Exploring Grade 10 Teachers' Mathematical Discourses During Euclidean Geometry Lessons in Johannesburg East District, South Africa

by

KAMBILA JOXE KYABUNTU

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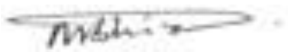
SUPERVISOR: **Dr HLAMULO WISEMAN MBHIZA**

As the candidate's supervisor, I have approved this thesis for submission.



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Supervisor: Dr Hlamulo Mbhiza

29/11/2023

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DECLARATION

I hereby declare that **Exploring Grade 10 Teachers' Mathematical Discourses During Euclidean Geometry Lessons in Johannesburg East District, South Africa** is entirely my own work and that all references to sources utilized or mentioned have been properly cited.

I additionally declare that the dissertation satisfies the acknowledged standards for originality after I submitted it to an originality checking software.

I additionally certify that I have never submitted this work, in whole or in part, for assessment at UNISA for another qualification or at any other higher education institution.

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Kambila Joxe Kyabuntu

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ABSTRACT

The current study sought to investigate the mathematical discourses used by Grade 10 teachers during Euclidean geometry lessons. To explore and understand teachers' classroom discourses during Euclidean geometry lessons, I researched with four Grade 10 mathematics teachers from Johannesburg East District in Gauteng, South Africa representing multiple cases. I espoused Adler and Ronda's Mathematical Discourse in Instruction (MDI) framework to investigate and understand the classroom discourses the four teachers used during the lessons on Euclidean geometry.

Within a qualitative research approach, I used semi-structured interviews, unstructured classroom observations, and video-stimulated recall interviews (VSRI) to generate data for the study. I used content analysis in relation to the MDI framework to analyse teachers' discourses and make summative judgments about their teaching of geometry concepts which allowed me to classify and categorise data in order to identify patterns and variances from the three data sources. The components of MDI were used to summarize the recorded lessons and all interviews were transcribed verbatim. The lessons were chunked into series of episodes, and I selected episodes that allowed for a detailed analysis of teachers' discourses using the component of MDI. I make sense of teachers' MDI and the rationale for the specific discourses they restricted while teaching by analysing the teaching shifts that took place in each chosen episode using the data they supplied via semi-structured interviews and VSRI. The findings demonstrate that the four components of Mathematical Discourse in

Instruction influenced each other during teachers' Euclidean geometry lessons. The teachers did not provide explanatory talk during the lessons, instead they used questions-and-answer discourse to get learners to name and legitimate Euclidean geometry concepts and principles. Learners' participation in all the observed lessons was limited to providing one-word answers and teachers did not prompt learners to provide justifications for their thinking.

Key words: Euclidean geometry, Grade 10, mathematical discourse, Mathematics Discourse in Instruction, Teaching

ABBREVIATIONS

CAPS – Curriculum and Assessment Policy Statement

DBE – Department of Basic Education

DSF – Directorate of Student Funding

FET – Further Education and Training

GDE – Gauteng Department of Education

MDI – Mathematical Discourse in Instruction

MKO – More Knowledgeable Other

NAPTOSA – National Professional Teachers' Organization of South Africa

NCTM – National Council of Teachers of Mathematics

NCS – National Curriculum Statement

RNCS – Revised National Curriculum Statement

SACMEQ – Southern and Eastern Africa Consortium for Monitoring Educational Quality

SADTU – South African Democratic Teachers' Union

TIMSS – Trends in International Mathematics and Science Study

UNISA – University of South Africa

VSRI – Video Stimulated Recall Interviews

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DEDICATION

To the Lord God Almighty, the One who is and who was, because you have taken your great power and have begun to reign in every aspect of this study and for providing me with the strength to reach this far.

To my son Kenan Jean-Abdias Kambila and my wife Tania Kyabuntu, for the inspiration they provide me each day.

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CHAPTER 1

UNDERSTANDING THE TEACHING OF EUCLIDEAN GEOMETRY IN SOUTH AFRICA

1.1 Introduction and background of the study

Euclidean geometry is viewed as one of the important topics in school mathematics curricula in many countries around the world (Naidoo & Kapofu, 2020; Mwadzaangati, 2015; Al-Khateeb, 2016). It prepares learners for mathematics, science, engineering, and technology professions that are at the heart of a country's economic development. Euclidean geometry sharpens our visual, logical, rational, and problem-solving abilities that we all need in different spheres of our lives (Ubah & Bansilal, 2019). However, despite numerous arguments for including Euclidean Geometry in secondary school mathematics curricula, the teaching of this topic has been characterized by perturbing pedagogical challenges in many countries including South Africa (Naidoo & Kapofu, 2020; Kanandjebo & Ngololo, 2017). Previous studies have demonstrated the need for learners to learn and own knowledge and skills for Euclidean Geometry (Machisi, 2021, Smith, 2020). One way of enabling learners' successful mathematical learning is through teachers' mathematical discourse during the lessons, relating the key concepts for topics and their relatedness (Adler and Ronda, 2017). This being the case, there is a scarcity of mathematics education research that explored teachers' mathematical discourse during Euclidean Geometry lessons, especially at the Grade 10 level within the South African context. Thus, the current study seeks to explore and understand Grade 10 teachers' mathematics discourse and address the identified research gap.

Notwithstanding the above discussion, of importance to note is that in South Africa, Euclidean Geometry was removed from the mainstream school mathematics curriculum in 2006 after a series of poor results in Grade 12 mathematics examinations (Engelbrecht &Phiri, 2010). The policy makers asserted that teachers did not have the required depth of content and pedagogical knowledge to effectively teach the topic and enable learners' understanding of the concepts (Bowie, 2009). The mathematics curriculum planners at the time argued that the poor results in the subject was sequel to the underteaching of the topic and pointed to lack of effective explanations that teachers offered during the lessons (Blaine, 2009; Bowie, 2009; Engelbrecht &Phiri, 2010). For instance, results from the Trends in International Mathematics and Science Study (TIMSS) of 2006 revealed that geometry was the area of mathematics where the performance of South African students was dismal, which further reinforced that the topic was difficult and should therefore be expunged from the curriculum (Ndlovu, 2013). According to Ntuli (2014), mathematics teachers had limited knowledge of Euclidean Geometry content and pedagogical knowledge which resulted in the under-teaching of the topic. These were some of the reasons that led curriculum planners to suggest that Euclidean Geometry should not be compulsory at Grades 10 to 12. As a result, the Revised National Curriculum Statement (RNCS) which came into effect in 2006 relegated Euclidean Geometry to an optional paper – Mathematics Paper 3, in which teachers were at liberty to either teach or not teach the topic during an academic year (Naidoo, 2013) with most schools and teachers opting for the latter (Naidoo, 2013). Of interest is the repercussion that came from this situation, in terms of most learners missing out on being exposed to the topic and its related concepts.

In addition to the above discussion, in the year 2010, researchers in South African universities reported that the exclusion of Euclidean Geometry as a compulsory topic in mathematics curriculum had increased the gap between secondary school and tertiary mathematics, for students enrolled in science and engineering programmes (Hlalele, 2020; Mouton, Louw & Strydom, 2012; Wolmarans, Smit, Collier-Reed & Leather, 2010). A study by the Mathematics Education Department at the University of the Witwatersrand highlighted a 37 percent drop in the June mathematics pass rate for first-year students in 2009 (Blaine, 2009). Blaine (2009) attributed lower pass rates to the lack of exposure that most of the students had in secondary school, which resulted in gaps in the foundational knowledge for the topic, which universities had to make up for. These findings were consistent with trends observed at other universities in South Africa, namely the University of Cape Town, the University of Stellenbosch, the University of Pretoria, the North-West University, the University of KwaZulu-Natal, and the Nelson Mandela Metropolitan University (Hlalele, 2020; Mouton, Louw & Strydom, 2012; Wolmarans, Smit, Collier-Reed & Leather, 2010). This discussion also highlights that there are mathematics teachers currently teaching in South African classrooms who were never exposed to Euclidean Geometry concepts as learners, and if universities did not manage to bridge the gaps, such teachers might not possess the adequate content to effectively help learners learn the topic with understanding. There is scarcity of mathematics education research that explored how the removal of the topic in 2006 impacted knowledge development for individuals who became teachers and are now expected to teach the topic.

In January 2012, the South African curriculum for Grades R–12 saw the introduction of a new curriculum referred to as the Curriculum and Assessment Policy Statement (CAPS), which was an amendment of the National Curriculum Statement (NCS) which came with the reintroduction of Euclidean Geometry as a compulsory topic for Grades 10-12 mathematics (Alex & Mammen, 2016). Although the decision to make Euclidean geometry compulsory again was commendable, the unions, including the South African Democratic Teachers' Union (SADTU) and the National Professional Teachers' Organization of South Africa (NAPTOSA) argued that teachers were not ready for the reintroduction of the topic in the curriculum. They cited the lack of adequate in-service teacher training prior to bringing back Euclidean geometry as a compulsory topic (Ntuli, 2014). South African mathematics teachers wondered why Euclidean geometry was reinstated into the mainstream mathematics curriculum when the difficulties that led to its removal in the mathematics curriculum had not been fully addressed (Ndlovu, 2013). One way of gaining insight into the nature of difficulties experienced in teaching and learning the topic is through researching with teachers, observing their classroom teaching practices with the purpose of understanding how they make Euclidean geometry concepts available for the learners. Considering that the concerns that led to making Euclidean Geometry optional for teachers and learners back in 2006 were related to the under-teaching of the topic, the current study explores Grade 10 teachers' mathematics discourse in instruction for the topic, to understand how they explain the contents for the topic to the learners, as well as their experiences of teaching the topic.

According to Department of Education,

> "Mathematics is a language that makes use of symbols and notations for describing numerical, geometric, and graphical relationships. It is defined as a human activity that involves observing, representing, and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves" (DBE, 2011, p. 11).

In view of this statement, of importance to note is that teachers' discourses during teaching plays a crucial role in ensuring that learners learn and own the mathematical language, skills, and knowledge (Mbhiza, 2021). This background further demonstrates the role teachers' content and pedagogical knowledge play in enabling learners understanding of concepts, not only for current learning, but for further education as well. The current study explored the nature of teachers' discourses during the lessons, to understand how their teaching of the topic enabled and/or limited opportunities for learners to gain epistemological access to mathematics contents related to Euclidean geometry.

1.2 Problem statement

One of the objectives of the National Curriculum Statement for Grades R–12 is to produce learners who can effectively communicate using visual, symbolic, or verbal skills in a variety of contexts (DBE, 2011). This objective is in line with the value of geometry, particularly Euclidean geometry, which is envisaged to teach learners how to solve geometrical problems using a variety of representational strategies and logical thinking. Bansilal and Ubah (2019) states that "The study of geometry provides opportunities for learners to visualise concepts that may be related to other areas of mathematics, including trigonometry, patterns and measurement" (p. 2). Effective teaching of Euclidean geometry has proven difficult, as previous studies have reported that many students have difficulty identifying, analysing, and thinking critically about spatial objects and images (Couto & Vale, 2014; Mabotja, 2017). Research indicates that few students tackle these questions and those who do perform poorly (DBE, 2018), even though Euclidean Geometry accounts for 50 ±3 of the marks on Grade 12 Mathematics Paper 2 (DBE, CAPS, 2011a). This being the case, of concern for the current study is that there is dearth of mathematics education research that explored the teaching of the topic at Grade 10 level within the South African context, especially studies focusing on teachers' discourses during teaching. Accordingly, this study sought to explore Grade 10 teachers' teaching of Euclidean geometry, to gain insight into how the discourses they inhibit during the lessons enabled and/or constrained learners' learning of the topic.

The Grade 12 annual examination reports reveal the difficulties associated with the teaching and learning of Euclidean geometry (Baiduri et al., 2020; Bonnie, 2016;

Bowie, 2009). Figure 1 depicts the data from a random sample of candidates for the years 2017 to 2019 on their performance in mathematics paper 2, in which Euclidean geometry is included (DBE, 2019). That is, I compiled the poster in Figure 1 from the analysis of different DBE reports, to demonstrate the performance of learners in Euclidean geometry.

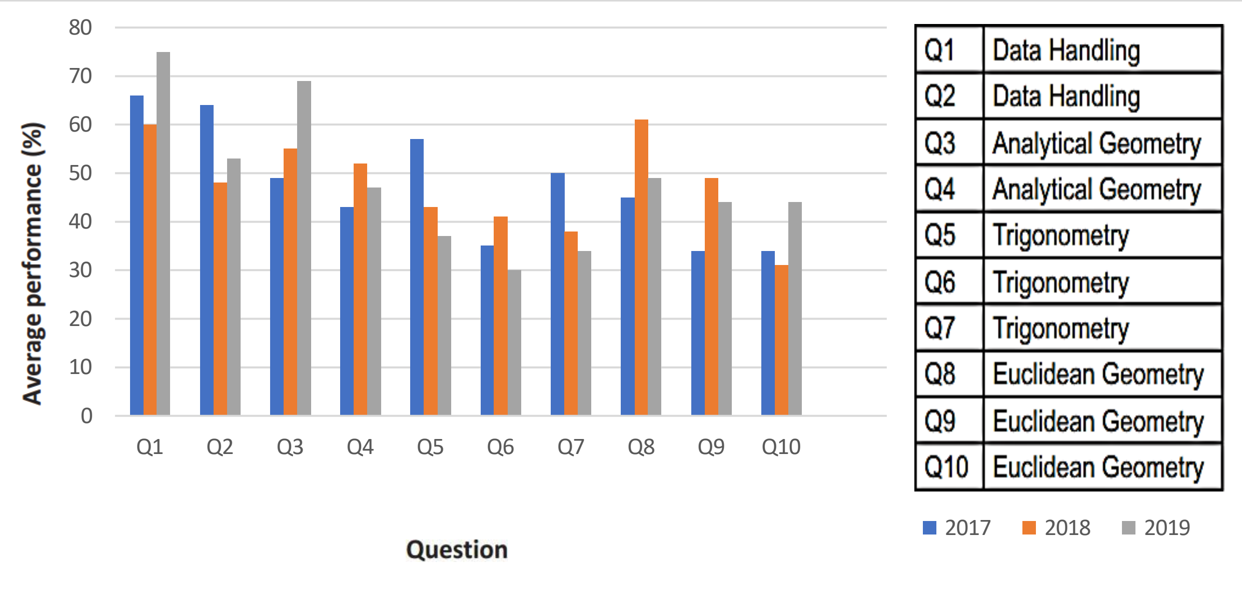


Figure 1. *Average Percentage Performance per Question for Paper 2 from 2017 to 2019 (DBE, 2017, 2018 and 2019)*

The depicted performance in Figure 1 demonstrates relative degrees of challenge of each question as experienced by candidates who wrote the Grade 12 mathematics examinations across those years. In relation to Euclidean geometry, the results demonstrate how poorly geometry performance has generally been over the three years. In this regard, the trend, as shown by the above graph, was that students performed well below 50% for questions 9 and 10, with some averages falling well into the 30% range, except for question 8 of 2018, especially when juxtaposed with other questions covering other concepts in paper 2. While the performance reveals the quality of learners' learning and understanding of mathematical concepts, of importance to note is that teachers' pedagogical reasoning and actions during the lessons plays a major role in enabling learners' knowledge retention for assessment. This being the case, the dearth of studies that explored the teaching practices for Euclidean geometry in South Africa has not offered insights into how teachers make the contents available for learners, to ensure high thinking quality and knowledge retention for assessments. Although various reports by the Department of Basic Education have not indicated what it is about the Euclidean geometry lessons learners find difficult, Marange and Tatira’s (2023) observation highlights some of the attributes that lead to difficulties in learning the topic. The authors reported that,

> The use of teacher-centered instructional approaches was mostly observed in most secondary schools. It was evident that mathematics teachers encouraged passive learning and made learners feel that they had nothing to contribute because they (teachers) were dominant throughout the lesson. The approach most teachers used led to boredom in class. (p. 2).

Thus, the focus of the current study was on how teachers explained concepts verbally, what they wrote on the board and notes they handed out as well as how these aided their mathematical explanations related to geometry concepts.

In addition to the above discussion, results from national tests also indicate learners' ongoing difficulties with mathematics, including their performance in Trends in International Mathematics and Science Study (TIMSS, 2011). Considering that Euclidean Geometry has the highest mark allocation in the South African Grade 12 mathematics paper 2, 50 out of 150 total marks (see Table 1) which is one-third of the total allocated marks for the paper, coupled with French’s (2004) assertion that learners’ conceptual rigor in other mathematics content areas depends on their understanding of geometry, could be the major attribute to high failure rates in the subject (French, 2004; Driscoll, 2010). Hence, I consider the current study to important part of the efforts to understand how teachers enable and/or constrain learners’ understanding of the concept and to configure strategies to improve learners’ performance in Mathematics across South African Schools.

Table 1.

Mark Composition in Grade 12 Mathematics Paper 2

Topic covered in the question paper	Marks composition
Euclidean geometry	50
Trigonometry	40
Analytic geometry	40
Statistics	20

This mark composition demonstrates that Euclidean Geometry is one of the key content areas of the South African mathematics curriculum. However, the marks that mathematics learners obtain in Grade 12 NSC examinations from Euclidean Geometry questions, are relatively low as indicated in Figure 1, reason research with Grade 10 teachers should be prioritised, to understand how teachers make the foundational knowledge available to the learners for future learning.

1.3 Rationale for the study

The rationale for this study is based on my own informal observations. I have observed in the school where I teach that if learners are well-oriented and guided, they stand a better chance to enjoy learning Euclidean geometry and understanding it. I have also observed in the same school that Grade 10 mathematics teachers teach learners a difficult approach to solving Euclidean geometry problems whereby learners are forced to memorise solutions of different examples done in class for them to reproduce the same steps when solving different Euclidean geometry problems in tests and examinations, instead of understanding the topic conceptually and relationally (Skemp, 1976). I have informally observed that after giving the proof of a particular theorem, teachers almost always show learners how to apply the theorem in solving Euclidean geometry problems. After attempting some examples, learners imitate the teacher's work examples to solve similar problems which are assumed to be effective as learners are expected to regurgitate memorised procedures in tests and examinations. This continues to an extent that whenever learners are given an opportunity to discuss a problem, they will repeat the same methods used by their teacher. If they fail to remember all steps correctly, they will be stuck unless they can rely on the teacher to help. While it is expected that the teacher becomes the source of information during teaching and learning, it becomes very difficult in this situation for learners to internalise the concepts and to solve the problems independently due to the overreliance on the teacher for steps to be reproduced. In this way, teachers are always happy and think that learners are interacting with them. The situation becomes more and more difficult during examinations or when learners are given homework that requires them to create meaning of the concepts and apply skills they learned in new mathematical situations.

I have also observed that learners seem to understand the concepts while completing their activities in class when the teacher is there to facilitate their learning and guide them through the work; but during their formal assessment tasks, it is the opposite. These informal observations led me to an assumption that teachers' discourses during Euclidean geometry lessons play a significant role in enabling or limiting effective mathematics learning and in turn learners' performance in the topic. Of concern for the current study is that within the South African context, there is dearth of research that explored teachers' discourses in instruction for Euclidean geometry, especially at Grade 10 level where learners are introduced to more complex geometric concepts (DBE, 2011). This phenomenon should be explored, to gain insight into the nature of teachers' classroom practices while teaching the topic, to understand how teachers enable and/or constrain learners' epistemological access to geometry concepts.

Notwithstanding that many scholars have conducted studies on different topics in Euclidean geometry (Mthembu, 2007; Van Putten, Stols & Howie, 2010; Dhlamini, 2012; Ubah & Bansilal, 2019), however, as stated earlier, there is dearth of research that explored teachers' explanatory talk while teaching the topic, especially at Grade 10 level in South Africa. Previous studies on Euclidean geometry within the South African context have focused on the teaching and learning of the topic at Grades 11 and 12 (Tachie, 2020), overlooking the importance of researching with Grade 10 teachers, to understand how they enable or constrain learners' epistemological access to the topic's concepts. The rationale for researching with Grade 10 mathematics teachers is also vested in the understanding that the effectiveness of the teachers' explanations or lack thereof in this grade can enable or hinder learners' future learning, considering that the work that is covered in grade is foundational for work to be covered in grades 11 and 12, as well as in future learning at institutions of higher learning.

Furthermore, the current study highlights the importance of comprehending and evaluating teachers' classroom discourses in mathematics, especially in the teaching of Euclidean geometry. Using a structured framework such as the Mathematical Discourse in Instruction, mathematics teachers can gain insights into how their discourses influence learners' learning and participation during mathematics learning and teaching. This study contributes to the broader body of research knowledge focusing on effective teaching practices in mathematics classrooms, arguing for the need for more interactive and explanatory discourses to improve learners'' learning of Euclidean geometry concepts.

1.4 Purpose of the study

The purpose of the proposed study is three-fold. Firstly, the study seeks to explore Grade 10 mathematics teachers' discourses during Euclidean geometry lessons. Secondly, to interrogate teachers' experiences of teaching Euclidean geometry at Grade 10. Thirdly, to examine factors that shape teachers' discourses during Euclidean geometry lessons.

1.5 Objectives of the study

The objectives of the study are as follows:

- To describe and critically interrogate Grade 10 teachers' discourses during Euclidean geometry lessons.
- To explore how teachers enable and/or constrain learner participation during Euclidean geometry lessons.
- To identify and analyse Grade 10 teachers' explanatory talk during Euclidean geometry lessons?

1.6 Research questions

The main research question that will guide the current study is:

What are teachers' mathematics discourses during Euclidean Geometry lessons in Johannesburg East District? To further explore the main research question, I have identified the following sub-research questions:

- What are teachers' discourses related to exemplification during Euclidean geometry lessons?
- How do Grade 10 teachers enable and/or constrain learner participation during Euclidean geometry lessons?
- What are teachers' explanatory talk during Euclidean geometry lessons?

1.7. Operational Definition of Terms

Table 2.

Operational Definition of Terms

Term	Operational Definition
Euclidian geometry	This is s branch of mathematics that focuses on the properties and relationships of points, lines, angles, and shapes in a two-dimensional space. In this study, Euclidean geometry refers to the topic taught in the second term of an academic year in Grade 10, which focuses on the following curriculum statements: "*1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles. 2. Investigate line segments joining the mid- points of two sides of a triangle. 3. Define the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium. Investigate and make conjectures about the properties of the sides, angles, diagonals and areas of these quadrilaterals. Prove these conjectures*." (Department of Basic Education (DBE), 2011, p. 25).
Grade 10	Grade 10 is the start of FET (Further Education Training) Phase of the South African Basic Education levels. The mathematics curriculum in this grade draws from the foundational knowledge and skills taught in earlier grades and introduces some more complex topics, including geometry, algebra, trigonometry, functions, and statistics.
Mathematical Discourse	According to Adler and Ronda (2015), mathematical discourse refers to the ways teachers and learners talk and communicate mathematical ideas, concepts and employ different problem-solving strategies. It encompasses the use of language, symbols, and gestures to convey mathematical understanding and reasoning. Within the context of this study, I locate the term "mathematical discourse" in Adler and Ronda's (2015) conceptualisation to not only ways of speaking, acting, interacting, thinking, and believing as well as ways of reading and writing, but also to mathematical values, beliefs, and points of view that Grade 10 teachers inhibits relating to Euclidean geometry teaching.
Mathematics discourse in instructions	Mathematics discourse in instructions refer to the teaching strategies and practices that teachers espouse and employ to facilitate the meaningful mathematical discussions and learning during the lessons (Adler & Ronda, 2015; Smith, 2020). Such strategies and practices are aimed at enabling active learner engagement, critical thinking, and mathematical communication during learning. For Adler and Ronda (2015), effective mathematics discourse in instructions encourage learners to articulate their reasoning, make conjectures and prove them, justify their solutions, and engage in collaborative dialogue about mathematical concepts.

1.8. Structure of the dissertation

This dissertation contains seven chapters. The main purpose of **chapter 1** is to provide the background of the study, highlighting the importance of the teaching of Euclidean geometry effectively within the South African context. This chapter also presents debates about the changes that took place in the South African curriculum regarding the teaching and learning of the topic. The problem statement, the rationale for conducting the current study, the purpose, specific objectives, and research questions are also provided. **Chapter 2** presents literature that addresses the teaching and learning approaches for Euclidean geometry. The chapter also provides literature associated with the teaching of Euclidean geometry in South Africa as well as challenges in teaching and learning of the topic.

In **chapter 3**, I present the Mathematics Discourse in Instruction (MDI) framework that is espoused to critically explore teachers' mathematical discourse during Euclidean geometry lessons. To do this, I discuss in detail the conceptualisation of the MDI framework and highlight its relevance and operationalisation for the current study. **Chapter 4** discusses the selected research methodology for the study. The selection and rationale for selecting the qualitative research approach, case study research design and the sampling strategy, which was purposive and convenience, as well as the sample size are discussed. I also provide details of the research processes in terms of the methods that I employed in this study, highlighting how issues of triangulation were addressed during data generation. In addition, details about data analysis technique that I used in this study are provided. Lastly, the ethical considerations made in this study, issues of credibility, transferability, dependability, and confirmability are also presented. **Chapter 5** presents the data that was generated using semi-structured individual interviews, non-participant unstructured classroom observations, and Video-Stimulated Recall Interviews (VSRI). Data are analysed and interpreted considering the reviewed literature and the espoused theoretical framework for the study. In **Chapter 6**, I present the findings and discussions of the themes that emerged from the analysis of the interviews, classroom observations and VSRIs. **Chapter 7** concludes this dissertation and provide recommendations for teaching as well as for future research studies. The limitations of the study are also highlighted in this chapter.

1.9. Chapter Summary

In this chapter I presented the introduction and background of the study by discussing the teaching and learning of Euclidean geometry within the South African context. Then, I outlined the problem statement for the study, highlighting the need to conduct studies focusing on the teaching of Euclidean geometry, particularly at Grade 10 level within the South African context. I also outlined the rationale for conceptualising the current study, the purpose, the research objectives, and the research questions underpinning this study. I have also provided the operational definitions of the key terms used in this study. Lastly, I presented the structure of this dissertation by detailing the focus of each chapter.

CHAPTER 2

MATHEMATICS DISCOURSE AND EUCLIDEAN GEOMETRY TEACHING

2.1 Introduction

In South Africa, there is still concern about how Euclidean Geometry is taught and learned at the school level. Without discounting the importance of the role that students play in the teaching and learning processes, the study's focus is on teachers' explanatory talk during Euclidean Geometry lessons in Grade 10. This chapter begins with a review of literature on the South African curriculum and Euclidean geometry, followed by conceptualisation of mathematics discourse. Then I focus on the discussion relating to understanding Euclidean geometry teaching and learning approaches. This is followed by a review of literature on the difficulties associated with the teaching of Euclidean geometry. Lastly, I review the literature on the teaching of Euclidean geometry within the South African schooling context.

2.2. Understanding the South African curriculum and Euclidean Geometry

According to Froneman and Hitge (2019), the field of Euclidean geometry has witnessed various changes within the framework of South Africa’s mathematics education curriculum. These alterations encompassed the adoption of Outcome-Based Education (OBE), Curriculum 2005, and Curriculum Assessment and Policy Statement (CAPS). These reforms were integral to the broader modifications applied across all subjects within the post-apartheid South African school curriculum. Froneman and Hitge (2019) categorized these changes based on outbound Grade 12 cohorts into the Tradition Knowledge-based Curriculum (TKC), also known as Curriculum 2005, for those matriculating up to 2007; the National Senior Certificate (NSC) with a constructivist approach through OBE for matriculants from 2008 to 2013 (Cilliers & Chetty, 2013); and the revision of OBE documented as CAPS for Grade 12 cohorts from 2014 to the present.

The transformation of the South African curriculum had repercussions for Euclidean geometry as a component of the mathematics curriculum. Notably, in 2008 (NSC, 2008), Euclidean geometry was excluded from the compulsory mathematics curriculum, with assessment relegated to an optional paper. The rationale behind this exclusion stemmed from the consistent poor performance of learners, attributed to educators' inadequate geometry content knowledge (Bowie, 2009). Machisi (2020) similarly argued that this exclusion resulted from a series of unsatisfactory Grade 12 mathematics results. The prevalent view is that the removal aimed to afford educators the time to enhance their capacity to teach geometry more effectively.

Criticism, however, surrounded the exclusion, with Siyepu and Mtonjeni (2014) contending that it disadvantaged learners pursuing engineering courses at the university level. Kearsley, cited in ASSAF (2009), asserted that it adversely affected learners' success in tertiary studies related to health sciences and mathematical engineering. Subsequent studies confirmed that learners who had not studied geometry in high school exhibited weaker mathematics skills at the university level (Tachie, 2020). Universities also argued against the exclusion, citing a lack of coherence in the study of shape and space, along with diminished opportunities for learners to work with proof (Bowie, 2009).

The ramifications extended beyond university mathematics departments, causing inconsistencies in high school geometry instruction. Geometry became a voluntary learning and teaching area, with learners choosing whether to write Paper 3, and educators exercising discretion in teaching it (Van Putten et al., 2010; Mabotja, 2017). This compromised the depth with which Euclidean geometry was addressed. The enrolment for Mathematics Paper 3 plummeted significantly in 2008, as schools lacking resources and capacity to teach geometry opted not to offer it. This decline drew criticism from education stakeholders. When geometry became optional, many learners opted out, missing out on crucial geometric reasoning (Ngirishi and Bansilal, 2019). The in-service geometry training for CAPS was deemed insufficient for educators, resulting in discomfort with the topic (Tachie,2020). Many educators in South Africa, who did not study geometry, were now expected to teach CAPS, leading to challenges in geometry conceptualization by learners (Govender, 2014; Naidoo &

Kapofu, 2020; Luneta, 2014). Implemented in 2012, CAPS brought about changes in the mathematics curriculum, reintroducing topics excluded in the OBE curriculum, such Euclidean geometry, aiming to provide students with more comprehensive understanding of mathematical principles and concepts (Herbst et al., 2017). Since then, Euclidean geometry has become a compulsory subject in the Further Education and Training (FET) phase (Brijlall, 2017), focusing on subject knowledge, excellence, and demanding higher standards of learners' performance (Maddock & Maroun, 2018). I have also observed that despite numerous curriculum reforms, learners' performance in Euclidean geometry remained poor and the ability of teachers to teach the subject is crucial. Thus, the current study aims to explore Grade 10 teachers' mathematical discourses during Euclidean geometry lessons.

2.3. Conceptualisation of Mathematics Discourse

The central premise of the current study is the position that Euclidean geometry teaching is a discursive activity (Adler & Ronda, 2015). According to Adler and Ronda (2015; 2017) and Sfard (2012), mathematics teaching entails processes whereby teachers and learners use different mathematics discourses, in which meaning related to mathematical objects are co-constructed through classroom interactions and language. This means that, mathematical objects derive their existence and associated meanings as teachers and learners interact with mathematical concepts and each other during teaching and learning to construct meanings for such concepts. This perspective posits that mathematics teaching involves engaging in mathematical discourse, whereby mathematical objects are constructed using language and communication (Adler & Venkat, 2014). This resonates with Lynch and Bolyard (2012) definition of mathematical discourse, which they view as the oral and/ or written communication of mathematical concepts or ideas during teaching and learning, the teaching of Euclidean geometry for the current study. One way of elaborating on this perspective is that it is envisaged that mathematics teachers explicitly articulate and create opportunities for discussions of the mathematical concepts to enable learners' mathematical learning (Gresham & Shannon, 2017). I concur with the authors cited herein and posit that teaching mathematics means creating opportunities for learners to meaningfully participate in this discourse. Similarly, Stein (2007) argues that it is through allowing learners to engage in critical discussions about the behaviour of mathematical objects that we can enable an in-depth understanding of mathematical topics and their relatedness. This being the case, it is not clear how teachers within the South African context encourages learners' analytical and critical thinking for Euclidean geometry, considering the dearth of studies that focused on the teaching of the topic, especially at the Grade 10 level.

In this study, I use the concept of mathematical discourse in line with Adler and Ronda (2015). Adler and Ronda (2015) averse that, to enable effective mathematics learning, teachers should create learning opportunities in which the learners engage in the process of evaluating and interpreting the perspectives or mathematical ideas of others, make conjectures and prove them, encouraging learners to engage in mathematical discussions to create mathematical stories about mathematical concepts. Drawing from Sfard (2008, p. 160), Adler and Ronda (2015) operationalise the term mathematical discourse as a unique form of communication, which is "made distinct by their tools, that is, words and visual means, and by the form and outcomes of their processes, that is, the routines and endorsed narratives that they produce". How then do mathematics teachers create opportunities for learners to participate in this discourse during lessons on Euclidean geometry? This question will be attended to in the next chapter as I demonstrate how MDI as the espoused theoretical framework enables the exploration and understanding of how teachers made Euclidean geometry concepts available for learners to learn within the context of the current study. Of interest for this study is how teachers designed Euclidean geometry lessons at Grade 10, in terms of the used examples and tasks during teaching, what they said and their observable actions during the lessons.

In addition to the above discussion, it is important to note that Adler and Ronda (2015) focus specifically on what teachers say and do during teaching and are silent about what teachers do not say and what they do not do during teaching. Thus, the current study also pays attention to what teachers do not say that they 'ought' to say as well as what they do not do. These aspects necessitate that I review the literature on the Euclidean geometry teaching approaches, to establish some of the envisaged ways of teaching the topic, which subsequently will be helpful in the delimitation of what teachers should say and/or do during the teaching of Euclidean geometry to facilitate the learning thereof.

2.4. Understanding Euclidean Geometry Teaching Approaches

Mathematics teaching, especially in Euclidean geometry should not be based on rote memorization and recalling and repeating previously learned steps and skills, but on critical understanding of the concepts and skills related to the topic (Heeralal & Dhurumraj, 2016). This is in line with the curriculum positionality of the CAPS (DBE, 2011), that envisage that a mathematics teacher should among other things position themselves as a mediator of learning, rather than being at the centre of teaching and learning in the classroom. While this is the case, previous research on the teaching of geometry in general have highlighted that teachers still employ a variety of teaching and learning strategies while educating children in Euclidean geometry (Bansilal & Ubah, 2019; Chimuka, 2017). In a study conducted by Tachie (2020), some teachers spoke and wrote throughout the teaching and learning process without allowing learners opportunities to internalise the contents presented and discussing their understanding thereof. The teachers taught through drill and regurgitation rather than seeking the learners to make observations, create conjectures and prove them to make generalisations about the mathematical objects.

The above discussion resonates with Chauraya and Brodie's (2018) contention that teachers need to be capacitated to develop the skills of eliciting learners' thinking relating to the errors they made. The authors found that when teachers delved into analysing and comprehending learners' errors in learning and assessment, the participating teachers showed increased expertise in mathematics and teaching methods (Chauraya & Brodie, 2018). This calls on teachers during Euclidean geometry lessons to go beyond just an identification and acknowledgement of correct and/or incorrect answers and make strides to shift towards actively using the incorrectly produced learner's answers productively as a one way to enhance in-depth understanding of the topic. It thus becomes imperative that studies with teachers' classroom discourses while teaching Euclidean geometry are conducted, to gain insight into how teachers make the contents available to the learners, especially how active classroom interaction is created and maintained during the lessons. This is in line with Chimuka's (2017) argument that teachers need to shift from teacher-centred approaches to learner- centred approaches, if the urgency to improve the performance in mathematics, particularly in Euclidean geometry is seriously considered. It is worth noting that, despite the ongoing calls for the need to actively engage with learners during geometry teaching and learning as co-constructors of mathematical knowledge, teachers continue to revert to traditional teacher dominant approaches (Tachie, 2020; Machisi, 2021). Although the studies above focused on teachers' teaching of geometry, there are dearth of studies that focused on geometry teaching at Grade 10 level, because most existing studies on the topic within the South African context were conducted at Grades 11, 12 or universities. To address this gap, the current study sought to explore and understand Grade 10 teachers' discourses during Euclidean geometry lessons.

Furthermore, Machisi (2021) found that Grade 12 teachers did not bother to inquire about their learners' understanding of the basic geometry concepts from earlier grades when employing traditional teaching approaches. The author demonstrated that the teachers delved directly into the geometry fundamentals for the current grade. Students were not given geometry theorems or axioms for autonomous research, observation, or discovery to make conjectures and in turn prove them. Mudhefi (2022) argues that,

> The use of educator-centred methods in this section of geometry and other concepts has been characterised by numerous challenges because the approaches present situations where learners are viewed as mere receptors of mathematical facts, formulas, principles and theorems through rote memorisation (p. 33).

This statement suggests that, unless given access to other teaching methods, teachers will most likely continue to teach Euclidean Geometry the conventional way. Wei et al. (2017) assert that pedagogy and instructional strategies used by teachers have an impact on how geometry is taught and retained. In support of the foregoing discussion, Luneta (2014) suggests that for teachers to ground their teaching of conceptual knowledge and in-depth understanding of Euclidean geometry, they should adopt pedagogical actions that allow learners to engage in investigation, exploration, discussion, and communication of geometric ideas. It was interesting to observe how teachers created and/or constrained learning opportunities to engage in these processes, to facilitate effective learning of geometric concepts.

Furthermore, other previous studies (Baiduri et al., 2020; Yew & Saleh, 2019) on effective teaching of geometry appraise Van Hiele's informed teaching as one of the effective ways for teaching Euclidean geometry, as compared to traditional teaching approaches. The rationale given for endorsing the Van Hiele's model of geometric reasoning is that it foregrounds the sequential understanding of geometry concepts, moving from inductive to deductive reasoning. Of importance to note is that the key tenets and premise of this model are located within the broader constructivist approach, which postulates the need for learners to be active participants in the process of meaning making during teaching and learning (Fujita et al., 2014). Through the adoption of Van Heile's theory, it is believed that teachers can address learners' common errors and misconceptions during teaching and learning. Figure 2 below depicts the Van Hiele's levels of geometric reasoning:

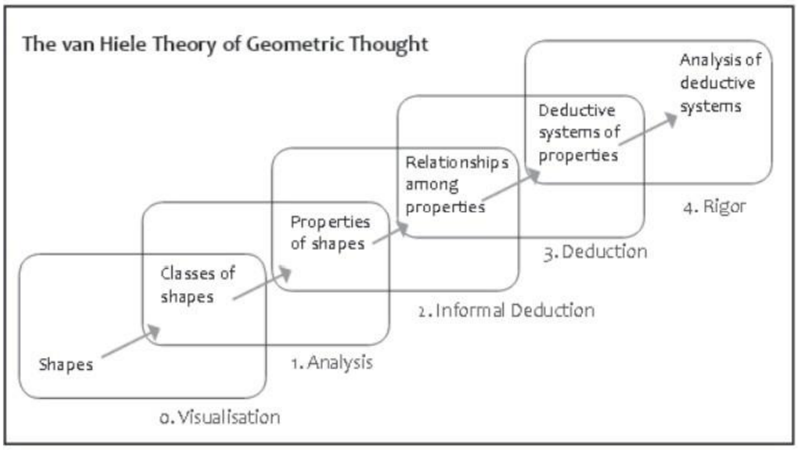


Figure 2.

Van Hiele's theory of geometric thought (Adapted from Van de Walle 2004, p. 347).

This figure is demonstrative that the learning and understanding of geometric concepts is cumulative and hierarchical and teachers are tasked with ensuring that learners can operate across the different levels, with rigor being the envisaged level for all learners to operate in.

As mentioned earlier, the CAPS curriculum emphasises the constructivist approach to teaching, including for the teaching of Euclidean geometry. It was important for this study to observe how teachers facilitated learners' learning of Euclidean geometry concepts through the discourses they inhibited during the lessons. The following section focuses on the difficulties that are associated with the teaching of Euclidean geometry.

2.5. Difficulties Associated with Teaching and Learning of Euclidean Geometry

Euclidean geometry instruction is still being neglected, which is a worry (Tutak & Adams, 2015). Wei et al. (2017) contend that teaching Euclidean geometry is challenging because it calls for the use of advanced cognitive abilities, which most teachers lack due to the lack of exposure to geometry concepts as learners and/or under-training at universities to teach the topic effectively. According to Wentzel (2016, p. 49), “Euclidean geometry instruction in South Africa is in a very bad state, and a great number of learners do poorly in Euclidean geometry.” This suggests that Euclidean geometry remains challenging to teach and learn and such challenges are attributed mainly to teachers’ teaching practices and the quality of content delivery during teaching (McAndrew et al., 2017; Utami et al., 2017; Tutak & Adams, 2015). Machisi (2021) argues that the difficulties that learners face in understanding geometric proofs are due to the continued overuse of traditional lack of learner-centred approaches during the lessons. Similarly, Sibaya (2020, p. 2) argues that “teachers need to be creative in designing geometric activities that would promote active learning” whereby learners are actively engaged in the process of meaning making during teaching and learning. While previous studies attribute the challenges associated with Euclidean geometry to teachers’ content knowledge and teaching practices, there is a scarcity of research that explored teachers’ teaching of the topic within the South African context, especially with Grade 10 teachers. This is due to the overfocus on learners’ errors and misconceptions on the topic, particularly in Grades 11 and 12.

As mentioned earlier, this study sought to explore and understand teachers’ discourses of Euclidean geometry at Grade 10 level, to address the identified research gap. In addition to the foregoing discussion, findings from Luneta (2015) revealed that most Grade 12 learners underperformed due to conceptual errors they committed when answering geometry questions. Other studies indicated that South African learners, Grade 12 learners are operating way below their envisaged levels, with geometry being one of the challenging topics (Ngirishi & Bansilal, 2019; Siyepu & Mtonjen, 2014). Alex and Mammen (2016) demonstrated that most learners operate at the concrete visualisation level rather than at the expected abstract level in geometry, which requires higher mental functioning for geometrical reasoning. Considering that most math teachers were not taught Euclidean geometry as learners as well as in teacher education, it becomes interesting how they facilitate learners' conceptual and procedural development to ensure that they reach higher mental functioning and operate at the abstract level of geometric reasoning (Ugorji & Alfred, 2017).

If students are taught by ineffective teachers, they have very limited chance of comprehending Euclidean geometry, as supported by Wei et al.'s (2017) argument that when teachers possess poor content and pedagogical knowledge of Euclidean geometry, learners are most likely to underperform in the topic. One way of understanding such expertise or lack thereof is through conducting research with teachers, as it is a central premise for the current study that teachers play a critical role in ensuring that learners learn and own knowledge and skills for Euclidean geometry. Researchers such as Ozkan et al. (2018) contend that learners at Grade 12 level experience challenges in Euclidean geometry as a sequel of limited basic skills in mathematics, but little has been done in mathematics education research within the South African context to explore how teachers make those basic skills available for learners during the lessons. The studies reviewed in this section demonstrate the existing challenges in the learning and teaching and teaching of geometry, but I am of the view that previous studies focused mainly on attributes to underperformance in geometry, overlooking researching with teachers as the key role players in ensuring that learners learn and own mathematical skills and knowledge of Euclidean geometry.

2.6. Chapter Summary

In this chapter I reviewed literature related to teachers' mathematical discourses during Euclidean geometry lessons. I started with the review of literature on the South African curriculum and Euclidean geometry. Then I discussed the conceptualisation of the concept of mathematical discourse. Then I focused on the research on the teaching approaches for Euclidean geometry. Lastly, I discussed the difficulties associated with the teaching and learning of Euclidean geometry.

CHAPTER 3

MATHEMATICS DISCOURSE IN INSTRUCTION: UNDERSTANDING THE TEACHING OF EUCLIDEAN GEOMETRY

3.1. Introduction

It is agreeable that for the findings of educational research to be more significant, reliable, and suited for generalizations and conclusions, the analytical processes should be firmly based on a well-defined theoretical framework (Kivunja, 2018; Varpio, Paradis, Uijtdehaage, & Young, 2020). According to Grant and Osanloo (2014), a theoretical framework is typically thought of as the blueprint or guide for research. In the current study, I adopted the Mathematical Discourse in Instruction (MDI) framework coined by Adler and Ronda (2015), because it enables a thorough investigation and understanding of mathematics teachers' classroom discourses, including teachers' explanatory talk and their efficacy or inefficacy during teaching and learning. This framework comprises of four interrelated components: Object of learning, exemplification, explanatory talk, and learner participation. In this chapter, I provide a comprehensive conceptualisation of MDI and demonstrate how the framework is operationalised within the context of the current study. MDI helped me with the language to describe what mathematically was made available or lack thereof during lessons on Euclidean geometry.

3.2. Mathematical Discourse in Instruction Framework

The MDI framework is deeply influenced by Vygotsky's (1978) sociocultural theory, which emphasizes the role of social interactions, cultural context, and historical factors in shaping cognitive development. This means that MDI draws on Vygotsky's ideas about how learning is mediated through social interactions and cultural tools, such as language, to foster the acquisition of scientific knowledge. As stated above, this framework comprises of four components: the first is the object of learning and focuses on the lesson goal and what the learners are expected to learn at the end of a specific lesson (Adler & Ronda, 2015). The authors state that the object of learning in a lesson could be a procedure, concept, or mathematical practice that the teacher intends for the learners to learn and own. The second component is exemplification, which refers to the selection and sequencing of examples together with accompanying task and representations teachers espouse during teaching to bring the object of learning into the fore with the learners (Adler & Ronda, 2015; Adler and Ronda, 2017). In the context of the current study, I pay attention to the way teachers used the variance and invariance in their example selections and sequencing to aid learners' generality and structure about Euclidean geometry concepts and skills. To operationalise the third component, explanatory talk, Adler and Ronda (2015) draw from Sfard's (2008) commognitive concept of word use to refer to teachers' naming of mathematical objects. This entails what teachers say and write during the lessons, as well as the substantiations of endorsed mathematical statements (what is considered mathematical knowledge). The last component is learner participation and focuses on the opportunities for learners to be co-constructors of mathematical knowledge. This component focuses on what learners say and do regarding the mathematics concepts and skills during the lessons. These four components and their relationship are depicted in Figure 3.

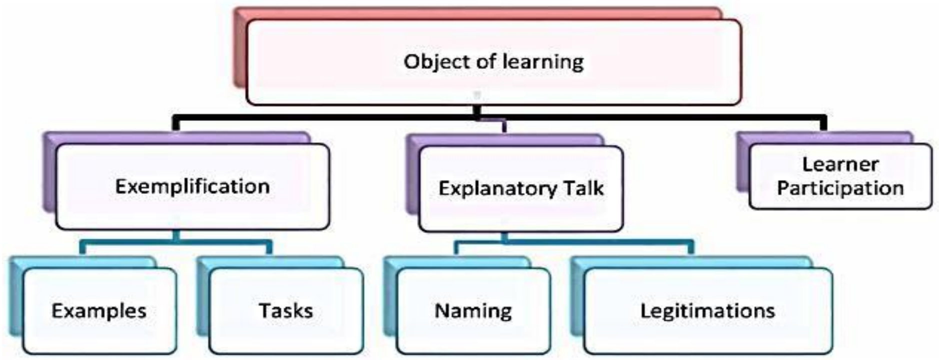


Figure 3. *MDI's components and how they interact (Adapted from Adler & Ronda, 2015, p.239).*

Due to the static nature of the graphic, it does not show how teaching and learning occur over time and sequentially. I therefore, pay attention to how each observed lesson developed over time and what can be spotlighted using the MDI framework. In the following sub-sections, I focus on the operationalisation of each component in exploring and understanding Grade 10 teachers' discourses during Euclidean geometry lessons in the current study.

3.2.1 Object of Learning

The core of teaching is defining what learners ought to learn and internalise at the end of the teaching and learning processes and Adler and Ronda (2015) refer to this as the object of learning. Rather than focusing only on the lesson's "objective," Adler and Ronda (2015; 2017) averse that it is fundamental to emphasize the relationship between the "object" and "learning," since this focuses attention on both the content and the abilities that learners are expected to possess in relation to that content at the end of the learning experiences. Making observations about whether the lesson's end goal has been attained or not requires an understanding of the teacher's intended learning objective(s). It is crucial to keep in mind that the teacher should be focused on the topic of learning and that the learners should be given clear instructions to ensure that they understand the lesson's purpose, which according to Vygotsky (1987) helps aid learners' internalisation of scientific concepts.

In addition, Alder and Ronda (2015) argue that an idea, method, algorithm, or meta-mathematical activity can all be learning objectives in a mathematics class. The term "object of learning" in the context of the current study refers to a particular Euclidean Geometry concept that the teacher gives special importance for the observed lesson. That is, in this study, I pay attention to what the teachers announced either verbally or as written texts on the board at the start of the lesson. For example, one of the participating teachers in the current study began the lesson by writing the following on the whiteboard: "Euclidean geometry – background", which signified that the teacher was going to focus on the introduction of Euclidean geometry concepts. With the complex nature of teaching and learning in mind, it is arguable that there can be situations where the object of learning is not clear to the researcher from teachers' verbal announcements or from what they write on the board. Thus, in this study, in cases where it was not clear what the object of learning for specific lessons were, I engaged in Video-Stimulated Recall Interviews (VSRI) post lesson observations with the teachers, to allow them to clarify what the intended object of learning was. The use of VSRI will be discussed in detail in Chapter 4. The following section focuses on the second component of MDI, exemplification.

3.2.2. Exemplification

The role of example in mathematics teaching across different levels of schooling has gained international attention in mathematics education research in the last two decades (Bills & Watson, 2008; Zaslavsky, 2017; Ball, 2017). According to Adler and Ronda (2015), exemplification can be defined as the teachers' presentation of examples and none-examples of the concept as they bring the object of learning into focus. The focus is on how teachers select and sequence examples during the lessons and how such selection and sequencing facilitates learners' generalisation regarding the behaviour of the mathematical object or concept, Euclidean geometry in the current study. Exemplification serves the role of demonstrating the irrelevant and relevant characteristics of a concept. The selection and sequencing of examples link closely with the object of learning because it is expected that a teacher should through lesson planning select examples they plan to use during teaching in advance, to ensure lesson coherence and limit learner confusion (Venkat & Adler, 2012). In this study, I focus on how teachers used examples and tasks to make Euclidean geometry concepts accessible for the learners during the lessons.

3.2.2.1 Examples

Alder and Ronda (2015, p. 4) contend that while research on examples sheds light on what teachers do and why, it does not give a comprehensive account of how examples build up to help learners focus on the object of learning or whether there is a shift towards generalisation in the set of examples selected and used during teaching. In this study, I am interested in the teachers' selection and sequencing of examples and how the examples accumulated within and across each teacher's episodes. I pay attention to what is variant and invariant in the teachers' selected examples and how their exemplification during Euclidean geometry lessons offered opportunities for learners to generalise the concepts. Similarity and contrast are two requirements for a series of examples to serve as a basis for generalization (Alder & Ronda, 2017).

According to Adler (2017), "if a set of examples brings attention contrast, and so what something is in relation to what it is not, or to a different class, opportunities are made available to recognise boundaries between classes of examples, and so further generalise" (p. 132). This suggests that contrast in exemplification comprises instances that focus on the differences or what remains variant in the used examples (Adler & Ronda, 2015). Thus, I consider how the examples teachers used during the lessons function both individually and jointly over the series of examples in a Euclidean geometry lesson to bring the objects of learning into focus in this study and how the teaching addresses curriculum standards as enshrined in CAPS (DBE, 2011).

In contrast, Adler (2017) draws from Marton and Tsui (2004) and states that "if a set of examples brings attention to similarity across examples, and so to that which is invariant, this offers opportunity to generalise" (p. 132). Concentrating on what something is by providing a range of related examples creates opportunities for generalizing the invariant, which Adler and Ronda refer to as similarity in exemplification. This tenet of the theory is used to understand instances where teachers use series of examples that have invariant features to draw learners' attention to the behaviour of Euclidean geometry concepts and create opportunities for learners to observe, make conjectures and prove them to reach generality (Adler & Ronda, 2015).

In addition to the above tenets, Adler and Ronda (2015) averse that opportunities for generalisation are enhanced when teachers use fusion in the examples that they use within and across different lessons. "When more than one aspect of an object of learning are fused, with simultaneous variance/invariance across an example set, generalisation is further enhanced" (Adler, 2017, p. 132). This means that, fusion occurs when multiple aspects of the object of learning are simultaneously changing or remaining constant across an example set and the authors believe that this, allows for even more opportunities for generality (Adler & Ronda, 2015). In the current study, this aspect of exemplification is useful, as it enables me to explore and understand how teachers use both invariance and variance within and across different examples to make the Euclidean geometry lessons available for the learners. Another important aspect of teachers' exemplification during teaching is the utilisation of learner tasks, considering that examples do not stand-alone in bringing the object of learning into focus (Adler & Ronda, 2015; Venkat & Adler, 2012).

3.2.2.2 Tasks

Explanations by examples are not sufficient. With every example, there is a task that guides what the students are expected to perform (Adler & Ronda, 2015). Clarke and

Roche (2018) contend that "teacher's choice and use of tasks are major determinants of the nature and quality of students' learning" (p. 95). Adler and Ronda (2015) define tasks as what learners are expected to complete drawing from the examples that teachers introduced during teaching and learning. Therefore, assignments are created to highlight specific capabilities, whilst examples are chosen as 'special cases' of the broad argument in consideration and for drawing attention to 'important qualities.' (Adler & Venkat, 2014). The aspect of tasks in this study enables the exploration and understanding of how teachers design and use tasks to bring skills for Euclidean Geometry to the fore, evaluate learners' internalisation of mathematical concepts or lack thereof. For example, one of the teachers in the current study marked the category of angles on the parallel lines and asked learners to find the relationship between them. Within the context of the current study, I focus on how teachers facilitated learners' engagement with the tasks and how they used the information gathered from learners' completed tasks for further learning. This discussion leads to the next tenet of MDI, explanatory talk, which focuses on teachers' explanations during teaching and learning, to enable learners' learning of the object of learning.

3.2.3 Explanatory Talk

This tenet of MDI focuses on the ways teachers make mathematical contents available to the learners through explanations they provide during the lessons (Adler & Ronda, 2015). Explanatory talk enables for the exploration and understanding of whether a teacher uses appropriate mathematical words and statements in explaining mathematical concepts and/or procedures during teaching. To operationalise the concept of explanatory talk, Adler and Ronda (2015) draw from Bernstein's concept of pedagogic discourse to account for what counts as mathematical in mathematics teaching and learning. That is, the role of explanatory talk during teaching is to name and legitimate the object of learning, the examples teachers select and use as well as how tasks are facilitated during classroom instruction. This tenet focuses on both what teachers write and say during teaching, to provide descriptions of the mathematics teachers make available for learners to learn and own through explanatory talk, as well as making summative judgments on naming and legitimating mathematics concepts and explanations as they accumulate within a lesson and across lessons (Adler and Ronda, 2015, p. 5). In this study, this tenet of MDI enables me to evaluate teachers' substantiations of mathematics related to the concept of Euclidean geometry as specialised knowledge, to understand what counted as mathematical knowledge in teachers' elucidations during teaching.

In relation to what counts as mathematically endorsed explanations during teaching, Venkat and Adler (2014) foregrounds the tension in how teachers navigate the complexity of managing the formal and informal ways of describing mathematical concepts. In this study, this tension was observable as teachers were tempted to simplify concepts and words during explanations, considering that the concept of Euclidean geometry is formally introduced for the first time in Grade 10, as a way of giving learners access to mathematical meanings, resulting in the usage of informal ways of describing geometry concepts. Venkat and Adler (2014) contend that often teachers become reluctant "to use formal mathematical language" during teaching as they consider the usage of formal mathematical words and statements as "abstract and the learners are put off by over reliance on formal talk with neglect of connecting mathematical ideas to colloquial meanings" (p. 132). Accordingly, the notion of explanatory talk enables me to unpack the mathematical discourses teachers inhabited during explanations, to understand how these facilitated learners' learning of Euclidean geometry lessons as they named and legitimated different concepts during the lessons.

According to Alder and Ronda (2015, p. 8), how the naming of the mathematical objects in focus in a particular lesson forms a central part of understanding what mathematics knowledge is made available for learners to learn within a lesson as well as across different lessons. Adler and Ronda (2015), drawing on Sfard's concept of naming of mathematical object as the utilisation of words that give meanings to other words, symbols in use in mathematical representations and ideas, or connections that are made during mathematical explanations. Within the context of this study, this concept allows me to classify naming within and across selected episodes as either non-mathematical or mathematical.

3.2.3.1 Legitimating criteria

Legitimation focuses on the "criteria for what counts as mathematics that emerge over time in a lesson and provide opportunity for learning geared towards scientific concepts" (Adler & Ronda, 2015, p. 243). This means legitimations refer to the substantiations that teachers make during teaching and learning, to describe the mathematical concepts and procedures that learners are expected to learn by the end of the lesson. In a classroom situation, a teacher is the one who make substantiations relating to the object of learning, hence making learners aware about why specific procedures are performed as well as what counts as important in a specificmathematical object (Adler & Ronda, 2015). In this study, I pay attention to the methods teachers use, the hints they provide about the different Euclidean geometry objects, which foregrounds the significance of the concept.

3.2.4. Learner Participation

Learner participation focuses on what learners are invited to say and do during the lessons, in relation to the mathematics they are learning (Adler & Ronda, 2015). According to Adler and Ronda (2015), "the main focus is to identify what learners are invited to say, whether and how learners have the opportunity to use mathematical language and engage in mathematical reasoning, as well as the teachers". During the teaching of the Euclidean geometry concepts, I focus on what teachers invited learners to say and how they gave answers to questions posed by the teacher during teaching, as well as what learners were expected to do with the mathematics that teachers present. In the analysis of the teachers' classroom discourses of Euclidean geometry, I pay attention to whether the teacher employed teacher dominant discourses in terms of the explanatory talk and classroom actions or learners were positioned and encouraged to be co-constructors of mathematics knowledge and participated actively in the process of meaning-making. The interactions I pay attention to in the current study are delimited to the interactions between teachers and learners and the way learners interacted with the examples the teacher presented during the lessons. That is, due to the focus of this study, I do not focus on the interactions between the learners during the analysis of the observed lessons.

3.3 Chapter Summary

This chapter provided a detailed conceptualisation and operationalisation of MDI in the current study, to explore and understand Grade 10 teachers' discourses of teaching Euclidean geometry. The chapter presented an account of what the object of learning is about, the process of exemplification, explanatory talk and learner participation and the relationship between these components of MDI. The next chapter provide a detailed account of the research processes that I engaged in to realise the study objectives and answer the predetermined research questions.

CHAPTER 4

Methodology: Studying Discourses of Euclidean Geometry

4.1 Introduction

According to McMillan and Schumacher (2010), a research methodology is a systematic and purposeful adventure organized to generate data on a particular research problem, with research methods used by the researcher to collect and analyse data. This chapter discusses the research paradigm, research approach, and research design that I employed to address the objectives and answer the research questions. The sampling strategy and study sample, research methods, and data analysis processes that I employed in this study are also discussed. This chapter also discusses how issues regarding trustworthiness through ensuring credibility, transferability, confirmability, and dependability were observed throughout the study. The last section of this chapter discusses ethical considerations that I made during the course of the study.

4.2. Research Paradigm

According to Rehman and Alharthi (2016, p. 51), a research paradigm refers to "a basic belief system and theoretical framework with assumptions about 1) ontology, 2) epistemology, 3) methodology, 4) methods … it is our way of understanding the reality of the world and studying it". My view of knowledge and reality is that they are subjective entities, and individuals within specific contexts interpret things differently based on their lived experiences, as well as their ways of thinking and knowing about the world. This study is located within an interpretive research paradigm, which focuses on understanding and interpreting subjective meanings that different people attach to their observations and experiences. The reason I located this study within the interpretive paradigm is that I focus on the subjective teachers' mathematical discourses and perspectives of what it means to make Euclidean geometry knowledge available for Grade 10 learners, which is located within the complexity of social constructions of what it means to teach mathematics in general.

In addition, in this study, I am concerned with exploring and understanding the Grade 10 teachers' discourses of teaching Euclidean geometry, which also involves exploring and analysing teachers' perspectives, beliefs, and understandings of teaching this topic. In the analysis of teachers’ discourses, I pay attention to underlying meanings that teachers attach to their teaching of the topic as well as providing interpretations on why they acted in particular ways during the lessons. The following section focuses on the research approach that I used in this study.

4.3. Research approach

In research, a particular research approach is adopted to serve as a guide on how data was collected, analysed, and reported (Bell, 2005). In this study, I espoused a qualitative research approach to explore and understand Grade 10 teachers’ mathematical discourses during Euclidean geometry lessons. Turner et al. (2021, p. 4) state that “qualitative research works under the universe of meaning, attitudes, aspirations, and values, which reacts to more in-depth expressions of relationships, interactions, and experience that cannot be haphazardly quantified and reduced to operationalized variables.” This means that, in qualitative inquiry, the researcher focuses on the in-depth exploration and understanding of the phenomena under study, which is located within the complex social relations that are located within specific contexts. This resonates with Babbie and Mouton’s (2001, p. 184) assertion that qualitative research is “an inquiry process of understanding a social or human problem based on building a complex holistic picture formed with words, reporting detailed views of information, rather than the explanation and prediction of human behaviour”.

Using the qualitative approach in the current study enabled me “to understand the situation under investigation primarily from the participants’ not the researcher’s perspective” (Hancock & Algozzine, 2016, p. 8). That is, to explore and understand Grade 10 mathematics teachers’ mathematical discourses during Euclidean geometry lessons, I immersed myself in their naturalistic teaching lives and understanding how they made geometry concepts available for the learners.

This approach enabled me to gain in-depth insights into the teaching of Euclidean geometry from the teachers’ perspectives instead of relying on my own assumptions as the researcher. I observed teachers’ lessons on Euclidean geometry and conducted

Video-Stimulated Recall Interviews (VSRI) to understand their pedagogical reasoning during the lessons that I observed. I also employed semi-structured interviews with teachers to understand their personal experiences of teaching Euclidean geometry. The use of the qualitative approach was helpful in this study due to its flexibility, which allowed me to engage in open-ended conversations with the teachers during both semi-structured interviews and VSRIs about their discourses of the topic (Creswell, 2012; Mbhiza, 2021). Denzin and Lincoln (2005) highlight the fact that qualitative researchers deploy a wide range of interconnected methods, always hoping to get rich and sufficient data on the subject under study. In this study, I researched with Grade 10 mathematics to understand their discourses of Euclidean geometry. To shed some light on teachers' discourses, it is necessary to obtain their personal views and beliefs regarding their teaching and factors that influence the discourses they inhibit during Euclidean geometry lessons.

4.4. Research design

McMillan and Schumacher (2014, p. 28) describe a research design as "procedures for conducting the study, including when, from whom, and under what conditions the data will be collected." Dhlamini (2012) emphasizes that research design is contingent upon the purpose of a research study, with most researchers regarding the research design as an architectural plan or a conceptual structure to map out and guide the research process. Within the qualitative research approach, there are various research designs that place emphasis on gathering data on naturally occurring phenomena, whereby most data gathered occurs in the form of words instead of quantitative measures (Creswell, 2013). In this study, I used a multiple case study research design to understand teachers' discourses of Euclidean geometry. According to Simons (2009, p. 21), a multiple case study design entails "an in-depth exploration from multiple perspectives of the complexity and uniqueness of a particular institution or system in a 'real-life' context". Similarly, according to Merriam (1998), "reality is not an objective entity; rather, there are multiple interpretations of reality", addressing the "embeddedness of social truths" in case study research design, which resonates with the interpretivist paradigm's position (p. 22). In this study, I researched with four different teachers from four different school sites, representing multiple cases. In analysing teachers' discourses during the lessons, I do not view the teaching of

Euclidean geometry as an objective process, rather I consider the multiplicity of interpretations of their teaching and consider that the teachers' pedagogical reasoning and actions during teaching are embedded within specific social truths. Thus, in this study, I view teachers' discourses of Euclidean geometry "as subjective, multiple, dynamic and complex" (Mbhiza, 2021, p. 75). Using multiple cases enables me to verify that teachers' discourses of Euclidean geometry are not merely the result of eccentricities of the context in which the study is conducted. The following section details the research sampling strategy and the resulting sample for this study.

4.5 Research sampling

Research sampling can be defined as being a group of subjects that is used to collect data. According to Cohen et al. (2013), there are two types of sampling strategies namely probability and non-probability sampling. In the former the process of selecting the participants is systematic and random while the latter is purposive (Cohen et al., 2013). There are different types of non-probability sampling namely, convenient sampling, quota sampling, dimensional sampling, purposive sampling and snowball sampling (Cohen et al., 2013). Within non-probability sampling, I used purposive sampling to select the schools and teachers for this study. According to Cohen et al. (2013), purposive sampling consists of handpicking of participants according to characteristics they have and required by the study, which includes their envisaged knowledge and abilities to answer the research questions. Thus, purposive sampling was employed in this study because participants were not only expected to possess the characteristics of a mathematics teacher but characteristics of a Grade 10 mathematics teacher who has experienced of teaching Euclidean Geometry.

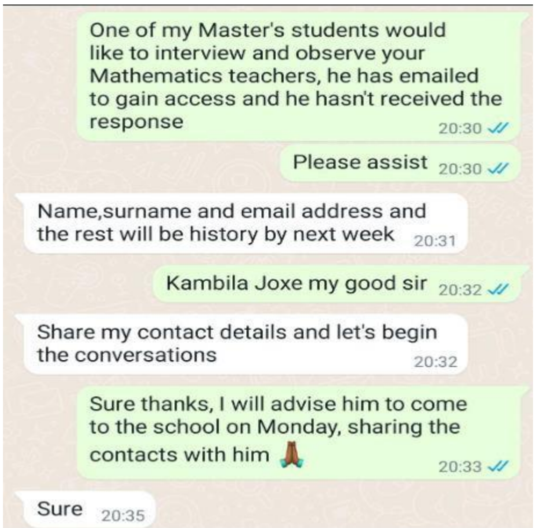
When it comes to selecting the region within which this study is located, a convenience sampling strategy was used. The schools that I worked with were selected based on their proximity to the region where I work and reside, as well as their proximity to each other. All the four participating schools are in proximity, about 500m apart, and this brought about convenience in terms of travelling to the schools. I could travel between the schools for any scheduled interview(s) and/or observations.

4.5.1. Challenges encountered during data collection.

I ran into a few obstacles when gathering my data, which could have affected how well it turned out. Convincing Grade 10 mathematics teachers to participate in the current study was difficult since some of them were averse to being investigated, and others appeared to have doubts about their knowledge of teaching Euclidean geometry. Additionally, even when I gave them the details of my research, several teachers still believed that it would have jeopardized their jobs, thus rejecting to become involved in the study. I went to a different public school within the region where the study is located to present the principals with my request to undertake research. Regretfully, some principals refused for their schools and teachers to participate in the study, even when I presented the approval from the Gauteng Department of Education research ethics office (Appendix B). After facing similar rejection in multiple schools, I decided to inform my supervisor about the situation. He assisted me in contacting a few institutions, and my requests were approved instantly (see Figure 4). This demonstrates the value of having a supervisor who has established relationships and rapport with different stakeholders in research. I lost a great deal of time before communicating the challenges with the supervisor and I realised that if I had communicated the challenges earlier, he could have helped me sooner.

Figure 4.

WhatsApp conversation between my supervisor and the head of one school



The following section discusses the research sample for this study.

4.5.2. Study Sample

As discussed earlier, I worked with four teachers from four different school sites. This was in line with Eisenhardt's (1989) suggestion that multiple case studies require that the researcher works with at least four cases but should not be more than 10 cases to enable in-depth exploration of the subject under scrutiny. In view of the challenges, I experienced detailed above, Figure 5 depicts the final sample for the current study.

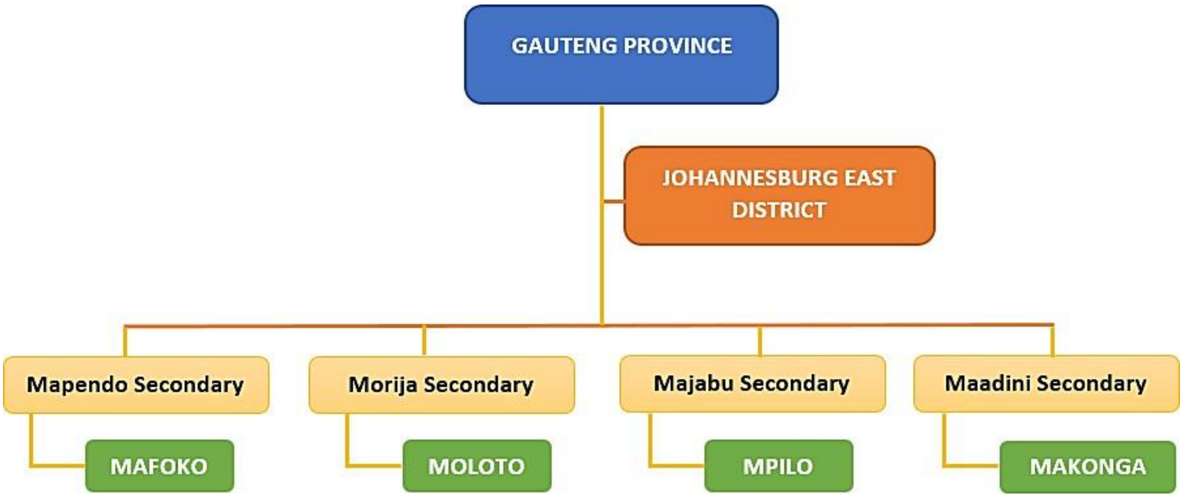


Figure 5. *Outline of the final sample*

As previously mentioned, case study research requires a bounded research context. For this reason, participating teachers and schools in this study were selected from various parts of the same Gauteng province district, as seen in the above Figure. Table 3 displays the participating teachers' biographical information.

Table 3.

Participants' biographical information

Pseudonym	Gender	Mathematics Education qualifications	Number of years teaching	Institution trained at to become a teacher
Mafoko	Male	Bachelor of Education	13 years	University of Zimbabwe
Moloto	Male	Honours Degree in Education	15 years	University of KwaZulu-Natal
Mpilo	Male	Diploma in Education	8 years	University of Zimbabwe
Makonga	Male	Bachelor of Education	2 years	University of Johannesburg

The present study included four males[1], as shown in the table above. The teachers are all qualified mathematics teachers with a combined teaching experience of two to fifteen years. While Moloto and Makonga received their teaching certifications in South Africa, as shown in the above table, Mafoko and Mpilo completed their teacher training in Zimbabwe. It was enthralling to research with this diverse group since their varying experiences and positions regarding general education and mathematics education influenced their discourses and methods of approaching Euclidean geometry in one way or another.

4.6. Research methods

Research methods refer to the different ways in which researchers generate data, including questionnaires, surveys, observations, document analysis, interviews to name but a few (Creswell, 2007; Sargeant, 2012). The selection of specific methods is based on the suitability to answer the research questions and address the research objectives (Creswell, 2014). In the current study, I selected and employed three research methods, to explore and gain insight into Grade 10 teachers' mathematical discourses during Euclidean geometry lessons. These methods were semi-structured individual interviews, non-structured and non-participatory classroom observations, and Video-Stimulated Recall Interviews (VSRIs). Using these methods maximised the credibility of the data I generated and ensured triangulation of the teachers' mathematical discourses of Euclidean geometry. Observations are the primary sources of teachers' discourses during Euclidean geometry lessons, with the other two methods acting as supplementary and complementary information in offering interpretive elaborations on teachers' observed discourses.

4.6.1. Unstructured classroom observations

Considering that qualitative research is aimed at providing an explicit interpretation of the broad patterns, order and the structure found among the study participants, observations are the best way to generate in-depth understanding of the nature of

[1] While this could be interpreted to be gender biasness, it should be noted that I could not get female mathematics teachers teaching Grade 10 within the bounded context to participate in the study.

events which participants engage in (Guthrie, 2011). To gain insight into teachers' mathematical discourses during Euclidean geometry lessons, observations "reveal classroom norms about teachers' authority, implicit rules about pupil participation, and the structure of classroom work and tasks" (Guthrie, 2011, p. 87). In this study, I used unstructured classroom observations that were non-participatory. This allowed me to understand teachers' discourses from their pedagogical patterns during the lessons. Instead of imposing my predetermined assumptions about teachers' discourses of Euclidean geometry, the unstructured nature of the observations allowed me to observe teachers' teaching naturalistically (Cohen et al., 2013). In other words, using the unstructured classroom observation technique enabled me to defer definitions and structures until a pattern emerged out of the discourses that the four teachers inhibited during teaching and learning (Bell, 2005). I adopted Mbhiza's (2021, p. 82) approach, which allowed for the reliable emergence of trends and patterns from observing how teachers acted, what they said during teaching, and how they interacted with learners, mathematical content and other physical artefacts in the classrooms while observing lessons on Euclidean geometry. As Mbhiza (2021) suggests, I did not engage in any active participation during teaching and learning such as interjecting. I focused on what I observed during the lessons and in turn made interpretations and summative judgements about teachers' MDI during Euclidean geometry lessons. All classrooms' observations were video recorded, to ensure that I captured sufficient information to understand Grade 10 teachers' discourses of Euclidean geometry.

My non-participatory presence during the lessons minimised disruptions during the teaching and learning of Euclidean geometry. This being the case, it should be noted that my presence in the classrooms during the lessons has somewhat impacted the naturalistic way teachers and their learners interact when there is no 'stranger' in the classroom space, especially the influence of having a video camera during the observations. Cohen et al. (2011) referred to this as particpants' reactivity, in which research participants alter their actions based on the presence of a 'stranger' in their setting, sometimes to impress the observer. Of importance to note is that, with time, the level of what I considered reactivity lessened, which was evidenced by the teachers and learners no longer focusing or giving gaze to the camera during classroom interactions. Table 4 depicts the number of lessons that were observed for each teacher, the time each lesson took as well as what the object of learning were in different lessons.

Table 4.

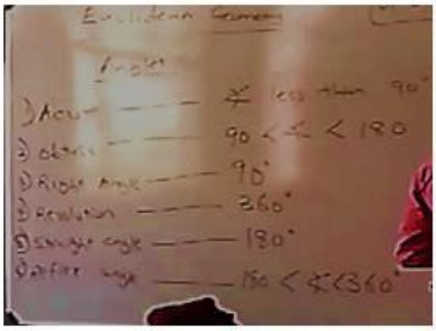
Information regarding the observed lessons for each teacher

Teacher's Name	Number of lessons observed	Duration	Object of learning
Mafoko	2	45 minutes	Properties of angles on parallel lines.
		45 minutes	Proof of the theorem that states that "the line segment joining the midpoint of two sides of triangle is parallel to the third side of a triangle and equal to twice the third side."
Moloto	2	45minutes	Background of Euclidean geometry (Geometry of straight lines and geometry of parallel lines)
		45 minutes	Proof of Theorem 1 that states that "the opposite sides and opposite angles of a parallelogram are equal."
Mpilo	2	45 minutes	Euclidean geometry background (Geometry of straight lines, triangles and angles on parallel lines)
		45 minutes	Solving problems that involve geometry of straight lines, triangles and angles on parallel lines.
Makonga	2	35 minutes	Conditions of congruency of triangles (SSS and SAS)
		35 minutes	Conditions of congruency of triangles (AAS and RHS)

The objects of learning were identified either from what teachers said and/or wrote on the board, mainly at the beginning of each lesson that I observed. Some of the challenges I encountered during observations included overcrowded classrooms, where I could barely find a space to position the camera and even to sit (see Makonga's case in section 5.4). Other classrooms were darker, making it difficult for the camera to clearly capture what the teacher wrote on the board, especially in cases where the board markers teachers used were faint (see Mpilo's and Mafoko's cases in sections 5.2. and 5.3. respectively). To ensure that all the information was captured, I used journal entries for each lesson to capture what teachers said and did during teaching. I also provide reconstructions of the images that I selected for specific information but are not visible as depicted in Figure 6.

Figure 6.

Example of reconstruction of invisible images



What is written in this blurred image:

Euclidean Geometry

1) Acute angle: $angle\ less\ than$ 90°
2) Obtuse angle: 90° $< angle <$ 180°
3) Right angle: $angle =$ 90°
4) Revolution: $angle =$ 360°
5) Straight-line angle: $angle =$ 180°
6) Reflex angle: 180° $< angle <$ 360°

This allows the reader to see the information that otherwise would be difficult to see. The following section discusses how semi-structured interviews were used in this study.

4.6.2. Semi-structured interviews

Alshenqeeti (2014, p. 40) views a qualitative interview as “an extendable conversation between partners that aims at having an ‘in-depth information’ about a certain topic or subject, and through which a phenomenon could be interpreted in terms of the meanings interviewees bring to it”. In this study, I used semi-structured face-to-face individual interviews to understand Grade 10 teachers’ positionality on what it means to teach Euclidean geometry, as well as the associated challenges of teaching the topic. The rationale for espousing semi-structured interviews is that they are flexible and allow researchers to tailor subsequent questions in cases where clarity is required based on the information provided by the participants. Using this method enabled me to gain insight into teachers’ biographical information, their views of what it means to teach mathematics, which in turn helped me understand some of the reasons why they taught the topic in particular ways. Through asking open-ended questions as well as allowing the participants to ask for clarity in the questions allowed me to gain in-depth understanding of the teachers’ experiences of teaching Euclidean Geometry in Grade 10, especially in cases where further elaboration was required (Creswell, 2013). As depicted in Table 5, the time it took to complete each interview differed from each teacher, which alludes to the nature of semi-structured interviews.

Table 5.

Time it took to complete each semi-structured interview.

Teacher's Name	Duration of the interview
Mafoko	47 minutes
Moloto	35 minutes 34 seconds
Mpilo	51 minutes 53 seconds
Makonga	48 minutes 19 seconds

To ensure that the data collected through the semi-structured interviews is credible, all interviews were audiotaped, as this provided me opportunities to listen and relisten attentively to what the teachers said, and not miss any information provided during the interviews. Using audio recordings during interviews enabled me to present accurate and credible information as well as in the data transcription and in the coding of the information given by the teachers (McMillan & Schumacher, 2014).

4.6.3. Video-Stimulated Recall Interviews

VSRI is a research method "whereby researchers show research participants a video of their own behavior to prompt and enhance their recall and interpretation after the event" (Paskins et al., 2017, p. 1) This method enabled me to gain insight into teachers' reasons for their observed classroom actions, to understand how and why they said certain statements and acted in particular ways during teaching. I identified and selected critical incidents in different lessons that represented some pedagogical and content knowledge issues that I needed teachers' reflections on. For example, the focus of some reflective conversations was on how teachers introduced the object of learning, while others focused on teachers' explanatory talk and the effectiveness of their exemplification on teaching Euclidean geometry. Thus, the MDI framework was helpful in framing the focus of the conversations that I had with the teachers. VSRIs

helped me to crosscheck what mathematics teachers perceived about mathematics teaching (from semi-structured individual interviews), against what they did during teaching in the classrooms, and this subsequently assisted in minimising teachers' superficial self-representations during interviews by directly confronting them with their actual actions of classroom practice (Mbhiza, 2021, p. 86).

The video recorded lessons were a "luxury of meta-analysis and reflection that was most likely to be absent in the original event" (Yinger, 1986, p. 271). Watching the recorded lessons gave teachers an opportunity to reflect on their own teaching, using the visual cues to offer explanations for what they said and did during teaching. The extract below is an example of the VSRI conversation that I had with Mpilo.

Researcher: *in this lesson, you started off by recapping the concept of angles on straight lines. This was good to see …*

Mpilo: *yes, I wanted learners to keep that understanding in mind, which would help them to work with angles when I introduced parallel lines. Our learners need you to do that, if you don't, they see a concept as a stand-alone.*

Researcher: *But I noticed that you did not make the links clear for the learners as to what the relevance of recapping that knowledge was.*

Mpilo: *I agree, and I could have told the learners what the links are. You see this thing of being able to watch your own lessons is helpful. I will sure tell the learners why I do certain things in class next time.*

The statement "I agree, and I could have …" demonstrate that using VSRI did not only supplement the information teachers provided during semi-structured interviews about their teaching of Euclidean geometry and their observed actions during teaching, it also presented educational opportunities for teachers to identify areas of their teaching of the topic that requires improvement. In chapter 5, these conversations are introduced in cases where interpretive elaboration is required, to elucidate on why teachers acted in certain ways, why they said what they said in the manner they uttered certain statements.

4.7. Data analysis

Maree (2007, p. 99) states that "qualitative data analysis is usually based on an interpretive philosophy that is aimed at examining meaningful and symbolic content of qualitative data.” He indicates that, to understand how participants make meaning of a phenomenon, it is important to analyse their perceptions, attitudes, knowledge, feelings, and experience.

According to Cohen et al. (2013, p. 462), qualitative data analysis is a process of “making sense of the data in terms of the participants definition of a situation, noting patterns, themes, categories and regularities.” This suggests that qualitative data analysis is concerned with reliable interpretation of the information provided by the participants. Within qualitative data analysis, this study used content analysis for all the data sources both individually and relationally. The rationale for choosing content analysis enabled me to interpret data through coding and categorising it to discern similarities and differences, thereby facilitating the interrogation of teachers’ discourses to formulate summative judgements about their teaching of Euclidean geometry (McMillan & Schumacher, 2014; Adler & Ronda, 2015. All the interviews were transcribed and recorded lessons were summarised using the components of MDI discussed in chapter 3. Photograph 1 are poster chats showing the process of generating the summative judgements for each teacher.

Makonga’s MDI poster chart

Mpilo’s MDI poster chart

Moloto’s MDI poster chart

Mafoko MDI poster chart

Photograph 1. *Teachers’ MDI poster charts*

As I watched different teachers' lessons, I chunked each lesson into episodes that are identified by different objects of learning. Chunking produced numerous episodes and I selected those episodes that allow for an in-depth analysis of teachers' MDI as presented in Table 6. I then examined each episode based on the MDI summative judgements tool, focusing on the ways teachers announced the object of learning, their explanatory talk, exemplification as well as how they encouraged or enabled learner participation (see Figure 7).

Figure 7.

Tool for summative judgements for teachers' MDI (Adapted from Adler & Ronda, 2016, p. 8).

Examples	Naming	Legitimating criteria
The set of examples provide opportunities in the lesson for learners to experience: **Level 1**: one form of variation i.e. similarity or contrast **Level 2**: at least two forms of variation: **S and** S OR **S and C** **Level 3**: simultaneous variation (fusion) of more than one aspect of the object of learning and connected with similarity and contrast within the example set. (**S, C, F**) **Level 0**: simultaneous variation with no attention to similarity and/or contrast	Use of colloquial and mathematical words within and across episodes is: **Level 1:** talk is *colloquial or non-mathematical* **(NM)** *e.g.* everyday language and/or ambiguous pronouns such as this, that, thing, to refer to what is being written or pointed at; *where Mathematical words are used, these are as names labels or to read a string of symbols* **(Ms)** **Level 2:** movement between **NM** and **(Ms)** and some *mathematical language used appropriately (Ma)* to refer to other words, symbols, images, procedures **Level 3:** movement between colloquial **NM** and formal math talk **Ma**	Criteria for what counts as mathematics that emerge over time in a lesson and provide opportunity for learning geared towards scientific concepts **Level 0:** all criteria are *non mathematical (NM)* and so either *Visual* **(V)**—e.g. cues are iconic or mnemonic; or *Positional (P)*—e.g. *a* statement or assertion, typically by the teacher, as if 'fact' or *Everyday (E)* **Level 1:** criteria include *Local* **(L)** e.g. a specific or single case (real-life or math), established shortcut, or convention **Level 2:** criteria extend beyond non mathematical and L to include Generality, but this is partial **GP** **Level 3: GF** math legitimation of a concept or procedure is principled and/or derived/proved

In analysing the teaching shifts that occurred in each selected episode, I draw from the information teachers provided during semi-structured interviews as well as during VSRI to make sense of teachers' MDI and reasons for particular discourses they inhibited during teaching. The following episodes were selected for each of the four participating teachers.

Table 6.

Selected episodes for each teacher

Teacher's Name	*Episodes*
Mafoko	**EPISODE 1** Learners' background knowledge about different angles
	EPISODE 2 Under-teaching of properties of angles on parallel lines that are equal.
	EPISODE 3 Under-teaching of properties of angles on parallel lines that are not equal
Moloto	**EPISODE 1** The lesson's introduction
	EPISODE 2 Moloto's assessment during a Euclidean geometry lesson
	EPISODE 3 Under-proving theorem 1
Mpilo	**EPISODE 1** Euclidean geometry background
	EPISODE 2 Solving of Euclidean geometry problems
Makonga	**EPISODE 1** "You need to be patient with them and try to create a positive learning environment".
	EPISODE 2 Conditions of congruency of triangles

4.8. Addressing issues of trustworthiness.

Lincoln and Guba (1985) suggest that the trustworthiness of a research study is vital to evaluate its worth or soundness. Trustworthiness involves establishing credibility, transferability, dependability, and confirmability, with each of these detailed below to demonstrate their operationalisation in this study.

4.8.1. Dependability

The notion of dependability is concerned with whether the researcher would obtain the same results if the researcher could use the same phenomenon and research methods again (Kumar, 2011, p. 172). Shenton (2004) argued that dependability in qualitative studies could be addressed when the research process is "reported in detail, thereby enabling a future researcher to repeat the work" (p. 71). Thus, in this study, I presented thick descriptions about the research processes that I followed to realise the study's objectives. In chapter 5, I present the teacher's observed lessons as series of episodes, providing thick descriptions and interpretations of their discourses during teaching of Euclidean geometry. I also present images as well as extracts of conversations between teachers and learners during the lessons, as evidence of the information I use to make summative judgements about participating teachers' discourses.

4.8.2. Credibility

In any empirical research, it becomes important for the researcher to ensure that the study findings are a true reflection of the information provided by the participants. Regarding this, Anney (2014) defines credibility of the study as "the confidence that can be placed in the truth of the research findings" (p. 276). Anney (2014) further suggests strategies to establish credibility of the study: "prolonged and varied field experience, time sampling, reflexivity (field journal), triangulation, member checking, peer examination, interview technique, establishing authority of researcher and structural coherence" (p. 276). In this study, I used member checking and triangulation to ensure the credibility of the data and findings from the study. In terms of member checking, I conducted VSRIs with the teachers, to allow them opportunities to reflect of their pedagogic reason and actions during the lessons, instead of relying on my own positionality about what they ought to have done during teaching. The use of classroom observations, semi-structured interviews and VSRIs did not only bring about the versatility of the data, but also ensure triangulation of the information of the teachers' discourses. For example, in cases where disjuncture was noted in what teachers said during the interviews compared to their classroom observed actions, I used VSRIs as opportunities to seek clarity from teachers regarding such disjuncture.

4.8.3. Transferability

Like the notion of dependability, transferability refers to "the degree to which the results of qualitative research can be generalized or transferred to other contexts or settings" (Kumar, (2011, p. 172). This effect is achievable in qualitative research studies when sufficient contextual information about the research site and its research activities are comprehensively publicized. Accordingly, to ensure transferability in the current study, I presented thick descriptions of all the research processes, including some of the challenges I encountered during data collection. In addition, I employed purposive sampling technique to select the participants, because it allows for the variety of information from participants to be gathered from different participants (Mbhiza, 2021).

4.8.4 Confirmability

Confirmability refers to the extent to which the research results can be "confirmed or corroborated by others" (Kumar, 2011, p. 172). Thus, to ensure confirmability in this study, my supervisor acted as an inquiry auditor to check whether the raw data, the analysed data presented, interpretations and findings represent truly the information provided by the participants. In cases where I was biased in analysing teachers' lessons, my supervisor redirected me to consider other possible interpretations of the information.

4.9. Ethical considerations

McMillan and Schumacher (2010) state considering that educational research focusses primarily on researching with humans, researchers are ethically responsible for protecting the rights and overall welfare of their study's participants throughout the study processes. Before data collection could commence in this study, I applied for ethical approval from the UNISA college of Education Research Ethics Committee as well as access to the schools from the Gauteng Department of Education and clearance was granted (see appendices I and J respectively).

In the current study, participants were given information sheets that informed them about what the study is about, methods of data collection and their right to withdraw from the study at any point in the study for whatever reason and that they would be no consequences for withdrawing (Babbie & Mouton, 2007). The information sheets included consent forms for agreeing to be audiotaped and video recorded (Appendix D). In addition, to ensure the concealing of the teachers' and their schools' true identities, I use pseudonyms for both teachers and schools in all writings of the study to ensure anonymity and confidentiality by not sharing what has been discussed in the interviews with other people.

4.10. Chapter Summary

In this chapter, I described the processes that I followed to generate data for the current study. The operationalisation of classroom observations, semi-structured interviews and VSRIs have been discussed, as well as the data analysis procedure. I have provided discussions on my methodological choices in terms of the research paradigm, research approach, research design, sampling technique and analytical approach. This chapter also discussed how I ensured the trustworthiness of the study and the ethical considerations that I made throughout the course of the study.

Chapter 5

Data Presentation and Interpretation: Looking into Teachers' MDI of Euclidean Geometry

5.1 Introduction

This chapter presents, analyses, and interprets selected classroom episodes for each participant, to understand their mathematical discourses of teaching Euclidean geometry. To ensure that I present and interpret each teacher's classroom discourses in detail, the cases are presented individually initially and then relationally. Sections 5.2 to 5.5 present analysis of Mafoko, Mpilo, Makonga, and Moloto respectively. For all the 4 teachers, I selected specific episodes from different lessons, which enabled me to determine what Euclidean geometry knowledge each of the 4 teachers made available for learners to learn based on the elements of Mathematics Discourse in Instruction discussed in the previous chapter as well as related literature on Euclidean geometry teaching. To present interpretive elaborations and clarity for some teacher's classroom discourses, I draw from the information that the participants provided during semi-structured interviews and VSRIs.

5.2. Data presentation and analysis – The case of Mafoko

The three episodes were selected from one of Mafoko's lessons. The episodes were selected based on the relevance of information that address the research objectives and the predetermined research questions. For this lesson, the object of learning was "angles on parallel lines". The identification of this object of learning is based on what the teacher announced explicitly at the beginning of the lesson: "*Good morning to you all, today we are going to learn about angles on parallel lines*", thereby drawing learners' attention to the mathematics they are expected to have learned at the end of the lesson. This object of learning relates to the mathematical knowledge that Mafoko expected the learners to learn and own in the selected lesson (Adler & Ronda, 2015).

Figure 8 represents the selected episodes from Mafoko's lesson 1 and are interpreted and discussed.

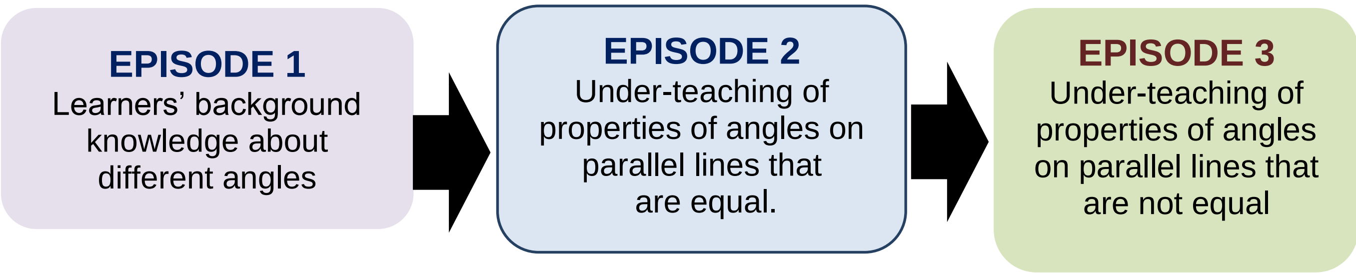


Figure 8. *Mafoko's selected episodes from one selected lesson.*

5.1.1. Episode: Learners' background knowledge and visualising parallel lines, transversal line, and different angles

Mafoko started the lesson by introducing the general types of angles which are acute angle, obtuse angle, right angle, revolution, straight-line angle, and reflex angle. To do this, he invited the learners to verbalise the names of any angles that they know, he said: "*what are the types of angles that you know*?" and different learners started verbalising some angles, and Mafoko wrote the names as he heard the learners calling them out. Consider the following extract:

1. Mafoko: *what are the types of angles that you know?*
2. Learner 1: *Acute angle*
3. Learner 2: *Obtuse angle*
4. Learners 3: *Right angle*
5. Learner 4: *Reflexive angle.*
6. Mafoko: *Not quite. Can someone help learner 4?*
7. Learner 2: *Reflex angle*
8. Mafoko: *Good! You have not listed all of the angles.*
9. Learner 5: *Straight angle*
10. Learner 6: *Revolution*

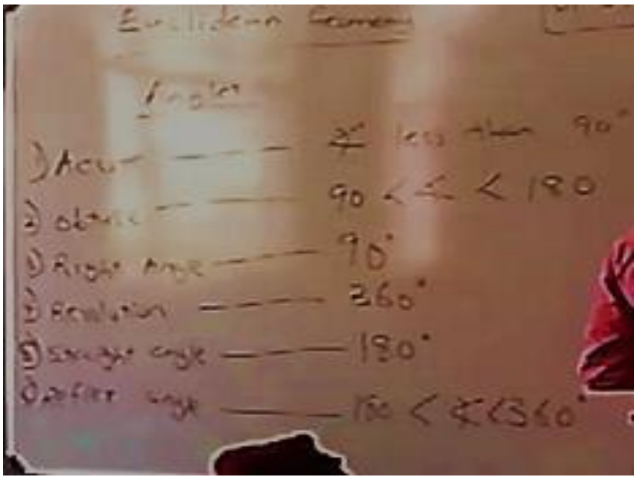
Accompanying actions: *As learners verbalised the names of the angles, Mafoko wrote the names on the board* (see image 1).

After the naming of the angles, Mafoko further asked the learners to verbalise the different characteristics for the six different angles on the board, for example, asking learners to state the size of angles that are regarded as Acute angles. The dialogue below demonstrates this discourse:

11. Mafoko: *Can you define each of the mentioned angles?*
12. Learner 2: *An acute angle is an angle less than 90°*
13. Mafoko: *Well-done. Next?*
14. Learner 4: *An obtuse angle is angle that is greater than 90° but less than 180°.*

15. Learner 7: *A right angle is an angle that add up to 180°.*
16. Mafoko: *No*
17. Learner 1: *It is an angle that is equal to 90°*
18. Mafoko: *That's great. Next?*
19. Learner 8: *A revolution are angles that add up to 360°.*
20. Learner 5: *A reflex angle is an angle that is greater than 180° but less than 360°*

Image 1 depicts the resulting writings from the exchange between Mafoko and his learners and it can be said that the writing of the angle size descriptions alongside the names of the angles acted as visual legitimating criteria.



What is written in this blurred image:

Euclidean Geometry

1) Acute angle: $angle\ less\ than\ 90°$
2) Obtuse angle: $90° < angle < 180°$
3) Right angle: $angle = 90°$
4) Revolution: $angle = 360°$
5) Straight-line angle: $angle = 180°$
6) Reflex angle: $180° < angle < 360°$

***Image 1**: Mafoko's explanation about general types of angles*

Considering that Mafoko wrote the types of angles and alongside wrote the distinguishing feature for each, this represented legitimation "criteria for what counts as valid mathematics in the lesson" (Adler & Ronda, 2016, p 12). The choice of words in lines 1 and 11 "… that you know?" and "can you define …" signify the invitation for learners to participate in the lesson. While this is the case, their participation was limited to the verbalisation of one-word answers, with no room for interpretive elaborations or clarity seeking opportunities for learners. This resulted in the discourse of right-answerism, which relates to what Sfard (2008) refers to as recalling discourse. This is exemplified by lines 15 through to 17, where the teacher only says "no" (Line 16) after the learner 7 got the answer wrong for the description of what a right angle is.

When focusing on Mafoko's mathematical talk (naming) during this episode, one can say that the naming from him was silent, considering that learners did the talking as he wrote on the board and the teacher did not offer any substantiations to learners' verbalisations. This being the case, apart from the incorrect answer given by learner 7, the overall classroom naming of mathematical objects involved the use of mathematical words as labels as well as naming of angles (Adler & Ronda, 2015). Of concern is that while Mafoko used the introduction of the different types of angles and their characteristics as an introduction of the lesson and drawing learners' attention to related concepts for the object of learning, he did not make connections of these mathematical definitions to the object of learning. Mafoko did not provide the explanatory talk to emphasise the importance of the definitions of angles he displayed in the board. For example, there were no substantiations on how learners are expected to use the revolution angle and the straight-line angle in this lesson. It is important for the teacher to make links between the introduction (background knowledge) and the current object of learning, so that learners do not see mathematics lessons as a series of disconnected events (Mbhiza, 2021).

To continue with the lesson, Mafoko drew a diagram as shown in the image 2 to explain key terms of the object of learning. He started by drawing two lines on the whiteboard, line AB and line CD on which he marked a symbol of an "arrow". Learners were asked to identify the nature of these two lines. The learners responded positively by giving to the teacher the correct answer which is "AB and CD are parallel line".

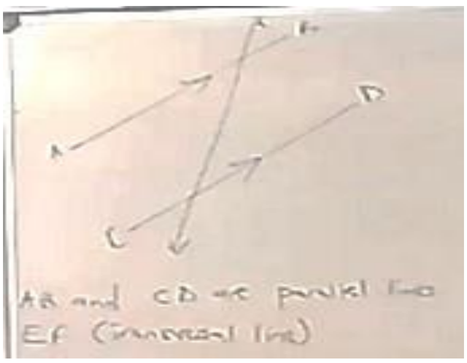


Image 2: Illustration of parallel lines and a transversal line.

As he drew the mathematical object in image 2, Mafoko engaged with his learners as represented in the exchange below:

21. Mafoko: *Let say we have these two lines AB and CD. How do you call them?*

22. Learners (collective answer): *Parallel lines*

23. Mafoko: *What if I draw another line that is cutting the two lines, say EF. How do you call it?*

25. Learners: (*No answers)*

26. Mafoko: *It is a special line which the name starts with the letter "T"*

27. Learner 1: *Transverse*

28. Mafoko: *Not quite. Try again.*

29. Learner 2: *A transversal line*

30. Mafoko: *Well-done! Let us clap for her. This transversal line together with parallel lines gives us angles with certain characteristics.*

The discourse of learners being invited to provide phrases or words in response to the questioned posed by the teacher, with no questions asked by the learners and no substantiations provided by the teacher, which is the same pattern of interaction as in lines 1 to 20. This resonates with Adler's (2017, p. 133) observation that these practices appeal "to the authority of the teacher or visual cues that produce a dependency on the teacher, on memory (this is what you must do); or on how things 'look'." The establishment of dependency on the teacher can be exemplified by the verbal cue that Mafoko gave in line 26 "… *name starts with the letter "T"",* prompting learners to verbalise the answers until the correct one is given, again appealing to the discourse of right-answerism. As Sfard (2008) and Vygotsky (1978) point out that this way of engaging learners during teaching is necessary for the learning of scientific concepts. However, this cannot be the endpoint for teaching and learning. It can be said that the lack of enunciations in Mafoko's classroom discourse resulted in limited opportunities for providing explanatory talks, especially to provide mathematical meanings to mathematical concepts such as 'parallel lines' and 'transversal line' in the extract above. This addressed missed opportunities for learners not only to use visual legitimation in naming mathematical concepts, but to also learn and internalise their meanings and relatedness, bringing the object of learning into focus.

afoko drew another line EF that cuts line AB and line CD as shown in the image 3. Then he asked learners once again to identify the nature of line EF. But this time around learners were not able to give a correct answer to the teacher until they were helped by the teacher. I have noticed that learners background knowledge was having some gaps which made it difficult for Mafoko to explain well different properties of angles**.** Mafoko marked different angles formed by the parallel lines AB and CD together with the transversal line EF using small letters a, b, c, d, e, f and g as shown in image 3. Then asked learners to identify angles that are similar or equal by providing suitable reasons. This is captured in the extract that follows below image 3.

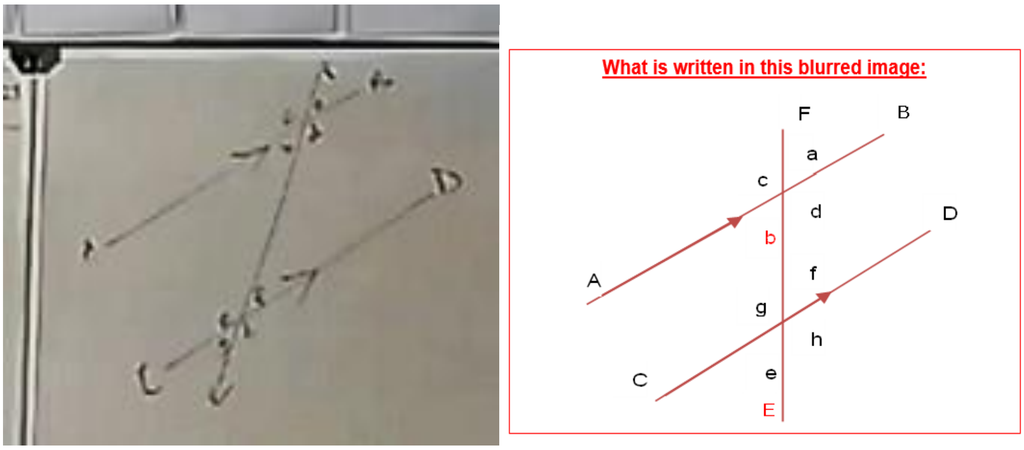


Image 3: Illustration of angles on parallel lines

32. Mafoko: *What can you say about angles **a** and **b**? You can also say something about **f** and **g***
34. Learners (collective answer): *Corresponding angles*
35. Mafoko: *No, Let start by saying if they are equal or what is the relationship between them?*
36. Learners (collective answer): *They are equal.*
37. Mafoko: *So, a = b and f = g. What is their name? or why are there equal?*
38. Learner 3: *Because they are vertically opposite angles.*

39. Mafoko: *So, vertically opposite angles are equal. Which angles also are equal?*
40. Learners (collective answers): ***c*** *and* ***d/ e*** *and* ***h***
41. Mafoko: *why are they equal?*
42. Learners: *They are vertically opposite angles.*
43. Mafoko: *Let us look for another relationship again.*
44. Learners: $b + c = 180°$ *and* $d + f = 180°$
45. Mafoko: *Why?*
46. Learner 4: *They are co-interior angles.*
47. Mafoko: *So, if angles add up to* $180°$*, they are special angles.*
48. Learners: *Like straight-line angles.*
49. Mafoko: *Yes, but these angles are not straight-line angles. How can you call such angles?*
50. Learner 5: *Supplementary angles.*
51. Mafoko: *Correct. Supplementary angles add up to* $180°$*. So, just for interesting sake they are also angles that add up to* $90°$*. How do you call them?*
53. Learners: *Complementary angles.*
54. Mafoko: *Okay. So far, we have identified the vertically opposite angles and co-interior angles. Let us look for the relationship between angles* $b + d$*. it is equal to what?*
56. Learners: $180°$
57. Mafoko: *Correct. Also angles* $c + a = 180°$*,* $b + f = 180°$*,* $h + f = 180°$*,… How do you call these angles?*
59. Learners: *Straight-line angles*
60. Mafoko: *So, straight-line angles add up to* $180°$*. They are also supplementary. Let us look for other properties.*
62. Learners: *angles* ***d*** *and* ***f***
63. Mafoko: *How are they called?*
64. Learners: *Alternate angles*
65. Mafoko: *Some people call them* ***Z-angles.*** *What can you say about these angles?*
66. Leaners: *They are equal.*
67. Mafoko: *Let us pick alternate angles from the diagram?*
68. Learners: *angles* ***b*** *and* ***f/ e*** *and* ***d***
69. Learner 6: *angles* ***a*** *and* ***b.***
70. Mafoko: *These angles are not* ***Z-angles.***
71. Learners: *What about corresponding angles?*
72. Mafoko: *Yes, they are angles that are in the same position. What are these angles?*
73. Learners: $a = f/c = e/d = h$
74. Mafoko: *How many properties did we get from this diagram?*
75. Learner 7: *5 properties*
76. Mafoko: *Correct. Corresponding angles are also called F-angles. Let me give you an activity.*

In line 32, Mafoko posed a question to his students about the relationship between angles *a* and *b*. However, he realised that some of the learners were struggling to comprehend the question or did not know the answer. To assist the learners in understanding the relationship between these angles, Mafoko asked them to consider the angles f and g. These two angles were strategically introduced as they are vertically opposite, allowing learners to visualize the positional relationship between *a* and *b*. This triggered their memory of the rules for corresponding, alternating, and co-interior angles. However, despite recalling these rules, the learners were unable to correctly determine the relationship between the angles and provided an incorrect answer.

In addition, in lines 47 to 53, Mafoko presented the concept of Supplementary and Complementary angles, which are important mathematical terms for learners to understand. However, the teacher could have further explained these concepts and used diagrams to aid in the explanation. This is because learners sometimes struggle to see the connection between supplementary angles and adjacent angles. Mafoko instructed the learners to investigate the relationship between angles $a + b$. Contrary, to this statement, Mafoko's approach lacked effectiveness, as it constrained learners to providing single-word responses. As a result, the learners may not have fully understood the concept being taught. It would have been more beneficial if Mafoko allowed the learners to engage in a discussion and explore the relationship between angles a and b in more detail as Adler and Ronda (2015) posit that learner participation plays a major role in ensuring effective learning of concepts and generality.

In view of the above analysis, it can be said that Mafoko used tasks of asking learners to identify the angles and their relationships in this excerpt as he did in the previous interactions with learners, making his discourse of teaching Euclidean geometry to be task bound. By task bound I mean, teaching situations in which the teacher calls on learners to verbalise or engage in mathematical actions and reasoning without providing explanations or exemplification of the concept, and where the legitimation criteria are the confirmation or disconfirmation that what learners are saying about mathematical objects is incorrect. Thus, it can be argued that the task-bound discourse that Mafoko exhibited in this section limited opportunities for learners to engage in meaningful and in-depth meaning about Euclidean geometry concepts. This is one aspect that MDI (Alder & Ronda, 2015) is silent about; what the implications for effective teaching are when one or some elements of mathematical discourse are not present in a lesson. Another notable discourse in Mafoko’s teaching is that he dismissed learners’ incorrect responses to questions and did not use such answers as opportunities for learning, by for example, asking leamers to provide further explanations to why they said what they said. This links closely with a teaching strategy that Moschovich (2015) deemed as routine answer, in which the teacher only listens for the correct answer during teaching and neglects to engage in discussions about the incorrect answers provided by the learners.

5.1.2. Episode 2: Under-teaching of properties of angles on parallel lines that are equal.

Episode 2 continues from the previous one, and Mafoko introduced the concept of corresponding angles. He started this episode by marking some angles on the parallel lines (see image 4) and asked learners to find the relationship between angles in sub-sections 1, 2 and 3.

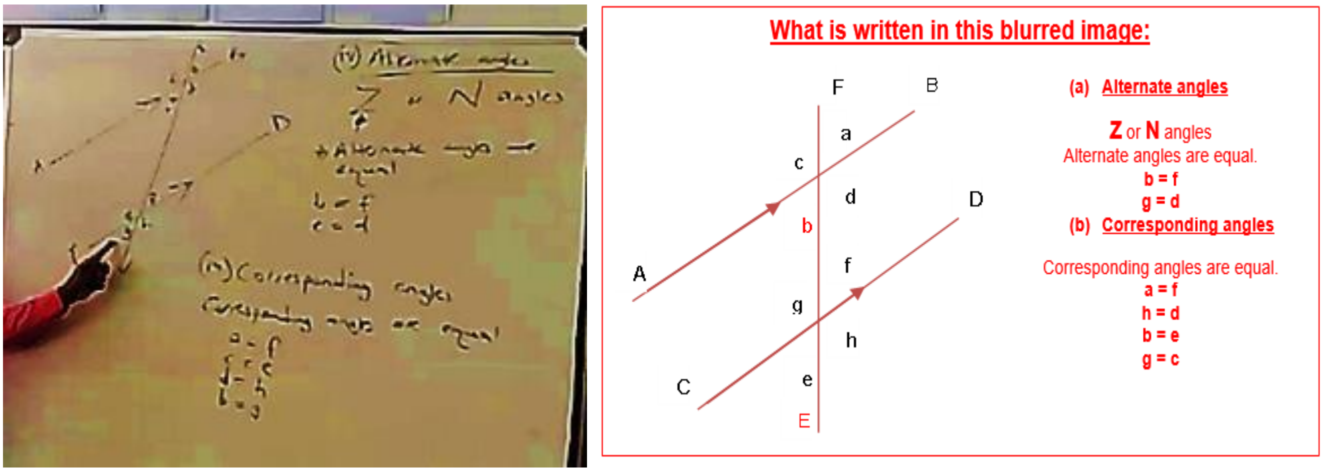


Image 4: Mafoko's explanation about alternates and corresponding angles

1) What can you say about angles a *and* ***b; c*** *and* ***d; e*** *and* ***f; g*** *and* ***h?***

Learners were confused by the question and most learners did not know what to say about the relationship between the angles they ought to focus on. Some answered "*vertically opposite angles*" but Mafoko was not pleased with the answer given by learners. Instead of building on the answers the learners gave to create a mathematics learning environment that does not only promote only one way of mathematising, Mafoko continued with his discourse of prompting the learners to keep on guessing until they verbalised responses that he expected. In this case, the teacher could have engaged further with the response given by the learners and ask them about the nature of vertical opposite angles. I regard this as a missed opportunity to foster in-depth learner participation during learning. Mafoko expected learners to provide the answer that "*these angles are equal*" ($a = b$; $c = d$; $\mathrm{e} = \mathrm{f}$; $\mathrm{g} = h$) then give a reason that "*vertically opposite angles are equal*". Like in the previous episode, he also used limited explanatory talk to make the concept of corresponding angles available for learners.

2) What can you say about angles a** and **f; b** and **e; c** and **g; d** and **h?

For these angles learners managed to provide the correct relationship and reason which is $a = \mathrm{f}, b = \mathrm{e}, c = \mathrm{g}$ and $d = h$ and the reason was that "corresponding angles are equal." Mafoko emphasized on these angles by telling learners that the best way to identify these angles they need always to look for "F shape". According to Adler and Ronda (2015), these kinds of phrases are visual legitimations, in which the representation (F-shape) acts as a mediational tool for learners to make links between the ideas of parallel lines and corresponding angles, which helps in building learners' mental images about the concept. It is difficult to see whether every learner understood the concept or not because every time Mafoko's learners answer questions collectively.

I observed that Mafoko did not extend the discussion by inviting learners to give comments to others' answers/output during the discussion, which limits learner participation during teaching and learning (Adler & Venkat, 2014). According to Adler (2017), although learner participation can sometimes be intimidating, it remains an important element of MDI in any lesson, to ensure that all learners are engaged in the processes of meaning making and co-construction of mathematical knowledge, encourages them to develop ideas, and requires learners to prove their observations or claims. I have also observed that predominately, learners were answering questions through chorusing, which arguably did not allow Mafoko to see the ability of his learners individually.

I have noticed that Mafoko's method of instruction was difficult for the students to follow and comprehend. Instead of providing examples to help students grasp his method of discussing the many properties of angles, he exclusively employed activities as a teaching tool. I got the chance to ask him about his teaching philosophy during VSRI. The conversation that follows exemplifies what we discussed:

> **Researcher**: *I have noticed that you did not use worked examples during the lessons. Why are you teaching without the use of examples?*
>
> **Mafoko**: *This is not how I teach all of my Grade 10 classes. I only employ this method of instruction in commercial classes when students struggle with mathematics comprehension and are not very strong.*
>
> **Researcher:** *Given their current situation, weak learners may find this technique challenging to understand. In my opinion, <u>using examples would have been far more beneficial for them</u>. I have observed that you were unable to provide them with a chance to address the board and voice their opinions for this reason and this limited their participation to one-word phrases.*
>
> **Mafoko:** *<u>The time is another reason</u> I utilize this. It is challenging for me <u>to complete the syllabus</u> in weaker classrooms, and at the end of the day, they have to take the same test as my other classes.*

Mafoko's reflective statements in this excerpt demonstrates that he was aware and intentional in his discourses of no exemplification and referenced time constraints as well as his positioning of the learners as weak for offering no examples during teaching. Driscol, Nikula and DePiper (2016) postulate that the best way to promote communication, productive learner participation and develop proficiency in mathematical reasoning during learning is to 'lighten the cognitive demand'. This suggests that learners should be given the learning opportunity to construct their own conjectures about mathematical objects, prove and disprove each other's conjectures, and arrive at solutions and thereby generality about the nature of mathematical concepts. Accordingly, the lack of exemplification and sustained learner engagement in Mafoko's discourse limited epistemological access to the concept of corresponding angles.

3) What can you say about angles b** and **f; d** and **g?

Learners were able to identify the relationship between these angles and provide the suitable reason because of Mafoko's explanation as shown in the image 4 below. That is: $b = f \; and \; d = \mathrm{g}$ and the reason for this is that "*alternate angles are equal*" (learners' chorused response). It is important to note that teacher's mathematical discourse remains essential because through it, learners are able to share and justify their own problem-solving strategies and analyse the reasoning of their peers as they work to solve problems collectively.

5.1.3. Episode 3: Under-teaching of properties of angles on parallel lines that are not equal.

Episode 3 continues from the previous one, and Mafoko after describing the properties of angles on parallel lines that are equal, he continued the same way to display and describe another category of angles, 'angles on parallel lines that are not equal' Mafoko marked the last category of angles on the parallel lines and asked learners to find the relationship between angles in sub-sections 1 and 2 below.

1) ***a*** and ***c*****;** ***b*** and ***d*****;** ***e*** and ***f*****;** ***g*** and ***h*****;** ***a*** and ***d*****;** ***c*** and ***b*****;** ***e*** and ***g*****;** ***f*** and ***h.***

Mafoko repeated the same statement asking learners to identify the relationships between the angles. This time around learners managed to say that $a + c = b + d = e + f = g + h = a + d = c + b = e + g = f + h = 180°$, but learners failed to provide an appropriate reason to this relationship which is "angles on a straight line add up to $180°$." Consider this excerpt:

77. Mafoko: *What can you say about angles **a** and **c**?*
78. Learner 1: $a + c = 180°$
79. Mafoko: *What is the reason to this statement?*
80. Learners: *No answers*
81. Mafoko: *How did we call these types of angles when we started the lesson?*
82. Learners: *No answers*
83. Mafoko: *You already forgot. We said that these angles are straight-line angles or supplementary because they add up to* $180°$*. Can you identify other straight-line angles?*
85. Learner 2: $b + d = 180°$
86. Mafoko: *Correct. Next?*
87. Learner 3: $e + f = 180°$
88. Mafoko: *Well-done.*
89. Learner 4: $g + h = 180°$
90. Mafoko: *That's great.*
91. Learner 5: $a + d = 180°$
92. Mafoko: *Yes. One more.*
93. Learner 6: $c + b = 180°$
94. Mafoko: *Correct.*
95. Learner 7: $e + g = 180°$
96. Mafoko: *Correct.*
97. Learner 8: $f + h = 180°$
98. Mafoko: *Okay. What about the following angles: **b** and **e**; **d** and **f**.*
99. Learners: $b + e = 180°$ *and* $d + f = 180°$
100. Mafoko: *Correct. What is the reason to this?*
101. Learners: *Co-interior angles*

One way of interpreting the reason why learners could not be able to provide the correct reason for these angles is because Mafoko introduced the lesson by presenting different properties of general types of angles, which he did not provide concrete examples to strengthen them. This being the case, it was interesting to note that in line 83, Mafoko reminded learner about the concept that was covered earlier in the lesson, to ensure that they think about the concepts relationally and not see the different ideas presented at different times as disconnected concepts (Adler & Venkat, 2014). It is important for the teacher not only to introduce different concepts in a lesson but offer explanatory talk during teaching to ensure that learners make links between the presented mathematical objects with mathematical explanations, to help them build mental images and mental definitions of concepts (Tall, 1981; Adler & Ronda, 2015).

2) ***b*** **and** ***e*****;** ***d*** **and** ***f*****.**

Mafoko did not take time to make connections between different angles which forced learners to stick to one way of answering questions. I have observed that once learners forgot some properties, it is difficult for them to find another of reasoning which is a limitation for them, and this made it challenging to give the answer for the angles. During VSRI, I asked Mafoko to reflect on this part of the lesson and he stated that he wanted the learners to give the answer as " $b + e = d + f = 180°$ *with reason that "co-interior angles are supplementary which means they add up to* $180°$*".* I have observed that even though learners could not remember the properties that "co-interior angles are supplementary", they could have derived the answer using properties of angles in (2) and (3) in the previous episode, to demonstrate continuity in their learning.

It is important for the teacher to challenge learners to formulate high level questions and arguments to provoke one another's thinking which is one way of giving learners opportunities to compare and contrast ideas and not only to stick to one way of thinking. Mafoko's naming and legitimating in his MDI of comprised of mainly positional (Mafoko's authority) criteria for confirming the Euclidean geometry knowledge and skills presented in different episodes. The table below represents the summary of Mafoko's mathematical discourses during Euclidean geometry teaching presented in the above episodes, based on the MDI components of mathematical discourse. Table 7 depicts the summary of Mafoko's MDI and Table 8 are the summative judgements relating to the teacher's explanatory talk, exemplification, and learner participation.

Table 7. *Summary of Mafoko's teaching episodes*				
Object of learning: Angles on parallel lines				
Teacher's explanatory talk		**Exemplification**		**Learner participation**
Naming	***Legitimating criteria***	***Examples***	***Tasks***	
Episode 1: Learners' background knowledge and visualising parallel lines, transversal line and different angles. - Mafoko used less everyday language to explain the different types of angles. He used appropriately mathematical language to explain used visual when drawing lines on the whiteboard and different angles. for example: Acute angle $angle\ less\ than\ 90°$ Obtuse angle $90° < angle < 180°$ Right angle $angle = 90°$ - Mafoko used equality symbol "=" and inequality symbol "<" to describe different angles. He also used visual when drawing different lines on the whiteboard.	**Legitimating criteria:** Mafoko did not use non-mathematical criteria. He used mathematical criteria that are general when providing specific definitions and properties of different angles, and no local mathematical criterial were used across this episode which means no convention was provided.	Example 1 Acute angle Example 2 Obtuse angle Example 3 Right angle Example 4 Straight-line angle Example 5 Revolution Example 6 Reflex angle All the examples that Mafoko introduced were treated as tasks, and the teacher did not use them to offer mathematical explanations. Mafoko used 1-6 which are only similar examples. No contrast examples or examples that bring fusion were used.	- Mafoko asked learners how many types of angles do we have? Then presented examples 1-7 to the whiteboard and asked to define each of the term or to give the meaning. The emphasis of examples 4-6 was that learners supposed to explain these examples in detail. - Mafoko drew two lines, line AB and line CD on the whiteboard on which he marked the symbol of an arrow and asked learners to identify the nature of these two lines. - Mafoko drew another line that cuts the line AB and line CD and asked learners to give the name of this line.	Learners were only invited to define different term and to identify the nature of different lines drawn on the whiteboard by Mafoko. Their participation was limited to uttering phrases in response to the questions the teacher asked throughout the lesson. This was not an isolated event a this was also observed in other lessons not included in this study due to space constraints. Mafoko did not give an opportunity to learners to engage in substantive conversations to demonstrate their knowledge and skills. He did not test learners on how to write definition of different angles using equality symbol or inequality symbol.

***Episode 2**: Under-teaching of properties of angles on parallel lines that are equal.* - Mafoko used less everyday language when pointing with his finger different angles he used the ambiguous pronouns “this” and “that” to refer to some angles on the diagram he designed on the whiteboard. He used mathematical language appropriately to explain the relationship between vertically opposite angles, corresponding angles and alternate angles when he used equality symbol “=” to express the following statements: 1. Vertically opposite angles are equal. 2. Corresponding angles are equal 3. Alternate angles are equal. **<u>Legitimating criteria:</u>** Mafoko used non-mathematical criteria when using mnemonics: the shape “**X**” to help learners remember how to identify vertically opposite angels. The shape “**F**” to help learners remember how to identify corresponding angles. The shape “**Z**” or “**N**” to help learners remember how to identify alternate angles. He also used mathematical criteria that are general when providing properties of these angles, and local mathematical criterial when showing to learners how to give a reason to a statement which is regarded as a convention.	Example 1 Vertically opposite angles Example 2 Corresponding angles Example 3 Alternate angles Mafoko wrote down on the whiteboard different letters that represent different angles as show on the diagram below. The task of learners was to identify the relation between these angles by filling in the gap with an appropriate mathematical symbol. 1. a _______b c _______d e _______f h _______g 2. c _______g a _______f b _______e d _______h 3. b _______f d _______g	Mafoko did not make some attempt to engage learners in in-depth discussions. The participation of learners was only limited to providing the relationship between angles. Mafoko did almost all the work by himself, and learners were subjected to being spectators to his performative actions. He should have allowed learners to identify angles that are vertically opposites, alternate angles and corresponding angles by themselves before asking them to provide the relationship between different angles. He asked mostly low-order questions (yes/no questions) that require simple factual responses. Mafoko did not use teaching strategies to develop critical and creative thinking or other higher-order thinking skills.

Episode 3: Under-teaching of properties of angles on parallel lines that are not equal. Mafoko did not use much everyday language. He used mathematical language appropriately to explain the relationship between co-interior angles and straight-line angles when he used following statements: 1. Straight-line angles are supplementary 2. Co-interior angles are supplementary The word “Supplementary” is in everyday language. After using this word, he then used mathematical language to explain this word by giving the following meaning: “Angles that add up to 180°” He also emphasized on the word “complementary” by providing to learners the following meaning: “angles that add up to 90°.” **Legitimating criteria:** Mafoko used non-mathematical criteria when using mnemonic: the shape “**U**” to help learners remember how to identify co-interior angels. He used mathematical criteria that are general when providing properties of these angles, and local mathematical criterial when showing to learners how to give a reason to a statement which is regarded as a convention.	Example 1 Straight-line angles Example 2 Co-interior angles The task of learners was to identify angles that are on the straight line and co-interior angles, then after that provide a mathematical relationship that links them. Learners where able to identify two pairs of co-interior angles and eight pairs of angles on the same straight line. The challenge was that some learners were not able to understand their task, and some were confused between straight-line angles and angles in a revolution which Mafoko did not take time to explain.	Learners’ participation where a bit better because Mafoko gave an opportunity to learners to go to the whiteboard and identify angles by themselves before providing any answer. He did give learners opportunities to compare and contrast ideas

Table 8. *Summative judgments of Mafoko's MDI*

EXPLANATORY TALK		EXAMPLIFICATION		LEARNER PARTICIPATION
Naming	**Legitimating criteria**	**Examples**	**Tasks**	
Level 1: **(NM)**: Less everyday language **(Ms)**: Inequality and equality symbols to express angles relationships **Level 2:** **(Ma)**: Mathematical language to refer to words and symbols **Level 3:** Less movement between **(NM)** and **(Ma)**	**Level 0:** **(NM)-(V):** Visual: Drawing lines on the board. **Level 1:** **(L):** Writing Euclidean geometry reasons for different statements in short form (convention) **Level 2:** **(GP):** Definitions and properties	**Level 1:** Use of examples that reflect **similarity** or **contrast** (One form of variation) **Level 2:** Use of examples with at least **TWO FORMS OF VARIATION.**	Limited to identifying objects, angles and properties verbally.	- **Only verbal participation** - **No physical participation**

5.2. Data presentation and analysis – The case of Mpilo

Concerning the teaching and learning of Euclidean Geometry in Grade 10, the principles begin in Grades 8-9. Learners are suffering with this topic because they lack a number of fundamental concepts in their background knowledge.

*(**Mpilo**, **semi-structured interviews**)*

Mpilo's statement above relates the importance of learners' background knowledge for the teaching and learning of Euclidean geometry. In this section, the analysis and interpretation of Mpilo's two observed lessons, together with the information from the VSRI are presented. Two episodes were selected from two of Mpilo's lessons. Figure 9 represents the selected episodes from Mpilo's teaching and are interpreted and discussed.

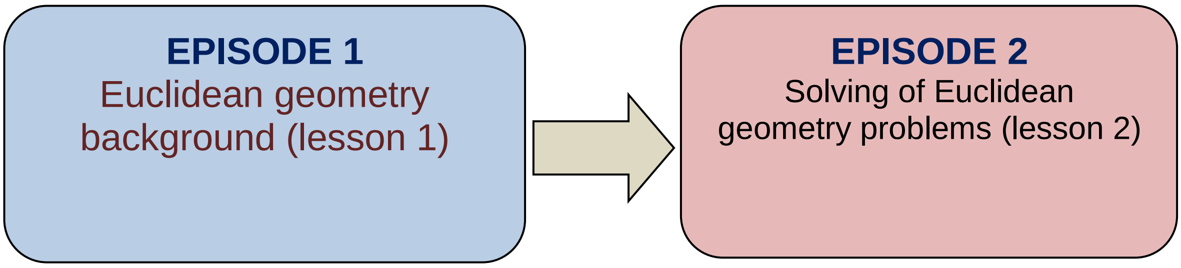


Figure 9. Mpilo's selected episodes from two selected lessons.

5.1.1. Euclidean Geometry background (Lesson 1)

Mpilo began the lesson by writing the following on the whiteboard: "Euclidean geometry – background" for learners to be clear of the object of the lessons (see Image 5). After writing the topic on the whiteboard, he gave to learners the meaning of background saying: "*background is everything that you know and that will assist you understand any concept*". The words Mpilo wrote on the board accompanied by his explanation of what he meant by background acted as the announcement of the object of learning. To make continuity links to the contents the learners have been introduced to in previous grades, Mpilo further told the learners that he was going to remind them about the basic geometrical concepts they have covered in Grades 8 and 9. This pedagogic action addressed Scott et al.'s (2011) suggestion that, before a teacher introduces new subject matter contents, they should create a classroom environment for engaging in "basic conversational techniques for building the future on the foundations of the past" (p. 15).

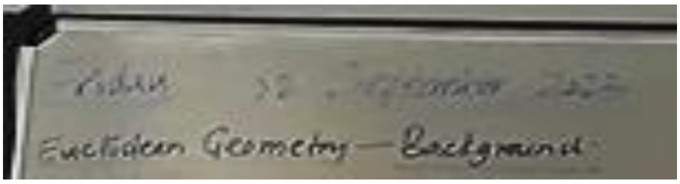


Image 5: The object of the lesson

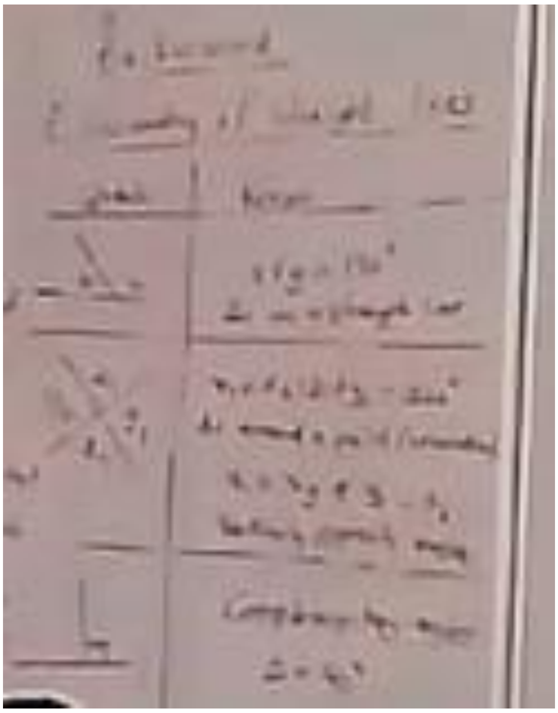


Image 6[2]: Mpilo's explanation on geometry of a straight line.

[2] As I had positioned the camera at the back of the classroom and due to the over crowdedness of Mpilo's classroom, the image of what he wrote on the board is blurry. Thus, in figure 8, I recreated what is written on the board. This is done for the images throughout this chapter, to provide ease of reading.

What is drawn and written in this blurred image:

Background

1) Geometry of straight lines

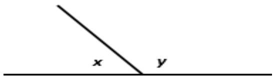
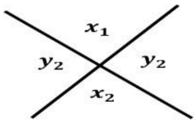
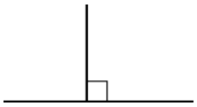
Sketch	Reason
 	$x + y = 180°$ $<_s$ *on a staight line*
 	$x_1 + x_2 + y_1 + y_2 = 360°$ $<_s$ *around a point (Revolution)* $x_1 = x_2$ & $y_1 = y_2$ Vertically opposite angles
	complementary angles $<= 90°$

Figure 10. *Recreation of what is written in image 6.*

Mpilo drew the three sketches to depict angles on straight lines, as depicted in image 6 (and Figure 10). Then he invited the learners to reflect on their knowledge of the angles on straight lines. Consider the following exchange.

24. **Mpilo:** *We are going to start with the geometry of straight lines. In Euclidean geometry, you need to remember that every statement needs to have appropriate reason. So, what can you say about angles x and y on the diagram? (See image above)*
25. **Learner 1:** $x + y = 180°$
26. **Mpilo:** *Correct. Who can help her with the reason?*
27. **Learner 2:** *because angles on a straight line add up to* $180°$.
28. **Mpilo:** *Look at the second diagram and tell the relationship between angles* $x_1, x_2, y_1,$ *and* y_2.
29. **Learner 3:** $x_1 + x_2 + y_1 + y_2 = 360°$
30. **Mpilo:** *Wonderful! What is the reason to this statement?*
31. **Learner 4:** *Revolution*
32. **Mpilo:** *What else can you use as a reason for the same statement?*
33. **Learner 5:** *Angles around a point.*
34. **Mpilo:** *Correct. What do you **remember** when you look at the third diagram?*
35. **Learners:** *A right angle*
36. **Mpilo:** *Correct. But how do you call angles that add up to* $90°$*?*
37. **Learner 6:** *Complementary*

This exchange reveals three elements of MDI in use: learner participation, visual legitimation criteria as well as positional legitimation. The positional legitimation in line 1 was effective in communicating to learners that for every mathematical claim that they make, they should provide accompanying justifications for their observations or statements. This is effective in ensuring that learners do not learn Euclidean geometry rules without accompanying rationales (Venkat & Askew, 2017). The questions and confirmations for learner correct answers in this extract demonstrate strides made by the teacher to not only involve the learners in making observations about the mathematical objects, but also shows how the teacher used the visual legitimation (image 6) together with positional legitimations in statements "*on the diagram" (line 1) and "Look at the second diagram"* (line 5) to help learners learn the rules associated with straight line angles.

Although learners managed to get the correct answers and reasons, the teacher's substantiations after the learners mentioned the reasons accompanying their mathematical observations could have enhanced opportunities for enabling learner epistemological access to mathematical concepts, especially learners who might be lost during learning. Thus, I argue that an element of explanatory talk should not only be employed in classroom situations where learners are struggling to get correct answers but should also be used for elaborations about mathematical concepts, their behaviour and relatedness. One could view Mpilo's question and answer strategy as effective, especially for promoting learner participation and reinforcing the foundational knowledge about angles on straight lines. However, the clear lack of balance between questioning and providing elucidations on learner answers limited opportunities for further learning. Thus, the teacher's discourse can be categorised as focusing solely on repeated recalls of mathematical concepts and associated rules, especially considering the nature of his questioning "*What do you **remember** when …"* (line 11). Having learner answers that are limited to one-word answers limits learner critical thinking about mathematical objects and concepts. Accordingly, there is need for substantive conversations during Euclidean geometry lessons, to help learners not to only rely on recall but internalise the meanings associated with different concepts in focus (see Adler & Venkat, 2014).

After Mpilo finished to give to recap the background on the geometry of straight line, he proceeded to introduce a new concept, parallel lines. Although Mpilo did not make pedagogical links between the part of the lesson on angles on straight lines, he wanted learners to apply their knowledge of this concept for further learning when he introduces the concept of parallel lines. This is evidenced by the following VSRI conversation we had about this.

Researcher: *in this lesson, you started off by recapping the concept of angles on straight lines. This was good to see …*

Mpilo: *yes, I wanted learners to keep that understanding in mind, which would help them to work with angles when I introduced parallel lines. Our learners need you to do that, if you don't, they see a concept as a stand-alone.*

Researcher: *But I noticed that you did not make the links clear for the learners as to what the relevance of recapping that knowledge was.*

Mpilo: *I agree, and I could have told the learners what the links are. You see this thing of being able to watch your own lessons is helpful. I will sure tell the learners why I do certain things in class next time.*

Mpilo's reflection resonates with Mason's (2011) assertion that it is challenging for teachers during teaching to make pedagogical action on elements of the lesdom they are not noticing. Image 7[3] depicts what Mpilo wrote on the board relating to angles on parallel lines.

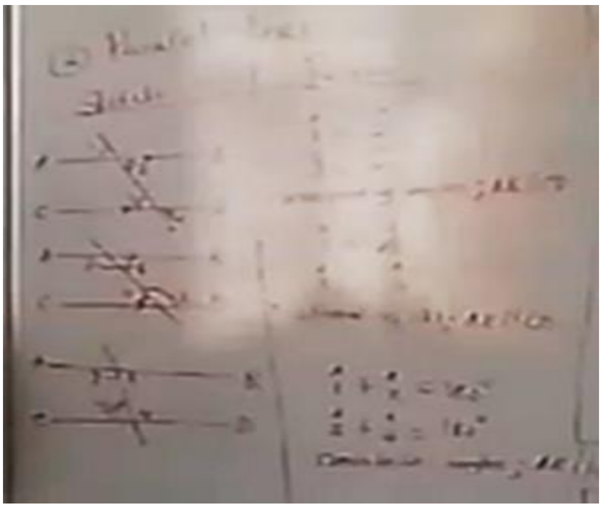


Image 7: Mpilo's explanation about properties of angles on parallel lines.

[3] The image is blurry, clarity on what was written on the board is presented on Figure 9.

What is drawn and written in this blurred image:

2) <u>Parallel lines</u>

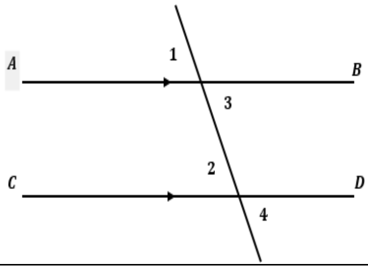
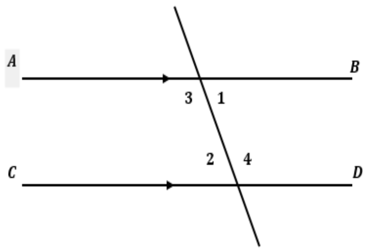
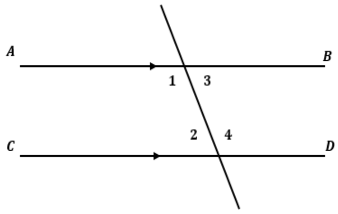
Sketch	Reason
	$\hat{1}=\hat{2}$ $\hat{3}=\hat{4}$ $corresponding <_s, AB \| CD$
	$\hat{1}=\hat{2}$ $\hat{3}=\hat{4}$ $Alternate <_s, AB \| CD$
	$\hat{1}+\hat{2}= 180°$ $\hat{3}+\hat{4}= 180°$ $Co - interior <_s, AB \| CD$

Figure 11. *Recreation of what is written in image 7.*

Below is the accompanying classroom exchange between Mpilo and the learners about what is presented on image 7.

38. **Mpilo:** *Before I draw different sketches of parallel lines on the board. What are parallel lines?*

39. **Learner 1:** *Line segments that are facing one another but will never cross paths.*

40. **Mpilo:** *What else do you still remember about parallel lines?*

41. **Learners:** *No answers*

42. **Mpilo:** *There are a lot of properties that you can deduced from parallel lines. Let us look at these properties. When you look at the first diagram, what can you say about angles 1, 2, 3 and 4.*

43. **Learner 3:** $\hat{1}=\hat{2}$

44. **Learner 4:** $\hat{3}=\hat{4}$

45. **Mpilo:** *Good! What is the reason to this statement?*

46. **Learner 5:** *They are all corresponding angles.*

47. **Mpilo:** Correct. They are also called F-angles. *What can you say about angles 1, 2, 3 and 4 on the second diagram?*
48. **Learner 6:** $\hat{1} = \hat{2}$
49. **Learner 7:** $\hat{3} = \hat{4}$
50. **Mpilo:** *Correct. What is the reason?*
51. **Learners:** *Alternate angles*
52. **Mpilo:** *Correct. They are also called Z-angles. Finally, what about angles 1, 2, 3 and 4 on the third diagram?*
53. **Learner 6:** $\hat{1} + \hat{2} = 180°$
54. **Learner 7:** $\hat{3} + \hat{4} = 180°$
55. **Mpilo:** That's great. *What reason can you give to this statement?*
56. **Learners:** *Co-interior angles*

It was interesting to see in this part of the lesson Mpilo opening the pattern of engagement to be more inviting to learners' knowledge of the concept. The choice of words "*Before I draw different sketches of parallel lines …*" signify that the teacher is aware that learners rely on the visualisation of the mathematical objects presented on the board and not necessarily construct meanings and internalise the concepts. Mpilo's move to this level of learner engagement created opportunities for learners to engage in meaning-making process, to offer a definition to parallel lines. This being the case, it is equally concerning that after learner 1 gave the answer, which seemingly was what Mpilo was expecting, he did not confirm or disconfirm whether the answer is correct of wrong (lines 15, 16 and 17). The lack of feedback has potential to make learners not to know whether their answer is correct or incorrect, which can impede on the effectiveness of their future learning (Vygotsky, 1987). In addition, it was interesting to note that when the learners failed to name parallel lines, Mpilo did not actually provide the definition himself, but reverted to demonstrating what the concept entails visually (line 19). During reflective conversation as we engaged in VSRI, I asked Mpilo why he did not provide a formal definition in line 19, considering that learners did not answer his question, which could be taken to mean they did not know the definition or at least how to expatiate on what learner 1 had said, and Mpilo said:

> *I did not prepare for this lesson. I only researched for the definitions of Euclidean geometry, axiom, and theorem; then I wrote some properties of angles and triangles down. I thought that since my object of learning was the background of Euclidean geometry, I will just use my experience to teach, and things will be easy for me which was not the case when I was in the lesson.*

According to Planas (2022, p. 2) contends that "Mathematics teaching and teachers need to support learners in their communication of the intended mathematical meaning, but also need themselves to successfully resolve their communicative intents of mathematical meaning”. Of importance to note is that for effective explanatory talk to occur, teachers should be prepared for the lessons to be able to work with learner’s ways of thinking and knowing during teaching. It can be said that Mpilo’s statement in line 19 was not intended to facilitate conceptual development but functioned to promote pedagogic issues in the lesson (Planas, 2022).

To continue the lesson, Mpilo presented some properties regarding triangles. His exchange with learners is presented below. Again, Mpilo made the content transition without communicating the relational links to the learners, which could promote the idea of learners treating Euclidean geometry concepts as disconnected (Scott et al., 2011).

Image 8: *Mpilo’s explanation about properties of angles on triangles*

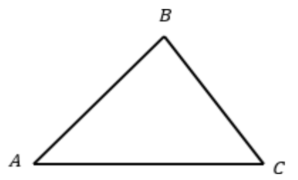
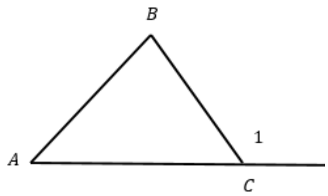
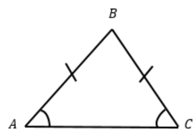
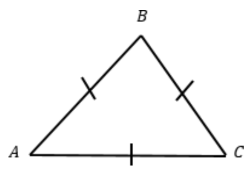
What is drawn and written in this blurred image: 3) <u>Parallel lines</u>	
Sketch	Reason
	$\hat{A}+\hat{B}+\hat{C}=180^\circ$ $Interior <_s \ of \ a \ triangle$
	$\hat{A}+\hat{B}=\hat{C}_1$ $Exterior \ < of \ a \ triangle$
	$\hat{A}=\hat{C}$ & $AB = BC$ $Isosceles \ triangle$
	$\hat{A}=\hat{B}=\hat{C}$ $AB = AC = BC$ $Equilateral \ triangle$

Figure 12. *Recreation of what is written in image 8.*

57. **Mpilo:** *Now, we are looking at the properties of angles and sides on triangles. Looking at the first triangle, what can you say about the angles?*
58. **Learner 1:** *They are not equal.*
59. **Mpilo:** *Not quite. What else can you say?*
60. **Learner 2:** *The interior angles add up to* $180°$
61. **Mpilo:** *Correct. Who can say this statement mathematically?*
62. **Learner 3:** $\hat{A} + \hat{B} + \hat{C} = 180°$
63. **Mpilo:** *Good! What do you know about angle* $\hat{C}$ *in the second triangle?*
64. **Learner 4:** *it is an exterior angle of a triangle.*
65. **Mpilo:** Correct. Who can give us the property of exterior angle of triangle*?*
66. **Learner 5:** *An exterior angle of a triangle is equal to the sum of the two opposite interior angles.*
67. Mpilo: *Correct. Someone to express it mathematically?*
68. **Learner 6:** $\hat{A} + \hat{B} = \hat{C}$
69. **Mpilo:** *Correct. If I draw the following, what type of triangle is this?*
70. **Learner 7:** *Is an isosceles triangle.*
71. **Mpilo:** That's great. *What properties can you extract from this triangle?*
72. **Learners:** *Two sides and two angles are equal.*
73. **Mpilo:** *Correct. Someone to express this mathematically?*
74. **Learner 8:** $\hat{A} = \hat{C}$ & $AB = BC$
75. **Mpilo:** *Good! Finally, what is the name of the last triangle?*
76. **Learners:** *Equilateral triangle*
77. **Mpilo:** *What are properties of an equilateral triangle?*
78. **Learner 9:** *All sides and all angles are equal.*
79. **Mpilo:** *What is Euclidean geometry?*
80. **Learners:** *No response*
81. **Mpilo:** *Euclidean geometry is the study of geometrical shapes, plane and solid of figures that are based on different theorems and axioms.*
82. **Learner 6:** What is the meaning theorem and axiom?
83. **Mpilo:** *I am going to break downs these words. Both are mathematical statement that are acceptable. A theorem needs to accompany by a mathematical proof or argument that makes it true while an axiom does not necessary require a mathematical proof.*

In this exchange, Mpilo continued with the process of having learners naming and legitimating mathematical objects using question-and-answer strategy until learner 6 (line 59) perforated the teacher's 'lesson framing' to ask the teacher to define the concepts of theorem and axiom. In line 60, it was interesting to note that the teacher did not provide the learner with the actual mathematical definitions of the two concepts, making his discourse to be non-mathematical. When asked to comment on this observable action during VSRI, Mpilo acknowledged again the lack of lesson planning and that he was caught off guard because his learners hardly asked questions:

> *Lesson planning is important. I really did not prepare for this lesson. I went blank there and I had to say something before the lesson ended or learners would see that this guy doesn't know the answer himself* (**Mpilo, VSRI**).

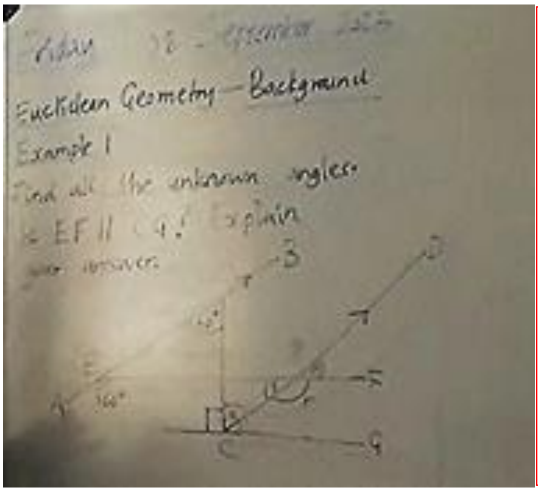
Overall, the teacher's lack of explanatory talk during teaching was influenced by lack of lesson planning, which limited what he could say and do with mathematical objects he presented to the learners.

5.1.2. Episodes 2: solving of Euclidean geometry problems.

Like in the first lesson, Mpilo began the lesson by writing the object of learning on the whiteboard. (See image 6). He commenced this episode by the following narrative:

> *Good morning boys and girls, last time we did Euclidean geometry-background whereby I managed to give you a lot of information which is the geometry of a straight line, angles, triangles and parallel lines. Having that information, we want now to apply this knowledge in the solving of Euclidean geometry problems.*

The utterance is helpful to enable the creation of the mathematical story about Euclidean geometry concepts, to help his learners follow how the concepts covered in the previous lesson relate with the current concepts to make a coherent and relational understanding of concepts (Scott et al., 2011). In the opening statement, the object of learning was communicated to be the application of the knowledge of angles covered in the previous lesson to solve "*Euclidean geometry problems".* This being the case, Mpilo did not differentiate between the current lesson and the previous lesson, to foreground the concepts and skill they would be using to solve Euclidean geometry lessons.



<u>What is written in this blurred image:</u>
<u>Euclidean Geometry – Background</u>
<u>Example 1</u>
Find the unknown angles. Is $EF \parallel CG$? Explain your answer.

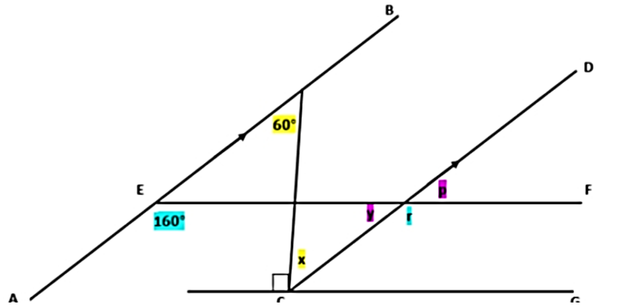


Image 9: The object of learning and Mpilo's example 1

As shown above, in image 9, Mpilo then proceeded to write '*example 1'* on the whiteboard to demonstrate to the learners what he meant by solving Euclidean geometry problems. I was waiting for him to ask learners to attempt this question since in the last lesson he took all the time allocated to the lesson to give learners background information about the geometry of straight lines, angles, triangles, and parallel lines but at my great surprise He said: *I will go through this example myself so that we can do it together.*

The over-reliance on a question-and-answer strategy that resulted in a lack of explanatory talk in Mpilo's mathematics discourse is concerning, especially if Planas (2022) iteration that "words and sentences in teacher talk mediate mathematical discourse practices with and between learners" (p. 3) is seriously considered. Although I hold the sentiment that learners should be given a chance to develop their Euclidean geometry talk to construct meanings to mathematical concepts and processes, teachers' Euclidean geometry talk plays a vital role in facilitating the improvement of learners' angle learning talk.

Before Mpilo gave the solutions, he asked learners the question: "*which section does the diagram above focus on?*" Learners could not be able to identify the section until the teacher himself told them that the diagram focuses on parallel lines. After that Mpilo explained the question and displayed the answers by asking learners to attempt first. He did this by presenting the following narrative:

> *Step 1, when answering questions used the properties of parallel lines to find all equal angles on the diagram. In this diagram your line of thing should be the properties of parallel lines and not properties of triangles. To do this, use different colour to mark all angles that are equal. Let us do that together.*

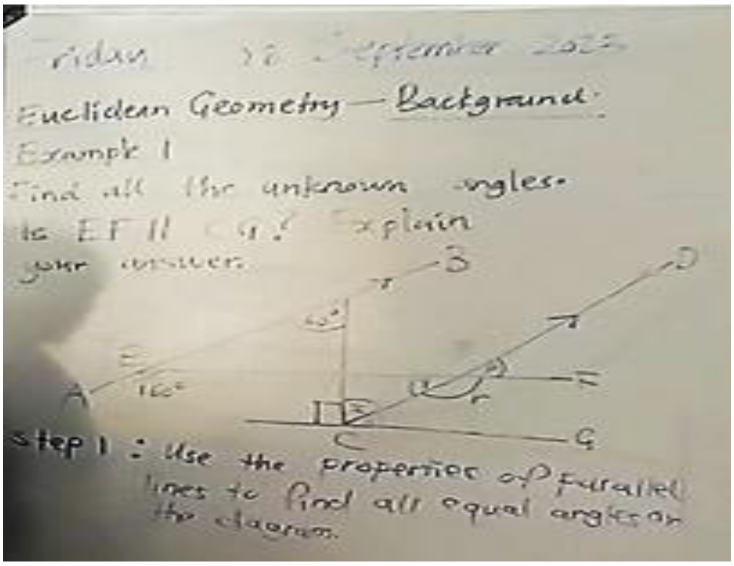


Image 10: Mpilo marking equal angles.

The written statement was the accompanying instruction for the learners, *"Use the properties of parallel lines to find all equal angles on the diagram"* (image 10). After this writing the line of thinking as indicated in step 1, he then asked learners to use their colours to mark all angles that are equal on the diagram.

> *Step 2: We are now going to determine the unknown angles.*

$\hat{x} = 60° \ (Alt < s =, AB \| CD)$

$\hat{r} = 160° \ (Corresp. < s =, AB \| CD)$

$\hat{y} + 160° = 180°$ $(Co - int.\ are\ Supp., AB \| CD)$
$\hat{y} = 180° - 160°$
$\hat{y} = 20°$

$\hat{p} = \hat{y} = 20°$ $(Vert. Opp. < s =)$

Step 3: We need now to show whether $EF \| CG$ *or not.*

$If\ EF \| CG\ then\ \hat{x} + 90° = \hat{r}\ , but\ \hat{r} = 160°\ and\ \hat{x} + 90° = 150°$
So, $EF \| CG$ *cannot be satisfied.*

From this narrative, we can see that Mpilo took more time to explain different ways of answering questions and how to write mathematical arguments to his learners forgetting that the object of learning was supposed to be the application of the previous lesson where learners supposed to demonstrate their level of understanding of the concepts learnt in the previous lesson. In addition, he demonstrated accurate and in-depth knowledge of most concepts in presenting the lesson but failed to give time to learners to apply their knowledge and responded to some learners' questions in a manner that did not attempt to be responsive to student developmental learning needs (See step 3 in the narrative above). After spending 27 minutes out of 45 minutes, learners were only left with 18 minutes to apply their knowledge when solving Euclidean geometry. This gives me an opportunity to recall one of Mpilo's responses of the semi-structured interview which is illustrated below.

Mpilo continued with the lesson by giving to learners the following activity:

Find the unknown angles in the diagram below and give reasons to your answers. He asked learners to discuss two by two.

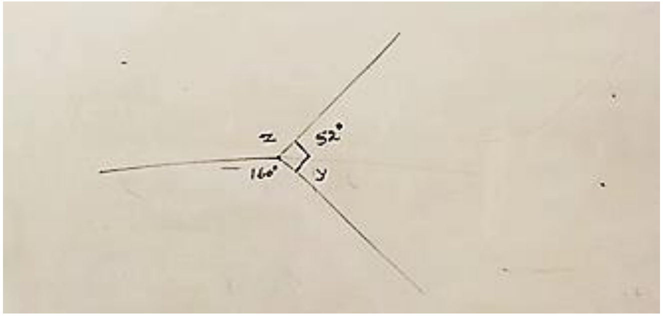


Image 11: Learners' activity

Looking at Mpilo's example 1 in image 9 and learners' activity in image 11, level of difficulty in both activities is different. Mpilo did the question with high level of difficulty than learners. I have observed that in some parts of the lesson, Mpilo has provided activities which address learners' critical thinking skills but failed to do so in this lesson where it was necessary for learners to display their critical thinking skills.

The solutions of the activity in image 11 are illustrated as follows:

Mpilo: *what is the value of the angle z?*
Learner 1: $160°$
Mpilo: *what is the reason for this?*
Learner 1: *Vertically opposite angles are equal.*
Mpilo: *wrong answer. Who can correct this?*
Learner 2: $z + 160° + 90° = 360°$
Mpilo: *what is the reason for this statement?*
Learner 2: *angles around a point add up to* $360°$.
Mpilo: *what is the value of angle z?*
Learner 3: $z = 360° - 160° - 90° = 110°$
Mpilo: *Correct. Let us clap for him. Now who can find the value of angle y?*
Learner 4: $y = 38°$
Mpilo: *how? Can you elaborate your answer?*
Learner 4: $y = 90° - 52° = 38°$.

From the above conversation, it is clear to see that learner 1 struggled to make a different between "angles around a point" and "Vertically opposite angles". This is due to lack of exposure to a variation of activities. In addition, I have observed that only some of the learners got an opportunity to apply their knowledge because the activity was very short and very simple for them. The table on the next page summarises the summary of Mpilo’s discourses in the above selected episodes, using the MDI components. Table 9 depicts the summary of Mpilo’s MDI and Table 10 are the summative judgements relating to the teacher’s explanatory talk, exemplification, and learner participation.

Table 9. *Summary of Mpilo's teaching episodes*

Object of learning 1: Euclidean geometry-background
Object of learning 2: Application on Euclidean geometry-background

Teacher's explanatory talk		**Exemplification**		**Learner participation**
Naming	***Legitimating criteria***	***Examples***	***Tasks***	
Episode 1: Euclidean geometry-background - Mpilo used less everyday language to define Euclidean geometry, a theorem and an axiom. He used mathematical language appropriately to describe different properties on geometry of a straight line, angles, triangles and parallel line. for example: *Mpilo defined Euclidean geometry as the study of geometrical shapes, plane and solid of figures that are based on different theorems and axioms.* - Mpilo used equality symbol "=" and inequality symbol "<" when writing down the reason of different statement.	**Legitimating criteria:** At this stage Mpilo did not use non-mathematical criteria. He used visual when drawing lines, angles and triangles on the whiteboard and mathematical criteria that are general when providing specific definitions and properties of different angles and triangles, and local mathematical criterial were used across this episode which means conventions were provided when writing reasons of different properties in short forms. For example: "Vert. opp. <s =" which means "vertically opposite angles are equal"	Mpilo did not give any example when he explained different concepts on geometry of straight line, angles triangles and parallel lines.	The task of learners was to listen and to offer tentative responses by maintaining silence as they watched the teacher, by responding mentally to theoretical questions posed by the teacher, and by copying the work of the teacher.	Mpilo did not give an opportunity to learners to go to the board and display their ability and skills. He did not test learners on how to write downs definition of different concepts and mathematical arguments. The interactions between Mpilo and his learners, and among learners, are inappropriate or insensitive.

***Episode 2**:* Under-problems solving that involve geometry of straight line, angles, triangles and parallel lines. - Mpilo used less everyday language when marking different angles on the diagram. - He used the ambiguous pronouns "this" and "that" to refer to some angles on the diagram he designed on the whiteboard. - He used mathematical language appropriately to give the solutions of different unknown angles. For example: $\hat{x} = 60° \ (Alt < s =, AB \parallel CD)$ $\hat{r} = 160° \ (Corresp. < s =, AB \parallel CD)$ $\hat{y} + 160° = 180°$ $(Co - int.\ are\ Supp., AB \parallel CD)$ $\hat{y} = 180° - 160°$ $\hat{y} = 20°$ $\hat{p} = \hat{y} = 20° \ (Vert. Opp. \ < s =)$ **Legitimating criteria:** When marking different angles on the diagram, Mpilo used non-mathematical criteria which are mnemonics: the shape **"X"** to help learners remember how to identify vertically opposite angels. The shape **"F"** to help learners remember how to identify corresponding angles. The shape **"Z"** or **"N"** to help learners remember how to identify alternate angles. He also used mathematical criteria that are general when providing properties of these angles, and local mathematical criterial when showing to learners how to give a reason to a statement which is regarded as a convention.	**Example 1** Find the unknown angles. Mpilo drew a diagram on the whiteboard as shown below and asked learners to assist him answer the example 1 and example 2. **Example 2** Is $EF \parallel CG$? Explain your answer. Learner's activity Find with reasons all the known angles. After these examples Mpilo then drew another diagram on the whiteboard and asked the learners to discuss the solutions two by two	- Mpilo took more time to display answer for example 1 and example 2 and did not make some attempt to engage learners in genuine discussion. - The participation of learners was only limited to answer oral questions. - He should have given enough time to learners first to attempts questions before he can give them feedback. - Mpilo did not use teaching strategies to develop critical and creative thinking or other higher-order thinking skills. He solved himself a high order problem and gave learner a simple problem.

Table 10. *Summative judgments of Mpilo's MDI*

EXPLANATORY TALK		EXAMPLIFICATION		LEARNER PARTICIPATION
Naming	**Legitimating criteria**	**Examples**	**Tasks**	
Level 1: **(NM)**: Less everyday language **(Ms)**: Inequality and equality symbols to write reasons in short form. **Level 2:** **(Ma)**: Mathematical language to describe properties and to give solutions for the unknown angles.	**Level 0:** **(NM)**: Use of mnemonics (X-angles, F-angles and Z or N-angles). **Level 1:** **(L):** Writing Euclidean geometry reasons for different statements in short form (convention) **Level 2:** **(GP):** General Mathematical criteria to provide properties of angles.	**Level 1:** Use of examples that reflect **similarity** or **contrast** (One form of variation) **Level 3:** Use of examples that reflect **simultaneous variation** (fusion) of more than one aspect of the object of learning.	Limited to: - Listening - Responding mentally to oral questions - Copying work from the board.	- **Only verbal participation** - **No physical participation**

5.3. Data presentation and interpretation – The case of Makonga

To improve students' attitude towards Euclidean Geometry, you must first understand that the affected kids are content with their attitude, and they will not change just because you have disapproved of it. Imploring them to change their attitude might not work very well. Cursing or threatening them will not work either. You need to be patient with them and try to create a positive learning environment.

*(**Makonga, semi-structured interview**)*

The above statement from the semi-structured interview shows that Makonga agreed that a positive learning environment plays a critical role in improving learners' attitude towards Euclidean geometry. In this section, the analysis and interpretation of Makonga's two observed lessons, together with the information from the VSRI and semi- structured interview are presented. Two episodes were selected from two of Makonga's lessons. These episodes were selected based on the relevance of information to answer the predetermined research questions. Figure 13 below represents the selected episodes from Makonga's teaching and are interpreted and discussed.

EPISODE 1

"You need to be patient with them and try to create a positive learning environment".

EPISODE 2

conditions of congruency of triangles

Figure 13. *Makonga's selected episodes from two selected lessons.*

5.3.1. Episode 1 (lesson1): "You need to be patient with them and try to create a positive learning environment."

The first episode of this lesson began with the illustration of Makonga's classroom as shown in image 12 below.

Image 12: Makonga's classroom

During the lesson, I noticed that Makonga struggled to assess learners' learning. He could not to attend to each learner and review their work properly, partly due to the crowded class where rows lacked space and also because he remained fixed to the small whiteboard throughout the lesson. I also observed that it was a big challenge for learners seating at the back of the classroom to see what Makonga's was displaying or explaining on the small whiteboard which led to a lot of misunderstanding, misconceptions and visualisation problems. It was also difficult for me when recording the video during classroom observation to have a clear picture of what was displayed on the board. In addition, Makonga did not keep the learning environment free from congestion and he did not facilitate activities that are appropriate within the physical learning environment for all learners to work productively within the allotted time.

During VSRI, I had time to interact with Makonga as follows:

Researcher: During the semi-structured interview, I asked you the following question: "what factors influence the teaching of Euclidean geometry in Grade 10?" and your answer was:

> *To have a manageable class with an appropriate number of students since overcrowded classes make it impossible for teachers to attend to every student.*

Since you are currently having an overcrowded class with a small whiteboard and based on your answer to the question above, how do you help learners with misunderstanding and misconceptions and those who seat at the back?

Makonga: *I always tried to give them hand out materials to assist them visualise what they cannot see from the board. To clear misunderstanding and misconceptions, I always take time to re-explain and repeat myself.*

From the discussions above, it is clear to see that Makonga did not use hands-on learning activities (activities that require physical participation of learners to construct, consolidate, or explain concepts). As a result, learners struggled with grasping Euclidean geometry. In addition, it is important for the teacher before every lesson to take care of the physical set-up of the learning environment, which generally includes the arrangement of chairs, tables, and other equipment in the classroom, designed to maximize learning.

5.3.2. Episode 2: Conditions of congruency of triangles

In episode 2, Makonga introduced and described the conditions of congruency of triangles. I have seen him taking almost 5 minutes checking in his textbook the page number on which the conditions of congruency of triangles are displayed while his learners were waiting. This demonstrates that Makonga was not ready for this lesson and the only thing he could do was to be attached to his textbook and to be glued to the whiteboard (See image 13).

Image 13: Makonga standing with his textbook before his learners.

He started the lesson by the following statement:

> *When we talk about congruency of triangles, you need to know that there are four conditions that need to be satisfied for triangles to be congruent. We are going to start with the first condition.*

Makonga started the lesson without doing a recap of the background knowledge. He explained and described the conditions of congruency triangles as illustrated below.

First condition: He started by drawing two triangles on the whiteboard and then engaged learners with some oral question as follows:

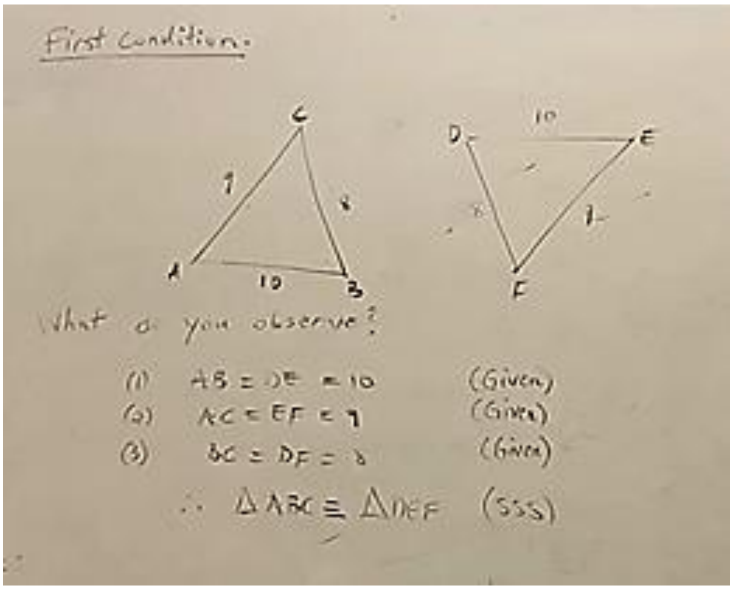


Image 14: Makonga's explanation about the first condition of congruency

After drawing these two triangles on the whiteboard, Makonga proceeded with the following discussion:

1 **Makonga:** *What do you observe in this diagram?*
2 **Learner 1:** *AB = DE = 10*
3 **Learner 2:** *AC = EF = 9*
4 **Learner 3:** *BC = DF = 8*
5 **Makonga:** *What conclusion can you give?*
6 **Learner 4:** $\Delta ABC \equiv \Delta DEF$
7 **Makonga:** *Why?*
8 **Learner 4:** *Because the corresponding sides of triangles are equal.*
9 **Makonga:** *How do you write this condition?*
10 **Learner 5:** *SSS*

The use of individual responses was interesting to watch because it illustrates that the questions that the teacher asked in those instances signalled a level of understanding, as learners had to provide answers relating to what they have observed in the diagram. This exchange further illustrates the observable action of drawing two triangles with their measurements has helped learners to see that the two triangles were congruent. However, I have observed in lines 2, 3 and 4 that Makonga did not ask learners to provide reasons to their statement even though the measurements were given on the diagram. He should have encouraged learners to provide the reason “GIVEN” so that they get used to the way of answering questions in Euclidean geometry.

Second condition: He used the same approach of starting by drawing two triangles on the whiteboard and then engaged learners with some oral question as follows:

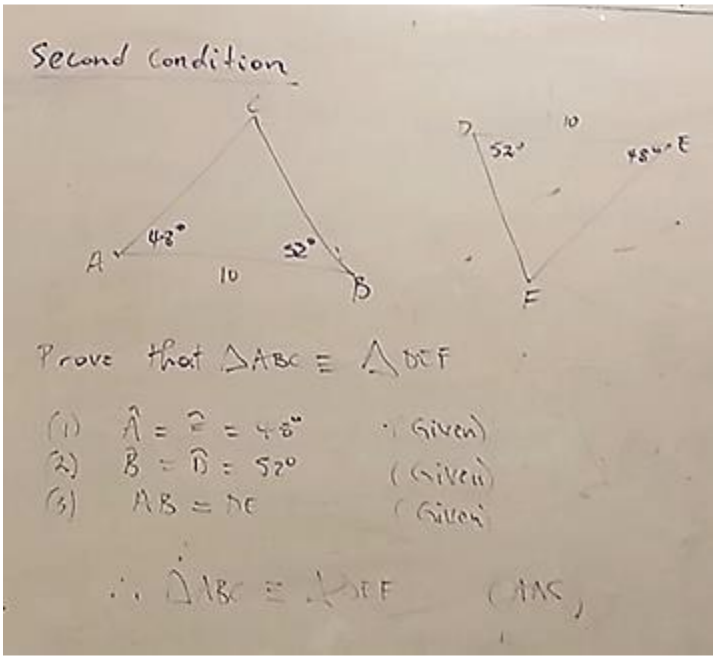


Image 15: Makonga’s explanation about the second condition of congruency

After drawing these two triangles on the whiteboard, Makonga proceeded with the same routine of engaging learners in a discussion as illustrated below.

11 **Makonga:** *What do you see?*
12 **Learner 1:** *The two triangles are congruent.*
13 **Makonga:** *How is that possible?*
14 **Learner 1:** *Because the look the same in terms of shape.*
15 **Makonga:** *No. Who can give us reasons that are more concrete?*
16 **Learner 2:** $\hat{A} = \hat{D} = 48°$
17 **Learner 3:** $\hat{B} = \hat{E} = 52°$
18 **Learner 4:** $AB = DE$
19 **Makonga:** *Correct. Now we can be able to say that* $\Delta ABC \equiv \Delta DEF$*.*
What reason can we give to support this statement?
20 **Learner 5:** *AAS*

21 **Learner 6:** *What about the following condition: AAA?*

22 **Makonga:** *It is not a condition for congruency, but this is a condition for similarity*

The exchange above demonstrated how Makonga provided some learning opportunities, which are somewhat aligned with the learning goals, and engaged majority of the learners to participate, to cooperate, and to collaborate in continued learning. However, I have observed that Makonga did not take time to respond immediately to learners’ responses. For example, in line 12, Learner 1 provided a correct answer by saying “*the two triangles are congruent”* but Makonga could not agree with the learner since the answer provided was not the one that he was expecting; in line 14, Learner 1 gave the reason why the two triangles were congruent by saying *“because they look the same in shape”.* That is, in lines 12, Learner 1 gave a correct answer that required mathematical justification.

To continue with learner interaction, Makonga responded positively by tailoring subsequent question, requiring the learner to explain their answer, but then dismissed their response in line 14 and asked for a more concrete explanation. Instead of dismissing the learner's response, the teacher could have scaffolded their learning by asking guiding questions as the More Knowledgeable Other (MKO) (Vygotsky, 1987), such as "What makes you think that they look the same?" This could have sustained the learner’s thinking about mathematical concepts in focus. This being the case, other learners in lines 16-18 provided the expected answers. This event demonstrates the need for teachers to maintain substantive engagements with the learners even in cases where learners give incorrect or answers that teachers did not expect. This is one-way teachers can extend and/or disrupt learners’ current learning and mediate their thinking for future learning (Mbhiza, 2021).

In my observation, the answer provided by the learner was not wrong in the fact that the learner used everyday language (*colloquial naming*) instead of mathematical language to describe his observations about the relationship between the two triangles. According to Adler and Ronda (2015), teachers are tasked with ensuring that learners learn and own skills to navigate between everyday and formal mathematical ways of thinking and speaking about mathematical objects. It is the role of the teacher to teach learners mathematical ways of answering questions, ensuring that pedagogical links are made to support connections between everyday ways of explanations and mathematical ways of explaining relationships in mathematics learning (Scott et al., 2011). Furthermore, Makonga did not comment on the responses that learners provided in lines 16, 17 and 18 even though the answers were mathematically correct, but they did not give any reasons to support their answers.

Furthermore, to support knowledge building, Makonga could have provide explanatory talk relating to the second condition of AAS or SAA in words and offered a counterexample for learners to make observations and move towards generality (Adler & Venkat, 2014). According to this condition, two triangles are congruent if two angles and one side of a triangle are equal to two angles and a ***corresponding side*** of the other triangle. Providing a counter-example can help in mitigating a common misconception where learners mistakenly consider two triangles congruent because of the two equal angles and one side. The naming in line 22 was also interesting, that Makonga juxtaposed between conditions for congruency and conditions for similarity but did not demonstrate the differences further for the learners through explanatory talk or using visual aids to show their differences. This could have enabled learners' in-depth understanding of the differences between the two and build relational links between the two concepts (Skemp, 1976).

Third condition: This condition is introduced similarly to the previous two conditions. In line 33, Learner 6 asked if the order of sides matters. The teacher emphasized that the order of sides does not matter. He drew once again two triangles on the whiteboard as depicted in image 16 and then engaged learners with some oral question as presented in lines 23-34:

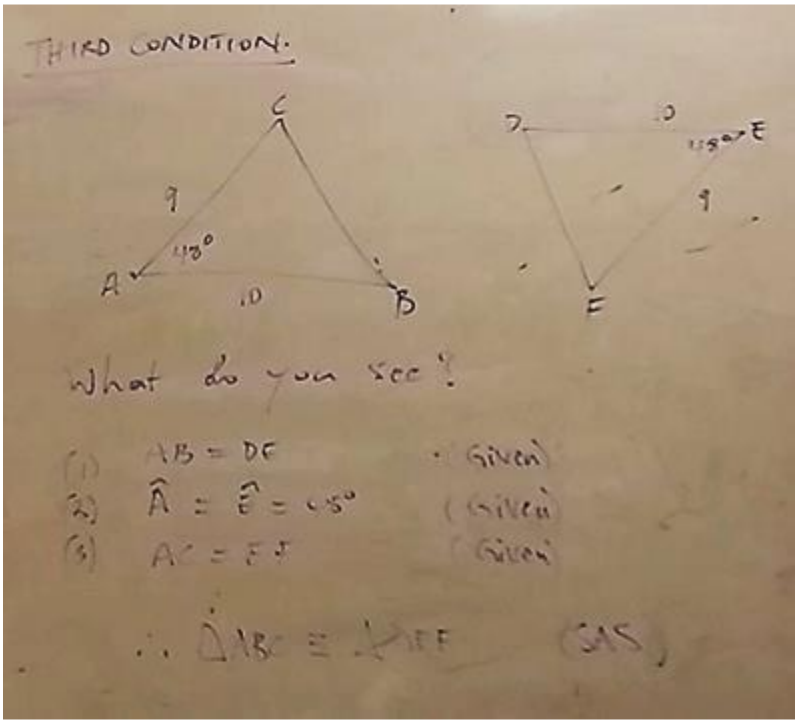


Image 16: Makonga's explanation about the third condition of congruency

23 **Makonga:** *What is your observation on this diagram?*
24 **Learner 1:** *AB = DE*
25 **Makonga:** *Correct. Anyone else?*
26 **Learner 2:** $\hat{A} = \hat{E} = 48°$
27 **Makonga:** *Good job. One more thing.*
28 **Learner 3:** *AC = EF*
29 **Makonga:** *Good. That is enough to conclude that* $\Delta ABC \equiv \Delta DEF$*. What reason can we give to support this statement?*
32 **Learner 5:** *SAS*
33 **Learner 6:** *Sorry Sir, does the order of sides counts in this condition?*
34 **Makonga:** *Yes, it does. You need to have two sides and one angle between those two sides. Nothing else is acceptable.*

In the conversation above, Makonga demonstrated accurate knowledge of key concepts both in terms of the knowledge of the content and knowledge of his learners and he responded to learners' questions or comments, creating a learning environment that allows dialogic interactions between the teacher and learners (Adler & Venkat, 2014). The only negative aspect I have observed is that Makonga did not once again encourage learners to provide a reason to the statements in lines 24, 26 and 28. In addition, he did not adapt and modify learning opportunities to create a supportive learning environment for learners to recognize each other's learning strengths, and value the contribution of others due to the structure of the classroom and the large number of learners in his class.

Forth condition: Makonga drew once again two triangles on the whiteboard. This time the two triangles where completely different from the previous cases and then engaged learners with some question-and-answer process (lines 35-52). It could be said that Makonga's discourse of teaching Euclidean geometry is the naming of concepts through question-and-answer strategy, which is common among all the participating teachers in this study. What sets Makonga's teaching from the other teachers' is that he made some movements towards engaging with leamers' questions during teaching and offered some explanations relating to the concepts he introduced.

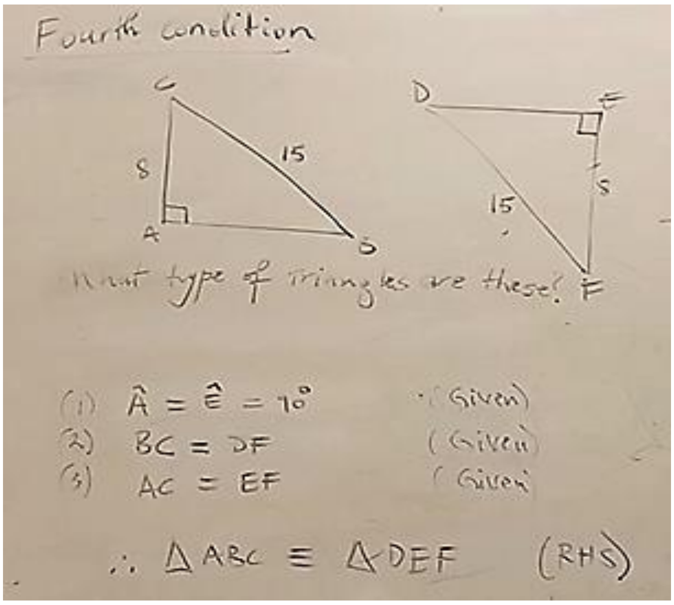


Image 17: Makonga's explanation about the fourth condition of congruency

35 **Makonga:** *Before describing the last condition. What types of triangles are displayed in the diagram?*
36 **Learner 1:** *Right-angled triangles*
37 **Makonga:** *Correct. What are the characteristics of such triangles?*
38 **Learner 2:** *They contain a right angle.*
39 **Makonga:** *Good job. What else?*
40 **Learner 3:** *It has two adjacent sides to the right angle and one longue side called hypotenuse.*
41 **Makonga:** *Do you still remember the last condition?*

42 **Learner 5:** *RHS*

43 **Makonga:** *Correct. Let us now prove that these triangles are congruence. What do you observe from the diagram?*

44 **Makonga:** *Yes, it does. You need to have two sides and one angles between those two sides. Nothing else is acceptable.*

45 **Learner 6:** $A = \hat{E}$

46 **Makonga:** *What is the reason?*

47 **Learner 6:** *They are both equal to* $90°$

48 **Makonga:** *What else do we have?*

49 **Learner 3:** *BC=DF*

50 **Learner 4:** $AC = EF$

51 **Makonga:** *Correct. Now we can be able to say that* $\Delta ABC \equiv \Delta DEF$*, with RHS as the reason.*

The last condition of congruency was well explained by Makonga with more interaction between him and the learners. I have observed that Makonga used only oral questions to engage learners during the lesson and did not give neither opportunities to learners to go to the board and demonstrate their ability and skills nor a chance for them to exchange ideas between themselves. It is important to note that even in this event, Makonga did not request learners to provide reasons for their mathematical statements, “so the criteria for how and why this was legitimate” was not created (Adler & Ronda, 2016, p. 14).

During VSRI, I managed to ask Makonga to clarify on some areas of the lessons. The first aspect was based on two questions that I asked him during semi-structured interview which link to some observable actions. I wanted to understand why he did not do exactly as he said when responding to these questions during the lesson. Below is the excerpt of the VSRI conversation:

Researcher: *During the semi-structured interview, I asked the following question: “What are some important aspects that you need to emphasize when teaching Euclidean Geometry in Grade 10?” and your response to this question was as follows:*

> *Some of the things that should be emphasized are vocabulary, shape properties, and how to apply these features when solving Euclidean Geometry problems.*

But, when I observed you throughout these lessons, you did not emphasize on the properties when learners were providing answers without reasons. What can you say about that?

Makonga: *I do not have a proper reason to that, but I thought since they are reading the answers from the diagram, it was not necessary for them to provide the reasons.*

Researcher: *Euclidean geometry as you have defined it early as "the study of shapes and solid objects based on different axioms and theorems"; it is important for learners to know that whenever answering questions in Euclidean geometry, they need at all time to provide a reason even if the statement or measurement is provided in the diagram the must at least use "GIVEN" as a reason to show that they did not solve for anything. By doing this, it will assist them as the progress with this topic to understand the structure on how to present arguments or answers in Euclidean geometry. The second question that I asked you during the semi-structured interview was: "What are some of the challenges, if any do you experience when teaching Euclidean Geometry in Grade 10?" and your answer was: I will say the theorems and properties because Euclidean geometry is the study of geometrical shapes (plane and solid) and figures based on different axioms and theorems.*

Why during your lesson, you do not have time to emphasize on properties and theorems knowing that your learners have challenges when it comes to their application?

Makonga: *It is just that I forgot to do that in this lesson. I normally do that and also looking at the number of the learners that I have in my class, I always have the tendency of moving fast which makes me sometimes forget to do certain things.*

Based on the discussion illustrated above, the learning environment and the number of learners in each Grade 10 class remain critical for effective teaching of Euclidean geometry. The second aspect that I have observed is that there was no individual task during the lessons, Makonga only asked collective questions for the class to respond as a group. He provided limited opportunities to engage learners individually in structured tasks to assess their learning about the different conditions he presented.

Table 11 depicts the summary of Makonga's MDI and Table 12 are the summative judgements relating to the teacher's explanatory talk, exemplification, and learner participation.

Table 11. *Summary of Makonga's teaching episodes*

Object of learning: Conditions of congruency of triangles

Teacher's explanatory talk		**Exemplification**		**Learner participation**
Naming	***Legitimating criteria***	***Examples***	***Tasks***	
Episode 1 (lesson 1): "You need to be patient with them and try to create a positive learning environment." This was only based on the learning environment which is considered here as one of the factors that affects the effectiveness of the teaching and learning of Euclidean geometry.				
Episode 2 (lesson 1 and 2): Under-the teaching of conditions of congruency of triangles - Mpilo used more everyday languages to explain and describe the conditions of congruency of triangles. He used less mathematical language appropriately when proving that two triangles are congruent. For example, in line 51 above: *Makonga said we can now say that* $\Delta ABC \equiv \Delta DEF$. - Makonga used equality symbol "=", degree symbol "°"and the angle symbol "ˆ" when writing down the mathematical arguments.	**Legitimating criteria:** Makonga did use non-mathematical criteria. He used visual when drawing triangles on the whiteboard and mathematical criteria that are general when providing specific arguments and properties of different conditions of congruency of triangles, and local mathematical criterial were used across this episode which means conventions were provided when writing reasons of the four different conditions of congruency of triangles in short forms. For example: "SSS" which means Side, Side, Side. "SAS" which means Side, Angle, Side. "AAS" which means Angle, Angle, Side. "RHS" which means Right angle, Hypotenuse, Side	Makonga gave learners four examples which include three similar examples and one contrast example on the conditions of congruency of triangles *Example 1* based on the first condition of congruency (SSS) as illustrated above *Example 2* based on the second condition of congruency (AAS) as illustrated above *Example 3* based on the third condition of congruency (SAS) as illustrated above *Example 4* based on the forth condition of congruency (RHS) as illustrated above	The task of learners was to observe two triangles drawn by Makonga on the white board, to listen him and to offer tentative responses by maintaining silence as they helped the teacher explaining and describing the four conditions of congruency of triangles, by responding mentally to theoretical questions posed by the teacher, and by copying the work of the teacher. - No structured task was given to the learners.	- Makonga did not apply any strategy to motivate learners to work productively and be responsible for their own learning. - He used poorly thought-out strategies that leave learners uninvolved and/or passive. - He did not give an opportunity to learners to go to the board and display their ability and skills. - He did not test learners on what he taught them in order to evaluate the application of the four conditions of congruency of triangles and also to verify how learners write mathematical arguments. - The interactions between Makonga and his learners, and among learners, were limited only to oral questions and answers.

Table 12. *Summative judgments of Makonga's MDI*

EXPLANATORY TALK		EXAMPLIFICATION		LEARNER PARTICIPATION
Naming	**Legitimating criteria**	**Examples**	**Tasks**	
Level 1: **(NM)**: More everyday language **(Ms)**: Equality, angle and degree symbols to write different reasons **Level 2:** **(Ma)**: Less Mathematical language to explain different conditions of congruency of triangles **Level 3:** More movement between **(NM)** and **(Ma)**	**Level 0:** **(NM):** Mnemonics: SSS, SAS, AAS and RHS to assist learners to remember to four conditions of congruency. **(V):** Visual: Drawing triangles on the board. **Level 1:** **(L):** Writing Euclidean geometry reasons of four conditions of congruency in short form (convention) **Level 2:** **(GP):** Definitions **Level 3:** Deriving the proof of two triangles being congruent.	**Level 1:** Use of examples that reflect **similarity** or **contrast** (One form of variation)	Limited to observing two triangles and answering oral questions.	- **Only verbal participation** - **No physical participation**

5.4. Data presentation and analysis – The case of Moloto

*Since the majority of Euclidean Geometry concepts in Grade 10 appear to be fairly new to students, I genuinely believe that the curriculum is too broad and extensive for students entering Grade 10. Since some of the topics they have included in Grade 10 are too complex, like the concept of proving theorems, I think the curriculum should be spread out a little more. It is a concept covered in Grade 9's congruency and similarity lessons, however students are unable to establish that a specific quadrilateral is a parallelogram. In general, the Grade 10 ATP's concept definitions are inadequate (***Moloto, semi-structured interviews***)*

Moloto's statement above demonstrates a concern about the curriculum specification and delimitation of Euclidean geometry in Grade 10. Moloto gave his point of view based on some challenges that he had experienced during his teaching career, highlighting that some topics should be covered in latter grades than in Grade 10. In this section, I presented the analysis and interpretation of three episodes selected from Moloto's two of observed lessons. Figure 14 below represents the selected episodes from Mpilo's teaching and are interpreted and discussed.

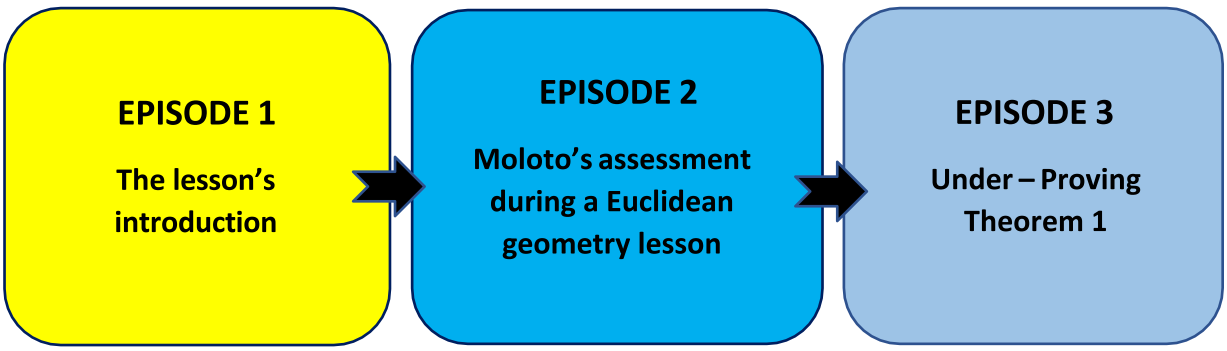


Figure 14. *Moloto's selected episodes from two observed lessons*

5.4.1. *Episode 1: The lesson's introduction*

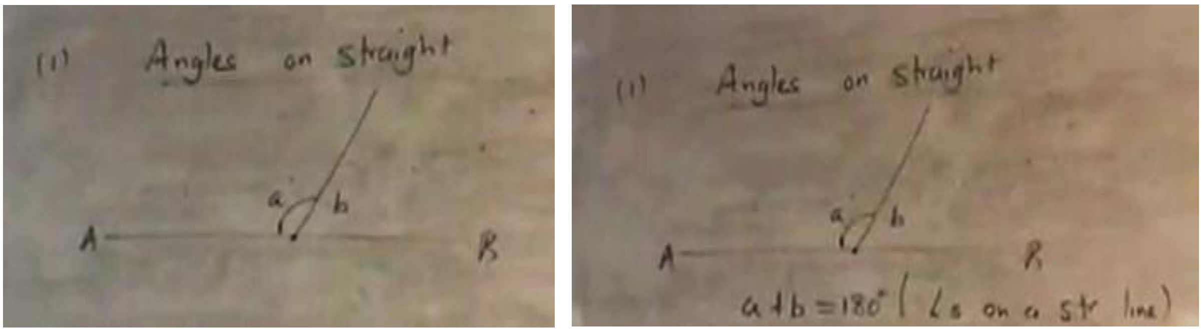
Moloto started the lesson by writing the object of learning on the whiteboard (See image 18). Then said to the learners the following statement: "today, we are going to look at the basic concepts of Euclidean geometry, thereby announcing the object of learning for the lesson.

Image 18: Moloto's Objection of Learning

Moloto continued the lesson with a recap of the rules for the following conditions: Angles on a straight line, angles around a point, vertically opposite angles and angles on parallel lines in order to assist learners remember something before giving them an activity. Moloto drew a diagram on the whiteboard as shown in image 19 (a), then engaged learners as follows:

84. Moloto: *What can you say about angles **a** and **b**?*
85. Learner 1: $a + b = 180°$
86. Moloto: *What reason can you give to this statement?*
87. Learner 2: *Angles on a straight line*
88. Moloto: *The reason is correct but you need to know that whenever you are Writing comments or reasons in Euclidean geometry, make sure to write them in short form. For example, this comment "Angles on a straight line" can be written in short as "$<_s$ $on\ a\ str.line$". Another thing you need to understand is that angles on a straight line add up to $180°$ which means they are supplementary.*

This extract, especially line 5 demonstrates that in his explanatory talk, Moloto used positional legitimation about angles on a straight line and the naming of angles on straight line and naming for supplementary angles was done using formal mathematical words and meanings. Thus, the legitimation criteria for Moloto moved towards generality for the concepts of angles on straight lines and supplementary angles. As evidenced by the extract above, Moloto gave learners an opportunity to express themselves and after that he took time to emphasize on how learners need to write reasons or comments (See image 10 (b)) to support their answers which is a positive aspect especially when you are teaching Euclidean geometry. That is, he used substantiations to guide learners on the nature of angles on straight lines and supplementary angles (Adler & Ronda, 2015). The representations in images 10 (a) and (b) acted a visual mediation for learners to see what the teacher means when he says angles on straight lines add up to 180 degrees.

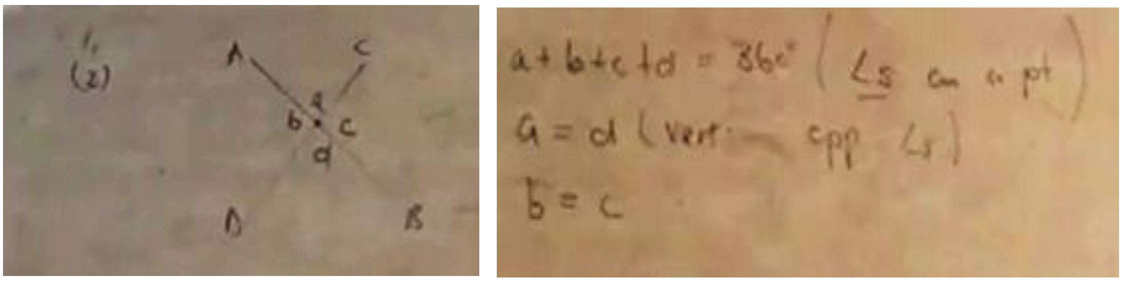


(a) **(b)**

Image 19: Recap of angles on straight line

Moloto continued with the recap by drawing another diagram on the whiteboard and then he engaged his learners once again as follows:

89. Moloto: *What can you say when you look at the diagram? (See image 11)*



(a) **(b)**

Image 20: Recap on angles around a point and vertically opposite angles

In the second point, I have observed that Moloto managed to assist leaners to remember the properties of angles around a point and vertically opposite angles. Learners were able to interact with him on a good note.

11. **Learner 1:** $a + b + c + d = 360°$
12. **Moloto:** *Why is this true?*
13. **Learner 2:** *Because they are angles around a point.*
14. **Moloto:** *What else can you say?*
15. **Learner 3:** Can I also say it is because these angles form a revolution?
16. **Moloto:** Correct. What else can you say about these angles?
17. **Learner 4:** $a = d \ and \ b = c$
18. **Moloto:** *Correct. How do you call these angles?*
19. **Learner 5:** *Vertically opposite angles*
20. **Moloto:** *That is correct. Don't forget always to write your reason in short form.*

Lastly, Moloto drew the diagram on the whiteboard as shown in image 21 below. Then engaged learners with oral question as follows:

Image 21: Parallel lines and transversal line

21. **Moloto:** *What type of lines are lines AB and CD?*
22. **Learners:** *No response*
23. **Moloto:** *These lines are called parallel lines. So, what is the name of the line that cuts lines AB and CD?*
24. **Learners:** *No response*
25. **Moloto:** *This is a transversal line. What can you say about angles* **a** *and* **b** *as shown in the diagram?* (See image 13).

Moloto asked the learners what type of lines AB and CD are (line 17). When no one answered[4] (line 18), he explained that these lines are parallel to each other (line 20),

[4] To avoid such confusion, teachers need to explain to learners how to identify parallel lines using either specification or arrows.

thereby naming the mathematical object he had presented as well as legitimising the concept of parallel lines for the learners (Adler & Ronda, 2015). It's possible that the learners didn't know how to identify parallel lines, and although Moloto did not refer to identifiers to why AB and CD are parallel, the statement itself draws learners' attention to the concept. Moloto then asked about the transversal line, which may have left the learners confused about why AB and CD are parallel.

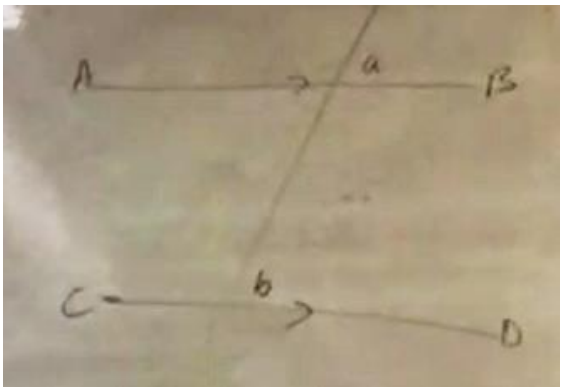


Image 22: Corresponding angles

In the discussion below, Learner 3 expresses this confusion caused by the teacher's failure to elaborate on the concept of parallel lines:

26. **Learner 1:** *The angle **a** is equal to angle **b.***
27. **Moloto:** *Correct. What is the reason?*
28. **Learner 2:** *Because corresponding angles are equal.*

Although some learners seem to be able to identify parallel lines, others were still struggling. Despite learner 3's incorrect answer, Moloto did not provide further explanation on how to identify parallel lines.

11. **Moloto:** *Correct. This is the way you can write it down:* $a = b\ (corresp. <_s, AB||CD)$ *Is a = b in the other diagram? Motivate your answer.* (See image 23).

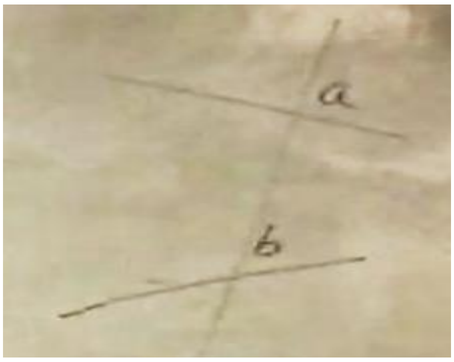


Image 23: Counter example of corresponding angles

12. **Learner 3:** *Yes, because they look exactly the same as in the previous case.*
13. **Moloto:** *This is not true. Who can give the correct answer?*
14. **Learner 4:** ***a** is not equal to **b** because there are no parallel lines like in the previous diagram.*
15. **Moloto:** *That is great. You need to understand that for you talk about corresponding angles, alternate angles and co-interior angles, there must be parallel lines but when talking about vertically opposite angles you do not necessarily need to have parallel lines* (See image 24).

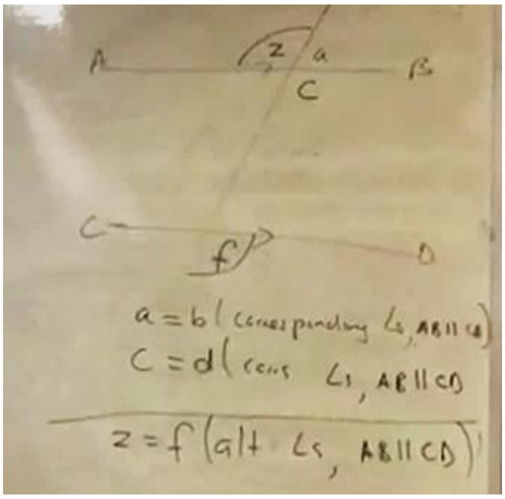


Image 25: Alternate and corresponding angles

From my observation, Moloto's introduction was effective, he started his lesson with a recap. During the recap, there was teacher to learners' interaction, he asked oral questions and developed his explanations based on the responses the learners provided. There was evidence that learners were on the same page with the teacher.

5.4.2. Episode 2: Moloto's assessment during a Euclidean geometry lesson

This episode displayed Moloto's assessment during a Euclidean geometry lesson and explored learners' answers and Moloto's feedbacks during and after assessment. After the introduction of the lesson, Moloto gave an activity to learners by writing the question and drawing the diagram on the whiteboard (See image 26). Then gave learners time to go through the question individually while he was going around checking the answers of learners. Some of the answers of learners are illustrated, interpreted, and analysed below.

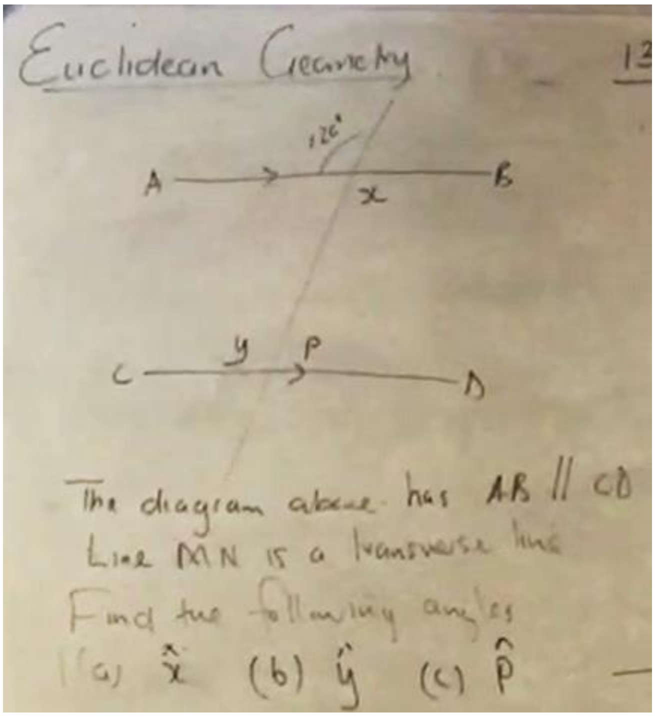


Image 26: Learners' activity

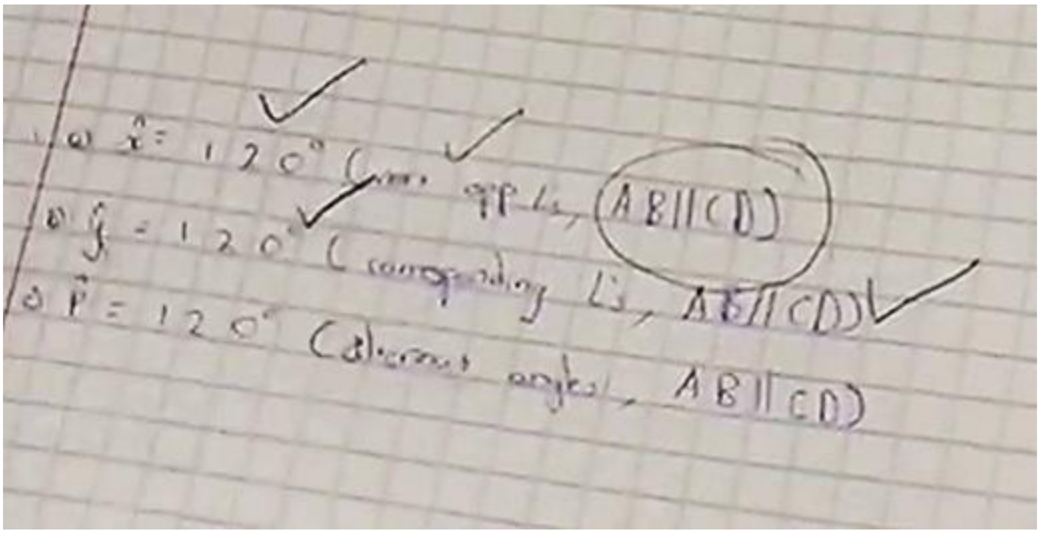


Image 27: Answers of Learner 1

From image 27, I have observed that Learner 1 was quick in getting the answers. For question (a), the answer was $\hat{x} = 120°$ which was correct. The only problem on this question was that the teacher when marking he did emphasize to the learner that when using the property "vertically opposite angles are equal", it is not necessary to indicate that the lines are parallel since this property does not need parallel lines in order to be satisfied. For question (b), Learner 1 managed to get both the size of angle ***y*** and the reason correct as shown in image 27. Lastly, Learner 1 did not manage to answer question (c) correctly. He said that $\hat{p} = 120°\ (alterna\ angles, AB||CD)$. This is incorrect since $\hat{y}$= $120°$ and both angles ***y*** and ***p*** are on a straight line and supposed to add up to $180°$ and not $240°$. Despite all this, I have observed that the teacher did not give this learner feedback, he rather marked the correct answers and left the learner in confusion. Moschovich (2015) averse that it is the responsibility of the mathematics teacher to engage with the errors that learners produce, to ensure that learners learn mathematical knowledge and skills beyond their current means.

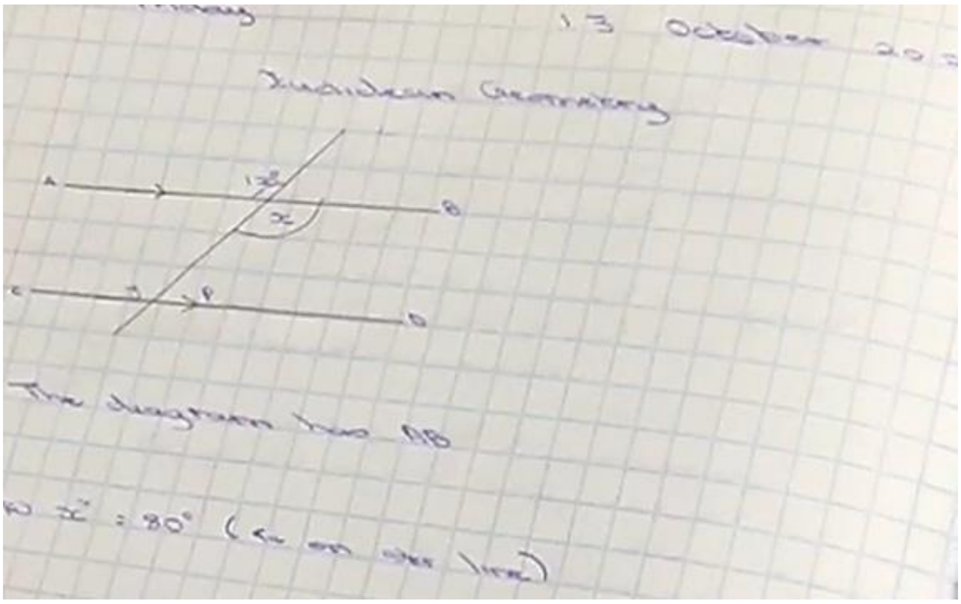


Image 28: Answers of Learner 2

Image 28 displayed the answer of Learner 2 for question (a). The learner managed to get $\hat{x} = 80°$ $(<_s\ on\ a\ str.line)$ which is wrong. I observed the reaction of Moloto when he saw this answer, the conversation is given below.

16. **Moloto:** *You can notice that angle **x** is vertically opposite to* $120°$ **.** *How did you get* $80°$ *? I don't understand, please explain to me.*
17. **Learner 2:** *I subtracted* $120°$ *from* $180°$
18. **Moloto:** *even though you did that the answer supposed to be* $60°$*. So, how did you get* $80°$ *?*
19. **Learner 2:** *I don't remember.*
20. **Moloto:** *What did we say about vertically opposite angles?*
21. **Learner 2:** They are equal.
22. **Moloto:** *So, what is the value of angle **x**?*
23. **Learner 2:** $120°$
24. **Moloto:** *Correct. That is how you suppose to answer question (a).*

From the above discussion, one can see that the learner was lost and was having misconceptions of the properties. In the exchange above, it is evident that Learner 2 was unable to obtain the correct value of x since they had forgotten the relationship between x and the given value (refer to Image 28: Answer of Learner 2). Some learners seem to lack visualization skills, as demonstrated by Learner 2 in this instance. Even though x equals 120 degrees due to the principle of vertically opposite angles being equal, Learner 2 was unable to recognize this. However, when the teacher asked about the relationship between vertically opposite angles, Learner 2 responded correctly, indicating that they could understand and remember the concept, but may lack visualization skills that the teacher through prompts and calling on the learner to focus on the visual cues was able to redirect. According to Barkay (2017), "The intention of scaffolding is to assist learners in thinking about the question by asking thought-provoking questions that maintain the complexity of the question" (p. 79).

During VRSI, I asked Moloto a couple of questions regarding this as represented by the exchange below:

Researcher: *What do you think could be the cause of misconception and confusion that Learner 2 has displayed considering the fact that your introduction was effective with a good recap?*

Moloto: *The challenge is that students are failing to relate arguments to various properties until a conclusion is reached. There is also the issue of students lacking logic in their reasoning when addressing Euclidean Geometry problems. Most students struggle with the mathematical method of developing arguments in Euclidean Geometry.*

Researcher: what solution would you give to this challenge?

Moloto: *The solution is for Grade 8-9 teachers to help learners with mathematical method of developing arguments when teaching geometry of straight lines and geometry of triangles. So, that when learners will be in Grade 10 will know how to reason mathematically and develop mathematical arguments.*

It is interesting to note that when asked about possible remedies to the knowledge and skills gaps, Moloto removes himself from the equation and shifts the teaching responsibility to teachers in previous grades. While contents covered in previous grades act as foundational information for learners to work with more complex concepts, it is equally important for current teachers to configure strategies to make up for the gaps. Image 29 illustrated the misconceptions of Learner 3 when solving the activity provided by Moloto. In the first line of his answer, the learner wrote that $x = p\ (corresponding\ \ <_s, AB \| CD)$ which is not correct.

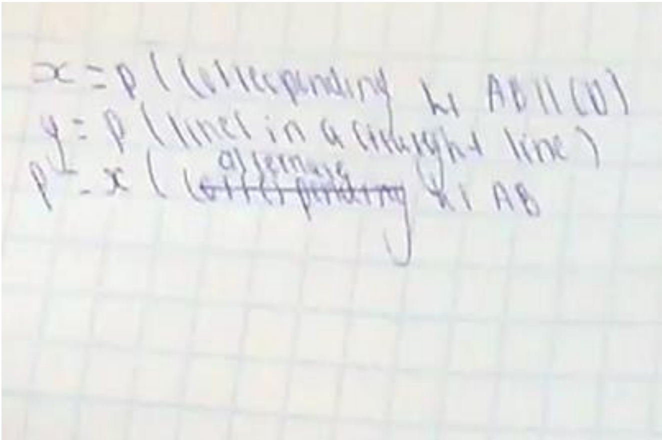


Image 29: Answers of Learner 3

The learner thought that the two angles look like and that they have the same size. This is the first misconception because angles x and p are not equal but supplementary (co-interior angles). In the second line of answers in image 19, Learner 3 wrote that $y = p\ (lines\ in\ a\ straight\ line)$ which the second misconception because the two angles are not equal but the add up to $180°$ and the reason “lines in a straight line” does not make any sense. The learner wanted to say “$<_s$ on a straight line”. Lastly, Learner 3 wrote in the third line that $p = x\ (alternate\ \ <_s, AB \| CD)$ which the same answer that the learner provided in the first line, but he only changed the reason. The misconception of the learner in this case is that angles *p* and *x* are alternate in the fact that they are in different locations, and they are facing different direction.

I have observed that Moloto when moving around checking answers of learners, he did give an opportunity to Learner 3 to explain how he got these answers and the reasons that he provided. I also observed that these misconceptions were caused by the conceptual understanding, and they are related to Moloto's explanatory talk relating to the different properties of angles on parallel lines. Another reason of these misconception is that Learner 3 found it difficult to differentiate between geometry and algebra because geometry requires spatial and logical skills while algebra needs analytical skills. To help Learner 3, Moloto could have used relevant vocabulary to describe relevant geometric properties and relationships between angles. To help learners visualise and make sense of the relationships between different angles on parallel lines and their properties, Moloto could have for instance used some mnemonics such "Z-angles" or "F-angles" to assist learners create mental images of these properties. After giving learners the time to attempt the task, Moloto stopped all learners from writing and started giving the corrections of the activity. I was expecting him to give an opportunity to learners to share their solutions on the whiteboard with their peers, but he did not. Image 30 displays the solutions of the activity given by Moloto.

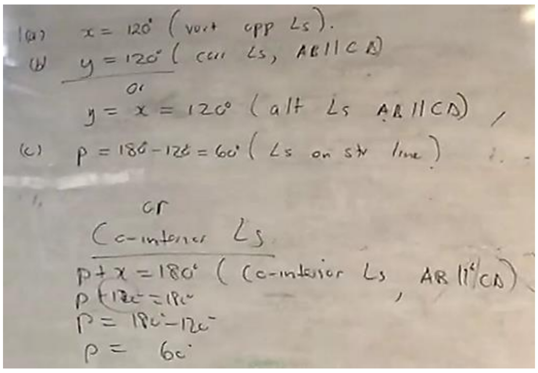


Image 30: Moloto's feedback

The positive aspect of Moloto's feedback is that he managed to give alternative ways of answering question and did allow learners to interact with him when giving the solutions. He also managed to emphasize on some misconceptions that learners were making when answering the questions.

5.4.3. Episode 3: Under-proving theorem 1

This episode was extracted from the second lesson of Moloto which was about proving theorem 1 that states that "the opposite sides and opposite angles of a parallelogram are equal". Moloto started the lesson by write the object of learning on the whiteboard and thereafter he wrote the statement of the theorem on the board as shown in image 31 below.

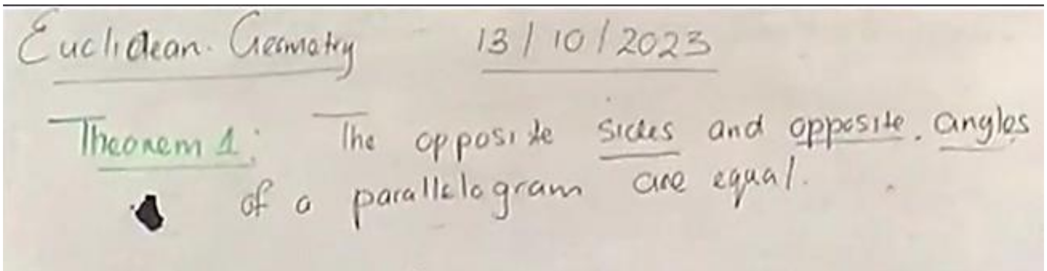


Image 31: Moloto's object of learning

Before giving the proof of the theorem, Moloto took time to give a recap to learners on the conditions of congruency of triangles by engaging learners in a short discussion as illustrated in the extract below.

25. **Moloto:** *How many conditions of congruency of triangles do we have?*
26. **Learner 1:** *4 conditions*
27. **Moloto:** *Correct. Let us list them.*
28. **Learner 2:** *SSS*
29. **Learner 3:** *SAS*
30. **Learner 4:** *AAS*
31. **Leaner 5:** *RHS*
32. **Moloto:** *That is very good. We are going to use congruency when proving theorem 1*

From the discussion above, Moloto managed to have a recap on the conditions of congruency of triangles that was the key concept required for the proof of theorem 1 and learners responded well to the recap. During the discussion, Moloto provided a recap on the conditions of congruency of triangles, which was the key concept required for the proof of theorem 1. The learners responded well to the recap, indicating that they had a good understanding of the necessary conditions for proving congruency. However, the recap was presented in an abstract manner[5], without any visual illustrations of the relationships. This relates to gaps in the teacher's discourse about the use of explanatory talk and substantive conversations for learners to unpack and understand what the four conditions entail and their associated processes in proving the theorem (Adler & Venkat, 2014).

Moloto continued the lesson by drawing a diagram of a parallelogram on the whiteboard and helped learners through explanatory talk relating to the outline of the proof as shown in image 32, which acted as both exemplification and visual legitimation of the concept for the learners.

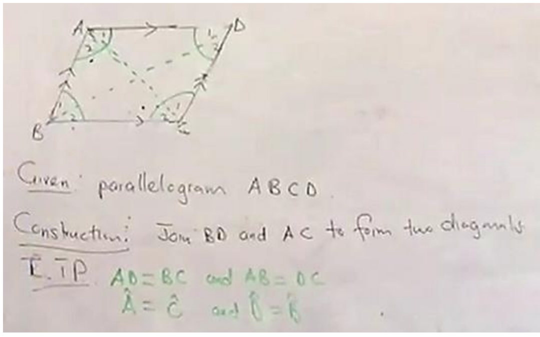


Image 32: The outline of the proof

After the outline of the proof, Moloto engaged learners into a discussion in order to prove theorem 1.

51. **Moloto:** *In order to prove this theorem, we need to prove congruency of two triangles. Would you please identify the two triangles that we need to prove congruency?*

[5] Such practices often pose challenges for learners when it comes to applying the concepts during a proof-type question.

52. **Learners:** *No answer*

53. **Moloto:** *The triangles are ΔABC and ΔCDA. What do you observe in terms of angles and sides in these two triangles?*

54. **Learner 1:** $\hat{D}_1 = \hat{B}_2$ $(alternate\ <_s, AD \| BC)$

55. **Moloto:** *It is correct, but these angles are not full angles of ΔABC and ΔCDA. Try again.*

56. **Learner 2:** $\hat{A}_1 = \hat{C}_2$ $(alternate\ <_s, AD \| BC)$.

57. **Moloto:** *Correct. We need another angle or a side.*

58. **Learner 3:** $\hat{A}_2 = \hat{C}_1$ $(alternate\ <_s, AB \| DC)$.

59. **Moloto:** *Correct answer. What else can you see?*

60. **Learners:** *No answer*

61. **Moloto:** *If you look at the diagram, you can see that AC is common side to both triangles. From what we have now, can we conclusively say that $\Delta ABC \equiv \Delta CDA$?*

62. **Learner 4:** Yes

63. **Moloto:** *What is the reason to that?*

64. **Learners:** *No answer*

65. **Moloto**: *We need now to use the conditions of congruency that we revised in our introduction. For this case we will use "AAS". We have proven that $\Delta ABC \equiv \Delta CDA$. Does it prove that the opposite angles and the opposite sides of a parallelogram are equal?*

66. **Learners**: *No answer*

67. **Moloto**: *Once you have proven that two triangles are congruent, it means that they are equal in sides and angles. So, we can say that AB=CD, BC=DA and $\hat{B}=\hat{D}$as shown in image 23. This will not be enough to prove the theorem, you will have to prove that $\Delta ADB \equiv \Delta BCD$. This will be your homework.*

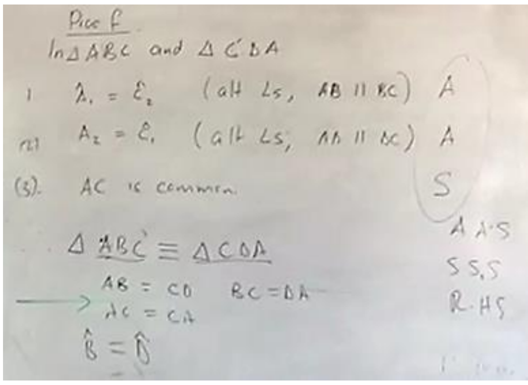


Image 33: Proof of the theorem

From my observation, learners struggled a lot during this lesson as illustrated in the discussion above. After Moloto did the construction and the outline of the proof, learners were completely lost as is the case in lines 51 and 52. Learners did not know where to start and which triangles to select to prove congruency as

Moloto as stated in line 51. Though Moloto revised the condition of congruency of triangles at the beginning of the lesson, learners forgot everything as they were going through the proof together with the teacher. The concept of proving theorem 1 proved to be difficult for them in the sense that most learners were never actually taught how to go about solving problems that require rigorous mathematical thinking. It was difficult for the teacher to completely interact with learners during this lesson because the concept was too broad and extensive. Moloto vocabulary was limited to help learners understand something during the lesson. Some learners were completely demotivated because they could not understand how to prove theorem 1.

Based on the above discussion, Moloto led the students through the proof of theorem 1, step by step. However, despite his efforts, the students found it challenging to follow him and were unable to respond to most of the questions he asked. The subject of geometry can be particularly demanding, involving abstract concepts that require logical and abstract thinking. Proving congruency, in particular, can be difficult, as it involves understanding and applying theorems and definitions, which can be more challenging than solving numerical problems. This requires a thorough understanding of the necessary conditions to prove congruency, such as the SSS (Side-Side-Side) or the SAS (Side-Angle-Side). It is vital for teachers to spend more time on the conditions of congruency to help learners grasp these concepts, which form the foundation of most of the proof in geometry and are essential for success in the topic.

Table 13 depicts the summary of Moloto's MDI and Table 14 are the summative judgements relating to the teacher's explanatory talk, exemplification, and learner participation.

Table 13. *Summary of Moloto's teaching episodes*					
Objects of learning: 1. Recap on geometry of a straight line 2. The proof of Theorem 1					
Teacher's explanatory talk		**Exemplification**		**Learner participation**	
Naming	***Legitimating criteria***	***Examples***	***Tasks***		
Episode 1: The lesson's introduction - Moloto used less everyday language when giving the recap on the geometry of a straight line. He used mathematical language appropriately to explain different properties of angles. for example: <u>Angles on a straight line</u> *see lines* 2 and 5 <u>Angles around a point</u> *see line* 7 <u>Vertically opposite angles</u> *see line* 4 - Moloto used equality symbol "=" and inequality symbol "<" to describe different angles. He also used visual when drawing different lines on the whiteboard.	<u>**Legitimating criteria:**</u> Moloto did not use non-mathematical criteria. He used visual when drawing lines on the whiteboard and mathematical criteria that are general when providing specific definitions and properties of different angles, and no local mathematical criterial were used across this episode which means no convention was provided.	Example 1 Angles on a straight line Example 2 Angles around a point Example 3 Vertically opposite angles Example 4 Angles on parallel lines	- Moloto drew a straight-line AB on the whiteboard and marked two angles on it as shown in image 10 (a). Then asked learners to identify the relationship between these two angles. - Moloto drew two lines, line AB and line CD on the whiteboard on which he marked angles a, b, c and d as shown in image 11 and asked learners to identify the relationship between these angles. - Moloto drew two parallel lines, line AB and line CD and another line that cuts these lines. After marking some angles on these lines and asked learners to identify the relationship between these angles.	Moloto did not give an opportunity to learners to go to the board and display their ability and skills. He rather asked them oral questions.	

Episode 2: Moloto's assessment during a Euclidean geometry lesson - Moloto used more everyday language when checking and marking learners' work. - Moloto used mathematical language appropriately to explain the relationship between vertically opposite angles, corresponding angles and alternate angles when he used equality symbol "=" to express the following statements: 1. Vertically opposite angles are equal. 2. Corresponding angles are equal 3. Alternate angles are equal. **<u>Legitimating criteria:</u>** - Moloto did not used non-mathematical criteria such as mnemonics when checking and marking learners' work. - Moloto also used mathematical criteria that are general when providing properties of angles, and local mathematical criterial when showing to learners how to give a reason to a statement which is regarded as a convention.	No examples Moloto gave to leaners an activity as shown in image 16 and gave learners time to attempt it. As for him he was moving around checking and marking the work of leaners.	Moloto gave an opportunity to learners to do the activity in their books. The participation of learners was only limited to attempting the activity in their books. Moloto did not allow learners to share their answers on the whiteboard with their peers. He only managed to move around and checking learners' answers from one learner to the other. He should have allowed learners to interact between themselves before giving them feedback.

Episode 3: Under-proving theorem 1 Moloto used everyday language when giving the object of learning, when writing the statement of theorem 1 on the whiteboard and when revising the conditions of congruency of triangles. He used mathematical language appropriately to explain the conditions of congruency of triangles and when providing the outline of the proof and when writing different mathematical arguments and different reasons. For example: $\hat{D}_1 = \hat{B}_2$ $(alternate\ <_s, AD \parallel BC)$ $\Delta ABC \equiv \Delta CDA$ **Legitimating criteria:** Mafoko used non-mathematical criteria when using mnemonic: 1. "**SSS**" to help learners remember the first condition of congruency of triangles. which means "Side, Side, and Side". 2. "**SAS**" to help learners remember the second condition of congruency of triangles which means "Side, Angle, and Side". 3. "**AAS**" to help learners remember the first condition of congruency of triangles which means "Angle, Angle, and Side". 4. "**RHS**" to help learners remember the fourth condition of congruency of triangles which means "Right angle, Hypotenuse, and Side". Moloto used mathematical criteria that are general when providing properties of different concepts and to present the proof of theorem 1.	No examples The task of learners was to identify angles and their properties as Moloto continued to display the proof of theorem 1. The challenge was that some learners were not able to understand their task, and some were confused. The task of learners was very simple because the teacher almost did everything by himself when proving the theorem.	Learners' participation was limited to teacher learners' interaction during the lesson. Learners did not have time to interact with their peers because everyone was interacting with the teacher when responding to different oral questions. Moloto did give learners opportunities to compare and contrast ideas.

Table 14. *Summative judgments of Moloto's MDI*

EXPLANATORY TALK		EXAMPLIFICATION		LEARNER PARTICIPATION
Naming	**Legitimating criteria**	**Examples**	**Tasks**	
Level 1: **(NM)**: More everyday language **(Ms)**: Inequality and equality symbols to express angles relationships. **Level 2:** **(Ma)**: Mathematical language to explain different properties of angles. **Level 3:** More movement between (**NM)** and (**Ma)**	**Level 0:** **(NM):** Mnemonics: SSS, SAS, AAS and RHS to assist learners to remember to four conditions of congruency. **(V):** Visual: Drawing lines on the board. **Level 1:** **(L):** Writing Euclidean geometry reasons for different statements in short form (convention) **Level 2:** **(GP):** Definitions and properties **Level 3:** Deriving the proof of Theorem 1.	**Level 1:** Use of examples that reflect **similarity** or **contrast** (One form of variation)	Limited to identifying angles and properties verbally.	- **Only verbal participation** - **Physical participation limited to attempting an activity**

CHAPTER 6
Findings and Discussion

6.1. Introduction

Chapter 5 focused on the data presentation, analysis, and interpretations of the four teachers' discourses that were generated by means of unstructured classroom observations, VSRIs and semi-structured individual interviews. The current chapter discusses the key discourses that emerged from the analysis of the three data sets. The components of MDI discussed in Chapter 3 undergird my thinking about teachers' teaching of Euclidean geometry. Table 15 depicts the themes that emerged from the analysis of the three data sources for each teacher and how they address the predetermined research questions. The study sought to answer the following main research question:

Main research question: *What are teachers' mathematics discourses during Euclidean Geometry lessons in Johannesburg East District?*

Table 15. Themes and their relation to sub-research questions

Theme	***Research questions addressed***
Explanatory talk is in exile, question-and answer at the center.	*What are teachers' explanatory talk during Euclidean geometry lessons?*
Exemplification to draw learners' attention to legitimate features.	*What are teachers' discourses related to exemplification during Euclidean geometry lessons?*
Learners' thinking is limited to one-word answers.	*How do Grade 10 teachers enable and/or constrain learner participation during Euclidean geometry lessons?*

These themes reveal that all the components of MDI need to work in unison and should all be pitched at high levels to ensure learners' effective learning of Euclidean geometry.

6.1.1. Explanatory talk is in exile, question-and answer at the center.

This theme focuses on teachers' lack of explanatory talk during Euclidean geometry lessons, in which what was considered legitimate was produced in learner responses to teachers' questions, with no accompanying elucidations from the teachers. This theme addresses the last sub-research question. That is, while the teachers allowed the learners to provide naming for various Euclidean geometry related concepts, specialised meanings for various concepts remained implicit due to lack of teachers' explanatory talk. The overuse of question-and-answer discourse during Euclidean geometry lessons resulted in lack of in-depth legitimation and naming of mathematical concepts related to Euclidean geometry. According to Sfard (2019, p. 1), "it is a common lore that teachers bear the main responsibility for what the students learn or fail to learn", suggesting their influence regarding learners' understanding or lack thereof for knowledge. The teachers in this study presented the information using visual cues and used the question-and-answer strategy to get learners to identify and name Euclidean geometry concepts and legitimate specific narratives. Of importance to note is that teachers' explanations play a significant role in bringing the object of learning into focus.

One thing that research that focus on the notion of instructional explanations in mathematics is silent about is that explanatory talk is not complete after the teacher just presented the information about the mathematical objects. Equally, just asking learners to recall and verbalise what they know about mathematical objects throughout the lessons without the teacher building on their responses for further elaboration limited the effectiveness of teachers' mathematical discourse in this study. This raised a question for me throughout this study, what does explanatory talk mean? What dominated the teachers' mathematical discourse in the current study were the lack of sustained explanations for teachers to present and legitimise Euclidean geometry concepts and processes. Thus, the lack of teacher explicit and engaged talk, beyond the questions and confirmations of learners' answers limited opportunities for foregrounding "what is to be known or done, and how" (Adler & Ronda, 2016, p. 6).

Although it could be argued that the discourse of question-and-answer that was predominant in the teachers' lessons resonate with the notion of teachers becoming facilitators of mathematical contents rather than being at the centre of teaching and learning, having no explanations, elucidations on how learners should work with specific concepts and associated rules can limit learners' epistemological access to geometry concepts, especially considering that the more abstract geometry concepts are introduced at Grade 10 (Charalambous et al., 2011). The four teachers overlooked the language level engagement in their exchanges with learners, that as much as learners are the ones who should learn and own Euclidean geometry concepts and skills, they ought to make naming and legitimating mathematical statements to guide learners about the nature of mathematics concepts and their relatedness (Wittwer & Renkl, 2008).

Teachers' discourse related to explanatory talk in this study are contrary to Tachie's (2020) findings that, some teachers spoke and wrote throughout the teaching and learning process without allowing learners opportunities to internalise the contents presented and discussing their understanding thereof. In the current study, teachers were the ones who engaged in peripheral teaching, in which the posed questions and prompted learner responses until the expected answers were verbalised. The teachers dominated the writing on the board like in Tachie's study, but they did not present explanations to guide learners on how to work with different geometry concepts that they introduced. Accordingly, I argue that questioning learners about mathematical objects and confirming or disconfirming their answers cannot be the endpoint of teaching. The teachers in the current study did not use 'responsive explanations', to unpack concepts that presented some difficulties for learners, as they focused primarily on using prompts until correct answers to questions were verbalised by learners.

6.1.2. Exemplification to draw learners' attention to legitimate features.

While the previous theme focuses on teachers' lack of explanatory talk during teaching, this theme is about how teachers made use of examples during teaching to bring the different objects of learning into focus, addressing the first sub-research question. The examples the four participants used during the lessons in the current study focused on the presentation of mathematical diagrams through which learners were required to identify the critical features of different Euclidean geometry concepts. As discussed in the previous theme, teachers used the question-and-answer strategies to prompt learners to identify specific features based on the questions that teachers asked. To lead learners to name and legitimate concepts and processes in this study, teachers draw geometric diagrams and asked learners to identify relationships between angles and/or lines. What can be said about the examples that teachers presented is that they were prototype or generic examples used for identifying relationships between angles and lines, and the talk about the features observable on the diagrams were limited to stating, 'which angle equates to which' and 'what the relationship were between the angles and lines that were presented'. The summative judgements for the four teachers' cognitive demands for the learners are at low level, because they did not create learning spaces for learners to complete tasks independently of the teacher. Across all the selected episodes, we rarely got to see learners given chances to engage in tasks or justify their work. Introducing more examples beyond the generic examples used to identify critical Euclidean geometry features could have allowed teachers to have in-depth conversations with the learners (Adler & Ronda, 2015; 2016).

This study also unearthed instances where teachers did not create spontaneous examples to respond to learners' current learning or difficulties as they verbalised answers to teachers' questions. Instead of ignoring incorrect answers, the teachers could have 'on their feet' created examples, including counter examples to learners' utterances to help them falsify their claims. Creating a teaching and learning space for more examples could have been one way of activating teachers' explanatory talk instead of the overuse of question-answer patterns in their mathematical discourse, within the space of limited examples. The transformation in teachers' explanations and exemplification is a critical condition for making the object of learning realised in mathematics teaching (Heyd-Metzuyanim, 2018).

6.1.3. Learners' thinking is limited to one-word answers.

Gresham (2018) argues that teaching and learning of Euclidean geometry is like teaching and learning a new language, and teachers should introduce learners to new words and their relatedness. Thus, it becomes important for teachers to allow learners to participate in learning, to present their observations and understanding of different Euclidean geometry objects and develop formal communication skills (Adler & Venkat, 2014). This theme answers the second research question which focuses on the nature of learner participation which was enabled during the lessons. The analysis of the selected episodes in chapter 5 revealed that learner participation was limited to giving one-word answers to teachers’ questions and teachers did not create a learning environment for learners to engage in elaborative communication about their observations. The participants did not create mathematics learning environments that encouraged learners to explain their observations, reasons or provide interpretive elaborations for different geometric principles or theorems. According to Adler and Ronda (2016), one-word answers do not encourage learners to think critically about mathematical objects and processes.

It was interesting to observe that there were instances where the teachers asked the learners to verbalise the relationship between angles and lines, and once the learners provided the correct identification of the relationships, the teachers did not prompt the learners to provide justifications for their answers. This limited learners’ participation to identification of the relationships without reasoning. In Euclidean geometry, it is important for teachers to create and maintain a classroom environment that allows learners to communicate their thoughts, justify those thoughts and engage in sustained conversations with both the teacher and their peers to construct meanings and deepen their understanding of geometric concepts. As demonstrated across different episodes for each teacher in chapter 5, Mafoko, Mpilo, Makonga and Moloto predominately encouraged the memorisation of Euclidean geometry concepts and associated principles instead of creating bridges between learners’ ways of thinking and mathematics curriculum goals. Teachers did not prompt learners to state the legitimate reasons in their verbalisations of the solutions to presented problems.

Overlooking the need for learners to engage in sustained conversations about Euclidean geometry concepts can limit their learning of such concepts, especially considering that this topic is loaded with numerous concepts that learners need to learn and internalise. In relation to thus, Mntunjani (2017) suggests that teachers should spend time in developing learners’ language use during Euclidean geometry lessons, to ensure legitimation of the concepts and their meanings. I argue that the development of geometric language can be limited when teachers only use questions-and-answer discourse without allowing learners to engage in extended conversations about geometric objects in focus. The findings highlight the need for teachers to adopt teaching strategies that allow learners to present their understanding and consider learners' backgrounds to use responsive explanations to disrupt and/or disrupt their knowledge of Euclidean geometry.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1. Introduction

In this study I explored Grade 10 teachers' mathematical discourses during Euclidean geometry lessons with four teachers from four different schools in Johannesburg East District, Gauteng Province in South Africa. To explore teachers' discourses, I employed Adler and Ronda's (2015) MDI framework as theoretical framing to analyse the data and answer the predetermined research questions. MDI allowed me to explore and understand the mathematics related to Euclidean geometry that the participating teachers made available for learners to learn, through paying attention to how they communicated the object of learning, how they explained concepts or lack thereof during teaching, how teachers used examples to bring different objects of learning into focus as well as how learners were encouraged to participate during teaching and learning. To generate data, I used three research methods: unstructured classroom observations, VSRIs and semi-structured interviews.

7.2. Summary of study findings

The findings of the study revealed that teachers did not engage in elucidation of the concepts during the lessons. Even in cases where learners asked clarity seeking questions, teachers did not draw from learners' current learning to provide explanatory talk to formally name Euclidean geometry concepts and legitimate them. The findings also show that, participating teachers used examples to demonstrate the critical features of the different concepts that they introduced, but they did not present examples to show variance and invariance for the concepts. In the following sub-sections, I present summaries of the findings from this study in relation to the predetermined research questions.

7.2.1. Explanatory talk is in exile, question-and answer at the center.

In this study, the four teachers allowed their learners to provide naming for different concepts they introduced in the lessons. However, teachers did not offer explanatory talks to bring the specialised meanings to the fore and legitimate concepts and meanings for the learners. The four teachers predominately used questions-and-answer strategies to engage with different topics. While this way of teaching can be a good strategy to get learners to become active co-constructors of Euclidean geometry concepts, teachers' explanations remain vital to ensure concepts clarification and legitimation for learners. Over-using the question-and-answer discourse during teaching resulted in lack of in-depth legitimation and naming of mathematical concepts related to Euclidean geometry which is concerning considering that this topic has numerous concepts that require clear operationalisation by the teacher for learners to learn and internalise such concepts. The teachers dominated the lessons by writing on the board, which is like the findings from Tachie's (2020) study. What makes the findings from the current study differ to those of Tachie's (2020) study is that the teachers in this study did not offer explanations to guide learners on how to work with different geometry concepts that they introduced on the board. Overall, lack of teacher explanations in this study limited opportunities for epistemological access to Euclidean geometry concepts.

7.2.2. Exemplification to draw learners' attention to legitimate features.

The study found that teachers introduced examples that were aimed at drawing learners' attention to specific critical features for Euclidean geometry. Teachers used single examples for each concept they introduced, missing opportunities to demonstrate similarities, contrasts and fusion as recommended by the MDI framework and to guide learners towards generalising Euclidean geometry concepts and principled. This finding resonates with the literature on the teaching of Euclidean geometry, particularly the study by Sinclair et al. (2016), which highlighted major threads of contributions in geometry teaching research. The finding relating to the missed opportunities to demonstrate similarities and contrasts of concepts for learners aligns with the broader discourse on effective teaching of Euclidean geometry (Adler &Ronda, 2015; Gresham & Shannon, 2017). Baccaglini-Frank & Mariotti (2010) highlighted that, in generating conjectures in Euclidean geometry, it is important to engage learners in sustained explorations to enhance opportunities for deeper geometric understanding, which links closely with the notion of missed opportunities for demonstrating fusion in the current study.

7.2.3. Learners' thinking is limited to one-word answers.

Relating to learner participation during the lessons, the findings highlighted that teachers limited learners' engagement to providing one-word answers and did not create classroom situations where learners were required to offer interpretive elaborations for their mathematical claims. The findings showed that there were no tasks that were created for learners to engage in independent learning to demonstrate their understanding or lack thereof. The findings highlighted that the teachers used a questions-and-answer discourse, limiting learner participation to one-word responses without encouraging elaborative justification for their answers in order to determine their current understanding and their reasoning (Hundeland et al., 2020).

7.3. Study limitations

The current study was restricted to exploring and understanding Grade 10 teachers' discourses of teaching Euclidean geometry. Only four teachers from four different secondary schools were sampled to participate in the study and the bounded context for the study was only one district, Johannesburg East in Gauteng. Accordingly, the data and findings were only generated from four teachers. Hence, the findings relating to the teaching of Euclidean geometry from this study may not be generalised. The study was only set to research on Euclidean geometry which is one of the ten topics making up the Grade 10 South African mathematics curriculum. Different findings might possibly be attained from other contexts, especially with larger sample sizes. Besides presenting VSRIs and semi-structured individual interviews as supplementing or complementing data due to space confinements, presenting data from these sources in separate chapters could have provided more elaborations for teachers' discourses during teaching.

7.4. Recommendations

In this section, I provide recommendations for teaching Euclidean geometry effectively, recommendations for future research as well as recommendations for teachers' professional development.

7.4.1. Recommendations for teaching Euclidean geometry effectively.

From the findings of this study, I have realised that teaching Euclidean geometry effectively requires effective mathematics discourse. I recommend that teachers should begin a lesson with fundamental concepts like points, lines, and angles to ensure that learners should have a solid foundational knowledge before moving on to more complex topics. This means that, teachers should make links between concepts covered in earlier grades to help learners with knowledge building and creation of the mathematics story related to the topic.

Teachers should also consider introducing Euclidean geometric concepts with real-world examples to make abstract ideas more tangible and relatable for learners. They should also limit the overuse of questions-and-answer discourse when teaching Euclidean geometry, especially at the beginning of the concept at Grade 10. Instead, teachers should use a variation of examples and offer explanatory talk to help learners develop the language for Euclidean geometry. Equally, learners should be encouraged to ask questions about Euclidean geometric concepts, and they should be able to model effective questioning techniques to stimulate curiosity and critical thinking. Teachers should pose challenging problems that require critical thinking and problem-solving skills and prompt learners to engage in sustained conversations about Euclidean geometry principles and theorems. This means that teachers should facilitate open-ended classroom discussions about Euclidean geometric concepts, encourage learners to justify their reasoning and engage in respectful debates to help them towards generality of the concepts and related processes.

Teachers should foster an environment where students explain concepts to each other. This not only reinforces their own understanding but also helps peers grasp the material. They should also actively address common misconceptions through classroom discussions and use learners' mistakes as opportunities for learning and clarification. In addition, they should foster a positive and interactive learning environment where learners can actively engage with and appreciate the principles of Euclidean geometry.

7.4.2. Recommendations for future research on Euclidean geometry

Based on the limitations identified in the preceding section, it is recommended that future research should replicate the study with a larger sample of schools from different districts across the country. Future research should explore how the different components of MDI influence each other in bringing the object of learning into focus. I also recommend that future studies should extend the study to other topics such as trigonometry and functions that are introduced for the first time in Grade 10. In addition, studies that focus on the nature of teachers' explanations in teaching Euclidean geometry should be conducted, to understand how teachers implement explanations in teaching sequences for the topic. Future studies should explore how teachers professionally notice their learners' mathematical thinking and communication of Euclidean geometry concepts and principles during teaching.

7.4.3. Recommendations for professional support

I recommend the use of Video-Stimulated Recall Interviews with teachers, for them to identify and reflect on critical incidences in their teaching and how they enable and/or constrain learners' epistemological access to mathematical concepts. The Department of Basic Education and teacher training institutions should popularise the use of VSRIs in teaching, to Foster a culture of reflective teaching. This can help teachers to engage in professional learning and configure alternative teaching strategies to ensure learners' effective learning. I also recommend the use of seminars and workshops to train teachers on how to teach Euclidean geometry effectively, particularly focusing on how to offer explanatory talk, use examples and encourage learner participation during teaching and learning.

7.5. Chapter summary

This chapter presented the summary of the research findings that emerged from the study and made some recommendations for effective teaching of Euclidean geometry, recommendations for future studies as well as recommendations for teacher professional development.

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APPENDICES

Appendix A: Teacher's Semi-structured Interview

Teacher A from school X
Question 1: Would you kindly introduce yourself.
Answer: I am teacher A from school X, a mechanical engineer with a PGCE in Education. I am currently teaching Physical Sciences and Mathematics in grades 10-12
Question 2: Why did you choose the path of teaching instead of one for engineering?
Answer: I am teaching because of lack of good opportunities in industry.
Question 3: How long have you been teaching?
Answer: I have been teaching since 2010, which is nearly 13 years of teaching experience.
Question 4: How would you describe your experience in the teaching field?
Answer: Mathematics and Physical Sciences being difficult subjects, in the beginning I faced a few challenges such as getting learners to be comfortable with my way of teaching was difficult which led to their poor performance, but as time went by and with more experience that I gained the enjoyment of teaching felt more natural.
Question 5: Is there any possibility (after 13 years in the teaching field) of you wanting or considering the idea of going back to your natural field of engineering?
Answer: No, because during my 13 years course of teaching my focus has been about Physical Sciences and Mathematics. I have only roughly worked on mechanical engineering during this time.
Question 6: Tell me about the performance of the learners you are currently teaching.
Answer: The level of performance varies among pupils depending on the subject matter; some students perform well while others perform poorly. The performance is generally below average. The majority of students continue to struggle.
Question 7: What intervention strategy do you normally use in order to improve the performance of your learners in mathematics?
Answer: I normally do organise extra lessons for weak learners after learning hours.

Question 8: What does it mean for you to teach mathematics?	**Answer: You cannot live without mathematics, thus teaching it to pupils benefits them in a variety of ways. Regardless of how you teach it, you will still retain the knowledge that will benefit you in other careers. One of the most crucial tools for influencing a learner's way of thinking is mathematics.**
Question 9: What does it mean for you the learn mathematics?	**Answer: I was able to pick up a lot of knowledge while studying mathematics. I have been able to solve several challenges that have come my way thanks to mathematics. I can claim that mathematics is universal and also something that everyone can do.**
Question 10: During your period as a learner did you have any history in Euclidean Geometry?	**Answer: Yes, although this topic; Euclidean Geometry was known as circle geometry.**
Question 11: What does the teaching of Euclidean Geometry mean to you?	**Answer: It is challenging to teach this topic. Euclidean Geometry fosters critical thinking in students. Critical thinking ought to, in my opinion, be one of the abilities that a student should have. Critical analysis and critical thinking are related, and Euclidean Geometry has more critical analysis. The teaching of this topic will assist students in developing new ways of thinking.**
Question 12: What can you say about the teaching of Euclidean Geometry at Grade 10 level? (Focus on curriculum specification and delimitation)	**Answer: I have been in teaching for 13 years and I have observed that learners are struggling a lot with Euclidean Geometry. The Annual Teaching Plan (ATP) is not well design in the fact that there is not enough time allocated to this topic. So, it is difficult to teach Euclidean Geometry concepts effectively.**
Question 13: What can you do to help improve the teaching of Euclidean Geometry in Grade 10?	**Answer: Euclidean Geometry requires a lot of time, thus I advise schools and instructors to plan extra lessons where teachers will have more time to thoroughly explain Euclidean Geometry concepts and also assist students in studying and revising past examination question papers.**
Question 14: What are some important aspects that you need to emphasize when teaching Euclidean Geometry in Grade 10?	**Answer: I always place a strong emphasis on shapes and their many properties when instructing Grade 10. In order to solve problems involving Euclidean Geometry, students need to be able to identify and comprehend several properties. This topic is challenging since many diagrams, theorems, and properties are not well understood.**

Question 15: What are some of the challenges, if any do you experience when teaching Euclidean Geometry in Grade 10?
Answer: The biggest difficulty in teaching Euclidean geometry is that students frequently use the wrong theorems and properties to solve problems. In order to prevent student from learning solely through hearing, I believe that more hands-on activities should be taught to them using visual aids. Many times, even though some students may already know the answer, it might be challenging for them to provide the right reasons to back up their claims.
Question 16: What factors influence your teaching of Euclidean Geometry in Grade 10?
Answer: From my point of view, time is the first factor that affects the teaching of Euclidean Geometry considering the way the Annual Teaching Plan is designed. The ATP is running fast and does not consider the fact that some concepts need more time to be comprehended and that in the classroom students do not have the same level of understanding. Secondly, learners in Grade 10 do not have a background of different shapes and their properties.
Question 17: How do you manage to work with these factors?
Answer: I always make sure that before I introduce a new concept, I will ask learners to go and conduct research in order for them to have a background information which will assist me to save time and be able the complete a particular concept within specified time.
Question 18: How do you encourage learners' participation during Euclidean Geometry lessons?
Answer: To encourage all learners to participate, I always put them in different groups to assist the slow learners to gain confidence and I will be moving around to monitor them.
Question 19: How do you introduce Euclidean Geometry to your Grade 10 learners?
Answer: To introduce Euclidean Geometry to Grade learners, I always start with the background where I will be teaching different shapes and their properties.
Question 20: How do your Grade 10 learners usually react to this new topic?
Answer: Some learners are excited to learn more about Euclidean Geometry while others are confused and afraid about the topic.
Question 21: Describe how you introduce Euclidean Geometry concepts in a lesson in Grade 10?
Answer: I normally draw a sketch on a chalkboard then I will use it to do a recap which will help me to know if learners do still remember something on it. I will finally use their answers to build up ideas that are related to the concept I am about to teach.

Question 22: Do you think this way of introducing a concept in a lesson will work for all Euclidean Geometry concepts?
Answer: No, for some concepts I usually use a statement that I will break downs with oral questions that I will be asking to learners in order to build up ideas around the concept I am about to teach. I also use daily life examples.
Question 23: How do you respond to learners' learning during Euclidean Geometry lessons in Grade 10?
Answer: By using different questions (oral) and check learners' responses which will assist me to know either they understand the concepts taught or they still need more help.
Question 24: How do you assess learners' learning during Euclidean Geometry lessons in Grade 10?
Answer: During a lesson, I use three different types of assessment: oral assessment, individual activities and group activities.
Question 25: Is there anything else you would like to share about the teaching of Euclidean Geometry in Grade 10?
Answer: I suggest that all schools must have projectors to help students visualize different aspects of Euclidean Geometry.

Appendix B: VSRI Conversations

VSRI: Teacher Mpilo

Researcher: In this lesson, you started off by recapping the concept of angles on straight lines. This was good to see …

Mpilo: Yes, I wanted learners to keep that understanding in mind, which would help them to work with angles when I introduced parallel lines. Our learners need you to do that, if you don't, they see a concept as a stand-alone.

Researcher: But I noticed that you did not make the links clear for the learners as to what the relevance of recapping that knowledge was.

Mpilo: I agree, and I could have told the learners what the links are. You see this thing of being able to watch your own lessons is helpful. I will sure tell the learners why I do certain things in class next time.

VSRI: Teacher Mafoko

I have noticed that Mafoko's method of instruction was difficult for the students to follow and comprehend. Instead of providing examples to help students grasp his method of discussing the many properties of angles, he exclusively employed activities as a teaching tool. I got the chance to ask him about his teaching philosophy during VSRI. The conversation that follows exemplifies what we discussed:

Researcher: Why are you teaching without the use of examples?

Mafoko: *This is not how I teach all of my Grade 10 classes. I only employ this method of instruction in commercial classes when students struggle with mathematics comprehension and are not very strong.*

Researcher: I disagree with you. Given their current situation, weak learners may find this technique challenging to understand. In my opinion, using examples would have been far more beneficial for them. I have observed that you were unable to provide them with a chance to address the board and voice their opinions for this reason.

Mafoko: *The time is another reason I utilize this. It is challenging for me to complete the syllabus in weaker classrooms, and at the end of the day, they have to take the same test as my other classes.*

Researcher: Well. During the semi-structured interview, I asked you a question which is "what can you do to help improve the teaching of Euclidean Geometry in Grade 10?" and your answer was:

> *Euclidean Geometry requires a lot of time; thus, I advise schools and instructors to plan extra lessons where teachers will have more time to thoroughly explain Euclidean Geometry concepts and also assist students in studying and revising past examination question papers.*

According to this answer, how do intend to thoroughly explain Euclidean Geometry concepts and assist learners who are struggling to improve as longer as you teach without using examples?

Mafoko: *Although it is a little challenging for me, I will ask the school's principal if he can let me spend more time with students on Saturdays or after school.*

Appendix C: LETTER TO THE PRINCIPAL and SGB Chair

26 April 2023

Dear Principal and SGB Chair

My name is Mr Kyabuntu Kambila Joxe; I am a master student and mathematics education researcher at the University of South Africa. I am doing research on ‘Exploring Grade 10 teachers’ explanatory talk during Euclidean Geometry lessons in the Johannesburg East District, South Africa’.

My research involves individual semi-structured interviews, classroom observations and audio-recording recall interviews with 3 teachers from 3 secondary schools in the Johannesburg East District. Video-recording and audio-recordings will be used during data collection. Both the semi-structured interviews and video stimulated interviews will take approximately 45 minutes to an hour and will take place after school hours.

I am inviting your school to participate in this research and for your teachers to share their experiences, knowledge, understanding and challenges of teaching Euclidean Geometry in Grade 10 classrooms. The research participants will not be disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this research for whatever reason without any consequences or penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

To ensure that the participants’ and your school’s true identities are protected, I will use pseudonyms to conceal both the true names of the schools and the participants in all writings of the study. The information provided by the participants will be used for the thesis and journal publications both locally and internationally. All research data will be destroyed after 5 years of completion of the research.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Mr Kyabuntu Kambila Joxe

Email: jkkyabuntu@gmail.com OR 17409926@mylife.unisa.ac.za

Cell phone: 082 762 7033

Appendix D: INFORMATION SHEET TEACHERS

26 April 2023

Dear Teacher

My name is Mr Kyabuntu Kambila Joxe; I am a master student and mathematics education researcher at the University of South Africa. I am doing research on 'Exploring Grade 10 teachers' explanatory talk during Euclidean Geometry lessons in the Johannesburg East District, South Africa'.

My research involves individual semi-structured interviews, classroom observations and audio-recording recall interviews with 3 teachers from 3 secondary schools in the Johannesburg East District. Video-recording and audio-recordings will be used during data collection. Both the semi-structured interviews and video stimulated interviews will take approximately 45 minutes to an hour and will take place after school hours.

I was wondering whether you would mind that I come and do a classroom observation while you teach your learners Euclidean Geometry. The information will be used to gain insight of Grade 10 teachers' explanatory talk during Euclidean Geometry lessons.

Your class routine will not be advantaged or disadvantaged in any way. You can ask me to leave the class at any time without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Your names and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed within five years after completion of the project. Please let me know should you require any further information.

Thank you very much for your help.

Yours sincerely,

Mr Kyabuntu Kambila Joxe
Email: jkkyabuntu@gmail.com OR 17409926@mylife.unisa.ac.za
Cell phone: 082 762 7033

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

UNISA university of south africa

Teacher's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow us to engage with you as one of the participants in the study titled: 'Exploring Grade 10 teachers' explanatory talk during Euclidean Geometry lessons in the Johannesburg East District, South Africa'.

I, ____

Circle one

Permission to be videotaped

I agree that my class can be videotaped during classroom observations.	YES/NO
I know that the videotapes will be used for this project only.	YES/NO

Permission to be interviewed

I agree to be interviewed for this study and audiotaped.	YES/NO
I know that I can stop the interview at any time and doesn't have to answer all the questions asked.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I can ask the researcher to leave the classroom at any time.
- I can ask for my class not to be part of any classroom observation.
- all the data collected during this study will be destroyed within five years after completion of the project.

Sign_____________________________ Date___________________________

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

UNISA university of south africa

Appendix E: INFORMATION SHEET PARENTS

26 April 2023

Dear Parent

My name is Mr Kyabuntu Kambila Joxe; I am a master student and mathematics education researcher at the University of South Africa. I am doing research on 'Exploring Grade 10 teachers' explanatory talk during Euclidean Geometry lessons in the Johannesburg East District, South Africa'.

My study involves coming into your child's classroom and observing their Grade 10 mathematics teachers teach. During the classroom observation a video-recorder will be used to record the teacher while teaching. During observations, your child may be captured by the video-recorder, therefore I am asking for permission from you to allow me to capture your child in the classroom.

I have chosen your child's class because my study seeks to work with Grade 10 mathematics teachers to gain insight into teachers' explanatory talk during Euclidean Geometry lessons. Your child will not be disadvantaged in any way during the course of the study. He or she will be assured that she can leave the classroom during observations without any penalty. There are no foreseeable risks in participating, and your child will not be paid for the study.

Since learners are not the primary participants in the study, I am going to protect their identities and throughout all the writings of the study, your child's true name will be concealed. His/her individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed after 5 years of completion of the research.

Please let me know if you require any further information. Thank you very much for your help.

Yours sincerely,

Mr Kyabuntu Kambila Joxe

Email: jkkyabuntu@gmail.com OR 17409926@mylife.unisa.ac.za

Cell phone: 082 762 7033

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

UNISA university of south africa

Parent's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called: 'Exploring Grade 10 teachers' explanatory talk during Euclidean Geometry lessons in the Johannesburg East District, South Africa'.

I, ________________________________the parent of ______________________________

Circle one

Permission to observe my child in class

I agree that my child may be observed in class. YES/NO

Permission to be videotaped

I agree my child may be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Consent

I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- he/she does not have to answer every question and can withdraw from the study at any time.
- he/she can ask not to be audiotaped, photographed and/or videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign______________________________ Date____________________________

Thank you very much for your help.

Yours sincerely,

Mr Kyabuntu Kambila Joxe

Email: jkkyabuntu@gmail.com OR 17409926@mylife.unisa.ac.za

Cell phone: 082 762 7033

UNISA university of south africa

Appendix F: INFORMATION SHEET LEARNERS

26 April 2023

Dear Learner

My name is Mr Kyabuntu Kambila Joxe; I am a master student and mathematics education researcher at the University of South Africa. I am doing research on 'Exploring Grade 10 teachers' explanatory talk during Euclidean Geometry lessons in the Johannesburg East District, South Africa'.

My study involves coming into your classroom and observing your Grade 10 mathematics teachers teach. During the classroom observation your teacher will be observed while teaching and you may be involved as well in your interaction during the observation, therefore I am asking for permission from you to allow me to capture you in the classroom.

I have chosen your class because my study seeks to work with Grade 10 mathematics teachers to gain insight into their discourses and approaches as they teach the subject. You will not be disadvantaged in any way during the course of the study. You may leave the classroom during observations without any penalty. There are no foreseeable risks in participating, and you will not be paid for the study.

Since learners are not the primary participants in the study, I am going to protect your identities and throughout all the writings of the study, you true name will be concealed. Your individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed after 5 years of completion of the research.

Please let me know if you require any further information. Thank you very much for your help.

Yours sincerely,

Mr Kyabuntu Kambila Joxe

Email: jkkyabuntu@gmail.com OR 17409926@mylife.unisa.ac.za

Cell phone: 082 762 7033

UNISA university of south africa

Learner Assent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called: 'Exploring the teaching of patterns, functions and algebra across rural Foundation Phase in Mpumalanga and Limpopo Provinces'.

My name is: ____

Circle one

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be videotaped

I agree to be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Assent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped or videotaped.
- all the data collected during this study will be destroyed 5 years after completion of the project.

Sign______________________________ Date____________________________

Thank you very much for your help.

Yours sincerely,

Mr Kyabuntu Kambila Joxe
Email: jkkyabuntu@gmail.com OR 17409926@mylife.unisa.ac.za
Cell phone: 082 762 7033

Appendix G: interview questions

1. How long have you been teaching?
2. What mathematics background do you have?
3. Tell me about the learners you are currently teaching.
4. What does the teaching of Euclidean Geometry mean to you?
5. What can you say about the teaching of Euclidean Geometry at Grade 10?
6. What are some of the challenges you experience when teaching Euclidean Geometry in Grade 10?
7. What factors influences your teaching of Euclidean Geometry in Grade 10?

a) How does the culture of your classroom enable learners to learn and understand Euclidean Geometry?

b) Would you say your classroom is learner-centred or teacher-centred during Euclidean Geometry lessons? Why?

8. How do you introduce Euclidean Geometry to your Grade 10 learners?
9. How do you respond to learners' learning during Euclidean Geometry in Grade 10?
10. How do you respond to learners' answers during classroom discussions for Euclidean Geometry in Grade 10?
11. In your view, what is the role of Euclidean Geometry in the life of the learners?

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

UNISA university of south africa

Appendix H: Stimulated-Recall Interview: Reflective interview on teaching approaches.

1. **Introduction**
   a) Explain to the teacher what SR interview is.
2. Focus: question asking and comments during the videotaped classroom observation (i.e., this is just a guideline for the interviewer; the interviewee will also ask questions and/or comment on any aspect they decide to).
3. Stimulated Recall Interview Rules:
   a) Either the interviewer or the participant can stop the video at anytime
   b) Distinguish between new observations and actual recall to the teacher
4. **Aspects to focus on:**
   a) The teacher briefly describes the purpose of the observed teaching episode
   b) Stimulated Recall seeks to address the following aspects:
      1. Teacher's perspective on what happened during the teaching episode
      2. The goal(s) the teacher aimed at achieving
      3. What prompted the teacher to act in certain ways during teaching.
      4. Teacher's perspectives on what they could have done better.
5. **Questions** to be asked each time the videotape is paused

a) Can you recall what motivated you to do this?
b) Did anything that occurred in the classroom influence your decision to teach this way/ask this question? Please explain.
c) What information did you base that decision on? (i.e., it could be a teaching approach or any form of interaction with the learners or teaching materials).
d) Was there anything else you wanted to say/do at that point but decided against?
e) Is there anything else you would like to share about this teaching episode?

6. Thank the teacher again for their time.

(The interview structure was adopted from Maloney, 2012).

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

Appendix I: Gauteng Department of Education Ethics Approval

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**18 May 2023**
Validity of Research Approval:	**08 February 2023– 30 September 2023** **2023/168**
Name of Researcher:	**Kyabuntu K.J**
Address of Researcher:	**47 6Th Avenue**
	Bezuidenhout Valley
	Jeppestown
Telephone Number:	**082 762 7033**
Email address:	**17409926@mylife.unisa.ac.za**
Research Topic:	**'Exploring Grade 10 teachers' explanatory talk during Euclidean Geometry lessons in the Johannesburg East District, South Africa'.**
Type of qualification	**Masters**
Number and type of schools:	**3 Secondary Schools**
District/s/HO	**Johannesburg East**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

18/05/2023

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below are met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix J: Unisa Ethical Approval

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE
REVIEW FEEDBACK

Date: 2023/05/10

Dear Mr KJ Kyabuntu

Thank you for the application for research ethics clearance submitted to the UNISA College of Education (CEDU) Ethics Review Committee (ERC).

The **medium** application was reviewed in compliance with the UNISA Policy on Research Ethics and the UNISA Standard Operating Procedure on Research Ethics Risk Assessment on 10th May 2023. **The application is referred back for revisions and clarifications.** The following comments, emanating from the reviews, are tabled for your attention, clarification and/or amendment:

Section 3.3 You need to acknowledge that minor learners will be present during your classroom observations, although they will not participate directly in your study. It is not necessary to "attend local and international conferences" to ensure the "ethical integrity of the study". Instead, the supervisor will also play a role during data analysis.
Section 3.4 It is unclear if the applicant is a teacher at one of the three schools or not. The applicant mentions "I have relationships with one of the schools in the Johannesburg East District as I am currently working in the same district", implying a possible conflict of interest. Therefore, the applicant needs to explain the steps he will take to maximise impartiality and ensure that the findings are not compromised by self-interests.
Section 3.6 The following statements are contradictory: "high schools teach Euclidean Geometry as part of their mathematics curriculum" and "exclusion of Euclidean Geometry from the basic mathematics curriculum".
International studies should be cited to motivate for the importance of the current study. Instead, the applicant focuses almost exclusively on South African research. This geographic focus is reflected in the main research question, "... Johannesburg East District" and should be reconsidered to ensure that the study has broader relevance and appeal.
The research sub-questions could be strengthened and could include a question related to the enhancement of Euclidean Geometry. However, these are research design rather than ethics considerations.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
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www.unisa.ac.za