EXPLORING SOUTH AFRICAN HIGH SCHOOL LEARNERS PROBLEM-SOLVING SKILLS IN EUCLIDEAN GEOMETRY

by

MATSHO STEPHENS KGALUSHI

32666691

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SUPERVISOR: DR S FALEYE

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RESEARCH DECLARATION

I, Matsho Stephens Kgalushi, with University of South Africa student number 32666691, hereby declare that the research work shall be entirely my work and the data collected shall be used only for the purpose of this research work and not for any other purposes.

………………………………………

Matsho Stephens K.
Principal Researcher

………………………………

Dr Faleye Sunday
Research Supervisor

DEDICATION

This study is dedicated to God the almighty, my late son **Ditshego**, my wife Mmamotsepe Debby, my children Keikantse, Bonolo, Tshegofatso and my granddaughter Tshwanelo for giving me inspiration to complete this study.

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ABSTRACT

This study investigated the effects of South African mathematics learners' problem-solving errors in Euclidean geometry on their performance in matric mathematics paper 2 of the National Senior Certificate (NSC) examination. The study also proposed Euclidean geometry teaching-learning continuum approach that may reduce the problem-solving errors and ultimately improve performance in the matric mathematics paper 2 examination. The study was underpinned by Newman problem-solving error analysis (Newman, 1977). To this end, the study was conducted in two phases: Phase I and Phase II. In Phase I, type of errors committed in the matric mathematics paper 2 and their effect on performance was investigated, while the Phase II of the study explored the efficacy of the teaching-learning continuum (EGPSLM-EGPSIA) approach proposed. Mixed method research approach was used in the study. This involved quasi-experimental and descriptive research designs. Quasi-experimental design used ex-post facto and pre-test post-test matching control research approach to collect data, while the descriptive research design adopted solution appraisal and classroom observation to collect data. In Phase I, 244 scripts from the 2020 mathematics paper 2 exam from three provinces and 432 scripts from the 2021 mathematics paper 2 exam from five provinces were used to collect data. In Phase II, random and convenient sampling were used to select three schools from the study population. Two of the selected schools were experimental schools and the remaining one was a control school. A total of 95 Grade 12 mathematics learners formed the study participants for the study in Phase II.

There were nine itemised findings but they are summarised as but not limited to (1) problem-solving errors in Euclidean geometry significantly affected the learners' matric mathematics paper 2 performance, (2) South African mathematics learners commit similar pattern of errors in Euclidean geometry yearly, (3) reading error was the leading error that were commonly committed in Euclidean geometry problem-solving, (4) EGPSLM-EGPSIA continuum intervention facilitate the learning of Euclidean geometry concepts, and (5)

EGPSLM-EGPSIA continuum intervention has the potential to improve the learners' performance in Euclidean geometry.

Key Terms: Euclidean Geometry; Problem-Solving Errors; Problem-Solving Skills; Performance in Euclidean geometry; Performance in NSC mathematics paper 2.

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LIST OF ABBREVIATIONS

ANA	-	Annual National Assessment
C2005	-	Curriculum 2005
CAPS	-	Curriculum Assessment Policy Statements
DBE	-	Department of Basic Education
EG	-	Euclidean Geometry
EGPSLM	-	Euclidean Geometry Problem Solving Learning Model
EGPSIA	-	Euclidean Geometry Problem Solving Instructional Approach
ERS	-	Error Rating Scale
IEB	-	Independent Examinations Board
LOLT	-	Language of Learning and Teaching
MEC	-	Member of the Executive Council
NCS	-	National Curriculum Statements
NDE	-	National Department of Education
NEA	-	Newman error analysis
NSC	-	National Senior Certificate
NSF	-	National Science Foundation
OBE	-	Outcome-Based Education
PCK	-	Pedagogy Content Knowledge
POPI	-	Protection of Personal Information
SACMEQII	-	Southern Africa Consortium for Monitoring Educational Quality
SCK	-	Subject Content Knowledge
SMSG	-	School Mathematics Study Group
SPSS	-	Statistical Package for the Social Science
TIMSS	-	Trends in International Mathematics and Science Study

CHAPTER 1

INTRODUCTION

1.0 Background to the study

This study was necessitated by the consistent poor performance of mathematics learners in mathematics examinations at all levels of education in South Africa, more especially in the Euclidean geometry branch of mathematics. The "Trends in International Mathematics and Science Study" (TIMSS) is a cross-national assessment of mathematics and science levels of learners from various participating countries. This evaluation is conducted in every four years. South Africa started participating in 1995 and has participated in the 1999, 2003, 2011, 2015, and 2019 assessments. However, it is disheartening to note that South Africa belongs to the bottom of the list of the assessment in all these assessment test. Table 1.1 shows South Africa learners' performance in 2019 TIMSS as an example.

Table 1.1: 2019 TIMSS Achievement in Mathematics

Country	Mathematics Mean (SE)	Country	Mathematics Mean (SE)
Singapore	616 (4.0)	Kazakhstan	488 (3.3)
Chinese Taipei	612 (2.7)	France	483 (2.5)
Korea, Rep. of	607 (2.8)	New Zealand	482 (3.4)
Japan	594 (2.7)	Bahrain	481 (1.7)
Hong Kong SAR	578 (4.1)	Romania	479 (4.3)
Russian Federation	543 (4.5)	United Arab Emirates	473 (1.9)
Ireland	524 (2.6)	Georgia	461 (4.3)
Lithuania	520 (2.9)	Malaysia	461 (3.2)
Israel	519 (4.3)	Iran, Islamic Rep. of	446 (3.7)
Australia	517 (3.8)	Qatar	443 (4.0)
Hungary	517 (2.9)	Chile	441 (2.8)
United States	515 (4.8)	Western Cape (9)	441 (4.4)
England	515 (5.3)	Lebanon	429 (2.9)
Finland	509 (2.6)	Gauteng (9)	421 (3.0)
Norway (9)	503 (2.4)	Jordan	420 (4.3)
Sweden	503 (2.5)	Egypt	413 (5.2)
Cyprus	501 (1.6)	Oman	411 (2.8)
Portugal	500 (3.2)	Kuwait	403 (5.0)
TIMSS Scale Centrepoint	500	Saudi Arabia	394 (2.5)
Italy	497 (2.7)	South Africa (9)	389 (2.3)
Turkey	496 (4.3)	Morocco	388 (2.3)

Kotze’ and Strauss (2007) reported on their investigation on South African Grade 6 learners performance in the mathematics test carried out by the Southern Africa Consortium for Monitoring Educational Quality (SACMEQII). One of the finding of the duo was that the overall learners’ achievement was at the lower end of the acceptable limits on SACMEQII benchmark. The duo informs that comparing the question item difficulty level and learners’ competencies reveal a significant low level numeracy, understanding and skills. If learners had poor mathematics skills from the primary and secondary level of education, it would always haunt them at the tertiary level of education. This was demonstrated in the study conducted by Faleye and Mogari (2009), which revealed that university students cannot carry out simple numeracy calculation as a result of early indulgence in the use of calculators. Also, the Annual National Assessment (ANA) (2012) reported that two thirds of the pupils in Gauteng leave primary school without the proper conceptual knowledge that is needed for the secondary school mathematics learning.

The researcher opines that a child's brain is like a blank page that are ready to be filled. If it is filled with garbage, then the brain will always produce garbage. That majority of our learners from the primary school have difficulties in conceptual understanding of mathematics clearly indicates that our classroom mathematics teaching and learning is problematic. Adler and Sfard's (2015) study was on the mathematics teaching and learning continuum from the primary school to the university. The duo reported that once mathematics learning deficiencies are concretised at the primary school level, the learners continue to battle to understand mathematical concepts throughout their academic careers. The mathematics poor performance continues to reflect in the future performance in mathematics examinations. For example, the yearly secondary school "National Senior Certificate" (NSC) examinations are reviewed next.

The 2017 NSC diagnostic report informs that there is very little upward pass percentage from 2014 to 2017. The report gave the pass percentages as 53,3%; 49,1%; 51,1% and 51,9% for 2014, 2015, 2016, and 2017, respectively. The learners performance in NSC mathematics is shown pictorially in Figure 1.1.

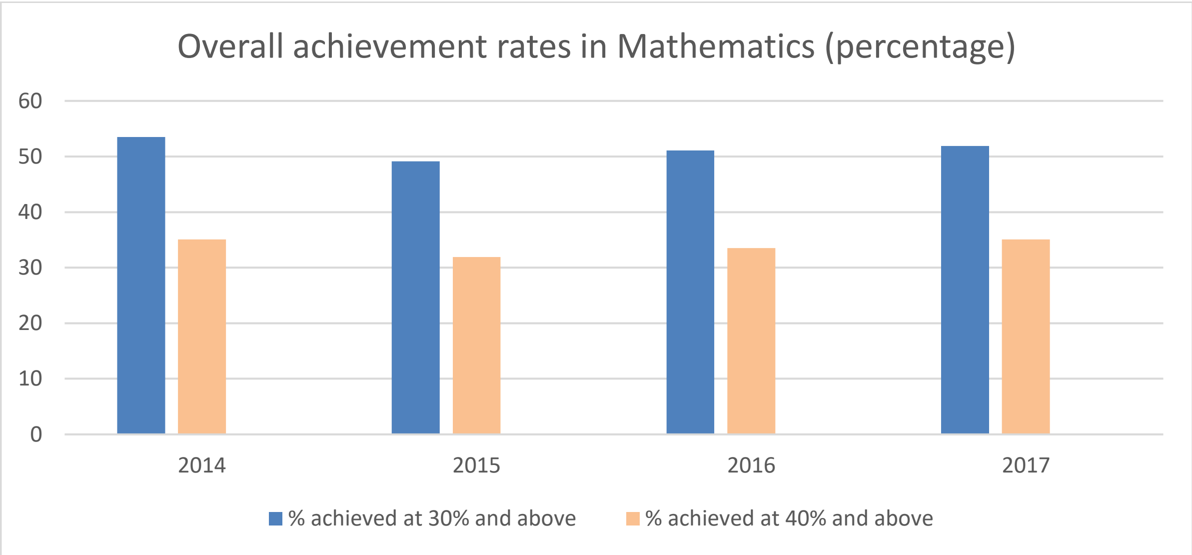


Figure 1.1: Overall learners' performance in NSC Mathematics

Mathematics education scholars have studied the possible factors that may be wrong with our classroom mathematics teaching and learning which results in unabated downward trend in performance of South African learners in mathematics, some of these factors are presented below.

1.1 Research proposed factors responsible for the poor mathematics learners' performance in mathematics in South Africa

Many mathematics education researchers such as Mji and Makgato (2006) have unveiled some factors that might have been responsible for the unabated continued poor performance of South African learners in mathematics. The researcher examined some of following factors:

(i) Underdevelopment of teachers during apartheid regime in South Africa.

Seroto (2004) informs that the type of education training given to the black student teachers during apartheid regime in South Africa was below the acceptable standard of qualifications. He pointed out that in 1988 there were 77,2% of black teachers in South Africa who had below standard qualifications. Some of these teachers had qualifications which were below standard ten or had three or two years certificate which was not in line with their white counterparts who were 100% professionally qualified teachers. According to Pillay (1984), poorly trained teachers will only create a circle of inferior teachers. Could it be that the chronic problem our mathematics learners are experiencing are as a result of the circle of incompetent teachers in our schools. If teachers are underqualified, they may not be able to deliver quality mathematics teaching that is necessary for producing learners that are sound in mathematics concepts. After all, an adage says 'you cannot give what you do not have.'

(ii) Frequent change in education policy.

As mentioned in 1.2(i), the teachers under the apartheid government were not properly trained, more especially, the black teachers. Therefore, the new incoming democratic government inherited a large number of teachers that were not qualified. This resulted in change of education policies and amendments of policies in a way to redress the inequalities created by apartheid government (Cross, Mugadi, & Rouchani, 2002). However, these educational policies were not given enough time before changing them, creating instabilities in the mathematics teaching and learning continuum.

After the inception of the democratically elected government in 1994, Cross et al. (2002) highlight that several documents were developed by the then National Department of Education (NDE) which cumulated into formation of the 1996 outcomes in education policy. The 1996 outcomes in education policy was named Curriculum 2005 (C2005) which was implemented in 1998 (DBE, 2012). The C2005 was sometimes referred to as Outcome-Based Education (OBE). Nonetheless, the OBE curriculum was changed into National Curriculum Statements (NCS) in 2004, citing structural problems in the OBE curriculum (Engelbrecht & Harding, 2008). The NCS was subsequently changed to Curriculum Assessment Policy Statements (CAPS) in 2012.

Each of the introduced policies was not given a test of time before changing them into another one. The researcher argues that frequent change in education policy may create learning instability in the mathematics classroom.

(iii) Old fashioned pedagogy.

Mji and Makgato (2006) cited old fashioned pedagogy as part of the problems causing mathematics learning difficulties in South African school learners.

Perhaps this problem might have stemmed from the large number of underdeveloped teachers from apartheid era, who were tagged ‘underqualified professional’ and the fact that teachers found it difficult to adapt to new pedagogical approaches.

In the first place, the underqualified teacher inherited from the apartheid government (Seroto, 2004) may not be able to teach mathematics properly, that is why they are called inferior teachers. This will continue to cause learning disabilities in the mathematics learners. This problem is complicated when these underqualified teachers train the mathematics student teachers, therefore, perpetuating the reproduction of underqualified mathematics teachers in this education system.

On the contrary, Howard and Mozejko (2015) indicate that teachers find it difficult to change to new method of teaching. The education system is dynamic; also is the classroom pedagogy. The teaching and learning continuum have moved from the old method of teacher-centred approach to learner-centred approach. The teacher-centred teaching and learning approach is characterised by teachers taking central role in mathematics classroom presentation while the mathematics learners are expected to be quiet, attentive and regurgitate everything said by the teacher. They (learners) are not allowed to express their own opinion but remain as an empty vessel that must be filled with mathematics knowledge (Mascolo, 2009).

Scholars like Piaget, Vygotsky, Bandura, van Hiele, among others, have all shown that knowledge is constructed and not transmitted. This formed the basis for the learner-centred pedagogy. In this pedagogical approach, learners take the lead in their learning, as they form and construct new knowledge. The teachers need to lead the learners into constructing their knowledge by posing leading conceptual questions.

(iv) Overcrowded classrooms

Anecdotally, the researcher observed that the past apartheid government in South Africa did not develop the black communities who constituted about 70% of South African population. The new democratically elected government born in 1994 inherited a huge task of providing classrooms in the existing schools, especially in the black townships and improving the standard of classrooms. Since these problems could not be overcome in one night, and the new democratic government's educational policy emphasised that no child must be denied the right to education, hence most of the mathematics classrooms are overcrowded, more especially schools in black communities.

Perhaps, this is why Marais (2016) mentioned that learners overcrowding in classrooms is a very serious challenge in South Africa. Overcrowded classrooms do not support effective mathematics learning since mathematics conceptual learning requires mental construction of knowledge which necessitates ability to concentrate while having deep thinking.

Mustafa, Mahmoud, Assaf, Al-Hamadi and Abdulhamadi, (2014) note that overcrowding of learners in a classroom has enormous negative influence on the learning of mathematics concepts. Learners' behaviour in an overcrowded classroom are uncontrollable, disruptive and hence, they cannot pay attention to what is taught in the class, and hence this type of behaviour impacts negatively on their academic progress (Benbow, Mizrachi, Oliver & Said-Moshiro, 2007).

Khumalo and Mji (2014) also assert that classroom overcrowding is not conducive for mathematics learning and may even have an adverse effect on the learners' health and facilitate the spread of contagious diseases. Furthermore, the duo added that true and thorough assessment may be impossible. Marais (2016), in his own investigation, discovered that classroom overcrowding make mathematics classroom management difficult and deprive the teacher from implementing various teaching strategies.

(v) Lack of adequate resources in the classroom

For a school to offer quality mathematics teaching, school and classroom resources are key factors to consider. Resources such as school library, electricity, computers, classroom charts, good whiteboard or black boards and toilets are very important components of quality educational deliveries. Inadequate resources are prone to learners' poor academic performance and large school dropout (Sedibe, 2011).

The school may also struggle to keep good, qualified mathematics teachers who may need to be using modern day classroom resources to facilitate their teaching. Most under-resourced schools are in the rural areas. Perhaps that is why most of the learners struggle to enrol in schools in the urban areas (Handal, 2013). This set of learners use collective arranged transport to and from school daily.

(vi) Shortage of skilled mathematics teachers

The researcher logically thinks that the cycle of inferior teachers that were initiated under the apartheid regime resulted in shortage of skilled mathematics teachers that mathematics classroom is experiencing today. Many mathematics learners who were not taught mathematics concepts properly end up mystifying mathematics and run away from it. Therefore, very few school leavers will be qualified to be admitted for a B.Ed. degree in mathematics. Even those ones that were able to be admitted for the course, they struggle to complete because of poor pedagogical background in mathematics.

All the education stakeholders in South Africa are worried about the underperforming problem that plaques the teaching and learning of mathematics in South Africa and as such, continue to look for appropriate remedy to this. In summary, some of the findings from mathematics education research, like Abakah (2019), show that mathematics' conceptual learning difficulties in our schools may be solved through understanding of Euclidean geometry. The findings reveal that if learners can succeed in conceptualising the geometry aspect of schools mathematics, it will assist in overcoming the conceptual difficulties the learners are experiencing in other areas of mathematics. South African primary and secondary schools' mathematics syllabus includes algebra, financial mathematics, trigonometry, calculus and geometry. As important as the geometry aspect of mathematics is, it was just introduced into the schools' mathematics curriculum through the CAPS curriculum which was only introduced in 2012. The teaching and learning of geometry is discussed in the next subsection.

1.2 The teaching of Euclidean Geometry in South African schools

1.2.1 Inclusion of Euclidean Geometry in South African school curriculum

As alluded earlier, the mathematics curriculum in South Africa includes the branches of mathematics such as algebra, financial mathematics, trigonometry, calculus and geometry. In 2012, Euclidean geometry was included in the CAPS curriculum. Golledge (1999) describes the study of Euclidean geometry as a fundamental instrument to gain entry to human knowledge and scientific sphere. Maybe this is based on the reason that Euclidean geometry is included in the CAPS curriculum. Perhaps the intention of the Department of Basic Education (DBE: 2012) is to include the learning of geometry in schools to unlock learners' mathematical thinking ability.

The NSC mathematics examination, which is also known as matric exam, consists of paper 1 and paper 2. Euclidean geometry was inculcated under paper 2. Hence, paper 1 consists of algebra, financial mathematics, number pattern, calculus and probability aspect of mathematics. Conversely, paper 2 includes data handling, trigonometry, and geometry (both analytical and Euclidean geometry) (see 1.3.3 for learners' performance in Euclidean geometry at the NSC exam).

The researcher considers all the foregoing factors as impediments on the path of learners' sound conceptual understanding in mathematics, particularly the Euclidean geometry aspect; hence improving the learning of geometry is paramount. This is the reason the study intends to investigate the students' problem-solving errors in Euclidean geometry in some secondary schools in South Africa. The researcher discusses the challenges facing the teaching of geometry in our schools in the next section.

1.2.2 The challenges facing the teaching of Euclidean Geometry in South African schools

As mentioned earlier, Euclidean geometry was introduced into the South African mathematics curriculum in 2012 into the CAPS school curriculum. The first set of CAPS matriculants was in 2014. The role that mathematics teachers play in the teaching and learning continuum cannot be overemphasised. The teaching of Euclidean geometry in South Africa is not without its own specific challenges. The researcher thinks that the key challenges are the required Euclidean geometry content knowledge by the teacher and the appropriate pedagogy content knowledges.

1.2.2.1 Required Subject Knowledge in Euclidean Geometry

The claim that 'one cannot give what he does not have' applies here. What mathematics teacher knows is what will impact how and what he/she does in the classroom (Turner-Bisset, 2005). It is, therefore, imperative for teachers to have a sound knowledge of the concepts they want to present in the class to learners so that they will be able to respond to any concept specific classroom cognitive demand and be adaptive in their classroom presentation to change the way the information is presented for it to be more understandable to the learners.

There are many mathematics teachers who, throughout their schooling days from primary to tertiary education, were not ready to be taught Euclidean geometry. This cohort of teachers might find the teaching of Euclidean geometry very difficult because they might not have the required subject content knowledge. Atebe (2011) avers that some teachers mystify the Euclidean geometry aspect of the mathematics curriculum; hence they avoid teaching this aspect of mathematics. The required concepts in CAPS curriculum from Grade 7 to Grade 12 is depicted in Table 1.2.

Table 1.2: Geometry Content Area under CAPS curriculum

GRADES	**MEASUREMENT (CAPS OVERVIEW)**	**SPACE AND SHAPE GEOMETRY (CAPS OVERVIEW)**
7	1. Areas and perimeters of 2D shapes 2. Surface area and volume of 3D objects	1. Geometry of 2D shapes 2. Geometry of 3D objects 3. Geometry of straight lines 4.Transformation geometry 5. Construction of geometric figures
8	1. Areas and perimeters of 2D shapes 2. Surface area and volume of 3D objects 3. The theorem of Pythagoras	1. Geometry of 2D shapes 2. Geometry of 3D objects 3. Geometry of straight lines 4.Transformation geometry 5. Construction of geometric figures
9	1. Areas and perimeters of 2D shapes 2. Surface area and volume of 3D objects 3. The theorem of Pythagoras	1. Geometry of 2D shapes 2. Geometry of 3D objects 3. Geometry of straight lines 4.Transformation geometry 5. Construction of

GRADES	MEASUREMENT (CAPS OVERVIEW)	SPACE AND SHAPE GEOMETRY (CAPS OVERVIEW)
		geometric figures
10	1.Properties of quadrilateral: kite, rectangle, rhombus, parallelogram, square and trapezium 2. Solve problems and prove riders using the properties of parallel lines, triangles and quadrilaterals 3. The midpoint theorem	1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of shapes. 2. Revise the volume and surface areas of right prisms, cylinders, spheres, right pyramids and right cones
11	1. Revision of properties of quadrilaterals	**1.CIRCLE GEOMETRY**
12		1. Conditions for polygons to be similar 2. Similarity and proportionality of shapes 3.Pythagoras theorem by similar triangles

The geometry content areas required by grade are displayed in Table 1.2. These areas of mathematics require sound geometric conceptual understanding for it to be taught appropriately.

1.2.2.2 Appropriate pedagogy content knowledge (PCK)

The learning of Euclidean Geometry requires abstract and deductive conceptual mental construction that is of deep mathematical thinking. This process needs extra effort and a special teaching approach compared to the teaching of other part of mathematics. Perhaps this was the thinking of van Hiele (1999), when he proposed his geometry instructional model. Van Hiele's instructional model proposes synthetic presentation of geometric concepts. The instructional model are interview, direct orientation, explanation, free performances and integration. Van der Sandt and Nieuwoudt (2003) concede that most South African teachers do not have any knowledge of any pedagogical approach involved in the teaching of geometry.

1.2.3 Learners' Performance in Euclidean Geometry at NSC under CAPS Curriculum

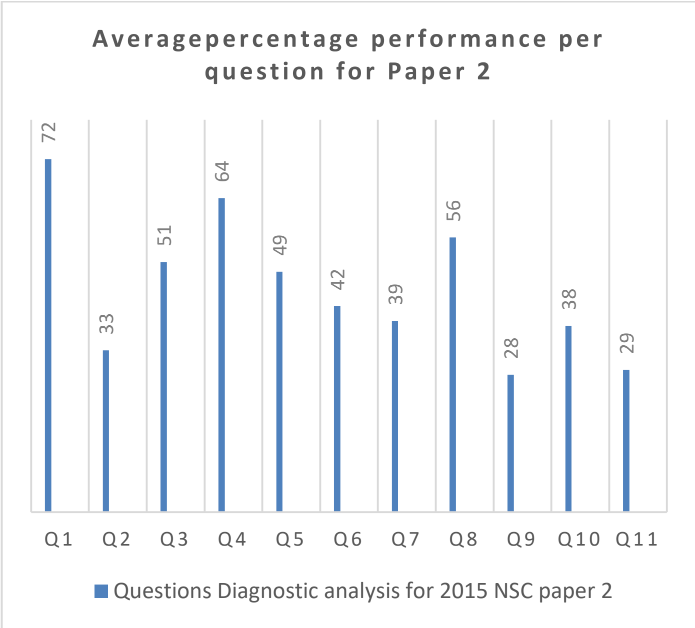
As mentioned in 1.3.1, NSC comprises paper (1 & 2). Paper 1 consists of algebra, financial mathematics, calculus, number patterns and probability, while paper 2 comprises statistics, analytical geometry, trigonometry and Euclidean geometry. Paper 1 has a total of 150 marks while paper 2 has a total of 150 marks in the NSC examination. We shall concentrate on learners' performance which is the focus of this study. Table 1.3 shows the marks distribution among the concepts that made up paper 2.

Table 1.3: Marks Distribution in Paper 2

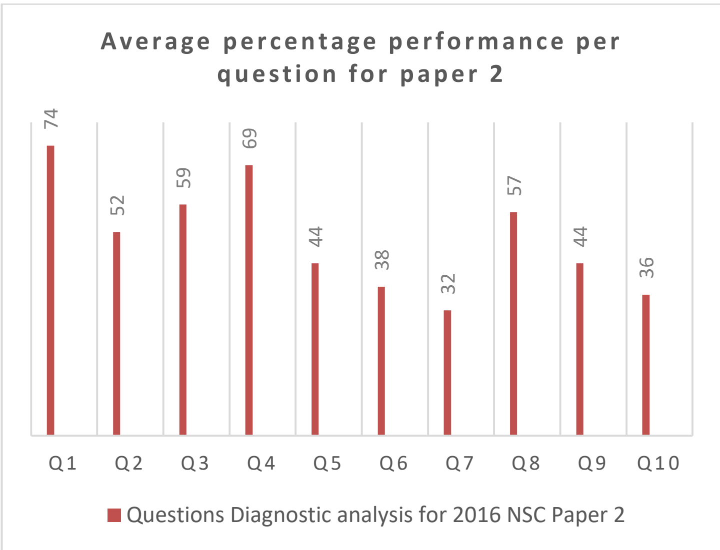
Topics	Marks composition
1. Statistics	20 ± 3
2. Analytical Geometry	20 ± 3
3. Trigonometry	40 ± 3
4. Euclidean Geometry and Measurement	50 ± 3
Total	**150**

From Table 1.3, Euclidean geometry is 50 marks out of the total of 150 marks for paper 2. This implies that Euclidean geometry's marks in paper 2 carries one third of the total marks for paper 2. The presentation in the following figures (1.2, 1.3 and 1.4) are the charts which show how learners performed in paper 2 from 2015 to 2017. The information given in figures 1.2, 1.3 and 1.4 are as found in the Department of Basic Education (DBE) diagnostic reports.

Figure 1.2: Average percentage performance per question for paper 2 (2015).

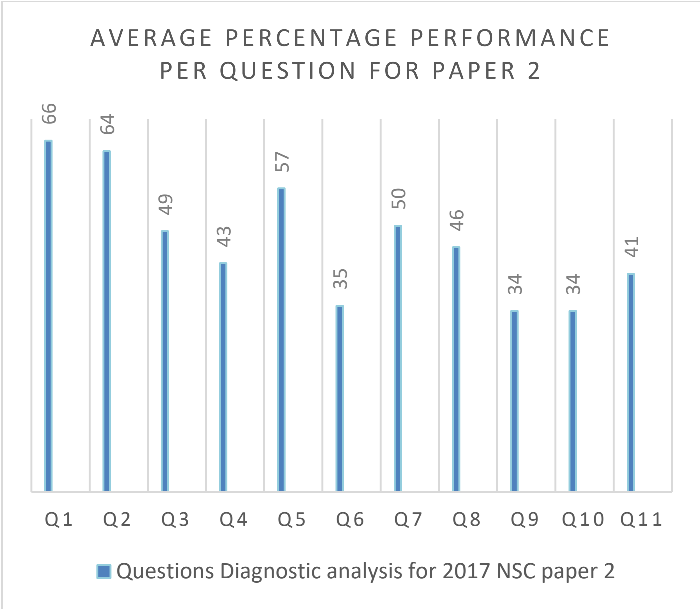


Q1	Data Handling
Q2	Data handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry
Q11	Euclidean Geometry



Q1	Data Handling
Q2	Data handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

Figure 1.3: Average percentage performance per question for paper 2 (2016).



Q1	Data Handling
Q2	Data handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry
Q11	Euclidean Geometry

Figure 1.4: Average percentage performance per question for paper 2 (2017)

The charts in figure (1.2, 1.3 and 1.4) show three consecutive years of poor performance in Euclidean geometry. It must be noted that learners' performance in Euclidean geometry has a significant impact on the pass or fail rates of learners in NSC paper 2 examination. The final mathematics result is based on calculating the addition of the marks from paper (1 and 2) and putting the result over 100%. Both papers have a total mark of 150 each. Hence a failure in paper 2, which may be as a result of scoring low marks in Euclidean geometry part of paper 2, often leads to failure in the NSC examination.

Table 1.3 depicts that Euclidean geometry carries one-third of the total marks of 150 marks in paper 2. Questions 8 to 10 represents Euclidean geometry questions in paper 2 consecutively in 2015, 2016 and 2017. The bar charts in these figures show that learners perform more poorly in geometry when compared to other areas of mathematics which constitutes the paper 2 in the three consecutive years.

Mathematics learners demonstrate poor Euclidean geometry's conceptual understanding in matric examinations annually. Mathematics is about problem-solving, including Euclidean geometry, which is an aspect of mathematics. For learners to be able to perform well in Euclidean geometry, they have to be able to demonstrate good problem-solving skill in Euclidean geometry. This is why the researcher is investigating the learners' problem-solving errors in the learning of Euclidean geometry in some South African schools with the aim of providing alternative learning models that may improve our mathematics learners' performance in Euclidean geometry and improve the overall performance in mathematics.

1.2.4 Theoretical consideration for the study

1.2.4.1 Learning Through consistent engagement

I had earlier given an anecdotal experience that I had as a secondary school mathematics teacher. I taught mathematics in Grade (11 & 12) in two secondary schools from 1996 to 2008. In my second school as a mathematics teacher, I felt so helpless with the manner learners were failing mathematics. Hence, I decided that I will be giving my learners class-test every Friday, mark the test over the weekend and give learners feedback the following Monday. The learners that did well in the test will be my friend for the week. In fact, sometimes I used to buy food for the best learner in the test during the break-time. To my surprise, learners began to do well in my subsequent tests, as a result the school got good matric results that year. My conclusion at that time was, I motivated the learners to consistently engage their studies, which resulted in facilitating their learning of mathematics concepts and improve their mathematics performance.

When I was thinking of how to develop intervention materials that could probably mitigate the problem-solving problem the learners that wrote the scripts used in Phase I had, I remembered my anecdotal intervention when I was a teacher. Therefore, I had an idea that 'consistent engagement of learning material might improve problem-solving difficulty'. This is what brought about development of 'Euclidean Geometry Problem-solving Learning Model' (EGPSLM) and 'Euclidean Geometry Problem-solving Instructional Approach (EGPSIA).

1.2.4.2 The Study Underpinned Theory

Van Dyck et al (2005) assert that teachers should conduct analyses of errors and implement alternative method to curb or alleviate the misconceptions and errors. This gives a direction on how the study was conducted. Therefore, this study investigated South African school learners' problem-solving errors in Euclidean geometry with a view to develop appropriate pedagogy that may help to correct these errors during problem-solving. Hence, the study is in two phases: Phase I investigated mathematics learners' problem-solving errors in Euclidean geometry and Phase II investigated the potency of the intervention in correcting the errors and improve learners' performance in Euclidean geometry. The study is underpinned by the Newman problem-solving error analysis (Newman, 1977).

Newman mathematics problem-solving error analysis

Newman mathematics problem-solving error analysis proposed five ways by which problem-solving error occur in learners' mathematics problem-solving procedure.

These are:

(1) Reading error

This is the error committed in learners' ability to read mathematical problems given and to identify sentences and mathematical symbols used. This type of errors emanated from the English language proficiency of the mathematics learners.

(2) Comprehension error

Learners' ability to understand the mathematics problem read. These errors have to do with not been able to link what was read and Euclidean geometry concepts.

(3) Transformation error

The learner's ability to convert the information given into determining the appropriate method of solution. This error implies that the learners are not versatile enough to decipher some core concepts (like theorems, injunctions, deductions and so on) in Euclidean geometry.

(4) Process skill error

This involves the learner's ability in processing the chosen method of solution in accordance with the correct procedure. This error implies that the learners are not versatile enough in problem-solving approach.

(5) Encoding error

The learner's ability to write according to the given question. That is, the learner finished solving the problem but misinterpreted what the answer meant. These errors have to do with calculation accuracy.

The researcher could not find any study that links Polya problem-solving theory which was first published in 1945 to Newman mathematics error analysis, which was first published in 1977. It appears, however, that Newman error analysis builds on Polya problem-solving theory. I would like to present the seemingly similarities between the theories in the Table 1.4.

Table 1.4: Similarities between Newman mathematics error analysis and Polya problem-solving theory.

Level	**Polya problem-solving theory**	**Newman Error Analysis**	**Comments**
1	Understand the problem	Reading Error	Both theories emphasise reading and understanding the mathematics problem given. Newman stresses failure to understand the problem in terms of error.
2	Devise a plan to solve the problem	Comprehension Error and Transformation Error	While Polya proposes that after understanding the problem, now devise how to go about solving the problem, Newman stresses the process of planning the solution approach, which includes comprehending the problem and transforming the

Level	Polya problem-solving theory	Newman Error Analysis	Comments
			problem into a solvable mathematics form. The errors that emanated in the process of devising the appropriate solution approach is what Newman termed comprehension error and transformation error.
3	Carry out the plan	Processing Skill Error	Next in both theories is to present a solution approach. The error that ensued in this process is what Newman termed processing skill Error.
4	Looking back	Encoding Error	Polya says check your solution answer and its process presented for correctness, while Newman says failure to arrive at the right answer is an encoding error.

Accordingly, Newman error analysis theory was applied in the study in Phase I to scrutinise the errors committed in solving the Euclidean geometry questions in paper 2 of the NSC matric examination and was used to analyse the type of errors committed and their associated error factors.

1.3 Significance of the study

Every year, matric pass rate in mathematics has been very poor. It is also academic fact that South African mathematics learners are always trailing in global ratings in mathematics. It should not be forgotten that mathematics is one of the very important subjects that are pre-requisite to university entrance into courses like medicine, engineering, accounting, actuary and economics. These courses are the economic backbone of any country.

In view of the foregoing, it is very important that everything should be done to ensure that South African mathematics learners are sound in the knowledge of mathematics. To this end, the researcher believes that this study has the potential to improve South African learners' performance in mathematics. Some universities in South Africa are currently running a pre-degree programme for some of their science students partly because they did not pass very well in mathematics. When learners' performance in mathematics is improved, then many matric students will inevitably pass mathematics. In addition, the quality of doctors, engineers, accountants, actuary experts and economist in the country will be of high standard.

1.4 Objective of the study

The objective of this study are as follows:

- To investigate the type of errors South African mathematics learners committed during problem-solving in Euclidean geometry.
- To investigate how the errors contribute to the study participants' performance.
- To propose a remedy that could help the learners overcome committing errors during problem-solving in Euclidean geometry.
- To improve the performance of mathematics learners in Euclidean geometry examinations.

1.5 The problem of the study

As stated in subsection 1.2, South African mathematics learners perform very badly in the Euclidean geometry aspect of any mathematics examination that contains Euclidean geometry (see subsection 1.3.3 of this study), as alluded to by Abakah, (2019). Many studies like Baful and Derequito (2022) have been conducted with the aim to solve the problem of South African school learners' poor performance in mathematics. However, the researcher believes that collecting data on the mathematics matric students' deficiencies in matric paper will the best approach to this problem so that intervention can be developed to remedy the problem. Therefore, the problem of this study was to investigate the type of errors South African mathematics learners commit while solving Euclidean geometry, how errors were committed and how to improve South African mathematics learners' performance in Euclidean geometry. This research approach is consistent with Hashemi, Abu, Kashefi, and Mokhtar (2015), and Rusyda, Kusnandi and Suhendra (2017), in which difficulty was found in the process of learning, and intervention was developed to mitigate students' difficulties. Hence, the research questions stated shall be used to find answers to the problem of the study.

1.5.1 The Research Questions

The research questions are stated according to the Phase in which they are applicable:

Phase I

- What are the types of errors committed by the South African mathematics learners in solving the Euclidean geometry problems aspect of the mathematics paper 2 examination used in this study?
- How does the errors committed in the research question (1) above affect the marks obtained in the Euclidean geometry aspect of the mathematics paper 2 matric examination used in this study?
- How does the marks obtained in Euclidean geometry aspect of the mathematics examination paper used in this study impact the total marks obtained in the mathematics paper 2 examination as a whole?
- How does the pattern of errors committed in the Euclidean geometry aspect of the 2020 NSC mathematics examination paper 2 compare to the pattern of errors committed in the 2021 NSC mathematics examination paper 2.

Phase II

- Does the intervention facilitate the study participants' learning of Euclidean geometry concepts?
- How does the study interventions impact the participants' performance in Euclidean geometry?

In addition, the following hypotheses stated in a null term at 0.05 probability significant level were used to guide the study:

Phase I

Hypothesis one:

H_0: There is no statistically significant effect when comparing the errors committed in solving the Euclidean geometry problems aspect of the mathematics examination paper used in this study with the marks obtained in the Euclidean geometry aspect of the examination.

H_1: There is statistically significant effect when comparing the errors committed in the process of solving Euclidean geometry problems aspect of a mathematics examination used for this study and the marks obtained in the Euclidean geometry aspect of the examination.

Hypothesis Two:

H_0: There is no statistically significant effect when comparing the marks obtained in the Euclidean geometry aspect of the mathematics examination paper used for this study and the marks obtained in the rest of paper 2 mathematics examination without the Euclidean geometry marks.

H_1: There is statistically significant effect when comparing the marks obtained in the Euclidean geometry aspect of the mathematics examination paper used for this study and the marks obtained in the rest of paper 2 mathematics examination without the Euclidean geometry marks.

Hypothesis three:

H_0: There is no statistically significant difference when comparing the errors pattern committed in the Euclidean geometry aspect of the 2020 NSC mathematics examination paper 2 to that of 2021 NSC mathematics examination paper 2.

H_1: There is statistically significant difference when comparing the errors pattern committed in the Euclidean geometry aspect of the 2020 NSC mathematics examination paper 2 to that of 2021 NSC mathematics examination paper 2.

Phase II Hypothesis:

H0: There is no statistically significant difference between the study participants' pre-test score and the post-test score in this study.

H1: There is statistically significant different between the study participants' pre-test score and the post-test score in this study.

1.6 Definition of key terms

1.6.1 Problem-solving

Polya (1985) defines problem-solving as the process by which the learner confronts and resolves a problem until a solution is reached. According to Polya (1945), problem-solving is the cornerstone of mathematics programmes, facilitating the acquisition of skills and concepts. Sternberg (2003) defines problem-solving as "a process of moving from a situation in need of resolution, overcoming any obstacles along the way." Sternberg (2003) categorised problem-solving into five steps: recognising that there is a problem; generating an approach for problem-solving; allocating resources for problem-solving; monitoring the problem-solving process; and evaluating the solution.

1.6.2 Constructivist

Constructivism is not merely a description of learning; it is a theory about learning. Learning, according to constructivist principles, requires invention and self-organisation from the learner's side, and it perceives learning as a developmental process (Fosnot, 2013). In the same vein, Duffy and Cunningham (2013) emphasised that learning is the active process of constructing new knowledge, while instruction is the facilitation of that construction. They further noted that learning necessitates activity and is influenced by contextual factors. According to Lesh and Doerr (2003), constructivism is a theory that recognises learners' construction of new knowledge and understanding, integrating it with their previous knowledge.

1.6.3 Constructionist

According to Papert and Harel (1993), constructionism is an educational theory in which learners are required to create physical artifacts to apply the learned content and to experience tangible results. They further elaborate that constructionism is often seen as learning by making.

1.7 Organisation of the study

Chapter one of this study gives the background and the problems associated to the teaching and learning of Euclidean Geometry in South Africa. It also includes the theorems underpinning the study and the research questions.

Chapter two discusses the main theme of the study. Problem-solving in Mathematics (Euclidean geometry). The theme is discussed in view of the nature of mathematics, the nature of teaching and learning and the associated theories.

Chapter three contains the review of similar studies on the subject matter, while chapter four gives the detailed methodology used in conducting this study research. This includes research design and data collection strategies employed in this study.

Chapter five explains the strategies used to analyse the data collected and presents results of the data analysis. While chapter six presents the summary of the study, discuss the findings from the study, discuss the implication and limitation of the study. Lastly, in chapter six, conclusion and recommendations are presented.

1.8 Conclusion of the Chapter

This chapter has described the problem facing mathematics learners in South Africa, more especially in the Euclidean geometry mathematics learners. It shows how these problems has held mathematics learners down to yearly poor performance in the NSC examination, otherwise known as matric examination in Euclidean geometry aspect of mathematics. By and large, it prepared the premise and the study underpinned theory, on which the study builds.

CHAPTER 2

CONCEPTUAL AND THEORETICAL FRAMEWORK

2.0 Introduction

This chapter discusses the geometrical conceptual and theoretical framework associated with this study.

2.1 Conceptual Framework

2.1.1 Background of Mathematics Concepts

Mathematics is a learning area that involves finding solutions to mathematics problems. I would like to define two important phrases from the preceding sentence. These are: 'mathematics problems' and 'find solution'. In this study, I refer to 'mathematics problems' as mathematics activity or a described situation that is implicit, which needs to be made explicit mathematically, while 'find solution' is the process of making explicit, the implicit 'mathematics problems.' This process is called problem-solving. Therefore, one can say that mathematics is all about problem-solving.

The process of making mathematics activity or a described situation that is implicit to be explicit explores the abstract thinking of the mathematics learners. Mathematics learners can link abstract thinking with the concrete application of implicit mathematics problems. This process builds the mathematical conceptual understanding of the learning domain. The concepts of 'mathematics problems' and 'find solution' are discussed in detail in subsection 2.2.

2.1.1.1 Mathematical Problem-Solving

Mathematical problem-solving entails thinking and following certain laid-down rules and patterns in the process of solving a mathematics problem. These rules and patterns are called *concepts*. For example, the rules and pattern that govern the operations in addition are different to that of division and multiplication, and vice-versa. Similarly, the rules and patterns that govern problem-solving in addition, division and multiplication are called the fundamental concepts of mathematics; they are also referred to as Arithmetic. Arithmetic is a mathematics concept which is of the lowest cognitive demand. Arithmetic concepts lay the foundation for other mathematics concepts which require higher cognitive demand. Arithmetic concepts are taught from the nursery and primary school educational levels, and the hierarchy of the cognitive demand of the mathematics rule and pattern progresses as the level of education progresses. Figure 2.1 depicts a brief summary of mathematics branches that took their roots from the Arithmetic concepts.

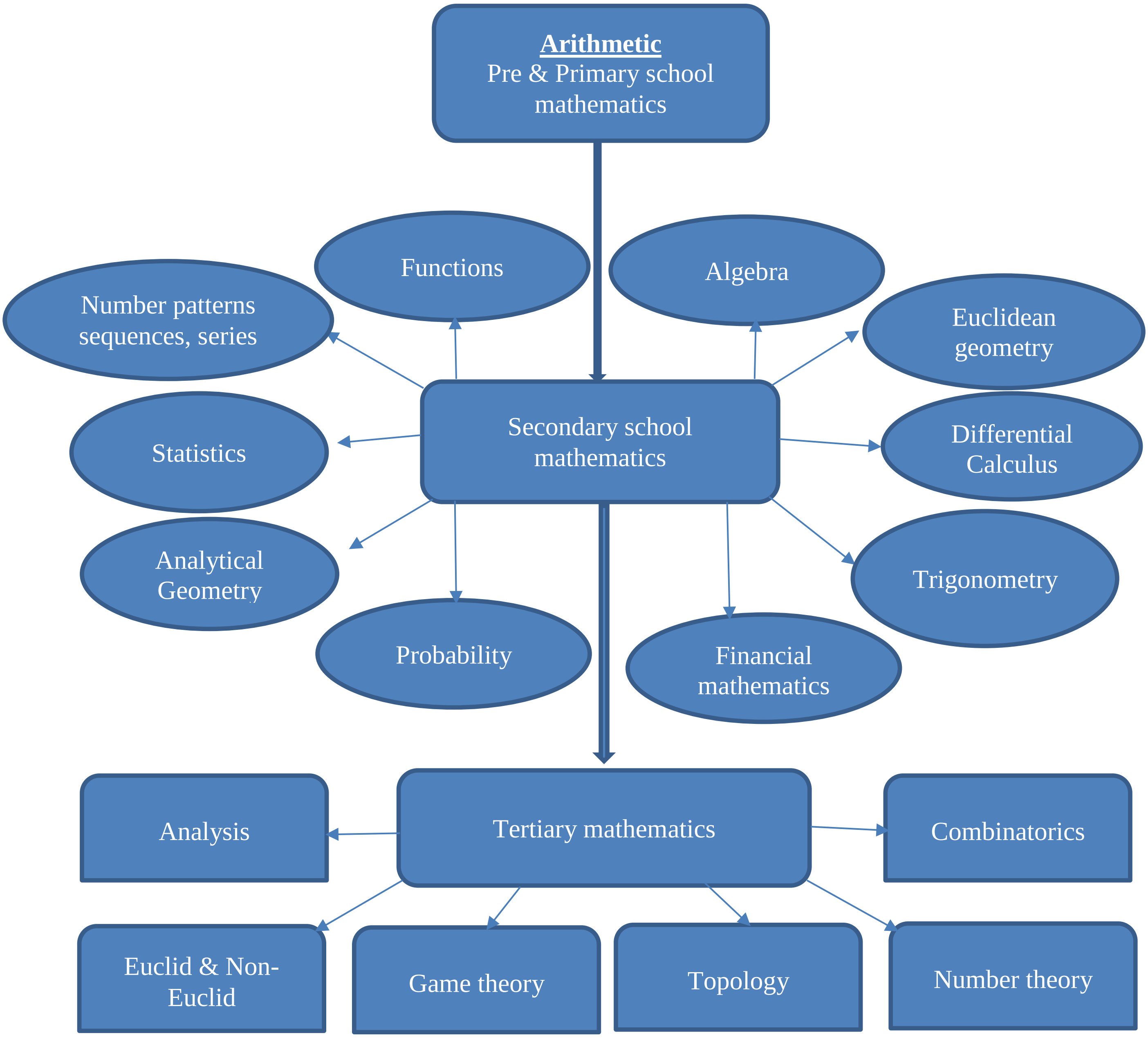


Figure 2.1: Brief summary of mathematics branches

The focus of this study is problem-solving in geometry aspect of mathematics, particularly the Euclidean geometry at the secondary school level.

2.1.1.2 Historical Background of Problem-Solving Concepts in Geometry

In this section, the researcher traced the development of geometry from the stone age to the emerging of Euclidean geometry and how it evolved to the present time. The researcher cannot claim to present every detail about how geometry evolved but he tried to present the most notable ones.

2.1.1.2.1 Early Concepts of Problem-Solving in Geometry

The reading of literatures on the history of geometry elucidates the fact that the early practice of geometrical problem-solving is contained in the Papyri. Papyri are the records of the books' journey from the oral tradition to the scrolls and to the codices. Hence, the ancient geometrical practices discussed in this work are what some authors are able to capture from some Papyri like Rhind Papyrus, Ahmes Papyrus, Moscow Papyrus and so on.

According to Byer, Lazebnik and Smeltzer (2010), geometry and the study of numbers were the two pre-modern age mathematics, while the earliest practice of geometry dated back to around 3000 Before Christ (BC). In Greek world, 'Geo' means 'the Earth' and 'metron' means 'measurement;' hence in Greek world, geometry means 'measurement of the earth' (Bayer, et al. 2010). Prabir Datta (2019) asserts that the earliest geometrical collection was about the discovery of obtuse triangle in the ancient Indus valley and the ancient Babylonia around 3000 BC, and that, most of the early practice of geometry was about the principles of length, angles, areas and volumes which were conceptualised to fulfil some needed activities in the area of craft, constructions and surveying. For example, both the Egyptians and the Babylonians were aware of what was later called 'Pythagorean theorem' long ago (say about 1500 years) before Pythagoras came up with the theorem around 800 BC. Some of these ancient geometrical principles are discussed in the next subsections. The discussions are according to the account of Forder (1958), Bayer et al. (2010) and Prabir (2019).

(A) Egyptian Early Geometrical Calculations

Bayer et al. (2010) report that Egyptians were expert in surveying and pyramid construction. For example, the great pyramid at Gizeh which is about 480ft tall, with a square base area of about 13 acres. The four sides of the pyramid were aligned with the four cardinal directions: east, west, south and north, within a fraction of a degree. As at 1650 BC, the Egyptians were able to calculate the area and volume of objects, (Prabir, 2019; Bayer et al. 2010). They used the following formula to calculate the area of a quadrilateral:

$$A = \frac{1}{4}\{(a+c)(b+d)\}$$

Where a, b, c and d represent the consecutive sides of the quadrilateral, and the following formular to calculate the volume of square based pyramid

$$V = \frac{\mathrm{h}}{3}(a^2 + ab + b^2)$$

Where a and b are the consecutive sides of the square base of the pyramid and 'h' is the height of the pyramid.

According to Rhind Papyrus in Bayer et al. (2010) and Forder (1958), the Egyptians were more interested in the amount of grains a cylindrical bin can take; hence they adapted the formula.

$$V = \frac{\mathrm{h}}{3}(a^2 + ab + b^2)$$

to get $A = \frac{256}{8}r^2$ for a circle of radius 'r' .

This gives π as $\pi \cong 3.1605$ which is close to today's value of $\pi = 3.1442$. Problem 30 in Ahmes Papyrus use this method to calculate the area of a circle. Moreover, the Papyrus that documented Egyptians' early mathematics revealed that the early Egyptian mathematician knew about Pythagorean Triples. Pythagorean Triples is using three integers to represent the lengths of the three sides of right-angled triangles.

(B) Babylonian Early Geometrical Calculations

Forder (1958) and Bayer et al. (2010) report that the early Babylonians mathematicians were well known for calculating in sexagesimal, that is in base 60. Furthermore, they used to calculate seven miles (11 kilometres) to measure the travel of the sun and acknowledge this as their time-mile. This time-mile was considered as the amount of time that was needed to walk a Babylonian mile (seven miles). This idea was used to divide a circle into 360 equivalent parts, (18, 22). They had an approximation for pie (π) as $\pi = 3.125$ and were using general rules of arithmetic and geometry to measure the areas and volume. For example:

(1) They calculate the area of a Cylinder as:

$$Area\ of\ Cylinder = base\ area\ \times height$$

(2) Frustum of a Cone:

$$Volume\ of\ the\ frustum\ of\ a\ Cone = \frac{1}{2}(Sum\ of\ the\ base\ area) \times height$$

(3) Square Pyramid:

$$Volume\ of\ the\ Square\ Pyramid = \frac{1}{2}(Sum\ of\ the\ base\ area) \times height$$

However, in the modern-day mathematics, equations 2 and 3 above are not correct.

(C) Greek Early Geometrical Calculations

Literature (Forder, 1958; Bayer, 2010; Prabir Datta, 2019) buttresses that around the 640 BC there was trader by the name Thales from Miletus, who used to trade far and wide across lands. He was the one who brought the Egyptian early mathematical geometric concepts to the Greeks. However, contrary to the Egyptian intuitional and experimental approach of developing the concepts, he took the concepts through deductive reasoning for validation. By so doing, he was able to add his own propositions to build on the Egyptian geometric concepts. Some of his propositions were:

- An angle inscribed in a semicircle is a right angle. (This theorem was recognised by the Babylonians since around 1400 years earlier).
- A circle is bisected by its diameter.

- The base angles of isosceles triangles are equal.
- The vertical angles formed by two intersecting angles are equal.
- Two triangles are equal if they have one side and two angles respectively equal. (Some have conjectured that Thales used this proposition to determine the distance from a ship to the shore).

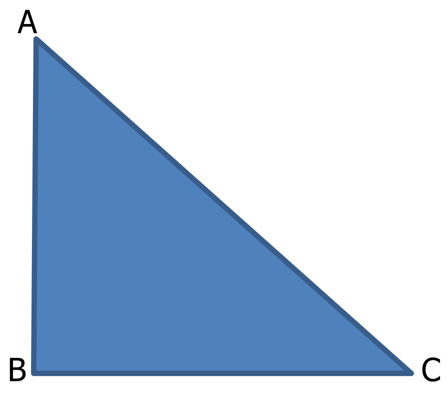
After Thales of Miletus, around 572 BC, the work of one his students by name Pythagoras became famous. Pythagoras believed that whatever, such as religion, matter, music or mankind that is of significant value should be able to be represented by whole numbers. Therefore, he found a school in which the focus subjects of learning are the number theory, geometry, music and astronomy. Most of the work of this school was dedicated to Pythagoras, including the famous Pythagoras theorem which states that:

> In a right-angled triangle, the square of the longest side is equal to the sum of the square of the two other sides.

I illustrate this with the diagram below:

Figure 2.2: Right angled triangle ABC

Given a right-angled triangle ABC



$$AC^2 = AB^2 + BC^2$$

The work done at Pythagoras school was able to formalise the concepts of geometry which was a premise on which the future geometry was built.

Around 460 BC, Hippocrates' work on putting propositions in a logical order, using alphabets letter to signify points and lines in geometrical figures, and using straight edge and compass in geometrical construction of geometrical figures like circles, double the cube and trisect angles. After Hippocrates, Plato the philosopher came on board around 427 BC. He also found a geometric academy. The rigorous academic work done in Plato's academy bridged the link between the early Pythagoreans geometrical concepts and later geometrical scholars.

Prabir Datta (2019) highlights that Eudoxus of Cuidos was a student from Plato's academy who came to be known around 400 BC for his work on two 'magnitude in proportion', and the discovery of irrational numbers. His work lead to been able to represent, for example, the length of the diagonals of unit square with a number.

(D) Problem-Solving in Geometry from Euclid to the Present Time

D(i) Euclid as reported by Bayer, et al.,(2010) and Fitzpatrick (2007).

Bayer et al. (2010) and Fitzpatrick (2007) indicate that between the 400 BC and 330 BC, there were many geometry scholars of which Euclid's geometry conceptual work became most famous. Euclid presented geometry as a visual representation of the physical world in which *terms* like *points, lines* and *plane* may be used to describe observable realities and how these *terms* interact with each other. His geometric intuition was to abstractly define a geometric object, provide an axiom that describes the object and use the rule of logic to develop other axioms about the object.

According to Fitzpatrick (2007), Euclid presented a 13-book treatise containing all known geometrical concepts at that time. This 13-book treatise was named '*Euclid Elements*'. This Euclid Elements contain all known axioms, definitions and theorems to form an axiomatic geometrical system which became a standard for about 2000 years. The title of each book contained in the Euclid Element Book are shown in Table 2.1.

Table 2.1: The Euclid Element 13-Book

Element Book Number	Title of the Book	Number of Definitions, Postulates, Common Notations, and Propositions
1	Fundamentals of Plane Geometry Involving Straight Lines	23 Definition, 5 Postulates, 5 Common Notations, 48 Propositions
2	Fundamentals of Geometric Algebra	2 Definition, 14 Propositions
3	Fundamentals of Plane Geometry Involving Circles	11Definition, 37 Propositions
4	Construction of Rectilinear Figures In and Around Circles	7 Definition, 16 Propositions
5	Proportions	18 Definition,25 Propositions
6	Similar Figures	3 Definition, 33 Propositions
7	Elementary Number Theory	22 Definition,39 Propositions
8	Continued Proportions	27 Propositions
9	Application of Number Theory	36 Propositions
10	Incommensurable Magnitudes	4 Definition,115 Propositions
11	Elementary Stereometry	28 Definition,39 Propositions
12	Proportional Stereometry	18 Propositions
13	The Platonic Solids	18 Propositions in which Propositions 2 is with Lemma, Propositions 13 is with Lemma, Propositions 16 is with Corollary, Propositions 17 is with Corollary, Propositions 18 is with Lemma.

Mathematicians after Euclid saw that Euclid Elements were not totally correct; hence, they continue to improve and modify the elements. These improvements and modifications gave rise to new geometrical concepts, even though the new concepts still share logical conceptual structure with the Euclid's Elements. For example, one of Euclid's Proposition that postulates that '*Any triangle is isosceles,'* this has been found not to be true. The proof and argument proposed by Euclid is given next:

Euclid's Proposition: Any triangle is isosceles

Proof: Consider a ΔABC, construct the angle bisector of the interior angle at A and the perpendicular bisector of segment $\overline{BC}$ at the point 'D'. If the angle bisector at A and the perpendicular bisector of segment $\overline{BC}$ are parallel, then ΔABC is isosceles. Otherwise, if the angle bisector at A and the perpendicular bisector of segment $\overline{BC}$ are not parallel, then they intersect at a point say 'P'. Let 'E' and 'F' be the feet of the perpendicular from 'P' to $\overline{BC}$ and $\overline{AC}$, respectively. Hence, we have what is represented in Figure 2.2.

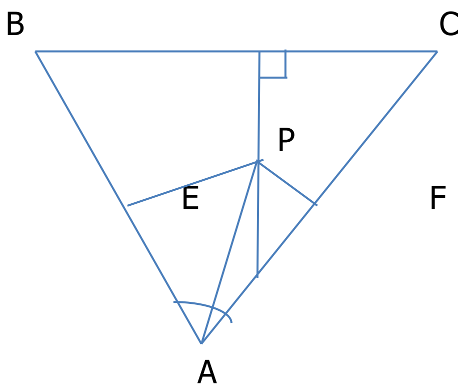


Figure 2.3: Any triangle is isosceles

From Figure 2.3, it follows that,

$\Delta AEP \cong \Delta AFP\ (AAS)$ and $\Delta BDP \cong \Delta CDP\ (SAS)$ from which it implies that $\overline{PE} = \overline{PF}$ and $\overline{PB} = \overline{PC}$, again since ΔBEP and ΔCFP are right angles, the third side must also be congruent so that $\Delta BEP \cong \Delta CFP\ (SSS)$. Thus,

$\overline{AB} = \overline{AE} + \overline{EB} = \overline{AF} + \overline{FC} = \overline{AC}$ and ΔABC is isosceles.

Some of those working on Euclid's Element propose that Euclid's argument and proof was logically wrong starting from the diagram. If the angle bisector at A and the perpendicular bisector of segment $\overline{BC}$ intersect at point 'P', then the point 'P' must lie outside of ΔABC, one of the 'E' or 'F' must lie inside of ΔABC, while the other lie in the exterior of ΔABC, hence the correct diagram should have been as given in Figure 2.4.

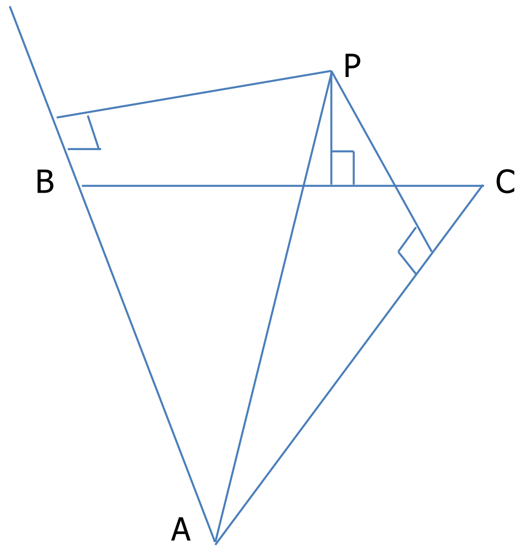


Figure 2.4: Assumption that any triangle is isosceles is not true.

Therefore, the assumption that any triangle is isosceles is not true.

However, van der Waerden (1903 – 1996) informed that regardless of all the imperfections noted in the Elements, it continued to influence the reasoning, theorems and geometrical methods until the advent of the non-Euclidean geometry and the new Euclidean geometry.

D(ii) David Hilbert's Contribution to Euclidean Geometry

According to Bayer (2010) and Fitzpatrick (2007), David Hilbert was a German mathematician who became prominent worldwide for his contribution to Euclid geometry. His intention was to expand Euclid's geometric ideas and to bring in some form of rigour and formalised logic which were missing in the Euclid treatise. He addressed the Euclid terms: *points, lines, plane, between, on, and congruency,* he grouped all axioms into five groups and give heading to each group.

These groups are:

- Axioms of connection (or incidence);
- Axioms of order;
- Axioms of congruence;

- Axioms of parallel; and
- Axioms of continuity.

D(iii) George Birkhoff's Contribution to Euclidean Geometry

After Hilbert, many other mathematicians sprung up and several axioms began to populate the Euclidean geometry platform. Notable of these mathematicians was the American George Birkhoff whose work in geometry put the points on any line in Euclidean geometry in one to one correspondence with the real number. This approach associates the real number properties with the points on any line.

D(iv) SMSG's Contribution to Euclidean Geometry

According to Bayer (2010), in the mid-19th century, the United States realised that they were behind the Soviet Union in mathematics, science education and research. To this end, National Science Foundation (NSF) was created. One of the strategies of the NSF was to establish a mathematics study group called school Mathematics Study Group (SMSG). SMSG was very productive; they proposed many axioms, some of which were rejected by other world mathematicians but many of their axioms were accepted and formed part of the axioms which are still in use by the present-day mathematicians.

This work focuses on the Euclidean geometry concepts in South Africa high school mathematics syllabus (see subsection 1.3.2.1 of this work for the detail of the concepts).

2.2 Theoretical Framework

2.2.1 Mathematical Problem-Solving

Mamona-Downs and Downs (2005) describe how difficult it is to define mathematical problem-solving. However, in this study, the researcher adopts the definition Polya offered for the definition of problem-solving. Polya defines problem-solving as finding a way around a difficulty or finding a solution to an unknown problem (Polya, 1945).

According to Hardin (2023), who enlightens that normally a problem has three components namely *given*, *goal* and *operation*. These components are explained next:

- Given: are pieces of facts or the information offered to illustrate the problem.
- Goal: how the problem looks like after been solved.
- Operations: are the actions that are needed to be conducted in the course of turning the problem into the desired goal.

A problem may be properly structured or ill-structured. A properly structured problem is explicitly presented, while ill-structured problems are implicitly presented. Implicitly presented problems may require higher cognition to reach the target goal because of the oblique nature of the problem.

In solving a mathematical problem, both declarative and procedural knowledge are required. Declarative knowledge is the knowledge of knowing that *something is,* and procedural knowledge is the knowledge of *how-to*. A problem-solver needs these two types of knowledge in solving a problem (Schoenfeld & Herrmann, (1982) in Hardin, 2023). The following diagram illustrates the problem-solving process.

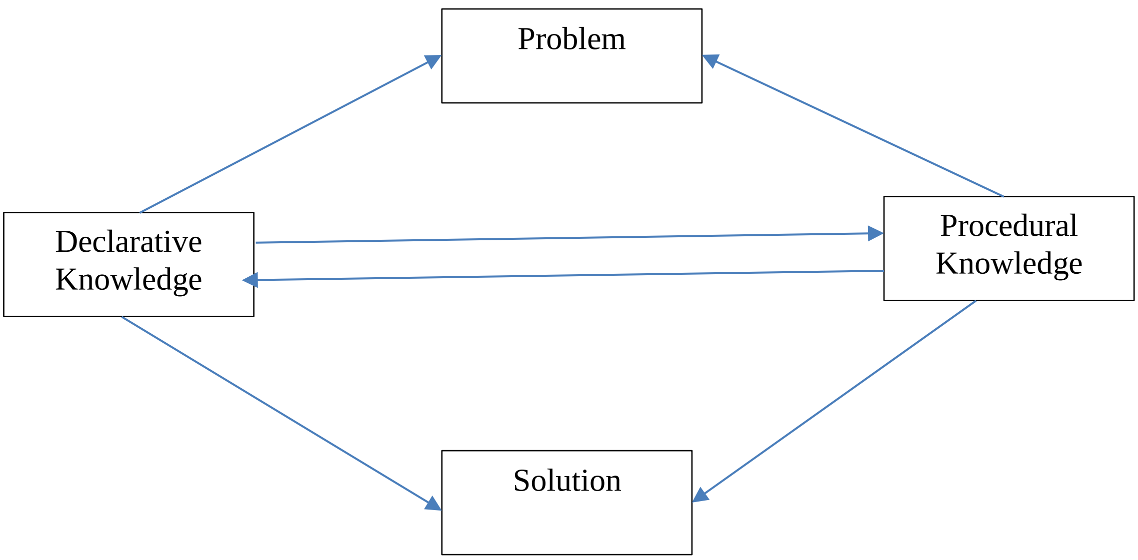


Figure 2.5: The Interaction between Declarative and Procedural Knowledge in Problem-Solving

In Figure 2.5, declarative and procedural knowledge interact with the problem to produce the solution.

2.2.2 Features of a would-be Successful Problem-Solver

For someone to be a successful problem-solver, he/she must have the conceptual understanding of the problem specific domain, have basic skills of the problem specific domain and the domain specific strategies (Schoenfeld & Herrmann, 1982 in Hardin, 2023).

(i) Conceptual Understanding of the Problem Specific Domain

Schoenfeld and Herrmann (1982) enlighten that conceptual understanding is the domain-specific information residing in the memory, and this information is arranged in the memory. Schema theory explains the arrangement of information in the memory.

Hardin (2023) explains that schema theory is a cognitive framework or concept that helps organised and interpret information. This cognitive framework is a mental model found in the long-term memory. Schema theories explain how the conceptual understanding of a domain by an individual can help make meaning of domain-specific situation or problem, based on the prior knowledge of that domain. Figure 2.6 is an illustration of the memory cognitive framework.

Human Memory

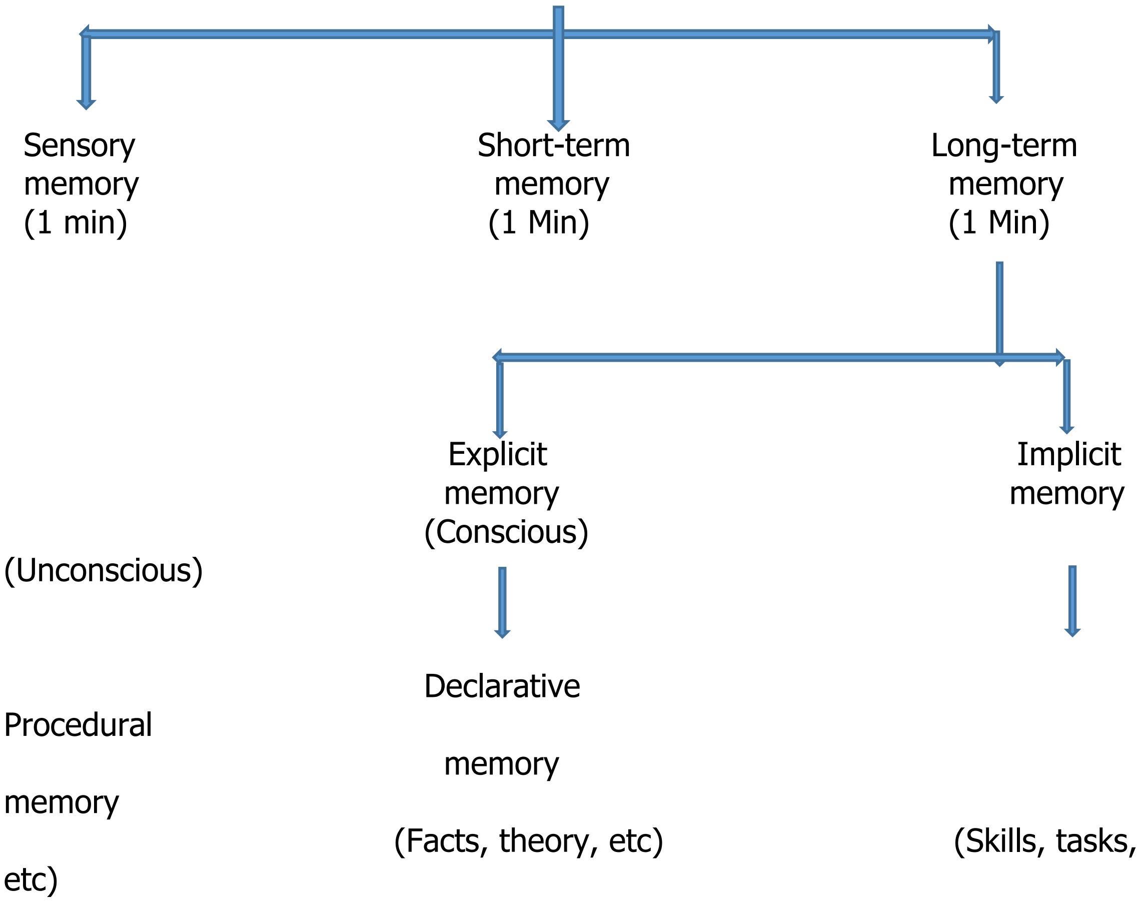


Figure 2.6: Illustration of the Memory Cognitive Framework

Online picture Available at

https://www.bing.com/images/search?q=illustration%20of%20the%20human%20memory

Piaget (1969) explains that schema is a building block of intelligent behaviour, a way of organising units of knowledge, such that each unit knowledge relates to one aspect of the world, like objects, actions, abstract concepts and so on. Piaget maintains that babies are born with genetically inherited basic schemata structure. It is on these basic schemata structure that subsequent learning and knowledge are based as the baby's schemata evolves. When a child is able to carry out an operation or can explain what he/she sees around him/her, then he/she is said to be in a state of cognitive equilibrium. A child who carries out an operation has sophisticated complex cognitive structure he/she uses to combine schemata in a logical way. As children grow, they improve on the complexity of the operations they can carry out and their abstract imaginations.

(ii) Basic Skills

Basic skills in a specific domain are the fundamental/primary skills that permits a problem-solver to carry out essential routine operations with ease. The skills are overlearned such that they become unconsciously implemented or habitually carried out automatically when the need arose. It helps in quick and accurate results-oriented problem-solving.

(iii) Domain Specific Strategies

These are the processes and procedures to follow in specific domain problem-solving. They are procedural in nature and consciously executed. The schemata that control this cognition are located in long-term memory. Unlike the basic skills, they are thoroughly thought of.

2.2.3 Mathematical Problem-Solving Theories

The researcher presents mathematical problem-solving theories from the behaviourist, cognitive and information processing point of view.

2.2.3.1 Behaviourist Theories

The behaviourist problem-solvers see problem-solving as a process that develops through positive and negative reinforcement mechanisms. They focus on the role stimuli-response interaction might play on problem-solving. Behaviourist like Thorndike (1911) in Faleye (2011) and Skinner (1966) link ability to solve a problem to the relevant previous experience, while Gestaltists believe that previous experience alone is not enough to solve a problem; the problem organisation is also required. The researcher discusses trial and error problem-solving theory and Hull's responses hierarchy problem-solving for example.

2.2.3.1(i) Trial and Error Problem-Solving Approach

Trial and error problem-solving approach was as a result of Thorndike stimuli-response (S-R), (1911) learning theory. Thorndike's learning theory is a learning process in which many attempts are tried out until when the problem is eventually solved. Hence, trial and error problem-solving approach involves trying multiple methods to find a solution; this continues until a solution is found. For example, children playing the game of jigsaw puzzle fixing exhibit this problem-solving method. When they are solving the jigsaw puzzle, they continue to try to fix different pieces into the same spot until they can finally find the piece that fits the spot.

2.2.3.1(ii) Hull's Responses Hierarchy Problem-Solving Hypothesis

Hull's problem-solving hypothesis involves learned responses that are applied to a situation in a hierarchical manner. The hypothesis posits that solution tendency increases with proximity to the solution (Hull, 1932). The hierarchy is based on the response for which habit strength is the strongest. Stimuli in a problem situation may evoke several different responses, and responses will be produced one at a time, in order of strength until the problem is solved or the organism exhausts its repertoire of responses.

2.2.3.2 Cognitive Theories

Cognitive problem-solvers view problem-solving as mental stages through which problem-solving proceeds. In fact, mental complex activities such as specialising, conjecturing and generalisation are manifestations of the problem-solvers' cognitive abilities as they explore solving problems.

Jean Piaget, one of the early cognitive theorist, accentuates that cognitive capabilities evolves in an orderly manner; hence certain prior thinking processes formed a foundation to the later ones (Piaget, 1977). Therefore, in simple terms, one can say that cognitive problem-solving involves a mental process that includes introspection, observation and the development of heuristics to solve a problem. The foundational cognitive problem-solving theories are discussed next.

2.2.3.2(i) Graham Wallas Creative Problem-Solving Approach

Wallas (1926) pioneered a complete creative step to problem-solving. The proposed steps are in four stages: preparation, incubation, illumination and verification. Each of these steps are explained as follows:

- Preparation – At this stage, the problem-solver takes time to properly understand the problem and what it might take to solve the problem.
- Incubation – At this stage, the problem-solver's mind subconsciously works on solving the problem.
- Illumination – After incubation, the problem-solver gets an idea on how to solve the problem. This is the metacognition stage of solving the problem.
- Verification – This is the last stage of the proposed problem-solving approach where the problem-solver checks if the proposed idea of how to solve the problem is correct.

The researcher observed that Walla's problem-solving approach provides a platform from which most of the subsequent mathematics problem-solving approaches were formulated. Mathematical problem-solving via Wallas' Four Stages of Creativity: Implication for the undergraduate classroom.

2.2.3.2(ii) Polya Problem-Solving Approach

After the publishing of Wallas' book in 1926, around 1957 George Polya published a book titled, "How to Solve It" (Polya, 1957). In this book, Polya presented four stages of solving a problem. The stages are understand the problem, device a plan to solve the problem, carry out the plan and look back. These steps are explained as follows:

- Understand the problem – The problem-solver should know what the known are and the required variables in a problem.
- Device a plan to solve the problem – The problem-solver is to engage in mental processes to explore how to solve the problem, including using

prior knowledge of solutions to similar problems, conduct introspection and observation.

- Carry out the plan – At this stage, a heuristics approach to solving the problem is presented and used in solving the problem.
- Looking back – At this stage, the problem-solver should check the correctness of each step of the solution.

From the 19th century to date, many other proposed problem-solving approaches have been coming out because of continuing research on the problem-solving approach by researchers in the field of mathematics education. But all of them have the form of the foundational problem-solving approaches framework discussed earlier. However, there are other theories that are not problem-solving theories per se, but they have a direct impact on problem-solving approach. The researcher is referring to theories like van Hiele learning theory, Newman error problem-solving analysis, and so on. Therefore, the researcher discusses van Hiele's learning theory and Newman error problem-solving analysis in the next subsection because they have some connection with this study.

2.2.3.2(iii) van Hiele Geometric Concepts Learning Model

Van Hiele learning model is not a problem-solving approach but a model that prescribes how mathematics learners can learn geometrical concepts according to their reasoning ability. In subsection 2.1.1 of this work, mathematics was defined as a learning area that involves finding solution to mathematics problems; hence, in mathematics we learn by solving problems. This definition makes van Hiele's learning model relevant to this study. The model postulates five levels that describe the cognitive reasoning ability of learners in geometry. The five levels are visualisation, analytical, abstract, deduction and rigour. For the sake of the readers, the researcher gives a brief explanation of each level.

Level 1 – Visualisation

Learners conceptualise only the appearance (figure) of shapes but do not know anything about their properties.

Level 2 = Analytical

Learners are able to conceptualise the appearance (figure) of shapes along with their properties.

Level 3 – Abstract

At this geometric conceptual understanding, learners understand relationship between shapes. They are able to conceptualise appearance (figure) of shapes, their properties, form abstract definition about the shapes, and understand the 'necessary and sufficient conditions' concepts in geometry.

Level 4 – Deduction

Besides understanding all the concepts prescribed for level 3, learners on level 4 understand the concept of deduction and deductive reasoning. They can provide conceptual proofs where necessary, understand the concept of axioms, postulates and theorems.

Level 5 – Rigour

Learners at level five understand every concept expected to be understood by learners at levels 1 to 4. They are able to solve complex geometrical problems that learners in level 4 may not be able to solve.

Van Hiele's learning model did not prescribe any particular order or steps in problem-solving but advice on the type of geometrical problems that can be given to learners according to their geometrical reasoning abilities. On other way round, learners' knowledge in geometry can be categorised to either low or high level, where van Hiele level 1 is low and van Hiele level 5 is high.

2.2.3.2(iv) Newman Error Analysis

Newman error analysis (NEA) was published in 1977 by M. Anne Newman, an Australian mathematics educator. NEA is a diagnostic procedure that emphasises the challenges a mathematics problem-solver must encounter in the process of solving mathematics problems (Newman, 1977). These challenges are in hierarchy as follows: reading, comprehension, transformation, process skills, and encoding (Newman, 1977: 1978). These hierarchy are explained as follows:

- Reading the problem;
- Comprehending what was read;
- Carry out a mental transformation from the words of the question to the selection of an appropriate problem-solving strategy;
- Use the correct heuristic approach as demanded by the selected strategy; and
- Encode the answer as demanded by the question.

Newman (1978) argues that if a problem-solver should fail at any level of the above stated hierarchy, he/she may not be able to obtain the correct answer. Casey (1978) avers that problem-solver often revisits the reading and comprehension levels in the process of problem-solving either to get more clarity of when confused or when feeling that what they are doing is wrong.

2.2.3.3 Information Processing Theories

From the cognitive problem-solving theories, the researcher observed that human cognitive problem-solving reasoning has limited abilities. Perhaps, that is why generally when mathematics problems increase in the level of difficulties then only fewer mathematics problem-solvers that can accomplish solving the problems. Hardin (2023) articulates that human cognitive ability in problem-solving depends on the working memory capacity, organisation of the long-term memory (Stamata), and the cognitive retrieval of relevant information. This might have necessitated the use of artificial intelligence (AI) in problem-solving. AI is the development and use of computer algorithms or software programs in problem-solving.

Newell (1972) noted the following basic principle in the development of computer algorithms for solving problems:

- The problem solution processes are in variant over the task.

- Characteristics of the problem are sufficient to determine the problem space (see the definition of problem space below).
- The structure of the task environment determines the possible structure of the problem space.
- The structure of the problem space determines the possible algorithms or computer programs that can be used for the problem solution.

Where the problem space is defined in this study as a vector β containing all the possible components, Å, that facilitates the solution of the problem. Let the components of β be:

- The initial state of the problem, h.
- A set of operators that can transform a problem state, j and
- A test for whether a problem state contains a solution, ρ.

Hence, Å is a subset of β, that is Å $\subseteq \beta$. Let $h, j, \rho \in$ Å, then
$h, j, \rho \in \beta$.

Similarly, let a solution space be a vector space V containing all the possible solutions to the given problem. That is, let V be a space over $\mathbb{R}$. Let $\mathcal{L}$ be a subspace of V, that is $\mathcal{L}\, c\, V$. There exist $v_{1,}\, v_{2,}\ \mathcal{E}\, \mathcal{L}$ such that

- $v_{1,} + v_{2,}\ \mathcal{E}\, \mathcal{L}$
- $av_i\ \mathcal{E}\, \mathcal{L}$ where $a\, \mathcal{E}\, \mathbb{R}$ and
- $av_{1,} + bv_{2,}\ \mathcal{E}\, \mathcal{L}$, where $a, b\, \mathcal{E}\, \mathbb{R}$ Hibert space

2.2.4 Theoretical Consideration for the Study

As mentioned in subsection 1.2.4.2 of this work, this study investigates South African school learners' problem-solving errors in Euclidean geometry with a view to develop appropriate pedagogy that may help to correct these errors during problem-solving in Euclidean geometry. Since then, there will be a need to look for types of errors committed during Euclidean geometry problem-solving, NEA is the most appropriate theory to adopt to the study of all the problem-solving theories discussed in this chapter. It will the best to guide the study into identifying the types of errors committed by the study participants as they solve Euclidean geometry problems.

2.3 Conclusion of the Chapter

It has been shown in this chapter that the concept of geometric problem-solving has been with man since time immemorial. Geometric natural problems that occur in man's live spans intuitive problem-solving, as the solution process to the problems were being refined and more sophisticated, the solutions to the problems were getting more accurate. The Papyrus was able to document the progressive sophistication of the solutions processes. Euclid brought a wider range of abstract geometric problems and more sophisticated solution approach which forms the basis of today's different geometric studies.

CHAPTER 3

LITERATURE REVIEW

3.0 Introduction

In this chapter, similar studies are reviewed. The review is divided into subsections that are relevant to this work. These subsections are problem-solving in Euclidean geometry, effect of problem-solving errors in Euclidean geometry, and subject content knowledge (SCK), pedagogical content knowledge in geometry (PCK) and learners’ attitude in the learning of geometry.

3.1 Problem-Solving in Euclidean Geometry

In chapter 2 of this study, the research described mathematics problem-solving as the process of making explicit, the implicit mathematics problems. Mathematics problems are usually in such a form that the problem-solver will need to seek the solution to the problem. The cognitive demand of the implicit state of these problems depends on the educational hierarchy. For example, the cognitive demand of the mathematics problems for primary school level will be different from that of the secondary school level, and as the educational hierarchy increases, so is the mathematics problem cognitive demand. The geometry aspect of mathematics is considered the most difficult learning area for mathematics learners (Atebe, 2008; Abakah, 2019). As a mathematics subject advisor, I can also anecdotally testify that problem-solving in Euclidean geometry area of mathematics has been difficult for mathematics learners. Abakah (2019) elucidates that most mathematics learners have Euclidean geometry phobia and avoid this aspect of mathematics. In fact, Rusyda et al.’s (2017) study was conducted to establish the fact that mathematics learners are scared of Euclidean geometry aspect of mathematics and are performing poorly in this area of mathematics..

Rusyda et al. (2017) researched about the ability of one secondary school learners in Euclidean geometry problem-solving. Learners were given geometry problems with context in Euclidean geometry mathematics and context in the main mathematics, excluding Euclidean geometry. Based on the findings, it was concluded that the learners' level of mathematics problem-solving ability was better than in Euclidean geometry, which is very low. It was, therefore, recommended that the learners' problem-solving ability can be increased by implementing the correct learning strategy, and learners should be constantly exposed to solving problems that measure their Euclidean geometry problem-solving ability. These results corroborate Atebe's (2008) finding.

From the foregoing findings, there is no doubt that the teaching and learning of geometry in schools need interventions that could help the learners to overcome their challenges in the learning of geometry. Since mathematics is all about solving problems, hence the focus of this study is about problem-solving in Euclidean geometry. In the next subsection, the researcher presents the work of some selected relevant mathematics education scholars who have attempted to facilitate the teaching and learning of mathematics through facilitated problem-solving approaches..

3.1.1 Polya Problem-Solving Hypothesis

Polya (1985) explains that the mathematics problem-solver (who is at the educational level of the problem) seeks the solution until it is found. In the same study, he proposed four problem-solving steps that could facilitate problem-solving in geometry. Mushlihuh and Sugeng (2017) clarified the four steps Polya suggested for solving mathematics problems. According to the duo, the first step is for the problem-solver to understand the problem, come up with a solution plan within the confinement of mathematical concepts applicable to the problem, the problem-solver is to follow his/her proposed solution plan, and lastly, re-examine whether every step is correctly worked out. This is largely applicable in facilitating problem-solving in Euclidean geometry. The researcher will review a few of these studies that have bearing with the current study.

Ersoy discovered, as part of his findings in Ersoy (2016), that not only that problem-solving can be learnt but it also impacts on development of learners' cognitive levels. The study was conducted by applying the four Polya (1985) problem-solving strategies to teach students problem-solving for 39 hours in 13 weeks. The main intention was to improve their problem-solving skills. It also emerged in the study that Polya problem-solving approach helped the students in the selection of the correct strategy when solving a problem. Ersoy (2016) maintains that the students that participated in his study could easily identify the given information than the unknown. These can be attributed to the fact that learners do not have the habit of cross-examining what is unknown in the question. Other studies like DeBellis (2006) and Fan and Zhu (2007) also reported similar findings as Ersoy (2016).

The researcher saw a link between Ersoy (2016) and Thambychik and Meerah (2010), concerning 'problem-solvers' cognitive ability'. The two studies have the potency to increase mathematics learner's cognitive ability of mathematics learners. Thambychik and Meerah (2010) conducted a study on mathematics skills and cognitive abilities in mathematics problem-solving. Part of the findings of the study was that learners are underperforming in information skills, visual-spatial and cognitive skills, which impedes mathematics problem-solving. They added that if learners' problems in problem-solving could be addressed, innovative programmes could be used to assist struggling mathematics learners in integrating mathematical concepts and skills during problem-solving processes.

Abakah (2019) is another study in which Polya problem-solving approach was used to improve the learning of Euclidean geometry. Abakah's (2019) study sought to improve the problem-solving skills of Grade 11 mathematics learners through the use of Polya (1985) problem-solving approach. Naturalistic classroom observation case study research design was used to conduct the study. Data were gathered for a period of two years. Part of the finding was that Polya problem-solving approach stimulated the study participants' desire and interest to learn Euclidean geometry. It also emerged that the intervention improved the study participants' performance in Euclidean geometry.

The aforementioned studies have shown that Polya's problem-solving approach is one of the antidotes that facilitate problem-solving in mathematics education.

3.1.2 van Hiele's Learning and Instructional Hypothesis

Pierre van Hiele and his wife researched into how to facilitate Euclidean geometry problem-solving in schools in their PhD work. The duo came up with a two-edged cognitive sword to fight the scourge of mathematics learners' incessant failure in the geometry aspect of mathematics as a result of their poor problem-solving abilities. The one face of the sword was to tackle learners' difficulties in the learning of geometry concepts, while the other face was meant to deal with the appropriate geometric concept knowledge transfer approach; that is, geometrical concepts knowledge transfer from the teacher to mathematics learners. Hence, the two faces of the sword were named: learning model and instructional approach. The learning model concerns the mathematics learners, whereas the instructional model is for the mathematics teacher.

The learning model highlights five cognitive levels required for learners to achieve to be proficient in geometrical problem-solving. The levels are visualisation, descriptive/analytical, abstract/relational, deduction and rigour. While the instructional approach highlights lesson presentation steps, this involves interviews, direct orientation, explanation, activities and integration. Many researchers have been trying to see how far these duo of intervention can facilitate different aspect of mathematics. An example of these studies are presented below.

Machisi (2021) selected four schools with Grade 11 to explore the experience of learning Euclidean geometry. Two discussion groups of learners were used to compile data. The first group was taught using van Hiele's theory-based approach while the second used conventional approach. The first group reported a productive learning experience in Euclidean geometry and second group outcome was negative. Learners struggle with Euclidean geometry because teachers continued to use conventional teacher-centred approach (Siyepu, 2014). Teachers using the traditional method do not bother about the level of understanding of the previous grade to evaluate whether learners have mastered the content or not. Instead, they go straight to content of the current grade without giving learners chance to investigate and discover theorems (Machisi, 2021).

Sadiki’s (2016) study investigated the effect van Hiele’s instructional approach in the teaching of congruent triangles in Grade 10 in Gauteng high schools. The study followed a mixed method research approach in which descriptive research design, involving the use of non-participant unscheduled classroom observations to collect qualitative data on the natural setting of the research field, while the pre-test post-test matching control causal comparative design was used to collect quantitative data. The intervention was the use van Hiele’s instructional approach in the teaching of congruent triangles. The findings of the study include that the intervention facilitated the learning of congruence of triangle concepts in the experimental groups; years of teaching experience in mathematics had a positive impact on lesson presentation; there was improvement in the geometrical problem-solving ability of the study participants in the experimental groups; the solution approaches of the study participants in the experimental groups improved. The achievement scores of the study participants in the experimental groups improved, while van Hiele learning level of the study participants increased.

Ngirishi and Bansilal (2019) used van Hiele’s hypothesis as an intervention in their study. The duo researched about the underperformance of learners in geometry. Van Hiel’s levels of thinking was used to excavate the understanding of basic geometry concepts. It was found out that learners have a challenge with linkage of properties, definitions of terms and shapes. Moreover, it was realised that learners were operating with lowest level of van Hiele’s levels of thinking. That is, they were struggling with identifying shape and their properties. Few of the learners had advanced to second level whereas most learners were operating at the first level of van Hiele's level of thinking (Atebe & Schafer, 2011).

3.1.3 Newman Error Analysis Hypothesis

Newman looked at how to facilitate Euclidean geometry problem-solving from the point of 'errors committed during the problem-solving'. He tagged his work 'Problem-Solving Error Analysis'. Newman believes that if the errors learners are committing can be identified and addressed, then the learners' problem-solving difficulties may reduce and thereby improving their performance (Newman (1983). Clements and Ellerton (1996) argue that Newman's framework for the analysis of errors was not put forward as a rigid information processing model of problem-solving but was meant to complement rather than to challenge descriptions of problem-solving processes such as those offered by Polya (1985). It is important to note that Polya (1985) focuses mostly on levels 2 and 3 of Newman error hierarchy, that is; the comprehension and transformation hierarchies.

Abdullah, Abu and Alhassora, (2017) are mathematics education scholars who studied the Newman error analysis and emphasized the following:

- The reading error - is the students' error in his ability to read the given mathematics problem and identify sentences and mathematics symbols used in setting the mathematics problem.
- Comprehension error - is the students' inability to understand mathematics problem.
- Transformation error - is the students' inability to be able to determine the appropriate method of mathematical solution.
- The process skill error - is the students' inability to correctly process the solution to the mathematics problem, and
- The students' inability to write the encoding error according to the given question.

These error factors are meant to analyse students' mathematics problem-solving skills with the view to improve the students' mathematics problem-solving skills (Abdullah et al, 2017).

Clements and Ellerton (1996) assert that Newman used the word 'hierarchy', since they argue that failure at any level of the above sequence prevents the problem-solver from obtaining satisfactory solutions (unless by chance, arrived at the correct solutions by faulty reasoning).

Abdullah et al. (2017) used Newman error analysis to conduct a small-scale research on evaluating creative thinking skills. The study involved a test that was designed to measure learners' performance in evaluating and creative problem-solving thinking skills. Learners' work were analysed and the results indicated that learners can only perform at the first stage of Newman's Model (reading error), that is, they were confronted with difficulties in the subsequent stages. This implies that the study participants could have been confronted with the error 1, which is coded 'reading error'. Reading error challenges include being unable to read and understand the presented problem, challenges with the geometric symbols and concepts. They could not continue to solve the problems presented because this error 1 is fundamental in continuing to successfully solve the problem.

Luneta, Makhubele and Nkhoma (2015) established the type of errors learners are committing when answering Euclidean geometric questions. van Hiele's levels of geometrical thought was used to evaluate learners' geometrical knowledge. Learners' scripts and content were analysed, and it was found out that learners incorrectly applied the rules, conceptual knowledge in geometry and problem-solving were very weak. According to Mudhefi (2022), learners' struggle in Euclidean geometry might be attributed to the current generation of mathematics educators that were not exposed to Euclidean geometry concepts during their high school days, even those who did it were not confident in solving geometric problem.

3.2 Effect of problem-solving Errors in Euclidean geometry

The researcher anecdotally noted that the leading error of all the errors Newman (1983) enumerated was the 'reading error'. Denfield, Nicolae and Beate (2014) accentuate that the language in which a mathematics problem is structured (maybe English language) has a predictive power on the mathematics scores. This implies that learners' challenges in Euclidean geometry problem-solving was not only in the core enterprise of mathematical logistics but also may be impaired in their ability to read Euclidean geometry problem and understand what to do. Perhaps this is why TIMSS (2019), in the diagnostic report, lamented about the inefficiency of our learners in the use of geometric language. The report emphasises that South African learners performed very badly in the test partly because of their reading errors. The researcher discussed the reading errors separately after which the other errors were discussed together. That is, errors were discussed under the subsection tittles: Reading error, and other errors.

3.2.1 The Effect of 'Reading Errors' in Euclidean geometry problem-solving

According to Vorster (2005), language is one major facet of mathematics culture, and for most learners in South Africa, the English language is their language of learning and teaching (LOLT) but not their home language. Vorster (2005) conducted a study about the impact of English in a school in which the English language was not the home language for all the learners. The test-instrument contained questions items which were structured in both English and home languages. The emerged findings show that because there was a 'home language' version of the question, that gave the learners better comprehension of the questions. Therefore, Vorster advice that teachers should be capacitated in the usage of new terminology and they should avoid code switching so that learners can be familiar with the terminologies.

Denfield et al. (2014) investigate the predictive power of English proficiency on mathematics scores, while controlling for gender, socioeconomic status, and grade level. Mathematics scores from the Florida Comprehensive Assessment Test for Grade 3-5 (N=177) were analysed using multiple linear regression. The result shows that English proficiency is a statistically significant predictor of mathematics scores. More importantly, mathematics scores increase simultaneously with English proficiency but inversely with grade level; the latter moderates the influence of English proficiency on mathematics scores, and lastly, gender and socioeconomic status have no significant moderating influence.

The researcher corroborates the findings of Denfield et al. (2014) with Yushau and Omar (2015). Yushau and Omar (2015) took the advantage of the Arab world gradually becoming English medium universities in place of their traditional Arabic medium. Hence, students are learning mathematics through the English language. To this end, the study investigated the relationship between language proficiency and mathematics performance among bilingual Arabic university students. The results of the data analysis indicate that the students' English proficiency levels affect the students' performance in mathematics courses.

In another study, Roux (2005) mentioned that language proficiency plays a pivotal role in mathematics achievement and is a driving factor in a higher order thinking skills. Appropriate use of language makes learning and teaching possible and hence deficient in language proficiency will jeopardise the learning of geometry. Van Hiele (1999) emphasised the importance of understanding geometric language in all teaching levels, as each level has its own language and symbols. Once the learner is unable to master a particular level, it is going to be difficult to understand the subsequent level as there are connections between levels.

Suglo, Bornaa, Iddrisu, Adams and Atepor (2023) suggest that vocabulary for mathematics education has an enormous impact on how learners perform in geometry as well. The study used achievement test, questionnaire and interview to collect data. The results found out that performance of learners relies on the circle theorem language proficiency and the intense knowledge of the teacher's geometry vocabulary. Words are a necessity to understand information. Therefore, learners should make sense of the words that convey content for improved knowledge and the capacity to grasp vocabulary influences all the terminology in classroom.

Bombio and Del Rosario (2022) focused on the correlation of learners' proficiency in English, especially reading comprehension and geometric proof skills. Test was given to students with the intention to measure English proficiency and geometric proof skills. The findings show that reading comprehension plays a pivotal role in geometric proof skills. Baful and Derequito (2022) corroborated this finding by reporting that if someone is not conversant with the English language, there is no way he/she can understand a text written in English. Geary (2004) added that language helps learners to manipulate and interpret information effectively in their memory.

3.2.2 The Effect of 'Other Errors' in Euclidean geometry problem-solving

Clements and Ellerton (1996) report that the results of Newman (1977a) shows that reading, comprehension and transformation errors made by 124 low-achieving Grade 6 pupils accounted for 13%, 22% and 12%, respectively of the total errors made. This translates to almost half the errors made occurred before the application of process skills. Similarly, in studies conducted in primary and secondary schools obtained results with about 50% of the errors occurred at the first three stages (reading, comprehension and transformation) of Newman's error hierarchies, which is constituent with Newman's results.

Kenys and Firda (2017) aimed at describing students' errors in problem-solving using NEA. The study was a qualitative study that used four students from the 2016 mathematics education course IKIP as study participants. Test and interview data collection techniques were used to gather the study's required data. The findings show that frequent mistakes were in transformation and skill processing hierarchy of NEA.

Sapire, Wilson-Thompson and Paulsen (2016) focused on the engagement of teachers regarding learners’ errors in mathematics and how teachers are using the identified error in teaching. A video of teachers offering lessons and learners interview were used as data for the research. It emerged that various errors committed by the learners, ranged from not able to read and understand what to be done in a mathematics problem, not being able to represent the problem with the right mathematics expression and calculation. It also emerged that teachers were avoiding the interaction with learners’ errors during the lesson and few of them were on point with the discussion of errors. Errors committed by the learners provide an insight of their geometric thinking; and hence, these errors can be used to develop our teacher’s knowledge and also the skill to analyse the incorrect solutions (Mosia, 2023).

3.3 Subject Content Knowledge (SCK), Pedagogical Content Knowledge in Geometry (PCK) and Learners’ Attitude in the Learning of Geometry

Van Putten, Howie and Stols (2010) found that there is a relationship between learners’ attitude in the learning of mathematics, performance, mathematics teachers’ SCK and PCK. Van Putten et al. (2010) investigated attitude and the degree of understanding in Euclidean geometry. Pre-post-test on geometry module was administered and some group of learners were interviewed before and after the administration of the module. The findings revealed that frustration and confusion of learners in mathematics was because of teachers who did not have appropriate SCK and hence, the PCK did not produce positive attitude towards the subject. It was also revealed that when such learners get a different teacher with a strong content knowledge their perception towards the subject changes. Jones (2002) acknowledged that if a teacher wants to teach geometry well, he/she must be able to recognise geometrical problems and theorems, and understand various contextual ways in which geometry is established.

The preceding claim is supported by Tachie (2020). The latter selected ten schools from one district and data were collected through group interviews and classroom observations. One of the major findings is that learners are not the only ones struggling with Euclidean geometry but also some teachers lack SCK and other practical strategies that are needed to be implemented during their teaching (PCK). In addition, the study also alluded to the fact that South African mathematics teachers especially those who did not study Euclidean geometry in their secondary schooling find it difficult to teach it. Moreover, their preparations were highly affected because they are preparing something that they do not have the confidence of understanding its content. Teachers who are uncomfortable in the particular topic compromise the understanding of geometric concepts and transfer of knowledge to learners.

Simon (2020) corroborated the preceding findings asserting that character and belief play a pivotal role in the success of the teaching career. Some teachers have a negative attitude towards teaching Euclidean geometry for reasons like, they did not do it in high schools, they do not have the confidence of the section in mathematics, or they have been forced by the Department of Basic Education (DBE) to teach it. Teachers also blame learners on areas that are difficult for them to teach.

Tsao and Tung (2022) explored learners' attitude towards geometry. A questionnaire which deals with different attitudes towards geometry was administered. The questionnaire dealt with confidence, enjoyment and the importance of learning geometry. The results indicated that learners have a negative attitude towards geometry. The duo explained that attitude is one behaviour that plays the main role in learners' geometry performance, their achievement in geometry is immensely affected when they displayed negative attitude (Farooq & Shah, 2008). However, Tsao (2018) notes that positive attitude of learners towards geometry yields good results because learners that participate in classroom activities achieve high marks in geometry.

As mentioned by Tsao (2018), positive attitude towards the learning of geometry produce good results; this was demonstrated by Mogari (2003). In addition, Mogari, (2003) investigated the correlation between achievement and attitude in Euclidian geometry. Enjoyment, significance of geometry and self-confidence were the dimensions used in the study. Attitudinal scale was conducted to steer clear of the influence that might arise because of the performance in the achievement test. The results indicated that learners displayed better attitude towards Euclidean geometry than what was anticipated. The teacher's character and teaching method contribute highly to the learners' positive attitude towards the topic, and also some learners develop the positive attitude because of the career they are intending to follow.

Even though it was not mentioned in Mogari (2003) that the teacher/s study participants were very good in terms of their SCK and PCK to have recorded a positive attitude from the study participants. The findings from other studies like Putten et al. (2010) confirm that in Euclidean geometry classes where the teacher lacks either or both SCK and PCK, their learners present negative attitudes. This was also corroborated by Jones (2002).

Sahin and Kendir (2013) researched on the effect of implementing metacognitive strategies for geometry problem-solving and attitude. Two groups were formed, that is the control and experimental group. Both groups were given a pre-test and post-test. The task comprised achievement test and attitude scale towards mathematics. Experimental group was taught geometry and the lessons were designed in a manner that it improves learners' potential to implement metacognitive skills for solving problems. Traditional method was used to teach control group. At the end, experimental group performed better than the control group and they also developed a better attitude towards geometry. Learners with better metacognitive strategies, irrespective of their grade, have higher achievement and positive attitude towards geometry (Kuiper, 2002).

All the studies noted that learners show negative attitude to mathematics learning generally, but the situation is worst when the teacher is not competent in the concepts of what he/she is teaching. Definitely when a teacher does not have a grasp of concepts what he/she present in the class, the teaching procedure will inevitably be affected. This problem will generate negative attitudes from the learners. In Euclidean geometry class, the fact that learners are struggling to understand the concepts may induce learners' negative attitude during the teaching of this aspect of mathematics. Kafele (2013) asserts that attitude is everything, and advice teachers to do what they can to bring positive change that they are motivated and excited when they enter into their classrooms. The study explains further that teachers' attitude towards learners is very important. Accordingly, teachers should create an environment that is conducive for learning, and regularly demonstrate and show their belief in all learners. The study also highlighted that teachers, apart from demonstrating belief in their learners, should also have passion for teaching, passion for children's role model for learners and passion for developmental growth.

Jojo (2016) explored the learners' conceptual understanding in geometry. A qualitative approach was used to explore the learners' conceptual understanding of Grade 8 pre-requisite geometric concepts on a preliminary test administered through questionnaires. Follow-up semi-structured interviews were further conducted with eight learners, four from each school, on the basis of their responses to get clarity on how they constructed geometric meaning. The researcher scrutinised the mental constructs learners made in the understanding of concepts, misconceptions and visual impact. The results showed that learners were lacking the understanding of basic geometric concepts and also incompetence in establishing mental construct that simplifies the geometric concepts.

3.4 Conclusion of the Chapter

Review of academic papers that are aligned to the current research was conducted under the subtitle: problem-solving in Euclidean geometry, Effect of problem-solving errors in Euclidean geometry, and the subject content knowledge (sck), pedagogical content knowledge in geometry (pck) and learners' attitude in the learning of geometry. Literature alluded to the fact that problem-solving in geometry has been a great problem to South Africa mathematics learners, especially the Euclidean geometry, to the extent that they now created phobia for the learning of geometry. The poor performance of South African mathematics learners in Euclidean geometry was traced to the numerous errors committed in the process of Euclidean geometry problem-solving. The literature reveals that attempts have been made by mathematics education scholars to track down the errors mathematics learners are committing during problem-solving in mathematics, particularly in geometry. However, the researcher thinks that a more authentic data will come from national senior certificate examination, where every mathematics learner put up their best in every subject they write. But there is no literature that support it, whether this has been done, at least not in South Africa. This created a big gap in the mathematics education research; hence, this study undertakes not only to carry out research on types of errors committed in Euclidean geometry problem-solving in the matric mathematics exam but also to offer a pedagogical solution that may remedy the problem.

CHAPTER 4

RESEARCH DESIGN AND METHODOLOGY

4.0 Introduction

This chapter describes the research design and methodological approaches that were used in this study. In the study, the researcher looks at research design as part of the research details that deal with the planning of a study, while the research methodology entails the procedures that are followed to conduct the study. These include the tools and instruments used in the study. Owing to the nature of this study, the research was conducted in two phases: Phase I and II. Phase I of the research intends to gain insight into the type of common errors South African mathematics learners commit in Euclidean geometry problem-solving procedures, while Phase II intend to proffer a solution that may mitigate committing such errors through intervention. Perhaps, this intervention may bring about the required solution to South African mathematics learners' problem-solving difficulty in Euclidean geometry. This study is in line with studies like Faleye (2011) and Hashemi, Abu, Kashefi and Mokhtar, (2015) in which difficulty was found in the process of learning, and intervention was developed to mitigate students' difficulties.

4.1 Research Design

Mixed method research approach was followed in this study (Faleye, 2011). The approach involved the use of both quantitative and qualitative approaches. The quantitative approach involved quasi-experimental research, while the qualitative approach involved the use of descriptive research design to collect the relevant data. Quasi-experimental research design adopted ex-post facto research design that involved solution appraisal in Phase I and pre-test post-test matching control research design in Phase II. In contrast, descriptive research design involves solution appraisal and classroom observation to collect data in Phase II of this study.

The mixed method research approach was employed to answer the following research questions and achieve the aims of the study:

- What are the types of problem-solving errors South African mathematics learners are committing while solving problems in Euclidean geometry?
- What leads them to committing these errors while solving problems in Euclidean geometry?
- To have a complete understanding of the South African learners' mathematics problem-solving skills difficulties in Euclidean geometry
- To be able to provide a way by which the identified problems could be overcome.

Similarly, the study was conducted in two phases for the following reasons: Phase I of the study was designed to find out the types of errors South African learners are committing while solving Euclidean geometry mathematics problems with a view to propose a teaching model remedy that may help the learners overcome the problem. Hence in Phase I, the researcher did not only collect data on the types of errors committed, and performance, but used the information from Phase I to develop a teaching model to address the problem-solving difficulties discovered.

In Phase II the researcher experiments the efficacy of the developed intervention which aimed at helping South African mathematics learners to overcome their difficulty in problem-solving discovered in Phase I. Table 4.1 shows how mixed method research was applied in this study.

Table 4.1: Summary of how Mixed Method Research was applied

Phase	Nature of Data Required	Research Design
I	(a) Quantitative Data (Type of errors committed, Performance in Euclidean geometry questions and in paper 2 as a whole)	Ex-post facto research design which involves solution appraisal approach

Phase	Nature of Data Required	Research Design
II	(a) Quantitative Data (Performance in Euclidean geometry questions class test in both pre and post-test) (b) Qualitative Data (Classroom observation data, and 'type of errors committed')	Pre-test Post-test matching control research design Descriptive research approach (Classroom Observation, and solution appraisal)

4.1.1 The Quasi-Experimental Design

Quasi-experimental design was in the form of ex-post facto research design which used solution appraisal approach to gather quantitative data in Phase I, and pre-test post-test matching control was applied in Phase II. In Phase I, ex-post facto research design made use of the existing record of the 2020 and 2021 NSC matric examination paper 2. Copies of these past matric NSC mathematics examination scripts from the study population were used to collect 'type of errors committed' and performance data. This is consistent with Luneta et al, (2015) in which learners' past scripts were used to collect data. It must also be mentioned that the researcher considers the NSC examination papers the best source of data for this study because the researcher wanted data that will represent the true abilities of South African mathematics learners' problem-solving skill in Euclidean geometry. Besides the fact that this examination is strictly supervised, the learners put in their best when preparing and writing the examination. In Phase II, quasi-experimental design was in the form of pre-test post-test matching control.

4.1.2 The NSC Examination Papers as Data Source

In South Africa, the final examination that leads to NSC is named matriculation examination and in a short form it is called the matric examination. It marks the culmination to 12 years of formal schooling. This examination is conducted by the DBE. There are minimum requirements attached to NSC examination that will imply that a matric candidate have passed the examination, these are: 40% in three subjects, one in home language and 30% in three subjects.

All NSC examination question papers are prepared, validated and quality checked by the Council for Quality Assurance in General and Further Education and Training (Umalusi), who is also responsible for the development and management of NSC qualifications. In addition, after the marking process have been completed, all the scripts are kept in a highly secured environment with 24 hours of security supervision by the DBE offices in each province. Scripts are regarded as confidential information and are kept for a period of six months before they can be destroyed. Owing to strict conditions surrounding the administration of scripts, the only way to access them is through the permission from the DBE Member of the Executive Council (MEC).

In fact, there are two government accredited examinations that marks the end of 12 years of secondary school schooling in South Africa – the NSC and Independent Examinations Board (IEB). Majority of the private schools in South Africa write the IEB examination, while majority of the public (government) schools in South Africa write the NSC examination. Averagely, about 98,5% of the school leavers candidates write NSC and about 1,5% write IEB examination. These examinations are both validated and quality assured by Umalusi.

Since about 98.5% of the South Africa school leavers write the NSC examination, this implies that NSC examination candidates are more representative of South Africa secondary school mathematics learners. Therefore, the NSC exam scripts and the learners in the public schools provided data for this research.

4.1.3 Descriptive Research Design

In Phase II, descriptive research design involved the use of classroom observation, and solution appraisal approach. It involved the use of classroom observation and solution appraisal approach to gather the required data.

4.2 Research Methodology

As mentioned in 4.1 above, this section shall give details of the research procedures involved in the study which includes study population, sampling procedure, instrumentation, pilot study, and data collection procedure.

4.2.1 Study Population

There are two Grade 12 secondary school exit-examinations in South Africa: the National Senior Certificate (NSC) examination, supported by the Department of Basic Education (DBE) and the Independent Examination Board (IEB) examination. Since an average of 98.5% school leavers write the NSC examination, while 1.5% school leavers write the IEB examination in South Africa, the NSC examination is more representative of the South African secondary school mathematics learners. Hence, the Grade 12 mathematics learners in the public schools in all nine provinces are the study population for this study.

4.2.2 Sampling

4.2.2.1 Phase I Sampling

The sample population for this study are the Grade 12 mathematics learners in the public schools in the five randomly selected provinces in South Africa. Convenient sampling method was used to select five provinces out of the study population (the nine provinces in South Africa). At the onset of this research study, letters were sent to the DBE in all the nine provinces in South Africa and a letter was sent to the national DBE office as well, seeking their permission to conduct the study and for their participation in the study. Because the research involved the use of past mathematics NSC scripts, and considering the strict protection accorded past NSC scripts (see 4.1.3), only five provinces agreed to participate in the study. It must be mentioned as well that two of the five province no longer had the 2020 scripts as they have been shredded. Hence, the researcher had scripts from only three provinces for the 2020 NSC examination. A total of 244 and 432 scripts of the 2020 and 2021 NSC examinations respectively were used to collect data in the Phase I of this study (see tables 4.2 and 4.3).

Table 4.2: 2020 Number of Scripts by Province

Province	**Schools**	**Number of Scripts**	**Cumulative number of scripts**
Northern Cape	NC_1	22	22
	NC_2	28	50
	NC_3	12	62
	NC_4	27	89
Sub-total	89		
Limpopo	L_1	14	103
	L_2	18	121
	L_3	28	149
	L_4	29	178
Sub-total	89		
Free State	F_1	26	204
	F_2	14	218
	F_3	09	227
	F_4	17	244
Sub-total	66		
Grand Total			**244**

Table 4.3: 2021 Number of Scripts by Province

Province	**Schools**	**Number of Scripts**	**Cumulative number of scripts**
Northern Cape	NC_1	30	30
	NC_2	30	60
	NC_3	7	67
	NC_4	27	94
Sub-total	94		
Limpopo	L_1	17	111
	L_2	24	135
	L_3	15	150
	L_4	20	170
Sub-total	76		
Free State	F_1	13	183
	F_2	17	200
	F_3	21	221
	F_4	18	239
Sub-total	69		
Mpumalanga	M_1	25	264
	M_2	29	293
	M_3	24	317
	M_4	24	341
Sub-total	102		
Gauteng	G_1	17	358
	G_2	30	388
	G_3	16	404
	G_4	28	432
Sub-total	91		
Grand Total			**432**

4.2.2.2 Phase II Sampling

In Phase II of the study, random sampling was used to select three provinces from the five provinces in the Phase I sample population, and convenient sampling was used to select three schools from the three randomly selected provinces, one school from each province. The conditions for the convenient selection were proximity to the researcher to facilitate classroom observation logistics, each selected school should be far apart to each other in order not to compromise the study, and each selected school should have Grade 12 mathematics learners and not only mathematics literacy learners. For the sake of research ethics and the Protection of Public Information (POPI) Act, the research used number code to represent each school where the study was conducted. Two schools were randomly selected as the experimental schools, while the remaining one was the control group. The two experimental groups were named school 1 and school 2, while the control group was name group 3. In school 1, there were 41 study participants (18 boys, 23 girls), in school 2, there were 33 study participants (15 boys 18 girls), and in school 3 there were 21 study participants (12 boys, 9 girls). Therefore, the study participants for Phase II were 95 Grade 12 mathematics learners.

4.3.1 Instrumentation

Under this subsection, the study variables and the instruments used in the data collection are explained, their validity and reliability were discussed.

4.3.1.1 The Study Variables

There were some variables, both independent and dependent, that influenced the outcome of this study. The independent variables act as the primary source; they do not change, while the dependent variables change as they interact with certain instruction or treatment. Examples of how certain instruction may influence the outcome of a dependent variable is illustrated as follows:

Let us consider a couple $(x, f(x))$, in which x is an independent variable, while $f(x)$ is a dependent variable. When x interact with f under some instructions, it produces different values of $f(x)$. For instance, let $(x: 1, 2, 3)$ and let $f(x)$ be define as $f(x) = 2x - 1$, when $x = 1$, then $f(x) = 1$, when $x = 2$, then $f(x) = 3$, and when $x = 3$, then $f(x) = 5$. Thus, for different values of x, we see different values of $f(x)$ under the instruction that $f(x) = 2x - 1$.

In this study, there was a need to identify the independent variable (x) and the dependent $(f(x))$ variables and put in place appropriate measuring instruments to measure the size of the change that ensued when $f(x)$ changes, because the change measured helped the researcher to answer the research questions. In view of the foregoing, in this study, problem-solving skill and learning were identified as independent variables, while "type of errors committed", and performance were identified as dependent variables. Therefore, appropriate measuring instruments were used to measure the changes.

Table 4.4: Variables used in this Study

Independent Variables (x)	Learning and Problem-solving skills
Dependent Variables $(f(x))$	Type of errors committed, and performance

When learning and problem-solving skills were manipulated by the intervention, they produce new number of committed errors (fewer or larger) and change in performance. In the Phase I of this study, since the researcher used the scripts of past mathematics exams, the mathematics teacher who prepared the matric candidates for the exams used in this study had tried to manipulate the learning and the problem-solving abilities (the independent variables in this study) of the candidates in the exam used to collect data. The research starts from measuring the results of the expected changes produced on the types of errors committed and the associated performance (the dependent variables in this study). Phase Phase II of this study proposes an intervention that will interact with the learning and problem-solving skills (dependent variables) to produce new committed errors and performance data to be measured as new results.

4.3.1.2 Validity and Reliability of a Measuring Instrument Explained

Validity of a research instrument refers to the level of accuracy of a research instrument in measuring what it is supposed to measure, while the reliability of a research instrument refers to how consistent is a measuring instrument in measuring what it supposed to measure. For example, if by 12 o'clock in the afternoon, a clock measures 12:05 and every day at the same time this clock measures 12:05, when actually the time is 12:00. This clock as a time measuring instrument is not valid but it is reliable. But if it gives different time measurement for different days when the time is supposed to be 12 o'clock, then the clock is neither valid nor reliable as a time measuring instrument. Validity has to do with the accuracy in measurement while reliability has to do with consistency of the measuring instrument. Therefore, all the instruments used were checked for accuracy and consistency, as mentioned earlier on the former is validity check and the latter is reliability check.

Validity check may be carried by using face-validity, content validity and construct validity. In contrast, instrument reliability check may be achieved through parallel forms, internal consistency, test and retest, and inter-scorer methods of testing the instrument reliability.

4.3.1.3 Instruments used in this Study

This study was conducted in two phases as mentioned above: Phase I and Phase II. The measuring instruments used in Phase I were, the 2020 and 2021 NSC mathematics paper 2 scripts and error list instrument, which were used to measure "errors committed", and performance. Conversely, in Phase 2, the measuring instruments used were pre-test instrument, post-test instrument, error list instrument, video recorder instrument, field notes instrument and classroom observation checklist instrument. They were used to measure classroom dynamics, errors committed and performance, while "Euclidean geometry Problem-Solving Learning Model, (EGPSLM)" and "Euclidean geometry Problem-Solving Instructional Approach Model, (EGPSIA)" were coupled and referred to as the intervention in Phase II in an attempt to correct all the Euclidean geometry conceptual errors revealed in Phase I. Hence, EGPSLM - EGPSIA were coupled to form a unit of intervention; they shall be referred to as "intervention." However, their development, validity and reliability are discussed separately.

The researcher discussed the development of the measuring instruments, including their validity and reliability.

4.3.1.4 The Development, Validity and Reliability of the instruments

4.3.1.4.1 Phase I Instruments

As mentioned in subsection 4.2.3.3, the measuring instruments used in Phase I were the 2020 and 2021 NSC mathematics paper 2 scripts and the error list instruments.

(i) National Senior Certificate (NSC) Examination Scripts

In this study, 2020 and 2021 scripts were used as measuring to identify the 'types of errors committed' and performance (see appendix 3).

a. Development of National Senior Certificate (NSC) Examination Scripts Instrument

The Council for Quality Assurance in General and Further Education and Training (Umalusi) is generally responsible for developing the NSC examination papers. As mentioned in subsection 1.3.1, mathematics paper 2 consists of data handling, trigonometry and geometry (both analytical and Euclidean geometry). Tables 4.5 gives the illustration of the number of questions allocated to each aspect of mathematics paper 2 and marks allocation composition of the 2020 and 2021 mathematics paper 2 matric exam.

Table 4.5: Composition of 2020 Grade 12 Mathematics NSC Paper 2

	Data Handling	**Analytical Geometry**	**Trigonometry**	**Euclidean Geometry**	**Total**
Question Number	1 (10 marks) 2 (9 marks)	3 (21 marks) 4 (19 marks)	5 (8 marks) 6 (24 marks) 7 (10 marks)	8 (17 marks) 9 (15 marks) 10 (17 marks)	
Marks Allocation	19 marks	40 marks	42 marks	49 marks	150 marks
% in the total Mark	12.67%	26.67%	28%	32.67%	100.01%

The composition of the 2021 mathematics paper 2 matric exam is similar to that of 2021 (see the composition of the 2021 mathematics paper 2 matric exam in Table 4.6.

Table 4.6: Composition of 2021 Grade 12 Mathematics NSC Paper 2

	Data Handling	Analytical Geometry	Trigonometry	Euclidean Geometry	Total
Question Number	1 (10 marks) 2 (10 marks)	3 (24 marks) 4 (15 marks)	5 (17 marks) 6 (15 marks) 7 (8 marks) 8 (10 marks)	9 (5 marks) 10 (13 marks) 11 (23 marks)	
Marks Allocation	20 marks	39 marks	50 marks	41 marks	150 marks
% in the total Mark	13.33%	26%	33.33%	27.33%	99.99%

b. Validity of NSC Examination Scripts Instrument

The Council for Quality Assurance in General and Further Education and Training (Umalusi) ensures the validity of this instrument.

c. Reliability of NSC Examination Scripts Instrument

The Council for Quality Assurance in General and Further Education and Training (Umalusi) ensures the reliability of this instrument.

(ii) Error List Instruments

This instrument consists of three sections; section 1 consists of the Newman Errors list, section 2 consists of the solution appraisal content, and section 3 consists of the error rating scale (ERS). All these three sections form the Error list instrument. The instrument was developed, validated and reliability checked as one instrument (see Appendix 11).

a. Development of Error List Instrument

Error list instrument consists of three sections as mentioned above: section 1 presents the Newman errors that underpin this study; section 2 presents how these errors were searched for in each script, coded and captured into the Number of errors template; section 3 presents how good or bad each error was committed (see appendix 3). Each section is explained below.

Section 1. According to Newman (1977), mathematics learners commit fundamental errors while solving mathematics. These errors were made available in Newman (1977), they are:

- Reading error – Not able to read mathematical problem given and to identify sentences and mathematical symbols used.
- Comprehension error – Not able to show some form of understanding of the problem given.
- Transformation error – Not able to determine the correct method of mathematical solution.

- Process skill error – Not able to provide correct mathematical processes involved in the solution method.
- Encoding error – Not able to write the solution according to the requirement of the question given.

These were the errors that the researcher looked for in each scripts.

Section 2 solution appraisal content was adapted from the Newman error list. They were used to identify and group each type of errors in the NSC scripts used in this study. It is also used to capture the marks obtained in the process of identifying errors committed. The solution appraisal content is given below:

The contents of the next table guide the researcher to identify each error type committed. The table is named Table 1 in Error list instrument.

S/N	Type of Errors
1	Not able to approach the problem in any correct way.
2	Not able to read mathematical problem given and to identify sentences and mathematical symbols used.
3	Not able to show some form of understanding of the problem given.
4	Not able to determine the correct method of mathematical solution.
5	Not able to provide correct mathematical processes involved in the solution method.
6	Not able to write the solution according to the requirement of the question given.

The errors garnered were captured into the number of errors template (table 2 in the Error list instrument) to facilitate data analysis.

Table 4.7: Number of Errors Template

School Name:		**Nov/Dec Matric examination**			**Year:**			
Scripts / Errors	Error 1	Error 2	Error 3	Error 4	Error 5	Total errors	Marks obtained	**%**
Script 1								
Script 2								
Script 3								
Script 4								
Script 5								
Script 6								
Script 7								
Script 8								
Script 9								
Script 10								
Script 11								
Script 12								
Script 13								

Section 3. Error Rating Scale (ERS)

The ERS is used to measure the intensity of the errors;

where:

1= Most fair: The error is committed as a result of mistake within a correct problem-solving approach.

2= Fair: Wrong problem-solving approach but able to manage the ensued mathematical calculations.

3= Bad: Not having any idea of how to answer the question, leave the question unanswered or wrong problem-solving approach coupled with poor mathematical calculations.

Table 4.8: Error Intensity Data Analysis Table
Scale: 1 = Good 2 = Fair 3 = Bad

School Name:		**Nov/Dec Matric examination**			
Scripts / Errors	Error 1	Error 2	Error 3	Error 4	Error 5
Script 1					
Script 2					
Script 3					
Script 4					
Script 5					
Script 6					
Script 7					
Script 8					
Script 9					
Script 10					
Script 11					
Script 12					
Script 13					
Script 14					
Script 15					
Script 16					
Script 17					
Script 18					
Script 19					
Script 20					

Any error discovered, no matter the number of times such error occurred, was multiplied by 5. For example, if ERS 1 was awarded to a particular error, no matter the number of times such error occurred, that error will be graded 5 in each occurrence in data analysis table. Therefore, five points was the rating most fair errors, while 15 was for bad errors. This is tabulated in the Error Data Analysis (see Appendix 3).

b. Validity of the Error List Instrument

The error list instrument was validated through face validity. Face validation evaluates whether a measuring instrument is relevant and appropriate for what is to measure. To this end, the error list instrument was checked for:

- The extent to which the instrument is relevant to grading 'the errors committed;' and
- How appropriate are the content constructs in the instrument?
- A validity rating form was prepared to facilitate the validation process (see Appendix 11).

The instrument and the rating forms were sent to three selected mathematics teachers that had a degree in mathematics education and had been teaching mathematics for more than eight years to carry out these exercises. It must be mentioned that the selected experts for these exercises were not involved in any of the research field in Phase II of this study. In addition, these experts were used to validate and conduct reliability check on all the measuring instruments used in this study.

The experts used the following rating scale:

For how relevant are the content items in the error list instrument:

1= Not/low relevance

2= Somewhat relevant

3= Highly relevant

For how appropriate are the item content constructs in the error list instrument:

1= Not appropriate

2= Somewhat appropriate

3= Very appropriate

When the mathematics experts completed their rating exercises, the researcher compiled the individual rating results and find the average of the rating results. Hence, the results from the experts gave 2.3 for the overall relevance rating and 3 for overall appropriateness. This implies that the measuring instrument is either somewhat or very well applicable, the item contents were highly relevant and that the content construct were highly appropriate.

(c) Reliability of the Error List Instrument

In this study, inter-rater reliability approach was adopted. Inter-rater reliability consistency is achieved when different experts rate the same instrument and the results are checked for rating consistency. The three mathematics experts that were used for the validity test were asked to reliability check the error list measuring instrument. The reliability rating results of the experts give: expert 1, expert 2, expert 3 gives 0.72, 0.70 and 0.71 respectively, with an average of 0.71. This gives somewhat a consistency rating, see appendix 12.

4.3.1.4.2 Phase II Instruments

As mentioned in subsection 4.2.3.3, the measuring instrument used in Phase II were error list, pre-test, post-test, field notes, video recorder and classroom observation checklist.

(i) Error List Instrument

Error list instrument was used in Phase II to collect 'error committed' data from the pre-test, classwork and post-test. The development, validity and reliability of error list instrument was discussed in subsection 4.3.1.4.1.

(ii) Pre-Test Instrument

Pre-test was used as a baseline test. Performance in the post-test was compared with the performance in the pre-test to measure the effect of the intervention in the study. In South Africa, the relevant Euclidean geometry topics to be covered by the mathematics learners are supposed to have been covered before Grade 12 classes. Teaching of Euclidean geometry in Grade 12 is a form of consolidation and revision. Hence, the researcher used a similar test instrument (the NSC mathematics examination paper 2 but of different academic year) that was used to collect data in Phase I to administer the pre-test in this study. To this end, the researcher used the Euclidean geometry questions from 2017 and 2018 past NSC mathematics paper 2 as a pre-test (see Appendix 1).

(a) Development of the Pre-Test Instrument

The researcher extracted the pre-test items from the Euclidean geometry questions aspect of 2017 and 2018 NSC mathematics paper 2 examination. The researcher did not add or subtract from any of the question's items.

(b) Validity and Reliability of the Pre-Test Instrument

As mentioned in subsection 4.1.2, Umalusi is responsible for the preparation of the NSC examination papers. They ensure the validity and reliability of all the questions in the mathematics paper 2 examination papers, including the 2017 and 2018 NSC mathematics paper 2 examination used as the pre-test.

(ii) Post-Test Instrument

The post-test was used to measure the improvement (or otherwise) in the study participants' performance and type of errors committed data.

(a) Development of the Post-Test Instrument

The researcher used the test instrument that was used for the pre-test to administer the post-test. The question items were shuffled such that the question item numbers in the pre-test and post-test were not the same. This was done to ensure that the study participants did not recognise the similarities in the two tests. Other than this, the pre-test and the post-test contained exactly the same question items (see Appendix 1 and Appendix 2).

(b) Validity and Reliability of the Post-Test Instrument

As mentioned under the pre-test, Umalusi is responsible for the preparation of the NSC examination papers as well as the validity and reliability of the test instrument.

(iii) The Intervention Instruments

The results of the data analysis from Phase I shows that many mathematics learners do not understand the concepts of Euclidean geometry in the way they presented their problem-solving approach. This instrument was developed after data collection and analysis in Phase I. All the errors enumerated by Newman

(1977) were discovered in Phase I. In this study, researcher intends to erase poor conceptual understanding in Euclidean geometry by South African mathematics learners in order to improve performance in Euclidean geometry aspect of mathematics in the matric examination. The researcher approached the problem of poor problem-solving approach which ultimately leads to underperforming of South African learners in Euclidean geometry in two facets which are: approach to learning Euclidean geometry, and how Euclidean geometry is taught. The Euclidean geometry problem-solving learning model (EGPSLM) and Euclidean geometry problem-solving instructional approach (EGPSIA) were developed to take care of the approach to geometry conceptual learning and how Euclidean geometry is taught. These two instruments were coupled to form the intervention instrument. The development, validity and reliability of the intervention instruments are discussed in the next subsection.

Euclidean Geometry Problem-Solving Learning Model (EGPSLM)

The EGPSLM instrument was used as part of the intervention instruments. It prescribes a method that a mathematics learner can follow to gain deep conceptual understanding in Euclidean geometry. EGPSLM is learner-centred approach; it is what the learner has to achieve by him/her self.

a. Development of EGPSLM

The results of the data analysis of the data collected in Phase I provided input for the development of the EGPSLM. This model contains three cognitive levels approach:

Level 1 - Learning and familiarising with glossaries of geometric terms, symbols and their meanings.

Level 2 – Gain deep understanding of the Euclidean geometry concepts.

Level 3 - Consolidation.

Solve Euclidean geometry problems personally without any assistance, starting with less cognitive demanding problems then to more cognitive demanding problems (see subsection 4.3.2 of this study for detail explanation on EGPSLM).

b. Validity of EGPSLM Instrument

The EGPSLM instrument was validated by content-validity process. This validity process involves sampling validity and item analysis. Sampling validity is about how well the measuring instrument covered the content to be measured, and the item validity is about the relevance and appropriateness of the instrument items it is supposed to measure. To this end, the EGPSLM instrument was checked for:

- The extent to which the instrument covers the Euclidean geometry concepts and symbols it is supposed to cover. This is sampling validity part of the content validity process.
- How relevant and appropriate are the content construct in the instrument. This is item analysis part of the content validity process.

A validity rating form was prepared to facilitate the validation process (see Appendix 13). The instrument and the rating forms were sent to the three selected mathematics teachers that validated the Error list instrument. The experts used the following rating scale:

For how far does the measuring instrument cover the Euclidean geometry concepts and symbols it is supposed to cover:

1= Not well covered

2= Somewhat covered

3= Very well covered

For how relevant are the content items in the EGPSLM instrument,

1= Not/low relevance

2= Somewhat relevant

3= Highly relevant

and

For how appropriate the content items in the EGPSLM instrument are

1= Not appropriate

2= Somewhat appropriate

3= Very appropriate

When the mathematics experts completed their rating exercises, the researcher compiled the individual rating results and found the average of the rating results. Hence, the results from the experts gave 2.5 as the overall expert rating for applicable, 2.7 for the overall relevance rating and 3 for overall appropriateness. This implies that the measuring instrument is either somewhat or very well applicable. The item contents were highly relevant and the content constructs were highly appropriate.

c. Reliability of EGPSLM Instrument

Inter-rater reliability approach was adopted to reliably assess the EGPSLM instrument. The same three mathematics experts that were used for the validity test were asked to reliably assess the EGPSLM intervention instrument. The researcher collated the results of the experts as: expert 1, expert 2, expert 3 gives 0.69, 0.70 and 0.70, respectively, with an average of about 0.70. This gives somewhat a consistency rating (see Appendix 14).

Euclidean Geometry Problem-Solving Instructional Approach (EGPSIA) Instrument

The EGPSIA compliments the EGPSLM in the process of acquiring deep conceptual understanding and versatile problem-solving skill in Euclidean geometry. While EGPSLM is learner-centred, the EGPSIA is teacher-centre. Therefore, EGPSLM- EGPSIA is a coupled intervention. EGPSIA proposes the way Euclidean geometry could be taught to facilitate the learning of Euclidean geometry concepts. EGPSLM proposes how learners could learn the concepts of geometry. The development, validity and reliability of the EGPSIA instrument are discussed next.

(a) Development of EGPSIA Instrument

The results of the data analysis of the data gathered in Phase I of this study, and the Newman (1983) errors remedy for problem-solving was adapted to develop the EGPSIA instrument. It is the proposed instructional steps to acquire the Euclidean Geometry problem-solving skills described in EGPSLM. The steps involved are given as follows:

Level 1 - Prior Knowledge

Level 2 – Euclidean Geometry Terms and Symbols

Level 3 – Concepts Learning

Level 4 - Internalisation

(see subsection 4.3.2 of this study for detail explanation on EGPSIA)

(b) Validity of EGPSIA Instrument

The instrument was validated by content-validity process as in validation of EGPSLM except that the validation process was to this end. The EGPSIA instrument was checked for:

- What the instrument covers against what it is supposed to cover as an instructional approach instrument. This is sampling validity part of the content validity process.
- How relevant and appropriate are the content construct in the instrument. This is item analysis part of the content validity process.

A validity rating form was prepared to facilitate the validation process (see Appendix 15). The instrument and the rating forms were sent to the three selected mathematics teachers that validated the EGPSIA instrument. The experts used the following rating scale used for:

How far does the EGPSIA measuring instrument cover what it is supposed to cover as an instructional approach instrument:

1= Not well covered

2= Somewhat covered

3= Very well covered.

How relevant are the content items in the EGPSIA instrument:

1= Not/low relevance

2= Somewhat relevant

3= Highly relevant

and

How appropriate are the content items in the EGPSIA instrument:

1= Not appropriate

2= Somewhat appropriate

3= Very appropriate

When the mathematics experts completed their rating exercises, the researcher compiled the individual rating results and found the average of the rating results. Hence, the results from the experts gave 2.3 as the overall expert rating for applicable, 2.6 for the overall relevance rating and 2.7 for overall appropriateness. This implies that the measuring instrument is either somewhat or very well applicable. The item contents were highly relevant and that the content constructs were highly appropriate.

(c) Reliability of EGPSIA Instrument

Inter-rater reliability approach was adopted to reliability assess the EGPSIA as in EGPSLM instrument reliability assessment. The same mathematics experts that reliability rated EGPSLM were asked to rate the EGPSIA intervention instrument as well, the rating was done exactly the same way as in EGPSLM instrument reliability assessment. The results are as follows: expert 1, expert 2, and expert 3, gave rating assessment of 0.68, 0.72, and 0.71 with an assessment average of 0.70. This assessment reliability rating was somewhat consistent, (see appendix 16).

(vi) Classroom observation checklist Instrument

As mentioned earlier, Phase II of this study involved the use of classroom observation to collect data. The intervention during the classroom observation in the two experimental fields are to teach the study participants Euclidean geometry concepts with the aid of the EGPSLM-EGPSIA, as an alternative approach to the traditional teaching approach of Euclidean geometry, while the control research field will not experience any intervention. Therefore, classroom observation was used to measure the efficacy of intervention in helping the study participants in Euclidean geometry problem-solving. To this end, a classroom observation check list was prepared to ensure compliance of the teaching procedure during the intervention.

(a) Development, Validity and Reliability of Classroom Observation Checklist Instrument.

The researcher did not develop a new checklist but used the checklist used by Abakah (2019). Therefore, the researcher did not carry out the validity and reliability checks, since it was already conceptualised in Abakah (2019).

(vii) Field Notes and Video Camera Instruments

Field notes and camera instruments were used to collect data during the classroom observation, but they neither need to be developed, validated nor reliability checked.

4.3.1.5 Pilot Study

Pilot study is subjecting the instruments meant to be used in research to efficacy test in a similar situation to the original research field. In addition, pilot study does not only test the efficacy of the instruments but also the intervention and research procedure. Pilot study saves unintended consequences that may compromise the study. Pilot test field should be small but have the capacity to allow all the instrument to be tested.

However, the conditions that surround the past matric scripts (see subsection 6.4 of this study) did not allow the researcher to conduct pilot study of the instruments used in Phase I.

4.3.1.5.1 Phase II Pilot Study Procedure

The instruments that were involved in the data collection procedure in Phase II were Error list, pre-test, post-test, classroom observation checklist, field notes, video camera and the intervention instruments. The data to be collected were baseline performance (pre-test) and post-test, and errors committed in the post-test.

The Phase II pilot study lasted for two weeks; it started on 31 January 2022 and ended on 04 February 2022. The researcher presented the intervention since the mathematics teacher for Grade 12 in the pilot school was not trained on the EGPSLM-EGPSIA which are the intervention instruments. At the end of the pilot duration, all the necessary data were collected.

4.3.1.5.2 Implication of the Pilot Study

The pilot study went very well according to the research procedure. Everything went as planned, and all the instruments were perfect for what they were supposed to measure. However, the researcher noticed the following issues:

- It was difficult to make the learners follow the precepts of EGPSLM because it is more of learners' centred concept. It needs the learners to work on it more in their private time.
- The researcher needs to incorporate a way to motivate the learners to work on EGPSLM.
- The EGPSIA subsection of the intervention runs very well.
- The "40-minutes a period" in the school timetable to teach Euclidean geometry appeared not enough when teaching through EGPSLM-EGPSIA teaching approach.

4.3.1.5.3 Remedy to the negative implication of the Pilot Study

- The teacher to give more compulsory homework to the learners in the EGPSLM concepts. This is to motivate the learners to work on EGPSLM concepts.

- The schools were encouraged to make their timetable a "double period" (a period was 40-minutes, double periods were 1hr 20 minutes) for mathematics. This was suggested so that there will be enough time for the teaching of Euclidean geometry.

4.4 Data Collection

As mentioned in subsection 4.1, this research was conducted in two phases: Phase I and Phase II. In Phase I, ex-post facto research design (using existing records: past NSC mathematics paper 2 scripts) was used to gather quantitative data, while descriptive research design was used to gather qualitative data. In Phase II, pre-test post-test research design was used to gather quantitative data, while descriptive research design was used to gather qualitative data.

4.4.1 Phase I data collection

In Phase I, ex-post facto research design was used to gather 'types of error committed' data from the NSC mathematics paper 2 examination, and the scores on each scripts in both Euclidean geometry aspect of the paper and the total marks scored in the whole paper.

4.4.1.1 Types of Error Committed and Performance Data Collection procedure

Two consecutive years (2020 and 2021) past mathematics paper 2 scripts in the NSC examination were requested from the DBE in each of the five participating provinces. In each of the five provinces, scripts from four different schools were collected. While the researcher did not have control over the school of choice of the scripts, he tried to get the 2020 and 2021 scripts from the same school. This implies that the scripts from 20 different schools participated in this research (see tables 4.2 and 4.3).

After collecting the scripts, type of error data was extracted by applying solution appraisal approach to scrutinising each question against the answer presented in the scripts. Each error found was coded (for example E_2, this means error 2), marked for easy identification and captured into a data capturing template (see appendix 3) already prepared for this purpose.

4.4.2 Phase II data collection

Phase II of this work pertains to when the researcher attempted to provide a remedy to the problem-solving difficulties the study participants in Phase I demonstrated in solving Euclidean geometry questions in paper 2 of the 2020 and 2021 NSC examination (see Chapter 5 for details of the Phase I findings). From Phase I findings, it was evident that there was a need to improve conceptual learning of Euclidean geometry and the problem-solving skills of South African mathematics learners. To this end, the researcher developed 'Euclidean Geometry Problem-Solving Learning Model' (EGPSLM) which took the study participants through approach to learning Euclidean geometry and developed 'Euclidean Geometry Problem-Solving Instructional Approach' (EGPSIA), which was an approach meant to facilitate classroom learning of Euclidean geometry.

Both qualitative data and quantitative data were collected. Classroom observation and solution appraisal approach were used to collect qualitative data, while pre-test and post-test were used to collect quantitative data. The qualitative data collection involved a non-structured, non-participant classroom observation approach and scrutinising of the study participants' classwork and post-test. As mentioned in subsection 4.2.2, three schools were selected from the sample population, two of the schools were experimental research fields, while one (1) was a control research field. Pre-test was used to take a baseline performance data which was used to measure achievement at the post-test. The EGPSLM and the EGPSIA are discussed next.

(i) Geometry Problem-Solving Learning Model (EGPSLM)

The researcher had intended to improve the learning of Euclidean geometry of the South African mathematics learners in Euclidean geometry concepts and their problem-solving skills in Euclidean geometry aspect of mathematics. The results of the data analysis of the data collected in Phase I provided input for the development of the EGPSM. This model contains three levels of cognitive approaches:

Level 1 – Learning and familiarising with glossaries of geometric terms, symbols and their meanings

At the study participants' private time, they engage the list of geometric glossaries that was given to them. At this level, it is proposed that Euclidean geometry learners should be grounded in the geometric terms, geometric symbols and their meanings before attempting to start solving problems in Euclidean geometry. This step is to familiarise Euclidean geometry learners with the Euclidean geometry languages after which they will be able to understand and comprehend questions in Euclidean geometry.

Level 2 – Gain deep understanding of the Euclidean geometry concepts

After familiarisation with the Euclidean geometry languages and other fundamental basics in Euclidean geometry, next is to learn the various concepts from the rudimentary to the expected cognitive level in matric examinations. In addition to the list of glossaries was the list of some important concepts. Learners were to devote time to study the Euclidean geometry concepts both through personal studies, classroom teaching and seeking help from either the mathematics teacher or peers, with the view to gain deep understanding of the Euclidean geometry concepts.

Level 3 - Consolidation

Solve Euclidean geometry problems personally without any assistance, starting with less cognitive demanding problems, then to more cognitive demanding problems.

EGPSLM is demonstrated in the form of a diagram depicted in Figure 4.1.

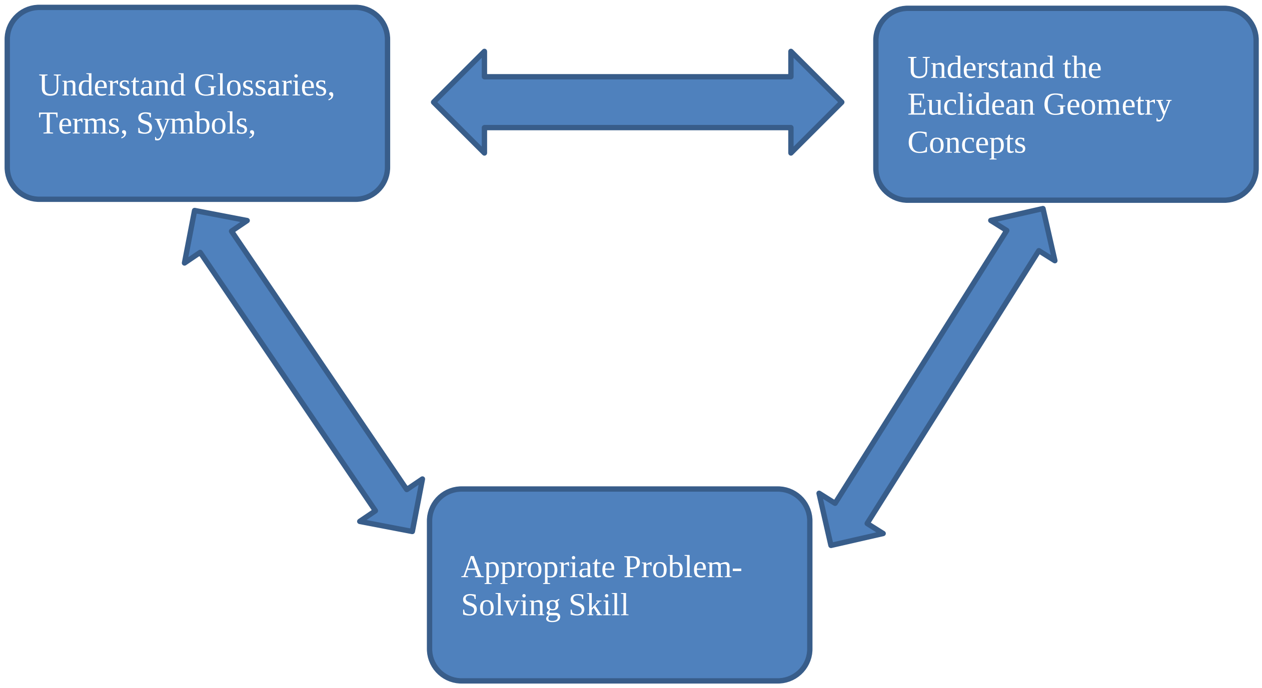


Figure 4.1: Figurative presentation of the EGPSLM

From Figure 4.1, the Euclidean geometry learning dynamics involve a forth and back learning continuum. Good understanding of the Euclidean geometry glossaries helps in its conceptual understanding and being grounded in the Euclidean geometry concepts helps in problem-solving. These three faces of Euclidean geometry learning interact with each other at every stage of the problem-solving process.

(iii) Euclidean Geometry Problem-Solving Instructional Approach, (EGPSIA)

EGPSIA is the proposed classroom learning of EGPSLM process. It is the proposed instructional steps to acquire the learning skills described in EGPSLM. The steps involved are given next.

Level 1 – Euclidean Geometry Terms, Symbols and Concepts Drilling

At the beginning of every lesson, the teacher asks various Euclidean geometry foundational questions on terms, symbols, concepts and listen to their responses.

This is to evaluate if the study participants were busy studying the list of Euclidean geometry glossaries and concepts given to them. The researcher believed that constantly engaging Euclidean geometry concepts will gradually help the study participants to eventually be properly rooted in the concepts. Thereafter, the teacher takes the study participants through Euclidean geometric terms, symbols and concepts from the hung charts and the Euclidean geometry glossaries and concepts list given to the study participants. The teacher explains and asks the study participants questions as he/she goes along (pointing to the hung Euclidean geometry charts. Moreover, the study participants also have their own copies; sometimes give classwork on the geometric terms and symbols.

Level 2 – Euclidean Geometry Concepts Drilling

The teacher writes and explains every concept involved in the topic to be learnt before applying the concepts. Linking everything to the terms, symbols and concepts just discussed.

Level 3 – Problem Solving

The teacher writes a question to be solved on the board. He/she explains to the study participants that the problem-solving stage is the application stage of the concepts learnt. The teacher emphasised the following steps:

- Read the problem;
- Comprehend what is read;
- Carry out mental transformation from the words of the question to the selection of an appropriate mathematics strategy;
- Apply the processing skills demanded; and
- Encode the answer in an acceptable written form (Newman, 1983).

At every lesson presentation, the teacher lay emphasis on the first three aforementioned items.

Level 4 – Consolidation

At this level, the teacher gives Euclidean geometry problems that are of more cognitive demand to learners with less supervision. He/she marks the classwork and gives compulsory homework to be submitted in the next lesson. He/she looks for errors committed during the problem-solving process and gives feedback to the learners.

Classroom Data Collection Procedure

Training of the Experimental Fields Teachers

In 2023 academic year, Euclidean geometry concepts were taught in April, which falls within term 2 of the academic year in South African schools. The two teachers in the experimental fields were trained on how to follow EGPSLM during problem-solving in Euclidean geometry and how to teach their learners, (who will be study participants during the intervention), EGPSM using the EGPSIA classroom approach. Two weekends (18-19) March 2023 and (25-26) March 2023), leading to the classroom use of EGPSM and EGPSIA, the teachers were taken through both EGPSM and EGPSIA, such that the researcher was sure that both teachers were good in the presenting EGPSIA. Therefore, the researcher ensured that the classroom teaching of Euclidean geometry in the mathematics classes of experimental fields' teachers followed EGPSM and EGPSIA in 2023.

The main data collection

In the 2023 school curriculum, Euclidean geometry was taught from 12 to 26 April. The intervention covered the entire curriculum allocated time for teaching Euclidean geometry. During the mathematics allocated period in each research field, on 12 April 2023, the pre-test (the baseline test) was administered in all the research fields.

Intervention started from the second lesson of the same day of writing the test, as all the three schools always had double period for mathematics, except on Fridays. According to the school timetable, each Grade 12 class had mathematics or mathematical literacy lessons five times in a week. However, the focus of this research was the mathematics classes. Three classroom observation were conducted separately in both experimental groups but was done only once in the control group. For experimental group 1, classroom observations were carried on the 13, 18 and 24 April 2023, while experimental group 2 had their own classroom observation on the 13, 17 and 21 April 2023. The control group had their classroom observation on the 13 April 2023. The researcher was always on the ground before the beginning of the lesson in all the classroom observation conducted. The intervention ended on the 25 April 2023 and the post-test was administered on the following day (26 April) during mathematics period in the all the three research fields. Video recorder and field-note were used to gather qualitative data. Solution appraisal was used on the pre-test to collect types of errors committed data.

4.4.3 Ethical Issues

Before University of South Africa (Unisa) gave the researcher permission to collect data from the field, the researcher got ethical clearance certificate (see Appendix 4). Furthermore, the researcher sought and got approval from the relevant authorities: the various provincial DBE offices and the national DBE office, the principals of the schools used in Phase II of the study and their learners (see Appendices 6, 7, 8, 9 and 10).

4.4.4 The consent of the participants

This investigation requires analysis of previous mathematics matric examination scripts in Phase I and the use of Grade 12 learners as study participants in Phase II. Permission to use examination scripts was sought and obtained from DBE, provincial offices (examination section) (see Appendices 6, 7, 8, 9 and 10). In Phase II of this study, the researcher was interested in investigating the cause of the study participants' problem-solving skill level and to offer remedy for the problem-solving skill deficiency detected in Phase I. To this end, after a school was chosen to participate in this Phase II of the study, a permission letter was delivered to district offices and to the schools. Creswell (2008) posits that research participation in social science should be voluntary. Hence, all the study participants in this study were voluntary and were allowed to withdraw (if need be) at any stage of the study.

4.4.5 Confidentiality

The researcher only had the photocopies of the Euclidean geometry part of the paper 2 of the NSC scripts used in Phase I of the study. The researcher ensured the confidentiality of the photocopied scripts, study participants and the classroom observation research fields. Personal names and information were anonymous.

4.5 Conclusion of the Chapter

This chapter gives the details of how the research was conducted. It described the research design and methodology used, the population of the study (both study and sample population), method of data collection and the instruments used in the data collection.

CHAPTER 5

DATA ANALYSIS AND PRESENTATION OF RESULTS

5.0 Introduction

In Chapter 4 of the study, the research design and method of data collection were presented. In this chapter, data analysis strategies of the data collected are presented, and the results are discussed. This study was conducted in two phases as mentioned in Chapter 4: Phase I and Phase II. The data analysis strategies and findings are presented in phases; that is, Phase I findings were presented first then followed by Phase II findings. In addition, the results are presented according to the research questions and the hypothesis. These results include the descriptive statistics, inferential statistics and the qualitative findings.

5.1 Data Analysis Strategies

As mentioned in subsection 5.1, Phase I data analysis strategies shall be presented first, followed by Phase II data analysis strategies.

5.1.1 Data Analysis strategies in Phase I

As mentioned in subsection 4.3.1, in Phase I, ex-post facto research design was used to collect data. From each script, marks obtained data was captured and also the solution appraisal section of the error list instrument was used to identify the problem-solving errors. Any error detected was coded, while the codes run from error 1 to error 5 (see data collection approach in section 4.3.)

Before presenting the data analysis strategies in Phase I, the researcher would like to discuss the preparation of the data for analysis.

5.1.1.1 Preparation of the Data for Analysis in Phase I

The data collected in Phase I were: the type of errors committed, marks obtained in the mathematics paper 2 for 2020 and 2021.

5.1.1.1 (i) Marks Obtained Data

The Euclidean geometry questions numbers were from 8 to 10 for 2020 and from 9 to 11 for 2021 (see 4.5 and 4.6). The marks obtained in the mathematics paper 2 for the two years were further separated into: the marks obtained in Euclidean geometry questions (EG marks obtained), marks obtained in the remaining questions without Euclidean geometry marks (Marks Obtained without EG marks), and marks obtained in the remaining questions together with the Euclidean geometry marks (Marks obtained with EG marks) by year in each of the three schools in a province and in each of the province. The researcher noticed that comparing the total marks obtained in the paper 2 examination with the marks obtained in the Euclidean geometry may amount duplication of the Euclidean geometry marks in the analysis, which may not produce correct result analysis.

5.1.1.1 (ii) Types of Errors Committed Data

The 'types of errors committed' data were coded, compiled and allocated under the main error components (that is: error 1, to error 5).

5.1.1.2 Phase I Data Analysis Strategies

It was depicted in Table 4.1 that in Phase I, ex-post facto research design was used to gather quantitative data (type of errors committed, performance in matric mathematics exam paper used and performance in Euclidean geometry aspect of the paper) in all the provinces. Quantitative data analysis techniques were used to analyse the data collected.

5.1.1.2.1 Quantitative Data Analysis Strategies

The data gathered were captured in Statistical Packages for the Social Sciences (SPSS) software. Exploratory and inferential analyses were conducted on the data captured.

5.1.1.2.2 Exploratory Data Analysis Strategies

Descriptive data analysis like mean, standard deviation, skewness and kurtosis were obtained from the "type of errors committed," and "performance" data. Charts are also presented to describe some of these attributes. These analyses illustrate the relationship, trends and tendencies among the data.

5.1.1.2.3 Inferential Data Analysis Strategies

Inferential data analysis was performed on the 'types of errors' and the marks obtained both in the Euclidean geometry questions and the remaining parts of the paper 2 for 2020 and the 2021 examination papers. The inferential strategies applied are:

- Regression analyses were performed on both the 'types of errors' and marks obtained. Test were conducted on the data to ensure goodness of fit considering the F-statistics and its associated p-value. Also, R-sq(adj) was used to explain the power of the variables used in the model.
- A t-test was performed to compare the effect of each type of errors on 2020 and 2021 examinations. In addition, the p-value was compared to the significance level of 5% to reject or accept the null hypothesis.

5.2 Phase II Data Analysis Strategies

It was mentioned in Table 4.1 that in Phase II, pre-test post-test matching control research design and descriptive research approach were used to gather both quantitative and quantitative data. Pre-test post-test matching control research design produced quantitative data, while descriptive research survey produced qualitative data: classroom observation and solution appraisal gave qualitative data. Quantitative data analysis techniques were used to analyse quantitative data collected, while qualitative data analysis techniques were used to analyse qualitative data collected.

5.2.1 Quantitative Data Analysis Strategies

The data gathered were captured into the SPSS software. Exploratory and inferential analysis were conducted on the data captured.

5.2.1.1 Exploratory Data Analysis Strategies

Descriptive data analysis like mean, standard deviation, skewness and kurtosis were obtained from the pre-test and post-test data. Charts were also presented to describe some of these attributes. These analyses illustrate the relationship, trends and tendencies among the data.

5.2.1.2 Inferential Data Analysis Strategies

Inferential data analysis was performed on both the pre-test and post-test scores data. Inferential statistical analyses were conducted on pre-test and post-test scores of both the control group and the experimental groups. Paired t-test analysis was performed to obtain the t-scores and to ascertain whether the intervention on study participants will significantly improve their post-test scores as compared to pre-test scores. The mean differential between the pre-test and post-test scores was conducted. In addition, the p-value was compared to the significance level of 5% to reject or accept the null hypothesis.

5.2.2.3 Qualitative Data Analysis Strategies in Phase II

The source of qualitative data in this phase of the study were the classroom observations, and solution appraisal applied to the classwork and the post-test scripts. The classroom data were to provide information on the efficacy of the intervention instruments to improve the problem-solving solving ability of the Phase II study participants when solving Euclidean geometry problems during the intervention period. This is important because the researcher would like to measure the effectiveness of the interventions (EGPSLM-EGPSIA) in improving the study participants' problem-solving skills while solving Euclidean geometry problems. In addition, the researcher sought to know if the intervention reduces errors committed during solving Euclidean geometry problems and increasing performance. Hence, the qualitative data analysis were in two folds: classroom data analysis and the analysis of the error list data.

(i) The classroom data collected were from video recorder and field notes. The analysis steps followed were:

- The data from video recorder were transcribed; the researcher conducted this for three times to ensure that no data was lost, or misrepresented.
- The data from the field notes were merged with the transcribed data.
- The data were arranged into different categories according to the research questions.
- The data were then coded. This entails assigning numerical values to all the categories so that they can be counted and tabulated.
- The last stage is tabulation according to the categories in a row and column format, such that it shows data relationships. This facilitates identifying common themes, patterns and relationships. Also, emerging themes we blocked to prevent themes from being repeated, emerging topics from each group were compared.

(ii) The error list data analysis involved scrutinising of the study participants' classwork and post-test scripts for errors 1 to 5.

5.3 Presentation of Results

As mentioned in subsection 5.1, the study was conducted in two phases: Phase I and Phase II. Presentation of the results were done in phases and in view of the research questions, starting with Phase I, then Phase II.

5.3.1 Presentation of Phase I Results

The research was conducted using 2020 and 2021 examination scripts to gather relevant data for this study. The 2020 results shall be presented first, followed by the 2021 results.

5.3.1.1 Presentation of 2020 Phase I Results

Research Question 1

What are the types of errors committed by the South African mathematics learners in solving the Euclidean geometry problems aspect of the mathematics paper 2 examination used in this study?

(a) Types of error committed results

As a result of the use of error list instrument, error 1 (reading errors), error 2 (comprehension errors), error 3 (transformation errors), error 4 (processing skill errors), and error 5 (encoding errors) were discovered in the 2020 matric mathematics paper 2 examination scripts used to collect data in this study. Table 5.1 shows the 'type of errors' findings for 2020. The number of mathematics scripts used for this study in 2020 was 244 and a total of 1464 error data were collected and analysed. Table 5.1 presents the type of errors committed results, the number of errors committed and the percentage representation of each error.

Table 5.1: 2020 Analysis of 'Type of Errors'

Error Type	**Number of Errors Committed**	**Percentage (%)**	**Cumulative Percentage (%)**
Error 1	426	29.08	29.08
Error 2	282	19.24	48,32
Error 3	252	17.20	65.52
Error 4	348	23.75	89.27
Error 5	156	10.64	99,91
Total	1464		

From the Table 5.1, the total number of errors committed and analysed were 1464 in the 2020 exam scripts used for this study. Error 1 (reading error) is the most commonly committed errors; they were 426 which represent 29.08% of the total error analysed. The next commonly committed error was error 4 (processing skills error); they were 348 and represent 23.75% of the total errors collected. Error 2 (comprehension error) was analysed to be the next commonly committed error; there were 282 of them. This makes 19.24% of the total errors committed. This was followed by error 3 (transformation error); there were 252 of this type of error and formed 17.20% of the total error committed. The least error committed was error 5 (encoding error); there were 156 of this type of error and formed 10.64% of the total errors committed.

The researcher presents the nature of the errors committed in the next subsection.

(b) Nature of errors committed

The errors were also analysed into how fair or bad the errors were to gain more insight into the nature of the errors committed. That is, whether it is just a mistake within a correct problem-solving approach or just not having any idea of how to go about the problem-solving approach. As mentioned in subsection 4.3.1.4.1, each error discovered were graded according to ERS scale, and analysed. The fairest errors committed earns 5, while the worst errors committed earns 15 (see appendix 3). The following charts give a descriptive illustration of the nature of the errors committed in Euclidean geometry questions of the exam paper used in this study.

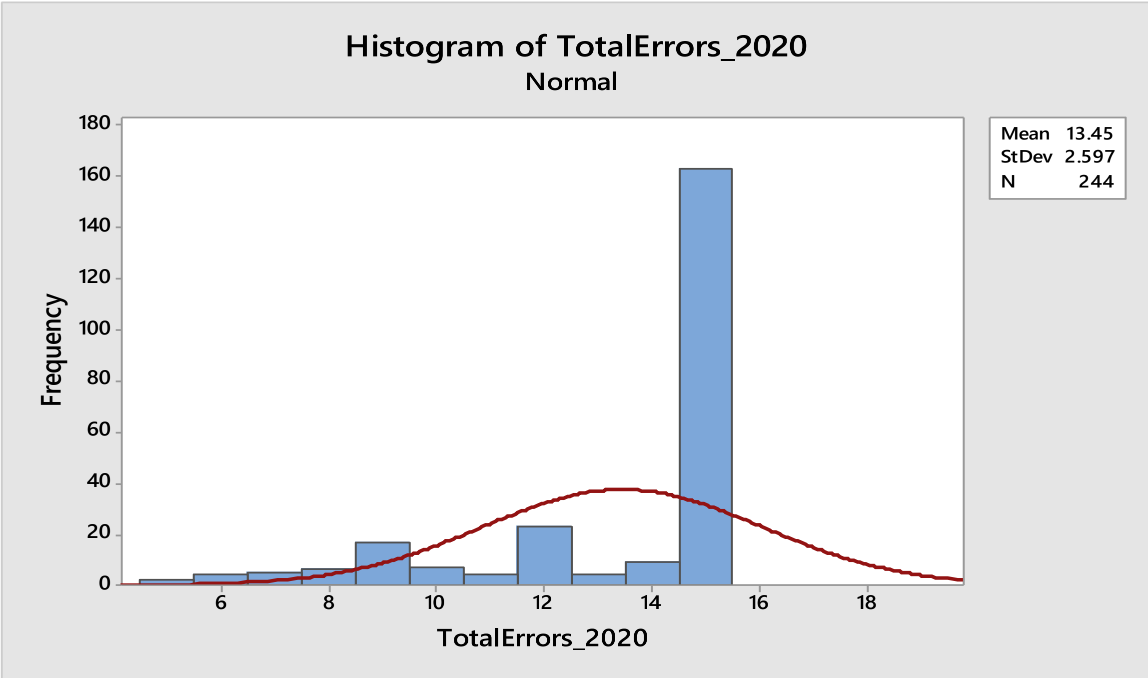


Figure 5.1: Histogram of the Nature of Error for 2020

Figure 5.1 shows that most of the errors committed were what the study categorised as bad. That is in most of the scripts analysed, the students did not have the right problem-solving skills to solve the Euclidean geometry questions in the exam paper used for this study or did not answer the questions.

Research Question 2

> *How does the errors committed in the research question (1) above affect the marks obtained in the Euclidean geometry aspect of the mathematics paper 2 matric examination used in this study?*

In 2020, mathematics paper 2 used in this study contained Euclidean geometry and some other aspects of mathematics. Other aspects of mathematics questions had a total mark of 101 while the Euclidean geometry questions had a total mark of 49. The total mark for the paper was 150. Table 5.2 presents the 'Marks obtained' in the Euclidean geometry descriptive statistics findings.

The descriptive findings of the type of errors committed in Table 5.1 shows that the most commonly committed errors are error 1 (reading error), 29.10% of the of the total errors committed, followed by error 2 (comprehension error), with 28.01% of the total errors committed, then error 4 (processing skill error), 21.06% of the total errors committed. During data collection, it was also noted that in many scripts, after these errors were committed, the problem-solving procedure ends there or proceed to present wrong solution procedure. Moreover, many Euclidean geometry questions were also avoided. This might have played a big role in the marks obtained in the Euclidean geometry aspect of the paper. The descriptive findings of the mark obtained in the Euclidean geometry of the paper used for this research is given below.

Table 5.2: General descriptive statistics 2020

		EG Marks
N	Valid	244
	Missing	0
Mean		8.93
Median		5.00
Mode		0
Std. Deviation		10.14
Skewness		1.41
Kurtosis		1.33
Minimum		0
Maximum		47
Range		47.00

The descriptive findings of the marks obtained data in the Euclidean geometry aspect of the paper are displayed in column two, the maximum mark over 49 was 47 and the minimum was 0, the mean was 8.93, the standard deviation was 10.14 and the mode is 0. The standard deviation of 10.14 implies that majority of the marks are between 1 and 19 in the Euclidean geometry aspect of the exam. This implies that majority of the study participants did not perform well in the Euclidean geometry aspect of the exam. Figure 5.2 shows the histogram of the marks obtained in the Euclidean geometry aspect of the exam.

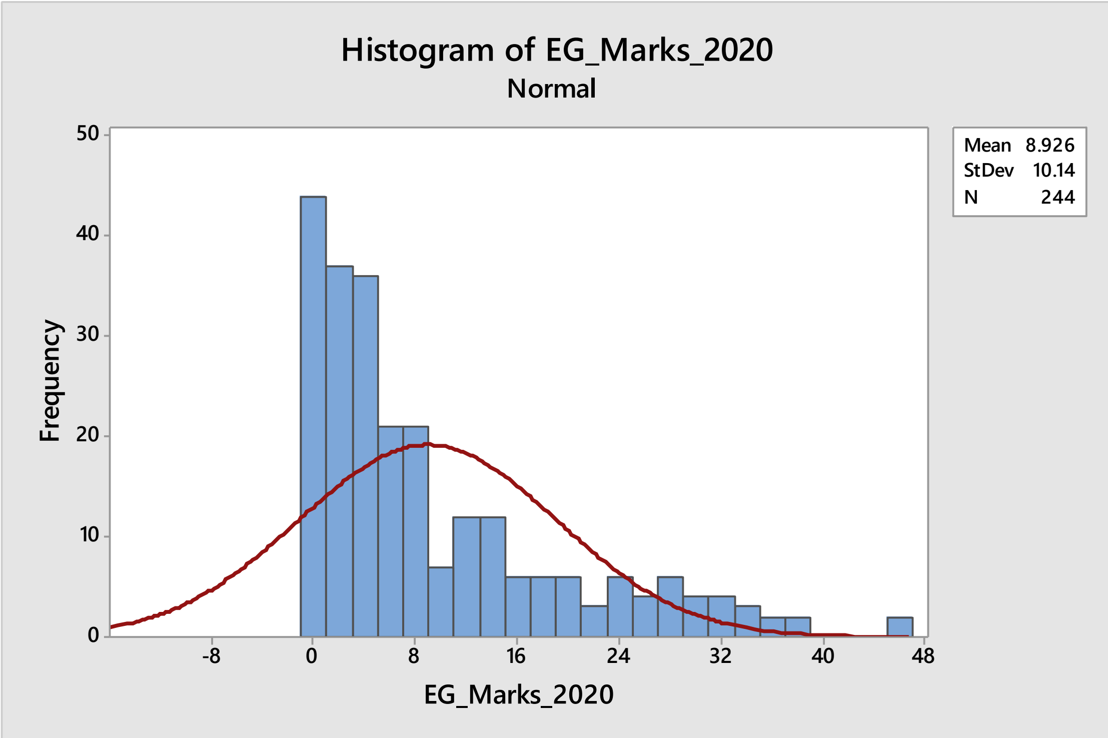


Figure 5.2: Histogram of the marks obtained in Euclidean Geometry

Figure 5.2 indicates that the marks obtained in the Euclidean geometry by most of the study participants are between 0 and 19 over 49. In addition, the researcher mentioned earlier that many of the study participants that committed these errors could not proceed to solving the problems in which the error was committed. These errors were graded 'Bad' errors in the ERS; these were found to be errors 1 and 2. Figure 5.3 shows the histogram of the nature of errors committed findings in the Euclidean geometry of the exam. Figure 5.3 is the same as Figure 5.1; it was only represented here for emphasis's sake.

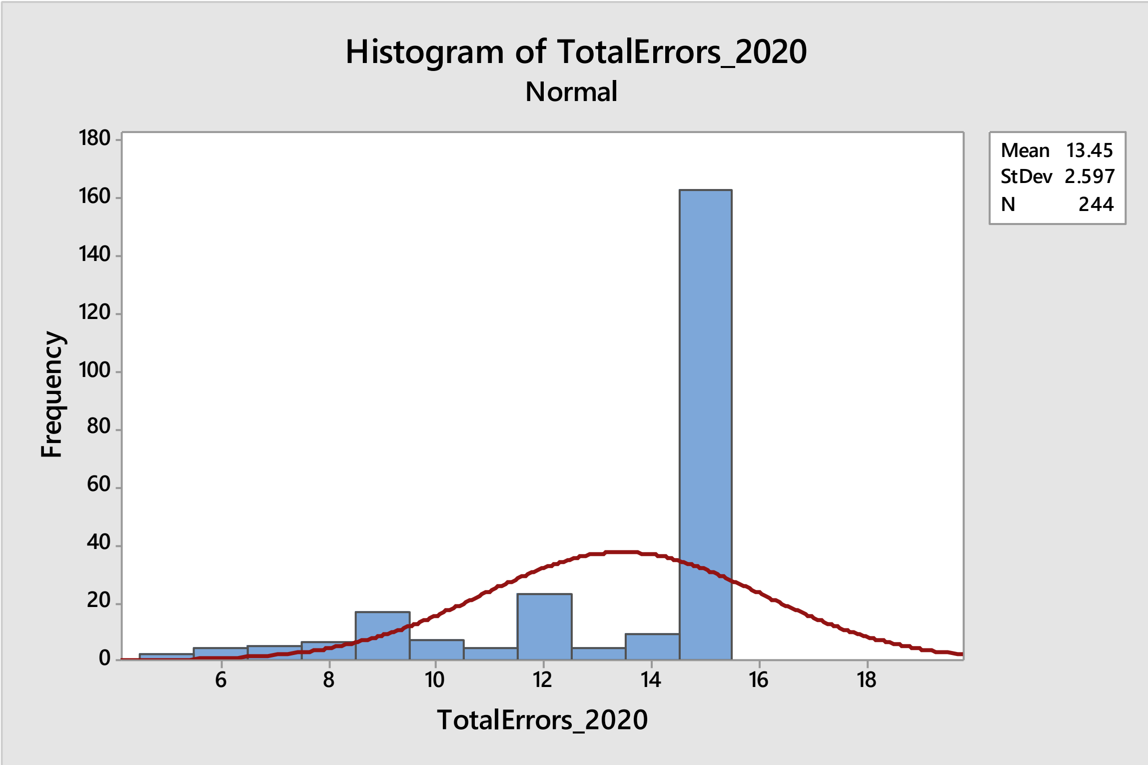


Figure 5.3: Histogram of the Nature of Error for 2020 (same as figure 5.1)

In Appendix 3, it was explained that 15 points is the highest rating of how bad the error committed is 15 and the lowest will be 5 points (Lowest error), in any case. This implies that 15 gives the worst error committed, and 5 error points implies that fewest error committed by individual study participant. Therefore, it is clear that the preceding chart illustrates that the summary of errors committed are categorize as 'Bad'.

The results of the hypothesis that tested the relationship between the errors committed and the mark obtained in the Euclidean geometry aspect of the exam are given next:

Hypothesis one:

H_0: There is no statistically significant effect when comparing the errors committed in solving the Euclidean geometry problems aspect of the mathematics examination paper used in this study with the marks obtained in the Euclidean geometry aspect of the examination.

H_1: There is statistically significant effect when comparing the errors committed in the process of solving Euclidean geometry problems aspect of a mathematics examination used for this study and the marks obtained in the Euclidean geometry aspect of the examination.

Regression analysis was used to investigate relationship between the errors committed data and the Mark obtained in Euclidean geometry data. The results of this analysis are presented in Table 5.3.

Table 5.3: Euclidean Geometry Mark Obtained and Error Committed Regression Model for 2020

Analysis of Variance					
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	104802	20960.5	660.83	0.000***
Error 1_2020	1	2486	2486.5	78.39	0.000***
Error 2_2020	1	449	448.6	14.14	0.000***
Error 3_2020	1	186	185.9	5.86	0.016***
Error 4_2020	1	570	569.8	17.96	0.000***
Error 5_2020	1	41	40.6	1.28	0.259
Error	237	7517	31.7	-	-
Lack-of-Fit	5	657	131.4	4.45	0.001
Pure Error	232	6860	29.6	-	-
Total	242	112320	-	-	-

Model Summary			
S	R-sq	R-sq(adj)	R-sq(pred)
5.6319	93.3%	93.2%	93.0%

Coefficients					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	118.91	3.12	38.13	0.000	-
Error 1_2020	-11.84	1.34	-8.85	0.000***	7.25
Error 2_2020	-7.88	2.10	-3.76	0.000***	12.49
Error 3_2020	-5.16	2.13	-2.42	0.016**	10.72
Error 4_2020	-9.71	2.29	-4.24	0.000***	8.65
Error5_2020	-2.87	2.54	-1.13	0.259	7.99

Based on Table 5.3, Error 1, Error 2, Error 3 and Error 4 are significant at 1% level. Based on the coefficient of errors, we can infer that Error 1 is the most variable affecting the 'EG_Percent_2020' with a coefficient of -11.84 followed by Error 4 with a coefficient of -9.71 and the least being Error5 with a coefficient of -2.87.

In view of the foregoing, H_0 is rejected and the alternative hypothesis is sustained.

Research Question 3

How does the marks obtained in Euclidean geometry aspect of the mathematics examination paper used in this study impact the total marks obtained in the mathematics paper 2 examination as a whole?

The answer to this question shall be drawn from descriptive statistics findings of the marks obtained in the Euclidean geometry aspect of the exam (EG Marks). It shall also be obtained from the total marks obtained in the exam without the Euclidean geometry (Total Mark Without EG) and the total marks obtained in the exam with Euclidean and other topics in the paper (Total Mark With EG). Charts are also used to corroborate the explanation. For the purpose of comparison among the three mark entities, the various marks were calculated over 100; this is what the researcher meant by standardisation of the marks.

Table 5.4: General descriptive statistics 2020 results

Frequencies: General descriptive province 2020

		EG Marks	Total Mark Without EG	Total Mark With EG
N	Valid	244	244	244
	Missing	0	0	0
Mean		19.11	31.33	27.04
Std. Deviation		21.50	20.64	19.84
Skewness		1.35	0.63	0.86
Kurtosis		1.11	-0.38	0.01

The descriptive data analysis depicted in Table 5.4 shows standardised analysed marks. The researcher will discuss the deductions from the comparison of the mean mark and the standard deviation of the results.

(a) The Mean Marks Effect Comparison

All the marks were standardised, that is, all the total marks in each case were converted to 100%. The descriptive statistics shows that the mean of the Euclidean geometry marks (EG), total marks obtained without the Euclidean geometry marks (Total Mark Without EG) and total marks obtained with the Euclidean geometry marks (Total Mark With EG) in the 2020 exam as 19.11, 31.33 and 27.04, respectively. The researcher noted that the highest mean marks (31.33) was the mean marks of (Total Mark Without EG) while the lowest mean marks (19.11) was the mean marks of (EG). It was evidence that the low marks (Total Mark With EG) had a negative effect on the mean mark of the (Total Mark With EG).

(b) Deductions from the Standard Deviation Results

The standard deviation of the (EG) was 21.50 and the mean was 19.11. This implies that majority of the marks were between 2 and 41. Likewise, the standard deviation of the (Total Mark With EG) was 19.84 and the mean was 27.04. This implies that majority of the marks were between 7 and 47. Also, the standard deviation of (Total Mark Without EG) was 20.64 and the mean was 31.33. This implies that majority of the marks were between 11 and 52.

As calculated earlier, the results show that majority of the study participants might have failed the 2020 exam because of their marks from the Euclidean geometry aspect of the paper. When Euclidean geometry marks were taken out of the total marks, more study participants passed the exam, but when the total marks include the marks from the Euclidean geometry, the results show that more study participants failed the exam.

The relationship between the marks obtained in the Euclidean geometry aspect of the 2020 exam and the total marks obtained in the 2020 exam was further tested by the following hypothesis:

Test of Hypothesis

Hypothesis two:

H_0: There is no statistically significant effect when comparing the marks obtained in the Euclidean geometry aspect of the mathematics examination paper used for this study and the marks obtained in the rest of paper 2 mathematics examination without the Euclidean geometry marks.

H_1: There is statistically significant effect when comparing the marks obtained in the Euclidean geometry aspect of the mathematics examination paper used for this study and the marks obtained in the rest of paper 2 mathematics examination without the Euclidean geometry marks.

Table 5.5: Total Mark Regression Model without Errors in 2020

Analysis of Variance					
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	95557.2	47778.6	102743.23	0.000***
EG_Percent_2020	1	3459.4	3459.4	7439.05	0.000***

Analysis of Variance					
Source	DF	Adj SS	Adj MS	F-Value	P-Value
OtherTopics_Percent_2020	1	14161.5	14161.5	30452.82	0.000***
Error	241	112.1	0.5	-	-
Lack-of-Fit	195	112.1	0.6	*	*
Pure Error	46	0.0	0.0	-	-
Total	243	95669.3	-	-	-

Model Summary			
S	R-sq	R-sq(adj)	R-sq(pred)
0.6819	99.9%	99.9%	99.9%

Coefficients					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.0540	0.0848	0.64	0.525	-
EG_Percent_2020	0.3170	0.0037	86.25	0.000***	3.26
OtherTopics_Percent_2020	0.6682	0.0038	174.51	0.000***	3.26

Regression Equation
Total_WithEG_Percent_2020 = 0.0540 + 0.3170 EG_Percent_2020 + 0.6682 OtherTopics_Percent_2020

Considering the F-statistic and its associated p-value, we can infer that there is statistical evidence of goodness of fit and we conclude that the data fits well the model developed at 1% significant level.

Based on Table 5.5, 'EG_Percent_2020' and 'Other Topics_Percent_2020' are significant at 1% level. Considering our variable of interest, 'EG_Percent_2020', it is significant and we can conclude that there is statistical evidence that 'EG_Percent_2020' impact 'Total_With EG_Percent_2020'. However, considering the coefficient of 0.3170, we can infer that 'EG_Percent_2020' has a negative impact towards 'Total_WithEG_Percent_2020'.

The R-sq(adj) helps to explain the power of the variables used in the model in determining 'Total_With EG_Percent_2020' and there is statistical evidence that the variables are very strong since they are able to explain 99.9% variation in 'Total_With EG_Percent_2020' and less than 0.5% cannot be explained by the current variables. In view of the foregoing, H_0 is rejected and the alternative hypothesis is sustained.

5.3.1.2 Presentation of 2021 Phase I Results

Research Question 1

What are the types of errors committed by the South African mathematics learners in solving the Euclidean geometry problems aspect of the mathematics paper 2 examination used in this study?

(a) Type of Error Committed

Table 5.6: 2021 Analysis of 'Type of Errors'

Error Type	**Number of Errors Committed**	**Percentage (%)**	**Cumulative Percentage (%)**
Error 1	664	30.74	30.74
Error 2	605	28.01	58.74
Error 3	275	12.73	71.47
Error 4	455	21.06	92.53
Error 5	152	7.04	99.57
Total	2160		

From the Table 5.6, the total number of errors committed and analysed were 2160 in the 2021 exam scripts used for this study. Error 1 (reading error) is the most commonly committed error; they were 664 which represent 30.74% of the total error analysed. The next commonly committed error was error 2 (comprehension error); they were 605 and represent 28.00% of the total errors collected. Error 4 (processing skills error) was analysed to be the next commonly committed error. There were 455 of them. This makes 21.06% of the total errors committed. This was followed by error 3 (transformation error). There were 275 of this type of error and formed 12.73% of the total error committed. The least error committed was error 5 (encoding error), there were 152 of this type of error and formed 7.04% of the total errors committed.

The researcher presents the nature of the errors committed in the next subsection.

(b) Nature of errors committed

As mentioned under the 2020 results presentation, the errors were also analysed into how fair or bad the errors were to gain more insight into the nature of the errors committed. The researcher would not like to repeat what was already mentioned under 2020 results presentation. The charts give a descriptive illustration of the nature of the errors committed in Euclidean geometry questions of the exam paper used in this study.

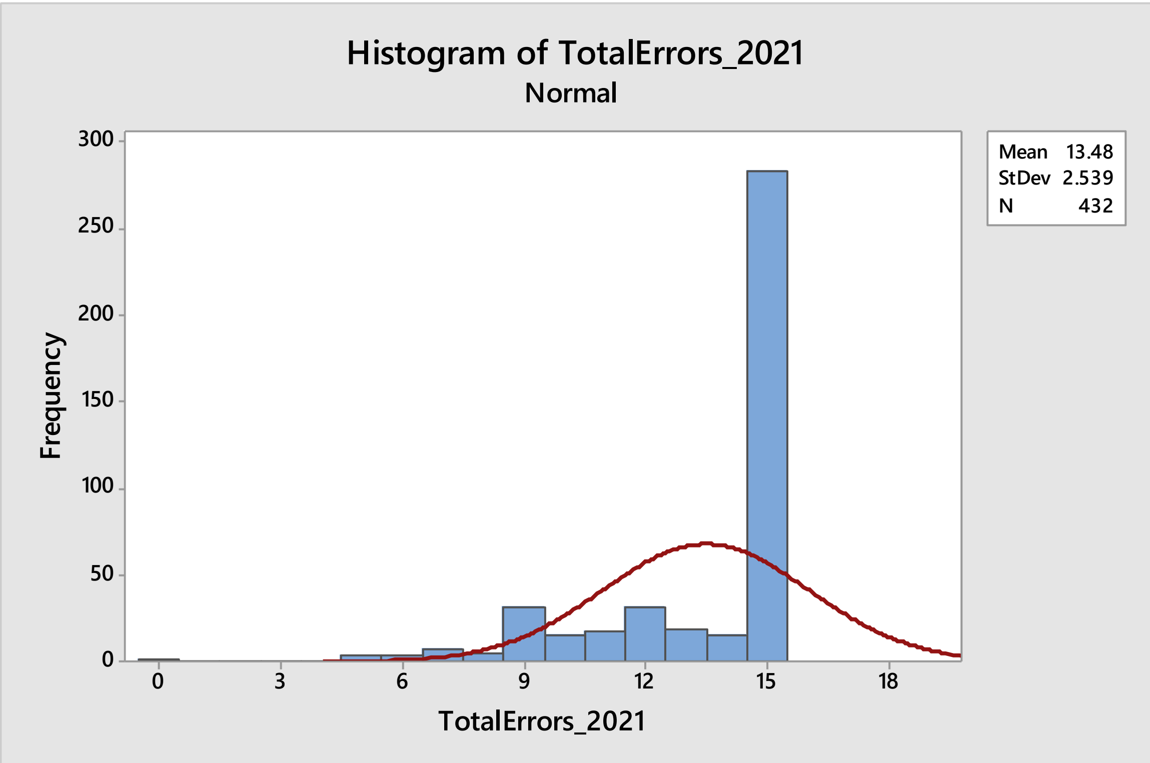


Figure 5.4: Histogram of the Nature of Error for 2021

Similar to 2020 error analysis results, Figure 5.4 shows that most of the errors committed were what the study categorised as bad. That is, in most of the scripts analysed, the students did not have the right problem-solving skills to solve the Euclidean geometry questions in the exam paper used for this study.

Research Question 2

> *How does the errors committed in the research question (1) above affect the marks obtained in the Euclidean geometry aspect of the mathematics paper 2 matric examination used in this study?*

In 2021, questions 9, 10 and 11 are the Euclidean geometry questions with a total mark of 41. Table 5.7 gives the 'Mark obtained' descriptive statistics results. The descriptive findings of the 'type of errors committed' data in Figure 5.6 shows that the most committed errors are error 1 (reading error), 30.74% of the study of the study participants committed this error, followed by error 2 (comprehension error); 28.01 of the study of the study participants committed this error, then error 4 (processing skill error), 21.06 of the study of the study participants committed this error. During the data collection, it was also noted that in many scripts that after these errors were committed, the problem-solving procedure ends there or proceed to present wrong solution procedure. Moreover, many Euclidean geometry questions were also avoided. This might have played a big role in the marks obtained in the Euclidean geometry aspect of the paper. The descriptive findings of the mark obtained in the Euclidean geometry of the paper use for this research is given next.

Table 5.7: General Descriptive Statistics 2021

		Euclidean Geometry Marks
N	**Valid**	**432**
	Missing	**0**
Mean		7.38
Median		4.5
Mode		0
Std. Deviation		8.16
Skewness		1.31
Kurtosis		1.18
Minimum		0
Maximum		40.00
Range		40.00

The descriptive findings of the marks obtained data in the Euclidean geometry aspect of the paper are displayed in column two, the maximum obtainable marks 41, the minimum was 0, and the maximum marks obtained was 40, the mean was 7.38 and the mode is 0. The standard deviation of 8.16 implies that majority of the marks are between 1 and 16 in the Euclidean geometry aspect of the exam. This implies that study participants did not perform well in the Euclidean geometry of the exam. Figure 5.5 shows the histogram of the marks obtained in the Euclidean geometry of the exam.

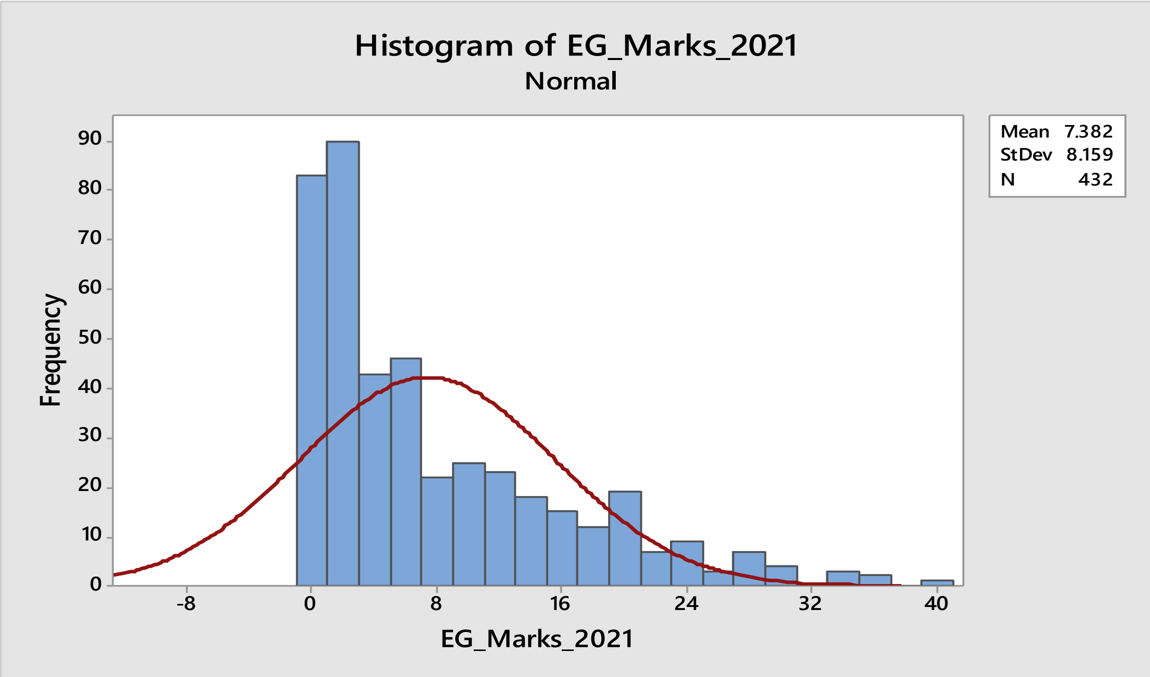


Figure 5.5: Histogram of Euclidean Geometry Marks in 2021

Figure 5.5 indicates that the marks obtained in the Euclidean geometry by most of the study participants are between 0 and around 16 over 41.

As the researcher mentioned in the findings for 2020, it was also discovered that many of the study participants that committed these errors could not proceed to solving the problems in which the error was committed. These errors were graded 'Bad' in the ERS; these were found to be errors 1 and 2. Figure 5.6 shows the histogram of the nature of errors committed findings in the Euclidean geometry of the exam.

Please note that Figure 5.6 is the same as Figure 5.4; it was only represented here for emphasis' sake.

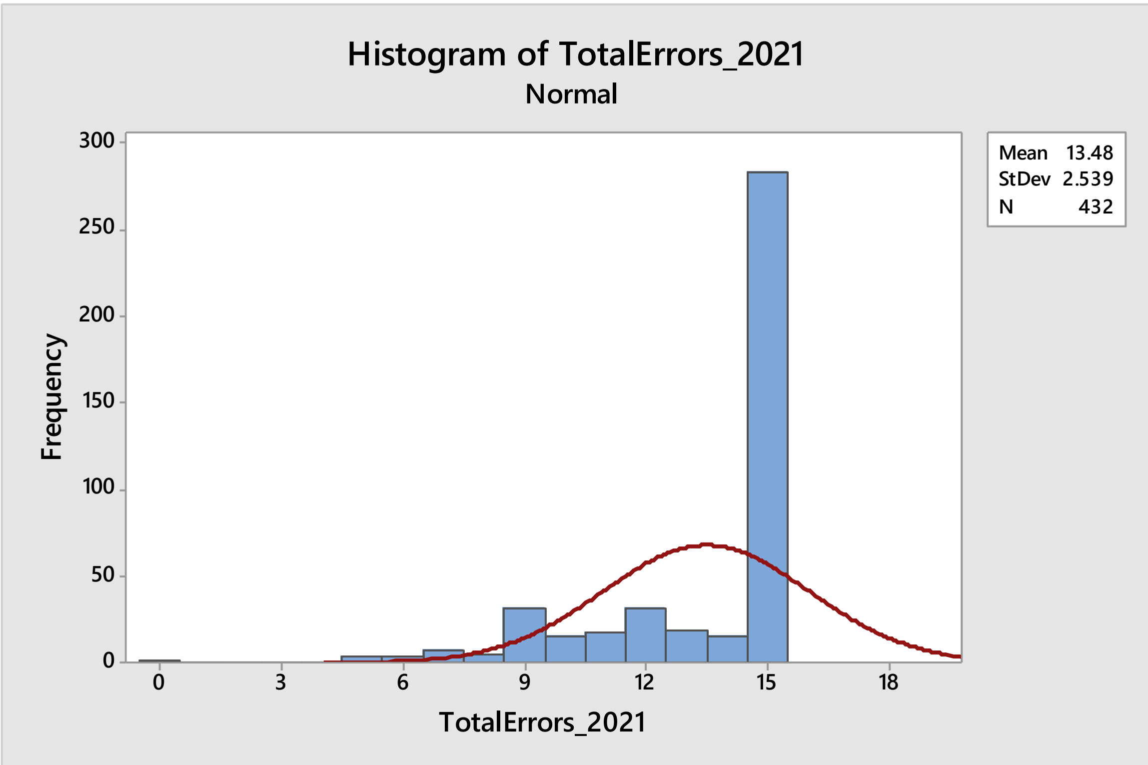


Figure 5.6: Histogram of the Nature of Error for 2021 (same as Figure 5.4)

In Appendix 3, it was explained that 15 points is the highest rating of how bad the error committed is, and the lowest will be 5 points (Lowest error), in any case. This implies that 15 gives the worst error committed, and 5 error points imply that fewest error committed by individual study participant. Therefore, it is clear that Figure 5.6 illustrates that the summary of errors committed are categorised as 'Bad'.

The relationship between the marks obtained in the Euclidean geometry aspect of the 2021 mathematics paper 2 exam and the total marks obtained in the 2021 mathematics paper 2 exam was further tested by the following hypothesis:

Hypothesis one:

H_0: There is no statistically significant effect when comparing the errors committed in solving the Euclidean geometry problems aspect of the mathematics examination paper used in this study with the marks obtained in the Euclidean geometry aspect of the examination.

H_1: There is statistically significant effect when comparing the errors committed in the process of solving Euclidean geometry problems aspect of a mathematics examination used for this study and the marks obtained in the Euclidean geometry aspect of the examination.

As done for 2020 data analysis, the regression data analysis was used to investigate the relationship between the errors committed data and the mark obtained in Euclidean geometry data. The results of this analysis are presented in Table 5.8.

Table 5.8: Euclidean Geometry Mark Obtained and Error Committed Regression Model for 2021

Analysis of Variance					
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	157098	31419.6	1001.65	0.000***
Error 1_2021	1	3445	3444.8	109.82	0.000***
Error 2_2021	1	1301	1301.1	41.48	0.000***
Error 3_2021	1	337	336.9	10.74	0.001***

Analysis of Variance					
Error 4_2021	1	910	910.2	29.02	0.000***
Error5_2021	1	169	169.5	5.40	0.021**
Error	425	13331	31.4	-	-
Lack-of-Fit	5	834	166.7	5.60	0.000
Pure Error	420	12498	29.8	-	-
Total	430	170429	-	-	-

Model Summary			
S	R-sq	R-sq(adj)	R-sq(pred)
5.60070	92.2%	92.1%	92.0%

Coefficients					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	115.86	2.29	50.62	0.000	
Error 1_2021	-10.48	1.00	-10.48	0.000***	6.67
Error 2_2021	-9.49	1.47	-6.44	0.000***	9.58
Error 3_2021	-4.85	1.48	-3.28	0.001***	8.41
Error 4_2021	-8.20	1.52	-5.39	0.000***	6.49
Error5_2021	-3.56	1.53	-2.32	0.021**	4.98

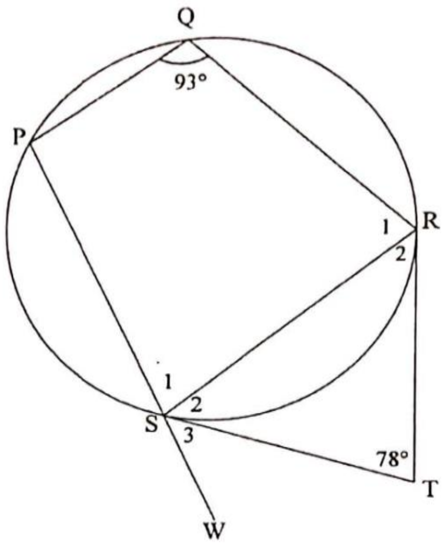
Based on Table 5.8, Error 1, Error 2, Error 3, Error 4 and Error 5 are significant at 1% level. Considering the coefficient of errors, we can infer that all the five errors do have a negative impact towards 'EG_Percent_2021'. Based on coefficients, 'Error 1_2021' is the most variable affecting the 'EG_Percent_2021' with a coefficient of -10.48, followed by 'Error 2_2021' with a coefficient of -9.49, followed by 'Error 4_2021' with a coefficient of -8.20, then followed by 'Error 3_2021' with a coefficient of -4.85 and the least being 'Error5_2021' with a coefficient of -3.56. Hence, the null hypothesis (H_0) is rejected and the alternative hypothesis (H_1) is accepted.

The researcher displayed a sampled scripts to support both 2020 and 2021 results though the sample script is taken from 2021 data.

Question 9 in the 2021 exam was used for this sample script.

QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. PS is produced to W. TR and TS are tangents to the circle at R and S respectively. $\hat{T} = 78°$ and $\hat{Q} = 93°$.



9.1	Give a reason why ST = TR.		(1)
9.2	Calculate, giving reasons, the size of:		
	9.2.1	$\hat{S}_2$	(2)
	9.2.2	$\hat{S}_3$	(2)
			[5]

Figure 5.7: Question 9 of the 2021 NSC Examination

This question is one of the simplest parts of the questions under Euclidean geometry in this exam, which many students could not answer correctly. The researcher presents an example of such study participants' answer scripts as follows:

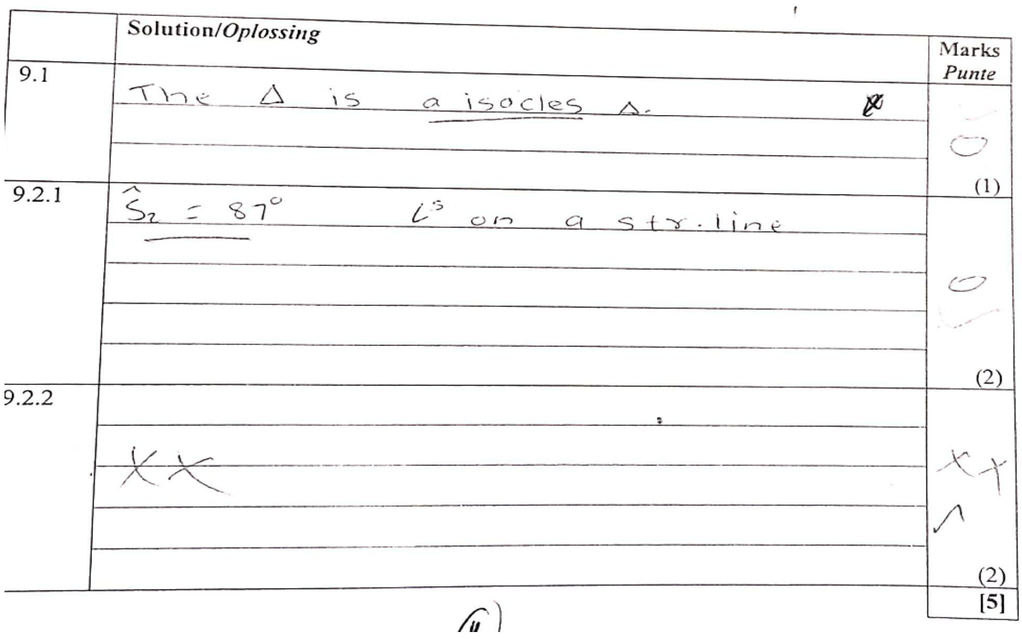


Figure 5.8: Answer Script Example

This paper presents an example of one of those scripts which were analysed as bad errors. Also, an example of those study participants that could not continue to present solutions after error 1 (reading error) or combined errors 1 to 3 (reading, comprehension and transformation errors) were committed.

Research Question 3

> *How does the marks obtained in Euclidean geometry aspect of the mathematics examination paper used in this study impact the total marks obtained in the mathematics paper 2 examination as a whole?*

As mentioned in the 2020 results presentation, the answer to this question shall be drawn from descriptive statistics findings of the marks obtained in the Euclidean geometry aspect of the exam, the total marks obtained in the exam without the Euclidean geometry and the total marks obtained in the exam including Euclidean geometry marks.

Please note that the marks obtained were standardised for the sake of comparison as done for the 2020 analysis.

Table 5.9: General descriptive statistics 2021 results

Frequencies: General Descriptive Province 2021

		EG Marks	Total Mark Without EG	Total Mark With EG
N	Valid	432	432	432
	Missing	0	0	0
Mean		18.005	40.69	34.49
Std. Deviation		19.90	22.92	21.11
Skewness		1.31	0.39	0.56
Kurtosis		1.18	-0.73	-0.48

The pattern of this 2021 exam descriptive data results is closely similar to that of 2020; hence, the results presentation shall follow the same procedure as in the 2020 result presentation to answer the preceding question.

(a) The Mean Marks Effect Comparison

All the marks were standardised; that is, all the total marks in each case were converted to 100%. The descriptive statistics show that the mean of the Euclidean geometry marks (EG), total marks obtained without the Euclidean geometry marks (Total Mark Without EG), and total marks obtained with the Euclidean geometry marks (Total Mark With EG) in the 2020 exam as 18.05, 40.69 and 34.49 respectively. The researcher noted that the highest mean marks (40.69) was the mean marks of (Total Mark Without EG), and the lowest mean marks (18.05) was the mean marks of (EG). It was evident that the low marks (Total Mark With EG) had a negative effect on the mean mark of the (Total Mark With EG).

(b) Deductions from the Standard Deviation Results

The standard deviation of the (EG) was 19.90 and the mean was 18.05. This implies that majority of the marks were between 2 and 38. Likewise, the standard deviation of the (Total Mark With EG) was 21.11 and the mean was 34.49. This implies that majority of the marks were between 13 and 56. Also, the standard deviation of (Total Mark Without EG) was 22.92 and the mean was 40.69. This implies that majority of the marks were between 18 and 64.

As calculated earlier, the results show that majority of the study participants might have failed the 2021 exam because of their marks from the Euclidean geometry aspect of the paper. When Euclidean geometry marks was taken out of the total marks, more study participants passed the exam, but when the total marks include the marks from the Euclidean geometry, the results show that more study participants failed the exam.

The relationship between the marks obtained in the Euclidean geometry aspect of the 2021 exam and the total marks obtained in the 2021 exam was further tested by the following hypothesis:

Test of hypothesis: Hypothesis Two

H_0: There is no statistically significant effect when comparing the marks obtained in the Euclidean geometry aspect of the mathematics examination paper used for this study and the marks obtained in the rest of paper 2 mathematics examination without the Euclidean geometry marks.

H_1: There is statistically significant effect when comparing the marks obtained in the Euclidean geometry aspect of the mathematics examination paper used for this study and the marks obtained in the rest of paper 2 mathematics examination without the Euclidean geometry marks.

Table 5.10: Total Mark Regression Model without Errors 2021

Analysis of Variance					
Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Regression	2	192071	96035.3	0.151	0.625
EG_Percent_2021	1	5269	5269.3	0.938	0.719
OtherTopics_Percent_2021	1	49388	49388.3	0.046	0.941
Error	429	0	0.0	-	-
Lack-of-Fit	318	0	0.0	*	*
Pure Error	111	0	0.0	-	-
Total	431	192071	-	-	-

Analysis of Variance					
Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**

Model Summary			
S	R-sq	R-sq(adj)	R-sq(pred)
0.495	4.60%	4.1%	3.8%

Coefficients					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-0.0001	0.00029	0.990	*	-
EG_Percent_2021	0.2733	0.0006	0.847	*	2.42
OtherTopics_Percent_2021	0.7267	0.0001	0.998	*	2.42

Regression Equation
Total_WithEG_Percent_2021 = -0.000000 + 0.2733 EG_Percent_2021 + 0.7267 OtherTopics_Percent_2021

Based on Table 5.10, 'EG_Percent_2021' and 'Other Topics_Percent_2021' are not significant at 10% level. However, considering the coefficient of 0.2733, we can infer that 'EG_Percent_2021' has a positive impact towards 'Total_With EG_Percent_2021'.
Hence, the null hypothesis (H_0) is rejected and the alternative hypothesis (H_1) is accepted.

Research Question 4

> *Is there any similarity when comparing the errors pattern committed in the Euclidean geometry aspect of the 2020 NSC mathematics examination paper 2 to the 2021 NSC mathematics examination paper 2.*

Answer to research question 4 requires that the results from both 2020 and 2021 be used to present answer to this question.

The descriptive data analysis of the 'Type of error committed' data in Table 5.1 for 2020 and Table 5.6 for 2021 show that the most committed errors, stated in the order of how large, are Error 1, Error 4, Error 2 and Error 1, Error 2, Error 4 for the 2020 and 2021, respectively. In contrast, the least error committed in both years are Error 5. Therefore, it then follows that the most Error committed are Errors 1, 2 and 4. In addition, the nature of the errors committed analysis charts in Figure 5.1 and Figure 5.4 are very similar. The researcher presents the error similarity correlation analysis results below.

Hypothesis one:

H_0: There is no statistically significant difference when comparing the errors committed in the Euclidean geometry aspect of the 2020 NSC mathematics examination paper 2 to that of 2021 NSC mathematics examination paper 2.

H_1: There is statistically significant difference when comparing the errors pattern committed in the Euclidean geometry aspect of the 2020 NSC mathematics examination paper 2 to that of 2021 NSC mathematics examination paper 2.

One-tailed t-test was used to compare each type of error committed from Error 1 to Error 5 in both 2020 and 2021 examination used in this study.

Error 1 2021 vs Error 1 2020				
	N	Mean	StDev	SE Mean
Error 1_2021	431	2.541	0.697	0.034
Error 1_2020	244	2.533	0.728	0.047

Difference = μ (Error 1_2021) - μ (Error 1_2020)

Estimate for difference: 0.0078

95% upper bound for difference: 0.1025

T-Test of difference = 0 (vs <): T-Value = 0.14 P-Value = 0.554 DF = 486

Based on t-statistic = 0.14 and p-value=0.554 which is greater than 0.05, we can safely conclude that the difference between the Error 1 2020 and Error 1 2021 means is not statistically significant, that is, there is no enough statistical evidence to infer that Error 1 2020 mean is different from Error 1 2021.

Hence, the researcher accepts the null hypothesis (H_0) and reject the alternative hypothesis (H_1).

Error 2 2021 vs Error 2 2020				
	N	Mean	StDev	SE Mean
Error 2_2021	431	2.647	0.567	0.027
Error 2_2020	244	2.639	0.609	0.039

Difference = μ (Error 2_2021) - μ (Error 2_2020)

Estimate for difference: 0.0080

95% upper bound for difference: 0.0865

T-Test of difference = 0 (vs <): T-Value = 0.17 P-Value = 0.567 DF = 475

Based on t-statistic = 0.17 and p-value=0.567 which is greater than 0.05, we can safely conclude that the difference between the Error 2 2020 and Error 2 2021 means is not statistically significant, that is, there is no enough statistical evidence to infer that Error 2 2020 mean is different from Error 2 2021.

Hence, the researcher accepts the null hypothesis (H_0) and reject the alternative hypothesis (H_1).

Error 3 2021 vs Error 3 2020				
	N	Mean	StDev	SE Mean
Error 3_2021	431	2.701	0.529	0.025
Error 3_2020	243	2.679	0.557	0.036

Difference = μ (Error 3_2021) - μ (Error 3_2020)

Estimate for difference: 0.0217

95% upper bound for difference: 0.0940

T-Test of difference = 0 (vs <): T-Value = 0.49 P-Value = 0.689 DF = 481

Based on t-statistic = 0.49 and p-value=0.689 which is greater than 0.05, we can safely conclude that the difference between the Error 3 2020 and Error 3 2021 means is not statistically significant, that is, there is no enough statistical evidence to infer that Error 3 2020 mean is different from Error 3 2021.
Hence, the researcher accepts the null hypothesis (H_0) and reject the alternative hypothesis (H_1).

Error 4 2021 vs Error 4 2020				
	N	Mean	StDev	SE Mean
Error 4_2021	431	2.789	0.452	0.022
Error 4_2020	244	2.791	0.464	0.030

Difference = μ (Error 4_2021) - μ (Error 4_2020)

Estimate for difference: -0.0021

95% upper bound for difference: 0.0586

T-test of difference = 0 (vs <): T-Value = -0.06 P-Value = 0.477 DF = 493

Based on t-statistic = -0.06 and p-value=0.477 which is greater than 0.05, we can safely conclude that the difference between the Error 4 2020 and Error 4 2021 means is not statistically significant. That is, there is no enough statistical evidence to infer that Error 4 2020 mean is different from Error 4 2021.
Hence, the researcher accepts the null hypothesis (H_0) and reject the alternative hypothesis (H_1).

Error 5 2021 vs Error 5 2020				
	N	Mean	StDev	SE Mean
Error5_2021	431	2.838	0.394	0.019
Error5_2020	244	2.824	0.403	0.026

Difference = μ (Error5_2021) - μ (Error5_2020)

Estimate for difference: 0.0138

95% upper bound for difference: 0.0666

T-test of difference = 0 (vs <): T-Value = 0.43 P-Value = 0.667 DF = 495

Based on t-statistic = 0.43 and p-value=0.667 which is greater than 0.05, we can safely conclude that the difference between the Error5 2020 and Error5 2021 means is not statistically significant, that is, there is no enough statistical evidence to infer that Error5 2020 mean is different from Error5 2021.
Hence, the researcher accepts the null hypothesis (H_0) and reject the alternative hypothesis (H_1).

Error 5 2021 vs Error 5 2020				
	N	Mean	StDev	SE Mean
Error5_2021	431	2.838	0.394	0.019
Error5_2020	244	2.824	0.403	0.026

Difference = μ (TotalErrors_2021) - μ (TotalErrors_2020)
Estimate for difference: 0.029
95% upper bound for difference: 0.369
T-test of difference = 0 (vs <): T-Value = 0.14 P-Value = 0.556 DF = 494

Based on t-statistic = 0.14 and p-value=0.556 which is greater than 0.05, we can safely conclude that the difference between the Total Error 2020 and Total Error 2021 means is not statistically significant, that is, there is no enough statistical evidence to infer that Total Error 2020 mean is different from Total Error 2021. Hence, the researcher accepts the null hypothesis (H_0) and reject the alternative hypothesis (H_1).

5.3.2 Presentation of Phase II Results

In this phase, there are three research fields, which the researcher called group 1, 2 and 3 involved in the study. Research field 1 and 2 are the experimental groups, while research group 3 is the control group. The results shall be presented group by group, starting with group 1.

It is also important to note that:

(1) The study participants were in Grade 12, meaning that the learning of Euclidean geometry concepts was not new to them at this stage of education level.
(2) The researcher ensured that he captured the first day of the intervention lesson presentation in the two experimental groups as a baseline point to be able to monitor learning progress as the intervention application progressed.

5.3.2.1 Group 1 Presentation of Results

Research Question 1

> *Does the intervention facilitate the study participants' learning of Euclidean geometry concepts?*

Learning facilitation was measured through the effect of intervention on the teaching and learning of Euclidean geometry concepts.

In group 1, classroom observation took place on 13 April 2023. The results of the classroom observations showed that the intervention was strictly followed by the study participants and the study participant mathematics teacher. As it was stated in subsection 4.3.1.3, the intervention consists of EGPSLM and EGPSIA. The former was meant to help the learners to learn basic Euclidean geometry concepts on their own, like Euclidean geometry terms and symbols, while EGPSIA was the proposed classroom Euclidean geometry teaching approach, which include hanging chats containing Euclidean geometry terms and symbols during every Euclidean geometry lesson throughout the intervention.

According to the findings, in the first lesson of the intervention, as the teacher entered the class, the study participants were moving in different directions: some were going to their seats, while some were grumbling. The teacher ordered everyone to get back to their seat and listen to him. The teacher gave each study participant a list of geometry terms and symbols to be read and internalise and mandated them to be bringing this list to the class during every Euclidean geometry lesson. He also motivated them by saying that his new teaching approach may help them pass their matric mathematics exam. This was learnt from the implication of the pilot study that the study participants may need to be motivated because mathematics learners already created negative attitude towards the learning of geometry (see subsection 4.3.1.5.1 of this study).

Again, part of findings of the first lesson of the intervention showed that the study participants were not enthusiastic about the lesson; they were grumbling and some even wanted to go out but the teacher did not allow them. Part of the findings from the field note showed that some of the study participants among those the teacher forced to stay in the class for the lesson said: "This is the part of mathematics I don't like at all."

It also emerged that most of the study participants were confused as a result of error 1 (reading Error), got most of the questions wrong and could not proceed in completing the problem-solving process, as seen in Phase I of this work. The researcher told the teacher to give compulsory homework to the study participants. This is one of the proposed pilot implication remedy mentioned in subsection 4.3.1.5.2 to make the study participant busy with their list of geometry terms and symbols at home.

It further emerged that by the third classroom observation that took place on 24 April 2023 of the intervention in this group, no study participant missed mathematics classes anymore. As soon as the teacher writes the topic of the day, most of the study participants started murmuring, talking to each other about the applicable terms and symbols concerning the topic. In addition, after the teacher had introduced the topic, as soon as he put a problem on the board, study participants were busy with trying on their own to solve the problem. The researcher noted that, all together, the classroom dynamics has changed after few days of the intervention. More importantly, the study participants had developed interest in the learning of Euclidean geometry.

It also emerged that most of the study participants were performing well in their classwork, solving problems of the board correctly. In most classwork, there were fewer problem-solving errors (see the excerpt from the class work.)

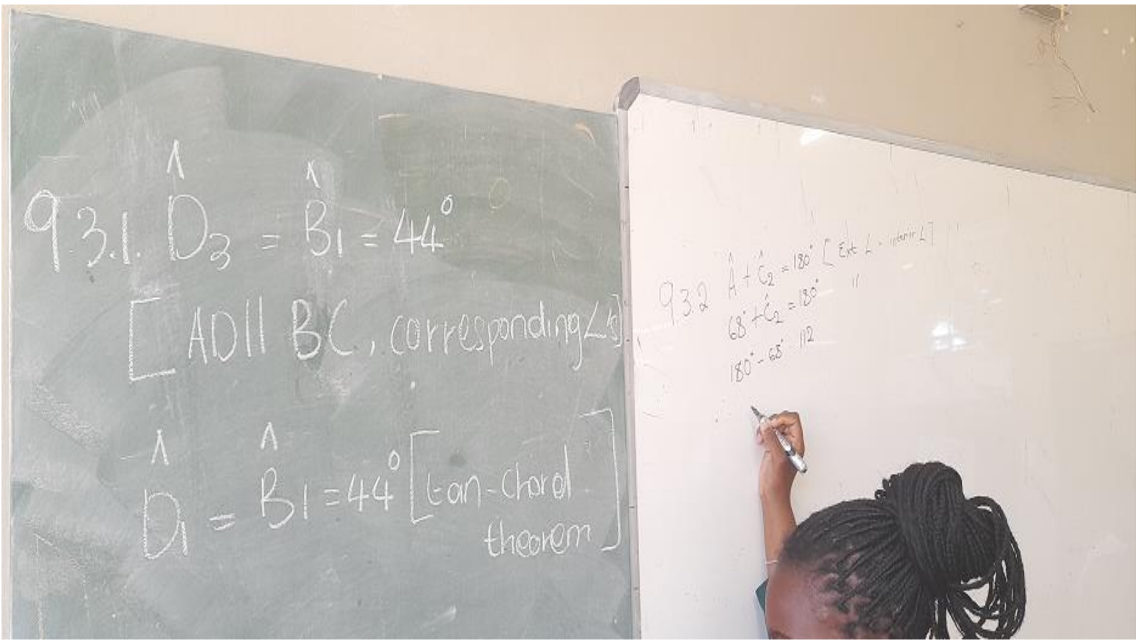


Figure 5.9: Example of learners' classwork participation.

The learner concluded the work as depicted in Figure 5.10.

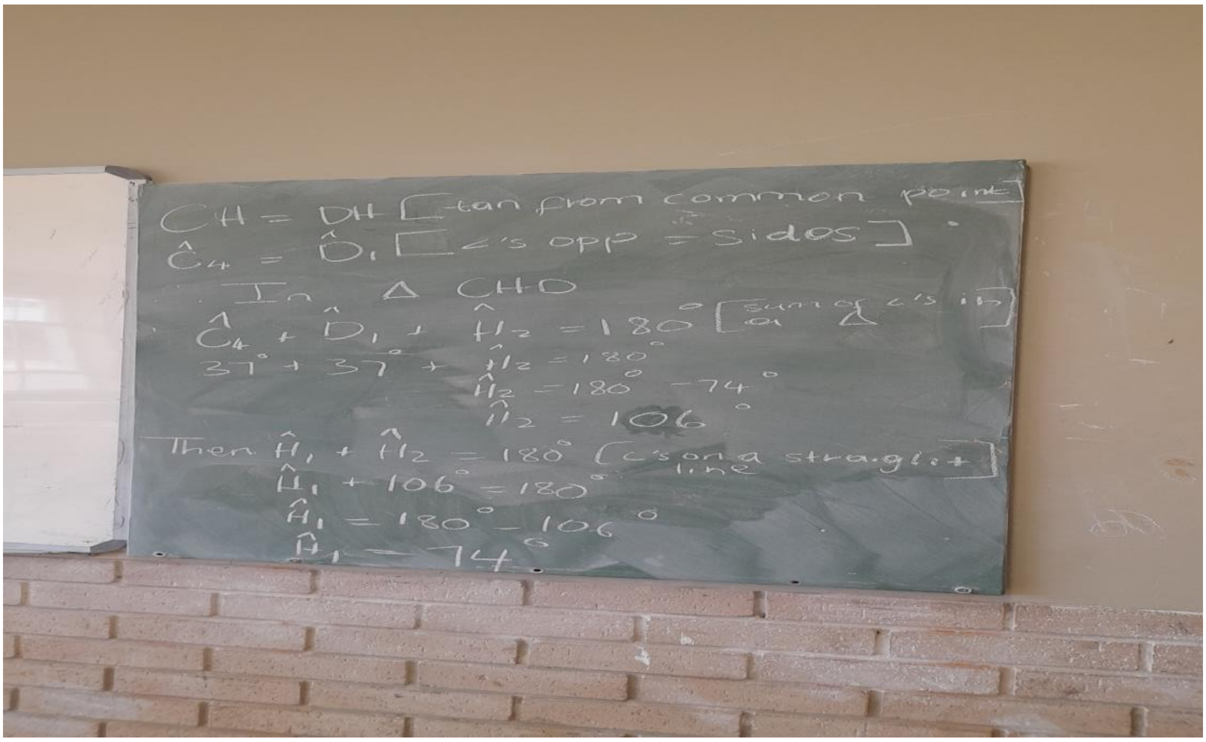


Figure 5.10: Example of learners' classwork

The foregoing excerpts from the classwork show that the problem-solving errors are minimised. The researcher deduced that the intervention might have made the learners to be interested in the learning of Euclidean geometry. As a result of this, the study of Euclidean geometry was facilitated and study participants' performance in Euclidean geometry improved.

The researcher presents the second classroom observation (which is the last observation for this group) as an example of the classroom procedure under the intervention. It should be noted that the names of the study participants mentioned were not their real names for ethical reasons. This particular classroom observation was conducted on 18 April 2023 from 8 am to 9 am. One period was 30 minutes long, hence a period of 8 am to 9 am was a double period. The researcher was already in the school before the observed lesson started, but did not enter the classroom; the lesson was a one-period lesson. The researcher was able to observe the attitude of the study participants while waiting for the teacher. They were discussing the homework they had from the previous lesson and checking similar problems in their mathematics textbook.

Immediately the teacher entered the classroom, all the discussions stopped and everyone went to their respective seats. The teacher hung all the Euclidean geometry concepts charts he brought to the class. He started talking. He called the class captain to distribute the study participants' workbook to the learners, while he waited patiently for the class to settle down. Then he continued talking.

Teacher: *I hope you all brought your list of Euclidean geometry terms and symbols?*

By this time those that had brought their own Euclidean geometry concepts list took them out.

Teacher: *Good. In case we have anyone without his/her own Euclidean geometry concepts list, then you have to use the hung ones.*

Study participant: *Thank you, sir.* Replied the study participants.

The teacher spent some time on asking the study participants various questions about the Euclidean geometry terms, symbols and concepts, to ensure that the study participants are revising Euclidean geometry at home with aim of been grounded in Euclidean geometry. This is a usual exercise before going into the day's work. After he was satisfied about how the class was shaping up in the learning of Euclidean geometry concepts, he then proceeded to ask about the last lesson.

This takes more time of the normal lesson presentation. It came out from the pilot study that EGPSIA teaching approach requires more teaching time than the traditional instructional approach.

Teacher: *Good. Can anyone remind us the concepts that we learnt in the yesterday?*
Many of the study participants raised up their hands to answer the question.

Teacher: Tebogo. The teacher called Tebogo to answer the question.

Tebogo: We solved problems on cyclic triangles.

'Right' the teacher said as he moved to…
Teacher: *Thank you Tebogo. Does anyone have any question or comment?*

Study participants: *No.*

The teacher wrote following question on the board, which was a continuation of the revision topic in the previous lesson.

> *Prove that the sum of the opposite angles of a cyclic quadrilateral is equal to 180°.*

To ensure that all the study participants overcome reading, comprehension and transformation errors, the teacher and the study participants discussed together on the demand of the question, meaning of terminologies involved in the question (terms like to prove, opposite angles, quadrilateral, and cyclic quadrilateral). Majority of the study participants actively participated in this discussion. This shows that the study participants were conversant with the Euclidean geometry concepts list because they studied it at home.

From the study participants-teacher discussion, the teacher noted that the study participants knew what to do and how it could be done. Then he proceeded to asking the class who will solve the question. Meanwhile, some of the students were already trying to solve the problem on their own.

Teacher: *Alright, now that we know what to do, who can solve the problem for us?*

Study participants: *Meeeee!*
There were many hands up as they wanted to solve the problem for the class.

Teacher: *Yes, Batseba come and try?*

Batseba went to the board and started solving the problem, while many of the other study participants were doing it together with her, until when the teacher cautioned them to allow Batseba to finish and not confuse her. Figure 5.11 depicts Batseba solving the problem.

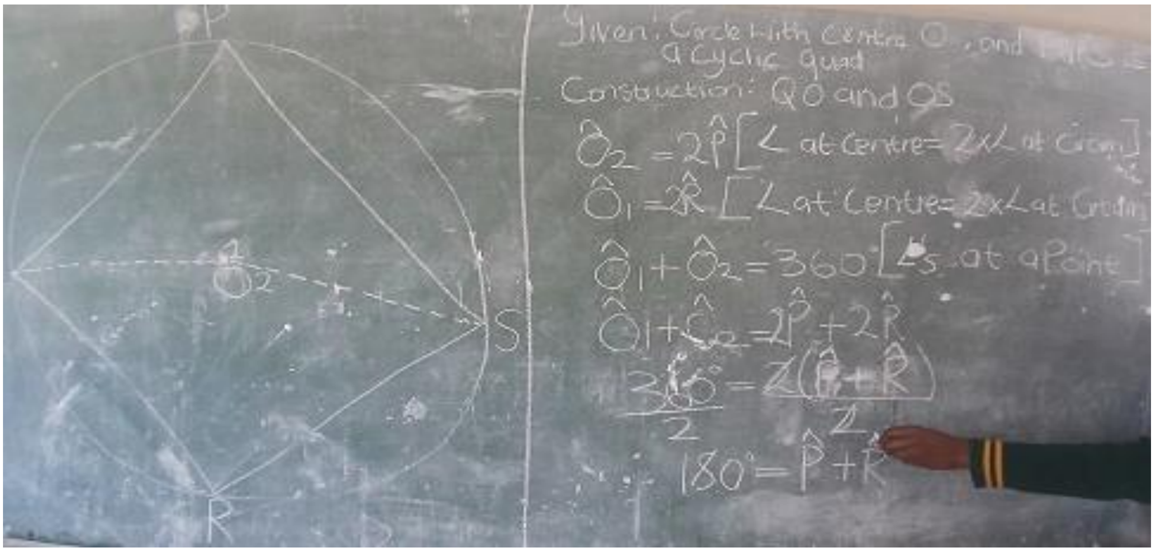


Figure 5.11: Study participants proving that opposite angles of a cyclic quadrilateral are supplementary.

When she finished with the proof, the teacher looked at me and I nod in happiness to see the improvement in the study participants' Euclidean geometry conceptual ability. At this time, the lesson was almost over. I wished the lesson could continue to see the treatment of another question.

Teacher: *Thank you Batseba, go back to your seat.* The teacher paused briefly and continued. *Who does not understand the proof?* He paused again to see the class response to his question and then continued. *Or any question.*

Study Participants: *We are fine, let's continue.* Answered the class.
The teacher looked at his wristwatch, proceeded to write two questions on the board and said:

Teacher: "*We cannot solve these questions today; they are your homework. We have four minutes to the end of the period; the time will not be enough to go to another question. But I must say that I'm happy with you guys. Answer these questions in your classwork book.*"

He stopped briefly and looked around *and* continued. "*Yes class captain, by tomorrow morning I want to see everyone's classwork book on my table. Let's call it a day.*"
The class ended.

The researcher noted that there were few study participants that were keeping quiet, not so much responsive. But the researcher did not want to interrupt the teaching proceedings; he would have loved that the teacher ask questions from those that were not so responsive. In any case, the researcher did not expect the whole class to be the same. In addition, after the lesson the researcher spent some time in the school to check the study participants' homework book. He randomly collected ten books to check how they did in their homework.

Research Question 2

How does the study interventions impact the participants' performance in Euclidean geometry?

The results of the pre-test and post-test shall be used to answer the foregoing research question. The descriptive statistics of the results shall be presented first, followed by the presentation of the hypothesis results. In addition, the result of the problem-solving errors from the solution appraisal in the error list shall be presented, and the answer ended with test of hypothesis.

(i) Results from the descriptive statistics

Table 5.11: Descriptive Statistics of the pre-post test results in group I

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
							Std. Error	Statistic	Std. Error
Pre_Test_1	41	2	42	20.02	11.195	.217	.369	-.874	.724
Post_Test_1	41	26	92	52.59	18.419	.507	.369	-.743	.724
Valid N (listwise)	21								

Pre-test

From Table 5.11, the mean of the pre-test is 20.02%, with a minimum mark of 2% and a maximum mark of 42%. The standard deviation of the marks in this test is 11.195. This implies that the marks are fairly disperse away from the 20% mark, which is the mean. In addition, the standard deviation of 11.20 implies that majority of the marks are between 9% and 32% marks. The skewness value of 0.217 and kurtosis value of -0.874 imply that the findings are fairly symmetrical (George & Mallery, 2010).

The histogram of the pre-test marks is given next:

(i) Results from the histogram chart

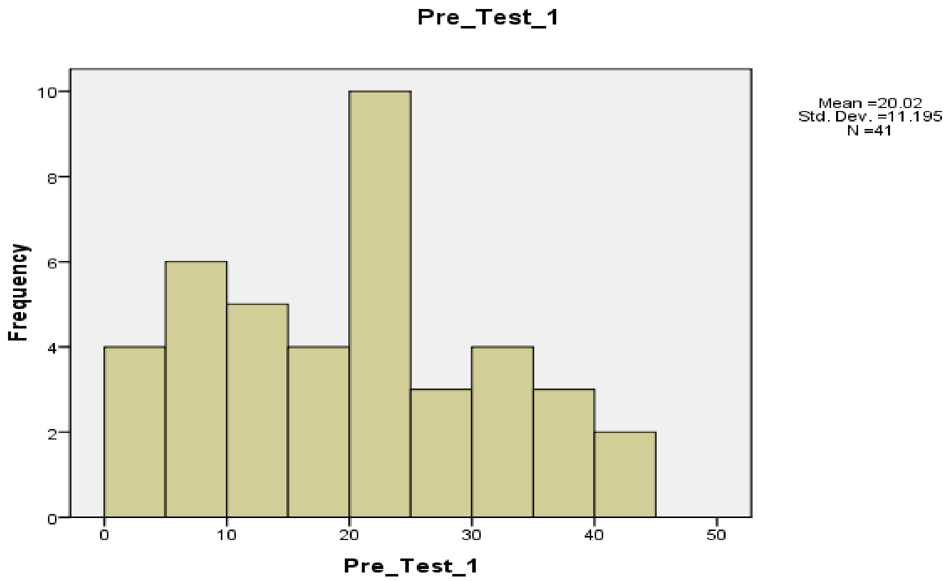


Figure 5.12: Histogram of the Pre-test Marks in Group 1

From Figure 5.12, it could be seen that the marks are fairly symmetrically distributed around the mean mark (20%), with majority of the marks between 9% and 32%.

(ii) Results from the problem-solving errors

The results of the solution appraisal in the Error list on the errors committed shows that out of 41 pre-test scripts, 11 (about 27%) of the student participants could not attempt any of the questions. These group of study participants might have been affected by reading error (error 1) since they could not even attempt any of the pre-test questions. Furthermore, 12 (29%) of study participants attempted few questions but were all wrong; they might have been affected by combination of errors 1 to 3 (these are reading, comprehension and transformation errors). About 18 (44%) of the study participants started many questions very well but they contained errors, could not finished some of the questions, or ended up with wrong answers but a few questions were answered correctly. These group of students might have been affected by 4 and 5, which are skill processing and coding errors. The researcher presents an example of the pre-test scripts in Figure 5.13.

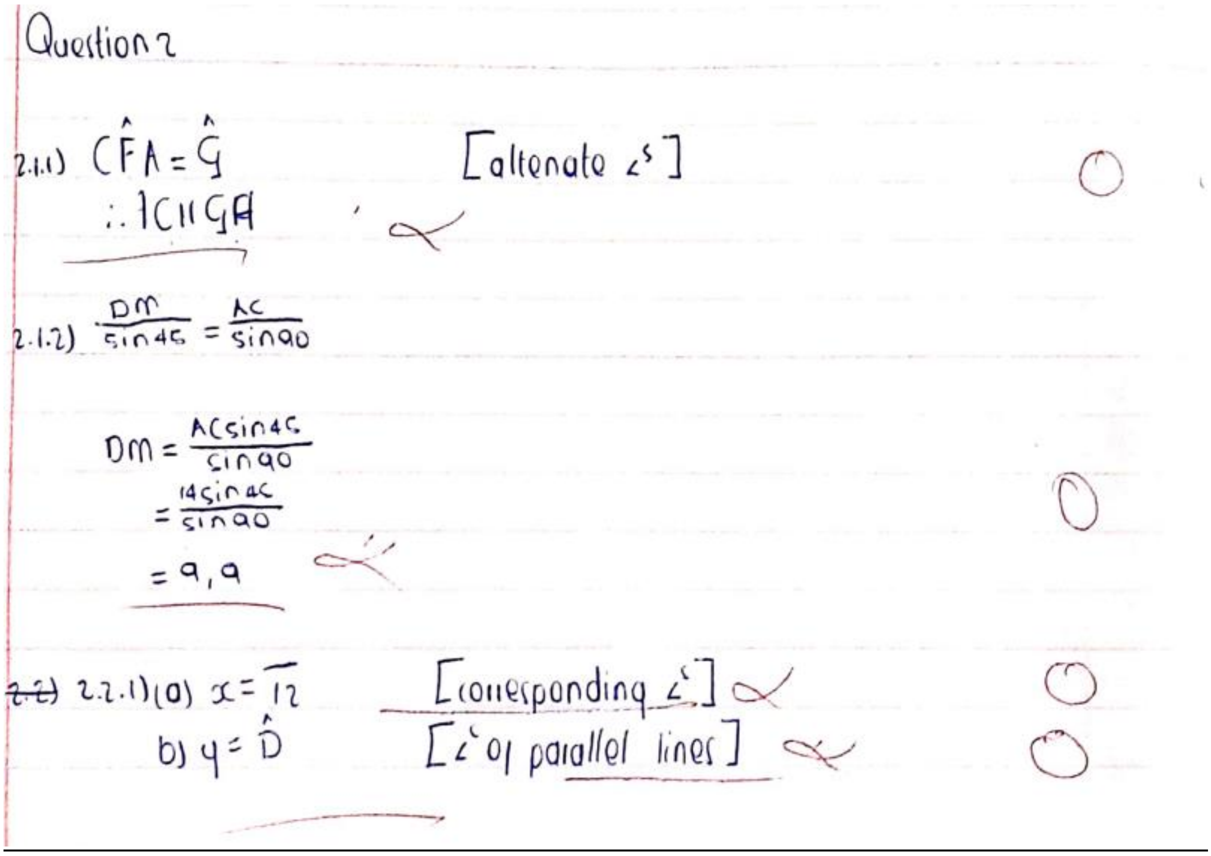


Figure 5.13. Example of Pre-test Scripts

Post-test

(i) Results from the descriptive statistics

Similarly, the mean of the post-test is 52.59%, with a minimum mark of 26% and a maximum mark of 92%. The standard deviation of the marks in this test is 18.419, which implies that the marks are fairly disperse away from the mean mark of 52.59%. In addition, the standard deviation of 18.42 implies that majority of the marks are between 34% and 71% marks. The skewness value of 0.217 and kurtosis value of -0.874 imply that the findings are fairly symmetrical.

(ii) Results from the histogram chart

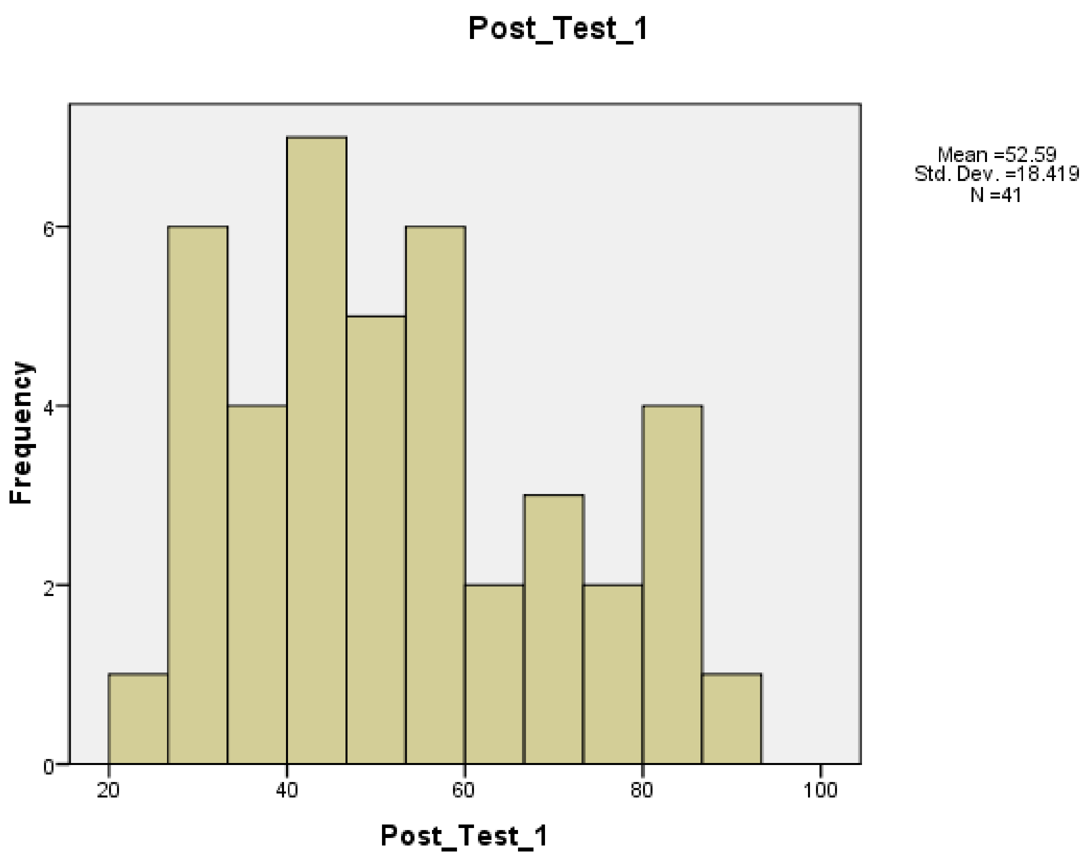


Figure 5.14: Histogram of the Post-test Marks in Group 1

From the chart, it could be seen that the marks are fairly symmetrically distributed around the mean mark (52.59%), with most marks between 34% and 71%.

(iii) Results from the problem-solving errors

The results of the solution appraisal in the Error list on the errors committed shows that out of 41 post-test scripts, there was no blank script, 7 (17%) of the study participants attempted few questions but were all wrong. The results imply that these group of study participants were still struggling with error 1 to 3, (reading, comprehension and transformation errors) and 34 (83%) of the study participants started many questions very well but they contained few errors, could not finish some of the questions or ended up with wrong answers but many questions were answered correctly. This implies that majority of the study participants in these group might have been grounded in the Euclidean geometry conceptual understanding and in the problem-solving processing skills and know how to properly put their answers. However, some (especially those that could not finish some of the questions) might still be having problem with problem-solving processing skills (error 4) and how to write answers properly (error 5). The researcher presents an example of the post-test scripts in Figure 5.15.

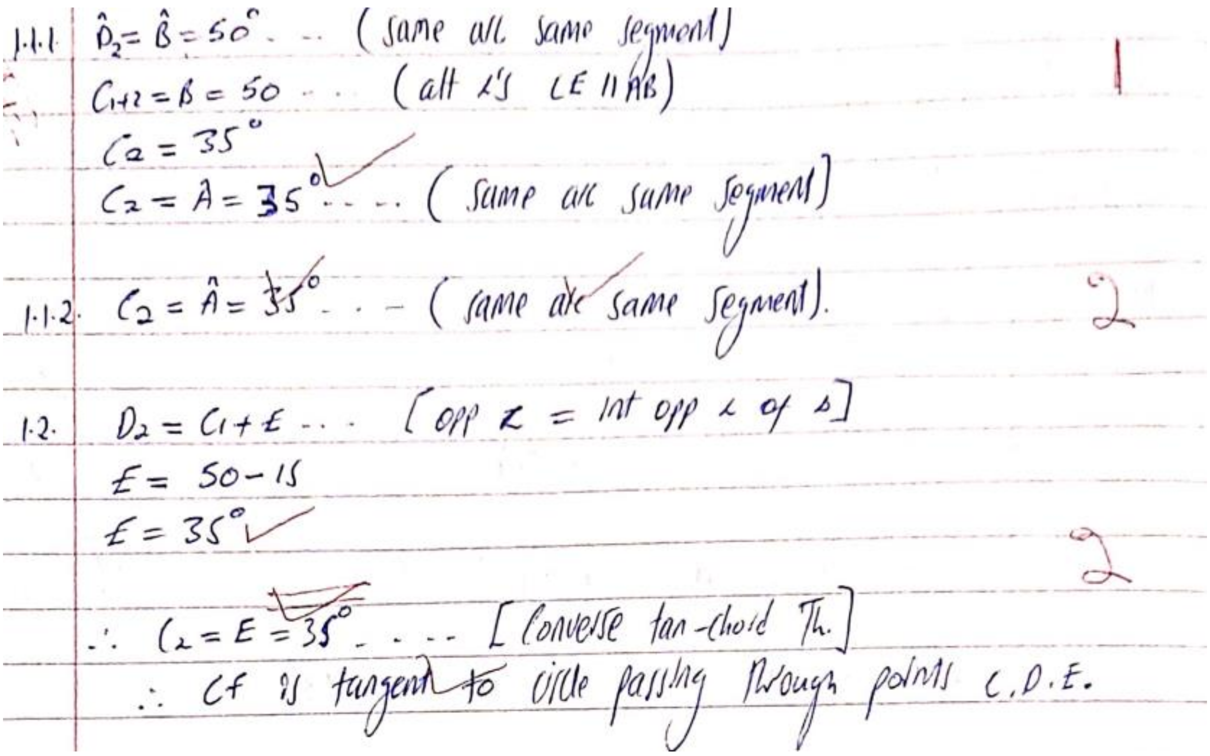

1.1.1 $\hat{D}_2 = \hat{B} = 50°$ ... (same arc same segment)
$C_{1+2} = B = 50$ ... (alt ∠'s CE ∥ AB)
$C_2 = 35°$
$C_2 = A = 35°$ .... (same arc same segment)
1.1.2 $C_2 = \hat{A} = 35°$ ... (same arc same segment).
1.2 $D_2 = C_1 + E$ ... [opp ∠ = int opp ∠ of Δ]
$E = 50 - 15$
$E = 35°$
∴ $C_2 = E = 35°$ .... [Converse tan-chord Th.]
∴ CF is tangent to circle passing through points C, D, E.

Figure 5.15: Example of Post-test Script

(iv) Test of Hypothesis

H_0: There is no statistically significant difference between the study participants' pre-test score and the post-test score in this study.

H_1: There is statistically significant different between the study participants' pre-test score and the post-test score in this study.

Table 5.12: T-test Result of Significant

One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	T	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Pre_Test_1	11.453	40	.000	20.024	16.49	23.56
Post_Test_1	18.281	40	.000	52.585	46.77	58.40

The t-test comparing the mean of the pre and the post-test mean at 95% significant level shows that the post-test mean (52.59) is statistically significant than the pre-test mean (20.02) with a p-value of 0,000 which is less than the 5% significant level. Therefore, the null hypothesis (H_0) is rejected and accept the alternative hypothesis (H_1).

5.3.2.2 Group 2 Presentation of Results

Research Question 1

Does the intervention facilitate the study participants' learning of Euclidean geometry concepts?

This group is the experimental group 2 in this study. The introductory part of the answer to this research question was done in group 1 presentation of results. Hence, the researcher will not waste time of the readers by repeating the same thing.

The findings in this group were very similar to the results that were obtained in group 1. In this group, classroom observations were conducted 13, 17 and 21 April 2023. The researcher observed the first lesson of the intervention. The results of the classroom observations data analysis in this group showed that the intervention was strictly followed by the study participants and the mathematics teacher. As was done in group 1, the group 2 mathematics teacher also hung the chats of Euclidean geometry terms and symbols in the classroom during the teaching of Euclidean geometry.

It emerged that in the first lesson of the intervention, the teacher gave the study participants instructions on how he will be conducting the teaching for the period of the intervention. He also gave each study participant a list of geometric terms and symbols to be read and internalised, and instructed them to always bring the list to the class during every Euclidean geometry lesson throughout the duration of the intervention. More importantly, the teacher also abided by the intervention lesson presentation procedure throughout the intervention. The results of the data showed that in the first lesson of the intervention, the study participants were not enthusiastic about the lesson. There were various problem-solving errors in the study participants' classwork and many of the study participants could not answer the questions in the classwork. They waited for the teacher to put the solution on the board.

It also emerged that as the invention lesson progresses, the study participants started to show interest in the learning of Euclidean geometry. There were increased study participants' participation in the classroom teaching procedure. As soon as the teacher wrote the topic of the day, most of the study participants will start to look at their list of terms and symbols while preparing to participate in providing answers. In addition, after the teacher had introduced the topic on the board, the study participants were busy: trying on their own to solve the problem, raising up their hands to either ask question, or raise comment. It emerged that the classroom dynamics during the lesson presentation had changed, compared to the results of the first day of intervention. The researcher noted that, altogether, most of the study participants had developed interest in the learning of Euclidean geometry. Furthermore, it also emerged that most of the study participants were performing well in their classwork, solving problems on the board correctly. Their classwork contains less problem-solving errors (see examples of study participants' classwork).

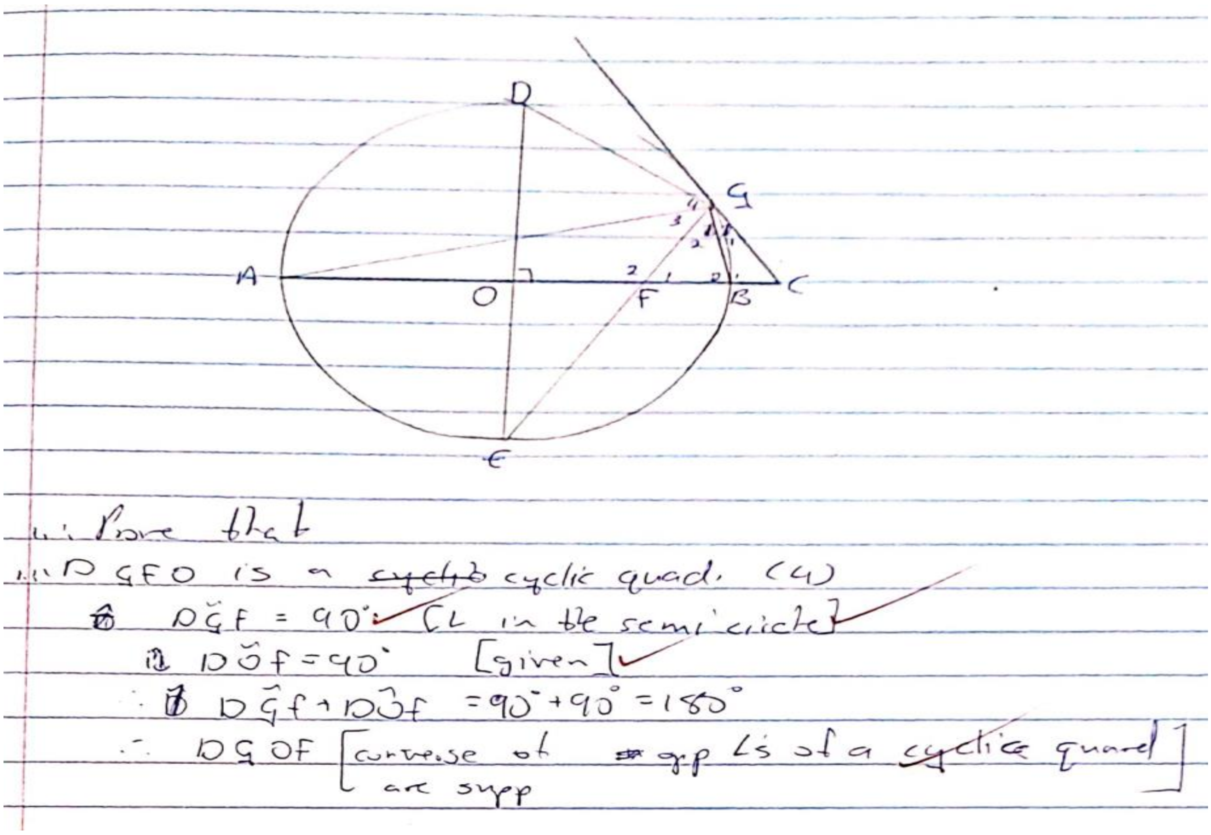


Figure 5.16: Example of classwork performance

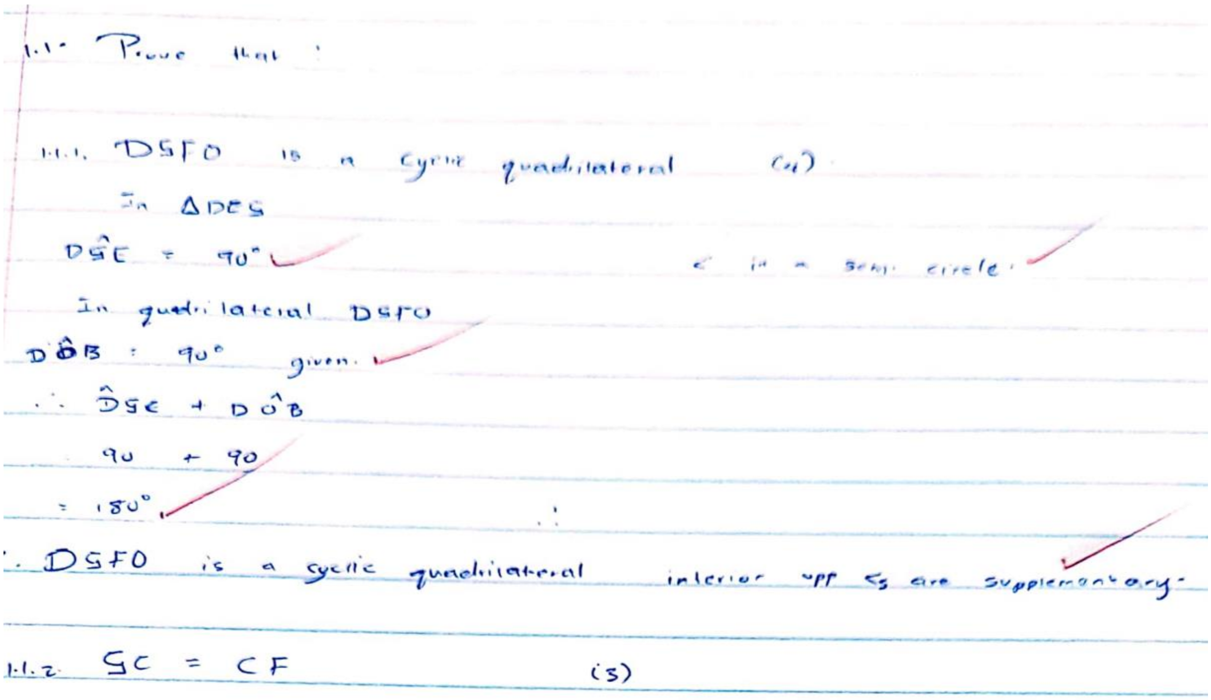


Figure 5.17: Learners' classwork

The researcher deduced that the intervention might have made the learners to be interested in the learning of Euclidean geometry.

Research Question 2

How does the study interventions impact the participants' performance in Euclidean geometry?

(i) Results from the descriptive statistics

Table 5.13: Descriptive Statistics of the pre-post test results in group 2

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
							Std. Error		Std. Error
Pre_Test_2	33	0	40	14.48	11.906	.686	.409	-.564	.798
Post_Test_2	33	12	84	41.64	17.575	.482	.409	-.097	.798
Valid N (listwise)	21								

Pre-test

From Table 5.11, the mean of the pre-test is 14.48%, with a minimum mark of 0% and a maximum mark of 40%. The standard deviation of the marks in this test is 11.91 implies that the marks are fairly dispersed away from the 14.48% mark which is the mean. It also predictive that majority of the marks are between 3 and 26 marks. The skewness value of 0.686 and kurtosis value of -0.564 implies that the marks distribution were not symmetrical but skewed to the left.

(ii) Results from histogram charts

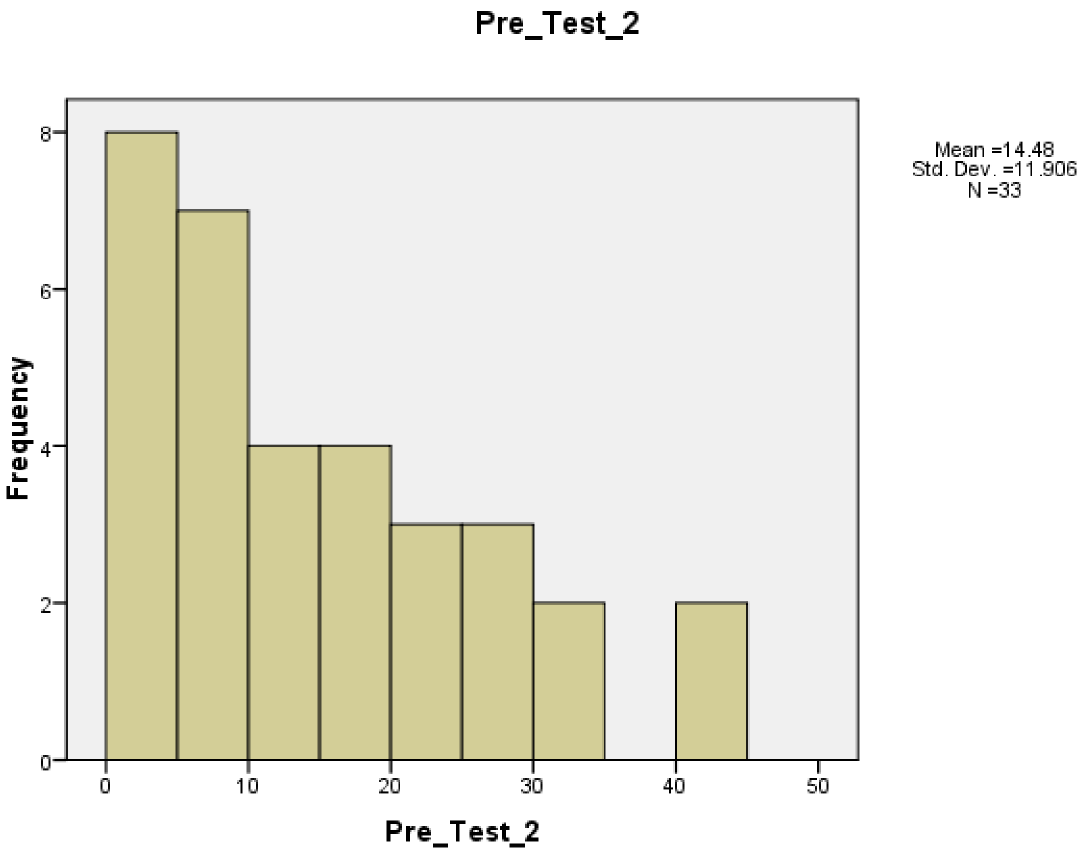


Figure 5.18: Histogram of the Pre-test Marks in Group 2

From the chart, it could be seen that the marks are not symmetrical but distributed more to the left of the mean mark (14.48%).

(iii) Results from the problem-solving errors

The results are like the group 1 pre-test result. The results of the solution appraisal in the Error list on the errors committed show that out of 33 pre-test scripts, 12 (36%) of the student participants could not attempt any of the questions. This group of study participants might have been affected by reading error (error 1), since they could not even attempt any of the pre-test questions, 10 (30%) attempted few questions but were all wrong, they might have been affected by combination of errors 1 to 3 (these are reading, comprehension and transformation errors) and 11 (33%) of the study participants started many questions very well but they contained errors, could not finish some of the questions, or ended up with wrong answers but a few questions were answered correctly. This group of students might have been affected by error 4 and error 5, which are skill processing and coding errors, respectively.

Post-test

(i) Results from descriptive statistics

Similarly, the mean of the post-test is 41.64%, with a minimum mark of 12% and a maximum mark of 84%. The standard deviation of the marks in this test is 17.58. This implies that the marks are fairly distributed away from the 41.64% mark which is the mean, and that, majority of the marks are between 24 and 59 marks. The skewness value of 0.482 and the kurtosis value of -0.097 implies that the findings are fairly symmetrical.

(ii) Results from histogram chart

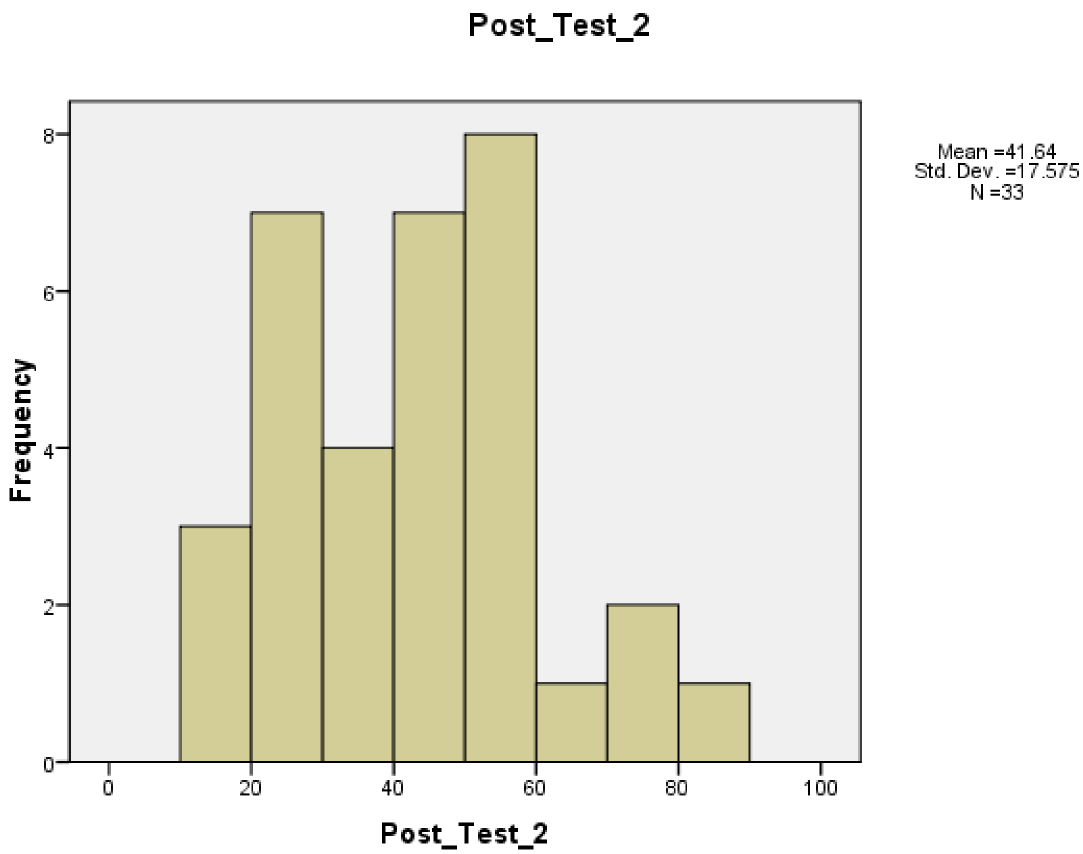


Figure 5.19: Histogram of the Post-test Marks in Group 2

From the chart, it could be seen that the marks are fairly symmetrically distributed around the mean mark (41.64%) and having majority of the marks between 24 and 59 marks.

(iii) Results from the problem-solving errors

The results presented here are similar to the post-test result of group 1. The results of the solution appraisal in the error list on the errors committed show that out of 33 post test scripts, there was no blank script. About 8 (24%) attempted few questions but were all wrong. The results, therefore, implies that these group of study participants were still struggling with error 1 to 3, (reading, comprehension and transformation errors) and 25 (76%) of the study participants started many questions very well but they contained few errors, could not finish some of the questions, or ended up with wrong answers but many questions were answered correctly. This implies that majority of the study participants in this group might have been grounded in the Euclidean geometry conceptual understanding and in the problem-solving processing skills and know how to properly put their answers, while some (especially those that could not finish some of the questions) might still be having problem with problem-solving processing skills (error 4) and how to write answers properly (error 5).

(iv) Test of Hypothesis for Group 2

H_0: There is no statistically significant difference between the study participants' pre-test score and the post-test score in this study.

H_1: There is statistically significant difference between the study participants' pre-test score and the post-test score in this study.

Table 5.14: Test of hypothesis descriptive statistics

One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	T	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Pre_Test_2	6.989	32	.000	14.485	10.26	18.71
Post_Test_2	13.610	32	.000	41.636	35.40	47.87

The t-test comparing the mean of the pre-test and the post-test at 95% significant level shows that the post-test mean (52.59) is statistically significant than the pre-test mean (20.02) with a p-value of 0,000 which is less than the 5% significant level. Therefore, the null hypothesis (H_0) is rejected and the alternative hypothesis (H_1) is accepted.

5.3.2.3 Group 3 Presentation of Results

Research Question 1

Does the intervention facilitate the study participants' learning of Euclidean geometry concepts?

This is the control group of the study. There was no classroom intervention that was administered to the study participants. The classroom teaching went in a traditional lesson presentation. The classroom observation was conducted only once in this group.

Classroom observation was conducted on 13 April 2023. This is the first day of the intervention. Also, the researcher would like to emphasise that this is the only observation that was done in this group. The results of the data analysis showed that the mathematics teacher taught Euclidean geometry in a traditional mathematics lesson presentation. After the classroom observation, the researcher enquired from the teacher why were some study participants were absent from the mathematics lesson on that day, and whether they know that Euclidean geometry will form part of the questions they will need to answer in the matric exam. The teacher said some of the study participants avoid Euclidean geometry part of the mathematics syllabus. It also emerged that in the lesson observed, some study participants were struggling to understand the content the teacher was teaching, while some looked confused and did not care. It also emerged that the type of questions the study participants were asking suggested that they did not understand even basic concepts of Euclidean geometry.

Research Question 2

How does the study interventions impact the participants' performance in Euclidean geometry?

Table 5.15 shows the descriptive statistics of the pre-post test results in the control group.

(i) Results from the descriptive statistics

Table 5.15: Descriptive Statistics of the pre-post test results in control group

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Pre_Test_Control	21	4	40	17.24	9.944	.928	.501	.349	.972
Post_Test_Control	21	8	52	27.33	12.253	.404	.501	-.173	.972
Valid N (listwise)	21								

Pre-test

From Table 5.15, the mean of the pre-test is 17.24%, with a minimum mark of 4% and a maximum mark of 40%. The standard deviation of the marks in this test is 9.94. It implies that the marks are fairly dispersed away from the 17.24%

mark, which is the mean. In addition, the standard deviation of 9.94 implies that majority of the marks are between 7 and 27 marks. The skewness value of 0.928 and the kurtosis value of 0.349 imply that the findings are fairly symmetrical.

(ii) Results from the histogram chart

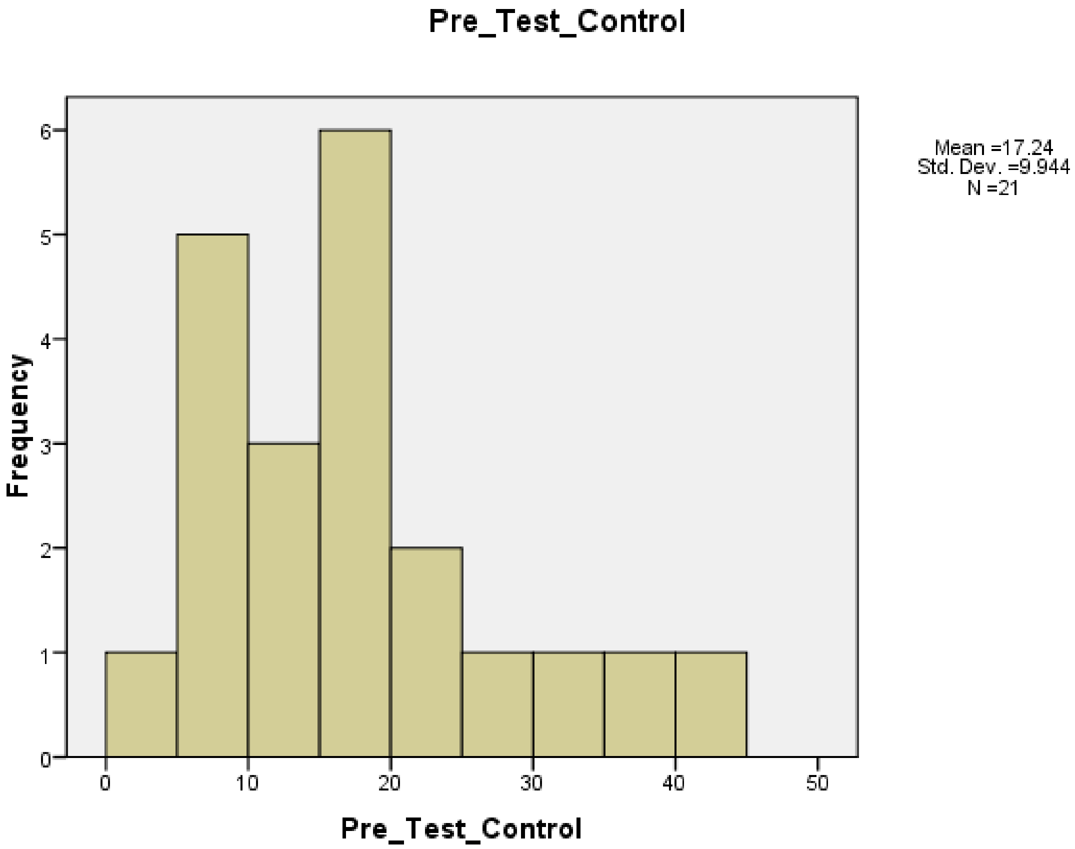


Figure 5.20: Histogram of the pre-test marks in control group

From the chart, it could be seen that the marks are fairly symmetrically distributed around the mean mark (17.24%) with light tails because of low values of kurtosis.

(iii) Results from the problem-solving errors

The results of the solution appraisal in the Error list on the errors committed shows that out of 21 pre-test scripts, 8 (38%) of the student participants could not attempt any of the questions. These group of study participants might have been affected by reading error (error 1) since they could not even attempt any of the pre-test questions, 7 (33%) attempted few questions but were all wrong. They might have been affected by combination of errors 1 to 3 (these are reading, comprehension and transformation errors) and 6 (29%) of the study participants started many questions very well but they contained errors, could not finished some of the questions, or ended up with wrong answers but a few questions were answered correctly. These group of students might have been affected by 4 and 5, which are skill processing and coding errors.

Post-test

(i) Results from descriptive statistics

Similarly, the mean of the post-test is 27.33%, with a minimum mark of 8% and a maximum mark of 52%. The standard deviation of the marks in this test is 12.25. It implies that the marks are disperse away from the 27.33% mark, which is the mean. In addition, the standard deviation of 12.25 implies that majority of the marks are between 15 and 40 marks. The skewness value of 0.404 and the kurtosis value of -0.173 imply that the findings are symmetrical.

(ii) Results from histogram

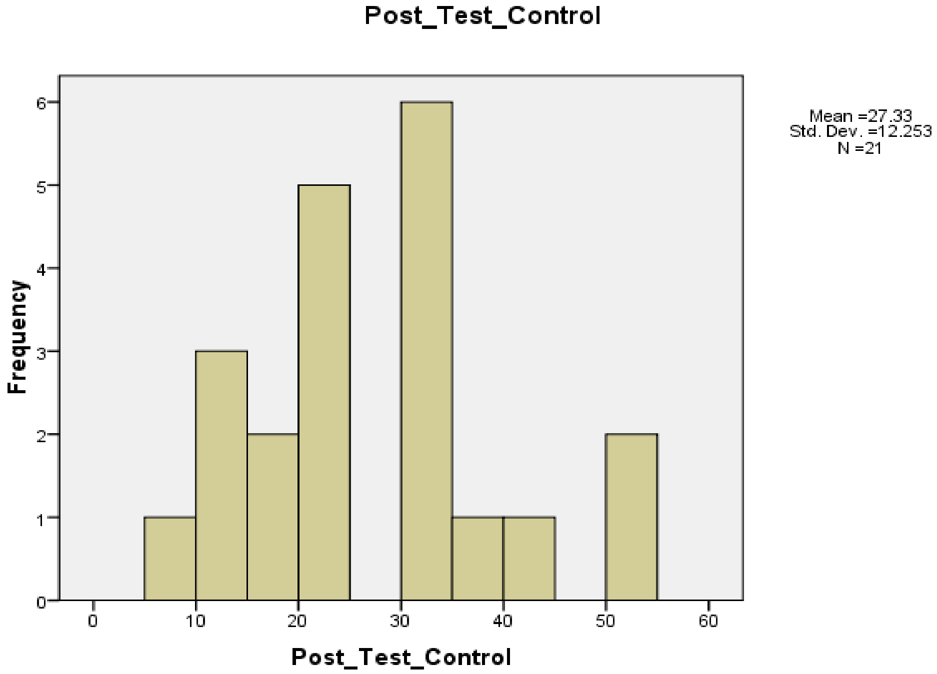


Figure 5.21: Histogram of the Post-test Marks in Group 1

From the chart, it could be seen that the marks are fairly symmetrically distributed around the mean mark (27.33%).

(iii) Results from the problem-solving errors

The results of the problem-solving errors of the post-test was not much different from that of the pre-test. The results of the solution appraisal in the error list on the errors committed show that out of 21 pre-test scripts, 7 (33%) of the student participants could not attempt any of the questions. This group of study participants might have been affected by reading error (error 1) since they could not even attempt any of the pre-test questions. In contrast, 7 (33%) attempted few questions but were all wrong; they might have been affected by combination of errors 1 to 3 (these are reading, comprehension and transformation errors) and 7 (33%) of the study participants started many questions very well but they contained errors, could not finished some of the questions, or ended up with wrong answers but few questions were answered correctly. This group of students might have been affected by error 4 and 5, which are skill processing and coding errors.

(iv) Test of Hypothesis for the Control Group

H_0: There is no statistically significant difference between the study participants' pre-test score and the post-test score in this study.

H_1: There is statistically significant difference between the study participants' pre-test score and the post-test score in this study.

Table 5.16: T-test results of significance

One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	T	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Pre_Test_Control	7.944	20	.000	17.238	12.71	21.76
Post_Test_Control	10.223	20	.000	27.333	21.76	32.91

The t-test comparing the mean of the pre-test and post-test at 95% significant level shows that the post-test mean (27.76) is statistically significant than the pre-test mean (12.71) with a p-value of 0,000 which is less than the 5% significance level. Therefore, the null hypothesis (H_0) is rejected and the alternative hypothesis (H_1) is accepted.

5.4 Conclusion of the chapter

This chapter presented the data analysis strategies and the results of the study. The study was carried out in two phases: Phase I and II. Phase I results show the type of errors the 2020 and 2021 matric candidates committed in the Euclidean geometry aspect of NSC mathematics paper 2 examination while Phase II shows the potency of the intervention to remedy the errors.

CHAPTER 6

SUMMARY, DISCUSSION, IMPLICATIONS, LIMITATION, CONCLUSION

AND RECOMMENDATIONS

6.0 Summary of the Study

This study investigated the type of errors South African mathematics learners commit in Euclidean geometry problem-solving and to propose interventions that could probably improve the problem-solving skills in Euclidean geometry in schools. The researcher thinks this study is a very important step to improving the learning and problem-solving skills in Euclidean geometry aspect of mathematics in our schools because it is believed that the mathematics matric data used in this study warrant the authentic ability of our mathematics learners in Euclidean geometry concepts. Moreover, the matric candidates demonstrate their utmost best at this junction of their education career. However, these data were very difficult to obtain because of the security measures guiding the scripts.

The study was divided into two phases: Phase I and Phase II. In Phase I, the study explored the errors committed in problem-solving in Euclidean geometry and the associated performance presented in the Euclidean geometry aspect of the 2020 and 2021 matric mathematics exam paper 2 in South Africa. Hence, the study follows ex-post facto research design which involves the use of solution appraisal approach to gather data. Newman mathematics error analysis hypothesis (Newman, 1977a, b) was used to identify each error types. The 2020 matric mathematics paper 2 exams scripts from four schools in three different provinces were used gather the required data for this study. The total of 2020 scripts used were 244. Similarly, 2021 matric mathematics paper 2 exams scripts from four schools in five different provinces were used gather the required data for this study. The total of 2021 scripts used were 432. The researcher would like to emphasise again that these matric papers were under very strong security and were normally destroyed after every two years. Consequently, the researcher did not have the opportunity to choose and select or sample which exam year, province or school to use for the study.

In Phase II, the researcher developed an intervention that may improve the problem-solving skills of mathematics learners in Euclidean geometry in South Africa schools. The 'types of errors' data collected in Phase I, Newman error remedial approach and other literatures were used to develop EGPSLM and EGPSIA which were coupled to form the intervention in the learning and teaching of Euclidean geometry. This phase was meant to test the efficacy of the intervention instrument that was meant to mitigate committing such errors discovered in Phase I and improve the problem-solving skills and performance of the Euclidean geometry learners in schools. To this end, random and convenient sampling were used to select three schools from the nine provinces in South Africa. Two of the schools were experimental schools, while the remaining one was a control school. A total of 95 Grade 12 mathematics learners forms the study participants for this study in Phase II.

Several statistical tools like, Pearson correlation tests, composite one way frequency tables with associated mean and standard deviation, and multiple comparison of means, were used to analyse data obtained in the study. The findings that emerged from the study are enumerated as follows:

- Reading error (error 1) was the most commonly committed error in both 2020 (29%) and 2021 (30%).
- Errors 1, 2 and 3, which were reading, comprehension and transformation errors, respectively, represent 65% of the errors committed in 2020 and 70% of the errors committed in 2021.
- Most commonly errors committed by percentage ranking were errors 1, 2 and 4, which were reading error, comprehension error and the processing skill error, respectively, in both 2020 and 2021. They represent 79.81% of the errors committed in 2020 and 2021 exams.
- The errors committed significantly affected the marks obtained in Euclidean geometry in both 2020 and 2021 exams.

- The marks obtained in Euclidean geometry significantly affected the overall marks obtained in the mathematics paper 2 of both 2020 and 2021.
- There was similarity in the types of errors committed in both 2020 and 2021.
- The intervention in Phase II of the study facilitates the learning of Euclidean geometry.
- The intervention in Phase II of the study improves the problem-solving skill of the study participants.
- The intervention in Phase II of the study improves the study participants' performance in Euclidean geometry.

6.1 Discussion

The foregoing research findings shall be discussed in view of the research questions and the hypothesis. Therefore, the discussion shall be done according to the following sub-headings.

- Type of problem-solving errors committed in the Euclidean Geometry aspect of the exam papers used in this study.
- Effect of problem-solving errors committed on performance in the Euclidean Geometry aspect of the exam papers used for this study.
- Effect of performance in the Euclidean Geometry aspect of the exam papers used in this study on the overall performance in the exam.
- Yearly similarity patterns in the problem-solving type of errors committed in the Euclidean Geometry aspect of the exam papers used for this study.
- Effect of intervention on classroom dynamics in this study.
- Effect of the intervention on the study participants' problem-solving skills.
- Effect of the intervention on the study participants' performance in the post-intervention test.

6.1.1 Type of problem-solving errors committed in the Euclidean Geometry aspect of the exam papers used in this study

The results of the data analysis show that all the Newman errors types are indicative in this study. They include the reading error, comprehension error, transformation error, process skills error, and encoding error. These were all found but in different degree, in the study (see tables 5.1 and 5.6). In the 2020 scripts, a total of 1464 error data were collected and analysed, while in 2021, the number of error data that were collected and analysed were 2160. In addition, the nature of the errors committed analysis in 2020 and 2021 exam shows that most of the errors were because of not having any idea of how to go about the problem-solving approach which the study categorised as 'bad error' in this study (see figures 5.1 and 5.4). The errors committed are discussed next in the light of NEA (1977a, b).

(i) The most errors committed

According to the findings of this study, the most committed are errors 1, 2 and 4, which are reading, comprehension and processing skills errors, respectively, in 2020 and 2021 exams. In the 2020 exam errors data analysis, they accounted for 72,07% of all the errors committed (see table 5.1). Similarly, in 2021, they accounted for 79,80% of all the errors committed (see table 5.6).

Newman (1983) argues that failure at any level of the five-error hierarchy prevents the problem-solver from obtaining satisfactory solution, unless by chance they arrive at correct solutions by faulty reasoning. What the researcher observed here is that the mathematics matric candidates that committed these errors in scripts used in this study had problem with reading and comprehending the Euclidean geometry questions in which the errors were committed, because they did not understand the demand of the question. Consequently, their solution approach wrong. Hence, van Hiele (1986) further elucidates that once the learner is unable to master a particular level in Euclidean geometry, it is going to be difficult to understand the subsequent level as there are connections between levels. This is also supported by the data analysis of the nature of the errors results which shows that most of the errors committed were bad errors such that the problem-solver might not be able to complete solving the problem wherever they occur (see Figure 5.8). This implies that, in both 2020 and 2021 exams, about two-third of the matric candidates that wrote the mathematics paper 2 exams might not have been able to complete solving the particular questions in which these errors occurred or they arrived at wrong solutions.

These findings are consistent with the literature cited in this work. Kenys and Firda (2017) report that the study shows that frequent mistakes were in transformation and skill processing hierarchy of NEA, which were errors 3 and 4. In foregoing findings narrated, most candidates had problem with error 2 (comprehension error). The findings from Sapire et al (2016) indicate that various errors committed by the learners ranged from not able to read and understand what to be done in a mathematics problem, not been able to represent the problem with the right mathematics expression and calculation errors. The error highlighted here are errors 1, 2 and 4, which the exact result was alluded to earlier.

(ii) The percentage errors 1 to 3 committed

Newman (1977a) informs that when a mathematics problem is presented to a mathematics learner, before trying to solve the problem, the learner must first read the problem. These should include the symbols and terminologies included in the problem, comprehend the expression of problem, symbols and terminologies in a way that they all make sense to him/her, then be able to choose appropriate processing skills to solve the problem, which is termed transformation. Hence errors 1 to 3 have to do with the mental cognition process that precedes physical presentation of solution. These set of errors set the trajectory of the problem-solving in each question. However, in this study, it was observed that the contribution of errors 1, 2 and 3, which include reading, comprehension and transformation errors were substantive compared to the total errors committed. The contribution of errors 1 to 3 in the 2020 exam was 65.52%

compared to total errors committed. Likewise, it was 71.47% in the 2021 exam. This implies that more than half of the errors were committed before even the application of the processing skills; that is, the physical presentation of the solution approach. As mentioned earlier, after committing an error, it is difficult to continue to solve the problem successively. This might be the reason that in many of the exam scripts analysed, the questions under the Euclidean geometry aspect of the exam were left unanswered or answered wrongly.

This finding is consistent with the reviewed literature. In Newman (1983), the study's outcome shows that about 50% of the errors occurred at the first three stages of Newman's error hierarchies in Newman (1977a). The first three stages are reading, comprehension and transformation. The same results were observed in Clements and Ellerton (1996) in which more than 50% errors were reported in the first three stages of Newman's error hierarchies.

(iii) Reading errors

The reading error is the most common error committed among all error types. It accounted for 29.08% and 30.74% in the 2020 and 2021 exams, respectively. Following one of the findings of Newman (1983) which informs that failure at any level of the five-error hierarchy prevents the problem-solver from arriving at correct answer. Perhaps, this is why one of the findings from the solution appraisal shows that in some scripts the matric candidates were unable to answer some parts of the questions in Euclidean geometry.

Vorster (2005) argues that language is one major facet of mathematics culture. The teaching, concepts and symbols are all delivered through a language; hence, learners must be proficiency in the LOLT. This was demonstrated by Vorster (2005) by making the test-instrument contained questions items which were structured in LOLT and home languages. The findings show that because the 'home language' version of the question gave the learners better comprehension of the questions. These findings suggest that maybe in South Africa we may consider putting a version of home language in mathematics questions.

When learners are not proficient in the LOLT in Euclidean geometry, they are confused and unable to process the Euclidean geometry concepts and symbols information. They will not be able to comprehend the information and definitely will have problem at the transformation stage of making sense of what to be learn. Denfield et al. (2014) demonstrate that language has a predictive power on mathematics scores. More importantly, language proficiency makes explicit the implicit state of Euclidean geometry problems. For reading errors to be reported in two consecutive years (2020 and 2021 matric exam years) as the most common errors in Euclidean geometry problem-solving, it shows that:

- This is the fundamental problem South African learners face in the learning of Euclidean geometry. If the research is extended to include about five consecutive years, it might still come out as the leading error.
- The reading error has influence on the comprehension and transformation error as explained in the first sentence of this paragraph.
- The reading error drags in comprehension and transformation errors as we see in subheading (ii) above, in which it was observed that errors 1 to 3 account for two-third of the errors put together.

In fact, Timss (2019), in the diagnostic report, lamented that South African learners performed very badly in the test partly because of their reading errors.

Many scholars have also lamented the effect of reading error on the learners' performance in mathematics in general and offer some kind of advice according to the findings from their studies. Vorster (2005) suggests that LOLT and test instrument should be in English and home languages. Denfield et al. (2014) and Yushau and Omar (2015) buttress that mathematics learners should improve their English language proficiency to be able to comprehend mathematics concepts.

6.1.2 Effect of problem-solving errors committed on performance in the Euclidean Geometry aspect of the exam papers used for this study

The study interrogated the effect of the errors committed on performance in the Euclidean geometry aspect of the matric candidates' 2020 and 2021 mathematics paper 2 examined by considering the following:

- the context of the 2020 and 2021 mathematics paper 2 exam used for this study;
- marks obtained in the Euclidean geometry aspect of the 2020 and 2021 matric mathematics paper 2 analysed; and
- the contribution of the errors committed to the marks obtained in the Euclidean geometry aspect of the 2020 and 2021 matric mathematics paper 2 analysed.

The discussion is ordered in the light of the two items enumerated above.

(i) The context of the 2020 and 2021 mathematics paper 2 analysed

The 2020 and 2021 mathematics paper 2 matric exam analysed contain questions in the statistics, trigonometry and the Euclidean geometry aspect of mathematics. The total marks obtainable in the exam was 150 marks. In 2020 exam, questions 8, 9 and 10 were Euclidean geometry questions with a total of 49 marks, which represent 32.67% of the total marks obtainable. Likewise, in 2021 exam, questions 9, 10 and 11 were Euclidean geometry questions with a total of 41 marks, which represent 27.33% of the total marks obtainable. This is in line with previous NSC matric papers (see Table 1.3 in this study).

(ii) Marks obtained in the Euclidean geometry aspect of the 2020 and 2021 matric mathematics paper 2 analysed.

In the Euclidean geometry aspect of the 2020 mathematics paper 2 analysed, the total marks obtainable was 49. The findings show that in the Euclidean geometry aspect of the paper, the minimum marks obtained in the exam was 0 and the maximum marks was 47 and the mean was 8.93. The standard deviation was 10.14 and the mode is 0. The standard deviation of 10.14 implies that majority of the marks are between 1 and 19 in the Euclidean geometry aspect of the exam. Likewise, in the 2021, the total marks obtainable in the Euclidean geometry aspect of the paper was 41 marks. The findings show that the minimum marks was 0, and the maximum marks was 40, the mean was 7.38, the mode is 0 and the standard deviation was 8.16. The standard deviation of 8.16 implies that majority of the marks are between 1 and 16 in the Euclidean geometry aspect of the exam.

The mode of 0 mark in the Euclidean geometry aspect of the paper in both 2020 and 2021 mean that majority of the marks were 0. This result points to a scenario in which majority of the candidates left Euclidean geometry questions entirely unanswered or answer them wrongly, or they do not understand Euclidean geometry concepts at all. This might have been because of reading error, that is, either they are neither proficient in the English language in which the problem was written, nor did they understand terminologies and symbols involved in constructing the problem (Vorster, 2005; Denfield et al., 2014) or combination of reading, comprehension and transformation errors (Kenys & Firda, 2017; Sapire et al., 2016). There are some of South African mathematics learners that created phobia for learning Euclidean geometry, perhaps, leading to these errors. This was also revealed in the qualitative data collection during classroom observation in Phase II of this study that some learners walked out of the class as the teacher was entering to teach Euclidean geometry. This result was also reported in Abakah (2019) which informs that most mathematics learners have Euclidean geometry phobia and avoid this aspect of mathematics.

The mean marks of 8.93 and 7.38 in the 2020 and 2021 exams, respectively, and the standard deviations 10.14 and 8.16 in the 2020 and 2021 exams, respectively, point to the fact that majority the matric candidates that wrote these exams might have performed poorly in the Euclidean geometry aspect of the exams. In fact, the mean marks and the standard deviations, implied that in 2020 majority of the marks are between 1.21 and 19.07 and in 2021 majority of the marks are between 0.78 and 15.54. This implies that majority of the study participants did not perform well in the Euclidean geometry of the exam. These can be seen in the histogram chart of the marks obtained in Figure 5.2 and Figure 5.5. The two histogram charts show that majority of the marks obtained were around 0 and 8.

Newman (1983) asserts that failure at any level of the Newman five-error hierarchy prevents the problem-solver from obtaining satisfactory solution, unless by chance, they arrive at correct solutions by faulty reasoning. In an exam that errors 1 (reading), 2 (comprehension), and 3 (transformation), accounted for 72.07% and 79.80% of the total errors in 2020 and 2021 in the mathematics paper 2 exams respectively, the results in this study only validate Newman's (1983) findings.

For the motives for the foregoing might be that many questions might have been left unanswered and even those ones that were answered were wrongly answered. All these might have had effects on the marks obtained by the matric candidates in the Euclidean geometry aspect of the 2020 and 2021 mathematics paper 2 examination.

(iii) Contribution of the errors committed to the marks obtained in the Euclidean geometry aspect of the 2020 and 2021 matric mathematics paper 2 analysed.

The results from the regression data analysis that was used to investigate relationship between the errors committed data and the Mark obtained in Euclidean geometry data are presented in Table 5.3 and Table 5.8 for 2020 and 2021, respectively. The results show that in both 2020 and 2021 exams, the effect Error 1, Error 2, Error 3 and Error 4 are statistically significant on the marks obtained in the Euclidean geometry aspect of the paper. Error 5 (coding error) may not be statistically significant in both years of the exams because very few candidates could get to writing the answer. Perhaps majority of those that got to the level of writing the final answers wrote the answers appropriately.

6.1.3 Effect of performance in the Euclidean Geometry aspect of the exam papers used in this study on the overall performance in the exam

First the deductions from the percentage composition of the marks allocation to each topics in the matric mathematics paper 2, and the mode marks in the Euclidean geometry (EG) will be discussed. This will be followed by the deductions from the mean and the standard deviation of the analysis of the results. Only Euclidean geometry marks obtained (EG), total marks obtained in the exam without including the marks from the Euclidean geometry (Total Marks without EG), and the total marks obtained in the exam, including the marks from the Euclidean geometry (Total Marks with EG), are used to discuss the effect of the performance in the Euclidean geometry aspect of both 2020 and 2021 matric mathematics paper 2 exams. These were used to conduct this study on the performance in the whole mathematics paper 2 exam in both years.

Tables 4.5 and 4.6 give the composition of the 2020 and 2021 mathematics paper 2 matric exams, respectively, which were used to collect data in this study. The total marks obtainable in both exams were 150 marks. Table 4.5 shows that in 2020, 32.67% of the 150 marks were allocated to Euclidean geometry questions, which were from questions 8, 9 and 10. Similarly, in 2021, Table 4.6 shows that 27.33% of the 150 marks were allocated to Euclidean geometry questions, which were from questions 9, 10 and 11. This implies that Euclidean geometry marks formed about one-third of total marks in the papers. It makes the second biggest topic after the trigonometry topic in terms of marks in the 2020 and 2021 mathematics paper 2 matric exams. Therefore, the marks from the Euclidean geometry may trigger a failure or a pass in the mathematics paper 2 exam in both 2020 and 2021 exams.

Tables 5.2 and 5.7, which give the descriptive statistics analysis results of mark obtained in Euclidean geometry in 2020 and 2021, respectively, show that the mode of the marks for both years was zero (0) mark. This means that the candidates that got 0 mark might have failed the exam because of the 0 mark they got from Euclidean geometry aspect of the paper. The analytical geometry aspect of the paper was 26.67% and 26% in the 2020 and 2021 exams, respectively. That these candidates got zero in the Euclidean geometry which shares some terminologies and concepts (like cords, segments, tangents) with the Euclidean geometry implies that they may not perform well in analytical geometry as well. These two topics made more than 50% in the two exams. Therefore, scoring zero in the Euclidean geometry and performing poorly in the analytical geometry can inevitably make a candidate to fail the mathematics paper 2 exams in both year 2020 and 2021.

The general descriptive statistics analysis results of performance in 'EG', 'Total Marks without EG' and 'Total Marks with EG' for both 2020 and 2021 exams are given in Table 5.4 and Table 5.9. In the 2020 exam, the mean of the 'EG', 'Total Marks without EG' and 'Total Marks with EG' are 19.11, 31.33 and 27.04 respectively. So also, in the 2021 exam, the mean of the 'EG', 'Total Marks without EG' and 'Total Marks with EG' are 18.05, 40.69 and 34.49, respectively. It is observed that in both 2020 and 2021 the mean mark for the EG is the lowest, while the mean mark for the 'Total Marks without EG' is the highest. This shows that the marks from the EG lowered the mean mark of the 'Total Marks with EG' when they are calculated together to get the final mark in the exam. The mean mark of the 'Total Marks without EG' shows the actual performance of the candidates in the other topics. These results are comparable to marks obtained in matric mathematics paper 2 in 2015, 2016 and 2017 displayed in Figure 1.2, 1.3 and 1.4, respectively.

The standard deviations of the 'EG', 'Total Marks without EG' and 'Total Marks with EG' give more information on the effect of the Euclidean geometry marks on the performance in the matric mathematics paper 2 in both 2020 and 2021. In 2020, the standard deviation of the EG was 21.50 and the mean was 19.11. This implies that majority of the marks were between 2 and 41. Similarly, the standard deviation of the 'Total Marks with EG' was 19.84 and the mean was 27.04. This implies that majority of the marks were between 7 and 47. Also, the standard deviation of the 'Total Marks without EG' was 20.64 and the mean was 31.33. This implies that majority of the marks were between 11 and 52. When the Euclidean geometry marks was added to the marks obtained from other topics of the paper, it brought down the total marks obtained, and more matric candidate failed the exam. Therefore, majority of the marks obtained were between 7 and 47. But if the Euclidean geometry was not added as it was in the 'Total Marks without EG' in which most of the marks were between 11% and 52%, and more candidate would have passed the exam.

This result is similar for the 2021. The marks for majority in EG was analysed to be between 2 and 38, the marks for majority in 'Total Marks with EG' was analysed to be between 13 and 56, and the marks for majority in 'Total Marks without EG' was analysed to be between 18 and 64. Again, it is observed that more matric candidates would have passed this exam if Euclidean geometry marks were not part of the final marks.

6.1.4 Yearly similarity patterns in the problem-solving type of errors committed in the Euclidean Geometry aspect of the exam papers used for this study

The researcher sought to know if there is consistency in the yearly types of errors committed by the South African mathematics learners in the Euclidean geometry problem-solving. From the descriptive findings depicted in Table 5.1 for 2020 and Table 5.6. For 2021**,** it is observed that in both 2020 and 2021 exams error 1 (reading error) to error 5 (coding error) were recorded. Of all the errors, error 1 was the most commonly committed; errors 1, 2 and 4 were the most commonly committed (arranged in terms of size of each error; in 2020, it was errors 1, 2 and 4, while in 2021, it was errors 1, 2 and 4). These errors made about two-third of the whole errors committed in each paper in each year; errors 1, 2 and 3 made more than half of the whole errors committed in each paper in each year.

One-tailed t-test was used to compare each type of error committed from error 1 to error 5 in both 2020 and 2021 examination used in this study to test the hypothesis. For each error type 1 to 5, the results show that each pair of errors were not statistically significant. For example, how error 1 was committed in 2020 was not statistically significant to how error 1 was committed in 2021. In addition, one-tailed t-test was used to compare total type of errors committed in 2020 to total type of errors committed in 2021. The results show that there was no statistically significant difference in the data.

The types of errors committed, and the size of each error committed relative to other errors perfectly fit in both 2020 and 2021 exams. Therefore, one might say that there seems to be a similar pattern in the ways the errors are committed. These results would have been more convincingly substantiated if the data were collected from about five consecutive years. The researcher tried to collect data from more than two years but the security approach guiding past matric papers was too strict. In fact, the researcher found out that matric past papers were to be destroyed after two years.

An adage says:

"He that does not know, and he does not know that
he does not know, ends up with a certificate
of stupidity."

In South Africa, we know that our mathematics learners have a problem with Euclidean geometry concepts. Perhaps we did not know how and where these problems occur, but they occur every year. But now this study has pinpointed (directed our attention) to 'how and where' the problem occurs. Targeted actions are now needed to confront the problems. The researcher proposed the intervention in the Phase II of this study as one of the actions needed to confront the problem.

6.1.5 Effect of intervention on classroom dynamics in this study

EGPSLM-EGPSIA was a coupled intervention in this study. EGPSLM proposes the guide to study participants' personal effort in the learning of Euclidean geometry, while EGPSIA proposes the instructional approach the teacher participant can use to effectively disseminate knowledge in Euclidean geometry in classroom. The intervention was administered only in the two experimental groups, while the control group was taught Euclidean geometry in the traditional form the teacher has been teaching before the study started. The researcher arranged to attend all first lessons in all the groups and collect the required data, including the control group. The findings attest that first lessons are very important; they are baseline results, that is, a reference point with which subsequent findings can be compared.

The findings of the classroom observations show, among other results, that in the first lessons of the three groups, some of the study participants were neither enthusiastic nor favourably disposed towards the learning of Euclidean geometry. As soon as the teacher participant entered the class to teach Euclidean geometry concepts, you could observe frustration and confusion among study participants. These types of attitude were not expected of Grade 12 learners who were preparing to write an examination that may dictate the trajectory of their life. Perhaps, they already created phobia for Euclidean geometry concepts and gave up as far as Euclidean geometry is concerned. These results also indicated that the teacher participants in the experimental schools managed to convince their learners to stay in the class with promises that their teaching of Euclidean geometry had changed. These came because of the suggestions that came after identifying some problems during the pilot study and the results show that the suggestion worked. These results might explain why some mathematics matric candidate avoided Euclidean geometry aspect of paper 2 as was seen during solution appraisal data collection.

Perhaps, this was owing to the instructional approaches of the mathematics teacher before the intervention. Before the intervention started after the selection of the schools that will participate in the study, the researcher went to meet the principal and the mathematics teachers in the selected schools to inform and solicit their support in the study. The researcher anecdotally observed that all the mathematics teachers were teaching mathematics in a traditional way of knowledge dissemination. The mathematics learners sit to listen to the teacher, some might be making noise, some might not boarder to attend the class and so on.

The results of the classroom observation during the intervention in the control group (though there was no intervention in the control group) gave the same result. Khan (2012) asserts that in traditional mathematics classrooms, mathematics students are like empty knowledge seekers, while the teacher is the direct and unilateral instructor. Khan mentioned that in such a mathematics class, students do not have the opportunity to initiate, question or argue their personal thought or interact with other learners. Stofflett (1998) describes a traditional mathematics instructor as a body of knowledge that must be taken without question. The traditional mathematics instruction classroom dynamics are very poor. Maybe that is why traditional mathematics learners show no interest in the learning of mathematics and always perform very poor in geometry. As seen in the Phase I of this study, the mode mark was zero and mean mark was below 20% in both 2020 and 2021 in the matric mathematics paper 2 used to collect data. Though the instructional approaches in the schools involved were not established but the results obtained align with the study consequences of traditional classroom teaching.

However, the findings showed that classroom interactions in the two experimental groups changed after the intervention was acquainted with, resulting in improved classroom dynamics. The study participants in these groups were very active in the class; they were asking questions that stimulate and demonstrate conceptual understanding, answering questions correctly in the class and solving problems on the board (see figures 5.10 and 5.11). The intervention must have impacted the interest of the study participants in the learning of Euclidean geometry. When mathematics learners are answering, asking questions in the class and solving problems correctly on the board, it demonstrates that the learners understand the mathematics concepts being taught. In addition, Fischer, Dobbs – Oates, Doctoroff and Arnold (2012) assert that learners' interest in the core learning improves academic achievement.

6.1.6 Effect of the intervention on the study participants' problem-solving skills in Phase II of the Study

The researcher did not collect data from the homework because of the doubt whether or not the study participant did the work by him/herself, but from the classwork and the post-test to measure how the intervention has mitigated the Euclidean geometry problem-solving skills difficulties that emerged in the baseline test (pre-test) data analysis and in the Phase I of the study. The results of the pre-test in both groups 1 and 2 under subsections 5.2.2.1, and 5.2.2.2 respectively, indicated that about 56% and 66% of the study participants in group 1 and 2, respectively, were affected by either error 1 or combine errors 1 to 3. Some could not answer any question while some made wrong attempts. About 44% in group 1 and 33% in group 2 made reasonable attempt but some could not finish their attempts owing to errors. These are either error 4 (processing skill errors) or error 5 (encoding error). Hence, the study participants displayed all the errors found in Phase I of the study. One of such study participants was the one that wrote the script in Figure 5.13.

The results of the post-test in both groups 1 and 2 under subsections 5.2.2.1 and 5.2.2.2. respectively, indicated that none of the study participant in both groups submitted blank answer sheets. Moreover, in group 1 and 2, 17% and 24% of the study participants attempted some questions, though with wrong approaches. This group of study participants might still have problem that were grouped under either error 1 or combine errors 1 to 3 in Phase I error analysis.

However, 83% and 76% of the study participants in group 1 and 2, respectively, presented problem-solving approaches that had little or no error. This group of study participants passed very well in the post-test. Figure 5.15 is an example of such scripts.

The results of the study show that in both experimental groups, there were remarkable improvement in how group 1 and 2 study participants presented their problem-solving approaches such that there was little or no error which enhance improved performance in the post-test compared to the pre-test.

6.1.7 The intervention in Phase II of the study improves the study participant's performance in Euclidean geometry

Tables 5.11 and 5.13 present the descriptive statistics of the pre-test and post-test in the experimental Group I and experimental Group II respectively in Phase II of this study. The mean of the pre-test in both experimental groups were below 20%, while the mean of the post-test in experimental group I was 52.59% and that of the experimental group II was 41.64%. These results show that there is improvement in the study participants' performance in the learning and problem-solving of Euclidean geometry concepts. The standard deviation marks shed more light on the spread of the study participants' performance in the pre-test and post-test.

It was further deduced from the tables 5.11 and 5.13 that because the standard deviations of the pre-test were 11.20 and 11.91 in the experimental Group 1 and 2, respectively, then majority of the study participants scores were between 9% and 32%, and 3% and 26% marks, in the experimental Group 1 and 2, respectively. While standard deviations of the post-test were 18.42 and 17.58 in the experimental Group 1 and 2, respectively, then majority of the study participants scores were between 34% and 71%, and 24% and 59% marks, in the experimental Group 1 and 2, respectively. Again, it is observed that in both experimental groups, more study participants passed the post-test compared to the pre-test in which none passed.

However, it was also observed that the post-test minimum marks were 26% and 12% in the experimental Group 1 and experimental Group 2, respectively, and maximum marks were 92% and 84% in the experimental Group 1 and 2, respectively. While there were some outstanding performances in both groups, the researcher noted that the minimum marks of 26% and 12% marks is of concern. Perhaps, this group of study participants might primarily not be proficient in the English language in which the questions were structured; this is part of reading error problem. Denfield et al. (2014) warned that language has a predictive power on mathematics scores. Language proficiency makes explicit the implicit state of Euclidean geometry problems.

In this case, study participants that are not proficient in English language. The language of teaching and learning will need to work on their language deficiencies to be able to harvest the full benefit of the intervention. The researcher did not expect that within the two weeks of the intervention, the concerned study participants will have overcome their language proficiency problem. The researcher think that this set of study participants were the ones that scored about 9% and 3% in pre-test, and in the post-test they were around 26% and 12% marks, which is also a form of improvement.

Lastly, tables 5.12 and 5.14 show that in both experimental groups, there is statistically significant difference between the baseline test (pre-test) and the post-test with a p-value of 0.000, which is less than the 0.05 significant level.

6.2 Implications of the Study

The results of this study aligned to the concepts of the NEA hypothesis and Newman Errors remedial hypothesis underpinning the study. Caution must, however, be taken not to assume that EGPSLM and EGPSIA as an intervention approach is a panacea for all the problem-solving ills in the study of Euclidean geometry in South Africa schools.

To guarantee sustainable efficacy of EGPSLM and EGPSIA as learning and teaching intervention that could roll away the problem-solving difficulties in the learning of Euclidean geometry in our schools, commitment from all stakeholders, mathematics learners, mathematics teachers, school administrators, curriculum planners and the DBE are very important. Effective learning is inspired by good teaching approach, and good teaching approach encourages the learners to learn. It is in the light of the foregoing that the implication of this study are enumerated and explained as follows:

- This study has been able to pinpoint exactly 'how and where' is the problem in our mathematics learners' problem-solving in Euclidean geometry.
- The main idea behind the conceptualisation of EGPSLM and EGPSIA is to encourage the learners to study Euclidean geometry on their own. However, the language of learning and teaching is a big part of the learners' problem-solving difficulties in Euclidean geometry according to the findings of this study, which needs to be sorted outside the ambit of mathematics domain. EGPSLM has proved to be reliable in breaking the Euclidean geometry concepts learning phobia created by mathematics learners. Killen (2007) accentuates that the knowledge learners gained by themselves is more valuable and lasting than that which is transmitted to them by someone else.
- EGPSLM as an instructional approach, links mathematics learners' personal learning effort to classroom learning participation through EGPSIA. It makes the teaching and learning of Euclidean geometry a learner-centred as suggested by Eyyam, Menevis and Dogrur (2010).
- Euclidean geometry dwells on multiple representation of Euclidean geometry concepts during the classroom teaching through hanging of the Euclidean geometry concepts charts in the classroom and also learners

bringing their own Euclidean geometry concepts lists to the class during Euclidean geometry lessons. This is in line with the findings of Taiwo, (2009) which indicate that multiple representation facilitate learning.

- It is important that the schools' timetable be adjusted such that it gives sufficient time to accommodate the EGPSIA. Obviously, the current traditional teaching approach of Euclidean geometry in the schools need less time to complete mathematics curriculum for the term, compared to the EGPSLM coupled with EGPSIA learning environment, where each learner presents their own solution to Euclidean geometry problems based on their personal Euclidean geometry concepts learning. More teaching time is needed to allow the teacher to be able to evaluate each learners' problem-solving progress through classwork, asking and answering questions during Euclidean geometry lessons.
- In view of the findings of the 'type of errors committed' data, it is proposed that the assessment procedure be more focused on the problem-solving skills presented rather than looking for the correct-answer driven assessment procedure.
- However, all the study teacher participants were grounded in the teaching of the Euclidean geometry, studies (like Atebe, 2011) have shown that mathematics teachers who are not proficient in the Euclidean geometry concepts lack confidence in teaching these concepts. Hence, the DBE should organise workshops, training and so on, for mathematics teachers to gain more conceptual understanding of Euclidean geometry concepts.

6.3 Limitations of the Study

This study was conducted by collecting data from the past mathematics matric scripts. The study was conducted despite intrinsic limitations. As a result of these unavoidable limitations, the findings and consequently, the conclusions drawn may have been affected in one way or the other. These limitations are enumerated and explained next:

- Accessing the past matric scripts was very difficult owing to security protection accorded the scripts and the procedures that need to be followed for the scripts to be released. The researcher spent three years before he could collect the few scripts used in this study.
- Besides this shortcoming, the past matric scripts are only kept for about two years before they are destroyed. This limited the number of consecutive years of past matric scripts one can get at any time. This complicated the obtainability of the past matric scripts.
- A bigger sample of at least consecutive five years of past matric mathematics paper 2 is preferred but owing to the aforementioned limitations, the researcher was restricted only the past 2020 and 2021 scripts, which implies that only two years of data were used to conduct the study.
- The complications attached to obtaining the past matric scripts affected the ability to conduct pilot study in Phase I of this study.

6.4 Conclusions of the Study

The objectives of these study, as mentioned in subsection 1.5, in Chapter 1 of this study are:

- to investigate the type of errors South African mathematics learners committed during problem-solving in Euclidean geometry;
- to investigate how these errors contribute to the performance of the learner in a given examination;
- to propose a remedy that could help the learners overcome committing errors during problem-solving in Euclidean geometry; and
- to improve the performance of mathematics learners in Euclidean geometry examinations.

The findings that emerged from Phase I of the study suggested that all the objectives of the study were not only accomplished but it also emerged that the type of errors learners commit are yearly repetitive, with the same pattern in nature. This calls for focus solution to learners' Euclidean geometry problem-solving difficulties in our schools. The Phase II findings suggested that EGPSLM and EGPSIA as an intervention has the potential to bring the required remedy to the learners' problem-solving difficulties in Euclidean geometry as intended by the study. The researcher, therefore, recommends the use of EGPSLM and EGPSIA in Euclidean geometry classes in South African schools.

6.5 Recommendations

Considering the types of errors that emerged in this study and the possibility that these errors may be occurring yearly, the proposed remedial intervention, and the fact that the objectives of this study were met, pave way for further research ideas. Therefore, the researcher suggests the following research ideas:

- This study has shown specifically the errors that may be plaguing South African mathematics learners' problem-solving in Euclidean geometry, resulting in incessant poor performance in the matric mathematics paper 2 exam. The two years data used for the study show similar pattern in the types of errors committed. Owing to the difficulty associated with getting the matric papers for research purposes, this study was limited to two years matric mathematics paper 2 exam data (2020 and 2021). The researcher recommends a nothing less than consecutive five years data to be able to ascertain the errors pattern. Perhaps this will help to bring an end to the South African mathematics learners' poor performance in Euclidean geometry.
- It emerged that error 1, which is reading error, is the biggest culprit that cause major difficulties for Euclidean geometry learners in both years 2020 and 2021 exams. However, the study did not separate the language effect of error but there are studies that proposed that home language should be added to the language of teaching and learning.

Investigation is needed to see how the language dimension in error 1 could be tamed.

- In this study EGPSLM coupled with EGPSIA as an intervention has proved to be a reliable remedial approach to improve the study participants' conceptual understanding in Euclidean geometry, minimise problem-solving errors and improve performance in Euclidean geometry. Investigation is needed to see how this intervention could be used to improve performance in the learning of analytical geometry in South African schools.

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APPENDIX 1: PRE-TEST AND MEMORANDUM

PRE –TEST

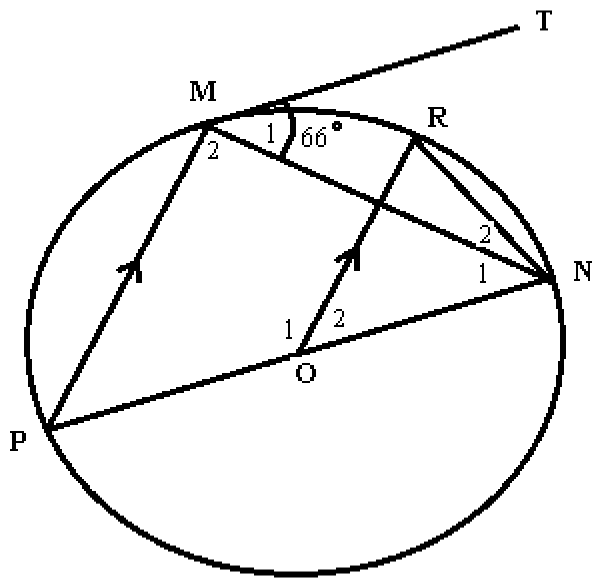
MARKS: 50
DURATION: 1HOUR

INSTRUCTIONS AND INFORMATION
Read the following instructions carefully before answering the questions.

1. Clearly show all your calculations, diagrams, graphs, etc. which you have used in determining the answers.
2. Diagrams are NOT necessarily drawn to scale.
3. Write neatly and legibly.

Question 1

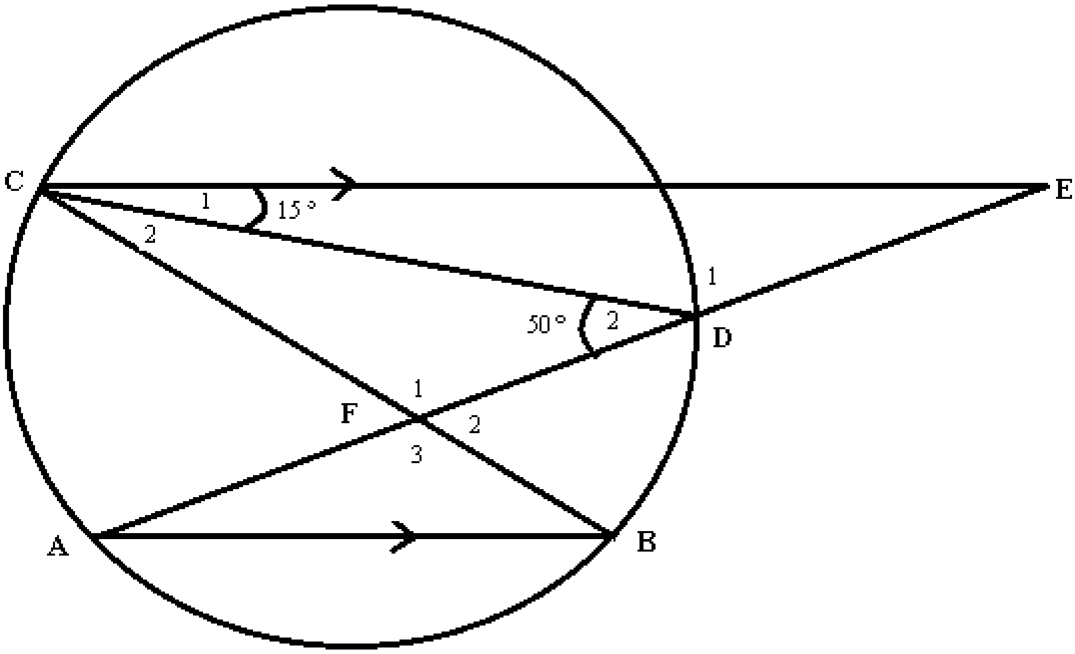
1.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR ∥ PM. NR and MN are drawn. Let $\widehat{M_1} = 66°$.



Calculate, with reasons, the size of EACH of the following angles:

1.1.1 $\hat{P}$ (2)
1.1.2 $\widehat{M_2}$ (2)
1.1.3 $\widehat{N_1}$ (2)
1.1.4 $\widehat{O_2}$ (2)
1.1.5 $\widehat{N_2}$ (4)

1.2 In the diagram, points A, B, D and C lie on a circle. CE ∥ AB with E on AD produced. Chords CB and AD intersect at F. $\widehat{D_2} = 50°$ and $\widehat{C_1} = 15°$.



1.2.1 Calculate, with reasons, the size of:

1.2.1.1 $\widehat{A}$ (3)

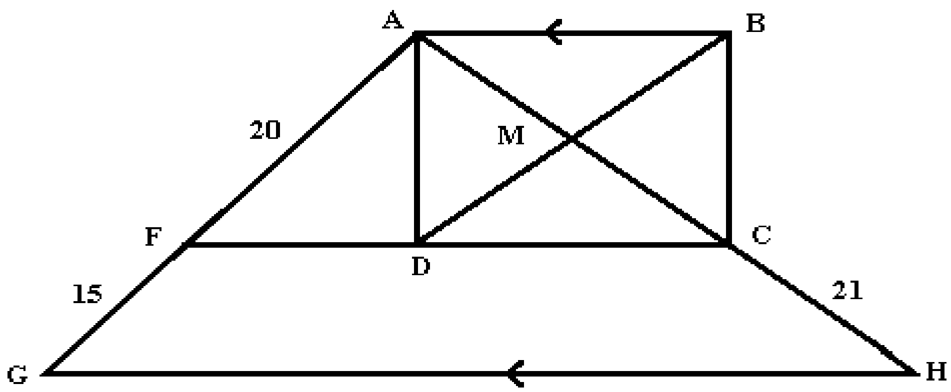
1.2.1.2 $\widehat{C_2}$ (2)

1.2.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)

[19]

Question 2

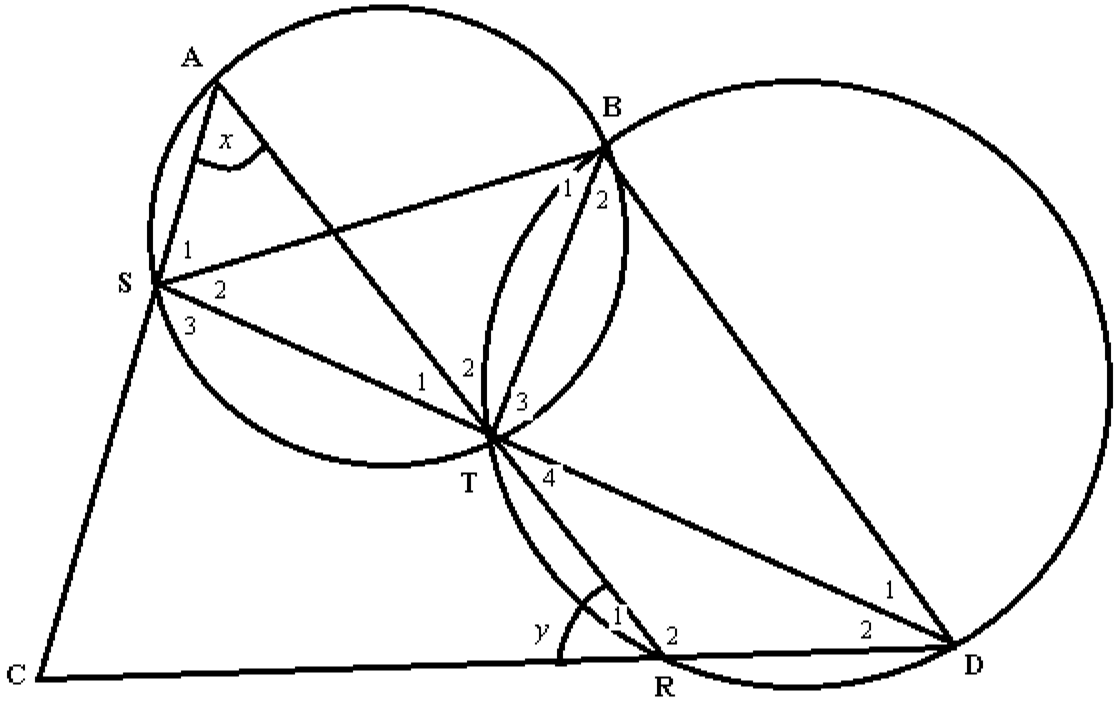
2.1 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that $AF = 20$ units, $FG = 15$ units and $CH = 21$ units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH,



2.1.1 Explain why FC ∥ GH. (1)

2.1.2 Calculate, with reasons, the length of DM. (5)

2.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\widehat{R_1} = y$.



2.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

(b) y (2)

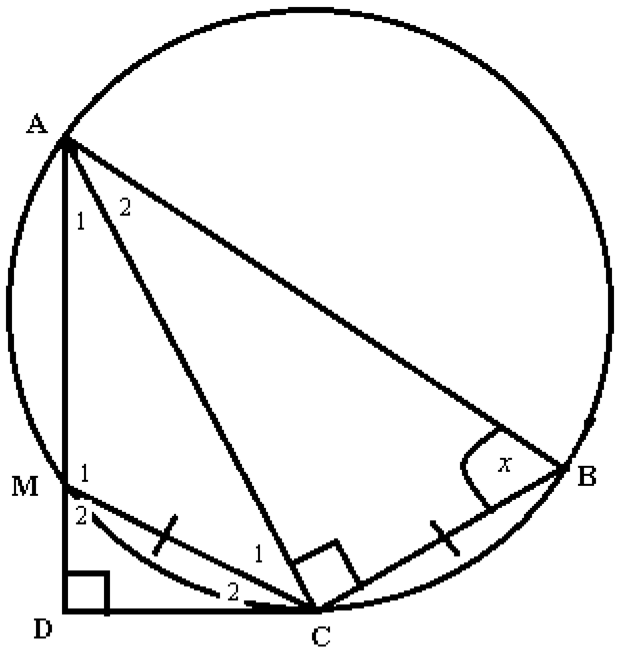
2.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

2.2.3 It is further given that $\widehat{D_2} = 30°$ and $A\hat{S}T = 100°$. Prove that SD is not a diameter of circle BDS. (4)

[17]

Question 3

In the diagram, ABCD is a cyclic quadrilateral such that AC ⊥ CB and DC=CB. AD is produced to M such that AM ⊥ MC. Let $\hat{B} = x$.



3.1 Prove that:

3.1.1 MC is a tangent to the circle at C. (5)

3.1.2 ΔACB ||| ΔCMD (3)

3.2 Hence, or otherwise, prove that:

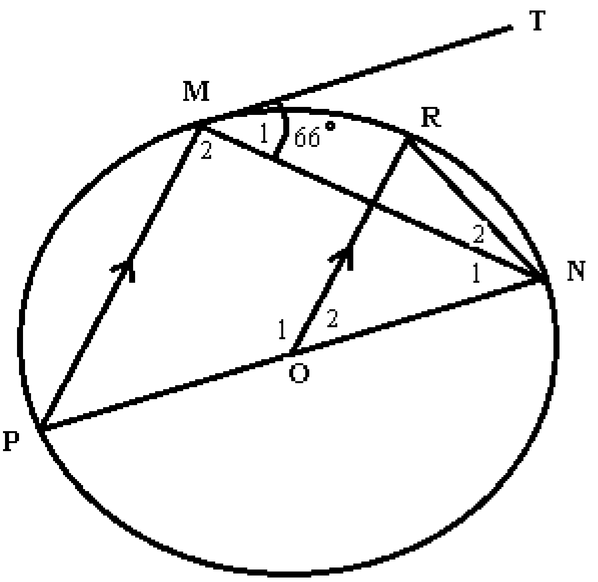
3.2.1 $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ (6)

[14]

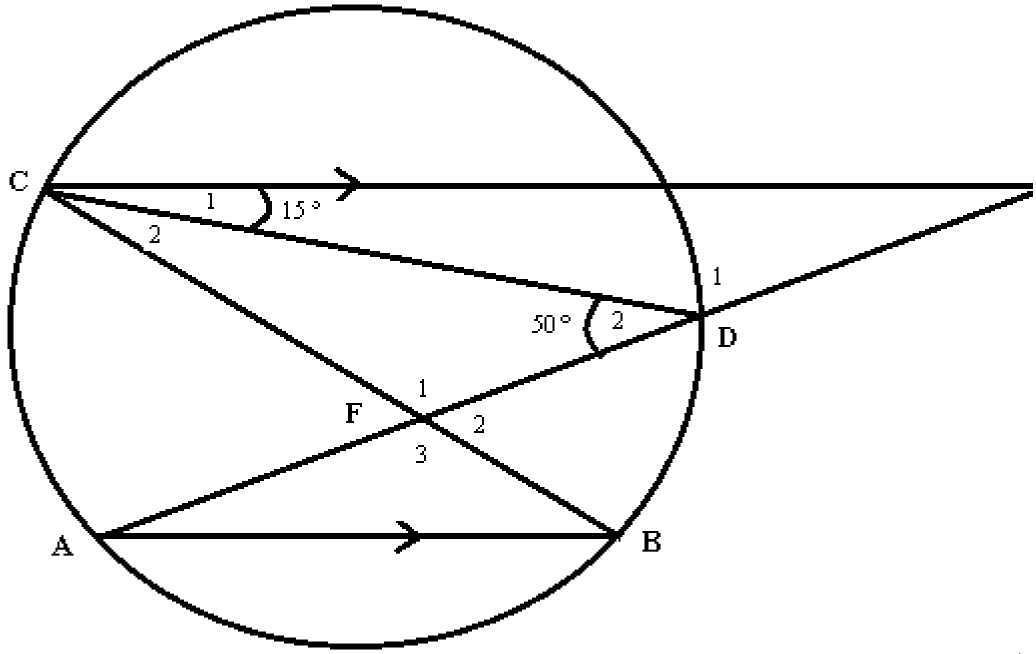
Total: 50 marks

Pre-Test Memo

Marking of Euclidean Geometry	
S	A mark for a correct statement (A statement mark is independent of a reason)
R	A mark for correct reason
S/R	Award a mark if statement and reason are both correct

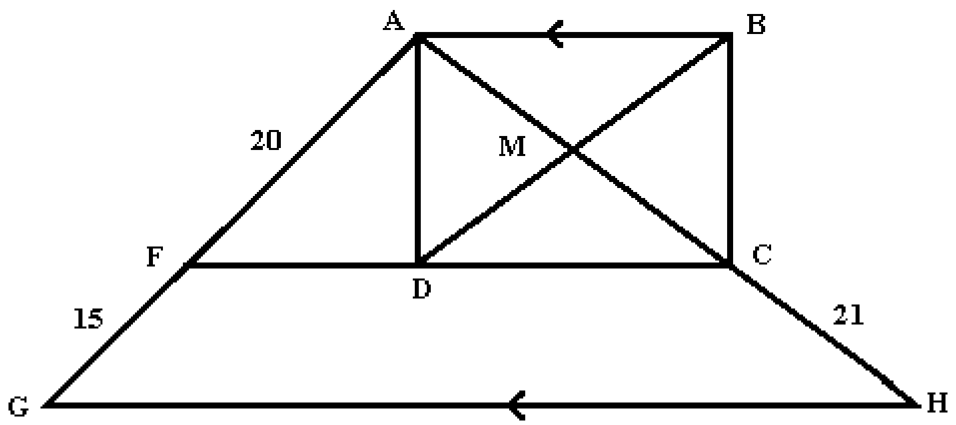
1.1			
1.1.1	$\hat{P} = \widehat{M_1} = 66°$ [tan chord theorem]	S √ R√	(2)
1.1.2	$\widehat{M_2} = 90°$ [$\angle's$ in semi circle]	S √ R √	(2)
1.1.3	$\widehat{N_1} = 180° - (90° + 66°)$[Sum of ∠ of ΔMNP] $= 24$	S √ R √	(2)
1.1.4	$\widehat{O_2} = \hat{P}$= 66° [corres. $\angle's$ PM‖ OR]	S √ R √	(2)
1.1.5	$\hat{R} + \widehat{N_1} + \widehat{N_2} = 180° - 66°$ [sum of $\angle's$ of ΔRNO]	S √	
	= 114°	S √	
	$\hat{R} = \widehat{N_1} + \widehat{N_2} = 57°$ [$\angle's$ opposite = radii]	R/S √	
	$\therefore \widehat{N_2} = 33°$	S √	(4)

1.2

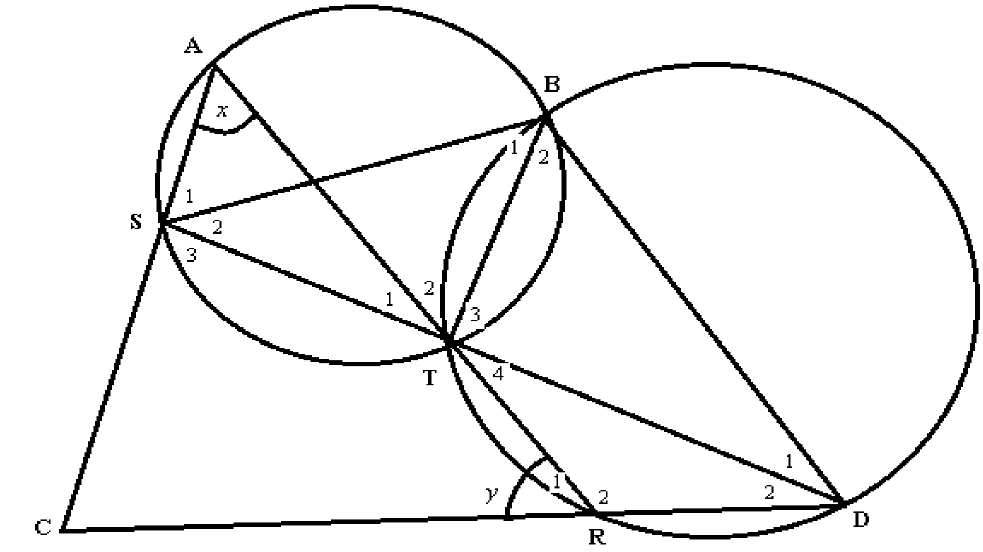


1.2.1.1	$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ]	S √
	$\hat{A} = 35°$ [alt. $\angle's, CE \parallel AB$]	S √ R √ (3)
	Or	
	$\hat{E} = 180° - (130° - 15°) = 35°$ [str. Line ∠'s of Δ]	S √
	$\hat{A} = 35°$ [alt. $\angle's, CE \parallel AB$]	S √ R √ (3)
1.2.1.2	$\widehat{C_2} = 35°$ [∠'s in same segment]	S √ R √ (2)
1.2.2	$\widehat{C_2} = \hat{E}$ [From 1.2.1.1 and 1.2.1.2]	S √
	∴ CF is a tangent to the circle	R √ (2)

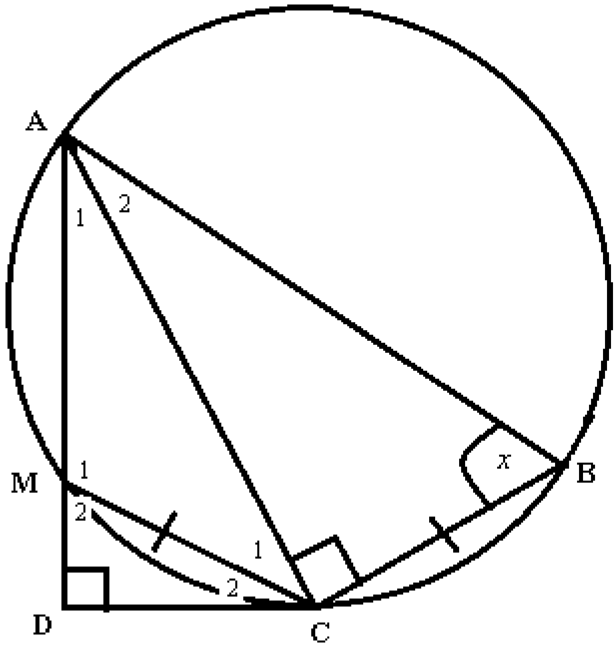
2.1



2.1.1	FC $\parallel$ AB $\parallel$ GH [Opp. Sides of rectangle]	R √ (1)
2.1.2	$\frac{AC}{CH} = \frac{AF}{FG}$ [line $\parallel$ one side of Δ] or [prop. theorem; FC $\parallel$ GH]	S √ R √
	$\frac{AC}{21} = \frac{20}{15}$	

	AC=$\frac{20\times21}{15}$	
	=28	AC √
	DC=AC=28 [Diagonals of rectangle =]	S √
	DM=$\frac{1}{2}$ DB=14 [Diagonals of rectangle bisect]	S √ (5)
2.2		
2.2.1(a)	$\widehat{B_1} = x$ [$\angle's$ in same seg]	S √ R √ (2)
2.2.1(b)	$\widehat{B_2} = y$ [ext. ∠ of cyclic quad]	S √ R √ (2)
2.2.2	$\hat{C}$=180°-(x+y)	S √
	S$\hat{B}$D +$\hat{C}$=x+y+180°-(x+y)	
	S$\hat{B}$D +$\hat{C}$=180°	S √
	∴ SCDB is a cyclic quad [converse opp. Angles of cyclic quad]	R √ (3)
	Or	
	$\widehat{S_1} = \widehat{T_2}$ [$\angle's$ in samesegment]	S √
	$\widehat{T_2} = \widehat{D_1} + \widehat{D_2} = B\hat{D}R$ [ext. ∠ of cyclic quad]	
	∴ $\widehat{S_1} = B\hat{D}R$	S √
	∴SCDB is cyclic quad [ext. ∠ of quad = opp ∠]	R √ (3)
2.2.3	$\widehat{T_4} = y - 30°$ [ext. ∠ of ΔTDR]	S √
	$\widehat{T_1} = y - 30°$ [vert opp $\angle's$ =]	S √
	$y - 30° + x100° = 180°$ [sum $\angle's$ of ΔAST]	
	∴ $x + y = 110°$	S √
	$S\hat{B}D = 110°$	R √
	∴SD not diameter	(4)
	Or	
	$A\hat{S}T = \hat{C} + \widehat{D_2}$ [ext. ∠ of ΔSCD]	S √
	$\hat{C} = 100° - 30° = 70°$	S √
	$S\hat{B}D = 180° - 70°$	
	=110°	S √
	∴SD not diameter [line does not subtend 90° ∠]	R √ (4)

3



3.1.1	$\widehat{A_2} = \widehat{A_1} = 90° - x$ [= chords subtend = $\angle's$]	S √ R √
	$\widehat{D_2} = x$ [ext. angle of cyclic quad]	S/R √
	$\therefore \widehat{C_2} = 90° - x$ [sum $\angle's$ of ΔDCM]	$\widehat{C_2} = 90° - x$ √
	$\therefore \widehat{C_2} = \widehat{A_1} = 90° - x$	
	∴ MC is a tangent to the circle at C [converse: tan chord thm]	R √ (5)
	Or	
	$\widehat{A_2} = \widehat{A_1} = 90° - x$ [= chords subtend = $\angle's$]	S √ R √
	$\widehat{C_2} + \widehat{C_1} = x$ [sum $\angle's$ of ΔACM]	$\widehat{C_2} + \widehat{C_1} = x$ √√
	$\therefore \widehat{C_2} + \widehat{C_1} = \hat{B} = x$	
	∴MC is a tangent to the circle at C.	R √ (5)
	Or	
	$\widehat{A_2} = \widehat{A_1} = 90° - x$ [= chords subtend = $\angle's$]	S √ R √
	$A\hat{M}C = A\hat{C}B = 90°$ [given]	
	$\therefore \widehat{C_2} + \widehat{C_1} = \hat{B} = x$	$\widehat{C_2} + \widehat{C_1} = x$ √√
	∴ MC is a tangent to the circle at C [converse: tan chord thm]	R √ (5)
3.1.2	In ΔACB and ΔCMD	
	$\hat{B} = \widehat{D_2} = x$ [proved or ext. ∠ of cyclic quad]	S √
	$\widehat{A_2} = \widehat{C_2} = 90° - x$ [proved or sum of $\angle's$ in Δ]	S √
	$\therefore \Delta\text{ACB} \parallel\!\mid \Delta\text{CMD}$ [$\angle, \angle, \angle$]	R √ (3)
	Or	
	In ΔACB and ΔCMD	
	$\hat{B} = \widehat{D_2} = x$ [proved or ext. ∠ of cyclic quad]	S √
	$A\hat{C}B = A\hat{M}C = 90°$ [given]	S √
	$\therefore \Delta\text{ACB} \parallel\!\mid \Delta\text{CMD}$ [$\angle, \angle, \angle$]	R √ (3)
	Or	

	$\widehat{B} = \widehat{D_2} = x$ [proved or ext. ∠ of cyclic quad]	S √
	$\widehat{A_2} = \widehat{C_2} = 90° - x$ [proved or sum of ∠'s in Δ]	S √
	$A\widehat{M}C = A\widehat{C}B = 90°$ [given or sum of ∠'s in Δ]]	
	$\therefore \Delta ACB \parallel\!\mid \Delta CMD$	S √ (3)
3.2	$\frac{BC}{MD} = \frac{AB}{DC}$ [ΔACB ∣∣∣ ΔCMD]	$\frac{BC}{MD} = \frac{AB}{DC}$ √
	$\frac{DC}{MD} = \frac{AB}{DC}$ [BC=DC]	
	$\therefore DC^2 = AB \times MD$	$DC^2 = AB \times MD$√
	In ΔAMC and ΔCMD	
	$\widehat{M}$ is common	S √
	$\widehat{A_1} = \widehat{C_2}$ [tan chord thm]	S √
	Or	
	$\widehat{C_1} + \widehat{C_2} = \widehat{B} = \widehat{D} = x$ [tan chord thm or ext. ∠ of cyclic quad]	
	ΔAMC ∣∣∣ ΔCMD [∠, ∠, ∠]	
	$\frac{AM}{CM} = \frac{CM}{MD}$	
	$\therefore CM^2 = AM \times MD$	$CM^2 = AM \times MD$√
	$\frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$	$\frac{AM \times MD}{AB \times MD}$ √
	$= \frac{AM}{AB}$	(6)
	or	
	$\frac{AC}{MC} = \frac{AB}{DC}$ [ΔACB ∣∣∣ ΔCMD]	$\frac{AC}{MC} = \frac{AB}{DC}$ √
	$\therefore CM \times AB = AC \times DC$	
	In ΔACB and ΔAMC	
	$\widehat{C} = \widehat{M} = 90°$ [given]	S √
	$\widehat{A_1} = \widehat{A_2}$ [proven]	S √
	Or	
	$A\widehat{C}M = \widehat{B} = x$ [proven]	
	ΔACB ∣∣∣ ΔAMC [∠, ∠, ∠]	
	$\frac{AC}{AM} = \frac{BC}{MC}$	
	$\therefore AC \times MC = AM \times BC$	$AC.MC = AM.BC$√
	$\therefore AC = \frac{BC.AM}{MC}$	
	$CM \times AB = \frac{BC.AM}{MC} \times DC$	Equating √
	$CM^2 = \frac{DC.AM}{AB} \times DC$ [BC=DC]	S √
	$\frac{CM^2}{DC^2} = \frac{AM}{AB}$	(6)

APPENDIX 2: POST-TEST AND MEMORANDUM

POST –TEST

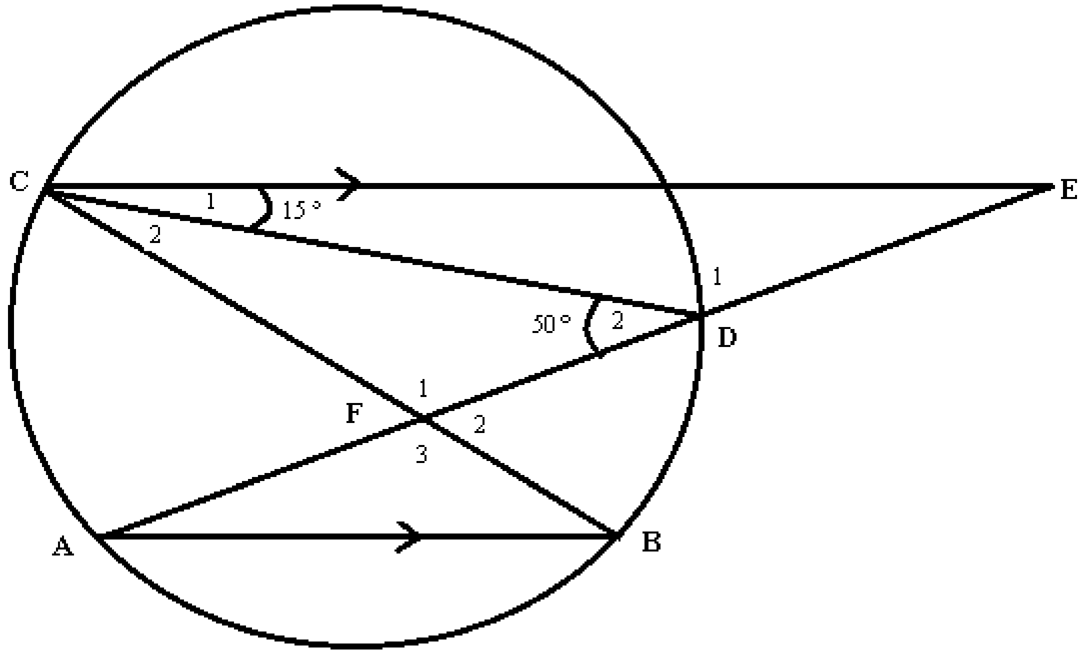
MARKS: 50
DURATION: 1HOUR

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. Clearly show all your calculations, diagrams, graphs, etc. which you have used in determining the answers.
2. Diagrams are NOT necessarily drawn to scale.
3. Write neatly and legibly.

Question 1



1.1 Calculate, with reasons, the size of:

1.1.1 $\hat{A}$ (3)

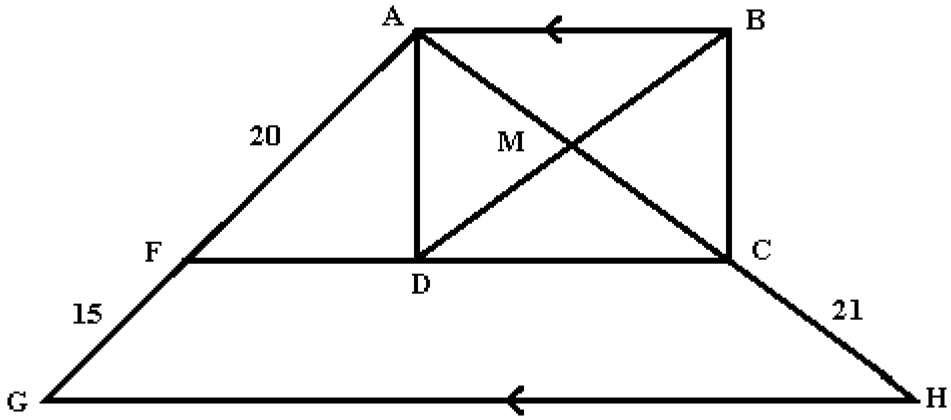
1.1.2 $\hat{C_2}$ (2)

1.2 Prove, with a reason, that CF is a tangent to the circle passing through points C,D and E. (2)

[7]

Question 2

2.1 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that $AF = 20$ units, $FG = 15$ units and $CH = 21$ units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH,



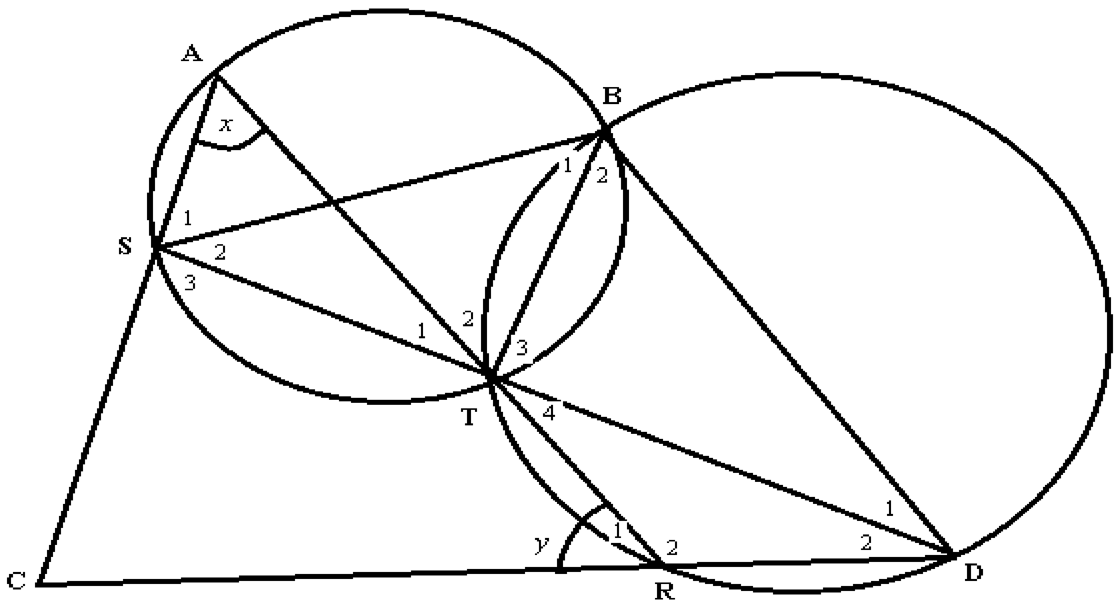
2.1.1 Explain why FC ∥ GH. (1)

2.1.2 Calculate, with reasons, the length of DM. (5)

[6]

Question 3

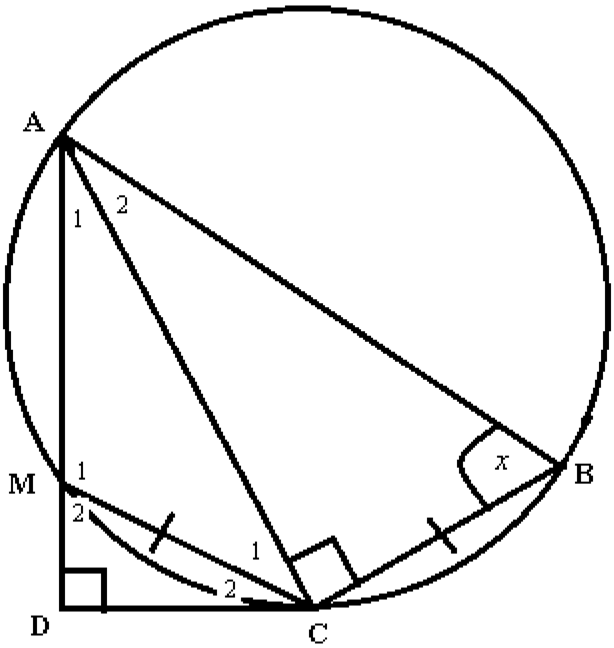
In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\widehat{R_1} = y$.



3.1 Name, giving a reason, another angle equal to:
(a) x (2)
(b) y (2)
3.2 Prove that SCDB is a cyclic quadrilateral. (3)
3.3 It is further given that $\widehat{D_2} = 30°$ and $A\hat{S}T = 100°$. Prove that SD is not a diameter of circle BDS. (4)
[11]

Question 4

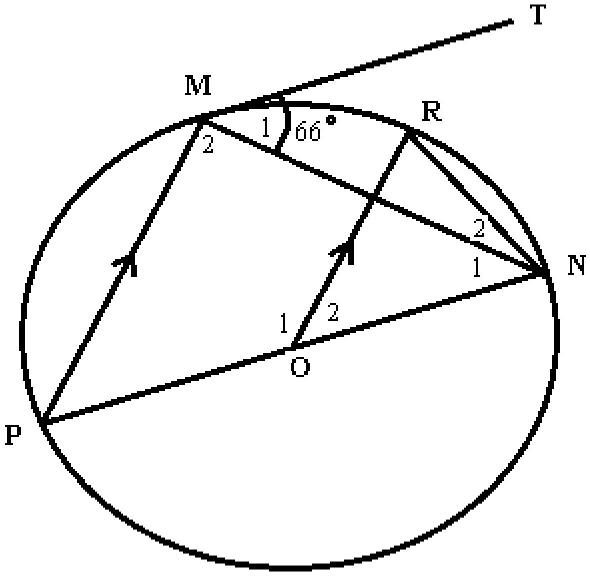
In the diagram, ABCD is a cyclic quadrilateral such that AC ⊥ CB and DC=CB. AD is produced to M such that AM ⊥ MC. Let $\hat{B} = x$.



4.1 Prove that:
4.1.1 MC is a tangent to the circle at C. (5)
4.1.2 ΔACB ||| ΔCMD (3)
4.2 Hence, or otherwise, prove that:
4.2.1 $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ (6)
[14]

Question 5

5.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR ∥ PM. NR and MN are drawn. Let $\widehat{M_1} = 66°$.



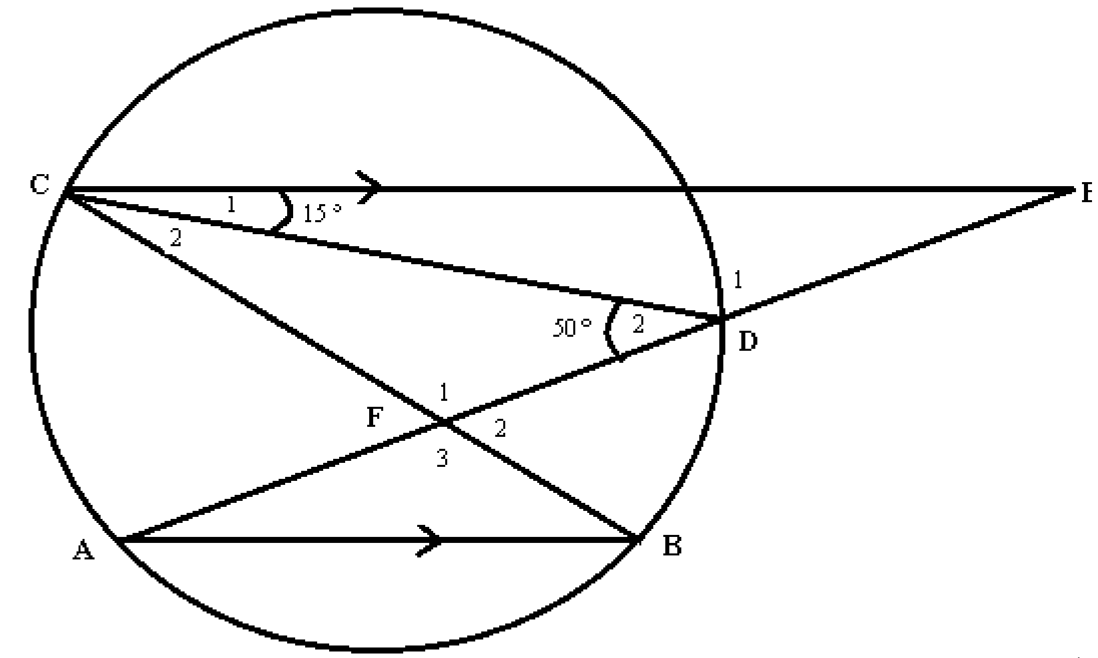
Calculate, with reasons, the size of EACH of the following angles:

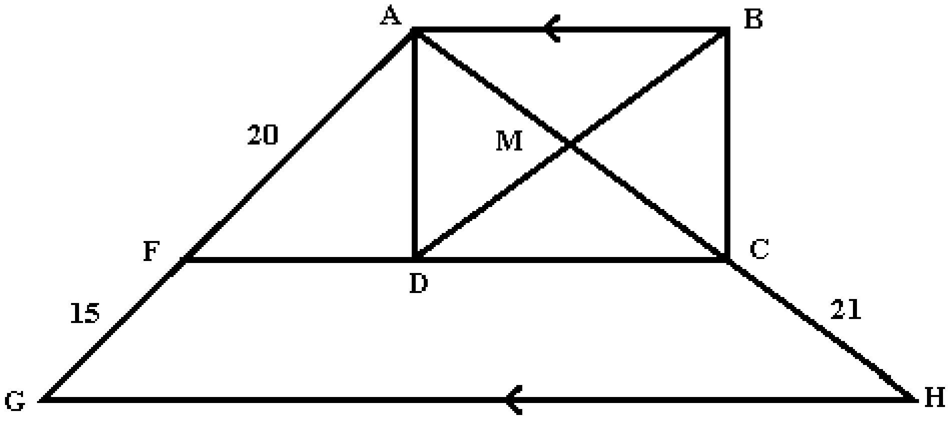
5.1.1	$\hat{P}$	(2)
5.1.2	$\widehat{M_2}$	(2)
5.1.3	$\widehat{N_1}$	(2)
5.1.4	$\widehat{O_2}$	(2)
5.1.5	$\widehat{N_2}$	(4)
		[12]

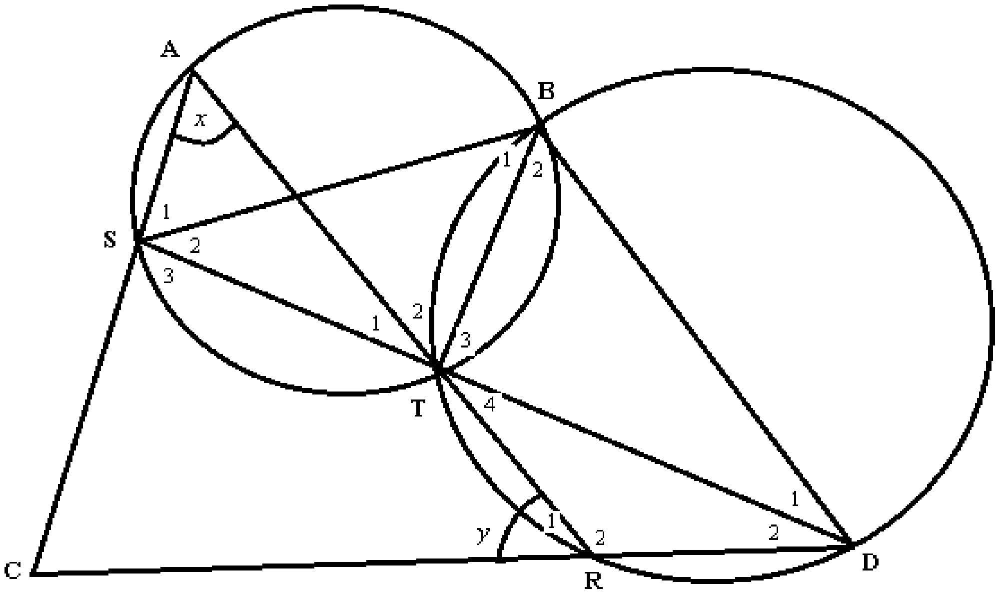
TOTAL: 50 MARKS

Post-Test Memo

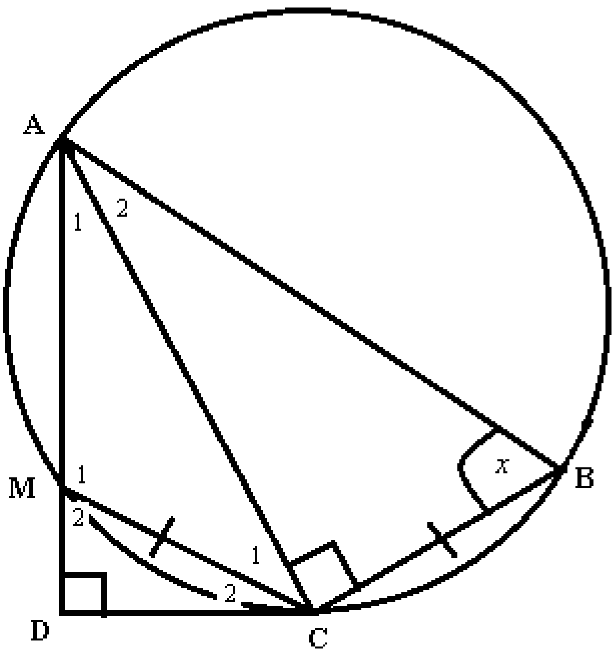
Marking of Euclidean Geometry	
S	A mark for a correct statement (A statement mark is independent of a reason)
R	A mark for correct reason
S/R	Award a mark if statement and reason are both correct

1.1.1		
	$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ]	S √
	$\hat{A} = 35°$ [alt. $\angle's, CE \parallel AB$]	S √ R √ (3)
	Or	
	$\hat{E} = 180° - (130° - 15°) = 35°$ [str. Line $\angle's$ of Δ]	S √
	$\hat{A} = 35°$ [alt. $\angle's, CE \parallel AB$]	S √ R √ (3)
1.1.2	$\widehat{C_2} = 35°$ [$\angle's$ in same segment]	S √ R √ (2)
1.2	$\widehat{C_2} = \hat{E}$ [From 1.2.1.1 and 1.2.1.2]	S √
	∴ CF is a tangent to the circle	R √ (2)

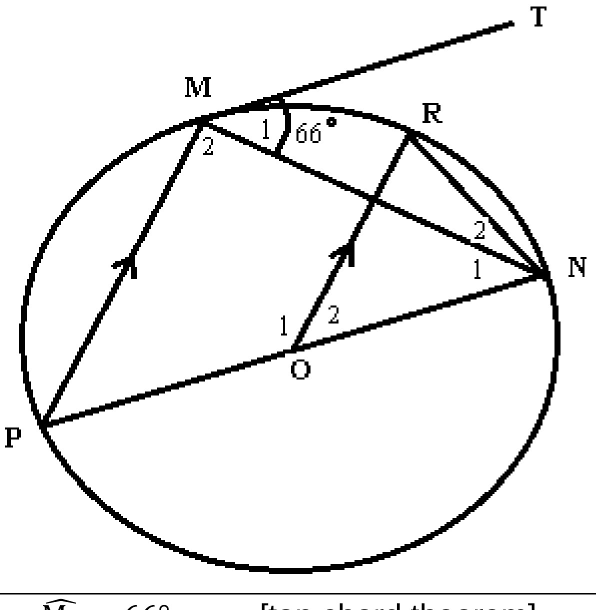
2		

2.1.1	FC|| AB|| GH [Opp. Sides of rectangle]	R √ (1)
2.1.2	$\frac{AC}{CH}=\frac{AF}{FG}$ [line || one side of Δ] or [prop. theorem; FC || GH]	S √ R √
	$\frac{AC}{21}=\frac{20}{15}$	
	AC=$\frac{20\times21}{15}$	
	=28	AC √
	DC=AC=28 [Diagonals of rectangle =]	S √
	DM=$\frac{1}{2}$ DB=14 [Diagonals of rectangle bisect]	S √ (5)
3		

3.1(a)	$\widehat{B_1}=x$ [∠'s in same seg]	S √ R √ (2)
3.1(b)	$\widehat{B_2}=y$ [ext. ∠ of cyclic quad]	S √ R √ (2)
3.2	$\hat{C}$=180°-(x+y)	S √
	S$\hat{B}$D +$\hat{C}$=x+y+180°-(x+y)	

	$S\hat{B}D + \hat{C} = 180°$	S √
	∴ SCDB is a cyclic quad [converse opp. Angles of cyclic quad]	R √ (3)
	Or	
	$\widehat{S_1} = \widehat{T_2}$ [$\angle's$ in samesegment]	S √
	$\widehat{T_2} = \widehat{D_1} + \widehat{D_2} = B\hat{D}R$ [ext. ∠ of cyclic quad]	
	$\therefore \widehat{S_1} = B\hat{D}R$	S √
	∴SCDB is cyclic quad [ext. ∠ of quad = opp ∠]	R √ (3)
3.3	$\widehat{T_4} = y - 30°$ [ext. ∠ of ΔTDR]	S √
	$\widehat{T_1} = y - 30°$ [vert opp $\angle's$ =]	S √
	$y - 30° + x100° = 180°$ [sum $\angle's$ of ΔAST]	
	$\therefore x + y = 110°$	S √
	$S\hat{B}D = 110°$	R √
	∴SD not diameter	(4)
	Or	
	$A\hat{S}T = \hat{C} + \widehat{D_2}$ [ext. ∠ of ΔSCD]	S √
	$\hat{C} = 100° - 30° = 70°$	S √
	$S\hat{B}D = 180° - 70°$	
	$=110°$	S √
	∴SD not diameter [line does not subtend 90° ∠]	R √ (4)
4		
4.1.1	$\widehat{A_2} = \widehat{A_1} = 90° - x$ [= chords subtend = $\angle's$]	S √ R √
	$\widehat{D_2} = x$ [ext. angle of cyclic quad]	S/R √
	$\therefore \widehat{C_2} = 90° - x$ [sum $\angle's$ of ΔDCM]	$\widehat{C_2} = 90° - x$ √
	$\therefore \widehat{C_2} = \widehat{A_1} = 90° - x$	
	∴ MC is a tangent to the circle at C [converse: tan chord thm]	R √ (5)

	Or	
	$\widehat{A_2} = \widehat{A_1} = 90° - x$ [= chords subtend = $\angle's$]	S √ R √
	$\widehat{C_2} + \widehat{C_1} = x$ [sum $\angle's$ of ΔACM]	$\widehat{C_2} + \widehat{C_1} = x$ √√
	$\therefore \widehat{C_2} + \widehat{C_1} = \hat{B} = x$	
	∴MC is a tangent to the circle at C.	R √ (5)
	Or	
	$\widehat{A_2} = \widehat{A_1} = 90° - x$ [= chords subtend = $\angle's$]	S √ R √
	$A\hat{M}C = A\hat{C}B = 90°$ [given]	
	$\therefore \widehat{C_2} + \widehat{C_1} = \hat{B} = x$	$\widehat{C_2} + \widehat{C_1} = x$ √√
	∴ MC is a tangent to the circle at C [converse: tan chord thm]	R √ (5)
4.1.2	In ΔACB and ΔCMD	
	$\hat{B} = \widehat{D_2} = x$ [proved or ext. ∠ of cyclic quad]	S √
	$\widehat{A_2} = \widehat{C_2} = 90° - x$ [proved or sum of $\angle's$ in Δ]	S √
	∴ ΔACB ||| ΔCMD [∠, ∠, ∠]	R √ (3)
	Or	
	In ΔACB and ΔCMD	
	$\hat{B} = \widehat{D_2} = x$ [proved or ext. ∠ of cyclic quad]	S √
	$A\hat{C}B = A\hat{M}C = 90°$ [given]	S √
	∴ ΔACB ||| ΔCMD [∠, ∠, ∠]	R √ (3)
	Or	
	$\hat{B} = \widehat{D_2} = x$ [proved or ext. ∠ of cyclic quad]	S √
	$\widehat{A_2} = \widehat{C_2} = 90° - x$ [proved or sum of $\angle's$ in Δ]	S √
	$A\hat{M}C = A\hat{C}B = 90°$ [given or sum of $\angle's$ in Δ]]	
	∴ ΔACB ||| ΔCMD	S √ (3)
4.2	$\frac{BC}{MD} = \frac{AB}{DC}$ [ΔACB ||| ΔCMD]	$\frac{BC}{MD} = \frac{AB}{DC}$ √
	$\frac{DC}{MD} = \frac{AB}{DC}$ [BC=DC]	
	$\therefore DC^2 = AB \times MD$	$DC^2 = AB \times MD$√
	In ΔAMC and ΔCMD	
	$\hat{M}$ is common	S √
	$\widehat{A_1} = \widehat{C_2}$ [tan chord thm]	S √
	Or	
	$\widehat{C_1} + \widehat{C_2} = \hat{B} = \hat{D} = x$ [tan chord thm or ext. ∠ of cyclic quad]	
	ΔAMC ||| ΔCMD [∠, ∠, ∠]	
	$\frac{AM}{CM} = \frac{CM}{MD}$	
	$\therefore CM^2 = AM \times MD$	$CM^2 = AM \times MD$√
	$\frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$	$\frac{AM \times MD}{AB \times MD}$ √
	$= \frac{AM}{AB}$	(6)

	Or	
	$\frac{AC}{MC} = \frac{AB}{DC}$ [ΔACB ||| ΔCMD]	$\frac{AC}{MC} = \frac{AB}{DC}$ √
	$\therefore CM \times AB = AC \times DC$	
	In ΔACB and ΔAMC	
	$\hat{C} = \hat{M} = 90°$ [given]	S √
	$\widehat{A_1} = \widehat{A_2}$ [proven]	S √
	Or	
	$A\hat{C}M = \hat{B} = x$ [proven]	
	ΔACB ||| ΔAMC [∠, ∠, ∠]	
	$\frac{AC}{AM} = \frac{BC}{MC}$	
	$\therefore AC \times MC = AM \times BC$	$AC.MC = AM.BC$√
	$\therefore AC = \frac{BC.AM}{MC}$	
	$CM \times AB = \frac{BC.AM}{MC} \times DC$	Equating √
	$CM^2 = \frac{DC.AM}{AB} \times DC$ [BC=DC]	S √
	$\frac{CM^2}{DC^2} = \frac{AM}{AB}$	(6)
5		
5.1.1	$\hat{P} = \widehat{M_1} = 66°$ [tan chord theorem]	S √__R√ (2)
5.1.2	$\widehat{M_2} = 90°$ [∠′s in semi circle]	S √ R √ (2)
5.1.3	$\widehat{N_1} = 180° - (90° + 66°)$[Sum of ∠ of ΔMNP] $= 24$	S √ R √ (2)
5.1.4	$\widehat{O_2} = \hat{P}$= 66° [corres. ∠′s PM∥ OR]	S √ R √ (2)
5.1.5	$\hat{R} + \widehat{N_1} + \widehat{N_2} = 180° - 66°$ [sum of ∠′s of ΔRNO]	S √
	= 114°	S √
	$\hat{R} = \widehat{N_1} + \widehat{N_2} = 57°$ [∠′s opposite = radii]	R/S √
	$\therefore \widehat{N_2} = 33°$	S √ (4)

APPENDIX 3: ERROR LIST INSTRUMENT

Section 1
According to Newman (1977) Mathematics learners commit fundamental errors while solving mathematics. These errors are

1. Reading error - Not able to read mathematical problem given and to identify sentences and mathematical symbols used.
2. Comprehension error - Not able to show some form of understanding of the problem given.
3. Transformation error - Not able to determine the correct method of mathematical solution.-
4. Process skill error - Not able to provide correct mathematical processes involved in the solution method.
5. Encoding error – Not able to write the solution according to the requirement of the question given.

The above errors were used to investigate the study participants' "errors committed", in the 2020 and 2021 matric mathematics paper 2 examination.

Section 2. Solution appraisal table was used to identify and group the errors.

Solution Appraisal

Table 1 Types of Errors

S/N	Type of Errors
1	Not able to approach the problem in any correct way.
2	Not able to read mathematical problem given and to identify sentences and mathematical symbols used.
3	Not able to show some form of understanding of the problem given.
4	Not able to determine the correct method of mathematical solution
5	Not able to provide correct mathematical processes involved in the solution method.
6	Not able to write the solution according to the requirement of the question given.

Table 2: Number of Errors Template

School Name:		Nov/Dec Matric examination			Year:			
Scripts / Errors	Error 1	Error 2	Error 3	Error 4	Error 5	Total errors	Marks obtained	%
Script 1								
Script 2								
Script 3								
Script 4								
Script 5								
Script 6								
Script 7								
Script 8								
Script 9								
Script 10								

Section 3: Error Rating Scale (ERS)

The ERS is used to measure the intensity of the errors;

where:

1= Most fair: The error is committed as a result of mistake within a correct problem-solving approach.

2= Fair: Wrong problem-solving approach but able to manage the ensued mathematical calculations.

3= Bad: Not having any idea of how to answer the question, leave the question unanswered or wrong problem-solving approach coupled with poor mathematical calculations.

Table 3: Error Intensity Data Analysis Table

Scale: 1 = Good 2 = Fair 3 = Bad

School Name:		Nov/Dec Matric examination			
Scripts / Errors	Error 1	Error 2	Error 3	Error 4	Error 5
Script 1					
Script 2					
Script 3					
Script 4					

APPENDIX 4: UNISA ETHICAL CLEARANCE

UNISA university of south africa

UNISA SCHOOL OF SCIENCE ETHICS REVIEW COMMITTEE

17 September 2021

Dear Mr SK Matsho

ERC Reference # : 2021/CSET/SOS/124
Name : Mr Stephens Kgalushi Matsho
Student # : 32666691

Decision: Ethics Approval from 17 September 2021 to 08 September 2026

Researcher(s):	Name:	Mr Stephens Kgalushi Matsho
	E-mail address:	32666691@mylife.unisa.ac.za
	Telephone #:	+27147181500
	Cell #:	+27767911571
Supervisor(s):	Name:	Dr Sunday Faleye
	E-mail address:	Efaleys1@unisa.ac.za
	Cell #:	+27734122114

Working title of research:

Exploring South African high school learner's problem solving skills in Euclidean Geometry

Qualification: PhD

Thank you for the application for research ethics clearance by the Unisa School of Science Ethics Review Committee for the above mentioned research. Ethics approval is granted until **08 September 2026**.

*The **low risk application** was **reviewed** by the School of Science Ethics Review Committee on 17 September 2021 in compliance with the Unisa Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa COVID-19 position statement on research ethics.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the *School of Science Ethics Review Committee*.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing, accompanied by a progress report.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data require additional ethics clearance.
8. No field work activities may continue after the expiry date (**08 September 2026**). Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.
9. Field work activities may only commence from the date on this ethics certificate.

Note:

The reference number ***2021/CSET/SOS/124*** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Yours sincerely,

Ms S Muchengetwa
Chair: School of Science ERC
Tel: 011 670 9253
E-mail: muches@unisa.ac.za

Prof Mantile Lekala
Director: Science
Tel: 011 670 9091
E-mail: lekalml@unisa.ac.za

Prof BB Mamba
Executive Dean: CSET
Tel: 011 670 9231
Email: mambabb@unisa.ac.za

APPENDIX 5: PERMISSION TO CONDUCT THE STUDY

492 Block
DD
Soshanguve
0152
04 April 2022

The Head of Department
(Name of province) Department of Education
Address

Sir/Madam

REQUEST TO CONDUCT RESEARCH IN THE PROVINCE

I Stephens Kgalushi Matsho, PHD student in Mathematics Education at Unisa request permission to conduct research using copies of Grade 12 Mathematics Paper 2 NSC scripts from 2019 to 2021.
My research topic is "Exploring South African high school learner's problem-solving skills in Euclidean Geometry". In this research , a researcher need to access 2019, 2020 and 2021 scripts of four different schools categorised as follows: (1) Two quintile 5 schools (2) Two quintile 1 or 2 schools.

The researcher main focus is the section where learners have answered Euclidean Geometry. Copies of scripts should be Euclidean Geometry response only in order to protect the identity of the learner.

Hoping my request is in order.

Yours faithfully
Matsho S.K
Contct Number: 076 791 1571 or 072 353 8888
Email address: kgalushim@yahoo.com

APPENDIX 6: FREE STATE DEPARTMENT OF EDUCATION ETHICAL CLEARANCE

Enquiries: M.Z. Thango
Ref: Research Permission: S.K. Matsho
Tel. 051 404 8808
Email: MZ.Thango@fseducation.gov.za

education
Department of Education
FREE STATE PROVINCE

492 Block DD
Soshanguve
0152

Dear Mr. S.K. Matsho

PERMISSION TO CONDUCT RESEARCH IN THE FREE STATE DEPARTMENT OF EDUCATION: FEZILE DABI AND LEJWELEPUTSWA DISTRICTS

This letter serves to inform you that you have been granted permission to conduct research in the Free State Department of Education within the Fezile Dabi and Lejweleputswa Education Districts. The details in relation to your research project with the University of South Africa are as follows:

Topic: Exploring South African high school learner's problem solving skills in Euclidean Geometry.

1. **List of schools involved:** Boitlamo SS, HTS Sasolburg, Noma SS and Welkom High SS.
2. **Target Population:** One official from the Head Office, Mr. Basjan, Director: Examinations to provide the researcher with Mathematics grade 12 paper 2 scripts from 2019 to 2021. The examination section is expected to provide eighty Mathematics learner's scripts per year, i.e. 2019 to 2021 for grade 12 NSC examinations at the selected schools. **NB: Please be advised that, the Department only keeps the scripts for six months, thereafter they are destroyed. Therefore, you might not get some scripts for particular years requested.**
3. **Period of research**: From the date of signature of this letter until 30 September 2022. Please note that the department does not allow any research to be conducted during the fourth term (quarter) of the academic year. Should you fall behind your schedule by three months to complete your research project in the approved period, you will need to apply for an extension. The researcher is expected to request permission from the school principals to conduct research at schools.
4. The approval is subject to the following conditions:

   4.1 The collection of data should not interfere with the normal tuition time or teaching process.

   4.2 A bound copy of the research document should be submitted to the Free State Department of Education, Room 101, 1st Floor, Thuto House, St. Andrew Street, Bloemfontein or can be emailed to the above-mentioned email address.

   4.3 You will be expected, on completion of your research study to make a presentation to the relevant stakeholders in the Department.

   4.4 The ethics documents must be adhered to in the discourse of your study in our department.
5. Please note that costs relating to all the conditions mentioned above are your own responsibility.

Yours Sincerely,

DATE: 25/04/2022

Mr. MZAMO W. JACOBS
DIRECTOR: QUALITY ASSURANCE, M&E AND STRATEGIC PLANNING

RESEARCH APPLICATION BY S.K. MATSHO, PERMISSION LETTER 25 APRIL 2022. FEZILE DABI & LEJWELEPUTSWA DISTRICTS
Strategic Planning, Research & Policy Directorate Private Bag X20565, Bloemfontein, 9300 - Thuto House, Room 101, 1st Floor, St Andrew Street, Bloemfontein

www.fsdoe.fs.gov.za

APPENDIX 7: GDE ETHICAL CLEARANCE

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**07 April 2022**
Validity of Research Approval:	**08 February 2022– 30 September 2022** **2022/121**
Name of Researcher:	**Matsho S.K**
Address of Researcher:	**492 Block DD**
	Soshanguve
Telephone Number:	**072 353 8888**
Email address:	**Kgalushimatsho@gmail.com**
Research Topic:	**Exploring South Africa high learners problem solving skills in Euclidean Geometry.**
Type of qualification	**PHD**
Number and type of schools:	**4 schools (2 Quintile 5 and 2 Quintile 1 or 2)**
District/s/HO	**HO**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

07/04/2022

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below are met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The letter would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools, or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.***
4. ***The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s has been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter/document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs, and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort to obtain the goodwill and cooperation of all the GDE officials, principals, and chairpersons of the SGBs, teachers, and learners involved. Persons who offer their cooperation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school program is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her research resources, such as stationery, photocopies, transport, faxes, and telephones, and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers, and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study, the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings, and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a summary of the purpose, findings, and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr. Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 07/04/2022

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX 8: MPUMALANGA DEPARTMENT OF EDUCATION ETHICAL CLEARANCE

education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

Ikhamanga Building, Government Boulevard, Riverside Park, Mpumalanga Province
Private Bag X11341, Mbombela, 1200.
Tel: 013 766 5552/5115, Toll Free Line: 0800 203 116

Litiko le Temfundvo, Umnyango we Fundo | Departement van Onderwys | Ndzawulo ya Dyondzo

Enq: N M Mazibuko
013 766 5148

Mr S K Matsho
Soshanguve
0152
Cell: 0767911571
Email: kgalushim@yahoo.com

RE: EXPLORING SOUTH AFRICAN HIGH SCHOOL LEANER'S PROBLEM SOLVING SKILLS IN EUCLIDEAN GEOMETRY

Your application to conduct research study was received and is therefore acknowledged. The tittle of your research project reads: **"Exploring South African high school leaner's problem solving skills in Euclidean Geometry".** I trust that the aims and the objectives of the study will benefit the whole department especially the beneficiaries. Your request is approved subject to you observing the provisions of the departmental research policy which is available in the department website. You are requested to adhere to your university's research ethics as spelt out in your research ethics.

In terms of the research policy, data or any research activity can be conducted after school hours as per appointment with affected participants and COVID -19 regulations to observed. You are also requested to share your findings with the relevant sections of the department so that we may consider implementing your findings if that will be in the best interest of the department. To this effect, your final approved research report (both soft and hard copy) should be submitted to the department so that your recommendations could be implemented. You may be required to prepare a presentation and present at the departments' annual research dialogue.

For more information kindly liaise with the department's research unit @ 013 766 5124/5148 0r n.madihlaba@mpuedu.gov.za

MRS LH MOYANE
HEAD: EDUCATION

06 / 04 / 2022
DATE

MPUMALANGA

APPENDIX 9: LIMPOPO DEPARTMENT OF EDUCATION ETHICAL CLEARANCE

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

CONFIDENTIAL

Ref: 2/2/2 Enq: Makola MC Tel No: 015 290 9448 E-mail: MakolaMC@edu.limpopo.gov.za

Matsho S.K
P.O Box 8.
Radium
0483

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.

2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"STUDENT'S PROBLEM SOLVING APPROACH IN EUCLIDEAN GEOMETRY "**

3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: MATSHO S.K

Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa - development is about people!

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

Dederen KO
Head of Department

18/10/2021
Date

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: MATSHO S.K

APPENDIX 10: NORTH-WEST DEPARTMENT OF EDUCATION ETHICAL CLEARANCE

education
Lefapha la Thuto la Bokone Bophirima
Noordwes Departement van Onderwys
North West Department of Education
NORTH WEST PROVINCE

Garona Building, Mmabatho
1st Floor, East Wing,
Private Bag X2044,
Mmabatho 2735
Tel.: (018) 388-3433
Fax.: 086-514-0126
e-mail: sgedu@nwpg.gov.za

OFFICE OF THE SUPERINTENDENT-GENERAL

Enq. : Dr T Phorabatho
Tel. : 018 388 3071/3433

To: Dr Faleye Sunday
University of South Africa
Faculty of Education

From: Ms S M Semaswe
Superintendent-General

Date : 03 July 2020

PERMISSION TO CONDUCT RESEARCH: MR MATSHO STEPHEN KGALOSHI

Your email received on the 02nd of July 2020 bears reference.

Permission is hereby granted to you to conduct research in the department as requested, subject to the following conditions:

- You contact the relevant School Principals for your target schools about your request with this letter of permission;
- Considering that your research will involve both Educators and Learners, the general functionality of the school should not be compromised by the research process.
- The participation in your project will be voluntary.
- The principles of informed consent and confidentiality will be observed in strictest terms, and
- The findings of your research should be made available to the North West Department of Education and Sport Development upon request.

Best wishes

Mrs S M Semaswe
Superintendent-General

14/07/2020
Date

COVID-19 (NOVEL CORONAVIRUS) | STAY AT HOME | WASH YOU HANDS OFTEN | WEAR MASK & GLOVES WHEN GOING OUTSIDE | KEEP TO YOUR DISTANCE

APPENDIX 11: INSTRUMENT VALIDATION FORM FOR ERROR LIST INSTRUMENT

The research instrument under consideration is designed to measure participants' type of errors committed, nature of the errors committed, and performance. This instrument consists of three sections; section 1 consists of the Newman Errors list, section 2 consists of the solution appraisal content, and section 3 consist of the error rating scale (ERS). The instrument is attached for full details of the instrument.

In order to have the appropriate items on the research instrument so that the researcher's desired aim would be achieved, your assistance is highly needed in this regard.

Please, judge each section on the instrument on its level of relevance and level of appropriateness to Grade 12 learners in Euclidean geometry. You are required to use the scale below:

Level of Relevance

1= Low/not relevant 2= somewhat relevant 3= highly relevant

Level of Appropriateness

1= Not appropriate 2= somewhat appropriate 3= highly appropriate

Personal information of Evaluator

Qualification: ……………………………… Status: …………………………………

Signature: ……………………………… Date: …………………………………

APPENDIX 12: INTER-RATER RELIABILITY TEST FORM FOR ERROR LIST INSTRUMENT

The research instrument under consideration is designed to measure participants' type of errors committed, nature of the errors committed, and performance. This instrument consists of three sections; section 1 consists of the Newman Errors list, section 2 consists of the solution appraisal content, and section 3 consist of the error rating scale (ERS). The instrument is attached for full details of the instrument.

Please, could you assist to conduct the reliability assessment of each section of the instrument. You are required to use the table below to record your rating:

Section	JUDGE	JUDGE	JUDGE
Section 1 Assessment			
Section 2 Assessment			
Section 3 Assessment			
Average Assessment			

Please note: Use only one column.

Personal information of Evaluator

Qualification: .... Status: ....

Signature: .... Date: ....

APPENDIX 13: CONTENT VALIDATION FORM FOR THE EGPSLM MATERIAL

The research instrument under consideration is designed as Euclidean Geometry learning model for Grade 12 mathematics learners, it forms part of the intervention material. This learning material consists of Euclidean geometry term, symbols and concepts appropriate for Grade 12 mathematics learners.

In order to have the appropriate items on the intervention materials, so that the researcher's desired aim would be achieved, your assistance is highly needed in this regard.

Please, learning material on its level of relevance, appropriateness and covered to Grade 12 learners in Euclidean geometry. You are required to use the scale below:

Level of Relevance

1= Low/not relevant 2= somewhat relevant 3= highly relevant

Level of Appropriateness

1= Not appropriate 2= somewhat appropriate 3= highly appropriate

Level of Covered

1= Not well covered 2= somewhat well covered 3= Very well covered

Personal information of Evaluator

Qualification: .... Status: ....

Signature: .... Date: ....

APPENDIX 14: INTER-RATER RELIABILITY TEST FORM OF EGPSLM LEARNING MATERIAL

The research learning material under consideration is designed as Euclidean Geometry learning model for Grade 12 mathematics learners, it forms part of the intervention material in my research. To have a reliable learning material as part of the intervention, so that the researcher's desired aim would be achieved, your assistance is highly needed in this regard.

Please, assess the learning material on its level of been a reliable learning material in this my research. Please assess the material on scale between 0 to 1.

Personal information of Evaluator

Qualification: ……………………………… Status: …………………………………

Signature: ……………………………… Date: …………………………………

APPENDIX 15: FACE VALIDATION FORM FOR THE EGPSIA MATERIAL

The research instrument under consideration is designed as Euclidean Geometry instructional approach for Grade 12 mathematics learners, it forms part of the intervention material. In order to achieve the desired aim in this research, your assistance is highly needed in this regard.

Please, rate the instructional approach material on its level of relevance, and appropriateness to Grade 12 learners in Euclidean geometry. You are required to use the scale below:

Level of Relevance

1= Low/not relevant 2= somewhat relevant 3= highly relevant

Level of Appropriateness

1= Not appropriate 2= somewhat appropriate 3= highly appropriate

Personal information of Evaluator

Qualification: ……………………………… Status: …………………………………

Signature: ……………………………… Date: …………………………………

APPENDIX 16: INTER-RATER RELIABILITY TEST FORM FOR EGPSIA INSTRUCTIONAL MATERIAL

The research instructional material under consideration is designed as Euclidean Geometry instructional approach for Grade 12 mathematics learners, it forms part of the intervention material in my research. To have a reliable instructional material as part of the intervention, so that the researcher's desired aim would be achieved, your assistance is highly needed in this regard.
Please, assess the instructional material on its level of been a reliable learning material in my research. Please assess the material on scale between 0 to 1.

Personal information of Evaluator
Qualification: ……………………………… Status: …………………………………

Signature: ……………………………… Date: …………………………………

APPENDIX 17: LANGUAGE EDITING CERTIFICATE

EDITING AND PROOFREADING CERTIFICATE

22 Osche Street

The Reeds

Centurion

0157

01 November 2024

TO WHOM IT MAY CONCERN

This certificate serves to confirm that I have edited Matsho Stephens Kgalushi's thesis titled, **"Exploring South African high school learners problem solving skills in Euclidean Geometry**."

I found the work easy and intriguing to read. Much of my editing basically dealt with obstructionist technical aspects of language, grammar and syntax, which could have otherwise compromised smooth reading as well as the sense of the information being conveyed. I hope that the work will be found to be of an acceptable standard. I am a member of Professional Editors' Guild.

Hereunder are my contact details:

Jack Chokwe (PhD – University of Leicester (United Kingdom)

Contact numbers: 072 214 5489

jackchokwe@gmail.com

Professional EDITORS Guild

APPENDIX 18: TURNITIN REPORT

Similarity Report

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