THE IMPACT OF 8PS LEARNING MODEL ON THE MATHEMATICAL PROBLEM-SOLVING PERFORMANCE OF GRADE 12 LEARNERS IN THE CONCEPT OF STATIONARY POINTS IN DIFFERENTIAL CALCULUS

by

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DEDICATION

For humanity entangled in everyday life problems and

Divinity the provider of required problem-solving skills

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ABSTRACT

Noting the centrality of problem solving to Mathematics and its capability to enhance learner performance in the subject, the study measured the impact of the use of 8Ps learning model on the mathematical problem-solving performance of South African Grade 12 learners in the concept of stationary points in differential calculus. The study explored how to apply the heuristic problem-solving model in the Mathematics classroom and the likely challenges its application could pose. The study sample comprised 253 Grade 12 learners and 8 teachers drawn from eight high schools in Tshwane West education district, Gauteng province. A mixed-method approach of quasi-experimental design involving non-equivalent control group and interpretive design were adopted. Constructivism and three problem-solving theories reinforced the study. Data were collected mainly using mathematical problem-solving achievement test, classroom observations and semi-structured interviews.

Quantitative data were analysed by descriptive and inferential statistical tools. The paired t-test result {t = 16.28, $p = .00001$, $p < \alpha = .05$} obtained reveals that the post-test result ($\bar{x} = 41.98,\ \sigma = 16.28$) of the experimental group is significantly higher than the post-test result ($\bar{x} = 16.01,\ \sigma = 6.15$) of the control group. The resulting eta-squared statistic, $\eta^2 = .69$, indicates a large effect size. Corroborating these results are the one-way ANOVA tests performed at $\alpha = .05$ significance level. For the high achievers, the experimental group achieved 70.6% while the control group produced 0%. The mean score ($\bar{x} = 51.25, \sigma = 6.24$) is thus statistically significant. For medium achievers, the $\{F(1,\ 48) = 22.058,\ p < .001\}$ obtained indicates a statistically significant difference in the mean scores of both groups. However, for the low achievers, the $\{F(1,\ 102) = .301,\ p = .584\}$ got yields a statistically insignificant difference. Overall, the quantitative data analysis result suggests that the 8Ps-oriented instruction applied in the experimental group was more effective than the traditional teaching methods adopted in the control group.

These results were triangulated by the qualitative data analysis results obtained. The classroom observations conducted yielded positive responses and attitudes from the participants towards the use of 8Ps-oriented Mathematics instruction. The interviews also recorded participants' favourable perceptions of the problem-solving learning method. This study therefore proposes that the 8Ps learning model be incorporated in the teaching and learning of Mathematics in South Africa.

Key Terms: 8Ps-oriented instruction, traditional instruction, mathematical problem solving, learner performance, mixed methods, constructivism, quasi-experimental design, interpretive design, differential calculus, stationary points

ABBREVIATIONS

ANCOVA: Analysis of covariance

ANOVA: Analysis of variance

CAPS: Curriculum Assessment Policy Statement

DBE: Department of Basic Education

DoE: Department of Education

FET: Further Education and Training

GDE: Gauteng Department of Education

H_1: Alternative Hypothesis

H_0: Null Hypothesis

NCTM: National Council of Teachers of Mathematics

NCS: National Curriculum Statement

NSC: National Senior Certificate

PISA: Program for International Student Assessment

TIMSS: Trends in International Mathematics and Science Study

Umalusi**:** The Council for Quality Assurance in General and Further Education and Training

UNISA: University of South Africa

ISTE: Institute for Science and Technology Education

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CHAPTER ONE

OVERVIEW OF THE STUDY

1.1 Introduction and Background to the Study

Following the abolition of apartheid in 1994, there were some significant strategic shifts in the education policies of South Africa, which had warranted the transformation of the school curriculum at one point or another. The remarkable reforms became necessary to put in place a unified national educational system premised on democracy, transparency and equity which would serve as an acceptable replacement for the discriminatory and fragmented educational system instituted by apartheid (Bantwini, 2010; DoE, 1996). The Department of Education (DoE) had set a vision of a South Africa that provides all the people fair access to training opportunities and life-long learning, the type that would improve the people's quality of life and result in a democratic, prosperous and peaceful South African society. The DoE added that the goal was to establish a country in which everyone, free from any form of discrimination, was entitled to a foundational education which included adult education (DoE, 1996; Wet & Wolhuter, 2009).

Among the various reform strategies taken into consideration to raise the standard of education for an average South African learner, the DoE had introduced the Outcomes-based Education (OBE) across South Africa as a crucial tool for educational transformation in 1994 (DoE, 2001). OBE was an educational philosophy that specified competencies and not just passes or fails. It was particular about the outcomes of learning rather than aims and objectives. Apart from representing a learner-centred education as opposed to a teacher-led classroom engagement, it attached due importance to the language of learning, the assessment criteria (which the learners had to be familiarised with right from the start of their learning period), selection of learning materials, the learners' varying backgrounds, and the usage of exemplars, among others (DoE, 2001; Warnich & Wolhuter, 2009).

An offshoot of the OBE was Curriculum 2005 – an educational policy designed to promote knowledge in local contexts yet aligned with global perspectives. The C2005, as often fondly referred to, indicated definite knowledge, skills and values which learners were expected to acquire in South African schools, which they were also expected to apply meaningfully to their individual lives (DoE, 2001). Launched in 1998, the educational policy was supposed to be fully implemented across all school grades by the year 2005. The C2005 had three particular features. First, it was typified by specific outcome statements – what the learners should know, value and apply. Second, integrated knowledge was emphasised as the content and the context formed the centrality of what was required to be learnt. Thus, Learning Areas (LA) were made to replace the subjects. Third, learner-driven teaching-learning strategies were promoted. The development necessitated that the teachers, learners and parents had to learn the language of the new curriculum. Incidentally, C2005 had faced strong criticism. It was believed that the school infrastructure was not adequate to cater for the educational policy; that there was lack of enough qualified teachers; that the available teachers had poor training and therefore would not be able to implement the policy; that each subject had a large number of outcome statements; that C2005 consisted of several assessment criteria; that the nature of assessing the outcomes was complex, among others (Asmal, 2005; DoE, 2001).

Consequently, in the year 2000, the DoE commenced a review process of the C2005. This gave rise to the Revised National Curriculum Statement in 2004 and eventually the National Curriculum Statement (NCS) in 2007. In the NCS, the various noticeable shortcomings of the previous education policies were pinpointed and addressed. For instance, the assessment criteria and the outcome statements were appropriately made fewer. Schools were put in phases whereby each phase selected suitable outcomes for each subject. Within each school phase, teachers did come together to determine the contents to be taught, the assessments, the expected outcomes, appropriate interventions to take care of learners' challenges etc., all in alignment with other phases and the National Curriculum Statement (DoE, 2001). In 2012, the

Curriculum and Assessment Policy Statement (CAPS) was introduced as the educational system in the democratic nation. This is seen as a restricted curriculum which has already pre-determined the content to be taught, how and the rate at which it is to be taught. CAPS, which has acquisition of the knowledge content as its main focus, is the education policy that is still in use in South Africa till date (DoE, 2001; DBE, 2011, DBE, 2012; DBE 2020; DBE 2021).

Hence, the teaching and learning of Mathematics had undergone several reform initiatives over the past few decades with the growing concerns and moves to engage learners of Mathematics in more concrete, more meaningful real-life contextual activities. Rather than continue to allow learners to acquire Mathematics knowledge by rote-learning, mere mastery of formulae and algorithms or by passive reception from external sources such as teachers and textbooks, attention is fast shifting towards engaging them effectively in active construction of Mathematics knowledge. For improved learner achievement in Mathematics, NCTM (1989, 1991, 2000, 2003) advocates that the primary objective of Mathematics teaching and learning should be problem solving and that every learner must be actively involved in problem-solving activities. The teachers' council strongly advocated for the creation of mathematical ideas by solving problems arising from Mathematics and other domains; adapting and applying a series of appropriate techniques for the purpose of the problem-solving process, allowing in-depth thinking about mathematical problem-solving processes, and adjusting to adapt to the situation.

The recommendation made by the NCTM has secured the backing of various researchers. Pimta, Tayruakham and Nuangchalerm (2009) observe that learners who are weak in Mathematics often find problem solving challenging. English and Sriraman (2010) therefore assert that such learners should be afforded the Mathematics classroom experience that can expose them to real-world problem situations and also be empowered to generate innovative mathematical knowledge, rather than just applying previously learnt concepts, rules, and techniques as is the case in many Mathematics classrooms. While Cobb, Yackel and Wood (2011) describe problem solving as the foundation of all Mathematics activities, Schoenfeld (2013) remarks that learners' knowledge of Mathematics improves as they solve Mathematics problems. Still in line with the NCTM recommendation, Aydogdu and Ayaz (2015) declare that problem solving should be the emphasis of the Mathematics curriculum because it contributes to Mathematics itself. The present study too declares a steady support for the NCTM recommendation and the various subsequent calls for investigations into applicable mathematical problem-solving skills and strategies that could improve learner achievements in Mathematics.

It is thus no overstatement that strengthening learners' problem-solving skills is a central tool that can improve their achievements in Mathematics (Bedada, 2021; Chirinda, 2013; Chirove, 2014; Dhlamini, 2012; Dhlamini & Mogari, 2011; English & Sriraman, 2010; Jupri & Drivers, 2016; Kodisang, 2022; Mogari & Lupahla, 2013; Ofori-Kusi, 2017; Palanisamy & Nor, 2021; Senthamarai, Sivapragasam & Senthilkumar, 2016; Zulyadaini, 2017). In accordance with this view widely expressed by researchers, the DoE in South Africa acknowledges problem solving as the heart of Mathematics and a process that enhances learner performance in the subject. Thus, it specifies clearly in its CAPS that it seeks to train students who can identify problems, find appropriate solutions to the problems and make decisions through thinking critically and creatively (DBE, 2011; DBE 2020; DBE 2021). Equally accepting that learning Mathematics through a problem-solving approach is capable of facilitating learners' understanding of the subject, the present study purposefully designed the 8Ps-oriented learning model to assess the Grade 12 learners' mathematical problem-solving performance in the concept of stationary points in differential calculus.

1.2 Statement of the Problem

Grade 12 learners' low understanding of differential calculus and their poor achievements in it notably in the NSC Mathematics Paper 1 has been a source of increasing concern for teachers, researchers and the Department of Basic Education in South Africa. As documented by DBE (2011) in CAPS Grade 12 Mathematics curriculum, only differential calculus takes 35± 3 marks (about 23%) of the total obtainable 150 marks for NSC Mathematics Paper 1. The implication of this is that the learners' under-performance in the topic contributes significantly to their failure in the entire subject. Talking about the cognitive levels of the same NSC Mathematics Paper 1 questions, the questions testing the learners' problem-solving skills are allotted 15% of the whole paper. As DBE (2011) further states, those questions on problem solving have to be non-routine problems that require higher-order reasoning and learners' ability to break down the given problems into constituent parts.

Studies (for instance, Areaya & Sidelil, 2012; Bedada, 2021; Brijlall & Ndlovu, 2013; Hashemi, Abu, Kashefi & Mokhtar, 2015; Lasut, 2015; Luneta & Makonye, 2010; Omoniyi, 2016; Pillay & Bansilal, 2014; Rabadi, 2015; Sahin, Cavlazoglu & Zeytuncu; 2015; Sebsibe, 2019; Yimer, 2019, to mention but a few) report that, an effective tool by which the learners can overcome their difficulties in calculus and improve their conceptual knowledge of it is by acquiring considerable problem-solving skills. There is therefore the need to find suitable learning methods to equip the learners with necessary problem-solving skills and strategies which can improve their achievement in the topic particularly and Mathematics in general.

In view of this, the current study designed the 8Ps learning model and set out to determine the effect of its use on the mathematical problem-solving performance of the Grade 12 learners in the concept of stationary points in differential calculus. The study sought to probe the nature and the level of the learners' mathematical problem-solving skills. The goal was to see how this informed their knowledge of the concept and their academic achievements in it. The study meant to obtain concrete explanations for what the learners' solution attempts were after being exposed to the 8Ps learning method; what mathematical reasoning they applied that led them to the solution moves; how and why they applied their solution strategies, and why their solutions came out the way they were.

1.3 Motivation for the Study

Problem solving is a fundamental component of Mathematics instruction. Hence, research into the teaching methods that improve students' learning in this area is necessary (Klang, Karlsson, Kilborn, Eriksson & Karlberg, 2021). This study was essentially prompted out of a concern to find measures that could remedy the learners' weak mathematical problem-solving performance. The researcher felt particularly touched by the annual Examiners' Diagnostic Reports on NSC examinations (as contained in DBE, 2014; 2016; 2020; 2021) lamenting that Grade 12 Mathematics learners often demonstrated weak mathematical problem-solving skills while solving questions on differential calculus. Every year, the NSC examiners did describe the responses given by candidates to questions on differential calculus as generally poor.

Moreover, from personal experience over the years as a Grade 12 Mathematics teacher and NSC Mathematics Paper 1 examiner, the researcher noticed that the Grade 12 learners found it difficult to grasp the concept of differential calculus. In support, anecdotally, in April 2017, he subjected 220 Grade 12 Mathematics learners of four secondary schools in Limpopo province (the secondary school where he was teaching then and three other ones nearby) to questions based on the concept of stationary points in differential calculus drawn from previous years' NSC examinations. The learners' responses to the questions also revealed that their mathematical problem-solving performance in the topic was rather deficient.

Another justification for this study is the discovery that differential calculus is one of the ten learning areas in the Grade 12 Mathematics curriculum that Mathematics teachers find difficult. Many of the teachers have not discovered appropriate methods that could make the learners acquire considerable conceptual understanding of the topic. Hence, the learners' performance in it has been noticeably low (Bezuidenhout, 2001; Brijlall & Ndlovu, 2013; Luneta, 2007; Luneta & Makonye, 2010; Sebsibe, 2019, among others).

Essentially, the current study can be considered as a contribution to previous research in Mathematics Education towards developing appropriate learning strategies that can equip Mathematics learners with necessary mathematical problem-solving skills, which will then boost their knowledge of differential calculus particularly and Mathematics as a whole. This study accepts the widely-held notion that problem solving is crucial to the learning of Mathematics and that applying suitable instructional methods to develop learners' mathematical problem-solving skills can enhance their understanding of the subject.

1.4 Aim and Objectives of the Study

The major aim of this study was to find out whether the use of the 8Ps learning model would have any effect on the problem-solving performance of Grade 12 learners in the concept of stationary points in differential calculus. To this end, the following research objectives were established:

1. To determine whether the use of 8Ps learning model in the teaching and learning of the concept of stationary points in differential calculus will have any effect on the learners' mathematical problem-solving performance.

2. To find out how the 8Ps learning model can be used in the teaching and learning of the concept of stationary points in differential calculus.

3. To determine what challenges, if any, that the use of the 8Ps learning model poses in the teaching and learning of the concept of stationary points in differential calculus.

1.5 Research Questions

The study raised and addressed the research questions below:

1. Will the use of 8Ps learning model in the teaching and learning of the concept of stationary points in differential calculus have any effect on the learners' mathematical problem-solving performance?

2. How can 8Ps learning model be used in teaching and learning the concept of stationary points in differential calculus?

3. What challenges, if any, does the use of 8Ps learning model pose in teaching and learning of the concept of stationary points in differential calculus?

1.6 Significance of the Study

In the everyday classroom context in South Africa till date, much attention is yet to be accorded administering Mathematics instruction to learners by problem solving (Dhlamini, 2012; Dhlamini & Mogari, 2011; Klang, Karlsson, Kilborn, Eriksson & Karlberg, 2021; Kodisang, 2022; Santos-Trigo, 2019). Hence, the current study came up as part of the ongoing efforts to promote learning Mathematics by problem solving in South African schools. The significance of this study is highlighted below:

- The study may serve as an avenue to address the issue of the learners' weak mathematical problem-solving performance as well as a means to provide suitable problem-solving skills and strategies to support the learners during mathematical problem solving.

- It may be useful in determining the nature and level of problem-solving skills of Grade 12 learners in tackling non-routine questions in the concept of stationary points in differential calculus in particular, and Mathematics in general.

- It may be a way to identify those difficulties that the learners experience in the process of solving problems based on the topic (and Mathematics generally) and as well a guide towards helping them overcome the difficulties.

- Besides, this empirical investigation may assist Mathematics educators in constructing helpful problem-solving instruction that can facilitate the learners' critical and problem-solving mathematical skills.

- It may also help Mathematics educators discover meaningful ways to engage learners in logical mathematical processes of reasoning needed for understanding mathematical concepts and solving Mathematics problems.

- The current study may equally make the educators understand the features typifying an instructional Mathematics environment that focuses problem-solving activities. By this, they can properly understand how learners' mathematical problem-solving performance can be explained, characterised and assessed.

- It may as well be a basis for accessing valuable participants' views about the essence of learning Mathematics through problem solving and usage of 8Ps-based learning method being proposed.

- Importantly, the study may open new research pathways on mathematical problem solving and also contribute to the on-going efforts at finding suitable problem-solving instructional methods for the teaching and learning of not only differential calculus but Mathematics in general in South African high schools.

1.7 Operational Definitions of Key Terms Used in the Study

With a view to facilitating the understanding of this study, the following key terms as used in the study are clarified as follows:

Problem: This is any difficult task or situation without an immediate clue of how to go about solving it.

Mathematical Problem: It is a mathematical task that needs a solution and which solution pathway is not obvious. The mathematical problem that this study centred on was a set of questions on stationary points in differential calculus, taken from recent past NSC Mathematics Paper 1.

Non-routine Mathematical Problem: This is a somehow complex, unfamiliar and challenging mathematical task which requires multiple solution methods and deep, critical mathematical reasoning before its solutions can be reached.

Routine Mathematical Problem: It is a simple, familiar and easy-to-solve mathematical question whose answer is quite predictable. It is a problem that can be solved by applying computational strategies and an already-known process.

Mathematical Problem Solving: It is viewed as the process of following some helpful heuristic steps and strategies to tackle challenging, well-structured, non-routine mathematical tasks whose answers are not instantly obtainable. Mathematical problem solving is used interchangeably in this study as problem solving in Mathematics.

Mathematical Problem-solving Skills: They are the mental processes or reasoning capabilities that enable a learner to choose the most suitable of the mathematical problem-solving techniques, go through the process of mathematical problem solving successfully and solve the mathematical problem correctly.

Mathematical Problem-solving Strategies: These are the steps or techniques a learner follows while thinking and navigating through a given Mathematics problem in order to arrive at the correct answer.

Stationary Points: These are the points on the graph where the gradient is zero. The graph's tangent is horizontal or parallel to the $\bar{x}$-axis given a stationary point. The three different categories of stationary points are the minimum point, maximum point and point of inflection.

8Ps Learning Model: It is a heuristic problem-solving framework purposely developed for this inquiry to probe the mathematical problem-solving performance of Grade 12 learners in South Africa. The acronym 8Ps refers to the eight phases of the problem-solving learning model and are namely: *Probing*, *Pinpointing*, *Patterning*, *Projecting*, *Prioritising*, *Processing*, *Proving* and *Predicting.*

8Ps Problem-solving Instruction: It refers to the series of lessons based on stationary points in differential calculus designed in line with the 8Ps learning model. This instruction was administered to only the experimental group in the study.

Traditional Instruction: It points to the direct, teacher-centred, *talk-chalk* teaching administered to the control group in the same series of lessons on stationary points in differential calculus.

Mathematical Problem-solving Performance: This refers to how well the participating Grade 12 Mathematics learners actually understood and solved correctly given mathematical problems on the concept of stationary points in differential calculus.

1.8 Structure of the Thesis

Chapter one provides the orientation to the study. It captures the introduction and background to the empirical inquiry; motivation for the study; problem statement; research aim and objectives; research questions, significance of the study, definitions of important terms and concepts used in the study; structure of the thesis and summary of the chapter.

Chapter two discusses the conceptual framework of the study. It centres mainly on three topical issues underpinning the study: learning of stationary points in differential calculus, mathematical problem solving and the description of 8Ps learning model.

Chapter three accounts for the theoretical framework of the study and also reviews literature relevant to the study.

Chapter four describes the research methodology followed in the study. This encompasses the research paradigm; the research design; research hypothesis, research population, sample and sampling procedure; instrumentation, development, validity and reliability of instruments; pilot study; practical classroom application of the 8Ps learning model; method of data collection and ethical clearance.

Chapter five focuses the analysis of the pilot study data and the results.

Chapter six deals with the analysis of the quantitative data for the study and the results of the analysis.

Chapter seven concentrates on the analysis of the qualitative data for the study and the results from the analysis.

Chapter eight consists of the summary of the study, discussion of results, conclusion and recommendations.

1.9 Reflection on the Chapter

The chapter has highlighted the various past educational policies in South Africa and CAPS, the one currently in use. It has pointed out that CAPS requires that an average South African learner have a considerable knowledge of Mathematics in order to progress in their education careers. It has mentioned that the Grade 12 learners under investigation have demonstrated weak academic achievements in the subject. Again, it has recognised problem solving as being central to Mathematics and that the learners' understanding of the subject will improve if taught through problem-solving technique. On account of this, the present study planned to evaluate the possible improvement that problem-solving instruction could bring to the learning of Mathematics through the application of the 8Ps learning model purposefully designed for the inquiry.

CHAPTER TWO

CONCEPTUALISING THE 8PS LEARNING MODEL FOR MATHEMATICAL PROBLEM SOLVING

2.1 Introduction

This chapter conceptualises the 8Ps learning model for mathematical problem solving by addressing the three major sections which underpin the study, namely: learning of stationary points in differential calculus as a vital area of Mathematics, problem solving as an integral part of Mathematics and the 8Ps learning model. The first section deals with: definitions of Mathematics, its relevance and interconnection to other subjects, and the place of Mathematics on the South African curriculum. The section talks about differential calculus as a core area of Mathematics, the concept of stationary points in differential calculus, and learners’ difficulties in learning differential calculus. The section also examines the unpopular traditional teaching method of learning Mathematics with its attendant unremarkable results.

In its second section, this chapter captures the meanings of problem and problem solving, and those skills and strategies that solving problems in Mathematics essentially requires. It describes mathematical problem solving; routine and non-routine mathematical tasks; well-structured and ill-structured mathematical questions; teacher’s essential role in learners’ mathematical problem-solving processes, and the challenges of teaching Mathematics through problem solving. Furthermore, the section discusses the poor problem-solving performance of Mathematics learners as a global phenomenon and the low mathematical problem-solving performance of South African learners particularly. In its third section, this chapter explores a number of existing problem-solving learning models. It then concludes by giving a vivid description of the 8Ps problem-solving learning model.

2.2 Learning of Stationary Points in Differential Calculus as a Vital Aspect of Mathematics

2.2.1 Definitions, Relevance and Interconnection of Mathematics to Other Disciplines

Being an old, broad, formal field of study, several scholars have defined Mathematics variously at different times. They have defined it based on its nature, varying uses, the types of problems it addresses, the methods it considers in addressing the problems, and the results it achieves. As observed by Mapaire (2016), Mathematics is regarded as a cornerstone for the future development and prosperity of a nation; it is essential to national growth in providing tools for understanding science, engineering, technology and economics; it is very valuable in the areas of public decision-making and contributions to the knowledge economy, and it provides learners with incredibly potent tools with which to explain, interpret and alter the world.

Mathematics is simply the language of science (Hussain, 2012) which uses notations and symbols to describe numerical, geometrical and graphical relationships. It is a distinctive human endeavour which entails observation, representation and investigation of qualitative links between mathematical objects as well as between physical and social phenomena (DBE, 2012). In the account of Elaine (2013), Mathematics is a discipline that is concerned with the logic of shape, quantity and order, one studying the relationships between things rather than the things themselves. Very important is how Mathematics defines and deals with concepts, and combines them into mathematical patterns and structures – noticeably abstract patterns and structures which show the connections between concepts and their structural behaviour. This study hoped to get further insight into the nature of mathematical problem-solving model.

Mathematics has proven to be a useful tool for developing ways of thinking, having abstract objects, deductive reasoning, and working with structural ideas organised in a logical structure (Chang & Beilock, 2016). Putting it differently, Mathematics is a pattern of thinking, organising and logical proof about shapes, structures, numbers, and concepts relating to everyday life (Zulyadaini, 2017). It helps in the development of thought processes which promote critical and logical reasoning, accuracy and problem-solving skills, all of which lead to good decision-making. Mathematical problem solving facilitates understanding our physical, social, and economic world and also teaches us innovative thinking (DBE, 2012). This had made the current study choose to prioritise learners' Mathematical problem-solving performance.

As a discipline that has continued to grow in breadth and depth, Mathematics is widely applied in several academic disciplines such as: computer, engineering, technology, economics, physics, chemistry, medicine, among others. As an interdisciplinary field of study, Mathematics is a crucial component of learning and doing in several academic disciplines. It is such a useful language and tool regarded as one of the fundamentals in our formal educational system (Moursund, 2006; 2012). Mathematics performs the role of coordinating phenomena in all other fields. It is a universal science, one typified as the most precise of all sciences (Irhamna, Amry & Syahputra, 2020; Lljedahl, Santos-Trigo, Malaspina & Bruder, 2016). Owing to the relevance and inter-connection of Mathematics to various other disciplines, this study set out to explore means to enhance the learners' mathematical problem-solving performance.

One major importance of teaching and learning Mathematics is to develop useful skills in learners by which to solve problems in Mathematics and in daily life. That may be why some people feel Mathematics is synonymous to solving problems and that the experience in solving Mathematics problems is capable of developing learners' thinking skills and helping them gain valuable skills for solving real-life problems (English & Sriraman, 2010; Senthamarai, Sivapragasam & Senthilkumar, 2016). A good understanding of Mathematics is therefore essential for young people's preparations for participation in and contribution to our present-day, scientific society. An increasing proportion of situations and problems being encountered in work contexts requires that they have some measure of knowledge of Mathematics before they can properly understand and successfully solve the problems. The youthful people, therefore, have to reckon with Mathematics as a critical tool they need to acquire as they encounter a wide variety of issues and difficulties in the different facets of their everyday lives (PISA, 2021). The present study also admits that, for the youth to be well-integrated into the present-day modern society, they require a considerable measure of understanding of Mathematics. Hence, this study came up to arm the youthful people with necessary mathematical problem-solving skills.

The different definitions and explanations of Mathematics given have emphasised the huge relevance of Mathematics to the sciences, various other disciplines and our everyday living. They have declared that we all require the knowledge of Mathematics individually and collectively as a society. They have pointed out that Mathematics is basically about solving problems, and that critical and logical thinking is required of a solver of mathematical problems. They have mentioned as well that the ability to think critically and solve mathematical problems can equip one with the ability to deal with problems in several other disciplines and life generally. In view of the great importance of Mathematics, Dermawan, Siagian and Sinaga (2021) thus remark that the subject has to be properly taught to learners for them to find easy how to carry out their present and future activities. Therefore, the present study sought to apply the 8Ps learning framework to gain insights into the nature and level of mathematical problem-solving performance of South African Grade 12 learners so as to equip them with problem-solving skills and strategies that can enhance their achievements in Mathematics.

2.2.2 The Place of Mathematics on the South African Curriculum

In post-apartheid South Africa, the teaching and learning of Mathematics has undergone a number of curriculum reviews and reforms, exposing teachers and learners to a variety of changes in terms of policy instructions, content topics, and theories, as each of the curriculum reformations requires the teachers to modify their pedagogical practices (Cobbinah & Bayaga, 2017). The different curriculum changes have involved either addition of new topics or removal of some topics or changes in the assessment criteria. Whenever new topics are added, regrettably, the educators are most times unsure of their level of content knowledge (Phasha, 2016).

The existing structure of South Africa's basic education is officially classified into two broad bands namely: The General Education and Training (GET) and the Further Education and Training (FET). The GET band serves as Grade R to Grade 9. This band is sub-grouped as: Foundation Phase (Grade R - 3), Intermediate Phase (Grade 4 - 6) and Senior Phase (Grade 7 - 9). The FET band, on the other hand, constitutes the Grade 10 - 12 learners. It is worth noting that, at every phase of education in the republic nation, Mathematics (or Mathematical Literacy) appears in the curriculum as a core subject that every learner has to study and pass to be able to proceed to the next class. Learners in the Foundation Phase, Intermediate Phase, Senior Phase and FET are supposed to offer Mathematics for 7, 6, 4.5 and 4.5 hours per week respectively. In fact, the time allocated to the teaching of Mathematics in all the cases is (about) the highest in the curriculum (DoE, 2003; DBE, 2012). This study chose to investigate Grade 12, being the final grade of the FET phase when the learners write their summative assessment (matriculation examinations) and being as well the terminal point of the band linking the high school education to the tertiary education.

Justifying the huge importance accorded Mathematics, the DBE comments:

> *Mathematics remains a core subject in the curriculum of any learner who desires to pursue a career in the physical, mathematical, computer, life, earth, space, environmental sciences or in technology. Through a thorough understanding and effective application of its knowledge and abilities, the study of Mathematics opens doors to rewarding activities and advances one's personal, social, scientific and economic growth* (DoE, 2003; DBE, 2012).

In view of this, the DBE recommends that all learners moving on to institutions of higher learning must be mathematically literate in order to succeed in whatever subjects they choose to pursue, as Mathematics in the FET Phase serves as the link between the FET Phase and the tertiary education band. Hence, this study looked into ways to improve learners' mathematical problem-solving performance.

In the NCS, the DBE mandates every Grade 10 - 12 learner in South Africa to take either Mathematics or Mathematical Literacy, believing that everyone needs mathematical skills in their everyday lives. Hence, it is essential that Mathematics instruction should foster learners' capacity for logical and critical thinking. It should produce capable problem-solvers who can confidently navigate the complex and fast evolving technological society (DoE, 2003). In its CAPS aims for Grade 10 - 12 Mathematics, the DBE expresses clearly the need to develop in learners cognitive and problem-solving skills that would enable them to recognise, analyse and solve problems critically and creatively. It asserts that instruction should include not only the *how* but also the *when* and the *why* of different problem-types. This is because learners cannot effectively use their knowledge in later life without completely grasping the learning procedures and proofs (DBE, 2012). As recommended by the DBE, critical, logical and analytical skills as a very crucial factor in mathematical problem solving, is considered important in the present study.

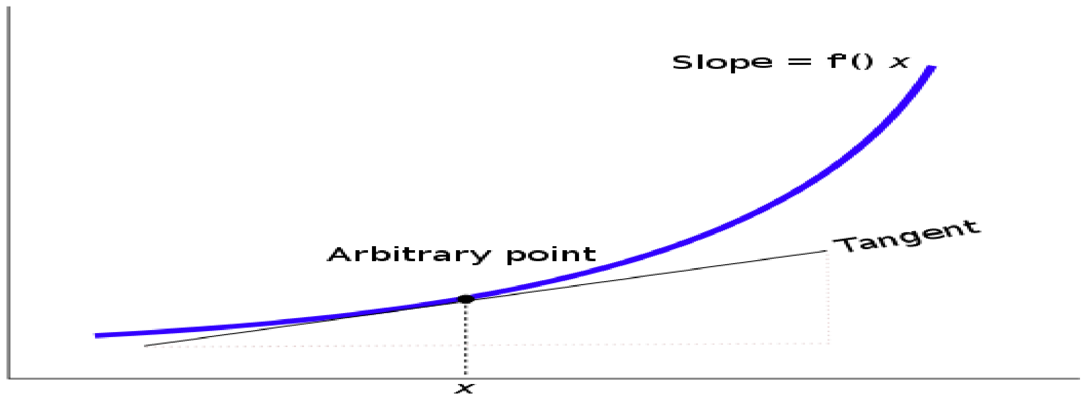
Having noted that Mathematics occupies a premium position on the South African curriculum, the current study deemed it appropriate to allow the subject its deserved importance. Thus, the study embarked on an exploration of possible effective strategies for teaching and learning the vital subject through the use of 8Ps problem-solving approach to see whether this can improve the Grade 12 learners' mathematical problem-solving performance.

2.2.3 Differential Calculus as a Core Area of Mathematics

Differential calculus is a mathematical aspect which considers how changes in one variable affect other variables (Nobre, Meireles, Rezende, Junior, Resende, Costa & Rocha, 2016). Differential calculus is an essential part of Mathematics as it is a basis for more advanced programmes in Mathematics and Engineering at secondary schools and tertiary institutions. It also has several applications in real life (Bedada, 2021). It is a pre-condition for several science, technology and engineering fields of undergraduate courses. The conceptual knowledge of calculus positively impacts learner performance in Mathematics and in the afore-mentioned fields. Differential calculus is a tool that enables people to attain greater measure of achievements than those branches of Mathematics preceding it (Roble, 2017; Sadler & Sonnert, 2016).

Apart from its centrality to Mathematics, the applications and generalisations of derivatives appear in different areas of Mathematics like functional analysis, complex analysis, algebra and geometry. Differential calculus is employed in conjunction with other mathematical fields for some important purposes (Loomis & Sternberg, 1990). For example, it is combined with linear algebra to obtain the most accurate linear approximation for a set of points in a domain. In probability theory, it is common practice to calculate a continuous random variable's probability using an underlying density function. In analytical geometry, particularly in the study of graphs of functions, calculus is used to find maxima and minima (i.e. high and low points), slope, concavity and points of inflection (Domanico, 2016; Garber, 2001; Saddler, Shea & Ward, 2011). The graph below is the derivative of a curve at a point:

Figure 2.1: Derivative of a Curve at a Point



Courtesy: *https://commons.wikimedia.org/wiki/File:Tangent_derivative_calculusdia.svg*

In fact, the uses of differential calculus go beyond the field of Mathematics. It stands as an area of Mathematics which has broad applications in a lot of disciplines such as:

science, engineering, business, economics, information systems and computer science (Mendezaba & Tindowen, 2018; Sahin, Cavlazoglu & Zeytuncu, 2015). Besides, calculus is well-applicable to real life. The many ideas in calculus have the potentials to define and model problems involving change in real-life circumstances (Yimer & Feza, 2019). Calculus is a beginning point for science and engineering students who are oriented through Mathematics in making sense of real-life problems (Boz-yaman, 2019). It is no overstatement that a considerable knowledge of the basics of differential calculus can facilitate solving real-life problems (Rohde, Jain, Poddar & Ghosh, 2012).

One of the reasons for exposing Mathematics learners to differential calculus is to train their minds in the scientific process of analysis. This is considered reasonable because science allows for the discovery of real-world problems, the generation of logical explanations for the problems, and the selection of rational solutions to the problems. The intention is for the learners to realise how to use their minds methodically to understand the world around them. Essentially, the goal of learning differential calculus is two-fold. First, it is to introduce the fundamental ideas of Mathematics for virtually any form of changing phenomena in controlled conditions. Second, it helps learners acquire practical scientific sense as well as valuable engineering problem-solving skills. It enables them to grasp how to reason rationally and break down even the most complex structures into few interconnected components (Hussain, 2012). Furthermore, differential calculus is used to understand the nature of space, time and motion more precisely. Mathematicians and philosophers spent years of research before resolving the paradoxes that came up while investigating motion and area, such as obtaining the sums of infinitely many numbers or cases requiring division by zero. The tools that clarified the paradoxes, in particular limits and infinite series, are eventually provided by differential calculus (Cheng, 2017).

Differential calculus, which was the focus of this study, is among the ten major topics in the Grade 12 Mathematics curriculum in South Africa. Usually taught for three weeks in May/June of Term 2 of each academic session, 4.5 hours being for each week, it weighs 35 ± 3 marks of the 150 total marks for NSC Mathematics Paper 1. As expected of them, the Grade 12 learners should be capable of, among other things, understanding and applying the principles of differential calculus to determine the rate of change of basic, non-linear functions and as well to solve simple optimisation problems (DBE, 2012).

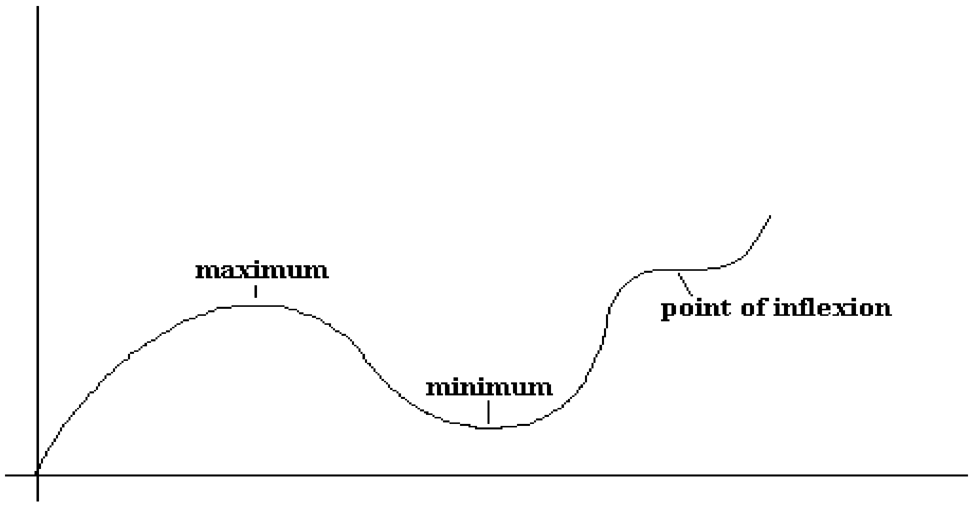
As outlined in CAPS, the policy statement currently in use for teaching and learning in all schools in South Africa, those areas of differential calculus that the Grade 12 Mathematics learners are to be exposed to are: limits, rate of change or gradient of a function at a point; derivatives of functions from first principles; applications of the product and quotient rules of differentiation etc.; equations of tangents to graphs; graphs of cubic and other suitable polynomial functions; stationary points, x-intercepts, and practical problems involving optimisation and rate of change (DBE, 2012). For a thorough investigation and also to make the study manageable, the present research has only focused on application of differentiation to obtain the stationary points.

The foregoing has emphasised the relevance and interconnection of differential calculus to other aspects of Mathematics, different other fields and real life. It has shown that the topic has the ability to expose learners to challenging tasks capable of equipping them with useful problem-solving skills. It can be safe then to appreciate the worthwhile decision taken by the DBE to include differential calculus in the list of the ten main topics for Grade 12 Mathematics learners. Based on this, the current study had got immense interest in making it a focus area for the investigation.

2.2.4 Stationary Points in Differential Calculus

In differential calculus, a point on the graph where the gradient is equal to zero is a stationary point. A stationary point can be a minimum or maximum point, or a point of inflection. At a stationary point, the tangent to the graph is parallel or horizontal to the x-axis; hence, $f'(x) = \frac{dy}{dx} = 0$ (HELM, 2008, 2015; Nicholas, 2004). The concept of stationary points has proven to be an important aspect of differential calculus for solving practical optimisation problems. When Mathematics is adopted to model our physical world, physical quantities are defined in the form of variables. Functions are therefore used to explain how the variables change. For example, engineers, scientists and economists are deeply interested in the *ups* and *downs* of a function, that is, its maximum and minimum values (also called its turning points). In various applications, the professionals too develop a high interest in these points for minimising costs or losses, and as well for maximising profit or power. Again, they make use of the concept of stationary points for the selection of the best element from among some available alternatives, considering certain criteria (Domanico, 2016; Saddler, Shea & Ward, 2011; Thomas, 1997). The graph below illustrates the three stationary points:

Figure 2.2: The Three Stationary Points



Courtesy: *https://revisionmaths.com/advanced-level-maths-revision/pure-maths/calculus/uses-differentiation*

The stationary point(s) of a function *f*(x), is obtained by first finding $f'(x) = \frac{dy}{dx}$. Thereafter, the zeroes of $f'(x)$ should be got, and then their corresponding values of *y*. After getting the stationary point(s), finding the nature of the stationary point(s) is another point of interest. That is, it remains to find out whether each stationary point is a maximum or minimum point, or a point of inflection. To do this successfully, one must find the second derivative, $f''(x) = \frac{d^2y}{dx^2} = 0$. Alternatively, the nature of a stationary point is determined by considering the gradient at either side of the stationary point. This obviously demands a proper knowledge of differentiation (Domanico, 2016; HELM, 2015; Saddler, Shea & Ward, 2011).

Particularly for being an important aspect of differential calculus that has practical applications and offers unquantifiable benefits, the current study selected the concept of stationary points as the area to work on. It dwelled on only this aspect of calculus since a limited scope could be covered for the research to be manageable, given the limited time and available resources.

2.2.5 Learners' Difficulties in Differential Calculus

In South African schools, differential calculus is introduced to learners for the very first time in Grade 12. This topic, which constitutes a main aspect of the Grade 12 Mathematics curriculum, had been taught for several years (Bansilal & Pillay, 2014; DoE, 2007). Differential calculus is one of the topics that the learners find challenging (Areaya & Sidelil, 2012; Bedada, 2021; Brijlall & Ndlovu, 2013; Omoniyi, 2016; Sebsibe, 2019). The DBE also observes that the Grade 12 Mathematics learners find the learning of differential calculus difficult; particularly solving problems on first principles and rules of differentiation, cubic function, stationary points and applications in optimisation (DBE, 2014; 2015; 2016; 2019; 2020; 2021).

The learners' difficulties in differential calculus may be due to how Mathematics teachers do concentrate on the procedural aspects without promoting the conceptual knowledge of the topic, which is only obtainable through deep learning – a learning with understanding as opposed to surface learning. The teachers do not realise that the two forms of knowledge follow a bi-directional process, meaning that one feeds the other (Bezuidenhout, 2001; Rittle-Johnson & Schneider, 2015; Zachiarides, Pamfilos, Christou, Maleev & Jones, 2007). Zachiarides et al. (2007) trace the causes to the learners' general perception of the topic as abstract and involving complex ideas. Artique, Batanero and Kent (2007) attribute the cause of the learners' difficulties in differential calculus to their inability to grasp the main concepts of the topic. In a comment by Brijlall and Ndlovu (2013), learners perform poorly in differential calculus because they depend mostly on procedures and isolated facts, and that the resource materials the teachers use tend to encourage the instrumental understanding of differential calculus concepts rather than its relational knowledge.

Many learners dread and fail calculus because of its complex and abstract nature and the methods used in communicating it to learners. As such, Mathematics learners require higher-order thinking skills to be able to cope with differential calculus (Sahin, Cavlazoglu & Zeytuncu, 2015). Another cause is that the teaching of differential calculus by the traditional method does not make learners understand its fundamental concepts (Axtell, 2006). The traditional approach does not really enable learners to reason critically and creatively, and has thereby contributed to the learners' inability to make meaningful achievements in calculus. Hence, teachers of Mathematics are implored to be very mindful of the learning strategies that accommodate learners' prior knowledge, their different learning styles and personal experiences (Arbin, Ghani & Hamzah, 2014). Owing to the challenging nature of the concept of calculus, the teachers need to pay special attention to careful selection of appropriate learning strategies for the learning of calculus lessons (Yimer, 2019).

Hashemi, Abu, Kashefi and Mokhtar (2015) declare that one means by which learners can achieve an improved conceptual understanding of calculus is the level of their problem-solving knowledge. Rabadi (2015) equally argues that problem solving might be a suitable and effective tool for overcoming learners' conceptual difficulties in calculus. Lasut (2015) then maintains that, since calculus poses as a Mathematics topic that learners do not grasp easily, suitable approaches and methods have to be discovered for active and effective learning of the topic. As their own contribution to the ongoing efforts towards discovering an appropriate pedagogy, Mendezabal and Tindowen (2018) recommend that the teaching and learning of differential calculus can be facilitated through focusing learners' conceptual understanding of the subject, which can best be achieved by developing their mathematical problem-solving skills. This recommendation tallies with the broad aim of the current study.

2.2.6 NSC Examiners' Diagnostic Reports on the Learners' Difficulties in Differential Calculus

The annual DBE examination diagnostic reports on learners' performance in differential calculus questions in NSC Mathematics Paper 1 attest to the learners' poor knowledge of differential calculus. In the NSC diagnostic reports annually released by the Department of Education (DBE, 2014; 2016; 2020; 2021), a lot of the examination candidates, in general, display a weak understanding of the concept of differential calculus especially in the concept of stationary points and therefore do not obtain encouraging marks in the questions set on it. The candidates' common errors and misconceptions in the topic as pointed out in the said examination diagnostic reports are highlighted below.

- Several candidates do mistake the x-intercepts for the turning points and do not remember to set the derivative as zero at a stationary point. In differentiating from first principles and also while applying the fundamental rules of derivatives, the candidates often commit notational and algebraic errors such as: incorrect substitutions; faulty expansions; addition of unlike terms; changing signs erroneously while multiplying; wrong use of formulas; failure to replace x with $(x + h)$; incorrect or omission of factorisation of the numerator or the part leading to derivative, especially one involving expansion of a cubic expression, etc. Some of the learners have a shallow knowledge of fractions and exponential laws which is necessary when working with calculus. Some learners cannot distinguish the function, $f(x)$ from its gradient, $f'(x)$. Some also fail to recognise the fact that the original function has to be in the differentiable form (for easy identification of the exponent, variable and coefficient) before applying suitable rules of differentiation. Their commonest mistake is their inability to know when to leave out the derivative notation. Also, some of them do not realise that the derivative of a constant such as π is zero.

- In questions set on the graphical application of knowledge of calculus, especially when a sketch is not given, most learners get confused. They do not find it easy to interpret higher-order questions to set up two simultaneous equations. Some learners usually forget to make the derivative equal to zero when calculating the x-coordinates of the turning points. Besides, they do not realise that the turning points of a function are the stationary points. Many learners do not understand the idea of a cubic function having a turning point on the x-axis or the function having a repeated root for the x-intercept. Some learners have difficulties identifying or reading from the graph the intervals for which a function is decreasing or increasing. The learners struggle to establish the connection among a function, its first and second derivatives. They depict a limited knowledge of significant aspects of a function such as: sketching, interpretation of a given equation or a given graph, obtaining equation(s) from given information and transformations, etc. Generally, the learners perform poorly in higher-order thinking questions on interpretation of graphs.

- Learners also have a poor understanding of the concept of concavity of functions. They cannot clearly explain when a function is concave down or concave up and the intervals for which this happens. They fail to realise that a change in concavity in the neighbourhood of a point has to take place for the point to be a point of inflection. They have a shallow idea of concavity of a function changing at a given point and the reason for the change relative to the given function. They struggle to link concavity to the second derivative; hence, they do not realise that on the interval $f''(x) > 0$ meaning positive (+ve), a function is considered concave up, but concave down on the interval $f''(x) < 0$ meaning negative (-ve). Particularly when given a cubic graph, they fail to recognise that, for some functions, concavity changes at the point of inflection. For some other functions, (e. g. $f(x) = x^4$), where this is untrue, they forget to show the difference in concavity on either side of the point of inflection (that is, by changing the sign in the second derivative). These learners also get confused whenever a graph of a cubic function has one stationary point.

- In problems requiring application of calculus to areas and volumes of shapes, some of the learners use incorrect formulas or wrong substitutions. Some are not able to simplify correctly the formula for volume or area or have problems with determining the derivative correctly. It is a common erroneous practice for some learners to equate the given expression of the area or volume to zero instead of the derivative of that expression. Furthermore, the learners do not exhibit enough skills when it comes to problems of optimisation, a very essential aspect of calculus which is an ideal opportunity for the real-world application of Mathematics. Most of them only see optimisation in the context of measurements, rather than also get exposed to optimisation of functions. They handle poorly questions on rates of change and questions asking them to differentiate with respect to variables other than x. In most cases, the candidates do not interpret correctly those questions which require applying calculus to practical problems.

The NSC examiners thus lamented that, on the whole, the candidates' responses to differential calculus questions are poor. In consideration of the foregoing, the current study decided to make differential calculus its focus area. It then designed the 8Ps-based learning model as the instructional method to apply. The motive was to determine whether the use of this learning model could bring about any significant improvement in the learners' mathematical problem-solving performance in the topic and, possibly, in their other aspects of Mathematics.

2.2.7 The Unpopular Traditional Teaching Method of Learning Mathematics and Its Attendant Unremarkable Results

Traditional teaching method is about the teacher transmitting knowledge to learners. It is a pedagogy that recognises the teacher as the exclusive source of knowledge and the learners as empty vessels that the teacher must fill, and who have to remain passive during the learning process. In traditional teaching method, learners only absorb the thoughts and information presented to them as correct and complete, and have to regurgitate them when required during examinations. Rather than being learner-oriented, this conventional instructional practice is a teacher-dominated approach largely based on memorisation and rote learning. Since the teacher is fully in control of the learning process, the learners do not have the opportunity to exercise independence. Besides, a typical traditional classroom maintains a structure that rests heavily on the use of textbooks, workbooks, worksheets, curriculum content and already established facts (Bedada, 2021; Cai, 2003; Cai, & Lester, 2010; Chirove, 2014; Dhlamini, 2012; Dhlamini & Mogari, 2012; Masilo, 2019; Ofori-Kusi, 2017; Omoniyi, 2016; Sebsibe, 2019).

While canvassing for the learner-oriented instructional method, Kalu (2012) portrays the traditional teaching approach as a single-person practice whereby the learners are noticeably uninvolved and treated as empty seekers of knowledge. He reports that the teacher's direct and unilateral instructions often rule the traditional classroom, and that the major aim of the teacher, as the information-giver, is to transfer knowledge to the inactive learners without necessarily giving due attention to learner-initiated questions, interactions or independent thoughts. With this prevailing arrangement in place, learners have to blindly accept ideas from the teacher without questioning them. This implies that the traditional approach presents an already proven body of knowledge which learners have to passively receive. Expressing a similar view, Tambara (2015) states that the traditional instructional method allows the teacher to direct and dominate the learning process but only assigns to the learners the duty of memorising rules, facts, definitions, procedures and algorithms.

For several years, the traditional approach which necessitated usage of mainly the textbooks and the chalkboard for Mathematics instruction was the practice. In a typical traditional Mathematics classroom, learners were involved in few classroom activities, with a substantial portion of the instruction time earmarked for individual learner seat-work and teacher-led class recitations. The curricula and instructions had been heavily based on transmission and absorption of pre-existing skills, concepts and facts. Thus, learners did passively accept the mathematical knowledge constructed by textbooks, the teacher and some other authority sources (Bedada, 2021; Cai, & Lester, 2010; Masilo, 2019; Ofori-Kusi, 2017; Omoniyi, 2016; Sebsibe, 2019)**.** Expressing his displeasure with the situation**,** Lasut (2015) laments that, in several calculus classes, the most common teaching strategy used by Mathematics teachers is still the traditional technique, which emphasises following computational procedures rather than grasping the true underlying Mathematics ideas.

As generally documented in research (for example, Dhlamini, 2012; Dhlamini & Mogari, 2012; Masilo, 2019; Mendezabal & Tindowen, 2018; Ofori-Kusi, 2017; Ukobizaba, Nizeyimana & Mukuka, 2021), the traditional teaching method has been demonstrations and practices adopting closed mathematical problems with predetermined answers. The learning method does not engage learners in active knowledge construction. It prioritises knowing answers over exploring questions; memorisation of rules, formulas and algorithms over logical and analytical reasoning; acquiring information in bits and pieces rather than understanding a concept in context and as a whole. The approach does not promote group work or learners interacting, contributing and sharing ideas with one another**.** It follows direct instruction and exposes learners to specific standard methods of solving particular Mathematics problems in specific sequences. Thus, a given mathematical task is taught and learnt in isolation rather than in relation to other aspects of Mathematics, to other subjects or to more complex projects. Summarily, research has widely acknowledged the following as the dominant characteristics of the traditional instructional method.

- The teacher is recognised as the sole transmitter of knowledge and major source of ideas. Thus, the method is purely teacher-driven and is governed by the teacher's unilateral, direct instruction.

- The lesson structure rests largely on textbooks as well as on content, skills and plans already predetermined.

- Learners are expected to absorb a proven body of knowledge. Hence, traditional teaching method prioritises already established mathematical concepts and ideas, thereby considering knowledge as being objective rather than subjective.

- Arriving at the solutions to Mathematics problems is the main focus of learning, not necessarily the process of having such solutions. Mathematics learners are therefore not fully involved in the process of deriving the solutions.

- Fact memorisation and mechanical learning are its most prominent traits. Hence, learners are required to regurgitate during examinations the ideas, explanations and procedures that the teacher had earlier transferred to them.

- The process of learning is treated as a passive one meant to be received. As a result, the focus has been on the teacher disseminating knowledge to learners who have to passively absorb it.

- It practically accords little or no importance to learners' interaction since whole-class instruction is a common method of learning.
- Traditional teaching method views learner assessment as being distinct from learning and so requires that learner assessment be carried out primarily through conventional tests. Hence, the teacher does not see the need to fully involve learners in the assessment.

The traditional, teacher-dominated, knowledge-transmitted, textbook-guided instructional approach is inadequate and has been unable to produce satisfactory learner achievement in Mathematics in South African schools (Axtell, 2006; Bedada, 2021; Chirove, 2014; Dhlamini, 2012; Dhlamini & Mogari, 2012; Lasut, 2015; Masilo, 2019; Mendezabal & Tindowen, 2018; Ofori-Kusi, 2017; Omoniyi, 2016; Sebsibe,2019). A relevant pointer to this is the low performance of the nation's Grade 12 Mathematics learners in their final examinations popularly referred to as the matriculation examinations. In her 2020 NSC examination report on the national performance of the Grade 12 learners in Mathematics (regarded as one of the *gateway* subjects), the Minister of Basic Education, Matsie Angelina Motshekga, condemned the significant decline in the pass rates (30% and above) for Mathematics which stand at: 49.1 (2015), 51.1% (2016), 51.9% (2017), 58% (2018), 54.6% (2019) and 53.8% (2020) (DBE, 2020).

Characterised by passive reception of knowledge, memorisation and rote, mechanistic learning, learning of isolated procedures and concepts, the traditional learning approach has yielded poor knowledge transfer, weak academic attainments and under-performances. It has churned out learners without satisfactory attainments in Mathematics and adequate problem-solving skills that can make them ably forge ahead in this dynamic, technological world. The learners who are products of the approach have unfortunately not been well able to relate the Mathematics knowledge acquired to real life. They are unable to see the inter-relationship between their school Mathematics knowledge and life's practical knowledge (Lasut, 2015; Mendezabal, 2018; Nobre, Meireles, Junior, De Resende, Da Costa & Da Rocha, 2016; Sebsibe, 2019; Ukobizaba, Nizeyimana & Mukuka, 2021).

Given this realisation, researchers and other stakeholders in Mathematics Education have re-evaluated the types of mathematical experiences to offer learners, in relation to the nature and depth of the learning content, the methods used to teach it, how to evaluate learners' progress as well as how to improve learners' access to quality Mathematics education (English & Watters, 2005). The popular move is no doubt a welcome development because, as technology grows rapidly in this 21st century, traditional instructional methods have proven to be outdated and unsuitable for the much-desired effective learning. Therefore, there should be meaningful adjustments to the existing learning methods and the development of new ones for the purpose of teaching and learning Mathematics (Bedada, 2021).

Like the various interest groups in the education sectors, the current study contends that the traditional pedagogical practice can neither produce desirable achievements in Mathematics nor adequately prepare learners for a worthwhile life after school. It believes a learner-centred instructional method is required; one which can actively involve the learners in the learning and assessment processes, and engage them in personal knowledge construction. It admits that, rather than continue to transfer or transmit knowledge to learners, the teacher should allow, encourage and guide them to create knowledge on their own so that they would be able to apply the knowledge meaningfully. With his utmost belief that learners can learn Mathematics effectively through problem solving, the current study proposed the use of the 8Ps problem-solving learning model.

2.3 Problem Solving as an Integral Part of Mathematics

2.3.1 Problem and Problem Solving

Van de Walle (2001) describes problem as an activity for which learners do not have memorised or prescribed rules or easily perceivable specific methods to arrive at its correct solution. Pehkonen (2008) calls it a task whose solution warrants that an individual handling it has to combine data previously known in a new way. İncebacak and Ersoy (2016) explain problem as a difficult situation to which a person needs to find a solution but does not know exactly how but still attempts to tackle it. To Adem and Qohar (2016), problem is a discrepancy between pretension and reality. Tonnsen (2021) buttressed the same idea by defining problem as the discrepancy between two states: the state given and the state wanted, such that the state wanted cannot be realised directly from the state given by using a strategy that easily guarantees a solution. For the present study, problem is seen as a situation or an activity whose solution strategies are not instantly available, one requiring some planned steps that must be consciously executed to find the solution. With this notion, this study carefully chose five recently past NSC questions on stationary points in differential calculus as the problems that the participating Grade 12 learners had to solve.

Problem solving stands as an advanced instructional strategy currently receiving advocacy in the field of Mathematics Education (Albay, 2019; Lee & Kim, 2005;

Sesanti & Triwahyuningtyas, 2021). It is a significant, ground-breaking concept forming an integral component of Mathematics (Ersoy & Guner, 2015). Problem solving ranks among the highest levels of cognitive skill. It is a higher-order thinking skill that is helpful in honing learners’ thinking, creativity and flexibility. It promotes cooperative skills among the learners and enables them to apply the knowledge acquired to various situations (Palraj, DeWitt & Alias, 2017). Problem solving remains a key aspect of human cognition which, for a very long time, has influenced the progress of human society (Voskoglou, 2021). It is a fundamental way to develop mathematical knowledge at any level of education (Catindoy, 2021). Therefore, problem-solving skill is reckoned with as a core skill in the learning of Mathematics in schools (Dermawan, Siagian & Sinaga, 2021).

Van de Walle (2001) defines problem solving as a major teaching strategy, which begins class work from the point where learners are, as against the other modes of teaching that start from the point where the teachers are, ignoring the previous knowledge learners bring with them to the classroom. Stigler and Hiebert (2004) explain problem solving as resolving a task for which learners do not have an available instant method. According to Woolfolk (2010), problem solving is a formulation of new answers, a step beyond the easy application of previously learnt rules to realise a goal. Expressing a similar view, Lester (2013) describes it as a systematic approach to working through a problem's details without having in sight an obvious solution method. For İncebacak and Ersoy (2016), problem solving is a process which provides an idea or a solution to a situation that is problematic. As their own contribution, Asoma, Ali, Adzifome and Eric (2022) view problem solving as critical to the learning of Mathematics as it gives learners the chance to participate in meaningful Mathematical discourse, which includes examining multiple representations and reasons for the solutions they come up with. They also mention that, while problem solving offers learners the opportunity to participate actively in learning activities, it equally allows the teacher to participate fully in the classroom as a learner.

The current study sees problem solving as a methodical means of obtaining a solution to a problem – that which demands that an individual solving a problem draws on their own experience and knowledge to produce a new understanding. Believing that the person solving a problem should be able to derive rational ways to meet the demands of the present unfamiliar situation, the current study considers problem solving as a systematic procedure by which the solver has to apply some mental processes to remove the gap between their present situation and their set goal. Viewing problem solving as eliminating discrepancies therefore places premium importance on understanding the process required for achieving this. This study holds that problem solving demands having a due knowledge of the process to follow while attempting to move from the present position to the goal, which has significant implications on the logicality of the solution the solver will obtain.

In the education sector and education research, problem solving was the main goal of the 1980s. In its 1980 publication entitled: *An Agenda for Action: Recommendations for School Mathematics of the 1980's*, the National Council of Teachers of Mathematics recognises the centrality of problem solving to the learning of Mathematics and then recommends that the main emphasis in Mathematics classes should be problem solving (NCTM, 1980). The NCTM consolidates on this submission in its 1989 publication, *Curriculum and Evaluation Standards for School Mathematics,* in which it identifies the primary goal of Mathematics as developing learners' mathematical power – that is, growing the individual learner's ability to investigate, hypothesise and think analytically, and also the ability to effectively apply various mathematical methods to solve non-routine problems. Again, in its 2000 publication, *Principles and Standards for School Mathematics*, an update of its 1989 publication, the NCTM renewed its commitment to problem solving by identifying problem solving as a vital component of Mathematics learning for all grades. It describes solving problems as not only a goal of learning Mathematics but also as a main means of learning it. The teachers' body believes that in the workplace and in everyday life generally, being a strong problem solver is highly advantageous. Thus, it views problem solving as a core component of Mathematics (NCTM, 2000).

Since the 1980s, there have been several calls that problem solving be accorded increased attention in the curriculum (NCTM 1980, 1989, 2000, 2002, 2006). One widely-given reason adduced to this view is that problem solving is central to the learning of Mathematics, and that it is a very essential process that learners have to understand to be able to perform well in Mathematics (Awuah, 2018; Brijlall & Ndlovu, 2013; Chirinda, 2013; Dhlamini, 2012; Dhlamini & Mogari, 2011; Dlamini, 2017; Kodisang, 2022; Masilo, 2018; Mogari & Luphala, 2013; Son & Ditasona, 2020; Sweller, Clark & Kirschner, 2010). Cai and Lester (2010) claim that the mathematical process called problem solving is potentially able to produce the mental skills necessary to support learners' mathematical development and understanding. Corroborating this, NCEE (2012) observes that problem solving is a beneficial life skill that entails a range of processes like: logical reasoning and interpreting, predicting, analysing, evaluating and reflecting, constructing creative arguments and developing innovative strategies that are useful not only for calculus, algebra and geometry but also the entire Mathematics curriculum. The NCEE thereby recommends that problem solving should be treated as an essential purpose of learning Mathematics and not as an isolated concept.

Problem solving is probably what makes us humans. With the various problems facing our competitive economy and society, every learner is supposed to possess certain problem-solving skills (Kale & Akcaoglu, 2020). Problem-solving methods are relevant in most human endeavours. In Mathematics specifically, activities like posing, identifying and defining problems, and finding various means to solve them are pivotal to the development of the subject. For some time now, the research agenda in Mathematics Education has often included the systematic study of the process of formulating and solving problems and how to structure problem-solving techniques for effective learning of Mathematics (Santos-Trigo, 2019). In the South African context, the idea of problem solving is envisioned by the DBE in its *Revised National Curriculum Statement for R-9 (Schools)* in which it regards problem solving as a major goal of all Mathematics instruction and as well as an integral part of all mathematical activities. Hence, learners are expected to adopt problem-solving techniques to explore and comprehend mathematical content (DBE, 2012).

In consideration of the foregoing, the present study takes problem solving as being crucial and central to the learning of Mathematics. It strongly posits that producing capable problem solvers who can competently use Mathematics in several practical ways should be the primary focus of Mathematics instruction. It further maintains that effective use of problem-solving instruction can engage learners in critical, creative and analytical thinking which will make them generate reasonable mathematical ideas. This in turn will enable them to have a proper understanding of mathematical concepts and processes. This study therefore advocates that all learners should be well-exposed to the problem-solving processes for them to have a good grasp of Mathematics.

2.3.2 The Skills and the Strategies of Mathematical Problem Solving

2.3.2.1 Mathematical Problem Solving

Problem solving has long been acknowledged as very essential to Mathematics, Mathematics teaching and Mathematics learning. It has permeated Mathematics curricula of many nations of the world so much so that several calls have been made for the separate teaching of problem solving and the teaching of Mathematics through problem solving. Therefore, problem solving has been an area of interest to researchers in Mathematics Education (Lljedahl, Santos-Trigo, Malaspina & Bruder, 2016). Mathematical problem solving is a complex cognitive activity that constitutes difficulties for learners (Zhang, Jackson, Hunt, Carter, Yang & Emerling, 2021). In the account of Burkhardt and Bell (2007), mathematical problem solving means tackling mathematical tasks that significantly differ from the ones the solver has learnt by heart. The main portion of the challenge lies in the solver deciding how to solve the problem, and how to choose the mathematical strategies to use. Burkhardt and Bell (2007) further argue that the learner solving the problem needs a rich, connected understanding of Mathematics, with the ability to see patterns of association and similarity, and the skills to execute the planned attack, and to ensure that the solution obtained is sensible in the context of the problem.

Mathematical problem solving is the process of dealing with mathematical tasks which can present to learners such intellectual challenges capable of enhancing their mathematical understanding and development. Such mathematical tasks do arouse learners' curiosity and interests, trigger their abilities to think and communicate mathematically, and facilitate their conceptual understanding of the subject. A learning environment that welcomes mathematical problem-solving practices offers learners a platform to present their solutions to their classmates in a manner comfortable to them. Such a learning environment welcomes learning Mathematics by negotiations, social interactions and shared understanding. By this, learners are allowed the opportunities to explain their thoughts and gain various perspectives of the Mathematics concept being learnt (Cai & Lester, 2010).

Problem solving in Mathematics improves learning gains and enhances deep, meaningful learners' understanding of Mathematics (Awuah, 2018; Dhlamini & Mogari, 2011, 2012; Mogari & Lupahla, 2013). Holding the same view, the NCTM advocates that problem solving should be made the main emphasis of Mathematics since it encompasses functions and skills that are vital to our everyday life (NCTM, 1980). In fact, it considers problem solving as a goal of learning Mathematics as well as a key tool for doing that. The NCTM adds that being an effective problem solver has benefits in both everyday life and in the workplace. It thus affirms that problem solving remains a core concept of Mathematics learning (NCTM, 2000).

For Mathematics learners to become competent problem-solvers, they are to be engaged in solving real-life problems in the classroom, such mathematical tasks not easily tackled by direct efforts or ones needing some creative insight to solve.

According to Liljedahl (2008), asking learners to solve routine problems already suggestive of solution methods obtainable by deliberate efforts cannot lead them to new and important discoveries. He therefore recommends such non-routine mathematical tasks that learners will try and fail by deliberate efforts, and which will make them rely on sudden inspirations or intuitions. Dhlamini (2012) also observes that an appropriate way to facilitate learner performance in Mathematics is to design instruction that can develop learners' problem-solving skills. Supporting this claim, the NCTM recommends that learners must be accorded opportunities to formulate, explore and tackle complex problems requiring some amount of effort and also have to be guided to always reflect on their thinking afterwards (NCTM 2000, 2009).

PISA (2021) also makes a strong case for mathematical problem solving. It categorically states that for learners to become mathematically literate, they would have to first apply their Mathematics content knowledge to be able to recognise the mathematical nature of a problem, particularly those ones encountered in daily life and then to represent the problem in mathematical terms. This transformation process from a messy, ambiguous, real-life situation to a properly defined Mathematics problem calls for mathematical reasoning. According to the document, as soon as the transformation has been successfully done, the resulting mathematical problem should be solved using the Mathematics concepts, procedures and transformation learnt. That demands taking some strategic decisions about the choice of the tools to adopt and the sequence of applying them. This action is a demonstration of mathematical reasoning. The PISA 2021 Mathematical Framework also reaffirms that learners have to evaluate their mathematical solutions by explaining the results in the context of the original real-life event.

Studies looking into the circumstances for fostering reasoning in mathematical problem-solving highlight that allowing learners to work on complex mathematical tasks rather than easy ones can go a long way to stimulate their mathematical reasoning (Francisco & Maher, 2005). The view tallies with the CAPS currently operational in South Africa which instructs Mathematics educators to consider problem solving and answering non-routine, unseen questions as an integral part of classroom teaching. Hence, mathematical problems are mathematical tasks that are capable of providing intellectual challenges that can facilitate learners' mathematical understanding and development. In the light of this, Mathematics educators have to be conscious of the appropriate ways to formulate mathematical problems, the kind of problems to pose to learners and how to engage learners in mathematical reasoning. They should as well be fully aware of the mathematical reasoning processes needed for understanding and solving mathematical problems; the qualities of a Mathematics classroom fostering problem-solving activities; suitable techniques of describing and evaluating learners' problem-solving skills and ways to inspire learners to develop problem-solving competencies (DBE, 2014).

Reflecting on the above submissions by researchers, the current study upholds the following ideas about mathematical problem solving. First, mathematical problem solving is an uneasy process that calls for deep mathematical thinking. Second, the mathematical problem-solving process is non-routine in nature, and is not based on the application of memorised rules or algorithms. Thus, a problem solver does not instantly know the means to arrive at the solution, but has to undergo some thinking processes to obtain the solution. Hence, a learner that will solve a Mathematics problem successfully is supposed to have previously acquired certain mathematical knowledge to apply to the current challenging and unfamiliar problem. This study also endorses the calls that problem solving be made an integral aspect of Mathematics learning for each grade level and each Mathematics topic.

2.3.2.2 Mathematical Problem-solving Skills

Several economies of the world have recognised possession of problem-solving skills as a key factor required in the present-day job markets and have resultantly called for its inclusion in school curricula (Awuah, 2018; Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016). Problem-solving skills are the learners' competencies in solving problems in a cognitive domain like Mathematics (Renkl & Atkinson, 2010).

Corroborating this, Ridwan, Retnawati, Hadi and Jailani (2021) describe problem-solving skills as reasoning capabilities to obtain conclusions based on the premises, determining suitable alternative solutions in decision making, creating alternatives to get keys, and thinking logically to assess the best solutions in answering problems. As noted by Dhlamini and Mogari (2011), problem-solving skills manifest as learners get involved in the process of learning by posing problems, evaluating variables connected to problems, executing solution-steps of problems, obtaining possible solutions to problems and assessing solutions to problems.

Therefore, as a means to make learners efficient Mathematics problem solvers, (Pohan, 2020) maintains that their skills need to be developed to understand Mathematics problems, create mathematical models and interpret solutions. The current study considers a mathematical problem-solving skill as a creative reasoning competence to determine conclusions based on the premises, to generate alternative solutions in decision-making and to map out logical solution-paths towards tackling a Mathematics problem. Notable ones among the necessary mathematical problem-solving skills identified by researchers (for instance, Awuah. 2018; Chirinda, 2013; McIntosh & Jarett; 2000, among others) are highlighted below:

- advance numeracy skills;
- skills for identifying and stating exactly what the mathematical problems are, including the assumptions made, at any given time;
- skills for clearly and correctly interpreting mathematical problems and relating them to real-life problems;
- analysing skills;
- skills for reasoning, constructing and presenting logical mathematical arguments;
- skills to deal with highly abstract concepts;

- necessary skills for breaking down a given problem into relevant sub-problems;
- computation and execution skills;
- skills for finding and presenting the solution, among others.

Taking cognizance of how crucial it is to have mathematical problem-solving skills, teaching these skills should be the cornerstone of any Mathematics curriculum. In fact, integrating problem-solving techniques into Mathematics education would offer learners a supportive atmosphere to examine real-world problems and discover means to obtain solutions. In later life, learners will still find the skills essential (Lee, 2007). Mathematics tasks should be based on learners' real-life experiences as this is capable of developing their problem-solving skills (DBE, 2011; Dhlamini, 2011). That was why the current study chose the concept of stationary points in differential calculus and the selected test questions (See Appendices A1 and A2) that relate to the participants' everyday experiences and also have practical, real-life applications.

The present study accepts accordingly that Mathematics learners should possess necessary mathematical problem-solving skills for meaningful achievements in the subject. It suggests that the skills will not only enable them to solve Mathematics problems correctly, but will also equip them with the confidence to successfully approach daily life problems. This study admits that mathematical problem-solving skills are capable of equipping learners with the much-needed critical thinking that can make them cognitively versatile in a lot of useful ways. It shares the same viewpoint with İncebacak and Ersoy (2016) who assert that acquisition of certain problem-solving skills is pivotal to leading a happy individual life and maintaining a healthy nation.

2.3.2.3 Mathematical Problem-solving Strategies

For over four decades now, mathematical problem solving has remained an interesting topic to researchers in Mathematics Education. It has been regarded as an advanced thinking ability which comprises different thinking processes (Codina, Cañadas &

Castro, 2015). The goal is to equip Mathematics learners with useful skills for solving problems in a series of issues in science, engineering, technology, finance, business, medicine and daily life (Akyüz, 2020). One of the essential constituents of problem-solving skills learners must possess is how to select suitable strategies that can enable them to successfully obtain the solution to a given Mathematics problem (İncebacak & Ersoy, 2016). With problem solving accorded prominence in most Mathematics curricula, incorporating different classroom experiences which require using problem-solving strategies is therefore an essential area of Mathematics Education (Aydin Güc & Daltaban, 2021).

Since application of problem-solving techniques to tackle mathematical tasks can be taught, Mathematics teachers are urged to carefully select those Mathematics problems that can develop learners' skills in the use of various problem-solving strategies. As documented by researchers (like: AydinGüc & Daltaban, 2021; Chrysikou, Motyka, Nigro, Yang & Thompson-Schill, 2016; Gigerenzer & Gaissmaier, 2011; Mogari & Chirove, 2017; Posamentier & Krulik, 1998; Sarathy, 2018, Tonnsen, 2021, to mention just a few), the following are the mathematical problem-solving strategies that may be helpful to Mathematics learners, depending on the nature and level of the problem situations they encounter:

- Critical thinking: An active and skilful way to conceptualise, apply, analyse, synthesise and assess the information gathered from experience, observation, reasoning, reflection etc. to obtain a solution.

- Brainstorming: Combining and developing a large number of ideas and solution moves until an optimum solution is obtained

- Algorithms: A set of sequential steps towards the solution, the use of a formula, for example.

- Divide and conquer: Simplifying and breaking down a challenging complex problem into a simpler, more manageable one.

- Lateral thinking: An indirect and creative way of obtaining a solution
- Analogy: Using a solution to an analogous problem. That is, first solving a simpler analogous question may give a clue to how to tackle a complex one.
- Abstraction: Tackling the problem first in a model of the system and later applying it to the real system
- Means-ends analysis: Selecting an action for every step to get close to the goal
- Reduction: Transforming the task into another task which solution exists
- Looking for a pattern: Checking if the solution to a problem provides a sequence, and then finding the rule for the terms of the sequence
- Research: Adapting or using existing solutions/ideas to solve similar problems
- Root-cause analysis: Determining the basic cause of a problem as a means to arriving at the required solution
- Insight: Applying a sudden solution, an instantaneous understanding of a complex situation or an unexpected recognition of a new idea, upon realising that the problem is similar to one earlier solved. The underlying mental processes which produce insight most probably occur outside of awareness.
- Morphological analysis: Evaluation of the output and the connections of the whole system
- Hypothesis testing: Assuming a possible explanation to a problem and attempting to justify or disprove the assumption, as the case may be
- Proof: This is a move to show that the problem has no solution. After discovering that it has a solution as expected, the starting point for solving the given problem will then be the point where the proof fails.

- Trial-and-error method: Making attempts to test different possible solutions until one obtains the correct one. This can also be called guessing and testing.

- Idea of focal objects: Synthesis of apparently non-matching properties of various objects into something new

- Heuristics: It is a mental rule-of-thumb strategy capable of simplifying complex problems and reducing several possible solution ideas and solutions to a more manageable set.

- Working backwards: In order to solve the problem, start with the solution and work your way backwards to the original concepts.

The current study regards mathematical problem-solving strategies as the steps or techniques that learners are supposed to follow while thinking and navigating through a given Mathematics task to arrive at the solution. As helpful as the strategies above can be, the present study opines that some of them may not be the best approaches to mathematical problem solving. Some of them may not be practical for several situations and some can be time-consuming especially in cases where there are several options to explore to reach the solution to a Mathematics problem. Besides, some of the techniques are only applicable to few or specific situations, while some do not guarantee correct answers. Even those of them that can lead to correct answers may not offer deep, sequential reasoning, and adequate procedural and conceptual knowledge of Mathematics. Some of the strategies may not arguably be applicable to school Mathematics given their high cognitive demand, complexity and sophistication. Thus, the present study developed and adopted the 8Ps learning model hoping it would be of great assistance to Mathematics learners in their quest to become confident and successful mathematical problem solvers.

2.3.3 Nature of Tasks Appropriate for Mathematical Problem Solving

2.3.3.1 Routine and Non-routine Mathematical Tasks

As noted in literature, mathematical problems can be classified into two groups depending on their degrees of complexity. These are routine and non-routine mathematical problems (Altun, 1998). Routine mathematical problems are simple, ordinary problems (Altun, 1998) that can easily be tackled by an equation, formula or an obvious method (Polya, 1957). On the other hand, non-routine mathematical problems are tasks that can engage learners in some mathematical reasoning which entails representing objects, recognising and investigating their attributes so as to discover relationships or invariants and means of supporting them (Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016). Non-routine mathematical problems cannot be solved with easily known formulas or methods; their solutions cannot be guessed in advance either. Solving them requires a combination of some suitable strategies such as: analysis, synthesis, looking for a relevant pattern, making a systematic list, trial and error, the use of a chart, diagram or model, higher-order reasoning and so on (Saygılı, 2017).

The mathematical tasks meant for learners to solve should be capable of challenging learners intellectually and also enhancing their mathematical understanding and development (Novita, Zulkardi & Hartono, 2012). They should be such activities that engage learners in thinking about the vital mathematical skills and strategies they are supposed to learn (Albay, 2019). This is because the success or otherwise of a problem-solving instructional approach rests on a wide range of factors like: the choice of tasks; the kind of problem-solving experiences or strategies applicable; the stage to apply the problem-solving experiences; the level of guidance the teacher is supposed to offer the learners, and the form of learner assessment the teacher considers (Lester, 2013). Hence, asking learners to solve unfamiliar or non-routine mathematical problems that require more than the ordinary applications of already known algorithms or learnt mathematical procedures can enhance their understanding of the fundamental concepts, widen their ideas of Mathematics and also expectedly take them through the problem-solving processes (English & Kirshner, 2015; Lesh & Zawojewski, 2007).

Researchers (such as Funke, 1995; Dorner & Wearing, 1995, among others) view non-routine questions as complex problems and routine problems as simple problems. They highlight some typical features of complex problems as follows:

- Complexity (must include large number of items, interrelation and need for decision-making);
- *Intransparency* (lacking in giving clarity of the situation and not easily showing commencement opacity and continuation opacity);
- Heterogeneity (must include some sub-problems or must test different areas);
- Enumerability (should be listed one by one and not lumped together, and should also not be too many);
- Connectivity (must be well-linked in terms of hierarchy, communication and allocation);
- *Polytely* (should have multiple goals and have to exhibit inexpressiveness, transience and opposition);
- Dynamics (must be indicative of time considerations, phase effects, temporal constraints, temporal sensitivity and dynamic unpredictability)

Within this framework, the current study declares that, for Mathematics learners to participate meaningfully in problem solving, the onus is on the teacher to prepare and present suitable Mathematical problems. The teacher should ascertain that the questions are non-routine. In other words, the questions should be challenging, unfamiliar and ones which have multiple solution methods. Although the questions have to address important Mathematics concepts, connect to learners' previous knowledge and be in line with the curriculum, they have to, importantly, be such that the learners cannot easily guess or predict their solutions or the solution strategies to apply. This study further argues that, should the learners instantly know or have easy methods previously established to use or can quickly discover the measures needed to complete or tackle the Mathematics tasks, such are routine tasks and cannot appropriately measure learners' mathematical problem-solving performance.

The present study also agrees that giving learners non-routine questions to solve will make them think deeply and critically before finally deciding on the solution procedure to follow among the different procedures and ideas competing for attention. It will deepen learners' understanding of a mathematical problem and its solution, and will equip them with the ability to solve similar or other Mathematics problems. As evident in research (for example, Cai & Lester, 2010; McIntosh & Jarrett, 2000), the current study states that, in addition to the Mathematical tasks being in accordance to the set goals, they should also be such that can capture learners' curiosity and interest, promote their conceptual understanding, and also foster their abilities to reason and communicate mathematically.

2.3.3.2 Well-structured and Ill-Structured Mathematical Tasks

A mathematical task well-structured offers majority of the required information and, in general, can be solved using few, simple rules. The problem contains everything learners should know and, relatively, the procedures for its solution are precise and clear (Voss, 2006). In other words, a properly structured problem has a convergent solution, clearly defined rules (Chen & Li, 2015) and somehow provides definite goals and steps that can guide the learners towards solving it (Nokes & Schunn, 2010). On the contrary, an ill-structured problem does not provide necessary information; its solution procedures and solution tend to be multiple (Voss, 2006). In their own observation, Chen and Li (2015) regards an ill-structured problem as a problem typified by improperly defined rules, numerous solution paths and several solutions. In a similar view, Nokes and Schunn (2010) describe an ill-structured problem as sometimes characterised by vague goals and ambiguity about its possible solution method. Byun, Kwon and Lee (2014) as well believe this kind of problem has unclear purpose, unpredictable rules and is without a limiting condition.

An ill-structured mathematical problem is a contextualised task which demands that learners define the problem and also determine the information and skills needed for solving it. This form of mathematical task often arises from specific contexts. It presents complex situations with some aspects of the situations not being concrete; it is not well-defined; it has openness and is based on real-life situations. It is a problem to clearly understand an ill-structured mathematical task. It is not easy searching for and choosing information useful for solving it. It is also uneasy to identify and justify different perspectives, organise information or determine the solution most appropriate for it (Avdiji, Elikan, Missonier, & Pigneur, 2018; Hong & Kim, 2016).

The three key attributes of an ill-structured problem are: authenticity, complexity and openness. An ill-structured problem is deemed authentic if it incorporates the context of daily life and has a substantial impact on the actual scenario. It is seen as complex if it presents incomplete rules. It is open as it allows a thinker to interpret and justify the problem in their own different ways (Hong & Kim, 2016; Santia, Purwanto, Sutawidjaja, Sudirman & Subanji, 2019). Basically, unknown elements, multiple concept links, several solutions, and solution paths are the major qualities of poorly structured mathematical problems. These issues call for the expression of human opinions due to their particular interpersonal activities (Abdillah & Mastuti, 2018).

Learners should not be given just any mathematical problems to solve as only well-designed mathematical tasks afford learners the opportunity to build on what they know and boost their learning (Cai, 2003). It is the teacher's obligation to provide a group of age-appropriate mathematical tasks that can encourage learner engagement, critical thinking, and the formation of cognitive connections. For a meaningful learner problem-solving experience, the teacher should challenge the learners with such mathematical problems that can address their individual needs and make them reason mathematically. The teacher should also offer support to the learners experiencing difficulties (Kaur, Yeap & Kapur, 2009). Again, instead of the teacher leading, dominating and providing the solution processes, he should allow learners to solve the well-structured Mathematics problems on their own. This allows for acquisition of deep and effective mathematical content knowledge and socio-emotional skills (Nieminen, Chan & Clarke, 2021). Under these circumstances, the current study decided to use the carefully prepared, non-routine past NSC Mathematics Paper 1 questions whose quality had been ensured by the DBE and the Umalusi Council for Quality Assurance in General and Further Education and Training.

2.3.4 Teacher's Role in Learners' Mathematical Problem-Solving Process

Over the last few decades, research conducted on mathematical problem-solving instruction has progressed significantly. Still, the need arises to explore appropriate ways by which Mathematics teachers can lend support to their learners in performing the complex activity (Lester & Cai, 2016). For Mathematics learners to be effective problem solvers, the teacher should present to them problem contexts that foster the production of meaningful mathematical concepts, rather than mere classroom experience based on applying previously taught concepts, rules and procedures, as evident in several Mathematics classrooms (English & Sriraman, 2010).

One of the teacher's key duties is to ascertain that he selects, prepares and presents suitable mathematical problems to the learners to solve. He has to engage the learners in challenging, well-structured, non-routine mathematical tasks. He should ascertain that the tasks are not above the grade of the learners in reference, especially as specified by the curriculum and that the tasks contain all information required for solving them. He also has to see to it that the tasks do not have easy or obvious solution methods or solutions. Importantly, the teacher has to be cautious enough not to remove or completely break down the mathematical complexities of the tasks for the learners to think and *struggle* to solve them. Mason (2010) remarks that increasing the complexities of the problems that learners would solve extends the learners' understanding of the concept being learnt. The mathematical tasks also have to be open-ended; they should address important Mathematics concepts, have multiple solution methods and answers, challenge and interest learners, and connect to learners' previous learning.

Mathematics teachers are also urged to give learners challenging problems capable of expanding their mathematical thinking and reasoning, while also assisting learners having difficulties in solving the problems. In order to improve the problem-solving method of teaching Mathematics, the teachers must be able to identify the sorts of mathematical tasks that can stimulate learners' engagement, critical thinking and development of cognitive links. The associated teachers' actions supporting application of mathematical problems such as this should take care of the individual learner's needs (Kaur, Yeap & Kapur, 2009). Mathematics teachers are further enjoined to use probing questioning to facilitate learners' reasoning and encourage them to engage in deep mathematical investigations (Sahin & Kulm, 2008).

Moreover, the teacher has to instill interest and curiosity for solving questions in learners, as well as courage and persistence to pursue every question to its logical conclusion. This is in consonance with the NCTM (2000) remark that, in order to build up learners' skills for problem solving, the teacher must stimulate learners' interest in mathematical problems and provide them numerous opportunities for imitation and practice. Pólya (1965) explains the form of guidelines that the teacher can offer learners during problem solving, like making learners interested in the subject, letting them learn guessing, and making them learn how to prove. Fauzan (2011) identifies three main responsibilities of the teacher while developing learners' mathematical problem-solving skills as: helping learners develop a series of problem-solving strategies; guiding them to master mathematical concepts, techniques and numeracy skills for problem solving and providing them opportunities to use those strategies in various wider situations.

Hähkiöniemi, Leppäaho and Francisco (2012) claim that there are three levels of teacher's guidance during learners' problem solving. First is the *surface-level guidance* during which the teacher does not reckon with some important areas of the learners' solutions. Second, the *inactivating guidance* allows him to reveal the potential investigation to the learners, and the third level is the *activating guidance*, when he guides the learners to investigate the essential aspect. Lester (2013) remarks that, for a problem-solving instructional method to be successful, the teacher should consider carefully a broad range of factors like: selection of problems that the learners will solve; the kind of problem-solving experiences to introduce; when to apply problem-solving instruction; the amount of guidance he is supposed to offer the learners and how to measure the learners' problem-solving progress.

Allevato and Onuchic (2008) suggest that, for learners to enjoy a productive mathematical problem-solving experience, the teacher is to:

- divide the learners into small, mixed-ability groups before the class activities;
- observe and encourage them. Instead of assuming the role of a knowledge transmitter, he observes and analyses the learners' problem-solving behaviour;
- stimulate collaborative work. He allows the learners to think and guides their thinking; he also encourages sharing of ideas among themselves;
- encourage the learners to apply their prior knowledge, or the strategies earlier acquired to solve the problems, and inspires them to select various strategies depending on the resources available;
- help the learners with their difficulties; he intervenes, carefully asks them useful mathematically challenging questions and follows their explorations;
- record solutions on the board for all the learners to analyse and discuss; he can also allow a representative of each learners' group to record solutions on the board. This is an opportunity for the learners to elaborate their thinking and justify their points of view to clarify any doubts that may arise. During this plenary session, the teacher serves as a guide and mediator in the discussions. He has to encourage active and effective learner participation;
- seek consensus. After taking time to address doubts, analyse resolutions and solutions got for the problems, he has to reach consensus with the learners on the correct solutions to the problems;

- formalise the content. He then presents formally the new content and concepts constructed, pinpointing the important operative properties and strategies.

The current study acknowledges the teacher's essential role in affecting, assessing and developing learners' mathematical problem-solving skills. It accepts that how well the teacher discharges this classroom duty contributes considerably to the level of effectiveness learners will attain in this regard. Drawing from Lester and Cai (2016), the current study therefore implores Mathematics teachers to focus on proper, learner-driven instructional practices capable of developing learners' mathematical problem-solving skills; to be strategic in their selection of suitable Mathematics tasks and to steer classroom discourse appropriately for full maximisation of learning opportunities. The current study further urges Mathematics teachers to engage learners in a variety of problem-solving exercises; explore different approaches to handling every Mathematics task; provide explanations and justifications for solutions to given Mathematics problems, and should also endeavour to generalise the solutions.

2.3.5 Challenges of Teaching Mathematics through Problem Solving

Employing problem-solving techniques to teach Mathematics is an effective means to equip learners with higher-order thinking skills and expose them to cross-curricular experiences connected to real-life. Applying specific problem-solving methods in learning Mathematics, learners can also successfully learn how to think through, approach and solve broad life problems (Szabo, Körtesi, Guncaga, Szabo & Neag, 2020). In fact, working on problems provides both learners and the teacher the opportunity to access the curriculum content, ideas, concepts and skills they are dealing with (Van de Walle, 2003). However, as central and helpful problem solving is to Mathematics, applying it as a method of teaching Mathematics comes with its attendant challenges.

One of such challenges that teachers face is how to productively develop suitable non-routine Mathematics problems that the learners will solve, and also how to effectively determine the suitability of such problems for the targeted grade level. The relevant mathematical tasks to be set or selected have to be unfamiliar and challenging to the learners. The problems also have to be capable of not only deepening the learners' understanding of Mathematics, but such that can as well take them through the problem-solving process. Importantly, mathematical tasks should be drawn from real-life situations. The aforementioned observation tallies with Kodisang (2022) who declares that proper mathematical problems have to be at learners' cognitive level while also highlighting the practical applications of Mathematics in everyday life. Reiterating the same view, Alsulami (2016) contends that knowledge application in practical settings fosters interactions between learners and society. In this regard, the ability of each Mathematics teacher to interpret, understand and apply correctly the evaluation criteria for setting tasks counts here. Briars (2014) points out some of the guidelines for analysing the nature of the Mathematics problems suitable for learners to solve as:

- The mathematical questions must support the learner's ability to master the content;

- The development of the content (upon which the mathematical questions are based) must be consistent with the laid-down *standards* and also has to be reflective of how learners learn the content in a most effective manner;
- The task must accord adequate priority to the critical topics of the curriculum recognised in the *standards.* It must be accurate, focused and coherent, for the learners to clearly see how ideas build on and connect to other ideas across subjects and grades.

Teaching Mathematics through problem solving has been found to be really demanding. As it is demanding on the teacher, so it is demanding on the learners. It poses difficulty for teachers mathematically, pedagogically and personally. To be able to understand learners' various approaches to given Mathematics problems and the relevance of those approaches, Mathematics teachers still need to acquire more mathematical expertise. Many of the teachers are unfamiliar with or do not have much idea about problem-solving instructional approach, which basically requires them to teach some mathematical concepts they themselves did not encounter while in school and also to teach such differently from the ways they were taught (Burkhardt & Bell, 2007; Schoenfeld, 1992; Van Velzen, 2016, among others). This study hopes to offer to Mathematics teachers helpful guides on the problem-solving strategies and processes they can follow in their Mathematics classrooms.

Some of the Mathematics teachers are still skeptical about the need to replace the traditional teaching approach with the problem-solving method. Thus, such teachers require some experience, confidence and self-awareness to do away with the traditional teachers' role and expertise they have been quite used to. At the pedagogical level, some challenges that teachers encounter include: how to determine the appropriateness of non-routine mathematical tasks and their degree of difficulty; the suitable time to give guidance to learners during problem solving, and the form of supports to offer learners to make them think mathematically and actively construct their own understanding of given Mathematical concepts. For these teachers to effectively teach Mathematics through problem solving, they will have to undergo additional training in terms of mathematical content, theory and methods (Burkhardt & Bell, 2007; Schoenfeld, 1992; Van Velzen, 2016).

For the learners too, solving non-routine problems is not an easy task, particularly open-ended, non-routine problems. To worsen the situation for the learners who, more often than not, battle in frustrations with solving unfamiliar, challenging non-routine Mathematics tasks, several available Mathematics texts provide just a few non-routine questions. Schoenfeld (2013) as well as Burkhardt and Bell (2007) condemn those school texts mostly filled with Mathematics questions that require learners to prioritise understanding of rule-based mathematical process above problem-solving procedures. Although it may seem difficult to select such mathematical thinking problems for use, researchers in Mathematics Education intentionally emphasise the essence of the learners working with such problems (Lupahla, 2014; Ofori-Kusi, 2017; Sullivan, Clarke & Clarke, 2013; Watson & Mason, 2006). The present study, through its 8Ps problem-solving learning model, also hopes to provide the learners with the critical-thinking learning strategies needed for solving non-routine Mathematics problems.

Moreover, the current research admits that teaching mathematical problem solving tends to be time-consuming. Sometimes, solving one or two challenging non-routine Mathematics questions may last a whole lesson period or more. Hence, the time often provided for Mathematics on the school time-table can hardly be enough to carry out thorough problem-solving activities. Then, if care is not taken, chances of achieving content coverage may not be feasible (Albay, 2019; Hsiao, Lin, Chen & Peng, 2017). On that note, the present study suggests some extra time be allocated to the process. This certainly will take its toll on the time allocations for other subjects. For judicious use of the limited time therefore, and to be able to integrate content and skills within the context of problem solving, this study also admonishes the teacher to set/select worthwhile Mathematics tasks, provide necessary guidance during problem solving and know when and how to expertly do so.

As demanding and challenging as teaching Mathematics by problem solving can be, the present study planned to show that a proper pedagogy effectively adopted can ease or remove the challenges posed by mathematical problem-solving processes. Towards realising this, the study made use of the 8Ps instructional model, hoping that if carefully applied in the classroom, it may impact positively on learners' mathematical problem-solving practices.

2.3.6 Poor Learners' Mathematical Problem-Solving Performance: A Global Concern

Problem solving is a very essential cognitive skill that is central to Mathematics. Nonetheless, this critical skill appears inadequate or non-existent among secondary school learners globally (Bush & Karp, 2013; İncebacak & Ersoy, 2016; Palraj, DeWitt & Alias, 2017). According to Curriculum 2013 still effective in the Indonesian educational system, Indonesian learners are required to be able to think innovatively, creatively and responsively. To this effect, they are trained and encouraged to be skilful in problem solving, including mathematical problem solving (Kirana, Lestari & Ristika, 2021). In spite of this, regrettably, the level of the learners' mathematical problem-solving skills is still below expectations. Their weak mathematical problem-solving performance has remained a noticeable difficulty that they do not find easy to overcome. This is of much national concern to education observers nationally and internationally. Past and recent reports by PISA and TIMSS equally have confirmed the weak achievements of the Indonesian learners in questions of international standard, especially ones demanding demonstration of mathematical problem-solving skills (Mullis, Martin, Foy, Olson, Preuschoff, Erberber, Arora & Galia, 2008; Ridwan, Retnawati, Hadi & Jailani, 2021).

In spite of the increased focus on problem solving instruction, American learners have continually recorded a low level of mathematical problem-solving performance relative to other developed countries (Jupri & Drivers, 2016; Vigdor, 2013). No exception is the case of the secondary school learners in Malaysia whose mathematical problem-solving skills have also been found to be weak. When faced with non-routine mathematical problems, the learners do not do well. It is thus necessary to introduce new and effective teaching and strategies for mathematical problem-solving (Palanisamy & Nor, 2021; Stephen, Lydia, Maria, Katherine, Westat & Judy, 2016). It is a similar situation with the Turkish secondary school Mathematics learners. Consequent upon a study, İncebacak and Ersoy (2016) conclude that the majority of the Turkish learners had difficulty solving non-routine mathematical problems. They notice that the learners are more successful with tackling mathematical problems that are related to the ones they already solved, or ones they had previously come across.

Namibian Grade 12 Mathematics learners too struggle to solve non-routine mathematical problems, probably owing to their poor usage of English Language and limited capacity to come up with their own strategies for problem-solving. The learners demonstrate lack of basic mathematical problem-solving skills (Fatokun, Hugo &

Ajibola, 2009; Hamukwaya & Haser, 2021; Mogari & Lupahla, 2013). Also, considering the results of private and public examinations of high schools in Nigeria, Akintade (2017) reveals that a lot of learners are yet to acquire necessary problem-solving skills required for success in Mathematics. The researcher therefore opines that unless the deficiency is rectified, the learners will continue to perform poorly in the subject. The PISA reports of 2003 and 2012 indicate that learners in the Netherlands equally record a better performance in mathematical literacy than in the aspect of solving mathematical problems. As a matter of concern for the nation, stakeholders in the education sector have called for a more problem-oriented Mathematics curriculum for the secondary school learners, one hinged on the problem-solving process.

The actual population of the current research – high school learners in South Africa – are not exempted from the generally low learners' mathematical problem-solving performance. In the last two decades, it was found out that South African learners exhibited a weak performance in Mathematics because they lack sufficient problem-solving skills and have limited knowledge of problem-solving strategies. The past and current reports from DBE, researchers and different national and international bodies evaluating curriculum, assessment and learner performance such as TIMMS and PISA generally indicate that the learners demonstrate low problem-solving skills because they are not adequately involved in problem-solving activities (Awuah, 2018; Bedada, 2021; Brijlall & Ndlovu, 2013; Chirinda, 2013; Chirove, 2014; Dhlamini, 2012; Dhlamini & Mogari, 2011; Kodisang, 2022; Luneta & Makonye, 2010; Mogari & Chirove, 2017; Ofori-Kusi, 2017; Yimer, 2019, etc.).

On the indicators of learner achievement in Mathematics, Maree, Aldous, Hattingh, Swanepoel and Vander Linde (2006) conclude that the mathematical problem-solving skills of South African learners are insufficient. They find out that, in Mathematics classes, teachers rarely encourage learners to pose questions and that learners cannot link Mathematics to everyday life. The findings by Wessels (2012) reveal that the primary cause of South African learners' low scores in Mathematics in TIMSS studies is their deficiencies in critical problem-solving skills. In his study, Dhlamini (2012) observes that Mathematics learners in South Africa exhibit lower problem-solving skills than Mathematics learners of other African countries. Dhlamini and Mogari (2011) also posit that South African learners' poor performance in Mathematics can be addressed within the framework of learners' problem-solving skills, and that strengthening these skills will help them overcome their difficulties in understanding Mathematics concepts. In fact, various studies conducted nationally and internationally have reported that South African learners demonstrate lower mathematical problem-solving performance than learners from most other African countries. In a bid to contribute towards addressing the poor mathematical problem-solving performance of learners globally, and of South African Grade 12 learners particularly, the idea of this study was conceived.

2.4 The Already Existing Learning Models and the 8Ps Learning Model

2.4.1 The Already Existing Learning Models

Determining learners' problem-solving skills in different Mathematics topics has become a huge concern for many researchers. This has brought about developing and adopting different mathematical problem-solving models (Awuah, 2018). Utilising learning models is vital for the facilitation of learners' mathematical problem-solving performance (Son & Ditasona, 2020). A learning model is a guideline for the designers of learning as well as a direction for teachers in implementing learning activities (Shoimin, 2014). It is an information-processing theory that focuses on figuring out the sequential steps of problem solving and the cognitive process that underlies those steps instead of separate skills required for the problem-solving process (Carson, 2007).

Zulyadaini (2017) explains the learning model as a practical instrument that aids learners in acquiring knowledge, skills and methods of thinking and expressing their thoughts. Various models (for example, Burton, 1984; Cherry, 2011; Kirkley, 2003; Mason, Burton & Stacy, 1982; Pólya, 1945; Schoenfeld, 1985; Wilson, Fernandez, &

Hadaway, 1993, to mention but a few) recommend different useful mathematical problem-solving processes while exploring ways to enhance the learning of Mathematics. The 8Ps heuristic model proposed for this study is hinged on the afore-mentioned and similar other problem-solving models.

2.4.1.1 George Pólya's Problem-Solving Approach (1945)

Pólya's (1945) problem-solving model appears the most enduring of all the learning models in Mathematics Education, as it has been recognised as the basis and standard for investigating competence in problem solving. The model has served as the foundation for other heuristic models developed afterwards (Awuah, 2018; Lljedahl, Santos-Trigo, Malaspina & Bruder, 2016). The four-step heuristic process, documented in Pólya's book entitled *How to Solve It,* requires that a problem solver has to: *understand the problem*, *devise a plan*, *carry out the plan*, and *look back to evaluate the plan*. The aim of Pólya's heuristics is to explore the processes of discovery and invention. His belief is that as learners participate in problem solving, they tend to discover and create new ideas and knowledge (Pólya, 1945).

As accounted for by Pólya (1945), understanding a problem means knowing what a mathematical problem requires in form of the mathematical concepts and procedures, evaluating previous knowledge and, sieving and separating vital information from the whole lot of apparently useful thoughts conceived. At this stage, Pólya's learning model expects a problem solver to provide answers to such questions as: What is the unknown in the question or what does the question ask me to find? Do I understand every word used in the question? Can I explain the question in my own words? What data are provided in the question? Can I come up with a diagram, chart, picture, graph etc. that can simplify the question? What is the condition given in the question? Will it be possible to meet the condition? Is the condition adequate and helpful enough to determine what is not known? Etc.

Devising a plan implies choosing suitable mathematical operations, strategies and processes to follow towards solving the problem. Among the various strategies suggested by Pólya (1945) are: the use of a formula, model, direct reasoning, symmetry, guess and check, etc. Others are: making an orderly list, considering special cases, solving a simpler problem, eliminating possibilities, looking for a helpful pattern, drawing a picture, working backwards, etc. Pólya recommends that the person solving a problem should establish a reasonable link between the unknown and the data provided. If that is not immediately possible, he has to consider auxiliary problems that can be of help. He urges the problem solver to answer questions like: Had I seen a similar or related problem having the same unknown before? Do I know of the same problem but of a slightly different form? Can I think of a theorem which can be useful? Can I first solve a related problem? Can I use the method or result of that related problem to solve the current one? Have I taken care of all the given data, condition(s) and other essential notions involved in the problem?

According to Pólya (1945), the third step to *carry out the plan,* meaning following the chosen procedure to tackle the task. The problem solver, at this stage, requires care, patience and persistence in making use of the devised thoughts and processes in obtaining a correct solution to the problem. He has to check each step followed and ensure that each is correct. If he finds out that the procedure is not being productive, he is expected to discard it and choose another one.

Lastly, after obtaining a solution to the problem, Pólya (1945) advises the problem solver to look back. He argues that looking back enables the problem solver to evaluate the result. In examining the solution obtained, the problem solver has to answer questions like: is the result logical? Can I prove that the result is correct? How reasonable is the argument that has produced the solution? Are there some other alternative means to obtain the solution? Is the solution, or the method, useful for solving some other problems? How do I interpret and also possibly generalise the result? Can I as well generate new problems to be solved? By doing so, the problem solver will be able to reflect on what strategies have worked and which ones have failed to help. He will also be able to predict what strategies or procedures can be useful for solving similar other future problems.

Pólya's four-step model, which has had a significant influence on how problem-solving is taught and learnt in schools for almost eight decades, is not spared from criticism. Lester (1980) considers the model as more of a proposal on methods of teaching learners how to solve problems than one actually describing how a successful problem solver thinks. Joseph (2011) argues that Polya's learning model only gives a framework on how to organise instruction of problem solving, without providing a guide on how to identify the difficulties that the problem solvers encounter, or an explanation of the thought processes they must go through to achieve success in problem solving. Rott (2012) faults the model as being seemingly very linear in nature. Instead of dwelling on the weak points of Pólya's model, the current study drew amply from its strong areas to develop the 8Ps learning model for this investigation.

2.4.1.2 Alan H. Schoenfeld Problem-Solving Process (1985)

The problem-solving model devised by Schoenfeld (1985) derives from Pólya's. However, in his own case, Schoenfeld recommends five stages of problem solving namely: *analysis* of the problem, *designing*/*planning* part of or the whole solution, *exploration* of the problem, *implementation* of a solution-plan and *verification* of the solution. It is Pólya's second phase, *devising a plan,* which Schoenfeld breaks down into the two stages of *design* and *exploration.* His argument is that a problem solver has to undergo the cycles of *analysis*, *design* and *exploration* before proceeding to the level of implementation. He further explains *design* as a phase for explicit planning and regulating the solution process and the *exploration* as a phase the problem thinker has to apply problem-solving heuristics, look at related problems and may even return to the analysis stage to understand the problem better.

Some researchers like Liljedahl, Santos-Trigo, Malaspina and Bruder (2016) feel that Schoenfeld explained his problem-solving principles at a practical and empirical level, unlike Pólya who discussed his majorly at a theoretical level. They submit that Schoenfeld gave a better and clearer understanding of how to teach problem solving and how to solve problems. They further note that Schoenfeld believes that as the entire process of solving a problem is a function of the problem solver's prior knowledge, current thoughts and attempts, it is also emergent and contextually-dependent. This, according to the critics, is different from Pólya's heuristic framework which centres around predefined and contextually-independent processes.

Schoenfeld (1985) five-phase problem-solving model also suffers attack. As Hähkiöniemi, Leppäaho and Francisco (2012) put it, Schoenfeld modified Pólya's (1945) straightforward model by including a stage he called *exploration* with no definite goal. They contend that Mason, Burton, and Stacey (1982) take care of this in their *attack* phase by showing that a problem solver may try out various ideas without necessarily creating or developing an explicit plan. Instead of this, Schoenfeld (1985) emphasises cycling between design and exploration. The present study equally found Schoenfeld's learning model relevant in the design of the 8Ps learning framework.

2.4.1.3 The Problem-Solving Approach by Mason, Burton and Stacey (1982/2010)

Mason, Burton and Stacey (1982/2010) present three phases of problem solving: e*ntry*, a*ttack* and r*eview*. In the *entry* stage, the problem solver is supposed to find answers to questions such as: *what do I already know*? *What is it that I actually want? What can I bring up? e*tc. This group feels questions such as these can help the solver take useful decisions about solving a problem. At the *attack* level, the problem solver can consider different solution strategies and plans. As soon as he gets a solution, he moves to the *review* phase, during which he considers ideas like: checking the resolution, reflecting upon the vital moments undergone, the ideas so formed in the process and extending this to a wider situation. Like James Wilson's group, John Mason and his colleagues do not subscribe to the processes of problem solving being strictly linear. They therefore advise a problem solver to move back and forth between the *entry* phase and the *attack* phase, believing that the solver may get stuck at any point while trying to devise or implement his ideas and so may need to go back to make a new entry for a proper understanding of the problem. They also recognise the essence of metacognition in the problem-solving process. Essentially, the problem-solving idea by Mason, Burton and Stacey (1982/2010) also partly strengthened the development of the 8Ps learning model.

2.4.1.4 Burton (1984) Problem-Solving Framework

Burton (1984) suggests a cyclic, four-phase problem-solving framework: *entry*, *attack*, *review* and *extension*, thereby adding another stage to the three-phase approach that he and his colleagues previously recommended. He considers these four stages as a cycle. According to Burton, a learner enters the process of problem solving by setting out to understand what a problem is all about and devising techniques for solving it. At this point, he has to answer questions like: What do I know that can be useful for solving the problem? What helpful ideas can I introduce? What am I required to do or obtain? Etc. He then attacks it by carrying out the strategies mapped out. In the process of attacking it, the solver may consider different plans. If one plan fails, he can rethink another one. He may even go back to the initial or entry phase perhaps to seek for more understanding of the problem. After he may have revised the initial strategies, he can re-attack the problem. After obtaining a solution, he reviews it and attempts to extend the problem by posing another problem. Hence, the cycle goes on and on.

Burton (1984) explains that as the problem solver engages a problem (or makes an *entry* into solving it), his curiosity gets aroused and so with confidence he *attacks* the problem, though he may withdraw from solving it if without enough confidence. He notices that the problem solver who can provide the solution to a problem develops a sense of achievement which causes him to look back to do the *review* of the solution process. Critics argue that, as appealing as Burton's ideas are, his learning principles are not buttressed by empirical tests and theories. They point out that, in most cases, after obtaining a solution to a given Mathematics problem, whether right or wrong, learners hardly take time to look back to *review* the solution obtained or the procedure followed. As plausible as that argument may sound, the current study argues that it does not necessarily rubbish or nullify the need to look back to ascertain the correctness of the solution obtained. For this reason, the 8Ps learning model for the current inquiry in its seventh phase (proving phase) also advised that, after getting the solution to a Mathematics task, the learner has to look back, and in its final phase (predicting phase) that the learner also has to generalise or apply the solution to other similar or related Mathematics problems (see subsection 2.4.2.7 - 2.4.2.8).

2.4.1.5 Problem-Solving Model by Wilson, Fernandez and Hadaway (1993)

Wilson, Fernandez and Hadaway (1993) propose a dynamic, cyclic analysis of Pólya's problem-solving phases. To them, instead of being necessarily linear in nature, the course of a problem-solving process should be a cycle. Therefore, they assert the possibility of an individual solving a problem jumping from a step to another possible step. As an addition to the four stages given by Pólya (1945), they added *managerial decision* as the fifth stage, which is the control centre of their graphical representation of the problem-solving process. The James Wilson group demonstrates the possible non-linearity of the problem-solving process explaining that, for instance, a problem-solver can proceed straight to the planning phase after engaging in thoughts to understand a problem. Or, if not satisfied with the plan made, he can as well attempt to go back to understand the problem better. In harmony with the observation made by Wilson and his colleagues, the 8Ps learning method developed by the present study is not strictly linear, it is also cyclical (see subsection 2.4.3).

2.4.1.6 The Problem-Solving Method by Kirkley (2003)

Kirkley (2003) considers problem solving as a multi-step process that problem solver has to follow to relate their past experiences to the current problem in order to find the solution. The writer therefore presents the phases in his approach as: recalling solution, representing problem, searching solution, implementing solution, succeeding, stopping and failing. The nature of his problem-solving approach is cyclic. According to him, a learner solving a problem can recall and make use of a useful solution plan he earlier used in solving a current problem. In case that fails to help, he can try to represent the problem in ways that can help him understand it and then search for its solution. If the solution is not reasonable enough, he advises the problem solver to find some other logical ways of representing the problem and/or of searching for the solution.

Kirkley (2003) believes that the cycle continues until the problem solver obtains a correct solution to the problem and also implements the solution successfully. Some critics contend that as useful as this problem-solving process seems, it is easy to argue that it encourages a problem solver to follow a short-cut by just recalling and applying a previous encounter/experience to solve a current problem instead of taking time to critically analyse the present problem and explore various possible solution procedures. The critics also feel Kirkley's short-cut method may not work for some complex mathematical tasks. The present study only draws on the strength of Kirkley (2003) problem-solving method, particularly with the 8Ps learning method being multiple-step and cyclic too like Kirkley's (see subsections 2.4.2 and 2.4.3).

2.4.1.7 The Problem-Solving Approach by Maccini and Gagnon (2006)

Maccini and Gagnon (2006) propose the problem-solving approach represented by the acronym STAR. The acronym is explained thus:

S – Search the problem

T – Translate the problem

A – Answer the problem

R – Review the solution.

The present research also tapped some strength from the STAR learning method. The first step (S – Search the problem) is synonymous to the first three phases (probing, pinpointing and patterning) of the 8Ps learning model. Its second step (T – Translate the problem) compares to phases four and five (projecting and prioritising) of the 8P approach. While the sixth phase (processing) of the 8Ps instructional method was guided by stage three (A – Answer the problem*)* of STAR learning framework, phase seven of the 8Ps learning model compares to step four of STAR (R – Review the solution. However, the 8Ps learning model moves a step further with its phase eight (predicting, subsection 2.4.2.8) not covered by the STAR method.

2.4.1.8 Cherry (2011) Problem-Solving Process

Cherry (2011) proposes a cyclic, seven-phase problem-solving method given as: identify the problem, define it, devise a strategy, organise information, allocate resources, monitor the progress and evaluate the solution. Despite arranging the cycle in this sequential order, Cherry remarks that a problem solver may not follow the order of the steps strictly. He can skip steps forwards and backwards as the situation demands until he obtains a satisfactory solution. Like Cherry's method, the 8Ps method also consists of multiple steps, and a problem solver can as well skip steps forwards and backwards depending on the mathematical problem situation.

2.4.1.9 The Problem-Solving Heuristics by Faucette and Pittman (2015)

Faucette and Pittman (2015) propose two separate problem-solving strategies which they describe as KNWS and SQRQ. These are explained as follows:

KNWS stands for:

K – What facts the learners already *know*

N – What information is *not* applicable

W – *What* a problem actually requires or wants the solver to discover

S – What *strategies* are applicable

SQRQ

S – *Survey* or *skim* the problem in order to have an idea of its nature.

Q – *Question* or ask to find out what the problem is about and what information it requires to get to its solution.

R – *Read* carefully to identify vital information, ideas, details and relationships needed to tackle the problem.

Q – *Question* or find out what steps must be taken to solve the problem; what information is available or provided; what strategies are required; what is unknown; what operation(s) is/are to be used with what number and in what sequence, etc. The problem-solving strategies recommended by Faucette and Pittman (2015) also influenced the construction of the 8Ps learning model.

2.4.2 The 8Ps Learning Model

The 8Ps learning model designed for this inquiry has eight phases namely: *probing, pinpointing, patterning, projecting, prioritising, processing, proving and predicting.* The phases are explained as follows.

2.4.2.1 Probing Phase

In this first phase, the Mathematics learner solving a problem has to scrutinise and carefully clarify the given question. He needs to examine it properly, visualising and figuring out what actually the question requires. He has to give a careful consideration and interpretation to the question posed. According to Lester (2013), if learners are motivated, inspired and involved in a conscious assessment of the problem at the beginning of the problem-solving process, they are likely to make progress with problem solving. This is the learner's first step towards stage one (*understanding the problem)* of Pólya's four-phase heuristics. It is also his initial move into the *entry level* of Mason, Burton and Stacey (1982/2010). Buttressing this, PISA (2021) describes this phase as one which demands that the learner should determine the mathematical structure of the problem by applying their Mathematics content knowledge.

2.4.2.2 Pinpointing Phase

Here, the problem solver takes a deliberate step to identify the key words, points, variables and conditions in the question. He tries to take hold of the important information in the question to work on so as not to work amiss. He needs to recognise the mathematical structure such as the relationships, regularities and patterns in the question. The solver can underline, highlight or note down such important elements or aspects of the question pinpointed. Still in his *entry level,* the learner takes another careful step towards further *understanding the problem* as he pinpoints the key elements in the questions which may be suggestive of the actual demands of the question. Maccini and Gagnon (2006) in the first stage of their four-step problem-solving approach tagged STAR call this phase *search the problem.* The learner's ability to *search* the question thoroughly will enable him to locate and pinpoint the key aspects of the question*.* Cherry (2011), in the stage one of his seven-phase problem-solving strategies, advises the problem solver in this phase to *identify the problem.* As their third step in problem-solving heuristics (KNWS), Faucette and Pittman (2015) equally note that this pinpointing phase can be a reasonable means to determine what is actually required by the problem or what the solver is expected to discover.

2.4.2.3 Patterning Phase

At this point, the problem solver forms a useful pattern that can assist in tackling the question. PISA (2021) calls this stage translating a problem into a mathematical representation or language. Representing a question as helpful patterns can take the form of an equation or a table, chart, picture, map, diagram, etc. Cherry (2011) explains this phase as *defining the problem*. Kirkley (2003) describes it as *representing the problem.* Maccini and Gagnon (2006) consider it as t*ranslating the problem.* The solver's effort to define, represent or translate the problem into meaningful patterns may help him discover some useful relationships and connections needed to solve the problem. This third phase serves as a complement to the first two in the problem solver's attempt at *understanding the problem as* Pólya (1945) recommended.

In fact, the first three phases of the 8Ps learning framework are what Schoenfeld (1985) summed up as the first stage of his own problem-solving model named *analysis* of the problem. PISA (2021) remarks that learners can make use of representations – whether symbolic, geometric, numerical, graphical, text-based or in programming code – to organise and express their mathematical reasoning. The student-assessment body (i.e., PISA) explains further that problem representations (named problem patterning in this study) enables learners to present a mathematical problem in a brief, clear way which can lead to efficient algorithms. It believes that representations are an inherent feature of mathematical modelling that allows learners to construct an idealised or streamlined formulation of a real-life problem.

2.4.2.4 Projecting Phase

The individual solving a problem draws up solution plans at this stage. He devises and comes up with ideas he can use in solving the problem. He harnesses and gives shapes to the valuable thoughts (gained from probing the question) and the patterns drawn. This can be called a brainstorming session, a juncture at which he has to figure out the resources at his disposal – a time to decide on the appropriate mathematical operations, assumptions, strategies and procedures to follow to solve the problem. This phase is what Pólya (1945) describes as *devising a plan*. Schoenfeld (1985) views it as the *design* and *exploration* stage, a period when the problem solver makes an explicit planning and regulation of the solution process. Kirkley (2003) describes this projecting phase as a period of *searching for solution* while Faucette and Pittman (2015) explain it as a time to find *strategies* which can be applied to solve the problem.

2.4.2.5 Prioritising Phase

The prioritising phase requires pruning or trimming down the different solution strategies formulated to a manageable portion. The problem solver arranges his solution ideas according to his perceived order of relevance and appropriateness to the question and sieves them. He then cautiously eliminates some less important ideas and keeps only those ones he feels directly relate to the question. Cherry (2011) captures this problem-solving stage as a time to *organise information.* The prioritising phase serves as the conclusive part of the solver's attempt at *devising a plan*, stage two of Pólya (1945) model. According to PISA (2021), it is a phase that necessitates taking strategic steps regarding the tools to use and the order of their applications. It is one calling for selecting the most appropriate ones from the list of strategies already developed in the projecting phase.

2.4.2.6 Processing Phase

In this phase, the problem solver executes his already prioritised solution plans. He carries out the plans as Pólya (1945) advised. He painstakingly processes all his thoughts, patterns, and chosen solution ideas to obtain a plausible solution to the question. This is another crucial stage of the problem-solving process when he has to put into effective use all the mathematical reasoning made in phase one to four for him to be able to solve the problem successfully. Schoenfeld (1985) calls this phase the *implementation* of the solution techniques and explains it as a stage of ultimate interaction and connection between the problem solver's previous knowledge, his various solution moves and thoughts all along. Mason, Burton and Stacey (1982/2010) label this phase the *attack* level of problem solving. For Cherry (2011), it is a time to *allocate resources,* a particular point the problem solver has to make use of all his efforts made in the first five phases in solving the problem. At this point, very applicable is the comment by PISA (2021) that, once the problem has been successfully transformed from an unclear, disorganised, real-world scenario into a properly defined Mathematics problem, relevant Mathematics concepts, algorithms and procedures should be used to solve the resulting mathematical problem.

2.4.2.7 Proving Phase

At this seventh phase, the problem solver is expected to evaluate, justify or establish the reasonableness of the solution produced in the processing phase. He reflects on the solution to test whether or not it is correct. He has to be able to check if the solution makes sense. He needs to find out why the solution comes out the way it does. For a benefit of the doubt, he may also have to re-examine his solution steps that produce the solution. This complies with Pólya's idea in his last problem-solving stage, *looking back to evaluate the plan* (Pólya, 1945). Buttressing this are the accounts of researchers like: Schoenfeld (1985) who describes the phase as *verification* of the solution obtained; Maccini and Gagnon (2006), as well as Mason, Burton and Stacey

(1982/2010) who tag it as a *review of the solution;* Cherry (2011) who terms it as *monitoring the progress* and *evaluating the solution.*

2.4.2.8 Predicting Phase

This final phase of the 8Ps problem-solving model aims to test the acceptability level and possible generalisation of the solution. It requires the solver to consider the possibility of generalising and extending his solution to other (similar) mathematical problems. This phase requires finding out whether the solution can be used to solve similar or related questions. Success at this level determines the degree of logicality and acceptability of his solution. This idea gains the support of Burton (1984), who in his four-phase problem-solving model recognises it as *extension* of the solution. The present study finds this final phase of the 8Ps learning model also significant in the sense that it emphasises the widely accepted idea that acquisition of mathematical problem-solving skills is not limited to solving only mathematical problems but has valuable applications in various other areas of specialisation and general life problems. During the intervention, the study participants were encouraged and guided on how to carefully carry out this last stage of the 8Ps learning framework.

2.4.3 Nature of the 8Ps Learning Model

Rather than being strictly linear in nature, the 8Ps learning model is both linear and cyclic. Its linearity lies in the sense that the eight stages of the problem-solving process (*probing, pinpointing, patterning, projecting, prioritising, processing, proving and predicting)* have to be sequentially followed (refer to Figure 2.3). The nature of the 8Ps learning model can also be cyclic (refer to Figure 2.4).

Figure 2.3: Linear Nature of the 8Ps Learning Model

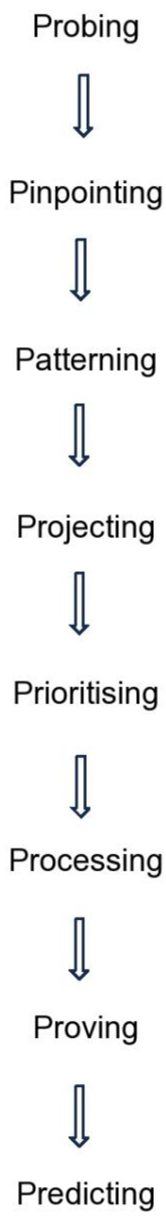
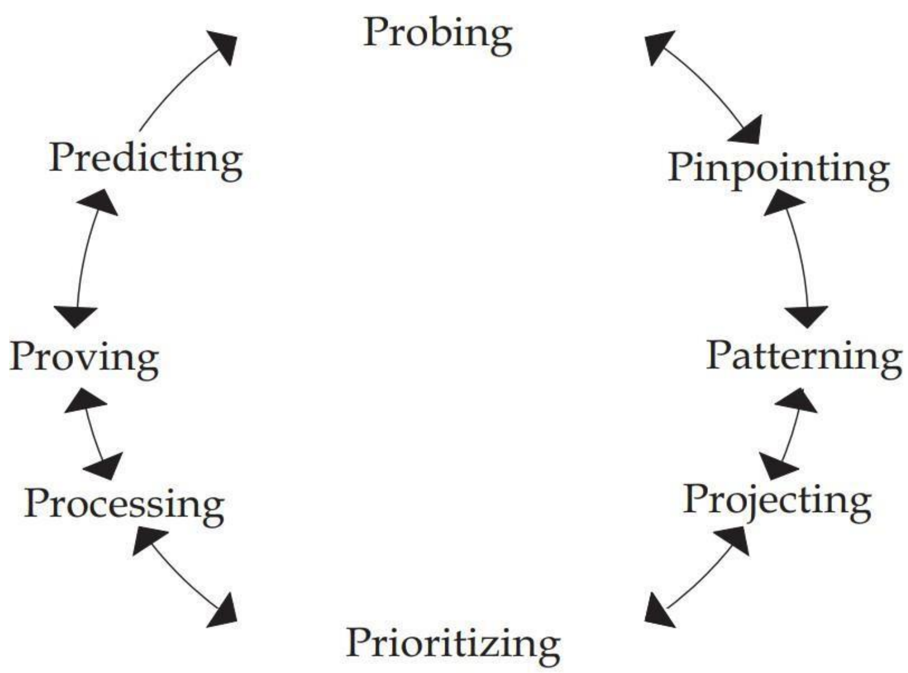


Figure 2.4: Cyclic Nature of the 8Ps Learning Model



The use of double-head arrows in-between the phases of the cyclic 8Ps learning model indicates that a problem solver can move back and forth along the phases in the course of finding solution ideas. In case he is confused or somehow stuck at a phase, he can move back to the previous phase(s) to seek more clarity.

For instance, if a problem solver has already probed a question and has also probably pinpointed the key areas of the question, and he is trying to translate the question into helpful patterns, he can still go back to probe the question again and/or ascertain he has correctly pinpointed or got hold of the essential requirements of the question before representing the problem with applicable patterns. Or after projecting and obtaining some seemingly sensible solution strategies, the problem solver can still go backwards to probe the question again, ensure he has pinpointed the actual word(s) or expression(s) the question requires or recheck his patterns, for him to be able to properly prioritise or consider his solution ideas in their order of relevance.

The prioritising phase appears central to this learning framework as the fruitful processing or solving of the problem hinges mostly on it. For the problem solver to be able to productively prioritise his solution procedures (that is, determine the level of suitability of his solution methods) before eventually choosing the one he considers most appropriate, it is advisable he quickly again refers to each of the previous phases as applicable to the problem. Hence, the first four phases, one by one, contribute towards the prioritising phase just as it also depends heavily on each of them. Besides, while the problem solver is trying to ascertain the logicality of the solution got at the proving phase, he can still go back to the processing phase to be sure no error is made in the course of processing the solution ideas (that is, while solving the problem). He can as well retrace his steps back to the first three phases for a better analysis and understanding of the problem. If need be, he can still double-check phase four to do a quick re-prioritisation of his solution moves.

In Figure 2.4, the use of a single-head arrow to link the predicting phase and the probing phase is worth noting. The indicates that the predicting phase only leads to the probing phase and not vice versa. In other words, after a problem solver may have used the solution obtained to a mathematical problem to predict the solution to another similar mathematical problem (generalisation of the solution), he can still return to phase one (probing the question) and subsequently to other phases in turn as the problem situation demands. To this end, the 8Ps learning model allows proper proving or authentication of a solution being generalised. However, movement along every other two phases of the model is conveniently vice versa.

2.4.4 Reflection on the Chapter

This chapter has captured the conception of the 8Ps learning model. This is done by recognising problem solving as a pedagogy capable of fostering learners' conceptual understanding of Mathematics, and so should be treated as a fundamental factor in Mathematics learning at each grade level and for each Mathematics topic. It has presented the definitions and relevance of Mathematics; the supreme position of Mathematics on the South African curriculum; differential calculus as a key area of Mathematics; learners' difficulties in differential calculus, and stationary points in differential calculus as the focus area of the study. The chapter has explained problem,

problem solving, mathematical problem solving, its required skills and strategies, and appropriate mathematical problem-solving tasks. It has expatiated on teachers' role in mathematical problem solving; possible challenges of teaching Mathematics through problem solving, and the low problem-solving performance of Mathematics learners in South Africa and globally. This chapter has described some existing problem-solving learning models and finally the 8Ps learning model designed for this investigation.

CHAPTER THREE

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

3.1 Introduction

This chapter explores the theories underpinning the study and reviews relevant literature. In a bid to provide due theoretical strength for the study, the first part of this chapter delves into the learning theory of constructivism and three problem-solving theories. Thereafter, the second section reviews previously conducted related studies for further insights into this study.

3.2 Theoretical Framework of the Study

The theoretical framework of a study is majorly about how the research theory explains and establishes the study (Varpio, Paradis, Uijtdehaage & Young, 2020). It is basically how the existing theories, frameworks or models guide the formulation of the research objectives, research questions, research hypotheses, research design, data collection and data analysis. Researchers make use of the theoretical framework to justify the conceptual framework of their research (Chukwuere, 2021). The theory of constructivism and three problem-solving theories informed and guided the direction of this heuristic problem-solving experiment. The theories helped in providing clear definitions and explanations for the key terms used in this study. They assisted in pointing out, describing and interpreting the important relationships among the major constructs of the study. They were also used to identify the gaps in knowledge of the applications of heuristic learning models to mathematical problem solving, and how such missing links could be bridged.

Particularly, the problem-solving theories were essentially adopted in this study to provide a theoretical base and justification for the choice, development and application of the 8Ps learning model. The problem-solving theories provided the researcher with more understanding about the nature and level of the problem-solving performance of the Grade 12 learners under study, and were as well useful in analysing the data that consequently emerged from the inquiry. Specifically, too, the constructivist learning theory was used to corroborate the problem-solving theories since both theories evidently share some common underlying assumptions that strengthened the goal of the current study. As such, more light was thrown on what a learner-driven, problem-solving Mathematics classroom should look like; how the researcher as the intervention Mathematics teacher in the experimental group was supposed to facilitate the problem-solving learning process; the level and quality of guidance, mentoring and support he was required to offer the learner participants until they obtained logical solutions to the assigned Mathematics tasks; the participants' reasoning, interactions, solution skills and strategies, the actual solutions and the logicality of those solutions.

3.2.1 Constructivism

Constructivism has been increasingly adopted as a theoretical rationale for learning and research so much so that several current education reforms identify with its philosophy. The present enquiry was partly approached from the constructivist perspective, believing that problem-solving processes hinge on some basic reasoning skills that fall within the constructivist framework, and more so that, considering real and authentic Mathematics tasks, constructivist learning centres on the active engagement of learners in problem solving and critical thinking (Brooks & Brooks, 1999; Shah, 2019). Constructivism is a learning theory which postulates that learners actively create their own knowledge by drawing on their prior knowledge and experiences. It is a learning philosophy that accords priority attention to personal construction of meaning by learners through experience, believing that the meaning so constructed is the influence of the interactions between the learners' prior knowledge and the current situation (Dev, 2016; Gallardo-Alba, Grüning & Serrano-Solano, 2021).

This constructivist learning principle of how learners actively construct knowledge based on their existing cognitive framework is a sharp contrast to the traditional perspective of learning which encourages the teacher's dominance of the learning process as the custodian of knowledge meant to transmit knowledge to the learners that must passively receive it (Chirove, 2014; Dhlamini, 2012; Elliott, Kratochwill, Littlefield & Travers, 2000; McLeod, 2019; Omoniyi, 2016; Sebsibe, 2019). Based on studies (such as: Brooks & Brooks; 1993; Honebein, 1996; Jonassen, 1994; Kalu, 2012; McLeod, 2019; Omoniyi, 2016; Opoh & Iwok, 2014; Sebsibe, 2019; Tam, 2000), the underlying principles of constructivism can be summarised as follows:

- Knowledge construction is strongly emphasised as opposed to knowledge reproduction. Knowledge is created by the cognising subject and not meant to be passively absorbed from the environment. Cobbinah and Bayaga (2017) believe that the responsibility for learning rests largely with the learners constructing their own understanding, while the teacher acts as the facilitator. According to them, during the learning process, teacher inputs have to be minimised, and learner participation maximised.

- Learners' prior knowledge and personal experience is accorded a premium place in the learning processes. Simonsmeier, Flaig, Deiglmayr, Schalk and Schneider (2021) hold that prior knowledge serves as a strong predictor for learning performance especially when positively mediated through appropriate processes. Brod (2021) therefore urges teachers to ensure that they consider evaluating learners' previous knowledge before teaching them a new content.

- Learning process is active and continuous. Rather than play dominance, or act as a sole-transmitter of knowledge to learners (erstwhile taken for *tabula rasa* or an empty vessel waiting to be filled), the teacher should actively involve the learners in the learning process and allow them to be responsible for their learning. Gallardo-Alba, Grüning and Serrano-Solano (2021) stress that the learning process has to be kept active and dynamic, and that learners should be allowed to contribute ideas and engage in discussions rather than being rendered inactive in the learning process.

- Knowledge is treated as a social activity, and so has to be socially constructed. That is, construction of new knowledge is supposed to be done through active learner-learner and learner-teacher interactions, and not by the teacher as the sole possessor of knowledge. The teaching-learning process is taken as an interactive session engaging learners in knowledge construction. During the interactive session, negotiations take place between the teacher and the learners, and resultantly, the teacher has to find it convenient to convey the learners towards a considerable level of understanding of the task being done (Dewey, 1938; Vygotsky, 1978). Expressing the same opinion, Gallardo-Alba, Grüning and Serrano-Solano (2021) recommend a cooperative classroom environment which welcomes constructive discussions and social interactions and also enables learners to challenge and share ideas.

- Knowledge is personal because every learner does hold distinctive views about situations or problems. Based on experience and previous knowledge, individual learners construct knowledge. This, therefore, makes it possible for learners to achieve different learning outcomes from the same classroom activity. Although exposed to the same learning experience, individual learners build their learning on the meaning and understanding personal to them. In support of this, some researchers (such as: Ertmer & Newby, 2013; Glattfelder, 2019; Loeng, 2020, among others) observe that a form of perception made of reality is an individual issue and that explains why learning becomes a product of personal experience and prior knowledge. This implies that learners come to class with different levels and qualities of prior knowledge. They make different reasoning and interpretations of the class instructions and tasks assigned to them. They also carry out the activities in different manners. Contrary to the position of some schools of thought on this matter, this however does not really mean that learners do not share knowledge through social interactions or that knowledge can still not be socially constructed. It only indicates that learners internalise knowledge in different ways. Buttressing this, Loeng (2020) states that learners still share knowledge despite that they have their own individual and personal ways of learning.

- Knowledge exists within the human mind and so may not match an existing reality. Learners constantly form their own understanding of the real world the way they perceive it. As soon as their perceptions and interpretations of reality or their new experiences change, they tend to modify, change or update their knowledge. This invariably implies that learning might involve some conceptual changes (Glattfelder, 2019).

The intervention administered to the experimental group conformed to the fundamental tenet of constructivism, which is mainly that learners learn by actively constructing knowledge by themselves based on their prior knowledge and personal experiences, and should therefore not be reduced to mere observers of the learning process or inactive absorbers of the knowledge that the teachers transmit to them. All the Mathematics lessons on the concept of stationary points in differential calculus recognised that the study participants brought their prior knowledge and personal experiences (whether correct or wrong) to each learning situation. The study participants were therefore allowed, encouraged, guided and supported to critique, re-evaluate and apply their prior knowledge and personal experiences to each lesson presented. They were given helpful support to repeatedly carry out the processes of articulating, interpreting, re-evaluating and applying their previous knowledge and experiences until they clearly showed that they understood the mathematical tasks being solved in each lesson.

3.2.2 Major Theories of Constructivism

This study adopts the theoretical perspectives of cognitive constructivism, social constructivism and radical constructivism. Although the three main theories of constructivism hold some apparently different views about learning, they still have common standpoints and are, to a reasonable extent, complementary to one another. Rather than dwell on their divergent view-points, the present study concerned itself with how their common notions foster learning by problem solving and the appropriate use of problem-solving learning models, particularly the 8Ps learning model, towards increasing learner achievements in Mathematics.

3.2.2.1 Cognitive Constructivism Centred on the Work of Jean Piaget

The cognitive constructivists claim that learners actively construct knowledge based on their cognitive structures. They believe that the level of cognitive development of a learner determines the extent to which such a learner will learn. Jean Piaget, the chief proponent and pioneer of cognitive constructivism, rejects the conventional classroom practice regarding the teacher as the sole custodian and transmitter of knowledge and learners as passive assimilators of the transmitted knowledge. He then proposes learning as an active and dynamic process that allows learners to make use of their prior knowledge and perceptions of the world to construct knowledge (Amineh & Asl, 2015; Schunk, 2012; Zhou & Brown, 2017). Research widely document the main Piagetian principles of constructivism as follows:

- Discovery learning should be promoted.
- Sensitivity to children's readiness is accorded priority.
- Individual differences exist, and
- Learners' ability to construct knowledge by themselves is a priority, as opposed to imposing knowledge on them.

In consideration of the admonition offered by Zuljan, Valenčič-Zuljan and Pejić-Papak (2021), for learners to acquire high-quality, permanent knowledge, it is advisable that the teaching of technical and scientific contents centres on cognitive constructivist approach. While administering the 8Ps-oriented intervention in the experimental group, the researcher allowed the participants to demonstrate their cognitive skills in the process of knowledge creation. Rather than try to transfer knowledge of the concept of stationary point in differential calculus to them, he guided and supported them as necessary. He gave them the chance to discover solution strategies and solutions on their own. He provided them opportunities to freely interact, discuss, make suggestions, ask and answer questions, seek for clarifications from fellow learners and also from him.

3.2.2.2 Social Constructivism Based on the Work of Lev Vygotsky

As a social constructivist, Lev Vygotsky placed a strong emphasis on learning through social interaction and learner's cultural background. He described learning as co-constructed since learners learn from one another. He did not accept Piaget's view that social and cultural contexts could be removed from the process of learning. For him and his fellow social constructivists, learning is a collaborative process during which knowledge develops while individual learners interact with their society and culture. Vygotsky noted that culture provides the cognitive tools the learner needs for development (Akpan, Igwe, Mpamah & Okoro, 2020; Amineh & Asl, 2015; Dorgu, 2015; Omodan, 2022).

Vygotsky (1978) observes that each function in a child's cultural development manifests two times – firstly on the social level and, secondly, on the individual level. He refers to the first occurrence, which is between people, as *inter-psychological,* and the second one that takes place right inside the child as *intra-psychological.* In Vygotsky’s view, this is not only applicable to voluntary attention, but also to logical memory and formation of concepts. He declares that culture provides the cognitive tools the learner needs for development. The required cognitive tools include the adults in the learner’s environment, cultural history, language, social context and, in recent times, access to electronic information. In respect of this Vygotskian principle, Liu and Matthews (2005) comment that Lev Vygotsky strongly recognises social and cultural interactions as mediums for knowledge construction, and thus sees learning as a social process which is basically context-oriented and situation-based.

The Vygotskian theory of socio-cultural learning also holds that the rate at which learners develop cognitively is determined by the Zone of Proximal Development (ZPD), a level of development that learners reach whenever they exhibit social behaviour. The theory further states that full social behaviour determines full ZPD level, and that the range of skills which learners can acquire with the teacher’s (or adult’s) guidance or peer collaboration is greater than what they can achieve all alone by themselves (Rieber, 2015; Vasileva & Balyasnikova, 2019). Believing in the intrinsic value of social interaction in cognitive development, Vygotsky (1978) argues that the community has a crucial role to play in how learners’ construct meaning because the environment influences what children think about and how they think.

Vygotsky (1978) states further that within the ZPD, social interactions from guided learning bring about cognitive development. Vygotsky notes that consciousness is the end-product of socialisation because learners master and interiorise the learning which comes about while interacting and collaborating with their peers and the teacher. He is of the belief that learning takes place within the ZPD, and that the ideas and concepts that learners cannot understand on their own, they can understand through the help received from adults (like the teacher) and more experienced learners. In their own perception, Mutekwe, Machingambi, Maphosa, Ndofirepi and Wadesango (2013), Vygotsky's socio-cultural method of learning offers virtually all learners the benefit to play an active role in the educational process.

As promoted by the social constructivists, the current researcher gave room for collaborative learning in all the experimental Mathematics instruction that he communicated to the experimental group. He appropriately structured, mediated and guided the peer interactions and learners' discussion groups. As the learners interacted and dialogued in their different collaborative peer and discussion groups, he welcomed and facilitated their cognitive strategies such as: questioning, criticising, predicting, suggesting, clarifying, analysing and summarising. Thus, the study participants were able to achieve a reasonable level of mathematical thinking and also gained some skills vital for mathematical problem solving.

3.2.2.3 Radical Constructivism Developed by Ernst von Glasersfeld

Glasersfeld (1974) theorises that knowledge is constructed and not perceived by the senses. He, like the other advocates of this idea, posits that learners create new knowledge upon the foundations of their already existing knowledge. They, however, do not agree that the knowledge so constructed by an individual has to do with reality, but that that only enables him to function in his environment. These proponents of radical constructivism hence uphold the idea that knowledge cannot be discovered, but it is rather invented.

Glasersfeld (1993) makes a submission that constructivism is a method of thinking that sees the natural environment as a source of knowledge. He argues that the external world is filled with events and objects that learners are supposed to learn about. According to him, neither scientists nor learners can totally understand reality. They can only form an approximate or likely notion of it. The notion so formed may or may not be the true reality. It may also not be complete. As this theorist observes, absolute truth is non- existent. He thus advises the learners to aim at how to form viable ideas about the world which they can apply to explain and understand nature. He believes such useful knowledge can assist in achieving their purposes and high quality of life. The implication of Glasersfeld's assertion is that reality depends on the mind to exist. Therefore, it is the mind that constructs knowledge (or reality), and there is no other exact copy of reality elsewhere.

Expressing a similar view, Glasersfeld (1995) regards knowledge as a completely individual construction, learning as an individual mental process and the individual learner as an independent investigator. For him, knowledge remains an independent issue and that it is situated in the mind. He views knowledge as a reality that the learner creates based on personal experience. While commenting on the principles of radical constructivism, Liu and Matthews (2005) state that Von Glasersfeld and his fellow radical constructivists place a strong emphasis on the learner as an independent creator of knowledge but only see the learner's surrounding community as a motivation for cognitive development.

The notion that the mind generates meaningful conceptions of reality offers advantages for education. As learners attempt to conceptualise reality, they have to first process it within them, organise it, and also reflect upon it. By so doing, learning is then an active process which draws from previous knowledge. In other words, what learners already know is deemed to have equal importance as what we anticipate they will know. This, in effect, has changed the role of the learner from being a passive receiver of knowledge to that of constructor and owner of knowledge. The present study placed due value on this standpoint by the radical constructivists during the conduct of the intervention. It ascertained that all the experimental Mathematics classrooms welcomed the various thoughts, diverse opinions, reflections, arguments, suggestions and contributions made by the study participants, and allowed these to logically lead to the discovery of the solution strategies and the solutions to assigned mathematical tasks.

3.3 Implications of Constructivist Learning Theories for Classroom Practice

From the foregoing (see subsection 3.2.2.1 - 3.2.2.3), the practical classroom applications of the three main constructivist theories (especially as relevant to the 8Ps learning model of the current study) can be summarised as follows:

- Learning processes must be learner-centred, not teacher-dominated. The Mathematics teacher is therefore advised to fully involve the learners in mathematical problem solving and critical thinking. Seen as a facilitator, guide and mentor, he is supposed to allow learners' views, suggestions, ideas, questions and contributions to drive and direct the Mathematics lessons. In essence, the teacher should welcome learner initiatives and autonomy and make the learners responsible for their own learning.
- Learning processes should focus more on mathematical procedures than on the solutions. With this, multiple mathematical perspectives, solution strategies and alternative solutions will be explored before obtaining a solution. This will go a long way to eliminate rote learning and memorisation of mathematical formulas and algorithms. Then, the mathematical reasoning, ideas and solutions arrived at by learners will simply be consequential of the learning processes.
- Effective learning means active construction of knowledge from the learners' prior knowledge and personal experiences. The teacher should allow learners to apply their prior knowledge and personal experiences to solve current mathematical problems, and ensure that he facilitates this appropriately.

- The Mathematics problems presented for learners to solve have to be tasks that are stimulating, real and authentic. This however does not mean the teacher has to set or select the tasks outside the specified Mathematics curriculum or that he would lower the cognitive complexity of the questions. Song (2011) remarks that real learning situated in a familiar context triggers the learner's previous knowledge. He further states that a learner recalls quickly the information which appears in a more familiar context. This becomes possible because the familiar context makes it easy for learners to activate their schemata, which are crucial for understanding and remembering new information. According to Kodisang (2022), a mathematical problem is meaningful if it is a real-life one and if learners see the essence of solving it.

- While the learners make personal creations of mathematical ideas using their previous experiences, individual opinions and cultural expositions, they can still collaborate and negotiate meanings and ideas through co-operations, communications, discussions, group work and interactions with fellow learners and their teacher. As the learners share ideas and make sense of the view-points of one another, they attempt to weigh the various perspectives and select the more plausible ones for use in determining the solutions to given mathematical problems. Through this, learners' understanding of a Mathematics concept becomes broadened and robust.

At this juncture, the present study holds as pertinent the remark made by Nayak (2012): The teacher's efforts in the classroom will be more effective if informed and guided by the understanding of how learners learn. Moreover, learning is likely to be more meaningful and rewarding if learners get the opportunity to explain and clarify their own ideas in the learning process. Then, it makes sense to observe the reflections of the concepts of constructivist learning in the classroom. In effect, the present study allowed the above practical applications of the constructivist learning principles to steer the conduct of the 8Ps-oriented intervention.

3.4 Problem-solving Theories

The three learning theories supporting problem solving considered in this study are: behaviourist learning theory, cognitive problem-solving theory and situational theory of problem solving.

3.4.1 Behaviourist Learning Theory

B. F. Skinner is often regarded as the chief proponent of behaviourism, while Ivan Pavlov, Edward Thorndike and John Watson are seen as its major advocates. Behaviourism believes learning results in changes to the form or frequency of observable performance. It holds that learning takes place when a particular environmental stimulus presented produces an appropriate response. The stimulus, the response, and the relationship between the two are the core elements in behaviourism. Moreover, the way the association between the stimulus and the response is produced, reinforced, and sustained is of utmost importance to the behaviourists. Besides, the behaviourists believe learners react to conditions in the environment as against participating actively in the discovery of the environment. To these theorists, the goal of instruction is to obtain a desired response from the learners consequent upon the presentation of a target stimulus (Ertmer & Newby, 2013; Fitriati, Fatmala & Anjaniputra, 2020).

One of the key learning techniques of behaviourism that encourage problem-solving performances is reinforcement. Reinforcement is a form of external motivation that aids learning. It is any consequence that strengthens behaviour. The common forms of reinforcement used in the classroom are: praise or some other types of verbal reinforcement, tangible rewards and token rewards. Reinforcement is applied in the classroom as a strategy to bring about desired academic performance or efforts. Hence, reinforcing learners in the classroom is important because it can increase learner achievement (Andre, 1986; Cotton, 1988; Fitriati, Fatmala & Anjaniputra, 2020; Kinyanjui, Aloka, Mutisya, Ndeke & Nyang'ara, 2015).

The behaviourists acknowledge the importance of positive and negative reinforcements in facilitating problem-solving skills and practices in learners. They describe positive reinforcement as the process of encouraging a pattern of behaviour with offers of reward immediately and every time the behaviour is exhibited. To them, the purpose of positive reinforcement is to increase the likelihood of the future occurrence of the behaviour. For instance, the teacher can positively reinforce desired learner behaviour by giving learners offers such as: praise, good remarks, applauses, gifts or even monetary rewards (Andre, 1986; Brunning, Schraw & Ronning, 1999; Chirove, 2014; Fitriati, Fatmala & Anjaniputra, 2020; Hardin, 2002).

Negative reinforcement, on the other hand, neither promotes negative behaviour nor implies negative punishment. It, however, refers to a method of strengthening a response or behaviour by withholding or taking away something good or, as the case may be, by adding or bringing in an unpleasant thing. If properly applied, the teacher can adopt negative reinforcement as a workable corrective measure to reduce undesirable behaviour or to motivate learners to change their unpleasant actions. As the teacher removes or keeps away certain things the learners like or enjoy multiple times, the learners tend to exhibit the target desired behaviour over time. For example, to encourage a particular learner behaviour, the teacher can relax or totally remove a set of restrictions earlier placed on the learners. The other side to negative reinforcement is that the teacher can consistently add or bring in certain things that the learners do not like or enjoy to make them avoid an unwanted behaviour and begin to depict the desired one (Andre, 1986; Brunning, Schraw & Ronning, 1999; Chirove, 2014; Fitriati, Fatmala & Anjaniputra, 2020). In either case, the behaviourists adopt both positive and negative reinforcements to influence behaviour.

Another problem-solving technique that the behaviourists also propose is trial-and-error. As a fundamental approach to mathematical problem solving, trial-and-error is typified by varied and repeated attempts at getting a desired mathematical solution. It allows a learner searching for the solution to a mathematical problem to make use of different mathematical methods until such obtains the actual solution. Although popular in its own way and commonly adopted by many present-day mathematical problem solvers, the behaviourists' idea of trial-and-error has been seriously criticised. Critics see it as being time-wasting while different attempts are being made for the correct solution. They argue that, rather than give room to deep mathematical reasoning and analytical problem-solving, trial-and-error technique allows cheap guesses and shallow mathematical thinking. Critics contend further that trial-and-error only gives the activity patterns for assessing a solution idea without explaining how the solution idea comes about. They also condemn it for being useful for only a class of problems (Kipman, 2020; Tonnsen, 2021; Wiesenfarth, 2019).

In the current study, while administering the 8Ps instruction to the experimental group, the researcher applied both positive and negative reinforcements effectively and accordingly. He made sure that the study participants were made to see that trial-and-error has limited usage. Hence, he guided them as to when to apply it and when not to, as they navigated through the eight phases of the 8Ps learning method to solve the Mathematics problems presented to them.

3.4.2 Cognitive Problem-solving Theory

John Dewey was a major historical adherent of cognitive theory of problem solving. Unlike Edward Thorndike who, in his problem-solving theory, supported the behavioural component of learning, Dewey focused both the conscious thinking ability (cognition) and behaviour of a problem solver as essential factors in problem solving (Bunning et al., 1999). Dewey argued that an individual attempting to solve a problem has to intentionally make use of thinking in the process. As recorded by some researchers (like Carson, 2007; Kulsum & Kristayulita, 2019; Williams, 2017, among others), Dewey's problem-solving method comprises the five mental stages highlighted below:

- Identifying or having awareness of the problem
- Defining or diagnosing the problem

- Gathering data and formulating tentative hypothesis
- Testing the hypothesis with a view to rejecting or accepting it
- Drawing and evaluating conclusions

The 8Ps problem-solving instruction administered to the experimental group was in consonance with Dewey's five mental steps. The phase one (probing) of the 8Ps learning model taught the study participants how to identify or become aware of a problem. Its phase two (pinpointing) and phase three (patterning) taught them how to diagnose or define the problem. The fourth (projecting) and fifth (prioritising) phases explained to them how to collect data and formulate tentative hypothesis required for solving the problem. While phase six (processing) described to the study participants how to test the tentative hypotheses so formulated in order to decide whether to accept or reject it, phases seven (proving) and eight (predicting) illustrated to them how to draw conclusions on the problem and evaluate such conclusions.

In his own cognitive problem-solving approach, the Gestalt psychologist, Kohler, acknowledged the role of insight in problem solving. He explained insight as the instantaneous awareness of appropriate or helpful relationships existing between means and objectives towards obtaining the solution to a given problem. Thus, a problem solver applies insight when he suddenly re-organises his mental elements into a structure which leads him to the solution to a problem. The problem solver can do mentally most of the problem-solving processes without necessarily keeping a record of the thinking processes undergone. It is only when the problem solver experiences an unexpected occurrence of insight to a problem that he is trying to solve that he may have to make notes on paper (Andre, 1986; Ash, Jee & Wiley, 2012; Kohler, 1925, 1959).

It is noted that the insight experience can be a predictor of accuracy; as such, the sudden intuitive solution (or solution got through sudden realisation or by insight) may be more likely correct than one got by conscious and incremental steps (Salvi, Bricolo, Bowden, Kounios & Beeman, 2016). The feelings of insight can also inform the problem solver of the potentially correct solution (Irvine, 2015). However, insight has been seen as a consequence of the learner's intelligence (learner's ability to mentally restructure the problem) and past experience (the information already acquired in the learner's memory) (Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016; Weisberg, 2015). Insight being a consequence of these two elements, the current study asserts that insight may not do much to help a Mathematics learner with low intelligence and/or weak past experience (poor previous knowledge of a Mathematics concept).

Kholer's cognitive idea of problem solving differs from Dewey's which consists of five mental steps that necessitate making notes on paper. Considering how demanding mathematical problem solving can be, Kholer's failure to break down his own method into steps capable of making communication on paper possible, may not make problem solving easy to do. In the current study, while the researcher administered the 8Ps-based treatment to the experimental group, he advised the participants to always note down on paper their solution thoughts and plans whenever they are to use insight. The essence of this is for them not to unnecessarily burden or overload their short-term memories and also to be able to regulate, monitor and review their solution processes.

Another Gestalt cognitivist, Graham Wallas, divided the problem-solving process into four steps: *preparation*, *incubation, inspiration* and *verification.* At the preparation stage, the problem solver does the analysis of the problem and collects useful information that may help in solving the problem. During incubation, the problem solver considers the problem subconsciously as he is busy relaxing, doing or thinking of another thing. At this point, the solver steps back from the problem, allowing his mind to contemplate and work through the problem. It is in the inspiration or illumination phase that the solution suddenly occurs to the solver – in a way similar to the insight experience proposed by the Gestalt cognitive psychologists. The verification stage is the time to check the solution and work out the details (Andre, 1986; Savic, 2016; Setyana, Kusmayadi & Pramudya, 2019; Wallas, 1926).

Brunning et al. (1999) comment that the peculiarity of Wallas' problem-solving approach lies in its inclusion of the incubation and the inspiration phases, which are not found in other problem-solving models developed by other researchers. Brunning et al. further state that analysing these two additional stages will not be easy because of their subconscious nature, unlike Dewey's approach which consists of a reasonable set of steps. The conduct of the current study also learnt from Wallas' problem-solving principles. The first three phases of the 8Ps learning framework fully explain Wallas' first step (*preparation*) and the second three phases can serve as Wallas' second step (*incubation*). The seventh phase (proving) and the eighth phase (predicting) take care of Wallas' last step, *verification*. In the current study therefore, the researcher advised the experimental group to communicate aloud and note down on paper any inspirations that occurred to them while making solution moves. He let the participants realise that this would give them the opportunity to regulate, monitor and review their solution strategies.

3.4.3 Situational Theory of Problem Solving (STOPS)

Kim and Grunig (2011) propose the situational theory of problem solving (STOPS) as an explanation to how and why a problem solver communicates during a problem-solving situation. They describe problem as perceptual discrepancies existing between actual conditions and experienced conditions in a setting that caused discomfort. Therefore, problem solving is an effort to eliminate the apparent discrepancies. According to Kim, Tam and Chon (2021), STOPS was premised on the idea that communicative behaviour is purposive and epiphenomenal to problem solving, and that communication is not merely what the sender does to influence the attitude and behaviour of the receiver, but what people do to deal with problematic life situations. In essence, these theorists present STOPS as a communication theory that explains people's motivated communicative actions in the context of problematic circumstances.

Kim and Grunig (2011) claim that the more committed an individual is to problem solving, the more such becomes acquisitive of information relevant to the problem, selective in handling the information, and transmissive in getting it across to others. They use the following four key variables to describe a problem solver's communicative actions: *problem recognition, constraint recognition, involvement recognition* and *referent criterion*.

- *Problem recognition:* It is the ability of the problem solver to perceive that something is missing and that no instant solution to the problem is available.

- *Constraint recognition:* When the individual solving a problem perceives obstacles in the problem to be solved, he has done *constraint recognition.* This action tends to reduce his ability to try to solve the problem. His communicative behaviour has thus been limited regardless of how properly he might have successfully undergone the *problem recognition* stage or how well he will fare in *involvement recognition.*

- *Involvement recognition:* This is a perceived connection between the problem solver and the problem situation.
- *Referent criterion:* It is a knowledge or subjective judgmental system which can affect the way the problem solver handles problem solving.

Kim and Grunig (2011) further describe problem solver's communicative behaviour while attempting to solve a problem. They note that as the individual is attempting to solve a problem, his communicative activeness rises in three domains of communicative action namely: *information acquisition, information seeking and information attending.* They explain the three domains as follows:

- *Information acquisition:* This refers to the communicative action that relates to the levels of information searching for problem solving. According to these theorists, *information acquisition* is done at two levels: *information seeking* and *information attending.* While *information seeking* is an active and planned process of exploring the information available for solving a problem, *information attending* is a passive and unplanned exploration of the information available for solving the problem.

- *Information selection:* This explains the extent to which the problem solver is directed or moved to acquire and share information capable of helping in solving the problem. This communicative behaviour is subdivided as *information forefending* and *information permitting. Information forefending* is an active process pointing to the extent to which a problem solver fends off some information in advance by assessing its value and relevance to a problem-solving task. *Information permitting,* on the other hand, is the extent to which a problem solver accepts any information relating to a given problem-solving task.
- *Information transmission:* It is the communicative behaviour that has to do with levels of educating others to make use of collective behaviour for problem solving. As in the previous two domains, *Information transmission* is carried out at two levels*, information forwarding* and *information sharing.* As explained by the two researchers*, information forwarding* is an active process based on how much of unsolicited information relevant to the solution of a problem which a problem solver makes available for others to access. They see *information sharing* as a passive process indicating the extent that a problem solver releases information only to those who request for his opinions, ideas or expertise about a problem.

During the intervention, the ideas offered by the proponents of situational theory of problem solving really assisted the researcher in having more understanding about how and why the study participants were communicating while solving the mathematical problems he posed to them. That was why in every 8Ps-centred Mathematics lesson, the researcher welcomed and encouraged the study participants to communicate their mathematical reasoning aloud to one another and to him as well. He noticed that the more the study participants became familiar with mathematical problem solving, the more they knew how to discover useful information necessary for tackling given mathematical tasks; and the better they were in handling the information and sharing it with fellow study participants and him as their teacher. With the knowledge gained from the assumptions of situational problem-solving theory, the researcher further understood how the study participants recognised the actual problem (the missing thing) in a mathematical task that must be found; how they perceived the constraints or obstacles in the mathematical tasks that prevented instant solution methods or solutions; how they looked for, acquired, shared, selected and processed the information to solve the mathematical tasks.

3.5 Implications of the Three Problem-solving Learning Theories for Classroom Practice

In the light of the above discussion (see subsection 3.4.1 - 3.4.3), the practical classroom applications of the three problem-solving learning theories, particularly as they support the use of the 8Ps learning model, are summarised as follows:

- For the purpose of determining where to begin instruction, Mathematics teachers must first conduct learner analysis, often known as pre-assessment of learners.
- They should make effective use of both negative and positive reinforcements to develop and enhance mathematical problem-solving skills and practices in the learners.
- The teachers should learn how to produce noticeable and quantifiable outcomes in learners through task analysis, specific objectives, criterion-referenced assessment, etc.
- While Mathematics learners can apply trial-and-error in solving some Mathematics problems, they should be guided not to rely heavily on it since it has limited usage.
- Mathematics learners need to do some conscious thinking in the mathematical problem-solving process which can be carefully facilitated by some step-by-

step heuristics.

- Since learner communication enhances mathematical problem-solving, learners should be encouraged to communicate their mathematical thinking and ideas aloud to fellow learners and the teacher while solving Mathematics problems in the classroom.

The intervention carried out in the experimental group was guided by the highlighted practical applications of the three problem-solving theories.

3.6 Review of Related Previous Studies

For a thorough understanding of the current study and to provide a solid background for it, relevant existing studies were reviewed. As noticed by Snyder (2019), building new research upon and relating it to existing knowledge stands as the building block of every academic research activity, irrespective of discipline. In their own observation, Maggio, Sewell and Artino (2016) comment that review of related past studies enables a researcher to have evidence of sufficient preparation, articulate clear research goals, choose suitable research methods, communicate relevant results and engage in thoughtful critique. Winchester and Salji (2016) also remark that the review will help the researcher develop his research idea, provide him with what is already known about the subject, make him identify any existing knowledge gaps and reveal how his own study can contribute to further understanding of the research topic.

Dhlamini (2012) examined how the context-based problem-solving instruction (CBPSI) could impact on Grade 10 learners' mathematical problem-solving skills. The study participants were 783 Grade 10 learners and four teachers from nine secondary schools in Gauteng province of South Africa. The study adopted the cognitive load theory as a theoretical basis for the enquiry. It followed a mixed-method research technique involving a non-equivalent control group design, which consisted of pre-test and post-test measures, classroom observations and semi-structured interviews. The researcher personally administered the CBPSI in five experimental schools while the Mathematics teachers of the four participating control schools communicated the normal traditional lessons to the learners. In administering the CBPSI, the researcher used context-based problem-solving tasks posed in the form of worked-out examples and worksheets handed out to participants in the experimental group. To determine the initial problem-solving performance of the learners before the intervention, he conducted a standardised functional Mathematics achievement test as a pre-test. After the intervention, he administered the same achievement test as a post-test.

The data collected was analysed by using mainly the one-way ANCOVA and the ANOVA. It was found out that CBPSI proved to be an effective instructional instrument to facilitate the problem-solving performance of the study participants. Dhlamini (2012) then charged future researchers to further investigate the learners' thinking process while solving Mathematics problems so as to find out how the learners move from elementary mathematical problem-solving skills to advanced problem-solving skills. The study also asked that further studies be conducted on some other instructional methods that can effectively develop the learners' mathematical problem-solving skills. Based on his recommendations, a need arose for the current study to assess the thinking process of learners during mathematical problem-solving in a different Mathematics topic in Grade 12 (another grade level of the high school) using a different problem-solving instructional method (i.e., the 8Ps learning technique).

Brijlall and Ndlovu (2013) conducted a qualitative case study with ten Grade 12 Mathematics learners of a rural school in Umgungundlovu District in KwaZulu-Natal, South Africa. It examined the learners' mental constructions of mathematical knowledge in the concept of optimisation problems in differential calculus. It collected data using structured activity sheets and semi-structured interviews. The participants were assigned three Mathematics tasks in the structured activity sheets. The tasks were done collaboratively in groups. Afterwards, each of the group leaders was interviewed. The data extracted from these instruments were analysed descriptively and interpretively. The researchers found out that the learners performed well in routine questions, which indicates that the learners were operating at an action level.

The interviews conducted revealed that the learners lacked conceptual understanding of the topic despite that they got some of the answers correctly. The study realised that learners usually created knowledge based on their previous mental assimilations. It also discovered that, when solving the given tasks, instead of using conceptual thinking, the learners mainly relied on procedural thinking. The study demonstrated that the learners' knowledge construction was mostly based upon isolated facts and procedures, and that this might be related to the manner in which the topic was taught and learnt. That is, the teaching and learning placed emphasis on the procedural aspects, neglecting the conceptual understanding of the topic. The study, described small-scale, can be expanded with both quantitative and qualitative methods and a bigger study sample. This opens doors to other studies like the current one.

Chirinda (2013) followed the mixed-method research technique hinged on constructivism to probe how improvement in the problem-solving skills of 57 Grade 8 learners of a school in Gauteng province of South Africa affected their achievement in Mathematics. The study participants in the experimental and control groups were 28 and 29 respectively. Using a problem-centred teaching-learning approach, the researcher created a teaching-learning environment that made use of some purposively prepared Mathematics problems and their corresponding solutions as the major deductive mathematical activities. The tools used for data collection were: questionnaire, direct observation of study participants, semi-structured interviews, learners' journals, compiled register of mathematical problem-solving skills, mathematical tasks and written pre-test and post-test. The data obtained were then analysed by the descriptive data analysis techniques.

In the findings obtained by Chirinda (2013), the experimental group improved their mathematical problem-solving skills after the intervention. The qualitative results then showed that the learners' improved mathematical problem-solving skills enhanced their performance and achievement in Mathematics. The study suggested that it is necessary that deeper research be done with a larger study sample, particularly to explore the learners' thinking processes during mathematical problem solving. It recommended further research that would reveal how learners proceed to advance mathematical problem-solving skills from elementary mathematical skills. It also recommended that further studies that incorporate effective design of Mathematics instruction to develop the mathematical problem-solving skills of South African learners be conducted. Thus, the recommendations gave rise to the conduct of the present research.

Makgakga and Makwakwa (2016) was a case study of some Grade 12 Mathematics learners who had difficulty in using first principles and rules of differentiation to solve differential calculus questions. The study sample, conveniently selected, was 37 learners and 1 Mathematics teacher of a secondary school in the Polokwane district of Limpopo province, South Africa. The investigation, which adopted the interpretive paradigm and the explanatory sequential mixed-methods design, employed diagnostic test and semi-structured interview schedule to gather data from the learner and the teacher participants. Quantitative data were generated through the diagnostic test of six test items administered to evaluate the learners' understanding of the derivatives of functions using first principles and rules of differentiation. Qualitative data were obtained from the semi-structured interviews conducted for seven randomly selected learner participants and the teacher. The aim was to elicit the mathematical reasoning behind the solutions provided by the learners in the diagnostic test and to make sense of the learners' difficulties while solving the test questions.

As found out by Makgakga and Makwakwa (2016), the diagnostic test indicates that: the learners presented wrong formulas and incorrect calculations in determining the derivatives of functions using first principles; they lacked conceptual and procedural knowledge of the topic. The semi-structured interviews corroborated the result that the learners' knowledge of differential calculus was inadequate. Noticing that the study did not propose its own learning method, the present study therefore came in to further this research using the 8Ps learning framework to see how it could impact the learners' problem-solving performance in the topic.

Dlamini (2017) adopted the document (script) analysis method to assess the causes of the low achievement of Grade 12 learners in calculus-based tasks. The goal was to find out the possible reasons for the learners' low performance particularly in cubic graphs and the applications of differential calculus in optimisation. The 171 learners involved in the study were purposively sampled from three secondary schools of Msukaligwa 1 Circuit in Gert Sibande District, Mpumalanga Province of South Africa. The research took the form of a qualitative research design as its data emerged from the analysis of the learners' scripts. The three formal tasks considered were: May common test, June mid-year examination and Trial/Preparatory examination. In the three tasks, only the questions set on cubic graphs and the application of calculus were analysed and interpreted. The content analysis of the learners' scripts was performed to find out their common mistakes, challenges, misconceptions and the nature of their responses to the mathematical tasks. The data gathered helped in understanding the potential causes of the learners' low performance in calculus. The research results revealed that the learners' weak performance in calculus-based tasks was due to their incompetence in some other Mathematics topics such as: inequalities, measurement, algebra and functions.

Dlamini (2017) considered the qualitative method as its only data-gathering technique, thus creating the need for further investigations, especially ones that will include the quantitative aspect. As pointed out by the researcher himself, using more than one data-gathering technique would further strengthen the research results. His remark is actually reasonable because applying more than one method would give room for further triangulation of the data collected and invariably of the results of the study. As observed by Yeasman and Rahman (2012), triangulation refers to a verification process which enhances the validity of the findings of a study through the incorporation of several methods and viewpoints. In other words, triangulation is the combined use of two or more theories, methods, data-types, data sources and investigators in the form of both quantitative and qualitative studies. Again, it is also noted that Dlamini (2017) study did not design a heuristic learning model for the investigation of the causes of the weak performance of Grade 12 learners in calculus. As part of its recommendations therefore, since the learners gave unsatisfactory responses to cognitive-level problem-solving questions (usually higher-order questions), there should be more emphasis on the use of an appropriate problem-solving approach to the teaching and learning of Mathematics. Consequent upon this, the current study emerged to further probe the learners' performance in calculus using multiple theories, multiple data sources, mixed methods and 8Ps problem-solving model particularly designed for that purpose.

The investigation carried out by Ofori-Kusi (2017) explored the use of a heuristic problem-solving instructional method to enhance the teaching and learning of Mathematics. The aim of the enquiry was to determine the effects of the heuristic approach on the achievements of Grade 6 learners in algebra. The two key theories that inspired the study are the modelling and modelling perspective, and APOS (Action, Process, Object and Schema). The investigator employed the modelling and modelling perspective theory to direct the development of modelling-eliciting activities used in the teaching method. He applied the APOS theory to guide the sequence of activities considered towards developing the learners' understanding of algebra. Applying a mixed-method approach, the heuristic problem-solving instructional method was conducted with 198 learners conveniently sampled from four primary schools in the Zululand district of KwaZulu-Natal. He made two of the schools the experimental group and the remaining two, the control group. While the qualitative part of the study consisted of the classroom observations of some Mathematics lessons of the four Mathematics teachers of the participating schools, the design and implementation of the quantitative component followed a non-equivalent control group design of pre-test and post-test measures.

For the Ofori-Kusi (2017) study, data were collected via an observation schedule, standardised achievement test in algebra and modeling-eliciting activities. The data were analysed with the use of the t-test, ANCOVA, Johnson-Neyman (J-N) formula and the effect size. The findings indicated that participation in the heuristic problem-solving instructional method improved the learners' scores in algebra. They also showed how the implementation of the heuristic problem-solving approach could be developed and applied in Grade 6 algebra Mathematics classroom. As acknowledged by the researcher, there is the need to conduct further studies on a larger scale as the investigation only covered the learning of algebra in four quintile 1 intermediate schools (out of the five quintiles) of a district. In addition, the researcher recommended that more studies be conducted on some other mathematical concepts different from algebra with a view to finding and improving alternative instructional methods for the teaching and learning of Mathematics. Hence, the present study was conducted to evaluate the problem-solving achievement of Grade 12 learners in differential calculus (a mathematical concept different from algebra) with a newly developed alternative instructional method.

Awuah (2018) employed a mixed-method research approach to conduct an enquiry into the problem-solving skills of Grade 12 learners in the concept of probability. It had 490 learners purposefully selected participants from seven South African high schools in the Nongoma education circuit of KwaZulu-Natal province of South Africa. It developed a cognitive test to collect quantitative data and performed the content analysis of the learners' scripts to gather qualitative data. Using Bloom's taxonomy as a framework, the research adopted descriptive and inferential statistics for data analysis. It eventually came up with the findings that the participants in fee-paying schools (quintile 4) recorded higher achievement scores than participants in non-fee-paying schools (quintile 1 to 3) at the levels of knowledge, comprehension, application, analysis and evaluation of Bloom's taxonomy. As part of its recommendations, Awuah (2018) charges future researchers to investigate more problem-solving teaching methods and resources using other Grade 12 Mathematics topics and other provinces of South Africa. It also urges future researchers to probe further into the learners' problem-solving performance by conducting interviews in order to obtain the reasons for learners' misconceptions in mathematical problem solving. Hence, the current research became necessary to cover the identified areas.

Mendezabal and Tindowen (2018) looked at the likely effects of using Microsoft

Mathematics to probe learners' attitudes, procedural skills and conceptual understanding of differential calculus. Following a quasi-experimental with pre-test, post-test, experimental-control group research design, it compared two Electrical Engineering classes of two different learning situations registered for a differential calculus course in a private university in the Philippines. The study also used the descriptive research design in which an attitudinal questionnaire was utilised to find out the effect of applying Microsoft Mathematics on students' attitude. The two classes, which consisted of 30 students each, were assigned randomly as experimental and control groups.

The experimental group learnt the differential calculus lessons through exploration and discovery of various concepts by making use of the Microsoft Mathematics embedded activity sheets. The control group was taught the same lesson contents by the traditional instructional approach. The pre-test administered before the conduct of the study revealed that the participants had a little understanding of differential calculus. After the intervention, the participants' performances improved significantly in the post-test. This portrays that the adoption of Microsoft Mathematics for learning differential calculus improves students' conceptual understanding and procedural skills. The results of the study indicating the ineffectiveness of the traditional teaching approach for the learning of differential calculus serve as a basis for further research since the main goal of the current study is to contribute to the ongoing efforts at finding an appropriate instructional method for the learning of the challenging topic and the entire Mathematics as a subject.

Masilo (2018) embarked on how to practically implement an inquiry-based learning method to improve the problem-solving skills of Grade 11 learners in Euclidean geometry. Acknowledging that inquiry-based learning approach is learner-centred, the researcher referred to it in the study as inquiry-based facilitation and treated it as such. The study which was hinged on the pragmatist perspective that constructivism is central to realism (meaning that in teaching and learning, inductive inquiry is supplementary to deductive inquiry), used embedded mixed-method approach. It followed convenience sampling in selecting a total of 97 learners from three Tshwane North district schools as experimental group and 69 learners from three Tshwane West district schools as control group. In response to the inquiry-based facilitation in the concept of Euclidean geometry administered to them, participants in the experimental group researched, weighed evidence, explored multiple perspectives, shared discoveries, engaged in cooperative learning and demonstrated self-regulated learning in solving the Mathematics problems assigned to them. In the control group, the topic was taught with the usual conventional method.

On the average, Masilo (2018) found that the study participants who received the inquiry-based facilitation acquired considerable problem-solving skills and improved more than their mates in the comparison group who did not experience the treatment. Having established that the traditional axiomatic approach encourages rote learning, as well as passive, deductive and algorithmic learning which is not in support of knowledge application in problem-solving, the study then inferred that learning by inquiry-based facilitation causes deep, authentic, self-regulated, non-algorithmic, learning that promotes problem-solving skills in the concept of Euclidean Geometry. At this point, it might be a welcome idea to carry out more studies that would further emphasise why the unsatisfactory traditional teaching approach has to be replaced. It might also be helpful to stress the need to learn Mathematics by problem solving and to also explore the possibility of discovering other suitable problem-solving instructional methods for this essential subject. As such, the present research was thought of.

Sebsibe (2019) applied a literature-informed intervention learning model to evaluate the conceptual knowledge of 295 Ethiopian Grade 12 Mathematics learners who had difficulties with the learning of calculus concepts. He adopted a design-based research approach administered in three phases and pre-test/post-test measures. He employed triangulated themes of learners' difficulties and rampart conceptual issues (which are the causes of the whole range of difficulties in calculus) as a foundation for the proposed intervention model. The study was conducted to emphasise the usefulness of the knowledge of calculus to undergraduate programmes in the fields of science and technology. It was also carried out to see the effect of the suggested learning model on the participants involved in the intervention and to devise ways to make them overcome their difficulties in learning the concept of calculus. Both the quantitative and qualitative analyses of the tests indicated that the intervention recorded a positive effect: the experimental group out-performed the control group particularly in their consistent conceptual reasoning, process level conception, qualitative justifications, few algebraic errors recorded and proficiency in symbolic manipulations achieved. Sebsibe (2019) then recommends that evaluating teachers' and learners' awareness and opinions about emerging theoretical and pedagogical frameworks are crucial issues demanding further research and attention. It also asked that further studies be conducted to replicate the study in a different context for possible generalisability of the results. From this perspective, the present study included the semi-structured interview schedule as one of its data sources to access information on the views and opinions of the study participants about the use of a problem-solving method for learning calculus. It was also conducted in a different research context with a different problem-solving learning framework.

Son and Ditasona (2020) investigated the effects of the applications of three different models namely: model of CORE RME (Model of Connecting, Organising, Reflecting and Extending with Realistic Mathematics Education), CORE model (Model of Connecting, Organising, Reflecting, and Extending) and the conventional model on the mathematical problem-solving skills of three intact groups of junior high school learners in Indonesia. The 145 study participants, who were all Grade 7 learners of 2018/2019 academic year, were distributed as: 50 for CORE RME model, 49 for CORE model and 46 for the traditional model. The underlying principles of the CORE RME learning model are: prior knowledge, guided reinvention, real context, metacognition, self-monitoring, self-developed models, interactivity and intertwining. The quantitative research, which followed a quasi-experimental approach, was intended to improve the learners' mathematical problem-solving skills and to compare the effects of the three models. The two classes of learners for CORE RME and CORE models constituted the experimental group while the group of learners who learnt by the traditional model formed the control group. The data-gathering instrument was a Mathematics problem-solving achievement test set in line with the Pólya's problem-solving indicators namely: understanding the problem; devising a plan; carrying out the plan, and looking back.

In that study, each learner's level of improvement in mathematical problem-solving skills was determined by a normalised gain test performed. The test results were then used in the one-way ANOVA test to obtain the differences in the improvements of the learners' mathematical problem-solving achievements. For a further validation of the one-way ANOVA, the post-hoc Scheffe test was done. Son and Ditasona (2020) consequently obtained that there were significant differences in the improvements recorded by the three groups of participants in their mathematical problem-solving achievements. They also discovered that the improvement in the mathematical problem-solving achievement of the group that learnt through the CORE RME model was better than that of the group that learnt through the CORE model. However, no significant difference in the improvements in the mathematical problem-solving achievements of participants who learnt through the CORE RME model and the conventional model. There was also no significant difference in the improvements in the mathematical problem-solving achievements of participants who learnt by the CORE model and the traditional model. As recommended by that study, solving mathematical problems requires the skills to apply knowledge in new ways, the skills to gain a thorough understanding of mathematical ideas and the need to consider more learning models capable of developing learners' mathematical problem-solving skills. Therefore, the present study might be found helpful in this regard.

The current research drew on the above past studies reviewed and similar other ones because it found them useful and relevant in terms of their research features such as: their thematic concerns basically focussing learners' mathematical problem solving; the heuristic, learner-centred instructional framework they applied; the mixed-method research approach followed; their theoretical stances revolving around constructivism and problem-solving theories, and importantly, the research purpose they all identify with – the need to develop learners' mathematical problem-solving skills for improved achievements in Mathematics. Through the various previous studies reviewed, the current study also identified some gaps in the literature and was conducted as an effort to bridge the gaps.

3.7 Reflection on the Chapter

This chapter has provided the theoretical support for this study and the review of related previous studies. By this, it has thrown light on the direction of the investigation. It has shown that both constructivism and the problem-solving theories used to justify the study are inter-related. The chapter has upheld that, as a learning theory that explains *how we know what we know*, constructivism views problem solving as the heart of learning, thinking and development. It has explained that while learners solve problems, they construct their own understanding making use of their existing knowledge. The chapter has also discussed the theories supporting the study, and has highlighted some practical classroom applications of the theories that Mathematics teachers can follow to boost the mathematical problem-solving performance of their learners. With the appraisal of relevant past studies done, the chapter has established why the current study chose the research direction it followed. The chapter has then noted that, if effectively utilised, problem solving can be a valuable cognitive learning tool that can enable learners to construct meaningful Mathematics knowledge.

CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 Introduction

Research methodology refers to the strategy or plan of action explaining the choice of specific methods for a study to obtain the desired outcomes. It points to how the researcher approaches the research problem and seeks answers to it. Basically, it is how research is carried out (Creswell & Creswell, 2017). Putting it differently, research methodology is a series of techniques followed in conducting research which provides the researcher with ways to gather, sort and analyse data so as to arrive at some logical conclusions (Walliman, 2017). The purpose and the nature of a research necessitate the choice of its methodology (Miles, Huberman & Saldana, 2014). Then, the selection of methodology informs and drives the research processes (Kivunja & Kuyini, 2017). This chapter explains in detail the research methodology used in the study. It captures the research paradigm, research hypothesis, research design, study population, sample and sampling procedure. It also covers the description of instruments for data gathering; strategies considered to ascertain the development, validity and reliability of the research instruments; pilot study, methods of data collection, the actual intervention and the ethical considerations during the study.

4.2 Research Paradigm

A research paradigm is a pattern or framework which guides a scientific study. It is a map that clearly defines appropriate theories and methods by which to solve a defined research problem (O'Donoghue, 2018). A research paradigm influences what and how a researcher thinks about a problem of interest because it represents the researcher's beliefs, values and thoughts about the world (Kamal, 2019). It assists the researcher in describing reality, truth and knowledge (Rahi, 2017). The methodological consequences of paradigm choice influence the research question(s), selection of study participants, data collection tools, processes and analysis (Kivunja & Kuyini, 2017). Davies and Fisher (2018) gave a similar view that a research paradigm is influential in the determination of the research questions and the data-gathering method. As recorded by Ngulube (2015), the major research paradigms are: positivism (realism), interpretivism (constructivism) and pragmatism (pluralism). Positivism posits that knowledge is relative and that only one objective reality exists. It claims that a research framework proceeds from the general to the specific, holding that an established theory can produce data. Research in line with positivism often follow the quantitative approach.

Ngulube (2015) goes further to say: Contrasting this is interpretivism, which, in its own case, views knowledge as a subjective reality that can be interpreted. The interpretivist epistemology accepts that knowledge exists, and that knowledge can be obtained, constructed and applied. It asserts that theory emanates from data, meaning research proceeds from the specific to the general. Studies following the paradigm of interpretivist epistemology are characterised by qualitative methodology. Pragmatism emerges as a paradigm combining positivism and constructivism (interpretivism), holding that knowledge is an objective reality that can also be subjectively interpreted. Research in pragmatism usually apply mixed methods. The present study is reinforced by the philosophy of pragmatism; hence, it employed mixed methods for the inquiry.

The pragmatic point of view was advanced in administering the intervention to help the Grade 12 learners under investigation construct knowledge and improve their mathematical problem-solving performance using the heuristic 8Ps learning framework. According to Creswell and Plano-Clark (2011), the origin of pragmatism is attributed to William James, John Dewey and Charles Sanders Peirce who generally believe that reality is continually debated, re-established, interpreted, and that the advisable method to follow is that which solves problem. The writers state further that pragmatism is a paradigm that encourages the use of mixed-method approach in research. Also supporting that several researchers embrace the pragmatic research worldview as a suitable paradigm for mixed-method research, Hall (2013) and Pearce (2012) comment that the frequent linkage of pragmatism to mixed-method research has really popularised it as a paradigm.

Rather than dwell on the debatable issues of truth and reality, pragmatism emphasises *what works* as the truth with regard to the research questions formulated for an investigation (Feilzer, 2010; Tashakkori & Teddlie, 2003). Pragmatism holds that there are singular and multiple realities that are subject to scientific inquiry and supports solving practical, real-world problems. By this, it saves a researcher from the practical and mental constraints dictated by the forced choice dichotomy between constructivism and post-positivism (Creswell & Plano-Clark, 2007). Thus, the current study enjoyed the benefit of not being restricted to a definite research method.

The paradigm of pragmatism considers the benefit of the outcome and selects suitable methods to see *what works*. It believes truth is constructed and not arbitrary because the paradigm itself is rooted in practice. The fact that pragmatism places emphasis on actions and their consequences separates it from interpretivism since it does not believe in interpreting our experiences anyhow or as one deems fit (as in relativism). Hence, pragmatism allows us to build our lives around experience linking together actions and their corresponding outcomes. John Dewey's work explains the broad outlines of pragmatism as a philosophy and its orientation to problem solving. It views knowledge as generated through action and evidence/experience and describes this as warranted assertions (Ngulube, 2015).

The choice of pragmatism for this study was to pave the way for a thorough explanation of the influence of applying the 8Ps heuristic learning model to measure the mathematical problem-solving performance of the learners under study. It was meant to show how the 8Ps problem-solving learning model would behave and develop while employing quantitative and qualitative data collection and analysis techniques for the investigation. It was chosen to see what the nature and level of mathematical problem-solving of the learners looked like. This is because pragmatism considers *what* and *how* to research and hinges its decisions on the outcomes of the research problem. Again, since pragmatism allows the use of multiple approaches for gathering and interpreting data, the present study was able to draw amply from quantitative and qualitative methods (Creswell, 2014). With the pragmatic approach, the current study essentially enjoyed the freedom to choose the research techniques, methods and procedures that could address its aims and objectives well. In effect, this study found the pragmatic paradigm appropriate for the investigation.

4.3 Research Hypotheses

To guide the direction of the study, the following null and alternative research hypotheses were expressed with the main problem statement as a basis:

Null hypothesis (H_0): There is no statistically significant difference in the mathematical problem-solving performance of the study participants exposed to the 8Ps problem-solving instruction and the study participants given the traditional instruction. (H_0: μ8Ps problem-solving instruction = μTraditional instruction)

Alternative hypothesis (H_1): There is a statistically significant difference in the mathematical problem-solving performance of the study participants exposed to the 8Ps problem-solving instruction and the study participants given the traditional instruction. (H_1: μ8Ps problem-solving instruction ≠ μTraditional instruction)

4.4 Research Design

Research design is an instrument or a guideline followed while attempting to address a research problem (Leavy, 2017). It provides the procedures for carrying out research, including aspects like: from whom? When? and under what circumstances are the data to be gathered? (Creswell & Poth, 2018). In other words, research design refers to the methods or the ways of approaching a problem and seeking solutions to it or a framework developed to provide answers for research questions (Taylor, Bogdan & De Vault, 2015). Each research has its own fundamental assumptions which determines its design (Flick, 2015). Understanding these underlying assumptions provides justification for the choice of research approach (May, 2011). The present study adopted a mixed-method approach consisting of the quasi-experimental design using a non-equivalent control group and a qualitative research method entailing the use of the action interpretive research design.

The quasi-experimental design of a non-equivalent, pre-test, post-test control group was adopted to probe the effects of using the 8Ps learning model on the learners' mathematical problem-solving performance in the concept of stationary points in differential calculus (research question 1). By following this design, the study was able to make comparisons between the pre-test and the post-test mean scores of the experimental and control groups. It was able to measure the effects of the manipulated or independent variable (the heuristic 8Ps instruction) on the dependent variable (the post-test). In the design, the pre-test and the post-test were taken as within-subject factors (repeated measures), while the experimental and the control groups as between-subject factors (between-groups). The following table is a synopsis of the non-equivalent control group design as used in this study.

Table 4.1: Non-equivalent Control Group Design

Experimental Group (EX)	O_1	X_1	O_2
Control Group (CL)	O_1	X_2	O_2

O_1 = observation preceding teaching O_2 = observation after teaching

X_1 = 8Ps-based instruction X_2 = traditional teaching

n (EX) = 128   n (CL) = 125   N = 253 (Refer to Section 4.6.2)

Various studies (such as: Awuah, 2018; Bedada, 2021; Chidziva, 2021, Chimuka, 2017; Dhlamini, 2012; Dimitriou-Hadjichristou, 2015; Masilo, 2018; Mendezaba & Tindowen, 2018; Ofori-Kusi, 2017; Omoniyi, 2016; Yimer, 2019) adopt the quasi-experimental design with non-equivalent control group which are useful in several situations where true experiments are unachievable. Since quasi-experimental design does not require any random assignment of the study participants into experimental and control groups, the intact groups of learners of the participating schools were taken as the study participants. The utilisation of quasi-experimental research designs in natural settings enables the researcher to assess subjects directly, determine the effects of the intervention introduced, and minimise the impact of unrelated variables. The researcher had no control over assignment of subjects to condition. Instead of randomisation, he made use of matching. In essence, he could not manipulate the variables but only observed categories of subjects. Particularly with the quasi-experimental design of the non-equivalent control group design, both control and experimental groups were compared and assigned based on convenience as opposed to randomisation (Singh & Singh, 2021).

As a complement to the quasi-experimental design and to offer opportunities for data triangulation, the current study also embraced the interpretive design of the participatory action research. In interpretive design, meaning is driven by understanding of the social world (Maree, 2007). Interpretive design provides a perspective for analysing a situation, and allows circumstances or external variables to have an impact on how a person interprets the world (Kodisang, 2022). Interpretivism seeks to advance knowledge by understanding people's unique points of view and the significance of those points of view (Creswell & Poth, 2018). The interpretive design being of the participatory action research (PAR), the study followed an approach that emphasised the importance of experiential knowledge for addressing issues occasioned by unequal social structures and for generating strategies that could produce potential solutions (Lenette, 2022).

The interpretive design endorsed by the current study entailed the use of classroom observation and semi-structured interview schedules. Through the classroom visits made and the interviews conducted with some selected study participants, the interpretive research approach helped gather further insights into how the proposed 8Ps learning model could be applied in a problem-solving Mathematics classroom (research question 2). Besides, during the classroom observations and the interviews, the interpretive research approach also made it possible to gather essential information about the participants' perceptions of mathematical problem solving and the 8Ps learning model as well as their attitudes to them. By this, the likely challenges that might confront the 8Ps learning model when being incorporated into the teaching and learning of Mathematics were revealed (research question 3).

4.5 Justification for Adopting Mixed-method Approach

The method of a research is a program or a series of procedures adopted for designing, conducting and reporting the research. It is the process or technique employed for the collection and analysis of data (O'Donoghue, 2018). The present study used mixed methods for the investigation believing that the approach can increase the credibility of the research results. A lot of studies have described the mixed-method approach using various terms like: synthesis, integrating, quantitative and qualitative methods combined, mixed methodology and multi-method (Creamer, 2017). Mixed-method research brings quantitative and qualitative methods together in a single study with the notion that both methods possess values that are complementary to each other. Both methods are therefore combined to answer the same research question(s) for greater credibility of a study and wider application of the research findings (Maxwell, 2016). As Dawadi, Shrestha and Giri (2021) point out, a mixed-method approach is a principled research method that is complementary to the conventional quantitative and qualitative research methods since it integrates post-positivism as well as interpretivism frameworks.

Adopting mixed methods has proven to be better than using a single method as that can provide richer and deeper insights into the research phenomenon, which may not be possible with the use of only qualitative or quantitative method. A mixed-method approach can combine and integrate multiple data sources together for the purpose of studying complicated problems (Poth & Munce, 2020). While the quantitative method allows the investigator to collect data from a lot of participants and, as such, increases the chances of generalising the research results to a broader population, the qualitative approach, in its own case, provides a deeper understanding of the issue(s) under study by including the attitudes, views and opinions of the study participants. Furthermore, using mixed methods allows for triangulation as the results emanating from the qualitative data are used to triangulate the findings from the quantitative data and vice versa. In research, triangulation is a technique which employs multiple methods or data sources to provide thorough insights into the research problem. It is also meant to test the research validity through the convergence of data from various sources (Carter, Bryant-Lukosius, DiCenso, Blythe & Neville, 2014). Applying mixed methods makes purposeful data consolidation possible; as such, the researcher would have a wide view of the study as he approaches the phenomenon of interest from various perspectives and research lenses (Shorten & Smith, 2017).

Creswell (2014) endorses the mixed methods for any research because a single method cannot reveal the whole picture of a problem because of the existence of multiple realities. Creamer (2017) feels usage of the mixed methods also promotes development which can bring about possible sequential occurrence of data collection and analysis. In a related observation, Creswell and Plano-Clark (2017) remark that, owing to the increasingly complex research problems in the social sciences, it is already becoming insufficient to employ only quantitative or qualitative techniques. Hence, the adoption of a mixed-method approach has proven to be necessary in most present-day research.

Appreciating all of the benefits derivable from the mixed-method approach, the conduct of the present study was done via the approach. As advantageous as the mixed-method approach has proven to be, it does not however go without its own challenges. Some of these challenges are: the need for broad data gathering, much time involved in the analysis of the wide data gathered and the researcher's obligation to be aware of the requirements of quantitative and qualitative methods. In the current study, as much as practicable, the researcher took time to get acquainted with the requirements of both quantitative and qualitative research methods, collected much relevant data and spent a lot of time analysing them.

4.6 Research Population, Sample and Sampling Procedure

4.6.1 Research Population

The population of a research is basically a substantial set of the persons with similar characteristics which constitutes the central point of an empirical study (Archibald, 2016). As a result of large population sizes, the researcher may choose a representative subsection of the population, often referred to as the study population. A researcher’s ability to select appropriate population relevant to his study and suitable respondents within the study population is crucial to the quality of data obtainable for the study (Martínez-Mesa, González-Chica, Duquia, Bonamigo & Bastos, 2016). In the current study, the Grade 12 Mathematics learners and Mathematics teachers of the Tshwane West education district of Gauteng province constituted the research population. The study involved Grade 12 Mathematics learners since the topic of interest is one of the ten content areas in the Grade 12 CAPS curriculum.

Of the nine provinces in South Africa, the selection of Gauteng Province for the conduct of the research was by convenience sampling. This is mainly because of its proximity and accessibility to the researcher. It was also to minimise costs and to facilitate other research logistics. Convenience sampling is a sampling technique which draws the study sample from a source easily accessible to the investigator (Andrade, 2021). It is a common form of sampling in which the researcher announces the intention to conduct a study and participants decide for themselves whether or not they wish to partake in it. Compared to other forms of sampling, convenience sampling is simpler, quicker and more affordable. It can be used to create goals and hypotheses for use in more rigorous research projects where other sampling techniques appear impractical (Stratton, 2021).

From the fifteen education districts in Gauteng, the researcher chose Tshwane West education district by purposive sampling. This education district falls in the rural areas of the province commonly experiencing inadequate teaching and learning resources, shortage of qualified teachers and insufficient infrastructural facilities. Being socially and educationally disadvantaged, the academic achievements in Mathematics of learners of the schools in the district are often adversely affected. It was these prevailing circumstances that had warranted the purposive choice of Tshwane West education district for the enquiry. Andrade (2021) sees a purposive sample as one whose characteristics well suit the research objectives. Buttressing this, Etikan, Musa and Alkassim (2016) explain purposive or judgmental **s**ampling as a sampling method whereby the researcher deliberately selects the participants that possess necessary information that can lead to the realisation of the research objectives and will be willing to share such. According to these researchers, purposive sampling is a non-random approach which requires no definite number of participants or any underlying theories. It mainly involves identification and choice of participants that are competent and knowledgeable about the relevant phenomena.

4.6.2 Sample and Sampling Procedure

From the study population (refer to 4.6.1), which is still very large, the researcher had to draw the study sample. Sampling is the method of choosing a limited number of cases from a large population of people (Walliman, 2017). As mentioned in subsection 4.6.1, the study sample was obtained by purposive sampling. The sampling strategy enables the researcher to select the participants that can provide relevant information and will be willing to share it (Chidziva, 2021; Kumar, 2014). Purposive sampling technique also allows the researcher to choose relevant participants based on the qualities they possess. However, the sampling can be disadvantageous in the sense that, if adequate precaution is not taken, the knowledge of the researcher about the information required for his study might influence his choice of the sample. As a check to this, the criteria for the selection of the study participants and the level to which the participants could contribute to the study should be predetermined (Orcher, 2016). To this end, the current study premised the choice of its sample on the below factors.

- Schools’ willingness to participate and offer necessary conveniences for the use of the researcher;

- Schools following normal teaching-learning process, not ones already involved in any intervention or tutorial programs funded by some agencies because this can influence the intervention results;

- Teachers' voluntariness to take part in the inquiry, and

- Availability of learners for the investigation.

Eight of the high schools in the education district constituted the study sample – four of them formed the experimental group and the remaining four were the control group. The use of purposive sampling also made it possible to ascertain that the eight schools have largely similar or comparable features especially in terms of their locations, infrastructure, teachers' qualifications, teaching-learning facilities, learner performance level and the same medium of instruction (English Language). Despite the fact that the researcher made efforts to ascertain that the selected schools had largely comparable characteristics, there might still be some unavoidable intrinsic differences in these features among the schools.

Rather than randomise the learners into groups, one intact classroom of the Grade 12 Mathematics learners per school was then randomly sampled from each of the eight schools. Hence, all the classrooms of Grade 12 Mathematics learners of each school had the opportunity to be selected for participation in the study. According to Noor, Tajik and Golzar (2022), as a sampling technique which is suitable for homogeneous populations and in which participants stand equal chances of getting selected, simple random sampling is frequently utilised in scientific studies. It then followed that all the eight Mathematics teachers for those classrooms randomly sampled were the teacher participants involved in the study.

The total number of Mathematics learners who took part in the study was 253 altogether and the number of teachers involved was 8. While the experimental group comprised 128 learners, the control group consisted of 125 learners. The number of the study participants could not be more than that considering the prevalent global pandemic (COVID-19) during the conduct of the study which warranted that each classroom of learners be split into two halves so as to maintain social distancing. Caggiano, Castelnuovo and Kima (2020) capture the sudden, helpless global circumstances in their submission that our lives have been drastically altered by the prevailing situation of uncertainties of the forced lockdowns and quarantines occasioned by the COVID-19 pandemic. For the continuation of the provision and utilisation of our educational services, we unavoidably have to make some emergency adjustments to our existing practices (Funa & Talaue, 2021; Gallardo-Alba, Grüning & Serrano-Solano, 2021; Talidong & Toquero, 2020). Since schools could not remain endlessly closed even as we were protecting lives, one of the emergency adjustments put in place then was splitting each classroom of learners in schools into two halves.

For the purpose of anonymity, the four experimental schools involved in the study were coded as EX1, EX2, EX3 and EX4 while the four schools constituting the control group were coded as CL1, CL2, CL3 and CL4. The labels Ex was used for experimental, CL for control and 1, 2, 3, 4 for the sequence of the schools. This indicates that EX1 is the experimental school 1; CL1 is control school 1, etc. As a way of protecting their identities, the 128 learners of the experimental group were assigned the codes: EX001, EX002, EX003 ... EX128 while the 125 learners of the control group were assigned CL001, CL002, CL003 … CL125. This system of identification used for the learners agrees with the declaration made by Coffelt (2017) that the ethical practice of keeping study participants' anonymity is meant to safeguard their privacy while gathering, analysing and interpreting data.

Noting that teacher’s profile is a key factor that could influence the outcome of the intervention, the researcher took cognisance of the academic qualifications and the years of teaching experience of the Mathematics teachers involved in the study, particularly the four teachers who applied the traditional teaching method in the control group. Basically, in South Africa, teachers’ qualifications range from the three-year college diploma to the university degree (DoE, 1996, 2007). All the four teachers of the control group were university degree holders. While two of them had taught

Mathematics for ten and twelve years, the other two had eleven and fifteen years of Mathematics teaching experience. As for the four teachers of the experimental schools, two were degree holders while the remaining two were studying towards their Master's degrees as at the time of the study. The four of them had at least ten years of Mathematics teaching experience at the high school level. Thus, all the participating teachers were rated comparatively equal in terms of their qualifications and years of teaching experience and were considered suitable for the study. Table 4.2 below is the summary of the study sample.

Table 4.2: Distribution of the Study Sample across Schools (at Pre-test)

Schools	Learners	Teachers	Sub-total	Total
EX 1	32	1	33	132
EX 2	31	1	32	
EX 3	34	1	35	
EX 4	31	1	32	
CL 1	33	1	34	129
CL 2	30	1	31	
CL 3	32	1	33	
CL 4	30	1	31	
Total	253	8	261	

4.7 Instrumentation

The main measuring instruments for this study are: mathematical problem-solving achievement test, classroom observation schedule and semi-structured interview schedule. The detailed discussion of the development, validity, reliability and administration of these main instruments is presented in section 4.7.1. The secondary data sources employed for the research are: 8Ps-based flip cards, 8Ps-based wall charts, 8Ps-based worked-out examples, mathematical problem-solving assessment form, learners' test scripts and transcripts from audio and video recordings. The explanation of how the secondary data-gathering tools were administered is given in section 4.9.4. The usage of various instruments for data collection allows for triangulation of the data. It also increases the likelihood of capturing multiple aspects of teaching and the depth of the study (Walan, McEwen & Gericke, 2016). Like Kodisang (2022) does, the current research also acknowledges that the data-gathering tools had both advantages and disadvantages and that their combined use made addressing the research questions easy.

The data-collection instrument employed for the quantitative aspect of the present study was the mathematical problem-solving achievement test. It was conducted as a pre-test and a post-test. For the qualitative component, classroom observation schedule and semi-structured interview schedule were adopted. The researcher had the opportunity to gather both quantitative and qualitative data simultaneously. He merged the data using both quantitative and qualitative data analysis methods and then interpreted the results together to provide a proper understanding of the phenomena of interest as advised by MacMillan and Schumacher (2006, 2014).

4.7.1 Development of the Instruments

4.7.1.1 Development of the Mathematical Problem-solving Achievement Test

The researcher drew the mathematical problem-solving achievement test from the 2017 - 2019 NSC examination questions set by the DBE on the concept of stationary points in differential calculus (Refer to Appendices A1 and A2). Although the past questions were already constructed in compliance with the assessment guidelines in the Grade 12 Mathematics CAPS document with their validity and reliability already taken care of, the researcher still made sure that the questions included in the achievement test cut across every aspect of the concept of stationary points as stipulated in the curriculum. He also ascertained that the questions had appropriate and varied degrees of difficulty, and that they required different strategies that could encourage learners to use a significant amount of mathematical thinking and reasoning. The primary purpose of the achievement test was to determine the effects (if any) of the use of the 8Ps learning method on the mathematical problem-solving performance of the Grade 12 learners in the concept of stationary points in differential calculus (research question 1). It would then be revealed whether or not the intervention had brought about any improvement in the learners' general understanding of the topic.

The achievement test was of a 90-minute duration. It consisted of five main questions in all, with each question having some sub-questions. The total marks allotted for it was 85 marks, which was later converted to percentage. On the answer booklet, provision was made for the individual learners to supply their genders, ages and the codes assigned to them for the purpose of the investigation. The learners' names were not required to protect their rights to confidentiality and privacy. The assigned codes were meant for possible interview call-ups later, and their genders and ages demanded for demographic analysis. Also, a marking memorandum was prepared to measure the learners' performance in the achievement test (see Appendix A3). The marking memorandum was drawn from the 2017 - 2019 NSC memorandums which contain the model solutions to all the questions selected for the achievement test.

Assessment of learners' performance is crucial to the teaching and learning process. Without it, teachers, learners, parents and other school stakeholders would not have a proper idea of how well or poorly learners perform in their studies (Mamolo, 2021). The use of achievement tests has been widely accepted as a way to measure the level of knowledge and skills acquired by learners after being exposed to some learning contents. Achievement tests are administered to evaluate learners' academic level, learning capability and performance. The major reason for constructing achievement tests is to measure competency in a specified domain (Hanif, Khan, Masroor & Amjad, 2017). Achievement tests, which can be teacher-set tests, curriculum-based measurement or standardised assessments, are also used to discover the weaknesses and strengths of a program (Schneider & Mather, 2015).

4.7.1.2 Development of the Classroom Observation Schedule

The current study modelled its classroom observation schedule on the classroom observation schedule developed by Dhlamini (2012). See appendix B for the classroom observation schedule and appendix D for the owner's adaptation consent granted. While probing the effect of context-based problem-solving instruction (CBPSI) on the problem-solving performance of Grade 10 learners not performing encouragingly in Mathematics, Dhlamini (2012) constructed the classroom observation schedule to examine how CBPSI could be implemented in the Mathematics classroom, and to access any potential challenges this could present for the teaching and learning of Mathematics. The current study found it appropriate to adapt the classroom observation schedule essentially because it shares similar goals with the Dhlamini (2012) enquiry; which is, basically seeking means to enhance learners' mathematical problem-solving skills.

In the present study, the main purpose of the adapted classroom observation schedule was to assess the approaches and strategies by which lessons on the concept of stationary points in differential calculus were offered to the learners in both experimental and control groups. It was essentially meant to measure the kind of attitude and response the participants would show to the 8Ps problem-solving instructional method. The items of the classroom observation schedule were developed in accordance with the research questions and the research objectives. They were structured to assess certain important issues during classroom teaching that would provide appropriate answers to research questions 2 and 3. In research question 2, the study purposed to figure out how the 8Ps learning method could be used to teach and learn stationary points in differential calculus. In research question 3, it aimed to determine whether there were any challenges that could be associated with the application of the 8Ps learning model to teach and learn the topic.

Without accurate evaluation of teaching and classroom practices, educational administrators cannot improve the teaching and learning process. Structured classroom observations are one method for capturing this complex relationship (Farah & Kimberley, 2018). According to Walliman (2017), some research questions are best answered by observing how the participants and the environment behave. Acknowledging the usefulness of classroom observation, Halim, Wahid and Halim (2018) state that, through classroom observation, teachers are exposed to new teaching methods which they might not have been aware of, and are provided constructive feedback capable of improving the quality and effectiveness of their classroom instruction, instructional methods and classroom management. Instead of focusing strictly on the teacher's efficiency or performance, some recent research reports (such as one from the World Bank Group) recommend that the essence of classroom observation should be the observational outcomes like the teacher's pedagogical practices, his use of instructional time and materials and ability to engage the learners effectively in the learning process (Bruns, Luque, De Gregorio & Rodrigues, 2015)

4.7.1.3 Development of the Semi-structured Interview Schedule

The semi-structured interview schedule for this study was likewise modelled on the semi-structured interview schedule designed by Dhlamini (2012) for the reason already stated in section 4.7.1.2. The semi-structured interview schedule (see Appendix C) was meant to provide additional answers to research questions 2 and 3. It was intended to further scrutinise and clarify the learners' conceptions, interpretations and mathematical reasoning which influenced their solutions to the achievement test questions, and to explain some gaps noticed in the test responses. The interview guide assumed a semi-structured design for possible slight deviation from the planned specific themes in practice. Although it comprised 16 question-items that were partly pre-determined, it was designed to still accommodate some relevant questions and ideas that the respondents' answers might generate during the interviews. This was unavoidable since the interviewees were not restricted in any way to certain controlled responses.

Interview is a popular data-gathering instrument often employed by social scientists to gain an in-depth understanding of how people think and act in social contexts. It, however, becomes questionable when the researcher considers only a few excerpts of an interview to draw inferences about the mental schema and social life of the research respondents (Philipps & Mrowczynski, 2021). Semi-structured interview is frequently used in qualitative research. It entails a conversation between the participant and the researcher, aided by an adaptable interviewing methodology and improved by follow-up questions about additional information and remarks. It enables the researcher to gather unstructured data, delve extensively into delicate and sometimes personal matters, and assess participants' thoughts, emotions and perspectives on a particular topic (DeJonckheere & Vaughn, 2019).

A semi-structured interview is an exploratory interview usually based on a guide (interview schedule). It basically centres on the main topic that provides a general pattern. In spite of the availability of the interview schedule for use, semi-structured interview still permits the interviewer to consider the topical trajectories in the conversation that may deviate from the schedule whenever such is deemed appropriate. This action enables the interviewer to go deep in the interview for other possible discoveries (Magaldi & Berler, 2020). Since semi-structured interviews are typically emergent, the interviews may not adhere strictly to questions as planned on the interview schedule (Hossain, 2011). Lots of researchers (for example, Chidziva, 2021; Chiphambo, 2017; Dhlamini, 2012; Mammali, 2015, to mention but few) also employed semi-structured interviews in similar studies. When using semi-structured interviews, some degree of openness should be provided to modify the sequence of themes and the depth to cover during interviews, depending on the interviewees' ideas and responses. Some of the question-items in the interview schedule are meant to address the various contexts and behaviour that the participants are likely to present during lessons. Every question should be structured in a way that allows for easy provision of feedback while addressing general themes (DeJonckheere & Vaughn, 2019). The construction of the interview guide for the present study took into account the series of recommendations from the aforementioned scholars.

4.7.2 Validity of the Instruments

The extent to which a research instrument measures what it is designed to measure determines its validity. For a study to yield productive results, a research tool must measure what it plans to measure. Using a validated measuring tool provides the assurance that the findings obtained from the analyses are valid (Dimitriou-Hadjichristou, 2015; Sürücü & Maslakçi, 2020). The current study viewed the validity of its research instruments as the extent to which the instruments produce data for the improvement of the participating learners' mathematical problem-solving skills in the concept of stationary points in differential calculus through the 8Ps-based intervention put in place. To this end, this study therefore followed the following procedures to validate the instruments.

4.7.2.1 Validation of the Mathematical Problem-solving Achievement Test

The mathematical problem-solving achievement test used in the study derived from past NSC questions which were already aligned with the curriculum and whose validity had also been ensured by the education department through its quality-control unit named Umalusi Council for Quality Assurance in General and Further Education and Training. In spite of this, the researcher still requested four Mathematics Education specialists to validate the achievement test. The four experts, tasked to conduct the content validity and construct validity of the test instrument, comprised a Subject Advisor in the FET phase of a district education department and three seasoned Grade 12 Mathematics teachers of about twenty years of classroom teaching experience. As commented by Heale and Twycross (2015), content validity demands that a research instrument sufficiently cover all the content expected in respect of the variable(s) being measured. This indicates that the instrument is supposed to assess the whole domain which relates to the variable(s) it is measuring. In the view of Ginty (2013), construct validity points to how well the measurements used genuinely test the theory or hypothesis they are meant to test. The construct validity of a certain test should show that the theoretical attribute it claims to predict is really predicted by the test results.

The inputs of these experts towards validating the achievement test were necessary to ensure the following: coverage of all necessary areas of the concept of stationary points in differential calculus as specified in the curriculum; suitability and clarity of language to the targeted participants, and relevance of the test to the objectives of the study. Initially, the mathematical problem-solving achievement test forwarded to the experts for validation comprised seven questions. The Subject Advisor recommended the removal of two questions "as they are measuring the same phenomena as two other questions in the tests". Also, one of the three high school Mathematics teachers reworded one of the questions "to make it clearer to the study participants". Based on the advice and recommendations of these Mathematics professionals, five questions incidentally emerged. The research tool was sent again to the four experts who confirmed that the instrument was a considerable representation of what it was meant to measure. The use of advice from experts to measure the validity of a test remains a popular phenomenon in academic research (See, for example, Awuah, 2018; Chirinda, 2013; Dhlamini, 2012; Ofori-Kusi, 2017; Zulyadaini, 2017, among others). Before the actual study was carried out, the mathematical problem-solving achievement test was pilot-tested to further strengthen its validity. Section 4.8 provides the explanation of the pilot experiment.

4.7.2.2 Validation of the Classroom Observation Schedule and the Semi-structured Interview Schedule

For the classroom observation schedule and the semi-structured interview schedule to be able to measure what they were purposed to measure, their contents were also aligned with the study objectives. After developing both schedules, to make sure they were properly structured, adequate and relevant, they were also sent to the same four seasoned Mathematics Education researchers for validation. The suggestions and advice from the experts warranted modifying, reconstructing and removing certain items of the instruments. After effecting the suggested changes, both measuring instruments were resent to the professionals who unanimously recommended that they were appropriate for the investigation. To further strengthen their validity, before the conduct of the main study, both schedules were also pilot- tested. See subsection 4.8 for the pilot testing.

4.7.3 Reliability of the Instruments

Reliability refers to the consistency of observed values derived from repeated measurements using the same measuring instrument under the same circumstances. Reliability is not only an attribute of a measuring instrument but also an attribute of the results produced by the measuring instrument (Sürücü & Maslakçi, 2020). These researchers contend further that owing to changes in the study population and study sample as well as differences in the times a measuring instrument is used, it might not be possible to have the same result every time; but that reliability is indicated by a strong positive correlation between the results. Sebsibe (2019) buttressed this by describing a research instrument as reliable if it is administered to some similar sets of respondents with similar situations and it produces similar results.

4.7.3.1 Reliability of the Mathematical Problem-solving Achievement Test

The achievement test was subjected to a test-test reliability check conducted on two separate occasions of an interval of two weeks with thirty-five Grade 12 Mathematics learners of a high school different from the eight schools for the main study. The two sets of test scores yielded the Pearson correlation coefficients $r = 0.76$ and $r = 0.78$ respectively. Studies generally affirm that 0.7 - 0.8 is an acceptable reliability coefficient range. Gay, Mills and Airasian (2012) remarked that a reliability test is a confirmation of the consistency of the scores that a test produces. In order to ascertain the internal consistency reliability of the achievement test, the Cronbach Alpha, α, of the pilot participants' post-test marks was computed. For the five test-items Q1, Q2, Q3, Q4 and Q5, α = 0.68, 0.69, 0.73, 0.72 and 0.67 respectively, thus making the average Cronbach Alpha, α = 0.698 ≈ 0.7. It is common practice in research that α = 0.7 represents an adequate measure of the internal consistency of a research instrument. This therefore suggests that the test-items of the achievement test adequately agreed and indicated that the measure was reliable.

4.7.3.2 Reliability of the Classroom Observation Schedule and the Semi-structured Interview Schedule

To ascertain the reliability of both the classroom observation schedule and semi-structured interview schedule, the internal consistency and reliability test was done. This was to make sure that each item of the two schedules consistently achieved the intended objectives relative to the overall objectives that the entire instruments set out to attain. As it is widely accepted that the reliability of a measuring tool is achievable by a process of repeated uses, the reliability of both research tools was further consolidated by the pilot test conducted. In the pilot run, a comparative check of the consistency of outcomes produced by the two schedules was done.

4.8 The Pilot Study

4.8.1 Conduct of the Pilot Study

The research instruments were pilot-tested on 82 Grade 12 Mathematics learners of a high school in the study population area. Although the high school was not among the eight schools selected for the main study, it shared similar socio-economic conditions with them, and was about 95 km far away from them. A school of that geographical location was chosen to guard against any form of interactions between learners of the pilot school and those of the (experimental) schools for the actual study. By convenience sampling, only two of the school’s four Grade 12 classes were involved in the pilot experiment which lasted two weeks (weeks 4 and 5 of Term 2 of the academic year 2019). The two intact classes A and B had 40 and 42 Mathematics learners respectively. Tables 4.3 and 4.4 contain information about the distributions of the pilot study sample according to their classes, ages and genders.

Table 4.3: Distribution of Pilot Study Sample into Classes across Ages and Genders

Age	Class A		Class B		Total
	Boys	Girls	Boys	Girls	
18	4	6	5	5	20
19	3	4	6	5	18
20	3	4	3	4	14
21	2	5	2	3	12
22	2	2	2	3	9
23	1	3	-	1	5
24	1	-	2	1	4
Total	16	24	20	22	82
$\bar{x} = 18.87$	$\sigma = 2.11$				

Table 4.4: Gender-distribution of Pilot Study Sample across Ages

Age	Total Participants	
	Boys	Girls
18	9	11
19	9	9
20	6	8
21	4	8
22	4	5
23	1	4
24	3	1
Total	36	46

A pilot study is a small-scale preliminary study conducted by the researcher to discover concerns that may develop in the main study (Crossman, 2020). It is the field-testing of the research instruments to determine how well they will work with a smaller sample of the population. This makes it possible for the errors that might have been made with sample selection and participation to be detected and corrected at the trial stage (Coughlan & Cronin, 2017). Pilot study enables the researcher to refine the research questions, determine suitable methods for answering them, and estimate the required resources and time for completing the study. It is also meant for putting the research tools for the main study through their paces (Bedada, 2021).

Essentially, pilot study is meant to find out the following: how appropriate, valid and reliable the research instruments are; how easy and achievable the administration of the research instruments will be; possible ways to improve upon the design of the research instruments and the methodology for the conduct of the actual study, and to assess the best time to administer the instruments (Hazzi & Maldaon, 2015). A pilot study is conducted in advance of the actual study to identify potential areas where the research protocols may not have been adequately observed, where the major study may fail, or whether the proposed instruments or methods are complicated or inappropriate (Kumar, 2019).

As specified by DBE (2011) in its CAPS for Grade 12 Mathematics, the pilot school allocated 4.5 hours per week on its timetable to the teaching of Mathematics. Each Grade 12 class has a Mathematics lesson period of one hour from Monday to Thursday and a 30-minute lesson period on Fridays. Having realised that the researcher is an experienced and qualified Grade 12 Mathematics teacher, the school made available to him the two Grade 12 intact classes to carry out the pilot study. The pilot experiment took place over a period of two weeks in May (Term Two) of the academic year 2019. The piloted sessions were audio- and video-taped. Tables 4.5 and 4.6 below explain how the pilot test was carried out.

Table 4.5: Week One of the Pilot Study

	Activity	
Day	Learner Participants	Teacher/Researcher
1	Wrote the pre-test	Introduced himself Talked briefly on the pilot study program Conducted and invigilated the pre-test
	Welcomed the teacher Moved to their groups Studied the 8Ps- based wall charts	Introduced lesson 1 Arranged learners in mixed-ability groups Pasted 8Ps-based charts on the walls
2 - 3	Studied the 8Ps-based flip cards and the worked-out examples Engaged in group discussion and interactions Engaged in problem-solving activities.	Distributed to learners the 8Ps-based flip cards and the worked-out examples Observed, guided and facilitated each group's discussion and interactions Observed, guided and facilitated the problem-solving activities
4 - 5	Stayed in their respective groups Went through the worked-out examples learning from the solution-steps Engaged in group discussion and interactions Engaged in problem-solving activities Participated actively in the correction and revision	Introduced and explained lesson 2 Directed them to make use of their worked-out examples Observed, guided and facilitated each group discussing and interacting Facilitating their problem-solving activities Involved learners fully in the correction and revision

Table 4.6: Week Two of the Pilot Study

	Activity	
Day	Learners (Participants)	Teacher/Researcher
	Remained in their different groups	Introduced and explained lesson 3
	Studied the worked-out examples to learn from the solution ideas	Referred learners to their worked-out examples
6 - 7	Discussed and interacted in their different groups	Observed and facilitated each group discussion and interactions
	Engaged actively in problem-solving activities	Observed and facilitated the problem- solving activities
	Participated fully in the correction and revision	Did correction and revision with learners actively participating
	Stayed in their respective groups	Introduced and explained lesson 4
	Went through the worked-out examples and learnt from the solution-steps	Directed them to their worked-out examples
8 - 9	Engaged in group discussion and interactions	Observed and facilitated the learners group discussion and interactions
	Engaged actively in problem-solving activities	Observed and facilitated the problem- solving activities
	Participated fully in the correction and revision	Did correction and revision with the active participation of learners
10	Wrote the post-test	Conducted and invigilated the post-test
	Four selected learners participated in short interview sessions	Interviewed four learners and two teachers

4.8.2 Highlights of the Pilot Study Results

The pilot study pre-test result {$\bar{x}$ = 22.67; **σ** = 7.67; N = 82} and the post-test result {$\bar{x}$ = 49.74; **σ** = 10.28; N = 82} indicate an improved mathematical problem-solving performance because the participants' post-test mean score was higher than the pre-test mean score. It remained to show that the improvement recorded was as a result of the 8Ps-based intervention put in place. In order to assess the efficacy of the 8Ps problem-solving instruction, the comparison of the mean scores of both the pre-test and the post-test was done using t-test at significance level of $\alpha = .05$. The t-test result {$t = -34.162$; $p = .001 < \alpha = .05$} implies that the participants' performance improved significantly after receiving the 8Ps-based instruction. It can therefore be deduced that the 8Ps learning framework was effective and had improved the mathematical problem-solving skills of the participants. Therefore, the research question 1 seeking to find out whether the use of the 8Ps learning model in the teaching and learning of the concept of stationary points in differential calculus has any effects on the learners' mathematical problem-solving performance has been answered. Analysis and comprehensive explanation of the descriptive statistics and t-test done which yielded the pilot study results are captured in Chapter 5.

There were follow-up interviews after the learners had finished writing the post-test. Only four of the learners and the two Mathematics teachers for the two intact classes A and B involved were interviewed. The four learners were selected from among those who got at least 30% in the post-test, 30% being the minimum pass marks for Mathematics in the NSC examination (DBE, 2011). The number of interviewees was limited to six to reduce the duration of the interviews and, at the same time, have a fairly reasonable time to discuss key topics about the phenomena of interest. Samples are frequently made small to facilitate thorough analysis that is essential to this kind of study. Qualitative samples are also purposively chosen; that is, they are chosen because they can provide comprehensive information about the topic being studied (Sebele- Mpofu, 2021; Vasileiou, Barnett, Thorpe & Young, 2018).

Furthermore, as advised by McGrath, Palmgren and Liljedahl (2018) that respondents in semi-structured interviews are free to express their minds and contribute any ideas they consider relevant, the researcher regarded the participants' views as very crucial. Adopting the semi-structured interview schedule prepared for the conduct of the interviews, he carefully noted, audio-taped and transcribed all the interview sessions. Following the idea of convergent validity, the researcher then compared and cross-validated the data got from his direct participant's classroom observations (as guided by the classroom observation schedule) and the data from the follow-up interviews. He found this easy to do because both instruments measured the same constructs. The result signified that there was a strong correlation between the data produced by the two instruments, which therefore implied a strong convergent validity.

This indicates that both the classroom observation schedule and the semi-structured interview schedule would be suitable and effective for the main study. According to Grobler and Joubert (2018), convergent validity is useful for determining how closely connected and consistent the data from interviews and classroom observations are. Moreover, Taherdoost (2016) states that convergent validity is the degree of correlation between two measures of theoretically related constructs. In effect, convergent validity ascertains that constructs which are supposed to be related are, in actual fact, related. Resultantly, the strong correlation or convergence of the results of both instruments well triangulated the above pre-test/post-test results. With these pilot-study results, it was expected that the 8Ps-oriented instruction would yield similar results in the main study since the pilot school was largely similar to the eight schools for the actual study in respect of its socio-economic situations. Besides, the actual study itself would be carried out under the same circumstances.

4.8.3 Practical Classroom Application of the 8Ps Learning Model

Further to the description of the 8Ps learning model provided in subsection 2.4.2, the explanation of how the learning model can be practically applied in a Mathematics lesson is given below.

Consider the following question below:

Given: $f(x) = 2x^3 - 5x^2 + 4x$. Calculate the coordinates of the turning points of the graph of f and draw the graph of f.

Explained below is how the eight phases of the learning model can be used to solve the question.

Phase 1: Probing

The probing stage of the 8Ps learning model expects the learner solving the Mathematics question to examine the question critically. To this effect, the learner must ask himself these questions:

What type of function is given in the question?

What does the question ask me to do particularly?

What is a turning point of a graph?

Is the coordinate of a turning point referring to only the y-coordinate of that point where the graph changes from increasing to decreasing or from decreasing to increasing?

How is the coordinate of a turning point correctly written?

Any other step(s) that I need to take to be able to draw the required graph of f?

If so, what is/are the other step(s)?

He must also be able to provide answers to the questions as follows:

It is a cubic function.

The question requires the turning points and the graph of f*.*

A turning point is a point on the graph where the graph changes from increasing to decreasing or from decreasing to increasing.

No, not referring to only the y-coordinate but also to the x*-coordinate inclusive*.

Simply because the word coordinate refers to a pair of ordered values with the x*-coordinate written before the y-coordinate, and both must be enclosed in brackets.*

A coordinate is correctly written as $(x; y)$.

Yes, there is one other step.

This is the calculation of the x*-intercepts and* y*-intercept.*

Phase 2: Pinpointing

As required by this phase, pointing out or identifying the key words, variables and conditions in the question can provide the problem solver with insight into the problem.

The key words in this question are: *calculate, coordinate*s, *turning points* and *draw.*

With reference to the identified key words, the problem solver may undergo a form of reasoning such as:

The question says *calculate*, not *estimate, guess* or *write down.* Thus, he has to do calculation and show all the solution steps.

It requires *turning points,* not *x- and y-intercepts, as may reason some others.*

It asks for *coordinates* of the turning points; that is, *both x-coordinates and y-coordinates,* not *the y-coordinates only.*

It also demands that the graph of f be *drawn,* not *sketched.* This indicates that all the necessary values involved in producing the graph must be accurately followed.

<u>Phase 3: Patterning</u>

To further understand how to tackle the problem successfully, the problem solver can check whether the question can be represented as helpful patterns. Using the question to form an equation, to draw a graph, table, chart, picture, map, diagram etc. or any other useful patterns may provide clues to useful solution strategies.

For example, the learner solving this problem can quickly deduce the following facts:

Useful hint	$f(x) = 2x^3 - 5x^2 + 4\text{x}$
Degree of function f	3
Type of function	cubic
Factorised form	$(2x - a)(x - a)$
Number of y-intercepts	1
Number of x-intercepts	3

Since the question is about a cubic function of the general form $ax^3 - bx^2 + \text{cx} + \text{d}$, it has two turning points, one is a (local/relative) maximum turning point and the other is a (local/relative) minimum turning point. The nature of these two turning points can be determined by their concavities as shown below:

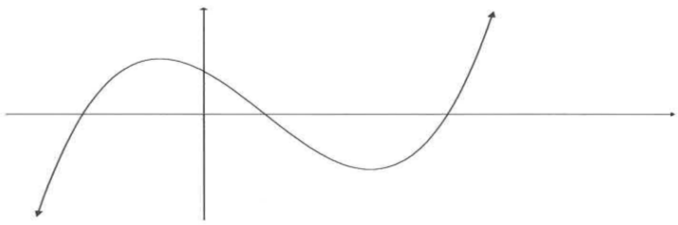
Maximum turning point	Minimum turning point
Concave upwards (i.e., gradient of the curve is moving from increasing to decreasing	Concave downwards (i.e., gradient of the curve is moving from decreasing to increasing)
∩ shape	U shape
$f''(x) < 0$	$f''(x) > 0$
$f'(x) = 0$	$f'(x) = 0$

Although the problem solver is supposed to *draw* the graph of f, he can first sketch it as a guide of what the actual drawing of the graph will look like.

The nature of constant a is very crucial to the shape of the graph of f.

For $2x^3 - 5x^2 + 4\text{x} = ax^3 - bx^2 + \text{cx} + \text{d}$, $a > 0$.

The graph can then be sketched as follows:



Phase 4: Projecting

Here, the problem solver has to come up with meaningful solution plans. From the mathematical reasoning made in phase 1 - 3, he has to decide on the applicable mathematical operations, assumptions, strategies and procedures to follow to solve the problem. Such suitable solution ideas for the question being solved here are provided as follows:

First, find the derivative, $f'(x)$, of $f(x) = ax^n$ using the general rule of differentiation expressed below: $f'(x) = \frac{dy}{dx} = an\text{x}^{\text{n-1}} = 0$. This is because, at a turning or stationary point, the gradient of a curve (i.e., the derivative) is always equal to zero.

Then, find the zeroes or factors of $f'(x)$, which are the x-coordinates of the turning point using one of the following formulas:

1. Factorisation
2. The general method, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
3. The completing the square method.

Next is to find their corresponding values of *y* (i.e., the *y*-coordinates) by substituting each *x*-coordinate in $y = f(x)$. Each of the required coordinates of the turning points will then be written as $(x; y)$.

Thereafter, the *x*- and *y*-intercepts have to be obtained using the following principle:

At *x*-intercept, y = 0; at *y*-intercept, x = 0.

Again, the *x*-intercept can be found by the following two methods:

1. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
2. $x = b^2 - 4ac$

<u>Phase 5: Prioritising</u>

In this phase, the problem solver arranges the projected solution ideas according to how useful and relevant they are. He can then consider only those ones that are most suitable and eliminate the less-important ones that are not directly related to the question. In this case, since $f'(x)$= 0 is factorisable, it is advisable that he adopts the factorisation method to obtain the *x*-coordinates, and drop the other two methods (the general method and the completing the square method) which tend to be time-consuming in this case. Furthermore, the idea of prioritising can make the problem solver decide to calculate the *x*-intercept by the formula $x = b^2 - 4ac$, which certainly is easier and quicker to use than $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Phase 6: Processing

At this point, the problem solver makes use of the already prioritised solution ideas to solve the question. He can do this as shown below:

$f(x) = 2x^3 - 5x^2 + 4\text{x}$

$f'(x) = 6x^2 - 10x + 4$

Set $6x^2 - 10x + 4 = 0$ (At a turning/stationary point, gradient of a curve = 0).

Dividing through by 2, we have $3x^2 - 5x + 2 = 0$.

Factorising, $x = \frac{2}{3}$ or $x = 1$ (x-coordinates)

For *y*-coordinates, substitute each *x*-coordinate in $y = f(x) = 2x^3 - 5x^2 + 4\text{x}$

When $x = \frac{2}{3}$,

$y = 2(\frac{2}{3})^3 - 5(\frac{2}{3})^2 + 4(\frac{2}{3}) = \frac{28}{27}$

Also, when $\text{x} = 1$,

$y = 2(1)^3 - 5(1)^2 + 4(1) = 1$

Turning points = $(\frac{2}{3}; \frac{28}{27})$ and (1; 1)

Now, the intercepts have to be calculated as follows:

At *y*-intercept, $x = 0$.

$y = f(x) = 2x^3 - 5x^2 + 4\text{x}$

$y = 2(0)^3 - 5(0)^2 + 4(0)$

Thus, y-intercept = 0

At x-intercept, y = 0.

$2x^3 - 5x^2 + 4x = 0$

$x(2x^2 - 5x + 4)$

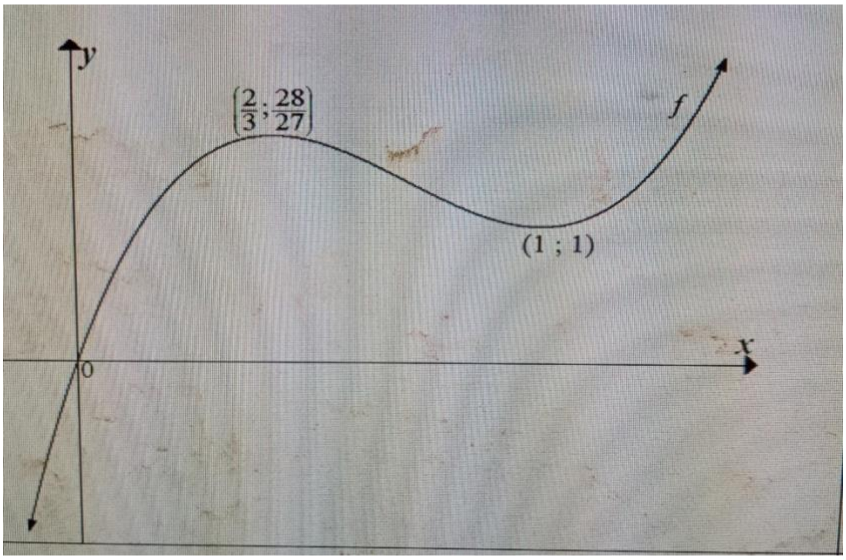
Therefore, x = 0 or $x = b^2 - 4ac$ (where $a = 2,\ b = -5$ and $c = 4$)

x = 0 or $x = (-5)^2 - 4(2)(4)$

x = 0 or $x = 25 - 32$

x = 0 or $x = -7$

The required graph of $f(x)$ is now accurately drawn below considering the values of the x-intercept, y-intercept and the two turning points:



Phase 7: Proving

After solving the mathematical problem, the proving phase expects the problem solver to reflect on the solution to check whether it is correct or not. In order to ascertain this, he may have to re-examine the solution strategies that lead to the solution. He may do this as follows:

First, check that $f(x) = 2x^3 - 5x^2 + 4\text{x}$ is correctly differentiated.

Then, substitute the x-coordinates $x = \frac{2}{3}$ or $x = 1$ in $f'(x) = 6x^2 - 10x + 4$ to see whether it will be equal to zero since $f'(x)$ = 0 at a turning/stationary point. This is carried out as follows:

When $x = \frac{2}{3}$, $f'(x) = 6x^2 - 10x + 4$

= $6(\frac{2}{3})^2 - 10(\frac{2}{3}) + 4$

= $\frac{24}{3} - \frac{20}{3} + 4$ = 0

When $x = 1, f'(x) = 6(1)^2 - 10(1) + 4$

= 6 – 10 $+ 4$ = 0

He can also double-check the calculations of *y*-coordinates, x- and y-intercepts, and ensure that the values are accurately plotted to produce the required graph of f.

Phase 8: Predicting

This last stage requires that the problem solver finds out whether the solution can be generalised or extended to other similar or related mathematical problems. That is, he needs to see whether the solution can be used to predict or obtain the solutions to other similar or related mathematical problems. By this, he wants to determine the level of acceptability of his solution. He can try that as explained below:

The problem solver can use the above solution to $f(x) = 2x^3 - 5x^2 + 4x$ to predict the solution to a similar function, $g(x) = x^3 - x^2 - x + 1$. Since $g(x)$ is also a cubic function like $f(x)$, it will have one y-intercept, three x-intercepts and has two turning points (one local/relative maximum turning point and one local/relative minimum turning point). Apart from this, $ax^3 - bx^2 + cx + d = x^3 - x^2 - x + 1$ indicates that $a > 0$. Hence, the shape of the graph of $g(x)$ will be of the same shape as the graph of $f(x)$. However, for function $h(x) = -5x^3 + 3x^2 - 2x - 1$, where $a < 0$, it will be a different graph shape. It is very important to mention here that, at times, some mathematical questions may not necessarily require making use of all the eight steps of the learning model. For more information about how the learning model is used, refer to subsection 2.4.2 - 2.4.3.

4.9 Data Collection Procedure

4.9.1 The Pre-intervention Classroom Observations of the Schools

Before the main study, the researcher carried out a two-week pre-intervention classroom visits to the selected eight schools. Spending a day in each of the schools, he observed at least two Grade 12 Mathematics lessons of about an-hour duration each. The purpose of this pre-intervention classroom observations was to evaluate the teaching methods followed by the eight schools. He planned to evaluate the standards, similarities and differences of their instructional methods so as to see whether any of the schools had some advantages over the other regarding how they taught and learnt Mathematics in Grade 12 classes. The researcher also purposed to compare and contrast the schools' methods of teaching Mathematics with the 8Ps problem-solving method. Notable among the objects of observations during his classroom visits were: each teacher's daily lesson plans; the language of communication used; the teaching-learning method and how effective such was to develop the learners' understanding of Mathematics; level of learner participation in the learning process; level of learner-learner and learner-teacher interactions during lessons; extent of teacher's dominance of the lesson and suchlike. The pre-intervention classroom observations indicated that all the eight schools adopted comparable traditional instructional methods, which were also different from the 8Ps learning method being proposed. Therefore, the schools were considered suitable to participate in the study.

4.9.2 Implementation of the 8Ps-based Intervention

4.9.2.1 The Researcher as the Implementer of the Intervention

As earlier explained in section 4.6.2, the study engaged eight schools – four of them as the experimental group while the other four were the control group. Only one intact classroom of learners per school was involved. In the four schools of the experimental group, the researcher took the teacher position and carried out the intervention by himself. Two of the experimental schools allowed him to take full charge of the normal Mathematics lesson periods to present the Mathematics topic to their learners. The concession was granted because it was realised that he is a certified and experienced Grade 12 Mathematics teacher, and also on the condition that the lessons were to be carried out under the close supervision of their Heads of Mathematics Department. The researcher agreed to that condition and was permitted to carry out the intervention. In the remaining two experimental schools where this was not possible, the researcher bargained with the school administrators asking for two hours per week in each of the schools. Apart from the time spent to administer the pre-test, post-test and the follow-up interviews, the researcher ensured that, altogether, he spent a total of nine teaching hours per experimental school within the two-month intervention which took place from May - June 2021. This was to ensure that the experimental group also covered the same period of time for the topic like their control group counterparts as officially allocated by the Grade 12 Mathematics curriculum. In order to control the possible personal biases, in all the intervention lessons he presented, the researcher was guided by the literature and relevant past studies as to what his appropriate duties were.

The researcher's decision to administer the intervention by himself saved him from the challenges associated with training the Mathematics teachers on how to implement the 8Ps learning framework. It was also a way to ensure the 8Ps instruction was communicated to the four experimental schools fully, consistently, thoroughly and at the same pace so that the true effect of the 8Ps learning pedagogy could be realised. This action conforms to the advice from Gay, Mills and Airasian (2012) that those variables that the experimental schools have to be exposed to, and which can possibly change the dependent variable, should be similar. Prior to the lessons, the researcher had to acquaint and acclimatise himself to the schools' settings and developed a sense of familiarisation with the learners. Although the four regular Mathematics teachers in the experimental schools did not administer the actual intervention, they provided necessary support to the researcher. Sometimes, they were around to observe the intervention lessons, noted down their observations and presented them to the researcher after the lessons. Their various observations and suggestions corroborated and enriched the researcher's participant classroom observations. The teachers' presence also promoted orderliness in the classroom and triggered the learners to engage fully in the study as they noticed that the research activities gained the support of their teachers. Table 4.7 below summarises the key components of each intervention lesson in the experimental group.

Table 4.7: Major Elements of Each 8Ps-based Intervention Lesson

Lesson Division	Mathematical Problem-solving Activity	
	Teacher Activity	Learner Activity
Introduction (10 minutes)	• Introduced the lesson topic in a whole-class format. • Explained the key terms and concepts of the lesson. • Related the topic to learners' real-life experience. • Asked learners relevant questions to evaluate their prior knowledge of the topic.	• Listened attentively to him as a class. • Asked questions and sought for clarifications. • Participated and made various contributions. • Answered the questions to demonstrate their prior knowledge of the topic.

	• Asked learners to mention sequentially the eight phases of the 8Ps learning method.	• Mentioned sequentially the eight phases of the 8Ps learning method.
Body (30 minutes)	• Arranged the learners in mixed-ability groups (each group having 3 - 5 learners). • Asked each learner to bring out the eight flip cards given them and study the key information about the model's eight phases on the flip cards. • Directed them to study the nature of 8Ps learning model displayed by the wall-charts. • Applied the 8Ps learning method to solve a question. • Provided learners with sheets of step-by-step, worked-out examples which demonstrated solution process with the use of the 8Ps. • Observed, encouraged and facilitated learners' group discussions.	• Stayed in their respective mixed-ability groups (each group having 3 - 5 learners). • Produced their individual eight flip cards, studied them carefully and noted the key information about the model's eight phases as displayed by the flip cards. • Studied carefully the nature of the model as displayed by the wall-charts. • Actively participated in the solution processes. • Discussed collaboratively in their respective groups the solution steps of the worked-out examples and shared their problem-solving skills and strategies among themselves. • Engaged in probing, pinpointing, patterning, projecting, prioritising, processing, proving and predicting.

Conclusion (20 minutes)	• Asked each group to present and reflect on their solutions • Assessed learners' solutions and sought to reach consensus with learners on the correct answers to the questions. • Assigned learners homework.	• Each group presented and reflected on their solutions. • Learners contributed group-by-group and individually too to justify the solutions arrived at and the solution steps followed. • Took home the assigned work.

From Table 4.7 above, the following points are noted about the 8Ps-based instructional method that the teacher used in the experimental schools:

- The teacher organised the learners into mixed-ability groups of 3 - 5 learners each. He did this so as to get the learners fully involved in the learning process. Active learner participation is commonly considered as an essential facilitator of mathematical problem solving. The learners were thus able to interact, express and share their various techniques for addressing the given mathematical problems with one another. He regrouped the learners on daily basis for them to learn from and share ideas with different other classmates.

- The teacher did not dominate the problem-solving processes. Instead, he monitored, guided and facilitated learners' group discussions and interactions.

- He asked the learners probing questions that prompted them to think and reason mathematically.

- He allowed learners to ask and answer questions; seek clarifications about whatever was not clear to them; make suggestions (whether right or wrong) and contribute their own ideas during problem-solving processes.

- The teacher supplied the learners with several step-by-step, worked-out examples to show them how to apply the 8Ps-oriented instructional method.

- Most of the Mathematics problems solved as worked-out examples were challenging, non-routine, well-structured and also well-connected to the learners' everyday experience.

4.9.2.2 The Researcher as a Non-participant Observer in the Control Group

The quasi-experimental design, characteristically, does not expose the control group to the intervention but uses it as a comparison to the experimental group. This was achieved by making sure that the four experimental schools were reasonably far apart (about 35 km) from the four control groups to guard against any form of interactions between the learners of both groups. That was done to ascertain that the control group learners had no access to the 8Ps problem-solving instruction so that the true effects of the 8Ps learning model were not contaminated. Howe, Keogh-Brown, Miles and Bachmann (2007) warn against exposing the control group to the intervention as the action can compromise the statistical significance of the study and reduce the observed differences between the experimental and control groups.

The two-month intervention was planned to coincide with the three-week period of Term Two (weeks 4, 5 and 6) officially earmarked by the DBE for the teaching and learning of differential calculus in Grade 12 classes. Hence, it was possible for the researcher to arrange, visit and observe four Mathematics lessons of each of the four control schools. In each control school, the learners' regular Mathematics teacher taught the same concept of stationary points in differential calculus through the usual traditional approach. The researcher only made scheduled visits to the schools and acted as a non-participant observer in the classroom activities. While observing and taking note of how the traditional instruction was being communicated by each teacher, he sat quietly at one corner of the class without interrupting or interfering with the class activities. He neither asked questions from the teacher nor interacted with the learners during or after the lessons. This idea agrees with Queiros, Faria and Almeida (2017) who explain observation as the process of gathering data while an event is happening, without necessarily interfering with the event occurring.

Table 4.8 below is a summary of the major components of each traditional lesson presented by the four Mathematics teachers of the control group.

Table 4.8: Major Elements of the Traditional Instruction

Lesson Division	Mathematical Problem-solving Activity	
	Teacher Activity	Learner Activity
Introduction	• Routinely introduced the topic. • Explained it briefly following their prepared lesson plans. • Made some illustrations of the topic. • Recalled he previous lesson briefly and link it to current lesson.	• Listened to the introduction • Listened to teacher's explanation without contributing to it. • Continued listening passively to the teacher • Gave the previous lesson topic on the few occasions the teacher asked for it.
Body	• Wrote on the white board two or three questions from the textbook and solved them as class examples following the step-by-step textbook explanation. • Asked learners few verbal questions occasionally while solving the questions; provided the answers most times where learners delayed responding or gave wrong answers. • Invited one or two learners to the white board to solve the mathematical problem on few instances.	• Watched and Listened to the teacher who was often standing in front of the class or by the white board while solving the questions. • Raised up their hands and waited to be called upon to orally answer the questions; often remained quiet when not sure of the answer or not confident enough to say the answers. • Stepped forward to the front of the class to solve the question on the white board; or not responding if unsure of answers.

	• Assigned two or three Mathematics tasks from the textbook to learners to do as class work. • Moved round the class or sat down to mark learners' work. • Did the corrections on the board, giving explanation and asking learners some questions in the process. • Asked learners to copy the corrections in their notebooks.	• Solved the given Mathematics tasks independently without any form of interaction with fellow learners. • Presented their work to the teacher for marking. • Learners watched and listened to him as he explained the corrections; also answered some of the questions clear to them. • Copied the corrections in their individual notebooks.
Conclusion	• Summarised lesson by reviewing salient points. • Assigned learners some Mathematics tasks as homework.	• Listened passively again to the teacher. • Wrote down the homework in their notebooks.

Essentially, the following deductions can be made from the table above:

- The four Mathematics teachers of the control group applied comparably similar traditional instructional methods to communicate the concept of stationary points in differential calculus to their learners.

- The traditional teaching pedagogy they employed was different from the 8Ps instructional method that the researcher adopted in the experimental group.

- The regular Mathematics teachers rarely engaged the learners in the mathematical problem-solving process. Most times, they kept the learners as passive absorbers of the knowledge who were only expected to ask/answer a few questions or contribute ideas once in a while.

- The teachers often directed and dominated class activities by doing much of the talking and transmitting the knowledge of the concept to the learners. In most of the teachers' lessons, there was little or no evidence of the definite or clear sequence of activities that the learners should carry out on their own towards acquiring the knowledge of the topic.

- It was usually more of a whole-class discussion with no problem-solving group discussions or learner-learner/learner-teacher interactions. It was often more of an individual acquisition of information since the learners sat individually and independently.

- Most of the mathematical tasks solved as class exercises, class examples and homework were not directly related to learners' real-life, contextual experience.

4.9.3 Administration of the Research Instruments during Intervention

4.9.3.1 Administration of Mathematical Problem-solving Achievement Test

Both experimental and control groups wrote the problem-solving achievement test which took the form of pre-test and post-test (Appendices A1 and A2). The pre-test was conducted in the first week before the start of the heuristic problem-solving 8Ps-oriented intervention to ascertain the learners' initial knowledge and understanding of the concept of stationary points in differential calculus. The post-test was administered during the last week of the intervention, following presentation of the 8Ps-based instruction to the experimental group and traditional instruction to the comparison group. The post-test was meant to assess the level of improvement that might have taken place in the mathematical problem-solving performances of the learners of both groups.

The two groups wrote the pre-test from Monday - Thursday of the first week of the inquiry, ahead of the intervention. Since the researcher could not cover the eight schools on a single day, the pre-test could not be conducted simultaneously in all the schools. Two schools wrote per day – one in the morning, and one in the afternoon.

The researcher organised it that way in order to be around personally during the conduct of the tests in all the participating schools. While administering the pre-test, the learners were requested not to indicate their names on the answer scripts but that they should use their individual assigned codes. Earlier on, the researcher had allocated a unique code to each learner. Learners in the experimental school were assigned codes: EX001, EX002, EX003 … EX128 while learners in the control group were allotted codes: CL001, CL002, CL003 … CL125. The learners were however instructed to indicate their gender and age on the answer scripts. As hectic and demanding as it was, the researcher made sure he completed marking of the learners' pre-test scripts with the prepared marking guideline (see Appendices A3) by that weekend. As a way to further ensure the equivalence of the eight participating schools, before carrying out the intervention, the pre-test results of all the schools were compared. It was seen that their pre-test results were more or less the same – poor and showing that both groups demonstrated no pre-knowledge of the concept of stationary points in differential calculus prior to the intervention.

The researcher also personally saw to the administration of the post-test in last week of the intervention. He administered it to three schools on Monday, three schools on Tuesday and the remaining two schools on Wednesday. He arranged it that way to make use of the remaining two days of the week to carry out interviews with some selected learners and the selected Mathematics teachers. It is noteworthy that, although the questions in the pre-test and post-test were the same, the questions in the post-test were rearranged and numbered differently. Questions 1, 2, 3 and 5 in the pre-test were numbered respectively as questions 2, 1, 5 and 3 in the post-test. Only question 4 retained its number in both tests. The same set of instructions for the pre-test (see the preceding paragraph) guided the learners of both groups in writing the post-test. Also, the same marking guide was used to mark the learners' pre-test and post-test scripts. The Subject Advisor who assisted with the content validation of the achievement test was asked to moderate the marking of the learners' answer scripts.

4.9.3.2 Administration of the Classroom Observation Schedule

The researcher employed the classroom observation schedule (Appendix B) to document the sequence of classroom events in both experimental and control groups. In the experimental classes, he adopted it as a structure and working framework for his intervention lessons. In the course of communicating the 8Ps problem-solving instruction to the learners, the researcher had the opportunity to directly observe the learners as they went through mathematical problem-solving processes. He found the observation schedule a handy tool for identifying and recording those important learning contexts and learner behaviour emerging during problem-solving processes. Although the regular Mathematics teachers of the experimental schools did not administer the intervention, they were occasionally around as support. They were given copies of the classroom observation schedule to use officially for assessing the intervention lessons they observed. The teachers later presented their observations (recorded on the classroom observation schedule) to the researcher who used them for necessary adjustment in subsequent lessons and also later as part of the thematic analysis of data from the classroom observations (see section 7.2).

Over the three-week period officially earmarked by the education department for the teaching and learning of differential calculus in Grade 12, the researcher paid scheduled classroom visits to the control schools to evaluate how the four regular Mathematics teachers adopted the traditional instructional approach to communicate the topic to their learners. For every classroom visit, he made use of the observation schedule to evaluate the nature and the quality of the conventional Mathematics lessons given by the teachers. He employed the observation schedule as a guide for pinpointing the necessary factors to look out for while observing the traditional instruction being given by each of the teachers. Some of these essential factors are: each teacher's daily lesson planning; level of learner participation allowed; the extent of presence of learner interactions and discussion; learner attitudes; mastery of the subject by the teachers; effectiveness of the instructional method used, etc. In each control school, the researcher was able to observe four lessons of an hour each. In order to avoid influencing the results of the classroom observations, the researcher restricted his role to that of a non-participant observer and carried out his duties without interrupting the lessons or interacting with the learners or the teachers.

4.9.3.3 Administration of the Semi-structured Interview Schedule

The semi-structured interview schedule (Appendix C) was applied to steer the directions of the interviews that the researcher conducted for eight learners purposively selected from both groups (one learner from each school) and four teachers (two teachers from the experimental group and two teachers from the control group) in the last week of the intervention. The interview schedule made it easy for the researcher to get valuable information on the participants' attitudes towards mathematical problem solving and particularly on the adoption of 8Ps instructional method to solve Mathematics problems. The researcher was also able to obtain information about their mathematical reasoning, cognitive processes, level of confidence and perseverance during mathematical problem-solving processes and the obstacles they had to get over to solve the Mathematics problems given to them. As an opportunity for the learners to externalise and verbalise their problem-solving thought processes, the gestures and expressions observed in the learners during classroom observations then became clearer to the researcher while he was interviewing them.

The current study was able to collect descriptive data in the interviewees' own words by using face-to-face, semi-structured interviews. It allowed the interviewees to respond freely to the open-ended questions the way they felt. Face-to-face, semi-structured interview is commonly considered in research as it offers flexible techniques for small-scale qualitative data (McGuirk & O'Neill, 2016). It is suitable for exploring the respondents' opinions and perceptions of even complex and sensitive matters. This interview type also allows the researcher to probe the respondents for further information and clarification of their responses (Naz, Gulab & Aslam, 2022). The semi-structured interview is as well advantageous in that it may be modified according to the interviewer's perception of what seems most appropriate. Therefore, it is not known to be as restrictive as a structured interview (O'Keeffe, Buytaert, Mijic, Brozović & Sinha, 2016). In numerous academic fields, interviews constitute a well-established research technique. To make analysis easier, such interviews are often orthographically transcribed. This can be done either manually (which can be tiresome and time-consuming) or using the computer to make transcription easier (Da Silva, 2021). In light of this, the interview sessions conducted for the present study were audio-taped and later transcribed for the purpose of analysis.

For the sake of confidentiality, the four learner and two teacher participants in the experimental group were coded as: EX-L001, EX-L002, EX-L003, EX-L004, EX-T001 and EX-T002. In their own case, the four learner and two teacher participants in the control group were coded as: CL-L001, CL-L002, CL-L003, CL-L004, CL-T001 and CL-T002. By interpretations, EX-L001 was the first learner in the experimental group interviewed; EX-T001 was the first teacher in the experimental group interviewed; CL-L001 was the first learner in the control group interviewed; CL-T001 was the first teacher in the control group interviewed etc. Although a learner was interviewed in each participating school, the three achievement groups were put into consideration in the choice of the eight learners (HL = 3, ML = 2 and LL = 1). The semi-structured nature of the interviews allowed all the twelve interviewees altogether to express their views freely and frankly for about thirty minutes allotted to each of them.

In order to maximise the time meant for each interview session, and to be able to elicit full information about the subject of discussion from the interviewees, the researcher provided the participants to be interviewed with the interview questions three days before the interviews and were advised to use the questions to prepare themselves. As for the teacher and learner participants of the control group who were also expected to respond to few questions on the 8Ps-based instructional method that they did not apply in teaching/learning the Mathematics topic, the researcher gave them a short explanation about the instructional method and also a one-page information leaflet containing the summarised explanation of the method to read three days to the interviews. The pre-arranged interviews were carried out at break times and an hour to the closing times of the days used.

The goal of the interviews was to corroborate the findings from the classroom observations and the content analysis of the participants' post-test scripts. By this, a deep insight into the phenomena under investigation could be obtained. The interview goal tallies with the idea from Bailey (2018) that classroom observation alone cannot provide sufficient data on the subject of interest, and that, normally, the investigator cannot capture everything in the classroom, understand everything observed or determine all the crucial things to observe. True to this claim, the researcher found out that some responses from the interviewees provided him with explanations for certain important actions and attitudes exhibited by the participants during classroom observations. All the interview sessions were audio-taped with the consent of the respondents, to ease the data analysis that would soon follow. The responses were later transcribed verbatim with the computer.

4.9.4 Administration of the Secondary Data Collection Instruments

4.9.4.1 The Learners' Background Information Form

In the South African economics of education literature, the relationships between learners' socio-economic situation and educational outcomes have been widely established. Learners' socio-economic circumstances bring their way a number of socio-economic hurdles that limit their abilities to achieve (Bayat, Louw & Rena, 2014). While discussing how socio-economic status can affect learner achievement, Li and Qiu (2018) maintain that learner background information has a significant influence on school activities and that socio-economic status is viewed as a strong predictor of learner achievement. Scrimin, Mastromatteo, Hovnanyan, Zagni, Rubaltelli and Pozzoli (2022) also believe that parents of high socio-economic status find it easy to prepare their kids for learning as they possess numerous resources to support them. However, parents having low socio-economic status frequently lack the financial, social and learning support for their children's education. Gobena (2018) investigates the influence of family socio-economic status on learners' academic achievement. He submits that, to sustain harmony among learners, socio-economic policies should be formulated in a way that allows learners from low-economic parents to enjoy equal opportunities as learners having high-economic status.

To be able to determine the possible influence of the participants' socio-economic circumstances on the 8Ps-oriented intervention results, the researcher assessed the socio-economic status of the study participants. He achieved this by developing a learner's background information form through which he gathered useful information for this purpose. Appendix E1 is a copy of the learner's background information form. The basic items of the form include: learner's age, gender, parentage status, education and employment status of learner's parent(s)/guardian and learner's accessibility to technology and other useful learning facilities at home. The researcher administered the forms by himself in the experimental schools during the intervention, while the learners' regular Mathematics teachers carried out the task on his behalf in the control schools. Analysis of the series of demographic information collected for the pilot test and the main study from the participants using the learner's background information form is provided in sections 5.2 and 6.2 respectively.

4.9.4.2 The 8Ps-based Wall Charts

A big learning chart conveying essential information about the 8Ps learning model was designed for the use of the experimental schools. The big chart (Appendix E2) hung on the wall displayed two different diagrams: one diagram illustrated the 8Ps learning framework as linear in nature and the other showed it as cyclic. Refer to section 2.4.3 for the description of linear and cyclic nature of the 8Ps learning model. The charts were pasted on the wall for reference – four charts per classroom, mounted conspicuously on the four walls of the classroom. Every now and then, the researcher reminded the learners to look at the wall chart regularly to take note of the sequence and descriptions of the eight problem-solving phases. He did remind them to learn from the arrows on the chart indicating the directions of movement along the model: forwards, backwards, upwards, downwards, cyclically and so on, while solving a given mathematical task.

A wall chart is a sizable card or piece of paper with text, diagram or image which the teacher can project on the wall or the blackboard or hold up for the class to see for the purpose of extended presentation or practice. It usually consists of combined verbal and visual materials (Kang, 2015). Utilising wall charts as a teaching tool promotes effective learning. It helps learners see ideas visually laid out in an organised manner, and also assists them to process content and make connections between concepts more easily (Evetsson, 2014; Maharani & Pudjobroto, 2012; Patria, Sudarsono & Rosnija, 2020). It was the belief of the present researcher/intervention teacher that, as the learners daily encountered the wall chart displaying the two linear and cyclic diagrams representing the nature of the 8Ps learning model, they would get familiar with the different phases of the model and their applications.

4.9.4.3 The 8Ps-based Flip Cards

Eight flip cards (each of the 4′ by 6′ postcard size) explaining the 8Ps learning model were designed and made available for use in the experimental classes. One flip card represented a letter P of the 8Ps problem-solving model and provided helpful ideas about the P it stood for. For easy reference and as an indication of the sequence of the eight phases of the learning model, the eight flip cards were labelled, P1, P2, P3, P4, …, P8 (see Appendix F). The researcher distributed the flip cards to the learners and instructed them to refer to them regularly as a guide during the mathematical problem-solving process. Each participating learner was given eight flip cards. The learners were also allowed to keep and use the flip cards later when solving practice questions and assigned homework. This visual tool was purposefully made to help the learners recall necessary information quickly about the 8Ps instructional method and to give them guidance about the use of each of its eight phases. It was meant to increase their ability to properly understand the learning model, retain essential ideas about it and give them helpful directions to follow in using the method.

Flip cards (also known as flashcards) are cards containing words and/or pictures and are often used as visual aids in the classroom. The use of flip cards is an easily implementable instructional strategy capable of helping learners reach higher processing levels (such as comprehension and application) in a self-driven way that enhances learner performance. Flip cards can enable the teacher to present learning materials in engaging and interesting ways, and can as well enhance learners' understanding of the teacher's explanations (Sartika, 2020). Flip cards ease the process of teaching and learning; teachers do not only need a proper instructional method but also a relevant media. The flashcard is one of such appropriate media. It acts as an effective aid to the teacher in presenting and explaining the subject matter to learners. It is a cheap and simple form of media. Flashcards will make terminology easier for them to learn by giving them the name or image of the object (Aulia, 2018). For effective classroom practice, usage of flip cards for learning can thus be recommendable since learners can take them almost anywhere and study them during their free moments. Using flip cards is as well fun and fast because it has multi-sensory appeal and it occupies only a short time during a lesson (Mathura & Zulu, 2021).

4.9.4.4 The 8Ps-based Worked-out Examples

Each learner in the experimental group was given some sheets containing the 8Ps-based worked-out examples (see Appendix G for a sample of the worked-out examples). The sheets contained some past NSC questions worked out as examples that the learners could learn from. Those questions solved had similarities with the ones in the achievement test. The 8Ps instructional method was applied to solve the questions with each step taken carefully shown and justified. It was intended that, as the learners studied and discussed the worked-out examples severally in the course of solving the Mathematics problems assigned to them, they would be acquainted with how the 8Ps learning method could be applied to solve related questions on the topic. The adoption of worked-out examples in teaching and learning Mathematics is a practice well-established (Ng & Dindyal, 2015). While studying the worked-out examples, learners can acquire a fundamental understanding of the domain concepts. This serves as a starting point for future problem-solving and is an effective condition for knowledge transfer (Renkl, 2017). As remarked by Yüce and Dost (2019), worked-out examples are a valuable part of mathematical thinking by analogy, teaching and learning process. The researcher's idea of providing the learners with related worked-out examples as a helpful guide and resources for solving mathematical tasks during the intervention is also in consonance with the prescription by NCTM (2000) that, for teachers to effectively develop learners' mathematical problem-solving skills, they must stimulate learners' interest in mathematical problems and provide them numerous opportunities for imitation and practice.

4.9.4.5 Mathematical Problem-solving Assessment Form

A mathematical problem-solving assessment form (Appendix H) was designed and administered to both the experimental and control groups. The assessment form was used after the intervention to scrutinise the participants' post-test scripts to measure and compare how much improvement that both instructional methods had made on the mathematical problem-solving performances of the two groups. The intention was principally to determine whether the 8Ps-based treatment had actually been of any positive impact.

The key criteria that the assessment form judged were the study participants' demonstration of the following problem-solving skills: identification and application of the correct formulas; correct interpretation of the question and translation of it to a meaningful equation; representation of the problem as helpful patterns such as pictures, diagrams, graphs, charts etc.; reasonable connection between the given problem and relevant previously learnt ideas; evidence of logical and sequential problem-solving steps; provision of correct justifications for solution-steps, and explanation and application of the solutions obtained. These assessment criteria are measured vis-à-vis the three-point scale namely: High-Level Problem Solving, HL (3 marks), Medium-Level Problem Solving, ML (2 marks) and Low-Level Problem solving, LL (1 mark). The content analysis of the participants' post-test scripts using the mathematical problem-solving assessment form is provided in section 7.3.

Deviating from the traditional assessment techniques which majorly focus on the correctness of learners' answers, a properly-designed performance rubric (called mathematical problem-solving assessment form in this study) provides valid and reliable information to the teacher about learners' performance in certain criteria, knowledge and processes. It thereby enables the teacher to award reliable scores as feedback on learners' progress of the specific criteria (Van de Walle, Karp & Bay-Williams, 2009). Corroborating this idea, Rosli, Goldsby and Capraro (2013) describe the performance rubric as an assessment tool for measuring students' learning holistically, the criteria of which emphasises the process of getting the solution thereby reflecting learners' conceptual understanding of the task assigned.

4.9.4.6 Transcripts from Video and Audio Recordings

The intervention lessons were video-recorded and the interviews audio-taped. The permission to do that was earlier obtained from relevant authorities and the participants. The services of a videographer were secured for that purpose since it was practically impossible for the researcher to combine that with his normal classroom and research duties. During the first two or three lessons, the learners slightly got distracted by the presence of the video recorder as that was apparently strange to them. Soon afterwards, they got used to it. After the intervention, the researcher did a careful transcription and analysis of the video and audio recordings. This action provided him with further insights into those learner actions and interactions that he could not have fully captured by himself while discharging the dual role of the teacher cum researcher. It produced valuable information about the gestures, postures and facial expressions of the learners (DuFon, 2002). It made available to him dense and more contextual data, such that can be stored away for a long time (Heidet, Tate, Divirgilio-Thomas, Kolanowski & Happ, 2010).

The conversion of audio or video to text is known as academic transcription. That is, academic transcription entails creating written documents from an oral or video recording. It is a useful tool for researchers to store ideas for later data analysis. Researchers use it to create a written version of an interview or a classroom observation because it is a practical method that most accurately conveys the speaker's words. The application enables users to revisit interviews and classroom observations they had previously completed to clarify and comprehend crucial actions and points they might have missed or not fully captured (Zafar, 2022). With the transcripts from video and audio recordings, the current researcher was able to do a thorough review and refinement of some of the observations he made as a participant observer during the intervention lessons and the interviews he conducted with the twelve selected participants.

4.10 Research Ethical Measures

Working with humans as participants in research often requires observing certain ethical measures on how well the study participants are treated, and how honestly the data is gathered, analysed and interpreted. It also demands that the participants be carefully and respectfully treated throughout the research period (Tsan & Nguyen, 2019; Walliman, 2017). Research ethics thus expects a researcher to full comply with some acceptable research norms, standards and practices. In conformity with the UNISA research ethical guidelines, policies and practices, before commencing the conduct of the present study, the researcher first sought and secured ethical clearance from the Ethics Review Committee of the Institute of Science and Technology Education (ISTE), UNISA (refer to Appendix O1). He then obtained the approval and consent of all relevant authorities for the research, namely: Gauteng Department of Education, school Principals, School Governing Boards, teachers, parents and learners of the eight schools (See Appendices O2, J1, J2, K1, K2, L3, L4, M3, M4, N3 and N4 respectively).

According to informed consent requirements, the researcher informed all the above stakeholders about the nature and essence of this research. He openly and honestly explained to the parents and the learners of the eight schools through the informed consent letters sent to them about the nature, objectives and benefits of the study (See Appendices J1, J2, K1, K2, LI, L2, M1, M2, N1 and N2). He let them know that taking part in the study was not obligatory, that every participant could decide to opt out whenever they wanted and that all of them would be duly protected from injuries throughout the research process. He also assured them he would treat with confidentiality their identities and the information they might supply. Both the parents and the learners signed the informed consent forms as an indication of their support for the study.

True to the assurances given to them, during the investigation, the researcher strictly followed all the ethical measures to see that the privacy and rights of the learners, teachers and schools involved in the study were in no way infringed upon. In compliance with the principle of privacy and confidentiality, he kept anonymous in the research report the names and identities of the study participants and their schools. Throughout the study, he made use of arbitrary codes in place of the names of the learners, educators, principals and schools. Additionally, the researcher ascertained that the data gathered were solely meant for the study. He also carefully stores away the research data under lock and key as expected of him. The measures taken to protect participants' privacy and keep their responses, behaviour and other information confidential are all in the participants' best interests (Holland, 2019).

4.11 Reflection on the Chapter

This chapter has covered the research methodology and procedures followed in the study for data collection. The study was informed by the mixed-method approach which allowed the researcher to collect data via quantitative and qualitative methods. The study samples and the sampling procedures have been explained. The research instruments adopted for data gathering have been described with the process of their development, validation and reliability also clarified. The data-collection procedures considered for the pilot study and the actual study have as well been elucidated. The chapter has concluded with the ethical measures taken in the conduct of the empirical experiment. The subsequent chapter therefore presents analysis of the pilot study data and the findings obtained.

CHAPTER FIVE

ANALYSIS OF PILOT STUDY DATA AND RESULTS

5.1 Introduction

The previous chapter discussed the mixed-method approach followed to gather data for the study. This chapter captures the analysis of pilot study data and the results it produced. Eighty-two (82) learners were involved in the pilot-run. The participants, who exhibited poor performance in Mathematics, were learners of a high school in the study population area associated with a deprived township status (see section 4.6.1). Research, generally, has noted that learners' socio-economic circumstances can influence their achievement outcomes (see subsection 4.9.4.1). In order to measure how the learners' socio-economic factors could impact this study and also to determine whether the learners were actually suitable to participate in the pilot study, the researcher collected and evaluated the learners' background information.

5.2 Evaluation of Pilot Study Participants' Background Information

The age and gender details of the pilot study participants were analysed as follows.

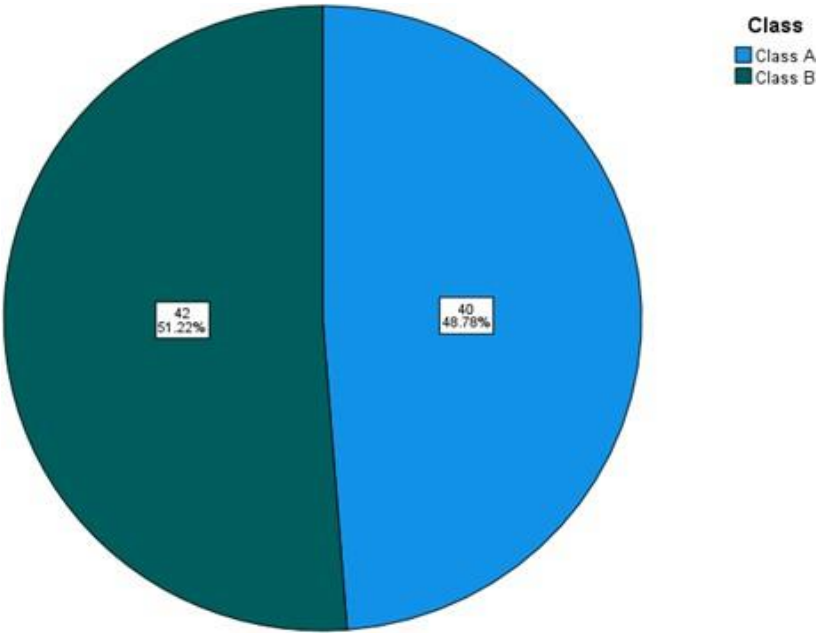
5.2.1 Distribution of Pilot Study Participants by Class

The eighty-two (82) participants in the pilot trial were distributed into two classes A and B as shown below. The choice and distribution of the participants into two classrooms or groups were not for any form of comparison but rather out of the researcher's belief that the two classrooms of 82 learners would be more representative of the main study sample than only a classroom of 40 or 42 learners.

Table 5.1: Frequency Distribution of the Pilot Study Sample by Class

	Frequency	Percent	Cumulative Percent
Class A	40	48.8	48.8
Class B	42	51.2	100.0
Total	82	100.0	

Figure 5.1: Pie Chart of Pilot Study Participants by Class



From Table 5.1 and Figure 5.1 above, 48.78% (n = 40) involved in the pilot study were learners from class A and 51.22% (n = 42) were learners from class B.

5.2.2 Distribution of Pilot Study Participants by Age

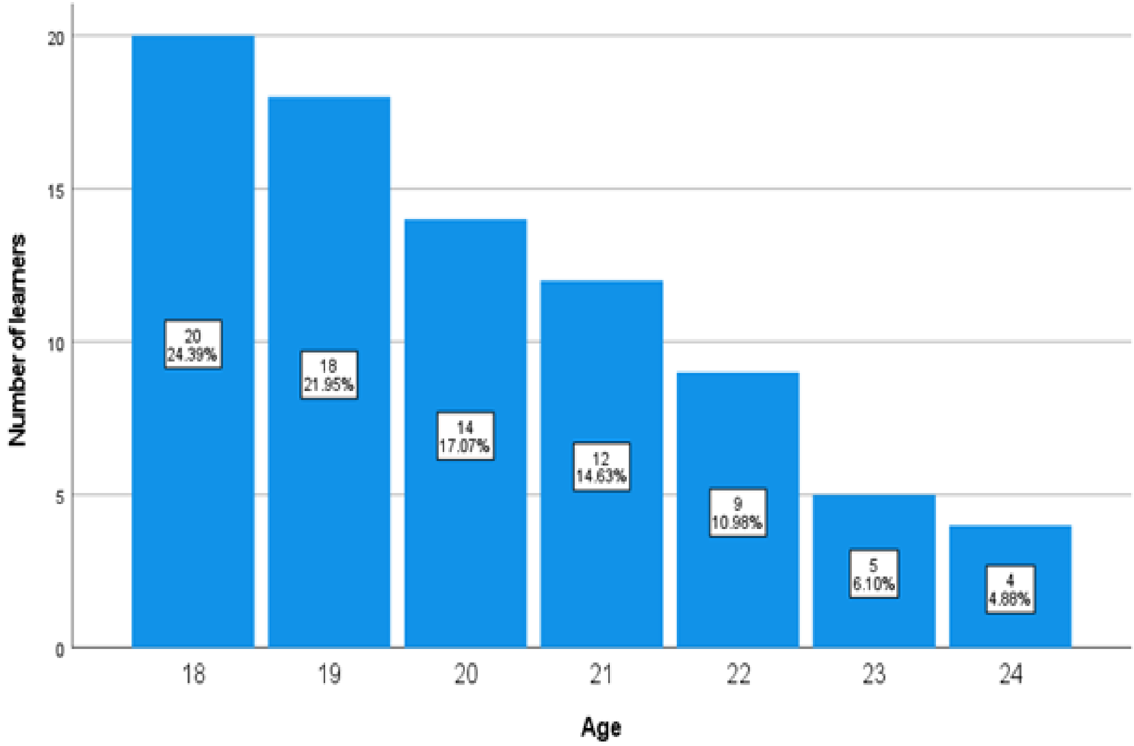
The pilot study participants' ages were recorded and analysed as follows.

Table 5.2: Age-distribution of Pilot Study Sample

Age	Frequency	Percent	Cumulative Percent
18	20	24.4	24.4
19	18	22.0	46.3
20	14	17.1	63.4
21	12	14.6	78.0
22	9	11.0	89.0
23	5	6.1	95.1
24	4	4.9	100.0
Total	82	100.0	$\bar{x}$ = 20.04; σ = 1.77

From Table 5.2, the age-range of the participants was 18 - 24 years. Majority of them (n = 64 = 78%) fell in the age-range 18 - 21 years. The mean age was 20.04 years with a standard deviation of 1.77 ($\bar{x} = 20.04;\ \sigma = 1.77$), and the median was 20 years. This implies that 50% of them were not more than 20 years of age. As supported by research (Dacey, 1989; Dhlamini, 2012; Lianghuo & Yan, 2000), age 18 - 21 years is appropriate for learners to understand and perform well in mathematical problem solving. The above information, as illustrated by Figure 5.2 below, gave the researcher further assurance that the participants were eligible for the pilot study.

Figure 5.2: Bar Graph of Pilot Study Participants by Age



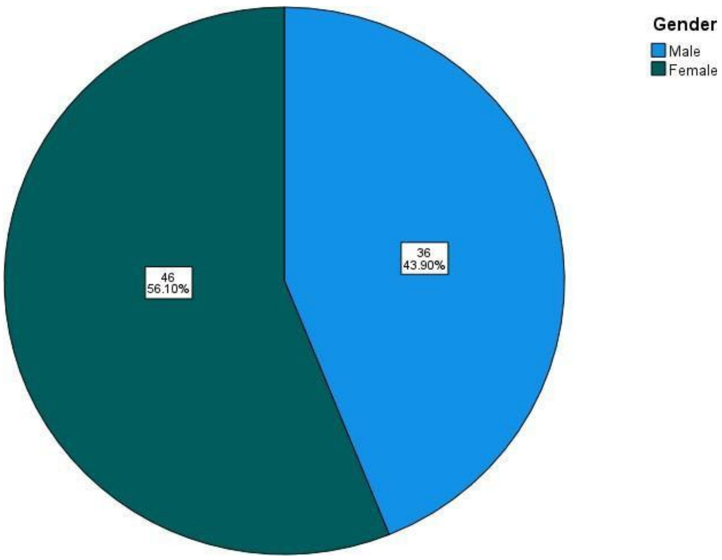
5.2.3 Distribution of Pilot Study Participants by Gender

The gender information of the participants gathered was analysed below.

Table 5.3: Frequency Distribution of Pilot Study Sample by Gender

Gender	Frequency	Percent	Cumulative Percent
Male	36	43.9	43.9
Female	46	56.1	100.0
Total	82	100.0	

Figure 5.3: Pie Chart of Pilot Study Participants by Gender



As shown by Table 5.3 and Figure 5.3, about 56.10% (n = 46) were females while 43.9% (n = 36) were males. The ratio of females to males was almost 5:4. Hence, both male and female learners were fairly represented as participants in the pilot study. Tannenbaum, Greaves and Graham (2016) advise that researchers should consider sex and gender in designing, conducting and reporting a study for the research results to apply to everyone. According to these writers, gender identity, roles and relations influence how the research implementation strategy works, and might bring about the discovery of some positive research outcomes, but if ignored, might result in some unintended consequences.

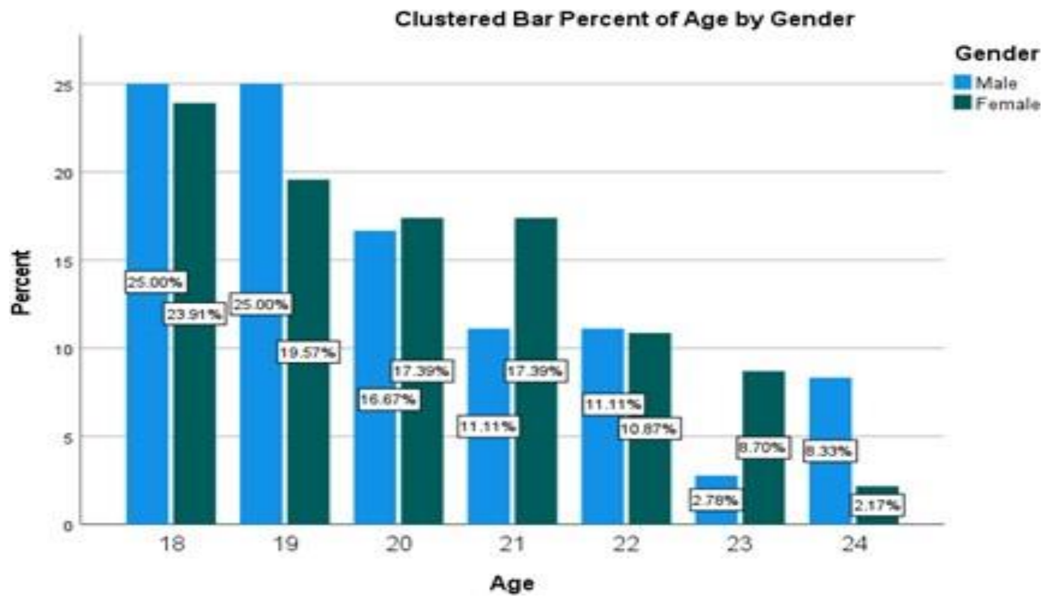
5.2.4 Age-distribution of Pilot Study Participants by Gender

The pilot test participants' ages were also distributed across genders. The information is given by Table 5.4 and Figure 5.4 below.

Table 5.4: Age-distribution of Pilot Study Sample across Gender

Age	Male	Percent	Female	Percent
18	9	25.0	11	23.9
19	9	25.0	9	19.6
20	6	16.7	8	17.4
21	4	11.1	8	17.4
22	4	11.1	5	10.9
23	1	2.7	4	8.7
24	3	8.3	1	2.2
Total	36	100.0	46	100.0

Figure 5.4: Clustered Bar Graph for Age-distribution of Pilot Study Sample across Gender



From Table 5.4 and Figure 5.4 above, for ages 18, 19 and 24 years, the percentages of male participants are greater than those of female participants. However, the percentages of female participants are greater than those of male participants in ages 21 and 23 years. For age 22 years, both percentages (11.11% and 10.87%) are almost the same. Generally, for all the ages, especially for age-bracket 18 - 21 years, the percentages of male and female participants are almost the same. Thus, gender balance across the ages was also achieved with the participants involved in the pilot test. Subsection 5.2.3 gives further explanation.

5.3 Descriptive Statistics of Pilot Study Data

The descriptive statistics was done to summarise the pre-test and post-test scores of both experimental and control groups in form of mean scores, standard deviation, variability and gain scores.

Table 5.5: Summary of Descriptive Statistics of Achievement Test Marks for Pilot study

Test	n	$\bar{x}$	Median	σ	Min	Max	Range	Q_1	Q_3	Coeffi. of Var
Pre-test	82	22.67	21.5	7.67	8	36	28	16	44	71.74%
Post-test	82	49.74	50	10.28	26	68	42	29.25	56.5	61.73%

Table 5.5 above reveals the pilot study participants' achievements in the pre-test as ($\bar{x}$ = 22.67; σ = 7.67; n = 82) and the post-test as ($\bar{x}$ = 49.74; σ = 10.28; n = 82) respectively. The result indicates a significant mean increase of 27.07% in the post-test mean score. It also gives a higher standard deviation in the post-test which signifies that the post-test results deviated from the mean more than the pre-test results. The significant difference between the post-test and pre-test mean scores is further emphasised as follows: sum of marks that the 82 participants got in the pre-test was 1859 while their total post-test marks was 4079 – about three times bigger than the total pre-test marks; the pre-test minimum and maximum marks were 8% and

36% respectively whereas the post-test minimum and maximum marks were 26% and 68% respectively, and their mode and median marks in the pre-test are 20% and 21.5% respectively while their mode and median marks in the post-test are 48% and 50% respectively.

5.4 Paired T-test Analysis of Pilot Study Data

The pilot-test was conducted mainly to try out the mathematical problem-solving achievement test. Having already shown in section 5.3 a significant increase of 27.07% in the post-test mean score over the pre-test mean score, it remained to consolidate the statistical significance of the mean difference. This was to prove that the impact of the 8Ps instruction produced the improvement in the learners' mathematical problem-solving performance. To this effect, both mean scores of the pre-test and post-test were compared using the paired t-test. This was run at α = .05 significance level with $p < \alpha = .05$ indicating that the post-test score was substantially different from the pre-test score, and $p > \alpha = .05$ showing the post-test score was the same as the pre-test score. Table 5.6 gives the summary of the t-test.

Table 5.6: Synopsis of Paired T-test Results of Achievement Test Marks for Pilot Study

Test	Participants	n	$\bar{x}$	σ	SEM	t	p
Pre-test	Grade 12 learners	82	22.67	7.666	.847	-34.162	p <.001
Post-test	Grade12 learners	82	49.74	10.283	1.136		

The paired t-test result {t = -34.162, p < .001, p < α = .05} points out that, on the average, the post-test result ($\bar{x} = 49.74,\ \sigma\ =\ 1.136$) is significantly higher than the pre-test result ($\bar{x} = 22.67,\ \sigma = .847$). The mean increase in the achievement test is 27.07 with a 95% confidence interval ranging from −28.650 to −25.496. The eta-squared statistic, $\eta^2 = \frac{t^2}{t^2+(n-1)} = .94$ obtained indicates a large effect size, meaning that 94% of the variability in the achievement tests is accounted for by the groups.

Hence, the null hypothesis based on equality of the mean scores of both groups is rejected. The foregoing shows that the mean score difference between the experimental and control groups is statistically significant. This further indicates that there is a significant improvement in the mathematical problem-solving performance of the experimental group given the 8Ps-oriented instruction.

5.5 Reflection on the Chapter

The chapter has explained how the pilot study data was analysed. It has started with the validation of the participants' sex and age in order to account for the effect that they could have on the outcome of the pilot test. The chapter has shown how the pilot study data were analysed using the descriptive statistics and the paired t-test. Finally, the chapter has been able to obtain that the research instruments, samples and procedures could be suitable for use in the main study.

CHAPTER SIX

QUANTITATIVE DATA ANALYSIS AND RESULTS

6.1 Introduction

The present chapter analyses the quantitative data for the main study which followed the quasi-experimental design of a non-equivalent control group. It also provides the results of the analysis. To this end, the quantitative data analysis method was applied. The aim of quantitative data analysis is to obtain precise and trustworthy measurements that can be analysed statistically. The present study used quantitative data analysis methods to measure the performances of the study participants in the pre- and post-tests to be able to determine whether the use of 8Ps-based instruction had any significant effect on the learners' mathematical problem-solving performance (research question 1). The quantitative data gathered were analysed using descriptive and inferential statistical methods. For the descriptive statistics, the numerical techniques followed were proportions, percentages, frequencies, means and standard deviations, while the graphical techniques applied were bar charts, pie charts, histograms and box-plots. The inferential statistics adopted were the paired t-tests, analysis of covariance (ANCOVA) and one-way analysis of variance (ANOVA). Since the ANOVA produced significant mean differences, graphical illustrations in the form of confidence interval error bars were used to determine where the differences existed. The analysis of the data which emerged from the main study, and the results got from them, are provided as follows.

6.2 Evaluation of Main Study Participants' Background Information

As a preliminary step to the quantitative data analysis, verification of the participants' background information was done. It was explained in subsection 4.9.4.1 that the participants' socio-economic status does affect the outcome of a study. To find out how the participants' socio-economic status could influence this study, and also to determine whether the participants were fit for this study, their background information was assessed. The information collected was the participants' ages and genders; participants' parentage status; education and employment status of participants' parents, and participants' access to technology and other learning facilities at home. The information is analysed in subsequent subsections.

6.2.1 Distribution of Main Study Participants by Group

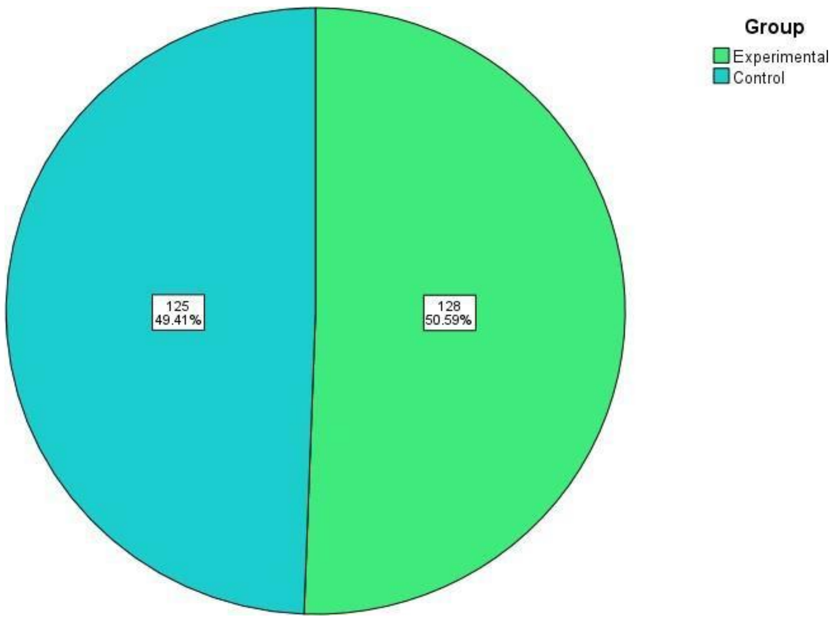
The main study involved a total of 253 participants – 128 for the experimental group and 125 for the control group. The percentage of participants, 50.6% (n = 128) in the experimental group was about the same as that of the control group 49.4% (n = 125) as indicated in Table 6.1.1 and Figure 6.1.1. Godby (2020) remarks that, in research, the control group serves as the standard to which comparisons are made. Thus, the control group should ideally be identical to the experimental group in every way except that it should not partake of the treatment administered to the experimental group. As Godby (2020) further comments that the inclusion of the control group substantially strengthens the researcher's ability to draw conclusions from a study, and also enables him to determine whether the intervention applied truly has a significant impact on the experimental group. Then, the possibility of the researcher drawing erroneous conclusions has been minimised.

Allen (2017) also supports this in his submission that the presence of the control group enables a researcher to claim that the research outcomes are due to the manipulation of independent variable rather than the influence of any extraneous variables. It states further that the control group consists of the participants not exposed to the treatment and is measured on the study's dependent variable. By comparing the differences in the dependent variable observed between the control group and one or more experimental groups, the level to which an independent variable is found to cause the dependent variable is then estimated.

Table 6.1.1: Distribution of Main Study Participants by Group

	Frequency	Percent	Valid Percent	Cumulative Percent
Experimental Group	128	50.6	50.6	50.6
Control Group	125	49.4	49.4	100.0
Total	253	100.0	100.0	

Figure 6.1.1: Pie-Chart of Main Study Participants by Group



6.2.2 Distribution of Main Study Participants by School

The 253 study participants were distributed into eight schools, four of which constituted the experimental group and the other four belonged to the control group. The numbers of participants in the eight schools were more or less the same because the percentage of learners of each school ranged from 11% - 13% (30 - 34 learners). Each participating school thus enjoyed fair representation in the study.

Table 6.1.2: Distribution of Main Study Participants by School

	Frequency	Percent	Cumulative Percent
Experimental Group 1	32	12.6	12.6
Experimental Group 2	31	12.3	24.9
Experimental Group 3	34	13.4	38.3
Experimental Group 4	31	12.3	50.6
Control Group 1	33	13.0	63.6
Control Group 2	30	11.9	75.5
Control Group 3	32	12.6	88.1
Control Group 4	30	11.9	100.0
Total	253	100.0	

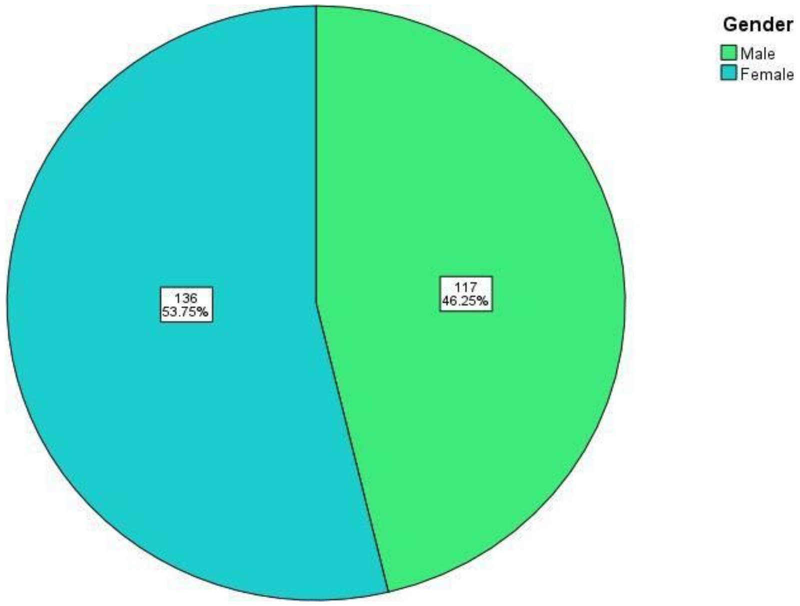
6.2.3 Distribution of Main Study Participants by Gender

About 53.8% (n = 136) learners were females while 46.2% (n = 117) were males. The same pattern observed in the pilot study was also evident in the main study where the ratio of males to females was almost 4:5. This shows that both genders were as well fairly represented in the main study (detail given in subsection 5.2.3).

Table 6.1.3: Distribution of Main Study Participants by Gender

	Frequency	Percent	Cumulative Percent
Male	117	46.2	46.2
Female	136	53.8	100.0
Total	253	100.0	

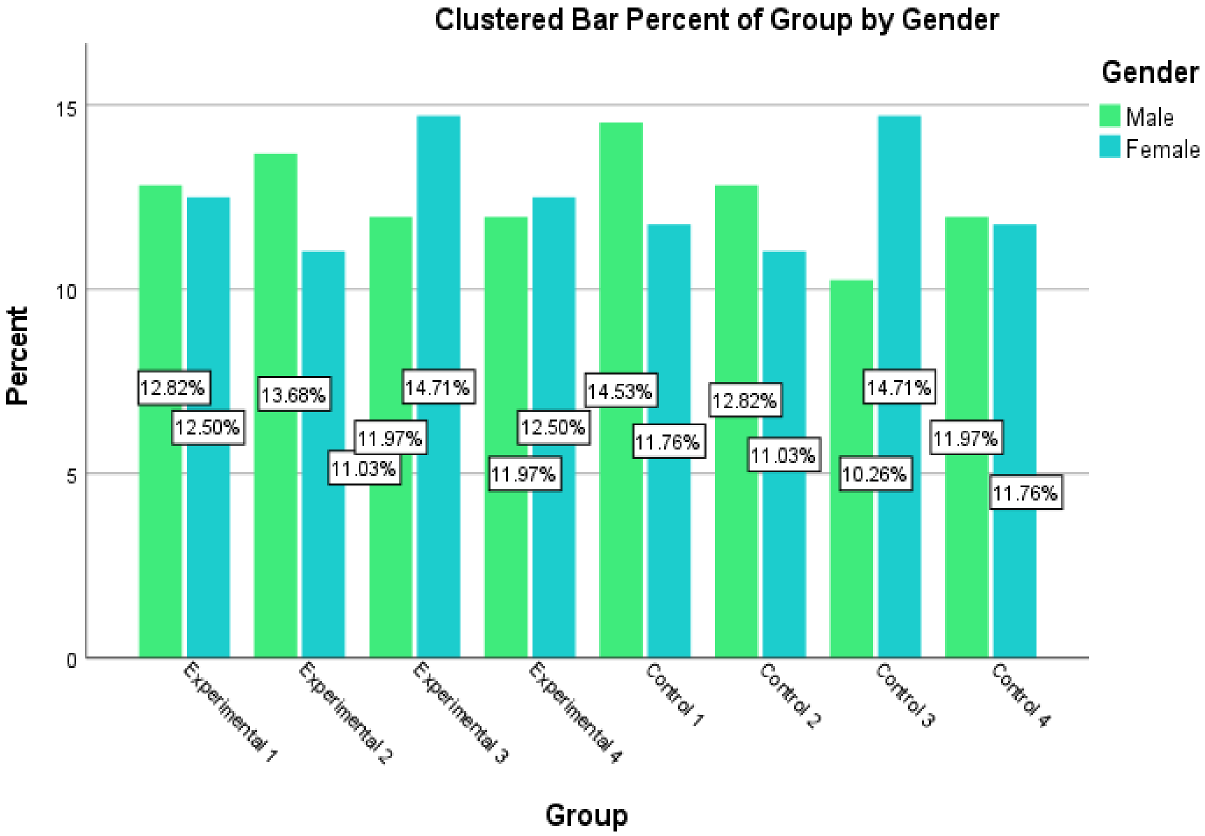
Figure 6.1.2: Pie Chart of Main Study Participants by Gender



6.2.4 Gender-distribution of Main Study Participants by School

Figure 6.1.3 gives the range of percentage male participants as 10.26% - 14.53% and that of percentage female participants as 11.03% - 14.71%. This is an indication that both genders had a fair representation in the main study.

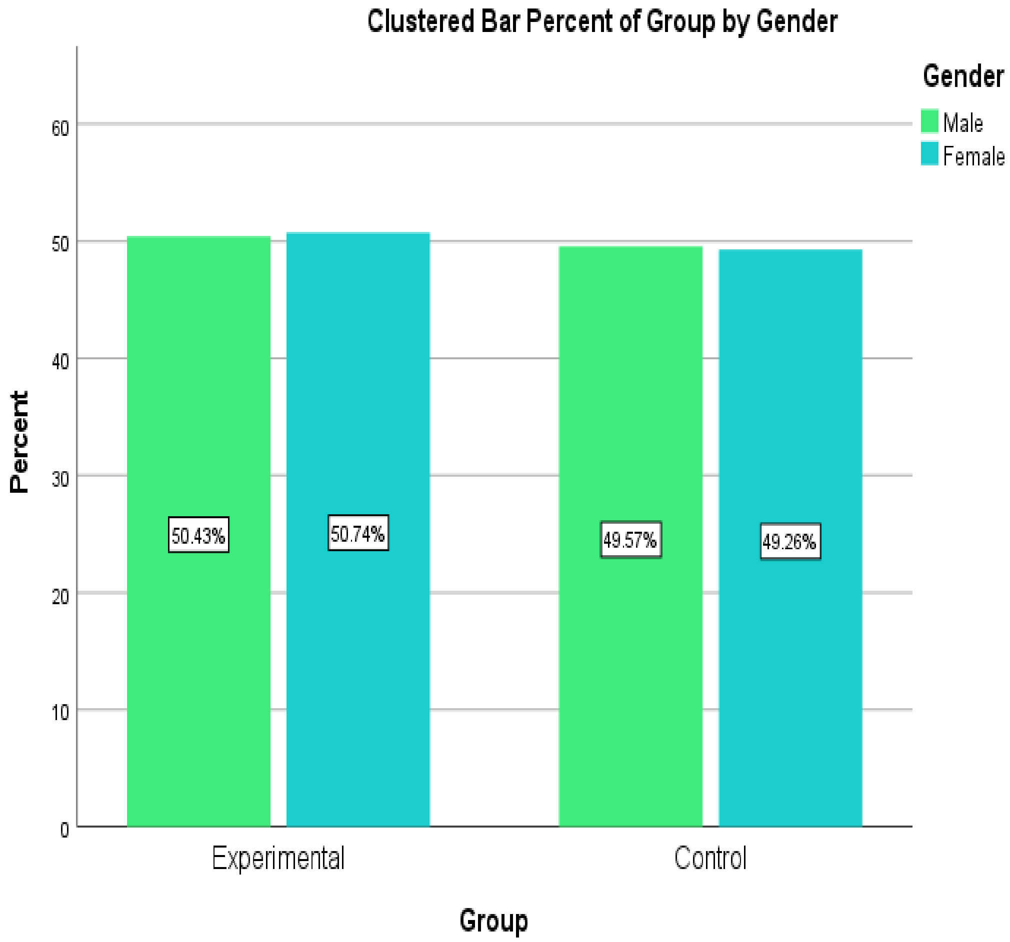
Figure 6.1.3: Clustered Bar Chart for Gender-distribution of Main Study Participants by School



6.2.5 Gender-distribution of Main Study Participants by Group

Figure 6.1.4 below reveals the percentages of male participants in the experimental group and the control group as 50.43% and 49.57% respectively. Both values are approximately the same as 50%. Also, the percentages of female participants in the experimental group and the control group are 50.74% and 49.26% respectively. This is just a slight difference of about 1.5%. Here as well, both groups enjoyed a fair gender-distribution of participants. Subsection 5.2.3 explains it further.

Figure 6.1.4: Clustered Bar Chart for Gender-distribution of Main Study Participants across Groups



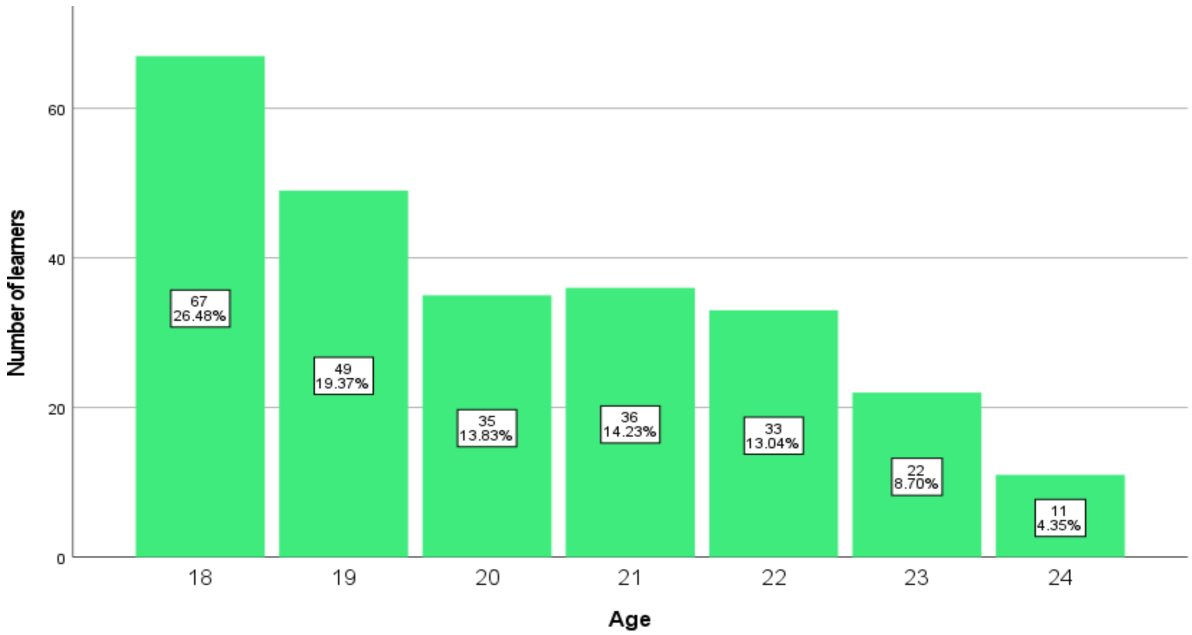
6.2.6 Distribution of Main Study Participants by Age

Most of the learners (73.9%) involved in the main study were aged 18 - 21 years. The largest proportion of learners were of age 18 years (26.48%), followed by those aged 19 years (19.37%). The mean age was 20.11 years with a standard deviation of 1.84 ($\bar{x} = 20.11;\ \sigma = 1.84$). The median was 20 years as depicted in the bar chart meaning that 50% of the participants were not more than 20 years. This is obtainable from Table 6.1.4 and Figure 6.1.5 below. It was earlier established in subsection 5.2.2 that the learners in the age-bracket 18 - 21 years are capable to learn and perform well in mathematical problem solving.

Table 6.1.4: Age-distribution of Main Study Participants

Age	Frequency	Percent	Cumulative Percent
18	67	26.5	26.5
19	49	19.4	45.8
20	35	13.8	59.7
21	36	14.2	73.9
22	33	13.0	87.0
23	22	8.7	95.7
24	11	4.3	100.0
Total	253	100.0	

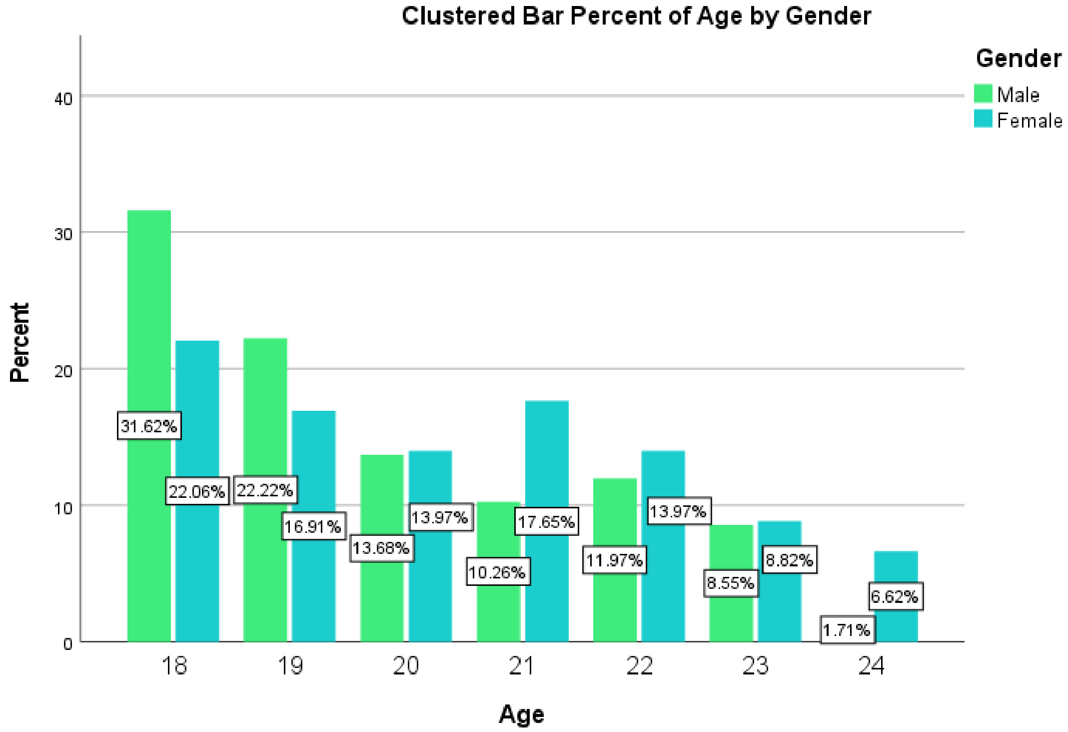
Figure 6.1.5: Bar Chart for Age-distribution of Main Study Participants



6.2.7 Age-distribution of Main Study Participants by Gender

The percentages of male participants are more than those of female participants in the ages 18 and 19 years, while those of female participants are more in the remaining ages. Ages 20 and 23 years have almost the same percentages of males and females participating in the main study. Gender-balance across all ages of the main study participants is also evident.

Figure 6.1.6: Clustered Bar Chart for Age-distribution of Main Study Participants by Gender



6.2.8 Distribution of Main Study Participants by Parental Details

Parental involvement has been viewed as a vital variable that positively impacts children's education (Lara & Saracostti, 2019). Evidence abounds that the level of interactions between the school and the parents affects the nature and rate of learners' success at school. In fact, active parental participation in school activities stands as a critical contributor to the learners' success at all grade levels of education. Parents who appreciate this essential role see themselves as their children's first educators and actively support their children’s learning at home by teaching them necessary skills and knowledge. Those parents reinforce school learning and foster respect for education. They encourage and develop in their children worthwhile school attributes such as: work habits, positive attitudes, cooperative behaviour, interests and motivation for school and suchlike (Brown, Hilukilua & Kambonde, 2014; Mutasa, Goronga & Gatsi, 2013).

The six major forms that parents’ participation in their children’s educational matters can assume are: parenting, decision-making, communicating, learning at home, volunteering and collaborating with the community. For a productive learner academic performance to be recorded, parents must be involved in all these six areas (Epstein, 1997). The more actively involved parents are in their children's learning, the better the achievement impacts are. This is because learners are more likely to respond and perform well in school when parents monitor their children’s homework; when they are active in parent-teacher associations, encourage participation in extracurricular activities and help children establish their future plans (Naite, 2021).

Based on the above research evidence, the current study appreciates the impact of parental participation in their children’s education. It accepts parental participation as an essential element that enhances learner academic achievements and which also has a favourable consequence on learner attitude and behaviour. Again, the current study acknowledges that parents’ involvement can influence the learner's attitude towards school, school attendance, classroom behaviour, motivation and self-esteem. That was why it collected and validated information regarding the parents and homes of the participants. The series of participants’ demographics gathered to this effect are: participants’ parentage status; education and employment status of their parents, and participants’ access to technology and other learning facilities at home. This is analysed as follows.

Table 6.1.5: Distribution of Main Study Participants by Parental Details

	Male (n = 117)	Female (n = 136)	Total (n = 253)
			$\bar{x} = 20.11$; σ =1.84
Learner's Parentage Status			
Living with Both Parents	22 (8.7%)	25 (9.9%)	47 (18.6%)
Living with a Single Parent	58 (22.9%)	61 (24.1%)	119 (47.0%)
Staying Alone – No Parents	19 (7.5%)	23 (9.1%)	42 (16.6%)
Staying with a Guardian	18 (7.1%)	27 (10.7%)	(17.8%)
			100.0%
Parent's Education Status			
Primary School	79 (31.2%)	92 (36.4%)	(67.6%)
High School: Below Grade 12	14 (5.5%)	21 (8.3%)	(13.8%)
Grade 12	9 (3.6%)	7 (2.8%)	(6.3%)
Diploma	8 (3.2%)	10 (4.0%)	(7.1%)
Degree and above	7 (2.8%)	6 (2.4%)	(5.1%)
			100.0%
Parent's Employment Status			
Employed in Public Service	25 (9.9%)	36 (14.2%)	(24.1%)
Self-employed	48 (19.0%)	59 (23.3%)	(42.3%)
Unemployed	44 (17.4%)	41 (16.2%)	(33.6%)
			100.0%
Access to Technology and Others			
Availability of Computer, Library, Maths. Games etc. for Home Use	7 (2.8%)	10 (4.0%)	(6.7%)
None of these at Home	104 (41.1%)	122 (48.2%)	(89.3%)
Not Sure	6 (2.4%)	4(1.6%)	(4.0%)
			100.0%

6.2.8.1 Distribution of Main Study Participants by Parentage Status

Information regarding the parentage status of the main study participants was obtained so as to determine the level of parental support, guidance and care they enjoyed in their educational matters. Akhtar, Ahmad and Saifi (2020) aver that the learning achievement of learners is largely influenced by the rate of parents' participation in the learners' education.

Figure 6.2.1: Bar Chart of Main Study Participants' Parentage Status.

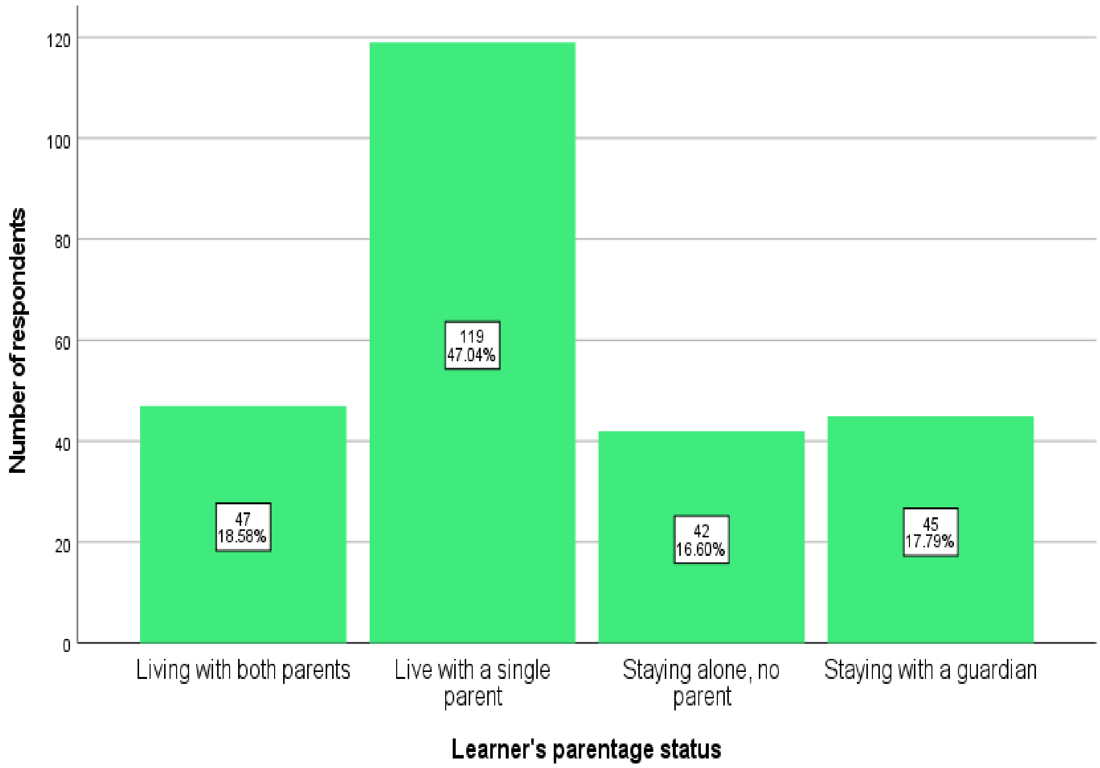
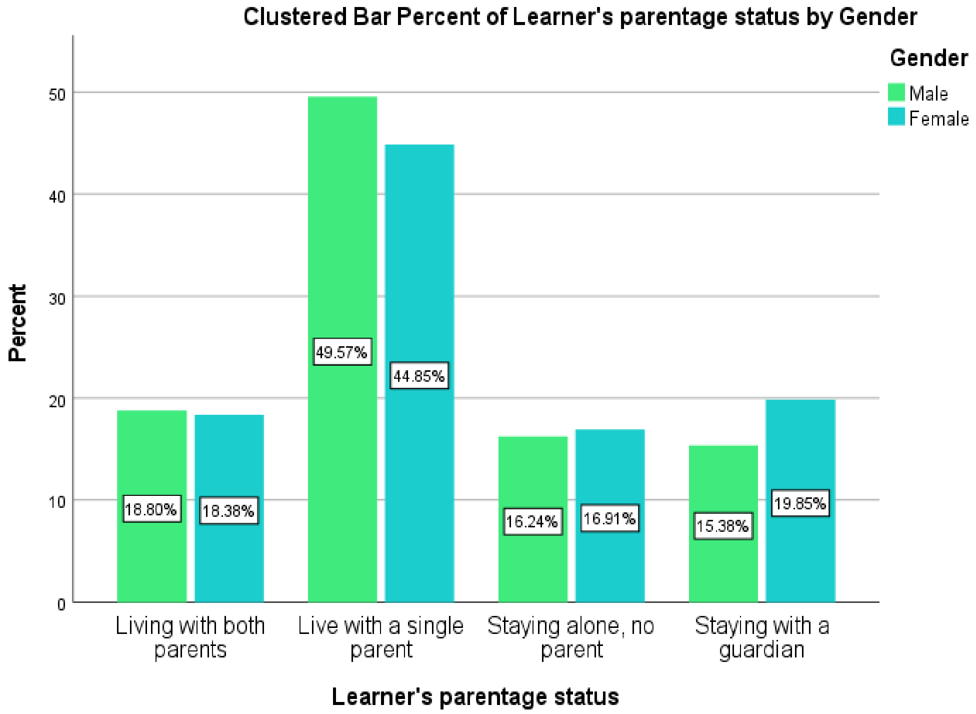


Figure 6.2.1 reveals that about half (n = 119 = 47.04%) of the total participants were living with single parents; 17.79% (n = 45) of them were staying with guardians and 16.6% (n = 42) were residing alone with no parents. Only 18.58% (n = 47) of the total participants in the main study were living with both parents. The implication of this is that only about nineteen out of every hundred participants (19%) enjoyed full parental care, encouragement and involvement in their education; hence, their weak academic attainments. The distribution of learner's parentage status was also done by gender. The information is shown as follows in Figure 6.2.2.

Figure 6.2.2: Multiple Bar Chart of Main Study Participants' Parentage Status by Gender

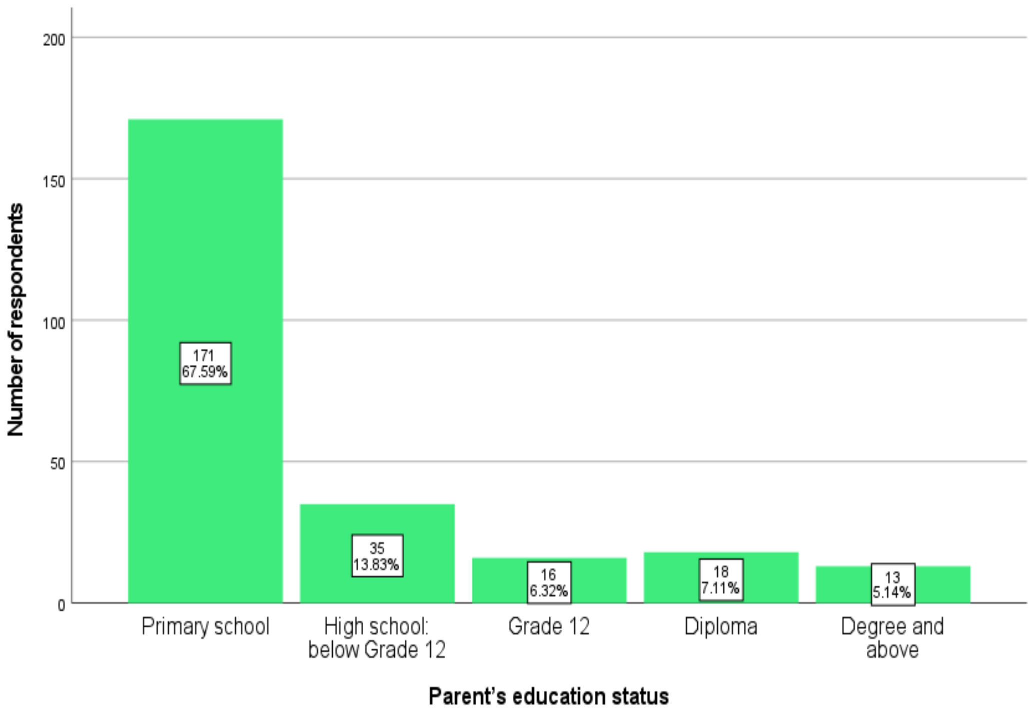


From Figure 6.2.2, the proportion of male participants to female participants was almost the same for those living with both parents and those staying alone without parents. For those living with single parents, the male participants took 49.57% compared to 44.85% of the female participants. About 19.85% of the female participants and 15.36% of the male participants were staying with guardians. Generally, the distributions of male and female participants in each category were more or less balanced.

6.2.8.2 Distribution of Main Study Participants by Parents' Education Status

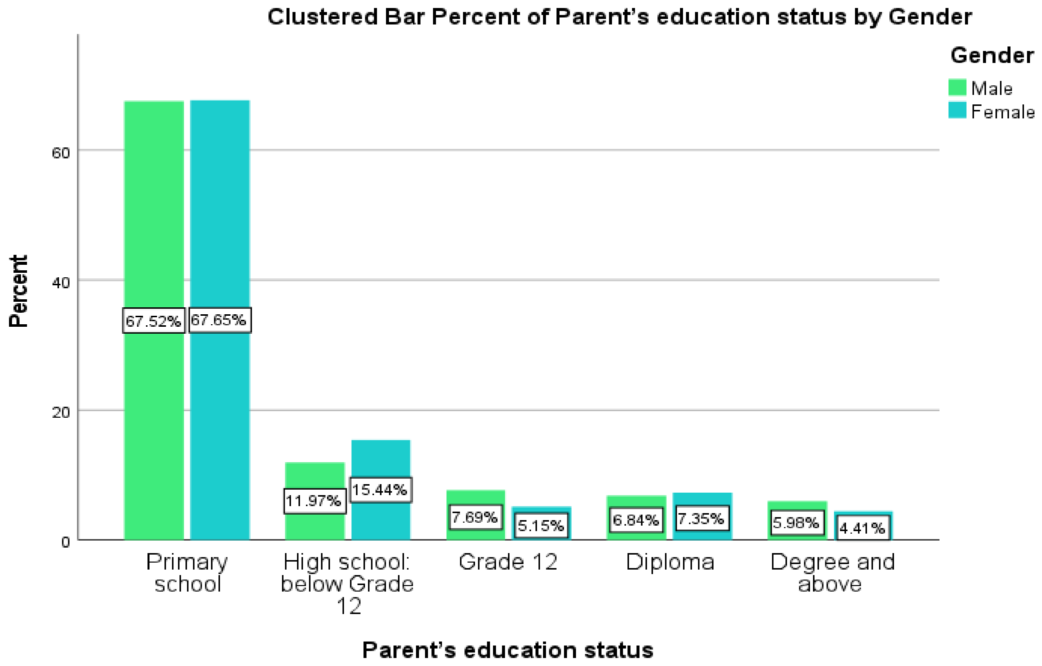
Information concerning the educational status of the participants' parents was also collected and analysed. There is a large correlation between parent's educational status and learners' academic performance as parent's level of education has significant influences on children's values, skills and knowledge (Idris, Hussain & Nasir, 2020). Highly educated parents often help their children in educational activities at home; they are more inclined to develop greater interest in their children's academic achievements and are more prepared to provide necessary items and fees required for their children to succeed in school than the less-educated ones (Atolagbe, Oparinde & Umaru, 2019). As the parent's educational attainment increases, the child's educational performance also increases because the parent has got improved skills with which to support the learner's education (Akhtar, Ahmad & Saifi, 2020). Jay, Rose and Simmons (2018) therefore implore parents to strive to earn a considerable level of qualifications that can enable them to have a proper impact on their children's education and to be able to train them with helpful mathematical problem-solving strategies.

Figure 6.2.3: Bar Chart of Parents' Education Status of Main Study Participants



According to Figure 6.2.3, the majority of the participants, 67.59% (n = 171) had parents having primary school certificates while 13.83% (n = 35) had parents holding the high school certificates less than Grade 12. Only 6.32% (n = 16) of them had parents with Grade 12 certificates as their highest qualifications; 7.11% (n = 18) had parents who were diploma certificate holders and 5.14% (n = 13) had parents with at least degree certificates. Thus, it is reasonably deducible that only few of the learners had the support and guide of parents who were highly educationally equipped.

Figure 6.2.4: Multiple Bar Chart of Parents' Education Status of Main Participants by Gender

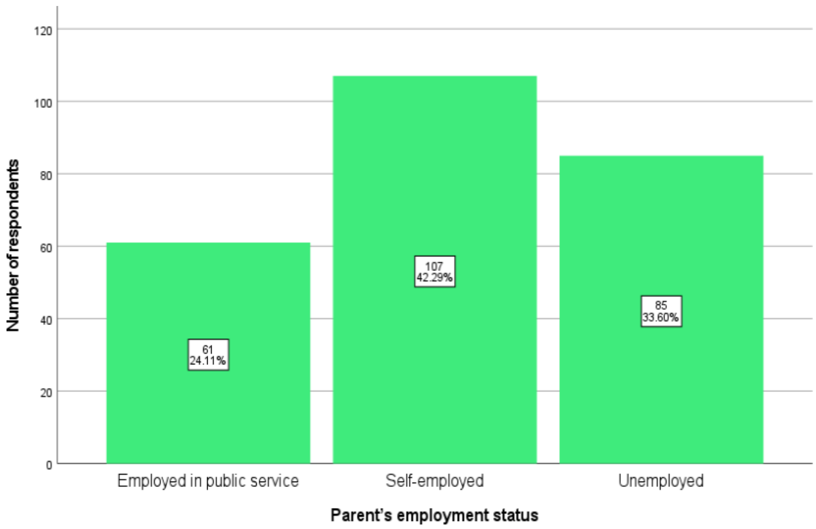


Considering Table 6.1.5, there were almost equal numbers of males and females who had parents who were holders of primary school and diploma certificates. More females than males had parents who had high school certificates below Grade 12. Then, as for parents whose highest qualifications were Grade 12 certificates and those holding minimum of first-degree certificates, the male participants were slightly greater in number than the female participants. Averagely, both genders were also well-represented in each of the five categories.

6.2.8.3 Distribution of Study Participants by Parent's Employment Status

The occupation of a learner's parent is the best indicator of the family's socio-economic position. Parents having occupations paying high salaries are more willing and more efficient in supplying their children's school needs than parents in occupations fetching low salaries. This has a high likelihood of influencing the learner's academic achievement. The parents engaged in office work or office-related jobs, particularly ones working in schools, are more likely to be committed to their children's educational development. Such parents do give close monitoring to their children's school activities and homework completion rates (Atolagbe, Oparinde & Umaru, 2019). To an extent, the parent's occupation determines the number of resources such will be able to invest in their children's education. Parental occupation is hence an indicator of the quantity of learning resources and opportunities parents will be able to offer their children. Thus, high academic achievers mostly come from families with high occupational status because the parents' high occupational level means good economic conditions which can result in material support for the children's education. On the other hand, low-achieving learners, in most cases, have traders, unskilled and semi-skilled workers as parents (Atolagbe, et al., 2019). Acknowledging the influence of parents' employment status on learner performance, the present study collected and evaluated information on the employment status of participants' parents. The information, as earlier shown in Table 6.1.5, is further described in Figure 6.2.5 below.

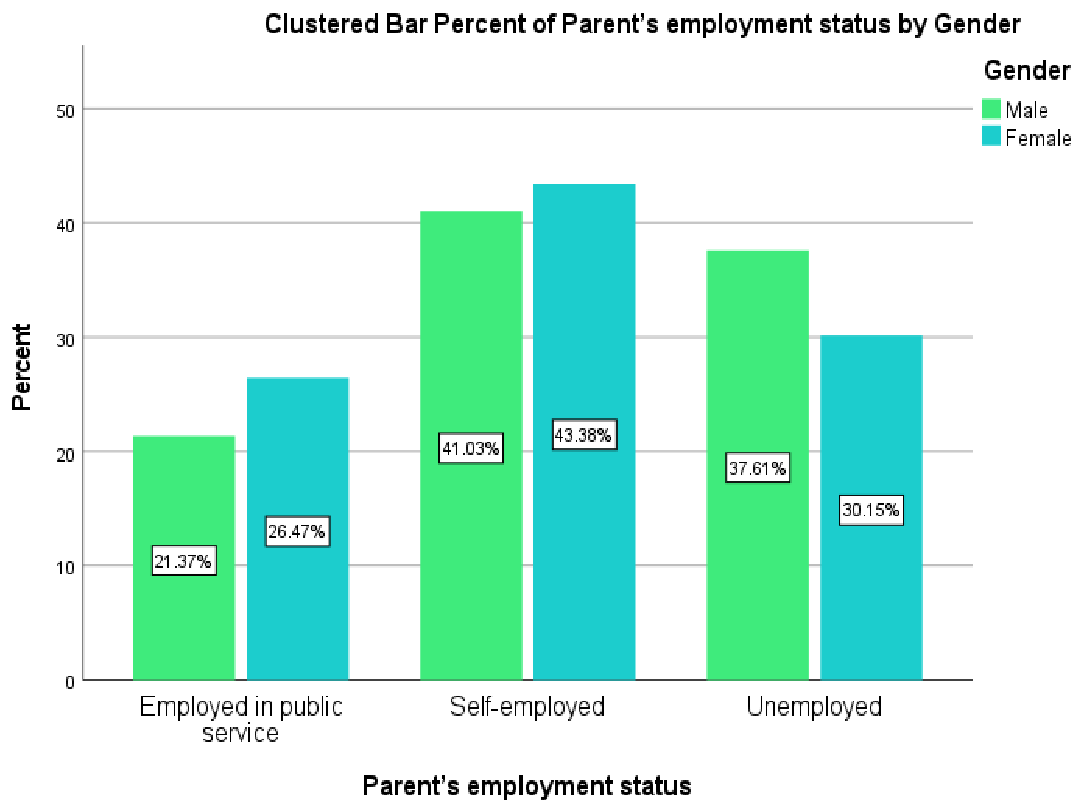
Figure 6.2.5: Bar Chart of Parents' Employment Status of Main Study Participants



About 42.29% (n = 107) of the participants had self-employed parents; 33.6% (n = 85) had unemployed parents while only 24.11% (n = 61) had parents employed in the

public service. It can then be inferred that few of the main study participants could boast of parents with a high economic capability. That could be another factor causing their low learning gains; hence, their weak mathematical problem-solving skills. The information on parents' employment status of participants by gender is illustrated by Figure 6.2.6.

Figure 6.2.6: Multiple Bar Chart of Parents' Employment Status of Main Study Participants by Gender

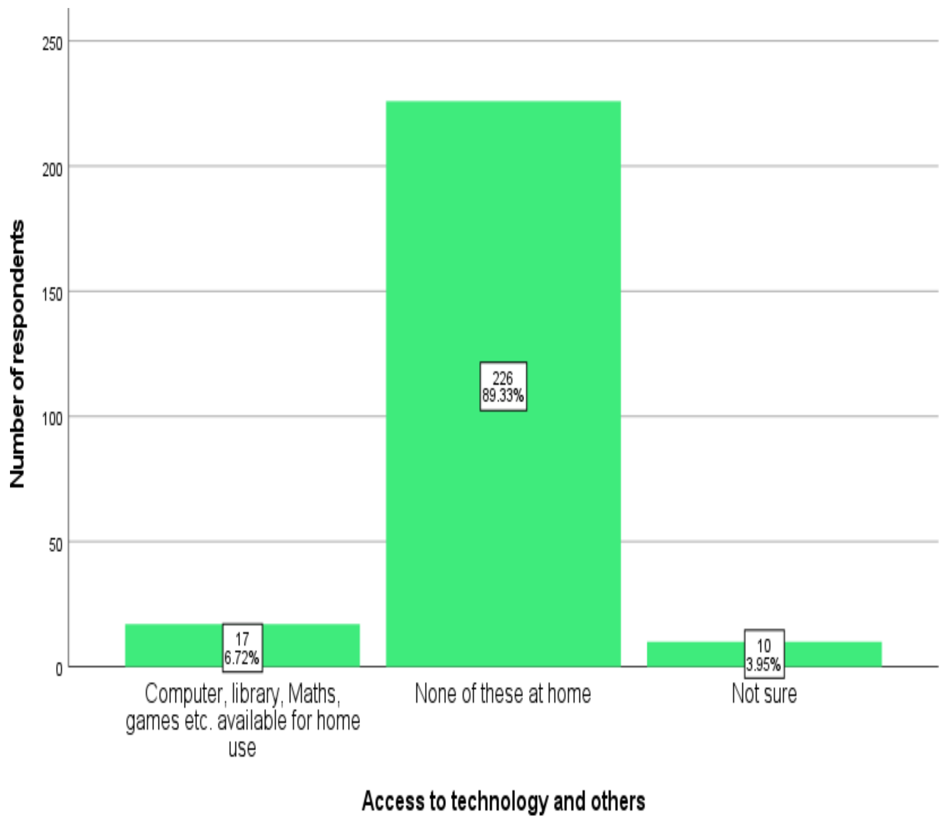


More males than females had unemployed parents while more females than males had parents that were employed in the public service or were self-employed. Averagely, the two genders were fairly represented in each category of parents' employment status.

6.2.8.4 Distribution of Main Study Participants by Access to Technology and Other Learning Facilities at Home

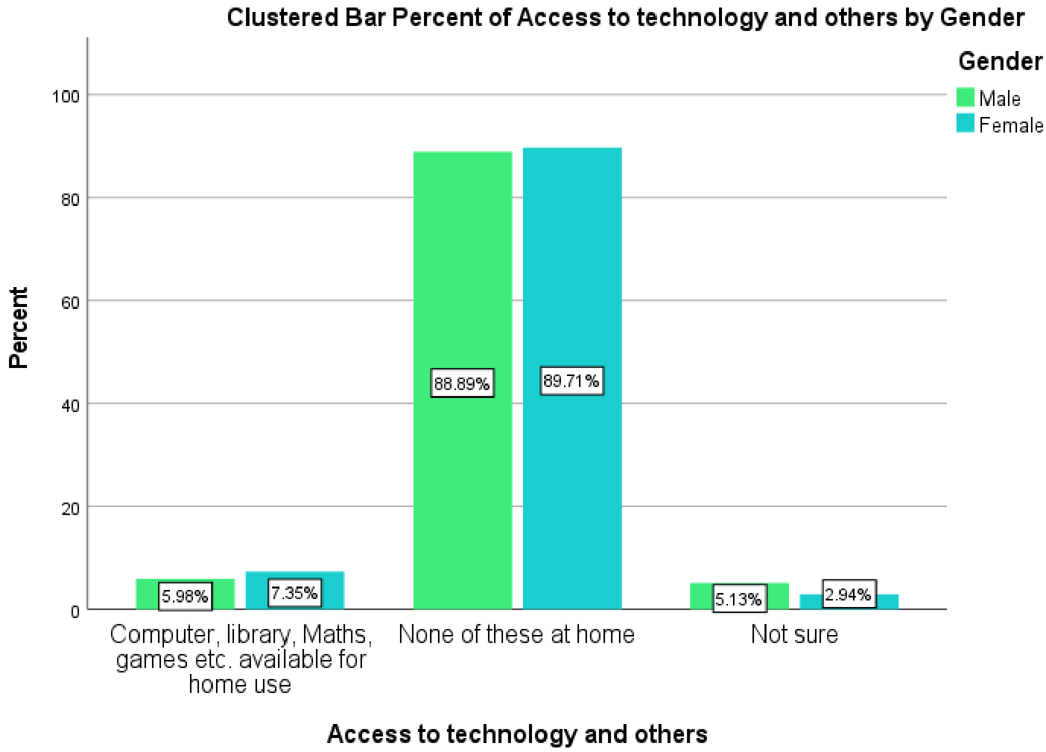
The information gathered on availability of technology and other learning facilities for the participants' use at home was analysed as shown below.

Figure 6.2.7: Bar Chart of Participants' Access to Technology and Other Facilities at Home



Majority of the participants (n = 226 = 89.33%) did not have access to technology and other learning facilities for their use at home. Only 6.72% (n = 17) indicated that they had the learning facilities for home use and 3.95% (n = 10) were not sure. The cross-tabulation of the participants' access to technology and other learning facilities at home across gender was done and the results are illustrated by Figure 6.2.8.

Figure 6.2.8: Clustered Bar Chart of Participants' Access to Technology and Other Learning Facilities at Home



From Figure 6.2.8 above, the percentages of male and female participants across all categories were almost the same for both male and female participants. A fair representation of both genders was also evident in this case.

6.2.9 Concluding Comments on Study Participants' Background Information

The foregoing has established that the main study participants in both experimental and control groups were fairly comparable considering the largely similar demographic variables characterising them. It has shown that both groups belonged to a community disadvantaged socio-economically, an observable factor adversely impacting on their learning attainments. Therefore, the study sample, n = 253, might be considered suitable for the investigation.

6.3 Descriptive Statistics of Main Study Pre-test and Post-test Marks

The descriptive statistics of pre-test and post-test scores for both experimental and control groups were calculated as follows in terms of mean scores, standard deviation, variability and gain scores

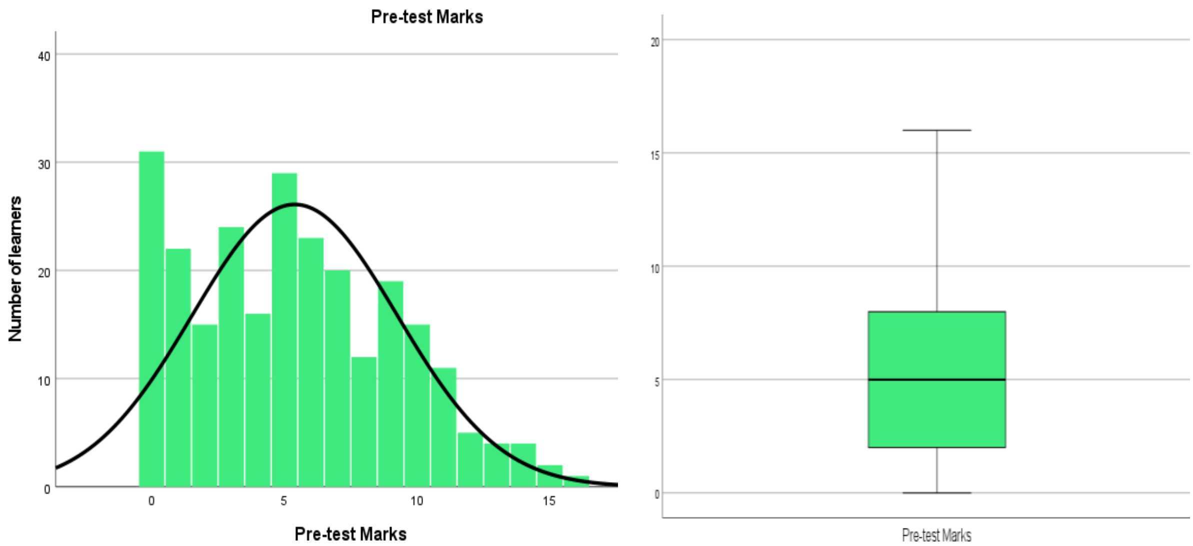
Table 6.2.1: Summary of Descriptive Statistics of Achievement Test Marks for Main Study

Test	n	$\bar{x}$	Median	σ	Min	Max	Range	Q_1	Q_3	Coeffi. of Var
Pre-test	253	5.36	5	3.87	0	16	16	2	8	71.74%
Post-test	238	29	22	17.9	3	62	59	14	48	61.73%

6.3.1 Descriptive Statistics of Main Study Pre-test Marks

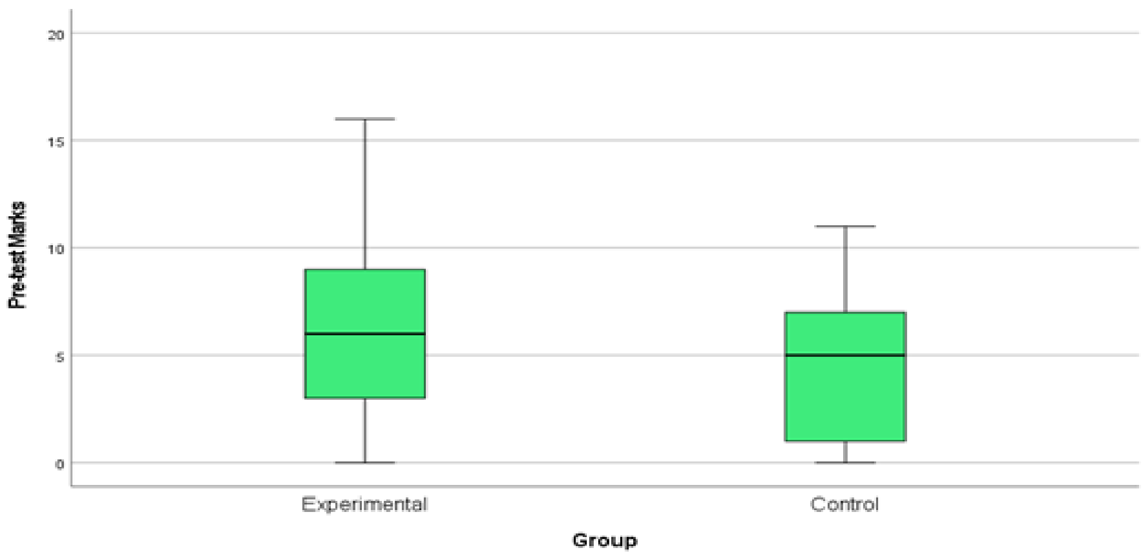
For the 253 study participants who wrote the pre-test, the pre-test marks ranged from 0% to 16%. The mean and standard deviation of the pre-test marks were 5.36% and 3.87% respectively. On the average, the participants obtained 5.36% in the pre-test. The median was 5%, meaning that half of the participants got marks not more than 5%. Using the empirical rule of the normal distribution, about 68.26% of the participants had marks that ranged from 1.52% to 9.26%. This shows that the participants performed poorly in the pre-test. Therefore, their initial ability level of the concept of stationary points in differential calculus is low. The coefficient of variation was 71.74% indicating that there was some variability in the pre-test marks. The pre-test marks are positively skewed as shown in Figure 6.3.1.

Figure 6.3.1: Histogram and Box-plot of Pre-test Marks for the Main Study



The fact that the histogram has a long tail to the right indicates that the data is positively skewed. The middle 50% of the participants had marks that ranged from 2% to 8%. The box-plot of the pre-test marks implies that there is a lot of overlap between the participants of both groups as shown in Figure 6.3.2.

Figure 6.3.2: Box-plots of Pre-test Marks by Group

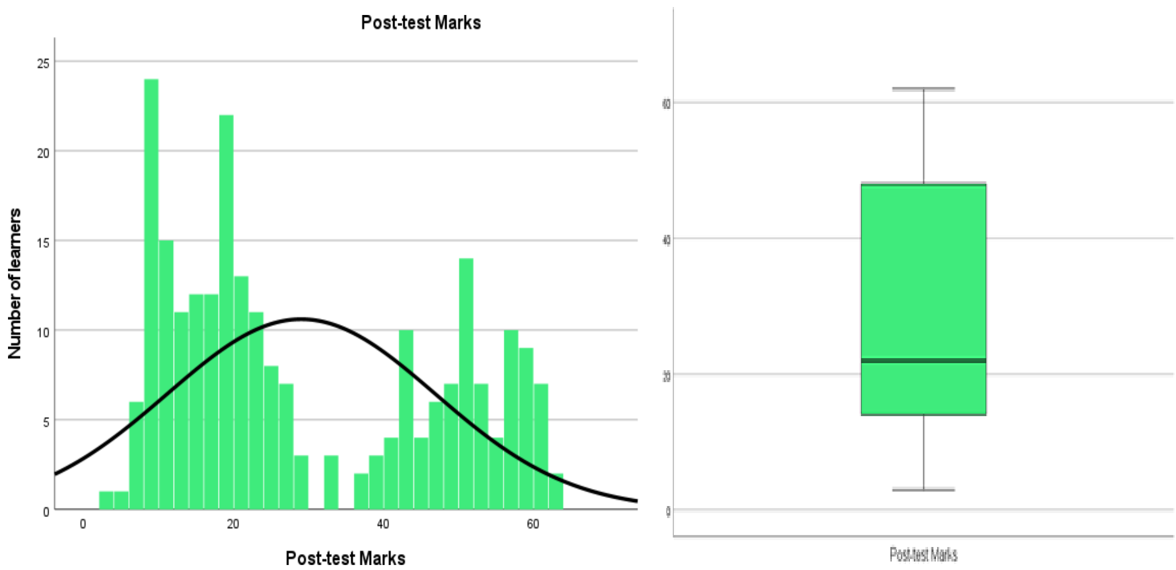


In Figure 6.3.2, the box-plots show that the medians of experimental and control groups are almost the same. However, the experimental group exhibited a larger variability than the control group. Experimental group having a longer tail to the right means positive skewness.

6.3.2 Descriptive Statistics of Main Study Post-test Marks

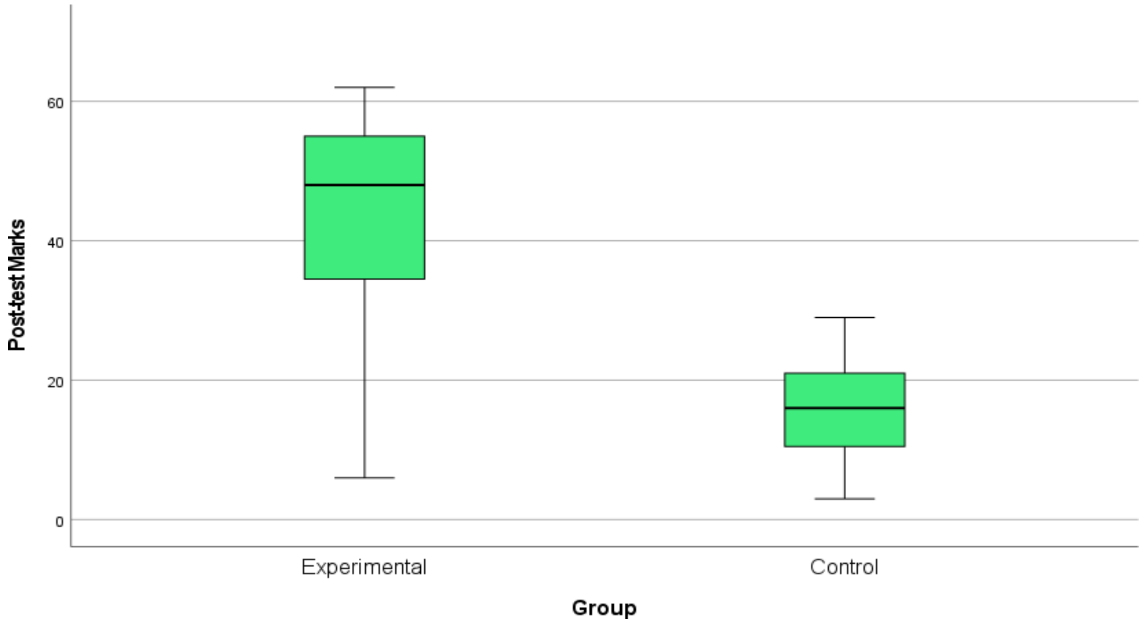
Out of the 253 study participants who wrote the pre-test, 15 participants could not make it to the post-test stage. Thus, only 238 participants wrote the post-test. The analysis of the post-test marks was therefore based on 238 participants, 119 of which belonged to the experimental group and the other 119 fell in the control group. The post-test marks had a minimum mark of 3% and a maximum mark of 62%, giving a range of 59%. The mean and standard deviation were 29 and 17.9 respectively. On the average, the performance was 29%. The median mark was 22% and thus half of the participants had marks not more than 22%. About 68.26% of the participants had post-test marks 11.10% and 36.90%. The coefficient of variation was 61.73% indicating that there is some variability. The histogram and box plot are as follows:

Figure 6.3.3: Histogram and Box-plot of Post-test Marks for the Main Study



The histogram presents two groups of participants, one which performed well (experimental group) and the other (control group) which did not. The box-plot shows that the data is positively skewed. The middle 50% of the participants achieved marks ranging from 14% to 48%.

Figure 6.3.4: Box-plots of Post-test Marks by Group



The box-plot representing experimental group post-test marks is negatively skewed while the box-plot for control group post-test marks is almost normally distributed. This means that, for the post-test, the mathematical problem-solving performances of both groups were different since the experimental group had a higher median than the control group. In fact, based on their post-test marks, all the participants in the control group were in the bottom 25% of the experimental group.

6.4 Comparing Post-test Performances of Both Groups

6.4.1 Descriptive Statistics of Post-test Marks of Both Groups

For comparison of the post-test performances of the experimental and control groups, the descriptive statistics for both groups were separately calculated and compared. The results obtained are summarised in Table 6.2.2 below:

Table 6.2.2: Summary of Descriptive Statistics for Post-test Marks of Both Groups

Post-test	n	$\bar{x}$	Median	σ	Min	Max	Range	Q_1	Q_3	Coeff. of Var
Exp. Grp.	119	41.98	48	16.28	6	62	56	33	55	38.79%
Cont. Grp	119	16.01	16	6.15	3	29	26	10	21	38.39%

As shown by Table 6.2.2, in the post-test, participants of the experimental group achieved ($\bar{x}$ = 41.98; σ = 16.28; n = 119) while participants in the control group made ($\bar{x}$ = 16.01; σ = 6.15; n = 119). This means that the experimental group given the 8Ps treatment recorded a mean score increase of 25.97% over and above the control group. The median of 48% made by the experimental group indicates half of the participants of the group did not get more than 48%. Similarly, the median of 16% attained by the control group indicates that half of the participants of the group got no more than 16%. Furthermore, the highest marks achieved by the experimental and control groups are 62 and 29 respectively. Table 6.2.2 generally reveals that the experimental group performed better in the post-test than the control group.

6.4.2 Paired T-test of Post-test Marks of Both Groups

In order to show that the mean score increase of 25.97% that the experimental group achieved in section 6.4.1 above the control group is statistically significant as a proof of the effectiveness of the 8Ps problem-solving instruction, the paired t-test for post-test marks of both groups was carried out at 5% level of significance (α = .05).The t-test was performed to test the null hypothesis stating that there is no significant statistical difference between the experimental and the control groups. In t-test, the value of the probability of error, p-value, which is smaller than α = .05 indicates that the post-test mean score of the experimental group is statistically significantly different from the post-test mean score of the control group. Hence, the null hypothesis is to be rejected. On the other hand, a p-value in t-test that is bigger than α = .05 signifies no significant statistical difference between both groups; thus, the null hypothesis is to be accepted. The t-test done is shown as follows:

Table 6.2.3: Synopsis of Paired T-test for Post-test Marks of Both Groups

Test	Group	n	$\bar{x}$	σ	SEM	t	p
Post-test	Experimental Group	119	41.98	16.28	1.49	16.28	$p < .00001$
Post-test	Control Group	119	16.01	6.15	0.56		

In table 6.2.3, the result of the paired t-test {t = 16.28, *p* < *.00001*, *p* < *α* = .05} reveals that the post-test result ($\bar{x} = 41.98,\ \sigma = 16.28$) achieved by the experimental group is statistically significantly higher than the post-test result ($\bar{x} = 16.01,\ \sigma = 6.15$) attained by the control group. This mean increase of 25.97% with a 95% confidence interval ranges from −24.15 to 27.79. The eta-squared statistic, $\eta^2 = \frac{t^2}{t^2+(n-1)} = .69$ produced yields a large effect size. Thus, 69% of the variability in the post-test is accounted for by the groups.

On the strength of that, the null hypothesis is rejected and the conclusion made that the mean difference is statistically significant. This means that the 8Ps-oriented instruction is effective. In other words, it signifies that the mathematical problem-solving performance of the study participants in the experimental group who received 8Ps-based intervention was better than that of their counterparts in the control group who were traditionally-instructed.

6.5 Analysis of Covariance (ANCOVA) for the Main Study

Analysis of Covariance (ANCOVA) is a statistical method serving as a hybrid between analysis of variance (ANOVA) and regression analysis. ANCOVA was performed to minimise the residual variations between the experimental and control groups at the pre-test stage of the enquiry. The dependent variable was the post-test while the covariate was the pre-test. In order to perform the ANCOVA, the researcher first tested the assumptions underpinning it. He tested the assumptions to determine whether he could proceed with the ANCOVA.

6.5.1 Levene's Test for Equality of Error Variances

To carry out the Levene's test, the null (H_0) and the alternative (H_1) hypotheses were considered. While the H_0 states that the error variances of the dependent variable are equal for both groups (meaning that the two groups are homogeneous), the H_1 states that the error variances of the dependent variable are different for both groups (meaning that the two groups are heterogeneous). Levene's test was performed in the current investigation to determine whether the error variances of experimental and control groups were comparable. The error variances in the two groups are seen as different if $p < .05$ significance level. That implies that the two groups are not homogeneous. However, if $p > .05$, the groups are homogeneous because their error variances would be the same. In performing the Levene's test for the current study, the groups were used as the independent variable; the post-test was taken as the dependent variable, and the pre-test served as the covariate.

Table 6.3.1: Summary of Levene's Test for Equality of Error Variances

F	df1	df2	Sig.
61.309	1	236	$p < .001$

The Levene's test for equality error variances is violated since $p < .001$, meaning that $p < \alpha = .05$ significance level. However, ANCOVA is robust to violations of the assumption of homogeneity of variances provided the ratio of the largest group variance is not more than three times the smallest group. In this case, the criteria were not met; so, the interpretation of ANCOVA could not be taken any further.

6.5.2 The Homogeneity-of-Regression Assumption

The assumption of equal slopes (or the homogeneity of regression assumption) was also tested. The current study tested this assumption to draw useful conclusions on the overall relationship between the covariate (pre-test) and the outcome/dependent variable (post-test). By this, it would be able to determine the overall mathematical problem-solving performance of the study participants. In case $p >$ α = .05 significance level, the relationship is insignificant, which implies that ANCOVA can be performed. However, it is a significant interactive relationship where $p <$ α = .05, that is, ANCOVA cannot be conducted. The slopes of the regression lines, formed by the covariate and the outcome variable, should be the same for each group. The assumption that there was no interaction between the outcome variable and the covariate was tested and the below result in Table 6.3.2 was obtained.

Table 6.3.2: Summary of Results of the Homogeneity-of-Regression Assumption

Source	df	F	Sig.
Group Pre-test Marks	1	21.384	P < .001

The interaction between the group and the covariate is statistically significant at $\{F\ (1,\ 234) = 21.384,\ p < .001\}$. This means that the regression slopes for the covariate differ between treatments and the homogeneity of regression slopes assumption is violated. Thus, a further interpretation of ANCOVA could not be made.

6.6 Simple Mathematical Analysis of Participants' Post-test Marks

6.6.1 Classification of Participants' Post-test Marks by Achievement Categories LA, MA and HA

For the purpose of this study, and for further analysis of the participants' post-test performances, participants' test scores were transformed and classified as low achievement (LA), medium achievement (MA) and high achievement (HA) as shown in Table 6.3.3 below.

Table 6.3.3: Classification of Learners' Post-test Marks as LA, MA and HA

Assessment Criteria	Code	Description
Low Achievement	LA	Below 19 marks
Medium Achievement	MA	20 - 39 marks
High Achievement	HA	40 marks and above

The groups were used to determine participants' achievement levels in the tests and also a one-way analysis of variance (ANOVA) was done with the post-test as the dependent variable.

6.6.2 Comparing Participants' Post-test Marks by Achievement Categories

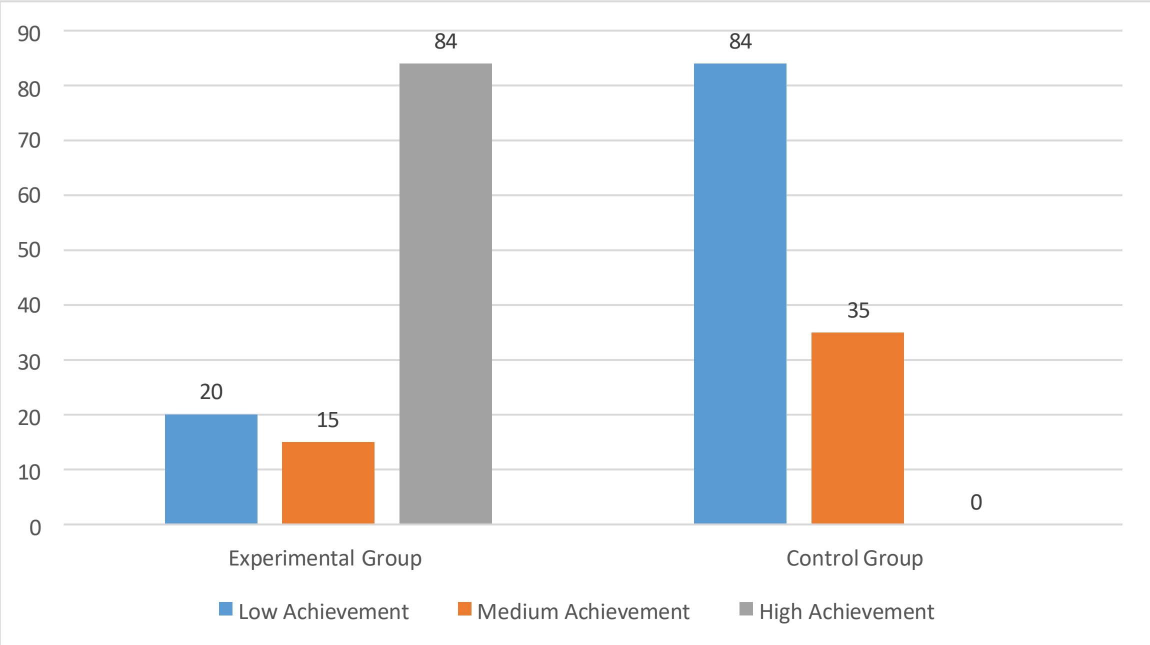
The analysis comparing the post-test scores of both groups by achievement categories is shown in Table 6.3.4.

Table 6.3.4: Summary of Participants' Post-test Marks by Achievement Categories

Groups	Level of Achievement	Post-test Marks
Experimental Group n(EX) = 119	LA	20 (16.8%)
	MA	15 (12.6%)
	HA	84 (70.6%)
Control Group n(CL) = 119	LA	84 (70.6%)
	MA	35 (29.4%)
	HA	0 (0%)

According to Table 6.3.4, in the post-test however, the majority of the control group participants (n = 84 = 70.6%) were low achievers. There were 35 (29.4%) medium achievers and no high achievers in the group. The post-test experimental group had 84 (70.6%) high achievers, 15 (12.6%) medium achievers and 20 (16.8%) low achievers. This generally reveals that the intervention conducted made a positive impact on the mathematical problem-solving performance of the participants in the experimental group. A graphical representation of the data is given in Figure 6.4.1.

Figure 6.4.1: Clustered Bar Graph of Participants' Post-test Marks by Achievement Categories



6.7 One-way ANOVA for Participants' Post-test Marks by Achievement Categories

The one-way analysis of variance (ANOVA) was performed for the comparison of post-test mean achievements in the categories: LA = Low Achievement, MA = Medium Achievement and HA = High Achievement. The post-test results were chosen to compare the 8Ps problem-solving instruction with the traditional instruction. Thus, an ANOVA test was conducted for each achievement group to determine the difference between experimental and control groups per achievement category. The mean scores and standard deviations for both groups for the post-test achievements in each category are shown in Table 6.4.1.

Table 6.4.1: Descriptive Statistics of Participants' Post-test Marks by Achievement Categories

Group		Total n	Mean $\bar{x}$	Standard Deviation σ	Standard Error (SEM)	95% Confidence Interval of the Mean Lower Bound	Upper Bound	Min	Max
LA	Experimental	20	12.30	4.169	.932	10.35	14.25	6	19
	Control	84	12.87	4.165	.454	11.97	13.77	3	19
	Total	104	12.76	4.151	.407	11.97	13.57	3	19
MA	Experimental	15	29.73	6.933	1.790	25.89	33.57	20	38
	Control	35	23.54	2.442	.413	22.70	24.38	20	29
	Total	50	25.40	5.107	.722	23.95	26.85	20	38
HA	Experimental	84	51.25	6.244	.681	49.90	52.60	40	62
	Control	-	-	-	-	-	-	-	-
	Total	84	51.25	6.244	.681	49.90	52.60	40	62

Looking at Table 6.4.1, the control group had no higher achievers in the post-test, but the experimental group made the mean achievement score ($\bar{x} = 51.25, \sigma = 6.24$). For the medium achievers, the mean score of the experimental group ($\bar{x} = 29.73, \sigma = 6.93$) is higher than the mean score of the control group ($\bar{x} = 23.54$, $\sigma = 2.44$). It is only in the low achievers' category that the mean score ($\bar{x} = 12.30$, $\sigma = 4.17$) of the experimental group is slightly lower than the mean score ($\bar{x} = 12.87, \sigma = 4.17$) of the control group. It can then be concluded that, in the post-test, the average mean score of the experimental group is greater than that of the control group. The ANOVA test results for both groups are summarised in Table 6.4.2 below.

Table 6.4.2: Summary of ANOVA of Participants' Post-test Marks by Achievement Categories

		Sum of squares	df	Mean square	F	Sig
LA	Between Groups	5.231	1	5.231	.301	.584
	Within Groups	1769.760	102	17.351		
	Total	1774.990	103			
MA	Between Groups	402.381	1	402.381	22.058	$p < .001$
	Within Groups	875.619	48	18.242		
	Total	1278.000	49			

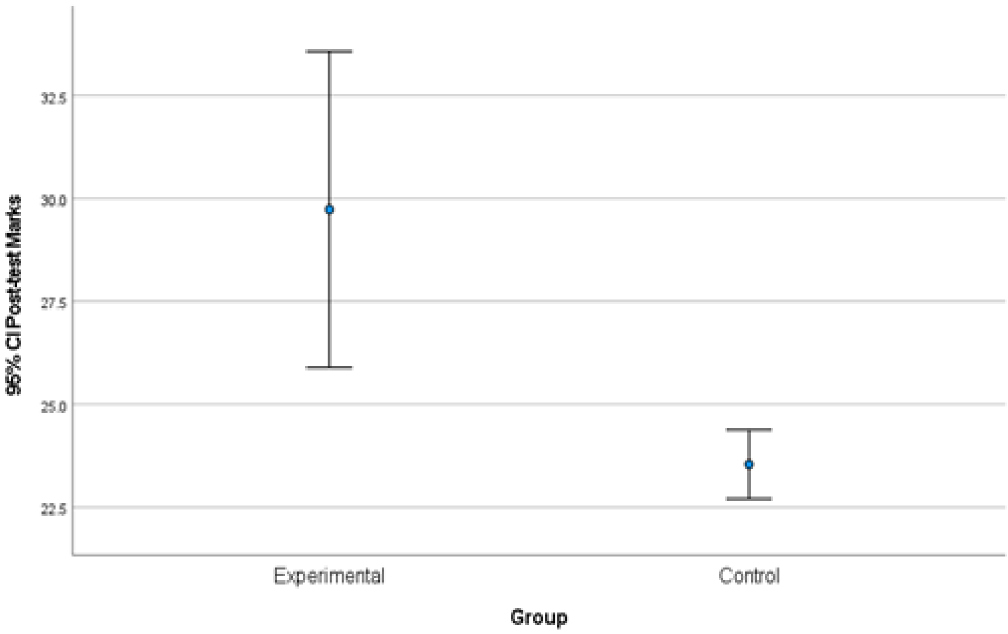
6.7.1 ANOVA for Low Achievers in the Post-test

From Table 6.4.2, the ANOVA result $\{F\ (1, 102) = .301, p = .584, p > \alpha = .05\}$ for the low achievers reveals that there is no significant statistical difference in the mean performance ($\bar{x}$ = 12.30, σ = 4.17) of the experimental group and the mean performance ($\bar{x}$ = 12.87, σ = 4.17) of the control group. Thus, there is no statistically significant difference between both groups for the low achievers.

6.7.2 ANOVA for Medium Achievers in the Post-test

The ANOVA result $\{F\ (1, 48) = 22.058, p < .001, p < \alpha = .05\}$ shows that, for medium achievers, the mean score ($\bar{x}$ = 29.73, σ = 6.93) for the experimental group is significantly higher than the mean score for the control group ($\bar{x}$ = 23.54, σ = 2.44). In other words, for the medium achievers, there is a statistically significant difference between both groups. The graph of the confidence interval error bars for the post-test marks of medium achievers of both groups is shown in Figure 6.4.2.

Figure 6.4.2: Confidence Interval Error Bars for Post-test Marks of Medium Achievers by Group



MacGregor-Fors and Payton (2013) postulate that if the sample sizes are roughly equal and the 95% confidence interval error bars do not overlap, the difference is statistically significant with p < $\alpha = .05$. In the current study, n (experimental group) = n (control group) = 119. Also, in Figure 4, the 95% confidence interval error bar for the experimental group does not overlap with the 95% confidence interval error bar for the control group. It may be safe to conclude that the mean score difference of both groups is statistically significant for the medium achievers at $p < .001$, $p < \alpha = .05$, but is statistically insignificant for the low achievers at $p = .584, p > \alpha = .05$.

6.7.3 ANOVA for High Achievers in the Post-test

Since the control group could not produce any high achievers while the experimental group recorded 70.6% (n = 84) high achievers (see Table 6.3.4 and Figure 6.4.1), the mean score ($\bar{x} = 51.25, \sigma = 6.24$) of the experimental group is statistically significant.

6.7.4 Concluding Comment on the ANOVA Test Results

On the whole, the ANOVA tests reveal that the post-test result of the experimental group is better than the post-test result of the control group. On the strength of that, the null hypothesis is hereby rejected and the conclusion drawn that the mean difference is statistically significant. This shows that the 8Ps-oriented instruction is effective. In other words, it signifies that the mathematical problem-solving performance of the study participants in the experimental group exposed to the 8Ps-based intervention is better than that of their counterparts in the control group who were traditionally instructed.

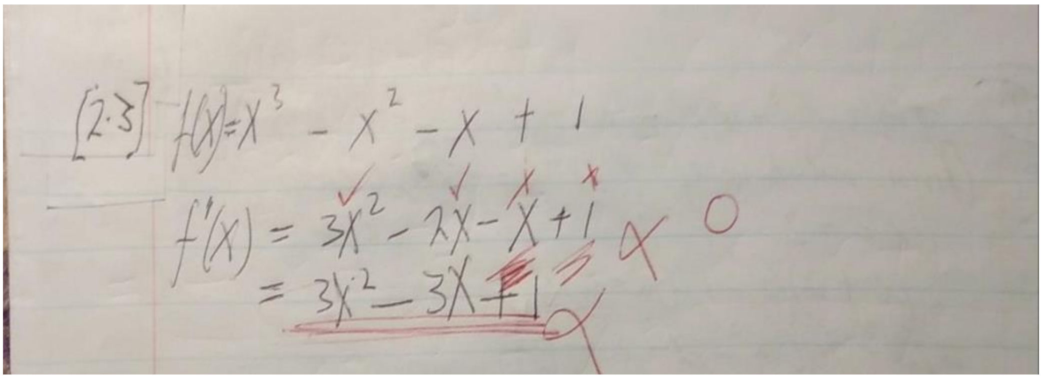
6.8 Analysis of Participants' Post-test Errors

Learners' poor mathematical problem-solving performance correlates to learner mathematical errors and misconceptions. Hence, as a means to improve learners' mathematical problem-solving performance, due attention has to be paid to the errors and misconceptions that learners make in their solutions to Mathematics problems (Chin & Pierce, 2019; Chiphambo & Mtsi, 2021; Hamukwaya & Haser, 2021; Luneta & Makonye, 2010; Makonye, 2011). In view of this, the present study identified the various forms of errors in the participants' solutions to post-test questions and classified them as Type A - E errors. It worked on only their post-test errors because, naturally, learner performance in the pre-test is expected to be poor with a lot of errors and misconceptions having not been taught the topic before the intervention. The explanation of the five major error-types noticed is as follows.

6.8.1 Type-A Error: Wrong differentiation of Given Equation

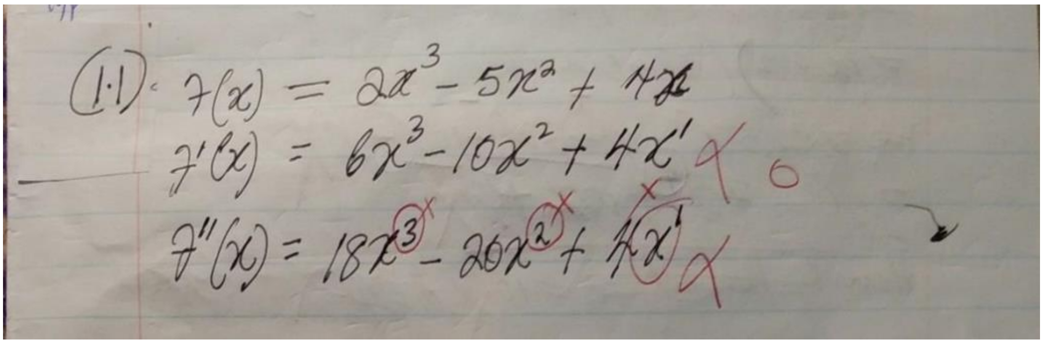
As the first step towards obtaining the coordinates of the x- and y-intercepts and also for calculating the coordinates of the turning points, the derivative of the equation given has to be obtained. Incidentally, some of the study participants did not do the differentiation correctly. Vignettes 1 and 2 (representing the cases of Type-A error committed by the participants) are excerpts from the post-test scripts of learners EX028 and CL096 respectively.

Vignette 1: Example of Wrong Differentiation by Participant EX028



From Vignette 1, participant code number 028 of the experimental group gave an incorrect derivative of $f(x)$. The participant did not remember that $f'(x)$ = x = 1 and also that the derivative of 1 or any constant is 0.

Vignette 2: Example of Wrong Differentiation by Participant CL096

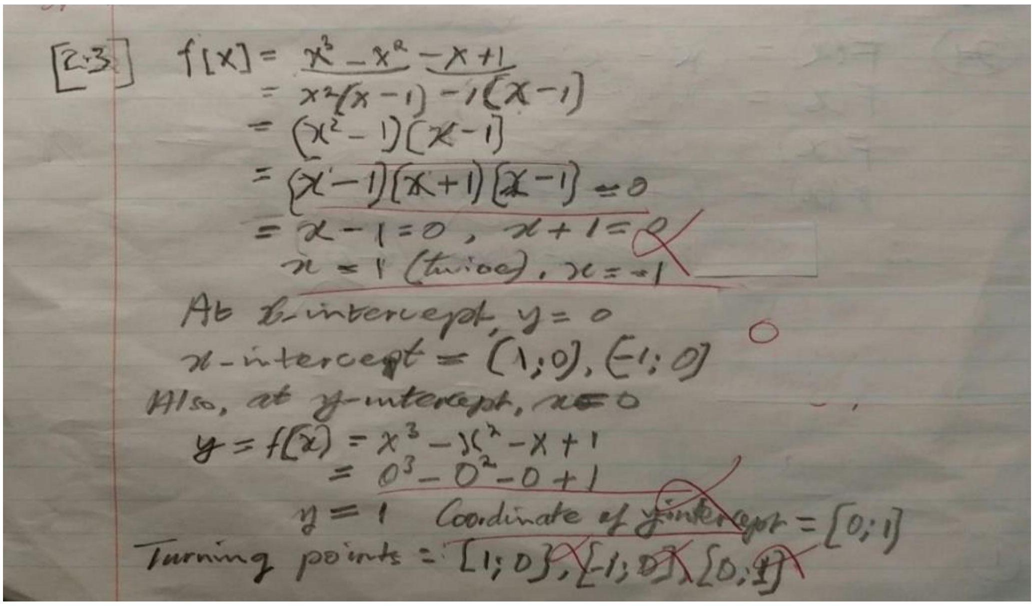


In Vignette 2, participant CL096 also gave a wrong differentiation of $f(x)$. After multiplying the exponent by the coefficient of x, the participant did not subtract 1 from the exponent of each of the three terms of the equation as required by the differentiation rule: $y = x^n$ which yields $f'(x) = nx^{n-1}$.

6.8.2 Type-B Error: Mistaking the X- and Y-intercepts for the Turning Points

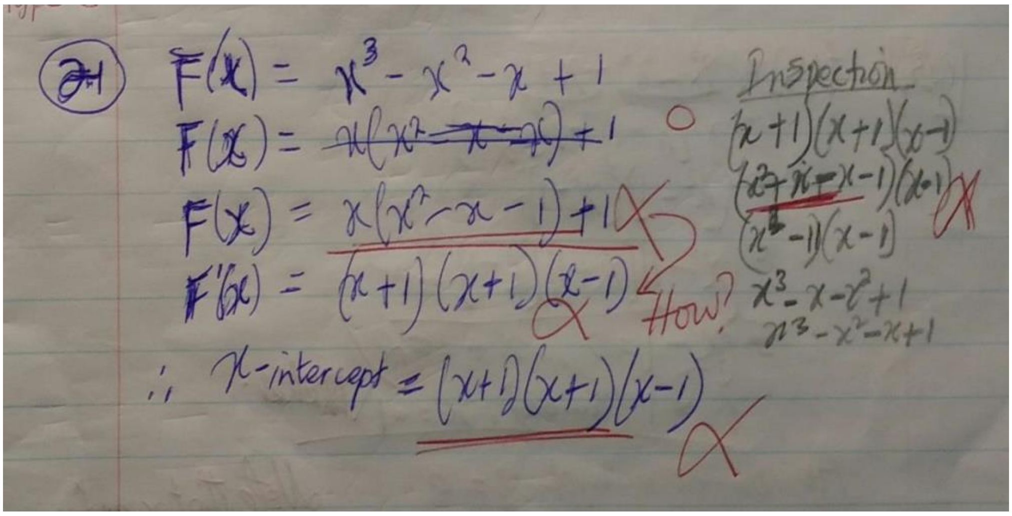
Another form of error made by some study participants was wrongfully taking x- and y-intercepts as the turning points. They failed to realise that, to obtain the stationary point(s), they would have to first differentiate the given equation; that is, find $f'(x)$ first, and thereafter set $f'(x) = 0$. Then, the x-coordinate(s) of the turning point would be obtained. The resulting x-value(s) would be substituted in $f(x)$ to obtain the y-coordinate(s) of the turning point(s). The following sample of such errors made (as shown by Vignette 3) is an excerpt from the post-test script of participant CL043.

Vignette 3: Confusing X- and Y-intercepts with the Turning Points



From Vignette 3, the participant factorised $f(x) = x^3 - x^2 - x + 1$ wrongly. He attempted to calculate x- and y-intercepts though, he even followed the right procedure for calculating the intercepts, he did not get the correct answers for the x- and y-intercepts because of the initial factorisation error made. He then erroneously concluded that the x- and y-intercepts found were the coordinates of the turning points. Those participants needed to be reminded that x- and y -intercepts are points on the x- and y-axes respectively where the graph touches x- and y-axes. A turning point, on the other hand, is a point where the graph changes sign (either from negative to positive or from positive to negative). A turning point can be maximum or minimum. In other words, the turning points of a function are stationary points.

Vignette 4: Another Confusion of X- and Y-intercepts with the Turning Points

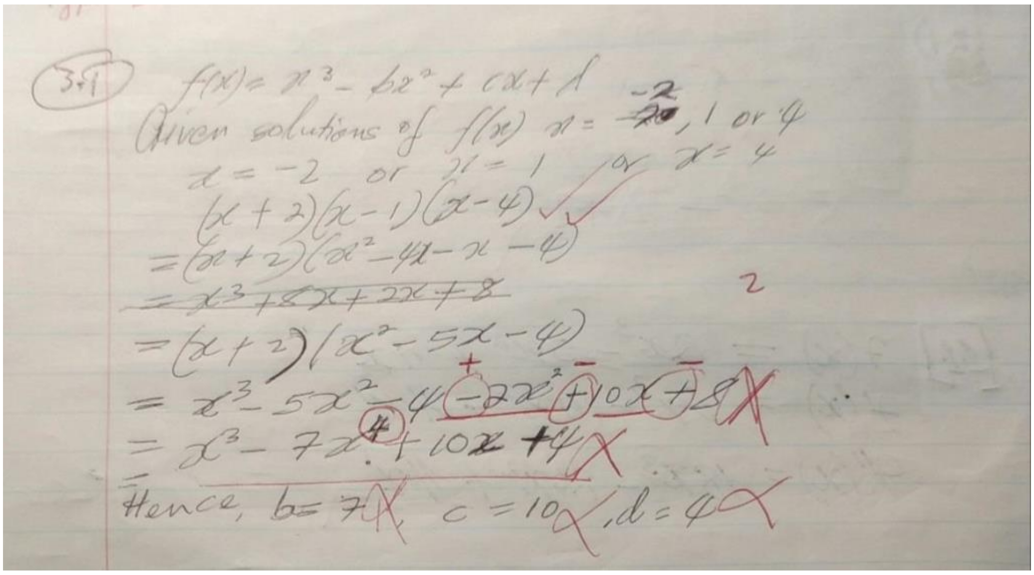


As contained in Vignette 4, question 2.1 of the post-test demanded that the participants calculate the coordinates of the turning points of the graph of $f(x)$. Like participant CL043 in Vignette 3 did, participant EX100 in Vignette 4 also calculated x-intercepts in place of the turning points required.

6.8.3 Type-C Error: Notational and Algebraic Errors

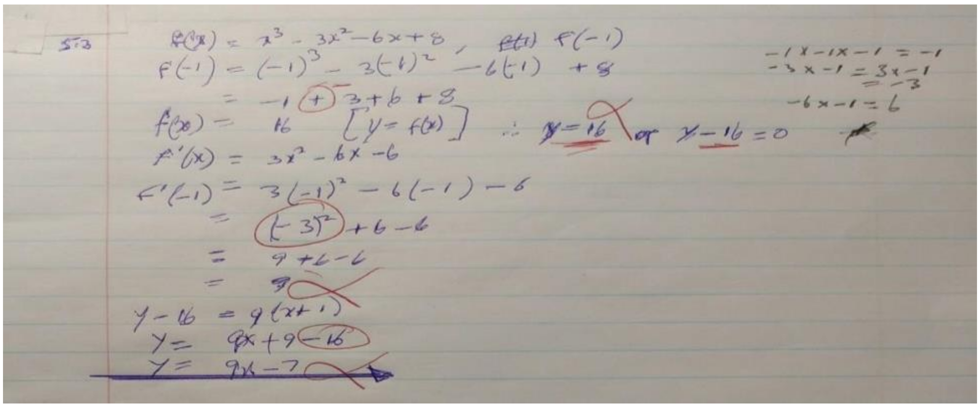
Some of the participants committed notational and algebraic errors like: incorrect substitutions; erroneous expansions; addition of unlike terms together; wrong change of signs in multiplication; misuse of formulae; faulty or omission of factorisation of the numerator or the part leading to derivative, especially one involving expansion of a cubic expression, etc. Some participants demonstrated a weak knowledge of fractions and exponential laws essential for understanding calculus. Some could not tell the difference between the function, $f(x)$, and its gradient, $f'(x)$. The most common error is their inability to know when to leave out the derivative notation.

Vignette 5: Notational and Algebraic Errors by EX126



Vignette 5 is a sample of the incorrect expansions and poor understanding of the product of negative and positive values demonstrated by the participants in the post-test. The participant started well by constituting the roots x = -2, 1 or 4 of the graph of $f(x)$ into three factors to be multiplied. He however could not carry out the expansion successfully. Hence, he did not get the correct equation to compare with the given expression $f(x) = x^3 - bx^2 + \mathrm{cx} + \mathrm{d}$ which could have made determining constants b, c and d possible. Also, in Vignette 6 below, participant CL022 made similar notational and algebraic mistakes while expanding $f(-1)$.

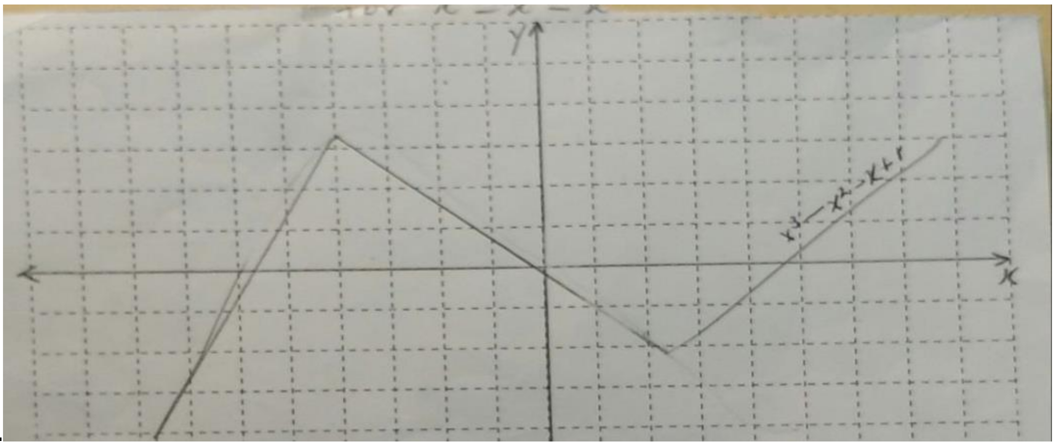
Vignette 6: Notational and Algebraic Errors by CL022



6.8.4 Type-D Error: Incorrect Graph Sketching and Indication of Stationary Points

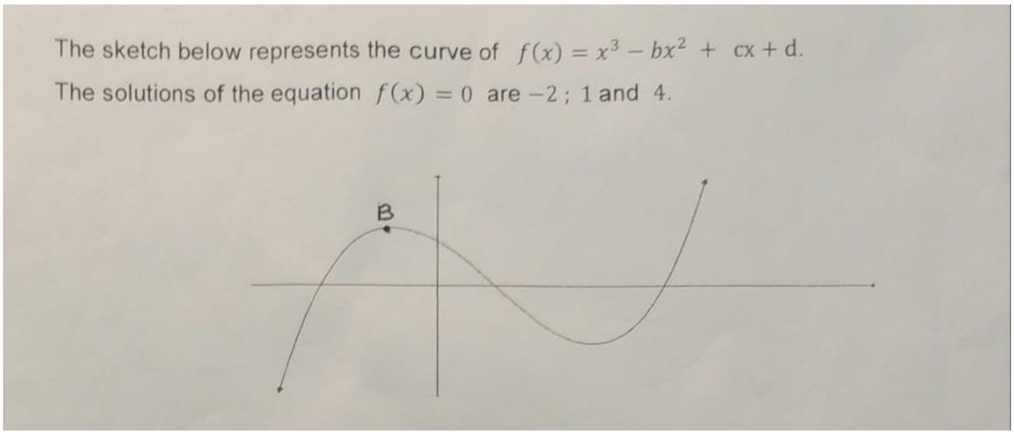
In the post-test questions 1.4, 2.3, 4.4, 4.5, 4.6 and 5.4 demanding the graphical application of knowledge of calculus, especially when a graph sketch is not given, many of the participants became confused. A number of the study participants could not properly draw the graph showing the stationary points. Some of them did not obtain the accurate values of the turning points from the graph. Drawing an appropriate graph indicating the correct values of the turning points, and x- and y-intercepts was also challenging to them. About half of the total study participants performed poorly in the higher-order thinking questions requiring interpretation of graphs. Some participants had difficulties identifying or reading from the graph the intervals for which a function is increasing or decreasing. They exhibited a shallow knowledge of the important aspects of a function such as: sketching, interpreting a given equation or graph, obtaining an equation from given information etc. Samples of the Type-D errors made by the participants are illustrated by Vignettes 7 and 8.

Vignette 7: Sketching the Graph of $f(x)$ without Considering Necessary Values



In Vignette 7, participant CL075 only drew the graph without giving due consideration to the x- and y-intercepts and the turning points. This is not acceptable as those values determine the nature of the graph.

Vignette 8: Misinterpretation of Given Graph

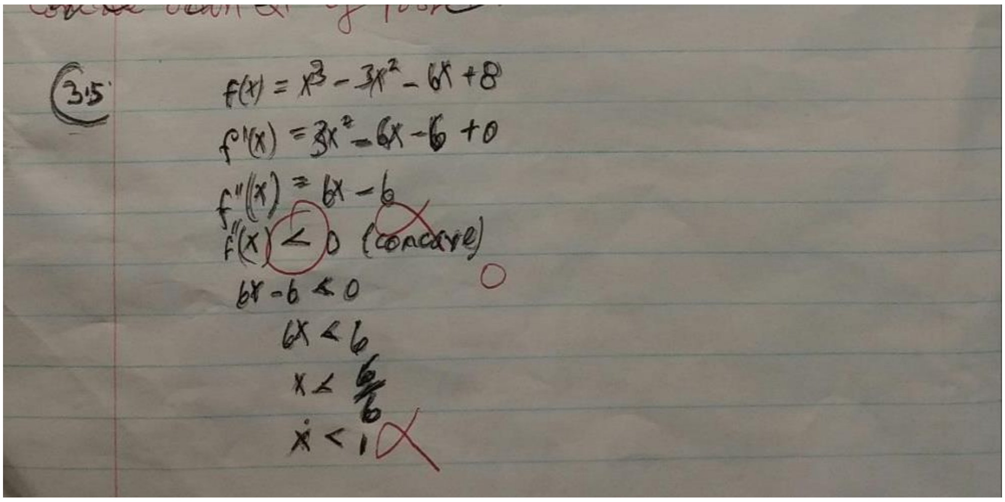

The sketch below represents the curve of $f(x) = x^3 - bx^2 + cx + d$.
The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4.

Vignette 8 presents the graph sketch (in question 3 of the pre-test and question 5 of the post-test) on which the participants were asked some questions. This warranted the participants’ abilities to give correct interpretation of the graph. Many of the participants, especially in the control group committed errors here owing to their poor interpretation of the graph. The affected participants did not understand that the given solutions of $f(x)$ are the x-intercepts of the graph (that is, the three points where the graph cuts the x-axis). Since they did not grasp the idea, they could not find the equation that could have helped them find constants b, c and d. Hence, many of them committed Type-D errors.

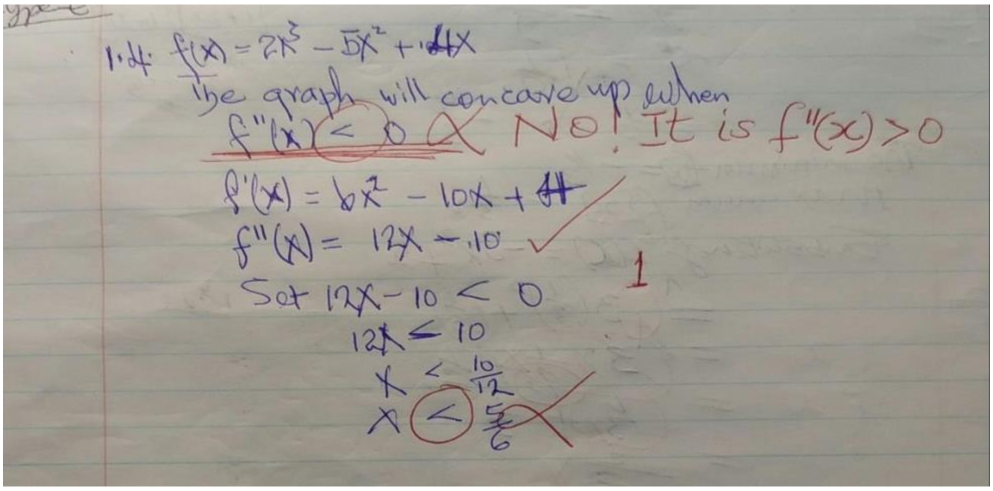
6.8.5 Type-E Error: Misunderstanding of Concavity, Minimal and Maximal Points

Some participants depicted a faulty understanding of the concept of concavity of functions. They could not clearly explain when a function is concave up or concave down and the intervals for which these happen. They did not realise that, to have a point of inflection, there must first be a change in concavity in the neighbourhood of a point. The concerned participants could not recognise that a function is concave upon an interval $f''(x) > 0$ (meaning positive, +ve) but concave down on an interval $f''(x) < 0$ (meaning negative, -ve). They got more confused when a cubic graph is given, forgetting that, for some functions, the concavity changes at the point of inflection. Vignettes 9 and 10 are examples of Type-E error made by CL005 and EX019.

Vignette 9: Faulty Idea of Concavity by CL005



Vignette 10: Faulty Idea of Concavity by EX019



As displayed by Vignettes 9 and 10, both participants in question, like some other participants, mistook the underlying principle guiding upward concavity [f″(x) > 0] for that of downward concavity [f″(x) < 0]. In Vignette 11 below, participant EX069 followed an incorrect notion of maximum point and so calculated wrongly the required value of q for which $f(x) + q$ to have a maximum point.

Vignette 11: Faulty Idea of Minimal and Maximal Points by EX069

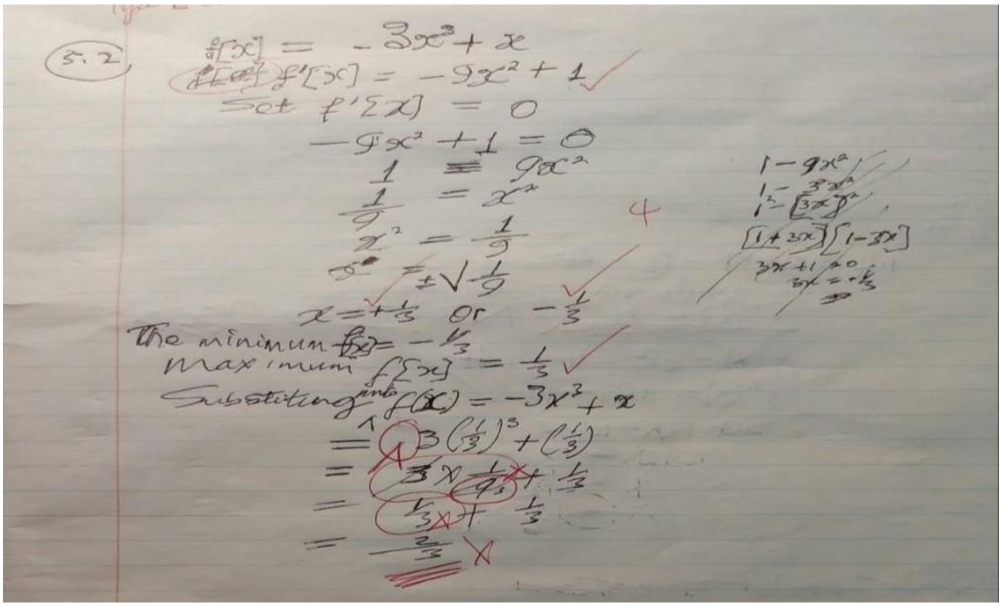


Table 6.4.3 and Figure 6.4.3 analyse the error-types and their frequencies of occurrence.

Table 6.4.3: Main Study Participants' Post-test Errors

Group	Post-test n	Frequency of Error-type				
		Type-A Error	Type-B Error	Type-C Error	Type-D Error	Type-E Error
Experimental Group	119	152	78	144	113	96
Control Group	119	452	333	361	312	223

The participants' post-test errors depicted in Table 6.4.3 are illustrated by the below clustered bar chart.

Figure 6.4.3: The Bar Chart of Participants' Post-test Errors

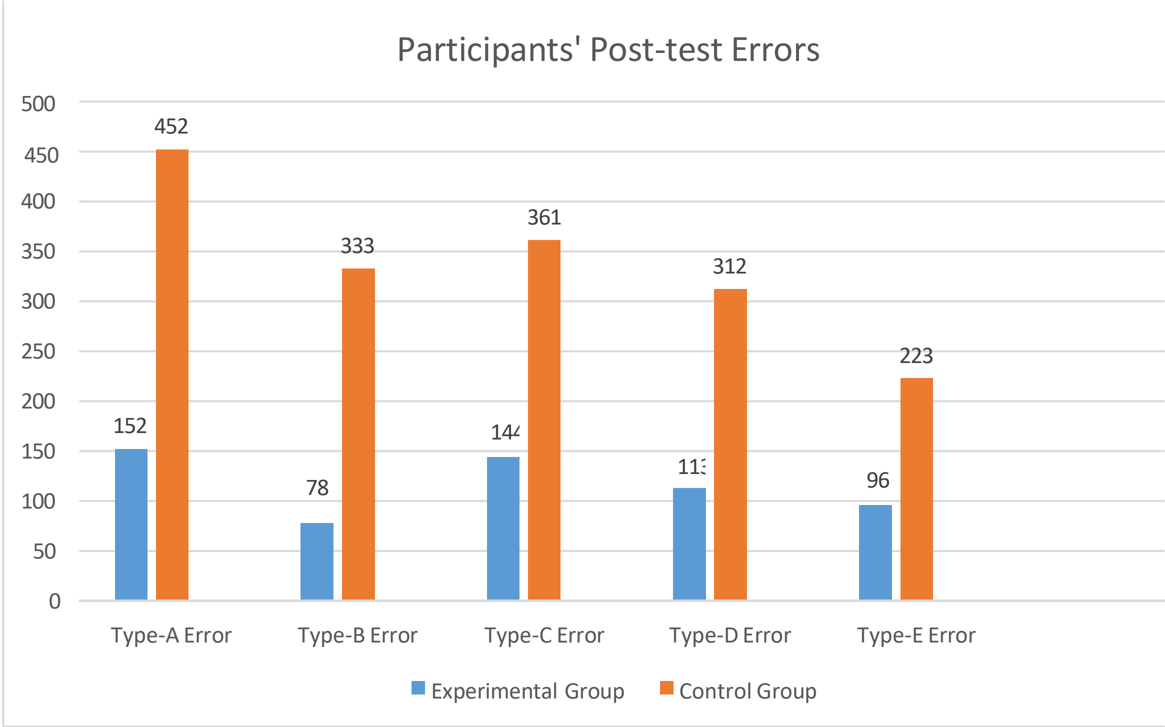


Table 6.4.3 and Figure 6.4.3 reveal that the experimental group made by far a smaller number of errors in the post-test than the control group. Altogether, the experimental group committed 583 errors while the control group committed 1781 errors. This indicates that the errors committed by participants of the control group is about three times the errors made by participants of the experimental group. This development is also an attestation that the experimental group exposed to the 8Ps-based instruction had recorded more improvement in its mathematical problem-solving skills than the control group instructed conventionally. The most common error-types committed by the participants in the post-test were types A, C and D

6.9 One-way ANOVA of Participants' Post-test Errors

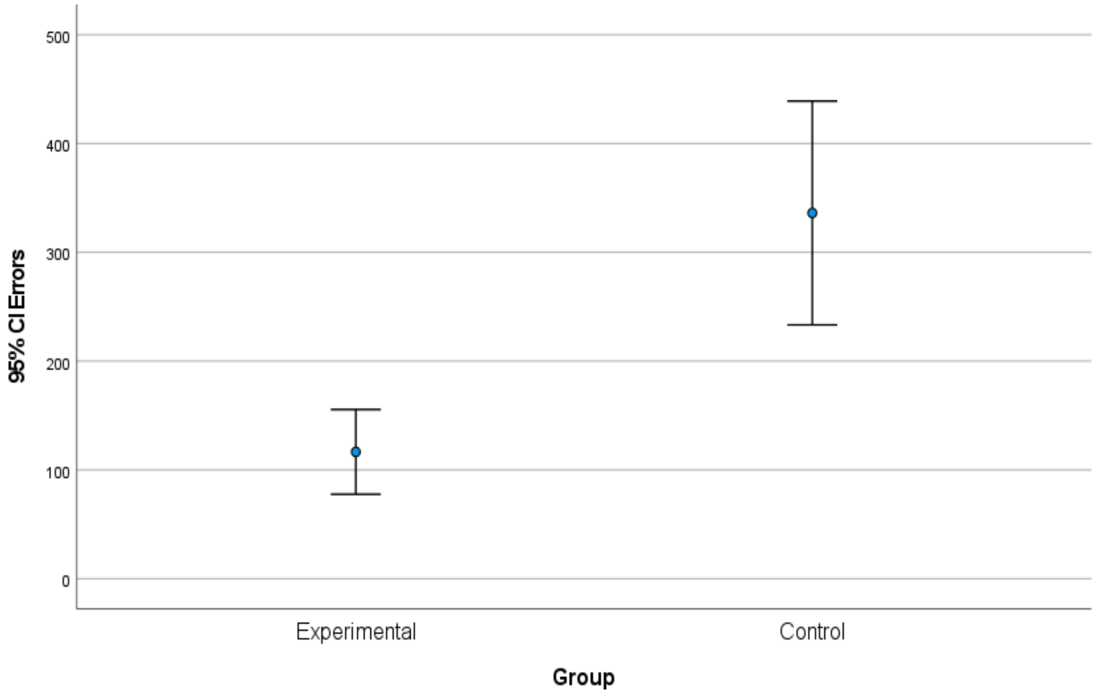
The participants' errors in the post-test were taken as the dependent variable while the participants' groups were regarded as the independent variable. One-way analysis of variances (ANOVA) was performed to determine whether the error mean committed by the experimental group was the same as the error mean recorded by the control group. The ANOVA test results for equality of error means are shown in Table 6.4.5.

Table 6.4.4: One-way ANOVA Participants' Post-test Errors

Source	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	120560.400	1	120560.400	30.749	$p < .001$
Within Groups	31366.000	8	3920.750		
Total	151926.400	9			

The ANOVA result $\{F(1,8) = 30.749, p < .001, p < \alpha = .05\}$ yields a statistically significant difference in the error means of the two groups. This points out that both groups were different in terms of their error means in the post-test. A large effect size of $\eta^2 =$ .79 was obtained. Thus, 79% of the variation in errors was accounted for by the groups. In the post-test, the experimental group had a mean error of 116.6 $(\bar{x} = 116.6,\ \sigma = 31.35)$ which was significantly lower than the mean error of control group of 336.20 $(\bar{x} = 336.20,\ \sigma = 82.81)$.This explains that, in the post-test, the experimental group had less errors than the control group. The confidence interval error bars for the results are illustrated in Figure 6.4.4 below.

Figure 6.4.4: Confidence Interval Error Bars for Post-test Errors by Group



Since the two 95% confidence interval error bars in Figure 6.4.4 do not overlap, and given the equal numbers of participants in both groups, the result $p < .001, p < \alpha = .05$ amounts to a statistically significant difference in the error means of the two groups (Subsection 6.7.2 explains this). The two non-overlapping confidence interval error bars further show that the post-test experimental group, which received the 8Ps-based treatment, had a lower mean error. This is, importantly, pointing to the effectiveness of the 8Ps instruction that the post-test experimental group received.

6.10 Reflection on the Chapter

The chapter has analysed the quantitative data obtained for the study by descriptive and inferential statistical techniques. It has begun by first validating the participants' demographics so as to determine their socio-economic status (generally believed to have an impact on the outcome of an investigation), and as well to ensure the participants' eligibility for the study. The chapter has established that the study participants of the eight schools involved were characterised by largely similar demographic variables and that their participation in the study was justifiable. On the whole, the chapter has found out that, both the descriptive and inferential statistical results obtained indicate that the post-test results were significantly higher than the pre-test results, which means that the experimental group differed significantly from the control group. That points out that the 8Ps learning method significantly improved the learners' problem-solving performance.

CHAPTER SEVEN

QUALITATIVE DATA ANALYSIS AND RESULTS

7.1 Introduction

The qualitative data gathered for this study was analysed by the qualitative data analysis method. The goal of the qualitative data analysis method is to understand a complex reality and the significance of actions in specific situations. While this analysis method is unconcerned with numerical representation, it seeks to deeply understand a given problem and provide illustrative, in-depth information for the explanations of the different aspects of the problem being analysed (Queirós, Faria & Almeida, 2017). Qualitative study is a detailed study employing face-to-face or observational methods to gather information from people while they are in their natural environments (McMillan & Schumacher, 2014). Qualitative data analysis method is interested in the universe of meanings, aspirations, motives, values, beliefs and attitudes, which corresponds to a larger space of relationships, phenomena and processes not reducible to operationalisation of variables (Maxwell, 2013). While quantitative research aims to explain, generalise and anticipate patterns through the analysis of variables, qualitative research is more concerned about comprehending and explaining the socially created reality around us. This shows that information is gathered from documents, observations and interviews, which is usually kept for analysis later (Bryman, 2016).

In the account of Flick (2018), qualitative data analysis has three general aims. The first aim is to describe a phenomenon in great detail. This phenomenon can centre on the subjective experiences of a particular person or group, or differences and similarities of several individuals or groups. The second aim is to identify the conditions on which such differences or similarities are based; that is, to provide justifications for the differences or the similarities. The third one is to develop a theory of the phenomenon being investigated by analysing scientific material. Flick (2018) further observes that qualitative data analysis follows three methods for analysing social phenomena. The first is by having subjective experiences as the focus. Data for this method are usually obtained from interviews with the participants or from documents written by the participants. The second method is about describing the making of a social situation. Data for this method often come from observation of the participants or from recording interactions with the participants. The third method delves into the implicit and unconscious aspects of a social phenomenon. Data for this method emanate from recording interactions with the participants and also from evaluating the phenomena beyond the awareness of the participants.

In the current study, the motive of using qualitative data analysis techniques was to see how the 8Ps-oriented learning method could be applied in the Mathematics classroom (research question 2). It was also adopted to find out the challenges, if any, that its use in the Mathematics classroom might encounter (research question 3). The qualitative data analysis results were basically meant to triangulate the results from the quantitative data analysis earlier obtained in chapter six. By this, a comprehensive understanding of the phenomena being investigated could be provided. To this end, the analyses of the data collected from the classroom observations, the study participants' post-test scripts scrutinised, and the interviews conducted with the selected twelve participants are presented as follows.

7.2 Analysis of Data from the Classroom Observations

The data arising from the classroom observations were subjected to thematic qualitative data analysis. Kiger and Varpio (2020) describe thematic analysis as a strong and adaptable technique for analysing qualitative data in a range of paradigmatic or epistemological perspectives. They further explain thematic analysis as a practical data analysis approach that sets out to understand thoughts, experiences or behaviour across a data-set. Their view is consistent with the idea conveyed by Braun and Clarke (2006) that thematic analysis is a qualitative method that examines a set of data for repeating patterns or themes, understanding them, and reporting them. These writers advise that, although a researcher enjoys a lot of freedom in choosing the themes to focus on, he should endeavour to recognise themes that offer significant insights that can help answer the research question(s). As a result of the flexible nature of thematic analysis, Kiger and Varpio (2020) also caution the researcher utilising it to clearly state their paradigmatic orientations and assumptions in order to make their findings and interpretations reliable.

Joffe (2011) remarks that theme analysis is ideally suited for constructivism since it explains how a social construct develops by analysing a significant quantity of data. According to him, constructivist thematic analysis seeks for more underlying, deeper themes in a set of data. Boyatzis (1998) admits that thematic analysis enables the interpretivists to achieve social creation of meaning that is expressed or presented in a manner that a representation of social facts or observations appears to occur with reliability and consistency of assessment. Since constructivism is one of the major theories establishing the current study (see section 3.2.1 - 3.3), and interpretive design is one of two research designs the current study followed (see section 4.4), the practice to explain the qualitative data for this study by thematic analysis thus aligns with the above submissions made by Joffe (2011) and Boyatzis (1998).

7.2.1 Classroom Arrangement in the Experimental Group

The researcher conducted all the intervention lessons with the learners assigned to small groups of three to five learners of mixed abilities (see subsection 4.9.2.1). The learners' desks and seats in each group were arranged in a manner that allowed them to sit face-to-face. The seating arrangement also provided the teacher appropriate classroom space to move around to monitor, guide and facilitate the activities of each group. The teacher was then able to see how each group was functioning and how every learner contributed to the mathematical problem-solving process. The classroom seating structure also made it possible for the learners to work together to accomplish the Mathematics tasks they were asked to do. It enabled them to interact, discuss, share ideas and learn from one another. By this, the learners got exposed to various perspectives of the concepts learnt since the grouping essentially made every learner realise that everyone had a vital role to play in class activities.

Group work is a widely adopted and broadly researched teaching approach which encourages learner collaboration to actualise shared learning goals and which has also proven to increase learner achievement, persistence, cooperation, motivation to learn, interdependence and positive learning attitudes. Learners working together and learning in groups achieve greatly because that is an opportunity for them to share their reasoning to one another, incorporate diverse viewpoints and to acquire communication skills as well as teamwork skills (Dhlamini, 2012; Kagan, 2014; Mammali, 2015; Wilson, Brickman & Brame, 2018).

7.2.2 Classroom Arrangement in the Control Group

In all the lessons the researcher observed in the four schools of the control group, the classroom structure took the usual conventional form whereby the learners' desks and seats were organised in rows and columns. Expectedly, this kind of seating arrangement only allowed whole-class teaching and learning. The learners could not be engaged in any form of group/pair discussions or collaborative mathematical problem-solving activities. The teachers were mostly in front of the entire learners close to the whiteboard (or at times pacing up and down the few available classroom isles) while explaining the lesson to the whole class. Once in a while, they did ask the learners a few questions and the learners too did respond to some of the questions understandable to them within the limited time the teachers allowed them.

7.2.3 Observation of Teacher Participants in Both Groups

In each of the four schools in the control group, the researcher observed four lessons (each being between 55 - 60 minutes duration). The number of lessons observed was limited to that in order to minimise the extent of interruptions of classroom activities and the structures in place in those schools. During each scheduled classroom visit, he observed both the teachers and the learners. Regarding the observation of the four schools in the treatment group, he had much time to observe the learners by virtue of his role as a participant-observer throughout the intervention period. While making use of the observation schedule designed for that purpose, he also got a notebook for taking field notes of important events as well as important learner/teacher actions and behaviour. He later put into use the field notes to enrich the data produced by the observation schedule. However, since he could not be a judge in his own matters, he arranged with the schools' four regular Mathematics teachers of the experimental schools to monitor and observe his lessons. Each of the four teachers formally observed his lessons two times and later provided him with their different, comprehensive written feedbacks.

When it was time for qualitative data analysis, the researcher read thoroughly the data emerging from the classroom observations of both groups. He took time to search for relevant patterns, differences and similarities in the data. Guided by the research questions and the research objectives, he carefully sorted and transcribed the data into common themes. The table below highlights the thematic analysis of the classroom observation data.

Table 7.1: Summary of Findings from Observation of Teacher Participants in Both Groups

Theme	**Implementation in the Experimental Group**	**Implementation in the Control Group**
Implementer of the exercise	The researcher acted as the teacher who implemented the intervention.	The learners' four regular Mathematics teachers carried out the conventional teaching.
The instructional method applied	8Ps-based instructional method	Traditional teaching method

Lesson planning and mode of delivery	Teacher's lesson plans and delivery conformed to the underpinning principles of 8Ps-based method.	Teachers' lesson plans were written and delivered in the usual traditional way.
Approach and strategies used to facilitate mathematical problem-solving process	Learners were divided into mixed-ability groups for small-group and collaborative learning.	Whole-class teaching and learning; no small-group discussions; use of routine solution rules and procedures
Integration of real-life examples and illustrations into the lessons	Adequate and appropriate use of real-life, contextualised examples and illustrations	Use of fixed examples and illustrations from textbooks and the curriculum, some of which were not directly related to learners' contextual, real-life experience.
Level of teacher-learner and learner-learner interactions during lesson	Learner-learner and learner-teacher interactions steered the learning processes.	Lessons dominated by the teachers: few learner-learner/ learner-teacher interactions.
Choice and application of teaching aids to enrich the lesson	Varied and suitable teaching resources adopted in most of the lessons	Few teaching resources used. The resources applied were only those ones specified in the textbook/ curriculum – some of which were not directly related to the learners' experience.
Mastery of the subject content	The teacher depicted mastery of the subject content as reported by the HOD and the regular Mathematics teachers.	The teachers also exhibited mastery of the subject content as observed by the researcher.

Challenges faced during mathematical problem-solving processes and the rate of overcoming them	Much time needed for the problem-solving processes Learners gradually overcame most of the challenges.	Since the traditional teaching approach lacks essential mathematical problem-solving heuristics, the teachers were challenged in this regard, despite their mastery of the learning content.
Attitudes towards teaching Mathematics by problem-solving	The teacher was positive and passionate about it.	The teachers were also positive about it as they also admitted it aids knowledge of Mathematics
Attitudes towards the instructional method applied	The teacher was passionate about the 8Ps-oriented instructional method.	The teachers' dispositions to it seemingly suggested they enjoyed using the traditional teaching method.

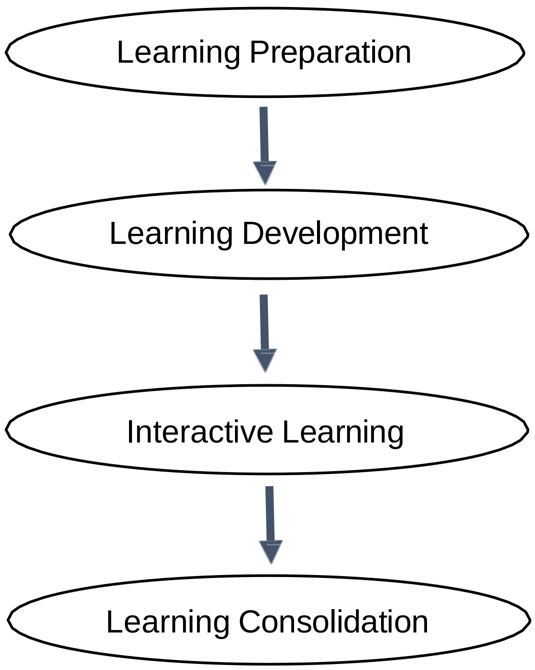
7.2.4 Concluding Comments on Observation of Teacher Participants

Table 7.1 clearly clarifies that the 8Ps instructional method that the researcher (as the intervention Mathematics teacher) applied to communicate the concept of stationary points in differential calculus to the experimental group is significantly different from the traditional methods that the learners' regular Mathematics teachers employed in the control group. The researcher's 8Ps mathematical problem-solving pedagogue followed the heuristic 8-phase principles namely: *probing, pinpointing, patterning, projecting, prioritising, processing, proving and predicting* (section 2.4.2 refers). On the other hand, the traditional teaching approach of the control group took the form of the conventional *talk-and-chalk*, textbook-dependent, whole-class, teacher-dominated instructional technique.

The table above also indicates that the researcher adopted the philosophy of the 8Ps-oriented learning method to structure his teacher's lesson plans, lesson delivery, learning-facilitating strategies, and selection of examples, illustrations and teaching aids used. However, the control group teachers adopted the conventional teaching practices to structure all of the above-mentioned variables. Another important point noted in the table is that, in the 8Ps treatment group, the teacher allowed the mathematical problem solving to be done through learners' active participation, group/pair discussions and learner-learner/learner-teacher interactions. In contrast to this, the control group teachers made use of none of these; they dominated the mathematical problem-solving processes and only involved the learners once in a while in the learning processes.

7.2.5 A Typical 8Ps-based Mathematics Lesson Plan

Figure 7.1: Schematic Representation of the 8Ps-based Lesson Plan



The traditional teaching method encourages teacher-dominated lecture method which rarely follows appropriate lesson planning. With this practice, full engagement of learners in classroom activities is impossible. Hence, a worthwhile instructional method that will lead to quality teaching and effective learning must reckon with a properly structured lesson plan (Cicek & Tok, 2022; Igbal & Siddique, 2021). In light of this, the researcher carefully planned each intervention lesson that he administered to the experimental group in the following four divisions: learning preparation, learning development, interactive learning and learning consolidation. He treated the four lesson stages as inter-related believing that they corroborate one another towards producing a meaningful and rewarding learning.

7.2.5.1 Learning Preparation

This first stage of the lesson commenced as the intervention teacher set up the learning environment in preparation for learning. As suitable for a mathematical problem-solving learning environment, the teacher divided the thirty learners that were present that day into six groups, with each group consisting of five learners of mixed abilities. For each group, he clustered some desks together, put the seats around them and made the learners sit face-to-face. The sitting arrangement was restructured that way to facilitate learner interactions and group discussions that would start afterwards. The rearrangement was also meant to provide him a reasonable space to move around the classroom to monitor the problem-solving activities in each group. Subsection 7.2.1 provides more explanation on the classroom setting of the experimental group. As part of the important operations of this initial lesson phase, the teacher pasted a big 8Ps-based wall chart on the conspicuous part of each of the four classroom walls. Each wall chart displayed the 8Ps learning model as both linear and cyclic in nature (see Appendix E2). He also gave each learner some sheets containing worked-out examples and eight flip cards bearing short information about the eight phases of the 8Ps learning model.

7.2.5.2 Learning Development

The teacher then proceeded to stage two of the lesson when he gave the lesson topic and a short introduction of it. He briefly connected the topic to a related Mathematics topic that the learners had previously learnt and stated the specific objectives of the lesson. The teacher then instructed the learners to study carefully and individually the solution steps that produced the solution to each of the two Mathematics problems solved as the worked-out examples, the strategies adopted and the explanation(s) provided for the solution steps. The teacher further instructed the learners that, all along as they would be doing this, they should, from time to time, look at the wall charts containing helpful guide on the sequence of the eight phases of the learning model and the flip cards bearing some useful information which could remind them of what each letter P (or each phase) stands for. He advised them to spend about ten minutes doing this individually first. The afore-mentioned activities that occurred in this lesson phase were for the development and initial facilitation of learning.

7.2.5.3 Interactive Learning

The actual problem solving and learning took place in this phase as the learners were afforded ample chance to exhibit and put into practice their prior knowledge and personal experience. To this end, the teacher stimulated active learner interactions and group discussions. He instructed the learners to first try to understand how the solutions to the two worked-out examples were arrived at, and that thereafter, they could start rubbing minds together on how to solve the other two similar questions assigned to them as class work. At this point, without interfering, the teacher allowed the learners to freely interact and discuss the work in their different groups. The learners thus had the opportunity to argue, comment, contribute and share ideas among themselves. He was only moving around, from one group to another, observing and analysing the learners' problem-solving behaviour. He was listening to their various contributions and monitoring each group to see that every learner was actively participating. He was checking as well whether the learners were consulting and making use of the 8Ps-oriented wall chart and the 8Ps-based flip cards as he had earlier instructed them.

This third phase of the lesson can be described as learners' interactive problem-solving stage. Although a little of teacher-learner interaction came up (as the teacher asked the groups few important questions, when necessary, as a way to offer them some guidance or as he answered some learners' questions seeking clarifications on certain problem-solving aspects), the solving of the two Mathematics problems was mainly done through learner-learner interactions and group discussions. Some of the challenging questions the teacher asked the learners (as applicable to each group) include: *What's your understanding of this problem? When you first encountered this problem, what idea came to your mind? Can you think of an alternative solution idea suitable for that? What idea or topic did you previously learn that you feel this mathematical problem relates to? Can you try to connect this mathematical problem to a situation in school, at home or in our community? How do you usually go about approaching a problem such as this at home?* In fact, this interactive learning phase is noted for its capability to stimulate and develop essential mathematical problem-solving skills in learners.

7.2.5.4 Learning Consolidation

This phase was earmarked for the learners to demonstrate their understanding of the solutions obtained. As an avenue for the learners to elaborate their thinking, justify their viewpoints and clarify any doubts that might have arisen during their interactive learning session, the teacher asked one representative from each group to explain the group's solutions to the class work. He later requested a volunteer to solve one of the questions on the whiteboard and explain to the class the process of doing so. The teacher and the learners then reached consensus on the solutions produced by the learners to the two questions of the class work. The teacher used this lesson phase to: determine and strengthen what the learners had learnt from the worked-out examples and solutions to the classwork; accord the learners the chance to showcase their skills and the knowledge newly acquired; clarify their doubts of any of the solution steps, strategies or the solutions themselves, and provide them with feedback on their performance. The teacher and the learners briefly reflected on the learning and reviewed the lesson objectives to check whether they had been accomplished. Finally, the teacher summarised the lesson, concluded it and assigned the learners another two similar questions as homework.

Table 7.2: A Synopsis of a Typical 8Ps-based Mathematics Lesson Plan

Lesson Phase	Lesson Activity (The Teacher…)
Learning Preparation (10 minutes)	Rearranged the sitting structure and made learners sit face-to-face. Divided learners into small, mixed-ability groups. Pasted the 8Ps-based wall charts on the walls. Distributed worked-out example sheets to learners. Gave eight flip cards to each learner.
Learning Development (10 minutes)	Presented the lesson topic and introduced the lesson. Connected the lesson to learners' prior knowledge. Stated the specific objectives of the lesson. Asked learners to study the worked-out examples individually. Instructed them to make use of the 8Ps-based wall charts and the 8Ps-oriented flip cards for the study.
Interactive Learning (20 minutes)	Stimulated learners' group interactions and discussions. Encouraged learners to apply their prior knowledge and personal experience to understand the two worked-out examples. Motivated them to do the same to solve the two given Mathematics problems. Guided and facilitated the groups' problem-solving processes. Moved around observing and analysing learners' problem-solving behaviour. Inspired each group with challenging questions. Assisted each group with their difficulties.

Learning Consolidation (20 minutes)	Asked a learner from each group to explain the group's solutions. Allowed a learner to solve one of the questions on the whiteboard and explain the process to the class. Reached consensus with the learners on the solutions. Provided learners with feedback on their performance. Summarised and concluded the lesson. Assigned homework to learners.

7.2.6 Observation of Learner Participants in Both Groups

The researcher had the opportunity to observe the learner participants of both groups directly in their natural settings. Allen (2017) views this kind of observation as a naturalistic one which basically involves gathering data from its naturally occurring contexts. It explains further that naturalistic observation entails observing the environments to be able to answer one's research questions. Salkind (2010) equally asserts that, in a naturalistic observation such as this, the researcher – whose presence the participants may or may not be aware of – observes and records the behaviour, events and other objects of interest. It added that naturalistic observation does not require any form of manipulation of the environment because the activities of interest are those that occur in everyday situations. Below is the analysis of data emanating from the researcher's naturalistic observations of the learner participants.

Table 7.3: Summary of Findings from Observation of Learner Participants in Both Groups

Theme	Events and Level of Occurrence in the Experimental Group	Events and Level of Occurrence in the Control Group
Mathematical problem-solving learning approach and strategies adopted	The heuristic, 8Ps-based learning approach and strategies; collaborative learning, group and pair discussions; learner-learner/learner-teacher interactions	Traditional learning method warranting the whole class to mostly listen to the teachers and asking/answering questions once in a while; use of algorithms, memorised

		formulas and rote learning
Level of learner involvement in the mathematical problem-solving processes	Learners participated fully and were opportune, encouraged and guided to discover the solution strategies and the solutions to the mathematical problems by themselves.	Learners were mostly passive listeners and receivers of the solutions to the mathematical problems. They asked and were asked few questions, and only made few contributions.
Ability to bring in previous knowledge to current learning	Learners brought in prior knowledge to knowledge construction.	Since the teachers did much of the knowledge creation, little or no learners' prior knowledge required.
Learners' response and adaptation rate to learning Mathematics by problem solving	Learners appreciated, responded positively and adapted (though gradually) to mathematical problem-solving processes.	The few responses learners were opportune to make indicated they would be willing to accept it.
Evidence of relating connecting mathematical problem solving to real-life situations	Well-demonstrated.	Little or no demonstration of this.
Observable impact of adopted instructional method on the learners'	Learners' mathematical problem-solving performance continued improving as the intervention progressed.	Little improvement observed in the learners' mathematical problem-solving performance.
Challenges faced during problem-solving process and rate of overcoming them	Much time and demanding efforts needed to discover higher-order mathematical reasoning leading to solution strategies. Learners gradually overcame most of the challenges.	Since the traditional teaching method does not encourage active learner participation, the learners were evidently challenged in this regard.

Learners’ reactions and attitudes to mathematical problem solving	Although mathematical problem solving was challenging to them especially initially, they showed interest and positive attitudes to it.	Learners demonstrated readiness and willingness to learn it.
Learners’ reactions and attitudes to usage of 8Ps problem-solving instruction	Learners welcomed and embraced it with positive attitudes	Not applicable to the learners here.

7.2.7 Concluding Comments on the Observation of Learner Participants

In summary, Table 7.3 reports that the learner participants in both groups learnt the concept of stationary points in differential calculus by two significantly different instructional methods. While the intervention group learnt it through the 8Ps-based problem-solving method, the control group learnt it by the traditional teaching pedagogy. The table also reveals that the experimental group found mathematical problem solving less challenging than the control group. It indicates that the experimental group got much more involved in the mathematical problem-solving process and depicted a more encouraging attitude to it than the control group. Importantly, the table shows that, at the different stages of the scientific enquiry, the learner participants in the experimental group considerably improved their mathematical problem-solving performance more than learners in the control group. The researcher observed this from the way and manner the learners of both groups performed in their responses and participation in class, how they tackled Mathematics tasks assigned them as class work, and their performances in homework, class work, pre-test and post-test.

7.3 Content Analysis of Participants’ Post-test Scripts

Morgan (2022) describes document analysis as a continuously underused but valuable qualitative research approach. It notices that while some researchers adopt it to complement other data sources; some use it as their only research method. While adducing reasons to why document analysis is not frequently applied in qualitative data analysis, Merriam and Tisdell (2016) maintain that several researchers ignore it

because they do not seem to be aware of how useful it can be. They assert that such researchers fail to realise that the approach is a means to explore the latent meanings in a data-set. The writers argue further that, in some circumstances, it is possible that some interviewees can be reluctant to release certain information crucial to a study because they are fearful, shy or they lack the confidence to do so, but that document analysis may be a source of obtaining such needed information. The writers also feel that researchers themselves may directly or indirectly influence the participants during observations or interviews, but that there is no way by which they can influence the pre-existing texts of a document for analysis. In a similar submission, Braun and Clarke (2013) comment that such researchers prefer focusing only the explicit meanings in a data-set instead of equally giving attention to identifying and interpreting the latent meanings within the data. They only make use of the data extracts as illustrative examples instead of attempting to analyse the extracts themselves.

In support of the conclusion drawn by Morgan (2022) in a study that carrying out a document analysis with already existing texts is a useful approach to qualitative research, the present study conducted the content analysis of the participants' post-test scripts. The motive was to investigate the definite aspects of the participants' poor mathematical problem-solving skills in order to identify particular areas needing improvement. Such specific aspects examined are: application of correct formulas and principles; translation of mathematical problems to suitable equations; representation of mathematical problems as helpful patterns; connection of current mathematical problems to prior knowledge; demonstration of logical and sequential solution-steps; reasonable justifications provided for solution-steps, and ability to correctly explain and apply the solutions obtained to similar or other mathematical problems. With this, it would then be easy to see whether the 8Ps-based intervention had any positive impact on the mathematical problem-solving performance of the experimental group.

Another important reason for performing the document analysis was to ensure that the study participants to be interviewed were selected from the different mathematical problem-solving achievement levels (high, medium and low). The current study believed that this would elicit diverse participants' opinions about mathematical problem solving and learning Mathematics through the 8Ps-based instruction. Following a similar practice in his study, Dhlamini (2012) notes that using classroom observations and document analysis as parameters for selecting the interview respondents would make the selection cut across all mathematical problem-solving achievement categories. In the present study, the content analysis conducted were performed via the below seven assessment criteria judged by the mathematical problem-solving assessment form (see Appendix E6 and subsection 4.9.4.6). Altogether, 238 post-test scripts were analysed – 119 of the experimental group participants and the other 119 of the control group participants (reason stated in subsection 6.3.2). The report of this is contained in subsection 7.3.1 - 7.3.7 below.

7.3.1 Application of Correct Formulas and Principles

All of the mathematical problems in the achievement test required that the study participants identify and apply certain applicable formulas and underlying principles before they could arrive at correct solutions. For this reason, how the participants skilfully recalled and applied important formulas and rules were evaluated and presented in Table 7.4 below. The participants' performance levels in the identified problem-solving strategies are rated High-Level Problem Solving (HL), Medium-Level Problem Solving (ML) and Low-Level Problem Solving (LL) as shown below.

Table 7.4: Level of Participants' Skilful Application of Correct Formulas and Principles

Object of Analysis	Group	High-Level PS	Medium-Level PS	Low-Level PS
Post-test scripts **n(EXP) = n(CON) =119**	Experimental	36.1% (43)	14.3% (17)	49.6% (59)
	Control	4.2% (5)	24.3% (29)	71.4% (85)

Concerning proper identification and application of suitable formulas and principles required to correctly solve the given Mathematics tasks, Table 7.4 reports that the experimental group performed better than the control group with a high-achievement level increase of 31.9%. It points out as well that half of the participants of the experimental group (49.6%) were low achievers while about three-quarters of the participants of the control group (71.4%) were low achievers in this aspect. It is only in the medium-achievement level that the control group had an increase of 10% above the experimental group. Generally, Table 7.4 indicates that the experimental group showed a better performance in this mathematical problem-solving area than the control group.

7.3.2 Translation of Mathematical Problems to Suitable Equations

The concept of stationary points in differential calculus also demands that the participants be able to interpret equations logically and also, in some cases, translate mathematical problems successfully to equations. Therefore, this problem-solving area was measured and the results obtained are given as follows.

Table 7.5: Level of Participants' Skilful Translation of Problems to Suitable Equations

Object of Analysis	Group	High-Level PS	Medium-Level PS	Low-Level PS
Post-test scripts **n(EXP) = n(CON) =119**	Experimental	37.8% (45)	16.8% (20)	45.4% (54)
	Control	5.9% (7)	29.4% (35)	64.7% (77)

According to Table 7.5, when it comes to correctly interpreting a given mathematical task and/or translating it to a suitable equation, the experimental group recorded a high-achievement level (37.8%) that is nearly seven times better than 5.9% of the control group. In the low-achievement category, the ratio of the performances of experimental and control groups stands as 5:7 respectively. However, the control group attained a medium-achievement level twice better than that of the experimental group. In general, the experimental group outperformed the control group.

7.3.3 Representation of Mathematical Problems as Helpful Patterns

Heuristic problem-solving models have commonly established that representing a mathematical problem in the form of patterns like: tables, charts, pictures, maps, diagrams, etc. can offer a problem solver a clue to the solution strategies suitable for a mathematical problem (see sections 2.4.1 and 2.4.2). Thus, the current researcher deemed it reasonable to assess how the study participants fared in this aspect too.

Table 7.6: Level of Participants' skilful Representation of Problems as Helpful Patterns

Object of Analysis	Group	High-Level PS	Medium-Level PS	Low-Level PS
Post-test scripts **n(EXP) = n(CON) =119**	Experimental	36.1% (43)	17.6% (21)	46.2% (55)
	Control	5.9% (7)	33.6% (40)	64.7% (75)

As evident in Table 7.6, in the aspect of skilfully representing assigned mathematical problems as patterns that could help solve the problems, the experimental group made 36.1% high-achievement level that is about six times that of the control group (which stands as 5.9%). The low-achievement levels of both groups follow a similar trend of ratio 5:7 observed in subsection 7.3.2. It is only the medium achievers of the control group that is about twice those of the experimental group. That notwithstanding, the experimental group generally recorded a higher performance here as well.

7.3.4 Connection of Current Mathematical Problems to Prior Knowledge

Prior knowledge plays a positive role in the construction of new knowledge (Brod, 2021; Omoniyi, 2016). Accepting this notion too, the present study assessed how effectively the study participants connected their previously learnt ideas to the learning of the concept of stationary points in differential calculus.

Table 7.7: Level of Participants' skilful Connection of Problems to Prior Knowledge

Object of Analysis	Group	High-Level PS	Medium-Level PS	Low-Level PS
Post-test scripts **n(EXP) = n(CON) =119**	Experimental	34.5% (41)	17.6% (21)	47.9% (57)
	Control	1.7% (2)	30.1% (36)	68.1% (81)

Regarding effectively relating prior knowledge to current learning, it is clear from Table 7.7 that the experimental group reached a high-achievement level (34.5%) that is twenty times higher than that of the control group. It also recorded a ratio of 12:19 (3:4, more or less) low-achievers against the control group. Although the ratio of medium achievers in the experimental and control groups is 3:5 respectively, Table 7.7 generally shows that the experimental group made a better achievement here.

7.3.5 Demonstration of Logical and Sequential Solution-steps

Like the 8Ps-based learning model, every heuristic mathematical problem-solving framework emphasises that the solution-steps logically and sequentially presented facilitates arriving at the correct solution to a mathematical task (sections 2.4.1 and 2.4.2 attest to this). Hence, there was the need to evaluate the participants' skills to do this effectively.

Table 7.8: Level of Participants' Demonstration of Logical and Sequential Solution-steps

Object of Analysis	Group	High-Level PS	Medium-Level PS	Low-Level PS
Post-test scripts **n(EXP) = n(CON) =119**	Experimental	34.5% (41)	24.5% (23)	46.2% (55)
	Control	0% (0)	26.9% (32)	73.1% (87)

As far as exhibiting evidence of successful use of logical and sequential problem-solving solution-steps to obtain solutions is concerned, Table 7.8 points out that the experimental group had 34.5% high-achievement level while the control group recorded no high achievements. It also shows that, while about half (46.2%) of the total participants in the experimental group were low achievers, about three-quarters (73.1%) of the total participants in the control group ranked as low achievers. Despite the ratio of about 3:4 medium achievers in favour of the control group, Table 7.8, on the average, shows that the experimental group made a better achievement in the use of this problem-solving strategy.

7.3.6 Reasonable Justifications Provided for Solution-steps

As explained in section 2.4.1 to 2.4.2, the process of obtaining an answer to a mathematical problem requires a problem solver to provide justifiable reasons for the solution-moves made. In this regard, Table 7.9 below depicts that the experimental group made high- and medium-achievement levels of 37% and 16.8% respectively, while the control group could only make a medium achievement of 37% but no high achievement. The groups' low-achievement levels produced by the table also indicate that for every five low achievers in the experimental group, there were seven low achievers in the control group. In all, the experimental group achieved more in this area than the control group.

Table 7.9: Level of Participants' Reasonable Justifications Provided for Solution-steps

Object of Analysis	Group	High-Level PS	Medium-Level PS	Low-Level PS
Post-test scripts **n(EXP) = n(CON) =119**	Experimental	37% (44)	16.8% (20)	41.2 % (55)
	Control	0% (0)	37% (44)	63% (75)

7.3.7 Ability to Explain and Apply the Solutions Obtained

Most of the heuristic problem-solving models expect that, after obtaining the solution to a mathematical problem, a problem solver should be able to explain and apply the solution to similar other problems and even real-life problems (see sections 2.4.1 and 2.4.2). Being of the same view, the current study also measured how the participants performed in this respect.

Table 7.10: Level of Participants' Abilities to Explain and Apply the Solutions Obtained

Object of Analysis	Group	High-Level PS	Medium-Level PS	Low-Level PS
Post-test scripts **n(EXP) = n(CON) =119**	Experimental	34.5% (41)	17.6% (21)	50.4% (60)
	Control	0% (0)	31.9% (38)	68.9% (81)

In the area of producing skilful explanations or justifications for solutions to the given mathematical problems, Table 7.10 reports that the experimental group attained 34.5% high-achievement level but the control group could not make any high achievement. The table further gives a ratio of 5:7 as the low achievers in the experimental and control groups. In spite of the fact that the number of medium achievers of the control group doubled that of the experimental group here, the experimental group is, in general, found better in the aspect of skilfully explaining and applying the solutions than the control group.

7.3.8 Concluding Comments on the Content Analysis Conducted

In general, the results of the content analysis of the participants' post-test scripts via the above seven assessment criteria as themes reveal that the experimental group exhibited a better use of mathematical problem-solving skills than the control group.

7.4 Analysis of Data from the Semi-structured Interviews

Thematic qualitative data analysis method was also employed to analyse the data from the semi-structured interviews conducted with the twelve selected study participants. As guided by Yin (2016), in reporting the analysis of the interview data for the current study, the researcher took the five-step process that guarantees a reliable thematic analysis. Before beginning to analyse the interview data, the researcher first did the compilation of the database by arranging the interview data. While doing this, he organised and prioritised the data, transcribed them verbatim using the computer and familiarised himself with the data by reading through them severally and thoroughly. He did this to have a proper understanding of the participants' views concerning their classroom experiences during the enquiry.

Thereafter, he moved to the second step, which is disassembling the data. At this point, the researcher broke down the compiled data into pieces, categorised and coded them. He did the disassembling repeatedly so as to refine the codes. He then proceeded to the third stage – reassembling the data. He repeatedly rearranged and recombined the disassembled pieces of data into categories to form common themes of interest for the analysis. While identifying the common patterns in the data-set to establish the themes, he pinpointed the errors, gaps and misconceptions. In the 5th phase meant for interpreting the data, he reviewed, recompiled, defined, refined and labelled the data for use in the report of the interview analysis. Maguire and Delahunt (2017) maintain that themes should be definite and coherent. They declare that in reviewing and establishing the themes for data analysis, the researcher should be able to provide answers to questions such as: are the themes logical? Do the data inform or match the themes? Are the themes overlapping or separate? Are there other themes that can be brought out of the data? Etc. The interview reports are analysed thematically below.

Table 7.11: Categorisation of Interview Questions into Themes

Theme Number	Theme	Questions per Theme
Theme 1	Understanding of stationary points in differential calculus	1; 2
Theme 2	Understanding of mathematical problem solving	3; 4; 5; 6
Theme 3	Notions about the problem-solving approach and strategies adopted during the lessons	7; 8; 9
Theme 4	Application of 8Ps-based instructional method in teaching and learning Mathematics	10; 11 12; 13
Theme 5	Possible challenges that application of 8Ps Learning Model may encounter	14
Theme 6	Possible attitudes to the use of 8Ps problem-solving instruction in Mathematics lessons	15; 16

Recall: *EX-L* means learner in the experimental group; *EX-T* means teacher in the experimental group; *CL-L* is learner in the control group and *CL-T* refers to teacher in the control group as earlier indicated on pages 146, 147 and 181.

7.4.1 Theme 1: Understanding of Stationary Points in Differential Calculus

Researcher: Asked the interviewees questions 1 and 2 below to measure their levels of understanding of the concept of stationary points in differential calculus.

1. *What is your understanding of differential calculus?*
2. *What does the concept of stationary points in differential calculus mean?*

The interchange below, quoted verbatim, represented interviewees' responses.

EX-L001: Differential calculus is derivative of a function. The stationary points are obtained when the derivative of a function is zero.

EX-L002: Differential calculus is f prime of x. Stationary points are minimum point, maximum point and point of inflection. These points are the same as $\bar{x}$ and y-

intercepts. We must solve the given equation to get the $\bar{x}$-intercepts and then substitute the $\bar{x}$-values in the equation to get the y-intercepts.

<u>*EX-L003:*</u> *It is called differentiation and its formula is f'(x) = $nx^{n\ +1}$. To obtain stationary points, we first differentiate the equation.*

<u>*EX-L004:*</u> *Differential calculus means* $\frac{dy}{dx}$*. At stationary points,* $\frac{dy}{dx} = 0$.

<u>*CL-L001:*</u> *Differential calculus is differentiation of a function. When we want to differentiate a function, we take the exponent to multiply the coefficient and then add 1 to the coefficient. f' (x) = $nx^{n\ +1}$. When we solve the value of f prime of x, we will get the stationary points.*

<u>*CL-L002:*</u> *To differentiate a function, we use the constant in the function to multiply the exponent and later subtract 1 from the function. The derivative of a variable like x is 0 and derivative of a constant is 1. Stationary points are three. They are minimum point, maximum point and point of inflection.*

Interviewees CL-L003 and CL-L004 did not respond to questions 1 and 2.

Drawing on the various responses above, the experimental group clearly demonstrates a better understanding of differential calculus and the concept of stationary points than the control group. Generally, the responses reveal that both groups have a limited understanding of the concept. This explains the reasons for some of the difficulties the learner participants had in the classroom while solving the mathematical problems given them as class work and the achievement tests. Below are the responses to the above two questions by the four teachers interviewed.

<u>*EX-T001:*</u> *Differential calculus, also called differentiation, refers to the derivative of a function, f* (x). It is symbolised as *f'* (x) or $\frac{dy}{dx}$*, and calculated by the formula f'* (x) = $nx^{n\ -1}$. Essentially, *differential calculus is all about how a change in one variable brings about changes in other variables. Stationary points are of three types:*

maximum point, minimum point and point of inflexion. These are points on the graph where the gradient must be equal to zero.

EX-T002: *Differential calculus, otherwise named differentiation or derivative and represented by $\frac{dy}{dx}$ or f′ (x) is given by the formula f′ (x) = nx^{n-1}. Stationary points are points on the graph where the tangent to the graph is horizontal to the x-axis. A stationary point can be minimum, maximum or a point of inflection.*

CL-T001: *Differential calculus deals with the rates of change of quantities. It is as well called derivative or differentiation. The symbol for it is $\frac{\Delta y}{\Delta x}$ or $\frac{dy}{dx}$ or $f'(x)$. Given a function, f(x), f′ (x) = nx^{n-1}. Stationary points are points at which the tangent to the graph is parallel to the x-axis. This implies that that, at stationary points, $f'(x) = 0$.*

CL-T002: *Differential calculus means finding the derivative of a function. Hence, the derivative of a function f (x) is obtained as f′ (x) = nx^{n-1}. Stationary points are the turning points of a function. These stationary points are maximum point, minimum point and point of inflection. To obtain the stationary points, we must first differentiate the function and set $f'(x)$ as 0. Then, we have to solve the equation to obtain the x-coordinate(s) of the turning point(s). We will later substitute the x-value(s) got in $f(x)$ to obtain the y-coordinate(s) of the turning points.*

The discussions with the four teachers imply that all of them possessed a sound knowledge of differential calculus and the concept of stationary points in differential calculus. The discussions particularly imply that the four control group teachers had considerable content knowledge to communicate the Mathematics concept to the learners as expected of them without putting their learners at a disadvantage, in spite of the fact that they applied the conventional teaching method. This lends support to the researcher's position in section 4.6.2 that all the teachers involved in the investigation were comparable and suitable to participate in the study.

7.4.2 Theme 2: Understanding of Mathematical Problem Solving

Researcher: Asked the interviewees questions 3, 4, 5 and 6 below to probe their levels of knowledge of mathematical problem solving.

3. *Can you briefly explain problem solving in Mathematics?*
4. *Do you feel problem solving is useful in Mathematics?*
5. *What strategies do you currently use in solving Mathematics questions*?
6. *How can learners acquire useful mathematical problem-solving skills in the classroom?*

The replies supplied by the learners to the questions are first presented below.

EX-L001: I can describe problem solving in Mathematics as a step-by-step method that makes us think deeply and very well when solving Mathematics questions in the class. Like the one you have been using to teach us Maths for some weeks now.

I won't lie to you Teacher; the method is super and wonderful. Although it has many steps to follow and it requires too much brainwork, I personally find it easy and clear. Teacher, I am happy to tell you that my interest in Mathematics is now increasing with the new method. Thank you, Mister.

Before, we were not learning the subject in groups. We did not interact or discuss in groups to contribute or share ideas in class. We were mostly listening to our Mathematics teacher and watching him solve the questions. But we did class work and homework most times.

I am sure if we can continue following your method, our mathematical problem-solving skills will improve and we will understand Mathematics better.

EX-L002: Wow, I guess you are referring to the method you used to teach us Maths in the past weeks (the learner spoke in excitement)*! It is the use of some sequential* *steps or strategies to obtain the solution to a Mathematics question. It also requires giving logical reasons for all the solution steps taken.*

It is a good method that is easy to follow. I quite appreciate it.

Before you came to our school, learners' seats and desks were usually arranged row by row, not brought face-to-face together to form different groups. No group discussions. Whenever the teacher asked us a question, a learner who understood it would raise up their hand and the teacher would call them to give the answer.

Meneer, if our Mathematics teacher can also start putting us into groups and allow us to learn in our various groups, I believe our mathematical problem-solving skills will develop.

<u>CL-L001:</u> We have never been taught problem-solving in Mathematics. But I think that's the idea you shared briefly with us. From your explanation and the information paper you gave us, I can say it is all about thinking very well to discover some strategies to apply to solve Mathematics questions.

Since it is a method that will allow learners to participate fully and contribute ideas in class, it must be a useful method.

But we don't apply that method in my class to learn Mathematics. After all, you saw it those times you came to my class that my Mathematics teacher often took the lead while we followed him. We listen to him. We watch him. Whenever he wants us to contribute ideas during a lesson, he asks us questions.

If we as Mathematics learners do our class work and homework regularly, I believe our mathematical problem-solving skills will increase.

Let me be sincere with you, Meneer, I don't find Mathematics easy. I can't actually say why I don't do well in the subject despite the effort I put in. In fairness to my Maths teacher, he tries his best. But I want to understand Mathematics. I like the subject.

With the little you explained to us about the method your research centres on, I think I am beginning to like the method. Maybe we can try it.

<u>CL-L002:</u> I am not sure my Mathematics teacher has taught me anything about mathematical problem solving. For that reason, please, permit me not to worry myself about that for now. I guess that is the method you briefed us about.

Respondents EX-L003, EX-L004, CL-L003 and CL-L004 also expressed similar views about questions 3, 4, 5 and 6 which are already captured in the above interview scripts. The interview scripts signify that learner participants in the experimental group had gained a fair understanding of mathematical problem solving during the intervention. The scripts also show that mathematical problem-solving idea was new to the learner participants in the control group. The little they could say about it was got from the brief explanation about the concept that the researcher earlier provided them. Both groups however admitted that learning Mathematics through problem solving can equip Mathematics learners with worthwhile problem-solving skills. The teachers as well answered questions 3, 4, 5 and 6. Their responses are as follows:

<u>EX-T001:</u> Mathematical problem-solving is a method of instruction that demands complex intellectual activity. It has to do with a mental process of thinking; in fact, it calls for a lot of reasoning and remembering.

Actually, I have been hearing about it for some time now. I had read a little about it before this research time. But sincerely, I have not tried using it for my Mathematics lessons. I am used to our traditional teaching method. But then, I do give a good introduction to each of my lessons by referring to the previous related lesson. I solve a mixture of simple and complex questions as class examples with the learners also contributing, not much or often though, truth must be told. I allow my learners to participate as much as the lesson time permits us. I give them challenging class work and homework too. I do whole-class teaching; I hardly use group work or group discussion anyway.

From my observation of the way you applied the 8Ps problem-solving method to teach the topic, there is no doubt that mathematical problem-solving is important and can really be helpful.

Sincerely, if learners are paired or grouped in the classroom, and are given some time for interactions and discussions while the teacher supports and monitors their activities the way you demonstrated that in your lessons, learners will develop useful mathematical problem-solving skills.

<u>*EX-T002:*</u> *Mathematical problem-solving technique follows a step-by-step approach and also gives logical reasons for all the solution-steps taken. The learning method requires that the mathematical tasks to be given to learners to solve must be challenging, non-routine ones, such that can make learners go through some mathematical thinking. They should not be simple, routine questions learners can easily solve.*

To be candid, I am aware of the existence of problem solving as an instructional Mathematics method. On two or three occasions, I had attended some workshops on the essence of problem solving in Mathematics in the past. However, I have not been using it. I still use the conventional instructional method.

Now, with some of your lessons I was privileged to observe, I think I now know and appreciate it better. Though using mathematical problem-solving techniques can be very demanding for both the teacher and the learners, I accept that problem solving is important to Mathematics. If we Mathematics teachers can really take pains to use it effectively, it can improve learners' understanding of Mathematics.

Mathematical problem-solving method can help learners acquire useful problem-solving skills if teachers allow learners to participate actively in the learning process. Just like you did in some of your lessons that I observed, dividing the learners into small groups for group discussions, interactions and collaborative learning will go a *long way to develop learners' mathematical problem-solving skills.*

<u>CL-T001:</u> *I must say mathematical problem-solving is a new idea. Not really new because I had heard and read about it at one time or another. But I can still call it new because, as far as I know, neither I nor any of my fellow Mathematics teacher friends apply the method to teach learners. Thank you for the brief orientation to the method that you gave us. With that, and the one-page information leaflet summarising the ideas of 8Ps learning method, I think I have some useful information on it. Mathematical problem solving is a method of learning Mathematics which advocates that learners should be allowed to discover the solution strategies and the solutions to Mathematical tasks by themselves while the teacher guides and encourages them. The teacher isn't expected to do it for them.*

In actual fact, I don't use the mathematical problem-solving method to teach my learners. I follow the normal, whole-class teaching method. But I try as much as I can within the limited time and available resources to encourage the learners to memorise required formulas and principles, and also to take time to do some individual Mathematical brainstorming for relevant solution ideas to use for solving a given mathematical task.

Though not easy to use, problem solving is essential for the learning of Mathematics. While learners are opportune to solve Mathematics problems on their own through group interactions and group discussions under the supervision of the teacher, learners will acquire helpful problem-solving skills.

<u>CL-T002:</u> *I think I have heard about this instructional method. But I don't use it, and I am yet to see a fellow Mathematics teacher around me adopt it either. From the little I knew about the method and the brief orientation you gave us, I should be able to talk briefly about it. Mathematical problem-solving is not in support of just applying the rules, principles and formulas to solve mathematical problems. It calls for the use of a sequence of certain mathematical skills and strategies. Also, it does not support the teacher dominating the learning process.*

Incidentally, we are just learning to possibly use the method in the Mathematics classroom. Based on a 3-day seminar on it that I attended sometime last year, and the brief orientation you gave us about the instructional method, I can say mathematical problem solving is beneficial and may enhance learners' understanding of Mathematics.

I assign my learners regular class work and homework. I don't forget to do corrections to the activities given to them. I also make sure I check and mark their work, though time does not permit me to do that regularly.

According to the short information you gave us, mathematical problem-solving method promotes dividing learners into small groups in the classroom for learning. It is purely learner-centred because the learners have to participate actively in the classroom activities by way of group discussions, learner-learner and learner-teacher interactions. It requires that learning be contextualised and related to real life. The method also involves assigning challenging, well-structured, non-routine Mathematics questions to learners and not some common questions which solution techniques and solutions learners can easily predict or obtain. Then, as not easily practicable as that method seems, I quite agree with you that learners can acquire useful problem-solving skills through it.

It emerged from the above deliberations that the teachers were aware there had been calls for the use of problem solving in teaching and learning Mathematics by stakeholders in Mathematics Education. It can also be inferred that the teachers also had some ideas about mathematical problem solving, although they had not been adopting it in their Mathematics lessons. Despite believing that learning Mathematics by problem-solving is demanding, all of them considered it helpful to the teaching and learning of Mathematics. They stated particularly that learners can acquire useful mathematical problem-solving skills if divided into small groups and are allowed to interact, discuss, share ideas and learn collaboratively in their various small groups.

7.4.3 Theme 3: Notions about the Problem-solving Approach and Strategies Adopted during the Lessons

<u>Researcher:</u> Asked the interviewees questions 7, 8 and 9 below to ascertain the kind of impressions the study participants had of the problem-solving approach and strategies adopted during the lessons in the course of the investigation.

7. *What instructional method did you use to teach/learn the concept of stationary points in differential calculus during this enquiry?*
8. *How did learners react and respond to the instructional method used in your class?*
9. *What do you feel about the 8Ps instructional method as a Mathematics teacher/learner?*

<u>*EX-L001:*</u> *We used the 8Ps learning method to learn the concept in my class.*

We are really happy with the method. Though it makes us think deeply before we can get solutions, it is worth it after all. The fact that the method expects us to show all the steps for our calculations makes the topic clear and understandable to us.

My mates and I love the method. We will not like it to end with your research. We must continue using it. After finishing your research, I know you will leave us because that's the way it does happen. Then, I wish my Mathematics teacher could continue adopting this new method you introduced to us.

<u>*EX-L003:*</u> *My class learnt the concept of stationary points in differential calculus by mathematical problem-solving method.*

We all love the instructional method. I could see that every learner in my class enjoyed it. Even after school hours, some of us did wait behind to discuss and learn together in groups. At home too, some of us still follow the small-group method of sharing ideas *and learning together.*

The idea is working and helping. We have to keep it!

<u>*EX-L004:*</u> *My class followed the principles of the 8Ps instructional method. We were divided into small groups. Each group had intelligent and slow learners brought together. So, the slow learners were learning from fast learners. We interacted, discussed, contributed and shared ideas, and learnt together in groups. All of us had the chance to participate in class activities. No longer the case of the teacher doing everything and spoon-feeding us. The way you guided and assisted us to think and discover solution ideas by ourselves is really okay.*

I love the method. All my mates are happy about it, every learner says so. Even those who were not showing interest in Mathematics before are now learning happily with us. We appreciate your effort, Mister.

I want the method. All of us want it to continue. The new method has come to stay! Sir, thank you.

<u>*CL-L002:*</u> *I don't know of any other method apart from the usual one my Mathematics teachers have been using since I got to this school.*

Learners gave mixed reactions to the method as they often do. I feel most of the learners did not like the method as they were always complaining they didn't actually understand the concept of stationary points in differential calculus.

I myself only partially like the method because I don't fully understand the topic.

<u>*CL-L003:*</u> *We followed the normal method that we always use to learn the concept. The teacher explained the topic. We listened to him. He solved some questions as class examples. He gave us some questions as class work, which we did. He corrected and marked our work. Then, he gave us some questions as homework which we must do and present in the next day's lesson. The same boring method, always!*

About half of the class did not seem to enjoy the method as they always frowned their faces during Mathematics lessons. This implied they didn't understand the topic through that method.

I myself also find the method boring and uninteresting because it does not allow learners to participate fully, interact, discuss in groups and share ideas like the method you hinted us about.

<u>CL-L004:</u> My Maths teacher did not change his method. He used his usual teaching method. His method did not involve some of the ideas of the new method you explained to us, like arranging the learners in groups to interact, discuss and share ideas on the Mathematics topic right in the classroom. He did teach all of us together as a class of one group.

Many of my classmates do not find the method okay because only few of us understand and perform fairly in Mathematics.

I too don't consider the method as the best one because my performance in Mathematics is only average or at times slightly below average.

EX-L002 and CL-L001 gave responses similar to the ones already captured above. The researcher's exchange with the learners suggests that the learner participants in the control group did not appreciate the traditional teaching approach that their teachers applied in presenting the topic to them. They blamed their inability to properly understand the concept of stationary points in differential calculus and Mathematics in general largely on the traditional approach. They therefore expressed their wish for a new method like the one the researcher briefed them about. The experimental group, on the other hand, were quite pleased with the 8Ps-oriented learning method introduced to them. They openly declared they would want to continue learning their other Mathematics topics by the method after the research might have ended. Next is the dialogue with the selected teacher participants.

CL-T001: *I employed the traditional teaching method.*

The learners had no other option: they had to learn to cope with the method since I have not introduced any other method to them. I must say I noticed some of them apparently did not enjoy it, much as I tried to simplify the topic via the method.

Yes, varying methods of instruction aids learning. More other methods can come in, I mean learner-centred methods, mathematical problem-solving methods like the one your research is introducing.

CL-T002: *I followed the traditional pedagogue.*

It's like learners showed some cold attitudes to it. They complained the method did not make them understand the topic.

I feel we can bring in constructivist learning methods; yes, mathematical problem-solving method such as the one you are working on.

The teacher participants in the experimental group were not asked to answer questions 7, 8 and 9 because their role in the investigation did not include communicating the Mathematics concept to the learners. Based on the dialogue above, teachers of the control group confirmed that they applied the traditional teaching method in their Mathematics lessons, just as the researcher observed during his classroom visits. They stated that they could notice that their learners were not excited about the conventional teaching method. They spoke further that the learners' negative body languages and the dull looks on most of their faces could be interpreted for a kind of longing for another method that could boost their understanding of Mathematics. Although, the teachers observed that applying mathematical problem-solving method would really be demanding, they declared that they themselves were open to any suitable learner-centred instructional method like the 8Ps learning approach that could improve learners' achievements in the subject.

7.4.4 Theme 4: Application of 8Ps Instructional Method in Teaching and Learning Mathematics

In the following interview scripts, the researcher posed questions 10, 11, 12 and 13 before the interviewees purposefully to find out whether they could support the possible incorporation of the 8Ps-based instruction in Mathematics lessons.

<u>Researcher:</u> Are you aware of the 8Ps mathematical problem-solving method?

*<u>All Respondents</u>: (*Answered separately) Yes.

<u>Researcher:</u> Can the method improve learners' mathematical problem-solving performance?

*<u>All Respondents</u>: (*Replied separately) Yes.

<u>Researcher:</u> Can you recommend the use of 8Ps-based instructional method for teaching and learning stationary points in differential calculus and possibly for other Grade 12 Mathematics topics?

*<u>All Respondents</u>: (*Answered separately) Yes.

<u>Researcher:</u> Why do you think the 8Ps learning method can improve learners' mathematical problem-solving performance and should therefore be recommended for use in the Mathematics classroom?

<u>EX-L001:</u> I like the way learners are fully involved and engaged through small-group discussions and interactions.

<u>EX-L002:</u> It allows learners to participate actively and fully in class activities.

<u>EX-L003:</u> The method does not allow the teacher to do everything. I appreciate the way it gives learners the chance to share ideas with each other. In fact, from

time to time, I gained a lot from some brilliant classmates put in my different groups. I gained ideas I couldn't have been able to think about only by myself.

<u>*EX-L004:*</u> *I appreciate the fact that the method gives helpful step-by-step techniques for solving Mathematical questions. With the other method we were using before your arrival, we did cram and memorise a lot. Believe me sir, with that other method, any learner who cannot memorise formulas, rules and even some solutions cannot survive.*

<u>*CL-L001*</u>*: From what you told us about the method, I can say any method which allows learners to participate fully in the class activities is fine and better, like that one you informed us briefly about.*

<u>*CL-L002*</u>*: The fact that the present method does not make me understand Mathematics as much as I want is enough for me to wish for a new one. I desire a better method that will put me and fellow learners first. From the short information you made available to us about the 8Ps instructional method as one that involves learners fully in class activities, I feel it should be able to help.*

<u>*CL-L004:*</u> *If the 8Ps-based instructional method actually engages learners actively in class by allowing them to interact, discuss and learn together in small groups just as you informed us briefly about it, I am fully in support of it. In our current method, we learners do little in class to get the solutions to mathematics questions. Teacher mostly solves questions on the board for us to write down in our notebooks. I think that's the major reason why we do not have much knowledge of Mathematics. Only a few learners are doing well in the subject. Sir, unfortunately, I am one of the learners who find Mathematics difficult.*

<u>*EX-T001*</u>*: Actually, the present method of instruction is teacher-centred. The teacher does much of the mathematical problem solving. But the 8Ps-based instructional method you are proposing is practically learner-oriented. I witnessed*

that in your lessons that I observed. Sincerely speaking, it is a better approach and will achieve an improved result.

EX-T002: *I give my support to the 8Ps-based instructional method because it promotes active involvement of learners in the learning process. Through the use of small-group interactions and discussions that it also encourages, learners can discover a lot of strategies to use for solving mathematics problems. By that, they can understand Mathematics better. Another thing I love about the method is that it encourages the use of real-life problems in the development and learning of Mathematics. Researcher, I must tell you I am overwhelmed by the potential of the 8Ps-based instructional method. If we teachers can try it and give it the much effort it requires, the method will enhance the problem-solving performance of our learners in Mathematics.*

CL-T002: *As I earlier mentioned, I still use the traditional teaching technique for my Mathematics lessons. But from the brief information you gave us about the 8Ps learning method, it can make learners do deep mathematical thinking to discover solution ideas by themselves with the guidance of the teacher. That's why I like it.*

The views expressed by CL-L003 and CL-T001 to the questions have been represented in the above responses. Considering the above conversation, it is evident that all the respondents supported the idea of incorporating the 8Ps-oriented instruction in Mathematics lessons, even as demanding and challenging as they described it.

7.4.5 Theme 5: Possible Challenges that Application of 8Ps Learning Model may Encounter

Researcher: Asked the interviewees question 14 below.

14. What challenges may confront the use of the 8Ps learning model in the Mathematics classroom?

EX-L001: The method is really helpful, but it needs you to think and think to be able to use it well. The deep mathematical thinking and reasoning has taught me how to discover different solution strategies for a question, and how to select the one that appears most suitable.

EX-L002: Eish, I don't think the instructional method is for lazy and unserious learners who cannot concentrate. (Eish is a common South African word to express worry*).*

EX-L003: Some learners who are shy or afraid to talk in class won't really appreciate the method. Well, with time, I hope they will adjust.

EX-L004: Good as the method is, it needs quite a lot of mathematical thinking.

CL-L002: According to the information you made available to us, the learning method requires several steps and strategies. I just hope all Maths teachers would be patient enough and create time to go through those steps.

CL-L003: From the short information that you gave us about the method, I am afraid that method will need a lot of effort. But I feel it appears better than the one we use at present.

EX-T001: No doubt about it, the learning method is very useful, but it is time-consuming. The 55 to 60 minutes for each lesson certainly can't be enough to apply the method effectively.

EX-T002: The main problem with the method of instruction is that it is seriously demanding on the part of both the educator and the learners. Now, how many teachers can and will actually take their time to follow the high demands of the 8Ps learning method? Again, I believe it will help serious learners more because it is only when learners learn well that they will have some quantifiable learning experience and previous knowledge to bring to the class that will help them construct new knowledge.

CL-T001: My own concern is how all Mathematics teachers will find it easy to use, and how they will create for every lesson adequate time to select or set Mathematics tasks that are connected to learners' daily experience. Remember that, at the same time, the Mathematical tasks also have to be age-appropriate and in line with the curriculum.

CL-T002: As helpful as the instructional method appears, how to execute all the sequence of steps in the method within the limited lesson period is what bothers me.

Interviewees CL-L001 and CL-L004 decided not to comment on the 8Ps problem-solving instruction on the ground that it was new to them. According to them, they would like to experience it first to see how it would work. Overall, almost all the respondents are optimistic about the method and admitted that, the likely challenges pointed out notwithstanding, the method has the potential to be effective.

7.4.6 Theme 6: Possible Attitudes to the Use of 8Ps-based Problem-solving Instruction in Mathematics Lessons

Researcher: Asked the interviewees questions 15 and 16 below to determine the kind of attitudes that the incorporation of 8Ps instruction in Mathematics lessons can receive from Mathematics teachers and learners.

15. *What may be the attitudes of teachers and learners of Mathematics towards the use of 8Ps problem-solving instruction in the Mathematics classroom?*

16. *Do you think the 8Ps problem-solving approach is in accordance with the National Curriculum Statement on the teaching and learning of Mathematics?*

In response to the questions, all the respondents expressed optimism about the 8Ps-based instructional method. Being of the opinion that the instructional method is helpful and in line with the current National Curriculum Statement, they stated further, in their separate replies, that most Mathematics teachers and learners would show positive and favourable attitudes to the use of the learning method in the Mathematics classroom. For instance, EX-L002 emphasised, "*Meneer*, a lot of Mathematics learners will love the 8Ps method, especially the serious ones. A lot of Mathematics teachers will appreciate it as well". EX-L004 said with excitement, 'sincerely speaking sir, though we learners have to do much thinking and concentrate very well to be able to use the method, it's a good method. As we like it here, I believe most Mathematics learners and teachers in many other schools will like it too". According to CL-L001, "Teacher, from your short explanation of the 8Ps learning method, the method appears okay. I feel it will help us learn Mathematics better. Please, bring it to us too as soon as possible. We want it. Many learners of other schools who want to learn Mathematics well will also like it".

Speaking in the same direction, EX-T001 spoke, "Mister, let me first congratulate you on this great idea of yours. To be candid, the 8Ps learning method will go far. It's such a very useful method that will gain the acceptance of several Mathematics teachers and learners. But being a method that is demanding, initially, some may be reluctant about it. However, I can assure you that it will eventually be accepted. Thank you for deciding to assist our numerous Mathematics learners who are seriously struggling to understand the subject". CL-T002 equally commented, "I have to tell you that, for proper understanding of Mathematics, both learners and teachers need an instructional method like the 8Ps-based one. From your short explanation and the brief written description of it that you gave us to read, the method is actually learner-centred and all about problem solving in Mathematics. It should be acceptable to many teachers and learners of Mathematics. Thanks Mister". Generally, the respondents expressed that they were hopeful that both teachers and learners of Mathematics would welcome the learning method with open arms.

7.4.7 Concluding Comments on Participants' Responses

All the respondents were given the opportunity to express their minds freely and frankly during the interviews. They were allowed to ask questions and seek clarifications about any questions unclear to them. They were also informed they could decide not to reply to any of the questions they were not comfortable with or ones they had no ready answers for. Finally, when asked whether they were satisfied with the whole interview process, they all answered in the affirmative.

7.5 Reflection on the Chapter

The chapter has given a detailed explanation of the processes, purpose and results of the qualitative data analysis for the study. The thematic procedures followed in analysing the data derived from the classroom observations, the content analysis of the participants' post-test scripts and the semi-structured interviews have been fully presented. All the common themes identified in each category have been carefully explored to provide necessary information about the phenomena of interest. Importantly, the entire qualitative data analysis has been done in consideration of the aim and objectives of the study, the research questions and the research hypotheses.

CHAPTER EIGHT

SUMMARY, RESULTS, CONCLUSION AND RECOMMENDATIONS

8.1 Introduction

In this chapter, the study is summarised; the results of the study are discussed; the conclusion is drawn and the recommendations are made.

8.2 Summary of the Study

The study began with the various transformations that the school curriculum in South Africa has experienced as occasioned by the significant shifts in the educational policies of the country for almost three decades now. The notable transformative strategies have been put in place purposely to provide everyone with equal training opportunities, life-long education, quality life and a democratic, prosperous and peaceful society. The study also explained the different reform initiatives that the teaching and learning of Mathematics particularly has witnessed over the years in order to discover how learners can learn this vital subject effectively and improve their achievements in it. Like several previous studies, the current study equally noted that, year in year out, learners' performances in Mathematics have been unsatisfactory. It observed too that the traditional teaching approach being used for a long time has not produced satisfactory achievements in this subject generally believed to be well connected and contributory to several other fields of study and our daily living.

Noted for promoting teacher's dominance of the learning process, reliance on teachers and other external sources as the sole source of knowledge, rote learning and mere mastery of formulas and algorithms, the traditional teaching approach has therefore become grossly unpopular. The need then has arisen for an appropriate alternative instructional method and strategies that can engage Mathematics learners in active knowledge construction. For about forty years now, the NCTM and many other researchers have acknowledged the important role that problem solving plays in Mathematics and have consistently advocated that problem solving be made the main goal of the Mathematics curriculum. They have prescribed that, if learners have to achieve meaningfully in Mathematics, learners have to learn the subject through problem solving. Admitting too that approaching Mathematics through problem solving can facilitate learners' conceptual understanding of the subject, the present study designed the 8Ps-oriented problem-solving framework premised on the other existing learning models to see whether it could improve learners' understanding of Mathematics.

The study chose Grade 12 as its research field, since it is the final grade of the FET band which serves as the link between the secondary education and the tertiary education that would eventually usher the learners to the world of work and life in general. The choice of this research field agrees with the researchers' widely-held idea that mathematical problem-solving skills are also required in several disciplines (programmes of studies in the tertiary institutions) and workplaces. The study also picked differential calculus as a topic of interest because of its relevance and interconnection to other areas of Mathematics, various other disciplines and real life. Apart from this, differential calculus is one the ten learning areas currently tested in the NSC Mathematics Paper 1 and it carries a weighting value of 35 ± 3 marks, the highest weighting allotted to any topic in the paper.

The aim of the study was to determine whether the use of the 8Ps learning model would make any impact on the low problem-solving performance of Grade 12 learners in the concept of stationary points in differential calculus. The study also sought to see how the problem-solving learning model could be applied in the Mathematics classroom and the possible challenges that could confront its classroom application. In order to achieve this aim and its six objectives (see section 1.4), the study raised three research questions (see section 1.5). In its second chapter, the study conceptualised the 8Ps learning model for mathematical problem solving. To this effect, the study provided detailed definitions of Mathematics, differential calculus and stationary points in differential calculus and how they could be learnt. It also discussed the concepts of problem, problem solving and mathematical problem solving; mathematical problem-solving skills and strategies; appropriate nature of tasks for mathematical problem solving; teacher's role in the mathematical problem-solving processes and challenges of teaching Mathematics by problem solving. The study pointed out that the issue of learners' challenge in mathematical problem solving is not limited to South African (high school) learners but a global one.

The learning theory of constructivism and three problem-solving theories provided justification for the study. The three major theories of constructivism (cognitive constructivism, social constructivism and radical constructivism) were found to be relevant to the study in spite of their apparently different views on learning. Rather than focusing on their divergent viewpoints, the present study only concerned itself with how their common notions about learning support the effective application of problem-solving learning models, particularly the 8Ps-based learning model, in the learning of stationary points in differential calculus to increase the learning gains of the participants. Besides, behavioural problem-solving theory, cognitive problem-solving theory, and situational theory of problem solving were the three problem-solving theories examined in this study. Previous studies relevant to the present study were as well reviewed.

Reinforced by the pragmatic paradigm, the conduct of the study followed the mixed methods involving both quantitative and qualitative data collection and analysis methods. The study assumed the quasi-experimental design of a non-equivalent, pre-test, post-test control group and the interpretive research design. This study made use of the mixed methods for the investigation believing that the approach would enhance its validity and reliability. With its mixed-method approach, the study was able to complement and triangulate the quantitative data collected with its qualitative data as an effort to increase the credibility of its results. The intact groups of 253 Grade 12 learners of eight schools constituted the study sample. Four of the schools formed the treatment group while the other four served as the control group.

The main measuring instruments for this study are: mathematical problem-solving achievement test, classroom observation schedule and semi-structured interview schedule. Prior to their usage in the main study, the research tools were properly developed, validated and their reliability ascertained. The secondary data sources adopted are: 8Ps-based flip cards, 8Ps-based wall charts, 8Ps-based worked-out examples, 8Ps-based mathematical problem-solving assessment form, learners' test scripts and transcripts from audio and video recordings. All the research instruments were pilot-tested and the pilot study results found them suitable for the actual study.

Prior to the conduct of the main study, a two-week pre-intervention classroom visits were made to the eight participating schools and it was established that all the schools were adopting comparable conventional instructional methods and could suitably participate in the study. During the main study, the researcher implemented the intervention by himself in the experimental group, while the four regular Grade 12 Mathematics teachers taught their learners the same learning content in the control group. As earlier mapped out, both quantitative and qualitative data were gathered during the two-month exercise. Making use of both quantitative and qualitative analysis methods, the data collected were analysed and the results obtained. Both descriptive and inferential statistics were used for this purpose (see chapter 5 - 7). The results of the analyses are discussed in the subsequent section.

8.3 Discussion of Results

The results for this study are discussed below based on the literature review, the research questions and the research objectives.

8.3.1 Discussing the Results Vis-à-vis the Literature

One of the data sources that contributed significantly to the current study is the literature. Through the related literature reviewed, this study has been able to establish the following results:

- That Mathematics is a core subject which has a huge relevance in the sciences, several other disciplines and real life; moreover, that knowledge of Mathematics appreciably enhances the understanding of the various other disciplines and produces the required critical and creative thinking that can help solve problems in other disciplines and everyday life. On this basis, this study has clarified the rationality of the decision taken by the DBE (like many other education departments globally) requiring that an average South African learner, irrespective of the education phase or grade, offer Mathematics (or at least Mathematical Literacy) in order to progress in their education career (refer to subsection 2.2.2).

- That differential calculus, the learning content focused in this study, is an essential aspect of Mathematics which forms a strong basis for most other fields of Mathematics and has broad practical applications in several other specialties (including the sciences, engineering, computer science, technology and economics) and the real life; furthermore, that acquisition of the conceptual knowledge of differential calculus facilitates learner performance in Mathematics and a lot of other fields, and generally assists learners in taking proper decisions in life. By this, this study has justified the appropriateness of DBE’s inclusion of differential calculus among the ten main learning areas of Grade 12 Mathematics curriculum and the choice of the concept of stationary points in differential calculus as the research focus area for the present investigation (refer to subsection 2.2.3 - 2.2.4).
- That South African Grade 12 Mathematics learners and numerous secondary school Mathematics learners globally have difficulties grasping the concept of differential calculus; also, that most of the learners’ difficulties, basically, are connected to their low mathematical problem-solving skills and the methods adopted by Mathematics teachers in teaching them the somehow complex topic. Therefore, this study has affirmed that how properly the teachers discharge this essential role of developing, affecting and assessing learners’ mathematical problem-solving skills contributes considerably to the level of

efficiency learners will attain in differential calculus and Mathematics in general.

- This study has further accepted that, for those Mathematics teachers to successfully attain this, they require more expertise and regular training on mathematical problem solving (see subsection 2.2.5 - 2.2.6; 2.3.6).

- That problem solving is crucial and central to the teaching and learning of Mathematics and more so that Mathematics learners require to display sound mathematical problem-solving performance for meaningful achievements in the subject and to be able to approach daily life problems confidently. As such, this study has shown that problem solving stands as the primary objective of Mathematics as declared by the NCTM, various researchers in Mathematics Education, lots of other interest groups in education and the DBE, South Africa (see subsection 2.3.1 - 2.3.2).

- That the teacher-driven traditional teaching pedagogy, which recognises the teacher as the major source of ideas and sole transmitter of knowledge, is grossly inadequate and has not yielded satisfactory learner achievements in differential calculus and Mathematics as a whole; but that the learner-centred problem-solving instructional method, that fully involves learners in the mathematical problem-solving processes, can significantly improve learners' understanding of Mathematics. Through this study's exploration of different heuristic problem-solving learning models and design of the 8Ps learning model for mathematical problem solving, it has offered some helpful mathematical problem-solving strategies capable of equipping learners with the much-needed critical and analytical thinking skills that can boost their knowledge of Mathematics and make them cognitively versatile in a lot of other useful ways (see subsection 2.4.1.1 - 2.4.3). The detailed comparison of the 8Ps learning method and the traditional teaching method is given in sections/subsections: 2.2.6, 2.3.5 - 2.3.6, 2.4.1, 2.4.3, 5.1 - 5.5, 6.1 - 6.9 and 7.1 - 7.5. Hence, the research objective 3 which sought to compare the 8Ps problem-solving method with the traditional teaching method is attained.

- That teaching Mathematics through problem solving is demanding on the teachers. That it is challenging to the teachers pedagogically and personally, since they must be able to do the following properly: set/select appropriate, well-structured, contextual, non-routine mathematical problems which equally have to relate to the learners' personal experience and the real life; employ suitable learner-oriented instructional approach that adequately involves the learners in the problem-solving processes; facilitate the learner-driven problem-solving processes without taking a lead or dominant role and, at the same time, determine the amount of classroom guidance to offer the learners and when to provide such, and importantly, allow the learners to actively construct knowledge by obtaining the solutions to the given mathematical problems by themselves (see subsection 2.3.4 - 2.3.5).

- Additionally, that teaching Mathematics through problem solving is also demanding on the learners. It is challenging to the learners in the sense that they have to be able to encounter non-routine mathematical problems; must display high problem-solving performance for them to be able to solve the unfamiliar, somehow complex mathematical problem; must be able to wade through the problem-solving processes and obtain applicable problem-solving strategies, and particularly, come up on their own with the required solutions to the problems assigned them with minimal teacher's guide (see subsection 2.3.4 - 2.3.5). Be that as it may, this study has pointed out categorically that mathematical problem-solving methods such as the 8Ps learning model effectively applied can ease or remove learners' difficulties in learning Mathematics. To this end, the study has suggested some useful problem-solving techniques that can assist both the teachers and the learners in carrying out the demanding tasks (see subsections/sections 2.3.2.2 - 2.3.2.3, 2.3.3.1 - 2.3.3.2, 2.3.4, 3.3 and 3.6).

- That the learning theory of constructivism and the three problem-solving theories adopted in this study strongly support teaching and learning Mathematics through problem-solving method as a way to improve learners' low mathematical problem-solving performance. From the standpoints of the three problem-solving theories, notable learning techniques that teachers can use to facilitate learner mathematical problem-solving performance include: learner analysis or pre-assessment of learners in order to determine where instruction should start; usage of task analysis, behavioural objectives and criterion-referenced assessment (among others) to produce observable and measurable outcomes in learners; positive and negative reinforcements; trial-and-error; consideration of learner's conscious thinking ability (cognition) and communicative behaviour during problem solving and insight. Through the constructivist perspective, it was majorly found out that the learners' mathematical problem-solving performance would improve: if learners are actively and continuously involved in mathematical problem solving and critical thinking rather than being reduced to passive listeners and absorbers of Mathematics knowledge; if they are allowed to construct their own Mathematics knowledge based on their previous knowledge and personal experiences; if they are encouraged and guided to engage in social construction of Mathematics knowledge which entails learner-learner and learner-teacher interactions, peer and group discussions and collaborative learning, and if the learning process centres more on mathematical procedures than on the solutions. By this, the learning theories have been used to understand and explain the learners' mathematical problem-solving performance (see section 3.2 - 3.5). Hence, the research objective 1 has been accomplished.

- That the learning theories have provided necessary theoretical base and justification for the choice, design and application of the 8Ps learning model for this enquiry. Specifically, the learning theories have shed light on important factors upon which the design and use of 8Ps learning model hinge. These include: what a learner-centred, problem-solving Mathematics classroom should look like; how Mathematics teachers are expected to facilitate the problem-solving process; the quantity and quality of guidance and support the teachers are required to provide the learners during problem solving towards arriving at logical solutions to the assigned Mathematics tasks; nature of learners' reasoning, interactions, solution moves and strategies, the actual solutions and the logicality of those solutions (refer to section 3.2 - 3.5). Therefore, the research objective 2 has been achieved.

- That the various related past studies reviewed have provided a thorough understanding of the current study and given it a solid background; that those past studies have supported and explained the goal of the present study, its choice of research methods, research design, research questions and research objectives. Again, that the appraised existing studies have contributed significantly to the development of ideas of the current study; provided what is already known about the present research topic; identified existing knowledge gaps, and revealed how the current study can contribute to further understanding of the research topic (see section 3.6).

- That, all said and done, the problem-solving instruction such as offered by the 8Ps learning model has the potential to foster the Grade 12 learners' conceptual understanding of the concept of stationary points in differential calculus and Mathematics generally.

8.3.2 Discussing the Results in Line with the Research Questions and the Research Objectives

<u>*Research Question 1*</u>*: Will the use of the 8Ps learning model in the teaching and learning of the concept of stationary points in differential calculus have any effect on the learners' mathematical problem-solving performance?*

The research question 1 is addressed as follows:

- Considering the descriptive statistics of the post-test performances of both groups as reported in section 6.4.1, experimental and control groups achieved ($\bar{x}$ = 41.98; σ = 16.28; n = 119) and ($\bar{x}$ = 16.01; σ = 6.15; n = 119) respectively. This result presents the experimental group as recording a mean score increase of 25.97% over and above the control group. The implication of their standard deviation values is that the marks obtained by the experimental group are spread out and away from the mean score whereas the marks got by the control group cluster around the mean score. Furthermore, the median marks of 48% and 16% made by the experimental and control groups respectively signifies that half of the participants of the experimental group did not get more than 48% while half of the participants of the control group got marks not more than 16%. Moreover, the minimum marks of 6% and maximum marks 62% obtained by the experimental group put its range of marks as 56%, while the minimum marks of 3% and maximum marks 29% obtained by the control group leave its range of marks as 26%. The foregoing is an indication that, in the post-test, the experimental group that was exposed to the 8Ps-based instruction outperformed the control group which received the traditional instruction.

- In order to show that the mean increase of 25.97% achieved by the experimental group above the control group is statistically significant, the paired t-test for post-test marks of both groups was performed at 5% level of significance. In t-test, $p < \alpha = .05$ shows that the two mean scores are different but $p > \alpha = .05$ signifies that both mean scores are equal. Also, a high t-value means a big difference between the two sample-sets, while a small t-score shows a considerable similarity between the two sample-sets. Given these circumstances, the result of the paired t-test {t = 16.28, *p < .00001, p* < *α* = .05} and the eta-squared statistic, $\eta^2 = \frac{t^2}{t^2+(n-1)} = .69$ which yields a large effect size, affirm that the mean score increase of 25.97% is statistically significant. Hence, the null hypothesis declaring that the post-test mean score of the experimental group is equal to the post-test mean score of the control group is hereby rejected. The conclusion is then drawn that there is a statistically significant mean difference between both groups. This is an indicator that the 8Ps-oriented instruction administered to the experimental group is effective (see subsection 6.4.2).

- According to the simple mathematical analysis done to compare the participants' post-test performances by achievement categories: LA (low achievement), MA (medium achievement) and HA (high achievement), the experimental group had 84 (70.6%) high achievers, 15 (12.6%) medium achievers and 20 (16.8%) low achievers. On the other hand, the control group participants produced 0% high achievers, 35 (29.4%) medium achievers and 84 (70.6%) low achievers (see subsections 6.6.1 and 6.6.2). The experimental group made this better attainment after they had received the 8Ps-based intervention.

- For further comparison of the post-test mean scores of both groups by the achievement categories LA, MA and HA, three sets of one-way analysis of variance (ANOVA) test were performed. An ANOVA test was carried out for each achievement category to determine the difference between experimental and control groups per achievement group. In the high achievers' category, the control group recorded 0% while the experimental group achieved 70.6%; the resulting mean score ($\bar{x} = 51.25, \sigma = 6.24$) is thus statistically significant (see section 6.7.3). For the medium achievers' category, the ANOVA result $\{F\ (1,\ 48) = 22.058,\ p < .001, p < \alpha = .05\}$ shows that there is a statistically significant difference in the mean scores of experimental and control groups. In other words, for the medium achievers, the mean score ($\bar{x} = 29.73, \sigma = 6.93$) of the experimental group is significantly higher than the mean score of the control group ($\bar{x} = 23.54$, $\sigma = 2.44$) (see section 6.7.1). However, only the ANOVA result $\{F(1, 102) = .301, p = .584, p > \alpha = .05\}$ of the low achievers produces an insignificant statistical difference in the mean score ($\bar{x} = 12.30, \sigma = 4.17$) of the experimental group and the mean score ($\bar{x} = 12.87$, $\sigma = 4.17$) of the control group (see section 6.7.2). On the whole, the ANOVA results indicate that the performance of the experimental group per achievement category is better than that of the control group. Thus, the null hypothesis claiming no statistically significant mean difference between the two groups is rejected.

- The comparison of both groups in terms of their post-test errors reveals that the experimental group made by far smaller the number of errors in the post-test than the control group. While the experimental group committed 583 errors, the control group committed 1781 errors, meaning that participants of the experimental group made about one-third of the errors committed by participants of the control group. Upon the calculation of their error means, the experimental group obtained an error mean ($\bar{x} = 116.60$, $\sigma = 31.35$) which was significantly lower than the error mean ($\bar{x} = 336.20$, $\sigma = 82.81$) attained by the control group. This is a pointer that the experimental group given the 8Ps intervention had recorded more improvement in its mathematical problem-solving performance than the control group instructed conventionally (check Table 6.4.3 and Figure 6.4.3 for details). In order to find out whether the error mean of the experimental group is the same as the error mean of the control group, a one-way ANOVA test for equality of error means was conducted. The result $\{F\ (1, 8) = 30.749, p < .001, p < \alpha = .05\}$ obtained and the large effect size of $\eta^2 = .79$ yield a statistically significant difference in the error means of the two groups (see section 6.9). Here as well, the null hypothesis is rejected, meaning that the experimental group exhibited a better mathematical problem-solving performance than the control group.

- The foregoing reveals that the use of the 8Ps learning model in teaching and learning the concept of stationary points in differential calculus has a significant quantitative effect on the learners' mathematical problem-solving performance (research objective 1). It also reveals that the socio-economically disadvantaged learners tend to demonstrate lower problem-solving performance than their socio-economically favoured mates. This is so because their deprived status does not support them with necessary resources that can equip them with helpful problem-solving skills. This is revealed by the achievements recorded by both groups in sections: 5.1 - 5.5, 6.1 - 6.9 and 7.1 - 7.5. The research question 1 is then attained.

Research Question 2: *How can 8Ps learning model be used in teaching and learning the concept of stationary points in differential calculus?*

The research question 2 is answered through the results from the thematic qualitative analyses of the data that emerged from the classroom observations, content analysis of the participants' post-test scripts and the semi-structured interviews conducted:

- During the classroom observations, both the learner and teacher participants of experimental and control groups were observed in their natural settings mainly by using the classroom observation schedule designed for that purpose (see Appendix C). In the experimental group, the researcher acted as a participant observer, but as a non-participant observer in the control group. The results of the classroom observations indicate that the 8Ps instructional method that the researcher administered to the experimental group is significantly different from the traditional methods employed to teach the control group. While the 8Ps learning method followed the heuristic 8-phase principles namely: *probing, pinpointing, patterning, projecting, prioritising, processing, proving and predicting* (section 2.4.2 refers), the traditional teaching approach adopted by the regular Mathematics teachers of the control group was of the conventional *talk-and-chalk*, textbook-dependent, whole-class, teacher- dominated instructional technique (see section 7.2).
- The content analysis of the participants' post-test scripts was performed via

seven assessment criteria specified in the mathematical problem-solving assessment form developed for the study. The post-test scripts analysed were 238 altogether – 119 of the experimental group participants and the remaining 119 of the control group participants. The goal was to investigate the definite aspects of the participants' poor mathematical problem-solving skills. Such specific aspects examined are: how the participants skilfully recalled and applied important formulas and rules; how they were able to interpret equations logically and/or translate mathematical problems successfully to equations; how reasonably they represented mathematical problems as patterns that could help solve the problems; how effectively the study participants connected their prior knowledge to current learning; evidence of successful use of clear and sequential problem-solving solution-steps to obtain solutions; provision of justifiable reasons for the solution-moves made, and how participants correctly explained and applied the solutions got to similar other problems and even real-life problems (see Appendix E6 and subsection 4.9.4.6). In all the seven problem-solving performance areas assessed, the experimental group was found to be more skilful than the control group (see section 7.3).

- Thematic qualitative data analysis method was also adopted to analyse the data from the semi-structured interviews conducted with the twelve selected study participants – eight learner participants and four teacher participants from both groups. Specifically, they were four learner participants and two teacher participants from each of the two groups. The semi-structuredness of the interviews allowed the twelve interviewees to express their views freely and frankly for about thirty minutes allotted to each of them. The primary purpose of the interviews was to corroborate the findings from the classroom observations and the content analysis of the participants' post-test scripts. The semi-structured interview schedule developed for the study was used. The sixteen questions of the interview guide were categorised into six themes for easy analysis. The analysis of the data from the semi-structured interviews yielded the following results:

- The learner participants had a limited understanding of the concept of stationary points in differential calculus. However, all the teacher participants possessed a rich knowledge of the concept. Despite that they applied the conventional teaching method, the four control group teachers demonstrated considerable content knowledge to communicate the concept to their learners without rendering them disadvantaged. It was also found out that the learner participants of the experimental group acquired a fair understanding of mathematical problem solving as a result of the intervention, and that reflected in their post-test performance. The control group, on the other hand, found mathematical problem-solving ideas new. The little that the group could say about it was got from the short explanation that the researcher earlier provided them. Both experimental and control groups however admitted that teaching and learning Mathematics through problem solving can raise learner performance in the subject.

- The participants' views and opinions about teaching and learning Mathematics through problem solving and the use of the 8Ps instructional method in teaching and learning Mathematics are summarised as follows:

    - The learner participants in the experimental group were quite happy with the 8Ps-oriented learning method introduced to them. Although they pointed out that using the method could be demanding, they openly declared that they would like to learn their other Mathematics topics through the method.

    - Rather displeased with the traditional teaching approach adopted by their teachers which they felt did not enable them to properly understand the concept of stationary points in differential calculus, participants in the control group expressed their wish for a new learning method like the 8Ps problem-solving method they were briefed about. Their teachers confirmed this by stating that they noticed the learners' dissatisfaction with the traditional teaching method from the way the learners frowned their faces and shifted uneasily during the lessons.

- Despite expressing that teaching Mathematics through problem-solving could be challenging, all the teacher participants considered the 8Ps learning method helpful to the teaching and learning of Mathematics.

- Being of the opinion that the instructional method can be useful and is in line with the current National Curriculum Statement, all the teacher participants stated that many Mathematics teachers and learners would show positive and favourable attitudes to the use of the learning method in the Mathematics classroom.

By and large, both the learner and teacher participants expressed favourable views about teaching and learning of Mathematics through problem solving and the incorporation of the 8Ps problem-solving instruction in Mathematics lessons. They equally showed positive attitudes to them (see subsection 7.4.1 - 7.4.6). Hence, research objective 2 has been realised. Generally, the results of the thematic qualitative analyses of the data that emerged from the classroom observations, content analysis of the participants' post-test scripts and the interviews conducted with the selected twelve study participants have clearly explained how the 8Ps learning model can be used to teach and learn the concept of stationary points in differential calculus (refer to subsection 7.2.1 - 7.4.5). Therefore, the research question 2 has been answered.

<u>Research Question 3:</u> *What challenges, if any, does the use of 8Ps learning model pose in teaching and learning the concept of stationary points in differential calculus?*

Subsection 7.4.5 identifies the challenges that application of the 8Ps problem-solving instruction may encounter in the Mathematics classroom. The challenges anticipated by the study participants are that the use of the 8Ps learning model:

- requires much mathematical thinking and much effort;

- necessitates that learners must be duly attentive when using it to solve Mathematics problems and must not be lazy, shy or afraid to talk or interact with fellow learners in class during the problem-solving processes;
- calls for several solution steps and strategies before arriving at the solution;
- is time-consuming and
- is demanding on both the learners and the teacher.

The current study, at this juncture, avers that every learning method or idea, new or old, has its own challenges; that what actually matters is that the challenges of any new method should be minimal and such that can be surmountable; that a new learning method or idea is likely to experience some challenges or cold attitude from prospective users initially, being that it is freshly introduced and because humans generally are often scared of switching to new situations. This study asserts that, in some cases, when users try out a novel idea and get familiar with it, they may incidentally discover it is easy and convenient to use. In spite of the probable challenges pointed out, most of the respondents were optimistic that the 8Ps learning method has the potential to be effective for teaching and learning Mathematics. They further mentioned that they were hopeful that both teachers and learners of Mathematics would welcome and receive the learning method with positive and favourable attitudes, more so that the instructional method is in line with the current National Curriculum Statement. The foregoing has shown that research question 3 has been addressed.

8.4 Limitations of the Study

Every study has its own limitations which, in one way or another, may influence its results and conclusion. The present study is not an exception in this regard as its conduct took place amidst its limitations. Highlighted below are the study's limitations:

- This study was restricted to only 253 participants who were Grade 12 Mathematics learners drawn from eight secondary schools of one province.

- The selection of the participating schools for the study majorly considered convenience and socio-economic factors. It did not follow any random assignment of the participants to experimental and control groups since it made use of the already established group of Mathematics learners in each of the eight schools involved.

- The study assumed that the sample was representative of all the Grade 12 learners in the province and, by extension, the entire Grade 12 learners of South Africa. This assumption was hinged on the following factors: all the secondary schools in South Africa (public and private) follow the same Grade 12 Mathematics curriculum provided by the DBE; the eight schools involved in the enquiry are mixed (having boys and girls); all the selected schools have their learners from different parental backgrounds and South African cultures, and they all use English Language as their medium of instruction.
- Again, the inquiry only investigated the problem-solving performance of the concept of stationary points in differential calculus taught for only two-and-a-half months altogether. The concept is just an aspect of one topic. The other aspects of differential calculus and the remaining nine learning areas of the Grade 12 Mathematics curriculum were not covered in the investigation.

- This duration of the investigation is also not long enough to significantly equip learners with enough mathematical problem-solving skills.

On account of the above factors, it may therefore be necessary to take caution in generalising the research results beyond the study participants. It may also be advisable to exercise care in extending the research findings beyond the socio-economically deprived communities where the inquiry was carried out.

8.5 Conclusion

As earlier specified in section 1.6, the current study is a conscious move to address the issue of the learners' weak mathematical problem-solving performance. It is an intended effort to understand the nature and level of the concerned learners' mathematical problem-solving performance, identify the challenges they encounter in the mathematical problem-solving process and provide them with suitable skills and strategies that can support them. In view of the results of the investigations discussed in subsection 8.3.1 - 8.3.2, it can be inferred that the study has accomplished its goal. The research results have shown that the 8Ps-oriented instruction can facilitate the problem-solving performance of the Grade 12 learners in the concept of stationary points in differential calculus. The results have also clarified how the 8Ps learning model can be used to teach and learn the concept, and the likely challenges that its use can encounter. It may thus be acceptable to say that the 8Ps-oriented instructional method has the potential to equip learners with useful mathematical problem-solving knowledge that can increase their learning gains in Mathematics. On this basis, this study proposes that the 8Ps instructional method be incorporated in the teaching and learning of the subject.

8.6 Recommendations

8.6.1 Recommendations for Improvement of Teaching and Learning of Mathematics

Towards facilitating the use of the 8Ps-based instructional method to bring about desirable improvements to the teaching and learning of Mathematics, the study makes the following recommendations:

- The DBE should place much more emphasis on teaching and learning of Mathematics by problem solving by restructuring the Mathematics curricula and the policies guiding the assessment of Mathematics in line with the principles of problem solving.

- It should provide regular training to Mathematics teachers in the form of seminars, workshops and other necessary support on the use of problem-solving instructional method.

- It should ensure that problem-solving instructional method is applied in every Mathematics classroom through monitoring and regular classroom visits to schools by inspectors from the department and the schools' respective Mathematics HODs.

- School managements should consider allocating additional time on the timetable, say 10 - 15 minutes, to each Mathematics lesson period.

- Rather than see it as an unnecessary time-wasting exercise, Mathematics teachers should endeavour to divide learners into small groups of mixed abilities for each lesson. They should facilitate the groups effectively and encourage learners to participate actively in their various interaction and discussion groups.

- Mathematics teachers should carefully plan each lesson period in such a manner that will apportion a reasonable length of classroom time to the learners to engage meaningfully in learner-learner and learner-teacher problem-solving interactions and discussions.

- In addition, Mathematics teachers have to ensure that Mathematics tasks to be solved as class examples, class work and homework are non-routine, well-structured, contextualised and related to learners' daily experience.

- Both the Mathematics teachers and their learners have to accept the simple truth that mathematical problem solving requires from them some mathematical thinking and reasoning. For them to be able to do it successfully therefore, they have to be consciously committed to it.

8.6.2 Recommendations for Further Research

The 8Ps-based learning model being proposed for the teaching and learning of Mathematics is a newly developed instructional method. As such, it is still open to appropriate modification based on relevant observations and suggestions that may arise when fully put into use in the Mathematics classroom. Again, as a way to increase the credibility of the problem-solving model, further research is necessary on it. Thus, it is recommended that future researchers:

- Investigate the effect of the use of 8Ps-based instructional method on learners' problem-solving performance in other aspects of differential calculus; the remaining nine learning areas of Grade 12 Mathematics curriculum, and in the learning of Mathematics generally in secondary schools and tertiary institutions;
- Replicate the study with participants from a research field that is not socio-economically deprived to ensure generalisation of results;
- Compare the effect of the use of 8Ps-based instruction to the effect of application of computer-assisted/ICT instruction on learners' mathematical problem-solving performance.
- Assess the effect of Mathematics curriculum restructure in relation to problem-solving principles on learners' mathematical problem-solving performance.
- Measure the effect of Mathematics assessment restructure in line with problem-solving principles on learners' mathematical problem-solving performance.

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LIST OF APPENDICES

Appendix A1: Pre-test

Learner's Code: ………… Learner's Class: A ☐ B ☐ (Tick as applicable)

Instruction: Answer all the questions showing clearly all your calculations.

Duration:1 hour 30 minutes

QUESTION 1 (Feb/March 2017 Q8)

Given: $f(x) = 2x^3 - 5x^2 + 4x$

1.1 Calculate the coordinates of the turning points of the graph of f (5)

1.2 Prove that the equation $f(x) = 2x^3 - 5x^2 + 4x$ has only one root (3)

1.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points (3)

1.4 For which values of x will the graph be concave up? (3)

[14]

QUESTION 2 (May-June 2017 Q9)

Given: $f(x) = x^3 - x^2 - x + 1$.

2.1 Write down the coordinates of the y-intercepts of f. (1)

2.2 Calculate the coordinates of the x-intercepts of f. (5)

2.3 Calculate the coordinates of the turning points of f (6)

2.4 Sketch the graph of f.clearly indicating all intercepts with the axes and the turning points. (3)

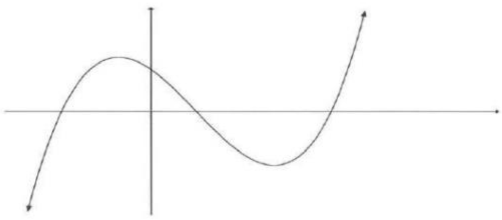
2.5 Write down the values for which $f''(x) < 0$. (2)

[17]

QUESTION 3 (Feb/March 2018 Q9)

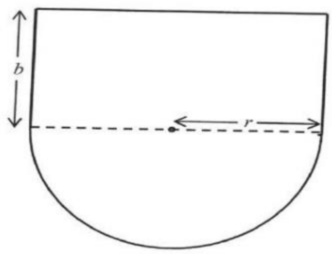
The sketch below represents the curve of $f(x) = x^3 - bx^2 + \text{cx} + \text{d}$.

The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4



3.1	Calculate the values of b, c and d.	(4)
3.2	Calculate the coordinates of B, the maximum turning point of f.	(4)
3.3	Determine an equation for the tangent to the graph of f at $x = 1$.	(4)
3.4	Sketch the graph of $f''(x)$ and clearly indicate the x- and y-intercepts.	(3)
3.5	For which value(s) of x is $f(x)$ concave upwards?	(2)
		[17]

QUESTION 4 (May-June 2017 Q10)



The figure is a design of a theatre stage in The shape of a semi-circle attached to a rectangle. The semi-circle has a radius r and the rectangle has a breadth b.
The perimeter of the stage is $60m$.

4.1	Determine the expression for b in terms of r.	(2)
4.2	For which value(s) of r will the area of the stage be a maximum?	(6)
	Given that $f(x) = 3x^3$. (Nov 2019 Q9)	
4.3	Solve $f(x) = f'(x)$.	(3)
4.4	The graphs f , f' and f'' all pass through the point (0; 0). For which of the graphs will (0; 0) be a stationary point?	(1)

4.5 Explain the difference, if any, in the stationary points mentioned in (4.3.2) (2)

4.6 Determine the vertical distance between the graphs of f' and f'' at $x = 1$ (3)

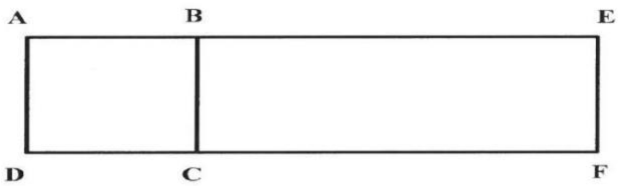
[17]

QUESTION 5

5.1 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at $(2 ; 4)$. Calculate the values of b and c. (May-June 2017 Q8.3) (7)

5.2 Given: $f(x) = -3x^3 + x$. Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$. (Feb/March 2018 Q10) (6)

5.3 A piece of wire $6\ m$ long is cut into two pieces. One piece, $x\ m$ long, is bent to form a square ABCD. The other piece is bent into a U-shape to form a rectangle BEFC when placed next to the square.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. (Feb/March 2017 Q9) (7)

[20]

TOTAL: 85

Appendix A2: Post-test

<u>Learner's Code:</u> ………… <u>Learner's Class:</u> A ☐ B ☐ (Tick as applicable)

<u>Instruction</u>: Answer all the questions showing clearly all your calculations.

<u>Duration</u>:1 hour 30 minutes

QUESTION 1 (May-June 2017 Q9)

Given: $f(x) = x^3 - x^2 - x + 1$.

1.1	Write down the coordinates of the y-intercepts of f.	(1)
1.2	Calculate the coordinates of the x-intercepts of f.	(5)
1.3	Calculate the coordinates of the turning points of f	(6)
1.4	Sketch the graph of f. Clearly indicate all intercepts with the axes and the turning points.	(3)
1.5	Write down the values for which $f''(x) < 0$.	(2)
		[17]

QUESTION 2 Feb/March 2017 Q8)

Given: $f(x) = 2x^3 - 5x^2 + 4x$

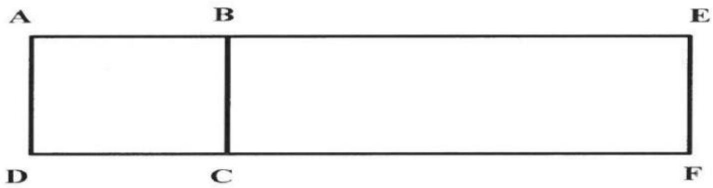
2.1	Calculate the coordinates of the turning points of the graph of f	(5)
2.2	Prove that the equation $f(x) = 2x^3 - 5x^2 + 4x$ has only one root	(3)
2.3	Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points.	(3)
2.4	For which values of x will the graph be concave up?	(3)
		[14]

QUESTION 3

3.1 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at $(2\,;4)$. Calculate the values of b and c (May-June 2017 Q8.3). (7)

3.2 Given: $f(x) = -3x^3 + x$. Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$ (Feb/March 2018 Q10). (6)

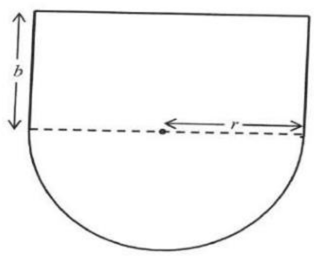
3.3 A piece of wire $6\ m$ long is cut into two pieces. One piece, $x\ m$ long, is bent to form a square ABCD. The other piece is bent into a U-shape to form a rectangle BEFC when placed next to the square.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum (Feb/March 2017 Q9). (7)

[20]

QUESTION 4 (May-June 2017 Q10)



The figure is a design of a theatre stage In the shape of a semi-circle attached to a rectangle. The semi-circle has a radius r and the rectangle has a breadth b. The perimeter of the stage is $60\ m$.

4.1 Determine the expression for b in terms of r. (2)

4.2 For which value(s) of r will the area of the stage be a maximum? (6)
Given that $f(x) = 3x^3$. (Nov 2019 Q9)

4.3 Solve $f(x) = f'(x)$. (3)

4.4 The graphs f , f' and f'' all pass through the point (0; 0).

For which of the graphs will (0; 0) be a stationary point? (1)

4.5 Explain the difference, if any, in the stationary points mentioned in (4.3.2) (2)

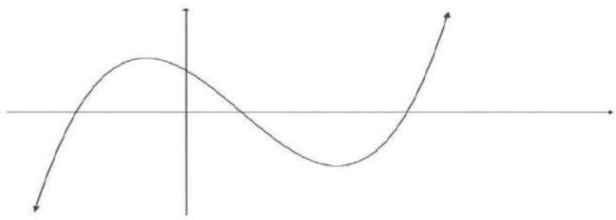
4.6 Determine the vertical distance between the graphs of f' and f'' at $x = 1$ (3)

[17]

QUESTION 5 (Feb/March 2018 Q9)

The sketch below represents the curve of $f(x) = x^3 - bx^2 + \text{cx} + \text{d}$.

The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4



5.1 Calculate the values of b, c and d. (4)

5.2 Calculate the coordinates of B, the maximum turning point of f. (4)

5.3 Determine an equation for the tangent to the graph of f at $x = -1$. (4)

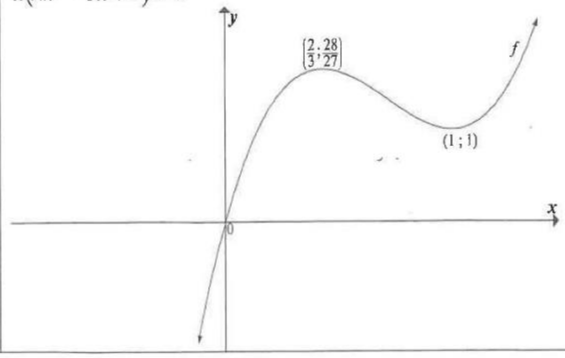
5.4 Sketch the graph of $f''(x)$ and clearly indicate the x- and y-intercepts. (3)

5.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

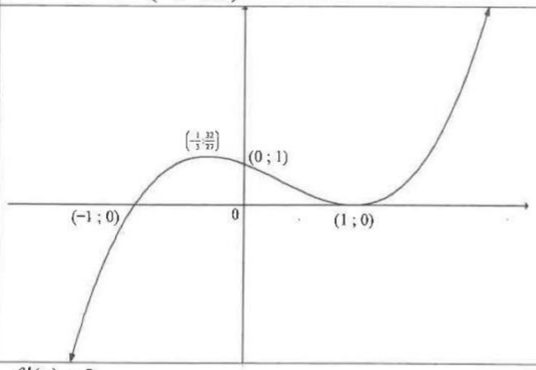
TOTAL: 85

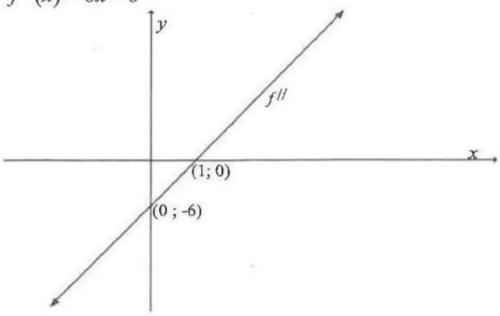
Appendix A3: Memorandum

1.1	$f'(x) = 0$ $6x^2 - 10x + 4 = 0$ $3x^2 - 5x + 2 = 0$ $(3x-2)(x-1) = 0$ $x = \frac{2}{3}$ or $x = 1$ $y = 2\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)$ $y = 2(1)^3 - 5(1)^2 + 4(1)$ $y = \frac{28}{27}$ or $y = 1$ Turning points are $\left(\frac{2}{3}; \frac{28}{27}\right)$ and $(1;1)$	✓ derivative ✓ derivative $= 0$ ✓ factors ✓ x-values ✓ y-values (5)
1.2	$2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0$ or $x = \frac{5 \pm \sqrt{25 - 4(2)(4)}}{4}$ $= \frac{5 \pm \sqrt{-7}}{4}$ No real roots / *Geen reële wortels* **OR / OF** $2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0$ or $b^2 - 4ac = 25 - 4(2)(4)$ $= -7 < 0$ No real roots / *Geen reële wortels*	✓ $x(2x^2 - 5x + 4) = 0$ ✓ $x = 0$ ✓ $\frac{5 \pm \sqrt{-7}}{4}$ (3) ✓ $x(2x^2 - 5x + 4) = 0$ ✓ $x = 0$ ✓ $b^2 - 4ac < 0$ (3)
1.3	$f(x) = 2x^3 - 5x^2 + 4x$ $x(2x^2 - 5x + 4) = 0$  	✓(0 ; 0) ✓turning points ✓shape (3)

1.4	$f(x)=2x^3-5x^2+4x$ $f'(x)=6x^2-10x+4$ $f''(x)=12x-10$ $f''(x)>0$ $12x-10>0$ $x>\frac{5}{6}$	✓ $12x-10$ ✓ $f''(x)>0$ ✓ answer (3)
	OR	
	Point of inflection: $x=-\frac{b}{3a}$ $x=-\frac{(-5)}{3(2)}$ $x=\frac{5}{6}$ The function is concave up for $x>\frac{5}{6}$ since $a>0$	✓ $x=-\frac{(-5)}{3(2)}$ ✓ $x=\frac{5}{6}$ ✓ $f''(x)>0$ (3)
	OR	
	Point of inflection: $x=\frac{\frac{2}{3}+1}{2}$ $x=\frac{5}{6}$ The function is concave up for $x>\frac{5}{6}$ since $a>0$	✓ $x=\frac{\frac{2}{3}+1}{2}$ ✓ $x=\frac{5}{6}$ ✓ $f''(x)>0$ (3)
		[14]

2.1	(0 ; 1)	✓ answer (1)
2.2	$f(x)=x^3-x^2-x+1$ $f(x)=x^2(x-1)-(x-1)$ $f(x)=(x-1)(x^2-1)$ $f(x)=(x-1)(x-1)(x+1)$ $f(x)=0$ $(x-1)(x-1)(x+1)=0$ x-intercepts: (–1; 0); (1; 0)	✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5)
	OR	
	$f(x)=x^3-x^2-x+1$ $f(x)=(x-1)(x^2-1)$ $f(x)=(x-1)(x-1)(x+1)$ $f(x)=0$ $(x-1)(x-1)(x+1)=0$ x-intercepts: (–1; 0); (1; 0)	✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5)

	$f(x)=x^3-x^2-x+1$ $f(x)=(x+1)(x^2-2x+1)$ $f(x)=(x+1)(x-1)(x-1)$ $f(x)=0$ $(x-1)(x-1)(x+1)=0$ x-intercepts: (–1; 0); (1; 0)	✓ $(x+1)$ ✓ (x^2-2x+1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5)
2.3	$f(x)=x^3-x^2-x+1$ $f'(x)=3x^2-2x-1$ $f'(x)=0$ $(3x+1)(x-1)=0$ $x=-\frac{1}{3}$ or $x=1$ $y=\frac{32}{27}$ $y=0$ $\left(-\frac{1}{3};\frac{32}{27}\right)$ $(1;0)$	✓ $f'(x)=3x^2-2x-1$ ✓ $f'(x)=0$ ✓factorisation ✓x value ✓x value ✓$y=\frac{32}{27}$ (6)
2.4		✓y– and x–intercepts ✓shape ✓turning points (3)
2.5	$f'(x)<0$ $-\frac{1}{3}<x<1$ **OR/*OF*** $(-\frac{1}{3}\ ;1)$	✓ $x>-\frac{1}{3}$ ✓ $x<1$ (2) ✓ $(-\frac{1}{3}$ ✓ 1) (2) **[17]**

3.1	$f(x)=(x+2)(x-1)(x-4)$ $=(x^2+x-2)(x-4)$ $=x^3+x^2-2x-4x^2-4x+8$ $=x^3-3x^2-6x+8$ $b=-3\ ;\ c=-6\ ;\ d=8$	✓✓ $f(x)=(x+2)(x-1)(x-4)$ ✓ expansion ✓ x^3-3x^2-6x+8 (4)
3.2	$f(x)=x^3-3x^2-6x+8$ $f'(x)=0$ $3x^2-6x-6=0$ $x^2-2x-2=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$ $=\frac{2\pm\sqrt{12}}{2}$ $x=-0{,}73$	✓ $f'(x)=0$ ✓ $3x^2-6x-6$ ✓ substitution into correct formula ✓ $x=-0{,}73$ (4)
3.3	$f(x)=x^3-3x^2-6x+8$ $f(-1)=(-1)^3-3(-1)^2-6(-1)+8$ or $f(-1)=(1)(-2)(-5)$ $=10$ $=10$ $f'(-1)=3(-1)^2-6(-1)-6$ $=3$ $y-10=3(x+1)$ $y=3x+13$	✓ $f(-1)=10$ ✓ $f'(-1)=3$ ✓ substitution ✓ $y=3x+13$ (4)
3.4	$f''(x)=6x-6$  	✓ $f''(x)=6x-6$ ✓x- intercept ✓y- intercept (3)
3.5	f concave upwards $f''(x)>0$ $6x-6>0$ $x>1$ NOTE: Answer only 2 / 2	✓ $f''(x)>0$ ✓ $x>1$ (2) **[17]**

4.1	$60 = 2b + 2r + \frac{1}{2}(2\pi r)$ $2b = 60 - 2r - \pi r$ $b = 30 - r - \frac{1}{2}\pi r$	✓ $60 = 2b + 2r + \frac{1}{2}(2\pi r)$ ✓ $b = 30 - r - \frac{1}{2}\pi r$ (2)
4.2	Area = area of rectangle + area of semicircle $A(r) = \text{length} \times \text{breadth} + \frac{1}{2}(\text{area of circle})$ $= (2r)\left(30 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}(\pi r^2)$ $= 60r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$ $= 60r - 2r^2 - \frac{1}{2}\pi r^2$ $= 60r - \left(2 + \frac{1}{2}\pi\right)r^2$ For a maximum, $A'(r) = 0$ $60 - 2\left(2 + \frac{1}{2}\pi\right)r = 0$ $60 - (4 + \pi)r = 0$ $r = \frac{60}{4+\pi}$ $= 8{,}40$ m	✓ $(2r)\left(30 - r - \frac{1}{2}\pi r\right)$ ✓ $\frac{1}{2}(\pi r^2)$ ✓ $60r - 2r^2 - \frac{1}{2}\pi r^2$ ✓ $A'(r) = 0$ ✓ $60 - 2\left(2 + \frac{1}{2}\pi\right)r$ ✓answer (6)
4.3	$3x^3 = 9x^2$ $\therefore 3x^3 - 9x^2 = 0$ ✓ $\therefore 3x^2(x-3) = 0$ ✓ $\therefore x = 0 \text{ or } 3$ ✓	(3)
4.4	$f(x)$ and $f'(x)$ since both have their derivatives ~~will be~~ as zero at $x = 0$ ✓	(1)
4.5	The stationary point on f is a horizontal point of inflection, not a turning point ✓ The stationary point on f' is a minimum turning point ✓	(2)
4.6	$f'(x) = 9x^2$ and $f''(x) = 18x$ $f'(1) = 9$ and $f''(x) = 18$ ✓✓ so the vertical distance between them is 9 ✓	(3) **[17]**
5.1	$3x^3 - 9x^2 < 0$ $\therefore 3x^2(x-3) < 0$ $\therefore x - 3 < 0$ ($3x^2$ is positive) $\therefore x < 3$ but $x \neq 0$	

	$y = x^3 + bx^2 + cx - 4$ $y' = 3x^2 + 2bx + c$ $y'' = 6x + 2b$ At point of inflection: $y'' = 6x + 2b = 0$ Substitute $x = 2$: $6(2) + 2b = 0$ $2b = -12$ $b = -6$ $y = x^3 - 6x^2 + cx - 4$ Substitute $(2;\ 4)$: $4 = 2^3 - 6(2)^2 + c(2) - 4$ $2c = 24$ $c = 12$ $y = x^3 - 6x^2 + 12x - 4$	✓ $y' = 3x^2 + 2bx + c$ ✓ $y'' = 6x + 2b$ ✓ $y'' = 0$ ✓ sub $x = 2$ into $y'' = 0$ ✓ value of b ✓ substitute $(2;4)$ ✓ value of c (7)

5.2	$f(x) = -3x^3 + x$ $-9x^2 + 1 = 0$ $x = \frac{1}{3}$ or $x = -\frac{1}{3}$ Maximum of f will be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)$ $= \frac{2}{9}$ Maximum of $f(x) + q$ will also be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) + q = \frac{8}{9}$ $\frac{2}{9} + q = \frac{8}{9}$ $q = \frac{6}{9}$ $= \frac{2}{3}$ For $f(x) + q$ to have a maximum of $\frac{8}{9}$ the value of q has to be $\frac{2}{3}$.	✓ $-9x^2 + 1 = 0$ ✓ $x = \frac{1}{3}$ *or* $x = -\frac{1}{3}$ ✓ Maximum at $x = \frac{1}{3}$ ✓ $f\left(\frac{1}{3}\right) = \frac{2}{9}$ ✓ $\frac{2}{9} + q = \frac{8}{9}$ ✓ $q = \frac{2}{3}$ **[6]**

5.3	Length of one side of the square / *lengte van sy van vierkant* $=\frac{x}{4}$ Length of the rectangle / *lengte van die reghoek*: $2l+x+\frac{x}{4}=6$ $l=\frac{6-\frac{5x}{4}}{2}$ $=\frac{24-5x}{8}$ $A=\left(\frac{x}{4}\right)^2+\frac{x}{4}\left(\frac{24-5x}{8}\right)$ $=\frac{x^2}{16}+\frac{24x-5x^2}{32}$ $=\frac{24x-3x^2}{32}$ $A=\frac{24x-3x^2}{32}.$ For minimum area / *Vir minimum oppervlaktte* $\frac{dA}{dx}=0$ $\frac{dA}{dx}=\frac{24-6x}{32}$ $6x=24$ $x=4$	✓ $\frac{x}{4}$ ✓ $\frac{6-\frac{5x}{4}}{2}$ or $\frac{24-5x}{8}$ ✓ $\left(\frac{x}{4}\right)^2$ ✓ $\frac{x}{4}\left(\frac{24-5x}{8}\right)$ ✓ $\frac{dA}{dx}=0$ ✓ $\frac{24-6x}{32}$ ✓ $x=4$ (7)
		[20] **TOTAL: 85**

Appendix B: Classroom Observation Schedule

Observation Focus	Focus Area
Teachers	1. Integration of real-life context-based tasks into the lesson 2. Integration of the 8Ps problem-solving model into the lesson 3. Challenges, if any, associated with the incorporation of the 8Ps model into the lesson 4. Problem-solving approach and strategies used in the lesson 5. Attitude towards problem-solving instruction 6. Attitude towards incorporation of 8Ps learning model in class

Control Group Learner Participants	1. Strategies for problem solving 2. Involvement and role during instruction/participation 3. Response to the incorporation of context-based material into instruction 4. Level of dependence on the teacher 5. Challenges, if any, related to context–based problem solving

Experimental Group Learner Participants	1. Problem-solving strategies (old/ new/ integrated) 2. Adaptation to new instruction 3. Innovativeness (how does a learner use context to facilitate problem? 4. Challenges, if any, related to context-based problem-solving instruction 5. Reaction and attitude to new instruction 6. Participation and involvement 7. How do learners use examples to facilitate understanding of new material?

Appendix C: Semi-structured Interview Schedule

Interview Questions for Both Teacher and Learner Participants

1. What is your understanding of differential calculus?
2. What does the concept of stationary points in differential calculus mean?
3. Can you explain briefly problem solving in Mathematics?
4. Do you feel problem solving is useful in Mathematics?
5. What strategies do you currently use in solving Mathematics questions?
6. How can learners acquire useful mathematical problem-solving skills in the classroom?
7. What instructional method did you use to teach/learn the concept of stationary points in differential calculus during this enquiry?
8. How did learners react and respond to the instructional method?
9. What do you feel about this instructional method as a Mathematics teacher/learner?
10. Are you aware of the 8Ps mathematical problem-solving method?
11. Can the method improve learners' mathematical problem-solving performance?
12. Can you recommend the use of 8Ps-based instructional method for teaching and learning stationary points in differential calculus and possibly for other Grade 12 Mathematics topics?
13. Why do you think the 8Ps learning method can improve learners' mathematical problem-solving performance and should therefore be recommended for use in the Mathematics classroom?
14. What challenges may confront the use of 8Ps learning model in the classroom?
15. What may be the attitudes of teachers and learners of Mathematics to the use of 8Ps problem-solving instruction in the Mathematics classroom?
16. Do you think the 8Ps problem-solving approach is in accordance with the National Curriculum Statement on the teaching and learning of Mathematics?

Appendix D

RE: I AM SEEKING YOUR CONSENT TO ADAPT YOUR RESEARCH TOOLS.

To: OMONIYI A A
Tue 2018/10/23 10:11

Dear Omoniyi A. A.

Sorry for taking long to come back to you.

I am giving you my consent to make use or adapt certain aspects of information in my data collection instruments to develop your instruments for your PhD.
I guess you will have a way to acknowledge properly in your thesis the adaptation and use of my instruments.
My permission is granted.

I take this opportunity to wish you well in your studies.

Kind regards
Dr Joseph J Dhlamini

Appendix E1: Learner's Background Information Form

Dear study participant, supply all the required information by ticking (√) in the appropriate portions just as applicable to you. Please, be assured that the information provided shall be used for the purpose of this research only. For the sake of your privacy, write the code assigned you instead of your name. Thank you.

Learner's Code: ________ Gender: (Male or Female): _______ Age: ______ years

Section A: Individual Learner's Parentage

Learner's Parentage Status	**Yes**	**No**	**You may pass a comment.**
I live with my both parents.			
I live with a single parent.			
I stay alone, not with my parent(s).			
I stay with my guardian.			

Section B: Educational Status of Learner's Parent(s)

Parent's educational status		**You may pass a comment here.**
Primary school		
High school: below Grade 12		
Grade 12		
Diploma		
Degree and above		

Section C: Employment Status of Learner's Parent(s)

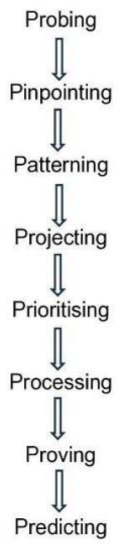
Parent's employment status		**You may pass a comment here.**
Employed in public service		
Self-employed		
Unemployed		

Section D: Availability of Technology and Other Learning Facilities for Home Use

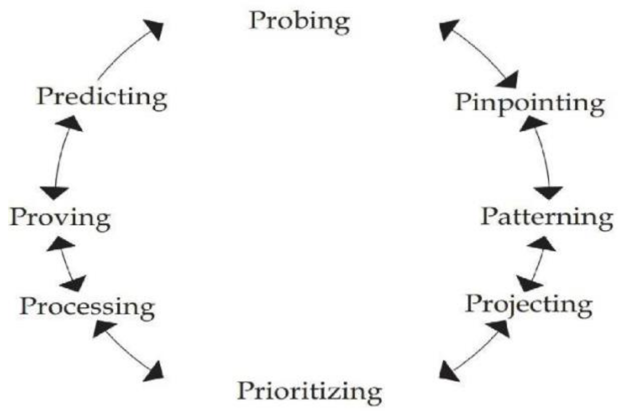
Access to technology and others		**You may pass a comment here.**
Computer, library, Mathematics. games etc. available for home use		
None of these at home		
Not sure		

Appendix E2: The 8Ps-based Wall Chart

THE LINEAR 8Ps LEARNING MODEL



THE CYCLIC 8Ps LEARNING MODEL



Appendix F: The 8Ps-based Flip Cards

P1

?

PROBING

Carefully examine the question to see what it actually requires.

P2

PINPOINTING

Identify the key words or demands of the question.

P3

PATTERNING

Represent the question as useful patterns, possibly.

(such as: equation, graph, pictures, map, diagram, table…)

P4

PROJECTING

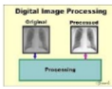
Draw up useful solution plans and strategies to follow.

P5

PRIORITISING

Arrange the solution ideas gathered in order of relevance.

P6



PROCESSING

Use the prioritised solution ideas to obtain a logical solution.

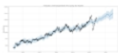
P7

PROVING

Check the correctness of the solution.

Does the solution make sense?

P8



PREDICTING

Find out whether the solution can be applied to similar or other questions.

Appendix G: A Sample of the 8Ps-based Worked-out Examples

Question 6: Consider $g(x) = x^3 - 6x^2 + 11x - 6$.

6.1 Write down the coordinates of the y-intercepts of g.

6.2 Calculate the coordinates of the x-intercepts of g.

6.3 Calculate the coordinates of the turning points of g.

Solution:

6.1 $g(x) = x^3 - 6x^2 + 11x - 6$

$y = g(x) = x^3 - 6x^2 + 11x - 6$ [Since $y = f(x)$]

At y-intercept, $x = 0$.

Substituting $x = 0$ in the equation, we have

$y = (0)^3 - 6(0)^2 + 11(0) - 6$

$y = -6$

∴ Coordinates of the y-intercepts of g = (0; -6).

6.2 At x-intercept, $y = 0$.

Substituting $y = 0$ in the equation, we have

$0 = x^3 - 6x^2 + 11x - 6$

Try $f(x) = \pm 1; \pm 2; \pm 3; \pm 4; \ldots$ to obtain zeroes/factors of the equation.

$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6$

$f(1) = 1 - 6 + 11 - 6$

$f(1) = 0$

This implies that $x = 1$, that is $(x - 1)$ is a factor of the equation.

To get the other two factors, use division method and $(x - 1)$ as a divisor.

Or try substituting $f(2)$ and $f(3)$, which are also factors of the equation.

The other two factors are $(x - 2)$ and $(x - 3)$.

You can also factorise $x^3 - 6x^2 + 11x - 6 = 0$ completely to get the three factors.

$\therefore\ x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3) = 0$

Either $x - 1 = 0,\ x - 2 = 0$ or $x - 3 = 0$

x-intercepts: $x = 1,\ 2$ and 3

Since at x-intercept, $y = 0$, the coordinates of x-intercepts of g are:

(1; 0), (2; 0), (3; 0).

6.3 $g(x) = x^3 - 6x^2 + 11x - 6$

Obtain the derivative of $g(x)$ as follows

$g'(x) = 3x^2 - 12x + 11$

At a turning/stationary point, gradient of a curve = 0

$\therefore 3x^2 - 12x + 11 = 0$

Since the equation is not factorisable, use the general method or the completing the square method to solve for x. Using the completing the square method,

$3x^2 - 12x = 11$

Divide through by 3 to have

$x^2 \ - 4x = \frac{11}{3}$

Now, take the half of the coefficient of the 2nd term, square it and add it to both sides. This means adding $(\frac{-4}{2})^2$. Having done this, the 2nd term, $-4x$, will not be included again as *it has finished its own role*.

$\therefore x^2 + (\frac{-4}{2})^2 = \frac{11}{3} + (\frac{-4}{2})^2$

$x^2 + (-2)^2 = \frac{11}{3} + (-2)^2$

Take note of how the brackets are open at the left side of the equation as follows in the process of completing the square. That is, open the brackets and multiply the two signs ($+ \times - = -$).

$(x-2)^2 = \frac{11}{3} + 4$

$(x-2)^2 = \frac{23}{3}$

Find the square root of both sides

$$x - 2 = \pm \sqrt{\frac{23}{3}}$$

$$x = 2 \ \pm \sqrt{\frac{23}{3}}$$

$x = 2 \ \pm \ 2.7689$

$x = 4.7689$ or -0.7689

$x \approx 4.8$ or -0.8 (x-coordinates of the turning points of g)

For *y*-coordinates, substitute each x-coordinate in

$y = g(x) = \ x^3 - 6x^2 + 11x - 6$

When $x = \ 4.7689$,

$y = \ (4.7689)^3 - 6(4.7689)^2 + 11(4.7689) - 6$

$y = 108.46 - 136.46 + 52.46 - 6$

$y = \ 18.46$

Also, when $x = \ -0.7689$,

$y = \ (-0.7689)^3 \ -6(-0.7689)^2 + 11(-0.7689) - 6$

$y = -0.4546 - 3.5472 - 8.4579 - 6$

$y = \ -18.46$

∴ Turning points = $(4.8; 18.5)$, $(-0.8; -18.5)$

Appendix H

Mathematical Problem-solving Assessment Form

Problem-solving Assessment Criteria	High-Level PS (3 marks)	Medium-Level PS (2 marks)	Low-Level PS (1 mark)	Total
Application of correct formula				
Translation of mathematical Problems to suitable equation				
Representation of mathematical problem as helpful patterns				
Connection of mathematical problem to prior knowledge				
Demonstration of logical and sequential solution-steps				
Use of reasonable justifications for solution-steps				
Ability to explain and apply the solutions obtained				
Total				

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APPENDIX J1

LETTER TO THE PRINCIPAL (EXPERIMENTAL SCHOOL)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Date: …………………………

Dr./Mr./Mrs.……………………………………..

The School Principal,

…………………………………………………….

…………………………………………………….

Dear Sir,

<u>Request for Permission to conduct Research in Your School</u>

I, A. A. Omoniyi (student researcher), am doing research with Prof. L D Mogari, Head, Institute of Science and Technology Education (ISTE) in the College of Graduate Studies/ISTE towards a PhD in Mathematics, Science and Technology Education at the University of South Africa (UNISA). We are inviting you to participate in a 5-week study entitled, *The Impact of 8Ps Learning Model on the Mathematical Problem Solving of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*. The main aim of the study is to evaluate the nature and level of mathematical problem-solving approach of South African Grade 12 learners, the challenges they may be encountering in their problem-solving processes and to explore suitable learning strategies capable of improving their mathematical problem-solving performance.

Your school has been purposively selected as one of the eight schools in the education district suitable for this study in terms of having largely similar or comparable features such as: the school location, infrastructure, teachers' qualifications, teaching-learning facilities, learners' performance level, English Language as the same medium of instruction, among others.

The study entails using 8Ps learning approach as the intervention in the experimental group and the conventional instructional method in the control group. It will employ problem-solving ability

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test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection.

Your school being one of the four schools chosen to be in the **Experimental Group**, I, as the researcher, will be the Mathematics teacher to administer the 8Ps instruction (intervention) to the intact group of your Grade 12 Mathematics learners on the Concept of Stationary Points in Differential Calculus. I will make comprehensive notes of the emergent themes observed during the class lessons. I will conduct the problem-solving ability pre-test (90 minutes in duration) with the learners on a suitable day a week to the 3-week intervention and also a 90-minute problem-solving ability post-test on another convenient day a week after the intervention. In addition, I will interview the Mathematics educator(s) and few selected learners (each for about 20 minutes) to obtain their views and opinions on the implementation and relevance of the 8Ps problem-solving model.

The study is capable of offering the following benefits: it will give the understanding of how the learners actually solve Mathematics problems, identify the difficulties they experience during problem-solving and, essentially, offer useful knowledge that can equip them with necessary mathematical problem-solving skills for obtaining logical solutions. The study can help the learners develop positive attitude and confidence required for solving Mathematics problems. It can as well assist Mathematics educators in designing and delivering helpful problem-solving instruction that can enhance the learners' mathematical critical and problem-solving abilities.

Please note that there is no inconvenience, discomfort, harm or risk of any form to be experienced by the participants. Be assured that all the research ethical measures will

be duly observed throughout the conduct of the research. Upon the completion of the study, its final submission to UNISA and its approval by the University for the Award of the doctoral degree, the Gauteng Department of Education, the Tshwane West education district office, your school and other stakeholders in the study will be given the feedback of the results of the study.

It is in view of the above that we solicit your support in this matter. We thank you in advance for your time to help us. We are confident you would be satisfied with the outcome of the investigation.

Yours sincerely,

……………………………….

Omoniyi, Adebayo Akinyinka.

(Student Number: 51940124).

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APPENDIX J2

LETTER TO THE PRINCIPAL (CONTROL SCHOOL)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Date: …………………………

Dr./Mr./Mrs……………………………………..

The School Principal,

…………………………………………………….

…………………………………………………….

Dear Sir,

Request for Permission to conduct Research in Your School

I, Omoniyi, A, A, (student researcher), am doing research with Prof. L D Mogari, Institute of Science and Technology Education (ISTE), College of Graduate Studies towards a PhD in Mathematics, Science and Technology Education at the University of South Africa (UNISA). We are inviting you to participate in a 3-week study entitled, *The Impact of 8Ps Learning Model on the Mathematical Problem Solving of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*. The main aim of the study is to evaluate the nature and level of mathematical problem-solving approach of South African Grade 12 learners, the challenges they may be encountering in their problem-solving processes and to explore suitable learning strategies capable of improving their mathematical problem-solving performance.

Your school has been purposively selected as one of the eight schools in the education district suitable for this study in terms of having largely similar or comparable features such as: the school location, infrastructure, teachers' qualifications, teaching-learning facilities, learners' performance level, English Language as the same medium of instruction, among others.

The study entails using 8Ps learning approach as the intervention in the experimental group and the conventional instructional method in the control group. It will employ problem-solving ability test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection.

Since your school is chosen as one of the four schools to be in the **Control Group**, your regular Mathematics teacher(s) will administer the usual traditional instruction to the intact group of the Grade 12 Mathematics learners in the school on the Concept of Stationary Points in Differential Calculus. I, as the researcher, on the other hand, will be around to observe some of the class lessons. I will conduct the problem-solving ability pre-test (90 minutes in duration) with the learners on a suitable day a week to the 3-week intervention and also a 90-minute problem-solving ability post-test on another convenient day a week after the intervention. Although the 8Ps learning model will not be applied in your school, to ensure that your school learners directly share of its potential benefits, at a time suitable at the end of the research, its application will be demonstrated to your Mathematics educators who can in turn use it to support the learners in mathematical problem-solving.

The study is capable of offering the following benefits: it will give the understanding of how the learners actually solve Mathematics problems, identify the difficulties they experience during problem-solving and, essentially, offer useful knowledge that can equip them with necessary mathematical problem-solving skills for obtaining logical solutions. The study can help the learners develop positive attitude and confidence required for solving Mathematics problems. It can as well assist Mathematics educators in designing and delivering helpful problem-solving instruction that can enhance the learners' mathematical critical and problem-solving abilities.

Please note that there is no inconvenience, discomfort, harm or risk of any form to be experienced by the participants. Be assured that all the research ethical measures will be duly observed throughout the conduct of the research. Upon the completion of the study, its final submission to UNISA and its approval by the University for the Award of the doctoral degree, the Gauteng Department of Education, the Tshwane West education district office, your school and other stakeholders in the study will be given the feedback of the results of the study.

It is in view of the above that we solicit your support in this matter. We thank you in advance for your time to help us. We are confident you would be satisfied with the outcome of the investigation.

Yours sincerely,

……………………………….

Omoniyi, Adebayo Akinyinka.

(Student Number: 51940124).

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APPENDIX K1

LETTER TO THE CHAIRPERSON SGB (EXPERIMENTAL SCHOOL)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Date: ……………………………

Dr./Mr./Mrs.……………………………………

The Chairperson, School Governing Board,

…………………………………………………….

…………………………………………………….

Dear Sir,

Request for Permission to conduct Research in Your School

I, Omoniyi, A A (student researcher), am doing research with Prof. L D Mogari, Head, Institute of Science and Technology Education (ISTE) in the College of Graduate Studies/ISTE towards a PhD in Mathematics, Science and Technology Education at the University of South Africa (UNISA). We are inviting you to participate in a 5-week study entitled, *The Impact of 8Ps Learning Model on the Mathematical Problem Solving of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*. The main aim of the study is to evaluate the nature and level of mathematical problem-solving approach of South African Grade 12 learners, the challenges they may be encountering in their problem-solving processes and to explore suitable learning strategies capable of improving their mathematical problem-solving performance.

Your school has been purposively selected as one of the eight schools in the education district suitable for this study in terms of having largely similar or comparable features such as: the school location, infrastructure, teachers' qualifications, teaching-learning facilities, learners' performance level, English Language as the same medium of instruction, among others.

The study entails using 8Ps learning approach as the intervention in the experimental group and the conventional instructional method in the control group. It will employ problem-solving ability test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection.

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Your school being one of the four schools chosen to be in the **Experimental Group**, I, as the researcher, will be the Mathematics teacher to administer the 8Ps instruction (intervention) to the intact group of your Grade 12 Mathematics learners on the Concept of Stationary Points in Differential Calculus. I will make comprehensive notes of the emergent themes observed during the class lessons. I will conduct the problem-solving ability pre-test (90 minutes in duration) with the learners on a suitable day a week to the 3-week intervention and also a 90-minute problem-solving ability post-test on another convenient day a week after the intervention. In addition, I will interview the Mathematics educator(s) and few selected learners (each for about 20 minutes) to obtain their views and opinions on the implementation and relevance of the 8Ps problem-solving model.

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Please note that there is no inconvenience, discomfort, harm or risk of any form to be experienced by the participants. Be assured that all the research ethical measures will be duly observed throughout the conduct of the research. Upon the completion of the study, its final submission to UNISA and its approval by the university for the award of the doctoral degree, the Gauteng Department of Education, the Tshwane West education district office, your school and other stakeholders in the study will be given the feedback of the results of the study.

It is in view of the above that we solicit your support in this matter. We thank you in advance for your time to help us. We are confident you would be satisfied with the outcome of the investigation.

Yours sincerely,

…………………………….

Omoniyi, Adebayo Akinyinka.

(Student Number: 51940124).

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APPENDIX K2

LETTER TO THE CHAIRPERSON SGB (CONTROL GROUP)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Date: …………………………

Dr./Mr./Mrs.……………………………………..

The Chairperson, School Governing Board,

…………………………………………………….

…………………………………………………….

Dear Sir/Ma,

Request for Permission to conduct Research in Your School

I, Omoniyi, A, A, (student researcher), am doing research with Prof. L D Mogari, Institute of Science and Technology Education (ISTE), College of Graduate Studies towards a PhD in Mathematics, Science and Technology Education at the University of South Africa (UNISA). We are inviting you to participate in a 3-week study entitled, *The Impact of 8Ps Learning Model on the Mathematical Problem Solving of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*. The main aim of the study is to evaluate the nature and level of mathematical problem-solving approach of South African Grade 12 learners, the challenges they may be encountering in their problem-solving processes and to explore suitable learning strategies capable of improving their mathematical problem-solving performance.

Your school has been purposively selected as one of the eight schools in the education district suitable for this study in terms of having largely similar or comparable features such as: the school location, infrastructure, teachers' qualifications, teaching-learning facilities, learners' performance level, English Language as the same medium of instruction, among others.

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Since your school is chosen as one of the four schools to be in the **Control Group**, your regular Mathematics teacher(s) will administer the usual traditional instruction to the intact group of the Grade 12 Mathematics learners in the school on the Concept of Stationary Points in Differential Calculus. I, as the researcher, on the other hand, will be around to observe some of the class lessons. I will conduct the problem-solving ability pre-test (90 minutes in duration) with the learners on a suitable day a week to the 3-week intervention and also a 90-minute problem-solving ability post-test on another convenient day a week after the intervention. Although the 8Ps learning model (intervention) will not be applied in your school, to ensure that your school learners directly share of its potential benefits, at a time suitable at the end of the research, its application will be demonstrated to your Mathematics educators who can in turn use it to support the learners in mathematical problem-solving.

The study is capable of offering the following benefits: it will give the understanding of how the learners actually solve Mathematics problems, identify the difficulties they experience during problem-solving and, essentially, offer useful knowledge that can equip them with necessary mathematical problem-solving skills for obtaining logical solutions. The study can help the learners develop positive attitude and confidence required for solving Mathematics problems. It can as well assist Mathematics educators in designing and delivering helpful problem-solving instruction that can enhance the learners' mathematical critical and problem-solving abilities.

Please note that there is no harm or risk of any form to be experienced by the participants. Be assured that all the research ethical measures will be duly observed throughout the conduct of the research. Upon the completion of the study, its final submission to UNISA and its approval by the University for the award of the doctoral degree, the Gauteng Department of Education, the Tshwane West education district office, your school and other stakeholders in the study will be given the feedback of the results of the study.

It is in view of the above that we solicit your support in this matter. We thank you in advance for your time to help us. We are confident you would be satisfied with the outcome of the investigation.

Yours sincerely,

…………………………….

Omoniyi, Adebayo Akinyinka.
(Student Number: 51940124).

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APPENDIX L1

INFORMATION FOR PARTICIPATING TEACHER: EXPERIMENTAL GROUP

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.**: 8/4/4/1/2

Research Title: *The Impact of 8Ps Learning Model on the Mathematical Problem- Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*

Dear Mr/Ms/Mrs………………………………….,

I am A. A. Omoniyi (student number: 51940124), doing research with Prof. L D Mogari of the Institute of Science and Technology Education (ISTE), College of Graduate Studies, towards a PhD. in Mathematics, Science and Technology Education at UNISA. We are inviting you to participate in the **experimental group** of the study.

Purpose of the study

The research sets out to evaluate how the Grade 12 Mathematics learners actually solve Mathematics problems, to identify the difficulties they may have during problem-solving and, essentially, to offer useful knowledge that can equip them with necessary mathematical problem-solving skills and strategies. The study can help the learners develop positive attitude and confidence required for solving Mathematics problems. It can also assist Mathematics educators in designing and delivering helpful problem-solving instruction capable of enhancing the learners' critical and problem-solving abilities in Mathematics.

Why you are being invited to participate in the study

Your school is purposively selected as one of the eight schools in the education district suitable for this study. Only one intact class of Grade 12 Mathematics learners of your school is needed to participate in the empirical research to be carried out during the school lesson periods. As a/the learners' Mathematics teacher, your participation is therefore required.

Nature of your participation in the study

The study entails using 8Ps learning approach as the intervention in the experimental group and the conventional instructional method in the control group. It will employ problem-solving achievement test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection. The researcher plans to play the role of the Mathematics teacher to administer the 8Ps problem-solving instruction on the concept of stationary points in differential calculus. This will hold for three weeks in April/May of Term 2 as officially planned in the Grade 12 Mathematics curriculum. Although your involvement as an educator in the experimental group is not to carry out the intervention, you are expected to give support to the researcher in the following areas: observe some of my intervention lessons and provide feedback; co-invigilate the pre-test and post-test with me; general co-ordination of the learners; answering some interview questions, among others.

Can you withdraw from this study even after agreeing to participate?

Yes, you can. Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are also free to withdraw from the study at any time if necessary and without giving a reason.

Are there any negative consequences for participating in the research project?

No, the participants will suffer no inconvenience, discomfort, harm or risk in the study.

Will any information you may supply and your identity be kept confidential?

Yes. Your name will not be recorded anywhere in the study. No one, apart from the researcher and the identified members of the research team, will know about your involvement in this research. Please note that your data may be used for other purposes such as: a research report, journal articles and/or conference proceedings. But your privacy will be duly protected. For instance, your responses will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

How the researcher will protect the security of data

The hard copies of your answers will be stored away by the researcher for five years in a locked cupboard in his personal library/reading room for future research or academic purposes. The electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval, if applicable. After the period of five years, the hard copies of the information will be burnt while its electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

Will you receive payment or any incentives for participating in this study?

Regrettably no payment or reward, financial or otherwise, will be offered to study participants. No cost whatsoever will be incurred by participants either.

Has the study received ethics approval?

Yes, the study has obtained written approval from the Research Ethics Review Committee of UNISA. You may request a copy of the approval letter from the researcher if you so wish.

How will you be informed of the findings/results of the research?

If you would like to be informed of the final research findings or require any further information or want to contact the researcher about any aspect of this study, please contact: A. A. Omoniyi on cellphone number +27744550208, fax number 0866081283 or e-mail address 51940124@mylife.unisa.ac.za. The findings will be made available after the thesis might have been submitted and approved for the award of the doctoral degree for which it is being conducted. Should you have any concerns about the conduct of the research, you may contact the researcher's Supervisor on 0123376168 or mogarld@unisa.ac.za. In case you have any ethical concerns, contact the UNISA research ethics chairperson.

Thank you for deciding to participate in this study.

………………………………………………………

Omoniyi, A. A. (Student Number: 51940124)
Researcher

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APPENDIX L2

INFORMATION FOR PARTICIPATING TEACHER: CONTROL GROUP

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.**: 8/4/4/1/2

Research Title: *The Impact of 8Ps Learning Model on the Mathematical Problem- Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*

Dear Mr/Ms/Mrs………………………………..,

I am A. A. Omoniyi (student number: 51940124), doing research with Prof. L D Mogari of the Institute of Science and Technology Education (ISTE), College of Graduate Studies, towards a PhD. in Mathematics, Science and Technology Education at UNISA. We are inviting you to participate in the **control group** of the study.

Purpose of the study

The research sets out to evaluate how the Grade 12 Mathematics learners actually solve Mathematics problems, to identify the difficulties they may have during problem-solving and, essentially, to offer useful knowledge that can equip them with necessary mathematical problem-solving skills and strategies. The study can help the learners develop positive attitude and confidence required for solving Mathematics problems. It can also assist Mathematics educators in designing and delivering helpful problem-solving instruction capable of enhancing the learners' critical and problem-solving abilities in Mathematics.

Why you are being invited to participate in the study

Your school is purposively selected as one of the eight schools in the education district suitable for this study. Only one intact class of Grade 12 Mathematics learners of your school is needed to participate in the empirical research to be carried out during the school lesson periods. As a/the learners' Mathematics teacher, your participation in is required.

Nature of your participation in the study

The study entails using 8Ps learning approach as the intervention in the experimental group and the conventional instructional method in the control group. It will employ problem-solving achievement test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection. While the researcher is to play the role of the Mathematics teacher in the experimental schools to administer the 8Ps problem-solving instruction on the concept of stationary points in differential calculus, you are expected to administer traditional instruction to your Grade 12 Mathematics learners on the same concept. This will hold for three weeks in April/May of Term 2 as officially planned in the Grade 12 Mathematics curriculum. During this period, I as the researcher, will pay you some scheduled visits to observe some of your class lessons. Kindly note also that you are to co-invigilate with me the pre-test and post-test; co-ordinate the learners; answer some interview questions and give me general support throughout the investigation period.

Can you withdraw from this study even after agreeing to participate?

Yes, you can. Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are also free to withdraw from the study at any time if necessary and without giving a reason.

Are there any negative consequences for participating in the research project?

No, the participants will suffer no inconvenience, discomfort, harm or risk in the study.

Will any information you may supply and your identity be kept confidential?

Yes. Your name will not be recorded anywhere in the study. No one, apart from the researcher and the identified members of the research team, will know about your involvement in this research. Please note that your data may be used for other purposes such as: a research report, journal articles and/or conference proceedings. But your privacy will be duly protected. For instance, your responses will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

How the researcher will protect the security of data

The hard copies of your answers will be stored away by the researcher for five years in a locked cupboard in his personal library/reading room for future research or academic purposes. The electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval, if applicable. After the period of five years, the hard copies of the information will be burnt while its electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

Will you receive payment or any incentives for participating in this study?

Regrettably no payment or reward, financial or otherwise, will be offered to study participants. No cost whatsoever will be incurred by participants either.

Has the study received ethics approval?

Yes, the study has obtained written approval from the Research Ethics Review Committee of UNISA. You may request a copy of the approval letter from the researcher if you so wish.

How will you be informed of the findings/results of the research?

If you would like to be informed of the final research findings or require any further information or want to contact the researcher about any aspect of this study, please contact: A. A. Omoniyi on cellphone number +27744550208, fax number 0866081283 or e-mail address 51940124@mylife.unisa.ac.za. The findings will be made available after the thesis might have been submitted and approved for the award of the doctoral degree for which it is being conducted. Should you have any concerns about the conduct of the research, you may contact the researcher's Supervisor on 0123376168 or mogarld@unisa.ac.za. In case you have any ethical concerns, contact the UNISA research ethics chairperson.

Thank you for deciding to participate in this study.

………………………………………………………

Omoniyi, A. A. (Student Number: 51940124)

Researcher

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APPENDIX L3

EDUCATOR'S CONSENT TO PARTICIPATE IN THE STUDY

(EXPERIMENTAL SCHOOL)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

I, ………………………………………………………………………………, confirm that the student researcher (A. A. Omoniyi: 51940124) requesting my consent to take part in this research has explained to me the nature, procedure, potential benefits and anticipated inconvenience of my participation.

I have read and understood the study as explained in the information sheets.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation **in the Experimental Group as a support to the researcher** is voluntary and that I am free to withdraw from the study at any time (if need be) without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I have received a signed copy of the informed consent agreement.

Participant's Name & Surname: ……………………………………………………………(Please print)

Participant's Signature: …………………………………………… Date: ………………………………….

Researcher's Name & Surname: ………………………………………………………………………………….

Researcher's Signature: ………………………………… Date: ………………………………..

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APPENDIX L4

EDUCATOR'S CONSENT TO PARTICIPATE IN THE STUDY (CONTROL SCHOOL)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

I, ……………………………………………………………………………………, confirm that the student researcher (A. A. Omoniyi: 51940124) requesting my consent to take part in this research has explained to me the nature, procedure, potential benefits and anticipated inconvenience of my participation.

I have read and understood the study as explained in the information sheets.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation **as a Mathematics teacher in the Control Group** is voluntary and that I am free to withdraw from the study at any time (if need be) without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I have received a signed copy of the informed consent agreement.

Participant's Name & Surname : ……………………………………………………………(Please print)

Participant's Signature : ……………………………………………… Date : ………………………….

Researcher's Name & Surname: ………………………………………………………………………. ……

Researcher's Signature: …………………………………………… Date: ………………………………….

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APPENDIX M1

INFORMATION FOR PARENT OF LEARNER PARTICIPATING IN THE EXPERIMENTAL GROUP

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Research Title: *The Impact of 8Ps Learning Model on the Mathematical Problem-Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*

Dear Parent,

I am A. A. Omoniyi (student number: 51940124), doing research with Prof. L D Mogari of the Institute of Science and Technology Education (ISTE), College of Graduate Studies, towards a PhD. in Mathematics, Science and Technology Education at UNISA. We are inviting your child to participate in the **experimental group** of the study.

Purpose of the study

The research sets out to evaluate how the Grade 12 Mathematics learners actually solve Mathematics problems, to identify the difficulties they may have during problem-solving and, essentially, to offer useful knowledge that can equip them with useful mathematical problem-solving skills and strategies. The study can help the learners develop positive attitude and confidence for solving Mathematics problems. It can also assist Mathematics educators to design and deliver helpful problem-solving instruction which can enhance the learners' critical and problem-solving skills in Mathematics.

Why your child is being invited to participate in the study

Your child's school is purposively selected as one of the eight schools in the education district suitable for this study. Only one intact class of Grade 12 Mathematics learners of the school is needed to participate in the empirical research to be carried out during the school lesson periods. Since your child is one of the school's Grade 12 learners for the investigation, his/her participation in the empirical research is being sought.

Nature of your child's participation in the study

The study entails using 8Ps learning approach as the intervention in the experimental group and the conventional instructional method in the control group. It will employ problem-solving achievement test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection. Please note that I, as the researcher, am to conduct the intervention (application of the 8Ps learning method) for the teaching and learning of the

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concept of stationary points in differential calculus in the experimental schools. In specific terms, as a learner participating in the **experimental group**, here below is his/her role in the study:

He/she, alongside other Grade 12 learners, is to participate in the writing of a 90-minute problem-solving achievement pre-test and post-test on the concept of stationary points in differential calculus on some convenient days before and after the research respectively. The purpose of this pre-test is to find out the learners' initial mathematical problem-solving skills in that aspect of Mathematics. On the other hand, the purpose of this post-test is to measure the improvement they may have made after being taught the topic.

He/she is also expected to attend and be fully involved in all the lessons on the topic which their regular Mathematics teacher using the usual traditional instruction. This will take place for three weeks in April/May of Term 2 as officially planned in the Grade 12 Mathematics curriculum. Please note again that this serves as their normal school lessons topic and so should be taken with utmost seriousness.

After the post-test, he/she will be requested to answer some questions in a short interview with me for about 20 minutes based on the problem-solving 8Ps instruction he/she has received. His/her availability is required here only if selected as one of the few learners for the interview.

Can he/she withdraw from this study even after having agreed to participate?

Yes, he/she can. Participating in this study is voluntary and he/she is under no obligation to consent to participation. If you decide to allow your child to take part, you will be given this information sheet to keep and be asked to sign a written consent form. He/she is also free to withdraw from the study at any time if necessary and without giving a reason.

Are there any negative consequences for his/her participation in the research project?

No potential inconvenience, discomfort, harm or risk whatsoever is to be experienced by the participants.

Will your child's identity and the information he/she may supply be kept confidential?

Yes, all the information he/she may supply as a participant in the enquiry and his/her identity will be kept strictly confidential. His/her name will not be recorded anywhere in the study and no one, apart from the researcher and the identified members of the research team, will know about his/her involvement in this research. Please note that his/her anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings. But his/her privacy will be well-protected in any publication of the information. As a way of doing this, his/her answers/responses will be given a code number or a pseudonym and will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

How the researcher will protect the security of data.

Hard copies of his/her answers will be stored by the researcher for a period of five years in a locked cupboard in my personal library/reading room for future research or academic purposes. The electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After a period of five years of the study has passed, the hard copies of the information will be shredded and burnt while its electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

Will he/she receive payment or any incentives for participating in this study?

Regrettably no payment or reward, financial or otherwise, will be offered to study participants and no cost whatsoever will be incurred by participants.

Has the study received ethics approval?

This study has received written approval from the Research Ethics Review Committee of UNISA. You can obtain a copy of the approval letter from the researcher if you so wish.

How will you be informed of the findings/results of the research?

If you would like to be informed of the final research findings or require any further information or want to contact the researcher about any aspect of this study, please contact: Omoniyi, A. A. on cellphone number +27744550208, fax number 0866081283 or e-mail address 51940124@mylife.unisa.ac.za. The findings will be made available after the thesis has been submitted and approved for the award of the doctoral degree for which it is being conducted. Should you have any concerns about the way the research has been conducted, you may contact the researcher's Supervisor on 0123376168 or mogarld@unisa.ac.za. In case you have any ethical concerns, contact the UNISA research ethics chairperson.

Thank you for taking time to read this information sheet and for deciding to allow your child to participate in this study.

………………………………………………………

Omoniyi, A. A. (Student Number: 51940124)

Researcher

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APPENDIX M2

INFORMATION FOR PARENT OF LEARNER PARTICIPATING IN THE CONTROL GROUP

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Research Title: *The Impact of 8Ps Learning Model on the Mathematical Problem-Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*

Dear Parent,

I am A. A. Omoniyi (student number: 51940124), doing research with Prof. L D Mogari of the Institute of Science and Technology Education (ISTE), College of Graduate Studies, towards a PhD. in Mathematics, Science and Technology Education at UNISA. We are inviting your child to participate in the **control group** of the study.

Purpose of the study

The research sets out to evaluate how the Grade 12 Mathematics learners actually solve Mathematics problems, to identify the difficulties they may have during problem-solving and, essentially, to offer useful knowledge that can equip them with useful mathematical problem-solving skills and strategies. The study can help the learners develop positive attitude and confidence for solving Mathematics problems. It can also assist Mathematics educators to design and deliver helpful problem-solving instruction which can enhance the learners' critical and problem-solving skills in Mathematics.

Why your child is being invited to participate in the study

Your child's school is purposively selected as one of the eight schools in the education district suitable for this study. Only one intact class of Grade 12 Mathematics learners of the school is needed to participate in the empirical research to be carried out during the school lesson periods. Since your child is one of the school's Grade 12 learners for the investigation, his/her participation in the empirical research is being sought.

Nature of your child's participation in the study

The study entails using 8Ps learning approach as the intervention in the experimental group and the traditional instructional method in the control group. It will employ problem-solving achievement test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection. Please note that your child's Mathematics teacher will use the traditional instructional method to teach the learners the concept of stationary points in

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differential calculus. In specific terms, as a learner participating in the **control group**, here below is his/her role in the study:

He/she, alongside other Grade 12 learners, is to participate in the writing of a 90-minute problem-solving achievement pre-test and post-test on the concept of stationary points in differential calculus on some convenient days before and after the research respectively. The purpose of this pre-test is to find out the learners' initial mathematical problem-solving skills in that aspect of Mathematics. On the other hand, the purpose of this post-test is to measure the improvement they may have made after being taught the topic.

He/she is also expected to attend and be fully involved in all the lessons on the topic which their regular Mathematics teacher using the usual traditional instruction. This will take place for three weeks in April/May of Term 2 as officially planned in the Grade 12 Mathematics curriculum. Please note again that this serves as their normal school lessons topic and so should be taken with utmost seriousness.

After the post-test, he/she will be requested to answer some questions in a short interview with me for about 20 minutes based on the problem-solving 8Ps instruction he/she has received. His/her availability is required here only if selected as one of the few learners for the interview.

Can he/she withdraw from this study even after having agreed to participate?
Yes, he/she can. Participating in this study is voluntary and he/she is under no obligation to consent to participation. If you decide to allow your child to take part, you will be given this information sheet to keep and be asked to sign a written consent form. He/she is also free to withdraw from the study at any time if necessary and without giving a reason.

Are there any negative consequences for his/her participation in the research project?
No potential inconvenience, discomfort, harm or risk whatsoever is to be experienced by the participants.

Will your child's identity and the information he/she may supply be kept confidential?
Yes, all the information he/she may supply as a participant in the enquiry and his/her identity will be kept strictly confidential. His/her name will not be recorded anywhere in the study and no one, apart from the researcher and the identified members of the research team, will know about his/her involvement in this research. Please note that his/her anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings. But his/her privacy will be well-protected in any publication of the information. As a way of doing this, his/her answers/responses will be given a code number or a pseudonym and

will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

How the researcher will protect the security of data.

Hard copies of his/her answers will be stored by the researcher for a period of five years in a locked cupboard in my personal library/reading room for future research or academic purposes. The electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After a period of five years of the study has passed, the hard copies of the information will be shredded and burnt while its electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

Will he/she receive payment or any incentives for participating in this study?

Regrettably no payment or reward, financial or otherwise, will be offered to study participants and no cost whatsoever will be incurred by participants.

Has the study received ethics approval?

This study has received written approval from the Research Ethics Review Committee of UNISA. You can obtain a copy of the approval letter from the researcher if you so wish.

How will you be informed of the findings/results of the research?

If you would like to be informed of the final research findings or require any further information or want to contact the researcher about any aspect of this study, please contact: Omoniyi, A. A. on cellphone number +27744550208, fax number 0866081283 or e-mail address 51940124@mylife.unisa.ac.za. The findings will be made available after the thesis has been submitted and approved for the award of the doctoral degree for which it is being conducted. Should you have any concerns about the way the research has been conducted, you may contact the researcher's Supervisor on 0123376168 or mogarld@unisa.ac.za. In case you have any ethical concerns, contact the UNISA research ethics chairperson.

Thank you for taking time to read this information sheet and for deciding to allow your child to participate in this study.

……………………………………………………

Omoniyi, A. A. (Student Number: 51940124)
Researcher

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APPENDIX M3

PARENTAL CONSENT TO CHILD'S PARTICIPATION IN THE STUDY

(EXPERIMENTAL GROUP: FOR A LEARNER LESS THAN 18)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.**: 8/4/4/1/2

I, ………………………………………………………, confirm that I consent to my child participating in the **Experimental Group** of the study (*The Impact of 8Ps Learning Model on the Problem-Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*) to be conducted by the student researcher (A. A. Omoniyi: 51940124). I have been informed about the nature, procedure, potential benefits and anticipated inconvenience of my child's participation.

I have also read and understood the study as explained in the information sheets.

I have sufficient opportunity to ask questions and hereby allow my child to take part in the study.

I understand that his/her participation is voluntary and that he/she is free to withdraw any time from the study (if need be) without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my child's participation will be kept confidential unless otherwise specified.

I have received a signed copy of the informed consent agreement.

Participant's Name & Surname: ………………………………………………………….(Please print)

Name & Surname of Participant's Parent: ……………………………………………. (Please print)

Signature of Participant's Parent: ……………………………… Date: ……………………………..

Researcher's Name & Surname: ……………………………… Date: ………………………………

Researcher's Signature: ……………………………………….. Date: ………………………………

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APPENDIX M4

PARENTAL CONSENT TO CHILD'S PARTICIPATION IN THE STUDY

(CONTROL GROUP: FOR A LEARNER LESS THAN 18)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.**: 8/4/4/1/2

I, ……………………………………………………, confirm that I consent to my child participating in the **Control Group** of the study (*The Impact of 8Ps Learning Model on the Problem-Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*) to be conducted by the student researcher (A. A. Omoniyi: 51940124). I have been informed about the nature, procedure, potential benefits and anticipated inconvenience of my child's participation.

I have also read and understood the study as explained in the information sheets.

I have sufficient opportunity to ask questions and hereby allow my child to take part in the study.

I understand that his/her participation is voluntary and that he/she is free to withdraw any time from the study (if need be) without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my child's participation will be kept confidential unless otherwise specified.

I have received a signed copy of the informed consent agreement.

Participant's Name & Surname: …………………………………………………………. .(Please print)

Name & Surname of Participant's Parent: ……………………………………………. (Please print)

Signature of Participant's Parent: ……………………………… Date: ………………………….

Researcher's Name & Surname: ……………………………… Date: ……………………………

Researcher's Signature: …………………………………………. Date: …………………………….

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APPENDIX N1

INFORMATION FOR PARTICIPATING LEARNER: EXPERIMENTAL GROUP

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Dear Learner,

My name is A. A. Omoniyi (student number: 51940124). I am doing research with Prof. L D Mogari, Head, Institute of Science and Technology Education (ISTE), College of Graduate Studies, towards a PhD. in Mathematics, Science and Technology Education at the University of South Africa (UNISA). We are inviting you to participate in the **experimental group** of the study entitled: *The Impact of 8Ps Learning Model on the Mathematical Problem Solving of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus.*

Purpose of the study

Some of the potential benefits of this study are: It can give the understanding of how the Grade 12 learners actually solve Mathematics problems, identify the difficulties they may be experiencing during problem-solving and offer useful knowledge that can equip them with necessary mathematical problem-solving skills and strategies for obtaining logical solutions. The study can help the learners develop positive attitude and confidence required for solving Mathematics problems. It can as well assist Mathematics educators in designing and delivering helpful problem-solving instruction that is capable of enhancing the learners' critical and problem-solving skills in Mathematics. Aside this, it can help Mathematics teachers discover appropriate problem-solving instructional methods for the teaching and learning of Mathematics.

Why you are being invited to participate in the study

Your school has been purposively selected as one of the eight schools in the education district suitable for this study. The study plans to involve all the Grade 12 Mathematics learners and is scheduled for the normal school lesson periods. You are therefore invited to participate in the study since you are one of the Grade 12 Mathematics learners of the school.

Nature of your participation in the study

The study entails using 8Ps learning approach as the intervention in the experimental group and the conventional instructional method in the control group. It will employ problem-solving ability test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection. Since your school is chosen as one of the four schools to be in the

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experimental group, I, as the researcher, will play the role of the Mathematics teacher to apply the 8Ps problem-solving instruction (intervention) to teach you the Mathematics topic. This will take place for three weeks in April/May of Term 2 as officially planned in the Grade 12 Mathematics curriculum. Specifically, your role in the study as a learner in the **experimental group** is as follows:

- You are to participate in writing a 90-minute pre-test a week to the intervention.
- You are expected to be fully involved in all the intervention lessons. Please note that this serves as your normal school lessons and so should be taken with utmost seriousness.
- Your general attention and co-operation as a learner is needed throughout the 3-week period in your school to ascertain the success of the study.
- You are also to participate in writing a 90-minute post-test in the last week of the study.
- After the post-test, you will be requested to answer some questions in a short interview with the researcher for about 30 minutes based on the problem-solving 8Ps instruction you have received if you are chosen as one of the learners for the interviews.

Can you withdraw from this study even after having agreed to participate?
Yes, you can. Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are also free to withdraw from the study at any time if necessary and without giving a reason.

Are there any negative consequences for participating in the research project?
The participants will not experience any form of inconvenience, discomfort, harm or risk.

Will your identity and the information you supply to the researcher be kept confidential?
Yes, all the information you may supply as a participant in the enquiry and your identity will be kept strictly confidential. Your name will not be recorded anywhere in the study and no one, apart from the researcher and the identified members of the research team, will know about your involvement in this research. Please note that your anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings. But your privacy will be well-protected in any publication of the information. Your answers/responses will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

How the researcher will protect the security of data

Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard in my personal library/reading room for future research or academic purposes. The electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After a period of five years of the study might have passed, the hard copies of the information will be shredded and burnt while its electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

Will you receive payment or any incentives for participating in this study?

Regrettably, no payment or reward, financial or otherwise, will be offered to study participants and no cost whatsoever will be incurred by participants.

Has the study received ethics approval?

This study has received written approval from the Research Ethics Review Committee of UNISA. You can obtain a copy of the approval letter from the researcher if you so wish.

How will you be informed of the findings/results of the research?

If you would like to be informed of the final research findings or require any further information or want to contact the researcher about any aspect of this study, please contact: Omoniyi, A. A. on cellphone number +27744550208, fax number 0866081283 or e-mail address 51940124@mylife.unisa.ac.za. The findings will be made available after the thesis might have been submitted and approved for the award of the doctoral degree for which it is being conducted. Should you have concerns about the way the research has been conducted, you may contact the researcher's Supervisor on 0123376168 or mogarld@unisa.ac.za. In case you have any ethical concerns, contact the UNISA research ethics chairperson. Thank you for taking time to read this information sheet and for deciding to participate in this study.

……………………………………………………

Omoniyi, A. A. (Student Number: 51940124)
Researcher

APPENDIX N2

INFORMATION FOR PARTICIPATING LEARNER: CONTROL GROUP

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

Dear Learner,

My name is A. A. Omoniyi (student number: 51940124). I am doing research with Prof. L D Mogari, Head, Institute of Science and Technology Education (ISTE), College of Graduate Studies, towards a PhD. in Mathematics, Science and Technology Education at the University of South Africa (UNISA). We are inviting you to participate in the **Control group** of the study entitled: *The Impact of 8Ps Learning Model on the Mathematical Problem Solving of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus.*

Purpose of the study

Some of the potential benefits of this study are: It can give the understanding of how the Grade 12 learners actually solve Mathematics problems, identify the difficulties they may be experiencing during problem-solving and offer useful knowledge that can equip them with necessary mathematical problem-solving skills and strategies for obtaining logical solutions. The study can help the learners develop positive attitude and confidence required for solving Mathematics problems. It can as well assist Mathematics educators in designing and delivering helpful problem-solving instruction that is capable of enhancing the learners' critical and problem-solving skills in Mathematics. Aside this, it can help Mathematics teachers discover appropriate problem-solving instructional methods for the teaching and learning of Mathematics.

Why you are being invited to participate in the study

Your school has been purposively selected as one of the eight schools in the education district suitable for this study. The study plans to involve all the Grade 12 Mathematics learners and is scheduled for the normal school lesson periods. You are therefore invited to participate in the study since you are one of the Grade 12 Mathematics learners of the school.

Nature of your participation in the study

The study entails using 8Ps learning approach as the intervention in the experimental group and the traditional instructional method in the control group. It will employ problem-solving ability test, classroom observation schedule and semi-structured interview schedule as measuring tools for data collection. Since your school is chosen as one of the four schools to be in the

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control group, your regular Mathematics teacher will apply the usual traditional teaching method to teach you the concept of stationary points in differential calculus. This will take place for three weeks in April/May of Term 2 as officially planned in the Grade 12 Mathematics curriculum. Specifically, your role in the study as a learner in the **control group** is as follows:

- You are to participate in writing a 90-minute pre-test a week to the intervention.
- You are expected to be fully involved in all the lessons. Please note that this serves as your normal school lessons and so should be taken with utmost seriousness.
- Your general attention and co-operation as a learner is needed throughout the 3-week period in your school to ascertain the success of the study.
- You are also to participate in writing a 90-minute post-test in the last week of the study.
- After the post-test, if you are chosen as one of the learners for the interviews, you will be requested to answer some questions in a short interview with the researcher for about 30 minutes based on the traditional instruction you have received.
- Although the 8Ps learning model will not be used in your school to teach you the concept, I will ensure that you as the Mathematics learners of the school also share of the potential benefits of the problem-solving model. After completing the research, I will find a convenient time to visit your school to demonstrate the application of the learning model to your Mathematics teacher who will in turn use it to support your mathematical problem solving. You only need to avail yourself of the opportunity at that time.

Can you withdraw from this study even after having agreed to participate?
Yes, you can. Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are also free to withdraw from the study at any time if necessary and without giving a reason.

Are there any negative consequences for participating in the research project?
The participants will not experience any form of inconvenience, discomfort, harm or risk.

Will your identity and the information you supply to the researcher be kept confidential?
Yes, all the information you may supply as a participant in the enquiry and your identity will be kept strictly confidential. Your name will not be recorded anywhere in the study and no one, apart from the researcher and the identified members of the research team, will know about your involvement in this research. Please note that your anonymous data may be used for other

purposes, such as a research report, journal articles and/or conference proceedings. But your privacy will be well-protected in any publication of the information. Your answers/responses will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

How the researcher will protect the security of data

Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard in my personal library/reading room for future research or academic purposes. The electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After a period of five years of the study might have passed, the hard copies of the information will be shredded and burnt while its electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

Will you receive payment or any incentives for participating in this study?

Regrettably, no payment or reward, financial or otherwise, will be offered to study participants and no cost whatsoever will be incurred by participants.

Has the study received ethics approval?

This study has received written approval from the Research Ethics Review Committee of UNISA. You can obtain a copy of the approval letter from the researcher if you so wish.

How will you be informed of the findings/results of the research?

If you would like to be informed of the final research findings or require any further information or want to contact the researcher about any aspect of this study, please contact: Omoniyi, A. A. on cellphone number +27744550208, fax number 0866081283 or e-mail address 51940124@mylife.unisa.ac.za. The findings will be made available after the thesis might have been submitted and approved for the award of the doctoral degree for which it is being conducted. Should you have concerns about the way the research has been conducted, you may contact the researcher's Supervisor on 0123376168 or mogarld@unisa.ac.za. In case you have any ethical concerns, contact the UNISA research ethics chairperson. Thank you for taking time to read this information sheet and for deciding to participate in this study.

……………………………………………………

Omoniyi, A. A. (Student Number: 51940124)

Researcher

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX N3

LEARNER'S CONSENT TO PARTICIPATE IN THE STUDY (EXPERIMENTAL GROUP)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

I, ……………………………………………………………………………., confirm that the student researcher (A. A. Omoniyi: 51940124) requesting my consent to take part in the study (*The Impact of 8Ps Learning Model on the Problem-Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*) **as a participant in the Experimental Group.** He has explained to me the nature, procedure, potential benefits and anticipated inconvenience of my participation.

I have read and understood the study as explained in the information sheets.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw from the study any time (if need be) without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I have received a signed copy of the informed consent agreement.

Participant's Name & Surname: ………………………………………………………… (Please print)

Participant's Signature : …………………………………… Date: …………………………………..

Researcher's Name & Surname: …………………………………..……………………………………

Researcher's Signature: ……………………………………. Date: …………….……………………

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Preller Street, Muckleneuk Ridge, City of Tshwane
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www.unisa.ac.za

UNISA university of south africa

APPENDIX N4

LEARNER'S CONSENT TO PARTICIPATE IN THE STUDY (CONTROL GROUP)

Ethics clearance Ref.: 2018_CGS/ISTE+006 **GDE Research permission Ref.:** 8/4/4/1/2

I,…………………………………………………………………………………, confirm that the student researcher (A. A. Omoniyi: 51940124) requesting my consent to take part in the study (*The Impact of 8Ps Learning Model on the Problem-Solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus*) **as a participant in the Control Group.** He has explained to me the nature, procedure, potential benefits and anticipated inconvenience of my participation.

I have read and understood the study as explained in the information sheets.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw from the study any time (if need be) without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I have received a signed copy of the informed consent agreement.

Participant's Name & Surname: ………………………………………………………… (Please print)

Participant's Signature: …………………………………… Date: ……………………………………

Researcher's Name & Surname: ……………………………………………………………………………

Researcher's Signature: …………………………………….. Date: ……………………………………

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Appendix O1

UNISA university of south africa

UNISA ISTE ETHICS REVIEW COMMITTEE

Date: 21 November 2018

ERC Reference # :2018_CGS/ISTE+006
Name : Adebayo Akinyinka Omoniyi
Student #:51940124

Dear Mr Adebayo Akinyinka Omoniyi

Decision: Ethics Approval from 21/11/2018 to 21/11/2023

Researcher(s): Name: Adebayo Akinyinka Omoniyi
Address: P. O. Box 40324, Arcadia 0007
E-mail address: 51940124@mylife.unisa.ac.za
Telephone #0782261874

Supervisor (s): Name: Prof Mogari
E-mail address: morgarld@unisa.ac.za
Telephone # 011 670 9422

Working title of research:

The Impact of 8Ps Learning Model on the Mathematical Problem Solving of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus

Qualification: PHD

Thank you for the application for research ethics clearance by the Unisa ISTE Ethics Review Committee for the above mentioned research. Ethics approval is granted for 5 years

*The **low risk application** was **reviewed** by the ISTE Ethics Review Committee on 20 September 2018 in compliance with the Unisa Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the ISTE ERC Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing, accompanied by a progress report.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data require additional ethics clearance.
7. No field work activities may continue after the expiry date. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.
8. Any amendments to the research instruments should be reported to the Committee in writing, accompanied by a progress report.

Note:

*The reference number **2018_CGS/ISTE+006** should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Yours sincerely,

Signature

Chair of ISTE ERC: Prof Keshnee Padayachee

E-mail:padayk@unisa.ac.za

Tel: (012) 337 6191

Signature :

Executive Dean: Prof Bheki Mamba

E-mail: mambabb@unisa.ac.za

Tel: (011) 670 9231

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Appendix O2

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	26 May 2021
Validity of Research Approval:	08 February 2021– 30 September 2021 2018/300A
Name of Researcher:	Omonyl AA
Address of Researcher:	102 Russel's Place
	213 Sophie de Bryne Street
	Pretoria Central
Telephone Number:	074 455 0208
Email address:	51940124@mylife.unisa.ac.za / akinomo2013@gmail.com
Research Topic:	The impact of 8Ps Learning Model on the mathematical Problem Solving of Grade 12 Learners in the Concept of Stationery Points in Differential Calculus
Type of qualification	PhD in Mathematics ,Science and Technology Education
Number and type of schools:	6 Secondary Schools
District/s/HO	Tshwane West

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

26/05/2021

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of COVID 19 pandemic researchers can ONLY collect data online, telephonically or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate. The approval letter will then indicate the type of arrangements that have been made with the school.***
4. ***The Researchers are advised to make arrangements with the schools via Fax, email or telephonically with the Principal.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

pp [signature]

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 26/05/2021

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix P: Turnitin Originality Report

The Impact of 8Ps Learning Model on the Mathematical Problem-solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus

ORIGINALITY REPORT

14%	12%	8%	3%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	hdl.handle.net Internet Source	3%
2	uir.unisa.ac.za Internet Source	2%
3	pt.scribd.com Internet Source	<1%
4	mail.mjltm.org Internet Source	<1%
5	researchspace.ukzn.ac.za Internet Source	<1%
6	Fonseca, Kathleen. "Learning to Become a Teacher of Mathematical Problem-Solving", University of Johannesburg (South Africa), 2021 Publication	<1%
7	Submitted to Midlands State University Student Paper	<1%

Appendix Q

LANGUAGE EDITING REPORT

Language Editing Report of the doctoral thesis entitled:

The Impact of 8Ps Learning Model on the Mathematical Problem-solving Performance of Grade 12 Learners in the Concept of Stationary Points in Differential Calculus

Student Researcher: Adebayo Akinyinka Omoniyi (51940124)

This is to affirm that I proof-read the doctoral thesis with the afore-indicated title and author. I edited the academic work for language errors such as: grammar, punctuation marks, spelling and formatting without necessarily tampering with the content and the researcher's intentions. I have to mention that I did not check the references and the tables used in the work for accuracy. I also need to state that the suggested corrections can still be further scrutinised and modified by the researcher and the Supervisor. Thank you for affording me the privilege to go through the write-up in the first place.

O.A.A. Stevenson, PhD
(Editor & Language Practitioner)
Deluxe Editing Centre: 0782261874, 0744550208