs to be fostered by giving them the necessary opportunities to develop. The mathematics program should ideally help learners to gain the power to think and act independently. This power helps learners to have control over their performance and

helps them to develop autonomy in learning. Just as in other countries, several factors influence the high school mathematics curriculum in South Africa. These include societal forces, the nature of mathematics as a subject and how children learn mathematics. Societal forces were dealt with in chapter one of this study. The way grade ten learners in three selected Catholic secondary schools learn mathematics, and in particular algebra, will be described in detail using data from observed lessons (see chapter four). The nature of the mathematics subject matter and to some extent how it is taught will be covered in this chapter.

One of the most critical variables in determining students' achievement is their opportunity to learn (OTL) (Thompson and Senk, 2009; PROM/SE Report, 2009). OTL is one of the most important factors influencing student achievement, (Kilpatrick, Swafford and Findell, 2001 and 2003). OTL is a powerful concept used principally to explain differences among students in comparative international studies of educational achievement and in some small-scale national research studies (Stevens, 1993). This study seeks to understand OTL and its implications, especially as it is examined in the context of the teaching and learning of grade ten algebra in selected Catholic schools in South Africa.

The concept of OTL is the subject of widely varied definitions. For example, OTL is described as: "the degree of overlap between the content of instruction and that which is tested" (Reeves and Muller, 2005), "equitable conditions or circumstances within the school or classroom that promote learning for all students" (Schwartz, 1995; Cooper and Liou, 2007), the "absence of barriers that prevent learning" (Mereku et al, 2005), "conditions or circumstances within schools and classrooms that promote learning for all students" (Cooper and Liou, 2007), "conditions that may benefit students' mathematics learning and achievement, provided for students by the educational system" (Gau, 1997). Wallace (2009) defines OTL as "what takes place in classrooms that enables students to acquire the knowledge and skills that are expected." The dictionary adds that OTL can include what is taught, how it is taught, by whom and with what resources.

From these definitions it is clear that OTL is concerned with factors that impact on learners' performance. Most of the factors depend on the educator in the classroom

for he or she can influence directly what students learn as well the conditions under which the learning takes place. The teacher can also prevent learners from making the most of their time in class by his or her attitude. If the teacher encourages the learners, they are motivated to learn and can make good progress. If on the other hand the teacher is discouraging, learners may develop a negative attitude towards the subject, become discouraged and not realise the opportunity to learn anything. The ideas implied in OTL help create a conducive environment that can benefit every learner. OTL can give all learners the chance to learn with understanding. In this study, there is resonance with Schwartz's (1995) definition which highlights that students must have access to high quality education in order to meet high standards. Though not expressing everything that this study is seeking to understand it gives the basis of where to look, that is, the classroom which is exactly where the data for this study will be gathered from.

The definition that expresses concern for what is assessed at the end of a course of study is limited in scope. If a test or examination contains what was taught it does not necessarily mean every learner had the opportunity to learn it. Ideally every learner should perform well since they were all exposed to the content before the test or examination, but the situation is not that simple. This assumption ignores the likelihood that they may not have possessed the necessary conditions under which to learn. It also depends on how the material was delivered to them by the teacher.

Though equitable conditions are not easy to specify to everyone's satisfaction, what is implied helps to point to something positive. If the environment is positive, it is reasonable to expect a high chance of learners having OTL. It is possible to observe conditions under which instruction takes place in the classroom and this helps in the understanding of OTL. Schwartz (1995) asserts that common sense dictates that in order for students to achieve, they must have appropriate opportunities to learn.

Generally barriers to learning come in different forms, they can be physical such as a disability that might hinder a learner from gaining access to the facilities open to other learners in his or her age group. Barriers could be cultural or social or both. A certain cultural practice may favour one portion of the population at the expense of another as is the case with gender issues where a course of study may be designed

specifically for boys or for girls. This would mean that one group is denied the opportunity to engage in whatever activity the other group is privileged to enter.

In South Africa the Department of Education released "White Paper 6" in 2001 which spelt out a policy of inclusive education. White Paper 6's position is that all children and youth can learn and that methodologies can be found to meet the needs of all learners. This is an idealistic view because it is not possible to meet the needs of all learners, but it does provide a direction to follow and an ideal to aspire for. The paper acknowledges that there are differences in learners resulting from various factors such as age, gender, ethnicity, language, class, disability and illness. Consequently, the teacher must target learners in different ways. There are many things between the learners and knowledge and the teacher must help the learners get the means to acquire, for example, mathematical knowledge.

Inclusive education also aims to maximise the participation of all learners in a culture and the curriculum of educational institutions and uncover barriers to learning. Implied in White Paper 6 is the view that all learners are to be given the opportunity to learn since it states that all children can learn and that their needs should be met by the system. Maximizing participation presupposes the existence or creation of the opportunity to learn. Starrati (2003) confirms the need for dealing with barriers if students are to have opportunities to learn. He further emphasizes the fact that when students have developmental, linguistic, cultural or other obstacles to learning, or encounter unfocused pedagogy, insufficient time-on-task and inappropriate curriculum material, a situation in which very little learning is possible exists.

In short OTL is concerned with the conditions under which learners have to learn and is positively associated with achievement. In this research OTL is considered in the context of the learning of algebra at grade ten level in selected Catholic secondary schools in South Africa.

2.5.1 The History of OTL

OTL was introduced as a means to ensure the validity and comparability of cross-national comparison in the First International Mathematics Survey in the early 1960s (McDonnell, 1995; Gau 1997; Boscardin, Aguirre-Muñoz, Stoker, Kim, Kim and Lee, 2005; Scherff and Piazza, 2005). McDonnell (1995) claims that opportunity to learn is rare among the many concepts that education researchers have used when depicting the complexity of the schooling process. It was introduced in studies of mathematics achievement conducted by the International Association for the Evaluation of Educational Achievement (IEA). OTL measured whether or not students had the opportunity to study the particular topic or learn how to solve a particular type of problem presented in the test (Husen, 1967). McDonnell (1995) agrees that ideas and concepts like opportunity-to-learn can play a critical role in defining policy problems and in framing solutions.

OTL standards differ from country to country and depend on a particular nation's economic and educational policies (Mereku et al., 2005). In cross-national studies the purpose of using OTL was to take into consideration the curriculum differences and the discrepancies in content coverage in comparing students' mathematics achievement across different national systems. Thus it was important and fair to consider what each country was offering its students before students' achievement could be compared. Indices of OTL in different schools, districts or regions within the same nation cannot be compared when those OTL standards are defined differently. To determine whether cross-national differences in students' mathematics achievement were caused by differences in students' learning experiences rather than in their ability to master the subject, measures were developed for quantifying the instruction that students had received in a subject prior to testing (Schwartz, 2005).

Knowing the stages of development of the concept as outlined above helps to put the study in context. The concept has developed and was adopted or used to answer the problems in the education system especially in racial equality related issues. The information on OTL that now exists can serve as reference while the term continues to gain acceptance and its application becomes more widespread and moves from

the developed world to the developing world.

2.5.2 Applications of OTL

2.5.2.1 OTL as a standard to measure school effectiveness

At first OTL was used simply as a yardstick to measure the effectiveness of federally-funded educational programs in the United States (Schwartz, 1995; Abedi et al., 2004). OTL standards are defined as the criteria for and basis of assessing the sufficiency or quality of the resources, practices and conditions necessary at each level of the educational system (schools, local educational agencies, and states) so as to provide all students with an opportunity to learn the material in voluntary national content standards or state content standards (Mereku et al., 2005; and Schott Foundation, 2008). OTL standards are viewed as equivalent to school delivery standards, as part of systemic reform, as input conditions, and as a reference for time available. Problems surrounding OTL standards include how to define them, how to measure them, when and how they can be applied, the incorporating of them into existing procedures and the potential for confusing policies and increasing legal issues (Ysseldyke, Thurlow, and Shin. 1995).

According to Scherff and Piazza (2005), OTL standards were a political development that received attention mostly in the 1990s. Scherff and Piazza (2005) based their explanation of the concept of OTL on the potential input resources (content, curriculum activities, and materials) to which students had access and exposure. These potential input resources are important when considering any educational endeavour because they are an essential part of any system whether effective or not. They do not, however, tell the whole story because they do not indicate who is responsible for what, when and how. OTL includes the multiplicity of factors that create the conditions for teaching and learning, such as curricula, learning materials, facilities, teachers and instructional experiences (Scherff and Piazza, 2005).

Scherff and Piazza (2005) expand the concept of OTL to include a new factor – the distribution of information. They also argue that the distribution of information is a

critical component of the school culture that creates conditions for school-wide student achievement. Considering the distribution of information provides one more way through which OTL can be improved or better understood. Broadening the concept of OTL provides us with a new framework to better understand the problems students face during the transition into high school. These include poor attendance, discipline related problems and a decrease in extracurricular activities (Cooper & Liou, 2005). These researchers conclude by saying that access to information is an important condition influencing a student's OTL.

Schwartz (2005) maintains that OTL includes the provision of curricula, learning materials, facilities, teachers and instructional experiences that enable students to achieve high standards. Reeves and Muller (2005) say that although OTL has received attention in international comparative studies such as TIMSS and in developed countries such as the United States of America, its use in the developing countries has been limited.

South Africa is a developing country where information on OTL is scarce (Revees & Muller, 2005). As such this study seeks to contribute understanding to the perceived benefits of engaging with the concept in the practice of mathematics education. It is possible for South Africa to adopt OTL with most or all its tenets as part of the policy in education. The concept of OTL provides a rich and vast reference that helps to address the present need for improving the performance of learners in the mathematics learning area in South Africa. Schwartz (2005) observes that in the USA, despite recent attention to OTL strategies, most schools do not view them as either standards to be met or as indicators of educational quality. In the South African situation, it remains to be seen whether some educators are even aware of such a concept let alone engage with it. OTL offers a means of describing school and classroom processes. Researchers can study how the subject materials are taught, and how students learn based on the framework of OTL. This helps to explain why students' achievement may vary within a classroom (school) and across classrooms (schools). Scherff and Piazza (2005) conclude that if schools are to be held accountable for the equal delivery of educational opportunities, the core of the education performance indicator systems should include the data of school and classroom processes.

In recent decades, escalating demands for accountability and higher standards of student performance have led to renewed interest in the concept of OTL and encouraged researchers to expand conceptions beyond the consideration of time to include the nature and quality of instruction and its prerequisites (Herman and Abedi, 2004).

2.5.2.2   OTL as a research concept

According to Boscardin et al. (2005), OTL as a research concept, was first introduced in the early 1960s, in the First International Mathematics Survey. By the mid-1980s the notion of OTL had gone through substantial revisions, for example the Second International Mathematics Study conceptualized OTL in terms of curriculum, (Gau, 1997). The implemented curriculum was termed OTL. By 1994 OTL was linked to accountability, that is, ensuring that all students have a guaranteed right to the core resources needed to provide a fair and substantive opportunity to learn. This is based on the manner in which OTL standards are defined (Schott Foundation, 2008). Accountability also entails that tools have to be put in place for monitoring potential differences in OTL among various groups of students. OTL can provide the necessary standard to study students' educational opportunity and to evaluate schools' provision of educational opportunities, as well as provide detailed explanatory information regarding student achievement (Schott Foundation, 2008).

Boscardin et al. (2005) identify three variables which show the impact of OTL on student outcomes. They are: curriculum content, instructional strategies and instructional resources. 'Curriculum content' focuses on how much students are exposed to specific subjects and topics that are being assessed. Different dimensions of curriculum content in OTL include content coverage, content exposure, and content emphasis (McDonnell, 1995). Content coverage is the most frequently studied dimension and has been measured in various ways, including teachers' self-reports, direct observations and analysis of the content of curriculum materials. Content exposure is usually measured by direct observation to see the amount of time a teacher spends covering the specific content. Content emphasis concerns the issue of how a content area is treated: as a major topic, a minor review

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or not taught at all. The more assessment tasks resemble the curriculum, the better the assessment will be at detecting the impact of instruction.

OTL issues addressed through instructional strategies include whether or not students have been exposed to the kinds of teaching and instructional experiences that would prepare them for success. This also includes the quality of instructional delivery, which is often measured by direct classroom observation.

OTL issues addressed through instructional resource variables are concerned about whether there are appropriate resources to prepare students for success and the achieving of standards. Aspects of this dimension focus on teacher preparation, including level of education, amount of experience, type of experience, participation in in-service professional development, and attitude. School resources continue to be seen as an important OTL indicator because they can enable or constrain a school's ability to provide a high-quality instructional program. Finally, unless results from these OTL studies can be used to inform policy decisions, the concept of OTL will remain a research concept with no direct impact on educational reform.

2.6   How to study OTL

Gau (1997) says that in the past the concept of OTL was often operationalized very narrowly as whether particular tested items were taught beforehand to the students who took the test. In those circumstances teachers' reports of coverage was the sole indicator of OTL. This was criticized for being too narrow, bound to the specific items and more representative of teaches' judgements of items rather than the content categories of which the item is an example (Gau, 1997). According to Gau (1997), despite the efforts made by researchers to broaden the operational definition of OTL, the results have not been that compelling. He concludes that there is room for improvement in studying the effects of OTL on students' achievement and that finding the best means of measuring OTL remains a matter of concern for investigators.

Gau (1997) seeks to understand the distribution and effects of a broadened conception of OTL on students' mathematics achievements. Although school is an

organisation intended to provide learners with OTL, this is not always realised in practical terms because OTL tends to vary from school to school as well as within any one school. Gau's study is of particular interest because it considered schools in the Catholic, non-Catholic and non-sectarian private sectors. The present study is concerned with OTL algebra in selected Catholic schools. It is possible that certain conditions may be the same as those found in Gau's study. The present study is not going to compare OTL in different categories of schools but only between three schools which are in the same sector but in two different environments: inner-city and township.

In Gau's (1997) study, three constructs, teachers' mathematical knowledge, content and level of instruction and school mathematical resources, were used to explain what was termed the expanded OTL. Gau points out that teachers' knowledge may influence the quality of instruction and hence the kind and quality of opportunities the students have. Accordingly, the content and the level of instruction to which students are exposed may affect their achievement. Furthermore, a school's mathematical resources influence the kind of classroom learning and instruction possible as well as the existence of extra curricula opportunity. Additionally other student characteristics such as gender and race and school characteristics may influence what is learnt from the opportunities provided. Gau's study reveals that the distribution of OTL is not equal throughout different categories of schools (Gau, 1997: 21).

According to PROM/SE (2009) direct observation of classrooms over an extended period of time would provide a rich source of information about the implemented curriculum. However such studies are very costly and time consuming so they are infrequently employed by researchers. Self-report measures such as daily teacher logs of instruction, interviews and questionnaires have been frequently used to determine content coverage and implementation data. Such self-reports have limitations because people can exaggerate things and they may not have the same frame of reference as the researcher.

2.6.1 Examples of OTL Studies

Reeves and Muller (2005) describe opportunity to learn as the degree of overlap between the content of instruction and what is tested. Underlying this OTL construct is the notion that curriculum frameworks and curriculum guides potentially act as inclusionary mechanism for ensuring that high status mathematical knowledge and skills are made equally available to all learners. Commenting on OTL in the current South African context, Reeves and Muller (2005) observe that given the recent revisions to the curriculum framework, it is plausible to anticipate that policy makers and others involved in schooling in the country will have a revitalized interest in the opportunities to learn that are being made available to low socioeconomic status (SES) populations of learners. Their observation is also that though OTL has received attention in international studies in developed countries, its use in developing countries has been limited. The present study is paying heed to the call to be involved in the consideration of OTL as a possible means of understanding how mathematics in general and algebra in particular is learnt.

Although this study has come at a very unstable time in the history of the education system in South Africa, when many changes are taking place, the consideration of OTL can never be out of date because of what it inherently promises. The current study seeks to contribute positively to the discussion of issues, problems and dilemmas associated with the teaching and learning of algebra, paying particular attention to the opportunities available to learners to accomplish the desired ends. An understanding of the way learners learn is necessary to widen and enrich the analysis of today's perceived problems in the learning of mathematics in general.

In a working paper of the United States Agency for International Development (USAID, 2008), it states that sixteen years after the launch of the first Education for All Conference, the impact in terms of educational outcomes and in particular student learning, has not been impressive. The paper argues that the reason why students are not succeeding is the lack of the opportunity to learn (OTL). The paper further argues that resources are not the only determinant for quality education since it was observed that schools outside the government systems showed that children were achieving higher learning outcomes with equal or less resources.

The USAID paper (2008) identifies eight crucial elements that create what they refer to as a basic opportunity to learn. These elements are: total instructional time, the hours in a school year and days that the school is open, teacher attendance and punctuality, student attendance and punctuality, the teacher-student ratio, instructional materials per student, and the classroom time spent on the tasks and skills taught per grade. Consequently, without a strategy to track these elements more closely, and direct funding to ensure that a minimum level is attained, children cannot be provided with a basic opportunity to learn. This working paper further argues that the basic OTL index starts from a relatively simple premise: learning is to some degree a function of time and effort. Without adequate time on task, no learning is possible (USAID, 2008:4). Investments in teachers, materials, curricula, and classrooms are wasted if they are not used for a reasonable period of time. A direct relationship between learning and OTL is assumed.

The USAID paper "posits that a basic OTL for developing countries needs to focus on a number of more fundamental measures before the above elements become relevant" (2008:4). The Fundamental Quality Level (FQL) implemented in a number of African countries in the 1990's system sought to establish the standards of inputs and infrastructure necessary to provide equality of school conditions. The FQL program provided a basis for dialogue about investment in education infrastructure. While extremely useful for informing investment decisions and choices, the FQL approach did not capture the management aspects of creating a genuine opportunity to learn. The paper further argues that the failure to focus on the fundamental OTL factors undermines all investments in higher level interventions (USAID, 2008:4).

Foundational elements were identified by USAID (2008). These are the inputs and management available (including instructional time), the nearness of the school and availability of teachers and students at appointed times of the day and the instructional materials for use by each student. The USAID paper reports that an OTL study in Ghana found that the overly ambitious curriculum was poorly aligned with teacher capability and that less than half of the material was actually covered during the school year.

Opolot-Okurut (2008) investigated factors that hinder pupils' opportunities to learn mathematics in primary schools in Uganda. He considered the challenges and problems that teachers face when they teach mathematics in primary schools. He identifies six relevant factors: (a) the personality of the teachers (b) the characteristics of the pupils (c) overcrowded classrooms (d) the nature of the curriculum and syllabus (e) government policies on education and (f) the learning environment and assessment methods. He explains that when teaching in large classes, teachers provide fewer exercises and practice so as to reduce the amount of marking that they have to do. There is also limited space to conduct group work that would ensure the effective coverage of content. Also, the overcrowded curriculum minimises the pupils' opportunity to learn mathematics as teachers try to cover too much content in too little time.

Opolot-Okurut (2008) recommends that these factors are examined and suggests that the implications of the problems and challenges identified in his study beg for further research, more focussed education policies and more support for teachers to improve pupils' opportunity to learn. Though Opolot-Okurut was considering primary school mathematics, the factors he identified can apply to secondary school mathematics as well. In this study these factors will inform the construction of the instruments used. All of the factors have a bearing when it comes to observation of learners engaged in the learning of algebra and also when undertaking a document analysis of the intended curriculum.

Scherff and Piazza (2005: 343) are of the opinion that "... now, more than ever, we need to talk about opportunity to learn" because, "while we hold students accountable to the same standards on high-stakes tests, a survey revealed unconscionable variation in the extent to which resources and instruction support their achievement." They also bemoan the fact that the indicator of whether a school is considered successful is usually student achievement scores, yet a single score can mask the complexities of teaching and learning, as well as the factors that impact test results. This makes the concept of opportunity to learn (OTL) a promising idea that can guide the assessment of schools and place them in proper perspective, especially when striving not to leave any child behind (Scherff and Piazza, 2005).

Scherff and Piazza (2005) contend that for curricular standards to be accurately measured by mandated assessments, educators must ensure that students have more choice, ownership and commitment to the educational enterprise. For this reason they see it fit to study students' perspectives as one viable way of shedding light on OTL, especially when considered against a social and political framework. Although students' self-reports address only one component of the complex phenomena that is OTL, when interpreted against a backdrop of legislative mandates for standards and assessment they can help to explain how variation in access and exposure to content and curricular tasks and materials can be used to evaluate state and local strategies and provide data on progress toward equal access to learning (Scherff and Piazza, 2005). They further point out that until the underlying issues in regard to OTL are considered, some inequality will continue to exist in U.S. school systems. It follows that the inclusion of the students' voice can serve as an important frame of reference for interpreting test scores and discussing the success or failure of schools. Scherff and Piazza (2005) stress that documenting factors associated with OTL and illuminating them in assessments and research studies such as theirs can provide a solid starting point.

Cooper and Liou (2007) suggest that in an effort to bridge the gap that characterises *racial achievement*, one factor that warrants further investigation is the opportunity to learn. They argue that though this is not a new concept it has been silenced in the current discourse on how to close the achievement gap. They concur with Gordon (1992) who suggests that it was immoral to compare student outcomes before any serious engagement in investigating the distribution of inputs, that is, the opportunities and resources essential for the development of intellect and competence.

Since 1988, OTL has been viewed as a viable tool to ensure equity in the distribution of educational resources and learning opportunities (McDonnell, 1995). OTL theory suggests that students' differentiated learning experiences, both within and between schools, can be attributed to unequal learning conditions rather than students' abilities to succeed (Schwartz, 1995). Research on OTL has focused on developing better ways to measure how elements of school culture such as school finance, student assessment and teacher quality are distributed and compared.

Cooper and Liou (2007) investigate whether all students have equal access to the type of information that can make the difference between dropping out of the educational system and staying in it. They view OTL as a powerful analytic tool that has the potential to enact the kind of progressive social policymaking that would transform the culture of schooling for children. They also recommend that the OTL framework be utilized within the context of reform efforts like No Child Left Behind (NCLB) so that policymakers and practitioners can better assess how learning opportunities are distributed between and across schools (Cooper and Liou, 2007). The NCLB Act requires states to develop assessments in basic skills to be given to all students in certain grades. Such an approach to reform requires the emphasis to be placed on exploring the ways unequal schooling conditions, including the distribution of high stakes information, serve as powerful indicators of the distribution of possibilities. At the National Opportunity to Learn Education summit held in 2008 it was concluded that:

> NCLB holds students, teachers, and administrators accountable for their performance. However, while NCLB may address the soft bigotry of low expectations, it does not address the hard bigotry of lack of resources for children facing de jure, but certainly de facto segregation in low-income communities.

Cueto, Ramirez and Leon (2005) say that the distance between what is intended and what is implemented is due to many factors. These include the fact that the curriculum is too long, the students do not master some of the competencies and so the teachers do not have sufficient time to cover them all, and that the teacher may have different priorities regarding what should be taught. Furthermore, the teachers do not master some of the competencies to be taught and so do not include them in their classes, or the teachers do not have the educational material needed to teach some competencies (Cueto et al., 2005).

In an investigation of Mathematics Textbooks and their use in English, French and German classrooms, Haggarty and Pepin (2002) conclude that learners in different countries are offered different mathematics and given different opportunities to learn that mathematics, both of which are influenced by textbooks and by teachers. In France, there is no grouping of pupils either by perceived ability or achievement

which means that all pupils in a particular class are given opportunities to learn the same mathematics, with each topic studied for the same amount of time by all pupils, from the same textbook supplied by the school. Teachers interviewed in France told them that they used the textbook most, if not all, of the time in their lessons, and as their main resource for lesson preparation. Textbooks are written by mathematics inspectors in France and therefore reflect the pedagogical concerns and emphases of those inspectors. All pupils in France were given opportunities to learn challenging mathematics and it seems to be their teachers' aims to select stimulating exercises for them in order to give them the opportunity to engage in the process of doing mathematics (as opposed to result-driven, closed learning).

Haggarty and Pepin (2002) claim that unless the learners have financial difficulties, the acquisition of German textbooks is the responsibility of learners. As a result of this arrangement, parents tend to pressurise teachers to stick to the same book every year to reduce the costs, especially when they have a number of children at school. This also means that pupils have access to the textbook both at school and at home since it literally belongs to them. The school itself decides on the textbooks to be purchased by parents from a list supplied by the Ministry of the *Land Germany*. Since teachers work from textbooks on an 'approved list,' they tend to assume that the books cover the National Curriculum sufficiently. The teachers rely mainly on the textbooks to determine what to teach.

Herman and Abedi (2004) investigate issues identified in assessing the opportunity to learn mathematics amongst learners whose first language is not English. They contend that fairness demands that English Language Learners (ELL) be given equitable opportunity to learn that upon which they are assessed, especially if those assessments carry significant consequences for their future. Accordingly, OTL data can help provide guidance in these areas. The reality is that an ELL is unlikely to improve unless or until students have more effective opportunities to attain the expected performance standards. They question whether the students were given such opportunities and also wonder what effective opportunities look like. They argue that in the absence of data on OTL, policy makers will be missing critical evidence on which to base their decision-making and schools will be missing critical feedback. They further stress that data on ELL OTL can focus attention, stimulate

schools' thinking about the strengths and weaknesses of their curriculum and course offerings, and encourage insight into priorities for professional development, materials acquisition and resource allocations just as with student performance data.

While acknowledging that there are innumerable other potential and important uses of OTL, Herman and Abedi (2004) considered their list sufficient to motivate their purpose of exploring selected issues in the measurement of OTL to provide preliminary findings on the relationships between ELL status and OTL and to raise questions for future study. They observed that while attention is paid to the definition of OTL and ways to potentially measure it, relatively little consideration is given to the quality of the measures and so little attention is paid to OTL for ELL. Their study only draws on research examining OTL for the general population and on specific teaching and learning issues relevant to ELL.

In the light of the strong relationship between OTL and student performance for ELL students, Herman and Abedi (2004) claim that their findings show that the relationship between language status and classroom-level OTL is significant. Their descriptive results indicate clear differences in OTL for ELL and non-ELL students in the study. Similarly the Hierarchical Linear Modelling (HLM) results show that the proportion of ELL students and OTL have important effects on student performance, even after controlling students' prior ability and background. Their preliminary observation findings also suggest inequities in OTL for ELL and non-ELL students. These data confirm the need for current debates about bias in testing for ELL students to shift towards giving at least as much attention to bias in OTL. The data of Herman and Abedi suggest that differential OTL may indeed play a role in the depressed performance of ELLs.

Herman and Abedi's (2004) findings regarding the relationship between language proficiency and OTL suggest that only examining exposure is limited. They found that exposure does *not* ensure effective access to curriculum and appropriate opportunities to learn. Herman and Abedi (2004) wonder if without such opportunities sufficient learning can occur. They encourage weighing the pros and cons of quantity versus quality and question whether it is desirable to pass as many as possible in the name of giving as many people as possible an opportunity. They conclude that

this means that quantity trumps quality. The many questions that Herman and Abedi (2004) asked have helped the present research to probe the same in the context of South Africa.

Mereku et al. (2005) are concerned with the influence of content coverage (which they saw as analogous to '*opportunity to learn*') on learners' achievement. They point out that the literature on content coverage can be separated into two main strands. One strand explores the influence of content coverage on learner's achievement and the other outlines studies which concern themselves with content coverage as part of a complex instructional component influencing the whole curriculum.

Sileo and van Garderen (2010) consider the fate of students with disabilities who are now educated in general classroom settings as a result of the implementation of the No Child Left Behind Act of 2001 and Individuals with Disabilities Education Improvement Act of 2004 in the United States of America. Many of these students struggle academically in various subject areas, including mathematics. Sileo and van Garderen (2010) encourage the creation of optimal learning opportunities for everyone since the emphasis in education is to ensure that all students learn. They suggest that the combination of research-based instructional practices in mathematics and co-teaching models may create powerful learning environments that enable all students to develop mathematical understanding. Co-teaching is a means of providing the desired learning and teaching outcomes that can benefit both students and teachers. They conclude that, although co-teaching structures can enhance student learning, it is also important to consider the subject matter. General and special educators can work together to blend their knowledge bases. This relationship is invaluable because it weds content and strategy specialists and allows teachers an opportunity to meet all students' mathematical learning needs (Sileo and van Garderen, 2010).

Though writing nearly two decades ago Bass (1993) made a point that rings true even today. He points out that the curriculum was still organised in ways that prevented many students from gaining access to the mathematics they need. Bass brought in an element which is not clear in other studies, that of what student's need rather than what is given to them irrespective of the relevance. The curriculum

should consist of what the learners find useful in their lives. This does not mean that the sole intention of the curriculum is to provide students with only what they need for that would make it narrow, but what they need can help to motivate them to want to learn. It can spark an interest in students to pursue mathematics further than the classroom. It is important to consider what the learners need so that they can find learning interesting and worthwhile.

In seeking to understand the significance of context, Rousseau and Powell (2005) adopt a framework that was explicitly tailored to the examination of equity in mathematics education. They sought a framework that would highlight the connections between equity and reform. For this purpose, they drew upon the work of Tate (2004) who outlined the important variables that shape students' opportunity to learn mathematics, namely *time, quality,* and *design.* Time and quality shape the vast differences in students' opportunities to learn, ensuring that students of colour and students from low-income backgrounds receive fewer opportunities to learn high quality mathematics than others.

According to Starratt (2003) it is unfair to test a student on material that they have not had enough time to prepare, especially when the failure of a test results in retention or the denial of a diploma. He views tests themselves as mere indicators of whether students have mastered the required material and says that it is up to teachers to identify the specific obstacles each student faces in his or her learning and to examine whether their own instruction is sufficiently reflective of curriculum standards to provide an adequate opportunity to learn. This shows the crucial role that teachers play in determining the future of their learners through preparing them for tests or examinations that channel them into different careers depending on their performance. Starratt (2003) concludes that the logical extension of the 'all children shall learn' policy is that no student should fail a test. In contrast to the above Starratt (2003) further stresses that judgments of learning should be based on multiple indices of learning and not be restricted to performance on standardized tests.

According to the Wikipedia Encyclopaedia, curriculum is a set of courses with a specific content and is the foundation of the teaching-learning process whether it is a school, college, university or training organisation. Curriculum may be partly or

entirely determined by an authoritative body. In this study the algebra curriculum is determined by the Department of Education. The curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well-articulated across the grades (NCTM, 2006). This is a good guide to use when considering OTL mathematics in the classroom. It is important that the program of work shows coherent ideas which are focused on materials and activities that are important to the learners and that ideas grow in logical sequence. Curriculum is taken as a central consideration whenever decisions on policies are made.

Stein, Smith, Henningsen, and Silver, (2000) claims that most debates about the improvement of mathematics teaching and learning centre on the curriculum, that is, what students learn, in what order and to what standards of proficiency. Cueto et al. (2008) say that theoretically, it is the implemented curriculum more than the intended curriculum that explains achievement. Achievement is referred to as the *attained curriculum* in the TIMSS framework.

Rousseau and Powell (2005) point out that there are various time-related factors that shape students' opportunity to learn such as time in class and planning time. These variables have in common their influence on content coverage and time-on-task in mathematics. In addition to influencing students' opportunity to learn, class time shapes teachers' efforts at reform. According to Rousseau and Powell (2005), teachers tend to stick to more procedural or rule-based teaching and learning, citing limited class time as the reason for them not to use reform-oriented practices. Both instructional and planning time shape teachers' responses to reform. Yet, this emerges as a potential equity issue insofar as the availability of class and planning time may be related to the teaching context. Therefore time is very important to consider since its management influences what, when and how material is covered.

Rousseau and Powell (2005) identify the adoption of a new curriculum as one of the key elements in the success of the 'The Mathematics: Application and Reasoning Skills' (MARS) project. Curriculum reform within this project began with the design of a new curriculum guide. Following the design of the curriculum guide, the district sought out and adopted instructional materials that supported the teaching and

learning of the objectives outlined in the curriculum guide. The curriculum guide and instructional materials were then used as key components of subsequent professional development efforts and served as crucial factors in the reform process (Rousseau and Powell, 2005). Also, access to high quality curricula and instructional materials is connected to issues of equity and fiscal adequacy. They cite a study where a strong relationship was found between students' economic status and the level of resources provided for their classroom experience. They claim that one of the concerns related to equity and curriculum is the likelihood that financial conditions will impact on the availability of high quality instructional materials, particularly in urban and/or high poverty schools.

The focus on high quality curriculum and instruction leads directly to another variable that must be considered in an examination of mathematics reform and equity-teacher preparation or teacher knowledge. According to Darling-Hammond (2000:37), "substantial evidence from prior reform efforts indicates that changes in course taking, curriculum content, testing or textbooks make little difference if teachers do not know how to use these tools well and how to diagnose their student's learning needs." Without adequately prepared and knowledgeable teachers, there is little chance that quality curriculum and instructional strategies will be effectively implemented. Barber and Mourshed (2009) pointed out that the best performing school systems across the world shared the following four clear lessons: The quality of education system cannot exceed the quality of its teachers; The only way to improve outcomes is to improve instruction; High performance requires every child to succeed; Every school needs a great leader. These lessons appear to suggest that learners should be provided with OTL.

While there are a myriad of professional development programs designed to promote mathematics reform, there are far fewer that have made an explicit effort to address issues of equity (Rousseau and Powell, 2005). According to Rousseau and Powell (2005), many people in mathematics education have yet to take seriously the significant differences between teaching contexts with respect to mathematics reform. It is one thing to say that all students can learn (including students in urban and high-poverty schools), but without trying to understand the influences of reform

in these contexts, it will be meaningless to say that opportunities to learn cannot be realized under such circumstances.

Like many countries of the world and particularly in developing countries, South Africa is faced with a critical shortage of mathematics teachers and a big challenge to develop a mathematically-skilled workforce in various fields (Moloi, 2005). On the supply side, statistics released by the Department of Education show that only three percent of all the students enrolled in institutions of higher learning in the year 2000 were in mathematical sciences as an area of specialization (Department of Education, 2005). In prefacing a National Strategy for Mathematics, Science and Technology for 2005-2009, the Department (2004: 10) took cognizance of this limitation and further expressed concern that the teaching of mathematics in schools was almost never a first choice to talented mathematics graduates. Consequently, mathematics was often taught by inadequately qualified teachers and this led to a vicious cycle of poor teaching, poor learner achievement and a constant under-supply of competent teachers. Evidently the current demand for mathematically competent workers in the country far outstrips both its supply and level of ability. Other than the 'dipstick' surveys that have pointed to low levels of achievement in numeracy, there has been no systematic attempt to research whether the curriculum is now accessible to all learners from different contexts (urban, rural and socio-economic strata) so that success through the schooling system can be guaranteed to all learners.

In South Africa in particular, broad participation (access) and quality achievement in mathematics have been prioritized for equity and general redress of historical inequalities (Department of Education, 2004). But educational equity is not likely without a range of opportunities to learn, opportunities that are wide enough to satisfy the diversity of talents of those who come to school (Eisner, 1994). According to Prediger (Undated), diversity of talent might manifest in the same classroom where it is possible to find that, while some students have already solved the given mathematical problems and are asking for further challenges, others have not even taken up their pencil. Whereas some students are trying to solve new mathematical problems with enthusiasm and creativity, others cannot even start working and have no confidence in their own capabilities. Arguably, this diversity of students is one of

the major challenges we meet in mathematics classrooms where opportunities to learn might seem equal but achievement is varied. This has major implications for teaching and learning mathematics and especially for the teacher who must create conditions that enable the diverse students to learn.

Mokhele (2007) conducted a study of Environmental Education OTL in the province of Mpumalanga in South African. She says that resources, both in terms of personnel and materials for Environmental education are inadequate and as a result the development of OTL Environmental Education is limited generally in the provincial schools. Schools in the province coped differently with the limitations imposed by provincial frameworks and this translated into differentiated OTL Environmental Education across the different schools. Mokhele (2007:127) has this to say, "In general, some schools have managed to create better opportunities for the learning and teaching of Environmental Education than others". Mokhele's study showed how some schools and teachers managed to create OTL Environmental Education in spite of the limitations in their own individual capacities. The mentioned study helps to shed light on OTL in the South African context.

Another study conducted on OTL in the South African context was carried out by Sehlola (2007), also in the area of Environmental Education. Sehlola (2007) explores how one primary school in the Gauteng Province of South Africa provides OTL about the environment, in the light of recent policy changes. Sehlola points out that one of the major limitations of the Department of Education's program of implementation of the new environmental learning policy is the inability to provide teachers with enough time to learn and implement the new ideas of the revised national curriculum statement.

Sehlola (2007) observes that the limiting factor at the school is the absence of properly qualified teachers in the field of Environmental education. However, though the teachers tended to limit the content of environmental lessons to topics covered at workshops, all the teachers observed made genuine attempts to include environmental themes and knowledge into their lessons. The school managed, through its own networks and initiatives, to create some latent capacity to provide significant OTL about the environment for its learners. For example, the school

formed partnerships with Non-Governmental Organisations to enhance professional development resulting in better delivery of lessons. Sehlola (2007) has therefore shown that it is possible to create OTL for learners even when the teachers are not all qualified. If the capacity building program (even through peer development) is in place then gradually the provision of OTL will improve.

While official curricula generally point to the ideals and aspirations of education systems, in the final analysis it is what is taught and learnt in the classroom (the implemented curriculum) that eventually translates into observable and measurable outcomes, intended or otherwise. Besides learner characteristics (e.g. gender, age, intelligence), access in terms of the availability, adaptability and acceptability of learning support materials constitutes one major determinant of the level and quality of learner achievement in the education process. But as the PROM/SE (2009) report observes, measurement of the curriculum that teachers implement in the classroom is fraught with complexity. Furthermore, a comprehensive view of the curriculum implementation may require us to not only determine what content teachers cover but what they emphasize, what achievement standards they use, the effectiveness of their pedagogical strategies, and the syllabi, textbooks and other resource materials employed to support learning. There is also awareness of how socio-economic contextual factors can hinder or support access to and success in the curriculum (UNESCO, 2005; Ross and Zuze, 2004).

2.7 The teaching and learning of mathematics.

It is common knowledge that learning mathematics has always challenged students the world over. Most students have a hard time to acquire mathematical skills. This calls for an intensive effort on the part of the teachers and learners to improve performance as much as possible. The call is not to remove the problem as this intention would be futile, but to find ways to overcome it, at least in part. In dealing with the learning process it is important to bear in mind that it is not only about the conditions prevailing in the school or classroom but also about content and exposure to that content.

In the process of mathematics education we have to consider what the learners are

learning and for what purpose. It is important to keep in mind the desired outcome such as the exit skills we hope to achieve. A host of factors must be taken into account, such as the context in which the teaching and learning takes place, the stakeholders involved, and the skills required by the market for which they are being developed. Assessment, both formative and summative is important to consider for it determines directly or indirectly how students learn and study. Formative assessment mainly involves what takes place in the classroom and includes exercises, assignments and portfolios which are used as evidence of learning. Activities outside the classroom, for example a visit to industry or any place that can expose a learner to practical mathematics, can also be part of the formative assessment which can help the learners to choose their future careers. Summative assessment involves mostly written examinations. All assessments must however support the development of mathematical proficiency (Kilpatrick and Swafford, 2002).

One of the most important ways of judging achievement in education is through assessment. Assessment and instruction are linked because assessment results have important implications for instruction. Porter (1995) says that school communities use assessment results in a formative way to determine how well they are meeting instructional goals and how to alter curriculum and instruction so that goals can be better met. (Although this is an old source it is still relevant today). But unless the content assessment and the format of assessment match what is taught and how it is taught, the results are meaningless, if not potentially harmful. The same is true if assessment tools are not of high quality. And there is also potential harm when decisions affecting students' futures are made based on results of assessments made with tools that are not appropriate for the purpose (Porter, 1995).

The language of instruction is another factor that impacts on curriculum. In South Africa the majority of the learners do mathematics in a language that is not their mother tongue. It is common knowledge that language is a major contributory factor in cognitive development as it is a vehicle for thought. Setati (2008) supports this idea and regards language as a tool for thinking and communicating. She investigates access to mathematics versus access to the language of power and exposes the struggle in multilingual mathematics classrooms. She reports that there

is a general view in South Africa that most parents want their children to be educated in English and that most learners would like to be taught in English. While there is no systematic research evidence, it is also widely held that many schools with an African student body choose to use English as the language of learning and teaching (LoLT) from the first year of schooling (Setati, 2008). Because the TIMSS results in South Africa were very poor, studies that have emerged from this argue that the solution to improving African learners' performance in mathematics is to develop their English language proficiency (Setati, 2008).

Setati (2008) asserts further that decisions about which language to use in multilingual mathematics classrooms are not only pedagogic but also political. Most research on mathematics education in multilingual classrooms has argued for the use of the learner's home language for learning and teaching mathematics. This provides a support while learners continue to develop proficiency in the other language of learning and teaching (e.g. English). Setati (2008) says the political role of language in mathematics education research and practice should not be ignored because doing so would imply that power relationships do not exist in society.

Many African teachers and learners investigated by Setati (2008) associate the English language with mathematics learning since it is the language of mathematics textbooks and assessment. Secondary school mathematics textbooks have never been published in African languages in South Africa. According to policy, African languages can be used as languages of learning and teaching but English and Afrikaans are the only possible choices for teaching and learning mathematics. What is interesting is that none of the teachers challenged the choice of English or the fact that textbooks and examinations are given only in English even though learners are still developing sufficient fluency in it (Setati, 2008).

Teachers and students in the Setati (2008) study have different views concerning the role of English in the teaching and learning of mathematics in South Africa. Some teachers are of the opinion that English should be used without question because it is an international language. According to these teachers using any other language would disadvantage the learners in the long run. One teacher argued that the mathematics classroom should be used as an opportunity for learners to gain access

to English. Other teachers believe that mathematics is a language in its own right and so it does not matter which language it is taught or learnt in. In other words the authority is in the mathematics, not in the language of instruction. Setati (2008) concluded by saying:

> Fluency in English, while necessary, is not a sufficient condition for improving performance or learning in mathematics. While successful learning of mathematics is only possible in contexts where the learners are fluent in the LoLT, it is also important to recognise the fact that success cannot only be attributed to the learners' proficiency in the LoLT. There are other factors such as the teacher's knowledge of the mathematics she is teaching, her knowledge of the learners and how she draws on the learner's fluency in the LoLT.

Cocking and Mestre (1997) point out two issues related to language: first, whether inadequate preparation in English impedes the learning of mathematics and second whether students who are bilingual develop cognitive systems that facilitate the learning of mathematics. They claim that one of the problems in the study of language is the confusion of language with other processes that also affect learning. The effects of language should be differentiated from other characteristics of culture and from patterns of interaction within classrooms.

2.7.1 Learning and teaching school algebra

Algebra is often described as a gateway to higher mathematics, not least because it provides the language in which mathematics is taught (Chick, Kendal & Stacey, 2004; MacGregor, 2004). But algebra is also known as a major stumbling block in school mathematics, both in the past and at present (Amerom, 2002). Accordingly, mathematics curricula all over the world are calling for greater understanding of the fundamentals of algebra and algebraic reasoning by all members of the society (Osta, 2004:3) Designing instruction to maximize learning opportunities also requires an in-depth understanding of the cognitive difficulties of learning algebra. This level of understanding can be achieved by studying the way that learners learn algebra from their point of view. Therefore, it is important to examine student and instructional factors related to algebra achievement in order to improve opportunities for success in mathematics. Ferrucci (2005) points out that teaching algebra and algebraic thinking is both complex and dynamic. Furthermore, various professional

organizations, researchers and educators have varied perspectives on the exact nature of the topics to be included in an algebra curriculum.

There is no consensus amongst researchers on what algebra is or how it should be taught and learned. Problems with algebra can be ascribed to external factors like the teaching approach and a poor image, but also to the intrinsic difficulties of the topic which prevent many students from making sense of it (Amerom, 2002). It appears the majority of students are not able to connect by themselves the knowledge domains that constitute manipulative algebra because of language obstacles. Some have argued that the difficulties children have in algebra relate to the abstract nature of the elements in algebra (Samo, 2008). Amerom (2002) points out that traditional school algebra is primarily a very rigid, abstract branch of mathematics which has few interfaces with the real world and it is often presented to students as a pre-determined and fixed mathematical topic with strict rules, leaving no room for their own input.

Traditional instruction begins with the syntactic rules of algebra, presenting students with a given symbolic language which they do not relate to (Amerom, 2002, Kaput, 1995). Students are expected to master the skills of symbolic manipulation before learning about the purpose and use of algebra (Stacey et al., 2004). In other words, the mathematical context is taken as the starting-point, while the applications of algebra (like problem solving or generalizing relations) are second place. Students are given little opportunity to find out the powers and possibilities of algebra for themselves. One can imagine that an average or below-average learner finds little satisfaction in practicing mathematics without a purpose or a meaning. Amerom (2002) points out that there is a rapid formalization of algebraic syntax in the traditional approach.

Even though we all have an immediate idea of what students learn when they learn school algebra, it is not an easy task to give a clear-cut definition (Amerom, 2002). Algebra is the first mathematical discipline that students encounter that uses variables (Saeman, undated). For the student, all previous mathematics problems have nothing that is that varied; previously they were presented with some numbers and an operator (a plus, minus, multiplication, or division sign) and had to come up

with the answer. With algebra, the situation is much more subtle. Instead of simple math problems, algebra students are presented with equations. Now the students must not only calculate variables, but they must also determine which operators to use. The algebra tutor must help the student overcome this paradigm shift. Algebra has been described as generalized arithmetic, as a study of procedures for solving problems, as a study of relationships among quantities and a study of structures.

Amerom (2002) lists typical topics of school algebra, these include simplifying algebraic expressions, the properties of number systems, linear and quadratic equations in one unknown, systems of equations in two unknowns, symbolic representations and graphs of different kinds of functions (linear, quadratic, exponential, logarithmic, trigonometric), and sequences and series. In most of the core activities we find aspects of *algebraic thinking* (mental processes like reasoning with unknowns, generalizing and formalizing relations between magnitudes and developing the concept 'variable') and *algebraic symbolizing* (symbol manipulation on paper). Generally it is agreed that students must acquire both competencies in order to have full algebraic understanding.

Algebra is regarded as the language of mathematics and has its own vocabulary, which makes it an essential part of all school mathematics (Stacey, et al, 2004). It could be said that working confidently with symbolism in algebra is akin to using a second language fluently. In South Africa the majority of black learners learn mathematics in a second language, so algebra can be perceived as a third language. As a language, algebra can be introduced as a medium for expressing relationships between two variables (MacGregor & Stacey, 1997). Because of the nature of generalization and abstraction, algebra is considered to be a difficult area of mathematics. There is no best approach to algebra because it is not possible to find an approach that suits all classroom situations. Since there is no agreement on what algebra is or what it should be, some have suggested that it is better to consider algebra in terms of its *roles* in different areas of application instead.

A variable that varies (as argument or parameter) is considered to be of a higher level of formality than the variable as generalized number or unknown, which is again more formal than the placeholder; at the top end we find the arbitrary symbol

(Amerom, 2002). This subtle variation of meanings of letters has been identified as one of the major obstacles in learning algebra.

Algebraic skills are directed at translating and generalizing given relationships among numbers (Amerom, 2002). The introduction to algebra usually involves the study of algebraic expressions, equations, equation solving, variables and formulas (Amerom, 2002). According to Kieran (1989, 1992), student's learning difficulties are centred on the meaning of letters, the change from arithmetical to algebraic conventions and the recognition and use of structure. Grade ten learners who are the main concern of this study would ideally have passed this stage, but those lagging behind might need reminding of these basic structures before they can proceed to new work. Students struggle to acquire a structural conception of algebra, which is fundamentally different from an arithmetical perspective (Amerom, 2002).

Learners who experience difficulties with conceptualising the structures or symbolism of early algebra would need special attention from the teacher if they are to build up their understanding to the desired level of competence at their grade level. Therefore, it is important to consider how missed opportunities to learn can be compensated for, bearing in mind that it is difficult to teach what has been missed over a long period and still hope to cover the work required in the current grade. If the needs of the learner who is struggling are not catered for, the learner will not realise the opportunities to regain what he or she has missed.

This and the next four paragraphs present a study by Panasuk (2010) who investigated learner's conceptual understanding in algebra using multiple representations. She wanted to find a consistent measure of student's conceptual understanding so as to be able to assess whether students learn algebraic concepts beyond procedures. To achieve this Panasuk launched a longitudinal study which led to the formation of the three phase ranking framework of conceptual understanding of linear relationships with one unknown. She looked at how students used different representations to a) extract information from a situation, b) represent the information in other forms, c) manipulate with representations, and d) interpret and test the solutions of the linear equations with one unknown.

According to Panasuk (2010) students who have a conceptual understanding grasp the full meaning of knowledge and can discern, interpret, compare and contrast related ideas of the subtle distinctions among a variety of situations. Conceptual understanding in algebra can be characterized as the ability to recognize functional relationships between the known and unknown, independent and dependent variables, and to distinguish between and interpret different representations of the algebraic concepts. It is manifested by competency in reading, writing and manipulating both number symbols and algebraic symbols used in formulas, expressions, equations and inequalities. Fluency in the language of algebra demonstrated by confident use of its vocabulary and meanings and flexible operation upon its grammar rules (i.e. mathematical properties and conventions) are also indicative of conceptual understanding in algebra.

Mathematical relationships, principles and ideas can be expressed in multiple representations including visual representations, verbal representations and symbolic representations. Each type of representation articulates different meanings of mathematical concepts. Representations are powerful communication tools for mathematical thinking. Each representational system contributes to effective communication of mathematical ideas by offering certain types of language. Mathematics students are continuously involved in the process of abstraction because they are engaged in transformation of their perceptions into mental images by means of different representations. If conceptual understanding is defined by the degree of abstraction, then the idea of adaptation to abstraction becomes critical, and the process of building mathematics conceptual understanding can be viewed as a transition between the levels of abstraction from lower to higher.

Panasuk (2010) speculates that the students who do not demonstrate conceptual understanding have not been exposed to the 'culture' of multiple representations of the same concept. The algebraic symbol system provides an avenue for expressing mathematical principles, concepts and ideas in general forms by using mathematical models. It is the only system that offers opportunity to logically investigate, justify, generalize and prove mathematical hypotheses. Yet, the algebraic symbol system is too sophisticated for some students. All the representations integrated into teaching and learning would contribute to the development of a big mental picture of the

concepts studied. Even the student who is advanced and capable, has developed strong procedural skills and achieved some level of operational conception will benefit from learning multiple tools to communicate his/her cognitive skills.

Panasuk (2010) concludes that the idea of multiple representations hardly needs advocacy to support its power, significance and vitality for mathematics and mathematics teaching. Given that mathematics is a science of patterns and multiple representations of these patterns, they should be an integral part of teaching and learning mathematics. Teaching through the multiple representations would require teachers to have a solid foundation in mathematical content, strong skills in a structural analysis of the concepts and tasks as well as sound knowledge in effective planning. Teachers skilled in seeing the big picture of the concept and small links between the sub-concepts are likely to be able to integrate multiple representations into their instructional practices and to use representations when teaching all students, rather than for remediation purposes only. Multiple representations availed to students are likely to stimulate in them the development of a conceptual understanding of mathematics. It is the duty of the teacher to expose students to multiple representations. With the encouragement and support of teachers, students will be able to internalize and integrate multiple representations into their cognitive structure and use the representations as a communications tool.

2.8   OTL conceptual framework

The problem of opportunities to learn mathematics can be understood by looking at the historical experiences that have shaped it over the years. Literature reviewed has shown that the problem is multi-dimensional and dynamic. Literature from other researchers has revealed that OTL holds a promise for improvement of educational performance all round. Most of the studies on the impact of OTL were carried out in the United States of America and their concern is mostly with the disadvantaged minorities. In South Africa the previously disadvantaged are in the majority.

The literature review has revealed that OTL can be used to aid the understanding of the teaching and learning process. It is clear that most researchers connect students' achievement with their opportunity to learn the content. Many researchers point out

that the study of students' opportunities to learn provides great insights into variation in student achievement. Researchers have described the opportunity to learn in a variety of ways: a teacher's reported content coverage, time allocated for instruction or instructional time that is actually used to deliver instruction. The achievement of students is not a simple issue of cause and effect for learners perform differently even under the same conditions. Some learners may not recognise the opportunities offered to them to learn because of other reasons that impact on them socially or otherwise.

An assortment of factors affects the learner during the process of education. Stevens (1993) observed that OTL is a conceptual framework developed from information obtained from a series of international and national research studies. Four OTL variables that are prevalent in research include content coverage, content exposure, content emphasis and quality of instructional delivery (Stevens, Wiltz and Bailey, 1998). However, when the OTL framework is used to determine whether or not students are provided with sufficient access and information to learn the curriculum at their age and grade level, it becomes a powerful concept of educational accountability.

Stevens (1993) claims that research involving OTL variables provides insight into why some students perform at or above grade level and others do not. Though useful information can be derived from studying the aforementioned variables one at a time, Stevens recommends studying them simultaneously. Doing so gives a clearer picture of the nature of the relationship between classroom practices and academic achievement. She concludes that the OTL conceptual framework provides a more comprehensive explanation that can be used to bring about a far more positive impact on students' academic achievement (Stevens, 1993). This can lead to more effective ways of improving teaching and learning. According to her the opportunity to learn the designated curriculum for a grade level or age group is a major equity issue for students who are at risk of not developing academically to their fullest potential. Classroom teaching practices are difficult to reform because educators and school administrators in the United States rarely use district-level assessment standards in their decision making (Stevens, 1993). Also, little or no data are collected or analysed about the activities that surround teaching and the

impact of classroom teaching practices on students' academic achievement outcomes. More research on teaching processes and practices apparently has not filtered down to or changed many of the teaching practices of teachers in poor urban school classrooms (Stevens, 1993).

Stevens goes on to blame teaching practices in many urban schools in the United States of America for not improving the already poor academic achievement of poor and minority students. Learners are given activities characterised by routines that do not allow room for creativity. Such practices deny learners the opportunity to learn the core curriculum appropriate at their age and grade level. Students in poor urban schools are described using terms that reveal their disadvantages emanating from their low socio-economic backgrounds which gives the impression that the schools are not responsible for their under-performance. Stevens (1993) argued that the ascribed descriptions have no relevance to the problem of low academic achievement since they cannot be changed**.**

Our understanding becomes clearer if student achievement is related to the opportunity to learn, regardless of family background. It becomes a more relevant issue to consider what happens in the classroom, that is, the teacher's teaching practices and the quality of the delivery of lessons for these can be changed or improved (Stevens, 1993). She observes that whereas learners from the middle class can get supplementary help from home, those from working class backgrounds depend on what is offered in the classroom. That is why it is important to consider teacher practices.

If teachers are assisted in implementing instructional models and programs that promote access to learning for poor and minority students, the result could be academic progress for those students (Stevens, 1993). If teachers themselves are not aware of the fact that their practices are unhelpful to the children's improved performance, the situation will continue unresolved. Stevens laments that reforming classroom practices is difficult because educators do not keep abreast with research or other official material that comes from the department of education or district office. The problem is how to encourage them to embark on good practices that increase the learner's opportunity to learn. Stevens (1993) uses a case study

approach to study "teachers learning and applying OTL assessment strategies" because of the nature of the questions posed, namely, asking *how* and *why* within real-life contexts when the boundaries between the phenomena and context are not clearly evident.

2.9   The theoretical framework

Though constructivism in general is the dominant discourse of mathematics teaching and has been interpreted as problem-based learning when it comes to the actual teaching and learning suggestion about how the knowledge is constructed, it is too general to reach the classroom directly. I think Duval (1995, 1999 and 2006) offers a practical approach that the educator can use in teaching for understanding. Duval is also a constructivist because he uses ideas from both Piaget and Vygotsky to produce a theory of how children learn mathematics. He developed a way of considering how children construct mathematical concepts based on the transformations of mathematical registers. Transformations refer to all manners of changes that a thing or person can undergo. States refer to the conditions or the appearances in which things or persons can be found between transformations. Piaget theorised that intelligence is active and constructive. Duval's theory of learning is also active for it involves transformations of registers of representations in the process of constructing knowledge.

According to Duval (1995) mathematical objects are a creation of the human mind; they do not exist out there. Drawing on Piaget (1973), he claims that children actively construct knowledge of the world by continually interacting with and adapting to their environment. Duval (1999) presents a framework that can be used to explain students' mathematical thinking. He says that in order to explain how students construct knowledge it is necessary to understand what he called cognitive functioning of mathematical thinking and conditions of learning. He argues that in mathematics conceptual acquisition necessarily passes through articulation of at least two semiotic representations. This articulation manifests itself by the rapidity and the spontaneity of the activity of conversion between registers (Duval, 1995).

In particular, conceptual acquisition in mathematics requires the learner to manage the following semiotic functions: the choice of the distinguishing features of the concept represented, treatment that is a transformation in the same register and conversion which is the changing from one register to another (Duval, 1995). In algebra, examples of treatment are: making 'x' the subject of a given formula or simplifying and rephrasing a question or sentence. In this study, it is important that students work with different registers or forms of representations of algebra concepts in order to learn algebra effectively. Conversions are transformations that change the system while maintaining the same conceptual reference such as going from a geometric to an algebraic representation of the difference between two squares. The squares are drawn and the difference in their areas shown by shading or some other marking. An algebraic representation can then be derived or simply deduced.

The combination of these three actions, formation, treatment and conversion, on a concept represents the construction of knowledge in mathematics but the coordination of these three actions is not spontaneous or easily managed (Duval, 1995, 1999; Hitt, 2002). Conceptual understanding is characterised by a person's flexible use of various representations of the same mathematical concept. The grade ten syllabus in South Africa aims to groom learners who can flexibly convert between the different representations algebra concepts. According to Duval (1995) the most difficult aspects of learning mathematics is the handling of conversions between registers.

In the classroom it is important for the teacher to ask the right questions, such as asking the learners to explain what they are doing. This enables the learners to make the necessary conversions in the process of building up the required mathematical concept. Once the learners are able to move back and forth between the registers, it is reasonable to say that they have understood that which they are supposed to learn. Formation does not require preparation because this is already done in the textbooks. Treatment and conversion are important and the teacher should be able to distinguish between them because there is a danger that the teacher can dwell on treatment, such as solving problems, without progressing to the conversion.

Duval (2005) points out that a mathematical object always has more than one semiotic representation attached to it. Also it is very easy to see the mathematical object as being one of its representations. There is a danger then that this mistaken identity may lead to misconception or no learning at all, for example, one can take the graph to be the object but it is actually only a representation of the object.

The implication for teaching is that the teacher should use a variety of representations of mathematical ideas, for example numeric, geometric, algebraic and oral. The reason is that various forms of representation are an integral part of doing mathematics and thus also of its teaching and learning (Hitt, 2002). Availability of multiple representations enables an entirely new perspective on a concept which is why the creation of a variety of representation of the same concept is essential if learners are to comprehend the required knowledge (Duval, 1995). It is from simultaneous access that representations in different registers provide multiple approaches to a problem.

If teachers understand Duval's structure of learning, that is, the three actions required in the construction of a concept, they can provide learners with the necessary experiences that lead to learning with understanding. By following the three conditions that Duval identified as essential for learning, the teacher can structure instruction logically. The way that an opportunity to learn is generated in the classroom is determined by the way instruction is structured and delivered. In the teaching of algebra the teacher has to facilitate the movement between different forms of representation. Algebra is based on the concept of variables which can be difficult for learners if different representations are not offered. Algebra is more than moving symbols around. Learners need to understand how the symbols come to be, how they are used to record ideas and gain insight into situations.

If the educator exposes learners to the distinguishing features of any mathematical concept, gives them enough practice (treatment) and leads them to make the appropriate conversion between different registers of representation, it is reasonable to expect them to learn with understanding. Considering the way the concepts are acquired is useful when planning for teaching and reflecting on lessons taught. Tailoring lessons to accommodate formation, treatment and conversion would give

learners a real opportunity to learn. Lessons in translating verbal and/or numeric representations to symbolic algebra rely heavily on the ability of the learners to make the necessary conversions between registers.

2.10   Conclusion

One category of factors can be termed instructional and includes instructional time which is time spent on preparation and delivery. This can influence or determine the mastery of the concept. Determining time allocated to particular topics officially, locally and individually by practitioners is important when considering OTL because time shapes both the teacher's and learner's efforts. Instructional practices are another factor for they shape or create the environment under which teaching and learning takes place. The practices are influenced by a teacher's content knowledge, beliefs and prior experience in teaching mathematics. This being the case, learners will experience different opportunities to learn because of a variation among the teachers in relation to their practices. This variation can become a serious concern in as much as it affects a student's performance.

Instructional materials support teaching and learning to a great extent as materials are connected to issues of equity. Provisions and the distribution of materials for use by students should take place in such a way that they reach all sections of society and so help to prepare students for success. In other words, a student's opportunity to learn will improve with the adequate provision of appropriate instructional materials.

The preceding literature review has revealed that OTL is an important concept when applied to the learning and teaching of mathematics. OTL has characteristics that if attended to can help improve the learning in mathematics classrooms. Variables that help to define OTL map out what can be said to be a beneficial practice.

Considering what psychologists say about acquisition of concepts has helped me understand how human beings learn and so apply it to the teaching and learning of algebra. Having an understanding of child psychology enables one to identify important factors that impact on the learning of children. For example, the psychologist Duval (1999) has contributed a lot to the understanding of how students

achieve mathematical understanding. He presents a framework that can be used to explain students' mathematical thinking. Duval (1999) states that mathematical understanding requires the coordination between at least two registers of representation. This informs teachers when they plan their lessons.

In much algebra teaching, conceptual understanding of the objects of algebra has tended to be segregated from the development of manipulative skill. Few have espoused the position that students' conceptual understanding grows as they engage in algebraic processes (Stacey et al., 2004:25). Algebra as a problem solving tool appears to be gaining significance. In various countries, problem solving by whatever means, has all but replaced traditional algebra. The hope was that, in focusing on algebraic understanding (however it might be defined) the techniques would take care of themselves. But it did not happen. Doing algebra is a process of acting on signs. Signs are the objects of the algebraic activity. Letter, expressions, graphs, written calculations, schemes, proofs, models and so on, are signs and are made of signs. Stacey et al., (2004) claimed that semiotics lies in the core of algebra, not that all algebra is semiotics. The point is that the way you teach algebra depends largely on what you believe algebra to be.

CHAPTER THREE

METHODOLODY

3.1   Introduction

This chapter describes the methodology adopted in the study. It is divided into three main parts. The first part is a review of the research questions answered by the study and challenges faced in this process. The second gives an overall view of the method used in conducting the study. Finally, a detailed description of the data collection process and analysis is provided. The process was flexible and open and I actively sought opportunities to revisit and revise the research design in order to address and add to the original set of research questions.

3.2   Review of research questions

As stated in chapter one, the first question in this study relates to the content and context of the official mathematics curriculum in grade ten. This question was answered largely through the review of documents containing the official or intended grade ten mathematics curriculum in South Africa. I was interested in uncovering the key content themes that are prescribed for grade ten learners. Justification for the identified themes was sought through literature and other means such as interviews with relevant subject area officials.

Next, I observed actual classroom lessons in progress to see how algebra was taught to grade ten learners in the selected schools. I wanted to explore how far teaching reflects the official curriculum. While observing the classes I paid particular attention to the way teachers provided the learners with OTL. The focus was on how OTL was characterized and how this could be explained from the official and enacted curricula. By answering the research questions the study hopes to contribute to the on-going debate on the teaching and learning of mathematics while assuming a positive relationship between opportunity to learn and achievement. The

study attempts to broaden knowledge and deepen the understanding of the role played by OTL in the teaching and learning of grade ten algebra.

3.3   Research Design

3.3.1 Qualitative Research

I used qualitative inquiry paradigm to investigate three grade ten mathematics educators teaching algebra. The claim that this study is qualitative stems from the understanding that qualitative research places emphasis on understanding through closely looking at people's words, actions and records, which is what this study sought to engage in (Denzin and Lincoln, 2000). Through observing lessons I discovered patterns of how each of the three teachers provided learners with opportunities to learn algebra. The patterns came through the actions and words that teachers used while teaching. I considered five factors to help me recognize when and how the teachers were providing learners with OTL. These were the teacher's approach to teaching, the questions they posed, the types of tasks they gave, their use of algebra specific terms and their use of different registers of representation of algebraic concepts. My task involved listening carefully to words said by both the teachers and the learners including observations of corresponding actions. These patterns have been presented for others to inspect. I tried in my descriptions to stay as close as possible to the construction of the world as the participants originally experienced it.

Denzin and Lincoln (1994) define qualitative research as a multi-method in focus, involving an interpretive, naturalistic approach to its subject matter. In line with this view, I examined official curriculum documents to explore the relationship between the intended and enacted grade ten algebra through observation of actual lessons at three different schools. The aim was to find out how the teachers created the opportunities for the learners to learn the expected algebra content. This took place in the usual classroom setting following the normal routine of the school day so as to avoid any stresses arising out of singling out some learners for the study. However, on one occasion one teacher offered to reschedule her lesson so that I would not

miss observation that day. I also reviewed literature on both the concept of OTL as well as on the teaching and learning of algebra.

Creswell (1994) defines qualitative study as an inquiry process of understanding based on distinct methodological traditions of inquiry that explore a social or human problem, where the researcher conducts the study in a natural setting. Qualitative research results are not sweeping generalizations but contextual findings. This process of discovery is basic to the philosophy underpinning a qualitative approach.

According to Schram (2003), qualitative inquiry is much more difficult to define than it is to identify for it appears in a myriad forms. Commonalities are however discernable, such as that it involves the studied use and collection of a variety of empirical materials, case studies, personal experience, introspection, life story interviews, observations, historical material, personal interactions and visual texts that describe routine and problematic moments and meanings in individuals lives.

I used the natural setting of the classroom to conduct my observation of the three different teacher's instruction and the classroom interaction. I then described in detail what I saw, heard and recorded. Creswell (1994) supports the presentation of a detailed view of the topic derived from a study of individuals in their natural setting arguing that if participants are removed from their setting, it leads to contrived findings that are out of context.

Qualitative research questions seek to find out what is going on so they often start with 'how' or a 'what' (Creswell, 1994). In this study I sought to find out what was going on in the selected class rooms by observing the teachers in action after having looked at what was stated in the official documents. When I went to the classrooms it was not with the intention to pass judgment but to learn from the experience so as to expand my own understanding of both teaching and the learning of algebra. I was then in a position to tell the story from the participant's view rather than as an "expert" who passes judgment on participants.

Rozycki (2009) stipulates that qualitative research is characterized by an emphasis on describing, understanding and explaining complex phenomena; on studying, for

example, the relationships, patterns and configurations among factors or the context in which activities occur. Consequently I have described in detail what took place in the classrooms I observed. I reveal though the descriptions and excerpts from the lessons how the teachers created OTL in their respective classrooms. In chapter four I describe what I perceived to the reasons behind their practices. I have recorded as far as possible what they said in their own words.

In trying to understanding how opportunities to learn algebra were provided or created I was aware that it was necessary to consider a host of influences such as the curriculum as defined by the department of Education, and how the educators interpret and enact it. This takes place outside and inside the classroom because the planning is usually done outside classroom. According to Hoepfl (1997) researchers can use qualitative methods to gain new perspectives on things about which much is already known. In this study I was aware that a lot was already known about opportunities to learn, especially in the United States of American context, but not much in the South African. OTL are also used to gain more in-depth information that may be difficult to convey quantitatively. Hoepfl (1997) further stresses that unlike quantitative researchers who seek causal determination, prediction and generalization of findings, qualitative researchers seek instead illumination, understanding and extrapolation to similar situations.

Another reason for selecting a qualitative approach is that in doing so, research problems tend to be framed as open-ended questions that can support the discovery of new information. The ability of qualitative data to more fully describe a phenomenon is an important consideration not only from the researcher's perspective, but also from the reader's perspective. Qualitative research puts the researcher in a better position to understand people by looking closely at their experience in the world from their own perspective, using their own words and actions. This provides data that is rich in detail that emanates from the participant's experiences of the world.

3.3.2 The design: case study

I selected the case study as the research method because as Stephens (2009: 46) says: "Case studies are a step to action. They begin in a world of action and contribute to it. Their insights may be directly interpreted and put to use." This study took place in the classrooms where action was observed and new knowledge sought. Another reason is that case studies are a common way of doing a qualitative inquiry (Denzin and Lincoln, 2005: 443).

Shuttleworth (2008) describes a case study as an in depth study of a particular situation rather than a sweeping statistical survey. It is a method used to narrow down a very broad field of research into one easily researchable topic. This study sought to understand how the opportunities to learn grade ten algebra were played out in selected Catholic schools. It was important to choose a design that allowed for deep descriptions of this process. The focus on process type questions coupled with a limited sample led to the selection of a case study as the preferred design. Also, case study research is flexible and enabled me make changes along the way, for example the pilot case became one of the main cases because I learned a lot from the experience.

Case studies are the preferred strategy when 'how' or 'why' questions are being posed, when the investigator has little control over events and when the focus is on a contemporary phenomenon within a real-life context (Creswell, 1994; Soy, 1997; Flick 2009; and Rozycki, 2009). This study has asked 'how' questions so the case study was appropriate. Also, a case study is especially appropriate when the boundaries between phenomenon and context are not clearly evident. I sought to find out the opportunities created in the classrooms by teachers for learners to learn algebra. Attending a lesson does not necessarily mean that OTL will arise.

The case study copes with the technically distinctive situation in which there will be many more variables of interest than data points. This is the situation in classrooms, where learners and teachers interact with each other and together with the subject matter. What one observes has to be taken in context because the story a case tells may or may not be useful unless the researcher explains the issues involved (Denzin

and Lincoln, 2005). It was necessary to use more than one way to get data for the cases because multiple sources of evidence strengthen the claims that I make in the end. Also, according to Soy (1997) a key strength of the case study method involves using multiple sources and techniques in the data gathering process. In this study I used interviews, documentation and other literature reviews and observations to collect data.

In order to avoid a total immersion in the setting or culture, sites, experiences and informants were sampled. In this study three schools were selected for participation. These were chosen because it was easy to access them without appearing to intrude. The exact classrooms that participated were identified for their relevance, that is, they were grade ten learners doing algebra.

The idea of using this method is supported by (Johnson, 1997) who points out that case studies allow the researcher to get in-depth understanding of the phenomena. Stevens (1993) also recommends that in order to see just what opportunity students are actually given to learn the curriculum, the case-study approach can be used. A case study is particularistic because it focuses on a specific phenomenon such as a program, event, process, person, institution or group (Creswell, 1997). For the current study the program was the learning of mathematics, the event was the particular topic of algebra taught to learners by their educators at their particular schools. The major purpose of this design is to describe a unit, rather than to test hypotheses. I present each of the three schools as a case. Each case tells a story of how the teacher provided OTL for the respective learners. Stake (2006) warns that even though many a researcher would like to tell the whole story, it is not possible because the whole story exceeds anyone's knowing and anyone's telling.

In this study the focus was on the opportunities to learn algebra as experienced by grade ten learners. This was used as an opportunity to learn more about the situation and so be able to explain the experiences of learners in relation to it. Soy (1997) stresses that case studies emphasize detailed contextual analysis of a limited number of events or conditions and their relationships.

The case study like any other design has strengths and weaknesses. A case study is low on control and representativeness. One can hardly differentiate cause from effect, and inferring from the intensive study of one or a few cases involves a high and generally unknown amount of risk (Labovitz and Hagedorn, 1976). (Although this is an old source what was said then is still relevant because differentiation between cause and effect is still problematic today). A frequent criticism of the case study methodology is its dependence on a single or few cases which renders it incapable of providing a generalizing conclusion (Soy, 1997; Shuttleworth, 2008). Soy (1997) observes that others feel that the intense exposure to study of the case biases the findings while some dismiss case study research as useful only as an exploratory tool. Yet researchers continue to use the case study research method with success in carefully planned and crafted studies of real-life situations, issues and problems. In this study the schools were chosen from one category of schools, that is Catholic Schools. These schools already have a distinctive ethos which is different from non-Catholic schools, but I don't think this necessarily handicaps the study because the learners come from the same communities. Experiences in the other schools, in my opinion, might not be different from what I observed in the Catholic schools. From personal experience of observing lessons in different school set ups (following student teachers on teaching practice), I have reason to believe that there are other reasons for differences other than type of responsible authority.

I was the instrument of research who evaluated what was going on in the classroom. I analysed the data with an eye to answering my research questions. Being in the classroom helped me get firsthand experience of what was going on in the classroom in terms of the teacher teaching and students learning.

The major advantage of a case study lies in the richness of its descriptive examples that results from the intensive study of one or a few units. In this way a lot can be learnt to augment what is already known. A case study provides more realistic responses than a purely statistical survey (Flyvberg, 2004; Shuttleworth, 2008). Furthermore, while a pure scientist is trying to prove or disprove a hypothesis, a case study might introduce new and unexpected results during its course, and lead to research taking new directions, because it is flexible. In this study I witnessed

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teaching activities that I had not known existed, for example, one teacher had ten learners working on problems on the chalkboard quite effectively.

Other advantages of a case study are its applicability to real life, contemporary human situations and its public accessibility through written reports. Case study results relate directly to the common reader's everyday experience and facilitate an understanding of complex real-life situations. The lengthy descriptions of the lessons presented in chapter four of this study give the reader a taste of what transpired in the visited classrooms. The reader can listen to the voices of both the learners and the teachers and experience what I experienced. But others feel that the intense exposure to the study of the case biases the findings.

Flyvberg (2004) defends case study on the grounds that it produces the type of context-dependent knowledge that research on learning shows to be necessary to allow people to develop from rule-based beginners to virtuoso experts. In a teaching situation, well-chosen case studies can help the students achieve competence, while context-independent facts and rules will only bring the student to a beginner's level.

3.4   Qualitative methods of data collection

People's words and actions represent the data of qualitative inquiry and this requires methods that allow the researcher to capture language and behaviour (Norman, Denzin and Lincoln, 2000). The key ways of capturing these are through observation (both direct and of the participant), in-depth interviews and group interviews, and the collection of relevant documents, photographs and video tapes.

3.4.1 The Interview

I chose to use interview because it is recommended as an integral instrument of data collection in qualitative research (Silverman, 2001). I used interviews to get information from teachers. It was also through interviews that I got clarifications and explanations from them (teachers) on certain issues that arose during the observations. As an interview is a joint product of what interviewees and interviewers talk about together and how they talk with each other, it was appropriate for this

study for I needed to understand what was going on. To this end, interviews were the best method available. Interviews permit observations to go beyond external behaviour and make it possible to explore feelings and thoughts (Patton, 2001). I then used data from the interviews together with that gleaned from the observations in my analysis and interpretations of the findings of this study.

Patton (1990) recommends the asking of probing questions during interviews. These include asking questions to get more detail, to fill out the picture of whatever it is we are trying to understand and to encourage the interviewee to tell us more. We indicate our desire to know more by gestures and verbal or non-verbal expressions. The interviewer can seek clarification from an interviewee without intimidating them. In this study structured (Appendix 1) and non-structured interviews were undertaken. Structured interviews were carried out with the three educators and one Department of Education mathematics subject official. These were involved because they play major roles in the enacted and intended curriculum respectively. I asked them questions about their experience as learners and teachers and also their opinions, ideas and justifications concerning the teaching of mathematics.

A voice recorder was used to record the conversation between us. Tape recording was useful in order to check against manually recorded responses, especially when direct quoting was required. One disadvantage of this mode of recording is that the presence of the tape recorder changes the interview situation to some degree (Borg and Gall, 1989). I had the advantage of using a very small recorder, not much bigger than an ordinary writing pen, so I do not think that the learners noticed it.

3.4.2 Observation

Cohen and Manion (1994) point out that observations lie at the heart of every case study. There are two principal types of observation: participant and non-participant observation. In this study the latter form was employed. I observed the lessons while the teachers were teaching algebra to a grade ten class. I sat at the back from where I could observe without being in anyone's way. I wanted to get first-hand information of what the learners were exposed to and to compare this input with the curriculum

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as laid down by the Department of Education with reference to the particular topic. At all times I asked myself what opportunities to learn do I observe?

Observations provided me with the opportunity to explore what people actually do (O'leary, 2005). Cueto et al. (2008) maintain that in studies where the main source of data is a teacher's self-reporting about their coverage of the competencies, there is a concern that they might over-report to impress the Ministry of Educational officials or a researcher. Yin (2011) supports the use of observation because he reckons that what you see with your own eyes and perceive with your own senses is not filtered by what others might have reported to you or what the author of some document might have seen. Observation, especially if it is prolonged like in this study, minimises the chances of that happening because one is bound to find out the truth if one observes and makes one's own assessment of what is happening rather than just relying on being told.

I listened to verbal and observed non-verbal exchanges between the learners and the educators at the same time. I witnessed chalk board exercises and other situations that arose from the unique experience of each classroom. I assured the teachers in writing and by word of mouth that the purpose of observing them was not to criticize them but to see first-hand what the learners were learning with the intention of gaining an understanding of the process. Before each observation session I briefly asked the teachers to tell me what they were going to teach and also if there was anything that they thought I should know beforehand. One of the teachers usually gave me lesson plans one or two days before the actual observation. However, I still asked her just in case she had changed her plans. This short period before the lesson helped me to establish an appropriate frame of reference.

After observation I talked to the teacher about the lesson to help me get their view of what had just transpired while it was still fresh in their memory. Unfortunately this was usually very brief for the teachers would be attending to other classes, except when the observation was at the end of the school day.

These observations took place over a period of seven calendar weeks. I observed lessons at the first school for one and a half weeks. After that I spent one week transcribing and studying the lessons and trying to make sense of what I had observed in preparation of what I thought then was the real field work. I later decided to use the results from the first school to build up one of my three cases. I then visited the other two schools to make final arrangements for my observations. It was then that I was informed that the grade ten classes at one of the schools had been amalgamated into one instead of the two I had initially asked to observe.

The observations at the other two schools took place during the same period. It was possible on most days for me to go from one school to other within forty minutes. When their times overlapped, I either missed the observation in one of the schools or the other school rescheduled. Qualitative methods allow for such flexibility. More details are given in the section on procedures.

3.5   Procedures

After I had identified OTL as a worthwhile variable in the study of the learning and teaching of algebra, I reviewed literature on the subject so as to understand what different researchers have said about it. The review confirmed the need for first-hand information in the context of my own observations in order to learn for my own improvement and so make a contribution to scholarship on the subject of teaching and learning algebra.

When I had reached the stage for going into the field, I identified the people with whom I had to communicate to gain access into the schools. I sent letters to the heads of the schools expressing my wish to carry out my research in their respective schools and so established lines of communication. I obtained signed permission from the school heads and the teachers (Appendix 2 &3) two months in advance of the observations. Permission for learners to participate in focus group interviews was sought from their parents or guardians but unfortunately I did not get them back. A covering letter explaining the purpose of the study accompanied the request. A copy of the objectives of the study was made available to all concerned parties in order to

acquaint them with the requirements of the study. The participants have remained anonymous throughout the research report.

I observed lessons and took notes. I audio taped what was happening in the classroom so that I could compare this with my manually recorded notes. During interviews with the teachers, I took notes and audio taped while the teacher answered my questions. I asked the teachers to tell me of their experiences in the teaching of mathematics in general and of their lessons that I had observed. I asked them how and why they might teach differently if they had to teach the lesson again. I asked the teachers what they thought about the student's learning.

The data to be presented first was a content analysis of the syllabus and other supporting documents from the Department of Education in South Africa. This was done to gauge the emphasis given to algebra in the curriculum and the time allocated in the guidelines to the particular topic. Facts were gleaned from the documents but it was also necessary to read between the lines and pursue collaborative evidence when that seemed appropriate.

Content analysis of documents is a non-intrusive form of research (Rozycki, 2009). This involves reviewing documents, memos or other pieces of written information for content and themes. By examining written words, the researcher studies one type of communication that occurs in the selected sample group. Other materials that are used at local levels, that is in particular schools or classrooms, were analysed for their part in affording learners the opportunity to learn algebra with understanding. The analysis of these documents helped in compiling a checklist of the materials that were used in the different classes observed.

Observations of the lessons and interviews with teachers followed this. Interviewing is a core method in qualitative research (Seidman, 2005). Through interviews, teachers gave an account of their situation, circumstances, feelings and perspectives in relation to the teaching and learning of algebra. I developed interview protocols for use with the teachers and the subject advisor (Appendix 1). Before carrying out the interviews I got support from my promoter and the student support system coordinator at the University of South Africa. However, during the interviews I did not

stick to the protocols because the interviewees sometimes volunteered information before I asked for it. Before using the protocols I practiced on my colleagues at work. This enabled me to develop a worthy instrument for the study of the input from these respondents.

I used a voice recorder for all the formal interviews and all the lessons observed. For informal interviews I relied on my handwritten notes. It did not seem appropriate to record while standing in the passage way or while walking away from the class room with the teacher preparing for the next class. Tape recording was useful to check against manually recorded responses especially when direct quoting was necessary in the report. The recorded interviews were transcribed as soon after the interview as was possible in order to minimise possible distortion in the report. Also some of the statements used as direct quotes in the report were verified with the respective respondents.

After the data was assembled in a presentable form, I explored these data sets further through the discussion of the major findings and their implications. I discussed how the three teachers provided their respective learners with opportunities to learn grade ten algebra concepts. Through the discussions, themes emerged that are important to this study. I then used the results of the current research to develop a new framework that helped define opportunities to learn in the classrooms using the idea of multiple representations of registers. With the new framework I tried to expand on the components of the other OTL frameworks and so link them with the new one. I then presented the conclusion and recommendations for further study.

3.6   Data Analysis

Unlike the analysis of quantitative data, there are few well established and widely accepted rules for the analysis of qualitative data (Bryman, 2008). Usually analysis of qualitative data consists of three parts which are noticing, collecting and thinking about interesting things. Interesting things are those occurrences, be they words or actions that resonate or strike a tune with the study topic. During my observation a

lot of activities took place and they could not all be recorded at the same time, but care was taken to focus on all what was pertinent to the study. After collecting data from the interviews and observations, I presented them as three cases. I provide comments on some of the activities using both the observation and interview data, as well as data gained from the literature reviewed. A discussion of the findings follows the presentation and sources of opportunities to learn as provided by the individual teachers are identified. Collating the findings was challenging because the three teachers presented very different ways of providing OTL. Soy (1997) stresses the need for researchers to anticipate key problems and events throughout the course of the study so as to attend to them when it becomes necessary.

3.6.1 Observation Data

From the daily observations, I drew conclusions and thought of new questions. These questions sharpened my eyes as I returned the next time. Observation data should be analysed as soon as possible after being collected to avoid being overwhelmed by masses of data at one time (Soy, 1997). I was not always able to analyse the entire lesson immediately after observation because I was also teaching and the second part of the observation sometimes entailed me observing two different classes on the same day. I did however make analytic comments besides my notes as I went along. Shuttlleworth (2008) says that analysing results for a case study tends to be more opinion based than statistical methods. The analysis in this study is organized around factors that emerged from the data as it affects OTL in the respective classes. After I began collecting data, I formulated more questions to ask teachers and to guide my observations.

The general questions considered during every observation were:
1. How do learners gain algebraic knowledge when:
    a) The teacher is teaching?
    b) They are engaged in class work activities?
    c) They are responding to particular types of questions?
2. What opportunities are provided for the learners to:
    a) Identify what concepts they are supposed to learn?
    b) Experience treatment within one register of representation?

    c) Convert from one register to another?

3. What are the indicators of opportunities to gain algebraic knowledge?

This framework helped me to focus my thoughts and find clues on what OTLs were available. After completing the observation it was not desirable to report on every lesson because of the sheer volume of material. I decided to report on two lessons per teacher. I chose the lessons that I perceived as representative of each teacher's way of teaching.

3.6.2 Interview Data

Mason (2002) says that the social world is always already interpreted because what we see is shaped by how we see it. Whatever form of interpretive reading a person adopts will be involved in reading through and beyond the data in some way, be it texts, artefacts or visual images. Interview data was presented together with data from observation because what the teachers said helped me to understand their way of teaching. During analysis I read and re-read the transcribed interviews and also listened to the tapes so as to keep as close as possible to the data sources. As observations formed, the main source of data (the interviews) were used to support or challenge what was observed in the classrooms.

3.7 Reliability and Validity

According to Golafshani (2003) and Stenbacka (2001), the terms reliability and validity are mostly defined with reference to quantitative research, so it is necessary to also redefine them in the context of qualitative research. Although reliability and validity are treated separately in quantitative studies, these terms are not viewed separately in qualitative research. Instead, terminology that encompasses both, such as credibility, transferability and trustworthiness is used. To understand the meaning of reliability and validity, it is necessary to present the various definitions of reliability and validity given by many qualitative researchers from different perspectives (Golafshani, 2003). Reliability and validity are conceptualized as trustworthiness, rigor and quality in a qualitative paradigm.

3.7.1 Reliability

Golafshani (2003) says that although the term 'reliability' is a concept used for testing or evaluating quantitative research, the idea is most often used in all kinds of research. If we see the idea of testing as a way of information elicitation then the most important test of any qualitative study is its quality. But Patton (2001) states that validity and reliability are two factors which any qualitative researcher should be concerned about while designing a study, analysing results and judging the quality of the study. In all studies whether qualitative or quantitative a question can be asked as to how an inquirer can persuade his or her audience that the research findings of an inquiry are worth paying attention to. The answer to the question depends on the nature of the research.

To ensure reliability in qualitative research, the examination of trustworthiness is crucial (Golafshani, 2003). To widen the spectrum of conceptualization of reliability and reveal the congruence of reliability and validity in qualitative research, Lincoln and Guba (1985) state that since there can be no validity without reliability, a demonstration of the former validity is sufficient to establish the latter reliability. Patton (2001) also states that reliability is a consequence of the validity of a study with regard to the researcher's ability and skill in any qualitative research.

To ensure reliability, I made use of different methods of data collection, that is, a literature review, document analysis, observations and interviews. Results from observation were verified through the interview method. I made an effort to ensure reliability by engaging pilot interviews to train myself and so refine the skills of observation and interviewing as suggested by Cohen and Manion (1994). The observation guide and the interview schedule were subjected to careful scrutiny by my thesis promoter and colleagues. The language used in the instruments was simple and straightforward.

3.7.2 Validity

According to Golafshani (2003) the concept of validity is described by a wide range of terms in qualitative studies. Although there is no common instrument to measure

validity in qualitative research, there is still the need to ensure the truthfulness and meaningfulness of data. According to Cohen and Manion (1994), the most practical way of achieving greater validity is to minimise the amount of bias as much as possible. To achieve this I carefully formulated questions for the interview. I also used multiple sources of data in order to minimise misperception and thus ensure the validity of my conclusions. I was careful not to mislead participants by the terms I used. For example, apart from appearing in the topic for the research the term OTL was not used in the interviews. That was done deliberately to avoid putting words into participant's mouths because it is not used in everyday language. It was my task to identify OTL from their words and actions. I think this added to the trustworthiness of my observations and conclusions. Also the combination of methods helps to strengthen validity. Patton (2001) says that each type and source of data has strengths and weaknesses, so using a combination of data types increases validity as the strength of one approach can compensate for the weakness of another.

Another paradigm in qualitative research is constructivism which views knowledge as being socially constructed and changeable depending on the circumstances (Healy and Perry, 2000). As qualitative research aims is to engage in research that probes for deeper understanding rather than the examining of surface features, constructivism is able to facilitate this aim (Johnson, 1997). Constructivism values multiple realities that people have in their minds and therefore, to acquire valid and reliable multiple and diverse realities, multiple methods of searching or gathering data are in order. Accordingly, I used observations, interviews and recordings to achieve a valid, reliable and diverse construction of realities.

Consultation with my promoter and colleagues at various stages of the study helped me develop ways of gauging whether or not I was in the right direction in terms of the expectations of the academic community. Consequently I think that what I have produced will be acceptable in academic circles. In this study data were sourced from official documents, educators and learners through document analysis, interviews and/or observations respectively. An effort was made to establish a chain of evidence both forwards and backwards. However, this study did not seek to generalize but to gain more understanding of the opportunities to learn algebra afforded grade ten learners at the selected Catholic secondary schools in South

Africa. Soy (1997) says techniques such as cross-case examination and within-case examination along with literature review helps ensure external validity.

In my presentation of the cases I used extensive quotations derived from field notes and transcripts of interviews, as evidence to help the reader enter into the world of the participants and so gain meaningful insight. In the course of writing the research report I went back to the participants and asked them if I had accurately recorded and interpreted their experiences. A few corrections were made.

Also we can find reliability in qualitative research by viewing videotapes several times alone and with others so as to capture as much as possible of what is pertinent to the study. This applies also to audio tapes. I listened to the recordings countless times while transcribing the tapes.

A common threat to internal validity is reliability. Threats to a study's validity and reliability exist at almost every turn in the research process. No one researcher can see all the potential problems, so a team approach to the discussion of validity and reliability during the development of a study design, and the creating and following of study protocols can help minimize the threats to validity. During my study I had a chance to present the first three chapters of my research at a student's conference. I received constructive comments, especially concerning my methodology. The contributions helped me refine my approach.

3.8   Research Ethics

The American Educational Research Association (AERA, 1992, 1996 and 2000) states that since education is aimed at the improvement of individual lives and societies, and research in education often directed at children and other vulnerable members of the population, the main objective of the code is to remind eduz\cational researchers to strive to protect these populations. The code encourages researchers to maintain competence by continually evaluating their research for its ethical and scientific adequacy by conducting all internal and external relations according to the highest ethical standards. This is sound advice even for this study.

AERA (1992, 1996 and 2000) stresses adherence to methodological perspectives relevant to research and keeping abreast with current criteria of adequacy by which research is judged. In this study an attempt was made to read and put into practice recommended procedures from my college without prejudice. I treated all participants with respect and was aware that their human rights had to be protected. I explained the aim of my research to them so that they could choose whether or not they wanted to take part. When they gave their consent they knew what was involved.

Participants were guaranteed anonymity and confidentiality. In the report I used pseudonyms for the schools, teachers and learners. I was sensitive to locally established policies, for example at one school I had to see the principal before every observation whilst in another school I could go straight to the classroom.

Observation is an important means by which we come to understand our world. I was keenly aware that while observation is a very useful means of gathering data on what was happening in the classroom and other learning situations, as a researcher I needed to be continually sensitive to ethical issues by acting with sincerity at all times. Soy (1997) encourages investigators to be aware that they are going into the world of real human beings who may be threatened or unsure of what the case study will bring. Though I did not detect such fears among the participants, I still explained what my research was about and they were free to ask me any questions relating to my research. When it was not convenient for the teacher that I observe the lesson, I did not insist. For example, on one day one of the schools found that the atmosphere was not conducive because of some social issues at the school. I postponed my observation to the following Monday since this happened on a Friday.

3.9   Challenges

I was challenged by the convergence of dual roles as an insider, that is, a Catholic nun and an outsider that is, a researcher. I have worked in Catholic institutions for eighteen years and so may be too familiar with them. I was aware of this possibility and tried as far as possible to be guided by research ethics. But the advantage of this long involvement is that I have formed a wide circle of contacts and colleagues

who have provided valuable advice. I was aware that the learners were pleased to have me in their classrooms because they were friendly and even offered to carry my bag. Some even sought me out to chat about studies in general. During the lessons I sometimes felt like taking an active part because of my natural impulse as a teacher to want to help and I had to restrain myself.

In the following chapters, I provide comprehensive descriptions of the setting in order to communicate the meanings of how the teachers provided their learners with opportunities to learn grade ten algebra. These rich descriptions help present the teacher's instructional techniques and behaviour as they assisted their learners to construct mathematical knowledge. I describe the learning environment that was created during the interactions among the teachers and students.

3.10   Conclusion

This chapter focused on the methodology of the study. The ways in which the research was carried out were clearly described. The rationale for choosing the qualitative research paradigm was given. A description of the research strategies adopted, methods of data collection and analysis and anticipated challenges were given. Data from the fieldwork are presented in the next Chapter.

CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.1   Introduction

In chapter one of the this study, several questions concerning the opportunity to learn algebra by grade ten learners in selected Catholic schools in South Africa were raised. Chapter two discussed the concept of opportunities to learn in order to establish what various researchers have argued on the subject so as to answer the question: "How can the opportunities to learn (OTL) grade ten mathematics in selected Catholic schools be characterised and explained from the official and enacted curricula?"

In the literature review section of this research, I discussed how most researchers in the field have argued that the rationale behind opportunities to learn is that students should not be held responsible for underperforming if they have not been given the opportunity to learn the material upon which the test or examination is based. A number of the studies demonstrate that student's learning opportunities vary and that variation in learning opportunities matters. In the USA, the OTL issue has been a major concern mainly for the minority communities and especially the African-American and Hispanics whose test scores were on average lower than those of the majority group. In the South African context, concern is largely with the majority African learners whose scores have been lower since the days of apartheid.

To answer the second and the third questions of the study- How is the curriculum enacted in the selected grade ten classrooms?, and How can the Opportunities to Learn (OTL) grade ten mathematics in selected Catholic schools be characterized and explained from the official and enacted curricula?, respectively- official documents from the Ministry of Education in South Africa are discussed in the following sections. After that, I present data from several observed grade ten algebra classes at the three Catholic schools that I visited. For clarity of definitions, what was actually done and taught in the classrooms is what is presented. The student's actual observed performance of mathematical activities in or outside of class defines the

achieved curriculum. The intended curriculum, on the other hand, is what is officially stated in the curriculum documents from the DoE.

4.2 Reports on classroom observations

The fieldwork for this study was conducted during the period February to March 2011. It consisted of classroom observations at three different grade ten classes at three schools (St Anne, St Bernard and Mt Carmel High Schools- not real names) and formal and informal interviews with the respective educators. Though not their real names, I will call the teachers Ann, Ben and Cherry and classrooms will be A, B and C respectively.

4.2. CASE 1

4.2.1 St Bernard Catholic High School.

This section of the report describes the first case of this study. It reports on lessons observed at St Bernard Catholic High School, from now on to be referred to simply as St Bernard. St Bernard is a co-educational school on the outskirts of a big city about thirty minutes' drive from the city centre. I observed a sample of five grade ten mathematics lessons over a period of two weeks; three of them had two periods each. Lessons at St Bernard are forty minutes long. A siren bell rings at the end of every forty minute interval. Learners move around when the bell rings while the teachers are usually in a fixed venue.

4.2.2 The teacher: Ben

Ben is one of two teachers who initially agreed to be observed for this study. The number of learners at the school has fallen so much over the years such that there is now only one class for grade ten mathematics at the school. Ben has been teaching for more than 8 years. He holds a Bachelor of Science Honours Degree in Applied Mathematics as well as a Post Graduate Certificate in Education (Senior Phase and F.E.T). He is currently studying for a Bachelor of Science (Honours) Degree in Financial Modelling. At the time of the research Ben was teaching Mathematics from

Grade eight to twelve, Physical Sciences from Grade ten to twelve and Computer Applications Technology from Grade ten to twelve. He is one of four mathematics teachers at St. Bernard.

I began our conversation by asking Ben to describe his experience as a teacher. Here is how he captured the trends over his eight years of teaching experience:

> *The first four years of my teaching career were exciting and enjoyable because I was involved with learners who were eager to learn and to succeed academically. Unfortunately, the past two or so years were so disappointing and stressful. Most of the learners that I have met have a negative attitude towards learning. The level of learner discipline is so pathetic in most of the schools that I have taught at.*

While depressed about the type of learners he has had to work with in recent years, Ben says that he enjoys teaching mathematics. He enjoys more when "in a lesson all the learners achieve all the lesson objectives." This is Ben's second year of teaching at St Bernard.

4.2.3 Classroom B

The classroom is at the end of the shorter side of an 'L' shaped building facing eastwards. The teacher's table is in front of the chalkboard and there is a metal cupboard against the wall on the left side of the chalkboard. Ben usually stands by this cupboard with his table on his right when he is teaching. Learners sit in almost horizontal rows with some of the desks touching. On the four walls, there are mathematical charts received from the Department of Education (Appendix 5 gives an illustration of the classroom and the sitting arrangements). On the first day of my observation, twenty six learners were present, 9 boys and 17 girls, with seven learners absent from the mathematics lesson. After a brief introduction to the class I sat at the back of the class near the door from where I had a good view of the whole class.

4.2.4 Lesson 1

This part of the report covers the first grade ten mathematics lesson that I observed at St Bernard during the first week of March 2011. I have chosen to report on this particular lesson because it was the first and gives a picture of Ben's mathematics teaching in general. This general picture will include several components of his mathematics teaching, especially his choices of registers of representation.

Learners came in slowly and settled a bit noisily in their seats. Ben already knew that some learners would be absent. His introductory remark as he cleaned the chalkboard was:

> *"If you can go, during the lesson for a meeting to go for a camp, it's up to you; you will get the results at the end".*

Ben asked the learners to tell him what aspect of the topic they were doing that day. This was because he had told them on the previous day what topic they would be doing. Together they identified the problems to be worked out on the chalkboard. The following conversation took place between Ben and the whole class to establish the problems to work out:

Ben: *So we are, we are now looking at what? literal what? ... Equations.*

Learners*: Equations (*in chorus).

Ben*: But we were supposed to be looking at simultaneous equations but we saw that when it comes now to let's say 'y' the subject of the formula some of you were not what? They were not; they were not able to what? To do so because ...You could not do so because of other constraints that I don't know. So we said let us look at what? At literal equations first then we will go back to what? To simultaneous equations. So I gave you some work to do.*

Learners: Yes

Ben*: It was number what? In class work.*

Learners*: It was number, number 1... (Inaudible)*

Ben*: Number what? Number 1 what?*

Learners: 1 g (sounded like).

Ben: Exercise 8 point what? 8.7 (Laridon, Barnes, Jawurek, Kitto, Myburgh, Pike, Scheiber, SigaBi, and Wilson, 2005:171)

Learners: 8

Ben: Number what?

Learners: 1d

Learners: 1e

Ben: F and what?

Learners: And h

Ben: And h and number 6.

Learners: ooh

It took over a minute to pin-point the number and item that they were going to do. To the learners Ben said that they could not proceed to doing simultaneous equations that day, "*because of other constraints that I don't know."* I pursued this issue during our discussion later to understand what he meant by "constraints." He told me that most of the reasons for the learner's present (poor) level of performance lay in the past. When I asked him about how he experiences the teaching of his grade ten mathematics class Ben responded by saying that:

> *It has not been easy teaching them because most of the learners never got the vital introduction of the topic at earlier grades. This means I have to teach the basics first before teaching actual outcomes for Grade ten.*

I was curious about the teaching of mathematics at the school in general and asked Ben about this. In response to my question on how he finds the teaching of mathematic at the school, he said that:

> *It is not so good considering the poor Matric results the school obtained in the previous year. Continuous changing of Mathematics teachers has contributed a lot to the poor learner performance in the subject. Learners on the other hand lack basic numerical skills which make it even more difficult for teachers to teach the subject.*

This elaboration enabled me to understand a little better what he meant when he referred to the "constraints" during the lesson.

Ben proceeded to do the corrections of the homework questions based on literal equations on the chalkboard. He had given the learners the homework on the previous day. To introduce this part of the lesson he gave the following instructions:

Ben: *So let's start with... So these are the corrections, so you take out your pencil and mark your work. Where you have every step that you get correct you put **a what?***

Learners: ***A tick.***

Ben: *Where you get it wrong you put an 'x' and then you use that pencil to correct where you have made it wrong. So let's look at number b. Number 1b it **was what?***

Learners: *It was $E = mc^2$*

Ben*: And you are supposed to make...?*

Ben: *m*

Learners: *c^2*

Ben: *And you are supposed to make?*

Learners: *c... m*

Ben: *m the subject of the formula. So what we said is we want to remain with what?*

Learners: *With m.*

Ben: *With m on its own. Like here it's on the right side. So it means we want it to remain with m alone on the right what?*

Learners: *Right side*

Ben: *Alone on the right side. So what are we supposed to do?*

Learner: *...then we divide by my (Inaudible)*

Ben: *Yaah, but then it will be a long what? A long step. It is like you wanted to come here, and you are standing there. You go around there and then you enter that way instead of you just moving there and there and you get in.*

Throughout the lesson Ben puts questions in the form of statements or phrases which require the learners to complete using one or a few words. Most of Ben's questions were in the form of the statements containing the word '**what**' either at the end or at the beginning. Learners largely chorused their responses. After the chorus Ben would repeat the learners' answer whole or in part. It was clear that Ben used only one form of representation. He wrote the algebraic expressions on the chalkboard and talked about them. What was written was not discussed. The problems were solved by the teacher. Solving problems is what Duval (1995) calls treatment within a register. Ben is teaching the learners how to simplify, emphasising how it is done and in so doing staying in the same register.

To construct new knowledge, conversion between the registers is necessary. Allowing change of register is what creates mathematical meaning (Duval, 1995). For Ben it seems more important to tell the learners what to do, he uses words to give learners clues to solve problems in a certain way.

Ben emphasised procedures that he wanted the learners to follow in order to get the answers. For example he said:

> *So what we do here, ...we just we want to remain with m on its own no matter it's on the left or right side. We just want to what remain with m using the shortest what?...*

Learners: Method

Ben: *Using the shortest means. So what do we do here? We just divide by c^2 because what?* **Here it is...?**

Learners: **Multiplied**

Ben: Multiplied **by what by what?**

Learners*:* **c^2**

Ben: *We are supposed to make m the subject of the formula. So what we said is we want to remain* **with what?**

 Learners*:* **With m.**

Ben*: With m on its own. Like here it's on the right side. So it means we want to remain with m alone on the* **right what?**

Learners*:* **Right side.**

The sentences cum questions that Ben used were largely pointing to very obvious words. Sometimes he asked questions and answered them before the learners could do so. In the excerpt above there are examples of some of the questions which were so pointed that common knowledge or sense alone would have enabled anyone to give the expected word(s). When Ben asked a different sort of question the learners hesitated to answer. An example of this is in the next excerpt. This follows immediately after the last line of the previous excerpt:

Ben: *M alone on the right side.* **So what are we supposed to do?**

Learner**: ...then we divide by my (Inaudibly)**

Ben: *Yaah,* (Approvingly) *but then it will be a long what? A long step. It is like you wanted to come here, and you are standing there. You go around there and then you enter that way instead of you just moving there and there and you get in. So what we do here is we just we want to remain with m on its own no matter it's on the left or right side. We just want to what? Remain with m. Using the shortest what?*

Learners: method

Ben: *Using the shortest means. So what do we do here we just divide by c^2 because what? H**ere it is...?***

Learners*: multiplied*

Ben: *Multiplied by what* **by what?**

Learners*: c^2*

The explanation about the merits of a short way of answering question seemed to strike a chord with the learners. This attempt to contextualise a short method into real life examples was appreciated by learners for they followed, with their eyes, the directions that the teacher was pointing to. Some were nodding their heads in agreement or approval. This kind of example gives learners an opportunity to relate to practical experiences that can help them to understand the reason for certain procedures or steps in the process of building their mathematical concepts. It provided a visual representation of a procedure. The justification for using the shortest method becomes meaningful. The completed worked example on the chalkboard was as given below:

$$E = m\ c^2$$
$$E/\ c^2 = m\cancel{c^2}/\cancel{c^2} = m$$
$$m = E/\ c^2$$

It took two and a half minutes to arrive at the final answer. The learners seemed satisfied with the teacher's working because when asked if anyone had any question regarding the example they unanimously said 'no'. But the 'no' did not indicate that the learners had the opportunity to learn, for the ability to say a word is no guarantee that the words are understood. Words are understood in the context in which they are spoken, what tone is used, the cultural interpretation added and the biases for / against the individual (Nagpal, 2011)

Also the meanings of words do not guarantee the grasp of the implications or the mathematical content. However, the teacher's comment after the learners had said that they had no questions was to the effect that their achievement was due to the fact that the problem was easy. This was a rather strange comment that can be discouraging to some learners who might take it that it was not worth their while. Ben said, *"It's because that was very easy. I don't know why I gave it to you."* In reality,

the problem was probably easy for the teacher and not necessarily for the learners. As I observed, there was no evidence that it was as easy for the learners.

A similar example was then worked out on the chalkboard using the same procedures. After the answer had been written down and the class was ready to move to another problem one learner asked the teacher, "*How do we know which one to pick?*" This question, coming as it did, after two worked examples which were still on the chalkboard, seemed to indicate that the learner was not following what was going on. Below is the exchange that took place:

Learner: Sir h*ow do we know which one to pick?*

There was an uneasy sort of laughing from the learners sitting near her but it was not booing.

Ben: *Yayaya, you what? We are told there.*

Learner*: Ohh... We are told... we get the answer (*talking fast)

Ben*: Where is your book?*

Another learner: *Where is your book?*

Learner*: Ehh. We are told?*

Yet another learner*: We are told.*

Ben*: Because you never understand if you are not eeh... referring to something. Because here we are told that we should make 'i' the subject of the formula.*

Learner: *Ohoo*. (Agreeing)

Ben: *So now where is 'i'? 'I' is there on the left side. We want to remain with 'i'. Since this 'v' is multiplying 'i' we divide by it both sides and they... they cancel out and this 'v' will be on the other side. So let us look and number b. We want to make what?*

Ben referred to the authority of the textbook. The textbook is where the formation of the concept is displayed. The situation did not seem to provide the learner who had asked the question with a satisfying response even though she seemed to agree with the teacher. For some learners, understanding comes gradually, but when one seems left behind the group of those who seem to understand, one might be discouraged to ask further and just allow the lesson to proceed. The fact that some

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members of the class also asked where her book was could have been embarrassing or intimidating for the learner so she let it pass. The learner got no real opportunity to learn how to identify the letter to be made the subject of the formula. No help was offered by the teacher or the peers through such an experience.

The teacher then quickly went over what they had just done pointing to steps as he spoke. He then immediately moved on to the next example. It could have been helpful to the learner had the teacher confirmed with her what she understood before moving on to the next example.

The teacher worked out another example using the same procedures as in the previous two. The difference was that now the letter that had to be made the subject of the formula was on the right hand side of the given equation. The teacher worked yet another example on the chalkboard. He emphasised the steps to be taken especially that easier steps had to be taken first. By easier steps he meant 'taking the terms to the other side'. During the process a learner suggested a step which the teacher denounced as below grade ten level even though he acknowledged that what the learner was suggesting was correct. The teacher asked the learners to suggest what to do if given:

v = u + at   and having to make 'a' the subject of the formula.

Here is what transpired:

Learner*: You say, v – u = u + at - u*
Ben*: Now you are now in grade ten.*
Learners*: Yes.*

*Ben: If you were in grade 8 or 9 I would say yes you are right. But now in grade ten you just say we take this 'u' to the other side. Since here it is positive it now becomes **what**?*

Learners: Negative.

*Ben: You are right but you know if someone can walk in here and see something like this; (He wrote; v – u = u + at – u), he will think that maybe this is a grade nine **what**?*

Learners: Class.

The learner concerned complained inaudibly and the teacher said.

Ben*: You are right. Did I say he is wrong?*
Class: *Nooo* (loudly)
Ben: *But now we must show maturity that you are now in grade **what?***
Learners: *Ten*... (and one said grade one).

The teacher did not seem to appreciate the correct answer because he thought that the format was below the standard expected of the grade level of the learner. But actually saying 'take *u* to the other side' does not really explain what is happening. It does not help the learner to understand why we are doing what we are doing. What the learner had suggested has justification in balancing the equation by performing the same operation on both sides. A teacher's attitude to a learner's contribution is important to the learner for it is a measure of his/her perceived on-the spot success.

Clearly, the learner felt let down by what he perceived to be an acceptable answer, particularly since it was actually correct. He was denied the opportunity to connect with his previous experience of similar work, from which he could then build his new knowledge. The experience might have thwarted further attempts by this particular learner to offer answers in future. According to the constructivist theory of teaching and learning, espoused by the Department of Education in South Africa, learners construct new knowledge based on prior knowledge in an environment that is created to make that possible. Though it seemed the teacher wanted the learner to adjust or modify his method, had this been accompanied by a more sympathetic attitude, I suspect the learner might have appreciated it better.

The teacher then stressed to the class the need to employ inverse operations in order to isolate the required terms. He used what he had just told the learners and reached the final answer. Below is an excerpt of what took place.

Ben: *It means that since this u is adding and we need to subtract it from both sides. Do you understand? We need to subtract it both sides because its adding so we just do the opposite. If it is adding we subtract, if it is subtracting we add it*

both sides. .. if it is multiplying we divide both sides and if it is dividing we multiply both sides. Do you understand?

Learners: *Yes*

Ben: *Yaah. You are right. We are just supposed to come here.* (Pointing to the answer)

Ben then confirmed with the learners the next problem to be done.

Ben: Eeeh the *next one is **what**?*

Learners: N*umber h.*

Ben: *Number h. H is a... Let us move fast.* This was the first time that Ben referred to speed.

The next example to be worked out brought in a new aspect, that is, it involved a fractional coefficient. The teacher explained how to work it out:

Ben: *So a is equal to what?* (Writing on the chalkboard)

Learners: *Half base times height.*

Ben: *So a = 1/2bh and we want to make which one...?*

Learners: *Height.*

Ben: *Height the subject of the formula.*

Ben: *And y*ou *see here you have got a fraction, this side. Every time when you are given an expression with a fraction you first you remove the fraction to make it **what**? The fraction. That will be the easier way. Because if you want to you want to remain with h and if you...*

Ben went through the procedure of simplifying using the same procedure of using inverse operations. During the process part of the dialogue is captured below:

Ben: *Something like this does not look nice so you must also remove this 2,* (Pointing to the 2 in this formula A=–). *So the first thing to do is to remove the what?* When Ben said 'does not look nice' some learners laughed*.*

Learner: *The half.* (very confidently)

Ben: *The fraction not the half because next time you will find there is 3/5. You first remove the fraction, we want remove this 2 at the bottom since this 2 is dividing...Let's see. You now this one is now ok you now understand what is going on even if you are going to calculate area. This is you are trying to find the height of a triangle when you are given the area.*

By saying, "*The fraction not the half"* Ben was paying attention to language. It was important to the teacher that the learners gave him the specific words that he wanted. It was uncomfortable when learners missed the exact words that the teacher was looking for. The particular words would help in the build-up of the skill he was imparting and thus give the learners the opportunity to learn. The teacher then announced that they were moving on to the next example but one learner who appeared on the verge of falling asleep said:

Learner: *Sir. Sir is it necessary to write all steps?*
Ben*: Yes, in maths... I told you that you will be told to make h the subject of the formula and how many marks will be there?*
Learners*:3*
Ben*: And how many marks will be there*
Learners*: 3*

Ben then explained in detail how marks were allocated according to the steps taken to reach the answer. According to him if an item was given three marks, two of those marks would be awarded for the working and the third mark for the final answer. He claimed that in mathematics the working is the answer, meaning that the working shown had a great weight on marks allocated. He encouraged the learners to be guided by the mark allocation so that they would not lose marks during examinations because they had omitted some steps in their working. This method appears to be intended to help learners to pass examinations, in other words, it is examination driven. Ben took some time to explain how the omission of steps would influence one's performance under examination conditions.

 Ben told the learners that "*In maths the working is the answer.*" He also pointed out that in the case of application of changing the subject of the formula, not all the steps would be necessary. They would have to simply substitute and then calculate. He distinguished between what was expected during class work and in examinations and used the example of when they had done factorisation where they were expected to show steps as opposed to when they had to solve quadratic equations. This was an attempt to connect with the previous experience of the learners. The

learners were very attentive; maybe because it was new information for them or simply because there was explicit reference to examinations and marks (assessment).

There was a long pause after the teacher had announced the next problem to be done. This pause gave the learners time to study the question more closely and to think about how to solve it. The question required them to make 'h' the subject of the formula if: s= 2πr (h+r). The teacher used the same mode of questioning as he had used in the previous examples, to solve this problem. Part of the exposition is given below:

Ben: *If you look on this function where is h...?*
Learners: *Inside the bracket*
Ben: *Its inside the bracket. But here is there any other h somewhere?*
Learners: *No*
Ben: *No. So there is no need for us to remove those brackets. So straight away we divide both sides by 2πr....*

He however used the word function to refer to the given expression, which was a bit above the heads of the learners. (There is no link between the word function and the variables in the formula). For a link of knowledge of the word function a conversion between the registers has to take place. What was important was for the learners to recognise that r, h and s were variables and that 'pi' and 2 were constants. There was no reason why they should have had that knowledge at that stage in grade ten. There were a lot of misconceptions in the explanation because functional notation had not yet been used. This was only a mathematical formula and dividing was only a form of treatment within the same register. Treatment only does not teach anything to anyone. Treatment is important to teach transformation inside one register but it is not the final objective of learning. According to Duval (1995) the construction of mathematical concepts depends strictly on the capacity to use several registers of semiotic representation of the same concept. The teacher declared that the second part of the same example was too easy and straight forward, so he would not do it for the learners. But he went into some detail about what values for 'pi' were acceptable. A learner asked about something that I did not catch and the teacher

said to him *"... has nothing to do with what we are doing so I think you are disturbing"*. The learner was silenced by the remark.

Ben then went on to what he called "*the more complicated ones.*" These were to be found on the same Exercise 8.7 the Classroom Mathematics textbook. The new examples had to do with square roots and squares. He again used the same procedure that he had used with the previous examples. At one stage he was explaining how a square and the square root are related when one learner said that they knew it and the teacher responded in the following words:

I am just reminding those guys who only remember maths when we are in the classroom. They don't study at all. They only do these things when we are in class. So I am just reminding them.... You want me to do everything for you.

The teacher then went over another example, and emphasised the steps to be taken. There were some of the questions that the teacher referred to as being grade nine material. Even though the teacher expected the learners to have some familiarity with the content from the previous grades, he nonetheless blamed the learners' previous exposure to required numerical skills as inadequate. Instead of creating opportunities for learners to consolidate their previous work with the present, the teacher made it seem as if nothing worthwhile had taken place in the learner's previous work with linear equations. Ben was still speaking when the bell rang and he immediately announced the homework to the class. The class was then dismissed.

4.2.5 Lesson 3

This lesson took place at the usual venue. There were thirty-two learners present for the lesson, one learner was absent. I am reporting on the third lesson because there was very little variation in the second lesson which was a single period. I took my seat at the back of the class and the learners seemed to have gotten used to being with me in the classroom. During the lesson I noticed that one learner sat alone at the back of the class and was not doing mathematics. The teacher noticed it and did not do anything about it. After the lesson I inquired about it from the teacher and he

told me that the learner had outstanding work for his portfolio in another subject that was why he had to do it then.

In this lesson work on linear inequalities was covered. Particular attention was paid to the effects of dividing both sides by a negative number and on how to handle fractional coefficients. To begin the lesson the teacher wrote 'LHS ≠ RHS', on the chalkboard and asked the learners to say what the expression meant. The learners responded correctly. The teacher pointed out that if an equal sign had been used it would have meant that the left hand side was equal to the right hand side (written as LHS=RHS), making it an equation. He explained pointing to this expression: 'LHS ≠ RHS', that it meant that the LHS was not equal to the RHS. Below is an excerpt of the explanation:

Ben: *LHS=RHS.*

Learners: Y*es.*

Ben: That is for an equation. If it is an inequality now... it means that. (Pause)... right hand side is not equal to the what?

Teacher and learners together*: Left hand side.*

Ben*: It means that one of them weighs more than the other what? The other side... either it's the right hand side which is greater than or vice versa. Do you understand?*

Learners*: Yes*

Ben*: If there is such a scenario... we said we call it a what? An inequality. Just like: x + 2 > 4. This is an inequality. It means that the left hand side is greater than what?*

Learners*: ... hand side. (inaudible)*

Ben*: The Left hand side. (Said wrong answer but wrote the correct expression). (long pause). So this is what we call an inequality when one side is greater than the other are what... That is the difference between an inequality and an equation. In an equation we are trying to make sure that both sides are what?*

Learners*: Equal*

Ben*: Equal. But if one side is greater than or less to the other that will be a difficult thing... So let us look at what? At the example that we are given there. Or let us look at exercise 2.1*

Learners: *8.21*

Ben: *8.21, 8.21*

There was a pause while learners opened their books to the page with Exercise 8.21.

Learners*: Yes (chorus)*

It is important to pay attention to words that are used in classroom communication. Fundamental to teaching and learning is a consideration of how communication takes place, of how meanings are shared Jarworski (1994). Jarworski also argues that in the teaching of mathematics it is also fundamental to ask what meaning and whose meaning. It is difficult to know what the teacher meant by "understand". It is also not clear whether the learners were responding to the same thing. Understanding and knowledge are often used interchangeably in everyday language. The notions are related; some understanding is necessary for knowledge and some knowledge is necessary for understanding.

The lesson proceeded as given below:

Ben: *Number 1, we are given: 1- 2x > x – 2.* (He wrote this on the chalkboard). *And they want you to solve for **what?*** (Laridon, et al; 2005:186)

Learners: For x.

Ben*: And they want you to show it on a **what**?*

Learners: *On the number line.*

Ben: *You did number line from grade **what**?*

Learners: *Grade 1.*

Ben: *Grade 1. So I cannot remind you about that for you have been doing it for the past ten years.*

Learners: *We have forgotten.*

Ben: *So it means first you collect?*

Learners: *Like terms*

*B*en: *Like terms*

Learner: *This one is a very good example* (was immediately contradicted by the teacher).

Ben: *It's a very bad example but don't worry. Ok let's start. Let's look at this one: x+2<4. We just take steps as the ones that we take when we are solving equations we collect like terms if you are having something like this...we take this 2 to the other side so it will be x < 4 minus what? Minus 2*

Learners: minus 2

Ben*: What is 4 minus two?*

Learners*: 2*

Ben*: Its 2. So it means this is our solution. 'x' is smaller than what?*
Learners*: Than 2*

In the above excerpt the learner who rated an example as being a very good one, was immediately contradicted the teacher, calling it a very bad example. Both did not explain their opinions. The learner was not allowed to express and try his own ideas coming from his evaluation of what was going on. According to the constructivist theory learners are not just receivers of knowledge but active participants in constructing their own meaning based on relevant experience. During the lessons that I observed I noticed that little time was given to the learners to evaluate their own or one another's statements. Chances to evaluate can create opportunities for learners to construct new knowledge or to consolidate what they already possess.

During this lesson the teacher was asking questions in the form of sentences that the learners had to complete and then he would repeat the answer after them. Posamentier and Jaye (2006) discourage such a practice for they say that it weakens the learners' interest in one another's answers. The questions seemed to have only one possible answer most of the time. The questions were directed to the whole class so they answered in chorus fashion. They did not seem to raise their hands to indicate that they had the correct response to offer. Actually, little time was given for thinking about the question and formulating an answer. The reason could be that it was unnecessary because of the nature of the questions. The teacher asked in a 'neat' way which would not allow any other answer but the one he was anticipating and the learners supplied it. The questions did not provoke much thinking and did not encourage the learners to answer in full sentences. Even good questions can lose their value if they are overused (Posamentier and Jaye, 2006).

According to Posamentier and Jaye (2006:53) "Questions should be constructed to stimulate different forms of thinking." According to them simple questions, such as those used by Ben in the observed lesson, merely test the recalling of data or procedures. They further point out that the teacher's questions shape learner's learning as the type of questions asked places emphasis on the process strands that are valued in the learning of mathematics. The questions used by Ben and the kind of responses they elicited did not give the learners the opportunity to think beyond

the obvious and they learned very little by it. If procedures are over stressed they can become routine and the learner can perform them well by rote not by understanding.

As reported above, the teacher mentioned that he was not responsible for filling in the gaps that might have resulted in the learners' knowledge because of their experience of frequent changes of educators. In this example he was not prepared to revise the number line because the learners were supposed to have used it since the first grade. The fact that he brought it up meant that he was aware of the need to remind them, but he was not prepared to do so. When the learners said that they had forgotten the teacher did not refer to the issue but proceeded with the other work. Sometimes even the learner who is capable may benefit from a quick reminder to give them an opportunity to connect with past work and learn new material. Posamentier and Jaye (2006) maintain that teachers should avoid giving learners the impression that the work they are doing is simple and that the reason for any difficulty they might be experiencing is their own.

The Department of Education requires that different or multiple representations of concepts be used when mathematics lessons are delivered and this section of algebra clearly calls for such treatment. Such an approach is useful for it is integral to the building up of mathematical concepts (Duval, 1995). The questions in the exercise that the class was doing required the use of two representations, that is, symbolic and graphical. After each problem had been worked out algebraically, the solution was represented on the number line. The teacher drew the number lines to illustrate the answers. It is expected that the symbolic would come after the graphical or pictorial representation but here it was the other way round. It would seem logical to start with the graphical or diagram before the symbolic form because a picture describes an object in its entirety and that seems to appeal to people. The exercise gave the opportunity to move from one register to other.

Below is an excerpt of how Ben explained how to illustrate the solutions using the number-line model:

Ben: *Smaller than 2. So when we illustrate this answer on a number line it does not mean that we need to draw a long number line. What it means is we just draw a straight line like that.* (He drew the line on the chalkboard)*.Your zero is there and your 2 is there and then just show that this is a number line by showing these arrows one is going that way and this is also going that way. And where is your 2? Your 2 is there. And then here we have got a what? A strictly less than 2. There is no equal or what? There is no smaller than or equal to so it means that we draw a small circle like that. And we don't shade it. And where are the numbers that are less than. Are they this side or that side?* (The teacher pointed to the sides on the number line)

Learners*: This side*

Ben*: This side so draw an arrow going this side. So this is x <2. So you also put what? You also put an x there to show that those are the values of what? Of x.* (A learner made a 'uugh' sound showing boredom or tiredness.) *Those ones that are less than two are from this 2 going that side to the left side and these ones going that side are greater than what?*

Learners*: 2*

Ben*: 2. So let's go to this one. First the thing we say is collect like terms, so let's take the as to this side and those constants to the other side. We are going to have minus 2, minus x greater than what. Minus2 minus 1 so it will be -3x>-3 and then now we want to remain with x so what we divide by what?*

Learners: *By minus 3*

Ben*: By minus 3, so it be minus 3x over minus 3 what and then minus three over what?*

Learners: *Minus 3*

Ben went on to work the next problem using the same procedure as above. The learners seemed to follow the procedures as the teacher expected. Everything seemed to be going on well until the teacher asked what direction was changed by dividing by a negative number and a learner said we were changing the direction of the sides:

Ben: *As long as you are dividing by a negative number if the coefficient of x is a negative number you can divide both sides to remain with x we change the direction of what? The sign. We change the direction of the what?*

Learners*: Of the sides.*

Ben*: Of the signs not of the sides*

Learners: *The sides*

The response of the learners showed that some of them were lost. The teacher completed the example and illustrated the solution on a separate number line and showed the learners the position of zero. The teacher stressed once more the change of direction of the sign as the result of dividing by a negative number. For the teacher this factor seemed to be very important and he repeated it many times. It may be that he saw this factor as giving the learners the opportunity to understand inequalities which involve negative numbers or else he perceived that his learners would need to hear it many times over before they could understand. Repeating it over and over would make it possible for the learners to understand it. The reason the rule worked was not given is because Ben wanted them to remember:

Ben: *So you must remember that if we are dividing by a negative number we change the direction of the what? The sign. Or if it was a smaller than we make it a bigger than and if it was a bigger than we make it a what?*

Learners*: Smaller than.*

Then Ben moved to the problems he called the "the challenging ones.' Some involved brackets and others contained fractional parts and or negative numbers. Ben explained how to 'remove' brackets as the first step in the process of simplifying the expressions to familiar forms. They could then follow the same procedures as before.

While Ben was explaining there was a sound of a message coming through one learner's mobile phone and it caused a stir. The teacher continued as if nothing had happened. Maybe if he had pursued the incident it would have taken longer to restore order. Learners are not allowed to use their mobile phones during lessons. The teacher continued to stress the influence on the sign of dividing by a negative number. In his usual way he asked what would happen if the expression: $-3x \leq 3$ was divided by -3

Ben: *Since also now, we are dividing by minus 3, or a negative number we interchange what?*

Learners: *The signs.*

One Learner: *4 minus* (Very loudly)

Ben: *No. Not on that side. So dividing by 3, this implies that x ≥ -1 and if you are going to represent it on the number line zero is there and minus one is on this side. We put what, a circle on top of minus one and then we are told that x is greater than or equal to what?*

Learners: Negative one.

The learner who almost shouted '4 minus' appeared to have made a random error. There did not seem to be any reason for dividing by negative four if that is what he meant, or if he wanted to subtract four from something which he did not have a chance to mention for he was cut short. The teacher's brief response to him did not help to clarify the misconception. The learner was not given the opportunity to develop his line of thinking. The fact that he said it loudly could have been a call for his own individual contribution. The learner got a short sharp answer that silenced him and prevented him from expressing himself any further. Instead the teacher proceeded as dictated below:

Ben: *Negative one, so you shade this one, you shade inside this circle and which numbers are greater than minus 1, these ones. So you draw an arrow going this towards, facing the positive numbers the arrow will be pointing towards the right.* (Learners were talking at the same time as the teacher so I did not catch what they were saying).*This is straight forward and very easy, because these are linear equations but when you go to grade eleven now you will be looking at quadratic. So if it is greater than or equal to, or less than or equal to, you shade this what?*

Learners: Circle:

Ben: *Small circle on the number line. But if it is strictly less than or greater than you leave it what?*

Learners: *Blank*

Ben: *Blank like that.* (Pause).*Ok let me just give you one example with a what? With a fraction. But there is nothing you must do because the way you solve this one is almost similar to the way you the what?*

Learners: The equations.

Ben: The equations, the linear equations.

There was a long pause while the teacher was choosing a suitable example and the learners seemed to be talking about their work to each other. Some learners were pointing to the work on the chalkboard.

Ben then reminded the learners that if an equation involved a fraction the first thing was to remove that fraction. He pointed to the fractional part that was to be removed. When he asked the learners to state the next step they hesitated to respond and the teacher rephrased his question in such a way that the answer became more obvious. He said:

Ben: *We said first look for the what?*

Learners*: The lowest common denominator.*

Ben*: The lowest common denominator. And what is the lowest common denominator, so the LCD is what?*

Learners*: 6*

Ben*: It's 6. So it means that we multiply all the terms by the what?*

Learners*: 6*

Ben*: By the LCD, so it will be* (the teacher wrote down the expression on the chalkboard and continued).

$$\frac{\quad}{\quad} - \frac{\quad}{\quad} < (x\text{-}2)$$ *(uugh-sounds of tiredness from some learners)*

3(4 –x) - 2 (2x -1) < 6x − 12 so it will be

12 − 3x - 4x + 2 < 6x -12. We collect the like terms. So it will be

-3 x - 4x - 6x < -12-12-2 (called it a bracket, just a slip of the tongue)

-13x < -26

Ben: *Since now eeh we divide by -13 both sides, we interchange the what?*

Learners: The signs. (Some of the responses were muffled, - not audible)

Ben*: We do not interchange we change the direction of the what? Of the, of the sign.*

The teacher paused, while the learners were 'discussing' their answers informally with their neighbours. The teacher did not tell them to discuss but he allowed it to take place. Learners need to talk about what they are doing to their peers for it provides them with the opportunity to communicate in appropriate words and symbols. Cooperative learning is a highly recommended way of engaging learners for it has been shown to influence learners' performance in a positive direction. The teacher then proceeded as follows:

Ben: *So it will be x is greater than?*

Learners*: 2.*

Ben*: 2 so on the number line you have you zero and your two there. You have x greater than what? Strictly greater than 2. So that is the representation on a what? On a number line. So as your class work I want you to do Exercise 8.21 1c, d, 2, 3*

In the above excerpt in particular the teacher was largely talking to himself. The class was getting restless. There was no motivation for them to pay attention. The teacher was posing questions in his usual manner of starting sentences for learners to complete but the learners were not supplying the required words or else they were hesitant to do so. The learners seemed to be tired and the content was probably too difficult for them. The teacher had talked for a long time. After this episode the learners were lively and seemed engaged in private conversations that might not have been on the subject of mathematics. Before the learners could start on their class work the teacher reminded them again of the need to find the lowest common denominator (lcd) and described the procedure again briefly.

As the learners were settling down to work the teacher asked if anyone had any questions. The majority replied in the negative and others did not respond immediately. There was a long pause during which the teacher expected all the learners to have started doing their class work but the majority had not. One learner called on the teacher to ask him something. The teacher went on to enquire if there was a problem or if there was something wrong on the chalkboard. The learners still did not settle to work. They seemed to be consulting each other. The teacher reminded them that they were to work individually since it was neither homework nor pair work. They continued to talk until he said this to them:

> *You do individual work, except if I say pair work, or group work or the whole class that is when you work together. So now it's individual work not together.*

They largely ignore his call for them to settle to individual work. One learner who had been quiet most of the time asked, "*Sir. Sir. Why over there we are dividing by a negative number but the sign doesn't change?*" The teacher searched the chalkboard for the example and asked the concerned learner to identify the one she meant. While the teacher was looking around the chalkboard for the error the learner realised that she had made a mistake and said so. The teacher thanked her. Some

learners continued to fidget and lively conversations on other topics that were not mathematics related were going on. Some learners wanted the teacher to confirm the numbers that he had chosen for them. He told the learners that the problems that he had given them did not require much time, in his own words, *"You do not need 5 minutes to do it."*

Immediately after the above statement the teacher started to explain again about the effects on the signs of dividing by a negative number. He went on to ask what they thought the effect of dividing by a positive number would be. He answered the question himself and told the learners that it would have been a good idea for him to have started by dividing by a positive number. But then he told them that they should have done the division by a positive number in grade nine.

Ben: *And you should have done this in grade nine.*
Learners: *Sir we didn't.*
Ben: *Maybe you were absent.*
Learners: Nooo.
One Learner: Sir we never did it.
Another learner: Never, never, never.
Ben: So how did you manage to be in grade ten?
Learners: Nobody fails in grade nine so we all passed...

When they told Ben that nobody fails in grade nine they laughed about it. It was as if they did not respect their grade nine qualifications. They were reminded that in grade ten there would not be any automatic promotion to the next grade so they had to work hard.

One learner had not started writing by then for she asked the teacher for the day's date. The other learners seemed amused and told her an inaccurate date, maybe just to confuse her more. The teacher's response was that she should be the one telling him what the date was on that day.

The teacher inquired from the learners about some pending portfolio work. They assured him that they were working on it. There were other conversations taking

place in the classroom. The teacher encouraged the learners to take the topic seriously for it might be their easy option when they write their examination. The bell rang and class was dismissed without mention of homework.

4.2.6 Additional comments on Ben's teaching

These comments are meant to give an overview of how Ben teaches algebra with an eye to identifying how he creates OTL. The literature reviewed pointed to the fact that OTL is concerned with the conditions under which learners have to learn and is positively associated with achievement. In this study OTL is considered in the context of the learning of algebra at grade ten level. OTL is determined largely by the way the teacher instructs the learners because what is learnt depends on what is taught (Kilpatrick et al., 2001). The way in which OTL are generated in the classroom differs from class to class. In this section I consider the way Ben affords his learners the opportunity to learn grade ten algebra by considering his choices of registers of representations. I will use the examples described above to highlight the way Ben attempts to give his learners OTL.

Mathematical knowledge is constructed in the mind of the individual. Students' construction of mathematical knowledge is greatly influenced by the experiences they gain through interaction with the teacher (Cobb and Steffe, 1983). Teachers decide upon the strategies to engage students. They create the opportunities for students to learn the knowledge and skills required in society and also in order to pass examinations so that they have something to show for their effort.

In his teaching Ben emphasises procedures. All teachers when they attend to a class of learners want to give them an opportunity to learn, each in his or her own way. For Ben learning is accomplished when learners complete given sentences using short phrases or single words. He starts the sentence spelling out a procedure and the learners complete it. For example he says, 'Here we are supposed to make 'h' the subject of the what? The learners respond, 'Formula.' Most of the questions were of this nature. Questions should be constructed to stimulate different forms of thinking. Ben's questions did not stimulate high order thinking and problem solving. A teacher's questions can shape a learner's learning as the type of questions asked

place emphasis on the process strands that are valued in the learning of mathematics (Posamentier and Jaye, 2006).

Communication with his learners was largely one sided, the teacher did most of the talking and all of the chalkboard working. Ben followed the examples as set in the Classroom Mathematics textbook. He worked out the examples in order of difficulty as arranged in the text book. However there are dangers associated with textbook driven instruction in that the questions might not apply to the context familiar to the learners. Learners naturally relate favourably to questions that mean something to them than to those that are far removed from their own experiences. Ben always repeated the answers that the learners gave him.

The role of the learners during the lessons was largely as passive listeners with almost no student-student interaction and only a few questions and comments directed at individual learners. Ben went through work already done by the learners as homework. During this exercise Ben seemed to put more emphasis on solving specific problems in a particular way.

Ben has little confidence in his learners learning potential. He asked and answered his own questions. He was talking to himself sometimes for he would answer the question before the learners could do so. His voice was the most powerful representation in his case. For him the students have to hear in order to understand.

A teacher's conception of mathematics influences every aspect of the teacher's teaching (Cobb and Steffe, 1983). Also a teacher's perception of student ability influences the way they engage students. Ben does not believe that the learners in his grade ten class can learn effectively. He has a poor opinion of his learners as indicated in his response during the interview. He said:

> *Most of the learners have a negative attitude because they often get frustrated when they attempt to solve story problems that involve algebra. They also believe that algebra is about techniques for solving those mysterious equations or other obscure problems.*

In response to a question about how he finds his teaching of mathematics at St. Bernard, Ben had this to say:

It is not so good considering the poor Matric results the school obtained in the last year. Continuous changing of Mathematics teachers has contributed a lot to the poor learner performance in the subject. Learners on the other hand lack basic numerical skills which makes it even more difficult for teachers to teach the subject.

What Ben is saying indicates that he is not responsible for the poor performance of the learners which he perceives as resulting from their previous experiences. The learners had been through five teachers in a space of just over two years. Ideally the teacher has to help every learner at the stage at which they are. It does not help matters to refer to previous omissions or work that should have been covered. Talking about it does not make it happen so action is required if the learners are to have the opportunity to learn.

Ben addresses the class as a whole and so does not give particular attention to individual learners unless they ask him a question. Class discussions were rare. Learners usually engaged in unsanctioned pair work which was discouraged in one of the lessons. But discussion in class enables the learners to rearrange their ideas and find new expressions and to communicate. When a person understands something, they are capable to say the same thing in different ways, in other words communicate it in more ways than one. Ben does not give space to learners and so they do not have anything to question because he has given them the answers. You ask when you have time to think about it. So if you have no time, you cannot ask.

The Department of Education in South Africa expresses in The National Curriculum Statement (2003b :11) that it is expected that grade ten learners are able to, "Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae). It is recommended in the same document that "the approach to the content of Learning Outcome 2 should ensure that learning occurs through... conversion between numerical, graphical, verbal and symbolic representations" (2003b: 18)

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The theory of learning as put forward by Duval, (1995, 1999, 2005), discussed in Chapter two of this study imply that for OTL to arise, the learner needs to be provided with the necessary representations that help in the construction of mathematical knowledge. The NCS is clear that the approach to the content of Learning Outcome 2 should come through conversion between the registers of representation.

The role of the teacher is to promote learning by giving different objects to learn. The object is not in the teacher or in the learners, it is in the problem. The problems are the objects of knowledge in the subject. When the learner reads the problem and thinks about it he or she is moving into the brain to get the knowledge to solve it. This is the moment when Piaget (1977) says there is disequilibrium because there are things they know about the problem and things they do not know about it. The teacher is a facilitator in the process. The learners have to create the object of the subject. Ben uses verbal representation and its written form to teach algebra. He gives the message to the learners that 'I am going to tell you and you are going to learn.' But no amount of verbal explanation can give any learner the concept of algebra because knowledge is constructed individually given the necessary conditions. According to the constructivist view of learning, based on Piaget's theory of assimilation and accommodation, learners must experience the concepts in order to build understanding. In Ben's class, learners sit passively and watched him perform actions to solve the problems for them. He asked leading questions and most of the time answered them himself.

Ben hardly used real life situations that have a meaning for the students. Learners have little chance to explore relationships or create solutions with their own methods because they are not presented with open-ended situations where there are many ways to find answers. Ben hurries through problems and concentrates on drilling rules and procedures. Drilling rules and procedures can lead to a stage where learners answer questions by routine, they can work automatically may be without understanding what they are doing.

Ben transmits a sense that mathematics is not difficult and that it is the learners that are not capable of doing it. Most communication was verbal yet there are many other ways of communicating. Ben seemed to be much more concerned about the procedures than the concept. Students cannot be expected to develop critical knowledge structures by practising procedures (Fennema and Romberg, 1999). To Ben, performing procedures was very important for the learners to practice. So he was exposing them to the procedures and expected them to do exactly as he told them. That was his way of providing OTL. He had mathematical knowledge which he wanted to give to the learners. He did not challenge them to think about the procedures but rather told them that the procedures worked and that they should just take it in. Ben hardly ever asked the question 'why' which would have provoked higher level thinking in the learners. Asking 'why' would have prompted the learners to explain and justify their own thinking. Talking about what they were doing in this manner would have given learners the opportunity to understand. From the first lesson he used the same mode of asking questions. His attitude seemed to expect little from his learners.

One factor that appears to have affected the way Ben provided OTL to his learners could be his workload. He is clearly overloaded (For his workload see section 4.2.2 above). If one is overloaded he or she cannot perform at their best because of the time factor for each load has its own demands and this can be overpowering. Time for research to improve quality and quantity is in short supply and this can adversely affect the OTL the teacher provides.

4.3   Case 2

4.3.1 Classroom C: Mt Carmel Catholic High School

This section of the report describes the second case of this study. It reports on lessons observed at Mt Carmel Catholic all girls' school. Mt Carmel is about eight minutes drive from the city centre. I observed eight mathematics lessons there.

The mathematics classroom is on the first floor. Adjacent to it is a small mathematics room where the mathematics teachers meet to share ideas or to wait to enter their classroom. It is in this room that we would talk about the lessons before and after they had taken place. As you enter the classroom you face the teacher's table which is aligned with the front desks of the learners.

There are thirty-two learners desks arranged in a column leaving a space allowing passage in any direction. There were seventeen learners doing grade ten mathematics. The empty places were not arranged in any specific order because the occupants of those seats, who are also in grade ten, moved to another venue because they do mathematical literacy.

4.3.2 The teacher: Cherry

Cherry is a teacher with twenty years of teaching experience, two of them abroad in the United States of America (USA). She has a degree in mathematics education and has taught at Mt Carmel for the last five years. Her two years abroad fostered an open attitude towards her teaching practice. She got used to being observed and as such does not mind it any time. She narrated this experience with visitors in the USA classrooms as follows:

> *There you can be visited any time of the day by any of the stakeholders, the parents or anybody interested. I got so used to it that it does not bother me whether I am being observed or not; I get on with my work. And I do believe that everybody should have access to observing lesson anywhere if they are studying.*

Of the three teachers whom I observed she was the only one who gave me her preparatory notes in advance of the lessons. I had two meetings with Cherry before I observed her teaching. She gave me her work schedule for the period from the beginning to the end of March. She indicated on the schedule what she would be covering each lesson. She also gave me her lesson plans the day before or on the day of the lesson. She would also give copies of all the worksheets distributed to the learners. In this sense, she was very helpful and exciting to work with.

4.3.3 The lesson.

For the present discussion, I present a set of examples gathered from two double-period grade ten algebra lessons at Mt Carmel. I discuss the two lessons together because they represent one continuous episode as one lesson progressed directly to another. I will illustrate how this happened later in this presentation. A copy of the lesson plan is included as Appendix 5. The first of the two lessons discussed here was actually the third in my observations at the school. I have chosen to report on it because in it the learners were much more involved in the activities than in the previous ones and, as I later found out, was fairly representative of Cherry's teaching of mathematics.

I directed my observations toward understanding how Cherry used different registers of representation to help her learners construct algebraic concepts. I focused on the questions that the teacher posed and the examples that she used. I examined the opportunities that Cherry provided for the students to learn and construct mathematics.

4.3.4 Topic of Lesson: Making y or any selected variable the subject of the equation

The above was written on the lesson plan but on the chalkboard the teacher wrote 'Rules.' The learners had not performed as well as Cherry would have liked in the end of term test they had written the previous day. Cherry had identified a weakness in the learner's way of changing the subject of a formula.

> Cherry: ...you mixed up the variables. The same rules apply. You know how it is if you have a driver's licence, right, it does not matter whether you are driving in Pretoria or Durban or Polokwane. It should not matter to you what city you are in. Just because it does not look like Pretoria does not mean that you can't drive. So we need to have those rules so fast in your head that you can do any expression any equation. Alright. Now let's just revise, and... in your book... just write down a few key concepts that we use.

This introduction gave the learners the opportunity to contextualise the meaning of 'Rules.' Mt Carmel is in one of the cities mentioned so learners could easily identify

with the real-life example given by the teacher. In this way the learners had a chance to relate mathematics to their everyday experiences. The activity of writing 'key concepts' was meant to provide the learners with the process of conversion; they had to think and when they wrote down their thoughts, they changed the register.

> Cherry: *We are talking about variables. Jot down the word variable and give me a nice little normal English but mathematically correct definition of what a variable is.*

The teacher wrote the word variable on the chalkboard and then moved into the learners' area to examine the learners' attempts. This teaching approach made me think of the constructivist dictum that people construct meaning for a new idea or process by relating it to ideas and processes that they already understand (Fennema et al., 1999). Here was the teacher giving the learners an opportunity to bring forward what they already knew so that they could build new knowledge from a familiar ground.

Learner: *Miss must we write it down.*

Cherry: *You tell me.* To the particular learner and then to the whole class she said: *And let's come up with a common nice definition. We always talk about them but if someone was to say to us define a variable. What's a variable? Ya.*

Learner: *It's like, it's like x it changes values. It does not have an x value.*

Cherry: *I like the second part of your definition more than the first, because it's not always x, it's a letter.*

A learner interjected with another attempt:

Learner: *Miss could we say that a variable is a figure with an unknown value?*

Cherry: *A figure?*

Learner*: It's not always x.*

Cherry*: It's not always x*

Learner: *Could be y.*

Cherry: *Could be y with unknown value. Does that mean... that it can be any value... in an equation?*

Learner: *...Given in an equation?*

Cherry: *In an equation I am asking.*

The learners discussed in unequal sizes of groups: in one corner there were three pairs, another two set of threes, one group of four and three students working individually. After saying the above the teacher went to stand next to one of the three and talked to her while waiting for the response from the whole class. The teacher had told me previously that the particular student that she visited often during her delivery of lessons was new to the school and was still trying to find her feet. Cherry tried to give her as much individual support as she could.

Cherry: *Listen carefully. We have come with a figure, a letter, which...?*

Learners: *With a value that changes.*

Cherry: *With a value that changes. It's better to say with a value that can change than to say it can take any value. Because remember in an equation there is one or two or three values which it can be. It can't be any value. Why?*

Learner: *Miss is it not what you get after working out?*

Cherry: *Ya. Can you be more specific?*

Learner: *...the one you are looking for.*

Cherry: *Ya. Be more specific?*

Learner: *You are looking for this one and you get that one.* (Sounded like)

Learners discussed the definition in low tones I could not catch what they were saying. Cherry kept prompting them to be more specific. Then she said to the whole group:

Cherry: *Alright, let's put our definition together we say,* it's *a letter it's a value, something in normal English. That can change. Vary variable.*

Learner: Uuh uh

Cherry: *The word va-ri-a-ble sounds like to vary. It's a letter something that will change... according to an equation now. We have worked with linear equations we have worked with quadratic equations. Don't you understand? In a linear equation how many values can that variable have?* (Said slowly)

The teacher tried to give the learners the opportunity to view the different ways in which a variable could be defined. The learners tried to understand through the various representations the idea of what a variable is. The idea of a variable is

central in the study of school algebra as discussed in the curriculum documents in section 10.2.1 (b), which states that grade ten learners are able to

Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).

There was a short pause presumably to give learners time to think about it and then the teacher continued:

Cherry: *How many possible solutions? If it's written like this: y=x+2. How many possible solutions? Lucy?*

Lucy*: Two.*

Cherry: *Two? Alright so the idea of a variable is something that changes. Now when we say solve an equation what we are asking you to do is to find.*

Learner: *The values.*

Cherry: *The value of the variables in that equation so that it makes it true. Right because we know that is how we test that our answer is correct here its 3 so we do our manipulation with the answer three. We test it by putting it in the variable's place and if left hand side is equal to the right hand side if it's a true statement we know we have solved it. So the idea of the variable the important thing to realise is that it can change but it depends. Now we have mentioned that often we use y's often we use x's.*

The terms were explained one by one through the guidance of the teacher. Each time, the teacher probed the learners to give more information and to refine their meanings. She would say something such as 'be more specific', 'does it mean then', 'come up with more' etc.

Below is part of the conversation between Cherry and the class while they were discussing a term:

Cherry*: We have been taught the process of solving like this. There's two, one is by inspection, what ...which means basically...?*

Learner: *Just looking at.*

Cherry: *Just looking at. We say aah it's going to be three aah and 2 so this is five so it can only be 2+3 that's inspection and are prepared to do it.*

This is what she wrote down: 5 = x + 2.

As she was pointing at the terms she was speaking aloud to the class what the corresponding thinking would be. The learners seemed satisfied because some were nodding in agreement. The teacher was linking the mental, verbal and visual representations to provide the learners with the opportunity to connect while they construct their own ideas.

Cherry: *Ok, now we come to the one that we fall in now ... it's with this manipulating now, write down another very important concept. When we talk about making something the subject, what exactly do we mean? Pause. What is the meaning in English ...when you are in your normal English classes when you talk about the subject and object? Is it related in any way?*

Learners*: I think so, I think so.* (In very low voice)

Cherry: *Eugh* (showing approval)

There was a moment of buzzing between the learners; I could not catch what they were saying because they were all talking at the same time. The teacher encouraged them to be more specific. Cherry asked the learners to distinguish the meaning of 'subject' in English and the following conversation was noted.

Ann: *The meaning is subject, like.*

Cherry: *Ann be more specific.*

Ann*: In English, the main subject is English, so we forego...*

Cherry*: Ok. So subject can mean topic.*

Learners*: eeh* (showing understanding)

Cherry*: In English when you are looking at sentences and we the subject, verb and the object, is it related in any way to our mathematical use of the word subject?*

Learners: *Miss there is someone at the door.*

Cherry: *It's not just someone, its Mr Tom.*

Mr Tom was admitted into the classroom, greeted, took what he wanted from one of the tables and left the room and the lesson proceeded.

Cherry: *Is it related? Come tell me about your English now subject, verb and object?*

Learner: *In English the subject is a thing or person that does the verb to the object.*

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Learners*: uuh, uuh.*

This was a loud approval from the class which meant the answer from that nominated learner was confirmed.

Cherry*: I love dogs. What is the subject?*

Learners*: I*

Cherry: Ok. *Now that's the common English structure that the subject comes at the beginning. Alright the person doing the action, then we have the subject at the end of a sentence.*

Learners: *Yes, yes.*

Cherry: *Mary?*

Mary*: Dogs are loved by me.*

Cherry*: Dogs are loved by me. Ok. It can swop... mathematically for us as well subject does not have to go there but there comes the difference. The subject in the mathematical sense is the variable that stands by itself that we need to find the value for.*

Pause

Cherry: *So it is related to the English sense of subject but it is not exactly the same.*

Learner: *Miss Can you repeat what you have just said?*

Cherry: *I could if could remember what I have just said.*

Other learners: *The subject is the variable that stands by itself that we need to find the value of.*

Cherry*: ... find the value of. The letter or the variable that stands by itself that we need to find the value of, that could be an a, an x, a y, a p, an h it can be anything. It can stand on the right hand side of the equal sign as well. It does not have to stand on the left but it must stand by itself. It's got to be alone. By habit we try and get it on the left hand side. That's just convention but it can be on the right hand side. It can swop it is not wrong. Alright now. Last concept, an equation and an expression. The difference between them?*

Learner: *I think an equation has an equal sign.*

Cherry: *Uuh, Ok. Can we come up with more, you are quite right An expression is a maths sentence, like an English sentence the subject, the verb, the noun, the adjective, ba, ba, ba there is different parts, different terms where as an expression has an equal sign somewhere in it.*

Learners: *An equation Miss.*

Cherry: *An equation I mean, I am very sorry.*

Pause

Cherry: *An expression is a maths sentence but an equation it does have an equal sign. This of critical importance because*

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Then the teacher drew a see saw showing two children one heavy and the other light and the see-saw tipped on the side of the heavy one. The learners where amused by the teacher's drawing, they laughed and seemed keen to know what was going to happen next. The pictorial representation of balance was appropriate and seemed to stimulate interest in learners.

Learners: *wooh, wooh (*And many other sounds to that effect were heard)

Cherry: *When we see the situation in the playground. Come on, the biggest little kiddie in the preschool trying to ride a see saw with a tiny kid.*

Learners: *aah Shame, shame* (Laughing)

Cherry: *Now this idea whole idea of a see saw or of a scale and I don't think you have ever seen an old fashioned scale, I wish we had one so I could put it up.*

The teacher drew a scale on the chalkboard as she spoke:

Cherry*: Aah. It has, it has like an arm with two hanging things and two bowls there and you decide how much weight you want to put on here, mind, you can put on a 2kg weight here this is what they did in the older system so try and understand. If you wanted two kgs of potatoes and you order from one of the regular shops in town. They put a two kg weight here. What is going to happen to the scale?*

Learner: *Like this, will go up*

Cherry: It's *going to go like this right and then they put your potatoes here until it comes back into perfect balance, like this.*

I could see the learners sitting near me drawing the scale in their books and talking about it.

Cherry: *Now ...If you can understand the concept of balance this is the secret to an equation. There are various manipulations that you can do. But you are never allowed this situation where, where what you do to one side puts a lot of weight on this side it's never allowed to go out of balance. What would you do to this one. To bring it back into balance?*

Telling learners that the balancing principle was the same as the one used in solving equations was encouraging learners to make the conversion necessary to finding solutions. Also, as she pointed out, if you can understand the concept of balance

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then solving the equation should not be a problem. In this way she was referring to conversion as well.

As she was talking Cherry matched the words with the action on the scale or the see-saw she had drawn on the chalkboard. The learners talked to their neighbours about what would happen to the scale. Some could be seen demonstrating the tilt with their hands. There were many voices and it was difficult to pick one as they were talking at the same time so I only caught a few words like: *It depends how much you make it.*

Cherry: *So I could take?*

Learners: *No Miss I should just.*

There were lively exchanges among the learners.

Learners*: No Miss not... can't you just put another child?!*

Learner: *Put another kiddie... there.*

Cherry: *Can't I just add another little kiddie there maybe*

Learners: *ooh*

Cherry: And *hopefully their weight together will balance this one. Now what manipulations are we allowed to do here in this equation here? Ya.*

Learner: *Let's just say that see saw that you just drew is multiplication eeh*

Cherry*: Yap. Come on*

Learner*: Eeh, the... Miss if... Isn't Miss what we do to the right hand side and the left hand side should equally balanc*e

Cherry: *should equally balanc*e. *Absolutely.*

Learner: *So you simply add the same on each side of the child*

Cherry: *That's correct... I just happen to have two kids and they are appropriate... and then one can't...*

Learner: *I won't be swinging ...I want to jump.*

Cherry: *And jump so fair. And now we have a balance and so if I am going put another one here I must also put another one there and.*

Learners: *And make sure they balance*

Cherry: *Exactly. If can just understand that that's fine When I am saying 'what can we do,' 'what can we add' I do not always mean we are plussing I am just saying if we are going to tip the scale in this direction by doing something there you have to do the same something here in a way. Ok. But let's get more specific here, and to get more specific and more accurate, this is just to understand what that balance means. But I am not speaking properly mathematically, but if you understand that that's good. Aah we have to one way*

to keep the balance... aah is by using inverse operations. Write that down. When we are talking about the inverse of something,

Pause

Cherry: *We are talking in a way, and I don't want to use vague terms but I just want to use terms that are normal English terms as well, you know when you knit, and you make a mistake.*

Learner: Yes.

Cherry: *You have got to pull it out right. You have got to undo it you pull it out. So in fact you are doing two opposite operations aren't you?*

Learners: *Aha.* (Agreeing).

Cherry: *You were knitting making it longer and you are reversing it. So we need to see inverses as a way they are operations that reverse another operation. They undo another operation so write down yourself a little definition for yourself. When we are doing inverses operations an inverse operation will and put it in inverted commas because I am aware that that I not speaking completely accurate, it's going to undo the another operation and to reverse it it's going to go in the opposite direction.*

Pause for about half a minute and the learners were quietly looking at their work

Cherry: *And this is... this is the second secret of equations the one is the idea of balance. The second one is the idea of undoing, going in the opposite direction meaning inverse operations. What are our operations? Our basic operations?*

Learners: *Addition.*

Cherry: Addition.

Learners: Subtraction

Cherry: Ok.

Learner: Multiplication and division.

Cherry: *And division. We have two pairs of inverse operations which are:*

Learners: Addition and subtraction

Cherry: *Adding and subtraction undo each other. Undoing. I am just... I am pulling it. Alright so the inverse operations addition and subtraction. Any the other ones?*

Learners: *Multiplication and division.*

There was a thirty second pause.

Cherry: *These are our most critical ones. Now there are other little tips of the trade* (sounded like) *... which are not basic operations but there other ways to undo them.*

Cherry: *They are not operations but they are fractions. What undoes the square root?*

Cherry wrote the square root sign on the chalkboard

$\sqrt{x} \rightarrow \qquad x^{1/2} \rightarrow \qquad$ and squaring $\rightarrow \quad x^2$

Learners: *Power of two.*

Cherry: *Power of two. Square root. OK. And also if something is squared then to undo it you square root it. Can you think of anything else? Think in terms of fractions?*

Learner: *Miss a cube root.*

$\overline{} \rightarrow \qquad x^{1/3} \rightarrow \qquad$ *and cubing.* $\rightarrow \qquad x^3$

Cherry: *A cube root and cubing something those are inverses. (Pause) This is related to this one and just think in terms of fractions I want to give one with x in the denominator, I can times. I am not showing the right hand side but to keep the balance, but to keep the balance I am just saying in the denominator because after all this means divide by x and divide and multiply and then we can do a cancelling. Do you remember when we started with equations we went a long way. This where we started I think in grade 8 and we said if we have an equation it's alright.*

Cherry wrote this on the chalkboard: $y - 3 = x$ and asked:

Cherry: *What was our procedure?*

Learners: *We added 3 to both sides*

Cherry: *So in fact we were using inverse operations, we had a minus and we did an inverse to it by adding the same amount because we knew that plus 3 and minus 3 we make nought. And we will left with y by itself. But there was the idea of balance. So we used our inverse operations here, added a 3 .. But we had to keep them balanced so if we are doing something to this side we must do exactly the same as to that side so we had to, add a 3 here as well. That's where we started originally. Then as we went on we said this is a pain having to do this every time, so let us just take it over right and this is where our idea of when it goes over the equal sign instead of being a minus 3 if we are going to take it over its going to change sign and become a plus three. But the fact that it becomes a plus 3 it's not just coincidental it's because we had a minus 3 that we wanted to get rid of so we had to plus 3 so we had to plus three there as well. But eventually we got quite good at this part and if we had y-3 = x we just said take it over.*

Learner: *Uuh*

This is what was written on the chalkboard as the teacher said the above.

$$y - 3 = x$$

$$+3 \quad +3$$
$$y = x + 3$$
$$y - 3 = x$$
$$y = x + 3$$

Cherry: *and we said but change the sign. But it's actually because we were applying inverse operations here now we must we can still, we must still just use this quick method. I just realised that what we have done in our heads is apply an inverse operations to make y the subject. Any questions before we go on with those basic, basic concepts. Ya?*

Learner: *Can't we just....*

Cherry: *Can't we what?*

Learner: *Can't we .... (*I could not catch the question but the teacher did for she responded)

Cherry: *Of course it is possible just because the letters. They behave in exactly the same way. So give me an example Lee.*

Lee: *2y +3x = 6*

Inviting learners to make up their own examples to be worked out on the chalkboard seemed to empower learners to own their learning. It can also be an indicator of how much they understand. Giving appropriate equation appears to show that the structure had become familiar and the learner is capable of converting from one register to the other. Cherry identified closely with the constructivist paradigm by giving the learners the opportunity to choose and make up their own problems to solve. This gave the learners ownership of their own learning.

Cherry*: So alright now pick what you want to make the subject.*

Learners*: x*

Cherry: *You want to make x the subject. Do you want to use the long or the short method?*

Learners: The long method.

Cherry: *We want to end up with x. There is a whole lot of things we must think about, we don't want you, we don't want you, and we don't want you. There is whole series of things we have to do. We need to, we can do step by step, but as we get used to it we can do it all in one step. Now you can start anywhere but my instinct tells is I am going to look at this one first why? Because it's easier to do addition and subtraction inverse*s.

The bell for the end of the first period

To get rid of the 2y on this side

$$-2y +2y +3x = -2y + 6$$

$$3x = -2y + 6$$

But consider the whole balance. If I am going to make this a divide by 3 then what I do to one thing I must to everything then I end up with a fraction here which I can move later that's no problem.

$$+3x/3 = (-2y + 6)/3$$

$$x = (-2y + 6)/3$$

$$x = -2y/3 + 2$$

Cherry: *But Its often easier just as a strategy if you have plus and minus something to remove that first before you start*

When Cherry asked the learners which method they preferred, short method or the long method, I found it strange that they unanimously chose the long method. But then both the methods were used to solve the same problem.

Cherry: *This is the problem for now. Now if we do it the long way, this is a plus as there is nothing in front of it, its plus its plus There is, the inverse of adding 2y is subtracting 2y And if I do it here using the idea of balance I must do it there.*

Learners were given a minute of working

Cherry: *Alright this is an implied plus So if I want to remove this one I must take away 2y because that will make it equal to zero but because it was in balance I am not allowed to upset the balance I must take away a 2y here as well. So now I am happy because I have got this one and could rewrite it. But I still want to get rid of the 3. This is a 3 times x. The reciprocal of the three times is to divide by three. If I do that here, that makes me very happy, because 3 divided by 3 gives me x. But if I am going to divide by 3 there I must divide everything, everything by three... as well. But is there a difference if I do that or that?*

Learners: *yes, no, no.*

Cherry: *Which one would be better?*

The learners were not in agreement on which one was better.

Learners: *This one? the other one?*

When learners struggle to come to grips with new knowledge, a state of disequilibrium presents itself. According to Piaget (1970), knowledge is built during the process of disequilibrium where individuals internally experience cognitive conflict when confronted with new information. During the disequilibrium, prior knowledge cannot explain new experiences. Therefore, through accommodating new knowledge and assimilating it with the prior knowledge, individuals form internal structures of knowledge unique to them (Piaget, 1970).

In the mathematics classroom, teacher and students continually use each other's' contributions to resolve disequilibrium and develop individual knowledge. However, during the process of negotiating and sharing with a knowledgeable teacher, students come to understand the mathematical meanings of the wider society, taken-as-shared-meanings (Cobb et al., 1991).

Cherry: *You see, now I aah... way this way in a way is a short cut because I said what I do to one side I must do to the other side. So in fact if I want to apply my own words strictly I must do to this side... and it has a common denominator then I can strictly say....*

$$(-2y + 6)/3 \text{ which is } x = -2y/3 + 2$$

So if I had chosen not to do it this way and said well I know it's just easier to get rid of this one first. It should not matter right. It shouldn't matter what I do first. So let's try it

$$2y + 3x = 6$$

And we still want to make x the subject. We want to get rid of y on this side. What if I did this, I divided the whole of the left hand side by three and the whole right hand side by three? Well can I do this: She was cancelling.

$$(2y + 3x)/3 = 6/3 \qquad \text{going to} \qquad x = 2y/3 + 2$$

Learners: *No-o. No-o , miss you also have to divide the 2...* (There were many voices)*.*

Cherry: *I can't just do that...*

Learners: B*ecause of the plus sign*

Cherry: B*ecause of the plus sign. But I can do this, I can split them into two terms and say that's 2y/3 plus 3x/3. And then based on that... and that one and here I have a two. Alright, now I still move this one to make x the subject. And then it just changes sign it goes over to the other side and I am still left with:*

$$x = -2y/3 + 2$$

which is exactly the same.

Learner: *So that our final answer?* She was referring to: x = -2y/3 + 2

Cherry: *Ya.*

Learners: *Ooh, uuh.* (Inaudible)

Cherry: *Alright. So we can do it and we should do it step by step, extremely careful, all the time say to myself how can get rid of this, how can I undo this? And do it step by step. Once you are very confident in this you can do it in one step. Then you can go straight… and do it in one step. But until then you need to work very careful, very self...*

Cherry quickly divided the chalkboard into ten slots and then announced the class work as Exercise 8.7.

Cherry: *Now the ones that I am going to give you they have very few actual constants its variables aah and the way we are going to do them is. We are all going to do all of them at the end of the day but for now I am going to give you one each. The first ten are going come up and put them on, on the board do it first Ok, come put it on the board and then we all agree. And then there is 7 that I am going to choose after this.*

Learner: *Miss are you giving us all... worksheets?*

Cherry: *I did make millions. I have made enough for the whole school. Aah alright exercise 8.7. Change the subject of each of the following formulae equations as indicated. The first one for example says:*

$$A = lb$$

And it wants you to make b the subject.

The papers were distributed by the teacher as she gave instructions. But when the exercise was about to begin some learners had different worksheets. The teacher apologised and handed them the correct ones and confirmed with the learners that they had the one with exercise 8.7. The teacher read out the first two questions the problems were all composite of letters with no numerals involved.

Cherry: *We are not working with numbers so that frees us to just work with inverse operations without making mistakes of six divided by three when you tell me it's 9. You just work with variables. The first one is very easy.*

During the allocation of questions the teacher jumped some learners who immediately alerted her that she had jumped them. She told them that she wanted to jump some and continued until the first ten were given out and then she attended to the remaining seven.

Cherry: *While they are doing this, Lee two d, e,...*(and so the remaining seven were allocated their individual questions as well)

Cherry: *Right, now the ones who are doing number two, those are slightly more complicated. So those are going to do one by one and you are going to explain to the class. The first ones a to j do it, and make sure you are right and then come put it on the board. As soon as you are ready order makes no difference Step by step slowly thoughtfully... thinking what you are doing and why you are doing it.*

The learners worked on their problems in complete silence for almost two minutes while the teacher was examining their work without commenting. The silence was broken as learners discovered parts of their working that needed clarification. As the teacher was going round learners could ask her to help, but it seemed she did not offer ready-made answers for all; I heard her say a couple of times "You tell me." She did entertain one who wanted to: '... *get rid of the power of five'* where she responded as follows:

Cherry: *Ok you take the fifth root. But your exponent rule says if you got x to the power n, right, and you want to get rid of power n, you just take it to the nth power that's why square root two means...*

Learner*: x to the power half.*

Cherry: *What do you want to make the subject?*

Learner: *It was x to the power n+1,*

Cherry: *I am ok with that. Come ladies come ladies come put it on. But if you are not sure bring it. Bring it. What do you want to make the subject what's the inverse of divide by three. Now do it step by step, this is making it more complicated than I wanted. To get rid of this and this you only need one step, try it, sometimes there are a number of steps, remember to multiply by 12 here, how about, lots of step by step. Ladies, as you are working, start looking I am not saying they are correct, but they are up there for your consideration and evaluation we are not saying they are correct. Question them if you say no I do not agree with you. You can't do this because these are not the answers these are their suggested workings.*

Learner: *Ah Jesus Christ* (sighing)

The bell rang to mark the end of the school day and the teacher's last words were:

Let's do the one we did not get as homework, then tomorrow we can do these. Thank you so much ladies.

This happened towards the end of the lesson because the other activity had taken a longer time than expected that is judging from the lesson plan. On the lesson plan the teacher had indicated that in question 1 a – j would be done by 10 learners on the board with explanation. It was an impromptu decision on the part of the teacher to vary the question, some were easy and others challenging.

The learners did not hesitate to go the chalkboard to attempt their individual problems. I took it that this was a practice that they were used to. The learners could consult each other, the teacher or any textbook if they needed to do so. There was a busy atmosphere and concentration was high. As it was the last lesson of the day, the learners were told that they would continue with finishing off the individual working and discussion of the solutions the following day. The learners were to finish their preparations for the presentations to be held on the following day, as part of their homework.

On the following day the learners immediately went to the chalkboard to complete their solutions. I gathered from the teacher that early that morning some learners had approached her to discuss their solutions with her. When the proposed solutions for the first ten items, that is, number one, a – j, were completed the teacher gave the learners time to look over the solutions individually and in silence. They were to note down anything that they picked up that would need clarification or correction. After that each learner was asked to present a verbal explanation of their solution, this encouraged them to think aloud. This activity took most of the double period lesson. They managed to complete all the questions. The interaction between the learners was very lively and positive. The teacher sat at the back and provided minimum direction and correction mainly in terminology or language used rather that in actual manipulation of the equations.

The activity described above gave the learners the opportunity to question each other's solution. Learners were given the opportunity to showcase their knowledge in front of their peers and educator. They all participated, but not in a competitive way. This exposed them to positive as well as negative criticism of the methods, thus giving them the chance to correct them, reconsider some of their own assumptions and have a chance to refine them if need be.

As a strategy of teaching the activity described above gave the learners the opportunity to display their knowledge and they could get immediate feedback from both their peers and teacher. Feedback encourages the learner to take control of their learning and also to organise their thinking appropriately. This kind of exposure trains them to be more open to criticisms and also gives them the chance to contribute in a positive way to the learning of others through their comments and other ways of communication made available by such a setting. This is in line with the expectations of the Department of Education which expects active participation and contribution from all learners during the learning process.

4.3.5 Comments on Cherry's teaching

To provide opportunities to learn, Cherry emphasized concepts and different registers of representation of algebraic concepts. She was aware of the necessity of using more than one register in presenting concepts. The syllabus encourages the use of different representations of concepts to the learners. The availability of multiple representations allows for new perspectives on a concept and so gives learners an opportunity to learn.

Cherry's teaching was focused on the process of thinking about connections between concepts and procedures. The lesson presented above is testimony to this where she went through the important definitions and procedures of changing the subject of formula with the learners. She gave learners a chance to contextualise their mathematics. During her introductions she gave the learners examples that related to their environment. Her learners recorded their work early in the lesson and throughout. She was very concerned that they learn to think for themselves and be able to articulate their thinking. In the first two minutes of the lesson she already directed learners to record:

> *Now let's just revise, and... in your book... just write down a few key concepts that we use.*

Writing down key concepts is conversion because the learners have to think about them and write them down. The learners have to change the register to communicate what they think. Cherry provided learners with opportunities to connect with the past so they could build new knowledge with the support of what they already knew. This is in line with the constructivist view of learning which state that it is necessary for the learners to build their knowledge on what they already know.

In an effort to help learners practice what they learnt, Cherry gave them the opportunity to display their efforts in front of their peers. She actively involved the learners in the learning process. She chose experiences that contributed to the achievement these goals. For example, during one of the lessons Cherry divided the chalkboard into ten sections and allocated the problems from the worksheet to ten different learners. She allocated the problems according to the capabilities of the learners. This was what she told me when I asked her after the lesson what criteria she had using in allocating the problems. The learners did not hesitate to go to the chalkboard to attempt their assigned problem. As they worked, the learners could consult each other, the teacher or any textbook if they needed to do so. There was a busy atmosphere and the level of concentration was good.

The above activity gave the learners a chance to examine and justify their solutions. When I saw ten learners going to the chalkboard I thought there was going to be chaos. I was proved wrong for the learners worked in an orderly way. I observed a number of them stand back and analyse their own work and make corrections before they sat down to look at their peer's efforts. There was a busy atmosphere and the teacher took a back seat though she could be consulted she encouraged the learners to help each other. This is in line with the social constructivist theory which claims that peer interactions stimulate student reflection about ideas that other students present.

By this exercise Cherry gave the learners the experiences of controlling their own learning and also enhanced their social skills of working together in a limited space and sharing resources. Piaget (1970) claimed that peer interactions stimulate student reflection about ideas that other students present. Explaining their solutions helps learners internalise the ideas for they have to make what they have worked out

intelligible to others. This gives them a great opportunity to revise the thinking and thus gain confidence in themselves as mathematics thinkers. Learners construct knowledge if they are actively involved in solving problems that they understand and want to solve (Vygotsky, 1978). Through social interaction, learners learn to interpret other's perspectives and fit them with their own interpretations. Learners learn to respect each other's ideas as well as their own.

Curriculum 2005 focuses on fostering learning that encompasses a culture of human rights and sensitivity to the values of reconciliation and nation building. Clearly the activity gave the learners the opportunity to receive and give assistance in an atmosphere that encourages tolerance and the building of each other's confidence, not only their own. It also gives them confidence in their own abilities to perform and contribute in an environment conducive to learning. The environment was designed to support and challenge the learners' thinking in a more direct way. In this situation mutual understanding is motivated and willingness to share without fear of judgement is also encouraged, at the same time the learners become resources for one another.

Cherry clearly attempts to adhere to Duval's (1995) recommendation that there is a need for more than one register of representation. She used many forms of representations in this particular lesson. She used voice, spoken and written language, and visual presentations. The learners had a chance to make mental representations by thinking about the problem and then put the mathematical representation on the chalkboard. They then used the mathematical representation to explain to their class mates how they solved the problem. All these: voice, language, written language and mathematical registers, gave the learners the opportunity to construct knowledge. The teacher made it possible for the learners to experience working in several registers. This is the important role of the teacher. The teacher provides the context for learning to happen.

Cherry's learners are equipped by their experience of making a move between representations in order to construct their own knowledge and representation. When they learn, the learners change the representation. If the representation makes sense to them they can change and use it in another similar context. When learners

are solving problems, this is known as treatment (Duval, 1995). While they are talking with each other, they are in the same register or they are solving problems which are also in the same register.

Cherry provided her learners with opportunities to understand that mathematics is about making sense of things. She guided their learning experiences in such a way that learners could actively construct correct mathematical meanings. For example in the lesson above she used promptings like 'Be more specific," "Add a bit more," "Does it mean then?" and 'How does that sound?" This gives the learners the opportunity to reflect upon their own thinking and if necessary to adjust and reform.

Cherry is flexible in her organisation of work and she adjusts her teaching to the learner's needs. She is not in a hurry and so gives her learners time to think over what they are doing and thus provide them with the opportunity to sort a concept out in their minds. Because she is flexible, she was able to adjust her timetable when I could not make it to observe her lesson. This meant that I did not skip a day of observation.

In Cherry's class, learners do not sit passively and watch the teacher perform actions to solve the problems. Instead they solve the problems themselves. According to the constructivist view of learning, based on Piaget's theory of assimilation and accommodation, learners must experience the concepts in order to build understanding. During the lesson Cherry walks around the room monitoring learner's work and gives assistance when it is required. She encourages the learners to compare their solutions with their neighbours. She gives the learners confidence by entrusting them with authority, saying "Work something between the two of you." Cherry believes that if learners select and use appropriate problem solving strategies together, they will learn to reason mathematically. Her comments are positive and affirming, for example if a learner gives a good answer she acknowledges this with words like "a brilliant idea" and "very good."

There were three learners in the class who were new to the school. Among the three was one who got particular attention from the teacher. Cherry nearly always went to the learner's desk to assist or just to look at the learner's work. The learner was

challenged by the work she had to do and was still trying to settle into Mt Carmel. Cherry follows her progress very closely. Though the learner is behind in the work, Cherry is providing her with the necessary support for her to catch up. She asks the learner to explain what she is doing and how she is solving the given problems. She guides learners on an individual basis to develop confidence and understanding of mathematic concepts and procedures. She talks to her learners in a non-threatening manner so that they are open to her and do not hesitate to seek her assistance when they feel they need it. She asks them to explain and justify their answers.

Cherry wants her learners to think mathematically and see the whole picture; how mathematical concepts can be thought of in different contexts.

Cherry observes her learners solve problems and listens to their strategies so that she can obtain information about their prior conceptual understanding. She then builds her instruction upon the learner's prior conceptual knowledge. She uses word problems to help learners form equations. She uses problems the learners can identify with. On the day that she taught about simultaneous equation she used examples that required learners read, identify variables, form linear equations, and solve and apply the derived solutions to answer related questions. One typical example of problems given to learners is reproduced below:

9. A maths test contains multiple choice questions worth 2 marks each and short questions worth 3 marks each. The test is out of 50 marks and there are 22 questions.
  a) Define two variables.
  b) Set up two linear equations.
  c) Solve the two equations simultaneously to determine the number of multiple choice questions;
  d) If the test was 1 hour long, how much of your time would you allocate to answering the short questions?

The above example depicts a process that leads the learners from formation, to treatment through to conversion and hence to the construction of the desired concepts.

Cherry guides lessons in such a way that the learners learn procedures in a conceptual context, such as in the example above and many others like it. Cherry

provides opportunities for learners to learn to extend knowledge into new situations. She provides them with the opportunities to construct mathematical concepts for themselves.

4.4   Case 3

4.4.1 St Anne Catholic High School

This section of the report describes the pilot study which I carried out at St Anne Catholic High School. St Anne is an all-girls inner city school where I observed seven periods of mathematics (algebra) teaching and learning. Byrne (2001) says that performing a pilot study is always a good idea regardless of which method is used, for it provides the researcher with experiential logistics from actual procedural implementation. The pilot study provided me with the necessary experience and training in observing learning and teaching in the capacity of a researcher as I needed to adjust to this new role. Through it I identified possible problems likely to be encountered when collecting data such as missing opportunities to record important incidents because of technical interruptions, intentional or accidental. In the end I was taking two recording devices just in case one of them failed me. However, though initially I had not intended the pilot study to be used as a case, I decided to present it as one because I learnt a lot from it. The pilot study helped me to plan my time well.

4.4.2 The teacher: Ann

Ann is one of two grade ten mathematics educators at St Anne. She holds a Bachelor Degree in Science specialising in Mathematics and Applied Mathematics and a Higher Educational Diploma. She also did many other subjects for non-degree purposes, such as Accounting 1 and 2, Business Law 1 and 2, Economics 1 and 2, Psychology 1, 2, and 3, four basic Psychology honours subjects and Operational research. She did all these courses to develop herself as a person, not necessarily because she is a teacher. Ann has taught for twenty seven years, twenty of them at a tertiary institution and the last seven at St Anne. Besides grade ten, Ann also

teaches grade eleven and twelve mathematics. She does not teach any other subjects.

Ann's mathematics classroom is on the first floor of a three story building. The building is over hundred years old. The adjacent classrooms are also mathematics classrooms of the other mathematics teachers. As you enter her classroom you face the teacher's table situated opposite the door. The chalkboard is slightly to the left of the teacher's table. There are charts on the other three walls, most of them of the demonstration type. The learners sit in rows and columns of two desks in each column.

Movement from one lesson to the other is determined by a siren bell. Learners and teachers are expected to be settled into the next lesson within five minutes. Teachers normally stay in their classroom while the learners move around. As soon as the bell rang I made my way to the classroom. Ann was in the classroom and she welcomed me and a student teacher who was also observing the lesson for other purposes. The grade ten learners filed busily into the classroom and settled down as they took their seats. The teacher greeted the learners who responded likewise. After a brief explanation of my presence in the classroom (this was because the learners already knew me as I am a teacher at the same school), I was asked where I would like to sit and I chose to sit at the back where I could have a good view of the whole class without being in anybody's way.

The learners were all present for the lesson. The lesson consisted of two thirty-minute periods following on from each other. All mathematics lessons at St Anne, except for one per week are double periods. Appendix 5 gives an illustration of the classroom and the sitting arrangements in the classroom.

4.4.3 The first Lesson: Generalising from number patterns

This was the first lesson that I observed. I have chosen to report on it because I wanted to understand how algebra is introduced to grade ten learners. I had asked Ann to invite me at such a stage when she would be introducing grade ten algebra. Below I describe the whole lesson in detail. At the end, I comment on Ann's teaching

in relation to the OTL algebra in her classroom. After observing the lesson I had a formal interview with her where she said the following about the teaching of algebra to grade ten:

> *... to teach from the concrete to the abstract because if you just teach from the abstract then you lose them. Like when you attended my class remember when we did the number patterns you know that would be concrete. You don't have any algebraic symbols. They see it by means of trial and error. They must see a pattern but that pattern can be converted to symbols.*

I was still setting up my voice recorder when communication commenced between the teacher and learners and so the first minute was not recorded. The teacher then called the attention of the learners to the lesson at hand:

> *Ok. Let me just get this on the board as fast as we can...*

The learners continued to talk to each other while the teacher was writing the following on the chalkboard.

$$\frac{\quad}{\quad} = 1 \quad = -$$

$$\frac{\quad\quad}{\quad\quad} = -$$

$$\frac{\quad\quad\quad}{\quad\quad\quad} = -$$

$$\frac{\quad\quad\quad\quad}{\quad\quad\quad\quad} = -$$

Ann: *Ok now they want to know... shhh... the next quotient of the factors. Now can you see what is happening?* (Pointing to the work on the chalkboard)

Learner: *Yes*

Ann*: We are adding natural numbers?*

Learners: *Yes*

Ann*: Now this one we have already added up to five so the next one will add up to?*

Ann together with the learners*: Up to six.*

And she wrote the following as she spoke and continued:

$$\underline{\hspace{3cm}} = -$$

Ann: *And what is happening to the denominators? We are dividing by a number that is one more than the last number there. Do you agree?*

Learners: *Yes*

Ann*: If we end at three we divided by four.*

Together*: By four*

When I say 'together' I mean that the teacher and learners spoke at the same time. This happened mostly when the question required the learners to complete a statement started by the teacher. The learners would anticipate the question and supply the missing word(s) simultaneously with the teacher.

Ann: If w*e stop at four we will divide by five*

Learners*: By five stop at five*

Ann: *.And this one if we stop at six we will divide by?*

Learners*: seven*

Ann*: Ok. Now they say write down the value of?* (Short pause).

Learners buzzed. It seemed that if the teacher paused the learners immediately talked/discussed with their neighbours without waiting to be told to do so. They talked and looked into each other's work. The teacher's pausing gave them an opportunity to consider the next move. During the above mentioned pause I heard one of the learners sitting near me saying to her neighbour '*Is it not one n is it a number?'*

Then the teacher gave the learners some information on what was required:

Ann: *Ok. Now what they want you to do is not to work it out on your calculators, you must find a pattern on the right hand side which gives you the value. Ok. Now let's see if we added up to three we divided by 3 over 2.*

$$\underline{\hspace{2cm}} = -$$

If we added up to four we divided by the last one over 2.

$$\rule{3cm}{0.4pt} \quad =\text{-}$$

If we added up to five we divided by the last term over 2. If we continue the pattern it will be last term over 2 which is four. You must not work it out on your calculator. Ya. You must work it according to the pattern.

One learner*: Yes*

Ann: *Ok. So now write down the value of?*

She spoke the figure as she was writing them down and continued:

$$\rule{4cm}{0.4pt} = \rule{1cm}{0.4pt} = 10$$

Ann: *Now according to the pattern on the right hand side it will be last term over 2.*

At that point there was a disturbance because one learner had a very loud hiccup, to which the teacher reacted thus:

Ann*: Mary! You must say excuse me. Ok.*

Another learner*: Excuuuse meee.*

This remark triggered some giggling from other learners and the teacher continued without commenting further on the matter.

Ann: *Ok. Now write down a conclusion from these examples. It's basically what I told you now ne?*

Learners: Yyes

As the learners were busy writing in their book Ann went on to say:

And that is... We are not looking for the formula yet. That's number six. Conjecture. We want it in words. (Pause) *Now tell me in words what we have done. What is all this? If we have a question of the sum of the natural numbers....* (Pause)*.*

When the teacher paused the learners consulted each other and were presumably trying to write down the conjecture but they did not offer any of their formulation until the teacher gave it to them in the following words:

If we have a question of the sum of the natural numbers, say up to n. We will and we divide by the next one which is n+1.

Learners: *n+1.*

Ann: *Then what will that sum be?* (Writing) *It will be n/2. Do you agree? If we add the sum of natural numbers up to a certain number which may or may not be n. If we divide that by the next natural number which is n+1 then that quotient will be n over 2. Last natural number over 2. That's what we see here, can you see?*

Learners*: Yes* (Very low)

Ann*: If we add up to 4 and we divide by the next one, what do we get? Last one over two.*

Ann repeated the same information in other words while pointing to the examples on the chalkboard. It seemed that she was concerned about pointing out the pattern to the learners and asking them if they agreed as if there was a chance for the learners to say no. She was pointing out the obvious, maybe in the hope that the learners would follow the pattern visually and then commit it to memory. Then the following exchange between the teacher and one of the learners took place.

Learner: *Mam isn't 1a just 3 over 3?*

Ann: *What?*

Learner: *Why did you write 2 over 2 for 1a?"*

Ann: *It is to show you that it's the same pattern, elsewhere does 1 go in?*

Learner: *But I mean why didn't you just write 3 over 3?*

Ann: *Because it is not 3 over 2, 3 over3 it's 3 over 2, Why? Because if we look at this whole pattern. What is the pattern? It is the last one over 2, it's the last one over two, it's the last one over two that's why there is two over two because my last one is two. It is just to illustrate the pattern. If we write 1 then you can't see what the pattern is.* (Pointing to examples on the chalkboard)

Learner*: Ohh* (Like expressing sudden insight but she continued to mutter something for the benefit of her neighbour. It was not loud enough for me to catch.)

It appears that the learner was asking in terms of equivalent fractions and could not understand why the teacher used $\underline{\quad}$ =1 =$\underline{}$ instead of $\underline{\quad}$ =1 =$\underline{}$ which seemed logical to her. The teacher's focus was on establishing the pattern but the learner was not following the teacher's line of argument which did not make sense to her at that moment. However the teacher continued to explain:

Ann: *For that one.* (Pointing to $\underline{}$) *Likewise if you just wrote the 2 then you would not have noticed the pattern. Ok.* (Pause) *So for conjecture we write in words. Now*

the general term. (Pause) *The formula. One plus two plus three plus... wara wara up to n. So what must we divide? Mary?*

Mary*: n+1 (There were other voices too)*

Ann*: n+1. The next natural number. And then according to the pattern on the right hand side, what will that be? Last natural number over two. And she wrote:*

$$-$$ on the chalkboard

$$\overline{\quad\quad\quad} = -$$

Ann: *The last one over 2, the last one..., not the next, not five, oho I see when you...*

Learners: ... *Add to 1... make it easier 4.*

In this segment, it seems like the teacher wanted the learners to move from a fractional number pattern to its general term. Her approach was to describe the numerator as a pattern on its own paying attention to the last term and then the denominator was the next number. The learners' responses were mixed in that many of the learners did not seem to be following what the teachers was doing or explaining. Though the learners had the examples showing the same pattern using a numerical end point such as 3 or 4 they were finding it difficult to cope with n and n+1. As a result many of the learners were talking at the same time. It seemed this part was causing some problems for them. Ann did say during one interview that she likes to challenge learners. These are her words:

> *I think you must challenge. Don't just teach what they give you, what is prescribed, you know. Challenge them; I give them a lot of difficult work sheets and encourage lateral thinking.*

At the same time Ann said that work should not be too difficult for the learners. She expressed this when she said:

> *You must not throw them with things that overwhelm them because that leads to... Immediately they believe they can't do maths and you have lost the other half you know. You mustn't. It must all be gradually, gradually.*

In line with the above view Ann gave the answer herself maybe because the learners seemed to be struggling and she did not want to overwhelm them.

Ann: *Ok that is a recursive number pattern, let me just see how we can write it. Aah T_{n+1} would be up to n plus one over n plus 2 and that will be (n+1)/2:*

$$T_{(n+1)=} \frac{\rule{3cm}{0.4pt}}{\rule{3cm}{0.4pt}} = \frac{\rule{1cm}{0.4pt}}{\rule{1cm}{0.4pt}}$$

T_{n+1} would be that. What if n+1 means it's the next term? You have written yours in terms of... How do you get the next one?

Learner: *No Mam...*

Despite the help given in the form of the above explanation, some learners still seemed to be struggling with the idea of the general term. They were finding it difficult to convert from the numerical register of representation to the symbolic. Duval (1995, 1999) claims that for effective construction of new knowledge it is necessary that the idea be expressed in more than one register of representation. Ann implied the same thing when she said the following describing how the learners should learn algebra:

> *They must see a pattern but that pattern can be converted to symbols. first in verbal then eventually in algebraic symbols.*

Without instruction from the teacher the learners were engaged in what appeared to be pair work. They appeared busy and concentrating until the teacher called for their attention:

Ann: Right. So we look at the next? Long pause, then she announced the question number and page. The learners continued to communicate with each other in low tones while opening their textbooks to find the exercise.

Ann: They say in the first question continue the pattern three more times. Now what do we see? First all what..? We increase the first one that we square by 1 and then we square it by one. So what will the next one be? So what will it be? It will be? 5 squared minus the previous one squared. And so what will that sum be? 4 plus 5

Learners: *4 plus five*

Ann*: Interesting number pattern Ne?*

Learners: *Yes*

Ann: *Then six squared minus 5 squared is equal to.*

Learners*: 6 +5 and the next one?*

Together*: 7 squared minus 6 squared equal to 7 +6.*

Learners*: Yes*

While saying the above the teacher was writing on the chalkboard as laid down below:

$$1^2 - 0^2 = 0 + 1$$
$$2^2 - 1^2 = 1 + 2$$
$$\text{Up to}$$
$$6^2 - 5^2 = 5 + 6$$

Ann: *So the difference between these two squares is the sum of these two numbers that we square. Ok And this one to make it more complete. You agree? This is actually what that was. To complete the pattern one pattern.* (She was pointing to the pattern on the chalkboard.)

Ann was interpreting the pattern for the learners and supplying them with the specific vocabulary and asking them whether they agreed with what she was writing when in fact they could not say no. She continued to show them as indicated below:

Ann: *Now let me show you one. Now they say prove that the pattern is true. Now this you must do you must work out the left hand side. Is equal to... 2 squared let me show you one. 2 square minus 1 that gives me 4-1 and that is 3. So the right hand side is 1+2 which is 3. So you can see that it is true for that one*

Learners: *Yes*

Ann: *So the next one... So the left hand side we say 9 minus 4 which is five. And the right hand side is equal to 2+ 3 which is 5. And that you must do for all. For this whole pattern.*

Learner: *What's that?* (To a neighbour)

Ann: *Now whenever you must prove something you know you are only grade ten for the first time. Hopefully, aah so...* (Ann broke into a little laugh).

Three girls giggled and the teacher went on:

Ann: *You are only in grade ten and you have not done these sums before prove, identity and all that. So whenever you have to prove one thing one equal to another. We cannot start with an equation like this and get a three here and get a three there ah ah. Whenever we have to prove something we say left hand side and you work on the left hand side only then right hand side you work with the equal to right hand side only. Whenever you have to prove something. Ok*

The way she spoke sounded like she was warning them to expect more difficult work to come their way henceforth. The learners spend about 20 seconds talking in low tones and the teacher called their attention to the next problem:

Ann: *Ok now they say number three use the number patterns to complete the following.* (Pause)

Ann: *So what will that be 8+9*

Learners: *8+9*

Ann: *And the next one 13 squared minus 12 squared*

Learners: *12 + 13*

Ann: *Now have you thought why this is true?*

Learner: *No-o*

Ann: *Did anybody come across a reason. It's because it's a, it's an interesting result ne? A result that we did not expect. Have you thought?*

Learner*: No no (low voice)*

Ann: *And could anybody come across why this is true?*

Learner: *It's the law*

Ann: *Also so is food, but it must have a reason. Now think back grade 9 No offence. You agree this is the difference between two squares so which is so would you agree with me.* (Pause)

She said this while writing and one learner said yes very loudly before Ann could finish her question. Other learners seemed to disapprove of her response but the teacher continued without commenting:

Ann: *Do you agree that I could factorise the difference of two squares? Yes.*

Learners: *Yes*

Ann: *And because these are consecutive numbers do you agree?*

Learners: *Yes*

Ann: *Do you remember what consecutive means?*

Some learners said no and others said yes and the teacher continued.

Ann: *The one follows the other.*

Learners*: Yes, the one follows the other.*

Ann*: The next one, if it's 13 the one is 14 and if it's 14 the next one is 15. Now what is the difference between any two consecutive numbers? One*

Learners*: One, yes.*

Ann*: And can you now see what we are now left with?*

Learner*: Definitely.*

The above response from a learner seems to suggest that the learner was reacting this way to make a point that the teacher was asking something too obvious and maybe insulting their intelligence.

Ann: *This is what we have there?*

One learner*: ooh (exclaimed as if something just became clear to her)*

Ann: *This was not part of your question. What is it?*

There was some noisy laughing and the teacher cautioned two learners.

Ann*: Tilde and Mercy.*

Tilde*: No Mam... and now you end up proving it.*

Ann*: Yes. But you were not supposed to do the proving. But I am just showing you why it works like this.*

At this point there was a lot of fidgeting, stretching and some yawning. It seemed that some learners were struggling to come to grips with the current work. They became restless but the teacher continued to explain.

Ann: The difference between 13 and 12 is 1 so it's one times the bracket and that is why it gave the right hand side 13-12 is one.

Learner*: ooh*

Ann*: The difference between two consecutive numbers, whole numbers. Let's start from zero. Ok*

Some learners continued to talk in the background while the teacher continued:

Ann*: Investigate this pattern in search of other consecutive numbers. So we now know that it does not matter if I say 41 squared minus 40 squared. We now know from recognizing the pattern it will be 40+41.*

There was a lot of fidgeting and Ann appealed to the learners:

Ann: *What do we do with these girls, they are bumming, they are yawning. Keep your body functions under control today.*

Learners*: ouch uhuh (and laughter)*

The above episode seemed to confirm Ann's approach to dealing with learners:

I think one of the most important things in teaching is rapport. You must have rapport, if you don't have that skill you've really lost half the children. You must have rapport with the children and then obviously you know more than they do hopefully.

Learner*: Mam*

Ann*: Ok they say experiment so we can take any two consecutive numbers. And we will find this. So now the next says formulate a conjecture. That means now that explain what you know this in words. It has to make sense in maths all the time. Ok would you agree that we could say that the difference between the squares of two consecutive whole numbers is equal to the sum of those two numbers? Do you agree?*

Learner*: I also say this.*

Ann: *The difference... Let's just repeat that. The difference between the sum of. The difference between the squares of two consecutive whole numbers is equal to the sum of those two numbers.*

Learner: *The squares of?*

Ann: *The squares of two consecutive whole numbers is equal to the sum.*

Learners*: The sum of what Mam?*

One Learner: *Of the numbers*

Ann: *Of those two numbers*

The teacher repeated the above conjecture as she wrote it on the chalkboard for the learners to copy into their books. They did so while some were repeating aloud parts of the conjecture. The teacher appealed to them to reduce the noise, but the learners continued to talk and the teacher proceeded:

Ann*: Now they say we must try and prove it. shhh shhh*

Learners continued to discuss loudly.

Ann: *Now they say we must try and prove it first. Ok. Shhh Shhh. Ok. So let's see try and prove now it they say so we must move to the general case which would be left hand side first.*

Talking continued among the learners. Talking on while the teacher was appealing for quiet was probably the learners' way of voicing the discomfort with the content. However, Ann continued to give them more information:

Ann: Do you agree it will be *(n+1)^2 – n^2* ?

One learner: *Yes.*

Ann: *It is the bigger. If the one number is n the consecutive one will be n+1. Right, and now what do we expect it to be? We expect it to be the sum of those two numbers ok. What I want you to do first is expand this bracket (n+1)^2 first. Remember how we did it last year. Foil. Except for food does foil ring a bell?*

Learners*: Yes*

First, Outer, Inner, and Last was said aloud by the whole class*.*

Ann*: Good. Foil this   (n+1)^2 – n^2 out for me.*

One learner said she had forgotten how to foil.

The teacher reminded by saying: *First, Outer, Inner, and Last* demonstrating with her hands. Then the difference between the squares of the numbers was found by expanding using the FOIL method as shown below. Lines were used to link terms being multiplied:

LHS = (n+1)^2 – n^2

$\quad$ *= (n+1)(n+1) – n^2*

$\quad$ *= n^2 + n+n+ 1 –n^2*

Learner: *Where does the one come from?*

Ann: *It's one times one.*

Learner: *Ooh*

Ann: *Now one times one. Which year did you do that exactly? Was it grade 1 or 2?*

Learners: Three

Ann: Three! Ok then...

$$= 2n + 1$$

RHS = n + n +1

$$= 2n +1$$

LHS = RHS

Ann: *Please tell me that you have remembered something from grade 9*

Learner*: Yes*

The first bell rang to mark the end of the first period. Learners talked to each other and could be seen referring to what they had written in their books. Possibly the teacher was giving the learners time to rest a little. She then resumed as follows:

Ann*: Well, remember what we said 'any generalisation is good if it makes life easy.'*

The learners continued to talk to each other and the teacher had to appeal to them.

Ann*: Come, come, come. Right. I think this is it for number patterns.*

Learners*: Yes*

Ann*: Why do you say yes?*

Learners*: No-o*

Learners: *Let's do maths* (Many were talking all at once)

Ann: *Chapter 5 Algebraic expressions* (Noisy) *Ex 5.3*

Learners*: Yes yes.*

Learner*: What page?*

Ann: *Page 104 in the new book and in the old book page 102.*

Ann: *Ok that starts exactly where it says foil it. Right. And hopefully you can still foil*

Learners: *Yes and No.*

Ann: *Don't be so naughty. Today you are like grade 9.*

The learners laughed loudly*.*

Ann: *Ok now.* (Pause) *Simplification. Do you recognise that? Would you like to foil it or can you do a shorter method. Yes*

Learner*: A short method*

Ann: *Yes. You can write it as these are the factors of the difference between two squares. Not so?*

Learners: So

Ann: So we can say *remember this. Do you remember the factors of the difference between two squares? So what will this be? (first term)^2-(last term)^2*

$(2x-3y)(2x+3y)= 4x^2 – 9y^2$ Sorry I have already squared in my head. You do not write down the steps if you can immediately write down the answer. You write the answer straight away. Remember, if you had put a foil it would have taken three times as long. So if you do not have a line you can foil until you are blue in the face and mind you...

The learners talked loudly.

Ann: *Ok. Shhh shhh. OK shhh shhh Ok Do you recognise that product as the factors of the difference between two squares. $(3a-4b)(3a+4b)$*

Learners*: Yes Mam.* (Loudly)

Ann: *Rose what would it be?*

Rose*: Bracket squared...*

Ann: *Ok simplify it.*

Rose:*$(3a-4b)(3a+4b)= 9a^2 – 16b^2$*

The FOIL method was used to remind learners of work covered in the previous year thus giving the learners an opportunity to connect with work done in the past and to prepare for new work. Learners had been given an exercise on simplifying the differences of two squares to do as homework. Two of the examples from the homework were worked out by the teacher on the chalkboard as shown below.

Ann: *Ok. Now we get to the bracket squared. There is also a shorter method.*

The teacher wrote $(a+b)^2$ on the chalkboard and said:

Ann: *Ok. Now foil this $(a+b)^2$ for me first then you tell me what you get and then we go to the short path.*

Time is given for foiling.

Ann: *Foil I said.*

Learners worked quietly.

Ann: *Now. There is one thing you must remember that any binomial squared will give you three terms. You will get a trinomial. As an answer. If you end up with two terms then you have made a mistake. So a binomial squared let's foil it out like you did for the other problem a-and... What do we get?*

$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2$ [Foiling]

So These are two like terms   ab +ba   Do you agree?

Learners: *Yes*

Ann: *So we add them and there we have $a^2 + 2ab + b^2$ a trinomial if we square a binomial.*

While the learners were still writing the teacher said to them:

Ann: *Right. Now we will look at the short cut because we do not want to foil and foil and foil. So if we have a binomial squared. It is first term squared whatever it might be. In this case it was only a it could have been 3x or y or whatever plus 2 times first term times second term plus second term squared...*

(Binomial)^2 = (First term)^2 + 2 (first term) x (second term) + (second term)^2

This is the short method which you already knew.

Two examples of straight forward substitution were worked out on the chalk board. By using the above procedure the teacher gave the learners the opportunity to get the right answers and hence an opportunity to pass should a similar problem come in the examination. If the learner memorises the formula he/she will get the correct answer even if they do not understand what is going on. It is common that some students have a tendency to use certain algebraic procedures without considering why the rules and procedures work. These students focus on the computational procedure rather than the conceptual and may get by and proceed to pass examinations without having had a real opportunity to learn.

Ann: *Binomial squared. Ok. Now let's see. Let's apply the rule. Ooh. We know we must get three terms. If you do the easy route you say 3x2 all squared plus y^2 ah ah wrong there must be a middle term. Right. The whole thing is not to write… don't write it like this now. You are going straight from there to there*

Learner: aah

Ann: *So you are going straight from there to that answer...*

Learner: *How on earth can you do that?*

Ann: *Do not always **resist change**. If the whole of south Africa resisted change we would not have a democracy now…*

Learner: *No.*

Ann: No. Ok. *So what do you get?*

Learner: *Oh wait*

Ann: *Ooh. Dana you are foiling out the long way. I don't want it.*

Learner: *Mam I don't get it*

Ann: *Aah. I don't want that.*

Learners were clearly struggling to come to terms with the shortcut method. There was some undertone complaining but the teacher was not deterred by that and so she continued to give the format of the answer again, maybe in a bid to win them over.

Ann: *(First term)^2 + 2 (first term) x (last term) + (last term)^2 . As soon as you are used to this way you will not even put this step you will go straight from there to there. So:* **$9x^2-6xy+y^2$**

Learners kept discussing the answer, some seemed unsatisfied with the suggested way of reaching at the final answer. The teacher appealed to them again:

*Ann: Do not always resist change**.***

One learner: *It's difficult.*

Ann: *Do you think so? I will not deduct marks if you foil.*

Learners: *Yes.*

Ann: *But I promise you, you will save a lot of time and apart from that remember when we did factorising last year? What did we do? We took out the common factor that is the one thing we did and the other one was the difference between two squares. Now from this year. Wait a minute. From this year I am going to give you a trinomial and I am going to say factorise. So you must work back from this to that. Now if you just foil you will not recognise this, if I don't show you this and I show you this only. So if you are not used to this method you will not recognise the trinomial. That's why it's better to do that. Ya.*

One learner: *Very interesting*

Ann: *For instance. Now if I write the y there outside the bracket it means times. If it makes you feel better you can put a dot there for times.*

Learner: *Mam what happens in the exam if I foil because it feels safer?*

Ann: *If you are going to-o?*

Learner: *Foil.*

Ann: Ooh*, I will not deduct marks. I won't penalise you if you foil. But manage but remember you have to factorise from a binomial as well. If you can manage your processes and reverse processes I am fine. That is this year's work. Ok Well please do it.*

Ann moved around the classroom as learners attempted the exercises and at the same time posed questions while some suggested solutions. They mostly asked questions which required the teacher to approve of their working or the final answer, such as, "Is this right Mam," "Am I right so far?" and "Should I continue?" etc.

One learner was called on intercom to report at the office immediately after school..

Ann: *Let's do it.* (Encouraging the learners).

Ann: *The negative belongs to the second term remember*

The learners worked quietly until the teacher called their attention:

Ann: *Ok what would that be? If you squared the bracket, you will only end up with two terms. It's the difference between two squares, not so? And can you see if you think this is a difference between two squares that would be the factors of the difference between two squares. Ok.*

Learners discussed for about a minute and then the teacher said:

Ann: *Ok see if you can do those.*

Work was done quietly and the teacher seemed pleased with the progress she was noticing as she moved round the class.

Ann: Ya. *You see you are getting there. Where is the first one? This one we will have three terms yes. Binomial squared, so binomial will give us three terms so:*

$$\text{(First term)}^2 + 2 \text{ (first term) x (last term)} + \text{(last term)}^2$$

$9x^2 - 12xy + 4y^2$ *and once again you can just write down that.*

Lucy: Mam

Ann: *Yes Lucy*

Learner: *Mam how do you get minus 12?*

Ann: *2 times 3 times is 6 times minus 2 is minus 12*

Learner*: Mam do we have to use this for the work that you give us?*

Ann: *I want you to practice. You can decide later whether you want to use it or not.*

Learner: *But I use...*

Ann: *How many times have I said today stop resisting change. You can always after a month's work you can always revert to your old habits but at least for now try and do this because it will help you when you get to factorisation of a trinomial. At least for now.*

Ann: *Do you agree that the next one will be the factors of the difference between two squares*

Learners: *Yes*

Ann: *So that will be:*

$$(\text{First term})^2 + 2 \text{ (first term) } x \text{ (second term)} + (\text{second term})^2$$

Ann: *Who got that?*

Ann: *Great and wonderful*

Ann: *I want you to use it and decide if you*

Ann: *How many times have I said to you today not to resist change?*

Ann: *Why do you suddenly think that those two must be suddenly the same. a+b is not the same as a-b. So we can apply that rule about binomial to this one. These rules are only for binomial squared. I don't see a binomial square there I see a product of the binomial. Not the same.*

One Learner: *That's exactly what I thought*

Ann: *Ok. Look at this one. Am I too fast?*

Learners: *Yes*

Ann: *Ok wait I will wait*

Ann: *Ya. That will do later. If you get a binomial cubed you will end up with four terms. You can actually work it out and find it but there is also a rule for that.*

Ok for now we are only interested in binomial squared. Ok. So the middle term is (First term)^2 + 2 (first term) x (last term) + (last term)^2

$$16b^4 + 8b^2 + 1$$

The learners cheered.

Ann: *Thank heaven for that.* (Laughter)

Learners: *Yes, yes, yes.*

Ann: *The pen is dropping. The pin is dropping!*

Ann: *Ok Homework*

One Learner: *Mam let us pray. Let's pray.*

Ann: *It is time to pray, but let me give you homework*

Learners: *Aah, aah Let's pray first*

Ann: *Go away,*

Learners: *Go away, go away.*

Ann*: Exercise 5.6, numbers: 10, 11, 12, 13, 16, 17,* (Laridon, et al, 2005:105)

Learners: *aah aah*

Ann: 19, *These sums are one liner, maybe two liners but not long. Come girls put away your books. Rejoice.*

Rejoice said the prayer.

The teacher asked the learners sitting near the windows to close them and just then the bell rang. The class was dismissed.

4.4.4 St Anne High School: Lesson 2

Lesson 2 took place two days after the one described above. Even though I observed the previous lesson it was taken by a student and so did not form part of my study. I observed it so that I could follow what the learners were covering. All the learners were present for the lesson which was a single period. After formal greetings the learners settled down and I took my place at the back of the classroom. In this lesson besides observing as much as I could of what was going on, I wanted to listen especially to the words that the teacher would use as she asked questions and gave instructions. I was interested in words that pointed to or suggested learning opportunities of any sort. Of course what one sees is no more than one's perception of what is going on in any situation. The words that the teacher uses can reveal the opportunities that the learners get to learn. Language plays a major role in all mathematical registers. So it is important to pay attention to the words that the teacher uses during the lesson. Language is critical to developing understanding.

4.4.5 The lesson

During the lesson the class went over homework problems from the previous day. The questions were done one by one with inputs from both the teacher and learners.

Part of the discussion process is captured below. The questions to be answered were 11, 12, 13, 16 and 17 (Laridon, et al, 2005:105)

To begin the lesson Ann wrote (a-3) and (a+3) on the chalkboard and facing the class she invited one learner by name to say what she recognised.

Ann: *What do you recognise that as...? What? Come Emma?*

Emma: *Mam, they are the factors of the difference between two squares*

Ann: *The factors of the difference between two squares. So what should it be?*

Emma*: Aah it is a^2 aah minus 9.*

Ann : Good. There was positive cheer from her peers as well and the teacher continued. *... and ...you could put in another step if you feel you need it. Right. But you will soon get used to just writing that straight away. Number?*

Learners: *...12, 12* There was some talking going on*.* Pause

Before moving to the next example which was number 12 the learners busied themselves trying to figure out what was required. They were consulting with each other. Then after a pause the teacher came in with the words:

Ann: *And again Kelly do you recognise that as the factors of the difference between two squares? So what should the product be Kelly?* She was pointing to:

$(2x - y)(2x + y)$ which she had written on the chalkboard.

Kelly did not give the answer, she appeared to be thinking deeply. The teacher then gave the answer herself writing it down as: $4x^2 - y^2$ while saying:

(first one)^2 - (second one)^2 (for emphasis). And continued:

Right. Everybody ok so far?

Learners: *Yes*

Ann: *Good. Rose I see you have a little sister now in grade 9....*The shared information was well received with laughter from the learners.

Ann: *number 13* (Pause)

The teacher wrote: $(3x^2 - y)^2$ on the chalkboard and asked:

Ann: *Ok so what would that be Jane?*

There was an attempt to answer but it was inaudible.

Ann: *This would be? This is. I just want you to tell me in words.*

Jane: ooh

Ann: *It's a binomial?*

Learners: *Squared* (Chorus)

Ann: Squared *and our answer must have?*

Learners: *Three terms (*Chorus*)*

Ann: Three terms. *So now you tell me?*

Jane: *It's $3x^2$ in brackets squared and then squared outside*

Ann: Ok

Jane: Plus 2 in brackets3 squared

Ann: Fine

Jane: $3x^2$ sorry eeh negative y

Ann: Good.

Jane: Plus negative y squared

Ann: Good.

Jane: $9x^2$

Ann: Ok.

Jane: minus $6x^2$ yes and y plus y^2

Ann: Good.

Other learners*: Minus y squared, minus y squared, minus y^2*

$(3x^2)^2 + 2(3x^2) + (-y)^2 + 9x^2 - 6x^2 - y^2$

Jane gave the answer slowly and was helped by her peers, especially on the signs and in particular the sign of the last term was changed three times before it could be agreed upon. The teacher was affirming Jane by saying 'good' after every correct term she pronounced, maybe the teacher felt that Jane needed such affirmation.

Ann: *Negative squared is?*

Learners: *Plus*

Ann: *Plus,*

Learners: Oh yes

Ann: Ok. *But now we can remember we are always look for shortcuts. So you will notice remember **I told you**, the negative is born of the second term. But you will notice that if we have something squared, binomial, the first term is always positive regardless of the signs because anything squared is positive. The last*

term is always positive, regardless of the signs and the middle term turns out to have the same sign as the binomial in the middle.

Learners: *Uuh (appreciating)*

Ann: *So you can double check once you get to your answer ok. Next one number?*

Learners: *16.*

Here the teacher was emphasising the structure of the answer so that in future the learners would recognise the format and just substitute the respective terms in order to get the correct answer. This procedure would enable the learner to get the correct answer whether or not they understood the difference between two squares. Thus they were provided with the opportunity to pass examinations should such questions appear on the paper. After explaining the procedure, Ann asked them if they were all ok to that point, to which there was a unanimous yes.

Another example number 16 was written on the chalkboard: $(3x^2 + 5y^2)(3x^2 - 5y^2)$

Ann: *And we recognise that as Mpho?*

Mpho: *The factors of the difference between two squares.*

Ann: *The factors of the difference between two squares. So what is it going to be?*

Mpho: *9x to the power 4 minus 25y to the power 4.*

Written: $9x^4 - 25y^4$

Ann: *Do you all agree?*

Learners: *Yes*

Ann: *Right and number 17*

Learners: *Yes Mam*

Ann: Now w*e have a binomial squared. Right? And you tell me what the signs will be even before we even start. Our final answer, first term is positive, last term is positive. and the middle one will be negative because this is negative. Right. So what do you get?*

Ann: *So what do you get?*

Learners: *x squared minus, x squared minus...*

Ann: *Ok. But first you are not so well versed in doing everything in one step so let's put in steps.*

Learner: *Minus y over 3*

$(x - y/3)^2$ $x^2 - 2xy/3 + y^2/9$

Learner: *That's true*

The teacher seemed pleased and humorously asked:

Ann: *Is the penny dropping?*

Learners: *Yes*

Ann: *Uuh thank heavens! And the last problem.* Pause. $(2 + b/3)^2$ *Binomial squared tell me about the signs before you even...*

Learners: *Positive, positive, positive.*

Ann: *And how many terms*

Learners: *Three terms*

Ann: *And what will it be Tracy?*

Tracy: $4+4b/3+b^2/9$

Ann: *Good. Do you agree?*

Learners: Yes.

Ann: *If you need to put an extra step fine do so but soon you will just write down the answer without going through all the steps. Tell me about the signs before you even do it.*

Learners: *Positive, negative, positive*

Ann: *How many terms?*

Learners: *Three*

Ann: *If you need to put in an extra step do so. But soon you just write down the answer. Ok Now we are going to have sums that are a bit more involved for instance. Do Exercise 5.7 from Classroom Mathematics. Let's do numbers 1, 3 and number 5.You must be very careful when you do this. This is a binomial square therefore it has three terms. Expand and put the signs. Ok. Now let's see why must you be very careful when you do this? This one we know it's a binomial square and we work it out three terms. This one is a binomial square and we work it out three terms. What must we be careful of?*

Learners: *The signs, the negative.* Many voices

Ann: *The negative, and so what must we do? Expand this and then you put a negative and a bracket when you expand that bracket and that negative will influence each and every term inside not only the first.*

Learner: *Mam why can't you multiply the negative by the a^2 first?*

Ann: *Well that's a good question. Why can't we multiply the negative by a^2 first? Because the bracket is raised to the power of?*

Learners: *2*

Ann: *Power of 2.So you cannot multiply inside a bracket that's raised to a power. You have to raise to the power first and then expand and use brackets else you are going to make a mistake. Ok. Go for it.*

Learners: *Mam number 5 or number 7* The learners were working individually but not quietly; they could consult their teacher or neighbours.

Learner: *Mam.*

Ann: *Yes.*

Learner: *Must we..?* (Inaudible)

Ann: *First you expand this and get three terms and then you write your negative and then you keep at the back of your mind this is negative must influence each and every term, so I better put a bracket. Then you expand this binomial squared inside the bracket then afterwards you multiply the negative with each and every term. After you have done that you add like terms.*

Pause

Ann attended to a fast learner individually and said to her:

> *If you are done let me see what you have done then you can try number 16*

And to the whole class she said:

Ok. And the rest of you, can you do it?

Learners: *No-o* (loudly)

Ann responded: *Not yet*

The learners were finding it difficult that is why they could not work fast.

Ann then moved to chalkboard and asked: *Phumi are you going tell us if you expand this, what do you get?*

Phumi: Mam it's e*qual to 4x2*

Ann: *4x^2*

Phumi: *4xy I mean minus*

Ann: *Minus 4xy*

Phumi: *Plus y^2*

Ann: *Plus y^2. Right Mary if you expand that what do you get?*

Mary: *x^2*

Ann: *x^2 speak up a bit.*

Mary: Minus y.

Ann: *Minus? What is this 2 times x times y is positive, ne? 2 times x times y and Mary?* 10 Seconds elapsed and the teacher prompted *'What the last term there?'*

Mary: *Plus y^2*

Ann: *Plus y^2. Good. The next step we must remove the brackets and this must be multiplied with each and every term. So we have 4x^2 just rewritten...*

(4x^2 -4xy+y^2)-(x^2 +2xy+y^2) just rewrite

4x^2 - x^{2-} 4xy-2xy +y^2-y^2 Do you agree?

Learners: Yes, no

Ann: *Can you see if you did not put a bracket you would have made a mistake? You would probably just remember to put minus x^2 and not the rest. Now add like terms and remember to keep your brackets bracket in order not to make a mistake.*

$3x^2$ - 6xy

Right number 16 try number 16.

In the meantime the majority of learners had completed the given class work. The teacher encouraged them to keep going.

Learner: *I love this!*

Nobody took any notice. I did not find out why they reacted that way.

Ann: *Put a bracket first, bracket.* Ok. *I think I want to do one last one. Shhh, shhh.*

This expression was put on the chalkboard. $(3a – 2b)^2 + (2a – b)^2$

Ann: *Ok. First product we recognise as? Bee?*
Bee: *Eeh*
Ann: *Ok. Tina what do we recognise this as?*
Tina: *The factors of the difference*
Ann: *The factors of difference between two squares. So what do you get?*
Learner: *3a all squared.*
Ann: *3a all squared which is $9a^2$*
Learner: *Minus $4b^2$*
Ann: *Minus $4b^2$. Do you all agree?*
Learners: Yes Mam, yes, yes.
Ann: *Ok. So we need to put a bracket because we need to forget about the negative. So expand that binomial squared what do you get?*
Learners: $4a^2$
Ann: *$4a^2$ that's good*
Learner: Minus *4ab*
Ann: Minus 4ab
Learner: Plus b squared
Ann: *Plus b squared*

$(9a^2 - 4b^2) - (4a^2 - 4ab + b^2)$

And now we need to remove the brackets. Pen?

Pen: Ok $9a^2 - 4b^2 - 4a^2 + 4ab - b^2$.

Ann: *Right, and then add like terms. Dawn what do you get?*

Dawn: It's aah $5a^2$

Ann: *Good.*

Dawn: *Aaah plus 4b I mean 4ab*

Ann: *Good*

Dawn: *And minus 3 aah aah minus 5b^2*

Ann: *Good. And that's it.*

Ann: Ok. And n*ow I want to show you another example.*

$(a - 1)(a+1)(a^2 + b)$

Ann: *Ok. If I ask you to do this you can see that it could take a very long time if we don't use a shortcut ne? You are going to multiply and foil out ok until you are blue in the face. What we start with is those two factors we recognise those two as the factors of the difference between two squares. So we keep this one because that one we know and now…*

Learners: *Factors of the difference between two squares*

Ann: *… and now we recognise it as the factors between two squares and again and that will give us*

Learner: $a^4 - b^2$

Ann: *Excellent and you would have multiplied and multiplied and multiplied ne? So we must always look for things that we can recognise. Ok. Now I can give you your homework*

Learners: *Aah aah Mam.*

Ann: *Aah what are you saying aah for? You are going to write a test tomorrow and what are you going to do over the weekend?*

Learners: *Worksheet Mam, Worksheet.*

Ann: *And homework as well*

Learners: *Uuh, aah Mam.*

Ann: Ok. *Number 14.and 17, and 18 and 19, and 20..*

Learners: *Mam?*

Ann: *14 17,18, 19, and 20 . Five sums.*

Learners: *Mam*

Ann: *Remember simplify you must multiply the brackets, factorise you must end up with factors,... and if I say simplify remove brackets and if I say factorise you must end up with a product.*

The bell rang while the teacher was saying the above

Learners: *Next Tuesday, Tuesday. Tuesday.*

Ann: *Ok let's make it Wednesday.*

Learners: *Yes, yes* (And some clapping of hands)

Ann: *Do you know that you are going to write a common exam?*

Learners: Yes, *No, when? When are we going to write it?*

Ann: *Next week in Maths and English. And we have to mark it. Two and a half hours of maths I think.*

Then there were noisy comments about the exam as the learners left the classroom.

4.4.6. Comments on Ann's teaching

Ann's teaching is a balance between attention to conceptual understanding and procedures, for example, after she had written an array of numbers on the chalkboard she asked the learners if they "noticed what was happening." There was a unanimous yes and she proceeded to describe the next two terms in parts, meaning the numerator then the denominator. The learners engaged in pairs to discuss their observations and in the process discovered and described relationships and patterns in order to form conjectures. "It is through the process of having students make and test conjectures that higher levels of reasoning and more complex learning will occur" (Making and Testing Conjectures, undated). Ann guided the learners to discover mathematical rules and procedures by asking them to describe the patterns they observed.

Though she did not always give them time to reflect and then describe, she reminded the learners to be on the lookout for patterns, obvious or hidden. She told them that observing patterns was important. When a pattern was identified the learners expressed it as a conjecture in words which they then chorused in its perfect form and recorded it in their class work books. One of the conjectures that was arrived at and chorused by the whole class was:

> *The difference between the squares of two consecutive whole numbers is equal to the sum of those two numbers.*

It is possible that when the learners repeated a conjecture accurately, she thought they had learned it. She asked the learners on several occasions to repeat conjectures and other statements that she deemed important. By this she demonstrated that mathematics has important procedures and statements that should be remembered by rote. During the lessons Ann said many times "Do not **resist change**" because for learning to take place some change must occur. It seems that Ann thought that some learners were not learning because they were resisting change. It was not because they were not understanding but because they were resisting change.

While teaching procedures, Ann gave learners time to ponder and compare their answers. Learner to learner interaction is recommended for it gives them a chance to compare answers in a non-threatening environment. Pupils can share ideas, justify their ideas and build new knowledge. When learners work cooperatively, the environment is conducive to the development of high mathematical thinking.

Ann tried to ensure that learners develop thinking processes that would give them correct answers. She guided the learners to reproduce correct procedures that led directly to correct answers. She brought their attention to a particular type of problem and provided them with the format for the answers. For example she gave them the format for the answer to a binomial squared as follows:

$(Binomial)^2 = (First\ term)^2 + 2\ (first\ term)\ x\ (second\ term) + (second\ term)^2$

Any learner can reproduce the answer to any binomial squared using this format with or without understanding what is going on.

Ann helped learners see how algebraic expressions were formed using number patterns. She showed learners how number patterns could be converted to algebraic expressions. According to Duval (1995) a concept can only be understood when at least two forms of its representation are used. In this case Ann used the numerical and algebraic representations. The ability to convert from one register to the other is essential for the construction of mathematical concepts. Ann's learners learnt how to convert a given number sequence into its algebraic form; I am not sure how successful she was at it. These experiences helped learners construct new

knowledge about the relationships between number patterns in two forms and this enabled them to move back and forth between registers providing them with OTL to find solutions.

Ann presented the problems or activities in a fixed way with little room for diversions or alternatives. I will use the excerpt from the first lesson to show how an alternative interpretation suggested by a learner failed to divert Ann's attention to the needs of the particular learner. On the chalkboard Ann had written:

$$\overline{} \quad = 1 \quad = \quad -$$

The learner wanted to know why Ann used 2/2 instead of 3/3 which the figures used seemed to suggest. The following is an excerpt of what took place:

Learner: *Mam, isn't 1a just 3 over 3?*

Ann: *What?*

Learner: *Why did you write 2 over 2 for 1a?"*

Ann: *It is to show you that it's the same pattern, otherwise where does 1 go in?*

Learner: *But I mean why didn't you just write 3 over 3?*

Ann : *Because it is not 3 over 2, 3 over 3 it's 3 over 2. Why? Because if we look at this whole pattern. What is the pattern? It is the last one over 2, it's the last one over two, it's the last one over two that's why there is two over two because my last one is two. It is just to illustrate the pattern. If we write 1 then you can't see what the pattern is.*

Ann was concerned about the sequence 2/2; 3/2; 4/2; 5/2 etc. and the learner was concerned about equivalent fractions, that is, 1=3/3 which is also 1=2/2. The learner wanted the link which Ann did not provide because she wanted the sequence in a particular form. This learner was denied the opportunity to use her previous knowledge about equivalent fractions in this new situation.

Ann guided her learners to work through individual parts that led in a definite direction toward finding the answers. She helped learners understand the steps and then looked for short cuts. This she did by advising them to skips steps wherever possible, especially when they had reached the stage of fluency and could easily discern the format of the answer. Is this still about conceptual understanding?

Ann said that she wanted her learners to develop lateral thinking skills that help them to solve almost anything. She also said that she gave difficult exercises to them so as to challenge them to think outside the box. Lateral thinking skills help learners in understanding mathematics and other subjects as well. Learners need to be challenged to analyse their thinking in depth to make connections and relationships. By giving learners very challenging exercises, Ann is giving them the opportunity to become inventive in the face of new material. Through sustained exposure to this approach, learners find their own ways to cope with and adjust to new situations. They develop habits favourable to the acquisition of mathematical concepts. They analyse and reflect on their own thinking and construct new conceptual knowledge which they can use it in new ways.

Ann has in-depth of knowledge of her subject matter. She has been teaching mathematics for twenty-seven years. This is evident in her teaching as she does not stick to the prescribed textbook. She has a vast collection of material from which to select the exercises that she gives the learners for homework.

To provide OTL, Ann emphasised procedures and answers. She wanted the learners to learn the procedures that would help them to get correct answers. Even though she taught them to go step by step to the answer, she also encouraged the learners to skip steps as they became fluent in obtaining correct expected answers. As with the first lesson, Ann emphasised procedures and answers and the chorusing of answers was used often. Though Ann tried to get the learners to contribute ideas, most of the time she ended up providing them with the information herself.

4.5 Conclusion

This Chapter presented the data gathered for the study. The analysis and interpretation of the data was done alongside the presentation. Data are presented in form of Portraits of teaching by three educators. The portraits are the empirical data for this study. In the next chapter more detailed analysis of data from the literature as well as from classroom observation is done.

CHAPTER FIVE

DATA ANALYSIS

5.1   Introduction

I now return to the question of 'How can the Opportunities to Learn (OTL) grade ten mathematics be characterised and explained from the official and enacted curricula?' The previous chapter presented detailed descriptions of three cases of algebra teaching in grade ten classes in three Catholic schools in the Gauteng province of South Africa. In this chapter I wish to explore these data sets further, through discussion of the major findings as well as the implications thereof. I will discuss how the three teachers provided their respective learners with opportunities to learn grade ten algebra concepts.

The review of the literature on OTL reveals that it is an openly defined term and is the result of a combination of school and learner characteristics, resources and curriculum characteristics. The word opportunity implies a set of circumstances that make it possible to do something. So OTL is associated and concerned with the circumstances under which learning takes place. The rationale behind OTL is that one cannot hold learners responsible for underperforming if they have not had the chance to learn that on which they are tested. OTL are viewed in terms of quality of instructional delivery, time allocated and spent on tasks, resources available to meet the demands of the syllabus, and the conditions under which learning takes place (Stevens, 1993; Schwartz, 1995; Scherff and Piazza, 2005; Cooper and Liou, 2007). In the USA, the NCLB Act requires states to develop assessments in basic OTL because what is learnt depends on what is taught (Kilpatrick et al., 2001). The literature also revealed that algebra is viewed as difficult among school mathematics topics (Wagner and Kieran, 1989). In South Africa, algebra is a topic in the mathematics learning area whereas in the United States, for example, it stands as a subject on its own and so receives relatively more attention from researchers.

Contributory factors which help to identify opportunities to learn in this study are: the nature and quality of instruction as manifested through teaching approaches, the

posing of questions, use of terminology, the type of tasks set by teachers, and how different registers of representation are used for concept building. The choice of these factors should not be interpreted as a definition of all possible OTL factors, but rather a focus on particular aspects of OTL. The interrelationships between these factors shape the opportunities that students have to learn algebra. Although in the previous paragraph I spoke about such things as resources, time and conditions of learning, I now focus only on those factors that I perceive to be more prominent in my study. Considering these factors is helpful in trying to understand how OTL are generated in the different classrooms. It offers a framework for discussion and reflection on some aspects of teaching mathematics that arose from the observations of the lessons. OTL as a framework for teaching and learning is highly conducive to framing analysis of lessons and becomes an important part of the shared repertoire of the practice. It helps to monitor the extent to which the enacted curriculum matches the intended.

My observations and the analysis which follows were done from a constructivist perspective. Students' construction of mathematical knowledge is greatly influenced by the experiences they gain through interaction with the teacher (Cobb and Steffe, 1983). I include excerpts from classroom discourses and interviews to explain how each teacher provided learners with OTL.

5.2 Commonalities

First I consider the commonalities among the teachers. Thereafter teaching approaches, the posing of questions, use of terminology and the different tasks required in each classroom will be discussed. All three schools used the same official curriculum for grade ten mathematics as prescribed by the Department of Education and specified in the National Curriculum Statement (NCS). They were all teaching algebra at the time of my observation. However, they were teaching different aspects of the topic, such as generalisation, simultaneous equations, and inequalities and modelling. The teachers worked with classes of different sizes, namely 30, 33, and 17 learners. The fact that all the teachers were qualified to teach mathematics at that level means that they had sufficient understanding of the mathematical concepts. However teaching is a personal undertaking which links with

each teacher's history of learning and teaching experience. To emphasise the differences that I perceived, I discuss each teacher separately and provide an analysis of the OTL in the respective classrooms.

Learning Outcome 10.2.4 of the Department of Education (2008) requires learners to manipulate algebraic expressions by multiplying a binomial by a trinomial, factorising trinomials, factorising by grouping in pairs, and simplifying algebraic fractions with monomial denominators. This shows that symbolic manipulation is at the centre of grade ten algebra in South Africa. Furthermore learning Outcome10.2.5 states that learners should solve linear equations, quadratic equations by factorisation, exponential equations of the form $ka^x+p = m$ (including examples solved by trial and error), linear inequalities in one variable and provide graphical illustrations of the solution, and linear equations in two variables simultaneously (numerically, algebraically and graphically).

By following the above syllabus, all the teachers dealt with the rules for manipulating algebraic expressions. The teachers gave their learners tasks that required them to manipulate first and second order algebraic expression and equations. Tasks used during lessons were largely taken from textbooks and occasionally from worksheets photocopied from different sources. Ben always gave learners work from the textbook because at his school photocopying was restricted to materials directly associated with learner-portfolios. Ann gave photocopied worksheets (collected over the last seven years) to the learners for homework. Cherry used photocopied worksheets when she wanted to give learners more work dealing with problem solving. The teachers all used the Classroom Mathematics textbook for grade ten but the textbook was used differently in each classroom.

Although it was not the intention of this study to compare the teachers, it has become necessary because of the different characteristics displayed; these make a significant contribution to the understanding of opportunities to learn. For example, Ben's learners used the textbook as their sole source of exercises during lessons and for homework while Ann's learners had the use of both that textbook and worksheets. From the point of view of variety of exercises, Ann's learners were in a better position. Variety offers learners the opportunity to interact with different

authors who differ in their choice of content, order and depth. This can contribute to a greater understanding and can motivate the learners. Ann told me that her motive for giving learners worksheets was to challenge them. While the Classroom Mathematics textbook has answers at the back so that learners can verify their work, worksheets do not have ready answers so the learners depend on the teacher or class discussions for verification of answers. Worksheet exercises provide learners with the chance to demonstrating their true understanding of the concepts involved because they had no ready answers for immediate feedback. Cherry's learners had the use of both the textbook and worksheets and, in addition to this, she asked learners to create their own problems. Creating their own problems offered learners an opportunity to verify their understanding by trying the different options open to them to produce something meaningful. This removes the textbook as the only source of exercises and helps to put the learners in a position where they can own their mathematics.

Algebraic manipulation was taught by all the teachers. Ann taught manipulation so that students could discover rules through observing patterns and then apply them *to pass examinations*. She dictated the rules and was more concerned about the answers that they yielded. Ben explained manipulation so that learners could answer given questions in a given way. He did all the working on the chalkboard while the learners listened. Cherry explained manipulation so that the learners could appreciate the rules and be able to apply them in different contexts. Below I will discuss in detail how each teacher in turn attempted to provide the learners with the opportunities to learn the designated algebra content.

5.3 How the teacher provided their learners with opportunities to learn algebra.

5.3.1. Ann's teaching approach

Ann's decision to start the lesson with number patterns suggested an attempt to implement what she believed would move from something concrete to an abstract-based approach to teaching algebra. Just what is concrete requires an explanation. A number pattern is not necessarily concrete because I believe that what an

individual knows is what is concrete for them. If you know it then it is concrete for you. So for some of the learners the number patterns may not have been concrete at all. However, Ann told the learners exactly how to identify patterns. She said that the learners must see a pattern and then convert it to symbols. In this respect she did provide them with the opportunity to see number patterns and led the learners to convert them to algebraic forms. Ann believed that her learners learned algebra that way.

Ann's mathematics instruction was focused on following procedures for arriving at answers. She guided the learners to reproduce correct steps that led directly to correct answers. However, the drawback of procedural oriented learning is that its emphasis is on doing rather than understanding. Learners can use certain algebraic procedures without considering why the rules and procedures work. They may focus on the computational procedure rather than the concept.

Ann brought the learners' attention to a particular type of problem and then provided them with the format for the answers. For example she gave them the format for the answer to a binomial squared as follows:

$(Binomial)^2 = (First\ term)^2 + 2\ (first\ term)\ x\ (second\ term) + (second\ term)^2$

Ann further stressed the number of terms in the expected answer:

Ann: *Now... there is one thing you must remember, that any binomial squared will give you three terms. You will get a trinomial as an answer. If you end up with two terms then you have made a mistake.*

Any learner could reproduce the answer to any binomial squared using this format with or without understanding what is going on. In this way Ann was giving her learners the opportunity to produce correct answers which they could even do under examination conditions. Ann guided her learners to work through individual parts that led them in a definite direction toward getting the answers. She helped learners understand the steps and then looked for short cuts:

Ann: *Ok. But now we can remember we are always looking for shortcuts. So you will remember I told you, the negative is born of the second term. But you will notice that if we have something squared, binomial, the first term is always positive regardless of the signs because anything squared is positive. The last term is always positive regardless of the signs and the middle term turns out to have the same sign as the binomial in the middle.*

It is reasonable to expect the learners to perform a binomial squared proficiently after such an explanation if they are given time to practise. The shortcut method which involved skipping steps in between implied that the answer was more important than the working. This is the message that Ann was giving to the learners.

However, Ann also paid attention to conceptual understanding, for example, after she had written an array of numbers on the chalkboard she asked the learners if they "noticed what was happening." This gave the learners the opportunity to observe, think and formulate an answer. The written form of the concept gave the learners an opportunity to convert the information to a mental form. This is because when learners observe, they are encouraged to think and attach meaning to what they see with the help of the teacher's words. What Ann wrote was always something that the learners were supposed to be familiar with, for example number patterns or factors. These were used as building blocks for further development of the topic. She used "deductive reasoning" to lead learners to discover. This was one of the ways Ann provided her learners with OTL.

Ann guided the learners to discover mathematical rules and procedures by asking them to describe the patterns that they observed or identified. First, the learners expressed it as a conjecture in words which they chorused in its perfect form and recorded it in their class work books. It is said that, "It is through the process of having students make and test conjectures that higher levels of reasoning and more complex learning will occur" (Making and Testing Conjectures, undated). This high level of thinking is what Ann was trying to help her learners to achieve. After establishing conjecture, the algebraic or symbolic form was then attempted.

Ann asked the learners on several occasions to repeat conjectures and other statements that she deemed important. By this she demonstrated that mathematics has important procedures and statements that should be remembered by rote. This

was one way of providing the learners with the opportunity to learn. When the learners repeated a conjecture accurately, the teacher thought they had learned it. In a way this approach does not seem to tally with that stated in C2005 (2003a, 19):

> As far as the mathematics learning area is concerned, learners are expected to acquire a functioning knowledge of the Mathematics that empowers them to make sense of society... competence in mathematical process skills such as investigating, generalizing and proving is more important than the acquisition of content knowledge for its own sake. Competence is not taught but it is developed by the individual learner's effort through relevant practice.

While teaching procedures, Ann gave learners time to ponder and compare their answers. Ann gave learners the opportunity to work together in pairs, discussing their observations and in the process discovering and describing relationships and patterns. Learner to learner interaction is recommended for it gives them the chance to air their views and answers in a non-threatening environment. Learners can share and justify their ideas and so build new knowledge. When the learners work cooperatively, the environment is conducive to the development of high mathematical thinking.

During the lessons, Ann said many times "Do not resist change" because for learning to take place some change must occur. It seems that Ann thought that some learners were not learning because they were resisting change. It was not because they did not understand, but because they were resisting change. Ann was furthering the view that learning of every sort changes who we are by changing our ability to take part, to belong and to negotiate meaning. If one resists this, one misses the opportunity to learn.

Generally Ann's teaching was very methodical, there was a lot of repetition and learners had to work through many exercises in order to reinforce the procedures which they could then reproduce possibly for examination purposes. She would have learners recite information in chorus to help them memorise for future use, for example:

Ann. *What I want you to do first is expand this bracket $(n+1)^2$ first. Remember how we did it last year. FOIL... Except for food does foil ring a bell?*
Learners*: Yes*

First, Outer, Inner, and Last was said aloud by the whole class.

Ann: *Good. Foil this $(n+1)^2 - n^2$ out for me.*

Although the mathematics instruction was dominated by the teacher speaking, mathematical content and conceptual understanding was focused on. For instance when Ann was introducing the generalisation of number patterns she *laboured* to get to the stage where learners could observe number sequences and establish patterns, verbalise them and then convert it into algebraic forms. This said, the fact that the learners could follow the given procedures does not necessarily mean that they learnt with understanding but that they could use the skills acquired in subsequent work. It is possible that after passing their examinations using the said procedures, they may not even be able to articulate what they have been doing let alone apply their knowledge elsewhere.

Ann believed in having rapport with the learners; she occasionally shared jokes with them and knew all her learners by name and endeavoured to find out, not in a nosey way, information about them so that she could know them better. This endeared her to them and helped create a friendly environment conducive to learning. The learners could express themselves freely without fear. For instance, one learner expressed herself very strongly when she did not follow what was going on. She asked "How on earth can you do that?" to which Ann responded by encouraging the learner not to resist change. Learners could challenge the teacher directly when seeking clarification or responding to social exchanges. The excerpt below shows a learner seeking clarification.

Learner: *Mam isn't 1a just 3 over 3?*

Ann: *What?*

Learner: *Why did you write 2 over 2 for 1a?"*

Ann: *It is to show you that it's the same pattern, elsewhere does 1 go in?*

Learner: *But I mean why didn't you just write 3 over 3?*

Ann: *Because it is not 3 over 2, 3 over3 it's 3 over 2, Why? Because if we look at this whole pattern. What is the pattern? It is the last one over 2, it's the last one over two, it's the last one over two that's why there is two over two because my last one is two. It is just to illustrate the pattern. If we write 1 then you can't see what the pattern is.* (Pointing to examples on the chalkboard)

The freedom to express themselves without fear gave the learners the opportunity to ask the teacher for guidance at any stage of the lesson and so with the opportunity to learn. However, the learners sometimes took advantage of their freedom and continued to hold private conversation when the teacher wanted them to pay attention instead.

Ann was thorough in her explanation and went into great detail to explain each and every step and term and wrote down most of what she said. The learners were free to copy the chalkboard work for reference. This gave the learners an opportunity to follow and connect ideas as they observed them develop systematically.

5.3.1.1   Ann's posing of questions

Since posing of questions plays an important role in the process of teaching and learning mathematics it is essential to note how the questions are formulated, to whom they are addressed and what knowledge is required in answering the questions. Ideally questions should be constructed to stimulate different forms of thinking. Posamentier and Jaye (2006) claim that the type of questions asked places emphasis on the process strands that are valued in the learning of mathematics. The fact that teachers ask the questions that are supposed to help them discover the knowledge implies that the performance of the learners is based on the teachers and not on the subject.

Ann directed questions to learners by name. Posamentier and Jaye (1999) recommend that the teacher select a student to answer a question rather than relying on volunteers. She asked questions which invited learners to make observations and/or recognise certain patterns. Mathematics can be said to be concerned about order and pattern. In this respect Ann wanted her learners to have the opportunity to recognise the patterns in given arrays of numbers or factors. That done, Ann expected learners to convert their observations into the required written forms, either to express it as a conjecture in words or in symbolic form. Recognition of patterns is an essential part of mathematics and the skill of looking for patterns is a good basis for extending the understanding of the number systems. This helps the

conversion into other forms, for example, algebraic. Ann guided the learners to the stage of expressing their thinking based on their observations.

Ann asked questions such as:

> *Now can you see what is happening? What do you recognise that as? And what is happening to the denominators? Now write down a conclusion from these examples. Now have you thought why this is true? Have you thought?*

Asking 'why' prompted the learners to explain and justify their own thinking. Ann created cognitive conflict for her learners by asking such questions to help them rethink their answers or resolve mathematical issues. Piaget (1970) called this conflict, disequilibrium. Disequilibrium happens when a student's current knowledge or cognitions does not help the student explain the new situation. The following excerpt indicates an occasion where learners were struggling to come to grips with new material:

Learner*: Mam what if n+1 over 2 ...because if you have...2*

Ann: *But it's not (n+1)/2. Always the last one over 2*

Ann was pointing to the previous examples.

Learner: *But is it true...we still get the answer...*

Ann: *The last one over 2, the last one..., not the next, not five, oho I see when you...*

Learners: *...Add to 1...make it easier 4*

Many learners were talking at the same time. It seemed this part was causing some problems for them. Ann did say during one interview that she likes to challenge learners. These are her words:

> *I think you must challenge. Don't just teach what they give you, what is prescribed, you know. Challenge them I give them a lot of difficult work sheets and encourage lateral thinking.*

Ann also asked rhetorical questions like:

Ann: *So These are two like terms*   **ab +ba**   *do you agree?*

Learners: *Yes*

And:

Ann: *Minus 4b^2. Do you all agree?*

Learners: Yes Mam, yes, yes.

Most of the time when Ann asked the kind of questions mentioned above, the learners responded positively. Having said this, it does not follow that the fact that they agreed meant they had understood, as on one occasion a learner responded to an unfinished question because she anticipated that the answer would always be yes to whatever the teacher was asking them. Such questions do not give the learners any opportunity to learn anything for they do not appeal to any reasoning before the utterance of the expected yes. On the rare occasion when a learner said "no" (usually in a low voice), the teacher did not appear to take any notice and just proceeded with the lesson, for example:

Ann: *Plus y^2. Good. The next step we must remove the brackets and this must be multiplied with each and every term. So we have 4x^2 just rewritten...*

(4x^2 -4xy+y^2)-(x2 +2xy+y^2) just rewrite

4x^2 - x^{2-} 4xy-2xy +y^2-y^2 do you agree?

Learners: *Yes, no.*

Ann: *Can you see if you did not put a bracket you would have made a mistake? You would probably just remember to put minus x^2 and not the rest. Now add like terms and remember to keep your bracket:*

13x^2 - 6xy is it.

Remember to keep your bracket in order not to make a mistake. Right number 16 try number 16.

5.3.1.2   Uses of terminology

Some of the specific terms covered in grade ten include algebraic expression, variable, product, factors, like terms, simplify, equation, sets of numbers, and patterns. Definitions of these terms lay a foundation for working with concepts.

From the above list, Ann used terms which were largely familiar to the learners such as natural numbers, next one, quotient, pattern and other operation related expressions. However, she did not define any of them. There were occasions where straight forward definitions could have benefitted the learners. An example is when

Ann used the term recursive number pattern; some of the learners clearly needed more information for them to understand.

Ann: *Ok that is a **recursive** number pattern, let me just see how we can write it. Aah T_{n+1} would be up to n plus one over n plus 2 and that will be (n+1)/2:*

$$T_{(n+1)=} \rule{4cm}{0.4pt} = \rule{1.5cm}{0.4pt}$$

T_{n+1} would be that. What if n+1 means it's the next term? You have written yours in terms of... How do you get the next one?

Learner: *No Mam.*

A lot of algebra was covered by the teacher in a short space of time, maybe too much for the learners to follow immediately. Information overload might not benefit the majority of learners and Ann was aware of it because she told me during the interview that the learners had to learn gradually. So on this occasion she acted against what she believed. But maybe it was not deliberate or she was guided by her wish to always challenge the learners and striking a balance can be tricky. Teachers sometimes find themselves in situations where they have to compromise one approach for the other because the classroom might demand spontaneity.

Ann was usually talking and writing at the same time. On some of those occasions what she recorded in writing did not correspond to what she was saying, for example:

Ann: *Ok. Now what they want you to do is not to work it out on your calculators, you must find a pattern on the right hand side which gives you the value. Ok. Now let's see if we added up to three we divided by 3 over 2.*

$$\rule{2.5cm}{0.4pt} = \text{-}$$

If we added up to four we divided by the last one over 2.

$$\rule{3cm}{0.4pt} = \text{-}$$

If we added up to five we divided by the last term over 2. If we continue the pattern it will be last term over 2 which is four. You must not work it out on your calculator. Ya. You must work it according to the pattern.

One learner*: Yes*

Ann: *Ok. So now write down the value of?*

Here we see a teacher who is secure in her own ability to do the mathematics she wants to teach but who is writing on the chalkboard something that does not completely correspond to what she is saying, for example:

...if we added up to three we divided by 3 over 2.   ——— =–

The statement and the fraction are not equivalent. She used the same misleading phrase several times, but no learner corrected her or challenged the meaning. This does not mean that she was mixed up, but it does indicate that she was concerned about the pattern.

Conjecture was another important term that Ann used but did not explain fully:

Ann: *Conjecture. We want it in words.(Pause) Now tell me in words what we have done. What is all this? If we have a question of the sum of the natural numbers.... (Pause)*

….

If we have a question of the sum of the natural numbers, say up to n. We will and we divide by the next one which is n+1.

Learners: *n+1.*

Ann: *Then what will that sum be? (Writing) It will be n/2. Do you agree? If we add the sum of natural numbers up to a certain number which may or may not be n. If we divide that by the **next** natural number which is n+1 then that quotient will be n over 2. Last natural number over 2. That' what we see here, can you see?*

Learners: Yes (Very low).

Instead of explaining the term fully for understanding, Ann appeared to expect learners to 'see' the pattern and convert it by trial and error into words which would later be translated into algebraic form. Learners were being encouraged or possibly forced to imitate a verbal pattern they did not necessarily understand.

$$\frac{}{} = -$$

From stating the conjecture to writing T_n almost in the same breath seemed too much for the learners to take in on one go, as indicated by the incomplete questions that they asked. Even the volume of their words went down. The teacher's explanation of the general term seemed aimed at persuading the learners to commit patterns and procedures to memory and hope for understanding to come later because Ann did say during the interview that the understanding has to come gradually. This said, it seemed like an information overload and the learners were clearly struggling to come to grips with the terms. Here the learners were receiving a message that learning mathematics involved mastering a series of difficult steps. The learners would perceive that their opportunity to learn this part of algebra lay in their ability to 'see' a pattern, try to describe it in words (conjecture) and attach general terms to it. In a space of five minutes, this was probably too much, hence the struggle.

5.3.1.3   The types of tasks

It has long been recognised that the context of learning mathematics and the context of mathematical tasks play an important role in the teaching and learning of mathematics (Ernest, 2011: 120). Ernest admits that there is an ambiguity in the term context for it can mean the social location of the task or the way in which the task is presented, the way it is dressed up as a problem with reference to objects and activities, and how it is communicated in a written or pictorial form such as a textbook problem (Ernest, 2011).

Ann gave learners exercises from the textbook and other worksheets for them to practice on. After working out some examples on the chalkboard, Ann always gave her learners some problems to work on individually but she did not seem to mind if the learners worked together. She presented the problems or activities in a fixed way with little room for diversions or alternatives but later on she told the learners that she did not mind them using other methods. This kind of teacher behaviour might stem from a belief that mathematics consists of strict rules to be followed and skills

to be practised. So it follows that to provide learners with the opportunity to learn, they must practice in a certain way, particularly the way suggested by the teacher.

On one occasion a learner wanted to know why Ann used 2/2 instead of 3/3 which the figures written on the chalkboard seemed to suggest. Ann kept her focus on the sequence 2/2; 3/2; 4/2; 5/2... and the learner was concerned about equivalent fractions, that is, 1=3/3 which is also 1=2/2. The learner wanted the link which Ann did not provide because she wanted the sequence in a particular form. This learner was denied the opportunity to use her previous knowledge about equivalent fractions in this new situation.

Having given learners problems to work on Ann sometimes dictated the procedures to be followed, for example:

Ann: *Ok. Now foil this $(a+b)^2$ for me first then you tell me what you get and then we go to the short path.*

This showed her reliance on procedures because dwelling on the kind of problems which the learners would have come across in earlier grades seemed to indicate that procedures were important and had to be reinforced through repetition. It could also be that Ann wanted to justify the short-cut that she was advocating by linking it to previously learnt procedures like foiling. (She used foil as if it was a verb instead of an acronym.) After instructing the learners to 'foil' Ann demonstrated on the chalkboard how to do it which I suspected some learners had long solved in their heads. This insistence on procedures seems to go against Ann's declaration that she wanted to develop the skill of lateral thinking which would help the learners to answer questions they had not met before.

While learners worked on the problems, Ann kept reminding them about what she wanted them to do. Insisting on certain ways of doing the problems seemed to be one of Ann's ways of ensuring that the learners had the opportunity to learn the concept or procedure in question. The excerpt below is an example of where Ann directed the learners to 'foil' as she wanted them to:

Ann: *Foil I said…*

Ann: *Now. There is one thing you must remember that any binomial squared will give you three terms. You will get a trinomial as an answer. If you end up with two terms then you have made a mistake. So a binomial squared let's foil it out like you did for the other problem a-and... What do we get?*

$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2$ [Foiling]

*So these are two like terms **ab +ba** do you agree?*

Learners: *Yes*

$(Binomial)^2 = (First\ term)^2 + 2\ (first\ term)\ x\ (second\ term) + (second\ term)^2$

This is the short method which you already knew.

What is clear is that Ann worked towards providing learners with the opportunity to pass examinations. The previous year's Matriculation results bears witness to the success of her endeavour where according to her account all the learners passed mathematics with marks above fifty percent. This is no mean achievement in a group where the overall average was way below fifty percent.

5.3.1.4   How Ann used different registers of representation

One way of identifying the OTL algebra is to use Duval's (1995) theory that a mathematical concept can only be understood when at least two forms of its representation are used and converted into each other. Ann showed her learners how number patterns could be converted to algebraic expressions. In this case Ann used the numerical and algebraic representations. Ann's learners learnt how to convert a given number sequence into its algebraic form. The ability to convert from one register to the other is essential for the construction of mathematical concepts. These experiences help learners construct new knowledge about the relationships between number patterns in two forms and this enables them to move back and forth between the registers providing them with OTL to find solutions.

When Ann was teaching the factors of the difference between two squares, she missed the chance to convert between algebraic and the geometric representations of the concept. What matters is not representation but their transformation.

5.3.2. Ben's teaching approach

I observed that Ben's approach to mathematics instruction was focused on imparting mechanical procedures necessary to achieve a correct answer. He would start a sentence spelling out a procedure and let the learners complete it. For example he said, *"Here we are supposed to make 'h' the subject of the what?"* The learners responded, "*Formula.*" He had the mathematics to 'convey' to the learners and he did so whether they understood it or not. It seemed as if it was enough for him to just fulfill his duty of passing on the knowledge to the learners.

However, his approach to instruction seemed to result from his lack of belief in the learners' ability to learn because of what he perceived to be their poor mathematical background. Normally if you expect little, little is what you will get from any situation. A teacher's conception of mathematics influences every aspect of the teacher's teaching (Cobb and Steffe, 1983). Also a teacher's perception of student ability influences the way they engage students. Below are Ben's exact words about how he perceived his learners' mathematical abilities:

> *Learners on the other hand lack basic numerical skills which makes it even more difficult for teachers to teach the subject.*

Ben did not feel responsible for the learners' situation because he blamed the circumstances that he perceived as contributing to it. He said:

> *Continuous changing of Mathematics teachers has contributed a lot to the poor learner performance in the subject.*

Ben did not have high expectations of the learners and said so. He told them he did not know why some of them could not solve simultaneous equations: "...*because of certain constraints that I do not know."* Sometimes he asked and answered his own questions before the learners could do so. It seemed that Ben had written the learners off from the start. But the fact that the learners had passed through many teachers could not be reversed. It was unfortunate that it impacted so negatively on the teacher's willingness to create opportunities for them to learn algebra. The learners had been through five teachers in a space of just two years. So teacher turnover can severely restrict the learners' opportunities to learn the subject.

Ben did not provide learners with the opportunity to develop a conceptual understanding of the ideas he was presenting to the learners. In Ben's class, learners sat passively and watched him perform actions to solve the problems for them, with almost no learner to learner interaction and only a few questions and comments directed at individual learners. In fact, he discouraged cooperative work when the learner engaged in it without him sanctioning it:

> *You do individual work, except if I say pair work, or group work or the whole class that is when you work together. So now it's individual work not together.*

The learners realised that they needed it and when Ben told them not to do it they ignored him and carried on. This is indicative of learners seeking opportunities to learn despite meeting resistance from their educator. The learners were probably not as poor as their teacher perceived them to be, they just did not have the opportunity to prove themselves. But discussion in class enables the learners to rearrange their ideas, find new expressions and communicate. When you understand something you are capable of saying the same thing in different ways, in other words you can communicate it in more ways than one. Ben did not give space for that to his learners, so they did not have anything to question because he gave them the answers, sometimes fully and sometimes partially. Normally you ask when you have time to think about something. So if you have no time, you cannot ask.

Ben indicated that he did not find it easy to teach this particular group:

> *It has not been easy teaching them because most of the learners never got the vital introduction of the topic at earlier grades. This means I have to teach the basics first before teaching actual outcomes for Grade ten.*

But there is a contradiction in this because when one learner used his previous knowledge Ben said that it was below the expected level even though the answer was correct he did not accept it.

Learner*: You say, $v - u = u + at$ -u*
Ben*: Now you are now in grade ten.*
Learners*: Yes.*

Ben: *If you were in grade 8 or 9 I would say yes you are right. But now in grade ten you just say we take this 'u' to the other side. Since here it is positive it now becomes* **what***?*

Learners: *Negative.*

Ben: *You are right but you know if someone can walk in here and see something like this;* (He wrote; v – u = u + at –u), *he will think that maybe this is a grade nine* **what***?*

Learners: *Class.*

The learner concerned complained inaudibly, maybe because he realised that he had just missed an opportunity to learn using something that he already knew and the teacher was not appreciative of it.

Ben's comment seems to indicate that his instruction was focused on processing the answers mechanically rather than understanding the concepts. A problem was taken as a sequence of things to be done such as adding, subtracting, dividing and multiplying. Learners were told to do the operation but they were not given reasons for doing so.

Ben usually worked through the problems the learners had already attempted as homework. During such exercises Ben seemed to put more emphasis on solving specific problems in a particular way. The problem with such an approach is that learners do not then have to understand or interpret the mathematics problem involved, they just follow the given sequence of steps towards a specific goal and that is it. Reproducing a sequence of steps does not address adequately the mathematical knowledge, processes, representational fluency and social skills the learners need in other contexts as recommended by the Department of Education.

5.3.2.1   Ben's posing of questions

Ben basically asked one type of question. He asked questions that pointed learners directly toward the correct answers. His questions did not stimulate high order thinking and problem solving because giving the correct response to leading questions does not necessarily mean that the respondent has understood. According to Fennema et al. (1999), understanding concepts involves more than accumulating

a set of facts and procedures. Ben hardly ever asked the question "Why?" Why questions invite learners to explain and justify their thinking and so give them the opportunity to understand what they are doing. If there are no 'why' questions no discovery can be made and real understanding cannot be expected.

Ben's leading questions were limited in scope requiring one or two word responses. Asking such questions did not create opportunities for learners to think about what they were doing. Ben's questions usually ended with the word what, for example:

Ben: *So we are, we are now looking at **what**? Literal **what**? ... Equations.*
Learners*: Equations (*in chorus).

Most of the questions were of this nature. After asking such leading questions Ben would often accompany the learners as they chorused the expected answers. If he did not chorus it together with them he would repeat the answer anyway after they had done so. He asked questions and faithfully answered them himself. He did this because he did not expect much from the learners who he described as lacking basic numerical skills. However, this does not justify denying them the opportunity to learn in a meaningful way where challenging questions are posed for their consideration instead of low order single word response questions as presented to them by their teacher.

In the excerpt below Ben asked the learners whether they understood and they all chorused "***yes***." Understanding that one side is greater than the other is taught as early as grade one, so in the context of grade ten it sounded trivial.

Ben*: It means that one of them weighs more than the other what? The other side... either it's the right hand side which is greater than or vice versa. Do you understand?*
Learners*: Yes*
Ben*: If there is such a scenario... we said we call it a what? An inequality. Just like: x + 2 > 4. This is an inequality. It means that the left hand side is greater than what?*

Below is another case where Ben asked very trivial question:

Ben: *...we take this 2 to the other side so it will be x < 4 minus what? Minus 2*

Learners: *minus 2*

Ben*: What is 4 minus two?*

Learners*: 2*

Ben*: Its 2. So it means this is our solution. 'x' is smaller than what?*

Learners*: Than 2*

Asking what four minus two confirms that Ben thought that the learners lacked basic numerical skills but here it seems rather extreme. It would have been more beneficial for the learners if he had asked them to work the problem out first before doing it himself on the chalkboard because I think most learners were capable of working out the answer in their heads. Ben demonstrated by writing on the chalkboard how to solve a problem which I expect some learners had already solved mentally, so it was probably a waste of time. There were other instances when similar comments could be made regarding the worthiness of spending time on what appeared to be obvious if not trivial.

Learners in Ben's class hardly raised their hands to offer answers. The teacher did not seem to expect it. The questions were not addressed to any one in particular so the learners tended to chorus back or together with the teacher. Ben addressed the class as a whole and so did not give particular attention to individual learners unless they asked him a question. When they had a chance to ask questions, leaners brought a little variety by posing different kinds of questions, for example: "*Sir how do we know which one to pick?*" and *"Sir is it necessary to write all steps?"*

5.3.2.2 Uses of terminology

Ben used some algebra specific terms but he did not explain them. Maybe he assumed that the learners knew them. At times Ben used words that had the potential of confusing learners, for example, he used word function to refer to the given expression, when there was no direct link between the word function and the variables in the formula. He also spoke of removing fractions as if they were tangible objects or quantities that could be handled physically. He referred to an expression

as not looking nice and so needing to be freed from its involvement with a fraction. It appeared as if the fraction was being removed for no other reason but that it did not look nice. This is quite misleading and could divert attention from the real problem of simplifying. Some learners appeared to be confused by the description of saying remove the fraction to the extent that one learner said "remove the half" when the teacher wanted her to say "remove the fraction." Immediately after saying remove the fraction the teacher went on to say we want to remove the two from the bottom. Using words in a consistent manner helps learners to understand the procedures while mixing them up can confuse them. The excerpt below shows what transpired during one lesson:

Ben: And y*ou see here you have got a fraction, this side. Every time when you are given an expression with a fraction you first you remove the fraction to make it* **what?** *The fraction. That will be the easier way. Because if you want to you want to remain with h and if you...*

Ben went through the procedure of simplifying using the idea of inverse operations. An excerpt of the process is captured below:

Ben: *Something like this does not look nice so you must also remove this 2,* (Pointing to the 2 in this formula A=÷   ). *So the first thing to do is to remove the what?* When Ben said 'does not look nice' some learners laughed*.*

Learner: *The half.* (very confidently)

Ben: *The fraction not the half because next time you will find there is 3/5. You first remove the fraction, we want remove this 2 at the bottom since this 2 is dividing...Let's see. You know this one is now ok you now understand what is going on even if you are going to calculate area. This is you are trying to find the height of a triangle when you are given the area.*

One could argue that the learner who said remove the half was correct according to the way the question was framed. This was not the only time that Ben rejected correct answers from the learners when the responses differed from the ones he wanted. By so doing the learners were missing the opportunity to display their own versions of the answers. This is unfortunate because learners get discouraged when their efforts are not given credit. By saying "*The fraction not the half because next time you will find there is 3/5"* the teacher was trying to convince the learner that the

half is not a fraction which is misleading. For the concerned learner this brief conversation was a missed OTL on her own terms, that is, through active participation.

Three other issues relevant to the enterprise arise in the above excerpt. One is the problem of what mathematics to teach and how to teach it. It appears Ben wanted to show that the problem was concerned with changing the subject of the formula from 'a' to 'h' and at the same time involving a fraction. For his contrived view of the learners' mathematical abilities, this was probably too much to handle in one example yet the formula for the area of a triangle is encountered by the learners as early as grade six if not earlier for some. The second issue linked to the first is that it appeared as if Ben had forgotten that the formula at hand was one that the learners were familiar with and so to say it did not look nice would probably confuse some learners. The third issue is that of authority in mathematics. Ben said "Every time **you are given** *an expression with a fraction..."* clearly this implies that the authority lay in the textbook and so the learners do not own their mathematics.

5.3.2.3   Types of tasks given by Ben

Ben gave learners tasks from the textbook to work on during class time as well as for homework. In this respect he gave the learners the chance to practice the prescribed skills and procedures. Communication with his learners was largely one sided, he did most of the talking and all of the chalkboard working. His voice was the most powerful representation in his case. For him the students had to hear in order to understand. He gives the message to the learners that "I am going to tell you and you are going to learn" but no amount of verbal explanation alone can enable the learner to attain any mathematical concept, because knowledge is constructed individually under favourable conditions. Verbal explanation is only one from of representation and is limited in scope.

5.3.2.4   Use of different registers of representation

In The National Curriculum Statement it is expected that grade ten learners are able to, "Recognise relationships between variables in terms of numerical, graphical,

verbal and symbolic representations and convert flexibly between these representations (Department o Education, 2002, 11). For the classroom the implication is that for OTL to arise, the learner needs to be provided with the necessary representations that help in the construction of mathematical knowledge. Ben did seem aware of this guideline. Ben moved from one representation to the other without allowing the learners time to think of or discuss the intermediary steps. Learners had little chance to explore relationships or create solutions with their own methods because they were not presented with open-ended situations where there were other ways to find and display answers. Ben tended to hurry through problems and concentrated on drilling rules and procedures. This is treatment which is not enough to teach anything to anyone. Treatment is important to teach transformation inside one register but it is not the final objective of learning. According to Duval (1995) the construction of mathematical concepts depends strictly on the capacity to use several registers of semiotic representation of the same concept.

5.3.3 Cherry's approach to teaching

I observed that Cherry's teaching was largely learner centred because she gave them time to engage in activities where learners' input dominated. For example in the lessons discussed above, of the four periods two and a half were allowed for learners to work out the problems and then explain to peers. Such an approach to teaching afforded her learners the opportunities to engage in meaningful mathematical discourse among themselves. Cherry wanted to help her learners make the connection between school mathematics and its application in the real world. She believed that putting mathematics in context made it easier for children to learn and more effective.

Cherry's teaching was focused on the process of thinking about connections between concepts and procedures. The lesson presented above is testimony to this where she went through the important definitions and procedures of changing the subject of formula with the learners. She gave learners a chance to contextualise their mathematics.

During the lesson Cherry walked around the room monitoring learners' work and gave assistance when it was required or requested. She encouraged the learners to compare their solutions with their neighbours. She gave the learners confidence by entrusting them with authority, saying "Work something between the two of you." Cherry believed that if learners select and use appropriate problem-solving strategies together they will learn to reason mathematically. Her comments were positive and affirming, for example if a learner gave a good answer she usually acknowledged it with words like, "a brilliant idea" and "very good."

Cherry catered for the individual needs of her learners. There were three learners in the class who were new to the school. Among the three there was one who got particular attention from the teacher. Cherry nearly always went to the learner's desk to assist or just to look at the learner's work. I asked Cherry why she was always visiting that particular learner and she told me that the learner was challenged by the work she had to do and was still trying to settle into the school. Cherry followed her progress very closely. She provided the learner with the support necessary to catch up. When she visited the learner she asked her to explain what she was doing. On the whole she tried to guide learners on an individual basis to develop confidence and the understanding of mathematics concepts and procedures. She talked to her learners in a non-threatening manner so that they were open to her and did not hesitate to seek her assistance when they needed it. She asked them to explain and justify their answers.

Cherry was flexible in her organisation of work and she adjusted her teaching to the needs of the learners. She was not in a hurry and so gave her learners time to think over what they were doing and thus provided them with the opportunity to sort concepts out in their minds. She was flexible in her planning and could easily accommodate me when it was necessary for me to change the day of my next observation.

5.3.3.1. Cherry's posing of questions

Cherry asked questions in long sentences where the first part would constitute an explanation put into context, for example:

Cherry: *We are talking about variables. Jot down the word variable and give me nice little normal English but mathematically correct definitions of what a variable is.*

She also asked questions which required the learners to reason out their answers. For instance she posed this question in response to the learner's definitions of a variable:

It can't be any value. Why?

She tried to follow the learners' thinking by posing questions that encouraged them to think further into the meaning of what they were saying. Meaning is created through participation in social activity (Murphy and Hall, 2008). Cherry gave her learners the chance to frame questions so that they could seek clarifications from their peers. On one hand this created opportunity for learners to justify their answers and on the other the experience of asking relevant questions. Sometimes Cherry asked the learners to formulate question for the whole class. Such an approach allowed the learners to see mathematics as a natural process in whose creation they actively took part. It brought mathematics nearer home for the learners. When learners present their thinking about ideas that they have initiated and discuss them with the class, they build confidence and are motivated. Cherry's learners showed a willingness to question each other's answers. She encouraged her learners, "*let's come up with a nice definition of*" and guided them towards a conceptual understanding of algebraic terms. The lesson helped learners construct concepts about variables and the required manipulations or solving.

She provided her learners with opportunities to understand that mathematics is about making sense of things. Cherry guided their learning experiences in such a way that learners could actively construct correct mathematical meanings. For example in the lesson she used promptings like "be more specific," "add a bit more," "does it mean then?" and "how does that sound?" This does not invite the learner to pay attention to the answer only but it gives them the opportunity to reflect upon their own thinking and if necessary to adjust and reform.

5.3.3.2 Uses of terminology

Cherry guided her learners to define relevant terms as well as establish relationships between them. She probed the learners to give more information and refine their meanings. She would say something such as "be more specific," "does it mean then," "come up with more" etc. Below is part of the conversation between Cherry and the class while they were discussing the term variable:

Cherry: *Listen carefully. We have come with a figure, a letter, which...?*

Learners: *With a value that changes.*

Cherry: *With a value that changes. It's better to say with a value that can change than to say it can take any value. Because remember in an equation there is one or two or three values which it can be. It can't be any value. Why?*

Learner: *Miss is it not what you get after working out?*

Cherry: *Ya. Can you be more specific?*

Learner: *...the one you are looking for.*

Cherry: *Ya. Be more specific?*

Learner: *You are looking for this one and you get that one.* (Sounded like)...

Cherry: *Alright, let's put our definition together we say,* it's *a letter it's a value, something in normal English. That can change. Vary variable.*

Learner: Uuh uh

Cherry: *The word va-ri-a-ble sounds like to vary. It's a letter something that will change... according to an equation now. We have worked with linear equations we have worked with quadratic equations. Don't you understand? In a linear equation how many values can that variable have?* (Said slowly)

It was worthwhile to spend time on building an understanding of the word variable because it is one of the most important concepts in algebra. Cherry guided her learners to establish the meanings of the algebra specific terms so that they would understand them when they come across them in questions. Referring to other meanings as well was probably an attempt to integrate across the curriculum.

Besides explaining the terms Cherry also drew 'pictures' on the chalkboard, for example, she drew a see-saw and a balance scale to reinforce the meaning of balance. She said she would have brought one to school if she had it. The concept of balance is used to reinforce the idea of equality. The learners clearly enjoyed their

teacher's attempts at drawing and will probably remember the concept because of association with this occasion.

5.3.3.3   Type of tasks

In an effort to help learners practice what they had learned Cherry gave them the opportunity to display their efforts in front of their peers. Cherry involved learners actively in the learning process. She chose experiences that contributed to the achievement of these goals. For example during one of the lessons Cherry divided the chalkboard into ten sections and allocated the problems from the worksheet to ten different learners. She allocated the problems according to the capabilities of the learners. This was what she told me when I asked her after the lesson what criteria she had used in allocating the problems. The learners did not hesitate to go the chalkboard to attempt their individual problems. As they worked the learners could consult each other, the teacher or any textbook if they needed to do so. There was a busy atmosphere and concentration was good.

The above activity gave the learners a chance to examine and justify their solutions. When I saw ten learners going to the chalkboard I thought there was going to be chaos. I was proved wrong for the learners worked in an orderly way. I observed a number of them stand back and analyse their own work and make corrections before they sat down to look at their peers' efforts. The atmosphere was conducive to independent as well as cooperative learning while the teacher took a back seat but she could be consulted at any time. She encouraged the learners to help each other. This is in line with the constructivist view which claims that peer interaction stimulates student reflection about ideas that other learners present (Piaget, 1970).

Through this exercise Cherry gave the learners an experience of controlling their own learning and also enhanced their social skills through working together in limited space and sharing resources. Explaining their solutions helped learners internalise the ideas for they had to articulate what they worked out in an intelligible way for others to follow. This gave them a great opportunity to revise their thinking and thus gain confidence in themselves as mathematics thinkers. Learners construct knowledge if they are actively involved in solving problems that they understand and

want to solve (Vygotsky, 1978). Through social interaction, learners learn to interpret others' perspectives and fit them with their own interpretations. Learners learned to respect each other's ideas as well as their own.

Curriculum 2005 focuses on fostering learning that encompasses a culture of human rights and sensitivity to the values of reconciliation and nation building. Clearly the activity gave the learners the opportunity to receive and give assistance in an atmosphere that encourages tolerance and the building of each other's confidence as well as one's own. It also gave them confidence in their own abilities to perform and contribute in an environment conducive to learning. The environment was designed to support and challenge the learners' thinking in a more direct way. In this situation mutual understanding is motivated and willingness to share without fear of judgement is also encouraged, at the same time the learners become resources for one another.

Cherry observed her learners solve problems and listened to their strategies so that she could obtain information about their prior conceptual understanding and offer help when it was needed. She then built her instruction upon the learners' prior conceptual knowledge in mathematics and related subjects such as the English language proper. She used word problems that the learners could identify with to help them form equations which they could then solve. On the day that she taught about simultaneous equations she used examples that required the learners to read, identify variables, form linear equations, and solve and apply the derived solutions to answer related questions. This kind of problem solving is indicative of process rather than procedure orientation to problem solving. One typical example of problems given to learners is:

9. A maths test contains multiple- choice questions worth 2 marks each and short questions worth 3 marks each. The test is out of 50 marks and there are 22 questions.

　　a) Define two variables.

　　b) Set up two linear equations.

　　c) Solve the two equations simultaneously to determine the number of multiple choice questions;

　　d) If the test was 1 hour long, how long, how much of your time would you allocate to answering the short questions?

This example has the features of building up concepts in a coherent way as opposed to bits and pieces to be put together at the end to form a whole. The sub-questions are linked in a meaningful way that appears to tell a story. Learners experience mathematics as a meaningful endeavour and they could easily identify with the question. Thus Cherry conducted her lessons in such a way that the learners learned procedures in a conceptual context, like the example above and many others like it. Cherry provided opportunities for learners to learn to extend their knowledge to new situations. She did that by creating opportunities for them to construct mathematical concepts for themselves.

5.3.3.4   Use of different registers of representation

Cherry clearly attempted to use different registers of representation during her lessons. She used spoken and written language, and graphical, symbolic and visual forms of representation. She helped her learners make mental representations by giving them exercises that demanded that they pay particular attention to what they were doing because explanations in front of the whole class followed thereafter. The learners had first to think about the problem, attempt it and then put the mathematical representation on the chalkboard. They then used the mathematical representation to explain to their class mates how they solved the problem. All these, voice, language, written language and mathematical registers, gave the learners the opportunity to construct knowledge. The teacher made it possible for the learners to experience working in several registers. Thus Cherry's equipped her learners by giving them experiences that made the movement between different registers possible while they constructed new knowledge.

5.4   Conclusion

Contributory factors which help to identify opportunities to learn in this study were discussed. These include the nature and quality of instruction as manifested through teaching approaches, the posing of questions, use of terminology, the type of tasks set by teachers, and how different registers of representation are used for concept building. Using the identified factors I discussed how the three teachers provided their respective learners with opportunities to learn grade ten algebra concepts. The

next chapter presents a detailed description of the new framework and provides guidelines on how it can be used as an aid for understanding OTL at a deeper level,

CHAPTER SIX

GENERAL DISCUSSION, SYNTHESIS AND CONCLUSION

6.1   Introduction

Literature reviewed on opportunities to learn (OTL) indicates that it is positively associated with achievement (Stevens, 1993, Gau, 1997; Kilpatrick et al., 2001 and PROM/SE 2009). If students are provided with an opportunity to learn they will do so (PROM/SE, 2009). But one of the problems with OTL is that it is open to multiple interpretations. Although OTL has received attention in international studies in developed countries, its use in developing countries has been limited (Reeves and Muller, 2005). Consequently this study seeks to contribute to the understanding and perceived benefits of engaging with the concept in the practice of mathematics education in South Africa.

Through the discussion of the findings, themes emerged that are important to this study. The themes do not stand completely detached from each other; they overlap in meanings and application. The themes are: choosing tasks that encourage discussion, giving space to learners, finding mathematical problems in the everyday experience of the learners, attending to learners individually, and using different registers of representation. I then used the results of the current research to develop a new framework that helps define opportunities to learn in the classrooms as I saw them arising. With the new framework I tried to expand on the components mentioned above and at different levels of achievement. This chapter presents a detailed description of the new framework, provides guidelines on how it can be used as an aid for understanding OTL at a deeper level, and discusses the shift in focus from general to specific components as far as mathematics learning and teaching is concerned that occurred during the course of the current research. It concludes with some suggestions for future research.

6.2   Defining Opportunities to Learn

After reviewing the relevant literature, it appears that four components of the OTL framework stand out. These are content coverage, content exposure, content emphasis and quality of instructional delivery. I have used the OTL framework to investigate the teaching of algebra in South Africa using three case studies and from this I was able to develop a new framework (or modified framework) that I now present.

Under the new framework content coverage is understood using indicators such as the type of tasks that are given to learners and whether there is evidence that the teachers choose **tasks that encourage discussion** and hence the building of new concepts. Similarly, a number of indicators can be used to investigate content exposure such as **finding mathematical problems in the everyday experiences of the learners** and asking questions that **give space to learners to think.** Indicators of content emphasis include giving learners the chance **to make the necessary conversion within and between different registers of representation** and the corresponding practice. The quality of instructional delivery is indicated by the approach that the teacher uses such as **giving attention to individual learners.** The widening of indicators broadens the scope for analysis.

Apart from building an expanded list of indicators that define the same concept, the new indicators identified in the new framework help to move broad indicators (such as content coverage, content exposure and content emphasis) into more specific ones that are readily observed in the context of a lesson. A broad framework is hereby transformed into a potentially manageable tool for investigating OTL in the classroom. However, since these indicators were derived from case study data, implementers of this framework should bear in mind that the indicators produced here may be more suited to conditions similar to the ones under which they were derived and that different indicators may need to be identified for different contexts.

In the section to follow I describe the new framework, and include a detailed explanation of the indicators, highlighting how they were derived and their potential

for use in analysis. The description is supported by evidence from current research as well as relevant literature.

6.2.1 Assessing content coverage

Content coverage is directly related to what is taught. In South Africa what is taught is determined by the Department of Education but its implementation is left to the teacher who decides what to present to the learners. It is the implemented curriculum more than the intended that explains achievement (Cueto et al., 2005). This shows clearly the important role that teachers play in the learning process. As they teach, teachers have to set tasks to reinforce concepts. This study has shown that teachers need to choose tasks **that encourage discussion as well as problem solving.** The tasks or activities in which learners are involved influence their thinking and the valuing of their subject. If the activities given to the learners are characterised by routines that do not allow room for creativity, such practices deny the learners the opportunity to learn the core curriculum for their age and grade level (Stevens, 1993).

Traditional instruction begins with the syntactic rules of algebra, presenting students with a given symbolic language which they do not relate to (Amerom, 2002). In the current study the insistence on rules was clear but the difference lay in how these rules were delivered. On one hand the learners were led from the definition of terms to their application while on the other the learners were simply told the rules to be followed. In what I call the 'middle learner group,' the learners were 'coerced' into accepting the rules as presented by the teacher. Presumably better understanding was to come from prolonged practice.

Learners are expected to master the skills of symbolic manipulation before learning about the purpose and use of algebra (Stacey et al., 2004b). In other words, the mathematical context is taken as a starting point, while the applications of algebra (like problem solving or the generalising of relations) come in second place. Students are given little opportunity to explore the powers and possibilities of algebra for themselves. One can imagine that an average or below-average learner finds little satisfaction in practicing mathematics without a purpose or a meaning. Amerom

(2002) points out that there is a rapid formalization of algebraic syntax in the traditional approach. In the classes observed, the transition from one representation to the other was often not clearly marked or else it was sudden and glanced over. For example, moving from algebra to its graphical representation of inequalities or from number patterns to algebraic form was sudden.

Algebra is the first mathematical discipline that students encounter that uses variables (Saeman, undated). The variables occur in different forms in grade ten: in expressions, systems of equations in one or two unknowns, in inequalities and in generalisations of number systems. This constitutes the content. The goal of school algebra is, therefore, to teach learners to manipulate the different compositions or forms in which variables are found. Manipulation means learners must perform operations or calculations, factorise, and describe and simplify different forms. Unlike arithmetic where the operations are straight forward, the students must now not only calculate variables, but also determine which operators to use. The algebra teacher must help the student overcome this paradigm shift. Generally it is agreed that students must acquire competency in algebraic thinking in order to have full algebraic capabilities like reasoning with unknowns, generalizing and formalizing relations and algebraic symbol manipulation on paper.

Generalising and formalising relations require a high level of mental action, therefore, teachers should **involve learners in activities that encourage them to construct their thinking at deep levels**. The kind of questions asked invoke different thought processes, for example, a question that asks a learner to explain 'why' and 'how' provides a strong opportunity to make connections between ideas. Being able to articulate a process or an idea encourages a deeper understanding of a process or idea because doing so involves movement between mental representation and verbal and/or visual representation. On the other hand, a question that asks for answers only provides a weak opportunity for learners to make connections in order to establish relationships that lead to conversions between ideas.

Teachers should ask learners to reason and use their reasoning to build theories that they can prove. **Learners should be engaged cooperatively and individually** in

exploratory lessons involving examples from their own real life experiences. Individual economic issues and group endeavours like fundraising activities that involve them in making decisions based on their experiences in real life are examples that can be used for this. Such exercises will acquaint learners with systems of variables which require the use of simultaneous equations and/or inequalities. These activities should be open-ended, challenging and problematic for learners so that they can be motivated to engage in them.

However, analysis of observation data reveals that teachers are primarily concerned with establishing rules that are used to solve problems from textbooks. Little attention is paid to learners' real life experience that could generate meaningful equations in variables that make sense to them.

6.2.2 Assessing content exposure

This study can be informative to curriculum developers. We can learn from Cherry's approach that learners can practice their work together on the chalkboard instead of on a one by one basis, thereby saving valuable class time. This is an example of **giving learners space**. When learners are involved directly in the learning of others as well as their own, it helps to create a conducive environment in which worthwhile learning can take place. In order to verbalize their thinking, learners must make a conversion. They think, talk and write on the chalkboard and this gives them the opportunity to learn with understanding. Classroom communication and interaction are primary issues for teachers to consider in constructing positive non-threatening learning environments (Cobb et al., 1990). This study has shown that when learners are given the opportunity to present before their peers, they are motivated to work and are eager to participate in a meaningful way **by researching and consulting with each other and the teacher**. By so doing they verify their ideas and construct valuable new knowledge. This is an example of **giving learners space** by encouraging classroom communication and interaction.

Teachers should not assume an all-knowing position and instead should **give space for learners to pursue their own line of thinking.** They should also not behave as if understanding is something that one either possesses or does not possess, for

understanding can take place at different levels for individual learners. In the case studies, the learners asked questions that showed that they expected more than the manipulation of expressions. Sometimes they wanted to stay on familiar ground and seemed to resist change but at other times they wanted to know if there were other ways to solve the problems.

It is a known fact that people learn from experience, therefore teachers should **find mathematical problems in everyday experience that are relevant for learners.** These problems should focus on the main concepts that teachers want learners to learn and should relate to their previous learning. For example, worksheets that tie algebra in with real world applications in sport, banking, food production, cell phone plans, population growth, cooking, borrowing money, life spans, music downloads, solar power, etc. could be produced together with the learners so that the solutions derived are not mysterious (simply found in the back of the textbook) but are real and meaningful. If for example, x=3 and y=4 was an answer to a given simultaneous equation, it would be better understood if it was given in the context in which the answer would be three oranges and four apples. Engaging in such exercises gives learners the opportunities of interpreting, articulating and applying mathematical concepts to familiar situations that make sense to them.

While they are interpreting the question they have to **translate between everyday experience and formal mathematical experience.** This experience is essential in the process of building a lasting understanding of learned concepts and links relationships that make sense to learners. As students work through the problems, they can use the mathematics skills and concepts they have learned in their mathematics curriculum and apply them to real-life situations.

6.2.3 Assessing content emphasis

The major purpose for teaching algebra should be to give learners the opportunity to make conversions between registers of representations in order to construct algebraic concepts for themselves. How do learners make the required conversions? Data from this research suggests that the learners are learning what the teacher is teaching, that is, from Ann they are learning that in algebra treatment is very

important in order to practice the prescribed manipulations and that once they have become efficient they can discard some steps and go straight to the answer. From Ben they are learning that algebra is a rigid set of steps that have to be strictly followed. From Cherry they are learning that algebra stems from real life experiences, that it has rules that are necessary to learn through practice (treatment) and cooperation.

The results of this study demonstrate that when learners are given opportunities to **practice in one register they become efficient and get correct answers.** Ann's learners became quite good at finding mathematical patterns and recognising and simplifying expressions. The questions the learners asked provide evidence that they were not following blindly but that they wanted what they were learning to make sense. But their teacher made the conversions for them and then gave them the opportunity to practice. When learners sit and listen passively as they watch the teacher perform the algebra, they do not have the opportunity to learn because they cannot make the conversions necessary for the construction of concepts. Ben's learners watched as he asked and mostly answered his own questions, for he distrusted their own mathematical abilities.

Cherry's learners were given the opportunity to form the concept by defining the algebra specific terms, and practicing and articulating in front of the class. The verbal explanations of their thinking and the writing of it on the chalkboard provided evidence that learners constructed meanings and communicated them. The articulation was indicative of their learning. The learners did not only learn algebra but also practiced social skills such as working with other people in a limited space, listening to others and being open to suggestions from others. They were also enthusiastic when their turn came to present their solutions on the chalkboard.

The fact that the official syllabus expects learners in grade ten to be able to recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae) should be taken more practically by the teachers. Conversion is essential in the building of mathematical concepts. It is important not just to know the correct procedures required to arrive at the answer but

to also know the thinking processes involved in getting a correct product and its conversion.

This study shows that all three teachers used the different registers of representation of algebraic concepts but did not always create opportunities for their learners to make all the conversions essential for the construction of those concepts. The teachers largely concentrated on the conversions within one register (treatment) and then moved straight to another representation without justification. For example, one teacher told the learners that after getting an algebraic solution for an inequality they should move directly to illustrating it on the number line. Reasons for doing so were not given except that the textbook said so. Another teacher stressed the number of terms and order of signs after the expansion of a binomial. Teachers need to be made aware that learners should make the conversions themselves and not just be told to do so.

6.2.4 Assessing the quality of instructional delivery

Constructivist teaching is about giving learners the opportunity to construct knowledge individually (Ernest, 2011). For instruction to be effective, learners must have, perceive and use their OTL (Kilpatrick, et al, 2001, 2003). The teachers should make it possible for learners to learn as individuals even when they are part of a group or the whole class. They can do this by not treating them as if they were all the same and in need of the same treatment. Concern should be with helping every learner in class learn efficiently at their own pace (Kilpatrick, et al, 2001, 2003). If teachers listen to individual learners explain their thinking, they provide them with OTL. Ann and Cherry showed us that it is possible to attend to individual learners in ways that do not exclude the other learners. Ann would always, while attending to individual learners, speak in a voice loud enough to be heard by everyone; her voice was a powerful representation that benefited all learners.

Every school situation, classroom and learner is different. The nature of each classroom evolves its own culture. The teacher who uses strategies that focus on helping students construct mathematical power must recognize the differences between each class of students and adapt instruction to fit this culture. Ben *refused*

to take responsibility for the poor academic background that he perceived as affecting his learners' *under*performance in mathematics. Cherry gives learners questions according to their abilities so that every learner can perform well. This helps to build confidence in the learners.

This study shows that the teachers emphasise answers even though they state the opposite. Ben told his learners that in mathematics the working was the answer and Ann introduced her learners to shortcuts to arrive at desired answer forms. Cherry seemed satisfied when two methods led to the same answer. They all said that the answer was not the most important end and yet in practice they seemed to attach the most importance to answers. Teachers have to decide what to emphasise and how to proceed with certain content.

Teachers must become more knowledgeable about how mathematical concepts are constructed so they can provide their learners with appropriate opportunities to do so. It is important that they be acquainted with the three actions of formation, treatment and conversion because these constitute the process by which mathematical concepts are constructed. If teachers know these actions, they will tailor their instruction to achieve them and so give learners the opportunity to learn.

In the new framework OTL are viewed in terms of whether the learners are given the chance to make the conversions for themselves or not. This framework considers the learning process much more closely. The three schools observed all had the elements usually defined as necessary for teaching and learning to take place: qualified teachers, recommended textbooks, timetabled mathematics lessons and the learner's willingness to learn. In the old framework these conditions define OTL so ideally the learners were supposed to learn. But this study has shown that this is not necessarily the case. Learners also need the exercise of movement between registers to make the necessary conversions.

However, when we look at how teachers created OTL for their learners, the differences appear to come from the way they used the different registers of representations of algebraic concepts. Now I will consider the three actions that are

deemed necessary for the construction of mathematical concepts. The actions are formation, treatment and conversion.

6.3   Synthesis

I now attempt to synthesize the teaching and learning of algebra using Duval's (1999) model of how learners construct mathematical knowledge. He claims that the conceptualisation of mathematical concepts relies on the articulation of at least two registers of representation (different ways working with mathematical knowledge). Teachers expose learners to a variety of representations of mathematical ideas including numeric, geometric and algebraic representations. Furthermore, in algebra they include number patterns and symbolical, graphical and verbal representations. If a learner can use more than one representation of a concept, then they have learned it. It follows that to provide OTL the teacher has to create the situation where the learners engage in activities that help them to distinguish the features of the concept (formation), practice in one register (treatment) and then change the representation into another register (conversion).

6.3.1   Considering formation

In the South African context, the formation of concepts has already taken place in the syllabus, textbooks and teacher's notes. All the schools used the same curriculum determined for them by the Department of Education. First and foremost, formation is the responsibility of the Department of Education which determines what is taught. This indicates that learners are given the same material to learn which ideally can be perceived as the same OTL. The teachers were all aware of these requirements and each was teaching some aspect of the above at the time of observation.

Following the above syllabus, all the teachers dealt with rules for manipulating algebraic expressions. They explained the rules. Verbal registers are important for the introduction of concepts but it is important to underline that a verbal register cannot exist on its own because it depends on the community of practice and on the different meanings that individuals usually give to words and ideas (Bagni, 2005).

The teachers gave their learners tasks largely from the Classroom Mathematics textbook which required the manipulation of first and second order algebraic expression and equations. So as far as formation is concerned, what is expected is clearly laid down in the curriculum documents though the teachers are at liberty to choose the order and depth to which they present the concepts. Therefore it is the other two actions that have a more direct bearing on the opportunity that the learners have of interacting with the expected algebra to the desired level of competence.

6.3.2 Considering treatment

Treatments are transformations inside a semiotic system or mode, such as rephrasing a sentence or isolating x in an equation. To meet the requirements specified by the department of education, teachers engage learners in multiplying, factorising, and simplifying algebraic expressions. The results of the study showed that symbolic manipulation is at the centre of grade ten algebra in South Africa. Learners in the respective classes were engaged in the process of manipulating algebraic expressions themselves or together with the teacher. In one extreme case the teacher did the manipulation for the learners while they sat and watched and then copied the work into their workbooks.

Treatment tended to dominate all classrooms. Learners were subjected to long exercises on practicing skills or performing particular calculations. They calculated values, expanded binomials, factorised trinomials, simplified expressions and solved equations and inequalities. All of these are done in the same register. But it is well-known that understanding is much more than practicing procedures. Understanding develops gradually over time through active engagement in mathematical thinking involving conversions between the registers.

During the lessons observed, teachers gave learners time to practice the concepts they had just been exposed to. Learners practiced individually or cooperatively. Understanding is expected to develop as learners practice. But understanding concepts involves more than knowing a set of facts and procedures. Treatment, though important in the process of concept building, is not enough because it is performed within one register of representation and there is a need to have more

than one register (Duval, 1995, 2006). Treatment leads to fluency in one register only and so offers a limited opportunity to learn for it does not prepare learners to use their knowledge beyond the particular type of problem being solved. It prepares them to perform the same procedure to solve similar problems but does not prepare them to construct new knowledge.

The results of this study show that the teachers were all concerned about treatment. One teacher, Ann, said in an interview that it was important to give learners challenging exercises to work through. The learners were given a lot of exercises to do in class and as homework. This practice is mirrored in the traditional curriculum where concept development is viewed as arising from computational proficiency with relevant procedures. Under such circumstances individual thinking or reasoning is given limited time. Sometimes the learners were told rules to perform certain procedures such as changing the subject of the formula, but no justification was offered for the rules. The learners were not engaged in reasoning about the structure or the need to perform calculations.

Ann wanted her learners to learn the procedures that would help them to get correct answers. Even though she taught them to go step by step to the answer she was also keen to have the learners reduce the number of steps as they became proficient in obtaining correct answers. Ann emphasised procedures and answers and the chorusing of answers was often used. Although Ann tried to get the learners to contribute ideas, most of the time she ended up telling them the answers.

In the first classroom when the teacher dealt with changing the subject of the formula, learners were told to take a term over to the other side of the equal sign and change the sign. It appeared as if they were performing some sort of trick or magic which presumably reduces algebra to "finding x" and no other meaning. Though such an action, taking the term over the equal sign, can yield a correct result it does not guarantee that the rule is understood. Without understanding, learners might not see a need to change the subject in the first place, let alone the need to transport terms across the equal sign. This shows that if treatment is taken as an end in itself, the result is the crippling of mathematical development in the learners because it is restricted to one register.

In the second classroom, rules were stressed as far as they led to the stage where a pattern or format of the answer was recognisable. Once the pattern was recognised (through the teacher's initiative), the learners were then encouraged to skip steps and just plant the answers using the identified format. One such example was when squaring a binomial. Learners were told to expect three terms in the answer where the signs connecting the terms could be predicted and put in place before the problems were worked out. Thus, when students are confronted with squaring a binomial, most will just start slotting terms into position. If they do not misread the terms, they will get the correct result.

In the third classroom, the same rules were explained but from a more practical point of view that linked with everyday experience, such as the fact that rules are essential and that once learned can be applied in other similar situations. The teacher used the example that once a person has learned to drive they can drive anywhere in the same country. It can be surmised that such learners are likely to be more motivated to understand the rules knowing that other occasions will arise when they may need to implement them.

6.3.3 Considering conversion

Conversions are transformations that change the system while maintaining the same conceptual reference, such as going from an algebraic to a geometric representation of a line in a plane. Limited opportunities are available for the learners to develop practices that involve converting between the registers by themselves. Ann showed her learners how number patterns could be converted to algebraic expressions. The discussions regarding the patterns helped learners predict the next terms after which they had to mentally coordinate the features of each sequence. In this way, Ann's learners learnt how to convert a given number sequence into its algebraic form. The ability to convert from one register to the other is essential for the construction of mathematical knowledge. So by engaging learners in the activity of converting from one register to the other, Ann provided OTL to her learners.

Conversion is essential for the building of mathematical concepts (Duval, 1999). The grade ten syllabus supports this view by stating that learners should be flexible and able to convert between different representations. It is expected that the conversion be done by the learners but in practice on the ground, this does not always take place. The learners are **shown or told** the different representations as if it was social knowledge, for example, the situation where a child is told "this is your aunt" and there is no room to ask for justification. So what the curriculum says is one thing and what happens in the class room, quite another. When I was teaching grade ten I did not pay any particular attention to this guideline about allowing learners to convert flexibly between different representations. I suspect that this may also be the case with many of my fellow teachers in the field. For me different representations were like objects to be introduced independently.

Ben's role was to verbalise the algebra while the learners listened to what he had to say. As far as Ben was concerned, to tell the learners the algebra was important and his words provided the clues to understanding. This was his way of providing his learners with the opportunity to learn. If the learners listened, then they were learning. And yet there was no motivation to pay attention for Ben started sentences and phrases which he expected the learners to complete using only one or two words. If they completed the statement correctly then they were learning. At times he would accompany them in supplying the required word. It is usual to start a question with the word 'what' but Ben seemed to nearly always put it at the end. He had mathematical knowledge which he wanted to give to the learners. He did not challenge them to think about the procedures. He simply told the learners that the procedures worked and expected them to accept the fact. This is treatment. But treatment though necessary is not sufficient for the construction of knowledge. The danger is that should the learners forget the procedure, they would have no conceptual basis for reconstructing it. If they had talked about what they were doing, the learners would have had the opportunity to understand better rather than engage in what appears to be a mindless recall of procedures.

Ben also used visual representation in his teaching of inequalities. He drew a number line on the chalkboard and illustrated the solutions sets on them. Though this graphical representation was appropriate and the learners copied it into their

work books, most had probably learned very little about the concept. This is because they did not have the opportunity to think through the reasons for doing so. They were not given the chance to question the movement from one form to the other. Reasoning leads learners to making a necessary conversion on their own. In this case, even though two different representations of the same concept were used, it does not translate to learning for it was still just treatment as the necessary conditions were not made available to the learners.

To provide opportunity to learn, Cherry emphasized concepts and different registers of representation of algebraic concepts. She was aware of the necessity of using more than one register of representation of concepts. The syllabus encourages the use of different representations of concepts to the learners. Availability of multiple representations can encourage new perspectives on a concept and so give the learners an opportunity to learn.

By asking learners to write down key concepts, Cherry was encouraging conversion because the learners had to think and write down. The learners had to change the register to communicate what they thought. Cherry provided learners with opportunities to connect with the past so they could build new knowledge with the support of what they already knew. This is in line with the constructivist view of learning which state that it is necessary for the learners to build their knowledge on what they already know.

During the lesson, when learners had to present solutions to problems in front of their peers, they had a chance to make mental representations by thinking about the problem and then put the mathematical representation on the chalkboard. The learners had to organise, link and communicate their solutions. They then used the mathematical representation to explain to their class mates how they solved the problem. All these: voice, language, written language and mathematical registers, gave the learners the opportunity to construct knowledge. The teacher made it possible for the learners to experience working in several registers. This is an important role of the teacher, to provide the context for learning to happen. The experience that the learners had is in line with the new curriculum in South Africa

which demands that teachers use learner-centred approaches that empower learners to take responsibility for their learning.

Kieran (1992) shows how content, teaching, and learning contributes to the difficulties that students have in learning algebra. Kieran pointed out that the cognitive demands placed on algebra students include treating symbolic representations which have little or no semantic content like mathematical objects and operating upon these objects with processes that usually do not yield numerical solutions. Mathematics does not lie in its symbols but in the ideas that these symbols represent.

Ideally you learn when you relate to the object of knowledge and the teacher is only the facilitator. You have to create the object of the subject. The object is not in the teacher or in the learners, it is in the problem. The problems are the objects of knowledge because when the learner reads the problem and thinks about that problem, he/she is moving into the brain to find a means of solving it. This is the moment when Piaget says there is disequilibrium because there are things that they know about the problem and things they do not know. There is no example of this taking place in Ben's lessons. The role of the teacher is to motivate the learners by giving them different objects to learn or setting up a situation aimed at providing chances for them to interact with each other. The teacher should offer a variety of representations of the object because the use of different representation registers allows the exact characterization of the mathematical object (Bassoi, 2006). This is so not only for treatments from the same object but also in the conversion of registers from a diversity of language forms (natural, arithmetic, algebraic, etc.).

All teachers face the challenge to engage learners in mathematics work, maintain their focused involvement in it, and assist them to take advantage of instruction to learn (Kilpatrick, et al, 2001). For the learners to be able to recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations as the syllabus demands, teachers need to present the work in ways that make it possible. Creation of situations where learners convert within one register was easy for the teachers but conversion between the registers was difficult. It demands that the teachers

themselves be keenly aware of the structure involved in the subject matter and that they choose appropriate aspects that contribute to the building of concepts they are to develop. The process requires teachers to have good mastery of the subject matter though this alone does not equip them with the capacity to deliver.

There are other constraints that hinder meaningful communication between teachers and learners. Time seems to have been a factor that influenced the amount of exposure that the teachers gave their learners to engage in the reasoning and thinking that was required for conversion to take place. It seemed that it would take too long for the learners to make the conversion themselves so most of the time teachers ended up making them for the learners. The results of the analysis show that learners do not get the full benefits of the process of conversion because the teachers tell them the answers. By doing so, they rob them of the excitement of discovery which leads to better and meaningful learning which is required for the effective construction concepts. Learners cannot be expected to organize themselves sufficiently to exploit this powerful action of converting between different registers.

In Chapter Two much of the focus was on general ways of interacting with OTL. Specific indicators of OTL as children learn in the classroom only formed a small part of the literature review. The shift in focus from the general to the particular that occurred during the course of the current study was a direct response to the conditions on the ground which suggested that to understand OTL better requires a certain level of knowledge of how learners construct mathematics concepts. This shift introduced me to Duval's theory (1995) about how conversion between registers was a prerequisite for the construction of mathematical concepts.

What emerged from the literature review is that OTL is important to consider in the teaching and learning of school mathematics as indicated by the many studies that have been undertaken in the hope of explaining differences in achievement within and between countries. Some of the factors found to impact on OTL include: the degree of overlap between the content of instruction and that which is tested (Reeves and Muller, 2005), equitable conditions or circumstances within the school or classroom that promote learning for all students (Schwartz, 1995), the absence of

barriers that prevent learning (Mereku et al., 2005), conditions or circumstances within schools and classrooms that promote learning for all students (Cooper and Liou, 2007), and conditions that may benefit student's mathematics learning and achievement, provided for students by the educational system (Gau, 1997).

These views help us to understand OTL in a general sense but when it comes to the classroom we need something more practical. It does not mean that if learners are placed in an environment conducive to learning, they will do so because if it is not supported by corresponding relevant practices, it does not necessarily follow that the learners have the opportunity to learn. It is not just about having a qualified teacher to instruct them, a clearly defined curriculum and up to date textbooks. Learners have to interact with the concepts in ways that allow them to connect between the registers and so build mathematical knowledge. Students learn if they are allowed to make the necessary movement between appropriate registers by themselves.

Much attention in the study of OTL is paid to the above mentioned factors, but these fall short because most of the necessary conditions for learning the subject matter also have unique conditions which must be fulfilled for learning to take place. Duval (1995, 1999, and 2006) has provided evidence to support the assertion that for a mathematics concept to be grasped at least two registers of representation are needed. What this suggests is that, for learners to have the opportunity to learn any mathematics concept, different registers need to be considered. To give the learners OTL it is vital that the learners be exposed to those registers that represent the required concept. Unless and until it is well understood that learners need to interact with different registers, some of the approaches where the goal is to have learners learn procedures and practice skills, will continue to characterise mathematics education to the detriment of meaningful learning. Acquaintance with different registers by the teacher should be a pre-condition for teaching mathematics.

6.4   Conclusions

Sources of opportunity that were perceived in this study include the intended algebra content given in the curriculum documents, lesson delivery and other classroom activities such as learners explaining to peers and/or the whole class. These

opportunities where characterised in terms of their availability and defined by learning experiences the learner was likely to take advantage of in constructing concepts. More directly, opportunities were seen in the teacher's approach, the kind of questions asked and tasks set and the way terms specific to the topic were used. For example, how the teachers decide what they are going to teach and which methods they will use affects what is made available to the learners. A teacher's conception of mathematics influences every aspect of the teacher's teaching (Cobb and Steffe, 1983). The teacher's perception of student ability influences the way they engage students. In this study one teacher asked leading questions that did not challenge or motivate learners to think and therefore provided a weak opportunity to learn.

Ann's approach was very systematic in her delivery of lessons giving the learners the experience of step by step ways of reaching desired answers. She cared more for process rather than content. She gave learners difficult exercises to challenge them to think outside the box. Through sustained exposure to this approach learners find their own ways to cope and adjust to new situations. They develop habits favourable to the acquisition of mathematical concepts. They analyse and reflect on their own thinking and construct new conceptual knowledge which they can use in new ways.

To Ben performing procedures was very important for the learners to practice. He exposed them to the procedures and expected them to do exactly as he told them. His questions did not provoke learners to think. He had already labelled them as being poor academically so there was no need for him to challenge them to think beyond the one or two word answers that he expected. He provided minimum opportunity to learn to his learners, not only by his attitude but also by the tasks he gave them. He would work out answers for them, ask questions and answer them himself, complain that they wanted him to do everything for them and yet he gave them no chance to showcase what they were capable of. Ben did not justify the rules that he taught nor did he engage learners in reasoning of any sort. Instead he sped through the exercises seemingly for the learner's benefit.

Cherry's lesson plans all indicated that problem solving was at the heart of her teaching. She gave her learners the experience of solving real life examples. Her

learners experienced taking control of their learning through learner to learner interaction and especially through explaining their solutions to the whole group. They gained knowledge and exercised social skills such as working together in harmony, contributing to the welfare of others, respecting the ideas of others which may or may not correspond to their own. All these are ingredients for good citizenship which is one of the critical aims of South African education. Furthermore the ability to communicate or articulate one's ideas is an important goal of education and it is also a benchmark for understanding (Fennema and Romberg, 1999).

All the teachers expressed a well-known view that many learners find algebra challenging. One of the reasons given in literature is that algebraic relationships encountered in learner's everyday experience will not present themselves as matters requiring symbolic manipulation. They will present themselves as decisions to make in situations such as financial planning and the selection of service providers. Teachers must prepare students, not to carry out algebraic procedures for their own sake. They must use algebra as a tool to solve problems and represent situations. Without conceptual understanding, procedures mean almost nothing. Connections make mathematics meaningful, memorable and powerful.

6.5   Recommendations

Any research reveals areas in which more research is needed in order to extend it. The present research is no exception and this section consists of some recommendations of the direction future research can take if OTL is to be better understood.

Recommendation 1
The current research has laid out the groundwork for future work in the area of providing opportunities to learn through the utilisation of different registers of presentation of mathematical concepts. There is also a need to develop this concept further in order to determine how best to present algebra concepts so that learners have the opportunity to learn them effectively.

Recommendation 2

We need replications of the present study in order to understand other teacher's approaches to mathematics teaching and learning. The present study identifies four aspects that help in the understanding of OTL. Perhaps other teachers would use different aspects or use the ones that I have identified in different ways. More models of mathematics teaching that are consistent with the provision of OTL are needed. Would all models look similar? Would teachers with different conceptions have similar approaches?

Recommendation 3

Teachers need to share their own experiences about how they create opportunities for their learners to learn algebra. Teachers need to talk about their teaching. This sharing can help others predict and better understand the experiences they might encounter. We need narratives of teachers who are trying to provide adequate OTL for their learners. It is crucial to document how other teachers have perceived their role in creating opportunities for their learners. These narratives could help teachers examine their beliefs about the ways children learn mathematics and show how these beliefs can impact on the learning opportunities they provide to their students.

What makes a teacher use a certain approach in the teaching of algebra? What means can be made available to teachers to help them create effective OTL? Investigation of questions like these will help teachers, teacher educators and researchers to work together to help create classrooms where students become mathematical thinkers if given the opportunity to do so.

Recommendation 4

Teacher education programs and universities need to investigate ways to help in-service teachers develop teaching strategies that are consistent with OTL. University faculties might ask the question: How can teacher educators develop programs that help pre-service and in-service teachers better understand OTL principles? More research would help us understand how OTL can be developed. Would these types of experiences help teachers understand what the practice of using OTL looks like?

Recommendation 5

The Department of Education needs to investigate alternate ways of making available to teachers explanations of the words used in the guidelines and what they mean in practical terms, for example the terms 'representation' and 'conversions.' How can teachers be assisted to understand the requirements of the specified outcomes?

6.5 General Conclusion

The performance of learners in mathematics is usually a subject of great concern for stakeholders such as parents, learners, educators, governments and the world at large. The question is asked why the performance is poor in some countries and solutions are sought. The findings of this study suggest that it is possible to develop and implement instruction based the theory of concept building through the framework suggested by Duval- that fits within a broader constructivist framework. The focus of this framework is on the cognitive functions of mathematical thinking and conditions of learning where conceptual acquisition realises through the articulation of at least two semiotic representations. The learning process goes through three actions: formation, treatment and conversion.

This study has shown the importance of opportunity to learn and in particular refers to the role of the teacher and the quality of instruction. The role of the teacher in providing grade ten learners with the opportunity to learn algebra through quality instruction is indispensible It is essential that the instruction be characterised by the three actions mentioned above and yet educators tend to concentrate on treatment at the expense or exclusion of conversion. It is important to note that that the more teachers practice the type of teaching involving the three actions, the better they become at providing opportunity to learn.

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Interview Schedule

Investigating the Opportunities to Learn algebra: Case Studies of two grade ten classrooms in selected Catholic secondary schools in South Africa.

Before the interview proper I will thank the interviewee for his time and assure him of the confidentiality that will be kept and also give him a little background of the study and why it is important.

1) Tell me about your own experiences as a learner in the secondary school and also as a teacher of mathematics.
2) When you were teaching what support did you get from the school in particular and from the Department of Education in general?
3) In the teaching of algebra are there any challenges that the teachers meet and how are they dealt with?
4) Now that you are the mathematics education officer, tell me about your experiences concerning the teaching and learning of algebra.
5) How can the teaching and learning of algebra be improved?
6) You may add any further comment about what we have been discussing in case we have overlooked something that you feel is important to consider.

THANK YOU

Appendix 2

Loreto convent

P. O. Box 287

Pretoria

24 November 2010

Dear Mr. Drake (not real name)

RE: <u>REQUEST FOR YOUR SCHOOL'S PARTICIPATION IN MY STUDY</u>

I am Sr Bernadette Chabongora of Loreto Convent School. I am working on a PHD programme with The University of South Africa. I am working on a thesis entitled: Investigating the Opportunities to Learn algebra: Case Studies of two grade ten classrooms in selected Catholic secondary schools in South Africa.

I am asking you to please allow me to observe some lessons for two weeks while grade ten learners are being taught during the first term of 2011. The information asked will be used for study purposes only. Names will remain confidential in the report. You are free to participate fully or only in part and can discontinue your participation at any time.

To give you some idea of what I am doing, please find below the objectives of my study:

Objectives of the study:
   a) To gain insights into how the concept of opportunities to learn can be used to explain the teaching and learning of grade ten-algebra.
   b) To explore the relationship between learner expectations and actual school practices in the mathematics learning area, particularly in algebra and how this affects the achievement of specified outcomes.
   c) To understand the structure and practice of mathematics teaching and learning in selected classrooms in South Africa with regards to the chances given to learners to learn.

May you please indicate you approval by providing your signature in the space provide below. You are free to ask for elaboration at any stage of the process of our working together.

Thanking you in advance.

Yours sincerely

Sister Bernadette Chabongora

Signature_____
_____

Loreto convent

P. O. Box 287

Pretoria

1 December 2010

The Principal

St Anne Secondary School

Pretoria

Dear Mr. Cape

RE: <u>REQUEST FOR YOUR SCHOOL'S PARTICIPATION IN MY STUDY</u>

I am working on a PHD programme with The University of South Africa. I am working on a thesis entitled:
Investigating the Opportunities to Learn algebra: Case Studies of two grade ten classrooms in selected Catholic secondary schools in South Africa.

I am asking you to please allow me to observe some lessons while one of your teachers is teaching algebra to grade ten learners during the first term of 2011. This will be my pilot study. The information they will be asked to provide will be used for study purposes only. All names will remain confidential and nobody will be quoted by name in the report. You are free to participate fully or only in part and can discontinue your participation at any time.

To give you some idea of what I am doing, please find below the objectives of my study:

Objectives of the study:
  a) To gain insights into how the concept of opportunities to learn can be used to explain the teaching and learning of grade ten-algebra.
  b) To explore the relationship between learner expectations and actual school practices in the mathematics learning area, particularly in algebra and how this affects the achievement of specified outcomes.
  c) To understand the structure and practice of mathematics teaching and learning in selected classrooms in South Africa with regards to the chances given to learners to learn.

May you please indicate you approval by providing your signature in the space provided below. You are free to ask for elaboration at any stage of the process of our working together.

Thanking you in advance.

Yours sincerely

Sister Bernadette Chabongora
Chabongorachabong@gmail.com

Signature_____

_____

Appendix 3

<div align="right">
Loreto convent

P. O. Box 287

Pretoria

24 November 2010
</div>

Dear Mr. Med

RE: <u>REQUEST FOR YOUR PARTICIPATION IN MY STUDY</u>

I am Sr Bernadette Chabongora of Loreto Convent School. I am working on a PHD programme with The University of South Africa. I am working on a thesis entitled: Investigating the Opportunities to Learn algebra: Case Studies of two grade ten classrooms in selected Catholic secondary schools in South Africa.

I am asking you to please allow me to observe some lessons while you are teaching algebra to grade ten learners during the first term of 2011. The information that you will be asked to provide will be used for study purposes only. Your name will remain confidential and you will not be quoted by name in the report. You are free to participate fully or only in part and can discontinue your participation at any time.

To give you some idea of what I am doing, please find below the objectives of my study:

Objectives of the study:
   a) To gain insights into how the concept of opportunities to learn can be used to explain the teaching and learning of grade ten-algebra.
   b) To explore the relationship between learner expectations and actual school practices in the mathematics learning area, particularly in algebra and how this affects the achievement of specified outcomes.
   c) To understand the structure and practice of mathematics teaching and learning in selected classrooms in South Africa with regards to the chances given to learners to learn.

May you please indicate you approval by providing your signature in the space provide below. You are free to ask for elaboration at any stage of the process of our working together.

Thanking you in advance.

Yours sincerely

Sister Bernadette Chabongora

Signature_____

_____
<div align="right">Loreto convent</div>

Appendix 4

P. O. Box 287

Pretoria
1 December 2010

The Mathematic Subject Advisor

Department of Education

Pretoria

Dear Mr. Tim

RE: <u>REQUEST FOR YOUR SCHOOL'S PARTICIPATION IN MY STUDY</u>

I am working on a PHD programme with The University of South Africa. I am working on a thesis entitled:
Investigating the Opportunities to Learn algebra: Case Studies of two grade ten classrooms in selected Catholic secondary schools in South Africa.

I am asking to interview you during the first term of 2011. The information you will be asked to provide will be used for study purposes only. Your All names will remain confidential and I will not quote you by name. You are free to participate fully or only in part and can discontinue your participation at any time.

To give you some idea of what I am doing, please find below the objectives of my study:

Objectives of the study:
   a) To gain insights into how the concept of opportunities to learn can be used to explain the teaching and learning of grade ten-algebra.
   b) To explore the relationship between learner expectations and actual school practices in the mathematics learning area, particularly in algebra and how this affects the achievement of specified outcomes.
   c) To understand the structure and practice of mathematics teaching and learning in selected classrooms in South Africa with regards to the chances given to learners to learn.

May you please indicate you approval by providing your signature in the space provided below. You are free to ask for elaboration at any stage of the process of our working together.

Thanking you in advance.

Yours sincerely

Sister Bernadette Chabongora
Chabongorachabong@gmail.com

Signature_____

_____

FAX 012 324 2780.
ATTENTION : SR BERNADETTE
FROM :

SCHEDULE Grade 10 Mathematics   C Pretorius   Iona Convent

Date	Time	Topic
Monday 7/03	12.55 - 13.50 (2 periods)	Problem solving using simultaneous equations.
Tuesday 8/03	10.55 - 11.55. (2 periods)	Problem solving - using simultaneous equations + graphical sol's.
Wednesday 9/03	8.55 - 9.25 (1 periods)	Linear inequalities - introduction.
Thursday 10/03	12.55 - 1.55 (2 periods)	Solving linear inequalities and representation on a number line.
Friday 11/03	8.55 - 9.25 (1 period).	Revision - plotting graphs of y = mx + c. + using to find simultaneous sol's
Monday 14/03	12.55 - 13.50 (2 periods)	Plotting non-linear equations - quadratics ⊛ → parabolas - introduction.
Tuesday 15/03	10.55 - 11.55 (2 periods)	TERM TEST
Wednesday 16/03	8.55 - 9.25 (1 period)	TEST REVISION + FILING.

Making y / any selected variable the subject of the equation

Planning sheet

GRADE : _____10_____   PERIODS: _2._   DATE: _16/03/2011._

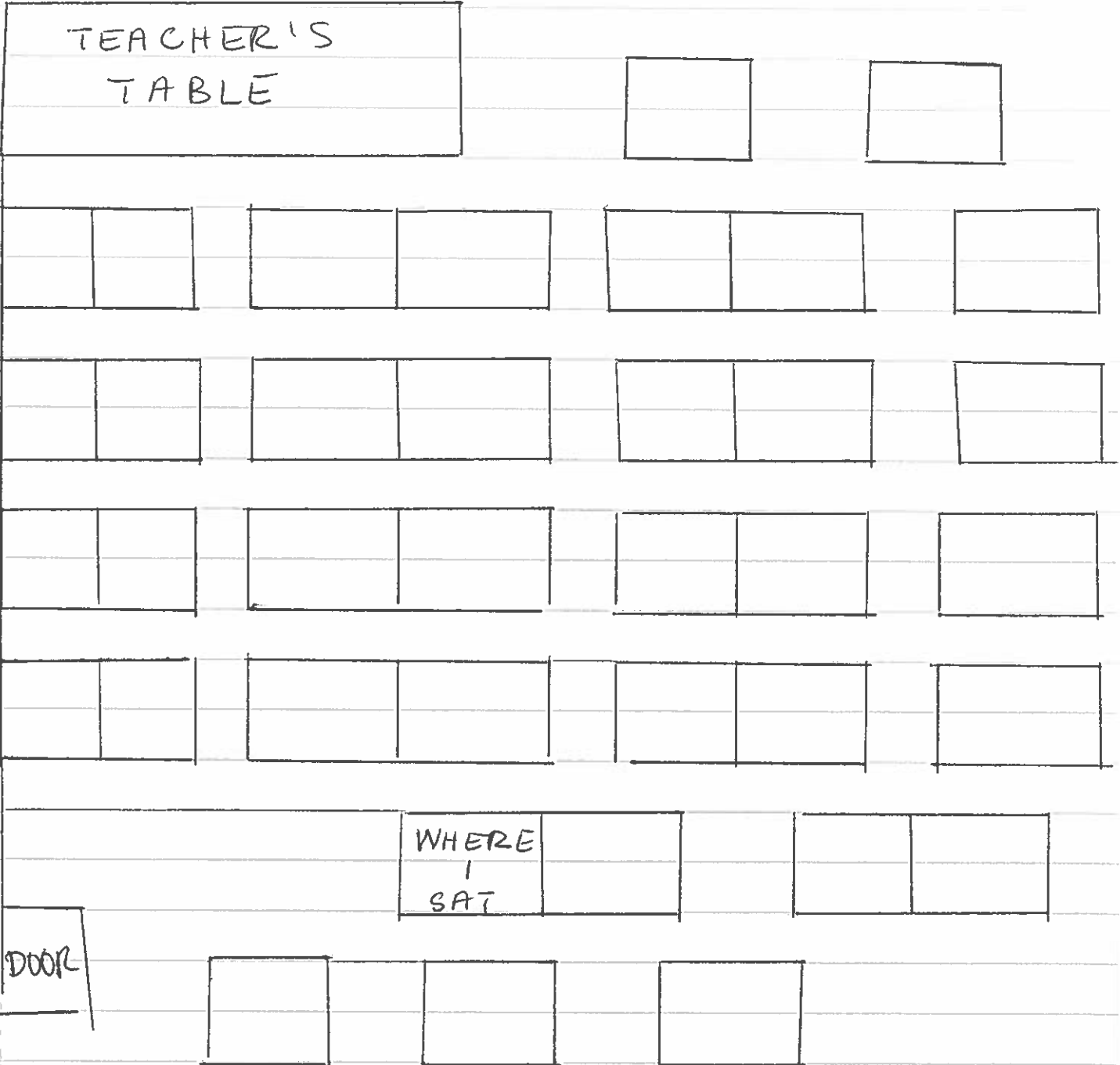| PRE-KNOWLEDGE/LINK	- Difference between expression and equation. - What is a "subject" in the Mathematical sense? - "Inverse" operations. - what does this mean? - "Solution" define - "Variable" - define. - "constant" - " - "co-efficient" - " (NB) Concept of 'Balance' (scales/see-saw as illustration).
NEW CONTENT	[Must be able to use in linear, quadratic, exponential contexts.] - Do exercise 8.7. (Classroom Mathematics) #1 (a) - (j) 10 learners do one each on board with explanation where needed/if challenged. - #2 (d) - (j) 7 learners do one each on board with explanations individually. - Question 4 - do as pairs - discuss (teacher on board). - Solving exponential equations Exercise 8.18. Selected examples. - Solving equations with $\sqrt{x}$ etc.
H/W / FOLLOW UP	ii⁄ⁱⁱ Exercise 8.7 #1s 6,7 and 8. Next → provide incorrect solutions - learners to identify errors, explain & provide correct solutions.

CLASSROOM B : ST BERNARD SCHOL
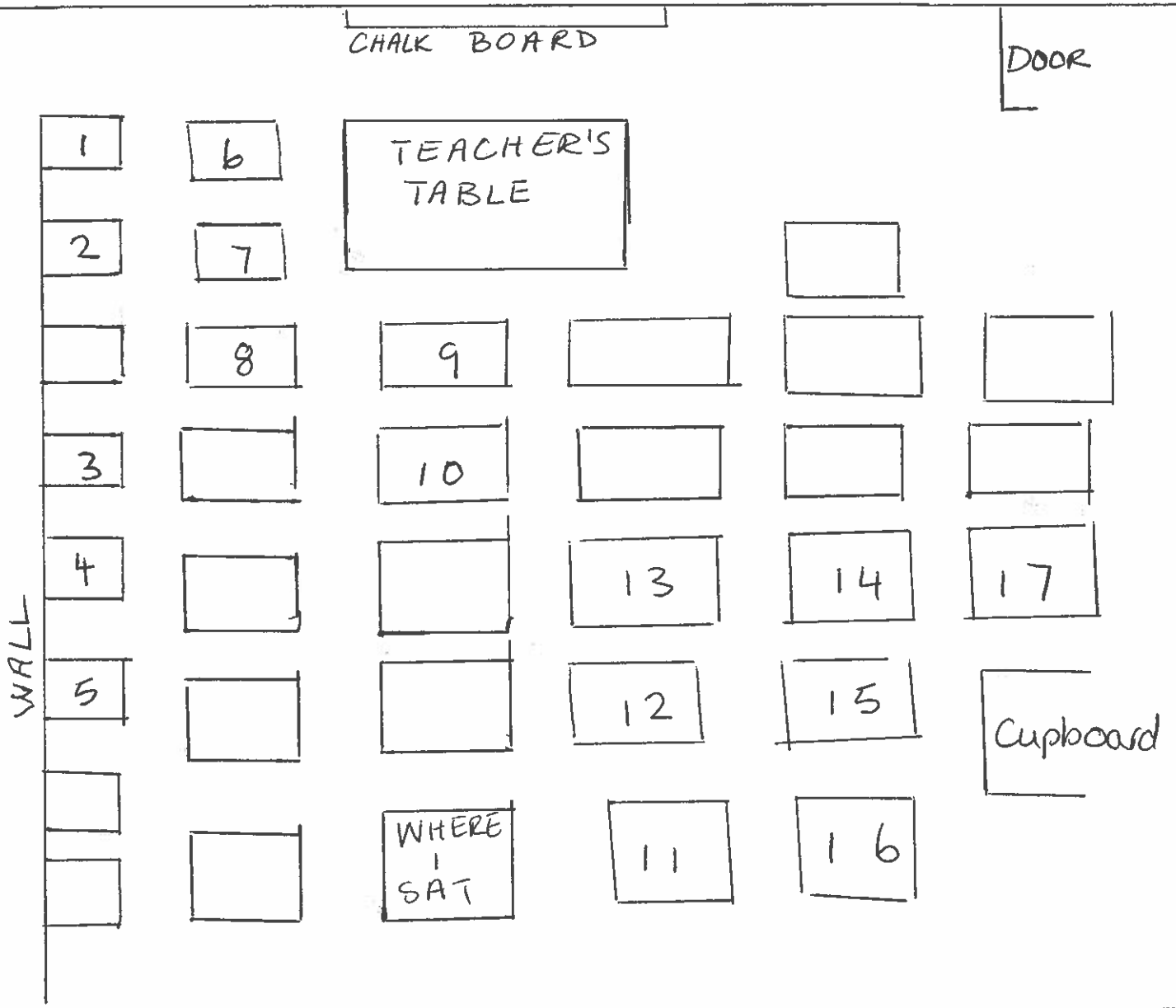
SITTING PLAN

CHALK BOARD

Cupboard

TEACHER'S
TABLE

WHERE
I
SAT

DOOR

CLASSROOM C : Mt CARMEL SCHOOL
SITTING PLAN

CHALK BOARD

DOOR

| 1 | 6 | TEACHER'S TABLE | | | |

WALL

2	7				
	8	9			
3		10		13	14	17
4			13	14	17
5			12	15	Cupboard
		WHERE I SAT	11	16	

CLASSROOM A : ST ANNE SCHOOL

SITTING PLAN

TEACHER'S
TABLE

CHAIR BOARD

DOOR

	WHERE I SAT					

DATE	TOPIC	CONTENT	LO/AS	ASSESSMENT	DATE COMPLED
TERM 1					
14/1 - 16/1	Real N's/Exponents/Surds/Error margins	• Real and non real numbers • Add, subtract, multiply and divide surds • Error margins	11.1.1/ 11.1.2		
19/1 - 23/1	Exponential/Number patterns	• Number patterns (constant second difference between consecutive terms)	11.1.3		
26/1 - 30/1	Number patterns	• Number patterns (general term is quadratic)	11.1.3		
02/2 - 06/2	Number patterns/Completion of squares	• Completion of square	11.2.4		
09/2 - 13/2	Fractions/Rational exponents/ Quadratic equations	• Fractions with binomial denominators • Rational exponents • Quadratic equations (factorisation, completing the square and quadratic formula)	11.2.2/ 11.2.5		
16/2 - 20/2	Formula/Inequalities/ Simultaneous equations	• Quadratic inequalities with one variable • Simultaneous equations in two variables, one linear and one quadratic	11.2.5		
23/2 - 27/2	Transformations	• Rotation around the origin through an angle of 90^0 or 180^0 • The enlargement of a polygon, through the origin, by a factor of k	11.3.4	Investigation	
02/3 - 06/3	Transformations/Analytical geometry	• The equation of a line through two points • The equation of a line through one point and parallel or perpendicular to a given line	11.3.3		
09/3 - 13/3	Analytical Geometry	• The inclination of a given line	11.3.3		
16/3 - 20/3	Modelling	• Mathematical models to investigate real-life problems	11.2.6		
23/3 - 27/3	Test/Exam			Test	
TERM 2					
15/4 - 17/4	Graphs-linear/parabola	• Relationships and conversions between variables: numerical, graphical, verbal and symbolical • $y = (x + p)^2 + q$	11.2.1/ 11.2.2	Assignment	
20/4 - 24/4	Graphs-hyperbola/exp/trig	• $y = \dfrac{a}{x + p} + q$ • $y = ab^{x+p} + q;\ b > 0$ • Trigonometry graphs (sin kx, cos kx, tan kx, sin$(x + p)$, cos$(x + p)$, tan$(x + p)$)	11.2.3		
28/4 - 30/4	Graphs-applications	• Practical problems	11.2.3		
04/5 - 08/5	Gradient/Ave rate of change	• Average gradient and gradient of a curve at a point	11.2.7		
11/5 - 15/5	Special angles	• Function values of the special angles 30^0, 45^0 and 60^0 (in surd form where applicable)	11.3.5		
18/5 - 22/5	Identities/Reduction formula	• Derivation and use of the identities $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$ • Derivation and use of reduction formulae for $\sin(90^0 \pm \theta)$, $\cos(90^0 \pm \theta)$, $\sin(180^0 \pm \theta)$, $\cos(180^0 \pm \theta)$, $\tan(180^0 \pm \theta)$, $\sin(360^0 \pm \theta)$, $\cos(360^0 \pm \theta)$, $\tan(360^0 \pm \theta)$, $\sin(-\theta)$, $\cos(-\theta)$, $\tan(-\theta)$	11.3.5		
25/5 - 29/5	Data: Dependant/Independent	• Measures of central tendency and dispersion in univariate numerical data by: o Five number summary (maximum, minimum and quartiles) o Box and whisker diagrams	11.4.1/ 11.4.4	Project	