AN INVESTIGATION INTO THE MECHANICS AND PRICING OF CREDIT DERIVATIVES

by

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Declaration

I declare that "An Investigation into the Mechanics and Pricing of Credit Derivatives",

is my own work and that all the sources that I have used or quoted have been

indicated and acknowledged by means of complete references.

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Direen Eraman   November 2009

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Contents

Abstract

With the exception of holders of default-free instruments, a key risk run by investors is *credit risk*. To meet the need of investors to hedge this risk, the market uses *credit derivatives*.

The South African credit derivatives market is still in its infancy and only the very simplistic instruments are traded. One of the reasons is due to the technical sophistication required in pricing these instruments. This dissertation introduces the key concepts of risk neutral probabilities, arbitrage free pricing, martingales, default probabilities, survival probabilities, hazard rates and forward spreads. These mathematical concepts are then used as a building block to develop pricing formulae which can be used to infer valuations to the most popular credit derivatives in the South African financial markets.

Key Terms:

Risk neutral probabilities; Arbitrage free pricing; Martingales; Default probabilities; Survival probabilities; Hazard rates; Forward spreads; Credit derivatives; Credit default swaps; Default digital swaps; Asset swap packages

Chapter 1

Introduction

For the past decade, we have witnessed a new, sophisticated and increasingly

popular development in the derivatives market – credit derivatives. Instead of the

normal derivatives where prices are dependent on prices of the underlying asset,

credit derivatives are instruments where their evaluations are driven by the credit risk

of commercial or government entities. Essentially, the underlying asset in credit

derivatives is the *credit risk* on an underlying bond, loan or other financial instrument.

1.1 Credit Risk

Definition 1.1.1: *Credit risk* is the risk that a borrowing entity will default on a loan, either through inability to maintain the interest servicing or because of bankruptcy on insolvency leading to inability to repay the principal itself.

When an entity defaults, the corresponding entity's bondholders suffer a loss as the value of their asset declines, and the potential greatest loss is that of the entire bond which they hold. The extent of credit risk fluctuates as the fortunes of the entity changes in line with their own economic circumstances and the macroeconomic business cycle. The magnitude of credit risk can be described by a firm's *credit rating.*

Definition 1.1.2: In personal finance, the term *credit rating* commonly refers to a score issued by a credit monitoring organisation. A person's credit rating indicates how creditworthy he or she is. In corporate finance, a credit rating is a "grade" assigned to a bond, bond issuer, insurance company, or other entity or security to indicate its riskiness.

Bond rating agencies like Moody's and Standard & Poor's (S&P) provide a service to

investors by grading fixed income securities based on current research and

prevailing market conditions. The rating indicates the likelihood that the issuer will

default either on interest or capital payments.

- For S&P, the ratings vary from AAA (the most secure) to C.

- For Moody's, the ratings vary from Aaa (the most secure) to D.

Only bonds with a rating of BBB or better are considered to be suitable for

investment by institutions. Anything below BBB, is considered to be junk, or below

"investment grade". Bond ratings are periodically revised based on most recent

available information. The following table gives a brief summary of the ratings issued

by Moody's and S&P.

Moody's	S&P	Meaning
Investment Grade Bonds		
Aaa	AAA	Bonds of the highest quality that offer the lowest degree of investment risk. Issuers are considered to be extremely stable and dependable.
Aa1, Aa2, Aa3	AA+, AA, AA-	Bonds are of high quality by all standards, but carry a slightly greater degree of long-term investment risk.
A1, A2, A3	A+, A, A-	Bonds with many positive investment qualities.
Baa1, Baa2, Baa3	BBB+, BBB, BBB-	Bonds of medium grade quality. Security currently appears sufficient, but may be unreliable over the long term.
Non Investment Grade Bonds		
Ba1, Ba2, Ba3	BB+, BB, BB-	Bonds with speculative fundamentals. The security of future payments is only moderate.
B1, B2, B3	B+, B, B-	Bonds that are not considered to be attractive investments. Little assurance of long term payments.
Caa1, Caa2, Caa3	CCC+, CCC, CCC-	Bonds of poor quality. Issuers may be in default or are at risk of being in default.
Ca	CC	Bonds of highly speculative features. Often in default.
C	C	Lowest rated class of bonds.
-	D	In default.

Source: Moody's and S&P Websites

Ratings agencies undertake a formal analysis of the various corporate entities, after which a rating is announced. The analyses on an entity by the rating agencies are fairly comprehensive and the issues considered in the analysis most frequently include the following:

- the financial position of the firm itself, for example, its balance sheet position and anticipated cash flows and revenues.

- other qualitative firm-specific issues such as the quality of the management and succession planning.

- an assessment of the firm's ability to meet scheduled interest and principal payments, both in its domestic and foreign currencies.

- the outlook for the industry and the competition within it.

- general assessments for the domestic economy.

Hence credit ratings can be used as an indication of the magnitude of an entities credit risk.

Another measure of credit risk (a market driven approach) is the credit risk premium, which is the difference between yields on the same-currency government benchmarks bonds and corporate bonds. If we consider a local investment in a local bond, government benchmark bonds can be assumed to be default-free, as the government can print more money in order to fund any shortfall in its obligations. This premium between a corporate bond and a similar government benchmark bond is the compensation required by investors for holding bonds that are not default-free. The credit premium required will fluctuate as individual firms and sectors are perceived to offer improved or worsening credit risk, and as the general health of the economy improves or worsens. For example, the graph below illustrates the variability in credit

spread premium in the South African corporate bond market. We can observe that

there is a reasonable spread differential between each of the credit ratings. This can

be associated to investors demanding higher yields for greater credit risk taken on.

Furthermore, the credit spreads in recent years indicate an increased credit risk

environment. This can be associated to the recent sub-prime credit crises

experienced offshore.

Local bond spreads per rating band
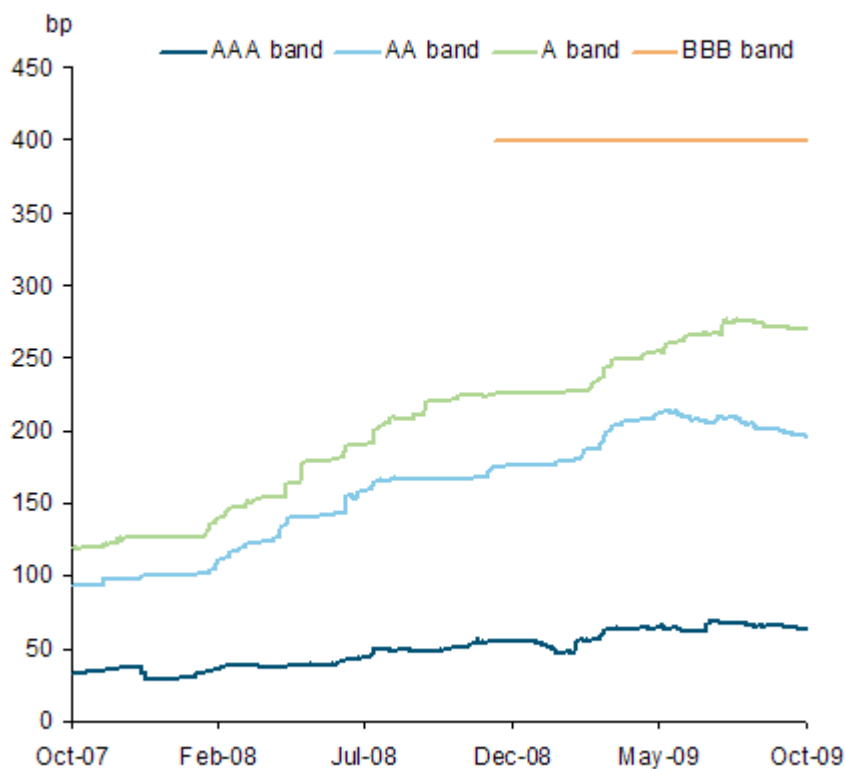
Source: RMB FICC Research, 1 October 2009

Some practical examples of credit risk are as follows:

- A holder of a corporate bond bears the risk that the market value of the bond will decline due to a deterioration in the credit quality of the issuer and hence a decline in the credit rating of the issuer.

- A bank may suffer a loss if a bank's debtor defaults on payments of the interest due and (or) the principal amount of the loan. This is typically dominant in a home loan or auto loan transaction.

Credit risk can be thought of as a subset of the vast ocean of financial risk. We now attempt to establish its position in the world of financial risk and provide a graphical breakdown of the constituents that make up credit risk as follows:
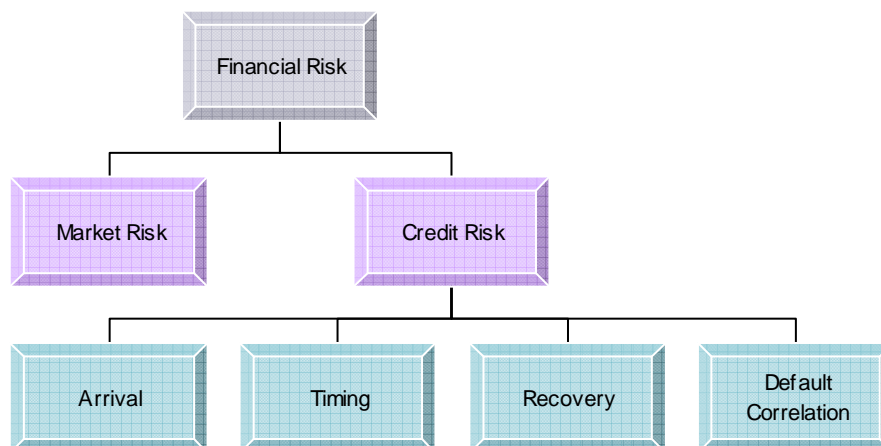
Financial risk is a measure of adverse changes in a financial position as a result of a changing economic environment. Broadly, financial risk may be divided into *market risk* and *credit risk*.

Definition 1.1.3: *Market risk* is the risk of changes in the market price of a defaultable asset, even if no default occurs.

Credit risk may be further categorised into the following four components:

- **Arrival risk:** this is a term for the uncertainty whether a default will occur or not. A common measure of arrival risk is the probability of default.

- **Timing risk:** refers to the uncertainty about the exact time of default. Timing risk is a more detailed and specific measure than arrival risk, as knowledge about the time of default includes knowledge about the arrival risk for all possible time horizons.

- **Recovery risk:** describes the uncertainty about the severity of the losses if a default has occurred. Market convention is to express the recovery rate of a bond or loan as the fraction of the notional value of the claim that is actually paid to the creditor.

- **Default correlation risk:** describes the risk that several obligors default together. This would imply joint arrival risk which is described by the joint default probabilities over a given time horizon, and joint timing risk which is described by the joint probability distribution function of the times of default.

1.2 Credit Derivatives

Before we formally introduce a credit derivative, a few key definitions are required:

Definition 1.2.1: A *credit event* is a general default event related to a legal entity's previously agreed financial obligation. In this case, a legal entity fails to meet its obligation on any significant financial transaction (e.g. coupon on a bond it issued).

Definition 1.2.2: *Reference entity* or *reference credit* refers to one (or several) issuer(s) whose defaults trigger the *credit event*. This can be one or several defaultable issuers.

Definition 1.2.3: *Reference obligations* or *reference credit assets* are a set of assets issued by the *reference credit*. They are required for the determination of the credit event and for the calculation of the recovery rate.

The events triggering a credit derivative are defined in a bilateral swap confirmation which is a transaction document that typically refers to an ISDA (International Swaps and Derivatives Association) master agreement previously executed between the two

counterparties. There are several standard credit events which are typically referred

to in credit derivative transactions:

- Bankruptcy

- Failure to Pay

- Restructuring

- Repudiation

- Moratorium

Definition 1.2.4: *Default payments* are the payments which have to be made if a

credit event has occurred.

Credit Derivatives were introduced to the market in the beginning of the 1990's.

Despite their short history their use and applications has grown rapidly. Credit

derivatives are now used not only by banks, but also by various funds, insurance

companies and even corporations, in order to separate and trade credit risk. The

term credit derivatives can be used for a broad range of securities. Schonbucher

(2003) gives one definition of credit derivatives which covers the different securities discussed here:

Definition 1.2.5: A credit derivative is a derivative security that has a payoff which is conditioned on the occurrence of a credit event. The credit event is defined with respect to a reference credit (or several reference credits), and the reference credit asset(s) issued by the reference credit. If the credit event has occurred, the default payment has to be made by one of the counterparties. Besides the default payment, a credit derivative can have further payoffs that are not default contingent. (Schonbucher 2003, p8)

Credit derivatives in their simplest form are bilateral contracts between a buyer and seller under which the seller sells protection against pre-agreed events occurring in relation to a third party (usually a corporate or sovereign) known as a reference entity. The reference entity will not (except in certain very limited circumstances) be a part to the credit derivatives contract, and will usually be unaware of the contract's existence.

One of the most popular credit derivatives in the South African financial markets is

the credit default swap (CDS). Hence, the last section of the introduction is

dedicated to understanding the cash flow dynamics of a CDS.

1.3 Credit Default Swaps (CDS)

Credit default swaps transfer the potential loss on a reference asset that can result

from specific credit events such as default, bankruptcy, insolvency and credit rating

downgrades. Marketable bonds are the most popular form of reference asset

because of their price transparency. While bank loans have the potential to become

the dominant form of reference asset, this is impeded by the fact that loans are more

heterogeneous and illiquid than bonds. Credit default swaps involve a protection

buyer, who pays a periodic or upfront fee to a protection seller in exchange for a

contingent payment if there is a credit event. The fee is usually quoted as a basis

point multiplier of the nominal value.

Some default swaps are based on a basket of assets and pay out on a first to default

basis whereby the contract terminates and pays out if any of the assets in the basket

are in default. Default swaps are the largest component of the global credit

derivatives market.

However structured, the credit default swap enables one party to transfer its credit

exposure to another party. Banks may use default swaps to trade sovereign and

corporate credit spreads without trading the actual assets themselves; for example

someone who has gone long a default swap (the protection buyer) will gain if the

reference assets' obligor suffers a rating downgrade or defaults, and can sell the

default swap at a profit if he can find a suitable counterparty. This is because the

cost of protection on the reference asset will have increased as a result of the credit

event. The original buyer of the default swap need never have owned a bond issued

by the reference assets' obligor. The maturity of the credit default swap does not

have to match the maturity of the reference asset and in most cases does not. On

default, the swap is terminated and default payment by the protection seller or

guarantor is calculated and handed over. The guarantor may have the asset

delivered to him and pay a pre-specified proportion of the nominal value, or may cash

settle the swap contract.

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┌──────────────┐   Periodic or upfront premium    ┌──────────────┐
│              │  ──────────────────────────────▶ │              │
│  Protection  │                                   │  Protection  │
│   Buyer      │  ◀ - - - - - - - - - - - - - - -  │   Seller     │
│              │   Payment contingent on credit event            │
└──────────────┘                                   └──────────────┘
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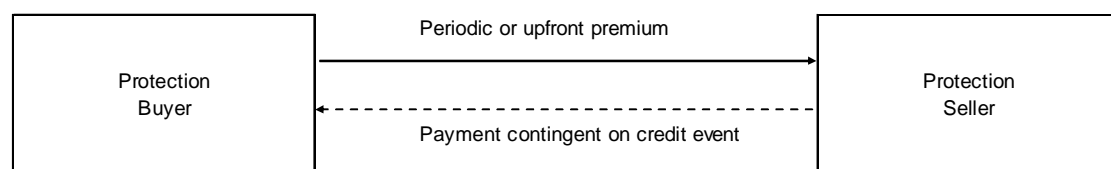
A Cash Flow Example:

As an example we use a hypothetical transaction between an investment company and XYZ bank. In this case the investment company sells protection (takes on credit risk) on R10 million General Motors to XYZ bank. Let's assume that credit protection is written on 100% of the nominal value.

The term of the transaction is 5 years. In return for the credit protection on General Motors, which the investment company is providing over this five-year period, XYZ bank agrees to pay a fixed fee of 1.6% (160 basis points) per annum payable quarterly.

Let's assume that settlement is physical. Thus, should a credit event occur, XYZ bank would be able to deliver any qualifying senior unsecured General Motors paper to the investment company in return for a R10 million payment and then the contract (and all future payments) would terminate. The charts below illustrate two potential scenarios for this transaction:
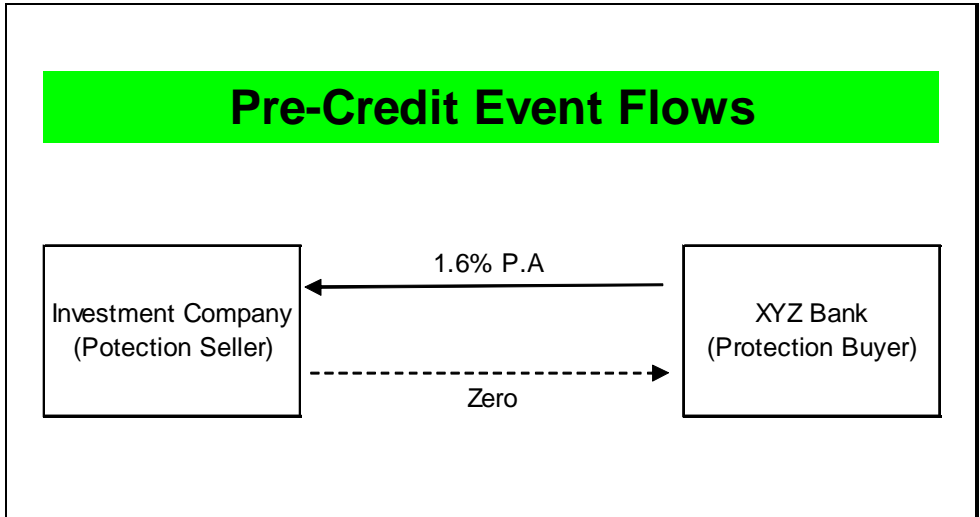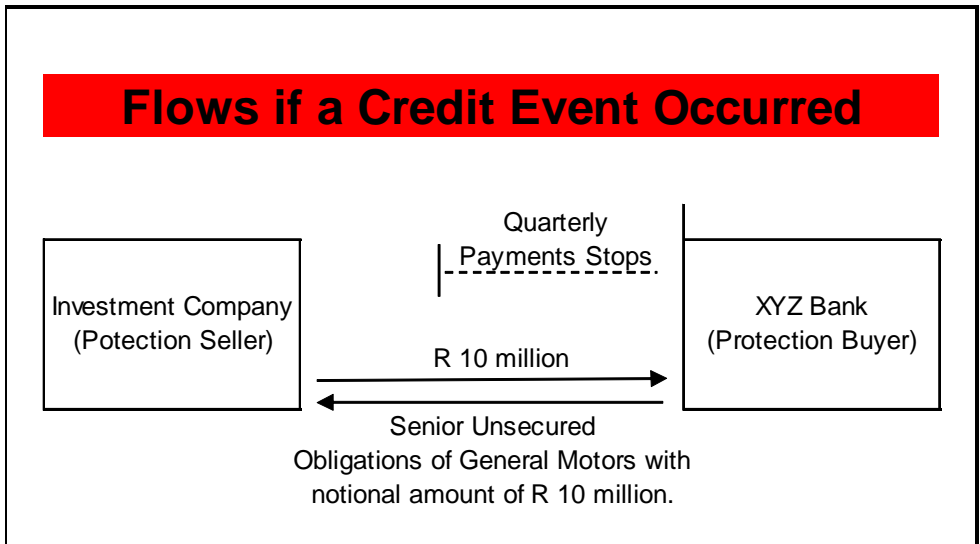
Chart A

Chart B

In chart A, no General Motors credit event occurs and XYZ bank simply continues to

pay the 160 basis points annual premium to the investment company. For XYZ bank

there is, therefore, a negative accrual relating to these payments.

Chart B however depicts a scenario in which a credit event occurs two years into the

transaction. In this case XYZ bank pays the investment company the premium of

160 basis points for the two years preceding the credit event and then receives R 10

million from the investment company in return for delivering any qualified senior

unsecured debt obligation with a notional amount of R 10 million. Following such

credit event, it is likely that General Motors' debt would be trading substantially below

par – and the investment company would be expected to bear the loss resulting from

the diminution of value. The table below summarises the cash flows:

Example Cashflows under "no default" and Credit Event At 2 Years Scenarios

	No Credit Event		Credit Event in 2 Years	
	XYZ Bank	Investment Company	XYZ Bank	Investment Company
Pays	160 bps for 5 years	zero	160 bps for 2 years, then General Motors Deliverable Obligation	R 10 million
Receives	zero	160 bps for 5 Years	R 10 million	160 bps for 2 years, then Recovery Value

Now that we have introduced the underlying terminology, we proceed to the focal

point of this dissertation, which is a logical argument to the mathematical concepts

required to develop credit derivative pricing models.

Chapter 2

Bonds, Credit Spreads and Implied Default

Probabilities

A Bond can be thought of as a securitised version of a loan. The major difference is

that bonds are tradeable in small denominations. This tradeability feature of bonds

opens access to a much larger number of lenders due to the following advantages

over conventional loans:

- each lender can lend small amounts.

- most bonds are more standardised than loans.

- lenders do not need to remain invested for the whole borrowing period as they can trade their bonds in the secondary market.

As a result the issuer can sell the bonds at competitive prices, which usually results in better conditions than bilateral negotiations with just a few potential loan creditors.

The price of defaultable bonds contains extremely important information about the markets perceived assessment of the issuer's default risk.

The three most significant and popular types of bonds are defined as follows:

Definition 2.1: *Fixed-coupon bonds* are bonds where the coupon amounts are fixed in advance, and the notional is repaid in full at maturity of the bond.

Definition 2.2: *Par floaters* are bonds where the coupon amounts are linked to a benchmark short-term interest rate plus a constant spread. Usually, the spread is chosen such that the price of the par floater is initially at par.

Definition 2.3: *Zero-coupon bonds* are bonds with no coupon payments. In order to compensate investors, they are usually issued at a discount to par.

We now begin to introduce some of the key mathematical concepts. We begin by setting the probabilistic foundations by defining the following:

Definition 2.4: A *sample space* or *universal sample space*, often denoted, Ω, of an experiment or random trial is the set of all possible outcomes.

Definition 2.5: A non-empty collection of subsets F of Ω is called a $\sigma-$algebra if:

1. $A \in F \Rightarrow \overline{A} \in F,$

2. for any countable sequence $\{A_i\}$ from F, $\bigcup A_i \in F,$

3. $\Theta \in F,$ where Θ denotes the impossible event.

Definition 2.6: Suppose F is a $\sigma-$algebra. If for any sequence $A_1, A_2,.., A_n,....$ of disjoint sets in F, one has:

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n),$$

we say that μ is countably additive or $\sigma-$additive.

Definition 2.7: A *measurable space* (Ω, F) consists of a sample space Ω consisting

of possible outcomes ω and a collection F of subsets of Ω called a $\sigma - $ algebra. A

countably additive mapping $\mu : F \to \Re^+$ is called a measure on (Ω, F).

Definition 2.8: A *probability measure* Q is a normed measure over a measurable

space (Ω, F); that is, Q is a real-valued function which assigns to every $A \in F$ a

number such that:

1. $Q(A) \geq 0$ for every $A \in F$;

2. $Q(\Omega) = 1$;

3. for any denumerable sequence $\{A_n\}$ of disjoint events from F,

$$Q\left(\bigcup_1^\infty A_n\right) = \sum_1^\infty Q(A_n)$$

The triple (Ω, F, Q) is called a *probability space*.

Definition 2.9: A collection $(F_t)_{t \in T}$ of sub-σ-algebras of F is called a *filtration* if

$F_s \subseteq F_t$ for all $s \leq t$. A stochastic process M defined on (Ω, F, Q) and indexed by

T is called *adapted* to the filtration if for every $t \in T$, the random variable M_t is F_t-

measurable.

The filtration can be thought of as a flow of information. The σ-algebra F_t contains

the events that can happen up to time t. An *adapted process* is a process that does

not look into the future. If M is a stochastic process, the filtration $(F_t^M)_{t \in T}$ is defined

by:

$$F_t^M = \sigma(M_s : s \le t).$$

We call this filtration the filtration generated by M, or the natural filtration of M.

Intuitively, the natural filtration of a process keeps track of the history of the process.

A stochastic process is always adapted to its natural filtration.

Definition 2.10: In general, if M is a random variable defined on a probability space

(Ω, F, Q), then the *expected value* of M, denoted by $E(M)$ or $E_Q(M)$, is defined

as follows:

$$E(M) = \int_\Omega M dQ$$

Definition 2.11: (*Martingale*): A stochastic process M_t adapted to F_t and

satisfying $E(|M_t|) < \infty$, $\forall t \in [0,T]$ is called a Q-*submartingale* if:

$$E_Q(M_T \mid F_t) \geq M_t \qquad \forall t \in [0,T],$$

and a Q-*supermartingale* if:

$$E_Q(M_T \mid F_t) \leq M_t \qquad \forall t \in [0,T],$$

M_t is a *martingale* if it is both a *submartingale* and a *supermartingale,* i.e., if

$$E_Q(M_T \mid F_t) = M_t.$$

A martingale can be thought of as a stochastic process such that the conditional

expected value (a definition can be found in Probability and Measure, page 445) of

an observation at some time T, given all the observations up to some earlier time t,

is equal to the observation at that earlier time t.

Definition 2.12: (*Martingale measure*): Let Q be a probability measure. If for every

dividend free traded asset with price process $p(T)$, the discounted price process

$p(T)/b(T)$ is a martingale under Q, then Q is called a *martingale measure* (or *spot-*

martingale measure). Here $b(T) = \exp\left(\int_t^T r(s)ds\right)$ is the value of the default-free

continuously compounded bank account at time T, and the default free-continuously

compounded discount factor from t to T is defined as $\beta(T) = 1/b(T)$.

Arbitrage free pricing is a fundamental assumption in deducing pricing equations for

derivatives. Arbitrage refers to the practice of taking advantage of a price differential

between two or more financial instruments; striking a combination of matching deals

that capitalizes upon the imbalance. If the market prices do not allow for profitable

arbitrage, the prices are said to constitute an arbitrage equilibrium or arbitrage-free

market. An arbitrage equilibrium is a precondition for a general economic

equilibrium.

The central importance of the martingale measure in mathematical finance is based

upon the fact that the existence of an equivalent martingale measure is equivalent to

the absence of arbitrage in the underlying market.

 (see Harrison and Pliska, 1981, for technical conditions under which this is true).

Definition 2.13: Let $\{F_t\}_{t \geq 0}$ be a filtration with respect to a non-null Ω space, which

belongs to the probability space (Ω, F, Q). Then the quadruple $(\Omega, \{F_t\}_{t \geq 0}, F, Q)$ is

said to be the *filtered probability space* with respect to filtration $\{F_t\}_{t \geq 0}$.

The analysis proceeding takes places in a filtered probability space $(\Omega, (F_t)_{t \geq 0}, F, Q)$

under the risk-neutral probability measure (spot martingale measure) Q, and all

probabilities and expectations are taken under the measure Q. In order to develop

the framework, we define as follows:

Definition 2.14:

Assume that A is any subset of F i.e. $A \in F$. The *indicator function* of A is denoted

by $1_{\{A\}}$. Hence,

$$1_{\{A\}}(\omega) = 1 \ \text{if} \ \omega \in A$$

$$\text{and}$$

$$1_{\{A\}}(\omega) = 0 \ \text{otherwise}$$

Definition 2.15:

We denote the time of default as τ, and the *survival indicator function* as $I(t)$, where

$I(t)$ is defined as $I(t) = 1_{\{\tau > t\}}$. Hence:

$$I(t) = 1 \ \text{if} \ \tau > t \ \text{(if default occurs after time} \ t \ \text{)}$$

$$\text{and}$$

$$I(t) = 0 \ \text{if} \ \tau \leq t \ \text{(if default occurs before or at time} \ t \ \text{)}$$

Definition 2.16:

We denote default free zero coupon bond (ZCB) prices for all maturities $T > t$ as follows:

$$B(t,T) = \text{Price of a ZCB at time } t \text{, which is paying a notional of 1 at } T$$

Definition 2.17:

We denote defaultable zero coupon bond prices for all maturities $T > t$, if $\tau > t$ as follows:

$$\overline{B}(t,T) = \text{Price of a ZCB at time } t \text{, if the time of default is beyond } t$$

In order to ensure the absence of arbitrage the following two conditions are required:

1. *Defaultable bonds are always worth less than default free bonds of the same maturity.*

   Hence,

   $$0 \leq \overline{B}(t,T) \leq B(t,T) \quad \forall t < T$$

Most defaultable bonds are not secured by collateral. Hence, investors of such bonds must assume not only interest rate risk but also credit risk of the issuer.

Depending on the credit quality and probability of default of the issuer, investors will

usually demand a discount in value to an identically traded default free bond. (usually

found in the form of a government benchmark bond). Hence, we don't restrict

ourselves by the above condition, as this is also an investor behavioural

phenomenon.

2. *Zero coupon bond prices are a decreasing, non-negative function of maturity,*

with $\bar{B}(t,t) = 1 = B(t,t)$

Hence,

$$B(t,T_1) \geq B(t,T_2) > 0 \text{ and } \bar{B}(t,T_1) \geq \bar{B}(t,T_2) > 0 \quad \forall t < T_1 < T_2, \ \tau > t$$

Similarly, this is also an investor behavioural phenomenon as investors demand

lower yields for bonds with shorter maturities, assuming a constant term structure of

interest rates. Hence, this would imply that bonds with shorter maturities will be

priced higher, since bond yields and bond prices and inversely related.

In order to further develop the framework, the following assumptions are made:

Assumption 2.1:

- At time t, the defaultable and default-free zero-coupon bond prices of all

  maturities $T > t$ are known. This assumption is not unreasonable as most

  bond exchanges throughout the world, would be able to provide this

  information as at close of any business day.

- The bond prices are arbitrage-free. This is also not an unreasonable

  assumption, as if arbitrage opportunities did occur, traders would take

  advantage of this momentarily and this would consequently result in

  arbitrage-free prices.

- The defaultable zero-coupon bonds have no recovery at default. The price at

  time t of a defaultable bond with maturity T can be represented as follows:

$$I(t)\overline{B}(t,T) = \overline{B}(t,T) \text{ if } \tau > t$$

and

$$I(t)\overline{B}(t,T) = 0 \text{ if } \tau <= t \text{ (since default has occurred and the recovery value is zero)}$$

Assumption 2.2:

The default-free interest-rate dynamics are independent of the default time, under the

probability measure Q. Hence,

$$B(t,T)\,|\,T \geq t \text{ and } \tau \text{ are independent under } (\Omega, F, Q).$$

We now state the first fundamental theorem of Asset Pricing, which will be critical to

formulate our results. A proof can be found in the seminal paper due to Harrion and

Pliska (1981, Stochastic Processes and Their Applications, 11, 215-260)

Theorem 2.1: (First Fundamental Theorem of Asset Pricing)

A financial market with time horizon T and price processes of the risky asset and

riskless bond given by $S_1, ...S_T$ and $B_1, ...B_T$, respectively, is arbitrage-free under

the probability P if and only if there exists another probability measure Q such that:

1. For any event A, $P(A) = 0$ if and only if $Q(A) = 0$. Hence, P and Q are

   equivalent probability measures.

2. The discounted price process, $X_0 = S_0 / B_0, ...X_T = S_T / B_T$ is a

   martingale under Q.

2.1 Pricing the default-free zero-coupon Bond

Using theorem [2.1], we can deduce that under the spot martingale measure, the

price of every contingent claim is given by the expected value of its discounted

expected payoff. Hence, we can price the default-free zero coupon bond as follows:

$$B(t,T) = E[e^{-\int_t^T r(s)ds}.1]$$ **Equation [1]**

$$= E[\beta(T)], \text{ where } \beta(T) \text{ is as defined in definition [2.12]}$$

where $r(s)$ is the default-free continuously compounded short rate.

2.2 Pricing the defaultable zero-coupon Bond

For a defaultable zero-coupon bond, the payoff is 1 only if the obligor did not default

by time T. If default did occur by time T, clearly, the value of the bond is zero.

Hence, $I(t) = 1_{\{\tau > T\}}$

$$I(T) = 1 \text{ if } \tau > T \text{ (if default occurs after time } T)$$

and

$$I(T) = 0 \text{ if } \tau \leq T \text{ (if default occurs before or at time } T)$$

Therefore, the price of the defaultable zero-coupon bond at time $t < \tau$ is:

$$\overline{B}(t,T) = E[e^{-\int_t^T r(s)ds} I(T)] \qquad \text{Equation [2]}$$

$$= E[\beta(T)I(T)], \text{ where } \beta(T) \text{ is as defined in definition [2.12]}$$

Using assumption [2.2] (independence), we can factor the expectation operator as a

product of expectations in equation [2]. Hence:

$$\overline{B}(t,T) = E[e^{-\int_t^T r(s)ds} I(T)]$$

$$= E[e^{-\int_t^T r(s)ds}]E[I(T)]$$

$$= E[e^{-\int_t^T r(s)ds}.1]E[I(T)] \qquad \text{Equation [3]}$$

Substituting equation [1] in equation [3]:

$$= B(t,T)E[I(T)]$$

$$= B(t,T)P(t,T) \qquad \text{Equation [4]}$$

where $P(t,T)$ in equation [4] is the implied probability of survival in the interval $[t,T]$.

The implied probability of survival will be discussed further in chapter 3.

Chapter 3

An extension to the Framework: Survival

Probabilities

This chapter is an extension to the framework developed in chapter 2. In formulating

equation [4], we briefly mentioned the *probability of survival*. We now proceed to

give a formal definition of the *probability of survival* as follows:

Definition 3.1: (Implied survival probability) Let the time of default, τ, be greater

than t. i.e. $\tau > t$

- The implied survival probability from t to T, where $T \geq t$, at time t is given by

  the ratio of the defaultable and default-free zero coupon bond prices, as

  follows:

$$P(t,T) = \frac{\overline{B}(t,T)}{B(t,T)}$$
Equation [5]

  where $B(t,T)$ and $\overline{B}(t,T)$ is as defined in equation [1] and equation [2]

  respectively.

- The *implied default probability* over the interval $[t,T]$ is denoted by,

  $P_{def}(t,T)$ and is defined as follows:

$$P_{def}(t,T) = 1 - P(t,T)$$
Equation [6]

If the prices of zero-coupon bonds for all maturities are available (as discussed in

assumption 2.1), then using definition 3.1, we can obtain the implied survival

probabilities for all maturities.

We now briefly mention some of the important properties of implied survival

probabilities $P(t,T)$:

- $P(t,t) = 1$ and $P(t,T)$ is non-negative and decreasing in T.

- $P(t,\infty) = 0$

- Normally $P(t,T)$ is continuous in its second argument, except if an important

  event scheduled at some time $t < T_1 < T$ has a direct influence on the survival

  of the obligor. An example of such an event would be a coupon payment

  date.

- Viewed as a function of its first argument, all survival probabilities for fixed

  maturity dates will tend to increase.

In order to analyse the default risk over a given time interval in the future, we need to

introduce the concept of a *conditional survival probability*. First, let's consider

conditional probabilities as follows:

Definition 3.2: (Conditional probability) Given a probability space (Ω, F, P) and two

events $A, B \in F$ with $P(B) > 0$, the conditional probability of A given B is defined by:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$
<div align="right">**Equation [7]**</div>

If $P(B) = 0$, then $P(A \mid B)$ is undefined (see Borel-Kolmogorov paradox).

We now define the following events:

- A : Survival until T_2

- B : Survival until T_1

Hence, $A \cap B$ will denote survival until T_1 and survival until T_2. If we assume

that $T_1 < T_2$, $A \cap B = A$, since event B is contained in A. In other words, if the

obligor survives until T_2, he must also have survived until T_1. Hence, applying

equation [7], the conditional probability of A given B is as follows:

$$P(A \mid B) = \frac{P(t, T_2)}{P(t, T_1)} \text{ , for } T_1 < T_2 < T$$

Similarly, we can now define the following:

Definition 3.3: (conditional survival probability) The conditional survival probability

over the interval $[T_1,T_2]$ as seen from t, given survival over $[t,T_1]$, is as follows:

$$P(t,T_1,T_2) = \frac{P(t,T_2)}{P(t,T_1)} \text{, where } t \le T_1 \le T_2 \qquad \textbf{Equation [8]}$$

As a consequence of the above definition, we can now define the *conditional default*

probability as:

$$P_{def}(t,T_1,T_2) = 1 - P(t,T_1,T_2) \qquad \textbf{Equation [9]}$$

Let's consider the following example, assuming zero recovery:

Define:

- $y(T)$: is the yield on a T-year *corporate* zero-coupon bond.

- $y_1(T)$: is the yield on a T-year *risk-free* zero-coupon bond.

- τ : is the random time of default

Using equation [2], the present value of a T-year *corporate* zero-coupon bond with a

principal of 100 can be quantified as follows:

$$\overline{B}(0,T) = 100e^{-y(T)T} \qquad \qquad \textbf{Equation [10]}$$

Similarly, using equation [1], the present value of a T-year *risk-free* zero-coupon

bond with a principal of 100 can be quantified as follows:

$$B(0,T) = 100e^{-y_1(T)T} \qquad \qquad \textbf{Equation [11]}$$

Next, the probability that the corporation will default between time zero and time T,

can be computed using equation [6] as follows:

$$P_{def}(0,T) = 1 - P(0,T)$$

$$= 1 - \frac{\overline{B}(0,T)}{B(0,T)} \text{, using the relationship in equation [5]}$$

$$= 1 - \frac{100e^{-y(T)T}}{100e^{-y_1(T)T}} \text{, substituting from equation [10] and [11]}$$

$$= 1 - e^{-[y(T) - y_1(T)]T}$$

<div align="right">**Equation [12]**</div>

Next, the conditional probability of default between T_1 and T_2 as seen from time 0,

can be computed as follows:

$$P_{def}(0, T_1, T_2) = 1 - P(0, T_1, T_2) \text{, using equation [9]}$$

$$= 1 - \frac{P(0, T_2)}{P(0, T_1)} \text{, using definition [3.3]}$$

$$= 1 - \frac{\dfrac{\overline{B}(0, T_2)}{B(0, T_2)}}{\dfrac{\overline{B}(0, T_1)}{B(0, T_1)}} \text{, using equation [5]}$$

$$= 1 - \frac{\dfrac{100e^{-y(T_2)T_2}}{100e^{-y_1(T_2)T_2}}}{\dfrac{100e^{-y(T_1)T_1}}{100e^{-y_1(T_1)T_1}}} \text{, using equation [10] and [11]}$$

$$= 1 - e^{-y(T_2)T_2} e^{y_1(T_2)T_2} e^{-y_1(T_1)T_1} e^{y(T_1)T_1}$$

$$= 1 - e^{-T_2(y(T_2) - y_1(T_2))} e^{T_1(y(T_1) - y_1(T_1))}$$

Consider the following numerical example: Suppose that the spreads over the risk-

free rate for 5-year and a 10-year BBB-rated zero-coupon bond are 130 and 170

basis points, respectively, and there is no recovery in the event of default. Using

equation [12], we can compute probabilities of default as follows:

$$P_{def}(0,5) = 1 - e^{-0.013*5} = 0.0629$$

$$P_{def}(0,10) = 1 - e^{-0.017*10} = 0.1563$$

In the example above, information on only the spreads was sufficient to infer default

probabilities. The actual yields on the underlying bonds were not required.

Chapter 4

An extension to the Framework: Hazard Rates

A popular concept in descriptive statistics and logistic regression is that of an *odds ratio.* A very basic definition of an odds ratio is as follows:

<u>**Definition 4.1:**</u> (odds ratio) The odds ratio of an event is the (expected) number of events divided by the (expected) number of non-events.

For example, an odds ratio of $1:3$ that a company will default at T_2 as seen from time t, mathematically means:

$$P_{def}(t,T_1,T_2):P(t,T_1,T_2)=1:3, \text{ where } t<T_1<T_2$$

Borrowing this line of thought, we can define the discrete hazard rate of default:

<u>Definition 4.2:</u> (discrete hazard rate of default) The discrete implied hazard rate of default over the interval $[T,T+\Delta T]$ as seen from time t is:

$$H(t,T,T+\Delta T)\Delta T = \frac{P_{def}(t,T,T+\Delta T)}{P(t,T,T+\Delta T)} \qquad \text{Equation [13]}$$

$$= \frac{1-\dfrac{P(t,T+\Delta T)}{P(t,T)}}{\dfrac{P(t,T+\Delta T)}{P(t,T)}}, \text{ using equation [8] and [9]}$$

$$= \frac{P(t,T)}{P(t,T+\Delta T)}-1 \qquad \text{Equation [14]}$$

Rearranging equation [14], we get the following relationship:

$$P(t,T) = P(t,T+\Delta T)[1 + H(t,T,T+\Delta T)\Delta T]$$ **Equation [15]**

Theorem 4.1: In the limit of $\Delta T \to 0$, the continuous hazard rate at time T as seen

from time t is given by:

$$h(t,T) = -\frac{\partial}{\partial T}\ln(P(t,T))$$ **Equation [16]**

provided that the default time $\tau > t$, and that the term structure of survival

probabilities is differentiable with respect to T.

Proof:

$$h(t,T) = \frac{\lim}{\Delta T \to 0} H(t,T,T+\Delta T)$$

$$= \frac{\lim}{\Delta T \to 0}\left\langle \frac{1 - P(t,T,T+\Delta T)}{\Delta T P(t,T,T+\Delta T)} \right\rangle \text{, from equation [13] and [9]}$$

- 48 -

$$= \frac{\lim}{\Delta T \to 0} \left\langle \frac{1}{\Delta T} \left[\frac{P(t,T)}{P(t,T+\Delta T)} - 1 \right] \right\rangle \text{ , from equation [14]}$$

$$= \frac{\lim}{\Delta T \to 0} \left\langle - \left[\frac{1}{P(t,T+\Delta T)} \right] \left[\frac{P(t,T+\Delta T) - P(t,T)}{\Delta T} \right] \right\rangle$$

$$= - \frac{1}{P(t,T)} \left[\frac{\partial}{\partial T} P(t,T) \right]$$

$$= - \frac{\partial}{\partial T} \ln P(t,T)$$

Chapter 5

An extension to the Framework: the relation to

forward spreads

This chapter explores the relationship that the survival probabilities and implied

hazard rates have to forward credit spreads. Let us give a formal definition of

forward rates.

Definition 5.1: (default-free simply compounded forward rates) Let $t \leq T_1 \leq T_2$. The

simply compounded forward rate over the period $[T_1, T_2]$ as seen from t is:

$$F(t, T_1, T_2) = \frac{\dfrac{B(t, T_1)}{B(t, T_2)} - 1}{T_2 - T_1}$$

Equation [17]

This is the price of the forward contract with expiration date T_1 on a unit par zero

coupon bond maturing at T_2. To prove, we re-arrange equation [17], and consider

the compounding of interest rates over successive time intervals as follows:

$$\frac{1}{B(t, T_2)} = \frac{1}{B(t, T_1)} \left(1 + F(t, T_1, T_2)(T_2 - T_1)\right)$$

Equation [18]

where:

$\dfrac{1}{B(t, T_2)}$, is the accumulation factor over (t, T_2)

$\dfrac{1}{B(t, T_1)}$, is the accumulation factor over (t, T_1) , and

$\left(1 + F(t, T_1, T_2)(T_2 - T_1)\right)$, is the simply compounded accumulation factor over (T_1, T_2).

Definition 5.2: (defaultable simply compounded forward rates) Assuming the notation

from definition [5.1], the defaultable simply compounded forward rate over (T_1, T_2) is:

$$\overline{F}(t,T_1,T_2) = \frac{\dfrac{\overline{B}(t,T_1)}{\overline{B}(t,T_2)} - 1}{T_2 - T_1} \qquad \textbf{Equation [19]}$$

Definition 5.3: (instantaneous continuously compounded forward rates) The default-

free and defaultable instantaneous continuously compounded forward rate for T as

seen from t are as follows:

$$f(t,T) = \lim_{\Delta t \to 0} F(t,T,T+\Delta t) = -\frac{\partial}{\partial T} \ln B(t,T) \qquad \textbf{Equation [20]}$$

$$\overline{f}(t,T) = \lim_{\Delta t \to 0} \overline{F}(t,T,T+\Delta t) = -\frac{\partial}{\partial T} \ln \overline{B}(t,T) \qquad \textbf{Equation [21]}$$

Proposition 5.1: The conditional probability of default per time interval (T_1, T_2) is the

spread of defaultable over default-free forward rates, discounted by the defaultable

forward rate:

$$P_{def}(t,T_1,T_2) = \frac{[\overline{F}(t,T_1,T_2) - F(t,T_1,T_2)](T_2 - T_1)}{[1 + \overline{F}(t,T_1,T_2)(T_2 - T_1)]}$$

Proof: Using equation [5] and equation [8], we get the following result:

$$P(t,T_1,T_2) = \frac{\overline{B}(t,T_2)B(t,T_1)}{B(t,T_2)\overline{B}(t,T_1)}$$

$$= \frac{1 + F(t,T_1,T_2)(T_2 - T_1)}{1 + \overline{F}(t,T_1,T_2)(T_2 - T_1)} \text{, using equation [17] and [19]}$$

But, from equation [9], we can deduce that:

$$P(t,T_1,T_2) = 1 - P_{def}(t,T_1,T_2)$$

Hence, this implies:

$$P_{def}(t,T_1,T_2)[1+\overline{F}(t,T_1,T_2)(T_2-T_1)]=[\overline{F}(t,T_1,T_2)-F(t,T_1,T_2)](T_2-T_1)$$

Re-arranging the equation, gives the result:

$$P_{def}(t,T_1,T_2)=\frac{[\overline{F}(t,T_1,T_2)-F(t,T_1,T_2)](T_2-T_1)}{[1+\overline{F}(t,T_1,T_2)(T_2-T_1)]}.$$

As a consequence, this can also be expressed as:

$$\frac{P_{def}(t,T_1,T_2)}{T_2-T_1}=\frac{\overline{B}(t,T_2)}{\overline{B}(t,T_1)}[\overline{F}(t,T_1,T_2)-F(t,T_1,T_2)]\qquad \textbf{Equation [22]}$$

Proposition 5.2: The discrete implied hazard rate of default is given by the spread of

defaultable over default-free forward rates, discounted by the default-free forward

rates:

$$H(t,T_1,T_2)=\frac{\overline{F}(t,T_1,T_2)-F(t,T_1,T_2)}{1+(T_2-T_1)F(t,T_1,T_2)}$$

Proof: From equation [13], we get:

$$H(t,T_1,T_2) = \frac{P_{def}(t,T_1,T_2)}{(T_2-T_1)P(t,T_1,T_2)}$$

$$= \frac{\overline{B}(t,T_2)[\overline{F}(t,T_1,T_2) - F(t,T_1,T_2)]}{\overline{B}(t,T_1)P(t,T_1,T_2)} \text{ , using equation [22]}$$

$$= \frac{B(t,T_2)}{B(t,T_1)}[\overline{F}(t,T_1,T_2) - F(t,T_1,T_2)] \quad \text{, using equation [5] and [8]} \quad \textbf{Equation [23]}$$

$$= \frac{\overline{F}(t,T_1,T_2) - F(t,T_1,T_2)}{1 + (T_2-T_1)F(t,T_1,T_2)} \text{ , using equation [17]}$$

Proposition 5.3: The implied hazard rate of default at time $T > t$ as seen from time t

is given by the spread of the defaultable over the default-free continuously

compounded forward rates:

$$\overline{f}(t,T) - f(t,T)$$

Proof: We obtain the above relation as follows:

$$h(t,T) = -\frac{\partial}{\partial T}\ln P(t,T))$$

$$= -\frac{\partial}{\partial T}\ln \frac{\overline{B}(t,T)}{B(t,T)}$$

$$= \overline{f}(t,T) - f(t,T)$$

The probability of default in a short time interval $[T, T + \Delta T]$ is proportional to the

length ΔT of the interval with proportionality factor $\bar{f}(t,T) - f(t,T)$. In particular, the

local default probability at time t over the next small time step Δt is approximately

proportional to the length of the time step, with the short-term credit spread as

proportionality factor. Hence, we can define as follows:

Definition 5.4: (local default probability) The local default probability at time t over

the next small time step Δt is given as follows:

$$\frac{1}{\Delta t} Q[\tau \leq t + \Delta t \mid F_t \wedge \{\tau > t\}] \approx \bar{r}(t) - r(t) = \lambda(t)$$

where $r(t) = f(t,t)$ is the risk free short rate and $\bar{r}(t) = \bar{f}(t,t)$ is the defaultable short

rate.

Chapter 6

An extension to the Framework: Recovery

Modelling

We now relax the assumption of zero-recovery as assumed in the previous chapters.

We view an asset with positive recovery as an asset with an additional positive payoff

at the time of default. The recovery value is the *expected value* of the recovery

shortly after the occurrence of a default.

There are many different setups that have been proposed to model the recovery of

defaultable assets. For purposes of this dissertation, I will present one particular

framework which is based upon the recovery of par model first presented by Duffie

[2](1998).

Definition 6.1: Let's define $e(t,T,T+\Delta T)$ at time $t < T$ as a deterministic payoff of 1

that is paid at $T+\Delta T$ if and only if a default happens in $(T,T+\Delta T)$:

$$e(t,T,T+\Delta T) = E\ [B(t,T+\Delta T)[I(T)-I(T+\Delta T)]\,|\,F_t] \qquad \textbf{Equation [24]}$$

Note that:

$$I(T)-I(T+\Delta T) = 1\text{, if default occurs in }(T,T+\Delta T)$$

and

$$I(T)-I(T+\Delta T) = 0\text{, otherwise}$$

Furthermore,

$$E\ [B(t,T+\Delta T)I(T)] = E\ [B(t,T+\Delta T)]E\ [I(T)]$$

$$= B(t,T+\Delta T)P(t,T) \qquad \textbf{Equation [25]}$$

And,

$$E\,[B(t,T+\Delta T)I(T+\Delta T)] = \overline{B}(t,T+\Delta T) \qquad \textbf{Equation [26]}$$

Since, $B(t,T+\Delta T) = \dfrac{\overline{B}(t,T+\Delta T)}{P(t,T+\Delta T)}$, from equation [5]

Hence, we get:

$$e(t,T,T+\Delta T) = B(t,T+\Delta T)P(t,T) - \overline{B}(t,T+\Delta T)\text{, using equation [24], [25] and [26]}$$

$$= \overline{B}(t,T+\Delta T)\left(\frac{P(t,T)}{P(t,T+\Delta T)} - 1\right)$$

$$= \Delta T\,\overline{B}(t,T+\Delta T)H(t,T,T+\Delta T)\text{, using equation [14]} \qquad \textbf{Equation [27]}$$

We now let the time step converge to zero in equation [27] in order to reach a

continuous coverage of the time axis. Hence, taking the limit $\Delta T \to 0$, we obtain,

$e(t,T)$ as defined as follows:

<u>Definition 6.2:</u>

$$e(t,T) = \frac{\lim}{\Delta T \to 0} \frac{e(t,T,T+\Delta T)}{\Delta T}$$

$$= \overline{B}(t,T)h(t,T) \qquad\qquad \text{Equation [28]}$$

$$= B(t,T)P(t,T)h(t,T) \text{, using equation [5]}$$

The value of a security that pays $\pi(s)$ if a default occurs at time s for all $t < s < T$ is

given by:

$$\int_t^T \pi(s)e(t,s)ds$$

$$= \int_t^T \pi(s)\overline{B}(t,s)h(t,s)ds \text{, from equation [28]}$$

Suppose the payoff at default is not a deterministic function $\pi(s)$ but a random

variable $\overline{\pi}$ which is drawn at the time of default τ. $\overline{\pi}$ is called a *marked point*

process. For a discussion on marked point processes, see Schoncucher 2003 (Page

91).

We denote by $\pi_e(t,T)$, the expected value (under martingale measure Q -

probabilities) of $\overline{\pi}$ conditional on default at T and information at t as follows:

$$\pi_e(t,T) = E_Q[\overline{\pi} \mid F_t \wedge \{\tau = T\}]$$

We can specify any distribution for the random variable $\overline{\pi}$. Then, conditional on a

default occurring at time T we can price a security that pays $\overline{\pi}$ at default as

$\pi_e(t,T)B(t,T)$. We discount the cash flow $\pi_e(t,T)$ by the risk-free discount factor.

If we do not know the time of default, the present value of the security can be

computed as follows:

$$\int_t^T \pi_e(t,s)B(t,s)P(t,s)h(t,s)ds$$

$$= \int_t^T \pi_e(t,s)\overline{B}(t,s)h(t,s)ds \text{ , from equation [28]}$$

Since, we do not know the time of default, we have to integrate these values over all

possible default times and weigh them with the respective probability of default

occurring, i.e. the density of the time of default in the continuous-time case.

Chapter 7

Application to pricing credit derivatives

This chapter focuses on the pricing dynamics of the most popular credit derivatives

seen in the South African financial markets.

In order to simplify the numerical implementation, we accept a discrete grid of time

points, the tenor structure:

$$0 = T_0, T_1, T_2, ..., T_K$$

The dates are indexed in increasing order $(T_k < T_{k+1})$ and the distance between two

tenor dates is denoted by $\delta_k = T_{k+1} - T_k$ for all $0 \le k \le K$.

The coupon and repayment dates for bonds, fixing dates for rates, payment and

settlement dates for credit derivatives all fall on T_k.

From

$$B(0,T_i) = \frac{B(0,T_{i-1})}{1 + \delta_{i-1} F(0,T_{i-1},T_i)}, i = 1,2,.......,k \quad , \text{ using equation [18]}$$

and

$$B(0,T_0) = B(0,0) = 1,$$

we obtain the prices of the default-free zero coupon bonds as follows:

$$B(0,T_k) = \prod_{i=1}^{k} \frac{1}{1 + \delta_{i-1} F(0,T_{i-1},T_i)} \qquad \text{Equation [29]}$$

Similarly, from

$$P(0,T_i) = \frac{P(0,T_{i-1})}{1 + \delta_{i-1}H(0,T_{i-1},T_i)} \quad , \text{ using equation [15]}$$

We deduce the price of the defaultable zero-coupon bonds as follows:

$$\overline{B}(0,T_k) = B(0,T_k)P(0,T_k)$$

$$= B(0,T_k)\prod_{i=1}^{k}\frac{1}{1 + \delta_{i-1}H(0,T_{i-1},T_i)} \qquad \textbf{Equation [30]}$$

Now, let $e(0,T_k,T_{k+1})$ be the value of 1 unit at T_{k+1} if a default occurred in (T_k,T_{k+1}).

Hence, from equation [27] on p.59:

$$e(0,T_k,T_{k+1}) = \delta_k H(0,T_k,T_{k+1})\overline{B}(0,T_{k+1}) \qquad \textbf{Equation [31]}$$

Note that the prices of the bonds given by equations [29], [30], and [31] above are

expressed in terms of two fundamental modelling quantities as follows:

- Term structure of default-free interest rates $F(0,T)$

- Term structure of implied hazard rates $H(0,T)$

Taking the limit $\delta_i \rightarrow 0$, for all $i = 0,1,2,...k$ in equations [29], [30] and [31]

respectively, we can express the prices in terms of continuously compounded

instantaneous forward rates and hazard rates: The results are as follows:

$$B(0,T_k) = \exp\left(-\int_0^{T_k} f(0,s)ds\right)$$

$$\overline{B}(0,T_k) = \exp\left(-\int_0^{T_k} [h(0,s) + f(0,s)]ds\right)$$

$$e(0,T_k) = h(0,T_k)\overline{B}(0,T_k)$$

7.1 Defaultable fixed-coupon bonds

A defaultable fixed-coupon bond has coupon payments of $\overline{c}_n$ at $T_n, n = 1,2,....N$. It is

possible for the individual fixed-coupon payments to differ slightly from each other

depending on day count conventions and the bond's specification. All the coupon

payments are known in advance.

Pricing formula:

The price of a defaultable fixed-coupon bond can be given as follows:

$$\overline{C}(0) = \sum_{n=1}^{N} \overline{c}_n \overline{B}(0,T_n) \text{, this term represents all the coupons}$$

$$+ \overline{B}(0,T_N) \text{, this term represents the principal payment}$$

$$+ \pi \sum_{k=1}^{N} e(0,T_{k-1},T_k) \text{, this term represent the recovery value.} \qquad \textbf{Equation [32]}$$

We can also represent the recovery payment as follows:

$$\pi \sum_{k=1}^{N} e(0, T_{k-1}, T_k) = \sum_{k=1}^{N} \pi \delta_{k-1} H(0, T_{k-1}, T_k) * \overline{B}(0, T_k)$$, using equation [31]

The recovery payments can be considered as an additional coupon payment stream

of $\pi \delta_{k-1} H(0, T_{k-1}, T_k)$.

7.2 Defaultable floater

In a defaultable par floater, the coupon payments at times T_n, are equal to the

reference interest rate plus a spread, denoted by s^{par}, assuming a nominal of 1.

Let's denote $L(T_{n-1}, T_n)$ as the reference interest rate applied over (T_{n-1}, T_n) at T_{n-1}, so

that $1 + L(T_{n-1}, T_n)\delta_{n-1}$ is the growth factor over (T_{n-1}, T_n). If we apply a no-arbitrage

argument, we can get the following result:

$$B(T_{n-1}, T_n) = \frac{1}{1 + L(T_{n-1}, T_n)\delta_{n-1}}$$
  Equation [33]

If this bond trades at a value greater than $B(T_{n-1}, T_n)$, this would imply that its yield is

less than $L(T_{n-1}, T_n)$. Hence, an arbitrage opportunity exists as a dealer can short

sell these bonds and invest in other money market securities which would yield

$L(T_{n-1}, T_n)$. At maturity of the bond, the dealer will pay interest to the bondholders at

a lower rate that what was earned on the invested money market securities.

Similarly, if this bond is trading at a value less than $B(T_{n-1}, T_n)$, this would imply that

its yield is more than $L(T_{n-1}, T_n)$. A dealer can borrow funds at a lower rate and

invest in the bond which offers a higher rate of return. Once again, an arbitrage

opportunity exists. Therefore, the relationship in equation [33] has to be true for no

arbitrage opportunities to occur.

The coupon payment at T_n can be represented as follows:

$$\delta_{n-1}[L(T_{n-1},T_n) + s^{par}]$$

$$= \left[\frac{1}{B(T_{n-1},T_n)} - 1\right] + s^{par}\delta_{n-1} \text{ , using equation [33]}$$

Let's consider the payment of $\left[\dfrac{1}{B(T_{n-1},T_n)}\right]$ at time T_n. The present value at T_{n-1} is

given as follows:

$$\frac{\overline{B}(T_{n-1},T_n)}{B(T_{n-1},T_n)}$$

$$= P(T_{n-1},T_n) \text{ , using equation [5]}$$

We use the defaultable discount factor, since the coupon payment is exposed to

credit risk over (T_{n-1},T_n). Seen at $t = 0$, the present value of $P(T_{n-1},T_n)$ now

becomes:

$$E_Q[B(0,T_{n-1})I(T_{n-1})P(T_{n-1},T_n)]$$

$$= B(0,T_{n-1})E_Q[I(T_{n-1})P(T_{n-1},T_n)] \text{ , using assumption [2.2]}$$

$$= B(0,T_{n-1})P(0,T_n) \qquad\qquad\qquad\qquad \textbf{Equation [34]}$$

Combine the above result with the fixed part of the coupon payment and observe the

following relation:

$$[B(0,T_{n-1}) - B(0,T_n)]P(0,T_n) = \left(\frac{B(0,T_{n-1})}{B(0,T_n)} - 1\right)\overline{B}(0,T_n)$$

$$= \delta_{n-1}F(0,T_{n-1},T_n)\overline{B}(0,T_n) ,$$

Pricing formula:

The model price of the defaultable floating-rate bond is:

$$\overline{C}(0) = \sum_{n=1}^{N}\delta_{n-1}F(0,T_{n-1},T_n)\overline{B}(0,T_n) \text{ , default reference rate}$$

$$+ s^{par}\sum_{n=1}^{N}\delta_{n-1}\overline{B}(0,T_n) \text{ , coupon spread}$$

$$+ \overline{B}(0,T_K) \text{ , principal}$$

$$+ \pi\sum_{n=1}^{N}e(0,T_{n-1},T_n), \text{ recovery} \qquad\qquad \textbf{Equation [35]}$$

7.3 Credit Default Swaps (CDS's)

CDS's are a product within the credit derivative asset class, constituting a type of

over the counter derivative. They are bilateral contracts in which a protection buyer

agrees to pay a periodic fee (called a "premium") in exchange for a payment by the

protection seller in the case of a credit event (such as a bankruptcy) affecting the

reference entity. The market price of the premium is therefore an indication of the

perceived credit risk related to the reference entity.

A credit default swap consists of two payment legs:

__Fixed leg:__ The protection buyer pays a periodic fee, $\bar{s}$, for credit protection. Hence,

the fixed leg consists of payments of $\delta_{n-1}\bar{s}$ at T_n if no default occurs until T_n. The

present value of the fixed leg is calculated by discounting the payments by the

defaultable zero coupon bond, as the fee payments are subject to credit risk.

$$\bar{s}\sum_{n=1}^{N}\delta_{n-1}\bar{B}(0,T_n)$$ **Equation [36]**

__Floating leg:__ The protection seller pays an amount of $(1-\pi)$ at T_n contingent on

default in (T_{n-1},T_n). The present value of the floating leg is calculated as follows:

$$(1-\pi)\sum_{n=1}^{N}e(0,T_{n-1},T_n)$$

$$= (1-\pi)\sum_{n=1}^{N}\delta_{n-1}H(0,T_{n-1},T_n)\overline{B}(0,T_n)\text{, from equation [31]} \qquad \textbf{Equation [37]}$$

where $e(0,T_{n-1},T_n)$ is the deterministic payoff of 1 that is paid at T_n if and only if a

default happens in (T_{n-1},T_n) and π is the recovery rate, as a percentage of the

notional.

Generally, in most CDS contracts, the market CDS swap rate is chosen such that the

fixed and floating leg of the CDS has the same value, i.e. the value of the contract to

the buyer must be zero when the contract is entered into. Hence, equating equation

[36] and equation [37] result in the following for the fair swap rate determined at

$t = 0$:

Pricing formula:

$$\overline{s} = (1-\pi)\frac{\displaystyle\sum_{n=1}^{N}\delta_{n-1}H(0,T_{n-1},T_n)\overline{B}(0,T_n)}{\displaystyle\sum_{n=1}^{N}\delta_{n-1}\overline{B}(0,T_n)}$$

Now, let's define weights as follows:

$$w_n = \frac{\delta_{n-1}\overline{B}(0,T_n)}{\sum\limits_{k=1}^{N}\delta_{k-1}\overline{B}(0,T_k)} \quad \text{where } n = 1,2,3,......N \qquad \textbf{Equation [38]}$$

so that

$$\sum_{n=1}^{N} w_n = 1 \qquad \textbf{Equation [39]}$$

Then the fair swap premium rate determined at *t* = 0 is given by:

$$\overline{s} = (1-\pi)\sum_{n=1}^{N} w_n H(0,T_{n-1},T_n) \qquad \textbf{Equation [40]}$$

We can make the following observations:

- Since, H is a function of P and $\overline{P}$ only, hence $\overline{s}$ depends only on the defaultable and default-free discount rates B and $\overline{B}$, which are given by the market bond prices.

- It is similar to the calculation of fixed rate in an interest rate swap (see Rebonato, 1998) as follows:

$$s = \sum_{n=1}^{N} w_n' F(0, T_{n-1}, T_n) \text{, (} s \text{ is the fixed interest rate)}$$

where

$$w_n' = \frac{\delta_{n-1} B(0, T_n)}{\sum_{k=1}^{N} \delta_{k-1} B(0, T_k)} \text{ where } n = 1,2,3,......N$$

When a CDS is traded, the value of the swap rate will change after the initial date, due to market movements. Let's assume that the original CDS spread was s and the new CDS spread is now $\bar{s}$. The marked-to-market value of the original CDS is as follows:

$$(\bar{s} - s) \sum_{n=1}^{N} \bar{B}(t, T_n) \delta_{n-1} \qquad \text{Equation [41]}$$

Equation [41] can be explained as follows: If an offsetting trade is entered at the current CDS rate $\bar{s}$, only the fee difference, $(\bar{s} - s)$, will be received over the life of the CDS. Should a default occur, the protection payments will cancel out, and the fee difference payment will also be cancelled. The fee difference stream is defaultable and is hence discounted by $\bar{B}(t, T_n)$. The diagrams below discuss this graphically.

Let's assume a CDS is purchased at time 0 and an offsetting trade is entered into at time t. Then, if no default occurs the cash flows can be represented as follows:

Protection Buyer Cash Flows with new CDS at time i: No Default						
Time Line	**0**	**1**	**2**	....	**t**	....	**N**
Initial CDS	$\overline{s}$	$\overline{s}$	$\overline{s}$		$\overline{s}$		
New CDS					s		
					$\overline{s}-s$		

Note, the net cash flows at each point in time are the difference in the swap rates.

Similarly, the cash flows if a default occurs at N are as follows:

Protection Buyer Cash Flows with new CDS at time i: Default at N						
Time Line	**0**	**1**	**2**	....	**i**	....	**N**
Initial CDS	$\overline{s}$	$\overline{s}$	$\overline{s}$		$\overline{s}$		$1-\pi$
New CDS					s		$-(1-\pi)$
					$\overline{s}-s$		0

Note, the net cash flows at each point in time are the difference in the swap rates. The payment at default cancels out. Hence, the only cash flows to consider are the difference in fee payments.

Hence, it can be seen that credit default swaps are useful instruments to gain exposure against spread movements, and not just against default arrival risk.

7.4 Credit Default Swaps (CDS's):   Market Valuation Approach

This section is dedicated to developing a logical argument for the most common

market valuation approach to CDS's. In particular, we consider the derivation of a

pricing formula with commonly assumed market conventions and practices. Given,

the popularity of the method, a practical example and implementation is given in the

appendix.

A typical CDS contract usually specifies two potential cash flow streams; a fixed leg

and a floating leg. On the fixed leg side, the buyer of protection makes a series of

fixed, periodic payments of CDS premium until the maturity, or until the reference

credit defaults. On the floating leg side, the protection seller makes one payment

only if the reference credit defaults. The amount of a contingent payment is usually

the notional amount multiplied by $1-\pi$, where π is the recovery rate, as a

percentage of the notional. Hence, the value of the CDS contract to the protection

buyer at any given point in time is the difference between the present value of the

floating leg, which the protection buyer expects to receive, and that of the fixed leg,

which he is expected to pay. Hence:

Value of CDS(to protection buyer) = PV(Floating leg) – PV (Fixed leg)   **Equation [42]**

In order to calculate these values, one needs information about the default probability

of the reference credit, the recovery rate in a case of default, and risk-free discount

factors.

Fixed Leg: On each payment date, the periodic payment is calculated as the annual CDS premium, s, multiplied by d_i, the accrual days (expressed in a fraction of one year) between payment dates. For example, if the CDS premium is 160 basis points per annum and payments are made quarterly, the periodic payment will be:

$$d_i s = 0.25(160) = 40$$

However, this payment is only going to be made when the reference credit has not defaulted by the payment date. So, we have to take into account the survival probability, or the probability that the reference credit has not defaulted on the payment date. For instance, if the survival probability of the reference credit in the first three months is 90%, the expected payment at t_1, or 3 months later, is:

$$P(t,t_1)d_i s = 0.9(0.25)(160) = 36 \ , \ t < t_1$$

where $P(t,t_1)$ is the survival probability at time t. Then, using the discount factor for the particular payment date, $B(t,t_i)$, the present value for this payment is $B(t,t_i)P(t,t_i)sd_i$. Summing up PV's for all these payments, we get:

$$\sum_{i=1}^{N} B(t,t_i)P(t,t_i)sd_i \ , \ t < t_1 < ..... < t_N \qquad \text{Equation [43]}$$

Finally, we need to account for the accrued premium paid up to the date of default when default happens between the periodic payment dates. The accrued payment

can be approximated by assuming that default, if it occurs, occurs at the middle of the interval between consecutive payment dates. Then, when the reference entity defaults between payment date t_{i-1} and payment date t_i, the accrued payment amount is $(sd_i)/2$. This accrued payment has to be adjusted by the probability that the default actually occurs in this time interval. In other words, the reference credit survived through payment date t_{i-1}, but not to next payment date t_i. This probability is given by:

$$P(t,t_{i-1}) - P(t,t_i)$$

Accordingly, for a particular interval, the expected accrued premium payment is :

$$(P(t,t_{i-1}) - P(t,t_i))(sd_i/2)$$

Therefore, the present value of all expected accrued payments is given by:

$$\sum_{i=1}^{N} B(t,t_i)(P(t,t_{i-1}) - P(t,t_i))(sd_i/2) \qquad \text{Equation [44]}$$

Now, we have both components of the fixed leg. Adding equation [43] and equation [44], we get the present value of the fixed leg:

$$PV(FixedLeg) = \sum_{i=1}^{N} B(t,t_i)P(t,t_i)sd_i + \sum_{i=1}^{N} B(t,t_i)(P(t,t_{i-1}) - P(t,t_i))(sd_i/2)$$

Equation [45]

Floating Leg: Assume the reference entity defaults between payment date t_{i-1} and payment date t_i. The protection buyer will receive the floating payment of $1 - \pi$, where π is the recovery rate. This payment is made only if the reference credit defaults and therefore it has to be adjusted by $(P(t, t_{i-1}) - P(t, t_i))$, the probability that the default actually occurs in this time period. Discounting each expected payment and summing up over the term of a contract, we get:

$$PV(FloatingLeg) = (1 - \pi) \sum_{i=1}^{N} (B(t, t_i)(P(t, t_{i-1}) - P(t, t_i))) \qquad \textbf{Equation [46]}$$

When two parties enter a CDS trade, the CDS swap rate is set so that the value of the swap transaction is zero. Hence plugging equation [45] and equation [46] into equation [42], we arrive at the following:

Pricing formula:

$$\sum_{i=1}^{N} B(0,t_i)P(0,t_i)sd_i + \sum_{i=1}^{N} B(0,t_i)(P(0,t_{i-1}) - P(0,t_i))(sd_i/2)$$

$$= (1-\pi)\sum_{i=1}^{N} B(0,t_i)(P(0,t_{i-1}) - P(0,t_i)))$$ **Equation [47]**

Given, all the parameters, s, the annual premium payment is set as:

$$s = \frac{(1-\pi)\sum_{i=1}^{N} B(0,t_i)(P(0,t_{i-1}) - P(0,t_i)))}{\sum_{i=1}^{N} B(0,t_i)P(0,t_i)d_i + \sum_{i=1}^{N} B(0,t_i)(P(0,t_{i-1}) - P(0,t_i))(d_i/2)}$$

7.5 Forward Start Credit Default Swaps

A forward start CDS is a CDS which is contracted at time T_0, but the actual fee payments and credit protection only start at a late time point T_{k0}, where $T_{k0} > T_0$. Should a default occur a some time before T_{k0}, the forward start CDS is cancelled with no cash flows being made.

The pricing argument for a forward start CDS is exactly as in pricing a CDS, except for the indices of the summation.

Pricing formula:

$$\overline{s} = (1-\pi) \frac{\sum_{n=k0}^{N} \delta_{n-1} H(0, T_{n-1}, T_n) \overline{B}(0, T_n)}{\sum_{n=1}^{N} \delta_{n-1} \overline{B}(0, T_n)} \qquad \text{Equation [48]}$$

7.6 Default Digital Swaps (DDS)

Digital is a term that is used to describe electronic technology that generates, stores and processes data in terms of two states: positive and non-positive. Positive is expressed or represented by the number 1 and non-positive by the number 0. In the context of DDS, the relevance applies to the floating leg having a payoff of either 1 or 0.

A default digital swap consists of two payment legs:

Fixed leg: Payment of $\delta_{n-1} \bar{s}^{DDS}$ at T_n if no default until T_n. The value of the fixed leg is:

$$\bar{s}^{DDS} \sum_{n=1}^{N} \delta_{n-1} \bar{B}(0, T_n) \qquad \text{Equation [49]}$$

Floating leg: A fixed payment of 1 is paid at default, independent of the actual recovery.

$$\sum_{n=1}^{N} e(0, T_{n-1}, T_n)$$

$$= \sum_{n=1}^{N} \delta_{n-1} H(0, T_{n-1}, T_n) \bar{B}(0, T_n) \qquad \text{Equation [50]}$$

As with CDS contracts, in most DDS contracts, the market DDS swap rate is chosen

such that the fixed and floating leg of the DDS has the same value. Hence, equating

equation [49] and equation [50] results in the following:

Pricing formula:

$$\overset{-DDS}{s} = \frac{\sum_{n=1}^{N} \delta_{n-1} H(0,T_{n-1},T_n) \overline{B}(0,T_n)}{\sum_{n=1}^{N} \delta_{n-1} \overline{B}(0,T_n)}$$

If we define weights as in equation [38] and equation [39], then the fair swap

premium rate is given by:

$$\overset{-DDS}{s} = \sum_{n=1}^{N} w_n H(0,T_{n-1},T_n) \qquad\qquad \textbf{Equation [51]}$$

Using equation [40] and equation [51], we can get the following result in terms of

DDS and CDS fee payments. Since, DDS is independent of recovery, while CDS's

are contingent on recovery, DDS's can be considered to be more expensive than a

corresponding CDS contract.

$$\overset{-DDS}{s} = \frac{1}{1-\pi}\overset{-}{s}$$

7.7 Asset Swap Packages

An asset swap package consists of a defaultable coupon bond $\overline{C}$ with coupon $\overline{c}$ and an interest rate swap. The bond's coupon is swapped into the reference interest rate plus the asset swap rate s^A. An asset swap package is usually sold at par.

Asset swap transactions are driven by the desire to strip out unwanted structured features from the underlying asset.

The payoff streams to the buyer of the asset swap package can be described as follows:

Time	Defaultable Bond	Swap	Net
$t = 0$	$-\overline{C}(0)$	$-1+\overline{C}(0)$	-1
$t = t_i$	$\overline{c}^*$	$-\overline{c}+L_{i-1}+s^A$	$L_{i-1}+s^A+(\overline{c}^*-\overline{c})$
$t = t_N$	$(1+\overline{c})^*$	$-\overline{c}+L_{N-1}+s^A$	$1^*+L_{N-1}+s^A+(\overline{c}^*-\overline{c})$

* denotes payment contingent on survival

We now define as follows:

$s(0) = $ Fixed-for-floating swap rate (the market quote)

$A(0) = \sum_{n=1}^{N} \delta_{n-1} B(0, T_n)$ This is the value of an annuity paying 1 unit, calculated based

on observable default free bond prices.

The value of an asset swap package is set at par at $t = 0$, so that :

$$\overline{C}(0) + \left(A(0)s(0) + A(0)s^A(0) - A(0)\overline{c} \right) = 1$$

The present value of the floating coupons is given by $A(0)s(0)$. The swap continues

even after default so that $A(0)$ appears in all terms associated with the swap

arrangement. Solving for $s^A(0)$:

$$s^A(0) = \frac{1}{A(0)}\left(1 - \overline{C}(0) \right) + \overline{c} - s(0)$$

Rearranging the terms,

$$\overline{C}(0) + A(0)s^A(0) = [1 - A(0)s(0)] + A(0)\overline{c} = C(0),$$

where the right hand side gives the value of a default free bond with coupon $\overline{c}$. Note that $[1 - A(0)s(0)]$ is the present value of receiving 1 unit at maturity t_N. We therefore obtain:

Pricing formula:

$$s^A(0) = \frac{1}{A(0)}[C(0) - \overline{C}(0)]$$

$$= \frac{C(0) - \overline{C}(0)}{\sum\limits_{n=1}^{N} \delta_{n-1} B(0, T_n)}$$

Chapter 8

Conclusion

The evolution of better models for credit risk measurement and better tools for credit

risk management are mutually reinforcing: traditionally, without the tools to transfer

credit risk, it was not possible to properly respond to the recommendations of a

portfolio model. Conversely, without a portfolio model, the contribution of credit

derivatives to portfolio risk-return performance has been difficult to evaluate.

However, as such technology becomes more widespread, as the necessary data

becomes more accessible and as credit derivative liquidity improves, the combined

effect on the way in which banks and others evaluate and manage credit risks will be profound. Banks have already adopted a more proactive approach to trading and managing credit exposures, with a corresponding decline in the typical holding period for loans. It is becoming increasingly common to observe banks taking exposure to borrowers with whom they have no meaningful relationships and shedding exposure to customers with whom they do have relationships to facilitate further business.

While it is true that banks have been the foremost users of credit derivatives to date, it would be wrong to suggest that banks will be the only institutions to benefit from them. Credit derivatives are bringing about greater efficiency of pricing and greater liquidity of all credit risks. This will benefit a broad range of financial institutions, institutional investors and also corporates in their capacity both as borrowers and as takers of trade credit and receivables exposures.

Given the growth potential of credit derivatives, this will initiate the development of a sounder regulatory framework for these instruments. No doubt, the quantitative technology used to price these instruments is an evolving process as the credit derivatives market becomes more complex and sophisticated.

Appendix

A pricing example of CDS's: Market Valuation Approach

The following is a pricing example of a CDS using the methodology in section 7.4.

Let's see how we can value a hypothetical CDS trade. Consider a 2-year CDS with

quarterly premium payments. Assume a spread of 200 basis points per annum, and

the discount factors and the survival probability for each payment date are as shown

below:

| | | Notional Amount | 1,000,000 | | | | | | |
| | | Recovery Rate | 30% | | | | | | |

	1	2	3	4	5	6	7	8	9	10
Month	Discount Factor	Survival Probability to the Period (%)	Fixed Periodic Payments (bps)	Expected Value of Fixed Payments (bps)	PV of fixed payments	Default proability for the period (%)	Expected Accrual Payment (bps)	PV of Accrued Payment	Expected Contingent Payment (bps)	PV of Contingent Payment
				[2] x [3]	[4] x [1]		[3]/2 x[6]	[7] x [1]	[1-30%] x [6]	[9] x [1]
0	1.00	100.0%	0	-	-	0.0%	-	-	-	-
3	0.99	99.9%	50	49.95	4,945	0.1%	0.02	2.47	7.00	693
6	0.98	99.6%	50	49.80	4,880	0.3%	0.08	7.35	21.00	2,058
9	0.97	99.1%	50	49.55	4,806	0.5%	0.13	12.13	35.00	3,395
12	0.96	98.4%	50	49.20	4,723	0.7%	0.17	16.80	49.00	4,704
15	0.95	97.5%	50	48.75	4,631	0.9%	0.23	21.38	63.00	5,985
18	0.94	96.4%	50	48.20	4,531	1.1%	0.27	25.85	77.00	7,238
21	0.93	95.2%	50	47.60	4,427	1.2%	0.30	27.90	84.00	7,812
24	0.92	94.0%	50	47.00	4,324	1.2%	0.30	27.60	84.00	7,728
				PV ->	37,268			141.48	PV ->	39,613

Fixed Leg	37,409.33	
Floating Leg	39,613.00	

Fixed Leg: Fixed, periodic payments

The present value of all expected fixed payments is found by multiplying each period's fixed payment by the respective survival probability, discounted at the risk-free rate and summed over the term of the CDS. The PV in the table of R 37,268, is the present value of the fixed leg for a notional of 1 million rands.

Fixed Leg: Accrual payments

Assuming that default occurs, if any, at the middle of the time interval between two payment dates, the value of the accrued premium payment if a default occurs is a half of 50 basis points, or 25 basis points. Then, the expected value of the accrued payment for each period is 25 basis points multiplied by the probability of default for that period, as in column [7] in the table above. Discount these values for all periods and summing them over the term of the CDS, we get a value of R 141.48 (sum of column 8), which is the present value of expected accrued fixed payments.

From above, we can see that the present value of the fixed leg over the 2 year term is (R 37,268 + R 141.48) = R 37,409.33.

Floating leg:

The expected value of the floating payment if a default occurs during each period is $1 - \pi$ multiplied by the probability of default for that period (column [9]). Assuming a recovery rate of 30%, the expected contingent payment is multiplied by each period's default probability. Discount this for each period and summing over the term of the

CDS, we get a value of R 39,613 as in column [10], which is the present value of the expected contingent payments.

Hence, we can find the value of this CDS to the protection buyer when the spread is 200 basis points per annum as:

R 39,613 – R 37,409.33 = R 2,203.68

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