Error Patterns and Underlying Misconceptions of Fractions among 15 Grade 6 Learners from a Public School in Gauteng

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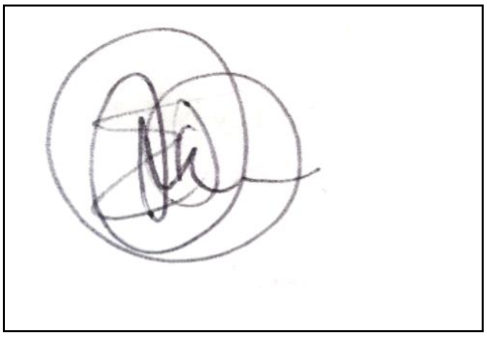
DATE: 04 NOVEMBER 2023

DECLARATION

I declare that **Error Patterns and Underlying Misconceptions of Fractions among 15 Grade 6 Learners from a Public School in Gauteng** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.



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Thapeli E.M　　　　　　　　　　Date

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This research would not be possible without the Almighty Lord for giving me strength and courage until the last day.

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LIST OF ABBREVIATIONS	
NCS	National Curriculum Statement
CAPS	Curriculum and Assessment Policy Statements
DBE	Department of Basic Education
ANA	Annual National Assessment
FET	Further Education and Training
ATP	Annual Teaching Plan
TIMMS	Trends in International Mathematics Science Study
NMAP	National Mathematics Advisory Panel
SASAMS	South African School Administration Management System

ABSTRACT

This study aimed to identify common error patterns concerning fractions for Grade 6 learners and the misconceptions associated with procedural, conceptual and factual errors, as well as subtraction and multiplication at a public school in Orange Farm, Gauteng. The study was based on the ideas of constructivism to understand learners' reasoning in developing constructions in the concept of fractions. Recent studies on errors and underlying misconceptions about fractions have indicated that most primary school learners struggle with fractions, affecting learners' future performance at the secondary level and their mathematics experience. The study used a qualitative case study approach to identify the errors made by Grade 6 learners. This approach provided a platform to investigate how Grade 6 learners solve fractions-related problems, allowing the researcher to uncover learners' misconceptions when dealing with fractions. The researcher chose Grade 6 since it is the exit level for the Intermediate Phase. The annual teaching plan, stipulated by the Curriculum Policy Statement policy of the Department of Basic Education (DBE) specifically for Grade 6 learners, focuses on the three operations: addition, subtraction and multiplication. Fifteen learners were purposively sampled and given a set of ten questions based on the operations mentioned above. The study's main data sources were a written test and interviews. The data collected during the written test and interviews were analysed using a qualitative approach and discussed using constructivism as a theoretical framework. Learners were allowed to explain their responses based on their incorrect answers. The results show that learners lack basic mathematical skills in fractions. They believe that numerator and denominator are the same; they add or subtract both denominators and numerators. When multiplying, learners add the numerators and denominators together and again, due to a lack of conceptual understanding of the multiplication of fractions, learners are challenged to deal with the multiplication of mixed fractions. Integers are multiplied separately from mixed fractions. In addition, some learners don't understand how they obtained their respective answers. Constructivism values prior knowledge as a basis for developing new knowledge. Teachers should not take for granted what has been covered before learners have dealt with fractions, as this can lead to misunderstandings. Therefore, reviewing the previous information before starting a new lesson is recommended. Another suggested measure is that learners should understand the difference between numerator and denominator and the correct applications of the three operations. The study recommends that appropriate teacher development training in specific mathematics topics be done at the primary school level, as this will lay a foundation for better transmission, not only on the basics of mathematics but also general mathematical knowledge.

Keywords: Fractions, mathematics, common errors, procedural errors, factual errors, conceptual errors, error analysis, cognitive architecture, misconception, perception

KGUTSUFATSO

Thuto ena e ne e ikemiseditse ho hlwaya mekgwa e tlwaelehileng ya diphoso tse mabapi le dikarolo tsa barutuwa ba Kereiti ya 6 le menahano e fosahetseng e amanang le diphoso tsa mokgwa, mohopolo le nnete ho tlatseletsa, ho ntsha le ho atisa dikarolwana sekolong sa mmuso Orange Farm, Gauteng. Thuto ena e ne e ipapisitse le mehopolo ya kaho ya tsebo ho utlwisisa menahano ya barutuwa ho ntshetsapele meaho mohopolong wa dikarolwana. Diphuputso tsa moraorao tse mabapi le diphoso le menahano e fosahetseng ya motheo mabapi le dikarolwana di bontshitse hore boholo ba barutuwa ba dikolo tsa pele ba sokola ka dikarolwana, tse amang tshebetso ya kamoso ya barutuwa sehlopheng sa sekondari le boiphihlelo ba bona ba dipalo. Patlisiso e sebedisitse mokgwa wa ho batlisisa wa maemo ho lemoha diphoso tseo entsweng ke barutuwa ba Kereiti ya 6. Mokgwa ona o fane ka sethala sa ho batlisisa hore na barutuwa ba Kereiti ya 6 ba rarolla mathata a amanang le dikarolwana jwang, ho dumella mofuputsi ho sibolla menahano e fosahetseng eo barutuwa ba nang le yona kapa ba e bontshang ha ba sebetsana le dikarolwana. Thuto ena e tla fana ka kenyelletso e ntle ka ho fetisisa ya algebra maemong a phahameng le a FET. Mofuputsi o kgethile Kereiti ya 6, kaha ke boemo ba ho tswa ho Mokgahlelo o Mahareng. Leano la ho ruta la selemo le selemo, le hlalositsweng ke pholisi ya Setatamente sa Pholisi ya Kharikhulamo la Lefapha la Thuto ya Motheo (DBE) ka ho qolleha bakeng sa barutuwa ba Kereiti ya 6, le tsepamisitse maikutlo hodima mesebetsi e meraro: ho eketsa, ho tlosa le ho atisa. Barutuwa ba leshome le metso e mehlano ba ile ba etswa disampole ka sepheo se nepahetseng mme ba fuwa dipotso tse leshome tse ipapisitseng le tshebetso e boletsweng ka hodimo. Mehlodi e meholo ya dintlha tsa thuto e ne e le teko e ngotsweng le dipuisano. Dintlha tse bokelletsweng nakong ya hlahlobo e ngotsweng le dipuisano di ile tsa hlahlojwa ka mokgwa wa boleng mme tsa tshohlwa ka mokgwa wa kaho ya tsebo e le moralo wa teori. Barutuwa ba ile ba dumellwa ho hlalosa dikarabo tsa bona ho latela dikarabo tsa bona tse fosahetseng. Diphetho di bontsha hore barutuwa ba haellwa ke tsebo ya mantlha ya dipalo ka dikarolwana. Ba dumela hore dipalo le dinomineitha di a tshwana; ba eketsa kapa ba fokotsa ka bobedi di- dinomineitha le dipalo. Ha ba atisa, barutuwa ba kopanya dipalo le di-dinomineitha mmoho, hape, ka lebaka la ho hloka kutlwisiso ya moelelo wa katiso ya dikarolwana, barutuwa ba phephetswa ho sebetsana le katiso ya dikarolwana tse tswakilweng. Dinomoro di ngatafaditswe ka thoko ho dikarolwana tse tswakilweng. Ho feta

moo, barutuwa ba bang ha ba utlwisise hore na ba fumane dikarabo tsa bona tse fapaneng jwang. Kaho ya tsebo e nka tsebo ya pele e le motheo wa ho ntlafatsa tsebo e ntjha. Matitjhere ha ba tshwanela ho nkela fatshe se ileng sa akaretswa pele ha barutuwa ba sebetsana le dikarolwana, ka hobane sena se ka baka ho se utlwisise hantle.

Ka hona, ho kgothaletswa ho hlahloba boitsebiso bo fetileng pele o qala thuto e ntjha. Tekanyo e nngwe e sisintsweng ke hore barutuwa ba lokela ho utlwisisa phapano pakeng tsa dinomoro le dinomineitha le tshebediso e nepahetseng ya ditshebetso tse tharo. Patlisiso e tsitsinya hore kwetliso ya ntlafatso e nepahetseng ya hlabollo ya matitjhere dihloohong tse tsebahalang tsa dipalo e etswe boemong ba sekolo sa mantlha, ka ha sena se tla theha motheo wa phetiso e ntle, e seng feela motheong wa thuto ya dipalo empa le tsebo ya kakaretso ya dipalo.

Mantswe a bohlokwa: **Dikarolwana, dipalo, diphoso tse tlwaelehileng, diphoso tsa tsamaiso, diphoso tsa nnete, diphoso tsa mohopolo, hlahlobo ya diphoso, meaho ya kelello, maikutlo a fosahetseng, maikutlo.**

ABSTRAK

Die doel van hierdie studie was om algemene foutpatrone rakende breukpatrone vir Graad 6-leerders en die wanopvattings wat verband hou met konseptuele en feitelike foute by optel, aftrek en vermenigvuldiging van breuke by ’n openbare skool in Orange Farm, Gauteng, te identifiseer. Die studie is gebaseer op die idees van konstruktivisme om leerders se redenasie in die ontwikkeling van konstruksies in die konsep van breuke te verstaan. Onlangse studies oor foute en onderliggende wanopvattings oor breuke het aangedui dat die meeste laerskoolleerders met breuke sukkel wat leerders se toekomstige prestasie op sekondêre vlak en hulle wiskunde-ervaring beïnvloed. Die studie het ’n kwalitatiewe gevallestudiebenadering gebruik om foute wat deur Graad 6-leerders gemaak is, te identifiseer. Hierdie benadering het ’n platform verskaf om te ondersoek hoe Graad 6-leerders breukverwante probleme oplos, wat die navorser in staat gestel het om die wanopvattings wat leerders het of openbaar wanneer hulle met breuke te doen het, te ontbloot. Die studie sal die beste inleiding tot algebra in die Senior en VOO-fases verskaf. Die navorser het Graad 6 gekies aangesien dit die uittreevlak vir die Intermediêre Fase is. Die jaarlikse onderrigplan bepaal deur die Kurrikulumbeleidsverklaring van die Departement van Basiese Onderwys (DBO) spesifiek vir Graad 6-leerders, fokus op die bewerkings: optel, aftrek en vermenigvuldiging. Vyftien leerders is in ’n doelbewuste steekproef gebruik en het ’n stel van tien vrae gekry wat gebaseer is op die bogenoemde bewerkings. Die studie se hoofdatabronne was ’n geskrewe toets en onderhoude. Die data wat tydens die geskrewe toets en onderhoude ingesamel is, is met behulp van ’n kwalitatiewe benadering ontleed en bespreek deur ’n teoretiese raamwerk van konstruktivisme. Leerders is toegelaat om hulle antwoorde op grond van hulle verkeerde antwoorde te verduidelik. Die resultate toon dat leerders nie basiese wiskundige vaardighede in breuke het nie. Hulle glo dat teller en noemer dieselfde is; hulle tel beide noemers en tellers op of af. Wanneer daar vermenigvuldig word, tel leerders die tellers en noemers bymekaar en, weer eens, as gevolg van ’n gebrekkige konseptuele begrip van die vermenigvuldiging van breuke, word leerders uitgedaag om die vermenigvuldiging van breuke te hanteer. Heelgetalle word afsonderlik van gemengde breuke vermenigvuldig. Daarbenewens verstaan sommige leerders nie hoe hulle hulle onderskeie antwoorde gekry het nie. Konstruktivisme besef die waarde van vorige kennis as ’n basis vir die ontwikkeling van nuwe kennis. Onderwysers moet nie dit wat gedek is as vanselfsprekend aanvaar voordat leerders met breuke te make het nie, aangesien dit tot misverstande kan lei. Daarom word daar aanbeveel dat vorige inligting hersien word voordat ’n nuwe les begin word. Nog ’n voorgestelde maatstaf is dat leerders die verskil tussen die teller en die noemer en die korrekte toepassings van die drie bewerkings moet verstaan. Die studie beveel aan dat toepaslike onderwyserontwikkelingsopleiding in spesifieke wiskunde-onderwerpe op laerskoolvlak gedoen word, aangesien dit ’n grondslag lê vir beter oordrag, nie net oor die basiese beginsels nie, maar ook algemene wiskundige kennis.

Sleutelwoorde: Breuke, wiskunde, algemene foute, prosedurefoute, feitefoute, konseptuele foute, foutanalise, kognitiewe argitektuur, wanopvatting, persepsie

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KEY WORDS AND DEFINITIONS

Fractions – numerical quantity that is not a whole number.

Common – occurring, found, or done often.

Error – a condition of being wrong in conduct.

Mathematics – the abstract science of number, quantity, and space

Procedural Errors – occur when different methods are used to answer the same question and slightly different answers are given

Conceptual Errors – mistakes that learners make when they do not fully understand mathematical concepts

Factual Errors – occur when the information in a problem is misunderstood.

Misconception – a view or opinion that is incorrect because it is based on faulty understanding.

Error Analysis – a method used to document the errors that appear in learners' language.

Perception – the ability to see, hear, or become aware of something through the senses.

Learner Knowledge – knowledge acquired by learners related to a good education and considerable learning.

CHAPTER 1
ORIENTATION OF THE STUDY AND RATIONALE

1.1 INTRODUCTION AND ORIENTATION

In South Africa, most learners do not major in mathematics at the secondary level due to certain difficult topics they cannot understand and always seem to fail. This gives learners a negative perception of mathematics. It is well known that learners perceive mathematics to be a difficult subject since certain mathematical concepts are difficult to grasp (Karao Yilmaz et al., 2018). Understanding fractions is important in mathematics achievement for learners' future success (Dogan-Coskun, 2019). In most countries, fractions are introduced in the first grade, and then extended in the higher elementary school levels. However, South African learners find it difficult to understand problems involving fractions because they lack basic mathematical skills. According to my experience as a mathematics teacher, what mainly contributes to this is how learners get to their answers in adding, subtracting and multiplying fractions; this leads to them exhibiting what appear to be unexplained error patterns.

This study identifies common error patterns made by Grade 6 learners in fractions at the primary school level. The research focused on adding, subtracting and multiplying fractions, including the underlying misconception associated with factual, conceptual, and procedural errors with the three mentioned operations. By understanding the mistakes learners make in solving mathematical problems, teachers can learn what causes those mistakes and what can be done to avoid such mistakes in the future. The dictionary definition of an error is an action which is inaccurate or incorrect; it also refers to the difference between the value which has been computed and the correct value. Not knowing how to solve fraction problems leads to poor learners' performance in mathematics, which also affects how they think, and eventually results in a negative attitude towards mathematics (Zakaria & Syamaun, 2017). Researchers have identified other common error patterns in fractions and have made recommendations to improve the teaching of different fractions-related concepts. However, there is still a need to identify error patterns since the problem persists.

The South African educational system must develop an integrated plan to assist learners in overcoming these errors related to fractions, thereby achieving better mathematics results. This will ultimately change most of the learners' perceptions of mathematics.

Chapter One - Orientation of the Study

This chapter focuses on the contextual background of the research, the problem statement, objectives, and research questions. It proposes the significance of the study, conceptual framework, theoretical framework, limitations, and delimitation.

Chapter Two – Review of Literature

This chapter presents the existing literature about the conceptual and theoretical framework the specific concepts on the topic and discusses the concepts of the topic and the study.

Chapter Three – Research Design and Methodology

This chapter focuses on the research philosophy design, sampling, data collection instruments' reliability, validity, trustworthiness and ethical considerations.

Chapter Four – Presentation of Data Results

This chapter presents the study's results, as obtained from the data collection instruments.

Chapter Five – Discussion of Findings and Recommendations

This chapter discusses the findings concerning the existing literature, makes recommendations concerning the purpose of the research, and mentions the limitations of the research and further research avenues.

1.2 BACKGROUND

In South Africa, learners in grades three and six performed poorly nationally on average in the system evaluation. The Trends in International Mathematics Science Study (TIMMS) in 2019 placed South Africa 49th of the 50 countries (Mullis et al., 2020). TIMSS is the most important international study of education, and it is sponsored by the International Association for the Evaluation of Education Achievement. According to TIMMS, approximately 50 countries participated in the study. It can be concluded that South African learners struggle far more than the rest of the world. It was also illustrated in the Annual National Assessment published in 2014 that learners underachieved in mathematics, and the Grade 4 mathematics average was 37 % (DBE, 2014).

Fractions are one of the most problematic areas in mathematics (Kor et al., 2018). Learners' thinking in relation to fractions display several different difficulties in understanding certain concepts and techniques to solve problems; therefore, any study seeking to understand how to improve learners' ability to solve fraction problems must ensure error patterns are well identified and then analysed according to their type; this includes the underlying misconceptions associated with factual, conceptual, and procedural understanding in addition, subtraction, and multiplication. As a mathematics teacher at the Grade 6 level, teaching and learning fractions in the classrooms is challenging; however, the errors are normally easy to spot and can be linked to a lack of basic skills and a misunderstanding of fractions, which results in misconceptions when faced with sums. Misconceptions can be defined as the misapplication of rules in mathematics (Hansen et al., 2020). Learners fail to connect with what they already know and what is being taught. This results in systematic errors (Baur, 2019); learners construct meaning internally by accommodating new concepts within their existing mental knowledge, only to find that information has been wrongly instilled. Knowing how learners commit errors can assist teachers in error analysis, providing opportunities to improve teaching and learning (Singh et al., 2017).

In South Africa, fractions are a topic of mathematics considered by primary school learners to be difficult and by teachers to be one of the most challenging to teach (Novitasari et al., 2018). However, early identification of these error patterns in fractions will improve learners' performance, enable teachers to transmit certain concepts more clearly, and improve attitudes towards mathematics in the classroom. This entails diagnosing the typical error pattern that learners make. Working with learners' errors diagnostically in context can strengthen the cognitive architecture of teachers' mathematics (Hugo, 2015). Since fractions are a complex but significant concept in mathematics, it must be given the proper attention and focus in the classroom (Alkhateeb, 2019).

Errors in fraction sums may occur for various reasons, including the learners' lack of basic knowledge, carelessness, and misunderstanding. In the South African primary school context, the language of instruction and a lack of basic skills related to solving fraction sums based on a tendency to confuse how to handle different operations with fractions tend to dominate. In the lower grades, such as Grade 6, comparing fractions, equivalent fractions and fractions operations appear as the most problematic concepts for learners, thus demanding that teachers understand this content thoroughly. At the level of mathematical method, learners find it hard to understand why they must follow certain rules or procedures to solve a particular problem, or they have been given incorrect information at the foundational level (Gabriel et al., 2013).

When teaching fractions, teachers should be aware of the common error patterns learners make and why they make them; this will ultimately assist teachers in transmitting basic knowledge correctly and help learners become aware of common mistakes and resolve them logically; this is crucial for their ongoing performance in the subject. The National Mathematics Advisory Panel (NMAP, 2008) determined that the knowledge of fractions provides a critical foundation for algebra. Therefore, it is very important that learners feel confident when approaching problems involving fractions, and teachers should take precautions to correct these errors first and arrange the teaching environment accordingly (Akkoyunlu & Soylu, 2006).

1.3 RATIONALE OF THE STUDY

This research identified common error patterns in Grade 6 learners' addition, subtraction, and multiplication of fractions. In addition, underlying misconceptions associated with factual, conceptual, and procedural errors in these three operations will also be analysed. Learning about fractions and approaching them effectively is core to success in mathematics (Siegler et al., 2013). In identifying these errors and analysing underlying misconceptions, the teachers will know the common error patterns and will, therefore, be able to deal with them accordingly. The Department of Education will also develop continual training sessions for teachers so that they will be able to present and explain concepts related to fractions more appropriately and effectively. When teaching mathematics, learners should become independent thinkers and better problem-solvers on their own; thus, a change in teaching methodology may be required (Barnes & Venter, 2008). Ways on how to ovoid some common errors in the South African primary school classroom under study will also be identified.

The topic of fractions is imperative in the mathematics curriculum, and it is learned concept-by-concept from the first grade until the end of the basic stage (Alkhateeb, 2019). In South Africa, according to the Curriculum and Assessment Policy Statement (CAPS) document for Intermediate Mathematics, being able to recognise and utilise equivalent fractions, solving problems involving fractions, calculating fractions, and describing and ordering fractions is emphasised (Department of Basic Education[DBE], 2014). Fractions occupy a central position in mathematics at all levels because they are used in everyday life in cases such as half prices, prices reduced by third, gradients of hills, or the division of food into quantities (Gabriel et al., 2013).

Fractions can also be used when calculating time and in more complex calculations in professions that use rates and percentages. This includes jobs in the health, science, engineering, farming and mechanics sectors (Gabriel et al., 2013). Dr Monica Neagoy, in her book "*Unpacking Fractions: Classroom-Tested Strategies to Build Learners' Mathematical Understanding*" offers three important reasons for teaching fractions in the United States. Firstly, fractions play a key role in activities such as recipes, calculating discounts, comparing rates, converting measuring units, reading maps, and investing money. Secondly, fractions are the foundation of algebra. Lastly, fractions also play a crucial role in learners' attitudes to mathematics.

According to the United States' National Mathematics Advisory Panel (2008), understanding fractions enhances learners' performance in mathematics and the skills acquired through fractions are a foundation for algebra (National Mathematics Advisory Panel, 2008). When learners understand fractions conceptually, it is easier to grasp algebra and geometry (Dogan-Coskun, 2019). Learners must be encouraged to become better problem-solvers as well as mathematicians to enable learners to become more independent thinkers; this demands a change in teaching methods (Barnes & Venter, 2008). Learners need to understand the relationship between a numerator and a denominator to accept fractions as quantities different from whole numbers (Petit et al., 2015). In analysing the common error patterns in Grade 6, learners answer questions involving fractions, and ways to overcome these errors may be identified; consequently, this study fulfils a strong educational need in South Africa.

1.4 STATEMENT OF THE PROBLEM

South African learners start engaging with fractions in the elementary grades. When preschool children develop mathematics knowledge, it may predict their success at the elementary level. Grade 6 is the intermediate phase, which exits the intermediate phase to the senior phase. The Annual National Assessment Diagnostic Report of 2012 found that when given two mixed numbers to add, learners tend to add the numerators and denominators separately to get an answer (Department of Basic Education, 2012); Figure 1,1 depicts an example from the report.

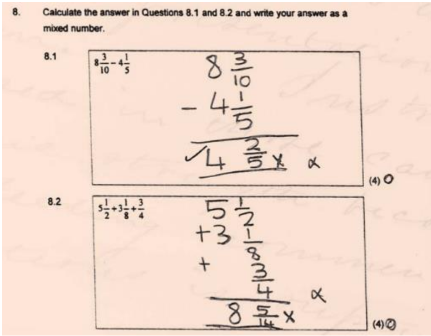


Figure 1.1: Example of learners adding numerators and denominators

The examples show that learners cannot add mixed numbers. In general, however, many learners have problems understanding the topic of fractions and learning about them (Dogan-Coskun, 2019). Teachers do not seem to understand the main reason for learners' poor performance in fractions and, therefore, require skills in developing strategies to enhance student learning. As a Grade 6 mathematics educator, I have discovered that learners have problems in three operations: addition, subtraction, and multiplication of fractions. When adding fractions, learners add the numerators and the denominators; for example, in addition, they incorrectly find $\frac{3}{7} + \frac{2}{7} = \frac{5}{14}$. When subtracting the fractions, learners subtract both numerators and denominators, incorrectly finding $\frac{7}{9} - \frac{3}{9} = \frac{4}{0}$, and when multiplying, learners incorrectly use the addition rule, finding $\frac{5}{9} \times \frac{4}{11} = \frac{9}{20}$. The study identifies common error patterns in addition, subtraction, and multiplication of fractions among a selected group of Grade 6 primary school learners in Gauteng. The most common mistake made by learners is that they add the numerators and the denominators together in solving the addition of fractions, meaning that learners lack the conceptual knowledge about reasoning in the addition of fractions, for example, incorrectly finding that $\frac{5}{9} + \frac{3}{27} = \frac{8}{36}$

(Lestiana et al., 2017). Therefore, the researcher wanted to understand the learners' misconceptions in response to their answers when adding, subtracting, and multiplying fractions, which would help educators develop different strategies when dealing with addition, subtraction and multiplying of fractions.

Dealing with fractions as a mathematics subject is viewed as greatly challenging (Karao Yilmaz et al., 2018). For problems that require the addition of fractions with the same denominator, learners add statements to the numerator and write them in the numerator and add statements to the denominator and write them in the denominator (Aksoy & Yazlik, 2017). It seems that when learners are required to answer a problem that needs multiplication with fractions, they grasp that multiplication is necessary, but they choose to do addition (Aksoy & Yazlik, 2017). Most learners are good at adding fractions with a common denominator but have problems with fractions with different denominators (Makhubele, 2021). The same study found that most learners are proficient at subtracting fractions with a common denominator but struggle with subtracting fractions with diverse denominators. (Makhubele, 2021). Ubah and Bansilal concluded that most pre-service and in-service teachers in primary schools in South Africa declared that fractions are one of the most challenging and complicated concepts for them to teach and for learners to understand (Ubah & Bansilal, 2018). This viewpoint means that from the intermediate up until the senior level, fractions seem to be a challenging concept in the mathematics classroom, and yet, the inability to master it adequately affects learners' overall performance in mathematics. Through this study, the researcher and educators teaching mathematics will be able to understand the underlying misconceptions associated with factual, conceptual, and procedural errors in fractions. The foundational concepts in fractions play an important role in algebra and advanced mathematics and thus must be grasped if learners are to pursue technological careers (Barbieri et al., 2020).

Knowledge of fractions in early grades will also help learners develop other mathematical skills such as algebra, geometry, and statistics (Siegler et al., 2013). However, it is often the case that learners don't know why they must follow specific rules in fractions. This brings us to conceptual, factual and procedural errors. It has also been perceived that learners also make mistakes when reading fractions (Karao Yilmaz et al., 2018). On a conceptual level, learners' primary complications in fractions is when the sum of parts is not equal to the whole, and procedural knowledge relating to comparing fractions (Shahbari & Peled, 2017). When analysing common error patterns, the researcher will apply error analysis to achieve different results.

The findings and recommendations of this study aimed to enable the education system to find appropriate teaching methods or strategies for teachers to overcome the common error patterns in addition, subtraction and multiplication of fractions. This includes the misconceptions associated with factual, conceptual and procedural errors in addition, subtraction and multiplication of fractions. In formulating strategies, it is hoped that teachers will increase their knowledge of fractions as a whole; they should also know the types of errors and the underlying misconceptions of these errors. Though fractions have already been investigated in other studies, little has been done to overcome the error patterns made by learners in fractions. The most important issues to focus on are problem-solving in fractions, teaching methods and fraction operations.

1.5 RESEARCH QUESTIONS

The following were the research questions.

1.5.1 Main Research Question

- What are common error patterns in addition, subtraction and multiplication of fractions among the selected group of Grade 6 learners in a public primary school in Gauteng?

1.5.2 Research Sub-Question

The sub-research question which guides the main question is as follows:

- What are the underlying misconceptions among these selected Grade 6 learners associated with the factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions?

1.6 OBJECTIVES OF THE STUDY

1.6.1 General Objective of the Study

- The study's general objective was to identify the common error patterns in the addition, subtraction and multiplication of fractions among the selected Group of Grade 6 primary school learners in Gauteng.

1.6.2 The Specific Objectives of the Study

- The study's specific objective was to explore the underlying misconceptions among the selected Grade 6 learners associated with factual, conceptual, and procedural errors in the addition, subtraction, and multiplication of fractions.

1.7 CLARIFICATION OF KEYWORDS

Several concepts that were used in the study will be briefly explained below. The definitions explain the meaning of the concepts within the content of the study. Some of the words are the researcher's own explanation.

- Common – something that is done or occurs often (Dictionary Unit for South African English, 2002)
- Error – is an action which is inaccurate or incorrect (own definition). It can also be explained as a human error due to a characteristic of psychological processes and perception (Woods et al., 2017).
- Fraction – is a number that represents a whole number that has been divided into a number of equal parts. This number consists of two parts: the numerator and denominator. Fractions can also be defined as several equal parts of a unit out of the total number of equal parts into which the unit has been divided. (Lamon, 2020)
- Error analysis - is the study of errors in learners' work to explain the reasons behind their mistakes. (Richards, 2015)
- Pedagogical content knowledge (PCK) – PCK is teachers` interpretations and transformations of subject matter knowledge in the context of facilitating student learning (Banks, 2019)
- Cognitive architecture – Cognitive architecture is a hypothesis about the fixed mental structures that enable mental systems to work with the necessary resources to attain intelligent behaviour when faced with a complex problem (Samsonovich, 2020)

- Misconceptions are incorrect views based on faulty thinking or understanding (Veloo et al., 2015).

1.8 RESEARCH METHODOLOGY

1.8.1 Research Approach

The qualitative research approach was used in this study. Thus, data collection, analysis, and report writing differed from the traditional, quantitative approach and were inductive, emerging and shaped by the researchers' experience in collecting and analysing the data (Creswell, 2014). Qualitative research studies behaviour as it occurs naturally (McMillan & Schumacher, 2014). The reason for the proposed research approach is that participants were studied in a closed but normal classroom setting. The researcher taught the Grade 6 learners mathematics for three years and could see that there was a problem in mathematics, especially with fractions; this affected all learners, including those who were top achievers in Grade 6. Therefore, in-depth knowledge and understanding of this problem is imperative. Furthermore, the chosen approach offered a unique advantage, allowing the researcher to explore deeper insights and identify the error patterns that might be overlooked. A few learners were chosen as participants with long-lasting contact to accurately identify the common error patterns made by learners in primary school. Learners had to explain their answers based on the incorrect answers they obtained.

The approach explored learners' attitudes, behaviour and experiences dealing with addition, subtractions, and multiplication. The chosen qualitative research approach employed terms such as credibility, transferability, dependability, and conformability. Arora and Stoner (2009) stated that researchers want to collect richer information and get a more detailed picture of issues and events related to the research objective using qualitative research. The qualitative method seeks answers to questions and involves the collection of evidence to answer the research question (Arora & Stoner, 2009).

1.8.2 Population and Sampling

The objective of the current study was to explore the underlying misconceptions among the selected Grade 6 learners associated with factual, conceptual and procedural errors in addition, subtraction and multiplication. For data collection, purposive sampling was used to select learners according to their academic performance in mathematics, specifically as a subject. The population consisted of 15 Grade 6 learners, boys and girls, aged between 10 and 12 years. As far as inclusion criteria were concerned, from the 15 learners selected, five were high achievers (HA), scoring between 70% and 100%; another five were average achievers (AA) scoring between 45% and 69%; and finally, five were lower achievers (LA) achieving between 45% and 0%. The researcher used this specific participation selection to avoid bias, ensure representative sampling, maintain openness about the purpose and criteria of the study, help build trust and encourage learners' participation. Due to the number of participants, in addressing the issue of biased conclusion, the researcher used a purposive sampling guided by the SASAMS, which has records of all grades, specifically with learners' performance; this also enhanced the reliability and validity of the study.

1.8.3 Research Design

A research design can be defined as the researcher's plan to investigate the data to answer the research questions best. The research approach employed in this study was the descriptive case study design. The case study design is a type of research inquiry that examines a real-life contemporary phenomenon (Thomas, 2021), the approach to gain a holistic understanding of the learners' perspective on fractions. This approach allows the researchers to collect a variety of data, in this case, the written tests with ten questions that included addition, subtraction and multiplication of fractions. Another approach used in this study was interviews based on the incorrect answers from the learners. The researcher obtained an in-depth look at an individual (Schoch, 2020). A case study is also defined as the in-depth exploration of a specific system with the help of different data collection tools (McMillan & Schumacher, 2010).

Therefore, the researcher explored and analysed fraction error patterns using learners' written tests and semi-structural interviews. The researcher chose this design because the evidence was collected systematically through written tests with ten questions and interviews based on incorrect answers. The written tests were scrutinised, and the incorrect answers from the test were analysed to identify common error patterns occurring in Grade 6 learners' responses. The learners were interviewed based on their incorrect answers, which helped the researcher obtain diverse perspectives and deepen the understanding of learners' misconceptions. The researcher captured the complexity of the fractions, resulting in a thorough analysis.

1.8.4 Instrumentation and Data Collection Techniques

Data was collected through questions included in their classroom activities. As mathematics educator, I used the “Mathematics Platinum Grade 6” learners’ book to develop the questions and enhance the test's accuracy and consistency. I ensured the validity and reliability of the content, which was important for the test to be effective and reliable. The study's general objective was to identify the common error patterns in addition, subtraction, and multiplication of fractions among a selected group of Grade 6 primary school learners in Gauteng. The questions formulated included addition, subtraction and multiplication of fractions. The test had addition and subtractions of three fractions with different denominators, addition and subtraction of two fractions with the same and different denominators and multiplication, and lastly, multiplication of fractions. Learners were given tests and worksheet forms to answer questions based on fractions, and showed their workings on the worksheet.

The 15 learners were put in one class and allocated one hour to answer the questions. The researcher used a one-hour period in a day for the learners to answer the questions. The worksheets from the 15 learners were marked and compared to group the common errors in categories. After determining the findings from the HA, AA and LA categories, learners were also interviewed based on the incorrect answers to identify their misconceptions. Responses were recorded, and errors were grouped according to factual, conceptual and procedural errors in addition, subtraction and multiplication of fractions and the interview responses.

1.8.5 Data Analysis and Interpretation

The general objective of the study was to identify the common error patterns in addition, subtraction and multiplication of fractions among a selected group of Grade 6 primary school learners. Furthermore, the researcher aimed to explore the underlying misconceptions among the selected learners associated with factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions. As explained in the previous section, the data collections were done through written test and interviews. The researcher used content analysis to gain valuable insight into identifying these common error patterns and underlying misconceptions, including examining the response's content to identify the underlying learners’ misconceptions. The researcher marked the learners’ responses, analysed those with incorrect answers and categorised errors based on their types. The ones with incorrect answers were grouped according to factual, conceptual and procedural errors in addition, subtraction and multiplication of fractions. Learners with incorrect answers were interviewed for the researcher to understand their perspectives. The learners were allowed to use their own home language because the current language of teaching and learning (LOLT) can be challenging to some learners.

1.9 ETHICAL CONSIDERATIONS

This investigation involved people; therefore, the study must adhere to the moral principles of ethics stipulated by the University of South Africa.

The researcher made sure that all the individuals' rights were respected. The researcher made sure that the nature of the study was disclosed to the participants, and they were informed of the benefit of the study, as well as being given the opportunity to ask questions. The participants were given consent forms with explanations therein. The learners were also given assent forms since the study was based on Grade 6 age learners, while their parents signed consent forms on their children's behalf. The researcher made sure to minimise and reduce the risk of harm to the participants by conducting the research during school hours and in one of the classes. The researcher ensured justice was done to everyone; 15 learners were selected using the purposive sampling method to ensure fairness.

The researcher ensured that there was no harm (physical, emotional or psychological) to the participants; this included giving learners the right to withdraw at any time in the data collection or research process. The researcher also allowed the learners the right to use the language of their preference. The following documents were used for the participants:

- Assent form for learners
- Consent form for parents
- Institutionally approved documents from the school
- Ethical Clearance from the University of South Africa

1.10 HOW MY INTEREST IN THIS RESEARCH TOPIC EVOLVED

Mathematics has always fascinated me for its logic and problem-solving skills. During my teaching experience, I realised that the topic with the most challenges in Grade 6 mathematics was the topic of fractions. Among various mathematical concepts, fractions are challenging to all learners, including those who perform on higher levels. I became interested in knowing more about the errors made by learners in fractions so that I could change the strategies used in teaching and learning fractions. The complexity and use of fractions in real situations piqued my curiosity. As mathematics education progressed, the complexity of fractions increased. The need to understand different methods used in addition, subtraction and multiplication of fractions, including mixed numbers or simplifying fractions, introduced new challenges. I noticed that many learners struggled with fractions due to various conceptual, procedural and calculation errors. Witnessing this problem made me want to find ways to solve these difficulties and promote learning. The need to find answers to conceptual fractions, student perceptions and resulting misconceptions motivated me to undertake this research, hoping to find solutions to the challenges of working with fractions. I am committed to advancing my research, contributing to the field, and ultimately improving mathematics education for all.

1.11 CHAPTER SUMMARY

In Chapter 1, the researcher presented the orientation of the study and an outline of the chapters. The most important aspect of this chapter was to provide a clear motivation for the study and set out the research context and objective. The research method was described, as well as the ethical considerations of the study. The researcher also briefly highlighted the main problems primary school learners face in solving fraction sums as a basis for further discussion. Lastly, the researcher discussed how she developed her interest in the current study.

CHAPTER 2
THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 INTRODUCTION

This section reviews some of the literature available on common error patterns in addition, subtractions and multiplications of fractions made by primary school learners. This section will discuss existing research and arguments related to common errors. An assessment of relevant literature is important for the present research (Snyder, 2019) since it provides collective evidence on common error patterns and misconceptions associated with factual, procedural and conceptual errors in addition, subtraction and multiplication of fractions. Theories underpinning the study are the constructivism learning theory, the behaviourism learning theory, the cognitive learning theory, the development learning theory and the humanistic learning theory. The section starts with the conceptualisation of fractions and then defines mathematics as a subject. These arguments are then followed by fraction definitions as this lays a foundation for the study and then touches on the evolution of diplomatic fractions, common errors in fractions and error analysis.

2.2 THEORETICAL FRAMEWORK

To comprehend the data acquired as part of the research study, the researcher employs a combination of theories relevant to the phenomenon under investigation. This combination of theories is known as the theoretical framework. It may comprise theorists' approvals for concluding the inquiry, or it may instead interpret the conclusions drawn from the data (Kivunja, 2018). Additionally, the theorists' recommendations can help the researcher conduct a more thorough analysis of the information gathered, analyse and present the results, and equip the researcher with crucial abilities needed to draw recommendations and conclusions (Kivunja, 2018). According to Osanloo and Grant (2016), the theoretical framework is the overall study's plan that directs, structures and supports the research while also laying out the researcher's methodology.

This study aimed to investigate the prevalent factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions among selected Grade 6 learners as well as the underlying misconceptions among the selected Grade 6 learners. The study was inspired by the researcher's observations of how learners struggle with fractions, the misunderstandings surrounding fraction addition, subtraction, and multiplication, and the general belief that this impacts learners' mathematical understanding as they progress through the grades. As a result, the researcher sought to uncover the common mistakes made by elementary school learners and the underlying assumptions that lead to factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions. Additionally, the researcher sought to discover strategies for addressing these misconceptions. Constructivism has been a pillar of the psychology of mathematics instruction largely due to Jean Piaget's impact.

Constructivism Learning Theory

This study is supported by Jean Piaget’s constructivism theory. Jean Piaget significantly inspired constructivism as a key ideology in the psychology of mathematics instruction (Paul, 1993; Piaget, 1960). According to Piaget, internal processes give people the capacity and desire to learn; hence, the human mind develops schemata (Simon, 2001). According to constructivism, learning occurs when a person engages with their surroundings to create their own knowledge. The learner is given the chance to investigate mathematical ideas and gain knowledge of them through practical exercises that follow the constructivist methodology. According to constructivist theory, the learner is an active creator of knowledge rather than a passive recipient of it (Olivier, 1989). Having prior experience with fractions means that a learner in Grade 6 can distinguish between what they comprehend and what they don't. In Grade 3, learners are given their initial introduction to fractions. They discover that fractions are numbers on a number line and that they divide a whole into equal parts. This material has been covered in other grades, beginning in third grade and continuing through sixth grade. Schemata are previously constructed notions seen as being connected to one another. Schemata are ideas or classifications that alter or adapt as the mind develops (Simon, 2001). Constructivists contend that learning should be seen as altering learners' schemata rather than their behaviour. According to constructivist theory, the learners are not submissive receivers of knowledge but rather active participants creating their knowledge (Olivier, 1989). Having existing knowledge of fractions means that Grade 6 learners can understand what they comprehend and what is beyond their understanding. Fractions are formally introduced in Grade 3, where learners learn that fractions are numbers on a number line and that they partition the whole into equal parts.

This is the knowledge which is carried from Grade 3 up until Grade 6 throughout other grades. Previously created concepts have been described as schemata that tend to be interrelated. Schemata are concepts or categories that adapt or change with mental development (Simon, 2001). The constructivists believe that learning cannot be viewed as a change in learner behaviour but rather a change in learners' schemata.

According to Piaget, assimilation and accommodation play a role in adaptation and organisation. Adult schemata differ from child schemata through the processes of adaptation and organisation; therefore, the intellectual growth process of both production and reconstruction is ongoing (Wardsworth, 1996). This procedure, in Piaget's view, involves assimilation and accommodation. Assimilation occurs, for instance, when a learner constructs knowledge and incorrectly treats the numerator and denominator as separate numbers in the following grade, contradicting what they have learned in earlier grades. This happens when a teacher explains the distinction between the numerator and denominator to a student. Sometimes, a different teacher needs to remind learners of the numerator and denominator, which they should understand fully from prior learning. As defined by Piaget, this accommodating process is how knowledge is produced. Misunderstandings are significant from a constructivist perspective because learners use their prior knowledge to comprehend new information. The risk is that, as mentioned, it might be confusing and deceptive to layer further new information on top of current knowledge.

This study focused on identifying common errors made by Grade 6 learners when adding, subtracting, and multiplying fractions. As a result, the constructivism theory suggests that as the learner actively participates in the construction of their knowledge, they can construct their own understanding and knowledge through experiences with fraction addition, subtraction, and multiplication. Piaget's idea of assimilation, according to Mohammed and Kinyó (2020), refers to the incorporation of new knowledge concerning previously known concepts or facts. This theory indicates that the newly absorbed knowledge is combined with the learner's prior knowledge. As a result, learners will have misconceptions that cause them to make factual, procedural, and conceptual errors when adding, subtracting, and multiplying fractions.

Constructivism is an epistemology with philosophical and psychological underpinnings. According to Piaget (1960), cognitive function is adaptive, and aids in organising the world of experience since knowledge is not passively received but actively constructed by the cognising subject. Consequently, learning is the process of gaining knowledge through experience. Learners know that when they must divide a pizza into portions for practical purposes, they must determine how many portions they will need to divide and distribute between them. As a result, when learners divide a pizza into equal portions, their experience and their concepts of fractions interact to produce knowledge and meaning. Constructivist learning is a personal endeavour because internalised ideas, guidelines, and general principles can be continuously used.

In addition to factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions, this study also highlights the underlying assumptions that Grade 6 learners have that lead to these errors. The term "mathematical misconception" refers to incorrect beliefs that learners have about mathematics due to unclear concepts, facts, or learning techniques (Kshetree et al., 2021). According to Im and Jitendra (2020), such misunderstandings could result from past exposure to unsuitable generalisations. Throughout my years of teaching, I've observed that learners learn best from their mistakes. They first attempt to solve a problem on their own using prior knowledge and make mistakes. After realising why they chose an incorrect answer, you should give them the proper procedure, facts, or concepts and explain how you arrived at the solution.

Constructivism holds that misconceptions are crucial to the teaching and learning process since they result from constructive conflicts that arise when developing new mathematical concepts (Kshetree et al., 2021). Such misunderstandings could result from past exposure to unsuitable generalisations (Im & Jitendra, 2020). Since learners construct their own knowledge, which results in misunderstandings, the notion of constructivism is crucial to this study's findings. According to constructivist theory, learners learn best when they try to grasp something on their own, using the teacher as a guide. According to constructivism, every student is viewed as an individual with specific needs and a complex past. Because of their individuality and complexity, teachers are encouraged to use them as a crucial component of the learning process. Minarni and Napitupulu (2020) concluded that constructivism-based learning improves mathematics skills, including connection, understanding, problem-solving, and creative thinking. Before introducing concepts, procedures, and methods for any operation while teaching fractions, a teacher must give learners time to reflect and build their own methods using their past knowledge.

The central idea of constructivism is that human learning is constructed and that learners construct new knowledge based on previous learning (Bada & Olusegun, 2015). Piaget's main focus of constructivism is on the individual and how the individual constructs knowledge; thus, the cognitive constructivism theory of Piaget proposes that humans construct their own knowledge, which they understand and use, with no given information (Piaget, 1952). It is in the construction process whereby misconceptions can occur, forming part of this study. Learners construct knowledge in solving problems by using procedures or concepts learned in addition, subtraction and multiplication of fractions. Naylor and Keogh state that the central tenets of constructivism are that learners can only understand new situations based on their existing understanding, and therefore, learning is an active process in which learners construct meaning by connecting new ideas with their existing knowledge (Naylor & Keogh, 1999, p. 93). Reconsidering mathematics can change learners' existing schematics by applying knowledge that learners already have. This application can make them vulnerable to misunderstandings. The constructivist approach is a successful method that provides the learner with meaningful learning experiences in the classroom (Brooks & Brooks, 1999).

According to Jean Piaget, learning occurs in processing information rather than reacting to an external stimulus. This viewpoint brings us to the theory related to constructivism, which partially contributes to the current study, namely cognitive learning theory. One of Piaget's stages in child development is cognitive development, and he is one of the key cognitive theorists who studied a child's mind (Clark, 2018). The study identified ways to avoid errors. Learners construct new ideas using existing knowledge and then process and reorganise information within a matrix. Cognitivists are aware of the role of environmental conditions in learning, but teacher explanations and conceptual demonstrations serve as environmental inputs to learners. Learning will be promoted by the practising of skills and correct feedback. Cognitive learning theorists emphasise the acquisition of knowledge and skills, forming mental structures, and processing information and beliefs (Schunk, 2012). The most important aspect is what learners do with information and how they attend to rehearsing, transforming, coding, storing, and retrieving new skills and knowledge. Cognitivists assume that an existing knowledge structure must be present to compare and process incoming information for learning (McLeod, 2003). The fact is that current learning builds upon the previous knowledge gained on any topic; this means that learners' prior knowledge is a key consideration before launching new concepts in the classroom. Therefore, this research studied the error patterns of Grade 6 learners in addition, subtraction and multiplication of fractions. In addition, the study also identified misconceptions associated with conceptual, procedural and factual errors in addition, subtraction and multiplication of fractions.

In general, learning theory assists teachers in many ways in developing a curriculum. These theories support teachers in setting educationally achievable objectives in their classes. Tyler (1949) stated that teachers can use educational psychology to monitor curriculum aims and then use them in curriculum planning to increase learning levels and adapt specific content to achieve specific educational objectives. Teachers use pedagogical learning theory to solve student challenges, such as whether to use punishment; moreover, knowledge of educational psychology helps teachers determine the type of knowledge that must be acquired, how to transmit it, and how this can be achieved when planning the teaching.

2.3 LITERATURE REVIEW

2.3.1 Conceptualisation of Fractions

Introducing fractions is a student's first experience in mathematics beyond the basic skills of addition, subtraction, and multiplication. For this purpose, this study aimed to identify common error patterns in fractions to improve the poor performance of mathematics in South African primary schools. Hasemann (1981) discovered that most learners solve problems by applying rules they can remember, but they cannot understand what they are doing. Understanding fractions is an important building block for understanding mathematics skills, and once this principle is understood, it becomes easier to understand other complex mathematical concepts, such as algebra (Barbieri et al., 2020).

2.3.1.1 What is a Fraction?

Steffe et al. (2013) define fractions as a number in the form of $\frac{a}{b}$ (a numerator and b denominator), where both numbers can be integers, and the denominator cannot be zero. Fraction derives from the Latin *fractio*, which means "to break into pieces," and denotes "part of a whole" (Long, 2001). Long further explains that the denominator reveals how many identical elements there are in the whole; the entire is divided into four equal parts if the denominator is 4 for $\frac{1}{4}$. The number of pieces being used is shown in the numerator; when the numerator is 2, for example $\frac{2}{4}$, there are two parts. Since fractions are numerical representations of parts of a whole number, the fraction concept also involves understanding non-numerical concepts and entails understanding the relations between the numbers in a fraction and, later, between numbers in different or equivalent fractions. The definition of a fraction adopted by this study is that a fraction is a part-whole, which refers to quantities divided into equal parts, where the difference between the number of selected equal parts and the number of components shall be expressed in fractions (Wijaya, 2017). The definition of fraction as explained alone makes it hard for learners to understand the notion of fractions. Therefore, learners must understand the meaning and form of fractions before doing fractional operations. Learning to plan fractions with realistic math instruction in elementary school

The meaning of a fraction refers to knowing that the numerator and denominator of a fraction work together to determine its size, not either number separately (Dyson et al., 2020). In reflecting on how learners are taught fractions, Steffe et al. (2013) emphasised different sets of elements whereby some elements are shaded while others are not. In their investigations, Spinillo and Bryant (1991) asked children to compare a model figure, whereby one has a dark side, and the other has a white side, with the same ratios and shades (Figure 2.1).

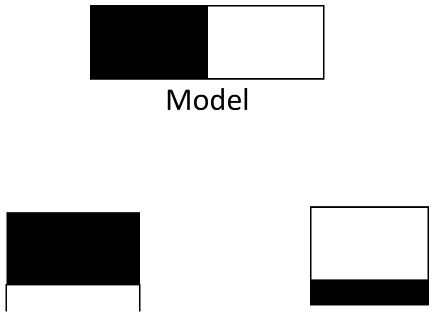


Figure 2.1: Comparison figures with fractions shaded.

When presented with the figures above, children can compare the ratios of the dark-coloured region successfully in solving problems, but they cannot compare the pair of inequalities in the same region. The model also showed the importance of children's proportional judgements, proving that there is a link between fractions and understanding proportions and ratios. Fractions are very important and play a central role in mathematics; however the theories in whole number development do not integrate fractions and whole numbers in one framework (Siegler et al., 2013).

2.3.1.2 Definition of Fractions

The concepts relating to fractions can be interpreted in several different ways, but they are generally known as sub-constructs. Charalambous and Pitta-Pantazi (2007) used modelling techniques to confirm these sub-constructs, which they named part-whole, ratio, quotient (division), measurement and operator. The following sections define the concepts known as sub-constructs relating to fractions.

Fractions as a relationship between part and the whole

Part-whole: Fractions are commonly used for representing part of a whole amount in everyday language. According to Wijaya (2017), fractions are considered from a part-whole point of view to give them meaning. Indeed, fractions in primary school are primarily defined as parts of a whole, whereby a whole can be anything divided into equal parts (Čadež & Kolar, 2018). This is the definition used most often by teachers when introducing the concept of fractions. A whole is divided into different equal parts, and if one part is selected from those equal parts, then we say that part is part of a whole (part-whole) – this is illustrated in Figure 2.2.

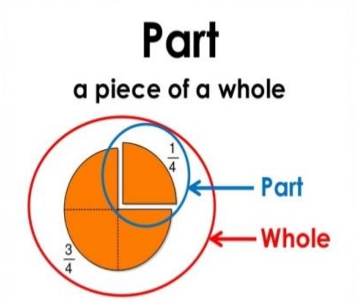


Figure 2.2: The concept of a fraction as part of a whole

When the total number of equal parts in the form matches the fraction denominator, pictorial representations of the part/whole area and measure models are referred to as "simple representations" (Wong & Evans, 2007), this is illustrated in Figure 2.3.

Figure 2.3: The concept of a fraction as part of a whole – simple presentation

Learners here count the parts whereby the shaded part is the numerator, and all the shaped parts are associated with the denominator.

Fractions as ratios

Ratio: The concept of a fraction as a ratio means that a fraction is the numeric presentation of a comparative relationship with the same or different quantities (Van de Walle et al., 2014). In $\frac{3}{2}$, we can say three as to two. Ratios can be expressed in two different ways. Part-part can be explained as the ratio of boys to girls in class, and part-whole can be explained as the ratio of boys to the whole class (Basturk, 2016).

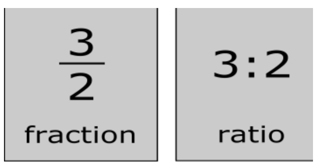


Figure 2.4: Fractions as an expression of a ratio

Fractions as operators

The concept of operators defines fractions in terms of their multiplicative operations. This definition can develop a conceptual understanding of fractions in their relationship to whole numbers or other fractions, mostly involving multiplication (Charalambous & Pitta-Pantazi, 2005). This means a fraction can cause a whole number to be enlarged or reduced by a certain quantity (Figure 2.5). For instance, we can take a picture and enlarge or reduce it in real life.

$$\frac{1}{2}\ of\ 24$$

Figure 2.5: A fraction as an operator

Fractions as measurement

Measurement: Fractions as a form of measurement determine the unit of measurement and divide the unit into smaller parts (Dogan-Coskun, 2019). Using fractions in this way can identify the measurement of an object to determine its overall length. The measurements here can be derived when the length is subdivided into smaller equal parts as seen in Figure 2.6.

Figure 2.6: Fraction as units of measurement

Fraction as quotient

The concept of a fraction as a quotient means that the fraction represents division; in other words, it is a number representing a numerator over a denominator (Dogan-Coskun, 2019), and its value is derived from dividing the numerator by the denominator. Thus, the outcome when it is a fraction represents a numeric value, rather than a representation of equal divided parts (see Figure 2.7).

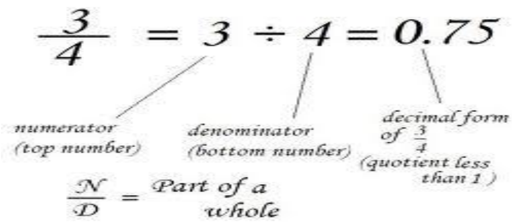


Figure 2.7: Fractions as quotients of division sums

This research focused on identifying error patterns of fractions made by learners actions in addition, subtraction and multiplication; the study also identified the misconception associated with factual, conceptual and procedural errors in the three mentioned operations. The above-mentioned sub-constructs were taken into consideration; however, the study provided few illustrative examples since the complete description of each is beyond the scope of this research.

2.3.2 Evolution of Fractions

In South Africa, basic fractions-related concepts are introduced in Grades 1 and 2 when learners are simply taught how to cut objects into equal parts. The concept of fractions becomes more formal in Grade 3 when learners learn about numerators, denominators, and definitions of the part-whole aspect. In Grade 4, learners begin working with simple fraction sums, adding fractions with the same denominator and different numerators. In Grade 4, learners then master comparing the magnitude of fractions using relational concepts such as greater than, less than and equal to; this also includes mixed fractions. After learners have solidified their understanding of whole numbers by the fourth grade, fractions become a primary focus of the mathematics curriculum (Namkung et al., 2018). In Grade 5, concepts such as equilateral fractions, adding, subtracting and multiplying fractions with the same denominators are explained to learners. Grade 6 learners used most of the concepts taught in the previous grade and now also deal with fractions with different denominators. Since the basic concepts of fractions form the cornerstone of learners' understanding of fractions, it is crucial to pay more attention to them in the early grades (Wijaya, 2017).

2.3.3 Importance of Fractions

Fractions are key parts of our everyday life. Fractions play a role in cases of half prices, prices reduced by a third, the gradient of hills or measuring the division of quantities of food, and even when stating the time, for instance, quarter to, half past, quarter pass etc. Mathematics is normally known as a discipline comprising a hierarchical structure of understanding, made up of consecutive, interfacing concepts; one of the most important concepts is that of fractions (Basturk, 2016). An understanding of fractions assists learners in knowing the nature of numbers and their interaction. Thus, fractions should be taught and learned by laying more complex concepts onto more simple ones meaningfully and in an articulated way.

Neagoy (2017) offers three important reasons for teaching fractions in the United States: they play a key role in activities such as recipes, for example $\frac{1}{3}$ cup of sugar; calculating discounts, for example, a 50% ($\frac{50}{100}$) decrease in shoes; comparing rates, for example, 5 miles per 3 hours; converting measuring units, for example, converting 100 mm to centimetres; reading maps, for example, distribution of natural vegetation and cultivated land; investing money, for example, investing $\frac{1}{4}$ of salary every month. Secondly, fractions are the foundation of algebra; for example, in solving for $\frac{2x}{y} = 10;\ y = 2$. Lastly, fractions also play a key role in learners' attitudes to mathematics. Moreover, fractions play a central role in the workplace, especially in fields involving numeracy and concepts such as percentages, rates, error rates, interest rates, employment rates, production levels, and so on. According to Hess and Handel (2016), 68% of adults at work use fractions and other rational numbers in their jobs. Fractions are also a standard reference for measurement or counting in units, whether in centimetres and cents, to basic amounts necessary for accurate food measurement (Tucker, 2008). The current study identifies misconceptions associated with factual, conceptual and procedural errors in addition, subtraction and multiplication of fractions.

Learners who are competent in fractions can better grasp advanced mathematics aspects such as algebra and calculus (Soni & OKamoto, 2020). Similarly, the National Mathematics Advisory Panel (2008) concluded that a strong foundation in fractions will help learners to succeed in mathematics. If one lacks an understanding of fractions, this will cause one not to understand mathematics, and it is more likely that one will ultimately lose interest in mathematics (Smith, 2002). Francis Fennell, President of the National Council of Teachers of Mathematics (2007), states that learning more advanced mathematics requires a fraction skill foundation, and fractions provide the best introduction to algebra in the middle school years. Learning fractions is an important skill in Grade 6, and learners who fail to learn fractions in the intermediate grades should be identified and assisted accordingly (Barbieri et al., 2020).

2.3.4 Mathematics

Mathematics is an intellectual system consisting of structures and correlations, including consecutive abstractions and generalisation processes (Karao Yilmaz et al., 2018). The Department of Education's (DoE) Revised National Curriculum Statement (DoE, 2002, p. 4) defines mathematics as a human activity involving observing, representing, and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects. Mathematics uses specialised language that involves symbols and notations to describe numerical, geometric, and graphical relationships. Through this definition, one may realise the goal of mathematics and its value in a child's life. Most importantly, understanding mathematical concepts and the value of mathematics at school lays a foundation for a successful knowledge of mathematics. Moreover, the Trends International Mathematics and Science Study's (TIMSS) findings indicate that most learners still struggle to solve mathematical problems. The concept of fractions in mathematics particularly, has been found to be difficult to teach and learn. Learners must grow and connect their conceptual and procedural knowledge to be competent in mathematics (Al-Mutawa et al., 2019). Furthermore, mathematical representations of fractions greatly improve the knowledge of fractional notions and the interpretation and solution of problems involving fractions (Supandi et al., 2018).

Fractions proficiency is required for a learner to approach more difficult and advanced mathematics successfully (Barbieri et al., 2020; NMAP, 2008). This study is proof that learning mathematics without recognising and addressing learners' misunderstandings and mistakes cannot be meaningful, and as a result, misunderstandings and mistakes are often only repeated. In addition, they constantly adjust new concepts to accommodate previous misunderstandings, which turns the entire learning process into a channel of misunderstandings and mistakes. (Mukanda et al., 2021). In all the studies, an ongoing need emerges to identify and analyse learners' problems related to fractions in mathematics. Tian and Siegler (2017) discovered that learners with mathematical difficulties lack understanding from middle school and lag behind the typical achievers in the same grade. Children’s understanding of fractions will improve their knowledge, skills and achievement in mathematics (Tian & Gunderson, 2020). According to Hasler and Akshoomoff (2019), the International Mathematics Union (IMU) and the African Mathematical Union (AMU) both stressed the need for every child to possess fundamental mathematical abilities in their studies of the value of math education. According to the 2015 TIMSS report, 61% of South African Grade 5 learners could not complete basic maths operations, making them the nation's learners with the worst performance among the participating nations. If learners become proficient in mathematics, they can understand and apply complex mathematical concepts such as fractions in their daily lives. Mathematical knowledge is important for teaching and learning mathematical concepts. Mathematical competence may be defined as the necessary skills to utilise mathematical concepts in lifestyle and tertiary education (Aytekin & Şahiner, 2020). In addition, its common content knowledge refers to the mathematical knowledge and competence that any well-educated individual, and certainly all mathematicians, holds and uses in contexts other than teaching (Scheiner et al., 2019).

The mathematical knowledge required for teachers to demonstrate mathematics was defined in two basic categories by Loewenberg Ball et al. (2008): common content knowledge and specialised content knowledge (numbers, processes and models, functions and algebra). Kholmatova (2020) stated that educational content should be presented in a way that maintains linkages between concepts, subjects, and sections within a single subject, as well as academic connections (Kholmatova, 2020), thus making mathematical connections when presenting lessons will assist learners in understanding the concept of concern.

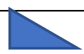
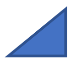
Mathematics educators must have a vivid relationship with mathematics and commit a significant portion of their professional lives to inspiring, observing, and assessing children's and learners' abilities in mathematical activities (Goos, 2020), which is certainly relevant in the context of the current study. While one cannot deny that topic matter knowledge is critical, only having this type of knowledge isn’t sufficient to be a good teacher. Boyd and Ash (2018) stated that learners may take the anchor problem provided by the textbook and introduced by the teacher and continue in multiple directions; for this, topic knowledge preparation is required (Boyd & Ash, 2018). Along with his studies on teacher knowledge, Shulman (1986, 1987) is also among the primary researchers concerning mathematical knowledge. In addition, only deliberate assimilation of knowledge allows for the long-term preservation of systematised knowledge, skills, and capacities among learners (Kholmatova, 2020).

2.3.5 Common Error Patterns of Fractions in Addition, Subtraction and Multiplication Of Fractions

An error is an action which is inaccurate or incorrect in a statement; in some cases, an error is synonymous with a mistake. On a more detailed level, Hansen (2006, p. 15) defines mathematical errors as mistakes made by learners as a result of carelessness, misinterpretation of symbols and text, lack of relevant experience or knowledge related to a particular mathematical topic, learning objective or concept, lack of awareness, or inability to check the answer given. Olivier (1989) defines error as answers arrived at through incorrect planning, yet this planning continued to be applied systematically and regularly in the same circumstances. Moru et al. (2014), argued that errors are indicators of the existence of misconceptions. Other errors arise under the general theme that procedures are not always consistent with whole number rules (Moru et al., 2014). The most common error in fractions is when learners fail to recognise that denominators define the size of the fractional part and that numerators represent the number of this part.

Studies show some common mistakes when adding, subtracting, interpreting, and comparing fractions (Lestiana et al., 2017). The literature reviewed stated that addition, subtraction, division, and multiplication of fractions are operations that learners struggle with (Baidoo, 2019; Ubah & Bansilal, 2018). Lestiana and colleagues indicated that most Grade 3 learners lack procedural and conceptual knowledge; when adding fractions, learners add numerators and denominators together, which indicates a conceptual error Similarly, when multiplying, learners multiply the numerator with the denominator of the other fraction, which indicates a procedural error (Lestiana et al., 2017). Brown and Skow (2016) determined the three categories of mathematics mistakes that learners commonly make, which are factual, procedural and conceptual mistakes. Table 2.1 below explains instances of each form of mathematical error and is taken from Brown and Skow's study.

Table 2.1: Mathematical Errors and Examples

Factual Mistakes	Examples
No comprehension of number facts	3 + 5 = 4 2 – 7 = 5
Misidentifies signs	3 x 2 = 5
Misidentifies the value of digits	6 – 2 = 7 (learners identify 6 as 9)
Counting errors	Miss a number or more in counting: 1, 2, 3, 5. Lack of mathematical terms. Learners do not know the meaning of numerator and denominator.
Lack of mathematical formulas	Learners do not know the formula for a triangle's area.
Procedural Mistakes	**Examples**
Regrouping errors	24 Learners forget to regroup the tens 47 + 61
Performing incorrect operations	4 x 5 = 9 Learners regard multiplication as addition
Fraction errors	Learners cannot find common denominator when adding and subtracting $\frac{1}{3}+\frac{2}{5}=\frac{3}{8}$ Errors in multiplying fractions $\frac{2}{4} \times \frac{3}{4}=\frac{6}{4}$ Errors in dividing fractions $\frac{6}{9} \div \frac{2}{3}=\frac{3}{3}$
Decimal errors	Learners do not align decimal places when adding or subtracting 2.34 + 47.1 = 70.5
Conceptual Mistakes	**Examples**
Misconception of place value	12 +9 = 102
Overgeneralization	In subtraction, student always put the greater number as the minuend. 10 – 24 = 14
Oversimplification	Right triangle  not a right triangle 

The present study identifies the common error patterns of Grade 6 learners in fractions at a government school in Gauteng. Alkhateeb (2019) has identified common errors learners make and grouped them into four. These four categories of errors include errors dealing with fractions as integers, errors related to basic concepts of fractions, errors due to confusion between fractions and fraction operations, and errors in executing the algorithms. The results also showed that learners ignore the integer in the mixed fraction and that more than 50% of learners' errors are associated with solution strategies. In their study of Grade 4s, Ghani and Maat (2018) discovered that learners frequently add fractional denominators incorrectly because they don't understand the basic concepts of fractions; the misconception is therefore associated with a conceptual error. Furthermore, the present study identifies the misconception associated with factual, conceptual and procedural errors in addition, subtraction and multiplication.

Liu (2018) discovered that limited instruction in fractions was directly visible in children's fraction representation, and their degree of understanding was at the same time also directly related to their whole number knowledge. Schumacher and Malone and Fuchs (2017) identified two types of whole number bias errors in describing fraction calculation errors among Grade 4 learners and determining error pattern differences as a function of problem type, orientation or mathematics achievement status. The first error in whole number bias is independent whole number bias, and the second is combination bias. The other error type reflects the misapplication of fraction procedures. Consequently, it might be said that the learner made careless, conceptual, and application errors; furthermore, most learners make both concept and application errors at the same time (Pala et al., 2019). Consequently, the learners had issues with the idea of identical pieces in the piece-whole relationship, according to Loc and colleagues, who looked into the fractional errors made by fourth- and fifth-grade learners (Loc et al., 2017).

Four Types of Common Errors

Putting the common errors learners make into categories will enable us to devise ways to overcome them. This will also provide a foundation on which to derive strategies to assist teachers. Four types of errors must be considered:

Careless errors

Careless errors are simple mistakes that can be immediately identified when evaluating the tests. Stay vigilant in looking for them as you examine the test to avoid this kind of error. (Pala et al., 2019). Careless errors are mistakes that have nothing to do with a lack of knowledge or proper understanding of a concept. They are self-inflicted wounds, the unforced errors that could and should have been avoided.

Conceptual errors

These errors are made by attempting to solve a problem without understanding the necessary qualities or principles (Pala et al., 2019). These are errors that learners make when they don't have a complete understanding of the mathematical concepts and end up making errors in the process. These kinds of errors are more common in multi-step word problems, multi-digit multiplication or long division, and fraction sums.

Application errors

These are errors you commit when you are aware of this concept but are unable to apply it to the situation (Pala et al., 2019). Application errors occur when a student cannot extend mathematical skills and knowledge gained in one context, in both familiar and new contexts.

Procedural errors

Procedural errors are errors you make in tests because of how you solve the problems in the test (Pala et al., 2019). These errors occur when learners do not follow the applicable method to solve a mathematical problem.

2.3.6 Misconceptions Associated with Factual, Procedural and Conceptual Errors in Addition, Subtraction and Multiplication of Fractions

In most cases, a misconception can be regarded as an incorrect application of a rule, a mathematical generalisation, or a product of incomplete comprehension (Deringöl, 2019). Due to a lack of clarity in concept development, mathematical misconceptions appear linked to incorrect concepts that learners develop in mathematics (Kshetree et al., 2021). Learners' misconception can have their origins in their prior knowledge, which they incorrectly generalised (Im & Jitendra, 2020). Common misunderstandings and errors in fractions include seeing the numerators and denominators as separate numbers, comparing the magnitudes of fractions based on knowledge of whole numbers (for example, $\frac{1}{5} > \frac{1}{2}$ because

5 > 2), and ranking fractions based on knowledge of whole numbers (for example $\frac{1}{2}, \frac{1}{8}, \frac{1}{12}$ because 2, 8, 12) (Namkung et al., 2018). Learners who study a mathematics-related subject wrongly or insufficiently encounter difficulties that affect their future educational careers. If these difficulties are not resolved, incorrect or incomplete learning develops into misunderstandings. Similarly, Doğan and Teremiz (2020) identified that learners have misconceptions about fractions; they make mistakes related to size, comparison and sequencing.

Biber et al. (2013) concluded that most Grade 5 learners have difficulty ordering, adding, subtracting and multiplying fractions. They identified three different misconceptions about adding fractions: learners couldn't separate the numerator and denominator, learners applied expansion only to the numerator, not to the denominator, and learners made errors in the summation of the coefficient of expansion with the numerator and the denominator. Other fraction errors related to learners applying addition rules in fractions while they were supposed to be multiplying. Learners also multiplied only the numerator without multiplying the denominator. Misconception when operating fractions discovered by Purwadi et al. (2019), found that learners think that adding fractions involves adding both the numerator and denominator of each respective fraction; they argue that $\frac{7}{12} + \frac{3}{12}$ can be transformed into $\frac{7+3}{12+12}$ because "+" can be applied in both numerator and denominator.

2.3.7 Causes of Common Error Patterns in Fractions

Maellasar and Jupri (2017) found that the causes of mistakes in fractions are: 1). implementation of irrelevant laws and strategies, 2). lack of understanding of the basic concept of multiplication and division of whole numbers with fractions, and 3). not having mastered the prerequisite skills required for fraction sums. In other words, learners do not know that integers can be expressed as fractions. Barbieri et al. (2018) focused on Grade 4 learners because the school curriculum strongly emphasises the importance of fractions. The researchers found that a few Grade 4 learners who had enough whole-number knowledge experienced difficulties in fractions, while those with weak whole-number knowledge experienced great difficulties in understanding fractions.

Literature on the learning of fractions at the primary school level has shown that fraction learning presents many challenges for learners. Kusuma and Retnawati (2019) defined the position of problems in Grade 6 classes when solving word problems in operations involving whole numbers, fractions and decimals. They found that there are four causes of learners' difficulties when working with word problems: Firstly, learners have difficulties in understanding the word problems; secondly, learners don't understand the fractions concept of operations, meaning they cannot solve the fractions properly; thirdly, learners lack basic numeracy skills; and lastly, learners are inaccurate because they are always in a hurry to finish. As a result, learners' inability to comprehend the idea of a fraction and how to use a fraction has prevented them from correctly responding to questions involving problem-solving (Mokhtar et al., 2019). Numerous fractions-challenged learners also have poor multiplication abilities (Rodrigues, 2017).

2.4 ERROR ANALYSIS AND LEARNING IN MATHEMATICS

Error analysis in this current study involves studying the uncertainties and inaccuracies associated with mathematical calculations and measurements. The error analysis helped the researcher to make a more informed decision regarding the reliability and validity of the findings. The approach also helped the researcher identify and categorise errors systematically, examining them and assessing their impact on teaching and learning. The approach can be integrated with the content analysis used in the current study to analyse the findings. Content analysis's purpose is to organise and interpret the qualitative data. One of the objectives of this study is to explore the underlying misconceptions among the selected Grade 6 learners that are associated with factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions. Regarding error analysis, learners write tests, and the researchers mark the test and classify the errors according to their nature. The basic task in error analysis is to examine learners' output, including correct and incorrect statements (Khansir, 2012). The researcher, therefore, determined the source of errors through learners' interviews to answer the researcher's questions effectively, as stipulated above.

There are four elements of error analysis, as explained by Sorto et al. (2014): awareness of an error, diagnostic reasoning of learners' thinking with regard to the errors, using the links in explaining errors, and multiple explanations of errors. Peng and Luo (2009) identified four keys to learners understanding mathematical problems: mathematical, logical, strategic and psychological skills. The study overlaps with Moru and colleagues, who identified errors related to content with which learners were familiar (Moru et al., 2014). Therefore, teachers must use everyday links in explaining errors because new knowledge is built on the already existing conceptual structures of which everyday experiences are a part.

The expansion of work was never ideal in teaching and learning mathematics. Herold et al. (2020) discovered that fraction comparison performance became worse when instruction was expanded, rather than initial instruction being laid down correctly; this most especially occurred in learners with learning difficulties (Herold et al. 2020). To determine the knowledge of learners and the type of errors made in their posed problems, teachers need to identify the correct problems themselves first (Dogan-Coskun, 2019). Teachers don't accommodate and consider their mistakes as a source of learning; therefore, such learning will contribute to their understanding of why the concept is so difficult. Considering learners' mistakes in their lessons, teachers are better equipped to prevent them from repeating the same errors (Alkhateeb, 2019).

Heemsoth and Heinze (2014) found that Grade 6 learners who were given examples of incorrect answers led to learners having a negative attitude about fractions. Only when learners had knowledge of fractions could they draw beneficial lessons from examples of errors; those with low prior knowledge learned more from examples of correct working. In their recommendations, they found that reflecting on errors improves learners' performance. However, error reflections are inconsistent because, in the error-based learning of the previous studies, examples of incorrect working had been combined with correct examples in different ways, the role of prior knowledge was not clear, and there was a lack of findings related to naturally valid school environments and relevant school curriculum topics.

Buchs et al. (2015) aimed at structuring cooperative learning to improve learners learning, especially that of average achievers. They proposed structuring interactions to improve the learning of average achievers in a mixed group of learners. These groups of learners would then understand the content in a group and be able to progress in fractions work. They further discovered that with this learning strategy, average achievers progressed from the baseline test. Even the high achievers did well in structured cooperative learning. Fuchs and colleagues assessed whether an intervention focus consisting of a combination of sums of fraction and decimal magnitude would add value in improving rational numbers over intervention focus when fractions were excluded (Fuchs et al., 2019). They discovered that fraction and decimal magnitudes do not add value to improving rational number understanding. According to these researchers, additional research is needed to test how combining fraction and decimal magnitudes improves rational number knowledge. While other studies concentrate on connecting teachers' beliefs and content knowledge, Samková (2019) used a tool known as Concept Cartoons to observe how future elementary school teachers solve problems and reason about fractions to learners. Their results showed that fractions are a complex subject for future schoolteachers as far as knowledge and pedagogical content knowledge are concerned. They suggested that cartoons are a universal tool for problem-solving activities and may also serve as a qualitative assessment tool.

Additionally, An et al. (2004) found that when learning a topic, learners gain conceptual and operational knowledge, while the teacher simultaneously becomes aware of learners' initial knowledge, forms of learning, and the difficulties and misconceptions they experience while learning the concepts. As a result, student knowledge (understanding of learners) is accepted as an important part of pedagogical content knowledge or teacher knowledge. Thus, it becomes necessary to determine what level of student knowledge teachers possess. Barbieri et al. (2020) evaluated the effectiveness of middle school fractions intervention and discovered that using a number line-centred mathematics intervention approach assisted struggling Grade 6 learners in improving their understanding of fractions. This APPROACH has been validated as a learning principle in teaching fractions.

However, Pavlovičová and Vargová (2020) investigated whether there was a connection between fractions as a measure and addition and equivalence and comparison of fractions, which learners should be aware of when learning fractions. They showed a positive connection between fractions as a measure and equivalent fractions, but there was a negative connection between fractions as a measure and addition of fractions. There are two ways in which teachers can approach learners' misconceptions:

Dyson et al. (2018) showed that Grade 4 learners used rule-based methods to reason about decimal magnitude before the lesson from the teachers, and this resulted in improved understanding of fractions and mathematical achievement within the intermediate grades.

Therefore, to spot errors, teachers' pedagogical content knowledge is key, as is the teachers’ perspective on mathematics education. The teachers’ knowledge of subject and teaching strategies and how they represent their lesson, assist in realising best practices in mathematics teaching and learning. Combining error analysis with content analysis in qualitative research increases the trustworthiness and reliability of the results. Researchers should integrate different strategies to identify and address potential errors, from bias detection to constant comparison and member checking. Practicing reflective practice and embracing methodological transparency are critical factors in successfully integrating error analysis into content analysis. By effectively combining these approaches, researchers can raise the level of their qualitative research and contribute to a deeper understanding of complex social phenomena.

2.5 SUMMARY

In chapter two, the theoretical framework is explained in detail. This chapter discussed the theory underpinning this study in detail. The researcher also reviewed the literature, presenting various studies that focused on the difficulties learners face in mathematical fractions, mostly in primary schools. Furthermore, the concept of fractions is discussed, including the meaning of fractions, the evolution of fractions, the importance of fractions, mathematics and common errors in fractions as previously identified by recent researchers. The researcher again discussed the misconception associated with conceptual, factual and procedural errors in addition, subtraction and multiplication of fractions. In addition, the causes of common errors were discussed in this chapter. The researcher also discussed the concept of error analysis and learning in mathematics. The methods used in this study are discussed in chapter three.

CHAPTER 3
RESEARCH METHODOLOGY AND RESEARCH DESIGN

3.1 INTRODUCTION

This chapter presents the research methodology and data-gathering methods utilised in this study. A test and interviews were used to collect the data. This chapter covers the study's methodology, population and sampling, instruments and data collection techniques, setting, participants, data gathering methods and analysis, and reliability and validity factors. Accordingly, the demographics and the environment is explored. This study aimed to discover the typical fraction addition, subtraction, and multiplication error patterns among a sample of Gauteng Grade 6 learners. Specifically, the research's specific goal was to explore underlying misconceptions among the selected Grade 6 learners associated with the factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions and the causes of these misconceptions.

3.2 RATIONALE OF THE RESEARCH APPROACH

This section of the report describes the route and methods used in the study.

3.2.1 Qualitative approach

The qualitative approach was used in this study. Creswell (2014) characterised the qualitative approach as inductive, emerging, and shaped by the researcher's experience collecting and analysing the data. The researcher aimed to understand the experiences of learners when dealing with fractions daily. As explained in Chapter 1, learners seek to understand the world in which they live in relation to fractions. The chosen approach helped the researcher achieve the goal of identifying and analysing the common error patterns occurring in Grade 6 learners’ fraction sums. The approach mainly focused on the individual experiences of the participants and the meaning that these participants attached to these experiences (Johnson et al., 2019). The behaviour studied occurred naturally (McMillan & Schumacher, 2014) since the participants were in a closed classroom setting without any distractions. The researcher reassured the participants of their rights and freedom to withdraw at any stage without incurring any penalties. This facilitated an environment that was helpful for the participants to explain how they got their incorrect answers using their own comfortable explanations. The participants spent adequate time with the researcher to enable proper identification of the errors and allow an accurate analysis of the data. The interviews were conducted interactively, one-to-one, in a manner approved by the interviewer and interviewee (Creswell & Creswell, 2018). Learners were given questionnaires in the form of tests and interviews based on the incorrect answers they provided.

3.2.2 Research Design

The research design selected for this study is the descriptive case study design. A case study enables a better understanding of learners’ error patterns and what they thought about their mistakes while also enabling insight into the teachers’ knowledge and beliefs concerning fractions. Robert Yin defined a case study as an empirical study that examines a contemporary phenomenon (the “case”) in depth in its actual context (Hollweck, 2015). These qualitative research methods allow the researcher to provide in-depth analysis and interpretation. This research design aimed to provide an in-depth understanding of the case (McMillan & Schumacher, 2010). The researcher planned to examine the learners' works using fractions and discover their misconceptions. The main source of information was the quality of student work; therefore, this qualitative study used a case study design.

3.3 Research Setting

This study was conducted in one of the primary schools of the Department of Basic Education in Orange Farm, in Circuit 3 of Johannesburg South district in Gauteng. Data collection was done during COVID-19 in term 3. The researchers complied with the South African National Curriculum and Assessment Policy Statement (CAPS), which is the comprehensive document introduced by the Department of Basic Education for all subjects listed in the National Curriculum Statement for Grades R-12. The following is the structure of Department of Basic Education to the school in concern.

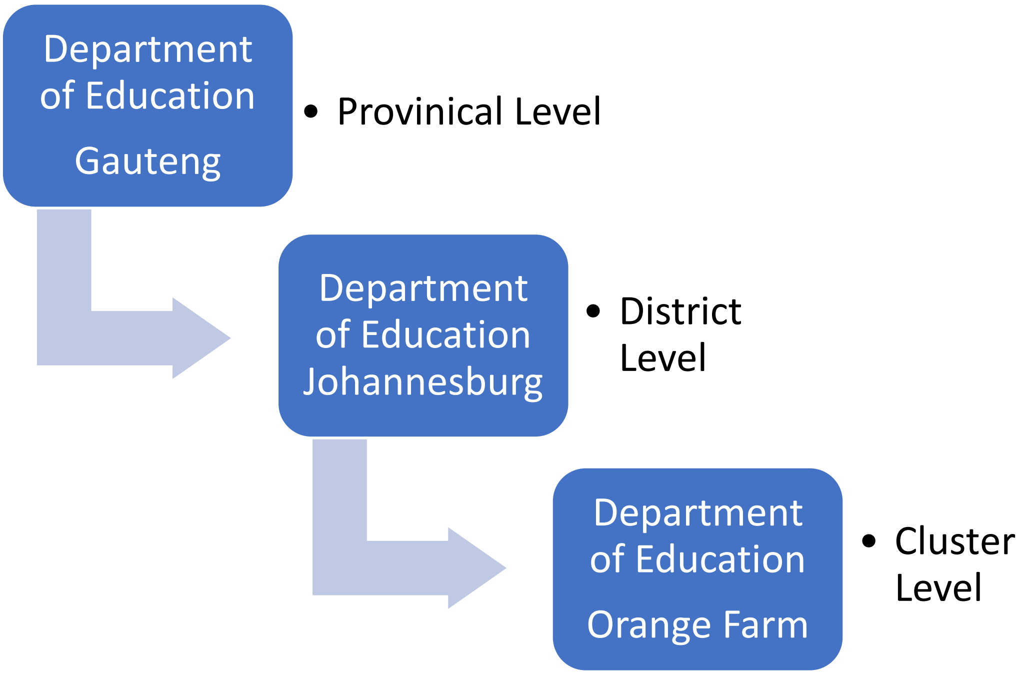


Figure 3.1: Department of Education Structures

3.4 Research Sample and Data Sources

The site for the research was a primary school in Orange Farm Township. Fifteen (15) learners were chosen from a population of 235 learners in Grade 6 classes. When one takes a small part of the population to be used for collecting data, it is called a sample (Polit & Beck, 2017). The researcher purposively chose 15 sixth-grade learners aged between nine and ten; therefore, purposive sampling was used for selecting the sample from the school. An interview was conducted with all learners who got their answers wrong in the test. Learners were grouped into three categories: high achievers (HA), average achievers (AA), and low achievers (LA). Learners were coded as HA, AA, and LA on their worksheets. The grouping selection came from the SASAMS school system, which lists all learners according to grades and performance.

3.5 Ethical Considerations

An ethical clearance request was submitted to the University of South Africa Ethics Committee for ethical approval before commencing the study. After receiving approval from the committee, the researcher submitted the proposal letter to the Govan Mbeki Primary School Principal for approval to conduct the research in the school. The proper protocol was followed to get clearances for all structures that govern the school up to the district level. The request to seek permission to conduct research at school was submitted to the principal; therefore, the principal made the stakeholders aware of the research, and the request was approved. After the permission was granted, the Grade 6 learners were informed of the study in time for them to decide whether to participate in the study or not without being forced to make specific decisions.

3.5.1 Principle of Autonomy

The autonomy of participants, their rights, and their dignity were respected in the research. The researchers ensured that the learners had the right to freely choose whether or not to participate in the study without coercion or pressure. A letter of parental consent was provided to the parents, and a letter of consent was provided to the learners, which played an important role in maintaining participants' autonomy. The learners were fully informed about the study's purpose, procedures, potential risks and benefits beforehand; they agreed to participate.

3.5.2 Principle of Beneficence

The research made a positive contribution towards the welfare of people. The principle of beneficence played a central role in conducting ethical and morally responsible research. In the context of this research, the researcher minimised the potential harm to the participants by ensuring that the classes were tidy and that the protocol of COVID-19 was observed. This principle guided the researcher in ensuring the well-being and safety of the learners. To preserve the benefits of research, the researcher thoroughly evaluated the potential risks and benefits before beginning the research procedure considering the possible physical, psychological and social effects of the research on the learners. In addition, the researcher took steps to protect the privacy and confidentiality of learners to preserve their dignity and general well-being. Informed consent was a key factor in the research practice because it enabled the learners to make informed decisions about their participation in the research. In general, promoting the benefit of others in research is critical to ethical research that prioritizes the well-being of learners and adheres to the principles of justice and respect.

3.5.3 Principle of Non-Maleficence

The principle of non-maleficence requires that there is no harm to participants in research. The research did not harm the research participants or anyone in general. The researcher ensured that the learners were protected throughout the research process. The researcher specifically ensured learners were protected from any physical, psychological or emotional harm; even when the interview was taking place; the researcher was vigilant in preventing inappropriate intervention that may adversely affect the learners.

3.5.4 Principle of Justice

The benefits and risks of research were fairly distributed among participants. The researcher ensured that the sampling of learners was fair and transparent without any bias or discrimination. The research included all categories of learners in the selection, such as high, average, and low performers in Grade 6. The researchers provided the learners with information about the research to ensure that learners could make an informed decision. The learners who had limited literacy or language problems were also accommodated. The learners were advised to use their own language when explaining their respective answers.

3.6 Instrumentation and Data Collection Technique

et al. (2015) referred to data as honest information that a researcher collects during a study. The qualitative approach used employed a case study. The researcher collected data through tests and semi-structured interviews. The researcher chose Grade 6, assuming they did common fractions in the previous grade. Learners were given a written test and wrote the test, which the researcher marked in terms of correct and incorrect answers, and recorded the marks on their written test. Interviews were conducted based on the incorrect answers of the learners. The researcher used an audio recorder and made notes while interviewing the participants. The recordings were then transcribed to ensure the researcher got the correct information. Each student was assigned codes used as references in this study if needed, for example, learner EM and learner LS. Here, there were no specific criteria used in coding.

The incorrect answers were grouped into categories, for example, the HA category (learners from high levels), the AA category (average achievers), and the LA category (low achievers). The chosen categories were taken from the SASAMS school system. Learners wrote the tests in one hour after the researcher explained what needed to be done and reminded them of everything about addition, subtraction, and multiplication of fractions. Selection was guided by learners' written responses, and selected learners were individually invited for an interview about their writing. Learners were interviewed based on their incorrect answers the following day. The various data collection techniques are discussed below.

The researcher adhered to basic ethical principles from the University of South Africa's policy on research ethics. The ethical principles are explained below, as promoted by UNISA.

3.6.1 Semi-structured interviews

A semi-structured interview is when the researcher has substantial control over the topic and asks the participants open-ended questions. In preparing the questions, the semi-structured interview allows questions to follow in a sequence that will promote knowledge access without strictly adhering to standardised questions. The learners explained the procedure used in their incorrect answers. Since the semi-structured interviews ensure flexibility when collecting data (Jensen & Laurie, 2016), learners where allowed to use their own language when explaining their responses. Although there is no strict adherence to standardised questions, semi-structured interviews assisted the researcher in identifying topics as participants responded, ultimately making data analysis easier (Johnson et al., 2019). The researcher conducted the interviews using relevant communication skills, such as verbal, documentation, and active listening skills, to enable easy data collection. Communication took place between the interviewer and interviewee to transfer information (Kaplan & Sadock, 2015).

3.6.2 Verbal Skills

Verbal skills apply when a researcher talks and speaks with participants in their own language and tone to deliver information (Lubwama, 2020). Participants were allowed to use their own language when communicating, which was then translated into English.

3.6.3 Documentation Skills

The researcher used technical writing to collect and record the data, and to convey information in technical terms (Lubwama, 2020). The researcher used question papers and wrote the responses on the paper.

3.6.4 Active Listening Skills

Active listening is when the researcher is speaking and the participants are responding to questions from the researcher, paying attention to the message (Lubwama, 2020). The researcher posed questions to the learners while learners listened attentively, and then the researcher processed the information as it was heard from the participants, confirming the content with the participants (Lubwama, 2020).

3.6.5 Written Tests

Tests form one type of assessment and entail specific procedures to measure learner-specific content (DBE, 2009). A test was administered to understand how learners think about the concept of fractions, and the test had ten questions. The test had different questions based on fractions' content, addition, subtraction and multiplication. Learners answered the questions in spaces provided on the test and were asked to show all their work. The research questions in the study corresponded to the administered test accordingly. The test included addition and subtraction of common fractions, addition and subtraction of mixed fractions as well as multiplication of common fractions and mixed fractions.

Name of the learner: ___________________ Date:____________________

ADDITION, SUBTRACTION, MULTIPLICATION OF FRACTION INCLUDING MIXED FRACTION

Show all your calculations.

1. $\frac{3}{9}+\frac{2}{3}+\frac{2}{8}$	6. $\frac{4}{12}+\frac{1}{12}$
2. $\frac{23}{24}-\frac{1}{4}-\frac{2}{12}$	7. $5\frac{3}{4}+3\frac{1}{5}$
3. $3\frac{3}{5}+1\frac{4}{5}$	8. $4\frac{1}{5}+3\frac{1}{2}$
4. $3\frac{4}{7}-2\frac{1}{7}$	9. $2\frac{5}{6}\ X\ 6\frac{2}{5}$
5. $\frac{4}{10}+\frac{1}{10}$	10. $\frac{2}{4}\ X\ \frac{5}{6}$

Figure 3.2: Written Test – Addition, subtraction and multiplication of fraction and mixed fraction

Rationale for the choice of questions

Question 1

This question test learners' understanding of the addition of fractions with three fractions having different denominators.

Question 2

This question with a numerator with two-digit numbers, tests the understanding of learners based on the addition of three fractions with different denominators.

Question 3

This question test learners' understanding of the addition of mixed fractions with the same denominators.

Question 4

This question tests learners' understanding of the subtraction of mixed fractions with the same denominators.

Question 5

This question tests learners' understanding of the addition of common fractions with the same denominator.

Question 6

This question tests learners' understanding based on the subtraction of common fractions with different denominators.

Question 7

This question tests learners' understanding based on the subtraction of common mixed fractions with different denominators.

Question 8

This question tests learners' understanding based on subtracting mixed numbers with different denominators.

Question 9

This question tests learners' understanding based on multiplying mixed numbers with different denominators.

Question 10

This question test learners' understanding based on the multiplication of common fractions with different denominators.

3.6.6 Interviews

The researcher will not be able to examine learners' written comments alone; this may not have been sufficient to achieve the goals of this study. Responses from the test from the learner will have limited the study. Therefore, learners' responses on how they followed certain steps in answering the test are very important, as this will assist the researcher in having more information based on what learners think and how they responded when attempting to answer the questions. Interviews are relatively better than written feedback for getting learners to reveal how much they know and collaborate to build their own knowledge. The researcher used interviews to gather rich information and draw more detailed conclusions.

The interviews gave the researchers an insight into how learners followed their respective procedures and their understanding of fractions. Through the interview process, the researcher unfolded the learners' misconceptions in addition, subtraction and multiplication of fractions. The semi-structured format used ensured that each learner with an incorrect answer was subjected to the same question in any order, starting with general questions. The learners gave their own understanding, skills, and perceptions concerning the concept of fractions. The correct approach was used by the researcher when conducting the interviews.

3.6.7 Data Analysis

Data analysis in qualitative research means that data collected will be interpreted (Polit & Beck, 2019). The researcher interpreted the data after conducting the interviews. The researcher continuously analysed data collected from the participants to ensure that all the relevant data was recorded. The researcher categorically arranged the data collected according to the types of errors. The tests were categorised according to the number of those with incorrect answers and grouped accordingly. The results obtained from the interviews have been kept safe in Word documents on a computer since the interview. The test was marked accordingly, and the researcher analysed question by question from the individual participant's solutions. The table analysed the number of correct and incorrect questions, including the percentages. The data provided by this table classified responses into three categories based on the level of performance of learners. Another table analysed the type of errors identified during data collection. The data provided by this table was used to disclose the incorrect solutions per question.

The researcher used the content analysis method to analyse the data and ensure the data was captured and correctly recorded. This procedure was to facilitate conceptual analysis of the data, identify the existing common error patterns in fractions, and determine how frequently learners made these errors. The common error patterns identified and the misconception associated with conceptual, procedural and factual errors were analysed and recorded in categories. The researcher evaluated data collected from all the participants to reach a conclusion. The researcher revised the data to minimise bias. Learners were allowed to use their own home language.

3.6.8 Trustworthiness

Trustworthiness in Qualitative research is like reliability in quantitative research. Stahl and King (2020) argue that establishing trust in qualitative research is crucial. Researchers now perceive trustworthiness to comprise credibility, transferability, dependability and confirmability. Guba and Lincoln first addressed the need for trustworthiness in qualitative research (Guba & Lincoln, 1986). In the current research, trustworthiness is crucial to ensure that results can be trusted based on practices and procedures.

3.6.8.1 Credibility

Credibility is synonymous with the concept of validity in quantitative research; however, it specifically pertains to internal validity. Ensuring the credibility of qualitative data involves incorporating diverse views throughout data collection to verify the appropriateness of the data. (Noble & Smith, 2015). The researcher is experienced educator, teaching Grade 6 learners which made the study more credible.

I ensured credibility in the research by using triangulation, utilising different methods to do the data collection to confirm the credibility of the data. Multi-method strategies were used, allowing the triangulation of data collection and enhancing the validity of the research conducted. The researcher used results obtained after marking the tests, and tape recorders were used during interviews. The process was used to provide accurate and relatively complete records. On the test completion, the researcher went through the test, marking the questions and grouping the errors for analysis. Learners were interviewed based on the errors made in the addition, subtraction and multiplication of fractions.

3.6.8.2 Transferability

Transferability in qualitative research is analogous to generalisability in quantitative research, meaning the degree to which the results may be applied to other contexts. Transferability pertains to the extent to which the results may be applied to comparable circumstances or persons rather than larger contexts. Transferability may be attained by a comprehensive and detailed account of the results obtained from various techniques of data collecting.

I ensured transferability by providing thick descriptions of the data obtained to allow other researchers to determine the applicability of the research in their contexts of interest. The study also includes a diverse sample of Grade 6 learners from different socio-economic backgrounds and school setting which makes it more transferable.

3.6.8.3 Dependability

Dependability speaks to the accuracy of the data collection and to what extent the data addresses the phenomena of interest. "To achieve dependability, researchers can ensure the research process is logical, traceable, and clearly documented" (Tobin & Begley, 2004, as cited in Nowell, 2017, p. 3). The study employed standard assessment Grade 6 test; therefore, I ensured dependability by using rigorous data collection techniques and applying best practices for the data analysis.

3.6.8.4 Confirmability

Confirmability addresses the concept of objectivity in qualitative studies. When confirmability is achieved, the likelihood that the findings will be validated or corroborated by other studies will be increased (Lincoln & Guba, 1986). I ensured confirmability in the study by ensuring that I kept my own bias in mind, checking and rechecking the data during the data collection process and using a clear coding schema.

3.7 SUMMARY

In this chapter, the researcher discussed the research design, approach, population and sampling, including the instrumentation and data collection techniques. Data was collected using written fractions tests, semi-structured interviews and naturalistic observation. Data analysis was done continuously during data collection to ensure that no data was missed for analysis. A brief discussion of the ethical principles related to the study proved that general ethical guidelines were adhered to. Data analysis processed explained in details and trustworthiness of the study explained. Detailed information on individual interviews and findings will follow in chapter four.

CHAPTER 4
DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

This chapter presents and analyses the findings. The study aimed to identify common error patterns in fractions and underlying misconceptions of fractions among 15 Grade 6 Learners from a public school in Gauteng. A written test was administered to learners in a classroom setting, and learners were interviewed according to their incorrect answers in the test, to understand their individual responses. The findings are presented accordingly, drawing from the analysis of collected data. This chapter presents the data analysis process followed by the tables summarising data obtained from the tests. This chapter will also analyse the interviews with the learners. The results of the interviews and the tests are reviewed. All these presentations were informed by the main research question and the other two secondary research questions in this study, which are:

The primary research question

- What are common error patterns in fractions at primary schools in Gauteng district?

The secondary research question

- What are misconceptions associated with factual, conceptual and procedural errors in addition, subtraction and multiplication of fractions?

4.2 DATA ANALYSIS PROCESS

The 15 learners were grouped into three categories, with five learners per category. Category 1 (HA) includes five learners, learners who are top or high achievers in class who are performing above other learners academically. Category 2 (AA) consists of five learners who are average achievers, l who are performing average academically, and Category 3 (LA) includes five learners who are low achievers academically in class. The reason for choosing learners from high, average, and low performers is to prove that fraction difficulties affect all levels in the classroom setting. All learners wrote the same test with ten questions. Table 4.1 elucidates the codes and learners' categories. Based on my experience, I have discovered that fractions affect all types of learners, whether the learner is a high, low, or average performer. Therefore, the researcher selected five high-, five average- and five low-level performing learners as participants. The researcher then categorised them accordingly using HA (as high achievers), AA (as average achievers) and LA (as average achievers) to identify different categories.

Table 4.1: Categories of learners and their codes

Learner Codes	Category (Level)	Venue	Date
AT	HA	Govan Mbeki Primary School	02/12/2021
EM	HA	Govan Mbeki Primary School	02/12/2021
DK	HA	Govan Mbeki Primary School	02/12/2021
GN	HA	Govan Mbeki Primary School	02/12/2021
JT	HA	Govan Mbeki Primary School	02/12/2021
LS	AA	Govan Mbeki Primary School	02/12/2021
HM	AA	Govan Mbeki Primary School	02/12/2021
FB	AA	Govan Mbeki Primary School	02/12/2021
KL	AA	Govan Mbeki Primary School	02/12/2021
IN	AA	Govan Mbeki Primary School	02/12/2021
TPN	LA	Govan Mbeki Primary School	02/12/2021
NC	LA	Govan Mbeki Primary School	02/12/2021
SN	LA	Govan Mbeki Primary School	02/12/2021
MS	LA	Govan Mbeki Primary School	02/12/2021
PL	LA	Govan Mbeki Primary School	02/12/2021

Table 4.2 below shows the categories of learners; the table has a column for all ten questions and a column specifying the percentage and the number of correct and incorrect solutions to all the questions of all 15 learners who wrote the test. Table 4.2 summarises the common error patterns identified by the researcher and the type of errors based on incorrect questions.

Table 4.3 states the number of correct and incorrect questions in learner categories as per the data collected. The table has a question column, high achievers column, low achievers column, total number of incorrect and correct answers and the overall percentage of incorrect and correct answers. The table also enlightens the percentage of incorrect and correct questions based on data collected.

Table 4.2: Categories of learners and number of correct and incorrect solution and their percentages

Question	HA	AA	LA	TOTAL	Percentage
1.	5 incorrect 0 correct	5 incorrect 0 correct	5 incorrect 0 correct	15 incorrect 0 correct	100% 0%
2.	5 incorrect 0 correct	5 incorrect 0 correct	5 incorrect 0 correct	15 incorrect 0 correct	100% 0%
3.	3 incorrect 2 correct	2 incorrect 3 correct	2 incorrect 3 correct	7 incorrect 8 correct	46% 54%
4.	1 incorrect 4 correct	2 incorrect 3 correct	2 incorrect 3 correct	5 incorrect 10 correct	33% 67%
5.	0 incorrect 5 correct	2 incorrect 3 correct	3 incorrect 2 correct	5 incorrect 10 correct	33% 67%
6.	0 incorrect 5 correct	1 incorrect 4 correct	1 incorrect 4 correct	2 incorrect 13 correct	13% 87%
7.	5 incorrect 0 correct	5 incorrect 0 correct	5 incorrect 0 correct	15 incorrect 0 correct	100% 0%
8.	5 incorrect 0 correct	5 incorrect 0 correct	5 incorrect 0 correct	5 incorrect 0 correct	100% 0%
9.	4 incorrect 1 correct	5 incorrect 0 correct	5 incorrect 0 correct	14 incorrect 1 correct	93% 07%
10.	3 incorrect 2 correct	1 incorrect 4 correct	3 incorrect 2 correct	7 incorrect 8 correct	47% 53%

Table 4.3: Factual, procedural and conceptual errors according to learners' categories

Summary of Findings

Question	High Achievers Average Achievers Low achievers	Type of Error	Example
1 $\frac{3}{9}+\frac{2}{3}+\frac{2}{8}$	Most HA, AA and LA, learners add numerators together and add denominators together.	Procedural and conceptual error	$\frac{3}{9}+\frac{2}{3}+\frac{2}{8}=\frac{7}{20}$
	Some miscount their answers, though the answer is incorrect.	Factual error	$\frac{3}{9}+\frac{2}{3}+\frac{2}{8}=\frac{7}{19}$
	Some HA learners misapply fraction procedures and miscount.	Conceptual error	$\frac{3\times2\times2}{9\times3\times8}=\frac{12}{27}$
2 $\frac{23}{24}-\frac{1}{4}-\frac{2}{12}$	Most HA, AA and LA learners subtract numerators together and subtract denominators together.	Conceptual error	$\frac{23}{24}-\frac{1}{4}-\frac{2}{12}=\frac{20}{8}$
	Some LAs executed the same procedure but swapped the numerator and denominator.	Conceptual error	$\frac{23}{24}-\frac{1}{4}-\frac{2}{12}=\frac{8}{20}$

Summary of Findings

Question	High Achievers Average Achievers Low achievers	Type of Error	Example
3 $3\frac{3}{5}+1\frac{4}{5}$	Most frequent HA, AA, LA procedural errors (use the incorrect procedure)	Procedural error	3 + 1 = 4 3 + 4 = 7 5 + 5 = 10 4 + 7 = 11 $=\frac{11}{10}$
	Most HA, AA, LA use the same procedure in adding numerators and denominators together.	Procedural error	$3\frac{3}{5}+1\frac{4}{5}=4\frac{7}{10}$
	Most AA and LA Conceptual errors (make the numerator the same, then add the two numerators)	Conceptual error	$3\frac{3}{5}+1\frac{4}{5}$ $3\frac{7}{5}+1\frac{7}{5}$ $4\frac{14}{5}$
4 $3\frac{4}{7}-2\frac{1}{7}$	Most HA, AA, LA omitted the integer, a careless mistake.	Conceptual error	$3\frac{4}{7}+2\frac{1}{7}=\frac{3}{7}$
	Most of LA correctly subtract the whole numbers and numerators but mistakenly add the denominators.	Procedural error	$3\frac{4}{7}-2\frac{1}{7}=1\frac{1}{14}$
5 $\frac{4}{10}+\frac{1}{10}$	Most of LA add numerators and denominators together	Procedural error	$\frac{4}{10}+\frac{1}{10}=\frac{5}{20}$
6 $\frac{6}{9}+\frac{3}{5}=\frac{9}{14}$	Most HA, AA, LA add numerators and denominators together.	Procedural error	$\frac{6}{9}+\frac{3}{5}=\frac{9}{14}$
7 $\frac{4}{12}-\frac{1}{12}=\frac{3}{12}$	All of the learners got it correctly.		$\frac{4}{12}-\frac{1}{12}=\frac{3}{12}$
8 $5\frac{4}{4}-3\frac{1}{5}$	HA, AA, LA learners subtract the whole numbers and numerators together, and subtract denominators.	Procedural error	$3\frac{4}{7}-2\frac{1}{7}=1\frac{3}{0}$
9 $2\frac{5}{6}\times 6\frac{2}{5}$	HA, AA, LA, Conceptual errors (multiply numerators together, denominators together, from the answer they get, they add them together and make it a numerator. Then they multiply whole numbers together and make the answer a denominator)	Procedural error	$5\times 2=10$ $6\times 5=30$ $2\times 6=12$ $\frac{50}{12}$
10 $\frac{2}{4}\times\frac{5}{6}$	HA, AA, LA conceptual errors (learners multiply numerators together but incorrectly add the denominators together).	Conceptual error	$\frac{2}{4}\times\frac{5}{6}=\frac{10}{10}$
	Most LA multiplies numerators together and multiplies denominators together but miscount.	Conceptual error	$\frac{2}{4}\times\frac{5}{6}=\frac{10}{12}$

Summary of Findings			
Question	High Achievers Average Achievers Low achievers	Type of Error	Example

The above tables show that most learners at all levels have problems with addition, subtraction and multiplication of fractions.

Tables 4.2 and 4.3 do not show how learners obtain their answers. It doesn't explain the answers in detail, as this is part of the interview process. The following discussion analyses how the fifteen chosen learners responded to each test and interview question. The researcher only interviewed learners with incorrect answers, and the data gathered showed correlations. The fifteen learners assessed only included those with incorrect answers who were interviewed. Their results were coded as per Table 4.1 above, and in the transcript, the interviewer will be identified as "interviewer".

4.2.1 Question 1: Factual, Conceptual and Procedural Errors in the Addition of Fractions With Different Denominators

$$\frac{3}{9} + \frac{2}{3} + \frac{2}{8}$$

Here, the researcher is trying to see if the learners can identify the common denominator or apply the correct procedure to work out the answer. It is evident from Table 4.2 that learners add the numerators together and then add the denominators together to get the answer. EM is a learner from the high achievers category. Her answers when attempting to answer Question 1 are reflected below. According to the data collected, all learners from high, average and low categories answered Question 1 incorrectly, meaning 100% of the learners answered Question 1 wrongly. Amongst these errors, the most common errors identified in Question 1 are procedural and conceptual errors in fractions.

4.2.1.1 Data from the High Achiever Category (HA)

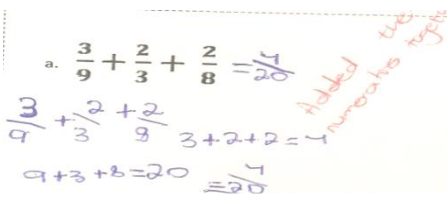


Figure 4.1: Example of Procedural and Conceptual Errors

3When interviewed based on her answer as per above, she said that she added 3 + 2 + 2 = 7 (numerators) and then obtained 7 as the answer, again, she added 9 + 3 + 8 = 20 (denominators). GN and SN also gave this response with the same explanation when they were interviewed. Their responses when interviewed are reflected below:

EM

Interviewer: How did you come up with your answer?

EM: My answer is $\frac{\mathbf{7}}{\mathbf{20}}$.

Interviewer: OK, how did you get that answer?

EM: I said three plus two equals to 5, then add 2, which gave me 7, then I wrote 7 on top. After I said 9 plus 3 equals to 12, then I added 8 equals to 20. My answer is $\frac{\mathbf{7}}{\mathbf{20}}$.

GN

Interviewer: How did you come up with your answer?

GN: I have added all these numbers [pointing at the all numerators].

Interviewer: You have added all numbers?

GN: Yes, Mam, I added $\frac{3}{9}$ *and* $\frac{2}{3}$ *and* $\frac{2}{8}$*, I added them together. No, man I actually said* $\frac{3}{9}$ *plus* $\frac{2}{3}$ *then got* $\frac{5}{12}$*, then* $\frac{5}{12}$ *plus* $\frac{2}{3}$ *. My answer is* $\frac{7}{20}$.

JT

Interviewer: How did you come up with your answer?

JT: I added the fractions, Mam.

Interviewer: OK, how did you get that answer?

JT: I said three plus two equals to 5. Then add 2, which gave me 7. Then I wrote 7 on top. After I said 9 plus 3 equals to 12, then I added 8, [which] equals to 20. M my answer is $\frac{\mathbf{7}}{\mathbf{20}}$**.**

The excerpt above clarifies that some learners do not apply the rule of identifying the Highest Common Denominator (HCD) when dealing with different denominators in addition fractions. According to my understanding of the data above, they think that they should add the numerator and also need to add the denominators to get the correct answer. Learners believe that numerators and denominators can be treated as separate whole numbers. Even when asked to clarify their answers, they still mention that they add all numerators together first and then add all the denominators together, after which they will get their answer. Learner SN was confused about signs; to her, subtraction and division was the same sign when she was explaining, but she wrote down the correct signs in the test. The above learners lack procedural and factual understanding. Their misconception is associated with procedural and factual errors in the addition of fractions.

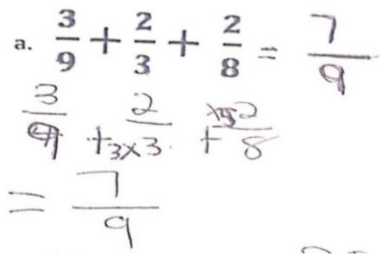


Figure 4.2: Learner DK's method of multiplying

Learner DK's calculations are reflected above; she remembers the method of multiplying the top and bottom of each fraction by the denominator to equalise the denominators to add the numerators using the common denominator. When asked to clarify her response, she said that she added three to the denominator of the second fraction and then got 9, after which she added the numerators. The following is her explanation:

DK

Interviewer: How did you come up with your answer?

DK: I wrote $\frac{3}{9}$*, then second fraction; I multiplied denominator by 3.*

Interviewer: Please explain why multiplied [the] second denominator by 3?

DK: OK, Mam, I wanted denominator to be the same.

Interviewer: Then?

DK: Mam, I then added 3 plus 2 plus 2, to give me 7. I took 9, and then my answer is $\frac{7}{9}$*.*

Clearly, from the answer and interview of DK, she remembers the method of equalising denominators so that she could take the common denominator and then add the numerators, but she missed the other denominator on the third fraction. DK's misconception, therefore, is associated with a conceptual error; she doesn't understand the algorithm of fraction addition.

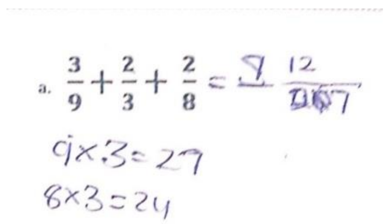


Figure 4.3: Learner's AT's response

Figure 4.3 above reflects learner AT's answer. The learner is very confused and doesn't even understand why she followed that specific procedure. When asked how she got that answer, she said that she multiplied 9 by 3 and multiplied 8 by 3 after that. When asked why she multiplied by 3, she said she wanted all denominators to be the same. My understanding here is that AT remembers that with different denominators, she must multiply but does not remember how the multiplying method works. Her explanation is below.

AT

Interviewer: How did you come up with your answer?

AT: Mam, in this fraction $\frac{3}{9}$ *I multiplied 9 by 3; then I also multiplied 8 by.*

Interviewer: Please explain why [you] multiplied the two denominators by 3?

AT: Here, Mam, I wanted denominators to be the same.

Interviewer: What about the numerators?

AT: Mam, I then added 3 plus 2 equals to 5; I then multiplied 5 by 2, to give me 7. Then my answer is $\frac{12}{27}$*.*

From the above responses, it is evident that learners with high-level achievements also experience difficulties when adding fractions with different denominators. They confuse the rules, and they are unable to identify the HCD. According to the data above, learners used only one method they could not fully understand.

4.2.1.2 Data from the Average Achievers Category (AA)

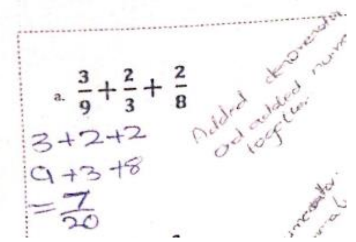


Figure 4.4: Learner HM'S response

The response in Figure 4.4 above is from learner HM, who is an average achiever. Learners IN and SN, also average achievers, made the same mistake. The learners also used the same incorrect method. They added the numerators together and then added the denominators together to get their answers. The following are the explanations for both the answers above:

HM

Interviewer: How did you come up with your answer?

HM: I added the numerators together: 3 plus 2 plus 2, then [I] got 7.

Interviewer: OK, [I] am listening.

HM: After I said 9 plus 3 equals to 12, then I added 8 equals to 20. My answer is $\frac{7}{20}$.

IN

Interviewer: How did you come up with your answer?

IN: I said 3 plus 2 again, plus 2. The answer is 7.

Interviewer: After that, what happened?

IN: Then I said 9 plus 3 equals to 12; then I added 8 equal to 20. My answer is $\frac{7}{20}$.

According to their explanation, it is evident that they lack procedural knowledge. The misconception thereof is associated with procedural knowledge.

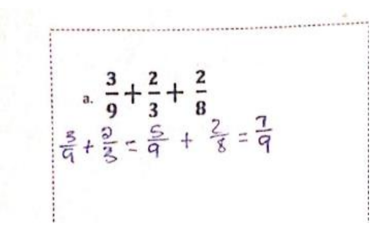


Figure 4.5: Learner FB's response

Learner FB's response is reflected above. He added all the numerators together and then chose the bigger denominator according to his explanation. This is how he explained her answer:

FB

Interviewer: How did you come up with your answer?

FB: I said 3 plus 2 again plus 2; the answer is 7.

Interviewer: Then?

FB: After I chose the bigger denominator and I got 9, and then write it as denominator in my answer, $\frac{7}{9}$.

From the above explanation of learner FB, it is clear that the conceptual knowledge is not clear to the learner. From my understanding here, factorising is also a problem because she didn't check if 8 is a factor of 9. I can conclude that her misconception is associated with a conceptual error.

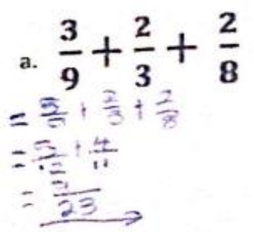


Figure 4.6: Learner KL's response

The answer in Figure 4.6 above is from learner KL. She added the first numerator with the second numerator and got 5, when she added the second numerator with the third numerator, it gave her 4, then she added 5 and 4 together and got 9 as the numerator. She did the same thing with denominators; she added 9 and 3, which gave her 12, then added 3 and 8, which gave her 11. After adding 12 and 11, she got 23 as the answer. Her response when asked how she got that answer is reflected below.

KL

Interviewer: How did you come up with your answer?

KL: In $\frac{3}{9}$ plus $\frac{2}{3}$, I have added numbers on top, 3 plus 2; then I got my answer as 5. Then [I] added numbers at [the] bottom, which is 9 plus 3. Then I got my answer as 12; I wrote it as $\frac{5}{12}$.

Interviewer: After that, what happened with one?

KL: Then in $\frac{2}{3}$ plus $\frac{2}{8}$ I started by adding 2 plus 2 on top. I got my answer 4, and then added numbers at [the] bottom, 3 plus 8, and then I got my answer as 11. My answer is now $\frac{4}{11}$.

Interviewer: How did you get your answer then?

KL: I now added $\frac{5}{12}$ plus $\frac{4}{11}$, then get my answer as $\frac{9}{23}$.

It is evident that learner KL doesn't understand the concept of the addition of fractions clearly. She is confusing the whole concept, treating the numerators and denominators as separate whole numbers. The misconception is, therefore, associated with a conceptual error in the addition of fractions.

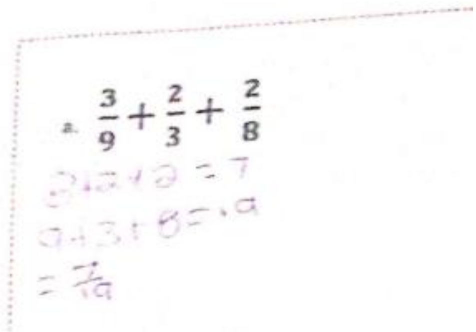


Figure 4.7**: Learner LS's response**

Figure 4.7 reflects learner LS's answer. She added the numerators and the denominators together but miscounted the answer from the denominators. The explanation of her answer is reflected below.

LS

Interviewer: How did you come up with your answer?

LS: I added the numerators together: 3 plus 2 plus 2, then got 7.

Interviewer: OK, [I] am listening

LS: After I said 9 plus 3 equals to 12, then I added 8 equals to 20; my answer is $\frac{7}{19}$.

It is clear from the above explanation that though she used the wrong method still he has a problem with addition, since he miscalculated the denominator. The error seen here is an application error and a misconception associated with conceptual error in the addition of fractions.

4.2.1.3 Data From The Low Achievers Category (LA)

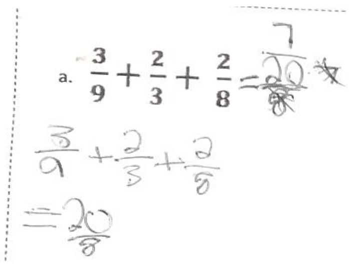


Figure 4.8: Learner SN's response

Figure 4.8 reflects the answer of learner SN, who is a low achiever. Learner MS also used the same method of addition of fractions. The learners added the numerators together and denominators together. When asked how they got their answers, they responded as follows:

SN

Interviewer: How did you come up with your answer?

SN: I said 3 minus 9 plus 2 minus 3, plus 2 minus 8.

Interviewer: OK, am listening, what is your answer then?

SN: My answer is $\frac{7}{20}$.

MS

Interviewer: How did you come up with your answer?

MS: I added 3 plus 2 plus 2, got my answer as 7.

Interviewer: Then after?

MS: Then I added 9 plus 3 plus 8, got my answer as $\frac{7}{20}$.

SN confuses the division (÷) and the subtraction (−)signs. This confusion shows that the learner lacks the basics; however, he still uses the same methods of adding numerators and denominators together; it looks like he also did not know how to write his fractions and manage to swap the numbers. Also, MS used the same method of adding numerators and denominators. This is the evidence of a conceptual and application error.

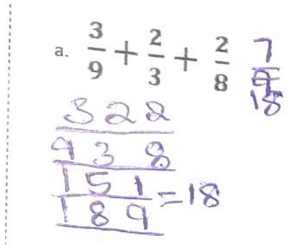


Figure 4.9: Learner TPN's response.

Figure 4.9 above reflects TPN's answer. According to this answer, he added the numerators together and the denominators together; however, he miscalculated the denominators. E

He explained how he got his answer:

TPN

> *Interviewer: How did you come up with your answer?*
> *TPN: I added 3 and 2; my answer is 5, then I added 2, which is 7.*
> *Interviewer: And this one, how did you get the answer?*
> *TPN: Then I added 9 plus 3, got 11, then plus 8, got 18, then my answer is* $\frac{\mathbf{7}}{\mathbf{18}}$.

TPN miscalculated and lost track of his calculations, providing an incorrect answer. The error seen here is an application error and a misconception associated with conceptual error in the addition of fractions.

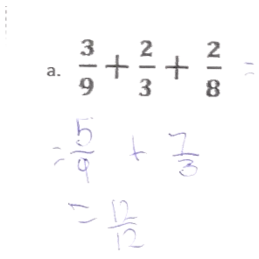


Figure 4.10: Learner PL's response

Figure 4.10 above indicates that learner PL is confused. She added the numerators of the first and second fractions to get the answer of 5, and then added all numerators to get 7. She then added 5 and 7 together to get 12 as the numerator. To determine the denominator, she only took the denominator of the first fraction and added it with the second fraction to make it 12. Her response to the interview is reflected below.

PL

Interviewer: How did you come up with your answer?

PL: I added 3 and 2 my answer is 5, then I added 2 which is 7

Interviewer: And then?

PL: Then I added 7 and 5 and got 12 as my answer

Interviewer: What about denominators? How did you get your answer?

PL: I added 9 and 3 then got 12, but my answer is $\frac{12}{12}$

Evidently, the learner lacks basics; she knows that she must add but gets confused about how to do it. The learners' misconception is associated with conceptual error.

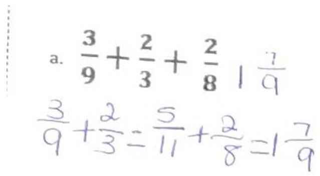


Figure 4.11: Learner NC's response

Figure 4.11 above indicated that learner NC added the numerators together, as shown in the picture and added the denominators together but miscalculated the denominators. She responded as follows on how she got her answer:

NC

Interviewer: How did you come up with your answer?

NC: I added 3 and 2; my answer is 5, then I added 2, which is 7.

Interviewer: OK, and what about the denominators?

NC: I said 9 plus 3 got my answer as 11, then plus 8 my answer is 19, and then removed 1 from 19 and make it [a] whole number, then put 9 as my denominator, my answer is $1\frac{7}{19}$.

Learner NC confuses the concepts; although she also did the same thing as others, she miscalculated and separated the numbers to make it a mixed fraction.

It is evident that even learners in a high-level performance category have difficulties in the addition of fractions. Learners cannot identify the Least Common Denominator (LCD) when adding fractions with different denominators. They cannot use the least common multiple (LCM) method or change the denominators to be all the same. Some mistakes are careless; other mistakes show that they still struggle with the concept of addition, and they don't have the foundations.

4.2.2 Question 2: Factual, Conceptual and Procedural Errors in the Subtraction of Fractions With Different Denominators

$\frac{23}{24} - \frac{1}{4} - \frac{2}{12}$

With this question, the researcher is trying to see if the learners can identify the common denominator or apply the correct procedure to determine the answer. It is evident from Table 4.2 that learners subtract the numerators and then subtracts the denominators to arrive at an answer.

4.2.2.1 Data From the High Achiever Category (HA)

EM is a learner from the high achievers' category; her answer when attempting to answer Q2 is reflected in Figure 4.12 below.

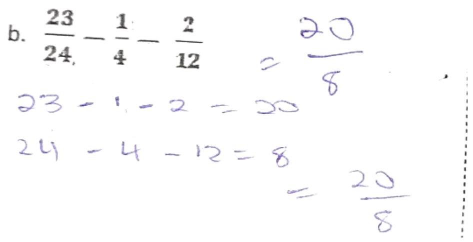


Figure 4.12: Learner EM's response

Explaining her answer as per above, she said that she subtracted 23 – 1 − 2 = 20 (numerators), then she subtracted 24 – 4 – 12 = 8 (denominators). EY and GN gave this same response when they were interviewed, responding as follows:

EM

Interviewer: How did you come up with your answer?

EM: I subtracted Mam.

Interviewer: OK, how did you subtract to get your answer?

EM: I subtracted 24 – 4 – 12 – answer is 8, then I got my answer as $\frac{20}{8}$.

GN

Interviewer: How did you come up with your answer?

GN: I have subtracted $\frac{23}{24} - \frac{1}{4}$ *and my answer is* $\frac{22}{20}$.

Interviewer: And then what else?

GN: Then I said $\frac{22}{20}$ *minus* $\frac{2}{12}$ *and my answer is* $\frac{20}{8}$.

The excerpt above makes it clear that some learners do not apply the rule of identifying the HCD when dealing with different denominators in subtracting fractions. To my understanding, based on the data above, they think that they should subtract the numerator and also need to subtract the denominators to get the correct answer. The learners believe the numerator and denominator can be treated as separate whole numbers.

Even when asked to clarify their answers, they still mentioned that they subtracted all numerators together and then subtracted all the denominators together, after which they would get their answer. The above learners lack procedural and factual understanding. Their misconception is associated with procedural and factual errors in the subtraction of fractions.

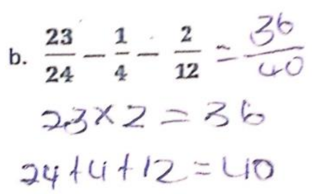


Figure 4.13: Learner AT's response

Figure 4.13 above depicts the response for learner AT. The learner is very confused and doesn't even understand why she followed that procedure. When asked how she got that answer, she said she multiplied 23 by 2 to get 36, then added 24, 4 and 12 together to get her response is reflected below.

AT

Interviewer: How did you come up with your answer?

AT: Mam, here I multiplied 23 by 2, which is equal to 36; then I said 24 + 4 + 12 equals to 4.

Interviewer: Then after.

AT: Then I got my answer as $\frac{36}{40}$.

From learner AT's response above, she doesn't seem to understand the nature of numbers and their interactions. This misunderstanding has interfered with the knowledge of operations. The researcher, therefore, concluded that the learners lack procedural and conceptual understanding of the subtraction of fractions.

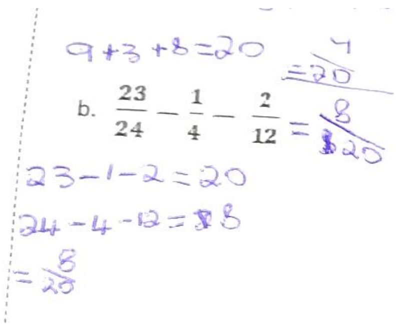


Figure 4.14: Learner JT's response

Figure 4.14 above reflects learner JT's answer. The learner subtracted the numerators and denominators together but swapped the answers to make the fraction a proper fraction. His explanation of how he obtained his answer is set out below.

JT

Interviewer: How did you come up with your answer?

JT: I subtracted, Mam; I said 23 subtract 1 subtract 2 equals to 20.

Interviewer: Then after?

JT: Then I said 24 subtract 4 subtract 12 equals to 8, my answer is $\frac{8}{20}$.

It is clear from the above response that learners mix up the parts and do not understand the concepts. This also indicates that learners have a problem simplifying the fractions. The misconception above is associated with conceptual errors in the subtraction of fractions.

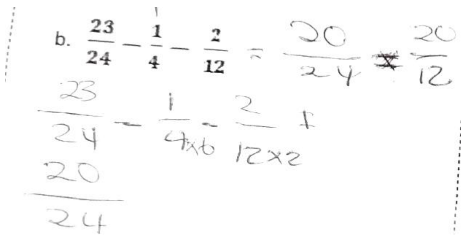


Figure 4.15: Learner DK’s response

Figure 4.15 above depicts learner DK’s response. She subtracted numerators together, and, when coming to the denominator, tried to make the denominator the same using the incorrect procedure. Her explanation of how she obtained the answer above is set out below.

DK

> *Interviewer: How did you come up with your answer?*
>
> *DK: Mam, I subtracted the numbers on top; I said 23 minus 1 minus 2 equals to 20.*
>
> *Interviewer: What about the numbers at the bottom?*
>
> *DK: Then I multiplied the second number below by 6, and the third number at the bottom by 2, to make it the same in all the numbers, then I got my number as* $\frac{20}{24}$.

It is clear from the above response that DK used the correct procedure but in the wrong way. She considers the numerators when multiplying, therefore the misconception associated with a procedural error in the subtraction of fractions.

The above data shows that even the high achievers struggle with subtracting fractions with different denominators. Learners confuse the procedures, don’t understand the concepts, and lack the basics. It is, therefore, concluded that learners make errors in the subtraction of fractions. They subtract the numerators and the denominators together; they don’t understand the process they apply and make careless mistakes when calculating and swapping the denominators without any reason to make the fractions improper fractions, meaning they cannot simplify the fractions.

4.2.2.2 Data is from the Average Achiever Category (AA)

According to the data collected, three learners from the average achievers' category made the same mistake. Learners FB, IN and LS subtracted the numerators together and denominators together in getting their answer. Below is their response when explaining their answer:

FB

Interviewer: How did you come up with your answer?

FB: I subtracted; I said 23 minus 1 minus 2 numbers on top.

Interviewer: Okay.

FB: Then I said 24 minus 4 minus 12. [I] got my answer as $\frac{20}{8}$.

IN

Interviewer: How did you come up with your answer?

IN: Okay, Mam. From 23, I took out 1, then took out 2 again to make it 20.

Interviewer: Then after?

IN: Then after, from 24, I took out 4, then took out 12 again, then got 8, and my answer is $\frac{20}{8}$.

LS

Interviewer: How did you come up with your answer?

LS: I subtracted, I said 23 minus 1, [I] got 22, and then I said again 22 minus 2, and then got my answer as 20.

Interviewer: What happened to denominators?

LS: I subtracted 4 from 24, then subtracted 12 again and got 8, my answer is $\frac{20}{8}$.

It is evident that learners' errors in subtraction of fractions involve subtracting the numerators together and subtracting the denominators together to get their answers. Learners do not look for the HCD to solve their problems. The misconception is, therefore, associated with conceptual errors in the subtraction of fractions.

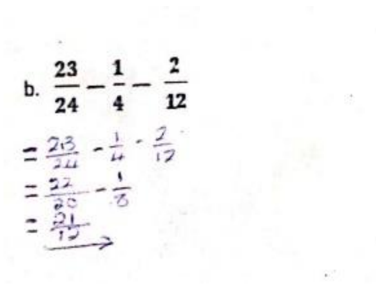


Figure 4.16: Learner KL's response

Figure 4.16 above reflects learner KL's answer. She subtracted the second fraction from the first fraction ($\frac{23}{24} - \frac{1}{4}$) and got $\frac{22}{20}$. She then subtracted $\frac{1}{8}$ from $\frac{22}{12}$ and the answer she got was $\frac{21}{12}$. She explained her answer as follows:

KL

Interviewer: How did you come up with your answer?

KL: I subtracted; I said 23 minus 1, [I] got 22, and then I said again 22 minus 1, and then got my answer as 21.

Interviewer: What happened to the denominators?

KL: I subtracted 4 from 24; my answer is 12, and I wrote my answer as $\frac{20}{12}$.

It is clear from the above comments that learners do not understand the fractional concept. Even dealing with procedures in fractions is a problem. The misconception above is associated with conceptual and procedural errors in the subtraction of fractions.

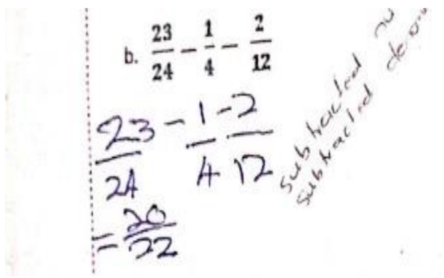


Figure 4.17: Learner HM's response

Figure 4.17 reflects learner HM's response. He subtracted 1 and 2 from 23 on the numerator and then got the numerator as 20. He then subtracted 4 from 24 but mistakenly wrote 22 as his denominator. Below is his explanation when responding to how he obtained his answer.

HM

Interviewer: How did you come up with your answer?

HM: I subtracted, I said 23 minus 1, got 22, and then I said again 22 minus 2, and then got my answer as 20.

Interviewer: What happened to the denominators?

HM: I subtracted 4 from 24; my answer is 22, and I wrote my answer as $\frac{20}{22}$.

It is evident from the above data that the learners do not follow the correct procedure in dealing with fractions. They also lack conceptual knowledge in the subtraction of fractions. The misconception is associated with conceptual and procedural errors in the subtraction of fractions.

Clearly, learners from the average category face challenges when dealing with the subtraction of fractions. Some of the errors are caused by miscalculations of fractions, treating numerators and denominators as separate and confusing the procedures.

4.2.2.3 Data from the Low Achiever Category (LA)

According to the data collected, three learners from the LA category, MS, NC, and SN, made the same mistake when subtracting fractions with different denominators. However, NC and SN swapped the denominators to produce an improper fraction.

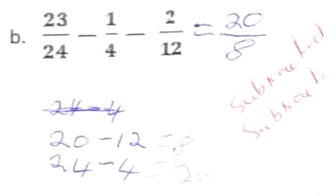


Figure 4.18: Learner NC's response

The learners subtracted the numerators together and subtracted the denominators together to get their answers. Below is their response when explaining their answers:

NC

Interviewer: How did you come up with your answer?

NC: I subtracted numbers on top, 23 minus 1 minus 2, and then my answer is 20.

Interviewer: OK [pause].

NC: I again subtracted the number at the bottom, 24 minus 4 minus 12; my answer is 8.

Interviewer: OK, please explain your answer.

NC: My answer is $\frac{20}{8}$*, but I cannot write it that way. I must put [the] small number in [the] top then the bigger number at the bottom* $\frac{8}{20}$*.*

SN

Interviewer: How did you come up with your answer?

SN: I subtracted, 23 minus 1 minus 2, and then my answer is 20.

Interviewer: And then?

SN: I again subtracted the number at the bottom, 24 minus4 minus 12; my answer is 8.

Interviewer: OK, please explain your answer.

SN: My answer is $\frac{8}{20}$, because 8 is smaller than 20.

MS

Interviewer: How did you come up with your answer?

MS: I said on the number on top, 23 minus 1, got my answer as 22, then 22 minus 2, [my] answer is 20.

Interviewer: Then after?

MS: Numbers at the bottom, I said 24 minus 4, got 20, then minus 12, got 8, my answer is$\frac{20}{8}$.

It is evident that learners' errors in subtraction of fractions involve subtracting the numerators together and subtracting the denominators together to get their answer. Learners do not look for HCD to solve their problems. Learners who swapped their answer it's clear that that they don't foundation in fractions. The misconception is therefore associated with conceptual errors in subtraction of fractions.

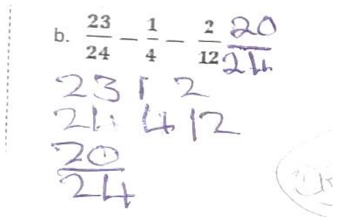


Figure 4.19: Learner TPN's response

Figure 4.19 above reflects learner TPN's answer. According to this answer, he subtracted the numerators together and then chose the HCD, the procedure was correct, but the application of the procedure became a problem. Below is his explanation of how he got his answer:

TPN

Interviewer: How did you come up with your answer?

TPN: I subtracted Mam, I said 23 minus 1 and again minus 2 my answer is 20.

Interviewer: And this one, how did you get the answer?

TPN: Then I to 24 as my denominator because is bigger than other denominators$\frac{20}{24}$.

It's clear that the learner remembered the procedure but couldn't apply it in the correct way. The error in concern is application procedural errors, but the misconception is associated with procedural errors in the subtraction of fractions.

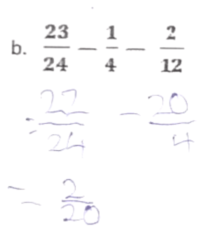


Figure 4.20: Learner PL's response

Learner PL above is totally confused. She subtracted the numerators together, got 20, then after she subtracted the numerator of the first fraction with that of the second fraction got 22 after she subtracted 20 from 22 got her answer as 2, with the denominator she only subtracted 4 from 4 and got 20 as her denominator. Below is her response on the interview.

PL

Interviewer: How did you come up with your answer?

PL: I subtracted, I said 23 minus 1 minus 2 my answer is 20.

Interviewer: And then?

PL: Then I said 23 minus 1 equals to 22, I then subtract 20 from 22, got my answer as 2.

Interviewer: What about denominators?

PL: I said 24 subtract 4 my answer is 20, $\frac{\mathbf{2}}{\mathbf{20}}$.

It is evident that the learners' lacks basics; she knows that she must subtract but get confused on how she must subtract. Learners' misconception is clearly associated with conceptual error.

4.2.3 Question 3: Factual, Conceptual and Procedural Errors in the Addition of Mixed Fraction With the Same Denominators

$$3\,\frac{3}{5} + 1\,\frac{4}{5}$$

The researcher here is trying to see if the learners will be able to work out denominator or apply the correct procedure to work out the answer. It is evident from Table 4.2 that learners add the whole numbers together, then add numerators together and then add the denominators together to get the answer. Some learners lose the next whole, whereas some don't how to simply the fraction in order to get the correct answer.

4.2.3.1 Data From the High Achiever Category (HA)

According to the data, four learners from the HA category managed to get the correct answer, except for JT.

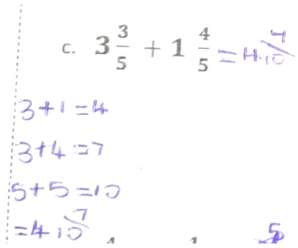


Figure 4.21: Learner JT's response

JT added the whole numbers together, then added the numerators, and then added the denominators. He explained how he obtained his answer:

JT

Interviewer: How did you come up with your answer?

JT: I added 3 and 1 my answer is 4; then I added 3 and 4 my answer is 7

Interviewer: After?

JT: The added 5 and 5 below my answer is $4\frac{7}{10}$.

It is clear from the answer below that the learner lack conceptual knowledge. Therefore the misconception associated with conceptual errors in addition of mixed fractions.

4.2.3.2 Data From the Average Achievers Category (AA)

Three learners from the AA category manage to get correct answer. Only two learners got the answer incorrect.

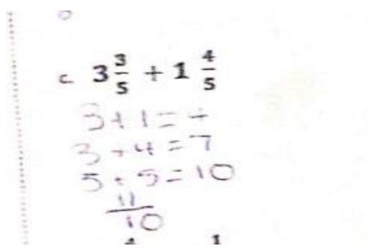


Figure 4.22: Learner LS's response

Learner LS got the procedure incorrect. She added the whole numbers together, and then added numerators to the answer she got from whole numbers and numerators. She then added the answers together and got 11; then she added the denominators together and got 10. She used 11 as her numerator and 10 as the denominator. The response below explains how he got his answer:

LS

Interviewer: How did you come up with your answer?

LS: I said 3 plus 1 got 4, and then I added 3 plus 4 from top then got 7,

Interviewer: Then after?

LS: After I added 4 plus 7, then got 11.

Interviewer: OK and what about the denominators?

LS: I said 5 plus 5 then got 10 my answer is $\frac{11}{10}$.

It is clear from the answer below that the learner lacks procedural knowledge. Therefore, the misconception is associated with procedural errors in the addition of mixed fractions.

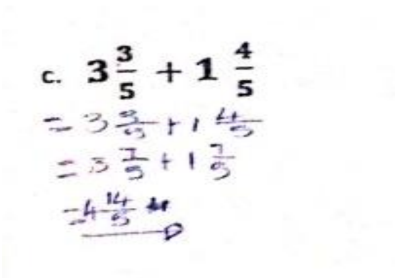


Figure 4.23: Learner KL's response

Figure 4.23 above reflects learner KL's answer. The learner added the whole numbers together got her answer as 4; she added the numerators together got the answer as 7, and then use 7 as common numerator for both fractions. She then added the two numerators together to make it 14. Below is her response when trying to explain her answer.

KL

Interviewer: How did you come up with your answer?

KL: I said 3 plus 1 got 4, and then I added 3 plus 4 from top, and then got 7.

Interviewer: Then after?

KL: I then make 7 to be the same in the two fractions so that I will add it.

Interviewer: Then what happen after?

KL: I said 7 plus 7 and it gave me 14.

Interviewer: OK and what about the denominators?

KL: I said 5 is my denominator because 5 is the same in all fractions, I got my answer as $4\frac{14}{5}$.

It is clear that KL doesn't really understand the procedures when adding mixed fractions; the learner knows that she has to add but not how. The misconception, therefore, is associated with a procedural error in addition of mixed fractions.

4.2.4 Question 4: Factual, Conceptual and Procedural Errors in the Addition of Mixed Fractions With the Same Denominators

$3\ \frac{4}{7} + 2\ \frac{1}{7}$

The researcher here is trying to see if the learners can apply the correct procedure to work out the answer. It is evident from Table 4.2 that learners subtract the whole numbers together, then subtracts numerators together, and then subtract the denominators together to get the answer. Some learners lose the next whole, whereas some don't know how to simplify the fraction to get the correct answer.

4.2.4.1 Data From the High Achievers Category (HA)

All learners for the high achievers category got this question correct.

4.2.4.2 Data From the Average Achievers Category (AA)

Three learners from the average category got this question correct.

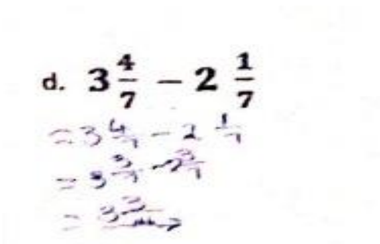


Figure 4.24: Learner KL's response

Figure 4.24 above reflects learner KL's answer. Based on her answer above, she didn't subtract the two whole numbers; instead, she only took the whole number of the first mixed fraction and then subtracted the numerators and added the denominators. Below is her explanation on how she got her answer:

KL

Interviewer: How did you come up with your answer?

KL: In 3 $\frac{4}{7}$ minus 2 $\frac{1}{7}$ I said 4 minus 1 got 3 then I added 7 and 7 from the numbers at the bottom I got 14.

Interviewer: After that what happened?

KL: Then I took 3 as my whole number and my answer is $3\frac{3}{14}$.

It is clear from the above explanation that the learner has a problem dealing with mixed fractions, and she carelessly overlooked the second whole in the second fraction. The misconception, therefore, is associated with a procedural error in subtracting mixed fractions.

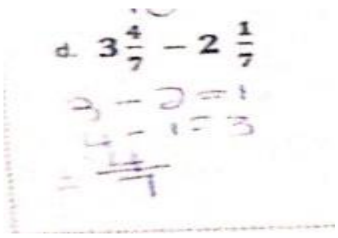


Figure 4.25: Learner LS's response

Figure 4.25 above reflects learner LS's answer. She subtracted the whole numbers and the numerators, then added the answers together and made it the numerator. She explained how she obtained her answer:

LS

Interviewer: How did you come up with your answer?

LS: I said 3 minus 2 its 1, then numbers on top I said 4 minus 1 got 3, then I added I and 3 I got 4.

Interviewer: OK, [I] am listening.

LS: After I took 7 put it the bottom because the numbers are all the same and my answer is $\frac{4}{7}$.

4.2.4.3 Data from the Low Achievers Category (LA)

Only one learner from the LA category got this question correct. According to the data collected, two learners NC, and SN made the same mistake when subtracting mixed fractions.

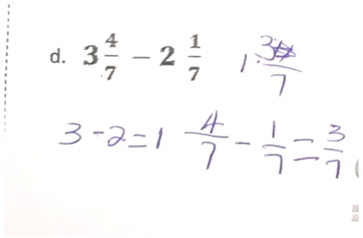


Figure 4.26: Learners NC's and SN's response

The learners subtracted the whole numbers and the numerators together to get their answers of $\frac{3}{7}$; they forgot to include the whole number. They explained how they obtained their answers:

NC

Interviewer: How did you come up with your answer?

NC: I said 3 minus 2 equals 1, and then said 1 $\frac{4}{7}$ *minus* $\frac{1}{7}$ *.*

Interviewer: What about the denominator?

NC: The denominator doesn't change Mam, and then I got my answer as $\frac{3}{7}$*.*

SN

Interviewer: How did you come up with your answer?

SN: I subtracted Mam, I said 4 minus 1, the numbers on top then my answer is $\frac{3}{7}$*.*

Interviewer: What about the whole numbers? What happened to them?

SN: I forgot to subtract the whole numbers, Mam.

It is evident that the learners made careless errors when dealing with fractions. They have omitted the whole numbers in their answers. The misconception is, therefore, associated with conceptual errors in the subtraction of mixed fractions.

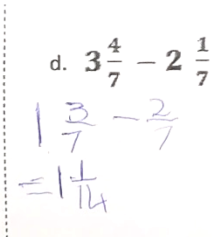


Figure 4.27: Learner PL's response

Figure 4.27 above reflects learner PL's answer. She subtracted the whole numbers, then subtracted the numerators and then added the denominators. Her comments during the interview are reflected below.

PL

Interviewer: How did you come up with your answer?

PL: I subtracted 2 from 3 got my answer as 1.

Interviewer: And then?

PL: Then I subtracted 1 from 4 got my answer as 3, after I said 3 minus 2 equals to 1.

Interviewer: What about denominators? How did you get your answer?

PL: I added 7 and 7 got 14, and my answer is $1\frac{1}{14}$.

Clearly, the learner mixed the operations; she subtracted and then had to add denominators. The misconception is associated with procedural errors in subtractions of mixed fractions.

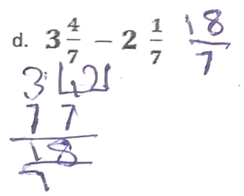


Figure 4.28: Learner TPN's response

Figure 4.28 above reflects learner TPN's answer. The learner's answer is confusing and I really could not understand how he got his answer. He explained as follows:

TPN

Interviewer: How did you come up with your answer?

TPN: I counted, 3 plus 4 plus 2 plus 1 plus 7, my answer is 18.

Interviewer: And this one how did you get the answer?

TPN: This number at the bottom is 7 my answer is $\frac{18}{7}$.

TPN used the incorrect procedure, and though he calculated his answers, they are still incorrect. The error seen here is an application error and the misconception is associated with a conceptual and procedural error in addition of fractions.

4.2.5 *Question 5: Factual, Conceptual and Procedural Errors in the Addition of Fractions With the Same Denominators*

$$\frac{4}{10}+\frac{1}{10}$$

The researcher is trying to see if the learners can use the common denominator or apply the correct procedure to find the answer. It is evident from Table 4.2 that learners add the numerators together and then adds the denominators together to get the answer.

4.2.5.1 Data From the High Achievers Category (LA)

According to data collected four learners from the HA category got the answer correct.

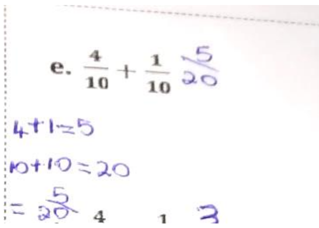


Figure 4.29: Learner JT's response

Figure 4.29 above reflects learner JT's answer. The learner added numerators and the denominators together. He commented as follows in the interview:

JT

Interviewer: How did you come up with your answer?

JT: I added Mam, I said 4 plus 1 equals to 5

Interviewer: Then after?

JT: Then I said 10 plus 10 equals to 20, and then my answer is $\frac{5}{20}$

4.2.5.2 Data From the Average Achievers Category (AA)

According to the data collected three learners in AA got Question 5 correct.

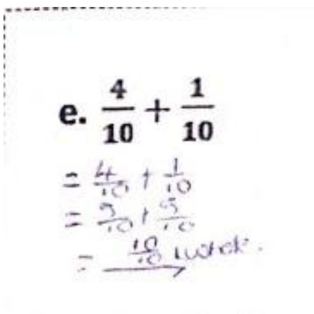


Figure 4.30: Learner KL's addition of numerators

Figure 4.30 above reflects learner KL's answer. Learner KL added the numerators together. He repeated the answer he obtained again by adding 5 and 5 together to get 10. He then added the denominator together to get 20. Below is his response to the above answer.

KL

Interviewer: How did you come up with your answer?

KL: I added 4 plus 1 is 5, I make 5 in both fractions.

Interviewer: After that what happened with one?

KL: Then I said $\frac{5}{10}$ *plus* $\frac{5}{10}$ *then my answer* $\frac{10}{10}$.

It is evident that learner KL doesn't understand the concept of the addition of fractions clearly. He is confusing the whole concept, treating the numerators and denominators as separate whole numbers. The misconception is, therefore, associated with conceptual errors in the addition of fractions.

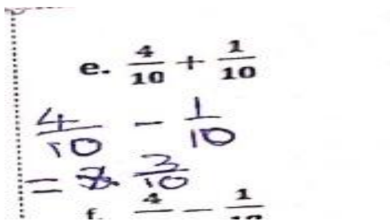


Figure 4.31: Learner HM's response

Figure 4.31 above reflects learner KL's answer. He subtracted the numerators instead of adding

HM

Interviewer: How did you come up with your answer?

HM: I subtracted Mam, I said $\frac{4}{10}$ *minus* $\frac{1}{10}$ *my answer is* $\frac{3}{10}$

Learner HM confused the operations. He carelessly changed the addition into subtraction. The above misconception is associated with factual error in subtraction of fraction with the same denominator.

4.2.5.3 Data From the Low Achievers Category (LA)

According to the data collected, four learners from the LA category got Question 5 correct.

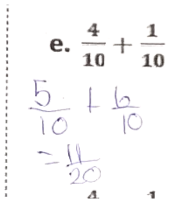


Figure 4.32: Learner PL's response

Learner PL above changed the numbers altogether. After changing the numbers, she added the numerators together and denominators together. Learner PL is totally confused. Below is her response during the interview.

PL

Interviewer: How did you come up with your answer?

PL: I added 5 and 6 my answer is 11.

Interviewer: And then?

PL: Then I added 10 and 10 and got 20 as my answer $\frac{11}{20}$.

Interviewer: What about denominators? How did you get your answer?

It is evident that the learners lacks basics; she knows that she must add but get confused on how she to do it. The learners' misconception is clearly associated with a conceptual error.

4.2.6 Question 6: Factual, Conceptual and Procedural Errors in the Subtraction of Fractions with the same Denominators

$\frac{\mathbf{4}}{\mathbf{12}} - \frac{\mathbf{1}}{\mathbf{12}}$

The researcher is trying to see if the learners can use the common denominator or apply the correct procedure to find the answer. It is evident from Table 4.2 that the learner subtracts the numerators together and then subtracts the denominators together to get the answer.

4.2.6.1 Data from the High Achievers Category (HA)

All learners from the HA category got Question 6 correct.

4.2.6.2 Data from the Average Achievers Category (AA)

According to the data collected, four learners from AA got Question 6 correct.

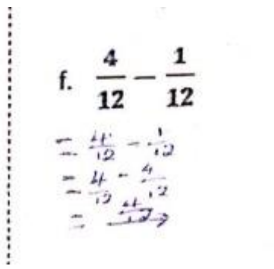


Figure 4.33: Learner KL's response

Figure 4.33 above reflects learner KL's answer. She subtracted the numerators but did not work out the answer correctly. She responded as follows on how she obtained her answer:

KL

Interviewer: How did you come up with your answer?

KL: In $\frac{4}{12}$ *minus* $\frac{1}{12}$*.*

Interviewer: After that what happened after?

KL: Then the number at the bottom is the same; I must use it as the number at the bottom.

Interviewer: Then what is your answer?

KL: My answer is $\frac{4}{12}$*.*

It is evident that learner KL doesn't understand the concept of addition of fractions. She confuses the whole concept and makes a careless mistake. The misconception is, therefore, associated with a conceptual error in the subtraction of fractions.

4.2.6.3 Data From the Low Achievers Category (LA)

Three learners from the LA category got Question 6 correct.

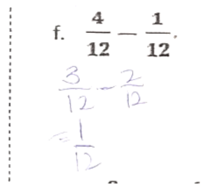


Figure 4.34: Learner PL's response

Figure 4.34 above reflects learner PL's answer. The learner subtracted the numerator. She got the answer by subtracting 2 from 3. The learner wrote 12 as the denominator. She explained how she obtained the answer:

PL

Interviewer: How did you come up with your answer?

PL: I subtracted, I said 4 minus 1 my answer is 3, and then I subtracted 2 from 3. My answer is 1.

Interviewer: And then?

PL: Then I wrote 1 on top and 12 at the bottom because 12 is the same, my answer is $\frac{1}{12}$.

It is evident that the learner lacks basics; she knows that she must subtract but become confused about how to subtract. The Learner's misconception is clearly associated with conceptual and procedural errors in the subtraction of fractions of the same denominators.

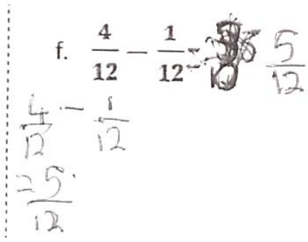


Figure 4.35: Learner SN's response

Figure 4.35 above reflects learner SN's answer. He added instead of subtracting. He explained how he obtained his answer as follows:

SN

Interviewer: How did you come up with your answer?

SN: I made a mistake by adding Mam.

It is evident that learners can make careless errors when dealing with fractions.

4.2.7 Question 7: Factual, Conceptual and Procedural Errors in the Subtraction of Mixed Fractions of Fractions With Different Denominators

$$5\,\frac{4}{4} - 3\,\frac{1}{5}$$

The researcher here is trying to see if the learners can identify the common denominator or apply the correct procedure to work out the answer. It is evident from Table 4.2 that learners make careless, application, factual, procedural and conceptual mistakes.

4.2.7.1 Data From the High Achievers Category (HA)

According to the data collected all learners in HA category got Question 7 incorrect. Below is learner EM.

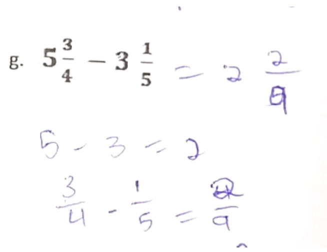


Figure 4.36: Learner EM's response

Figure 4.36 above reflects learner EM's answer. She subtracted the whole numbers together, subtracted the numerators together and then added the denominators together.

EM

Interviewer: How did you come up with your answer?

EM: I subtracted the whole numbers, 5 minus 3 equals to 2, 3 minus 1 equals to 2.

Interviewer: OK then, what about the denominator?

EM: I said 4 plus 5 equals to 9, and my answer is 2 $\frac{2}{9}$.

Learner EM mixed the operation all together. The problem is that she didn't know how to look for the HCD. Therefore, the learners' misconception is associated with factual and procedural errors in the subtraction of mixed fractions.

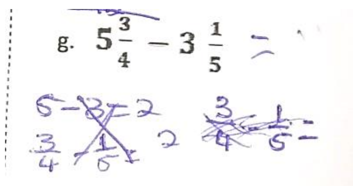


Figure 4.37: Learner GN's response

Figure 4.37 above reflects learner GN's answer.

GN

Interviewer: How did you come up with your answer?

GN: I have added all these numbers [pointing at all THE numerators].

Interviewer: You have added all numbers.

GN: Yes Mam, I added $\frac{3}{9}$ and $\frac{2}{3}$ and $\frac{2}{8}$, I added them together, no man I actually said $\frac{3}{9}$ .plus $\frac{2}{3}$ then got $\frac{5}{12}$, then $\frac{5}{12}$ plus $\frac{2}{3}$ my answer is $\frac{7}{20}$

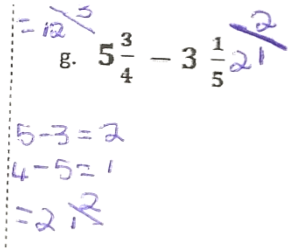


Figure 4.38: Learner JT's response

Figure 4.38 above reflects learner JT's answer. The learner subtracted the whole numbers, then subtracted the denominators together and subtracted the numerators together. The learner's comments during the interview are reflected below.

JT

Interviewer: How did you come up with your answer?

JT: I have subtracted Mam, 5 minus 3 equals 2, subtracted the numerators 3 minus 1 equals to 2, and then I subtracted 4 minus 5 equals to 1.

Interviewer: And how did you come to your answer?

JT: My answer, Mam is 2 $\frac{2}{1}$.

Clearly, learner JT assumed that since she is subtracting, she will have to subtract in all steps without looking for an HCD as per the rule. The above misconception is associated with a procedural error in subtracting mixed fractions with different denominators.

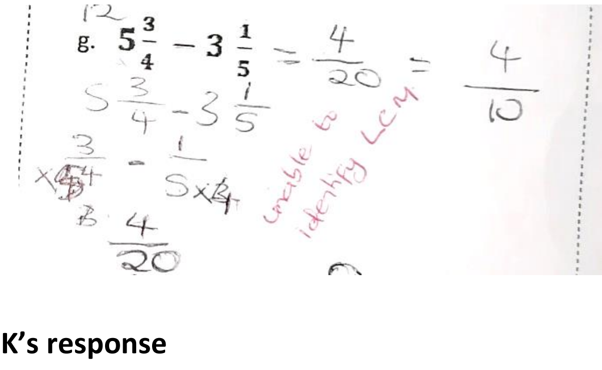


Figure 4.39**: Learner DK's response**

Figure 4.39 above reflects learner DK's answer. She multiplied the denominators together and got 20, then added the numerators together and got 4. She explained how she obtained her answer:

DK

Interviewer: How did you come up with your answer?

DK: Mam, I multiplied the denominators. I said 5 times 4, in the first fraction, and then I said 4 times 5 with the second fraction to get 20.

Interviewer: And what about the numerators?

DK: I said 3 plus 1 to get 4, and my answer is $\frac{4}{20}$.

It is evident that learner DK understands that she has to make the denominators the same but uses the incorrect procedure. The misconception, therefore, is associated with a procedural error in the subtraction of fractions.

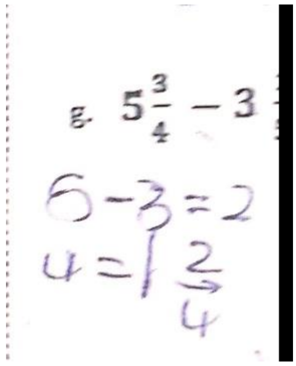


Figure 4.40: Learner AT's response

Figure 4.40 above reflects learner AT's answer. The learner is very confused and doesn't even understand why she followed that specific procedure. When asked how she got that answer, she said she multiplied 9 by 3, and then again multiplied 8 by 3. When asked why she multiplied by 3, she said she wanted all denominators to be same. To my understanding, AT remembers that when having different denominators, she must multiply but does not remember how the multiplication method works. Her explanation of how she obtained the answer is reflected below.

AT

Interviewer: How did you come up with your answer?

AT: Mam, in this fraction $\frac{3}{9}$ *I multiplied 9 by 3, and then I also multiplied 8 by 3.*

Interviewer: Please explain why [you] multiplied the two denominators by 3?

AT: Here, Mam, I wanted denominators to be the same.

Interviewer: What about the numerators?

AT: Mam, I then added 3 plus 2 equals to 5, I then multiplied 5 by 2, and to give me 7, then my answer is $\frac{12}{27}$.

From the above responses, it is evident that learners with high level achievement also experience difficulties when adding fractions with different denominators. They confuse the rules, and they are unable to identify the HCD. According to the data above, learners used only one method they cannot fully understand.

4.2.7.2 Data From the Average Achievers Category (AA)

According to the data collected, all learners from the average category got this question incorrect.

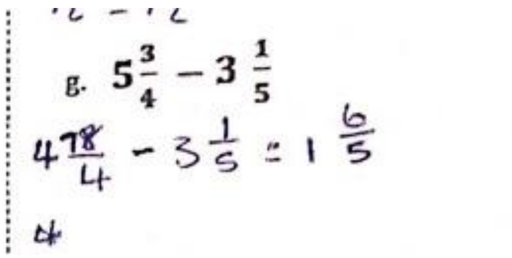


Figure 4.41: Learner FB's response

Figure 4.41 above reflects learner FB's answer. The learner is very confused. The learner responded as follows in the interview:

FB

Interviewer: How did you come up with your answer?

FB: I took 1 from whole number (5) then add 3, it gave me $\frac{7}{4}$.

Interviewer: Okay.

FB: Then I said 4 minus 3 equals to 1, I then said 7 minus 6 equals to 1.

Interviewer: I don't understand.

FB: Mam, my denominators are $\frac{3}{4}$ *and* $\frac{1}{5}$, *I took the bigger denominator Mam, which is 5, my answer is* $1\frac{6}{5}$.

FB is confused and lacks basics; it seems he doesn't have prior knowledge of fractions. The misconception is, therefore, associated with factual, procedural and conceptual errors.

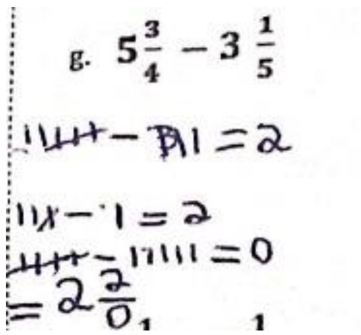


Figure 4.42: Learner IN's response

Figure 4.42 above reflects learner IN's answer. The learner subtracted the whole numbers, and then subtracted the two numerators and the two denominators. The learner responded as follows in the interview:

IN

Interviewer: How did you come up with your answer?

IN: Okay Mam I said 5 subtract 3 my answer got 2, then I took numbers on top and said 3 subtract 1 answer is 2

Interviewer: Then after?

IN: Then after I said 4 minus 5, you can't so my answer will be 0 after I got my answer as 2 $\frac{2}{0}$

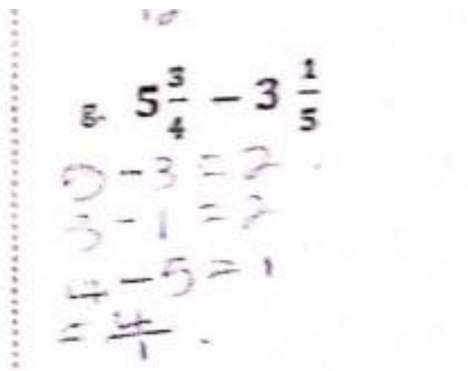


Figure 4.43: Learner LS's response

Figure 4.43 above reflects learner LS's answer. The learner subtracted the whole numbers after subtracting the numerators and then the denominators. The learner explained how the answer was obtained:

LS

Interviewer: How did you come up with your answer?

LS: I subtracted, I said 5 minus 3 got 2, then 3 minus 1 got 2 the 4 minus 5 got 1

Interviewer: Okay then your answer?

LS: I then added 2 and 2, got 4 so my answer is $\frac{4}{1}$

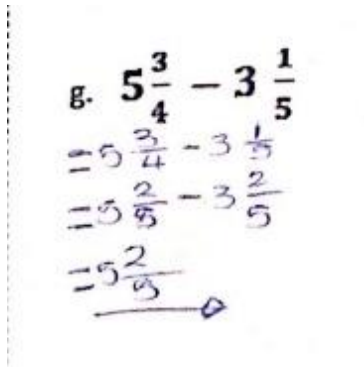


Figure 4.44: Learner KL's response

Figure 4.24 above reflects learner KL's answer. She subtracted the numerators and then used 2 as a common numerator and took 5 as the denominator. Here comments during the interview are reflected below.

KL

Interviewer: How did you come up with your answer?

KL: I subtracted, I said 3 minus 1 got 2.

Interviewer: What happened to the denominators?

KL: I wanted the number below to be the same; then I put 5 in both fractions, then my answer is 5 $\frac{2}{5}$.

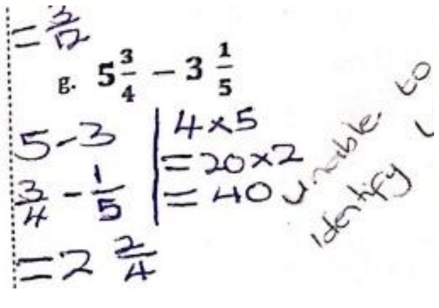


Figure 4.45**: Learner HM's response**

Figure 4.45 above reflects learner HM's answer. The learner subtracted the whole numbers and then subtracted the numerators. The learner responded as follows in the interview:

HM

Interviewer: How did you come up with your answer?

HM: I subtracted, I said 5 minus 3, got 2, and then I said 3 minus1 got 2.

Interviewer: What happened to the denominators?

HM: I took 4 as the number below 2 $\frac{2}{4}$.

It is evident from the above data that learners do not follow the correct procedure in dealing with fractions. They fail to identify the LCD in all problems. They also lack conceptual knowledge of the subtraction of fractions. The misconception is associated with conceptual and procedural errors in the subtraction of fractions.

4.2.7.3 Data From the Low Achievers' Category (LA)

According to the data collected, all learners form the Low Achievers category failed to answer Question 7.

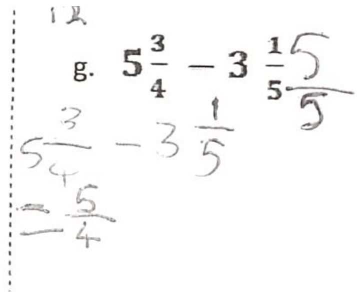


Figure 4.46: Learner SM's response

Figure 4.46 above reflects learner SM's answer. The learner is very confused and seems not to understand the basics of fractions. The learner responded as follows in the interview:

SN

Interviewer: How did you come up with your answer?

SN: I took 5 because is bigger put it on top

Interviewer: What happened with the other numbers?

SN: I did not write them

Interviewer: Okay, then how did you get your answer?

SN: I put 5 on top the 4 at the bottom

The learner doesn't have the basics at all and she does not understand the required procedures.

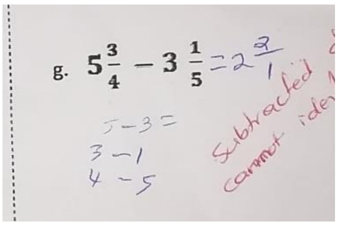


Figure 4.47: Learner MS's response

Figure 4.47 above reflects learner MS's answer. The learner subtracted the whole numbers, the numerators and the denominators. The leaner commented as follows during the interview:

MS

Interviewer: How did you come up with your answer?

MS: I said 5 minus 3, got 2, 3 minus 1, got 2, 4 minus 5, got 1.

Interviewer: Then after?

MS: My answer is 2 $\frac{2}{1}$.

The learners' misconception is associated with conceptual and procedural errors.

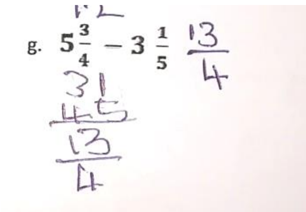


Figure 4.48: Learner TPN's response

Figure 4.48 above reflects learner TPN's answer. The learner is very confused and doesn't understand what is required. He explained as follows on how he obtained the answer:

TPN

Interviewer: How did you come up with your answer?

TPN: I added Mam.

Interviewer: How did you add?

TPN: Then I added the numbers then my answer is $\frac{13}{4}$.

The error seen here is an application error and misconception associated with conceptual errors in the subtraction of fractions.

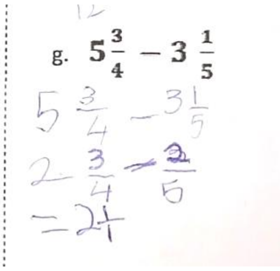


Figure 4.49: Learner PL's response

Figure 4.49 above reflects learner PL's answer. Learner PL above subtracted the numerators together and then subtracted the whole numbers together before finally subtracting the denominators. She explained how she obtained the answer:

PL

Interviewer: How did you come up with your answer?

PL: I said 5 minus 3 got 2, 3 minus 2 got 1 and then 4 minus 5 got 1.

Interviewer: And then?

PL: My answer is $2\frac{1}{1}$.

It is evident that the learner lacks the basics; she knows that she must subtract but gets confused on how. The learner's misconception is clearly associated with conceptual error.

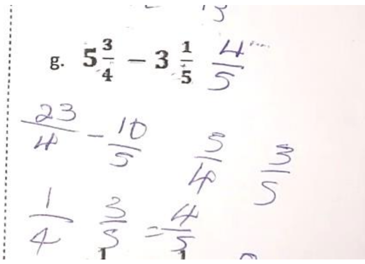


Figure 4.50: Learner NC's response

Figure 4.50 above reflects learner NC's answer. Learner NC above is confused and doesn't understand the basics of fractions. The learner responded as follows in the interview:

NC

Interviewer: How did you come up with your answer?

NC: I put 3 on top then 5 at the bottom.

Interviewer: Why?

NC: So that I get my answer

Learner NC confuses concepts, it's clear that she doesn't understand fractions and the basics. The misconception is associated with factual and conceptual errors in the subtraction of fractions.

4.2.8 Question 8: Factual, Conceptual and Procedural Errors in the Subtraction of Mixed Fractions of Fractions With Different Denominators

$4\,\frac{1}{5} - 3\,\frac{1}{2}$

In Question 8, the same applies as in Question 7. The researcher is trying to see if the learners can identify the common denominator or apply the correct procedure to determine the answer. It is evident from Table 4.2 that learners make careless, application, factual, procedural and conceptual mistakes.

According to data collected, learners in all categories responded the same way as they did with Question 7. Some subtract the whole numbers, subtract the numerators and subtract the denominators, as set out below:

$= 4 - 3 = 1,$

$= 1 - 1 = 0,$

$= 5 - 2 = 3,$

$= 1\,\frac{0}{3}$

It is clear that their misconception is associated with conceptual, factual, procedural errors in subtractions of fractions.

4.2.9 Question 9: Factual, Conceptual and Procedural Errors in Multiplication of Mixed Fractions With Different Denominators

$2\,\frac{5}{6} \times 6\,\frac{2}{5}$

The researcher here is trying to see if the learners will be able to convert the mixed numbers into improper fraction first before they can calculate or apply the correct procedure to work out the answer. It is evident from table 4.2 that learners do careless, application, factual, procedural and conceptual mistakes.

4.2.9.1 Data From the High Achiever Category (HA)

All learners in the High Achievers category got Question 9 incorrect; below are their responses.

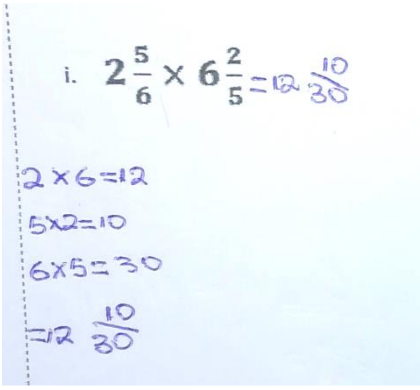


Figure 4.51: Learner JT's response

Figure 4.51 above reflects learner JT's answer. The learner multiplied the whole numbers together, then multiplied the numerators together and then multiplied the denominators together; all other learners did the same procedure.

JT

Interviewer: How did you come up with your answer?

JT: I multiplied Mam, I said 2 times 6 my answer is 12, and then 5 times 2 got 10.

Interviewer: What about the denominator?

JT: I said 6 times 5 got 30 then my answer is 12 $\frac{10}{30}$.

EM

Interviewer: How did you come up with your answer?

EM: I multiplied whole numbers first 6 times 2 got 12, then 2 times 5 got 10, the numbers below 6 times 5 then got 30.

Interviewer: OK.

EM: My answer is 12 $\frac{10}{30}$.

GN

Interviewer: How did you come up with your answer?

GN: I have multiplied Mam, 6 times 2 equals to 12, 5 times 2 equals to 10 then 6 times 5 equals to 30

Interviewer: You have multiplied all numbers:

GN: Yes Mam, and my answer is 12 $\frac{\mathbf{10}}{\mathbf{30}}$

DK

Interviewer: How did you come up with your answer?

DK: 6 times 2 equals to 12, 5 times 2 equals 10 and then 6 times 5 equals to 30

Interviewer: And your answer is?

DK: My answer is 12 $\frac{\mathbf{10}}{\mathbf{30}}$

AT

Interviewer: How did you come up with your answer?

AT: I said 6 times 2 equals 12, then 5 times 2 equals 10, then 6 times 5 equals 30.

Interviewer: And then?

AT: Then my answer is 12 $\frac{\mathbf{10}}{\mathbf{30}}$.

4.2.9.2 Data From the Average Achievers Category (AA)

All the learners from the average achievers' categories got Question 9 incorrect.

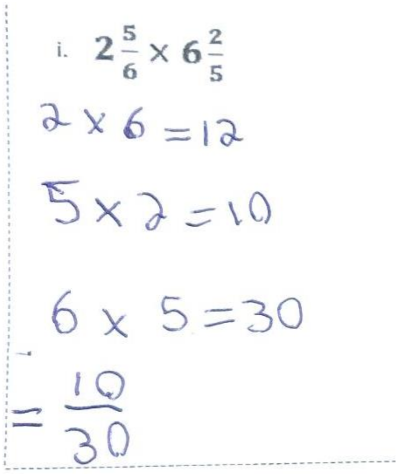


Figure 4.52: Learner IN's response

Figure 4.52 above reflects learner IN's answer. The learner multiplied the whole numbers and then multiplied the numerators and denominators. The learner explained how he obtained his answer:

IN

Interviewer: How did you come up with your answer?

IN: I said 2 times 6 equals to 12, then 5 times 2 equals to 10 then 6 times 5 equals to 30.

Interviewer: After that what happened?

IN: my answer is $12\frac{10}{30}$*, sorry Mam I forgot to put 12.*

It is clear that learners cannot convert mixed fractions into improper fractions when multiplying the mixed fractions. The misconception is, therefore, associated with factual and procedural errors in multiplication of mixed fractions.

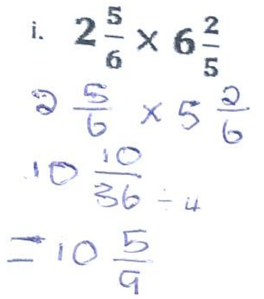


Figure 4.53: Learner FB's response

Figure 4.53 above reflects learner FB's answer; he mistakenly changed the second whole number and then multiplied the two whole numbers and multiplied the numerators after multiplying the denominators. This is how he explained his answer.

FB

Interviewer: How did you come up with your answer?

FB: I made a mistake, Mam; I was supposed to write 6 here.

Interviewer: Then?

FB: I then said 2 times 5 equals to 10 then 5 times 2 equals to 10 after 6 times 6 equals to 36.

Interviewer: My answer is $10\frac{5}{9}$*.*

It is clear from above explanation of learner FB that her conceptual knowledge is inadequate. The learner cannot convert improper fractions into proper fractions to do cross-calculations. I can conclude that her misconception is associated with a procedural and factual error.

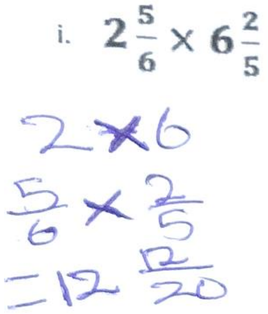


Figure 4.54:Learner HM's response

Figure 4.54 above reflects learner HM's answer. Learner HM is from the average achievers. The learner multiplied the whole numbers together and then multiplied the numerators and denominators together but miscalculated his answers. He explained his answer as follows:

HM

Interviewer: How did you come up with your answer?
HM: I said 2 times 6 got 12, and then 5 times 2 got 12 then after 6 times 5 got 20
Interviewer: OK, [I] am listening.
HM: My answer is 12 $\frac{\mathbf{12}}{\mathbf{20}}$.

From the above explanation of learner HM, it is clear that his conceptual knowledge is not clear, and he cannot do basic calculations. The learner cannot convert improper fractions into proper fractions to do cross-calculations. I can conclude that her misconception is associated with procedural and factual errors.

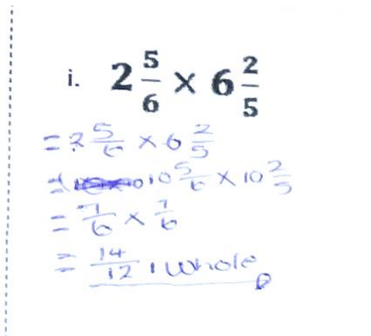


Figure 4.55: Learner KL's response

The answer here is from learner KL. She added the first numerator with the second numerator then got 7, after she added first denominator and whole of a second fraction and got 12. Below is her response when asked how she got that answer.

KL

Interviewer: How did you come up with your answer?

KL: I said 5 plus 2 equals 7.

Interviewer: After that what happened with one?

KL: Then I said 6 plus 6. [The] answer is 12.

Interviewer: How did you get your answer then?

KL: I then put 14 on top and 12 at the bottom $\frac{14}{12}$.

It is evident that learner KL doesn't understand the concept of multiplication of fractions clearly. She's confusing the whole concept; changing the operation is a major concern. The misconception is, therefore, associated with conceptual error in multiplication of mixed fractions.

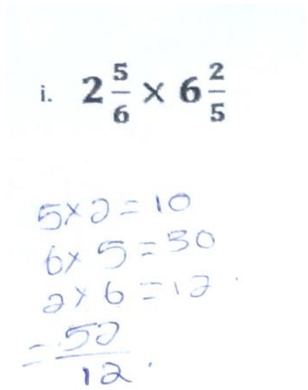


Figure 4.56: Learner LS's response

Figure 4.56 above reflects learner LS's answer. She multiplied the numerators and the denominators together and then added the answers she got. She explained her answer as follows:

LS

Interviewer: How did you come up with your answer?

LS: I said 5 times 2 equals to 10.

Interviewer: OK, am listening.

LS: After I said 6 times 5 equals to 30, then 2 times 6 equals 12.

Interviewer: What is your answer?

LS: I added Mam I said 10 plus 30 plus 12 equals to 52 then I put 52 on top then the number at the bottom is 12. My answer is $\frac{52}{12}$.

The learner confuses the concepts. The misconception is associated with a conception error in the multiplication of mixed fractions.

4.2.9.3 Data From the Low Achievers Category (LA)

According to the data collected, only four learners from the Average Achievers group got Question 10 correct.

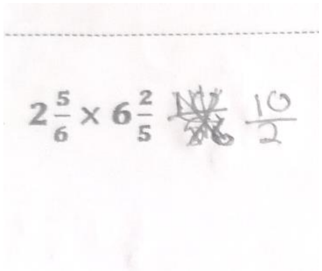


Figure 4.57: Learner SN's response

Figure 4.57 above reflects the answer from learner SN, who is in the lower achievers category. The learner multiplied the numerators together, putting it as the numerator, and took 2 as the denominator. The learner explained how the answer was obtained:

SN

Interviewer: How did you come up with your answer?

SN: I said 5 times 2, got 10.

Interviewer: OK.

SN: Then [the] number on top is 10 I took this 2 (pointed first whole number) and put it under 10, and my answer is $\mathbf{\frac{10}{2}}$.

The problem here is conceptually and procedurally. The misconception is associated with conceptual and procedural errors in the multiplication of fractions.

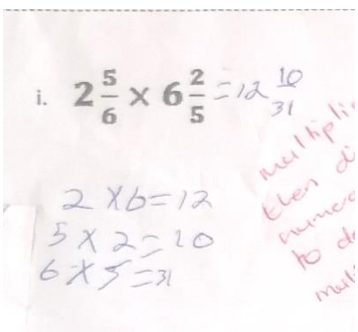


Figure 4.58: Figure MS's response

Figure 4.58 above reflects MS's answer. Learner MS multiplied the whole numbers together, and then multiplied the numerators together before finally multiplying the denominators together, but miscalculated the denominator.

MS

Interviewer: How did you come up with your answer?

MS: I multiplied Mam, I said 2 times 6 got 12, 5 times 2 got 10 then 6 times 5 got 31.

Interviewer: Then after?

MS: Then my answer is 12 $\frac{\mathbf{10}}{\mathbf{31}}$.

The learner clearly doesn't have procedural knowledge; therefore, the misconception is associated with a procedural error in the multiplication of fractions.

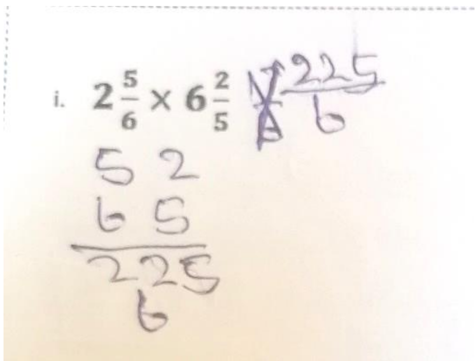


Figure 4.59: Learner TPN's response

Figure 4.59 above reflects learner TPN's answer. Here, the learner is totally confused. He doesn't know the basics. Below is his explanation of how he got his answer:

TPN

Interviewer: How did you come up with your answer?

TPN: I have multiplied, Mam.

Interviewer: How did you multiply?

TPN: I multiplied. I took 5, 2, 6 and 5, then my answer is $\frac{\mathbf{225}}{\mathbf{6}}$.

According to the above explanation, the learner does not understand the concept. The misconception is, therefore, associated with all three types of errors: factual, procedural and conceptual.

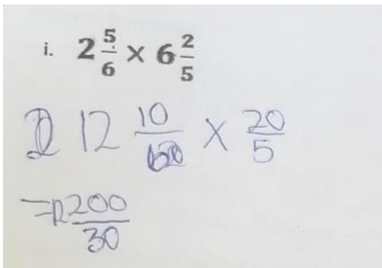


Figure 4.60: Learner PL's response

Figure 4.60 above reflects learner PL's answer. She multiplied the two whole numbers, the numerators and the denominators. She responded as follows in the interview:

PL

> *Interviewer: How did you come up with your answer?*
>
> *PL: I multiplied; I said, 2 times 6 got 12, and then 5 times 2 got 10, and then 6 times 5 got 60.*
>
> *Interviewer: And then?*
>
> *PL: Then I said 10 times 20.*
>
> *Interviewer: What about denominators? How did you get your answer?*
>
> *PL: I added 9 and 3, then got 12, but my answer is* $\frac{12}{12}$.

It is evident that the learner lacks basics; she knows that she must add but gets confused about how. The learners' misconception is clearly associated with conceptual error.

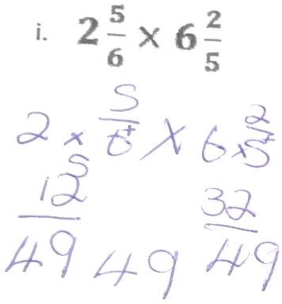


Figure 4.61: Learner NC's response

Figure 4.61 above reflects learner NC's answer. Learner NC above multiplied the whole numbers with the denominators and miscalculated all her answers. Her response on how she got her answer is reflected below:

NC

> *Interviewer: How did you come up with your answer?*
>
> *NC: I said 2 times 6, got 49, and then on the second fraction, I said 6 times 5, got 49 for numbers at the bottom*
>
> *Interviewer: OK, and [pause]?*
>
> *NC: I got my answer as* $\frac{32}{49}$.

Learner NC doesn't understand the procedure and the concept of multiplication. The misconception is, therefore, associated with procedural and conceptual errors in the multiplication of fraction.

4.2.10 Question 10: Factual, Conceptual and Procedural Errors in Multiplication of Fractions With Different Denominators

$$\frac{2}{4} \times \frac{5}{6}$$

The researcher is trying to see if the learners will do a cross multiplication or apply the correct procedure to determine the answer. It is evident from Table 4.2 that learners make careless, application, factual, procedural and conceptual mistakes.

4.2.10.1 Data From the High Achievers Category (HA)

Based on the data collected, four learners from the high achievers' category got Question 10 correct.

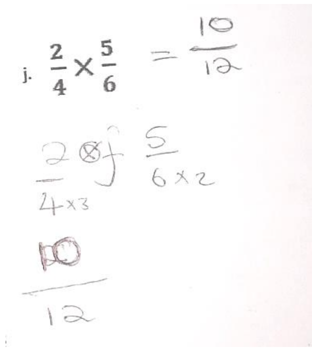


Figure 4.62: DK's response

Figure 4.62 above reflects learner DK's answer. Learner DK multiplied the numerators together, and when coming to the denominator, DK tried to make the denominators the same. Below is her response:

DK

Interviewer: How did you come up with your answer?

DK: I said 2 times 5, got 10, and then made denominator[s] the same.

Interviewer: How?

DK: I multiplied 4 by 3 to make it 12, then multiplied 6 by 2 to make it 12 again, so that I will have 12.

Interviewer: What is your answer?

DK: My answer is $\frac{10}{12}$.

DK is confusing the procedures. She is using addition and subtraction procedures in solving multiplication problems which is incorrect. The misconception is, therefore, associated with procedural error in multiplication.

4.2.10.2 Data From the Average Achievers Category (AA)

Based on data collected, four learners from the average category got Question 10 correct.

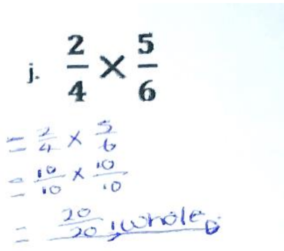


Figure 4.63: Learner KL's response

Figure 4.63 above reflects learner KL's answer. The learner multiplied the numerators together and then got 10. He then set both the numerator and denominator in the two fractions to 10. His explanation is below:

KL

Interviewer: How did you come up with your answer?

KL: I multiplied the number on top, and my answer is 10.

Interviewer: After that, what happened with one?

KL: The answer is 10, and then I put 10 on top here and at the bottom to make it the same.

Interviewer: What about this fraction here?

KL: I put 10 on top and 10 at the bottom to make it the same.

Interviewer: Then what about your answer:

KL: I added 10 plus 10, got 20 on top and then added 10 and 10 at the bottom to make it 20. My answer is $\frac{\mathbf{20}}{\mathbf{20}}$*, equals to 1, whole.*

KL doesn't understand the procedure. He understands that he must multiply but not how to do it. The misconception is associated with procedural errors in the multiplication of fractions.

4.2.10.3 Data from the Low Achievers Category (LA)

According to the data collected, only one learner from the Low Average category got Question 10 correct. All four other learners got the answer incorrect.

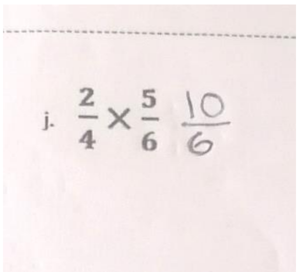


Figure 4.64: Learner SN's response

Figure 4.64 above reflects learner SN's answer. The learner multiplied the numerator only and took the denominator from the second fraction. The learner's comments during the interview are reflected below.

SN

Interviewer: How did you come up with your answer?

SN: I said 2 times 5, got 10, and then my answer is $\frac{\mathbf{10}}{\mathbf{6}}$**.**

Interviewer: What about your denominator?

SN: I took 6, because it's bigger than 4.

The learner knew that she must multiply but used an incorrect procedure when coming to the denominator. The misconception is, therefore, associated with a procedural error in multiplication.

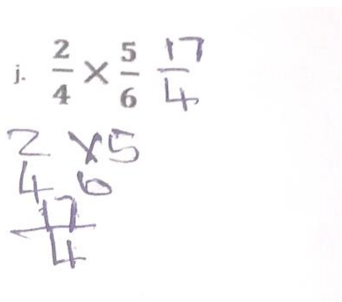


Figure 4.65: Learner TPN's answer

Figure 4.65 above reflects learner TPN's answer. The learner is very confused and doesn't understand what he is doing. His explanation of how he got his answer is below.

TPN

Interviewer: How did you come up with your answer?

TPN: I said 2 plus 5 is 7, I then multiplied 2 times 5, [which] is 10.

Interviewer: And then?

TPN: Then I added the numbers 7 plus 10, I got 17.

Interviewer: How did you get this 4?

TPN: I then took 4 and put it at the bottom, and my answer is $\frac{\mathbf{17}}{\mathbf{4}}$**.**

The learner confuses the operations. The procedure used is totally incorrect. The misconception is, therefore, associated with a procedural error in multiplication.

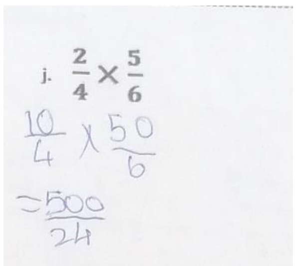


Figure 4.66: Learner PL's response

Figure 4.66 above reflects learner PL's answer. Learner PL multiplied the numerators, and after getting the answer, she first multiplied the same answer with 5 and then multiplied the denominators.

PL

Interviewer: How did you come up with your answer?

PL: I said 2 times 5, my answer is 10, and then 10 times 5, got 500.

Interviewer: Why multiply your answer with 5?

PL: Because my teacher showed me to do it this way.

Interviewer: Okay and then how did you get your answer?

PL: I then said 4 times 6, got 24, and my answer is $\frac{\mathbf{500}}{\mathbf{24}}$.

Evidently, the learner lacks basics; she knows she must multiply but doesn't know the correct procedures. The misconception is, therefore, associated with procedural errors in the multiplication of fractions.

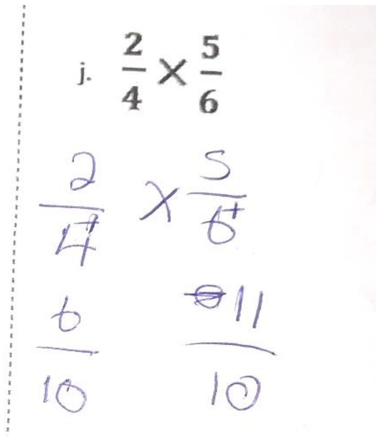


Figure 4.67: Leaner NC's response

Figure 4.67 above reflects learner NC's answer. Learner NC above was confused on addition and did not understand the basics of fractions. The learner's responses during the interview is reflected below.

NC

Interviewer: How did you come up with your answer?

NC: I took 10 and put it on both fractions and then put 6 on top on [the] first fraction and 11 on second fraction.

Interviewer: Why?

NC: So that I get my answer.

Learner NC confuses concepts; it's clear that she doesn't understand fractions and the basics of fractions. The misconception is associated with factual and conceptual errors in the subtraction of fractions.

Clearly, learners from all three categories have problems when it comes to the multiplication of fractions. However, in high achievers, the problem is not that much compared to other categories. Most of the misconception is associated with procedural and conceptual errors in the multiplication of fractions.

4.3 SUMMARY

This chapter presented the research findings and data analysis. The results of the written tests were summarised in tables. Codes were used to present the participants in Table 4.1. HA in Table 4.1 represents high achievers, those learners performing on a high level academically. AA in Table 4.1 represents average achievers, those learners who perform at an average level academically. LA represents low achievers, those learners who perform on a low level academically. The frequency of correct and incorrect solutions in each test unit is given in percentages in Table 4.2. The chapter had a section on analysis based on what the tables portrayed.

Excerpts from the learner's written test scripts and the oral responses from the interviews were analysed. The chapter presented interview reports of all the incorrect solutions from the interviewees and the corresponding analysis. The analysis was done simultaneously from both the written test and the interviews. The results showed that learners do have misconceptions about the subject of fractions. The misconceptions identified are associated with procedural, conceptual and factual errors in the addition, subtraction and multiplication of fractions. A conclusion and recommendations based on the study findings are found in the next chapter (Chapter 5).

CHAPTER 5
FINDINGS, RECOMMENDATIONS AND CONCLUSION

5.1 INTRODUCTION

This chapter discusses the research findings, recommendations and conclusions. The discussion starts with an overview of the study, followed by the findings according to the research questions. I answer the research questions by deliberating on the findings in relation to the existing literature and the underlying theory. The chapter then answers the two secondary questions followed by the primary question in the study. The section on recommendations discusses recommendations for training and practice, including recommendations for the CAPS policy, the limitations of the study, its strength, and potential contribution. I discuss the limitations of the study, followed by the potential contributions of the study. Lastly, the conclusions are discussed in the next subsection. I then reflect on the journey I took to complete this study. This chapter concludes with a review of each objective, the relevant data collected and its analysis. The results are explicitly presented before they are discussed.

For ease of reference, I restate the research questions that guided my study:

MAIN RESEARCH QUESTION

- What are common error patterns in addition, subtraction and multiplication of fractions among the selected group of Grade 6 learners in a public primary school in Gauteng?

RESEARCH SUB-QUESTION

- What are the underlying misconceptions among these selected Grade 6 learners associated with factual, conceptual, and procedural errors in addition, subtraction, and multiplication of fractions?

5.2 OVERVIEW OF THE STUDY

The study aimed to identify common error patterns in Grade 6 learners' addition, subtraction and multiplication of fractions in a public primary school in Gauteng Province. By identifying these common error patterns, teachers can address underlying misunderstandings and provide targeted instruction to help learners better understand the dialect. In the world of mathematics education, there are often common misconceptions that learners develop as they move through their learning. The study also explores the underlying misconception among the Grade 6 learners associated with factual, conceptual and procedural errors in addition, subtraction and multiplication of fractions.

The researcher discovered that the most dominating misconceptions are associated with conceptual and procedural errors in addition, subtraction and multiplication of fractions. Many underlying misconceptions among these Grade 6 learners can hinder their progress in understanding and mastering mathematics concepts. The study also determines how these common errors can be avoided. Fractional errors can be avoided by following a few simple strategies. It is necessary to understand the basic principles of fractions thoroughly. These include the ability to add, subtract, multiply and divide fractions correctly. In achieving the study's objectives, the researcher used a qualitative case study approach and purposively selected 15 learners from Grade 6, who are specifically from high-, average- and low-performance levels.

5.3 FINDINGS ACCORDING TO THE RESEARCH QUESTION

This study aimed to analyse common errors in fractions made by learners at a public school in Gauteng province. It was envisaged that the study's findings would present the common error patterns from learner and experienced educator perspectives. Furthermore, insights gained could help educational psychologists plan interventions to assist learners. Through these findings, educators will have ways to assist learners in avoiding these errors when dealing with fractions at the primary level, hoping that this will motivate those working at a more complex level with fractions at the secondary level.

5.3.1 Answering Secondary Research Question

One of the objectives of this study was to identify the underlying misconceptions among the selected Grade 6 learners that are associated with factual, conceptual and procedural errors in addition, subtraction and multiplication in the concept of fractions. The secondary questions helped to explore aspects related to the main research question and provide additional insight into the topic; therefore, this required conducting additional analysis and gathering additional information.

A test was administered to obtain information on this topic from the learners’ answers. Interviews with learners who had incorrect solutions added additional oral input to the written feedback collected, and the next section describes how Grade 6 learners responded, providing incorrect solutions, and demonstrating misconceptions about fractions as a concept.

5.3.1.1 Misconceptions Associated With Procedural Errors in Fractions

When learners add three fractions with different denominators, they add all the numerators together and then add the denominators together to get their answers. Learners failed to find the common denominator and follow the correct procedure. Learners assume that adding a fraction is equivalent to adding a whole number. One of the learners also miscalculated the numbers instead of giving the sum when adding numerators and denominators.

The procedure used was incorrect. Learners cannot identify the common denominator when the denominators are the same. When the denominators are different, they cannot find the LCD, making it hard to get the correct answers.

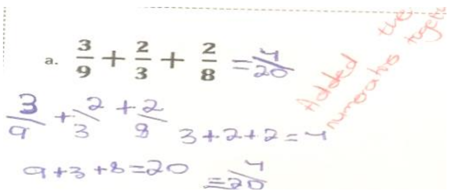


Figure 5.1: Different denominators

Although this calculation can be done by using either the common multiples of the denominators or by multiplying the two denominators to find a common denominator, this indicates a lack of understanding at the basic level of multiplication and factorisation concepts. Therefore, learners don't have a procedural understanding of addition and subtraction of fractions with the same and different denominators. This can be seen in Question 1 above; all the learners got Question 1 incorrect. The same applies when learners subtract fractions with different denominators; learners subtract the numerators together and the denominators together to work out their answers. This can be seen in Question 2; all learners got Question 2 incorrect. When adding the mixed fractions with the same denominators, 46% of learners got Question 3 incorrect. Most learners managed to add the whole numbers, the numerators, and the denominators together, as seen in Question 3. When subtracting mixed fractions with the same denominators, 33% of learners got

Question 4 wrong; learners applied the same method, most of the learners managed to subtract the whole numbers, but they subtracted the numerators together and then subtracted the denominators together, as can be seen in Question 4.

When learners add or subtract mixed numbers, they overlook or ignore the whole numbers and focus on the numerators and denominators only; they add or subtract the numerators together and do the same with denominators to get their answers.

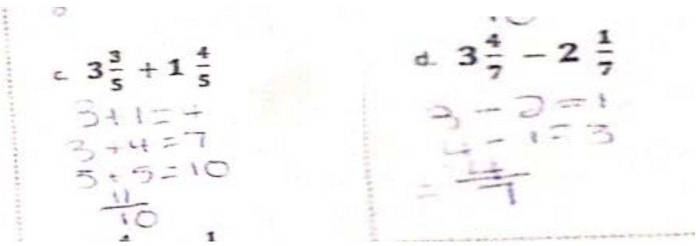


Figure 5.2: Error on adding and subtracting mixed numbers

Another common error pattern identified is that learners add or subtract the whole numbers, add or subtract the numerators, and then do the same with the denominator to get the answer.

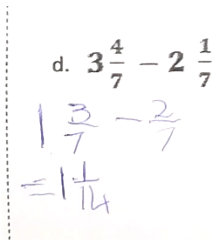


Figure 5.3: Addition or subtraction of denominators

Figure 5.3 indicates that learners are unaware that mixed numbers must first be converted into improper fractions. To do so requires both an understanding of fractions as a concept and multiplication skills. However, learners explained that they had not been taught these principles and how to apply them. They confused the procedure and lacked the procedural knowledge in addition and subtraction of mixed numbers.

When adding or subtracting fractions with the same or different denominators, learners add numerators together and add the denominators together to get the answer.

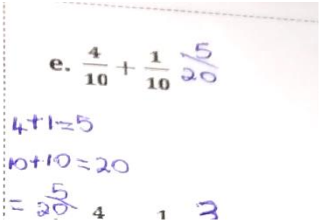


Figure 5.4: Example of addition of denominators

This results in an inability to understand the basics of solving sums involving fractions. When asked, the learners said that their teachers had taught them to solve the sums using this method. Besides not knowing multiples, some learners did not know what a numerator and denominator meant. They would take each as a whole number and not understand the relationship between the two and that the digits are part of a fraction. They don't know the procedure adequately and lack procedural knowledge of fractions.

When adding two fractions with the same denominators, 33% of learners got Question 5 incorrect. They added the numerators and the denominators together. When subtracting mixed fractions with different denominators, all learners got Question 8 wrong. Learners subtracted the whole numbers together; they subtracted the numerators and denominators together. When multiplying mixed fractions with different denominators, learners multiplied the whole numbers together, and then multiplied the numerators and the denominators together; all learners got Question 9 incorrect.

5.3.1.2 Misconceptions Associated with Conceptual Errors in Fractions

When adding the three fractions with different denominators, some learners changed the operation; this means that learners confused the addition and multiplication signs, which proves the lack of understanding in adding fractions, as can be seen in Question 1. When subtracting three fractions with different denominators, most learners subtracted the numerators together then subtracted the denominators together, and in some cases, they swapped the denominators to numerators to make the fractions proper fractions, as can be seen in Question 2; all learners got Question 2 incorrect. When adding mixed numbers with the same denominators, most learners managed to add the whole numbers but changed the numerators to be the same, and then added the numerators together and kept the denominators as they were, as is evident from Question 3. When subtracting mixed fractions with the same denominators, learners managed to subtract the whole number but then subtracted the numerators. However, when it came to the denominators, they added them together.

Following this demonstration of a lack of basic understanding and skills, when multiplying mixed numbers, learners multiplied the whole numbers together, and then multiplied the numerators followed by the denominators.

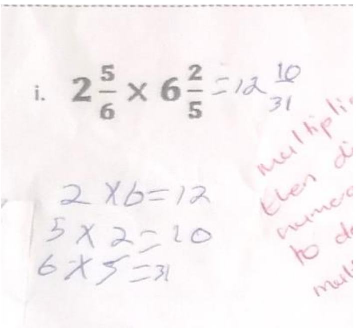


Figure 5.5: Multiplication errors

Some wrote the answer according to the incorrect method they employed, while others didn't write a response that indicated consideration of the whole numbers. Their answers indicated that they only focused on multiplying the fractions. Learners didn't know that they had to change the mixed numbers into improper fractions first before multiplying the fractions.

5.3.1.3 Misconceptions associated with factual errors in fractions

With regard to factual mistakes, there are several common misconceptions about addition, subtraction, and multiplication. One common misconception was that all mistakes in these actions were due only to a lack of skill or intelligence. However, errors could be caused by many different factors, such as calculation errors; for example, when adding, the learners miscalculated the denominators through using the incorrect procedure, misinterpretation or misunderstanding of the problem, or even a simple lack of concentration. See Figure 5.6 below.

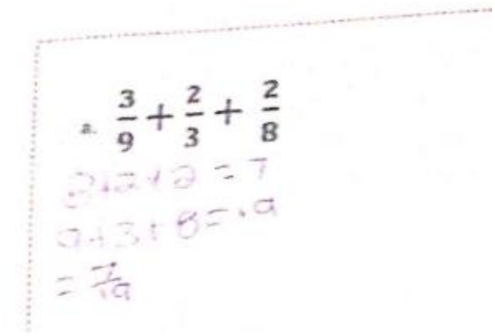


Figure 5.6: Miscalculation of denominators

Another common misconception was that mistakes in these activities were always a sign of a lack of understanding. While accuracy is important, focusing on the process and strategy used to arrive at a solution is equally important. Identifying and addressing these misconceptions can foster a more supportive and growth-oriented environment for learning and acquiring these basic math skills.

5.3.2 Answering the Primary Research Question

When dealing with fractions, some common error patterns can occur in addition, subtraction, and multiplication. Common errors discovered in the selected group of Grade 6 learners are forgetting to find the common denominator before adding or subtracting fractions. This omission can lead to incorrect results because fractions cannot be combined correctly without a common denominator. Another error is adding or subtracting the numerators incorrectly without changing the denominators, resulting in an incorrect fraction. Likewise, when multiplying fractions, it is important to multiply numerators and denominators separately, but Grade 6 learners often make mistakes when multiplying both numerators and denominators. This mistake can lead to a completely different and incorrect product. In addition, forgetting to simplify a finite fraction can also be a common mistake. It is important to check whether the fraction can be further reduced to its simplest form. Being aware of these common error patterns can help learners identify and correct their own mistakes, which leads to a better understanding and mastery of fractional operations.

5.4 RECOMMENDATIONS

5.4.1 Recommendation for Training of Teachers

Based on the findings discussed above, the following recommendations on training and practice for teachers and learners in the teaching and learning fractions are made:

1. Teacher development through effective training and workshops is very important as this will assist teachers in developing effective teaching and learning in primary school education.
2. Mathematical topics should be dealt with in detail and, in the case of fractions, broken down into even smaller components.
3. Teachers should try to give learners expanded activities; furthermore, teachers should agree on one consistent method to use as learners move through the grades.

Based on findings formulated during the data analysis, teachers need more knowledge in teaching fractions, especially in the lower grades, and must teach factorisation so that learners will have the proper basics required for solving fraction sums. This study brought forward the most critical issue that educators tend to overlook, which is learners' prior knowledge, leading to incorrect teaching strategies (Peng & Luo, 2009). This omission must be considered when introducing the fractions and throughout the lesson preparation. Teachers need to understand the learners' errors and then try to base their teaching on different strategies. According to Luo (2004), error analysis is an essential task imperative to mathematics teaching, although it may simultaneously perplex teachers. Although not part of this investigation, it also emerged that learners have problems reading questions and understanding the instructions; some will not even write according to the instructions. The Department of Basic Education should try to emphasise reading and assist teachers through workshops in teaching learners to read more.

5.4.2 Recommendation of Good Practices

When it comes to teaching and learning, teachers can use many good practices to ensure effective learner understanding and engagement. First, teachers must create a safe and supportive learning environment where learners are encouraged to ask questions and make mistakes without fear of judgment. This can be achieved through open and positive communication, fostering a growth mind-set and providing constructive feedback. In addition, teachers should use various teaching strategies such as hands-on exercises, visual aids, and real-life examples to help learners understand abstract concepts related to fractions. Breaking down complex ideas into smaller, more manageable chunks and matching instructional frameworks to individual learner needs are also important strategies.

Finally, incorporating technology such as interactive applications or virtual manipulative can make learning segments more engaging and accessible to learners.

By implementing these good practices, teachers can advance learners’ understanding and appreciation of fractions, helping them succeed in mathematics and beyond. Teachers’ preparedness in presenting fractions also greatly impacts learners' learning of fractions. When teachers come to class unprepared, they simply do not know what to teach learners when it comes to fractions. This situation is exacerbated when teachers and learners arrive late, setting a negative learning environment. How fractions are introduced to learners is the key to their grasp of the subject. When teachers introduce fractions, they should ask learners questions to establish their level of understanding and identify gaps in prior knowledge. In the interviews, learners pointed out that teachers in previous grades used different techniques when teaching fractions, and learners became confused; therefore, they weren’t using any rules – they are simply adding, subtracting and multiplying numbers. Their knowledge of fractions is basically non-existent in solving problems with fractions. This indicates that learners should be taught using consistent methods, and teachers in earlier grades must employ methodologies in common with teachers in Grade 6. From the above, it can be concluded that Grade 6 mathematics teachers in primary school must introduce fractions basics every time they introduce the fraction topic. Learners must understand the difference between the numerator and denominator and not treat both separately.

Visual models are also an effective way of introducing fraction concepts. Teachers should have one common technique or method in teaching aspects of fractions and build them into complexity. There should be consistency in the presentation of fraction rules and terminology used, so that learners don’t get confused by different teaching techniques.

Prior to learning fractions, learners must grasp the concepts of factorisation and lowest common multiples (LCM). These skills are core to fractions and algebraic sums in senior grades. Thus, learners must have a strong foundational understanding of multiples and factors before being taught fractions so they do not get confused between them. Different mathematical problems involving factors and multiples should be practised from the basic to more challenging levels, and games may be employed. Learners should reach a point where they can identify the LCM of two or even three numbers before exploring fractions.

Once they have grasped this, they could easily reach a common denominator to solve subtraction and addition sums involving fractions with different denominators.

When starting fractions, learners should be taught the parts of fractions by being allowed to make practical examples, such as those of a paper "pizza". They should be taught the names of the parts of the fractions and the different types of fractions. They must learn that there is a difference between proper fractions, improper fractions and mixed fractions. This will lay the groundwork for learners to grasp the difference between the mathematical functions of the numerator and the denominator and that together, they form a mathematical concept called a fraction; they need to know not to treat both the same way when adding and subtracting fractions.

Once the learners understand this, the teacher may build on this groundwork by teaching the rules of adding and subtracting fractions. The first step in this is to get learners to focus on the denominator as a starting point and check if the denominators are the same or different. Some practise can be done with maths cards using two fractions with the same or different denominators; the teacher can hold them up to the learners, who can then shout out "same" or "different". Once this knowledge is established, addition and subtraction sums involving fractions with the same denominator should be done first until knowledge is laid down. Then, the learners' knowledge of multiples and factors is revised. Some practise in finding the LCM should be done. Finally, sums involving different denominators may be demonstrated and practised. Considerable time should be spent practising these fraction sums.

Lastly, in sums involving mixed fractions, a clear understanding of the components of mixed fractions is crucial. Learners must be taught that in a mixed number, there is both a whole number and a fraction. Then, teachers should teach learners how to change mixed numbers into improper fractions. Exercises should be done on this concept until understanding is established. This is the starting point; it is simply an extra step added onto the process of solving fractions sums, and after that, learners can proceed as per a normal fraction problem. Learners then begin with multiplying fractions. Learners must understand that the rules of adding and subtracting fractions no longer apply, and new rules apply for multiplication sums. They should be given practise in multiplication; they multiply the numerators and then the denominators to reach the answer. The final lesson is then in simplifying the fraction to reach its most simple form, which is a return to the concept of factorisation. Learners should do exercises involving simplifying fractions composed of large numerators, denominators, and mixed fractions. Once learners clearly understand the above, in this order, it is more likely that the common errors will be reduced if not eliminated.

1. Teachers should understand the reasons for errors to assist learners in avoiding those errors.
2. Once the teachers understand their learners' abilities, they should adapt their teaching planning and plan activities that may facilitate group learning and other dynamics that accommodate all the learners in class.
3. The Department of Basic Education should at least ensure to equip schools fully with mathematics resources, which will help teachers to be able to use different ways to present fractions and adapt if there are learners who are struggling to understand the subject on their first attempt.
4. This study should be presented to the Department of Educational Psychology to create awareness of learners' difficulties in learning fractions.
5. Future research should investigate the effectiveness of teacher development by the Department of Higher Education, primarily to determine what training is most effective in assisting teachers for curriculum purposes in mathematics at the primary school level regarding core concepts like fractions.
6. Most primary school teachers teach subjects they have not been professionally trained at university level. This has meant that children in underprivileged schools have been let down, especially in the subject of mathematics. Since mathematics is a specialised and often complex subject, the Department of Higher Education should make sure that those who teach mathematics and have not been trained for it at University are given special training to equip them with the required skills and passion to transmit difficult topics such as fractions in a competent way.
7. Continued professional development should be made available to teachers, especially mathematics teachers; new techniques should be explored, and if

necessary, they should be trained up to use them in specialist areas such as in the teaching of fractions.

5.4.3 Recommendation of Policy

The study, therefore, proposes a review of the National Department of Basic Education's curriculum on the on-the-job training of primary school educators in mathematics in a manner that is broad enough to improve the required skills needed for better achievement of learners in mathematics. The department should do training courses and award educators with mathematics certificates and other recognition that will motivate teachers, especially those who were not trained to teach mathematics. As far as quality management is concerned, the Department of Basic Education should conduct follow-up inspections with primary school mathematics educators on a monthly basis to monitor the progress of learners in mathematics, especially in Grades 3, 6 and 9, as these are exit grades.

Several key areas should be addressed per the National Curriculum and Assessment Policy Statement's (CAPS) Section on Mathematics Education Policy for Grade 6.

1. First, it is important to ensure that curricula are aligned with the latest research and best practices in mathematics education. This includes, for example, incorporating problem-solving, critical thinking and real-world applications into the curriculum to promote a deep understanding of mathematical concepts.
2. In addition, the institution should prioritise the professional development of mathematics teachers and provide them with on-going training and resources to improve their teaching practices. Collaborative learning should also be encouraged so learners can work together and engage in meaningful discussions to deepen their mathematics.
3. Additionally, the institution should explore the use of technology in the classroom and provide learners with tools and resources that can enhance their learning.
4. Finally, regular assessments should be implemented to monitor student progress and identify areas for improvement. By focusing on these recommendations, the Department of Basic Education Policy can create a

positive learning environment that promotes mathematics skills and prepares 6th graders for success in mathematics.

5.5 LIMITATIONS OF THE STUDY

The importance of the qualitative study is its methodology and the ability of other teachers in similar primary schools in South Africa to learn from it. However, the investigation had other complexities – challenges for learners, including the English language used as the language of instruction. For the validity of these findings, learners had to use their home language to explain how they came to their answers. Furthermore, due to COVID-19 protocols being in place in the year 2020, learners were not supposed to be more than 15 in class, and the school implemented a rotational time-table; the number of participants limited the generalisability of the research findings; however, the researcher overcame this problem by adopting strategies that improve the representativeness of the sample and the use of appropriate statistical measures. However, regardless of the limitation of the rotational timetable or the size of the population, the results still contributed to the study's validity. The learners were all Grade 6 learners who are recorded to have passed Grade 5 the previous year, and their views contributed to the study's trustworthiness. Theoretically, they were taught fractions in the previous year within the public educational system, and their contribution to the study assisted the researcher in knowing or identifying the problems they faced when answering questions based on addition, subtraction and multiplication of fractions. Had a wider variety of schools been studied, including those in the private sector, more interesting results and different conclusions would have been obtained.

5.6 POTENTIAL CONTRIBUTIONS OF THE STUDY

Identifying common mistakes in fractions by Grade 6 learners can make a big difference in their understanding and mastery of this basic mathematics concept. By identifying and correcting these mistakes early, teachers will help learners build a strong foundation in fractions and prepare them for success in more complex mathematics topics. By actively identifying and correcting these errors, teachers will provide targeted interventions and corrections to help learners correct their misconceptions and develop a deeper understanding of fractions. This approach can improve problem-solving, self-confidence, and overall mathematics ability. Additionally, identifying these errors allows teachers to adjust their teaching strategies and lesson plans to target and avoid these misconceptions, ultimately improving student learning for all learners. Furthermore, understanding the common errors learners make in fractions can materially impact the Department of Basic Education and its approach to mathematics education. Avoiding common mistakes when using fractions can greatly impact the learning journey of Grade 6 learners, thereby improving learners' positive perception of mathematics.

5.7 CONCLUSIONS

The literature and the research findings demonstrate that learners lack the basic understanding required to avoid simple errors in fraction sums. Moreover, teachers differ in how they teach fractions. Another underlying factor in learners' failure is that most teachers present their lessons without any preparation, leaving them unable to cover what is required, leaving learners with a deficit of both knowledge and eagerness to learn mathematics. Learners' difficulties in mathematics in primary schools affect their performance at the secondary school level. Grade 6 is the exit grade from the Intermediate Phase to the Senior Phase. The National Development Plan stated that learners in Grades 3, 6 and 9 should achieve 50% or more in mathematics (National Planning Commission, 2012). This is far from being reached. If this goal is to be recovered, complex and fundamental concepts such as fractions must be revisited, and teachers must adapt new approaches and skills to enable learners to achieve the required percentage for progression.

Teachers' knowledge of their teaching methods differs. Inadequate use of correct skills is insufficient regarding teaching of fractions, as is evident in the fact that this study finds that most learners don't even understand why they must follow certain rules in fractions. They tend to apply one rule in all fraction operations: addition, subtraction and multiplication. When teachers understand learners' mistakes in mathematics problem-solving, they could identify why and how these errors occurred and derive solutions (Mohammad, 2019). Due to their difficulties and inability to pass mathematics, learners tend to have negative perceptions towards mathematics and end up not taking mathematics as their major subject at the secondary level. This has impacted the number of people studying technical, scientific and engineering subjects, which in turn impacts the skills level in South Africa. The problem persists to such an extent that the South African education system ultimately lacks a decent supply of mathematics educators. Schools then end up taking any teacher at the primary level to teach mathematics, simply because they did mathematics at the secondary level.

REFERENCES

Akkoyunlu, B., & Soylu, M.Y.. (2006). A study on learners' views on blended learning environment. *The Turkish Online Journal of Distance Education.* 7.

Aksoy, N. C., & Yazlik, D. O. (2017). Student Errors in Fractions and Possible Causes of These Errors. *Journal of education and training studies*, *5*(11), 219-233.

Alkhateeb, M. A. (2019). Common Errors in Fractions and the Thinking Strategies That Accompany Them. *International Journal of Instruction*, 12(2), 399-416.

Al-Mutawah, M. A., Thomas, R., Eid, A., Mahmoud, E. Y., & Fateel, M. J. (2019). Conceptual Understanding, Procedural Knowledge and Problem-Solving Skills in Mathematics: High School Graduates Work Analysis and Standpoints. *International journal of education and practice*, *7*(3), 258-273.

Altiparmak, K., & Palabiyik, E. (2019) Analyzing Learning Objectives to Content Domain of Fractions, Operations with Fractions and Decimal Notation in Grades 1-8 by the Revised Bloom's Taxonomy. *Elementary Education Online 18*(1), 158-173

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the US. *Journal of Mathematics Teacher Education*, 7(2), 145-172.

Arora, R., & Stoner, C. (2009). A mixed method approach to understanding brand personality. *Journal of Product and Brand Management*, *18*(4), 272-283.

Aytekin, C., & Şahiner, Y. (2020). An investigation of preservice mathematics teachers' teaching processes about procedural and conceptual knowledge related to division with fractions. *Elementary Education Online*, *19*(2), 958-981.

Bada, S. O., & Olusegun, S. (2015). Constructivism learning theory: A paradigm for teaching and learning. *Journal of Research & Method in Education*, *5*(6), 66-70.

Baidoo, J. (2019). Dealing with grade 10 learners' misconceptions and errors when simplifying algebraic fractions. *Journal of Emerging Trends in Educational Research and Policy Studies, 10*(1), 47-55.

Banks, S. R. (2019). *Strengthening Pedagogical Content Knowledge in Division of Fractions through a Three-Part Professional Development Series: A Multiple Case Study* (Doctoral dissertation, Baylor University).

Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2020). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. *Journal of Educational Psychology, 112*(3), 628. https://doi.org/10.1037/edu0000384

Barnes, H., & Venter, E. (2008) Mathematics as a social construct: Teaching mathematics in context. *Pythagoras*, 68, 3-14.

Basturk, S. (2016). Primary student teachers' perspectives of the teaching of fractions. *Acta Didactica Napocensia*, *9*(1), 35-44.

Baur, N. (2019). Linearity vs. circularity? On some common misconceptions on the differences in the research process in qualitative and quantitative research. *Frontiers in Education*. https://www.frontiersin.org/articles/10.3389/feduc.2019.00053/full

Biber A. C, Tuna A., & Aktas O. (2013). *Learners' misconceptions of fractions and its effect on solving fractions problems*. *Trakya University Journal of Education, 3*(2), 152-162.

Boyd, P., & Ash, A. (2018). Mastery mathematics: Changing teacher beliefs around in-class grouping and mindset. *Teaching and Teacher Education, 75, 214-223.*

Braithwaite, D. W. & Siegler, R. S. (2021). Putting fractions together. *Journal of Educational Psychology*. *113*(3), 556–571.

Brooks, M. G., & Brooks, J. G. (1999). The courage to be constructivist. *Educational leadership*, *57*(3), 18-24.

Brown, J. & Skow, K. (2016). *Mathematics: Identifying and Addressing Student Errors.* https://www.coursehero.com/file/51722012/Iris-Center-Mathematics-Identifying-and-Addressing-Student-Errorspdf/

Buchs, C., Wiederkehr, V., Filippou, D., Sommet, N., & Darnon, C. (2015). Structured cooperative learning as a means for improving average achievers' mathematical learning in fractions. *Teaching Innovations*, *28*(3), 15-35.

Čadež, T. H., & Kolar, V. M. (2018). How fifth-grade learners reason about fractions: A reliance on part-whole subconstructs. *Educational Studies in Mathematics*, *99*(3), 335-357.

Castro-Rodríguez, E., Pitta-Pantazi, D., Rico, L., Gómez, P. (2016). Prospective teachers' understanding of the multiplicative part-whole relationship of fraction. *Educational Studies in Mathematics*, 92(1), 129-146.

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study learners' understandings of fractions. *Educational Studies in Mathematics*, *64*(3), 293.

Clark, K. R. (2018). Learning theories: cognitivism. *Radiologic Technology*, *90*(2), 176-179.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research Methods in Education*. Routledge.

Creswell, J. W. & Creswell, J. D. (2018) *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches.* Sage.

Creswell, J. W. (2014). *Research Design: Qualitative, quantitative and mixed method approaches* (4th ed.). SAGE Publications.

Department of Basic Education. (2009). *Curriculum and Assessment Policy Statement: Grades 7-9: Mathematics.* https://www.education.gov.za/Portals/0/CD/National%20Curriculum%20Statements%20and%20Vocational/CAPS%20SP%20%20MATHEMATICS%20GR%207-9.pdf?ver=2015-01-27-160141-373

Department of Basic Education. (2012). *Report on the annual national assessments. 2012.* https://www.education.gov.za/Portals/0/Documents/Reports/ANA%20REPORT%202012.pdf?ver=2012-12-04-151536-000 Date of Access: 10 Nov. 2023.

Department of Basic Education. (2014). *Annual national assessment of 2014: Diagnostic report first additional language and home language.* https://www.education.gov.za/Portals/0/Documents/Reports/2014%20ANA%20DIAGNOSTIC%20REPORT%20FAL.pdf?ver=2015-02-11-112930-187 Date of Access: 10 Nov. 2023.

Department of Education. (2002). Revised National CurriculumStatement: Grades R -9, Schools. https://www.education.gov.za/Portals/0/CD/GET/doc/overview.pdf?ver=2006-11-21-100143-000

Deringöl, Y. (2019). Misconceptions of Primary School Learners about the Subject of Fractions. *International Journal of Evaluation and Research in Education*, *8*(1), 29-38.

Dictionary Unit for South African English. (2002). *South African Concise Oxford Dictionary*. Oxford University Press Southern Africa.

Doğan, A. & Tertemiz, N. I. (2020). Fraction models used by primary school teachers. *Elementary Education Online*, 19(4), 1888-1901.

Dogan-Coskun, S. (2019). The Analysis of the Problems Posed by Pre-Service Elementary Teachers for the Addition of Fractions. *International Journal of Instruction*, *12*(1), 1517-1532.

Dyson, N. I., Jordan, N. C., Rodrigues, J., Barbieri, C., & Rinne, L. (2020). A fraction sense intervention for sixth graders with or at risk for mathematics difficulties. *Remedial and Special Education*, *41*(4), 244-254.

Fennell, F. (2007). Fractions are Foundational. *National Council Teachers of Mathematics NCTM News Bulletin.*

Fuchs, L. S., Fuchs, D., Malone, A. S., Seethaler, P. M., & Craddock, C. (2019). The role of cognitive processes in treating mathematics learning difficulties. In D. C. Geary, D. B. Berch, & K. M. Koepke (Eds.), *Cognitive foundations for improving mathematical learning* (pp. 295–320). Elsevier Academic Press.

Gabriel, F. C., Coché, F., Szucs, D., Carette, V., Rey, B., & Content, A. (2013). A componential view of children's difficulties in learning fractions. *Frontiers in Psychology*, *4,* 715.

Ghani, S. N. A., & Maat, S. M. (2018). Misconception of fraction among middle grade year four learners at primary school. *Research on Education and Psychology*, *2*(1), 111-125. https://doi.org/10.4236/ce.2018.914168

Goos, M. (2020). Communities of practice in mathematics teacher education. *In Encyclopedia of mathematics education (pp. 107-110).*

Hansen, A. (Ed.). (2006). Children's errors in mathematics: Understanding common misconceptions in primary schools. Learning Matters.

Hansen, A., Drews, D., Dudgeon, J., Lawton, F., & Surtees, L. (2020). *Children's Errors in Mathematics*. Sage Publications.

Hansen, R. A., Greene, S. B., Williams, C. E., Blalock, S. J., Crook, K. D., Akers, R., & Carey, T. S. (2006). Types of medication errors in North Carolina nursing homes: a target for quality improvement. *The American Journal of Geriatric Pharmacotherapy*, *4*(1), 52-61.

Hasemann, K. (1981). On difficulties with fractions. *Educational Studies in Mathematics*, *12*(1), 71-87.

Hasler, H. M., & Akshoomoff, N. (2019). Mathematics ability and related skills in preschoolers born very preterm. *Child Neuropsychology*, *25*(2), 162-178.

Heemsoth, T. & Heinze, A. (2014). The impact of incorrect examples on learning fractions: A field experiment with 6th grade learners. *Instructional Science*, *42*(4), 639-657.

Herold, K. H., Bock, A. M., Murphy, M. M., Mazzocco, M. M. (2020). Expanding task instructions may increase fractions problem difficulty for learners with Mathematics learning disability. *Learning Disability Quarterly*, *43*(4), 201-213.

Hess, R. D., & Handel, G. (2016). *Family worlds: A psychosocial approach to family life*. Routledge.

Hollweck, T. (2015). Robert K. Yin.(2014). Case Study Research Design and Methods. *Canadian Journal of Program Evaluation*, *30*(1), 108-110. https://doi.org/10.3138/cjpe.30.1.108

Hugo, J. P. (2015). *Support strategies by principals to enhance job satisfaction amongst male teachers in the Ehlanzeni School District, Mpumalanga* (University of South Africa, Doctoral dissertation).

Im, S., & Jitendra, A. K. (2020). Analysis of proportional reasoning and misconceptions among learners with mathematical learning difficulties. Journal of Mathematical Behavior, 57(2020), 1-20

Jensen, E., & Laurie, C. (2016). Doing real research: *A practical guide to social research.*

Johnson, R. E., Grove, A. L., & Clarke, A. (2019). Pillar integration process: *a joint display technique to integrate data in mixed methods research. Journal of Mixed Methods Research, 13(3), 301-320.*

Kaplan, H., & Sadock, B. J. (2015). *Comprehensive textbook of psychiatry. Philadelphia: Lippincott Williams & Wilkins.*

Karao Yilmaz, F. G., Özdemir, B. G., Yasar, Z. (2018) Using digital stories to reduce misconceptions and mistakes about fractions: an action study. *International Journal of Mathematical Education in Science and Technology*, 49(6), 867-898.

Khansir, A. A. (2012). Error Analysis and Second Language Acquisition. *Theory & Practice in Language Studies*, 2(5), 1027-1032.

Kholmatova, S. K. (2020). Didactical principles of initial teaching in mathematics. *Academic Research in Educational Sciences, 1(4).*

Kivunja, C. (2018). Distinguishing between theory, theoretical framework, and conceptual framework: A systematic review of lessons from the field. *International Journal of Higher Education*, 7(6), 44-53.

Kor, L. K., Teoh, S. H., Mohamed, S. S. E. B., & Singh, P. (2018). Learning to make sense of fractions: Some insights from Malaysian primary 4 learners. *International Electronic Journal of Mathematics Education*, *14*(1), 169-182.

Korstjens, I., & Moser, A. (2017). Series: Practical guidance to qualitative research. Part 2: Context, research questions and designs. *European Journal of General Practice*, *23*(1), 274-279.

Kshetree, M. P., Acharya, B. R., Khanal, B., Panthi, R. K., & Belbase, S. (2021). Eighth Grade Learners' Misconceptions and Errors in Mathematics Learning in Nepal. *European Journal of Educational Research*, *10*(3), 1101-1121.

Kusuma, U. I. & Retnawati, H. (2019, October). Analysis of sixth graders' difficulties in solving Mathematics word problems on whole numbers, fractions, and decimals. *Journal of Physics: Conference Series, 1320*(1), 012008.

Lamon, S. J. (2020). Teaching Fractions and Ratios for Understanding: Essential *Content 83 and Instructional Strategies for Teachers. Routledge.*

Lestiana, H. T., Rejeki, S., & Setyawan, F. (2017). Identifying learners' errors on fractions. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *1*(2), 131-139.

Lin, C. P., Chen, W., Lin, C. C., Su, Y. T., & Xie, W. (2014). Group scribbles to support "fraction" collaborative learning in a primary school. *Research and Practice in Technology Enhanced Learning*, *9*(3).

Lincoln, Y.S ., Guba E. G. (1986). But is it rigorous? Trustworthiness and authenticity in naturalistic evaluation*. New Directions for Program Evaluation Evaluatio* (30), 73–84.

Liu, Y. (2018). Fraction magnitude understanding and its unique role in predicting general mathematics achievement at two early stages of fraction instruction. *British Journal of Educational Psychology*, *88*(3), 345-362.

Loc, N. P., Tong, D. H., & Chau, P. T. (2017). Identifying the concept fraction of primary school students: The investigation in Vietnam. *Educational Research and Reviews*, *12*(8), 531-539.

Loewenberg Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of teacher education, 59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Long, L. (2001). *Fabulous Fractions: Games and Activities That Make Math Easy and Fun*. John Wiley & Sons.

Lubwama, R. (2020). Communication Skills. *In The Inside Track to Excelling As a Business Analyst (pp. 53-77). Apress, Berkeley, CA.*

Maellasar, E., & Jupri, A. (2017), Analysis of student errors on division of fractions. *Journal of Physics Conference Series*, 812, 012033.

Makhubele, Y. E. (2021). The analysis of grade 8 fractions errors displayed by learners due to deficient mastery of prerequisite concepts. *International Electronic Journal of Mathematics Education*, *16*(3), em0645.

Malone, A. S., & Fuchs, L. S. (2017). Error patterns in ordering fractions among at-risk fourth-grade students. *Journal of Learning Disabilities*, *50*(3), 337-352.

Malone, A. S., Fuchs, L. S., Sterba, S. K., Fuchs, D., & Foreman-Murray, L. (2019). Does an integrated focus on fractions and decimals improve at-risk learners' rational number magnitude performance? *Contemporary Educational Psychology*, *59,* 101782.

McLeod, G. (2003). Learning theory and instructional design. *learning matters*, *2*(3), 35-43.

McMillan, J., & Schumacher, S. (2010). *Research in education evidence based inquiry (7th int. ed.). Harlow: Pearson Education.*

Minarni, A., & Napitupulu, E. E. (2020). The role of constructivism-based learning in improving mathematical high-order thinking skills of Indonesian learners. *Infinity Journal*, *9*(1), 111-132. https://doi.org/10.22460/infinity.v9i1.p111-132

Mohammed, S. H., & Kinyó, L. (2020). The role of constructivism in the enhancement of social studies education. *Journal of critical reviews*, *7*(7), 249-256.

Mokhtar, M. A. M., Ayub, A. F. M., Radhiah, R., & Said, S. S. M. (2019). Analysis of Year Four Learners' Difficulties in Solving Mathematical Problems Involving Fraction. *International Journal of Academic Research in Business and Social Sciences*, *9*(11), 1560-1569.

Moru, E. K., Qhobela, M., Wetsi, P., & Nchejane, J. (2014). *Teacher knowledge of error analysis in differential calculus. pythagoras, 35(2), 1-10.*

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science.* TIMSS & PIRLS International Study Center website. https://timssandpirls.bc.edu/timss2019/international-results.

Namkung, J. M., Fuchs, L. S., Koziol, N. (2018). Does initial learning about the meaning of fractions present similar challenges for learners with and without adequate whole-number skill? *Learning and Individual Differences*, *61,* 151-157.

National Mathematics Advisory Panel. (2008). *National Mathematics Advisory Panel. Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. Department of Education, Washington, US. https://files.eric.ed.gov/fulltext/ED500486.pdf

National Planning Commission. (2012). *National Development Plan 2030. "Our future - make it work".* https://www.gov.za/sites/default/files/gcis_document/201409/ndp-2030-our-future-make-it-workr.pdf

Naylor, S., & Keogh, B. (1999). Constructivism in classroom: Theory into practice. *Journal of Science Teacher Education*, *10*(2), 93-106.

Neagoy, M. (2017). *Unpacking Fractions: Classroom-Tested Strategies to Build Learners' Mathematical Understanding*. Alexandria, VA: Association for Supervision and Curriculum Development.

Noble, H., & Smith, J. (2015). Issues of validity and reliability in qualitative research. *Evidence-based nursing*, *18*(2), 34-35.

Novitasari, N., Lukito, A., & Ekawati, R. (2018). Slow learner errors analysis in solving fractions problems in inclusive junior high school class. In *Journal of Physics: Conference Series,* 947 (1), 012035.

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. International journal of qualitative methods, 16(1), 1609406917733847. https://doi.org/10.1177/1609406917733847

Olivier, A. (1989). Handling learners' misconceptions. *Pythagoras,* 21, 10-19.

Osanloo, A., & Grant, C. (2016). Understanding, selecting, and integrating a theoretical framework in dissertation research: Creating the blueprint for your "house". *Administrative Issues Journal,* 4(2), 12-26. Available at: https://files.eric.ed.gov/fulltext/EJ1058505.pdf (Accessed 5 April, 2022)

Pala, R. H., Herman, T., & Prabawanto, S. (2019, February). Learners' error on mathematical literacy problems. In *Journal of Physics: Conference Series* (Vol. 1157, No. 2, p. 022125). IOP Publishing.

Pavlovičová, G., & Vargová, L. (2020). Investigation of selected aspects of fraction understanding. *TEM Journal*, 9(2), 702-707.

Peng, A., & Luo, Z. (2009). A framework for examining mathematics teacher knowledge as used in error analysis. *For the Learning of Mathematics*, 29(3), 22-25.

Petit, M. M., Laird, R. E., Marsden, E. L., & Ebby, C. B. (2015). *A focus on fractions: Bringing research to the classroom. Routledge.*

Polit D.F., Beck C.T. (2019). Resource manual for nursing research: Generating and assessing evidence for nursing practice. *Lippincott Williams & Wilkins.*

Purwadi, I., Sudiarta, I., & Suparta, I. N. (2019). The Effect of Concrete-Pictorial-Abstract Strategy toward Learners' Mathematical Conceptual Understanding and Mathematical Representation on Fractions. *International Journal of Instruction*, *12*(1), 1113-1126.

Resnick, I., Rinne, L., Barbieri, C., & Jordan, N. C. (2019). Children's reasoning about decimals and its relation to fraction learning and mathematics achievement. *Journal of Educational Psychology*, *111*(4), 604-618.

Richards, J. C. (2015). *Error Analysis: Perspectives on Second Language Acquisition*. Routledge.

Rodrigues, J. (2017). Developing fraction screeners to identify children at risk for mathematics difficulties (Unpublished doctoral dissertation). University of Delaware, Newark

Samková, L. (2019) Investigating subject matter knowledge and pedagogical content knowledge in mathematics with the Concept Cartoons Method. *Scientia in Educatione*, 10(2) p. 62–79.

Samsonovich, A. V. (2020). Socially emotional brain-inspired cognitive architecture framework for artificial intelligence. *Cognitive Systems Research, 60, 57-76.*

Scheiner, T., Montes, M. A., Godino, J. D., Carrillo, J., & Pino-Fan, L. R. (2019). What makes mathematics teacher knowledge specialized? Offering alternative views. *International Journal of Science and Mathematics Education, 17(1), 153-172.*

Schoch, K. (2020). Case study research. *Research Design and Methods: An Applied Guide for the Scholar-practitioner, 245-58.*

Schunk, D. H. (2012). *Learning theories an educational perspective sixth edition*. London: Pearson.

Shahbari, J. A. & Peled, I. (2017). Modelling in primary school: Constructing conceptual models and making sense of fractions. *International Journal of Science and Mathematics Education*, *15*(2), 371-391.

Shulman, L. S. (1986). Those who understand: A conception of teacher knowledge. *American Educator*, *American Educator*, *10*(1), 9-15, 43-44.

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences*, *17*(1), 13-19.

Simon, S. D. (2001). From neo-behaviourism to social constructivism: The paradigmatic non-evolution of Albert Bandura. *Unpublished thesis, Bachelor of Arts Honors, Division of Educational Studies, Faculty of Emory College of Emory University*.

Singh, C. K. S., Singh, A. K. J., Razak, N. Q. A., & Ravinthar, T. (2017). Grammar errors made by ESL tertiary learners in writing. *English Language Teaching*, *10*(5), 16-27.

Smith, J. P. III (2002). 'The development of learners' knowledge of fractions and ratios' in Making Sense of Fractions, Ratios, and Proportions, *2002 Yearbook, National Council of Teachers of Mathematics*, 2002. 3-17.

Snyder, H. (2019). Literature review as a research methodology: An overview and guidelines. *Journal of Business Research*, 104, 333-339.

Soni, M., & OKamoto, Y. (2020). Improving children's fraction understanding through the use of number lines. *Mathematical Thinking and Learning*, 1-11.

Sorto, M. A., Shalem, Y., & Sapire, I. (2014). Teachers' explanations of learners' errors in standardised mathematics assessments. *Pythagoras*, *35*(1), 1-11.

Spinillo, A. G. & Bryant, P. (1991). Children's proportional judgments: The importance of “half”. *Child Development*, 62(3), 427-440.

Stahl, N. A., & King, J. R. (2020). Expanding approaches for research: Understanding and using trustworthiness in qualitative research. *Journal of Developmental Education*, *44*(1), 26-28.

Steffe, L. P., Nesher, P., Cobb, P., Sriraman, B., Greer, B. (Eds.). (2013). *Theories of mathematical learning*. London: Routledge.

Supandi, S., Waluya, S. B., Rochmad, R., Suyitno, H., & Dewi, K. (2018). Think-Talk-Write Model for Improving Learners' Abilities in Mathematical Representation. *International Journal of Instruction*, *11*(3), 77-90.

Thomas, G. (2021). *How to do your case study*. London: Sage Publications.

Tian, J. & Gunderson, E. (2020). Teaching Fractions to Young Children. *YC Young Children*, 75(4), 62-67.

Tian, J. & Siegler, R. S. (2017). Fractions learning in children with mathematics difficulties. *Journal of learning disabilities*, 50(6), 614-620.

Tucker, A. (2008). Fractions and units in everyday life. *Calculation vs. Context*, 75.

Tyler, R.W. (1949). Basic Principles of Curriculum and Instruction. IL: The University of Chicago.

Ubah, I. J. A., & Bansilal, S. (2018). Pre-service primary Mathematics teachers’ understanding of fractions: An action–process– object–schema perspective. *South African Journal of Childhood Education, 8*(2), 1-12. https://doi.org/10.4102/sajce.v8i2.539

Van de Walle, J., Karp, K. S., Bay-Williams, J. M. (2014). *Elementary and Middle School Mathematics: Teaching Developmentally (8th Edition).* London: Pearson, 417-431.

Veloo, A., Krishnasamy, H. N., Wan Abdullah, W. S. (2015). Types of student errors in mathematical symbols, graphs and problem-solving. *Asian Social Science*, 11(15), 324-334.

Vygotsky, L. (1978). Interaction between learning and development. *Readings on the Development of Children*, 23(3), 34-41.

Wadsworth, B. J. (1996). *Piaget's theory of cognitive and affective development: Foundations of constructionism.* Longman Press.

Wijaya, A. (2017). The relationships between Indonesian fourth graders' difficulties in fractions and the opportunity to learn fractions: A snapshot of TIMSS results. *International Journal of Instruction*, 10(4), 221-236.

Wong, M., & Evans, D. (2007). Learners' conceptual understanding of equivalent fractions. In *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia* (pp. 824-833). Adelaide, SA: MERGA.

Woods, D. D., Dekker, S., Cook, R., Johannesen, L., Sarter, N. (2017). *Behind Human Error*. London: CRC Press.

Zakaria, E. & Syamaun, N. (2017) The effect of realistic mathematics education approach on learners' achievement and attitudes towards mathematics, *Mathematics Education Trends and Research* (1), 32-40.

6. Annexures

6.1 Annexure A: Ethics Certificate

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/11/11

Ref: **2020/11/11/36041904/45/AM**
Name: Ms M Thapeli
Student No.: 36041904

Dear Ms M Thapeli

Decision: Ethics Approval from 2020/11/11 to 2023/11/11

Researcher(s): Name: Ms M Thapeli
E-mail address: 36041904@mylife.unisa.ac.za
Telephone: 072 3236705

Supervisor(s): Name: Prof Kamleshie Mohangi
E-mail address: mohank@unisa.ac.za
Telephone: 012 337 6169

Name: Prof France Machaba
E-mail address: emachamf@unisa.ac.za
Telephone: 012 429 8582

Title of research:

Error Patterns and Underlying Misconceptions of Fractions among 15 Grade 6 Learners from a Public School in Gauteng

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/11/11 to 2023/11/11.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2020/11/11 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.

Open Rubric

University of South Africa
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www.unisa.ac.za

6.2 Annexure B: Turnitin Report

turnitin

Digital Receipt

This receipt acknowledges that Turnitin received your paper. Below you will find the receipt information regarding your submission.

The first page of your submissions is displayed below.

Submission author:	Evelyn Mpho Thapeli
Assignment title:	Complete dissertation/thesis DRAFT
Submission title:	Error Patterns and Underlying Misconceptions of Fractions a...
File name:	TurItInNovember2023-15.docx
File size:	4.76M
Page count:	135
Word count:	35,965
Character count:	198,601
Submission date:	16-Nov-2023 11:38AM (UTC+0200)
Submission ID:	2229955417

Error Patterns and Underlying Misconceptions of Fractions among 15 Grade 6 Learners from a Public School in Gauteng

By

EVELYN MPHO THAPELI

(STUDENT NUMBER: [illegible])

DISSERTATION

submitted in partial fulfilment of the requirements for the degree of

MASTER OF EDUCATIONAL PSYCHOLOGY

in

FACULTY OF EDUCATION

at

UNIVERSITY OF SOUTH AFRICA

SUPERVISOR:

PROFESSOR FRANCE MACHABA

PROFESSOR KAMLESHIE MOHANGI

DATE: 04 OCTOBER 2022

1

6.3 Annexure C: Request for permission to conduct research at Govan Mbeki Primary School

Title of the research: **Error Patterns and Underlying Misconceptions of Fractions among 15 Grade 6 Learners from a Public School in Gauteng**

Date: 10 October 2020

Mr M.M Kele

Principal

Tell: 082 552 5153 Email Address: bramikemoeti@gmail.com

Dear Mr Kele

I, Mpho Evelyn Thapeli am doing research under supervision of **Professor Fance Machaba Professor** in Department of Mathematics **and Professor Kamleshie Mohangi,** Profess in the Department of **Psychology of Education** towards a Master's Degree in Education at the **University of South Africa.** We are inviting you to participate in a study entitled, **Error Patterns and Underlying Misconceptions of Fractions among 15 Grade 6 Learners from a Public School in Gauteng** The aim of the study is:

- What are common error patterns in addition, subtraction and multiplication of fractions among the selected group of Grade 6 learners in a public primary school in Gauteng?

Your school has been selected because the researcher is working as educator and will love to formulate strategies in subject matter. The study will entail selection of group of learners in **Grade 6** classes. **Participants** will be **15 learners** and allowed to participate only in consent of the parents since they are minors.

The benefits of this study are that learners will be aware of their mistakes in fractions through error analysis, and will be able to learn from their mistakes; teachers will also state their challenges in fractions, through all collection of data the teachers will be assisted in formulating strategies in dealing with fractions. There will be no risk in the research. There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will entail making correction of worksheet given to learners and word of mouth to teachers who has been interviewed.

Yours sincerely

____________________________ (researcher) Date: 10 October 2020

Mpho Evelyn Thapeli

Educator (Researcher)

6.4 Annexure D: Letter requesting parental consent for minors to participate in a research project

Dear Parent

Your ___________________________(son/daughter/child) is invited to participate in a study entitled, **Analysis of common errors made by learners in mathematical fractions; Strategies to overcome errors in Gauteng Province Primary Schools**

I am undertaking this study as part of my **Master's** research at the University of South Africa. The purpose of the study is to identify fraction common errors made by learners in primary school and the possible benefits of the study are the improvement of Mathematics. I am asking permission to include your child in this study because grade 6 learners exit from intermediate to senior, they need to understand the fractions and be able to identify their mistake in order for them to learn from their mistakes. I expect to have 15 other children participating in the study.

If you allow your child to participate, I shall request him/her to:

- Complete a test with 10 questions in two hours within two week cycle in Govan Mbeki Primary School.
- Be interviewed on the incorrect answers

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are high performance in Mathematics, identifying their mistakes and learning from them. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available.

In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

The benefits of this study are improvement in Mathematics results; learners will identify their mistakes and learn from them.

There is no risk and there will be no reimbursement or any incentives for participation in the research.

If you have questions about this study please ask me or my study supervisor, **Professor France Machaba and Professor Kamleshie Mohangi** Department of Psychology and Department of Mathematics, College of Education, University of South Africa. My contact number **072 323 6705** and my e-mail is mphothapeli@gmail.com .Permission for the study has already been given by **Mr Kele M.M (Principal)** and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:

Sincerely

________________________________ _________________________

Parent/guardian's name (print) Parent/guardian's signature: Date:

Mpho Thapeli 10 October 2020

Researcher's Researcher's signature Date:

6.5 Annexure E: Letter requesting assent from learners in Govan Mbeki primary school to participate in a research project

Dear learner Date_______________________

My name is mam **Mpho Evelyn Thapeli** and would like to ask you if I can come and do mathematics I am trying to learn more about how children do mathematics in class. If you say YES to do this, I will come and give you worksheet to answer based on fractions and you will enjoy the activities. We will do a fun game where you have to answer some questions for me. I will not ask to you to do anything that may hurt you or that you don't want to do. I will also ask your parents if you can take part. If you do not want to take part, it will also be fine with me. Remember, you can say yes or you can say no and no one will be upset if you don't want to take part or even if you change your mind later and want to stop. You can ask any questions that you have now. If you have a question later that you didn't think of now, ask me next time I visit your class. Please speak to mommy or daddy about taking part before you sign this letter. Signing your name at the bottom means that you agree to be in this study. A copy of this letter will be given to your parents.

Regards

Mam Thapeli 10 Octomber 2020

Your Name	**Yes I will take part** 	**No I don't want to take part** 
Name of the researcher		
Date		
Witness		

6.6 Annexure F: Participant information sheet

Date: ________________________

Title: Analysis of common errors made by learners in mathematical fractions; Strategies to overcome errors in Gauteng Province Primary Schools

DEAR PROSPECTIVE PARTICIPANT

My name is **Mpho Evelyn Thapeli** and I am doing research under the supervision of **Professor Kaino L.M**, a Professor in the Department of **Psychology of Education** towards a **Master's Degree** at the University of South Africa. We are inviting you to participate in a study entitled Analysis of common errors made by learners in mathematical fractions; Strategies to overcome errors in Gauteng Province Primary Schools

WHAT IS THE PURPOSE OF THE STUDY?

This study is expected to collect important information that could identify the common errors made by learners. Learners could also overcome these errors in knowing their mistakes made in fractions. Find the view from teachers on difficulties faced by learners in fractions.

WHY AM I BEING INVITED TO PARTICIPATE?

You are invited because grade 6 is exit grade from intermediate phase to senior phase

I obtained your contact details from the school. Total number of participants in this study is 15 grade 6 learners.

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?

Describe the participant's actual role in the study.

The study involves questionnaires. The questionnaires will be in test format with 10 questions. The questions will be based on addition and subtraction of fractions. The research will take two hours in two a week cycle.

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE?

Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent / assent form. You are free to withdraw at any time and without giving a reason

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

You are helping the teachers to be aware of learner's errors in fraction and how to overcome the mistakes. Learners will also improve their performance in mathematics.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?

There are not negative consequences for your participation.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?

You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research. Your name will not be recorded anywhere and no one will be able to connect you to the answers you give. Your answers will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

Your data may be used for other purposes, such as a research report, journal articles and /or conference proceedings. A report for the study may be submitted for publication, but individual participants will not be identifiable in such a report

HOW WILL THE RESEARCHER(S) PROTECT THE SECURITY OF DATA?

Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

No payment or reward will be offered, financial or otherwise. The researcher will take care of any costs incurred by the participant in adherence with the principle of fair procedures (justice).

HAS THE STUDY RECEIVED ETHICS APPROVAL

This study has received written approval from the Research Ethics Review Committee of the Psychological Department at Unisa. A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?

If you would like to be informed of the final research findings, please contact Mph Evelyn Thapeli on 072 323 6705 or email mphothapeli@gmail.com. The findings are accessible for 5 years. Should you require any further information or want to contact the researcher about any aspect of this study, please contact **Mpho Thapeli on 072 323 6705** email mphothapeli@gmail.com.

Should you have concerns about the way in which the research has been conducted, you may contact Professor Kaino L.M, email muganyizikaino@gmail.com.

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

__________________________ Date: 10 October 2020

Signature

Mpho Evelyn Thapeli

6.7 Annexure G: Consent/assent to participate in this study (Return slip)

I, __________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the questionnaires in test format

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) ____________________________________

____________________________ ___________________________________

Participant Signature Date

Researcher's Name & Surname (please print) Mpho Thapeli

10 October 2020

Researcher's signature Date

6.8 Annexure H: Grade Six Research Test

Name of the learner: ___________________ Date:____________________

ADDITION AND SUBTRACTION OF FRACTION AND MIXED FRACTION

Calculated the following fractions and show all your calculation on a space provided.

1. $\frac{3}{9} + \frac{2}{3} + \frac{2}{8}$	5. $\frac{4}{10} + \frac{1}{10}$
2. $\frac{23}{24} - \frac{1}{4} - \frac{2}{12}$	6. $\frac{4}{12} + \frac{1}{12}$
3. $3\frac{3}{5} + 1\frac{4}{5}$	7. $5\frac{3}{4} + 3\frac{1}{5}$
4. $3\frac{4}{7} + 2\frac{1}{7}$	8. $4\frac{1}{5} + 3\frac{1}{2}$

MULTIPLICATION OF FRACTIONS AND MIXED FRACTION

9. $2\frac{5}{6} \times 6\frac{2}{5}$	10. $\frac{2}{4} \times \frac{5}{6}$

6.9 Annexure I: Language letter

Member South African Translators' Institute
www.language-services.online

12 November 2023

TO WHOM IT MAY CONCERN

The dissertation "Error Patterns and Underlying Misconceptions of Fractions among 15 Grade 6 Learners from a Public School in Gauteng" by Evelyn Mpho Thapelo has been proofread and edited for language by me.

I verify that the manuscript is ready for publication or public viewing regarding language and has been formatted per the prescribed style.

Please note that no view is expressed regarding the document's technical contents or changes made after the date of this letter.

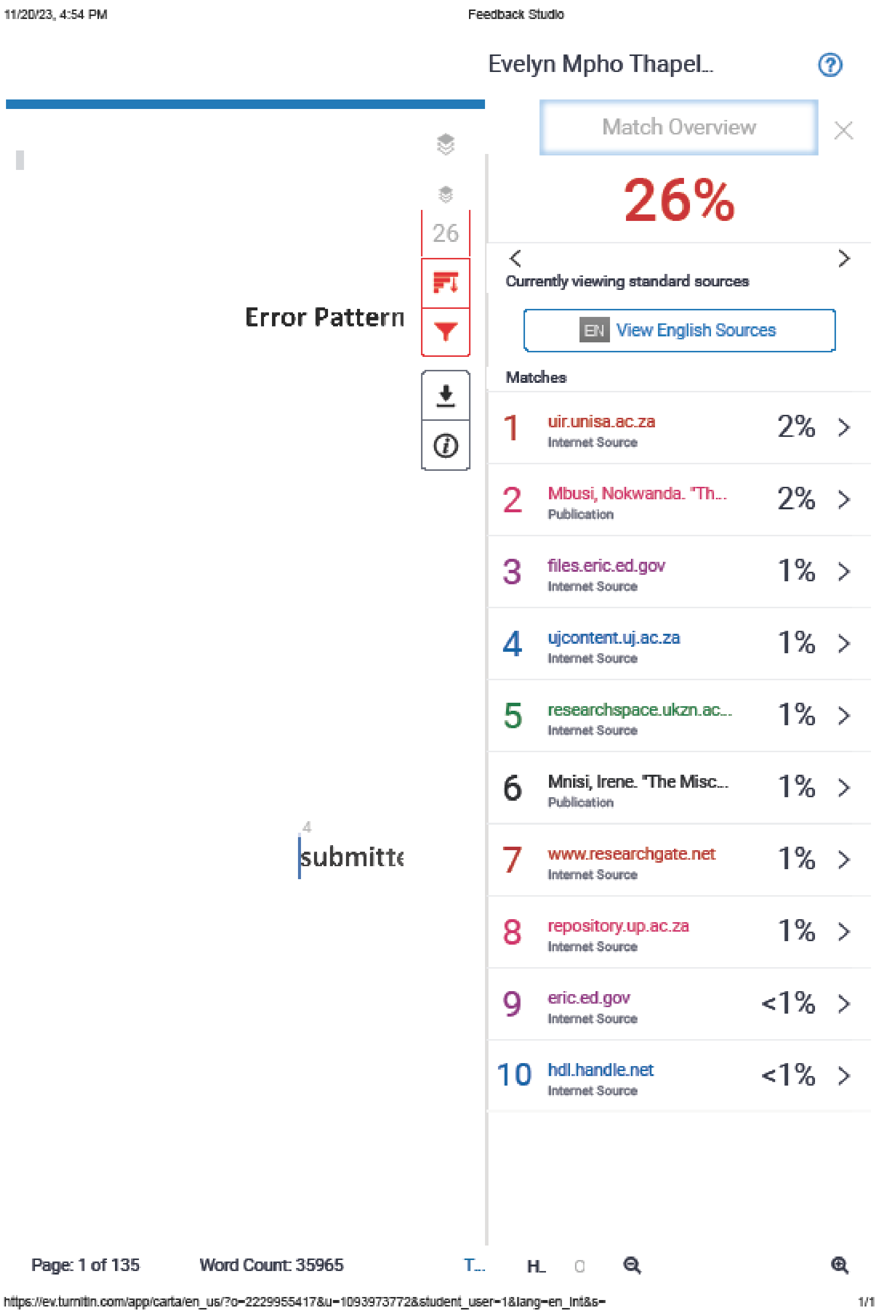
Kind regards

Anna M de Wet

SATI MEMBER 1003422

BA (English, Afrikaans, Latin) (Cum Laude), University of Pretoria.
BA Hons ((Latin) (Cum Laude), University of Pretoria.
BA Hons (Psychology), University of Pretoria.

6.10 Annexure J: Turn-It-In Similarity Index



6.11 Annexure K: Sample of an interview Transcript

1st

UNISA | college of education

Interview to learners DK (CH)

Questions based on incorrect answers from learners in grade 6

Researcher's questions 1: Explain how you got your answer?

Learners Response:

a. $\frac{3}{9}$ ke e Hotelletse then kare $\frac{2}{3\times3}$ so that it will be the same as $\frac{3}{9}$. the $\frac{2}{8}\times5$, then I added numerators together answer $\frac{7}{9}$.

b. I changed denominator to be $\frac{23}{24}-\frac{1}{4\times6}-\frac{2}{12\times2}$, I subtracted numerators answer $= {}^{20}/_{24}$

d. ke entse mistake ka numerator suppose to be $1\frac{3}{7}$.

e. denominator ac the same

Researcher's questions 2: Can you explain the rule or process followed?

Learners Response:

g. I subtracte whole numbers $(5-3)$ then subtracted numerator $(3-1)$, denominator I changed it to 20 by $\frac{3}{4\times5}$ and $\frac{1}{5\times5}$ and is $\frac{4}{20}$

h. subtracted whole $(4-3)$ the numerator $(1-1)$ then $\frac{1}{5\times2}$ and $\frac{1}{2\times5}$ answer $\frac{0}{10}$.

i) I multiplied whole (2×6) then (5×2), numerator (6×5) the $\frac{2}{5\times6}$ and answer $12\frac{7}{30}$

j) $(2\times5)=10$ and I said den $\frac{2}{4\times3}$ and $\frac{5}{6\times2}$ answer $\frac{10}{12}$.

H

UNISA | college of education

Name: Learner: DK **Date:** 30 November 2020

1. ADDITION AND SUBTRACTION OF COMMON FRACTIONS

Calculate the following fraction and write answer in its simplest form

a. $\frac{3}{9}+\frac{2}{3}+\frac{2}{8}=\frac{7}{9}$

$\frac{3}{9}+\frac{2}{3\times3}+\frac{2}{8}$

$=\frac{7}{9}$

b. $\frac{23}{24}-\frac{1}{4}-\frac{2}{12}=\frac{20}{24}\neq\frac{20}{12}$

$\frac{23}{24}-\frac{1}{4\times6}-\frac{2}{12\times2}$

$\frac{20}{24}$

c. $3\frac{3}{5}+1\frac{4}{5}=4\frac{7}{5}$

$3\frac{3}{5}+1\frac{4}{5}$

$3+1=4$

$=\frac{7}{5}$

d. $3\frac{4}{7}-2\frac{1}{7}=1\frac{2}{7}$

$3\frac{4}{7}-2\frac{1}{7}$

$3-1=2$

$4-1=3$

$=\frac{7}{7}$

know the rule but don't know how to place ans in its place

e. $\frac{4}{10}+\frac{1}{10}=\frac{5}{10}$

$\frac{4}{10}+\frac{1}{10}$

$4+1=5$

$\frac{5}{10}$

f. $\frac{4}{12}-\frac{1}{12}=\frac{3}{12}$

$\frac{4}{12}-\frac{1}{12}$

$4-1=3$

$\frac{2}{12}$

g. $5\frac{3}{4}-3\frac{1}{5}=\frac{4}{20}=\frac{4}{10}$

$5\frac{3}{4}-3\frac{1}{5}$

$\frac{3}{4\times5}-\frac{1}{5\times4}$

$\frac{4}{20}$

unable to identify LCM

h. $4\frac{1}{5}-3\frac{1}{2}=\frac{0}{10}$

$4\frac{1}{5}-3\frac{1}{2}$

$\frac{1}{5\times2}-\frac{1}{2\times5}$

$\frac{1}{10}$

unable to identify LCM

2. MULTIPLICATION OF COMMON FRACTIONS

Calculate the following fraction and write answer in its simplest form

i. $2\frac{5}{6} \times 6\frac{2}{5}$ $= \frac{7}{30} = \frac{7}{15}$ $2\frac{5}{6\times5}$ of $6\frac{2}{5\times6}$ $12\ \frac{7}{30}$ Unable to change mixed numbers into common errors	j. $\frac{2}{4} \times \frac{5}{6}$ $= \frac{10}{12}$ $\frac{2}{4\times3}$ of $\frac{5}{6\times2}$ $\frac{10}{12}$

~~~~~~~~~~~~~~~~~~~~~~~~END~~~~~~~~~~~~~~~~~~~~~~~~
~~~~~~~~~~~~~~~~~~~~~~~~

Dk (H) 1st

Researcher: Ha o ache lebitso la hao, e tlo explainela pele hore how did you get the answer, neh mmh

learner: yes mam uhm, ke etsere ke $\frac{3}{9}$ and then katla ka nona ka multiplier ka 3 kamo, ya tlo nya 9 kamo ka multiplier ka 2 ka 5

Researcher: okay ooh we itse 3 over 9 and then, monana, bate bone hantle mona

learner: antere mo ke 2 ae 3 ke e and then kare 3x3

Researcher: okay, ware 3x3

learner: yes mam

Researcher: okay, monana

learner: ke $\frac{2}{5}$ antere, kare 2 x 5

Researcher: ooh jwale kabatla hore o explainele hore hobaneng we itse 3x3 mo

learner: antere ena etlo mpha answer ele 1 kaofela hore e tshware

Researcher: okay monana ke x by enana o isetae

learner: efeng

Researcher: o s oo ke ena 2 over 8 ere ke ngole he, weitse 3 over 9 ($\frac{3}{9}$) we e-tlhohelele ele jwalo neh

learner: yes mam

Researcher: wabona kenna ke ngola language ya hao

learner: yes mam

Researcher (silence) ke yona ntho ena we entseng mona akere

learner: yes mam.
Researcher: Okay (silence) so that etlo tshwana ka
learner: yes mam (silence)
Researcher: mmh, then morena, kanje we itseng
kana mona we itseng
learner: ke itse ke $\frac{2}{5}$ anke kare 2 x 5
Researcher: mmh and then
learner: mam
Researcher: and then o bo etsang hape lare
explaine re qete hee
learner: mam
Researcher: ee $\frac{2}{8}$ o itse 2 x 5, 5 x 2
learner: yes man
Researcher: yes man
learner: and then ke di addile di number
tse kaofdaltse ka o dimo, t
Researcher: mmh
learner: tsa mpha number eleng $\frac{7}{9}$
Researcher: (silence) tsa ofa ansa ee
learner: yes mam
Researcher: areye ko B
learner: B mam anke kamo ke $\frac{23}{24}$ so
moo anke ke minus, ke qadile ka gata kano
ka fatshe hore di number di tshwane, so
ke itse 4 ka x 6
Researcher: Okay, hore etlo ofa bokae
learner: ya tlo nfa 24
Researcher: mmh
learner: and then ka mo ka kare kar 10 x 2
ya mpha 24, haqeta moo kamo ka hodimo
ka minus tse ya mpha 2

answer ya teng ke 20 over ntho 724 $\left(\frac{20}{724}\right)$
Researcher: (silence) then answer o be o etsa
jwang obo o subtractor
learner: yes mam (learners talking)
Researcher: (silence) (dont forget to put your code code
answer keng, 00, weitse o subtracthile neh
learner: yes mam
Researcher mmh
learner: number 1, nubmber moo mam
Researcher: ~~ke~~ mm
learner: ke addile no ka adda 3 plus 1
it gave me 2 aa I mean 4 and then
I added 3 plus 4 it give me 7 and this
denomenator I put then down so it gave
me 4 whole $\frac{7}{5}$ $\left(4\frac{7}{5}\right)$
Researcher: mmh okay that one its straight
forward mmh (talking one ~~the~~ the learners)
ware disturba nguanaka hareye ka (d)
learner: number (d) mam ke subtraction
so ke subtractile 3 minusa 2 it gave me 1
1 whole,
Researcher mmh
learner: and 4 minus 1 it gave $\frac{2}{7}$
Researcher: 4 minus 1, number (D) weitse
3-2 atere
learener: yes mam
Researcher: ya ofa 1, then
learner: and I said 4 minasa 1 ya mpha
te ke 3 te ngotse 2
Researcher mistake
learner: yess mam

Researcher: enang wentseng
learner: moo
Researcher: mm
learner: mona mam, I added 4 plus 1 it gave me $\frac{5}{10}$
Researcher: mmh, why over 10
learner: mam
Researcher: hobaneng o ngotse 10
learner: Because denomenator are the same
Researcher:(silence) mmh
learner: f mam, it 4 − 1, I minus 1 from 4, it gave $\frac{3}{12}$
Researcher: mmh
learner: yes mam, so I wrote 12 because the numerator are the same
Researcher: m
learner: ke etse ee g mam I subtract 3 from 5 it gave me 2, it gave me 2 it gave me 2 then I subtract 1 from 3 it gave me 4 over, and here I must multiply by 5 here 4 x 5 it gave me 20 and here I multiply 5 by 5 it also gave me 20, so my numerator is 20
Researcher: okay, ke batla o qale u subtracted (silence) akere that wat u said
learner: mam
Researcher: you subtracted the whole number okay then
learner: and then I subtracted ee fractions,
Researcher: mmh

Researcher: yah ke batla ho ngola subtracted a
~~learner~~ numerator tsenang 3-1 akere
learner: okay
Researcher: okay, then
learner: and then, the denominator I ~~-8~~ multiply
4 by 5 It gave me 20, and I also multiply
5 by 4, it also gave me 20
Researcher: multiply by 5 akere o chollo
jwalo
learner: yes mam
Researcher: multiply enang
learner: $\frac{1}{5}$ I multiplied by 5
Researcher: ee
learner: yes mam
Researcher: mara o ngotse 4
learner: nekere ke ngola 5
Researcher: 5 x 4,
learner: by 5
Researcher: by 5
learner: yes mam
Researcher: then answer
learner: an answer is $\frac{4}{20}$
Researcher: huu, okay ansa ya hao ke
$\frac{4}{20}$ neh, 2 ee e ile kae, akere weitse
2 5 minus 3, akere we itse 5 minus
3
learner: yes mam
Researcher: mmh, 2 ya teng ekae, and then
ware 3 minus 4, 3-4, I mean 3-1
(silence), re e fete
learner: yes. mam
Researcher: okay

Researcher: vite okay explaining (h) how did you get answers for (h)

learner (h) I subtracted 4, 4 minusa 3 it gave me $\frac{1}{4}$ mam I didnt write it

Research: mmh

learner: and I said $3\frac{1}{5} - \frac{1}{2}$, I ~~said~~ subtracte 1 minus 1 it gave me 0, (silence)

Researcher: yah, then after

learner: I gave me zero mam

Researcher: mmh

learner: and then here, deno, denomenator-numerator uuh, I said 5 x 2

Researcher: mmh

learner: and it gave me 10, and this side I did 2 x 5 it also gave me 10, so my answer was $\frac{0}{10}$

Researcher: mmh okay, hareye ko (talking to learners.) then ko (I), explain how you got your answer ko (i), (shhh.) explain how you got your answer in (i)

learner: in (i) mam I, I I multiplied 2 over 6, mam most of the wholes I didnt write them, cause I only looked here,

Researcher: oooho

learner: okay I, I multiplied 2 over 6 and it gave me 12, (silence), mam I multiplied 2 over 6 it gave me 12, and here I also multiplied 5x2 it gave, it gave me 10 and here the numerator I said 6 x 5 it gave me 30 and I also said 5 x 6 it also gave me 30

Researcher: you multiplied 2 and 5

learner: no mam 5 plus 2
Researcher: La qalle fatshe pele, weits* o qadile kang
learner: ke qadile ka di whole
Researcher: yah wa renge
learner: kare 2x6 ya nfa 12 ~~and~~
Researcher: yah
learner: and I said 5x2 its 10
Researcher: mmh.
learner: and then the, the numerator I said 6x5, it gave me 30 (silence) and I also said 5x6 it also gave me 30 so my answer
Researcher: you also said 5x6, which one, yes mam
learner: yes mam
Researcher: mmh.
learner: my answer is $12\frac{2}{30}$
Researcher: yah, answer $12\frac{2}{30}$
learner: $12\frac{7}{30}$
Researcher: ooh $12\frac{7}{30}$ (shu) hareteke mona
learner: I said 2x5, it gave me 10 a my numerator I said 4x3 it gave me 12, and I said 6x2, it also gave me 12 so my answer became 10 over 12 ($\frac{10}{12}$)
Researcher: please explaine again. o. you said, I didnt get it you said
learner: I, I said 2x5, I multiplied 2 by 5, it gave me 10, and I said 4 my den, my numerator I said 4x3 it gave me 12, and I also did the same 6x2

it also gave me 12, so my answer is $\frac{10}{12}$

Researcher. Okay now second question

~~Rete~~ ~~Re~~ and then ko (I) and (j) o sekedi process efeng,

learner. I used multiplication,

Researcher. oooh, thank you

6.12 Annexure K: Sample of coding of the transcript

Translations

Participant 5

DK (High Category)

Researcher: you don’t say your name, you just going to explain how did you get your answers neh, mmh

Learner: yes mam uhm, here its $\frac{3}{9}$ and then I multiplied 3, then it gave me 9 this side I multiplied 2 by 5

Researcher: okay ooh you said 3 over 9 and the here? I can’t see properly

Learner: here it’s 2 over 3, this one and I said 3 X 3

Researcher: okay, then you said 3 X 3

Learner: yes mam

Researcher: okay, dear

Learner: it’s $\frac{2}{8}$ I said 2 X 5

Researcher: ooh then I need you to explain why you said 3 X 3 here

Learner: this one is going to give me 1, all they must be the same

Researcher: okay dear multiplied this one by this one, where did you find this one

Learner: which one mam

Researcher: oo.. 8…. Oo its 2 over 8, let me write it, you said 3 over 9, you left it like that neh

Learner: so $\frac{3}{9}$ plus $\frac{2}{3}$ plus $\frac{2}{8}$ equals to $\frac{7}{20}$

Researcher: mm is that your answer

Learner: yes mam

Researcher: you see am writing your language here

Learner: yes mam

Researcher: this is what you did here neh

Learner: yes mam

Researcher: okay (silence)so that this will be the same as this one

Learner: yes mam

Researcher: then here, what did you say

Learner: I said, its $\frac{2}{8}$ I said 2 X 5

Researcher: mmhh and then

Learner: mam

Researcher: again explain here what did you do?

Learner: mam

Researcher: this $\frac{2}{8}$ yousaid 2 X 5 or 5 X 2

Learner: yes mam, then I added all these numbers on top

Researcher: mmhh

Learner: they gave me $\frac{7}{9}$ (silence)

Researcher: (silence) they all gave you this answer

Learner: yes mam

Researcher: let's go to b

Learner: b mam we have $\frac{23}{24}$ this side so this is subtract, I started with numbers at bottom so that they will all be the same so I said 4 X 6

Researcher: okay, to give you what answer?

Learner: it gave me 24

Researcher: mmh

Learner: and then I said 12 X 2, it gave me 24, then after I subtracted number on top which is 2, then my answer is $\frac{20}{24}$

Researcher: (silence) then what did you do with your answer did you subtract it?

Learner: yes mam

Researcher: mmh

Learner: number ..number here

Researcher: mmh

Learner: I have added 3 and 1, it gave me 2, no I mean 4, and then I added 3 plus 4 it gave me 7, and this denominator, I have put them down so it gave me 4 whole $\frac{7}{5}$

Researcher: mmh okay that one its straight forward mmh, learners you disturbing , okay lets go to d

Learner: number d, mam its substraction, so I subtracted 1 from 3, it gave me 1 whole

Researcher: mmh

Learner: and 4 subtracted by 1 it gave me $\frac{2}{7}$

Researcher: 4 subtract 1, number d you said 3 – 2

Learner: yes mam

Researcher: then it gave you 1, then

Learner: and I said 4 subtract 1 and it gave me 3 I wrote 2

Researcher: that's a mistake

Learner: yes mam

Researcher: here what did you do

Learner: here

Researcher: mmh

Learner: here I added 4 and 1 it gave me $\frac{5}{10}$

Researcher: mmh, why over 10

Learner: because the denominators are all the same

Researcher: (silence) mmh

Learner: f mam, it 4 – 1, I subtracted 1 from 4, then got $\frac{3}{12}$

Researcher: mmh

Learner: yes mam, so I wrote 12 because the numerators are the same

Researcher: mmh

Learner: should I do g mam, I subtracted 3 from 5, it gave me 2, it gave me 2, it gave me 2, then I subtracted 1 from 3 it gave me 4 over, and here I must multiply by 5, 4 X 5 it gave me 20 , so my numerator is 20

Researcher: okay, I want you to start, you subtracted (silence) that's what you said

Learner: mam

Researcher: you subtracted the whole number, okay then

Learner: and then I subtracted fractions

Researcher: mmh, I want to write, you subtracted numerators 3 – 1

Learner: okay

Researcher: okay the

Learner: and then the denominator I multiplied 4 by 5, it also gave 20

Researcher: multiplied 5, you said that

Learner: yes mam

Researcher: multiply this one

Learner: $\frac{1}{5}$ I multiplied by 5

Researcher: ee

Learner: yes mam

Researcher: but you wrote 4

Learner: I wanted to write 5

Researcher: 5 X 4

Learner: by 5

Researcher: by 5

Learner: yes mam

Researcher: then answer?

Learner: answer is $\frac{2}{20}$

Researcher: huu okay, your answer is $\frac{4}{20}$, where did this one goes to, you said 5 subtract 3, you said 5 subtract 3, neh

Learner: yes mam

Researcher: mmh, where is this 2, and then you said 3 subtract 3, I mean 3 – 1 (silence) should we go on

Learner: yes mam

Researcher: okay, explain h, how did you get the answer

Learner: h I subtracted 4, 4 – 3, it gave me $\frac{1}{4}$

Researcher:mmh

Learner: and I said $3\frac{1}{5}$ - $2\frac{1}{2}$ I subtracted 1 from 1, it gave me 0 (silence)

Researcher: yah then after

Learner: it gave me 0

Researcher: mmh

Learner: and the here, deno, denominator, numerator uuh, I said 5 X 2

Researcher: mmh

Learner: it gave me 10, and this side, I did 2 X 5 it also gave me 10, so my answer was $\frac{0}{10}$

Researcher: okay lets go to i, explain how you got your answer at i, (shhh), explain how you got your answer in i

Learner: i mam I, I I multiplied 2 over 6, mam most of the wholes I did not write them, cause I only looked here

Researcher: oohoo

Learner: okay I, I multiplied 2 over 6 and it ga over 6 it gave me 12 and here the gave me 12, (silence), mam I multiplied 2 over 6 it gave me 12, and here also I multiplied 5 by 2 it gave me 10 and here the numerator, I said 6 X 5, it gave me 30 and I also said 5 X 6 it also gave me 30

Researcher: you multiplied 2 and 5

Learner: no mam 5 plus 2

Researcher: after, start again you said, or started with

Learner: I started with whole numbers

Researcher : yah then

Learner: I said 2 X 6 it gave me 12

Researcher: yah

Learner: and I said 5 X 2 its 10

Researcher: mmh

Learner: and then, the numerator, I said 6 X 5, it gave me 30 (silence) and I also said 5 X 6 it also gave me 30 so my answer

Researcher: you also said 5 X 6, which one

Learner: yes mam

Researcher: mmh

Learner: my answer is $12\frac{2}{30}$

Researcher: yah your answer 12 $\frac{2}{30}$

Learner: 12 $\frac{2}{30}$

Researcher: ooh, 12 $\frac{2}{30}$ (shh) lets go to this one

Learner: I said 2 X 5 it gave me 10 my numerator, I said 4 X 3 it gave me 12 so my answer became 10 over 12 ($\frac{10}{12}$)

Researcher: please explain again, you said, I did not get you, you said

Learner: I, I said 2 X 5, I multiplied 2 by 5, it gave me 10, and I said 4 my then, my numerator I said 4 X 3 it gave me 12, and I also did the same 6 X 2 it also gave me 12, so my answer is $\frac{10}{12}$

Researcher: okay now second question, what procedure did you use to both i and j

Learner: I used multiplication

Researcher: ooh, thank you

Learner: the 2 X 6 is equals to 12

Researcher: mmh

Learner: 6 X 5 is equals to 30

Researcher: mmh 5 X 2

Learner: is equals to 10

Researcher: mmh

Learner: 6 X 6 is equals to 30

Researcher: mmh

Learner: then my answer is 12, answer is 12 $\frac{10}{30}$

Researcher: mmh, here

Learner: number j number j is $\frac{2}{4}$ X $\frac{5}{6}$

Researcher: mmh

Learner: then 2 X 5 is equals to 10

Researcher: mmh

Learner: then 4 X 6 is equals to 24

Researcher: mmh

Learner: the my answer is $\frac{10}{24}$

Researcher: okay good