THE EFFECT OF PROBLEM-SOLVING TEACHING APPROACH ON LEARNING FRACTIONS IN GRADE 8

By

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DECLARATION

I declare that the thesis titled: The effect of problem-solving teaching approach on learning fractions in Grade 8, is my own work and that all the sources that I have used and quoted have been acknowledged with complete references. I have not previously submitted the same work for a qualification at/ in other university/ faculty. I hereby cede copyright to the University of South Africa.

O.E. AGADAGBA DATE

DEDICATION

I dedicate this thesis to Jehovah God the Almighty, my creator, creator of heaven and earth. He travelled with me throughout the journey; we have safely reached the destination. I also dedicate this work to my immediate family Joy, Benaiah, and Bethel.

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ABSTRACT

Problem-solving teaching approach is critical in improving learners' cognition and problem-solving skills in different content areas in mathematics. Therefore, this quantitative study evaluated the effect of the problem-solving teaching approach on learning Fractions in Grade 8. This study draws from Polya's Problem-Solving framework that consists of sequential learning phases applied by the facilitator for intervention in guiding learners during their learning process. This study's null hypothesis, which was rejected, stated that there is no difference between the problem-solving teaching approach and traditional teaching approach on learning addition and subtraction of fractions (H_0): μproblem-solving approach =μtraditional approach. The positivist view in this study emphasised that the complexity of teaching and learning is understood through a scientific approach and numerical measurement. Participants were educators and learners in Mopani East and West District of Limpopo, categorised into experimental and comparison groups. The quasi-non-equivalent pre and post-test design, and questionnaire were used to engage the participants to access the empirical evidence. The 363 learners' who took part were 175 experimental group learners' and 188 comparison group learners. Participants were conveniently sampled, and no random assignment of participants was done. Data analysis combined both descriptive analysis and inferential Kruskal-Wallis and Sample Wilcoxon Signed Rank measurements. Learners were exposed to problem-solving learning in the experimental group and to traditional teaching and learning in the comparison group. In response to problem-solving teaching and learning, learners engaged in understanding the problem phase, devising the plan phase, carrying out the plan phase, and evaluating the solution phase through self-centred, group and self-regulated learning to build knowledge and skills. Findings revealed that learners who learned through problem-solving gained problem-solving skills and improved their performance. The results generated from SPSS, Kruskal-Wallis, Wilcoxon Signed Rank through paired t-test data analysis according to learners' performance in experimental group indicates significant p-value of $p=0.000$ with large effect size $H= 0.163$, $p<0.05$. The large effect size implies problem-solving contributed significantly towards the improvement of learners' problem-solving learning, skills, and achievement of moderate and advanced scores on learning addition and subtraction of fractions.

Key terms: Fraction; Problem-solving approach; Traditional teaching approach; Mathematics class; Fractions model; Learning Strategies; Cognition; Algorithms; Heuristic; Concepts.

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ACRONYMS

S/N	ACRONYMS	MEANING
1	AA	Advanced Achievement
2	ACT-R	Adaptive Control of Thought - Rational
3	ACCSSF	America Learners Common Core State Standard of Fractions
4	ANOVA	Analysis of Co-variance
5	ANA	Annual National Assessment
6	CA	Correct Answer
7	CAPS	Curriculum Assessment Plan Strategy
8	CCSSIE	Common Core Standard States Initiative Expectations
9	CGS1-3	Comparison Group School 1-3
10	CORE	Consortium on Reaching Excellence in Education
11	CPA	Concrete-Pictorial-Abstract
12	CRA	Concrete Representation Abstract
13	DoE	Department of Education
14	DBE	Department of Basic Education
15	EA	Empty Answer
16	EGS1-3	Experimental Group School 1-3
17	GET	General Education and Training
18	ICA	Incorrect Answer
19	INCA	Incomplete Answer
20	LA	Low Achievement
21	LCM	Lowest Common Multiple
22	LDE	Limpopo Department of Education
23	MA	Moderate Achievement
24	MACLP	Moderate Achievement Cognitive Level Performance
25	MD	Mean Difference
26	NAEP	National Assessment of Educational Progress
27	NCTM	National Congress of Educators of Mathematics
28	NECT	Nation Education Collaboration Trust
29	NQF	National Qualification Framework
30	OBE	Outcome-Based Education
31	PSL	Problem-solving Strategy Learning
32	SAIDE	South Africa Institute for Distance Education
33	SANSCEDR	South Africa National Senior Certificate Examination Diagnostic Report
34	SAQA	South African Qualification Authority
35	SDL	Self-Directed Learning
36	STD. D	Standard Deviation
37	SPSS	Statistical Package for Social Science
36	STT	Learners’ Talking Time
37	TIMSS	Trends in International Mathematics and Science Study
38	TTM	Traditional Teaching Model

39	TTT	Educators' talking time
40	UNISA	University of South Africa

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CHAPTER ONE

STUDY OVERVIEW

1.1 INTRODUCTION

Fractions make up a key part of mathematical knowledge, and they are also connected to other mathematics topics and branches (Yang et al., 2022). The application of fractions is evident across most other mathematics topics like finance, trigonometry, calculus, probability, and algebra. These mathematics topics include fractions mostly at computation stages. Fractions constitute an important topic that learners should understand to be successful in the application of mathematics concepts and theorems (Yang et al., 2022). However, this is an area in which learners struggle with most (Namkung & Fuchs, 2019).

Furthermore, previous studies have suggested reasons why most learners in Grade 8 think, fractions are a challenging topic (DeWolf & Vosniadou, 2015; Lewis, Mathews & Hubbard, 2016; Mathaba, 2019; Makhubele, 2021). However, they acknowledged the reason most learners in Grade 8 classes are learning high school mathematics fractions content for the first time. In addition, learners who are admitted in Grade 8 still lack basic knowledge of fractions (Makhubele, 2021). Furthermore, learners build on their prior knowledge, meaning that when encountering fraction problems, they naturally relate it to what they know in whole numbers before trying to solve the problem as fractions (Weisstein, 2022). The other reason learners struggle with fractions is that fractions can express different meanings and definitions (Weisstein, 2022). Fractions are present in unique forms and learners at times overgeneralise their knowledge of whole numbers (Lee & Boyadzhiev, 2020). However, learners' challenges in learning fractions can be minimised if educators could guide learners to understand the link and the difference between fractions and whole numbers (Arrahim, Sugiharti, & Damayanti, 2020).

The teaching approach that minimises learners’ challenges, enhances problem-solving skills, and develops cognition on addition and subtraction of fractions based on the focus of this study. The current study implemented the teaching of addition, subtraction, and equivalence of fractions through Polya's problem-solving framework in Grade 8. All relevant literature supporting the teaching and learning of fractions through the problem-solving approach were cited. For data collection, this quantitative study used a self-designed questionnaire, pre and post-test. The questionnaire, pre- and post-test findings were used to evaluate learners' problem-solving skills, cognition of the Fractions content, and performance achievement scores on learning addition, subtraction, and equivalence of fractions on learning through problem-solving teaching and traditional teaching approach.

1.2 BACKGROUND OF THE STUDY

According to Weisstein (2022), fractions can express different meanings and definitions. Thus, fractions are defined and interpreted according to scholars' studies field. As a result, misconceptions are raised in learners' minds due to a lack of proper explanations (Makhubele, 2021). Furthermore, learners are experiencing difficulties because they do not understand the concept of Fractions. This implies that learners are not able to relate fractions concepts to real-life situations (Ubah, 2021). Nonetheless, Grade 8 learners are experiencing difficulties because of insufficient mathematics teaching and learning background (Weisstein, 2022). Moreover, most mathematics educators are not exposed to various teaching approaches like problem-solving, problem-centred, and problem-based which guide learners' self-learning and probe their understanding by questioning (Diputra, Suryadi, Herman & Jupri, 2023). Furthermore, because of the disparity in mathematics education from the apartheid regime, most educators can only teach using traditional methods (DBE, 2015). Consequently, learners lack mathematical background that is centred on strategies that develop problem-solving skills, knowledge and understanding of fractions concepts through active involvement rather than concepts memorising (Wiest & Amankhonah, 2019).

The mathematics background in most schools favours traditional teaching and learning, where learning of mathematical concepts knowledge is acquired through memorisation, documentation, and imitation of educators' procedures (Baloyi, 2018). Furthermore, this approach motivates learners to become passive receivers of information, resulting in a poor understanding of fraction concepts due to solving problems that are based on sets of algorithm routine exercises (Wiest & Amankhonah, 2019). Grade 8 secondary schools' mathematical background of learners in South African is like that of Saudi Arabia's secondary schools where mathematics is given the least preference, and teaching and learning are educator centred (TIMSS, 2015). This implies that the teaching and learning process starts and ends with the educator's instructions (TIMSS, 2019). In addition, implementation of teaching approaches that enhance learners' problem-solving skills and using real-life situations in learning mathematics contents are challenging for most educators. According to Trend in International Mathematics and Sciences Study TIMSS (2019), South Africa learners underperform in mathematics. Furthermore, learners have challenges in solving most mathematics questions because of their mathematical background and their inability to use the problem-solving approach in solving mathematical fractions. Thus, most learners struggle when adding and subtracting fractions with unlike denominators like $\frac{1}{2} + \frac{1}{3}$ reaching the wrong conclusion that $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$ (Nyambe, 2020). In addition, Makhubele (2021), acknowledged South African Grade 8 learners and beyond have difficulties in using the lowest common denominators (LCDs) when adding and subtracting fractions. These challenges are due to inadequate fractions content knowledge because of the dominating traditional teaching approach in most schools (Dhlamini & Kibirige, 2014; Graph, 2022). Furthermore, most learners are inadequately prepared to solve fraction problems at an early primary stage. Thus, Grade 8 learners lack basic fractions content concepts knowledge in their earlier phase (Adendorff & Moodley, 2014). The Department of Basic Education (DBE) (2014) reports that learners in Grades 3-8 cannot perform basic operations in addition and subtraction of fractions through problem-solving. Thus, educators were encouraged to adopt and teach through problem-solving textbooks (DBE 2012-2014). Nonetheless, Department of Education (DBE, 2014) results obtained based on the problem-centred approach in Grades 3, 7, and 8 showed average marks for fractions as 10%, 17%, and 14% in third grade 18%, 21%, and 32% in fifth grade and 15%, 14% and 22% in eighth grade. Furthermore, Makhubele (2021), acknowledged that the South African mathematics background in problem-based learning results is inadequate when compared to the USA, Singapore, Japan, Hong Kong, and China. In contrast to the South African background, countries like China, Singapore, the USA, New Zealand, and Nigeria showed higher scores in mathematics achievement because their mathematical background adopted a problem-based approach to teaching and learning (TIMSS, 2019). For example, the Nigeria Ministry of Education (2014) reported an improvement in mathematics achievement from 24% in 2009 to 68% in 2014 (Sa’ad, Adamu & Sadiq, 2014; Rasheed & Ogundokun, 2016). Similarly, data accumulating from the USA, National Assessment of Educational Progress (NAEP) reported increases in learners' performances from (2009-2017). The NAEP (2019) reported a 62% improvement in Grade 4 learners' performances in addition and subtraction fractions when compared to 25% of the learners in the same grade in 2009 before the adoption of a problem-solving-based approach to teaching and learning across the grades. Furthermore, TIMSS (2019) reported Singapore to be in first position with 43%. South Africa on the other hand the lowest scores. In addition, South African performance is lower when compared with countries such as Japan, Hong Kong, and China (Jordan, Resnick, Rodrigues, Hansen & Dyson, 2017).

In addition, Jordan et al. (2017) described South African grades and assessment as different to those of countries such like Singapore, China, and the USA. These countries' classroom settings are different from South African classroom settings. For example, in China, most learners start to learn addition and subtraction of fraction concepts in the first and second grades just after the whole numbers and basic operations concepts. Similarly, United States of America (USA) learners experienced fractions in the early first grade, and in first and second grade, the American Learners Common Core State Standard of Fractions (ACCSSF) includes partition of shapes (Mesitti & Chan, 2022). They learn fractions by shading, colouring equal amounts of fractions "equal shares." In third grade, learners learn fraction concepts using signs and symbols unit of fractions (fractions with numerator and denominator) and comparisons of fraction values. In fourth grade and beyond, learners learn fraction equivalent concepts and how to find solutions to fraction problems using the four basic operations (National Mathematics Advisory Panel [NMAP], 2008; Wilkins & Norton, 2018). Because of early learning, learners in the USA, are very proficient, competent, and advanced in solving fractions problems because of early content knowledge background (Wilkins & Norton. 2018). On the contrary, South African mathematics classroom were introduced to problem-based and fractions concepts in fourth grade and beyond (DBE, 2017). Thus, South African learners' mathematics backgrounds are different from those of many developed countries. However, the high mathematics achievements of the other countries in the TIMSS competition were attributed to problem-solving (DBE, 2019; TIMESS, 2019). For example, Singapore's achievement was attributed to the Singapore Ministry of Education introducing the use of a concrete pictorial abstract (CPA) approach in teaching and learning. Singapore's curriculum was built on teaching practices based on concrete, pictorial abstract (CPA) a type of problem-based approach built on George Polya's four heuristic strategies of mathematics problem-solving teaching and learning approach, which is referred to as Polya's problem-solving strategies. In addition, DoE (2019) results showed that 54% of learners in General Education and Training (GET), are not exposed to fractions content, 48% are not exposed to problem-solving and more than 63% cannot apply problem-solving in solving fractions. Thus, there is a need to enhance learners' problem-solving skills when we compare Singapore's mathematics background to South Africa (Albay, 2019).

Similarly, TIMSS (2019) recommended that South African educators engage their learners more in mathematics fractions and problem-solving because it encourages learners to be involved and develop essential problem-solving skills required to achieve. Nonetheless, in mathematics, South Africa was ranked position 38 out of 39 countries in both 2015 and 2019 Grade 8 Trends in International Mathematics and Science Study (Naidoo & Hajaree, 2021). According to Naidoo (2021), out of the thirteen test items in the TIMSS (2019) eight items addressed basic concepts of fractions, and five items focused on operations involving fractions and the application of problem-solving skills in solving fractions.

Table 1.1: Five Top Rank Countries in Mathematics Achievement (TIMSS, 2019)

Countries	**Average scale scores**	**Standard error**
Singapore	621	3.2
Republic of Korea	606	2.6
Chinese Taipei	599	2.4
Hong Kong SAR	594	4.6
Japan	586	2.3
Five Underperforming Rank Countries in Mathematics Achievement (TIMSS, 2019)		
Countries	Average scale scores	Standard error
Botswana	391	2.0
Jordan	386	3.2
Morocco	384	2.3
South Africa	372	4.5
Saudi Arabia	368	4.6

Extracted from TIMSS 2019, highlights of Mathematics and Science achievement of Grade 8 South African Learners IEA (TIMSS, 2019).

Thus, looking at Table 1.1. Conclusion can be reached according to TIMSS, 2019 reports that there are gaps in problem-solving knowledge when comparing the overall levels of average scale scores and standard error of Singapore learners to South African learners. Hence, this could be a substantial reason for the Department of Education (DoE) in South Africa to re-design or re-adopt the framework for teaching the learners fractions using the problem-solving approach (DoE, 2011; Aksoy & Yazlik, 2017). Thus, it is vital to investigate, focusing on the effects of problem-solving teaching on learning fractions in Grade 8 classes. The current background and understanding knowledge of Grade 8 learners on fractions shown in Trends in International Mathematics and Science Study (TIMSS, 2019) implies that South African Grade 8 learners find mathematics fractions concepts difficult to understand through the traditional approach and lack problem-solving in learning mathematics fractions.

1.3 PROBLEM STATEMENT

Nyambe (2020) acknowledged that learners experienced difficulties in solving fractions and argued that learners can minimise these challenges if the teaching and learning of fractions is made through the problem-solving approach, especially in the Grade 8 classes. Furthermore, Nyambe (2020) acknowledged that not much literature on problem-solving in learning fractions had been published. In addition, misconceptions, difficulties, and common errors, are commonly researched, and published in most studies conducted on topics relating to fractions and problem-solving (Dhlamini & Kibirige, 2014). Therefore, learners are experiencing more difficulties due to educators' teaching approach, they make mistakes and have misconception on addition, and subtraction, and in understanding knowledge of fractions concepts and fractions equivalent (Ubah, 2021).

The uses and application of fractions vary in our everyday life and are important to our society not only for mathematical applications (Nyambe, 2020). According to Yang et al (2022), knowledge of fractions is applicable in everyday activities context like trade (i.e., purchasing and discounting in shopping), and measurement (i.e., knowing the required amount of foodstuff in recipes when baking cookies and understanding of time concept). In addition, in this advanced society where technology is the order of the day, knowledge of fractions plays a vital role to properly understand modern technology. Hence, a proper understanding of fractions is important, and educators are expected to teach using appropriate teaching approaches and learning strategies to enable the learners to understand fractions concepts better (Nyambe, 2020). However, these is the area in which Grade 8 learners in South Africa struggle with most (Makhubele, 2021).

Filimowicz (2018) suggested that practical demonstrations of fractions should be conducted when teaching concepts from fractions schemes. They enable learners to conceptualise concrete and abstract concepts. Educators should teach learners to conceptualise existing knowledge schemes and help learners understand scenarios where fraction schemes link with whole numbers (Dube & Hlalele, 2018). Furthermore, Filimowicz (2018) acknowledged this as a problem, because most lessons on fractions do not help learners differentiate between fraction numbers from whole numbers. In addition, Makhubele (2021) argued that Grade 8 learners cannot solve elementary fraction questions by themselves without using a calculator. Hence, educators should guide learners in deeply understanding concepts and not memorising. Lack of practical demonstration, experimenting and teaching on fractions contributed to a high rate of mistakes, and misconceptions about learning fractions in the minds of most Grade 8 learners (Makhubele, 2021; Da-Ponte et al., 2022).

The South African Department of Basic Education (DBE, 2020) acknowledges that most Grade 8 learners have insufficient basic knowledge of fractions in their early grades. Thus, learners make mistakes, and conceived misconceptions because of insufficient knowledge of fractions content learned. These resulted in inadequate skills, leading to poor learning and misinterpretation of concepts (Makonye & Fakude 2016). According to Makonye and Fakude (2016), most teaching approaches encouraging learners to build on prior knowledge, lead to building on false concepts and erroneous principles and contribute to their mistakes, and misconceptions (DBE, 2020). These contribute primarily to underperformance fractions-related topics, therefore, in mathematics in general (Makonye & Fakude, 2016; NSCEDR, 2017). In addition, Dube and Cias (2019) acknowledged that the lack of adequate problem-solving teaching skills is affecting most learners' mathematics achievement in South Africa.

Furthermore, Makhubele (2021) acknowledges that learners' lack of skills to solve fractions problems contributed to under performances in mathematics and this has resulted in progression instead of promotion of most learners from Grade 7 primary school into Grade 8 high school. Hence, most learners who failed to obtain 40% standard pass rate and above in mathematics were not promoted into Grade 8 high schools instead, they were progressed or condoned (DBE, 2021). Most Grade 8 learners' when in Grade 7 score below 40% mathematics pass rate (DBE, 2021). In addition, the Department of Basic Education DBE (2021) diagnostics report stated that most learners under-performed in mathematics scores in the National Senior Certificate Examinations, and this is connected to their inability to answered mathematics questions connecting to topics that link with fractions concepts and problem-solving. These difficulties facing learners in solving mathematics fractions concepts have concerned the Department of Basic Education (DBE) of South Africa to lower the standard achievement rate in mathematics in the Further Education Training (FET) to 30% (DBE, 2017). Table 1.2 shows consecutive three-years (2019 – 2021) percentages achieved in mathematics topics tied to fractions and problem-solving and not tied to fractions and problem-solving approaches.

Table 1.2: Learner's Achievement in NCS Mathematics Fractions Related Topics

Comparing fractions and non-fraction question topic	**2019**		**2020**		**2021**	
	Non-PS Fraction	Fraction on PS	Non-PS Fraction	Fraction on PS	Non-PS Fraction	Fraction on PS
Percentage achieved	% score	% score	% score	% score	% score	% score
Analytical geometry	69	47	57	47	57	43
Calculus	37	30	66	25	38	8
Euclidean geometry	49	44	52	43	34	24
Finance	-	57	-	48	-	49
Functions and graph	62	39	55	45	77	48
Number pattern	70	33	66	40	72	50
Trigonometry	37	30	47	28	36	21
Probability and counting	26	21	30	18	-	27

Extracted from National Senior Certificated Diagnostics Reports: Subjects Contents 2019 - 2021

The results in Table 1.2 reveal that learners performed below average across mathematics topics tied to fractions and problem-solving. These substantial findings showed that South African learners are experiencing problems in solving fractions through a problem-solving approach. The implication is underperforming in mathematics (NSCEDR, 2018). This study has shown how learners reduces their mistakes and increase in their knowledge of fractions concepts on learning fractions using Polya's mathematical problem-solving approach.

1.4 RESEARCH QUESTION AND HYPOTHESIS

Varna et al., (2023) explained research questions as focus questions. The main questions guide the overall study. They further outline the two types of researched questions to be: I) quantitative research questions, and II) qualitative research questions. Therefore, this study's questions focus on quantitative research questions. Hence, comparative reasoning, empirical findings, and data measurements were the main evidence presented for verifying the hypothesis and answering the study research question.

1.4.1 Research questions

The main research question clarified in this study is 'What is the effect of the problem-solving teaching approach on learning fractions in Grade 8?'

The following sub-questions were answered in this study.

- What challenges do learners experience in solving problems on addition and subtraction of fractions?
- How does the problem-solving teaching approach enhance learning of fractions in Grade 8?
- What problem-solving skills are essential to enhance cognition in the addition and subtraction of fractions?

The descriptive pre and post-test analysis based on problem-solving knowledge and understanding is used to clarify the main research question: What is the effect of the problem-solving teaching approach in learning fractions in Grade 8? Furthermore, descriptive pre and post-test analysis triangulated through questionnaire analysis was used to elucidate the following sub-research question: What challenges do learners experience in solving problems on addition and subtraction of fractions? How does problem-solving teaching enhance the learning of fractions in Grade 8? What problem-solving skills are essential to enhance cognition in the addition and subtraction of fractions?

1.4.2 Research Hypothesis

Appropriate teaching and learning approaches are important for proper knowledge and understanding of fractions concepts (Varna et al., 2023). The argument presented is that a proper teaching approach should demonstrate using real-life situations and proper learning approach should involve learners' active participation in their classroom activities (Graphy, 2022). Proper teaching approach led to proper learning approach and a proper learning approach yields correct content learned and the content determines the amount of knowledge and skills acquired. Furthermore, most educators in South African schools are using the traditional teaching approach which develops educators' explanation skills and learners' memorization, incitation, and repetition skills in learning (Mesiti Artigue, Hollingsworth, Cao, & Clarke, 2021). However, Graph (2022) acknowledged that this approach enhances learners' understanding of learning and further argued that the approach is easy to apply and saves lesson time preparation. However, Fadipe, Sepeng and Rankhumise (2021) observed that the problem-solving approach enhances learners' cognition of learning. In addition, they higlight that teaching using the problem-solving approach enables the educator to teach at an informal level where learners can relate what they are learning in the classroom with their social reality. Therefore, this study tested the problem-solving teaching approach and traditional teaching approach to learning fractions in Grade 8.

The following hypothesis was set up:

- The null hypothesis **(H_0):** There is no difference between the problem-solving teaching approach and the traditional teaching approach on learning addition and subtraction of fractions.

H_0: µproblem-solving approach =µtraditional approach.

- The directional hypothesis **(H_1)**: Problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions.

H_1: µproblem-solving approach ≠ µtraditional approach.

The pre-, post-test, and questionnaire statistically significant value paired sample, Kruskal-Wallis and Wilcoxon Signed Rank test SPSS inferential verification result clarified the study's null hypothesis **(H_0):** There is no difference between the problem-solving teaching approach and the traditional teaching approach on learning addition and subtraction of fractions **H_0:** µproblem-solving approach =µtraditional approach.

1.4.3 Aim and Objectives

This study aimed to evaluate the effect of the problem-solving teaching approach on learning fractions in Grade 8. The objectives of this study were to:

- Evaluate the challenges experienced by learners in solving problems on addition and subtraction of fractions.
- Determine how problem-solving approach enhances the learning of fractions in Grade 8.

- Determine the problem-solving skills essential to enhance cognition in the addition and subtraction of fractions.

1.5 RATIONALE OF THE STUDY

The importance of fractions as argued previously is that: (1) fractions are useful and applicable in the context of purchasing, discounting in shopping, knowing the required number of recipes in baking cookies, and telling time; (2) the natural part of common fractions is essential in helping learners to gain experience and ability in comprehending and applying concrete materials and activities; and (3) common fractions can be applied in more fields of study (Educators' Lab, 2017). According to Thurtell (2019), learning fractions can help learners in diverse ways like experience with concrete materials can assist learners with abstraction, and proportional reasoning understanding is necessary to help learners understand their natural realities, and ratio problem-understanding enhances knowledge about types of models and representation. Lastly, an advanced understanding of fractions problems helps learners to learn through the problem-solving approach (Ubah, 2021).

The benefits of learning fractions through the problem-solving approach are outlined in DBE (2017). For example, learners apply critical and creative thinking skills, perceive constructs, apply model representation, and solve problems using problem-solving skills. Learning fractions through problem-solving enables learners to improve their creative and reasoning skills to advance more problem-solving cognition in fractions. Fractions knowledge improves proportional reasoning. It provides opportunities for learners to study related courses at higher education and follow related career paths and occupations like mathematical thinking, finance, trigonometry, and civic work (Arrahim, Sugiharti, & Damayanti 2020).

Proportional reasoning and skills relevant to fractions computations are important in all fields that require fraction knowledge (Ubah, 2021). Hence, learners must develop good skills to do fractions computations on problem-solving, build comprehension on fractions schemes between fractions concepts, and link knowledge of fraction to other subjects and topics using real-life situations. This study supports ongoing literature in applying problem-solving teaching and learning in Grade 8. It helps educators adjust their teaching on fractions where necessary and improves learners' understanding of active learning. These significant benefits enabled the researcher to embark on this practical investigation of the effects of the problem-solving approach in teaching Grade 8 fractions.

1.6 MEANING OF FRACTION TERMS AND CONCEPTS

- A fraction is a mathematical expression that stands for a ratio of two numbers. For example, $\frac{5}{7}$. The top number (numerator) stands for the part of the whole, and the bottom number (denominator) stands for the whole. In mathematics, fractions are used to perform operations such as addition, subtraction, multiplication, and division. This study uses fractions expression that involves addition, subtraction, and fractions equivalent to facilitate Polya's mathematics problem-solving strategy (Ubah & Bansilal, 2018).
- Problem-solving – A problem is a situation or challenge that requires a solution. Problems can be simple or complex and can exist in a wide range of areas including mathematics topics such as fractions and everyday life. Problem-solving is the process of finding a solution to a problem by a means which was not immediately obvious (Albay, 2019). According to Lein et al. (2020), problem-solving is a process of finding a solution to a mathematical question using different approaches such as: (a) finding the problem; understanding what the problem is and what needs to be solved. (b) Gathering information: collecting relevant data and information to help understand the problem and solution. (c) Generating options: producing a list of potential solutions to the problem. (d) Evaluating options: assessing the pros and cons of each solution to figure out the best course of action. (e) Implementing a solution: putting the chosen solution into action and taking the necessary steps to carry it out. (f) Evaluating the outcome: checking to see if the solution has effectively solved the given questions. Good problem-solving requires critical thinking, creativity, and decision-making skills (Polya 1887-1985; SAIDE, 2017).
- Mathematical Modelling- is an abstract conceptual subject taught at most SA schools to equip learners with mathematics knowledge and to develop their thinking and problem-solving skills (Koskinen & Pitkaniemi, 2022). Modelling implies knowledge representations formulated around physical objects and visualisation grounded on the construction of symbolic representations of concepts and the establishment of relations (Mokotjo, 2017).
- The fractions model is a visual representation of fractions that helps to understand the concept of dividing a whole into equal parts. Examples of fraction models include. (a) Number line models: a number line divided into equal parts, with the fraction represented by the length of a

segment on the number line. (b) Area models: a rectangle divided into equal parts to represent the fraction. The numerator represents the number of parts that are shaded, while the denominator represents the total number of parts in the rectangle. (c) Set models: a set of objects, such as blocks or candies, can represent fractions. The numerator represents the number of objects in the set, while the denominator represents the total number of objects in the set. (Purwadi, Sudiarta & Suparta, 2019). Fraction models can help Grade 8 learners understand the concepts of fractions and perform addition and subtraction operations with fractions. They are often used in mathematics education to help learners visualise the relationship between fractions and to build a deeper understanding of mathematical concepts (Purwadi et al., 2019).

- Teaching Approach - an alternative method used by educators based on their teaching experience in teaching fractions (Ubah, 2021). Similarly, they were able to devise teaching approaches that enable learners to gain a thorough conceptual understanding of fractions.
- Tradition Teaching Approach, also known as the educator-centred approach, has been the dominant method of teaching for years. In this approach, the educator is the primary source of information, and the learners are passive listeners. The educator delivers a lesson, explains the topic and the learners memorise and apply the information (Masilo, 2018). The educator is the centre of attention, the learners take notes, ask questions, and complete the assignments.
- Learning Strategy: a learning strategy is a plan or approach that helps learners to effectively learn and retain information (Yang et al, 2022). There are different learning strategies, and the best approach for an individual learner may depend on their learning style, subject matter, and the goal of the learning. Examples of common learning strategies include: (a) practice and repetition: repeating materials multiple times to help embed them in long-term memory. (b) Active learning: engaging in hands-on activities or discussions to actively process information and make connections between new and existing knowledge. (c) Note-taking: writing down key information to help reinforce understanding and organised thoughts. (d) Visual aids: using visual aids, such as images, videos, and diagrams, to help process information and make connections between concepts. Developing an effective learning strategy can help Grade 8 learners improve their memory, increase their understanding of fractions, and be more successful in their studies. It is important to experiment with different strategies and find what works best for each learner (Vetter, Orr, Dwyer, & OConnor, 2020).

1.7 RESEARCH DESIGN

The research design in this study refers to the study plan outlining the methods and procedures for collecting and analysing data. The quantitative methodology, quasi-non-equivalent pre- and post-test designs were used in this study. The design enables users to interact in an already arranged group and does not involve the random assignment of participants to a group. This design is appropriate for this study because, it is not ethical or feasible to randomly assign participants to a group (Creswell, 2014). The design uses the non-comparison group pre and post-test data collection process. Moreover, the main purpose of adopting a quasi-experimental design should be to control variables using a teaching approach to evaluate the effects on the other variables (McMillan & Schumacher, 2014).

1.7.1 Data Collection Process

Six schools were sampled conveniently from two districts (Mopani East and West) to participate in this study. The comparison-grouped schools comprised 188 Grade 8 learners from Mopani West and the experimental-grouped schools comprised 175 Grade 8 learners from Mopani East. The two districts are approximately 35 km apart. Data collection procedures started with test administration in both groups. The pre-test data were collected before teaching of fractions in the comparison group and intervention in the experimental group. The researcher taught fractions through the problem-solving approach in the experimental group. Educators of the comparison group used the traditional approach to teach. The researcher was further involved with non-participating observation as confirmation of the lesson in the comparison group. The post-test and questionnaire data were collected after teaching interventions and teaching observation confirmation in the comparison group. In addition, observation in the comparison group was necessary to ensure the educator covered all learning content on fractions before the learners took the post-test. The questionnaire was used to collect data with the aim of understanding the challenges or/and benefits of using the problem-based approach. McMillan and Schumacher (2014) observed that questionnaires enable the researcher to understand the degree of learners' understanding knowledge of fractions.

1.7.2 Pre; post-test and questionnaires

Experimental group	Pre-test	Intervention	Post-test	Questionnaire
	O----------------	**X**----------------	**O**--------------------------	Z
Comparison group	Pre-test	Observation	Post-test	Questionnaire
	O----------------	**X**----------------	**O**--------------------------	Z

Figure 1.1: Non-equivalent-group pre and post-test Design

The pre and post-test comprised written work on addition and subtraction of fractions that served as a pre-test for all the learners in both groups. The same written work (pre-test) also served as the post-test. The purpose of the post-test was to verify the effect(s) of the intervention. The design was the non-equivalent-group pre-, and post-test design.

1.7.3 Data Analysis

McMillan and Schumacher (2014) explained data analysis as a systematic procedure for describing and evaluating data and purposefully discovering useful information. This study's data analysis was used to draw the conclusion that supports the hypothesis and clarifies the research questions. Inferential SPSS non-parametric techniques and descriptive analysis were applied in this study. The analysis in this study focused on presenting and evaluating the questionnaire, pre- and post-test findings. Kolmogorov-Smirnov, Shapiro-Wilk's normality statistical test, Kruskal-Wallis Wilcoxon Signed-Rank test, and paired t-test were used to verify the hypothesis. The descriptive analysis was used to clarify the research questions. In addition, one-way ANOVA analysis questionnaires analysis was triangulated to support the findings in the pre-, and post-test data (McMillan & Schumacher, 2014).

1.8 RELIABILITY AND VALIDITY

The ability of the instrument to yield the same result consistently under comparable conditions when administered independently is known as reliability (Desta, 2019). McMillan and Schumacher (2014) define reliability as "consistency of measurement". They further explain that validity is the degree to which meaningful and useful inferences and uses of data based on numerical scores are allowed. A research tool that measures exactly what it is designed to measure is valid. The learners' written work (pre-test and post-test) was assessed through an analytical scoring scale to ascertain its reliability. Pilot testing of the pre and post-test conducted ensured the reliability and validity.

1.8.1 Ethical Issues

The ethical clearance was granted by the University of South Africa, College of Education. Ethics deals with beliefs about what is right or wrong, proper, or improper, and good or bad. Openness and honesty were considered in this study. All the participants including parents were informed on time about the plans and aspirations of the study. The researcher assured participants of their confidentiality and anonymity. In addition, the researcher informed participants that participation was voluntary, meaning no monetary value or gift for participating in this study. Furthermore, permission to conduct research was granted by Limpopo department of education, and schools principals also gave permission to conduct research in their various schools.

1.9 SCOPE AND STUDY ASSUMPTIONS

The scope of this study was limited to six public high schools in Mopani East and West District of Limpopo Province. Schools were sampled conveniently according to the educator teaching approach, number of classes, and learners' participants. Activities were limited to teaching and learning fractions, writing tests, and completing questionnaire questions. The researcher assumed that teaching fractions using Polya's mathematics problem-solving assists in improving achievement and enhances problem-solving skills in fractions and mathematics in general. It would help learners acquire basic knowledge of fractions. Hence, the researcher posits that this study would help build learners' confidence in problem-solving learning and evaluate the effect of the problem-solving approach on learning Grade 8 fractions. In addition, the researcher believes that this study will assist learners in having basic knowledge of fractions and help educators to reflect on their teaching approach. Thus, teaching and learning through the problem-solving approach would assist in initiating problem-solving teaching and learning strategies in class. This study assumed that all participants in this study learnt from Grades R-7 and were experiencing Grade 8 high school class for the first time. Thus, are full-time Grade 8 learners in the Mopani district of Limpopo Province, South Africa.

1.10. CHAPTER OUTLINE

Chapter 1: Overview of the study overview

Chapter 2: Literature review and theoretical framework

Chapter 3: Research methodology, paradigms, ethical issues, validity, and reliability

Chapter 4: Quantitative findings, results presentation, triangulation, and discussions

Chapter 5: Interpretation, discussion, conclusion, and recommendations

1.11. CHAPTER SUMMARY

This chapter provided the introduction and background to the dissertation. It presented an overview of teaching and learning of fractions. The chapter focused on the problem-solving approach and traditional teaching and learning of fractions in Grade 8. The problem statement, objectives, hypothesis, and research questions were presented in this chapter. The chapter discussed the quasi-non-equivalent design of the quantitative methodology used in this study. Questionnaires, pre- and post-test instruments were used to collect data. The data analysis used both descriptives and inferential analytical techniques. The key concepts in the study were defined to provide the necessary study content. The next chapter provides the literature review and theoretical framework for this study.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

Skills to do fractions computations are required to solve mathematics fraction problems (Rahman, 2019). Furthermore, problem-solving skills are enhanced when using problem-solving teaching and learning approach. According to Desta (2019), when learners are taught to solved mathematics using real-life words problems in mathematics, they enhance their problem-solving skills. This chapter discusses themes aligned to problem-solving teaching and learning in Fractions.

The first section of the literature review has the following focus headings: (a) previous studies on problem-solving teaching and learning (b) teaching and learning in mathematics classrooms; addressing traditional and problem-solving teaching approaches (c) difference between problem-solving and traditional teaching methods (d) the impact of mathematics classrooms (e) effort implemented in enhancing an effective classroom. The second section presents an overview of fractions with the following headings (a) learners' knowledge of fractions (b) learner performance in fractions (c) educators' knowledge of fractions (d) challenges associated with teaching and learning of fractions (e) solution in enhancing teaching and learning of fractions. The third section discusses the problem-solving theoretical framework. The components that follow are Anderson's, Verschaffel, Hmelo Silver and George Polya's problem-solving framework (Anderson, Burnham, Thompson, 2000). The theoretical framework section discusses the rationale behind Polya's problem-solving framework under the following headings (a) Polya's four phases of mathematics problem-solving (b) the role of the educator in a problem-solving classroom (c) the role of question and assessment (d) group learning and reflection in a problem-solving classroom and the evaluation of problem-solving teaching on learning fractions in Grade 8. Thus, this study does not compare problem-solving and traditional teaching and learning but it evaluates the effect of problem-solving teaching learning fractions in Grade 8 and the effects in enhancing learner problem-solving skills. Liu, Yi, and Wang (2022) agree that the effectiveness of teaching on learning should be evaluated, and a comparison should be done between an independent and dependent variable. Hence, the study adopted an experimental group (teaching and learning through the problem-solving approach) and a comparison group (teaching and learning through the traditional approach) to evaluate the effects of the problem-solving teaching approach on learners' learning of fractions in Grade 8.

2.2 PROBLEM-SOLVING IN MATHEMATICS

Previous studies have suggested reasons why most learners considered fractions as challenging especially to Grade 8 learners. For example, Siegler Thomson and Schneider (2011) agree that the lack of effective development of instruction materials on fractions for teaching learners in kindergarten through Grade 8, hindered learners' understanding of learning fractions. This means that educators should use real objects like paper, and iron bars to assist learners in understanding instructions when teaching fraction concepts. McNamara and Shaughnessy (2010) agreed that improper teaching strategies reduce learners' fraction sense. By extension, learners can easily lose interest in learning when the educators' strategy does not link with the fraction concepts. Therefore, educators must adopt appropriate teaching strategies when conducting a demonstration and explaining operations on fraction concepts. Malone and Fuchs (2017) acknowledge that error patterns in ordering fractions are common mistakes learners make in learning fractions. Hence, educators emphasise fractions order and equivalent concepts when teaching fractions. DeWolf and Vosniadou (2015) state that improper representation of fraction magnitudes and the whole number bias hinder learners' understanding of fractions. By extension, learners are quick to apply their knowledge of whole numbers when solving, representing fractions. Therefore, when teaching fractions, educators should assist learners in understanding the difference between whole numbers and fraction numbers. Lewis Mathews and Hubbard (2016) argued that the nature of the learners' neurocognitive architectures and non-symbolic foundations of understanding fractions can hinder learners' knowledge and understanding of fractions. Hence, educators should be sensitive to the learners' natural cognitive development before posing them with fraction problems. Mathaba (2019) argued that a lack of problem-solving skills can hinder learners' understanding of fractions. learners need to experience fraction problems from the earliest grade before Grade 8 and beyond.

Nevertheless, most studies that focused on teaching through problem-solving acknowledged 'that the use of the problem-solving teaching approach enhanced learners' understanding of mathematics and problem-solving skills' (Mceleli, 2019; Nyembe, 2020; Lamon, 2020). In addition, Fadipe Sepeng and Rankhumise (2021) agree that teaching through a problem-centred approach enhances learners understanding of quadratic equations. This evidence is substantial enough to confirm that fraction lessons can be made easy through problem-solving teaching to enhance cognition on addition, subtraction and equivalent and improved understanding, and learners' achievement (Albay, 2019). Furthermore, learners can achieve these with advanced help from knowledgeable facilitators (educators/educators) and peers (Ibrahim, Shak, Mohd, Zaidi & Yasin, 2015). According to Ibrahim et al. (2015), teaching and learning using problem-solving, fosters group and collaborative learning activities. Such group settings foster creativity through active participation. Thus, active participation by learners reduces educators talking time and increases learners' talking time and attention can shift from the educator chalkboard interaction to learners' group work and collaborative learning activities.

Furthermore, this study confirmed that learning through problem-solving reduces mistakes, and misconceptions commonly made by learners when learning mathematics fractions. It enhances educators' teaching and equips educators' content knowledge of fractions. For example, Mathaba (2019) believes that teaching through problem-solving improves educators' and learners' understanding knowledge of algebra and reduces learners' procedural errors. It is therefore advisable for educators to teach using the problem-solving approach in their classes. Malone and Fusch (2017) observe that teaching through problem-solving helps learners to reduce mistakes when presenting fractions order. This agrees with Mdaka (2011), who observes that learners make mistakes because of the learning approach learn from the educator teaching approach. Therefore, educators should adopt a teaching approach that best teaches mathematics fractions because learners have trouble solving fractions; and educators possess insufficient content knowledge of fractions (Mamba, 2012; Dhlamini & Kibirige, 2014; Luneta, 2015).

Mohyuddin and Khalil (2016) acknowledge that learners' understanding can be enhanced through problem-solving teaching and learning. Furthermore, Dhlamini and Kibirige (2014) acknowledge that learners experience difficulties when solving fractions and that a proper teaching approach can assist in minimising common errors and misconceptions. In this study, learners' difficulties with fractions were acknowledged and minimised using the intervention. In addition, Mathaba (2019) argues that teaching through problem-solving helps learners differentiate between algorithm problems from heuristic mathematical problems. Algorithms mathematical problems are mathematical questions with solutions that require learners to apply set rules to solve the problem while heuristic problems are mathematical questions whose solutions are not obvious to learners (Yapatang & Polyiem, 2022).

Similarly, the problem-solving approach helps learners identify algorithms and heuristic variables connected with fraction concepts in mathematics (Yapatang & Polyiem, 2022). Thus, in this study, learners solve both algorithm and heuristic fractions problems. Teaching through the problem-solving approach assists learners in performing basic calculations on addition, subtraction, multiplication, and division involving fractions operation effectively (Nyembe, 2020). Nyembe (2020) further explained that teaching and learning through problem-solving enhances learner problem-solving skills. Filimowicz (2018) argues that educators can accommodate different learners' learning styles using problem-solving.

The South Africa Institute for Distance Education (SAIDE, 2017) argues that teaching through problem-solving builds learners' reasoning and transforms them from novice problem-solver to competent problem-solver. The effects of teaching using the problem-solving approach were visible in the learners' post-test scores in this study; learners acquired knowledge of fractions that enabled them to reason proportionally. Problem-solving approach help to obtain effective teaching and learning and enables learners to acquire adequate content knowledge of fractions (Fadipe, Sepeng, & Rankhumise, 2021). learners acquire advanced knowledge of fractions through problem-solving in Grade 8. Participants were able to demonstrate advanced learning approach using problem-solving and enhanced problem-solving skills involving fractions. Understanding learners' diversity and backgrounds, the researcher used problem-solving teaching to encourage learners' involvement during mathematics classes in achieving higher cognitive learning fractions. This supports the TIMSS (2019) report which indicated that teaching through problem-solving enhances learners' higher order of critical thinking and help them to solve problems at both abstract and concrete levels. Thus, it supports extant literature, which reveals that problem-solving teaching minimises learners' errors in learning fractions.

2.2.1 Mathematics Classes

Koskinen and Pitkaniemi (2022) explained mathematics classrooms as a teaching environment where mathematics concepts and mathematics computations skills are taught and demonstrated by educators. Just like other settings, mathematics fraction classes are complex, teaching seeks to understand the learning that reflects and accommodates the complexity (Koskinen & Pitkaniemi, 2022). Fraction concepts are complex and fundamental in mathematics (Mesitti & Chan, 2022). They consist of a numerator (the top number) and a denominator (the bottom number). The numerator represents the part of the whole that is considered, while the denominator represents the total number of parts that make up the whole (Shimizu Kaur, Mesiti & Chan, 2022). Alexakos (2020) defines two major mathematics classes where teaching and learning fractions are experienced. In traditional teaching classes, the educator instructs the learners using the sequence of lessons designed in mathematics. Contrastively, in problem-solving classes, the educator assumes the role of facilitator and guides learners' learning using mathematical concepts that allow them to explore and solve mathematical problems by themselves (Abdulrahim & Orosco, 2020). Nevertheless, the Grade 8 mathematics content is illustrated through any of the two teaching and learning approaches (Koskinen & Pitkaniemi 2022). However, the activities of educators and learners are the main factors that define a mathematics class (Chan & Moate, 2022). According to Emanuelsson and Sahlström (2022), traditional teaching classes focus on building a solid foundation of learners' memorisation and developing a skill set for manipulating learning through repetition, recitation, and documenting mathematical fraction concepts. learners gained confidence and mastered exercises on fractions through repetition. Educators present material in lectures and learners document examples and answer similar problems (Kaur, 2022). Thus, exercises are to reinforce mathematical concepts of knowledge and understanding.

Problem-solving teaching classes contrast with traditional classes, in problem classes educators focus on helping learners to develop critical thinking skills that would enable them to apply their knowledge using real-life situations (SAIDE, 2017). Teaching and learning are based on the demonstration and experimentation of manipulation using concrete visual representation modelling (Zhang, Cao, Chan & Wan, 2022). Hence, this study uses Polya's mathematical problem-solving strategies to enhance learner understanding knowledge of the problems, devising a plan, carrying out a plan and evaluates their solution(s), when solving abstract and concrete fractions concepts, and to accommodate learners different learning styles (Zhang et al., 2022).

2.2.1.1 Traditional Teaching Classes

Mesiti et al. (2021) define traditional teaching classes as an environment where learners receive instruction from their educators. Traditional teaching classes have dominated schools for centuries and still dominate currently in most South African schools (Ubah, 2021). Traditional teaching is primarily educator-centred, learners are required to listen carefully, take notes, and memorise the teaching instructions and information (Graph, 2022). Teaching and learning materials involve notebooks, chalk, textbooks, and chalkboards. Assessment includes classwork, homework, and formal and informal tests (Graph, 2022). The educators teaching in the traditional classes focused on class management using a set of rules with high discipline to instruct learners' learning and behaviours. This approach has gained recognition in most SA schools over the years in imparting knowledge and skills to learners (Tumkaya & Ulum, 2020). Graph (2022) explains that traditional teaching and learning have advantages, which include time management. It saves lesson preparation and learning time. The approach is easy to implement in classes with little resources. This is confirmed in this study. Furthermore, the application of traditional teaching and learning requires little mathematical content knowledge. The educators only assess learners' conceptual understanding knowledge through explanation, repetition, recitation, and documentation of concepts. These assist the learners to remember the lesson's rules and concepts quickly (Tanujaya et al., 2017). In addition, teaching using the traditional approach enables educators to solve problems quickly (Egodawatte & Stoilescu, 2015). These effects were observed in comparison classes. Learners completed the instrument before the duration actual time.

Nonetheless, this approach was criticised for being too rigid and not adaptable to different learners' learning strategies or individual learners' needs (Clarke, Keitel, & Shimizu, 2006; MacDonald, 2012; Yassin & Almasri, 2015). Furthermore, the educators might have saved the teaching and learning time. However, learners would lack an environment that challenges their curiosity or motivates willingness to explore their hypothesis or confirm their prediction. (Di Leo, Muis, Singh & Psaradellis, 2019). Therefore, traditional teaching does not emphasise how, when, and why. According to Di Leo et al., (2019), it is an ill-equipped knowledge if learners undergo all learning processes and procedures without understanding why, how, or when to apply, knowledge learned in the classroom to the real world.

2.2.1.2 Problem-solving Teaching Classes

According to Lein et al. (2020), problem-solving is a process of finding a solution to a mathematical question using different approaches. It is applied in different areas, including classrooms for teaching, learning, and solving problems involving addition and subtraction of fractions (Polya 1887-1985; SAIDE, 2015). Recently, educational institutions have focused on teaching and learning approaches that are more learner-centred and interactive (SAIDE, 2017; UNISA, 2018; DBE, 2022). It is generally believed that the traditional teaching and learning approach does not engage learners in learning effectively, to become active participants in their learning process (DEE, 2022). South Africa Revised National Curriculum Statement for Grades R-9 emphasises that the primary purpose of educating a child is to solve problems. Therefore, problem-solving is the primary goal for mathematics instruction and integrates parts of all mathematical activities (DBE, 2018). In addition, SAIDE (2017) obliged educators to make problem-solving the primary purpose of educating learners on fractions. Furthermore, teaching using problem-solving creates a learning environment where learners can solve real-world mathematical problems and enhances essential solving skills learners need to succeed when solving mathematics fractions and beyond (Yang et al., 2022).

According to Yang et al (2022), problem-solving classes help learners with learning planning and improves problem-solving learning and method. In addition, problem-solving provides learners with an opportunity to think freely and be creative. Problem-solving teaching fosters collaborative learning, and learners are encouraged to work together in groups to solve problems (Ubah & Bansilal, 2018). Learners develop skills required to solve complex fraction problems and strategies like breaking complex mathematical problems down into smaller parts to identify patterns that enable them to solve through multiple methods and make sense of the problem (Amador, 2016; Weisstein, 2022).

Chan et al. (2022) state that testing various hypotheses that depend on proportional reasoning can only be experimented within problem-solving classes. Four phases of Polya's mathematics problem-solving are problem understanding, devising a plan, carrying out the plan, and evaluating the solution that can be verified through problem-solving (Polya 1887-1985; SAIDE, 2017). Thus, teaching and learning progress from uncertain to certain between external objects and perception in problem-solving (Zhang et al.,2022). The problem-solving classes and the traditional classes are two distinct teaching models (Chan, Clarke, & Cao, 2018). Thus, this study evaluates the effects of teaching using the problem-solving approach on learning fractions in the experimental group and comparing the effect of the traditional approach in the comparison group. The differences between problem-solving and traditional class are summarised (Cf Table 2.1).

Table 2.1: Summary: Problem-solving and Traditional Teaching class

difference	Problem-solving Approach	Traditional Approach
Learning Approach	Teaching focuses on helping learners enhance problem-solving skills and critical thinking abilities. It involves self-learning grouped discussions, cooperative learning, experimenting, and consultation of various source materials like textbooks and the Internet.	Teaching focus on helping learners to memorise, imitate and document educators' instructions and information. Consultation sources are learners textbooks and the educator's guide. It involves listening, copying notes on a chalkboard, and committing activities in memory.
Role of Educator	The educator acts as a facilitator, monitors group work, sets questions and motivates learners to reflect on their solutions.	The educator is the centre of the classroom activities, teaches, instructs, and provides learners with classwork and homework.
Curriculum	The curriculum is re-structure around specific problems that require learners to apply knowledge creatively. Examples relate to the real world.	The curriculum is often pre-determined with a set of topics that cover a specific order. Examples are numerical values based on fraction problems.
Assessment	Assessment focuses on learners' ability to solve problems and think critically and enhance learners' problem-solving skills.	Assessment consists of exams and quizzes that measure learners' ability to memorise, imitate, and recite Concepts.
Classroom Activities	Activities are based on active solving of fraction problems and collaboration.	Learners sit passively, listen to lectures and complete worksheets or exercises.

2.2.2 Efforts implemented in enhancing achievement in mathematics

Learners' mathematics achievement and performance are of interest to parents and all mathematics stakeholders in education (DoE, 2022). In South Africa and globally, efforts are being channelled toward improving the performance and understanding of mathematics and proper teaching and learning approaches (TIMSS, 2019). South Africa's education department has witnessed an education transition from the apartheid regime times when mathematics was taught in higher standard grade levels (SAIDE, 2017). During apartheid, mathematics education was not learnt in most South African schools (UNISA, 2014). Currently, South Africa is experiencing an education system which is inclusive of all who learn and teaching that has brought diverse methods towards teaching and learning mathematics education throughout the country (DBE, 2014). Mathematics is now a compulsory subject in all schools. Nevertheless, The Department of Basic Education is still concerned with what and how educators are teaching mathematics in their classes (DBE, 2018). This is because the outcomes-based education teaching approach has a singular comprehensive National Curriculum and Assessment Policy, which implies that a single assessment is conducted for all subjects. Consequently, the curriculum has been dominated by the traditional teaching method (DBE, 2014).

Thus, the Department of Education encourages schools to adopt textbooks that outline problem-solving teaching and learning, which encourages continuous assessment of learners (DBE, 2012). Furthermore, there have been adjustments in the National Curriculum Statement through new Curriculum Assessment Policy Statements (CAPS) (DBE, 2016). Educators' assessment and qualifications and evaluation bodies have been established, for example, the South African Qualification Authority SAQA and the South African Council of Educators were established to evaluate foreign qualifications and to create standards in the National Qualification Framework (NQF). Educators' teaching training, workshops, and internship programs were introduced to equip educators with learning content. Both National Education Content Trust and the South African Council of Educator workshops were established to equip mathematics educators (DBE, 2022).

These efforts were implemented by the South African Department of Education to enhance mathematics classes. Nevertheless, more efforts are still needed. For example, in the Department of Education diagnostic reports (2019-2022), matric results indicated that 70% of matriculated learners did not achieve a 50% bachelor pass rate in mathematics across the country (DBE, 2022). This further confirms the TIMSS (2019) report that South African learners, especially those in Grade 8, need more fraction and problem-solving skills. Therefore, more efforts and studies are required to enhance learners' ability to solve problems in mathematics.

2.2.3 The importance of effective mathematics classes

An effective mathematics class provides support engagement and promotes active learning (Chan & Moate, 2022). It motivates learners to engage in the process of developing their mathematical thinking and problem-solving skills (Chan & Moate, 2022). In these classes, the educator's role is crucial as the facilitator who guides learners in their process of discovering mathematical concepts and applying them to the real world (Yapatang & Polyiem, 2022). Learners engaged in this environment during intervention administration increased their performance in solving fraction questions. According to Mesiti et al. (2022), educators must conduct teaching and learning of fractions in an effective mathematical class for a better understanding of complex mathematical problem-solving. It fosters the physical arrangement of objects. Physical class seating arrangement may hinder or support learners' learning. As observed in this study, chairs in classes were rearranged and learners re-grouped to foster learner-to-learner talking time and minimise educator-to-learner talking time during learning (Yapatang & Polyiem, 2022). It addressed individual learner needs and allowed brighter learners to share their knowledge with others in the group (Thomas, 2019). Educators' effective classes provide clearer and more concise explanations of mathematical concepts when teaching; they use different teaching strategies like drawing, visual representations, and modelling objects; they use real-life situations and the learning environment to support positive feedback; they encourage collaboration and promote mindset growth (Zhang et al., 2022). Fraction-solving skills require precision (Da-Ponte et al., 2022). Therefore, targeted feedback helps learners identify and correct mistakes, and encourages positive activities like collaboration and teamwork (Chan & Moate, 2022).

In addition, learners solve advanced problems; they are free to use different learning styles. Educators understand that learners are motivated to learn, and they feel comfortable in taking risks, and making mistakes that are crucial to learning using problem-solving (Yapatang & Polyiem, 2022). Furthermore, critical thinking is enhanced in an effective classroom environment because learners are motivated and encouraged to ask questions, challenge assumptions, and consider multiple perspectives using real-world examples (Ubah & Bansilal, 2018). Therefore, effective classrooms are important to enhance Grade 8 learners' problem-solving skills and learning and solving fraction problems. Thus, effective classes help learners to become effective fractions and mathematics problem-solvers (Ubah, 2021).

2.3 OVERVIEW OF FRACTIONS

The word fraction originates from the Latin word 'fractus' which means 'broken.' The knowledge of fractions originates with the ancient Egyptians (Weisstein, 2022). Knowledge of Fractions are used to determine events like telling time, for purchasing during shopping and even to determine the amount of recipe needed for cooking and baking (Yang, Zhao, Liu, Tang, Liu, Tang & Cheng, 2022). Recently most scholars have defined fractions as parts of a whole expression with a numerator and a denominator, and more generally, as equal parts when used in daily languages (Ubah, 2021; Yang, et al., 2021; Graph, 2022). Fractions describe the parts of an object (Ubah & Bansilal, 2018). According to Weisstein (2022), fractions are classified as; proper fractions$\frac{2}{3}$, improper fractions, $\frac{5}{3}$ and mixed fractions $5\,\frac{2}{3}$. Fractions can be equivalent forms of decimals and percentages. Fractions are complex pieces of items assembled to yield a whole (Siegler & Lortie-Forgues, 2015). The complexity of fractions consists of four multiple constructs: ratios, operations that perform quantities, quotients, and units of measurement. In addition, the part of the whole sub-construct plays a vital part in understanding the four fractions' sub-constructs (Deringol, 2019). Furthermore, Deringol (2019) outlined five sub-constructs of fractions namely, part of a whole, ratio, operator, quotients, and measure.

Similarly, fractions in this study covered these five sub-constructs because they are essential across the Grade 8 mathematics curriculum. For instance, in the lower grades, most educators explain common fractions as part of a whole; in most cases, the explanation involves sharing and dividing an object among learners (SAIDE, 2017). However, the knowledge of fractions is essential in higher grades FET phase. Its application is critical in content areas like trigonometry, probability, quadratic formulas, financial mathematics, and algebraic equations (Yang et al., 2022; Nyembe, 2020). Therefore, learners should understand and be able to apply the knowledge of fractions to succeed in advanced mathematics. Educators need to teach distinguishing sub-constructs of fractions.

Nevertheless, fractions constitute an important topic in mathematics that learners need to comprehend but, some are reluctant to develop an interest in learning fractions. This may be among other reasons, why little attention is focused on research studies on Grade 8 fractions, particularly in the context of problem-solving (Makhubele, 2021).

2.3.1. Approach for Teaching Fractions

According to Langoban (2020), mathematics concepts are difficult to inculcate because educators are not exposed to different strategies. The teaching of most fraction concepts is challenging to most educators (Makhubele, 2021). However, proper teaching approaches from the educator helps learners to understand fractions concepts better (Makhubele, 2021; Yang et al., 2022). Here are strategies most scholars believe can help in teaching fractions (Nyembe, 2020). Teaching using visual aids such as real-world objects to represent fractions helps learners understand fractions better (Desta, 2019). For example, a pizza pie chart can used to illustrate the part of a whole construct and fractions scheme (Nyembe, 2020). Furthermore, educators can demonstrate fractions concepts by using guide to stream approach, that is by using real-life situations to demonstrate fractions problems and solutions (Quigley & Herro, 2019). For example, if four learners are to share a dough of pizza equally the following question can be asked to the learners: What portion does each learner get from the dough of the pizza? The example emphasises sharing that leads to learners' understanding that $\frac{1}{4}$ is a fraction, where one whole is the numerator and four is the parts' denominator. This is illustrated in Figure 2.1 below.

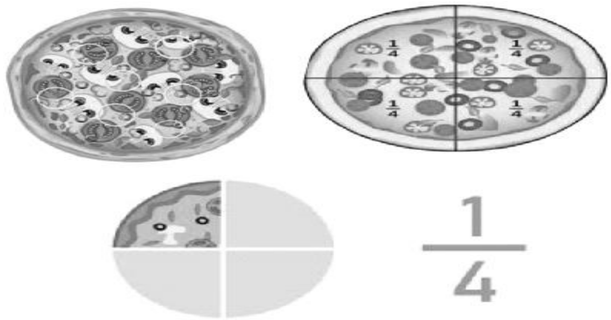


Figure 2.1: A dough of pizza divided into four equal parts

The educator can draw fraction wall diagrams to enhance learners' understanding of how to find quarters in a half, and what results from a quarter from two quarters. This assists in accommodating learners' learning styles during the teaching of fractions. For example, $(\frac{1}{2} - \frac{1}{4})$ $\frac{1}{2} - \frac{1}{4} = .\frac{2-1}{4} = \frac{1}{4}$. *Dough of pizza divided into four equal parts extracted from (*Nyembe, 2020*).*

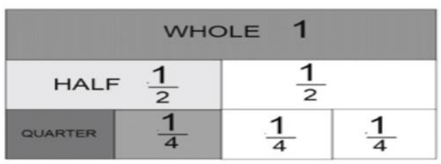


Figure 2.2: Illustrating mixed fractions extracted from (Nyembe, 2020)

Figure 2.2 reveals$\frac{1}{2} - \frac{1}{4} = .\frac{2-1}{4} = \frac{1}{4}$. Learners visualise the solution diagram and see that the first box presents the whole which is then divided into two; the two boxes represent half of the whole box and each one of the half boxes is further divided into two representing a quarter of the whole, meaning half of the half of a whole. Notably, learners who learn using physical visual objects understand and achieve better scores than those who do not (Desta, 2019).

Educators can encourage discussion among learners as a learning strategy. According to Smith and Mancy (2018), exploring the relationship between metacognitive and collaborative talks in-group mathematical problem-solving encourages learners to discuss their problem-solving strategies with each other. This can help learners to learn from each other and develop a deeper understanding of fractions. Active learning involves physical activity which means educators can use manipulatives like fraction bars, tiles, or cubes when teaching to assist learners visualise fractions and understand how they work (Vetter Orr, O'Dwyer, & O'Connor, 2020). Multiple visual representation models like number lines help learners understand fraction diversity. Educators can use various manipulatives (e.g., fraction concept displays) to improve learners' understanding when teaching fractions.

Different manipulatives used in representing length model fractions extracted (Pearson, 2014).

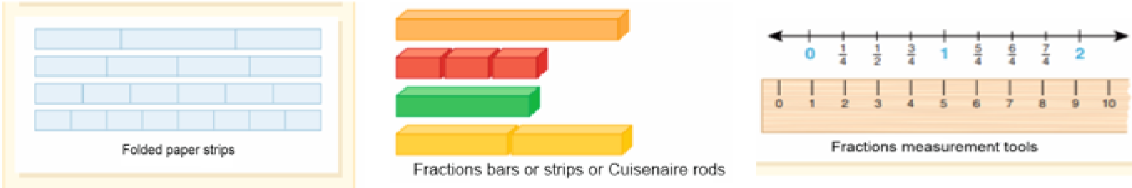


Figure 2.3: Manipulates use in representing length or measuring models for fractions

Teaching using various manipulatives assists learners to recognise that fractions are numbers that expand beyond the whole number system (Hannula, Haataja, Löfström, Moreno-Esteva, Salminen-Saari & Laine, 2022). For example, the length model has pieces lengths of 1-10. The strips model can produce learner-made fractions (Lamon, 2020). According to Lamon (2020), the teaching fractions scheme can assist in teaching fractions effectively. Furthermore, fraction schemes include using equivalent concepts to help learners understand that fractions can be different but equal. For example, $\frac{1}{2}$ is equivalent to $\frac{2}{4}$ and $\frac{3}{6}$ these would help learners to practice different exercises and problems (Hannula et al., 2022). Teaching fractions through problem-solving fosters the ability to solve open-ended problems. The ability to solve open-ended problems assists learners in developing their own learning strategies for solving problems (Aksoy & Yazlik, 2017). For example, open-ended questions like if a pizza is cut into eight slices and 3 slices are eaten, what number of ways can the remaining slices be divided among 2 people? Such questioning provides learners with different options to solve the problem and allows each learner to provide his or her own solutions (Nyembe, 2020). Learners can reflect and think on their strategies and develop metacognitive understanding.

According to Hannula et al. (2022), educators provide the support and guidance needed to help learners understand the problems, and gradually remove scaffolding, and encourage learners to solve problems independently. In addition, Hino and Funahashi (2022) agreed that the teaching approach can guide learners to focus on the lesson's objectives. Educators can assist learners in developing a better learning approach by using appropriate teaching methods (Desta, 2019).

2.3.2 Challenges in Solving Fractions

Learners' challenges with fractions vary depending on factors like age, grade, teaching and learning approach, and learners' previous knowledge (Nyembe, 2020). Nonetheless, fraction concepts are challenging to learners in mathematics topics (Mathaba, 2019; Makhubele, 2021).

Learners' difficulties involve adding, subtracting, dividing, and multiplying (Mdluli 2013; Dhlamini & Kibirige 2014; Mathaba, 2019). These challenges are experienced not only by South African learners but also by learners in other countries. According to Chan et al. (2018), Australian learners experienced challenges in solving basic addition and subtraction of fractions. Furthermore, calculation and procedures are challenges facing Egyptian learners when solving fractions (Aksoy & Yazlik, 2017). These challenges in learning fractions have an impact on learners' performance in mathematics, especially when they face higher mathematics topics (DEB, 2020). Research conducted in South Africa on learners having misconceptions about addition, subtraction, multiplication, and division of fractions (Mdluli, 2013; Dhlamini & Kibirige 2014; Mathaba, 2019) confirmed errors and misconceptions in addition of fractions are common challenges troubling learners in Grade 8 (Dhlamini & Kibirige, 2014). Learners' challenges include identifying equivalent fraction numbers, fraction order values and fraction sub-constructs, and parts of a whole (Fazio, Kennedy & Siegler, 2016). Learners could identify $\frac{1}{2}$, $\frac{1}{3}$ to be fraction numbers but could not identify $\frac{1}{2}$, $\frac{2}{4}$ and$\frac{3}{6}$, as equivalents (Fazio, Kennedy & Siegler, 2016). According to the National Education Collaboration Trust (NECT), (DBE, 2022), 16% of Grade 8 learners cannot identify fractions greater or less, or equal to $\frac{1}{2}$. Makhubele (2021) argued that calculations and procedural mistakes are common errors identified when learners solve fractions with different denominators (Makhubele, 2021). However, Polya (1945) believes that learners have difficulties understanding mathematical concepts because they are not guided to understand the problem. When learners do not understand the fraction problems deductive way in which educators teach concepts, they find it difficult to answer the questions (Daulay & Ruhaimah, 2019).

2.3.2.1 Learners' Challenges Involving Additions and Subtraction of Fractions

One challenge learner face in solving addition and subtraction fractions is using the same procedures for solving addition and subtraction of whole numbers (e.g., 2 + 3 = 5 and 3-2= 1), for solving addition of fractions (e.g.,$\frac{3}{5} + \frac{2}{3} = \frac{3+2}{5+3} = \frac{5}{8}$) (Dhlamini & Kibirige, 2014; Makhubele, 2021). Learners' misconceptions of addition and subtraction of fractions include adding a numerator to a numerator and a denominator to a denominator (e.g.,$\frac{3}{5} + \frac{2}{3} = \frac{3+2}{5+3} = \frac{5}{8}$) (Makonye & Fakude, 2016; Mathaba, 2019). According to Mathaba (2019), the mistakes learners make show lack of conceptual understanding of addition and subtraction. Conceptual understanding refers to an integrated and functional grasp of fraction concepts (Nyembe, 2020). In pursuance of this, it is believed that educators emphasise the importance of procedures and memorisation more than conceptual understanding (Fadipe, Sepeng, & Rankhumise, 2021). Furthermore, learners challenge related procedural fluency when adding and subtracting fractions. Furthermore, Mathaba (2019) believes that a lack of procedural proficiency leads to the poor conceptualisation of addition and subtraction of fractions. From the argument, one may conclude that learners that make mistakes lack conceptual understanding of adding and subtracting fractions because of improper teaching and learning approaches (Mathaba, 2019). Similarly, learners having trouble adding and subtracting fractions are using similar procedures. They add numerators and denominators, and subtraction, as observed in this study ($e.g. \frac{3}{4} - \frac{1}{2} = \frac{3-1}{4-2} = \frac{2}{2} = 1$). The researcher observed learners' challenges with the addition and subtraction of fractions. However, these challenges vary depending on individual learners and backgrounds. Most learners struggle more than others and require additional support to understand fraction concepts. Based on these arguments the researcher implemented problem-solving teaching on learning fractions in the Grade 8 class.

2.3.2.2 Challenges in Teaching and Learning of Fractions

Arum, Kusmayadi and Pramudya (2018) observed that learners are experiencing difficulties in problems-solving involving fractions. Instead of understanding mathematical concepts associated with problems, learners attach importance to the educator's approaches, procedures, and methods used to solve fraction problems. This learning style is a challenge as learners can only relate solving fractions to the movement of whole numbers (Naghavi, 2015). This learning style adopted by learners makes it difficult for them to solve basic addition and subtraction problems (Weisstein, 2022). Most educators do not expose learners to problem-solving approaches, which affects learners’ understanding (Banji, 2017; Yang et al., 2022).

Furthermore, learners often have trouble associating the addition and subtraction of fractions with prior knowledge of adding and subtracting whole numbers, and educators often relate their explanation of the addition of fractions with whole numbers (Mathaba, 2019). Makhubele (2021) argued that this is the primary root of misconceptions about teaching and learning fractions. For example, the researcher observed learners adding and subtracting fractions as whole numbers (e.g., $\frac{3}{5} + \frac{2}{3} = \frac{3+2}{5+3} = \frac{5}{8}$). Learners apply an addition scheme associated with the numerator as a whole number like, $\frac{3+2}{5+3} = \frac{5}{8}$. Educators showed a tendency to use the Lowest Common Denominator (LCD) as the only method for solving addition fraction problems. For example, if learners were to solve $\frac{3}{5} + \frac{2}{3}$, the educator expects learners to identify the LCD (e.g., $\frac{3}{5} + \frac{2}{3}$ LCD = 3×5=15 and $\frac{9+10}{15} = \frac{19}{15} = 1\frac{4}{15}$). However, the educator who is exposed to alternate approaches helps learners convert the common fractions to equivalent decimal fractions before adding the decimals, e.g.,$\frac{3}{5} = 0.6 + \frac{2}{3} = 0.66$, (0.6 + 0.667 = 1.27, which equal to $1\frac{4}{15}$. However, since educators are not exposed to different approaches to teaching, the teaching of fractions to learners becomes challenging (Tursucu et al., 2020). According to Fuchs and Malone (2021), learners' difficulties in learning fractions are associated with the educator's knowledge of equivalents. When educators engage learners in equivalent fractions, they directly influence learners to recall diverse types of fractions (e.g., proper, improper, and mixed fractions). Thus, learners who struggle with conversion from improper to mixed fractions and decimals and percentages may not identify equivalent fractions (e.g., $\frac{1}{2}$ = 0.5 = 50) (Wang, Fuchs, Fuchs, Gilbert, Krowka & Abramson, 2019). Most educators teach concepts based on ontological knowledge. For instance, a educator with a realistic worldview may likely transmit his/her knowledge to learners based on a realistic knowledge viewpoint (Resnick, Jordan, Hansen, Rajan, Rodrigues, Siegler, & Fuchs, 2016; Namkung, Fuchs & Koziol, 2018). By extension, most educators approach focus algorithms concepts that help learners with memorisation (Fadipe, Sepeng, & Rankhumise, 2021).

2.3.2.3 Educators' knowledge of fractions as a challenge

The educator's knowledge of fractions depends on training, experience, and professional development (Mothibeli, 2018; DBE, 2020). Educators should have a strong conceptual knowledge of fraction sub-constructs (e.g., parts of a whole, division, and ratios) operations (Luneta, 2017). According to Luneta (2017), educators have limited pedagogical content knowledge to effectively teach fractions. Luneta (2017) further explains that educators' limited content knowledge of fractions consequently deters them from using fraction manipulatives and problem-solving teaching strategies that help learners understand fractions.

Similarly, Desta (2019) acknowledges that most high school educators could not explain fraction sub-constructs with terms that could help learners develop a deeper understanding of fraction concepts. Most educators could not explain the meaning of fractions such as $\frac{2}{3}$ as part of the whole sub-construct. In his study, Luneta (2017), half of the educators could not use manipulatives to represent $\frac{2}{3}$. Only 32% expressed the part-whole sub-construct, 21% expressed the quotient sub-construct, 19% expressed the ratio sub-construct 14% expressed the operator construct, and 14% expressed the measure sub-construct. Luneta (2017) thus concluded that most educators' challenges are related to inadequate knowledge of fraction sub-constructs. This further confirms that most educators make use of the examples in mathematics textbooks and use approaches from these textbooks without proper understanding.

2.3.3 Solution to Enhance Effective Teaching of Fractions

This section discusses the core component solutions and answers to challenges discussed previously. The component is teaching fractions using prescribed textbooks, fraction manipulation models, problem-solving, and fraction schemes. The components discussed include the following:

2.3.3.1 Teaching Fractions using Prescribed Textbooks

According to Siegler and Lortie-Forgues (2015), mathematics textbooks can enhance the teaching and learning of fractions. Educators can enhance learning of fractions by using examples set in prescribed mathematics textbooks. Moreover, schools are obliged to apply a teaching approach in the mathematics textbooks-based problem-solving approach (DBE, 2015). Young educators teach using examples from the recommended mathematics textbooks (Mothibeli, 2018). According to Mothibeli (2018), educators should use their curriculum knowledge which is knowledge from textbooks to motivate learners' interest in supporting their prospective and ontological beliefs when teaching (Mothibeli, 2018; DBE, 2020). Curriculum refers to the plan and design explored in teaching and learning, the goals, content objectives, instructional strategies, assessment methods, and resources used to support teaching and learning (Mothibeli, 2018; DBE, 2020). In this study, curriculum is related to teaching content, teaching, and learning of fractions in the prescribed mathematics textbooks.

2.3.3.2 Teaching Fractions using Fractions Manipulation Models

Using fraction manipulation models helps learners understand fraction problems better (Nyembe, 2020). Manipulatives provide learners with alternative learning, learners see and touch concrete objects that are present in abstract form in the problem (Mudaly & Naidoo, 2021). An example of a manipulative is paper folding. Learners touch the concrete objects that represent the area covered (Nyembe, 2020). For example, $1\frac{2}{3}$ can be represented using an area model.

Rectangular area model illustrating $1\frac{2}{3}$ *partition modified from (*Nyembe, 2020*).*

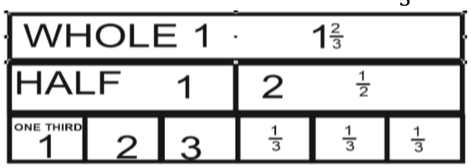


Figure 2.4: Partitioning $1\frac{2}{3}$ using rectangular region (Extracted from Nyembe, 2020*)*

The above is a rectangular area model of a whole partition. The rectangular diagram is a visual drawing representing abstract$1\frac{2}{3}$, this assists the learners in learning and understanding better than $1\frac{2}{3}$ is divided into one, two and three partitions. Modelling aids learners' understanding of fractions. Table 2.4 below summarises different manipulatives for teaching and learning fractions in Grade 8 (Nyembe, 2020).

Table 2.2: Summary of Models used in Representing Fractions

Model	**Description**
Area models	In an area model, a fraction is represented as a shaded part of a whole region or rectangle. This model presents and expresses fractions as a numeric embodiment, relating to parts-area. Weisstein (2022) argues that this model best defines the part-whole sub-construct. Weisstein (2022) further advises that when introducing the part-whole sub-construct, the model should be introduced appropriately. Therefore, educators should carefully consider all criteria. The optimal focus should be on implementing the area fractions model. When using the area model, the educator needs to use a symmetrical object or figures that are divided evenly. He avoids the use of a circle when modelling odd number denominators. For example, he represents a half by shading half of the rectangle.
Set models	In a set model, a fraction is represented as a part of a set of objects. For example, if there are ten marbles in a bag and three of them are red, then the fraction of red marbles is 3/10. This model expresses and presents fractions numerically, as a certain number of objects corresponding to given objects that fall within a cluster. The cluster corresponds to the whole objects inside the cluster while the objects in the group that are equal to the sub-constructs of the cluster become the main clusters that form the fractions. This model can be effective only when learners develop the ability to group objects into a cluster. However, learners find this highly challenging because at most they cannot differentiate fractions as they usually pay attention to objects' size rather than their number (NCTM,

	2000). Thus, learners need to have full knowledge of divisions and skills to effectively understand the cluster fraction model. While the objects inside the cluster form the whole of the cluster, a group of objects that comprise the sub-cluster of the main cluster form the fraction. Successful use of this model depends on learners' ability to group the cluster objects. Learners mostly fail to distinguish between fractions since they tend to focus on the size of the objects rather than their number (NCTM, 2000).
Length models	In a number line model, fractions are represented as points on a line between 0 and 1. The length model makes use of the distance between two objects provided. The model presents scenarios using number line lengths that are divided into different numerical units on the line to represent the fraction. Hence, the given fraction numbers are then identified at the appropriate point on the line. Tangible but real numbers as decimals are written as fraction numbers. Studies support the length model as effective in facilitating learners' understanding on knowing fractions as numbers and boosting their knowledge of learning other concepts of fractions (Nyembe, 2020). It also allows learners to gain knowledge about fractions by positioning them on the number line and considering the size of the fractions based on their distance, particularly the distance between half and one.
Strips and Circles	Fraction strips are paper, or plastic divided into equal parts to represent different fractions. Learners manipulate the strips to compare and order fractions and to add and subtract fractions. Fraction circles are circular shapes that are divided into equal parts to represent different fractions. Learners can use these circles to visually compare fractions and to learn how to add and subtract fractions.

2.3.3.3 Teaching Fractions using Problem-solving

In section 2.3.1, it has been argued that teaching and learning fractions using problem-solving is an effective way of helping learners understand addition and subtraction fractions better (Langoban, 2020). Manipulative application when teaching fractions requires problem-solving skills. For example, the educator who wanted to instil creativity in two classes gave Classes A and B a problem that involved real-life situations using problem-solving teaching strategies such as hands-on activities (Nyembe 2020). Problem-solving provides learners with more practice opportunities and feedback which helps learners develop confidence in solving fractions (Quigley & Herro, 2019). Class A: three-metre (3M) wire to construct sunglass and class B; two metre (2M) wire to produce a fork. To construct sunglasses, Class A had to use $1\frac{1}{2}$ M wire, and to construct a fork, Class B had to use $\frac{1}{2}$ M wire. What number of sunglasses can Class A form from the 3M wire? What number of forks can Class B form from the 2M? (Nyembe 2020). To respond to the questions above, the educator in Class A used the length model to illustrate to learners $1\frac{1}{2}$M of 3M.

Class B educator also used the length model to illustrate to learners $\frac{1}{2}$M of 2M. Class A: $1\frac{1}{2}$ M. Educators demonstrated to learners using divisions like $3 \div 1\frac{1}{2} = 2$. Conclusion: Two sunglasses can be formed from 3M wire. Representing the solution on the length model helped the learners visualise and understand the problem better without dividing the three-meter wire by the mixed number $1\frac{1}{2}$ M as follows. Class A: length model $1\frac{1}{2}$M wire (Cf Figure 2.4).

Number line length model illustrating $1\frac{2}{2}$ *of 3M partition modified from* (Nyembe, 2020).

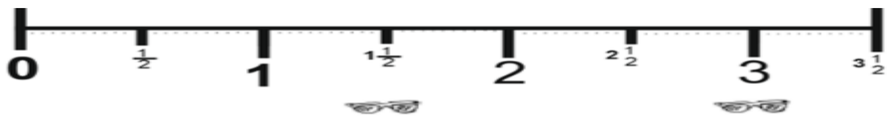


Figure 2.5: Illustrating length model (Extracted from Nyembe, 2020)

Class B: $\frac{1}{2}$M wire was demonstrated to learners using division, $2 \div \frac{1}{2} = 4$. The conclusion was that four forks can be formed from 2M wire. Representing the solution on the length model helped learners understand the problem even without dividing the two with the proper fractions $\frac{1}{2}$**.** Group B used the length model in modelling $\frac{1}{2}$M wire in Figure 2.5 below. *Number line model manipulative illustrating* $\frac{1}{2}$*M of 2M partition modified from (Nyembe, 2020)*

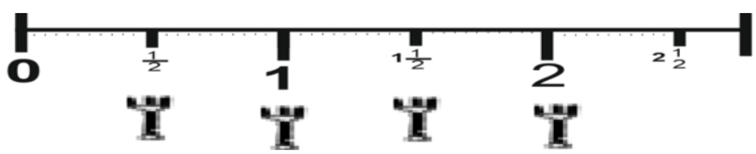


Figure 2.6: Illustrating Length model (Extracted from Nyembe, 2020)

2.3.3.4 Teaching fractions using a fraction scheme

Teaching fraction schemes is an effective way to help learners understand mathematics fraction concepts (Namkung et al., 2018). These schemes are equivalents, simplifications, addition and subtraction, multiplication, division, and conversion; fraction schemes emerge in learning and teaching fractions (Namkung et al., 2018). Similarly, understanding equivalent fractions concepts assists learners in connecting mathematical concepts. Educators can start by teaching equivalent fractions concepts, helping learners to identify, create and practice equivalent fractions (Resnick et al., 2016). For example, educators can help learners understand that $\frac{1}{2}$ = $\frac{2}{4}$ and $\frac{3}{6}$ are equivalent fractions because they represent half of a whole. According to Alghazo and Alghazo (2017), simplified fraction schemes teach learners to reduce fractions by finding the highest common factor of the numerator and denominator. The educator provides learners with problems to practice and helps them understand to check answers. For example, $\frac{4}{8}$ simplified to $\frac{1}{2}$ by dividing both the numerator and denominator by four (Arrahim, Sugiharti & Damayanti, 2020). Similarly, addition, subtraction, multiplication, and division schemes are an effective way of teaching fractions. The educator demonstrates to learners how to add and subtract fractions with different denominators by finding a common denominator. For example, $\frac{1}{3}$ + $\frac{2}{3}$= $\frac{3}{3}$=1 and $\frac{3}{4} - \frac{1}{4}$ =. $\frac{2}{4} = \frac{1}{2}$. Fractions are multiplied by multiplying the numerator and denominator separately. For example, $\frac{1}{2} \times \frac{2}{6} \times \frac{2}{6} = \frac{1}{3}$. In addition, fractions can be divided by multiplying the first fractions by the reciprocal of the second fractions for example $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$. Finally, the demonstration of the conversion fraction scheme is an effective method for teaching fractions. Incorporating these schemes into teaching helps learners develop a better understanding of fractions. Fractions are critical concepts used extensively in mathematics and various fields like engineering, science, and finance. Effective teaching of fractions would help learners apply knowledge of fractions to solve real-world problems in various fields.

2.4 PROBLEM-SOLVING FRAMEWORKS

Various models and theories on the problem-solving framework in teaching and learning have been defined by Polya, Vygotsky, Hmelo-Silver, Anderson, Verschaffel, and others (Polya's, 1945/1973; Human 1992; Murray, Olivier & de-Beer 1999; Hmelo-Silver, 2004; Mayer, 2009; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013). Therefore, a proper framework can be objective to content and environmental needs. In South Africa, just like in other countries, understanding heuristic mathematical problems can help one to become good problem-solving solver involving fractions (Alghazo & Alghazo, 2017). Heuristic mathematics problem is an essential part of fractions in the mathematics curriculum (DBE, 2015).

2.4.1 Anderson et al., (2000) Problem-solving Framework

Anderson et al., (2000) problem-solving framework focuses on adaptive control of thought-rational (ACT-R) of cognitive science and depicts that learning involves the acquisition of declarative knowledge of new situations using search and analogy. Compilation of domain-specific productions strengthens declarative and procedural knowledge. Anderson's framework is based on developing learners' cognition. Cognition means to understand conjectures and theorems in schemas along with knowledge about the function of theorems. Procedural knowledge means 'knowing how', and which production systems or sets of condition-action pairs to use. According to Andersons et al., (2000), declarative knowledge is about facts and unequivocal understanding that individuals are knowingly aware of and can account for. Andersons et al., (2000) believes that procedural knowledge is inherent knowledge of how to do things, as well as knowledge about how to perform cognitive activities.

2.4.2 Verschaffel Problem-solving Framework

Verschaffel's problem-solving framework focuses on understanding problem text (De corte et al., 2004). The framework trains learners to decode text problems. Thus, learners become active readers. It impacts success when solving mathematical word problems in fractions (Lewis, Matthews & Hubbard, 2016). In addition, it develops learners to become good problem-solvers who relate mathematical problems to situation models like translating the main text character in the problems into an expression that makes sense to the problem-solver (Lewis, Matthews, & Hubbard, 2016).

2.4.3 Hmelo-Silver Problem-solving Framework

Hmelo-Silver's (2004), framework focuses on self-directed learning (SDL), it helps learners to make sense of the problem by articulating and analysing the problem to identify the relevant variables (Banji, 2017). Furthermore, it enables learners to identify and solve needed information in the mathematics problem and check if their answers are correct (Banji, 2017). In the context of this study, the framework means that learners can attempt to provide answers by selecting the vital information in a mathematical problem. If they excel in providing the right solution, alternative methods can help learners check if their answer is correct (Fadipe, 2017). Hmelo-Silver (2004) acknowledges that learners' insufficient knowledge of identification may affect their learning process. Furthermore, learners need to reflect on current information as a focal point of learning. In addition, Banji (2017), agrees that the following aspects aid learning; acquisition of flexible knowledge, increase in effective problem-solving skills, SDL skills towards their learning goals, effective and efficient development of group working skills, and self-encouragement for advanced achievement.

2.4.4 Polya's Problem-solving Framework

Polya's problem-solving framework defines problem-solving as a process that begins with problem understanding and interpretation. Problems are solved by following four strategy stages: (1) understand the problem (2) devise a plan (3) carry out the plan (4) evaluate the solution (Fadipe, 2017). Polya's framework focuses on four hierarchy knowledge. Its objective is to break down complex mathematical problems into smaller understandable phases. These strategies were adopted in this study's theoretical framework. Thus, the findings of this study were evaluated in line with Polya's framework.

2.5 THEORETICAL FRAMEWORK

A theoretical framework constitutes concepts, assumptions, and propositions in explaining a phenomenon (Filimowicz, 2018). It provides structural connections and relationships between different variables and the underlying principles that govern them. Furthermore, it serves as the foundation that guides the development of hypotheses and the selection of appropriate methods and techniques (Arrahim, Sugiharti, & Damayanti, 2020). This study's theoretical framework is based on existing theories of George Polyas' mathematics problem-solving strategies as concepts for the study used to verify the hypotheses and research methods. It helps understand the relationship between variables and the underlying variables that govern them (Arrahim, Sugiharti, & Damayanti, 2020).

2.5.1 George Polya's Theoretical Framework

George Polya's theoretical framework consists of four phases, which are: understand the problems, devise a plan, carry out the plan, and evaluate the solution. This framework tool is used to enhance learners' problem-solving skills and break down complex problems into smaller forms for clarity and simplicity (Filimowicz, 2018). According to George Polya (1945), in his book, *How to solve it*, learners do not know how to solve problems. He further argued that difficulties are not associated with learners not knowing mathematics but with guiding learners using proper problem-solving approaches. According to Filimowicz (2018), this design assists individual learners in solving problems logically and systematically. It breaks down complex problems into smaller manageable parts. The phases outlined the reasons why this study adopted the theory. It addresses problems related to addition, subtraction, and equivalent fraction problems and emphasises the importance of breaking complex mathematical problems down into smaller parts (Arrahim, Sugiharti, & Damayanti, 2020). In addition, the aspiration of this theory is in line with what this study envisages, namely, learners should be able to enhance problem-solving skills such as understanding problems, devising a plan, carrying out the plan, and evaluating solution which is what the theory is advocating (Chin et al., 2019). The primary data findings in this study are from activities based on teaching equivalent, addition, and subtraction fractions. The theory requires learners to acquire problem-solving skills in learning.

2.5.1.1 Understand the Problem

Understanding a problem requires learners to read the questions carefully. Learners can synthesise the problem, break them into smaller parts, and identify the key information (Chin, Blair, Wolf, Conlin, Cutumisu, Pfaffman & Schwartz, 2019). According to Chin et al., (2019), understanding problems and interpretation are the primary assignments of educators and learners. Learners should understand what the question is about and be able to identify essential information from the question (Naghavi, 2015). Understanding abstract information from a given question is crucial for heuristic problem solvers to advance knowledge of how to connect previous solutions with current situations (Chin et al., 2019). Furthermore, problem understanding enhances learners' critical thinking, and enables them to ask questions such as, what is this problem about? What information can I use to make sense of this problem? What fraction schemes can solve this problem? This is probed by using fraction strips and length model manipulatives to demonstrate equivalent fractions. For example, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$, and length model $1\frac{1}{2} + 2\frac{1}{2} = 4$ for a better conceptual understanding of teaching addition and subtraction schemes (Weisstein, 2022).

2.5.1.2 Devise the Action Plan

Once learners understand the problem, they proceed to the next phase of devising a plan. To devise a plan, learners synthesise the problem down to smaller parts. They re-organise the information on a model diagram like a chart or similar scheme previously used to solve similar problems (Pritchard, 2017). According to Pritchard (2017), planning is essential to the problem-solving process. A plan can effectively be devised only if resources are available; unavailability of resources may result in poor planning (Naghavi, 2015). The educator ensures learners are given the appropriate information to devise a plan like data identification and interconnectivity (Mavridis, Katmada, & Tsiatsos, 2017). This is proven by demonstrating solving fractions using conversion. Learners understand how a single fraction can used to represent decimals and percentages. For example, $\frac{1}{2}$ = 0.5 = 50% and $1\frac{1}{2} = \frac{3}{2}$ = 1.5 = 150%. This can enable learners to use a variety of learning styles (Naghavi, 2015).

2.5.1.3 Carrying out the Plan

Carrying out the plan means plans in action. Calculations, experiments, and diagrams can be used to solve problems (UNISA, 2014). According to Rahman (2019), educators should assist learners in using the best procedure from the entire previous set plan. Individual observation of progress and self-correction play a vital role in this level (Tursucus et al., 2020). Banji (2017) believes that most learners' initial plans do not yield the needed results at first time. Therefore, learners should realise that most initial plans do not solve the problem. However, with proper interpretation, one can solve problems correctly. Learners should solve problems carefully and avoid discouragement when a plan fails; they should be cautious and work Phase 1 step at a time until their goal is accomplished (Nyembe, 2020). Learners should be aware that failure in a plan demands starting over again as one cannot proceed without completing the previous phase. This is probed by demonstrating fraction models like the area model and strategies for converting fractions to equivalent decimals and percentages. For example, $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$. Alternatively, $\frac{1}{8} = 0.125 + \frac{2}{8} = 0.25 = \frac{3}{8} = 0.375$. Therefore, 0.125 + 0.25 = 0.375.

Pie chart manipulative illustrating $\frac{1}{8}+\frac{2}{8}=\frac{3}{8}$. Partition modified from (Nyembe, 2020).

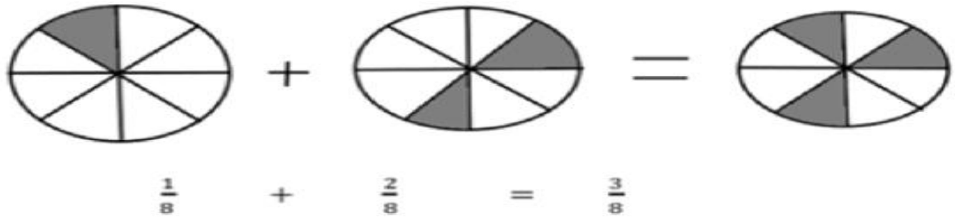


Figure 2.7: Illustrating Area model of $\frac{1}{8}+\frac{2}{8}=\frac{3}{8}$ (Nyembe, 2020)

2.5.1.4 Evaluate Solution

According to Albay (2019), learners need to reflect on their solutions. Learners may self-evaluate solutions by asking: Do my answers make sense? Can I provide an alternative approach to arrive at this answer? Can I represent this answer in terms of the problem statement? These questions can assist learners in reflecting on their answers and willingness to try alternative approaches. Polya’s problem-solving framework model consists of four steps. The theoretical framework tool can help learners develop problem-solving skills (Filimowicz, 2018). Figure 2.8 illustrates the model of George Polya’s four mathematical problem-solving strategies below.

Illustrating Polya’s four mathematical Problem-*solving strategies modified from Fadipe (2017)*

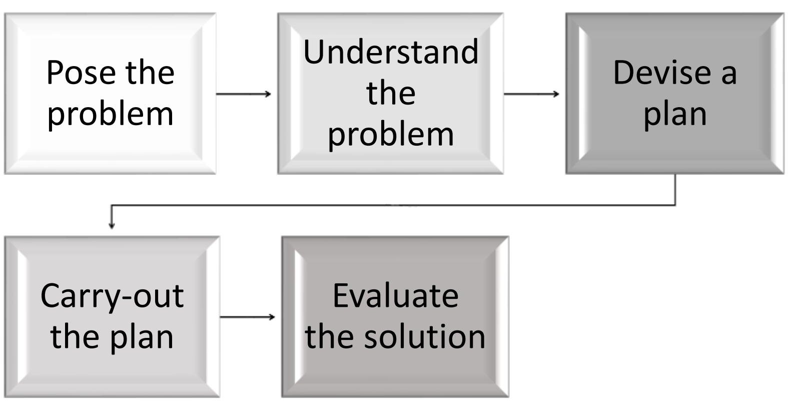


Figure 2.8.: Illustrating Polya’s problem-solving four-phase model

Table 2.3: Summary of Polya’s problem-solving Approach framework

Phase I	**UNDERSTAND THE PROBLEM**
Probing	Can you restate the problem in your own words? Can you determine what is known about these types of problems? Is there missing information that, if known, can allow you to solve the problems? What is the goal of the problem?

Phase II	**DEVISE A PLAN**
Probing	Make a list of the known information. Make a list of the information that is needed Draw a diagram. Make an organised list that shows all possibilities. Make a table chart. Look for a pattern. Write an equation and define what each variable represents.
Phase III	**CARRY OUT THE PLAN**
Guide	Work carefully. Keep an accurate and neat record of all your attempts. Work backwards. Try to solve a similar but simpler problem. Perform an experiment. Guess a solution and then check your result.
Phase IV	**EVALUATE THE SOLUTION**
Guide	Ensure that the solution is consistent with the facts of the problem. Interpret the solution in the context of the problem. Ask yourself whether there are generalisations of the solution that could apply to other problems.

2.6 WHY POLYA'S PROBLEM-SOLVING TEACHING APPROACH

Polya's mathematical problem-solving approach is a well-known teaching and learning framework used for solving mathematical problems (Albay, 2019). The ontological perspective claims that problem-solving is an iterative and dynamic process that involves both the problem-solver and the problem itself (Nasution, Harahap & Samosir, 2019). Thus, these ontological perspectives are key reasons the approach is appropriate in this study, which are: (I) Problem space: This refers to the domain in which the problem exists. (II) Problem solver: This is an active agent within the problem space. (III) Problem representation: This refers to how the problem is represented and understood by problem-solver (Daulay & Ruhaimah, 2019). Furthermore, Daulay and Ruhaimah (2019) agree with this approach that problem-solvers use to navigate the problem space and find a solution. Moreover, Polya's problem-solving approach is applied to various mathematical topics, including fractions. It enables an application set of hierarchy strategies, like understanding the problem, devising a plan, carrying out the plan, and evaluating the solution. This four-step phase process encourages learners to think critically and systematically when solving problems. These phases are applied in teaching fractions in this study (Daulay & Ruhaimah, 2019).

It is important to consider phases separately. Understanding the problem phase fosters the educator's careful involvement in informing the learners, how to understand fraction problems. Thus, learners start by reading the questions thoroughly. This enables learners to understand the text in the problem as envisaged by Verschaffel’s problem-solving model (Lewis & Mayer, 1987; Pape, 2004). Learners identify the needed information to solve the problem and the constraints or limitations that may hinder them from solving the problem before proceeding to the second step. According to Banji (2017), once educators have ensured that learners have a clear understanding of the problem, they should devise a plan to solve it. This phase emphasises identifying the necessary steps to follow. Hmelo-Silver (2004) supports this problem-solving as self-directed learning. Thus, learners determine the materials needed to devise the plan before moving to the third step (Nasution et al., 2019).

In the carrying out the plan phase, learners execute the plan that they developed in devising the plan phase, using mathematical procedures to solve the problem. Learners apply each step outline in their plan before proceeding to the next step. Evaluate solution: after solving the problem, learners check the solution to ensure it is accurate and makes sense and whether there are alternative methods they can use to solve the problems (Banji, 2017; Daulay & Ruhaimah, 2019). This enables mathematical problems to be solved as real-world problems and each phase enhances learners' problem-solving skills evaluated in this study on solving complex fraction problems (Daulay & Ruhaimah, 2019). Furthermore, the approach accommodates individual learners' learning styles and supports teaching strategies like brainstorming, working backward using previous knowledge, and assuming solutions to similar problems (Ndayambaje & Ralph, 2020). It benefits small groups and cooperative learners who collaborate. For example, learners can learn mathematical concepts as peers and groups without the educator’s involvement (Schwartz, 2015; Tumkaya & Ulum, 2020). Nasution et al (2019) observed that using Polya’s problem-solving approach reduced mistakes and misconceptions in solving fractions because it enables complex problems to be broken down into smaller units and fosters monitoring of smaller groups in class sessions. These arguments substantiate reasons why Polya’s problem-solving is considered an appropriate approach for evaluating the teaching of fractions in Grade 8 in this study (Daulay & Ruhaimah, 2019).

2.6.1 Ontological Perspective of Polya’s Problem-solving Approach

The ontological perspective of Polya’s problem-solving approach emphasises the dynamic and interactive nature of problem-solving, and the interaction between the problem solver and the problem space (Nguyen, Thuan, & Giang 2023). Breaking down a problem into its constituent parts through Polya's theory provides a useful framework for teaching and learning mathematical fractions (Daulay & Ruhaimah, 2019). The ontology breaks down complex mathematical problems into key components. (I) Problem space: This refers to the domain in which the problem exists. It includes the problem itself, any constraints or limitations, and the available tools or resources used to solve the problem. (II) Problem solver: This is an active agent within the problem space, who brings knowledge, skills, and strategies to bear the problem and interact with the problem space to find a solution. (III) Problem representation: This refers to how the problem solver represents and understands the problem. This includes the metal models or visual representations that the problem solver constructs to understand the problem (Vye et al., 1997; Vetter, Orr, O'Dwyer & O'Connor, 2020). The strategies are approaches the problem-solver uses to navigate problem space. Polya's approach uses four strategies: problem understanding, devising a plan, carrying out the plan, and evaluating the solution (Daulay & Ruhaimah, 2019). For example, the problem 'What is the percentage increase if the price of a bus ticket of R$\frac{1}{2}$ is increased to R$\frac{3}{4}$? How can this be solved?

Question: What is the percentage increase if the price of a bus ticket of R$\frac{1}{2}$ increased to R$\frac{3}{4}$?

Table 2.4: Applying Polya's problem-solving theoretical framework in solving fractions

Phase I	**UNDERSTAND THE PROBLEM** Learners understand the problem by reading, decoding the text meaning and re-stating the problems in their own words, and identifying key information in the questions.
Probing by questioning	Do you understand the meaning of percentage increase in bus price? What does the R stand for in R$\frac{1}{2}$? Can you identify the bigger fractions? What operation would be used to solve the question? What fraction form is this? Identify $\frac{1}{2}$ as a proper fraction.
Phase II	**DEVISE A PLAN** Learners convert to decimals, draw manipulatives, and plan using addition and subtraction operations. Modify the plan if addition does not yield the results.
Probing by questioning	Can you convert to equivalent decimal and percentage? $\frac{1}{2}$ = 0.5 = 50%. Would it be easy to sum after converting to an equivalent decimal and percentage? $\frac{3}{4}$= 0.75

	= 75% would you add or subtract? Which fraction is the biggest? $\frac{1}{2}$ = 0.5=50% from bigger $\frac{3}{4}$ = 0.75=75%.
Phase III	**CARRY OUT THE PLAN** Calculations using different approaches
Checking process	**Approach Option 1** Direct common fraction approach $\frac{3}{4} - \frac{1}{2} = \frac{3-2}{4} = \frac{1}{4}$ **Approach Option 2** Equivalent decimal: R0.75 − R0.5 = R0.25 **Approach Option 3** Equivalent percentage: 75% - 50% = 25%
Phase	**EVALUATE THE SOLUTION** Use self-evaluation questions such as, which solution option makes sense? Which options are reasonable? Which option is accurate? In addition, which option best answers the question? Learners were asked to convert fractions to decimals. Which fraction is the biggest? Explain in your own words what the increase and percentage stand for. Can bus tickets be paid with R$\frac{1}{2}$ to R$\frac{3}{4}$? Have I used a bus ticket?
Guide	**Evaluating solution Option 1:** $\frac{3}{4} - \frac{1}{2} = \frac{3-2}{4} = \frac{1}{4}$. This option does not make sense because the problem is in the context of money and percentages; hence, money is not denoted as $\frac{1}{4}$ as well as a percentage. Conclusion: Option 1 does not answer the question. **Evaluating solution Option 2:** R0.75 – R0.5 = R0.25. This option makes sense because the problem is in the context of money. However, does not answer the question of what percentage increased; percentage is not expressed as R0.25. **Evaluating solution Option 3:** 75% - 50% = 25%. This option answers the problem. Calculate the percentage increase if the price of a bus ticket of R$\frac{1}{2}$ increased to R$\frac{3}{4}$. Therefore, the bus ticket increased by 25% in bus price from R$\frac{1}{2}$ to R$\frac{3}{4}$. This option answers the question.

2.6.2 Enhancing Learners’ Problem-solving Skills

Abtahi (2018) emphasises the need to increase learners' abilities towards problem-solving. Teaching through Polya’s problem-solving approach enhances the following learners’ problem-solving skills: (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and (4) evaluating solution skills (Daulay & Ruhaimah, 2019). In addition, learners acquire critical and proportional thinking skills in mathematics when learning using problem-solving (Nasution et al 2019). The objectives are achieved when teaching through problem-solving (DBE, 2017). Enhancement in learners' problem-solving skills helps them focus on the given task and concentrate on the problem which enables them to think better about the process and solution (Da-Ponte Quaresma & Mata-Pereira, 2022). In addition, the enhancement of learners' planning skills helps learners in organising, analysing collecting, and recording vital information from the problems (Hodgen, Foster, Marks, & Brown, 2018). Similarly, enhancement when carrying out plan skills develops learners' calculation and procedural approaches. Likewise, enhancement in evaluating solution skills is consistent with interpreting the solution in the context of a fraction problem; self-evaluating the solution can be generalised (Hodgen et al., 2018).

2.6.3 Component of Problem-solving Teaching and Learning

This section discussed the roles of the following components when using the problem-solving teaching approach. These components are the role of the educator, questions, group learning, assessment and reflection in problem-solving teaching and learning.

2.6.3.1. The Educator’s Role

The educator in problem-solving classes assumes the role of facilitator (Kaur, 2022). S/he eases the learning process and possesses the responsibility to design questions and crucial tasks, ease the teaching using problem-solving, and effectively control the class (Kaur, 2022). The educator's content knowledge and professionalism are vital when facilitating lessons on fractions and when working with learners in a group (Hino & Funahashi, 2022). S/he assesses and evaluates learners’ prior knowledge and understanding of fractions. He identifies misconceptions and addresses them as early as possible (Brown, Skow, & IRIS Centre, 2020). The expertise of the facilitator should be experienced in guiding learners to devise a good approach toward learning and thinking (Mesiti et al., 2021). Fractions are abstract concepts, so it is essential to use concrete objects like fraction bars, circles, and pizzas to help learners understand the fraction concepts. The educator should create an enjoyable environment and make the learning process more engaging and exciting for learners (Mesiti et al., 2021).

Educators' differential instructions and mentoring skills are required to facilitate problem-solving (Hmelo-Silver, 2004). Not all learners learn in the same way or at the same pace. Educators should provide differentiated instructions, such as group work, to meet the individual needs of all learners. Educators should design systematic patterns to help them stop giving support to learners on observing that learners are now proficient in problem-solving learning (Chan, Clarke, & Cao, 2018). They can focus attention on other duties like listening to learners' discussions, ideas and reasoning when solving problems, encouraging learners' engagement by providing opportunities for learners to work collaboratively, share their ideas, and ask questions. This helps learners develop better deeper understanding of the concepts and enhances their problem-solving and communication skills.

The educator's role as a facilitator is important in assisting the learners to understand the problems, devise a plan, carry out the plan and evaluate their solutions, and assist them in harnessing problem-solving skills, by providing scaffolding, questioning, and brainstorming during fractions lessons through problem-solving (Nissim, 2020). Educators should provide feedback to help learners understand their strengths and areas for improvement. This will help learners monitor their progress and make necessary adjustments to their learning strategies (Quigley & Hero, 2019).

2.6.3.2 The Role of Questions

Problem-solving questions afford educators opportunities to recognise the valuable properties of questions (Nissim, 2020). According to Nissim (2020), for questions to meet the standard criteria that enable learners to think in different dimensions and be self-motivated, the questions need to be real to learners, open-ended and not simple to answer. Well-formulated questions enable learners to analyse, evaluate, and apply correctly both their previous and new knowledge and develop their reasoning faculties toward their learning approaches (Chan, Clarke, & Cao, 2018). Problem-solving questions enable learners to formulate hypotheses and arouse logical and critical thinking. In addition, questions are asked in a way that raises learners' expectations toward increasing learners' learning abilities and promotes active participants' involvement (Schwartz, 2015).

2.6.3.3 Group Learning

Group learning is an essential component in problem-solving classes (Demitra & Sarjoko, 2018). According to Khumalo (2018), questions that may look complex to an individual learner can be solved by other learners in group discussions. Group learning fosters learner-to-learner interaction increases learners' talking time (STT) and reduces the educator-talking time (TTT) (Novotna & Hospesova, 2022). It gives opportunities to brilliant learners as it affords them time to share knowledge with struggling learners (Sari et al., 2019). It is well-known that in group learning, learners seize the opportunity to express their understanding and challenges of the question; learners benefit through active involvement (Emanuelsson & Sahlström, 2022). Sometimes with the facilitator's help, the group can solve problems that each learner finds challenging to answer. Thus, the facilitator should ensure that every learner in the group has his own part to play during group learning (Vetter, Orr, O'Dwyer & O'Connor, 2020). Learners' collaboration during group learning produces new knowledge. It is essential to ensure that all learners participate actively during group work (Riyadi, 2021).

2.6.3.4 The Assessment

Brown and Koch (2020) defined assessment as a process of gathering, analysing, interpreting, and documenting data to measure learners' knowledge, skills, attitudes, and beliefs. Examining the definition, assessment is a continuous event to acknowledge the stage at which learners are in terms of learning and understanding. Assessment is not a single event, rather it is an integral part of learning and should not be at the level of evaluating learners' work at the end of the learning program, rather should be a continuous and daily process (Lin, Yin, & Wang, 2022). The purpose of assessment is to evaluate the learning outcomes and level of learners' abilities to solve problems and to evaluate problem-solving skills (Daulay & Ruhaimah, 2019). However, problem-solving assessment helps learners reflect on the decision concerning the effects of learning styles (Lin, Yin, & Wang, 2022). Further, it guides learners' learning and informs educators how well learners understand the lesson assessment (Naidoo & Hajaree, 2021).

2.6.3.5 The Role of Reflection

To formulate meaningful knowledge, there must be a reflection during and after both the teaching and learning process (Hmelo-Silver, 2004). Reflection helps both the learners and the educator to meditate. It helps learners to meditate on their learning styles and procedures in solving fraction problems and helps educators to meditate on their teaching approaches. (Amador, 2016).

Reflection influences how learners interpret knowledge. Therefore, problem-solving tasks should be considered an end to achieve the learning objective. Reflection can enable learners to connect newly gained knowledge to prior understanding. It helps learners develop critical thinking skills that further enhance the development of other problem-solving skills during learning (Clarke, Stephens, & Sullivan, 2021).

2.7 CHAPTER SUMMARY

This chapter discussed previous studies in the teaching and learning of fractions through problem-solving. It highlighted the importance of teaching and learning fractions through the problem-solving framework. It presented an overview of fractions and familiar challenges experienced by learners when solving fractions. In addition, it discussed five key reasons why learners find fractions to be difficult. The use of fractions models and teaching strategies was emphasised among other viable solutions that can help both educators and learners to overcome difficulties associated with teaching and learning of fractions. The problem–solving framework was further discussed as an effective teaching and learning approach that can help both educators and learners solve and understand fractions and mathematics concepts better in their various mathematics classes. Thus, teaching using Polya’s problem-solving teaching model affects learners' problem-solving skills and learning of fractions in Grade 8 in this study.

CHAPTER THREE

RESEARCH DESIGN

3.1 INTRODUCTION

This chapter discusses the overall research process and approach used in conducting the study, from research questions to the data collection process and data analysis. The methodology was executed based on the nature of the research questions and hypothesis. Verification and clarification of the research questions and hypothesis were based on the quasi-non-equivalent pre- and post-test design. This methodology corresponds with Polya's framework in nature by following a systematic process and evaluating the effects of problem-solving teaching on learning fractions. The design supports collection and data analysis and considers ethical issues like obtaining informed consent and protects confidentiality. Thus, the design process ensured the criticality of validity and reliability of the data findings, results from interpretations, and significant implications for verification of the theory.

3.2 RESEARCH PARADIGMS AND PHILOSOPHICAL ASSUMPTIONS

Kamal (2019) defines a research paradigm as a set of beliefs, assumptions, and methodologies that shape how researchers approach a particular field of study and a given research question (Kamal, 2019; Varna et al, 2023). It encompasses the theoretical framework, research design, methods, and techniques that guide the researcher's evaluation, investigation, and interpretation (Khupe & Keane, 2017). In addition, Khupe and Keane (2017) outline the following paradigms, including positivism, interpretivism, critical theory, and postmodernism. Positivism is a scientific approach that assumes that reality exists objectively and can be measured and observed by empirical research methods (Ryan, 2018). Interpretivists emphasise the importance of object experience and interpretation in understanding social phenomena (Gannon, Taheri, & Azer, 2021). Critical theory seeks to identify and challenge power imbalances in society, while postmodernism questions the idea of absolute truth and objective reality (Zyphur & Pierides, 2019). The paradigm choice irrespective of discipline, ideals and assumptions concerning the nature of knowledge depends on research questions, the nature of the phenomenon being studied, and the researcher's beliefs and values (Ryan, 2018). Each paradigm offers a unique perspective that enriches the research process (Ryan, 2018). Nonetheless, this study is associated with the positivist paradigm assumption because of its quantitative nature and research questions (Gannon, Taheri, & Azer, 2021).

3.2.1 The Positivism Philosophical Assumption

According to Ryan (2018), the positivist research paradigm focuses on empirical process observation. Thus, the positivist researcher understands the world using scientific approaches and ideas on numerical measurement. Emphasis is on empirical evidence; positivists reject speculative theories and subjective interpretations (Zyphur & Pierides, 2019). Zyphur and Pierides (2019) acknowledged that positivism is rooted in the idea that all knowledge is derived from sensory experience and scientific methods. Furthermore, scientific methods are the most reliable means of obtaining accurate knowledge (Hwang, 2019). Despite the criticism of the focus on empirical evidence, reductionism, and rejection of knowledge (Corry, Porter & McKenna, 2019), Shukla (2020) believes that positivist knowledge is significant and impacts on variety of findings, including psychology, sociology, and economics. The positivist paradigm has played a vital role in developing scientific disciplines, including natural sciences, and social sciences (Corry, Porter, & McKenna, 2019). Hwang (2019) believes in objective realities, that there are universal laws that govern the natural world meaning scientific methods are appropriate to obtain reliable knowledge and verify theories and hypotheses. Positivist beliefs improve society by providing solutions to practical problems (Ryan, 2018). The reasons for adopting the positivist paradigm in this study are explained below.

The positivist paradigm stresses objectivity and uses scientific methods to obtain reliable knowledge (Hwang, 2019). It ensures that findings are not influenced by personal biases and subjective interpretations, it places high value on empirical evidence obtained through observations and experimentation to verify hypotheses and theories systematically and rigorously (McMillian & Schumacher, 2014). In line with this perspective, this study collected tests and questionnaire numerical scores to verify the hypothesis. Shukla (2020) acknowledges the quantitative predictive power associated with the positivist paradigm. Therefore, it supports quantitative research methods as they allow the collection of numerical data and analysis using statistical techniques that enable the identification of relationships between variables. Positivists seek to identify universal laws of the natural world used to make predictions and develop practical solutions to problems (Hwang, 2019). Positivism also contributes to scientific progress by encouraging the development of new hypotheses and theories. This has led to vital important discoveries and advancements in various fields (Ryan, 2018). Therefore, the nature of this study is outlined with a positivist philosophical perspective by focusing on evaluating the effect of a problem-solving teaching approach on learning fractions in Grade 8. Problem-solving teaching was conducted in the experimental group. This conforms to the positivist belief that research claims are tested according to variables (Hwang, 2019). Therefore, this study developed lenses that focused on experimental and observation measurement of the objective reality that exists in teaching and learning of fractions in Grade 8 classes. Conclusively, positivist paradigm procedures are systematic; they start with hypothesis assumptions, followed by collecting data to test the hypothesis, analysing the findings to support or discard the hypothesis, and verifying the theories. Similarly, the study evaluates Polya's mathematical problem-solving theorem on learning fractions in Grade 8.

3.3 RESEARCH METHODOLOGY

A study's methodology is a systematically organised approach that includes research design, data collection process, analysis, and interpretation (Scotland, 2012; Shukla, 2020). The research approach focuses on answering the research question and hypothesis (Creswell, 2014; Shukla, 2020). Khoo-Lattimore, Mura & Yung, (2019) define the three research methodologies as the mixed method, qualitative and quantitative methodology. They further explained the mixed method as a process that involves the collection of both quantitative and qualitative data. The qualitative method approach explores the social beliefs of individuals and society. In addition, the approach focuses on understanding human social problems. It involves questioning and interviews while the quantitative approach stresses objectivities, testing hypotheses and verifications of theories by evaluating the relationship between variables, measured by instruments (Khoo et al., 2019). The analysis in this study used statistical tools (Myers, 2020). The objectiveness of the quantitative approach conforms with this study by numerical collection of data using test and questionnaire instruments through quasi non-equivalent experimental design research process.

3.3.1 Quasi Experimental Design

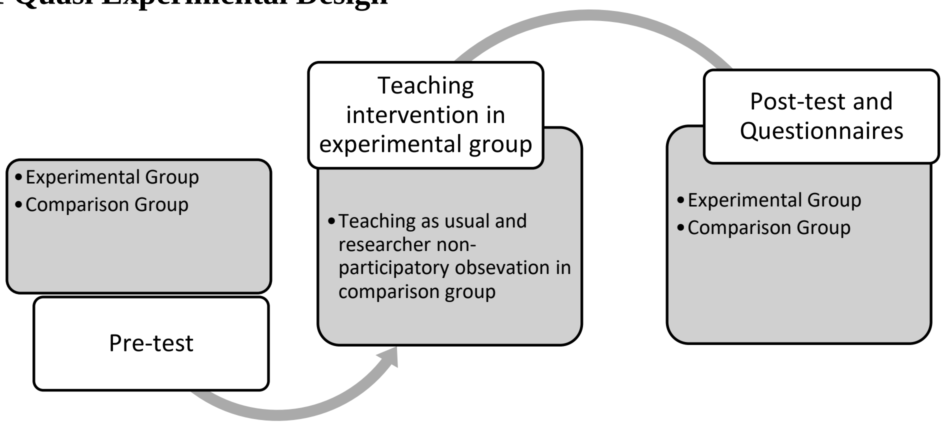


Figure 3.1: Quasi-experimental design approach (Lauren Thomas, 2020)

Table 3.1: Types of research approaches

Research Approach	Description	Purpose
Experimental design	It involves the manipulation of one variable to determine the effect of an outcome.	To establish cause-and-effect among variables
Quasi-experimental design	The design is like an experimental design but has no random assignment.	To establish cause and effect using non-random assigned.
Correlation design	It involves measuring the degree of association between two or more variables.	To establish a relationship between variables
Survey design	This design involves collecting data from large group questionnaires or interviews.	To obtain information about a particular population.
Case study design	This design involves an in-depth analysis of a small group of cases.	To gain an understanding of a particular phenomenon.
Longitudinal design	It involves collecting data from the same participants over an extended period.	To investigate changes in participants over time.
Cross section design	It involves collecting data from different groups of people at the same point in time.	To compare differences and similarities among groups.

Extracted from (Schumacher & McMillan, 2014)

The quasi-non-equivalent pre and post-test design is used to probe already assigned participants (Myers, 2020). According to Myers (2020), a quasi-non-equivalent design is experimental in nature. However, it lacks pure experimental control over the independent variable. Therefore, there is no random assigning of participants into groups. Hence, it uses pre-existing groups and makes comparisons between groups (Schumacher & McMillan, 2014). It is used to evaluate the effect of problem-solving teaching on learners' learning (Schumacher & McMillan, 2014). Its components are (I) non-equivalent groups (pre and post-test) (II) participants from six high schools that are in Grade 8 classes who have not received lessons on fractions using problem-solving (III) grouping participants into experimental and comparison groups (IV) Researcher administered pre-test in both groups, baseline assessment to evaluate the degree of non-equivalent understanding of fractions and problem-solving (V) eased intervention in experimental group using problem-solving and teaching in comparison group through traditional method (VI) evaluated intervention effects through post-test and questionnaire in each group (VI) analysed and discussed questionnaire and test findings using inferential and descriptive tools.

3.3.1.1 Why quasi-non-equivalent pre and post-test design

The quasi-non-equivalent pre and post-test design supports the use of intact, already assigned Grade 8 classes without randomly assigning participants to groups (MacMillan & Schumacher 2014). According to Malmqvist, Hellberg, Möllås, Rose, & Shevlin (2019), considering issues surrounding ethics, it is not feasible to randomly assign participants into groups (MacMillan & Schumacher, 2014). Furthermore, quasi-non-equivalent pre and post-test design enables the use of a convenience sample and assists in assigned comparison and experimental groups. The design supports the data collection using pre and post-test instruments which is in line with this study. The process starts with a pre-test instrument, followed by an intervention, and concludes with the administration of the post-test and questionnaire instrument. Design model (Cf Figure 3.2).

Experimental group	Pre-test	Intervention	Post-test
	O----------------X----------------O		
Comparison group	Pre-test	Observation	Post-test
	O----------------Y----------------O		

Figure 3.2: Non-equivalent-group pre and post-test Design

Figure 3.2 reveals (O) representing pre and post-test as similar instruments. Furthermore, reveals X and Y interventions and observations. The (X) in experimental represents intervention while (Y) in comparison represents confirmation of teaching observation.

3.4 RESEARCH APPROACH

The research approach focuses on the quasi-non-equivalent group pre-test and post-test design. It uses a convenient sampling method. Learner participants were from six schools in Mopani East and West District. A total of 205 Grade 8 learners from three schools in Mopani West were assigned to the comparison group and 201 Grade 8 learners from three schools in Mopani East were assigned to the experimental group. The two groups are in proximity of about 35km apart. The researcher eased intervention in the experimental group and observation to confirm teaching in the comparison group.

Questionnaires, pre- and post-tests were the data collection instruments. The questionnaire instrument was for triangulation purposes. According to Hwang (2019), triangulation of instruments provides reliability of the data instrument. Hence, triangulating the questionnaire instrument along with pre and post-test instruments enhances the study instrument's reliability. The post-test instrument collects findings per performance and achievement stance and the questionnaire instrument collects findings per understanding knowledge of fractions stance. For example, if the questionnaire indicates that participants have advanced levels of understanding knowledge on learning fractions, the post-test measured actual numerical scores achievement. Hence, if both show positive results, it strengthens the validity of the findings (Mertens & Hesse-Biber, 2012; Hwang, 2019). Thus, triangulation of the questionnaire is necessary in this study to increase validity, reliability and reduce bias, provide comprehensive credibility, and increase the integrity of the results (Malmqvist, et al., 2019). The Researcher administered the pre-test instrument before introducing problem-solving teaching in the experimental group because, the post-test and questionnaire instruments were administered after teaching of addition, subtraction, and equivalent fractions in each group. Below is the research model (Cf Figure 3.3) of the research approach and data collection process.

Experimental group	Pre-test	Intervention	Post-test	Questionnaire
	O-----------------X----------------O---------------------------Z			
Comparison group	Pre-test	Observation	Post-test	Questionnaire
	O-----------------Y----------------O---------------------------Z			

Figure 3.3: Research process model

Figure 3.3 shows (O) representing pre and post-test and Z representing the questionnaire instrument. It indicates pre-, post-test, and questionnaire are the same for both groups.

Furthermore, the figure used X and Y to represent intervention and observation. This indicates that the experimental group received intervention and observation of teaching confirmation in the comparison group.

3.4.1 Piloting

According to Malmqvist et al. (2019), piloting is a key step in research, it refers to the process of testing a research instrument on a small sample before it can be used in the main study. In addition, piloting provides the reliability of the research instrument by ensuring clarity, feasibility, and validity of the design. It builds confidence in the collection instrument before conducting the main study (Malmqvist et al., 2019). In this study, piloting was used to ensure that pre-, post-tests, and questionnaires were feasible, clear, valid, and reliable and that they would measure the intended purpose that they were designed to measure. For example, when testing for clarity and feasibility of the instruments, the researcher administered the instrument to a group of fifteen learners. However, the findings result showed that the instrument was not clear to learners because learners could not decode one-third and complete the test questions within an hour duration.

Furthermore, the test instrument comprised four questions only on addition and subtraction fractions. Hence, did not measure equivalent and all content knowledge of fractions in Grade 8 fractions (Sari et al., 2019). Thus, the findings enabled the researcher to identify confusing spelling, ambiguous questions, and instructions and acknowledged that excess time had to be allocated to the initial instrument. Furthermore, learners did not understand one-third as $\frac{1}{3}$ and complete the test and questionnaire before the duration. Moreover, the questionnaire had open-ended questions thus, learners did not understand and did not complete all sections. Hence, the piloting helped the researcher to re-adjust the time to 50 minutes. The researcher corrected spellings and ambiguous words by rewriting one-third as $\frac{1}{3}$. In addition, all feasibility issues were adjusted, and after the piloting, the researcher discovered that learners did not complete all sections and a prequestionnaire was not needed since learners had not experienced teaching.

Furthermore, another instrument covered the equivalent, addition, and subtraction of fractions with five questions and one questionnaire. To further ensure the testing of the validity and reliability of the instrument, these instruments were administered to the same learners and further administered in another school with thirty-four learners. These 34 learners were not part of this study. Their responses reveal that the instrument is feasible, and error-free, and time allocation is reasonable. This was consistent with the time taken to complete the questions in the main study. Thus, this second instrument was adopted from past Grade 8 mathematics questions and textbooks available to schools for teaching fractions. Therefore, the researcher-built confidence in the instruments, methods, and design. It helped save time and resources and reduce mistake, risks, and issues by testing the instrument on a small scale before investing time and resources into the main study.

3.4.2 Data Collection Instruments

The data collection instruments were the questionnaire, pre- and post-tests. The researcher ensured that the instruments addressed all issues relating to ethics. Moreover, the researcher had an overview of the number of learners, school locations, educators, and teaching approaches. This information enabled the researcher to make informed decisions. The purpose of collecting this information was for hypothesis verification and responding to research questions. Therefore, the systematic process of data collection process used led to accurate hypothesis testing and research question responses (McMillan & Schumacher, 2014)

3.4.2.1 Pre-test and Post-test

McMillan and Schumacher (2014) define a test as an instrument that demands participants to complete a cognitive task by responding to a standard set of questions. The pre- and post-test **(O)** are type-out standardising achievement tests with questions on equivalent, addition, and subtraction of fractions (Cf Appendix G). The test scores were over twenty-five and converted to a hundred. The duration was 50 minutes. The test focused on Grade 8 fractions. The post-test data were collected after the problem-solving teaching in the experimental and confirmation of teaching in the comparison group. The test evaluated learners’ achievement and performance scores on solving fractions. The post-test findings were used to evaluate the effects of problem-solving teaching on learning fractions in the experimental group. The result analysis was based on Polya’s framework and marks were recorded based on correct answers (Ca) incomplete answers (Ica) incorrect answers (Inca) and Empty answers (Ea).

3.4.2.2 Questionnaire

A closed-ended questionnaire was administered to all learner participants. However, an open-ended questionnaire was administered to educators to assist them in reflecting on their teaching and problem-solving (Cf Appendix J). The learner questionnaire consists of ten questions divided into sub-sections A B, C, D, and E with each section comprising two questions (Cf Appendix H). It evaluates learners' understanding of knowledge and problem-solving skills acquired while learning fractions content. The comparison group educators completed a questionnaire. There was no need for educators in experimental schools to complete a questionnaire since they did not take part in the study. However, they assisted by introducing the facilitator to their classrooms and shared their teaching timetable with the researcher.

3.5 POPULATION SITE AND SAMPLE

This study was conducted in the Mopani District that comprised of approximately 250 secondary schools. The district had schools classified as the Mopani East and West, where each classification had approximately half of the total number of the whole district. The two classifications were identified in this study as comparison (Mopani West) and experimental groups (Mopani East).

Table 3.2: Sample (Comparison and Experimental School)

Comparison Grouped schools					**Experimental Grouped schools**			
School Code	class	Learner	Educator		School code	Class	Learners	educators
CGS1	1	69	1		EGS1	1	52	0
CGS2	1	76	1		EGS2	1	79	0
CGS3	1	66	1		EGS3	1	73	0
TOTAL	**N=3**	**N= 211**	**N = 3**		**TOTAL**	**N=3**	**N=204**	

Table 3.2 indicates the overall number of sampled participants in this study. Six schools were conveniently sampled on the number of learners, location, and accessibility to resources like time and distance proximity (Shukla, 2020 Etikan, Musa, & Akassmi, 2016). Three schools were assigned to the experimental group in Mopani East and three schools were assigned to the comparison group in Mopani West. The two groups were approximately 38km apart Shukla (2020) defines convenience sampling as opportunity sampling which easily accesses groups or

participants used in the study. Thus, the convenience sampling method is appropriate in this study because, it enables the collection of data from schools to be easily accessible and available to the researcher (Etikan, Musa, & Akassim, 2016). In addition, it enables the researcher to choose participants from available schools willing to participate in the study. Moreover, due to ethical considerations, it is not feasible to sample schools that are not viable and ready to participate in the study (Dougherty, 2021). The comparison group is denoted by the acronym CGS1-3. The CGS1 group comprised of 3 schools, that is, schools 1-3 (with 69, 76, 66 learners respectively), a total of 211 learners. The number of educators at comparison schools was three. Similarly, EGS1-3 represents experimental group schools 1-3 (with 52, 79, 73 learners respectively), a total of 204 learners. The educators that assisted for control purpose in the experimental group were three educators. The total of learners sampled was 415, however, only 363 learners completed the course of research, that is, they participated in pre-test, attended the lessons, and wrote the post-test. In the CGS group 188 learners completed the cycle and in the EGS group 175 learners completed the cycle. Therefore, results are based on 363 learners. The educator participants sampled included comparison group educators; they taught fractions using the traditional approach. However, the experimental group educators did not take part in teaching but introduced the me as the intervention facilitator to the respective experimental group classes.

3.6 RESEARCH PROCESS

The process comprises activities in comparison and experimental groups. The research process began with the researcher's visitation to schools and administration of ethical documents and data instruments in comparison schools. During the first visit to schools, the researcher was given permission to conduct research and administer consent letters to the educators and parents, and assent letters to learners. During the second visit, the researcher administered the pre-test to all learners. During the third visit, he observed learning and confirmed that the teaching of Fractions took place in the comparison group. During the fourth visit, he administered the post-test and questionnaire.

The same process was conducted in the experimental schools. However, after the administration of the pre-test, comparison learners continued learning through the traditional approach, with their educators facilitating lessons as normal. Learners in the experimental group were engaged in learning through the problem-solving process, where I facilitated lessons using the problem-solving stages to engage learners in learning Fractions. The post-test and questionnaire were administered after learning observation and confirmation of teaching using traditional teaching and ensuring learners learnt all fraction concepts content before being allowed to write the post-test and questionnaires in the comparison schools.

3.6.1 Data Collection and Observation Procedure (Comparison Group)

The comparison schools CGS1-3 are in the same circuit. Tests and questionnaires were administered during the mathematics period duration of 50 minutes. The school's mathematics period was an hour hence learners completed the instrument within the period. Below is the summary of the research process in CGS1-3.

Table 3.3: Data collection program (Comparison group)

Day 1	Activities
Introduction and request for approval from the school's management. Request for educators' consent. Planned with educators. Checked timetables to mark mathematics periods and make alternative arrangements with other educators where necessary. The educator's role in the comparison group was to teach and control the class while the researcher only observed lessons as a non-participant observer in all lessons.	
Day2	Activities
Class visit. Self-introduction to the learners. Explanation of the study rationale, ethical issues, and their rights as volunteer participants. Distribution of consent and assent forms.	
Day 3	Activities
Collection of signed consent and assent forms. Administration of pre-test.	
Lesson	Observation
L1	Definition, forms, and conversion into decimal and equivalent.
L2	Addition and subtraction of proper and improper fractions.
L3	Addition and subtraction of improper and fractions.
Day 7	Administration of the post-test and questionnaires by the researcher.

3.6.1.2 Pre-test Data Collection Process (Comparison Group)

The researcher administered the pre-test in the comparison schools. However, due to the proximity of schools, the researcher could not administer the pre-test in all three schools. However, two schools were covered in one day and the third school was covered on another day. Learners sat 15 minutes before the starting time. Programs were held by the school and educator while the researcher invigilated the pre-test in all comparison schools. Furthermore, all question booklets and writing material during the process were made available by the researcher. The researcher ensured answer booklets were collected from learners as soon as they were done writing. Furthermore, the researcher ensured that booklets were not exposed to educators since the same questions would be administered as the post-test. Learners wrote their real names and surnames for easy identification. Names were coded in the booklet, for example, a learner at School 1 could be coded as CGS1.1 in the answer booklet for the pre-test (Cf Table 3.4).

Table 3.4: Pre-test Learner Participants (Comparison Group)

School code	Number of Learners	Number Absent	Number wrote
CGS1	69	2	67
CGS2	76	4	72
CGS3	66	0	66
TOTAL	211	6	205

3.6.1.3 Learning observation and teaching confirmation in comparison group schools

The researcher was involved in non-participatory observation in comparison classes. Observation was necessary for teaching and learning confirmation in comparison schools and learners were taught through the traditional approach. Thus, the researcher moved beyond the opinion-based findings obtained and used questionnaires to discover the possible reasons for the learners' performance in the given test.

3.6.1.4 Post-test and Questionnaire Procedure (Comparison Group)

The same process in the pre-test was used in administering the post-test and questionnaires. However, an additional 10 minutes were added to enable learners to complete the questionnaires.

Table 3.5: Post-test learner participants (Comparison group)

School code	Number wrote pre-	Number absents	Number wrote
CGS1	67	5	62
CGS2	72	6	66
CGS3	66	6	60
TOTAL	205	17	188

N=205 learners wrote the pre-test and N=188 wrote the post-test and questionnaire. Seventeen (17) attrition because of the missing variable. SPSS table system generated N=188 in comparisons. Thus, 188 learners completed the pre-test, post-test, and questionnaire scores were analysed.

3.6.2 Data Collection and Intervention Procedure (Experimental Group)

The experimental three schools are in the same circuit. The pre-test was administered in all schools EGS1-3. Table 3.7 presents a summary of the research process and activities.

Table 3.6: Data Collection Program (Experimental Schools Group)

Day 1	Activities
Introduction and request for approval from the school's management. Request for educators' consent. Plan with educators. Checked timetables to mark mathematics periods and made alternate arrangements with other educators where necessary. The educators' role in the experimental schools was to introduce the researcher to the class.	
Day2	Activities
Class visit. Self-introduction to the learners. Explanation of the study rationale, ethical issues, and their rights as volunteer participants. Distribution of consent and assent forms.	
Day 3	Activities
Collection of signed consent and assent forms. Administration of pre-test.	
Lesson	Facilitating the lessons on Fractions (3 lessons)
L1	Definition, forms, and conversion into decimal and equivalent.
L2	Addition and subtraction of proper and improper fractions.
L3	Addition and subtraction of improper fractions.
Day 7	Administration of the post-test and questionnaires by the researcher.

3.6.2.1 Pre-test Data Collection Process (Experimental Group)

The pre-test process is the same as the comparison group process in Section 3.7.1.2.

Table 3.7: Pre-test Learner Participants (Experimental Group)

School code	Number of Learners	Number Absent	Number wrote
EGS1	52	0	52
EGS2	79	3	76
EGS3	73	0	73
TOTAL	204	3	201

3.6.2.2 Intervention Process

The researcher implemented the intervention, teaching fractions through Polya's problem-solving framework by assuming the role of facilitator in all classes. He eased learning using problem-solving. However, he assumed the facilitator's role because educators in the experimental groups were not exposed to problem-solving teaching and did not opt for a training workshop because of their class teaching commitments. Thus, EGS2 and EGS3 declined because of their teaching commitments with other classes but, EGS1 accepted the training workshop invite. However, to maintain consistency in experimental classes, the researcher facilitated problem-solving teaching in experimental schools. He maintained consistency by using the same-prepared lessons throughout the schools.

The process began with an introduction, learner discussions and creating an encouraging environment to motivate interaction lessons. During learner-to-learner discussion group work, the researcher probed learners' understanding by questioning to clarify learners' better understanding of the concepts before proceeding to the next learning phase. Learners were challenged to create an encouraging environment that would allow full learner participation (Hmelo-Silver, 2004). Polya's problem-solving framework addresses the following in learning sessions.

1. Understanding the problem.
2. Devising a plan.
3. Carrying out the plan.
4. Evaluating the solution.

Learner-led groups were employed throughout sessions. The facilitator eased each (A, B, C, D, and E) for ten minutes probing learners' knowledge by questioning and demonstration and assigned exercises to each group to discuss and go to the board and demonstrate and explain solutions in front of the other groups. The researcher encouraged learners to critique each solution and suggest alternative solutions for solving the problem. The facilitator managed and guided all group discussions and posed all relevant questions by ensuring learners understood the problems before proceeding to the next level. There was no formal assessment during the intervention session, however, individual and group work was given.

3.6.2.2.1 Implementing Problem-solving Strategy

Polya's four-step problem-solving framework, which was implemented during the process is as follows:

1. Understanding the problem.
2. Devising a plan.

3. Carrying out the plan.
4. Evaluating the solution.

The above steps eased the learning of fractions. In each session, learners solved addition, subtraction, and equivalent fractions, and converted fractions to equivalent proper, improper, and mixed fractions.

Table 3.8: Polya's Problem-solving Phases

Problem-solving: Phase I	
Strategy	**UNDERSTAND THE PROBLEM**
Reading	The facilitator encouraged learners to read to comprehend the problem before solving it. He probed learning understanding by questioning learners to identify types of fractions and asked learners the following questions: Can you write the equivalent fractions in decimal and percentage? Do you understand what the question is about? In addition, identify operations to be performed. Can you re-state the problem in your own words? Can you determine what is known about these types of problems? Is there missing information that, if known, would allow you to solve the problem? After ensuring that the learners understand the first session, the facilitator proceeded to the next session of devising a plan.
Problem-solving Phase II	
Strategy	**DEVISE A PLAN**
Demonstration	The facilitator demonstrated to learners the use of manipulation for solving fractions. For example, simplification of fractions using paper folding and converting decimals and percentages using iron length. In addition, the facilitator demonstrated the conversion of fractions from improper to mixed fractions using paper folding and length model. Bars and circles were drawn to assist them with adding, subtracting, dividing, and multiplying fractions. The facilitator eased learner strategies in devising a plan by encouraging learners to make a list of known information. For example, drawing a model, making a list that shows possibilities, looking for a pattern, writing necessary equations, and guessing a solution. After confirming that learners had devised a plan to solve fractions, the facilitator proceeded to the third phase. The

	facilitator encouraged learners to stay motivated because most initial plans do not yield desired results. He encouraged learners to complete each step before moving to the next.
Problem-solving Phase III	
Strategy	**CARRY OUT THE PLAN**
Explanation	Learner-led group work was employed throughout the sessions in Groups A, B, C, D, and E. The facilitator eased each group in 10minutes probing their process. He checked individual learners' work and group exercises. After the discussion, each group demonstrated their group work on the chalkboard. The facilitator encouraged learners to critique each other's solutions and suggest alternative procedures and solutions. The facilitator managed and guided group learning and asked relevant questions. There was no formal assessment in the intervention sessions. However, individual tasks and group work were given. Learners were encouraged to work carefully and keep accurate and neat records of all attempts and be ready to modify the plan and work backwards, solve similar, simpler fraction problem, and check if the solution conformed to the context. On completing the carry out plan phase, the facilitator proceeded to evaluate the solution.
Problem-solving Phase IV	
Strategy	**REVIEW THE SOLUTION**
Conceptualise	The facilitator ensured that learners' solutions were consistent with the facts of the problem. He interpreted the solution in the context of the problem and checked for over-generalisation solutions that could apply to other problems.

3.6.2.2.2 Implementation of Lesson

The lesson's objective was to evaluate and understand the problem, devise a plan, carry out the plan, and evaluate solution skills and learners' knowledge acquired on fractions.

Table 3.9: Lesson Plan and Notes

	Activities
Day 1	Lesson 1: Definition, forms, and conversion into decimal and equivalent

Activities	This session does not involve a problem-solving strategy hence the facilitator directs subjects to their mathematics textbook and workbook to read and understand the rules used in converting from decimal to a percentage, the types of fractions, and the definition of fractions.
Day 2	**Activities**
L2	Addition and subtraction of proper and improper fractions
activity	Calculate the percentage increase if the price of a bus ticket of R$\frac{1}{2}$ is increased to R$\frac{3}{4}$.
	Protocol Day 2
Phase 1	**Understand the Problem**
Activity	The researcher probes the following: What type of fractions are R$\frac{1}{2}$ and R$\frac{3}{4}$? Can you convert them to decimals? Which fraction is the biggest? Do you know the meaning of percentage? Have you used a bus ticket before?
Phase 2	**Devise a plan**
Activity	Convert proper fractions into decimals, draw a model, and try addition and subtraction operations. Modify the plan and determine the increase difference in descending order by subtracting the smaller value from the bigger value.
Phase 3	**Carry out the plan**
Activity	Convert into decimal fractions R0.5 and R0.75. Subtract to determine the difference. R0.5 - R0.75 = R-0.25: Convert the value into a percentage. -0.25 x 100 = -25%, **Modify plan:** Subtract to determine the difference. R0.75 - R0.5 = R0.25, Convert the value into a percentage. 0.25 x 100 = 25%.
Phase 4	**Review the solution**
Activity	Inspect the solution in the context of the problem. The problem is based on money hence the value of money cannot be negative, go back and modify the plan because the plan does not interpret the solution in the context of the problem. The solution is in the context of the problem. Yes, it makes sense that the solution must be positive as money has positive value only in real-world applications.
Day 3	Lesson 3: Addition and subtraction of improper fractions
Activity1	Mr. Xolo a sales representative promised to discount Bongani R$\frac{1}{2}$ if she bought a shirt sold for R$\frac{6}{5}$ and shoes sold at R$\frac{9}{10}$. What is the selling price for both the shirt and shoes, and what is Bongani's cost price if she buys both the shirt and shoes?
Activity2	Lesson 3: Addition and subtraction of mixed fractions
	Mrs Joy has $5\frac{1}{8}$mm candy, gave $2\frac{17}{24}$mm to Bethel, but Bethel says $2\frac{17}{24}$cm is too much and returns $\frac{7}{6}$cm to Mrs Joy. Adam approached Mrs Joy and demanded $2\frac{11}{12}$cm. How much candy did Mrs Joy have left after sharing with, (a) Bethel, and (b) Adam?
Phase 1	**Understand the problem**
Activity	Understand the fractions as proper and improper fractions, and understand the words discounts, selling price, and cost price. Identify words not needed to solve the problem.

Phase 2	Devise a plan
Activity	Convert sales prices to decimal: shirt R.1.2 and shoes R0.9. Add to get the total sales price. Convert the discount value into decimal: R0.5. Bongani cost price = total sales price, subtract discount value.
Phase 3	**Carry out the plan**
Activity	Convert into decimal fractions. R1.2 and R0.9. Add to determine the total sales price: $\frac{6}{5}+\frac{9}{10}=\frac{21}{10}$ decimal R1.2 + R0.9 = R2.1. Solution (a) total sales price =R $\frac{21}{10}$ = R2.1 decimal. Solution (b): Bongani cost price = sales price minus discount. $\frac{21}{10}-\frac{1}{2}=\frac{8}{5}$. Decimal = R2.1 - R0.5 = R1.6
Phase 4	Review the solution
Activity	Review the solution. The solution is in the context of the problem. Yes, it makes sense that the solution must be positive as money has positive value only in real-world applications.

By the end of the intervention, the facilitator ensured that the learners enhanced and demonstrated the following problem-solving skills.

- Understanding of problem skills
- Devising of plan skills
- Carrying out of plan skills
- Evaluating of solution skills

Problem understanding skills. Learners demonstrated an understanding of problem skills by reading, decoding meaning, and restating questions before performing operations.

Devising of plan skills. Learners demonstrated enhancement by identifying fractions' order, creating details, and organising course of action like adding or subtracting to achieve the plan.

Carrying out of the plan skills. Learners demonstrated enhancement in carrying out plans by calculating and integrating using procedure techniques required in solving the problem.

Evaluating of solution skills. Learners enhance their knowledge by reflecting on their solving before and after finding the solution and checking the solution for consistency with the plan. Furthermore, learners had the opportunity to draw a conclusion and indicate that the problem had been solved.

3.6.3 Fidelity of the Intervention

According to Sandra and Siedlecki (2021), fidelity is defined as the extent to which the intervention is delivered. Thus, fidelity intervention is based on five components which are intervention design; provider's training; intervention delivery; receipt of intervention and enactment of skills acquired from the intervention. Intervention delivery pertains to methods used to ensure the standardisation of lessons. The facilitator adhered to and used a common schedule throughout lessons in experimental schools. In addition, the same duration and consistency of facilitation in lessons were adhered to in all classes. The learning focuses on Grade 8 fractions content. Furthermore, receipt of intervention accounted for in this study since participants in the intervention group acquired knowledge in various sessions and skills acquired applied in the next session. For example, learners solved equivalent fractions and converted them to decimals and percentages. Assessment recorded improvement. In addition, learners in the experimental group expressed positive views toward the intervention as an appropriate learning strategy that effected changes in everyday learning. Intervention design is relevant in the study since intervention sessions were in experimental schools and not in comparison schools. In addition, an intervention was conducted on Polya's problem-solving framework. In addition, Polya's problem-solving framework was probed as the major constituent problem-solving intervention with all its four levels. Similarly, training on Polya's framework was avoided to avoid inconsistencies. For this reason, the researcher became the facilitator.

3.6.4 Experimental Post-test Process

The same process in the pre-test administration was followed. The purpose of conducting a post-test was to evaluate the effects of the intervention. The evaluation of the intervention was feasible when improvement in the post-test was tracked against the pre-test because conducting a pre-test only would not have been useful.

School code	Number wrote pre-	Number absent	Attrition rate	Number wrote

EGS1	52	3	0	49
EGS2	76	8	6	62
EGS3	73	7	2	64
TOTAL	201	18	9	175

Table 3.10: Pre-test population (Experimental group schools)

N=201 wrote the pre-test. However, N=175 wrote the post-test and questionnaire. Due to learners missing variables, the SPSS inferential system generated N=175 learners completed the pre-, post-test and questionnaire in experimental schools. The analysis is therefore based on the 175 learners who participated.

3.6.5 Overall Questionnaires Pre-test and Post-test Summary

The study starts with N=221 learners, N= 33 was absent and N= 188 completed the pre-test, post-test, and questionnaire in comparison schools. No record of attrition because no subject was out of the study due to any reason. Thus, N= 188 variables were recorded. While experimental learners start with N= 204, however, N= 21 absent and N= 8 attritions, that is, learners who willingly withdrew from the study. N= 175 completed all pre-, post-test, and questionnaires. The attrition rate contributes to the reduction of participants N= 415 to 363 as indicated in the study population and sample summary (Table 3.11).

Table 3.11: Overall comparison group and experimental group data collection

Comparison Group Participant					**Experimental Group Participant**				
Code	**Subjects**	**Absent**	**Attrition**	**Wrote**	**Code**	**Subjects**	**Absent**	**Attrition**	**Wrote**
CGS1	**79**	**17**	**0**	**62**	**EGS1**	**52**	**3**	**0**	**49**
CGS2	**76**	**10**	**0**	**66**	**EGS2**	**79**	**11**	**6**	**62**
CGS3	**66**	**6**	**0**	**60**	**EGS3**	**73**	**7**	**2**	**64**
TOTAL	**221**	**33**	**0**	**188**	**TOTAL**	**204**	**21**	**8**	**175**

The following responses and marks were informed through categories responses: Correct answer (Ca), Incomplete answer (Ica), Incorrect answer (Inca), and Empty answer (Ea). Thus, to conform to the scale of zero scores, variables were treated as empty answers. Total number sample of participating learners was N= 363.

3.7 DATA ANALYSIS

According to Shukla (2020), data analysis refers to ways in which data is collected and modelled to support decisions. Thus, the pre-test, post-test, and questionnaire data scores findings were marked and interpreted to represent learners' cognitive achievement and performance. The analysis focused on inferential statistics and descriptive analysis. Thus, hypothesis verifications through the SPSS deduced statistically significant p-value and descriptive statistics were used to answer achieve data analysis. Furthermore, the descriptive analysis evaluated learners' knowledge, understanding, and problem-solving skills acquired. The paired sample t-test was used to analyse learners' average performances. Through the SPSS, the Kruskal-Wallis test was used to analyse the performance of learners in learning fractions in each group and the Wilcoxon Signed-Rank test was used to analyse the effect of learners' performance on learning using problem-solving and traditional teaching approaches. Furthermore, the questionnaire's yes/agree, and no/disagree responses were analysed using the dependent ANOVA. In addition, questionnaire descriptive analysis was used to analyse learners' understanding and knowledge of learning fractions.

3.8 RELIABILITY AND VALIDITY OF THE STUDY INSTRUMENT

3.8.1 Reliability of the Instrument

The reliability of the research instrument is vital to the research study. According to Surucu (2020), reliability assures that results are consistent, dependable, trustworthy, error-free, and not forged. It increases confidence that the result obtained is accurate and should be used for valid conclusions (Grgic, Lazanica, Schoenfeld & Pedisic, 2020). Thus, reliability is the consistency of the instruments, design, and procedure. The crucial questions that required reliability are 'Does the test measure aspects that are central to learning and teaching fractions? Are all concepts of fractions involving addition, subtraction and equivalents of fractions taught?

The researcher ensures reliability by using multiple data collection instruments like test and re-test methods (pre and post-test). Test and retest ensure the reliability and consistency of instruments (Grgic et al., 2020). This method ensures consistency of the test instrument by administering the same test as pre-, and post-tests to the same learners at different times to measure consistency of the results. Furthermore, the researcher conducted piloting on the instrument before using it in the main study. Piloting is a key step to ensure an instrument's reliability. According to Malmqvist et al., (2019), piloting refers to the process of testing an instrument on a small sample before it is used in the main study. Thus, this study used a pilot to ensure clarity, feasibility, and reliability of the questionnaire, and pre-, and post-test instruments. Thus, it builds confidence in the study design (Cf Section 3.4.1).

Hwang (2019) acknowledges that appropriate and well-established instruments ensure internal reliability. Thus, an instrument validated in previous studies is appropriate to ensure reliability because such an instrument would have undergone rigorous testing and would have been proven to produce consistent and reliable results. The researchers adopted the test instruments from Grade 8 previous assessment questions and mathematics textbooks and the questionnaires from previous studies (Appendices I and K). Furthermore, internal reliability was ensured in each group by administering the same test as pre- and post-test without the participants' prior knowledge. Thus, scores obtained in pre-, and post-test were correlated. In addition, administering the same standardised achievement test in Grade 8 fractions content learning areas to both groups of learners ensured stability and reliability. Furthermore, using multiple instruments to score and rate the same behaviour increases reliability (Surucu, 2020). The researcher ensured inter-ratter reliability by using both questionnaires and test instruments to ensure the teaching and learning of fractions covered in each group involving addition, subtraction, and equivalence of fractions understood by learners on learning fractions. In addition, Hwang (2019) acknowledges that statistical tests can be applied to measure the internal consistency of the instrument's reliability. It assesses the degree to which an item on the instrument measures the same construct. Cronbach's alpha statistical and Spearman-Brown's statistical tests were used to ensure the reliability of the test instrument. Cronbach's alpha verified the questionnaire's reliability and Spearman-Brown's statistical test verified the reliability of the pre- and post-test. The Cronbach's alpha questionnaire statistical value of 0.87 and Spearman-Brown's pre and post-test value of 0.78 are substantial evidence that these instruments are reliable and consistent.

3.8.2 Validity of the Instrument

Surucu (2020) explains that the main validity components of a study focus on cognitive tests. Thus, learners' responses to cognitive tests and questionnaires were summarised and converted to obtain a numerical data value that befits quantitative properties and interpretation using criterion standards. The standardised achievement tests were used to evaluate learners' problem-solving skills and performance achievement in learning fractions (Surucu, 2020). In addition, validity, as the extent to which instruments are accurately measured is explained as appropriate judgment used in measuring specific decisions that arise with scores generated by the instrument (McMillan & Schumacher, 2014). Therefore, validity is established by presenting evidence that the inferences are appropriate, and quantitative validity refers to the standard of findings obtained from data collection instruments and conclusions drawn from the analysis (Creswell & Plano, 2016).

Quantitative validity types are external, content, criterion-related, and construct validity. According to Surucu (2020), external validity refers to the extent to which results and findings can be generalised to larger populations. It is considered a threat because the study used only six Quintile 1 schools. This means that the study focused on small schools with little resources located in rural Limpopo province. Hence, comparing these schools to other schools in townships or the number of participating learners to Grade 8 populations in South African schools could not be valid. Thus, generalisation to a larger population may be done with caution. However, the threat was reduced because the study adopted a larger sample population of three hundred and sixty-three (363) sampled learner participants from six government secondary schools. Thus, generalisation can be to an extent, especially with schools having similar conditions that obtain in this study.

According to Grgic et al., (2020), content validity means accepting data collection instruments by ensuring the instruments cover all the content learning areas. This study's teaching and learning content areas are equivalence, addition, and subtraction. This is considered a threat since the instrument does not cover all learning areas like division and multiplications of fractions. However, this threat was reduced because the instruments collected data on equivalence, addition, and subtraction of Fractions content only that the learners were taught. According to Shukla (2020), when gathering evidence on test content, expert(s) should be invited to judge the criticality contents of the instruments and evaluate the important parts of the instrument. Experts should judge the content according to research-based, accurate representation, and adequate representation of the framework. The researcher ensured that content did not pose a threat in this study by ensuring the instrument's contents were validated by an expert in mathematics education, to critique the instrument and review its contents to ascertain its adequacy to measure fractions knowledge based on Polya's problem-solving framework. Furthermore, content validity was ensured in the experimental group by ensuring that all fractions' contents and concepts were facilitated, and all learners were present during lessons and learned fractions concepts using Polya's problem-solving framework.

Similarly, content validity was ensured by ensuring that learners did not know that the same pre-test would be administered as the post-test. In addition, content validity was ensured in the comparison group by non-participatory teaching observation. The observation was necessary for confirmation that educators taught all aspects of the fractions content, learners were present, learnt, and learners did not know that the same test would be administered as a post-test before writing the post-test. Thus, these ensured that the threat to content validity was eliminated from this study. McMillan and Schumacher (2014) explained criterion-related validity to whether the scores are related to external standards. This includes the correlation of experimental and comparison scores using the same instrument. Thus, the threat was minimised in this study. In addition, criterion-related validity does not pose a threat in the study as instrument criterion is standard set-in Grade 8 learner mathematics textbooks and from past Grade 8 questions, and marking was done and categorised according to the learners' responses.

Similarly, construct validity was ensured. Surucu (2020) acknowledges that constructive validity revolves around the data collection process. Hence, the construct involves searching for evidence that the instrument accurately measures an abstract trait. This was not a threat to this study because teaching using different methods and approaches was used to strengthen learners' knowledge of fractions. For example, during interventions, learners used the problem-solving learning approach in experimental schools and the traditional learning approach in comparison schools. In addition, triangulation of the questionnaire responses validates the test confirming that the learners' knowledge, understanding, achievements and performances correspond with knowledge and understanding skills (Purwadi, et al., 2019). Furthermore, the study ensured internal validity. According to Ndayambaje and Ralph (2020), internal validity is the extent to which the researcher can conclude that there is a cause-and-effect relationship among variables. This was a threat to this study since the adoption of an experimental group is a threat to internal factors because of the comparison group (Ndayambaje & Ralph, 2020). However, the research design, the quasi-non-equivalent group's pre- and post-test design helps to ensure that the groups are comparable at the onset of the study. This reduced the possibility that differences between comparison and experimental group are due to pre-existing differences among participants. In addition, the pilot test helped to ensure that the instruments used were reliable and valid, error-free, and accurately reflected the constructs it intended to measure. Furthermore, a large sample size of three hundred and sixty-three (363) used, confirms that the effects are not due to chances of random variability, therefore, increasing generalisation of findings and reducing the threat to internal validity by eliminating extraneous variables like bias using blind procedures. Hafen and Köhler (2017) refer to the blind procedure as a procedure when participants are not aware of certain information about the study that could bias their responses. Hence, the following blind procedure was used to ensure internal validity: (I) ensured that both groups of learners did not know that the same pre-test would be used as the post-test; (II) the comparison and experimental groups were in different districts, 35km apart, and schools were further apart from each other; (III) the researcher was accompanied to the classes by experimental educators to confirm that the experimental facilitator was not biased or expressed conflict of interest in his group. Furthermore, since the researcher was responsible for facilitating problem-solving teaching in experimental classes it was crucial, he observed, to confirm educator teaching is in accord with the Grade 8 equivalent addition and subtraction of fractions and through traditional methods in comparison schools and not according to the test and questionnaire instrument. Thus, this study ensured internal validity and confirmed that effects confidently attributed to the independent variable under evaluations.

3.9 ETHICAL ISSUES

This study acknowledges the importance of ethical issues. According to Thurtell (2019), ethical issues are vital considerations in any research study, and it is the responsibility of the researcher to protect the rights and well-being of participants and ensure that the research is conducted ethically and responsibly. The researcher addressed the following ethical issues in this study.

- Permission to conduct the research study
- Informed consent and assent
- Privacy and confidentiality
- Risk of harm

- Fairness and respect
- Data integrity and honesty

Permission to conduct research:

Before approaching the schools, the researcher obtained an approval ethical letter certificate from the UNISA Ethics Committee (Cf Appendix B) and obtained an approval letter document to conduct research in public schools from the Limpopo Department of Education (Appendix C). Both documents were presented to the school principals together with the request to conduct research letter to obtain permission to conduct research in the selected schools (Cf Appendix D).

Informed consent and assent:

The researcher ensured that educators in each group consented by completing and signing the consent letter before being assigned to the class comparison or experimental group schools. Similarly, parents completed and signed to consent before their child(ren) participated (Appendix E). Furthermore, each learner assented by completing and signing an assent form before being approved as a participant (Cf Appendix F). The researcher collected all signed forms from all involved.

Risk of harm:

This study is classified as a Type D risk study. Hence, risks are exceptionally negligible. However, the researcher documented consent and assent forms and informed learners of their right to withdraw if the need arose and there would be no gifts or monetary benefits for participating in the study. In addition, learners were responsible for safeguarding their private belongings and responsible for securing, protecting, and reporting any risk of losing any items and properties.

Privacy and confidentiality:

The researcher addressed these issues concerning participants' privacy and confidentiality by informing participants that their personal information would be kept anonymous, protected, and respected. According to Alshwaikh (2017), privacy, anonymity, and confidentiality should be granted to learners in research findings published. The researcher ensured that the information obtained through data collection could not be traced back to the participant and verified and replicated by third parties. Thus, learners' details like names, schools, and locations were coded on the answer booklet to keep their personal information anonymous and confidential. However, learners registered were kept identifying learners' groups. Thus, ethical issues surrounding privacy and confidentiality were ensured.

Fairness and respect:

The researcher addressed this ethical concern by ensuring that all participants were treated fairly and with respect. He ensured that their culture, religion, and personal beliefs were respected. For example, learners were informed that their participation was voluntary and that no awards or gifts would be provided in this study and informed of their right to withdraw from the study when their circumstances changed. In addition, the researcher ensured that all learners were given an equal opportunity to ask questions and get feedback. The researcher also ensured that all learners were from similar traditional classes and were not exposed to multiple learning strategies before the intervention.

Data integrity and honesty:

This was addressed by the researcher by ensuring that data were collected accurately and honestly. For example, responses/answers were marked using the same themes and memo criterion for both group schools. The data analysed used techniques that ensured only participants with complete instrument data were analysed and reported. In addition, the researcher avoided falsification or manipulation of data by ensuring all participants' instruments were stored in USB in electronic form for additional moderation and verification. Thus, ethical issues were addressed, and the researcher is confident that this study was conducted ethically and responsibly; and that participants' rights were protected.

3.10 CHAPTER SUMMARY

This chapter presented a positivist perspective on single reality. Discussions focused on research design and methodology. The quasi-non-equivalent pre-test post-test design was adopted in this chapter. Polya's problem-solving strategy was adopted as an intervention to enhance learners' skills in experimental groups; it constitutes the core objectives of the test. The skills covered in this chapter included understanding the problem, devising a plan, carrying out the plan, and evaluating the solution. The intervention assists learners in applying problem-solving learning strategies to solve fraction problems effectively. In addition, to ensure high validity and reliability, both inferential and descriptive statistically significant analytical tools were used in collecting and analysing the data.

CHAPTER FOUR

FINDINGS

4.1 OVERVIEW

This section presents data analysis and the study findings. Chapter 3 presented the questionnaire, pre- and post-test as the instruments used for the data collection process. Hence, this section focuses on analysing the data and presenting the findings. The inferential analysis reports the performance of learners' pre-test scores as normality statistically significant results. Furthermore, differences in learner performance before and after teaching (pre-, post-test, and questionnaire) were analysed using parametric and non-parametric techniques. The overall inferential and descriptive analyses involved calculations, and presentations using tables, graphs, and charts.

4.2 INFERENTIAL STATISTICS

The normality test analyses learners' pre-test score performance. The Kolmogorov-Smirnov and Shapiro-Wilk normality test were used to evaluate data distribution and choose parametric or non-parametric techniques for hypothesis verifications (Ndayambaje & Ralph, 2020). Normal distribution data is symmetrical, that is, the mean, median, and mode is equal. Hence, the normality test was conducted for pre- and post-test distributions in experimental and comparison groups.

According to Bustamante et al. (2022), the Kolmogorov-Smirnov and Shapiro-Wilk tests can be vital in evaluating non-equivalent group performances for compatibility. However, the Kolmogorov-Smirnov test is appropriate for data ($N > 100$, that is, 188 and 175) and the Shapiro-Wilk test is appropriate for class data ($N<100$ that is 49 to 66) (Bustamante et al, 2022). Thus, both tests are appropriate for this study because group participants are greater than a hundred and class participants are less than a hundred. The result reports a statistically significant p-value. For example, Kolmogorov-Smirnov threshold p-values ($p\text{-value} < 0.05$) were statistically significant. Thus, the null hypothesis was rejected, and the directional hypothesis was retained. Similarly, the Shapiro-Wilk ($p\text{-value} < 0.01$) was statistically significant. Thus, the null hypothesis was rejected, and the alternate hypothesis was retained as a verified summarised decision.

4.2.1. Analysis Based on Learners Pre-test Performances

Testing for pre-test normality distribution. The test had the following hypotheses:

(H_0): The pre-test performances are normally distributed in both groups.

(H_1): The pre-test performances are not normally distributed in both groups.

Table 4.1: Experimental and Comparison Group Pre-test Result

Group Name	Code	N	Test	Mean	Std.	%Mean	T-stat	P-value
Comparison	CGS	205	Pre-test	9.03	4.58	4.40	2.865	0.096
Experimental	EGS	201	Pre-test	8.87	5.44	4.40	2.545	0.096

Not statistically significant p=0.096 on count 3, and average 4.4% (Cf Table 4.1). Hence, retained null hypothesis: **(H_0)**: The pre-test performances are normally distributed in both groups and rejects directional hypothesis **(H_1)**: The pre-test performances are not normally distributed in both groups. Additional information (Cf Figure 4.1) is presented below.

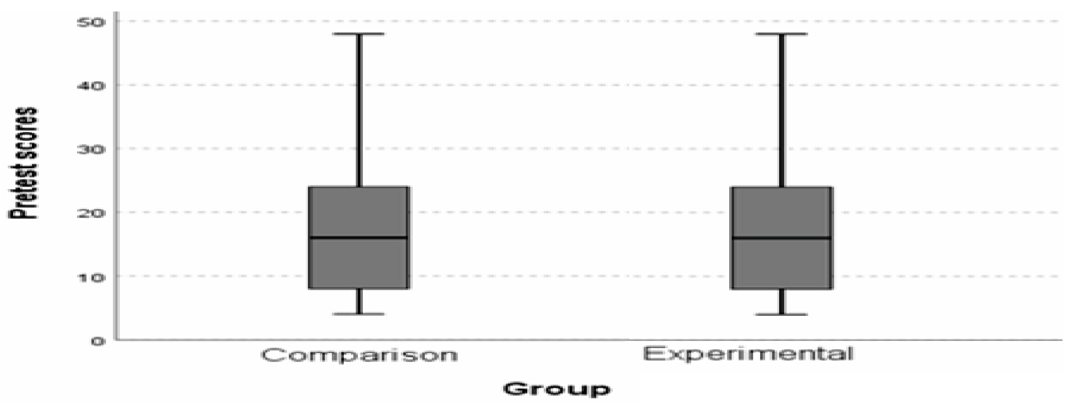


Figure 4.1: Box Diagram Illustrating Learners' Performance Before Teaching (Pre-Test)

The boxes are symmetrical as they revealed equal performances in each group. This implies groups are compatible and learners are at equal levels of knowledge and understanding of fractions.

4.2.2 Analysis Based on learners' pre and post-test performance

For evaluating learners' pre- and post-test performance for normal distribution, the test had the following hypotheses:

(H_0): The pre-and post-test performances are normally distributed in each group.

(H_1): The pre-and post-test performances are not normally distributed in each group.

The Shapiro-Wilk test result revealed a statistically significant value of p=0.000 in each group on a larger difference of 0.836. Hence, rejecting the null hypothesis **(H0):** The pre-and post-test performances are normally distributed in each group, and retaining directional hypothesis **(H_1)**: The pre-and post-test performances are not normally distributed in each group. This implies that there is a difference in pre-, and post-test performances in each group after teaching and learning fractions (Cf Table 4.2) as illustrated below.

Table 4.2: Experimental and Comparison groups, pre- and post-test Performance Results

			General statistics			**Kolmogorov-Smirnov[a]**			**Shapiro-Wilk**		
School	Code	Test	N	Mean	STD.	DF	T-stat	P-value	T-stat	DF	P-value
Overall	CGS	Pre-test	188	9.23	5.703	186	2.865	0.096	0.812	186	0.000
		Post-test	188	29.36	17.423	186	2.818	0.096	0.836	186	0.000
	EGS	Pre-test	175	8.91	5.399	173	2.545	0.096	0.619	173	0.000
		Post-test	175	38.88	19.510	173	1.806	0.099	0.819	173	0.000
School 1	CGS1	Pre-test	62	9.23	4.520	61	0.931	0.112	0.851	61	0.000
		Post-test	62	30.77	18.338	61	0.867	0.112	0.825	61	0.000
	EGS1	Pre-test	49	8.57	6.047	48	0.852	0.126	0.765	48	0.000
		Post-test	49	38.20	17.889	48	1.000	0.126	0.933	48	0.008
School 2	CGS2	Pre-test	66	8.12	5.115	65	0.893	0.109	0.768	65	0.000
		Post-test	66	25.69	14.590	65	0.899	0.109	0.809	65	0.000
	EGS2	Pre-test	62	8.71	4.459	61	0.915	0.112	0.855	61	0.000
		Post-test	62	31.42	18.454	61	0.999	0.112	0.832	61	0.000
School 3	CGS3	Pre-test	60	10.47	7.008	59	0.787	0.114	0.817	59	0.000
		Post-test	60	32.00	18.762	59	0.805	0.114	0.875	59	0.000
	EGS3	Pre-test	64	9.38	5.667	63	0.811	0.111	0.837	63	0.000
		Post-test	64	46.63	18.751	63	0.984	0.111	0.960	63	**0.034**

When each group was considered separately, the comparison pre-, and post-test performances were not statistically significant. However, since Experimental School 3 indicated a statistically significant value of p=0.034, the null hypothesis **(H_0)**: The pre-and post-test performances are normally distributed cannot rejected in EGS3. The implication is that no difference in pre- and post-test performances in School 3's experimental and comparison group post-test Shapiro-Wilk test results.

However, Kolmogorov-Smirnov not statistically significant p-values> 0.05 (Cf Table 4,2) implies that non-parametric Kruskal-Wallis and Wilcoxon Signed-Rank test analysis are appropriate for analysing the study's null hypothesis. However, EGS3 indicates a not statistically significant p=0.034. This implies that the parametric paired t-test analysis is appropriate. Hence, both parametric paired-sample-test and non-parametric Kruskal-Wallis and Wilcoxon Signed-Rank tests were used to verify the study's null hypothesis. However, the paired sample-t-test analysed differences in groups' average performances, and the Kruskal-Wallis test analysed learners' performances in each group while the Wilcoxon Signed-Rank test analysed the effect on learners' performances through problem-solving and traditional teaching approach Additional information illustrated (Cf Figure 4.2-3) below.

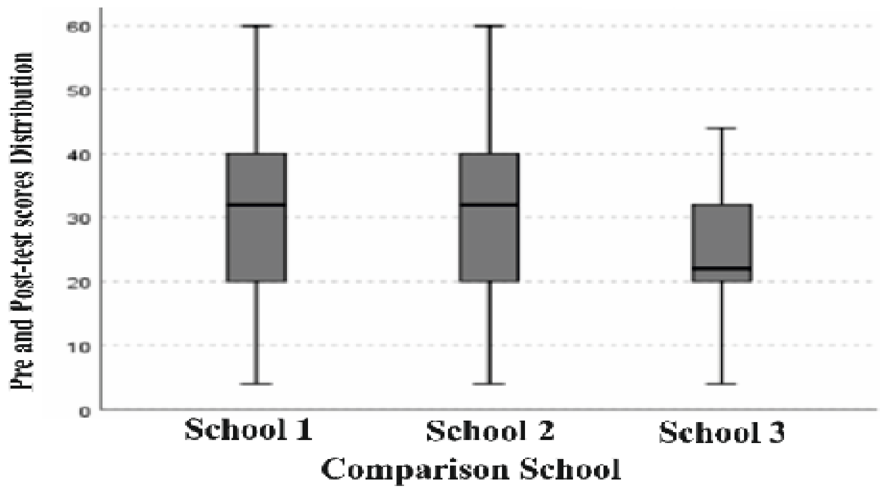


Figure 4.2: Box Diagram of Learners' Performances before and after Teaching in CGS

The non-symmetrical box across schools implies pre- and post-test performances are not normally distributed. Hence, performances before and after teaching in the comparison schools are different.

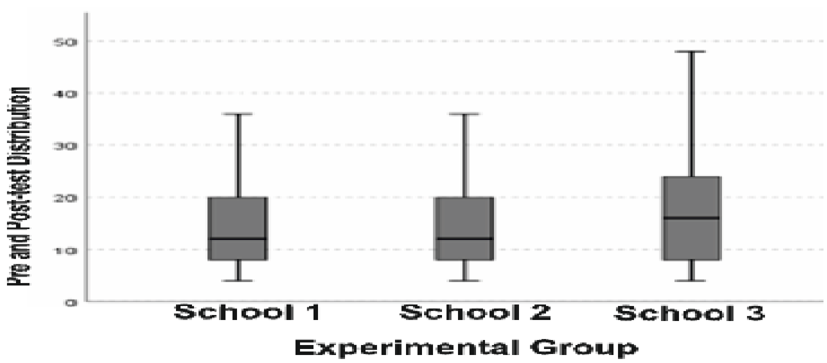


Figure 4.3: Box Diagram of Learners' Performances before and after Teaching in EGS

The unsymmetrical box in Schools 1 and 2 implies pre- and post-test performances are different. However, EGS 3 box in School 3 implies post-test performances are equal in experimental and comparison School 3. However, overall performances indicate statistically significant p=0.000.

4.2.3 Analysis Based on Learners' pre- and post-test Average Scores Performances

The paired sample t-test was used to assess learners' pre- and post-test average scores performances. Thus, a statistically significant p-value>0.05 implies that the null hypothesis is rejected if the p-value is greater than 0.05. The test had the following hypothesis:

(H_0): There is no differences in pre-, and post-test average score in each group.

(H_1): There is differences in pre-, and post-test average score in each group.

Table 4.3: Difference between Experimental and Comparison Groups' Pre- and Post-Test Result

Comparison group

School	Test	N	MD	STD	SEM	95%C	T-stat	DF	P-value
School 1	Pre, & Post-test	62	-21.54	17.55	2.229	4.45	-9.66	61	.043
School 2	Pre, & Post-test	66	-17.57	13.88	1.709	3.41	-10.27	65	.044
School 3	Pre, & Post-test	60	-21.53	16.76	2.164	4.33	-9.95	59	.044
Experimental group									
School	Test	N	MD	STD	SEM	95%C	T-stat	DF	P-value
School 1	Pre, & Post-test	49	-29.20	16.83	2.40	4.83	-12.32	48	.044
School 2	Pre, &Post-test	62	-22.70	18.29	2.322	4.64	-9.77	61	.042
School 3	Pre, & Post-test	64	-37.25	17.44	2.18	4.35	-17.08	63	.042

Overall difference between pre- and post-test scores performances

N	Group name	Code	Test	Mean	Variance	DF	T-stat	P-value
188	Comparison group	CGS	Pre-test	9.23	32.69	361	0.54	0.96
175	Experimental group	EGS	Pre-test	8.91	29.13			
188	Comparison group	CGS	Post-test	29.38	307.21	350	-4.86	0.000
175	Experimental group	EGS	Post-test	38.88	382.83			

Statistically significant p= 0.043 and 0.044 and T-Stat= -9.66; -10.27 and -9.95 and pre- and post-test average difference = -21.24; -17.57 and -23.53 in comparison schools.

In addition, the research recorded a statistical significance of p=0.044 and 0.046 and T-stat = -12.32; - 9.77 and -17.08 and pre-, and post-test average difference =-29.200; -22.70 and -37.25 in experimental schools.

Similarly, overall, statistical value p=0.000 and statistically different -4.86 was recorded for learners' average performance achievement. Hence, the decision is rejecting the null hypothesis **(H_0)**: There are no differences in pre-, and post-test average performances in each group and the study retained the directional hypothesis **(H_1)**: There are differences in pre-, and post-test average performance in each group.

The implication of statistically significant p=0.000 indicates better performance in experimental schools. In addition, individual learners in experimental performances are better in post-test scores compared to comparison learners. Thus, the average score of 22.21% (38.88) of experimental learners' performances is higher than comparison average performances of 15.63% (29.38). This confirms that individual learners in the experimental group yield high average performances in the post-test. Conclusion on paired sample t-test p=0.000 statistically t-stat=-4.86 and difference 6.58% (9.5). The decision is rejecting the null hypothesis **(H_0)**: There is no difference between problem-solving teaching approach and the traditional teaching approach on learning addition and subtraction of fractions and retaining the directional hypothesis **(H_1)**: Problem-solving teaching approach enhances Grade 8 learners' cognitive development addition and subtraction.

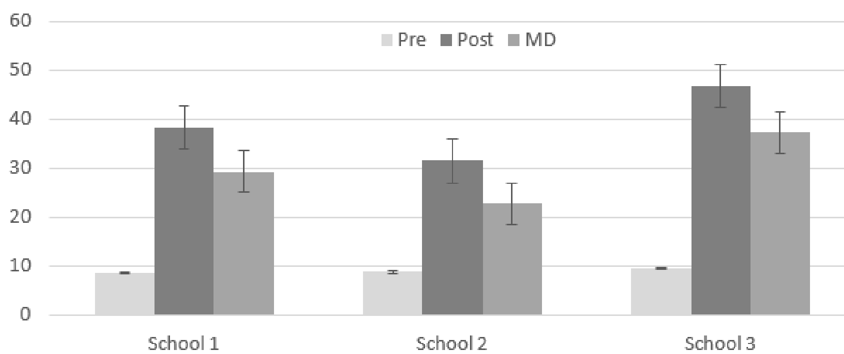


Figure 4.4: Illustrating differences in comparison group pre- and post-test mean scores

Unequal bars represent average different performances in Schools 1-3. Thus, learners' performances are different in comparison schools' pre- and post-test scores.

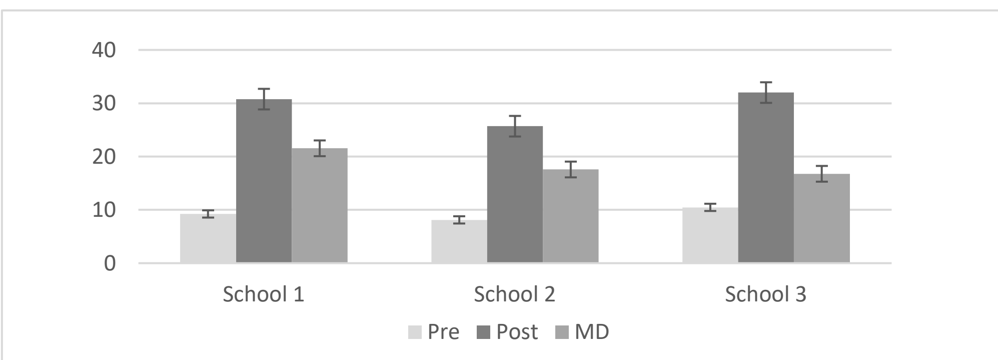


Figure 4.5: Illustrating Differences in Experimental Group Pre- and Post-Test Mean Scores

Unequal bars illustrate the difference in performances in Experimental Schools 1-3. Thus, learners' performances are different in experimental schools' pre- and post-tests.

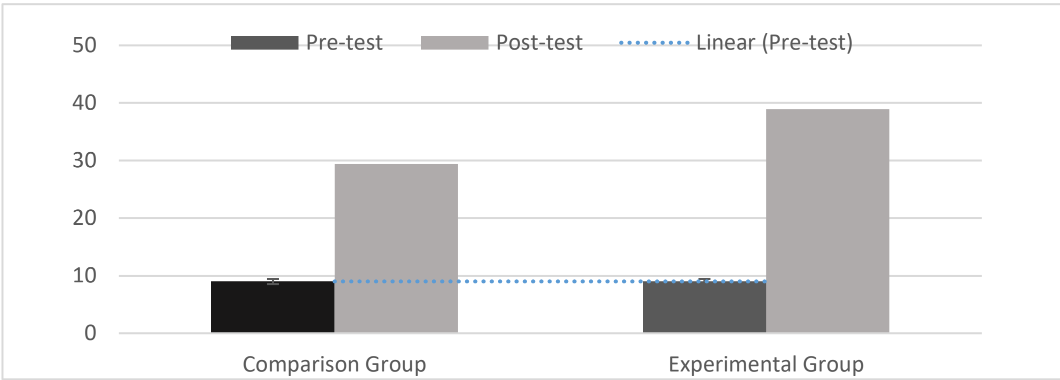


Figure 4.6: Overall Test Differences in Comparison and Experimental Group Pre- and Post-Test

The equal black bars in the graph represent no statistical significance in pre-test performances and the unequal grey bars in post-test performances represent statistical significance. In addition, post-test performances yielded better performances in the experimental group. This implied that most individual learners' performances were better in the experimental group. Thus, problem-solving enhanced Grade 8 learners' performances and cognition on learning addition, subtraction, and equivalent fractions.

4.2.4 Analysis Based on Learner Performances on Learning Fractions using each approach

The Kruskal-Wallis test assesses the performances of learners learning through the traditional approach and problem-solving learning approach. A statistically significant value ($p < 0.05$) implies rejecting the null hypothesis. The test evaluates and assesses learners' performances in comparison to learning using a traditional approach and experimental learning using the problem-solving intervention.

The test had the following hypothesis.

(H_0): There are no differences in learners' performances on learning using each approach in school.

(H_1): There are differences in learners' performances on learning using each approach in school.

Table 4.4: Learners' Performances before Learning: Traditional and Problem-Solving Results

Pre-test Comparison group schools						**Pre-test Experimental group schools**					
School	Code	N	Rank Sum		Mean	School	Code	N	Rank sum		Mean
School 1	CGS 1	62	6192.0		99.870	School 1	EGS1	42	3964.0		80.897
School2	CGS 2	66	5451.0		82.590	School2	EGS2	62	5590.0		90.161
School3	CGS 3	60	6123.0		102.05	School3	EGS3	64	5846.0		91.343
Paired Wise Comparison: Number of comparisons Three											
Pre-test Comparison group schools						**Pre-test Experimental group schools**					
Code	N	MD	Limit		P-value	Code	N	MD	Limit		P-value
CGS 1&2	128	17.28	23.004		0.2164	EGS 1&2	111	9.263	23.132		1.000
CGS 1&3	122	21.79	23.554		1.000	EGS 1&3	113	10.44	22.972		0.829
CGS 2&3	126	19.45	23.201		0.133	EGS 2&3	126	1.182	21.565		1.000
Overall Pre-test						Post-test					
(N)	(H)	C. Rank	S^2	DF	P-value	Overall	(H)	C. Rank	S^2	DF	P-value
188	**4.920**	**0.996**	**5.99**	**2**	**0.0845**	**175**	**1.354**	**0.995**	**5.99**	**2**	**0.051**

The test yielded no statistically significant value in learners' pre-test performances before learning using traditional and problem-solving approaches. Thus, p=0.0845 and Rank = 0.99, and p=0.051 on rank differences R=0.99. However, the decision cannot be reached based on pre-test performances since both learners are from similar traditional teaching classes and learners have not yet been introduced to the form of teaching and learning approach. Thus, the hypothesis cannot be verified using pre-test performances.

Table 4.5: Learners' performances after learning traditional and problem-solving results

Post-test Comparison group schools						**Post-test Experimental group schools**					
School	Code	N	Rank Sum		Mean	School	Code	N	Rank sum		Mean
School 1	CGS1	62	4348.5		87.26	School 1	EGS1	42	5410.0		88.744
School2	CGS2	66	4071.5		71.37	School2	EGS2	62	4710.5		65.669
School3	CGS3	60	6980.0		127.43	School3	EGS3	64	7645.5		109.02
Paired Wise Comparison: Number of comparisons Three											
Pre-test Comparison group schools						**Pre-test Experimental group schools**					
Code	N	MD	Limit		P-value	Code	N	MD	Limit		P-value
CGS1&2	128	15.88	21.474		0.195	EGS1&2	111	23.07	23.118		0.050
CGS1&3	122	40.17	21.988		0.000	EGS1&3	113	20.31	22.958		0.102
CGS2&3	126	56.05	23.132		0.000	EGS2&3	126	43.39	21.552		0.000
Overall Pre-test						Post-test					
(N)	(H)	C. Rank	S^2	DF	P-value	Overall	(H)	C. Rank	S^2	DF	P-value
188	23.181	0.991	5.99	2	0.065	175	30.347	0.994	5.99	2	0.000

The test recorded no statistically significant value in post-test performances in traditional learning performance achievement scores. Comparison group post-test p-value=0.065 and Rank H=23.18 and rank different Rank=0.11. However, a statistically significant value was obtained in problem-solving learning performance achievement scores in the experimental group. That is, the experimental group post-test p-value=0.000 H=30.347 and different rank different=0.163.

It is evident from Table 4.5 that most of the learners in the experimental group (problem-solving learning approach) scored higher in the post-test compared to the pre-test at the following effect size for the Kruskal-Wallis test:

The eta squared based on the H-statistic $Eta\ [H]= \frac{(H-K+1)}{(N-1)}$

$$Eta\ 2[H] = \frac{(30.347-3+1)}{(175-1)} = 0.163.$$

Where H is the value obtained in the Kruskal-Wallis test, K is the number of groups, and N is the number of observations. The interpretation values are H=0.01 – $<$0.06 (small effect), 0.06 – $<$0.14 (moderate effect), and $>$ =0.14 (large effect). Therefore, the significant value was recorded as p=0.000, $p< 0.05$, at a large effect size of 0.163. The results implication is that in learning problem-solving in the experimental group, learners' performances in post-scores yielded better ranks than comparison learners.

Learners in the comparison group on learning traditional performances still yielded similar scores. Thus, indicating no statistical difference. However, learner performances are statistically significant in experimental post-test scores after receiving problem-solving teaching and learning intervention on problem-solving. Additional information (Cf Figure 4.7).

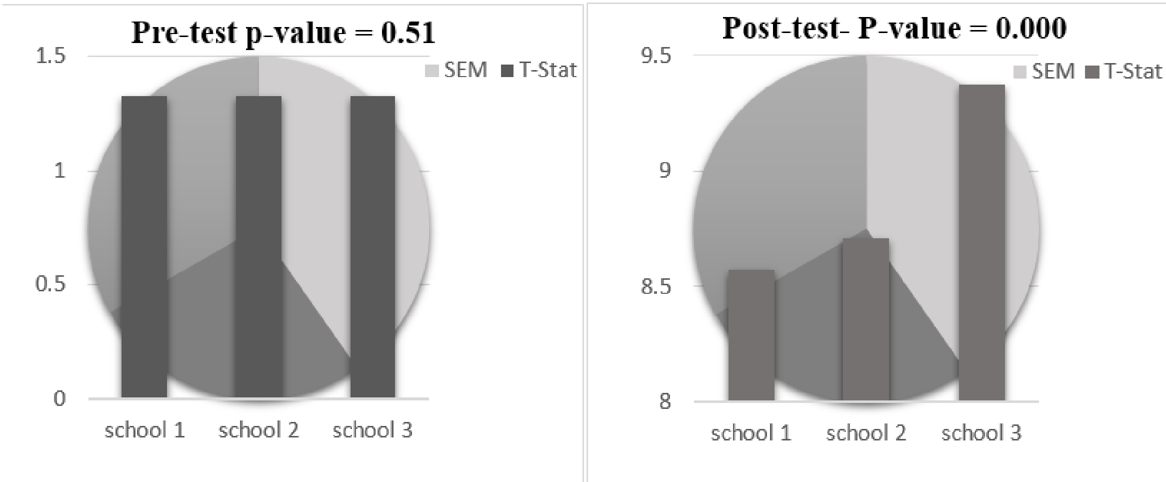


Figure 4.7: Illustrating Effects of Problem-Solving Intervention in the Experimental Group

In Figure 4.7, equal bars represent no effects in pre-test and unequal bars in post-test represents effects in post-test performances. Thus, the problem-solving effects change in learners' performances in learning fractions.

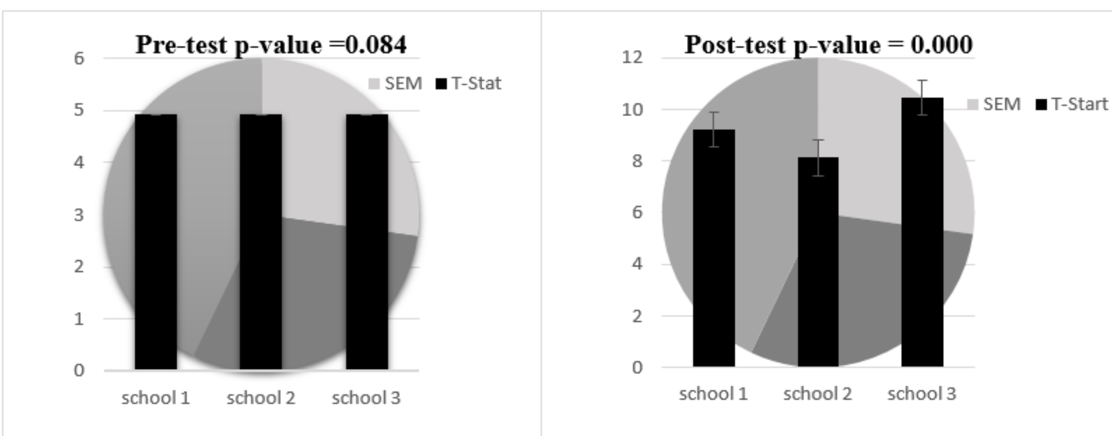


Figure 4.8: Illustrating the Effect of Teaching Using the Traditional Approach in the Comparison Group

Figure 4.8 Equal bar in the pre-test presents no effects and the unequal bar in the post-test presents effects in the post-test. Thus, the traditional teaching approach effects change in performances in the comparison group.

Table 4.6: Overall effect results (Tradition vs. problem-solving)

Group name	Test	N	Mean	% Mean	T-stat H	DF	Effect Size	P-value
Comparison group	Pre-test	188	9.28	5%	4.920	3	0.016	0.0845
Experimental group	Pre-test	175	8.91	5%	3.354		0.008	0.396
Comparison group	Post-test	188	29.6	16%	23.181	3	30.347	0.000
Experimental group	Post-test	175	38.88	22%	30.347		0.163	0.000
Total		363	Moderate difference between group is 0.05					

Table 4.6 showed statistically significant post-test p=0.000 H=30.347 on moderate different 0.05.

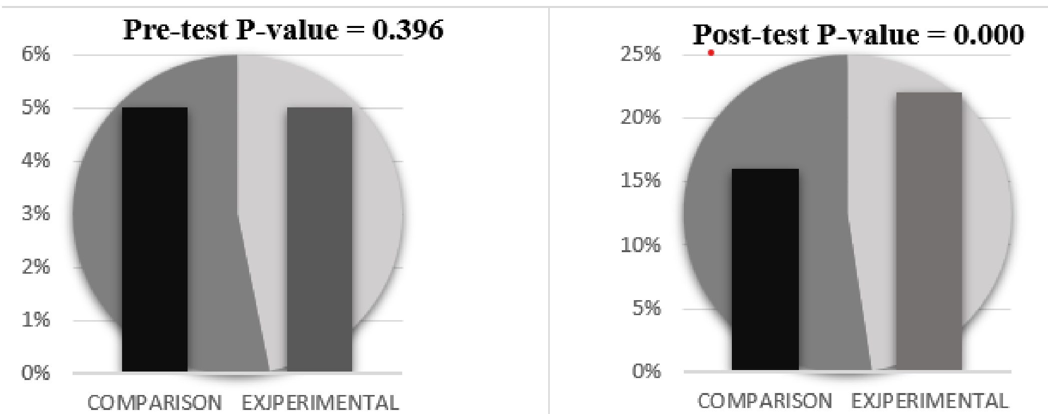


Figure 4.9: Illustrating overall performance achievement according to the teaching approach

Table 4.7: SPSS generated post-test scores summary results

Hypothesis Test Summary			
Null Hypothesis	Test	Sig.	Decision
The distribution of Posttest is the same across categories of Group.	Independent-Samples Kruskal-Wallis Test	0.000	Reject the null hypothesis.
Asymptotic significances are displayed. The significance level is .050.			

Kruskal-Wallis Test Summary	
Total N	363
Test Statistic (Kruskal Wallis H)	30.347[a,b]
Degree of Freedom	3
Asymptotic Sig. (2-sided test)	.000
a. The test statistic is adjusted for ties.	
b. Multiple comparisons are not performed because there are less than three test fields.	

The post-test pie chart (Cf Figure 4.9) shows unequal bars representing increased performances in the experimental group on learning using the problem-solving approach. This implies 22% average learner performance in comparison group 16% average performance. Conclusion and decision reached based on Inferential SPSS generated statistically significant value (Cf Table 4,7) rejecting the study null hypothesis **(H_0)**: There is no difference between problem-solving teaching and traditional teaching on learning fractions and retained directional hypothesis **(H_1)**: Problem-solving teaching enhances Grade 8 learners' cognitive addition and subtraction of fractions.

Table 4.8: Summary of Kruskal-Wallis Pre- and Post-test Results Decision

	Null Hypothesis	Test	Sig.	Decision
1	No effect on learners' pre-test scores in the experimental group distribution before the problem-solving intervention	Independent-Samples Kruskal-Wallis Test	.51	Retain the null hypothesis
2	No effect on learners' post-test scores after intervention in the experimental group distribution after intervention	Independent-Samples Kruskal-Wallis Test	.000	Reject the null hypothesis
3	No effect on learners' pre-test scores in the comparison group distribution before problem-solving intervention	Independent-Samples Kruskal-Wallis Test	.084	
4	No effect in learners' post-test scores after intervention in the comparison group distribution after intervention	Independent-Samples Kruskal-Wallis Test	.000	

4.2.5 Effects of Traditional and Problem-solving Teaching on Learner Performance

The Wilcoxon Sign Rank test analysed the effect of traditional teaching and problem-solving teaching on learner performance by evaluating the number of learners with high-performance scores in the traditional class. The test predictions are, I) the median difference between the overall pre and post-test scores is zero, II) the median difference between the overall pre and post-test scores is not zero.

Table 4.9: Effectiveness of Traditional Teaching Approach Result after Teaching and Learning

Ranks						**Test Statistics**	
SCHOOL	Group	Pre - Post-test	N	Mean Rank	Sum of Ranks	z-value	p-value
Overall sample	Comparison Group	Negative Ranks	16[a]	42.61	383.50	-9.272[d]	0.000
		Positive Ranks	148[b]	69.81	8796.50		
		Ties	24[c]				
		Total	188				
School 1	Comparison Group	Negative Ranks	3[a]	13.00	39.00	-5.341[d]	0.000
		Positive Ranks	53[b]	23.20	951.00		
		Ties	6[c]				
		Total	62				
School 2	Comparison Group	Negative Ranks	5[a]	14.50	72.50	-5.128[d]	0.000
		Positive Ranks	51[b]	24.60	1008.50		
		Ties	10[c]				
		Total	66				
School 3	Comparison Group	Negative Ranks	8[a]	15.00	15.00	-5.711[d]	0.000
		Positive Ranks	44[b]	23.18	1020.00		
		Ties	8[c]				
		Total	60				
a. Post-test < Pre-test							
b. Post-test > Pre-test							
c. Post-test = Pre-test							
d. Based on negative ranks.							
e. Wilcoxon Signed Ranks Test							

Table 4.9 showed rank sum positive, negative, and tie. The test statistics present the z-value and p-value. The legend presents the overall sample 16[a] learners obtained pre-test greater than post-test, and 148[b] learners obtained post-test scores greater than pre-test and 24[c] learners obtained ties scores.

Considering each school separately. In School 1, 3[a] learners obtained pre-test scores greater than post-test scores, and 53[b] learners obtained post-test scores greater than pre-test and 6[c] learners obtained ties in pre-test and post-test. In School 2, 5[a] learners obtained pre-test scores greater than post-test scores, and 51[b] learners obtained greater post-test scores and 10[c] learners obtained equal pre and post-test scores. In School 3, 8[a] learners obtained pre-test scores greater than their post-test scores, and 44[b] learners obtained post-test scores greater than pre-test and eight 8[c] learners obtained equal pre- and post-test. Furthermore, the results indicate a statistically significant value of p=0.000. The decision is to reject the null hypothesis **(H_0)**: there are no differences in learners' performances in traditional teaching and retained **(H_1)**: there are differences in learners' performances in traditional teaching. Conclusion: 78.72% (148) of learners improved in performance after teaching in traditional classes.

Table 4.10: Effectiveness of the Problem-Solving Approach Result after Teaching and Learning

Ranks						**Test statistics**	
SCHOOL	**Group**	**Pre & Post-test**	**N**	**Mean Rank**	**Sum of Ranks**	**z-value**	**p-value**
Overall sample	Experimental group	Negative Ranks	10[a]	31	31	-10.434[d]	0.000
		Positive Ranks	157[b]	73.79	10700		
		Ties	8[c]				
		Total	175				
School 1	Experimental group	Negative Ranks	3[a]	0	0	-6.037[d]	0.000
		Positive Ranks	44[b]	24.5	1176		
		Ties	2[c]				
		Total	49				
School 2	Experimental group	Negative Ranks	5[a]	9.5	9.5	-6.073[d]	0.000
		Positive Ranks	53[b]	25.83	1265.5		
		Ties	4[c]				
		Total	62				
School 3	Experimental group	Negative Ranks	2[a]	0	0	-6.039[d]	0.000
		Positive Ranks	60[b]	24.5	1176		
		Ties	2[c]				
		Total	64				
a. Post-test < Pre-test							
b. Post-test > Pre-test							
c. Post-test = Pre-test							
d. Based on negative ranks.							

e. Wilcoxon Signed Ranks Test

Table 4.10 presents pre- and post-test rank sum positive, negative, and tie in the experimental group. The overall sample revealed that 10[a] learners obtained pre-test scores greater than the post-test score, 157[b] obtained post-test greater than the pre-test, and 8[c] Learners' scores were equal. Considering each school separately. In School 1, 3[a] learners obtained pre-test scores greater than the post-test and 44[b] learners obtained post-test scores greater than the pre-test, and 2[c] learners tied. In School 2, 5[a] learners obtained pre-test scores greater than the post-test and 53 learners obtained post-test scores greater than the pre-test, and 4[c] learners tied. In School 3, 2[a] learners obtained pre-test scores greater than the post-test while 60[b] learners obtained post-test scores greater than the pre-test, and 2[c] learners tied. The results further indicate a statistically significant value of P=0.000. Thus, the decision is to reject the null hypothesis **(H_0)**: there are no differences in learners' performances after problem-solving teaching and retain directional **(H_1)**: there are differences in learners' performances after problem-solving teaching.

Conclusion: 89.71% (157) of learners improved after teaching and learning using the problem-solving approach. Additional information in Figure 4.9.

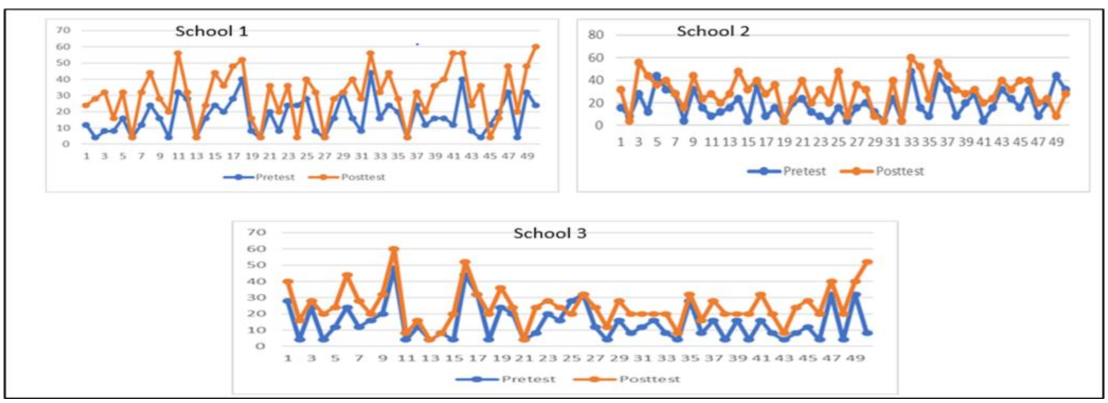


Figure 4.10: Learners performance before and after traditional teaching approach

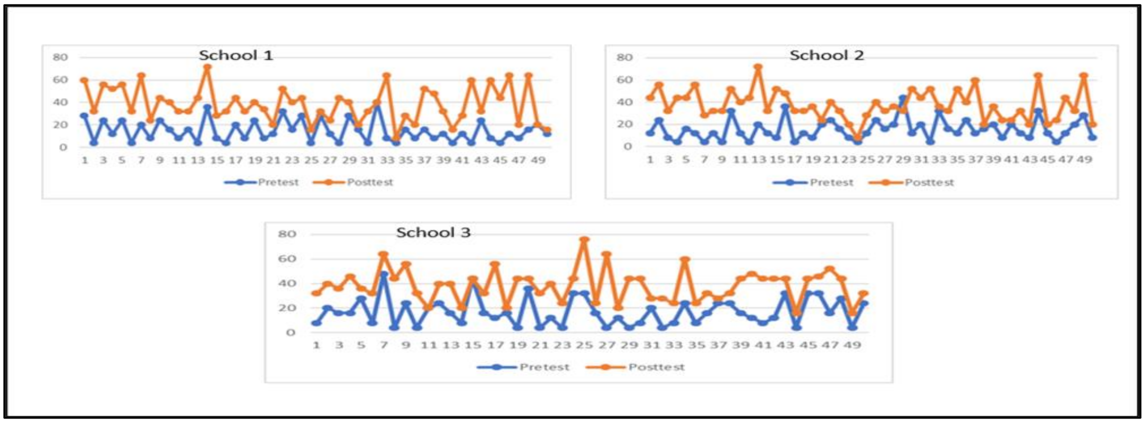


Figure 4.11: Learners (Problem-solving Teaching Approach)

4.2.5.1 Learners' Performances in Traditional vs Problem-solving Teaching

Both approaches are statistically significant p=0.000 and learners' performances increased after teaching and intervention in both groups. Nevertheless, problem-solving teaching in the experimental group yielded higher performances of 89.71% (157) which is greater than the traditional teaching in comparison group performance of 78.72% (148) on learning fractions. Therefore, the problem-solving teaching approach yielded higher effects on teaching Grade 8 fractions. Therefore, the Wilcoxon Signed Rank Test statistically significant p=0,000 decision is to reject the study's null hypothesis. **(H_0)**: There is no difference between problem-solving teaching and traditional teaching approach on learning addition and subtraction of fractions and retain the study directional hypothesis **(H_1)**: Problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions on learning Grade 8 fractions (Cf Table 4.11 below).

Table 4.11: Summary of Results Decision in Comparison and Experimental Group

	Null Hypothesis	Test	Sig.	Decision
1	The median of differences between the **Overall** Pre-test and Post-test equals 0.	Related-Samples Wilcoxon Signed Rank Test	.000	Reject the null hypothesis.
2	The median of differences between the pre-test and post-test scores in **School 1** equals 0	Related-Samples Wilcoxon Signed Rank Test	.000	Reject the null hypothesis
3	The median of differences between the pre-test and post-test scores in **School 2** equals 0	Related-Samples Wilcoxon Signed Rank Test	.000	Reject the null hypothesis.

4	The median of differences between the pre-test and post-test scores in **School 3** equals 0	Related-Samples Wilcoxon Signed Rank Test	.000	Reject the null hypothesis

4.2.6 Summary of Pre- and Post-test Analytical Inferential Reports

The Kolmogorov-Smirnov test was used to evaluate learners' pre-test performance and knowledge of fractions before teaching. Results (Cf Table 4.1) present no statistically significant p=0.096 and T-stat$\approx$ 3 for both groups. This implies learners had the same level of knowledge and performance before teaching. In addition, the groups are compatible and can be compared.

Furthermore, the Kolmogorov-Smirnov and Shapiro-Wilks tests were used to evaluate learners' pre- and post-test performances. Results (Cf Table 4.2) present statistically significant P=0.000 and T-stat (T>0.05). The implication is that a non-parametric statistical technique is appropriate to verify the study hypothesis. However, Experimental School 3 reveals no statistical significance at P=0.034. This implies that the parametric statistical technique is appropriate. Nonetheless, since the overall results revealed statistical significance at p=0.000, the decision was to use the non-parametric Kruskal-Wallis Test and Wilcoxon Signed-Rank Test technique for testing the study null hypothesis and parametric dependent Sample T-Test due to the element of no statistical significance in School 3.

The paired sample t-test was analysed for differences in learners' pre- and post-test score performances. Results (Cf Table 4.5) present statistically significant P=0.000 in pre- and post-test performances. The implication is that learners performed better in the post-test than in the pre-test. In addition, experimental learners yielded better results than learners in the comparison group. Thus, based on paired sample t-test analysis the decision is to reject the null hypothesis **(H_0)**: There is no difference between problem-solving teaching and traditional teaching on learning addition and subtraction of fractions and retain the directional hypothesis **(H_1)**: Problem-solving teaching enhances Grade 8 learners' cognitive development in addition and subtraction of fractions on learning Grade 8 fractions.

Furthermore, the Kruskal Wallis test was used to evaluate and assess the performance achieved by learners after learning using traditional and problem-solving approaches. The result (CF Table 4.7) presents statistically significant p=0.000 and H=30,347. This implies that experimental group learners yielded better performances in learning using problem-solving. In addition, 22% of the average performances in the experimental group are higher than the 16% average performances in the comparison group. Similarly, the decision is to reject the study's null **(H_0)**: There is no difference between the problem-solving teaching approach and traditional teaching on learning addition and subtraction of fractions and retain the directional hypothesis **(H_1)**: The problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions.

The Wilcoxon signed-rank test was used to ascertain the effectiveness of the approach in teaching and learning in both the experimental and comparison groups. Results (Cf Tables 4.9 and 4.10) present statistically significant p=0.000. This implies that problem-solving teaching yielded more effective performances in learning compared to traditional teaching. In addition, (Cf Table 4.10) presents 89.71% (157) of N=175 experimental learners with better performances after receiving the problem-solving intervention and 78.72% (148) of N=188 comparison learners with better performances. Similarly, the decision to reject the study's null **(H_0)**: There is no difference between the problem-solving teaching approach and traditional teaching on learning addition and subtraction of fractions and retain the directional hypothesis **(H_1)**: The problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions.

4.3 DESCRIPTIVE ANALYSIS

This section presents learners' achievement scores according to problem-solving knowledge and understanding of fractions and problem-solving learning skills and mistakes committed.

4.3.1 Analysis Based on Learners' Pre and Post-test Average Achievement Scores

Table 4.12 presents ties in pre- and post-test Min and Max achievement scores. The pre-test scores ranged from 10.47 to 8.12 and the post-test scores ranged from 25.69 to 32.0. Thus, the average post-test scores are greater than the pre-test scores. This implies that problem-solving and traditional teaching improved learners' average achievement. However, the overall average pre-test scores ranged from 4 to 32 and 8 to 84 in the post-test in the comparison group. The pre-test scores ranged from 4 to 24 and 8 to 84 in the post-test in the experimental group. Furthermore, the average pre-test achievement in the comparison group was 24% (9.23), less than the average post-test 76% (29.36). Similarly, the average pre-test achievement was 19% (8.91), less than 81% (38.88) in the experimental group.

Thus, post-test achievement in the comparison group was 76% (29.36) less than 81% (38.88) in the experimental group. Thus, the experimental group's average achievement was higher than the one achieved by the comparison group. Thus, problem-solving enhanced learners' achievement in obtaining higher scores on learning fractions (Cf Table 4.12).

Table 4.12: Comparison of Experimental and Comparison Groups' Average Test Results

SCHOOL	Code	Test	N	Mean	STD	Min	Max
Overall	CGS	Pre-test	188	9.23	5.703	4	32
		Post-test	188	29.36	17.423	8	84
	EGS	Pre-test	175	8.91	5.399	4	24
		Post-test	175	38.88	19.510	8	84
School 1	CGS1	Pre-test	62	9.23	4.520	4	20
		Post-test	62	30.77	18.338	8	84
	EGS1	Pre-test	49	8.57	6.047	4	24
		Post-test	49	38.20	17.889	8	84
School 2	CGS2	Pre-test	66	8.12	5.115	4	24
		Post-test	66	25.69	14.590	8	76
	EGS2	Pre-test	62	8.71	4.459	4	24
		Post-test	62	31.42	18.454	12	84
School 3	CGS3	Pre-test	60	10.47	7.008	4	32
		Post-test	60	32.00	18.762	8	76
	EGS3	Pre-test	64	9.38	5.667	4	24
		Post-test	64	46.63	18.751	12	84
Total scores			363				

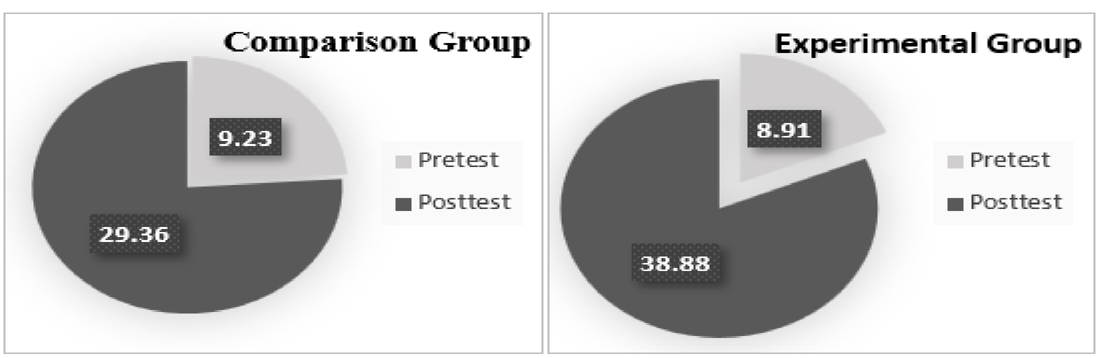


Figure 4.12: Illustrating Learners' Overall Average Test Scores Achievement

Conclusion: EGS's 81% (38.88) average score is greater than CGS's 76% (29.36) average score. Hence problem-solving yielded higher average achievement scores on learning fractions.

4.3.2. Analysis Based on Learners' Pre- and Post-test Achievement Score Levels

Masilo (2018) categorised three achievement levels as Low, Moderate, and Advanced. Thus, this study classified learners' scores achieved according to Masilo's (2018) achievement levels which are low achievement (LA 0– 30), moderate achievement (MA 31-60), and advanced achievement (AA 61 – 100). Table 4.13 below presents results based on learners' achievement levels.

Table 4.13: Experimental and Comparison Group Test Achievement Level Results

Group Name	Categories	Interval	Pre-test %	Post-test %	Difference	Comment
Comparison	LA	0 -30	98.94% (186)	62.77 (118)	36.17% (68)	Decrease
	MA	31-60	1.06% (2)	27.13% (51)	26.06% (49)	Increase
	AA	61--100	0.00% (0)	10.11% (19)	10.11% (19)	Increase
Experimental	LA	0 -30.	100.0% (175)	38.29% (67)	61.71% (108)	Decrease
	MA	31--60	0.00% (0)	44.00% (77)	44.00% (77)	Increase
	AA	61---100	0.00% (0)	17.71% (31)	17. 71% (31)	Increase

Learners decrease in low, increase in moderate, and further increase in advanced scores performance achievement levels from pre- to post-test (Cf Table 4.13). Before (pre-test) traditional teaching, learners achieved 186 (98.94%) in the comparison group's low-level achievement. However, after teaching (post-test) in the comparison group, the number of low achievements decreased from 98.94% (186) to 62.77% (118). Similarly, in the experimental group before (pre-test) problem-solving intervention learners achieved 100% (175) low achievement level. However, after intervention, the number of low scores decreased from 100% (175) to 38.29% (67). Hence, 36.17% decrease in the low achievement level in the comparison group and a 61.71% (108) decrease in the low achievement level in the experimental group (Cf Table 4.13).

Furthermore, 1.06% (2) learners achieved moderate scores achievement levels in the comparison group (pre-test) before traditional teaching. However, after traditional teaching (post-test) the number increased from 1.06% (2) to 27.13% (51) in learners with moderate achievement levels. However, in the experimental group (pre-test) before the intervention, none of the learners achieved a moderate achievement level. However, after intervention (post-test) the number increased to 44.0% (77) in moderate scores achievement level. Thus, 26.06% (49) achievement increased in the moderate achievement level in the comparison group, and 44.00% (77) scores achievement increased in experimental achievement.

Similarly, no learners in each group achieved an advanced achievement level in the pre-test. However, (post-test) after traditional teaching number increased by 10.11% (19) scores in the advanced achievement level in the comparison group, and the number achieved in the experimental group increased to 17.71% (31) scores achievement in the advanced level after the intervention.

Table 4.14: Comparison of Achievement Level Percentage Decreased and Increased Results

Achievement Level	Code	percentage	Comparison group	Experimental group
Low Achievement	**LA**	**Decrease**	36.17%	61.71%
Moderate Achievement	**MA**	**Increase**	26.06%	44.00%
Advances Achievement	**AA**	**Increase**	10.11%	17. 71%

The statistics differences in pre- and post-test scores percentage achievement level (Cf Table 4.14) indicate 36.71% and 61.71% decreased in each group low scores achievement level. This implies that 36.71% of learners in the comparison group and 61.71% of learners in the experimental group with low achieved scores enhanced knowledge and understanding after teaching and achieved higher scores and moved from a low achievement level to a higher level. Similarly, after teaching and intervention learners with moderate scores achievement levels enhanced better understanding (Cf Table 4.14). Thus, a 36.17% decrease in the comparison group is less than a 61.71% decrease in the experimental group in low scores achievement level. Similarly, a 26.06% increase in moderate score achievement level in the comparison group is less than 44% moderate scores in the experimental group moderate scores achievement level. Problem-solving yielded high achievement. Similarly, a 10.11% increase in advanced achievement level is less than a 17.71% increase in the experimental group's advanced achievement level therefore, problem-solving intervention in the experimental group yielded more increase in advanced achievement. Additional information (Cf. Figure 4.15).

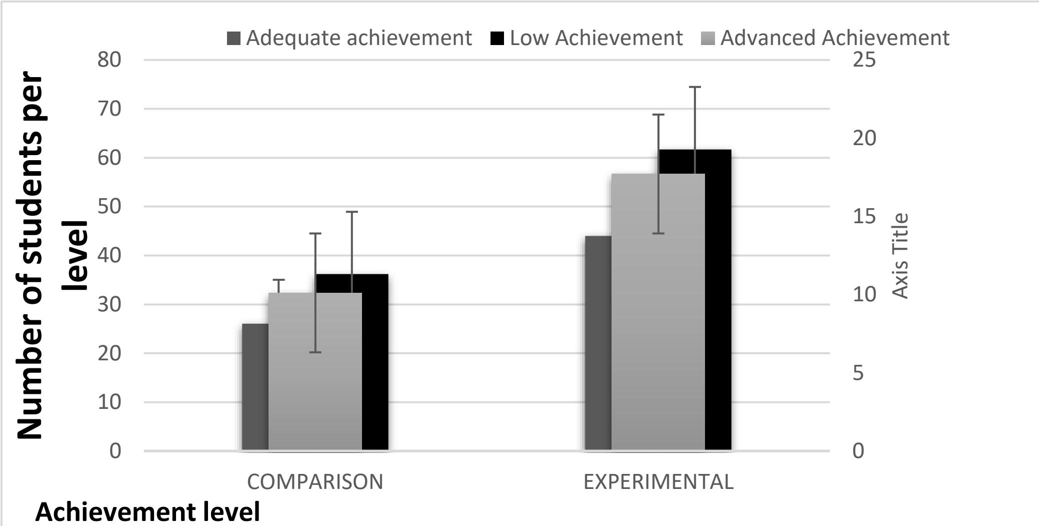


Figure 4.13: Cognitive achievement test level (Experimental and comparison groups)

Conclusion: Reached 61.71%, 17.71%, and 44% decrease and increase achievement levels in the experimental group. This implies that problem-solving teaching enhanced learners' achievement score levels. Thus, the research question 'How does problem-solving teaching approach enhance the learning of fractions in Grade 8?' is answered by the evidence that shows that the problem-solving teaching approach enhances learners' learning of fractions in Grade 8 in the achievement of higher scores.

4.3.3 The Test Questions

The pre- and post-test questions (Cf Appendix G) are the same problem. Questions 1-5 are based on equivalent, addition, and subtraction fractions. Question 1 addressed learners' levels of problem understanding of fractions as shown below (Cf Table 4.15).

Table 4.15: Experimental Pre and post-test question 1

Question 1: Farmer Ben counted all his livestock yesterday. $\frac{1}{4}$ of the animals were pigs, $\frac{2}{3}$ of the animals were cows, and $\frac{1}{3}$ were chickens. Which of these animals did Ben have the most? 

This question is posed as an abstract and deductive question. This is how questions are presented in most Grade 8 textbooks for assessment purposes with the expectation that pictures would assist learners to have a clearer understanding of the questions and recall their prior memorised solution. To test learners' knowledge of equivalent fractions, the questions would be posed as "Convert $\frac{1}{4}$, $\frac{2}{3}$ and $\frac{1}{3}$ to decimal and percentage." Alternatively, learners can be asked to "arrange the fractions in ascending order." However, if the questions were posed as suggested previously and learners were to answer the question from their prior knowledge and understanding through memorisation, no critical thinking and analysis skills would have been necessary. Thus, straight questions would produce direct responses. Furthermore, in this study, most learners had in their memory that bigger fraction numbers like the numerator and denominator stand for bigger values. Thus, these misconceptions can be minimised in learning if learners are guided using proportional reasoning and questioning to build critical facts leading to the logical conclusion that bigger numerators and denominators do not represent fraction values. Moreover, a smaller numerator and denominator order of fractions can be equal to and greater than a fraction with a bigger numerator and denominator. For example, $\frac{50}{100}$ equal $\frac{1}{2}$. Therefore, questions should be structured to help learners understand mathematical processes, that is, to display the connection between mathematical expression and equation systems and to construct mathematical simplification. Furthermore, learners should search for structure, patterns, and relationships and avoid the generalisation of the whole number in their solution. Secondly, questions were structured for learners to be able to apply mathematical problem-solving skills, decipher meaning, and engage in sense-making activities, rather than applying a straightforward formula and substitution. Questions were structured in a way that enhanced concept discovery through sense-making and exploring the relations and patterns. In addition, critical thinking and self-discovery of concepts and relationships were encouraged. Learners were expected to understand the questions through exploration and self-discovery. Therefore, enhancement of mathematical problem-solving skills is crucial to assist learners to be successful in self-discoveries of fractions mathematical concepts.

4.3.3.1 Test analysis per problem-solving knowledge and understanding of fractions

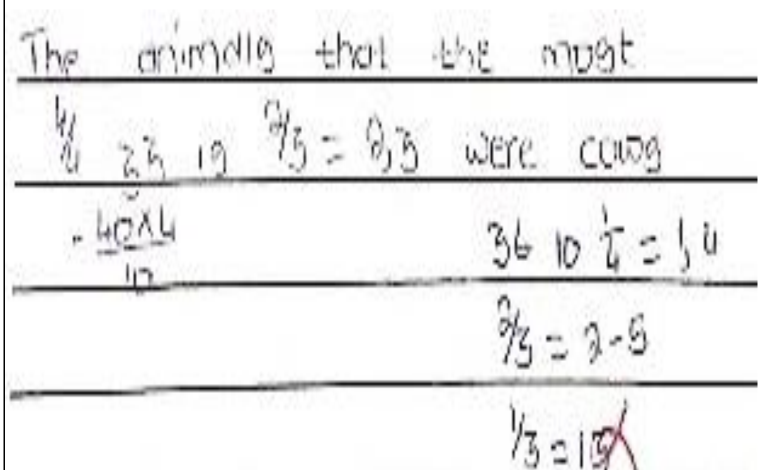
This section reports learners' pre- and post-test achievement on problem understanding, devising a plan, carrying out the plan, and evaluating the solution according to Polya's (1957) framework.

Furthermore, learners' answers are categorised into Correct (C), Incomplete (Inco), Incorrect (Inc), and Empty (E).

4.3.3.1.1 Understand the Problem Phase

Problem understanding is evaluated using Question 1 (cf. Appendix G). Learners were to read the question carefully, relate the picture information to fraction values and draw conclusions based on the fraction. They would then write the ordinate value of fractions and connect associations in the picture. Prior knowledge was necessary. Learners were required to show the problem understanding skills. However, pre-test answers showed that learners were inexperienced and faced challenges in applying problem-solving skills to answer Question 1. Learners' responses are as follows:

Table 4.16: Learner EGS3.1's answers: pre- and post-test

Pre-test (Before intervention)	Vignette 4.1  The animals that the most 4/4 23 is 2/3 = 0,3 were cows - 40x4/10 36 10 1/4 = 1,0 2/3 = 2-5 1/3 = 15

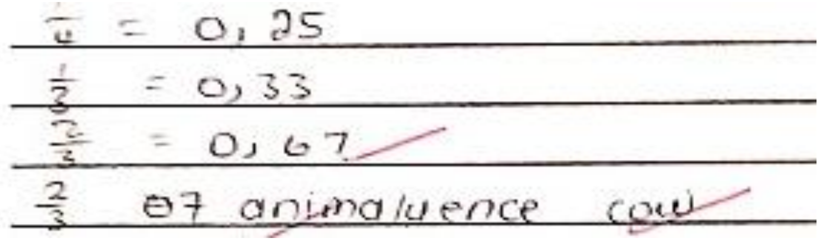
Post-test (After intervention)	Vignette 4.2 

Table 4.17: Phase 1: Learners' Results According to Understanding of the Problem

CODE	**TEST**	**(CA)**	**INCA**	**ICA**	**EA**
CGS	Pre-test	2.13% (4)	19.68% (37)	73.40% (138)	4.79% (09)
EGS	Pre-test	2.29% (4)	44.00% (77)	41.71% (73)	12.00% (21)
CGS	Post-test	20.74% (39)	30.34% (57)	43.09% (81)	5.85% (11)
EGS	Post-test	46.29% (81)	24.57% (43)	25.71% (45)	3.43% (6)

Findings in the pre-test understanding problem phase showed the number of learners who understood Question 1. The pre-test correct answers are the same in both groups (Cf Table 4.16). However, findings in the post-test showed learners in the experimental group improved in knowledge and understanding of problem skills after the intervention and were able to answer the question. Learners in the experimental group achieved more correct post-test answers (cf. Table 4.16). This implies that problem-solving intervention enhanced learners' problem-solving skills. Thus, most individual learners in experimental classes achieved advanced achievement scores level in the post-test. The problem-understanding phase achieved a 44.29% increase in correct answers in the experimental group and 10.11% in the comparison group.

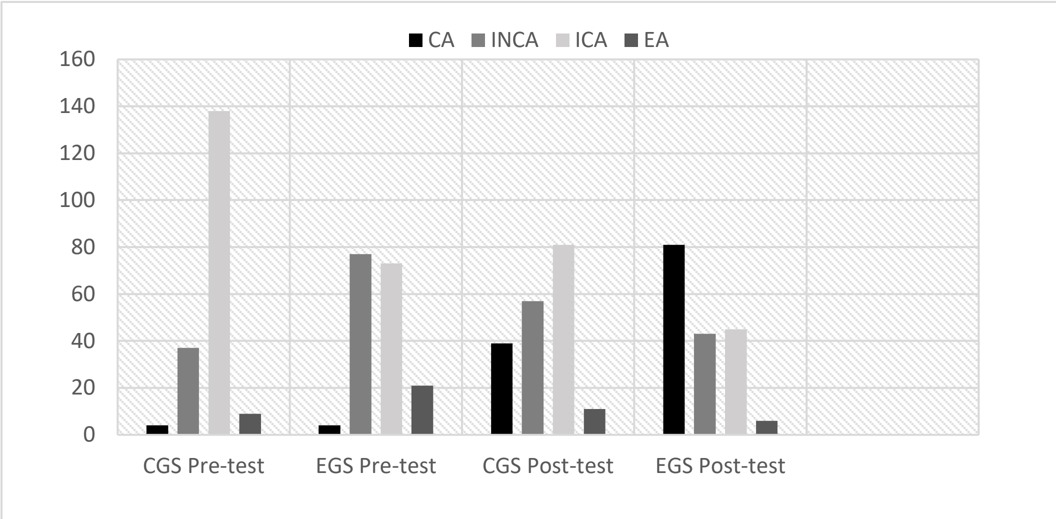


Figure 4.14: Illustrating Learners' Knowledge of Problem Understanding in Phase 1

4.3.3.1.2 Devise Plan Phase

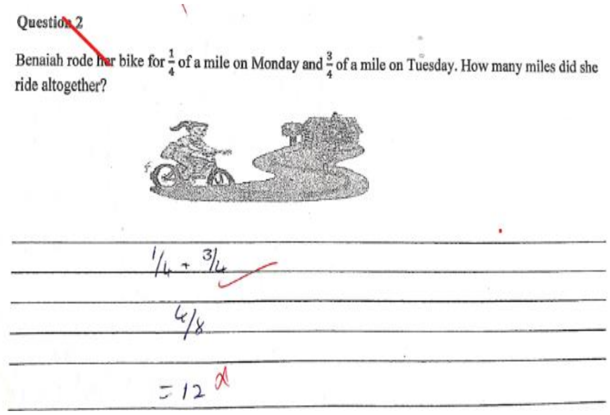
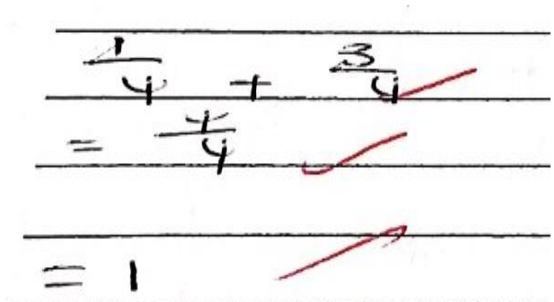
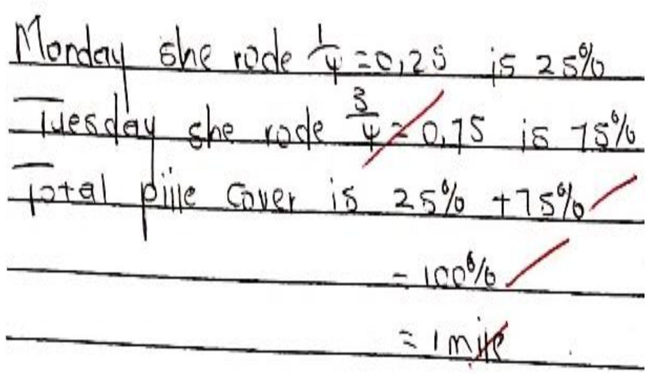
The Devise plan phase evaluated Question 2. Skills that were envisaged in solving this problem were critical thinking skills, drawing, guessing, and checking through patterns. In addition, the phase involved creating expressions between known fractions and operations. In addition, Question 2 assessed devising plan skills and the ability to interpret fraction problem operations. Thus, learners had to apply various methods like conversion, equivalent, decimal and percentage. They applied prior knowledge and used common factors to draw a division line. However, the pre-test findings showed that most learners had challenges identifying and devising plans (cf. Table 4.19).

Table 4.18: Phase 2: Learners' Results According to Devise Plan

CODE	**TEST**	**(CA)**	**INCA**	**ICA**	**EA**
CGS	Pre-test	5.85% (11)	37.78% (71)	51.06% (96)	5.31% (10)
EGS	Pre-test	2.29% (04)	56.57% (99)	24.00% (42)	17.14% (30)
CGS	Post-test	39.89% (75)	45.75% (86)	9.05% (17)	5.31% (10)
EGS	Post-test	54.86% (96)	28.57% (50)	13.14% (23)	3.43% (06)

Findings in this phase showed learners devised plans in the experimental group and learners' empty answers decreased in both groups. Correct and incomplete answers increased in the comparison group. However, post-test findings showed 54.86% (96) correct answers in the experimental group and 39.89% (75) in the comparison group. Furthermore, the number of empty answers declined in the post-test. This confirms learners' challenges eased in calculations and application of problem-solving procedures and previous mistakes were minimised after the problem-solving intervention.

Table 4.19: EGS2.16's answer: pre – post-test

Pre-test (Before intervention)	Vignette 4.3  Question 2 Benaiah rode her bike for $\frac{1}{4}$ of a mile on Monday and $\frac{3}{4}$ of a mile on Tuesday. How many miles did she ride altogether? $1/4 + 3/4$ $4/8$ $= 12$
Pre-test (After intervention)	Vignette 4.4  $\frac{1}{4} + \frac{3}{4}$ $= \frac{4}{4}$ $= 1$
Post-test (After intervention)	Vignette 4.5  Monday she rode $\frac{1}{4}$ = 0,25 is 25% Tuesday she rode $\frac{3}{4}$ = 0,75 is 75% Total bike cover is 25% + 75% = 100% = 1 mile

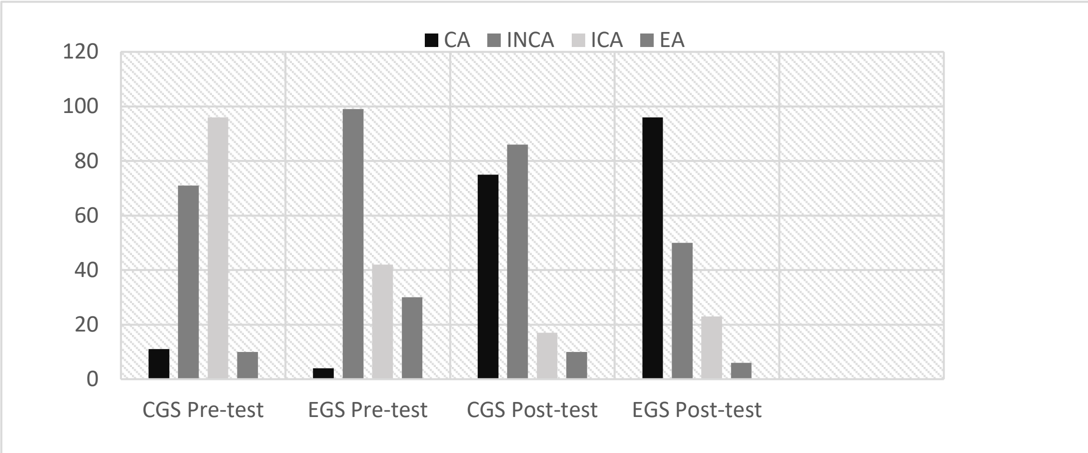


Figure 4.15: Illustrating Learners' Knowledge of Phase 2 (Devise Plan)

4.3.3.1.3 Carry out the Plan Phase

This phase was evaluated through Question 3. Learners had to add unlike fractions. For example, $\frac{2}{3}+\frac{5}{6}$. Learners had to demonstrate procedure calculation knowledge and proper planning to complete this phase. However, learners who did not devise plans were not able to proceed and committed procedural mistakes. Characteristics required when learners carried out their plans were: (1) patience, (2) persistence, and (3) readiness to devise an alternate plan if their first plan did not yield the expected results. Thus, this phase required different solving strategies using present operation concepts to convert fractions to other equivalent decimals and percentages. For example, using the length model to differentiate the distance between $\frac{2}{3}$ and $\frac{5}{6}$. However, pre-test findings showed that learners could not use manipulatives to represent fractions. This implies that learners could not link abstract concepts to concrete objects. They could not decode words to symbolic operations. They could relate addition operations. In their attempt to *carry out the plan*, learners could specify the concept but could not state how and where they applied it in problem-solving. Thus, pre-test findings showed learners lacked *carrying out the plan* skills by wrongly applying addition as a multiplication operation, as shown in learners' responses below (cf. Table 4.19).

Table 4.20: CGS1.17's Answers: Pre- and Post-Test

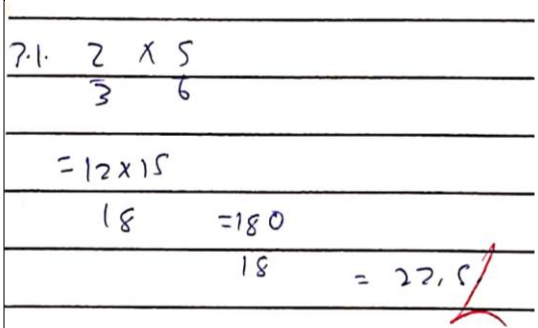
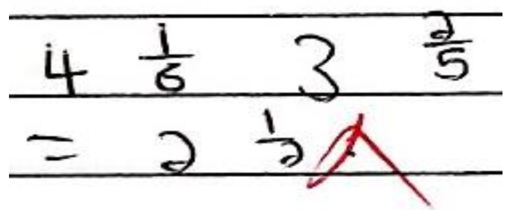
Pre-test (Before intervention)	Vignette 4.5 
Post-test (After intervention)	Vignette 4.6 

Table 4.21: Phase 3 Learners' Results in Carrying Out the Plan

CODE	**TEST**	**(CA)**	**INCA**	**ICA**	**EA**
CGS	Pre-test	2.66% (05)	22.34% (42)	66.49% (125)	8.51% (16)
EGS	Pre-test	0.00% (00)	8.00% (14)	64.00% (112)	28.00% (49)
CGS	Post-test	12.23% (23)	29.78% (56)	48.94% (92)	9.04% (17)
EGS	Post-test	15.43% (27)	30.88% (54)	44.57% (78)	9.14% (16)

Furthermore, the pre-test showed that learners were unable to carry out their plans. Thus, showing an inability to perform basic addition operations and interpret concepts to achieve answers (Cf Table 4.21). Furthermore, the pre-test showed calculations were not performed accurately due to a lack of essential visual concrete model and problem-solving carry out the plan problem-solving stage. However, post-test findings showed learners increased low, moderate, and advanced achievement score levels. Thus, 12.23% (23) achieved correct answers in the comparison and 15.43% (27) achieved in the experimental group.

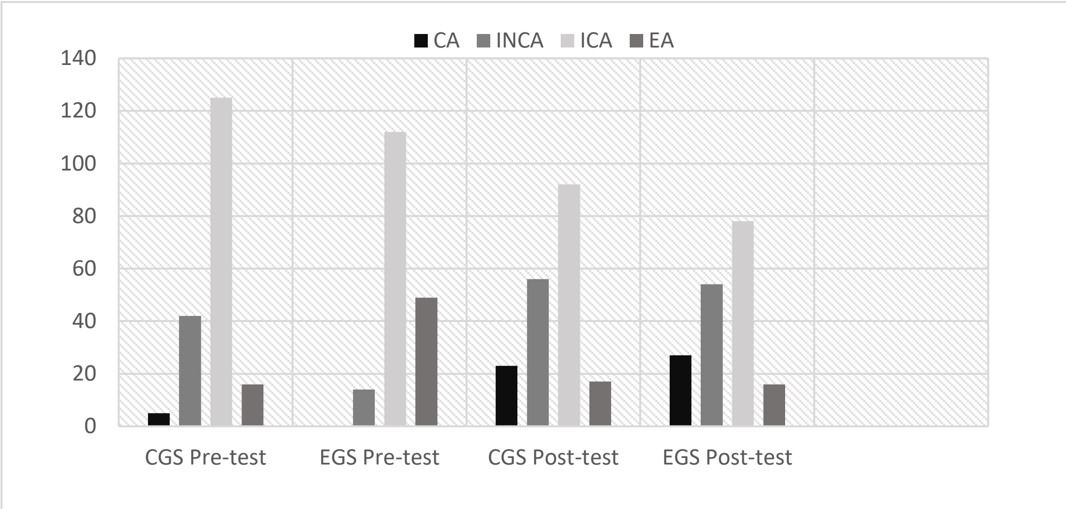
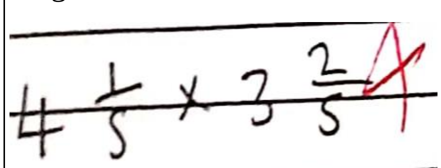


Figure 4.16: Illustrating Learners' Knowledge of Carrying out the Plan Phase 3

4.3.3.1.4 Carry out the plan phase

Furthermore, Question 4 addresses Phase 3. However, the question structure was for learners to be able to apply previous knowledge. Learners were expected to display a higher level of procedural skills. The expectation was to relate the number of pizzas eaten to solve the problem using addition or subtraction operations. However, pre-test findings showed errors like that of Question 3. This posed challenges that learners would not progress to the next phase if they could not complete a phase.

Table 4.22: EGS2.32's answer: pre – post-test

Pre-test (Before intervention)	Vignette 4.7 

Post-test (After intervention)	Vignette 4.8 

Table 4.23: Phase 4 learners' results in carrying out the plan

CODE	**TEST**	**(CA)**	**INCA**	**ICA**	**EA**
CGS	Pre-test	3.19% (06)	44.28% (87)	42.55% (80)	7.98% (15)
EGS	Pre-test	0.00% (0)	34.29% (60)	33.71% (59)	32.00% (56)
CGS	Post-test	27.66% (52)	51.60% (97)	17.02% (32)	3.72% (07)
EGS	Post-test	34.28% (60)	43.43% (76)	10.86% (19)	11.43% (20)

The pre-test findings showed that most of the learners failed to attempt Question 4 (Cf Table 4.23) because 3.19% (6) completed pre-tests and none in the experimental group. The analysis showed that learners possessed insufficient knowledge of basic calculation, procedures, problem-solving and carrying out the plan skills. This substantiates the fact that the learners lacked problem-solving and carrying out the plan skills. Hence, learners could not carry out the plan and solve the question correctly. However, post-test findings showed 34.28% (60) correct answers achievement in the experimental group. This shows learners acquired problem-solving and the carrying out the plan skills during the intervention. Findings showed an increase across CGS and EGS. The increase in the experimental group indicates an enhancement in learners' problem-solving and carry out plan skills on learning fractions.

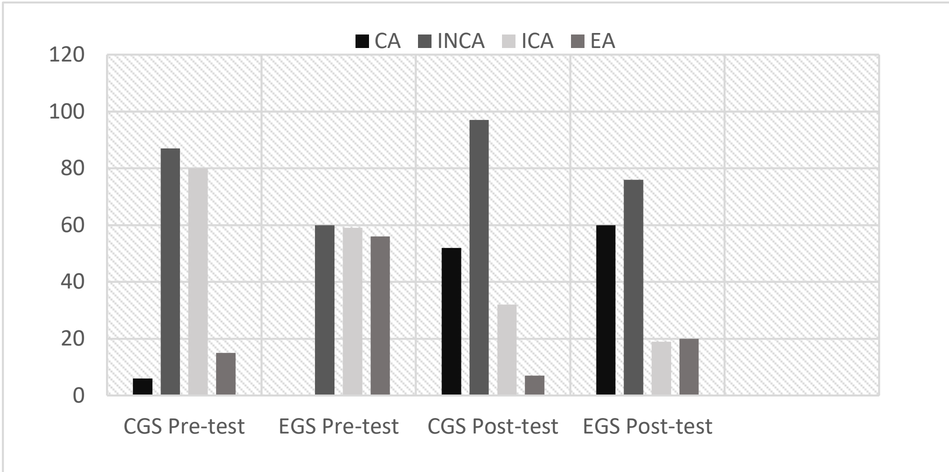


Figure 4.17: Illustrating Learners' Knowledge in Carrying out the Plan Phase 3

4.3.3.1.5 Evaluate the Solution

This phase was assessed through Question 5. It assessed learners' problem-solving evaluation skills. Learners conceptualised concepts by evaluating solutions. The question required learners to reflect on the process, and how they obtained solutions and expressed their answers. However, pre-test findings showed that learners lacked basic problem-solving and solution evaluation skills because most of them could not evaluate their problems. Thus, none of the learners evaluated the solution in the pre-test (Cf Table 4.25). Thus, a high percentage of empty responses showed that learners lacked basic evaluation skills before the intervention.

Table 4.24: EGS1.29's answers: pre- and post-test

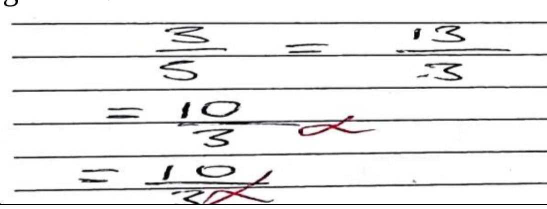
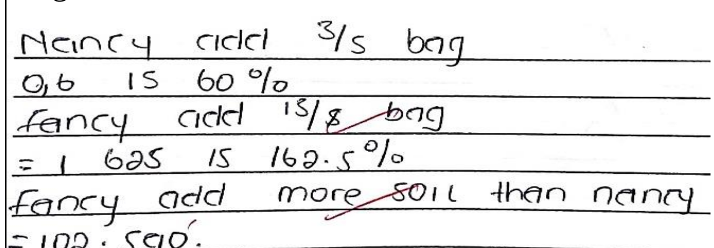
Pre-test (Before intervention)	Vignette 4.7 
Post-test (After intervention)	Vignette 4.8 

Table 4.25: Phase 5 Learners' Results in Solution Evaluation

CODE	TEST	(CA)	INCA	ICA	EA
CGS	Pre-test	0.00% (0)	23.40% (44)	58.51% (110)	18.09% (34)
EGS	Pre-test	0.00% (0)	3.43% (06)	50.29% (88)	46.29% (81)
CGS	Post-test	0.00% (0)	32.98% (62)	40.43% (76)	26.60% (50)
EGS	Post-test	24.57% (43)	28.00% (49)	35.43% (62)	12.00% (21)

Post-test findings showed learners' achievements in evaluating solutions in the experimental group and no achievement in the comparison group. This implies that this phase is not easy to achieve in classes taught using the traditional approach. However, post-test findings showed 24.57% answers in the experimental group. This achievement implies that learners enhanced their evaluating of solution skills. In addition, learners transitioned from low achievement levels of problem understanding to higher levels of knowledge. Hence, learners declined tremendously in misconceptions errors. The finding justified learners lacking problem-solving and evaluation of solution skills. Additional information is illustrated (Cf Figure 4.18).

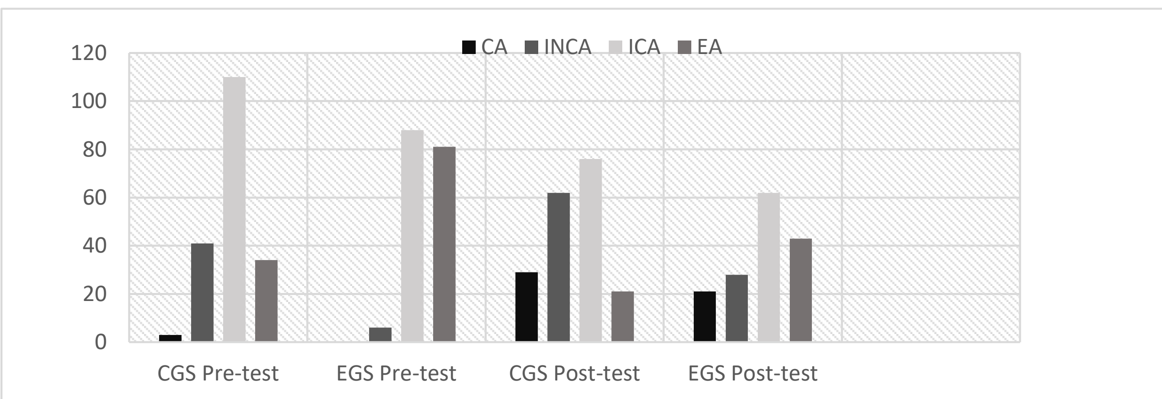


Figure 4.18: Illustrating Learners' Responses in the Evaluation of the Solution

Table 4.26: Overall Summary of Skills Acquired per Level of Knowledge and Understanding

Question and phase level		**Problem-solving phase**	**Comparison group**			**Experimental group**		
			Pre-	Post	Diff	Pre-	Post-	Diff
1	**L1**	Understand the problem	2.13%	20.74%	18.61%	2.29%	46.29%	44%
2	**L2**	Devise plan	5.85%	39.89%	34.04%	2.29%	54.86%	52.57%
3	**L3**	Carry out Plan	2.66%	12.23%	9.57%	0.00%	15.43%	15.43%

4	**L3**	Carry out plan	3.19%	27.66%	24.74%	0.00%	34.28%	34.28%
5	**L4**	Evaluate the solution	0.00%	0.00%	0.00%	0.00%	24.57%	24.57%

Table 4.26 above shows improvement in all phases in the experimental group. In addition, they achieved a higher percentage than learners in the comparison group. The phase of evaluating the solution seemed challenging to both groups. Learners in the traditional classes were not exposed to problem-solving and evaluation of solution skills. However, after problem-solving intervention, 24.57% was recorded in the experimental group and 0.00% in the comparison group. Therefore, learners improved in problem-solving and solution evaluation skills. Additional information is provided in Figure 4.19 below.

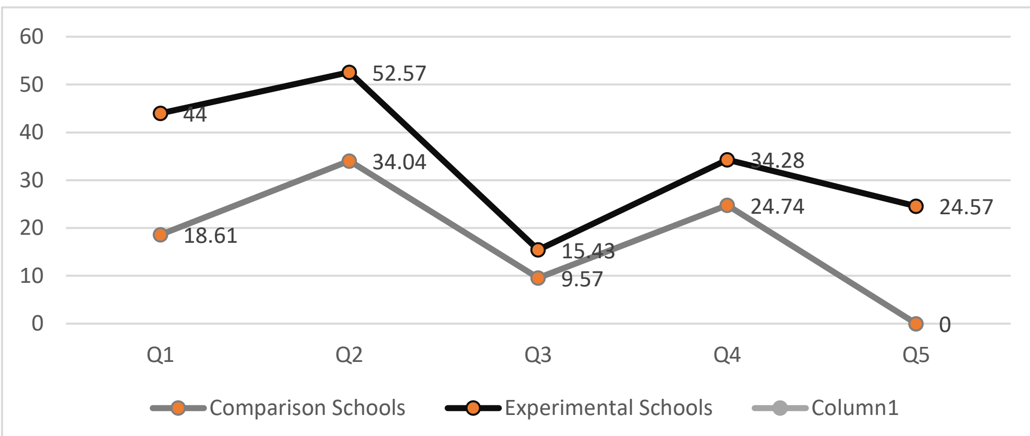


Figure 4.19: Overall Comparative Group's Content Knowledge and Understanding

4.3.4 Errors Committed Before Learning Fractions

This section discusses errors committed by learners with examples. It presents the challenges that learner participants in this study experienced when solving addition and subtraction of fractions. The research question 'What challenges do learners experience in solving problems on addition and subtraction of fractions?' is clarified (Cf Table 4.26) by the summary below.

Table 4.27: Summary of errors committed in pre-and post-test

Error type	Example

Grammatical, lexicon, and irrelevant words Learners' challenges included not writing conclusions on evaluating the solution stage and not simplifying answers.	Question 1 conclusion: the that. $\frac{2}{3}$. Question 2 conclusion: miles that she will ride altogether is. $\frac{4}{4}$. Question 5. adding. $\frac{10}{3}$. soil
Calculation Error	**Referring to addition and subtraction**
Misconception on addition procedures.	Question 2. $\frac{1+3}{4} = \frac{3}{4}$. Question 3. $\frac{2}{3} + \frac{5}{6} = \frac{1}{6}$. Question 4. $\frac{1}{3} - \frac{1}{4} = \frac{1-1}{3-4} = \frac{4}{2}$..
Operation Error and writing out of context	**Wrongfully using operation**
Applying multiplication instead of addition **and** calculating instead of listing or choosing.	Question 1. $\frac{1}{4} + \frac{2}{3} + \frac{1}{2} = \frac{4}{10}$. Question 2. $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$.
Forcing solution and procedural error.	Writing out of the context Question1.$\frac{1}{4} + \frac{2}{3} + \frac{1}{2} = \frac{4}{10} = 4 \div 2$. Question 1. $\frac{1}{4}\ pigs, \frac{2}{3} cows, \frac{1}{3} chicken$ 4 pigs + 1 cows 3 chicken = pigs
Errors due to problem-solving. Unable to read and understand the question.	Empty response
Error due to misconceptions and lack of procedural knowledge.	Question 3: $\frac{2}{3} + \frac{5}{6} = \frac{2+5}{3+6} = \frac{7}{9}$. Question 4: $4\frac{1}{5} + 3\frac{2}{5} = \frac{10}{5} + \frac{9}{5} = \frac{10+9}{5+5} = \frac{19}{10}$

4.4 QUESTIONNAIRE ANALYSIS

The questionnaire analysis, inferential and descriptive analyses' components are: (I) understanding based on the importance of fractions, (II) understanding based on fractions' meaning, (III) understanding based on the teaching approach, (IV) understanding based on learning approach, and (V) understanding based on fractions concept.

4.4.1. Analysis Based on Learners' Knowledge and Understanding of Fractions

Below are Shapiro-Wilk test results for learners' understanding based on yes/agree and no/disagree response distribution. The findings (cf. Table 4.28) reveal that learners' yes/agree, and no/disagree responses were distributed normally thus, one-way ANOVA was adopted to analyse learners' understanding.

Table 4.28: General Statistics Normality Yes/Agree and No/Disagree Distributions

Overall	Code	Response	N	C	Mean	Med	Std.	K	T-stat	P-value	Decision
School	CGS	Yes/agree	188	15	90.89	87.0	9.279	1.924	0.943	0.416	Normal
		No/disagree	188	15	31.07	31.0	9.896	2.509	0.963	0.747	Normal
		Not certain	188	15	3.93	4.0	1.982	3.646	0.891	0.068	Normal
	EGS	Yes/agree	175	15	84.72	85.0	12.75	2.247	0.981	0.975	Normal
School		No/disagree	175	15	29.0	29.0	10.60	2.095	0.968	0.835	Normal
		Not certain	175	15	2.4	2.0	1.624	2.820	0.937	0.344	Normal

Statistically significant P>0.05 for learners' responses average percentage performances 48.35% (90.89) and 48.41% (84.72) imply that responses from each group are compatible and can be compared on normality significance of 0.050 for statistical significance. Furthermore, this implies that the number of responses from each group is equivalent and can be compared for analysis. Additional information (Cf Figure 4.33).

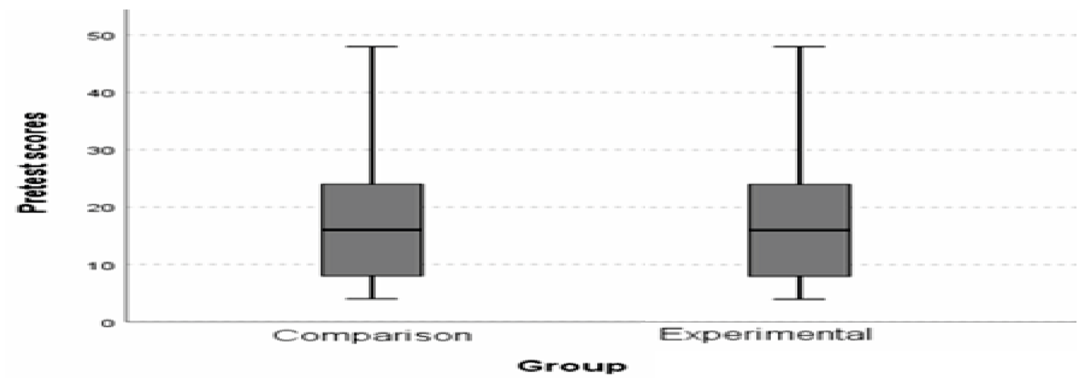


Figure 4.20: Boxplot Diagram Illustrating Group Normality Distribution

The boxplots are symmetrical, this implies that yes/agree and no/disagree responses are from normal distribution groups. The implication is that responses are compatible with 0.096.

4.4.2 Analysis per Experimental and Comparison Learners' Knowledge and Understanding

The one-way ANOVA assesses for differences in learners' yes/agree and no/disagree responses. The test had the following hypothesis: **(H_0)**: There is no difference in learners' understanding of yes/agree and no/disagree knowledge distribution. **(H_1)**: There is a difference in learners' understanding of yes/agree and no/disagree knowledge distribution.

Table 4.29: Comparison and Experimental groups' knowledge and understanding results

Theme	**Yes/Agree responses**				**No/Disagree responses**				
Group	**N**	C	M	SEM	**Group**	**N**	C	M	SEM
CGS 1	62	5	94.4	5.269	**CGS 1**	62	5	94.4	5.269
CGS 2	66	5	88.6	4.885	**CGS 2**	66	5	88.6	4.885
CGS 3	60	5	85.8	1.828	**CGS 3**	60	5	85.8	1.828
Sources	**Sum-square**	**Mean square**	**DF**	**Fiscal**	**Sources**	**Sum-square**	**Mean square**	**DF**	**Fiscal**
Between	192.400	96.2	**2**	1.050	**Between**	326.933	163.27	**2**	1.718
Within	1099.200	91.6	**12**	**P-value**	**Within**	1142.00	95.17	**14**	**P-value**
Total	1291.6			0.379	**Total**	1468.933			0.221
Experimental Group									
Theme	**Yes/Agree responses**				**No/Disagree responses**				
Group	**N**	C	M	SEM	**Group**	**N**	C	M	SEM
EGS 1	49	5	73.2	4.140	**CGS 1**	62	5	22.4	4.308
EGS 2	62	5	87.0	4.219	**CGS 2**	66	5	33.4	4.523
EGS 3	64	5	95.8	4.695	**CGS 3**	60	5	31.2	5.323
Sources	**Sum-square**	**Mean square**	**DF**	**Fiscal**	**Sources**	**Sum-square**	**Mean square**	**DF**	**Fiscal**
Between	1297.733	648.87	**2**	6.832	**Between**	338.800	169.40	**2**	6.832
Within	1139.6	94.97	**12**	**P-value**	**Within**	1347.200	112.27	**14**	**P-value**
Total	2437.333			0.010	**Total**	1686.000			0.260

The results in Table 4.29 show no statistically significant values of p= 0.379 & 0.122 SEM = 5.26, 4.88, and 1.82 in the comparison group's responses yes/agree and no/disagree. The implication is not statistically significant, there are no differences in yes/agree and no/disagree response distribution. However, statistically significant values of p= 0.010 SEM = 4.140, 4.219 & 4.695 in the experimental group's responses yes/agree and not statistically significant effects of P = 0.260 SEM = 4.308, 4.523 & 5.323 in learners' no/disagree responses distribution. This implication is that there is a difference between yes/agree and no/disagree responses in understanding of fractions. Thus, the experimental group learners showed understanding knowledge, and the comparison group learners did not. This further, showed that experimental learners agreed based on their understanding. See additional information (Cf Figure 4.25).

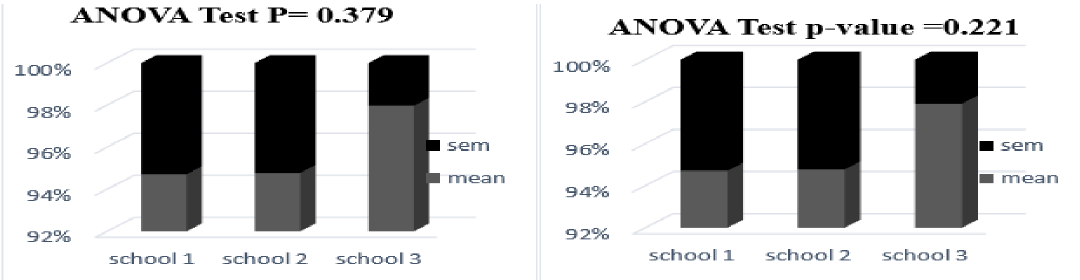


Figure 4.21: Comparison group yes/agree and no/disagree ANOVA test distribution

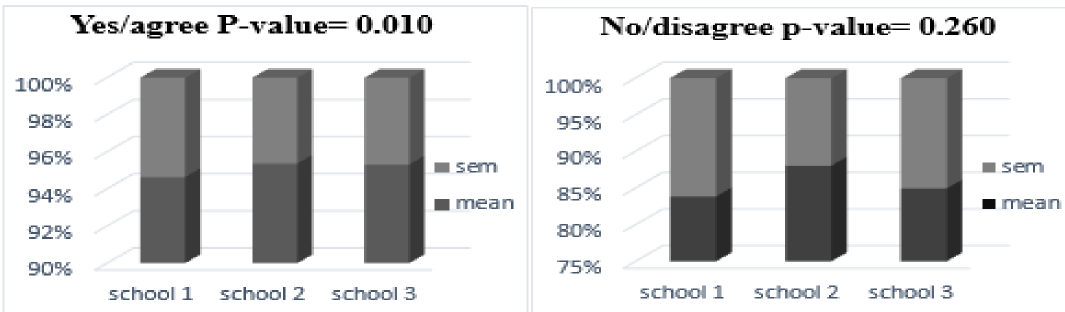


Figure 4.22: Experimental Group School ANOVA Yes/Agree Test-Distribution

Findings in (Cf Figure 4.25) showed learners' yes/agree and no/disagree responses distribution are equal which does not tally the questionnaire's yes/agree and no/disagree. This implies that learners answered the questionnaire questions without understanding the questions. Therefore, yes/agree is equal to no/disagree. Therefore, traditional teaching does not enhance learners' knowledge and understanding of fractions. However, the experimental group's yes/agree and no/disagree knowledge and understanding of the questions tally with the context. This implies that the problem-solving intervention enhanced learners' knowledge and understanding of fractions. Thus, the main research question 'What is the effect of problem-solving teaching approach in learning fractions in Grade 8?' was clarified based on one-way ANOVA statistically significant values p=0.010 and 0.260.

Table 4.30: Summary of one-way ANOVA Decision per effects on Teaching Approach

Null Hypothesis	Test	Sig.	Decision

1	The distribution of yes/agree and no/disagree responses is the same in the comparison group.	Dependent One-Way ANOVA	0.379 and .022	Retain the null hypothesis.
2	The distribution of yes/agree and no/disagree responses is in the same experimental group.	Dependent One-Way ANOVA.	.010 and .260	Reject the null hypothesis.
Asymptotic significance displayed. The significance level is .050.				

4.4.3 Questionnaire Descriptive Analysis

4.4.3.1 Analysis Based on Learners' Understanding of Problem-solving and Fractions

The analysis is based on learners' understanding of fractions and problem-solving enhancement. It is categorised into five components which are shown in Table 4.31 below.

Table 4.31: Learners' Knowledge and Understanding of Fraction Results

Questionnaire responses results		**Comparison**			**Experimental**		
S/N	QUESTIONS	Yes	No	none	Yes	No	none
A: Understanding Based on the Importance of Fractions							
1	Do you think it is important for learners to learn fractions in Grade 8?	152	34	02	167	08	00
2	Can the knowledge of fractions apply in the real world?	85	89	14	108	62	05
B: Understanding Based on Fractions Meaning							
3	Do you think $\frac{3}{5}$ is a proper fraction?	155	31	2	153	20	02
4	Can five (5) friends share three candies among themselves?	134	45	9	115	50	10
C: Understanding Based on Teaching Approach							
5	Is it good for educators to explain fractions using real-life stories like telling of time or sharing of objects?	107	78	3	116	56	03
6	Do you think fractions can be understood in group study?	142	41	05	150	34	1
D: Understanding Based on Learning Approach							
7	Do you think solving complex exercises in fractions assists in developing our minds for us to answer other complex questions in another mathematics topic?	151	33	04	139	30	06
8	Do you think it is okay to try solving questions over and again if you do not get the answer to the question/problem?	147	32	09	153	19	03
E: Understanding Based on Fraction Concepts							
9	Do you understand half has $\frac{3}{2}$?	**140**	46	02	**41**	131	02
10	Do you think $\frac{3}{2}$ can be present in any fraction model?	**142**	37	09	**13**	153	09
	Total	**1355**	466	59	.1155	563	43

Results (Cf Table 4.31) showed a total of 1355 yes/agree and 466 no/disagree responses in the comparison group. Thus, 282 yes/agree do not tally with the questionnaire's understanding and implies that, 20.81% (282) yes/agree to questions without proper understanding, and they did not enhance their understanding knowledge in the comparison group. Furthermore, the table shows that a total of 1155 yes/agree and 563 no/disagree in the experimental group. Thus, 4.68% (54) answered yes/agree to questions that disagreed with the questionnaire understanding. The implication is that 4.68% (54) of learners in the experimental group responded yes/agree without understanding the questions' context and did not improve in problem-solving.

In conclusion, 4.68% (54) of learners in the experimental group demonstrated inadequate knowledge of fractions in learning. Thus, it is less when compared to 20.81% (282) in the comparison group. The implication is that 95.32% of learners in the experimental group acquired understanding, knowledge, and problem-solving skills, and 79.19% in the comparison group acquired understanding knowledge. Thus, the research question 'What problem-solving skills are essential to enhance cognition in addition and subtraction of fractions?' is clarified, 4.68% (54) and 95.32% (1101) in the experimental group that understanding the problem, devising a plan, carrying out a plan and evaluating a solution are essential problem-solving skills required for learners to achieve better understanding.

4.4.4 Questionnaire Summary

The Shapiro-Wilks test was conducted on each group's yes/agree and no/disagree responses to determine parametric analysis. The results showed yes/agree and no/disagree responses distributed normally. Thus, the one-way ANOVA technique was adopted for analysing the questionnaire's findings. The one-way ANOVA results (Cf Table 4.29) showed no statistically significant values (P= 0.379 & 0.122 SEM = 5.26, 4.88 and 1.82) in the comparison group's yes/agree and no / disagree distribution. Moreover, it showed statistically significant (P= 0.010 SEM = 4.140, 4.219 & 4.695) in yes/agree and not statistically significant (P=0.260 SEM= 4.308, 4.523 & 5.323) from no/disagree findings in the experimental group. The implication is that learners' yes/agree, and no/disagree responses distribution was enhanced by the problem-solving teaching intervention. Furthermore, the descriptive analysis showed, 4.68% (54) learners which revealed the number of learners with limited understanding of problem-solving in the experimental group and 20.81% (282) in the comparison group. Findings showed that more learners in the experimental group acquired high problem-solving understanding knowledge and skills. In addition, 95.32% (1101), of the experimental group's learners improved in problem-solving skills when compared to 79.19% (1070) in the comparison group.

Conclusion: Based on 95.32% (1101) in the experimental group, the problem-solving teaching approach enhances learners' understanding of learning fractions. The research question 'What problem-solving skills are essential to enhance cognition in addition and subtraction of fractions?' was clarified (Cf Table 4.29 and 4.31) based on 4.68% (54), 95.32% (1101) and problem-solving understanding problem, devise a plan, carry out the plan and evaluate solution problem-solving skills.

4.5 OBSERVATION REPORT

This section presents reports based on results emanating from teaching and learning observed in the experimental and comparison groups. Observation of learners' learning process was not feasible in the experimental group because learners were guided by the researcher through their learning process. Moreover, intervention teaching through Polya's problem-solving approach seemed to be essential and interconnected as observed in this study because the learning strategies of learners yielded skills that enabled them to acquire learning content informed by knowledge. For example, problem-solving intervention strategies enhanced learners' problem-solving learning strategies, and through problem-solving learning strategies, learners acquired problem-solving skills that enabled them to achieve higher levels of mathematical fractions content knowledge (Cf Table 4.26). Similarly, the teaching approach observed in the comparison school (traditional approach) yielded learning strategies that enabled learners to improve in solving fractions and acquire mathematical fractions content knowledge and understanding. Thus, the approach used in teaching is the main determiner of learning strategies and skills learners apply to fractions content knowledge. Table 4.32 is a summary of observation findings observed from teaching, learning, skills, and content acquired in each group.

Table 4.32: Observation Report

Observation	Experimental group	Comparison group
Teaching strategies	Facilitated using questioning to probe. Conducted demonstration and experimentation of class activities. Guided learners on how and when to use manipulatives to support solutions,	Explained using concept repetition. Class activities were adopted from textbook exercises. Instructed learners on the chalkboard to show operations

	and by writing on the chalkboard for emphasis.	and procedures that enable learners to solve exercises from the textbook.
Learning strategies	Group discussion, group learning, and presentations. The use of manipulative and real-world examples in problem-solving textbooks. Cross-examining solutions to note findings. Completed tasks to commit activities to memory.	Listened to the educators without any distractions. Documented by coping the procedures notes or examples as written by the educator on the chalkboard. Completed similar exercise activities to commit content to memory.
Skills acquired	Understanding problems, devising plans, carrying out of the plans, and evaluating of the solution skills. In addition, proportional and critical thinking skills linked relationships between fraction concepts, knowledge gained using calculations, integration, exploration, identifying, classifying, solving problems, and conducting activities logically.	Memorisation, recitation, explanation, and documentation skills were acquired. Learners imitated educators' examples, procedures, operations, and concepts. Documentation by copying educators' examples from the chalkboard and reviewing content from the textbook individual completing activities.
Content learned	Equivalent fractions, examples from real-world activities. *Types of fractions; proper and mixed numbers relating to common fractions. *Addition and subtraction of fractions relating to common fractions. *Conversion of common fractions into decimal and percentage fractions.	Equivalent fractions, examples from textbook exercises. *Types of fractions; proper and mixed numbers relating to common fractions. *Addition and subtraction of fractions relating to common fractions. *Conversion of common fractions into decimal and percentage fractions.

4.5.1 Analysis Based on Teaching and Learning Model

Table 4.33: Traditional and Problem-solving effects on Learning Fractions Results

Comparison group					
Stage	**level**	**Traditional teaching phase**	Pre-test	**Post-test**	**Differences**
1	**L1**	**Explanation**	2.13%	20.74%	18.61%
2	**L2**	**Documentation**	5.85%	39.89%	34.04%
3	**L3**	**Repetition / recite**	2.66%	12.23%	9.57%
4	**L4**	**Repetition / recite**	3.19%	27.66%	24.74%
5	**L5**	**Memorisation**	0.00%	0.00%	0.00%
Experimental group					
Phase	**level**	**Problem-solving phase**	Pre-test	**Post-test**	**Differences**
1	**L1**	Understand the problem	2.29%	46.29%	44%
2	**L2**	Devise plan	2.29%	54.86%	52.57%
3	**L3**	Carry out Plan	0.00%	15.43%	15.43%
4	**L3**	Carry out plan	0.00%	34.28%	34.28%
5	**L5**	Evaluate the solution	0.00%	24.57%	24.57%

4.5.1.1 Analysis Based on Traditional Teaching Effects on Learning Fractions

The traditional teaching observation approach model is presented in Figure 4.24 below and its effect on learning fractions in the comparison group is presented in Table 4.33 above.

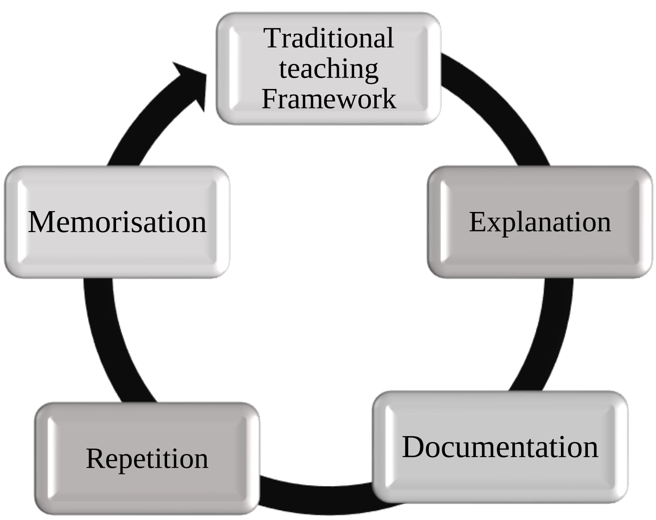


Figure 4.24: Traditional Teaching Observation Approach Model

Stage 1: Explanation

The teaching and learning in the comparison groups were observed to begin with educators' explanations of fractions' meanings, concepts, and definitions. Probing by questioning was not initiated. The learners were instructed to stay quiet; no discussion or distraction of any form was allowed during this stage. Thus, knowledge and understanding were not probed for better proportional reasoning. After the explanation, the educator confirmed by asking learners if they understood the explanation before proceeding to the second stage. The effects were evident (Cf Table 4.33). Before, the explanation (pre-test), 2.13% of correct answers were recorded. However, after the explanation (post-test) the number increased to 20.78% and a change of 18.61% increase was recorded in traditional teaching effects on learning fractions.

Stage 2: Documentation

In the second stage, documentation, learners were allowed time to copy what the educators wrote on the chalkboard. The educators encouraged learners to copy neatly and correctly. In addition, the educators ensured that all learners accurately copied examples, formulas, and procedures. This helped learners commit to activities to memorise and help them solve similar related questions, before teaching (pre-test), 5.82% answered correctly. Thus, after teaching the number increased to 39.89% effect of 34.04%. Thus, traditional teaching improved learners' understanding by 34.04% in Phase 2.

Stage three: Repetition of concepts

In this phase, the educator instructed learners to solve and practise simple similar examples. Educators provided learners with more and similar exercises from the textbooks, these exercises were presented as classwork and homework. The objectives at this stage are to help learners commit work to memorise, when learners repeat or recite the same questions, it enables them to commit procedure and explanation to memory (Weisstein 2022). In this stage, the researcher observed learners flipping through their notes pages checking to ensure that they did not deviate from the educator's procedure. The results of the pre-test findings showed 2.66% correctly answered. However, in the post-test conducted after teaching the number increased to 12.23%, which is a statistical difference of 9.57%. Thus, the traditional approach yielded 9.57% effects on learners' understanding knowledge of learning fractions.

Stage 4: Memorisation

This phase was not emphasised by the educators. However, learners consciously memorised the educators' examples and procedures knowing that similar problems would demand solving. Thus, learners were not exposed to different strategies that they could use to solve the same questions. Therefore, learners could not evaluate the solutions or conceptualise their answers with the questions' context. The results evident (Cf Table 4.33) show that before teaching (pre-test) none of the learners (0.00%) evaluated the solution, and after teaching (post-test), there was no effective difference (0.00%). Hence, traditional teaching as applied in this study does not enhance the learners' understanding and knowledge in evaluating or conceptualising the solution.

4.5.1.2 Analysis Based on Problem-solving Effects on Learning Fractions

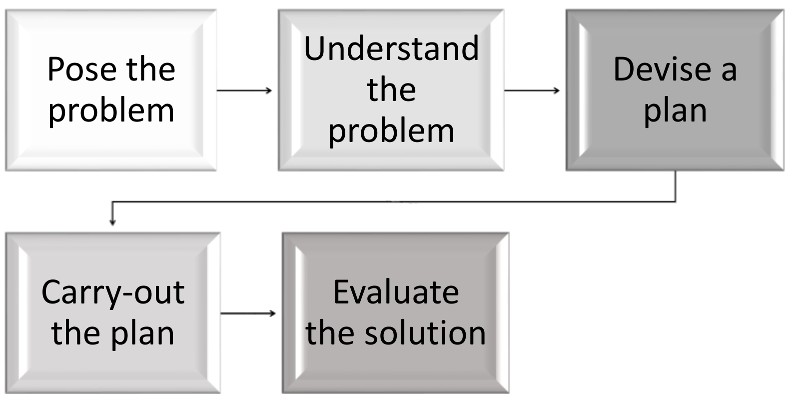


Figure 4.23: Polya's Problem-Solving Teaching and Learning Model

Phase 1: Understand the Problem

This phase used probing by questioning learners. The facilitator asked learners questions like: Can you restate the questions given in your own words? Can you state what is known about this problem? Is there missing information that, if known, would allow you to solve the problem? What is the purpose and why do you need to provide the answers to this question? Furthermore, the educator stressed the importance of identifying equivalent fractions and emphasised by writing examples on the chalkboard for better understanding. The effects were evident (Cf Table 4.33), before the intervention (pre-test), 2.29% of the learners understood the questions. However, after the intervention (post-test), the number increased to 46.29% statistical difference of 44%. Therefore, the problem-solving approach enhances learners' understanding and knowledge of learning fractions. Thus, a 44% effect in learners' understanding knowledge of learning through the problem-solving approach was recorded.

Phase 2: Devise a Plan

Learners were guided on how they could devise a plan to solve the given problem. They were guided using the following statement: make a list of known information, write fraction numbers and identify the values, translate operation words like differences, altogether, of, and over to appropriate operational symbols, like ($-, + \times,$ and $\div$), draw a diagram, using an organised list that shows all possibilities, represent information on tables chart, look for a pattern then write the mathematical expression. Learners were guided through experiential, investigating, and discovering processes using unconventional materials to communicate new knowledge. Additional resources like rectangular regions, geo-board, paper folding, and pattern blocks were provided for learners to use to devise a better and more logical plan. The effects are shown in Table 4.33. Before the intervention, (pre-test) 2.29% of the learners managed to devise a plan. However, after the intervention, 54.86% of the learners managed to devise a plan. Thus, a 52.57% increase was recorded. Therefore, the problem-solving approach enhances learners' knowledge and understanding and develops problem-solving skills such as devising plans for learning fractions.

Phase 3: Carry out the plan

The facilitator encouraged and motivated learners to demonstrate a positive character and attitude toward achieving appropriate solutions during Phase 3. Learners were guided to solve problems patiently and to be ready to start over again if errors or mistakes were observed in their initial plan or if the plan did not yield the expected solutions. Furthermore, learners were advised to keep accurate and neat records of all attempts and other solving skills like calculation, integration, evaluation, and proportional reasoning skills. Thus, learners could reflect on their calculation and simplification processes. The effects are evident (Cf Table 4.34), before the intervention, 0.00% of the learners could carry out the plan. However, after the teaching intervention (post-test), 34.38% of learners could carry out the plan, resulting in a 34.38% effect recorded. Therefore, problem-solving enhances learners' understanding.

Phase 4: Evaluate the Solution

The fourth phase of the problem-solving requires learners to evaluate the solution. In this study, the facilitator emphasised and guided learners through self-evaluation, learners reflected on the solution to ensure that their solution was consistent with the facts of the problem. In addition, answers were interpreted based on the problem content. Furthermore, learners could connect the link between whole numbers and fractional numbers to avoid the following: over generalisation of the solution, false guessing answers, and unnecessary repetition of words that did not associate with the solution. The effects are evident in Table 4.34. Before the intervention, none of the learners evaluated the solution. However, after the intervention (post-test), 24.57% of learners were able to evaluate the solution. The result confirmed that problem-solving learning enhances learners' problem-solving skills in learning fractions.

4.6 DISCUSSION OF TRIANGULATION OF RESULTS

The test findings indicate an improvement in each group's post-test. However, Table 4.2 showed a significance of p=0.000 in the experimental group's average performance. This implies that problem-solving intervention improved learner performance in learning fractions. Learners performed better after the intervention than before the intervention (Cf Table 4.3). Thus, the experimental average performance yielded a better significant value p= 0.000 statistic -4.86 and an average percentage of 22.21% (38.88) greater than the comparison average score of 15.63% (29.38) with an average moderate effective size of 0.0658 statistical difference effectiveness. A similar improvement was evident in learners' performances based on their learning approach. Kruskal-Wallis results (Cf Table 4.4) showed improvement in the group's performance after teaching and intervention. However, the experimental group performance was significant at p=0.000 and H= 30.347, large effective size eta [H]= 0.163. The implication is that learning through problem-solving yielded better performances in learners than the traditional teaching and learning approach. In addition, the average effect distribution of 22% in the problem-solving intervention was observed to be better than the 16% effect on the traditional learning approach. Thus, a statistical difference effect of 0,06 was recorded in the experimental group.

Furthermore, analysing learners' performance was based on the problem-solving approach and traditional approach used in each group. The findings (Cf Table 4.10) in the experimental group revealed a significant value of p=0.000 and an average performance of 89.71% (157) in teaching problem-solving in the experimental group. This is greater compared to the 78.72% (148) traditional teaching approach value. Therefore, problem-solving teaching intervention yielded better teaching of Grade 8 fractions. Therefore, based on a significant value of p=0.000, a statistical average score performance of 0.066, a learners' learning performance score of 0.06, and an effective teaching delivery score of 0.12, were recorded. The decision based on statistical performances is to reject the study's null hypothesis **(H_0)**: There are no differences between the problem-solving teaching approach and the traditional teaching approach on learning fractions and retain the directional hypothesis **(H_1)**: The problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions.

Triangulating the descriptive analytical reports showed a similar improvement. Analysing learners' average scores achievement (Cf Table 4.12) an achievement score of 81% (38.88) in the experimental group was observed than 76% (26.36) achievement scores in the comparison group. In addition, individual learners in the experimental group were observed to achieve higher achievement levels. Thus, 61.71%, 17.71%, and 44% decrease and increase in percentage achievement levels imply that problem-solving teaching enhances learners' understanding achievement of higher scores on learning fractions. Constant improvement in achievement levels was observed in experimental analysis based on problem-solving knowledge and understanding of fractions (Cf Table 4.25). For example, there was no achievement in the evaluation solution phase in both groups during the pre-test. The implication is that all learners were similar in the traditional teaching class. However, an achievement of 24.57% was obtained in the experimental group, and no achievement was recorded in the comparison group. Therefore, traditional teaching does not enhance learners' problem-solving evaluation skills.

Similarly, triangulating questionnaire results showed no statistically significant value p= 0.379 and 0.122 SEM = 5.26, 4.88, and 1.82 in comparisons between yes/agree and no/disagree responses (Cf Table 4.29). The implication is that the traditional teaching approach does not improve learners' knowledge and understanding of fractions. It implies that a proper understanding of fractions cannot be obtained through explanation only. Thus, learners' memorisation does not contribute to the understanding of fractions (Cf Table 4.29). A not statistically significant value of p = 0. 379 and 0.122 was recorded. In contrast, the experimental group showed significant values p= 0.010 on yes/agree and not significant p= 0.260 on no/disagree. This implies that problem-solving teaching enhances learners' understanding of fractions. On significant p= 0.010, the rank larger size of 0.26 was recorded. Hence, problem-solving teaching and learning enhanced learners understanding and knowledge of learning fractions.

Furthermore, observation reports (Cf Table 4.33) showed similar constant improvement in problem-solving skills across the framework model phase. However, the comparison group's achievement is less in comparison to the experimental group at a rate of $18.61\% < 44\%$, $34.04\% < 52.57\%$, $9.59\% < 15.53\%$, $24.74\% < 34.38$ and $0.00\% < 24.57\%$. In addition, no comparison learners achieved in evaluating solution self-reflection. The implication is learners lack concrete reasoning as they possess abstract reasoning only because most learners are learning through writing and memorisation of concepts in traditional classes. Performance in the comparison group was unsatisfactory because there was no achievement in advanced levels. This supports the literature that educators teach at formal reasoning levels that do not relate what learners are learning in the class to the real world. Thus, a significant value of p=0.000 substantiates the rejection of the study's null hypothesis **(H_0)**: There are no differences between problem-solving teaching and traditional teaching approach to learning fractions. The conclusion was to retain the directional hypothesis **(H_1)**: The problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions.

4.7 CHAPTER SUMMARY

The findings indicate that the experimental group learners have performed better and achieved high scores on learning fractions compared to the comparison group learners across all categorised knowledge and understanding. Therefore, the objectives were achieved. The null hypothesis **(H_0)**: There are no differences between the problem-solving teaching approach and traditional teaching approach on learning fractions was rejected and the directional hypothesis **(H_1)**: The problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions is retained.

CHAPTER FIVE

CONCLUSIONS, RECOMMENDATIONS, AND LIMITATIONS

5.1 INTRODUCTION

This chapter discusses verification, clarifications and implications of this study's research questions and hypothesis, based on the data findings and observations discussed in Chapter 4. Furthermore, it evaluates the objectives and presents the contribution, recommendations, and limitations of the study.

5.2 VERIFICATION OF THE HYPOTHESIS

The study's null hypothesis **(H_0)** that there are no differences between problem-solving teaching and traditional teaching on learning fractions and the directional hypothesis **(H_1)** that the problem-solving teaching approach enhances learners' problem-solving skills and cognitive development in learning fractions is verified.

Learners' pre-test performances showed a not statistically significant $p=0.096$ on average normality score of 4.40% (9.03) and 4.40% (8.87) for each group (Cf Table 4.1). This implies, that both groups' performance was equivalent, and can be compared according to the findings obtained. Furthermore, learners' test (pre and post-test) performances showed a significant value of $p=0.000$. The results are based on significant, non-parametric Kruskal-Wallis and Wilcoxon Signed Rank adopted for hypothesis verifications. However, EGS3 findings were not significant $p=0.034$ based on not significant, paired t-tests were also adopted for analysing learners' average post-test performances.

Verifying this study hypothesis according to learners' average in pre and post-test performances, the performances were significant in the experimental group's post-test and not significant in the comparison group's post-test. Thus, the decision based on paired sample-t-test on analysing learners' average performances was to reject the null hypothesis (Cf Table 4.3) on the count that significant value $p<0.05$ which is $p=0.000$. However, an improvement was observed when the school groups were considered separately, in EGS1-3 showed, $p= 0.044$, differences T-stat=-12.32; - 9.77 and -17.08, and average differences -29.200; -22.70 and -37.25 and average difference performances MD=-29.200; -22.70 and -37.25. Performances in EGS are greater than

CGS MD=-21.24; -17.57 and -23.53. The implication is that more individual learners in the experimental group achieved higher post-test scores compared to learners in the comparison group. Thus, the study's null hypothesis was verified based on paired sample t-test average learners' significant performances value p=0.000 and statistic difference of -4.86 on a large scale of 0.067. The decision is to reject the null hypothesis **(H_0):** There is no difference between problem-solving and the traditional approach to learning addition and subtraction of fractions. The decision is to retain the directional hypothesis **(H_1):** The problem-solving teaching approach enhances Grade 8 learners' cognitive development in addition and subtraction of fractions.

Furthermore, verification of the study's null hypothesis was based on learners' performance according to the problem-solving intervention and traditional teaching and learning approaches. In the comparison schools, no significant statistical values in learners' performance on learning using traditional teaching were recorded (Cf Table 4.4 and 4.5). Thus, results indicate no statistically significant value of p= 0.0845 before teaching and no statistically significant value of p=0.065 after teaching on small rank effect size H=0.11 and statistical rank limit H=4.920 and 23.181. The implication is that learners' learning strategies were the same in all comparison schools. However, performance scores achieved increased slightly on learning through the traditional approach in the pre-test and post-test in comparison schools (Cf Table 4.6). However, the null hypothesis verification in the experimental schools' (Cf Tables 4.4 and 4.5) results showed a no significant value of p=0.051 before learning and a statistically significant value after learning of p=0.000 with a large effect size =0.163 on a statistical rank difference H=30.347 learning fraction using the problem-solving approach. The implication is that learners used different learning strategies after receiving the problem-solving intervention in the experimental school. Similarly, SPSS generated Kruskal-Wali's test decision summary showed a significant value of p=0.000 on degree freedom count of 3 and large effect size H=0.163 $p<0.05$. Thus, learners' performances based on learning using the problem-solving intervention yielded better effects on learning fractions. Therefore, the verification decision of the null hypothesis based on findings from SPSS Kruskal-Wallis's generated decisions p=0.000, H =30.3476, N = 363, DF = 3 is rejecting the null hypothesis **(H_0):** There is no difference between problem-solving teaching and traditional teaching approach on learning fractions and retain the directional hypothesis **(H_1):** Problem-solving teaching approach enhances Grade 8 learners' cognitive development on learning fractions.

Similarly, verification of the hypothesis is based on problem-solving teaching and traditional teaching approaches. Learners' performances in traditional teaching (Cf Table 4.9) showed overall comparison group schools' performances as follows; sixteen (16) learners obtained a pre-test score greater than the post-test score and 148 learners obtained post-test score greater than the pre-test scores and twenty-four (24) learners obtained tie scores resulting in 79% (148) improvement on learning fractions using traditional teaching. In contrast (Cf Table 4.10), in the experimental group, ten (10) learners obtained a pre-test score greater than the post-test 157 learners obtained greater post-test scores and eight (8) learners obtained tie scores resulting in 89.71% (157) improvement in learners. Thus, an improvement of 89.71% (157) in the experimental group is greater than 78.72% (148) in the comparison group. In addition, the 5.7% (8) decline in learners' performance in the experimental group is less than the 8.5% (16) decline in learners' performances in the comparison group. Thus, based on the Wilcoxon Signed Rank test significant value of p=0.000 and rank difference of R=0.109 on statistic 0.050, the decision was that problem-solving teaching yields better performance than traditional teaching. Thus, the verification decision rejects the null hypothesis (H_0): There is no difference between the problem-solving teaching approach and traditional teaching on learning addition and subtraction of fractions. and the verification decision retains the directional hypothesis (H_1): The problem-solving teaching approach to learning enhances Grade 8 learners' cognitive understanding of fractions.

Triangulating questionnaire yes/agree and no/disagree results (Cf Table 4.28), yields the observation that analysing learners' knowledge and understanding of fractions showed no statistically significant value of p= 0.379 and 0.122 in yes/agree and no/disagree comparison group responses. This implies that learners' yes/agree, and no/disagree responses do not reflect the questionnaire's yes/agree and no/disagree. The implication is that learners cannot interpret the knowledge and understanding of fractions in a real-world context. This led to responding falsely to yes/agree which requires no/disagree in the questionnaire. Thus, traditional teaching does not enhance learners' knowledge and understanding of fractions. In contrast, results (Cf Table 4.28) indicated a statistically significant effect of p-value$<$0.05, p= 0.010 SEM = 4.140, 4.219 and 4.695 yes/agree and further revealed not statistically significant P = 0.260 SEM = 4.308, 4.523 & 5.323 in learners no/disagree in the experimental group. This implies that the interpretation of learners' questionnaire responses shows that there was improvement in understanding fractions. Thus, responses tally with the questionnaire knowledge. The problem-solving intervention enhances learners' cognition in addition and subtraction of fractions in real-world contexts (Loveluck ,2012). Thus, problem-solving learning skills enhances learners’ problem understanding through devising a plan, carrying out the plan, and solution evaluation skills of fractions. Hence, based on this study's findings and verification of the study’s null hypothesis (H_0)**:** There is no difference between problem-solving teaching and traditional teaching on learning addition and subtraction of fractions. The decision is to retain the directional hypothesis (H_1)**:** Problem-solving teaching on learning enhances Grade 8 learners’ cognitive understanding of fractions on Kruskal-Wallies significant value p=0.000.

5.3. RESPONDING TO RESEARCH QUESTIONS

5.3.1. How does the problem-solving teaching approach enhance the learning of fractions in Grade 8?

Learners’ average achievement level descriptive analytical findings provide answers to sub-research question 1. Learners' average achievement scores before learning range from 8.12 to and 10.47 after learning, range from 25.69 to 32.0 (Cf Table 4.12). Thus, learners' achievements after learning are greater than achievements before learning in each group. However, the overall achievement of 76% (29.36) in comparison to the traditional class is less than the 81% (38.88) achievement in the problem-solving experimental class. Therefore, 81% of the learners enhanced their learning of fractions using the problem-solving approach. In addition, this question can be re-stated as, how does problem-solving teaching compare to the traditional teaching approaches and to what extent does problem-solving learning enhance learners' problem-solving skills? The descriptive analytical report based on learners' pre-, and post-test achievement levels (Cf Table 4.13) showed improvement in all problem-solving achievements. When considering each group separately, 36.1%, 26.06%, and 10.11% decrease and increase in low, moderate, and advanced achievement levels in traditional classes less compared with 61.71%, 44%, and 17.71% decreases and increases in low, moderate, and advanced achievement levels from the problem-solving classes. Looking at each group's achievement scores, problem-solving yielded better achievement. Hence, responding to Sub-research question 1 ‘How does problem-solving teaching approach enhance the learning of fractions in Grade 8?’ is answered based on descriptive analytical achievement level 61.71%, 44%, and 17.71% decrease and increase in the experimental group that, problem-solving learning enhances learners' learning in decreasing, low achievement scores level and increases learners moderate and advanced scores achievement level on learning fractions in Grade 8.

5.3.2. What problem-solving skills are essential to enhance cognition in learning fractions?

The descriptive analysis report based on problem-solving knowledge and understanding of fractions provides a clearer answer to sub-research question 2 be restated as 'what problem-solving skills are essential to enhance learners' cognition in learning addition, subtraction, and equivalence of fractions?' This study affirms that understanding the problem, devising a plan, carrying out the plan, and evaluating the solution are essential problem-solving skills required to enhance learners' cognition on learning addition and subtraction fractions (Cf Table 4.26). The results (Cf Table 4.26) of learner problem-solving knowledge and understanding showed improvement in group post-test scores achievements and a tremendous reduction in mistakes and misconceptions after the intervention. For example, before the intervention (pre-test) none of the learners N=363 possessed the solution skills. The effect is visible as learners could not answer question 5 correctly. However, after problem-solving intervention in the post-test, 24.57% of the learners N=175 evaluated the solution while none evaluated the solution in comparison schools of N=188 learners. This implies individual learners in the experimental group acquired problem-solving skills in learning fractions. Furthermore, the degree of skills acquired in the experimental problem-solving class is greater compared to the comparison traditional classes as shown by the statistics that 44%>18.61%, 52.57% >34.04%, 15.43%>0.00%, 34.28%>0.00%, and 24.57%>0.00%. Thus, understanding the problem, devising a plan, carrying out the plan, and evaluating of the solution are essential problem-solving skills required by learners to enhance cognition in addition and subtraction of fractions.

Furthermore, triangulating questionnaire descriptive analysis report of the learners' knowledge and understanding of fractions results (Cf Table 4.31) yielded a similar improvement. For instance, only 4.68% (54) of learners in the experimental group showed inadequate knowledge and understanding of fractions after acquiring learning using the problem-solving approach. This contrasts with the comparison group's 20.81% (282). Thus, data findings revealed that more learners in the experimental group acquired essential solving skills required to enhance knowledge and understand the addition and subtraction of fractions. Thus, 95.32% (1101) of experimental learners showed enhanced problem-solving skills. Hence, sub-research question 2 'What problem-solving skills are essential to enhance cognition in addition and subtraction of fractions?' is answered on 95.32% (1101) test achievement and 44%>18.61%, 52.57% >34.04%, 15.43%>0.00%, 34.28%>0.00%, and 24.57%>0.00% problem-solving knowledge and understanding that understanding the problem, devising a plan, carrying out the plan and evaluating the solution are essential skills to enhance learners' cognition in addition and subtraction of fraction.

5.3.3. What challenges do learners experience in solving addition and subtraction fractions?

The research sub-question 3 can be restated as 'What mistakes are committed by learners when solving addition and subtraction of fractions?' In both groups, it was observed that learners committed mistakes. However, after the intervention, experimental learners' mistakes decreased tremendously while mistakes persisted in the comparison group after learning fractions as outlined in Table 4.27. Test findings revealed that learners committed mistakes like false guessing solutions, calculation mistakes, procedure errors, and misconceptions relating to the addition and subtraction of fraction numbers. Thus, after the intervention, post-test findings showed that experimental group learners did not commit the mistakes previously committed in the pre-test (Cf Vignette 4.1) while comparison group learners still committed similar mistakes. Thus, the tremendous reduction in mistakes can be attributed to learners' understanding of basic problem-solving interventions and the development of problem-solving skills in learning fractions in Grade 8.

For example, Vignettes 4.1 to 4.8 and Table 4.28 revealed that learners do not understand how to identify bigger fractions within $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{2}{3}$ which led to false guessing of answers and providing solutions out of context. Similar mistakes were observed in the comparison learners' solutions after teaching. Thus, 18.61% achievement is low as observed in the traditional learning group. However, problem-solving, after the intervention showed tremendous improvement and reduction in errors and mistakes that led to 44% achievement. This finding substantiates that learners should understand what the questions are all about, be able to outline key information needed to answer the questions correctly and understand how to connect previous solutions with current problems.

Thus, problem understanding should be the primary assignment of learners, and educators should guide learners through this stage (Koziol, 2018). Furthermore, findings revealed that learners could not devise a plan for adding and subtracting fractions in the pre-test (Cf Vignette 4.4 and Table 4.27). For example, in Question 2, learners were asked to add $\frac{1}{4}+\frac{3}{4}$, and the pre-test findings revealed that only 2.29% and 5.85% of learners in the experimental and comparison groups added fractions correctly. However, post-test scores showed an increase from 5.58% to 34.04% in the comparison group and a tremendous increase from 2.29% to 52.57% in the experimental group. Thus, individual learners in the experimental group added fractions correctly after the intervention. This can be attributed to problem-solving interventions as learners learned various approaches like converting fractions to decimals. For example, $\frac{1}{4} = 0.25\ \frac{1}{3} = 0.33$ and $\frac{2}{3} = 0.67$. Such approaches made it easier to add and avoid calculations and procedural errors (Cf Vignette 4.3 and Table 4.27). Similarly, the challenge of carrying out the plan and evaluating solutions for solving fractions eased in the experimental classes. Findings (Cf Table 4.27) showed 3.19%, none 0.00% from comparison, and 0.00% of learners in the experimental group could carry out the plan and evaluate the solution. Hence, evaluating the solution was observed to be a serious challenge in traditional classes since all learners achieved 0.00% at this level and learners at this level were from similar traditional teaching classes. However, after teaching, 24.74% and 0.00% improvements were recorded in the comparison group while 34.28% and 24.57% improvements were recorded in the experimental group. Thus, learners in the comparison group did not achieve the evaluation level. Hence, sub-research question 3 answered on findings that learners have challenges of understanding problems, devising a plan, carrying out the plan, and evaluating solutions for solving addition and subtraction fractions. Thus, problem-solving minimised learners' mistakes in addition and subtraction fractions.

Furthermore, the main research question posed in this study *'**What is the effect of problem-solving teaching approach on learning fractions in Grade 8?'*** was answered**.** Descriptive findings showed that problem-solving teaching and learning enhances problem understanding, devising a plan, carrying out of the plan, and solution evaluation skills. Subsequently, enhanced cognitive performance and higher achievement scores were realised and learners' mistakes in addition and subtraction of fractions were minimised. The post-test scores findings revealed that experimental learners committed fewer errors and mistakes (Cf Table 4.26). Learners were able to overcome calculation errors by converting fractions to decimals and percentages, avoiding false guessing solutions, and reducing inappropriate repetition (Cf Figure 4.18). Findings support the fact that when learners make mistakes and fail to apply rules correctly in learning it is because learning occurs procedurally (traditional learning). However, conceptual understanding is evident when a learner sees why the rule does not apply to the problem (Jordan & Luneta, 2017). The constant improvement in achievement level was evident that problem-solving enhanced learners problem-solving learning of fractions in Grade 8. Learners minimised mistakes and acquired an understanding of the problem, and how to devise a plan, carry out the plan, and evaluate the solution, which enhanced their performances and helped them achieve the cognitive level.

Furthermore, triangulating questionnaire findings, provides a comprehensive answer to the main research question; *What is the effect of a problem-solving teaching approach on learning fractions in Grade 8?'* yes/agree, and no/disagree responses confirmed that learners' learning in the traditional class cannot relate activities in class to real-life situations. Thus, not statistically significant values of p= 0.379 and 0.122 imply that yes/agree and no/disagree findings implied that the traditional teaching and learning approach does not enhance learners' understanding and knowledge of learning fractions. Thus, learners' learning strategies are built on concept memorisation without understanding. Thus, 74% (140) of learners could not relate the question to a real-life context. For example, when asked 'Do you think half is $\frac{3}{2}$?' 74% (140) agreed with the question 'Do you think $\frac{3}{2}$ cannot be presented on fractions model'? 75.53% (142) disagreed (Cf Table 4.32). The results show that 20.81% (282) of responses in traditional classes do not relate activities in class to a real-life situation. Hence, traditional teaching does not enhance learners' conceptual knowledge and understanding of learning fractions. Thus, learners' achievement after teaching is attributed to learners' ability to memorise and repeat concepts. This agreed with Namkung et al., (2018) that repetition of facts and concepts improves recognition but reduces the reliability of representations in memory. This confirmed the hypothesis that repetition elicits similarities of non-identical memory, and memorisation disadvantages learners' learning understanding because most items in memory cannot be recalled. Memorisation does not guarantee long-term retention of information in the memory, in emphasis, memorisation only teaches learners procedures without knowing where, how, and when to use the procedures (Namkung et al., 2018). This was evident, in 79.19% of the learners who could not see why the rule does not apply when responding to the questionnaire questions, and 74% (140) and 75.53% (142) of the learners could not relate questions to real-life context for better knowledge and understanding in traditional classes after learning.

In contrast, experimental learners' findings showed 23.43% (41) agreed with the same question 'Do you think half is $\frac{3}{2}$?', and 7.43% (13) of learners disagreed with 'Do you think $\frac{3}{2}$ cannot be presented on any fractions model'? Furthermore, only 4.68% (54) could not relate class activities to the real-life context in problem-solving classes. This implies that only 95.32% of learners using problem-solving learning can relate class activity questions to real-life context for better understanding in addition, 95.32% showed that the intervention discussions created learners' awareness that fractions can be understood better in a real-life context. Thus, it is evident that learners in traditional classes still have challenges in conceptual understanding, relating fractions' abstract problems to concrete objects. In contrast, findings revealed that learners in problem-solving classes no longer have these challenges. This agreed with Van de Walle et al., (2014), that using manipulatives enhances understanding of the problems and learning opportunities. Based on the findings above, the main research question ***'What is the effect of problem-solving teaching approach on learning fractions in Grade 8?'*** was answered based on Kruskal-Wallis large effect size eta [H] =0.163, which implies that learners' increase in performance and achievement and understanding of learning addition and subtractions of fractions to a large effect size of 0.163 through the problem-solving teaching and learning approach (Albay, 2019).

5.4 PROBLEM-SOLVING FRAMEWORK

This study showed that problem-solving teaching enhanced learners' understanding of fractions. It enhances learners' problem-solving skills, fosters cognition of addition and subtraction fractions, and reduces learners' mistakes and misconceptions about learning fractions. The Kruskal Wallis statistical significance value p =0.000 on average= 22% (38.88), standard error = 2.44, and a larger effect size of 3 implies that rather than ignoring the gap between learners' levels of knowledge and understanding, problem-solving learning can bridge the gap from low achievement to higher achievement scores level in Grade 8. This study showed that educators should provide and use unconventional materials and resources to engage learners in understanding problems. The materials provided should help interpret and relate questions from abstract to concrete real-life situations. Learners should be allowed to relate their own opinions on problems in group discussions. Educators should probe learners' understanding of the questions by asking them questions that connect their perceptions of the problem to their own physical world. It enhances problem understanding and helps learners in devising plans. Findings (Cf Table 4.34) showed that 2.29% of learners understood the problem and after the intervention, a 46.29% statistic and 44% were achieved in Phase 1. Similarly, the devise a plan phase encourages learners to collect ideas that connect and associate the abstract problem with the mental and physical visual world. Thus, the facilitator presented the exercise patiently by allowing learners enough time to produce a plan, helped identify patterns, and illustrated their information using drawings like tables and graphs. This phase enhances self-regulated learning and helps learners to understand fraction concepts and procedures. The effects were evident (Cf Table 4.37), 2.29% devised a plan before the intervention, and after the intervention, 54.86% devised a plan. Thus, a 52.57% increase was recorded. Similarly, the carry out the plan phase was assessed and evaluated using concept relationships. This stage required information to understand and decode text to symbols and operations like calculating, integrating concepts procedures that emerged from drawing conjectures based on relations of concepts. Learners worked carefully and kept accurate and neat records of all attempts. Thus, learners reflected on their calculations and simplification process. The effects showed none 0.00% of the learners carried out the plan before teaching. However, after the intervention, 34.28% of learners carried out the plan. Therefore, problem-solving enhances learners' understanding of fractions.

Through searching for information and supporting facts, inferences and evaluations were made. Conjectures were tested, confirmed, and validated (Masilo, 2018). Conclusions from the evaluation of the solution were emphasised in Phase 4. Learners did not conceptualise knowledge and understand the origin of concepts that formed the problems. Problem-solving teaching and learning influenced learners' understanding by directing orientation to understand the problem, coordinating the conceptualisation of devising the plan, executing conjecturing application, and directing conceptualisation for drawing conjecturing that evaluates solutions and validates conclusions. Thus, the facilitator should follow (Cf Figure 4.23) the problem-solving framework chronologically when facilitating problem-solving. In addition, the facilitator should validate learners' conclusions and ideas relating to reasoning. Learners should be allowed enough time for thinking, searching, analysing, conjecturing, testing, inference, conclusions, and applications. The facilitator should guide learners to think logically. Thus, experimental learners in this study enhanced their understanding of problems, devising of plans, carrying out of the plans, and evaluating the solutions skills (Cf Table 4.33). After the intervention post-test, 24.57% of learners achieved correct answers on evaluating the solution. These results confirmed that the problem-solving teaching and learning framework model helps to successfully implement the problem-solving teaching approach and enhance learners' understanding and achievement in learning fractions. The model helps to enhance learners' problem-solving skills and encourages active participation in discovering and creating own knowledge on solving fraction problems in both formal and informal educational systems.

5.5 EVALUATING THE STUDY OBJECTIVES

5.5.1. Evaluate how problem-solving teaching enhances learning fractions in Grade 8

Learners' pre- and post-test scores achievement results evaluate the objective of 'how problem-solving teaching enhanced learners learning of fractions in Grade 8 (Cf Table 4.13). For example, before the intervention in the experimental class, learners achieved 100% (175) in the low achievement level. The implication is that learners' knowledge and understanding of fractions are low. None of the learners understood how to interpret and relate problems to prior knowledge and real-life context. Hence, learners lacked problem understanding and were unable to answer questions correctly. However, after the intervention, learners decreased in low achievement and increased in moderate and advanced achievement levels. For example, the low achievement level decreased from 100% (175) to 38.29% (108), the moderate achievement level increased from none 0.00% to 44%, and the advanced achievement level increased from none 0.00% to 17.71%. Thus, learners' learning was evaluated using performance achievement levels. Thus, the study objective was achieved through problem-solving intervention in the experimental group. The findings in this study agree with Makhubele (2021) who states that learners' mistakes and errors can be attributed to difficulties experienced in learning and insufficient background understanding of fractions at lower grades. Thus, this study confirms that learners are only exposed to low score achievement levels in Grade 8. In addition, pre-test findings in this study showed that none of the learners achieved advanced score levels on learning using the traditional method. The implication is that, in the lower class, advanced levels of knowledge and understanding were not addressed through traditional teaching. There was an enhancement of understanding of the problems after the intervention as learners progressed to moderate and advanced achievement levels. Therefore, problem-solving enhanced learners' problem-solving skills at a moderate level. Thus, this study asserts that good performance scores and achievement depend on problem-understanding levels. Thus, advanced levels are attainable on condition that low achievement levels are addressed through the problem-solving learning approach. Furthermore, educators should ensure that learners understand problems and scores should be achieved based on performance and achievement levels using problem-solving cognitive phases like understanding the problem, devising a plan, carrying out the plan, and evaluating the solution. If these phases are followed in the learning of fractions learners will be able to retain knowledge and understanding in their long-term memory.

5.5.2. Evaluate learners' challenges in solving addition and subtraction of fractions

The mistakes committed by learners showed the challenges experienced in solving addition and subtraction of fractions. However, problem-solving learning enhances learners' problem-solving skills and minimises learners' mistakes and errors committed in the pre-test. Therefore, this contributes to learners' achievement of moderate and advanced achievement levels. For example, average pre-test scores (Cf Table 4,12) revealed 5.09% (8.91) experimental achievement before the problem-solving intervention. Thus, learners' errors and mistakes committed can be attributed to learners' low achievement levels. However, after the intervention, post-test findings showed 22.22% (38.88). The implication is that learners reduced mistakes committed previously and progressed to moderate and advanced achievement levels. Therefore, problem-solving intervention enhances learners' performance and activates learners' problem-solving skills. The skills acquired helped to minimise mistakes and difficulties when solving addition and subtraction of fractions. Understanding problem skills helped learners minimise mistakes and misconceptions. Hence, the study objectives were achieved based on an achievement performance score of 22.22% (38.88). Thus, understanding the problem, devising the plan, carrying out the plan, and evaluating the solution skills help learners overcome challenges in solving addition, subtraction, and equivalent fractions.

5.5.3. Determine problem-solving skills essential to enhance cognition in learning fractions

The problem-solving framework enhances learners' learning and subsequently evaluates learners' problem-solving skills. Polya's problem-solving framework evaluates the essential problem-solving skills in enhancing cognition in learning fractions. Understanding the problem, devising the plan, carrying out the plan, and evaluating the solution problem-solving skills were crucial for the achievement of this study's objectives (Cf Table 4.33). Understanding the problem yielded a 25.39% improvement. Furthermore, data findings revealed a 24.57% improvement in the experimental group's evaluation of the solution phase. The tremendous improvement in experimental learners' achievement after the intervention can be attributed to their problem-solving skills. Problem understanding, devising a plan, carrying out the plan, and evaluating the solution skills foster an understanding of the interaction of real-world contexts. Learners interact with concrete objects, which connect mental abstracts with the real world. Furthermore, Phase 2 enhances learners' skills and their daily observation of the use of fraction concepts in everyday activities. Learning through problem-solving enhances learners' conceptual understanding of learning fractions. Phase 3 emphasises real-life observations and the development of modelling of concepts through constructions and drawings used to make sense of the problem. Learning model strategies in teaching fractions assist learners in formulating conjectures and predicting relationships between fraction concepts and figures. Modelling real-life objects is experienced through concrete conceptual knowledge and abstraction. Resources like charts-based facilitation enhance learners' learning; and concrete interaction increases their understanding. This study showed that the use of Polya's theoretical problem-solving framework in the experimental group and learning using real objects enhanced learners' cognition in learning addition and subtraction fractions in Grade 8.

5.6. STUDY IMPLICATIONS ON PRACTICE

5.6.1 Theoretical Implications

This study's findings support the problem-based chronological and hierarchy teaching phase. It is crucial that educators reconsider the traditional approach and promote problem-solving teaching and learning. Positivism perspectives support experimental knowledge and bridge memorised learning. The problem-solving approach promotes long-term knowledge and understanding of fractions and assists learners to retain information longer when involved in learning (Siegler et al., 2011; Norton & Boyce, 2013). Thus, the teacher is not regarded as a source of information in this study. However, due to the lower achievement level of learners' knowledge of adding and subtracting fractions, the teacher guided all processes of learners' learning. Thus, the traditional approach was not ignored completely but used to guide learners through the problem-solving phase. Furthermore, it helped learners to be aware of their progress and where they fell short in the process of acquiring problem-solving knowledge and skills.

5.6.2 Implications of Methodology

The philosophical assumptions of positivism paradigms express the importance of objectivity when interpreting reality. References on objectivity provide this study's framework reference. Positively, this study addresses quantitative reasoning. In addition, it addresses arguments in context-based and generalisation can be transferred to similar settings like the context of this study. The questionnaire was embedded as an instrument for data collection and was administered after intervention alongside the post-test in each group. The SPSS inferential data findings analysed through paired t-test, Wilcoxon rank signed test and Kruskal Wallis's test address quantitative hypothetical quantitative significant and objective stance, and descriptive questionnaire and non-participatory observation data analysis address research questions' objectives. The two stance objectives were triangulated for comprehensive viewpoints of this study's positivism stance objectives.

The inferential data findings and analytical results led to the decision to reject the null hypothesis **(H_0):** There is no difference between problem-solving teaching and traditional teaching on learning addition and subtraction of fractions and retain the directional hypothesis **(H_1):** The problem-solving teaching approach enhances Grade 8 learners' cognitive understanding of fractions. In addition, questionnaires, pre- and post-test problem-solving framework data findings provide substantial support for the directional hypothesis **(H_1):** The problem-solving teaching approach to learning enhances Grade 8 learners' cognitive understanding of fractions. In addition, questionnaire data findings complement the test findings. They increase this study's findings' credibility. Thus, pre- and post-test data findings provide clarity to the research's main questions. Hence, problem-solving teaching influenced problem-solving learning of addition and subtraction fractions. Hence, the positivist stance of this study asserts that research needs to move back and forth objectively to contribute towards change.

5.6.3. Implications on Pedagogy Knowledge

This study confirms that the problem-solving teaching approach enhances learners' problem-solving learning and skills in solving fractions and enhances learners' cognition of addition and subtraction fractions. Learners' performance and achievement scores improved in problem-solving learning groups compared to the traditional teaching achievement scores on learning fractions. Learning through problem-solving minimised learners' mistakes and errors in learning addition and subtraction fractions.

5.7 CONCLUSION

This section discusses the conclusion based on (I) the problem-solving teaching framework, (II) the literature review, and (III) the methodology of the study.

5.7.1. Conclusion Based on the Problem-Solving Teaching Framework

Findings in the pre-test and questionnaire data confirm that learners provide answers to questions by falsely guessing solutions without a basic understanding. The findings show that learners depend on memorised answers in their prior knowledge. In this study, learners tried recalling prior similar solutions that were irrelevant to the current problems. However, these difficulties were overcome after the problem-solving intervention in the experimental group. Learners' dependence on prior knowledge and generalisation of whole numbers was immensely reduced. Problem-solving intervention guides learners on how to understand the problem. It advanced learners' understanding of identifying terminology and demonstration of concepts. The post-test showed that learners understood the given problem, and the number of empty responses decreased in the post-test. The plans generated after the intervention were relevant to the questions and learners evaluated their solutions by drawing conclusions and they minimised false guessing of whole numbers as solutions. Learners connected differences between whole and fraction numbers and avoided overgeneralising whole numbers as solutions. Learners substituted, simplified, and calculated values using the devise a plan skill. They checked if the answers were relevant to the questions. In addition, the evaluating solution phase was challenging and not fully developed, and this stage required learners to conceptualise their solution to the problem relevant to their context. However, most responses in the comparison learners did not capture this stage. Thus, it is recommended that solutions be interpreted in the problem-solving context.

5.7.2. Conclusion from the Literature Review

Understanding fraction concepts poses a serious challenge to learners' learning of fractions because the traditional teaching approach explains fraction concepts at a formal level that does not encourage learners' involvement (Rose & Björling, 2017; Nyembe, 2020). Moreover, due to the disparity in mathematics education in South Africa, most mathematics educators adopted the traditional teaching approach (Makhubele, 2021). However, this study showed that problem-solving learning enhances learners' problem-solving skills, and subsequently enhances learners' performance and achievement, and understanding of learning of addition and subtraction of fractions. The knowledge applies to other branches of mathematics. Procedural knowledge and declarative knowledge are argued as important in the acquisition of content learning. Learners find learning fraction concepts challenging because they possess only procedural knowledge. Thus, the interpretation of declarative knowledge is a missing basic requirement. Hence, literature recommends learning through the problem problem-solving approach as a means of closing the gap between declarative and procedural knowledge. Thus, learning through problem-solving develops learners' self-learning, and instead of providing a straight and simple routine to solutions, it engages learners in making sense of activities that cause them to apply proper and critical thinking in solving problems. Problem-solving teaching strategies direct educators to design problems correctly and use probing techniques in questioning to guide learners' learning. When learners perfect the process of learning through problem-solving, they acquire problem-solving skills like problem understanding, devising a plan, carrying out the plan, and solution evaluation. These skills assist learners in developing declarative knowledge. Thus, post-test findings showed enhancement in learners' learning understanding, performance, and achievement. In addition, it helps learners integrate problems by relating meaningful parts of the problem to their social reality. The intervention process assisted learners with understanding, planning, executing, and evaluating solutions. The problem-understanding skills help learners advance procedural knowledge where applicable in devising a plan and carrying out the plan. Learners engage with sense-making activities like the application of evaluation knowledge in advancing procedural knowledge and linking fraction concepts to other mathematical concepts. Thereby, networking addition and subtraction of fractions. In addition, fractional processes and concepts advance when learners possess skills such as problem-understanding, devising a plan, carrying out the plan, and solution evaluation. The problem-solving teaching process in this study included the administration of test and questionnaire to learners using the problem-solving framework. It focused on an advanced understanding of problems and the development of learners' cognitive load. However, if learners have not mastered the basics, it may contribute to learners' cognitive load. Furthermore, educator reasoning is higher than the learners' levels of knowledge and understanding. This may aggravate the cognitive load on learners if not guided. Each level of the framework addresses the needs of that level according to learners' learning styles. Diverse levels of knowledge and influenced by diverse learning experiences like action learning and verbal learning are carried out to evaluate solutions of mental learning rather than verbal learning. Problem-solving learning involves explanation and demonstration. Thus, problem-solving requires diverse resources like manipulatives and modelling diagrams. These resources help learners to learn at diverse levels of knowledge and understanding.

5.7.3. Methodological Conclusion

The methodological conclusion was presented based on learner test, questionnaire, and confirmation of teaching observation. The approach enhances learners' understanding of learning fractions and shows that explanation, demonstration, and experimental learning supplement each other in teaching fractions. The increase in post-test performance and achievement levels in the experimental group indicated that learners gained problem-solving skills in accordance with Polya's problem-solving framework. These skills assisted learners in answering questions they could not answer in the pre-test. In addition, the skills contributed to individual learner achievement and performance, and a decrease in the low achievement level and increases in moderate and advanced achievement levels (Cf Table 4.14). Hence, an increase in experimental learners' achievement performances and enhancement in problem-solving skills is attributed to the problem-solving intervention. Thus, the Wilcoxon Rank Signed test significant p-value of 0.000, and Kruskal Wallis's Test statistically significant p=0.000, and overall achievement scores in the experimental group were greater than the comparison group. This means that the problem-solving teaching approach enhanced the learning of fractions in Grade 8.

5.8 LIMITATIONS

The following limitations were observed in this study. Foremost, only Grade 8 learners from six schools participated in this study. This study was conducted in selected districts in Limpopo purposefully. Thus, conclusions can be generalised to an extent, especially to situations and conditions like the schools investigated in this study. This study was limited to quantitative and positivist perspectives. The study population. was conveniently sampled based on availability of required resources like time, access to locations and the number of learners in each school. Moreover, there are other schools in Limpopo whose conditions differ from schools in the study. Secondly, the intervention included addition, subtraction, and equivalent concepts of Grade 8 fractions. Hence, emphasis was placed on problem-solving learning and learners' problem-solving skills. However, the knowledge gained on learning addition, subtraction, and equivalent fractions in this study may not apply to division and multiplication or other sections of fractions. The research design, quasi-non-equivalent pre-, and post-test, did not randomly select participants to groups, instead, it used the intact arranged participant classes, which was a threat to internal validity because absolute control of the research environment was difficult to maintain. Thus, higher internal validity and internal reliability cannot be claimed, because of test-retest measure. Situational factors such as absenteeism, and lack of learners' interest especially in the experimental group, contributed to high attrition during the post-test period. Hence, pre-test and post-test results were limited to learners who wrote the test and completed the questionnaire. Furthermore, limitations are acknowledged in the consideration of results in all research, this study is not an exception. The schools selected in this study are Quintile 1 schools, lacking basic amenities and resources due to rural locations; with an overcrowded classroom size of over forty-nine learners in a classroom, learners' performance in such classes could have been negatively affected by large learner numbers during teaching and learning processes.

5.9 RECOMMENDATIONS FOR FURTHER RESEARCH

This study recommends the following: foremost the researcher observed that educators from the comparison group schools could not implement the problem-solving approach. They taught using the traditional teaching and learning approach. Hence, this study recommends that various teaching approaches should be used during teaching. Thus, the Department of Education should organise professional training like workshops and exchange programmes to train educators in various teaching approaches, especially, problem-solving. Secondly, schools in this study do not have enough resources and concrete materials like iron bars, paper folds, and pie charts that can be used to demonstrate the teaching of addition and subtraction of fractions. Concrete objects should be used during lessons to help learners link abstract concepts to real-life situations. This research concluded that teaching fractions through problem-solving enhances learners' problem-solving skills and performance achievement. Therefore, educators should practically implement teaching using the problem-solving approach. In addition, it helps learners to answer real-world questions and fosters learners' active learning. Furthermore, the problem-solving approach supports reflection and critical thinking. Hence, the government should provide more funds to support professional development activities, especially resources required to facilitate the problem-solving approach in Grade 8 mathematics.

The Department of Basic Education once obliged schools to use curriculum, teaching, and learning styles that promote problem-solving. Thus, the government should provide schools with a learning environment that supports problem-solving, to better equip learners to challenge learning space and to participate in lifelong learning that involves gaining skills that foster an understanding of problems and help extract vital information from any task. Thus, this study recommends mathematics textbooks that support problem-solving learning strategies. It further recommends that teaching and learning activities, tasks, assignments, tests, and exams should be based on real-life situations. Schoolwork should relate to real-life activities and surface learning, that is, repetition of facts should be prohibited. In addition, educators should be encouraged to use Polya's problem-solving framework as a teaching strategy. In this study, Polya's problem-solving framework equipped learners with adequate problem-solving learning skills required to answer questions on fractions. Furthermore, post-test findings revealed that mistakes and errors commonly committed by learners were minimised in learning through Polya's problem-solving strategies. This study observes that the problem-solving framework in this study is effective in helping educators support learners to acquire fraction content knowledge and skills that enhance cognition in addition and subtraction fractions. As such, the framework can be recommended to mathematics educators for the teaching of Grade 8 addition and subtraction fractions. Furthermore, learners should be encouraged to learn through mistakes, errors committed, task and assignment corrections, self-correction, self-assessment, collaboration, peer teamwork, and group discussion.

Further studies are therefore recommended to verify the findings of the current study to strengthen the development of research data, based on the problem-solving teaching approach in this study. A larger sample size that is spread over a wider geographical area is further recommended as a possibility that would yield more insights into greater generalisation. Also, the sampled schools being public schools, and not private schools may have influenced the findings due to limited resources, and this may have hindered the facilitator from demonstrating more teaching through the problem-solving approach in schools and may have affected the participants' responses in one way or the other.

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APPENDICES

APPENDIX A: ETHICAL CERTIFICATE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2019/11/13

Ref: **2019/11/13/56991223/56/AM**
Name: Mr EO Agadagba
Student No.: 56991223

Dear Mr EO Agadagba

Decision: Ethics Approval from 2019/11/13 to 2022/11/13

Researcher(s): Name: Mr EO Agadagba
E-mail address: 56991223@mylife.unisa.ac.za
Telephone: 0846727064

Supervisor(s): Name: Dr MM Masilo
E-mail address: masilmm@unisa.ac.za
Telephone: 012429 6154

Title of research:

The effect of teaching grade 8 common fractions through the use of problem-solving model

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2019/11/13 to 2022/11/13.

*The **low risk** application was reviewed by the Ethics Review Committee on 2019/11/13 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date **2022/11/13**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number ***2019/11/16/56991223/56/AM*** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

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ACTING EXECUTIVE DEAN
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APPENDIX B: LPD APPROVAL LETTER TO CONDUCT RESEARCH

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

Ref: 2/2/2 Enq: Mabogo MG Tel No: 015 290 9365 E-mail:MabogoMG@edu.limpopo.gov.za

Agadagba EO
Box 3589
Giyani
0826

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"THE EFFECT OF TEACHING GRADE 8 COMMON FRACTIONS THROUGH THE USE OF PROBLEM –SOLVING MODEL."**
3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: AGADAGBA EO

CONFIDENTIAL

Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa - development is about people!

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

MRMao

Ms NB Mutheiwana
Head of Department

03/12/2019
Date

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: AGADAGBA EO

CONFIDENTIAL

APPENDIX C: PRINCIPAL'S PERMISSION LETTER

College of Education
Department of Mathematics Education
UNISA
Date....................................
The principal
(Name of School)

Dear Sir/Madam

Research title: The effect of problem-solving teaching approach on learning fractions in Grade 8. My name is Emmanuel Agadagba. I am a student at the University of South Africa, studying for master's degree in mathematics education. I am conducting a research study that will involve mathematics educators and learners in Grade 8 at your school. I request your permission to conduct research at your school. The research study will focus on teaching strategies applied in Fractions to improve the learners' mathematical fractions solving skills.

Your school is one of the selected schools representing your district in Mopani, classified as experimental group. Your role in the research study will involve allowing the researcher to execute pre- and post-test in one Grade 8 Mathematics classroom and to observe learners as they will be engaging in classroom activities. Your permission would enable me to teach grade 8 learners using problem-solving teaching approach, conduct a test, and administer questionnaire in class. Further, I request permission to allow the subjects educator to accompany me whenever am attending the period only to observe progress and tracked during the lesson. I am looking forward to spending one or two weeks at your school in a very productive interaction with one educator and his/her mathematics class in Grade 8.

This research study will contribute to the improvement of the educator's teaching strategies as well as learners' critical thinking skills in Fractions. Please note that if you allow one educator and his/her Grade 8 Mathematics class to participate in the research study, the following ethical values will apply: the educator and learners' participation is voluntary; all information will be treated with confidentiality and anonymity in order to ensure that no harm or bad effect will be caused to participants by the research study; all observation videos and interview recordings if any, will be destroyed at the end of the study; participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

I will avail to you the summary of the study results at the time of completion if you would wish to have the summary.

Thank you in advance for your support.

Yours sincerely

Agadagba Emmanuel.

(Please complete the consent form below and return to me) Yours sincerely.

Declaration

Permission to conduct research.

I...................................... the principal of .... understand the context of the research study and I grant permission that the research study (title: The effect of problem-solving teaching approach on learning fractions in Grade 8.) be conduct at the school. I am aware that the educator and learners' participation is voluntary; all information will be treated with confidentiality and anonymity in order to ensure that no harm or bad effect will be caused to participants by the research study; all observation videos and interview recordings if any, will be destroyed if any, at the end of the study; participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

Principal's signature: ....

Date: ....

APPENDIX D: EDUCATORS CONSENT LETTER

College of Education

Department of Mathematics Education

UNISA

The educator---

(Name of School) ---

Date....................................

Dear Educator

Re: Request for your participation in a research study

My name is Agadagba Emmanuel. I am a student at the University of South Africa, studying for master’s degree in mathematics education. I request you to participate in a research study titled: The effect of problem-solving teaching approach on learning fractions in Grade 8. Your school is group as an experimental. Your interaction with the researcher will involve allowing the researcher to execute questionnaire, pre- and post-test in your classroom, to collaborate with you in administering problem-solving learning in your classroom. I will further request you to always accompany me to your class for you to track the lesson and to confirm that I did not expose or teach learners the test or questionnaire questions. I am looking forward to spending two weeks in your classroom engaging in a very productive interaction with your learners.

Your participation is voluntary. You may discontinue participation at any time if you so wish. You may also refrain from answering interview questions when there is a need to do so. I am looking forward to your participation in the research study. I request you to sign the consent form provided if you accept my request to participate.

Thank you in advance for your support.

Yours sincerely

Agadagba Emmanuel

(Please complete the consent form below and return to me)

Educator’s participation consent

I..

(Name & Surname)

Understand the context of the research study titled: The effect of problem-solving teaching approach on learning fractions in Grade 8. I am aware that optionally I can allow video and audio recording during participation. I am aware that anonymity and confidentiality will be adhered to in this study. I informed that I may withdraw my consent to participate at any time without penalty by advising the researcher. I agree on my free will to participate in the research study.

Participant’s signature: .... Date: ....

APPENDIX E: PARENT/GUARDIAN CONSENT LETTER

College of Education

Department of Mathematics Education

UNISA

Date....................................

The parent/guardian --

(Name of School) --

Dear Parent/Guardian

Re: Request for your child to participate in a research study

My name is Agadagba Emmanuel. I am a student at the University of South Africa, studying for master's degree in mathematics education. I request you to allow your child to participate in a research study titled: The effect of problem-solving teaching approach on learning fractions in Grade 8. Your child's role in the research study will be to participate in problem-solving activities. His/her progress he observes during participation in problem-solving. I will also conduct pre- and post-test to track the progress of your child. Further, intervention which is teaching using problem-solving would administer to find out how your child experienced the problem-solving activities during lessons. At the end of the research study your child expected to show improved critical thinking skills in Fractions.

If you allow your child to participate in the research study, take note that the following ethical values will apply: your child's participation is voluntary; he/she may discontinue participation at any time if a need arise; he/she may also refrain from answering some interview questions when there is a need to do so. I am looking forward to your child's participation in the research study. I request you to sign the consent form provided if you give permission that your child may participate.

Thank you in advance for your support.

Yours sincerely

Agadagba Emmanuel

(Please complete the consent form below and return to school)

Participation consent

I.................................., parent/guardian of .... understand the context of the research study titled: The effect of problem-solving teaching approach on learning fractions in Grade 8. I am aware that optionally I can allow video and audio recording of my child's participation during problem-solving sessions. I am aware that anonymity and confidentiality will be adhered to in this study. I informed that I may withdraw my consent for my child to participate at any time without penalty by advising the researcher. I agree on my free will that my child will participate in the research study.

Parent's/Guardian's signature: .... Date: ....

APPENDIX F: LEARNERS' ASSENT LETTER

College of Education

Department of Mathematics Education

UNISA

Date....................................

The learner ---

(Name of School) ---

Dear Learner

Re: Request for your assent to participate in a research study

My name is Agadagba Emmanuel. I am a student at the University of South Africa, studying for master's degree in mathematics education. I request you to participate in a research study titled: The effect of problem-solving teaching approach on learning fractions in Grade 8. Your role in the research study will be to participate in classroom problem-solving activities. Your progress will be monitor during participation in problem-solving. I will also conduct pre- and post-test to track your progress of. Furthermore, you would complete a questionnaire for me to understand your experienced on problem-solving activities during the lesson. At the end of the research study, you are expecting to improve in critical thinking skills in Fraction.

If you agree to participate in the research study, take note that the following ethical values will apply: your participation is voluntary; you may discontinue participation at any time if a need arise; you may also refrain from answering questionnaire questions when there is a need to do so. I am looking forward to your participation in the research study. I request you to sign the assent form provided if you agree to participate in the study.

Thank you in advance for your support.

Yours sincerely

Agadagba Emmanuel

(Please complete the assent form below and return to school)

Participation assent

I ...., understand the context of the research study titled: The effect of problem-solving teaching approach on learning fractions in Grade 8. I am aware that optionally I can allow video and audio recording of my participation during problem-solving sessions. I am aware that anonymity and confidentiality will be adhered to in this study. I informed that I may withdraw my assent to participate at any time without penalty by advising the researcher. I agree on my free will to participate in the research study.

Learner's signature: .... Date: ....

APPENDIX G: PRE AND POST-TEST QUESTIONS

Research Title:

THE EFFECT OF PROBLEM-SOLVING TEACHING APPROACH ON LEARNING FRACTIONS IN GRADE 8

Grade---

School code---

Learner code--

Age--

Gender---

Date---

Objectives: The objective of this test is to expose you to solving common fraction questions using problem-solving approach. By the end of this test, you should be able to apply the following skills while solving problems that involve common fraction.

- Calculating skills
- Integrating skills.
- Evaluating skills

DURATION: 50 MINUTES

Question	Marks allocated
Question 1	Three marks
Question 2	Four marks
Question 3	Five marks
Question 4	Six marks
Question 5	Seven marks
TOTAL	**Twenty-five marks**

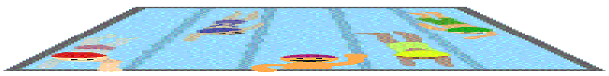
<u>Instructions:</u>

- Answer all questions in the best viable way you can.
- Use any method you think is the best to answer the questions.
- Last answer can be in fraction, whole number or two decimal placed value.
- Do not use a calculator.

Question 1: Farmer Ben counted all his livestock yesterday. $\frac{1}{4}$ Of the animals were pigs, $\frac{2}{3}$ of the animals were cows and $\frac{1}{3}$ were chickens. Which of these animals does Ben have the most?

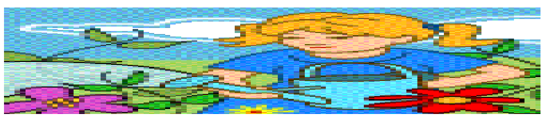
Question 2: Benaiah rode her bike for $\frac{1}{4}$ of a mile on Monday and $\frac{3}{4}$ of a mile on Tuesday. What are the total miles that she rides altogether?

Question 3: Bethel covers a distance of $\frac{2}{3}$ at the swimming pool in the morning and $\frac{5}{6}$ in the evening. How much farther did Bethel swim in the morning than in the evening?



Question 4: At a pizza party, Joy and his friends ate $4\frac{1}{5}$ cheese pizzas and $3\frac{2}{5}$ pepperoni pizza. How much of pizza did they eat in total…?

Question 5: Joy added $\frac{3}{5}$ soil to her garden. Her Daughter Benaiah added $\frac{13}{8}$ soil to the same garden. Who added more soil to the garden and what quantities of soil was add more?



APPENDIX H: LEARNERS QUESTIONNAIRE

Research Title:

THE EFFECT OF PROBLEM-SOLVING TEACHING APPROACH ON LEARNING FRACTIONS IN GRADE 8

Grade---

School code---

Learner code---

Age---

Gender---

Date--

Objectives: by the end of this questionnaire learners should be able to express their feelings toward: (a) Learning of mathematics (b) Learning of fractions (c) Teaching and Learning approach (d) Problem solving model (e) Traditional modelling approach.

DURATION: 50MIN

Marks: 100

SECTIONS	Marks allocated
SECTION A	20marks
SECTION B	20marks
SECTION C	20marks
SECTION D	20marks
SECTION E	20marks
TOTAL	**100marks**

LEARNERS QUESTIONNAIRE RESPONSES		**Yes/agree**	**No/disagree**
S/N	QUESTIONS		
SECTION A: Understanding based on the important of fractions			
1	Do you think it is important for you to learn mathematics fractions in Grade 8?		
2	Do you think, we can interpret or used $\frac{1}{2}$ or 0.5 fractions concepts knowledge in real life situation or discussions?		
SECTION B: Understanding of Fractions			
3	Do you think $\frac{3}{5}$ is a proper fraction?		
4	Do you think it can be possible for five friends to share three objects like $\frac{3}{5}$?		
SECTION C: Understanding per teaching and approach			
5	Do you think that you will understand better when your educators explain fractions by using real-world examples like time tell or buying of objects?		
6	Do you think that you can understand fractions and learn better if you allowed to study or discuss with your friends that understand better?		
SECTION D: Understanding per learning approach			

7	Do you think solving complex exercise in fraction can help us to develop our mind to understand questions in other mathematics topic?		
8	Have you learnt how you can solve mathematics questions even when you have never taught or given the example before?		
SECTION E: Understanding of fractions concepts.			
9	Do you think $\frac{3}{2}$ of fractions meaning is half of an object?		
10	Do you think $\frac{3}{2}$ of fractions can represented by fractions model?		
	Total		