Using erroneous examples for teaching Grade 9 Algebraic Linear Equations at a School in Johannesburg Central District

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DECLARATION

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DEDICATION

This work is dedicated to my wife, Eleanor Gwenzi, my sons Prevost and Ngonidzashe and my daughters, Melody and Tatenda Nyashadzashe.
May God abundantly bless them.

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ABSTRACT

The importance of mathematics is well known the world over, and its applications in commerce, science, technology and everyday life situations cannot be emphasised. Algebra is an abstract and challenging branch of mathematics to teach and learn. Teachers struggle to teach it, and learners struggle to understand it, hence they develop a plethora of misconceptions and associated errors. Passing mathematics at FET is a prerequisite for admission into tertiary institutions to study science, engineering, technology and mathematics. To pass mathematics, one has to have a comprehensive understanding of algebra at an early stage of learning. This study's purpose was to explore the impact and effectiveness of erroneous examples for teaching Grade 9 algebraic linear equations. Learners' performance in mathematics in Grade 9 is crucial because, in South Africa, learners use their results to choose the subject combination they will take at FET. This study was underpinned by constructivism, which recognises the role of the individual in making knowledge. This study followed an explanatory embedded mixed-methods research design. Qualitative data for this study was collected through lesson observations and semi-structured interviews with two Grade 9 mathematics teachers. A pre-test was used to collect quantitative data from both the experimental and control groups (N=31) and (N=28) respectively. A post-test was also administered to both the experimental group (N=12) and the control group (N=24). Excel was used for data management. Statistical data analysis was done using IBM SPSS version 28. The experimental and control groups were compared using a t-test at the 95% confidence limit (2-sided). The results were declared statistically and significantly different for all p-values of 0.05 or less. The post-test results after the intervention showed an improvement in performance for the two study groups, with the experimental group improving from $\bar{x} = 1.12903$ in the pre-test to $\bar{x} = 18.83333$ in the post-test, an increase of $\bar{x} = 17.7043.$ In contrast, the control group improved from $\bar{x} = 12.82143$ in the pre-test to $\bar{x} = 23.00000$ in the post-test an increase of $\bar{x} = 10.17857.$ The greater increase in the mean differences by the experimental group suggests that the intervention positively impacted learners' performance. Qualitative data obtained from semi-structured interviews and lesson observations were coded and identified according to different themes. Erroneous examples positively impacted the TEG's teaching practice and improved the learners' performance.

Keywords: algebraic linear equations, errors, erroneous examples, intervention, misconceptions

LIST OF ABBREVIATIONS AND ACRONYMS

CK	Content Knowledge
DBE	Department of Basic Education
FET	Further Education and Training
NSC	National Senior Certificate
PCK	Pedagogical Content Knowledge
SACMEQ	Southern and Eastern African Consortium for Monitoring and Educational Quality
SRCK	School Related Content Knowledge
TCG	Teacher in the control group
TEG	Teacher in the experimental group
TIMSS	Trends in International Mathematics and Science Studies
ZAD	Zone of Actual Development
ZPD	Zone of Proximal Development
EG	Experimental Group
CG	Control Group
L1	Home language
L2	Second language
SCE	Senior certificate examination

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CHAPTER 1: ORIENTATION TO THE STUDY

1.1 INTRODUCTION AND BACKGROUND

Mathematics is a subject that stimulates national growth and economic development worldwide. As a result, countries prioritise teaching mathematics to every school-going child (Capuno et al., 2019, p.547; DBE, 2019). The (ibid) further asserts that the skills and knowledge that students acquire in mathematics are important for overcoming real-life problems, hence the need by national governments to ensure that learners understand the basic fundamental mathematics concepts at early stages.

One of the aims of the mathematics curriculum (DBE, 2017, p.5) is to provide an education comparable in quality, breadth and depth to those of other countries. To achieve this, South Africa (SA) has taken part in international and regional science and mathematics tests, namely, the Trends in International Mathematics and Science Studies (TIMSS) and the Southern and Eastern Consortium for Monitoring Education Quality (SACMEQ). Among 39 countries participating in the latest TIMSS tests, SA came out second last (Bowie et al., 2022, p.2). Only 41% of SA Grade 9 learners who participated in the TIMSS 2019 tests showed acquisition of basic mathematics knowledge. Thus, 59% of the participants did not show an understanding of some basic mathematics knowledge. At the regional level, mathematics results have also been unsatisfactory, out of 15 countries, SA obtained position 14 in the SACMEQ IV tests (DBE, 2017, p.5).

Nationally, there has been growing concern regarding the number of learners taking mathematics at the National Senior Certificate (NSC) level (DBE, 2019). In 2019, the number of learners who registered for pure mathematics dropped by over 11,800 compared to those who registered for pure mathematics in 2018 (DBE, 2019). Furthermore, the DBE (2022, p.197) revealed that there had been a significant number of learners performing below 50% in the 2019-2022 period. The table below gives a summary of learners' performance. The least pass mark in SA at FET level is 30%. Table 1 shows large numbers of learners passing in the range 30% to 40%. However, learners who pass in this range do not qualify for enrolment at tertiary institutions to study mathematics and other mathematics related subjects. Hence, even though the pass rate maybe high, few learners qualify to study mathematics at tertiary institutions. Nationally, the pass rate for the past four (4) consecutive years that is, 2019, 2020, 2021 and 2022 was 54.6%, 53.8%, 57.6% and 55.0%.

Year	**Learners wrote**	**0-9. 9 %**	**10-19, 9 %**	**20-29, %**	**30-39, %**	**40-49, %**	**50-59, %**	**60-69,9%**	**70-79, %**	**80-89, %**	**90-100%**
2019	222 034	9,2%	17,2%	19,0%	19,6%	14,7%	9,6%	5,6%	3,1%	1,5%	0,4%
Number of learners		20 428	38 190	42 187	43 519	32 639	21 316	12 434	6 884	3 331	8 89
2020	233 315	12,1%	16,5%	17,5%	18,2%	13,2%	9,1%	6,1%	3,9%	2,3%	0,9%
Number of learners		28 232	38 497	40 831	42 464	30 798	21 232	14 233	9 100	5 367	2 100
2021	259 143	9,7%	15,3%	17,4%	19,9%	14,7%	9,7%	6,4%	3,8%	2,1%	0,8%
Number of learners		25 137	39 649	45 091	51 570	38 095	25 137	16 586	9 848	5 443	2 074
2022	269734	10,8%	16,4%	17,8%	19,0%	13,9%	9,5%	6,1%	3,8%	2,0%	0,7%
Number of learners		29 132	44 237	48 013	51 250	37 492	25 625	16 454	10 250	5 395	1 889

Table 1.1: ***Performance distribution in Mathematics: 2019-2022 (percentage)*** (Source: DBE (2022, p.197)

Provincial results have also been unsatisfactory. The percentage of learners who passed mathematics with 40% in the Gauteng province for the past four (4) consecutive years that is, 2019, 2020, 2021 and 2022, was 67.8%, 65.4%, 68.2%, and 62,7% respectively. In addition, performance in the Johannesburg Central District over the same period was 59.8%, 59.6%, 64.6%, and 61.0%. The Department of Basic Education (DBE) suggests that most learners who performed poorly at the 30% and 40% levels in the NSC examinations show that they had not mastered the elementary algebraic mathematical skills (DBE, 2019, 2020, 2021 & 2022). Moreover, over the same period, it was reported that most learners exhibit common errors in algebra, which include, among others, failure to factorise algebraic expressions, being unable to expand a binomial expression and the inability to solve simple algebraic equations (DBE, 2021, p.185). The above raises concern about whether learners successfully transitioned from General Education and Training (GET) to Further Education and Training (FET) and whether misunderstandings of algebraic processes occurred as reflected by the common errors exhibited in national examinations. Hence, the researcher concluded that there is a need to address learners' misconceptions and errors early.

1.2 PROBLEM STATEMENT

The problem investigated in this study was the impact and effectiveness of erroneous examples for teaching Grade 9 algebraic linear equations to improve learners' performance. One of the aims of the curriculum is to produce learners who can use critical and creative thinking to solve problems (DBE, 2012, p.5). Critical and creative thinking skills could be taught to learners through algebra. In SA, learners are introduced to algebra at the Senior Phase (SP) level, Grades 8 to 9. However, due to using symbols and variables to represent numbers, algebra, as general arithmetic, is abstract and complex for many learners and learners develop misconceptions and associated errors when learning algebra. As a result, the majority of learners fail mathematics in Grade 12. The high failure rate in mathematics in Grade 12 has a ripple effect in that many learners fail to enrol at institutions of higher learning to further their studies in mathematics, physics, engineering and accounting, to mention a few. When few learners enrol to study these critical subjects, it implies that the country does not have enough skilled labour to run the economy. As a result, most industries would rely on expatriate workers. There will also be not enough mathematics teachers in the schools, resulting in a high teacher-learner ratio, thereby forcing the DBE to look for mathematics teachers from the international community.

The DBE, in its Annual Diagnostic Reports (DBE, 2019, 2020, 2021& 2022), suggest that learners perform poorly in mathematics is due to inadequate fundamental skills of algebra, which they should have acquired at the Senior Phase Level. Most learners who fail mathematics in Grade 12 lack an understanding of algebra's basic skills, for example, failure to factorise a quadratic trinomial and apply the distributive rule when simplifying brackets. The researcher proposes that by exposing learners to their misconceptions and errors through error analysis, they may be able to avoid such errors in Grade 12 examinations.

1.3 PURPOSE STATEMENT

The purpose of this study was to explore the impact and effectiveness of erroneous examples for the teaching and learning of grade 9 algebraic linear equations to improve learners' performance. This study explored two (2) mathematics teachers' pedagogical approaches in teaching Grade 9 algebraic linear equations to understand how they influence learners' errors and misconceptions. The researcher designed this study to better understand learners' difficulties when solving algebraic linear equations and how to overcome them. Additionally, the researcher exposed the TEG to a new teaching strategy of erroneous examples when teaching algebraic equations to make learners aware of the common errors and misconceptions that may impede learning at higher levels. Furthermore, the researcher was able to understand the TEG's perceptions of erroneous examples as an intervention strategy.

1.4 RESEARCH QUESTIONS

The researcher used the primary and secondary questions to better understand the investigated phenomenon. These research questions assisted the researcher in gaining a deeper understanding of learners' errors and misconceptions in algebraic linear equations and the impact and effectiveness of erroneous examples in teaching and learning algebraic linear equations in the Johannesburg Central District in Gauteng.

1.4.1 Primary research questions

1. What is the impact of using erroneous examples on the effectiveness of teaching and learning Grade 9 algebraic linear equations?
2. How do the testing outcomes explain Grade 9 learners' difficulties in mathematics?

1.4.2 Secondary research questions

Given below are the secondary research questions used to unpack the primary research questions.

1. What strategies do teachers use when teaching Grade 9 algebraic linear equations?
2. What errors do learners commit when solving Grade 9 algebraic linear equations?
3. What are the benefits of using erroneous examples to teach Grade 9 algebraic linear equations?
4. How can teachers use erroneous examples for teaching Grade 9 algebraic linear equations to improve learners' performance?

1.5 OBJECTIVES

1. To explore Grade 9 teachers' pedagogical approaches when teaching algebraic linear equations.
2. To explore the errors that Grade 9 commits when solving algebraic linear equations.
3. To determine the benefits of using erroneous examples for teaching Grade 9 algebraic linear equations.
4. To suggest how teachers may use erroneous examples to improve Grade 9 learners' performance when solving algebraic linear equations.

1.6 RESEARCH HYPOTHESES

$H_{0\,a}$: There is no significant difference between pre-test and post-test results when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

$H_{1\,a}$: A significant difference exists between pre-test and post-test results when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

$H_{0\,b}$: There is no significant difference in learner performance between the experimental and control groups when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

H_{1b}: A significant difference exists in learner performance between the experimental and control groups when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

1.7 CONCEPTUAL FRAMEWORK

Underpinning this study is zone of proximal development (ZPD, scaffolding (Vygotsky), Bandura's observational learning theory and Newman's Error Analysis model. According to Qin (2022, p.139), a child's ZPD is the distance between the level of actual development, which is determined with the help of independently solved tasks, and the level of possible development, defined with the help of tasks solved by the child under the guidance of adults or in cooperation with the more intelligent peers. However, Zaretsky (2021, p.41) states that the ZPD is what a child can do in collaboration with adults. The (ibid) contends that while working with an adult today, a learner can solve a problem, and tomorrow, the learner can solve the problem alone. Furthermore, learners' problem-solving capacity develops when teachers create appropriate learning conditions. In this study, a written test was used to ascertain the learners' ZPD, after which an intervention strategy was implemented.

According to Qin (2022, p.139), scaffolding is a process whereby the teacher sets up a situation that makes it easy for the learner to solve a problem. The teacher gradually withdraws support as the learner shows the capability to manage the situation. Kusmaryono et al. (2021, p.343) describe scaffolding as actions by teachers and peers in supporting, facilitating, assisting and accelerating learners' learning tasks. Scaffolding is the temporary support given to learners while solving problems (Lei, Xin, Morita-Mullaney & Tzur, 2020, p.127). In this study, the researcher used erroneous and correct examples for scaffolding.

Observational learning is when someone acquires new responses by observing others (Greer, Dudek-Singer & Gautreaux, 2020, p.487). Observational learning occurs when people learn through observing others' behaviour. In this study, learners observed erroneous examples from which they learned how others committed errors, as exemplified in erroneous examples. In addition, learners learned how other learners effectively solved problems, as indicated by incorrect examples. When studying erroneous examples, learners identified their errors and misconceptions, which they managed to control in subsequent exercises.

According to the Newman's error analysis model, learners exhibit six types of errors when solving problems namely: reading, comprehension, transformation, processing and encoding errors. The

Newman's error analysis model was used to analyse learners' written responses to understand learners' difficulties in learning algebraic linear equations.

1.8 RESEARCH METHODOLOGY

Patel and Patel (2019, p.48), define research methodology as a set of steps and techniques researchers use to collect and analyse data and report on the outcome. This study follows an explanatory embedded mixed methods approach, utilising qualitative and quantitative methods. According to Marutha (2020. p.421), researchers integrate qualitative and quantitative methods in a single study using mixed method research (MMR). Moreover, Ngulube (2020, p.426) describes MMR as research that fully answers a research question by combining qualitative and quantitative approaches in one study. Using qualitative and quantitative approaches in one study, researchers aim to maximise the benefits of both approaches while minimising their disadvantages. MMR is a research approach with philosophical assumptions that guide how qualitative and quantitative data is collected, analysed, integrated and interpreted in a single study (Creswell & Creswell, 2018, p.5). The researcher used MMR to take advantage of the strengths of qualitative and quantitative methods in answering the questions in this study.

1.9.1 Population and sampling method

A population is defined as all the items or people a researcher wishes to understand (Naseri, Hussin, Esa, Azizi and bin Nordin , 2021, p.654). The population for this study was Johannesburg Central District, comprising sixty-eight (68) secondary schools. Since studying whole populations is impossible, researchers usually choose small sections of the populations using sampling techniques (ibid, p.654). Convenience sampling was used to sample learners from the experimental and control group. Convenience sampling occurs when the researcher conveniently selects participants who could provide the most relevant information for the study (Emerson, 2021, p.76). Participants in this study were Grade 9 learners and their teachers. The learners in the experimental group were not performing well and were willing to participate and benefit from the intervention. Therefore, convenience sampling was considered an appropriate sampling method for this study.

The researcher conveniently chose three (3) Johannesburg Central District schools for this study. One (1) school was used in a pilot study. A pilot study is used to enquire as to whether the researcher should continue with the study and, if so, in which manner (In, 2017, p.601). A pilot study assesses the feasibility of implementing the proposed intervention in the main study and ensures that the researcher gains experience using the research tools (In, 2017, p.601). Through the pilot study, the researcher may identify possible future problems in the study and, as such, address them before the full-scale study (Malmqvist, Hellberg, Mollas, Rose & Shelvin, 2019, p.1).

From the other two (2) schools, one (1) served as an experimental group (EG), (N=31) in the pre-test and (N=12) in the post-test and the other as a control group (CG), (N=28) in the pre-test and (N=24) in the post-test.

1.9.2 Data collection procedures

Quantitative and qualitative methods were used to collect data for this study. Pre- and post-tests were administered to collect quantitative data from learners, while semi-structured interviews and lesson observations were used to collect qualitative data from the teachers. The pre-and post-test comprised algebraic linear equations and word problems involving linear equations. This study was carried out in three (3) phases (Phase 1, 2 and 3). Phase 1 investigated how teachers teach linear equations before the intervention strategy in the experimental school. The researcher' focus was on the teacher's pedagogical approaches, learner assessment and learner-learner and teacher-learner interactions. In Phase 2, the study focused on the impact and effectiveness of the strategy in the experimental school. Phase 3 was a continuation of the successes of Phase 2.Before implementing the intervention, the researcher and TEG discussed how to use erroneous examples when teaching guided by literature. However, it was not expected that the teacher would get it perfectly right the first time, and iterations of the method became necessary. The researcher conducted twelve (12) lesson observations, of which four (4) lesson observations were before, during and after the interventions. Lesson observations afforded the observed teacher a chance to display their ability to demonstrate that they can link theory and practice (Weber, Waxman, Brown & Kelly, 2016, p.93). Also, the researcher had the chance to understand the teacher's pedagogical approaches. The researcher adapted Seeping's (2010, p.4) observational instrument for this study. The researcher compiled notes during lesson observations. Additionally, the researcher recorded interviews and stored them in a memory stick and a code-controlled computer.

1.9.3 Data analysis and interpretation

Data analysis involves searching for meaning from collected data (Ngulube, 2015, p.131). Pre-and post-test quantitative data was analysed using descriptive statistic (mean) to determine whether the intervention produced a significant difference in learners' performance. IBM SPSS version 28 software was used to analyse quantitative data.

Qualitative data analysis involves transforming raw data into meaningful information that is easy to interpret (Ngulube, 2015, p.132). The researcher transcribed, coded and categorised audiotaped data guided by research questions and objectives. The researcher employed IBM SPSS version 28 statistical software to analyse quantitative data and used research questions to interpret qualitative data (Creswell & Creswell, 2018, p.307).

1.9.4 Reliability and validity

Validity of research is mainly about what a study researches in that it should research what it is designed to research in the best way possible (Surucu & Maslakci, 2020, p.2695). To enhance validity, two (2) academics from UNISA's Department of Mathematics Education, moderated the test questions used in this study.

Reliability refers to stable and consistent research findings (Surucu & Maslakci, 2020, p.2695). An instrument is reliable when it produces consistent results anytime, anywhere and when used by anyone. Additionally, when an instrument measures with great accuracy, it is reliable (ibid). The researcher used various data collection methods to enhance reliability, including pre-and post-tests, semi-structured interviews and lesson observations (Adler, 2022, p.601). Consistency of results of an instrument determines the reliability of research (Rose & Johnson, 2020, p.4). The test questions used in this study were piloted to ensure consistency and reliability of results. This study was also peer and supervisor reviewed to ensure reliability.

1.9.5 Trustworthiness

The trustworthiness of research is determined by four (4) significant criteria: credibility, dependability, transferability and confirmability (Adler, 2022, p.599). In this study, data triangulation that uses different data collection methods was used to minimise bias and enhance credibility (ibid, p.601). Semi-structured interviews and lesson observations were the methods used to collect this study's qualitative data.

Dependability refers to the stability of research findings (Kalu & Bwalya, 2017, p.51). To ensure dependability, I took field notes to make sure detailed information was provided. Obtaining quality information from participants depends on the relationship between the researcher and participants. When participants have confidence in the researcher, reliable data can be collected (Kyangas, Mikkonen and Kaariainen, 2019, p.51). The researcher made an effort to establish rapport with participants to gain their confidence, which helped as they opened up during interviews.

Transferability refers to how research findings fit situations outside the current study (Noble & Smith, 2015), as cited in Kalu and Bwalya (2017, p.50). In this study, I provide complete details and the context of erroneous examples for teaching linear equations in Grade 9, and this will help anyone who wants to transfer the research results to decide on the sensibility of the transfer.

Confirmability means establishing that research interpretation resulted from data analysis, not the researcher's predispositions. In any qualitative research, confirmability is when credibility, dependability, and transferability are satisfied (Guba & Lincoln, 2005).

1.9.6 Research ethics

Research ethics generally refers to all ethical considerations related to science and research (Kyangas et al. ,2019, p.50). The researcher applied for an Ethical Clearance Certificate from the University of South Africa's Ethics Committee in the College of Education (CEDU) to access the schools. Thereafter, the researcher requested permission to do the research with the following stakeholders: the Province, District and Principals. Participants agreed to take part in this study by signing informed consent forms. Since all learners were minors below 18, the researcher sought permission from their parents before the study commenced. The researcher explained the participants' roles and rights in research (Bos, 2020, p.41).Pseudonyms instead of actual learners' and school names were used to protect participants' confidentiality. Participants were informed that all information they shared with the researcher would be used only in this research.

1.10 CHAPTER LAYOUT

Chapter 1 gave the background and introduction, statement of the problem, aim and objectives of the study, research questions and research methodology. Also discussed are the ethical considerations.

Chapter 2 deals with the study's conceptual framework.

Chapter 3 reviews related literature that includes misconceptions and errors displayed by learners when solving linear equations.

Chapter 4 presents research methodology, design, sampling methods, data collection and analysis techniques. In addition, it also discusses issues of validity, reliability, trustworthiness and research ethics.

Chapter 5 presents data analysis.

Chapter 6 presents the research results.

Chapter 7 covers research limitations, delimitations, conclusions and recommendations.

1.11 CHAPTER SUMMARY

SA learners' poor performance at both international and local levels was a concern that motivated the researcher to undertake this study. The DBE suggests that learners' poor performance in the SCE is because they lack mastery of basic mathematical concepts and skills in Grades 8 and 9. As a result, the researcher concluded that there is a need to explore different teaching strategies, particularly the use of erroneous examples, to try and improve learners' performance and academic achievement in Johannesburg Central District 14 of the Gauteng Province.

This chapter presented the background and introduced learners' misconceptions and errors that lead to poor performance in Johannesburg Central District 14. The study' purpose and problem statement are also stated. Furthermore, the proposed research methodology, research design and ethical considerations are stated. Chapter 2 discusses conceptual framework underpinning this study.

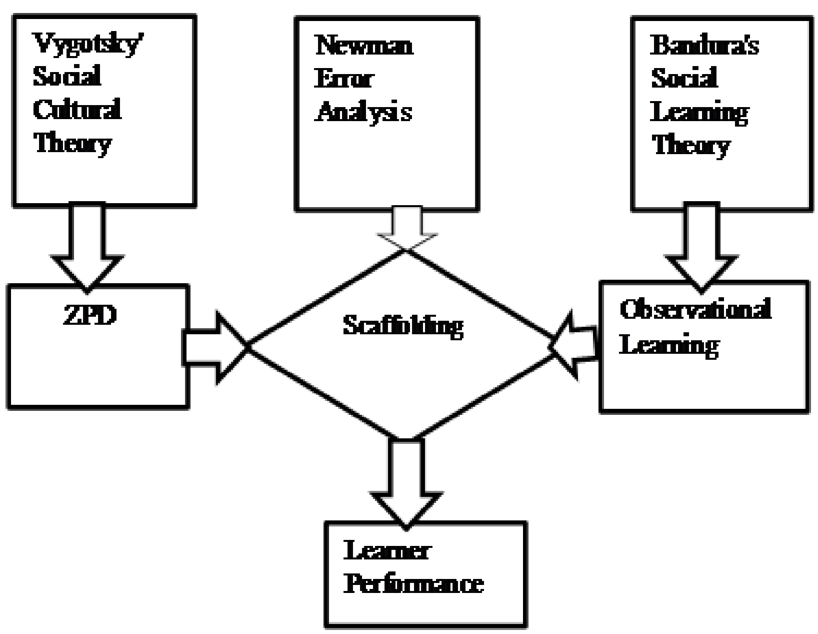
CHAPTER 2: CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

Chapter 1 discussed this study's background, including the problem statement, the purpose of the study, aims and objectives and a summary of the chapters. This chapter discusses the conceptual framework underpinning the study.

A conceptual framework guides a researcher in his/her investigation by specifying important variables to be studied (Miles, Huberman and Saldana, 2014), as cited in Ngulube (2020, p.28). When researchers use some aspects of a theory in their investigation, the result is a conceptual framework (Ngulube, 2020, p.29). Vygotsky's sociocultural theory (SCT), particularly the zone of proximal development (ZPD) and scaffolding in conjunction with Bandura's observational learning theory and Newman's error analysis guided this research. This study's conceptual framework is as shown the figure below.

Figure 2.1: ***The conceptual framework for the study***



2.2 VYGOTSKIAN SOCIOCULTURAL THEORY

The Vygotskian theory of learning hypothesised that people develop mentally by interacting socially in their environment (Alkhudiry, 2022, p.2117) and mediated by signs and tools (=, ÷, $\times, -, +$) (Abtahi, 2021, p.18). In this study, the signs, as envisaged by Vygotsky, play a great role as learners learn algebra, and this is elaborated on in the next chapter. Vygotsky proposed that the first phase of a child's cultural development takes place between the people as the child interacts with adults and later within the child him/herself mediated by signs (Abtahi, 2021, p.18). According to constructivism, assimilation, accommodation, and equilibration are the basic mechanisms for learning and development (Wahyudi, Suyitno & Isnarto, 2020, p.33). When a learner is able to incorporate a new experience into the one s/he already has, that is assimilation, whereas accommodation is the process whereby the learner has to restructure their current knowledge so that new information can fit better. In most cases, learners develop misconceptions during the process of assimilation.

Disequilibrium is a contradiction between what the learner already knows and what s/he is learning. This would force the learner to look for ways to strike a balance in their conceptual understanding. Cognitive constructivism may explain why learners develop misconceptions because for learning to occur, learners need to create or construct their own knowledge (Kshetree, Acharya, Khanal, Panthi & Belbase, 2021, p.1101). Constructivists believe that even the most articulate verbalising and explanation cannot transmit knowledge directly from teacher to learner. The teacher's role is to help the learner by providing mental and physical models that s/he can use to abstract mathematical meaning.

According to Vygotsky's theory, historical and sociocultural conditions determine what the individual should learn and how they should learn it. Vygotsky further argues that children acquire their knowledge through induction to the cultural knowledge that society has accumulated over several centuries. Thus, sociocultural theorists assert that human beings are the same substance as the culture in which they live (Radford, 2016, p.2). That is to say, people's culture is not just a stimulus to which they should adapt; rather, culture affects how people think, behave, interact and feel (ibid p. 2). However, what the child learns from society is not a copy of what society has given him/her; rather, it would be the child's personal meaning of what they have learnt. Vygotsky asserts that a child's learning is assisted through tools, signs and cultural objects and by a more knowledgeable other (MKO) who may be an adult or a peer. In this study, the tools for learning are the erroneous examples, and the MKO is the teacher. Vygotsky named the stage where a child requires assistance while learning the "zone of proximal development" (ZPD).

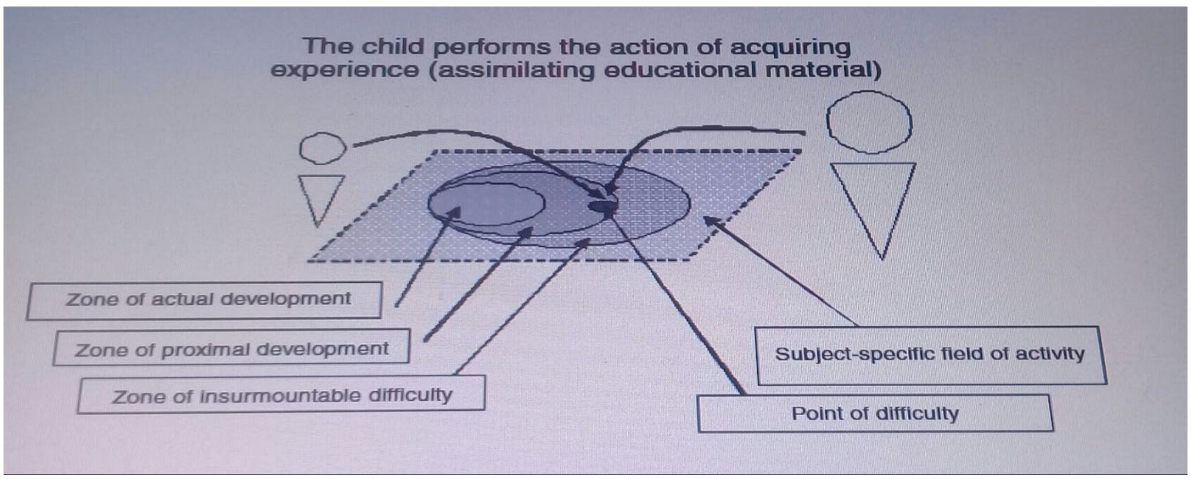
2.2.1 Zone of proximal development

Vygotsky asserts that all learning begins when learners fail to solve problems independently, and this shows the learners' ZPD. According to Qin (2022, p.139), the ZPD is the distance between the child's actual level of development, determined by their ability to solve problems independently and the level of possible development, detrained by their ability to solve tasks under the guidance of adults or in cooperation with the more knowledgeable peers. However, Margolis (2020, p.17) describes the ZPD as the type of assistance the teacher provides learners to solve tasks they cannot solve alone. The ZPD, defined by Qin (2022, p.139), is the one adopted for this study.

Zaretsky (2021, p.41) contends that the range of activities that learners are able to do with help from adults or in collaboration with them is the learner's ZPD. The ZPD is the zone of potential development of the learner. Thus, the range of activities that learners are capable of doing independently defines their zone of actual development. Zaretsky (2021, p.43) further asserts that the fine line separating the actual zone of development (ZAD) and the ZPD is the first difficult activity the learner fails to do independently, that is the point at which they need an adult's help or help from a more knowledgeable learner. The ZPD comprises those actions that learners understand but are not able to execute, that is, the zone within which learners act with understanding and awareness when they receive help from an adult (ibid, p.43). This implies that if learners cannot interact meaningfully when solving a task, any help from an adult would not be successful. I, therefore, propose that it is in their ZPD that learners develop misconceptions as they attempt to accommodate what they are learning in line with their prior knowledge. They create their own understanding of the concept they are studying. Zaretsky (2022, p.41) asserts that teachers must determine learners' ZPD. Teachers can achieve this by analysing learners' written or spoken responses to questions. If the learner's ZPD is accurately determined, the learner's prior knowledge is established, enabling the teacher to design and implement appropriate scaffolding. However, the teacher needs to have the requisite knowledge for him/her to identify effectively learners' ZPD and design appropriate intervention strategies. Thus, the teacher has to have requisite knowledge of content (CK) and pedagogical content knowledge (PCK), mathematics pedagogical content knowledge (MPCK) and other relevant skills to help learners migrate in their ZPD. These requisite skills are explained in the next chapter.

By analysing learners' responses in the test items, the researcher managed to ascertain the learners' zones of actual development and their respective ZPDs. Vygotsky's concept of the ZPD allowed the researcher to design intervention strategies to assist learners in overcoming their errors and misconceptions when solving algebraic linear equations.

Figure 2.2: ***Zone of Proximal Development*** (Adapted from Zaretsky, 2021, p.41)



The ZPD is dynamic, that is, it does not disappear, and its borders shift, resulting from the expanding actual zone of development, while at the same time, the ZPD itself expands. Cognitive theorists contend that development only occurs when learners confront difficulty that is when they fail to manage an activity independently. At this stage, a need arises to get a means of action and an adult or peer is that source of help. The ZPD is important because, without it, scaffolding is impossible (Makgakga, 2016, p.38). The ZPD played a fundamental role in phase 1 of this study. It assisted in determining the kind of activities to implement to help (scaffolding) learners overcome their errors and misconceptions.

2.2.2 Scaffolding

According Darlind-Hammond, Flook, Cook-Harvey, Barron and Osher (2020, p.97), for learners to progress from one level of development to the other in learning, they need assistance and guidance to independently solve problems. Cho et al. (2020, p.275) describe scaffolding as a mechanism for assisting learners accomplish that which they cannot achieve on their own. Additionally, Krishnan (2019, p. 809) states that scaffolding is a process by which learners can solve a problem and achieve a task they would not solve without assistance. Krishnan (2019, p.811) further contends that those activities that move learners progressively towards higher levels of understanding and greater independence in the learning process constitute scaffolding. However, Cho et al. (2020, p.275) describe scaffolding as a temporary, intentional, just-in-time support that helps learners move towards new skills, concepts or levels of understanding. This means that teachers withdraw scaffolding as soon as the learner shows some degree of freedom in carrying out tasks. Thus, the teacher's objective when scaffolding allows learners to achieve independence and self-regulation and become problem-solvers, the teacher should gradually withdraw scaffolding (Makgakga, 2016, p.36).

In this study, the researcher used erroneous and correct worked examples as a scaffolding strategy to enable struggling Grade 9 learners to solve algebraic linear equations and extend their zone of actual development while extending their ZPD. The implementation of scaffolding can take various forms, of which giving tasks is one of them. The following are possible ways of effecting scaffolding.

1. Assessing learners' prior knowledge of the task at hand,
2. Stimulating the learners' interest in the task at hand,
3. Simplifying tasks into manageable activities for learners,
4. Giving learners some direction for achieving their goals,
5. Clearly identifying the differences between the learners' work and the standard or desired solution,
6. Reducing the learners' frustration during algebra lessons, and
7. Clearly defining and modelling the task's expectations learners should form.

The figure below shows how to implement scaffolding (Rahma et al., 2020, p.55).

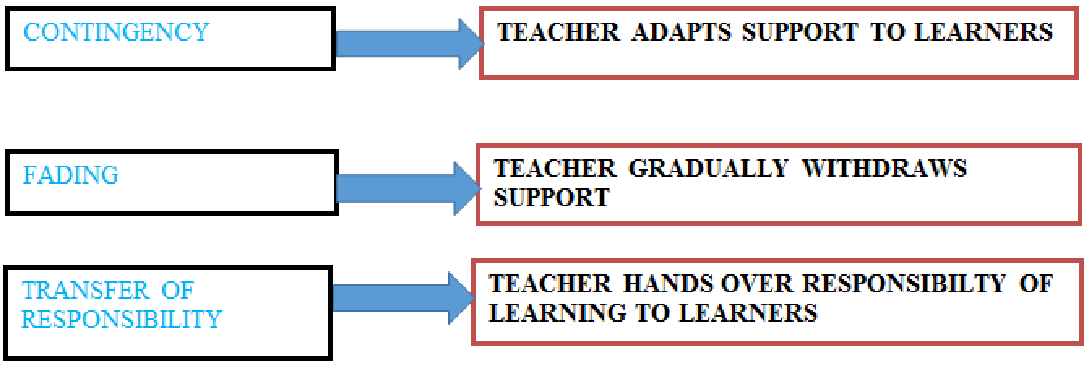


Figure 2.3 Scaffolding (Adapted from Rahmat et al. 2020)

After the pre-test, the TEG discusses learners' errors and misconceptions with the whole class (CONTINGENCY). Learners then worked in pairs or small groups (FADING) before and during (TRANSFER OF RESPONSIBILITY) after the intervention exercise, learners showed fewer or no errors and misconceptions in solving algebraic linear equations.

2.3 BANDURA'S OBSERVATIONAL LEARNING THEORY

Observational learning is the cornerstone of Bandura's (1986) Social Learning Theory. According to Bandura, learning occurs through the interplay of cognitive and social processes: learners develop new skills through observation. Panganiban and San Miguel (2023, p.212) state that observational learning is when someone acquires a skill or response by observing other people's behaviour or actions. Ramsey et al. (2021, p. 478) describe observational learning as any instance in which someone learns a new skill by observing or modifying a previously learned skill. Learning through observation is beneficial, especially when active involvement is impossible (Ramsey et al., 2021, p.478). According to Ahn, Hu and Vega (2020, p.1), observational learning has four (4) main tenets: attention, retention, replication and motivation. Paying attention is crucial in any learning situation, and learners need to pay attention during lessons.

Additionally, learners should stay motivated during the lesson to retain what they learn. However, He (2022) argues that selective attention is key in observational learning and determines what learners observe and what they will learn. Furthermore, the characteristics of the object of observation are also important in that learners may have an interest in observing novel characteristics or those similar to themselves (ibid). Observational learning played an important role in this study and helped the teacher and learners at the experimental school observe the researcher, demonstrating how to use erroneous examples to help learners overcome their errors and misconceptions when solving algebraic linear equations.

This study focused on finding out if learners would benefit from erroneous examples. The researcher was the model for the TEG, while the erroneous examples were the models for the LEG. Learning from models is similar to learning from examples. Examples show all the process steps required to arrive at the solution of a problem. However, the examples used should appeal to learners' interest if they are to remain focused.

2.4 NEWMAN ERROR ANALYSIS

Newman's error analysis is a comprehensive theory that categorises learners' errors when solving problems. According to Herawati and Marfuah (2021, p.3240), Newman's error analysis comprises five (5) categories: Reading error, Comprehension error, Transformation error, Processing error and Encoding error. The types of errors and their indicators are shown in Table 2.1 below.

Error type	Indicator
Reading error	1. Learner cannot read the key words or symbols in the question
Comprehension error	1. Learner can read but is unable to understand the meanings of words. 2. Learner does not understand fully what is being asked.
Transformation error	1. Learner understands what is known in the question, but fail to identify the required operations, or apply wrong order of operations. 2. Learner does not know the formulas used to solve the problem. 3. Learner does not know the arithmetic operations to use in the problem. 4. Learner is unable to make a model or formula from the given information in the problem.

Processing skill error	1.	Learner knows correct operations, or order of operations, but cannot carry out the correct procedures.
	2.	Learner does not understand the procedures for solving the problem.
	3.	Learner is unable implement steps correctly.
Encoding error	1.	Learner can get the correct answer but fails to write the answer in the correct form.
	2.	Learner fails to get the correct answer using the steps used.
	3.	Learner fails to write the final answer after solving the problem.
	4.	Learner fails to use the correct units required in the final answer.

Table 2.1: ***Newman's Hierarchy of Mathematical Task Execution***

Using Newman's error analysis, the researcher categorised learners' errors and analysed them to determine their frequency and possible causes. The error analysis results helped in the design of the intervention targeting the errors.

2.5 CONCLUSION

This chapter discussed the notion of a conceptual framework and its relevance to the study. Additionally, Vygotsky's sociocultural learning theory, the ZPD and scaffolding, Bandura's observational learning theory, and their relevance were discussed and explained. Finally, Newman's error analysis and its application in categorising learners' errors in test items were also discussed. The review of related literature is presented in the following chapter.

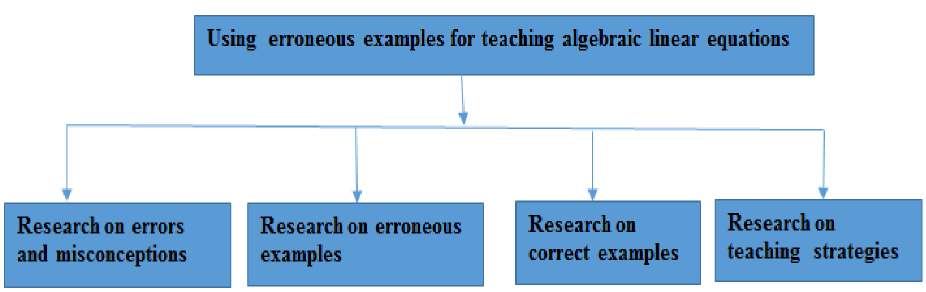
CHAPTER 3: LITERATURE REVIEW

3.1 INTRODUCTION

A literature review is a means by which the researchers identify the knowledge relevant to a particular field of study (Paul & Criado, 2020, p.1). The (ibid) further contend that a literature review clarifies the current information, explains the implications of the problem under analysis, links theory and practice, highlights gaps in current literature and places the study within the field's research agenda. Given the above, this chapter will locate this study compared to previous research. The previous chapter discussed this study's conceptual framework. It explained the sociocultural theory of Vygotsky (i.e., the ZPD and scaffolding), and their relevance to this study. The chapter also explained Bandura's observational learning in alleviating learners' errors and misconceptions, and finally, how Newman's error analysis assisted in categorising learners' errors and designing the intervention for this study. This chapter discusses a literature review guided by the themes formulated from the research objectives, informed by the problem of the study. Figure 3.1 below shows the literature review map followed by the study.

This study is focused on learners' errors and misconceptions, and as such, the researcher will first define errors and misconceptions in mathematics generally and algebra in particular. The researcher will also explain the types of errors and their causes. Furthermore, the researcher will discuss the pedagogical approaches for teaching mathematics generally and Grade 9 algebra in particular. Finally, it suggests possible strategies for alleviating learners' errors and misconceptions in solving Grade 9 algebraic linear equations.

Figure 3.1: ***Literature review map***



3.2 TYPES OF ERRORS IN MATHEMATICS

As a teacher of mathematics for several years, I know that learners give wrong answers to questions daily, be it in class exercises, tests, assignments or homework. While answering questions, learners exhibit different types of errors for various reasons. According to Pan, Sana, Samani, Cooke and Kim (2020, p.1105), an error is a mismatched fact or process for any standard. Mulungye, O'Connor and Ndethiu (2016, p.31), describe an error as a mistake or discrepancy from the truth when applying a procedure to solve a mathematical problem. Errors may be due to learners' failure to see the relationship between what they already know and what they are learning (Enu & Ngcobo, 2020, p.48). Mathaba and Bayaga (2019, p.26) state that learners who fail to connect what they already know and the new concept studied or cannot use algebraic rules commit many errors. Most learners overgeneralise what they learned in arithmetic when studying algebra (Aliustaoglu, Tuna & Bider, 2018, p.591). Concurring with Aliustaoglu at al. (2018), Mulungye, O'Connor and Ndethiu (2016, p.31) state that the major factor behind learners' errors is that they overgeneralise, oversimplify and overspecialise algebraic equations and/or expressions. As an example of an overgeneralisation, when learners are asked to solve for the equation, $(x + 5)(x + 6) = 20$ learners commit this error, $x + 5 = 20$ or $x + 6 = 20$ leading to getting wrong answers of $x = 15\ or\ x = 14$. The rule that the learner used here is when the right-hand side equals zero. This rule applies to the equation, $(x + 5)(x + 6) = 0$, of which the answers are $x = -5\ or\ x = -6$.

Makonye and Khanyile (2014, p.68), argue that there are three categories of learners' errors namely, errors, random errors and careless errors. Errors that result from a lack of complete understanding of the basic information about a concept is a systematic error (ibid). In other words, systematic errors are learned errors. Learners take time constructing conceptions, which they think best represent what they are learning and as such, systematic errors are indicators of misconceptions (Makonye & Khanyile, 2014, p.68). Learners tend to commit the same type of error in a similar situation and this shows that the learner learned the error (Barbieri, Miller-Cotto & Booth, 2019, p.381). Systematic errors are methodically constructed errors, they are wrong answers that learners produce over time. Systematic errors indicate a flawed line of thinking and some researchers call them misconceptions (Luneta & Makonye, 2010, p.36). Errors are visible and we find them in learners' written work and speech. Luneta and Makonye (2010, p.36) argue that systematic errors occur when learners attempt to get the answer to the problem "$5x - 12x$" and give the answer as $7x$ instead of $-7x$. Such responses result from prior knowledge in lower grades, where learners learned that they could not subtract bigger numbers from smaller ones. Since no borrowing is possible, the learner resorts to subtracting the smaller number from the bigger one but fails to put the correct sign to the answer.

On the other hand, careless errors are no repeated wrong answers, which are easily corrected by learners on their own. Carelessness errors commonly result when learners carelessly provide answers without applying reason (Makonye & Khanyile, 2014, p.68), i.e., when learners simplify the expression like $15x - 3x$ and get the answer as $2x$. However, learners can easily correct this error when revising their work. The individual can easily correct careless or random errors. Learners could make errors for many reasons, possibly due to an error in transformation or over- or under generalisation, among others (Hansen, Drews, Dudgeon, Lawton & Surtees, 2020, p.1).

Transformation errors are due to learners' failure to observe the rules. For instance, learners may incorrectly apply the order of operations (BODMAS) when simplifying an expression involving two or more operations. The study by Makonye and Hantibi (2014, p.1565) observed that learners confused the multiplication of negative numbers with subtraction. For instance, $-4 \times -8 = -12$, and failure to observe the order of operations results in getting answers like $2 - 2 \times 4 = 0$.

Interference of new knowledge may also act as a source of errors. In the case of learning how to simplify algebraic expressions immediately after learning the laws of exponents (Gumpo, 2014, p.15), learners may apply the laws of exponents inappropriately when simplifying expressions lik$e$ $2y + 4y = 8y^2$ instead of $6y$. Parwati and Suharta (2020, p. 104) categorised errors into technical and substantial. An example of a technical error is making a wrong calculation or misusing an algorithm. However, learners may easily correct such errors when they re-check their work. Substantial errors result from no concepts or insufficient understanding of symbols, incorrect application of processes and even inability to plan how to solve the problem (Parwati & Suharta, 2020, p.104).

3.3 LEARNERS' MISCONCEPTIONS IN MATHEMATICS

Researchers use the term 'misconception' when describing learners' prior knowledge that contradicts the core concepts of a discipline (Verkade et al., 2017, p.4). The (ibid) further contend that misconceptions cause systematic errors, which impede learning. Since misconceptions impede the acquisition of new knowledge, teachers must pay special attention to learners' errors and address the misconceptions early to enable effective learning of new concepts. When learners strongly believe their conception is correct, correcting that misconception by standard means of instruction is difficult. Parwati and Suharta (2020, p. 104) describe a misconception as a consistent conception error used in solving different problems. For instance, a learner may get the correct answer to $(3^2)^2 = 3^4$ and get a wrong answer to $(3^2)^3 = 3^5$. From the above example, it is clear that the learner accidentally got the correct answer to the first by adding exponents, and this was difficult to detect had the teacher not given the second question.

Verkade et al. (2017, p.4) classified misconceptions as factual and ontological. Factual misconceptions are beliefs about incorrect information that one develops through environmental interaction. The basis of such information may be family members, friends or even the teachers. Through personal experience, learners develop ontological misconceptions. Ontological misconceptions are common-sense beliefs resulting from an individual's subjective experience of trying to understand the world around them when there is no fundamental knowledge from formal education (Verkade et al., 2017, p.5). According to Soeharto, Csapo, Sarimanah, Dewi and Sabri (2019, p.248), some misconceptions are more resistant to correction than others are.

How learners understand the equal sign, negative numbers and variables is important in solving algebraic equations (Booth, McGinn, Barbieri & Young, 2017, p.64). Most learners understand an equal sign to mean that the answer comes next. This understanding results in learners failing to complete the number sentences like 4+6 = ___+ 7, with most learners writing 10 or 17 instead of 3. The negative sign is another source of learners' errors because of its abstract nature (Booth et al., 2017, p.64). Most learners believe the negative sign is an operation for subtraction and face difficulties understanding negative numbers (i.e.,-9).

Regarding variables, learners' most common misconception is that the letter in a number sentence represents a real object or a label (Rich, Franklin, Strickland, Isaacs & Eatinger, 2022, p.213). This misconception is in the classic error example of the 'Student and Professor' problem in which learners are asked to write a number sentence representing the phrase, 'six times as many students as professors' most learners wrote $6s = p$. Accordingly, learners believed that the letter s is the label for students instead of a variable representing the number of students. Booth et al. (2017) further argue that some learners ignore the variable. For example, when asked to solve $(x + 7) + 8$, learners write 15. Another source of error is learners' knowledge of mixed numbers. This confuses when working with algebra. For instance, learners may write $x + 7 = 7x$, $thinking\ what\ is\ done\ in\ 6\ plus\ 1\ half\ equals; 6 + \frac{1}{2} = 6\frac{1}{2}$ also applies in algebra.

Conjoining is another source of errors, in which case learners add variables and constants. In their study, Alshwaikh and Adler (2017) found that learners added variables and constants when simplifying algebraic expressions like $2x + 4$ as $6x$. Conjoining is a common error exhibited by learners in Grades 8 and 9 and can persist into higher grades if not properly addressed. The (ibid) argue that conjoining is an error showing that a learner does not understand the basic algebraic principles. Teachers should know the errors and misconceptions of learners so that they can plan proper interventions targeting the roots of these misconceptions (Verkade et al., 2017, p.7). To achieve this important role, the teacher should have the requisite knowledge for teaching and content: knowledge of subject content (SCK) and knowledge of teaching (PCK). A full discussion of PCK and SCK is in section 3.6.

3.4 RESEARCH ON ERRONEOUS EXAMPLES

Several studies have been conducted on errors in algebra and algebraic equations (e.g., Rushton, 2018; Enu & Ngcobo, 2020). An empirical study in the United States of America (USA) observed that learners who compared incorrect examples with correct examples when solving equations outperformed their counterparts who only studied proper examples (Durkin & Rittle-Johnson, 2012, p.207). In another study in Kenya by Mulungye, O'Connor and Ndethiu (2016, p.31), it was found that when teachers engage purposively with learners' algebraic ideas, they create rapport in the classroom. These researchers observed that many teachers could not utilise learners' mathematical ideas to help them overcome misconceptions when solving equations. Metcalfe

(2018, p.468), in a review, found that Japanese learners outperform others in international tests because, as a norm, they engage with errors and misconceptions before formal instruction. However, most of the earlier studies on the use of erroneous examples for teaching took place in Europe and the USA, and there is little evidence showing the use of erroneous examples as a teaching strategy in SA schools. Thus, after reading about some success stories in Japan on the effects of using errors to promote academic performance, the researcher explored the effectiveness of erroneous examples for teaching algebraic linear equations to improve academic performance in SA schools.

According to Richey, Miguel, Bray, Mogessie, Scruggs, Andres, Star, Baker and McLaren (2019, p. 173), incorporating incorrect examples can facilitate learners' cognitive development. An erroneous example is typically a worked example with one or more steps intentionally made wrong. Erroneous examples may be the way to address learners' misconceptions in solving algebraic equations. Additionally, studying erroneous examples may motivate learners who might be unwilling to participate in class for fear of making errors. When such learners see errors presumably made by others, they may feel that they are not the only ones struggling with a particular topic. Also, seeing errors and knowing how they arise and how to correct them may foster conceptual understanding when learners have misconceptions.

Rushton (2018) argues that when learners study the hypothetical errors of others, they reflect on their errors, which helps them reconstruct and correct them. The above results contradict the behaviour theory that showing learners' incorrect examples strengthens the existing misconception (Richey et al., 2019, p.174). Moreover, studying erroneous examples might enable learners to think about their own thinking (metacognition) when examining why erroneous examples might be wrong. Thus, studying erroneous examples can assist learners to revise and correct their understanding of concepts (Richey et al., 2019, p. 174).

However, some studies have found that when learners study erroneous examples they get confused and frustrated at first but later learn better from their confusion and frustration (van Peppen, Verkoeijen & Heijltjes, 2021, p.750). Additionally, not all learners might be able to overcome their initial frustration and confusion when studying erroneous examples (Richey, 2019, p.176). In phase two of this study, the TEG was intentionally planting errors in examples or use learners' errors to activate discussion, promote analysing skills and correct misconceptions.

3.5 RESEARCH ON WORKED/CORRECT EXAMPLES

From time immemorial, teachers have used work examples for teaching (Sweller, van Merrienboer & Paas, 2019, p.265). In mathematics, a fully worked-out problem or task showing all the necessary steps leading to the required solution is a worked example. This study uses the term **correct example** instead of a **worked example.** The proponents of the worked example effect (Sweller et al., 2019, p.265) discovered that learners who studied worked examples performed better in post-tests than learners who practised solving similar problems. The authors suggest that worked examples reduce the extraneous working load, improving working memory capacity. However, some studies (e.g. Kalygua, Chandler, Tuovinen & Sweller, 2001, p.580) found that most worked examples do not provide reasons for each step; they need good structuring to be more effective. According to Sweller et al. (2019, p.265), correct examples may not be effective for learners with better prior knowledge. The authors further assert that designing good work examples is challenging because they should not integrate several concepts into one example.

Several researchers have demonstrated that learning from correct examples is applicable across many domains, from mathematics to visual arts, and is more effective than conventional problem solving when learning novel materials (Retnowati, Ryres & Sweller, 2017, p.2). Chi (2018, p.261) concurs that worked examples are the primary resources that textbook writers use to teach learners how to solve problems. Zhu and Simon (1987) showed that when correct examples are used for teaching and learning, a 3-year mathematics course could be covered in 2 years. Laboratory studies have shown that learners prefer studying correct examples (Chi, 2018, p.251). Learners commit to memory knowledge they obtain from worked examples. It is from work examples that most learners acquire problem-solving skills. However, laboratory studies showed that learners who studied worked examples often fail to solve problems that deviate slightly from the original example solution (Chi, 2018, p.261). Citing Sweller and Cooper (1985), Chi (2018, p.261) argues that learners who studied correct examples outperform learners who do not learn worked examples on similar questions; however, there is no significant difference in post-test results when learners solve dissimilar questions.

Research has shown that correct examples only give learners algorithms to study and follow in similar cases without acquiring a deeper understanding of concepts (van Peppen et al., 2020, p.749). The (ibid) observed that learners who studied worked examples could not solve problems that differed slightly from those studied. The above suggests that correct examples fail to provide enough rationale for applying the steps, making it difficult to transfer knowledge acquired from studying examples to other situations (Chi, 2018, p.262). For correct examples to be more effective, learners should be prompted to self-explain steps in the worked example (Chi 2018, p.265). Instead of trying to obtain new information through problem search and discovery, textbooks provide learners with work examples to study (Retnowati et al., 2017, p.666). Accordingly, worked examples allow learners to concentrate on the solution steps rather than the problem (van Peppen et al., 2020, p.749).

Loibl and Leuders (2019, p.1) observed that learners who studied worked examples outperformed their counterparts who solved practice problems in post-tests. The (ibid) state that worked examples increased learners' working memory by reducing extraneous cognitive load. Extraneous load is presentation material, which has nothing to do with the learning principles, and this takes up memory space needed for effective learning. Learners who engaged in problem solving experienced a working memory load, reducing their working memory. Reducing cognitive load enabled learners to transfer knowledge to long-term memory (Retnowati et al., 2017, p.668). The instructional guidance in worked examples allows learners to pay attention to the problem solving principles and their applications (van Peppen et al., 2020, p.749). However, these researchers observed that not all learners use their freed memory to focus on applying a worked example in unfamiliar situations since they process worked examples superficially, thereby limiting their effect.

3.6 PROFICIENCY IN MATHEMATICS

Mathematical proficiency comprises five interdependent and connected strands of understanding (Kilpatrick, Swafford & Findell, 2001, p.5). They are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (ibid,). These strands are explained below.

3.6.1 Conceptual understanding

When a learner is able to understand the operations used in mathematics and how they are related, the learner has conceptual understanding (Kilpatrick et al., 2001, p.5). Conceptual understanding links the network of ideas and information, allowing the individual to notice how knowledge is related and connected (Hiebert & Lefevre, 1986, p.12). Accordingly, when solving algebraic linear equations, learners should clearly understand how different chunks of knowledge are related and connected. Equipped with conceptual knowledge, learners should be able to navigate back and forth along mathematical perspectives (Brezovszky, et al., 2019, p.63). Implicit in this statement is that when learners fully understand concepts, they can use their prior knowledge to make connections with the current knowledge they would be learning. Conceptual understanding helps learners logically organise knowledge and make connections between what they know already and new ideas and (ibid).

3.6.2 Procedural fluency

Procedural fluency is when a learner can demonstrate his/her ability to effectively, correctly, and properly execute all the steps in solving a mathematical problem (Kilpatrick et al., 2001, p.5). Garg (2017, p.1) asserts that the knowledge and application of techniques and guidelines for doing mathematical activities constitute procedural fluency. When a learner is able to solve problems by applying reasoning and understanding concepts developed through active engagement in mathematics, the learner has procedural fluency (Al-Mutawah, Thomas, Eid, Mahmoud & Fateel, 2019, p.258). The (ibid) posit that learners may not make sense of mathematics by inactively listening to how other learners make sense of mathematics, suggesting that learners should be actively involved in doing mathematics to learn mathematics. Asking learners to identify errors in erroneous examples and letting learners find correct ways of solving the problems may be one way of making learners actively engaged in learning.

3.6.3 Strategic competence

A learner's ability to design a mathematical strategy for solving mathematical challenging situations measures their strategic competence (Kilpatrick et al., 2001, p.124). Equipped with strategic competence, a learner can decide on his/her strategy for solving a mathematical challenging situation (Garg, 2017, p.1). In concurrence, (Hayati & Kamid, 2019, p.118) assert that learners demonstrate strategic competence when they choose the best approach that enables them to solve the problem.

3.6.4 Adaptive reasoning

According to Kilpatrick et al. (2001, p. 5), when a learner is able to reason, reflect, and describe situations he/she possesses adaptive reasoning. Through adaptive reasoning, learners reflect on and evaluate their own work (Garg, 2017, p.1). Adaptive reasoning can help learners to connect their prior knowledge and the current knowledge (Kilpatrick et al., 2001, p.129).

3.6.5 Productive disposition

When learners take time to think and reason about a suitable strategy for solving a problem, they demonstrate a productive disposition (Garg, 2017, p.1). Accordingly, Kilpatrick et al. (2001, p.5) suggest that effective disposition helps learners make practical sense and value of mathematics learning. Learners with productive disposition persevere when faced with challenging mathematical situations and believe in their efficacy (Kilpatrick et al., 2001, p.129).

3.7 TEACHERS' KNOWLEDGE FOR TEACHING MATHEMATICS

Teachers have an important role of interpreting and implementing curriculum programs. However, the decisions they make and the applications they use depend on the knowledge they possess. Shulman (1986, p.8), proposed that for teachers to effectively execute their role they should possess three fundamental components of knowledge, namely, knowledge of subject content (CK), knowledge of teaching methods (PCK) and curriculum knowledge (CK). Although these components are separate, they are related. Content knowledge is the awareness of the core mathematical concepts and operations and their relation. On the other hand, PCK includes knowledge and methods necessary to make learners understand mathematical concepts. Thus, PCK comprises knowledge of the learner, unique teaching methods and approaches for measurement and assessment. Knowledge of the learner requires the teacher to understand how learners think about their learning challenges and be aware of what learners already know and the possible misconceptions they may have (Booth et al., 2017).

The teaching of content, on the other hand, involves many, among other things, examples, analogies and presentations that a teacher uses to make content meaningful and accessible to learners. To do this, the teacher has to have the requisite skills and knowledge, which include content knowledge (CK), mathematical content knowledge (MCK), specialised content knowledge (SCK) and school related content knowledge (SRCK), to mention just a few. The following section deals with these concepts.

3.7.1 Importance of teachers' mathematics content knowledge.

Dreher, Lindmeier, Heinze and Neim (2018, p.319) argue that a mathematics teacher should possess a general knowledge of mathematics that would allow him/her to communicate with other mathematicians. However, s/he should possess specific knowledge in specific fields of mathematics, like what engineers or astronomers do. Sepeng (2014), as cited in Makgakga (2016, p.44), argues that a mathematics teacher should understand subject content knowledge well for effective teaching and learning. According to Dreher et al. (2018, p.326), content knowledge (CK) is subject matter knowledge that does not include how that subject matter is taught. Furthermore, CK is the body of knowledge, including the information taught and what learners should learn in that particular content area. CK generally refers to the facets, concepts, theories, and principles taught and learned in specific academic subjects or school courses (ibid, p.16). Shulman defines CK as the amount of knowledge in the teacher's mind and how it is organised. The (ibid) further asserts that CK is knowledge that includes the facets and concepts of a particular discipline. Jacob, John and Gwany (2020, p.16) state that CK represents the teachers' understanding of the subject matter. Jacob et al. (2020, p.16) further assert that CK includes both the teachers of what to teach – *"knowing what"* and procedural knowledge – "*knowing why*" of a discipline. From the various definitions of content knowledge, it is apparent that mathematics teachers should understand the nature of knowledge: the epistemological and ontological aspects of the subject they teach. Teachers should demonstrate that they can use various teaching methods and profoundly appreciate their subject matter. The researcher and the TEG shared knowledge of using erroneous correct examples to help learners reduce/eliminate errors and misconceptions in solving algebraic linear equations.

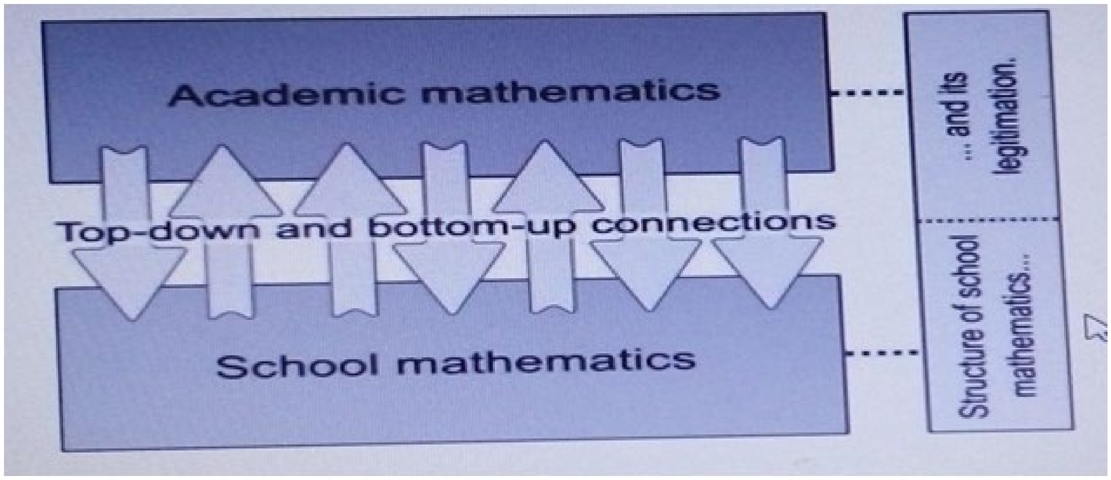
According to Carrillo-Yanez et al. (2018, p.338), teachers should possess specialised content knowledge (SCK) for them to teach effectively. According to Livy and Vale (2011, p 23), SCK is a subgroup of knowledge of mathematics content. Analysing content knowledge for mathematics, Ball, Thames and Phelps (2008, p.399) identified three components namely, common content knowledge (CCK), specialised content knowledge (SCK), and horizon content knowledge (HCK). The (ibid) further demonstrates that specialised content knowledge is the unique mathematical content the teacher possesses for his/her daily teaching duties.

In contrast, common content knowledge is any adult's knowledge (Ball, Thames & Phelps, p.399). Accordingly, these researchers suggest that for a mathematics teacher to be more effective in his/her teaching responsibilities, s/he should have a firm background of mathematical knowledge over and above that of an ordinary adult. Teachers with good content and knowledge can demonstrate to learners how mathematical concepts are connected (Livy & Vale, 2011, p.23). Showing mathematics connections when using erroneous and correct examples while teaching how to solve algebraic linear equations is important as this will enable learners to reduce or eliminate errors and misconceptions.

Dreher et al. (2018, p.328) argue that mathematics teachers should be able to simplify academic mathematics into school mathematics by organising academic mathematics in a way learners can easily understand by learners. Dreher et al. (2018) further argue that since teachers use textbooks for teaching content, they should be able to verify that textbooks' contents are easy for learners to understand. Teachers should know how to simplify definitions in school textbooks to match learners' levels of understanding (Ball & Bass, 2003).

Furthermore, secondary school teachers should possess specific mathematics content knowledge that would allow them to connect academic mathematics with school mathematics. This special kind of mathematics is called school related content knowledge (SRCK) (Dreher, 2018, p.329). SRCK is special mathematics content knowledge that secondary school mathematics teachers should possess which comprises knowledge about school mathematics and academic mathematics. There is a blending between SRCK and pedagogical knowledge. Hence, knowing learners' misconceptions is unnecessary (Dreher et al., p.330).

Figure 3.2: ***Overview of school related content knowledge*** (Adopted from Dreher 2018)



As the figure above shows, SRCK has three facets: academic knowledge, bottom-up and top-down connections, and school mathematics. Dreher et al. (2018, p.330) contend that teachers must know which mathematics concepts should learners understand in one level for use in the next level. For instance, which aspects of algebra should learners fully understand when introduced so that they will understand the concepts of abstract algebra they will study in future (p.331). In the bottom-up direction, teachers' knowledge of how academic mathematics and school mathematics are intertwined encompasses the teachers' ability to perceive mathematical ideas in learners' remarks. This includes the teachers' interpretations of learners' definitions, proofs and theorems as reflected in learners' marks. The researcher observed how teachers teach algebraic linear equations and unpack the concept of algebraic linear equations. The researcher also demonstrated how using erroneous and correct examples can help learners reduce/eliminate errors and misconceptions about solving algebraic linear equations.

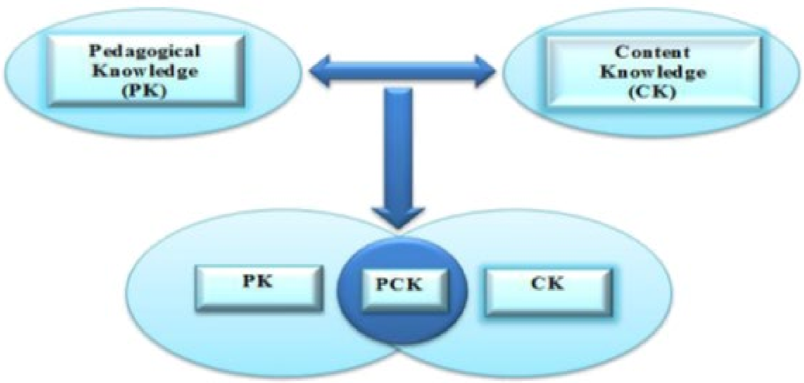
3.7.2 The importance of pedagogical knowledge (PK) of mathematics teachers

Scholars do not seem to agree on what constitutes pedagogical content knowledge (PK). Shulman (1986, p.8) proposed that PK involve teaching precepts and strategies in classroom management and organisation. It includes knowledge of learners by the teacher, knowledge of assessment and the purpose of education. Jacob, John and Gwany (2020, p.16) define PK as the common knowledge about pedagogy, teaching approaches, how learners learn, methods of assessment and the knowledge of the various theories about learning. Shulman (1986, p.8) proposed that PK is any theory or belief about teaching and the learning processes a teacher may possess and influence the teaching. Jacob et al. (2020, p.16) further argue that PK is the knowledge of how to teach. PK also includes the teacher’s knowledge of teaching methods, learners' learning, and assessment techniques.

3.7.3 The importance of the mathematics teacher’s PCK

The figure below depicts the relationship between the PCK, PK and CK as applied in mathematics teaching.

Figure 3.3: ***Overview of PCK*** (Adopted from Jacobs et al., 2020)



The teacher’s way of how s/he presents and formulates the subject matter knowledge to make learning possible is PCK (Shulman, 1986. p.8). Thus, it is the teacher’s ability to simplify content to the level of understanding of learners. It also includes the teacher’s understanding of how topics and strategies in a particular subject matter are understood or misunderstood by learners. It is a unique knowledge that bundles content knowledge with knowledge of learners, learning and pedagogy (Jacob et al., 2020, p.19). PCK links pedagogical and content knowledge. PCK includes the teacher’s knowledge of how learners learn specific content, how learners know what they learn, how they think and what teaching strategies and curriculum resources can be utilised to teach content (Jacob et al., 2020, p.19).

Ball et al. (2014) further contend that PCK includes the teacher’s capability to answer constructively to learners’ questions and, at the same time, present questions and problems that are productive to learning. The teacher’s ability to understand where and how learners make errors and how s/he is prepared with alternative solutions and models constitutes their PCK (Ball et al., 2014). A teacher with good PCK can unpack subject matter into chunks that learners can understand and make sense of meaning. S/he can explain concepts or procedures at the learners' level of understanding (Jacob et al., 2020, p.20). The (ibid) further state that with good PCK, a teacher can understand areas where learners have trouble learning the subject, and s/he should present concepts to understand their structure, errors and misconceptions.

The above information assisted the researcher in identifying the CK and PCK of teachers in the experimental group. The same helped the researcher assess how these teachers apply their CK and PCK in addressing learners' errors and misconceptions. There seems to be a strong relationship between CK and PCK in that teachers with good CK know which concepts are difficult to understand. As such, if they have good PCK, they can unpack the content to match the learners' level of understanding.

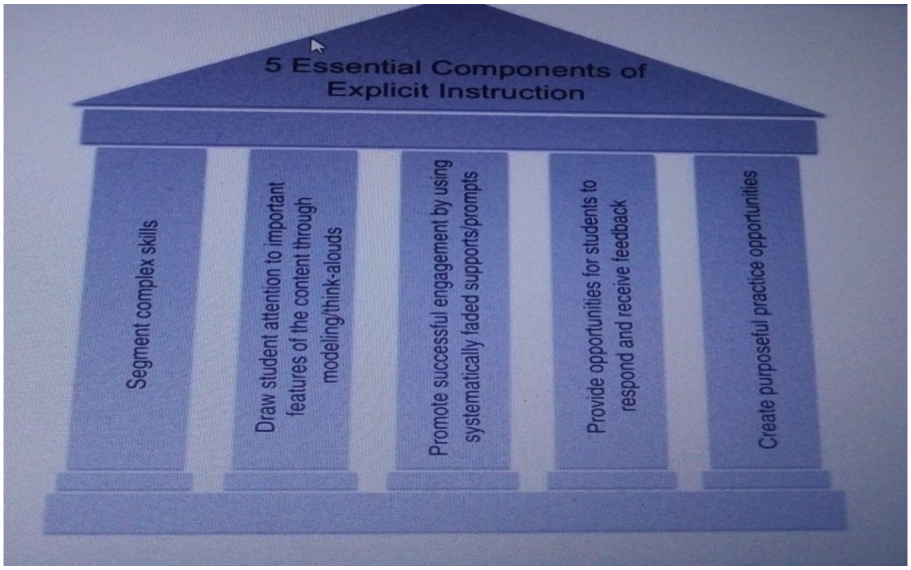
3.8 BEST PRACTICES IN MATHEMATICS TEACHING

Teachers should use different teaching approaches to meet different learners' individual learning styles. No one teaching approach is suitable for all learning styles (Wan, 2017, p.284). The relevant teaching approaches for this study are discussed in this section.

3.8.1 Explicit instruction within erroneous examples

Explicit instruction is a combination of research based instructional behaviours used by teachers to design and deliver instructions that support learners to learn successfully by providing clear objectives and reducing the cognitive load (Hughes et al., 2017, p.4). Explicit instruction promotes active learner participation by ensuring learners work in pairs or small groups and receive regular feedback on their progress. Makgakga (2016, p.52), citing Steedley et al. (2008, p.4), states that explicit instruction is a teaching approach that allows teachers to interact with learners, providing an interdependent relationship. The teacher should give concise instructions with clearly defined, achievable objectives in this learning relationship. Learners' prior knowledge is vital if teachers are to succeed in customising teaching instructions (Makgakga, 2016, p.52).

Figure 3.4: ***Five major components of explicit instruction*** (Adapted from Hughes et al., 2019)



Hughes et al. (2017, p.1), in their review, found that direct instruction has five major components, as depicted in the figure above. These components are: Segment complex skills, Drawing student attention to important features of content through modelling /think aloud, Promoting successful engagement by using systematic faded feedback supported prompts, Providing opportunities for learners to Respond and Receive feedback and Creating Purposeful Practice Opportunities.

For learners to understand complex skills or concepts, the teacher should break down complex concepts into simple, manageable chunks that are easier to understand and teach these chunks separately but in a logical, sequential manner. This calls for a teacher's CK, PK and PCK. As discussed earlier, it is clear that only teachers with sound CK can unpack teaching content to match the learners' level of understanding. Teachers should use chunks to prepare multistep strategies and teach them separately so learners can fully understand concepts (Hughes et al., 2017, p.2). To draw learners' attention to focus in class, teachers should make sure that they provide learners with clear learning objectives at the beginning of each lesson, present content using simple language understood by all learners and model answers so that learners see what and how to solve problems(ibid). The third component of explicit instruction demands that teachers provide learners with successful engagement through scaffolding. Teachers should support learners through correct examples, which they can study before solving problems. Monitoring learners' task responses can let the teacher know when to withdraw support so learners can work independently.

Feedback is crucial in explicit instruction (Hughes et al., 2017, p.1). Timeously provided feedback increases learner attention and engagement. At the same time, the teacher learns how learners understand what is being taught (Kuyyogsuy, 2019, p.76). Learners can work in pairs, small groups or as individuals. The teacher can provide scaffolding through prompts, re-voicing or think aloud to ensure learners fully understand new concepts or skills. The teacher should ascertain learners' prior knowledge to identify gaps in knowledge with the new concept. The researcher in this study observed how the teacher organised learners during lesson observation during the second phase. The main aim of explicit instruction is to ensure learners acquire and retain knowledge. As such, teachers should create purposeful practice to test learners' understanding, retention, fluency and knowledge transfer. Teachers should provide timely feedback (Hughes et al., 2017, p.1).

3.8.2 Cooperative learning within erroneous examples

Cooperative learning is an instructional strategy where teachers put learners with different abilities into small groups (heterogeneous) so that they can understand content by helping one another (Johnson & Johnson, 2018, p.59). Yusuf et al. (2019, p.1401) describe cooperative learning as a teaching strategy that uses the cooperative learning instincts of humans in which learners work in small groups, helping one another to achieve a group goal. In addition, Silalahi and Hutauruk (2020, p.1684) describe cooperative learning as a teaching approach in which learners work in small groups to solve problems and complete tasks to accomplish a goal. Makgakga (2016, p. 54) posits that is effective when the teacher designs a variety of learning activities that keep learners engaged.

In cooperative learning, learners learn how to share ideas and depend on one another. They learn that the success of one member is the success of the group. As envisaged by Johnson and Johnson (2018, p. 59), cooperative learning has five major elements: Individual and Group Accountability, Positive Interdependence, Direct Positive Interaction, Social and Interpersonal Group Skills and Group Processing. Through positive interdependence, learners learn to acknowledge that individual success depends on the group's success. Learners also develop interpersonal and social skills, which is the hallmark of teamwork. They learn to share responsibilities and be accountable for their success. Cooperative learning also teaches learners to reason critically and provide feedback to group members. Through group processing, learners develop self-esteem and positive attitudes towards learning. Cooperative learning is a learner-centred teaching and learning approach, which has several variations which include Teams Games Tournament (TGT), Student Teams-Achievement Divisions (STAD) and Teams Assisted Individualisation (TAI).

In STAD, the success of an individual depends on the success of the group (Takko, et al., 2020, p.209). In TGT unlike in STAD, tournaments replace individual quizzes and Najmi et al.'s (2021, p.248) study observed that learners taught by TGT outperformed their counterparts taught by conventional methods. Panggabean et al. (2021, p.4) concur that cooperative learning affords learners great opportunities for discussing and solving problems. TAI is another variant of cooperative learning involving heterogeneously selected groups (Silalahi & Hutauruk, 2020, p.1688). Novalinda et al. (2020, p.2975) state that TAI, as a collaborative learning strategy, considers the learner's differences and their environment. Learners get individual assistance as and when required.

The above discussion shows that cooperative learning affords learners to develop real-life skills of cooperation, communication, resilience, hard work and conflict resolution. In this study, learners discussed erroneous examples in groups and assisted one another in identifying errors in the erroneous examples. Learners also discussed how to solve equations correctly after identifying erroneous steps. In addition, learners discussed the possible misconceptions of the errors in the erroneous examples. Since learners worked in heterogeneously selected groups, they benefited from each other. They also developed cooperation and were motivated to learn. Learners learned how to work like detectives when looking for erroneous steps in erroneous examples. Learners also realised that there are common misconceptions in algebra and solving equations, which resembled their misconceptions, which helped them adjust their understanding. The cooperative learning strategy also benefited the teacher in the experimental group when he participated in designing intervention activities with erroneous examples for assignments and homework and how to implement the same daily.

3.8.3 Problem solving in the context of erroneous examples

Problem solving is another active learning approach where the teacher presents learners with a problem without an immediate solution or a rule to get the answer (Rahman, 2019, p.72). The proponent of problem solving, Polya (1973), states that it consists of four steps, namely, (1) Comprehending the problem, (2) Making a plan, (3) Executing the plan, and (4) Checking.

At first, learners should read and understand to extract relevant information needed to solve the problem; hence, learners must have good reading skills. They must list all the given information and what they must find. Learners may have to state the problem in their own words to show understanding. The second step is devising a plan of action to solve the problem. Learners may have to convert mathematical sentences into mathematical equations or develop some heuristics for arriving at a solution. In this step, learners discuss extensive strategies or models to apply. In the third step, learners apply the best strategy or model they selected to solve the problem. The fourth step is where learners reflect on their solution, checking all steps from the first to the last. Learners have to explain their solution in light of the original problem.

Tambunan's (2019, p.1), study investigated the effectiveness of problem solving and scientific approach to enhance students' mathematical capabilities in higher order thinking skills and observed that problem solving was more effective than the scientific approach. However, the effectiveness of problem solving depends on the teacher's ability to design challenging activities that allow learners to discuss in small groups to achieve a common understanding of concepts (Simamora & Saragih, 2019, p.62).

Additionally, for learners to benefit from problem solving, they should have a good understanding of mathematics and the ability to reason and acquire strategies for solving non-routine problems (Siagan, Saragih & Sinaga, 2019, p.333). Utilising problem solving, explicit instruction and erroneous examples may afford learners the best opportunity to benefit from learning mathematics. The previously mentioned strategies equip learners with critical and creative skills the qualities that are most required in industry and commerce (Rahman, 2019, p.72).

3.9 SUMMARY

The literature review was presented in this chapter, and the types of errors and misconceptions were discussed. The literature indicated that erroneous examples were used in other countries like the USA and Japan, but little attention has been given in South Africa. This study was conducted at the teacher level when teaching Algebraic Linear Equations therefore, the content and pedagogical content knowledge of teachers were reviewed to understand their position in using erroneous examples. Teaching practices and resources were also examined to understand how they affect learners' performance in mathematics.

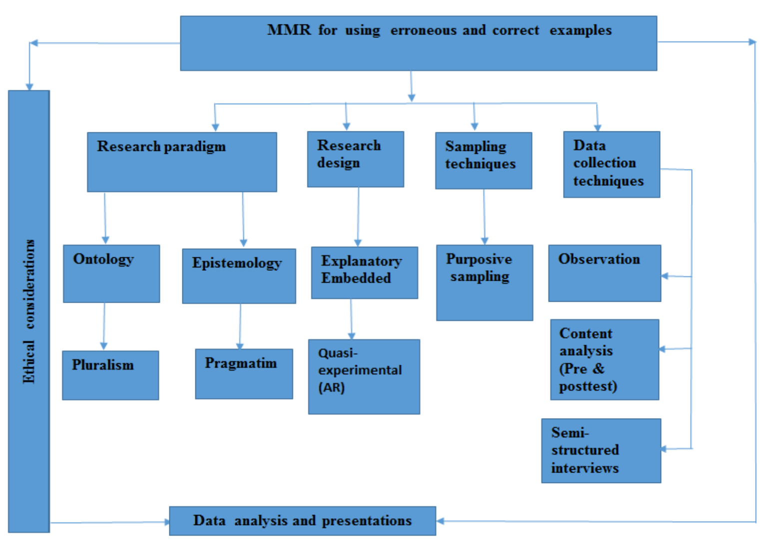
CHAPTER 4: RESEARCH METHODOLOGY

4.1 INTRODUCTION

This chapter discusses and explains the research methodology, research design, research paradigm, research approaches, data collection and analysis methods underpinning this study. Additionally, it discusses the population and sampling techniques used. Furthermore, it discusses the issues of reliability, validity, and trustworthiness, which entails credibility, dependability, confirmability and transferability. Finally, the issue of research ethics is presented.

According to Mishra and Alok (2022, p.1), methodology is a scientific research method. Through research methodology, the researcher explains all the steps that s/he undertook in studying a research problem. Research methodology is a systematic way of finding answers to research problems (Kothari, 2017, p.8). The (ibid) further contends that methodology entails research methods and the logic for selecting them. It allows the researcher to display logic and a systematic way when conducting research (Kivunja & Kuyini, 2017, p.28). Research methodology comprises research paradigms, research design, sampling techniques, data collection and analysis instruments, data collection procedures and analysis and ethical considerations. The methodology map for this study is depicted in Figure 4.1 below.

Figure 4.1: ***Map for methodology*** (Adapted from Ngoako & Modiba, 2021)



4.2 RESEARCH PARADIGM

Kivunja and Kuyini (2017, p.26), state that a paradigm is a school of thought or beliefs common to a group of people from which they can construct meaning or interpretation of research data. The (ibid) further contend that a paradigm is the researcher's lens, which s/he uses to determine the methodology to use in the study and to decide on the research methods suitable for collecting and analysing data. Guba and Lincoln (1994, p.105) define a paradigm as those beliefs researchers use in an investigation. Through a research paradigm, researchers articulate what they consider important and simultaneously indicates what lies within and outside the boundaries of legitimate research (Guba & Lincoln, 1994, p.108). Makgakga (2016, p.83) argues that a paradigm comprises the phenomenon being studied, the research questions to be answered and how they should be analysed and interpreted in light of the data collected. Furthermore, Creswell and Creswell (2018, p.44), argue that a paradigm is a worldview constituting a set of beliefs that researchers hold, which guide their actions about ontology, epistemology and methodology. All research paradigms depend on some ontology, epistemology and methodology (Creswell & Creswell, 2018; Ngulube, 2020). This study adapted Ngulube's (2020) definition of a research paradigm.

4.2.1 Ontology

According to Willig (2019, p.4), ontology refers to those taken-for-granted things which form the basis of how we understand the world. The study of the nature of reality is ontology (Al-Ababneh, 2020, p.75). Of prime concern to ontology is what existing things are made up of (Willig, 2019, p.4). Through ontology, researchers inquire about the reality in the world (Ngoako & Modiba, 2021, p.498). Thus, ontology helps the researcher decide which data collection methods will enable him/her to answer the research questions. Ontologically, MMR uses pluralism (Romm, 2018). The ontological worldwide viewpoint that best suits MMR is pluralism (Ngulube, 2019; Ghiara, 2020). Pluralism is an ontological perspective that allows researchers to use a variety of approaches in a single study (Ghiara, 2020, p.14). Thus, researchers can use both quantitative and qualitative data for the enhancement of the study's validity and reliability (ibid). The use of pluralism and pragmatism in research aims to link constructivism/positivism and positivism/realism as ontological and epistemological stances in research (Ngulube & Ngulube, 2022, p.2).

4.2.2 Epistemology

Epistemology is the study of the nature of knowledge and how humans create and communicate it among themselves (Willig, 2019, p.4). Saunders, Lewis and Thornhill (2019), posit that, epistemological questions guide researchers in debating the desirability and possibility of subjectivity, objectivity, validity and causality in research. Creswell and Plano Clark (2018), state that pragmatism is the most suitable epistemological perspective to use in mixed methods research (MMR). The problem of dualism in research is resolved using pragmatism (Ngoako & Modiba, 2021, p.498). Thus, with a pragmatic stance, researchers can use both quantitative and qualitative social science perspectives in debates to arrive at an agreeable solution (Babbie, 2013). The rivalry between quantitative and qualitative researchers is ended by using pragmatism (Creswell & Creswell, 2018, p.52).

Pragmatic researchers focus on what is practically possible in research. They study research problems in varied ways that are appropriate to them. Dawadi et al. (2021, p.27) contend that researchers may adopt a pragmatist stance in a study to use multiple methods for gathering enough data for answering research questions. Pragmatism is a paradigm that advocates using the mixed research approach, paying particular attention to the research problem and applying different methods in answering the research question (Kivunja & Kiyini, 2017, p. 35). Maarouf (2019, p. 5) argues that pragmatism justifies using a mixed research approach because it allows researchers to select the best methods from different paradigms to answer their research questions. Accordingly, the researcher adopted a pragmatic paradigm for this study because it caters to the research hypotheses and questions.

4.3 RESEARCH APPROACH/DESIGN

Research design comprises the plans and procedures used in research based on researchers' philosophical assumptions (Creswell & Creswell, 2018. p.40). The major research approaches are quantitative, qualitative and mixed methods, as explained below.

4.3.1 Quantitative research approach

Quantitative research relies on methods that produce numerical data and facts (Ahmad, Wassim, Irfaan, Gogoi & Srivastava, 2019, p.1). Quantitative research uses mathematical and statistical methods to establish a relationship between research variables. Because of its accuracy and precision of measure, quantitative research is empirical research (Ahmad et al., 2019, p.1). Furthermore, quantitative research focuses on collecting data that is measurable and is most effective at answering the "what" or "how" questions (Goertzen, 2017, p.12). A quantitative research approach only responds to the hypotheses for this study, leaving the research questions unanswered because they require qualitative information.

4.3.2 Qualitative research approach

Researchers use qualitative research to explore and understand participants' perspectives about the phenomenon under investigation (Creswell & Creswell, 2018, p.41). Goertzen (2017, p.12) describes qualitative research as research that motivates how participants feel and think about the situation under investigation. Since people think and feel differently in any given situation, qualitative researchers believe in multiple realities (Creswell & Creswell, 2018, p.41). To better understand human attitudes, behaviour, and experiences (Ahmad et al., 2019, p.1), constructivist researchers use qualitative methods, which include observations and structured or unstructured interviews. In this study, lesson observations and semi-structured interviews helped the researcher answer the research questions.

4.3.3 Mixed methods research approach

According to Ngulube and Ngulube (2022, p.2), a mixed research approach is the best approach for researching complex problems that cannot be thoroughly investigated using one methodology. Assessing the impact and effectiveness of erroneous examples is a complex problem, which cannot be accomplished by one approach. The researcher utilised MMR to achieve the study's aim and objectives. Creswell and Plano Clark (2018), state that a mixed research approach uses different philosophical assumptions that allow data collection and analysis from various sources in one study. Mixed methods research designs offer many benefits in positivism and interpretivism frameworks (Dawadi, Shrestha & Giri, 2021, p.27), combining quantitative and qualitative data to explain research issues meaningfully. The (ibid), argue that quantitative data provides breadth to the study while qualitative data contributes depth. MMR uses several research designs, which include sequential exploratory, sequential explanatory, convergent and embedded research designs.

4.3.3.1 Sequential exploratory design

In the sequential exploratory approach (Creswell & Creswell, 2018, p.52), the researcher initially explores participants' perceptions in Phase one. The researcher analyse the data in this study and use the information in Phase two. The researcher uses the qualitative phase to develop an instrument for the study's second phase.

4.3.3.2 Sequential explanatory design

Creswell and Creswell (2018, p.52) describe explanatory design as a design that is sequential and is executed in two phases. In Phase 1, quantitative data is collected and analysed. In Phase 2, qualitative data is collected based on quantitative analysis in Phase 1. According to Ngulube (2020), the researcher may use the explanatory sequential method starting with quantitative data followed with qualitative data. After analysing quantitative data, the researcher understands research questions and other possible questions s/he might ask when collecting qualitative data (Makgakga, 2016, p.93).

4.3.3.3 Convergent mixed-methods design

Subedi (2016, p.572) states that in a convergent mixed methods design, quantitative and qualitative data are collected and analysed separately. The results are then compared and interpreted to make conclusions as to whether the results contradict or support each other. With this design, the researcher can directly compare the two datasets to provide convergence of data sources. Creswell and Creswell (2018, p.52), state that in a convergent mixed methods design, the researcher combines quantitative and qualitative data to better understand the research problem.

4.3.3.4 Embedded design

According to Creswell and Creswell (2018, p.52), an embedded design enables the researcher to simultaneously or sequentially collect quantitative data first followed by qualitative data. The qualitative data is collected to support the quantitative data. Using an embedded design, a researcher can mix different data sets at the design level (Bastable, Meng, Fsalcon & Mclntosh, 2023, p.202), allowing him/her to answer several research questions using quantitative and qualitative data. Asenahabi (2019, p.85) indicates that an embedded design assumes that one data set is not enough to provide information to fully understand the phenomenon being investigated, and as such, merging different data sources is the answer. In this study, learners' written scripts provided the primary data and interviews and observations provided the secondary data. The embedded design has several variants, and the quasi-experimental mixed methods design is one of them.

Figure 4.2: ***Embedded Design*** (Adopted from Subedi, 2016, p.57)

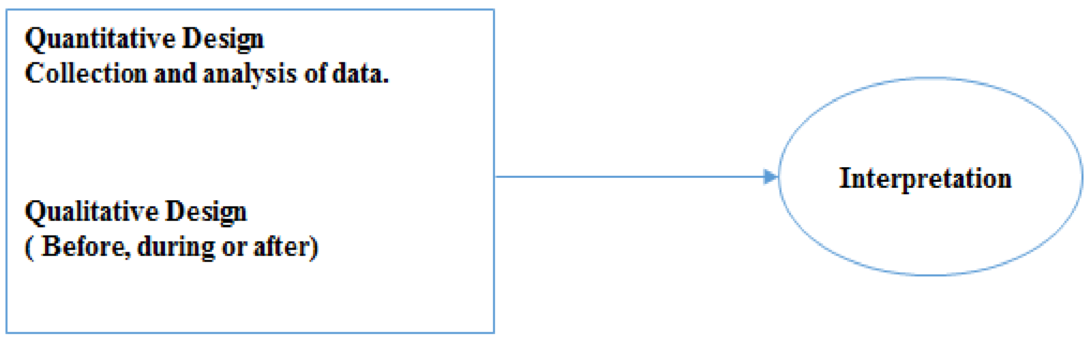


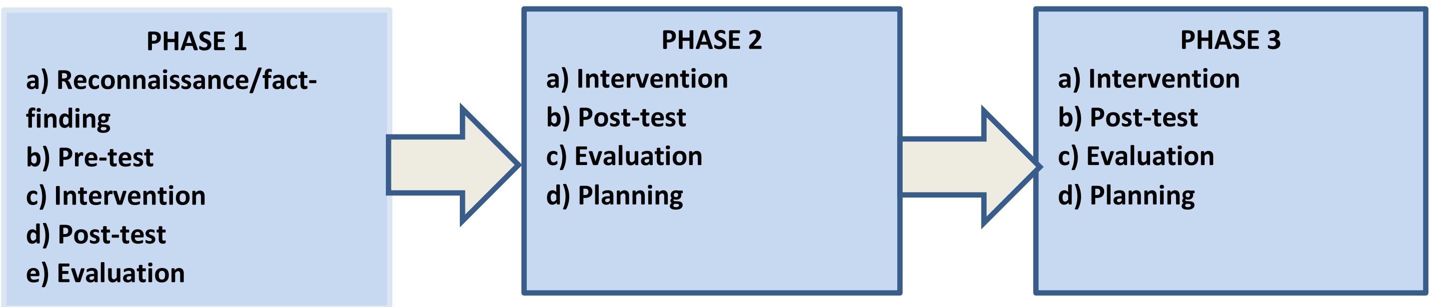
Figure 4.2 Embedded Design (Adapted from Subedi 2016 , p. 57

4.3.3.4.1 Embedded mixed methods action research design

In an embedded mixed methods design, researchers use the experimental and control groups without assigning participants into groups (Creswell & Creswell, 2018, p.52). Researchers first collect quantitative data, followed by the collection of qualitative data. Qualitative data supports the quantitative data (Asenahabi, 2019, p.85). An experimental and control groups are required to achieve this, making an explanatory embedded research design suitable for this study. Since this study aimed to improve learner performance in solving algebraic linear equations, the researcher implemented the explanatory embedded research design in the form of action research (AR).

AR is research in which researchers systematically combine and connect two forms of activity, action and research (Al-Obaydi & Rahman, 2021, p.232). AR is learning by doing, whereby an individual or a group of people learn from solving problems they encounter (ibid, p.232). Kamarudin and Mat Noor (2023, p.2) describe AR as an activity carried out by teachers, principals or other interested people in education to collect data on their schools' operations. AR utilises MMR data collection methods. (Morales, 2016, p.158) describes AR as an inquiry where the primary goal of the researcher is not to produce theoretical knowledge but to improve the researcher's capacity and subsequent practices. From the above definitions, AR can be described as an organised approach through which people work on their challenges to improve their situations. The AR stages adopted for this study were as shown below.

Figure 4.3: ***Action Research Phases***



PHASE 1 - Run-in period

As depicted in the figure above, the first step was identifying a problem during the reconnaissance period. A run-in period, or pre-intervention washout period (Hassain et al., 2020, p.2), is the duration allowed to eliminate the carry-over effects of the confounding variables to determine the intervention's true effect. During this period, participants are either given the same placebo or no treatment (Jin et al., 2020, p.1). The run-in period also allows participants time to decide whether to participate or not (Rees et al., 2016, p.2). In this study, participants were given the chance to decide whether to participate or not in the research. All learners who decided to participate wrote pre-test before the intervention strategy was implemented. In a reconnaissance exercise, the researcher carried out three classroom observations at both the experimental and control schools to find out how teachers deliver lessons and interact with learners.

PHASE 2 AND 3

By nature, AR is iterative, so three intervention episodes were implemented. After marking the first post-test, the researcher noted learners' errors. The experimental school's teacher and researcher designed an intervention strategy focusing on learners' errors. After each intervention cycle, learners wrote a post-test, which was marked, and learners' errors were noted and planning for the next cycle was done. According to Kamarudin and Mat Noor (2023, p.2) educational practitioners select innovative strategies to improve their practice and enhance learners' performance. The intervention terminated after the third post-test in Phase 3 when fewer learners committed fewer errors and a washout period was allowed before learners wrote the final post-test to assess the impact of the intervention.

According to Verma (2021, p.429), a washout period is when participants are taken off the intervention to minimise the effects in the post-intervention trials. The (ibid) argues that stopping or withholding the prescribed intervention is the common washout period method in research trials. For this study, a washout period of two (2) weeks was allowed. The TEG used traditional teaching methods during the washout period.

4.4 LOCATION

This study was carried out in two secondary schools in the Johannesburg Central District in the Gauteng Province of SA. The experimental school was in a township where the learners speak various home languages (Zulu, Sepedi, etc.), although they were taught in English. The control school is in a semi-urban area, although most learners are from surrounding townships and speak different home languages, they were also taught in English. The learners in the township school were not performing well in algebraic linear equations, while those in the semi-urban school were performing well. Most of the learners at the two schools use public transport to and from school.

4.4.1 Profile of participating teachers

TEG and TCG were the teachers teaching the experimental and control groups respectively. Both schools are in the Johannesburg Central District. The schools were more than five kilometres apart. The experimental school had three Grade 9 classes, while the control school had five Grade 9 classes.

The number of learners at the experimental school was 641 from Grade 8 to Grade 12, of whom 335 were boys and 306 were girls. There were 171(92 boys and 79 girls) Grade 9 learners. The control school had 1148 learners from Grade 8 to Grade 12; 459 were boys, and 689 were girls. There were 200 (90 boys and 110 girls) Grade 9 learners.

Teacher	Qualifications	Teaching experience
TEG	1. MBA (General) 2. BED (Hons) Leadership 3. BED (Hons) Mathematics Education 4. ACE Mathematics Education 5. Secondary Teachers' Diploma	1. 29 years teaching mathematics and accounting. 2. 5 years teaching Grade 9 mathematics.
TCG	1. ACE Mathematics Education	1. 13 years teaching mathematics. 2. 10 years teaching Grade 9 mathematics. 3. MST lead teacher. 4. PLC coordinator.

Table 4.1: Teachers' profiles

4.5 POPULATION AND SAMPLING TECHNIQUES

Rahi (2017, p.3) and Naseri, Hussin, Esa, Azizi and bin Nordin (2021, p.654) defines population as all the items or people a researcher wishes to understand in a study. The population for this study is Johannesburg Central District, comprising 68 secondary schools. However, it was impossible to study the whole population, so the researcher chose a small section of the population through a sampling technique. Sampling is selecting a small part of the population (Rahi, 2017, p.3; Ngoako & Modiba, 2021, p.500) concur when stating that sampling is the method of selecting sample members. Sampling establishes a small group representing the population under investigation (Leedy & Omrod, 2015; Ngoako & Modiba, 2021, p.500). Sampling aims to match the sample to the research aims and objectives (Campbell, et al., 2020, p.653). The (ibid) further states that as a result, the rigour of the study is improved together with the trustworthiness of the collected data and results thereof.

MMR researchers can use several sampling techniques, among which are the following: identical sampling, multi-level sampling, parallel sampling, nested sampling, purposive sampling, and convenience sampling. Each sampling technique has its pros and cons and some researchers, due d to time constraints, choose the easiest techniques. This study utilised convenience sampling.

Convenience sampling occurs when a researcher selects participants from a conveniently accessible source (Andrare, 2021, p.86). When a researcher conveniently chooses participants who fit the study's criteria, that sample is a convenience sample (Emerson, 2021, p.76). Convenience sampling was used for this study because the researcher could easily access the schools, and the learners' characteristics were suitable to the study's aim and objectives. The participants were Grade 9 learners (N=31 and N=12) and their teacher for the experimental group in the pre-and post-test respectively and (N=28 and N=24) and their teacher for the control group in the pre-and post-test respectively. The schools are approximately ten kilometers from each other, and it was not likely that participants from the two schools would share information about the intervention.

The population for this study consists of 68 secondary schools in the Johannesburg Central District, with only 3 secondary schools being conveniently selected. The criteria for selecting the schools were: (1). Low performance for at least 4 consecutive years and being in a high-density township and willingness to participate in the study (2). High performance for at least 4 consecutive years, multi-racial being in previously advantaged community and willingness to take part in the study. The high performing school was chosen to serve as a benchmark for the intervention, which was aimed at improving the performance of the low performing school to the level of the control group..

The Group Areas Act of 1950 barred people from living together, and the government removed Africans, Indians and Coloureds from areas classified for White occupation. Under Apartheid, Indian learners received a better education than African learners did and Coloured learners did. In 1994, the government repealed the Group Areas Act, learners could attend school anywhere in the country, and many African learners from Soweto chose to attend school in previously Indian schools in Lenasia. During Apartheid, Indian learners received better education than African learners. Of the 3 schools selected, 1 school served as a pilot study, and 2 schools served as the main study's sample.

4.6 INSTRUMENTS AND METHODS OF DATA COLLECTION

4.6.1 Classroom observations

According to Barrett and Ywycross (2018, p.63), observation is a powerful tool for collecting qualitative data and enables researchers to gather wide-ranging information within a setting, including verbal and non-verbal conversations, environmental characteristics and actions. Through observation, researchers obtain personal experience of the happenings at a research site. In this study, the researcher used an observation schedule adapted from Sepeng (2010, p.4) to get first hand experiences regarding how the teachers at the 2 schools in the main study carry out their day-to-day teaching business. The researcher conducted lesson observations at both schools before the intervention and after the intervention at the experimental only. The researcher compiled notes during observations and collected data directly by interacting with the teacher and learners and seeing their behaviour in their natural setting (the classroom) (Creswell, & Creswell, 2018, p.52). The researcher focused on understanding learners' meaning about solving algebraic equations and not concentrating on the meaning that the researcher brings from literature or his/her own meaning (Creswell & Plano Clark, 2018). The researcher played the role of participant observer during lesson observations (Renjith, Yesondharan, Noronha, Ladd & George, 2021, p.1). As a participant observer, the researcher was able to participate as a regular member in the undertaken activities. The researcher was able to plan, discuss, and identify the strategy to use during the teaching and learning of algebraic linear equations to improve learners' performance.

Using Sepeng's (2010) observation schedule, the researcher focused on the following aspects:

1. **Classroom practice**

The researcher observes the teacher's algebra pedagogical approaches. Furniture arrangement of the teacher's responses to learners' correct and incorrect responses and how the teacher utilises learners' errors during teaching.

2. **Learners' activities**

The researcher observes how learners engage in problem solving and apply their decision making skills in problem solving.

3. **Evaluating skills**

The researcher observed whether learners could evaluate their own work and identify their own or other learners' errors.

4. **Reflection**

The researcher followed how the teacher responded to learners' errors and s/he addressed learners' challenges in solving algebraic equations.

5. **Learner-learner interaction**

The researcher observed whether the teacher encouraged learner group discussions, paired activities or group presentations.

6. **Learner responses**

The researcher observed how learners presented their work, whether they presented work as individuals, in groups, on the chalkboard, verbally, etc.

Sepeng's (2010) observation schedule uses a four-point scale, and the researcher used this scale in all classroom observations. Using this schedule, the researcher collected data at the experimental school using the above stated points.

CLASSROOM SURROUNDINGS	INSTRUCTIONS
- Setting up a learning culture	- Using question and answer techniques
- Directing classroom operations	- Involving learners in learning process
- Directing learners' behaviour	- Using formative assessments
- Arranging physical place	- Showing responsiveness and flexibility

Table 4.2: Classroom surroundings and instructions (adapted from Sepeng, 2010)

A conducive learning environment is paramount for effective teaching and learning (Baafi, 2020, p.124). During fewer observations, the researcher focused on how the teacher created a conducive learning environment. The researcher was also able to observe how the teacher-learner and learner-learner interact. Additionally, the researcher observed how teachers utilised learners' errors and misconceptions as learning opportunities.

4.6.2 Pre-test and post-test

A pre-test is a tool researchers use to measure a dependent variable before introducing an intervention in an experiment. At the same time, a post-test is a tool used to measure the dependent variable after some treatment in an experiment (Pan & Sana, 2021, p.1). Thus, an instrument researchers use to determine the characteristics of the dependent variable prior to administering a treatment is called a pre-test. When researchers use the instrument after administering some treatment, it becomes a post-test. The pre-test and post-instrument were divided into two parts: algebraic equation (AE) Q1 to Q6 and word problems (WP) Q7 and Q8. The researcher piloted the test items before they were used in the main study.

A pilot study is undertaken to test the accuracy, reliability and appropriateness of research methods and instruments before launching a full-scale study (In, 2017, p.601). A pilot study enables the researcher to gain experience using the research tools (ibid, p.601). At the pilot stage, the researcher noticed that the pre-test items needed adjustments to meet the study objectives. There were a few erroneous examples in the first draft, making it difficult to assess a variety of learners' errors and misconceptions. Three members from the Mathematics Department at the University of South Africa (Unisa) moderated the revised instrument, and the researcher incorporated their suggestions.

The test items met the Curriculum and Assessment Policy Statement (CAPS) requirements as explained in (DBE, 2012). Using the pre-test, the researcher became aware of learners' challenges in solving algebraic linear equations and their general performance in mathematics. Using the post-test results, the researcher was able to assess the impact of the intervention. Using a pre-test, the researcher measured the learners' performance in algebraic equations before administering an intervention. This helped the researcher understand the learners' difficulties in solving algebraic equations before administering an intervention. The researcher used Newman's error analysis protocol to analyse learners' responses. After the first cycle, learners wrote a post-test, which was meant to measure learners' performance and the effectiveness of the treatment strategy. To allow for uniformity and consistency, the learners at both schools wrote the tests concurrently and the two teachers served as invigilators at their schools. The learners at the low-performing school served as the experimental group, and those from the high-performing school served as the control group. Figure 4.4 below shows how to administer a pre-and-post-test in research settings.

Figure 4.4 ***Diagram for the experimental design*** **(Adapted from Babbie, 2010, p.234)**

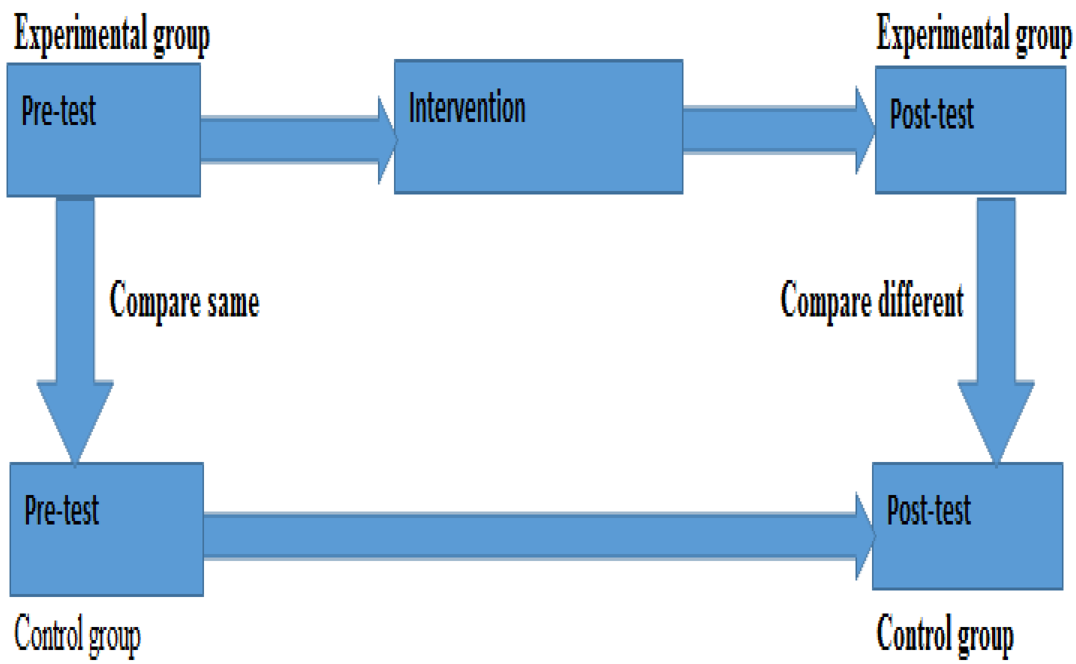


Table 4.3: ***The structure of the pre-test and post-test for this study***

ITEMS	MEANING
SECTION 1 Algebraic equations (AE)	1. Solving equations with the variable on one side of an equal sign 2. Solving equations with variables on both sides of equal sign 3. Solving equations involving brackets 4. Solving equations involving fractions 5. Identifying errors in a given solution and solving the equation correctly
SECTION 2 Word problems (WP)	1. Forming equations from word problems and solving them.

The test questions comprise two sections. Section 1 is on algebraic equations (AE), section 2 is on word problems (WP), and there are 8 questions. The researcher concurrently administered the pre-tests to the two groups without intervention. Teachers at the participating schools assisted with the invigilation of the test. Later, both study groups concurrently wrote the post-test after the intervention at the experimental school.

4.7 SEMI-STRUCTURED INTERVIEW WITH TEACHER

The researcher conducts interviews to understand participants' perceptions of their experiences and the meanings they attach to these experiences (Naz, Gulab & Aslam, 2022, p.42). Two interviews were conducted in Phase 1, one with each teacher. Two interviews were conducted in Phase 2 with the teacher at the experimental school. The researcher used semi-structured interviews and open-ended questions to probe for more information or clarity on previous responses (Queiros, Faria & Almeida, 2017, p.378). With permission from the participants, the researcher recorded and saved the interviews on a memory stick with some backups on the computer. The researcher also took field notes. The researcher compared information during analysis using field notes and interview transcriptions for greater reporting accuracy (ibid). At this stage, there may be no constructs, so the researcher used the interview responses to analyse the emerging constructs inductively before introducing the intervention. Both semi-structured and open-ended questions were used in this study. The following unstructured questions were used to clarify some responses whenever the need arose. The researcher asked the following questions to the teacher at the experimental school before, during and after the intervention.

1. In your opinion, what could be the reasons for learners' poor performance and why?
2. How do you encourage active learner participation during lessons?
3. What teaching resources do you use in your classroom?
4. What classroom assessments do you use in mathematics, and how?
5. How do you support learners with challenges in mathematics?
6. How do you promote a teaching and learning culture in your classroom?

Interview responses after the pre-test helped design the intervention strategy at the experimental school. During the intervention, the researcher asked the teacher at the experimental school the following questions.

1. Do you think using erroneous examples is appropriate for teaching Grade 9 algebraic linear equations and why?

2. What are the challenges of using erroneous examples for teaching algebraic linear equations?
3. What are the benefits of using erroneous examples for teaching Grade 9 algebraic linear equations?
4. Do you think erroneous examples would be suitable in the future for teaching Grade 9 algebraic linear equations and why?

The responses during the intervention assisted the researcher in understanding the benefits and challenges of using erroneous examples for teaching and learning algebraic linear equations. After the intervention, the researcher asked the teacher at the experimental school the following questions:

1. How is learner participation now in comparison to the time before the intervention?
2. Do you think erroneous examples can be used on a daily basis and why?
3. According to you, was using erroneous examples an effective strategy for improving learners' performance and why?
4. How can we make the use of erroneous examples more effective?

The responses from the interview allowed the researcher to get an understanding of the teacher's experiences with the intervention strategy for teaching and learning algebraic equations and their impact and effectiveness on learners' performance. The researcher sorted all interview responses into themes discussed in the qualitative analysis section. Lesson observations triangulated interview responses.

4.8 DATA ANALYSIS

Data analysis transforms raw information into sensible or meaningful data (Creswell & Creswell, 2018, p.222). In this study, data analysis involved representation, transcription and interpretation of the test outcomes lesson observations and semi-structured interviews to make sense of how teachers teach algebraic equations and how learners' test outcomes explain why learners struggle in mathematics.

4.8.1 Analysis of the quantitative data

The researcher coded learners' test responses according to Newman's Error Analysis coding system, as in the table below.

Category	Code	Meaning
1	CA	Correct response
2	RE	Reading Error
3	CE	Comprehension Error
4	TE	Transformation Error
5	PSE	Process Skills Error
6	EnE	Encoding Error
7	BL	No Response

Table 4.4: ***Newman's Error Analysis coding system***

Quantitative data were organised into groups using responses from learners' scripts. Table 4.4 above shows the codes used in this study. Putting learners' responses into groups enabled the researcher to understand how Grade 9 learners solved algebraic linear equations and the type of errors they committed. The researcher responded to the research objectives using the pre- and post-tests descriptive statistics. Using the independent t-test, the researcher tested the statistical significance of the data, using a $p\text{-value} < 0.05$ at the 95% confidence limit. The researcher also used the IBM SPSS statistical software package to analyse pre-test and post-test data and to compare the two study groups.

4.8.2 Analysis of qualitative data

Creswell and Creswell (2018, p.52), state that the analysis and gathering of qualitative is done concurrently. Using Creswell's steps of analysis, the researcher analysed qualitative data as shown in Figure 4.4 below.

Figure 4.5 ***Analysis of data in qualitative research*** (Adapted from Creswell, 2014, p.197)

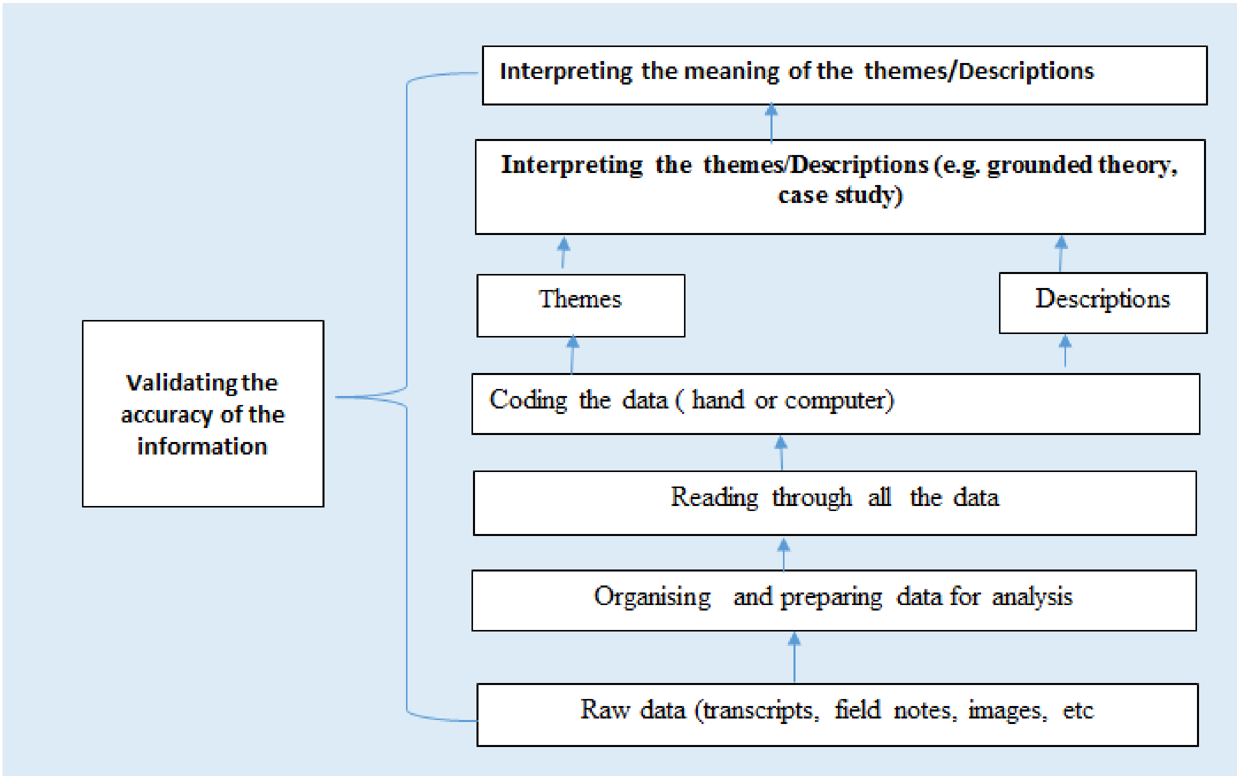


Figure 4.5 above depicts a linear progression of events from the bottom to the top. However, in real-life situations, the events' order may differ (Creswell & Creswell, 2018, p.52). Raw data was derived from classroom observations and semi-structured and open-ended interview questions. The researcher transcribed all interviews and scanned all field notes and classroom observations. After scanning and transcribing, the researcher read the data to understand the information. Reading the information enabled the researcher to create themes. After numbering and abbreviating, the researcher used codes to identify themes and put them into other groups.

Extant data will be re-coded if need be. Guided by Creswell's (2014, p.197) coding system, the researcher analysed data to achieve the following:

1. Come up with codes that readers expect based on existing information and literature
2. Showing codes that the researcher did not expect at the beginning of the analysis
3. Showing codes that engulf outstanding theoretical perspectives in the research.

4.9 VALIDITY AND RELIABILITY

This study used both quantitative and qualitative methods. Validity in quantitative research is measured differently from validity in qualitative research. Any valid research investigates what it is designed to investigate in the best way possible (Heale & Twycross, 2015, p.66). Quantitative data was collected using pre-and-post-test items. Surucu and Maslakci (2020, p.2695) define validity as how good an instrument measures what it is designed to measure and reliability as an indicator of how stable it produces the measurements it should be. Thus, validity is how accurately an instrument measures what is measured, while reliability is the instrument's reputation for producing accurate measurements of what should be measured. Several measures were undertaken to enhance the validity and reliability of the study, including acknowledging biases in sampling methods and meticulous record-keeping to demonstrate a clear decision trail to ensure transparent and consistent interpretation of data. In addition, respondent validation was used to ensure the validity and reliability of the research. The researcher invited participants to comment on interview transcripts to ensure the researcher's interpretation was according to the participant's responses.

4.9.1 Pilot study

A pilot or feasibility study is a trial version of a study or a research instrument before conducting the main study (Gani, Imtiaz, Rathakrishan & Krishnasamy, 2020, p.140). Piloting a research instrument for this study was important because it ensured that the instrument measured what it intended to measure in the main study. Gani et al. (2020) state that any instrument is considered good when the data collected using, the instrument is valid and reliable. An instrument's validity and reliability depend on whether it produces the same results when used on several occasions with different participants by different researchers. A good data collection instrument has to pass the validity and reliability tests, this is important because the research conclusion depends on the information collected using the instrument (Gani et al., 2020, p.140).

Piloting the study before the main study enables the researcher to detect areas where the main study could meet future problems (Malmqvist, Hellberg, Mollas, Rose & Shelvin, 2019, p.1). A pilot study also helps show if the research instrument is adequate for collecting data for the main study. According to Gani et al. (2020), conducting a pilot study before the main study adds value and credibility to the research. Conducting a pilot study also enables the researcher to determine if the pre-test is suitable for collecting data for the main study (Lowe, 2019, p.117). By piloting the pre-test for this study, the researcher adjusted some of the difficult test items for most learners to understand. In addition, piloting the research instrument enabled the researcher to adjust the time allocation for the pre-test. More time was needed to complete the pre-test, and this was adjusted from 45 minutes to an hour.

Pilot studies are usually small-scale undertakings than the main studies, and as such, they have the potential to provide limited information on the actual studies. This means that some weaknesses may only surface in the main study. Apart from being a miniature study, a pilot study also has contamination as a limitation (McDonnell, et al., 2020, p.12). When data is collected from the same respondents for both the pilot study and the main study, information tends to be biased or contaminated. However, participants in this study come from a school that did not take part in the main study. Thus, the issue of contamination does not apply in this study.

Moreover, the pilot study provided important information on data collection and analysis for this study. To ensure the validity of the instrument, 3 experts from UNISA's Faculty of Mathematics Education in the College of Education moderated the test questions. Learners' scripts in this study were analysed to determine the type of errors Grade 9 learners commit when solving algebraic linear equations. The table below shows the error categories used in the pilot study. These were recorded as Correct Response (CA), Reading Error (RE), Comprehension Error (CE), Transformation Error (TE), Process Skills Error (PSE), Encoding Error (EnE), and No Response (BL).

The teacher at the pilot school administered the pre-test under strict examination guidelines, ensuring that learners did not share answers. The researcher marked the learners' scripts, taking note of errors using the codes as shown in Table 4.5.

4.9.2 Analysis of the pilot study

The pilot study revealed that the original time allocation was insufficient since most learners did not complete the task. The researcher adjusted the time allocation from 45 minutes to 60 minutes. Additionally, the researcher noted that questions 7 and 8 were difficult to understand for most learners since only one learner managed to get the correct for question 8.1. These questions posed a comprehension challenge because most learners spoke English as a second language. The researcher replaced questions 7 and 8 in the main study with easier questions.

Question	PILOT TEST											
	CA		CE		TE		PSE		EnE		BL	
1.1	5	71%	2	29%	0	0%	0	0%	0	0%	0	0%
1.2	7	100%	0	0%	0	0%	0	0%	0	0%	0	0%
2.1	5	71%	2	29%	0	0%	0	0%	0	0%	0	0%
2.2	6	86%	0	0%	0	0%	1	14%	0	0%	0	0%
3.1	5	71%	2	29%	0	0%	0	0%	0	0%	0	0%
3.2	4	57%	0	0%	0	0%	3	43%	0	0%	0	0%
4.1	4	57%	2	29%	0	0%	0	0%	0	0%	1	14%
4.2	5	71%	0	0%	0	0%	1	14%	0	0%	1	14%
5.1	4	57%	1	14%	0	0%	0	0%	0	0%	2	29%
5.2	2	29%	4	57%	0	0%	0	0%	0	0%	1	14%
6.1	2	29%	4	57%	0	0%	0	0%	0	0%	1	14%
6.2	0	0%	3	43%	0	0%	0	0%	0	0%	4	57%
7.1	0	0%	3	43%	0	0%	1	14%	0	0%	3	43%
7.2	0	0%	0	0%	0	0%	5	71%	0	0%	2	29%
8.1	1	14%	3	43%	0	0%	0	0%	0	0%	3	43%
8.2	0	0%	4	57%	0	0%	0	0%	0	0%	3	43%
Total	50	44.64%	30	26.79%	0	0.00%	11	9.82%	0	0.00%	21	18.75%

Table 4.5: ***Distribution of learners' responses from the pilot study***

Table 4.5 shows that 50 responses were correct, representing only 44.64 % of the expected 112 correct responses. The table also reveals that all learners managed to correctly answer question 1.2. All learners could answer only question 1.2 correctly, indicating that most had difficulties solving algebraic linear equations. The table also reveals that 26.79% of the learners committed comprehension errors. The pilot study also revealed that no learner committed transformation and encoding errors. However, 9.82% of the learners committed process skills errors and 18.75% did not answer some questions. Learners performed poorly from questions 5.2 to 8.2. There were no correct responses for questions 6.2, 7.1, 7.2, and 8.2, confirming that learners have difficulties solving algebraic linear equations. The pilot study results confirm the DBE (2022, p.197) report findings that learners in lower grades struggle to solve algebraic questions, affecting their performance when they get to Grade 12. The pilot study also revealed that only 44.64% of the learners' responses were within the learners' zone of actual development (ZAD).

4.9.3 Semi-structured interviews with the teachers

According to Naz, Gulab and Aslam (2022, p.42), using semi-structured interviews, researchers can know the experiences, opinions and research subjects' perceptions relating to the area of investigation. Since qualitative research has no absolute reality, individuals construct multiple realities in their cultural and social contexts. With semi-structured interviews, researchers get to know about these various realities. In qualitative research, validity is described using several terms, namely credibility, dependability, authenticity, truth-value, generalizability, trustworthiness and legitimation, which are ambiguous and difficult-to-define terms (Rose & Johnson, 2020, p.9). The researcher used semi-structured interviews following the Naz et al. (2022) protocol. The researcher probed teachers to get clarity and a deeper understanding of their perceptions regarding learners' errors and misconceptions in algebraic linear equations.

4.9.4 Classroom-observation schedule

The researcher used Sepeng's (2010) observation schedule in this study. See Table 4.2

4.10 TRUSTWORTHINESS

Trustworthiness is about the readers' confidence in the researcher's data collection methods and interpretation of the findings (Connelly, 2016, p.435; Stahl & King, 2020, p.26). Stahl and King (2020, p.26) state that there are four criteria for trustworthiness, which determine the quality of any qualitative research: credibility, dependability, confirmability and transferability.

Credibility refers to the readers' confidence that the study is a true account of the research's findings (Connelly, 2016, p.435). According to McGill, McCloskey, Smith, and Veitch (2023, p.6), credibility is a measure of plausible information obtained from participants, whether the researcher correctly interprets the participants' original views. To ensure credibility, the researcher interacted with participants for a long time to build rapport between researcher and participants. Additionally, this study was peer and supervisor-reviewed to ensure credibility. Member checking is another means the researcher used to ensure the authenticity of transcripts for this study. Data triangulation (Kalu & Bwalya, 2017, p.51) strengthens a study's credibility. The researcher used three different data collection methods to triangulate data collection, that is, pre-test and post-test, semi-structured interviews, and classroom observations. The researcher also reported negative findings if encountered during the analysis process.

Dependability refers to the stability of research findings (Kalu & Bwalya, 2017, p.51). According to McGill, McCloskey, Smith and Veitch (2023, p.6), dependability refers to how stable the research findings are over a period. Dependability also involves the notion that the researcher does not use his/her imagination when evaluating, interpreting and recommending research findings, but gives a true reflection of the data received from research participants (Stahl & King, 2020, p.27). To ensure dependability, the researcher kept notes of all activities for this study. Establishing rapport between the researcher and participants ensured that participants provided dependable information. In addition, the researcher probed participants for clarity and made follow-up questions where necessary to obtain dependable information.

Confirmability refers to whether research findings are repeatable by others (Connelly, 2016, p.435). (Korstjens & Moser, 2018, p.121) state that confirmability refers to how far research findings can be confirmed by others. The researcher maintained detailed notes of decisions taken during the analysis and progress of the study. In addition, the researcher conducted member checking with participants to ensure confirmability. The researcher also presented the study during tutorial lessons, received feedback and implemented suggestions in consultation with the supervisor.

Transferability refers to the usefulness of research findings in similar situations (Connelly, 2016, p. 435). According to Korstjens and Moser (2018, p.121), transferability is the degree to which research findings can be transferred to other settings or contexts. A thick description of the context of the study is given. In addition, the researcher ensured transparency during the analysis process.

4.11 ETHICAL CONSIDERATIONS

According to Kyngas, Mikkonen and Kaariainen (2020, p.50), research ethics refers to all ethical considerations that relate to research and science. The main aim of ethics in research is safeguarding the anonymity, confidentiality and protection of participants. Research ethics protects both the participants and the researcher. The researcher first applied for clearance to do research from UNISA's Ethics Research Committee. Upon getting the clearance certificate, the researcher sought permission to research in the Gauteng Province from the Gauteng Department of Education (GDE). Upon receiving an approval letter from the GDE, the researcher sought permission to do research in Johannesburg Central District from the District Director. Equipped with the relevant approval letters, the researcher sought permission to do research from the Principals and School Governing Body of selected schools.

In this study, the researcher asked for informed consent from the Grade 9 mathematics teachers of selected schools to participate in the research. The aim and objectives of the study were explained to all the participants. Their roles and rights in participating and not participating were also explained (Khan et al., 2021, p. 3821). The researcher sought parental permission for learners' participation. The researcher informed participants they would not receive any financial benefit from participating in this study. Participants' confidentiality and anonymity throughout the study were guaranteed. The researcher informed participants that they could withdraw from the study at any point and time without any penalties. To ensure anonymity, participants were informed that pseudonyms would be used instead of their real names, and all information that may lead to identifying participants would be coded. Teachers signed the consent forms and learners signed the assent forms to acknowledge having agreed to participate in this research. This ensured the protection of both the researcher and participants. The participants were informed that no personal data would be shared with unauthorised people. The researcher ensured that the services of a counsellor from the school or the district were available should there be any emotional discomfort among learners.

4.12 SUMMARY

The researcher discussed the research methodology of this study. The methodology section discusses the research paradigm, approaches, research design, instruments and strategies for data collection and analysis, and all the research processes. The researcher also discussed data collection and analysis procedures. Also discussed were issues of validity, reliability and trustworthiness, together with ethical issues in this chapter. The next chapter discusses the data analysis.

CHAPTER 5: PRESENTATION OF FINDINGS

5.1 INTRODUCTION

The previous chapter discussed the research methodology used in this study, a mixed-methods approach. This chapter reports the quantitative and qualitative data collected using the pre-test and post-test items, the lesson observations and the semi-structured interviews. The following subsections are found in this chapter: the analysis of interviews with the teachers before, during and after the intervention with the experimental group. The chapter also reports on the classroom observations before, during and after the intervention, the quantitative analysis of the pre-tests, the post-tests, the statistical analysis of the pre-tests, and post-tests and finally, the conclusion of the chapter.

This study intended to answer the following research questions:

1. What pedagogical approaches do teachers use to teach Grade 9 algebraic linear equations?
2. What are the types and possible causes of learners' errors in Grade 9 algebraic linear equations?
3. What are the benefits and challenges (if any) of using erroneous examples to teach Grade 9 algebraic linear equations?
4. How can teachers use erroneous examples for teaching Grade 9 algebraic linear equations to improve learners' performance?

Additionally, the study intends to answer the following hypotheses:

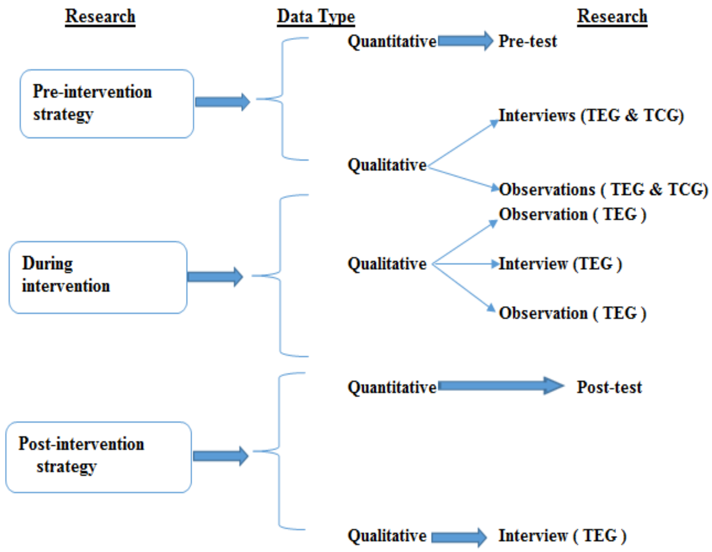
$H_{0\,a}$: There is no significant difference between pre-test and post-test results when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

$H_{1\,a:}$ A significant difference exists between pre-test and post-test results when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

$H_{0\,b:}$ There is no significant difference in learner performance between the experimental and control groups when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

$H_{1b:}$ A significant difference exists in learner performance between the experimental and control groups when teachers use erroneous examples to teach Grade 9 algebraic linear equations.

Figure 5.1: ***The structure for data collection for this study*** (Adopted from Makgakga 2016, p.114)



5.2 QUANTITATIVE ANALYSIS OF PRE-TEST RESULTS

This study used two public schools in the Johannesburg Central District of the Gauteng province. In the experimental group, $n = 31$, while in the control group $n = 28$. The ages of learners ranged from 14 to 16. There were 12 boys and 19 girls in the experimental group while the control group comprised 13 boys and 15 girls. All learners in the experimental group were Africans, while the control group had 13 African learners, 13 Indian learners and 2 Coloured learners. The researcher expected to have an equal number of learners in each group, 40 per group. However, some learners and their parents declined the invitation to participate.

Furthermore, the study intended to understand how learners' testing outcomes help explain why grade 9 learners experience difficulties in mathematics. To achieve this, the researcher administered a pre-test in both the experimental and control schools. The researcher implemented an intervention strategy in the experimental school to reduce or eliminate learners' errors when solving algebraic linear equations using the intervention strategy. The analysis of quantitative data for the pre-test is shown in Tables 5.1 to 5.8.

In this study, EG and CG codes are for the experimental and control groups, respectively. The researcher assigned these code names to the participating schools for anonymity as espoused in the research ethics code of conduct.

The researcher used Newman's error hierarchical order to categorise learners' errors when solving algebraic linear equations. Newman's error analysis assigns learners' errors into five categories: Reading errors, Comprehension errors, Transformation errors, Process skills errors and Encoding errors.

The following codes were used to categorise learners' responses to questions from Q1.1 to Q8.2: Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) where learners did not write anything in the answer space.

The pre-test was used to measure learners' performance in solving algebraic linear equations involving a variable on one side of the equal sign, equations involving variables on both sides of the equal sign, equations involving brackets on one side and both sides of the equal sign, equations involving fractions and algebraic linear equations derived from word problems. To distinguish between the responses of learners from the experimental group and those from the control group, the researcher coded from LE1 to LE31, for learners' responses from the experimental group, where LE1 is learner number 1 and LE31 is learner number 31. The codes LC1 to LC28 were used for learners' responses from the control group, where LC1 is learner number 1 and LC28 is learner number 28. The test questions for this study are shown in Appendix L. For analysis, questions 1.1 and 1.2 are grouped as one question and the others, which are 2.1 and 2.2, 3.1 and 3.2 up to 8.2.

5.2.1 Learners' answers in question 1

Table 5.1 below indicates data generated from Q1 items in which learners were required to identify the errors in Q.1.1 and solve Q1.2. Question 1.1 required learners to identify the error a hypothetical learner committed in the first step when solving the equation $2x + 3 = 15$, where s/he simplified the equation to $5x = 15$ in step 1 of the solution method. Learners were expected to indicate that the fictitious learner committed an error by adding unlike terms (conjoining) (Sanders & Pournara, 2019, p.94) on the left side of the equal sign, $2x + 3$ to give $5x$. In Q 1.2, learners were supposed to use knowledge of additive inverses to collect all numbers to one side of the equals sign before dividing by 2 to get the value of the unknown.

Table 5.1: Learners' answers in Q1.1 and Q1.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the experimental and control groups.

		PRE-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
1.1	EG	0	0%	0	0%	30	96.80%	0	0%	0	0%	0	0%	1	3.20%
	CG	10	35.71%	0	0%	18	64.29%	0	0%	0	0%	0	0%	0	0%
1.2	EG	0	0%	0	0%	30	96.80%	0	0%	0	0%	O	0%	1	3.20%
	CG	13	46.43%	0	0%	14	50%	0	0%	0	0%	1	3.57%	0	0%

Table 5.1 reveals that 0% of learners from the experimental group got a correct response for Q1.1 and Q1.2, whereas 35.71% and 46.43% of learners from the control group obtained correct responses for Q1.1 and Q1.2, respectively. Table 5.1 also shows that 96.80% of the learners in the EG committed comprehension errors on Q1.1 and Q1.2, whereas 64.29% and 50% of the learners from the CG committed comprehension errors on Q1.1 and Q1.2, respectively. The table also revealed no reading, transformation, process skills, or encoding errors on Q1 items. However, only 3.20% of the learners in the EG left Q1.1 and Q1.2 blank. Most of the learners from the two groups did not understand the demands of the question. The learners' responses revealed that most learners from both groups lacked relational thinking (Muchoko, Jupri & Prabawanto, 2019, p.2). Learners from both groups also lack the relational understanding of the equal sign's meaning and variables' meaning in the equation (ibid, p.2). Learners' responses showed they understood the equal sign as a symbol, signifying "doing something" to get the answer. For example, LE29's responses to Q1.1 below confirm most learners' difficulties in understanding the meaning of the equal sign.

Figure 5.2 Learner LEG29's response to Q1.1

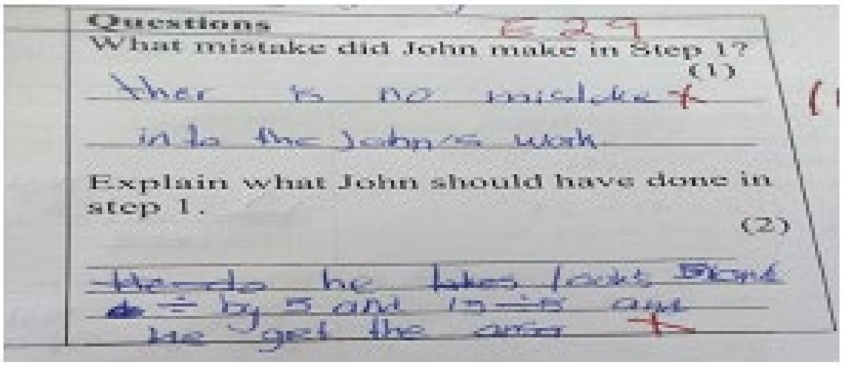


Fig 5.2 LEG 29's response

In the above sample, LE29 reveals that s/he found nothing wrong with simplifying $2x + 3$ to $5x$. This learner said there is no error in adding $2x + 3$ to get $5x$. This learner, EL29, like most learners in the EG, displayed a lack of understanding of operation with like and unlike terms, and s/he added different terms and joined them (conjoin) (Sanders & Pournara, 2019, p.94). When most learners see an addition sign, they think of adding terms (Ngoveni & Mofolo-Mbokane, 2019, p.240). Most of the learners in EG simply added terms on the left side of the equal sign and, therefore, could not get a correct answer to Q1.2. According to Newman's error analysis, the most common error committed by learners in this question was the comprehension error. Unlike terms, most learners did not understand that they should not add.

Figure 5.3 Learner LEG29's response to Q1.2

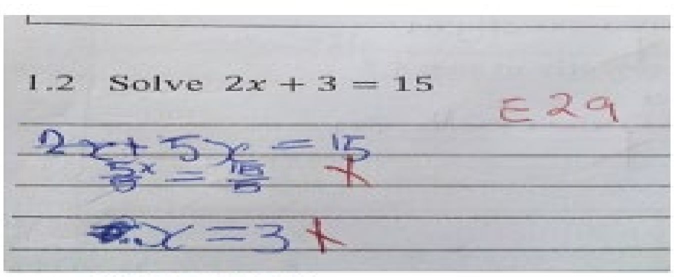


Fig 5.3 LEG29's response

In the excerpt above, this learner, LE29, collected unlike terms on the left-hand side of the equal sign and added them, $2x + 3$ and obtained $5x$. As stated earlier, learners tend to add different terms whenever they see an addition sign between them (Ngoveni & Mofolo-Mbokane, 2019, p.240). To get the correct answer, learners were supposed to subtract 3 from both sides of the equal sign to separate numbers from variables, that is $2x + 3 - 3 = 15 - 3$, which simplifies to $2x = 12$, giving the value$x = 6$.

5.2.2 Learners' answers in question 2

Table 5.2 below indicates data generated from Q2 items in which learners were required to identify the errors in Q.2.1. In Q2.1, learners were required to identify the error in solving the equation $4x - 3 = x + 15$ in which the first solution step was given as $4x - 3 - 3 = x + 15 - 3$. The error in the first solution step was reported in Pournara's (2020, p.2) study as a *switching addends error* in which the learner "moves" a term to the other side of the equal sign but fails to change the sign. This Q2.2 required learners to solve the equation $4x - 3 = x + 15$, showing all the necessary steps.

Table 5.2: **Learners' answers in** Q2.1 and Q2.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the experimental and control groups.

		PRE-TEST													
		CA		**RE**		**CE**		**TE**		**PSE**		**EnE**		**BL**	
Question	**Group**	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
2.1	**EG**	1	3.20%	0	0%	28	92.32%	0	0%	0	0%	0	0%	2	6.45%
	CG	4	14.29%	0	0%	22	78.57%	0	0%	0	0%	0	0%	2	7.14%
2.2	**EG**	0	0%	0	0%	29	93.55%	0	0%	2	6.45%	0	0%	0	0%
	CG	6	21.43%	0	0%	18	64.30%	0	0%	3	10.71%	0	0%	1	3.57%

Table 5.2 reveals that only 1 learner 3.20% the EG was able to answer question 2.1 correctly and 92.32% of the learners committed comprehension errors, whereas 14.29% of the learners in the CG were able to get correct answers for Q2.1 and 78.57% committed comprehension errors. The table also shows that 93.55% of learners from the experimental school committed comprehension errors in question 2.1 whereas 64.30% of the learners from the control group registered

comprehension errors. Only 6.45% and 10.71% of learners from the EG and CG respectively committed process skills errors. The majority of learners were not able to identify the error in Q2.1. However, in their second step most of the learners added unlike terms thereby committing a conjoining error. This can be seen in LC7's response below. The learner added $x + 12$ to get $12x$ on the right hand-side of the equal sign.

Figure 5.4 Learner LCG7's response Q2.2

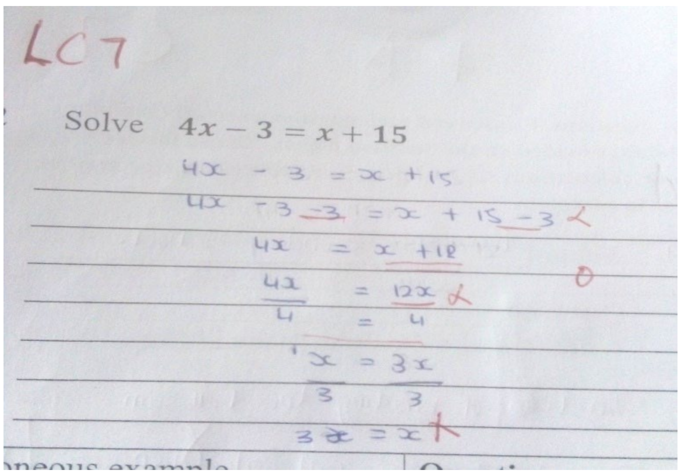


Figure 5.4: ***LCG7's response to Q2.2***

The learner was then faced with an unfamiliar situation where s/he had to solve the equation$x = 3x$. The learner divided both sides by 3 to reduce the equation to a familiar structure in which a similar error was found in Pournara's (2020, p.2) to get $3 = x$, ignoring the x in $\frac{x}{3}$ on the left side. Most of the learners committed the same problem when solving this equation. Pournara (2020, p.2) argues that when learners are faced with an unfamiliar structure in their solution steps, they "force" answers to fit the form $x = k$ which they are most familiar with.

Learners were supposed to add 3 to both sides of the equal sign to remove -3 from the left side and then subtract x from both sides to separate numbers from variables. That is $4x - 3 + 3 = x + 15 + 3$, giving $4x = x + 18$ and then $4x - x = x - x + 18$, giving $3x = 18$, resulting in $x = 6$. From the table, it can be seen that 0% of learners from the EG were able to solve question 2.2, while 21.43% of learners from the CG were able to get correct answers for question 2.2. Most learners committed the *switching addends error* (Pournara, 2020, p.2) when solving this problem.

The switching addends error results from learners' failure to understand the question and as a result, learners commit comprehension errors (Pournara, 2020, p.2).

5.2.3 Learners' answers in question 3

Table 5.3 below indicates data generated from Q3 items in which learners were required to identify the errors in Q3.1. A fictitious learner solved the equation, $2(x + 4) = 24$ and in his/her first step of solution, wrote $2x + 4 = 24$. The fictitious learner committed a carelessness error by failing to distribute 2 to both terms in the brackets. Learners were required to use their knowledge of the distributive laws to identify the error committed in this solution step. In Q3.2, learners were required to solve the equation, $2(x + 4) = 24,$ showing all the necessary steps.

Table 5.3: **Learners' answers in** Q3.1 and Q3.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the experimental and control groups.

		PRE-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
3.1	EG	1	3.20%	0	0%	30	96.80%	0	0%	0	0%	0	0%	0	0%
	CG	7	25%	0	0%	21	75%	0	0%	0	0%	0	0%	0	0%
3.2	EG	0	0%	0	0%	30	96.80%	0	0%	1	3.20%	0	0%	0	0%
	CG	11	39.30%	0	0%	16	57.14%	0	0%	1	3.75%	0	0%	0	0%

Table 5.3 shows that there were no blank pages meaning that all learners attempted answering all parts of Q3.1 and Q3.2. However, only 3.20% of the learners from the EG were able to get correct answers to question 3.1 while 25%% of the learners from the CG obtained correct answers to question 3.1. The table also shows that 0% of learners from the EG was able to answer question 3.2 correctly while 39.30% of learners from the CG managed to get correct responses. The table also shows that 96.80% of the leaners from the EG committed comprehension errors on Q3.1 and Q3.2 while 75% and 57.14% of the learners from the CG committed comprehension errors on Q3.1 and Q3.2 respectively. In Q3.2 learners were required to solve the equation $2(x + 4) = 24.$ However, most of the learners were not able to remove brackets correctly by distributing 2 to both terms inside the brackets. Most of the learners wrote $2x + 4 = 24$ as their first step in the solution

leading them to get incorrect responses. Since most of the learners failed to remove the brackets correctly by distributing the factor 2 to both terms inside the brackets, hence, learners committed a comprehension error. The table also shows that only 3.20% and 3.75% of learners from the EG and CG respectively, committed process skills errors on Q3.2. One of the learners wrote:

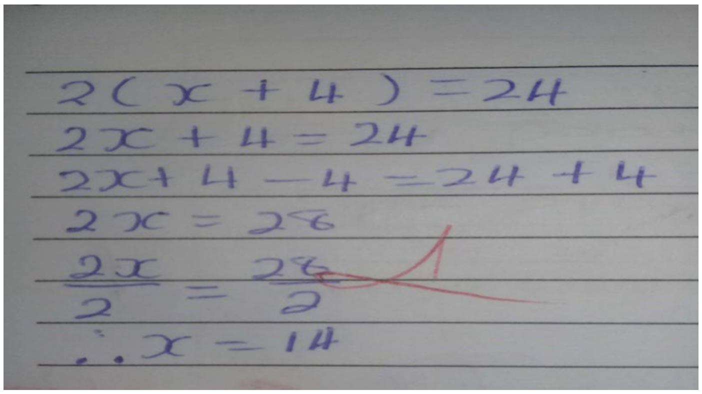


Figure 5.5: ***Learner LEG29's response to Q 3.2***

The most common error committed by learners from the two study groups is the comprehension error which is caused by insufficient conceptual understanding. Using conceptual knowledge, learners are able to understand the relationship between concepts and how ideas can be used to solve problems (Kilpatrick et al., 2001, p.118).

5.2.4 Learners' answers in question 4

Table 5.4 below depicts the data obtained from learners' responses to Q4.1 and 4.2 in which learners were required to identify the errors in Q4.1 and then solve Q4.2. In Q4.1, learners were required to state the error in solving the equation $2(x-4)=3-2(x+2)$, where the first step in solving this equation erroneously is $2x-4=1(x+2)$. The first step shows a comprehension error on removing brackets on the left side and a process skills error on subtracting before distribution on the right side. Q4.1 tested learners' understanding of the application of the distributive property to the left side of the equation and the application of the order of operations on the right hand-side. Q4.2 required learners to solve the equation $2(x-4)=3-2(x+2)$ showing all the necessary steps.

Table 5.4: **Learners' answers in** Q4.1 and Q4.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the experimental and control groups.

		PRE-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
4.1	EG	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	4	14.29%	0	0%	22	78.57%	0	0%	0	0%	0	0%	2	7.14%
4.2	EG	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	0	0%	0	0%	20	71.43%	0	0%	7	25%	0	0%	1	3.57%

From Table 5.4 above, it is clear that learners from two study groups found questions 4.1 and 4.2 challenging. 0% of learners from the EG solved question 4.1 correctly, while 14.29% of learners from the CG were able to answer question 4.1. The table also shows that 0% of learners from EG and CG could not answer question 4.2 correctly. The table also shows that 100% of the learners from the EG recorded comprehension errors in Q4.1 and Q4.2, whereas 78.57% and 71.43% of the learners from the CG recorded comprehension errors. Only 25% of the learners from the CG recorded process skills errors. Most of the learners from both groups simply copied the erroneous example in 4.1 as the answer to question 4.2. The most common error in this question was the comprehension error since most learners simply copied the erroneous example as the answer because they did not understand what to do in the question.

Learners' answers in questions 4.1 and 4.2 revealed that learners face challenges when solving algebraic linear equations involving brackets. Most of the learners could not distribute the term outside the bracket to all the terms in the bracket on the left-hand side. For the expression, $2(x - 4)$ most learners wrote $2x - 4$ instead of $2x - 8$. On the right-hand side, most learners first subtracted 2 from 3 and then distributed 1 to remove brackets and got $x + 2$ instead of first distributing -2 to get $3 - 2x - 4$. For example, LEG3's response is shown below:

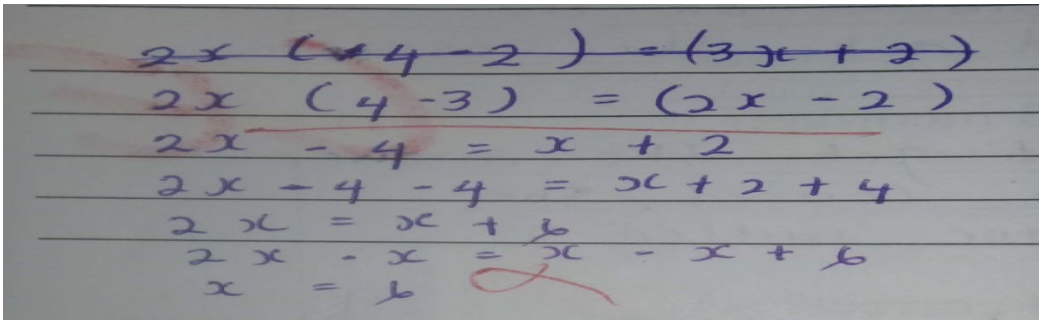


Figure 5.6: ***LEG3's response to Q4.2***

Learners' performance in Q4.1 and 4.2 revealed that most of the learners at the study schools lack procedural knowledge, which could be why they performed poorly. In their study on learners' misconceptions of algebra, researchers Ngoveni and Mofolo-Mbokane (2019, p.234) observed that learners who lack procedural knowledge perform poorly in tests and assignments.

5.2.5 Learners' answers in question 5

Table 5.5 below shows data collected from learners' answers in Q5.1 and Q5.2. Q5.1 required learners to justify the procedures to solve the equation $\frac{6x}{2} - 5 = 4$. In Q5.2, learners were required to solve the equation using a different approach. For example, simplifying $\frac{6x}{2}$ to $3x,$ thus reduce the equation to $3x - 5 = 4$, which is less complicated than the previous strategy. Alternatively, learners could have added 5 to both sides to get $\frac{6x}{2} = 9$ and then multiplied by 2 on both sides to get $6x = 18$ and proceeded to find the answer, $x = 3$.

Table 5.5: **Learners' answers in** Q5.1 and Q5.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the experimental and control groups.

		PRE-TEST													
		CA		**RE**		**CE**		**TE**		**PSE**		**EnE**		**BL**	
Question	**Group**	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
5.1	**EG**	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	3	10.71%	0	0%	23	82.14%	0	0%	0	0%	0	0%	2	7.14%
5.2	**EG**	0	0%	0	0%	30	96.80%	0	0%	1	3.20%	0	0%	0	0%
	CG	9	32.14%	0	0%	15	53.57%	0	0%	0	0%	0	0%	4	14.29%

Table 5.5 above indicates that 0% of learners from the EG were able to get a correct response to Q5.1 and Q5.2, whereas 10.71% and 32.14% of the learners from the CG recorded correct answers on Q5.1and Q5.2 items. The table also shows that no learners from the two groups committed reading, transformation and encoding errors. However, the most common error committed by learners in this question was the comprehension error, in which 100% and 96.80% of the learners in EG recorded this error in Q5.1 and Q5.2, respectively. In comparison, 82.14% and 53.57% of the learners in the CG recorded this type of error in Q5.1 and Q5.2, respectively. Learners from the EG attempted both Q5.1 and Q5.2, even though no one obtained a correct answer. The table also shows that 7.14% and 14.29% of the learners from the CG did not answer Q5.1 and Q5.2, respectively. For instance, LEG9's response is shown below:

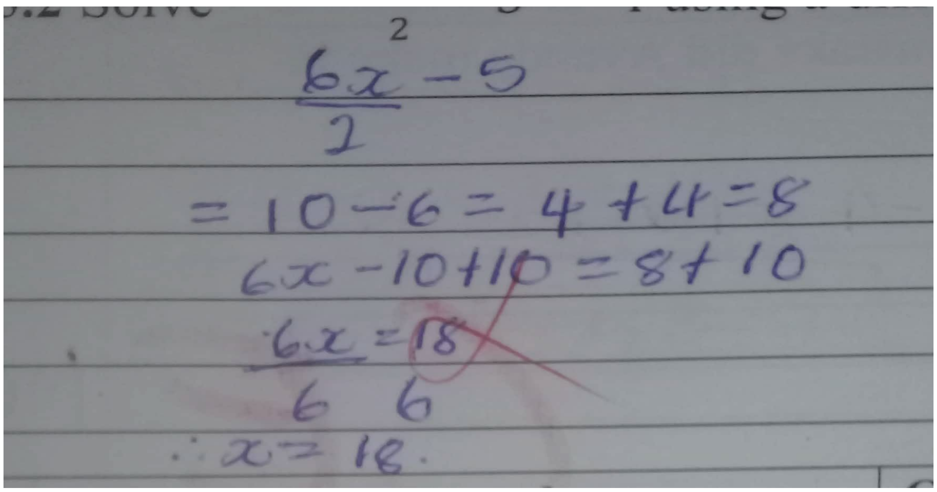


Figure 5.7: ***LEG9's response to Q5.2***

Question 5.1 required learners to state what was done correctly in steps 1 and 3, and most of the learners were unable to explain what was done correctly. This could be due to language constraints since most learners speak isiZulu as their L1 and English as L2. In support, Cross et al. (2019, p.150) state that learners' understanding of the written language determines their success in academic subjects.

Question 5.2 required learners to demonstrate conceptual understanding and procedural flexibility (Al-Mutawah et al., 2019, p.263). However, most learners revealed that they did not know any other approach to solving the problem. Thus, they committed a comprehension error because they just copied the work, as shown in the example. According to the authors, when learners are able to use several solution methods for a particular problem, they demonstrate conceptual understanding. However, most of the learners from both groups showed that they had no alternative strategy for solving the problem revealing that learners had limited conceptual knowledge. This could have been due to the teacher's pedagogical strategies whereby the teacher did not expose learners to different methods for solving one problem.

5.2.6 Learners' answers in question 6

Table 5.6 depicts a collection of data from learners' responses to Q6.1 and Q6.2. This Q6.1 required learners to identify errors committed in the erroneous example when solving the equation involving fractions with different denominators, whereas Q6.2 asked learners to solve the equation, showing clearly all the necessary steps. In Q6.1, learners were supposed to identify the error in solving the equation $\frac{x}{2}+\frac{x}{3}=\frac{1}{6}$, in which the first solution step was shown as $\frac{2x}{5}=\frac{1}{6}$.

Table 5.6: **Learners' answers in** Q6.1 and Q6.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the experimental and control groups.

		PRE-TEST													
		CA		**RE**		**CE**		**TE**		**PSE**		**EnE**		**BL**	
Question	**Group**	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
6.1	**EG**	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	5	17.89%	0	0%	18	64.29%	0	0%	0	0%	0	0%	5	17.89%
6.2	**EG**	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	3	10.71%	0	0%	19	67.89%	0	0%	2	7.14%	1	3.57%	3	10.71%

As reflected in Table 5.6 above, 0% of learners from the EG were able to answer Q6.1 and Q6.2, while 17.89% and 10.71% of the learners from the EG were able to get correct answers to Q6.1 and Q6.2, respectively. The table shows that 100% of the learners in the EG recorded comprehension errors in Q6.1 and Q6.2, whereas 64.29% and 67.89% of the learners in the CG recorded comprehension errors in Q6.1 and Q6.2, respectively. Both groups recorded no reading, transformation, process skills, and encoding errors on Q6.1. However, 7.14% and 3.57% of the learners in the CG recorded process skills and encoding errors in Q6.2.

The most common error committed by learners on Q6 was comprehension error. For instance, most learners wrote $\frac{x}{2}+\frac{x}{3}=\frac{1}{6}$ and simplified it to $\frac{x}{5}=\frac{1}{6}$. This showed that learners did not understand that in the question, what is common in this question are the numerators on the left-hand side. However, learners treated the question as having the same denominators, revealing that they lack conceptual knowledge of solving equations involving fractions with different denominators. Learners require conceptual knowledge to understand concepts and operations, which will enable them to solve problems (Al-Mutawah et al., 2019, p.259). The authors further argue that the majority of learners did not perform well in mathematics due to lack of conceptual understanding.

5.2.7 Learners' answers in question 7

Table 5.7 shows data collected from learners' responses to Q7.1 and Q7.2. In Q7 learners were required to find the dimension of a rectangle given that the length of the rectangle is twice the size of the width and that the perimeter of the rectangle is 24cm. Learners were required to demonstrate strategic competence in order to solve Q7. To demonstrate strategic competence a learner should be able to transform information given in word problems into equations or diagrams that would help them to solve the mathematical problems (Al-Mutawah et al., 2019, p.259). Question 7 tested learners' understanding of the concept of perimeter and how they could use algebraic reasoning to find the dimensions of the rectangle. Learners were asked to identify the error committed by a hypothetical learner in solving the problem in Q7.1.

Table 5.7: **Learners' answers in** Q7.1 and Q7.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the experimental and control groups.

		PRE-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
7.1	EG	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	4	14.29%	0	0%	15	53.57%	3	10.71%	1	3.57%	0	0%	5	17.89%
7.2	EG	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	2	7.14%	0	0%	15	53.57%	4	14.29%	1	3.57%	1	3.57%	5	17.89%

Table 5.7 above reveals that 0% of the learners in the EG were able to answer questions 7.1 and 7.2 correctly, while 14.29% and 7.14% of the learners in the CG were able to get correct answers

on Q7.1 and Q7.2 respectively. Table 5.7 also shows that 100% of learners in the EG registered comprehension errors in Q7.1 and Q7.2, while 53.57% of the learners in the CG recorded comprehension errors in Q7.1 and Q7.2. The percentages of transformation errors for the CG ranged from 10.71% to 14.29% in Q7.1 and Q7.2, respectively. Only 3.57% of the learners in the CG recorded process skills errors and encoding errors in q7.1 and Q7.2, respectively. Most of the learners in the two study groups did not understand how to relate the given dimensions of the rectangle to the perimeter, and as a result, most of the learners just added the length and the width and equated to the perimeter. Due to a lack of conceptual knowledge (Kilpatrick et al., 2001, p.124), learners were not able to represent the given information diagrammatically by sketching a rectangle, which could have helped them realise that a rectangle has two pairs of equal sides, which should add up to 24cm. The table showed that most of the learners in the two study groups lack understanding of perimeter; hence, they could not transform the words and statements in the question into equations and diagrams to help them solve the problem.

5.2.8 Learners' answers in question 8

Table 5.8 depicts data obtained from learners' responses to Q8.1 and Q8.2. In Q8.1, learners were required to identify the error committed by a hypothetical learner when finding the current ages of two boys, given that one of them is five times as old as the other and that after four years, the older boy will be three times the age of the younger one. Q8.2 required learners to solve the problem, showing clearly all the necessary steps.

Table 5.8: **Learners' answers in** Q8.1 and Q8.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the EG and CG.

		PRE-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	**Group**	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
8.1	**EG**	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	2	7.14%	0	0%	17	60.71%	2	7.14%	0	0%	0	0%	7	25%
8.2	**EG**	0	0%	0	0%	31	100%	0	0%	0	0%	0	0%	0	0%
	CG	0	0%	0	0%	18	64.29%	3	10.71%	0	0%	0	0%	7	25%

The quantitative analysis of learners' responses to questions 8.1 and 8.2, as shown in Table 5.8, reveals that 0% of learners from the EG were able to respond correctly to questions 8.1 and 8.2, whereas 7.14% and 0% of the learners in the CG were able to correct answers to Q8.1and Q8.2 respectively. The table also showed that 100% of the EG committed comprehension errors on Q8 items, while 60.71% and 64.29% of the learners in the CG committed comprehension errors on Q8 items. Learners in the EG recorded no transformation, process skills, or encoding errors, while 7.14% and 10.71% of the learners in the CG committed transformation errors in Q8 items. Table 5.8 shows that the most common error committed by learners in both groups was the comprehension error. Most learners did not understand the meanings of the words and statements in the question. For instance, most of the learners did not understand statements like "five times as old as", "after four years", and "three times as old as", hence most of the learners just copied the erroneous example as their answers.

The table shows that learners at the two study schools struggle to solve word problems involving algebraic linear equations. Most of the learners at the two schools did not understand the information in the question, showing that they lacked conceptual understanding, which is flexible and generalisable (Rittle-Johnson et al., 2017, p.7). Learners' failure to solve Q8 items could have been due to learners' lack of strategic competence (Muchoko et al., 2019, p.2). Learners with strategic competence are able to come up with equations and expressions that they can use to help them solve mathematical problems (ibid).

The learners' responses to the test items help to explain why learners experience difficulties in mathematics. Most of the learners lacked an understanding of the demands of the questions, and this explains why learners performed poorly in the pre-test.

5.3 TEACHERS' SEMI-STRUCTURED INTERVIEWS BEFORE INTERVENTION

Immediately after giving pre-tests at the participating schools, the researcher interviewed the two teachers to understand of their perceptions regarding the learners' poor performance in algebraic linear equations. The researcher utilised Bandura's theory to analyse the data, adopting motivation and attention, as the framework for analysis.

The results of the pre-intervention interviews are discussed here. The rationale for these interviews was to know the problems faced by the two mathematics teachers. The researcher asked the two teachers the same questions. Pseudonyms TEG and TCG were used to name the two teachers where TEG and TCG represent teacher at experimental school and teacher at control school respectively. All questions were asked and answered in English.

5.3.1 Pre-intervention analysis of interviews

The two teachers were asked the italicised categories of questions in the same order.

Mathematics challenges: In your opinion, what could be the reasons for learners' poor performance and why?

Interaction in classroom: How do you encourage active learner participation during lessons?

Teaching resources: What teaching resources do you use in your classroom?

Assessment: What classroom assessments do you use in mathematics, and how?

Supporting learners: How do you support learners with challenges in mathematics?

Classroom culture: How do you promote a teaching and learning culture in your classroom?

The section below presents some of the extracts used as examples in thematic form. The first interview question required the teachers to provide their perception of learners' challenges when solving algebraic linear equations and the possible reasons. Teachers' responses were organised under the following themes: Comprehension of linear equations, Motivation for learning, Teaching resources, Assessment of learning outcomes, Learner support and Learner motivation.

5.3.1.1 Comprehension of linear equations

Transcript 1: In your opinion, what could be the reasons for learners' poor performance and why?

The response by the teacher from the experimental group to the first interview question is shown in Extract 5.1 below.

Extract 5.1 TEG: *The possible reasons could be misconceptions and inheritance of mistakes from previous classes. In addition, learners are not in a position to write work that is homework.*

Extract 5.2 TCG: *First, …maybe the foundation is poor, I think if they don't have the foundation skills, they can't. If they don't understand the basic simple things like HCF and LCM which they have done at primary school and they still can't do it. I think the foundation was bad. Also, …sometimes learners know what to do they don't just understand the question. They can't pick up keywords in the question. So, they mix equations with algebraic expressions. They mix up HCF with prime factorisation because they don't pick up keywords so they can't answer the questions correctly. What they also do is, they start with an expression, they turn it into an equation, and they add unlike terms. So, they combine all the knowledge they have that's how they come to the answer. I think that's why they do bad.*

According to the TEG, learners' poor performance in mathematics is because of misconceptions and mistakes they acquire in earlier classes and learners' unwillingness to write their homework for practice because they lack comprehension of what they read. Learners' unwillingness to write homework could be due to a lack of motivation to learn, possibly because they think mathematics is for the talented few. This concurs with Varaidzai, Makondo and Makondo (2020, p.10), who assert that learners' poor performance is attributed to lack of motivation and poor attitude towards mathematics. This supports Bandura's (1986) theory of learning, which states that motivation plays an important role in learning because learners are most likely prepared to learn any material that is within their liking and interest. TEG's response also points to the fact that learners have poor backgrounds in mathematics when he says it "*could be the inheritance of mistakes from previous classes".* This confirms learners' poor performance in the item that required learners to demonstrate their understanding and application of knowledge of fractions, which they learned in Grade 8.

Like TEG, TCG attributes learners' poor performance to poor background, "*maybe the foundation is poor, I think. If they don't have the foundation skills, they can't".* Additionally, TCG believes that learners perform poorly because they cannot differentiate between concepts; "*They mix equations with algebraic expressions. They mix up HCF with prime factorisation because they don't pick up keywords, so they can't answer the questions correctly*". TCG's response implies that learners have difficulties comprehending questions. According to Newman's Error Analysis, comprehending a question is key to answering the question. This supports that some learners at CG performed poorly on questions requiring them to remember the most basic concepts they learned in Grade 8. Due to poor comprehension skills, learners *"add unlike terms. So, they combine all the knowledge that they have..."* to answer questions, suggesting TCG is one of the reasons why learners do poorly in tests. Additionally, according to TCG, learners' failure to "*pick up keywords*" results in them performing poorly.

5.3.1.2 Motivation for learning

Transcript 2: How do you encourage active learner participation during lessons?

Extracts 5.3 and **5.4** below describe how TEG and TCG get learners involved during lessons.

Extract 5.3 TEG: *We usually give them individual attention as a mechanism to realise our goal. We also use question and answer so as to arouse interest and make them focus.*

Extract 5.4 TCG: *For me, my classroom is always very personal. So, I always want to know what is happening in their lives, and I always talk with learners so that I know their weaknesses and strengths. For weaker children, I always ask them to do the easiest sum on the board so that they can be confident, and then I always praise them, but I also tell them when they are not doing well. But I like learners to come up to the board and show me what they are doing so that I can see if they know what they are doing.*

TEG's response indicates that teachers at the experimental school use different teaching styles which allow active learner participation during lessons and said, *"We usually give them individual attention as a mechanism to realise our goal"*. In addition, TEG suggests that teachers at the experimental school make an effort to try to get learners motivated and focused to learn by using the question-and-answer approach.

From Extract 5.4 above, it is evident that TCG is passionate and wants learners to perform well in class. The teacher makes an effort to know the learners' background, as is indicated in this statement, "*So, I always want to know what is happening in their lives, and I always take time to get to know the learners so that I know their weaknesses and strengths".* TCG also involves learners in teaching and learning by allowing weak learners to come to the board and solve the easiest problems to boost their confidence. Additionally, TCG praises learners when they do well to encourage them to display what they know. However, she also states that she highlights learners' mistakes when they go wrong. TCG wants learners to feel confident and gives them a chance to solve problems on the board. The statement, "*I like learners to come up to the board and show me what they are doing so that I can see if they know what they are doing",* reveals that the teacher is passionate and wants to see learners performing well in class. TCG encourages cooperative learning by pairing weak learners with more capable learners.

5.3.1.3 Teaching resources

Transcript 3: What teaching resources do you use in your classroom?

Extract 5.5 below describes the types of teaching materials that TEG uses for teaching mathematics.

Extract 5.5 TEG: *Looking at our digital divide, one will always use the traditional method, that is, the chalkboard. Today we experienced load shedding. It would be a futile exercise to make use of smart boards and other materials. So whiteboards and teaching boards are the ones that are more reliable, although we experience problems of lack of resources such as markers and budgetary constraints.*

Extract 5.6 TCG: *I think the first material is the board and myself because we don't have resources to teach mathematics, and none of our classes have smart boards. We can't play YouTube videos because our learners cannot access the internet. Therefore, it is just very basic. It is chalk and board and textbooks, which I have. Yes, I use all the textbooks, which is Platinum. I don't make copies of anything they use the* *textbook. If I want some materials, I send it to the group, and they go through it, but I think because of a lack of resources, that's all we have: the textbook.*

From Extract 5.5 above, it is clear that TEG uses ICT in the form of smart boards. However, the teacher mentions that mathematics teaching is affected when there is load shedding because they cannot use smart boards. TEG also laments the shortage of markers because of budgetary constraints. Ultimately, TEG's response shows a shortage of learning and teaching materials (LTSM) at the experimental school, which could be the reason for learners' unsatisfactory performance in the pre-test.

From Extract 5.6 above, it is evident that the resources for teaching at CG are chalkboards and textbooks. However, TCG occasionally uses WhatsApp groups to send extra materials to learners. Access to the Internet is a challenge, so the teacher cannot use YouTube videos. The teacher uses a chalkboard and textbooks. The use of textbooks by learners at CG provided them the chance to learn strategies for solving problems (Sievert, van den Ham, Neidermeyer & Heinze, 2019, p.2), and they performed better than the learners at EG did.

5.3.1.4 Assessment of learning outcomes

Transcript 4: What classroom assessments do you use in mathematics, and how?
Extracts 5.7 and **5.8** below are TEG and TCG responses on the types of assessments they use in mathematics.

Extract 5.7 TEG: *We use all types of assessments depending on what we want to achieve. One type of assessment, ...especially when you are to determine learners' backgrounds, is diagnostic analysis. We also use formative and summative depending on the goals we want to reach or realise. Sometimes, we also use a baseline to ascertain language barriers, not language but learning barriers of learners, what they know and have acquired.*

Extract 5.8 TCG: *Informal assessment…, often we give them informal tests so they can continuously practice, and I can monitor them to see what they are* *doing. Taking past papers is a good way to get them to practice. So, I do previous papers every Friday and walk around and see what they are doing. I also play games, but that's not a formal assessment, but ah…, yes, informal assessment. …but in the formal assessment, there are investigations, assignments, projects and tests.*

Researcher: How do you administer them?

Extract 5.9 TCG: *The informal ones I write on the board. If I have extra paper, I give it to them. They answer the questions. I give them about 15 minutes, then I start going around, and I will pick up all the mistakes they make. About 10 minutes before the end of the period, we just give it to our partners and then mark it.*

Researcher: And for the formal ones, the assignment?

Extract 5.10 TCG: *Assignment, …they do in school. They are allowed to use resources. They can use their textbooks and notebooks; if they know what they are writing, they can bring in other resources. And once they are done, it comes to me, and I mark it. After marking, I give it back to learners so they can make corrections and check if marks are added correctly.*

TEG acknowledges that assessments are given depending on their goals. EG did not specify the type of assessments, for instance, *assignment, investigation and project.* However, TEG says, "*When you want to determine the background of the learner, it is diagnostic analysis"*. TEG also mentioned using baseline assessment to ascertain the learning barriers. TEG's responses show that the teacher is concerned about learners' poor performance and, as such, identifies learners with learning barriers. TEG's response indicates that scaffolding (Vygotsky) assists learners in their ZPD.

TCG, like TEG, revealed using all types of assessments. TCG uses informal tests every Friday to assess learners' understanding of all the work covered for the week so that learners can continuously monitor their progress. The teacher indicated that past papers are used as informal tests for learners to practice. TCG moves around when learners write informal tests, checking learners' work to see if they make mistakes and how they can be helped. TCG uses peer marking for all the informal tests: "And *about 10 minutes before the end of the period, we just give it to our partners and then mark it."* TCG's extract shows that the teacher prepares learners for formal assessments by giving them informal tests every Friday. TCG also makes an effort to help learners work on their mistakes in informal tests, which could be why some learners did well in the pre-test. However, some learners might not take informal tests seriously and do not perform well in formal tests, as seen in the pre-test.

TCG indicates that learners can use their textbooks, notebooks and other necessary resources to answer questions for formal assessments. This aligns with departmental policy on assessment, especially for assignments, investigations and projects. TCG also gives learners immediate feedback (Jug, Xiang & Bean, 2019, p.244). As the extract indicates, "*Then you give it back to learners, and they do some corrective work, check if the marks are added up correctly".* According to Jug et al. (2019, p.244), immediate feedback allows early correction of mistakes and timely reinforcement of good performance.

5.3.1.5 Learner support

Transcript 5: How do you support learners with challenges in mathematics?

The extracts below reflect the teachers' perceptions of how they support learners facing challenges in mathematics.

Extract 5.11TEG: *The support could be one-on-one. Sometimes, we profile them. When we profile them, it gives us an opportunity to do face-to-face interaction. In addition, one gives them extra work, from simple to complex, and monitors their progress.*

Extract 5.12 TCG: *Because of our class size, giving each child individual attention is quite difficult. Perhaps teaching and re-teaching is the support we can give them. Revising past papers. Let them do a lot more homework activities I think that will help them. However, in terms of our class sizes, getting to* *every child is very difficult, but our school offers intervention. There is an extra period, so we try to sort out the problems during that period, however, it is not enough either. Like one period a week doesn't cover all learners' problems. Difficulties for learners: I think they needed more time. So, there should be an intervention support programme maybe every day. So, learners may come in and do their work. And if they don't understand, perhaps there should be a teacher to assist them. Also, peer learning is also important. If learners don't understand something, I put them next to a strong learner, and I will ask that child to explain it to them because sometimes they are comfortable asking their friends questions, and it works. In the class, if I find that someone is very weak, I place them with a strong learner, not for them to copy, but for them to feel more comfortable and understand better. But sometimes that can go wrong because they always copy from their friend.*

TEG's response indicates that teachers at the experimental school support learners with difficulties in mathematics. "*Sometimes we profile them. When we profile them, it gives us an opportunity to interact face-to-face".* TEG's response indicates that learners do not receive regular support. In addition, only when teachers profile learners do they interact face-to-face with the teacher and receive graded extra work designed to monitor their progress. Teachers at the experimental school occasionally provide face-to-face learner assistance because most learners do not do their homework due to a lack of motivation, as indicated in Extract 5.1 above.

Extract 5.12 reveals that TCG supports learners with difficulties in mathematics.
TCG acknowledges that it is a challenge to provide individual attention to learners with difficulties in mathematics and indicated that re-teaching is the support that could be given. The teacher also indicated that past papers are revised with learners, and learners are also given more homework activities. TCG mentions an intervention once a week for each class, even though there is not enough time to address all problems in one period. According to TCG, if the school could hire an intervention teacher to assist learners daily, the problem might be sorted. TCG also indicates that peer teaching is sometimes used by placing a weak learner with a more competent learner. This is in line with Vygotsky's scaffolding. TCG is cognisant that some learners feel comfortable and

learn from their peers. However, the teacher cautioned that the weaker learner should be monitored so that s/he may not just copy from the friend.

The above extracts show that at both of the study schools, some support is given to learners with difficulties in mathematics, even though teachers should do more to assist them.

5.3.1.6 Learner motivation

Transcript 6: How do you promote a teaching and learning culture in your classroom?

Extract 5.13 TEG: *The culture is established through inviting parents to interact with learners, and to interact with the school activities. For the culture, we always set up a mechanism, code of conduct, and policies that will always guide learners to adhere to the ways. In building culture, we try to look at the vision. That vision will then inform the mission, and the mission will help in building a strategy. By so doing, I think this will help build a culture of learning.*

Extract 5.14 TCG: *So, first of all, discipline is most important. The learners need to be prepared. When I walk in, they must know we [gona] do mathematics. Their textbooks must be out, and their notebooks must be out. All homework must be marked because, at the end of our textbooks, we do have answers. I make sure the children mark it unless it is from the board, and then I will go through that. When I go back, I ask them if the same children would like me to re-show them, and I do that. And we start with the work, the topic. And when we do the work, there is a lot of interaction with learners in the classroom. So, it's not just me to do the work. I explain, and I question them to see whether they understand or not. Also, it is important to do revision so that you know, perhaps let us say you start with algebraic expressions, just remind them what [was.] Tell me about the things that you did a long time ago. What's a prime number what is? Just that they constantly remember. That's why I think we have* *a test every Friday, and you test them on everything. Then, it helps them remember because learners forget very easily.*

TEG's response reveals that learners are introduced to the teaching and learning culture at the experimental school by getting parents involved in their children's learning. Teachers can get parents engaged in school activities by ensuring that parents monitor learners' homework activities (Hussain et al., 2022, p.452). Parental engagement might also enhance learners' motivation to learn and overcome their attitude towards learning (ibid). TEG states that the school has a code of conduct that guides learners on conducting themselves in class and interacting with each other outside the classroom.

TCG's first response emphasises learner discipline. According to the teacher, discipline and orderliness are important facets of effective learning. Learners should have everything for the lesson ready before the teacher enters the classroom, as revealed in the statements, "*When I walk in, they must know we [gona] do mathematics. Their textbooks must be out, and their notebooks must be out".* TCG also believes learners should take responsibility for their learning by participating in the learning process. Learners at CG should mark their work and identify where they have difficulty understanding before coming to school so that the teacher can revise and explain before teaching new work. TCG also encourages a culture of learning by interacting with them during lessons and says, "*So it's not just me to do the work. I explain, and I question them to see whether they understand or not".* suggesting that question and answer method is used during learning and teaching.

From extract 5.14, it is clear that TCG utilises learners' prior knowledge during teaching and learning (Maryam et al., 2020, p.81). TCG's question and answer teaching approach supports learners (scaffolding) (Maryam et al., 2020, p.81). Using the question and technique, TCG is able to ascertain what learners still remember from previous lessons and related to the current lesson. In this way, the teacher helps learners to relate what they already know to what they are currently learning. TCG demonstrates to learners how concepts are interconnected.

5.4 LESSON OBSERVATION AFTER PRE-TEST

Before implementing the intervention, the researcher held three observation sessions with both teachers to investigate their pedagogical approaches to teaching Grade 9 algebraic linear equations.

The researcher used Sepeng's (2010) observational schedule to collect the following information: teachers' methods of teaching methods, interaction in classroom, resources for teaching and learning, and classroom assessment. Theoretical resources of the ZPD and scaffolding were used to analyse the data.

5.4.1 Teaching methods applied

TEG's teaching approach in the experimental school was predominantly the "telling method". In this approach, the teacher owns and distributes information in the classroom. TEG used the chalkboard all the times and the teaching was teacher-centered. TEG would tell learners what they were going to learn on the day and wrote the topic on the chalkboard. For instance, the teacher said "*Today we are going to learn about how to solve an equation with variables on the left and right sides of the equal sign"* and writes $2x + 5 = x + 9$. TEG did not make use of the learners' prior knowledge. TEG could have asked learners to solve the equation using what they already know about solving algebraic linear equations with a variable on one side before demonstrating the solution steps for solving one with variables on both sides to allow learners to learn from their mistakes (Loib & Rummel, 2014, p.74). TEG systematically demonstrated how to solve the algebraic linear equation involving variables on both sides of an equal and learners copied the work into their notebooks. Through demonstrations on the chalkboard, TEG provided scaffolding (Margolis, 2020, p.18).

Learners in the EG worked individually and the teacher did not walk around to check on the learners' progress. However, TCG would introduce lessons by referring to the previous lesson's concepts after marking homework for the previous day. TCG applied scaffolding (Margolis, 2020, p.18) using the learners' previous knowledge as a starting point for the lessons. TCG introduced lessons by asking learners the difference between an expression and an equation. Most of the learners could not state the difference and the teacher wrote both the equation and expression on the board equation: $x + 5 = 15$; expression$x + 5$. TCG demonstrated to learners that in an equation the variable has a specific value that will make the statement true whereas in an expression the variable can take any value. This was in support of the statement in the interview where it was stated that, "*Tell me about the things that you did a long time ago. What's a prime number what is etc., just so that they constantly remember"*. Learners in the CG actively participated in the learning process. Learners were paired for them to assist one another when they face challenges. TCG involved learners in the learning process by applying the ZPD in guiding learners having difficulties in solving equations on the chalkboard.

5.4.2 Classroom interaction

Getting learners to participate is crucial for achieving learners' satisfaction and understanding of concepts (Martin, Parker & Deale, 2020, p.227). The (ibid) argue that when learners participate in class, they are likely going to perform well. TEG did not allow learners to be active participants in the learning process because his teaching was teacher-centred. Learners did not get the chance to interact with each other to share ideas and help one another. Each learner was working individually. TEG arranged desks in single rows and each learner sat alone doing his/her work.

Unlike TEG, TCG allowed learner interaction by allowing them to sit in pairs. Learners at CG had the chance share ideas. Learners could compare their answers and in so doing, learn from one another. The CG allowed learners to display their understanding by presenting their solutions on the chalkboard and solve problems. In one instance TCG asked a learner to solve the equation $2(x+3)=12$ on the chalk board. The TCG randomly chose learners to come to the board and demonstrate their understanding. TCG did not focus only on learners who volunteer answers, but would encourage all learners to volunteer so that they may learn from their own mistakes. When a learner showed that s/he was facing difficulties solving a problem, TCG would ask others to contribute towards solving the problem.

TCG showed eagerness to motivate learners to learn from their mistakes by encouraging them to volunteer answers. TCG always engaged learners by encouraging them to volunteer answers (Havik & Westergard, 2020, p.448). The (ibid) assert that motivated learners pay attention in class, participate in classroom discussions and show interest in learning. According to Havik and Westergard (2020), when learners work in groups, they discuss, ask each other and their teacher questions, critically listen to each other and present arguments using their own examples from experience.

5.4.3 Teaching and learning materials

Teaching resources are crucial for the effective accomplishment of the curriculum. From time immemorial, textbooks had been the major resource for the mathematics teacher (Ulusoy &

Incikabi, 2020, p.1). As envisaged in the curriculum and instruction, the goal is to use all influence on the design and production of textbooks (Ulusoy & Incikabi, 2020, p.1). The authors further opine that textbooks serve the function of a mediator between the teacher and the curriculum. According to Gracin and Matic (2016, p.351), textbooks are specifically developed resources for classroom teaching and learning. TEG and TCG used textbook as their main resource for teaching and learning. They complied classwork and homework activities from textbooks. This concurs with Ulusoy and Incikabi (2020, p.1) that teachers use textbooks to select tasks and guide instruction.

However, in addition to the textbook, TCG used learners' workbooks as an alternative source for classwork and homework activities. This is supported by Utami, Aminatun, and Fatriana (2020, p.7), who state that student workbooks are important teaching and learning resource. The (ibid) further contend that workbooks benefit learning because they provide learners with explanations that simplify learning material, making it easier to understand concepts. Both teachers used the DBE mathematics workbooks. In addition to textbooks and workbooks, both schools had smart boards. However, the researcher did not observe how the teachers use smart boards due to load shedding.

5.4.4 Classroom assessment

Both TEG and TCG gave learners classwork. Learners at CG exchanged their notebooks, whereas learners at EG marked their own work during revision. By letting learners exchange their workbooks, TCG enabled learners to see each other's solution methods, thereby allowing them to develop strategic competence since they would compare their working approaches. Durkin, Rittle-Johnson, Star & Loehr (2023, p.1) state that allowing learners to share and compare how they solve particular problems is a reform pedagogy in practice. Taylor et al. (2020, p.271) state that letting learners mark their own work or each other's is an effective method of encouraging learners to self-regulate and promote self-efficacy and improvement. Both teachers used classwork as a form of formative assessment that allowed learners to monitor their understanding of concepts. Written work also allows the teacher to evaluate the extent to which learners understand the topic and make adjustments where necessary (Taylor et al., 2020, p.271).

In addition to classwork, both teachers gave their learners homework to allow them to retain concepts taught (Bandura, 1986) in class through further practice at home (William & William, 2021, p.210). The (ibid) further argue that homework promotes learner autonomy because learners work on their own, so they take responsibility for making their own knowledge. Hussain et al. (2022, p.452) assert that homework gives learners a chance to put more effort where they need to improve and, at the same time, enhances parental involvement. Also, through homework, parents monitor their children's progress (ibid).

5.5 INTERVENTION STRATEGY: USING ERRONEOUS EXAMPLES

Immediately after the interview, the researcher and TEG discussed how to implement the intervention strategy to improve learners' performance in solving algebraic linear equations. The researcher noticed that most learners from the experimental group repeated the errors shown as erroneous examples in test questions. Prior to the intervention, the researcher observed that TEG did not allow learners to discuss questions in groups. Additionally, it was also observed that TEG did not focus on learners' errors and related misconceptions, rather, TEG's focus was on learners' correct responses. Each time a learner responded to a question, TEG would ask other learners to confirm whether the answer was correct or wrong before giving the correct response. The researcher also noticed that TEG usually used whole-class discussions, which did not enable learners who struggled to participate. During whole-class discussions, teachers have a tendency to interact more with stronger learners at the expense of learners who struggle (Mosser et al., 2022, p.4). Shery (2019, p.1) concurs that in whole-class discussions, teachers' follow-ups evaluate learners' answers as right or wrong, and this discourages learners' extended participation.

Through a reconnaissance process (Mapotse, 2018, p.232), the researcher collected information regarding learners' performance in solving algebraic linear equations. The researcher observed and interviewed the TEG to collect information about the challenges that learners faced. Moreover, the researcher used data collected from the pre-test to inform the intervention strategy.

5.5.1 Implementing erroneous examples as an intervention strategy for teaching algebraic linear equations

The intervention was implemented in phase 2 of this study. After some discussions, the researcher and TEG agreed on using erroneous examples to teach Grade 9 algebraic equations for the improvement of learners' performance. The researcher demonstrated how erroneous examples can be incorporated into the teaching of algebraic linear equations on a daily basis. The main purpose of implementing the strategy was to achieve the following objectives, namely:

1. to find out possible barriers to using erroneous examples for teaching Grade 9 algebraic linear equations;
2. to determine the benefits of using erroneous examples for teaching algebraic linear equations and
3. to suggest how teachers might use erroneous examples to enhance learners' performance in solving Grade 9 algebraic linear equations.

Initially, TEG was reluctant to use erroneous examples for teaching for fear that learners may master the erroneous procedures as shown in erroneous examples and reproduce them in their responses to questions in future (Metcalfe, 2018, p.465). The researcher observed that TEG was using whole-class discussions as opposed to cooperative learning in which learners are able to share ideas and discuss problems (Silalahi & Hutauruk, 2020, p.1684). According to Khasawneh, Al-Barakat and Almahmoud (2023, p.1), when teachers use whole-class discussions, only the high-performing learners participate, while low-performing learners are sidelined and, as such, sit passively in class, and others engage in disruptive behaviour. Before the implementation of the intervention strategy, most learners sat quietly during class discussions.

The intervention followed Bandura's (1986, p.459) observational learning. The TEG participated as an observer when the researcher demonstrated how to use erroneous examples for teaching Grade 9 algebraic linear equations. Despite earlier doubts on how to use erroneous examples for teaching on a daily basis, TEG was keen to implement the strategy as demonstrated by the researcher. Following the demonstration, TEG first used correct examples, which he clearly indicated as "correct examples" (Rushton, 2018, p.5) when first introducing a topic or sub-topic and demonstrated to learners step-by-step how to arrive at the answer, explaining what is done at every stage and why. Thereafter, TEG assigned learners to their respective groups to analyse erroneous examples based on earlier correct examples.

5.5.2 Cooperative learning during intervention

During the first cycle of the intervention, the researcher and TEG agreed that instead of having whole-class discussions, cooperative learning should be implemented by placing learners into small groups of five to make it easy for the teacher to monitor learner participation and ensure that every learner takes part. To prevent learners from choosing their friends as group members, TEG wrote numbers 1-5 on cards, and learners who picked the same card numbers belonged to the same group. Learners were given group rules, which included, the need to respect each other, to be active participants in the learning process by getting involved in group discussions since everyone makes their own knowledge through making their own mistakes and asking questions where they do not understand (Retnowati, Ayres & Sweller, 2017, p.668).

Since most textbooks do not have erroneous examples, TEG and the researcher prepared worksheets in which fictitious learners made errors when solving algebraic linear equations. During group discussions learners would identify errors in the given examples, discuss the possible causes of the errors and then solve the problems explaining to each other how they solved the problems (Rushton, 2018, p.5). Unlike what was observed before the intervention, TEG walked around the class while learners were working in their respective groups and asked learners to show the errors they identified and to explain how they would solve the problems. TEG, also walked around the classroom to ensure that learners remain attentive during the lesson. Attention (Bandura, 1986) is an important tenet of observational learning. TEG also asked learners to suggest different approaches for solving the same problems. When asking learners to explain their answers, TEG wanted to ascertain that learners understood what they were doing and find out what challenges learners experienced in solving the problems. Bandura (1986) argues that when learners understand what they are learning, they will be able to reproduce information when answering questions. Where learners were not clear on what to do, TEG provided hints on the way forward so that they understand the concept or skill they are learning. For instance, in one lesson where learners were solving the problem, $6(x + 3) = 24$, most of the learners could not come up with an alternative approach besides first distributing the coefficient 6 (Coppersmith & Star, 2022, p.383). TEG showed learners that besides using the standard algorithm, recognising the structural features of the problem could help in reducing the number of steps when solving the problem. TEG guided learners on how to solve the problem by first finding the highest common factor (HCF) of 6 and 24 and dividing each side by the HCF reducing the problem to $x + 3 = 4$ obtaining $x = 1$. By exposing learners to multiple solution methods, TEG broadens learners' procedural flexibility

(Coppersmith & Star, 2022, p.383). This allows learners to choose the most efficient strategy for solving problems in their encounters.

Learners were given a quiz at the end of every week to monitor their progress in mastering the concepts taught. After the second week of the intervention, there was a slight improvement in the errors learners committed when solving algebraic linear equations. Some learners were found to be committing the same errors they used to before the intervention. This prompted TEG and the researcher to try using learners' own errors in the erroneous examples for group discussions during the second phase of the intervention.

Despite having assigned learners into groups randomly using numbered cards, during the first phase of the intervention, it was observed that learners were not put into heterogeneous ability groups. In phase two, TEG reshuffled the groups to allow for mixed ability and gender balanced grouping. In addition, TEG allowed learners to go outside the classroom for group discussions, and most learners enjoyed doing mathematics outside the classroom. In phase two, the researcher and TEG agreed to use learners' own errors in the erroneous examples so that learners could explain how they arrived at their answers, which would enable them to correct their errors and misconceptions (Metcalfe, 2018, p.471). The (ibid) contends that using learners' errors helps them identify their knowledge gap, which helps in closing the gap. TEG engaged all learners during discussions, unlike before the intervention. It was observed that during group discussions, learners worked cooperatively, sharing ideas and asking questions on how to solve problems. However, the researcher noticed that most learners struggled to express themselves in English. TEG allowed learners to code switch during group discussions. Mixed ability grouping enabled the less capable learners to get assistance from the more capable ones (Zambrano, Kirschner, Sweller & Kirschner, 2019, p.2). Group discussions allowed learners to share ideas, negotiate mathematical properties with each other, discuss problems and show respect for each other's opinions (Hansen, 2022, p.813).

5.5.3 Problem solving during the Intervention

When learners engaged in group discussions, TEG offered a scaffolding (Margolis, 2020) by probing them for further information. One example was when learners solved a word problem as a class activity regarding ages of two boys. The problem stated that there are two boys, and the older boy is three times the age of the younger boy. In four years' time the older boy would be twice the age of the younger boy. Learners were required to find the boys' current ages. Most of the learners were confused about the question and could not figure out how someone who is three times the age of the other would be twice as old. Learners could not use the keywords "twice" and "three times" in formulating an equation which would enable them to solve the problem. TEG probed learners on their understanding of the words "twice", on which some would say "plus two". TEG clarified to learners that "twice" simply means "times two", the same as "three times" means "times three".

TEG assisted learners in coming up with the required equation to solve the problem by saying "let us say the younger boy is x years old now, if the older boy is three times old, how old is he in terms of x". Most learners suggested that the older boy would be three years while others would say $x + 3$ years. Through further probing, explanation and discussions of keywords (Xin et al., 2020, p.106), learners formed the equation $3x + 4 = 2(x + 4)$. Solving this equation was another challenge. Most learners failed to distribute 2 to both x and 4, resulting in them writing $3x + 4 = 2x + 6$. TEG guided learners to remove brackets on the right to give $3x + 4 = 2x + 8$. After collecting terms like terms on either side of the equal sign, learners obtained that x is 4, showing that the younger boy is four years old while the older boy is twelve years old.

The researcher noticed that Grade 9 learners at the experimental school had difficulties expressing their ideas in English. This could be the reason for learners' poor performance in the test. TEG allowed learners to code-switch during group discussions.

In cycle three, TEG and the researcher agreed to continue using learners' own errors for teaching and continued with the same group setup used in the second phase since it seemed to achieve the study's objective. TEG emphasised to learners that they should always take note of keywords, especially when solving word problems since keywords convey special meanings which would enable them to formulate equations. TEG gradually reduced assisting learners in phase three, allowing them to take control of their learning (Gouws, 2019, p.47).

In phase three, the predominant activity was group presentations, in which learners presented their solution methods to the whole class through their group representatives. TEG emphasises to learners that it is imperative for them to actively participate in learning because this provides learners with the opportunity to identify any knowledge gaps that they might have and provide an opportunity to close their knowledge gaps (Metcalfe, 2018, p.471).

5.5.4 Assessment during the intervention

TEG usually gave learners a quiz at the end of the week to assess progress in understanding the concepts taught during that week. TEG marked the quiz by highlighting learners' errors (Rushton, 2018, p. 5). TEG used learners' highlighted errors for group discussions during revision the following week. TEG used classwork and homework as monitoring instruments for the learners' progress during the intervention. TEG designed worksheets for both classwork and homework activities using the textbooks and learners' workbook.

5.6 TEG SEMI-STRUCTURED INTERVIEW DURING INTERVENTION

Soon after conducting intervention classroom observations, the researcher interviewed the TEG to get his perception of the benefits and challenges of using erroneous examples for teaching and learning algebraic linear equations. Below are the samples of transcriptions of TEG's responses to the interview questions, organised under the themes: Comprehension of linear equations, Challenges of using erroneus examples, Benefits of using erroneous examples, and Feasibiity of using erroneus examples.

5.6.1 Comprehension of linear equations

Transcript 1: Do you think using erroneous examples is appropriate for teaching Grade 9 algebraic linear equations and why?

Extract 5.15 TEG *It is not effective for Grade 9 learners because of language barriers. Learners cannot articulate themselves in English when prompted to answer the questions.*

TEG's response in the extract above confirms what the researcher noticed when learners were discussing in groups. TEG allowed learners to code-switch during group discussions, allowing them to express their ideas well and helping them understand the concepts and skills discussed.

According to Maluleke (2019, p.1), most SA learners start learning English in Grade 3, suggesting that code-switching would help learners understand new concepts and skills in mathematics.

5.6.2 Challenges of using erroneus examples

Transcript 2: What are the challenges of using erroneous examples for teaching algebraic linear equations**?**

Extract 5.16 TEG: *There are two major challenges for using erroneous examples, especially in Grade 9. Firstly, it is time consuming. As per Departmental requirements, we must cover a certain number of topics per term, and using erroneous examples may not allow us to complete the scheduled work per term. Secondly, the language barrier makes it challenging to implement. Most learners struggle to speak English, and explaining the errors and their possible causes in English is a big challenge for the majority of learners.*

Most of the learners at the experimental school struggled to express themselves in English when discussing in groups. This is confirmed by TEG's response in extract 5.16 above when he said , most *learners struggle to speak English, and explaining the errors and their possible causes in English is a big challenge for the majority of learners.* However, learners were allowed to switch between languages to clearly express their ideas (Maluleke, 2019, p.1).TEG also indicates that erroneous examples need more time to prepare and implement, noting that teachers have to complete a stipulated number of topics termly as a departmental requirement, suggesting that TEG is forced to complete the syllabus even if learners may not fully understand the concepts or skills taught. This is supported by Tran (2019, p.299), who asserts that most teachers complain that the strategy needs more time to prepare and implement.

5.6.3 Benefits of using erroneous examples

Transcript 3: *What are the benefits of using erroneous examples for teaching Grade 9 algebraic linear equations?*

Extract 5.17 TEG: *It's an alternative method of teaching which encourages a sense of ownership and belonging since learners discover mistakes and errors and come up with solutions. It also allows reinforcement of the knowledge already taught.*

TEG's response confirms what the researcher noticed during the intervention when learners were discussing in groups. Even though learners struggled to express themselves in English, they enjoyed working in groups, identifying errors in erroneous examples and working together to find solutions to the problems. Learners felt a sense of belonging when they identified themselves with the members of their groups. They paid positive attention when identifying errors and helping each other to collectively solve problems (Di Tommaso, et al., 2021, p.6). Reinforcement of concepts taught was observed when some learners committed fewer errors when solving algebraic linear equations in the weekly quizzes during the intervention, suggesting that learners were able to avoid previous errors (Metcalfe, 2018, p.465)

5.6.4 Feasibility of using erroneous examples

Transcript 4: Do you think erroneous examples would be suitable in future for teaching Grade 9 algebraic linear equations and why?

Extract 5.18 TEG: *It can be used in future, particularly for remedial purposes and consolidation of concepts. It can be used in collaboration with other methods, e.g., narrative method, question and answer, and heuristic methods, to be more effective***.**

From the extract above, it can be seen that TEG acknowledges that the strategy can be used in future. However, TEG suggests that for the strategy to be more effective, it should be used in conjunction with other teaching methods like narrative, question-and-answer and heuristic methods. TEG's response suggests that for effective implementation of erroneous examples, teachers should have a piece of differentiated and integrated pedagogical content knowledge for them to use the most effective strategy to teach mathematics (Jacob, John, Gwany & Okonka, 2020, p.14).

5.7 CLASSROOM OBSERVATION AFTER INTERVENTION

The researcher carried out one post intervention classroom observation to evaluate the effectiveness of the intervention strategy. I wanted to determine whether TEG continued using the strategy for teaching and learning algebraic linear equations.

5.7.1 Teacher-learner interaction

It was observed that TEG continued to use erroneous examples, as demonstrated during the intervention (Bandura, 1986, p.459; Govindaraju, 2021, p.12). Unlike prior to the intervention, TEG was found to use the question-and-answer approach during lesson introductions. This was done to find out if learners had the requisite knowledge before introducing new knowledge. According to Bandura's observational learning theory, it can be concluded that TEG gained knowledge on how to use erroneous examples for teaching and learning. As a result, TEG also provided support (scaffolding) to learners to progressively build their knowledge (Vygotsky, 1978, p.89). As a teaching strategy, scaffolding enables learners to connect new knowledge with prior knowledge (Khatri, 2021, p.33). Unlike what TEG was doing prior to the intervention, TEG asked learners some questions that enabled him to understand what learners knew in relation to the current concept to be learned. For instance, in a lesson when TEG was introducing solving equations involving fractions, he asked learners to solve fraction problems with different denominators. The teacher wanted learners to show that fractions may only be added or subtracted if they have the same denominators. It was observed that some learners were simply adding or subtracting numerators and then adding or subtracting denominators. Using learners 'erroneous responses, TEG explained how the lowest common denominator is used to form equivalent fractions before adding or subtracting fractions. Building on the learners' knowledge of solving fraction problems with different denominators, TEG explained how to solve equations involving fractions with different denominators.

After the intervention, TEG was using more learner-centred teaching strategies. Before the intervention, TEG marginalised low performing learners during whole-class discussions. However, after the intervention, no learner was marginalised as TEG guided all learners in constructing their own knowledge by paying attention to their mistakes as well as those of their peers (Di Tommaso et al., 2021, p.6). Learners shared ideas and discussed problems in groups.

5.7.2 Learner-learner interaction

After the intervention, it was observed that TEG allowed learners to share ideas by putting them into small groups. It appeared that TEG had realised small group discussions enable all learners to participate actively during lessons. Using this strategy, TEG allowed learners to share ideas in solving problems and provided assistance when needed (Di Tommaso et al., 2021, p. 6). Compared to the interaction before the intervention, this seemed effective, as no learner was seen to be passive during the lessons (Khasawneh, Al-Barakat & Almahmoud, 2023, p.1). Learners were challenging one another when solving problems. Most learners asked comprehension questions during lessons and were found to elaborate on their ideas and those of their peers (Abramczyk & Jurkowski, 2020, p.296). In support, Havik and Westergard (2020, p.490) argue that when learners work in small groups, they remain engaged, ask their teacher and each other questions, pay attention and use their own examples and those of their peers to build their own knowledge.

5.7.3 Resources for teaching and learning

Apart from textbooks and learners' workbooks, TEG also used worksheets during lessons. The worksheets were designed based on learners' errors and misconceptions. TEG appeared to have realised that learners learn better from erroneous examples if the errors resemble their own mistakes (Metcalfe, 2018, p.471).

5.7.4 Classroom assessment

TEG used classwork and homework as formative assessments to monitor learners' understanding of concepts and to provide scaffolding (Kyaruzi, Strijbos, Ufer & Brown, 2019, p.281). Before the intervention, TEG gave learners both classwork and homework; however, in most cases, learners did not do the homework as expected due to a lack of motivation. This is supported by (Jacob, John, Gwany & Okonka, 2020, p.16), who assert that highly motivated learners take responsibility for their learning and are always a pleasure to teach, while unmotivated learners make teaching difficult and painful. Lack of motivation to learn could be the reason for poor performance by most of the learners before the intervention (Bandura, 1986). After the intervention, most of the learners were motivated to learn and were doing classwork and homework activities on time (Jacob, John, Gwany & Okonka, 2020, p.16).

5.8 POST-INTERVENTION SEMI-STRUCTURED INTERVIEW WITH TEG

Soon after post-intervention classroom observations, the researcher held an interview with the teacher at the experimental school. The interview was held to get the teacher's perception of the intervention and whether there were any benefits. The teacher's responses were organised under the themes: Motiation, Challenges, Benefits and Possible ways of using erroneous examples. The following excerpts are samples of TEG's responses to the interview questions.

5.8.1 Motivation

Transcript 1: How is learner participation now in comparison to the time before the intervention?

Extract 5.19: TEG *In comparison to the time before the intervention, learners'participation has improved. Learners are able to detect errors in the examples, and they seem to enjoy looking for errors and learning how to correct them. Some learners seemed pleased when they found out that they were not the only ones committing certain errors. Even the weak learners are active during lessons.*

TEG's response confirmed what the researcher observed. Learners were motivated to learn (Tran, 2019, p.12) and enjoyed working in small groups, sharing ideas and respecting each other's opinions.

5.8.2 Challenges

Transcript 2 Do you think erroneous examples can be used on a daily basis and why?

Extract 5.20: TEG *I think erroneous examples can be used for remedial purposes as it is time consuming or requires more time to implement.*

TEG's response suggests that teachers may need extensive professional development to implement a new teaching strategy. Preparing erroneous examples for teaching might be a challenge as teachers have limited resources (Jacob, John & Gwany, 2020, p.14). Besides time constraints, TEG might have found his role short-changed from controlling learners in class to managing groups of learners (Tran, 2019, p.12).

5.8.3 Benefits

Transcript 3 According to you, was using erroneous examples an effective strategy for improving learners' performance and why?

Extract 5.21: TEG *Yes, it was effective as it addresses misconceptions and common errors in learning algebraic linear equations***.**

Extract 5.21 confirms what the researcher noticed during the classroom observation. Learners could identify common errors misconceptions in erroneous examples. Improved learners' performance in the post-test further suggests that the intervention exposed learners to common errors commited when learning algebraic linear equations, and were avoided in their responses (Metcalfe, 2018, p.465).

5.8.4 Possible ways of using erroneous

Transcript 4 How can we make the use of erroneous examples more effective**?**

Extract 5.22: TEG *It can be made more effective if it is also used for remediation,revision and consolidating concepts that have been taught.*

TEG's response suggests that the strategy might be more effective if teachers use learners' errors during lessons to familiarise learners with their common errors (Rushton, 2018, p.5). Discussing erroneous responses during revision sessions might help learners identify their knowledge gaps and allow them to repair their reasoning and understanding (Hansen, 2022, p.813).

5.9 POST-TEST QUANTITATIVE ANALYSIS

This section discusses the post-test results in line with the research questions and objectives. Learners' errors were classified using Newman' error analysis. Test outcomes explained why learners experince difficulties in mathematics. The following codes were used to categorise learners' responses to questions from Q1.1 to Q8.2: Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) where learners did not write anything in the answer space.

Table 5.9: Learners' answers in post-test Q1.1 and Q1.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 1 items for the EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
1.1	EG	6	50%	0	0%	4	33.33%	0	0%	0	0%	0	0%	2	16.67%
	CG	8	33.33%	0	0%	16	66.67%	0	0%	0	0%	0	0%	0	0%
1.2	EG	10	83.33%	0	0%	0	0%	0	0%	0	0%	1	8.33%	1	8.33%
	CG	18	75.00%	0	0%	4	16.67%	0	0%	1	4.17%	1	4.17%	0	0%

Table 5.9 shows the results of learners' performance for all categories, with CA ranging from 50% to 83.33% for the EG and 33.33% to 75% for the CG. The experimental group showed some performance improvement in Q1.1 and Q1.2 by obtaining 50% and 83.33%, respectively, in comparison to 33.33% and 75% for the CG, respectively. Improvement in learners' performance in Q1.1 and Q1.2 suggests that most EG learners could understand the questions after the intervention.This suggests that learners paid attention during the intervention and they were able retain and reproduce what they learned (Bandura, 1986). 0% of learners in the EG correctly answered Q1.1 before the intervention, whereas 50% of learners got correct answers in Q1.1 after the intervention. This improvement in learners' performance suggested that the intervention positively impacted learners' performance. Learners' performance remarkably improved after the intervention, in which 33.33% of the learners in the EG recorded comprehension errors compared to 96.80% of comprehension errors recorded prior to the intervention. There were no comprehension errors recorded for learners in the EG on Q1.2. after the intervention. This indicated that the intervention assisted learners in identifying the common error associated with solving linear algebraic equations involving one variable on one side of the equation. The results also revealed that 66.67% and 16.67% of the learners in the CG recorded comprehension errors in Q1.1 and Q1.2, respectively. Performance in the CG also improved suggesting that learners had also gained knowledge.

Table 5.10: Distribution of learners' responses to post-test questions Q2.1 and Q2.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) in question 2 for EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	**Group**	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
2.1	**EG**	3	25.00%	0	0%	7	58.33%	0	0%	0	0%	0	0%	2	16.67%
	CG	7	29.17%	0	0%	16	66.67%	0	0%	0	0%	0	0%	1	4.17%
2.2	**EG**	7	58.33%	0	0%	2	16.67%	0	0%	0	0%	3	25%	0	0%
	CG	15	62.25%	0	0%	4	16.67%	0	0%	4	16.67%	1	4.17%	0	0%

Table 5.10 shows that the range in performance of learners in the experimental school is from 25% to 58.33%, whereas that of learners in the control school is from 29.17% % to 62.25% in the CA category. The CG performed better than EG. However, there was a marked improvement in learner performance for the EG in the CA category in the post-test in comparison to the pre-test in which 25% of the learners recorded correct answers on Q2.1 as compared to only 3.20% prior to the intervention and 58.33% of learners recorded correct answers on Q2.2 as compared to 0% prior to the intervention. The results also indicate that learners' performance ranged from 16.67% to 58.3% for the EG in the CE category and 16.67% to 66.67 % for the CG in the same category. The results also indicate no reading, transformation, process skills or encoding errors for both study groups in Q2.1. Only 25% of the learners in the EG and 16.67% in the CG recorded process skills errors in Q2.2, while only 4.17% recorded encoding errors. 16.67% and 4.17% of the EG and CG learners did not answer Q2.1, respectively. The results also reveal that 16.67% and 4.17% of the learners in the CG committed process skills errors and encoding errors, respectively.

Although learners at the experimental school did not perform well in Q2.1, there was an improvement in Q2.2 after the intervention, suggesting that the strategy allowed some learners to identify the common errors associated with solving algebraic linear equations with variables on both sides of the equal sign. Learners in the experimental school showed an improved understanding of the mathematical procedures when solving algebraic linear equations. However, a few learners in the EG still committed encoding errors after the intervention. For instance, this LEG2's answer to Q2.2, $4x - 3 = x + 15$ was:

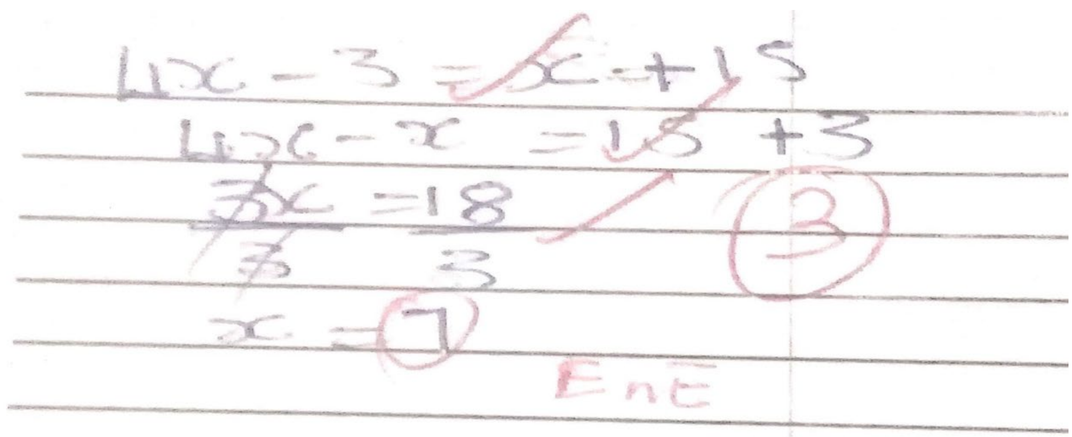


Figure 5.8: ***LEG2's response to Q2.2***

The learner understood the question's demands and carried out all the necessary procedures well, but was careless in writing the final answer and therefore committed an encoding error.

Table 5.11: Learners' answers to post-test questions Q3.1 and Q3.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 3 for EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
3.1	EG	7	58.33%	0	0%	3	25.00%	0	0%	0	0%	0	0%	2	16.67%
	CG	14	58.33%	0	0%	10	41.67%	0	0%	0	0%	0	0%	0	0%
3.2	EG	8	66.67%	0	0%	2	16.67%	0	0%	1	8.33%	1	8.33%	0	0%
	CG	18	75.00%	0	0%	4	16.67%	0	0%	2	8.33%	0	0%	0	0%

Table 5.11 above shows that performance percentages for the experimental group ranged from 58.33% to 66.67%, whereas the performance percentages for the control group ranged from

58.33% to 75% in the CA category. Both study groups performed equally in Q3.1, recording 58.33% correct answers. However, the CG outperformed the EG in Q3.2. The results also show that 75% CG learners correctly answered Q3.2, while 66.67% in the EG obtained correct answers. In the CE category, the performance percentages ranged from 16.67% to 25% for the EG, while that of the CG ranged from 16.67% to 41.67%, showing that the EG recorded fewer comprehension errors than the CG. The two groups performed the same in the PSE category, where both groups recorded 8.33% process skills errors each. The EG also recorded 8.33% in the EE category and 16.67% in the BL category. There was a significant improvement for the EG in the post-test, showing that the strategy positively impacted learners' performance. The EG recorded 3.20% correct answers in the pre-test and then recorded 58.33% correct answers after the intervention. However, 16.67% of the learners in the EG did not answer Q3.1 in the post-test. The most common error in this question was comprehension error followed by process skills errors. For example, a response to Q3.2, solve the equation $2(x + 4) = 2$, from one of the learners was as shown in the excerpt below:

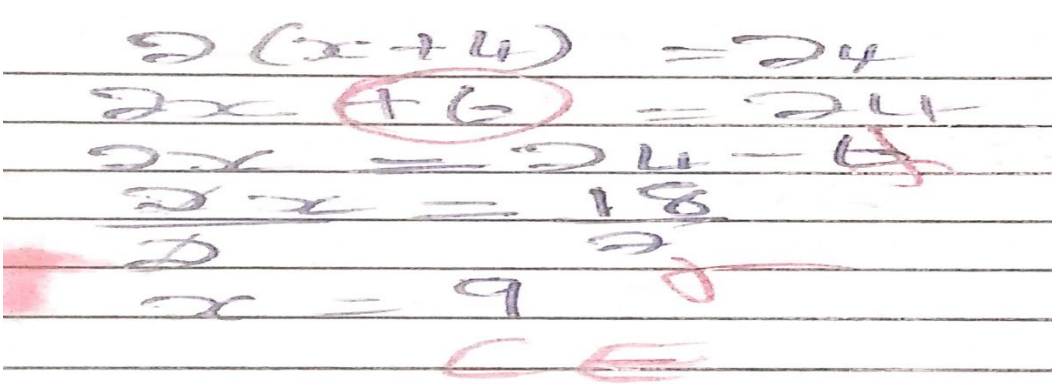


Figure 5.9: ***Learner LCG 12's answer to Q3.2***

The learner knew what to do but made a process skills error of adding 2 to 4 instead of multiplying 2 by 4. However, a wrong operation was used in answering to the problem.

Table 5.12: Learners' answers to post-test Q4.1 and Q4.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 4 for EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
4.1	EG	2	16.67%	0	0%	7	58.33%	0	0%	1	8.33%	0	0%	2	16.67%
	CG	6	25.00%	0	0%	4	16.67%	0	0%	14	58.33%	0	0%	0	0%
4.2	EG	0	0%	0	0%	3	25.00%	0	0%	7	58.33%	1	8.33%	1	8.33%
	CG	3	12.50%	0	0%	0	0%	0	0%	21	87.50%	0	0%	0	0%

The learners' responses to post-test questions 4.1 and 4.2 shown in Table 5.12 above reveals that learners' performance in the CA category ranged from 0% to 16.67%% for the EG and 12.50% to 25% for the CG. In the CE category, performance percentages ranged from 25% to 58.33% for the EG, while that of the CG ranged from 0% to 16.67%, showing that the EG still recorded more comprehension errors after the intervention. However, there was some marked improvement for the EG on Q4 items in the post-test, where 16.67% of the learners got correct answers to Q4.1 compared to 0% before the intervention. This improvement suggested that the intervention positively impacted learners' understanding of how to avoid common errors in solving algebraic linear equations. The results also indicate no reading and transformation errors on Q4 items for both study groups. However, in the PSE category, the performance percentages ranged from 8.33% to 58.33% for the EG, while for the CG, performance ranged from 58.33% to 87.50%, showing that the EG experienced fewer errors than the CG. There were no encoding errors for the EG, while the CG recorded 8.33% encoding errors. The high percentages in the CE and PSE categories are a sign that learners at the two schools still struggle to solve equations with brackets on opposite sides of the equal sign.

Learners at both schools showed a lack of conceptual and procedural knowledge (Coppersmith & Star, 2022, p.383). For instance, this learner, LEG9 had this answer to Q4.2: Solve for *x*: $2(x-4)=3-2(x+2)$ was as shown in the excerpt below:

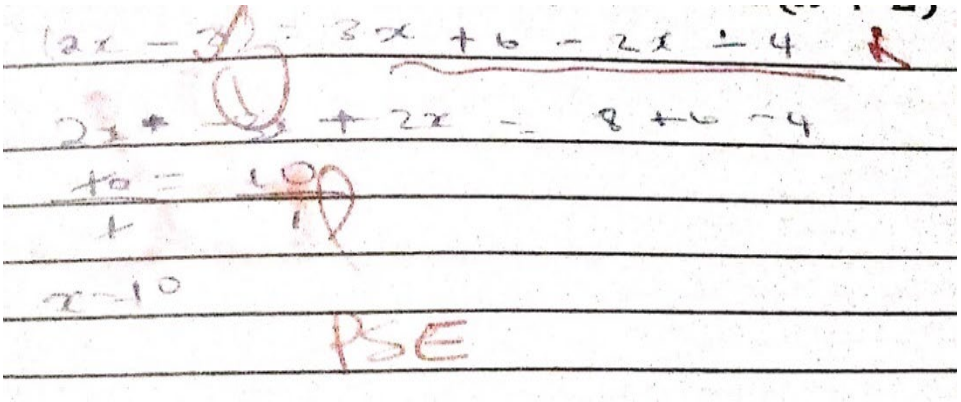


Figure 5.10: ***LEG9's response to Q4.2***

The learner's solution strategy shows a process skill error. The learner treated the right-hand side as if it was a product of two binomials, thus s/he distributed both 3 and -2 into $(x + 2)$.

Table 5.13 Learners' answers to post-test Q5.1 and Q5.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 5 for the EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
5.1	EG	3	25%	0	0%	5	41.67%	0	0%	0	0%	0	0%	4	33.33%
	CG	9	37.50%	0	0%	13	54.17%	0	0%	0	0%	0	0%	2	8,33%
5.2	EG	2	16.67%	0	0%	3	25.00%	0	0%	0	0%	0	0%	7	58.33%
	CG	7	29.17%	0	0%	15	62.50%	0	0%	1	4.17%	0	0%	1	4.17%

Table 5.13 above shows that the percentages of CA responses for the EG ranged from 16.67% to 25%, and for the CG, they ranged from 29.17% to 37.50%% for questions 5.1 and 5.2. The percentages of CE responses ranged from 25% to 41.67% for the EG, while that for the CG ranged from 54.17% to 62.50% for the CG. The results also indicated no TE and EnE errors for both study groups. However, there were more blank spaces in post-test on Q5 than in pre-test. The BL's percentages ranged from 33.33% to 58.33% for the EG, while the percentages for the CG ranged

from 4.17% to 8.33%. However, improved performance for the EG in the post-test is shown, where 25% of learners registered correct answers on Q5.1 instead of 0% on the same question before the intervention. Moreover, 16.67% of the learners in the EG managed to get correct answers to Q5.2 compared to 0% on the same question in the pre-test. This improvement in learners' performance suggested that the intervention assisted learners in identifying common errors in solving involving fractions.

Nevertheless, the high percentages in the CE and BL categories reveal that learners at the two schools still struggle to come up with different strategies for solving equations involving fractions. Most of the learners in both groups just copied the correct worked example, indicating that learners failed to comprehend the demands of the question. Most learners in the two study groups committed comprehension errors because they copied the correct work example instead of using a different method, as requested in Q5.2.

Table 5.14: Learners' answers to post-test questions Q6.1 and Q6.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 6 for the EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
6.1	EG	4	33.33%	0	0%	4	33.33%	0	0%	0	0%	0	0%	4	33.33%
	CG	14	58.33%	0	0%	9	37.50%	0	0%	0	0%	1	4.17%	0	0%
6.2	EG	0	0%	0	0%	6	50.00%	0	0%	0	0%	2	16.67%	4	33.33%
	CG	8	33.33%	0	0%	4	16.67%	0	0%	4	16.67%	7	29.17%	1	4.17%

Table 5.14 shows that the performance percentages in the CA category ranged from 0% to 33.33% for the EG while the percentages ranged from 33.33% to 58.33% for the CG. In the CE category, the percentages ranged from 33.33% to 50% for the EG while the percentages ranged from 16.67% to 37.50% for the CG. There were no RE, TE and PSE errors for both groups on Q6.1. The percentages in the BL category ranged from 0% to 4.17% for the CG while the percentages for the

EG was 33.33%. The outcome showed that most EG learners still experience difficulties solving equations with different denominators. Most learners added the numerators first and then added the denominators later showing a lack of both procedural and conceptual understanding (Manandkar, Pant & Dawadi, 2022, p.2). Most of the learners from both groups showed that they still struggle with solving equations involving fractions involving uneqal denominators. Some of the CG learners added denominators and multiplied numerators. A case in point is this example from LCG3's response to Q6.2, Solve for x: $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$, was:

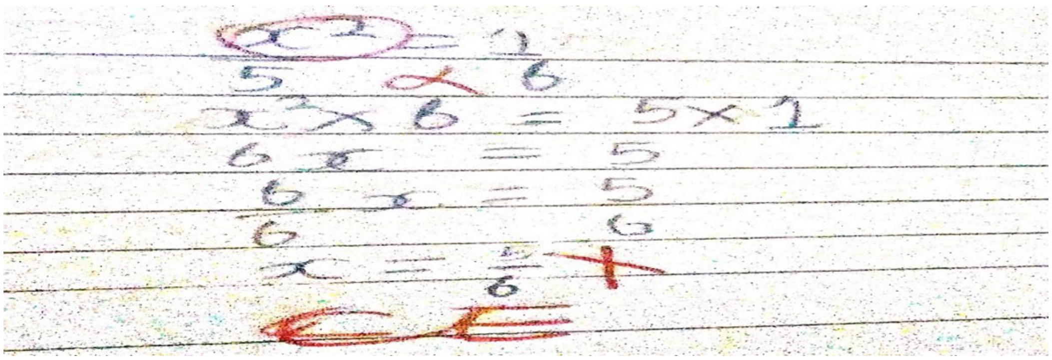


Figure 5.11: ***LCG3's response to Q6.2***

The learner's working shows that s/he did not comprehend the question and as such applied the knowledge of exponents incorrectly. There was a comprehenaion error when s/he simplified $x^2 \times 6 = 5 \times 1$ to $6x = 5$. Another indication of lack of comprehension can be seen in LEG3's resonse as shown here.

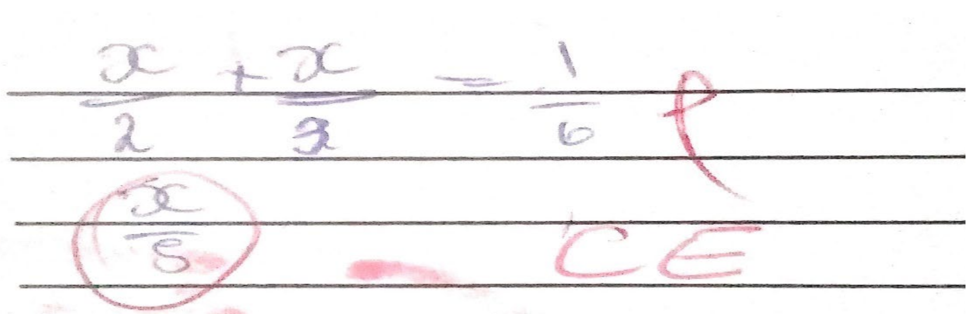


Figure 5.12: ***LEG3's response to Q6.2***

The learner might have thought of the variables as denominotors and since they are the same s/he kept them the same and added 2 and 3 as is done when adding frcations with the same denominators. However, the learner did not know what to do thereafter and abandoned the work. Most of the learners committed both comprehension and carelessness erors in this question. However, learners' performance improved after the intervention, suggesting that the strategy might have impacted learners' performance positively.

Table 5.15: Learners' answers to post-test Q7.1 and Q7.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 7 for the EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
7.1	EG	0	0%	0	0%	7	58.33%	0	0%	0	0%	0	0%	5	41.67%
	CG	0	0%	0	0%	20	83.33%	0	0%	0	0%	0	0%	4	16.67%
7.2	EG	0	0%	0	0%	4	33.33%	0	0%	0	0%	1	8.33%	7	58.33%
	CG	0	0%	0	0%	19	79.17%	0	0%	0	0%	0	0%	5	20.83%

Table 5.15 above reveals that 0% of learners from both groups were able to correct answers for Q7. This shows that the experimental and control school learners still have difficulties solving word problems. Question 7 tested learners' ability to form an algebraic linear equation that could be used to determine the dimensions of the rectangle. To form this equation, learners were supposed to demonstrate conceptual understanding by making a sketch diagram to represent the scenario and show the connection between concepts (Kenedi, et al., 2019, p.70). The table also shows that learners' responses in the CE category ranged from 33.33% to 58.33% for the EG, while that of the CG ranged from 79.17% to 83.33%, suggesting that most of the learners at both schools still struggle to solve word problems. The performance percentages in the BL category ranged from 16.67% to 20.83% for the CG and from 41.67% to 58.33% for the EG. The results also revealed no TE and PSE errors committed by learners in Q7. However, the high BL percentages for both groups showed that both groups of learners still struggle to solve word problems. Most of the learners from both groups could not remember the formula for finding the

perimeter of a rectangle or make a sketch diagram of a rectangle, which they could have used to decide on the dimensions of the sides of the rectangle. Most of the learners from the two study groups revealed that they still made comprehension errors, which resulted in them failing to transform the words and statements in the question to sketch diagrams or formulate equations to use in solving the mathematical problem. In Q7, learners were given the width and length of the rectangle in terms of *x*, but most did not remember that a rectangle has two equal and opposite sides. Most learners added the length and width of one side, equating the sum to 24*cm.* For instance, this learner, LEG7's answer was as shown in the excerpt below:

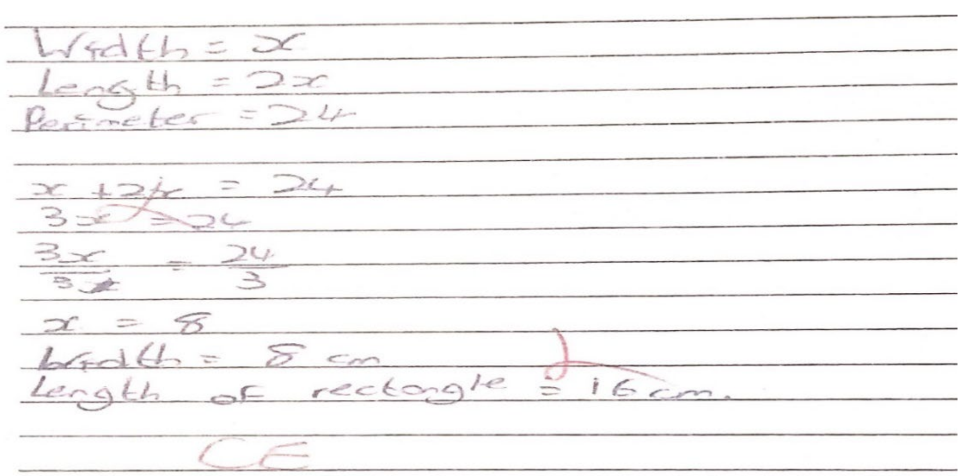


Figure 5.13: ***LEG7's response to Q7.2***

The learner did not understand the question. If the learner had checked his/her answer, s/he would have found that the answer was incorrect and could have tried another strategy. Most learners in the two study groups just copied the erroneous example in Q7.2. This shows that learners did not comprehend the question, so most of the learners committed comprehension errors.

Table 5.16: Learners' answers to post-test Q8.1 and Q8.2 showing actual numbers and percentages of Correct Answers (CA), Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL) on question 8 for the EG and CG.

		POST-TEST													
		CA		RE		CE		TE		PSE		EnE		BL	
Question	Group	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
8.1	EG	0	0%	0	0%	6	50.00%	0	0%	0	0%	0	0%	6	50.00%
	CG	2	8.33%	0	0%	17	70.83%	0	0%	0	0%	0	0%	5	20.83%
8.2	EG	0	0%	0	0%	3	25.00%	0	0%	0	0%	0	0%	9	75.00%
	CG	1	4.17%	0	0%	14	58.33%	0	0%	1	4.17%	2	8.33%	6	25.00%

From Table 5.16 above, it can be seen that learners' responses in the category CA ranged from 4.17% to 8.33% for the CG, whereas the EG recorded 0% for both Q8.1 and Q8.2. The performances in the category CE ranged from 25% to 50% for the EG, while that for the CG ranged from 58.33% to 70.83%. The high percentages in the CE category suggested that both study groups still struggle to comprehend the information given in word problems in the form of words and statements. The results also indicated no RE and TE errors for both groups. However, there were high percentages in the category BL, ranging from 20.83% to 25% for the CG, while that for the EG ranged from 50% to 75%. The high percentages in the BL and CE categories showed that many learners at both schools still struggle to understand word problems. Most learners performed poorly in Q8, like in Q7, showing they still struggle to solve word problems. Learners were not able to answer Q8 because they did not understand the problem. Learners could not make an equation connecting the boys' current ages to their ages four years later, showing a lack of conceptual and procedural knowledge (Kenedi et al., 2019, p.70). Most of the learners were not able to transform the words and statements in the word problem into expressions and equations that could have assisted them in solving the problem. Most of the learners still struggle to make sense of terms like "five times as old as", "after four years" and "three times as old as". Most learners just copied the erroneous example, showing they did not comprehend the question. Some of the learners thought that five times as old meant 5×5 and thought that John is 25 years old, as shown in the solution by LEG12 below:

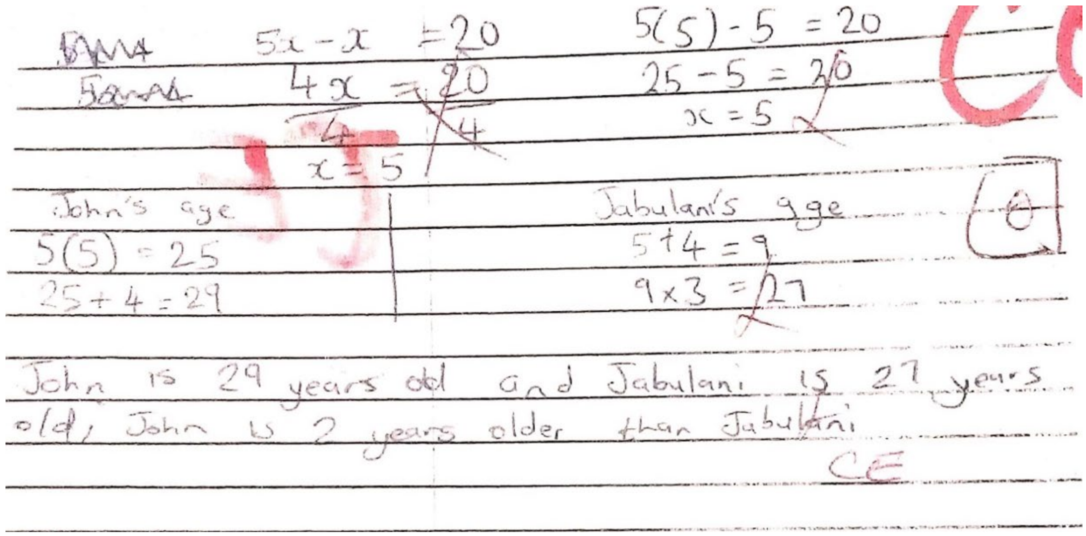


Figure 5.14: ***LEG12's response to 8.2***

The above excerpt shows that the learner did not understand the question, hence his/her failure to interrogate the answer. The learner should have checked if the work matched what is meant in the question, that John's age is five times the age of Jabulani. The learner made a comprehension error, so s/he could not verify their answer. Due to a lack of comprehension, most learners could not transform the word problem into a mathematical equation.

The learners' responses to the test items indicte that learners experince difficulties in mathematics. Most of the learners exibited comprehension, tranformation and process skill errors.

5.10 STATISTICAL ANALYSIS

Data collected quantitatively from the pre-and posttest were descriptively analysed, and results are presented below to fulfil this study's objectives. IBM SPSS version 28 and Excel were used for statistical analysis and data management respectively. A t-test is a statistical method that assumes that the individual observations being analysed are independent of one another and that they follow a normal distribution (Liang, Fu, & Wang, 2019, p. 21). The (ibid) also assert that the t-test is used when the data satisfies the conditions of independence, normality and homogeneity. The quantitative data for this study was obtained from two distinct groups of the same type. To test for normality, the (ibid) state that the mean and standard deviation (SD) are compared, and a much smaller mean than the SD suggests that normality does not exist and that a t-test and that a t-test is may not be used to compare means. The ratios of the mean to the SD for the pre-test quantitative data for the EG and CG are 1.129/2.963 and 12.821/13.875 respectively. Since the difference between the means and standard deviations is small, the t-test was used to compare the means of the two study groups.

A t-test is also used to test hypotheses in order to determine the effectiveness of an intervention strategy in research. There are three t-tests: one sample t-test, paired t-test and independent samples t-test. An independent samples t-test is used to compare the mean values of two sets of information to find out if they are from one population; for instance, if learner samples are selected from two groups, Group A and Group B, of different sizes, it is not expected to get the same mean and standard deviation (Mishra et al., 2019, p.408). This study used an independent samples t-test because the samples were of different sizes. The researcher used the mean scores to evaluate the impact and effectiveness of the intervention. A 95% confidence limit (2-sided) was used to interpret the results. The results were also considered statistically significant for all p-values below 0.05. Learners' performance on a question-by-question basis before and after the intervention is given in this section.

A p-value of less than 0.05 was used to interpret results as significantly different. Thus:
If $p < 0.05$, shows significant results, whereas,
If $p \geq 0.05$, shows insignificant results.

5.10.1 Pre-and post-tests results analysis

Pre-and post-test results for questions in this study were analysed and discussed separately on a question-by-question basis from Q1 to Q8. The mean ($\bar{x}$) score and the p-value were used to compare the groups.

Table 5. 17: ***Pre-and post-test results analysis of Q1: IBM SPSS Statistics 28***

	SETTING	GROUP	N	MEAN	P-*value*	CONCLUSION
QUSETION 1		**EG**	31	0.0000		
	PRE-TEST	**CG**	28	2.50000		
					< 0.001	significant
		EG	12	4.58333	0.104	

POST-TEST	**CG**	24	3.45833		not significant

Table 5.17 shows the analysis of Q1 results and revealed that the two groups performed significantly different, where the ($p - value < 0.001$) which is less than ($p - value = 0.05$) at the 95% confidence limit. Moreover, the CG with ($\bar{x} = 2{,}5000$) compared to the EG with ($\bar{x} = 0{,}00000$) performed better than the EG before the intervention. The results showed that before the intervention, learners in the EG could hardly solve the basic algebraic linear equations. For instance, learners struggled to solve the basic algebraic linear equation $2x + 3 = 15$. This is confirmed by TEG's response in Extract 5.1 who states that, *The possible reasons could be misconceptions and inheritance of mistakes from previous classes* (P86). Most learners in the EG combined unlike terms indicating that they lacked knowledge of basics algebra.

However, the results revealed an insignificant difference in performance for the two groups after the intervention with the (p-value = 0.104) greater than ($p - value = 0.05$), 95% confidence limit. Nevertheless, the EG outprerformed the CG by recording a mean score ($\bar{x} = 4{,}5833$) as compared to ($\bar{x} = 3{,}4583$) for the CG. The results indicated that the EG's performance significantly improvement after the intervention, where it recorded a mean score ($\bar{x} = 4.5833$) compared to the ($\bar{x} = 0.00000$) in the pre-test suggesting that the use of erroneous examples was effective in improving learners' performance in Q1.

Table 5.18: ***Pre-and post-test results analysis of Q2: IBM SPSS Statistics 28***

	SETTING	GROUP	N	MEAN	P-*value*	CONCLUSION
QUSETION 2	**PRE-TEST**	**EG**	31	0.25806		
		CG	28	1.28571		
					0.007	significant
	POST-TEST	**EG**	12	3.50000		
		CG	24	3.62500		
					0.835	not significant

The analysis of Q2 before and after the intervention for the EG and the CG, as shown in Table 5.18 showed that the EG and the CG performed significantly different before intervention ($p - value = 0.007$)which is less than ($p - value = 0.05$), 95% confidence limit. The CG's mean score ($\bar{x} = 1.28571$) against EG's mean ($\bar{x} = 0.25806$) for the EG, showed that the CG outperformed the EG on Q2 before intervention. Most of the learners at the experimental school had difficulties solving the equation $4x - 3 = x + 15$.

Although the CG performed better than the EG on Q2 before interventioin, the analysis indicated an insignificant difference after intervention for the same question ($p - value = 0.835$) which is greater than ($p - value = 0.05$), 95% confidence limit. The analysis of the post-test for the CG and EG also showed CG and EG recorded the means of ($\bar{x} = 3.62500$) and ($\bar{x} = 3.50000$) respectively indicating that the CG outperformed the EG on Q2 after intervention. The analysis also showed that there was an improvement in prerformance for the EG from a mean ($\bar{x} = 0.25806$) in the pre-test to a mean ($\bar{x} = 3.50000$) after intervention suggesting that the strategy positively impacted learners' performance. Improved performance by the EG corroborates TEG's response in Exttact 5.19 who staed that, *learners participation improved and learners were able to detect errors in given examples helped them to avoid the same errors when they encounter similar questions* (P103). This is in confirmation of Bandura's (1986) theory that when learners are motivated, they pay attention, retain and reproduce what they learn.

Table 5.19: ***Pre-and post-test results analysis of Q3: IBM SPSS Statistics 28***

	SETTING	GROUP	N	MEAN	P-*value*	CONCLUSION
QUSETION 3		**EG**	31	0.16129		
	PRE-TEST	**CG**	28	2.89286		
					0.001	Significant
		EG	12	5.16667		
	POST-TEST	**CG**	24	5.50000		
					0.738	not significant

The results analysis outcomes of Q3 as shown in Table 5.20 revealed that EG and CG performed significantly different before intervention with the p-value ($p = 0.001$) which is less than p-value

$(p - value = 0.05)$, 95% confidence limit. The CG outperformed the EG before intervention recording a mean$(\bar{x} = 2.89286)$ compared to the mean $(\bar{x} = 0.16129)$ for the EG. The low mean for the CG in Q3 is supported by the TCG's response in Extract 5.2 who stated that, […] *they don't understand the question.* [...] *So they mix equations with algebraic expressions. What they do is, they start with an expression, they turn it into an equation and they add unlike terms* (P86).

The results for Q3 after intervention showed that EG and CG performed significantly different $(p - value = 0.738)$ which is greater than $(p - value = 0.05)$, 95% confidence limit. Nevertheless, after intervention, the CG outperformed EG recording a mean score$(\bar{x} = 5.50000)$ higher that EG's mean score $(\bar{x} = 5.16667)$.

Table 5.19 also indicated that the EG outperformed after intervention by recording a mean score $(\bar{x} = 5.16667)$ compared to a mean score $(\bar{x} = 0.16129)$ before intervention. The higher mean after intervention suggests the strategy effectively impacted learners' performance in solving algebraic linear equations involving brackets. After the intervention, some EG learners could solve the equation $2(x + 4) = 24$.

Table 5.20: ***Pre-and post-test results analysis of Q4: IBM SPSS Statistics 28***

	SETTING	GROUP	N	MEAN	P-*value*	CONCLUSION
QUSETION 4	**PRE-TEST**	**EG**	31	0.12903		
		CG	28	0.82143		
					0.018	significant
	POST-TEST	**EG**	12	1.75000		
		CG	24	2.91667		
					0.077	not significant

Table 5.20 showed the results analysis of Q4 indicating a significant difference in perfermance betweenEG and CG before intervention $(p - value = 0.018)$ which is less than $(p - value = 0.05)$, 95% confidence limit. Moreover, the CG with a mean $(\bar{x} = 0.82143)$ which is higher than

the EG's mean ($\bar{x} = 0.12903$) performed better than the EG. Although CG outperformed EG before intervention, the low mean scores in the analysis showed that learners in both study groups struggled to solve algebraic linear equations with brackets.

The analysis of results after intervention for Q4 showed EG and CG performred insignificantly ($p - value = 0.077$) which is greater than ($p - value = 0.05$), 95% confidence limit. However, despite no significant difference in performance, the CG performed better than the EG by recording a mean ($\bar{x} = 2.91667$) which is higher than ($\bar{x} = 1.75000$) for the EG. Nevertheless, the low means after intervention for both groups indicated that learners at the two study schools still had difficulties solving algebraic linear equations with brackets. Most learners struggled to solve the equation $2(x - 4) = 3 - 2(x + 2)$.

Table 5.20 also revealed improved performance for EG and CG after intervention. EG improved from mean ($\bar{x} = 0.12903$) before intervention to mean ($\bar{x} = 1.75000$) after intervention, suggesting that the intervention enhanced learners' performance in Q4. The CG's performance also improved from a mean of ($\bar{x} = 0.82143$) to a mean of ($\bar{x} = 2.91667$) after EG received intervention.

Table 5.21: ***Pre-and post-test results analysis of Q5: IBM SPSS Statistics 28***

	SETTING	GROUP	N	MEAN	P-*value*	CONCLUSION
QUSETION 5	**PRE-TEST**	**EG**	31	0.12903		
		CG	28	1.64286		
					< 0.001	significant
	POST-TEST	**EG**	12	1.25000		
		CG	24	1.62500		
					0.455	not significant

The analysis of results for Q5 before intervention, as shown in Table 5.21, showed that EG and CG performed significantly different ($p - value < 0.001$) which is less than ($p - value = 0.05$), 95% confidence limit. The CG outperformed the EG before intervention, recording a mean score ($\bar{x} = 1.64286$), which is higher than the mean score ($\bar{x} = 0.12903$) for the EG. However, the low mean scores indicated that both study groups had difficulties finding an alternate method to solve algebraic linear equations involving fractions. Many learners could not find an alternate strategy to solve the equation $\frac{6x}{2} - 5 = 4$.

The analysis of results also showed that EG and CG performed insignificantly different after intervention, p-value ($p - value = 0.455$) which is greater than ($p - value = 0.05$), 95% confidence limit. Moreover, the low means of ($\bar{x} = 1.25000$) for the EG and ($\bar{x} = 1.62500$) for the CG revealed that learners still had difficulties solving equations with fractions after intervention. Nevertheless, the CG outperformed the EG when comparing the mean scores before and after intervention. Table 5.21 also indicated that the EG's performance improved, recording the means of ($\bar{x} =$ 1.2500) and ($\bar{x} = 0.12903$) before and after intervention respectively, indicating that the intervention positively impacted learners' performance in Q5.

Table 5.22: ***Pre-and post-test results analysis of Q6: IBM SPSS Statistics 28***

	SETTING	GROUP	N	MEAN	P-*value*	CONCLUSION
QUSETION 6		**EG**	31	0.16129		
	PRE-TEST	**CG**	28	2.00000		
					< 0.001	significant
		EG	12	1.83333		
	POST-TEST	**CG**	24	4.12500		
					0.002	significant

The results before intervention shown in Table 5.22 above indicated EG and CG performed significantly different ($p - value < 0.001$) which is less than ($p - value = 0.05$), 95% confidence limit. Moreover, the higher mean ($\bar{x} = 2.00000$) for the CG compared to ($\bar{x} = 0.16129$) for the EG showed that the CG outperformed the EG before intervention. However, the mean mark of ($\bar{x} = 2.00000$) showed that even though the CG outperformed the EG, learners at the experimental school struggled to solve equations involving fractions with different denominators.

The analysis of results after intervention for Q6 revealed that EG and CG performed significantly different ($p - value = 0.002$) which is smaller than ($p - value = 0.05$) at the 95% confidence limit. Moreover, the CG's greater mean ($\bar{x} = 4.12500$) compared to EG's mean ($\bar{x} = 1.83333$) showed that the CG outperformed EG after intervention. The analysis also indicated that the EG improved after intervention by recording a mean of ($\bar{x} = 1.83333$) compared to the mean of ($\bar{x} = 0.16129$) before intervention suggesting that the strategy positively impacted learners' performance in solving algebraic linear equations involving fractions with different denominators. For instance, a reasonable number of learners were able to solve the equation $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$ after intervention.

Table 5.23: ***Pre-and post-test results analysis of Q7: IBM SPSS Statistics 28***

	SETTING	GROUP	N	MEAN	P-*value*	CONCLUSION
QUSETION 7		**EG**	31	0.09677		
	PRE-TEST	**CG**	28	1.03571		
					0.038	significant
		EG	12	0.58333		
	POST-TEST	**CG**	24	0.54167		
					0.908	not significant

Table 5.23 showing analysis of results before intervention indicated that EG and CG performed significantly different ($p - value = 0.038$) which is less than ($p - value = 0.05$), 95% confidence limit.

The analysis of results for Q7 after intervention revealed that the two groups performed insignificantly different ($p - value = 0.908$) which is greater than ($p - value = 0.05$), 95% confidence limit. The low means scores ($\bar{x} = 0.58333$) and ($\bar{x} = 0.54167$) for the EG and CG respectively indicated that learners at the two schools still have difficulties solving word problems. However, the results also show that while there was an improvement for the EG, there was a decline in performance for the CG. The improvement in performance for the EG suggested that the intervention enhanced learners' performance in solving word problems.

Table 5.24: ***Pre-and post-test results analysis of Q8: IBM SPSS Statistics 28***

	Setting	Group	N	Mean	P-*value*	Conclusion
QUSETION 8		EG	31	0.19355		
	Pre-test	CG	28	0.64286		
					0.094	not significant
		EG	12	0.16667		
	Post-test	CG	24	1.20833		
					0.010	significant

Table 5.24 showed analysis of results for the pre-test results indicating that the two study groups did not perform significantly different ($p - value = 0.094$) which is greater than $p - value = 0.05$) at the 95% confidence limit. The low mean scores ($\bar{x} = 0.19355$) and ($\bar{x} = 0.64286$) for the EG and CG respectively, showed many learners had difficulties solving word problems.

The analysis of results, as shown in Table 5.25, also showed that the EG and CG performed significantly different after intervention ($p - value = 0.010$) which is less than ($p - value = 0.05$), 95% confidence limit. The CG outperformed the EG recording means of ($\bar{x} = 1.20833$) and ($\bar{x} = 0.16667$) respectively. The results also revealed an improvement for the CG, while the EG showed a decline after intervention. The results suggested that the strategy did not positively impact learners' performance in Q8. The low mean scores for both study groups before and after intervention revealed that learners still struggled to solve word problems.

Table 5.25: ***Pre-and post-test results from Q1 to Q8: a summary comparison of means***
Before and after intervention comparison of EG and CG.

	Experimental group			Control group		
Item	**Pre-test**	**Post-test**	**p-value**	**Pre-test**	**Post-test**	**p-value**
	31	12		28	24	
Q1	0.00000	4.58333	< 0.001	2.50000	3.45833	0.134
Q2	0.25806	3.50000	< 0.001	1.28571	3.62500	< 0.001
Q3	0.16129	5.16667	< 0.001	2.89286	5.50000	0.001
Q4	0.12903	1.75000	0.007	0.82143	2.91667	< 0.001
Q5	0.12903	1.25000	0.016	1.64286	1.62500	0.971
Q6	0.16129	1.83333	0.005	2.00000	4.12500	0.002
Q7	0.09677	0.58333	0.179	1.03571	0.54167	0.272
Q8	0.19355	0.16667	0.858	0.64286	1.20833	0.201

Table 5.25 above shows how EG and CG performed before and after intervention from Q1 to Q8. The results revealed different mean values for Q1 before and after intervention. The outcome showed the EG performed significantly different before and after intervention in Q1 (p-value < 0.001), less than (p-value $= 0.05$), 95% confidence limit. The results showed that the learners in the EG improved significantly in Q1 from a mean ($\bar{x} = 0.00000$) before intervention to a mean ($\bar{x} = 4.58333$) after intervention. The significant performance improvement in Q1 for the EG suggested that the intervention positively impacted learners' performance. However, the analysis of results also indicated that the CG did not perform significantly different in the pre-and posttest ($\bar{x} = 2.50000$)and ($\bar{x} = 3.45833$) respectively, for Q1 (p-value= 0.134), which is greater than (p-value = 0.05), 95% confidence limit. However, the CG performed better in post-test versus pre-test.

The analysis of results for Q2 showed that both the EG and the CG performed significantly different (p-value < 0.001) and (p-value < 0.001) in the pre-test and post-test, respectively. Both groups of learners improved performance in solving algebraic equations involving variables on opposite sides of the equal sign. Moreover, the analysis also revealed that the EG performed better than the CG, suggesting that the intervention positively affected learners' performance.

Data for Q3 indicated that the EG performed significantly different after intervention than before intervention (p-value< 0.001), which is less than (p-value$= 0.05$), 95% confidence limit. Leaners in the EG were able to solve algebraic linear equations involving brackets. Similarly, the CG also performed significantly different in the post-test in comparison to the pre-test (p-value < 0.001) that is less than (p-value$= 0.05$). Although both study groups significantly improvement in Q3, the EG recorded a greater improvement than the CG, suggesting that the intervention strategy positively affected the performance of learners.

The results for Q4 depicted that the EG performed significantly different before and after intervention (p-value $= 0.007$) that is less than (p-value $= 0.05$), 95% confidence limit. Comparably, the CG also performed significantly different in the pre-and post-tests (p-value < 0.001) which is less than (p-value$= 0.05$) at 95% confidence limit. The outcomes for Q4 suggested that the intervention positively impacted EG learners' performance.

The results for Q5 indicated that the EG performed significantly different in the two tests (p-value$= 0.016$), which is lower than (p-value $= 0.05$) at 95% confidence limit. The higher mean score ($\bar{x} = 1.25000$) for after intervention in comparison to the mean score ($\bar{x} = 0.12903$) before intervention indicated that the EG improved performance after intervention. The better performance for the EG after intervention, suggested that the strategy was effective in enhancing learners' performance in solving algebraic linear equations involving fractions. However, the analysis of results for Q5 depicted that the CG performed insignificantly different in the two tests (p-value $= 0.971$) which is higher than (p-value$= 0.05$) at 95% confidence limit.

The results for Q6 showed that the EG performed significantly different in the two (p-value$=$ 0.005) which is less than (p-value 0.05) at 95% confidence limit. Correspondingly, the results for Q6 revealed a that CG performed significantly different in in the two tests (p-value$= 0.002$) which is less than (p-value$=0.05$). Although both study groups performed significantly different in the two tests, the CG showed a better improvement with a post-test mean ($\bar{x}=4.12500$) compared to the post-test mean ($\bar{x}=1.83333$) for the EG. Moreover, the higher mean score ($\bar{x}=1.83333$) for the EG after intervention compared to the mean ($\bar{x}=0.16129$) before intervention suggested that the intervention positively impacted learners' performance.

The results analysis for Q7 indicated that the EG performed insignificantly different in the two tests (p-value $=0.179$) which is higher than (p-value $=0.05$) at 95% confidence limit. Equally same, the results analysis showed that at 95% confidence limit, the CG performed insignificantly different in the two tests (p-value $=0.272$), which is greater than (p-value $=0.05$) at 95 % confidence limit. Although the EG perfumed insignificantly different in the two tests, there was an improvement after the intervention, showing a higher mean score ($\bar{x}=0.58333$) compared to the mean score ($\bar{x}=0.09677$) before the intervention. The higher mean score after intervention recorded by the EG suggested that the strategy positively affected learners' performance. TEG in Extract 5.21 confirmed that the intervention had a positive impact on learners' performance and stated that; *yes, it was effective as it addresses misconceptions and common errors in learning algebraic linear equations* (P104). When learners are able to fix their misconceptions and errors, it implies that they have developed conceptual knowledge (Kilpatrick et al. 2001). The results analysis for Q7 also showed a decline in performance for the CG from a mean score ($\bar{x}=1.03571$) to a mean score ($\bar{x}=0.54167$). The low mean scores in the results analysis for Q7 revealed that learners in the EG and CG still have difficulties solving word problems.

The data analysis for Q8 depicted that the EG performed insignificantly different in the two tests (p-value$=0.858$), which is greater than (p-value $=0.05$) at 95% confidence limit. However, there was a decline in performance for the EG (mean score $\bar{x}=0.16667$) after intervention compared to the mean score ($\bar{x}=0.19355$) before the intervention, suggesting that the strategy did not positively impacted learners' performance in Q8. Although the CG performed insignificantly different in the two tests, there was improvement from mean ($\bar{x}=0.64286$) in the pre-test to mean ($\bar{x}=1.20833$) in the post-test suggesting that learners gained knowledge from traditional teaching strategies.

Table 5.26: ***Pre-and post-test performance comparison between experimental and control groups***

Performance for experimental and control groups

Group	N	Setting	Mean	P-value
EG	31	Pre-test	1.12903	
	12	Post-test	18.83333	< 0.001
CG	28	Pre-test	12.82143	
	24	Post-test	23.00000	0.004

The table 5.26 above showed the pre-and post-test performance comparison between EG and CG. The results indicated that the EG performed significantly different in the two tests (p-value < 0.001) which is less than (p-value $= 0.05$) at 95% confidemce limit. The EG recorded a significant improvemt from a mean ($\bar{x} = 1.12903$) before the intervention to a mean ($\bar{x} = 18.83333$) after intervention. This higher mean score after intervention, suggested that stratregy helped learners to enhance their performance in solving algebraic linear equations. Similarly, the CG performed significantly different (p-value $= 0.004$) which is less than (p-value$= 0.05$) at 95 % confidence limit. The CG's higher mean score in the post-test ($\bar{x} = 23.00000$) compared to the pre-test mean score ($\bar{x} = 12.82143$) suggested that learners gained knowledge from traditional teaching strategies.

Table 5.27: ***Pre-and post-tests performance summary of experimental and control groups***

Pre-test				**Post-test**		
Group	**N**	**Mean**	**P-value**	**N**	**Mean**	**P-value**
EG	31	1.12903		12	18.83333	
CG	28	12.82143	< 0.001	24	23.00000	0.195

The analysis of results, as shown in Table 5.27 above, revealed that the EG and CG performed significantly different before intervention (p-value < 0.001) which is less than (p-value $= 0.05$)

at 95% confidence limit. The CG showed a higher mean score ($\bar{x} = 12.82143$) compared to the EG's mean score ($\bar{x} = 1.12903$). 3is greater than (p-value $= 0.05$) at 95% confidence limit. However, although the two study groups did not perform significantly different after intervention, the CG outperformed EG. Even though the CG outperformed the EG in both tests, the EG improved better, suggesting that the strategy enhanced the learners' performance in solving algebraic linear equations.

5.11 CHAPTER SUMMARY

This chapter presented both quantitative and qualitative data collected using pre-and post-tests, and lesson observations and semi-structured interviews respectively. The data was collected to explore the impact of erroneous examples on the effectiveness of teaching Grade 9 algebraic linear equatons. The pre-and post-tests, lesson observations and semi-structured interviews were analysed using literature and the study's conceptual framework.

Newman Error Analysis was used to analyse and discuss the pre-test and post-test results using the categories Correct Answers (CA), Reading Errors (RE), Transformation Errors (TE), Process Skills Errors (PSE), Encoding Errors (EnE) and Blank Responses (BL).

The study's outcomes revealed that the EG's performance improved significantly after the intervention (p-value < 0.001) which is less than (p-value $= 0.05$) at 95% confidence limit. The higher mean score ($\bar{x} = 18.83333$) recorded by the EG after the intervention compared to the mean score ($\bar{x} = 1.12903$) before the intervention showed a significant improvement for the EG. However, the outcomes of the analysis revealed that the EG and the CG did not perform significantly different in the post-test (p-value $= 0.195$) showing that the scores for the EG and CG were not significantly different. However, the CG recorded a higher mean ($\bar{x} = 23.00000$) in the post-test compared to ($\bar{x} = 18.83333$) for the EG.

The outcomes of the qualitative analysis indicated that the strategy motivated learners to participate actively during both class and group discussions. The outcomes of the quantitative analysis also revealed that the most common errors committed by Grade 9 learners when solving algebraic linear equations are comprehension errors. The analysis also indicated that learners were able to identify errors in erroneous examples and share ideas on how to solve the problems. The analysis also revealed that the TEG provided scaffolding to learners, which enabled learners to identify common errors committed when solving algebraic linear equations.

Since the EG performed statistcally significant after the intervention (p-value < 0.001), the study rejected the null hypothesis which states that there is no significant difference between pre-test and post-test results when teachers use erroneous examples for teaching Grade 9 algebraic linear equations. The study accepted the alternative hypothesis which states that a statistically significant difference exists between pre-test and post-test results when teachers use erroneous examples to teach Grade 9 algebraic linear equations. Furthermore, there was no statistically significant difference in performance between the EG and the CG in the post-test (p-value > 0.195) which is greater than 0.05 at 95% confidence limit. However, the EG's performance improved more than CG's performance suggesting that the intervention was effective in improving learners' performance. The study accepted the null hypothesis, which states that there is no significant difference in learner performance between the experimental and control groups when teachers use erroneous examples to teach Grade 9 algebraic linear equations.
Therefore, the intervention managed to improve the performance of the experimental group to the level of performance of the control school.

The next chapter discusses the study's quantitative and qualitative findings using both theory and literature that guided this research.

CHAPTER 6: DISCUSSION OF RESEARCH FINDINGS

6.1 INTRODUCTION

This chapter discusses the quantitative and qualitative findings presented in Chapter 5. The findings of classroom observations carried out with the EG before, during and after implementing the intervention and interviews with the two teachers are also presented. Results analysis of quantitative data derived from the tests are discussed using the categories indicated in section 4.7. The conceptual framework underpinning this study, theories and the literature, guided the interpretation of findings to answer research questions and hypotheses.

6.2 QUANTITATIVE FINDINGS

Quantitative data for the EG and CG was collected using pre-and post-test questions. Data analysis and management was done using IBM SPSS version 28 and Excel respectively. The t-test was used to compare learners' performance in the EG and CG. The mean scores before and after the intervention with the EG were used to interpret the impact and effectiveness of the intervention and to compare the EG and CG.

6.2.1 Results of pre-and post-tests

Both the EG and CG wrote a pre-test which showed learners' knowledge before implementing the intervention. Both the EG and the CG wrote a post-test which showed the EG's final performance after the intervention (Bohari, 2020, p.75) and how the CG performed in the post-test. An erroneous example is a worked example in which one or more steps in the working are deliberately made incorrect and learners are required to identify the incorrect step(s) and then solve the problem, whereas a correct example is one in which all steps are correctly laid out for learners to follow and implement when solving similar problems in future. Using erroneous examples involves assigning learners into small groups of three to five learners each and asking them to work cooperatively to identify errors and discuss ways of solving the problems correctly. The teacher provides assistance (scaffolding) where necessary by prompting learners to explain what is done incorrectly in the erroneous example and suggest possible strategies for solving the problem.

6.2.1.1 Results before and after intervention for question 1

The result of Q1 shows that EG and CG performed significantly different in the pretest ($t = -5.384,\ p < 0{,}001$). The CG recorded a higher mean of ($\bar{x} = 2.50000$) compared to EG with the mean of (0.0000). Learners in the EG showed a lack of conceptual knowledge (Kilpatrick, Swafford & Findell, 2001, p.5) in solving the equation $2x + 3 = 15$. In this question, learners were supposed to collect like terms on the side using inverse operations. However, most learners simplified the left-hand side by adding the unlike terms, indicating that they lacked procedural knowledge of solving algebraic linear equations. Additionally, due to inadequate conceptual knowledge, learners could not check the accuracy of their answers, otherwise, they could have found out that the answer was incorrect and would have tried using other strategies. Kilpatrick et al. (2001, p.117) describe *procedural fluency* as knowledge of skills how to apply them correctly and flexibly.

The findings revealed that 96% of the learners in the EG committed CE in Q1.1 and Q1.2, whereas 64% and 50% of the learners in the CG committed CE in Q.11 and Q1.2, respectively. The findings revealed that 0% of the learners in the EG committed TE, PSE and EnE in Q1.1 and Q1.2, whereas 0% of the learners in the CG committed TE and PSE and 3.5% committed EnE in Q1.1 and Q1.2, respectively. The most common error for both groups was CE.

The results after implementing the intervention strategy showed that the EG and the CG did not perform significantly different ($t = 1.685\ , p = 0.104$). However, the EG recorded a higher mean score ($\bar{x} = 4.58333$) compared to the CG's mean score of ($\bar{x} = 3.45833$). Moreover, the EG recorded a higher mean difference of $\bar{x} = 4.58333$ compared to $\bar{x} = 0.95833$ for the CG, suggesting that the intervention positively impacted learners' performance.

After the intervention, 50% of the learners in the EG were able to collect like terms using inverse operations, in comparison to 0% before the intervention, indicating that the strategy assisted learners in avoiding the error of combining unlike terms when solving algebraic linear equations. Moreover, the results analysis also indicated that 33% and 0% of the learners in the EG committed CE in Q2.1 and Q2.2, respectively, while 66.7% and 16.7% of the learners in the CG committed CE. There was no TE, PSE and EnE for the two study groups in Q2.1. However, while the learners in the EG committed no TE and PSE errors, 8.3% of these learners committed EnE.

6.2.1.2 Results before and after intervention for question 2

Pre-test results for Question 2 revealed a significant difference in performance for EG and CG ($t = -2.850, p = 0.007$). The CG outperformed the EG with mean scores of $\bar{x} = 1.28571$ and $\bar{x} = 0.25806$ respectively. Learners at the EG struggled to solve equations of the form $4x - 3 = x + 15$. In this question, learners were supposed to use additive inverses to collect like terms and then apply multiplicative inverses to find the value of the unknown. Some of the learners were able to use additive inverse operations to solve the problems. However, some of the learners, even though they were able to apply inverse operations, were careless in writing their final answers, thereby committing encoding errors. An encoding error occurs when a learner writes an incomplete and imprecise final answer (Rachmawati, Sah & Hasanah, 2023, p.1).

The analysis of the findings revealed that only 3% and 0% of the learners in the EG were able to solve Q2.1 and Q2.2, respectively, while 14.3% and 21% of their counterparts were able to solve Q2.1 and Q2.2, respectively. The results also reveal that 92% and 94% of the learners in the EG committed CE in Q2.1 and Q2.2, respectively, whereas 79% and 64% of the learners in the CG committed CE in Q2.1 and Q2.2, respectively. Learners in the two groups did not commit TE, PSE and EnE in Q2.1. However, 0%, 6% and 0% of the learners in the EG committed TE, PSE and EnE, respectively, in Q2.2, while 0%, 11% and 0% of the learners in the CG committed TE, PSE and EnE, respectively in Q2.2 respectively. Most learners showed lack of knowledge on how to collect like terms using inverse operations. The most common error for both groups was CE.

The two groups performed insignificantly different in Q2 ($t = -0.210,\ p = 0.835$). The CG recorded a greater mean score of $\bar{x} = 3.62500$) in comparison to the EG's mean score of $\bar{x} = 3.50000$. However, the results indicated that the EG showed great improvement by recording a higher mean difference of $\bar{x} = 3.24194$ after the intervention versus $\bar{x} = 2.33929$ for the control group, suggesting that the intervention helped learners improve their performance by being able to identify common errors committed when solving algebraic equations with variables on both sides of the equal sign. The results support Rushton's (2018, p.8) findings that when learners study erroneous examples, they gain knowledge significantly. Moreover, 50% and 58% of the learners in the EG were able to solve Q2.1 and Q2.2 in the post-test versus 3% and 0% in the pre-test, while 29% and 62% of the learners in the CG were able to solve Q2.1 and Q2.2 after intervention compared to 14.3% and 21% before intervention.

However, the findings analysis also showed that 58% and 16.7% of the learners in the EG still committed to CE in Q2.1 and Q2.2, respectively. There was no TE, PSE and EnE for both study groups in Q2.1, yet there was 0% TE, PSE and 25% EnE for the EG in Q2.2 and 0%, 16.7% and 4% TE, PSE and EnE in Q2.2 for the CG.

6.2.1.3 Results before and after intervention for question 3

Pre-test results for Q3 revealed that the EG and the CG performed significantly different ($t = -4.810,\ p < 0.001$). The CG recorded a higher mean score of $\bar{x} = 2.89286$) compared to the mean score of $\bar{x} = 0.16129$ for the EG. The CG Learners showed conceptual knowledge for solving equations of the form $2(x + 4) = 24$. In this question, learners were supposed to use the distribution rules to remove brackets when solving the equation $2(x + 4) = 24$.

The low mean score recorded by the learners in the EG revealed that learners struggled to solve equations involving brackets. Most of the learners in the EG revealed that they lacked procedural and conceptual knowledge for solving algebraic equations involving brackets. Most failed to apply the distributive rule, revealing they lacked procedural and conceptual knowledge. Most learners only partially applied the distributive rule to the first term in the brackets. Lack of conceptual knowledge prevented learners from checking if the answers satisfied the conditions of the equation. This is supported by Kilpatrick et al. (2001, p.118), who state that when learners have conceptual knowledge, they can check their answers and correct mistakes on their own.

Additionally, the analysis of the findings of Q3 revealed that only 3.2% and 0% of the learners in the EG were able to solve Q3.1 and Q3.2, respectively, whereas 25% and 39% of their counterparts were able to solve Q3.1 and Q3.2 respectively. Neither study group had TE, PSE, or EnE in Q3.1. However, both study groups recorded 0% transformation and encoding errors and 3% and 4% process skill errors in Q3.2 for the EG and CG, respectively. The most common error for both groups was CE.

Results for posttest Q3 revealed EG and CG did not perform significantly different ($t = -0.340, p = 0.738$). However, the CG recorded a higher mean score of $\bar{x} = 5.5000$, whereas the EG recorded a mean score of $\bar{x} = 5.16667$. Although the EG recorded a lower mean score than the CG, the EG improved significantly by $\bar{x} = 5.00538$ in the post-test, as compared to $\bar{x} =$

2.60714 for the CG, suggesting that the intervention helped learners in the EG to acquire some conceptual and procedural knowledge (Kilpatrick, 2001, p.118) for solving equations involving brackets.

The post-test results analysis of Q3 also revealed that 58% and 67% of the learners in the EG were able to solve Q3.1 and Q3.2 compared to 3.2% and 0% in the pre-test. The CG recorded 58% and 67% correct answers in Q3.1 and Q3.2 respectively. Neither study group had TE, PSE, and EnE in Q3.1, while the EG recorded 0% TE, 8.3% PSE and 8.3% EnE in Q3.2. Q3.2, the CG recorded 0% TE and EnE and 8.3% PSE. The most common error for both groups was CE.

6.2.1.4 Results before and after intervention for question 4

The results before intervention for Q4 depicted that there was a significant difference in performance between the two groups ($t = -2.486, p = 0.018$). The findings indicated that the EG recorded a mean score of $\bar{x} = 0.12903$ whereas the CG recorded a mean score of $\overline{x} = 0.82143$. The CG performed better lhan the EG before intervention in Q4. The CG learners were able to solve equations of the form $2(x-4) = 3-2(x+2)$, whereas learners in the EG struggled to solve algebraic linear equations of this form. In this question, learners were supposed to use the rules of distribution on the left side and the rules of order of operations and rules of distribution on the right side.

No learner in the EG could solve this problem before the intervention. All learners in the EG copied the erroneous steps shown in the erroneous example. Their responses were as stated earlier, because learners lacked conceptual and procedural knowledge, they could not verify the answer's accuracy. Despite being clearly indicated as an erroneous example, learners did not believe what they read. Instead, they just copied the work as it was. The pre-test results analysis revealed that 100% of EG learners committed CE in Q4.1 and Q4.2, while 78% and 71% of the learners in the CG committed CE in Q4.1 and Q4.2, respectively. Both study groups Recorded 0% TE, PSE and EnE in Q4.1. However, in Q4.2, the CG recorded 25% PSE while both groups recorded 0% TE and EnE in Q4.2. The most common error committed by both groups was CE.

The results of the post-test for Q4 revealed no significant difference in performance for the EG and the CG ($t = -1.845, p = 0.077$). The findings indicated that the CG performed better than the EG, with mean scores of $\bar{x} = 2.91667$ and $\bar{x} = 1.75000$, respectively. Despite no significant difference in performance in the post-test, the EG showed some improvement by recording a mean difference of $\bar{x} = 1.62097$, while the CG improved by $\bar{x} = 2.09524$. Additionally, 16.7% of the learners in the EG obtained correct answers in Q4.1 after the intervention, compared to 0% before intervention. The findings indicated that the EG's learners improved performance after implementing the intervention, suggesting that the intervention helped learners improve their performance.

The findings also revealed that 58% and 25% of the learners in the EG still committed CE in Q4.1 and Q4.2, respectively, while 16.7% and 0% of the learners in the CG committed CE in Q4.1 and Q4.2, respectively. There were no TEs for both groups in Q4.1 and Q4.2. However, 8.3% and 58% of the learners in the EG committed PSE in Q4.1 and Q4.2, while 58% and 87.5% of the learners in the CG committed PSE. For the EnE, the EG recorded 0% in Q4.1 and 8.3% in Q4.2, while the CG recorded 0% EnE in Q4.1 and Q4.2. The most common error for both groups was CE.

6.2.1.5 Results before and after intervention for question 5

The results before intervention for Q5 revealed that the EG and the CG performed significantly different$(t = -3.918, p < 0.001)$. The results indicated that the CG outperformed the EG. The CG recorded a mean score of $\bar{x} = 1.64286$, and the EG recorded a mean score of $\bar{x} = 0.12903$. The findings showed that the EG learners could not solve equations of the form $\frac{6x}{2} - 5 = 4$, indicating that learners lacked procedural knowledge. The Q5 required learners to demonstrate their flexibility in solving equations involving fractions. Learners could have first simplified $\frac{6x}{2}$ to $3x$ making it easier to apply additive inverses to solve the problem. However, most of the learners just copied the work as shown in the worked example revealing that learners lack procedural fluency. Procedural fluency refers to learners' knowledge of the where and when to use these procedures flexibly and accurately and appropriately (Kilpatrick, Swafford & Findel, 2001, p.121).

The results analysis revealed that no learner 0% of the learners in the EG were not able to explain why the approach used to solve the problem was correct, while 10.7% of the learners in the CG were able to explain why the method used in the example was correct. The results showed that 100% and 96.8% of the learners in the EG committed CE in Q5.1, and Q5.2 respectively, while 82% and 53, 6% of the learners in the CG committed CE in Q5.1 and Q5.2 respectively. There was 0% TE, PSE and EnE for the EG in Q5.1 while there were 0% TE and PSE and 7% EnE for the CG in Q5.1. Additionally, there were 0% TE and EnE committed by the EG in Q5.2 while there were 0% 0% TE and PSE and 14% EnE for the CG. The most common error committed by both groups was CE.

The post-test results findings for Q5 indicated that the two study groups did not perform significantly different $(t = -0.760,\ p = 0.455)$. The findings revealed that the EG recorded a mean of $\bar{x} = 1.25000$ while the CG recorded a mean of $\bar{x} = 1.62500$. However, the EG showed a greater improvement by recording a mean difference of $\bar{x} = 1.12097$ versus a mean difference of $\bar{x} = -0.01786$ for the CG suggesting that intervention positively impacted learners' performance. Learners gained procedural flexibility because they were able to get alternative strategies to solve equations involving fractions (Coppersmith & Star, 2022, p.383). The findings also revealed that 25% and 16.7% of the learners in the EG were able to solve Q5.1 and Q5.2 after the intervention versus 0% in both questions before the intervention, while 37.5% and 29.3% of the learners in the CG were able to solve Q5.1 and Q5.2.

There were 41.7% and 25% of the learners in the EG who committed CE in Q5.1 and Q5.2 respectively, while 54.2% and 62.5% of the learners in the CG committed CE in Q5.1 and Q5.2 respectively. There were 0% TE and PSE and 33.3% EnE committed by learners in the EG in Q5.1, while there were 0% TE and PSE and 8. 3% EnE committed by learners in the CG in Q5.1. The findings also revealed that 0%, 58.3% and 0% of the learners in the EG committed TE, EnE and PSE in Q5.2 respectively, while 0% and 4.2% of the learners in the CG committed TE and PSE and EnE errors respectively in Q5.2. The most common error committed by both groups was CE.

6.2.1.6 Results before and after intervention for question 6

The pre-test findings of the results of Q6 depicted that the two groups performed significantly different $(t = -3{,}916,\ p < 0.001)$. The findings revealed that the CG outperformed the EG recording a mean score of $\bar{x} = 2.0000$ whereas the EG recorded a mean score of $\bar{x} = 0.16129$. The findings of the results indicated that learners in the EG struggled to solve equations involving fractions with different denominators. Learners in the EG had difficulties solving equations of the form $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$. In this question, learners were supposed to remove the fractions by multiplying by the lowest common denominator. However, most of the learners in both added numerators and then denominators. Some of the learners multiplied the numerators and added the denominators revealing lack of conceptual knowledge in solving algebraic fractions. Learners could not verify the validity of their answers due inadequate conceptual knowledge (Kilpatrick et. al., 2001, p.118).

The analysis of the results revealed that 0% of the learners in the EG was able to answer both Q6.1 and Q6.2, while 17.9% and 10.7% of the learners in the CG were able to answer Q6.1 and Q6.2 respectively before the intervention. The results also revealed that 100% of the learners in the EG committed CE in Q6.1 and Q6.2, while 64.3% and 67% of the learners in the CG committed CE in Q6.1 and Q6.1, respectively. The results also showed that both study groups recorded 0% TE, PSE and EnE in Q6.1. The EG recorded 0% TE, PSE and EnE in Q6.2 while the CG recorded 0% TE and 7% PSE and 3.5% EnE in Q6.2. The results analysis revealed that the most common error committed by learners in both study groups is CE. Very few learners committed TE, PSE and EnE.

The post-test findings for Q6 indicated that the EG and CG performed significantly different($t = -3.442, p = 0.002$). The CG performed better than the EG recording a mean score of $\bar{x} = 4.12500$ as compared to $\bar{x} = 1.83333$ for the EG. The EG recorded a mean difference of $\bar{x} = 1.67204$ versus a mean difference of $\bar{x} = 2.12500$ for the CG. Moreover, 33.3% and 10% of the learners in the EG were able solve Q6.1 and Q6.2 in the post-test as compared to 0% in both questions prior to the implementation of the intervention suggesting that the strategy positively impacted learners' performance on solving algebraic linear equations. However, the low mean score after the intervention showed that the learners in the EG still lack procedural and conceptual knowledge for solving equations with multiple different denominators. Moreover, the results analysis depicted that 33.3% and 50% of the learners in the EG committed CE in Q6.1 and Q6.2 in the post-test as compared to 37.5% and 16.7% of the learners in the CG committed CE in the same questions. There was no TE, PSE and EnE recorded by the EG in Q6.1 while the CG recorded 0% TE and PSE and 4.2% EnE in Q6.1. The results also revealed that there were 0% learners in the EG who committed TE, PSE and 16.7% who committed EnE in Q6.2 whereas there was 0%, 16.7% and 29% of the learners in the CG who committed TE, PSE and EnE in Q6.2 respectively.

6.2.1.7 Results before and after intervention for question 7

The findings of pre-test results for Q7 depicted that the EG and the CG performed significantly different ($t = -2.173, p = 0.038$). The findings showed that the CG outperformed the EG with mean scores of $\bar{x} = 1.035714$ and $\bar{x} = 0.09677$ respectively. The low mean scores for the groups indicated that learners did not enough conceptual knowledge to form equations from word problems they could use to solve the problem. Conceptual knowledge enables learners to make different representations of mathematical situations and knowing how to use these representations for different purposes (Kilpatrick, et al., 2001, p.119). Question 7 required learners to find the dimensions of a rectangle given that the length of the rectangle is twice the size of its width and the perimeter is 24cm. In Q7.1, learners were required to explain the errors in the erroneous example and Q7.2 required learners to solve the problem showing all the necessary steps.

In this question, most of the learners repeated the error in the erroneous example and never checked their answers to determine the accuracy of the answer. The results of the findings depicted that the learners in the EG failed to see that the first step and fourth steps in the working are incorrect because they lack conceptual knowledge. According to Kilpatrick et al. (2001, p.118), learners with conceptual knowledge know facts and methods and understand where and when to apply certain facts and methods.

The findings revealed that 0% of the learners in both study groups were unable to get a correct answer in Q7.1 and Q7.2 in the pre-test. The findings also showed that 100% of the learners in the EG committed CE in Q7.1 and Q7.2 while 53.6% of the learners in committed CE in both Q7.1 and Q7.2. There were 0% TE, PSE and EnE for the EG in Q7.1 while there were 11%, 4% and 0% TE, PSE and EnE respectively for the CG in Q7.1. The findings also indicated that there was 0% TE, PSE and EnE for the EG while there were 14.3% TE and 4% PSE and EnE for the CG. The most common error committed by the two groups is CE.

The finding of the post-test results for Q7 revealed that the EG and CG did not perform significantly different ($t = 0.118, p = 0.908$). However, the EG outperformed the CG recording a mean score of $\bar{x} = 0.583333$ compared to $\bar{x} = 0.54167$ for the CG. Moreover, the EG recorded a great mean difference of $\bar{x} = 0.48656$ after intervention against a mean difference of $\bar{x} = -0.49404$ for the CG suggesting that the intervention positively impacted learners' performance in Q7. Contrary to expectation, the CG's performance decreased in the post-test. The findings also revealed that some learners in the EG gained conceptual knowledge (Kilpatrick et al., 20001, p.119) as a result of the intervention since they were able to formulate an equation from the word problem which helped them to solve the problem.

Even though there were some improvements in performance in the learners in the EG, the findings also revealed that the learners still face some difficulties in solving word problems, the main challenge being comprehending the question. There were 58.3% and 33% CE recorded for the EG in Q7.1 and Q7.2 respectively and there were 0% TE and PSE for the EG in both Q7.1 and Q7.2 while there 0% and 8, 3% EnE for the EG in Q7.1 and Q7.2. The results also showed that the CG recorded 83.3% and 79% CE in Q7.1 and Q7.2 respectively while there 0% TE, PSE and EnE for the CG in both Q7.1 and Q7.2. The findings also revealed that the most common error committed by the two study groups was CE.

6.2.1.8 Results before and after intervention for question 8

The findings of the results of pre-test Q8 showed that the EG and CG performed insignificantly different$(t = -1.720, p = 0.094)$. The CG outperformed EG recording a mean score of $\bar{x} = 0.64286$ versus $\bar{x} = 0.19355$ for the EG. However, the low mean scores indicated that learners at both schools lacked strategic reasoning (Kilpatrick et al., 2001, p.129). In Q8.1, learners were supposed to explain errors in the erroneous example and in Q8.2, learners were supposed to solve Q8 showing all the necessary steps.

Q8. John is five times as old as Jabulani. After four, John will be three times as old as Jabulani. What are their present ages? To solve this problem, learners were supposed to form expressions for the boy's current ages and their ages after four years.

The findings revealed that no learner in the EG could obtain a correct answer in Q8.1 and Q8.2, and only 7.4% of the learners in the CG could obtain correct answers for Q8.1 in the pre-test. There was 100% CE for the EG in both Q8.1 and Q8.2 in the pre-test, while 60.7% and 64.3% CE were for the CG in Q8.1 and Q8.2, respectively. There was no TE, PSE, or EnE for the EG in both Q8.1 and Q8.2 in the pre-test, while there were 7% and 11% TE for the CG in Q8.1 and Q8.2, respectively. The CG also recorded 0% PSE and EnE in Q8.1 and Q8.2.

The post-test findings for Q8 depicted that the EG and the CG performed significantly different$(t = -2.755, p = 0.010)$. The CG outperformed the EG, recording a mean score of ($\bar{x}$ = 1.20833), whereas the EG recorded ($\bar{x} = 0.16667$). The CG recorded a mean difference of $\bar{x} = 0.56547$ against $\bar{x} = -0.02688$ for the EG. The findings revealed an increase in performance for the CG and a decrease in performance for the EG, suggesting that the intervention strategy did not positively enhance learners' strategic reasoning in this question.

Although the CG recorded an increase in mean difference, the low mean scores recorded by both study groups revealed that learners still have difficulties in understanding word problems, which require strategic reasoning. According to Kilpatrick et al. (2001, p.119), learners with strategic reasoning are able to understand texts, identify connections between concepts in mathematical situations and apply reasoning to solve question.

6.3 SUMMARY OF RESULTS OF THE PRE-AND POST-TEST

The pre-and post-test results were summarised using the totals for the EG and the CG. Table 6.1 below depicts the results of the summary.

Table 6.1: P*re-test results summary*

Group	N	Mean	t - score	p-value
EG	31	1.12903		
CG	28	12.82143	- 4.370	< 0.001

Table 6.1 above revealed that the EG and CG performed statistically different before the intervention ($t = -4.370, p < 0.001$) at 95% confidence limit, contrary to expectations. The CG recorded a higher mean of $\bar{x} = 12.82143$ compared to $\bar{x} = 1.12903$ for the EG. The low mean score recorded by the EG indicated that learners were struggling to solve algebraic linear equations before the intervention. Most of the learners in the EG did not understand the demands of the questions.

Table 6.2: ***Post-test results summary***

Group	N	Mean	t - score	p-value
EG	12	18.83333		
CG	24	23.00000	-1.329	0.195

The results of the post-test revealed that the EG and CG did not perform significantly different ($t = -1.329, p = 0.195$) at 95% confidence limit, contrary to expectations. The CG performed better than the EG. Although the two groups did not perform significantly different contrary to expectation after the EG had received some treatment, the EG's mean score increased from $\bar{x} = 1.12903$ before the intervention to $\bar{x} = 18.83333$ after the intervention, an increase of $\bar{x} = 17.7043$ compared to the CG's mean scores which increased from pre-test ($\bar{x} = 12.82143$) to post-test ($\bar{x} = 23.00000$), an increase of $\bar{x} = 10.17857$. The results indicated that the EG gained more knowledge on solving algebraic linear equations, suggesting that the intervention was effective in enhancing learners' performance.

6.4 QUALITATIVE FINDINGS

The qualitative data for this study were obtained from classroom observations and semi-structured interviews with participating teachers. Lesson observations were conducted to find out whether the intervention had an impact on teaching and learning of algebraic linear equations. Furthermore, semi-structured interviews helped to understand the views of the TEG on the impact of the intervention for teaching Grade 9 algebraic linear equations.

6.4.1 Classroom observations

The observations conducted before, during, and after implementing the intervention are shown below.

Pre-intervention findings

The findings of the results of this study prior to the intervention revealed that the teacher followed a traditional way of teaching, which was teacher centred and focused much of his attention on high-achievers at the expense of those learners who struggled in mathematics. Learners worked individually and the teacher did not walk around to check on the learners' progress. The teacher used the lecture method, a teaching approach that is teacher centred (Akcadag, 2021, p.348). Moreover, the teacher used whole class discussions that prevented low-achievers from participating in the learning process. The teacher did not promote learners' interaction by letting each learner work individually (Khasawneh, Al-Barakat & Almahmoud, 2023, p.1).

It was observed that the TEG used the textbook as the main source for both classwork and homework activities. This is supported by Ulusoy and Incikabi (2020, p.1), who contend that textbooks are major teachers' sources for classwork, homework and major instruction guides. Textbooks are specially designed resources for use by teachers and learners and serve as mediators between the curriculum and the teacher (Gracin & Matic, 2016, p.351; Ulusoy & Incikabi, 2020, p.1). Textbooks are essential resources because they contain the syllabus topics and guide teachers in sequencing topics (Mwisa, Wafula & Marwa, 2022, p.6). Textbooks are an important resource because they provide learners with exercises for practice and questions on related topics. It is essential that every learner have a textbook because textbooks are central to learners' performance (Mwisa, Wafula & Marwa, 2022, p.6). Learners use textbooks to study for tests and examinations and acquire knowledge for self advancement (Rezat, Fan & Pepin, 2021, p.1198). Learners at the two schools had access to textbooks. Each learner was issued a mathematics textbook and workbook from the Department of Basic Education. However, despite having enough textbooks, learners did not perform well an indcation that learners did not effectively use the textbooks.

Although the TEG gave learners homework, most did not complete their work before the next school day, indicating that most learners were not motivated to learn and did not want to take responsibility for making their own knowledge. Some of the learners did not complete homework and would offer excuses such as having failed to take down the task or forgetting the book at home, while other learners felt like it was too much work for them to do after school. This is supported by Keane and Heinz (2019, p.1), who observed that when learners are given too much homework, they fail to complete it and offer many excuses, for instance, failing to understand what they were supposed to do.

Findings during the intervention

Most learners appeared motivated to participate in the learning process during the intervention (Bandura, 1986). The findings suggested that the intervention promoted learner-centred teaching since learners were put into small mixed-ability groups and worked cooperatively to solve problems. Assigning learners into small mixed-ability groups ensured that even the low performing learners were no longer side-lined from the teaching and learning process. While working in small mixed-ability groups, learners can develop communication, critical thinking and analytical skills

(Abramczyk & Jurkowski, 2020, p.1). The learner-centred teaching approach ensured that the learners became actively involved in creating their own knowledge under the teacher's guidance. The teacher ceases to be the distributor of knowledge and becomes the teaching and learning coordinator. Learners involved in the lesson share ideas and respect each other's opinions. While working in small groups, learners identified errors in erroneous examples and assisted each other in solving the problems correctly. The findings also suggested that the TEG paid attention when the researcher demonstrated how to implement the intervention strategy because he was able to put into practice what he learned (Bandura, 1986). The findings also revealed that most of the learners were completing their homework before the next school day because they realized the importance of doing homework. This is supported by Keane and Heinz (2019, p.1), who posit that , when teachers emphasise the importance and value of homework, learners are motivated to complete the homework tasks. When the TEG emphasised to the learners that doing homework would provide an opportunity to correct their misunderstandings and help them curate their own knowledge, a number of learners completed their homework. Learners were excited to work like detectives when they looked for errors and found how to correct them.

Findings after the intervention

As observed during the intervention, TEG paid attention to the researcher's demonstration of intervention strategy because he was able to apply the strategy after the intervention. The TEG continued to use small mixed-ability groups (Bandura, 1986). The findings suggested that TEG changed his teaching approach after having been exposed to the intervention. During the intervention, the TEG was introduced to a new teaching approach of using learners' errors as opportunities for learning and how to help learners monitor their learning processes so as to close the gaps between their prior knowledge and new concepts (Abramczyk & Jurkowski, 2020, p.3). After the intervention, the TEG was found to have developed an interest in wanting to know how learners arrive at their answers by probing them to understand their thinking. The findings also showed that TEG prepared worksheets for learners in addition to getting questions from the textbook. The TEG used learners' erroneous solutions to design worksheets for group discussions.

To respond to the research questions, data was collected from observations before, during and after the intervention. The research objectives, are as outlined below:

1. to explore how teachers, apply pedagogical strategies to improve learners' performance in Grade 9 algebraic linear equations
2. to identify the possible challenges that teachers encounter in trying to enhance learners' performance using erroneous examples
3. to understand the possible advantages of using erroneous examples to improve learners' performance in solving algebraic linear equations
4. to determine how teachers can use erroneous examples for teaching Grade 9 algebraic linear equations to improve learners' performance.

The data collected before, during and after the intervention from observations are presented in the following sections using the themes that emerged.

6.4.2 Teaching strategy

The TEG's style of teaching before the intervention was more teacher-centred and not motivating to learners. The teaching approach was predominately the telling method. For instance, in lesson two, TEG using the telling method (van Lehn et al., 2019, p.459) would say, we subtract 4 from both sides, then subtract x from both sides to group like terms together and lastly, we divide by 2, when solving the equation $3x + 4 = x + 10$. TEG did not explain the reasons for all the actions taken to solve the problem, contrary to van Lehn et al. (2019, p.459), who states that teachers should not just tell learners what to do to solve problems. Instead, they should guide learners in thinking using prior knowledge to get answers. However, the TEG was more concerned about completing the syllabus on time.

Moreover, in the interview, TEG indicated that by policy, they are required to complete the syllabus on record and said, *"...as per departmental policy, we are required to complete the syllabus on record."* The approach was used in order to complete the syllabus. In addition, TEG focused mostly on the correct answers and would not ask learners to explain how they got their answers. Learners worked individually and they marked their own work during revision sessions. The teacher did not walk around to check on the learners' progress.

The findings showed that TEG did not support learners with mathematics challenges. Even though TEG provided scaffolding (Margolis, 2020, p.18) through worked examples, this approach only benefited the high-achievers. Low-achieving learners just copied the work from the board without understanding, and this could have contributed to learners' poor performance before the intervention. TEG did not make use of learners' prior knowledge; he would not ask learners about what they did the previous day before introducing the new work. According to Cordi, Schreiner and Rasch (2023, p.1), prior knowledge enables learners to encode what is being taught and makes it easier to understand new concepts.

Unlike TEG, the TCG started her lessons by making reference to the previous lesson's concepts as soon as learners complete marking homework. For instance, TCG asked learners to state the difference between an expression and an equation, to which most of the learners gave an affirmative answer. By allowing learners to be active participants during lessons, TCG was able to identify learners' ZPD and supported learners who had learning difficulties through scaffolding (Margolis, 2020, p.18).

During the intervention, TEG was able to support both high and low-achieving learners during group work activities. TEG encouraged learners to explain their answers and demonstrate their answers on the board so that the teacher would see what learners understood and what they were still struggling to understand. During group discussions, learners identified errors in erroneous examples and assisted each other in finding correct solutions. This is supported by Abramczyk and Jurkowski (2020, p.57), who state that when learners are organised to work in small groups, they interact, share ideas and support each other's learning processes. TEG walked around monitoring learners' progress and would provide scaffolding by prompting learners to give possible reasons for the errors in the erroneous examples and guiding them on how to solve the problems correctly. TEG encouraged learners to identify keywords when solving word problems. In other words, TEG encouraged learners to understand the problem before attempting to solve it. This is supported by Goodrich and Namkung (2019, p.257), who state that learners' ability to solve word problems depends on their ability to comprehend the problem narrative and understand the meanings of keywords in the problem. Most of the learners struggled to clearly explain their ideas in English. However, TEG allowed learners to code-switch (Maluleke, 2019, p.1) wherever necessary but encouraged learners to understand the meanings of the keywords in word problems (Goodrich & Namkung 2019, p.257).

6.4.3 Classroom interaction

Learners can get satisfaction from teaching and learning if they are actively involved during lessons (Martin et al., 2020, p.227). These researchers argue that they might succeed only when they become active participants during lessons. However, learners at the experimental school were not seen to be active participants before the intervention. Learners had no chance to interact with one another because they sat individually at their respective desks. The seating arrangement might have made it easy for the teacher to control the class. This might have contributed towards learners' poor performance in the pre-test.

The teacher at the control school allowed learners to show their answers on the board. She allowed learners to work in pairs and learn from one another. TCG involved both the high achievers and low achievers in the teaching and learning process by randomly asking any learner to answer questions and to demonstrate their answers on the board. TCG encouraged her learners to volunteer their answers so that they could learn from their mistakes if their answers were wrong and boost their confidence if their answers were correct. By encouraging learners to volunteer their answers, TCG managed to keep learners motivated and engaged during teaching and learning. Motivated learners pay attention in class, participate in class discussions and are interested in learning (Havik & Westergard, 2020, p.448).

During the intervention, it was found that the TEG was keen to get learners to participate actively during lessons by organising them to work in small groups, monitoring and providing them with support whenever needed. Learners were found to be motivated to take part in group discussions even though some learners struggled to express themselves clearly. The teacher at the experimental school encouraged learners to use vernacular (i.e., isiZulu) when necessary but reminded them to always ascertain the meanings of keywords in the problems. According to Erath, Ingram, Maschkovick and Prediger (2021, p.256), language is an important means and resource for the learning of mathematics, and as such, learners should be given the opportunity to explain mathematics terms and justify procedures in their own language. While working in small groups, learners at the experimental school were able to share ideas and experiences on solving algebraic linear equations. This was in support of Havik and Westergard (2020, p.448), who argue that when working in small groups, learners discuss, share ideas and ask each other and their teachers some questions. While working in small, manageable groups, learners also listen critically to each other and present meaningful arguments using their own examples (ibid).

6.4.4 Teaching and learning resources

According to Ulusoy and Incikabi (2020, p.1), it is impossible to implement the curriculum without teaching and learning resources. The main resource for teaching mathematics has been the textbook from time immemorial. The authors further contend that textbooks serve as a mediator between the teacher and the curriculum because they are specially developed for classroom teaching and learning.

The findings revealed that both TEG and TCG used textbooks as their main resource for teaching and learning. The two teachers also used the textbook as a resource for both classwork and homework activities. Both teachers also used textbooks to choose tasks and get instruction guidance. The findings revealed that both teachers used the DBE mathematics learners' workbooks for further practice. The findings also revealed that TCG used other learners' workbooks as additional homework and classwork activity sources. This is supported by Utami, Aminatun and Fatriana (2020, p.7), who state that learners' workbooks are valuable sources of classwork and homework activities. In addition to textbooks and DBE workbooks, the TEG designed worksheets using learners' erroneous solutions. Learners at the experimental school used worksheets during group discussions, which helped them reduce the gaps in their understanding between what they already knew and the new concepts.

Furthermore, the findings revealed that despite having smart boards in their classrooms, neither teacher was able to use them due to the unavailability of electricity during shedding. According to Das (2019, p.24), the use of ICT in mathematics classrooms can enhance learners' understanding of solving equations and proving some geometric problems, which in turn helps learners develop their critical thinking. Moreover, when teaching and learning resources are effectively utilised they enhance learners' performance (Mbugua, Tanui, Kirui & Maina, 2021, p.18). Accordingly, lack of and underutilisation of teaching and learning resources impacts learners' performance negatively (ibid, p.18). Unlike the TEG, the TCG was able to use the whiteboard. The TEG was unable to use the whiteboard because of the unavailability of markers due to a lack of funds.

6.4.5 Classroom assessment

The findings revealed that both teachers gave learners classwork. Before the intervention, the experimental learners marked their own work during revision sessions. By letting learners mark their work, the TEG allowed them to self-regulate and develop self-efficacy and improvement (Taylor et al., 2020, p.271). However, not all learners are able to self-regulate. Some learners were found to mark some of the incorrect answers to avoid writing corrections. The findings revealed that both teachers used classwork as a formative assessment, which enabled them to evaluate the extent to which learners understand the content being taught and then make adjustments when necessary. Classwork also enabled learners to monitor their understanding of concepts.

In contrast to the practice at the experimental school, learners at the control school exchanged their notebooks and marked each other's work. The TCG allowed learners to exchange their workbooks to see each other's solution methods, thereby allowing learners to develop strategic competence since they would compare their working approaches. Rittle-Johnson, Star and Durkin (2017, p.19) argue that allowing learners to share and compare their solution strategies is a reform pedagogy in practice internationally, and as such, teachers are encouraged to put it into practice. Taylor et al. (2020, p.271) also state that letting learners mark each other's work effectively enables learners to self-regulate and promote self-efficacy, allowing learners to learn each other's reasoning. When learners mark each other's work, they learn different approaches to solving problems (Rittle-Johnson, Star & Durkin, 2020, p.599).

The findings also indicated that both teachers gave learners homework. Homework promotes learner autonomy and responsibility for their own work (Hussain et al., 2022, p.452). Homework also gives learners a chance to do extra work outside of class time and this allows learners the chance to retain those concepts taught in class through further practice (Keane & Heinz, 2019). The (ibid) further argues that through homework, learners take responsibility for making their own knowledge. In support of the above, Hussain et al. (2022, p.452) assert that homework gives learners the opportunity to exert more effort where they need to improve, and at the same time, homework enables parental involvement in their children's learning. Through homework, parents are able to monitor their children's progress (ibid). However, the findings showed that most of the learners at the experimental school did not do their homework prior to the intervention. After the intervention, the results revealed that most of the experimental group learners were completing their homework before lessons, and this suggested that learners were motivated to learn and take responsibility for gaining and making their own knowledge.

6.4.6 The intervention strategy: Using erroneous and correct examples

Since the intervention for this study aimed to improve learners' performance in solving algebraic linear equations, it was action research. Through a reconnaissance process (Mapotse, 2018, p.232), the researcher collected information regarding learners' performance in mathematics. The researcher used classroom lesson observations and open and semi-structured interviews to collect information from TEG about learners challenges in mathematics. Moreover, the researcher used data collected from the pre-test to inform the intervention strategy.

Before the intervention, the researcher observed that the TEG did not focus on learners' errors and related misconceptions; rather, TEG's focus was on learners' correct responses. Each time a learner responded to a question, TEG would ask other learners to confirm whether the answer was correct or wrong before giving the correct response. The researcher also noticed that using whole-class discussions, the TEG did not enable learners who struggle in mathematics to participate in the learning process. Mosser et al. (2022, p.4) observed that during whole-class discussions, most teachers have a tendency to interact more with high achieving learners at the expense of learners with learning difficulties. This is echoed by Shery (2019, p.1), who argues that in whole-class discussions, teachers' follow-ups tend to evaluate learners' answers as either right or wrong and discourage learners' extended participation.

The researcher implemented the intervention in three cycles. In the first cycle, the intervention focused on cooperative learning as opposed to whole-class discussions, which TEG had been using prior to the intervention. Learners were given roles to play in their respective groups. In the second cycle, the focus on creating mixed-ability and gender-balanced groups and using learners' errors as erroneous examples during group discussions in which learners assisted one another in identifying errors and suggested possible ways of solving the problems correctly. In the last cycle, learners applied what they learned during the second cycle and presented their solutions to the whole class. The TEG allowed all learners to participate during lessons by implementing cooperative learning. Through cooperative learning, the teacher was able to monitor all learners' progress and was also able to assist learners who struggled with mathematics.

6.4.7 Implementation of erroneous examples as an intervention strategy

Immediately after interviewing the TEG, the researcher held discussions with the TEG and decided to use erroneous examples as an intervention strategy to improve learners' performance.

During the first cycle of the intervention, the findings revealed that learners were assigned into homogeneous ability groups and as such, not all learners were actively involved during group discussions. Only high achieving learners participated during group discussions and benefited. Cooperative learning did not benefit some learners because learners who faced difficulties were found to be in the same group and could not get support from their peers. The TEG frequently assisted low-achieving learners during group discussions since the majority of the learners could hardly comprehend most of the questions. The findings of the first cycle revealed that there was no improvement in learners' performance and that homogeneously grouping learners could not produce the desired goals.

During the second cycle, learners were assigned to mixed-ability groups. During the second cycle, the findings revealed that learners were working cooperatively, sharing ideas and learning from each other's errors (Smale-Jacobse, Meijer, Helms-Lorenz & Maulana, 2019, p.4). The TEG also monitored learners' progress as he walked around the class, visiting each group of learners and asking questions on how they identified errors in the erroneous examples. The TEG asked questions to ascertain what learners knew so that he could provide scaffolding during the intervention. The TEG also probed learners to explain their solution strategies, which enabled the teacher to get an understanding of their thinking in coming up with their solution methods. The findings also revealed that apart from getting assistance from their peers, the TEG offered scaffolding to learners where they faced difficulties. Additionally, the findings showed that using worksheets that were created using learners' errors assisted most learners in learning from their mistakes and, therefore, managed to reduce the gaps between what they already knew and the new concept. The findings in the second cycle indicated that there was some improvement in both learners' participation and performance in solving algebraic linear equations.

The findings in the third cycle indicated that TEG emphasised that learners should look out for the meanings of keywords in word problems since keywords have special meanings which help learners solve problems. Learners are able to solve word problems if they can to read and understand the narrative in the word problem, which depends on the learners' mathematics vocabulary (Goodrich & Namkung, 2019, p.257). For instance, learners should understand the meanings of words like "twice" and "as much as" for them to be able to solve word problems involving age.

Working together in small groups, learners assisted each other in identifying errors in erroneous examples. However, most learners were still struggling to understand the meaning of some keywords and in such situations, TEG offered scaffolding, for instance, by prompting learners to explain what they understood to be the meanings of the words "twice" and "double" when used in a word problem. The findings in the final cycle also suggested that most of the learners were actively involved in the learning process by participating in group discussions and making contributions during group presentations. The findings in the third cycle also revealed that peer and teacher scaffolding was gradually reduced, and learners were allowed to take charge of their learning.

6.5 POST-INTERVENTION CLASSROOM OBSERVATION

The teacher used the question-and-answer approach to introduce lessons. This allowed the teacher to ascertain learners' prior knowledge and identify gaps between the current concept and what the learners already know. Knowledge of learners' prior knowledge enables teachers to adjust their lessons to meet learners' needs and allows them to make connections between concepts.

6.5.1 Interaction between teacher and learners

Data collected from the lesson observation suggested that the teacher at the experimental school wanted learners to link what they already knew with what they were currently learning by asking learners questions related to the previous lesson. The findings revealed the teacher at the experimental school wanted learners to gain conceptual knowledge by making connections between concepts. The question-and-answer method used by the teacher enabled him to get an understanding of the challenges that learners might be facing and also to gauge their progress. According to Rohid et al. (2019, p.21), when teachers monitor learners' talk, they will be able to identify those who participate and why and also identify those who do not participate and why and in so doing, they will be able to offer scaffolding when required.

The findings revealed that most learners were actively involved in teaching and learning. Learners participated actively during group discussions and group presentations, allowing them to demonstrate what they understood. Learners were also allowed to ask questions for clarification from the teacher or their peers. When learners are allowed the chance to ask each other and their teacher questions during teaching and learning, they develop an interest in the teaching and learning process, enhancing their understanding. Peer-to-peer and learner-to-teacher communication is crucial in the learning of mathematics because it enables learners to build confidence and also allows learners the chance to consolidate their knowledge and make connections between concepts (Rohid et al., 2019, p.21)

Data collected also revealed that the TEG supported during lessons through scaffolding. The TEG guided learners in the identification of keywords and assisted learners in getting the meanings. The TEG ensured that he did not passively pass information to learners but allowed learners to be creators of their own knowledge by being actively involved in the learning process. When learners explain their thinking to others, they recognise their misconceptions, correct their understanding, and strengthen connections between what was taught and the new information (Webb et al., 2019, p.177). The data showed that the TEG was using a learner-centred teaching approach instead of the teacher-centred approach he used before the intervention.

6.5.2 Learner-learner interaction

The data collected suggested that learners learn cooperatively when grouped into mixed-ability groups. Learners were able to assist each other in identifying errors in erroneous examples and worked cooperatively to find strategies for solving the problems. Learners were actively participating during lessons. When learners interact with each other, they get the opportunity to reconstruct their knowledge and understanding (Webb, et al., 2019, p.177). No learners were seen to be disengaged during teaching and learning. This was in contrast to the situation prior to the intervention, where some learners were seen passively sitting during class discussions. Learners were able to ask comprehension questions and challenged each other during group discussions.

The findings indicated that cooperative learning allowed learners to develop teamwork skills, respect for each other's opinions, self-respect, accountability and responsibility for making their knowledge from their mistakes. Most learners were motivated to learn and showed interest in participating in group discussions where they demonstrated their understanding and lack of concepts and were willing to learn one from the other.

6.5.3 Teaching and learning resources

The findings revealed that the TEG and the TCG had similar teaching and learning resources. Both teachers used textbooks as the main resource for classwork and homework activities. They also used the textbook for guidance and instruction to implement the curriculum, as supported by Ulusoy & Incikabi (2020, p.1), who opine that textbooks mediate between the teacher and the curriculum. Gracin and Matic (2016, p.351) mentioned that a textbook is a specially developed resource for classroom teaching and learning. When resources are available, they enhance the schools' effectiveness and positively impact learners' academic performance, provided they are used effectively (Sirajo & Abdullahi, 2023, p.122). The findings suggest that the teacher of the control school used textbooks more effectively than her counterpart because her learners performed better than those at the experimental school.

However, during and after the intervention, the TEG managed to improve learners' performance utilising worksheets. The TEG designed worksheets using common errors in solving algebraic equations in the first cycle of the intervention, and this did not yield the desired improvement in learners' performance since most of the learners could not suggest how the errors in the erroneous examples were generated. When the TEG designed worksheets based on learners' own erroneous solutions, learners were able to explain their reasoning in coming up with those errors, and this helped learners identify the gaps between what they already knew and the new concept they were studying. The findings revealed that the TEG continued the use of worksheets, an idea he observed from the researcher during the intervention. The findings also showed that most of the learners were motivated to learn when they used worksheets with errors resembling their own errors, enabling them to correct their understanding.

6.5.4 Classroom assessment

Data collected indicated that the TEG continued to use both classwork and homework to monitor learners' progress. The use of class work and homework allowed the TEG to provide assistance to learners when needed, and this helped learners not develop knowledge gaps. The findings revealed that most learners completed their homework before class time, unlike the period before the intervention, where most did not. It suggests that most learners were motivated to learn and complete their homework after the intervention.

The results of findings suggested the intervention positively impacted learners' attitudes towards learning. The fact that learners discovered that committing errors is not unique to individual learners might have motivated learners to persevere when solving problems, resulting in learners improving their performance.

6.5.5 Semi-structured interviews

The researcher conducted interviews with both the TEG and the TCG, as mentioned earlier in Chapter Four. The interviews were conducted to understand of what the two teachers believed causes learners' poor performance in solving algebraic linear equations. The researcher conducted face-to-face semi-structured interviews with each of the teachers before implementing the intervention. The findings revealed that both teachers believed that learners' poor performance was a result of a lack of understanding and misconceptions inherited from previous grades.

The researcher also conducted semi-structured interviews with the TEG during and after the intervention to understand the benefits and challenges of using erroneous examples to improve learners' performance in solving algebraic linear equations. The duration of the interviews was between fifteen and twenty minutes per session per teacher. The findings indicated that designing worksheets was time consuming and challenging at the beginning of the intervention. The TEG pointed out that most of the low-achieving learners could not explain how most of the errors in erroneous examples were committed due to language difficulties. For example, learners in the EG could not explain what was wrong in the solution to $4x - 3 = x + 15$, where the first step in the solution was shown as: $4x - 3 - 3 = x + 15 - 3$. For all learners at the experimental school, English is second language and expressing themselves in the language was a challenge. The TEG encouraged his learners to code-switch so that learners do not get stuck during discussions. However, learners were encouraged to understand the meanings of keywords, expressions and statements in word problems.

The findings revealed that the strategy needed more time to plan and implement. The TEG was complaining about completing the syllabus as per departmental requirement. However, the TEG acknowledged that the intervention exposed learners to common errors and enabled them to fix their own mistakes. The teacher also contended that the intervention allowed learners to make connections between concepts and enhanced their understanding. When learners receive explanations as to why erroneous examples are incorrect, they correct their misconceptions and the connection between prior knowledge and new information is strengthened (Webb et al., 2019, p.177). The findings indicated that the TEG was more comfortable to use erroneous examples to consolidate learners' understanding after using other instructional strategies to impart knowledge.

6.5.6 Performance of Grade 9 learners in mathematics

The findings showed that poor performance by learners in solving algebraic linear equations is as a result of several factors which include among others learners' negative attitude towards mathematics, and teachers' pedagogical practices (Mazana et al., 2019, p.207), and matheamics anxiety (Namkung, Peng & Lin, 2019, p.460). In addition lack of resources for teaching and learning, language of instruction and poor comprehension of questions contribute significanly towards learners' poor performance in mathematics. The learners' performance at the experimental school significantly improved even though it was not satisfactory. The EG showed an increase in mean scores from $\bar{x} = 1.12903$ before intervention to ($\bar{x} = 18.83333$) after intervention, revealing an increase of $\bar{x} = 17.7043$ suggesting that the strategy positively impacted learners' performance.

6.5.7 Learners' knowledge of mathematics

The findings revealed that most learners at the two study schools lacked conceptual knowledge. Conceptual knowledge assists learners in organising their work and make connections between their prior knowledge and what they are currently taught (Webb et al., 2019, p.177). However, through active participation during group discussions and assistance from both the teacher and peers, The EG seemed to have improved conceptually.

The TEG provided learners with some scaffolding during the intervention and this motivated learner to be actively participate in making their own knowledge during lessons. Learners were not passive recipients of knowledge. The learners at the experimental school gained knowledge on how to solve algebraic linear equations, respect each other learners' opinions and how to be team players during and after the intervention. Learners were able to solve equations like $2x + 3 = 15$ which they could not solve before the intervention (Rushton, 2018, p.3). Because learners actively participated during lessons, they were able to create their own mathematics knowledge by learning from their own mistakes (Ouko, 2021, p.7).

6.5.8 Learner support in Mathematics

The findings prior to the intervention revealed that the TEG was not supporting learners with difficulties in mathematics because his teaching strategy was mainly teacher-centred. This showed that TEG did not apply the principles of the ZPD. However, the TCG applied the principles of the ZPD by assisting learners who experienced learning difficulties by seating them with their more capable peers and allowed learners to compare solution strategies by allowing learners to mark each other's work.

Data gathered during and after the intervention revealed that the TEG used cooperative learning successfully by allowing learners to be active participants in the teaching and learning process. The TEG was able to apply the principles of the ZPD by assisting learners who experienced learning challenges by placing learners into mixed-ability groups. Learners assisted each other to solve equations like $4x - 3 = x + 15$ applying inverse operations which they could not do before the intervention. Learners were given the chance to display their understanding and challenges during group presentations (Rohid et al., 2019, p.21). Learners were afforded the chance to learn from each other because they were placed into mixed-ability groups where those who are more knowledgeable would assist those who are struggling.

6.5.9 Pedagogical content knowledge

Data collected before the intervention revealed that the TEG's pedagogical strategy did not carter for learners' individual differences. The TEG mainly focused on high achieving learners at the expense of those learners with learning difficulties. The teachers' pedagogical content knowledge (PCK) allows him/her to teach effectively by simplifying complex concepts into chunks that are easily understood by learners. PCK also enables teachers to create a conducive environment for learning that benefits all learners. The TEG seemed to have a challenge in applying a teaching strategy that creates an enabling environment for all learners. However, during and after the intervention the TEG indicated that he was able to change his teaching strategy so as to accommodate learners with different learning abilities. During and after the intervention, the TEG was able to provide support to learners who were struggling with mathematics. The TEG was able to assist learners to use their prior knowledge in creating new knowledge by linking what they already know to what they were learning. This helped learners to develop conceptual understanding, which assisted them in solving problems after the intervention. The data collected revealed that the TEG wanted to understand learners' understanding by asking probing questions during the intervention. For instance, TEG would say, what would you do to collect like terms in the equation $3x + 4 = x + 8$? TEG wanted to understand the learners' understanding of how to solve equations with variables on both sides of the equal sign. This is in support of Mason (2020, p.134), who states that teachers ask learners probing questions to check on their understanding.

6.6 SUMMARY OF QUALITATIVE AND QUANTITATIVE RESULTS

As indicated earlier, the strategy involved assigning learners into small mixed-ability groups where they are given the chance to work cooperatively and to identify errors in erroneous examples and collaborate in looking for possible strategies for solving the problems. Learners learn to be team players as they work together to find strategies of solving problems. Furthermore, learners develop communication skills and critical thinking in addition to developing a sense of belonging to the community of learners. When learners engage in group activities, they become actively involved in making their own knowledge rather than passively listening to others.

6.6.1 Teacher's expertise experience

The findings of results revealed that the strategy enabled the learners to identify their mistakes by comparing their solution strategies with those of their peers thereby making their own knowledge from their mistakes. The results showed that through identifying where they went wrong in solving problems, learners were able to develop conceptual knowledge, which assisted them in solving problems in the future (Kilpatrick, Swafford & Findel, 2001, p.118). Additionally, the results revealed that the TEG could understand the difficulties that learners encounter while solving algebraic linear equations, enabling him to provide scaffolding to assist learners who struggle with mathematics.

6.6.2 Motivation when using erroneous examples

When using erroneous examples learners are assigned into mixed ability groups, where they can assist each other and get assistance from the teacher when necessary. When learners compare their solution methods with their peers they are motivated to learn when they realise that they are not the only ones facing difficulties in solving algebraic linear equations. While discussing in small manageable groups rather than whole-class discussions, learners ask comprehension questions to peers and their teacher and, therefore, develop thinking and reasoning skills (Webb et al., 2019, p.177). Learners also develop communication and collaboration skills. The findings revealed that inspired the TEG, and this changed his teaching practice, which positively impacted learners' performance as reflected in the improved learners' average marks in the post-test.

6.6.3 Improvement in learners' performance

This study investigated the impact and effectiveness of erroneous examples as a strategy of instruction teachers could use to improve Grade 9 learners' performance in solving algebraic linear equations. As such, it seemed that erroneous examples enabled learners in the EG to improve performance in solving algebraic linear equations (Rushton, 2018, p.2). The findings revealed that before intervention, most of the learners struggled to solve the most basic algebraic equations due to inadequate skills. For instance, learners in the EG struggled to solve the basic equation $2x + 3 = 15$. The majority of simply copied the erroneous examples when solving algebraic linear equations because they did not understand the questions.

As earlier stated, the results after intervention revealed the difference in mean scores between the experimental and control groups. The post-test results revealed that the EG and CG did not perform statistically different ($t = -1.329,\ p = 0.195$) at 95% confidence limit. The CG performed better than the EG. However, the EG's mean score increased from ($\bar{x} = 1.12903$) to ($\bar{x} = 18.83333$) by $\bar{x} = 17.7043$ whereas the CG's mean scores increased from ($\bar{x} = 12.82143$) to ($\bar{x} = 23.00000$) by $\bar{x} = 10.17857$. The findings of the results indicated that the EG gained more knowledge on solving algebraic linear equations suggesting that the intervention was effective in enhancing learners' performance.

6.7 ANSWERING RESEARCH QUESTIONS

The primary research questions this study was carried out to answer, as stated in Chapter 1, are: 1) What is the impact of using erroneous examples on the effectiveness of teaching and learning grade 9 algebraic linear equations? 2) How do the learners' testing outcomes help explain why grade 9 school learners experience difficulties in mathematics? The secondary research questions were designed to unpack the primary research questions by responding to the objectives of the study. The following section is dedicated to answering the secondary research questions.

6.7.1 Pedagogical strategies

From the lesson observations carried out before intervention with both the teacher of the TEG and TCG, the findings revealed that the two teachers used different teaching approaches. The control group teacher used a learner-centred teaching approach where she allowed learners to work collaboratively in mixed ability pairs. However, the TEG used the teacher-centred approach, where together with the high achieving learners dominated class discussions. The findings revealed that the TEG did not support those learners who struggled with solving algebraic linear equations. He was mainly focused on getting correct answers from learners without worrying about how they got answers. When learners gave wrong answers, the TEG would write and explain the correct answer on the chalkboard for learners to copy into their workbooks as corrections. This was in contrast to van Leeuwen and Jansen (2019, p.72), who state that teachers should ask learners to elaborate and clarify their answers to show the level of their understanding. The TEG offered scaffolding using correct work examples, which learners copied into their notebooks for reference in the future when solving similar problems. However, not all learners might have understood what they copied, and the TEG did not allow them to learn from their peers since each learner was observed sitting at their individual desks. The TEG's teaching strategy before intervention did not allow learners to make their own knowledge.

The TEG allowed learners to share ideas by marking each other's work during revision. Learners from the control group were given the chance to compare their answers; in that way, they could learn one from the other. The learners from the control group practised cooperative learning, and they were allowed to explain their answers. In contrast, those from the experimental group did not practice collaborative learning. Prior to the intervention, learners from the experimental group were passively receiving information from the teacher and were not actively involved during lessons.

The findings revealed that both teachers used textbooks and learners' workbooks as sources of both classwork and homework exercises. According to (Sievert, van den Ham, Neidermeyer, & Heinze, 2019), textbooks are a crucial resource for teaching mathematics because they teach the sequencing of content and the selection of tasks.

During the intervention, teaching and learning with the experimental group were implemented using action research. In cycle one; learners were given the chance to work in small groups where they followed group rules agreed upon between the researcher, the TEG and the learners. Working in groups of learners allowed each learner to be actively involved during lessons. As researcher and the TEG we developed worksheets with erroneous examples for learners to discuss in groups. Learners identified errors in erroneous examples and discussed methods of solving the problems correctly. However, during the first phase, not many learners benefited from discussing in group because learners were not grouped into mixed ability groups in which the more knowledgeable learners would assist those who struggle with mathematics. In the second cycle, learners were assigned into mixed-ability groups, and the researcher and the TEG agreed to use learners' errors as erroneous examples during group discussions. Learners worked cooperatively during group discussions. Learners assisted each other during group discussions and were offered a chance to learn from their own mistakes and those of others. Some learners were motivated to learn when they learned they were not the only ones committing specific errors when solving algebraic linear equations. Learners received support from both the teacher and their peers. Learners were offered the opportunity to ask questions among themselves in their respective groups and also ask the teacher. Learners were actively involved during since they no longer receive information while passively sitting at their desks.

In phase 3, we agreed to continue using learners' own errors during group discussions since this assisted most of the learners in checking their understanding of concepts and accordingly adjusted the gaps in their understanding. In the third cycle, learners presented their solutions to the rest of the class, showcasing their knowledge of the concepts. However, the findings revealed that not all learners were articulate in expressing themselves in English. The TEG allowed learners to code-switch whenever necessary for a smooth flow of discussions and presentations.

After the intervention, the TEG prepared worksheets using learners' errors, and learners presented their solutions to the whole class. The TEG seemed interested in using the teaching strategy for enhancing learners' performance. Learners also appeared to have gained knowledge on solving algebraic linear equations, as reflected by their improved performance in the post-test. The TEG motivated learners to participate during lessons since this allows learners to learn from their mistakes and helps them make their own knowledge.

6.7.2 Types of errors when solving algebraic linear equations

The results of the pre-test and post-test revealed that learners commit various types of errors when solving algebraic linear equations. The types of errors learners committed included comprehension, transformation, process skills, and encoding errors. No reading errors were recorded for this study since learners were not interviewed. The findings before and after the intervention revealed that Grade 9 learners generally have problems comprehending questions when solving algebraic linear equations. Most of the learners were observed to commit comprehension errors, while a few committed transformation, process skills and encoding errors. The results revealed that most of the experimental and control group learners committed comprehension and transformation errors.

Additionally, there were very few encoding and processing skills errors committed by learners from the two study groups. Understanding the questions' demands was revealed as the major challenge experienced by learners when solving algebraic linear equations. When a learner fails to understand questions, s/he will hardly get correct answers. Learners should understand what they read to identify the information given and then decide on what is required to solve the problem. The findings of the results indicated that most of the learners from both study groups had insufficient conceptual and procedural knowledge, the type of knowledge that should assist learners to understand what they read. Therefore, most of the learners committed comprehension errors.

6.7.3 Challenges of using erroneous examples

The results of the qualitative findings revealed that developing worksheets using erroneous examples requires the teacher to have a good PCK. The TEG hesitated to use erroneous examples for teaching because he thought exposing learners to errors might reinforce learners to commit errors. However, after some discussions and demonstrations, the TEG was willing to implement the intervention strategy. Another challenge is that for the strategy to be effective, learners should be put into heterogeneous groups in terms of ability and gender. Otherwise, learner interaction would not be effective. Additionally, it was observed that learners' proficiency in language of instruction is important, for instance, English, explaining errors in erroneous examples might be difficult. Erroneous examples need more time to prepare worksheets and time to implement, which might affect the effective completion of the syllabus on time.

6.7.4 Benefits of using erroneous examples

The findings indicated that erroneous examples enabled learners to identify their gaps in understanding concepts. While learners work co-operatively and collaboratively in identifying errors and finding strategies for solving the problems, learners are actively engaged during lessons. Learners become players and develop communication skills, respect each other's opinions, and be responsible for making their own knowledge. Additionally, the findings showed that erroneous examples exposed learners to the common errors that Grade 9 learners commit when solving algebraic linear equations and that learners are more likely to avoid such errors in the future. Learners might also be motivated to learn when they observe other learners commit the same errors.

When learners work co-operatively to identify errors in erroneous examples, they might develop critical thinking skills as they work like detectives. Additionally, identifying errors in erroneous examples might enable learners to develop creative thinking and conceptual and procedural understanding.

6.7.5 Developing the effective use of erroneous examples

The results indicated that when learners are assigned into small mixed-ability groups they had a chance to interact among themselves and with their teacher and they participated during lessons. Learners become responsible for making their own knowledge. Group rules should guide learners during their interaction with group members. Learners should not be passive observers waiting to absorb information without understanding what they are learning.

To effectively implement erroneous examples for teaching, the teacher should know all learners' levels of performance so that when learners are assigned into groups. Learners with low mathematical ability will benefit when assigned to a group with more knowledgeable peers. Learners should be encouraged to participate in group discussions, ask comprehension questions where they do not understand and become active participants during group discussions. Learners are encouraged to be responsible for creating their own knowledge. Learners should be encouraged to work cooperatively and collaboratively to create knowledge.

6.8 CHAPTER SUMMARY

The quantitative and qualitative findings from the collected data were discussed in this chapter. The results analyses of the two tests, the semi-structured interviews and lesson observations were integrated with a literature review and theoretical perceptions. The quantitative analysis of data before intervention indicated CG outperformed EG. The major challenge that learners from both study groups experienced was a lack of understanding of the demands of most of the questions. Many learners committed comprehension errors, among other types of errors, which included transformation, process skills and encoding errors. However, the findings revealed that both study groups performed unsatisfactorily. The lesson observations' results before the intervention indicated that most of the learners from both study groups had difficulties solving algebraic linear equations. Lesson observation results also revealed that the TEG was using teacher-centred strategies before intervention. The results of the findings showed that learners could not interact with each other as the learners sat passively at their desks during class discussions, which high performing learners dominated at the expense of those learners who had difficulties in solving algebraic linear equations which were not given the necessary support they needed most. The findings of the results revealed that the TCG provided learners with the necessary support they needed by allowing them to share ideas in pairs and assisting learners when she walked around during lessons.

The results during the intervention showed that TEG was able to change his teaching strategy. He involved most of the learners during lessons by grouping them into mixed ability groups in which they assisted each other in identifying errors in erroneous examples and cooperated to solve the problems correctly. Learners were allowed to discuss mathematics problems and share ideas on solving algebraic linear equations. The TEG was found to use learners' prior knowledge during teaching and learning, which allowed learners to see the connection between what they already knew and the new concept they would be learning.

The results of the statistical analysis prior to the intervention indicated that the EG and CG performed significantly different before intervention. The CG outperformed the EG. The analysis of the results after the intervention revealed that learners from both study groups performed better after the intervention. The findings revealed that the EG and the CG performed insignificantly different in the post. The CG outperformed the EG. However, the EG recorded a higher improvement rate than the CG, suggesting that the intervention positively impacted learners' performance.

CHAPTER SEVEN: CONCLUSION AND RECOMMENDATIONS

7.1 INTRODUCTION

This study was carried out to investigate the impact and effectiveness of erroneous examples to enhance Grade 9 learners' performance in solving algebraic linear equations. Extensive research abounds, indicating that algebra provides many learners with challenges, resulting in many failing to pursue their dreams of furthering education in mathematics related careers (McGinn, Lange & Booth, 2015, p.27). Research also indicates that when learners are afforded the chance to acquire a deeper understanding of the basic concepts in algebra, they have a better chance to succeed later (Star & Rittle-Johnson, 2009, p.14). However, not all teachers are able to implement strategies that enhance learners' understanding of the basic concepts in algebra. This study was carried out in two schools: one underperforming school, which was used as the experimental school, and the other a high-performing school, which was used as the control school.

This study used erroneous examples as the intervention strategy for improving learners' performance in solving Grade 9 algebraic linear equations. Implementing erroneous examples as a teaching strategy involves assigning learners to small groups of five or fewer. They co-operate in identifying errors in erroneous examples and sharing ideas on finding solutions to the problems. The intervention strategy used was reconnaissance action research involving three cycles: first, second and third. In the first cycle, learners were assigned into groups using picking cards numbered from one to five, and all learners who chose the same number belonged to the same group. The TEG, the learners and the researcher established the rules for the groups, and the learners respected and followed the rules. Learners showed respect for each other's opinions, worked co-operatively during group discussions, and took responsibility for their learning by making mistakes and asking comprehensive questions where they failed to understand. However, during phase 1, fewer learners benefitted from whole class discussions because learners were not grouped into mixed ability groups. The low-achieving learners could not understand the fictitious errors in the erroneous examples, and this resulted in the TEG assisting them often.

In the second cycle, learners were grouped into mixed ability groups and learners' own errors obtained from diagnostic testing were used in erroneous examples. The low-performing learners could get assistance from their more knowledgeable counterparts, and all learners participated during group discussions. Learners became aware of their own errors and those of their peers and discussed methods of solving the problems. Most learners became motivated to learn when they realised other learners committed the same errors. Learners actively participated when they presented their solution methods to the whole class during group presentations. However, not all groups were able to present their work due to time constraints.

Mixed ability groups continued in the third cycle because they proved effective in getting every learner to participate during lessons. Group discussions and presentations continued during the third cycle, enabling most learners to actively participate in the teaching and learning process. Learners shared ideas and learned social skills through co-operative learning. Learners experienced how to create knowledge from their mistakes. The teacher also learned how to keep learners engaged in the learning process through group discussions. The teacher was able to assist most of the learners whenever they faced challenges during group discussions by walking around and monitoring learners' progress.

In this chapter, the rationale for using the research design is reflected upon to conclude the impact and effectiveness of erroneous examples for teaching and learning Grade 9 algebraic linear equations. In this chapter, the study's limitations are discussed, and recommendations for future research are also suggested.

7.2 SIGNIFICANCE OF THE STUDY

As stated earlier, erroneous examples were used as an intervention strategy for teaching and learning algebraic linear equations. The use of erroneous examples involved co-operative learning in which learners were grouped into small groups of five or fewer learners. As such, collaborative learning is a learner-centred teaching approach with an emphasis on co-operation, collaboration and teamwork. Learners worked in groups to identify errors in erroneous examples and share ideas on achieving learning objectives (Retnowati et al., 2017, p.667). Erroneous examples allow learners to recognise erroneous steps in solving problems, and this assists learners in identifying the difference between erroneous and correct procedures. Learners' effort in identifying errors in erroneous examples and coming up with possible solutions teaches them to persevere in solving problems (McGinn, Lange & Booth, 2015, p.29; Rushton, 2018, p.12).

This study aimed to explore the impact and effectiveness of erroneous examples as an instruction method, to investigate how Grade 9 teachers teach algebraic linear equations, explore the challenges of erroneous examples and suggest ways of using erroneous examples to enhance learners' performance in solving algebraic linear equations. The researcher used a three-cycle reconnaissance action research to implement the strategy with the experimental group. To measure the effectiveness of erroneous examples as an intervention strategy, pre-test and post-test results were used to compare learners' performance before and after the implementation of the intervention with the experimental group.

The research design for this study was explanatory embedded mixed methods research using a pre-test and post-test to explore the impact and effectiveness of erroneous examples as an intervention strategy for teaching Grade 9 algebraic linear equations. This study used the pre-test to compare learners' performance from the experimental and control groups before the intervention. The researcher used a post-test with the experimental and control groups to compare learners' performance in solving algebraic linear equations after the intervention. The results before intervention revealed that both groups performed unsatisfactorily however, the results were statistically different. The control group outperformed the experimental group before intervention. The post-test results showed the two study groups did not perform statistically different. Despite performing insignificantly different, the control group outperformed the experimental group. However, the experimental group improved more than the control group by obtaining a greater difference of means between the test after intervention and the test before intervention.

Semi-structured interviews were used with the TEG and the TCG to understand what the teachers perceived as the possible reasons for poor performance by Grade 9 learners when solving algebraic linear equations. The researcher conducted lesson observations with the TEG before, during and after the intervention to see how the teacher utilised his teaching strategies. As stated earlier, the researcher administered a test to both study groups after the implementation of the intervention with the experimental group to compare learners' performance and determine the impact and effectiveness of the intervention strategy.

7.3 THE MAIN FINDINGS OF THE STUDY

The quantitative results of the before the intervention showed that the control group outperformed the experimental group, and there was a statistically significant difference in performance between the two study groups. However, although the control group outperformed the experimental group, both groups' performance was unsatisfactory. The results of the test after intervention revealed that the two study groups did not perform statistically different. Moreover, both groups improved their performance in the test after the intervention. However, post-test results indicated that the experimental group recorded an improvement that was statistically significant ($p - value < 0.001$). This significant improvement suggested that when erroneous examples were used for teaching algebraic linear equations might have positively impacted the TEG's teaching strategy, as the teacher was exposed to the new teaching practice during the intervention.

Data collected quantitatively and qualitatively before implementing the intervention indicated that learners from both study groups had comprehension difficulties, possibly due to poor conceptual knowledge of mathematics (Kilpatrick, Swafford & Findell, 2001, p.118). Before the intervention, The TEG's PCK mainly explained concepts to learners, and most of the time, learners sat passively, taking notes. Most learners committed comprehension errors in the pre-test, indicating they did not understand what they read. Poor comprehension affected most of the learners from the experimental group who simply copied the erroneous examples when responding to questions before intervention. The results also indicated that prior to the intervention, the TEG's teaching strategy was teacher-centred, and the main focus was on getting correct answers, which came from high-performing learners who dominated class discussions. Even though the TEG provided learners with correct examples as a way of scaffolding, the teacher did not assist low achieving learners who might not have understood the examples since learners were not given a chance to discuss in groups. Before the intervention, the results showed that the TEG did not use learners' prior knowledge and did not ask questions linking the previous concepts to the one currently studied. It seems like the TEG was not aware of learners' abilities before the intervention. Teachers should be mindful of learners' abilities and use teaching strategies that accommodate all learners.

During the first cycle of the intervention, the use of worksheets complemented textbooks and workbooks, which motivated learners in the experiment to learn and complete their homework, unlike before the intervention. This is supported by Bandura's (1986) theory which states that when the model motivates learners, they will pay attention be able to reproduce the act. The worksheets motivated learners to do homework. This suggests that learners' performance may be improved when resources are available and used appropriately. The results of the first cycle indicated that when learners are grouped according to ability, erroneous examples would benefit only the high performing learners. Low performing learners struggled to identify errors. In most instances, the teacher had to assist these learners since they could not get assistance from their group members.

Phase 2 results showed when learners work in mixed-ability groups, they assist each other and are better motivated to learn. Learners actively participated during lessons. They respected each other's opinions, collaborated and cooperated in group discussions. The results also revealed that when learners analyse erroneous examples representing their own mistakes, they learn better and can fill their knowledge gaps. During the second cycle, the data collected also revealed an improvement in learners' participation during lessons.

The data collected in the third cycle suggests that learners can benefit more from studying erroneous examples when working in small mixed-ability groups and when analysing errors that are similar to their errors. Learners who struggled in mathematics were able to get assistance from both the teacher and their peers. The teacher gradually reduced assisting learners, allowing them to take responsibility for their learning. Most learners were motivated to learn because even the low achieving learners were found to be active during group and class discussions.

After the intervention, the post-test results revealed the EG and the CG performed insignificantly different ($t = -1.329$, p-value $= 0.195$). However, the control group performed better than the experimental group. The results analysis of the post-test revealed that the control test outperformed the experimental group. The results also showed that both study groups improved in performance. The EG's mean score improved by $\bar{x} = 17.7043$ after the intervention compared to the CG's mean score which improved by $\bar{x} = 10.17857$. The improvement in mean score marks suggest that erroneous examples when used as an intervention strategy positively impacted learners' performance. The improvement also suggests that the TEG's teaching strategy changed, indicating the intervention positively impacted the teacher.

The findings suggested that the study achieved its goal of exploring the impact and effectiveness of erroneous examples as a teaching strategy for change to enhance Grade 9 learners' performance in solving algebraic linear equations. Before the intervention, the TEG used only correct examples to deliver lessons and used whole-class discussions as the only platform to engage learners in teaching and learning. However, only high-performing learners benefitted from the strategy. The TEG used erroneous examples to allow learners to participate actively during lessons. Learners were assigned into small mixed-ability groups when they identified errors in erroneous examples and shared ideas on solving the problems. Learners appeared motivated to learn, willing to participate in group discussions and respected each other's opinions.

7.4 THEORETICAL FRAMEWORK AND THE RESEARCHER'S WORDS

The results of this study provided understanding on how erroneous examples may be used to teach Grade 9 algebraic linear equations. The results revealed that while learners work cooperatively in small mixed-ability groups, they can assist each other in identifying errors in erroneous examples and that when learners analyse errors that are similar to their own errors, they correct their own misconceptions and reduce the gaps between their prior knowledge and the new concepts they are learning. As indicated earlier, using erroneous examples for teaching and learning promotes active learner participation in the learning process. When learners participate actively during lessons, they become creators of their own knowledge, which may improve their motivation to learn.

There experimental group's performance improved greatly after the intervention, suggesting that using erroneous examples as an intervention strategy effectively improved learners' performance in solving algebraic linear equations. When learners analyse and correct erroneous examples, they connect what they know already and what they are learning and this promotes deeper understanding (Rushton, 2018, p.12). Teachers with better content and pedagogical content knowledge may be able to effectively use erroneous examples daily to enhance learners' performance by transforming their expertise into knowledge that can be easily understood by learners (Kutluca, 2021, p.744; Jacob, John & Gwany, 2020, p.15).

Teaching using erroneous examples was characterised by learners working co-operatively in small groups, sharing ideas on how to identify errors in erroneous examples, and collaborating in solving the problems. Working in mixed-ability groups allowed learners to learn one from the other. Most learners were motivated to learn, knowing they were not the only ones having difficulties solving algebraic linear equations. The results indicated that if learners co-operatively study erroneous examples in small mixed-ability groups, they can overcome learning difficulties and improve their performance. Additionally, the results revealed that when learners are allowed to demonstrate their understanding through group presentations, their confidence in mathematics is improved, which improves their performance. The results also showed that by applying the principles of ZPD and scaffolding, the teacher might enhance learners' understanding of solving algebraic linear equations.

7.5 PROPOSED TEACHING STRATEGY

Because of this study, the literature review and the theoretical framework underpinning the study, a teaching strategy can be suggested. For the effective use of erroneous examples for teaching, it is proposed that there should be enough orientation of the novice teacher by the expert teacher. The expert teacher should provide clear guidelines on how to plan lessons with erroneous examples. Firstly, there is a need to identify a lesson objective and list the common errors associated with the objective. After identifying an objective to be achieved, the next step is to choose one error for each example. After choosing one error, create worksheets focusing on the error and let learners identify the error and discuss the possible ways of solving the problem, explaining all the necessary steps for solving the problem (McGinn, Lange & Booth, 2015, p.31). Creating erroneous examples using learners' own errors was more effective than using fictitious errors because learners quickly identified their own misconceptions, and this helped them to close their own knowledge gaps (Metcalfe, 2018, p.471).

Observational learning is proposed as an effective means for use with erroneous examples for teaching and learning. In observational learning, the expert teacher will be the model, and the novice teacher(s) should learn through observation during orientation (Bandura, 1986, p.459). Observational learning encourages expert and novice teachers to learn from each other by sharing teaching strategies within social contexts. However, for observational learning to be effective, the observer should be motivated to learn and pay attention to the model.

7.6 THE STUDY'S LIMITATIONS

As stated before, the study's participants were chosen from disadvantaged schools of Johannesburg Central District of Gauteng Province. Only two out of sixty-eight (68) secondary schools in the district were selected for this study. The experimental school was in Soweto, where all learners were Africans, while the control school was in Lenasia, where most learners were Indians and a small proportion were Africans and Coloured learners. Only two teachers of mathematics and their learners participated in this research. Suppose other teachers and their learners from other districts were selected, in that case, their responses might have been different because different environments and social backgrounds affect and influence lessons differently.

This study focused on exploring the impact and effectiveness of erroneous examples for teaching Grade 9 algebraic linear equations. The researcher used learners' answers from the pre-and post-tests, teachers' responses from the semi-structured interviews and lesson observations as the main data sources for this study. If different data collection instruments were used, the researcher might have collected more and different data, which could have yielded different results. It is impossible to generalise the findings of this study to the entire population due to the reasons stated above.

7.7 SUGGESTIONS FOR FUTURE STUDY

The major findings of this study seemed to suggest that erroneous examples are effective when learners work cooperatively in small mixed-ability groups and when learners analyse errors that are similar to their own errors. Additionally, the findings suggest that erroneous examples enabled most learners participate actively during lessons when analysing their own errors. This study used only one teacher to teach the experimental group. For other future research to investigate the impact and effectiveness of erroneous examples, it is suggested that involving schools in a different environment to see if the study can yield the same results. Another suggestion is to investigate how learners perceive the use of erroneous examples for teaching and learning. Further research can be conducted on the impact and effectiveness of erroneous examples at the learner level, as this study was more on the teacher level.

7.8 CONCLUSION

This study explored the effectiveness of erroneous examples for teaching Grade 9 algebraic linear equations. The teacher of the experimental group and researcher suggested grouping learners into small mixed-ability groups in which learners were given the chance to analyse their own errors, which appeared to have positively impacted learners' academic performance. This study enabled the experimental group's teacher to get learners participate during lessons through group discussions and group presentations. Moreover, the study allowed the experimental group's teacher to assist learners who had difficulties solving algebraic linear equations during teaching and learning.

The benefits of erroneous examples for teaching Grade 9 algebraic linear equations obtained in this study will encourage other teachers and schools to implement the strategy to improve learners' academic performance and change teachers' practices. Furthermore, this study's results will assist the participating teacher to realise the benefits and challenges of using erroneous examples to impact learners' academic performance in general.

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APPENDICES

APPENDIX A: UNISA ETHICS CLEARANCE CERTIFICATE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2022/07/06

Ref: **2022/07/06/57344574/22/AM**
Name: Mr J Gwenzi
Student No.:57344574

Dear Mr J Gwenzi

Decision: Ethics Approval from 2022/07/06 to 2025/07/06

Researcher(s): Name: Mr J Gwenzi
E-mail address: 57344574@mylife.unisa.ac.za
Telephone: 0839489751

Supervisor(s): Name: Dr T.P. Makgakga
E-mail address: makgatp@unisa.ac.za
Telephone: 072 475 9363

Title of research:

Using erroneous and correct examples for teaching Grade 9 algebraic equations at two schools in Johannesburg Central District

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2022/07/06 to 2025/07/06.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2022/07/06 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

Open Rubric

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2025/07/06**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2022/07/06/57344574/22/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof Mpine Makoe
ACTING EXECUTIVE DEAN
qakisme@unisa.ac.za

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APPENDIX B: PROVINCIAL CLEARANCE LETTER

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**26 January 2023**
Validity of Research Approval:	**08 February 2023– 30 September 2023** **2022/289A**
Name of Researcher:	**Gwenzi J**
Address of Researcher:	**2807 Webb Street**
	Toekomsrus Ext 2
	Randfontein
Telephone Number:	**083 948 9751**
Email address:	**gwenzijulius196@gmail.com**
Research Topic:	**using erroneous and correct examples for teaching Grade 9 algebraic equations at wo secondary schools in Johannesburg Central District.**
Type of qualification	**Master of Education**
Number and type of schools:	**3 Secondary Schools**
District/s/HO	**Johannesburg Central**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

26/01/2023

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below are met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The letter would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools, or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.***
4. ***The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s has been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter/document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs, and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort to obtain the goodwill and cooperation of all the GDE officials, principals, and chairpersons of the SGBs, teachers, and learners involved. Persons who offer their cooperation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school program is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her research resources, such as stationery, photocopies, transport, faxes, and telephones, and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers, and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study, the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings, and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a summary of the purpose, findings, and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Dr. Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: ...26|01|2023...................

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX C: DISTRICT APPROVAL LETTER

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

Enquiries: Caroline Tladi
File no: 3/2/1
Tele: 011 983 2135
Email: Caroline.Tladi@gauteng.gov.za
Chief Directorate: Education Planning and Research

MR J GWENZI
2807 WEBB STREET
TOEKOMSRUS EXT 2
RANDFONTEIN
1759

By Email: gwenzijulius196@gmail.com

Dear Mr Gwenzi

SUBJECT: ACKNOWLEDGEMENT OF RESEARCH APPROVAL

Correspondence received from you on 26 July 2022 titled: *Using erroneous and correct examples for teaching Grade 9 algebraic equations at two schools in Johannesburg Central District* refers.

Thank you for informing the District Office of your intended research in three secondary schools namely, Nghunghunyani Comprehensive Secondary, Thabo Secondary and Trinity Secondary.

The following attachments are acknowledged and received:

1. GDE Research Approval letter from the Knowledge Management and Research Directorate at GDE Head Office – 13 July 2022.
2. Letter to the District Director
3. Research Proposal
4. Ethics Clearance
5. List of schools
6. Pre-Test and Post-Test items

You are welcome to proceed with your research however, in the execution thereof; kindly be reminded to adhere to the COVID-19 protocols when rolling out your research plans for data gathering. In the interest of protecting the safety and well-being of learners and staff, please be mindful of condition 3 and 4 of the GDE Research Approval letter.

The District Office wishes you great success with your research and reminds you to comply with the terms and conditions against which approval has been granted by the Knowledge Management and Research Directorate.

Yours sincerely

Seate

MS B. L. T. SEATE
DISTRICT DIRECTOR
DATE: 29/07/2022

Tel: (011) 355 0000
17 Simmonds Street, Johannesburg, 2001 | P.O. Box 7710, Johannesburg, 2000

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APPENDIX D: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN GAUTENG PROVINCE

Title of your research: The use of erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District

Date: 7 July 2022
The Head of Department
Gauteng Department of Education
Contact details of the person: 0113550000

Dear Head of Department
I, Mr. J.Gwenzi am doing research for a master's degree with the University of South Africa. I am inviting you to participate in a study entitled "Using erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District".

Your department has been selected because this research is a follow-up study from my thesis entitled: "**Using erroneous and correct examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District"**.

The purpose of this study is to explore the effectiveness of using erroneous examples for teaching Grade 9 algebraic linear equations in South African schools in Johannesburg Central District.

The study will entail Action Research in collective case study research design in a pragmatic research paradigm. Convenience sampling of participants will be conducted. Three qualitative data collection instruments namely, interviews, documents analysis and observation, two quantitative instruments namely, pre-test, and post-test will be used to collect data.

The benefits of this study are that teachers will be introduced to a new teaching strategy and learners may improve their academic performance in mathematics and therefore contribute to the province's achievement.

The low-risk and medium category was selected because the participants are adults/minors who will participate voluntarily, will sign consent letters, and may withdraw from participation if they so wish without penalties.

There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will entail sharing of articles, scholarly books, book chapters and papers presented in local and international conferences. Articles will be accessible in Google Scholar and Research gate online platforms. Links will be provided for all articles and book chapters published online.

Yours sincerely

Any unfolding information regarding this study, you may contact my chair of Department:

Name: Prof JJ Dhlamini
Contact details: 012 429 2023
Email: dhlamjj@unisa.ac.za

APPENDIX E: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT JOHANNESBURG CENTRAL DISTRICT

UNISA college of education

Title of my research: **USING ERRONEOUS EXAMPLES FOR TEACHING GRADE 9 ALGEBRAIC LINEAR EQUATIONS AT A SCHOOL IN JOHANNESBURG CENTRAL DISTRICT**

Date: 7 July 2022
Name of the person to whom you address the request: The District Director
District of the person: Johannesburg Central District D14
Contact details of the person: 011 983 2234

The District Director

I, Mr. J. Gwenzi, am researching for a master's degree at the University of South Africa. I am requesting permission to carry out research at three schools in your district.

Your district has been selected because this research is a follow-up study from my thesis entitled: "USING ERRONEOUS EXAMPLES FOR TEACHING GRADE 9 ALGEBRAIC EQUATIONS AT A SCHOOL IN JOHANNESBURG CENTRAL DISTRICT". The purpose of this study is to explore the effectiveness of using erroneous examples for teaching Grade 9 algebraic linear equations in South African schools in Johannesburg Central District.

The study will entail Action Research in a collective case study design in a pragmatic research paradigm. Convenience sampling of participants will be conducted. Three qualitative data collection instruments will be used to collect data: interviews, document analysis and observation, and two quantitative instruments, namely, pre-test and post-test.

The benefits of this study are that teachers will be introduced to a new teaching strategy, and learners may improve their academic performance in mathematics and, therefore, contribute to the district's achievement.

The low-risk and medium categories were selected because the participants are adults/minors who will participate voluntarily, will sign consent letters, and may withdraw from participation if they so wish without penalties.

There will be no reimbursement or any incentives for participation in the research.

The feedback procedure will entail sharing articles, scholarly books, book chapters and papers presented at local and international conferences. Articles will be accessible on Google Scholar and Research Gate online platforms. Links will be provided for all articles and book chapters published online.

Yours sincerely

Any unfolding information regarding this study, you may contact my chair of Department:

Name: Prof JJ Dhlamini
Contact details: 012 429 2023
Email: dhlamjj@unisa.ac.za

APPENDIX F: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT SCHOOL 1

Title of my research: **USING ERRONEOUS EXAMPLES FOR TEACHING GRADE 9 ALGEBRAIC LINEAR EQUATIONS AT A SCHOOL IN JOHANNESBURG CENTRAL DISTRICT**

Date: 7 July 2022
The Principal

Dear Sir/Madam

I, Mr. J. Gwenzi am doing research for a master's degree with the University of South Africa. I am requesting permission to do research at your school.

The purpose of this study is to explore the effectiveness of using erroneous examples for teaching Grade 9 algebraic linear equations in South African schools in the Johannesburg District. Your school has been selected because this research is a follow-up study from my thesis entitled: "**Using erroneous examples for teaching Grade 9 algebraic equations at a school in Johannesburg Central District**".

The study will entail Action Research in collective case study research design in a pragmatic research paradigm. Convenience sampling of participants will be conducted. Three qualitative data collection instruments namely, interviews, documents analysis and observation, two quantitative instruments namely, pre-test, and post-test will be used to collect data.

The benefits of this study are that teachers will be introduced to a new teaching strategy and learners may improve their academic performance in mathematics and therefore contribute to the school's achievement.

There low-risk and medium category was selected because the participants are adults/minors who will participate voluntarily and will sign consent/assent letters and may withdraw from participation if they so wish without penalties.

There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will entail sharing of articles, scholarly books, book chapters and papers presented in local and international conferences. Articles will be accessible in Google Scholar and Research gate online platforms. Links will be provided for all articles and book chapters published online.

Yours sincerely

Any unfolding information regarding this study, you may contact my chair of Department:
Name: Prof JJ Dhlamini
Contact details: 012 429 2023
Email: dhlamjj@unisa.ac.za

APPENDIX G: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT SCHOOL 2

UNISA | college of education

Title of my research: **Using erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District**

Date: 7 July 2022
Name of the person to who you address the request: The Principal
Dear Sir/Madam

I, Mr. J. Gwenzi am doing research for a master’s degree with the University of South Africa. I am requesting permission to do research at your school.

The purpose of this study is to explore the effectiveness of using erroneous examples for teaching Grade 9 algebraic linear equations in South African schools in Johannesburg District. Your school has been selected because this research is a follow-up study from my thesis entitled: “Using erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District”

The study will entail Action Research in collective case study research design in a pragmatic research paradigm. Convenience sampling of participants will be conducted. Three qualitative data collection instruments namely, interviews, documents analysis and observation, two quantitative instruments namely, pre-test, and post-test will be used to collect data.

The benefits of this study are that teachers will be introduced to a new teaching strategy and learners may improve their academic performance in mathematics and therefore contribute to the school’s achievement.

There low-risk and medium category was selected because the participants are adults/minors who will participate voluntarily and will sign consent/assent letters and may withdraw from participation if they so wish without penalties.

There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will entail sharing of articles, scholarly books, book chapters and papers presented in local and international conferences. Articles will be accessible in Google Scholar and Research gate online platforms. Links will be provided for all articles and book chapters published online.

Yours sincerely

Any unfolding information regarding this study, you may contact my chair of Department:

Name: Prof JJ Dhlamini

Contact details: 012 429 2023

Email: dhlamjj@unisa.ac.za

APPENDIX H: INVITATION OF TEACHERS TO PARTICIPATE IN THE STUDY

UNISA college of education

Date: 7 July 2022

Title: Using erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District

DEAR PROSPECTIVE PARTICIPANT

My name is Mr. Gwenzi. I am doing research for a master's degree with the University of South Africa. I am inviting you to participate in a study entitled "Using erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District"

WHAT IS THE PURPOSE OF THE STUDY?

This study is expected to collect important information that could assist in answering the main research question on the use of erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District where conclusions will be drawn from the findings and recommendations made to schools and department of education.

WHY AM I BEING INVITED TO PARTICIPATE?

You are invited because of your expertise and experience in the phenomenon under investigation. I obtained your contact details from your principal as your immediate line manager. The approximate number of participants is one mathematics teacher and forty Grade 9 learners conveniently sampled from your school.

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?

Your role will be to participate in face-to-face in-depth interviews or telephonic interviews depending on the Covid-19 protocols at the time of interview session.

The study involves semi-structured interviews dominated by essay-type questions. The interview session will not exceed the duration of 45 minutes and may be scheduled in-person or on Telephone the researcher giving a call. You are requested to participate in an individual interview.

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE?

Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw from participation without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The benefits of this study are that participants will be introduced to a new teaching strategy that may help learners better understand mathematics and hence improve their academic performance. The implications and recommendations in articles will help in the improvement of the teachers' own practices in mathematics.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?

There are no negative consequences for you if you participate in research, as the research title does not contain sensitive issues that may cause harm or discomfort. However, if it may happen that any participant experiences a feeling or psychological or emotional discomfort, first aid assistance will be offered by the researcher and participant referred for counselling by senior education official or /and psychologist depending on the nature of discomfort.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?

You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research as means of maintaining confidentiality. Your name will not be recorded anywhere, and no one will be able to connect you to the answers you give in order to adhere to the code of research ethics known as anonymity. Your answers will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings to maintain confidentiality of your contribution to the research. The main researcher who will abide to all code of research ethics will access the data. Your answers may be reviewed by people responsible for making sure that research is done properly, including the transcriber, external coder, and members of the Research Ethics Review Committee. Otherwise, records that identify you will be available only to the researcher, unless you give permission for other people to see the records. The participants' anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings. The participants' privacy will be protected in any publication of the information.

HOW WILL THE RESEARCHER(S) PROTECT THE SECURITY OF DATA?

The researcher will store hard copies of your answers for a period of five years in a locked cupboard/filing cabinet at my home for future research or academic purposes; electronic information will be stored on a password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. Hard copies will be shredded and electronic copies will be permanently deleted from the hard drive of the computer with a relevant software programme.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

There will be no payment or reward for voluntary participation in the study.

HAS THE STUDY RECEIVED ETHICS APPROVAL

This study has received written approval from the Research Ethics Review Committee of the College of Education, (UNISA.) A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?

If you would like to be informed of the final research findings, please contact Mr. J. Gwenzi on 0839489751 or email gwenzijulius196@gmail.com. The findings are accessible for a period of three years.

Should you have concerns about the way in which the research has been conducted, you may contact my Chair of Department, Dr. Prof JJ Dhlamini at 0764950067 or dhlamjj@unisa.ac.za.

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

Yours sincerely

Any unfolding information regarding this study, you may contact my chair of Department:
Name: Prof JJ Dhlamini
Contact details: 012 429 2023
Email: dhlamjj@unisa.ac.za

ASSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, __________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and I am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interview sessions.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) ____________________________________

____________________________ ___________________________________

Participant Signature Date

Researcher's Name & Surname (please print) ____________________________________

_____________________________ __________________________________

Researcher's signature Date

APPENDIX I: REQUEST LETTER TO PARENT /GUARDIAN FOR CHILD /WARD PARTICIPATION

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Mr.J. Gwenzi

2807 Webb Street

Toekomsrus,

Randfontein, 1759

Cell : 0839489751   work : 011 852 2202

Email address: gwenzijulius196@gmail.com

Dear Parent,

Re: Request for your child to participate in research

My name is Mr. J. Gwenzi from the University of South Africa in the Department of Mathematics Education. I am doing research for a master's degree at the University of South Africa. I am requesting to give your child permission to participate in a study entitled: "Using erroneous examples for teaching Grade 9 algebraic linear equations at a school in Johannesburg Central District". The purpose of this study is to explore the effectiveness of using erroneous examples for teaching Grade 9 algebraic linear equations in South African schools

If you allow your child to participate in this research, he/ she will participate in a series of activities related to this research. I will be administering a pre-test and a post-test, and conduct interviews and to observe your child when learning mathematics. Results from these tests will only be used to identify areas learners may be having difficulties and will not count against your child's grade.

The benefits of this research study consist of identify areas in which teachers and learners need to focus on in order to improve their performance in mathematics. Participation is completely voluntary. Your child's name and program results will not be released without your permission. I am only interested in seeing how to provide your child with the best education.

If you have any questions, please feel free to contact me on my cell number: 0839489751

Hoping to hear from you soon.

Your sincerely,

Any unfolding information regarding this study, you may contact my chair of Department:
Name: Prof JJ Dhlamini
Contact details: 012 429 2023
Email: dhlamjj@unisa.ac.za

Parent/Legal guardian Consent form return slip

Please sign and return the bottom portion of this consent form as soon as you have read the letter above.

I, the parent/ legal guardian of ...., acknowledge that the researcher has explained to me the need for this research, explained what is involved and offered to answer any questions. I freely and voluntarily consent to my child /ward's participation in this research. I understand all information gathered during the research will be completely confidential.

Name of learner: ....

Signature of parent/ legal guardian: ....

Date: ....

APPENDIX J: INVITATION OF LEARNERS TO PARTICIPATE IN THE STUDY

Date: 7 July 2022

Title: using erroneous examples for teaching Grade 9 algebraic linear linear equations at a school in Johannesburg Central District

DEAR PROSPECTIVE PARTICIPANT

My name is Mr. J. Gwenzi. I am doing research for master's degree with the University of South Africa. I am inviting you to participate in a study entitled "Using erroneous examples for teaching Grade 9 algebraic linear equations a school in Johannesburg Central District"

WHAT IS THE PURPOSE OF THE STUDY?

This study is expected to collect important information that could assist in answering the main research question on the use of erroneous examples for teaching Grade 9 algebraic linear equations a school in Johannesburg Central District where conclusions will be drawn from the findings and recommendations made to schools and the department of education.

WHY AM I BEING INVITED TO PARTICIPATE?

You are invited because of your expertise and experience in the phenomenon under investigation. I obtained your contact details from your principal as your immediate line manager. The approximate number of participants is one mathematics teacher and forty Grade 9 learners purposefully sampled from your school.

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?

Your role will be to participate in face-to-face in-depth interviews or telephonic interviews depending on the Covid-19 protocols at the time of interview session. Furthermore, you will write a pre-test and post-test. The study involves semi-structured interviews dominated by essay-type questions. The interview session will not exceed the duration of 45 minutes and may be scheduled in-person or on Telephone the researcher giving a call. You are requested to participate in an individual interview.

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE?

Participating in this study is voluntary and you are under no obligation to assent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written assent form. You are free to withdraw from participation without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The benefits of this study are that you will be introduced to a new teaching strategy that may help you to better understand how to solve algebraic equation and improve your performance. The implications and recommendations in articles will help in the improvement of the learners' performance in mathematics generally and algebraic equations in particular.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?

There are no negative consequences for you if you participate in research, as the research title does not contain sensitive issues that may cause harm or discomfort. However, if it may happen that any participant experiences a feeling or psychological or emotional discomfort, first aid assistance will be offered by the researcher and participant referred for counselling by senior education official or /and psychologist depending on the nature of discomfort.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?

You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research as means of maintaining confidentiality. Your name will not be recorded anywhere, and no one will be able to connect you to the answers you give in order to adhere to the code of research ethics known as anonymity. Your answers will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings to maintain confidentiality of your contribution to the research. The data will be accessed by the main researcher who will abide to all code of research ethics. Your answers may be reviewed by people responsible for making sure that research is done properly, including the transcriber, external coder, and members of the Research Ethics Review Committee. Otherwise, records that identify you will be available only to the researcher, unless you give permission for other people to see the records. The participants' anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings. The participants' privacy will be protected in any publication of the information.

HOW WILL THE RESEARCHER(S) PROTECT THE SECURITY OF DATA?

Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet at my home for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. Hard-copies will be shredded and electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

There will be no payment or reward for voluntary participation in the study.

HAS THE STUDY RECEIVED ETHICS APPROVAL

This study has received written approval from the Research Ethics Review Committee of the College of Education, (UNISA). A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?

If you would like to be informed of the final research findings, please contact Mr. J Gwenzi on 0839489751 or email gwenzijulius196@gmail.com.

The findings are accessible for a period of three years. Should you have concerns about the way in which the research has been conducted, you may contact my Chair of Department, Dr Prof JJ Dhlamini at 0764950067 or.

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

Yours sincerely

Any unfolding information regarding this study, you may contact my chair of Department:

Name: Prof JJ Dhlamini
Contact details: 012 429 2023
Email: dhlamjj@unisa.ac.za

ASSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, __________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interview sessions.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) ______________________________________

______________________________ ____________________________________

Participant Signature Date

Researcher's Name & Surname (please print) ______________________________________

______________________________ ____________________________________

Researcher's signature Date

APPENDIX K: CONSENT FORMS TO THE PRINCIPAL AND TO ALL THE PARTICIPATING TEACHERS

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I ……………………………… (please print your name in full) the principal/ a grade 9 mathematics teacher agree to be a participant in the research conducted by Mr. J. Gwenzi in which he will be exploring the effectiveness of using erroneous and correct examples for teaching Grade 9 algebraic equations.

I give consent to the following: My school to participate in the research.

Yes □ or No □ (use a cross to indicate your selection)

- To give lessons in my classes for context-based problem-solving activities.

  Yes □ or No □ (use a cross to indicate your selection)
- To administer an achievement test in my classes.

  Yes □ or No □ (use a cross to indicate your selection)
- To be interviewed.

  Yes □ or No □ (use a cross to indicate your selection)

To be observed during lessons.

Yes □ or No □ (use a cross to indicate your selection)

Signature of the Principal: ________________________________

Signature of the teacher:___________________________________

Date:_______________________________________

APPENDIX L: PRE-POST TEST ITEMS

Dear Learner

This is a diagnostic test, which comprises eight questions. Please read each question carefully before you answer. Answer all the questions in the spaces provided on the question paper. Marks for each question are shown in brackets ( ). Show all your calculations. A non-programmable calculator may be used. This paper will take you about 1hour to complete.

NAME__________________________ **School**______________________________

1.1 Erroneous example	**Questions**
John solved this problem incorrectly. This is his working: $2x + 3 = 15$ Step 1. $5x = 15$ Step 2: $\frac{5x}{5} = \frac{15}{5}$ Step 3: $\therefore x = 3$	What mistake did John make in Step 1? (1) ______________________________ ______________________________ Explain what John should have done in step 1. (2) ______________________________ ______________________________ ______________________________ ______________________________

1.2 Solve $2x + 3 = 15$ (3)

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2.1 Erroneous example	**Questions**
Kagiso solved this problem incorrectly. This is his working: $4x - 3 = x + 15$ Step 1: $4x - 3 - 3 = x + 15 - 3$ Step 2: $4x = 12$ Step 3: $\frac{4x}{4} = \frac{12}{4}$ Step 4: $\therefore x = 8$	Why is step 1 incorrect? (2) ______________________________ ______________________________ ______________________________ ______________________________ What error did Kagiso make to get the answer 8 in step 4? (1) ______________________________ ______________________________

2.2 Solve $4x - 3 = x + 15$ (4)

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3.1 Erroneous example	**Questions**
Mohammed solved this problem incorrectly. This is his working: $2(x+4) = 24$ Step 1. $2x + 4 = 24$ Step 2. $2x + 4 - 4 = 24 + 4$ Step 3. $2x = 28$ Step 4 $\frac{2x}{2} = \frac{28}{2}$ $\therefore x = 14$	What is done incorrectly in step 1? (2) ______________________ ______________________ ______________________ What is done incorrectly in step 2? (1) ____________________ ____________________ What is done correctly in step 4? (1) ______________________________ ______________________________ ______________________________ ______________________________

3.2 Solve $2(x+4) = 24$ (4)

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4.1 Erroneous example	**Questions**
Ayanda solved this problem incorrectly. $2(x-4)=3-2(x+2)$ Step 1. $2x-4=1(x+2)$ Step 2. $2x-4=x+2$ Step 3. $2x-4-4=x+2+4$ Step 4 $2x=x+6$ Step 5. $2x-x=x-x+6$ Step 6. $\therefore x=6$	What mistakes did Ayanda make in step1? (2) ______________________________ ______________________________ ______________________________ ______________________________ What mistake did Ayanda make in step 3 (2) ______________________________ ______________________________

4.2 Solve $2(x-4)=3-2(x+2)$ (5)

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5.2 Solve $\frac{6x}{2}-5=4$ using a different method. (3)

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5.1 Correct example	**Questions**
Vuyo solved this problem correctly. This is his working: $\frac{6x}{2}-5=4$ Step 1. $\frac{6x\times 2}{2}-5\times 2=4\times 2$ Step 2. $6x-10=8$ Step 3. $6x-10+10=8+10$ Step 4. $6x=18$ Step 5. $\frac{6x}{6}=\frac{18}{6}$ Step 6 $\therefore x=3$	Why did Vuyo multiply throughout by 2 in Step 1? (1) ______________________________ ______________________________ Why is step 3 correct? (1) ______________________________ ______________________________ ______________________________

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6.1 Erroneous example	Questions
Ruth solved this problem incorrectly. This is Ruth's working: $$\frac{x}{2}+\frac{x}{3}=\frac{1}{6}$$ Step 1. $\frac{2x}{5}=\frac{1}{6}$ Step 2 $2x \times 6 = 1 \times 5$ Step 3 $12x = 6$ Step 4 $\frac{12x}{12}=\frac{6}{12}$ $\therefore x = \frac{1}{2}$	Why is Step 1 incorrect? (1) __________________________________ __________________________________ __________________________________ Why is Step 2 correct? (1) __________________________________ __________________________________ __________________________________ Why is Step 3 incorrect? (1) __________________________________ __________________________________ __________________________________ Why is Step 4 correct? (1) __________________________________ __________________________________

6.2 Solve $\frac{x}{2}+\frac{x}{3}=\frac{1}{6}$ (4)

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7. The length of a rectangle is twice as long as its width. Determine the lengths of the sides of the rectangle if its perimeter is 24cm.

7.1 Incorrect example	Questions
Let width of rectangle be x Length will be $2x$ Step 1 $2x + x = 24$ Step 2 $3x = 24$ Step 3 $\frac{3x}{3x} = \frac{24}{3x}$ Step 4 $x = 8$ Width of rectangle = 8 cm Length of rectangle = 16 cm	Why is Step 1 incorrect? (2) ________________________ ________________________ ________________________ What is the error in Step 3? (1) ________________________ ________________________ ________________________ ________________________ ________________________

7.2 Solve question 7 in the space below. (4)

8. John is five times as old as Jabulani. After four years, John will be three times as old as Jabulani will. What are their present ages?

8.1 Incorrect example	**Questions**
Boys' ages now Let Jabulani's age now be X Step 1: John 5X yrs Jabulani 5yrs Boys' ages 4 years later Step 2: John 5X+4 = 20X Jabulani 5+4 =20 Step 3: 20X = 20 Step 5: $\frac{20X}{20} = \frac{20}{20}$ Step 6: X = 1 ∴ Jabulani 1 year old now and John is 5 years old.	Why is Jabulani's age in step 1 incorrect? (1) _________________________________ _________________________________ What mistakes are shown in step 2? (2) _________________________________ _________________________________ _________________________________

8.2 Solve Question 8 on the space below (5)

APPENDIX M: PRE-POST TEST MEMORANDUM

Dear Learner

This is a diagnostic test, which comprises eight questions. Please read each question carefully before you answer. Answer all the questions in the spaces provided on the question paper. Marks for each question are shown in brackets ( ). Show all your calculations. A non-programmable calculator may be used. This paper will take you about 1hour to complete.

NAME MEMORANDUM **School** ______

1.1 Erroneous example	**Questions**
John solved this problem incorrectly. This is his working: $2x + 3 = 15$ Step 1. $5x = 15$ Step 2: $\frac{5x}{5} = \frac{15}{5}$ Step 3: $\therefore x = 3$	What mistake did John make in Step 1? (1) He added unlike terms. Explain what John should have done in step 1. (2) He should have subtracted 3 from both sides.

1.2 Solve $2x + 3 = 15$ (3)

$2x+3 = 15$

$2x+3-3 = 15-3$

$\frac{2x}{2} = \frac{12}{2}$

$\therefore x = 6$

2.1 Erroneous example	**Questions**
Kagiso solved this problem incorrectly. This is his working: $4x - 3 = x + 15$ Step 1: $4x - 3 - 3 = x + 15 - 3$ Step 2: $4x = 12$ Step 3: $\frac{4x}{4} = \frac{12}{4}$ Step 4: $\therefore x = 8$	Why is step 1 incorrect? (2) 3 was subtracted from both sides instead of being added to both sides What error did Kagiso make to get the answer 8 in step 4? (1) He subtracted 4 from 12 instead of dividing 12 by 4.

2.2 Solve $4x - 3 = x + 15$ (4)

$4x - 3 = x + 15$
$4x - 3 + 3 = x + 15 + 3$ ✓
$4x = x + 18$
$4x - x = x - x + 18$ ✓
$\frac{3x}{3} = \frac{18}{3}$ ✓
$\therefore x = 6$ ✓

3.1 Erroneous example	**Questions**
Mohammed solved this problem incorrectly. This is his working: $2(x+4) = 24$ Step 1. $2x + 4 = 24$ Step 2. $2x + 4 - 4 = 24 + 4$ Step 3. $2x = 28$ Step 4 $\frac{2x}{2} = \frac{28}{2}$ $\therefore x = 14$	What is done incorrectly in step 1? (2) 2 is not distributed to both x and 4 on removing brackets. ✓ What is done incorrectly in step 2? (1) 4 is added on the right-hand side instead of being subtracted as was done on the left side. What is done correctly in step 4 ? (1) Dividing both sides by the coefficient to get the value of the variable, x. ✓

3.2 Solve $2(x + 4) = 24$ (4)

$2(x+4) = 24$
$2x + 8 = 24$ ✓
$2x + 8 - 8 = 24 - 8$ ✓
$\frac{2x}{2} = \frac{16}{2}$ ✓
$\therefore x = 8$ ✓

4.1 Erroneous example	Questions
Ayanda solved this problem incorrectly. $2(x-4) = 3 - 2(x+2)$ Step 1. $2x - 4 = 1(x+2)$ Step 2. $2x - 4 = x + 2$ Step 3. $2x - 4 - 4 = x + 2 + 4$ Step 4 $2x = x + 6$ Step 5. $2x - x = x - x + 6$ Step 6. $\therefore x = 6$	What mistakes did Ayanda make in step1? (2) 2 was not distributed to both x and −4 on the L.S.H and subtraction was done before multiplication on the RHS. What mistake did Ayanda make in step 3 (2) She subtracted 4 on the LHS, instead of adding 4 on LHS.

4.2 Solve $2(x-4) = 3 - 2(x+2)$ (5)

$2x - 8 = 3 - 2x - 4$
$2x - 8 + 8 = 3 - 4 + 8 - 2x$
$2x = 7 - 2x$
$2x + 2x = 7 - 2x + 2x$
$\frac{4x}{4} = \frac{7}{4}$
$\therefore x = \frac{7}{4}$

5.1 Correct example	Questions
Vuyo solved this problem correctly. This is his working: $\frac{6x}{2} - 5 = 4$ Step 1. $\frac{6x\times 2}{2} - 5 \times 2 = 4 \times 2$ Step 2. $6x - 10 = 8$ Step 3. $6x - 10 + 10 = 8 + 10$ Step 4. $6x = 18$ Step 5. $\frac{6x}{6} = \frac{18}{6}$ Step 6 $\therefore x = 3$	Why did Vuyo multiply throughout by 2 in Step 1? (1) To remove the denominator on the left-hand side. Why is step 3 correct? (1) Additive inverse of −10 is used to group like terms.

5.2 Solve $\frac{6x}{2} - 5 = 4$ using a different method. (3)

$\frac{6x}{2} - 5 = 4$

$3x - 5 = 4$ ✓

$3x - 5 + 5 = 4 + 5$

$\frac{3x}{3} = \frac{9}{3}$ ✓

$\therefore x = 3$ ✓

OR $\frac{6x}{2} - 5 = 4$

$\frac{6x}{2} - 5 + 5 = 4 + 5$ ✓

$\frac{6x}{2} = 9 \Rightarrow \frac{6x}{6} = \frac{18}{6}$ ✓

$x = 3$ ✓

6.1 Erroneous example	**Questions**
Ruth solved this problem incorrectly. This is Ruth's working: $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$	Why is Step 1 incorrect? (1) Numerators and denominators are added on the left-hand side. ✓
Step 1. $\frac{2x}{5} = \frac{1}{6}$	Why is Step 2 correct? (1) Cross multiplied to clear denominators. ✓
Step 2 $2x \times 6 = 1 \times 5$	Why is Step 3 incorrect? (1) Addition instead of multiplication was done on the right-hand side $1 \times 5 \neq 6$ ✓
Step 3 $12x = 6$	
Step 4 $\frac{12x}{12} = \frac{6}{12}$	Why is Step 4 correct? (1) Dividing both sides by the coefficient of x to get the value of x. ✓
$\therefore x = \frac{1}{2}$	

6.2 Solve $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$ (4)

LCD = 6

$6\left(\frac{x}{2}\right) + 6\left(\frac{x}{3}\right) = 6\left(\frac{1}{6}\right)$ ✓

$3x + 2x = 1$ ✓

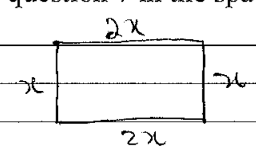
$\frac{5x}{5} = \frac{1}{5}$ ✓

$\therefore x = \frac{1}{5}$ ✓

7 The length of a rectangle is twice as long as its width. Determine the lengths of the sides of the rectangle if its perimeter is 24cm.

7.1 Incorrect example	**Questions**
Let width of rectangle be x Length will be $2x$ Step 1 $2x + x = 24$ Step 2 $3x = 24$ Step 3 $\frac{3x}{3x} = \frac{24}{3x}$ Step 4 $x = 8$ Width of rectangle = 8 cm Length of rectangle = 16 cm	Why is Step 1 incorrect? (2) The left-hand side should be multiplied by 2 $2(2x+x) = 24$ What is the error in Step 3? (1) The variable was used to divide on both sides instead of using the coefficient only.

7.2 Solve question 7 in the space below. (4)



$2x + x + 2x + x = 24$

$\frac{6x}{6} = \frac{24}{6}$

$x = 4$

The Width = 4 cm
Length = 2 (4cm)
= 8cm.

or

$2(x + 2x) = 24$

$2x + 4x = 24$

$\frac{6x}{6} = \frac{24}{6}$

$\therefore x = 4$

Width = 4cm
Length = 2(4cm)
= 8cm

8. John is five times as old as Jabulani. After four years, John will be three times as old as Jabulani will. What are their present ages?

8.1 Incorrect example	Questions
Boys' ages now Let Jabulani's age now be X Step 1: John 5X yrs Jabulani 5yrs Boys' ages 4 years later Step 2: John 5X+4 = 20X Jabulani 5+4 =20 Step 3: 20X = 20 Step 5: $\frac{20X}{20} = \frac{20}{20}$ Step 6: X = 1 ∴ Jabulani 1 year old now and John is 5 years old.	Why is Jabulani's age in step 1 incorrect? (1) Jabulani is not 5yrs now. He is X years old now ✓ What mistakes are shown in step 2? (2) Multiplication was done instead of addition. ✓ $5X + 4 \neq 20X$ and $5+4 \neq 20$ ✓

8.2 Solve Question 8 on the space below (5)

Now
John Jabulani
5X X

4 years later:
John Jabulani

5X + 4 X + 4 ✓

$5X + 4 = 3(X + 4)$ ✓
$5X + 4 = 3X + 12$
$5X + 4 - 4 = 3X + 12 - 4$
$5X = 3X + 8$

$5X - 3X = 3X + 8 - 3X$
$\frac{2X}{2} = \frac{8}{2}$ ✓

$X = 4$ ✓

∴ Jabulani is 4 years old now

John's age = 5(4yrs)
= 20 yrs ✓

APPENDIX N: SAMPLE PRE-TEST LEARNER SCRIPT

8/60 LEGS 5

Dear Learner

This is a diagnostic test, which comprises eight questions. Please read each question carefully before you answer. Answer all the questions in the spaces provided on the question paper. Marks for each question are shown in brackets ( ). Show all your calculations. A non-programmable calculator may be used. This paper will take you about 1hour to complete.

NAME [illegible] **School** [illegible] Secondary

1.1 Erroneous example	**Questions**
John solved this problem incorrectly. This is his working: $2x + 3 = 15$ Step 1. $5x = 15$ Step 2: $\frac{5x}{5} = \frac{15}{5}$ Step 3: $\therefore x = 3$	What mistake did John make in Step 1? (1) ~~He should have solved~~ He did not make ~~any~~ ~~for the exponent first~~ mistake in Step 1 ✗ Explain what John should have done in step 1. (2) He should have first solved for x and then work out the exponent. ✗

1.2 Solve $2x + 3 = 15$ (3)

~~$2x + 3 = 15$~~
~~$2x + 3 - 3 = 15 - 3$~~
~~$2x = 12 = 2$~~
~~$x = \sqrt{12}$~~
~~$2x = $~~

$2x + 3 = 15$
$2x + 3 - 3 = 15 - 3$ ✓
$2x = 12$ ✓ (2)
$2^2 = 12$

2.1 Erroneous example	**Questions**
Kagiso solved this problem incorrectly. This is his working: $4x - 3 = x + 15$ Step 1: $4x - 3 - 3 = x + 15 - 3$ Step 2: $4x = 12$ Step 3: $\frac{4x}{4} = \frac{12}{4}$ Step 4: $\therefore x = 8$	Why is step 1 incorrect? (2) He was surposed to add ✓3 and not subtract 3 (2) What error did Kagiso make to get the answer 8 in step 4? (1) He added +2 + 4 and then ~~de~~ divided by 2 ✗ 0

2.2 Solve $4x - 3 = x + 15$ (4)

$4x - 3 = x + 15$

$4x - 3 + 3 = x + 15 + 3$ ✓ (1)

$4x = x + 18$

$\frac{4x}{4} = \frac{18x}{4} = x = \frac{9x}{2}$ ✗

$x = 18x$

3.1 Erroneous example	**Questions**
Mohammed solved this problem incorrectly. This is his working: $2(x + 4) = 24$ Step 1. $2x + 4 = 24$	What is done incorrectly in step 1? (2) He took the brackets away in the question. ✗
Step 2. $2x + 4 - 4 = 24 + 4$ Step 3. $2x = 28$	What is done incorrectly in step 2? (1) He added 4 by 24+4 ✓ (1) He should of subtracted it
Step 4 $\frac{2x}{2} = \frac{28}{2}$ $\therefore x = 14$	What is done correctly in step 4? (1) He did step 4 correctly $\frac{2x}{2} = \frac{28}{2}$ ✓ $\therefore x = 14$ (1)

3.2 Solve $2(x + 4) = 24$ (4)

$2(x + 4) = 24$

$2(4x) = 24(4)$

$\frac{8x}{8} = \frac{96}{8}$

$x = 12$ ✗

4.1 Erroneous example	Questions
Ayanda solved this problem incorrectly. $2(x-4)=3-2(x+2)$ Step 1. $2x-4=1(x+2)$ Step 2. $2x-4=x+2$ Step 3. $2x-4-4=x+2+4$ Step 4 $2x=x+6$ Step 5. $2x-x=x-x+6$ Step 6. $\therefore x=6$	What mistakes did Ayanda make in step1? (2) He did not solve for the brackets in ~~the~~ step 1 What mistake did Ayanda make in step 3 (2) He added 4 instead of subtracting, He should of put x+6 together

4.2 Solve $2(x-4)=3-2(x+2)$ (5)

$2(x-4)=3-2(x+2)$

$2(4x)=3-2(x+2)$

$2(4x)=1(2x)$

$8x=2x$

$\frac{8x}{8}=\frac{2x}{8}$

$x=\frac{1}{4}x$

0

5.1 Correct example	Questions
Vuyo solved this problem correctly. This is his working: $\frac{6x}{2}-5=4$ Step 1. $\frac{6x\times2}{2}-5\times2=4\times2$ Step 2. $6x-10=8$ Step 3. $6x-10+10=8+10$ Step 4. $6x=18$ Step 5. $\frac{6x}{6}=\frac{18}{6}$ Step 6 $\therefore x=3$	Why did Vuyo multiply throughout by 2 in Step 1? (1) He multiplied because he thought he did the inverse operation. Why is step 3 correct? (1) It is ~~incorrect because~~ correct because he added to instead of subtracting

5.2 Solve $\frac{6x}{2} - 5 = 4$ using a different method. (3)

$\frac{6x}{2} - 5 = 4$

$5 + 3x = 4 \rightarrow 4 \times 8 = 32$

$\frac{3x}{2} - \frac{4}{8} = 1 \quad 3 = 32$ ✗ (0)

6.1 Erroneous example	**Questions**
Ruth solved this problem incorrectly. This is Ruth's working: $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$	Why is Step 1 incorrect? (1) He did not divide by the same denominator ✗ (0)
Step 1. $\frac{2x}{5} = \frac{1}{6}$	Why is Step 2 correct? (1) It is ~~incorrect~~ correct because he did the inverse operation.
Step 2 $2x \times 6 = 1 \times 5$	
Step 3 $12x = 6$	Why is Step 3 incorrect? (1) ~~He's~~ did not 1×5 is not 6 its 5 ✓ (1)
Step 4 $\frac{12x}{12} = \frac{6}{12}$	Why is Step 4 correct? (1) He divided 12 by 12 and 6 by 12 ? (0) Why?
$\therefore x = \frac{1}{2}$	

6.2 Solve $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$ (4)

$\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$

$\frac{3x}{2\times3} + \frac{2x}{3\times2} = \frac{1}{6}$

$\frac{3x}{6} + \frac{2x}{6} = \frac{1}{6} = \frac{1x^2}{6} - \frac{6}{1}$

$x = 6x^2 - 6 = x^2$

$\therefore x^2 = 1$ ✗

 P.T.O

7 The length of a rectangle is twice as long as its width. Determine the lengths of the sides of the rectangle if its perimeter is 24cm.

7.1 Incorrect example	**Questions**
Let width of rectangle be x Length will be $2x$ Step 1 $2x + x = 24$ Step 2 $3x = 24$ Step 3 $\frac{3x}{3x} = \frac{24}{3x}$ Step 4 $x = 8$ Width of rectangle = 8 cm Length of rectangle = 16 cm	Why is Step 1 incorrect? (2) Step He did not write x^2 ✗ What is the error in Step 3? (1) He divided by $2x$ when he was supposed to divide by 3

7.2 Solve question 7 in the space below. (4)

$2x + x = 24$

$\frac{3x^2}{3} = \frac{24}{3}$

$x^2 = 8$ ✗

$\therefore x^2 = 8$

0

P.T.O

8. John is five times as old as Jabulani. After four years, John will be three times as old as Jabulani will. What are their present ages?

8.1 Incorrect example	Questions
Boys' ages now Let Jabulani's age now be X Step 1: John 5X yrs Jabulani 5yrs Boys' ages 4 years later Step 2: John 5X+4 = 20X Jabulani 5+4 =20 Step 3: 20X = 20 Step 5: $\frac{20X}{20} = \frac{20}{20}$ Step 6: X = 1 ∴ Jabulani 1 year old now and John is 5 years old.	Why is Jabulani's age in step 1 incorrect? (1) He said that Jabulanis age is x not ~~not~~ Johns ✗ (0) What mistakes are shown in step 2? (2) Johns age is not supposed to be x. ✗ (0)

8.2 Solve Question 8 on the space below (5)

John 5yrs Jabulani 5xyrs

5yrs + 4yrs = 9 5xyrs + 4yrs

9 x yrs

$\frac{9}{9}$ $\frac{9x}{9}$

1 = x ✗ (0)

APPENDIX O: SAMPLE POST-TEST LEARNER SCRIPT

46/60 LCG 11

Dear Learner

This is a diagnostic test, which comprises eight questions. Please read each question carefully before you answer. Answer all the questions in the spaces provided on the question paper. Marks for each question are shown in brackets ( ). Show all your calculations. A non-programmable calculator may be used. This paper will take you about 1hour to complete.

NAME ______ **School** ______ Secondary

1.1 Erroneous example	**Questions**
John solved this problem incorrectly. This is his working: $2x + 3 = 15$ Step 1. $5x = 15$ Step 2: $\frac{5x}{5} = \frac{15}{5}$ Step 3: $\therefore x = 3$	What mistake did John make in Step 1? (1) He added the 3 to the 2x to make it 5x Explain what John should have done in step 1. (2) He needed to say ~~2x~~ ~~2x+15-3~~ 2x+3-3 = 15-3

1.2 Solve $2x + 3 = 15$ (3)

$2x + 3 - 3 = 15 - 3$
$\frac{2x}{2} = \frac{12}{6}$
$x = 6$

2.1 Erroneous example	**Questions**
Kagiso solved this problem incorrectly. This is his working: $4x - 3 = x + 15$ Step 1: $4x - 3 - 3 = x + 15 - 3$ Step 2: $4x = 12$ Step 3: $\frac{4x}{4} = \frac{12}{4}$ Step 4: $\therefore x = 8$	Why is step 1 incorrect? (2) He said 4x-3-3 = x+15-3 it is supposed to be 4x-3+3 = x+15+3 What error did Kagiso make to get the answer 8 in step 4? (1) He subtracted the 4 from the 12

2.2 Solve $4x - 3 = x + 15$ (4)

$4x - 3 = x + 15$

$4x - 3 + 3 = x + 15 + 3$

$4x = x + 18$ ✓

$4x - x = x - x + 18$ ✓

$3x = 18$

$\frac{3x}{3} = \frac{18}{3}$ ✓

$x = 6$ ✓ (4)

3.1 Erroneous example	Questions
Mohammed solved this problem incorrectly. This is his working: $2(x + 4) = 24$ Step 1. $2x + 4 = 24$ Step 2. $2x + 4 - 4 = 24 + 4$ Step 3. $2x = 28$ Step 4 $\frac{2x}{2} = \frac{28}{2}$ $\therefore x = 14$	What is done incorrectly in step 1? (2) He never multiply the 2 and 4 to get the answer of 8 ✓ (2) What is done incorrectly in step 2? (1) He added 4 to the 24 when it needed ~~but it was~~ to be subtracted ✓ (1) What is done correctly in step 4 ? He divided both sides by 2 (2)

3.2 Solve $2(x + 4) = 24$ (4)

$2(x + 4) = 24$

$2x + 8 = 24$ ✓

$2x + 8 - 8 = 24 - 8$

$2x = 16$ ✓

$\frac{2x}{2} = \frac{16}{2}$ ✓

$x = 8$ ✓ (4)

4.1 Erroneous example	Questions
Ayanda solved this problem incorrectly. $2(x-4) = 3-2(x+2)$ Step 1. $2x-4 = 1(x+2)$ Step 2. $2x-4 = x+2$ Step 3. $2x-4-4 = x+2+4$ Step 4 $2x = x+6$ Step 5. $2x-x = x-x+6$ Step 6. $\therefore x = 6$	What mistakes did Ayanda make in step1? (2) She never distributed the -2 into the bracket but rather subtracted it from the 3. (1) What mistake did Ayanda make in step 3 (2) she subtracted the 4 ~~it~~ on the LHS ~~but added the~~ when ~~it~~ ~~he 4 needed to be added~~ (1)

4.2 Solve $2(x-4) = 3-2(x+2)$ (5)

$2(x-4) = 3-2(x+2)$

$2x-8 = 3-2x-4$ ✓

$2x+2x-8 = 3-2x+2x-4$ ✓

$4x-8+8 = -1+8$ ✓

$\frac{4x}{4} = \frac{7}{4}$ ✓ (5)

$x = \frac{7}{4}$ ✓

5.1 Correct example	Questions
Vuyo solved this problem correctly. This is his working: $\frac{6x}{2} - 5 = 4$ Step 1. $\frac{6x \times 2}{2} - 5 \times 2 = 4 \times 2$ Step 2. $6x - 10 = 8$ Step 3. $6x - 10 + 10 = 8 + 10$ Step 4. $6x = 18$ Step 5. $\frac{6x}{6} = \frac{18}{6}$ Step 6 $\therefore x = 3$	Why did Vuyo multiply throughout by 2 in Step 1? (1) 2 is the LCD ✓ (1) Why is step 3 correct? (1) He added 10 on both sides to make the equation ~~bala~~ balanced ✓ (1)

5.2 Solve $\frac{6x}{2} - 5 = 4$ using a different method. (3)

$\frac{6x}{2} - 5 = 4$

$\frac{6x}{2} - 5 + 5 = 4 + 5$

$\frac{6x}{2} \times 2 = 9 \times 2$

$6x = 18$

$\frac{6x}{6} = \frac{18}{6}$

$x = 3$ (3)

6.1 Erroneous example	**Questions**
Ruth solved this problem incorrectly. This is Ruth's working: $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$	Why is Step 1 incorrect? (1) The $\frac{x}{2}$ and $\frac{x}{3}$ is added to mak $\frac{2x}{5}$ (1)
Step 1. $\frac{2x}{5} = \frac{1}{6}$	Why is Step 2 correct? (1) The denominator of the opposite fraction was multiplied but the fraction to mak it balanced (1)
Step 2 $2x \times 6 = 1 \times 5$	
Step 3 $12x = 6$	Why is Step 3 incorrect? (1) The 1 and 5 is added to get a 6 (1)
Step 4 $\frac{12x}{12} = \frac{6}{12}$	Why is Step 4 correct? (1) both sides were divided by 12 (1)
$\therefore x = \frac{1}{2}$	

6.2 Solve $\frac{x}{2} + \frac{x}{3} = \frac{1}{6}$ (4)

LCD = 6

$\frac{6}{2}(x) + \frac{6}{3}(x) = \frac{6}{6}(1)$

$3x + 2x = 1$

$\frac{5x}{5} = \frac{1}{5}$

$x = \frac{1}{5}$ (4)

7 The length of a rectangle is twice as long as its width. Determine the lengths of the sides of the rectangle if its perimeter is 24cm.

7.1 Incorrect example	**Questions**
Let width of rectangle be x Length will be $2x$ Step 1 $2x + x = 24$ Step 2 $3x = 24$ Step 3 $\frac{3x}{3x} = \frac{24}{3x}$ Step 4 $x = 8$ Width of rectangle = 8 cm Length of rectangle = 16 cm	Why is Step 1 incorrect? ~~it is correct~~ (2) ~~There are 2 sides that need to be the with~~ 16 is correct What is the error in Step 3? (1) They both sides were divided by 3x (1)

7.2 Solve question 7 in the space below. (4)

$2x + x = 24$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

Width = 8

Length = 16

(0)

8. John is five times as old as Jabulani. After four years, John will be three times as old as Jabulani will. What are their present ages?

8.1 Incorrect example	Questions
Boys' ages now Let Jabulani's age now be X Step 1: John 5X yrs Jabulani 5yrs Boys' ages 4 years later Step 2: John 5X+4 = 20X Jabulani 5+4 =20 Step 3: 20X = 20 Step 5: $\frac{20X}{20} = \frac{20}{20}$ Step 6: X = 1 ∴ Jabulani 1 year old now and John is 5 years old.	Why is Jabulani's age in step 1 incorrect? (1) We do not know what is his age and it must be x (1) What mistakes are shown in step 2? (2) They multiplied the ~~5~~ and 4 on both sides ~~to~~ get 20 (2)

8.2 Solve Question 8 on the space below (5)

~~$5x+4 = 20x$~~
~~$4 = 20x - 5x$~~
~~$4 = 15x$~~

John $5x+4 = 9$ Jabulani $5+4=9$

$5x+4-4 = 9-4$

$\frac{5x}{5} = \frac{5}{5}$

$\frac{5x}{5} = \frac{5}{5}$ ✗

$x = 1$ (0)

∴ Jabulani is 1 year old and john is ✗5

APPENDIX P: LESSON OBSERVATION SCHEDULE

DEMOGRAPHIC DETAILS

1. Name of School: ____

2. Physical Address of School: ____

____

3. Postal Address of School: ____

____

4. Tel: ________________________ Fax: ________________________

5. Name of Principal: ___________________________

Male		Female	

6. Name of Teacher: ___________________________

Male		Female	

7. Grade Observed: ____________ 8. Number of Learners: ____________

OBSERVING CLASSROOM PRACTICE

1. How does teaching and learning of Mathematics occur? (Please list e.g., *whole class*)

(i) ________________________ (ii) ____________________

(ii) ________________________ (iv) ____________________

2. How is the classroom arranged? (*Furniture*) _________________________________

3. What methodology/approach is being used? ________________________

4. Which resources are used?

____

OBSERVING TEACHING AND LEARNING IN THE CLASSROOM

5. How does the teacher deal with correct or incorrect responses?

The PEER system underlies the lessons in a classroom situation. It might not be possible to incorporate all of them in a particular lesson but each lesson will contain some aspects of this system. Please tick (O) your rating.

A	**PRODUCTIVE SKILLS**	Excellent	Good	Average	Needs More	Not Applicable
1.	Learners are able to do reading on the concept being taught.					
2.	Learners write notes on the concept taught.					
3.	Learners are able to solve problems given as exercises.					
4.	Learners are able to relate and apply the concept in real life problems.					
5.	Learners are able to use their knowledge of and experience in the concept in formulating their own responses.					
6.	Learners are able to accomplish work given on the concept independently					
7.	Learners are able to define and describe learned terms encountered when dealing with the concept.					
8.	Learners are able to follow the steps in solving exercises based on the concept.					
9.	Learners competently use technology (calculators) in areas where it is required in the concept.					

10.	Learners are able to deal with problems in real and abstract context using the concept.					
11.	Learners' ways of making decisions in problem solving is enhanced.					
B	**EVOCATIVE SKILLS**	Excellent	Good	Average	Needs More Attention	Not Applicable
1.	Learners ask questions for clarification.					
2.	Learners ask questions to consolidate their understanding of the concept					
3.	Learners are puzzled by certain areas of the concept and hence very inquisitive.					
4.	Learners are able to interpret new information on the concept.					
5.	Learners ask critical questions to ensure that methods used are appropriate.					
6.	Learners use their referencing skills to acquire better understanding of the concept.					
C	**EVALUATIVE SKILLS**	Excellent	Good	Average	Needs More Attention	Not Applicable to
1.	Learners are able to do self-assessment tasks in the concept learned.					
2.	Learners are capable of evaluating their own work on the concept.					
3.	Learners are able to evaluate procedures followed in problem solving in the concept.					

4.	Learners are able to identify errors committed when dealing with the concept.					
5.	Learners are able to discuss pros and cons in using specific methods to solve problems.					
6.	Learners are able to identify incorrect ways of solving problems.					
7.	Learners have alternative ways to solve problems based on the concept.					
D	**REFLECTIVE SKILLS**	Excellent	Good	Average	Needs More Attention	Not Applicable
1.	Learners are constantly engulfed in the world of "exploration in errors."					
2.	Learners reflect on errors committed in solving problems and work towards eliminating those errors.					
3.	Learners are able to respond to questions testing their comprehension of the learned concept.					
4.	Learners are able to select and use appropriate methods in solving problems.					
5.	Learners are able of hypothesizing in problem solving.					
6.	Learners can reflect on the decision they made in solving a particular problem.					

Please provide examples of errors corrected when dealing in the topic being evaluated

____

____

____

____

____

____

____

APPENDIX Q: SEMI-INTERVIEW QUESTIONS BEFORE INTERVENTION

Phase 1: Pre-intervention semi-structured interview questions

Q1. In your opinion, what could be the reasons for learners' poor performance and why?

Q 2. How do you encourage active learner participation during lessons?

Q3. What teaching resources do you use in your classroom?

Q4. What classroom assessments do you use in mathematics, and how?

Q5. How do you support learners with challenges in mathematics?

Q6. How do you promote a teaching and learning culture in your classroom?

APPENDIX R: SEMI-INTERVIEW QUESTIONS DURING INTERVENTION

Phase 2: semi-structured interview questions, during intervention

Q1. Do you think using erroneous examples is appropriate for teaching Grade 9 algebraic linear equations and why?

Q2. What are the challenges of using erroneous examples for teaching algebraic linear equations**?**

Q3. *What are the benefits of using erroneous examples for teaching Grade 9 algebraic linear equations?*

Q4. Do you think erroneous examples would be suitable in future for teaching Grade 9 algebraic linear equations and why?

APPENDIX S: SEMI-INTERVIEW QUESTIONS AFTER INTERVENTION

Phase 3: semi-structured interview questions, after intervention

Q1. How is learner participation now in comparison to the time before the intervention?

Q2. Do you think erroneous examples can be used on a daily basis and why?

Q3. According to you, was using erroneous examples an effective strategy for improving learners' performance and why?

Q4 How can we make the use of erroneous examples more effective**?**

APPENDIX T: STATISTICAL ANALYSIS FOR PRE-POST TESTS

Excel was used for data management, and IBM SPSS version 28 was used for data analysis. T-test was used to compare the results of the two study groups that is, the experimental and the control group. The results were interpreted at the 95% confidence limit (2-sided). The results were declared significant if the p-value was less than 0.05.

This means that:

If $p < 0.05$, results are significant.

If $p \geq 0.05$, the results are not significant.

Question 1
T-Test

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.00000	.000000	.000000
	CG	28	2.50000	2.457038	.464337

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	159.895	<.001	-5.670	57
	Equal variances not assumed			-5.384	27.000

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	<.001	<.001	-2.500000	.440882
	Equal variances not assumed	<.001	<.001	-2.500000	.464337

The above table indicates that in the pre-test, the experimental and control groups' performances were significantly different ($p < 0.001$).

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-3.382851	-1.617149
	Equal variances not assumed	-3.452740	-1.547260

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	95% Confidence Interval Upper
GROUP	Cohen's d	1.691050	-1.478	-2.052	-.895
	Hedges' correction	1.713715	-1.459	-2.024	-.883
	Glass's delta	2.457038	-1.017	-1.588	-.432

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test **Question 1**

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	4.58333	1.781640	.514315
	CG	24	3.45833	2.084710	.425540

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	1.275	.267	1.598	34
	Equal variances not assumed			1.685	25.500

Independent Samples Test

		t-test for Equality of Means			
		Significance			
		One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference
GROUP	Equal variances assumed	.060	.119	1.125000	.704176
	Equal variances not assumed	.052	.104	1.125000	.667536

The above table indicates that in the post-test, the experimental and control groups' performances were not significantly different (p = 0.104).

The rest of the tables are interpreted like the above tables.

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-.306058	2.556058
	Equal variances not assumed	-.248450	2.498450

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	1.991711	.565	-.145	1.267
	Hedges' correction	2.037034	.552	-.142	1.238
	Glass's delta	2.084710	.540	-.176	1.244

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 2

T-Test
[DataSet3]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.25806	.773207	.138872
	CG	28	1.28571	1.760832	.332766

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	42.897	<.001	-2.952	57
	Equal variances not assumed			-2.850	36.235

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.002	.005	-1.027650	.348162
	Equal variances not assumed	.004	.007	-1.027650	.360581

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-1.724833	-.330467
	Equal variances not assumed	-1.758778	-.296522

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
GROUP	Cohen's d	1.335414	-.770	-1.296	-.236
	Hedges' correction	1.353312	-.759	-1.279	-.233
	Glass's delta	1.760832	-.584	-1.113	-.045

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test
[DataSet4]

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	3.5000	1.31426	.37939
	CG	24	3.6250	2.24214	.45767

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equa Means
		F	Sig.	t
GROUP	Equal variances assumed	5.715	.023	-.178
	Equal variances not assumed			-.210

Independent Samples Test

t-test for Equality of Means			
Significance			
One-Sided p	Two-Sided p	Mean Difference	Std Diff

GROUP	Equal variances assumed	.430	.860	-.12500	.70352
	Equal variances not assumed	.417	.835	-.12500	.59448

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-1.55473	1.30473
	Equal variances not assumed	-1.33455	1.08455

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
GROUP	Cohen's d	1.98986	-.063	-.755	.631
	Hedges' correction	2.03515	-.061	-.739	.617
	Glass's delta	2.24214	-.056	-.748	.638

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 3

T-Test
[DataSet5]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.16129	.734701	.131956
	CG	28	2.89286	2.922952	.552386

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	76.553	<.001	-5.034	57
	Equal variances not assumed			-4.810	30.081

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	<.001	<.001	-2.731567	.542581
	Equal variances not assumed	<.001	<.001	-2.731567	.567929

Independent Samples Test

t-test for Equality of Means
95% Confidence Interval of the Difference

		Lower	Upper
GROUP	Equal variances assumed	-3.818067	-1.645067
	Equal variances not assumed	-3.891300	-1.571833

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	2.081127	-1.313	-1.873	-.743
	Hedges' correction	2.109020	-1.295	-1.848	-.733
	Glass's delta	2.922952	-.935	-1.496	-.359

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet6]

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	5.16667	2.886751	.833333
	CG	24	5.50000	2.536687	.517799

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	.386	.539	-.355	34
	Equal variances not assumed			-.340	19.727

Independent Samples Test

		t-test for Equality of Means			
		Significance			
		One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference
GROUP	Equal variances assumed	.362	.725	-.333333	.938684
	Equal variances not assumed	.369	.738	-.333333	.981102

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-2.240969	1.574302
	Equal variances not assumed	-2.381691	1.715024

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
GROUP	Cohen's d	2.654999	-.126	-.818	.569
	Hedges' correction	2.715417	-.123	-.800	.556
	Glass's delta	2.536687	-.131	-.824	.564

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 4

T-Test
[DataSet7]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.12903	.499462	.089706
	CG	28	.82143	1.389206	.262535

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	20.962	<.001	-2.597	57
	Equal variances not assumed			-2.496	33.265

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.006	.012	-.692396	.266575
	Equal variances not assumed	.009	.018	-.692396	.277438

Independent Samples Test

t-test for Equality of Means

95% Confidence Interval of the Difference

		Lower	Upper
GROUP	Equal variances assumed	-1.226203	-.158589
	Equal variances not assumed	-1.256678	-.128115

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	1.022476	-.677	-1.200	-.149
	Hedges' correction	1.036180	-.668	-1.184	-.147
	Glass's delta	1.389206	-.498	-1.022	.034

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet8]

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	1.75000	1.712255	.494286
	CG	24	2.91667	1.931808	.394329

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	.001	.977	-1.771	34
	Equal variances not assumed			-1.845	24.677

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.043	.086	-1.166667	.658885
	Equal variances not assumed	.039	.077	-1.166667	.632308

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-2.505683	.172349
	Equal variances not assumed	-2.469794	.136461

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	1.863609	-.626	-1.330	.087
	Hedges' correction	1.906018	-.612	-1.301	.085
	Glass's delta	1.931808	-.604	-1.312	.117

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 5

T-Test
[DataSet9]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.12903	.427546	.076790
	CG	28	1.64286	2.003964	.378714

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	65.972	<.001	-4.107	57
	Equal variances not assumed			-3.918	29.221

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	<.001	<.001	-1.513825	.368565
	Equal variances not assumed	<.001	<.001	-1.513825	.386420

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-2.251864	-.775786
	Equal variances not assumed	-2.303883	-.723766

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	1.413670	-1.071	-1.614	-.519
	Hedges' correction	1.432618	-1.057	-1.593	-.513
	Glass's delta	2.003964	-.755	-1.298	-.200

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet10]

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	1.25000	1.356801	.391675
	CG	24	1.62500	1.468880	.299834

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	.096	.758	-.740	34
	Equal variances not assumed			-.760	23.766

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.232	.464	-.375000	.506847
	Equal variances not assumed	.227	.455	-.375000	.493264

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-1.405036	.655036
	Equal variances not assumed	-1.393577	.643577

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	1.433578	-.262	-.955	.436
	Hedges' correction	1.466201	-.256	-.934	.426
	Glass's delta	1.468880	-.255	-.949	.444

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 6

T-Test

[DataSet11]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.16129	.522607	.093863
	CG	28	2.00000	2.434322	.460044

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	29.800	<.001	-4.106	57
	Equal variances not assumed			-3.916	29.249

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	<.001	<.001	-1.838710	.447851
	Equal variances not assumed	<.001	<.001	-1.838710	.469522

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-2.735515	-.941904
	Equal variances not assumed	-2.798634	-.878785

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	1.717779	-1.070	-1.614	-.519
	Hedges' correction	1.740802	-1.056	-1.592	-.512
	Glass's delta	2.434322	-.755	-1.298	-.200

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test
[DataSet12]

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	1.83333	1.642245	.474075
	CG	24	4.12500	2.290102	.467465

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	1.126	.296	-3.083	34
	Equal variances not assumed			-3.442	29.467

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean	Std. Error
		One-Sided p	Two-Sided p	Difference	Difference
GROUP	Equal variances assumed	.002	.004	-2.291667	.743332
	Equal variances not assumed	<.001	.002	-2.291667	.665786

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-3.802300	-.781034
	Equal variances not assumed	-3.652416	-.930918

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
GROUP	Cohen's d	2.102461	-1.090	-1.822	-.343
	Hedges' correction	2.150305	-1.066	-1.782	-.336
	Glass's delta	2.290102	-1.001	-1.742	-.241

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 7

T-Test
[DataSet13]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.09677	.300537	.053978
	CG	28	1.03571	2.268662	.428737

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	18.453	<.001	-2.284	57
	Equal variances not assumed			-2.173	27.856

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean	Std. Error
		One-Sided p	Two-Sided p	Difference	Difference
GROUP	Equal variances assumed	.013	.026	-.938940	.411030
	Equal variances not assumed	.019	.038	-.938940	.432121

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-1.762014	-.115866
	Equal variances not assumed	-1.824306	-.053574

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
GROUP	Cohen's d	1.576549	-.596	-1.116	-.071
	Hedges' correction	1.597680	-.588	-1.101	-.070
	Glass's delta	2.268662	-.414	-.933	.112

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet14]

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	.58333	1.164500	.336162
	CG	24	.54167	.508977	.103895

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	2.795	.104	.150	34

	Equal variances not assumed	.118	13.144

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.441	.881	.041667	.277031
	Equal variances not assumed	.454	.908	.041667	.351851

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-.521329	.604662
	Equal variances not assumed	-.717613	.800946

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	.783563	.053	-.640	.746
	Hedges' correction	.801394	.052	-.626	.729
	Glass's delta	.508977	.082	-.612	.774

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 8

T-Test

[DataSet15]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	.19355	.542792	.097488
	CG	28	.64286	1.282771	.242421

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	10.842	.002	-1.783	57
	Equal variances not assumed			-1.720	35.601

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.040	.080	-.449309	.252034
	Equal variances not assumed	.047	.094	-.449309	.261289

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-.953998	.055380
	Equal variances not assumed	-.979434	.080816

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	95% Confidence Interval Upper
GROUP	Cohen's d	.966702	-.465	-.981	.055
	Hedges' correction	.979659	-.459	-.968	.055
	Glass's delta	1.282771	-.350	-.867	.172

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet16]

Group Statistics

	POST-TEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	.16667	.389249	.112367
	CG	24	1.20833	1.768791	.361053

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	13.108	<.001	-2.002	34

	Equal variances not assumed	-2.755	27.139

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.027	.053	-1.041667	.520269
	Equal variances not assumed	.005	.010	-1.041667	.378134

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-2.098981	.015648
	Equal variances not assumed	-1.817348	-.265985

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	95% Confidence Interval Upper
GROUP	Cohen's d	1.471544	-.708	-1.416	.010
	Hedges' correction	1.505030	-.692	-1.384	.010
	Glass's delta	1.768791	-.589	-1.296	.130

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Comparisons between the pre-test and post-tests

Question 1

T-Test
[DataSet17]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.00000	.000000	.000000
	POSTTEST	12	4.58333	1.781640	.514315

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	44.027	<.001	-14.608	41
	Equal variances not assumed			-8.912	11.000

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	<.001	<.001	-4.583333	.313753
	Equal variances not assumed	<.001	<.001	-4.583333	.514315

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-5.216969	-3.949697
	Equal variances not assumed	-5.715334	-3.451333

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
SETTING	Cohen's d	.922836	-4.967	-6.219	-3.696
	Hedges' correction	.940157	-4.875	-6.104	-3.628
	Glass's delta	1.781640	-2.573	-3.809	-1.305

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet18]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	2.50000	2.457038	.464337
	POSTTEST	24	3.45833	2.084710	.425540

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	2.645	.110	-1.502	50

	Equal variances not assumed	-1.522	49.998

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	.070	.139	-.958333	.637934
	Equal variances not assumed	.067	.134	-.958333	.629835

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-2.239661	.322994
	Equal variances not assumed	-2.223396	.306729

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	2.293287	-.418	-.967	.135
	Hedges' correction	2.328419	-.412	-.953	.133
	Glass's delta	2.084710	-.460	-1.016	.106

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 2

T-Test

[DataSet19]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.25806	.773207	.138872
	POSTTEST	12	3.50000	1.314257	.379393

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	5.964	.019	-10.046	41
	Equal variances not assumed			-8.024	14.053

Independent Samples Test

		t-test for Equality of Means			
		Significance			
		One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference
SETTING	Equal variances assumed	<.001	<.001	-3.241935	.322695
	Equal variances not assumed	<.001	<.001	-3.241935	.404011

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-3.893632	-2.590239

	Equal variances not assumed	-4.108149	-2.375722

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
SETTING	Cohen's d	.949139	-3.416	-4.399	-2.412
	Hedges' correction	.966954	-3.353	-4.318	-2.368
	Glass's delta	1.314257	-2.467	-3.666	-1.235

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet20]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	1.28571	1.760832	.332766
	POSTTEST	24	3.62500	2.242136	.457674

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	1.356	.250	-4.212	50
	Equal variances not assumed			-4.134	43.411

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean	Std. Error
		One-Sided p	Two-Sided p	Difference	Difference
SETTING	Equal variances assumed	<.001	<.001	-2.339286	.555429
	Equal variances not assumed	<.001	<.001	-2.339286	.565861

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-3.454898	-1.223674
	Equal variances not assumed	-3.480141	-1.198430

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	1.996694	-1.172	-1.758	-.575
	Hedges' correction	2.027281	-1.154	-1.731	-.567
	Glass's delta	2.242136	-1.043	-1.657	-.412

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 3

T-Test
[DataSet21]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.16129	.734701	.131956
	POSTTEST	12	5.16667	2.886751	.833333

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	36.871	<.001	-9.077	41
	Equal variances not assumed			-5.933	11.556

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	<.001	<.001	-5.005376	.551444
	Equal variances not assumed	<.001	<.001	-5.005376	.843716

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-6.119040	-3.891712
	Equal variances not assumed	-6.851542	-3.159210

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	1.621955	-3.086	-4.018	-2.134
	Hedges' correction	1.652398	-3.029	-3.944	-2.095
	Glass's delta	2.886751	-1.734	-2.694	-.739

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet22]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	2.89286	2.922952	.552386
	POSTTEST	24	5.50000	2.536687	.517799

Independent Samples Test

Levene's Test for Equality of Variances		t-test for Equality of Means	
F	Sig.	t	df

SETTING	Equal variances assumed	2.304	.135	-3.406	50
	Equal variances not assumed			-3.443	49.988

Independent Samples Test

		t-test for Equality of Means			
		Significance			
		One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference
SETTING	Equal variances assumed	<.001	.001	-2.607143	.765539
	Equal variances not assumed	<.001	.001	-2.607143	.757130

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-4.144773	-1.069512
	Equal variances not assumed	-4.127893	-1.086393

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
SETTING	Cohen's d	2.752012	-.947	-1.519	-.367
	Hedges' correction	2.794171	-.933	-1.496	-.362
	Glass's delta	2.536687	-1.028	-1.639	-.399

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 4

T-Test

[DataSet23]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.12903	.499462	.089706
	POSTTEST	12	1.75000	1.712255	.494286

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	18.815	<.001	-4.843	41
	Equal variances not assumed			-3.227	11.732

Independent Samples Test

		t-test for Equality of Means			
		Significance			
		One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference
SETTING	Equal variances assumed	<.001	<.001	-1.620968	.334697
	Equal variances not assumed	.004	.007	-1.620968	.502360

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-2.296901	-.945034

	Equal variances not assumed	-2.718296	-.523639

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
SETTING	Cohen's d	.984438	-1.647	-2.394	-.884
	Hedges' correction	1.002916	-1.616	-2.350	-.867
	Glass's delta	1.712255	-.947	-1.703	-.158

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet24]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	.82143	1.389206	.262535
	POSTTEST	24	2.91667	1.931808	.394329

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	.408	.526	-4.535	50
	Equal variances not assumed			-4.423	41.040

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	<.001	<.001	-2.095238	.462038
	Equal variances not assumed	<.001	<.001	-2.095238	.473730

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-3.023269	-1.167207
	Equal variances not assumed	-3.051927	-1.138550

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	1.660966	-1.261	-1.855	-.658
	Hedges' correction	1.686411	-1.242	-1.827	-.648
	Glass's delta	1.931808	-1.085	-1.704	-.448

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 5

T-Test
[DataSet25]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.12903	.427546	.076790
	POSTTEST	12	1.25000	1.356801	.391675

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	31.839	<.001	-4.162	41
	Equal variances not assumed			-2.809	11.855

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	<.001	<.001	-1.120968	.269354
	Equal variances not assumed	.008	.016	-1.120968	.399131

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-1.664939	-.576996

	Equal variances not assumed	-1.991777	-.250158

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
SETTING	Cohen's d	.792247	-1.415	-2.141	-.675
	Hedges' correction	.807117	-1.389	-2.101	-.662
	Glass's delta	1.356801	-.826	-1.560	-.063

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet26]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	1.64286	2.003964	.378714
	POSTTEST	24	1.62500	1.468880	.299834

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	4.473	.039	.036	50
	Equal variances not assumed			.037	48.902

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean	Std. Error
		One-Sided p	Two-Sided p	Difference	Difference
SETTING	Equal variances assumed	.486	.971	.017857	.494577
	Equal variances not assumed	.485	.971	.017857	.483037

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-.975531	1.011245
	Equal variances not assumed	-.952891	.988605

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
SETTING	Cohen's d	1.777940	.010	-.535	.555
	Hedges' correction	1.805177	.010	-.527	.547
	Glass's delta	1.468880	.012	-.533	.557

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 6

T-Test

[DataSet27]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.16129	.522607	.093863
	POSTTEST	12	1.83333	1.642245	.474075

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	31.230	<.001	-5.118	41
	Equal variances not assumed			-3.460	11.873

Independent Samples Test

		t-test for Equality of Means			
		Significance			
		One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference
SETTING	Equal variances assumed	<.001	<.001	-1.672043	.326710
	Equal variances not assumed	.002	.005	-1.672043	.483278

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-2.331847	-1.012239

	Equal variances not assumed	-2.726270	-.617816

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
SETTING	Cohen's d	.960947	-1.740	-2.497	-.967
	Hedges' correction	.978984	-1.708	-2.451	-.949
	Glass's delta	1.642245	-1.018	-1.790	-.214

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet28]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	2.00000	2.434322	.460044
	POSTTEST	24	4.12500	2.290102	.467465

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	.012	.912	-3.225	50
	Equal variances not assumed			-3.240	49.540

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean	Std. Error
		One-Sided p	Two-Sided p	Difference	Difference
SETTING	Equal variances assumed	.001	.002	-2.125000	.659015
	Equal variances not assumed	.001	.002	-2.125000	.655869

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-3.448670	-.801330
	Equal variances not assumed	-3.442654	-.807346

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	2.369072	-.897	-1.466	-.320
	Hedges' correction	2.405364	-.883	-1.444	-.315
	Glass's delta	2.290102	-.928	-1.527	-.313

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 7
T-Test

[DataSet29]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.09677	.300537	.053978
	POSTTEST	12	.58333	1.164500	.336162

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	13.690	<.001	-2.183	41
	Equal variances not assumed			-1.429	11.572

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	.017	.035	-.486559	.222921
	Equal variances not assumed	.090	.179	-.486559	.340468

Independent Samples Test

		t-test for Equality of Means 95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-.936758	-.036360

	Equal variances not assumed	-1.231432	.258314

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
SETTING	Cohen's d	.655676	-.742	-1.423	-.052
	Hedges' correction	.667983	-.728	-1.397	-.052
	Glass's delta	1.164500	-.418	-1.097	.279

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet30]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	1.03571	2.268662	.428737
	POSTTEST	24	.54167	.508977	.103895

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	8.294	.006	1.043	50
	Equal variances not assumed			1.120	30.142

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean	Std. Error
		One-Sided p	Two-Sided p	Difference	Difference
SETTING	Equal variances assumed	.151	.302	.494048	.473587
	Equal variances not assumed	.136	.272	.494048	.441145

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-.457181	1.445276
	Equal variances not assumed	-.406713	1.394809

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
SETTING	Cohen's d	1.702484	.290	-.259	.837
	Hedges' correction	1.728565	.286	-.255	.824
	Glass's delta	.508977	.971	.350	1.575

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

Question 8

T-Test
[DataSet31]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	.19355	.542792	.097488
	POSTTEST	12	.16667	.389249	.112367

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	.206	.652	.156	41
	Equal variances not assumed			.181	27.980

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	.438	.877	.026882	.172098
	Equal variances not assumed	.429	.858	.026882	.148762

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-.320678	.374441
	Equal variances not assumed	-.277854	.331618

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	.506190	.053	-.614	.719
	Hedges' correction	.515691	.052	-.602	.706
	Glass's delta	.389249	.069	-.599	.734

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet32]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	.64286	1.282771	.242421
	POSTTEST	24	1.20833	1.768791	.361053

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	3.202	.080	-1.332	50
	Equal variances not assumed			-1.300	41.267

Independent Samples Test

		t-test for Equality of Mean			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	.094	.189	-.565476	.424409
	Equal variances not assumed	.100	.201	-.565476	.434888

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-1.417926	.286973
	Equal variances not assumed	-1.443577	.312624

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	1.525693	-.371	-.919	.181
	Hedges' correction	1.549065	-.365	-.905	.178
	Glass's delta	1.768791	-.320	-.869	.237

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

TOTALS

Comparison of performance of control and experimental groups in pre-and post-test

T-Test

[DataSet33]

Group Statistics

	EG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	31	1.12903	2.963578	.532274
	POSTTEST	12	18.83333	8.166821	2.357558

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	27.687	<.001	-10.559	41
	Equal variances not assumed			-7.325	12.138

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	<.001	<.001	-17.704301	1.676684
	Equal variances not assumed	<.001	<.001	-17.704301	2.416898

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-21.090433	-14.318169
	Equal variances not assumed	-22.963617	-12.444986

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	4.931607	-3.590	-4.602	-2.559
	Hedges' correction	5.024171	-3.524	-4.517	-2.511
	Glass's delta	8.166821	-2.168	-3.266	-1.036

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet34]

Group Statistics

	CG	N	Mean	Std. Deviation	Std. Error Mean
SETTING	PRETEST	28	12.82143	13.875247	2.622175
	POSTTEST	24	23.00000	10.116710	2.065065

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
SETTING	Equal variances assumed	2.193	.145	-2.977	50
	Equal variances not assumed			-3.050	48.828

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
SETTING	Equal variances assumed	.002	.004	-10.178571	3.418740
	Equal variances not assumed	.002	.004	-10.178571	3.337708

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
SETTING	Equal variances assumed	-17.045313	-3.311830
	Equal variances not assumed	-16.886543	-3.470600

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
SETTING	Cohen's d	12.289920	-.828	-1.393	-.256
	Hedges' correction	12.478192	-.816	-1.372	-.252
	Glass's delta	10.116710	-1.006	-1.615	-.380

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet35]

Group Statistics

	PRETEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	31	1.12903	2.963578	.532274
	CG	28	12.82143	13.875247	2.622175

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	44.010	<.001	-4.582	57
	Equal variances not assumed			-4.370	29.226

Independent Samples Test

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	<.001	<.001	-11.692396	2.552045
	Equal variances not assumed	<.001	<.001	-11.692396	2.675653

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-16.802778	-6.582015
	Equal variances not assumed	-17.162882	-6.221911

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	9.788636	-1.194	-1.746	-.634
	Hedges' correction	9.919831	-1.179	-1.723	-.626
	Glass's delta	13.875247	-.843	-1.394	-.278

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

T-Test

[DataSet36]

Group Statistics

	POSTTEST	N	Mean	Std. Deviation	Std. Error Mean
GROUP	EG	12	18.83333	8.166821	2.357558
	CG	24	23.00000	10.116710	2.065065

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
GROUP	Equal variances assumed	.952	.336	-1.237	34
	Equal variances not assumed			-1.329	26.808

		t-test for Equality of Means			
		Significance		Mean Difference	Std. Error Difference
		One-Sided p	Two-Sided p		
GROUP	Equal variances assumed	.112	.225	-4.166667	3.369231
	Equal variances not assumed	.097	.195	-4.166667	3.134099

Independent Samples Test

		t-test for Equality of Means	
		95% Confidence Interval of the Difference	
		Lower	Upper
GROUP	Equal variances assumed	-11.013767	2.680434
	Equal variances not assumed	-10.599465	2.266132

Independent Samples Effect Sizes

		Standardizer[a]	Point Estimate	95% Confidence Interval	
				Lower	Upper
GROUP	Cohen's d	9.529624	-.437	-1.135	.267
	Hedges' correction	9.746481	-.428	-1.109	.261
	Glass's delta	10.116710	-.412	-1.111	.295

a. The denominator used in estimating the effect sizes.
Cohen's d uses the pooled standard deviation.
Hedges' correction uses the pooled standard deviation, plus a correction factor.
Glass's delta uses the sample standard deviation of the control group.

APPENDIX U: EDITING CERTIFICATE

Unit 3 West Square Business Park
407 West Avenue
Randburg
2194

22 December 2023

TO WHOM IT MAY CONCERN

This serves to confirm that I have edited and made the necessary corrections and emendations to the thesis:

USING ERRONEOUS EXAMPLES FOR TEACHING GRADE 9 ALGEBRAIC LINEAR EQUATIONS AT A SCHOOL IN JOHANNESBURG CENTRAL DISTRICT

by

JULIUS GWENZI

Sincerely

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Publisher, editor and translator

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APPENDIX V: TURNITIN REPORT

TURNITIN ORIGINALITY REPORT

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