Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng

By

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Declaration

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I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the dissertation to originality checking software and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at Unisa for another qualification or at any other higher education institution.

29/11/2022

SIGNATURE DATE

Dedication

This dissertation is dedicated to my children, Mackenzie, and Skylar. Do your best, even when you feel like giving up. It will all be worth it.

This dissertation is also dedicated to my husband, Vaughn. You push me to be better, and I know that you will always support me.

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Abstract

The study was motivated by the fact that learners are not performing well in the topic of fractions due to the lack of conceptual understanding of the concept. The purpose of this qualitative study based on an interpretive paradigm, which draws upon Kilpatrick et al.'s (2001), was to explore how Grade 6 mathematics teachers develop learners' mathematical proficiency in the addition and subtraction of common fractions topic. The aim of the study was to answer the following question: *How do Grade 6 mathematics teachers develop learners' proficiency when teaching the addition and subtraction of common fractions?* Four Grade 6 mathematics teachers, from Tshwane South District in Gauteng, were purposively selected, and data was collected through classroom lesson observations (observing and analysing the video lessons) and semi-structured interviews. Three key findings were revealed: Firstly, teachers provided opportunities for developing procedural fluency and productive disposition. Secondly, teachers provided limited opportunities for learners to develop conceptual understanding and strategic competence. Lastly, it was a challenge for teachers to understand that mathematical proficiency is developed over time, working consistently with learners, and ensuring there are more equal opportunities for the development of each strand. It is essentially recommended in this study that a developmental intervention workshop be conducted and should focus on making sense of what it means to teach fractions for proficiency.
Keywords: Common Fractions, Mathematical Proficiency, Addition, Subtraction, Denominator, Development, Teachers

List of acronyms

ANA	• Annual national Assessments
CAPS	• Curriculum and Assessment Policy Statement
DBE	• Departmet of Basic Education
TIMSS	• Trends in Mathematics and science Study
GDE	• Gauteng Department of Education
ATP	• Annual Teaching Plan

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1 CHAPTER ONE: INTRODUCTION TO THE STUDY

1.1 Introduction and Background

Fractions are discussed and described in the South African curriculum in the content area Numbers, Operations and Relationships (Department of Basic Education [henceforth DBE], 2011). This concept also features in contexts occurring within many other content areas, such as algebra, probability, trigonometry, and geometry. For example, in the Further Education and Training (FET) band, fractions are used in trigonometrical ratios, in theorems when proving the similarity of triangles and in algebra when dealing with algebraic functions. Fractions also link to the Science, Technology, Engineering and Mathematics (STEM) field as well as medicine, health care and construction (Bruce et al., 2013).

Studies have noted the importance of fractions and indicate that they are also necessary for life after school as they are linked to business and technology courses as well as opportunities in many pursued careers such as bookkeeping, accounting, sales agents and travel agents, amongst others (Bruce et al., 2013; Tian & Siegler, 2017). According to Fazio et al. (2016), the concept of fractions is considered important as they are considered a foundational mathematical concept. Fractions are used in later mathematical concepts such as algebra, one of the areas of advanced learning of mathematics, which implies that the fractions learned earlier on, in the foundational grades, are of utmost importance (Lortie-Forgues et al., 2015).

Although fractions are regarded as important in our everyday lives, Umugiraneza et al. (2017); Tsanwani et al. (2014) had many questions about the efficiency of the teaching and learning of Mathematics in South Africa. Van der Walle et al. (2016) also argued that the emphasis on the use of algorithms does not promote a conceptual understanding of fractions. This must be seen in the light of (Gabriel et al., Coche, Szűcs, Carette, Rey & Content (20, 2015), who highlighted that there seems to be more emphasis on procedural knowledge than conceptual knowledge.

My experience as a Grade 6 mathematics teacher allowed me to teach different groups of learners and interact with many mathematics teachers around my area. During this interaction, I realised that learners do not understand why $\frac{1}{5}$ is smaller than $\frac{1}{3}$. What immediately came to my mind was that learners do not have a conceptual understanding of fractions (Braithwaite & Siegler, 2021; Deringöl, 2019; Nahdi & Jatisunda, 2020; Trivena et al., 2017). This understanding suggests that learners still see the denominators as wholes. This might also be a problem with how learners name fractions. For example, if the learner says, "one over five", then the notion of a fraction describing parts of a whole is not comprehensible. That is why it is so difficult for learners to add and subtract because they only see numbers but not the meaning of the number. This finding suggests that learners are not mathematically proficient and struggle to understand fractions and their related concepts (Fennell et al., Kobett & Wray, 2014). Undertaking this study will provide a solid basis for formulating thoughts and views about developing learners, proficiency in the addition and subtraction of common fractions.

1.2 Purpose and Rationale for the Study

The study aimed to explore how Grade 6 mathematics teachers develop learners' proficiency when teaching addition and subtraction of common fractions. The need for the study and the rationale for the study grew from my own experience as the Grade 6 mathematics teacher. I have observed that the learners are not mathematically proficient, and, as such, they struggle to understand fractions and their related concepts (Fennell et al., 2014).

Understanding how teachers made sense of the addition and subtraction of fractions and how they developed the skill of solving such problems has illuminated the challenges experienced. This served as the reason for me to investigate and explore the strategies used and the challenges teachers face that lead to learners not being proficient in fractions. Such challenges were translated into guidelines and provided clues to teachers and curriculum specialists on what to consider when teaching fractions for proficiency. The findings, implications, and recommendations in this study have provided some hints and helpful suggestions on how teachers could empower themselves to understand the concept of fractions better. This would assist them in adapting their classroom instructional practices while equipping learners with the necessary skills to become proficient in the addition and subtraction of common factions. The recommendations provided in this study will also assist in articulating the applicability of the theoretical ideas discussed in the Mathematics Teaching and Learning Framework for South Africa (henceforth, MTF) (DBE, 2018).

1.3 Statement of the Problem

The foundational levels of mathematics are important to grasp so that learners can further their mathematical studies (Olanoff et al., 2014). Primary school plays a pivotal role in shaping young learners as what they learn during this time is built upon in later years. In the intermediate phase, grades 4-6, mathematics covers five key areas: numbers, operations and relationships, patterns, functions and algebra, space and shape (geometry), measurement and data handling (DBE, 2011). As the Curriculum and Assessment Policy Statement (henceforth, known as CAPS) document stipulates the various topics, it is the teachers' responsibility to transition from one topic to the next so that learners aren't caught off guard. A possible problem that may be the cause of fractional difficulty is the transition from whole numbers to rational numbers (Lortie-Forgues et al., 2015). In my seven years of experience teaching mathematics, I found that learners do not understand that there are numbers between the whole numbers as teachers (myself included) focus on emphasising the whole numbers and not mentioning the numbers between the whole numbers. They are then suddenly introduced to fractions. For learners to transit from whole numbers to fractions is one area that needs serious attention.

One other critical point is that the concept of common fractions, which is taught in Term 1 in the Intermediate Phase, Grade 6, is allocated ten hours and covers a wide scope in the Numbers, Operations and Relationships content area (DBE 2011, p. 34). This equates to 16,6% of term one being dedicated to common fractions. Teachers must cover various concepts during this time, as set out in the CAPS document (DBE, 2011, p.231). Fractions are covered in other countries such as Japan, Taiwan, Singapore and Korea (Kaur, 2008; Watanabe, 2006; Kaur, 2014;).

The Human Sciences Research Council (HSRC) in South Africa released the Grade 5 results of the 2015 and 2019 Trends in International Mathematics and Science Study (henceforth, TIMSS) (Isdale et al., 2017; Reddy et al., 2019). TIMSS is an international assessment that analyses the mathematics and science knowledge of Grades 4 and 8 learners from selected countries worldwide. In 2015, Grade 5 learners participated in TIMSS and showed South Africa was one of the underperforming countries, coming second last overall (Isadle et al., 2017). In 2019, South Africa once again came out as an underperforming country by coming third last (DBE, 2020). From the TIMSS 2015 and the TIMSS 2019 Grade 5 reports, it can also be seen that learners are underperforming in the "reasoning" skill (which is conceptual knowledge) (DBE, 2020; Isdale et al. 2017).

The National Senior Certificate diagnostic report (DBE, 2018, p.128), highlighted that common errors and misconceptions identified are "some candidates could not use discount and selling prices as fractions to determine the percentage". It was indicated that "percentages, proper fractions and decimal fractions need to be revised on a regular basis in Grade 12" (p.124) as a suggestion for improvement. I would add to these findings and suggestions that regular revision might not be enough unless fractions at the elementary level are taught for conceptual understanding. Furthermore, a study by Aksoy and Yazlik (2017) found that learners have certain misconceptions in relation to fractions such as addition and subtraction of fraction with unlike denominators. This is echoed by Lestiana et al. (2017), who stated that learners cannot find a common denominator when adding and subtracting fractions.

South Africa had an assessment strategy called the Annual National Assessment (ANA) which takes place yearly and was discontinued in 2013. In this assessment, earners performed poorly due to various reasons. According to the ANA Analysis Reports of 2011-2013, conducted by the Department of Basic Education, (DBE, 2011, 2013), the learner performance in ANA, particularly in fractions, reveal the following:

In 2011, learners' ideas of fractions were limited or distorted (DBE, 2011). According to the results of the 2011 ANA, the learners were unable to arrange fractions in ascending or descending order. This seems to suggest that learners cannot comprehend the size of fractions and understand that the bigger the denominator, the smaller the fraction. Furthermore, learners were unable to solve equations with mixed numbers, such as subtracting "three-and-one-quarter" from "five-and-one-eight" ($5\frac{1}{8}$ - $3\frac{1}{4}$)" (DBE, 2011).

In 2012, the same problems of learners not being able to arrange fractions in ascending or descending order occurred. Furthermore, learners could not add mixed numbers (DBE, 2013). It is evident from the 2012 analysis, that learners still struggle with the concept of addition and subtraction of mixed numbers, implying that teachers may not have utilised strategies to improve their teaching of the concept.

In 2013, learners were still struggling with mixed numbers. Furthermore, learners applied incorrect mathematical rules and strategies to manipulate the denominators and numerators. *The following challenges were revealed:*

- *"Using a column method when adding mixed fractions.*
- *Adding the numerators and denominators"* (DBE, 2013, p. 35).

Research has echoed the reports from the ANA stating that learners do struggle with ordering fractions as they have the misconception that a smaller denominator implies a smaller fraction (Fazio et al., 2016; Fuchs & Malone, 2016; Gabriel et al., 2013; Van Steenbrugge et al., 2015).

Although important and used in everyday life, fractions are still regarded as a difficult concept for children to understand (Gabriel et al., 2013). Furthermore, different hindrances lead to these learning difficulties with fractions. These hindrances are whole number bias, different meanings of fractions and conceptual and procedural understanding. Although Van Steenbrugge et al. (2015) emphasise that conceptual understanding includes knowing the procedural steps, they also regard making connections and utilising a higher level of thinking as important. For example, a deep connection in the problem $\frac{1}{3} + \frac{2}{6}$ is learners knowing you must work with equivalent fractions to solve the problem. It means teachers should ensure that learners can apply the knowledge of equivalent fractions in solving related problems. This view is supported by Kilpatrick et al. (2001), who emphasised that varying from the focus on rote procedures to understanding the structure of mathematics is critical in becoming mathematically proficient. Proficiency in fractions at the elementary level is critical as it forms the basis for concepts like algebraic fractions and trigonometry taught in secondary schools. Fractions are also integral to conceptual understanding of probability. Despite the emphasis on the importance of fractions, it appears that there is still a need to re-look at how teachers develop learners' conceptual understanding in teaching and learning fractions. This study focuses on attempting to explore teachers' practices in developing learners, mathematical proficiency in the addition and subtraction of fractions.

I felt that there is a need to explore further how teachers develop learners' mathematical proficiency in fraction calculation. This topic within the numbers content area is eminent in mathematics (Naidoo & Hajaree, 2020), and a need for a better understanding of it has occurred. If learners are not taught fractions properly in the foundational years, they will always have a problem with the topics that incorporate the understanding of fractions.

1.4 The Aims and Objectives of the Study

1.4.1 Aim of the Study

The aim of the study is to outline and substantiate a broad conceptualisation of how teachers develop learners' proficiency when teaching the addition and subtraction of common fractions. The aim was achieved by pursuing the objectives set out in the next section.

1.4.2 Objectives of the Study

- To explore Grade 6 mathematics teachers' understanding of common fractions
- To describe the strategies used by teachers to develop learners' proficiency when teaching addition and subtraction of common fractions
- To suggest possible strategies to enhance the development of learners' proficiency in the addition and subtraction of common fractions

The study was done to answer the research questions in the following section.

1.5 Research Questions

1.5.1 Main Research Question

How do Grade 6 mathematics teachers develop learners' proficiency when teaching the addition and subtraction of common fractions?

1.5.2 Secondary Questions

- How do Grade 6 mathematics teachers describe common fractions?
- What strategies do Grade 6 mathematics teachers use in developing learners' proficiency[1] when teaching the addition and subtraction of common fractions?

[1] Proficiency refers to developing conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. These concepts are further explained in Chapter 2.

- How can Grade 6 mathematics teachers enhance their classroom practices in developing learners' proficiency when teaching the addition and subtraction of common fractions?

1.6 Definition of Key Terms

Mathematics: Mathematics is defined as a language that makes use of symbols to describe numerical, geometric, and graphical relationships (DBE, 2011). Mathematics is the subject wherein you will find the fractions content area.

Common Fractions: Common fractions are rational numbers that can be expressed in the "a/b" form, where a (numerator) and b (denominator) are integers and where $b \neq 0$, rather than parts of wholes (Tsai & Li, 2017):

Grade 6: The exit level of the intermediate phase. The final grade within the intermediate phase in the South African education system

Mathematical Proficiency: Kilpatrick et al. (2001) explore five strands that are interwoven and interdependent. These five strands are: Conceptual understanding; procedural fluency; strategic competence; adaptive reasoning and productive disposition.

1.7 Literature Review and the Conceptual Framework

This section presents a concise version of the literature review presented in Chapter 2. The purpose of this study, as mentioned in Section 1.4, is to explore how Grade 6 mathematics teachers develop learners' proficiency when teaching addition and subtraction of common fractions. I reviewed the literature paying particular attention to experiences around teaching fractions. Creswell (2012, p. 28) defines a literature review as a "framework for establishing the importance of the study as well as a benchmark for comparing the results of a study with other findings." On the other hand, Maree (2012, p. 33) points out that the literature review "provides a knowledge base for the research to be conducted, enabling the researcher to better understand the research problem and field of study." These definitions point to three important areas, namely, providing knowledge on the topic, identifying established research on the topic and identifying gaps within the research. These areas are confirmed and elaborated by Rocco and Plakhotnik (2009) who assert that without a literature review, the research would have no basis and no connection which would imply no need for the study.

According to Copur-Gencturk (2021), Oluyinka et al. (2020), Ubah and Bansilal (2018), Newton (2008), Tsai and Li (2017), Stewart (2005) and Van Steenbrugge et al. (2015) some teachers find fractions difficult to teach. In turn, learners find them difficult to learn. Learners who lack fraction knowledge are unable to perform fraction calculations and may struggle with later mathematical concepts such as algebra (Brown & Quinn, 2006; Oluyinka et al., 2020). This may lead to poor mathematical performance and therefore discourage learners from pursuing careers in mathematical fields.

From the perspective of how learners experience learning fractions, Lestiana et al. (2017) and Fauzi and Suryadi (2020) stated that there are various reasons why learners may struggle with fractions. These reasons include that fractions are not seen in everyday life, and there are many rules associated with fractional equations, which may imply that teachers focus more on rote procedures instead of conceptual understanding (Bruce et al., 2013). Likewise, Veloo and Puteh (2017) note the reason for mistakes may include focusing on procedural knowledge rather than conceptual knowledge. Difficulties noted by Fauzi and Suryadi (2020) are lack of foundational knowledge or methods and approaches used by teachers who lack content knowledge. Furthermore, Ubah and Bansilal (2018) and Tsai and Li (2017) identify a few sources of learners' difficulties with fractions. These difficulties include:

1) A full curriculum, which leaves little time to develop conceptual knowledge
2) An assumption that the properties of fractions are the same as whole numbers
3) The concept of equivalent fractions.

Jigyel and Afamasaga-Fuata'I (2007) support these claims that the concept of equivalent fractions is important, and learners lack this knowledge. This study drew on Kilpatrick et al.'s (2001) five strands of mathematical proficiency to frame the study.

1.8 Research Methodology

1.8.1 Research Paradigm

According to and Rehman Alharthi (2016), a research paradigm is a set of principles on how the world is observed and a framework that guides the researcher. Similarly, Kivunja and Kuyini (2017) state that a research paradigm is a lens through which a researcher looks at the world and outlines what is acceptable and how to conduct research. Furthermore, Hughes (2010, p. 35) asserts that a paradigm is “a way of seeing the world that frames a research topic” and influences the way that researchers think about the topic. My study is underpinned by an interpretivist paradigm wherein I try to understand particular situations or phenomena (Adom et al., 2016) while focusing on the meanings embedded in documented and oral accounts, and generally involves the analysis of material such as documentary sources and/or oral and personal histories and narratives gathered through data collection strategies such as interviews (Bisman &Highfield, 2012). I chose this paradigm because I wanted to understand how teachers develop mathematical proficiency within learners and gain a deeper understanding of this (Creswell, 2007).

1.8.2 Research Design

Creswell (2014) states that research design “focuses on data collection, analysis, and writing, but they originate out of disciplines and flow throughout the process of research”. Research design is the plan for conducting the study. Maree (2012) define case study as an in-depth exploration of a particular group within a particular setting to gain an in-depth understanding of an issue in its real-life situation. I chose to use the case study design because it involved detailed and in-depth analysis of each teacher’s practices in the classroom settings and to understand their perspectives in those settings. To achieve that, I went to the site to experience the reality by exploring how four Grade 6 mathematics teachers (the case) developed learner mathematical proficiency when teaching fraction addition and subtraction of fractions. I also understood that the conditions of the classrooms were integral to my research.

1.8.3 Research approach

In this study, I followed a qualitative research approach because I wanted to personally understand teachers’ opinions, views, and their experiences in developing the fractional concepts (Creswell, 2014; Rahman, 2020). To make it much clearer, Creswell (2007, p. 73) describes it as the approach in which the "researcher explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audio-visual material, and documents and reports), and reports a case description and case-based themes." In this case study, I got closer to the phenomenon under study, and collected data through lesson observations (analysing the video lessons) and face-face interviews.

1.8.4 Population and Sampling

Maree (2012) and Cohen et al. 2007) define a sample as a subset of a population while the population consists of all the people the researcher is interested in studying. The sample will represent the population group (Creswell, 2010). For this study, I employed non-probability sampling as the sampling technique in the qualitative research design. Parveen and Showkat (2020) define non-probability sampling as using nonrandomised methods to draw the sample. This implies that the probability of selecting a particular person is unknown.

I utilised two sampling techniques, namely purposive and convenience sampling. I conveniently chose schools that were accessible and close to me in terms of their geographical location and willingness to participate in the research (Etikan et al., 2016). When sampling participants, I chose four primary schools and one Grade 6 mathematics teacher in each school. The criteria for choosing teachers were based on the teaching experience, qualifications and the grade performance for the past three years. I analysed a small sample as I embarked on a "small-scale research" with four teachers, where no attempt to generalised the findings was desired (Cohen et al., 2007). The sample chosen enabled me to get an understanding of how teachers develop learner mathematical proficiency when teaching fraction calculations in additive conceptual field.

1.8.5 Instrumentation and Data Collection Techniques

Kabir (2016) states that data collection is the process of gathering and measuring information in a systematic way that enables one to answer stated research questions. Data collection takes place because it is necessary to summarise a large amount of data (Maree, 2012). I collected data through observations and semi-structured interviews (Kabir, 2016; Parveen & Showkat, 2020). Maree (2012) argues that interviews are important when undergoing qualitative research as the researcher will ask open-ended questions that will acquire rich and descriptive information from participants. Furthermore, interviews are meant to record and analyse people's opinions, experiences, beliefs and ideas on relevant topics (Parveen & Showkat, 2020). I gained rich and descriptive information, first-hand, from the Grade 6 mathematics teachers to determine how they develop learners, mathematical proficiency when teaching addition and subtraction of fraction. I conducted face-to-face, semi-structured interviews using open-ended questions so the participants could respond in their own words (Gay et al., 2009). Through observations, I described situations as they existed using five senses, presenting a sketch of a situation under study (Parveen & Showkat, 2020) and without questioning and communicating with participants (Maree, 2012).

1.8.6 Data analysis and Interpretation

Cohen et al. (2007) describe data analysis as the process of bringing order, structure and meaning to the mass of collected data and entails breaking down the research into basic parts to answer the research question. Data analysis is essential to make sense of the data collected (Maree, 2012; Ngulube, 2015).

Creswell (2015, p. 156) indicated that coding is "the process of analysing qualitative text data by taking them apart to see what they yield before putting the data back together in a meaningful way." In this study, an analysis of the data involved identifying distinct concepts, finding similar concepts and focusing on identifying emergent patterns for categorisation. This process enabled me to select and assimilate categories of organised data from axial coding in organised and meaning-filled expressions (Williams & Moser, 2019).

I have used a six-phase framework as suggested by Clarke and Braun (2006). This framework provided some guidance on how to coalesce the patterns into categories. The process can be summarised as follows: The first step is about transcribing, reading collected data several times and listening to audio recording more than once to get a grip on what it entails. The second step is the initial stage of organising the data meaningfully. In the third step, the data is coded and arranged into categories. The fourth step relates to how the categories are grouped to help answer the research questions. I have used the deductive approach to interpret and analyse data and come up with detailed information to help answer the research questions. The views of the participants were captured verbatim to support the arguments made in the study

1.9 Delimitation and Limitations of the Study

According to Theofanidis and Fountouki (2018), limitations are potential weaknesses of a study, and they encompass all the factors that are impossible to avoid, and which may affect the internal validity of the research. Furthermore, Creswell (2007) defines limitations as deficiencies that the researcher identifies in a study. This research involved four primary schools situated in Gauteng Province under the Tshwane South District. I observed the lessons of four Grade 6 mathematics teachers and interviewed them.

I do acknowledge that the research sample is small and might have weakened the study with an inability to draw enough data. The selected schools are from the same district/area. The study's findings cannot be generalised, but they provide a richly contextualised understanding of how teachers could be supported to enhance their classroom practice to develop learners' proficiency when teaching addition and subtraction of common fractions.

The study duration was too short, so the information obtained could be a fraction of many more things occurring in schools regarding the teaching of addition and subtraction of common fractions. This is confirmed by Akanle et al. (2020), who said that limitations could be based on time constraints that could hinder the collection of rich data. The quality of data depended on the teachers available.

1.10 Ethical Considerations

Cohen et al. (2007) argue that research ethics refers to moral principles or rules of behaviour that researchers must consider before conducting research, particularly when the research involves humans. I was therefore obliged to protect the participants' rights and welfare. I have received ethical clearance from the ethics committee at the University of South Africa (UNISA) (Annexure A). Before carrying out the study, permission was also obtained from the school principals (Annexure B). Subsequently, I requested permission from participants (Annexure C) to research them. In compliance with the principles of informed consent, I gave participants consent forms (Annexure D). In the form of a letter, I explained to the participants the study's aims, purpose and educational benefits. I was open and honest with the participants. Signing the consent form will protect teachers and safeguard their well-being and interests during and after the research. Lessons were observed during school hours, and interviews were conducted after hours because I did not want to interfere with the smooth running of the schools.

According to Denzin and Lincoln (2011), anonymity and confidentiality are academic research's foundations. The names of the participants were not disclosed to ensure confidentiality. The schools, teachers, learners and principals remained anonymous, and their personal information did not appear in the report. The data gathered was strictly used for the study. Confidentiality was used to encourage respondents to provide truthful responses by guaranteeing the confidentiality of the information provided. I utilised pseudonyms for the participants. I closely followed the practices of the protection of personal information (POPI) act.

1.11 Credibility of the Study

Credibility: Credibility deals with the question of how consistent the findings are with reality (MacMillan & Schumacher, 2006; Shenton, 2004). This is essential in ensuring trustworthiness. Merriam (2009) states that a research study is trustworthy to the extent that there has been some consistency in carrying it out. I was immersed in the study and interacted with the participants while collecting data. Furthermore, I constantly examined the data, analysed it, theorised and revised theories where necessary. This ensured credibility and trustworthiness.

Trustworthiness: Maree (2012) asserts that trustworthiness refers to how the researcher can convince the readers that the findings in the study are of high quality and can therefore be trusted. According to Lincoln and Guba (2000), this implies that the aim of trustworthiness in qualitative research is to support the argument that the question's findings are worth noting. Maree (2012) states that trustworthiness can be stated in terms of credibility, confirmability, transferability and dependability.

Confirmability: Confirmability refers to the objectivity of the researcher and the data (Maree, 2012; Shenton, 2004). Findings were based on the experiences and ideas of the participants to ensure confirmability. This implied that the findings were not based on my preferences or experiences. Furthermore, Maree (2012) argues that to verify confirmability, external researchers should confirm the study's procedures to allow for data verification. I was objective and did not make assumptions about how Grade 6 mathematics teachers develop learners' mathematical proficiency when teaching.

Transferability: Shenton (2004), Merriam (1998) and Maree (2012) describe transferability as the extent to which results can be transferred and generalised to other similar contexts. I provided as much information and thick descriptions regarding the context, occurrence and experience of the study as possible, and validated the research process with teachers who were not part of the study to see if the information made sense to them.

Dependability: Dependability indicates the stability and consistency of the research process and methods (Maree, 2012) if the study were to be repeated with the same or similar participants or in a similar contextual setting (Shenton, 2004). The quality of data recording, the procedures used to acquire the data, the documentation of data and the transcription of interviews were monitored to ensure dependability (Maree, 2012).

Triangulation: I used triangulation to ensure dependability. Triangulation involves the use of various sources of information (more than one teacher), methods (observations, interviews) and data sources (documentation, recordings).

1.12 Organisation of the Dissertation

Chapter 1: Introduction and background of the study

This chapter sets the context of the study and provides an overall argument for why the study was instigated. I have introduced the study and discussed its background. Other aspects discussed were a statement of the problem, purposes of the study, research questions and assumptions, the study's rationale, the definition of key terms, and limitations.

Chapter 2: Conceptual framework and literature review

In this chapter, I introduce and review the literature based on how teachers make sense of common fractions and how they develop learners' proficiency in teaching the addition and subtraction of common fractions at the Grade 6 level. In particular, it exposits gaps identified in the literature and the conceptual framework used in the study. Concepts of the theory are explored to establish their practical implications concerning the teaching of addition and subtraction of common fractions. Models for representing fractions and gaps will also be identified, which justifies the need for this research to be conducted.

Chapter 3: Research design and methodology

This chapter focuses on the qualitative research methods used to carry out this study. These include the research design, paradigm and approach. Population and sampling techniques, data collection techniques, data collection procedures and data analysis are also discussed. The chapter also considers technical issues that are considered.

Chapter 4: Data presentation and analysis of Findings

In this chapter, data is presented, and findings are thematically analysed. These findings are analysed and interpreted in line with the study's objectives to answer the research questions.

Chapter 5: Discussion of results

This chapter concludes by highlighting and summarising key aspects concerning processes and conclusions made in this study. This section also includes the answers to the research questions. Recommendations that might encourage teachers to adapt their teaching strategies are made, and finally, the chapter makes suggestions for further research.

1.13 Chapter Summary

The study explored teachers' practices in developing learners' proficiency in the addition and subtraction of common fractions. This chapter highlighted the importance of fractions for grasping mathematical concepts in the more advanced grades and learners' difficulty with fractions. The aims and objectives of the study were emphasised to explore teachers' development of learners' proficiency when teaching addition and subtraction of common fractions. The study aimed to explore Grade 6 mathematics teachers' understanding of common fractions, describe the strategies teachers use to develop learners' proficiency when teaching addition and subtraction of common fractions and suggest possible strategies to enhance the development of learners' proficiency in the addition and subtraction of common fractions. I alluded to mathematical proficiency that needed to be taught for learners to be capable mathematics learners. Furthermore, I presented an outline of the dissertation, the limitations of the study, a definition of key terms, the organisation of the study and the conclusion. The next chapter reviews the work of other scholars.

2 CHAPTER TWO: CONCEPTUAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

This section explores scholarly work relating to and debates surrounding the teaching of fractions in mathematics. The goal is to outline and substantiate a broad conceptualisation of how teachers develop learners' mathematical proficiency when teaching fraction calculations, focusing on the addition and subtraction of common fractions. To achieve the outlined goal, I discuss the notion of mathematical proficiency and how mathematical proficiency is reflected in the addition and subtraction of common fractions in literature.

To delineate how the learners' mathematical proficiency is developed in the addition and subtraction of common fractions, I offer some insight by discussing the following topics: meaning of fractions, types of fractions, the importance of fractions and the representation of fractions. Furthermore, this section explores content coverage as per the CAPS document, strategies used by teachers to teach addition and subtraction of common fractions, challenges experienced by teachers and how such challenges are addressed. The next section presents the conceptual framework used in this study.

2.2 Conceptual Framework

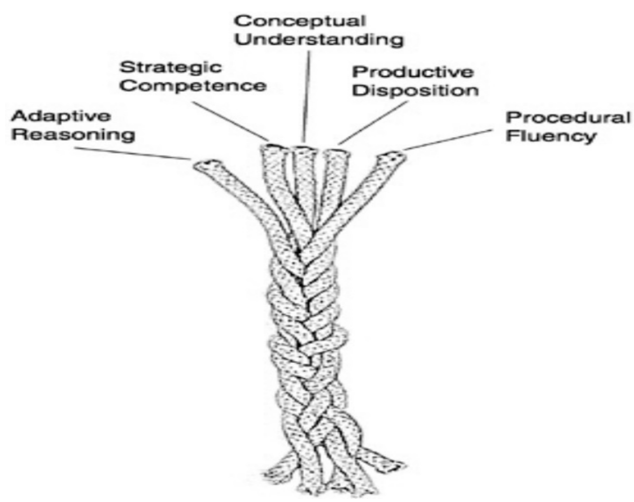
This study is framed and guided by Kilpatrick et al.'s (2001) five strands of mathematical proficiency. These strands provide a structure for understanding how they serve as a lens for exploring how teachers develop learners' proficiency in the addition and subtraction of common fractions. In other words, this section provides an overview of how ideas are organised to achieve this research study's purpose and analyse the findings. Kilpatrick et al.'s (2001) strands of mathematical proficiency are espoused as relevant conceptual tools for the current study as they provide the vocabulary to explain the participants' views (Kodisang 2022). What follows is the description of mathematical proficiency.

Mathematical proficiency

Awofala (2017, p. 489) defines mathematical proficiency as the "quality of being skilled and exhibiting expertise, competence, knowledge, beliefs, and facility in doing mathematics and becoming proficient problem solver with high productive disposition." For anyone to learn mathematics and be successful, one needs to be mathematically proficient (Kilpatrick et al., 2001). Furthermore, mathematically proficiency combines learners' knowledge, skills, abilities and beliefs with teacher support, a curriculum and a learning environment (Hamid et al., 2020). In the context of this study, mathematical proficiency is defined in relation to the organising framework that consists of five strands as set out by Kilpatrick et al. (2001).

The strands are conceptual understanding; procedural fluency, strategic competence; adaptive reasoning and productive disposition. Kilpatrick et al. (2001) emphasise that varying from focusing on rote procedures to understanding the structure of mathematics is critical to becoming mathematically proficient. These strands serve as the lens to explore how teachers develop learners' mathematical proficiency in the addition and subtraction of common fractions. It is, therefore important to highlight that this review draws our attention to the gaps found in literature and the characteristics of mathematically proficient learners. The strands of mathematical proficiency are presented in Figure 1.

Figure 1: Five strands of proficiency



From Kilpatrick et al. (2001, p. 5)

In the context of this study, the notion of mathematical proficiency is represented by five intertwined strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Awofala, 2017; Kilpatrick et al., 2001). The term intertwined means that these five strands are closely connected and cannot be separated. In other words, to attain proficiency, teaching and learning activities must address them all at once (Suh, 2007). To stress how integrated or intertwined the strands are, Suh (2007), Moodley (2008) and Samuelsson (2010), recommend that teachers need to ensure that activities given to learners build upon these five strands.

The clarification, in this case, is illustrated by providing a succinct description of each mathematical proficiency strand as set out by Kilpatrick et al. (2001) concerning its place within the teaching and learning context. Each strand is adopted as the theoretical basis for mathematical proficiency and reflects skills that learners should demonstrate to become proficient. They are discussed below.

Conceptual Understanding

Explaining conceptual understanding in meaningful language and terms that are specific, Kilpatrick et al. (2001) summarised it as an understanding of concepts, operations and relationships. In developing number sense, DBE (2011) suggests the following activities as the focal points: the meaning of different kinds of numbers, the relationship between different kinds of numbers, the relative size of different numbers, the representation of numbers in various ways, the effect of operating with numbers and the ability to estimate and check solutions (p.10)

Teaching for conceptual understanding requires teachers to go beyond just teaching a method; learners need to make sense of the method, understand how it connects, when to use it and explain why it has been used (Kilpatrick et al., 2001). Van der Walle et al. (2016) point out that this strand includes how concepts are represented and interpreted using manipulatives, tables, diagrams and pictures. This definition can potentially assist learners in understanding different ways of representing fractions. Furthermore, the MTF (DBE, 2018, p9) argued that “Conceptual understanding enables learners to see mathematics as a connected web of concepts.” Drawing from these definitions, for example, fraction calculation requires a clear understanding of fraction concepts in terms of what they mean, how they are represented, and how they could be applied in real-life situations. If learners lack an understanding of fraction calculations with like denominators, they are likely to fail to understand fraction calculations with unlike denominators (Bruce et al., 2018).

DBE (2011, p8) captures the specific aims of teaching mathematics that relates to mathematical proficiency as set out below. Learners need:

- *To have a deep conceptual understanding in order to make sense of Mathematics*
- *Acquisition of specific knowledge and skills necessary for–*
    - *the application of Mathematics to physical, social and mathematical problems*
    - *the study of related subject matter (e.g. other subjects)*
    - *further study in Mathematics.*

These specific aims and skills link to Kilpatrick et al.'s (2001) definition and explanation of the five strands of mathematical proficiency. Mathematically proficient learners accomplish all these skills and aims as stipulated in the Mathematics CAPS document.

Procedural Fluency

Kilpatrick et al. (2001) refer to knowledge of procedures, that is, the knowledge of when and how to use them appropriately and skill in performing them flexibly, accurately and efficiently. Furthermore, the MTF (DBE, 2018, p. 9) defines procedural knowledge of mathematics as "the processes through which mathematics is done." Learners need to have the skill to solve fractional problems and use the strategies effectively and efficiently. The learners need to know when they have to use a particular strategy. This suggests that teachers should be able to design activities in such a way that they assist them in connecting procedures with underlying concepts. These activities provide learners with opportunities to recall and practice strategies and justify their procedures, to be able to create their own informal strategies and procedures and use formulae

Learners will use their previously learned procedural knowledge to complete the two equations by connecting the procedure to the concept of addition and subtraction of unlike denominators. Furthermore, they can practice strategies learned during class and create their own strategies and procedures. Within schools, many teachers focus on procedural knowledge, which is superficial, as learners only know and practice procedures associated with operations (Suh, 2007). To further clarify what procedural fluency entails, Gravemeijer (2016) indicated that another way to develop this strand is to understand procedures and rules, how to complete fractional equations, and why they are using that rule or that procedure to complete the fractional equation.

Adaptive reasoning

Adaptive reasoning refers to the capacity to think logically about the relationships among concepts and situations. Awofala (2017) states that learners are proficient in adaptive reasoning when they can think logically about the existing problems, estimate and reflect on the problems, and justify solving them. Teachers should be able to support learners, and design activities in such a way that they develop mathematical thinking by asking effective questions such as:

- Why is that true? How did you reach that conclusion? How would you prove that? Why did you decide to use this method? Can you think of another method that might have worked?

Once the teacher has given learners a problem to solve, they can pair them up and ask them to explain and justify their method of solving the problem. Teachers can also go around the classroom and interact with the learners asking them questions that will develop their adaptive reasoning.

Strategic competence

Strategic competence is the ability to formulate mathematical problems, represent them, and solve them (Kilpatrick et al., 2001). This strand is similar to what has been called problem-solving and problem formulation, as “learners should be able to identify and use appropriate strategies and to devise their own strategies to solve mathematical problems” (DBE, 2018, p. 9). Teachers should be able to support learners, design activities in such a way that they can formulate a mathematical problem, represent the mathematical problem through a drawing, equation or other form of concrete representation, use mathematics to solve the formulated problem, evaluate the solution used and relating real-life situations to mathematics.

Productive disposition,

Kilpatrick et al. (2001) describe productive disposition as the tendency to see sense in mathematics, to perceive it as both valuable and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. This definition suggests that teachers should be able to support learners in such a way that they evoke positive emotion toward the subject and enjoyment in its engagement; have a greater emphasis on effort, learning, understanding and recognition; relate real-life situations to mathematics; promote a positive classroom atmosphere as well as increase the level of enthusiasm created and recognised and encourage persevering with a task.

An example of this would be for teachers to create positive and enjoyable classrooms. When a student arrives at the incorrect answer, ask them to explain their reasoning so that you can pinpoint where they have gone wrong and then assist them from there.

If learners see that mathematics is used in everyday life, they will begin to appreciate and see its need. An activity that could help them is asking them to bring a real-life mathematics problem to class every week to solve (such as we had a cake that was cut into 7 pieces. My mom ate 1 piece, I ate 2 pieces and my brother and dad ate the remaining pieces equally. How many pieces did dad and brother eat?). Teachers also need to provide lessons that are interactive and allow the learners to be actively involved in their own learning

Suh (2007) asserted that the five strands are interconnected (Kilpatrick et al., 2001), and learners need to develop all five strands to be considered mathematically proficient. She concentrated her teaching on the five strands of mathematical proficiency and discovered that her learners changed their attitude toward mathematics. Using all five strands within her teaching, she realised that she was reaching more learners as adapting her teaching to the five strands allowed her to differentiate her lessons.

Groves (2012) argued that teachers must develop these five strands of mathematical proficiency in learners by adapting their questioning strategies. In addition, Groves emphasised that teachers need to examine their pedagogy and move away from traditional teaching methods to include activities, questions, tasks, etc., that will develop all five strands of mathematical proficiency. This suggests that teachers need to assist learners with developing these skills by exposing them to as many opportunities as possible.

Additionally, Ball (2003) affirms that student opportunities to develop mathematical proficiency are shaped within the classrooms through interaction with teachers and specific content. Chi and Wylie (2014) argued that learning is more effective if learners are actively involved in their own learning. Figure 2 presents an example to show that learners are not only taught how to solve mathematics problems but to think fluidly to apply their math skills to a wide variety of problems.

Figure 2: Applying mathematics in a real-life situation.

Class activity: Moila spent a third of the money her grandparents gave her on mathematics cooking books. She also spent 1/9 of the money on a bag of hair bands.

a) What fraction of the payment has Moila got left?

b) Explain to your desk mate how you got your answer.

With your desk mate, find another way to solve the problem.

The above example shows a question with fractions of different denominators and is an addition of a common fractions word problem. For learners to understand this question and how to solve it fully, they need to have a procedural and conceptual understanding. Conceptual understanding in this context means learners should be able to understand what a fraction of payment means. They also need to know which basic concepts are needed to assist them in solving this problem. Furthermore, they could use various representations like percentages to reinforce their understanding of how much a fraction is. This understanding will enable learners to know that they need to use equivalent fractions to solve the problem. Once learners have converted the fractions, they can then solve the problem.

Furthermore, learners will need to apply strategic competence and adaptive reasoning when working with their neighbour to explain and justify their solution and devise another way to solve the problem. For learners to confidently tackle this task, they must show a positive disposition toward mathematics.

To reiterate the connectedness of the strands, Groves (2012) also highlighted that the strands are not independent but related, thus leading to the notion of intertwined connectedness. This suggests that one cannot focus on only one or two strands and expect learners to be mathematically proficient. It might also mean that learners need to conceptualise fractions and use their strategic competence to become fluent in solving problems. From these views, one might argue that mathematical proficiency is more than just knowing concepts in mathematics but also about knowing mathematics and becoming mathematically proficient, as discussed in the next section.

To conclude this section, it is imperative to emphasise the implications for classroom practices. Teachers are expected to grasp and understand the content before teaching it to ensure that when it is being taught, they are transferring the correct knowledge and can assist learners who do not understand. Teachers must set aims and lesson objectives that utilise fraction vocabulary when planning. The well-formulated objectives implicitly suggest teaching, learning and assessment activities that promote student proficiency in fractions (Kodisang 2022, p.33). I argue that the model discussed in this section guides teachers' practices to proficiency in fractions. The next section summarises and synthesises the existing scholarly research on what it means to develop learners' proficiency in fractions.

2.3 Literature Review

The literature review serves as a knowledge base for researchers to conduct their research (Maree, 2012) and a framework for authenticating the importance of the study (Creswell, 2012). These definitions point to three important areas: providing knowledge on the topic, identifying established research on the topic and identifying gaps within the research. These areas are confirmed and elaborated by Rocco and Plakhotnik (2009), who assert that without a literature review, the research would have no basis and no connection, implying no need for the study.

2.4 Basic Fraction Concepts

This section explores fundamentals to teach addition and subtraction of common fractions. According to CAPS (DBE, 2011), Grade 6 learners are expected to do addition and subtraction of fractional concepts with like and unlike denominators. For learners to achieve this, they need to have a foundational understanding of basic fraction concepts. Basic concepts, in this context, refer to the meaning of fractions, the types of fractions, their importance, and how they are represented and applied in real-life situations.

2.4.1 Meaning of Fractions

According to Ubah and Bansilal (2018), (DBE, 2011) and Loc et al. (2017), the concept of a fraction is defined as the number of equal parts of a whole. The word fraction originates from the Latin word *fractus,* meaning "broken" (Rothstein et al., 2006). The concept of fractions can be interpreted in five ways, namely:

a) **Part-whole. This refers to a whole being divided into equal parts,** e.g., Sihle eats one-half of a chocolate bar. The remainder is equally divided between two friends. How much does each one get? (Show your answer in a drawing).
b) **Measure. This is about a fraction length being represented on a number line,** e.g., Thando needs $2\frac{2}{10}$ metres of rope to make a basket. How many baskets can she make with $28\frac{1}{5}$ metres of rope?
c) **Operator. This refers to a fraction acting as a function,** e.g., Calculate $\frac{2}{3} \times 336$.
d) **Ratio. This represents fractions as ratios-comparing two quantities,** e.g., $\frac{2}{5}$ cup of milk is needed to make 40 biscuits. How many cups are needed for 2 000 biscuits? Or, is 10 litres of milk enough to bake 2 000 of these biscuits?
e) **Quotient. This refers to fractions as division,** e.g., $3 \div 4 = \frac{3}{4}$.

Based on the examples given, a fraction can have multiple interpretations. The five constructs are explained below as outlined by Tsai and Li (2017).

The ***part-whole*** construct is the foundation for ordering and comparing fractions. This construct can be used where visual or concrete representation is present so learners can see and experience fractions. This part-whole construct is where teachers need to begin (Yun & Flores, 2008). Learners' understanding and mastery of the part-whole construct will then serve as the basis for the ratio construct. The ratio construct deals with the relationship between two quantities. This construct is necessary for later mathematical concepts. After the ratio construct, we find the ***quotient*** construct, which lends itself to algebraic equations and expressions. Following the quotient construct, the ***operator*** construct is found. This operator construct is essential to understand a fraction of a quantity. Finally, the ***measure*** construct revolves around developing proficiency in additive operations in fractions, and if learners have difficulty with this construct, they will have difficulty with fractions on a number line. If learners and teachers focus on the part-whole construct, the other constructs will be underdeveloped, and learners may not develop the other constructs (Tsai & Li, 2017).

The part-whole construct is seen as an early pre-school concept where children share with each other (Fuchs et al., 2013), and this is the construct that is considered the most natural to learners as they are doing this from a young age. Teachers need to consider these connections that learners have made with real-life situations. Teachers must specify fractions' importance and how they relate to everyday experience. For example, when you bake a cake, you use 2 eggs, which is $\frac{1}{6}$ of a dozen. If we did not use fractions, we would either have a very large cake (imagine using 12 eggs instead of 2) or a cake that tastes horrible. This will also pique the learners' interests and develop a desire to develop this fractional concept.

To mathematicians, fractions are rational numbers that can be expressed in the $\mathrm{a/b}$ form, where $b \neq 0$, rather than parts of wholes (Tsai & Li, 2017). Fractions are sometimes represented by two numbers, separated by a horizontal line called the

$$\frac{2}{5}$$

fractional bar. Looking at the example alongside, the number above the fractional bar line is a numerator (2) which represents the total number of equal parts of the whole being considered, while the number below the fractional bar line is the denominator (5) which represents the total number of equal parts, into which a whole can be divided.

If we look at Figure 3, an example of a question could read as follows: "What is the fraction of the orange shaded part?" and the answer would be $\frac{5}{12}$. The circle is divided into 12 equal parts (denominator), and the number of parts being considered, the orange parts, are 5 of those 12 parts.

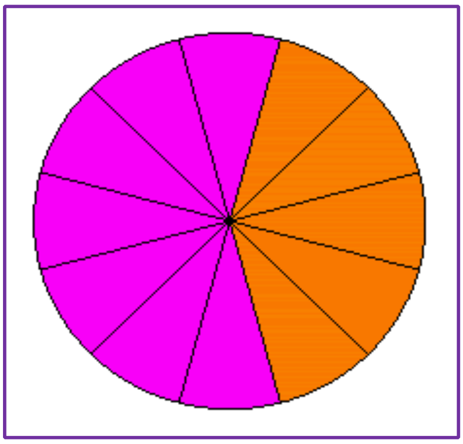


Figure 3: Circle divided into 12 equal parts

According to Jordan et al. (2013), understanding fractional concepts include both procedural and conceptual knowledge. This suggest that procedural fluency can only be informed by a conceptual understanding of fractions. Knowledge of the relationship between the numerator and denominator, as opposed to them being seen as whole numbers, would be considered conceptual knowledge. Once learners understand this relationship, they will be able to be involved in the addition and subtraction of fractions which is procedural knowledge.

2.4.2 Types of Fractions

According to Luneta (2015) and the mathematics CAPS document for grades 4-6 (DBE, 2011, p. 16-17), the different types of fractions are common fractions, decimal fractions and mixed numbers. Examples of proper fractions are $\frac{1}{2}$, $\frac{3}{4}$ and they have a value of less than 1. Examples of improper fractions are $\frac{3}{2}$, $\frac{6}{5}$ and examples of mixed numbers are $1\frac{2}{3}$, $2\frac{1}{3}$. These fractions have a value of greater than 1. Mixed numbers consist of a whole number and a fractional part and can be expressed as improper fractions. Decimal fractions can have a value between any two whole numbers, for example, 0,5 or 1,5. The area of common fractions can further be broken up into like fractions – these are fractions that have the same denominator, such as ($\frac{2}{3} + \frac{1}{3}$) – and unlike fractions ($\frac{1}{2} + \frac{1}{4}$), these are fractions that have different denominators.

2.4.3 Importance of Fractions

According to Copur-Gencturk (2021) and Fazio et al. (2016), the concept of a fraction is considered important as it is considered one of the foundational mathematical concepts. Fractions are used in later mathematical concepts such as algebra, one of the areas of advanced mathematics learning. An example of fractions in algebra looks like this: $\frac{x}{5}$ + $\frac{y}{2}$ =. Although teachers describe fractions as "a difficult concept to learn and to teach, they are of significant value within a rich mathematics curriculum in all of its complexity and realizations" (Brown, 2013, p. 7). This is further echoed by Oluyinka et al. (2020), who stated that fractions are difficult to teach and learn as concepts, and if misunderstandings and misconceptions start from a young age, learners will likely struggle with them later in life too.

The significant value of fractions can be seen in the cross-curricular impact they have. Fractions can be seen in music (note values) or geography (scaling graphs and cross-sections), changes in measure (population, GDP) or art (scaling, colour-blocking, quilting, perspective, geometric and symmetrical designs and portioned space) (Tucker, 2008; Fazio & Siegler, 2011; Lamon, 2020; Tangkui & Keong, 2021. That is how fractions cut across the subject in the curriculum. Thus, teachers' fractional knowledge is important as it will inform their teaching of the important topic needed for more complex concepts later in mathematics, such as algebra. The next section discusses how fractions are represented.

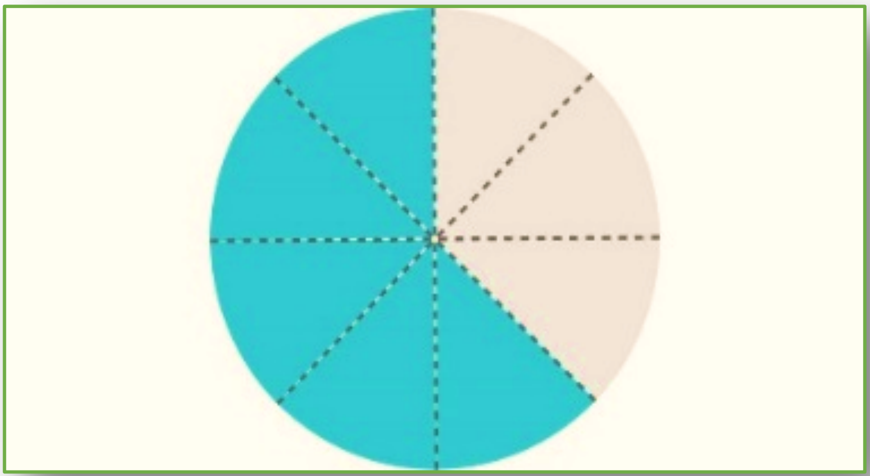
2.4.4 Representation of Fractions

2.4.4.1 Terminology in representing fractions

Language is a structured system of communication that humans use. According to the CAPS document (DBE, 2011:8), mathematics is seen as a language "that makes use of symbols and notations to describe numerical, geometric and graphical relationships." A learner should "develop the correct use of the language of Mathematics" to develop crucial mathematical skills (DBE, 2011, p. 8). Learning should be contextualised, and proper language is encouraged to achieve this. It is vital for teachers to encourage learners to use the correct mathematical language, drawing from the understanding of mathematical proficiency,.

Ratio, relationship, whole, numerator, and denominator are examples of terms used in fractions (DBE, 2011). For example, part of a whole refers to a fractional number where the numerator is identified in terms of the number of shaded or selected equal parts, while the denominator is the total number of equal parts that the whole has been divided into (Ubah & Bansilal, 2018). For example: What is the fraction of the blue shaded part? Learners will count the number of pieces the circle is divided into and then have to count the number of pieces shaded in blue. Their answer should be $\frac{5}{8}$. (See Figure 4).

Figure 4: Circle divided into 8 equal parts



Source: https://www.dkfindout.com/uk/maths/fractions/

Learners could also identify shaded (or unshaded parts) and then write the fractional expressions, such as, $\frac{2}{8} + \frac{3}{8}$ (or $\frac{1}{8} + \frac{1}{4}$) Furthermore, when teaching fractions, teachers must use the correct terminology so learners are aware of it and can use it. As Bruce et al. (2013) explained, if teachers use incorrect terminology, learners might consider the denominator and numerator as two whole numbers rather than one fractional quantity. For example: $\frac{1}{2}$ is read as 'half' but learners are often taught to read it as "one over two" or "one out of two". If learners are taught the latter, they will consider the denominator and numerator as two separate whole numbers instead of dividing a whole into two parts (Siebert & Gaskin, 2006).

2.4.4.2 Models for representing fractions

Mathematical concepts could also be represented in different ways; for example, learners could represent fractional equations as numbers, words, pictures or symbols. According to the National Council for Teachers of Mathematics Principles and Standards (NCTM) (2000), these representations will enable learners to understand and work with fractions better. Furthermore, Lamon (2001) stipulates that representational models are important and play a role in teaching and learning fraction concepts. Fractions may be represented using models, namely, area models, set models and length models.

Area models are defined as a whole being partitioned or sectioned into equal parts (Hull, 2005). This is the most common type of representation of fractions. An example of this model is seen in Figure 5.

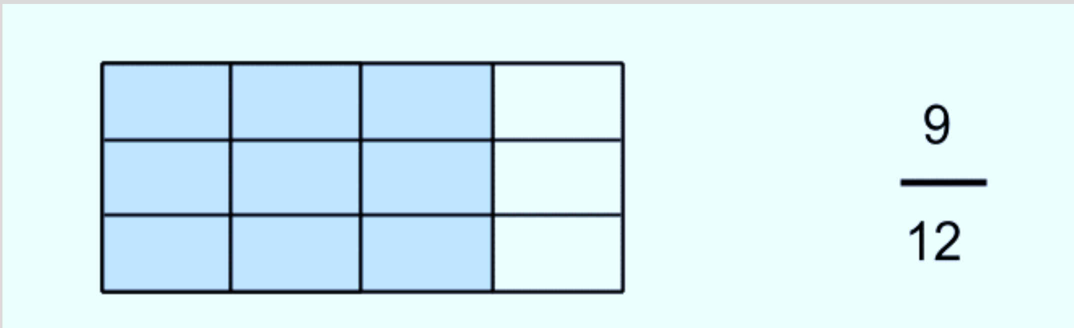


Figure 5: Example of Area Model

Source: https://davidwees.com/content/one-model-adding-fractions/

It was highlighted in DBE (2018) that with **set models,** learners are expected to understand that a group of objects can be considered a whole and that the individual objects are parts of the whole. An example of this model can be seen in Figure 6.

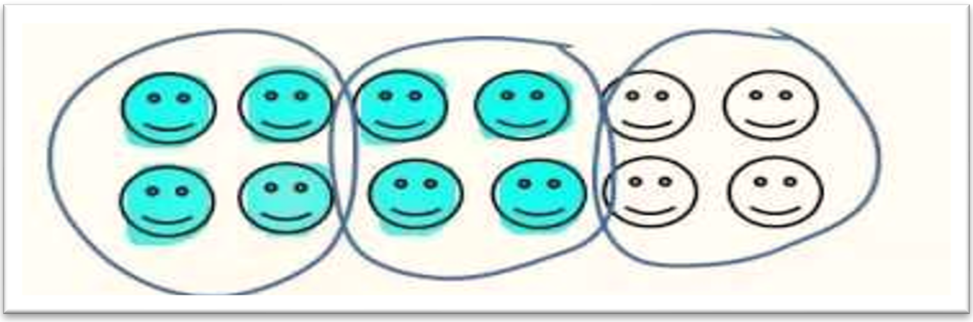


Figure 6: Example of Set Model

Source: https://learnzillion.com/resources/47000/

The shaded part in Figure 6 represents $\frac{2}{3}$ of the set

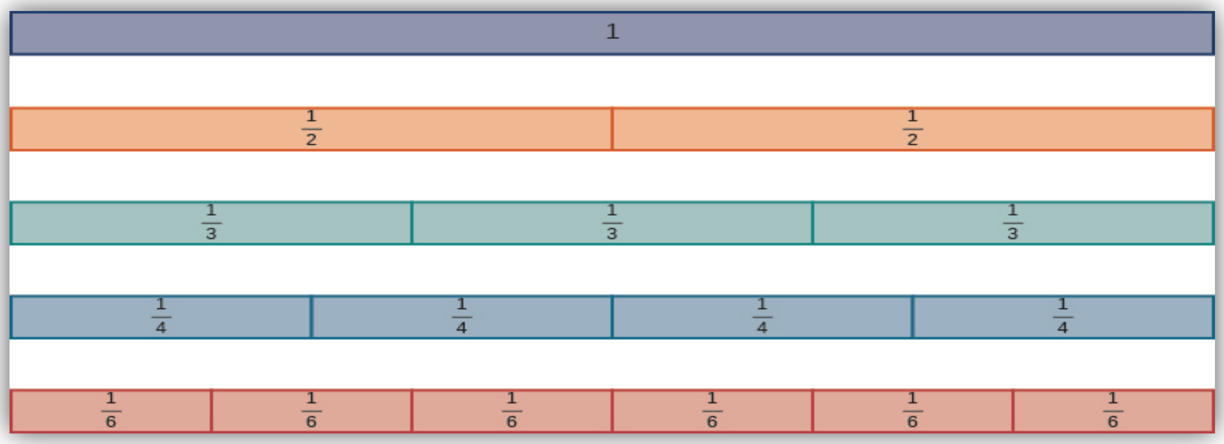


Figure 7: Example of Length Model

Source: https://cnx.org/contents/yqV9q0HH@1.41:_oNnfiWA/Visualize-Fractions

Length models are different to set and area models. This model relates to the number line. The length of the whole is divided into equal parts (NCTM, 2000). An example of this model can be seen in Figure 7.

Teachers could use this model to teach the addition and subtraction of common fractions, equivalent fractions and also for the comparison of fractions. For example, learners could use the fraction wall to find equivalent fractions. If learners are given the following question, $\frac{1}{3} + \frac{2}{6}$, they can use the fraction wall to find the equivalent fraction for $\frac{1}{3}$ and solve the equation in its simplest form.

2.4.5 Content Coverage on Fraction Calculation

According to the DBE (2011), the aim of teaching mathematics is to develop an awareness of how mathematical relationships are used and the important role that mathematics plays in one's everyday life. This aim suggests there is more to mathematics than just attaining and using the knowledge procedurally. More to mathematics might also refer to how knowledge is applied in our everyday life. Table 1 illustrates the spread of content throughout the phase.

Table 1: Specification of content for common fractions

Topics	Grade 4	Grade 5	Grade 6
1.2 **Common Fractions**	**Calculations with fractions:** • Addition of common fractions with the same denominators • Recognize, describe and use the equivalence of division and fractions **Solving problems** • Solve problems in contexts involving fractions, including grouping and equal sharing	**Calculations with fractions:** • Addition and subtraction of common fractions with the same denominators • Recognise, describe and use the equivalence of division and fractions **Solving problems** • Solve problems in contexts involving common fractions, including grouping and sharing	**Calculations with fractions:** • Addition and subtraction of common fractions in which one denominator is a multiple of another **Solving problems** • Solve problems in contexts involving common fractions, including grouping and sharing

Adopted from DBE (2011, p. 16

Drawing from the understanding of mathematical proficiency, Table 1 presents the breakdown of fraction calculation in terms of the concepts of skills to cover per grade. Some of the main characteristics in the table relate to how fractions are described, ordered and calculated. Although the key concepts cover quite a number of skills, the study will focus on how teachers develop learners' mathematical proficiency in the addition and subtraction of common fractions in Grade 6.

Table 1 has been included for several reasons: It identifies key concepts covered (Describing and ordering fractions and calculations with fractions), it specifies content covered, it shows the progression from one grade to the other, and it also provides some guidance on the type of teaching and learning activities that could be implemented in the classroom. Activities describe and clarify key concepts and skills covered in each grade. Table 1 explains how the key concept of calculations with fractions develops from Grade 4 to Grade 6.

The purpose of this study is to explore how calculation with fractions is unpacked by teachers, focusing on the addition and subtraction of common fractions in which one denominator is a multiple of another. Looking at Table 1, learners are expected to master the fraction skills as follows: In Grade 4, the focus is on the addition of common fractions with the same denominator, such as, $\frac{2}{8}+\frac{3}{8}$, in Grade 5, the focus is on the addition and subtraction of common fractions with the same denominator, such as $\frac{9}{16}-\frac{3}{16}$ whereas in Grade 6 the focus is on addition and subtraction of common fraction in which one denominator is a multiple of the other, such as $\frac{2}{8}+\frac{16}{24}$.

This difference in focus suggests that the focus, as highlighted in the previous statement, will form part of the prior knowledge for Grade 6 activities. The understanding is that other skills for Grades 4 and 5 will also form part of the prior knowledge. If learners lack an understanding of fraction calculations with like denominators, they are likely to fail to understand fraction calculations with unlike denominators (Abdul Ghani & Maat, 2018; Bruce et al., 2013). This emphasises the importance of recognising prior knowledge or foundational concepts for a better understanding of the new concepts.

The CAPS document stipulates what needs to be taught and the order in which concepts must be taught. Examining the Mathematics CAPS document for Grade 6 common fractions, it is seen that there is a progression of concepts from Grade 5. A teacher must build upon this Grade 5 knowledge of common fractions and connect with learners' prior knowledge. Engaging with learners' prior knowledge will ensure that new connections can be made (DBE, 2011).

Foundational knowledge, which is the mathematical skills, terminology, methods and approaches of reasoning essential for more advanced or independent learning in mathematics, is vital for further fractional studies and other mathematical concepts. Teachers should build upon prior knowledge and use scaffolding to develop learners' mathematical proficiency. According to the DBE (2011), the sequencing of fractions should first be approached from the addition and subtraction of common fractions with like denominators, followed by the addition and subtraction of common fractions whereby one denominator is a multiple of the other denominator and lastly addition and subtraction of common fractions with unlike denominators. The greater learners' expertise in common fractions, the more requisite knowledge they will have on how to deal with the addition and subtraction of mixed numbers.

As explained in the previous paragraphs, fraction calculation in this case involves the addition and subtraction of common fractions in which one denominator is a multiple of the other, as well as the addition and subtraction of mixed numbers and fractions of whole numbers. This study has explored teachers' practices on the calculation of fractions at the Grade 6 level, with a special focus on addition and subtraction of common fractions in which one denominator is a multiple of the other, for example, $\frac{2}{3} + \frac{1}{9}$.

The addition and subtraction of common fractions are the beginning stages of calculations regarding fractions (Robinson, 2014). According to Piaget (Huitt & Hummel, 2003), there are many cognitive development stages that learners will move through to grasp concepts in the world around them. As learners progress through the fraction concept, teachers will begin with the concrete conceptualisation stage, allowing learners to work with concrete objects such as cutting an apple into halves or quarters. Teachers will then move on to the next stage, which is considered semi-concrete representational conceptualisation. During this stage, teachers use visual representations, such as drawing a picture of an apple which has been cut into equal parts. Visual representation refers to physically drawing the problem at hand, such as the example in Figure 8 where the pizzas have been drawn. The example in Figure 8 below provides a visual representation of the addition of common fractions with the same denominator.

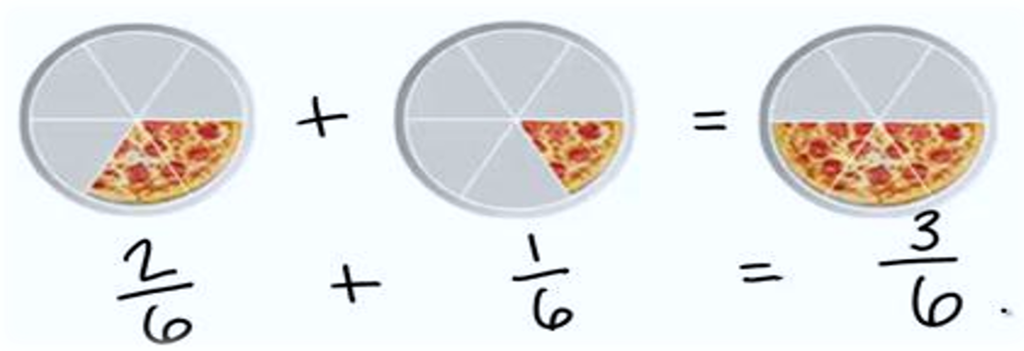


Figure 8: Visual Representation of addition of common fractions with like denominators.

Source: https://learnzillion.com/lesson_plans/8191-add-and-subtract-fractions-with-like-denominators

Once learners have worked with visual representations and mastered the semi-concrete representational conceptualisation, they can then move on to semi-abstract representational conceptualisation questions. In this stage, teachers will ensure that they teach the representation of fractions in a model, diagram, line or area. After this stage has been mastered, learners will move onto the abstract conceptualisation stage, which entails doing calculations and manipulations of fraction symbols, for example, $\frac{2}{3} + \frac{1}{9}$. Problems on fractions also focus on the addition and subtraction of fractions with different denominators, for example: $\frac{1}{2} + \frac{1}{4}$ or $\frac{5}{6} - \frac{1}{3}$. Problems involving the calculation of fractions with unlike denominators might be a challenge for learners as they would need to undergo a process to find the lowest common denominator (LCD) and then equivalent fractions so that the unlike denominators can be treated as like denominators, as this is one of the ways way learners can add or subtract fractions (Ubah & Bansilal, 2018).

From my experience as a mathematics teacher, we sometimes used the introductory concepts of one grade as activities for expanded opportunities of the previous grades. This implies that if there are above-average learners in a particular grade, we normally use the activities of the next grade to accommodate such learners. Calculations require a clear understanding of fraction concepts in terms of what they mean, how they are represented, and how they could be applied in real-life situations.

The addition and subtraction of common fractions are basic operations that learners must perform. These operations are taught first before learners move on to multiplication and division of common fractions. With the addition and subtraction of fractions, learners are taught the formal steps to follow to complete the algorithm. Before learners perform these algorithms, they need to identify whether they are working with like or unlike denominators, as there are different procedures for the two.

In 2013, Ally and Christiansen undertook research that proposed a rubric for assessing the teacher's instruction to develop mathematical proficiency in classrooms. The results from the research revealed that opportunities to develop procedural fluency are common, but generally of a low quality; that opportunities to develop conceptual understanding are present in about half the lessons, but are also not of high quality; and that overall opportunities to develop mathematical proficiency are limited, because learners are not engaging in adaptive reasoning, hardly have any opportunities to develop a productive disposition, and are seldom given the opportunity to engage in problem-solving which could develop their strategic competence.

This study's findings suggest teachers should clearly understand what it means to develop learners' mathematical proficiency. For example, as I have indicated from my experience, the understanding of mastering procedures in the addition and subtraction of common fractions relates to identification and grouping of like terms followed by use of basic operations. My experience suggests that the skills in performing procedures flexibly, accurately and efficiently (Kilpatrick et al., 2001) were not fully developed.

Again, from my experience, learners can complete the addition and subtraction calculations with like denominators as they are expected just to add the numerators and leave the denominator as is (for example: $\frac{6}{9} + \frac{2}{9} = \frac{8}{9}$), however, learners struggle with addition and subtraction calculations with unlike denominators. Many learners I have encountered just add the numerators and add the denominators (for example: $\frac{1}{4} + \frac{1}{8} = \frac{2}{12}$) as opposed to finding the lowest common denominator by using equivalent fractions (for example: $\frac{1}{4} + \frac{1}{8} = ?$, learners will have to find an equivalent fraction for $\frac{1}{4}$ that has a denominator of 8 (as we need to have like denominators to complete the equation). Therefore, their "new" equation looks as follows: $\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$).

As a Grade 6 mathematics teacher, I have found that the concept of times tables, multiples and factors were lacking among my learners, so finding the lowest common multiples and equivalent fractions (Petit et al., 2010) was challenging. Therefore, this experience suggests that teachers need to use alternative strategies like manipulatives, games, etc., to assist the learners in understanding the challenging areas.

Samuelsson (2010) undertook a study that used the five strands of mathematical proficiency as a framework for assessing the impact of teaching approaches on Swedish learners' mathematical proficiency over five years. The study examined the success of traditional and problem-solving approaches to teaching. The learners were presented with problems that tested each of the five strands. His results indicated that different teaching approaches do indeed positively affect the learners' development. Although the study encourages teachers to use different strategies, the challenge is that such strategies depend on how many learners are in the classrooms, the sitting arrangements, and the type of resources accessible to learners.

2.5 Strategies Used by Teachers to Teach the Addition and Subtraction of Fractions

A study at a Zimbabwean school to investigate the methods used by teachers to teach fractions (Chinyoka et al., 2012) discovered that teachers were influenced by their perceptions and beliefs, which means that if they were taught in a certain way, that is how they would teach. Teachers taught to focus on procedures and rules, were most likely to focus on that during their lessons. This can be of concern as they are then not teaching to develop learners' mathematical proficiency. The next section presents multiple ways of representing and teaching the addition and subtraction of fractions.

- Manipulatives/concrete objective/fraction lab

As teachers move away from traditional teaching methods, there has been great encouragement to utilise hands-on activities, getting learners involved in their own learning (Kukey et al., 2019). According to Abramovich et al. (2019), learners learn best by being involved in the learning process. Furthermore, Harris and Hofer (2011) argued the importance of hands-on activities. These activities are created by the teacher and involve the learners being actively involved in their own learning; such activities could include the use of blocks, marbles, paper, pizza, the list is endless as long as learners can touch and feel (as well as partition) the object with which they are working. These activities will enhance the learners' learning experience and make the lesson exciting for both the student and the teacher (Furner & Worrell, 2017). Using these types of manipulatives may help learners visually see the problem, but learners may have a problem if they do not develop an understanding of fractions to also complete the problem abstractly.

- Use of different models

Lamon (2001) and Niemi (1996) stipulate that representational models are important and play a role in teaching and learning. Furthermore, Petit et al., (2015) and Monson, Cramer & Ahrend (2020) agree with the previous statements and argue that different models are used to enhance and develop fraction knowledge. Fractions are represented using different models. These models are either continuous or discrete models. Area and length models are known as continuous models, consisting of wholes that can be broken into smaller parts (Rapp et al., 2015). One will utilise these models at the introduction of the fraction concept and during the concrete teaching and learning phase before learners move on to abstract calculations. An example of this model would be to divide a pizza into 6 equal parts and then shade "one sixth" or $\frac{1}{6}$, as shown in Figure 9 below

Figure 9: A circle divided into 6 equal parts

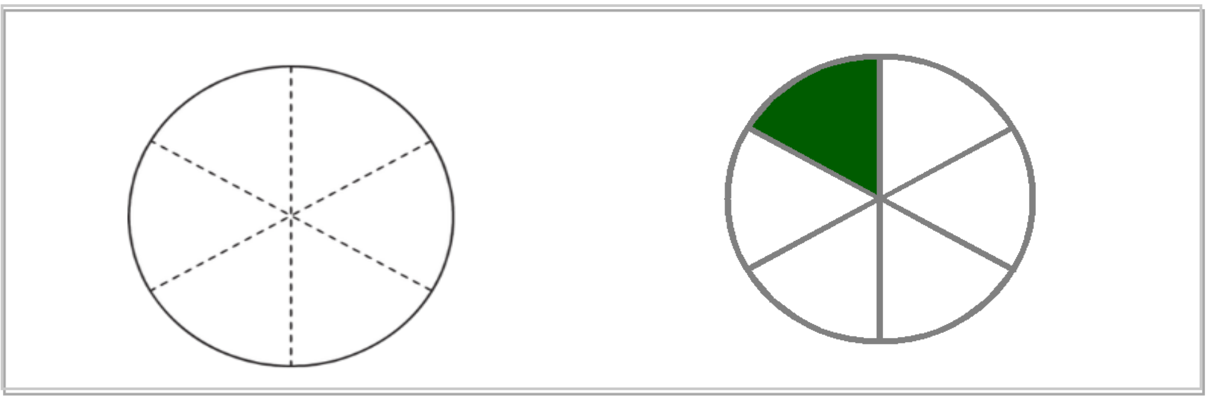
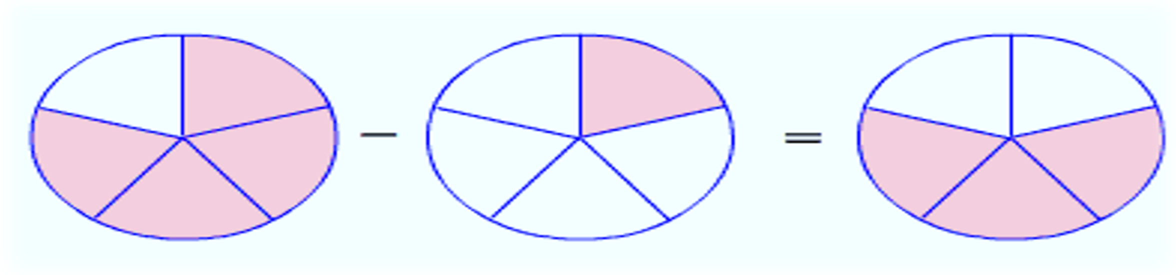


Figure 10 below depicts a subtraction calculation using continuous models:

Figure 10: Example of Continuous Models



$$\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$$

In these two examples (Figures 9 and 10) above, learners can visually see the representation of the fraction. The challenge that learners may encounter when using the continuous model might arise when working with fractional calculations of unlike denominators because it might be too challenging for learners to see this visually (see the example below):

$$\frac{1}{2}+\frac{2}{5}$$

In the above example, learners must find equivalent fractions to solve this problem. Continuous models will promote mathematical proficiency if learners can fully grasp the concept of fractional calculations instead of learning rote procedures and steps (Ubah & Bansilal, 2018).

The **discrete model** or set model is considered to be confusing as learners are expected to understand that a group of objects makes up a whole (as opposed to a single object being divided into parts), and that each individual object would be a part of that whole (DBE, 2018b). If learnt, this model may add to mathematical proficiency of fractions as learners extend and deepen their knowledge of fractions. Research by Behr et al. (1988) asserted that learning this model would provide a greater understanding of fractions.

- **Games, rhymes and songs**

Technology has brought people all over the world together, and using game-based applications in the mathematics classroom could interest learners. A study by Simsek (2016) showed that participants' fractional skills increased as they played the fraction game (Motion Math: Fractions!). Furthermore, Vankus (2008) asserts that learners' motivation to do mathematics increases when they are involved with game-based applications. Using this method may be advantageous to certain learners who enjoy the "gaming life". However, if the game is too challenging for learners, they may not want to play and can become more demotivated to be involved in learning mathematics.

To stress the importance of using different strategies to develop learners' proficiency in mathematics, Fazio et al. (2016) developed a game, based on previous research, called "Catch the Monster with Fractions", where Grade 4 and 5 learners are actively playing with various fractional concepts, and they are given feedback. In this game, learners took a pre- and post-test, and they had improved from the pre-test to the post-test. They completed another study where children did not receive feedback, and they did not improve (Fazio et al., 2016). This implies that children require constructive feedback to be aware of their mistakes and how to rectify them.

- **Mathematics mnemonics**

Mnemonics can be used when learners have difficulty retaining information. This strategy can help learners improve their memory. These mnemonics can be visual or verbal. Research by DeLashmutt (2007) asserted that mnemonics assisted some learners with mathematics. There is no specific research on fractional mnemonics and their ability to enhance student learning. A mnemonic I enjoyed using in my classroom was KFC when dividing fractions. **KFC** can be explained as **Keep** the first fraction, **Change** the sign from divide to multiply, **Flip** the last fraction. The learners really enjoyed it, and when I see them, they still remind me of that mnemonic.
Using this strategy may help learners struggling with remembering, although, everyone may not use this strategy. If learners utilise this strategy, they may be able to be mathematically proficient as they will be able to use the strategy in all contexts relating to fractions.

- **Use of standard algorithm**

In the beginning, when completing fraction calculations, learners will start by adding or subtracting fractions that have like denominators (e.g. the sum of $\frac{1}{7}$ + $\frac{3}{7}$). In the example, learners can add the numerators (1 + 3 = 4), and the denominator remains the same, so the solution is $\frac{4}{7}$. When working with adding and subtracting fractions that have unlike denominators, learners have to be aware of the mathematical procedure as they will need to get the denominators to be alike (they do this by finding a Lowest Common Multiple of the two denominators–this is all about working with equivalent fractions). If learners are unaware of the conceptual understanding of like fraction calculations, they will not be able to complete unlike fraction calculations. This strategy should be combined with other strategies to ensure effective teaching and learning of fractions as well as developing mathematically proficient learners.

Empson (2003) asserts that a conceptual understanding of common fractions and a focus on scaffolding will help learners move from what they know to what they should know. Learners can make connections when they move from the known to the unknown. These connections are made when learners are actively involved in their learning, and their attention is captured (DBE, 2011). To be successful in the topics mentioned above, like algebra, learners should know how to describe fractions using the correct language, how to represent fractions and how to apply fractions in context.

2.6 Challenges in Teaching the Addition and Subtraction of Fractions

According to Copur-Gencturk (2021), Oluyinka et al. (2020), Ubah and Bansilal (2018), Newton (2008), Tsai and Li (2017), Stewart (2005), Van Steenbrugge et al. (2015) and Wu (1999), some teachers find fractions difficult to teach, and, in turn, learners find them difficult to learn. Learners who lack fraction knowledge cannot perform fraction calculations and, in turn, they may struggle with later mathematical concepts such as algebra (Brown & Quinn, 2006; Oluyinka et al., 2020). This difficulty may lead to poor mathematical performance and therefore discourage learners from pursuing careers in mathematical fields.

According to Hasemann (1981), there are several reasons for learners having difficulty with fractions. This is echoed by Lestiana et al. (2017) and Fauzi and Suryadi (2020), who stated that there are several reasons that learners may struggle with fractions. These reasons include that fractions are not seen in everyday life, and there are many rules associated with fractional equations, which may imply that teachers focus more on rote procedures instead of conceptual understanding (Bruce et al., 2013). Likewise, Veloo and Puteh (2017) note the reason for mistakes may include focusing on procedural knowledge rather than on conceptual knowledge. Difficulties noted by Fauzi and Suryadi (2020) are a lack of foundational knowledge or methods and approaches used by teachers who lack content knowledge. Furthermore, Ubah and Bansilal (2018) and Tsai and Li (2017) identify a few sources of learners' difficulties with fractions. These difficulties include:

- A curriculum that is full, which leaves little time to develop conceptual knowledge
- The assumption that properties of fractions are the same as whole numbers
- The concept of equivalent fractions.

Jigyel and Afamasaga-Fuata'I (2007) support these claims that the concept of equivalent fractions is important, and learners lack this knowledge.

Studies have been conducted concerning teachers' conceptual and procedural knowledge of fractions and found that although both are present, procedural knowledge dominates (Copur-Gencturk, 2021; Ward & Thomas, 2006). Teacher-led explanations of fractions have differed from teacher to teacher. Some teachers focus on conceptual information, while others focus on procedural information (Ward & Thomas, 2006). Furthermore, teachers may also struggle with conceptual knowledge, so they do not know what they are doing, which indicates that they cannot explain to learners why they do certain steps or follow specific sequences (Copur-Gencturk, 2021). This implies that the teachers focus on procedural knowledge and will focus on procedures during their teaching. (Olanoff et al., 2014).

Teachers, in some cases, have limited subject content knowledge and therefore are not equipped with the necessary knowledge to teach learners the concepts (Ward & Thomas, 2006). Pienaar (2014) argues that teachers need to broaden their knowledge of fractions and understand the intricacies of the concept. As learners and teachers seem to have difficulty with fractions, this impedes the teaching and learning process.

There are many misconceptions concerning the addition and subtraction of common fractions, such as the addition and subtraction of common fractions with unlike denominators (Trivena et al., 2017). The difficulty of addition and subtraction of common fractions can arise when teaching the concept with unlike denominators, as teachers and learners need to use equivalent fractions by finding the lowest common denominator. As a Grade 6 teacher, I found that learners struggled with the procedure of using equivalent fractions as their foundational times table knowledge was lacking. Therefore, they could not list multiples of the denominators to find a common lowest multiple. The explanation below on fractional procedures may shed some light on the difficulty of understanding fraction operations:

> *When we add or subtract fractions, we have to find a common denominator, but not when we multiply or divide. And once we get a common denominator, we add or subtract the numerators, but not the denominators, despite the fact that when we multiply, we multiply both the numerators and denominators, and when we divide, we divide neither the numerators nor the denominators*. (Siebert & Gaskin, 2006, p. 394)

Now this may seem easy for those who conceptually understand fractions, but it can prove difficult for those who are learning fractions procedurally. Most mistakes seem to be procedural and conceptual (Lestiana et al., 2017). Teachers make learners to memorise rules, formulas and algorithms associated with fractions. It is rarely a consideration for teachers to share with learners the reason behind the use of operations related to fractions (Duzenli-Gokalp & Sharma, 2010; Murray & Newstead, 1998). Learners may not always memorise correctly and therefore it is imperative that learners understand what they are doing and why they are doing it. In the example above, learners may add (or subtract) the numerators and then add (or subtract) the denominators instead of using equivalent fractions. For example: in $\frac{1}{2} + \frac{1}{4}$, learners may say 1 + 1 = 2, therefore the numerator is 2, and 2+ 4 = 6, therefore the denominator is 6, so their answer would read as follows: $\frac{2}{6}$.

2.7 How are Such Challenges Addressed?

Teachers should understand fractions concepts thoroughly to help learners develop mathematical concepts related to fractions (McDiarmid & Wilson, 1991). They need to be aware of the language they use when teaching fractions. For instance, according to Bruce et al. (2013), the fraction $\frac{3}{7}$ can be read as three sevenths, additionally, this fraction can also be read as three out of seven or three over seven. In this instance, learners see the numerator and denominator as two whole numbers rather than dividing the whole into seven equal parts (Siebert & Gaskin, 2006).

Mathematics is seen as a structured communication system, and it is described as a "language that makes use of symbols and notations to describe numerical, geometric and graphical relationships" (DBE, 2011, p. 8). It is a universal language, as the symbols and equations are understood in every corner of the world. To be a functioning literary member of society, one needs to be proficient in the language used within any given society.

For learners to understand the fraction concept, learning should be contextualised by using proper language. It is therefore important to understand the language of fractions to develop the necessary skills and knowledge required, which improves problem-solving, connecting and reasoning process skills.

Furthermore, when teaching fractions, it is essential that teachers use the correct terminology, so learners are aware of it and can use it. Further, as Bruce et al. (2013) explained, if teachers use incorrect terminology, learners might consider the denominator and numerator as two whole numbers rather than one fractional quantity. For example: $\frac{1}{2}$ is read as "half" but learners are often taught to read it as "one over two" or "one out of two". If learners are taught the latter, they will consider the denominator and numerator as two separate whole numbers instead of dividing a whole into two parts (Siebert & Gaskin, 2006).

Regarding teachers' practices, İskenderoğlu (2017) and Van Steenbrugge et al. (2017) observe that the method for teaching the addition and subtraction of common fractions was necessary for further fractional equations and studies. This includes multiplication and division of fractions as well as fractional problem-solving. When providing instruction to learners around fractional concepts, teachers should have a sound knowledge of the topic and provide a stimulating and challenging classroom. Understanding the areas that learners struggle with will also equip a teacher with a starting point for helping learners and clarifying their misconceptions or difficulties (NCTM, 2000).

Drawing from the literature, a focus on foundational concepts in earlier grades should be emphasised as this will greatly help learners in later grades (Fuchs et al., 2013). Concepts such as times tables should have great importance placed upon them as they will help learners with equivalent fractions and, subsequently, the addition and subtraction of common fractions with unlike denominators. An emphasis on conceptual and procedural knowledge can assist with the challenges related to the addition and subtraction of common fractions as learners will develop an understanding of why and how a procedure is utilised (Veloo & Puteh, 2017).

On the other hand, Newton (2008) indicated that:

- Prospective teachers had low confidence and high anxiety when it came to mathematics.
- It may also be helpful to send teachers on courses and workshops relating to fractions and mathematics.
- The study revealed that pre-service teachers had low confidence in fractions, however, after the course with a focus on developing pre-service teachers' conceptual understanding of fractions, their confidence increased.

We can also assume that when learners receive constructive feedback when assessed on their learning and are taught using various strategies, then they can pinpoint where they went wrong and use that feedback to better their understanding. Thus, it is important that teachers are aware of their learners who struggle so that they can provide feedback and help where necessary (Empson, 2003).

Teachers can create various classroom activities, such as those created by Suh (2007), to develop mathematical proficiency in learners. These activities included, "Modelling mathematics meaningfully" to teach for and assess conceptual understanding. The activity, "Maths Curse" emphasises a productive disposition towards mathematics. She explained reading a book called "Maths Curse". "Mathematics Happening" was another activity listed. This activity sought to allow learners to become more familiar with the concept of problem-solving. The last two activities mentioned, "Convince me" and "Poster Proofs", were created to develop strategic competence and adaptive reasoning.

2.8 Conclusion

I have discussed, through literature, the meaning of mathematical proficiency, the basic fraction concepts and strategies used to develop learners' mathematical proficiency concerning the teaching and learning of common fractions. It is clear from the studies reviewed that learners are struggling with the concept of addition and subtraction of common fractions.

Although the literature has clarified how to develop learner mathematical proficiency in fractions to a certain extent, it is still not clear, in relation to the addition and subtraction of common fractions, how teachers should support learners to understand why they are following certain procedures to perform fraction tasks, engage in problem-solving activities in fractions, formulate their own problems and justify and explain the reasoning behind their choice of problem-solving strategies. The next section discusses the theory that underpinned this study

3 CHAPTER THREE: RESEARCH METHODOLOGY

3.1 Introduction

In this chapter, I describe the methodological choices used in the study. Furthermore, full details of the data collection and analysis processes and how I gained access to the field are given. The methodological processes were used to achieve the objectives of the study, which were:

- To establish how Grade 6 mathematics teachers understand the fraction concept
- To describe how teachers develop learners' mathematical proficiency in the addition and subtraction of common fractions
- To suggest possible strategies to enhance the development of learners' proficiency in the addition and subtraction of common fractions.

This chapter presents the methodological choices in six sections. The first section describes the research paradigm, approach and design used in this study; the second details the sampling procedures; the third explains how data was collected, followed by how it was analysed. The last two sections focus on credibility and ethical considerations observed in this study. The chapter ends with a conclusion.

3.2 Methodological Choice

3.2.1 Research Paradigm

According to Rehman and Alharthi (2016), a research paradigm is a set of principles of how the world is observed and serves as a framework that guides the researcher. Similarly, Kivunja and Kuyini (2017) state that a research paradigm is a lens through which a researcher looks at the world and outlines what is acceptable and how to conduct research. Furthermore, Hughes (2010, p. 35) asserts that a paradigm is "a way of seeing the world that frames a research topic" and influences the way that researchers think about the topic. This study's research paradigm and design are within the interpretivist paradigm, which is used to understand situations or phenomena (Adom et al., 2016). In other words, an interpretive position offers a perspective of examining the phenomenon methodically and in detail, typically to explain and interpret it.

The research philosophy that underpinned this study was social constructivism. Riegler (2012), Jones and Brader-Araje (2002) and Tamur and Juandi (2020) define constructivism as creating knowledge through what one sees as opposed to passively acquiring knowledge. It is important to highlight that Amineh and Asl (2015) understand social constructivism as a process whereby meaning is constructed. This view is supported by Pouliot (2004), who said constructivism is a means of constructing reality. Drawing from these views, I constructed realities of how teachers make sense of the fraction concept and interpreted the realities as subjective for every individual teacher. I watched each video lesson three times to develop the concepts based on the experiences I encountered. These concepts were further clarified during the interviews with the participants, wherein they further unpacked their experiences.

Furthermore, constructivist research focuses on the meanings embedded in documented and oral accounts and generally involves the analysis of material such as documentary sources and/or oral and personal histories and narratives gathered through data collection strategies such as interviews (Bisman & Highfield, 2012). I was involved in constructing meaning from what transpired in the classroom and during the interviews.

From the ontological perspective, I used more than one data collection method to explore multiple realities. An ontological perspective refers to the idea that there are no shared realities and that reality and social phenomena are constructed by individuals (Al-Saadi, 2014; Scotland, 2012). Epistemology refers to the idea that knowledge is discovered by exploring and understanding people's world (Al-Saadi, 2014; Scotland, 2012). I interacted with the participants. This involves subjectivity as I must understand the world using my own understanding and the participants' understanding. In this study, I observed video-recorded lessons and engaged with the participants through an interview to gain a rich, descriptive understanding. Thus, this research project explored various ways participants attempted to develop learners' proficiency in the fraction concept.

Research approach

This section examines the approach I used to tackle the research questions. Groves (2015, p. 1) defines the research approach as "the plans and the procedure for research that encompass the steps from broad assumptions to detailed methods of data collection, analysis, and interpretation." The research approach used in this study followed a qualitative approach to allow me to interact with respondents and question them on their reasoning and justification for certain strategies employed when unpacking the fraction concept.

Creswell (2014) asserts that a qualitative approach is used to explore and understand the meaning of individuals or groups regarding a social or human problem. This approach is non-numerical and presents facts collected in narration (Maree, 2012). Furthermore, Berg and Lune (2012) support this statement by Maree (2012), stating that qualitative research is characterised as meanings, a definition, and a description of things. Various authors believe that qualitative research is used to improve researchers' understanding of individuals' cultures, beliefs and values, human experiences and situations, and to develop theories that describe these experiences (Creswell, 2011, 2014; Daniel, 2016; Maxwell, 2012; Rahman, 2017). Merriam (1998) affirms that qualitative research is based on individuals interacting with their social world to construct a reality.

According to Merriam (2009, p. 5), qualitative research is "interested in understanding how people interpret their experiences, how they construct their worlds and what meaning they attribute to their experiences". In light of this, I became the primary vehicle for the data collection and analysis processes. Thus, there was a need for a relationship to be developed between the researcher and participants. Qualitative research aims to develop an understanding of participants and how they make sense of their lives. In this study, I ensured that the participants' actions were observed in the settings in which they occurred. For example, I watched the video recorded in the real setting, which, in this case, was the classroom. I could record the issues that emerged during the lesson presentation and probe participants on such issues. All the interpretations and analyses were made descriptively, not using numbers.

3.2.2 Research Design

Creswell (2014) states that research design "focuses on data collection, analysis, and writing, but they originate out of disciplines and flow throughout the process of research". The research design is the plan for conducting the study. The research design used in this study was a case study design. Maree (2012) defines a case study as an in-depth exploration of a particular group within a particular setting to gain an in-depth understanding of an issue in its real-life situation. I chose this design to enable me to study each case's realities and context.

Furthermore, Creswell (2007, p73) concurs with this statement by Maree (2012) who indicated that the approach in which the "researcher explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audio-visual material, and documents and reports), and reports a case description and case-based themes", is seen as a case study. Using the case study design allowed me to explore and analyse the rationale behind using particular strategies in developing the fraction concept.

3.3 Research Sample and Data Sources

3.3.1 Population and Sampling

Maree (2012) and Cohen et al. (2007) define a sample as a subset of a population, while the population consists of all the people the researcher is interested in studying. The sample represents the population group (Creswell, 2010). I employed non-probability sampling, enabling me to analyse and embark on "small-scale research". For example, in this study, I engaged four participants where no attempt to generalise was desired in the case study (Cohen et al., 2007). Parveen and Showkat (2020) define non-probability sampling as using non-randomised methods to draw the sample. This implies that the probability of selecting a particular person is unknown.

I utilised two sampling techniques, namely purposive and convenience sampling. A purposive sampling technique is utilised as participants are chosen keeping in mind the purpose of the study (Rai & Thapa, 2015; Etikan Musa & Alkassim, 2016; Campbell, et al., 2020), which means that the participants were chosen to obtain relevant and rich information about the teaching of common fractions. Furthermore, convenience sampling was also used as schools were chosen within Tshwane South District as it is in close proximity to the researcher (Etikan Musa & Alkassim, 2016; Stratton, 2020). When sampling participants, I chose four primary schools and one teacher in each school. The criteria for choosing the teachers were based on the teaching experience, qualifications and grade performance for the past three years. I used a case of four primary school teachers in different schools. The sample chosen enabled me to engage with Grade 6 mathematics teachers to determine how they develop learner mathematical proficiency when teaching fraction calculations in the additive conceptual field. The participants' demographics are discussed in the next chapter.

3.3.2 Instrumentation and Data Collection Techniques

Kabir (2016) states that data collection is the process of gathering and measuring information in a systematic way that enables one to answer the stated research questions. Data collection takes place because it is necessary to summarise a large amount of data (Maree, 2012). There are various types of data collection techniques, such as interviews, observations, documents, pictures, and photographs (Kabir, 2016; Parveen & Showkat, 2020). In this study, data was collected using observations (Appendix B) and interviews (Appendix C)

Lesson Observations: I presented a sketch of the realities under study and described the situations as they existed using my five senses (Parveen & Showkat, 2020). The lesson observation protocol was designed to elicit information on teachers' practices when teaching the addition and subtraction of common fractions to Grade 6 learners. This included how teachers make sense of the fractional concept, how the explain the fractional concept and how the develop mathematical proficiency. Furthermore, Maree (2012) concurs with the previous statement stating that observations are a systematic collection process that relies on the researchers' ability to use the five senses without questioning and communicating with participants. With Kilpatrick et al. (2001) as my guiding framework, I observed the Grade 6 mathematics teachers, through watching the video lesson, as they developed learners' mathematical proficiency while teaching the concept of addition and subtraction of common fractions (See Appendix B for the Observation Schedule).

Interviews: Maree (2012) argues that interviews are essential when undergoing qualitative research as the researcher will ask open-ended questions that will acquire rich and descriptive information from participants. Furthermore, interviews are meant to record and analyse people's opinions, experiences, beliefs and ideas on relevant topics (Parveen & Showkat, 2020). I gained rich and descriptive information first hand, from the Grade 6 mathematics teachers to determine how they develop learners' mathematical proficiency when teaching addition and subtraction of fractions. I conducted face-to-face, semi-structured interviews using predetermined interview questions, set out in a specific order. Open-ended questions were asked so that the participants could respond in their own words (Gay et al., 2009). As discussed in Chapter 2, I used Kilpatrick et al.'s (2001) model to facilitate the understanding of concepts used in this study and provide guidance on formulating items used in the data collection instruments.

3.4 Data Analysis Methods

3.4.1 Data Analysis and Interpretation

Cohen et al. (2007) describe data analysis as bringing order, structure and meaning to the mass of collected data. Furthermore, Cohen et al. (2007) argue that data analysis entails breaking down the research into basic parts to answer the research question. This implies that data analysis is used to obtain usable and useful information. Data analysis is essential to make sense of the data collected (Maree, 2012; Ngulube, 2015). The interviews and observations were the main data sources during this study.

In this study, I mainly chose the inductive approach because I had to search for patterns from lesson observations and interviews (Thomas, 2003). Probing questions were also developed from the patterns seen and observations made during the lesson observation. This study is conceptualised with the interpretive paradigm. It was important for me to interpret data as it emerged. I chose an inductive approach to make sense of the data collected; however, the points of reference in Kilpatrick et al.'s (2001) strand of mathematical proficiency assisted in interpreting the findings and helped make conclusions from the results. The strands of mathematical proficiency were used as the themes to discuss and summarise the findings.

Creswell (2015, p. 156) indicated that coding is "the process of analyzing qualitative text data by taking them apart to see what they yield before putting the data back together in a meaningful way." In support of this, Elliott (2018) asserts that coding is the breaking down of data collected. Once data is collected, a cyclic process consisting of open coding, axial coding and selective coding ensues (Williams & Moser, 2019). They go on to further explain the process by defining each stage. Open coding is the first level of coding, where the researcher identifies distinct concepts and themes for categorisation. I sorted through the data and found similar concepts amongst the data. Axial coding is the second level that enables the construction of linkages between data and focuses on identifying emergent categories and further refines, aligns, and categorises emerged patterns. This step is in preparation for selective coding. Selective coding is the third level of coding and enabled me to select and assimilate categories of organised data from axial coding in organised and meaning-filled expressions (Williams & Moser, 2019). Creswell (2015) validates this explanation from William and Moser (2019) by asserting that researchers will move from a large number of codes to a small, refined number of codes. The case study was understood and analysed from its natural setting and acknowledged its context and complexity (Maree, 2012). The conceptual model used in this study served as a foundation to describe and analyse findings. Figure 11 summarises the methodological processes involved in this study

Figure 11: Sequence of Stages of Data Collection

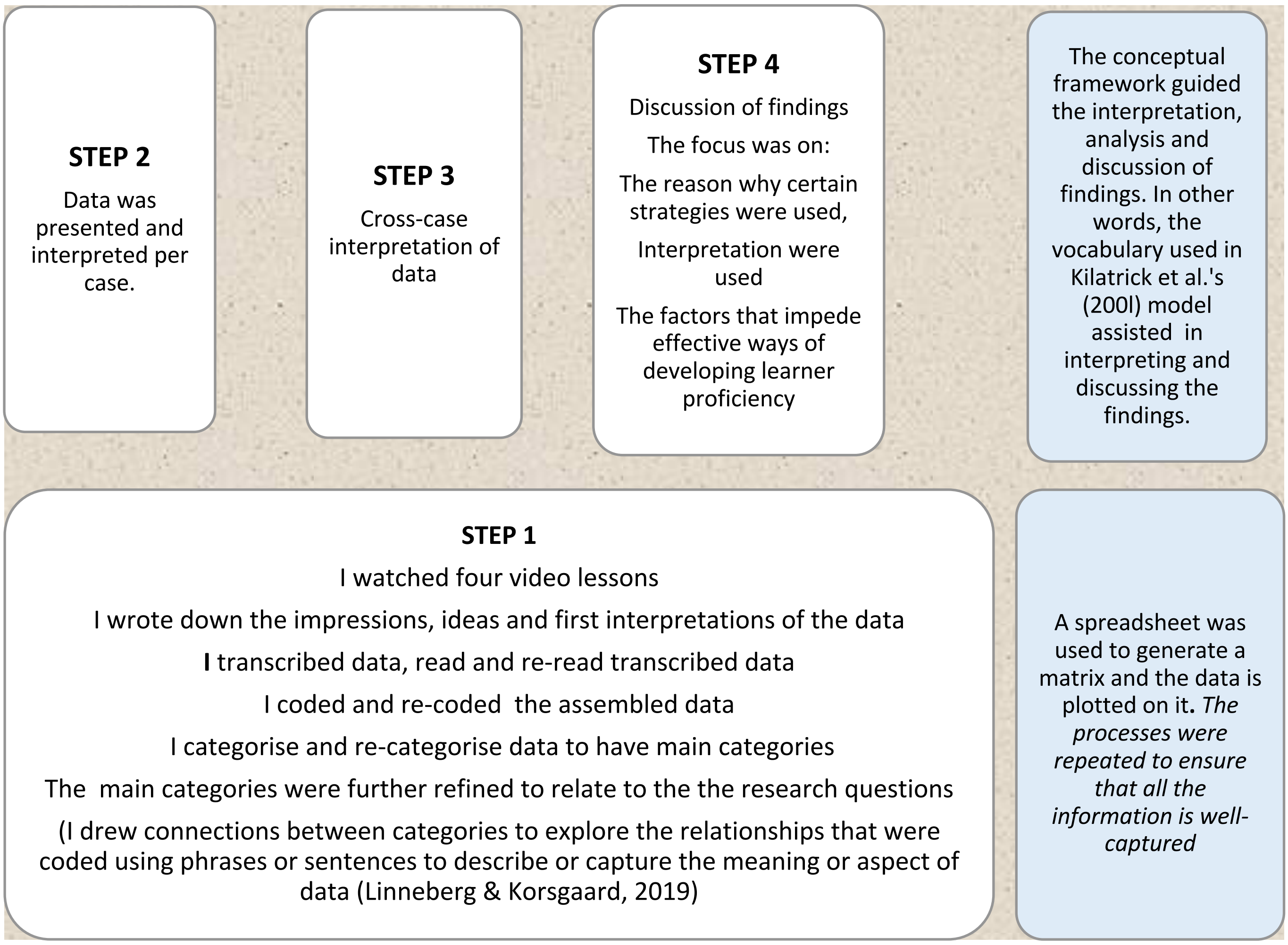


Figure 11 shows a progression or a sequence of stages in a circular flow and emphasises the interconnectedness of pieces. I first started by watching the video lessons twice for each participant, transcribed data, and watched the video for the third time. I ensured that the data was captured as presented in the video. I started familiarising myself with data. Read data three times. I looked at the patterns that emerged from the data and started the coding process. I then reviewed all the coded data and went on to rearrange data. All these steps were repeated more than once to ensure that the field data was not missed and that data was categorised so that it helped answer the research questions. This study's inductive approach involved categorising data into codes and main categories.

Although the video is a resource that objectively displayed key information and facts about an activity, there were some disadvantages:

- Some parts of the video focused on specific areas and could not reach all classroom areas. Therefore, this seems to suggest that thorough planning is needed in the future, starting by identifying actions to be captured or recorded to support and enhance feedback processes.

3.5 Methodological Norms

Credibility: Credibility deals with the question of how consistent the findings are with reality (MacMillan & Schumacher, 2006; Shenton, 2004). This is essential in ensuring trustworthiness. Merriam (2009) states that a research study is trustworthy to the extent that there has been some consistency in carrying it out. I was immersed in the study and interacted with the participants while collecting data. Furthermore, I constantly examined the data, analysed it, and theorised and revised theories where necessary. This process ensured credibility and trustworthiness.

Trustworthiness: Maree (2012) asserts that trustworthiness refers to how the researcher can convince the readers that the findings in the study are of high quality and can therefore be trusted. According to Lincoln and Guba (2000), this implies that the aim of trustworthiness in qualitative research is to support the argument that the question’s findings are worth noting. Maree (2012) states that trustworthiness can be used in terms of credibility, confirmability, transferability and dependability.

Confirmability: Confirmability refers to the objectivity of the researcher and the data (Maree, 2012; Shenton, 2004). Findings were based on the experiences and ideas of the participants to ensure confirmability. This implies that the findings were not based on the researchers' preferences or experiences. Furthermore, Maree (2012) argues that to verify confirmability, external researchers should confirm the procedures of the study to allow for data verification. I was objective and did not make assumptions about how Grade 6 mathematics teachers develop learners' mathematical proficiency when teaching.

Transferability: Shenton (2004), Merriam (1998) and Maree (2012) describe transferability as the extent to which results can be transferred and generalized to other similar contexts. I provided as much information and thick descriptions regarding context, occurrence and experience of the study as possible to ensure transferability.

Dependability: Dependability indicates the stability and consistency of the research process and methods (Maree, 2012) if the study were to be repeated with the same or similar participants or in a similar contextual setting (Shenton, 2004). There was monitoring of the quality of the data recording, the procedures used to acquire the data, the documentation of data and the transcription of interviews to ensure dependability (Maree, 2012).

Triangulation: I used triangulation to ensure dependability. Triangulation involves the use of various sources of information (more than one teacher), methods (observations, interviews) and data sources (documentation, recordings). In this study, I used video lesson observations and semi-structured interviews to collect data from four different Grade 6 mathematics teachers.

3.6 Ethical Considerations

Cohen et al. (2007) argue that research ethics refers to moral principles or rules of behaviour that researchers must consider before conducting research, particularly when the research involves humans.

Informed consent: According to Creswell (2014), during data collection, the researcher requested and obtained the permission of individuals in authority to acquire access to study participants at research sites. The researcher obtained an official letter of acceptance from the university and school principals before the study was carried out to adhere to research ethics. Subsequently, the researcher sought the participants' permission to conduct research on them. In compliance with the principles of informed consent, the researcher gave consent forms to all the participants. The researcher explained the study's aims, purpose and educational benefits to the participants in a letter. The researcher was open and honest with the participants. Signing the consent letter protected teachers and safeguarded their well-being and interests during and after the research.

Voluntary participation: The participants were aware that their participation in the study was voluntary, that they had the freedom to withdraw from the study at any time without any unfavourable consequences, and they would not be harmed as a result of their participation or non-participation in the study.

Protection of human dignity and rights of participants: Ethical principles include the right to privacy, confidentiality and anonymity, the right to withdraw or terminate participation, and the right to access information. Furthermore, it is important to protect the participants from any harm the research might cause. In doing so, the researcher complied with ethical issues of confidentiality, anonymity and privacy.

Anonymity and confidentiality: According to Denzin and Lincoln (2011), anonymity and confidentiality are the foundations of academic research. The names of the participants were not disclosed to ensure confidentiality. The schools', teachers', learners' and principals' names remained anonymous and did not appear in the report. The data gathered was strictly used for the study. Confidentiality was used to encourage respondents to provide truthful responses by guaranteeing the confidentiality of the information provided. The researcher utilised pseudonyms for the participants.

Additionally, research ethics involves compliance with acceptable research norms, morals, standards and principles.

3.7 Limitation and Delimitation of Study

All studies will have limitations and deficiencies relating to the research methods and strategies used. Limitations are potential weaknesses of a study, and they encompass all the factors that are impossible to avoid and may affect the internal validity of the research (Baloch, 2011). Furthermore, Creswell (2007) defines limitations as deficiencies that the researcher identifies in a study.

The following factors limited the study:

Researcher's Knowledge: This was the researcher's first-time collecting data as a master's student. As an inexperienced researcher, this could hinder the efficacy of the study and the data collected.

Time: The teachers used the amended Annual Teaching Plans (ATP). Common Fractions were scheduled to be taught in Term 2 (as opposed to Term 1, prior to COVID-19). There was also one teacher that taught the concept of fractions at a later date than the other three teachers.

Video recorded lessons: Due to COVID-19, the GDE stipulated that no external persons were allowed on campus, so I had to watch lessons online. This was a problem as I was not able to get a full view of the classroom- the camera was either pointed at the teacher or the blackboard; therefore, I was uncertain as to which learners were answering and whether or not the learners were the same ones who answered before or different learners.

Sample size: A small sample size may weaken the study with the inability to draw enough data to form generalisations.

Withdrawal from the study: Participants may withdraw at any time, and this forced the researcher to focus on a smaller sample size

3.8 Chapter Summary

This chapter discussed the nature of the study and the approaches to data collection and analysis. I outlined all the methodological processes involved in collecting and analysing data. Furthermore, the chapter elucidated methodological norms, the study's credibility, ethical considerations, limitations and delimitations. The limitations of the study were also discussed. All these elucidations were explained in the context of my study

4 CHAPTER FOUR: DATA PRESENTATION AND INTERPRETATION

4.1 Introduction

This chapter provides the analysis and interpretation of the data collected from the participants. The study aimed to explore how Grade 6 mathematics teachers develop learners' proficiency when teaching addition and subtraction of common fractions. A classroom observation (video-recorded lessons) and interviews with four Grade 6 teachers were employed to gather data from the participants. In this study, I report the findings directly as observed. These findings are discussed and summarised in the next chapter. The lesson observation and interviews conducted were intended to answer the following research questions:

Main research question

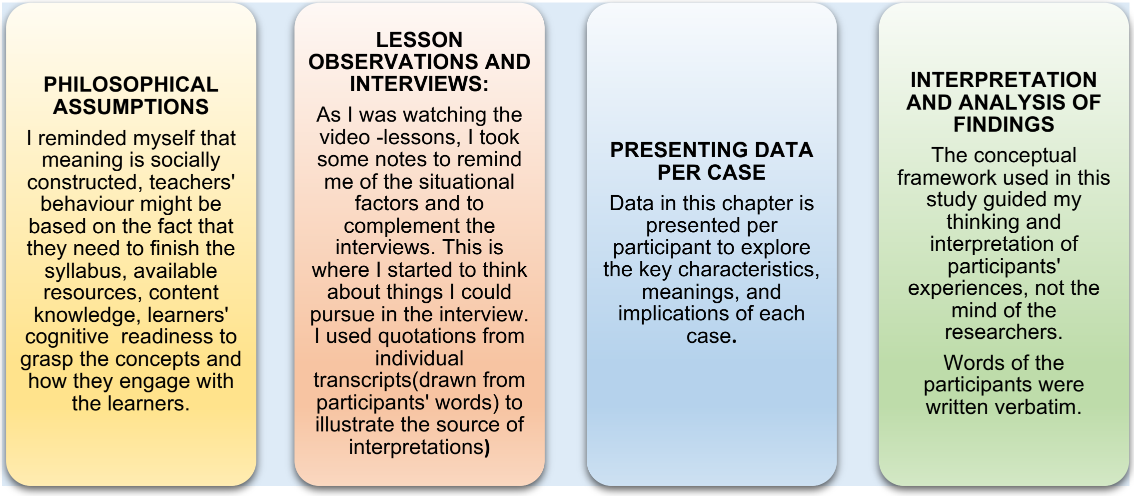
- How do Grade 6 mathematics teachers develop learners' proficiency when teaching addition and subtraction of common fractions?

Secondary research questions

- How do Grade 6 mathematics teachers describe common fractions?
- What strategies do Grade 6 mathematics teachers use in developing learners' proficiency when teaching the addition and subtraction of common fractions?
- How can Grade 6 mathematics teachers enhance their classroom practices in developing learners' proficiency when teaching the addition and subtraction of common fractions?

The findings of the study are presented systematically. In other words, data is presented as a coherent body of ideas or principles. Kilpatrick et al.'s (2001) model guided my thinking when interpreting and analysing data. I want to highlight that the lesson observations were recorded, and the camera was focused on the participants and the blackboard; therefore, I am uncertain whether the learners responding to the questions were the same or different. Furthermore, the participants took pictures of examples, which will be seen as extracts (labelled 1-9) in this document. Figure 12 presents a summary of the framework that guided the data collection and analysis processes for effective communication in this chapter.

Figure 12: Framework of Data Presentation



4.2 Biographical Information of Teachers who Participated in the Study

Data was collected from four different schools. I observed four video lessons, one per school within the Tshwane South District of Gauteng, on various dates that coincided with the addition and subtraction of fractions being taught. Teachers' lessons were observed, and then an interview was conducted with each teacher to delve deeper into their lessons and teaching. Table 2 reflects teachers' biographical information.:

Table 2: Teachers' biographical information.

Participant	Years teaching experience	No. of years teaching Grade 6	Qualification	Gender
Participant 1 in School 1	6 years	2 years	Bachelor of Education-Intermediate/Senior Phase majoring in Maths Education	Female
Participant 2 in School 2	8 years	4 years	Bachelor of Education- Intermediate Phase majoring in Maths and Science Education	Female
Participant 3 in School 3	7 years	5 years	Bachelor of Education – Senior Phase, majoring in English and Maths Education	Female
Participant 4 in School 4	3 years	3 years	Bachelor of Education – Intermediate/Senior Phase, majoring in Maths and Social Science Education	Female

Table 2 shows that all participants had experience teaching mathematics at Grade 6 level and had, on average, more than two years of teaching experience. What is also worth noticing is that all the participants did the B. Ed degree with a specialisation in Intermediate Phase and majoring in mathematics. Participant 3 is the most experienced, and Participant 1 is the least experienced in teaching Grade 6 Mathematics. Table 3 presents the date and time of the lessons observed, the topics taught and the date and time of the interviews with participants.

Table 3: Date and Time of Lessons Observed and Date and Time of Interviews.

	Lesson 1	Time and date	Lesson 2	Time and dates	Interviews Time and dated
Participant 1	Introduction to Fraction and Equivalent Fractions	21/04 11:20-12:20	Addition and subtraction of common fractions.	22/04 11:20-12:20	07-07-2022 at 12:15
Participant 2	Introduction to Fraction and Fraction wall	25/04 08:45-09:45		26/04 08:45–09:45	07-07-2022 at 19:00
Participant 3	Introduction to Fractions and Comparing Fractions.	25/04 10:45-11:45		26/04 10:45-11:45	06-07-2022 at 08:15
Participant 4	Introduction to Fractions and Equivalent Fractions.	06/06 10:45-11:45		07/06 10:45-11:45	08-07-2022 at 15:00

It can be seen from the data in Table 3 that Participants 1-3 followed the Annual Teaching Plan (henceforth known as ATP) as they taught mathematics during the planned time. In contrast, Participant 4 only taught the fraction topic a month later. According to communication with Participant 4, she said: “we are currently two weeks behind the planning, and we have our test/exam week from the 23-31 May 2022. So hopefully, fractions will be taught in the week of the 31st May 2022".

It is also noticeable from Table 4.2 that the focus of all the lessons was on the addition and subtraction of common fractions. Participants 1 and 4 focused on Introduction to Fraction and Equivalent Fractions, whereas Participant 2 focused on the Fraction Wall and Participant 3 focused on Comparing Fractions. Although all participants introduced fractions, they focused on different aspects during their first lessons. This indicates that participants planned their lessons according to their class and learners and what they thought their learners needed to understand the addition and subtraction of common fractions best.

4.3 Demographics of the Schools

Four Primary School Mathematics teachers were observed and interviewed. The primary schools were all public schools within the Tshwane South District of Gauteng. Participant 1 had a class size of 38 learners, with an even number of boys and girls. The language of instruction was English. Participant 2 had a class size of 35, with 20 boys and 15 girls. The language of instruction was English. Participant 3 had 35 learners, there were 19 girls and 16 boys, and their language of instruction was English. There were 12 girls and 14 boys in Participant 4's class, and she had 26 learners. The language of instruction was also English. The teachers attended various workshops and training run by the Gauteng Department of Education regarding mathematics and the implementation of CAPS. Furthermore, they collaborated across the phase to share ideas and resources within their school. The Mathematics Head of Department is an important role player in supporting the mathematics teachers. All four Grade 6 mathematics teachers said that their heads of department were a great source of knowledge and support. In the next section, I will elaborate on the context of the study

4.4 Context of the Study

This study involved four primary schools within the Tshwane South District of Gauteng. The participants were four Grade 6 mathematics teachers, one teacher from each of the four primary schools. The sample population for the research was taken from one district because this allowed for an effective collection and comparison of the data collected. The study involved teachers who were teaching Grade 6 mathematics.

Different schools were selected to compare and evaluate the collected data and allow for the generalisation of the methodology used but not the results. These schools were chosen because they are all public schools and teach mathematics for six hours over five days. This interaction frequently occurs due to the importance of mathematics within the education department.

I believe that using these schools as the basis of the research was appropriate for the following reasons: First, the schools are public schools and use the CAPS document and Annual Teaching Plans as guidance which implies that all content is regulated, and the schools are teaching the same content. Second, the schools were in the Tshwane South District, implying that the workshop and training received were the same for all four teachers from the four schools within the study. So far, limited research has been undertaken using the same strategy of lesson observation and semi-structed interviews to gain an understanding of the development of learners' proficiency within the content of fractions.

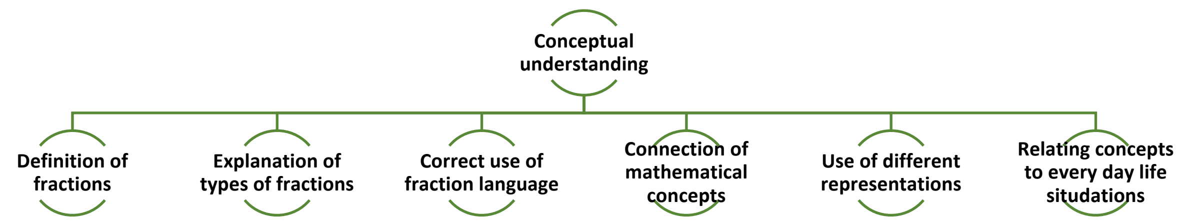
Due to the COVID-19 pandemic, all lessons were observed online via various online platforms such as MSTEAMS, Google Classroom and ZOOM. All these schools had access to internet and devices such as computers, laptops, or cell phones. As with all online endeavours, the participants had challenges with stable internet connectivity. Sometimes, something had to be re-explained so that I could hear or see. Other challenges relate to the difficulty of seeing around the classroom, as my focus was how the teacher taught the content. As one of the teachers was teaching, she could not point her camera in the direction of the class; however, I could hear what the learners were saying when answering and interacting with the teacher. The next section presents a synopsis of the strands of mathematical proficiency.

4.5 A Summary of the Conceptual Framework That Formed the Basis of my Research

This section describes concepts (points of reference, indicators) that helped me derive insights from the data collected and to justify the existence of issues that emerged in this study. These concepts assisted me in interpreting and analysing the data. The strands of Kilpatrick et al.'s (2001) theory form a concrete basis for my research problem. Further, they gave me a better understanding of the content studied in this research. For instance, they created a reasonable basis for my research into teachers' strategies to develop learners' proficiency in the addition and subtraction of common fractions. The vocabulary used in these strands also assisted in probing teachers and the explanations they gave to help me better understand the research problem. These strands are conceptualised within the perspective of what it means to teach fractions for proficiency. The points of reference for each strand are set out in the following sections.

4.5.1 Conceptual Understanding

Figure 13: Conceptual Understanding points of reference



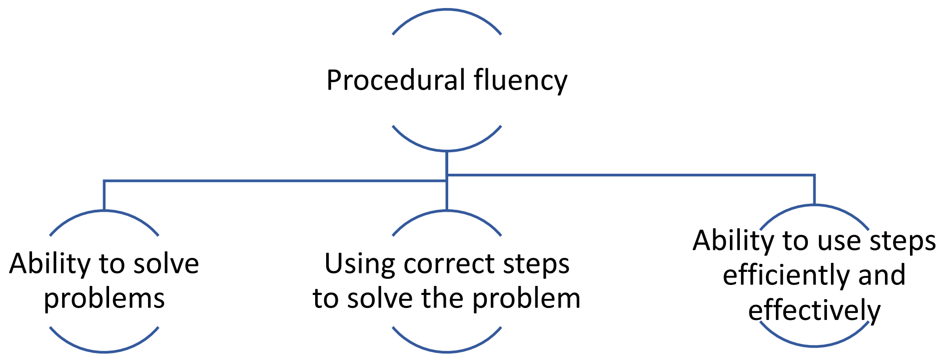
Conceptual Understanding is summarised as understanding concepts, operations and relationships (Kilpatrick et al., 2001). Furthermore, Kilpatrick et al. (2001) stated that teaching for conceptual understanding requires teachers to go beyond just teaching a method; learners need to make sense of the method, understand how it connects, when to use it and explain why it has been used. This implies that when teachers are teaching, they are expected to introduce the topic, define, and explain fractions and link it with prior knowledge of fractions. Moreover, teachers are expected to represent fractions in various ways. Teachers should teach so that learners connect prior knowledge with existing knowledge and ensure that concepts are scaffolded.

To develop crucial mathematical skills, a learner should "develop the correct use of the language of Mathematics" (DBE, 2011:8). To achieve this, learning should be contextualised, and the use of proper language is also encouraged. Drawing from the understanding of mathematical proficiency, it is important for teachers to encourage the learners to use the correct mathematical language. Furthermore, teachers are expected to use the correct mathematical language to ensure that learners are using that correct language too.

There are various ways fractions can be represented, and teachers could use these multiple representations to accommodate the different learning styles of the learners in their class. The various ways of representation include numerical representation, diagrammatical representation, or narrative representation. Fractions can also be represented using various models such as the area model or the length model. Figure 14 presents a summary of procedural fluency

4.5.2 Procedural Fluency

Figure 14: Procedural Fluency points of reference

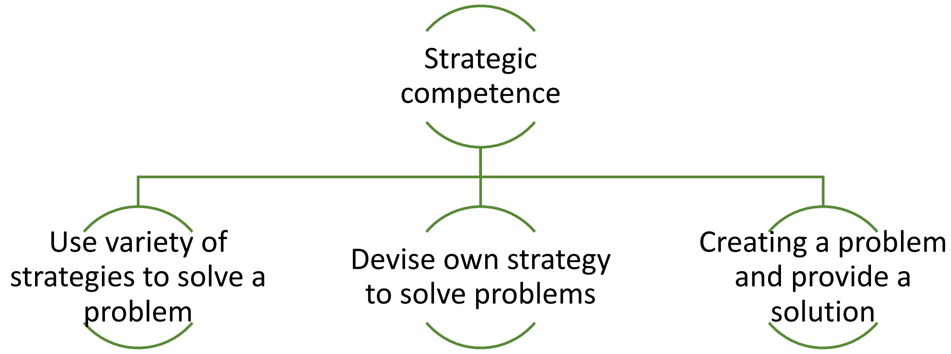


Procedural Fluency is knowledge of procedures, that is, the knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently (Kilpatrick et al., 2001). Learners need to have the skill of solving fractional problems and using the strategies effectively and efficiently. The learners need to know when they have to use a particular strategy. The main question is how teachers facilitate learning to build procedural fluency.

Teachers are expected to communicate the steps of a procedure clearly and explain in detail how it must be used. The procedures are described step by step, and learners can use the appropriate strategy and provide a justification for that strategy. Furthermore, teachers should allow learners the opportunity to practice strategies that were learned during class as well as create their own strategies and procedures. In addition, teachers need to help learners to build on familiar procedures as they create their own informal strategies and procedures. Figure 15 describes tools which relate to strategic competence.

4.5.3 Strategic Competency

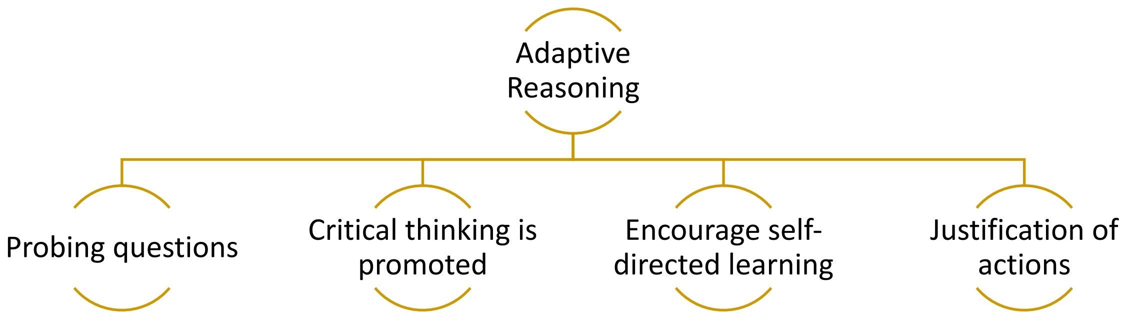
Figure 15: Strategic competence points of reference



Strategic Competency is the ability to formulate mathematical problems, represent them, and solve them (Kilpatrick et al., 2001). Teachers should create opportunities that allow learners to identify and use appropriate strategies and devise their own strategies to solve mathematical problems. Furthermore, teachers should allow learners to solve problems and create problems they can solve. Also, teachers should encourage learners to use different strategies in solving problems. Figure 16 discusses indicators that are pertinent to adaptive reasoning.

4.5.4 Adaptive Reasoning

Figure 16: Adaptive Reasoning points of reference



Adaptive Reasoning is the capacity to think logically about the relationships among concepts and situations (Kilpatrick et al., 2001). Awofala (2017) states that learners are proficient in adaptive reasoning when they can think logically about the existing problems, estimate and reflect on the problems, and justify solving them. Teachers are expected to create opportunities for learners to explain their procedures and provide justifications for their procedures. The class environment should be one where teachers ask probing questions such as *why, how*, and *what* happens next and promote critical thinking. Table 4 shows how teachers develop a productive disposition in mathematics lessons.

4.5.5 Productive Disposition

Table 4: Tools that provide a clear vision of what Productive Disposition entails

Productive disposition	*Involvement of learners in making decisions, demonstrations etc*
	Encourage learners to take initiative in solving problems
	Keep learners motivated on task
	Build learners' confidence
	Build sense of responsibility in learners
	Build learners' curiosity
	Show learners the power of persistance
	Encourage learners not to disrupt discussions and presentation
	Encourage learners to cooperate
	Create positive climate for learners

Productive Disposition is the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that a steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics (Kilpatrick et al., 2001). Teachers should create opportunities where learners are involved in their own learning. Furthermore, the class environment should evoke positive emotion toward the subject and enjoyment in its engagement, emphasising effort, learning, understanding and recognition. It should relate real-life situations to mathematics, promote a positive classroom atmosphere, increase the level of enthusiasm created and recognised, and encourage persevering with a task. The following representations were used when transcribing the teachers' interviews: *Researcher-Interviewer* **(R-I)** and *Teacher-Participant* **(P).** The next section discusses the pilot of my observation schedule at two schools; the findings are presented below: The findings of a preliminary small-scale study conducted to assist in the modification of data collection instruments are discussed in the next section.

4.6 Pilot Study

Before starting the main study, I conducted a pilot study to test the feasibility of the data collection instrument. I piloted the observation schedule with two Grade 6 mathematics teachers, none of whom were part of the main study. The pilot study participants did not have any contact with the main study participants, which ensured that the observation schedule and data collected during the main study was unbiased. The pilot study's purpose was to evaluate the observation schedule and determine whether the appropriate data would be collected from the observation schedule. I observed two participants presenting one double (60 minutes) mathematics lesson each. During the pilot study, I found that one double period would not be enough to ascertain the necessary information. Furthermore, I found that my observation schedule was repetitive and certain items in the observation schedule were unnecessary. This information allowed me to adapt the observation schedule for the main study. The profile of the teachers who participated in the pilot study is presented in Table 5.

Table 5: Demographics of Pilot Study Teachers

Participant and Age Category	**Qualifications**	**Teaching Experience**	**Grades Taught**
Teacher 1 **46 Years Old**	B. Paed (Prim. Ed)	23 Years	Grade 3-10
Teacher 2 **64 Years Old**	B.A. and HDipEd	34 Years	Grades 5, 6 and 7

4.7 Presentation and Interpretation of Findings From the Lesson Observation and Interview

The following sections will present the findings of the lesson observation and interview of each participant.

The findings are reported per participant to form a unified whole. I started with my own interpretation of the findings, followed by a quotation to illustrate my description. The next section presents data for Participant 1(P1)

4.7.1 Participant 1's Lesson Observation and Interview

I observed Participant 1 presenting a lesson from 11:20-12:20 on the first day, and she taught the topic: *Introduction to Fraction and Equivalent Fractions*. The next day from 11:20-12:20 she focused on *addition and subtraction of common fractions*. The interview with Participant 1 immediately after the two lessons is presented. To incorporate learners' prior knowledge, P1 engaged the learners as follows:

> *P1: Good morning, boys and girls. We are going to be moving into our next topic of fractions. You have covered this in previous years, so let's do some revision. Who remembers what a fraction is?*
>
> *Learner: Parts of a whole.*
>
> *Learner: A whole broken up.* [the definition was not fully defined to include 'into equal parts']
>
> *Learner: One-half.*

Without probing learners on their responses that appear to be too general (parts of the whole) and with insufficient information (a whole broken up), particularly the last one, P1 commented and continued the lesson as follows:

> *P1: I'm glad to see that you remember what fractions are. Right, so what are the parts of a fraction?*
>
> *Learner: Uhm, what do you mean parts?*
>
> *P1: So, a fraction has a top number and a bottom number. Wait, let me write a fraction on the board (writes fraction on the board- $\frac{1}{2}$).*

Although P1 tried to use a different representation to explain the concept of fractions, she used a half, which learners gave, as an example. Looking closely at learners' responses, it gives the impression that learners always relate a fraction to half. After writing half on the board, the class exclaimed and said:

> *Learner: Oh!*

P1: What do we call the top number? What do we call the bottom number? And what does it represent?

Learner: Top number is the numerator, and the bottom number is the denominator.

Learner: Denominator tells us the total amount of parts, and the numerator tells how many parts the question is looking at– so shaded or not shaded. Whatever the question asks.

P1 managed to draw from learners' prior knowledge to define a fraction; however, in defining the fraction concept, the emphasis was more on naming the top and bottom parts of a fraction. Once the learners provided the teacher with a definition for fractions, the teacher asked them to read aloud about the different types of fractions in the DBE book to involve them, encourage social-emotional learning, and improve listening and reading skills, increasing their vocabulary and sharpening their focus. She said:

P1: Okay great. Let's turn to Page 26 in our DBE workbooks. Can some[one] read for me the different types of fractions?

[Different learners read from the DBE book, page 26]

Figure 17 depicts an extract from the Grade 6 mathematics DBE workbook.

Figure 17: Extract from the DBE Book Page 26

Proper Fraction
A proper fraction is a fraction in which the numerator (the top number) is smaller than the denominator (the bottom number). It is less than one. Examples: $\frac{1}{3}, \frac{2}{5}, \frac{5}{7}$.

Improper Fraction
An improper fraction is a fraction in which the numerator (the top number) is greater than or equal to the denominator (bottom number).
Examples: $\frac{4}{3}, \frac{5}{2}, \frac{7}{5}, \frac{2}{2}$.

Mixed Fraction
A mixed fraction is a whole number and a proper fraction combined into one "mixed number". It is larger than one. It is also called a mixed number.

Common Fraction
A common fraction is a fraction in which the numerator and denominator are both integers, as opposed to fractions. It is also called a vulgar fraction.

The purpose of reading from the workbook was unclear, as P1 did not comment or ask questions based on what they read. She carried on with the lesson and said:

P1: Right. Let us look at Page 30. A Fraction Wall. What can we use a Fraction Wall? What does it show us?

Learner: How one whole is divided into parts.

Learner: Different types of fractions.

Learner: The size of different type of fractions.

Learner: Uhm, which fractions are equal.

The next extracts illustrate how P1 encouraged and guided learners to use the correct mathematical language. P1 asked the questions, and the dialogue happened as follows:

P1: Great. Can anyone remember what term we use for fractions that are equal?

Class: [silence]

P1: It starts with an 'e' and has equal as part of the word.

Learner: Equal Fractions?

P1: Try again.

Class: Equiv..

P1: On the right track.

Class: Equivalent Fractions.

Furthermore, to show how learners were guided without telling them the correct answer, the teacher said:

P1: Good. You remembered. And what does that (equivalent fractions) mean?

Class: [silence]

P1: Can anyone remember?

Class: [shaking their heads]

P1: Okay. Let me refresh your memories. Equivalent fractions are fractions of the same value but different numerators and denominators. Let's look at our Fraction Wall. Look at $\frac{1}{2}$ and $\frac{2}{4}$.

Class: [Looks at Fraction Wall]

P1 tried to explain to the learners; however, it was not clear if the learners understood the concept of “same value”. She was able to use the steps efficiently and effectively to introduce the equivalent fraction, and that was illustrated through the following conversation:

P1: What do you notice? Take out a ruler and put it on the line by one-half.

Class: Two quarters is the same as one-half.

P1: Good. So, who can tell me what other fraction is equal to one-half?

Class: Three-sixths.

Class: Four-eighths.

P1: Good.

Class: Six-twelfths.

P1: Great. Let’s complete the activity at the bottom of page 30 and the work through page 31-35. I will hand out the memo tomorrow, and you can mark then.

Learners were involved and kept motivated on the task, and that seemed to motivate learners. This activity was important to introduce learners to factors and multiples used to solve problems involving addition and subtraction of common fractions. The learners used the ruler to help them answer the questions. However, during the interview, P1 acknowledged that during her lesson, she could have used various ways to represent fractions; however, she did not do anything about it during the lesson. She said:

P1: Yes, we could make use of cardboard, food items, you know visual representation.

During the lesson, P1 ensured that there was a connection between mathematical concepts such as multiplication, multiples and times tables. This understanding is justified by the following conversation:

Class: *Uhm, we need to get the denominators to be the same so we need to change on Fraction*

P1*: Okay. So important, denominators need to be the same. How do we change* *fractions? Can we just change them any way we like?*

Class: *No. We need to look for a number that both denominators can go into.*

P1: *So, in the example of the board, if we look at the denominators [points or the 2 and the 4], can they go into any number?*

Class*: Well, 2 can go into 4.*

P1*: Okay. Two times what number gives me 4?*

Class*: Two times 2*

Teacher: Great. So, if I change the 2 denominators to 4, must I change the 4 denominator?

Class*: No.*

P1: Why not?

Class: *They're the same now, and when they're the same, you can add*

P1: *Okay, great, so how do I change the 2 denominator?*

Class: Multiply

In this conversation, learners could draw from their prior knowledge (*getting the denominator to be the same, factors and multiples),* and P1 encouraged the connection of mathematical concepts by making learners aware that they could use simple linear equations to determine multiples of numbers. The conversation also helped learners realise they could use different strategies to solve a problem. I have also observed that the language used by the teacher might confuse or might not be clear to the learners. For example, as indicated in the previous conversation with learners, P1 said:

Great. So, if I change the 2 denominators to 4, must I change the 4 denominator?

Two denominators, in this case, might mean the number of denominators. Furthermore, when P1 was asked the question: 'What concept should Grade 6 learners understand in order to develop an understanding of addition and subtraction of common fractions? During the interview she responded and said:

P1: Definitely, multiplication and division of whole numbers (I have added this). This will aid them to do the addition and subtraction, and they will use division *to be able to simplify fractions. Also, before that it's good to explain equivalent fractions by making use of a fraction wall.*

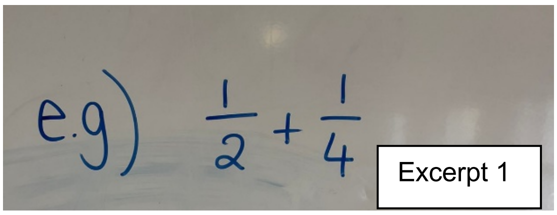
This response seems to highlight the importance of prior knowledge relevant to navigating the new knowledge. P1 explained that learners struggle with their times tables, which is why addition and subtraction of common fractions are challenging.

R: Do you believe that Grade 6 learners find it difficult to understand the concept of addition and subtraction of common fractions? Why?

P1: I would say in general Fractions is a topic that is fun to teach, but when it comes to adding and subtracting with different denominators, and due to the fact that our learners struggle with their times table it becomes challenging for them to actually use the timestable to get a common denominator and that's where I feel they lose interest, or they don't follow through with the steps correctly

Using the example, $\frac{1}{2} + \frac{1}{4}$, the teacher explained the sum in the following way:

P1: Okay. Everyone seems to understand. Now, what happens if we have different denominators? Let us go back to this sum (points to $\frac{1}{2} + \frac{1}{4}$ which was written on the board earlier). Can someone tell me what we need to do here?



Excerpt 1

Class: Uhm, change it

P1: How? Can someone explain? Do you know? Must I remind you?

In this instance, P1 probed the learners to think deeply and give their insight into the problem. The types of questions asked by P1 encourage learners to come up with their own appropriate strategies to solve the problem. Secondly, the teacher wanted learners also to explain how they solved the problem. The class responded, and the conversation continued as follows:

Class: *Uhm, we need to get the denominators to be the same, so we need to change on Fraction*

P1*: Okay. So important, denominators need to be the same. How do we change fractions? Can we just change them any way we like?*

Class: *No. We need to look for a number that both denominators can go into*

P1: So, in the example of the board, if we look at the denominators (points on the numbers 2 and 4), can they go into any number?

Class: *Two can go into 4.*

P1*: Okay. Two times what number gives me 4?*

Class: *Two times 2*

P1: *Great. So if I change the 2 denominator to 4, must I change the 4 denominator?*

Class*: No.*

P1: *Why not?*

Class: *They're the same now, and when they're the same, you can add*

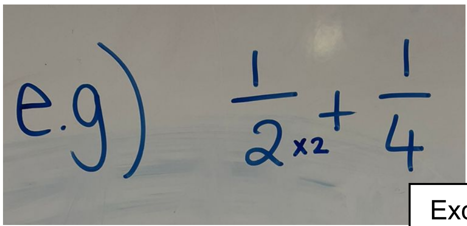
P1: *Okay great so how do I change the 2 denominator?*

Class: *Multiply.*

P1: *What must I multiply? Can someone come show me on the board?*

Class: *(writes on board- pictures to follow)*

P1*: Okay, so he says we must multiply the 2 denominator by 2.*



Excerpt 2

Class: Yes.

P1*: And then? Are we done changing that fraction? Does it become one quarter?*

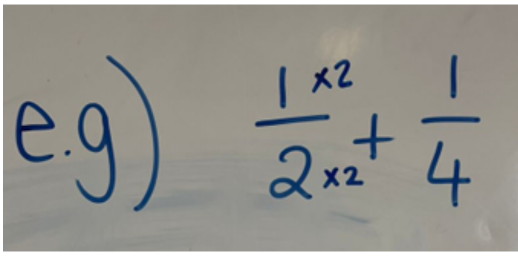
Class: No.

P1*: What did I forget? What is the golden rule?*

Class: What you do to the bottom, you must do to the top.

***P1:** What does that mean?*

Class: If we multiply the denominator by 2, we must multiply the numerator by 2.

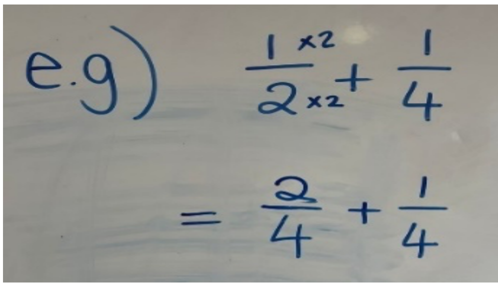


Excerpt 3

P1: Okay. So, what is our new Fraction?

Class: Two-quarters

***P1**: Okay so our sum looks like this? (writes on board) Is this right?*



Excerpt 4

Class: Yes, and now you can add the numerators.

***P1**: Why?*

Class: Because the denominators are the same

***P1**: Good. So two plus one is..*

Class: Three.

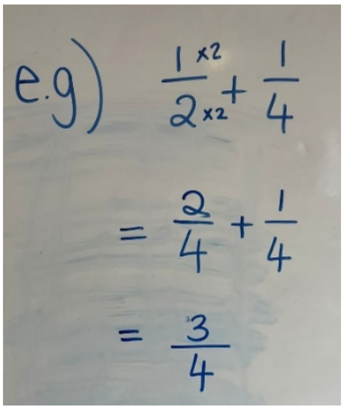
***P1**: And what is my denominator?*

Class: Four.

Class: Quarters

Teacher: So my answer is..?

Class: Three-quarters



Excerpt 5

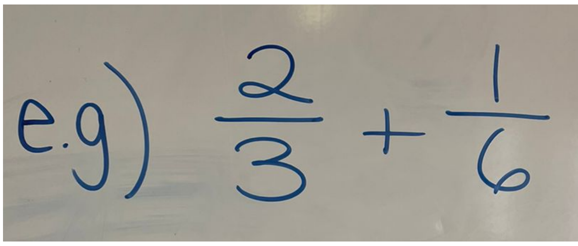
In this engagement and conversation, the core idea of the ZPD was shown when a more knowledgeable person (P1) could enhance a learner's learning by guiding them through a task slightly above their ability level or building from prior knowledge. However, this engagement seems to focus more on using procedures to develop ideas to solve specified design problems.

P1 ensures that she involves the learners in each step of the procedure and requires them to justify and provide reasons, which promotes logical reasons for their actions. During the interview, P1 emphasised that when learners explain their reasoning, she can see their thinking and assist them if they are not on the right track: The following extract serves as a case in point.

> *R: How do you gauge whether your learners understand or not?*
>
> *P1: Yes. I find that when they explain out loud, then firstly, I know that they're thinking but also, as you say, I can see where they are and where I can assist them.*

P1 worked through the first examples with her class, expecting learners to explain their procedure choice and the following steps. In the next two examples, she allowed the learners an opportunity to first complete the sum before asking why she chose to do the procedure that way. This develops the Strategic Competence strand as the learner is solving the problem on their own and providing a solution. This can be seen in the example below:

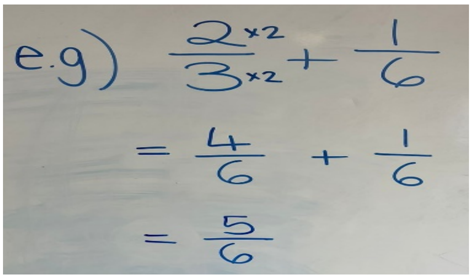
> ***P1**: Good. Let's try the next one (writes of the board). Who would like to come up and try?*



Excerpt 6

Class: [hands go up]

Class: [girl comes to the board, works through the sum and writes a final answer]



Excerpt 7

P1: *Okay. Let's look at what you've done. So she says we must multiply the 3 denominator by 2. Is she on the right track?*

Class*: Yes.*

P1*: Why did you do that?*

Child*: Three times 2 is 6, and the other denominator is 6.*

P1*: Okay. What golden rule has she followed?*

Class*: What you do to the bottom, you do to the top.*

P1: *Good. Remind me what that means again?*

Class*: She multiplied the 3 denominator by 2, so she must multiply the numerator by 2.*

P1: *Good. So what is our new fraction? Or rather, our equivalent fraction?*

Class: *Four-sixths.*

P1*: Well done. What happens next?*

Class: *Because our denominators are the same, we can add the numerators.*

Class: *And leave the denominator as is.*

***Class**: So our answer is five-sixths.*

***P1:** Do you think the method will be different with subtraction?*

***Class**: No.*

My interpretation of the above example is that P1 is asking for justification for her actions, which means she is testing the conceptual understanding of fractions, and she is not stopping at getting the right answer. This can also be seen in the following example:

P1: So, what now? I see A (a learner in class) has multiplied by the denominator and numerator by 2. Why?

Class: The one fractions denominator is 14. 7 times 2 is 14. We need the denominators to be the same.

P1: Why?

Class: Because if they're not then we can't add the fractions.

Justification of actions allows the learners to explain their reasoning. During the lesson, P1 explained the procedure that must be followed to solve the problem. She did not allow learners to devise their own strategy to solve the problem, however, during the interview, she explained that learners could use their own strategy, and she chose to provide them with a strategy to ensure those who struggle have steps to follow. She echoed and said:

R: Do you feel that there is a need for learners to come up with their own strategies for solving calculations with fractions?

P1: I would say that learners are given freedom in finding their own way to get to the answer, so yes, it is allowed but not forced upon so if learners understand it the way its taught, then they can carry on with it if they come up with their own way, which leads them to the same answer, they are allowed to use it.

P1 allows the learners to try without interrupting them. She asks for an explanation and justification when they have completed the sum. If something is incorrect, she reminds learners that we can make mistakes and that we learn from mistakes. The discussion proceeded as follows o provide a guide as to where more effort was needed:

P1: Good. So, who notices where A (student) went wrong?

Class: She added instead of subtracted.

P1: Yes. So, our answer is [writes on the board). Because we must subtract. Boys and Girls, we are all human and we can all make mistakes. We learn from our mistakes. But it is so important that you play close attention to what you are doing.

The class environment was safe and conducive for the learners; even those who were not as confident could participate in the lesson. During the interview, P1 explained the following:

R: Do you see your learners making sense and having fun of what they are learning?

P1: I think you have to create a safe space for learners. As you say, I don't know everything, you don't know everything so we can learn from each other.

P1 explains that there are certain things that learners enjoy doing and do not enjoy doing in fractions. She explained as follows:

R: So what do you think that learners like about fractions, and what do they dislike about fractions?

P1: Children like working with fractions by identifying them through diagrams, finding equivalent fractions, drawing diagrams to represent a fraction, and I think they dislike simplifying fractions and working with improper and mixed fractions.

She also noted that learners do find the concept of fractions to be a challenging one because they do not have that foundation times tables knowledge:

R: Do you believe that Grade 6 learners find it difficult to understand the concept of addition and subtraction of fractions? Why?

P1:I would say in general, fractions is a topic that is fun to teach, but when it comes to adding and subtracting with different denominators, and due to the fact that our learners struggle with their times table, it becomes challenging for them to actually use the times table to get a common denominator and that's where I feel they lose interest, or they don't follow through with the steps correctly.

When learners are feeling despondent and losing interest, P1 believes that she could make the lesson more interesting and more fun by adding videos or coming up with fun ways for learners to remember the steps:

R: During your lesson, were there other strategies you could have used during your lesson?

***P1**: I would use videos to enhance the lesson as that is visual and auditory, and fun. I think learners give more attention to videos that use bright colours and songs, so I think by using videos it gets learners to pay attention and take in what is being said rather than a traditional method of teaching.*

As an acknowledgement that learners lack knowledge of the times table, she remarked and said:

R: What could you do to improve your teaching of addition and subtraction of fractions?

P1: I would have looked up more ways to get learners to find the common denominator, and I would have made the multiplication the key point of the lesson. Looked for fun ways for kids to remember all the steps in getting the answer.

The next section explains Participant 2’s lesson observation and interview data.

4.7.2 Participant 2’s Lesson Observation and Interview

The interview with Participant 2 was conducted after introducing fractions, the fraction wall and addition and subtraction of common fractions, during which the teacher provided opportunities for learners to develop their mathematical proficiency. During the interview, I delved deeper into her practices in facilitating the lesson.

P2 started her lesson with a video to help learners remember fractions. In her interview, she explained that in her classroom, when a video or song is played, her learners are more likely to remember it:

> *P2: And especially if it's a song, and in the culture that I work with, anything that I give to them in a song or dance form, they just thrive on it. So, they love it.*

Furthermore, she goes on to explain that the video was used to refresh the learners' memory and remind them of what they had previously learned about fractions (line 225-228):

> R: *Why did you structure the lesson in that way?*
>
> ***P2**: I, the video was used to awaken the knowledge of fraction that was already taught in grade four and five, so it was just for them to remember fractions first before I built on their current knowledge that they had. So it's just to get their attention to the lesson.*

Once the video was done, she asked learners to tell her anything they could remember about fractions and engaged the learners as follows:

> ***P2**: Okay, great. So, what are proper and improper fractions?*
>
> ***Learner:** Proper fractions are fractions where the numerator is smaller than the denominator.*
>
> ***Learner:** And improper fractions are fractions where the numerator is bigger than the denominator.*
>
> ***Learner**: And improper fractions can be changed into mixed numbers.*

During her initial discussion with her class, she asked probing questions that allowed the learners to stimulate their thinking and activate their prior knowledge. This approach allowed the teacher to have a springboard from which she could move forward and enable learners to connect prior knowledge with existing knowledge. She continued the lesson as follows:

> ***P2**: Nice. You guys remember a lot. What are mixed numbers? What are mixed numbers made up of?*
>
> *Student: A whole number and a fraction.*

As learners provided her with answers, she probed them to explain more, as can be seen below:

> ***Learner**: Proper and improper fractions*
>
> ***P2**: Okay, hold up. If I write the fraction, $\frac{3}{7}$, on the board, what is my numerator and what is my denominator?*
>
> ***Learner:** The 3 is the numerator.*
>
> ***Learner**: The 7 is the denominator.*

After the learners had provided P1 with answers, she allowed them to read and highlight the definition as provided by the Department of Basic Education (DBE) workbook:

> ***P2**: Yes, so fractions can also represent parts of a set. Okay, let's take out our GDE books and a highlighter and turn to page 26. Right, who would like to read for us?*
>
> *Class: [hands shoot up]*
>
> ***P2**: Thank you (learner's name)*
>
> *Class: [learner begins reading]*

Proper Fraction
A proper fraction is a fraction in which the numerator (the top number) is smaller than the denominator (the bottom number). It is less than one. Examples: $\frac{1}{3}, \frac{2}{5}, \frac{5}{7}$.

Improper Fraction
An improper fraction is a fraction in which the numerator (the top number) is greater than or equal to the denominator (bottom number).
Examples: $\frac{4}{3}, \frac{5}{2}, \frac{7}{5}, \frac{2}{2}$.

Mixed Fraction
A mixed fraction is a whole number and a proper fraction combined into one "mixed number". It is larger than one. It is also called a mixed number.

Common Fraction
A common fraction is a fraction in which the numerator and denominator are both integers, as opposed to fractions. It is also called a vulgar fraction.

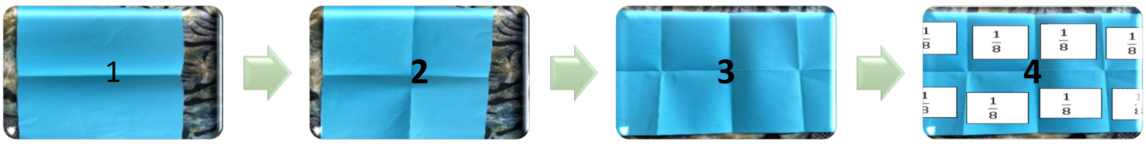
Figure 18: DBE Workbook Page 26

> ***P2:** Please highlight the headings and the sentences underneath the headings. Okay, let us do a quick activity at the bottom of page 26. It's the same as the classroom example we went through now.*

Reading and highlighting the definitions in the DBE workbook allowed learners to hear and see the different types of fractions and consolidate the introduction of the lesson, catering for different learning styles. Once the class provides the teacher with a definition and explanation of the type of fractions, she does an activity with them:

> ***P2**: Excellent. I'm going to hand out a piece of paper to everyone.*
>
> *[hands out paper]. Okay. Everyone folds it in half. This whole piece of paper is divided into how many parts?*

Figure 19: Folding Paper activity



*(**Picture 1**) Class: Two.*

***P2**: What is the fraction of each part?*

Class: One-half.

*(**Picture 2**) Teacher: Fold the page in half again. How many parts do we have now?*

Class: Four.

***P2**: And what is the fraction of each part?*

Class: One quarter

*(**Pictures 3 & 4**) **P2**: Fold your piece of paper one more time. How many pieces do we have now?*

Class: Eight.

***P2**: And what fraction is each piece?*

Class: one-eighth.

This concrete visual representation allows learners to physically see that fractions are a whole being divided into parts. Along with the video, learners experience both visual and auditory teaching from the teacher. Furthermore, learners are experiencing concrete examples with which to work. As P2 explains in her interview, some learners need physical examples to understand fully:

R: What concepts should Grade 6 learners understand in order to develop their understanding of addition and subtraction of common fractions?

***P2:** They should understand that a fraction is a part of a whole. And I think because most teachers do not teach with a physical example, when they teach it, then the children can't understand. If I have two-halves, I always use a slab of chocolate when I start teaching the fractions. So they break in even pieces that even pieces part is also very, very difficult for them to comprehend. And then if we add all those fractions together, we get one whole, whether it's a paper, one whole, so I think that concept they should know. And understand.*

As the lesson progresses, P2 moves on to equivalent fractions, asking the learners questions before providing them with answers. If they are uncertain, she aids them:

***P2**: Okay great. What does Equivalent mean?*

Class:[learners keep quiet]

***P2:** Listen to the word. Equi-Valent. What word can you hear in there?*

Class: Equal

***P2**: So can anyone tell me what that has to do with fractions?*

Class: Uhm, equal fractions?

Class: Fractions that are equal?

Class: Same size [relating them to the size or area and this might be informed by the practical activity done in class] fractions?

***P2**: Awesome. That's great. You're all on the right track. So, what are Equivalent Fractions?*

Class: Fractions that are equal?

Class: Fractions that are the same size?

***P2:** Can someone give me an example?*

Class: [no response from learners]

Once the teacher sees that the class needs help, she allows them to open their books and look at a fraction wall and work through equivalent fractions using the fraction wall:

P2: Okay. Let's turn to page 30 and look at the fraction wall. If we look at the fraction bar that has been split into two parts, what fraction is each piece of the bar?

Class: One-half (write the fraction on the board).

***P2**: Now, which fraction bar is equal to the one-half fraction bar?*

Class: The green one.

Class: Two of the green ones.

***P2:** Okay, let's check. How many pieces is the whole bar split into?*

Class: Four.

***P2**: So one piece of the bar is what fraction?*

Class: One-quarter.

***P2**: Lovely. So how many of those green pieces equal one-half?*

Class: Two.

***P2**: So, what fraction is equal to one-half?*

Class: Two-quarters.

P2 asked probing questions to activate prior knowledge so she could build on that prior knowledge. Once she had established prior knowledge, explained the concept of equivalent fractions, and worked through some examples, P2 gave the learners an activity on equivalent fractions to establish whether they understood.

During her lesson, she asks the learners what they remember and asks for an explanation. Her learners were actively involved in her lesson, answering questions. When they were uncertain of the answers, learners still attempted to answer questions. P1 moves from addition and subtraction of common fractions with the same denominators to addition and subtraction of common fractions with different denominators. The teacher asks them how to solve *the problem:*

P2: Okay, let's move on from there. Today we are going to be looking at adding and subtracting fractions. Does someone remember how to add and subtract fractions?

Class: I think you add the numerators.

Class: And leave the denominators.

P2: Wow. Grade 6s, I see you remember quite a lot from last year. I will let your Grade 5 teacher know that she did a very good job.

Class: [laughs] yeah, Mrs X is the best.

P2: Okay, moving on. So, when we add and subtract fractions with denominators that are the same, we add the numerators and keep the denominator the same. I'm going to do an example of the board [writes on the board]

$$\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$$

P2: So, can we see that we add the numerators and we keep the denominators as they are. Okay, one more.

$$\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$$

Once this has been established, she moves on to addition and subtraction of common fractions with unlike denominators. The teacher first investigates what her learners remember by probing and guiding them.

***P2**: Okay. When the denominators are the same, just add the numerators. So, what happens when the denominators aren't the same?*

Class: We add the numerators.

***P2**: And what happens with the denominators?*

Class(no response) …

***P2**: (writes an example of the board):*

$$\frac{1}{2} + \frac{1}{4} =$$

***P2**: The denominators aren't the same so what do we do? It has something to do with Equivalent fractions.*

Class: Uhm, do we have to find equivalent fractions? So that our denominators can be the same?

***P2**: Yes, on the right track. So, we have to find equivalent fractions so that the denominators can be the same. How do we do that?*

Class: Multiply

***P2**: Multiply what?*

Class:[no response].

When learners are uncertain, P2 provides them with foundation bits of information that they can use which refresh their memory:

***P2**: Okay, we have to find something called the LCM. Who remembers what LCM stands for?*

Class: Lowest Common Multiple

***P2** Yes. So, what is something you have to know to find your LCM?*

Class: Times tables.

***P2**: So when you have fractions with different denominators, you need to take the denominators and list the multiples of each denominator. Once you do that, you find the common denominator. You then need to ask yourself what are you going to do to the current denominator to get to the new multiple, which will be your new denominator.*

Class: [looks confused]

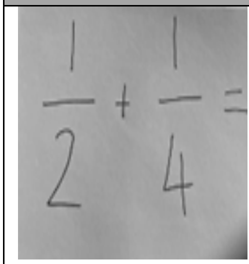
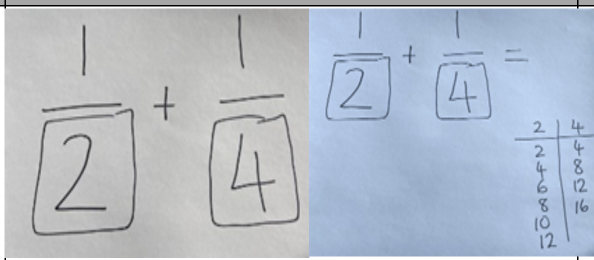
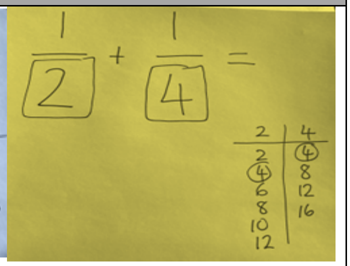
P2 explained during the interview that learners find fractions difficult as they do not have a good grasp of their times tables: This is what she said:

R: Do you believe that Grade 6 learners find it difficult to understand the concept of addition and subtraction of common fractions? Why?

***P2**: I do think that they find it difficult, because, and I'm not sure where the problem comes in, but I don't understand when it's like and unlike the denominator. If it's unlike when to change it, where the problem comes in* *they, they struggle with times tables, so they can't find the equivalent fractions.*

During the explanation of LCM, the class was very confused. P2 then decided to explain again, using an example on the board. The teacher will always make sure that she digs deeper to identify learners' challenges.

Figure 20: Example done on board

Step 1	**Step 2**	**Step 3**
		

*(**Step 1**): **P2**: Let me do an example on the board [writes on the board].*

***(Step 2**) **P2**: So, our sum is one-half plus one-quarter. Our denominators are 2 and 4, so we list the multiples of 2 and 4.*

*(**Step 3**) **P2:** Once we've listed the multiples, we look for multiples that are the same in each list.*

The engagement with learners continued as follows:

***P2**: If you look at the list of multiples, you will see that 4, 8, 10 and 12 are all common BUT we need to look for the LOWEST multiple on each list. So out of 4, 8, 10 and 12, which one is the smallest?*

Class: Four.

***P2**: Yes. So, if we look at the denominators. We can see that one of the fractions already has a denominator of 4, so we do not have to do anything to that fraction. We leave one quarter as one quarter. But the other fraction has a denominator of 2, so we have to change that fraction. We have to find an equivalent fraction where the denominator is 4.*

Class: How do we do that?

__P2__: So you have to ask yourself the question, how do we get our current denominator to a denominator of 4. What do we do to the 2?

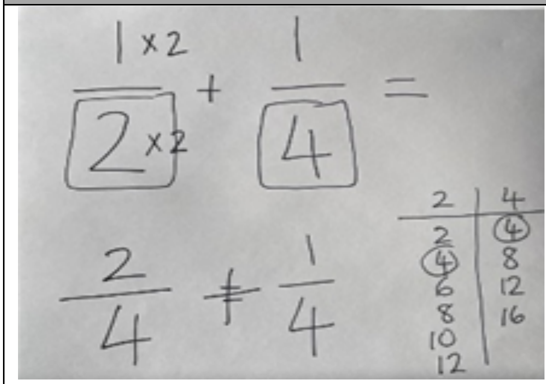
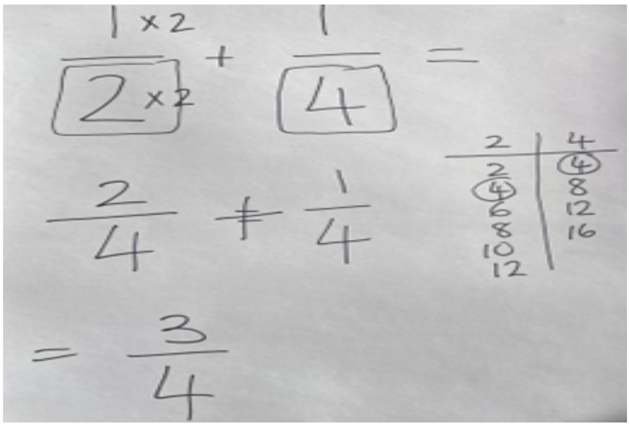
Class: We could add 2

__P2__: We could yes. But the rule is multiply. So, 2 multiplied by what gives me 4?

Class: Two times 2.

***(Step 4) P2:** Yes 2 times 2 is 4. But numerators are very jealous so what you do to the denominator, you must do to the numerator. So if we multiply the denominator by 2, we must multiply the numerator by 2.*

Figure 21: Continuation of example done on board

Step 4	Step 5
	

***(Step 5) P2**: Remember that the numerator is very jealous. So whatever we do to the bottom, we must do to the top. So, our equivalent fraction is two quarters. Now our sum is two quarters plus one quarter which equals?*

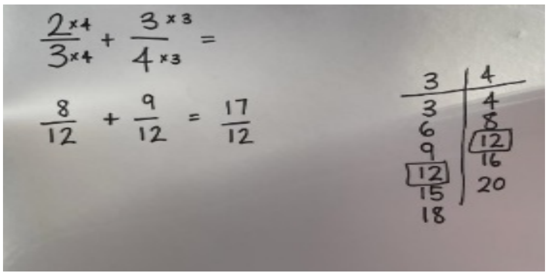
Class: Three-quarters

For this example, even though P2 was explaining, she involved her learners in the problem, asking probing questions, and guiding them. When asked if she would allow learners to create their own strategies to solve a problem, she had the following to say:

R: Do you feel that maybe you could have let the kids come up with their own strategies for solving calculations?

P2: I could have most probably, but I am a little bit of a control freak. So I struggle, letting them find their own ways. The confession made by the teacher]. But I do understand that it is maybe a better way for them to grasp and understand the methods to use. But yeah, I'm sure I could if maybe, but I didn't.

As a way of assessing learners' understanding, P2 asks the learners to solve the problem as individuals and one of the learners presented the solution as follows:



Excerpt 8

Teacher: Okay, B. Explain what you've done.

B: I see that the two denominators are 3 and 4, so I list the multiples of 3 and 4. Once I list the multiples, I find a common multiple which in this case is 12. So, I think I need to ask myself how I get my denominators to 12.

***P2:** You're on the right track B. So how do I get my denominators to 12?*

B: Well, 3 times 4 equals 12 and 3 times 4 equals 12.

***P2**: That's good. What happens to the numerator?*

B: Well, numerators are very jealous, so what we do to the bottom, we do to the top. That's why I multiplied the numerator by 3 and by 4. So, our equivalent fractions are eighth twelfths and nine twelfths. And our answer is seventeen twelfths.

***P2**: Well, done. What do we call a fraction where the numerator is bigger than the denominator?*

Class: Improper fractions.

During the interview, P2 reiterated how she asks questions so that learners explain their thinking either to her or their peers:

R: In your classroom, how do you try your best to accommodate learners with different learning styles or different levels of learning?

***P2**: So they will. I will give them the steps and then they should repeat it. Or I'll ask him, what should I do next? Like, teach the teacher. And then, if we have time, I do have like a buddy system, where I let them explain it to each other. So to break up in little groups and have like a study a little self-study? Or peer study rather?*

P2's lesson had a variety of teaching styles, and she explains in her interview that you need to try and reach the majority of your learners:

R:In your classroom, how do you try your best to accommodate learners with different learning styles or different levels of learning?

***P2**: I, when I prepare, I make sure that I prepare for all the learning styles. So those who need to read to understand, I have a visual, the worksheets. And then, for those who are more visually inclined, I will have a video. And then obviously, the ones who are audio will listen when as the teaching goes on.*

During the interview, she explained that although she does not force the learners to answer, she does encourage them to participate and checks on the learners by walking past them while they are working and being available if they need any help:

R: While you are presenting your lesson, how do you check learners' understanding of the lesson?

***P2:** I do examples on the board. And then, I asked them to guide me through it. And if I see some learners are not putting up their hands, I do make sure they know it, depends on the learners. If I know the learners does not like the attention, I will not put pressure on them. But then I will when they start doing the worksheets that said, I'll walk past them and see if they can understand or* *if they do understand so that's my, my way of checking how's the participation within the lesson*.

The next section presents Participant 3's lesson observation and interview data.

4.7.3 Participant 3's Lesson Observation and Interview

After the presentation and observation of the lessons, an interview was conducted to deepen my understanding of how the teacher provides opportunities to develop the learner's mathematical proficiency during the addition and subtraction of common fractions. She started her lesson by allowing the learners a few minutes to complete a mental mathematical activity unrelated to the topic of the lesson (See Figure 22).

Date: ____________ Compare and order common fractions, including specifically tenths and hundredths

Mental mathematics:

6 + 4 =		50 + 50 =		90 + 10 =	
98 + 2 =		75 + 25 =		900 + 100 =	

Figure 22: Mental Maths Activity

After that, before explaining anything, she allows the learners to work with concrete examples, manipulating and folding the paper accordingly:

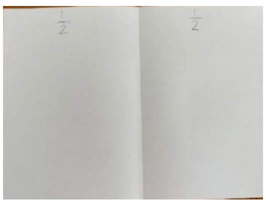
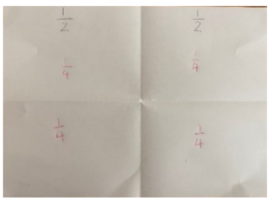
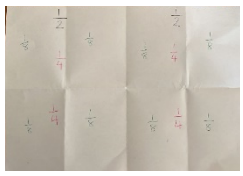
Step 1	Step 2	Step 3
		

Figure 23:Folded paper

(Step 1) ***P3:*** *(marks the mental maths- learners give the answers). Okay, on page 59, you will see we are going to start with fractions. That's what the white paper is for. Take that* *white paper and fold it in half (shows the class how to do it). Like this. Okay when we open the page, what did we do except fold the page in half. What did we do with this paper?*

Class: We divided it

Class: We divided it into two [teacher did not complete the sentence or correct the learners to further expand their knowledge]

***P3**: Okay, so if we had to write a fraction for this one-half, what would our fraction be?*

Class: One-half.

***P3**: Okay, I want you to take your pen and write one-half, one-half*

Class: Must be write it big?

Class: Must be write one-half in each block?

Class: Can we use colour?

Class: Where must we write it?

(Step 2) ***P3**: Write one-half in each block, preferably at the top. You can use colour. You can make it about that size (shows the size with her fingers). Okay, so now we took this whole paper and we divided it into a half and we said this is one-half and this is one-half. So how many of these blocks do I need to make a whole paper?*

Class: Two

***P3**: Two (shows two with her fingers). So I need 2 blocks. So one-half plus one-half gives me one whole page. One-half plus one-half equals two over two (*this should be two-halves*), which is 1 whole page. Okay, so now I want you to take this page and fold it in half again. Fold it like this (shows them). You folded it like this (shows them), now you need to fold it again like this [shows them].*

[learners fold the papers]

***P3**: So when you open it, you have 4 blocks (opens her folded page to show the learners). Okay, so now, if you look at these blocks and I ask you what is the fraction of one block, what will you say?*

Class: One-fourth

***P3**: Or you can say one quarter. So now I want you to take a different colour and write one* *quarter in each of the blocks. One quarter, one quarter, one quarter, one quarter [teacher points to each block as she says one quarter]*

Class: Where should we write it?

***P3**: Anywhere in the blocks. So you said that one of these blocks is one-quarter or one-fourth. If I ask you what is the fraction for two of these blocks, how will I write it? What will you tell me?*

Class: A half

***P3**: Okay, yes, that is simplified.*

Class: Two quarters

(Step 3) *Teacher: Well done. So, two over four [*The correct language should be two quarters or two parts of four parts] *and then we simplify it into a half. Now I want you to take the page, you folded it into a half, then into quarters, now I want you to fold that into a half like this [shows the learners]. how to do it) so that when you open it (opens it), you got a few blocks (shows them what it should look like). Okay, if I ask you what fraction will one of these blocks be?*

Class: One over six [language learned from the teacher].

***P3:** Look carefully. Count.*

Class: One over eight

Teacher: Okay, if I were to ask you what fraction five of these blocks will be, what will you say?

Class: Ma'am must I simply?

***P3**: No, you don't have to simplify, just give me the original*

Class: One over five

Class: Five over eight

***P3**: Five over eight because its five of the eight blocks. If I were to ask you for the fraction of three of the eight blocks, what would you say?*

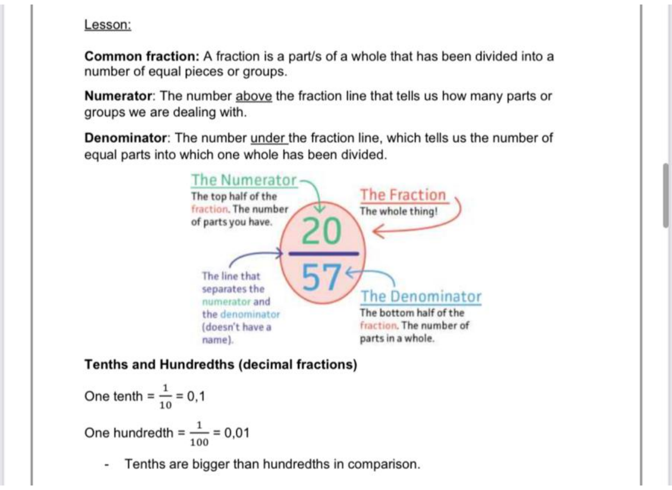
Class: Three over eight.

***P3**: Okay, do you understand that this piece of paper is cut into eight pieces which is my denominator. And as the question gets asked, your numerator changed. Your* *number at the top changed. Did you notice?*

Class: Yes

When P3 had completed the concrete manipulation example, she provided the learners with a definition for fractions

***P3**: Okay. Is there anyone that doesn't understand? Do you understand that fractions if one whole that has been divided into parts or pieces. Okay, page 59. I want you to take out how many ever highlighters you have, you going to rotate them. Example, pink, orange, yellow, green, pink, orange, yellow, green*



Lesson:

Common fraction: A fraction is a part/s of a whole that has been divided into a number of equal pieces or groups.

Numerator: The number above the fraction line that tells us how many parts or groups we are dealing with.

Denominator: The number under the fraction line, which tells us the number of equal parts into which one whole has been divided.

Tenths and Hundredths (decimal fractions)

One tenth = $\frac{1}{10}$ = 0,1

One hundredth = $\frac{1}{100}$ = 0,01

- Tenths are bigger than hundredths in comparison.

***P3**: Okay, I want you to take one colour and highlight common fractions. Right after common fractions, it says reads the definition]. Okay, highlight the parts that says parts of a whole that has been divided into a number of equal parts. Did you notice when we folded our piece of paper that we had equal sizes of blocks, not one part bigger than the other. Fractions are made up of a numerator, a number on top of the line and a denominator, a number at the bottom of the line. D for down, d for denominator. Okay, so take a highlighter and highlight numerator and highlight above, above the fraction line and then you highlight 'how many parts we are dealing with' So, if we look at the questions I was asking you. When I said what is the fraction of one of the four blocks, you said?*

P3 started by paper folding that led to the definition of a fraction; however, there was no further explanation of what denominator and numerator mean in the context of fractions

Class: One-quarter.

***P3**: When I asked you what is the fraction of two of the four blocks, you said?*

Class: Two-quarters.

***P3**: The numerator changes as the question changes. When I asked you five of the eight blocks, you said five-eighths. When I asked you for three of the eight blocks, you said three-eighths. Does everyone understand that?*

Class: Yes.

***P3**: Next colour. Please highlight denominator. The denominator is under the fraction line, highlight that and then highlight the number of equal parts int which the whole has been divided. Any questions so far? No? All good? Okay, so if you look at the diagram you will see it has a numerator and denominator and the line that separates them, which is a fraction line.*

P3 took much time on fraction representation, leading to a definition of fractions. Furthermore, she connected prior knowledge with new knowledge by touching on decimal fractions and place values. However, as this was not needed for the addition and subtraction of common fractions, she quickly moved on to comparing fractions, where she asked them how they compare fractions:

***P3**: Let's carry on. [writes on the board]. If you have three-eighths versus five-twelfths, just by looking at those two, will you be able to tell me which one is bigger? What do you think?*

Class: Five over twelve?

Class: Three eights.

The learners used their knowledge of fractions and the size of fractions to deduce which fraction could be the biggest; however, there were mixed answers, and P3 continued with the lesson as follows:

***P3**: So, we [are] getting mixed reviews. So, what we do when we compare fractions, we have to make them the same size by making the denominator the same. That way, we can make 100 per cent sure. So, then we can compare them easily. So how do we get the denominators to be the same? Who can remember?*

Class: We subtract 8 from 12

Class: We can add 4 to 8 to get 12.

Class: You times 8 by 12.

P3 allowed the learners the opportunity to provide her with answers where they had to use their prior knowledge to explain how to compare fractions. Once learners had given a variety of answers, P3 guided the learners through an explanation by describing the following:

***P3**: Okay, there's the first correct answer. You can look at the denominators and multiply them together, but that means that you will work with very big numbers. We always try to work with very small numbers, so we ask ourselves, can we times 8 by something to give me 12, is that possible?*

Class: No.

***P3**: Can we multiply 12 by something to give me 8?*

Class: No!!!

***P3**: So we can definitely multiply the two numbers together. Or we can look for the LCM of the two numbers. What is the LCM?*

Class: Lowest Common Multiple.

***P3**: Okay, let's list the multiples of 8 and 12. Count with me– 8, 16, 24, 32, 40, 48.*

Class: Eight, 16, 24, 32, 40, 48

***P3**: Okay, I'm going to stop by 40. Then 12, 24,36, 48.*

Class: 12, 24, 36, 48.

***P3**: Okay, I'm going to stop there. Now, remember we are looking for the LOWEST common multiple, which is what?*

Class: Twenty-four.

***P3**: Can you see that they also have 48 in common? But we are looking for the smallest number, the lowest common multiple. So our denominator is going to be 24. Okay, so I will make both denominators 24. So I got to go back to my original fraction and ask myself what must I do to 8 to get to 24?*

Class: Times 3.

***P3**: Good, and what's that thing I must do? What I do at the bottom?*

Class: I must do at the top.

Teacher: Good I'm glad that you remember that. Twelve times what will get me 24?

Class: Two.

***P3**: Okay. So three times 8 is 24, and three times 3 is 9. And two times 12 is 24, and two times 5 is 10. So one page has nine of the 24 blocks coloured in, and the other page has 10 of the 24 blocks coloured in. Which has the most pieces coloured in?*

Class: The ten out of 24.

***P3**: Yes. 10 over 24, so you go back to the original fraction and say that five-twelfths is the bigger fraction. Any questions on comparing fractions? So first, we make the denominators the same by finding the LCM.*

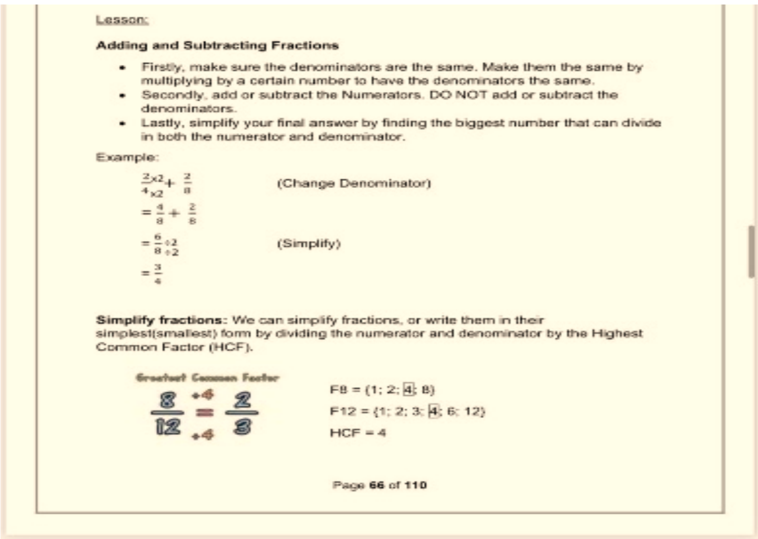
During this part of the lesson, P3 asked guiding questions to assist learners when they could not remember. The learners were actively involved with the lesson; when the teacher asked a question, they answered the question. As the teacher explains LCM and compares fractions, she touches on equivalent fractions but does not spend much time on them. During her interview, she alluded to the fact that she relies on the lower grades to have taught certain things:

R: Which concepts should Grade 6 learners understand in order for them to understand addition and subtraction of common fractions?

***P3**: I sort of rely on the lower grades, especially the grade fours. By the time they get to me, I feel like the difficulty of the fractions just basically increase. They should already understand it.*

The learners do a small activity on equivalent fractions before moving on to addition and subtraction of common fractions. P3 provides them with the steps at the very beginning of the lesson and doesn't allow learners to develop their own strategies:

***P3**: Okay lets mark (marks the activity). Okay, take out your highlighters, we going to highlight.*



Lesson:

Adding and Subtracting Fractions

- Firstly, make sure the denominators are the same. Make them the same by multiplying by a certain number to have the denominators the same.
- Secondly, add or subtract the Numerators. DO NOT add or subtract the denominators.
- Lastly, simplify your final answer by finding the biggest number that can divide in both the numerator and denominator.

Example:

$\frac{2_{\times 2}}{4_{\times 2}} + \frac{2}{8}$ (Change Denominator)

$= \frac{4}{8} + \frac{2}{8}$

$= \frac{6_{\div 2}}{8_{\div 2}}$ (Simplify)

$= \frac{3}{4}$

Simplify fractions: We can simplify fractions, or write them in their simplest(smallest) form by dividing the numerator and denominator by the Highest Common Factor (HCF).

Greatest Common Factor

$\frac{8}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{2}{3}$

F8 = (1; 2; 4; 8)

F12 = (1; 2; 3; 4; 6; 12)

HCF = 4

Page **66** of **110**

Figure 24: Page 66 of P3's Booklet

***P3**: Highlight the steps that you need to know when adding and subtracting fractions. What is important to remember is that when you are adding and subtracting fractions, the denominators have to be the same; otherwise, you cannot do any calculations. When we are multiplying and dividing fractions then you can calculate with any denominators. But for today, our denominators need to be the same. Why do you think they need to be the same?*

Class: So that the pieces are the same size

***P3**: Think about if you did a puzzle, your pieces need to be the same size; otherwise, your puzzle will not fit, and you won't be able to complete the puzzle.*

Class: So when a fraction has the same denominators, they are the same size. We can compare them, and we can add/subtract them.

***P3**: That is correct. So a few things to remember when adding or subtracting the denominators: the denominators must be the same so that you can only add or subtract the numerators. You do not add or subtract the denominators. Where possible, simplify.*

Although P3 provides her learners with the steps to follow, she does ask questions where they are expected to reason, as seen below:

> ***P3**: But for today, our denominators need to be the same. Why do you think they need to be the same?*

This allows learners to think critically and engage their reasoning capabilities. As a class, they look at a numerical example together on the board together, as follows:

> ***P3**: Let's look at an example:*

$$\frac{2}{4} + \frac{2}{8}$$

$$= \frac{2 \quad \times 2}{4 \quad \times 2} + \frac{2}{8}$$

$$= \frac{4}{8} + \frac{2}{8}$$

$$= \frac{6 \quad \div 2}{8 \quad \div 2}$$

$$= \frac{3}{4}$$

> ***P3:** So, the steps are as follows:*

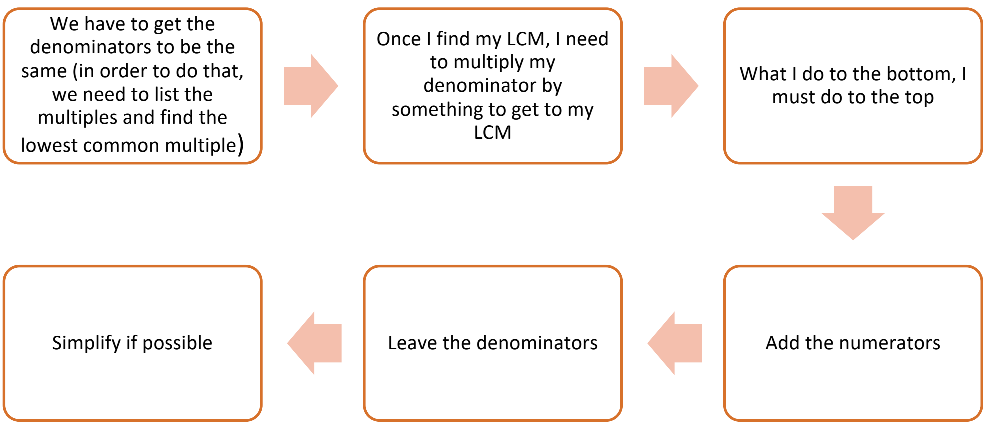


Figure 25: P3's Steps for solving Fraction equations

P3 provides the steps (as seen in Figure 25) for the learners and ensures that they understand the order in which the steps must be followed, allowing learners' Procedural Fluency to develop. When prompted about allowing learners to develop their own strategies, she had the following to say (line 681-682):

> ***P3:** Yes, if it, if it works, you know, if, if it's a viable method, do you mean that they can use any method that they understand?*

When further prompted, P3 realised that she was not allowing the learners the opportunity to develop their own strategies and conceded that maybe it was a good idea to let them try to develop their Strategic Competence more:

> ***R:*** *Do you feel that maybe you could have let your learners come up with their own strategy for solving equations or solving fractions?*
>
> ***P3****: And that's more of a learner-led class, you know? So I'll allow it. I don't have any problem with it. I just haven't thought of actually applying it. Within my classroom. Having the children figuring [pause]. Yeah, no, having the, the children figure out the method.*

Further in her lesson, P3 allows the children the opportunity to work through an example and allows them to explain what they have done:

> ***P3****: Okay, let's do one more together. I am going to give you 4 minutes to work through it yourself, and then we will check it together.*
>
> *(4 minutes pass)*
>
> *Teacher: Who would like to take us through their steps?*
>
> *Class: Okay, so we have the equation:*
>
> $$\frac{3}{5}+\frac{3}{10}$$
>
> *Class: We have to look at our denominators, and we can see that they are not the same. So we have to list the multiples of each denominator. The multiples of five are 5, 10, 15, 20, and the multiples of 10 are 10, 20, 30.*
>
> *Class: We can then see that the common numbers are 10 and 20.*
>
> *Class: But we must look for the smallest number, which is 10.*
>
> ***P3****: So what is our denominator?*
>
> *Class: 10*
>
> *Class: Right so then we have to ask ourselves 5 times what will give us 10? We don't have to worry about the other denominator as it is already 10. So 5 times 2 equals 10.*
>
> $$\frac{3 \quad \times 2}{5 \quad \times 2}+\frac{3}{10}$$
>
> *Class: What we do to the bottom, we must do to the top. Then we just add the*

numerators and leave the denominators as 10.

$$\frac{6}{10} + \frac{3}{10}$$

$$\frac{9}{10}$$

***P3**: What happens next?*

Class: We simplify

Teacher: Can we simplify nine-tenths?

Class: Uhm [pause].

***P3**: Well on way to check is to list the factors of 9 and 10 and see if there is a common factor that we can use to divide by. So the factors of 9 are 1, 3 and 9, and the factors of 10 are 1, 2, 5 and 10. Is there a common factor?*

Class: 1

Teacher: Yes, that is true, but 10 divided by 1 is?

Class: Ten.

***P3**: And 9 divided by 1 is?*

Class: Nine.

***P3:** So our fraction will be nine tenths. SO it is in its simplest form*

As the class explains, P3 does not ask for justification or reasoning behind the steps used, so it is difficult to determine if the learners understand or if they are just following the example and the steps set out at the beginning of the lesson. In her interview, P3 alludes to the fact that she has to get through content and time is limited:

> *R: Do you feel that maybe you could have let your learners come up with their own strategy for solving equations or solving fractions?*
>
> ***P3**: But no, I haven't thought about doing it like that, because I'm so used to. That's the way I teach, you know, I have this content that I need to teach them. But I think I think that also comes down to the time that we have to teach.*

Furthermore, P3 explains that during her fraction lessons, she focuses more on providing the information as she knows some learners struggle:

> ***R:** Do your learners see the point in learning fractions?*
>
> ***P3:** I haven't done it with fractions, I think because I, as a teacher, see maths as a difficult topic. I think that I tend to focus more on the explanation of the*

topic and trying to make them understand than I do them providing me with examples, because I feel overall, a lot of them struggle.

When the class moved on to the third example, P3 involved the class more and asked them to provide reasons for their answers:

P3*: Okay, we are moving on. Next example:*

$$\frac{9}{10} + \frac{3}{4}$$

P3*: Okay. Can we add?*

Class: No.

P3*: Why not? I can just add the numerators and add the denominators.*

Class: Nooooooooo. We can't add denominators. We can only add numerators.

P3*: Okay so I can just add 9 and 3? What will my numerator for the answer be?*

Class: (laughs) Noooooooooo. We have to get our denominators to be the same. Then we can add the numerators.

P3*: (laughs) Okay, so how do we do that?*

Class: [lists the steps as previously stated]

$$\frac{9}{10} + \frac{3}{4}$$

$$\frac{9 \quad \times 2}{10 \quad \times 2} + \frac{3 \quad \times 5}{4 \quad \times 5}$$

$$\frac{18}{20} + \frac{15}{20}$$

$$\frac{33}{20}$$

10	4
20	8
30	12
40	16
	20
LCM: 20	

P3*: What happens now? Are we done?*

Class: Yes.

P3*: Well, we could be, but it looks messy because our numerator is bigger than our denominator. So let's convert into a mixed number. What is a mixed number?*

Class: A whole number and a fraction.

P3*: Good. So how do we convert from improper to mixed numbers?*

Class: We have to divide 20 into 33.

P3*: Okay, good. So how many times can 20 go into 33?*

Class: Once

Teacher: And then? What now?

Class: We have 13 left over.

P3*: Yes so what do we do with that 13? What does it become?*

Class: Our numerator of our fraction part?

P3*: Yes so our mixed number is?*

Class: One and thirteen-twentieths

P3*: Well done. Everyone seems to be on the right track.*

P3 involved her learners more by asking questions where they were expected to explain their reasoning (she even made a mistake at one point, and the learners rectified her), such as:

P3: Okay, we are moving on. Next example:

$$\frac{9}{10} + \frac{3}{4}$$

P3: Okay. Can we add?

Class: No.

P3: Why not? I can just add the denominators together and the numerators together?

Class: [laughs] No. We don’t add denominators. We only add numerators.

P3: Okay do I, can just add the 9 and the 3? What will my answer be?

Class: No. We can only add numerators if the denominators of the two fractions are the same.

P3: Okay, so how do we do that?

Class: [lists the steps as previously stated.]

Again, the learners follow the steps she has provided (Figure 25), and the teacher does not ask for justification during the steps. There is very little opportunity for learners to develop their Adaptive Reasoning, but their Procedural Fluency is developed more.

During the interview, P3 spoke of different ways that she could represent fractions which involved concrete manipulations:

R: Is there any other way that you could have used during your lesson to represent fractions?

***P3**: Um, yes, what I've done before is I'll have actually a printed pizza on the paper. And they will cut the pizza in half. So I've done that. I haven't done the clay, I thought of people of the children bringing clay to school, and then they can divide the clay and actually see, you know, half, half and quarter and what not.*

P3 ensures that her classroom environment is safe so that learners can answer without being judged:

***R:** How do you differentiate and accommodate learners with different learning styles and needs? How do you handle those who are shy or scared to answer questions?*

***P3**: So I will then say, Why are you guys doing that? You know, you made a mistake; it happens. We're all human. So I try to, to bring that judgment, if I can call it that. I tried to bring that down in my classroom. So that if I ask a learner* *that struggles, and they can't answer me that they don't get judged, that people just think, okay doesn't understand. And if I see you really under doesn't understand, then I'll move on. I won't put too much emphasis on the child because I don't want him to feel like sometimes when kids get put on the spot, they also sort of panic, and they can't answer, you know?*

This safe and positive environment allows learners to be involved, which is an opportunity for a Productive Disposition to develop, but it also allows learners the space to build their confidence within the subject as the teacher encourages participation, does not force you but will still attempt to get an answer from all learners.

The next section presents the data for Participant 4’s lesson observation and interview.

4.7.4 Participant 4’s Lesson Observation and Interview

Data on Participant 4’s practices used to develop mathematical proficiency within learners during the addition and subtraction of common fractions were gathered from the interview conducted after the lesson observations on introduction to fractions, equivalent fractions and addition and subtraction of common fractions.

P4’s lessons were based on mixed numbers, and during her lesson, she was able to teach converting between mixed numbers and improper fractions. She started the lesson off by revising what they had done and asked them to remind her of what they had learnt about fractions:

> ***P4**: Good morning Grade 6s. Remember I told you yesterday that ma’am will be watching our lesson today? Please say good morning to ma’am.*
>
> *Class: Good morning ma’am. Good morning ma’am.*
>
> ***P4**: Okay. Let’s do some revision from yesterday. We looked at our Fraction Wall. What did we use our fraction wall for?*
>
> *Class: To show that fractions are parts of a whole*
>
> *Class: To look at fractions that are the same size*
>
> ***P4**: So what are fractions? What are the parts of a fraction?*
>
> *Class: Fractions are a whole divided into parts*
>
> *Class: Fractions have a denominator and a numerator*

Class: The numerator is the top number, and the denominator is the bottom number.

***P4**: Good. What did we say about comparing fractions?*

Class: Uhm, when we compare fractions, they have to have the same denominators.

Teacher: What happens when our fractions are not the same? What do we do then?

Class: We have to find equivalent fractions

***P4**: How do we find equivalent fractions?*

Class: [pause]

***P4**: It's three letters. The three letters stand for something*

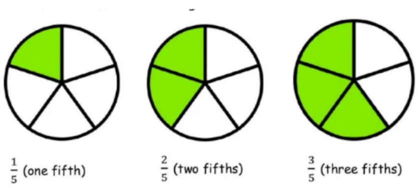
Class: Lowest denominator?

Class: LCD

Class: Lowest Common Denominator

***P4**: Well done. You guys remember a lot. We have to find the LCD and equivalent fraction fractions in order to compare fractions. Today, we are going to cover addition and subtraction of fractions and mixed numbers. Let's start.*

(P4 wrote down following fraction in two different representations): $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$



Her lesson started by showing the learners a visual representation. She believed that the learners enjoyed that and had fun doing it, however, she noted in the interview that she could have used more concrete manipulation:

R: Do you think that there are other ways that you could have represented fractions?

***P4**: So on, well, in a more physical way, couldn't use paper, could have used cardboard, could have torn pieces up, could have drawn in cardboard? Definitely, not just via technology could have been something a little bit more handheld or solid. Yeah, definitely.*

The class covered equivalent fractions and the LCD, which is needed when adding and subtracting fractions. As P4 progresses through her lesson, she starts with adding mixed numbers with common denominators and explaining the steps to be followed when solving a problem. When asked why she started that way, she had the following to say:

> *R: Why did you structure the lesson in that way?*
>
> ***P4**: Yeah, yeah. I think that [I] wanted to kind of give them a good introduction. So to explain it first. Because I find that if we start doing something together, especially if it's a writing exercise, they don't concentrate. So I wanted to do something where they [were] all just listening first and looking, and then we discuss, and then they try some of the examples with me, or they try and link together how they understand the topic, and then they do the work. I always, I like to actually explain something, and then they do the work. Because then it gives me a chance to walk around and actually see what they [are] struggling with; it gives me that spare time. Whereas if I teach they work, teach work, it can be a little bit disjointed. So I prefer to have quite a set structure that definitely.*

Once she had moved on from addition and subtraction of mixed numbers with like denominators to the addition of mixed numbers with unlike denominators, her learners were more involved:

> ***P4**: So when we add or subtract fractions, they must have the same denominator. And when we don't have the same denominator, we have to find [pause].*
>
> *Class: Equivalent fractions*
>
> ***P4**: Let's turn to page 32 in our workbooks. We are going to work through Question 17, sub-questions 1, 3 and 5. Look at the examples and explanations at sub-question 2 and 4. I'm going to give you about 12 minutes to complete it. You will see sub-question 1, is just finding the LCD of the two numbers. Who remembers how to find the LCD?*
>
> *Class: Something to do with multiples.*

***P4**: Yes, I showed you how you can list the multiples of each denominator and then find the SMALLEST number in both the lists. Do you remember?*

Class: Yes.

***P4**: Sub-question 1 is listing the LCD. Sub question 3 and 5 is adding and subtracting, unlike denominator fractions. What do we do when we have unlike fractions?*

Class: Equivalent fractions.

Class: Make them the same.

***P4**: What should be the same?*

Class: The denominators.

***P4**: Why?*

Class: If the denominators are the same, then we can add or subtract them. If they aren't the same, then we cannot add or subtract them.

Class: Then we have to find equivalent fractions.

***P4**: Good. So let's look at one example, and then you can work on your own, and we will mark. Sub-question 2, example a:*

$$\frac{1}{2}+\frac{1}{4}$$

$$\frac{2}{4}+\frac{1}{4}$$

$$\frac{3}{4}$$

2: 2,4,6,8,10, 12
4: 4,8,12
LCD is 4

P4 ensures that her learners are being probed with guiding questions so she can see if they understand. She also allows them to give reasons when asking why the denominators should be the same:

***P4**: What should be the same?*

Class: The denominators

***P4**: Why?*

Class: If the denominators are the same, then we can add or subtract them. If they aren't the same, then we cannot add or subtract them.

Allowing learners to give reasons will develop their Adaptive Reasoning and show the teacher whether the learner has fully understood what has been explained. When a learner was uncertain, she explained why she did something and how she did it:

***P4**: Good. So let's look at one example, and then you can work on your own, and we will mark. Sub-question 2, example a:*

$$\frac{1}{2} + \frac{1}{4}$$

$$\frac{2}{4} + \frac{1}{4}$$

$$\frac{3}{4}$$

2: 2,4,6,8,10, 12
4: 4,8,12
LCD is 4

Class: How did you get two-quarters from one-half?

***P4**: Good question. We can see in our list of multiples that there is a number that appears in both lists, 4. Can you see that?*

Class: Yes.

Class: I also see 12.

***P4**: Yes good. But remember, we are looking for the LOWEST, the SMALLEST number in the list. In both lists. Which is pause]?*

Class: Four.

Teacher: Okay. Once we find our LCD. We can see that one of our fractions already have a denominator of four, so we do not have to change that fraction, but the other fraction has a numerator of 2 and that fraction we have to get to a denominator of 4. So how do we do that?

Class: I think we have to multiply 2 by 2 which is 4.

***P4**: Good. So if we multiply the denominator?*

Class: We must multiply the numerator. [Learners get up, start doing a little dance movement that they remember from last year.

***P4**: Okay great. Does anyone know why we multiply the top and bottom by the same number? How it is an equivalent fraction?*

Although the learners knew that the numerator and denominator had to be multiplied by the same number, she wanted them to explain why that was the case and why the fraction was thus equivalent. When they did not know, she showed them and explained once again:

***P4**: Okay great. Does anyone know why we multiply the top and bottom by the same number? How it is an equivalent fraction?*

Class: Uhm [pause].

***P4**: Okay, let me show you.*

$$\frac{1 \quad \times 2}{2 \quad \times 2}$$

***P4**: If we look at that, what did we say about fractions that have the same numerator and denominator?*

Class: It equals to one.

***P4**: Yes. That's why it is equivalent and doesn't change the value because we are actually multiplying by 1. One times anything is the number itself and equivalent fractions have the same value, just different numerators and denominators.*

Class: Oh. That makes sense.

***P4:** Yes. So, whatever you multiply the bottom by, you must multiply the top by so that the fraction you are multiplying by is equal to one. Got it?*

Class: Yes, got it.

***P4:** Okay, you can now work through the rest of page 32. We will mark in 20 minutes. Remember to write down all your steps. You get marks for your steps. And if you have steps, I can see where you went wrong, if it is wrong and what your thought process is.*

Class: Yes ma'am.

[learners work through page 32 while teacher walks around helping those who need help].

This explanation showed how the fractions could be equivalent and why we multiply the denominator and numerator by the same number. This develops Conceptual Understanding as they can understand what they are doing instead of just doing it (just following the steps). It is evident from the interview with P4 that learners need to understand that for fractions to be compared (and later added and subtracted), their denominators must be the same:

R: *What concepts do the Grade 6 learners need to understand to becoming proficient in addition and subtraction of fractions?*

P4*: I would definitely say sizing of fractions, like knowing or understanding the size of fractions and their value, knowing what different fractions are. Being able to compare them looking at are half and a quarter, and being able to visualise, okay, this is what a half looks like, this is what the quarter looks like. I can't combine them without making them the same thing. So maybe just understanding of the value of a fraction by itself, and then equivalent fractions being able to see how they equal or how they different.*

During the interview, P4 explained that she loves getting her learners involved in her lessons, and she can see their thought processes when they are explaining and when they are involved, which allows the opportunity for learners to develop their Productive Disposition:

R: In your lesson, do you see the kids making sense and having fun with what they're learning?

P4*:And then after that, I think for me, I love interaction. I really like it when learners interact. Because lessons can drag, a triple can take as you pass could take forever. But I think, I think my learners enjoy fractions the most when they interact with me and interact with each other; I've got a smart board. And I don't think I did it on the day that you came, but they often come up, and they use a smart board. And I find that is when they have the most fun.*

In her classroom, the teacher also welcomes mistakes and encourages learners to learn from mistakes. There is a safe space within her classroom that allows learners to make these mistakes but learn from them. This further allows for the development of a Productive Disposition:

> ***P4**: Which numbers are the same in both lists?*
>
> *Class: Fifteen and 30*
>
> ***P4**: And which of those two numbers is the lowest? The smallest?*
>
> *Class: Thirty. Oh. The LCD is 15. Sorry.*
>
> ***P4**: That's alright. We are allowed to make mistakes. We learn from mistakes.*

To ensure that learners are solving problems and understanding what they are working through, the teacher allows them the time to work through the sums and then she marks the sums with them. The learners are expected to explain, step by step, what they have done and why they have done it that way:

> ***P4**: That's alright. We are allowed to make mistakes. We learn from mistakes. Okay great. Well done guys. Let's move on. We looked at sub-question 2 and worked through example a. You would have spent some time looking at example b and c. When you give me the answer to Question 3, remember to explain step by step what you have done.*
>
> *Class:*

$$\frac{1}{4} + \frac{1}{8}$$

$$\frac{1 \quad \times 2}{4 \quad \times 2} + \frac{1}{8}$$

$$\frac{2}{8} + \frac{1}{8}$$

$$\frac{3}{8}$$

4: 4,8,12,16
8:8,16
LCD = 8

> *Class: We need to list the multiples of each denominator and then find the smallest common multiple in both list. We can see that in both lists we have 8 and 16.*

Class: But the smallest between 8 and 16 is 8.

Class: One denominator is already 8, so we leave that fraction as is. But the other fraction has a denominator of 4 of so have to multiply the 4 by 2 and because we did that at the bottom, we must do it at the top, so we multiply the 1 by 2. Our equivalent fraction for one quarter is two-eighths.

Class: We can then add the two fractions because they both have a denominator of 8. So our answer is three-eighths.

***P4:** Well done. That is a very thorough explanation. Let's carry on marking.*

As the learners worked through the examples, showing the steps, developing their Procedural Fluency and Strategic Competence, they were also providing reasons and explaining why they had chosen those steps and why they were doing certain things which provided an opportunity for them to develop their Conceptual Understanding and Adaptive Reasoning.

Furthermore, P4 explains that she could have allowed the learners to develop their own strategies to solve the problems:

R: Do you think that you could have allowed for learners to come up with their own methods?

***P4**: No, definitely. I think that's a great idea. But that's, that's exactly it. It's like I don't it's almost like I don't allow room for discovery, which is frustrating, because you, you [are] so set on this is the method I'm going to teach. This is the method I'm going to assess. So it's no I agree with you. And it would be nice if they could talk to each other, and then figure out together what they could use?*

During P4's lesson, her learners engaged with her, and she explained the reasoning behind equivalent fractions. Furthermore, she allowed the space for learners to explain their procedures and justify their reasoning. Her classroom was a safe space, and learners had no fear when answering (even if they were wrong). Participant 4 did not allow learners to create their own problems, which she knows and looks to implement in the sections going forward.

This chapter covered the presentation of data from each of the participants' classroom observations and interviews. The next chapter will analyse the data and provide recommendations.

5 CHAPTER FIVE: DISCUSSION OF RESULTS AND FINDINGS

5.1 Introduction

In the previous chapter, the data was presented and interpreted. This chapter discusses the results and findings from the data collected from classroom lesson observations and participant interviews. Furthermore, recommendations are given. The aim of the study was to answer the following question: *How do Grade 6 mathematics teachers develop learners' proficiency when teaching the addition and subtraction of common fractions?* This question involves developing learners' conceptual understanding, procedural fluency, productive disposition, strategic competence and adaptive reasoning regarding the concept of fractions. There were no emerging themes in this study. I used the concepts of Kilpatrick et al. (2001) to analyse and interpret the data (Thomas, 2003).

5.2 Summary of the Study

This section discusses the findings from semi-structured interviews and lesson observations. I used the qualitative approach to explore how teachers develop learners' mathematical proficiency when teaching addition and subtraction of common fractions. Triangulated results discussed in this chapter are supported with reference to the existing literature. The chapter commences with a response to the research questions, theoretical implications, the limitations of the study and concludes with recommendations and suggestions for further research. The purpose of conducting this study was expressed in two key research questions. What follows is a discussion of how the findings relate to and answer the research questions.

5.3 Responding to the Research Questions

In this chapter, I discuss how the four participants delivered their lessons and justified their actions. During the interview, participants were probed to gain greater insight and uncover why issues emerged during lesson presentations. The contextually grounded information produced will contribute to the body of knowledge and enhance the recommendations made in this study. I drew on the existing literature and Kilpatrick's model to discuss the findings, answer the research questions and make conclusions. This model was discussed in Chapter 2 and summarised in Chapter 4.

5.3.1 How do Grade 6 Mathematics Teachers Describe Common Fractions?

Findings analysed in this study show that teachers had a clear vision of what they wanted to achieve with their lessons and what learners should learn as lessons were presented. What was also revealed in this study is that all the teachers taught relevant content as prescribed in the South African Curriculum and Assessment Policy Statement. How teachers describe the common fractions has been demonstrated in different ways. Firstly, all the participants define a fraction as part of the whole. However, in this definition, none of them has emphasised that the parts must be equal, which means that the whole must be broken into equal parts. If that is not emphasised, then the concept of sharing will not make sense. This practice implies that learners may believe that a whole divided into parts, equal or unequal, is a fraction concept.

Regarding the definition of common fractions, none of them has explained what it entails. P2 asked learners to read and highlight the definition from the DBE workbook, but no further explanation was given. P3 did not explain the definition of the common fraction per se, but as she presented, she asked learners to define concepts such as mixed fractions, which form part of the definition. In the interview, the teachers attributed their lack of explanation of common fractions to the fact that the learners should have learnt that in previous grades. For example, P3 indicated that she relies on Grades 4 and 5 to impart basic fraction knowledge to the learners.

This section concludes by directly responding to my research question: *How do teachers describe common fractions?* Teachers describe fractions as part of the whole; however, they still need to refine their definition of fractions to include "equal parts". This definition implies that describing a fraction could be demonstrated through multiple representations, giving contrasting examples and most importantly, explaining the contextual meaning of a numerator and denominator.

5.3.2 What strategies do Grade 6 mathematics teachers use in developing learners' proficiency when teaching addition and subtraction of common fractions?

This question refers specifically to teachers' approaches to develop learners' conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition in fraction concept. To answer this research question, I have broken it down into five sub-questions to solidify the understanding of each strand of mathematical proficiency.

How do teachers develop learners' conceptual understanding of the fraction concept?

The findings in this section are discussed in terms of how teachers draw on learners' prior knowledge to build on the new knowledge, the use of fraction language and how fractions are represented.

Prior Knowledge

All the teachers ensured that they established prior knowledge before they moved to a new topic. They would recap the foundational knowledge of fractions focusing on the definition of a fraction and the types of fractions. This practice is also justified by McGowen and Tall (2010), who state that concepts are better remembered when building on prior knowledge. Information that is not built upon prior knowledge will be easily forgotten. Similarly, Allsopp et al. (2016) and Witzel (2016) believe that engaging with learners' prior knowledge and making connections will enhance conceptual understanding of concepts and provide a solid foundation for learning. In other words, starting the lesson with a question is a good base. Asking questions stimulates prior knowledge and connects what they already know and what they will learn.

To draw attention to the importance of recognising knowledge that stems from experience, Dong et al. (2020) alluded that learners with more prior knowledge will interact more than those with less prior knowledge. Therefore, it is important to remember what is being taught. I also realised that learners first had prior knowledge of fractions; however, they needed to be reminded and guided to answer questions. Secondly, the teachers did not give the learners all the answers; they steered them by asking probing questions. What also emerged during the lesson presentations was that learners failed to comprehend fractions as "parts making up a whole" until teachers visually represent the concept.

Just to give few examples, P1 predominantly asked learners to define a fraction and identify and name parts of a fraction. P2 played a video to excite the learners and jog their memories. After the video, she asked the learners what they remembered about fractions and equivalent fractions. Participant 3 started her lesson by having the learners actively involved and folding a piece of paper; however, rather than asking them what they remembered, she told them, and they read the information from a sheet she created. This approach also involved a comparison of fractions. Participant 4 asked learners to find equivalent fractions of given fractions.

The use of language

During the lessons, the some of the teachers used the correct mathematical language such as “denominator” instead of the bottom number, “numerator” instead of the top number, “equivalent fractions” and “lowest common denominator” or “lowest common multiple”. However, during classroom observation, it was noted that P3 used terms such as “over” instead of “so many parts of” or for example “fifth/three quarters” and her learners also used this vocabulary. She did not correct them. P 2 and 4 used terms such as “five sixteenths”, and “five parts of sixteen parts”, and the learners used the same language. Using the correct language allows learners to understand that the numerator and denominator are not whole numbers but instead equal parts of a whole. Proper fractional language usage is needed to develop crucial mathematical skills (DBE, 2011).

Using the correct language also allows for consistency within the school and outside the school. Siebert and Gaskin (2006) reiterate that fraction terminology should focus on “two quarters” as opposed to “two out of four” as the latter implies that the numerator and denominator are whole numbers and not thought of as parts of a whole number. This is supported by Tsai and Li (2016) and Simon et al. (2018) who stated that using the terms “three out of four” or “three over four”, encourages learners to see fractions as whole numbers and not parts of a whole. Learners need to understand that fractions are parts of a whole to develop the concept of comparing fractions and equivalent fractions. Participants 2 and 4 used the correct terminology, whereas Participant 3 used the incorrect terms such as “*fourteen over sixteen”*, initially, and when her learners used the same terms, such as, “*one over eight”, “five over eight”* and *“three over eight”* which are incorrect mathematical terms, she did not correct them.

Lamon (2001) advises that teachers should develop and encourage the correct use of terminology when developing fraction language. When teachers were busy with the paper-folding activity, verbally expressing the fractions allowed learners to hear and use the correct fraction language. Learners could name the fractions, such as "one-quarter of the whole A4 paper". Riccomini et al. (2015) argued that learners should use the correct mathematical language as this will develop their mathematical proficiency. Adding to Ricommini et al. (2015), Adams et al. (2016) advise that proper mathematical language usage will limit misconceptions and misunderstandings and further develop the learners' mathematical skills.

Representation of fractions

Moloto & Machaba (2021) and Siegler et al. (2013) state that using different representations of fractions can help learners with a deeper understanding and help them learn better and more effectively. During the lesson observations, teachers made use of a fraction wall (Participants 1, 2 and 4), circles or pizzas divided into equal parts (Participants 3 and 4) and paper folding activities (Participants 2 and 3) to diagrammatically represent fractions. The lessons represented fractions in different ways; however, some teachers believed that they could have used more visual representation, such as P1, who said, "*Yes we could make use of cardboard, food items, you know visual representation*" (line 136 from Interview 1) and more concrete materials to represent fractions. P1 and P4 further indicated that the use of such material allows the learners to be involved in a tactile way and physically experience the fraction.

Furthermore, teachers wrote the numerical fractions on the board for learners to see and used the correct fractional terminology when presenting the lessons. Dreher et al. (2015) concur with this view and argue that exposure to multiple fraction representations will construct and deepen their conceptual understanding. This understanding was also supported by Van der Walle (2016), who indicated that using different types of representations would ensure that learners could visualise fractions and that when they grasped these concepts, they were more likely to perform better.

To conclude this section and directly respond to my research question: *How do teachers develop learner conceptual understanding when teaching addition and subtraction of fractions?* In developing conceptual understanding, teachers managed to draw from learners' prior knowledge; they represented fractions in different ways and used the correct fraction language. However, they still needed to refine their definition of fractions to include that fractions are broken into **equal** parts, and they needed to emphasise the use of proper vocabulary, such as “three quarters” instead of “three over four” or “three out of four”.

How do teachers develop learners’ procedural fluency in fraction concepts?

During the lessons, some teachers taught procedures and rules while engaging their learners in the lesson and explanation; however, some teachers chose to engage simply in explaining the steps without engaging the learners. For example, during the lesson presentation, although teachers explained the steps to solve the problem, P1, P2 and P3 did not elaborate on why the numerator and denominator needed to be multiplied by the same number. This omission could lead to a lack of understanding, and learners may forget to multiply the numerator and denominator by the same number as they do not understand why they should (they’ve only been taught to get the denominators to be the same). Learners who understand can modify and adapt procedures to accommodate nonroutine problems.

To emphasise the importance of assessing learners’ understanding, Participant 1 stated in her interview that if she allowed the learners to explain their procedures, she could get a glimpse into their level of understanding and correct any misconception by re-explaining (line 107-108, Interview 1). Furthermore, P2 explained that she expects her learners to guide her through what they have done and explain what they have done (line 283-289 Interview 2).

Participants 3 and 4 concurred with Participants 1 and 2 who stated in their interview that they asked their learners to explain what they had done, come to the board and work through an example. Teachers’ views confirm Atyekin and Sahiner’s (2020) and Guner’s (2020) understanding, arguing that memorisation of steps will lead to less retention, and learners will most likely forget what they have learned. Also, Liu et al. (2019) agree and state that learners who memorise without understanding are less likely to remember and cannot apply what they have learned to address non-routine problems. If learners are explaining the steps and working through them with the teacher, the teacher can see whether they understand or not, and they can assist straightway, and then learners will not carry on solving the problems incorrectly.

To conclude this section and directly respond to my research question: *How do teachers develop learner procedural fluency when teaching addition and subtraction* *of common fractions?* There were numerous opportunities for learners to develop their procedural fluency during the observed lessons. Most of the lessons found the teacher demonstrating and developing the procedure (although allowing learners to interact and answer questions posed). During the lesson, teachers outlined the steps necessary to solve fraction problems. The lessons were controlled and allowed for minimal exploration of procedures and methods. Additionally, teachers need to ensure that they develop learners' conceptual understanding as well; providing numerous opportunities for the development of procedural fluency without providing opportunities for the development of conceptual understanding will be futile.

How do teachers develop learners' strategic competence in fraction concepts?

During the lessons, learners could not devise their own strategies to solve the problems. The learners were only introduced to one way to solve problems and that was how they had to complete the problem. If learners are not given the opportunities to use their own strategies to solve problems, it might suggest that the procedures given to them are not meaningful. This view is supported by Syukriani et al. (2017), who argue that strategic competence is vital to developing the knowledge, skills, competence and beliefs needed in mathematics. To add to this, Copur-Gencturk and Doleck (2021) and Özdemir and Pape (2012) stated that strategic competence allows learners to be flexible as had to use what they know and understand and adapt it to the given problem.

Furthermore, the learners were told what method to use, and there was little to no room for learners to devise their own strategies to solve problems. Participants 1, 2, 3 and 4 gave the learners a set worksheet to complete and provided them with the procedures to follow to complete the worksheet's problems. For example, the worksheets focused on equations ($\frac{7}{12} + \frac{2}{4}$ =) rather than problem-solving (*Example of problem-solving question: Rachel took* $\frac{1}{2}$ *of an hour to paint a table and* $\frac{1}{3}$ *of an hour to paint a chair. How much time did she take in all?).* This example focuses on routine problems but allows minimal creative and critical thinking. This is the strand teachers need to work on the most, as there was very little room for learners to create their own problems, represent them and solve them. Problem-solving was one-directional. Teachers emphasised that they were aware that they did not allow learners to explore and investigate as they were pressed for time and were worried that some learners would learn the “wrong” way to do something. They might only remember that or some learners would do nothing and wait for the answers.

As learners venture out into the world, they are faced with situations where they may run into problems. This seems to suggest that allowing learners to formulate their own problems within the classroom setting and then solve those problems will equip learners with strategies they may need for real world problem formulation and problem solving. This could be one of the strategies to help leaners make sense of the fraction concept. To show a need for teachers to develop the skill for learners to be creative and competent in coming up with their own strategies to solve problems,

To conclude this section and directly respond to my research question: *How do teachers develop learner strategic competence when teaching addition and subtraction of common factions?* Teachers can provide learners with many activities in which they can solve equations or routine problems, however, they allow very little room for problem-solving and even less room for problem formulation and creating strategies to solve problems so, the teachers did not develop the learners' strategic competence.

How do teachers develop learner-adaptive reasoning in fraction concepts?

During the observed lessons, the teachers sought justification and reasoning when solving the set problems. The strategies used by participants to unpack the fraction concepts, enabled learners to make conclusions based on what they saw, engaged with and how they made sense of the explanations given to them. To build on this understanding Wibowo (2016) and Susilawati et al. (2021) stated that adaptive reasoning was one of the core elements of mathematics. Rizki et al. (2018) and Muin et al. (2018) apprehended inductive reasoning as a way of reaching a general conclusion based on observed data to emphasise the importance of reasoning in mathematics. In contrast, deductive reasoning was a conclusion based on agreed rules. This study found that teachers provided learners with rules on how to solve the problems. In other words, teachers encouraged learners to reason deductively. This implies that there was more development of Procedural Fluency as learners were expected to reason using rules they had been taught. There was little development of deductive reasoning as learners were given no opportunity to devise their own rules.

During the lessons, Participants 1, 2, 3 and 4 involved the learners in their explanation and asked for justifications and reasons why certain things were done. When teachers started by revising addition and subtraction of common fractions with like denominators, learners could use their prior knowledge to explain what needed to be done to solve the problem. When teachers asked them how to solve addition and subtraction of common fractions with unlike denominators, they could reason using their prior knowledge. To show that this was a good practice from the side of teachers, Susilawati et al. (2021), Muin et al. (2018), and Rizki et al. (2018) endorsed adaptive reasoning as promoting the ability to reflect, explain, justify, and think logically about a problem.

Allowing learners to explain their steps and provide justification further develops their Adaptive Reasoning. For example, the teachers P2, P3 and P4, involved the learners in their lessons which allowed them to gauge where the learners were. If they saw that the class was not understanding, they would try and explain in a different way or more in a visual way. They ensured that her classroom was a safe environment, and they encouraged all learners to try so that they could build the leaners' confidence.

To conclude this section and directly respond to my research question: *How do teachers develop learner adaptive reasoning when teaching addition and subtraction of common fractions?* Teachers provided opportunities for learners to explain and justify their reasoning for steps taken to solve fraction problems. However, teachers could further develop this by allowing learners to engage in problem-solving and adapt their thinking to problem-solving scenarios.

How do teachers develop learners' productive disposition of the fraction concept?

All four participants agreed that fractions is a concept that most, if not all, Grade 6 learners struggle with and find challenging. In line with the participants' views, Pavlovičová and Vargová (2020) and Sartono and Karso (2020) emphasised that learners find fractions difficult and even more so if fractions are found in problem-solving (Istiqomah & Prabawanto, 2019). I have observed that developing a productive disposition is extra challenging when most learners struggle with a concept. During their lessons, the participants created opportunities for learners to be involved by asking questions, ensuring they were provided with a platform to interact; and allowing learners to come to the board, write on the board and explain their procedures of how to solve a problem, and for learners to persist in their problem-solving. When learners weren't feeling as confident, teachers would assist their learners, guide them, support them, and help them get on the "right track". For example, P1 would say, "look carefully", "we are allowed to make mistakes", and "we learn from mistakes". When learners have a teacher that has a positive attitude that provides them with support and encouragement, their attitude toward mathematics, and particularly fractions, will not waiver.

Research has shown that teachers should extend problem-solving into real-world problems as this will help learners see that mathematics is useful and found in everyday life. Haji et al. (2019) support this view by arguing that we can improve learners' productive disposition by allowing them to be involved in real-life problem-solving or extending mathematics problems into real-life. During the interview, P3 echoed the same sentiments and said:

> *P3: "They need to understand where to apply maths and why do they need to learn about maths... Yeah, so I think they enjoy it a little bit more, if they know where it is applicable and how it's applicable to their lives."*

A study by Awofala et al. (2022) found a relationship between learners' productive disposition and achievement. As productive disposition increases, achievement increases. As learners gain more confidence in fractions, they are more likely to persevere and not just give up.

Teachers need to develop learners conceptual understanding and provide opportunities for learners to discover fraction problem-solving and investigate different methods as this will allow learners to have a feeling of self-assurance from an appreciation of one's abilities or qualities. To support and add to this view, Çiftçi and Yildiz (2019) indicated that more confident learners, will persevere and will also try their best. Creating understanding within learners will allow them to solve routine and non-routine problems. The lack of opportunities for problem-solving can be taken as "something" to work on. Teachers need to, within their fraction lessons, allow learners to be involved in problem-solving and problem formulation.

To develop Productive Disposition, for example, P3 and P4 allowed their learners to complete an example independently without disruptions. As the teachers were explaining the example, they established the procedure that the learners needed to follow to complete the sum. This ensured that learners had the correct steps to solve the problem, and as seen later, the learners were able to use the steps efficiently and effectively. This shows the development of Procedural Fluency. Therefore, it is important to provide opportunities for productive disposition to develop so that learners could have positive attitude and habit to see mathematics as something logical and useful for life.

To conclude this section and directly respond to my research question, which states: *How do teachers develop learner productive disposition when teaching addition and subtraction of common factions?* Teachers tried to provide opportunities for learners to see mathematics as worthwhile, sensible and useful by creating safe learning environments where they were not afraid to answer; they still needed to extend fraction problems to real-life problems and so provide learners opportunities to be creative in coming up with strategies to solve problems. This approach could build learners' confidence and further develop a productive disposition.

To answer the main research question: What strategies do teachers use to develop learners' proficiency in the addition and subtraction of common fractions? The findings can be summed up as follows: Teachers created an environment for learners to develop mathematical proficiency by connecting prior knowledge to existing knowledge and creating a springboard for new knowledge. Furthermore, teachers teach procedures and methods and expect learners to explain their reasoning behind using a specific method. Teachers also ensured that there is a safe space within the classroom that allowed learners to make mistakes but learn from them. There are certain things that teachers could do more to ensure that mathematical proficiency is further developed, like ensuring that learners use the correct terminology, different representations, and are engaged with concrete manipulatives when learning fractions. Moreover, learners should be allowed the freedom to create problems and find solutions to problems.

5.4 Theoretical Implications of This Study.

Teachers' practices in developing learners' proficiency in the addition and subtraction of common fractions match the points made in references describing each strand of mathematical proficiency. With Kilpatrick et al. (2001) framework as a guide in this study, I constructed my lesson observations, looking for certain aspects within the lesson that would speak to the specific strand being developed. The interpretation and analysis of lesson observations and interviews were also guided by this framework. As these strands do not work in isolation, during my observation, I found that some things within the lesson could fit into more than one strand.

5.5 Limitations of the Study

As per the Gauteng Department of Education regulations, I was not allowed to observe the teachers and learners within their classroom; I had to do it via an online platform. This made it difficult at some points to see who was answering and sometimes to hear what was being said.

Furthermore, I had difficulty finding public schools to participate, as each teacher had their challenges. Over and above that, the management team at various schools stated that their teachers were under enough stress with their workload and did not need an extra set of eyes on them.

The time I spent with the teachers was limited, and I would have liked to have observed one or two more lessons (perhaps even to have viewed the learners' tests and journals) to see how the teachers went further and how they consolidated the topic of fractions.

5.6 Recommendations and Suggestions for Further Research

In light of the findings, this study recommends that teachers make use of a variety of models or representations of fractions, such as length models, area models, set models, as well as visual representation, auditory representation, and numerical representation. Ubah (2021) and Dreher et al. (2015) asserted that being exposed to different models and representations will further develop learners' understanding. School management teams should purchase equipment and charts related to fractions to enhance the teaching and learning within their school. If schools have the equipment, teachers are encouraged to use the equipment.

Teachers are sources of knowledge, and it is recommended that teachers share their knowledge. Many of the teachers attend mandated workshops from the GDE, but other than that, there is very little collaboration between teachers at schools, across schools and provinces. This collaboration will ensure that best practices are shared and that strategies and tools are discussed and communicated to ensure that the learners receive the best possible teaching. When teachers feel there is support, they will then be able to address challenges and ask for help if they need it, instead of allowing misconceptions or frustrations to fester and grow. It is recommended that teachers develop procedural fluency along with conceptual understanding, as procedures that are memorised and not understood are less likely to be retained. As was stated by Pedersen and Bjerre (2021), understanding the concept of fractions and equivalent fractions is more effective than memorisation. Based on the data collected, it is recommended that teachers develop all five strands of mathematical proficiency within learners as they are intertwined. When planning lessons, teachers should plan and cater for the development of all five strands within the concept of the fraction. These recommendations will ensure the effective teaching of fractions and the development of learners' mathematical proficiency within fractions.

5.7 Conclusion

The study aimed to discover how teachers develop mathematic proficiency within learners regarding the addition and subtraction of fractions. Throughout the lesson observation and interviews, it was evident that teachers had an understanding of fractions and of what they wanted to get across to their learners during their fraction lessons. They tried their best to impart knowledge onto their learners, however, they did not always use the correct terminology. It seemed that the teachers were trying to get the content across in a way that their students would understand. There are five intertwined strands, and each one of these five strands must be developed to be mathematically proficient. Teachers need to provide opportunities for learners' mathematic proficiency to develop. My study highlighted that the fraction concept should, first, be understood before moving on to procedures or methods. The memorisation of rules will lead to less retention of the rules, and learners will be unable to use them and adapt them to unfamiliar problems.

It is also beneficial for learners to be actively involved in their learning and have the opportunity to explore and take control of their own learning. Learners who are actively involved in the lesson will not be reliant on the teacher and can think creatively and adapt their thinking. I am of the opinion that teachers should plan in such a way that allows for reflection so that learners (and teachers) know and develop a way forward in order to enhance and further develop mathematical proficiency within fractions.

In developing learner mathematical proficiency, this study's findings revealed that teachers rely more on procedures and rules in solving problems related to the addition and subtraction of common fractions. During the lessons, less time was spent developing the other strands. Despite the fact that the results cannot be generalised because of the small sample used, the study could benefit mathematics teachers because it deals with approaches on how to unpack the fraction concept for proficiency. Teachers need well-planned training on how to develop learners' proficiency in mathematics topics.

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6 APPENDICES

6.1 APPENDIX A: OFFICIAL LETTER OF REGISTRATION CONFIRMATION

UNISA university of south africa

2335

LENDIS A P MRS
1122 BESEMBIESIE ROAD
MONTANA PARK
0182

STUDENT NUMBER : 59293047

ENQUIRIES TEL : 0861 670 411
FAX : (012) 429-4150
eMAIL : mandd@unisa.ac.za

2022-04-22

Dear Student

I hereby confirm that you have been registered for the current academic year as follows:

Proposed Qualification: MED MATHS EDUC (90061)

CODE	PAPER	S	NAME OF STUDY UNIT	NQF crdts	LANG.	PROVISIONAL EXAMINATION EXAM.DATE	PROVISIONAL EXAMINATION CENTRE(PLACE)
Study units registered without formal exams:							
@ DFMED96			MEd - Mathematics Education (Dissertation)	**	E		
DFMED96			MEd - Mathematics Education (Dissertation)	**	E		

@ Exam transferred from previous academic year

You are referred to the "MyRegistration" brochure regarding fees that are forfeited on cancellation of any study units.

\# Your attention is drawn to University rules and regulations (www.unisa.ac.za/register).
Please note the new requirements for reregistration and the number of credits per year which state that students registered for the first time from 2013, must complete 36 NQF credits in the first year of study, and thereafter must complete 48 NQF credits per year.
Students registered for the MBA, MBL and DBL degrees must visit the SBL's ESOnLine for study material and other important information.
Readmission rules for Honours: Note that in terms of the Unisa Admission Policy academic activity must be demonstrated to the satisfaction of the University during each year of study. If you fail to meet this requirement in the first year of study, you will be admitted to another year of study. After a second year of not demonstrating academic activity to the satisfaction of the University, you will not be re-admitted, except with the express approval of the Executive Dean of the College in which you are registered. Note too, that this study programme must be completed within three years. Non-compliance will result in your academic exclusion, and you will therefore not be allowed to re-register for a qualification at the same level on the National Qualifications Framework in the same College for a period of five years after such exclusion, after which you will have to re-apply for admission to any such qualification.
Readmission rules for M&D: Note that in terms of the Unisa Admission Policy, a candidate must complete a Master's qualification within three years. Under exceptional circumstances and on recommendation of the Executive Dean, a candidate may be allowed an extra (fourth) year to complete the qualification. For a Doctoral degree, a candidate must complete the study programme within six years. Under exceptional circumstances, and on recommendation by the Executive Dean, a candidate may be allowed an extra (seventh) year to complete the qualification.

BALANCE ON STUDY ACCOUNT: 0.00

Yours faithfully,

Prof M S Mothata
Registrar

0100 O 00 0

UNIVERSITEIT VAN SUID-AFRIKA PRETORIA UNIVERSITY OF SOUTH AFRICA

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

6.2 APPENDIX B: OBSERVATION SCHEDULE

CLASSROOM OBSERVATION SHEET

1.

School		Educator	
Grade		Date	
Topic		Sub-topic	
Class enrolment		Number of learners present	
Time:		To	

2. PHYSICAL SETTINGS

A. CLASSROOM ENVIRONMENT	
Student seating arrangement (Rows; Pairs; Small groups; Others) Drawing of classroom	

Classroom adequate size for learners	
Classroom walls have enough for display (posters- displayed and promote learning) Availability Textbooks and printed worksheets Availability of other resources and manipulatives	
B. USE OF TECHNOLOGY	
Calculators (scientific or non-scientific)	
Computers	
Interactive white board; projector and camera	
None	

3. LESSON EFFECTIVENESS

A. LESSON DESIGN	
Structure Lesson introduction; Presentation & Investigation; Conclusion & Summary of the lesson, aims and objectives clearly stated, Consolidation and reflection **Types of Teaching and learning Activities** Routine or non-routine problems/problem solving or complex procedures, Application to real-life situations)	
B. CONTENT -DELIVERY	
Time managed at each stage of the lesson delivery	
C. UNDERSTANDING OF THE CONCEPT OF FRACTION	

Observation focus: **Conceptual understanding**	**Indicators of the development of each skill from the teachers' perspective**	What emerged	What seems to suggest
	Definition of fractions		
	Explanation of types of fractions		
	Correct use of fraction language		
	Connection of mathematical concepts		
	Use of different representations		
	Note all the follow-up questions you need to ask in the interview based on what has been observed during the lesson		

Observation focus: **Procedural Fluency**	**Indicators of the development of each skill from the teachers' perspective** Ability to solve problems, using correct steps to solve a problem, ability to use steps efficiently and effectively	How is it promoted (Actual Data emerged)	What seems to suggest from the observation
	Note all the follow-up questions you need to ask in the interview based on what has been observed during the lesson		

Observation focus: **Strategic Competence**	**Indicators of the development of each skill from the teachers' perspective** solving of problem, creating a problem and providing a solution, using various strategies to solve a problem, devise own strategies	How is it promoted (Actual Data emerged)	What seems to suggest from the observation
	Note all the follow-up questions you need to ask in the interview based on what has been observed during the lesson		

Observation focus: **Adaptive Reasoning**	**Indicators of the development of each skill from the teachers' perspective** probing questions; critical thinking promoted, encourage self-directed learning, Justification of actions.	How is it promoted(Actual Data emerged)	What seems to suggest from the observation
	Note all the follow-up questions you need to ask in the interview based on what has been observed during the lesson		
Observation focus: **Productive disposition**	**Indicators of the development of each skill from the teachers' perspective** Involvement, take Initiative, motivated and on-task,	How is it promoted(Actual Data emerged)	What seems to suggest from the observation

	cooperation, confidence, curious, no disruptions, positive climate prevail, sense of responsibility, Persistence		
	Note all the follow-up questions you need to ask in the interview based on what has been observed during the lesson		
Instructional/teaching/facilitation strategies (Connection to prior knowledge; Appropriate to develop a concept. Most frequently used strategies. Justification of actions. Address misconception, use of games, manipulatives, stories, etc. Verifying solutions, drawing conclusions, move around to observe, assistance provided, use differentiated instruction; Incorporate varied	What emerged	What seems to suggest	

assessment, inclusivity, Instructional resources incorporation (appropriately used) Guiding students; Leading students through discussion; Incorporating resources; Promote inquiry learning, moving around to help learners; opportunities for students to justify; Appropriateness of grouping arrangement		
Questioning strategies Higher and low-order questions; Open-ended questions provide opportunity for all students to respond; promote learner understanding;	What emerged	What seems to suggest
	Note all the follow-up questions you need to ask in the interview based on what has been observed during the lesson	

4. CONSOLIDATION

Achievement of lesson outcomes **(W**ere the objectives achieved, elaborate)	What emerged
Summary **(**How was it done? Did it emphasise main points of the lesson? Did it give insight to what may be done next lesson?**)**	What emerged
Reflection on the lesson presented **(**Ability to reflect on the practices**)**	Note important and critical issues that will form part of the recommendations

6.3 APPENDIX C: INTERVIEW SCHEDULE

1. How many years have you been teaching grade 6 mathematics?
2. How long have you been a teacher?
3. What do you believe are prerequisites for effective teaching of addition and subtraction of common fractions?
4. Do you believe that Grade 6 learners find it difficult to understand the concept of addition and subtraction of common fractions? Why?
5. Which key concept should Grade 6 learners understand in order to develop an understanding of addition and subtraction of common fractions?
6. How can teachers be supported to enhance their practices in developing learners' mathematical proficiency in the addition and subtraction of common fractions?
7. What strategies do you use in developing learners' mathematical proficiency in the addition and subtraction of common fractions?
8. Do you consider the teaching of addition and subtraction of common fractions as important?
9. How do you check learners understanding during your lesson presentation?

6.4 APPENDIX D: REQUESTING PERMISSION FROM THE PRINCIPAL

Request to principals for permission to conduct research in schools

31 January 2022

Dear Sir/Madam

Subject: Request to conduct research in your school

My name is Ashley Pearl Lendis, and I am student at the University of South Africa. I am presently enrolled for the MEd with a specialization in mathematics education. To complete the requirements for the degree, I need to conduct research that is related to the area of my specialization. My research project is entitled: **Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng**. The purpose of the study is to explore how teachers develop learners' mathematical proficiency (conceptual understanding; adaptive reasoning; strategic competence, procedural fluency, and productive disposition) when teaching fraction calculations. My supervisor is Ms Sophy Kodisang, a lecturer in the Department of Mathematics Education. Her office telephone number is 012 429 3964.

I will be an observer for lessons on fraction calculations that will be presented by grade 6 mathematics teacher. A semi-structured interview will be conducted with the participant after the second lesson observation at a time and place that is convenient for both the participant and researcher. The interview will be audio recorded for a verbatim transcription. The participants will therefore be requested to sign a consent form provided. All data collected will be analysed and a report regarding the study will be written. This research study will not only benefit the institution involved but will contribute to Mathematics teaching in the district. Please note that if you allow one teacher and his/her grade 6 mathematics class to participate in the research study, the following ethical values will apply:

- the teacher and students' participation is voluntary

- all information will be treated with confidentiality and anonymity to ensure that no harm or bad effect will be caused to participants by the research study
- participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

Due to the Covid-19 pandemic where social distancing is emphasised as one of the regulations, the lesson observations and interviews will be conducted remotely. Teachers who give a consent to participate will be requested to capture their lesson presentation on a video camera using their cell phones and send the recordings to the researcher or use google classroom platform. The contents of the video will always be kept confidential, and no one will be invited in the google classroom besides the researcher. After analysing the lessons maximally, the video recordings will be deleted permanently. The interviews will be conducted telephonically, through Zoom or Microsoft Teams. The participant and the researcher will be the only people having access to any of the platforms. The interviews will be recorded, with the permission of the participant. The recordings will be deleted permanently after being maximally used.

I will avail to you the summary of the study results at the time of completion if you would wish to have the summary.

Thank you in advance for your support.

Sincerely yours,

Ashley Lendis

Please complete the consent form below

CONSENT FORM FOR PRICIPALS

I.. the principal of
.... understand the context of the research study and I grant permission that the research study (title: **Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng**) may be conducted at the school. I am aware that the teacher and learners'

participation is voluntary; all information will be treated with confidentiality and anonymity to ensure that no harm or bad effect will be caused to participants by the research study; all observation videos and interview recordings will be destroyed at the end of the study; participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

_______________________________ _________________________

Principal Date

School Stamp

6.5 APPENDIX E: REQUESTING PERMISSION FROM THE SCHOOL GOVERNING BODY

Request to the school governing body for permission to conduct research in schools

31 January 2022

Dear Sir/Madam

Subject: Request to conduct research in your school

My name is Ashley Pearl Lendis and I am student at the University of South Africa. I am presently enrolled for the MEd with a specialization in mathematics education. To complete the requirements for the degree, I need to conduct research that is related to the area of my specialization. My research project is entitled: **Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng.** The purpose of the study is to explore how teachers develop learners' mathematical proficiency (conceptual understanding; adaptive reasoning; strategic competence, procedural fluency, and productive disposition) when teaching fraction calculations. My supervisor is Ms Sophy Kodisang, a lecturer in the Department of Mathematics Education. Her office telephone number is 012 429 3964.

I will be an observer for lessons on Fractions that will be presented by grade 6 mathematics teacher. A semi-structured interview will be conducted with the participant after the second lesson observation at a time and place that is convenient for both the participant and researcher. The interview will be audio recorded for a verbatim transcription. The participants will therefore be requested to sign a consent form provided. All data collected will be analysed and a report regarding the study will be written. This research study will not only benefit the institution involved but will contribute to Mathematics teaching in the district. Please note that if you allow one teacher and his/her grade 9 mathematics class to participate in the research study, the following ethical values will apply:

- the teacher and students' participation is voluntary

- all information will be treated with confidentiality and anonymity to ensure that no harm or bad effect will be caused to participants by the research study
- participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

Due to the Covid-19 pandemic where social distancing is emphasised as one of the regulations, the lesson observations and interviews will be conducted remotely. Teachers who give a consent to participate will be requested to capture their lesson presentation on a video camera using their cell phones and send the recordings to the researcher or use google classroom platform. The contents of the video will always be kept confidential, and no one will be invited in the google classroom besides the researcher. After analysing the lessons maximally, the video recordings will be deleted permanently. The interviews will be conducted telephonically, through Zoom or Microsoft Teams. The participant and the researcher will be the only people having access to any of the platforms. The interviews will be recorded, with the permission of the participant. The recordings will be deleted permanently after being maximally used.

I will avail to you the summary of the study results at the time of completion if you would wish to have the summary.

Thank you in advance for your support and I would appreciate an early reply to the request.

Sincerely yours,

Ashley Pearl Lendis

Please complete the consent form below

CONSENT FORM FOR SGB

I.. the SGB member of

.... understand the context of the research study and I grant permission that the research study (title: **Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District**

of Gauteng) may be conducted at the school. I am aware that the teacher and learners' participation is voluntary; all information will be treated with confidentiality and anonymity to ensure that no harm or bad effect will be caused to participants by the research study; all observation videos and interview recordings will be destroyed at the end of the study; participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

_____________________________   _______________________

SGB Member   Date

6.6 APPENDIX F: REQUESTING PERMISSION FROM THE TEACHER

Request to teachers for permission to conduct research in schools

31 January 2022

Dear Sir/Madam

My name is Ashley Pearl Lendis, and I am student at the University of South Africa. I am presently enrolled for the MEd with a specialization in mathematics education. To complete the requirements for the degree, I need to conduct research that is related to the area of my specialization. My research project is entitled **Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng**. The purpose of the study is to explore how teachers develop learners' mathematical proficiency (conceptual understanding; adaptive reasoning; strategic competence, procedural fluency, and productive disposition) in fraction calculations.

My supervisor is Ms Sophy Kodisang, a lecturer in the Department of Mathematics Education.

I would like to be a non-participative observer for lessons on addition and subtraction of fractions that will be presented by you. I hope to then conduct a semi-structured interview with you after the lesson observation at a time and place that is convenient for both of us. The interview will be audio recorded for a verbatim transcription. The participants will therefore be requested to sign a consent form provided. All data collected will be analysed and a report regarding the study will be written.

I believe that the work I am doing could be relevant to mathematics education in District. The information collected will be treated with confidence and I hereby give undertaking that:

- Your participation is voluntary.
- You may discontinue participation at any time if you so wish.
- You may also refrain from answering some interview questions when there is a need to do so.

Due to the Covid-19 pandemic where social distancing is emphasised as one of the regulations, the lesson observations and interviews will be conducted remotely. You are requested to capture your lesson presentation on a video camera using your cell phone and send the recordings to the researcher or be observed using google classroom platform. The contents of the video will always be kept confidential, and no one will be invited in the google classroom besides the researcher. After analysing the lessons maximally, the video recordings will be deleted permanently. The interviews will be conducted telephonically, through Zoom or Microsoft Teams depending on your preference. You and the researcher will be the only people having access to any of the platforms. The interviews will be recorded, with your permission. The recordings will be deleted permanently after being maximally used.

I am looking forward to your participation in the research study. I request you to sign the consent form provided if you accept my request to participate. Thank you in advance for your support

Sincerely yours,

Ashley Lendis

Please complete the consent form below

CONSENT FORM FOR TEACHER

I, ... (Name and Surname), understand the context of the research study titled: **Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng).** I am aware that I should video record my lesson presentation and send to the researcher, furthermore, there will be audio recorded interviews telephonically, through Zoom or Microsoft Teams. I am aware that anonymity and confidentiality will be adhered to in this study. I am informed that I may withdraw my consent to participate at any time without penalty by advising the researcher. I agree on my free will to participate in the research study.

______________________________ ________________________

Teacher's Signature Date

6.7 APPENDIX G: REQUESTING PERMISSION FROM THE PARENT

Request to parents for permission to conduct research in your child's classroom

April 2022

The Parent of grade 6 Mathematics Learner

Hillside Primary school

Tshwane South District

Dear Parent/ Guardian

Re: Request for your child to participate in a research study

My name is Ashley Pearl Lendis. I am a Mathematics education student at the University of South Africa. I request you to allow your child to participate in a research study titled: Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng**.** Your child's role in the research study will be to participate in the lesson.

If you allow your child to participate in the research study, take note that the following ethical values will apply:

- your child's participation is voluntary.
- he/she may discontinue participation at any time if a need arises.
- he/she may also refrain from answering some interview questions when there is a need to do so.

Due to the Covid-19 pandemic where social distancing is emphasised as one of the regulations, the lesson observations and interviews will be conducted remotely. Teachers who give a consent to participate will be requested to capture their lesson presentation on a video camera using their cell phones and send the recordings to the researcher or they will be observed through google classroom platform. The contents of the video will always be kept confidential, and no one will be invited in the google classroom besides the researcher.

I am looking forward to your child's participation in the research study. I request you to sign the consent form provided if you give permission that your child may participate. Thank you in advance for your support

Yours sincerely
Ashley Lendis

Please complete the consent form below and return to school

CONSENT FORM FOR PARENT

I...
parent/guardian of .... understand the context of the research study titled: **Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng**. I am aware that video and audio recordings of my child's participation will be conducted. I am aware that anonymity and confidentiality will be adhered to in this study. I am informed that I may withdraw my consent for my child to participate at any time without penalty by advising the researcher. I agree on my free will that my child will participate in the research study.

______________________________ ________________________

Parent/ Guardian's Signature Date

6.8 APPENDIX H: REQUESTING PERMISSION FROM THE LEARNER

Dear grade 6 Learner

REQUEST FOR YOUR ASSENT TO PARTICIPATE IN A RESEARCH PROJECT

I am a Deputy Principal at Crawford International. I am studying for a Master's degree at the University of South Africa in the Department of Mathematics Education under the supervision of Dr SM Kodisang

I hereby ask you to take part in my research project. If you agree to take part, you will only sit in the class and continue with the lesson with your teacher. Due to the Covid-19 pandemic where social distancing is emphasised as one of the regulations, the lesson observations will be conducted remotely. Your teacher who gives a consent to participate will capture the lesson presentation on a video and send the recordings to the researcher or the teacher will use google classroom platform. The contents of the video will always be kept confidential, and no one will be invited in the google classroom besides the researcher. After analysing the lessons maximally, the video recordings will be deleted permanently.

My research topic is: Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng.

I hope that at the end of the research there may be answers to how mathematics teachers develop your mathematical proficiency in addition and subtraction of fractions. If you take part in this research, I will only use the answers that you give to the teacher and the recorded videos. I promise not to disturb your teacher while teaching if they are teaching through MSTeams or Zoom.

If you agree to take part in this research, I promise that the videos will not be shown to anyone, and no one will know who took part. Your Mathematics teacher will also be asked to take part in the study. You are not forced to take part in the study. You can stop taking part at any time you want, and you will not be punished for stopping to take part.

Yours sincerely
Ashley Lendis

Please complete the consent form below and return to school

ASSENT FORM FOR LEARNER

I____ hereby agree to take part in this research. I am aware that the results of this research will be used to promote teaching and learning and may be used to help future learners. I am also aware that my teacher will video record the lesson during the presentation and the records will be kept confidentially.
In addition, I understand that I can decide to stop taking part in this study at any time and that I will not be punished.

Signed___________________________ Date _____________________

6.9 APPENDIX I: EXTRACT OF TRANSCRIPT

Interviewer: What do you think the kids really like about fractions? And what do they really dislike about fractions?

Participant 4: So I, after UK, we did more of addition and subtraction of different denominators. And I was very surprised to see a change of attitude the first day it's like it's a bit wow, you know what's going on. And then the second day, we made a bit of like, we would say the steps together, and they really enjoyed that and we added colour, but I think they definitely love the mixed numbers changing mixed numbers into improper fraction. It's, I don't know it's just and we did a story. You dive under the water you can't back up they just enjoy that. I think they I notice what's interesting is the mixed numbers that process is easier for them than turning the improper fraction into a mixed number. But they've had a block about division for a while, I really had to crack through with division when I did it in term one it was, it's soul destroying to try to get them to be excited about division. So I think it's definitely that route of division, it's a more difficult operation, whereas the mixed numbers they liked, they liked multiplying, they like adding. So yeah, I think mixed numbers, they really enjoy that. That's definitely their favourite.

Interviewer: This awesome. Now, I think, you know, as you talk to people and you observe lessons, you sort of see that everyone is struggling with the same thing. You know, it's not you; they're struggling with your fractions, or division. It literally everybody I've had my interviews with is in the same boat and you all from different schools,

Participant 4: That’s such a (in audible) for you to see.

Interviewer: No, it is, it is. And that's why I am really enjoying this study. And I'm hoping you know whatever comes out from this study I’ll be able to sort of implement or give advice to people.

Participant 4: I also want to use it, I’m also excited.

Interviewer: Well you know at the end when I submit and pass, you will get a copy of this study. So…

Participant 4: It's always nice to get people hate observation and criticism or constructive criticism. I love it. I really do. Like people hate their IQ (in audible), their Q&A style. Now that its (inaudible). And they can't stand class visits. And I'm like, why? It's so nice. It's great to have what all your hard work put on display, and then have people kind of assess it and tell you, Okay, this is going well, this is what you can work on, because you can't self-reflect while you're busy in the nest. And it's not that easy. Yeah so.

Interviewer: So this is, we moving now on to the topic of, ag sorry, the section about your lesson. So the first one is what were the objectives of your lesson. So by the end of that hour and a half, what did you want to achieve?

Participant 4: So basically, on a very fundamental basis, I wanted them to complete a certain amount of work. So at the beginning of the day, I said, Okay, well, these learners haven't yet learned, or they've learnt about mixed numbers. And they've learned about improper fractions, but they haven't actually done a recap with me yet, because that was part of the topics that I had to cover. So I thought to myself, which topics do I have to cover? I have to cover improper fractions, mixed numbers, adding and subtracting with mixed numbers, as well. So I all I wanted to do is I wanted to give a base, give an introduction to those topics, give examples, do exercises with them. And then via that complete the work in the textbook. So it was just to make sure that they have an understanding of those topics. Luckily, I knew the book will cover them again. So that was just the basis because I know we're going to practice the topics again later. So it was more just an introduction, I would say it had introduction, an introductory aid.

Interviewer: How do you accommodate learners with different learning styles, different learning abilities in your classroom? So looking at differentiation?

Participant 4: So with understanding the topic or basically doing the work in teaching?

Interviewer: Yeah, teaching.

Participant 4: Understanding in teaching. So I know that a lot of learners don't like speaking or contributing to what they do and don't understand. I know that a lot quieter than others. So I think the first the first type of aspects that I do cover is verbal, I definitely do with the learners who learn via people talking and discussing the topic. I do cover that because they can all chat and discuss visually, I do try and use visual aids. But via technology. I don't use a lot of models, which is one thing I did address that I want to do more. So visual aids, verbal. What else? Also, I don't know if this is even a branch of it, but they physically do it. They come up and they actually partake in understanding the work. So I think those aspects I do cover but not models. Unfortunately, auditory, I'd love to get more into videos. But once again, it's finding something that is worth taking up the time in your lesson. So that's definitely something I could also get into.

6.10 APPENDIX J: ETHICAL CLEARNACE FROM THE UNIVERSITY OF SOUTH AFRICA

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2021/04/14

Ref: **2021/04/14/59293047/02/AM**
Name: Mrs AP LENDIS
Student No.:59293047

Dear Mrs AP LENDIS

Decision: Ethics Approval from 2021/04/14 to 2024/04/14

Researcher(s): Name: Mrs AP LENDIS
E-mail address: ashleylendis@yahoo.com
Telephone: 0760546851

Supervisor(s): Name: MRS SM KODISANG
E-mail address: kodissm@unisa.ac.za
Telephone: 012 429 3964

Title of research:

Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2021/04/14 to 2024/04/14.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2021/04/14 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
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6.11 APPENDIX K: ETHICAL CLEARANCE FROM THE GAUTENG EDUCATION DEPARTMENT

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**24 January 2022**
Validity of Research Approval:	**08 February 2022– 30 September 2022** **2022/28**
Name of Researcher:	**Lendis AP**
Address of Researcher:	**56 Elnita Avenue**
	Northwold
	Randburg
Telephone Number:	**076 054 6851**
Email address:	**ashleylendis@yahoo.com**
Research Topic:	**Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions in Tshwane South**
Type of qualification	**Masters**
Number and type of schools:	**4 Primary Schools**
District/s/HO	**Gauteng North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

24/04/2022

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

6.12 APPENDIX L: LANGUAGE EDITORS LETTER

Member South African Translators' Institute
www.language-services.online

20 Mama Street
Eldo Park
0157

28 November 2022

TO WHOM IT MAY CONCERN

The manuscript "Grade 6 mathematics teachers' development of learner mathematical proficiency in addition and subtraction of common fractions, in the Tshwane South District of Gauteng" by Ashley Pearl Lendis has been proofread and edited for language by me.

I verify that it is ready for publication or public viewing regarding language and style and has been formatted per the prescribed style.

Please note that no view is expressed regarding the document's subject-specific technical content or changes after this letter's date.

Kind regards

Anna M de Wet

SATI MEMBER 1003422

BA (Afrikaans, English, Classical Languages) (Cum Laude), University of Pretoria.
BA Hons (Latin) (Cum Laude), University of Pretoria.
BA Hons (Psychology), University of Pretoria.

6.13 APPENDIX M: TURNITIN REPORT

59293047_Dissertation_Lendis

by Ashley Pearl Lendis

Submission date: 19-Nov-2022 10:31AM (UTC+0200)
Submission ID: 1958568905
File name: Dissertation_Lendis_November_2022_Final.docx (2.93M)
Word count: 47513
Character count: 252969