THE EFFECT OF PROBLEM BASED TEACHING AND LEARNING ON THE ACHIEVEMENT OF HIGH SCHOOL MATHEMATICS LEARNERS

By

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Declaration

I declare that ***THE EFFECT OF PROBLEM BASED TEACHING AND LEARNING ON THE ACHIEVEMENT OF HIGH SCHOOL MATHEMATICS LEARNERS*** is my work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete reference.

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Abstract

This research project, which was modeled on the constructivist theory of acquiring knowledge, and on recent recommendations regarding classroom instruction, investigated the impact of problem-based teaching and learning (PBTL) on the performance of high school mathematics learners. The effect of an intervention programme on an experimental group of students was compared with a control group by assessing their mathematical knowledge and skills; their procedural proficiency and conceptual development on varied mathematical contents; their skills in managing and controlling learning; and their views towards their learning programmes. The results obtained indicated that the experimental group indicated a higher conceptual understanding, an improved use of procedures, a higher level of managing and controlling learning, and positive views towards the programme and interest in the fundamentals of learning mathematics. An analysis of the data obtained by the on-progress instruments employed for the experimental group also indicated improved quality of participation and reflective thinking practices.

Key Terms: Authentic assessment; Cooperative learning; Curriculum; discovery; Mathematical achievement; Mathematical performance; Meta-cognition/Self-regulation; Problem based; Problem solving.

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CHAPTER ONE

INTRODUCTION AND ORIENTATION

1.1 Introduction and rationale for the study

Much time has been spent in describing the complex nature of teaching and learning since the time of Socrates. Several scholars have argued that these acts are complex and need to be understood as an interaction of various kinds of knowledge resources and practices (Brodie, 2001:17). Philosophers and psychologists have worked for centuries to analyze these acts and they have come up with various learning theories that attempt to describe the process (Pollard, 2002). Two theories have had a particular influence on teaching and learning (Biggs, 1996; Black, 1999; Cobb 1999). The first is *behaviourism,* which advocates rule-bound traditional rote learning. The second is referred to as *constructivism*, which promotes a more learner-oriented approach to learning.

Scholars believe that it is unthinkable to attain critical goals in mathematics by means of rote learning, as outlined by the *Curriculum Framework for Ethiopian General Education* (Ministry of Education [MoE], 2007a). Therefore a major classroom shift was proposed by the *School Improvement Programme of Ethiopia* (MoE, 2007b), away from rule-bound, content- and teacher-dominated teaching which places the focus on “transmission of information” (Armour -Thomas and Allen, 1993). Fortunately however, international research on the effects of these former methods shows that only those who have been taught by ‘open methods’ (e.g., problem-centered) could recognise the applications of mathematics in their daily lives (Boaler, 1997; Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti & Perlwitz, 1991), as it leads to effective teaching and learning with understanding (*Principles and Standards for School Mathematics* [NCTM], 2000).

The focus of this study is a problem-based teaching and learning (PBTL) approach to mathematics, underpinned by the belief that the learners are active and innovative individuals, having their own interest in and capacity for knowledge and self-development (Felder & Brent, 2002; Leu, 2002). Furthermore, “…the approach attempts to establish individual and social procedures to monitor and improve the nature and quality of knowledge construction” (Murray, Olivier, & Human, 1998:270). PBTL aims to blend specific instructional strategies and other essential elements such as problem solving, cooperation, inquiry, discovery, and metacognitive strategies in learning in any mathematics curriculum (Harris, Marcus, McLaren, & Fey, 2001; Leu, 2002).

The benefits of making the subject 'problematic' are widely supported by research (e.g., Cobb *et al*., 1991; Murray *et al.*, 1998), and many constructivist scholars recommend the use of problematic situations to enable students to construct their own knowledge by means of 'doing' mathematics, solving problems and organizing the subject matter (Dossey, 1992; Freudenthal, 1971; Polya, 1988). To achieve higher order cognitive goals, courses should be built around problems in such a way that students can spend much of their time discussing problems in groups instead of constantly waiting for the teacher to explain it to them. "The criterion for choosing a problem is that it has the capacity to engage students in the class in making and testing mathematical hypotheses" (Fitzgerald & Bouck, 1993:253). In such cases, students engage in complex mathematical tasks chosen carefully by the teacher that will help to develop mathematical power (Fitzgerald & Bouck, 1993; Lampert, 1988; NCTM, 2000). Hiebert and his colleagues termed this as problematizing mathematics. To this end, "…problematic situations help learners to think critically about why things are true, to inquire, to search for solutions, and to resolve incongruities" (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne, 1999:151). In facilitating such learning, the *Professional Standards for Teaching* (NCTM, 1991) and the *Principles and Standards for School Mathematics* (NCTM, 2000) recommend the teacher's skilled guidance in orchestrating oral and written discourses in ways that contribute to the students' understanding of mathematics.

In spite of this positive trend in mathematics education globally, recent government reports in Ethiopia show evidence of poor quality education with low overall performance and achievement by students at all levels (MoE, 2003, 2007b). Many official documents (e.g., *Curriculum and Evaluation Standards* [NCTM], 1989; *The National Research Council*, 1989) report that much of the failure in school mathematics is the result of a tradition of teaching that is inappropriate to the way most students learn. This failure in part may occur because the mathematics curricula are quantity sensitive and the instructional processes are procedurally oriented (MOE, 2007a).

To solve the above problem in mathematics education, the *National Council of Teachers of Mathematics* (NCTM), in some of its documents (NCTM, 1989; NCTM, 1991; NCTM, 2000) mentioned in earlier paragraphs, propose a new vision and possibilities for a mathematics curriculum through PBTL. The study therefore, focuses on the effect of this approach, PBTL, in the Ethiopian context.

1.2 Problem statement

1.2.1 Problem analysis

Over the past 15 years concerted efforts have been made by concerned bodies, among others the Ethiopian Government, to make substantial improvements to the mathematics curriculum (MoE, 2003). As part of this reform, there have been attempts to change teachers' beliefs about learning and teaching. However, many problems still remain. The school mathematics curriculums are still quantity sensitive and teacher dominated. Moreover, teachers hardly ever encourage students to learn mathematics by means of problem solving, communication, connection, and proof and reasoning (MoE, 2007a). Similarly, according to the researcher's observation and experience, most current classroom practices are still accompanied by procedurally oriented instruction. This instruction could be characterized as "chalk and talk", that is, the traditional transmission lecture. However, a quantity sensitive curriculum and procedurally oriented instruction restrain life-long learning, higher order thinking, and problem-solving in mathematics (Madsen & Baker, 1993:260).

Owing to the problems stated above, a number of high school students appear to be developing negative attitudes towards mathematics, are becoming less confident in solving mathematical problems and less able to communicate or reason mathematically. Besides, they seem to fail to learn and do mathematics in a sprit of inquiry and 'figuring it out'. Their abilities to formulate and solve problems, and judge/evaluate results, are not vivid. Their motivation to develop independent, cooperative and democratic attitudes towards learning mathematics seems to be low. They also seem to exert only the minimum effort to enhance their understanding, planning, implementation, assessment and evaluation skills in learning mathematics.

In summary, many of the learning problems indicated so far can be attributed mainly to the lack of problem-based teaching and learning as a result of transmission, teacher-centered mathematics teaching, that is, in teaching-learning situations where students are not engaged in co-operative and self-regulated problem-solving activities.

1.2.2 Formulation of the research problem

The problem that will be researched by means of this study is the perceived effect of the traditional teacher-centered, transmission teaching on mathematics teaching and learning, resulting in mechanistic learning, and the memorization and learning of isolated concepts and procedures. A further effect of transmission teaching is the perceived lack of transfer, poor achievement and under-performance. The entire teaching and learning process remains teacher-directed and regulated and metacognitive strategies and student self-regulation remain underdeveloped. Therefore, the broad research problem may be formulated as: What is the effect of problem-based teaching and learning on student performance and achievement in mathematics?

In order to obtain answers to the above-mentioned research problem the following research questions were formulated:

- How will the introduction of PBTL in the mathematics classroom affect the overall performance and achievement of students?
- What will the effect of PBTL be on the ability of students to identify, formulate and solve problems, and to evaluate results?
- To what extent will the introduction of PBTL affect student self-regulation and meta-cognitive strategies?
- What will the overall effect of PBTL be on students' motivation and self-confidence?

1.3 Aims of the research

The aim of this study is to assess the effect of the introduction of PBTL into the traditional, transmission-based mathematics classroom. Having this general objective, the specific objectives are:

- to investigate the effect of PBTL on mathematical performance and achievement;
- to determine the effect of PBTL on the ability of students to identify, formulate, and solve problems and evaluate results;
- to indicate the extent to which PBTL will affect student self-regulation and meta-cognition; and
- to test the effect of PBTL on students' motivation and self-confidence.

1.4. Operational definitions

In this study,

- ***Problem-based teaching and learning*** refers to a classroom situation in which students are expected to be active and creative individuals with the will and ability to seek knowledge and self-development.
- ***Achievement in mathematics learning*** refers to students' successful accomplishment of learning goals.
- ***Performance in mathematics*** refers to the number of marks a student scores out of 130 in a mathematics test, and out of 50 for each of the procedural and conceptual questions in the 'reference test', after the delivery of an intervention programme.

1.5. Research design

To observe the effect of problem-based teaching and learning (PBTL) of mathematics, first of all, a literature study was undertaken. By consulting a wide variety of sources relevant to this study, theoretical knowledge was gained on the influence of PBTL on the learning of mathematics. This was followed by an empirical investigation.

In the empirical investigation an educational experiment was conducted. Experimental research was considered an appropriate mechanism to verify the following hypothesis that was formulated from the research problem:

- The introduction of problem-based teaching and learning (PBTL) into the traditional mathematics classroom will result in improved student performance and achievement in mathematics.

An intervention programme was designed in which problem-based teaching and learning (PBTL) was implemented and practised. The experimental group as well as the control group consisted of 50 grade ten students. Both groups were enrolled at Ayider High School during the 2006/2007 academic year. These students were used as subjects because they form a representative sample of the students in public schools in Ethiopia. After the intervention period a comparison was made between the performances and achievements of the two groups of students, namely, the experimental group on the one hand, and the control group on the other.

Various techniques were used to analyze the data, including pre-test/post-test results, students' self-assessments and teacher assessments. The SPSS statistical package was, *inter alia*, utilised in this regard.

1.6. Chapter division

The final dissertation report is organized as follows:

Chapter 1*:* Introduction and orientation.

Chapter 2*:* Literature survey – the nature and possibilities of PBTL.

Chapter 3*:* Research design and research instruments.

Chapter 4*:* The processing of the data.

Chapter 5*:* Summary, conclusions and recommendations.

CHAPTER TWO

THE NATURE AND POSSIBILITIES OF PROBLEM BASED TEACHING AND LEARNING

2.1. Introduction

Problem-based teaching and learning (PBTL) has been a point of discussion for a long time; it is nothing new. For example, as far back as 1971, Freudenthal advocated reinvention of mathematical ideas by posing problems as opposed to teacher-telling. Over the last two decades as well, research publications have provided evidence of the benefits of this approach (e.g., Cobb *et al.,* 1991:25; Harris *et al.,* 2001:310; Hiebert *et al.,* 1999:152-153;Murray *et al*., 1998:270; NCTM, 2000:3).

Instruction by means of problem-based teaching and learning has the advantage of determining what students know and understand, and then selecting and posing tasks or problems that will help them develop the desired new knowledge (Fendel, Resick, Fraser, & Alper, 1997:11; Lappan & Briars: in Van de Walle, 1998:13). This could, therefore, be contrasted to the traditional view of mathematics which tenaciously advocates mathematics as consisting of a fixed set of rules and procedures (Hoffmann, 2001:2). Problem-based instructional strategies focus on helping students to 'figure out' the process of obtaining rules and procedures by means of their own endeavour and enthusiasm and not to rely on mere memorization to decide which rule or procedure to use (Cangelosi, 1996:51-52; Van de Walle, 1998:13).

However, there are potential downsides to PBTL. One is the belief held by students about mathematics (Emenaker, 1996:75; Hart, 2002:9; Hoffmann, 2001:2; Schoenfeld, 1992:358-359; Wong, Marton, Wong, & Larn, 2002:2). Another is the influence of the traditional formal transmission teaching on students throughout their schooling. To overcome these challenges, a study was undertaken of the literature and research reports related to PBTL. PBTL is in agreement with the constructivist view of learning (Felder & Brent, 2002:5; Leu, 2002:22), and comprises of a number of learning principles. In this chapter, the researcher is going to review some of the theoretical foundations that PBTL is based on. In the discussion the researcher will focus mainly on the concept that PBTL is inclusive of several specific instructional strategies.

Following this, PBTL will be contrasted to the traditional formal transmission teaching. The key to PBTL, namely the social nature of learning, will be discussed at some length. Then an analysis of the principles of the curriculum, together with the development of the syllabus in relation to PBTL, will follow. In PBTL, instructional methods and strategies effective to the context also merit discussion. Finding a mechanism to follow up on students' ongoing progress is extremely important. This will also be dealt with under the heading 'authentic assessment'. Finally, it will be necessary to take an in-depth look at some examples as manifested in a real-life classroom.

2.2 Theoretical frameworks as a foundation for practice

PBTL was based on the cognitive constructivists' view that a student reorganizes experiences to resolve a new problem situation (Ernest, 1996:346-347; Rogoff, 1999:79; Von Glasersfeld, 1996:312), and also on the social constructivist view that an individual's knowledge and meaning construction is shared within a community of learners (Vygotsky, 1978:86). Thus, the teaching material was designed in an activity-based manner that was potentially problematic to learners at a variety of conceptual levels.

Problems in the activities ranged from very easy to problems that call for more detailed and sophisticated thinking. Both routine and non-routine problems were presented in the context of the teaching material. These problems were designed to help students learn concepts and skills in both algebra and geometry. Furthermore, the questions in each of the five chapters of the teaching material were organized in ways that would help to develop both procedural and conceptual knowledge (Hiebert & Carpenter, 1992:77-78). Apparently it is fundamentally based on Polya's (1988: xvi-xvii) classification of problems from a pedagogical perspective and Bloom's (1984) *Taxonomy of Educational Objectives* applied to questions in mathematics, ranging from the lowest level (knowledge) to the highest level (evaluation) (Brumbaugh, Ashe, Ashe, & Rock, 1997:74-76; Cangelosi, 1996:73). Polya (1988) classifies questions into four major types according to increasing difficulty or educational value. This includes, *one rule under the nose*; *application with some choice*; *choice of a combination*; and *research level.* These types of questions ranged from requiring the simple mechanical application of a rule in the first type, to a novel combination of rules or examples and plausible reasoning in the fourth type. Polya (1988) argues that both the degree of difficulty and the educational value in relation to teaching students to think increase as one goes from type 1 to type 4.

Bloom's taxonomy applied to mathematics is worth mentioning. Questioning skills are central to the repertoire of effective teaching and meaningful learning. Questions can be categorized by type and level. For instance, a distinction could be made between 'open' and 'closed' questions. Unlike 'closed' questions which usually have one correct answer, 'open' questions can have a number of right answers. Another essential distinction can be made between 'higher order' questions and 'lower order' questions. Higher order questions call for reasoning, problem-solving, analysis and evaluation. Lower order questions, on the other hand, involve simple recall or comprehension (Brumbaugh *et al.*, 1997; Leu, 2002:30; NCTM, 1989:23; Van den Berg, 2004:285). A bulk of the literature indicates that teachers overwhelmingly ask more 'closed' and lower order questions than 'open' and higher order questions (Brumbaugh *et al.*, 1997:80; Cangelosi, 1996:77-78; Kyriacou, 1998:35).

During the intervention programme, students were considered as active beings who were able to construct their own knowledge by interacting between their thinking and experience. This is acknowledged in Piaget's accounts, namely that when learners encounter a new experience they both 'accommodate' their existing thinking to it and 'assimilate' aspects of the experience. For Piaget (Cathcart, Pothier, Vance, & Bezuk, 2001:23-25), learning is based on three types of knowledge: (1) physical (obtained from concrete experience), (2) social (obtained from interaction), and (3) logical (obtained through reflection and abstraction).

The theory advanced by Vygotsky that concerns the development not only of individual knowledge and meaning but also the shared meanings within a community, is equally valuable. In this approach, the instructional activities are developed in a manner that encourages learners to start from their experiential learning of mathematics rather than from their formal mathematics learning (Skemp, 1971:32). PBTL-based curricula are guided by the application of particular instructional strategies, motivational problems, clear goals, expectations, assessment, and support for the participants. In this respect, some educational materials were used as resources in the development of the teaching material in an activity-based form (*e.g.*, Cangelosi, 1996; Coxford, Usiskin, & Hirschhorn, 1991; Dietiker, 1997a, 1997b; Dolan & Williamson, 1990; Dolan, Williamson, & Muri, 1997; Hoey, & Wotton, 1994a, 1994b; Martin, 1995; Posamentier & Stepelman, 1996; Sobel & Maletsky, 1988; Swanson, Swenson, Musser, & Burger, 1988).

The PBTL approach was based on both the Piagetian and Vygotskian theories. This is because the researcher believes that neither of these theories could be complete and self-sufficient for every context in the instructional process. In response to this, Cobb (1999:140) argues that mathematical learning must be seen as both a process of active individual construction and of enculturation into the mathematical practice of the wider society. Furthermore, Murray *et al.*, (1998:270) maintain that the construction of mathematical knowledge is in the first place an individual, and in the second place, a social activity. Social interaction encourages reflective thinking which constructivists firmly believe is the source of knowledge at all levels. In this regard, criticisms toward the Piagetian theory from two directions are worth mentioning:

- research reports assure that children's intellectual abilities are far greater than those reported by Piaget's intellectual development; and
- the over-emphasis of the Piagetian approach to self-discovery learning, while at the same time ignoring the social context in which learning takes place (Pollard, 2002:139).

The Vygotskian theory can contribute to alleviate the deficiencies in the above-mentioned theory. This theory is based on two fundamental themes. They are:

Theme 1: The language and forms of understanding that are embedded in particular contexts and social practices are important 'resources' that are available to a learner from the outset (Vygotsky, 1987:110-111).

Theme 2: Experienced/competent participants can play the role of inducting less-competent learners, and in 'mediating', 'scaffolding' and extending their understanding (Vygotsky, 1978:86).

In a problematic situation, the themes in the Vygotskian approach have the following bearings on student thinking:

1. Ideas, language, and concepts derived from interaction with others could structure, challenge, enhance or constrain thinking.
2. Teachers must engage with students' existing cultural and conceptual understandings (and misunderstandings) before attempting further instruction.

3. Initial understanding is essential for later concept construction. Students who fail to engage in initial understanding may fail to grasp new information and concepts, or may learn merely for tests, which hamper their capacity to transfer their learning to new situations (*National Research Council*, 1999:25).

It should be noted that culture and the social context of the student influence his/her understanding (Bruner, 1995: 332; Boaler, 1993:342). Thus, when students experience this culture and context, they assimilate particular cognitive skills, strategies, knowledge and understanding (Dunn, 1988).

To sum up, in implementing a constructivist view in classroom mathematics instruction, the following description of the learning cycle becomes pertinent (Heddens and Speer, 1995:10). This cycle provides both teacher and student with the opportunity to: (1) *engage* and access prior knowledge; (2) *investigate* problems via ‘’hands-on’’ and ‘’minds-on’’ activities; (3) *develop*, clarify, and construct meaningful explanations; (4) *expand* and apply new understandings; and (5) use authentic assessment to *judge* the new understandings.

2.3 Defining problem-based teaching and learning (PBTL)

According to Anthony (1997:365), for the constructivists’ view to be successful, active learning should be operationalized by cognitive, meta-cognitive, affective and resource management. Recent research findings provide evidence that the perception of a student has changed from that of a passive recipient to that of an active constructor of knowledge (Silberman, 1996: ix; Thompson & Maguire, 2003:9, 23).

Current learning perspectives incorporate three important assumptions:

- learning is a process of knowledge construction, not of knowledge recording or absorption;
- learning is knowledge-dependent; people use their current knowledge to construct new knowledge;
- the student is aware of the processes of cognition and can control and regulate them; this is self-awareness, or meta-cognition (Anthony, 1997:349).

The learning processes of students are strongly influenced by their meta-cognitive knowledge (Flavell, 1987:232) and the interpretation of their learning environment (Anthony, 1997:350). To that end, self-regulation and active learning cannot be seen in isolation and are highly integrated elements of learning. To encourage learning through active involvement, instructional methods and strategies play a key role. PBTL ranks as one of the best approaches with new views and possibilities in mathematics teaching and learning, advocated by several research works and documents (*e.g.*, Cobb et al., 1991; NCTM, 1989, 1991, 2000). The formal originator of this idea, namely constructivism as problem-centered approach to learning, is John Dewey (Bereiter, 1992:348).

Dewey (in Hiebert *et al.*, 1999:156) identified three fundamental features: (1) problems are identified (which include defining the problem); (2) resolutions are sought (problems are studied through active engagement via the collecting of data, formulating a hypothesis, interacting with the problem and observing the results); and (3) conclusions are reached when problems are partially or totally resolved.

Current definitions of PBTL are compatible with Dewey's definition. PBTL also seems to have a strong link with the genetic approach advocated by well-known mathematicians such as Wittman, Polya and Freudenthal who give due attention to the retracing of original work by discoverers or inventors (De Villiers, 2003:13). Problem-based teaching and learning (PBTL) is, therefore, the type of classroom situation needed to support a constructivist approach to teaching-learning. Leu (2002:22) states that problem-based teaching and learning derives from the conviction that the student is an active and creative individual with the will and ability to seek knowledge and self-development. The main purpose of PBTL is to make learning innovative, lifelong and retainable, providing the student with the capacity to

- understand and control his/her own knowledge;
- develop a problem-solving ability;
- develop self-knowledge and self-confidence; and
- broaden his/her understanding of concepts.

Leu (2002:22) further argues that PBTL encourages learning through a form of scientific method, namely identifying a problem and searching for a possible solution. In its most explicit form PBTL involves all, or usually some combination of the following steps. The teacher presents a problem or scenario and asks the students, in small groups, to

- write a problem-definition or statement;
- build hypotheses or make conjectures to initiate a resolution process;
- list what is known, what needs to be known, and what needs to be done to solve the problem (update periodically);
- generate possible solutions and identify the best one; and
- complete the best solution and define it *(*Felder & Brent, 2002:5; Leu, 2002:22).

During the formulation of the problem, teachers should be certain that the problem selected is relevant and appropriate to the area of study and that it is understandable. In formulating the problem, the teacher should support the students in eliminating that which is not tenable before they begin to work. A hypothesis could be developed based on available data. During this time students are not given any information; rather, they acquire the information on their own. After the data have been interpreted, students communicate openly or discuss whether to accept, modify or reject the hypothesis, by means of which students develop their reasoning skills, such as deductive and inductive reasoning, which could be construct-a-concept or discover-a-relationship (Cangelosi, 1996: 86-112; Leu, 2002:22).

Accordingly, PBTL is in a sharp contrast to the traditional lecture method which was employed in mathematics for a long time in the history of mathematics education. The latter could essentially be characterized by a two-step process:

1. the teacher uses examples to show how to solve a particular type of problem or how to do an assignment; and
2. the students routinely imitate the given procedure mechanically to find answers to a great number of similar examples (Heddens & Speer, 1995:7).

Currently, the attention seems to shift toward mathematics programmes that promote an active learning process, that is, in which the student is mentally, physically, and emotionally involved. The outcome of such an active learning process is empowering in respect of reasoning, experimenting with alternative techniques, focusing on the process rather than the product, making use of learned material in new situations, and gaining confidence by making own discoveries instead of trying to recall memorized rules.

However, Mason (1994:80) argues that students' success in realizing given aims will depend not only on the teaching style adopted, but also on a host of other factors that one should be aware of.

These include, a predisposition to involvement in problematic questions; the general peer attitude to learning; the teacher's attitude to learning and teaching, his/her interest, and commitment; student ability with automated skills; the extent to which the students' own initiatives are employed in the learning process; and the extent to which the students share the teacher's goals. The benefit of this approach is that a classroom environment conducive to learning is created in which students can:

- wrestle with new information and experiences in seeking meaning that results in meaningful learning;
- utilize past experiences to confront new challenges; and
- build new meanings through personal effort, giving due attention to individualized learning, learning preference and pace of learning.

Through interaction, involvement and socialization, students need to understand both the risks and the rewards of searching for new knowledge and understanding. Students who pass these processes successfully will end up with "…the potential residue to learn important mathematics", referred to by Hiebert *et al.*, (1999:161). They posit three theoretical foundations to deal with and describe the notion of a mathematical residue, namely:

(1) Dewey's idea that knowledge is the fruit of activity that resolves problematic situations;
(2) Brownell's observation that understanding is rather seen as a by-product of activity than a direct target of instruction; and
(3) Davis' formulation that understanding is a residue that gets left behind when students solve problems. The 'residue' helps students to demonstrate understanding after an activity is over.

For Hiebert *et al.*, (1999: 161-163), three kinds of residues are available as a result of engaging in a mathematical task. The first is *insight* into the structure of the subject matter. The second is *strategies* for solving problems and the third is *disposition* toward mathematics. However, the nature of residue will depend on entry behavior and the nature of the problem. A residue that is worthwhile is likely to occur when activities focus on broad concepts and contexts. To ensure this, Brown, Hewitt and Mason (1994:86) suggest that mathematics at all levels should include opportunities for:

- exposition by the teacher;
- discussion between the teacher and the students, and among the students themselves;
- appropriate practical work;
- consideration and the practice of fundamental skills and routines;
- problem solving, including the application of mathematics to everyday situations; and
- investigational work.

2.4 The social nature of mathematics learning

Recently educationists have claimed that one of the shortcomings of the traditional form of classroom instruction is that it gives virtually no attention to students' interaction. Clearly, learning is enhanced through communication and meaningful discourse (Goulding, 2004:53-54; Knuth, 2001; NCTM, 2000). Acquiring understanding and mastery in learning could be promoted when students discuss their ideas with their fellow-students and when they and their peers are involved in the teaching activity. This 'involvement' is at the centre of social learning. The relevance of this statement is echoed by the following quotation from Gardner (1999:99):

> "In several cases it is erroneous to conclude that the Knowledge required to execute a task resides completely in the mind of a single individual. …that is, the successful performance of a task may depend upon a team of individuals, no single of whom possesses all of the necessary expertise but all of whom working together are able to accomplish the task in a reliable way."

This argument is strongly supported from many perspectives. Mathematics is considered to be a product of a social process. In relation to this, Lakatos described how mathematical concepts are stabilized or changed through a process as a result of social interaction (Voigt, 1996:23). This philosophical point of view rejects the conception that the truths of mathematical statements are absolute. Negotiation of meaning and ideas through communication is central to teaching and learning in a socio-cultural approach. It is also believed that it has a positive impact on students' understanding and their potential for engaging in explaining and justifying their thinking (Steele, 2001:404). In accordance with this, Kilpatrick (1985:10) describes that the mathematics classroom is a social situation jointly constructed by the participants, in which teacher and students interpret each other's actions and intentions in the light of their own agendas. More explicitly, Hiebert and colleagues (1998) assert that the central ingredient of relational understanding is communication. "Communication involves talking, listening, writing, demonstrating, watching . . . participating in social interaction, sharing thoughts with others, and listening to others share their ideas" (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1998:5). Communication has the power of making mathematical ideas objects of reflection, refinement, discussion, and amendment (*Standards and Principles for School Mathematics*, [NCTM], 2000:60). Solomon (in Voigt, 1996:28) argues cognitive development is the "progressive socialization of the child's judgment" and learning in mathematics lessons is the initiation into social interaction. Furthermore, classrooms as mathematical communities should engage students in generating mathematical ideas, and in validating and modifying these ideas by means of inductive and deductive reasoning (Cangelosi, 1996:8).

In summary, the classroom is a place of social beings. Thus, the doing of mathematics becomes a social activity (Bell, 1993:20). It can furthermore be seen as a social activity with roots in the cultural and societal environment (Schoenfeld, 1992:340). Social interaction as the theory of 'teaching and learning' has essential practical applications. Therefore, social interaction serves, at least, the following purposes in PBTL classrooms:

(1) the negotiation of mathematical meanings and the sharing of essential ideas;
(2) the fostering of positive relationships between the teacher and the students, and among the students themselves; and
(3) the development of a team spirit and team-work.

2.5. Curriculum principles and syllabus development

2.5.1 Describing underlying principles

The problem-based teaching and learning (PBTL) lesson revolves around a central problem or theme. PBTL is the type of classroom organization needed to support a constructivist approach to teaching and learning (Felder & Brent, 2002:5). A well-planned PBTL constructivist lesson has three fundamental components: (1) a task, a situation, or problem presented to the students for consideration; (2) an opportunity for students to work on the task; and (3) an opportunity for discussion and reflection on the work done by the students after accomplishing the task (Van de Walle 1998:483). Besides this, the model regards the many assumptions about knowledge, learning, teaching, learner, teacher, curriculum, expectations, the nature of knowledge, and the roles of the participants and the characteristics of classroom activities. The description of each aspect below is based on the works of Huetinck and Munshin (2000), Pollard (2002), NCTM (2000), and others.

1. ***Knowledge*** - a changing body of knowledge constructed individually and socially. It is built on what the student produces and what the participants contribute (Huetinck & Munshin, 2000:57).

2. ***Learning*** - the active construction of prior knowledge and the collaborative construction of socially defined knowledge and values. It happens by means of multiple opportunities, diverse processes, and socially constructed opportunities, to connect to what is already known. Furthermore, it can come about through independent teaching or the interdependence of the teacher and the students. In this context learning involves learning with understanding, actively building new knowledge from experience and prior knowledge (Ernest, 1996:347; Huetinck & Munshin, 2000:57; NCTM, 2000:20-21; Pollard 2002:145).

3. ***Teaching*** - is considered to be challenging students, guiding thinking towards a more complete understanding. Teaching plays the role of providing an opportunity for the student to gradually construct knowledge and skills through experience, interaction, and adult support. It also includes understanding what students ought to know and need to learn (Huetinck & Munshin, 2000:57; Pollard, 2002:145; NCTM, 2000: 16-19).

4. ***The role of the teacher*** - The teacher has three central roles: (a) organizer of the learning environment, (b) assessor of students' thinking; and (c) initiator of group activities. In the first place the teacher acts as a facilitator, a guide, and a co-participant. In the second place the teacher plays the role of active listener. He listens to students' conceptions, their ideas, individually and socially, and co-constructs different interpretations of knowledge (Huetinck & Munshin, 2000:57; Marsh, 2004:43; Pollard, 2002:145).

5. ***The role of the student*** - The student plays the role of active constructor, with others and with him-/herself. The student is thought of as an active thinker, explainer, interpreter, questioner, and as working as an individual or active social participant. He/she is motivated intrinsically and socially to construct his/her own knowledge (Huetinck & Munshin, 2000:57; Marsh, 2004:43; Pollard, 2002: 145).

6. ***The role of peers*** - Peers are believed to stimulate thinking by raising questions, and they are part of knowledge construction (Huetinck & Munshin, 2000:57).

7. ***Characteristics of student activity*** - The activities involve a class, group or individual discussions with an adult or other peers. Tasks should be designed to encourage group problem-solving (Hopkins, 2002:43; Pollard 2002:145).

8. ***Equity principle -*** Mathematics learning calls for high expectations from and strong support for all students. The curriculum is designed for heterogeneous classes. In PBTL almost everyone is expected to gain a deep understanding of the curriculum and to make valuable contributions as member of a learning group (NCTM, 2000: 12-14).

9. ***Curriculum***. The curriculum is built around problems which are context-rich and interesting. The focus is on broad principles and investigation with less focus on mechanical skills. In this case, "…a curriculum is more than a collection of activities" (NCTM, 2000:14-16). Students are encouraged to work independently and cooperatively.

10. ***Assessment*** - To support the learning of mathematics authentic assessment furnishes useful information to the programme stakeholders (NCTM, 2000:22-24).

11. ***Summary ideas*** - The approach encourages collaboration and language development on the one hand, and on the other, challenges are restructured to clarify thinking and extend meaningful understanding. Students make use of their learning experiences and explore in their own way and at their own pace, a process through which confidence and understanding are built.

To sum up, the process that models a problem-based teaching and learning experience for students describe the following actions (Davis, 1996:291-293; Felder & Brent, 2002:5; Harris *et al.,* 2001:311; Hopkins, 2002:158-161; Leu, 2002: 22; Savoie & Hughes, 2006), namely to

- identify a problem suitable for the students;
- connect the problem to the context of the students' world so that it presents authentic opportunities;
- organize the subject matter around the problem, not the discipline;
- give students the responsibility to define their learning experiences and planning to solve the problem;
- encourage collaboration by creating learning teams; and

- expect all students to demonstrate the results of their performance.

2.5.2. Curriculum material supporting PBTL

Researchers like Davis (1992:226-228) and Wong *et al.,* (2002:9) indicated that the traditional view of the teaching and learning of mathematics has had a negative impact on student performance and achievement. Mathematics was thought of as a set of rote procedures for symbol manipulation in a certain predetermined way. Consequently, in the eye of students, mathematics was regarded as a body of absolute truth (Fleener, 1996), associated with certainty (Lampert, 1990) and viewed as a set of rules with symbols for playing around (Kloosterman, 1991). In this regard, Lampert (1990:32) claims that students' views about mathematics are ill-conceived in many ways. For example, *doing* mathematics means following rules laid down by the teacher and a textbook, *knowing* is associated with remembering and applying the correct rule and getting the answer quickly, and *mathematical truth* is determined when the answer is ratified by the teacher. The traditional school experience together with the associated teaching practice and the curriculum in use are, at least in part, to blame for these misconceptions. In the end, this type of traditional practice provides students with little opportunity for making sense out of mathematics (Cathcart et al., 2001: 388; Dougherty & Wilson, 1993:295).

Several research studies repeatedly commented on the above approach by proposing new possibilities in curriculum as well as in instructional methods (*e.g.,* Cobb, *et al.* 1991, Lampert, 1988; Murray, *et al.*, 1998). These research reports are based on the recommendations and vision set by the documents of the *National Council of the Teachers of Mathematics* on different occasions (NCTM, 1989:246, 1991:35-36, 2000:3).

PBTL is strongly supported by cognitive theories (Harris *et al*., 2001:310). It has the potential to provide teachers with rich contexts of appropriate instructional approaches. Research on teaching and learning for discovery, as well as the constructivist view on mathematics learning, provides strong support for the argument that students learn well when they are actively involved in the search for understanding. The NCTM standards proposed fundamental shifts and changes in the traditional teaching approach (teacher *telling* and students *listening)* and suggested a coherent, focused, and well-articulated curriculum (Harris *et al*., 2001:311). Keegan (1995:3), amongst others, suggests a judicious appropriate mixture of the didactic, discovery and enquiry methods relevant for each particular teaching and learning context.

The emerging view proposes that students learn important aspects of mathematics more effectively when they receive the opportunity to use methods other than the traditional ‘learning-by-example’ didactic method. In this respect Davis (1996:292) proposes a mathematical classroom situation that will call for students spending their time solving novel problems. After the students have invented solutions, they could be given the opportunity to reflect on what they have just done. Once students have developed their mathematical power through such innovative, reflective approaches, there should also be a classroom environment where they will be encouraged to pose additional problems, probably extending or modifying the original problem that they have just solved or posing their own problems (Fendel *et al*., 1997:11). According to Davis (1996:293), such an approach originally resulted in the historical invention of mathematical methods and procedures (*e.g*., Fourier Functions) in response to specific practical problems, methods of which the power have later become recognized as more generally applicable.

In contrast to “mathematics as something you are taught first and apply second”, the NCTM in its *Principles and Standards for School Mathematics* (NCTM, 2000:3) suggested a new vision for school mathematics:

> “The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding . . . . Students confidently engage in complex mathematical tasks . . . . They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to conform or disprove these conjectures.”

The document gives due attention to teacher preparation and empowerment for effective teaching. It outlines three tenets for effective teaching:

(1) knowledge and understanding of mathematics, of students as learners, and pedagogical strategies;
(2) a challenging and supportive classroom learning environment; and
(3) a continual search for improvement (NCTM, 2000:17-19).

This document also asserts that the relationships among teachers, students, and mathematical ideas will develop appropriately where there is a fundamental change in teacher and student conceptions of their roles in the classroom. Curriculum materials that enhance student learning-by-doing mathematics as opposed to the traditional curriculum, which encourage much information-gathering and 'learning-by-example', need to be articulated. Currently, many documents and research-based resources are available that can assist educators in decision-making in their efforts to transform mathematics education. The NCTM (2000:55-56) suggested that high-quality mathematics education should feature a coherent curriculum in which mathematical ideas are integrated and built on one another in ascending complexity. In an engaging curriculum, mathematics ideas will focus on central themes that will allow students to deepen their knowledge and understanding which will in turn, enable them to apply mathematical ideas in context-rich situations.

To achieve the visions and shifts recommended in mathematics is to employ appropriate instructional methods and context-rich tasks that create opportunities for mathematics learning (Cangelosi, 1996: 86-112; Davis, 1996:292; Harris *et al.*, 2001:310) (Table 2.1 illustrate the case).

Table 2.1: Blends of instructional strategies and the nature of good tasks

Blend of instructional strategies	***Nature of good tasks***
• *Motivational strategies* • *Group-work* • *Cooperative/collaborative work* • *Inductive and deductive reasoning* • *Discovery method* • *Inquiry method* • *Problem solving skills* • *Reflective practices and metacognitive skills* • *Authentic fair and reliable assessment* • *Activities that call for more student participation* • *The use of wait -time for critical thinking*	• *Present problems or puzzles that interest students.* • *Help teachers get an insight into student prior knowledge.* • *Provide appropriate levels of challenge and support for students.* • *Encourage students to collaborate in resolving difficulties.* • *Lead students to the discovery of important concepts and problem solving techniques.*

Source: Harris *et al*, (2002:310).

The use of these instructional methods and the employment of context-rich tasks require teachers to have a thorough understanding why a particular instructional strategy is relevant for a particular task and for the mathematics embedded in the problems, and to be able to pose insightful questions to guide student thinking (Harris *et al.,* 2001:316). Problematic situations will help students to devise different solution strategies and explanations which must be acknowledged from the instructional point of view.

Teaching in such a situation calls for reflective thinking, that is, thinking thoroughly about the tasks students are engaged in, the nature of the questions and the questioning techniques employed, and also, thinking about the ways students use information and manage resources to solve specific mathematical problems.

2.6. Methods of instruction and strategies effective to the context

2.6.1 Overview

The distinction between rote learning and learning with understanding could be related to the type of instructional strategy employed (Black, 1999:120). Learning with understanding is dependent on teaching for understanding and rote learning is related to transmission teaching. Teaching is effective when it results in important learning by students (Hopkins, 2002:146-148). Learning is important only when students can use the outcomes, when they can make sense of it in relation to their daily lives, or when it contributes to their learning (Leu, 2002:9). According to the *Principles and Standards for School Mathematics* (NCTM, 2000:20-21), conceptual understanding is an important component of proficiency, and learning with understanding as an essential ingredient to enable students to solve the new kinds of problems they will inevitably face in the future.

However, these ideas call for appropriate instructional strategies and curriculum materials that fit the context. A key to achieving the above objectives is to promote reflective thinking, to communicate effectively about tasks, and to reflect on the process of learning (Van de Walle, 1998:494). Moreover, students should be taught mathematics by means of problem-solving, and should be taught how to make use of information from their environment and from other sources

(Davis, 1992:237). Creating a mathematical environment, posing worthwhile mathematical tasks, employing cooperative learning groups, using models and calculators as thinking tools, encouraging discourse and writing, requiring students to justify their responses and listening actively, could play an important role in achieving the above goals (Van de Walle, 1998:34). This will have the effect of upgrading students' potential to investigate, to understand the world around them, to analyze, to draw conclusions and to communicate effectively (Brown, Hewitt & Mason, 1994:82-83; Webb & Welsch, 1993:303).

This notion is supported from many perspectives. For example, the realm of meaningful learning should be based on the 'doing' of mathematics (Dossey, 1992:44); applying 'meaning method' as opposed to a 'mechanical method' (Brownell: in Kloosterman & Gainey, 1993:5); teaching with meaning as opposed to rote teaching (Skemp 1976:23); and problematizing mathematics and enhancing reflective thinking (Hiebert *et al*., 1999:156-158). To the contrary, the traditional model which is characterized by 'talking and telling', encourages memorization of facts, rules, and other mathematical concepts which does not result in long-lasting understanding (Cruikshank & Sheffield, 1988:8).

2.6.2 Cooperative learning groups

Cooperative learning is based on the presence of student peers. It also encourages student-to-student interaction, and establishes a systematic relationship among team members, and merits benefit for both the students and the teacher (Huetinck & Munshin, 2000:15). However, a simple collection of individuals may not constitute a cooperative learning group (Posamentier & Stepelman, 1996:4). Although there are a range of strategies that a cooperative group teaching model comprises of, they could be underpinned by the following principles:

1. *Positive interdependence*. This is related with feeling connectedness in the accomplishment of a common goal, and the success of every individual for the success of the group.
2. *Individual accountability*. This is closely connected with holding every member of the group responsible for learning.
3. *Face-to-face interaction*. This is getting in close proximity to members of the group which results in dialogue that promotes continued progress.

4. *Social skills*. This includes skills that enable groups to function effectively (*e.g.,* taking turns, encouraging, listening, giving help, clarifying, checking, understanding, probing). Such skills enhance communication, trust, leadership, decision-making, and conflict-management.
5. *Processing*. This focuses on assessing the group's collaborative efforts, and targeting improvement.
6. *Heterogeneous groups*. This refers to including students of genders, social backgrounds, social skills and physical attributes (Fitzgerald & Bouck, 1993:251; Hopkins, 2002:158; Thornton & Wilson, 1993:282).

Recent works also indicate that students in cooperative learning groups will be more effective when they have developed desirable group behaviors, when the teacher employs group interaction methods that enhance active learning, and when they work with less group conflict by setting group contracts (ground rules) (Hillier, 2002:173-174; Hopkins, 2002:157;MoE, 2006:97-98; 103-104; Rhodes, Stokes, & Hampton, 2004:63-73). Working in cooperative groups could be organized into distinct phases, and in teacher behavior as well (Arends, 2000:332; Basic Educational Support for Tigray [BEST], 2002). Table 2.2 illustrates this last aspect.

Table 2.2: The phases of teacher behavior in cooperative learning activities

Phase	***Teacher behavior***
Phase 1*: Provides objectives*	• *Teacher goes over objectives for the lesson and establishes learning set.*
Phase 2*: Presents information*	• *Teacher presents information to students either through verbal presentation or with text readings.*
Phase 3*: Organizes students in learning teams*	• *Teacher explains to students how to form learning teams; helps groups make efficient transition.*
Phase 4*: Tests*	• *Teachers tests knowledge of learning materials; organizes groups, presents results of their work*
Phase 5*: Recognizes achievement*	• *Teacher finds ways to recognize both individual and group-effort and achievement.*

Source: *Adapted from* Basic Educational Support for Tigray (BEST, 2002)

In summary, cooperative learning serves the following purposes in PBTL classrooms, namely it

- improves the learning of academic content and promotes higher level thinking skills;
- provides students with the opportunity to think logically and creatively;
- fosters students' achievement and improves their ability solve mathematical problems;
- improves students' strategies for acquiring information;

- develops personal and social skills;
- boosts students' self-esteem;
- improves students' ability to work with others during learning;
- helps students experience self-reliance;
- increases gender relations; and
- allows students' own decision-making (Artzt & Armour-Thomas:137; Hopkins, 2002:155-156; Marsh, 2004:46; McGlinn, 1991:14;).

2.6.3 Problem-solving and meta-cognition in the learning of mathematics

Recent works propose "…problem solving as an integral part of all mathematics programmes" (NCTM, 1989:23, 2000:52; Goulding, 2004:5). In fact, the formal proposal of problem-solving in mathematics learning dates back to the 1940s, and much is credited to George Polya (Masingila, Lester, & Raymond, 2002:19). Polya proposed a four-step problem-solving process with identifiable strategies. They are: (1) *understanding the problem,* which calls for familiarization with the problem; (2) *design a plan,* which involves the identification of strategies applicable to the context*;* (3) *carry out the plan,* which is closely related to the implementation of the particular strategy or strategies in solving the problem; and (4) *look back*, which involves the reviewing of the original problem and the process, and the generation of new ideas in extending and improving the original problem.

Several scholars propose the learning of mathematics through problem-solving. For example, Polya (in Kroll & Miller, 1993:58) declared that the first duty of a teacher of mathematics is to help his/her students develop problem solving abilities. Cockroft, (1994:50) also claims that the primary focus at all levels of learning mathematics must be on problem-solving. Thus, all meaningful learning is achieved by means of problem-solving (Fehr & Phillips, 1967:375).

However, Schoenfeld, (1992:348), claims that besides the knowledge of mathematical ideas, three other aspects of cognition are essential for students to become 'good' problem-solvers. These include: (1) problem-solving strategies; (2) meta-cognitive processes; and (3) beliefs and attitudes. To that end, Polya's problem-solving strategies prompt meta-cognitive abilities in mathematics learning (Kilpatrick, 1985:10). "Meta-cognition refers to conscious monitoring (being aware of how and why you are doing something) and *regulation* (choosing to do something or deciding to change your own thought processes)" (Van de Walle, 1998:51). Furthermore, White and Mitchell (1994:26) argue that training in meta-cognition improves students' control over their learning. For them, "…meta-cognition is the opposite of a lack of control: knowledge of the processes of thinking and learning, awareness of one's own, and the management of them". In relation to regulating one's own learning like goal-setting, self-reinforcement, self-recording, and self-instruction, much research has been done on its theoretical foundations and its impact on academic achievement (see Zimmerman, 1989:1-25). To this end, Olaussen and Braten (1999:410) argue that:

> "Self-regulated learners are purposeful, strategic and persistent in their learning. They generate and direct their own learning experiences rather than act in response to external control. Self-regulated learners also possess the ability to evaluate their own progress in relation to the goals they have set and to adjust their subsequent behavior in the light of these self-evaluations."

Black (1999:122) observed that low-achieving students lack self-regulation skills, and when trained, demonstrate rapid progress.

A summary of the meta-cognitive processes in mathematics learning, and in particular in problem-solving is given in Table 2.3 below.

Table 2.3: Metacognitive processes in mathematics learning

Meta-cognitive processes	***Description of the component elements***
Management aspect	*Means (a) making sure that you understand what a problem is all about before you hastily attempt a solution; (b) planning; (c) monitoring, or keeping track of how well things are going during a solution; (d) allocating resources, or deciding what to do, and for how long, whilst working on the problem* (Schoenfeld, 1987).
Meta-cognitive components	*Involve planning, evaluating, and monitoring problem-solving activities* (Van de Walle, 1998)
Meta-cognitive strategies	*This calls for connecting new information to former knowledge; deliberately selecting thinking strategies; and planning, monitoring and evaluating processes* (Dirks, 1985).

The third key aspect of cognition mentioned by Schoenfeld (1992:448) that influences problem-solving in mathematics is the affective factors that intervene during the process. One's success in problem-solving activities is clearly dependent on the attitudes and beliefs about problem-solving, about oneself as a problem-solver, and about the ways to go about solving problems (Boekaerts, Seegers, & Vermeer, 1995:242; Kroll & Miller, 1993:62-65; Renga & Dalla, 1993:25). Those who believe they are able to and enjoy solving a problem tend to become better problem-solvers. On the contrary, students who harbor negative attitudes about mathematics are not going to do well in problem-solving (Van de Walle, 1998:58). Moreover, Schoenfeld, (1992:352-353), argues that poor problem-solvers are unable to explain why they employ the strategy they have selected. As a matter of fact, a meta-cognitive checklist that is periodically completed by students will help them develop their meta-cognitive processes (See Van de Walle, 1998:78).

Finally, an effective problem-solving programme would be considerate of the knowledge, belief, attitude, and of the control factors which play a role over an extended period of time. To this effect, Charles and Lester, (1982:5) suggest incorporating three essential components in the programme. They are: (1) suitable problem-solving tasks; (2) clear roles for the teacher and students; and (3) guidelines for organizing and implementing instruction.

2.7. Authentic assessment practices: portfolio of evidence

2.7.1 Overview

PBTL calls for a major change in the way mathematics and mathematics learning are defined. In line with this change an overall renovation in how we measure students' success is suggested (Jones, 1994:16; Schmidt & Brosnan, 1996:18). To ensure intensive, high-quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption (Cathcart *et al.*, 2001:73; NCTM, 2000:22-24; Reichel, 1994:24; Robinson & Bartlet, 1993:330). The best way to achieve this is to employ authentic assessment, the reason being that the tasks undertaken by the students are more practical, realistic and challenging than the traditional paper-and-pencil tests (Kantrov, 2000:4; Marsh, 2004:57). Darling-Hammond, Ancess and Falk (1995:2) argue that during authentic assessment students are involved in more meaningful, context-bound activities, organizing their energies on "…challenging, performance-oriented tasks that call for analysis, integration of knowledge, and invention". This at least, includes four elements of the portfolio as an assessment dimension and authentic assessment practice. A discussion of each will be done below.

2.7.2 Self-assessment

Self-assessment (SA) is nothing other than students assessing themselves (Cathcart *et al*., 2001:75). It is aimed at developing the capacity and willingness to assess their own progress and learning (Van den Berg, 2004:283). Furthermore, it helps to develop mathematical power and indicates how much one knows and what to learn more (Stenmark, 1989:26).

Student self-assessment is mostly based on three major areas: the first is *content* self-assessment; the second is *disposition* self-assessment*;* and the third is *behaviour* self-assessment. Each of these areas answers specific questions that are helpful to student-learning. Student self-assessment also merits several benefits for themselves. The first is the development of meta-cognitive abilities. The second is the consideration of 'intelligence-fair' instruments (Gardner, 1999:101-102). And the third is the great share it contributes to evidence-assessment from varied resources. There are a number of ways that self-assessment data can be gathered, amongst others by means of questionnaires, checklists, and in writing (Van de Walle, 1998:80; Williams, 2000:133-134). Self-assessment is one of the writing tools used as vehicles to communicate in mathematics (Burns, 1995:3).

2.7.3 Peer-assessment

The peer-group can develop clear criteria to assess their colleagues' work, for instance during presentations (Race, Brown, & Smith, 2005: 130). According to Tanner and Jones, (1994:413) peer assessment helps to involve students in recursive, self-referential learning processes which supports the explicit development of meta-cognitive skills. This approach then helps to enhance student communication and interaction during classroom instruction (Van den Berg, 2004:283).

2.7.4 Group-assessment

Teachers' group observations, based on a standard format, can provide authentic assessment data (Reichel, 1994:23). According to Van de Walle (1998:492), group-work makes the gathering of information much easier and ensures that data are gathered from all students, not only from a few. Group-work also has the following advantages: (1) it provides much more information in any given period; (2) substantial group-work reduces the amount of paper that is necessary for assessment (Van de Walle, 1998:492).

Group-assessment data can serve the following purposes: (1) to promote students' learning via constructive feedback; (2) to inform about the effectiveness of instruction; (3) to evaluate and grade students' work; and (4) to evaluate a programme (Van de Walle, 1998:*Ibid*).

2.7.5 Teacher-assessment

Kyriacou (1998:115) notes that feedback to students about their progress is of immense importance in contributing to their motivation and further progress. Clearly, authentic assessment is an ongoing feedback system that helps to monitor and record student learning and outcomes. It may come with many possibilities and access to the teacher. Among these include that the teacher forwards constructive comments on the portfolios of evidence on elements such as self-assessment, peer assessment and others. In the actual classroom the teacher could also employ checklists and formats help follow up students' degree of and quality of participation. A typical example is the cooperative behaviour checklist (CBC) (Huetinck & Munshin, 2000:22)

2.7.6 Portfolio assessment

Portfolio assessment provides the access for the presentation of students' performance, including their strengths and also areas for improvement (Lee & Silverman, 2001:103; NCTM, 2000:23). It also helps to maintain records of students' progress demonstrated by means of different mathematical competencies – mathematical activity behavior and social interactive behavior (Black, 1999:131).

Crowley (1993) states that a portfolio is a collection of students' work. This authentic assessment mechanism can provide an excellent form of communication among stakeholders (Van de Walle, 1998:83). The use of portfolios promotes students' self-assessment, and helps to, amongst others, communicate their understanding of mathematics with a higher level of proficiency, and emphasizes the role of the student as the active mathematician with the teacher as the guide (Lambdin & Walker, 1994:318). Portfolios are excellent devices that indicate more clearly and intensely than letter grades students' performance and understanding. The issue of assessing portfolios of evidence can be resolved by a rubric/scoring guide (Kuhs, 1994:334; MoE, 2006; Reichel, 1994:22).

2.8 Classroom examples

2.8.1 The learning process

PBTL deals with activities that encourage exploration and discovery in classroom instruction. It also provides confident teachers with the opportunity to treat mathematics classroom instruction as a setting for mathematical inquiry similar to mathematicians exploring a set of new ideas (Fitzgerald & Bouck, 1993:252).

According to Harris *et al.* (2001:316) and Davis (1996:293), interesting problems serve as a foundation for instruction. In the PBTL process, four key phases are identified. These include, introducing the context, exploring the problem, sharing insights and reasoning, and extending the concepts.

The first phase involves the introduction of the problem situation that motivates the lesson and provides an initial setting for investigation. Here the teacher plays the role of moderating classroom discussions and of clarifying directions for small group investigations.

During the second phase students work in small groups to investigate the problem using various strategies. While working, they are encouraged to gather important data, look for patterns, construct patterns and meanings, and make and verify conjectures. In this regard the teacher plays certain key roles such as of facilitating, circulating around the room to provide support, guiding and encouraging through the clarification of task directions, asking probing questions, and giving hints, and thereby ensures that all students are participating and contributing.

In the third phase students articulate their thinking, first in small groups and then to the larger class. Furthermore, they clarify one another's thinking as they advance in understanding. This also helps them to see multiple and workable ways to approach a problem. They even have the opportunity to deal with open questions and appreciate the existence of more than one valid answer.

In the fourth phase they work to extend their understanding of newly learned concepts and skills and apply them. This helps them to further see connections within mathematics, between areas in mathematics, and its applications.

The four phase process employed by Harris *et al.*, (2001:311-316) in solving context rich and engaging problems, is closely related to the four stage process (experimenting; reflecting and explaining; hypothesizing and articulating; and verifying and refining) as suggested by Cangelosi (1996:100) in a discover-a-relationship lesson. Cangelosi argues that students hardly achieve the application level (solving real-life problems) without first having achieved the discover-a-relationship objective by incorporating inductive learning activities.

2.8.2 Real classroom manifestations

1. ***Enquiry type*.** An example from an actual classroom discussion is given below.

The question was adapted from Huetinck and Munshin (2000:460). It deals with years and exponents (powers with large exponents). The detail is as follows:

Mathematical content: **Exponents, Number Sense**

Material needed: **Problem statement, calculator** (Optional)

Problem: If $2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = k \cdot 2^{1995}$, then what is the value of k?

List of required background knowledge

Constructivists argue that students build knowledge when there is an interaction between their experiences and the new knowledge structure. Thus, it seems necessary to identify the background knowledge in tackling this problem before one starts doing the problem, such as rules, procedures, properties, theorems and so on.

Abrha: I think this could be solved using a calculator only.

Abrar: I think this results in $2^{1998+1995} - 2^{1997+1996} = k \cdot 2^{1995}$ then $k = 0$

Selam: I think it is 3.

The teacher writes on the board: $2^{1998}-2^{1997}-2^{1996}+2^{1995}=k\cdot 2^{1995}, k=0?\ 3?$ Only by means of the calculator?

Teacher: Abrha, what is your reason to say so?

Abrha: Because the numbers are very big.

Teacher: Do you agree with him, Dagim?

Dagim: No, I rather agree with Selam. We can make divisions on both sides by 2^{1995} to get

$$k=\frac{2^{1998}-2^{1997}-2^{1996}+2^{1995}}{2^{1995}}=\frac{2^{1998}}{2^{1995}}-\frac{2^{1997}}{2^{1995}}-\frac{2^{1996}}{2^{1995}}+\frac{2^{1995}}{2^{1995}}=3$$

Teacher: Does your answer work now, Abrar?

Abrar: I saw it. It is only when multiplication we add the exponents.

Teacher: What is your idea about the solutions process, Senait?

Senait: The answer is the same as given by Dagim, but the method is different.

Teacher: Can you explain it?

Senait: Yes, first I have used addition rule to collect the powers with exponent 1995 and apply the rule of exponents to get a common factor with exponent 1995 on the other side?

Teacher: how could this help you solve the problem?

Senait: I will apply distributive low to get common factor 2^{1995} and thus, k-1=2 or k=3.

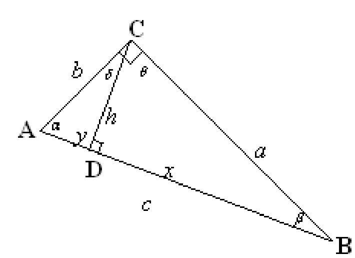
Amina: Mine is also k =3, but I reduce powers to as a product of 2^{1995} and eliminate factors of 2^{1995} to yield k = 3.

Ermias: I think Dagim, Senait, and Amina did it correctly. There are different routes to the solution.

<u>2. Discovery type: Discover a relationship</u>

Mathematical content: **<u>Pythagoras theorem</u>**

Material needed: **<u>Problem statement and a diagram of a right angled triangle with sides *a, b*; hypotenuse *c*; and altitude to hypotenuse *h*; and segment division on the hypotenuse by *h* given by *x* and *y*</u>:** (Rhoad, Milauskas, & Whipple, 1991:378).



The solution process carries on by working on the sub-problems as follows:

List of required background knowledge

At this stage students are required to identify what is known and what is unknown before they rush on to solve the problem.

Sub-problem one: Use the properties of the complementary angles and a right-angled triangle to form systems of equations that relate α, β, δ, and θ from the figure above.

Sub-problem two: At this stage students are encouraged to solve the systems of equations in sub-problem one and state the relationships, for example, $\beta = \delta$.

Sub-problem three: The students are now asked to identify similarity theorem/s that is/are relevant to list similar triangles. For instance, AA similarity theorem could be used to list as $\Delta BDC \sim \Delta CDA$ as one among the similar pairs of triangles.

Sub-problem four: At this stage students are encouraged to use the similar triangles so far identified in sub-problem three to find important ratios and products or relationships. For example, from (1) $\Delta BDC \sim \Delta CDA$, it was obtained $\frac{BD}{CD} = \frac{DC}{DA} = \frac{BC}{CA} \Rightarrow (DC)^2 = BD \cdot DA$, or $h^2 = x \cdot y$

The Pythagorean theorem: Sub-problem four was helpful for students to state important mathematical relationships and conjectures as in the box on the right below. The most important relationship, namely, the Pythagorean theorem, is obtained from the last two, that is, (3) and (4): $a^2 + b^2 = c \cdot x + c \cdot y = c(x+y) = c \cdot c = c^2$.

At this stage, students are required to conjecture as many propositions and relationships as possible. After the introduction of more approaches to the Pythagorean theorem, such as Euclid's elegant proof and the Chinese approach, students were encouraged to prove the theorem using deductive reasoning or formal proofs.

(1) $h^2 = x \cdot y$
(2) $h \cdot c = a \cdot b$
(3) $a^2 = c \cdot x$
(4) $b^2 = c \cdot y$

3. Problem solving: A typical example of problem-solving is the problem given below (Billstein, Libeskind & Lott, 1997:556-557).

Problem: *How many diagonals does a 20-gon have*?

Understand the problem

Restatement by students: ____

Carry out the plan

Strategy/strategies: For example, students identify a workable strategy.

(1)____________________ (*Examine related simpler cases*)

(2) ____________________ (*Pattern development*)

At this stage students were helped to make use of the first four or five polygons to see the number of diagonals emanate from each vertex of the polygon and look for a pattern.

Carrying out the plan. At this level students were helped if they could note that a diagonal is determined by two non-consecutive vertices and counted twice. This idea was helpful to students to reach the conclusion that there are about (20) (20-3)/2 or 170 diagonals in a 20-gon.

Looking back

The reasoning was essential to arrive at the conclusion: (extension) $\frac{n(n-3)}{2}$. However, an alternative approach should also be tried out if there is any. For example, the notion on combinations may help to discover the number of lines determined by the vertices but without the sides, that is, $\frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$ which is consistent with the previous result.

4. *Others*. Deal with open-ended questions, investigational or project problems.

2.9. Concluding remarks

Mathematics is best grasped in the context of meaningful learning and interesting problems (Fendel *et al.*, 1997:11). To create an effective learning situation in the classroom, Van de Walle (1998:13) identifies three important conditions: (1) the atmosphere that facilitates the exploration of meaning; (2) a continual search of new information and experiences by learners; and (3) creating an environment for personal discovery. A classroom climate that is purposeful, task-oriented, relaxed, warm and supportive and has a sense of order best facilitates student learning (Kyriacou, 1998:64-65). Moreover, this is effective in establishing and maintaining students' attitudes and motivation towards the lesson.

To achieve the above goals, teachers must be knowledgeable about teaching and learning styles. For example, the use of the teacher-centered, exposition-dominated traditional approach may not as such be effective. In sharp contrast to this ‘formal teaching style’ is the ‘informal teaching style’. This method emphasizes student-centered activities and the use of more open-ended tasks.

Also, the traditional skill-based curriculum is in a sharp contrast to the new views and possibilities that call for a problem-based and concept-based curriculum. The latter curriculum demands teaching and learning methods that blend specific instructional methods and essential elements of learning. For example, PBTL blends fundamental instructional methods and strategies, such as cooperative and inquiry learning, problem-solving, discovery learning, and meta-cognitive strategies.

CHAPTER THREE

THE RESEARCH DESIGN AND RESEARCH INSTRUMENTS

3.1. Introduction

In order to determine whether PBTL has an effect on students' performance in mathematics, an empirical study was undertaken. In planning the empirical study, it was necessary to state the problem, the basic questions and a hypothesis clearly, to identify an appropriate methodology, to design appropriate instruments, to schedule the research activities, as well as to select suitable participants for the research.

This chapter will report on the way the data were gathered and processed, together with illustrations. A description will be given of the research design and why it was selected, and of the procedures followed in having the research project acknowledged by parents, school officials, and school districts. In order to reach valid conclusions a systematic and thorough organization, analysis, classification, and consolidation of the data were done, all of which will be explicated in this chapter.

3.2. Research problem and hypothesis

The traditional teacher-centered transmission approach regarding mathematics teaching and learning apparently results in mechanistic learning, memorization and the learning of isolated concepts and procedures. Furthermore, it has major pitfalls, amongst others, regarding students as a collection of individuals instead of acknowledging their abilities to construct their own knowledge, based on the interaction of their experiences with the world around them. A further outcome of transmission teaching is the lack of acquiring knowledge and skills, and poor performance and achievement. The entire teaching and learning process remains teacher-directed and regulated, and meta-cognitive strategies and student self-regulation remain undeveloped.

Hence, the broad research problem was stated as follows:
What is the effect of problem-based teaching and learning on students' performance and achievement in mathematics?

Consequently, the intervention programme was designed around the following research questions:

- How will the introduction of PBTL in the mathematics classroom affect the overall performance and achievement of students?
- What will the effect of PBTL be on the ability of students to identify, formulate and solve problems, and on self-evaluation?
- To what extent will the introduction of PBTL affect the students' self-regulation and meta-cognitive strategies?
- What will the overall effect of PBTL be on the students' motivation and self-confidence?

In this study, PBTL was compared to the traditional teaching approach. It was anticipated that problem-based teaching and learning would improve the students' performance and achievement in mathematics. Students involved in problem-based teaching and learning (PBTL) intervention were involved in problem-solving, inquiry, discovery, self-regulating strategies, meta-cognitive skills, and cooperative learning activities.

Hence, the following hypothesis was formulated:
The introduction of problem-based teaching and learning (PBTL) into the traditional mathematics classroom will result in improved student performance and achievement in mathematics. A further outcome will be the development of meta-cognitive strategies resulting in improved student self-regulation.

3.3. Experimental design

It appeared that an educational experiment with a pre-test-post-test design would be the most appropriate in conducting the empirical investigation.

3.3.1. Subjects of the study

The subjects of this study were 50 grade ten students who served as the experimental group and another 50 grade ten students who served as the control group. Both groups were enrolled at Ayider High School in Mekelle city during the academic year 2006/2007. These students were selected from 1082 students by using simple random sampling. The students' names were arranged in alphabetical order and coded with four digit numbers 0001 through 1082. However, a complete set was obtained for only 47 members of the experimental group and 45 members of the control group, totaling 92.

3.3.2 Procedures

The researcher first obtained permission from the district and the school principals as well as from the parents of the subjects to collect data and carryout the research. The researcher consulted the subjects' parents using letters. At the beginning all the stakeholders of the programme who involved were informed about what the intervention programme is entailed. The subjects were told that they would probably be benefited from the programme. Moreover, the notion of informed consent was highlighted to make subjects feel confident about their involvement in the programme. After all these things have been settled, scheduled activities were begun to be practiced and implemented.

The mathematical test (MT) and Learning and Study Strategies Inventory - High School version (LASSI-HS) were administered during the first and last weeks of the implementation of the programme. During the first administration of the LASSI-HS, translations into the mother tongue and rephrasing were possible whenever the respondents were not adequately clear about an item. This was to avoid language barriers when answering the questionnaire. The reference test (RT) was administered as additional instrument during the third week of the last month in the intervention programme for both groups. This was for two major reasons: the first is that the nature of questions required students to employ their procedural and conceptual skills, and hence the teacher (the researcher in this case) needed time to analyse the students' answers; the second reason was that the programme was near completion and the other instruments were administered during the last week, and students may have been bored. The Programme End Evaluation Questionnaire (PEEQ) was administered during the last week of the intervention programme for both the experimental and the control groups.

The on-progress instruments, namely the Portfolio of Evidence (PE) and the Cooperative Behaviour Checklist (CBC) had their own patterns of administering. The PE was the cumulation of the chapter-end self-assessments arranged at the end of the five chapters of the teaching material. The CBC was employed throughout the intervention programme; however, the data used were only those gathered during the first and last weeks of the intervention programme. These instruments were unique to the experimental group. This was for three major reasons: the *first* is because the socio-constructivists suggest authentic assessments such as self-assessment, peer-assessment, and others in order to develop students' meta-cognitive strategies which are relative to problem-solving. The *second* reason is that assessment is an integral part of the instructional process and it was done by means of different mechanisms. Amongst others, by means of the observation checklist (the CBC) to help the teacher (the researcher in this case), to improve his teaching and facilitate students' learning. This was helpful to identify misconceptions and immediately intervene, which is in line with the constructivist view, particularly the socio-constructivist view of learning. The *third* reason is that it may be worthwhile to take note of students who make significant progress or who experience significant difficulties in learning mathematics. The PBTL approach which is based on the constructivist approach, embraces these components as workable tools of the programme. During the administration of the CBC, five teacher educators were involved. The teacher educators were aware of the concepts *listening, on-task, and supporting* before they were committed to the survey. The reason behind this was to obtain as much accurate and valid data as possible.

The intervention lasted for six months, starting from the first week in June to the last week in December, 2006. Much of the survey was done during the students' vacation and break (from July to October), which was actually more of a summer season. During this time more than 75% of the material was covered. Both the control group and the experimental group were involved for two hours each day from Monday to Sunday, that is, 7x2=14 hours a week for three months. In total, in these three months the respondents attended the intervention programme for a minimum of 3×4×14=168 hours. In the second week of October the schools reopened for the new academic year. The respondents agreed to attend classes for about two to three hours every Saturday and Sunday, that is, for a minimum of 4 hours a week for the next two and half months (10 weeks) from the second week in October to the third week in December, which is more than 4×10 = 40 hours. The total minimum instructional hours were about 168+40 = 208 hours. Each of the six instructional months was considered to consist of nearly 35 hours. The time allotted for the five chapters of the course material was nearly the same, except for chapter three. Thus, all four chapters were allotted approximately 38 hours each and the third chapter (Trigonometry) about 56 hours. All of the research took place at Mekelle College of Teacher Education where the researcher is working. Moreover, the institution provided two classrooms and every resource needed for the purpose of the research.

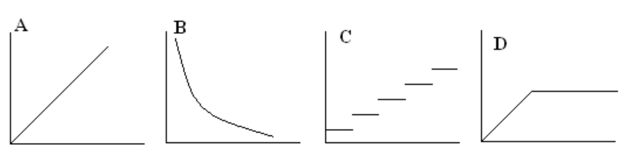
3.3.3 Data collection

In the research project, six major instruments were used for the collection of the data. These included the mathematical test (MT), the reference test (RT), the cooperative behaviour checklist (CBC), the portfolio of evidence (PE), the Learning and Study Strategies Inventory (LASSI-HS), and the programme end evaluation questionnaire (PEEQ).

3.3.3.1 The mathematical test (MT)

This is a 130 item multiple-choice question test. The questions were collected from standardized reading materials (*e.g*., Dahnke & Pavlovich, 1990; Posamentier & Stepelman, 1996; Sobel & Maletsky, 1988; Swanson *et al*., 1988). This test covers all types of mathematical concepts starting from the middle grades level to the end of grade 9. The questions included called for students' knowledge of *procedures*, *properties*, *problem-solving skills*, *representations* and *applications*. However, the results were not calculated separately for each of the above-stated elements, rather as a sum total out of 130. The questions included all types of questions: *word-problems*, *non-algebraic*, *geometry* of two and three dimensions, and *algebra* of all types, up to grade nine-level. (See Table 3.1 below, and for more detail, see Appendix 1).

Table 3.1: Sample questions from the mathematical test

Q. No	***Question***	***Relevance***
93	*Which of the following is **NOT** a property of integers?*	*Knowledge of basic number properties*
91	'*What is **FALSE** about the data 2,2,5,4,2,2?*'	*Knowledge of algorithms, rules, or procedures*
85	*Find the **NEXT** number in the sequence: 4, 10,18,28,40,...*	*Problem-solving*
97	*What theorem or assumption provides the **best explanation** for the fact that when you saw a board, the edge of the cut is a straight line?*	*Application*
76	*Which graph **best** represents the cost of a phone cell?* 	*Representations*
8	*Sue has 2 brothers. She is 3 times as old as Michael, her youngest brother. The age of her other brother, John, is the difference between Sue's and Michael's ages. If the sum of all their ages is 36, how old is John?"*	*Translating, symbolizing and forming equations*

3.3.3.2 The reference test (RT)

This test consisted of two categories of questions, namely, procedural and conceptual. The procedural questions call for simple procedures, algorithms, or rules. On the other hand, the conceptual questions call for students' understanding of concepts. The RT was developed, based on the work of Cobb *et al*. (1991), Brumbaugh *et al.,* (1997), and on Van de Walle's (1998) suggestions in advancing a procedural question to a conceptual level. Table 3.2 gives an example. (For more details, see Appendix 2).

Table 3.2: Sample questions from the reference test

Procedural question	***A question advanced to conceptual***
1. *If p, q, and r are propositions with p≡T, q≡F, and ¬r≡T, (T stands for truth-value **true** and F stands for truth-value **false**) then find the truth-value of* $[(p \wedge q) \Leftrightarrow (\neg r \Rightarrow (p \vee q))]$	1'. *For p and q propositions, if* $\neg(p \Leftrightarrow q) \equiv F$, *then find the truth-value of* $(q \Rightarrow p) \Leftrightarrow (q \wedge \neg p)$. *F- Stands for truth-value **false**. Justify your solution process.*

3.3.3.3 The cooperative behaviour checklist (CBC)

The CBC was adapted from research by Huetinck and Munshin (2000). This test was based on the three major areas to measure the quality of student participation during cooperative learning, namely *on-task, listening,* and *supporting.*

On-task **–** making every effort to make the team successful by accomplishing assigned tasks and responsibilities. This act involves being duty-conscious and feeling responsible for the success of the team. This implies that individual members either work independently or with somebody else, in response to the teacher and colleagues, *etc.* (Johnson, 2002:102). Alternatively, students who are talking to somebody, doodling, working on something else not related to the assigned task, making noise, and walking around without any defined purpose, are considered to be off-task.

Listening **–** implies to be a good listener, namely giving attention when others present their ideas, taking advantage of others' ideas and the results of their investigations (Hiebert *et al.,* 1999; McGlinn, 1991).

Supporting – refers to sharing ideas (negotiate meanings) with members of the team, and supporting other teams and team members in completing assignments and tasks (Huetinck & Munshin, 2000).

Initially, there were ten teams, each consisting of 5 members. There were three withdrawals and a rearrangement was made to form 7 teams consisting of 5 members each, and 2 teams consisting of 6 members each.

A complete description is given in 3.3.1. An example of the CBC Checklist is indicated in Table 3.3, and for more details see Appendix 3.

Table 3.3: The CBC Checklist format (+ and – marks are used to indicate quality of participation)

Team	**Student Code**	**On-task**							**Listening**							**Supporting**						
	No	1	2	3	4	5	**6**	**7**	1	2	3	4	5	6	7	1	2	3	4	5	6	7
	1																					
A	2																					
.	3																					
.	4																					
.	5																					
Total	Positive																					
	Negative																					

3.3.3.4. The learning and study strategies inventory, high school version (LASSI-HS)

This questionnaire is designed to measure students' self-regulating strategies and in particular to find out how students study, how they learn, and how they feel about learning and studying. This questionnaire is described in Weinstein and Palmer, (1990). The ten subscales that make up the LASSI-HS are the *attitude scale*, the *motivational scale*, the *time management scale*, the *anxiety scale*, the *concentration scale*, the *information processing scale, selecting the main idea scale*, the *study aid scale*, the *self-testing scale* and the *test strategies scale*.

Table 3.4 below illustrates descriptions with examples (Olaussen & Braten, 1999: 412-13). For each of the 76 items the students were to indicate, on a 5-point scale, how the statement fits them. (For details, see Appendix 4).

Table 3. 4: Sample items and descriptions from the LASSI-HS

Scale	*Description and sample items*
Attitude	*Addresses students' general attitude and interest toward mathematics* *e.g. I don't care if I finish maths in high school as long as I can get a job*
Motivation	*Focuses on students' motivation and responsibility for performing the specific task related to success.* *e.g. I work hard to get good marks in maths, even when I don't like the maths being done*
Time management	*Measures the degree to which students create and use schedules to organize and control work progress* *e.g. I only study maths when I have to write a test*
Anxiety	*Addresses the degree to which students worry about the study and their performance* *e.g. I am very tense when I study maths*
Concentration	*Focuses on students' ability to concentrate and direct their attention to academic tasks, including study activities* *e.g. I find it hard to pay attention during a maths lesson*
Information processing	*Addresses to what extent students use strategies to elaborate and organize information, monitor comprehension and relate new material to prior knowledge* *e.g. I tried to connect between the maths I am learning and what I already know*
Selecting Main Ideas	*Measures students' skills at selecting the main ideas in the content they are given* *e.g. I can tell the difference between more important and less important information in a maths lesson*
Study Aid	*Measures the use and generation of diverse technical solutions and materials aimed at supporting and increasing meaningful learning and retention* *e.g. I use symbols, key words, diagrams, or tables in summarizing my maths*
Self-Testing	*Addresses to what extent students monitor or check their own understanding as they go through and review study material* *e.g. I check to see if I understand what my teacher is saying during a maths lesson*
Test-Strategies	*Focuses on students' strategies during both test preparation and test taking* *e.g. When studying for a maths test or exam, I think of questions that I think might be asked*

3.3.3.5 The portfolio of evidence (PE)

The PE is based in five major areas, *reflective activity*, *summary-making and journal writing*, *contribution to groups*, *portfolio assignments*, and *attendance and punctuality*, as indicated in table 3.5. These elements, for each of the five chapters of the teaching material, were developed based on the *College Preparatory Mathematics* (Dietiker, 1997a, 1997b; Hoey & Wotton, 1994a, 1994b) and the *Higher Diploma Programme* (MoE, 2006). The scoring guide/rubric adapted from the MoE (2006) with modifications, was used to assess students' reflective practices.

Table 3.5 and table 3.6 below show a part representation of the chapter-end self-assessment format. (For more details see Appendix 5).

Table 3.5: Partially represented chapter-end self-assessment format

AREA	**PROGRAMME SELF ASSESSMENT- CHAPTER-1** *Circle one word to assess yourself in each area*				**Teacher Ass't (1-4)**
Reflective Activity/Diary	**POOR:** *Little reflection; very brief comments; not all reflective activities completed*	**FAIR:** *Some reflection; some longer comments with little relevance to the topic of discussion; some reflective activities completed*	**GOOD** : *Good ability to reflect and to criticize; most reflective activities completed with good quality; discussion directly related to topic*	**EXCELLENT:** *Well developed reflection, critical ability and self analysis; discussion relevant and related to the topic; all reflective activities completed*	
. **Continues**					
TOTALS	**POOR (1)**	**FAIR (2)**	**GOOD (3)**	**EXCELLENT (4)**	**Teacher Ass't**
Self Assessment Out of 20______/20 Self Assessment Total ________%					____/**20** _____**%**

Provision was made for students to assess themselves out of 20 (converted to 100) based on the above areas. The teacher is to evaluate students' self-assessment for each chapter to confirm the assigned value or change it, by providing constructive feedback. For each of these areas students used a 4-point scale with performance indicators ranging from poor (1) to excellent (4). The detail is given in Appendix 6.

Table 3.6: Sample items on the areas of chapter end self-assessment

Area	Example
Summary making	*With your study team, quickly list as many mathematical ideas or topics you remember from this unit. Don't go back to the text or your notes! You will have time for that. This exercise is to see what you can remember. Be attentive to any suggestions offered by your teammates. Each member needs to write the list down. Then identify big ideas as topics, demonstrate the main idea and* • *Solve each problem and show all work, especially the sub-problems.* • *Finally, explain the method you used to arrive at your solution. Use complete sentences and be as descriptive as you can.*
Reflective activity/diary	• *What new skills, applications, properties and modes did you learn in this chapter?* • *What part of the chapter did you enjoy most? Why?* • *What pat of the chapter did you find most difficult? Why?*
Journal writing	▪ *Imagine one of your friends was absent in time when logarithms were held due to unavoidable reasons. Suppose he/she firmly argues with you as* $\log_2^{-8} = -3$*.How would you convince him/her logically?*
Portfolio assignment	*Complete the following questions. Part of the homework in this unit received ideas from relations. For each idea,* • *Choose a representative problem and show its complete solution.* • *Write a brief explanation of how each kind of problem is solved.* • *Make a list of the new terms you learned in this unit. Write the definition, with figures if appropriate, next to each term and use this to help learn them. Hand in a photocopy of it.*

3.3.3.6. The programme end evaluation questionnaire (PEEQ)

The PEEQ is based on seven categories, namely, the *programme*, *learning, assessment, time, resource, objective achievement*, and *programme overall evaluations* which were aimed at assessing students' views on the programme. This instrument is an adaptation, with modifications from the *Higher Diploma Programme* (MoE, 2006). Table 3.7 below shows examples of each of the categories. The detail of the questionnaire is given in Appendix 6. Students were required to respond on a three-point scale given as: *Yes* (**2**), *Not sure* (**1**) and *No* (**0**) to demonstrate their views on the learning programme for the first six categories and poor (**1**) to excellent (**4**) for the last category.

Table 3.7: Sample items from programme end questionnaire

Category	*Sample*
Programme	*Was the programme well organized?*
Learning	*Were you actively involved in your own learning?*
Assessment	*Were you aware of the different continuous assessment methods?*
Time	*Was enough time given for what you were expected to do?*
Resource	*Was the resource helpful?*
Objective achievement	*Has the programme achieved its aim?*
Programme overall evaluation	*How would you rate the intervention programme overall?*

3.3.3.7 Ethical measures, and ensuring validity and reliability

Ethical measures. Specific ethical measures were introduced in order to respect the integrity and humanity of the participants. To do this effectively, the guidelines for informed consent, the *Institutional Guide to DHEW Policy*, (1971) was strictly adhered to.

*Ensuring validity and reliability***.** There are at least four major stages in any research project. These include the designing stage, the data obtaining stage, the data analysis stage, and the data reporting stage (Cohen, Manion, & Morrison, 2000:115-117). Accordingly, the researchers suggested a means to minimize the threat to validity at each of the four stages mentioned above. In response to this, at the first stage, sheer chance was used to assure an appropriate sample, a pre-test/post-test design was selected as it assumed to answer the research questions, and appropriate instruments were used to obtain data. In addition, everything possible was done to ensure the availability of adequate resources in delivering the intervention programme. At the second stage, every effort was made to reduce drop-out rates and the non-return of questionnaires. Furthermore, a considerable amount of time was spent to motivate the respondents, and an appropriate time interval was set between the pre-test and the post-test.

At the data analysis stage, due attention was given to avoid the subjective interpretation of data. To this end, the statistical software package, SPSS, was used to analyze the data and to produce statistically correct results. In the end careful analyses were made to avoid a Type I-error (the neglect of a true null hypothesis) or a Type II-error (accepting a false null hypothesis), and the degrading of the data. Finally, at the reporting stage, the data were used as it happened to present valid information. The intention was to ensure that the research questions were answered by means of the appropriate use of the data available.

3.4. Data analysis

The mathematical test (MT) was designed to measure students' understanding of mathematics from a wider perspective. The results received by the students in the experimental group were compared to those of the students in the control group, both at the beginning and at the end of the intervention period. To ascertain whether the differences in the results were significant, a one way ANOVA analysis was applied. The reference test (RT) consisted of 20 procedural questions and 20 conceptual questions which were correspondingly advanced from the procedural questions. The reference test was administered to both groups at the end of the programme. Once again a one-way ANOVA was run to compare the performances of the experimental and the control groups.

The information obtained by means of the questionnaires was also tabulated and analyzed by means of a one way ANOVA. It was done for each of the ten subscales of the LASSI-HS. The portfolio of evidence was organized in the form of percentiles (out of 100) under the students' self-assessment (SA) and teacher assessment (TA). Correlations between the students' self-assessment (SA) and teacher assessment (TA) were calculated for each of the five chapters of the teaching material. Using this, the students' progress in their reflective thinking along the consecutive chapters was assessed. Furthermore, the *Programme End Evaluation Questionnaire* (PEEQ) was administered to both groups at the end of the programme to ascertain students' views on different components of the programme, areas of strength and areas for improvement, and its success as a whole. The closed questions were categorized, tabulated and analyzed under seven headings, namely, the *programme*, *learning*, *assessment*, *time*, *resource*, *objective achievement*, and *programme overall* ev*aluation.* For the open-ended questions a descriptive approach was employed and responses were organized in relation to the nature of the question. This means that responses were organized under the categories of the three major questions, namely, *best aspects of the programme, areas for improvement,* and *comments on the programme*. Finally, the cooperative behaviour checklist (CBC) was marked with (-) and (+) to indicate the quality of student participation in the three major areas mentioned. However, for analysis purposes, the results of the first and the last weeks of the intervention programme were used. In the end, the number of positives and negatives were counted for each category and analyzed, using a one-way ANOVA. For doing the analyses, the statistical software package SPSS, version 13 was used.

3.5 Summary

Six instruments were used to verify the research hypothesis and to obtain answers to the broad research problem and the basic questions derived from the problem statement. The instruments were designed to measure the students' knowledge, skills, attitudes, and views on the programme, the quality of their participation, their meta-cognitive and self-regulating strategies, and their ability to make conjectures, to formulate problems and to evaluate the results. For example, the LASSI-HS was designed to measure the students' ability in applying their meta-cognitive and self-regulating learning mechanisms. The CBC was designed to measure the degree of the students' progress in respect of the quality of participation during cooperative learning. The PE, as an authentic assessment mechanism, was introduced to determine the correlation between student assessment (SA) and teacher assessment (TA) in their reflective thinking. Finally, the PEEQ was applied to investigate the programme's success in general, and to identify areas of strength and areas for improvement, in particular. The MT and RT were developed from standard documents. These instruments were designed to purely measure the mathematical knowledge and the skills of the students.

CHAPTER FOUR

THE PROCESSING OF THE DATA

4.1. Introduction

The empirical data for this study were collected using different instruments that were applied at different occasions. The first group of instruments includes instruments that were administered either at the beginning of the experiment, or at the end, or at both. This group of instruments was administered for both the experimental and the control group of students. The second group includes instruments such as the cooperative behaviour checklist (CBC) and the portfolio of evidence (PE). These instruments were administered throughout the intervention programme for only the experimental group of students.

The data collected by means of the above instruments will be organized, analyzed, classified, and consolidated in this chapter with the aim to identify areas of agreement and disagreement with research previously done. Also will the hypothesis presented in the study be tested, and conclusions will be made on the outcome of the results.

The central question guiding the experimental research was: what is the effect of problem based teaching and learning (PBTL) on students' performance and achievement in mathematics? In trying to determine this effect, the researcher asked specific questions. They were:

(a) How will the introduction of PBTL to the mathematics classroom affect the overall performance and achievement of students?
(b) What will the effect of PBTL be on the ability of students to identify, formulate and solve problems, and to evaluate the results?
(c) To what extent will the introduction of PBTL affect the students' self-regulation and meta-cognitive strategies?
(d) What will the overall effect of PBTL be on the students' motivation and self-confidence?

The research hypothesis was also formulated from the research problem. It was stated as follows:

The introduction of problem-based teaching and learning into the traditional mathematics classroom will result in improved student performance and achievement in mathematics.

4.2 Data analysis and results

4.2.1 The mathematical test (MT)

The mean scores which the experimental and the control groups obtained in the mathematics pretest (MT) were compared by running a one-way ANOVA. The results of this analysis are given in table 4.1. The F-value, $F(1,90)=0.25$, indicates that there was no significant difference in the performance of the two groups in the pre-test. This implies that the experimental and the control groups could be considered comparable concerning their level of mathematical knowledge and skills.

The same test (MT) was administered as a post-test at the end of the intervention programme. Once again an ANOVA was run to compare the performance of the two groups. From the results it is evident that there was a significant difference in the post-test with the experimental group out-performing the control group [$F(1,90) =37.79$, $p<0.0001$]. The results of the ANOVA analysis are also included in table 4.1.

It is clear that both groups made a tremendous improvement in their mathematical performance during the intervention period. However, when the two groups were compared, the experimental group achieved a significantly higher mean score on the MT than the control group.

The test was totally objective covering a large content area of the subject, consisting of core ideas of the middle grades to the end of grade nine. (For more details, see appendix 1.)

Table 4.1: Differences in mathematical achievement between the experimental and the control groups (MT)

Group	*Pre-test*		*F*	*Post-test*		*F*
	M	***SD***	*(1,90)*	***M***	***SD***	*(1,90)*
Control	38.34	8.66		56.93	10.47	
Experimental	37.43	8.85	0.25	69.98	9.77	*37.79* ***

n= 47 for experimental and n=45 for control
***p< 0.0001

4.2.2 The reference test (RT)

The RT was administered at the end of the intervention programme as an additional instrument for both the experimental and the control groups. This test was designed to measure students' ability in evaluating results, making conjectures and solving problems. (For more details, see appendix 2). A one-way ANOVA was run to compare the performances of the two groups on the twenty procedural questions (converted to 50), twenty conceptual questions (converted to 50) and the total sum of the procedural and conceptual questions (added up to 100). The results of this analysis are given in table 4.2. From the results it is apparent that there was a significant difference between the experimental and control groups in both the procedural and conceptual question sections with the experimental group out-performing the control group at [$F(1,90) = 11.77$, $p<0.001$ and $F(1,90) = 32.00$, $p<0.0001$] respectively**.** The results further indicate that the means of the total (sum of the scores on the procedural and conceptual questions) are significantly different for the two groups with the experimental group performing better than the control group [$F(1,90) = 22.62$, $p<0.0001$]. (Refer to Table 4.2.)

Table 4.2: Differences in mathematical achievement between experimental and control groups (RT)

Test type	*Group*				**F**
	Control		*Experimental*		
	M	*SD*	*M*	*SD*	*(1,90)*
Procedural	*27.81*	*7.09*	*33.12*	*7.72*	*11.77***
Conceptual	*20.42*	*6.51*	*29.49*	*8.66*	*32.00****
Total score	*48.23*	*12.59*	*62.61*	*16.1*	*22.62****

n=47 for experimental and n=45 for control.
***p<0.001*
****p<0.0001*

4.2.3 The learning and study strategies inventory (LASSI-HS)

The LASSI-HS was administered at the beginning and at the end of the intervention programme as an additional instrument for both groups. (For more details, see appendix 3.) Students' responses to each of the ten scales of the LASSI-HS were compared by running a one-way ANOVA. The F-test results of the LASSI-HS for each of the ten scales are listed in table 4.3 for both the control and the experimental groups. The F-values, $F(1,90)= 0.44$ for the attitude scale, $F(1,90)= 0.22$ for the motivation scale, $F(1,90)= 0.07$ for the time management scale, $F(1,90)= 0.24$ for the anxiety scale, F (1,90)= 0.03 for the concentration scale, F(1,90)= 0.07 for the information processing scale, F(1,90)= 0.01 for the selecting the main ideas scale, F(1,90)= 0.00 for the study aid scale, F (1,90)= 0.00 for the self-testing scale and F(1,90)= 0.75 for the test strategies scale, indicate that there was no significant difference in the self-regulating abilities and meta-cognitive skills of the two groups at the beginning of the programme. This shows that the experimental and control groups could be taken as comparable concerning their self-regulating abilities and their skills on each of the ten self-regulating scales.

Table-4.3: Difference in self-regulation abilities in mathematics learning between the experimental and the control groups (LASSI-HS)

Scale	*Group*								*F (1,90)*	
	Control				*Experimental*					
	Pre-test		*Post-test*		*Pre-test*		*Post-test*			
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*Pre-test*	*Post-test*
Attitude	2.83	1.34	2.59	1.27	2.62	1.29	3.50	1.25	0.44	11.95**
Motivation	2.69	1.24	2.67	1.20	2.57	1.22	3.26	1.3	0.22	5.11*
Time management	2.70	1.25	2.56	1.21	2.77	1.30	3.06	1.28	0.07	14.59**
Anxiety	2.61	1.20	3.28	1.15	2.73	1.21	2.58	1.21	0.24	7.94*
Concentration	2.57	1.21	2.48	1.26	2.53	1.24	3.50	1.32	0.03	14.18**
Information processing	2.56	1.28	2.62	1.19	2.63	1.27	3.49	1.17	0.07	12.19**
Selecting main ideas	2.64	1.27	2.57	1.21	2.61	1.31	3.53	1.17	0.01	15.24**
Study aid	2.48	1.24	2.57	1.15	2.48	1.26	3.31	1.29	0.00	8.30*
Self-testing	2.57	1.22	2.62	1.12	2.57	1.28	3.52	1.20	0.00	13.96**
Test strategies	2.63	1.20	2.66	1.18	2.86	1.34	3.59	1.24	0.75	13.60**

n=47 for experimental and n=45 for control
**p<0.05*
***p<0.001*

A one-way ANOVA was also run to compare the students' improvement in the LASSI-HS at the end of the programme. From the results displayed in table 4.3, it is obvious that there was a significant difference in each of the ten scales of the LASSI-HS between the means of the two groups, with the experimental group showing more improvement than the control group. Evidently, the F-values, [F(1,90)= 11.95, p<0.001] for the attitude scale, [F(1,90)= 5.11, p<0.05] for the motivation scale, [F(1,90)= 14.59, P<0.0001] for the time management scale, [F(1,90)= 7.94, p<0.05] for the anxiety scale, [F(1,90)= 14.18, p<0.001] for the concentration scale, [F(1,90)= 12.19, p<0.001] for the information processing scale, [F(1,90)= 15.24, p<0.001] for selecting the main ideas scale, [F(1,90)= 8.30, p<0.05] for the study aid scale, [F(1,90)= 13.96, p<0.001] for the self-testing scale and [F(1,90)= 13.60, p<0.001] for the test strategies scale, indicate that the students from the experimental group demonstrate a significant improvement on the LASSI-HS scale.

4.2.4. On-progress instruments

4.2.4.1. The cooperative behavior checklist (CBC)

The instrument was used throughout the intervention programme for the experimental group only. This instrument was designed to measure the quality of participation in cooperative learning groups on three measures, namely, *on-task*, *listening*, and *supporting*. The analysis was done only during the first and the last weeks of the intervention programme by running a one-way ANOVA. The means of the positive and negative counts of these two weeks were compared for the three measures. It should be clear that 'positive' means 'in parallel with the description of the measures' and 'negative' means an act manifested by a student that is not in line with the description of the measures. (For more details, see appendix 4.) The analysis of the cooperative behavior checklist is given in table 4.4.

Table 4.4: Shifts in the quality of participation of the experimental group (CBC).

		First week		***Last week***		**F**
Measure	*counts*	***M***	***SD***	***M***	***SD***	***(1,46)***
Listening	*Positive count*	13.14	3.58	41.57	3.64	216.80***
	Negative count	33.86	3.58	5.43	3.64	216.80***
On-task	*Positive count*	16.43	15.13	42.71	2.21	20.68**
	Negative count	29.71	16.53	4.29	2.21	16.27**
Supporting	*Positive count*	12.29	4.75	40.00	3.11	166.78***
	Negative count	34.71	4.75	7.00	3.11	166.78***

n=47 for the experimental group.
**p<0.001
***p<0.0001

The F-test results indicate positive shifts in each of the three measures. It is evident that for *listening* there was a significant increase in the positive count from the first week to the last week of the intervention programme [F (1, 46) = 216.80, P<0.0001] and *vice versa* for the negative counts. Similarly, the positive [F(1,46)= 20.68, P<0.001] and negative counts [F(1,46)= 16.27, P<0.001] on the *on-task* measurement show that there was a significant shift in the number of students engaging in tasks in the first and last week of the intervention programme. Finally, the F-value [(F(1,46)= 166.78, p<0.0001] for positive counts and [(F(1,46)= 166.78, p<0.0001] for negative counts on the *supporting* measure clearly indicates that there was a significant shift in the provision of support experienced during the first week of the intervention programme to the provision of high quality support during the last week of the intervention programme.

4.2.4.2. The portfolio of evidence (PE)

The portfolio of evidence (PE) comprises of five chapter-end progress self-assessments and other basic elements. Each chapter-end progress self-assessment is based on five areas of evaluation: (1) *reflective activity and summary making*; (2) *journal writing/communication*; (3) *contribution to group-work*; (4) *portfolio assignment;* and (5) *attendance and punctuality*. (For the details of these areas, see appendix 5.)

A rating scale/rubric ranging from poor to excellent: **poor** =1, **fair** = 2, **good** = 3 and **excellent** = 4, was employed to assess each chapter-end progress self-assessment both by the teacher and the students themselves. The relationship between student self-assessment (SA) and teacher assessment (TA) was determined by calculating the Pearson correlation coefficients for the experimental group using the statistics package SPSS, version 13. The results of this analysis are given in table 4.5.

Table 4.5: The relationship between student-assessment (SA) and teacher-assessment (TA)

	SA		***TA***		ρ
Chapter	***M***	***SD***	***M***	***SD***	***(2,46)***
1	93.62	8.52	76.81	7.55	-0.070
2	84.68	8.10	79.15	6.20	0.103
3	79.68	4.82	82.55	5.50	0.154
4	78.83	4.20	83.83	5.63	0.469**
5	82.45	4.76	88.51	4.65	0.413**

n=47 for experimental
** p < 0.01 (2-tailed)

The Pearson correlation value, ρ[2, 46] = -0.070 for the first chapter indicates that there was no significant correlation between student self-assessment (SA) and teacher assessment (TA). The Pearson correlation value, [ρ[2, 46] = 0.103] for chapter 2 has nearly the same implication as in the case of chapter 1. The [ρ[2, 46] = 0.154] in the third chapter also indicates that there was no significant relationship between SA and TA. However, a moderate but significant correlation was obtained for chapters 4 and 5 between SA and TA [ρ(2, 46) = 0.469 for chapter 4] and [ρ(2, 46) = 0.413 for chapter 5].

The significant relationship between SA and TA in the last two chapters indicated that the students' self-evaluation was more consistent with the teacher's evaluation and that the students progressed from giving themselves false hope to critically evaluating themselves. This implies that the teacher assessment got closer to the student assessment as compared to the first three chapters. The lack of correlation between the student and teacher assessments in chapters 1 to 3 could be a result of fear that they will be excluded from the programme or that they may be considered stupid by their peers and their teacher.

4.2.5. The programme end evaluation questionnaire (PEEQ)

The PEEQ focuses on programme evaluation, namely strengths and areas for improvement on different aspects of the programme. It is divided into two major parts: a closed-ended part aimed at the evaluation of several aspects of the intervention programme, and an open-ended part aimed at evaluating the nature of the programme: areas of strength, improvement and additional comments. (For further information, see appendix 6) This questionnaire was administered at the end of the intervention programme, because the aim was to measure the students' overall view and attitude towards the programme.

The closed-ended questions were categorized into seven basic areas. In doing so, a three-point scale: ***Yes*** (**2**), ***Not sure*** (**1**), and ***No*** (**0**) was used in analyzing and comparing the results of the first six categories. However, a four-point scale was used to evaluate the programme's overall effectiveness: ***Poor*** (**1**), ***Fair*** (**2**), ***Good*** (**3**) and ***Excellent*** (**4**).

- ***Programme***: This focuses on the organization, content challenge, effectiveness, usefulness of elements, making a difference in learning, helpfulness in learning, effectiveness in developing reflective thinking, and usefulness in the role-model development of the programme.

- ***Learning***: This refers to the role of the programme in developing self-learning and responsibility, making use of varied strategies in learning, in changing attitude toward cooperative learning, and in encouraging students to actively be involved in their learning.
- ***Assessment***: This focuses on the students' understanding of the role of continuous assessment, helpfulness of self-assessment mechanisms, usefulness of continuous assessment, and the role of the programme in the continuous awareness of one's performance.
- ***Time***: This focuses on the appropriate allocation of time for task accomplishment.
- ***Resource:*** This refers to the helpfulness of the resources employed in the intervention programme.
- ***Objective achievement:*** This focuses on whether the aim of the programme was achieved. The aim was stated as: To improve the quality of mathematics learning by means of a programme that will develop their basic skills, knowledge and attitudes towards mathematics.
- ***Programme evaluation****:* This is an evaluation of the overall effectiveness of the programme.

Table 4.6: The Programme End Evaluation (PEEQ)

	Control		**Experimental**		**F**
Category	**M**	**SD**	**M**	**SD**	**(1,90)**
Programme	1.32	O.11	1.98	0.02	4.4**
Learning	0.49	0.22	1.91	0.06	15.78***
Assessment	0.22	0.07	1.90	0.09	24.00***
Time	1.68	O.48	2.00	0.00	5.33**
Resource	1.19	0.40	1.89	0.32	14.00***
Objective achievement	1.19	0.04	1.96	0.20	15.40***
Programme overall evaluation	1.64	1.05	3.87	0.34	24.78***

n=47 for experimental group and n=45 for control group
***p<0.001*
****p<0.0001*

The mean scores of the views of the experimental and the control groups on the different categories of the PEEQ were compared by running a one-way ANOVA. The results of this analysis are given in table 4.6. The results indicate that there was a significant difference in the views of the experimental and the control group, with the experimental group showing a more positive inclination towards the programme by means of which they were taught for each of the seven categories.

From table 4.6, it is evident that the experimental group of students evaluated the programme based on the PBTL approach as well-organized, the content was challenging, effective, useful in making a difference in learning, helpful in learning, effective in developing reflective thinking, and useful in role-model development [$F(1,90) = 4.4$, $P<0.001$]. Similarly, a significant difference was observed in the category *learning* [$F(1, 90) = 15.78$, $P<0.0001$]. This shows that the PBTL helped the experimental group of students in developing their learning strategies and their responsibility, in making use of varied strategies in learning, in developing positive attitudes toward cooperative learning, and in being actively involved in their learning.

A significant difference [$F(1,90) = 24.00$, $P<0.0001$] was also observed in the category *assessment*, with the experimental group of students evidencing a better understanding of the role of continuous assessment, helpfulness of self-assessment mechanisms, usefulness of continuous assessment, and the role of continuous awareness of one's performance. It is also clear that the experimental group evaluated the programme as effective in allocating sufficient time for task accomplishment [$F(1, 90) = 5.33$, $P<0.001$].

Furthermore, a significant difference [$F(1,90) = 14.00$, $P<0.0001$] was also observed in the category *resources*. In this regard, the experimental group evaluated the resources employed in the programme as more effective and important than the control group did. Significant differences were also observed in the last two categories evidencing confirmation of objective achievement and positive view on the programme. The experimental group confirmed that the intervention programme based on the PBTL approach was effective in developing basic skills, in improving performance, and in the level of mathematical knowledge.

The second part of the questionnaire consisted of three open-ended questions which were analyzed in descriptive form. The first question requested an answer on the *best aspects of the programme*, the second question required identifying *areas for improvement*, and the third question called for *comments on the programme*. A summary of the responses is given in table 4.7. From the responses it is evident that the views of both the control and the experimental groups were quite incomparable. Clearly, the experimental group focused on the fundamental ideas of the learning programme, as compared to the control group who focused on routine and short-term goals. The conclusion here is that the PBTL-based intervention programme benefited the experimental group in developing positive attitudes towards mathematics and the learning of mathematics.

Table 4.7: Students' views after the intervention programme (PEEQ)

Question number and category	View	
	Experimental group	Control group
1. ***Best aspects of the programme***	• *Developed a culture and spirit of cooperative/team-working.* • *Helpful in identifying strategies of solving perplexing problems.* • *Increased confidence to perform better in mathematics.* • *Confronted with challenging questions that stimulate critical thinking.* • *Improved attitude towards mathematics and thereby self-confidence in mathematics.* • *Encouraged a multi-approach in solving mathematical problems rather than a route to it.* • *Featured free student involvement, free discussion and high interaction.* •*Sensing real assessment: assessing oneself and others, based on honest judgment.* • *Developed accountability, responsibility, and risk taking for one's own learning.*	• *Nothing new, however, helped to save time in the actual academic year.* • *It is an additional programme, so it could help to consolidate what we have in the actual classroom.* • *This is what we experienced in the school.* • *The teacher was doing everything for us, so we thank the teacher..* • *It was supplemental programme in spare time, and this was really helpful for us, saving time for other subjects because we have covered the math that we have to cover in the academic year.*
2. A***reas for improvement***	• *The sustainability of the programme should be assured.* • *Include other subject areas as well.* • *The chapter-end self test exercises need to be explored further with the help of the teacher.* • *Needs more homework and project works.* • *Enough time should be given in dealing with chapter-end self-test exercises.*	• *The teacher should do more problems that will be helpful in the national examinations.* • *This programme is helpful in saving time and hence it will be very valuable if other subjects are also included* • *The textbook is not enough for more information*
3. ***Comment on the programme***	• *The programme should be institutionalized.* • *Opportunities should be given for other students as well.* • *An excellent programme, a different programme than we experienced at school.* • *Teaching material should be kept up as it is.* • *Teachers at school should be well acquainted with these methods and approaches of teaching/learning strategies.* • *More technology should be included, like the computer.* • *The programme is more important than what we experienced at school, so schools should experience it as well.*	• *The programme should be presented every summer season.* • *The programme should focus on ways that are helpful to succeed in the national exams.* • *The information and the problems in the textbook are not enough. Therefore, the teacher should provide us with other ways that could help in solving problems, and come with some important problems and solve them for us.* • *The programme should focus on the areas were national level questions are usually available.*

4.3. Discussion

4.3.1 The students' performance on the MT

A comparison of the students' performance in the mathematical test (MT) indicated that the experimental group performed better than the control group after the intervention programme. Apparently, these two groups of students were both the outcomes of the traditional teacher-dominated and transmission-based instructional approach. Furthermore, these students were selected from the same public government school. During the intervention programme, the experimental group was involved in discovering relationships, inquiry mathematics, self-regulated problem-solving, reflective practices, and so on, which are very helpful in developing critical thinking. On the other hand, the control group was taught by means of the traditional transmission-based approach, called 'chalk and talk'. This implies that their focus in learning was more on the memorization of facts and procedures and on the mastering of rules.

The MT covered a wide area of the mathematical content, as well as a broad area of mathematics problems. For example, it covered problem-solving, applications of mathematical ideas, basic properties, representations, algorithms, rules, and procedures. However, the data analysis and interpretation were not based on these contents and areas. There were two major reasons for this: the first, due to the range of the data and the challenge to manage it; the second reason, was to rather focus on the contents and areas mentioned as elements of the mathematical test and analyses thereof to reach a conclusion. Triangulation was believed to be a better means to come up with reliable and valid results.

Although an improvement was observed within the groups themselves, a comparison between the results of the two groups revealed a significant difference. It therefore seems reasonable to conclude that the intervention programme on PBTL had a positive impact on students in the experimental group as far as the following aspects are concerned: the construction of sophisticated conceptual understandings that help to improve their problem-solving abilities, the use of representations, applying mathematics to the context, and the real understanding of algorithms, rules and procedures.

4.3.2 The students' performance on the RT

A comparison of the two groups of students indicated that the experimental group out-performed the control group on the procedural, the conceptual, and the sum of both the procedural and conceptual parts of the test. However, the difference in performance is much more relevant in the conceptual part than in the procedural part of the test.

The questions on the conceptual part were correspondingly advanced from their procedural counter parts as was proposed by van de Walle (1998), Brumbaugh et al. (1997), and Koehler and Prior (1993). The results further indicated that students who were taught by means of the informal approach had a better chance to approach problems in the procedural part by moving back from the conceptual part. This implies that conceptual understanding helps learners to move back and forth, and relates the conceptual and procedural items and put the procedural questions as an element of the conclusion of the conceptual part.

In summary, it seems reasonable to conclude that the experimental group was helped by the intervention to develop their conceptual understanding to a larger extent as compared to those students who were taught by means of the procedural oriented approach. Moreover, the experimental group seemed to be successful in the procedural part as well, due to their conceptual understanding creating opportunities for them to tackle procedural problems.

4.3.3 The students' self-regulation

The F-test results listed in table 4.3 indicate that more significant changes were observed in the experimental group than in the control group on each of the ten scales of the LASSI-HS regarding a particular item concerning the way the student learns, studies, and feels about learning and studying.

In this regard, the experimental group of students demonstrated a significant improvement on the 'anxiety' scale. This improvement suggests that students taught by means of the PBTL were helped to release tension and develop a liking for mathematics. A significant improvement on the *attitude* scale was also observed, indicating that the experimental group developed a more positive attitude towards mathematics than the control group.

The significant difference between the experimental and the control groups on the *concentration* scale indicated that the experimental group of students was helped by the PBTL approach to improve their concentration when doing mathematics, and also with their studies. Similarly, a significant improvement was observed in the experimental group's *information processing.* This means that the experimental group improved by making use of strategies to elaborate and organize information, monitor comprehension and to relate new material to prior knowledge.

The significant increase in the *motivation* scale scores indicated that the PBTL approach and material encouraged students' motivation and helped them to shoulder the responsibility for their learning. Furthermore, the experimental group's significant improvement in the *study aid* scale suggested that the PBTL was helpful in improving the students' use and generation of diverse technical solutions and materials aimed at supporting and increasing meaningful learning and memorization. In the *selecting main ideas* scale as well, the experimental group made significant progress in selecting and identifying important ideas in the content they were dealing with, as compared to the control group.

A comparison of students' improvement in the *self-testing* scale indicated that the experimental group made more significant progress as compared to their control group counterparts. The result showed that students in the experimental group were much more conscious of monitoring or checking their understanding as they studied and revised the study material than the control group. It seems reasonable to conclude that the PBTL-approach helped the experimental group in making progress on this scale as well. Also, a significant difference was observed between the two groups in the *time management* scale. The result indicated that the experimental group was assisted by the PBTL-approach to make significant changes in creating and using schedules to organize and control their work progress.

In the *test strategies* scale, the results indicated that the experimental group significantly employed important strategies during both test-preparation and test-taking, as compared to the control group.

4.3.4 The students' quality of participation

This on-progress instrument was employed for the experimental group of students only, because the assurance of quality participation when students are engaged in cooperative learning is not in accordance with the traditional teacher-dominated and transmission-based instructional approach which was used with the control group.

The F-test results for the cooperative behaviour checklist (CBC) are listed for the three areas *listening, supporting,* and *on-task* during the first and the last weeks of the intervention programme, as displayed in table 4.4. These results indicate positive shifts in each of the areas. Although this instrument was used throughout the intervention programme, that is, from the first to the last week, the analysis and the interpretation of the data was carried out only for the first and the last weeks.

Listening. The students demonstrated a significant improvement from the first week through the consecutive weeks to the last week. This indicates that the students progressed from poor participation to being active listeners. This means that the students gave attention when others forwarded ideas, and took advantage of others' ideas and the results of their investigation (McGlinn, 1991; Hiebert *et al.*, 1999). Thus, the CBC incorporated in the PBTL-approach was also helpful in improving students' quality of participation in group discussions and cooperative learning.

On-task. Significant improvement was also observed in this area. It was obvious that the PBTL-approach contributed to a shift from a negligent type of involvement towards being active participants. This implies that students became duty conscious and felt responsible for the success of their group or the team in which they were working, and also for themselves.

Supporting. A significant shift was observed in the quality of participation from the first week to the last week of the intervention programme, indicating that the students developed a sense of team sprit in working in teams. Furthermore, students preferred working with others, negotiating ideas with team members and other teams. This also includes supporting others in the process of accomplishing tasks. Table 4.8 below summarizes the outcomes of the CBC.

Table 4.8: Outcomes of the CBC on the students' quality of participation

Students' experience in the first week	*Students' experience in the last week*
• *Nearly all members preferred to be passive listeners.* • *Students' participation rate was inconsistent* • *Resistance to contribute and share ideas in the group and with group members.* • *Fear to take risks in their learning and less confident in their learning of mathematics.* • *They pretend to be attentive but without initiating ideas and without interest.* • *Less courage to support others and to work with them.*	▪ *Avoid dominance.* ▪ *Create an environment that calls for a fair share of task accomplishment.* ▪ *Develop a sense of shouldering responsibility and accountability for own learning.* ▪ *Increased participation by giving up dominance and silent listening.* ▪ *Practice managerial skills, develop the required group behavior like facilitating, guiding, problem-solving, critical thinking by minimizing undesired group behavior like being a silent listener, saboteur, etc.*

4.3.5 The Portfolio of evidence: the students' self-assessment and teacher-assessment

This on-progress instrument was also employed throughout the intervention programme for the experimental group only. The reason is that authentic assessment like the portfolio of evidence is not commensurate with the traditional teacher-dominated and transmission-based approach. Rather, this method is closely related to the constructivist (socio-constructivist) view of assessment. The ρ_values listed in table 4.5 indicate evidence of an increase in the degree of similarity between the teacher's assessment and students' self-assessment. In the first chapter, the teacher critically evaluated the students' chapter-end self-assessment. On the other hand, the students' assessment of their performances was relatively high. It should be remembered that these students were taught by means of the traditional approach and they would not be critical of this new approach in chapter one. The reason was that they feared that if they critically assessed themselves and assigned themselves lesser marks their teacher or peers may consider them dull.

Efforts were made to convince the students to assess the results that would represent their real ability. Though there was more of a correlation as from the second chapter, there was still an inclination to persist in not honestly evaluating themselves in chapters two and three. In chapters 4 and 5 the students' assessment (SA) and the teacher's assessment (TA) were moderately correlated. This indicates that the students developed to the point of being more able to assess themselves honestly and critically.

In conclusion, the results in table 4.5 indicate that PBTL helped the students to be able to critically evaluate their work and to become critical at self-evaluation.

4.3.6 The programme effectiveness evaluation

The results in tables 4.6 and 4.7 indicate that there was a significant difference between the experimental and control groups in each of the seven areas, namely, *programme*, *learning, assessment, time*, *resource*, *programme overall evaluation*, and *programme objective achievement* as described in section 4.3.5. It is evident that the experimental group was more aware of the areas and developed a more positive outlook towards the programme.

The responses to the open-ended questions given by the experimental and control groups could be categorized as 'fundamental' versus 'routine', respectively (see table 4.7). The researcher chose these concepts because the responses of the two groups were influenced by their experiences. The experimental group of students focused on fundamental ideas that are critical to learning. These students evaluated the programme from a psychological and philosophical point of view. For example, they evaluated the programme from the point of experiencing involvement and interaction, of meaningful learning and effective teaching, and of long range planning. On the other hand, the control group evaluated the programme in respect of immediate and short-term goals, such as good grades. However, this does not lead to and encourage lifelong learning. It relates to immediate achievement. For instance, most responses of the students in the control group seemed to originate from success in the exams, either at the local level or at the national level. In the discussion of the responses to three open-ended questions, it should be noted that the two groups came from the same school and were taught through the same instructional process, the 'talk and chalk' approach, throughout their school career.

4.4 Summary

The research was intended to answer four fundamental questions that were stated at the beginning of the research. The data analysis and results seem to warrant several conclusions in relation to these research questions. The first conclusion is that a problem-based teaching and learning (PBTL) approach to the teaching of mathematics in high schools *does* have a positive impact on the mathematical performance and achievement of the students. Significant improvements were observed when comparisons were made between the mean scores of the experimental and control groups.

The second conclusion is that the PBTL-approach to the teaching of mathematics to high school students has a positive impact on the students' ability to identify, formulate, and solve problems, and to evaluate results. This was observed from the data obtained by means of the reference tests and the students' work in the portfolios of evidence.

The third conclusion is that a PBTL-approach has a positive effect on the students' self-regulation and meta-cognitive strategies. The data obtained by means of the LASSI-HS and the analysis thereof suggest that the experimental group showed a significant improvement on the LASSI-HS, evidencing a higher degree of improvement than the control group, in all of the ten scales.

The fourth conclusion is that a PBTL-approach has a positive effect on students' motivation and self-confidence. The data obtained from the LASSI-HS indicated that there was a significant difference between the control group and the experimental group regarding motivational beliefs. Furthermore, the analysis of the PEEQ data suggests that a change took place in their self-confidence to work with others, to do mathematics, and to be responsible and accountable for their learning. They also developed the confidence to shoulder responsibilities in their learning and to be able to take risks in their studies.

The results of the PE analysis also evidenced that the PBTL-approach to mathematics improved their reflective thinking practices. Furthermore, the CBC results from the first and the last weeks of the intervention programme indicated that the students made great progress in their listening skills, their desire to support and to work with others, and to engage in mathematical tasks with concentration. It is clear that these two instruments were important devices in developing their meta-cognitive skills and their reflective thinking in mathematics learning.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1. Introduction

The purpose of this study was to investigate the impact of PBTL on the traditional transmission-based mathematics classroom. To reach this aim a six month long intervention programme was introduced to an experimental group, while a control group was not exposed to this programme. Fundamental elements of the research project were identified at the beginning of the programme.

The first of these was formulating the following research question:

What is the effect of problem-based teaching and learning on student performance and achievement in mathematics?

In order to obtain an answer to this question specific research questions and accompanying aims were formulated.

After theoretical knowledge was sought on the influence of PBTL on the learning of mathematics an educational experiment was conducted. An intervention programme was designed where problem-based teaching and learning (PBTL) were implemented and practiced.

In this chapter the findings of the research will be summarized, and conclusions and recommendations will be made.

5.2 Summary

5.2.1 Summary of the literature review

Research reports gave due attention to the constructivist approaches to teaching and learning mathematics (Thornton & Wilson, 1993:277). This theory claims that students learn when there is interaction between their thinking and experience via a sequential development of more cognitive structures (Pollard, 2002:138). It also emphasizes self-discovery and the social context in which learning takes place. The contribution of peers, the teacher, and other adults to support the student's learning is accentuated. As a model of instruction, a student-centered approach for the teaching of mathematics was recommended.

Student-centered learning is fundamentally based on the active involvement of students in their own learning. In other words, in a learner-centered environment students are challenged, they are given an introduction of what is to be expected, are given choices and control, are encouraged to work cooperatively, are given material that is motivating and applicable, are made confident about their personal competence to thrive, are given individualized consideration pertaining to their learning preferences, and finally, are given the power to contribute to the standards and methods of what would be employed for evaluation (Aggarwal, 1996).

Student-centered teaching and learning in mathematics could take different forms. Amongst the most well-known is the problem based teaching and learning (PBTL) approach (Felder & Brent, 2002; Harris *et al*., 2001; Leu, 2002). The approach has fundamental principles that enhance self-discovery and knowledge-construction by making connections between new aspects of information and the learner's existing network through self-regulated problem-solving (Koehler & Prior, 1993:288-295; Murray *et al.,* 1998;). PBTL incorporates several specific instructional strategies, discussed in earlier chapters. It seems to be the best approach to achieve the general goals of learning mathematics, as outlined by the National Council of the Teachers of Mathematics in its *Curriculum* and *Evaluation Standards* (NCTM, 1989). These goals are: (1) learning to value mathematics, (2) becoming confident in their ability to learn mathematics, (3) becoming mathematics problem-solvers, (4) learning to communicate mathematically, and (5) learning to reason mathematically (NCTM, 1989:5).

The fundamental underlying principles that describe PBTL as an approach in teaching mathematics are multifaceted. The first of these are curriculum principles. In this regard, mathematics is thought to be learned best in the context of sense-making and engaging problems that interconnect mathematics concepts, also with applications in other subject areas (NCTM, 2000). The emphasis is on principles and methods of investigation, and not on mechanical skills. Concepts and skills are learned in context through a variety of problems, which are both routine and non-routine. Furthermore, there is an opportunity for students to work both independently and cooperatively on long-term projects and explorations (Harris *et al.,* 2001; Fendel *et al.,* 1997).

Another of the principles mentioned is the employment of an appropriate instructional strategy. This includes cooperation during learning, particularly in cooperative learning groups. Learning is a social process and hence students learn best through sharing and negotiating mathematical ideas (Bell, 1993; Clarke, 1997; Voigt, 1996). The teacher, students, and peers could be considered a community of learners with the teacher playing a key role such as observing, facilitating, guiding, and supporting the students, and leading discussions (Clarke, 1997; Huetinck & Munshin, 2000; Pollard, 2002). Another basic element is communication in learning mathematics. The skill of communication involves sharing both triumphs and frustrations in working on mathematical problems; an opportunity to write about mathematical thinking, to reflect on what students have done, and to make oral and written presentations to one another about their work (NCTM, 2000; Hiebert et al., 1998).

The PBTL-approach also considers assessment to be an integral part of instruction, and not an interruption (NCTM, 2000). Fundamentally, it places the emphasis on on-progress assessment tools to evaluate what the students are learning. This includes students' written and oral presentations, teacher assessment of student interactions during cooperative group-work, and students' evaluation of their own and one another's work, including their portfolios. In doing so, the focus of the assessment is on the process, and not on the product, as element of the portfolio of evidence, including self-, peer-, and group-assessment. These elements are thought to develop the students' meta-cognitive skills (Tanner & Jones, 1994; Van den Berg, 2004).

Interactions with others are considered to be the heartbeat of mathematics learning (Koehler & Prior, 1993). Improving classroom interaction involves essential elements, amongst others, questioning at an appropriate level, responding to student-initiated interactions such as questioning and problem-solving, monitoring peer group interactions (exchange among students) and finally, considering classroom realities (respecting students and providing support when necessary).

5.2.2 Summary of the findings of the empirical investigation

The data analyses lead to several findings. The first is that problem-based teaching and learning (PBTL) has a positive effect on the performance and achievement of mathematics students. It was observed that the experimental group significantly outperformed ($p<0.0001$) the control group on the mathematical test (MT) after they had been subjected to the PBTL intervention programme. This finding is consistent with the reports of Cobb *et al.,* (1991) about “assessment of problem-centered for second graders”, Murray *et al.*, (1998) about “learning through problem-solving”, Lampert (1989), about “teaching for understanding and developing mathematical power” in the intermediate grades, and Hiebert and Wearne (1992) about “conceptually based” instruction of the first graders.

The second aim of this study was to determine the effect of PBTL on the ability of students in identifying, formulating and solving problems, and in evaluating results. According to the results obtained, the experimental group of students performed significantly better ($p<0.0001$) than the control group in this regard. This research result is compatible with the findings reported by Carpenter, Fennema, Peterson, Chiang, and Loef (1989), Soled (1990), Fennema, Carpenter, and Peterson (1989), Heibert and Wearne (1992), and Cobb *et al.*, (1991). The latter observed that an experimental group of students outperformed the control group of students on measures of conceptual understanding and higher order applications. These results are indicative of the rejection of traditional teaching methods, and instead, calling for a constructivist learning theory.

The third aim was to determine how PBTL will affect students’ self-regulation and meta-cognition. The data analysis showed a significant improvement in the experimental group regarding self-regulated strategies, that is, ways of managing and controlling-*(*planning, monitoring and evaluating) processes. In this regard, significant improvement in mean scale values were observed for all ten the scales, namely, motivation, anxiety, study-aid, attitude, time-management, concentration, information-processing, selecting main ideas, self-testing, and test strategies.

The data analysis of the portfolio of evidence (PE) showed that the percentage of variance between the students’ assessment (SA) and the teacher’s assessment (TA) increased through the chapters 1 to 5, indicating an improvement in the students’ reflective practices, and their self-judgment being more realistic. This finding supports Van den Berg’s (2004) suspicion that the assessment dimensions (self-, peer-, and group assessment) as components of the PE, could stimulate reflection and meta-cognition. It had an impact on the improvement of the students’ self-regulating skills and in their knowledge of controlling their cognition, that is, in developing meta-cognition.

The fourth aim was to test the effect of problem-based teaching and learning (PBTL) on students’ motivation and self-confidence. Analyses of the data obtained by means of the Cooperative Behaviour Checklist (CBC) and the Programme End Evaluation Questionnaire (PEEQ) were useful in coming to conclusions in this regard. It was found that there was a significant improvement in students' quality of participation. Significant shifts in mean scale values were obtained for each of the three measures: *listening, on-task* and *supporting* when comparisons were made between the first week and the last week of the intervention programme. This indicates students' increased motivation and self-confidence to work with team members. The results of the analysis on the PEEQ indicated that the experimental group showed a significant higher degree of positive confirmation in each of the seven areas (*programme, learning, assessment, allotted time, resource, objective achievement and programme overall evaluation*) and in the open-ended part of this instrument, as compared to the control group.

The above-mentioned results on the CBC and the PEEQ seem to be consistent with the findings reported by Cobb *et al.*, (1991). The report evidences that the experimental group of students held stronger beliefs about the importance of understanding and collaborating in mathematics. They indicated a positive attitude towards cooperative learning in mathematics, with less emphasis on test-oriented learning. This finding seems also to be reconcilable with the CGI-led project reports which mentioned that the students showed significant progress in problem-solving and mathematical confidence compared to those students taught conventionally (Carpenter *et al.*, 1989). In line herewith, a number of well-known and successful projects such as IMPACT (*Increasing the Mathematical Power of All Children and Teachers)* which is based on Cognitively Guided Instruction (CGI) (Carey, Fennema, Carpenter, & Franke, 1995), QUASAR (*Qualitative Understanding: Amplifying Student Achievement and Reasoning*) (Silver, Smith, & Nelson, 1995:10), and SDMED (*The San Diego Mathematics Enrichment Project*) (Bezuk, Armstrong, Ellis, Holmes, & Sowder, 1993) are witnesses of the success stories.

The teachers' summary reports about the intervention programme were practically consistent with Clarke's (1997) conceptual framework of what the teacher should do, and his/her related views about the teaching and the learning of mathematics in a reformed classroom. The teachers commented on the nature of tasks, the context of teaching and learning, instructional strategies, modes of communication, focus of learning, and the assessment methods employed.

5.3. Recommendations

From the perspective of the findings of the research project the following recommendations are made:

1. *Students should comment on their own learning*. Students should be given the opportunity to reflect on their own learning, because this was found essential in developing their meta-cognitive and self-regulating skills. There are several methods that can be used to provide students with the opportunity to comment on their learning, such as students' portfolios of evidence, including the reflective diary, checklists, chapter-end progress self-assessment, and the completion of questionnaires.

2. *Every effort should be made to develop positive attitudes towards mathematics*. Teachers should do their utmost to ensure that students develop positive attitudes towards mathematics by focusing on motivational skills, developing strategies for effective learning, developing resource management skills, developing skills to work with peers/colleagues cooperatively and, in general, teaching students to "learn how to learn". As a result of the intervention programme students developed positive attitudes, felt proud and confident in learning and doing mathematics, were interested in solving problems, and in the end, promised to carry on doing mathematics.

3. *School teachers and teacher educators should be involved in in-service training programmes*. There must be a transition from the traditional skill-based curriculum to a problem-based curriculum which enhances the students' conceptual development. Teachers should be given the opportunity to master new instructional strategies. To make this transition, they need to be involved in in-service training programmes such as workshops and refresher courses, they should be given adequate preparation time, opportunities for team teaching, as well as opportunities for sharing their experiences with colleagues.

The importance of the above-mentioned needs was evident from the comments of the students and the five teacher educators who were involved in the programme. They suggested that school teachers should be acquainted with the methods and philosophies of the programme (PBTL) they were engaged in.

4. *There should be scheduled programme moderation*. Such an action would help to identify areas that need improvement, areas of strength, and directions that are helpful for future instructional processes. In relation to this, the moderator of the intervention programme commented as follows on the programme and, in particular, on students' involvement, on the appropriateness of tasks, and on the organization and management of the portfolios:

> "My comments focus on students' portfolio of evidence. I learnt that the portfolios meet the principles suggested by Reece and Walker (2003). To say it in other words, the portfolios are valid as they are directly related to the standard. They have the right authority as the evidences relate to the students' own abilities. Moreover, the evidences are up-to-date. Last but not least, the evidences are sufficient enough. My second comments focus on the authenticity of assessment practices. I learnt that students have made self-assessments so as to show their progress. In other words, the students got an opportunity to reflect on their own learning. This shows they were responsible and accountable in selecting tasks, in assembling materials, in demonstrating and showing their thinking power and expressive skills. They were able to put their insight on things they know and they do not know in a balanced and sensible way. All these make the portfolios informative enough. These are correct evidences of students' progress and life learning. My final comment is, the programme should be built and planned in a sustainable manner for future generations."

5. *Involve as many stakeholders as possible in the education community to build an intervention programme and to participate in the delivery thereof*. Involve all the school community members in planning and running an intervention project and indicate the contribution expected from these bodies. The students involved in the experiment commented on this aspect and the literature also emphasizes its importance (see Projects SDMED, QUASAR and IMPACT). All of these stakeholders, the teacher, the students' parents and other educational communities, can play a leading role in planning and running a project of this kind with the accompanying improvement of education.

6. The mathematics curriculum has to focus on all-encompassing and integrated ideas. This kind of curriculum calls for an effective articulation between the theory and practice available in the realm of mathematics. It should be based on new views and approaches rather than on the traditional positivist and transmission viewpoints. It is proposed that the curriculum must focus on an 'informal teaching style' which is in sharp contrast to a 'formal teaching style.' The informal teaching style emphasizes student-centered activities and the use of more open-ended tasks which are negotiated with students. In this regard, the curriculum should blend teaching and learning methods that comprise several specific instructional methods.

In such a curriculum, themes such as mathematical communication, reasoning and proof, connections, and problem-solving are all treated as an integral part of the curriculum. Related activities have to encourage students to explore, develop, investigate and construct their mathematics ideas and concepts.

5.4 Limitations of the study

This study was a small-scale research project, investigating the effect of PBTL on the achievement of students in mathematics. This work is a stepping-stone for larger scale research, and can be regarded as initial research that may provide information on new views and possibilities in the areas of the mathematics curriculum and instruction. Further research, involving more participants and researchers, will provide and consolidate results that will have fundamental benefits for the educational community at large.

5.5 Conclusions

The programme which was developed and modeled on the philosophy of problem-based teaching and learning (PBTL) and the constructivist view of learning had a positive impact on students' performance and achievement in mathematics. Its major benefits included providing students with positive experiences in mathematics and increasing their future learning and involvement in mathematical activities. It also contributed to the development of notions concerning the cognitive, meta-cognitive, and affective domains, and self-regulating skills, and the improvement of affective factors influencing students' learning.

It is believed that the findings obtained by means of this research project could have important implications for the educational system. The intervention programme evidenced that it is possible to reach nearly all students in the teaching of mathematics and to develop their confidence in their ability to do mathematics. This has the impact of empowering students to accept responsibility and accountability for their own learning, whereby showing their willingness to accept academic challenges and to grapple with the perplexing dilemmas they are confronted with.

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Appendix 1 (A1): Mathematical Test

Code Number: ___________

General Direction:

1. Attempt all questions.

2. Put your answer on the space provided arranged immediately before the question number

3. You should rely completely on yourself.

4. Time allotted is 4 hours.

5. Don't use colored pens (red, green) and pencil.

Choose the best answer from the alternatives given and put on the space provided on the left of each question number.

_______ 1. *Let* $P \equiv \sqrt{2}$ *is irrational and* $q \equiv 3+7=11$. *Which one is an equivalent symbolic representation for the statement " Neither* $\sqrt{2}$ *is irrational nor 3+7= 11" ?*

A. $\neg p v \neg q$ B. $\neg p \wedge \neg q$ C. $\neg p \wedge q$ D. $\neg(p \wedge q)$

_______ 2. *If A and B are sets, A is a proper subset of B, and A is equivalent to B, then which must be* ***True***?

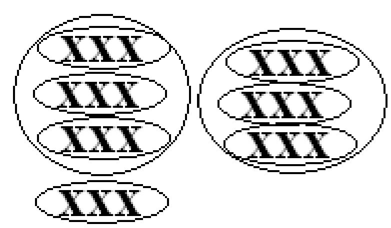
A. $B \subseteq A$ B. $A = B$

C. B is an infinite set D. None of A, B or C

______ 3. *If A = {0} and B = {0, {0}}, which one is* ***NOT TRUE****?*

A. A∩B= A B. $A/B = \phi$ C. B/A= {0} D. $A \Delta B = B$ $E.AUB = B$

_______ 4. *The number for this set suggested by the grouping (in base 3) has:*



A. A 1 in the third place from the right

B. A 1 in the second place from the right

C. No 1in its numeral

D. None of A, B or C

_______ 5. *A number is perfect if the sum of its proper divisors is equal to the number. Which number is perfect?*

A. 12 B. 15 C. 18 D. 28

_______ 6. *How many zeros are required to be able to write* $4.8^7 + 6.8^2 + 2$ *in it standard base 8 representations:*

A. 4 B. 5 C. 6 D. None of A, B, or C

_______ **7.** *A total of 20 children and dogs are playing in the park. If you counted their legs, you would get 56 in all. How many children and how many dogs are there?*

A. 12 and 24 C. 12 and 8

B. 2 4 and 20 D. 14 and 6

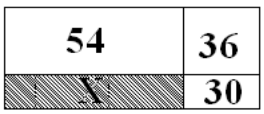
_______ **8.** *Sue has 2 brothers. She is 3 times as old as Michael, her youngest brother. The age of her* *other brother, Jon, is the difference between Sue's and Michael's ages. If the sum of all their ages is 36, how old is Jon?*

A. 6 B. 12 C. 18 D. None of A, B or C

_______ 9. *When 983_{12} is changed to base 10 numeral, the hundreds digit is:*

A. 1 B. 2 C. 7 D. None of A, B or C

_______10. *What is X equal to for X represents area of the shaded region in the figure and all shapes are rectangles?*



A. 24 B. 45

C. 54 D. None of A, B, or D

_______ 11. ***N*** *people met at a business meeting. If each person shook hands with every other person exactly once and there were 66 such shakes, how many people where there in the business meeting?*

A. 11 B. 13 C. 12 D. 14

_______ 12. *Using the set union definition of the addition of whole numbers, which of the following pairs of sets could be used to show that 2+4 is 6?*

A. {a, b} and {a, c, d, e}

B. {2} and {4}

C. {x, y} and {x, b, z, w}

D. {1, 3} and {2, 4, 6, 8}

_______ 13. *Using this addition table, find (A+B)+C*

A. A B. B

D. D C. C

+	A B C D
A	A B C D
B	B C D A
C	C D A B
D	D A B C

_______ 14. *If **a** divided by **b** gives a quotient of **c** and a remainder **d**, then*

A. ac+d = b

B. bc = a+d

C. bc = a-d

D. ac-d = b

_______15. *A survey was taken in a school cafteria. Of the 125 students surveyed:*
47 *liked hanbergers*
30 *liked pizza*
12 *liked both pizza and hamburgers Identify the **FALSE** statement.*

A. The number of students who liked either hamburgers or pizza is 65.
B. The number of students who liked either hamburger or pizza but not both is 53.
C. The number of students who liked neither hamburger nor pizza is 72.
D. The number of students who didn't like pizza or hamburgers is 123.

_______ 16. *How many ways can you make a change for 35 cents using only **nickels**, **dimes**, or **quarters**?*

A. 6 B. 8 C. 9 D. 12

_______ 17. *Which one is **NOT TRUE**?* (a an integer)

A. $0.99\overline{9} = 1$

C. $a.\overline{9} = a + 1 \text{ for } a > 0$

B. $-7.\overline{9}=-6$ D. $a.\overline{9}=a-1 \; for \; a<0$

_______ **18**. *Fill in the correct values for this intermediate addition algorithm. Then find the sum of the numbers in the three boxes.*

	4	8	9	2
+	7	6	5	9
			1	1
		1	□	0
	□	4	0	0
1	1	□	0	0

A. 6

B. 5

C. 5

D. None of A, B, or C

_______ **19**. *When the following numbers of stars as a base three numeral are expressed it equals:*

*********** A. 102 B. 103 C. 1022 D. 1202

_______ **20**. *Which of the following are **EQUAL**?*

(i) $37_8 + 124_8$ (ii) $354_8 - 156_8$ (iii) $328 \times 5_8$

A. ii and I B. i and iii

C. ii and iii D. None of A, B, or C

_______ **21**. *Which one of the following is **NEVER** true for a function f?*

A. f(x).f(y)= f(x+y)
B. f(x.y)=f(x)+f(y)
C. f(x+y)=f(x)+f(y)
D. f(x/y)=f(x)-f(y)
E. None of these

_______ **22**. *Which of the following **DOES NOT** divide 1357924860?*

A. 9 B. 4 C. 6 D. 8

_______ **23**. *If **a**=$2^7.3^3.5^2$, which of the following is **FALSE**?*

A. 15 divides ***a*** C. 21 divides ***a***

B. 12 divides ***a*** D. 75 divides ***a***

_______ **24**. If *the six digits number **23k, 168**, is divisible by 3, then which one is **NOT** possible value of k?*

A. K=1 C. k=7

B. K= 4 D. k=10

_______ 25. *The LCM is $2^2.3^3.5^2.13$ and the GCF is $2^2.3^2.13$, one number is 1404, what is the **SUM** of the digits of the other number?*

A. 9 B. 8 C. 7 D. None of A, B or C

_______ **26**. *If p and q are primes (but neither is 2), then*

A. 2p is prime C. p+q is prime

B. p + q is composite D. p+q is an odd number

_______ **27**. *If the number of divisors of the number $p=5^{\alpha}.17$ is 6, then the natural number represented by p is:*

A. 85 B. 425 C. 170 D. None of A, B, or C

_______ **28**. *Which one is **TRUE**?*

(i) If a and b are unit digits ab+ba is a multiple of 11

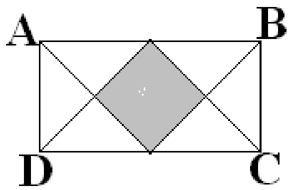
(ii) If a and b are unit digits ab-ba is a multiple of 9

(iii) if a, b and c are unit digits abc-cba is multiple of 99

(iv) If a and b are unit digits then ab9-ab is a multiple of 90

A. I and iv b. iv only C. ii and iii D. i, ii, and iii

_______ **29**. *If ABCD is a rectangle and angle bisectors intersecting on the sides what is the area of the shaded region if a and b are its dimensions?*



A. ab

B. $\frac{1}{2}$ ab

C. ab/4

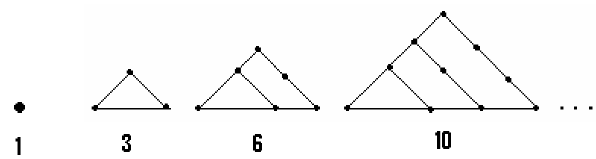
D. a^2-b^2 E. None of these

_______ **30**. *Consider the following dots that represent numbers called triangular. How many dots will have the number in the fifth pattern?*

A. 15 C. 22

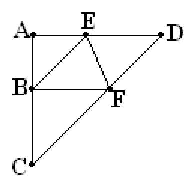
B. 18 D. 23

E. None of A, B, C, or D



_______ **31**. *If BC+BE=AD, BE=EF which one is* ***TRUE****?*

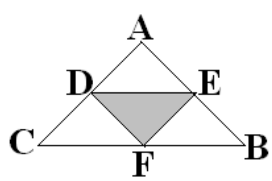
A. BC+EF=AD
B. BC+EF=CD
C. BC+BF=AD
D. AB+BC= AD



_______ **32.** *ABC below represents Equilateral triangle and D,E and F are midpoints. What is the probability of tossing a coin to fall on the shaded region and facing head up?*

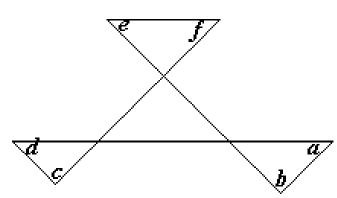
A. 1/8 C. 1/4

B. 1/2 D. 1/16



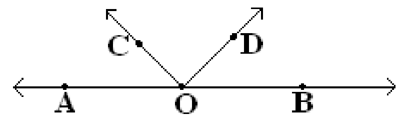
_______ **33**. *The sum of the angles* ***a*** *through* ***f*** *is* __________

A. π B. 3π
C. 2π D. 4π



________**34**. *In the figure right below, m (∠ AOC) = 7x-10, m(∠ DOB)= 8x+5 and m(∠ BOC)= 125º,* x= ______

A. 20 B. 140 C. 160 D. 130



_______ **35**. *If the following numbers are arranged from* ***least to greatest****, which one will be in the middle?*

3/7, 2/9, 5/13, 7/11, 5/12

A. 3/7 B. 5/12 C. 5/13 D. 7/11

_______ **36**. *Which of these fractions is* ***between*** *7/15 and 8/13?*

A. 3/5 B. 2/5 C. 2/3 D. None of A, B, or C

_______ **37**. *If the expanded form of a number is* $3(10^2)+7+4(1/10)+5(1/10^3)$, *then the numeral representing that number has how many digits?*

A. 4 B. 5 C. 6 D. None of A, B, or C

_______ **38**. *Express* $0.\overline{367}$ *as a fraction in simplest form. The sum of the numerator and denominator of this fraction is:*

A. 367 B. 999 C. 1366 D. 27

_______ **39**. *If* 0.0000004205 is divided by 0.00000006001, *the answer is approximately:*

A. 0.7 B. 0.07 C. 0.007 D. 0007

_______ **40**. *If* $15.00 *shirt is reduced* 35% *and* a $24.00 *pair of pants is reduced* 15%, *what is the total sale of the two items?*

A. $8.85 B. $19.50 C. $30.15 D. None of A, B, or C

_______ **41**. *An item was marked up 25% and then this price was marked down 20%. The net effect of these two markings is a*

A. 0% mark up
B. 2.5% mark up
C. 5% mark up
D. Not enough information to work the problem.

_______ **42**. *Which is the* ***best buy****?*

A. 13 OZ for 70 cents C. 24 Oz for $1.36
B. 27 Oz for $1.68 D. 32 Oz for $2.00

_______ **43**. *Miss Kelley and Mr. Peabody have 25 and 36 children in their classrooms, respectively. The ratio of boys to girls in Miss Kelley's classroom is 3:2 and the ratio of girls to all the children in Mr. Peabody's classroom is 1:3. If the two classes are combined, the ratio of girls to boys is:*

A. 3:5 B. 2:7 C. 22:39 D. 19:42

_______ **44**. *Three baseball players, Chuck, Mark, and Paul, play for Chicago, Montreal and Philadelphia. No player's city starts with the same letter as his name. Paul has never been to Chicago. Who plays for Philadelphia?*

A. Chuck B. Mark C. Paul D. Mark and Paul

_______ 45. Which one is **FALSE**?

(i) $\pi = 22/7$

(ii) $22/7 = 3.1428571$

(iii) $6.36\overline{0}$ is both rational and terminating

A. i and ii B. ii only C. ii and iii D. iii only

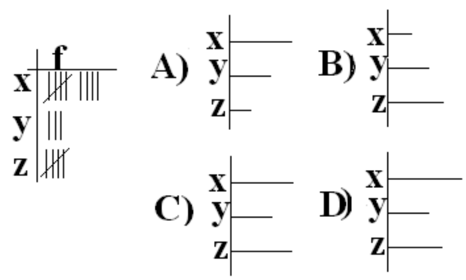
_______ 46. *Let f(x)= 4^x and g(x) = $\log_4^x$, x>0. Then*

A. g(f(x))= x, x ∈ ℜ
B. f(g(x))=x, x>0
C. f(g(-2))=-2
D. g(f(-1))=-1
E. Domain of g(f(x))={x/x ∈ ℜ }

_______ 47. *I am thinking of a number. It is a* ***prime*** *number* ***less than 100****. The* ***sum*** *of its digits is* ***16****. If you* ***reverse*** *the digits, the resulting number is also* ***prime****. The number is* ____________

A. 79 B. 97 C. 87 D. 93 D. A and B

_______ 48. Which ***bar graph*** best represents the *frequency* table?



_______ 49. *The numerical value of tan 15°+tan 75° is*

A. $2+\sqrt{3}$ B. 4 C. $2-\sqrt{3}$ D. –4 E. 3

_______ 50. *Which is* ***TRUE*** *when comparing two sets of scores?*

(i) The larger standard deviation comes from the set with the larger mean.

(ii) If the means are equal, so are the variances

A. i only B. ii only C. i and ii D. Neither is true

_______ 51. *When the complex fraction* $1-\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{x}}}$ *is simplified the result is:*

A. 1 B. x C. x+1 D. 1-x E. –x

_______ 52. A person walking along a straight road observes that at two consecutives points A and B 2kms apart, the angles of elevation of a hill in front of him are 15^0 and 30^0 respectively. The height of the hill is:

A. $2(2-\sqrt{2})km$ B. $(2\sqrt{3}-1)km$ C. $\dfrac{2-\sqrt{3}}{2}km$ D. None of the above

_______ 53. *There are 26 names in a hat, 13 girls' names, and 13 guys' names. What is the probability that 2 girls' names will be drawn?*

A. 4/25 B. 1/4 C. 6/25 D. 1/3

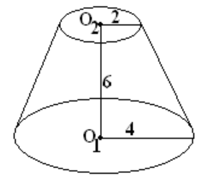
_______ **54**. *The shape given below is the frustum of a right circular cone. If the radii are 4cm and 2cm, and is 6cm thick, what is the* ***VOLUME*** *of the cone obtained by extending having the upper cross-section as a base?*

A. $8\pi cm^3$

B. $64\pi cm^3$

C. $56\pi cm^3$

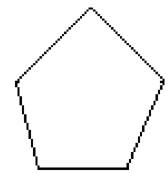
D. $28\pi cm^3$



_______ **55**. *Add up the number of lines of reflection symmetry and the number of centers of rotation symmetry for this regular polygon.*

A. 5 B. 6

C. 10 D. 11



____ **56**. *Which of the following is (are)* ***TRUE***?

(i) If a figure is a rectangle, then it is also a parallelogram.

(ii) If a figure is rhombus, then it is also a square.

(iii) If a figure is a square, then it is a rectangle and a rhombus.

A. i and ii only C. ii and iii only

B. i and iii only D. i, ii and iii

_______ **57**. *Which of the following is the* ***LARGEST***?

A. Measure of a vertex angle of a regular 5-gon.

B. Measure of a vertex angle of a regular 10-gon.

C. Measure of a central angle of a regular 3-gon.

D. Measure of a central angle of a regular 6-gon.

_______ **58**. *Which one of the following is* ***FALSE***?

(i) A 5-gon could be used to tile a floor

(ii) A regular 7-gon has a vertex angle sum of 900^0

(iii) A hexagonal prism has a total face of 8.

(iv) A pentagonal prism has a total edge of 10.

A. i and ii B. ii and iii C. i only D. iv only

_______ **59**. Which of the following areas is **EQUIVALENT** to 40,000m^2?

A. 4 hectares B. 4 Ares C. 40km^2 D. None of A, B, or C

_______ **60**. *A sheet of plywood is 2.4m long, 1.2 m wide, and 1.5 cm thick. What is its* ***VOLUME*** *in* cm^3?

A. 43,200cm^3 B. 420cm^3 C. 43.2cm^3 D. 420,000cm^3

_______ **61**. *The water in a jug weighs 4.7kg. What is the capacity of the jug in milliliters?*

A. 4.7 B. 0.0047 C. 470 D. 4700

_______ **62**. *The solution set of the equation* $7\theta\cos 5\theta + \sin 7\theta \sin 5\theta = \frac{1}{2},\ for\ 0 \le \theta \le 2\pi$

is:

A. $\{\frac{\pi}{6},\frac{11\pi}{6}\}$ B. $\{\frac{-\pi}{3},\frac{\pi}{6}\}$ C. $\{\frac{\pi}{6},\frac{-5\pi}{6}\}$ D. $\{\frac{-\pi}{6},\frac{11\pi}{6}\}$ E. None of these

_______ **63**. *The solution set of the system of the equations*: $\begin{cases} 2^x.3^y = 18 \\ 2^y.3^x = 12 \end{cases}$

A. {(1,2)} B. {(2,1)} C. {(-1,2)} D. {(-1,-2)}

_______ **64**. *Which one is **FALSE**?*

(i) If the area of a circle is equal to its circumference, then r=2

(ii) If the volume of a sphere is equal to its area then r=3

(iii) If the area of a circle is twice the its circumference, then the radius of the circle is 4.

(iv) If the surface area is twice as large as its volume for a sphere, then r=4

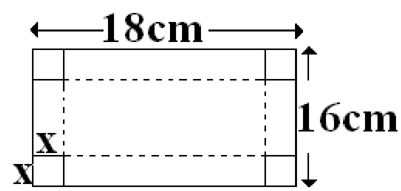
A. i only C. iv only

B. i, ii, iii and iv D. None of A, B, or C

_______ **65**. *Imagine cutting out the square corners of this rectangular sheet of paper. Then fold up on the dotted lines to make a box. Which value of x will lead to the box of greatest volume?*

A. X=1 C. X=3

B. X=2 D. X=4

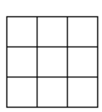


_______ **66**. *The **LARGEST** possible cone is cut from a wooden rectangular prism 1 by 1 by 2 units. Similarly, the largest possible sphere is cut out from a wooden cube 1 by 1 by 1. Which has the **LARGER** volume?*

A. Sphere C. They're equal in volume

B. Cone D. Impossible to determine

_______ **67**. *How many **squares** of **all** sizes can you find in this array?*



A. 9 C. 14

B. 13 D. 15

_______ **68**. *Which of the following is **NOT** an appropriate congruence relation for showing that triangles are congruent?*

A. ASA B. SSS C. SAS D. AAA

_______ **69.** *If $(223)_x = (133)_{x+1}$, the **VALUE** of **x** is _____*

A. 4 B. 5 C. 6 D. 7

_______ **70**. *Which of the following is* ***FALSE****?*

(i) The *centroid*, *cirucumcenter*, *incenter* and *orthocenter* could lie on a side.

(ii) The *centroid* always lies on the side of the triangle.

(iii) The *orthocenter* and *circumcenter* could lie outside the circle.

A. i and ii C. ii and ii

B. i only D. iii only

_______ **71**. *Which of the following regular polygons cannot be constructed with a compass and straight edge?*

A. 36-gon B. 85-gon C. 60-gon D. 64-gon

_______ **72**. *To construct the circle inscribed in a given triangle, each of the following construction steps would be applied except:*

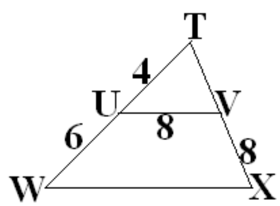
A. perpendicular bisector

B. perpendicular to a line through a point not on the line

C. Angle bisector

D. Circle

_______ **73**. $\overline{UV} \, // \, \overline{WX}$ find $\overline{WX}$



A. 12 C. 18

B. 20 D. 16

_______ **74**. *At a certain time of day, a flagpole casts a shadow 21 meters long on a horizontal ground. At the same time, a vertical pole 4 meters high casts a shadow 6 meters long. How tall is the flagpole?*

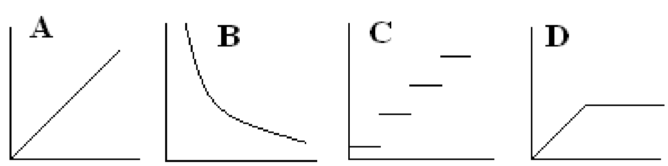
A. 10 meters C. 14 meters

B. 11.6 meters D. 31.5 meters

_______ **75**. The area of the region bounded by the graph of h(x) = $|x-1|$ and y = 4 is:

A. 16 B. 32 C. 8 D. 24

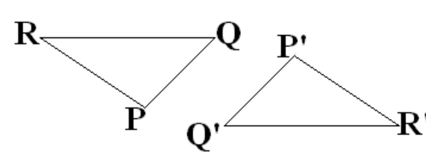
_______ **76**. *Which graph best represents the cost of a phone cell?*



_______ **77.** *In how many ways can 8 men form a committee if at least 3 men are to be on the committee?*

A. 255 B. 7 C. 15 D. 36

_______ **78**. *Which transformation will map ΔPQR to $PQ'R'$?*



A. Translation C. Reflection

B. Rotation D. Glide reflection

_______ **79**. *Which of the following statements is* ***TRUE****?*

A. Translations and reflections preserve the orientation of a figure.

B. Magnification preserves the shape and size of a figure.

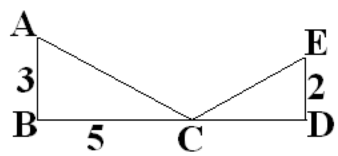
C. If two polygons are congruent, then there exists a translation, rotation, reflection, or glide reflection that maps one to the other.

D. If two polygons are similar, there exists a translation, rotation, reflection, or grid reflection that maps one to the other.

_______ **80**. *Given: that $\Delta ABC \sim \Delta EDC$, what is the scale factor of the magnification involved in mapping ΔABC to ΔEDC?*

A. 2/3 C. 2/5

B. 3/2 D. 5/2



_______ **81**. *ΔABC is congruent to $\Delta A'B'C'$ and $\overline{AB} \parallel \overline{A'B'}$. Point P is the midpoint of $\overline{AA'}$ and point Q (different from P) is the midpoint of $\overline{BB'}$ and $\overline{PQ} \perp \overline{AA'}$. Which of the following statements is* ***necessary TRUE****?*

A. A translation maps ΔABC to $\Delta A'B'C'$

B. A reflection maps ΔABC to $\Delta A'B'C'$

C. A glide reflection maps ΔABC to $\Delta A'B'C'$

D. Either a translation or a reflection maps ΔABC to $\Delta A'B'C'$

_______ **82**. *Triangles ΔABC and $\Delta A'B'C'$ are congruent and have the same orientation. The perpendicular bisectors of $\overline{AA'}$ and $\overline{BB'}$ intersect. Which transformation will map ΔABC to $\Delta A'B'C'$?*

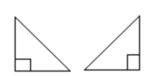
A. Translating C. Reflection

B. Rotation D. Gild reflection

_______ **83**. *Which of the following doesn't map every line ℓ to a line ℓ' such that $\ell \parallel \ell'$.*

A. Reflection C. Rotation of 180^0

B. Translation D. Magnification



_______ **84**. *Here are two congruent right triangles. How many figures can be formed by arranging these triangles in different ways?*

A. 2 B. 3 C. 4 D. 5

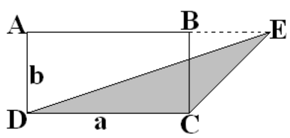
_______ **85**. *Find the* ***NEXT*** *number in the sequence: 4, 10, 18, 28, 40,* _____

A. 48 B. 54 C. 60 D. None of A, B or C

_______ 88. When $\frac{(3.2^{20}+7.2^{19})52}{(13.8^{4})^{2}}$ is ***SIMPLIFIED*** the result is equal to ____

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{16}$ D. $\frac{1}{2}$

_______ 89. *If ABCD is a rectangle and E is on the line AB extended, what is the* ***AREA*** *of the* ***shaded region (ΔDCE)****?*



A. $\frac{1}{2}$ab C. a^2b^2

B. ab D. a^2+b^2

_______ 90. *Which of the following is most* ***affected*** *by one* ***extremely*** *low score?*

A. Mean B. Median

C. Mode D. All are equally affected

_______ 91. *Which is* ***FALSE*** *for the data 2, 2, 5, 4, 2?*

A. The mode equals the median

B. The mean is greater than the median

C. The median is 3

D. The mode is less than the mean

_______ 92. *What can be concluded from the following "****Proof****"?*

0=-6(7+(-7)) = (-6)7+(-6)(-7) = **-42**+(**-6**)(**-7**)

A. 7-7= 0 B. (-6)7=-42

C. (-6)(-7)=42 D. (-6)0=0

_______ 93. *Which of the following is* ***NOT*** *a property of integers?*

A. Distributivity C. Multiplicative inverse

B. Additive Inverse D. Multiplicative Identity

_______ 94. *Which of the following is* ***NOT*** *helpful in simplifying this computation?*

(5/7.13/19) 7/5

A. Commutativity of multiplication

B. Associativity for multiplication

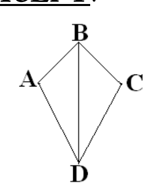
C. Inverse for multiplication

D. Distributivity

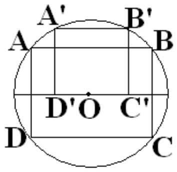
_______ 95. *Given: Kite ABCD with diagonal* $\overline{BD}$*. In order to show that the diagonal divides the kite into two congruent triangles, each of the following statements is* ***justifiable*** *and* ***useful*** ***EXCEPT****.*

A. <A≅ <C C. $\overline{BD} \cong \overline{BD}$

B. $\overline{AB} \cong \overline{CB}$ D. $\overline{DA} \cong \overline{DC}$



_______ **96**. *The area of the square ABCD inscribed in circle 0 is 30cm². What is the area of square* $A'B'C'D'$ *inscribed in a semicircle of circle 0?*



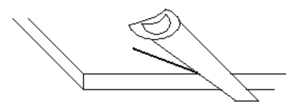
A. 15cm²

C. $\sqrt{15}$ cm²

B. 12 cm²

D. $2\sqrt{3}$ cm²

_______ **97**. *What theorem or assumption provides the best explanation for the fact that when you saw a board, the edge of the cut is a straight line?*



A. Two distinct non-parallel lines intersect at a point.

B. Two distinct non-parallel planes intersect at a line.

C. A plane and a line insect at a line.

D. Two parallel planes that intersect the same line will form a plane containing the two.

_______ **98**. *Which two of the following models represent the same multiplication problem?*

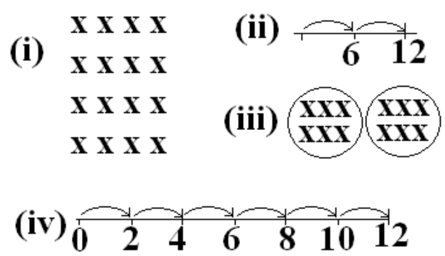
A. i and ii

C.

ii and iii

B. ii and iv

D. iii and iv



_______ **99**. *Which of the following* ***DOESN'T*** *model division?*

A. There are n objects. You can place them in equivalent piles. How many will be in each pile.

B. There are n objects. You can place the objects in a rectangular array with m rows. How many columns are there?

C. There are n objects. You can make m piles. How many objects will each pile contain?

D. There are n objects. You can make piles of m objects each. How many piles will you obtain?

_______ **100**. *The quotients* **6÷3** *and* **3÷6** *can be used to show that*

(i) Division of whole numbers is not associative

(ii) Division of whole members is not commutative.

(iii) The whole numbers are not closed under division.

A. i only

B. iii only

C. i and iii only

D. ii and iii only

_______ **101**. If x and 5 are relatively primes and LCM of (x, 5) = 120, then GCF (3x, 15) is:

A. 3

C. 12

B. 5

D. 10

_______ **102**. If $1-\frac{4}{x}+\frac{4}{x^2}=0$, then $\frac{2}{x}$ *e*quals

A. -1 B. 1 C. 2 D. -1 or 2 E. -1 or -2

_______ **103**. *If four times the reciprocal of the circumference of a circle equals the diameter of the circle, then the area of the circle is*

A. $\frac{1}{\pi^2}$ B. $\frac{1}{\pi}$ C. 1 D. π E. π^2

_______ **104**. *For all non-zero numbers x ands y such that* $x=\frac{1}{y}$, $(x-\frac{1}{x})(y+\frac{1}{y})$ *equals:*

A. $2x^2$ B. $2y^2$

C. x^2+y^2 D. x^2-y^2 E. y^2-x^2

_______ **105**. *If* a =1, b =10, c =100 and d=1000, then

(a+b+c-d) + (a+b-c+d) + (a-b+c+d) + (-a+b+c+d) *is equal to*:

A. 1111 C. 3333

B. 2222 D. 1212 E. 4242

_______ **106**. *Four boys bought a boat for $60. The first boy paid one half of the sum of the amounts paid by the other boys; the second boy paid one third of the sum of the amounts paid by the other boys; the second boy paid one third of the sum of the amounts paid by the other boys; and the third boy paid one fourth of the sum of the amounts paid by the others boys. How much did the fourth boy pay?*

A. $10 B. $12 C. $13 D. $14 E. $15

_______**107**. *The number of district pairs (x,y) of members satisfying both of the following equations:*

$x=x^2+y^2$

$y=2xy$ *is*

A. 0 B. 1 C. 2 D. 3 E. 4

_______ **108**. *Opposite sides of a regular hexagon are 12 inches apart. The length of each side, in inches is:*

A. 7.5 B. $6\sqrt{2}$ C. $5\sqrt{2}$ D. 9/ $2\sqrt{3}$ E. $4\sqrt{3}$

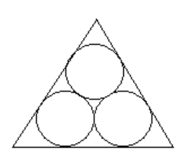
_______ **109**. *Al's age is 16 more than the sum of Bob's age and Carl's age, and the square of Al's age is 1632 more than the square of the sum of Bob's age and Carl's age. The sum of the ages of Al, Bob and Carl is:*

A. 64 B. 94 C. 96 D. 102 D. 140

_______ **110**. *How many pairs (m, n) of integers satisfy the equation* $m+n = mn$?

A. 1 B. 2 C. 3 D. 4 E. More than 4

_______ **111**. *Each of the three circles in the adjoining figure is externally target to two of the circles. If each circle has radius three, then the perimeter of the triangle is:*



A. $36+9\sqrt{2}$ C. $45+9\sqrt{3}$

B. $36+6\sqrt{3}$ D. $18+18\sqrt{3}$ E. 45

_______ **112**. *Three fair dice are tossed at random, (i.e., all faces have the same probability of coming up). What is the probability that the three numbers turned up can be arranged to form an arithmetic progression with common difference one?*

A. $\frac{1}{6}$ B. $\frac{1}{9}$ C. $\frac{1}{27}$ D. $\frac{1}{54}$ E. $\frac{7}{36}$

_______ **113**. *If* $y = (\log_2^3)(\log_3^4)\ldots(\log_n(n+1))\ldots(\log_{31}^{32})$ *then*

A. 4<y<5 C. 5<y<6

B. y=5 D. y=6 E. 6<y<7

_______ **114**. *Let* **E** *be the point of intersection of the diagonals of convex quadrilateral* **ABCD**, *and let* **P,Q,R** *and* **S** *be the centers of the circles circumscribing triangles* **ABE**, **BCE**, **CDE** *and* **ADE**, *respectively. Then*

A. PQRS is a parallelogram

B. PQRS is a parallelogram if and only if ABCD is a Rhombus.

C. PQRS is a parallelogram if and only if ABCD is a Rectangle.

D. PQRS is a parallelogram if and only if ABCDE is a rectangle.

E. None of A, B, C or D

_______ **115**. *A motorist covers a distance of 360 km. He could cover the same distance in one hour less time by increasing his average speed by 30 km/hr. The average speed of the motorist is*:

A. 90km/hr C. 180km/hr

B. 120km/hr D. None of A, B, or C

_______ **116**. *Two teams of wood cutters working together completed their assignment in eight days. How many days would it take each team to fulfill this work if team one works twice as fast as the other?*

A. 6 B. 12 C. 24 D. 48 E. None of A, B, C or D

_______ **117**. When the seventh term of the following pattern is divided by the first term the resulting number is:

9x12,345,679 = 111,111,111

18x12,345,679 = 222,222,222

27x12,345,679 = 333,333,333

36x12,345,679 = 444,444,444 . . .

A. 5 B. 6

C. 7 D. 8

_______ **118**. *The equation V+F=E+2 is known as* ***Euler's formula****. Where V, F and E are the number of vertices, number of faces and number of edges respectively. If 20 equilateral triangle faces form a solid, how many vertices does the solid have?*

A. 12 B. 30 C. 60 D. None of A, B, or C

_________ **119**. Let $f(x) = a^x$ and $g(x) = -a^x$ be exponential functions. One of the following is <u>NOT TRUE</u>, which one?

A. f(x) is increasing for a>1

B. f(x) is decreasing for 0<a<1

C. g(x) is increasing for a>1

D. *g(x) is increasing for 0<a<1*

_______ **120**. *Which one of the following is* ***NOT TRUE*** *of the following decimal?*

0.454454445444454444454444445 ...

(i) There are 5149 digits preceding the hundredth 5

(ii) There are more 4's than 5's

(iii) There are only 5050 4's preceding the hundredth 5

(iv) There are 99 fives preceding the hundredth 5.

A. i and ii only

B. ii only

C. i, ii and iii

D. ii, iii and iv only

_______ **121**. *For a set of data, the mean square is found to be 20 and the mean of the square of the individual scores is found to be 425. Then the standard deviation of the data is:*

A. 5 B 25 C. $\sqrt{825}$ D. None of A, B, or C.

_______ **122**. *Which of the following properties is* ***NOT USED*** *in the computation below?*

A. Distributivity

B. Associativity

C. Commutativity

D. None of A, B, or C

$5(37+43)=5((30+7)+43)$

$= 5((30+(7+43))$

$= 5(30+50)$

$= 5.30+5.50$

_______ **123**. *Which of the following is* ***NOT AN EXAMPLE*** *of the commutative property?*

A. (3+4)+5 = 5+(3+4)

B. 9+8 = 8+9

C. (2+4)+(5+6)=(5+6)+(2+4)

D. (6+3)+2=6+(3+2)

_______ **124**. *Which statement is* **FALSE**? **W** *is the set of whole numbers.*

A. a-0=a for all a∈W

B. a-b = b-a for some a,b∈W

C. (a-b)-c=a-(b-c) for all a,b,c∈W

D. a-b∈W for some a,b∈W

_______ **125**. *Which of the following is* ***FALSE***?

A. The opposite of an integer is always negative

B. There exists the least integer.

C. For every integer there is an integer less than it.

D. The opposite of a positive integer is always negative.

E. Every negative integer is the opposite of an integer.

_______ **126**. *What of these is* ***TRUE***?

A. ((a-b)-c)d=((a-c)-b)(-d)

B. (a-b)(-c)=(b-a)(-c)

C. (-a-b)c=(-b-a)c

D. None of A, B, or C

_______ **127**. **20%** *of a number x* is **0.8**. *What is the solution set for y in the equation* $4y^2-x = 0$?

A. {4} B. {2} C. {-2 , 2} D. {-1 , 1}

_______ **128**. *A segment* **30cm** *long is noted in the ratio* **2:3:5.** *What is the length of the longest part of the segment in the ratio*?

A. 6 B. 9 C. 15 D. 25

_______ **129**. *Which one of the following is* ***TRUE***?

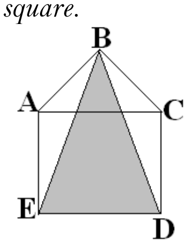
(i) the sum of the first n odd natural numbers is given by n^2.

(ii) the sum of the first n natural numbers is given by $\frac{n(n+1)}{2}$.

(iii) the sum of the first n even natural numbers is given by $n^2 + n$

A. i and ii B. ii and iii C. i and iii D. all of i, ii and iii

_______ **130**.*In the figure ΔABC at right is equilateral of side* ***a*** *with ACDE is a square. Which one is* ***NOT TRUE*** *of this shape?*



A. *ΔABE≅ΔCBD*

B. ΔBDE is isosceles

C. The area of shape ACDE is equal with the area of ΔABC.

D. The area of the shaded region (ΔBDE) is $\frac{a^2}{4}(2+\sqrt{3})$

E.. None of these.

Table A1.1: The pretest- posttest raw data for the experimental group on MT.

No	Pretest	Posttest
1	46	81
2	38	57
3	30	57
4	36	71
5	44	56
6	37	68
7	31	61
8	31	80
9	37	63
10	32	60
11	45	60
12	31	65
13	35	62
14	35	78
15	27	58
16	48	83
17	26	55
18	57	76
19	36	81
20	33	64
21	31	72
22	34	65
23	38	79
24	35	73
25	37	74
26	34	62
27	35	63
28	25	57
29	35	76
30	39	74
31	47	91
32	24	62
33	23	56
34	38	80
35	56	93
36	44	69
37	31	76
38	39	63
39	40	75
40	32	70
41	43	76
42	38	86
43	36	71
44	37	63
45	35	75
46	47	70
47	71	82

Table A1.2: The pretest–posttest raw data for the control group on MT

No	Pretest	Posttest
1	33	57
2	33	63
3	44	71
4	32	52
5	31	57
6	32	54
7	34	38
8	35	47
9	71	85
10	46	52
11	32	57
12	45	70
13	43	52
14	34	56
15	38	60
16	29	43
17	58	80
18	38	63
19	51	61
20	31	35
21	55	69
22	45	70
23	37	57
24	40	64
25	48	57
26	35	56
27	39	67
28	43	67
29	37	54
30	21	39
31	30	52
32	38	65
33	33	65
34	43	61
35	34	47
36	35	47
37	39	51
38	36	63
39	37	55
40	40	55
41	34	47
42	32	45
43	32	47
45	34	52

Appendix 2 (A2): The Reference Test (RT)

Procedural-set of questions

1. If p, q and r are propositions with $P \equiv T, q \equiv F$ *and* $\neg r \equiv T$, *find the truth-value for the compound proposition* $[(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)]$

2. If A and B are sets such that $n(A) = 4$, $n(B) = 8$, *and* $n(A \cap B) = 2$, *then find* $n(A \cup B)$.

3. Find the area of a rectangle whose diagonal is 5 units and one of its legs is 4 units.

4. Find the volume of a right circular cylinder with base radius 4cm and height 3cm.

5. The number of diagonals of a convex polygon is given by the formula $\frac{n(n-3)}{2}$. *If the number of diagonals of a convex polygon is 20, then find the number of sides, n.*

6. If for acute angle θ, $Cos\theta = \frac{4}{5}$, *then find* $\sin\theta$.

7. Find the area A, of the shaded region in figure 1 bellow if AC = 5 cm.

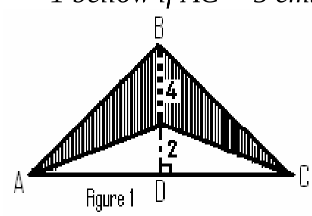


Figure 1

8. If A and B are mutually exclusive and if P(A)=0.3 and P(B)=0.4, what is P(A $\cup$ *B)?*

9 If the sine of angle A is 0.5, in a right triangle ABC find the lengths of the legs, a and b, if the hypotenuse, c= $10cm$.

10. Find h =HG in figure 2 if EF=4units long.

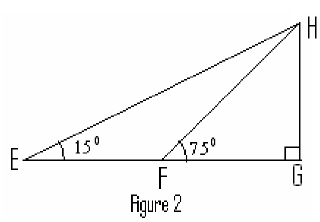


Figure 2

11. Solve $\log_2 x(x-4) = 5$

Conceptual-set of questions

1'. *For p and q propositions, if* $\neg(p \Leftrightarrow q) \equiv F$, *then find the truth-value of* $(q \Rightarrow p) \Leftrightarrow (q \wedge \neg p)$. *Justify your solution process.*

2'. *If A and B are sets such that* $n(A) = 4$ *and* $n(B) = 8$; *then find the minimum and maximum of* $n(A \cup B)$. *Provide a clear justification.*

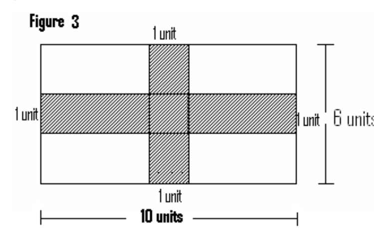
3'. *Of all rectangles with perimeter 8 units, which one has the greatest area?*

4'. *A circle may be considered a "many-sided" polygon. Use this notion to describe the relationship between a prism and a cylinder. Use this idea to develop the volume of a cylinder.*

5'. *There are 10 points on a plane. How many lines can be formed by these points if any three of them are not collinear? Generalize the case for n points.*

6'. *If* $Cos\theta = \frac{4}{5}$, *in right triangle, does it necessarily mean one leg is 4 units and the hypotenuse 5 units? Justify your answer and support it with a diagram.*

7'. *Find the area of the shaded region if all quadrilaterals are rectangles in figure 3.*



8'. *Explain whether events A and B can be mutually exclusive if P(A)=0.8 and P(B)=0.9. Justify it!.*

9'. *An angle of a right triangle has a cosine of 0.9375. Which is longer, the leg adjacent to the angle or the leg opposite to the angle? Justify your conclusion.*

10. Three points A, B, and C lie in a straight line on a level ground. A tower CD whose foot is at C is such that the angles of elevation of D from A and B are 30^0 *and* 45^0 *respectively. If the distance AB is* $400m$, *find the height CD of the hill. Use diagram to solve the problem.*

11'. *What is wrong with the statement "the logarithm of the product of two numbers is the sum of the logarithms"? Justify your answer!*

Procedural-set of questions

12. *Evaluate* $\log_8 64$

13. *Sketch the graphs of* $y=2^x$ *and* $y=\left(\frac{1}{2}\right)^x$

14. *Are the lines* $l_1: 2x+4y+5=0$ *and* $l_1: 3x+6y-4=0$ *parallel?*

15. *A line makes an angle of* 30^0 *with the positive x-axis. Find the slope of the line.*

16. *Find the mean, median and mode of the set of data: 90, 95, 85, 90, and 20.*

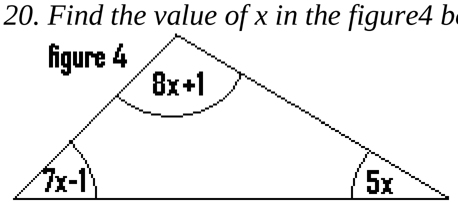
17. *Solve the system of equations:*

$$\begin{cases} 2x-y=7 \\ 3x+y=13 \end{cases}$$

18. *If you have 1000 Jelly beans in a jar and you know that* $P(Blue)=\frac{4}{5}$ *and* $P(\mathrm{Re}\,d)=\frac{1}{8}$, *then find the probability of getting a Blue bean or a red bean?*

19. *Which floor ceramic is more liable to the danger of breaking, the ones tiled on a leveled floor or the ones tailed on the unlabelled floor?*

20. *Find the value of x in the figure4 below.*



Conceptual-set of questions

12'. *Does* $\log_1 6$ *make sense? Why or why not?*

13'. *Use the concept on the graph of* $y=2^x$ *to sketch the graph of* $y=-2^x$. *Elaborate how these two functions are similar and different!*

14'. *Explain in more than one way why two lines with the same slope are parallel.*

15'. *Explain how you might find a measure of an angle of a staircase that will describe the staircase's steepness.*

16'. *John had scored 90, 95, 85, and 90 on his first four tests and scored a 20 on his fifth exam. Which of the three averages would John want the instructor to use to compute his grade? Why? Which measure is affected the most by an extreme score? Justify it!*

17'. *What is the physical meaning of the solution to the system of two equations with two variables? Consider different cases for the solution and give your own interpretation!*

18'. *If you have 1000 Jelly beans in a jar and you know that* $P(Blue)=4/5$ *and* $p(\mathrm{Re}\,d)=1/8$, *list several things you can say about the beans in the jar.*

19'. *Suppose you were to tile the floor at your home with some beautiful ceramics, however you were not sure whether the floor is leveled or not. How would you check it using local materials? If you found it unleveled what would you do?*

20'. *In figure 5 below, AC>AB, m (∠ B)+m (∠ C)<* 180^0, *m(∠ B)=6x-45, m(∠ C)=15+x, find a restriction on the value of x.*

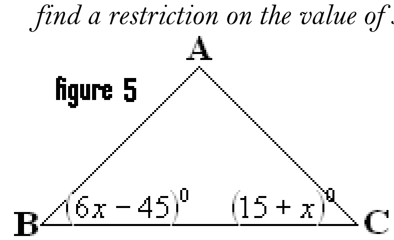


Table A2.1: 'Reference test' raw data for experimental group

S.N0	Procedural (50%)	Conceptual (50%)
1	42	38
2	27	24
3	23	18.5
4	29	24.5
5	39	32
6	31	28.5
7	22.5	14.5
8	45	38
9	20.5	18
10	18.5	10
11	25	20
12	32	26.5
13	27	24.5
14	37.5	36.5
15	21	16
16	38.5	34
17	17.5	12
18	30	28
19	36.5	36
20	24	18.5
21	37	33.5
22	28	22.5
23	41.5	38
24	33.5	34
25	35	34
26	32.5	30
27	32	20.5
28	27	21
29	37.5	38
30	40.5	38
31	43	42
32	21	18.5
33	25	28
34	38.5	39.5
35	41	38
36	31	34
37	41	30
38	30	28
39	41.5	38.5
40	42	38.5
41	38	36
42	42.5	36
43	39.5	40
44	30	23.5
45	38	29.5
46	41.5	38
47	42	40.5

Table A2.2: 'Reference test" raw data for the control group

S. N0	Procedural (50%)	Conceptual (50%)
1	26.5	24
2	32	22
3	41	24
4	20	12.5
5	23	15
6	24	18
7	18	15
8	24	18
9	41	35.5
10	24	20
11	24.5	20
12	38.5	30
13	28	24
14	26	18.5
15	30.5	30
16	24	18
17	42.5	36
18	35	25.5
19	32.5	24
20	20.5	18
21	34.5	18
22	38.5	28
23	24.5	15
24	35	25.5
25	30	22.5
26	28	22.5
27	32.5	23
28	34	2
29	26.5	20
30	19	15
31	25	17
32	36	25.5
33	36	30
34	28.5	17
35	18	12.5
36	18	10
37	25	17
38	32.5	25.5
39	28	24.5
40	24	15
41	17.5	17.5
42	15	12.5
43	18	18
44	24	17.5
45	28	20

Appendix 3(A3): Cooperative Behavior Checklist

(+ and - marks are used to indicate quality of participation)

Team	Student	On-task							Listening							Supporting							Comments
		1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	
Team-1																							
Team-2																							
Team-3																							
Team-4																							
Team-5																							
Team-6																							
Team-7																							
Team-8																							
Team-9																							

Table A3.1: CBC raw data for the first week of the intervention programme

Team	S.	On-Task							Listening							Supporting						
1	**Code**	1	2	3	4	5	–	–	1	2	3	4	5	6	7	1	2	3	4	5	6	7
		–	–	–	–	–	–	–	–	+	+	–	+	+	–	–	–	–	+	–	—	–
		–	–	–	–	–	–	–	–	–	–	+	+	–	+	–	+	–	–	–	+	–
		–	–	–	+	–	–	–	–	–	–	–	+	–	–	–	–	–	–	–	–	–
		–	–	–	–	–	–	–	+	+	–	–	–	–	–	–	+	–	–	–	–	+
		–	–	–	–	+	+	–	–		+	–	–	+	+	–	–	–	–	+	–	–
2		–	–	–	–	+	–	+	–	–	–	+	–	–	–	+	–	+	–	–	–	–
		–	–	–	–	–	–	+	–	–	–	+	–	+	–	–	–	–	–	+	–	+
		–	–	–	–	–	–	–		+	+	+	+	–	–	+	–	–	–	+	–	–
		–	–	–	–	–	–	–	+	–	–	–	–	+	–	–	–	–	+	+	–	–
			+	–	–	+	–	–	–		–	+	+	–	–	–	+	–	+	+	+	–
3		–	–	–	–	–	–	–	–	–	–	–	–	+	–	–	–	–	–	–	–	+
		–	–	–	–	–	–	+	–	–	–	–	–	+	+	–	–	–	–	–	–	–
		–	+	–	+	–	–	+	–	+	+	+	–	–	–	–	–	–	+	+	–	–
		–	–	+	–	+	–	–	+	–	+	+	–	–	+	+	+	–	–	+	–	—
		–	–	–	–	+	–	–	+	–	–	–	–	–	–	–	–	–	+	+	+	–
4		–	–	–	–	–	–	–	–	+	–	–	+	–	–	–	+	–	–	–	–	–
		–	–	+	–	–	–	+	–	–	+	–	+	+	–	–	–	–	–	–	–	–
		–	–	–	–	–	+	–	–	–	+	–	–	+	–	–	–	–	–	+	–	–
		–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	+	+	–	–	–
		–	+	–	–	–	–	–	–		+	–	—	+	–	–	–	–	–	–	–	–
		–	–	+	–	+	–	–	–	+	–	–	–	+	–	–	–	+	–	+	–	+
5		–	+	–	+	–	–	–	+	–	–	+	–	–	–	+	–	–	–	+	–	–
		–	–	–	–	–	–	–	–	–	–	–	–	–	+	–	–	–	–	+	–	–
		–	+	–	–	–	–	–	–	–	+	–	+	–	–	–	+	–	+	–	+	–
		–	+	+	–	–	–	–	+	–	+	–	–	+	+	+	–	+	+	–	–	–
6		–	–	–	–	–	–	+	–	–	+	+	+	–	–	–	–	–	+	–	–	–
		–	–	+	–	–	+	–	–	–	+	–	–	–	–	–	–	–	–	+	+	–
				–		+	–		–	–	–	+	–	–	–	+	–	–	–	+	–	–
		–	–	–	–	–	–	+	+	–	–	–	–	–	–	–	–	–	+	–	–	–
		–	+	–	–	–	–	–	–	+	–	+	–	+	–	–	+	–	+	–	–	–
7		–	–	–	–	–	–	–	–	–	–	+	+	–	–	+	–	+	–	+	+	–
		–	–	+	–	–	–	–	–	–	–	–	–	–	–	–	+	–	–	+	–	–
		–	+	–	–	–	–	+	+	–	–	–	+	+	–	–	+	–	–	–	–	+
		+	–	–	–	+	+	–	–	–	–	+	–	–	+	+	–	–	+	–	–	–
		–	–	–	+	–	–	+	+		+	–	+	+	+	–	–	–	+	–	+	–
8		–	–	–	–	–	–	–	–	–	–	–	+	–	–	+	+	–	–	–	–	–
		–	–	–	–	–	–	–	+	–	–	+	–	–	–	+	–	–	–	–	–	+
		–	–	–	–	–	+	+	+	–	–	+	–	–	+	–	–	–	+	–	–	–
		–	–	–	–	–	–	–	–	–	–	–	–	+	+	–	–	+	–	–	+	+
		–	–	+	–	–	+	–	–	+	–	–	+	+	–	+	–	–	+	–	–	–
		–	–	–	–	–	–	–	–	–	–	–	–	+	+	–	–	–	–	+	–	–
9		–	–	–	+	–	–	–	–	–	–	+	+	–	–	–	—	+	–	–	+	+
		–	–	–	–	+	–	+	+	–	+	–	+	–	–	+	–	+	–	+	–	–
		–	–	+	+	–	+	+	–	+	–	+	–	+	+	–	–	+	–	+	–	–
		–	+	–	+	–	–	+	–	–	–	–	–	–	–	–	–	–	–	+	+	–
		–	–	–	–	+	–	–	–	–	+	–	+	–	–	+	–	–	–	+	–	–
		+	–	–	–	+	_	-	–	+	–	+	–	–	+	–	–	–	+	–	–	–
+ve		2	9	8	7	13	11	7	12	10	14	8	17	18	13	13	10	9	16	21	9	8
-ve		45	38	39	40	34	36	40	35	37	33	39	30	29	34	34	37	38	31	26	38	39

Table A3.2: CBC raw data for the last week of the intervention programme

Team	S.	On-Task							Listening							Supporting						
1	Code	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
		+	+	+		+	+	+	+	+		+	+	+	+	+	+			+	+	+
		+	+	+	+	-	+	+	+	-	+	+	+	+	+	-	+	+	+	+	+	+
		+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		+	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+
		+	+	+	+	+	-	+	+	+	-	+	+	+	+	+	+	+	-	+	-	+
2		+	+	+	+	+	-	+	+	+	+	+	+	+	+	+	-	-	+	-	+	-
		+	+	-	-	+	+	-	+	-	+	-	+	+	+	-	+	-	+	+	+	+
		+	-	+	+	+	+	+	+	-	+	-	+	+	+	+	+	-	+	+	+	+
		+	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-	+	+	-
		+	-	+	+	+	+	+	+	+	-	+	+	+	+	+	-	+	-	+	+	+
3		-	+	+	+	+	+	-	+	-	+	-	-	+	+	+	+	+	-	-	+	+
		+	+	+	+	+	-	+	+	+	+	-	+	+	+	+	+	+	-	-	+	+
		+	+	+	-	+	+	-	+	+	+	+	-	-	+	+	+	+	-	+	+	+
		+	+	-	+	+	+	-	+	+	+	+	+	+	+	+	-	+	+	+	+	+
		+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	+	+
4		+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	+	+
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		+	+	+	+	+	-	+	+	+	+	-	+	+	+	+	+	+	+	+	+	-
		+	+	+	+	+	-	+	+	+	+	-	+	+	+	+	+	+	+	-	+	+
		+	+	+	+	+	+	+	+	+	+	-	+	+	+	-	+	+	+	+	+	-
		+	-	+	+	+	+	-	+	-	+	-	+	+	+	-	+	-	+	-	+	+
5		+	+	+	+	+	+	+	-	+	+	-	+	+	+	+	-	+	+	-	+	+
		+	+	+	+	+	+	-	+	+	+	+	+	-	+	+	+	+	+	+	+	+
		+	-	+	+	+	+	+	+	-	-	+	+	+	+	-	+	-	+	+	-	+
		+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
6		+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		+	+	+	+	+	+	-	+	+	+	-	+	+	+	-	+	+	-	+	+	+
		+	+	+	-	+	+	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+
		+	+	+	-	+	+	+	+	-	+	+	+	+	-	+	+	+	-	-	+	+
		+	+	-	+	+	+	+	+	+	-	+	+	+	+	-	+	+	-	+	+	-
7		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		+	+	+	+	+	+	-	+	+	-	+	+	+	-	+	+	+	-	+	+	+
		+	+	+	+	+	+	+	+	+	-	+	+	-	+	+	+	-	+	+	+	+
		+	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	+
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
8		+	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+
		+	+	+	+	-	+	+	+	+	+	+	-	+	+	+	+	+	+	-	+	+
		+	+	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		+	+	+	-	+	+	+	+	+	+	+	+	-	+	+	+	+	+	+	+	+
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
9		+	+	-	+	+	+	+	+	-	+	+	+	+	-	+	+	+	+	+	-	+
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		+	+	+	+	+	+	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+
		+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	+	+	+	+	+	+
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+
+ve		46	**44**	**43**	**40**	**44**	**42**	**40**	**46**	**39**	**41**	**35**	**43**	**43**	**44**	**40**	**40**	**40**	**35**	**38**	**45**	**42**
-ve		1	**3**	**4**	**7**	**3**	**5**	**7**	**1**	**8**	**6**	**12**	**4**	**4**	**3**	**7**	**7**	**7**	**12**	**9**	**2**	**5**

Appendix 4 (A4): LASSI-SH Learning and Study Strategies Inventory - High School Version

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Adapted for Ayider High school Grade 10 students for the purpose of Research Work by H.N. Weldeana.(Obtained by means of Unisa)

DIRECTIONS

The Learning and Study Strategies Inventory - High School Version (LASSI-HS) is designed to find out how you learn, how you study, and how you feel about learning and studying. On the following pages you will find 76 statements about learning and studying of mathematics. Read each statement and then mark one of these choices on the answer sheet:

1. **Not at all like me**
2. **Not very much like me**
3. **Somewhat like me**
4. **Fairly much like me**
5. **Very much like me**

To help you decide which choice to mark, we will explain what is meant by each one.

By **Not at all like me,** we do not necessarily mean that the statement would never describe you, but that it would be true of you only rarely. Cross out number 1 for this choice.

By **Not very much like me,** we mean that the statement generally would not be true of you. Cross out number 2 for this choice.

By **Somewhat like me,** we mean that the statement would be true of you about half of the time. Cross out number 3 for this choice.

By **Fairly much like me,** we mean that the statement would generally be true of you. Cross out number 4 for this choice.

By **Very much like me,** we do not necessarily mean that the statement would always describe you, but that it would be true of you almost all the time. Cross out number 5 for this choice.

Cross out the number that describes you best.

Example

1	2	3	4	5

Try to answer according to how well the statement describes you, not how you think you should be or what others do.

There is no right or wrong answer to these statements. Please work as quickly as you can without being careless and please answer all the items.

S.N	Statement	Rate				
		1	2	3	4	5
1	*I am worried that I will fail maths at school.*					
2	*I can tell the difference between more important and less important information in a maths lesson.*					
3	*I find it difficult to stick to a study timetable for maths.*					
4	*After a maths class, I look over the work we did in class to help me better understand it.*					
5	*I don't care if I finish maths in high school as long as I can get a job.*					
6	*I think of other things during the maths lesson and don't really listen to what is being said in class.*					
7	*I use special study aids, such as main headings and words printed in italics or bold face, that are in my maths textbook.*					
8	*I try to identify the main ideas or most important information in a maths lesson while the lesson is being presented.*					
9	*I get discouraged because of low marks for a maths test or examination.*					
10	*I am up-to-date in my maths assignments.*					
11	*Problems outside of school such as financial problems, conflict with parents, dating (being in love), etc. cause me to not do my maths.*					
12	*I try to think through a topic while doing maths and decide what I am supposed to learn from it.*					
13	*Even when some parts of the maths are dull and not interesting, I manage to keep working until I finish.*					
14	*I feel confused and undecided as to what my objectives of my maths studies should be.*					
15	*I learn new maths terms and concepts by visualizing a situation in which they occur.*					
16	*I come to a maths class unprepared.*					
17	*When studying for a maths test or exam, I think of questions that I think might be asked.*					
18	*I would have preferred not to take maths in school.*					
19	*The notes I take as I read my maths textbook are helpful when I review the work.*					
20	*I do poorly on maths tests because I find it hard to plan my work within a short period of time.*					
21	*I try to think of possible test questions when studying work done in the maths class.*					
22	*I only study maths when I have to write a test.*					
23	*I change the maths I am studying into my own words.*					
24	*I compare my maths class work/homework with other students to make sure it is correct.*					
25	*I am very tense when I study maths.*					
26	*I review the maths of the previous lesson before the next lesson.*					

27	*I have trouble summarizing the maths that I have just heard or done in class.*					
28	*I work hard to get good marks in maths, even when I don't like the maths being done.*					
29	*I often feel like I have little control over what happens to me in the maths class.*					
30	*I stop often while doing maths and think over or review what I have been doing.*					
31	*Even when I am well prepared for a maths test, I feel very upset when writing it.*					
32	*When I study a topic in maths I try to make the ideas fit together and make sense.*					
33	*I talk myself into believing some excuse for not doing a homework assignment in maths.*					
34	*When I study maths, I have trouble figuring out just what to do to learn the work.*					
35	*When I begin a maths test, I feel pretty sure that I will do well.*					
36	*I check to see if I understand what my teacher is saying during a maths lesson.*					
37	*I do not want to learn a lot of maths in school. I just want to learn what I need to get a good job.*					
38	*I am sometimes unable to keep my mind on my maths work because I am restless or moody.*					
39	*I try to find connections between the maths I am learning and what I already know.*					
40	*I set high standards or goals for myself in maths.*					
41	*I end up "cramming" (learning a lot of maths in a very short period) for almost every test.*					
42	*I find it hard to pay attention during a maths lesson.*					
43	*I pay special attention to the first and/or last parts of most paragraphs when reading my maths text book.*					
44	*I only study the sections of maths I like.*					
45	*I am very easily distracted from the maths I'm doing.*					
46	*I try to find connections between the maths I am studying and my own experiences.*					
47	*I make good use of study hours after school to also study maths.*					
48	*When doing maths which is difficult for me I either give up or study only the easy parts.*					
49	*I make drawings or sketches to help me understand the maths I am studying.*					
50	*I dislike most of the maths done in class.*					
51	*I have trouble understanding just what a test question in maths is asking.*					
52	*I use symbols, key words, diagrams, or tables in summarising my maths.*					

53	*While I am writing a maths test, worrying about doing poorly gets in the way of keeping my mind on the test*					
54	*I don't understand some sections of maths because I do not listen carefully.*					
55	*I use my maths text book to prepare assignments.*					
56	*I feel very panicky when writing an important maths test*					
57	*When I decide to do my maths homework, I set aside a certain amount of time and stick with it.*					
58	*When I write a maths test I realize I have studied the wrong material.*					
59	*It is hard for me to know which parts of my maths text book are important to remember.*					
60	*I pay attention fully when studying maths.*					
61	*I use the headings of paragraphs and sections inscribed in blocks as guidelines for important ideas in my maths text book.*					
62	*I get so nervous and confused when writing a maths test that I don't answer questions to the best of my ability.*					
63	*I memorize mathematical rules, formulas, techniques, etc., without understanding them.*					
64	*I test myself to be sure I know the maths I have been studying.*					
65	*I put off the maths I'm suppose to do more than I should.*					
66	*I try to see how the maths I am studying would apply to my everyday living.*					
67	*My mind wanders a lot when I do maths.*					
68	*In my opinion, the maths I learn at school is not worth learning.*					
69	*I go over homework assignments when reviewing maths done in class.*					
70	*I have a hard time knowing how to study for different parts of maths.*					
71	*Often when doing maths I seem to get lost in details and can't remember the main ideas.*					
72	*When they are available, I go to review sessions or extra classes in maths.*					
73	*I spend so much time with my friends that my maths suffers.*					
74	*When writing maths tests or doing other work in maths, I find I have not understood what is required of me and lose marks because of it.*					
75	*I try to make connections between the various ideas in what maths I am studying.*					
76	*I have a hard time finding important aspects of the work done in the maths class.*					

Thank you for completing the Questionnaire.

Table A4.1: LASSI-HS Pretest raw data for the experimental group

S.	Rate				
No	1	2	3	4	5
1	6	8	17	10	6
2	11	13	12	6	5
3	5	12	6	10	14
4	16	13	9	3	6
5	7	8	12	11	9
6	3	5	9	16	14
7	20	11	7	5	4
8	18	16	5	3	5
9	5	11	5	10	16
10	14	19	4	5	5
11	3	5	21	8	10
12	15	12	7	9	4
13	19	13	10	3	2
14	8	9	7	9	14
15	15	13	9	7	3
16	7	5	7	15	13
17	10	7	9	13	8
18	2	7	9	22	7
19	9	20	7	9	2
20	4	7	8	20	8
21	15	17	6	5	4
22	4	10	9	19	5
23	7	20	8	7	5
24	10	20	10	4	3
25	4	5	20	10	8
26	10	12	15	7	3
27	4	8	14	11	10
28	5	13	18	6	5
29	8	9	9	11	10
30	13	17	10	5	2
31	4	5	12	17	9
32	10	14	10	7	6
33	7	12	13	10	5
34	2	4	12	20	9
35	13	16	11	5	2
36	10	10	12	10	5
37	2	9	15	10	11
38	4	8	13	12	10
39	10	11	9	9	8
40	13	16	11	5	2
41	5	10	12	10	10
42	2	9	15	10	11
43	5	13	12	9	8
44	3	10	13	13	8
45	4	7	9	14	13
46	9	13	15	6	4
47	7	13	11	9	7
48	6	3	10	17	11
49	12	9	17	6	3
50	3	5	13	19	7
51	3	7	14	13	10
52	8	10	17	5	7
53	2	7	13	20	5
54	5	5	13	17	7
55	10	17	13	7	0
56	5	10	13	11	8
57	5	10	17	9	6
58	8	8	20	5	6
59	5	4	10	13	15
60	13	11	17	3	3
61	15	8	12	9	3
62	4	7	18	11	7
63	4	11	12	12	8
64	9	11	10	8	9
65	3	9	14	13	8
66	10	15	10	7	5
67	7	4	13	10	13
68	9	8	10	11	9
69	13	15	9	3	7
70	7	6	13	11	10
71	5	2	20	13	7
72	9	14	13	7	4
73	8	9	12	10	8
74	9	10	12	9	7
75	11	9	19	5	3
76	5	2	15	12	13

Table A4.2: LASSI-HS posttest raw data for the experimental group

S. No	Rate				
	1	2	3	4	5
1	4	18	13	8	4
2	4	6	14	16	7
3	14	13	13	5	2
4	3	5	16	14	9
5	15	18	9	3	2
6	12	17	8	7	3
7	3	7	12	13	12
8	3	8	9	13	14
9	14	12	11	8	2
10	1	8	13	18	7
11	2	7	9	12	17
12	4	7	9	12	15
13	2	3	10	19	13
14	12	16	10	5	4
15	3	7	13	15	9
16	13	15	7	5	7
17	3	7	9	16	12
18	10	22	9	4	2
19	3	5	7	20	12
20	12	19	11	3	2
21	4	5	6	17	15
22	10	19	9	5	4
23	7	5	7	20	8
24	3	4	10	20	10
25	10	18	13	4	2
26	3	8	10	14	12
27	14	12	10	7	4
28	6	13	17	6	5
29	4	25	14	3	1
30	2	5	10	17	13
31	9	17	12	5	4
32	5	7	10	16	9
33	7	12	13	10	5
34	9	20	12	4	2
35	2	3	13	16	13
36	2	7	12	14	12
37	13	15	10	5	4
38	13	15	10	5	4
39	1	7	12	16	11
40	2	5	11	16	13
41	8	15	12	7	5
42	17	15	9	4	2
43	2	7	12	15	11
44	11	13	13	6	4
45	14	16	9	5	3
46	4	6	13	15	9
47	5	8	10	13	11
48	7	15	13	7	5
49	5	7	12	13	10
50	12	18	7	7	3
51	9	20	10	5	3
52	3	6	11	16	11
53	5	17	14	8	3
54	15	13	9	5	5
55	2	5	8	12	20
56	12	18	8	5	4
57	3	7	20	9	8
58	4	3	10	17	13
59	14	15	10	5	3
60	4	5	8	16	14
61	3	9	9	14	12
62	15	15	9	5	3
63	11	13	11	7	5
64	4	7	9	13	14
65	13	14	9	6	5
66	3	8	9	13	14
67	17	12	9	5	4
68	11	15	13	3	5
69	4	8	14	11	10
70	13	15	9	5	5
71	9	15	16	5	2
72	4	3	14	16	10
73	5	7	15	13	7
74	14	17	8	5	3
75	2	5	18	15	7
76	7	20	11	7	2

Table A4.3: LASSI-HS pretest raw data for the control group

S. No	Rate				
	1	2	3	4	5
1	1	3	17	11	13
2	6	10	18	7	4
3	6	12	9	9	9
4	11	10	16	6	2
5	5	9	9	12	10
6	4	7	9	13	12
7	12	11	16	3	3
8	4	12	12	10	7
9	2	6	9	20	8
10	9	14	9	7	6
11	3	6	20	7	9
12	8	18	10	6	3
13	11	14	7	6	7
14	5	10	6	9	15
15	14	9	11	8	3
16	5	10	13	11	6
17	13	15	14	2	1
18	3	9	14	11	8
19	14	11	7	9	4
20	2	4	10	17	12
21	11	12	13	6	3
22	4	8	8	20	5
23	9	14	14	5	3
24	10	20	9	2	4
25	1	3	13	18	10
26	7	12	11	11	4
27	6	12	11	8	8
28	19	13	9	2	2
29	2	7	18	10	8
30	12	16	10	5	2
31	4	6	12	15	8
32	13	20	3	4	5
33	9	15	12	3	6
34	3	4	19	10	9
35	8	8	10	12	7
36	8	10	10	9	8
37	7	8	10	12	8
38	4	8	13	11	9
39	13	18	5	5	4
40	12	15	11	5	2
41	2	7	9	19	8
42	3	5	9	15	13
43	10	14	10	6	5
44	4	6	17	12	6
45	3	8	14	10	10
46	7	18	8	7	5
47	12	12	11	6	4
48	3	7	18	11	6
49	15	12	8	4	6
50	8	9	7	9	12
51	3	6	6	15	15
52	8	12	20	2	3
53	6	12	11	9	7
54	5	6	13	16	5
55	6	13	20	2	4
56	7	11	16	7	4
57	4	9	17	9	6
58	8	9	20	5	3
59	6	4	9	14	12
60	8	16	12	5	4
61	14	12	9	7	3
62	2	7	13	19	4
63	3	5	18	9	10
64	10	8	12	10	5
65	3	9	14	12	7
66	14	13	9	4	5
67	3	3	17	11	11
68	7	8	11	10	9
69	14	12	10	5	4
70	7	6	13	12	7
71	11	16	12	6	0
72	10	13	11	7	4
73	11	10	13	7	4
74	7	10	12	10	6
75	6	13	10	8	8
76	3	5	7	11	19

Table A 4.4: LASSI-HS posttest raw data for the control group

S. No	Rate				
	1	2	3	4	5
1	4	9	13	12	7
2	4	15	16	7	3
3	5	7	6	16	11
4	7	14	15	6	3
5	4	5	14	10	12
6	9	12	11	9	4
7	9	13	17	4	2
8	6	13	12	8	6
9	3	2	7	20	13
10	8	17	10	6	4
11	2	6	7	16	14
12	7	20	14	2	2
13	11	14	8	7	5
14	9	7	11	15	3
15	13	12	9	9	2
16	5	12	17	9	2
17	11	14	15	3	2
18	2	5	10	15	13
19	11	14	9	7	4
20	1	4	16	12	12
21	12	9	14	6	4
22	3	4	8	20	10
23	9	14	16	4	2
24	7	18	16	4	0
25	2	3	10	16	14
26	6	9	15	12	3
27	5	10	13	12	5
28	16	11	12	4	2
29	3	7	19	7	9
30	6	21	12	4	2
31	3	7	17	13	5
32	11	18	7	5	4
33	2	16	15	7	5
34	2	8	8	22	5
35	4	11	13	11	6
36	7	11	12	8	7
37	6	6	9	12	12
38	10	16	10	7	2
39	4	4	10	16	11
40	13	16	10	4	2
41	2	6	9	21	7
42	3	2	10	17	13
43	8	17	10	5	5
44	2	7	11	16	9
45	3	6	7	16	13
46	4	18	12	6	5
47	7	13	15	7	3
48	3	7	22	10	3
49	8	14	13	3	7
50	3	8	11	14	9
51	2	8	7	20	8
52	5	12	23	3	2
53	5	7	9	13	11
54	14	17	5	5	4
55	6	16	18	3	2
56	5	6	10	20	4
57	4	12	15	8	6
58	6	9	18	6	7
59	4	5	10	10	16
60	16	14	6	5	4
61	9	13	14	7	2
62	0	6	14	20	5
63	3	4	19	11	8
64	7	12	13	8	5
65	2	8	8	14	13
66	10	17	10	4	4
67	3	6	10	18	8
68	7	12	14	8	4
69	9	13	13	5	5
70	6	8	7	12	12
71	2	5	12	19	7
72	6	14	14	5	6
73	9	14	15	6	1
74	3	9	12	17	4
75	3	13	12	9	8
76	2	1	12	20	10

N.B: ***n = 45 for the control and n=47 for the experimental group***

AREA	***Appendix 5 (A5): Chapter End Self-assessment*: Chapter_____ (1, 2, 3, 4, 5)** *Circle one word to assess yourself in each area*				***Teacher As'mnt*** **(1-4)**
Reflective Activity/Diary	***POOR:*** *Little reflection; very brief comments; not all reflective activities completed.*	***FAIR:*** *Some reflection; some longer comments with little relevance to the to topic of discussion; some reflective activities completed.*	***GOOD*** *:Good ability to reflect and to criticize; most reflective activities completed with good quality; discussion directly related to topic.*	***EXCELLENT:*** *Well developed Reflection; critical ability and self analysis; relevant and related to the topic; all reflective activities completed.*	
Summary making and Journal writing	***POOR:*** *Little thought and time given to summary; little understanding of the process of summary making. Inappropriate use of mathematical notions and terminology; little explanation of the tasks; no use of multiple representation; little communication of concepts, ideas, and definitions.*	***FAIR:*** *Some thought & ability to summary making; some ability to be self critical; some direction and initiation to summarize. There is a clear use of mathematical notation and terminology; provides some quality explanation for the given tasks; communicates concepts, ideas, and definitions clearly to some extent; provides some multiple mathematical representations.*	***GOOD:*** *Good grasp of summary making techniques; good improvement in making summary. There is a clear use of mathematical notation and terminology; provides quality explanation for most of the given task; communicates concepts, ideas, and definitions clearly most of the time; provides most multiple mathematical representations for tasks.*	***EXCELLENT*** *High degree of summary making skill; accurate selection of big ideas during summary making. There is a clear use of mathematical notation and terminology; provides quality explanation for the given task; communicates concepts, ideas, and definitions clearly; provides multiple mathematical representations for tasks.*	
Contributions to group work	***POOR:*** *Often lets others get on with the task; rarely participates constructively in discussion; often working in isolation within the group.*	***FAIR:*** *Takes a reasonable share of group tasks, usually when asked to do so; listens to others & makes occasional contributions to discussion.*	***GOOD:*** *Willingly takes on work within the group, occasionally helping to organize; is often a good active listener & contributes readily to discussion; often helps the group to move forwards.*	***EXCELLENT:*** *Shows clear ability to help the group get on with the task & to involve group members; encourages others to speak & makes them feel involved; good leadership skills; gets on with tasks reliably.*	
Portfolio assignment	***POOR:*** *fail to select a good representative problem; words, drawings, and diagram not relevant to the problem, but evidence an effort to cope up with the idea.*	***FAIR:*** *The selected problem may be a slight representative; work is incomplete and not clear solution process.*	***GOOD:*** *Solid understanding in selecting a representative problem; appropriate mathematical language with figure describing the term, but may have missed minor elements. Understands feature of problem, explores it, selects appropriate strategy and reviews, revises, extends.*	***EXCELLENT:*** *Clear understanding of problem selection; complete understanding of the features of the problem, detailed understanding in problem exploration; selecting appropriate and workable strategy; reviews, revises, and extends the problem with appropriate figure.*	
Attendance & punctuality	***POOR:*** *Attended less than 80% of lessons; late for 20% or more lessons; work completed late; may have to out of participation after discussion with the teacher.*	***FAIR:*** *Attended 80% of lesson with absence for unavoidable reasons accepted by the teacher; late for 10% or more lesson; work sometimes late.*	***GOOD:*** *Attended 90% of lesson with absence for unavoidable reasons; usually punctual; work usually done on time.*	***EXCELLENT:*** *Attended every lesson; always punctual for lesson and with work submission.*	
TOTALS	**POOR (1)**	**FAIR (2)**	**GOOD (3)**	**EXCELLENT (4)**	**Teacher Ass'ment**
				Self Assessment Out of 20______/20 Self Assessment TOTAL %_______%	_____/**20** _____**%**

GENERAL OBSERVATIONS

What have you found most useful in this chapter?

Write a reflective comment on your overall performance and achievements in this chapter

Would you say that for your overall performance in this chapter you should receive a
Poor **Novice** **Apprentice** or Distinguished? **Circle one**.

Name and Signature of student:
Date:

OBSERVATIONS AND COMMENTS FROM THE TEACHER

General comments on achievement in this chapter.

Comment specifically on areas where the teacher disagreed with the self-assessment.
Comment on what the student should do in the next chapter to improve.

The overall performance of the student in this chapter: **Poor** □ **Novice** □
Apprentice □ or Distinguished□? **make a thick on one**
Name and Signature of the teacher: ______________________________
Date:____________________________

Source: (MoE, 2006; Huhs, 1994; Jones, 1994)

Table A5.1: Raw data for chapter end students' self-assessments (SA) and teacher assessment (TA)

Chapter-1		Chapter-2		Chapter-3		Chapter-4		Chapter-15	
SA %	**TA %**	**SA %**	**TA %**	**SA %**	**TA %**	**SA %**	**TA %**	**SA %**	**TA %**
100	80	90	80	80	85	85	85	85	90
100	75	85	75	75	80	80	85	80	85
95	70	90	75	85	85	85	90	75	80
100	75	95	70	80	80	85	85	75	75
80	80	80	80	85	80	80	85	80	85
100	65	90	70	85	75	75	85	75	90
75	80	85	85	85	85	70	75	75	90
100	75	90	75	75	90	75	85	80	85
100	90	85	90	70	90	80	90	75	95
90	65	75	70	80	80	80	90	85	95
80	85	80	85	75	80	75	90	80	90
100	70	100	75	80	85	75	85	75	85
100	75	75	75	80	75	85	80	85	85
100	75	75	75	75	75	75	75	80	80
80	75	80	75	75	75	75	70	80	85
100	90	85	90	80	85	85	90	85	85
100	75	100	80	85	85	80	90	85	90
80	75	75	80	75	90	80	85	85	90
95	70	80	75	80	80	75	80	80	85
100	75	85	75	85	85	80	80	90	95
100	85	95	85	80	80	75	80	85	95
100	75	80	80	85	80	75	75	85	95
85	80	80	80	80	85	75	70	80	95
100	85	80	85	80	90	80	85	85	90
100	80	90	85	90	90	85	85	85	90
80	70	75	75	75	70	80	90	85	95
100	65	75	70	75	80	80	80	90	95
80	75	80	75	80	85	80	85	85	90
100	70	75	75	75	80	85	85	85	85
90	90	85	90	80	90	85	90	85	85
80	70	75	75	75	80	75	80	80	85
100	80	95	75	80	80	75	85	80	90
100	95	100	90	90	95	90	95	90	90
85	80	80	80	75	80	80	85	85	85
100	80	80	85	85	85	80	85	85	85
80	85	70	85	75	80	75	85	80	90
90	85	85	85	80	80	80	85	95	90
95	70	75	80	70	85	75	90	85	90
90	75	85	80	80	80	75	90	80	90
100	70	95	65	75	80	80	90	85	95
100	90	95	90	75	95	75	85	85	90
100	65	85	85	85	80	75	85	85	90
100	65	95	75	85	85	80	85	85	95
100	80	100	85	85	85	75	75	70	85
80	80	80	80	75	85	75	80	80	85
90	***75***	***85***	***75***	***85***	***80***	***80***	***80***	***85***	***85***
100	***70***	***85***	***75***	***80***	***70***	***80***	***75***	***80***	***85***

Appendix 6 (A6) Programme End Evaluation Questionnaire (PEEQ)

PART ONE

Mathematics Intervention programme for ten graders at Ayider High School in Mekelle, Tigray.

Aim: To improve the quality of mathematics learning through the programme that will develop the basic skills, knowledge and attitudes toward mathematics

Has the Programme achieved its aim? Circle one of: Y (yes) NS (not sure) N (no)

	Please answer the questions below for the entire programme by ticking the column for Y (yes) NS (not sure) or N (no)	*Y*	*NS*	*N*
1	*Was the programme well organized?*			
2	*Were you actively involved in your own learning?*			
3	*Was the content sufficiently challenging?*			
4	*Were you kept aware of how well you were doing on the programme?*			
5	*Was enough time given for what you were expected to do?*			
6	*Was the way in which the programme was taught effective?*			
7	*Was the resource helpful?*			
8	*Was the programme useful to developed self learning and responsibility?*			
9	*Was the programme useful to develop a role model characteristic?*			
10	*Do you have a better understanding of how to use different strategies in your learning?*			
11	*Were you aware of the different continuous assessment methods?*			
12	*Do you have a better understanding of working with mates cooperatively?*			
13	*Was the programme helpful to do the different self-assessment mechanisms?*			
14	*Were the different elements of the programme useful?*			
15	*Was the programme helpful to continuously assessed?*			
16	*Has the programme changed the way you learn?*			
17	*Has the programme made you a reflective learner?*			
18	*Do you feel the programme has helped you?*			

How would you rate the Intervention Programme overall?
Circle one. **poor** 1 **fair** 2 **good** 3 **excellent** 4

PART TWO

Open ended observations
What would you say were the best things about the programme/ intervention?
What are one or two things that could improve the programme?
Please add any other comments you would like to make

Source: (MoE, 2006)

Thank you for your comments and writing the questionnaire.

Table A6.1: Raw data for programme end evaluation questionnaire (PEEQ)

Statement No	***Control***			***Experimental***		
	Yes (2)	***Not sure (1)***	***No (0)***	***Yes (2)***	***Not sure (1)***	***No (0)***
1	23	14	8	46	1	0
2	6	24	25	45	1	1
3	21	12	12	47	0	0
4	0	7	38	44	2	1
5	19	9	17	47	0	0
6	18	13	14	47	0	0
7	3	11	31	40	5	2
8	8	6	31	43	2	2
9	22	15	8	45	2	0
10	4	5	36	46	1	0
11	0	9	36	46	1	0
12	6	5	34	42	3	2
13	5	9	31	45	2	0
14	24	16	5	46	1	0
15	3	7	35	41	3	1
16	19	15	11	45	2	0
17	20	13	12	47	0	0
18	21	14	10	45	2	0
Objective Achievement	7	30	8	45	2	0

Statement	**Experimental**				**Control**			
	Poor (1)	***Fair (1)***	***Good (3)***	***Excellent (4)***	***Poor (1)***	***Fair (1)***	***Good (3)***	***Excellent (4)***
Programme overall evaluation	0	0	6	41	0	32	10	3