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Reclaiming Africa's Intellectual Futures

BOUNDARIES BETWEEN MATHEMATICS AND MATHEMATICAL LITERACY: DOES SPECIALISATION MATTER?

PROF MF MACHABA INAGURAL LECTURER

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Open Rubric

STRUCTURE OF THE PRESENTATION

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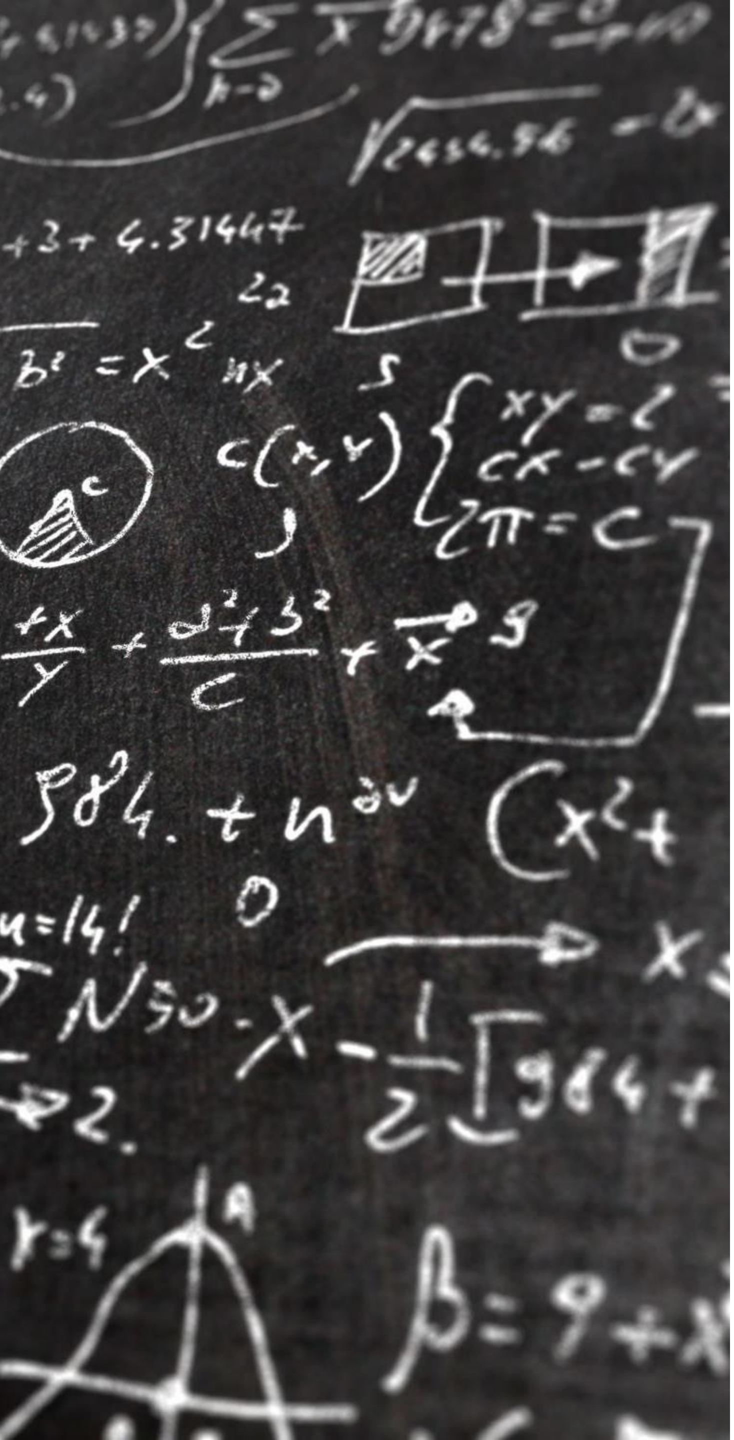


Introduction

- This inaugural lecture emerges from a bigger study which analysed teachers' and facilitators' (subject advisors) views on the pedagogical practices used in Mathematics (M) and Mathematical Literacy (ML) in South Africa.
- Mathematical Literacy was introduced in 2006 in the Further Education and Training (FET) band (Grades 10-12) in South Africa as a subject that runs parallel to M.
- Thus, in South African schools, learners from Grades 10 to 12 take either M or ML.
- These are two separate learning areas from the exit band of the South African secondary education system.
- This means that there is a boundary which has been officially placed between M and ML.
- The possibility exists that by the time learners reach Grades 11 and 12, they would have acquired different levels and different nature of mathematical knowledge and problem-solving skills.

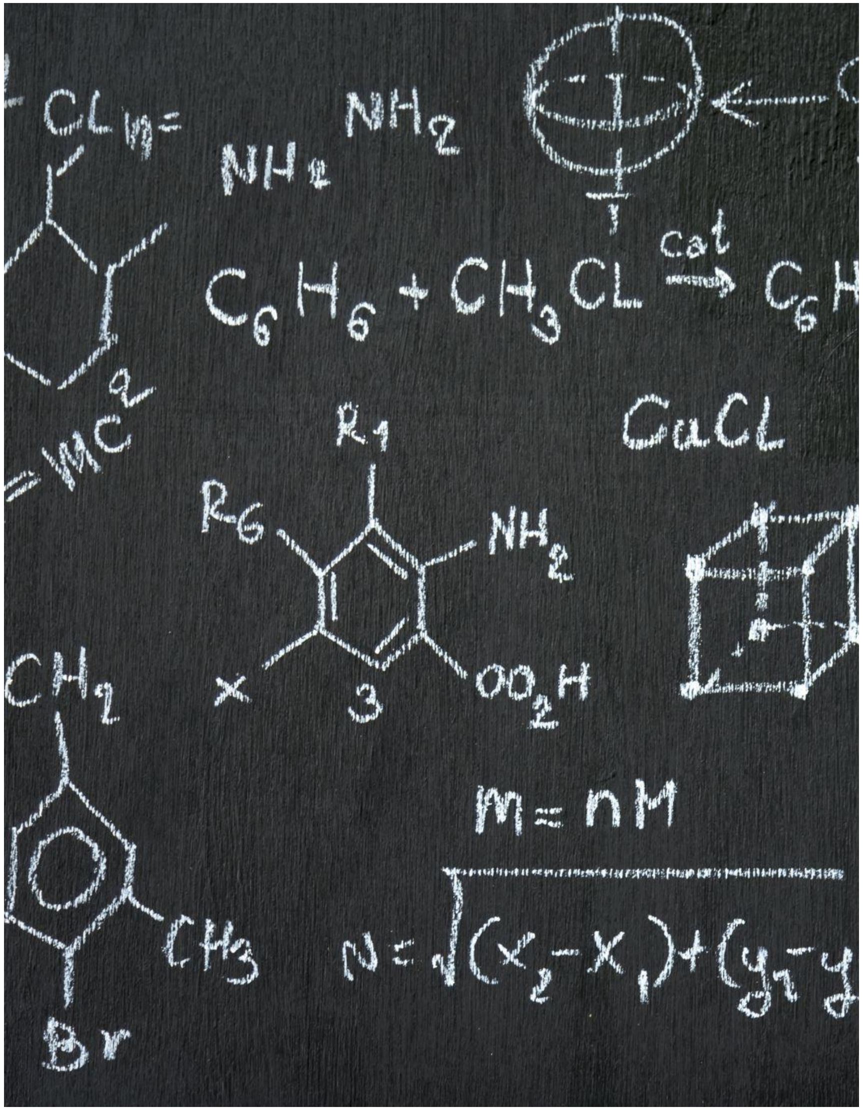
Why Mathematical Literacy

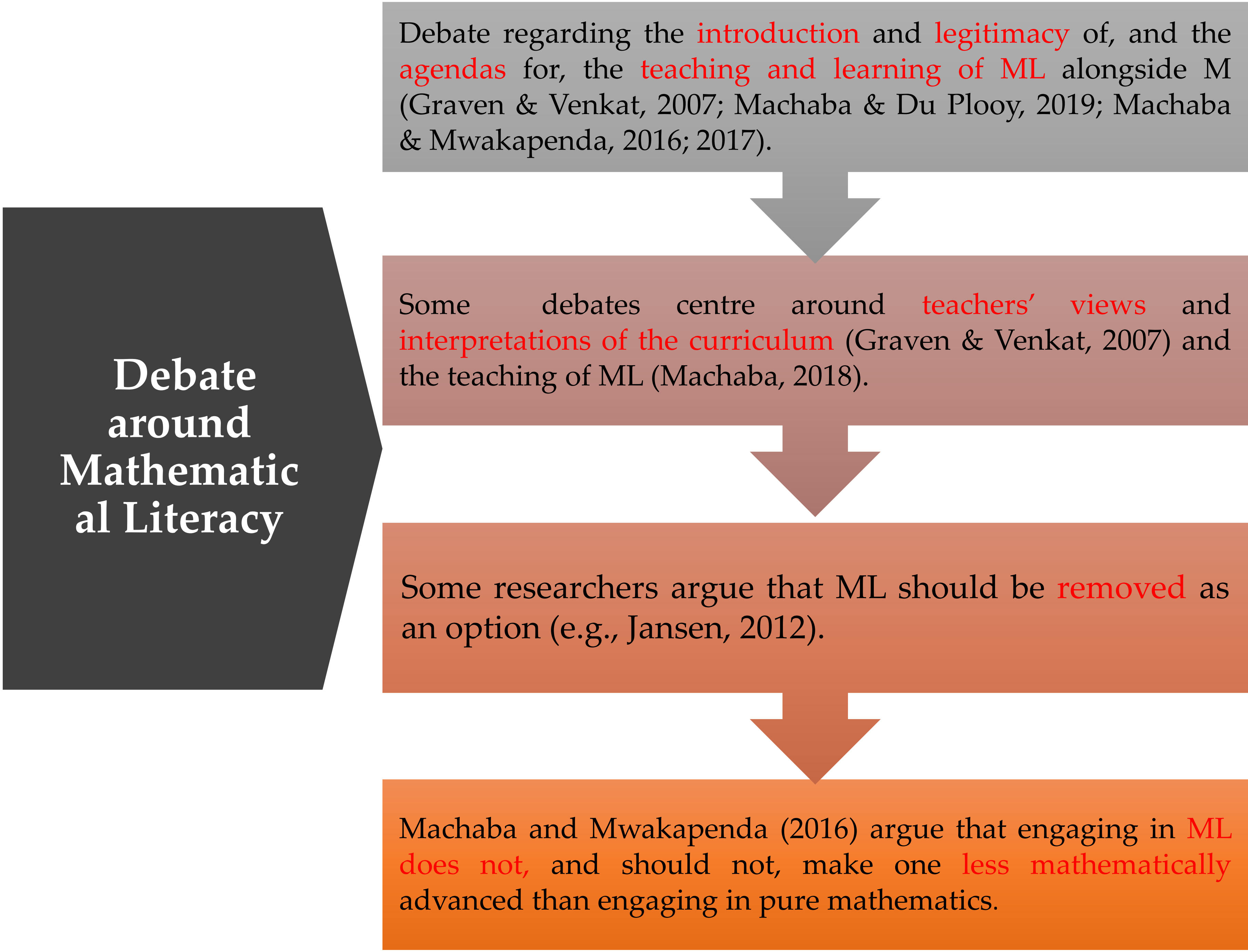
- To make a mathematical–oriented course available and compulsory for all learners in South Africa.

- Democratisation of mathematics – greater access to mathematics for more people

- Mathematics for democracy - a push by the government to enable more people to use mathematics.

- To nullify Hendrik Verwoerd's ideology: "What is the use of teaching the Bantu child mathematics when it cannot use it in practice"?




Debate around Mathematical Literacy
Debate regarding the introduction and legitimacy of, and the agendas for, the teaching and learning of ML alongside M (Graven & Venkat, 2007; Machaba & Du Plooy, 2019; Machaba & Mwakapenda, 2016; 2017).
Some debates centre around teachers' views and interpretations of the curriculum (Graven & Venkat, 2007) and the teaching of ML (Machaba, 2018).
Some researchers argue that ML should be removed as an option (e.g., Jansen, 2012).
Machaba and Mwakapenda (2016) argue that engaging in ML does not, and should not, make one less mathematically advanced than engaging in pure mathematics.

Mathematics and Mathematical literacy orientations

The Curriculum and Assessment Policy Statement Mathematics is defined as

"a language that makes use of symbols and notations for describing numerical, geometrical and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute to decision-making. Mathematical problem solving enables us to understand the world (physical, social, and economic) around us, and most of all, to teach us to think creatively" (DBE, 2011, p. 8).

Mathematical Literacy is defined as:

"a subject that develops competencies that allow learners to make sense of, participate in and contribute to the twenty-first century world – a world characterized by numbers of different ways. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology" (DBE, 2011, p. 10)

My Argument

- M and ML are dialectically linked, and viewing these seemingly contradictory mathematical pathways from multiple perspectives suggests that they are, in fact, completely reconcilable.
- M and ML are not only complementary but that any dichotomy between the two subjects cannot be justified.
- In doing this, I explore teachers' and facilitators' views on M and ML pedagogical practices to gain a better understanding of this possible tension around whether the teaching approach for the two domains should be different or similar.
- Taking a socio-cultural perspective and in particular drawing on the concept of funds of knowledge (Civil, 2016; Moll, Amanti, Neff, & González, 1992),
- Kilpatrick et al.'s (2001) five strands of mathematical proficiency,
- and Bernstein's (1996) constructs of rules of recognition and realisation,

Theoretical Orientations

- The key theoretical constructs informing this lecture come from the field of the Sociology of Education, as articulated by Bernstein.

- Bernstein (2000), there are essentially two types of boundaries (classifications) concerning curriculum structuring: weak and strong boundaries.
- Bernstein (1982, p. 59) refers to classification as:

"the nature of differentiation between contents. Where classification is strong, contents are well insulated from each other by strong boundaries. Where classification is weak, there is reduced insulation between contents, for the boundaries between contents, are weak and blurred".

- Mathematics appears to be strongly classified (the boundary between Mathematics and everyday context is strong) and represents the collection curriculum code.

- On the other hand, ML appears to be too weakly classified (the boundary between mathematics and everyday context is weak) and represents the integrated curriculum code (Bernstein, 1996).

Theoretical Orientations cont….

- Framing refers to the “form of the context in which knowledge is transmitted and received and refers to the specific pedagogical relationship between the teacher and the taught” (Bernstein, 1982, p. 59).

- A strong framing is a pedagogical practice where the transmitter has explicit control over the selection of subject matter, sequencing of the lesson plan or work schedule, pacing of subject matter, and criteria for assessment and classroom interactions.

- In a weak framing, the “learner has apparent control over the modalities of communication” (Bernstein, 2000, p. 13).

- Bernstein alludes to framing as a form of control that legitimises the selection of communication, sequencing of that selection, pacing and criteria used to control communication.

Theoretical Orientations cont….

- The concepts of classification and framing, according to Bernstein, yield to concepts of recognition and realisation rules.

- Bernstein (1996, p. 31), recognition rules "at the level of the acquirer", are the means by "which individuals can recognize the speciality of the context they are in".
- The recognition rule, essentially, enables the appropriate realisation of putting things together.
- Parker indicted that recognition rules are criteria (special relationship) for making distinctions, for distinguishing the speciality of a thing / a practice / a specialisation / a context; what makes it what it is.
- They are principles for recognising the "legitimate text", the voice to be acquired, and are determined by the classification principle at work (relation between different knowledge discourses and practices).

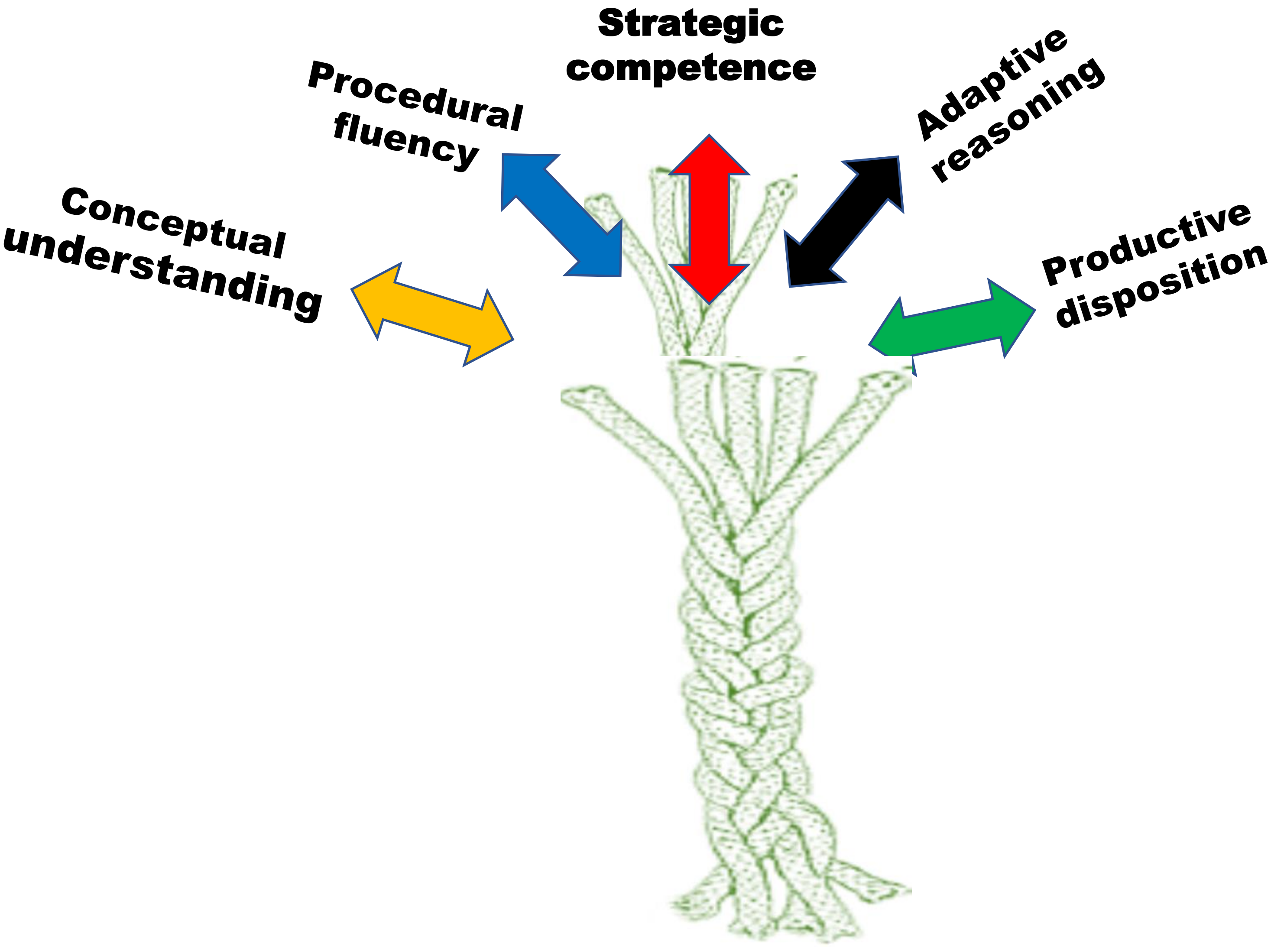
Theoretical Orientations cont….

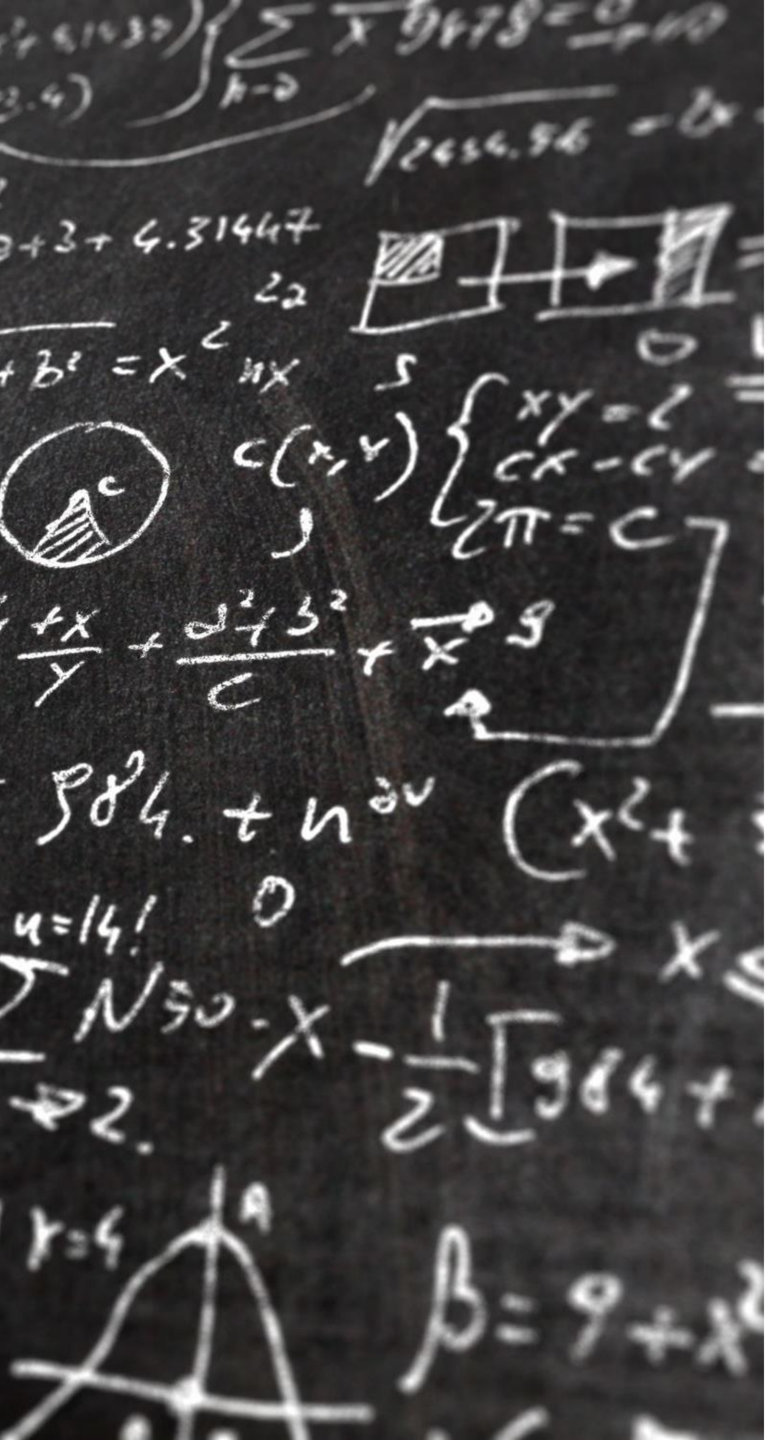
- The realisation rule determines "how we put meanings together and how we make them public" (Bernstein, 1996:32).
- The realisation rule means that the acquirer (teacher) is able to produce a legitimate text in the required discourse.
- "Text" refers to anything that attracts evaluation; for example, the way one talks about the mathematics and ML pedagogical practices.
- Possession of the realisation is reflected in the ability to produce (act, speak or write) the expected (legitimate) text, in this context, of mathematics and ML pedagogical practices.\
- The acquisition of the recognition and realisation rules for a specific practice, for example, teaching mathematics or ML, will depend on the evaluation rules of the pedagogic discourse, that is, the criteria of what is seen to be the "legitimate text" in mathematics or ML teaching practices.
- Therefore, different specialised consciousness – orientations to meaning – could be acquired, depending on the selection and organisation of knowledge contents of mathematics or ML and how they are made available to teachers (what is recognised as legitimate knowledge and practice).
- In her description of recognition and realisation rules, Parker (2008:79), drawing from Bernstein, indicated that:
    - It is through the evaluative rules (rules of recognition and realisation) that specific pedagogic knowledge and practices are constituted as legitimate in practice and orientations to meaning are acquired.

Funds of Knowledge

- In deliberating on the unique identity of ML, the construct 'funds of knowledge' appealed to me as potentially underpinning the intentions of ML.
- The knowledge and experiences in everyday life should become relevant and meaningful, especially and specifically for the teaching of ML.
- Moll et al. (1992, p. 72) "historically developed bodies of knowledge and skills essential for household or individual functioning and wellbeing".
- Civil (2016) used it to develop teaching innovations that build on the background, knowledge and experiences of students, their families and their community

Strands of mathematical Proficiency





Theoretical Orientations cont….

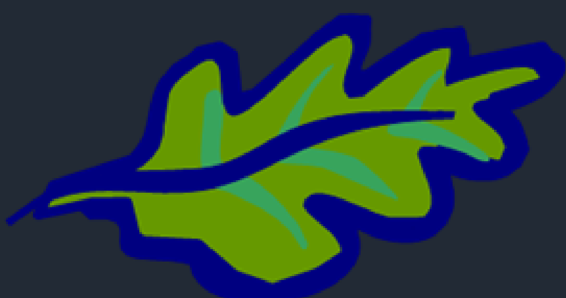
- In this lecture, these theoretical constructs are used to consider learners and teachers' professional dispositions.
- How do teachers think and speak about their M and ML knowledge, students, pedagogic practice and the relationship between themselves and their students.
- The use of 'dispositions' in 'professional dispositions' is related with Bourdieu's notion of habitus - the embodiment of culture, and it provides the lens through which the world is interpreted.
- Habitus predisposes (but not determines) thoughts, actions, and behaviors.
- I use this to refer to the "tendencies people have to respond to particular context in habitual, if not predictable ways" (Cooper 1998, p. 526)

LEARNERS EXAMPLES

AND

DICUSSIONS

LEAF EXAMPLE

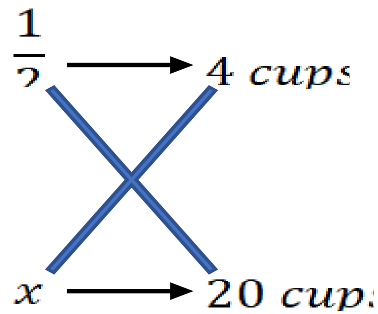


- In my initial work (Machaba, 2016), I gave learners a leaf and asked, "Does this leaf have an area? If yes, how will you find out what it is? Work out its area".
- Many learners indicated that no, the leaf does not have an area because there is no length and breadth.
- Others said they do not think the leaf has an area, because the leaf is not a rectangle and does not have length and breadth.

EXAMPLE : MASTERS STUDENT

If you need 1/2 cup of sugar and 4 cups of flour to bake a cake, how many cups of sugar will you need if you want to use 20 cups of flour?

Learners’ solution strategies

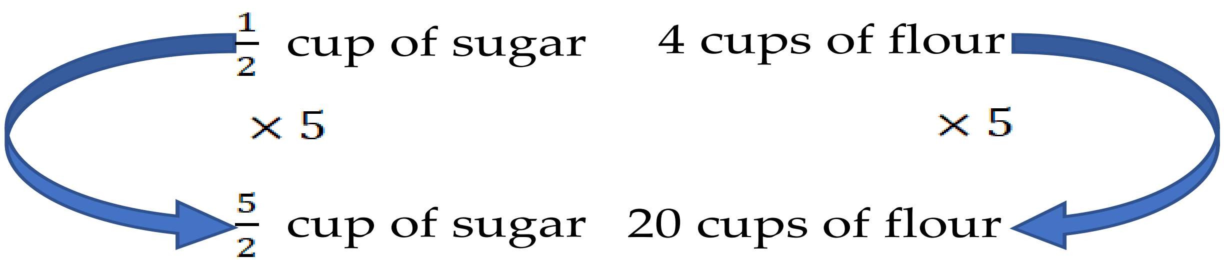


$$4 \times x = 20 \times \frac{1}{2}$$

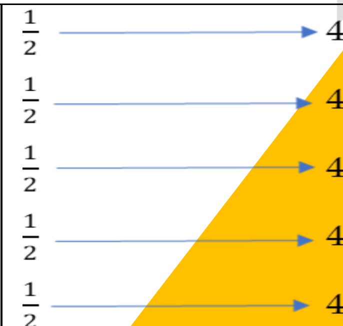
$$4x = 10$$

$$x = 2\frac{1}{2}$$

Solution strategy 1



Solution strategy 2



Answer: $2\frac{1}{2}$

Solution strategy 3

Many learners showed strategy 1 than strategies 2 and 3.

It is a procedural, formulaic approach, the academic, classic approach, privileged school - looking method ‘solving for x’, strategies which may not make sense to learners.

Solution strategies 2 and 3 are context-specific methods, non-formulaic ways of working, meaning-making, problem-solving task strategies.

“working out the problems the long way [is] tedious and stupid”, “[algebra] is a better way to go about it than prehistoric way” (Civil, 2016)

There is a valorisation of strategy 1 (‘algebraic method’) and de-valorisation of strategy 2 and 3 (non-algebraic approaches).

High status associated with being good at algebra (Machaba, 2018; Machaba & Du Plooy, 2019),

M learners seem to be associating themselves with strategy 1 and ML learners with strategies 2 and 3.

I argue that M, although highly valued as a school subject, does not necessarily translate into M applicability or the ability to apply mathematical concepts in the everyday.

Privileging an algebraic way of working does not mean that students are necessarily proficient at using algebra or that they appreciate its power of abstraction and the concept of generalisation.

ML learners seemed aware that they had to think more when using nonformulaic methods

The issue of the valuing of school M, the formulaic way of working, solving for x and a devaluing of everyday mathematics (ML) became central in shaping this lecturer.

Research Design and Methodology

For this lecture, I report on the data of only four M and ML teachers from two schools (two M teachers and two ML teachers) and two facilitators (subject advisors)—one M facilitator and one ML facilitator—from one district in Gauteng.

Thus, two M teachers (MTS1 and MTS2), two ML teachers (MLTS1 and MLTS2) from two schools, and one facilitator from M (MF) and one from ML (MLF) were purposefully chosen for an interview.

An in-depth interview of an hour was conducted with each of them. :

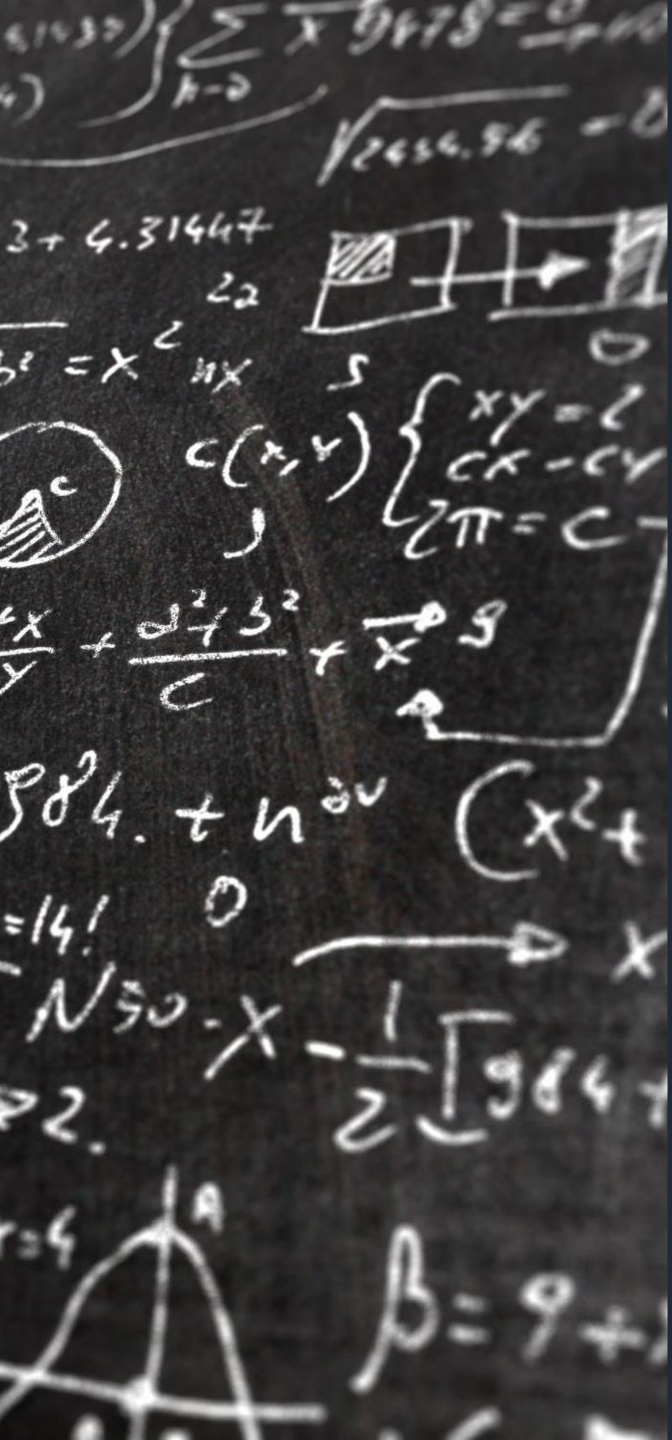
Pedagogical practices and mathematical knowledge in M and ML

The issue discussed in this section concerns pedagogical practices related to the teaching of M and ML.

Key to the pedagogy relates to mathematical rules, for example, those linked to the teaching of fractions.

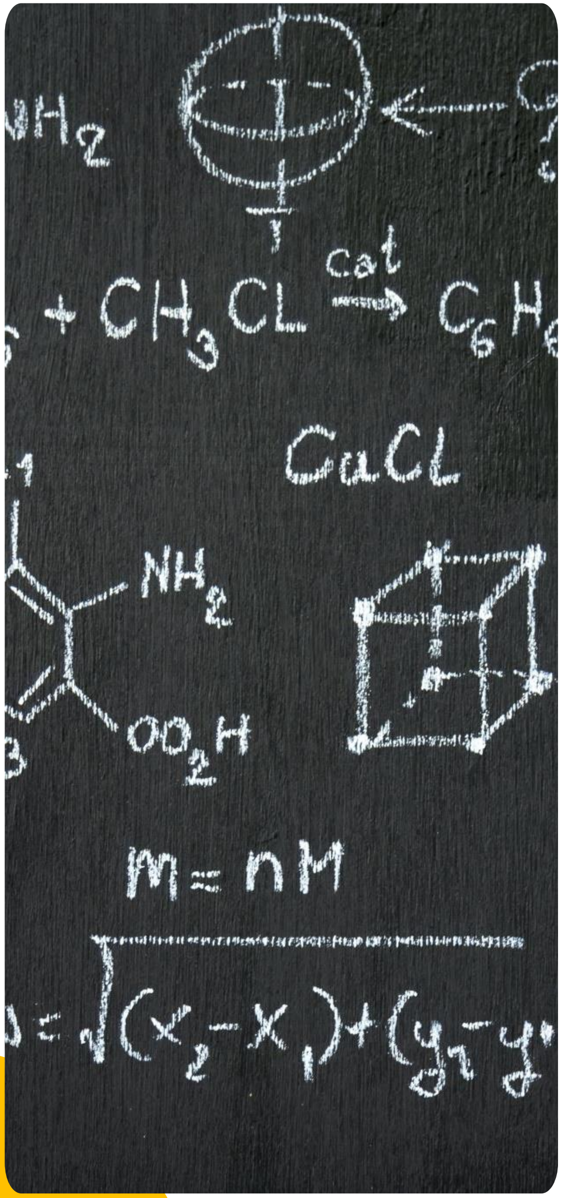
ML is also associated with reasoning, and problem-solving strategies while M is seen as a discipline that deals primarily with the application of rules.

This difference suggests an issue needing further discussion, i.e., that the strategy for teaching may be domain specific



Mathematics for direct teaching

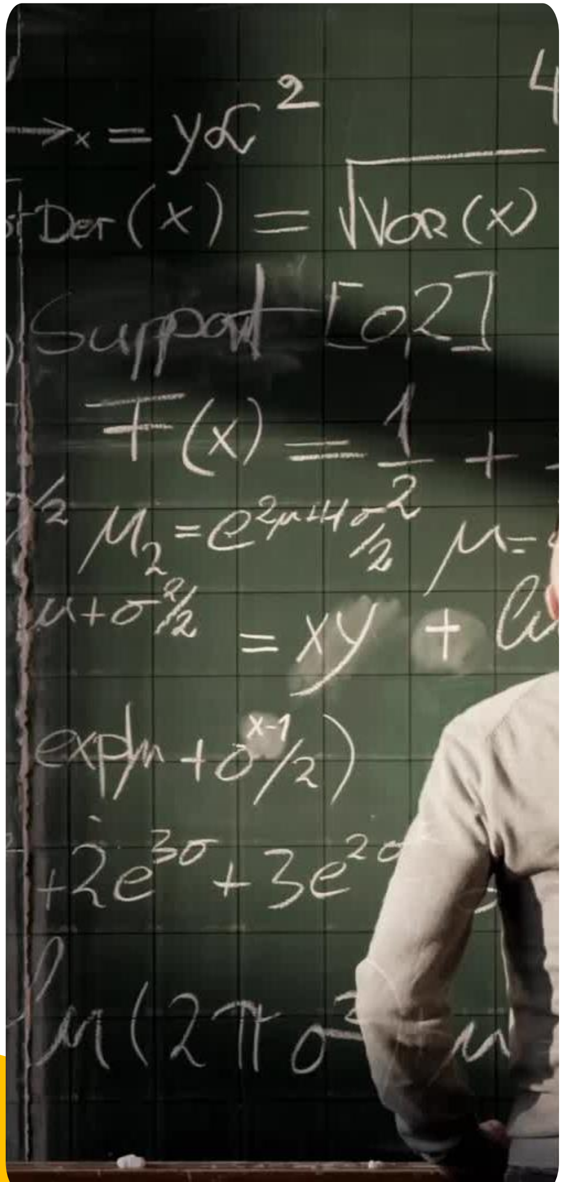
- When asked about how to teach mathematics, MLTS2 said:
  - to be a good M teacher needs one to be able to *explain maths concepts,* dedicated and *able to control learners' work and give feedback,* able to *transfer your understanding of maths concepts* to learners. Let learners *do lots of maths practice*.
- Similarly, when asked to describe and explain what makes one a good M or ML teacher, MF and MLF said the following:
  - In M, as an educator, *give a lot of activities* for learners to practise, supervise them, give them *extra work, an extra lesson* on Saturday and tell learners that M is simple (MF).
- MLF said:
  - [In mathematics it] is the teacher who let learners *practise a lot, …* M does not need one to talk but *a lot of practice*; you talk few minutes by *providing an example* and giving learners work to solve. You give them a *similar problem* of the same concept but not *different problems* at a time.



- The extracts above imply that mathematics is taught by giving a "lot of practice" to learners.

- The fact that in M teachers expected to "transfer their understanding of maths concepts to learners" suggests that mathematics appears to require direct teaching (Selling, 2016) as opposed to a learner-centred approach (Brodie, 2008).

- This is in contrast to reformed instructional practices which encourage learners to construct mathematical knowledge by themselves through investigations and discovery of knowledge which enhance conceptual understanding, (Hiebert et al., 1996; Moyo & Machaba, 2021).



- It appears that in terms of Bernstein's ideas both classification and framing are expressed as being strong.
- It seems that teachers have more control than learners over how knowledge is selected, sequenced and evaluated in the classroom.
- This suggests that neither mathematics teachers nor facilitators have acquired and developed new images of 'good practice' for mathematics teaching (recognition rules), and new pedagogic identities (forms of consciousness) that enable them to carry out these practices (realisation rules).
- Teachers and facilitators are still reflecting on "old" recognition and realisation rules for teaching mathematics.

Mathematical Literacy for problem-solving and Mathematics for procedures

- On the other hand, MLTS1 views M to be taught through rules, procedurally using steps, while ML is taught through a problem-solving approach. MLTS1 said:

  “but if an M teacher steps into an ML class and starts to teach ML learners like he/she is teaching mathematics learners steps 1, step 2, are not going to work. In mathematics, problems are given, and steps followed the application of rules—never get into the problem-solving. In ML I let learners figure things out,

  MLF added:

  > in M learners are not in a problem-solving approach, but they are taught steps, procedures rules, and how to get to the answer.

-

- Mathematics is taught/presented procedurally, step by step, following rules, with the main objective being to arrive at the answer, whereas ML is taught using a problem-solving approach and real-life problems are given.

- A "nugget" approach seems to characterise ML. To help learners get into the problem, they are "give(n) some nuggets on how to get the problem and what the problem is all about".

- MLTS1 identifies ML with a "problem-solving approach", and M with a "procedural" approach.

- MLF considers it problematic to use a procedural approach to ML.

- It appears that time is also a key aspect that distinguishes how the two subjects are approached.

- As MTS2 said:

  "When I teach ML, I facilitate learning more than when I teach M. I give the problem to learners, let them work on their own, and go around and check what and how they are doing the problem".

  "When I start teaching M I had these wonderful ideas that I am going to change, but when I got into the class is a different situation altogether. …… in M the syllabus is so huge I cannot facilitate learning, whereas in ML we have a lot of time so I can facilitate learning. And so, in M you are forced to give learners rules, and content and let them work at home so that you cover the syllabus".

Mathematics for rules and Mathematical Literacy for reasoning

When MF was asked to comment on Task 1, we ended up focusing on the discussion of the addition of fractions since she was talking about the Lowest Common denominator (LCD). I gave her an example, to add $\frac{1}{2}+\frac{3}{4}$, She said the following:

> If learners learn about fractions, they know that there is always a numerator and denominator, so a good M teacher will say, "Let's look at the LCD, you multiply each fraction by the LCD".

So, according to MF, a mathematics teacher would use an LCD Maths rule strategy to add $\frac{1}{2} + \frac{3}{4}$. When asked what other strategy could be used to add the above fractions, she said: "No, I don't have, there is no other strategy". I asked her to read $\frac{1}{2} + \frac{3}{4}$. She read, "Half plus three quarter". So, I asked, "What is half of the bread plus three-quarter of bread?" (Little did I know by then that through asking this question, I was drawing from her household knowledge.) She said, "One loaf of bread and a quarter of bread, which $1\frac{1}{4}$". I said, "Does it need an LCD"? It appears that MF started to make sense of the problem when I drew from everyday knowledge.

ALTERNATIVE STRATEGIES WITH MF

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{4} = 1\frac{1}{4}$$

OR

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1\frac{1}{4}$$

Figure 3. Decomposed Strategy

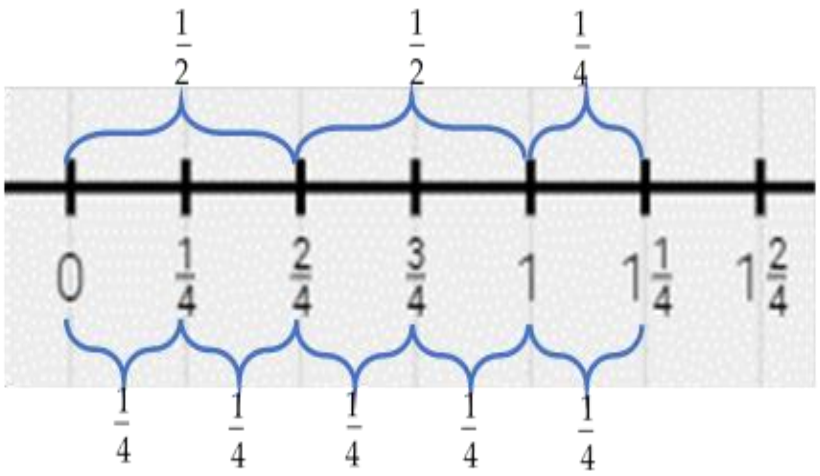


Figure 4. Number line strategy.

In fact, after spending a long time trying to come up with alternative strategies, MF did not believe that those strategies could be used in the M class, but rather that they are ML strategies. She said:

> ML deals [s] with the context, like for example the one we were dealing with now of $\frac{1}{2} + \frac{3}{4}$, to say how many quarters therein $\frac{3}{4}$ and $\frac{1}{2}$, I think this is an ML method because it is the context, this is how the ML people teach.

- MF considers a way of reasoning which regards "$\frac{1}{2}+\frac{3}{4}$" as asking how many quarters are there in "$\frac{3}{4}$ and $\frac{1}{2}$" (as in Figure 3 and 4) as an approach for ML but not for M.
- It appears that MF characterises M approaches as consisting of rules that must be followed.

- At some stage, she said, "In M we have rules and those rules I am telling you must be followed to the latter. M is about rules".

- The fact that ML is about context and the cultural nature of mathematics is not valued by MF.

- This seems to suggest that teaching and learning strategies are domain specific.

- MF1 views the use of the LCD as a strategy suited for M and the splitting (decompressing) strategy and the number line strategy (Figure 3 and 4) for ML.

- ML strategy appears to make more sense because the emphasis is more on understanding the concepts and making sense of the problem through drawing from everyday context, its strategies are still devalued when compared to mathematics algorithmic, procedural strategies which do not make sense (Civil, 2002; 2016).

- In Bernstein's (2000) terms, it appears that mathematics teachers' recognition and realisation rules (orientations to meanings based on how they have been socialised and inducted into the M and ML discourses) seem to be reflecting the old Apartheid curriculum of recognising and teaching mathematics as rules and procedures without understanding.

- It appears that teachers and facilitators have not yet institutionalised the bias and focus of the required mathematical official knowledge. That is, the "legitimate" text—that is accepted as good mathematics teaching practice.

The issue of associating strategies to a specific domain of knowledge in this study raises several questions:

- How do we define competence in M and ML?
- How do M and ML tasks and their classroom settings affect engagement?
- What is the role of language and cognition in M and ML settings?
- Which strategies can be said to be exclusively for M and which ones can be said to be for ML, and which ones for both?
- Also, what are the implications of having one strategy for M and another for ML?
- What are teachers' perceptions of who knows and who does not know Mathematics?
- Do teachers and learners believe in a preferred way to do mathematics?
- Who determines which ways are the preferred ones?

Conclusion and recommendations

- The data discussed reveals that learners' and teachers' strategies are domain specific.
- In other words, there are teaching strategies that are associated with M and others associated specifically with ML.
- This implies that in Bernstein's terms, the pedagogical practices (evaluative criteria) in M are viewed as rules and application of procedures.
- Bernstein refers to this type of teaching as visible pedagogy, which is associated with a performance-based curriculum.
- On the other hand, pedagogical practices (evaluative criteria) for ML are viewed as for reasoning and problem-solving

Conclusion and recommendations

- Bernstein refers to this type of teaching as invisible pedagogy, which is associated with a competency-based curriculum.
- Thus, the evaluative criteria as indicated by teachers and facilitators are inconsistent with those which are espoused in the mathematics curriculum document which seems to suggest that mathematics should be a learner–centred.
- This implies that teachers and facilitators have not yet developed the required "legitimate" text, practices, and criteria (recognition and realisation rules) for these specialised forms of consciousness in M.
- Their reflections on the recognition and realisation rules still demonstrate unreformed pedagogical practices.
- One would argue that the recognition and realisation rules reflected in ML by teachers and facilitators seem to be the required "legitimate" text as advocated for mathematics by researchers in mathematics education.

Conclusion and recommendations

- The idea of associating strategies with a certain learning area has an implication in learning and teaching M and ML.

- For learning, it suggests that ML and M learners participate in different discourse practices.

- ML learners are expected to act, think and believe differently from M learners because they are from different communities of practice and therefore their participation in their community of practice would be different.

- For teaching, it raises critical questions linked to the issue of M and ML being considered 'separate' subjects. Does it mean that they are inherently different discourses, and therefore require different identities of teachers?

- Does it mean that for one teacher to work productively with M and ML, the discourses in M and ML should be consistent (not in conflict) with the identity(ies) of the teacher? When would it be important for these discourses to be consistent?.

- I suggest that the differences between the two subjects (M and ML) should not be inconsistent (in conflict).

- If they were in conflict, what does it mean for the teacher who is teaching both?

- The teacher would have split identities, which can make the task of teaching very difficult.

- Again, for the teacher who is teaching both subjects, should not have discourses which are in conflict, otherwise, it would require ways of behaving which are different from the identity of the teacher.

Machaba's family

The End

NDO LIVHUWA !!

NDAA!!!