EXPLORING THE EFFECTS OF CONCEPT-BASED INSTRUCTION IN THE TEACHING AND LEARNING OF MATHEMATICS: A CASE OF ALGEBRAIC EXPRESSIONS

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MILDRET NCUBE

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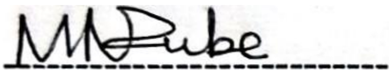
DECLARATION

I, Mildret Ncube, declare that: EXPLORING THE EFFECTS OF CONCEPT-BASED INSTRUCTION IN THE TEACHING AND LEARNING OF MATHEMATICS: A CASE OF ALGEBRAIC FUNCTIONS is my own work and has not been previously submitted for examination at UNISA for another qualification or to any other institution. All the sources that I cited or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the thesis to originality checking software and falls within the accepted requirements.



Mildret Ncube Date: 28/12/2021

Student number: 48809969

ABSTRACT

The study explored the effects of concept-based instruction in the teaching and learning of mathematics. Continuous poor performance of learners in mathematics motivated this study. From the reviewed literature, it was established that poor performance was mostly owing to the use of teaching and learning approaches that were not motivating and learner engaging. The purpose of the study was to investigate if concept-based instruction could improve learners' performance in mathematics through equipping them with conceptual understanding. To achieve this objective, a sequential explanatory design in which data were analysed quantitatively and qualitatively was employed. The mixed method design adopted pre and post-tests, questionnaire and semi-structured interviews. The theory of constructivism underpinned this study in a bid to bring up the importance of creating knowledge for oneself through linking new information to prior knowledge.

A one group pre-test-post-test-design involving 35 learners from one township high school in Mopani District, Limpopo Province of South Africa was adopted. The single group was observed before and after intervention. A purposive and convenient sample was opted for, as it was accessible to the researcher and the learners were already grouped according to level of performance therefore would be easy for identification of improvement in performance. All the 35 learners wrote the two tests and completed a questionnaire each. Six (6) of the learners were selected for interviews for clarification of how they had arrived at their solutions and provision of more information. The selection was influenced by the solutions they had provided in the tests.

Data analysis started with coding and categorising emerging themes from the data sources that were involved. Tables and graphs were used to illustrate data. A dependent t-test was carried out to identify changes in performance by learners in the two tests that were administered. From the findings, learners got engaged in the learning process and enjoyed the way lessons were conducted. The study found that the concept-based instruction allowed learners to build their own knowledge and enhanced conceptual understanding. The approach enabled them to relate new and prior knowledge. Learners managed to grasp aspects to do with algebraic functions and linked them to other topics which demonstrated connectivity. The t-test revealed that the intervention had positive gains. Both quantitative and qualitative results confirmed that concept-based instruction has the capability of improving learners' performance in mathematics.

The study recommends that concept-based approach of teaching and learning be extended to other mathematics topics and grade levels hoping it would be able to bring success the way it did to grade 11 learners of Mopani District in the topic of algebraic functions. The study therefore, recommends a paradigm shift from traditional teaching and learning approaches to the ones that target concept formation and building.

The study developed concept-based teaching and learning guidelines that were meant to promote teaching for conceptual understanding. The guidelines were designed to help learners take active roles in their learning and be in a position to link and connect knowledge.

KEYWORDS AND THEIR DEFINITIONS:

Concept: an idea, understanding of something, meaning, characteristic or thought driven from specific instances.

Conceptual understanding: is knowledge of abstract ideas.

Procedural understanding: is mastery of computational skills and knowledge of procedures.

Teaching and learning approach: is a broad range of processes from the organisation of classroom resources to the activities engaged by teachers and learners to facilitate learning.

Concept-based instruction: a teaching and learning approach that better targets conceptual understanding.

Constructivism: is a teaching and learning theory where learner centred approaches are encouraged to ensure active participation.

DEDICATION

This thesis is dedicated to my parents, my late father Josaya Juru Maramba and my late mother Mhurai Ngonidzashe Tafireyi. It is also dedicated to my late sister Precious Mushati. Their memories continue to inspire me in all my academic work. My father and mother, you made sure you left me with something to hold unto. Thank you for ensuring that I received the best education and intellectual development when you were still on this earth. My sister Precious, you were encouraging me every time to never give up. May this achievement honour your memory.

The three really appreciated education during their days. I salute them.

May their souls rest in eternal peace.

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Table of Contents

DECLARATION i
ABSTRACT ii
DEDICATION iv
ACKNOWLEDGEMENTS v
LIST OF TABLES x
LIST OF FIGURES xi
CHAPTER 1: THE PROBLEM AND ITS SETTING 1
1.1 Introduction 1
1.2 Rationale for the study 2
1.3 Background of the study 3
1.4 The research problem 9
1.5 Research questions 10
1.6 Research aims and objectives 10
1.7 Purpose of the study 11
1.8 Significance of the study 11
1.9 Structure of the thesis 12
1.10 Summary of chapter 13
CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK 14
2.1 Introduction 14
2.2 Concept 14
2.3 Understanding 15
2.3.1 Procedural understanding 16
2.3.2 Conceptual understanding 16
2.4 Teaching and learning approaches 19
2.4.1 Traditional teaching and learning approach 20
2.4.2 Concept-based instruction teaching and learning approach 21
2.5 Other studies on teaching and learning approaches 24
2.6 Literature gap 29
2.7 THEORETICAL FRAMEWORK 29
2.7.1 Introduction 29
2.7.2 Overview of constructivism 30
2.7.3 Social constructivism 31
2.7.4 Constructivism and concept-based instruction 32
CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY 37
3.1 Introduction 37

3.2 Research methodology 37
3.2.1 Quantitative research 38
3.2.2 Qualitative research 39
3.3 Research design 40
3.3.1 Sequential explanatory design 40
3.3.2 One group pre-test-post-test design 41
3.4 Data sources and data collection techniques 42
3.4.1 Negotiating access to data collection 42
3.4.2 Research instruments 43
3.5 Intervention 51
3.6 Content for the study 52
3.7 The pilot study 52
3.7.1 Pilot test of pre and post-tests 53
3.7.2 Pilot test of questionnaires 53
3.7.3 Pilot interviews 53
3.8 Population and sampling techniques 55
3.8.1 Study participants 55
3.9 Sampling 56
3.9.1 Convenience and purposive sampling 57
3.9.2 Sampling techniques 58
3.10 Description of site 59
3.11 Reliability 59
3.12 Validity 60
3.12.1 Content validity 60
3.12.2 Construct validity 60
3.13 Trustworthiness in this study 61
3.13.1 Credibility 61
3.13.2 Transferability 62
3.13.3 Dependability 62
3.14.4 Confirmability 62
3.15 Data analysis 63
3.15.1 Data analysis from interviews 63
3.15.2 Data analysis from questionnaires 64
3.15.3 Data analysis from pre and post-tests 65
3.16 Triangulation 66
3.17 Ethical considerations 66

3.17.1 Ethics certificate 67
3.17.2 Informed consent 67
3.17.3 Confidentiality 67
3.18 Summary of chapter 68
CHAPTER 4: DATA ANALYSIS AND PRESENTATION OF RESULTS 69
4.1 Introduction 69
4.2 Data presentation and analysis 69
4.2.1 RESEARCH QUESTION 1: What effect does concept-based instruction have on the teaching and learning of mathematics? 70
4.2.2 RESEARCH QUESTION 2: What advantages does concept-based approach have over other teaching learning approaches? 88
4.2.3 RESEARCH QUESTION 3: What changes are brought about in learners by the use of concept-based instruction teaching and learning approach? 96
4.2.4 RESEARCH QUESTION 4: What challenges can be addressed by the implementation of concept-based teaching and learning in mathematics? 106
4.3 Summary of the findings 114
4.4 Conclusion of the chapter 116
CHAPTER 5: SUMMARY OF FINDINS, CONCLUSION AND RECOMMENDATIONS 117
5.1 Introduction 117
5.2 Summary of the major findings 117
5.2.1 Research question 1 118
5.2.2 Research question 2 120
5.2.3 Research question 3 122
5.2.4 Research question 4 124
5.3 Recommendations of the study 125
5.3.1 Recommendations to teachers 126
5.3.2 Recommendations for pre-service and in-service training 126
5.3.3 Recommendations to policy makers 127
5.4 Contribution of the study 127
5.4.1 Implications of the guidelines 127
5.4.2 Implementation of the guidelines 129
5.6 Summary of the study 130
5.7 Suggestions for further study 131
5.8 Conclusion of the study 131
REFERENCES 133
APPENDIX A: ETHICS CLEARANCE CERTIFICATE 184

APPENDIX B: QUESTIONNAIRE 189
APPENDIX C: TEST 191
APPENDIX D: MARKING GUIDELINES 195
APPENDIX E: INTERVIEW GUIDE 199
APPENDIX F: INTERVIEW TRANSCRIPTION FOR ONE PARTICIPANT 201
APPENDIX G: LESSON PLANS 203
APPENDIX H: LETTER TO THE DEPARTMENT OF EDUCATION 215
APPENDIX I: RESPONSE FROM THE CIRCUIT MANAGER 217
APPENDIX J: LETTER TO THE PRINCIPAL 218
APPENDIX K: RESPONSE FROM PRINCIPAL 219
APPENDIX L: CONSENT LETTER 220
APPENDIX M: LETTER TO THE LEARNER 223
APPENDIX N: TURNITIN REPORT 225
APPENDIX O: LANGUAGE EDITING CERTITICATE 226

LIST OF TABLES

Table 3.1: Summary of the descriptions of the test questions 48

Table 3.2: Sample demographic characteristics 56

Table 4.1: Overview of themes and categories meant to explore effects of concept-based instruction teaching and learning approach 70

Table 4.2: Measures of dispersion and central tendency 96

Table 4.3: One-Sample Kolmogorov-Smirnov Test for normality 98

Table 4.4: Paired Samples Test 100

Table 5.1: Concept-based teaching and learning guidelines 129

LIST OF FIGURES

Figure 4.1: Learners enjoying algebraic fractions 71

Figure 4.2: L7's performance before concept-based approach 74

Figure 4.3: L7's performance after concept-based approach 74

Figure 4.4: L31's assessment of the quality of teacher's explanation 76

Figure 4.5: Learners who adjudged the teacher to be clear during teaching 76

Figure 4.6: L4's performance in identifying domain and range of a hyperbola 77

Figure 4.8: L29's performance prior to conceptual understanding 79

Figure 4.9: L29's performance after intervention 79

Figure 4.10: Confirmation of concept building questions during the lessons 80

Figure 4.11: Pre-test performances on questions 1 and 2 by L7 82

Figure 4.12: Post-test performances on questions 1 and 2 by L7 83

Figure 4.13: Pre and post-test results 84

Figure 4.14: Working shows evidence of conceptual understanding by L5 85

Figure 4.15: Learners' position on being given a chance to explain and describe mathematical phenomenon 90

Figure 4.16: Learners free to ask questions 91

Figure 4.17: Performance of L34 showing lack of knowledge on the subject 92

Figure 4.18: Performance of L34 after lesson on concepts displaying improvement 93

Figure 4.19: Was it easy to understand the topic? 94

Figure 4.20: L7's performance on Q15 before concept-based lesson 95

Figure 4.21: L7's performance on Q15 after concept-based lesson 96

Figure 4.22: One can learn mathematics better by memorising facts and procedures 101

Figure 4.23: The important aspect in mathematics is to get correct answers 105

Figure 4.24: Mathematics concepts are difficult to understand 105

Figure 4.25: Attitudes and feelings about the learning of mathematics 108

CHAPTER 1: THE PROBLEM AND ITS SETTING

1.1 Introduction

Mathematics is problematic to learners in most of the countries globally (Chand, Chaudhary & Prasad, 2021). Some of the problems as highlighted by Yong, Gates and Chan, 2018; Wilkerson and Fenwick, 2017; Singha, Goswamal and Bharali (2012) include, mathematics is too complex to understand, is not interesting as learners are not aware of its applicability, has many formulae to be memorised, is time consuming, is unaffordable because the subject needs extra coaching, learners do not get expected marks, and there is lack of basics. Delivering content in such a way that learners can connect to new and old information has been another challenge in mathematics. These problems result in learners not performing well in mathematics. Performance in mathematics is below standard as witnessed by Mazana, Suero Montero and Olifage, 2019; Khalid, 2017; Singh (2015); Human, van der Walt and Posthuma (2015). Furthermore, in South Africa, the situation seems to be worse as indicated by numerous researchers.

Low achievements in mathematics remain worrisome in South Africa (Naidoo & Kapofu, 2020; Fleming, 2020; Jojo, 2019; Tachie & Molepo, 2019; Schulze & Bosman, 2018; Ramrathan, 2017; Spangenberg, 2017; Salami & Oleke, 2017; Naicker, 2017; Siyepu, 2013). Changwe and Mulenga, 2018). Ngoepe (2014) also stressed the continuous poor achievements in mathematics by South African learners. According to Moodley (2014), South Africa trails behind the rest of the world in terms of mathematical performance. Arends, Winnaar and Mosimege (2017) note that the performance results of South African learners are poor as witnessed by the national and international studies such as Annual National Assessment (ANA) and Trends in International Mathematics and Science Study (TIMSS). Furthermore, Ominiyi (2016) laments that the pass rates of mathematics in South Africa remain the least of all other subject pass rates. Robertson and Graven (2020) and Visser, Hannan and Juan (2019) indicate that the South African government acknowledges the dire state of underperformance in mathematics.

Taylor (2019), Jojo (2019), and Alex and Juan (2017) observed the teaching of mathematics in South African schools as being among the worst in the world. According to them, South Africa participates in a number of local and international tests like ANA and TIMSS, respectively, and the results as cited by Isdale, Reddy, Juan and Arends

(2017) indicate that the majority of learners are seriously underperforming. Howie (2001) indicates that the majority of learners are seriously underperforming. The quality of mathematics and science education has been continuously reported to be bad. In the ANA tests given to Grade 9, the results show a 13%, 14% and 10% pass rates for 2012, 2013 and 2014 respectively. TIMSS administers its tests to Grade 8 learners. However, South Africa in 2007 and 2011 fielded Grade 9 learners for the mathematics tests. The reason, according to Spaull (2013), was that the tests were too difficult for her Grade 8 learners. However, this did not improve the situation as seen in the TIMSS 2011 results. South Africa after having Grade 9 learners instead of Grade 8 learners was still at the bottom together with Honduras and Botswana (Reddy, Janse van Rensburg, Van der Berg & Taylor, 2012). Again, out of the 48 countries which participated in TIMSS 2015, South Africa ranked second from last (Saal, Van Ryneveld & Graham, 2019). Matric results from 2008 to 2020 as indicated by the National Senior Certificate (NSC) Exam Reports for each year, show poor pass rates respectively as follows: 45.7%, 46,0%, 47,4%, 46,3%, 54.0%, 59,1%, 53,5%, 49,1% 51,1%, 51,9%, %58%, 54,6% and 50, 8%.

Baliyan and Khama (2020) and Kriek and Grayson (2009) identified poor performance in mathematics to be a cause of concern and established ineffective teaching approaches as one of the main causes of this deplorable situation. The researcher is of the opinion that teaching and learning approaches have a major role to play in learners' academic performance in mathematics. In line with the researcher's view, Ngoepe (2014) accentuates instructional practices used by teachers as having profound influences on learners' attitudes and performance in the subject. All these findings suggest that there is a problem in the teaching and learning of mathematics. The researcher therefore decided to explore the effects of concept-based instruction teaching and learning approach. This was proposed as an attempt to find a sustainable solution to improving learners' performance in mathematics.

1.2 Rationale for the study

Poor performance in mathematics is a major concern to mathematics teachers, subject advisors, curriculum specialists, district and provincial senior managers, learners, parents and the entire nation. The researcher having been a high school mathematics teacher for almost three decades is equally concerned about learners' poor performance in the subject in question. Most learners find the subject difficult and a worrisome number of learners opt to drop the subject. Since mathematics is one of the most crucial learning areas and the cornerstone for the development of any country, its poor results is a serious issue in any nation.

Learning difficulties in mathematics are not number-based, but mainly concept and competency based (Chinn, 2020; Novriani & Surya, 2017; Chamundeswari, 2014). Learners lacking conceptual understanding seem to have a high risk of misconceptions. The development of concepts is challenging but extremely important and according to Tsvigu (2007), the way information is presented affects the way learners select, attend, organise, and integrate information. There are situations where learners are found having many facts, rules and skills but lacking conceptual understanding. In fact, most mathematics lessons are henpecked by rottenly learnt rules and procedures. In such cases, the educator takes centre stage and learners receive information from their educator. Learners in these kinds of situations find it difficult to solve problems through connections and/or applications. The understanding gained by learners from such situations is not rich, as learners cannot apply the gained knowledge to constrained situations. The researcher appreciating the importance of conceptual understanding in mathematics decided on the need to investigate if concept-based instruction could bring about better changes in learners' performance in mathematics.

1.3 Background of the study

The teaching of mathematics highly depends on developing concepts. A concept is an abstract or generic idea generalised from particular instances (Bomer & Maxin, 2018). A concept is simply an idea of something. For one to have an idea of something, there should be some connections or links. Mathematical tasks need to be developed in such a way that they are mathematically rich, giving learners opportunities to hook and connect the learner to the real world (Ball, 1993). Conceptual knowledge, which results in knowing concepts, theories and allowing reasoning, is what is needed for improved and quality performance in mathematics. According to Selvianiresa and Prabawanto (2017), conceptual knowledge is vital to a learner as it gives him/her higher chances of retaining the information and the ability to relate and use it in unfamiliar situations. Learners with conceptual understanding appreciate the importance of a mathematical idea and where to use it as well as justifying its applicability. Lack of conceptual understanding as suggested by Chepkirui (2020) and Bantubonse (2019) is mainly attributed to the kind of teaching and learning instruction used by the educator.

South Africa introduced the Curriculum and Assessment Policy Statement (CAPS) as a was a way of trying to come up with active learning, critical and creative thinking, more knowledge and skills (Department of Basic Education, 2011). This was a way of trying to come up with teaching for conceptual understanding. However, Jojo (2019) declares unacknowledged poor mathematics teaching in most South African schools. Most educators use teaching methods that only allow them to cover the content specified in the work schedules. Teachers use weak but rapid instructional teaching and learning approaches to pass assessments instead of teaching for conceptual understanding (De Zeeuw, Craig & You, 2013). Furthermore, completion of tasks is therefore preferred at the expense of conceptual understanding. Mathematics aspects are taught as if they are disconnected, making it difficult for the learners to make connections and applications to other situations. This is also may be due to the fact that the educators do not receive thorough training for them to be able to successfully teach for conceptual understanding.

The right kind of instruction helps learners attain conceptual understanding (Leinhardt, 2019; Leitzel, 2018; Perkins, 1998). However, the gap between teaching for conceptual understanding and what happens in the classes is difficult to close because teachers are mostly worried about covering the required content because of time and lack of better approaches. Educators just employ teaching and learning approaches that are convenient to them but not conducive and beneficial to learners for conceptual understanding (Barkley, 2010). This in turn negatively affects learners' abilities and poor results are inevitable. There is therefore an urgent need to explore teaching and learning approaches which are hoped to bring better conceptual understanding so that attempts to design effective intervention strategies can be implemented.

Many strategies like direct instruction, differentiation, personalised and collaborative learning have been established to try to improve the performance in mathematics. However, they have not been fruitful due to the fact that they do not bring conceptual understanding which is rich to learners as it can be connected to other situations. The attempts as observed by Mochesela (2007) seemingly have low impact as learners still struggle and perform poorly in mathematics. Most of the South African High Schools offer mathematics lessons after school hours, on weekends and during school holidays as observed in almost all provinces. Mlachila and Moeletsi (2019), Ogbonnaya and Awuah (2019) and Schulze and Bosma (2018) revealed that there was no marked improvement in the country's mathematics results every year despite more time spent on teaching learners. This is evidenced by the matric results from 2008 to 2020 which range from 45% to 59% and the pass rates are continuously fluctuating and to add onto that, the greater number of the learners considered to have passed have marks below 50%.

According to Canagarajah (2018), Bereczki and Karpati (2018) and Molefe and Brodie (2010), several studies on mathematical practices have been conducted worldwide. Regardless of these conducted studies, learners are continuously reported to be finding mathematics difficult meaning to say that the results from the studies are not being used to bring changes in the teaching and learning of mathematics. Having cited this problem of poor performance in mathematics, the researcher anticipated reasons why learners continuously find the subject difficult. The researcher deduced failure to understand mathematics concepts as one of the main reasons contributing to this problem. The researcher then attributed teaching and learning approaches used by educators as one of the main causes of failure to perform well in mathematics. Faulkner, Earl and Herman (2019); Niss and Hojgaard (2019); Simon (2017) view understanding in mathematics as a learner's ability to justify why a given mathematical claim or answer is true or why a mathematical rule makes sense. The researcher opines that it is not what learners are taught but the way they are taught which makes them understand concepts.

The method of instruction is extremely important in the way learners grasp mathematical knowledge and skills. According to Henderson and Landesman (1995), mathematics should not be taught using top-bottom approaches that are from educator to learners all the time. Rieckmann (2018) accentuates that child-centred approaches intensify performance of learners. The researcher, being in the mathematics teaching field for a long time, has discovered that learners find the subject difficult and boring, and have negative attitudes towards the subject to an extent that some believe that they will never pass the subject. Through learners' solutions in given tasks, discussions and also asking them probing questions the researcher discovered that failure to grasp the complex concepts delivered to them by educators is one of the main reasons causing learners to encounter problems in the subject. Learners indicated that the subject is too difficult to understand making it boring. According to learners, there are many things to be memorised and they cannot afford that.

Also informed by research, poor performance is linked to learners' negative attitudes towards learning of mathematics, unavailability of teaching resources, lack of motivation, teacher quality and performance, social economic status and teaching strategies (Varaidzaimakondo & Makondo, 2020). Most of the learners spend most of their time in the classrooms with mathematics being a set of arbitrary rules and procedures to be memorised (Ball, Lubienski & Mewborn, 2001). Teachers should teach in line with Feldman (2016)'s views of teaching concepts rather than rote process to help learners discover their own mathematical abilities. Dewi and Primayana (2019) also view teaching for conceptual understanding as a way of enhancing mastering of concepts and enabling learners to explain each of the steps used in a working and to connect to related concepts. Lester and Charles (2003) added that learners with conceptual understanding know more than isolated facts and can identify important mathematical ideas and can connect and retain them. The stated properties actually depict deep understanding which characterises learners' ability to see how given things or situations are related to each other. This is actually needed in mathematics as the subject is cumulative.

In most cases, mathematics educators teach learners using the traditional teaching approaches, which emphasise drilling and memorisation of facts (Giddens, 2008). The traditional views about mathematics as suggested by Knowles (1980) see the role of the teacher as to demonstrate manipulation and emphasise rules and algorithms that learners need to master. According to Romberg and Kaput (1999), learners are expected to memorise facts and practise procedures until they have mastered them. Furthermore, Van Heuvelen (1992) notes that with traditional direct instruction, it seems the educators are trying to pour knowledge into their learners' minds, while the learners are passively listening to the educator, or watching demonstrations and little of the knowledge is retained. Mathematics educators opt for teaching methods that are manageable but might not be conducive for learners to understand concepts let alone see the bigger picture of the lesson (Angelo & Cross, 1993). Teachers should teach learners for understanding. Hiebert (2013) believes that something is understood if one can see it related or connected to other things already known. Hiebert (2013) gave two types of understanding: procedural understanding and conceptual understanding. According to him, procedural understanding is attained when learners know what to do and conceptual understanding exists when the reasons for doing something are known. Skemp (1989) also mentioned two types of understanding: instrumental understanding and rational understanding. Instrumental understanding involves having mathematical rules and ability to apply and manipulate them and rational understanding involves having a mathematical rule, knowing how to use it and the reason why it works. Therefore, instrumental understanding and procedural understanding are related as they both focus on just knowing what to do and how to do. Rational understanding is linked to conceptual understanding as they both emphasise on ability to apply rules and give reasons why those rules are applicable.

Traditional teaching approaches make learners possess procedural understanding, which does not last as there is no meaning making by learners (Ball, 1990). Procedural learning does not benefit learners in the long run. Bergsten and Frejd (2019) and Tytler (2020) emphasise that 21st century teachers should encourage learners to have conceptual understanding of mathematics. Therefore, learners must be taught to understand concepts entailing learning as more than being shown, memorising and repeating. According to Deane and Asselin (2015), a concept is a mental construct that is abstract, timeless, universal and broad. Al-Qatawneh (2009) asserts that a concept can be shown through a variety of examples. Erickson (2008) sees concepts as broad ideas that transcend the perspectives and limits of any specific subject area. If one teaches learners targeting them to grasp these general ideas, then they will be teaching them for conceptual understanding. Teaching for conceptual understanding brings deep understanding and learners can apply knowledge to other situations.

Learners with conceptual understanding know more than isolated facts and methods and they are able to learn new ideas already known (Copley & Wesley, 2008). Conceptual understanding facilitates learners to remember and retain ideas. Reys, Lindquist, Lambdin and Smith (2014) see conceptual understanding as requiring learners to be more active in thinking about relationships and making connections along with adjusting accommodate the new learning with previous mental structures. Conceptual understanding of mathematics according to the National Council of Teachers of Mathematics (2000) encourages learners to be more independent and confident. Korn (2014) believes there is lack of this conceptual understanding that characterises the core business of mathematics.

There is therefore a need for teachers to identify and use teaching and learning approaches that will help learners gain conceptual understanding. If no alternative instructional methods are used, it will negatively affect the ability of learners to use discipline-related concepts, not just in presenting their understanding of the discipline but also in the workplace where practical application of such concepts may be required. The traditional teaching methods do not teach learners for deep understanding as they often necessitate textbook-based instructions that accentuate procedures, rules and laws targeting learners to just pass examinations. They focus more on covering content at the expense of conceptual understanding. Knowledge without sufficient understanding does not last. Peterson and Barrett (1987) accentuate that regurgitation of facts is not important but the development of powers of the mind. In other words, content knowledge is not enough to solve problems in the mathematics department but the issue is how to integrate deeper understanding of concepts.

To attain conceptual understanding there are some teaching and learning approaches that can make it happen. Giddens (2007) recommends the concept-based instruction approach as being capable of tackling the current problem of learners lacking conceptual understanding. Concept-based instruction as observed by Erickson (2007) is a teaching and learning approach that brings 'real world' meaning to content knowledge and skills. Erikson adds that concept-based instruction connects learners to their previous experience, bringing relevance to learning and facilitates deep understanding of content knowledge. Schill and Howel (2011) see concept-based instruction as leading learners to think about content and facts at a much deeper level. Erickson, Lanning and French (2017) add that concept-based instruction as a way of teaching and learning that goes beyond regurgitation of factual knowledge in a way promoting conceptual understanding.

Instead of the traditional teaching and learning approaches that emphasise recalling specific aspects in isolation, concept-based instruction concentrates on the understanding of broader principles that can be applied to a variety of specific examples. Scholars like Erickson (2007), Erickson, Lanning and French (2017), Schill and Howell (2011) and Robertson (2013) support concept-based instruction as better teaching approach for concept understanding where the learners take a centre stage in finding new information and presenting it to other peers in a way that enables them to see a bigger picture of the topic. In the traditional teaching approaches, learners do not get a chance to discover for themselves. With this traditional direct instruction approach, educators take centre stage, learners do just the role of listening and taking down notes, contrary to contemporary teaching approaches like constructivism. Educators just want the learners to receive information without knowing how it is driven. If learners get a chance to discover key content on their own; it helps them to have a deeper understanding. Educators need to assist learners to reach effective concept understanding. Bloom (1956) in his taxonomy postulates that educators should not stop at awareness level only but follow it up to understanding and meaning creation to ensure that effective learning takes place. Burner (1966) believes that all subjects have a certain logic or conceptual structure associated with them. As such, it is the responsibility of the educators to help learners gain a full understanding of that subject's underlying conceptual structure as the gateway of making good use of foundational knowledge to solve complex problems.

As the responsibility of learning is transferrable from educators to learners, mathematics learners struggle to comprehend the learning style and teaching methodologies. EL Miedany (2019) and Biggs and Moore (1993) contend that the learner should take an individual transformation approach to learning from passive to active learning to enable conceptual understanding. Tetlock (2019) and Killen (2010) believe that learning becomes more effective when learners have the opportunity to think, then reason and debate their own understanding of concepts. Learning is a search for meaning and coherence in one's life where emphasis is placed on what is learnt and its personal significance to the learner, rather than how much is taught (Polman, Hornstra & Volman, 2021; Candy, 1991). As such, teaching methods that require learners' active participation and engagement in acquiring the knowledge are the best for conceptual understanding.

Having gone through the views of other researchers like Biggs and Moore (1993), Killen (2010), Copley and Wesley (2008), Erickson (2007) Schill and Howel (2011), Robertson (2013) and many others' views, the researcher discovered that teaching and learning approaches have a role to play on the way learners grasp concepts. A large number of students find mathematics boring, irrelevant and unrewarding (Batool, Akhter & Kasoom, 2020; Dele-Ajayi, Strachan & Pickard, 2019; Yingprayoon, 2017; Colgan, 2014). There is need for teachers to use teaching approaches that instil motivation in learners and help them to attain conceptual understanding. It is apparent that learners' conceptual understanding is enhanced by teaching and learning approaches that involve learners participating in activities, which make them discover on their own rather than just being spoon fed by the educators. The concept-based instruction teaching and learning approach, which is a teaching and learning approach involving organising ideas to make sense of facts and the real world, seems to be one of those approaches whose effects can bring better conceptual understanding. Hence, the researcher needed to explore the effects of concept-based instruction on the teaching and learning of mathematics.

1.4 The research problem

Various reasons for poor performance and lower pass rates in mathematics such as learners' and teachers' attitudes, learners' lack of key mathematical concepts, unavailability of teaching and learning resources, lack of training and experience of teachers have been identified (Varaidzomakondo & Makondo, 2020; Arivina & Retnawati, 2020). Lack of understanding of key concepts seems to be the major cause. Researches by McIntyre (2005), Bush (2011), Mamba (2013) and Mashazi (2014) highlighted this in their investigation on learners' misconceptions in mathematics. In their studies, they discovered that learners make errors in owing to misconceptions. These misconceptions according to them are owing to lack of conceptual understanding. Learners lack conceptual understanding because their learning is based on memorisation and routine drilling (Mendezabal & Tindowen, 2018; Mochesela, 2007).

There is, therefore, a need for educators to adapt to the current teaching and learning styles that focus on concept-based understanding. This study was designed to establish whether or not concept-based instruction teaching and learning approach could positively affect learners' achievement in mathematics. To achieve that goal, this study will seek answers to the following research questions.

1.5 Research questions

- What effect does concept-based instruction have on the teaching and learning of mathematics?
- What advantages does concept-based instruction have over other teaching learning approaches?
- What changes are brought about in learners by the use of concept-based instruction teaching and learning approach?
- What challenges can be addressed by the concept-based instruction in the teaching and learning of mathematics?

1.6 Research aims and objectives

1.6.1 Research aims

- To gain understanding of the effects of concept based instruction approach on learners' conceptual understanding.
- To bring out effects of concept based instruction in the teaching and learning of Mathematics.

1.6.2 Research objectives

- To discover the benefits of concept-based instruction approach in the teaching and learning of mathematics.
- Identify the advantages of using concept-based instruction teaching/learning approach.
- Identify changes in learners' performance and interests owing to use of concept-based instruction approach.
- To make recommendations regarding the implementation of concept-based instruction approach in the teaching and learning of mathematics.

1.7 Purpose of the study

This study explored the effects of concept-based instruction in the teaching and learning of mathematics with the intention to bring positive changes in learners' understanding, performance and attitudes in the subject. According to Baran, Fioravera, Marchisio and Rabellino (2017), mathematics teaching has to prepare adaptive learners who are able to apply what they learnt in school in different and challenging situations that they may encounter in their lives and at workplace. This means that the teaching of mathematics should enable learners to gain knowledge and apply it to other situations. Genc and Erbas (2019) aver that mathematics teaching and learning should help learners to develop confidence and competence necessary to deal with any mathematical situation without being hindered by fear of mathematics. Teaching and learning approaches greatly affect learners' understanding, hence affect their performance. In support of this, Everaert, Opdecam and Maussen (2017) stated that teaching approaches have a great impact on learners' performance. In the same vein, Wentzel (2002) also pointed that choices of teaching approaches bring important differences in learning. Errors and misconception owing to lack of conceptual understanding, as suggested by Qian and Lehman (2017) and Owusu (2015) are results of educators' instruction approaches.

The researcher opines that concept-based instruction approach is an approach that can assist in having these beliefs fulfilled. Concept-based instruction approach can make mathematics have meaning to learning as it enables learners to improve their thinking skills (Saez-Lopez & Sevillano-Garcia, 2019; Erickson, 2012). This is because this teaching and learning approach involves learners, as they do not just receive information as the educator passes it onto them. In concept-based instruction, learners formulate concepts for themselves and it enables them to link and connect this to other situations.

The study intended to eplore the effects of concept-based instruction and use the results to improve learners' conceptual understanding in mathematics through implementation of the concept-based instruction by providing guidelines. The purpose of carrying this study was to help both educators and learners by bringing an improvement in the performance in the performance in mathematics and allow learners to perceive the subject in a better way.

1.8 Significance of the study

The whole significance of research is to release the findings to the public so that information can be used to bring better changes to the society (Strydom, 2011). The study on effects of concept-based instruction teaching and learning approach was worth carrying out because the results of the study could inform educators, subject advisors, curriculum planners and textbook writers to broaden their understanding of how teaching approaches can affect learners' conceptual understanding. Detailed information on concept-based instruction in the teaching and learning of mathematics together with the guidelines are hoped to greatly contribute to educators' classroom instruction.

Few studies in South Africa as well as globally focused attention on the concept-based instruction teaching and learning approach and worse, none of the studies directly explored the effects of concept-based instruction on the teaching and learning of mathematics. It was against this background that the researcher felt the study on exploring the effects of concept-based instruction on the teaching and learning of mathematics was worth carrying out.

1.9 Structure of the thesis

To present this study and its findings the researcher used the following organisation:

The **first chapter** describes the problem and its setting. In addition, it includes the motivation for the study, followed by background of the study is the researcher's offset. The chapter goes on to give the research problem, its purpose and significance. Research questions, limitations as well as the study's aims and objectives also form part of this chapter.

Chapter 2 presents a detailed review of related literature together with the theoretical framework underpinning the study. The chapter gives the basis of the study by examining the previous works done by other researchers. The chapter has a section describing the research gap. There is also a detailed discussion on the theoretical framework.

Chapter three describes the methodological constructs of the research. Detailed explanations of the research design and data collection methods employed are outlined. The sample, sampling methods, the participants of the study, the research instruments, reliability, validity, ethical considerations and limitations of the study are also discussed in this chapter.

Chapter four is a presentation of the data collected through the following: pre-test, post-test, questionnaire and interviews. Data collected were tabled and graphically represented then analysed.

Chapter five is a discussion of the findings, summarising the key findings, development of teaching and learning guidelines and giving recommendations. The chapter then concludes the study.

1.10 Summary of chapter

This chapter has given the background, the statement of the problem, the aim of the study and its accompanying objectives together with the research questions. The chapter also outlined the purpose and the significance of the study as well as the chapter outline. The next chapter discusses a review of the related literature concerning concept-based instruction as well as traditional teaching approaches in general. The intention of the literature study is to gain more understanding of the problem.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

There has been a plethora of research on the teaching and learning of mathematics and in particular teaching and learning approaches. Numerous studies, as will be seen in the sections to follow bring out many views about teaching and learning approaches. However, there is limited research on the concept-based instruction teaching and learning approach. According to Chiphambo (2017), literature review allows the researcher to position his/her study in relation to related academic studies. Furthermore, literature review involves exploring, locating and summarising research studies about a research topic (Creswell, 2014). This chapter is meant to give reviews of related literatures in an attempt to provide foundation and ground theory for an organised study of types and effects of teaching and learning approaches in mathematics. It also gives the theoretical framework that underpins this study.

2.2 Concept

Leggett (2011) defines a concept as an accepted collection of meanings or characteristics associated with certain events, objects, conditions, situations or behaviours. Furthermore, Leggett (2011) maintains that concepts are created by assigning and relegating objects and happenings that have common properties beyond any single observation. Consequently, concepts can be viewed as a collection of objects or events having common aspects. This means a concept is a term given to generalisations or observations made about events. Thompson (2006) views a concept as an abstract based on characteristics of perceived reality. In this case, a concept can be seen as an idea that has been generalised and stipulated by many people. A concept is therefore a general idea, understanding of something, meaning, characteristic or thought driven from specific instances.

A concept is not something that a learner can easily attain; s/he has to attain it through a process of pure thought (Skemp, 1964). There has to be some reflective thinking with sifting of existing ideas in order to come up with ideas that give meaning to new ideas. Concepts are the strands, threads or unifying themes that faculty have identified to shape, organise and implement the curriculum in some logical focused way (Valiga & Bruderdle, 1994, p. 17). One can also view concepts as a collection of social, cultural, historical constructions and ideas that maintain similar forms and patterns overtime (Hardin & Richardson, 2012). Concepts as explicated by Ominiyi (2016) are constructs which are timeless, universal and transferrable across time and situations. Dreher, Lindmeier, Heinze, and Niemand (2018) define a concept as a fundamental idea of mathematics that is the basis of solving problems. A concept can therefore be seen as a collection of generalised ideas that one can appropriately apply, and be in a position to justify the mathematical thinking behind those ideas.

According to Lerman (1983), concepts do not just develop from nowhere; they are discovered and Grossman adds that knowledge through self-discovery has more meaning to learners than guided discovery. The self-discovery approach challenges learners to think more deeply about concepts and to create representations and explanations that connect with their prior experience in a personally meaningful way (Simon, 1985). Learners' discoveries help them to retain concepts better than learners who are spoon fed by teachers (Sada, 2019). This is because discovery facilitates the process of building cognitive structures. It is therefore, important for the educator to know the strategies to employ for learners to be able to build conceptual understanding through discovering either on their own or guided.

2.3 Understanding

For effective learning, learners have to understand concepts. Understanding can be explained as a situation where one can see how something is related or connected to other things already known (Hiebert, 1988). Understanding involves connection of ideas rather than just knowing isolated facts (Maynard 2019; Grossman 1986). According to Mwakapenda (2004), understanding concepts is one of the most important traits associated with educational goals. Omoniyi (2016) is of the opinion that if learners understand, they want to learn more and have desire for deep understanding which promotes learning. He also believes that if learners do not understand they feel discouraged, forcing them to give up learning. Understanding breeds confidence and engagement while not understanding leads to disillusion and discouragement (Chinamasa, 2012).

Understanding enables memory transfer and it greatly influences learners' attitudes, beliefs, motivation, interest and achievement in a subject (Stern, Ferraro & Mohnkern, 2019; Lewis, Lange & Gills, 2005; Greenfield, 1996). Understanding as described by Nickerson (1985) depends on the amount of knowledge one has about concepts. Arends, Winnaar and Mosimege (2017) see educators as the influencers of the understanding of mathematical concepts. However, mathematics is a subject area well known for having problems in terms of understanding (Ali & Reid, 2012). In understanding, a learner might have either procedural or conceptual understanding. Procedural understanding and conceptual understanding are terms used to denote two forms of mathematical understanding (Boroody, Clements & Sarama, 2019; Ocal, 2017; Hiebert & Lefevre, 1986; Wearne & Hiebert, 1988). These terms will be elaborated in the next two subsections.

2.3.1 Procedural understanding

Procedural understanding involves mastering of rules (Nahdi & Jatisunda, 2020). Atkinson (1997) views procedural understanding as mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms and definitions. This kind of understanding results in memorisation of rules and algorithms and also recitation of facts. Procedural understanding involves memorised operations with little or very limited understanding of underlying meanings (Duarte & Nogueira, 2019; Arslan, 2010), and this type of understanding does not last long in the mind (Mutsvangwa, 2016). This is because the learners do not search for the how and why part in the learning process when procedural knowledge is imparted. Procedural understanding, as explained before can also be referred to as instrumental understanding. Procedural understanding can be taken as recognition of symbols and ability to follow rules to solve problems. It is therefore sequential in nature. This understanding has specific situations where it is applicable.

The main problem with procedural understanding is that it is not transferrable as it just puts emphasis on rules, facts and procedures used for solving mathematical problems. Rules, facts and procedures as explained by Mushipe (2016) are used for specific situations or things making it difficult to generalise owing to their inflexibility. Memorisation of rules and algorithms is not adequate for conceptual development (Machaba & Moloto, 2021; Osterman & Brating, 2019; Sinay & Nahornik, 2016). Rubin (2005) also argues that there is little value in knowing a set of procedures without having the underlying meanings. Teaching content and facts is not enough for learners as they do not promote conceptual understanding. There is a need for a better understanding which brings deeper understanding of underlying concepts and allows learners to relate and interconnect. Research work by Rizvi and Lawson (2007), Protheroe (2007) Ghani, Ibrahim and Yahaya (2017) underscores the importance of conceptual understanding. However, most educators prefer procedural knowledge because it is ease to instil.

2.3.2 Conceptual understanding

Conceptual understanding is defined by Yurekli, Stein, Correnti and Kisa (2020), Semilarski and Laius and Rannikmae (2019) and Rittle-Johnson and Schneider (2015) as knowledge of abstract ideas. In harmony, Ojaleye and Awofala (2018), Kenedi, Helsa, Ariani, Zanil and Hendri (2019) see conceptual understanding as knowledge of underlying structure of mathematics and consisting of relationships and interconnections of ideas that explain and give meaning to mathematical procedures. Williams (2020), Zengin (2019) and Van de Walle (2004) specifically describe conceptual understanding of mathematics as involving logical relationships constructed internally and existing in the mind as part of the network of ideas. Setyaningrum (2018), Liu and Chen

(2018) and Kilpatrick, Swafford and Findel (2001) consider conceptual understanding as an integrated and functional grasp of mathematical ideas. Schuster, Cobern, Adams, Undreiu and Pleasants (2018) and Grossman (1986) describe conceptual understanding as the grasping of ideas which involves knowing more than isolated facts and methods. In line with this, Borghi, Fini and Tummolini (2021) and Skemp (1976) posit that an individual possessing conceptual understanding knows how to process something and the reasons behind the way s/he processes it that way. A learner having conceptual understanding in mathematics provides a holistic education for himself/herself (Korn, 2014). In mathematics, Thurtell, Forrester and Chinnappan (2019), Hinton and Flores (2019), Setyaningrum (2018) and Andamon and Tan (2018) see conceptual understanding as knowledge that involves a thorough understanding of underlying and foundational concepts behind mathematical algorithms.

According to Awofala (2017), and Ghazali and Zakaria (2011), conceptual understanding allows learners to recreate formula and proofs without the rote process and enabling them to solve mathematical problems in various forms as well as in novel situations. They went on to indicate that being in possession of conceptual understanding is a gateway to better achievement in mathematics. This is because conceptual understanding requires careful development which enhances understanding of key concepts. When teaching for conceptual understanding, learners are given a chance to connect procedures to underlying concepts, justify their process and give self-explanation (Rittle-Johnson & Schneider, 2015). Bandalos (2018) and Mosimege (2018) add that learners with conceptual understanding know the how and why parts of given statements. This depicts that learners can generalise on their own and come up with their own algorithms. Mostly, learners develop and discover conceptual knowledge for themselves (Husni,2020). Conceptual knowledge is therefore flexible and it gives learners an allowance to think creatively. A learner with this type of understanding, as supported by Berry, Waite, Dear, Capon and Murray (2018), Lindquist, Suydam and Reys (1995) and Hiebert and Lifevre (1986) can think about relationships make connections along with adjustments to support new and prior mental structures. The emphasis is that conceptual understanding is about relationships of ideas. Learners in possession of this kind of understanding can compare, link and interconnect ideas and can apply concepts in problem-solving situations.

Frederick and Kirsch (2011) believe that when learners understand the meaning and underlying principles of mathematics, it is an indication that they have conceptual mathematics knowledge. According to Naicker (2017), learners who are developed to acquire such understanding have a sound grounding and understanding of mathematical concepts. If a learner possesses conceptual understanding then the learner understands ideas and generalisations that connect mathematical concepts (Kaiser, 2020; Widada, Herawaty & Lubis, 2018). Comprehension of mathematical ideas, operations and relations is what conceptual understanding is all about in mathematics. Learners possessing conceptual understanding can link and connect ideas thereby putting themselves in a better position of dealing with misconceptions. Teaching mathematics becomes meaningful if there is conceptual understanding (Maass, Geiger, Ariza & Goos, 2019; Leitzel, 2018; Korn, 2014).

Conceptual understanding as observed by Mayer (2019), Ferraris, Mazzoleni, Devalle and Couturier (2019); Hiebert and Lifevre (1986) is achieved in two ways: the construction of relationship between pieces of information and by the creation of relationships between existing knowledge and new information entering the system. Piaget (1977) refers to these processes as accommodation and assimilation respectively. These two processes describe how an individual adjusts the mind to new experiences and be in a position to take new data (Kim & Park, 2019; Ncube, 2016). Assimilation occurs when a new idea is interpreted in terms of an existing schema (Milad, 2019; Hanfstingl, Benke & Zhang, 2019; Leitzel, 2018; Moodley, 2014). A new mental structure is therefore created based on an existing mental structure. Saracho (2021), Azizah, Masykuri and Prayitno (2018) and Moodley (2014) also describe accommodation as occurring when there is incorporation of new ideas which are not related to the existing schemas. This comes about because the existing schema may not be enough to assimilate new ideas. Therefore, existing schemas need to be modified or else new schemas have to be created so that the new experience can be taken care of (von Glasersfeld, 2019; Zhu, Wright, Wang & Wang, 2018; Ncube, 2016). Therefore, conceptual knowledge requires the learner to be active in thinking about relationships and making connections and also adjusting possessed knowledge to accommodate new ideas.

The key point behind conceptual understanding is that it helps learners to reason and present their arguments logically. Ozrecberoglu and Caganaga (2018) and NCTM (2000) view conceptual understanding as encouraging learners to be independent and confident preparing them for any situation. Learners possessing conceptual understanding are able to attack mathematical questions with completely new scenarios. Taking into consideration a situation where two points A (-2; 0) and B (5; 0) and a learner is asked to determine AB, learners lacking conceptual understanding will struggle to see that it is 7 units. In fact, some of them may even add 5 and (-2) to get 3 units. They will not even realise that the answer they are giving is even smaller than OA. In mathematics, learners with conceptual understanding understand key ideas and common sense of those ideas. Therefore, learners need to possess conceptual understanding of mathematical ideas and their significance in given situations. Learners' knowledge mostly consists of unrelated facts, formulae and equations poorly organised for retention and future use (Akhter & Akhter, 2018; Acharya, 2017; Heuvelen, 1991; Mestre, 1991). These are results of lack of conceptual understanding.

Achievement of conceptual understanding is described by Mayer (2019), Qi and Chau (2018) and Hiebert and Lifevre (1986) as a learner's ability to construct relationship between pieces of information and create relationship between existing knowledge and new knowledge. Rays, Suydam and Lindquist (1995) are also of the opinion that a learner possessing conceptual knowledge can actively think about relationships, can make connections and can adjust to accommodate new and previous mental structures. Wojsik (2017) also sees conceptual understanding as something that cannot be attained through direct teaching. To attain conceptual understanding, concepts have to be learnt, understood and then applied to other situations. Blais (1988) believes that lack of conceptual understanding results in shallow knowledge that is difficult to retain. There is a need to challenge learners to think so that they develop conceptual understanding.

According to Niss and Jablonka (2020), Letwinsky (2017) and Kilpatrick, Swafford and Findell (2001), conceptual understanding is a critical component of mathematical literacy that is necessary for anyone to learn mathematics. Conceptual understanding has been described as the kind of understanding needed by mathematics learners for improved performance because of its characteristics. In my thinking, both procedural and conceptual knowledge are required by the learner depending on the situation in which the learner is in. However, even though procedural knowledge is important in some cases, conceptual knowledge outweighs it. Conceptual knowledge allows learners to understand basic concepts or key ideas and it gives learners the ability to connect and apply it to other situations. How then can this conceptual understanding be developed in learners? The answer comes from the type of teaching and learning approaches that the educator employs. This researcher is now going to describe two different teaching and learning approaches.

2.4 Teaching and learning approaches

Teaching learning approaches refer to a broad range of processes from the organisation of classroom and resources to the activities engaged by teachers and learners to facilitate learning (Khalil, Meguid & Elkhider, 2018). Poor quality approaches lead to poor performance in mathematics (Retnawati, Kartowagiran, Arlinwibowo & Sulistyaningsih, 2017; Spaull, 2013). The use of poor teaching and learning approaches does not promote acquisition of adequate basic concepts and skills. Therefore, there is great need to use approaches that are fun and unthreatening to learners in order to achieve better results.

2.4.1 Traditional teaching and learning approach

Traditional teaching and learning approach begins by defining the concepts and then focus on developing an understanding of a systematic sequenced procedure through use of definition and examples (Bond, 2020; Van Merrienboer & Kirschner, 2017). Norqvist (2018) argues that traditional teaching and learning approach emphasises procedures and algorithms with facts and rules learnt by rote and stored as bits of data in the memory. It encourages memorisation at the expense of logical thinking. Onyancha (2017) summates that there is a possibility that there is no understanding of learnt content, therefore making it difficult for learners to relate to other content. Learners involved in the traditional approach lack conceptual understanding and problem-solving skills as they compensate their lack of understanding by memorising mathematical procedures (Chinn, 2020; Lambert & Spinath, 2018; Simon, 1985). The approach is content-based and teacher-centred, and its attention is focused on fostering lower order thinking skills. It places no emphasis on conceptual understanding.

The traditional teaching and learning approach is described as a one person show with the educator dominating while learners are observed as empty knowledge seekers (Gambari, Shittu, Ogunlade & Osunlade, 2018; Inuwa, Abdullah & Hassan, 2018; 18; Kalu 2012; Omoniyi, 2016). In the class of an educator using traditional approach, learners absorb transmitted sets of established facts, skills and concepts without regurgitation. In the traditional approach when teaching exponents, learners are just told that any number or term to the power zero is one. There is no explanation of how the one is obtained. The approach employs the chalkboard method to explain problem-solving (Jeske, Jones & Stanford, 2019). According to Thomas, Cassady and Heller (2017) and Wei and Eisenhart (2011), exam pressures affect teaching and learning approaches. Educators in most cases target the ability of learners to answer exam questions not taking into consideration the use of the knowledge in any other situation. Learners cannot apply learnt material to new situations. Moreover, the learnt material is easily forgotten. There is no link to real life situations. This simply means that the knowledge acquired by the learners involved in this approach is not meaningful. Awidi and Paynter (2019) add that learners from classes where traditional approach is employed do not perform well because they are not fully equipped with necessary critical problem solving skills.

The traditional approach has been extensively noted by education critics for its failure to recognise learners as knowledge constructors (Mutsvangwa, 2016). This teaching and learning instruction involves assigning and correcting homework and emphasising repeated practice. According to Arsaythamby and Cut (2014), the approach does not give learners the opportunity to develop their own understanding as they do not have much time for independent thoughts or even initiated questions. The learners are expected to accept the educator's information without questioning (Omoniyi, 2016). Eventually, learners end up losing interest because of lack of time for active participation. The approach's main target is to get correct answers. The educator using the traditional based approach worries about content to be covered not what has to be understood. The traditional approach is mainly characterised by learning of isolated facts and memorisation.

The teaching and learning of mathematics has been heavily dominated by the traditional approach that depends on the chalkboard and the textbook with limited class activities (Onyancha, 2017). The traditional approach dominates most mathematics classes with the educator as the main person to deliver knowledge through lectures and learners being in the passive mode (Alexander 2017; Tanujaya & Prahmana 2017; O'neill & McMahon 2005). Lecturing is the main method of teaching and learning in the traditional approach and this method does not allow learners to reflect. The lessons that are conducted using this approach are exam-oriented with little or no emphasis on conceptual understanding. There is no opportunity for learners to learn at their pace and there is no time for interaction or negotiation (Panthi & Belbase, 2017).

Insufficiency of the traditional mathematics teaching leads to low performance in the subject (Stigler and Hiebert, 2004). There is much involvement in teaching by imposition which needs replacement by teaching by negotiation. More importantly, learners should play a major role in learning for them to develop conceptual understanding. Mishra, Gupta and Shree (2020), Hyun, Ediger and Lee (2017) and Simon (1985) support the need to shift from traditional teaching and learning approach of lecturing and demonstrating to one that demands new skills in planning and facilitates learner participation during the learning process. Selection of the right teaching and learning approach makes learning funny and appreciated (Wahyudi, Joharman & Ngatman, 2017). It is assumed that if a lesson is funny, then learners get involved and concentrate which in turn brings better performance. Therefore, having discussed the characteristics of the traditional approach and came up with mostly unfavourable qualities, there is need to bring in another teaching and learning approach.

2.4.2 Concept-based instruction teaching and learning approach

A concept needs to be introduced using approaches that give a learner the chance to grasp it well and avoid misconceptions. The teacher has to use teaching and learning approaches that bring conceptual understanding in learners. Not every method can be used to develop learners' conceptual understanding (Brown & Palinscar, 2018; Korn, 2014). Accordingly, concept-based instruction is one of the teaching and learning approaches that can be used for learners to understand and have meaningful learning as it continually concentrates on moving learners towards deeper conceptual understanding. Brussow, Roberts, Scaruto, Sommer and Mills (2019), Deane (2017) and Al-Qatawneh (2012) encourage educators to shift from traditional teaching and learning approach to concept-based instruction approach that is conceptually oriented. The intention of this approach is to develop conceptual understanding.

Concept-based instruction was first proposed by Jerome Bruner the cognitivist. Bruner (1961) postulates that learners are supposed to construct their own knowledge through organising and categorising information. Concept-based instruction approach is a teaching and learning approach that better targets conceptual understanding (Erickson, 2012), allows connections and encourages learners to think at more elevated levels bringing high retention levels (Ross & Myers, 2017). It is an approach used to bridge learners' thinking approaches from factual knowledge to conceptual level of understanding. An instruction that is concept-based is a teaching and learning approach that creates a way of organising ideas for learners for them to make sense of facts, link and apply them in other situations. Concept-based instruction enables learners to understand how and why each of the ideas and relationships work the way they do (Giddens, 2019; Higgins & Reid, 2017; Skemp, 1976). This is because this teaching and learning instruction involves comprehension of operations and relations.

Concept-based instruction deepens understanding of knowledge and helps learners to structure their learning (Nielsen, Lanciotti, Garner & Brown, 2021; Fazilatfar, Jabbari & Harsij, 2017; Erickson, Lanning & French, 2017). It allows learners to develop critical thinking skills that are crucial in learning (Ross & Myers, 2017). Moreover, it creates relevance to learning and brings deeper understanding of content. Concept-based instruction enables learners to link their prior experiences to new experiences. It increases learners' critical thinking and influences their perception of issues. Nielsen (2016) views concept-based instruction as bringing connection between topics as well as subjects. According to Lanning and Brown (2019), Shemilt and Howson (2017) and Schill and Howell (2011), concept-based instruction enables learners to put learning in a bigger picture which in turn brings deeper conceptual understanding. It therefore helps learners to raise their performance. This kind of instruction is one in which the educator has to set objectives which are big ideas that have to be achieved by the end of the lesson.

Concept-based instruction is a teaching and learning approach that mandates more high order and critical thinking skills (Lee & Wilson, 2018; Erickson, 2012). Taking into consideration a situation where a learner has this question: If the second difference of a quadratic sequence is two, does the sequence have a maximum or a minimum value? In this case, the learner has to demonstrate an understanding of the effect of the sign in the coefficient of the squared independent variable in a quadratic function. The learner in this case has to do critical thinking. The sequence being quadratic has a parabolic graph whose maximum or minimum is determined by the coefficient of the independent variable squared. The learner has to apply the fact that the second difference of a quadratic sequence equals to the value of the coefficient twice. This is an application of high order and critical thinking to answer this question.

When using concept-based instruction, the main objective is to develop concepts before introducing procedures and algorithms. Stern, Ferraro and Mohnkern (2017), Baron (2017) and Al-Qataweneh (2012) see concept-based instruction as bringing global enduring understanding as it emphasises the involvement of learners in the learning process relating concepts and skills to their background knowledge. There is meaningful learning with new knowledge acquired by the learner not lined up or stocked on top of previously acquired knowledge (Deane & Asselin, 2015). Previous knowledge and experience are integrated with new knowledge as a way of understanding new interlocking concepts. Concept-based instruction fosters mathematical understanding and confidence in learners through collaboration of ideas and exploration of concepts (Akben, 2020; Erickson et al., 2017; MacMath, Wallace & Chi, 2009). In support Lopez-Faican and Jaen (2020) and Parris and McInnis-Bowers (2017) stress the fact that collaboration of ideas brings confidence and feelings of unity.

Wagganer (2015) views learners involved in this approach ending up being co-constructors of knowledge through asking questions, justifying their work, communicating their ideas to one another, and comparing and contrasting ideas. Teaching learners using the concept-based approach enables them to develop, communicate, justify their thinking processes, and to connect to the real world contexts making them see mathematics as accessible and enjoyable (Lantolf & Esteve, 2019; Awofala, 2017). The knowledge developed by learners through concept-based instruction is flexible as it can be adapted to new situations. It can be used to discover and learn new things. In this approach the teacher facilitates discussions and involves learners in activities prompting for information. The concept-based instruction is a learner-centred approach developing reasoning skills using concepts and skills (Hautemo, 2017; Wood, 2017). It allows transformation of ideas, which according to Mondal, Majumder and Mandal (2019), together with the taxonomies of Bloom (2001), occurs when learners can combine facts and ideas, synthesise, generalise, explain, hypothesise or make interpretations, conclusions and/or generalisations.

Kenedi, Helsa, Ariani, Zainil, and Hendri (2019) view mathematics as a subject that enables learners to solve problems, investigate, represent and communicate mathematical concepts and ideas. Mishra, et al. (2020) underscore the need for teaching and learning approaches that allow learners to come up with globally minded ways that can solve various problems using several approaches and be in a position to deal with difficult situations. The concept-based teaching and learning approach is one such instruction capable of making this happen. It allows learners to acquire knowledge and have reasons for ‘why’ as opposed to the traditional approach that focuses on the ‘how’ part (Mayer, 2019; Goldman, 2019; Selden & Selden, 1995).

2.5 Other studies on teaching and learning approaches

Umugiraneza and Bansilal (2017) carried an exploration on teachers’ practices in teaching mathematics. The study involved 75 mathematics teachers from KwaZulu-Natal. A questionnaire was administered to the participants, Grade 4 to Grade 12 teachers, in which they were supposed to indicate teaching methods that they used. The results indicated that almost all teachers who were involved in the study used teacher-centred approaches. The study recommended that professional development to help teachers improve their expertise on teaching strategies was a necessity.

Chimuka (2017) carried out an investigation on the effect of integration of geogebra into the teaching of circle geometry on Grade 11 South African learners from Limpopo province. The study sought to explore the extent to which technology inspired techniques and strategies impact on learners’ achievement in mathematics. The results of the investigation indicated better achievements in performance in the post-tests from the students who used geogebra as compared to the ones who were instructed with traditional teaching method.

Ntow and Hissan (2021) explored the impact of concept-based instruction on senior high school Ghanaian students’ achievement in circles theorems. An investigation was carried on the influence of concept-based instruction and traditional teaching method. The findings of the study established significance difference in mathematics achievement of students. Students who were exposed to concept-based instruction performed better than their counterparts who were in the class of conventional instruction.

Anyamene, Nwokolo, Anyachebelu and Anemelu (2012) investigated the effects of computer-assisted instruction package on the performance of Nigerian senior secondary students in mathematics. The study examined the significance of retention achievement scores of students taught using computer-based instruction and conventional method. From the findings, students taught used computer based packages performed better than those who were taught using conventional method of instruction.

Erickson (2012) conducted a study on concept-based instruction in the International Baccalaureate programmes. She discovered that concept-based instruction teaching and learning approach increases motivation for learning, encourages constructivist learning, values collaborative thinking and provides opportunities for meaning making. Her results also indicated that concept-based instruction allows learners to structure their learning as well as deepening their understanding.

Al-Qatawneh (2012) used two groups of Grade 10 English learners in his study on the effects of concept-based instruction. One group was a control group and the other one was an experimental group. The results that he got depict positive effects of concept-based instruction on conceptualisation and motivation. Furthermore, Al-Qatawneh (2012) also discovered positive effects of concept based instruction in teaching and learning and in his recommendations, advised educators to consider teaching for conceptual understanding.

Chappel and Kilpatrick (2003) investigated the effects of instructional environment. In their study, they used college level calculus students and their instructors to view effects of concept-based versus procedural based instructional approaches. Several achievement measures were used to determine mastered concepts and procedures. It was discovered that the students who were exposed to the conceptual approach had significantly higher scores in both conceptual and procedural tasks than those who were in the procedural group. The conclusion was that concept-based instruction helps learners develop conceptual understanding without disturbing their procedural skills.

Leonard, Gerace and Dufresne (1999) also conducted a research for concept-based instruction in physics where they established that understanding concepts simplifies physics to learners. Lee (2012) as well conducted his study on concept-based problem solving to assist learners to solve complex problems in linguistics and had same establishments as the trio. These last four researches were based on subjects that are deemed difficult by society based on concept-based curriculum and instruction's three-dimensional design model that frames factual, content and skills with disciplinary concepts, generalisations and principles. The findings of the researcher indicated positive results from implementing concept-based instruction approach.

Ghazali and Zakaria (2011) carried out an investigation on the relationship between conceptual and procedural understanding in learners. They gave learners an algebra test consisting of questions that needed conceptual and procedural ideas. It was revealed that learners' procedural understanding was higher than their conceptual understanding. However, they realised that procedural and conceptual understanding complement each other. Their recommendation was that educators should use teaching and learning approaches that improve conceptual understanding and minimise use of algorithms and memorisation.

Gurbuz, Catlioglu, Birgin, and Erdem (2010) investigated fifth grade students' conceptual development of probability through activity-based instruction. They divided 50 learners into two equal groups, namely; experimental and control groups. A pre-test was administered to both groups before intervention. During intervention, learners in the experimental group were given hands on tasks. Learners in the control group were passive in the lessons with the educator being in control of the learning process, lecturing to learners. After intervention, the two groups were given another test to enable the researchers to compare the results of the two groups in the two tests. It was discovered that learners in the experimental group performed better and they had better conceptual understanding as indicated by the way they answered questions that needed application and comprehension.

Zulnaidi and Zakaria (2009) investigated the effectiveness of teaching methods. They discovered that methods that emphasised conceptual understanding were important to learners as they provided power to connect or link new and old information. According to the two, a positive relationship existed between conceptual understanding and learner achievement. To them, conceptual understanding enables learners to solve mathematical problems in various forms.

McCoy and Ketterlin-Geller (2004) explored the difficulties associated with learning facts without conceptual understanding. They discovered that learners who used the conceptual approach were better off in terms of performance than the ones who used the traditional approach. Their findings indicated that concept-based teaching approach had many benefits, including improved critical thinking and perceptions in the topic. The two researchers also proved that concept based instruction approach helps learners to diversify, understand and interconnect ideas.

Williams, Abraham and Negueruela-Azarola (2013) conducted a study that explored teachers' perceptions of the potential of the concept-based instruction as an effective approach to teaching and learning. According to the trio, the concept-based teaching approach involves the use of conceptual strategies where concepts are used as tools for understanding. Their findings indicate that the coalition of curriculum, instruction and assessment is crucial for educators. They recommended that future studies have to focus on teachers' professional training, emphasising on how they teach as well as the effect of their teaching instructions on learners.

Owusu (2015), in his investigation on the impact of constructivist-based teaching in South Africa discovered that concept-based instruction contradicts the traditional approach of memorisation of information with the teacher taking charge of the intellectual work in the classroom. McCoy and Ketterlin-Geller (2004) assert that concept-based instruction provides a shift from content-centred learning to learning-centred approach where focus is on concepts, principles and generalisations, using related facts and skills as tools to gain deeper understanding of disciplinary content, trans-disciplinary themes and interdisciplinary issues, and to facilitate conceptual transfer. Conceptual transfer can occur when a learner has to solve for x given: $5^{tan\,x} = 0{,}2.$ The learner has to identify the type of equation first, which in this case is trigonometric and exponential. Solving exponential equations requires one to create a common base on both sides. Therefore, 0,2 has to be changed to a common fraction in its lowest terms to $\frac{1}{5}$ which is the same as 5^{-1}. Exponents are then equated and the trig-equation can now be solved easily. Conceptual transfer enables the learner to simplify the equation first, which makes it meaningful. This becomes an indication of deep understanding of concepts.

Merriam, Caffarella and Baumgartner (2007) believe that in this 21st century, teaching and learning should be centred on lifelong learning where both traditional and contemporary learning theories and methods are fused to reach set teaching and learning objectives as, "no single teaching strategy is effective all the time for students" (Killen 2006). Zimmerman and Schunk (2001) postulate that learners should be self-directed and regulated, that is, set personal goals for themselves, take initiatives and modify learning approaches to suit their needs. The responsibility of learning has to shift from the educator to the learner. However, without educators' support, concept understanding will be difficult. This support can come in many ways, for example, using a varied combination of teaching strategies like inquiry methods, case studies, direct instruction and role-play to involve or empower learners act the part and cooperative learning to evaluate the outcome.

In a learner-centred approach, the educator encourages learners to transform and reflect in order to construct their own meaning with special focus on deep and permanent understanding not just information gathering (Fink 2013). The educator acts as a facilitator. In the learner-centred approach, it is the duty of learners to monitor, summarise, elaborate, and explain the concept. Allowing learners to play an active role in learning gives them a chance to identify and resolve their personal misunderstandings and it allows them to apply what they learn to relevant situations. This indicates an increase in learners' level of understanding.

When educators use concept based instruction, they determine the concept that is the target of instruction, organise graphics to illuminate the concept for the learners and learners' success in mastering the concept, that is determined by applying it across instances using increasingly complex critical thinking measures. This represents an educator-centred approach as opposed to the learner-centred approach where learners need to take ownership of their learning in order to reach the lifelong learning objectives of the 21st century. However, this instruction method as much as it might appear to be educator-centred is learner-centred. Concept-based instruction help learners to substantiate information about concepts and have an in-depth understanding before they can apply them to real life situation or in problem solving (Garfield, 1995). The educator's duty must be to encourage learners to solve problems in a way that is meaningful to them and give them a chance to explain how and why they came up with their solutions.

In learning, learners should be prepared to look for additional information on their own, navigate through unfamiliar text until they create their own knowledge. The more learners look for information the better they improve on their research skills, take responsibility of their learning and reach deeper understanding of concepts than they would have from listening in a lecture. More importantly, learners get motivated as they play an active role in their learning. Concept understanding is a key lifelong asset to learners as they can give appropriate responses to questions, create solutions to problems, construct meanings, and participate in shared activities without misconceptions (Erickson, 1999; Roid & Haladyna, 1982). According to Romberg (1998), the setup of a class using concept-based instruction must be such that it allows learners to make conjectures, present their arguments and also discuss strategies, therefore, helping them to be able to gain understanding and empowering them to present their arguments logically.

Education research is a powerful tool for change and improvement in a cyclical, dynamic and collaborative process in which people address social issues affecting their lives (Cohen, et al. 2007). Therefore, this research had intended to provide better ways of improving learners' conceptual understanding. An effective instruction in a mathematics lesson is one that brings conceptual understanding in learners (Shellard & Moyer, 2002). The educator as the mediator of learning is tasked with conceptualising effective teaching strategies that will improve conceptual understanding in the teaching and learning of mathematics that support other teaching methods that are normally used in creating foundation knowledge.

The success of learners in any learning environment as suggested by Omoniyi (2016) depends on the instructional strategies employed by the educator to achieve predetermined instructional objectives. Erickson (2007) adds that well-designed instructional approaches raise learners' interest while ill-designed instructional approaches lower learners' engagement. Alfieri, Brooks, Aldrich, and Tenenbaum (2011) contend that learners have to be exposed to manipulation of variables, exploration of phenomena and application of principles to afford them with opportunities to identify patterns and discover underlying causalities. The educator therefore has to go deeper into other theorists' views in order to come up with better teaching approaches.

Conceptual understanding of mathematics allows learners to be independent and confident (NCTM, 2000). Teaching for conceptual understanding might be time consuming and tiresome but it equips the learner with valuable knowledge. Teachers do not normally teach for conceptual understanding. They use traditional teaching and learning approaches in most cases because they just target to finish the content specified in the work schedules and are more worried about learners obtaining just pass marks. According to Yu (2013), teachers mostly use weak rapid instructional teaching and learning approaches to pass assessments instead of preparing connections for them to fit into the real world. The curriculum in most cases is also designed in a way that it discourages teachers to engage learners in active construction of knowledge but emphasises on rote learning for higher pass rates. Learners do not get the opportunity to get excellent scores as the world just focuses on getting correct answers and not on how they are obtained.

Learners learn mathematics well through active construction of meaning of concepts, reorganisation, representation, reconstruction and social negotiation with peers and elders (Belbase, 2016). Therefore, teaching and learning approaches that allow learners to create their own constructions and look for alternative methods of solving problems should be encouraged. Conceptual development might be challenging and demanding but it is extremely important. Having discussed other researchers' suggestions, views and establishments, the researcher realised some of the areas that were left out. The following is an elaboration of the literature gap.

2.6 Literature gap

The researcher has been provided with better understanding of the teaching and learning approaches by the review of literature of other studies. Though evidence has been gathered that a plethora of researches have been carried out on the effects of concept-based instruction, few of them focus attention on mathematics. To my knowledge, there has not been enough research on concept-based instruction in South Africa focussing attention on functions. Considering the gaps that have been elucidated, the researcher felt that exploring the effects of concept-based instruction in the teaching and learning of mathematics might greatly influence the education system and other researchers.

2.7 THEORETICAL FRAMEWORK

2.7.1 Introduction

Theories are important because they direct researchers' attention to a specific relationship and provide meaning for the phenomena that is studied. Theories also rate the relative importance of the research questions and place individual studies research findings within large context. A framework is a set of supposes and constructs that are shared by special theories (McShane, 1991). The purpose of a theoretical framework is to backup and give direction to a study. It guides the study by providing views and explanations to phenomena. A framework also assists in the choice of the research design and methodology. Theories support teaching for conceptual understanding as they also consider conceptual understanding more valuable to learners. This brings us to theoretical framework binding this study. This study is corroborated by constructivist theory. This theory was chosen because it develops skills such as critical thinking, analysis, evaluation and creation which are in line with the core business of concept-based instruction.

2.7.2 Overview of constructivism

Constructivism is a paradigm that greatly contributes to the shift of responsibility from the educator to the learner (Watling & Ginsburg, 2019; Yasmin, Sohali, Sarka & Hafeez, 2017; Omoniyi, 2016). The concept-based instruction is primed in education research that uses constructivism-teaching methods to ensure that the three-dimension curriculum model: concept, knowledge and skills are considered to aid learners improve their high order thinking skills and sharpen their critical thinking. Kaufman (2018), Edinyang (2016), Jonassen (2011), and Kalu (2012) believe that constructivism facilitates, interprets and explains learners' building of conceptual understanding. Swanson (2020) and Cevikbas and Kaiser (2020) discovered that it promotes intellect for an increased complex and changing world by enabling learners to connect the dots and see the bigger picture of content covered in class. Yu and Singh (2018), Schukajlow, Rakoczy and Pekrun (2017), and Whitehead (1948) allude that failure to grasp mathematical concepts is related to the way the subject is taught. His emphasis seems to be that educators should target learners to gain conceptual understanding rather than targeting procedures only. Hence, it requires that educators shift their focus from lecture methods to teaching conceptual ideas using facts as supporting tools (Erickson, Lanning & French 2017).

Constructivism is a teaching and learning theory, where learner-centred teaching approaches are encouraged to ensure active participation of learners. A learning environment that uses the constructivist approach has learners playing an active role in the learning process while the educator assumes the duty of a facilitator assisting learners to get their own understanding of a concept (Clark, 2018; Nugroho, 2017; Doolittle, 1997). The beliefs of the constructivists as suggested by Vintere (2018), Amin and Mariani (2017), and Glasserfield (1995) are that learners build mathematical concepts through active cognitive and adaptive processes. Constructivism is a theory that depends on observation and scientific study about how knowledge is gained by learners (Hidayat & Rostikawati, 2018; Brandon & All, 2010). The theory is applicable for this study because it values learner's inquiry. Constructivism is one of the best theories of teaching and learning because it increases learners' conceptual understanding (Adak, 2017; Mutsvangwa, 2016).

According to Surya and Syahputra (2017, Surya and Putri (2017) and Piaget (1973), learners acquire mathematical knowledge not through internalising rules enforced from outside but by constructing them internally through their own thinking abilities. Moreover, learners construct meaning based on their own experiences and this involves individual interpretation (Carless & Bound, 2018; Clark & Veale, 2018). The constructivist approach opposes the traditional approach which according to Hardman and Heliyon (2019) emphasises the learning of rules and tricks to solve mathematical problems. In the same vein, Fenwick and Tennant (2020), Bhattacharya (2017) and Fosnot and Perry (1996) concur that knowledge is constructed based on personal experiences and hypothesis of the surroundings. The learner is seen as an information constructor while the educator facilitates the gathering of a series of facts. Following the steps of the constructivists, the educator does not tell learners what to do but gives them a chance to construct their own knowledge. Learning should not be viewed as a process of knowledge acquisition but an active constructive process. According to the beliefs of the constructivists, teaching has to support the learners' constructive processes rather than be a way of delivering information to learners.

2.7.3 Social constructivism

According to Brussow, Roberts, Scaruto, Sommer and Mills (2019), Garcia (2018) and Giddens (2007), concept-based instruction approach complements constructivist philosophy by promoting active learner-centred approach. Theory and concepts play a critical role in social science research in generating ideas, formulating and evaluation hypothesis and building new theories for analysis (Nyoni, 2008). Constructivist learning is not a simple passive process of receiving from the surrounding environment but a dynamic process in which learners interact (Aljohani, 2017). Belbase (2016) affirms that learners learn mathematics through active construction of the meaning of concepts individually and by social negotiation with peers and elders. This depicts the fact that knowledge construction has individual input backed up by social interactions. Therefore, the constructivist paradigm contributes greatly to the shift of responsibility from the teacher to the learner (Alt & Itzkovich, 2019; Suhendi, 2019; Wilson, 2017; Ominiyi, 2016; Brooks & Brooks 1999). The central perspective of constructivism as reviewed by Appiah, Ozuem, Howell and Lancaster (2019) stresses the importance of participants' views and the environment in which it is expressed.

Some of the theorists like Patton (2019) and Neutzling, Pratt and Parker (2019) believe that learning is a result of interactions with members of a particular culture. Vygotsky is one of the social constructivists who strongly believes that learning is a social process where cognition is brought about by socialisation. People have shared meaning and understanding that can be negotiated through discussions where they acquire multiple realities because knowledgeable peers can perform same function as educators if not more (Vygotsky, 1978). Deslauriers, McCarty, Miller, Callaghan and Kestin (2019) and Mattar (2018) believe that learners understand better when they are actively involved in their own learning. In line with these views are Alexander and Boud (2018) who assert that knowledge and understanding come from a social content with learning occurring when deep understanding and support is observed. This deep understanding depicts the presence of conceptual understanding in learners. Hoyles (1985) also asserts that learners' ideas in discussions can suggest modifications to one's own thoughts. These views are in line with the social constructivists who aver that construction of knowledge is through social interactions. Following the social constructivists' explanations, the educator is a mediator there to guide the learners in the achievement of their goals.

2.7.4 Constructivism and concept-based instruction

Constructivism and concept-based instruction help learners foster or improve their critical thinking and also enable them to connect and apply concepts. According to Makonye and Nhlanhla (2014), the constructivist theory implies that learners do not come to a new educational level as empty vessels but bring pre-knowledge from previous studies. The constructivists highly believe in learners are not supposed to wait for knowledge to be deposited into them. This contradicts traditional teaching approaches, which according to Gonczi (2020), Fomunyam (2018) and Omoniyi (2016) view learners as empty vessels needing to be filled with knowledge. Baroody and Clements (2019) and Young and Collin (2004) view the traditional teacher as an information giver to passive learners promoting memorisation at the expense of logical thinking. This does not support the key aims of mathematics of producing critical thinkers that can raise and present arguments logically. Constructivism approach assumes that during learning, learners interact with the environment, their cognitive structures placed in conflict, and conceptual knowledge is negotiated (Huang & Liaw, 2018; Berns, Mota, Rube & Dodero, 2018; Savery & Duffy, 1995). Constructivism postulates that learners do not act as receivers of whatever is brought to them by the educator. Instead, it makes learners connect what they learn to real life situations. However, the teacher cannot be dismissed completely, s/he has to be available to guide learners construct their knowledge.

Celik (2018), Mochesela (2007) and Tambaoan and Gaylon (2019) argue that learners undergoing the traditional teaching and learning approaches which do not emphasise constructivism do not perform satisfactorily in mathematics as they lack conceptual understanding needed for critical solving skills. Constructivists teaching and learning situations involve hands-on activities. According to Bridgers, Jara-Ettinger and Gweon (2020) and Phillips (1995), it is good when learners discover and make their own inferences of prevailing situations instead of explaining to them what is happening. In support of this view, Bada and Olusegun (2015) suggest that teachers should not simply transmit knowledge to learners but learners have to actively construct their own knowledge by discovering, transforming information, checking new information against old information. However, Cohen and Rodgers (2021), Smith (2017) and Driscoll (2000) posit that knowledge exists within the human mind and does not need to match any real world reality.

Using teaching and learning approaches that rely on the constructivist approach has more benefits as far as academic achievement is concerned. The learners taught using these approaches have better reasoning capacities that allow them to argue and present their arguments logically. Learners involved in these approaches have many ways of solving problems. Active learning which is the main characteristic of the constructivists to the learning process enhances remembering as learners gain understanding as they get involved in the learning (Chang & Hwang, 2018).

Teaching and learning theories are employed in this study. However, learners and educators should not be held by learning theories but use them to think creatively and see the bigger picture as suggested by the concept-based instruction teaching and learning approach. A graphic organiser used in concept-based instruction is meant to explicitly illustrate the structure and organisation of information (Lantolf & Esteve, 2019; Erickson et al., 2017; Hudson, Lignugaris-Kraft, & Miller, 1993). The visual display of relevant content material helps learners to link prior knowledge with new learning; this deepens their level of understanding the material (Renkl & Scheiter, 2017; Calvo-Ferrer 2017; Pedrotty Bryant, Ugel, Thompson, & Hamff, 1999). Visual displays can be seen useful when the concept of a graph being shifted is taught. If learners actually draw the graph and shift it accordingly, they can easily identify the effects and be able to come up with the equation of the new graph.

This study will use Grade 11 mathematics learners as its participants. Grade 11 learners have already covered some basic mathematical concepts in high school (existing knowledge) but they have to learn more concepts in depth using new teaching and learning strategies to prepare them for tertiary education. The social constructivist theorist Vygotsky (1978) together with Seufert (2019); Eisteban-Guitart (2018) believe that learning occurs when a learner's prior knowledge is raised from a lower level to a higher level called the Zone of Proximal Development (ZPD). According to Vygotsky, learning takes place owing to social interaction; that is when a learner interacts with other people, either adults or more knowledgeable peers in his/her social or cultural setting (Abtahi, Graven & Lerman, 2017; Kansellar, 2002).

In concept-based instruction, the educator has to provide brief essential knowledge in pictures or graphics to place learners in the ZPD and then provided them with sources where they can acquire more information that is detailed. The concept of an inverse function reflecting a function in the line $y = x$ can be deduced by learners on their own after being guided to determine an expression for the inverse function and plot it. Learners having plotted the graph of a function and its inverse for 3 or more functions can easily detect the reflection property. This raises learners' understanding to a higher level. Vygotsky (1978) and Rassaei (2019) believed that when a learner is in the Zone of Proximal Development (ZPD) for a particular task, providing the appropriate assistance will give the learner enough motivation to achieve the task.

Constructivists postulate that active knowledge construction is done by learners themselves (Altmeyer, Kapp, Thees & Malone, 2020; Darsih, 2018; Naidoo & Naidoo 2009). Something seen can be easily remembered and understood better than something just heard as heard things can be forgotten easily (Rosenthal, (1995). In contrast, seeing enables learners to retain taught things better than just hearing. That is why anyone who understands the beauty of mathematics knows that it is a subject that involves more of learners doing things for themselves instead of being passive recipients. The teacher's duty as suggested by Widodo (2018), DeSutter and Stieff (2017) is therefore, to stimulate thought and mental activities that help learners achieve in mathematics. The educator also has to make learners aware of the fact that what they learn is related to their personal life. This instils self-motivation and makes them mindful and attentive. According to Castiglioni and Gaj (2020), Huang, Spector and Yang (2019), Kaufman (2018) and Medlock (2017)), conceptual understanding from the constructivist perspective characterises constructing, restructuring and situating in contexts. In the same vein, Haag and Haag (2020), Adler and Oktem (2018), Yang, Wen, Wang and Clark (2017) and Mochelesa (2007) buttress that conceptual understanding involves reconstructing prior knowledge.

Constructivists explain that construction of knowledge through existing schemas to new ideas takes place through accommodation, assimilation and equilibration. Assimilation is described as an individual's ability to match new knowledge to already possessed knowledge (Siegler 2005; Chmielewska & Gilanyi 2018; Widodo, Nayazik & Prahmana 2019; Schlaile, Zeman & Mueller 2021). Assimilation therefore occurs when a new situation confirms to prior knowledge and increases existing mental network. If new information fails to match prior information, then there is disequilibrium. In trying to fix disequilibrium, existing schema is replaced so that new information fits well. This process of fixing disequilibrium is known as accommodation. Accommodation as speculated by Bada and Olusegun (2015) entails reframing the world and new experiences into the mental capacity already present. It is the ability to make changes in prior knowledge so that it fits new knowledge whereas equilibration involves interaction of new and old knowledge. Lang (2021), Cash (2017) and Reys, Suydam and Lindquist (1995) see conceptual understanding as requiring learners to be active in thinking about relationships and making connections along with adjusting accommodate the new learning with previous mental structures.

However, this conceptual understanding, which characterises the core business of mathematics, is lacking (Perignat & Katz-Buonincontro 2019; Korn, 2014). The DBE (2012) sees mathematics as developing mental processes that enhance logical and critical thinking, accuracy and problem solving that contributes to decision making. Social constructivists believe that mathematical knowledge does not exist out in the universe waiting to be discovered but is constructed (Cederman, 2021; Shah, 2019; Yucel, 2018; Davis, Maher & Noddings, 1990). Furthermore, Sekwena (2019), Hammond (2017) and Fox (2001) see the constructivist theory as emphasising the importance of context, prior knowledge and concept-based instruction approach to learning with learners being active participants and dismissing the passive reception of knowledge and memorisation. However, constructivism is not a teaching approach but a theory of teaching and learning that informs rather than prescribing practice (Taber, 2017; Bearman & Nestel, 2017; Major & Mangope, 2012).

Erickson (2012:42) argues that educators should teach for deep understanding of conceptual knowledge, not just for remembering isolated and small bits of factual knowledge. With concept-based instruction, learners can generalise concepts across multiple instances and connect dots in different ways hence the multidisciplinary of themes come into effect. Connectivism is a learning theory that sees understanding as based on connecting knowledge to experience and real life (Downes, 2019; Harasim, 2017; Cormier, 2008). To ensure that effective concept understanding is achieved, educators should plan every detail about their intended teaching process. This will also stimulate independent learning and resourceful thinking (Miklikova, 2018; Reddy, 2018) as the educator can ask provoking questions, request interpretation, explanation and hypothesis from students. However, learners who are not able to manage abstract learning are not be able to learn effectively with this method. Killen (2015) argues that learner-centred teaching approaches require learners to seek feasible ways to solve contextualised complex issues thereby motivating them to first understand theory and then use it to solve problems in a cooperative way. Learners having been taught laws of exponents and how to solve quadratic inequalities may be asked to solve simultaneous equations with one of the equations being exponential and not simplified. In addition, learners will have to create a simplified linear equation first then solve the equations simultaneously. The educator will only highlight the key concepts related to the topic at hand as a way of pointing learners to the direction they should focus on. As indicated by Fry, Ketteridge and Marshall (2009), educators should not tell learners what and how to think but encourage them to think for themselves in a broad academic rationale.

The constructivists are against traditional teaching and learning approaches where learners submissively accept content (Mutsvangwa, 2016). They want a shift from traditional approach that involves direct instruction of teaching to one that has the educator as a facilitator of learning. Proposals for change are advocated for teaching and learning approaches that are based on constructivism for the development of mathematical concepts (Hwa, 2018; Irvine, 2017; Atkinson, 1997). According to Atkinson (1997), there is a feeling that more active learning involved in the constructivist approach are necessary to provide learners with the requisite knowledge and skills.

These current reforms greatly support the development of conceptual understanding. The constructivist perspective is more appropriate than other learning theories since it facilitates the development of understanding and meaning in students (Carless & Boud, 2018; Jack, 2017; Osei, 2019). On the other hand, Alharthi and Alsufyani (2020) and Von Glasersfeld (1989) assert that a constructivist teacher tends to explore how students see the problem and why their path towards a solution seemed promising to them. Salari, Roozbehi, Zarifi and Tarmizi (2018), Johnson (2017) and Hendry, Hays, Challinar and Lynch (2017) believe that a shift for teaching and learning approaches based on constructivism may help in achieving better performance.

2.8 Summary of the chapter

The chapter has given an overview of what other researchers have done, deduced and the theoretical framework binding this study. The literature review revealed lack of conceptual understanding by learners as the reason for poor performance in mathematics. From the review, it was gathered that the constructivists emphasise the shift of learning responsibility from the teacher to the learner. The review of the existing literature accentuated the need for better teaching and learning approaches to improve conceptual understanding in mathematics. Furthermore, the literature gap was highlighted as a way of providing directions for future research. Having supported the study by other researchers' views, the researcher, in the next chapter, is going to describe of the research design and methodology.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter describes the design and methodology used in the study to explore the effects of concept-based instruction in the teaching and learning of Mathematics. There is also a description on the targeted population, sample and sampling techniques, research instruments, validity, reliability and data analyses procedures. The chapter concludes by describing ethical considerations and the pilot study.

The main purpose of the chapter is to explain the methodology employed in trying to answer the following research questions:

- What effect does concept-based instruction have on the teaching and learning of Mathematics?
- What advantages does concept-based instruction have over other teaching and learning approaches?
- What changes are brought about in learners by the use of concept-based instruction teaching and learning approach?
- What challenges can be addressed by the implementation of concept-based instruction in the teaching and learning of Mathematics?

3.2 Research methodology

Sileyew (2019) defines research methodology as the steps followed by researchers to conduct their research. Babbie and Mouton (2001) assert that research methodology focuses on explaining the intended type of study and the expected results. This section explicates the research strategies, research approach and the data collection methods as well as sampling techniques. McMillan and Schumacher (2010) add that the purpose of a research methodology is to specify a plan for generating empirical evidence to be used to answer the research questions. The study intended to present the participants' conceptual understanding in a precise way. Moreover, participants presented solutions to the pre-test and post-test which the researcher used to gather information about their thinking processes. The researcher got information of how learners think from the way they presented solutions and also their responses during the interviews and in the questionnaires. The interviews with the open-ended questions unearthed more information which could not be identified from the test solutions. During the interviews, the researcher had a chance to keep on asking questions for clarity on certain aspects. For example, the participants were asked to give pellucidity on why they had used certain methods like the use of the midpoint formula to get the x-coordinate of a turning point. They could explain that it was because the turning point was lying on the axis of symmetry.

The study is an evaluation research since its main aim was to explore the effects of concept-based instruction. The researcher targeted the identification of the advantages of employing the concept-based instruction in the teaching and learning of Mathematics as one way of exploring the teaching and learning approach in question. Evaluation research as viewed by McMillan and Schumacher and (1997) and Mertens and Wilson (2018) determines the merit and worth of a particular practice. In this case, the researcher tried to come up with advantages of teaching and learning using the concept-based instruction. The findings of the study determined the importance of concept-based instruction in the teaching and learning of Mathematics.

As explained earlier, the study employed the sequential explanatory research design that expended both the quantitative and the qualitative methods. Qualitative data was used to explain and explore quantitative data in way trying to gather information on how learners built their conceptual understanding. An elaboration of the composites of quantitative and qualitative data is provided below.

3.2.1 Quantitative research

Apuke (2017), Marten (2010), Aliaga and Gunderson (2000) view quantitative research as explaining phenomena by collecting numerical data that are analysed using mathematically based methods, in particular statistics. In the same light, Basias and Pollalis (2018), Apuke (2017) and Maxwell (2013) view quantitative research as involving studying a phenomenon through variables that are measured and compared across contexts. In the current investigation, the exploration of the concept-based instruction was done through the statistical data that was collected in the post and pre-test and also from questionnaires. Descriptive, inferential and exploratory data analyses were carried out in a bid to come up with answers to the research questions of this study. The mentioned data analyses methods are discussed under data analysis.

According to Hennink, Huttter and Bailey (2020), Sewell, Desai, Mutsaa and Lottering (2019) and Wyse (2011), quantitative research involves quantifying the problem by way of generating numerical data or data that can be transformed into useable statistics to formulate facts and identify patterns about opinion, attitudes and behaviour. After collection of data in a study, there are descriptions, explorations and explanations done to establish a logical interpretation of data. In the present study, the researcher collected test marks and also information from other parts of the questionnaire that included yes/no and true/false questions and analysed them quantitatively using descriptive and inferential statistical methods. Frequencies were tabulated and data were graphically represented and descriptively interpreted. The dependent t-test was used to inferentially analyse tests results to examine changes. In support of these views, Queiros, Faria and Almeida (2017) and Gregar (1994) describe quantitative research as an approach that tests theories by measuring the relationship among variables. The researcher was also interested in coming up with the relevance of constructivists' views that underpin this study.

3.2.2 Qualitative research

Merriam and Grenier (2019), Rashid, Rashid and Warraich (2019) and Manson (2006) view qualitative research as supporting an interpretivist orientation since it enables the researcher to understand and explore the richness, depth, context and complexity within which participants in the study operate. The inquirer was interested in gathering information about learners' conceptual understanding in Mathematics for her to explore the effects of the concept-based instructional teaching and learning approach, and therefore employed qualitative research approaches.

Qualitative research, unlike quantitative research deals with non-numerical data (Longo, 2019; Moalusi, 2020). Kilicoglu (2018), Banks (2018), Van de Wiel (2017) and White (2005) describe the data involved in qualitative research as principally verbal. Data from interviews and from open-ended questions responses from the questionnaire were qualitatively analysed. Qualitative research is explained by Hennink, et al. (2020), Skinner, Edwards and Smith (2020), King, Horrocks and Brooks (2018), Flick (2018), Mohajan (2018) and Denzin and Lincoln (2005) as involving inter naturalistic approach to the world. The researcher involved Grade 11 learners at an established high school so that the participants could operate under their normal settings. The researcher aimed to gather rich information on the development of conceptual understanding from the participants through interviews and questionnaires.

Qualitative researchers analyse participants' experiences of the world around them by having 'a close engagement' with the collected data, through insightful strategies that illuminate authentic meanings (Naicker, 2017; Bazeley, 2013; Mitchell, Mouratidis & Weale, 2007). There was close engagement with collected data and data analysis was carefully done. More than one data source was involved in the exploration of the concept-based instruction. Moreover, thematic analysis was engaged to gain holistic understanding of the findings and come up with answers to the study's research questions.

Qualitative research was involved to reveal hidden aspects, as in this case where a follow-up of a learner's conceptual understanding was done through interviews, there was need to probe the learners to explain how they came up with solutions in the test. In the questionnaires as well, learners freely expressed their attitudes and feelings about the teaching and learning of

Mathematics. Qualitative analysis of data enabled the researcher to come up with information that was rich and had a deeper insight into learners' conceptual understanding.

3.3 Research design

Creswell and Plano Clark (2007) define a research design as the procedures for collecting, analysing, interpreting and reporting data in a research study. According to McMillan and Schumacher (2010), the purpose of a research design is to specify a plan for generating empirical evidence used to answer the research questions. The research design elaborates how an investigation occurs and the conditions under which it is conducted. The objective of the research design was to explain the plan that was proposed for gathering information to come up with answers to the research questions. Durrheim (2006) explains a research design as a strategic plan with a broad outline and key features of the work to be undertaken including data collection methods and analysis. The intention of the researcher was to implement a design that would result in drawing the most valid and credible conclusions from the answers to the research questions.

The intention of this study was to explore the effects of concept-based instruction on the teaching and learning of Mathematics. The sequential explanatory design in which data were collected and quantitatively and qualitatively analysed was undertaken by the researcher to elicit results to the inquiry. The researcher as well exploited the one group design to explore the effects of the concept-based instruction.

3.3.1 Sequential explanatory design

The sequential explanatory design is a combination of quantitative and qualitative research that involves the consolidation of collected data. Qualitative and quantitative research methods were triangulated to seek convergence, corroboration and correspondence of results. The results from one method were used to clarify the results from another method. Interviews were used to get interpretations of some of the solutions that were provided in the tests. Bowen, Rose and Pilkington (2017) and Mbewe (2013) point out that qualitative data explains quantitative data and it is brought in to explore quantitative data in depth. Du Plooy-Cilliers, Davis and Bezuidenhout (2014) suggest the need for researchers to find explanations for why certain things come about for them to be able to find solutions to remedy situations. Therefore, sequential explanatory design was habituated as a means of finding explanations to the research problem and solutions to the problem. Consequently, sequential explanatory design, which allows qualitative results to further explain the quantitative findings was selected with credence that mixing quantitative and qualitative techniques would enable clarification of how and why concept-based instruction affects conceptual understanding.

Chiphambo (2018) concurs that the philosophical and epistemological foundation for employing mixed methods in association with a research study is done to obtain different, but complementary data on the research problem. For this study, the researcher wanted to understand the effects of concept-based instruction on the teaching and learning of Mathematics with the aim of improving conceptual understanding in learners. The mixed methods approach was employed to give room for thorough data analysis as the analysis was from different angles of focus. There was a simultaneous generalisation of results to gain deeper understanding of the concept-based instruction approach.

The findings from the quantitative data determine the type of data one has to collect in the qualitative phase (Guetterman & Fetters, 2018; Brannen, 2017; Gay, Mills & Airasian, 2006). This simply means that the qualitative phase is informed by the quantitative phase. In the present study, most of the interview questions were based on the tests which were administered before and after intervention. The test informed the researcher where participants appeared to have understood or not understood. While the quantitative research method helped to identify improvements in participants' scores and concepts that learners did not understand, the qualitative method which involved interviews helped to deepen focus and explain more on those experiences.

The sequential explanatory design was selected for this study because of being a mixed method approach. The researcher triangulated research methods high for quality results. The two methods complemented each other in a manner to cover up for any gaps created by the other one in an effort to get solutions to the research problem.

3.3.2 One group pre-test-post-test design

A one group pre-test and post-test experimental design was followed in this study. It can simply be referred to as a single group design. In this data gathering instrument, a single group of participants is observed and measured prior to and after the intervention has taken place (Gusnedi, Ratnawulan & Triana, 2018). This design as postulated by Alam (2019) involves only one group of participants in the study. There is no control group used for comparisons of changes that take place as a result of the treatment or the intervention. This design subjects all participants to one condition.

The one group pre-test-post-test design clearly shows change providing description of the process before, during and after data gathering. For this inquiry, a single group was put under observation with the same test administered before and after intervention to investigate participants' improvements in conceptual understanding.

It is of advantage to use the single group design because it is easy to implement and in the current study, the researcher was able to control all processes that were involved alone. This design minimises information-processing bias. The other reason for involving one group was to avoid a control group as its involvement would disadvantage other learners.

However, the single group design has limited application as it can only be applied to a particular group. Its application to one group makes the generalisability of obtained effects to be unreliable in other circumstances. There is lack of control in the design as the data gathering techniques have the limitations of irreversibility. This is because once mistakes are noticed it is difficult to restart as it might affect the respondents.

3.4 Data sources and data collection techniques

The researcher targeted to use one already established Grade 11 class from a high school in Giyani, within Mopani District. The class had lessons delivered using the concept-based instructional teaching approach. Monitoring the class allowed the researcher to determine the effects of the concept-based instructional approach and to deduce if it was worth implementing in the teaching and learning of Mathematics.

The participants were from the school where the researcher was teaching because of accessibility and reduction of research costs. The class which participated in this study had 35 learners. One class was chosen to participate in this study for efficiency. The researcher opted for the Grade 11 class that had learners who struggled in Mathematics. Grade 11 class was opted for because there was hope that the intervention could help them to improve their understanding of Mathematical concepts and this would ultimately improve their performance in Mathematics. These learners were identified and put in that class using their term 1 marks that were extremely low compared to the performance of other learners. The researcher chose Grade 11 because it is an important stepping stone to obtain the National Senior Certificate. Owusu (2015) considers this grade level critical as it has the potential to impact on Grade 12 performance and beyond. In the same vein, Masilo (2018) also considers Grade 11 to be extremely important as it prepares learners for the last level in the Further Education and Training (FET) phase. The researcher's experience in teaching high school Mathematics for more than 20 years has revealed that most basic matric concepts are introduced in that grade. In addition, doing well in Grade 11 boosts a learner's confidence and that prepares him/her well for the final year in high school.

3.4.1 Negotiating access to data collection

The researcher asked for permission to conduct the study at the school from the school principal (Appendix J) who then sent a recommendation to the Department of Basic Education (DBE).

Permission to conduct the study in the school was given by the circuit manager for Man'ombe Circuit (Appendix I).

3.4.2 Research instruments

Data collection is a process whereby the researcher gathers and measures information on variables that s/he has interest on. The data collected have to enable the researcher to answer research questions and evaluate outcomes. For the researcher to collect data there is need to use research instruments. Research instruments are defined by Binet, Gavin, Carroll and Arcaya (12019), Albino (2017), Gall, Gall and Borg (2007) as tools used to solicit data from respondents in a research. In other words, research instruments refer to measuring tools that are created to gather information on a topic to be investigated.

Data for this study were collected through the following instruments: questionnaires, tests and interviews. Firstly, the participants wrote a pre-assessment test on already taught concepts to assess their level of conceptual understanding. This was followed by the intervention which was meant to bring changes in conceptual understanding of participants. After intervention a post-test was conducted to ascertain changes that had been brought about by the intervention. Questionnaires were later on distributed to collect participants' opinions on issues concerning the concept-based instruction that had been used during intervention and also solicit perceptions on the teaching and learning of Mathematics in general. Lastly, interviews were held for explanations, further understanding and also opinions to get detailed information from participants. All these were involved as a way of trying to gather enough information to answer the research questions of the study. The study's research instruments are discussed in detail next.

3.4.2.1 Interviews

An interview is one of the most main data collection tools in qualitative research (Madungwe, 2018; Punch, 2005). This is because an interview gives the researcher detailed information about the participant's feelings, perceptions, attitudes, and opinions. Furthermore, McGrath, Palmgren and Liljedahl (2019), Powney and Watts (2018), Adhabi and Anozie (2017) and Fox (2011) view an interview as an important data gathering technique involving verbal communication between the researcher/interviewer and the participant/interviewee. In the interviews that were conducted for the current study, there was exchange of information between the researcher and each of the interviewees and data were obtained through questions and answers.

According to Wolcott and Lobczowski (2021) and Arksey and Knight (1999), interviews help people to make explicit things that may have been implicit, enunciating their hidden perceptions, feelings and understandings. In the study, the researcher gave participants a chance to explain how they came up with their solutions and thereby obtaining valid and reliable information concerning thinking processes. In the interviews, participants reflect on events without committing themselves to paper (Madungwe, 2018). Respondents were free to give their own views freely as there were no involving questions which needed answers in pen and paper which might have been a stressful undertaking to them.

The aim of interviews in qualitative research, according to Glaw, Inder, Kable and Hazelton (2017) and Nieuwenhuis (2007) is to see the world through the eye of the participant and also learn more about the participants' behaviours, beliefs and views. Information on the effects of concept-based instruction on the teaching and learning of Mathematics was freely gathered from participants through face-to-face interviewees. Learners at times do not understand why certain procedures work or even when they should consider alternative or equivalent procedures that may be more appropriate to solve Mathematics problems (Meyer, 2017; Tularam & Hulsman, 2015). Therefore, the researcher wanted to be sure if learners had conceptual understanding or were just using procedural knowledge without logical reasoning. Interviews helped to explore the effects of concept-based instructional teaching and learning approach on learners' conceptual understanding.

One hallmark of mathematical understanding is a student's ability to justify why a given mathematical claim or answer is true or why a mathematical rule makes sense (Mata-Pereira and da Ponte, 2017). Similarly, MacMath, Wallace and Chi (2009) found that students may develop procedural knowledge, but lacking the deep conceptual understanding necessary to solve new problems or make connections between mathematical ideas. The researcher requested participants to justify their answers as a way of checking the degree of their conceptual understanding. The interviews were executed to backup questionnaire responses and test solutions.

Interviews consist of questions that are planned before the interview and these are called structured interviews. The opposite of these are the unstructured interviews which have no questions prepared in advance. In a bid to amalgamate the two extreme types of interviews, we have the semi-structured interviews which the inquirer preferred. Semi-structured interviews have part of the questions planned before the interview while the other questions are created as the interview proceeds to follow up on participants' responses. Interviews for this study were semi-structured in order to generate quality data using open-ended questions. Detailed information on semi-structured interviews is given next.

3.4.2.1.1 Semi-structured interviews

A semi-structured interview as postulated by Koshy (2005) is a qualitative method of inquiry that combines a pre-determined set of open-ended questions that gives exploration opportunities to the interviewer. In the present study, the predetermined questions were on the interview guide (Appendix E). The other questions were created as the interview progressed and this provided an opportunity of smooth conversation between the interviewer and the interviewee.

Low (2019), Gray (2021), McMillan and Schumacher (2010) and Madungwe (2018) add the fact that semi-structured interviews allow probing of views and opinions when interviewees are asked to expand, illustrate or explain their answers on the spot. For the current inquiry, new ideas came up from both the interviewer and the respondent during the interviews. In accordance, Seidman (2006) adds that semi-structured or unstructured, open-ended interviewing serves the purpose of understanding the experience of the participants and the meanings they make out of that experience. The interviews gave room for identifying respondents' experiences and also elaboration of unclear responses by interviewees. The semi-structured interviews increased the researcher's chances of getting reliable information from the respondents.

Naicker (2017) views these semi-structured interviews as having an advantage of accommodating open-ended questions. The inclusion of open-ended questions made the collected data rich as interviewees answered freely without any pressure like the one they are exposed to in the tests. The open-ended questions allowed the interviewee to talk in depth using their own words and this helped the researcher gain insight into the respondents` understanding.

3.4.2.1.2 Structure of the interview

The construction of the interview guide was manoeuvred by the research questions as well as literature. After identifying the research questions, the researcher proceeded to look for the advantages and disadvantages of the interviews and also the types of interview to be used. It was seen fit to utilise the semi-structured interview so that a follow-up to the reasons behind the solutions provided in the tests could be made and also for clarifications on points of interest. The researcher then decided on the number of participants to be involved and the criteria used to choose them for the interviews. She decided that six out of the 35 participants would be selected to take part. This selection was based on the type of solutions that had been provided in the tests. These involved among other properties: correct answers from wrong methods, first stages correct ending with wrong answers, other methods used to show deep understanding as well as signs of misconceptions.

An interview guide (Appendix E) consisting of the main items that the researcher planned to cover in the interviews was created. However, there was no specific order to be followed in asking the questions as some of the questions were created during the interview, with most of them being follow-up to participants' solutions in the tests or to what they would have said in the interview. From the interviews, data were recorded by the researcher and written up as a transcript (interview questions and answers). This data collection technique in agreement with McInnes, Peters, Bonney and Halcomb (2017) and Bengtsson (2016) provided the researcher with an opportunity to deepen the discussion and gather more information from the participants.

3.4.2.1.3 Advantages of interviews

According to Fritz and Vandermause (2018), Gill and Baillie (2018), Kilinc and Firat (2017) and Creswell (2014), interviews are flexible data collection tools. The semi-structured interviews that took place in this study allowed probing of views and opinions. The interviews gave respondents a chance to bring in their own views on critical issues. The researcher also had a chance to get clarification on issues missed when learners express themselves in writing.

3.4.2.1.4 Disadvantages of interviews

Interviews might be time consuming. The presence of the researcher might also affect or intimidate the respondents as some of the questions might make them feel uncomfortable. However, the researcher pilot tested this instrument in trying to identify time consuming items. The researcher realised that, during the interview, writing down everything that the interviewee would have said would require too much time. To reduce the time to be spent per interviewee, the researcher decided to take down only the main points and leave the rest of the capturing to the video recorder. The interviewer explained the purpose of the interview and then started with a neutral question for the researcher to feel free and comfortable to provide necessary information.

3.4.2.2 Questionnaires

McMillan and Schumacher (2010) and Suresh (2018) view the questionnaire as the most widely used technique for obtaining information from subjects. The two writers went on to describe a questionnaire as a written set of questions that are the same for all the subjects. For this study, the questionnaire consisted of written or printed series of questions which respondents had to answer for the purpose of gathering information. The questionnaire was selected to be one of the instruments because it is a fast, cheap and efficient way of getting information from participants.

The questionnaire consisted of both closed and open-ended questions. Open-ended questions were there to allow participants' full answers and to give them allowance for suggestions and expression of their feelings.

Questionnaires were distributed to the 35 participants of the study. This was done just after the intervention so that learners could express their experiences and challenges in the learning of algebraic functions and also reveal their feelings and attitudes about that topic in particular and the subject Mathematics in general. Following the writings of Arafa, Anzengruber, Mostafa and Navarini (2019), Abessolo, Rossier and Hirschi (2017), Lynch and Lynch (1996), Nunan (1999), Gillham (2000), and Brown and Principles (2001), the questionnaire best fitted the study because it provided respondents with anonymity which made them to share information more easily. Anonymity stepped up rate of answers and improved provision of genuine responses. Taking into consideration the fact that the researcher opted for the mixed methods research design, the questionnaire was contrived accordingly to collect both qualitative and quantitative data. As a result, the questionnaire contained closed and open-ended questions. The questions which were involved in the questionnaire were guided by the research questions and literature.

The questionnaire that was designed (see Appendix B) had four sections. The first section had three fill in parts about participants' identity code, gender and age. The second section had six yes/no questions concerning the way the teaching and learning of algebraic functions had been conducted during intervention. A seventh question that was open-ended was at the end of the second section and it wanted respondents to explain their challenges and give acknowledgements of the topic that had been taught. The third section contained four True/ Partly True/ False/ Partly False/ Don't know type of questions which were on general learning of mathematics. The last section was an open-ended question that called for respondents' feelings and perceptions about mathematics and their suggestions on what could be done to improve performance in the subject. The questionnaire was short with questions constructed using simple and accessible English. Closed and open-ended questions were constructed to allow triangulation of research methods.

3.4.2.2.1 Advantages of questionnaires

It is of advantage to use questionnaires as it is inexpensive in the sense that they can be self-administered and there would be no need to hire research assistants to help in conducting the research. In the current study, the researcher administered the questionnaire on her own and it was economical. Data can be collected faster since the completion of the questionnaire does not require the presence of the researcher. The questionnaire saves time in that it covers the questions of the researcher and allows the respondents to give answers at once. For this inquiry, it took less fortyeight (48) hours for the learners to return the questionnaires to the researcher. The questionnaire provided anonymity to the respondents thereby ensuring truthful and honest responses.

3.4.2.2.2 Disadvantages of questionnaires

However, questionnaires can be of a disadvantage as they cannot provide face-to-face, one-on-one session between the respondents and the researcher such that the respondents may find it hard to interpret the research questions which might result in skewed answers. To avoid misinterpretation of the questions, simple and straight forward questions were used in the questionnaire. Respondents may provide false information as they try to portray positive image of themselves. Fortunately, in the current study respondents were not supposed to use their names on the questionnaire for anonymity reasons.

3.4.2.3 Pre and post-tests

Pre and post-testing involve participants writing a test before and after an intervention with the aim of determining the effects of the intervention. Learners had already been taught the topic functions in the previous term using the traditional teaching and learning approach by another teacher for one week. Chuang, Weng and Chen (2018) and Leedy and Ormond (2005) believe that pre-test and post-tests are used for measuring result changes. Pre and post-tests were conducted with the aim to assess learners' conceptual understanding before and after the intervention. This design strongly suggests that intervention brings changes. In this study, the researcher used the concept-based instructional teaching and learning approach during intervention to bring changes in learners' conceptual understanding. The pre and post-tests were conducted to detect changes.

The same test (Appendix C) was used before and after intervention to investigate the learning gains that were brought about by the concept-based instructional teaching and learning approach. The test consisted of 13 questions that were developed in line with Mathematics National Curriculum Statement (2011). The questions tested participants' knowledge, comprehension, application, synthesis, and evaluation that came from different sources including past exam papers and text books. Questions 1, 2, 4, 9, 10 and 11 required facts and basic information whilst 3, 5, 8.1, 11 an12 were testing ability to analyse and interpret. Critical analysis, evaluation and opinin were demanded by questions 6, 7, 8.2 and 9. The main source was the Grade 11 Maths handbook and study guide. The other questions were also created by the researcher. A summary of the descriptions of test questions is tabled next.

Table 3.1 Summary of the descriptions of the test questions

Question number	Skill tested	What the question required the participant to do
1	Graph interpretation	Determining equations of asymptotes
2	Graph interpretation	Determining lines of symmetry
3	Drawing graphs	Sketching and labelling graphs
4	Graph interpretation	Identifying domain and range
5	Graph interpretation	Determining points of intersection of graphs
6	Graph interpretation	Identifying where graph is above y-axis
7	Graph interpretation	Making sense of the given graphs
8	Graph interpretation	Determining distance between 2 graphs
9	Graph interpretation	Determining effects of translations
10	Graph interpretation	Determining effects of reflections
11	Graph interpretation	Substitution and simplifying
12	Notation interpretation	Substitution and simplifying
13	Calculations	Determining gradient
14	Application	Determining gradient of a point on a curve
15	Substitution and calculation	Solving quadratic equations with irrational roots

The test question paper consisted of blank spaces where participants would write their answers. One of the instructions on the question paper was that participants were to show all their workings. The pre-test was given at the beginning of April and the post-test was conducted at the end of the same month and both were marked manually using marking guidelines (Appendix D). Some of the aspects which the researcher took note of were the following: how correct answers were obtained, correct answer and no working, attempted and failed to finish well as well as use of inappropriate strategies.

3.4.2.3.1 Pre-test

A pre-test, as postulated by Sharma (2018) and Niaz (1998), is given to brief the researcher about the general structure of learners to be involved in the study. This test is written before intervention. In this study, a test was administered to find out what the participants already knew and it was written before participants got a treatment or an intervention. The pre-test was given so that the researcher could identify the level of conceptual understanding of the participants. The main purpose of the pre-test was to measure participants' existing knowledge of algebraic functions. The other reason for having the pre-test written was for the researcher to identify aspects that needed emphasis as well as being in a position to incorporate some of the participants' interests in the research based on participants' previous knowledge.

3.4.2.3.2 Post-test

A post-test, as elaborated by Namaziandost, Shafiee and Hashemifardnia (2019), is a trial test or a preliminary test that is given to participants to assess the effectiveness of some activity. This kind of test is given at the end of an experiment to measure the changes brought about by the research experiment. For the current study the test was written after completion of the intervention programme in order to bring out changes in participants' thinking capacities. The participants' achievements within the two tests were collated to come up with effects of the concept-based instruction.

3.4.2.3.3 Advantages of pre and post-tests in a study

The implementation of pre and post-test is the merest way of determining the effectiveness of an intervention. The two tests were employed to measure the changes that were brought about by the intervention that involved the use of concept-based instruction as a teaching and learning approach. There was exploration of changes in participants' conceptual understanding. Writing the same test before and after intervention made conducting the test and analysing the results easy. The same test was used before and after intervention to strengthen validity. According to Noble, Scheinost and Constable (2019) and Siegelman, Bogaerts and Frost (2017), the repetition of the test under similar conditions improved reliability and this is further reinforced through the writings of Healy and Perry (2000) who stated that consistency is achieved by test-retesting.

3.4.2.3.4 Disadvantages of pre and post-tests

The use of the same test for both the pre and post-test cannot be totally trusted as some participants can take advantage of memorising procedures from the first test. However, participants spent over a month from the time they wrote the pre-test to the time they set for the post-test which made the researcher believe that the gap between the two tests allowed the results to be as valid as they would have been even without the pre-test. The changes are not always a result of the intervention. The same applies to the present study. The issue of the testing threat could not be ruled out.

3.5 Intervention

Intervention is an interference provided to modify a process on a situation. It is a purposive strategy that is meant to bring about desired outcomes. Intervention is a face-to-face process meant to obtain valid and reliable information. The intervention is a 'treatment.' Easton and McColl (1997) view a treatment as what the researcher applies to the experimental units in order to observe a change in a variable. In this study, intervention was conducted in a way trying to bring conceptual understanding and also according to Mulford and Robinson (2002) expose participants to situations where their incorrect knowledge would not work.

The intervention was conducted for four weeks starting from 2 April 2019 with each lesson stretching for an hour. The educator (researcher) taught 35 participants using the concept-based instructional approach. Intervention lessons were conducted on the topic: Algebraic functions as portrayed by the lesson plans (Appendix: G). The lessons were in line with the Mathematics National Curriculum Statement (2011) which state the following as skills expected to be achieved by learners: drawing graphs, investigating, reasoning interpreting, comparing, demonstrating, communicating, describing and analysing.

The researcher started the intervention by briefly explaining the purpose of the process and content that was to be covered. This was done to let the participants know exactly what was expected of them. The concept-based instruction teaching and learning approach was employed in the intervention. During intervention, participants were given instructions and a chance to relate to prior knowledge, identify patterns and make connections thereby solving problems independently including tackling the novel ones. The main idea was for the participants to be able to gain conceptual understanding through active participation in the learner-centred activities that were provided. Individual, pair and group written work, discussions and demonstrations were some of the intervention activities.

Participants were encouraged to use a variety of strategies to solve given problems. The educator asked follow-up questions during the lessons probing clarification and deepening conceptual understanding. Furthermore, the participants were urged to ask questions for clarification. They were given time to explicate and rationalise their reasoning. A variety of illustrations were done by participants and at times by the educator. These lessons were mainly targeted to improve participants' conceptual understanding.

3.6 Content for the study

The study concentrated on algebraic functions in its undertaking to explore the effects of concept-based instruction in the teaching and learning of mathematics. The researcher chose algebraic functions after realising that it is one of the crucial topics in mathematics. Functions involve most of the challenging concepts and they play a central role in all fields of mathematics. The topic of functions has been highlighted in previous reports as the one where learners achieve low marks in the final Grade 12 examinations (Clement, 2001; Mpofu, 2016; Mushipe, 2016). Mushipe (2006) suggests that this could be a result of commonly used traditional teaching methods which do not promote learner understanding. The other reason could be that functions involve a connection and application of other concepts like transformations and differential calculus. Despite these problems, Carraher and Schliemann (2017), and Yerushlamy and Shternberg (2001) assert that functions play a vital role in high school mathematics. According to Gravemeijer (2004), functions are the backbone of Algebra. Gronmo (2018) then describes algebra as a language of mathematics; playing a major role in students' opportunities to pursue many different types of careers in a modern society. Learners had been taught the topic before for two weeks, but were struggling a lot so the researcher thought that may be bringing in a different instructional approach would help learners attain conceptual understanding. Therefore, the researcher decided to use this topic with the hope that after the intervention and discussions, there could be ways of improving conceptual understanding in learners.

The researcher focused attention on the linear, parabola, hyperbola, and exponential functions. Axes of symmetry, asymptotes, domain, range, graphs, translations, reflections, lengths, distance between graphs and average gradient are the aspects which were dealt with during data collection. The researcher consulted the Grade 11 Mathematics work schedule for enough information on the content to be covered. Among the Grade 11 text books that were consulted are: The Maths handbook and study guide, Classroom Mathematics, Platinum Mathematics and Clever Maths. Past exam question papers from different provinces were also referred to.

3.7 The pilot study

A pilot study as described by Ismail and Kinchin (2018) and Bless, Higson-Smith and Kagee (2006) is a small study conducted prior to a larger piece of research to determine whether the methodology, sampling and analysis are adequate and appropriate. In the current inquiry, the pilot study was therefore a rehearsal of the actual study meant to evaluate the intended research approach. Madungwe (2018), Cohen, Manion and Morrison (2002), Opie (2004) and Mbewe (2013) see the pilot study as meant to assess the feasibility of the study and to pre-test the research instruments prepared for the study. The researcher administered the pilot study in order to find out the shortcomings of each research instrument before embarking on the actual study. The pilot study imparted knowledge of the main study and tested the validity of the instruments.

More importantly, the researcher used the pilot study to refine the instruments and to identify possible unforeseen problems that could disturb the results of the research. The tests, interviews and questionnaires were pilot tested. The pilot studies that were carried out are elaborated in the next sub sections.

3.7.1 Pilot test of pre and post-tests

The test which was used before and after intervention was pilot tested twice on ten learners who were not participants of the study. The process started with ten learners given instructions of what was supposed to be done. The learners then wrote the test twice with a brief intervention that lasted for one week conducted between the two tests. Pilot testing was done to check the validity of the questions in terms of their ability to expose learners' level of conceptual understanding. In this case, the researcher wanted to expose the deficiencies of the test. The test was found to be correct with no unclear or ambiguous questions. The time allocated for the test was also discovered to be enough as participants were able to complete answering questions in the stipulated time. The other reason for testing the instrument was to identify and discard inappropriate questions as well as any other problems associated with the test. Fortunately, there were no problems discovered; so, modifications were not necessary.

3.7.2 Pilot test of questionnaires

After the two tests, the ten learners who wrote the tests were given questionnaires. Instructions were given and they completed the questionnaires which they submitted on the same day. The researcher discovered that there were many blank spaces indicating that learners had not been given enough time to respond to all the questions and also maybe some of them had forgotten what had taken place during lessons. She then gave them more time to fill in the remaining spaces and then decided that it would be necessary to let participants have enough time to complete the questionnaire and also give participants questionnaires just after intervention. The testing was to check the applicability of the included questions or the validity of the questions. The responses from the questionnaire allowed the researcher to check the clarity of the questionnaire items. The provided responses indicated that the questions were clear and relevant to the study. Besides the time factor, no adjustments were made to the questionnaire.

3.7.3 Pilot interviews

Pilot interviews are mini-versions of interviews which were held with two learners from the group of learners who were involved in the pilot study. The researcher used the interview guide (Appendix E) taking note of the proceedings of the interviews and checking how respondents felt about the questions based on their responses. The researcher was able to improve her way of asking questions. An example of a situation when the researcher felt the need for changing her words was on the question, 'Explain how you got this answer....' It was realised that the question was a bit harsh or impolite. The respondents acted as if they were being burdened. The researcher decided to change the question to, 'Do you still remember how you got this solution?'

This is roughly how the pilot interview was conducted:

[Note: **R** represents researcher and **L** is for participant]

R: *I want you to go through your solutions in the two papers for Question 2.*

R*: In the first test you could not determine the axes of symmetry for the hyperbola, but in the second test you got it right. What was your problem in the first test?*

L*: I just remembered the formula, y = x+c but did not know how to* get c.

R: ***Explain how you got the answer in the second test.***

R: ***In fact, do you still remember how you got the solution?***

L: *First, I knew the hyperbola had two axes of symmetry that both pass through the point of intersection of the asymptotes of the function.*

R: *That's an excellent explanation. Now can you proceed to tell me how we ended up with these two equations?*

L: *Because I knew the gradients to be 1 and negative 1, I therefore substituted the point of intersection for the asymptotes to get c for each.*

From the mini-interview, the researcher also discovered that writing down everything that was expressed by interviewees as responses to her questions would demand more time for each interview session. In fact, it was difficult to take down everything correctly. The decision that was made was that, only main points would be jotted for follow-up. The rest of capturing would be for the video recorder.

The other aim of mini-show test was to get experience of conducting an interview and also to determine the appropriateness of the questions. I managed to get a rough idea of the follow-up questions that I would ask in the actual study.

3.8 Population and sampling techniques

Sharma (2017), Rahi (2017), Chiromo (2006) and Best and Kahn (2006) define a population as all individuals, units, objects or events that will be considered in a research project. Furthermore, Nurun Nabi and Dip (2017), Keyton (2010), Nachmias and Nachmias (1996) view a population as the aggregate of all cases that conform to some designated set of specifications. A population can therefore be simply taken as a group of people sharing a certain characteristic of interest. In the present study, the population involved shared a common characteristic of being Grade 11 Mathematics learners. The targeted population as described by Teeroovengadum and Nunkoo (2018) and Greenland (2005) is the population about which information is wanted or the totality of elements which are under discussion and about which information is desired. In this study, the population consists of Mathematics learners and the targeted population was Grade 11 learners from Mopani District of South Africa. A class of 35 learners was purposively and conveniently selected from one school for this study. The researcher was the only teacher who participated in the study. Details of the selected class and how it was selected are given below.

3.8.1 Study participants

One Mathematics Grade 11 class of low performing learners was selected to participate in the study. A class of the 35 learners belonged to a school in Giyani. The school had an enrolment of around 1500 learners (both boys and girls). There were 248 Grade 11 learners at that school, with 158 of them studying mathematics and the remainder were studying mathematical literacy. There were four mathematics classes which were allocated according to subject combinations. The school had a policy that screened Grade 10 learners at the beginning of every year according to the Grade 9 performance in mathematics and natural science. Learners would then proceed to the next grade in their respective classes. The classes were therefore, of mixed ability. The researcher then used term one marks and took the bottom thirty-five learners in mathematics. The reason for taking those learners to participate in this study was the hope that the intervention would benefit those learners in one way or the other. The researcher also thought that if there was going to be an improvement in conceptual understanding and/or performance, it would be easy to identify the changes. Table 3.2 provides the demographic characteristics of the participants in terms of age and gender.

Table 3.2 Sample demographic characteristics

Age distribution of the Respondents	**Frequency**	**Percentage**
16-18 years	16	45.7%
19-21 years	13	37.1%
22-24 years	6	17.1%
Total	35	100.0%
Gender	**Frequency**	**Percentage**
Male	15	42.9%
Female	20	57.1%
Total	**35**	**100%**

Analysing the age distribution one can see that the greater percentage of learners were above 18 years which is above the average age for Grade 11 learners. Normally, a learner in Grade 11 should be around 17 years. This issue of ages is part of evidence that the learners in that class were low performers. They took time to reach that grade probably because they had repeated other grades or were repeating that grade.

The researcher selected that class of least performers. In fact, the principal of the school requested the researcher to consider that class hoping that the intervention would benefit the learners and the school in one way or the other. Therefore, the selection was not random; it was purposive and convenient.

3.9 Sampling

For the researcher to draw valid conclusions in a study, there is a need to gather data from a well-defined sample (Ncube, 2016). It was necessary to use the best sampling method in order to come up with valuable data. According to Brink (1991), Rahi (2017), and Moser and Korstjens (2018), sampling refers to a process of selecting the sample from a population to obtain information regarding the phenomena. The grade 11 class that was chosen provided information that was required by the researcher and it represented the other mathematics learners. Du Plooy-Cillers et al. (2014) refer to a sample as participants or respondents that the researcher can gain access to within the population. The researcher could not involve all learners from the targeted population; so, she decided to choose a representative group (sample) from which she would gain in-depth data from the central phenomenon. The sample was conveniently and purposively selected as explained below.

3.9.1 Convenience and purposive sampling

A purposive and convenient sample was used for this study. It consisted of a Grade 11 class of learners from a high school in Mopani District, Limpopo Province, South Africa. Etikan and Babtope (2019), Mishra and Alok (2017), and Nachmias and Nachmias (1996) describe purposive sampling as based on subjective judgement of the researcher in order to come up with a sample that best represents a population. In this case, the researcher had learners' performance in mind so she had to consider the class of low performers. Convenience sampling which Adeniran (2019), Fathima (2018), and Lewis, Zhang and Utaaker (2018) refer to as opportunity sampling, is a sampling technique where the researcher chooses a sample that is easy to reach or convenient to work with; yet fitting the criteria the researcher is looking for and available at the time the study is being carried out (Farrugia, 2019; Cohen, Manion & Morrison, 2011; Bertram & Christiansen, 2014). Again, the selected class was convenient and cost effective as the researcher was working at the school where the participants were learning. The participants were selected on purpose, as their class consisted of less gifted learners or learners who had challenges in learning; so, they needed help most.

The sample was easily accessible to the researcher and at the same time, the participants were selected based on level of performance for easy identification of improvement in performance. Therefore, it helped the researcher by minimising transport costs and saving time. The way participants were selected had its own advantages and disadvantages which are elaborated below.

3.9.1.1 Advantages of convenience and purposive sampling

It is easy to conduct as it has fewer governing rules of how the sample should be selected. Making generalisations from a sample that has been selected conveniently and purposively is simple since participants have common characteristics. The researcher felt that getting low performing learners participate could help them benefit academically from the teaching and learning instruction in question. The sampling method also saved both money and time during data collection as it was easily accessible to the researcher.

3.9.1.2 Disadvantages/Limitations of convenience sampling

However, selecting participants because of easy accessibility could have left out some needy learners. This kind of selection has high risk of bias in it because there is no randomness in the selection of participants. According to Jager, Putnick and Bornstein (2017), this lacks clear generalisability. The other problem could be data manipulation during collection by participants.

It is extremely difficult to evaluate the expert's reliability in purposive sampling. False assumptions could have been made because of this sampling method.

3.9.1.3 Mitigation of shortcomings of convenience sampling

Although the chosen sampling method has the mentioned shortcomings, the demographic information given in Table 3.1 above and the description of the target population together with the background of the school from where the research was carried out are well defined to reduce sampling bias. More importantly, the researcher also used a class which she was not teaching to avoid bias. Triangulation of research methods and data were also conducted as a way of trying to cover up for the chances of the problems that emanate from this kind of sample.

3.9.2 Sampling techniques

Once the problem statement has been identified, the next task is to select the unit of analysis (Merriam, 1988). As described by Kitay and Callus (2018) and McMillan and Schumacher (2010), the unit of analysis is the object to be studied in terms of research variable that constitutes the constructs of interest. In the current study, effects of the concept-based instructional teaching and learning approach on learners' conceptual understanding were explored. The unit of analysis in this case was the effects of concept-based instruction on the teaching and learning of mathematics. Having the unit of analysis at hand, the researcher proceeded to identify the possible participants for the study. One Grade 11 class of low performing learners from one high school participated in this study. The researcher decided to use that class so that after administering the concept-based instruction, she could possibly identify if there were changes in conceptual understanding, and hence improvement in performance. The class was selected for convenience because it was easy to reach and learners of that class needed much more attention. On top of that, the research was carried out during school study period, but after normal lessons to ensure that school business was not interrupted.

Using single subject methodology and avoiding control was an advantage as control groups have chances of creating artificial situations. When one uses control groups, data could end up being skewed or being corrupted to fit whatever the researcher needs. A single group was used to compare, control and explain measurements. All participants in the group study were exposed to the same conditions. Therefore, all learners in the selected class had a chance to benefit from the intervention as everyone was a participant. Their levels of conceptual understanding were observed before and after introducing experimental factors.

3.10 Description of site

The research was conducted at a school in Mopani East District, Limpopo Province of South Africa. The school enrols more than 1500 learners, both boys and girls. It is a non-paying fee public school with roughly seven classes per grade running from Grade 8 to Grade 12. The school is situated in a township area. Mosoge, Challens and Xaba (2018) and Dhlamini (2012) indicate that performance of learners from township schools is least impressive. Awuah (2018) and Dlamini (2017) add that most learners from township schools fear mathematics and therefore opt to do mathematical literacy. Nevertheless, the school that participated in the study had the majority of learners doing mathematics. The reason for opting to do mathematics is maybe because most learners are now enlightened. They know the advantages mathematics bring when it comes to selection of courses at tertiary institutions. They are guided by visions of what they want to pursue post-matric. However, the learners at this school are streamed according to performance in mathematics the moment they get into Grade 10 and they maintain those classes until they write their matric examinations.

3.11 Reliability

Taber (2018), Mohajan (2017), Cohen, et al. (2013) posit that for a research instrument to be reliable, it must demonstrate consistency. In addition to this view, reliability deals with the ability to come up with the same view of a given phenomenon when a review is conducted under the same conditions (Streiner, 2020; Kivunja & Kuyini, 2017; Gertz, 1973). In short, reliability refers to consistency and stability of variables or stability of the findings. For the present study, the researcher expected to obtain significant results that would not be biased.

On the same note, Mohajan (2017) and Opie (2004) view reliability as concerned with the faith that one has in the data obtained from the use of an instrument. In this inquiry, reliability was ascertained by a pilot study. In the pilot study, the researcher checked if the instruments used would really bring out the desired outcomes. It was discovered that after making necessary alterations like allocating enough time on questionnaires and improving questioning techniques in interviews, the instruments were reliable. The reliability of a measuring instrument reflects the consistency with which results can be obtained when it is administered repeatedly (Onyancha, 2017; Akintade, 2017). The use of the same test before and after intervention together with the use of the same participants had findings that displayed closeness in results, which is an indication of consistency. The triangulation of research methods which involve quantitative and qualitative data collection together with the use of three different data gathering instruments was also meant to improve the reliability of the study.

3.12 Validity

Validity as explained by Mohajan (2017) concerns what an instrument measures and how well it does so. Test validity refers to the extent to which inferences based on instruments are reasonable (McMillan and Schumacher 2010). In line with these views, validity can be described as the extent to which the selected instrument actually reflects the reality of constructs that are being measured (Du Plooy-Cilliers et al., 2014). In other words, it is a measure of the degree to which explanations of an event match reality. Validity does not depend on the data, but the interpretation of the data. A test can be regarded as valid if it serves its intended purpose well (Mbewe, 2013). In short, validity is the degree of credibility, genuineness and believability of the research results.

Mulder (1989) highlights the existence of various types of validity and indicated that the type of validity depends on the purpose of the instrument. Content validity and construct validity were found to be more appropriate in this study. They are discussed next.

3.12.1 Content validity

Baghestani, Ahmadi, Tanha and Meshkat (2019), Xie (2018) and Creswell (2007) define content validity as the degree to which the content of an instrument covers the extent and depth of the topic it is supposed to cover. If a test is to be valid, it has to be aligned to what learners are expected to learn (Onyancha, 2017). The Curriculum Assessment Policy Statement (CAPS) for Grade 11 Mathematics was adhered to in this study. The questions were set in line with the expected learning outcomes. According to Osterlind (1989), content validity can be checked by a panel of evaluators who are skilful to rate if content is in line with curriculum standards. In this study, before the pilot study, the test was given to three colleagues and a mathematics curriculum advisor to check if they were up to standard and they collectively approved it.

3.12.2 Construct validity

Awuah (2018) sees construct validity as the relationship between content of instruments and the constructs destined to measure. Therefore, construct validity simply refers to the ability of a test to measure what it is intended to measure. The results obtained from the test provided answers to the research questions. Therefore, the effects of concept-based instruction were identified. The same test was used twice in this research, which was a way of strengthening validity. There was a change in results owing to the intervention which is what the researcher wanted to determine in terms of validity of the test. The questionnaire was valid because the required information from the respondents was obtained. The interview also brought out more information concerning learners' conceptual understanding level and also their attitudes and feelings towards mathematics, which was good for determining the effects of concept-based instruction.

Generally, the use of the three data sources (tests, questionnaire and interviews) facilitated the corroboration of findings. The weaknesses of one instrument, if any, was compensated by the other instruments. Triangulation increases confidence in research findings. Research methods were triangulated to ensure validity.

3.13 Trustworthiness in this study

Bless, Higson-Smith and Sithole (2013) and Cypress (2017) describe trustworthiness as dealing with how much trust one can give to the research processes and its findings. Trustworthiness in a study is concerned with the asperity and rigidity from the validity of a research process. Trustworthiness is used as a measure of reliability and validity in a study. For this study, trustworthiness was achieved by ensuring the following: credibility, transferability, dependability, and conformability of a study.

3.13.1 Credibility

Credibility as explained by Abdalla, Oliveira, Azevedo and Gonzalez (2018) refers to the accuracy with which the researcher interprets data collected from the participants. Credibility is a quality of being worthy to be trusted. Triangulation of data sources enhanced credibility in this research. Mishra and Rasundram (2017) articulate that the main purpose of triangulation is to increase the credibility and validity of results as it strengthens conclusions about the findings and reduce risk of false interpretations. By triangulating the data sources, the intention was to invigorate the research findings and to lower the chances of false renditions. Triangulation of research methods was meant to reduce deficiencies and preconceptions driven from one rammed peer examination.

Peer debriefing was involved as well to enhance credibility. Gill, Gill and Roulet (2018) and Anney (2014) describe peer debriefing as a process whereby the researcher seeks support from other professionals. The researcher got most of her guidelines from her supervisor (Professor Ngoepe) and was also guided by three scholars with doctoral degrees (Maziriri, Kativhu and Makuku). These guidelines were meant to improve the quality of this study. The researcher also held discussions with other colleagues of the same academic background. Suggestions were given on data collection and analysis to improve the quality of the research findings.

Padgett (2016) suggests performing member checking which according to Thomas (2017) involves validating the research findings by seeking participants' feedback. An example of seeking participant feedback was observed in the interviews when open-ended questions were involved giving room for the researcher to probe for more information on issues of interest. This was another way of increasing the credibility of the study.

3.13.2 Transferability

As stated by Munthe-Kaas, Nokleby and Nguyen (2019) and Collis and Hussey (2013), transferability is the degree to which results and analysis can be applied beyond specific research project. Transferability bore on the applicability of research findings to other situations. Moon, Brewer, Januchowski, Adams, and Blackman (2016) observed that the methodology of a research together with qualitative data analysis must show why research can be transferrable. Following this writing, a detailed research methodology was given and thematic analysis was involved to strengthen the findings so that the results can be used by other researchers as well as bringing changes to the teaching and learning of mathematics.

According to Kyngas, Kaariainen and Elo (2020) and Anney (2014), a researcher can strengthen transferability by providing detailed information of the research process and its participants so that other researchers can have detailed information about the study. Transferability was enhanced in this study through provision of detailed description of the research design and methodology, data collection and analysis.

3.13.3 Dependability

Korstjens and Moser (2018) view dependability as the stability of findings over time. Shenton (2004) asserts that dependability can be viewed as the quality of the process of integration that takes place between the data collection method, data analysis and the theory generated from the data. Dependability therefore is about the way a study is carried out to ensure consistency. The process of triangulation of research methods and data instruments was a way of ensuring consistency. Researchers can achieve dependability by ensuring a logical research process that is traceable and clearly documented (Nowell, Norris, White & Moules, 2017; Tobin & Begley, 2004). An audit trail of the whole research process which includes test scripts, questionnaires, interview notes, voice recordings and transcriptions as supporting evidence for this study is available.

3.14.4 Confirmability

Confirmability has to do with establishing that the researcher's findings and interpretations are clearly driven from data (Kalu & Bwalya, 2017; Moon, 2019; Tobin & Begley, 2004). Furthermore, confirmability brings out aspects of neutrality. There is a need for the researcher to bring out that the objectivity of the research not the researcher's views. In the study, there was demonstration of how data were interpreted and how conclusions were reached to achieve easy follow-up of the process and replication. Data were audited which means that the researcher (after putting down the research methodology) continuously referred to literature and checking her data for maintaining the right direction in coming up with sound findings.

Triangulation of data sources and research methods enhanced confirmability by facilitating data validation through substantiation from several sources. On the other note Ryan, Coughlan and Cronin (2007) and Guba and Lincoln (1989) aver that confirmability can be established by achieving credibility, transferability and dependability. In the study, the trio were attained as explained in this section. Therefore, confirmability was instituted.

3.15 Data analysis

Data analysis involves an orderly review and stimulation of data aiming to make deductions and conclusions to support decision-making (Bloomberg & Volpe, 2008). On the same note, Merriam and Tisdell (2015) describe data analysis as a process of meaning making of what has been said by people and what the researcher has observed and read. Data analysis in this study involved breaking down the data into manageable patterns or categories in order to understand and make sense of the data. Hatch (2002) simply describes data analysis as a systematic search for meaning. The main reason for data analysis as supported by Castleberry and Nolen (2018) was to get answers for the research questions. An explanation for the analysis of data for each research instrument is given in the following subsections.

3.15.1 Data analysis from interviews

Thematic analysis was used to analyse data from the interviews. Thematic analysis is defined as a meticulous process of identifying, analysing and reporting themes that emerge from a qualitative study (Maziriri, Madinga, & Lose, 2017). Braun and Clarke (2006) consider thematic analysis in a research study as providing the opportunity to identify, analyse and report the emerging themes within the collected data. The researcher wanted to bring order, structure and meaning to the data collected by breaking it up into wide themes, patterns, trends, and relationships.

Firstly, there was transcription of data collected for each interviewee from the audio tapes. The researcher went through the transcribed data for familiarisation and generated codes. Coding is done to identify data that had to do with the study (Elliott, 2018; Hesse-Biber & Leavy, 2010). These codes, according to Cheng, Fu, Sun, Bilgihan and Okumus (2019) and Gray (2015) entailed location of themes. Themes and categories were deduced from the codes. In the construction of themes, the researcher read the transcriptions several times to identify relevant data. Data related to the study topic, research questions, literature and also data that had new important meaning were identified for theme formation. The following themes emerged: Learning atmosphere, improvement in conceptual understanding, aptitude competency, learner engagement with lessons, mathematics as a cumulative subject, and mathematics challenges.

Each theme was discussed individually, followed by substantiating quotes. Interpreted data was compared with existing literature. The researcher had to make sense of the data in order come up with answers to her research questions.

3.15.2 Data analysis from questionnaires

The analysis of responses from questionnaires was done through descriptive statistics and graphs. Both quantitative and qualitative data were collected using the questionnaires. Firstly, the researcher concentrated on the multiple choice questions (yes/no and true/false). The frequencies of the responses were obtained and were expressed as percentages. Secondly, attention was given to the open-ended questions where the researcher needed more time to read and identify common concepts in the provided responses. A grid was then prepared for collating the data from the responses.

3.15.2.1 Graphical analysis

According to Slutsky (2014), graphical representation is an efficient way for the audience to organise and interpret data that is numerous or complicated in a short period of time. Graphical representation of answers was found to be a suitable method of analysing data as it conveys information more quickly to the audience. The reader can find understanding of information easy when s/he visualises through graphs.

The percentages of each response per question were represented on pie charts and bar graphs with each representation of a response titled. Pie charts make comparison of responses easy with each response category represented as a percentage of the whole. The bar graph was also used for comparison, but mainly it was for tracking changes. In a bar graph, bars represent proportions for each response category. The pie chart and the bar graph are both visually simpler than most of the graphs.

3.15.2.2 Descriptive analysis

There were two open-ended questions in the questionnaire which were thematically analysed. Thematic analysis is described as a framework used to classify, organise and describe data according to key themes (Castleberry and Nolen 2018; Lewis, Ritchie, Ormston and Morrell 2003). Mihas (2019), Mohajan, 2018, Maree (2012) emphasise that a researcher must establish how the participants make meaning of a specific phenomenon by analysing their perceptions, attitudes, understanding, knowledge of feeling and experiences. In this study, the researcher wanted to explore the effects of concept-based instruction on the teaching and learning of mathematics. Therefore, she created a questionnaire that was targeted to address all the stated properties in an attempt to get answers to the research questions.

The researcher read the responses that were provided in the questionnaires to develop general sense of data and came up with codes. She then organised the codes into categories and finally developed themes from the categories. Three themes were deduced and are as follows: Learning atmosphere, challenges in learning mathematics and teacher as a facilitator. The multiple choice questions were grouped according to the mentioned themes as well. The themes were related to previous studies following the views of Castleberry and Nolen (2018) and Lewis et al., (2003) that emphasise the checking of the relationship between themes and previous studies.

3.15.3 Data analysis from pre and post-tests

Data from the pre and post-tests were analysed descriptively and inferentially. Descriptive statistics as explained by Salvatore (2021) and Woodrow (2014) simply describes the data provided by participants while inferential statistics can lead to conclusions about the population under study. The main reason for using descriptive statistics was to describe and summarise the test results in a way determining the effects of the intervention on participants. Inferential statistics, on the other hand, was meant to come up with conclusions not only for the participants but beyond the study.

3.15.3.1 Descriptive statistics

Descriptive statistics helped the researcher to summarise the test results and to compare performance before and after intervention in a way trying to determine the effectiveness of concept-based instruction. The arithmetic mean, standard deviation, median, range, quartiles, maximum and minimum scores were computed for the two tests. Overall performance in each test was analysed and comparisons between the results of the two tests were done. Tables, pie charts, scatter plots and bar graphs were utilised to present the data. The researcher preferred to use distribution tables, pie charts, and bar graphs for ease interpretation of information.

3.15.3.2 Inferential statistics

Inferential statistics gives chance to make inferences about populations using collected data. In this study the inference involved the dependent t-test which was ran to examine change. The study was carried out to explore the effects of the concept-based instruction. Researchers often want to cause change through intervention (Lawrie, 2017). The researcher then wanted to check if the intervention had really brought some changes. The detection of the change was done using the average-based change approach. The latter approach as explained by Grissom and Kim (2012) uses pre and post-test means to describe changes in a single group study.

These changes were checked using the differences between the means of the pre-test and the post-test. The researcher had to create the null hypothesis assuming that there was no difference in the means for the two tests, one test written before intervention and the one after intervention. It was measured against the alternative hypothesis which assumed that the mean for the post-test was not equal to the mean for the pre-test. The test was done at 95 % confidence interval.

3.16 Triangulation

Flick (2018), Moon (2019), Olsen (2010), Cohen and Manion (2011) define triangulation as use of more than one strategy to work on a single research problem or investigation in order to understand deeper issues. In the opinion of Noble and Heale (2019), Jespersen and Wallace (2017), Guion, Diehl and McDonald (2002), triangulation is a method used to check and establish validity in a study through analysing research questions from multiple perspectives. In this research, different research methods (quantitative and qualitative) were used together with three different research instruments (questionnaires, interviews and pre and post-tests). The use of different sources of data as proposed by Fusch, Fusch and Ness (2018) and Maxwell and Loomis (2003) enhances validity. Furthermore, Johnson, Adkins and Chauvin (2020), Daniel (2019), Jick (1979) postulate that triangulation ensures trustworthy outputs of the study and increases credibility of the study (Noble and Heale 2019; Hussein 2009). Triangulation therefore, ensured rich, robust and dependable results for this study.

Data triangulation, as supported by Hussein (2009), Abdalla et al., (2018), and Freeman (2020) render the use of multiple data sources in the same study for confirmatory, completeness and validation purposes. The following data sources: pre-tests, post-tests, questionnaires together with semi-structured interviews were used as a way of gathering rich data. Triangulation explicates the complexity of a problem by studying it from different points of views (Dzwigol, 2020; Flick, 2018; Cohen & Manion, 2007). In this study, the researcher believed that the combination of tests, questionnaires and interviews was best, as the data sources complemented each other to bring out the effects of concept-based instruction on the teaching and learning of mathematics.

3.17 Ethical considerations

Ethical issues involve protecting the rights of research participants. Participants have to voluntarily take part in a study. The participant has the right to know what s/he is supposed to or not do. There is also need for a clear understanding of the procedures as well as risks before giving consent to participate. Madungwe (2018) highlights that voluntary participation means that people should not be coerced into taking part in a research study. Participants for this study were clearly informed that their participation was voluntary and were free to opt out anytime. Harriss, MacSween and Atkinson (2019) add that the requirement of ethical standards is that participants are not put in a situation where they might be at risk of harm as a result of participation. For this study, the researcher made sure participants would not write their names on test papers or questionnaires for the sake of confidentiality. Participants were also notified about how information and data they had provided would be used, processed, shared, and disposed. The Unisa Research Ethics Committee granted approval of the research study and awarded a certificate which indicates that the proposal was meeting the ethical standards.

3.17.1 Ethics certificate

The researcher applied for ethical clearance to the Unisa Research Ethics Committee and a clearance certificate reference **2018/10/17/48809969/19/MC (Appendix A)** was issued. Caution is that whenever human beings are the epicentre for an enquiry, researchers need to find out the ethical entailments of what they intend to study (Cortazzi; Jin, 2021; Leedy & Omrod, 2015). The purpose of the ethical clearance was to ensure that the study was conducted in a responsible and ethically accountable way.

3.17.2 Informed consent

Cohen (2011) defines informed consent as a principle on ethical behaviour in research which declares participants have the right to know what they are researched on. Palan and Schitter (2018) and Neuman (2014) further highlight that it is not just enough to get permission from people; they also need to know why they are being asked to participate so that they can make informed decisions. Informed consent documents (Appendices L & M) gave explanation of the purpose of the study, the procedures to be followed in terms of time, place and activities. It also discussed issues concerning remuneration and confidentiality. Learners who participated in this study together with their parents/guardians were asked to sign assent and consent letters respectively for them to make the decision to participate voluntarily without force or coercion.

Lee, Holmes, Neri and Kushida (2018) and De Vos, Strydom, Fouche and Delport (2011) propounded that deception involves withholding information, or offering incorrect information in order to ensure the participation of subjects when they would otherwise possibly have refused it. To avoid deception, information was provided to the respondents regarding the nature and content of the study and did not promise them any favours in exchange for participating in the study. Permission for learners to participate in the study was requested from the school principal as well as from the DBE. The researcher also strongly adhered to Unisa Ethics Policy.

3.17.3 Confidentiality

Confidentiality and anonymity are ethical principles that the researcher has to ensure (Zimmer, 2018; Harriss, MacSween & Atknson, 2017; Mamba, 2013). Confidentiality and anonymity were guaranteed to the respondents by the researcher. To maintain this standard, the participants did not use their names on questionnaires and in the tests. The study was conducted taking into consideration the guidelines from the Research Ethics Committee as suggested by Sumrin and

Gupta (2021) Mohajan (2017); Du Plooy-Cilliers et al. (2014) to ensure reliability and validity of information.

The researcher ensured that consent documents had to be clarified and signed before participants took part in the study. This was meant to minimise and give learner respondents an opportunity to ask any questions and get clarity of research-related issues. Again, to maintain privacy and confidentiality, the consent forms, data sources and findings of the study are kept in a securely locked cabinet. The soft copies of the study records are kept in password-protected files.

3.18 Summary of chapter

The chapter has given clarifications of the research design and methodology that were employed for data collection and analysis in this study. Explanations of the quantitative and qualitative approaches that were detailed. The target population, sampling and sampling techniques were discussed. Data instruments which involved questionnaire, pre-test, post-test and interviews that were used to enable the researcher to come up with conclusions on how the concept-based instructional teaching and learning approach affects learners' conceptual understanding in Mathematics were also described in this chapter. Furthermore, the chapter concluded with explanations of the pilot study as well as data analysis. The issues regarding validity, reliability and the strategies that were used to ensure ethical standards were also elaborated. The next chapter is going to present collected data and analyse the key findings of the study in order to get answers to the study's research questions.

CHAPTER 4: DATA ANALYSIS AND PRESENTATION OF RESULTS

4.1 Introduction

The preceding chapter gave a detailed description of the research design and methodology of the study. This chapter presents the results obtained through tests, questionnaires and interviews. Interpretation and discussion of the results is done simultaneously to enable understanding of themes emerging from the concept-based teaching with reference to algebraic equations. A discussion of the results in relation to the literature reviewed is presented to address the following research questions.

- What effect does concept-based instruction have on the teaching and learning of mathematics?
- What advantages does concept-based instruction have over other teaching and learning approaches?
- What changes are brought about in learners by the use of concept-based instruction teaching and learning approach?
- What challenges can be addressed by the implementation of concept-based instruction in the teaching and learning of mathematics?

4.2 Data presentation and analysis

Sections 4.2.1 to 4.2.4 consist of presentation, analysis and discussion of the findings with respect to each research question. A summary of each research question is provided and the chapter then ends with concluding remarks.

The following table presents the themes and categories that emanated from the data analysis.

Table 4.1: Overview of themes and categories meant to explore effects of concept-based instruction teaching and learning approach.

THEMES	CATEGORIES
Class atmosphere	• The reception by learners • Mastering of concepts in learning • Quality of teachers' explanations • Learner-teacher interaction/engagement • Conceptual understanding in learners • Pace of comprehension
Teacher as a facilitator	• Encouraging independence • Freedom to express oneself
Comparative analysis	• A comparative approach
Mathematics as a cumulative subject	• Importance of prior knowledge
Challenges in learning mathematics	• Lack of preliminary background in mathematics • Too many rules and long steps involved • Boredom and attitude • Teachers' pace vs. learners' pace

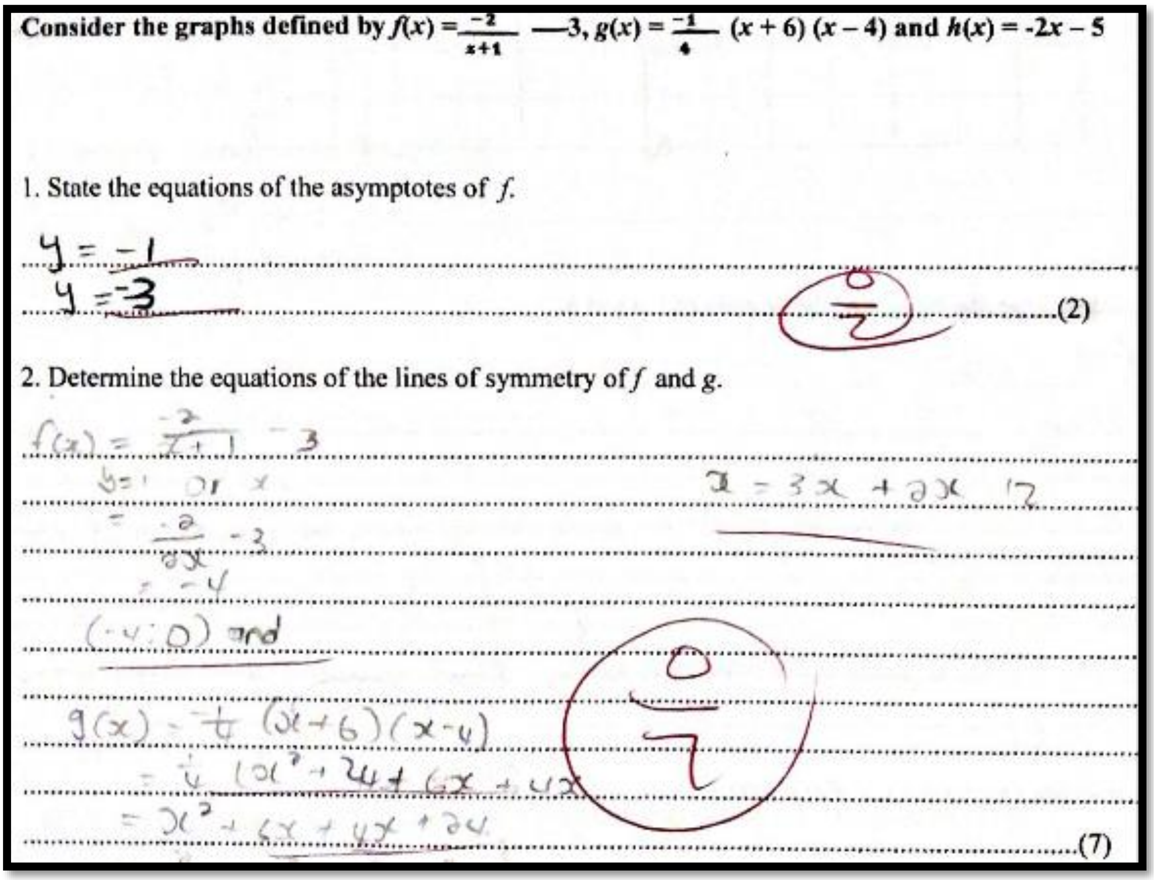
The themes and categories that emerged were then used to explore the effects of concept-based instruction on the teaching and learning of mathematics and their role in identifying the effects will be examined in the research findings presented in the next sub-sections. For each research question there is acknowledgement of the categories that the researcher assumed would give answers to that question. The following sub-sections contain the presentation and analysis of data per research question.

4.2.1 RESEARCH QUESTION 1: What effect does concept-based instruction have on the teaching and learning of mathematics?

This research question had intended to determine the effect of concept-based instruction on the teaching of mathematics. In trying to determine the effect of the teaching and learning instruction in question, learners' attitudes, perceptions, motivation, capabilities, skills and conceptual understanding can be explored (Ahmed, Chandran, Klobas & Linan, 2020; Yu & Singh, 2018; Stephens & Ormandy, 2018; Kaya & Geban, 2011; Niemi, 2010). In line with the views of these authors, the following sub-sections present data under the following subthemes: Learners' emotions towards the teaching and learning approach; mastering of concepts in learning; comparative approach; quality of teachers' explanations; educator-learner interaction; conceptual understanding in learners; and learners' perceptions about instructional strategies.

4.2.1.1 Comparative analysis of results using tests pre and post-test marks

The success of the intervention can best be demonstrated by a comparative analysis of the results of the participants in the tests before and after the intervention. The purpose of the tests conducted was to determine if there were changes in learner performance attributable to the intervention. The pre-test was meant to give the researcher an idea of the participants' level of understanding of the subject under focus. The post-test was a summative assessment to evaluate and compare the participants' level of understanding after intervention. Pre and post-tests are given to determine the status of the learners with regards to some skill, aptitude, or achievement as a basis for judging the effectiveness of an intervention (Mohseni, Seifoori & Ahangari, 2020). Figure 4.1 displays how one participant attempted questions 1 and 2 in the pre-test.


Consider the graphs defined by $f(x) = \frac{-2}{x+1} - 3$, $g(x) = \frac{-1}{4}(x+6)(x-4)$ **and** $h(x) = -2x - 5$

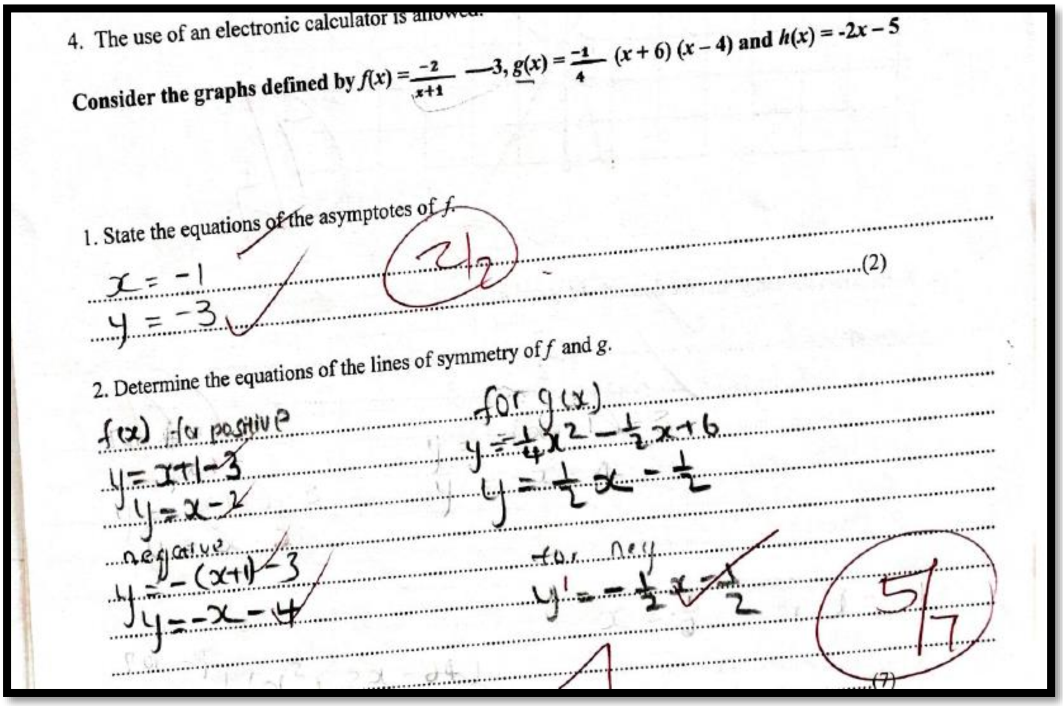
1. State the equations of the asymptotes of f.

$y = -1$
$y = -3$ ....(2)

2. Determine the equations of the lines of symmetry of f and g. ....(7)

Figure 4.1. Pre-test performances on questions 1 and 2 by L7

In Figure 4.1, L7 failed to identify the asymptotes. The learner could not link, apply or connect the question properly according to what had been learnt before. After concept-based intervention, the performance of the learner showed a marked improvement in the post-test. This is displayed in Figure 4.2.


4. The use of an electronic calculator is allowed.

Consider the graphs defined by $f(x) = \frac{-2}{x+1} - 3$, $g(x) = \frac{-1}{4}(x+6)(x-4)$ **and** $h(x) = -2x - 5$

1. State the equations of the asymptotes of f. ....(2)

$x = -1$

$y = -3$

2/2

2. Determine the equations of the lines of symmetry of f and g.

f(x) for positive

$y = x + 1 - 3$

$y = x - 2$

negative

$y = -(x+1) - 3$

$y = -x - 4$

for g(x)

$y = -\frac{1}{4}x^2 - \frac{1}{2}x + 6$

$y = \frac{1}{2}x - \frac{1}{2}$

for neg

$y' = -\frac{1}{2}x - \frac{1}{2}$

5/7 ....(7)

Figure 4.2. Post-test performances on questions 1 and 2 by L7

The learner marks pushed from level 1 at 12% to a post-test mark of 64% in level 5. For all the participants, in post-test marks there was no learner who was in level 1, below 30%. Almost half of the learners scored higher than 80%. The responses by the participants with respect to test solutions indicate improvement when compared to the pre-test marks. The marks for the 35 participants in the two tests were recorded and graded according to levels.

Figure 4.3 shows the distribution of marks across all levels for all the learners in the pre and post-tests.

BAR GRAPH ILLUSTRATING PRE AND POST-TEST MARKS DISTRIBUTION

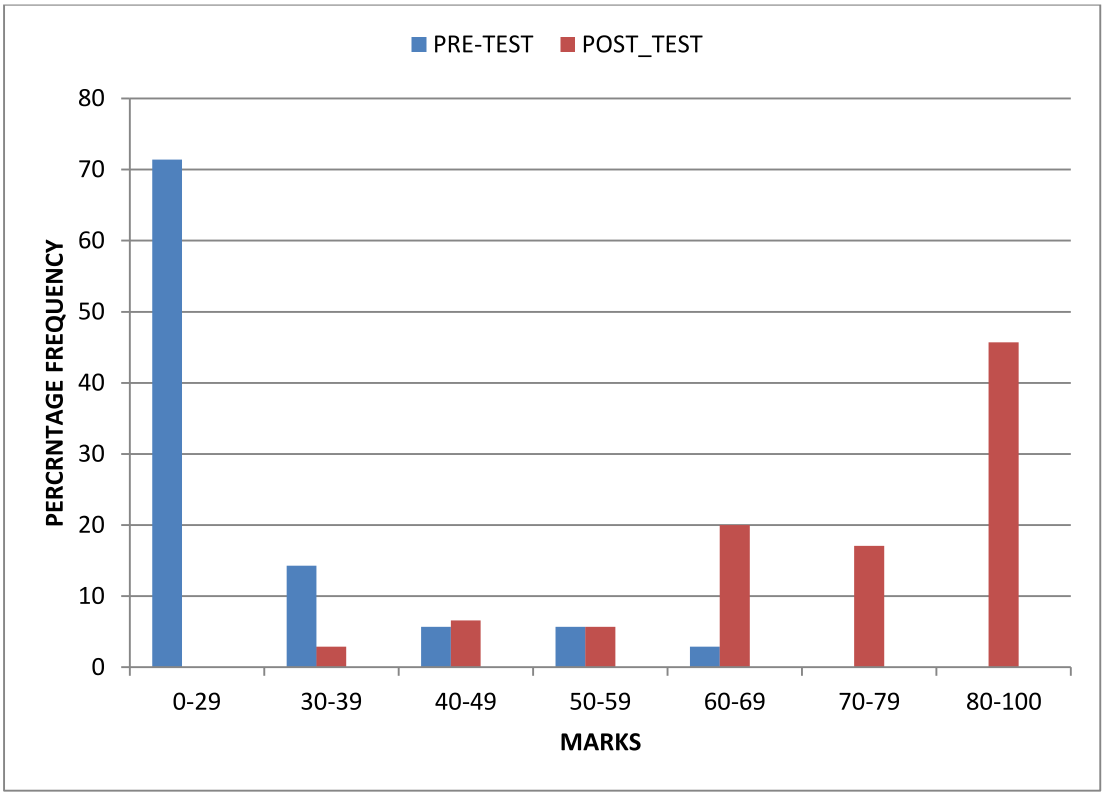


Figure 4.3. Pre and post-test results

The distribution of marks in the pre-test is positively skewed. The marks being skewed to the right was a sign that there were a few high test marks in the pre-test. Evidently, the participants did not do well in the pre-test. The performance was very poor. Almost 75% of the learners were in level 1, which is 0-29% range. Ten per cent (10%) of participants obtained marks above 40%. This poor performance can be attributed to lack of conceptual understanding as the majority of learners failed to interpret, apply, analyse, link or connect questions to what they had already learnt.

The performance in the post-test shows the participants had high marks; only a few attained below 60%. The lowest mark in the pre-test was 2% compared to 38% in the post-test. Similarly, the highest mark increased from 52% to 100%. The average mark in the pre-test was below 30% and moved up to more than 70% in the post-test. The results in the pre and post-tests demonstrated positive gains. The distribution of marks was skewed to the left opposing the one for pre-test results which was skewed to the right. This can be interpreted to mean that the concept-based instruction was effective as the learners managed to achieve better comprehension of the subject after the intervention. The responses by the participants with respect to the second test showed that the participants had gained conceptual understanding.

4.2.1.2 Learners' emotions towards concept teaching approach

To ascertain mathematical enjoyment of the function concept, participants were asked in 3.1.1 of the questionnaire (see Appendix B), whether they enjoyed the topic of algebraic functions. Of the 35 learners that responded to this question, 82.9% (N=29) agreed that they enjoyed the topic, while the remainder of the respondents 17.1% (N=6) indicated that they did not enjoy the topic of algebraic functions. For immediate clear analysis, responses of participants to this question are displayed in Figure 4.4.

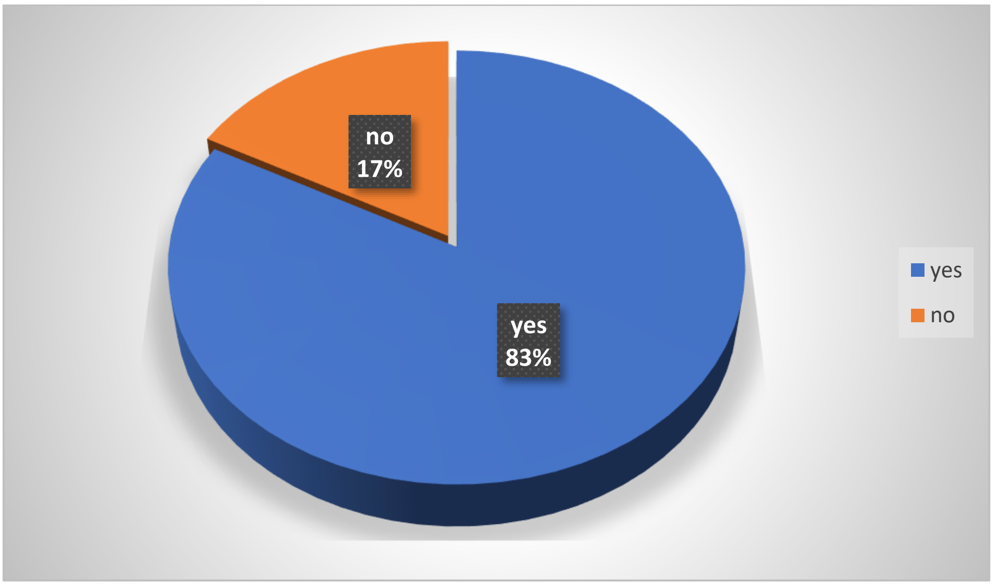


Figure 4.4. Learners enjoying algebraic fractions

The knowledge about whether the learners enjoy the topic is important for the teacher to devise intervention strategies in the teaching method. Enjoyment, according to Schukajlow (2015), is a positive activating emotion that affects learners' engagement with content. Bourne (2018) defines mathematical enjoyment as having positive attitudes towards mathematics and willingness to attempt involved activities as well as feeling confident in one's own mathematical ideas. The learners' enjoyment of algebraic functions should be reflected by their ability to solve problems in that section and also be backed by interviews for clarity. Therefore, the emotion of enjoyment should be associated with the ability to tackle algebraic functions. This result corroborates the finding by Wilkie and Sullivan (2018) who consider mathematical enjoyment as significant in addressing learners' disengagement in lessons. They found that learners who are engaged tend to enjoy the lesson and those who are disengaged lack the interest and zeal for the lesson under discussion.

The intervention that was concept-based was established to improve the learners' comprehension of the algebraic functions. During the intervention, the researcher gave the participants a chance to discover the parameters under investigation in the algebraic questions and present their findings. The learners managed to determine the equation of the graph in situations where they were given the turning point of a parabola and any other point. They could determine the equation of that graph and represent it graphically. They also managed to determine the range and axis of symmetry. The learners proffered their experiences after intervention through the questionnaire as given next:

> **L5**: *I absolutely enjoyed the lesson. It was fun and engaging. Previously, I could not see the connection between turning of a parabola and its range.*
>
> **L17**: *It was really awesome. I liked the fact that the teacher started from the basics and moved to the complex.*

The remark by L5 suggests associating fun with engaging. The researcher regarded having fun in a lesson to be a result of engagement with the lesson. This is in line with the findings by Bordbar (2019), Volet, Seghezzi and Ritchie (2019), and Ainley and Ainley (2011), that positive activity-related emotions such as enjoyment promote learners' interest, effort and engagement. L17 acknowledges that the teacher started from the basics, which infers that the teacher started with a concept that was easier to understand. The learning started from simple aspects to concrete ones, therefore from sketching and interpreting straight line graphs then to the other three functions in order of complexity (see Appendix G - Lesson Plans). The concept-based instruction that was employed during intervention can be attributed to having influenced the learners' pleasure in learning algebraic functions. The researcher believes that the learners' pleasure meant that concept-based instruction has a positive effect on the learners' understanding of algebraic functions involving turning points and lines of symmetry among other related parameters.

The participants were given half an hour to work on their own then later on another 30 minutes to present their findings, discuss answers and ask questions freely.

> **L13:** *The teacher gave us ample time to understand the basic concept which she demonstrated clearly. The building of the knowledge around the issue was made easier as we followed with ease as bigger examples were given. I can now determine range of the functions.*

Some learners also indicated how much they appreciated the ample time to understand how to construct tenets of algebraic functions. The learners utilised that time to relax and build their knowledge as well as clearing misinterpretations.

An interview by the researcher (T) with one learner (L) confirmed an enhanced level of confidence:

> **T:** *So, if you are given any other function, are you able to identify the domain and the range.*
>
> **L5:** *Yes, I am extremely happy I now know where to focus my attention on to identify them.*
>
> **T:** *Well, what are you saying?*
>
> **L5:** *I mean to say that if it is a parabola, I check its turning point because it gives me the minimum or maximum y values and if it is a hyperbola, I know I should exclude the asymptotes.*
>
> **T:** *That's great. How about if it is an exponential?*
>
> **L5:** *I use the asymptote as a boundary for the range.*
>
> **T:** *So, did you enjoy the teaching and learning approach that was used during those lessons?*
>
> **L5:** *Yes, I really enjoyed. I discovered that I can do some of the mathematics problems on my own if I am given enough time.*

Participants also enjoyed determining the link between the turning of a parabola and its range. Prior to intervention, most of the learners failed to determine the range of most of the functions. L13's assertion that bigger examples could be followed with ease relates to the introduction of basic concepts and gradual move to more complex ones. This is in line with the views of Booth, McGinn, Barbieri and Begolli (2017), Selvianiresa and Prabawanto (2017), Pardimin, Ninsiana and Dacholfany (2018) who assert that learning which is organised in a simple to complex way enhance learners' understanding power.

This enjoyment nurtures their long-term memory and improves cognitive, physical and social development. This then improves their problem solving skills as they become creative and cooperative. This gives learners higher chances of grasping concepts and building conceptual understanding.

4.2.1.3 Mastering of concepts in learning

The learners were asked to explain how they came up with their solutions to specific questions in the first test, and then in the second test. L7 responded to Q8.1 as shown in Figure 4.5:

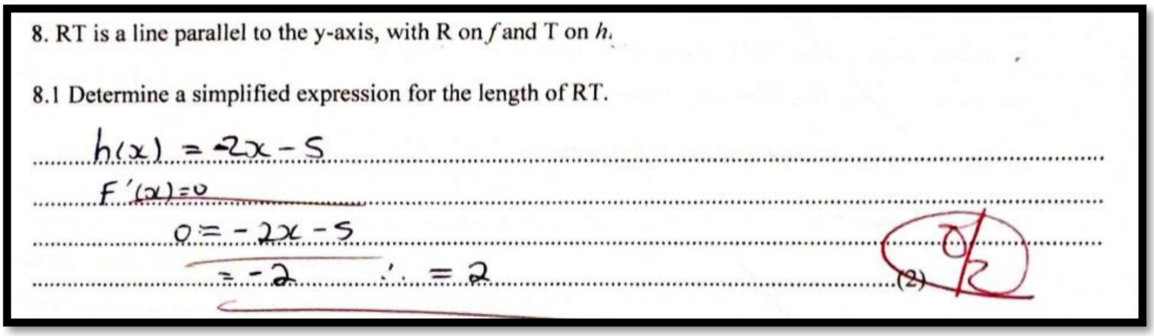


Figure 4.5. L7's performance before concept-based approach

Before being taught through concept-based approach, L7 failed to determine a simplified expression of RT as shown in Figure 4.5. The learner explains:

> **L7**: "*In the first test I did not understand what the question was talking about. I did not know what to do.*"

The teacher engaged the learners on the same topic using a concept-based approach and administered the exercise again to the class for them to apply their newly acquired knowledge. The performance of L7 is shown in Figure 4.6.

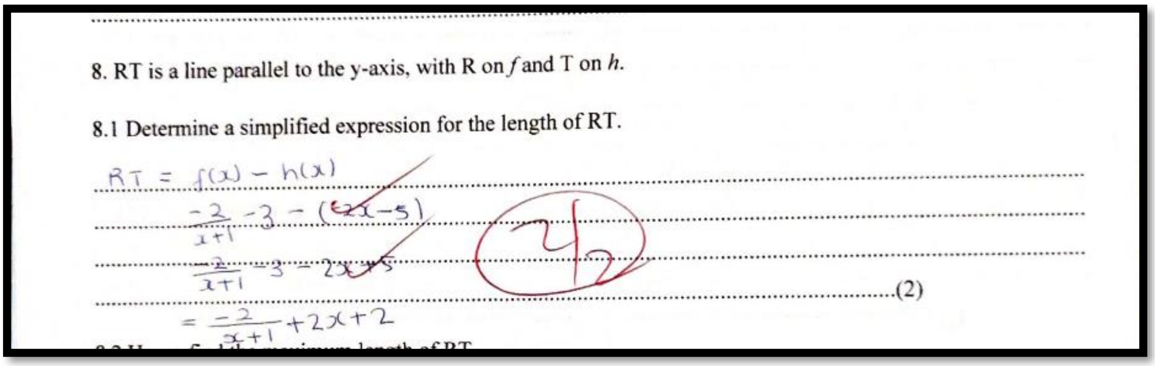


Figure 4.6. L7's performance after concept-based approach

The learner managed to get a simplified expression for the length of RT. The researcher probed for an explanation from L7.

> **T**: *The first question needed an expression for the length of RT. Now in the first test your answer was 2. In the second test you got it right. Explain how the expression is determined?*

> **L7**: *Since the line RT joins the 2 graphs and is parallel to the y-axis then RT represents the difference between the y-values of the 2 graphs which in this case is g(x)-h(x).*

All the six learners interviewed gave explanations that are similar to that given by L7. This was an indication that the learners had gained conceptual understanding. This means there is enhanced building of knowledge when there is conceptual understanding. This finding corroborates what Smith (2017) and Jones (2012) found that conceptual knowledge is rich because it interconnects by requiring the learner to be active in thinking about relationships and connections that foster adjustments to accommodate the new learning with previous mental structures.

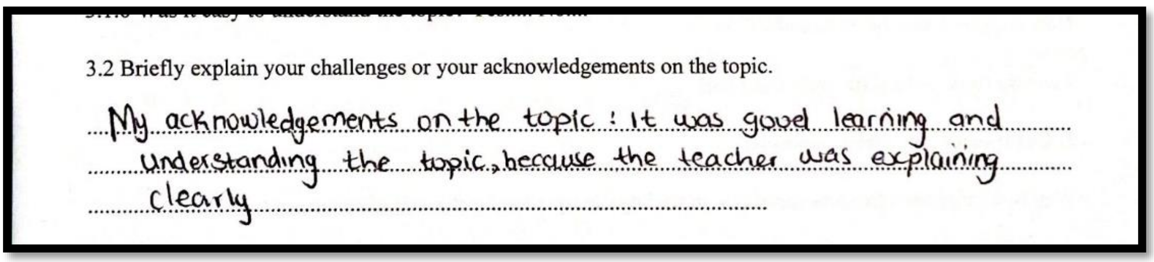
During the interview session, learners were asked to explain the way they had understood the concepts after being taught through concept-based instruction. The responses indicated that the teaching and learning approach enabled the learners to construct their knowledge, which means knowledge was not created for learners. This is confirmed by the following narration from L24:

> **T:** *In general, how were the lessons on algebraic functions?*
>
> **L24**: *They were better as we were given time to think and find friends to explain what I understand. During the explanation to friends the concepts even became clearer to me and I was able to think of other examples that clarified the concepts learned.*

The narration given by L24 demonstrates that understanding the meaning and underlying principles of mathematical concepts indicates the possession of conceptual knowledge in mathematics (Genc & Erbas, 2019; Frederick & Kirsch, 2011). The finding is in line with the fact that when learners understand the meaning and underlying principles of mathematical concepts, they can apply these concepts to other situations in the field of mathematics. This finding also de-emphasises the memorisation of learnt things in line with Broman (2021), Koul, Lerdpornkulrat and Poondej (2018), who posit that with rote learning, the application and retention of concepts are difficult. Therefore, the concept-based instruction brought positive effects to the teaching and learning of mathematics with learners constructing meaning based on their involvement during intervention.

4.2.1.4 Quality of the teacher's explanations

The questionnaire requested participants to rate the potency and quality of the teacher's explanations. Thirty (30) of the 35 participants, yielding to 85.7%, indicated that the teacher's explanations were clear. L31 wrote his experience in Figure 4.7:



3.2 Briefly explain your challenges or your acknowledgements on the topic.

My acknowledgements on the topic : It was good learning and understanding the topic, because the teacher was explaining clearly

Figure 4.7. L31's assessment of the quality of teacher's explanation

Clarity in teaching by the teacher is therefore important for the learner to understand. The teacher must be open to questions by learners who may require further clarification on the concept being taught. In this case, reinforcing by repeating the concepts is useful for the learner to grasp the concepts. The teacher also needs to probe the learners on whether they understand the concepts as he or she explains them to the class. This is important in eliminating the portion that may fail to understand through alternative examples. In this study, 29 participants with an equivalence of 82.9% of the total participants revealed that the teacher's explanations were clear. This is illustrated in Figure 4.8:

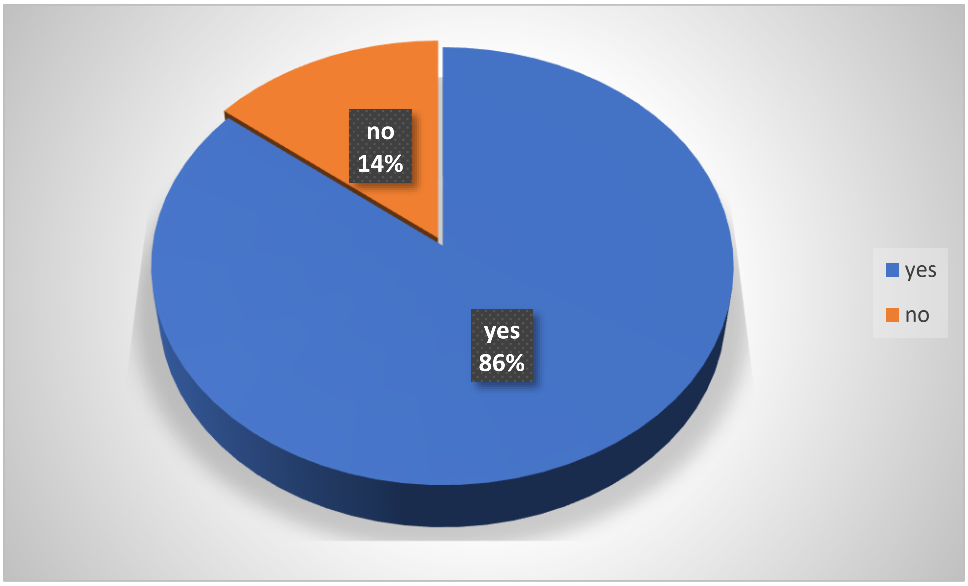


Figure 4.8. Learners who adjudged the teacher to be clear during teaching

The majority of the class who found the explanation by the teacher as clear are in line with the findings by Simamora and Saragih (2019) and Lachner and Nuckles (2016) who found that giving learners support for them to understand mathematics is essential in teaching and learning. Hussain and Cambria (2018) aver that explanations pave way for building conceptual understanding which is extremely important in the teaching and learning of mathematics. Explanations enhance the integration of existing and new knowledge.

4.2.1.5 Educator-learner interaction

One of the most vital strategies that enhance concept-based instruction in the teaching and learning of mathematics is the active involvement of learners as part of teaching and learning. In this study the researcher checked if there was interaction between the educator and the learners so that she could deduce the effects of the interaction on producing conceptual understanding. Asking questions and providing explanations during lessons improve learner participation and also promote active learning. The educator asked learners questions during the intervention and their responses enabled her to identify their misconceptions and shortcomings and to explain to clear misconceptions. One of the misunderstanding from participants was that of identifying the domain or range of the hyperbola. Most of the participants could not determine the domain and the range in the pre-test. However, there was a mark able improvement on that aspect as well as the rest of the test aspects in the post-test. The following two extracts will demonstrate the variance in the two tests' solutions. Figure 4.9 shows the performance of L4 in the test prior to intervention.

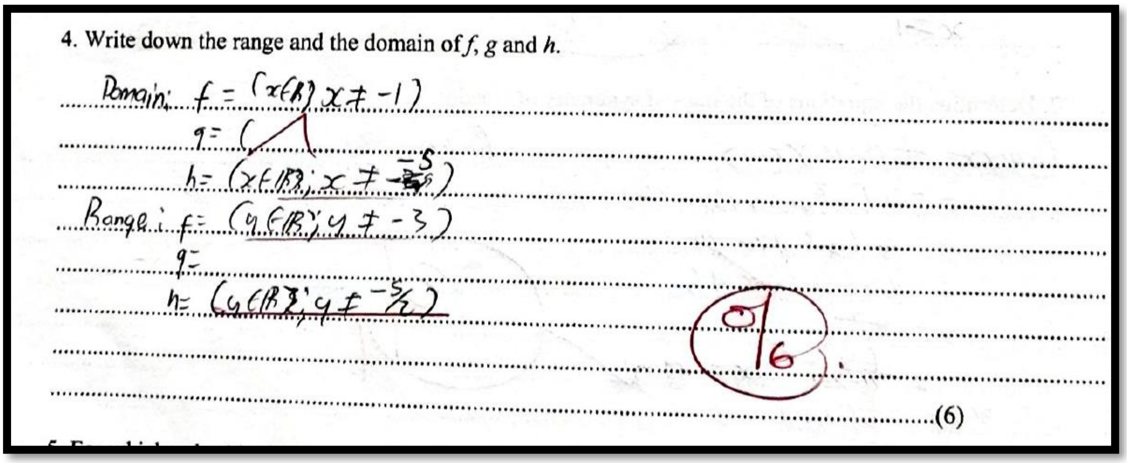


Figure 4.9. L4's performance in identifying domain and range of a hyperbola

The performance of the learner showed lack of understanding on both the domain and range in the parabola that was given.

Figure 4.10 shows L4's performance after concept-based intervention by the teacher.

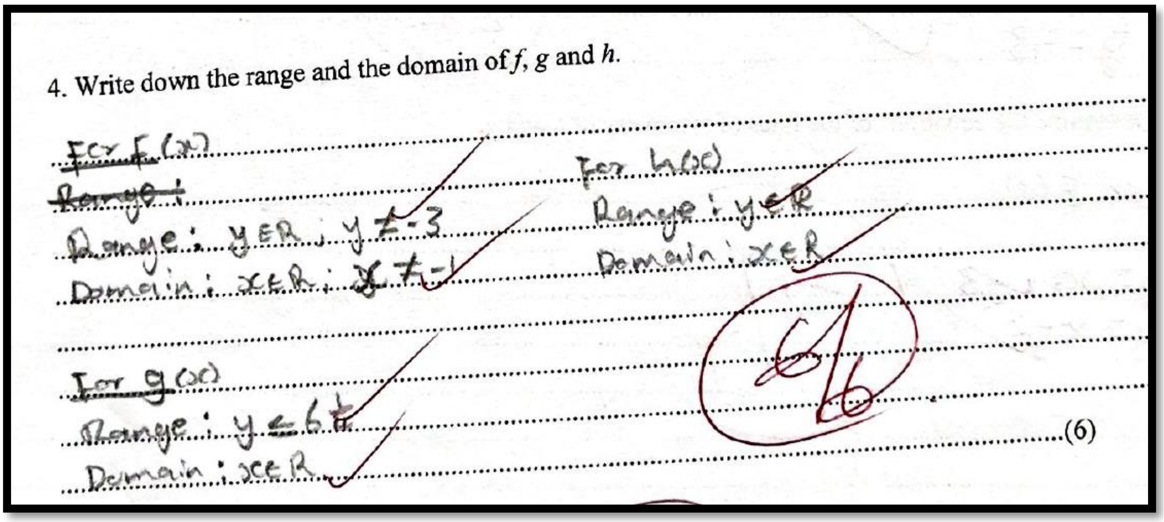


Figure 4.10. L4's performance after concept-based intervention by the teacher

The teacher made an intervention lesson that was concept-based and L4 understood and could determine the domain and range of a hyperbola as shown by full marks obtained in Figure 4.10. The performance after intervention agrees with Assefa (2016) who found that lack of conceptual understanding undermines a learner's performance in interpreting mathematical procedures.

Asking questions helps to identify the degree of understanding by learners. This also assists learners in communicating facts and ideas, making connections as they link new information to prior knowledge. It therefore aids them to gain conceptual understanding.

The interview with L13 reveals the importance of asking probing questions.

> **T:** *I want us to go through your solutions in the two papers for Question 2. In the first test you could not determine the axes of symmetry for the hyperbola but in the second test you got it right. How did the teacher help you?*
>
> **L13:** *The teacher helped me through the questions that she asked. These questions gave me a direction of understanding the steps to take. I can say I got a mind-map through the questions she asked during the procedure of solving the functions.*
>
> **T:** *What was your problem in the first test?*
>
> **L13:** *I just remembered the formula, $y = x+c$, but did not know how to get c. I had memorised the formula, and I was unable to apply it.*
>
> **T:** *So, in the second test how did you get the c?*
>
> **L13:** *First, I knew the hyperbola had 2 axes of symmetry that both pass through the point of intersection of the asymptotes of the function. This became easy as the teacher explained to us what to do, even asking myself questions like* '*What am I given in the diagram, and what is needed.*' *I provided myself with the answers made it clear to me to solve for c.*

The responses by L13 show that questions help develop conceptual understanding. The questions asked by the teacher enabled conceptual understanding as they demanded the learner to create an answer instead of simply recalling and making use of algorithms. This is illustrated in performance before and after intervention by L29 in question 11 in Figures 4.11 and 4.12. The question involved was meant to probe learners' conceptual understanding of transformations in functions.

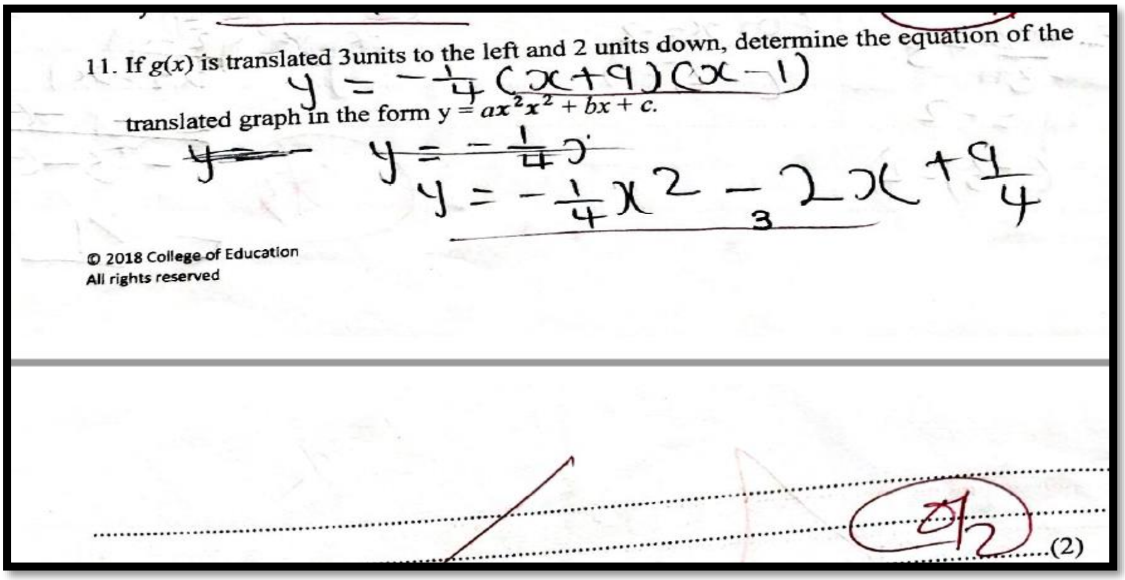


Figure 4.11. L29's performance prior to conceptual understanding

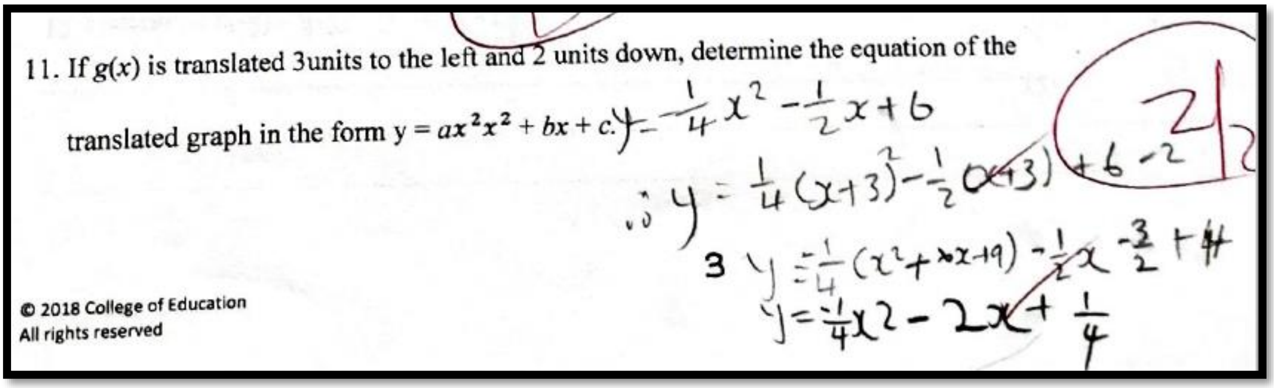


Figure 4.12. L29's performance after intervention

Of the 35 participants who were involved, 32 of the yielding 91% indicated that the teacher asked them concept building questions during the lessons. There are 9% learners who failed to recognise concept building questions during the intervention lesson. The responses to this question are illustrated in Figure 4.13.

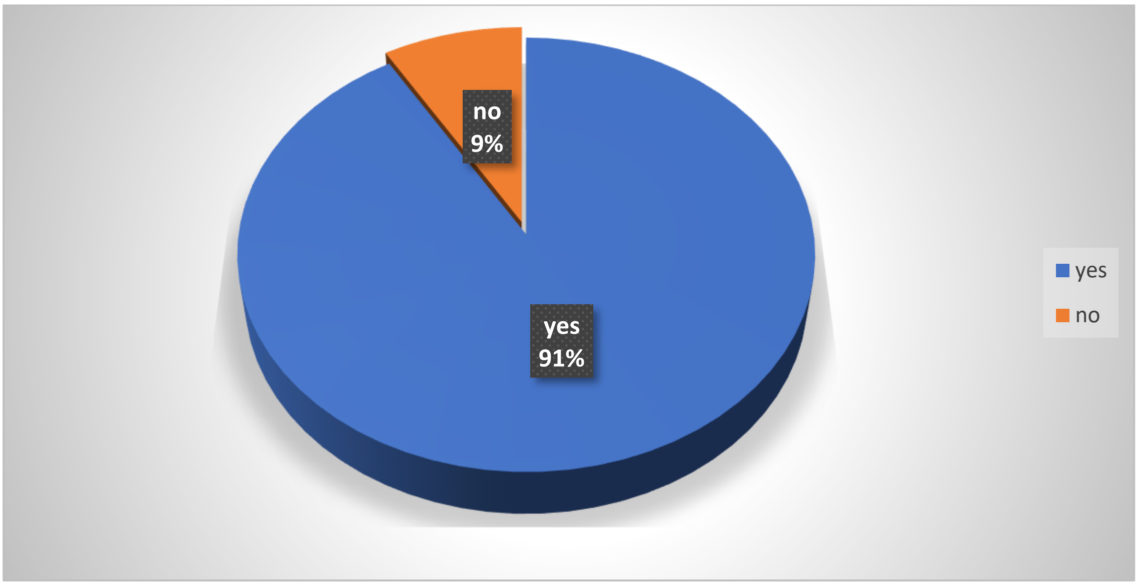


Figure 4.13. Confirmation of concept building questions during the lessons

The responses provided by the participants helped the researcher to determine the effectiveness of the concept-based teaching and learning approach. L25 narrates the confidence gained after the intervention by the teacher:

T: *After the recent lessons on algebraic functions, did you get any changes as far as building of knowledge is concerned?*

L25: *I really got some changes which I hope to pass on to my friends.*

T: *Can you briefly explain anything that you can emphasise about a turning point?*

L25: *I know that the gradient is zero as there is no increase or decrease at that point.*

Constant checking by the teacher on whether the learners understand the concept being put across is important. The following narrations confirm what transpired in the intervention session.

T: *What did you notice about the teaching style during the intervention session?*

L21: *At every step of the concept, the teacher stops and checks our level of understanding before proceeding.*

L3: *Yes, the teacher always seeks to inquire whether we would be following her explanations by asking us to give what we would have understood.*

The teacher engages learners by asking concept building questions. This also assures the learners that they are being taken care of by the teacher. The learners are bound to open up to the teacher's probing thereby opening up to better levels of understanding. This finding is in line with the assertion by Collier (2011) and Paul and Elder (2001), who found that questioning derives critical thinking which is vital to knowledge generation on any given subject.

4.2.1.6 Evidence of conceptual understanding in post-test

The points that were used in the interviews to justify answers provided in the second test reveal some understanding of algebraic functions concepts. Figure 4.14 gives post-test working of question 4 by L5.

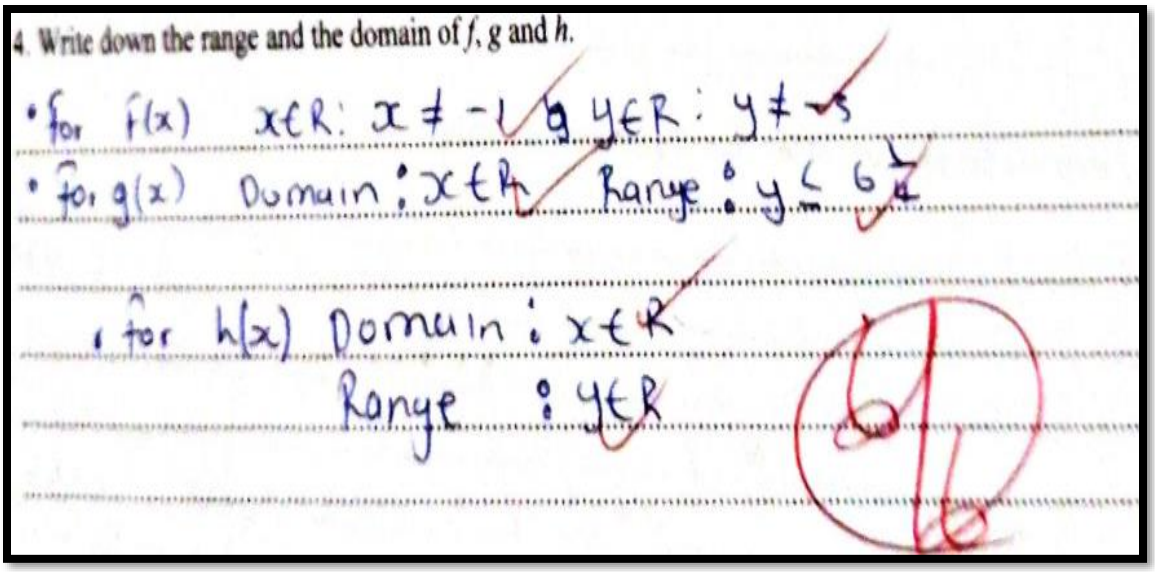


Figure 4.14. Working shows evidence of conceptual understanding by L5

The researcher interviewed L5 and she proffered the following explanation regarding one of her solutions to the test:

> **T:** *Based on your solutions for Question 4 in the two tests. At first you did not get the domains and ranges for all the functions then this time you got all of them correct. What was the problem in the first test?*
>
> **L5:** *In the first test I did not know what the question was referring to.*
>
> **T:** *You mean to say you did not know what range and domain meant?*
>
> **L5:** *No, I didn't.*
>
> **T:** *So, can you now briefly explain the two terms.*

L5: *Domain is the set of all possible values of the independent variable whilst range is for dependent variable.*

T: *So, if you are given any other function, are you able to identify the domain the range.*

L5: *Yes, I now know where to focus my attention on to identify them.*

T*: Well, what are you saying?*

L5: *I mean to say that if it is a parabola, I check its turning point because it gives me the minimum or maximum y values and if it is a hyperbola, I know I should exclude the asymptotes.*

T: *That's great. How about if it is an exponential?*

L5: *I use the asymptote as a boundary for the range.*

L5's explanation during the discussion is a logical presentation of arguments. In one response L5 said she knows where to focus her attention in order to find the domain and range of any given function. The participant was able to link and connect knowledge, enabling her to provide justification to her answers.

L24 gave the following testimony on concept-based learning:

T: *Explain the sudden change in performance in relation to getting the axis of symmetry for the parabola in the first test.*

L24: *I did not know what to write in the first test that is why I left a blank space. However, after the teacher clearly explained the axis of symmetry concept, I then understood that the line of symmetry as a line dividing the given item into similar halves so it should pass through the turning point. That line should be parallel to the y-axis so its equation is given by x = to the x-value of the turning point.*

For L24, the explanation reflects thorough understanding of the axis of symmetry. All the learners interviewed after the post-test gave plausible knowledge on the section on functions which showed they understood concepts presented during intervention regarding functions. The researcher interpreted this to mean concept-based teaching provides learners with adequate understanding on how to work their mathematical functions.

4.2.1.7 Learners' perceptions about instructional strategies

The study also assessed perceptions of learners on different teaching methods or strategies. The researcher conducted intervention lessons which gave learners the opportunity to build their own knowledge individually or with their peers. The following is an extract from lesson 8 (see Appendix G) indicating some of the activities that were conducted in the lessons:

- Learners individually determine an expression for the length between two graphs and the actual length.
- Learners in groups interpret and make deductions from given graphs.
- Learners given a chance to ask questions.
- Learners present solutions of given problems on the board giving explanations to support their solution

The intervention lessons were planned such that learners had the opportunity to build their own knowledge individually or with their peers (see Appendix G). The participants were given time to reason and present their arguments through question and answer sessions and also through presentations. The lesson plan showed how the teacher gave participants time to construct their knowledge under the following themes:

- Teacher asks learners to describe and explain an asymptote.
- Teacher gives learners a chance to identify the domain and the range.
- Teacher guides learners to determine axes of symmetry.
- Teacher gives learners a chance to ask questions on the learnt aspects.
- Teacher allocates time for illustrations of individual tasks.

The process of identifying levels of understanding of learners helps in preparing for a lesson as it enables the teacher to match with learners' paces. The activity undertaken by the teacher is in line with Kassing and Jay (2020), Matunga (2018), Mahoney (2003), who found that the teacher's duty is to provide a variety of learning activities from which learners can select what suits their individual needs. Matching tasks together with matching instructional strategies are then designed to suit individual learner needs (Cross, Joanisse & Archibald, 2019; Kroesbergen, 2002). The participating learners noticed the improvements brought about by the teaching and learning approach that was employed by the teacher during intervention. Their responses, discussed below, after the intervention indicate positive effects of the teaching and learning instruction.

After the lessons, the researcher asked how the learners perceived the concept-based approach.

T: *What perception do you have on concept-based approach over other methods you have been exposed to in your mathematics lessons?*

The learners gave the following responses:

L7: "*It was fine and great. Many different activities were involved.*"

L11: "*Teacher must continue teaching us with these methods. It was good.*"

L23: "*We were taught slowly as if we were playing. It was enjoyable. I had time to get answers on my own.*"

L33: "*The teacher taught us patiently giving us activities to work alone and discuss to come up with solutions.*"

It can be deduced from the interview responses that the participants appreciated the teaching and learning approach that was used during intervention. The results given by learners about their perceptions add to findings from other research projects in different aspects. Students may feel intimidated by information presented inappropriately and may lose interest in a particular subject. It seems necessary to pay more attention to choosing and carrying out the instruction method (Mayer, 2019). The choice of instructional strategy can influence students' attitude towards mathematics (Celik, 2018; Guido, 2018; Hosack, 2006). Therefore, learners require special instruction adapted to suit their needs as noted by Makela and Vellonen (2018); Kroesbergen (2002). To make sure that learning and understanding of mathematics occur in the classroom, it is essential to identify powerful learning styles and strategies.

4.2.1.8 Summary on research question 1

The goal of the question was to determine the effects of the concept-based instruction on the teaching and learning of mathematics with reference to the topic: algebraic functions. The rationale behind teaching is for learners to understand what they are taught. It was therefore necessary to check the degree to which participants had achieved these properties by paying attention to learners' emotions and perception towards the topic, the quality of teacher's explanations, educator-learner interaction, and appreciation of used teaching and learning approach. A comparative analysis on the performance in the pre and post-tests has been done. Enjoyment, according to Bourne (2018); Hartley (2006), is an emotion and is about how one feels, not what we think. Learners' feelings greatly affect their work at school (Lou & Restall, 2020). Unfavourable attitudes may be experienced as a result of negative feelings towards a subject. Mathematical enjoyment is considered by Cai and Leikin (2020), JenBen, Gierlinger and Eilerts (2021) to be vital in addressing learners' disengagement in that subject. Learners put effort in mathematics if enjoyable activities are involved in teaching and learning. Joy in learners was measured through self-reports that were presented in the questionnaire as responses to one of the questions. In the interviews, participants attested that they enjoyed how the lessons were carried out. Joy was also measured through behaviours that involved gratitude and appreciation. Learners enjoyed learning and engaged well in given activities during intervention. Absence of enjoyment is one of the causes of failure to achieve one's potential (Blackman, 2020; Kumar, 2020). Improvement of enjoyment during mathematics lessons is therefore a key strategy to address subject fall back (Brown, Brown & Bibby, 2008; Barnes, 2021).

Results in this research question have shown that the concept-based instruction is capable of capturing learners' enjoyment. Moreover, the teacher's explanations were provided to attest comprehension and the explanations were used to increase conceptual understanding. The interactions gave learners a sense of belonging which enhanced their engagement in class activities. This finding corroborates that of Hiebert and Grouws (2007) that teaching involves classroom interactions among educators and learners around content facilitated to achieve learning goals. The questionnaire and interview responses indicate that questions were asked by the educator during intervention. The questions were designed to enable learners to construct conceptual understanding. The intervention was learner-centred, giving participants a chance to construct their own knowledge. Deeper understanding was facilitated. Concept-based instruction had a positive effect on the participants in terms of performance and attitudes towards learning mathematics, giving credit to the development of conceptual understanding in learners. The post-test results depict favourable effects of the concept-based instruction to the teaching and learning of mathematics.

4.2.2 RESEARCH QUESTION 2: What advantages does concept-based approach have over other teaching learning approaches?

In this section, the researcher interrogated how concept-based instruction as a teaching and learning approach can be relied upon over other teaching and learning approaches. The researcher used questionnaire responses, tests results and interviews to come up with advantages of the concept-based instruction teaching and learning approach. The following categories guided discussions in an attempt to get answers to the preceding research question: freedom to express one self, learner engagement with lessons, and pace of comprehension, need to explore oneself and comparative approach using test marks.

4.2.2.1 Freedom to express oneself

In item 3.1.4 of the questionnaire, participants were asked if they were given a chance to explain and describe mathematical phenomenon. In this sub-section the intention was to find out if participants had a chance to describe and explain mathematical ideas and patterns as well as give reasons to support their inputs. According to data collected from the questionnaires, 35 participants (83%), indicated that they were given a chance to do so. The remaining 17% of the participants stated that they did not get a chance to describe and explain mathematical ideas and patterns as well as give reasons to support their inputs.

This is illustrated in Figure 4.15:

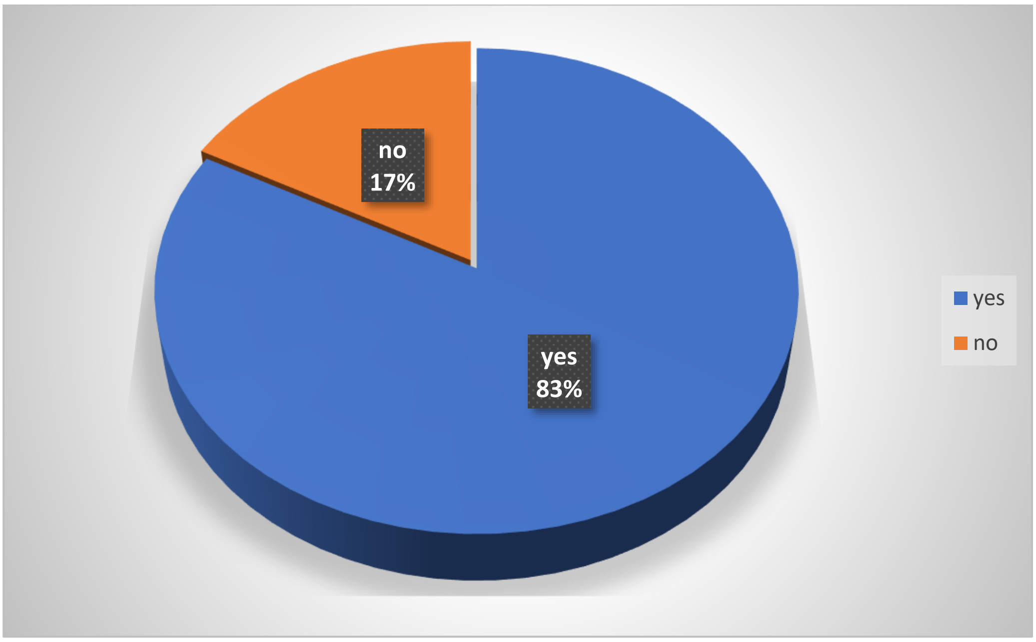


Figure 4.15. Learners' position on being given a chance to explain and describe mathematical phenomenon

The interview with L9 who acknowledged that they were given a chance to explain and describe mathematical phenomenon is as follows:

> **T:** *After learning algebraic functions for the second time, did you find any changes in your understanding of concepts?*
>
> **L9:** *Yes, I did. I found myself being able to link many things.*
>
> **T:** *What led you to be able to link many things?*
>
> **L9:** *It was because the teacher gave us chances to do things on our own. We had time to discover and see relationships. The teacher was not spoon feeding us this time.*
>
> **T:** *So what do you think must be done to make learners understand mathematical concepts?*
>
> **L9:** *Teachers must not spoon feed us and make us memorise maths stuff. Connecting mathematics to what we do and experience in our lives can make it easier for us to grasp.*

The above interview excerpt shows that participants disclosed that they were given a chance to describe and explain mathematical phenomenon. There was a de-emphasis of memorisation of learnt things. This finding is in line with that of Onsea, Soentjens, Djebara and Merabishivili (2019), that with rote learning both application and retention of concepts are difficult. The participant indicated that he felt good when discovering and coming up with his own solutions rather than always relying on the teacher. This is in line with the finding by Genc and Erbas (2019) and Frederick and Kirsch (2011) that understanding the meaning and underlying principles of mathematical concepts imply possession of conceptual knowledge in mathematics.

4.2.2.2 Learner engagement with lessons

Participants were asked if they were free to ask questions during intervention lessons. The majority of the learners, 71% revealed that they had the freedom to ask questions during lessons, which is a good sign of the prevalence of learner-centred learning. Learners were given a chance to engage with the teacher through the process of asking questions. Participants' responses to the mentioned question are illustrated in Figure 4.16.

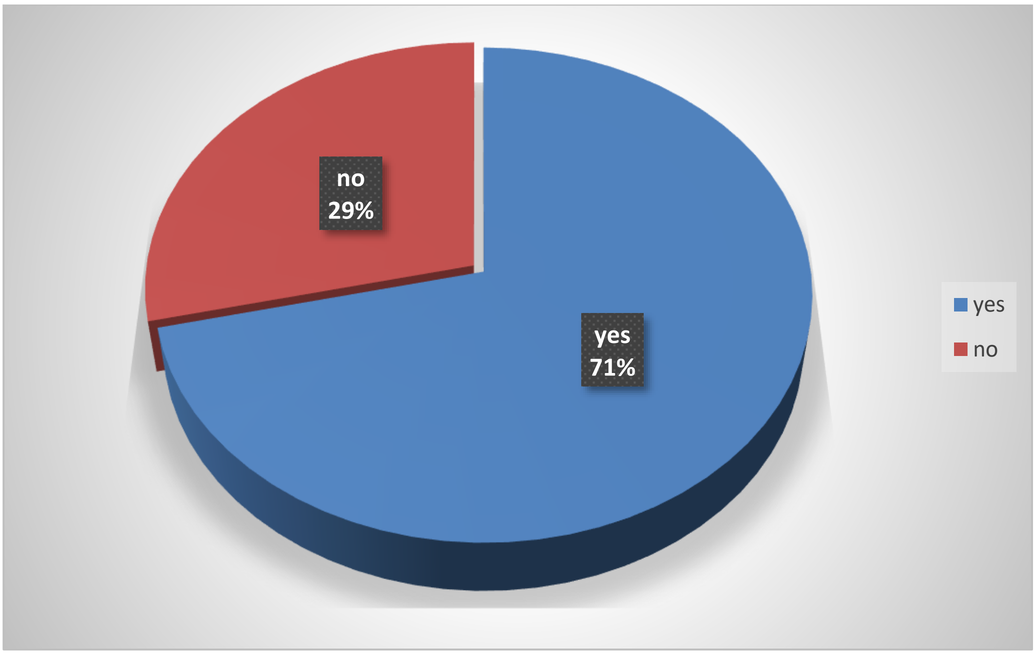


Figure 4.16. Learners free to ask questions

The displayed responses show that 71% of the participants were free to ask questions during intervention. The percentages suggest that quite a number of learners were free to ask questions in class during the intervention. Learner L34 asked the following questions:

L34: *Madam my basic problem is on understanding what you mean by function f, h or g on the same axis. Even last year I did not understand from my grade 10 mathematics lessons? Now when you introduce the word asymptote and its equation I am lost.*

L34's performance on asymptote equation is shown in Figure 4.17.

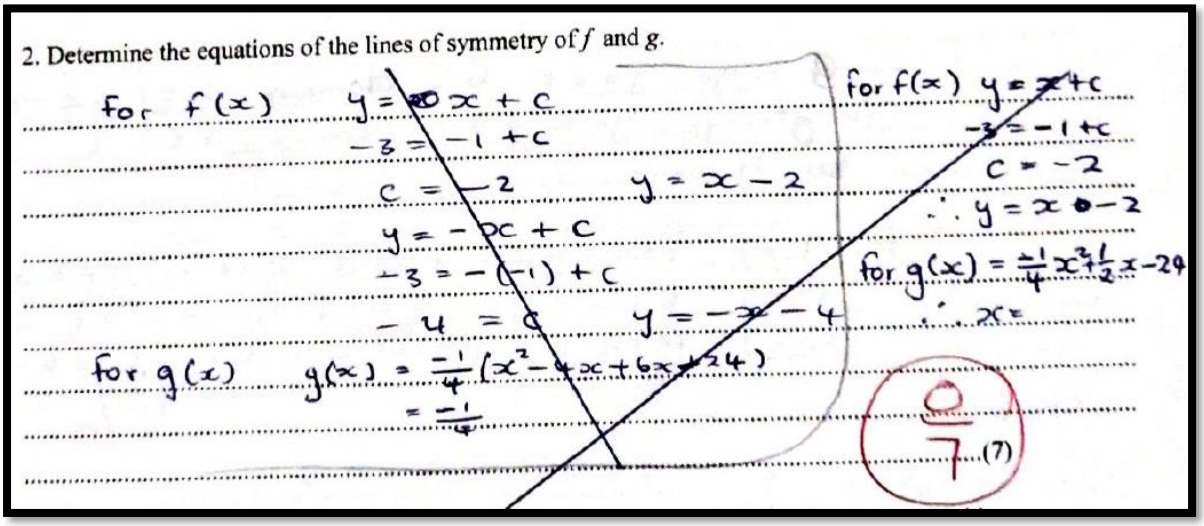


Figure 4.17 Performance of L34 showing lack of knowledge on the subject

There was an intervention input by the teacher for the whole class. Other learners indicated that they had the same misunderstanding. The teacher interacted with the class explaining the concepts in response to L34:

T: *Thank you for asking. A function is a relationship which maps each and every x value to one and only y value. It means that from the domain the set of x values each value must have only one corresponding value in the range or set of y values. Two or more x-values can be mapped onto one y-value but one x-value cannot have 2 or more corresponding y-value. This is called a many-to-one relationship. Let us look at our African culture of marriage. We get situations of one wife one husband and more than one wife to one husband but not many husbands to one wife.*

Another real life situation is when each person must visit one town at a time. One person one town, 2 or more people one town, but not one person to more than one town.

On the same axis it means you are using the same Cartesian plane to draw the function f, g and h. In other words, the y-axis is common to all the three functions. You may have come across two or three goats each tethered by a rope to the same pole and they will be grazing the grass on the ground around the pole. The goats can be sharing the same axis, the ground.

Asymptotes are lines that graphs do not touch. On your calculator if you substitute the asymptote into a given function, the calculator will report Maths ERROR, which means the answer, is undefined.

The teacher explained what a function and same axes are and employed concepts in real life to aid in the understanding. After the intervention, the performance of L34 is shown in Figure 4.18:

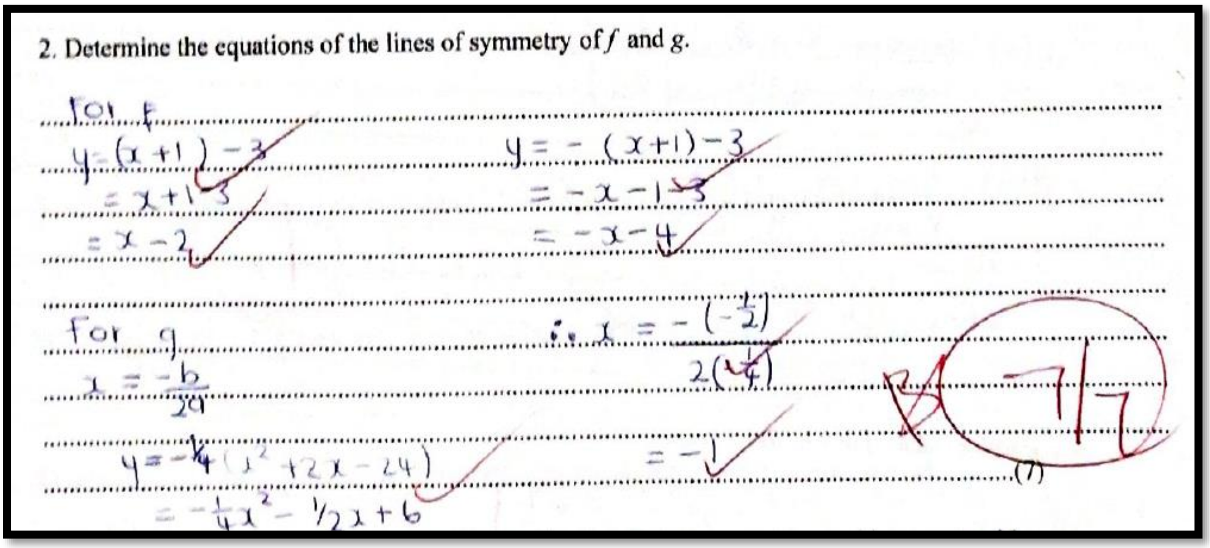


Figure 4.18. Performance of L34 after lesson on concepts displaying improvement

The majority of the learners were able to determine the equations of lines of symmetry after the process of engaging through asking questions. They were able to tackle question 2 the way L34 did in Figure 4.16. The process of asking questions can be interpreted as a sign that learners had been following up what was being presented to them. It also signified that they were trying to link what was being presented to their prior knowledge. Therefore, the researcher could conclude that the knowledge gaps were closed as learners asked questions and their learning efforts were enhanced. The questions asked by learners allowed the teacher to diagnose their understanding which then served as formative assessment. This finding corroborates that of Barnes and Todd (2021) Jabbarova (2020), Wachira (2016) who found that learning should encourage learners to discuss and communicate their ideas as well as that of Adler and Sfard (2016) that teachers have to create environments which are free of hierarchies as these encourage collaborations among learners. There was mind growth in learners when they asked questions and it gave them independent self-esteem. The approach allowed learners to be active in their learning process, revealing their knowledge gaps and above all, strengthened their understanding. This made the concept-based instruction advantageous over other teaching approaches like rote learning.

4.2.2.3 Pace of comprehension

The researcher ascertained the pace at which the learners grasped concepts to establish the efficacy of concept-based instruction in the teaching and learning of mathematics. Learners were asked whether it was easy to understand the topic algebraic functions and the results displayed in Figure 4.19 illustrate the distribution of the learners' responses regarding the aforementioned.

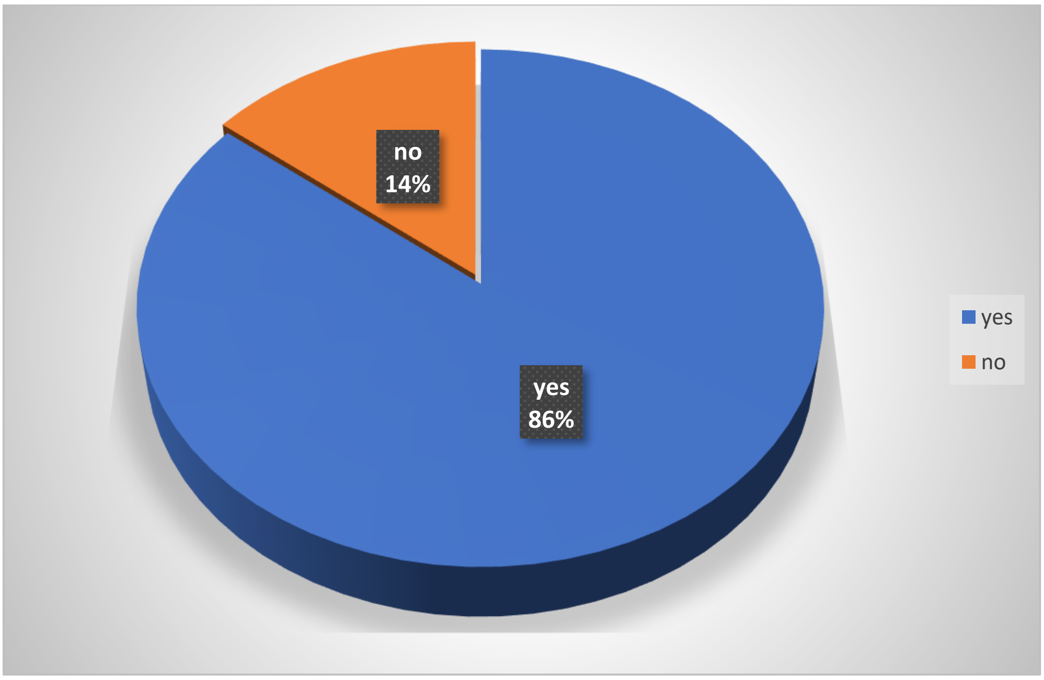


Figure 4.19. Was it easy to understand the topic?

A total of 30 participants, comprising 86%, indicated that they understood the topic while the remaining 5 indicated a lack of understanding of the topic. The number of learners who found it easy to understand in concept-based instruction is impressive. This evidence gives credit to the concept-based instruction that was used during the intervention. Understanding meant learners grasped the fundamental ideas. This is in line with the views of Janvid (2018); Goransson, Orraryd and Fiedler (2020) who see grasping of concepts as the key to understanding. Therefore, concept-based instruction was viewed as effective as it managed to bring understanding in learners.

4.2.2.4 The need to explore oneself

In concept-based teaching and learning, a learner can explore him/herself. This means a learner has the ability to evaluate his/her capability in tackling a problem. Each time a concept is understood and applied, it gives motivation for the next attainment. Academic curiosity is ignited and thrusts the learner into a mode to pursue more knowledge in mathematics. This is confirmed by L7:

T: "*What do you perceive to be different in the method that I have used to teach functions?*"

L7: "*Given time to do and think by myself has helped me to bring most of the answers on my own.*"

The disclosure by the learner about doing and thinking by himself signifies he had the opportunity to explore his capabilities. This is in line with the finding by Tsimane (2020), Lapidow and Walker (2020), Bass, Shafto and Bonawitz (2018), and Machisi (2013) that learners learn by exploring and making their own inferences, discoveries and conclusions rather than being told what will happen. That sequence of learning builds the self-confidence prerequisite to master the subject.

The performance of L7 before concept-based learning given in Figure 4.20, and Figure 4.21 shows performance on the same question after exposure to concept-based teaching.

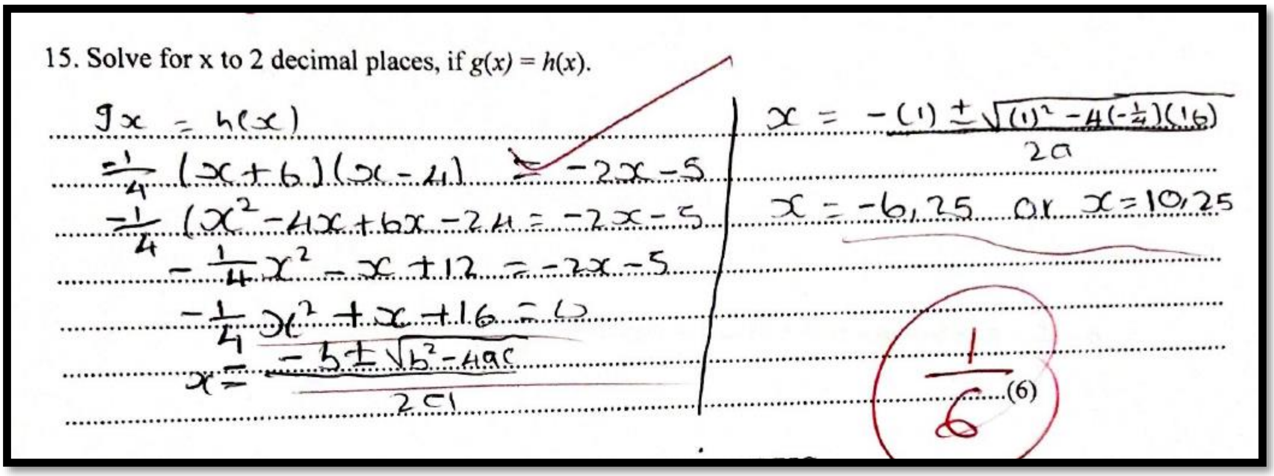


Figure 4.20. L7's performance on Q15 before concept-based lesson

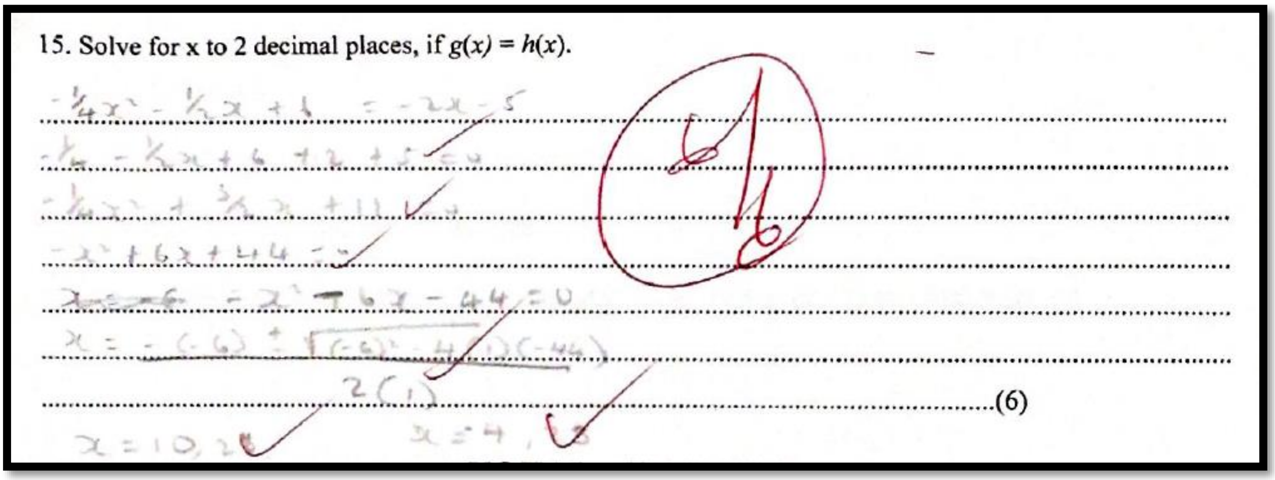


Figure 4.21. L7's performance on Q15 after concept-based lesson

The ability to excel in solving the problem in Figure 4.21 can be interpreted as an indication of the learner's desire to explore his capabilities and propel his performance to improve levels of achievement. This corroborates the findings by Archer-Kuhn, Lee, Finnessey and Liu (2020); Wisker (2018); Wagganer (2015) that learners involved in concept-based learning end up being co-constructors of knowledge through asking questions, justifying their work as they exert themselves to perform better than previously. Deducing for oneself is better than having deductions made by the teacher. Therefore, the duty of a mathematics teacher is to be a mediator, guiding learners in their discoveries.

4.2.2.5 Comparative approach using test marks

The marks obtained by participants in the two tests are presented in Table 4.1. The minimum, maximum, range, mean, standard deviation and the quartiles for the two tests were computed.

Table 4.2 Measures of dispersion and central tendency

Test	Minimum	Maximum	Range	Mean	St dev	25%	50%	75%
Pre-test	2	52	50	20.26	14.12	7.5	17	28.5
Post-test	38	100	62	75.23	16.73	66	75	88

The mean mark in the post-test (75.23%) is much higher than that of the pre-test (20.26%). The range of marks in both tests is almost 50%. The bottom 25% of the participants obtained below 8% marks while in the post-test the upper boundary for the bottom quarter of participants was 66% of marks. Half of the participants scored below 18% in the pre-test and in the post-test half of the learners scored a minimum mark of 75%. Three quarters of the participants scored below 29% in the pre-test whereas in the post-test 75% of the participants scored above 66%. The top quarter in the post test scored above 85%. There is a great variation between the minimum and maximum marks for the 2 tests. Minimum 2 in pre-test compared to 38 in the post-test, then maximum of 52 in pre-test versus 100 in post-test (see Appendix N). The standard deviations and the ranges are high in both tests indicating that participants' performance varied much in both tests. The results show a marked improvement in post-test performance which might be interpreted as an improvement in conceptual understanding.

4.2.2.6 Summary on research question 2

The results show that learners taught using the concept-based teaching and learning approach perceived mathematics as easier to understand than before. The learners took a more active part in their learning. This supports the finding by Davadas and Lay (2017) that learners taught using concept-based approach view mathematics as an active and inquiry-based discipline. The improved marks in post-test support that conceptual understanding leads to creation of meaning and construction of systems of meaning imply that what was learnt was stored in the memory system (Chipambo, 2018). Learners were able to build their own knowledge and skills allowing an effective understanding of concepts, which in turn improved their retention of information.

Results from the post-test compared to the pre-test attest marked improvement in scores. The interview responses reveal that participants' levels of thinking were transformed. According to Summermann, Sommerhoff and Rott (2021), Lischka, Lai, Strayer and Anhalt (2020), Loong and Herbert (2018), this transformation occurs when learners combine facts and ideas, and synthesise, generalise, explain, hypothesise or arrive at a conclusion, generalisation or interpretation. The concept-based instruction teaching and learning approach allowed participants to reason and present their arguments logically as evidenced by the way they responded. In the questionnaires, participants indicated that they were given a chance to describe and explain mathematical phenomenon which helped them to gain a better understanding of concepts. They were given a chance to express themselves in class, which helped them to be open-minded, flexible, curious and motivated. This gave them confidence and power to argue and present their facts systematically. It was advantageous to employ the concept-based instruction approach in the intervention as revealed by the aforementioned benefits. The researcher therefore concluded that the concept-based instruction is a better teaching and learning approach than traditional approaches.

4.2.3 RESEARCH QUESTION 3: What changes are brought about in learners by the use of concept-based instruction teaching and learning approach?

This research question was utilised in this study to help the researcher identify changes that the concept-based instruction could bring among learners. The following categories guided discussions in an attempt to get answers to the above stated research question: comparative analysis and perceptions on the following: learning mathematics by memorisation, prioritising to get correct answers, understanding mathematics concepts, and the role of a teacher as a facilitator.

A comparative analysis was conducted using the PSPP software. The analysis was conducted to detect changes using the Average Based Change method (ABC). The ABC method made use of the statistics that described the centre of the distribution (Estrada, Ferrer & Pardo, 2019; Grisson & Kim, 2012). The dependent t-test was chosen to check if the difference between the achievements in the two tests written before and after intervention was statistically significant. Before the application of the t-test it was necessary to check if the results from the test would be valid. The researcher had to test if the marks from the two tests were normally distributed. The distributions of the marks in the two tests were tested for normality using Kolmogorov-Sminorv test. The normality test was conducted under the following hypotheses:

***Null hypotheses,* H_0**: Data come from normal distribution.

***Alternative hypothesis,* H_1**: Data are not from a normal distribution.

The following results were obtained after running the normality test.

Table 4.3 One-Sample Kolmogorov-Smirnov Test for normality

	Post-test		*Pre-test*
N	35		35
Normal Parameters Mean	75.23		20.26
	16.97		14.33
Most Extreme Differences Absolute	.13		.14
	.07		.14

	-.13		-.10
Kolmogorov-Smirnov Z	.75		.80
Asymp. Sig. (2-tailed)	.620		.539

For both tests, the p-values were found to be greater than 0, 05 (p=0, 62>0,05 and p=0,539>0,05). The goodness of fit test was in favour of the alternative hypothesis. Therefore, the assumption was that the marks from both tests followed a normal distribution. Since the goodness of fit test brought favourable results, the researcher applied the paired samples t-test to check if the change in performance in the two tests was statistically significant.

4.2.3.1 Dependent t-test

The dependent or matched pair t-test compared the mean scores for the tests before and after intervention. The researcher tested the null hypothesis of no significant difference in the level of understanding before and after intervention against the alternative hypothesis of existence of a significant difference. The test was done at 95% confidence interval. The hypotheses that guided the test can be summarised as:

Null hypothesis/H_0**:** $u_1 = u_2$

Alternative hypothesis/H_1 **:** $u_1 \neq u_2$; where u_1 and u_2 are the mean scores for the pre-test and the post-test respectively.

Table 4.3 presents the results that were obtained from matched-pairs t-test distribution that was carried out following two tailed conditions**.**

Table 4.4 Paired Samples Test

Pair	Paired Differences					*t*	*df*	*Sig. (2 tailed)*
	Mean	*Std. Deviation*	*Std. Error Mean*	95% Confidence Interval of the Difference				
				Lower	*Upper*			
Post-test Pre-test	**54.97**	**9.81**	**1.66**	**51.60**	**58.34**	**33.16**	**34**	**.000**

The results from the t-test that was carried out had a p-value that was extremely small ($p<0.05$) so the null hypothesis was rejected; hence, the alternative hypothesis was accepted. Therefore, there was statistically enough evidence to confirm that the intervention brought changes in participants' achievement. There was a marked change in participants' achievement as was portrayed by the t-test results. Improvement in performance was attributed to the understanding of concepts. This finding supports that of Erickson (2012) who concluded that concept-based instruction allows learners to structure their learning and deepen their understanding, leading to improved performance. The finding also corroborates that of Chappel and Kilpatrick (2003) who investigated the effects of instructional environment in college level calculus students and their instructors to view effects of concept-based versus procedural-based instructional approaches. It was discovered that students who were exposed to the conceptual approach had significantly higher scores in both conceptual and procedural tasks than those who were in the traditional methods group.

4.2.3.2 Perception on learning mathematics by memorising

The study identified changes that had been brought about by the concept-based teaching and learning approach by the questionnaire where participants were asked (see Appendix B) to rate if they could learn mathematics better by memorising. Figure 4.22 illustrates the distribution of the responses.

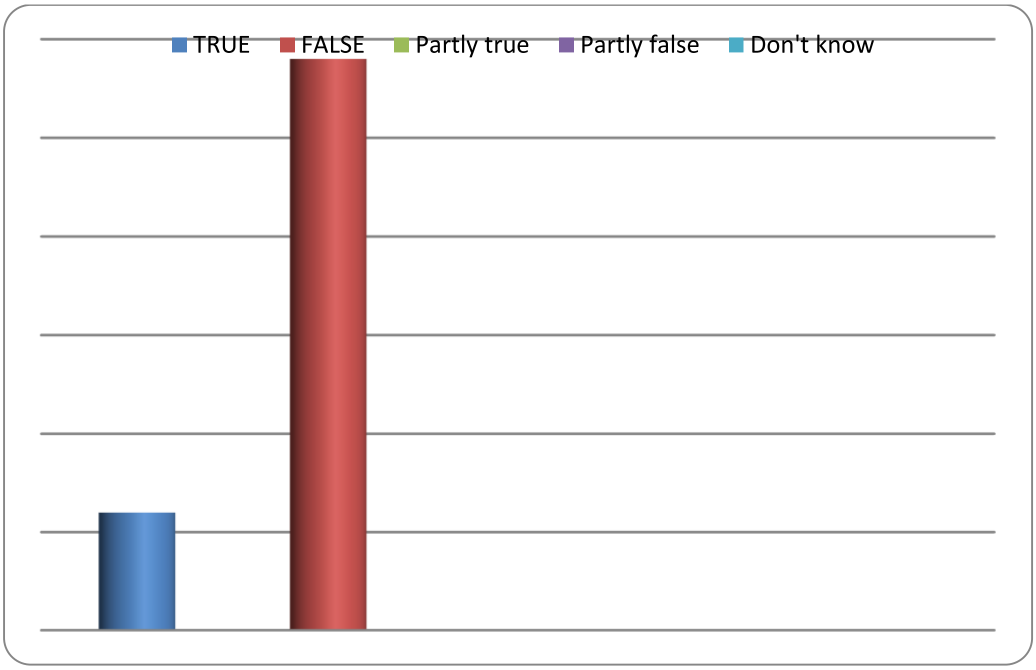


Figure 4.22. One can learn mathematics better by memorising facts and procedures

Figure 4.22 shows 82.9% interviewees responded that it was false and 17.1% indicated that it was true. This finding contradicts that of Mochesela (2007) who found that there is a belief that mathematics is best learnt through memorisation of facts and procedures. The result in Figure 4.20 is based on responses after the intervention lesson which used concept-based teaching and learning approach. The success of the concept-based intervention is likely to have influenced the outcome shown in Figure 4.12 since the learners scored better marks in the post-test. The interviewed learners contrasted their views on learning the traditional way and the concept-based way and some of the responses are given next:

> **T:** *You are aware of the traditional method of the teacher discussing examples on the board and emphasising on the procedure and memorising it for one to arrive at the answer. How do you compare it with the concept based approach?*
>
> **L13**: *Knowing what to do in mathematics is better than memorising facts.*
>
> **L18**: *In maths, cramming is bad because one must know and be able to link facts to what you already know.*
>
> **L25**: *Not really for the idea of cramming because it does not help me plus it's easy to confuse crammed maths formulas and procedures.*

The learners' sentiments show that they have recognised the importance of concept-based approach as opposed to rote learning. The finding is in line with that of Ahmad, Ahmad and Bakar (2018); Ahlstrom, Nilsen, Benzein Behm and Wallerstedt (2018); Zulnaidi and Zakaria (2012) who investigated the effectiveness of teaching methods and found that methods that emphasise conceptual understanding are important to learners as they provide the power to link new and old information. The learners' sentiments are also supportive of the finding by Owusu (2015), that concept-based instruction is opposed to the traditional approach of memorisation of information in which the teacher takes charge of the intellectual work in the classroom. When L18 remarks that he must 'know and be able' to connect new facts to old knowledge, it means the learner is aware of the fact that the knowledge must reside within his faculties. He must be able to recall and manipulate or reconfigure the knowledge to solve problems.

The following conversation from the interview also stresses the importance of gaining conceptual understanding. It discredits cramming as follows:

> **T:** *Then for 8.2 there was need to determine the maximum length of RT. I can see you added the two functions in the first test but in the second test you again got it right. Why did you add the two functions in the first test?*
>
> **L18:** *I added because I just remembered that when I see questions like there is somewhere where you use the two functions and I couldn't remember well whether to add or subtract.*
>
> **T:** *Were you having a reason for that?*
>
> **L18:** *No. It's something which I had crammed.* **T:** *In the second you used a method which*
>
> *Is used when calculating the y value for a turning point. Is that so?*
>
> **L18:** *Yes Mam.*
>
> **T:** *Why?*
>
> **L18:** *Because my expression was for a quadratic function with a<0 so would give a maximum point. So that is why I applied that method.*

The responses by the participant with respect to test solutions show that the participant had gained conceptual understanding. The learner's justification of answers provided in the second test was characterised by a logical presentation of arguments during the discussion. The participant was able to link and connect knowledge, which enabled him to provide justification to the answers.

4.2.3.3 Perception on prioritising to get correct answer

The other notion that the study assessed is that the important aspect in mathematics is to get correct answers, therefore completely disregarding the potency of concept-based instruction (see Appendix B). The responses are displayed in Figure 4.23:

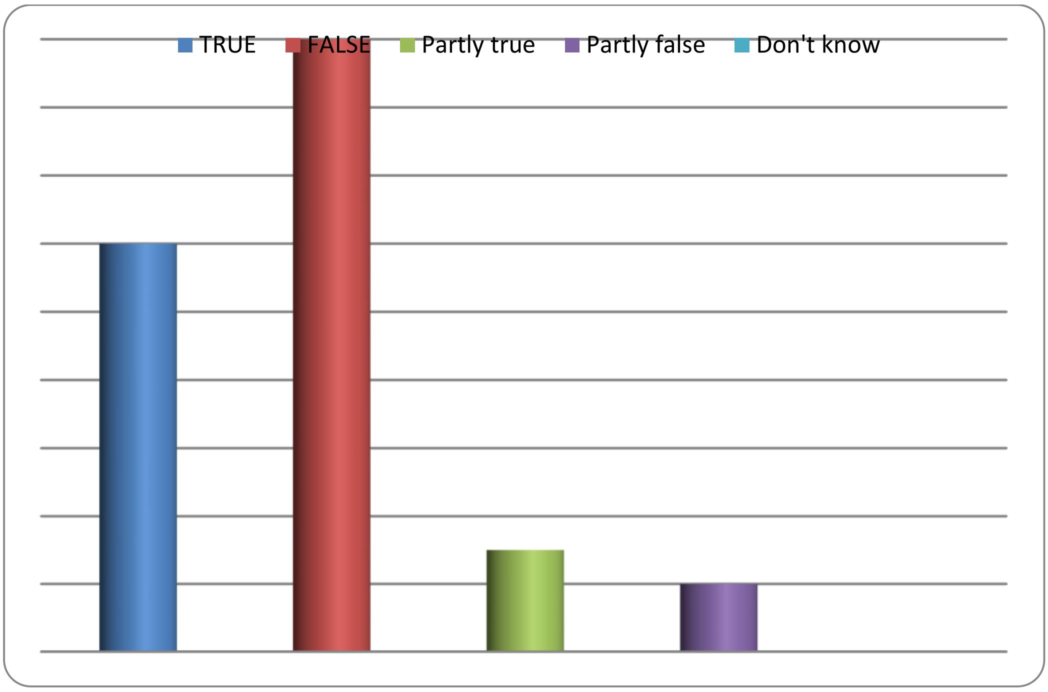


Figure 4.23. The important aspect in mathematics is to get correct answers

The results displayed in Figure 4.23 show that the majority of learners, 18, equivalent to 51.4% reported that it was false and two learners equivalent to 5.7% indicated that it was partly false. This gives a combined majority of 57.1% who say it is a false notion that the most important aspect in mathematics is to get correct answers. These participants seemed to have realised that possessing conceptual knowledge increases one's chances of connecting, linking and relating ideas as well as getting correct answers. The concept-based intervention placed more emphasis on discussing and focusing more on solution attempts and less on answers (Moschkovich & Zahner, 2018; Planas, 2018; Garofalo & Lester, 1985). The remaining 34.3% and 8.6% indicated that it was true and partly true, respectively, making a combined total of 42.9%. This is quite substantial and may be interpreted to mean that it may take a while for learners to appreciate the importance of attaining conceptual understanding. Some of the participants who were interviewed responded as follows:

> **T:** *What is your perception on the belief that the important aspect in mathematics is to get correct answers?*
>
> **L18:** *Understanding is good. Correct answers come from understanding.*
>
> **L25:** *Getting answers is no longer important to me. I want to understand more things.*

The learners' responses show the basis of concept-based learning. The learners would rather know the terrain they are navigating so they can get to the ultimate goal, which is the answer. The acquisition of knowledge is greater than the answer. This finding supports that of Wade and Kidd (2019); Leighton (2019); Kay and Kummerfeld (2019); Kilpatrick et al. (2001) who stressed the importance of learners knowing why their answers are right; they need conceptual understanding.

4.2.3.4 Perception on understanding mathematics concepts

The researcher interrogated participants' beliefs on conceptual understanding. She asked if mathematical concepts are difficult to understand (see Appendix B). The responses are displayed in Figure 4.22. The majority of participants, 57.1%, indicated that it was false that mathematical concepts are difficult to understand. A further 11.4% opted to indicate that the notion was partly false. The responses were given after conducting the concept-based intervention. The majority are professing that it is not difficult to understand concepts in mathematics.

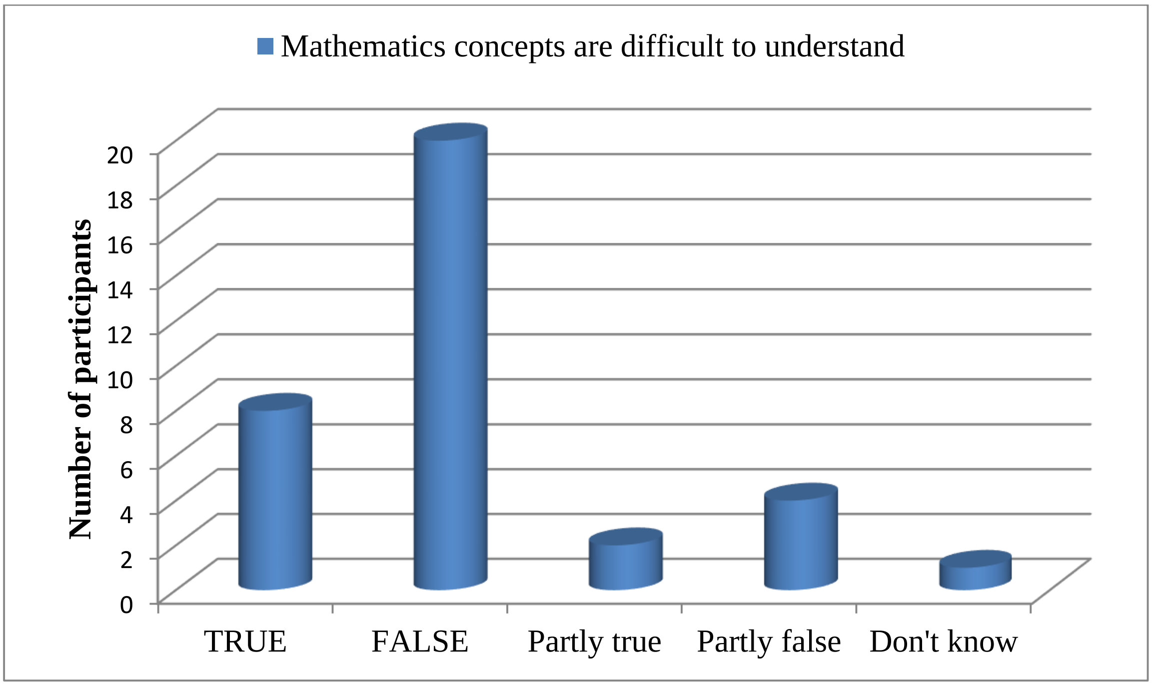


Figure 4.24. Mathematics concepts are difficult to understand

The other participating learners, at 22.9% and 5.7%, indicated it was true and partly true, respectively, that mathematical concepts are difficult to understand. This can be interpreted to mean that it takes time for all learners to shift their mind set from rote learning to concept based learning. Mathematics is considered to be a difficult subject by most of the learners (Gafoor & Kurukkan, 2015). During interviews, the learners expressed their perceptions on understanding mathematics concepts:

T: *Are mathematical concepts difficult to understand judging from your performance in algebraic functions?*

L33: *It's not difficult madam. The second phase of your teaching using concepts has made it easier to understand from situations in real life which we experience. I now give myself time to work on problems on my own and also ask for assistance from the teacher or my friends.*

L13: *Maths is not difficult. When you approach it through concepts it becomes clearer to understand. The subject is linked to most of the things that we have already done in other grades; so, when answering questions, we apply some of those things.*

These two learners posit that the understanding of mathematics has been made less burdensome as the teacher approached it by way of concepts. This finding agrees with Driessen, Knight and Smith (2020); Hoellwarth, Moelter and Knight (2005) who assert that active-learning instructional strategies promote conceptual understanding because learners make sense of what happens during the process of teaching and learning. However, there are some learners who will continue to face challenges with mathematics as typified by the following response:

L7: *Eish, this subject is difficult for me. It has a lot of stuff and I forget it.*

L7 declares that mathematics is difficult for him as it contains too much information which he forgets. This means the learner is failing to connect and link new knowledge to what he already knows. At grade 11 this may be understandable if in the previous years and grades he was exposed to formulas and memorising style of teaching and learning devoid of concept-based approach. This has been observed by Vogler and Burton (2010) who found that teachers mostly use weak rapid instructional teaching and learning approaches to pass assessments instead of preparing connections for learners to fit into the real world. They posit that the curriculum in most cases is also designed in a manner that discourages teachers to engage learners in active construction of knowledge but emphasises on rote learning for higher pass rates. This study is taking the position that concept-based instruction which is introduced as soon as children enter school when they learn counting with examples of objects like fruits, huts or animals must be maintained throughout their school years. This would make mathematics, hopefully, enjoyable at the advanced school year like grade 11 which participated in this research.

4.2.3.5 Perception on the role of the teacher as a facilitator

The study examined the role of the teacher as a facilitator in enabling the teaching and learning of mathematics through concept-based instruction. The researcher posed a question on how the learners expect the teacher to play his or her role when teaching them mathematics.

T: *How do you like to see the teacher play his or her role in teaching you mathematics?*

L13: *I like attempting and doing it first on my own then ask for the teacher's help when I get stuck. Because I am happy when I do it for myself as it helps me to understand better and also check if I understand.*

L25: *I do not like it when the teacher gives us solutions. I feel like working out problems first on my own then will consult teacher when I get stuck.*

L29: *Yes, I realised that there is more understanding when one is given a chance to discover and identify relationships on his/her own.*

L33: *I like being given time to discover and make connections on my own first then when I get challenges, I get the teacher's help.*

L18: *We were given a variety of class activities... this time we were given time to derive our own solutions. We discussed among ourselves and managed to arrive at solutions.*

The responses by the learners indicate their preference to attempt questions first before seeking the teacher's assistance. L13 highlights that she is happy if she gets an opportunity to work on mathematics problems on her own as she is satisfied with acquiring skills. The same sentiments were given by the other learners, which show that learners recognise that knowledge must accumulate in their faculties. The role of the teacher is therefore to facilitate. This finding corroborates the one by Amponsah, Kwesi and Ernest (2019) and Landsberg (2005) who found that a real teacher is a facilitator who creates a classroom environment that is conducive for learners to be able to make sense of the knowledge, skills and values being taught. It is also in line with Ntsohi (2005) who opines that learners gain courage, confidence and their motivation is sustained when they are given a chance to explore mathematical phenomenon on their own. Ntsohi suggests that learners must be given the opportunity to use their own methods to get solutions, which reinforces the concepts learned.

4.2.3.6 Summary on research question 3

This section is a summary of results on research question 3, which is about the changes that were brought about by the concept-based instruction in learners. The results from the dependent t-test suggest an improvement in conceptual understanding of learners. The intervention was conducted to help learners to gain conceptual understanding. The performance of learners in the post-test showed much improvement which the researcher attributed to the concept-based instruction. The learners did not enjoy getting answers from the teacher as the majority of them disclosed that they wanted to solve problems on their own. Participants seemed to have realised the importance of learning to grasp concepts instead of just getting correct answers. They wanted the teacher to assist them in building their own knowledge. According to Jonsson, Granberg and Lithner (2020) and Gabler and Ufer (2021), if teaching and/or learning strategies are solely based on rote learning, students will be prevented from developing their ability to struggle with important mathematics. The responses provided by interviewees also indicated changes in perception of the subject and teaching and learning approaches

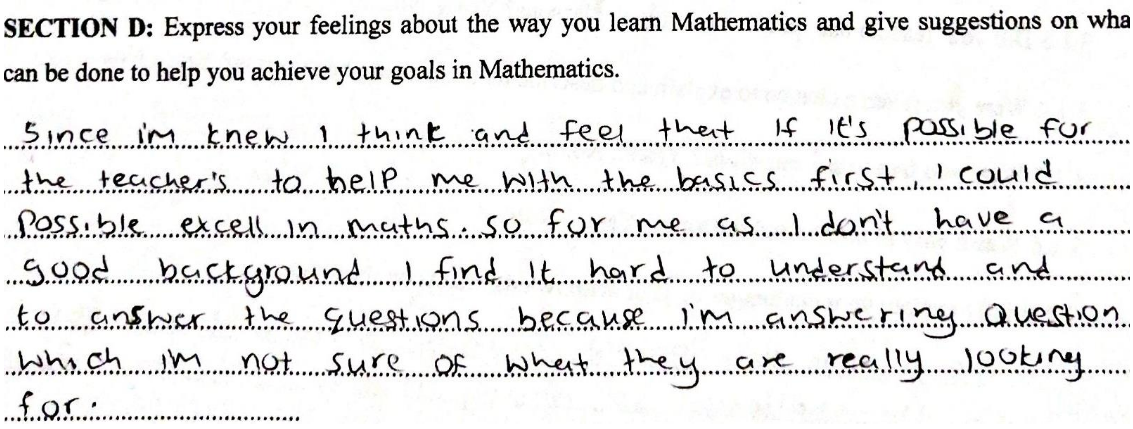
4.2.4 RESEARCH QUESTION 4: What challenges can be addressed by the implementation of concept-based teaching and learning in mathematics?

Learning challenges impact performance of learners as well as their self-esteem (Filippello, Buzzai & Messina, 2020; Metsapelto, Zimmermann & Pakarinen, 2020). Teaching and learning mathematics has faced many challenges worldwide either in delivery methods and students' participation (Li & Schoenfeld, 2019; Mazana, Montero & Olifage, 2019). The researcher, therefore, decided to determine the challenges that learners face in mathematics in order to provide recommendations of eliminating the symptoms.

The study covered the mathematics topic: Algebraic functions. The learners were asked to explain the challenges they experienced in the mentioned topic as well as in the subject mathematics in general. Findings from participants' emotions, attitudes and feelings that were aired through the questionnaire and interviews were used to evaluate the challenges faced by learners when learning mathematics. From the challenges that were deduced, the following categories are discussed: poor mathematics background; too many rules and long steps involved; boredom and attitude; educator's pace versus learners' pace; and lack of variety of activities to reduce boredom.

4.2.4.1 Poor mathematics background

The main characteristic of conceptual understanding, according to Kleine, Staarman and Ametller (2019), is the rich network of connecting pieces of information. The development of conceptual understanding is achieved by constructing relationships between pieces of information that are stored in memory or between an existing piece of knowledge and newly learnt information (Gutknecht & Wibral, 2021; Naidoo, 2011). In simple terms, for the development of conceptual understanding to occur, there is need to establish connections between new knowledge and existing knowledge. Ornstein, Shapiro, Clubb, Follmer and Baker-Ward (2018) stressed that relationships between knowledge cannot be built if prior knowledge does not exist. Responding to section D of the questionnaire (see appendix B), one of the participants had this to say:


SECTION D: Express your feelings about the way you learn Mathematics and give suggestions on wha can be done to help you achieve your goals in Mathematics.

Since i'm knew I think and feel that if it's possible for the teacher's to help me with the basics first, I could possible excell in maths. so for me as I don't have a good background I find it hard to understand and to answer the questions because i'm answering question which im not sure of what they are really looking for.

Figure 4.25. Attitudes and feelings about the learning of mathematics

The participant was worried about his lack of prior knowledge. Learning and learner achievement are greatly influenced by prior knowledge (Simonsmeier, Flaig, Deiglmayr, Schalk & Schneider, 2018; Yang, Chen & Chen, 2018; Achmetli, Schukajlow & Rakoczy, 2019). In their study, Hailikari, Kajuvuori and Lindblom-Ylanne (2008) discovered that prior knowledge contributed much in the learning of mathematics. This depicts that even when functions were taught, there was need for learners to have a background of algebra and graphs for them to link to the new knowledge. Participants in the current study expressed their worries concerning lack of preliminary background. Here are some of the participants' interview responses as part of their challenges:

> **L10:** *It is difficult for me to master concepts of mathematics in grade 11 because I lack sufficient background of mathematics which are taught in grade 8 and 10*.
>
> **L27:** *The sad part and the embarrassing facts are that I even fail to do the basic things in mathematics that some grade 8 and 9 learners can do*.

The concerns from participants indicate that learners themselves can see that lacking mathematical background affects them in grasping new concepts. The most worrying factor is that most of these learners are demotivated by their poor background because they will not understand the basic concepts in the lesson while those with the background will be able to quickly grasp and understand. It makes them feel bad and, in most cases, hesitant to participate during discussions. Genc and Erbas (2019); Kenedi, Helsa, Ariani, Zainil, and Hendri (2019), Masingila, Olanoff and Kimani (2018), Reddy, Juan, Isdale and Fongwa (2019) indicate that most of the learners in any grade level lack mathematical knowledge and skills expected of them. These challenges of poorly laid mathematical foundations are extremely difficult to deal with in higher grades. This is supported by Spaull (2013) who indicated that children who fall behind in mathematics in primary school find themselves faced with an insurmountable task in attempting to keep up with the pace of work in high school. These views are in line with the notions of the constructivists who believed in the impact of prior knowledge to the learning process (Piaget & Cook, 1952; Stapleton & Stefaniak, 2019; Neutzling, Pratt & Parker, 2019; Minarni & Natitupulu, 2020).

In an attempt to get further substantiation on the efficacy of concept-based instruction in the learning and teaching of mathematics, learners' proffered views, reactions and solutions which were aired through questionnaires and interviews were taken into cognizance.

Some of the responses from questionnaires were as follows:

> **L3:** *My background in Mathematics is poor and makes me struggle, but I am happy I am improving.*
>
> **L4:** *The teacher explains but I do not understand easily because of my poor Mathematics background.*
>
> **L5:** *I had a lot of challenges in the topic of which most of them were due to my poor background that gave me problems to understand with others.*
>
> **L13:** *It's not good to memorise Maths. One must understand basics well.*
>
> **L23** *I could answer some of the questions using some of the things I already knew about functions. Good background helps.*
>
> **L12** *Poor mathematical background hinders us learners to do well in Mathematics.*

Some of the responses from interviews that were vivid testament to the usefulness of prior knowledge in the use of concept-based instruction were as follows:

> **L29:** *I managed to pull through because what we were learning called for Mathematics basics. I applied my basics and it helped me.*
>
> **L33** *We need good teachers from lower grades so that we start and finish well. The way we learnt this time is what we should have been doing all the time.*

The learners' responses suggest that for better comprehension of concepts, it is essential to understand the basics in mathematics. The participants realised that some of the problems they were facing were created by lack of basic concepts which could make it easy for one to perform mathematics tasks successfully. It is extremely difficult for a learner to grasp new concepts if the basics are lacking as there is nothing to connect the new information to. The educator also finds it difficult to introduce aspects which do not have a background. There has to be connections and links in the construction of new knowledge. Any new skill or aspect in mathematics in most cases depends on one's understanding of prerequisites. The first mathematical skill that a learner has to acquire is the concept of numbers.

There is need for learners to be able to solve simple problems that are based on numbers. In this case, learners should be able to understand and make use of the four main operations (addition, subtraction, multiplication and division). Learners who possess a strong mathematics foundation acknowledge that the subject feared by most learners is not difficult. This is mainly because it becomes easy for them to relate new knowledge to the already possessed concepts through assimilation and/or accommodation. Mathematics background, as explained by Newell and Mallik (2011), is extremely important as it determines success in grasping new information and construction of conceptual understanding. Lack of background knowledge in learning resulted in poor connection between concepts (Lawrence, 2018; Sodeman, 2007). Hence, there is need to find ways of equipping learners with a strong mathematical background in order to do away with the obstructions. Concept-based instruction is a teaching and learning approach that can help learners gain a much better background of mathematics. The teaching and learning approach can then allow learners to identify patterns in concepts and be in a position to make connections to other concepts, topics as well relating to other subjects.

4.2.4.2 Too many rules and long steps involved

Another recurring challenge that emerged from the interviews was that of rules and long calculation steps involved in mathematics. The next extract indicates one of the interviewee's responses.

T: *How do you rate yourself in Maths?*

L7: *I am not good I know.*

T: *But is Maths a difficult subject?*

L7: *Yes and even if I put extra effort, I don't get it right most of the time, the calculations are long and it's boring.*

T: *What do you think makes the subject to be difficult?*

L7: *It has too many rules to be followed. If one cram s/he forgets them.*

T: *Is cramming bad?*

L7: *In Maths it's bad because one must know a lot of things and be able to link?*

These responses coincide with the results obtained by Guner (2020); Meng, Qasem and Shokri (2020) Kieran (2007); Demby (1997) who posited that mathematics involves too much terminology and rules which offer little meaning to many learners. Mbewe (2013) argues that learners misuse previously learnt procedures and rules in situations where they are not applicable. Ncube (2016) and Watson (2007) add that learners overgeneralise and misapply the mathematics rules because the subject involves too many rules. Too many rules, according to Ncube (2016) are difficult for learners to remember well and apply appropriately. The reason for these unfortunate encounters is lack of conceptual understanding. Learners need to be taught using teaching and learning approaches that help them gain retainable and applicable understanding. The concept-based instruction is a recommended approach that can address this challenge.

Some of the comments made by the participants in the interviews on the issue of too many rules and long steps involved in mathematics include:

> **L10:** *To be honest I feel that this is too much as they are many steps to reach to the answer*.
>
> **L24:** *I wish there can be a short cut which is manageable to reach to the answer*.
>
> **L30:** *I have discovered that I am failing to master the entire steps due to the fact that I have no knowledge of the other topics that are related to the current topic of algebraic functions*.

From the participants' responses, one can deduce that learners are afraid of questions which call for long calculations involving many rules and applications like when learners make use of the method of completing the square to determine the coordinates of a turning point. The responses suggest that mathematics learners are afraid of the calculation steps involved in the subject. In fact, it can be deduced that these learners lack conceptual understanding and want to make use of memorised procedural knowledge and they get lost along the way. A learner who has mastered the concepts can answer questions well even if the questions involve many steps to be followed because s/he knows the meaning and relevance of each step. This is because conceptual understanding allows linking and transferring of ideas (Roling, Choksi & Abild-Pedersen, 2019; Erickson, 2012). This boils down to the need to use approaches that foster conceptual understanding in learners, of which concept-based instruction is one of them.

4.2.4.3 Boredom and attitude

Another chronic challenge that emerged from interviews was that of boredom and attitude. The theme identified in this study is in line with the works of Mutodi and Ngirande (2014) which assert that students who struggle with mathematics perceive the amount of material in the subject to be overwhelming, therefore, making it difficult to absorb. It is alluded that most of the learners find mathematics boring, mostly irrelevant and unrewarding (Kunwar, Shrestha & Sharma, 2021; Colgan, 2014). The following narrative indicates that some of the students have a negative attitude towards mathematics and this ultimately leads to boredom. The following responses from questionnaires bring out these attitudes towards mathematics:

> **L2:** *Boring, I hate Maths. The teacher explained a lot of things at the same time.*
>
> **L3:** *A lot of homework which I don't know is given. It's boring.*
>
> **L9:** *Maths is not everyone's cup of coffee; it's difficult.*
>
> **L24:** *Mathematics is just a boring subject boring and dreaded by most of the people so also find all topics boring*.

Participants indicated that they were bored by the subject. The problem with this kind of attitude towards the subject is that it becomes difficult for one to put effort in something which s/he finds boring. Furthermore, the general misconceptions about mathematics being a difficult subject may cause anxiety that may mutate into a phobia, resulting in lack of interest in the subject.

In line with the discussion above, Puspitarini and Hanif (2019); Brown and Crippen (2017); Colgan (2014) suggest that educators should strive to use resources and strategies that motivate and also capture learners' interests. Most learners have a negative attitude towards mathematics and feel that they are not good at it (Celik, 2018; Mata, Monteiro & Peixoto, 2012). Yet Colgan (2014) sees having a positive attitude in mathematics as vital in a learner's achievement. There are higher chances of these negative attitudes being instigated by lack of understanding because learners get motivated when they understand what they are learning. Consequently, it is important to find ways of helping learners to attain conceptual understanding so that they can find pleasure in learning the subject. Hoellwarth, Moelter and Knight (2005) see active-learning instructional strategies as promoting conceptual understanding. This is because learners make sense of what happens during the process of teaching and learning. The concept-based instruction enhances active learning, therefore, worth recommending for effective teaching and learning of mathematics.

4.2.4.4 Educator's pace versus learners' pace

Learners who have difficulties in learning mathematics need help with the evaluation of the effectiveness of chosen solution strategies (Siagan, Saragih & Sinaga, 2019; Ozrecberglu, Caganaga, 2018; Kroesbergen, 2002). There is need for differentiation when teaching mathematics. Learners need to be given tasks according to their level of understanding and if the class is of mixed ability te educator has to check learners' abilities and give them tasks accordingly. If weak learners are given challenging problems, then they may be demoralised. The high flyer learners also get bored when continuously given tasks that are not challenging. In fact, learners need to be grouped and be given activities and tasks that match their level of understanding. To the high flyers, the method of completing the square can be used to determine the coordinates of a turning point but for the weak learners using a formula can be a better way to go. At the same time, if learners of mixed ability are put in one group at times it works as the competent ones help the struggling ones with explanations, but at times this situation affects the poor ones as they may feel looked down upon by the better performing learners. To get a clear picture of this situation, there is need to check challenges that were raised by the participants in the questionnaires.

Some of the participants gave the following sentiments:

> **L8:** *It feels bad if you do not know what other learners know in a class.*
>
> **L19:** *I cannot ask if the others seem to know what I do not know.*
>
> **L34** *…did not find a chance to get someone to explain for me to understand.*

Martin and Evans (2018), Dignath and Buttner (2018), Jones, Wilson and Bhojwani (1997) recommend explicit instruction for learners who have difficulties in learning mathematics. This is a way of showing the need to have special teaching instructions for the less gifted learners so that they also grasp the concepts at their own pace. Learners do not understand what is being taught at the same time; so they need to be treated differently. Some need faster paces of teaching while others need slow paces. The educator should be able to identify the needs of these learners and group them accordingly, then find suiting instructional practices for the different groups. Concept-based instruction is highly recommended as it gives both the educator and learners chances to determine the pace of the lesson. The educator then paces the lesson in such a way that all learners get actively involved giving them a chance to understand conceptually. The pace of the lesson has to focus on conceptual understanding not on completing given tasks.

4.2.4.5 Lack of variety of activities to reduce boredom

Learning mathematics is not only a matter of thinking and reasoning, but is also dependent on the attitudes of the learners towards the subject (Mazana, Montero & Olifage, 2019; Mensah, Okyere & Kuranchie, 2013; Kele & Sharma, 2014). Hence, there is need for the educator to prepare and implement activities that can capture the interests of the learners. The educator has to vary his/her teaching methods, approaches and techniques to produce better outcomes. The teaching and learning of mathematics should be practical and exciting to prevent negative attitudes among learners.

Educators should employ a variety of activities to facilitate learning in order to alleviate learners' engagement level and confidence in the learning of mathematics (Iji, Abah & Anyor, 2018; Attard, 2012; Kele & Sharma, 2014). Sullivan and McDonough (2007) suggest that teachers find ways of encouraging learner engagement and confidence in learning mathematics. This can be achieved by implementing meaningful activities embedded in real-life contexts (Balta, 2021; Kacerja, 2012).

Taking into consideration these responses:

> **L3:** *Our teachers must help us to develop concepts by giving us a lot of different activities.*
>
> **L4:** *We need different activities in our mathematics lessons.*
>
> **L24:** *I really enjoyed the way we were taught this time because there were a lot of activities involved.*

One can deduce that varying activities during lessons will help in capturing learners' interests. If there is a variety of activities and one activity is not interesting, then the other one/s can be interesting. In the teaching and learning of algebraic functions, learners can draw graphs; interpret drawn graphs and also apply knowledge from other topics like algebraic equations and inequalities. Conceptual understanding can therefore be attained as among the activities there are chances that at least one of them will enhance conceptual understanding.

To motivate learners and make them appreciate the subject, learners need to be involved in activities that capture their minds. Learners must not get tired of a subject's activities because once they become tired, they lose interest. If interest is lost, then teaching and learning becomes tough. Therefore, there is a need to identify and give learners activities that do not overwhelm them. Such activities should make learners active and gain confidence. Activities should enhance attainment of conceptual understanding in learners. Situations whereby answers just come from unclear circumstances should be avoided. Therefore, teaching and learning approaches like the concept-based approach that allows use of a variety of activities to give room for conceptual understanding should be employed.

4.2.4.6 Summary on research question 4

According to Gafoor and Kurukkan (2015), the type of instruction used for teaching and learning affects learners' attitudes towards mathematics. Teaching and learning has to be through approaches that promote positive attitude towards mathematics and enhance conceptual understanding. The approaches should encourage independence so that learners have an opportunity to assess their thinking capabilities. A variety of activities has to be included to reduce boredom. Selection of tasks should be carefully done to match learners' level of understanding so that all learners get involved in the learning process.

This study also had an important objective of deciphering how educators build on prior knowledge of learners in teaching concepts like algebraic functions. It was gathered during the interviews that learners were facing a superfluity of multifaceted challenges which they attributed to lack of a firm background in mathematical concepts. The majority of participants blamed their foundational backgrounds claiming that it was very weak. They indicated that they many a times fell short of basic concepts which they were supposed to have acquired right from their lower grades, making it extremely difficult for them to grasp new concepts. This lack of background knowledge meant poor retention of concepts learnt before. The main issue lies on equipping learners with conceptual understanding. Once learners attain deep understanding of concepts, then problems of getting tired with long steps and too many rules will not arise as learners will know exactly what is supposed to be done and the meaning of what they will be doing. The implementation of the concept-based instruction with its properties of targeting learners on gaining conceptual understanding can bring desirable changes in the teaching and learning of mathematics.

Prior to being taught through concept-based approach, some learners did not understand algebraic functions. Their challenges were solved by the use of a different teaching and learning approach that targeted learners' acquisition of conceptual understanding. The concept-based instruction brought positive effects to the teaching and learning of mathematics, with learners constructing meaning based on their involvement during intervention.

4.3 Summary of the findings

The concept-based instruction that was used during the intervention was discussed in chapter 3 seems to have helped participants to gain rich knowledge which allowed them to be able to solve mathematical problems on their own. According to Kristiyajati and Wijaya (2019); Sahidin, Budiarto and Fuad (2019), if a learner understands the meaning and underlying principles of mathematical concepts, s/he has conceptual knowledge in mathematics. There was a de-emphasis of memorisation of learnt things. This is because with rote learning, application and retention of concepts are difficult (Bressington, Wong, Lam & Chien 2018; Langton, 1991). The participants indicated that they felt good when discovering and coming up with their own solutions rather than always relying on the teacher. One of the participants responded as follows during the interview session:

T: *When learning, what do you prefer between getting solutions on your own and being given solutions by your teacher?*

L: *I like attempting and doing it first on my own then ask for the teacher's help when I get stuck.*

T: *Why do you like that?*

L: *Because I am happy when I do it for myself as it helps me to understand better and also check if I understand.*

The participants realised the importance of constructing knowledge themselves, which is in line with the ideas of constructivism, the theory that guides this study. The responses indicate that there can be advantages in using concept-based instruction in the teaching and learning of mathematics as it enables learners to construct better quality of knowledge.

The responses by the participants with respect to elaborations to their test solutions also suggest the merits of the teaching and learning approach that was used during the intervention. The approach, according to the findings, allows learners to think and connect and ultimately come up with solutions rather than waiting for the teacher to provide solutions. Participants' achievements in the tests markedly improved, which was a sign of changes in conceptual understanding and this was attested during the interviews. The interviewees had no problems in accounting for their solutions. There was a great improvement in terms of learner confidence and ability to relate and connect what they had learnt.

Learners become worried and bored by long steps in mathematics. This is because learners in most cases are taught with emphasis on procedural understanding. This type of understanding is difficult for learners to retain and also difficult to be applied or linked to other situations. There is therefore need to teach learners for conceptual understanding. If learners have this kind of understanding, then there will be less or no complaints about long steps or too many rules involved in the subject. The concept-based instruction had positive effects in the current research.

Construction of one's own knowledge is crucial according to constructivists' views. However, building knowledge can only exist when there is conceptual understanding. Vincensi (2019)) points out that there is interconnection in conceptual understanding. Moreover, conceptual understanding requires the learner to be active in thinking about relationships and making connections, along with adjusting accommodate new learning with previous mental structures (Fletcher, Hicks, Johnson, Laverentz, Phillips, Pierce & Gay, 2019; Young & Legister, 2018;

Lindquist, Suydam & Reys 1995). Therefore, the teaching for conceptual understanding through concept-based teaching and learning approach in mathematics is vital as it brings valuable knowledge.

4.4 Conclusion of the chapter

This chapter presented results which address the four research questions of the current study. The study determined the effects of the concept-based instruction on the teaching and learning of mathematics with reference to the topic: algebraic functions. The rationale behind teaching is for learners to understand what they are taught. The results show that learners taught using the concept-based teaching and learning approach perceived mathematics as easier to understand than before. More importantly, participants realised the importance of learning to grasp concepts instead of just getting correct answers. The next chapter will present a summary, conclusion and recommendations of the study.

CHAPTER 5: SUMMARY OF FINDINS, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter is the epilogue of the study. The focal point of this study was to explore the effects of concept-based instruction on the teaching and learning of mathematics. This chapter provides a summary of the findings, contribution of the study towards addressing the issue of poor performance in mathematics and recommendations based on the findings from the previous chapter. Limitations of the study are also discussed, conclusions that have been drawn are given and the chapter concludes up with suggestions for further studies and the contribution of the study to the field of mathematics education.

5.2 Summary of the major findings

The effects of concept-based instruction on the teaching and learning of mathematics were explored through the following research questions:

Research question 1: What effect does concept-based instruction have on the teaching and learning of mathematics?

Research question 2: What advantages does concept-based instruction have over other teaching and learning approaches?

Research question 3: What changes are brought about in learners by the use of concept-based instruction in the teaching and learning of mathematics?

Research question 4: What challenges can be addressed by the implementation of concept-based instruction in the teaching and learning of mathematics?

This section presents the major findings of the current study in line with the aforementioned research questions. Data were collected from learners from one high school in Mopani District, Limpopo Province of South Africa. Thirty-five (35) learners participated in the study. Data were collected through tests, questionnaires and interviews. The collected data was tabled, diagrammatically represented and statistically analysed. Data were also coded into themes and categories. A mixed method approach involving quantitative and qualitative data analysis was adopted to address the four research questions that guided this study.

The data gathered highlight the effects of concept-based instruction on the teaching and learning of mathematics. The following subsections give a summary of the major findings of each research question.

5.2.1 Research question 1

What effect does concept-based instruction have on the teaching and learning of mathematics?

Determining the effects of concept-based instruction on the teaching and learning of mathematics was one of the main focus of this study. A succinct of what was established by the study in answering this research question is outlined from 5.2.1.1 to 5.2.1.2

5.2.1.1 Learners' emotions towards concept teaching approach

The majority of the respondents enjoyed the topic of algebraic functions under the concept-based method of teaching (see figure 4.1 chapter 4). The researcher interpreted the fact that learners enjoyed the algebraic functions to mean that they understood functions as they were taught during intervention. This was evidenced by improved performance after intervention the lessons using the concept-based teaching as portrayed by Figure 4.11 and Table 4.1 in chapter 4. Deductions from the interview responses showed that the participants appreciated the teaching and learning approach that was used during the intervention. The responses given by learners about their perceptions revealed that students may feel intimidated when inadequate information, which excludes concepts, is presented during lessons.

It was established that enjoyment during lessons nurtured long-term memory and improved engagement. This relationship between enjoyment and learner engagement is supported by Guo (2021); Mazana, Suero Montero and Olifage (2019); Ntibi and Edoho (2017); Ainley and Ainley (2011); Jamison (2003), who indicated improvement in performance as one of the results of enjoyable lessons. Cognitive, physical and social development were also enhanced in learners. All these mentioned properties lead to higher chances of learners building strong conceptual understanding.

5.2.1.2 Quality of teacher's explanations

Of the 35 participants, 30 of them which yields to 85,7% indicated that the teacher's explanations were clear. The success of any lesson solely depends on how much the teacher knows and his or her effectiveness in delivering the lesson and engaging with learners. Explanations actively engage learners, stimulate them to connect information, reinforce learning and retention (Donelan, 2003; Chauraya & Brodie (2018). Schnitzler (2021) asserts that explanations help in clearing misconceptions. It is vital for a learner to understand what is being taught. Clarity in teaching by the teacher was found to be important for learners to understand, especially if the teacher is open to questions from learners who require further clarification on the concept being taught. There was an indication that the teacher's explanations during intervention enhanced learners' conceptual understanding. Clear explanations intensified the integration of existing and new knowledge. If explanations are clear, then construction of conceptual knowledge is possible. Therefore, there is a need for clear explanations to facilitate conceptual understanding in learners.

5.2.1.3 Mastering of concepts in learning

The learners did not understand the questions and did not know how to tackle algebraic functions prior to introduction of the concept-based instruction. After the teacher engaged the learners on the same topic using a concept-based approach and re-administered the exercise for them to apply their newly acquired knowledge, performance was much improved (see Figures 4.6 & 4.7). This was inferred to mean there is enhanced building of knowledge when there is conceptual understanding. According to Permana, Hindun and Rofiah (2019), mastering of concepts effectively contributes towards learners' critical thinking skills. Participants discovered a logical presentation of arguments during the concept-based discussions. One of the interviewees was asked to explain how to determine an expression for the distance between two graphically represented functions. After giving an explanation that the two expressions representing the functions are to be subtracted, it was further asked to give a reason why they are to be subtracted. The participant managed to explain convincingly because he had mastered the concepts.

All the learners interviewed after the post-test gave plausible knowledge on the functions section, which showed they understood concepts presented during intervention regarding functions. The researcher interpreted this to mean concept-based teaching provides learners with adequate understanding on how to solve mathematical functions. The teacher gave learners time to explain to their class mates the way they understood the concepts after teaching through concept-based approach. Learners were active in the building of their understanding which is believed by Anwar and Rahmawati (2017) to fulfil the theory of constructivism. The researcher observed that the opportunity given to learners to explain concepts to fellow learners enabled them to construct their knowledge. The concept-based instruction brought positive effects to the teaching and learning of mathematics. It enabled learners to construct meaning based on their involvement during lessons.

5.2.1.4 Educator/learner interaction

The teacher, through the process of asking questions during intervention lessons, managed to identify the learners' misconceptions as well as their shortcomings. Asking questions helped to identify the degree of understanding by learners. The questioning assisted in communicating facts and ideas as the learners made connections between new information and prior knowledge. It aided them to gain conceptual understanding.

Learner involvement in class activities promoted conceptual understanding as emphasised by the constructivists in the literature review. The interaction between the educator and learners, according to Hanun (2017), made learners to be more active in their learning process. More importantly, concept-based instruction had positive effects on the participants in terms of performance and attitudes towards learning mathematics, giving credit to the development of conceptual understating in learners. Most of the learners indicated that the teacher asked them concept building questions during the intervention lessons. The learners were receptive to the teacher's probing questions thereby achieving higher levels of understanding.

5.2.1.5 Comparative analysis of pre and post-test marks

The success of an intervention can be demonstrated by a comparative analysis of the participants' results in the tests before and after intervention (Kashi'ie, Said, Zainal & Miswan, 2017; Fancher, 2013). The tests were meant to determine the success of the intervention through changes in learners' performance. The results in the pre and post-tests demonstrated positive gains (see Figure 4.13). In the first test almost 75% of the participants got below 30% while in the post–test no participant got below 30%. The solutions provided by learners in the post-test showed that learners had gained conceptual understanding. Minimum, maximum and average marks obtained in the post-test showed improved scores (see Table 4.1). This was attributed to the concept-based approach's effectiveness as the learners managed to achieve better comprehension of the subject after the intervention. The feedback by the participants with respect to the second test showed that the participants had gained conceptual understanding.

5.2.2 Research question 2

What is the advantage of concept-based instruction over other teaching and learning approaches?

This research question's objective was to ascertain the superiority of the concept-based instruction and establish if it could be relied upon over other teaching and learning approaches. What the study deduced is outlined in 5.2.2.1 to 5.2.2. 5 as follows:

5.2.2.1 Freedom to express oneself

The majority of the participants (83%) (see Figure 4.15) revealed that concept-based teaching and learning gave them a chance to explain and describe mathematical phenomenon. Mathematical phenomena include facts, skills, ideas and patterns involved in mathematics. Explaining as described by Ingram, Andrews and Pitt (2019) helped the learners to develop new understanding of mathematical ideas, link and connect information. There was a de-emphasis of memorisation of learnt things. The majority of the learners' revelation that they had the freedom to ask questions during the lessons attests learner-centred teaching and learning processes. There was intellectual development in learners when they asked questions and it gave them high self-esteem and self-confidence.

5.2.2.2 Learner engagement with lessons

Learners were given a chance to engage with the teacher through the process of asking questions. More than 70% of the participants as illustrated by Figure 4.16 revealed that they had the freedom to ask questions during the intervention lessons. The results show that learners taught using the concept-based teaching and learning approach perceived mathematics as easier to understand than before. As a result, learners took a more active role in their learning. Learner engagement as explained by Halverson and Graham (2019) correlated with important educational outcomes like increased conceptual understanding and, according to Funnell (2017), maximised participation and interaction. The improved marks in the post-test support the assertion that conceptual understanding leads to creation of meaning in mathematics. The participants indicated that they felt happy and satisfied when discovering solutions rather than always relying on the teacher.

5.2.2.3 Pace of comprehension

Most participants (86%) indicated that they understood the topic after introduction of the concept-based approach in the learning process. The participating learners grasped the aspects to do with algebraic functions and could link them to other topics and this demonstrated grasping of fundamental ideas as well as connectivity. Siagian, Suwanto and Siregar (2021) describe mathematical connections as important in bringing understanding of concepts. One of the revelations of the capability of linking algebraic functions to other topics was when participants realised they could use the midpoint formula from analytical geometry to determine the axis of symmetry for a parabola.

5.2.2.4 Need to explore oneself

Participants indicated in the interviews that they were given time to think by themselves and find their own solutions (see L7 in 4.2.2.4). The disclosure by the learners about independently solving mathematics problems signified that they had the opportunity to self-explore and make conclusions by themselves which according to Loannidou (2017); Yang, Luk, Webster, Chau and Ma (2016); Machisi (2013) enhanced their concept building. There was an improvement in solving problems and that was interpreted as desire to explore their capabilities and propel performance to higher levels. The process built self-confidence in the learners which is a prerequisite to mathematics knowledge and understanding, which according to Hackett and Betz (1989) improves mathematics performance. The approach allowed learners to evaluate their capability in tackling problems. The duty of a mathematics teacher is then regarded to be that of a mediator, guiding learners in their building of conceptual knowledge.

5.2.2.5 Comparative approach

The ability to excel by the majority of learners shown in the post-test has been attributed to the positive effects of concept-based instruction. A minimum mark of 2 in the pre-test was witnessed and compared to 38 in the post-test. Maximum mark of 52% in pre-test and 62% in post-test and mean of 20,26% and 75,23% respectively were also observed. The performance in general indicated a marked improvement in performance by learners which, in a way depicts the creation of conceptual understanding because of the concept-based instruction. This means that there was creation of meaning and construction of systems of meaning. Therefore, the concept-based instruction enabled the learners to reason and present their arguments logically.

5.2.3 Research question 3

What changes are brought about in learners by the use of concept-based instruction teaching and learning approach?

The study utilised the preceding research question to be in a position to identify the changes that would be brought by the application of the concept-based teaching and learning approach. Sections 5.2.3.1 to 5.2.3.5 provide a summary of what was established in line with the above stated research question.

5.2.3.1 Comparative approach

The study found out that there was statistically enough evidence to believe that the intervention brought changes in participants' achievement. A comparative analysis through the use of program for statistical analysis called PSPP was carried out. A marked change in participants' achievement was portrayed by the t-test results (see Table 4.3). The results from the test had a p-value that was extremely small ($p < 0,05$) and the null hypothesis $H_0 : u_1 = u_2$ which suggested equality in the mean for the pre-test and the average of the post-test was rejected in favour of the alternative hypothesis H_1: $u_1 = u_2$. There was statistically enough evidence to accept that the intervention had brought changes in participants' performance. Improvement in performance was attributed to the understanding of concepts. The findings in accordance with Andamon and Tan (2018); Borji, Radmehr and Font (2021) suggest that conceptual understanding improves performance sustainability in mathematics.

5.2.3.2 Perception of learning mathematics by memorising

In the questionnaire, 82,9% of the participants attested that they did not believe in memorising mathematics facts and procedures (see Figure 4.2.2). Also, the majority of participants' interview responses indicated that the assertion that they could learn mathematics better by memorising was false. Participants seemed to have realised that memorisation would not bring understanding of concepts. They added that crammed information can neither be linked nor connected easily to other information. The justification of solutions during the interview session portrayed that learners had gained conceptual knowledge and they even gave praise to possession of conceptual understanding over memorisation of facts. The participants' views were in line with Miller, Perrotti, Silverthorn Dalley and Rarey (2002) who emphasised that success in learning is hinged on grasping concepts not memorisation of facts. Conceptual understanding paved way to linking prior and new information.

5.2.3.3 Prioritising on getting correct answers

The majority of participants (see figure 4.2.3) responded in interviews that it was false to assert that the important aspect in mathematics is to get correct answers as this disregarded the potency of concept based-instruction. They placed more emphasis on solution attempts than on just getting answers as established by Moschkovich (2018) and Planas (2018) in their discoveries. These participants seemed to have realised that possessing conceptual knowledge increases one's chances of connecting, linking and relating ideas, as well as getting correct answers. Participants appeared to have followed the ideas of Kilpatrick, Swafford and Findell (2001) who stressed that conceptual understanding allows facts and methods to be connected, therefore easier to remember and use. It was established that acquisition of knowledge is greater than just getting baseless answers.

5.2.3.4 Perception on understanding mathematics concepts

Most of the participants as shown in Figure 4.22 indicated that it was false that mathematics concepts are difficult to understand. Hiebert and Carpenter (1992) indicated gaining conceptual understanding as one of the most widely accepted ideas in mathematics education. In this study, learners disclosed that the understanding of mathematics was made less burdensome as the teacher approached it by way of focusing attention on concept building. Participants realised that understanding concepts allowed them to link new and old information, thereby enhancing their knowledge. This is a mind shift in favour of concept-based learning. Therefore, concept-based instruction was established to bring favourable changes in learners' attitudes and perceptions about mathematics.

5.2.3.5 Perception on role of teacher as a facilitator

The learners accepted the role of a teacher as a facilitator in enabling the teaching and learning of mathematics through concept-based instruction. The learners prioritised attempting on their own before requesting the teacher to help if they encountered a problem in solving mathematical problems. The teacher's role as described by Landsberg (2005) and Amponsah, Kwesi and Ernest (2019) was to facilitate by creating a classroom environment that was conducive for learners to make sense of knowledge, skills and values of what was being taught. With the teacher as a facilitator, it was established that learners gained confidence as they got a chance to explore mathematical phenomenon.

5.2.4 Research question 4

What challenges can be addressed by the implementation of concept-based instruction in the teaching and learning of mathematics?

This research question was brought in to establish the learning challenges that impacted the performance and self-esteem of mathematics learners. The reason for considering this question was to be in a position to come up with recommendations for eliminating or reducing poor performance in mathematics. An outline from 5.2.4.1 to 5.2.4.5 provides what was established in an attempt to answer this research question.

5.2.4.1 Poor mathematics background

The lack of preliminary background was identified as one of the main challenges for learners in grasping new concepts, particularly in linking new and prior knowledge. Participants in this study expressed their worries concerning lack or poor mathematics background (see Figure 4.25). In the interviews, hapless comprehension of concepts was linked to poor mathematics background as well. The findings of this study revealed that a strong mathematical foundation in learners which according to Michael (2015) involves more than the rote application of procedural knowledge, could be established by employing teaching and learning approaches that allow learners to build conceptual understanding.

5.2.4.2 Too many rules and long steps involved in mathematics

The study found that learners were demotivated by too many rules and long steps involved in solving mathematical problems. Guner (2020), Meng, Qasem and Shokri (2020), Kieran (2007) and Demby (1997) support the fact that mathematics has a plethora of terminology and rules. These rules are misapplied if learners lack conceptual understanding. The concept-based approach, as an approach enhancing conceptual understanding, can help in doing away with that notion as it enables learners to see the reason behind each step. Concept-based instruction was recommended as an approach that could help learners in gaining retainable and applicable understanding.

5.2.4.3 Boredom and attitude

There was also the issue of learners getting bored and developing a negative attitude towards the subject. Kunwar, Shrestha and Sharma (2021) alluded that mathematics is boring to most of the learners. According to Nett, Goetz and Daniels (2010), boredom can be highly detrimental. In the findings of this study, it was discovered that the subject is considered boring by most of the learners. Mostly, learners hated the subject because of poor performance. Lack of variety of activities was also another cause of boredom in the subject. The study established that mathematics, being a cumulative subject, makes it difficult for learners to gain new knowledge after failing to link it to prior knowledge and learners end up finding themselves disengaged and hating the subject.

5.2.4.4 Educator's pace versus learners' pace

Most of the participants indicated that it was false that mathematical concepts are difficult to understand. Understanding of concepts was found to be partly affected by the way teachers taught learners of different abilities in the same class. The South African curriculum (CAPS) allocates time for each topic which in most cases makes it difficult for educators to match the pace of learners with different levels of understanding. Learners disclosed that the understanding of mathematics becomes burdensome if teacher approaches without matching the pace of the lesson to the level of learners' understanding. Differentiation was therefore established to be a better method of helping learners to grasp concepts which match the views of Martin and Evans (2018). The concept-based instruction is highly recommended to give the teacher and learners a chance to use favourable paces in teaching and learning.

5.2.4.5 Lack of variety of activities to reduce boredom

It was established that lessons lack variation of activities of which if there is a variety it improves learner engagement and then attitude. The study found out that it is extremely difficult to engage in a lesson where there is no variation of activities. According to Mazana, Montero and Olifage (2019), learning highly depends on attitude of learners towards a subject; hence, the need for teacher to prepare and implement a variety of activities that capture their interests. To prevent negative attitudes among learners, teaching and learning was discovered to need involving practical and exciting activities. The concept-based instruction with its variety of activities that encourages learner engagement is therefore highly recommended.

5.3 Recommendations of the study

This study explored the effects of concept-based instruction, the advantages that it has over other teaching and learning approaches and the changes brought about by its implementation. The study also went on to determine some of the challenges that learners face when learning mathematics so that they could be addressed by implementing concept-based instruction if possible. Conceptualisation of this study was against the background of poor performance in mathematics. From the findings of this study and the preceding discussion recommendations were proffered to different stakeholders for deliberation as presented in the next section.

5.3.1 Recommendations to teachers

The findings of this study established the need to employ concept-based instruction in the teaching and learning of mathematics. Therefore, some recommendations were made to teachers so as to add value to the implementation of the approach. The study recommends mathematics teachers to:

- de-emphasise memorisation of learnt mathematical facts as concept-based teaching makes it easier to understand abstract facts as these are presented in real life situations.
- ask questions about the learners' emotional response when discovering and coming up with solutions as feelings are indicative of understanding what is being taught. This facilitates self-esteem and independent learning.
- strive to demonstrate connectivity of concepts under discussion to other topics as well as other subjects so that the topic is not viewed in isolation and without relevance to real life.
- avoid hurrying through a lesson and give learners time to solve mathematics problems and think for themselves in a process of self-exploration. The teacher should also guide learners to build concepts and apply them to real life examples so that knowledge develops and lodges in their mental faculties.
- perform the role of a mediator or midwife in guiding learners to discover each new knowledge frontier.
- probe for associated meaning in learners as part of assessing the level of understanding.
- stress that, emphasis is not on the answer, but that a correct path traversed leads to a correct answer.
- encourage this hands-on approach that is steeped in constructivism.

5.3.2 Recommendations for pre-service and in-service training

The quality of teaching and learning highly affects learners' performance. Teachers, therefore, require coaching and mentoring to improve their quality of teaching instruction and enhance their capacity to teach for conceptual understanding. The workshop should be meant to allow teachers to acquire new or better knowledge to improve skills that would enable them to render effective and efficient lessons. This will also give an opportunity for teachers to appreciate the power of conceptual understanding. The teachers will also attain knowledge for personal development and career advancement. Therefore, there is need for:

- development of pre-service and in-service programmes to help mathematics teachers acquire and update their skills in teaching using approaches that bring conceptual understanding in learners.

- provision of professional development opportunities to give a chance for teachers to acquire or increase their knowledge in teaching and learning approaches that develop conceptual understanding.
- provision of more information to teachers on teaching for conceptual understanding through workshops and seminars to equip them with practical skills and activities so that they become confident and their classroom presentations become effective.

5.3.3 Recommendations to policy makers

Curriculum is planned in such a way that the teaching has to follow certain ways and levels (Wortham, 2006). The study then recommends that:

- Curriculum developers should design curriculum statements in such a way that does not compel teachers to focus on completing the syllabus at the expense of learners' understanding.
- Curriculum to be designed in such a way that it enhances and facilitates conceptual understanding in learners. This is because of the fact that mathematics is a cumulative subject. Therefore, the Annual Teaching Plan should arrange topics logically in such a way that they connect well.

5.4 Contribution of the study.

The findings of this study were meant to address the issue of poor performance in mathematics. The literature reviews together with the findings of this study established that poor performance was mainly owing to lack of conceptual understanding in learners. Learners' achievement as supported by Law, Geng and Li (2019) and Wu, Yu and Gu (2020) depends on the quality of the teaching and learning instruction that the teacher uses. From the findings and the literature that was reviewed, the researcher developed guidelines to help both educators and learners in the application of concept-based teaching and learning. The guidelines present how conceptual understanding can be ensured in the teaching and learning of mathematics.

5.4.1 Implications of the guidelines

Table 5.1 gives a brief summary of the guidelines for teaching and learning for conceptual understanding.

Table 5.1: Concept-based teaching and learning guidelines

CONCEPT-BASED TEACHING AND LEARNING GUIDELINES			
TEACHER'S INPUT	LEARNERS' RESPONSE		LEARNERS' OUTPUT
ACTION	**MIND STATE**	**EFFECT**	**RESULT**
Proffer concepts based on lesson being taught	Show pleasure in absorbing concepts	Enjoyment nurtures their long term memory and improves cognitive development	Become creative and cooperative
Create sufficient time to learners to work on their own	Appreciate the relaxed time to understand how to construct important tenets of the lesson	Linking new knowledge with old knowledge; apply these concepts to other situations in the field of mathematics	Able to present their findings, explain and describe mathematical phenomenon
Probe the learners through questions whether they understand the concepts	Gives evidence of the teacher's care	Improves learner participation, promotes active learning	Streamline and consolidate understanding of concepts
Open to questions by learners	Builds self-confidence	Questions during lessons improve logical presentation of arguments	Remove misconceptions; Reorient attitude toward mathematics; Create understanding
Assess by test, investigation and/or assignment	Eager to show new knowledge	Better performance	Await new topic

The researcher developed the guidelines as her contribution towards improving mathematics performance by learners and also a contribution to the body of knowledge. The guidelines were meant to enhance the quality of teaching and learning of mathematics in Grade 11 and hence improve learners' performance. The guidelines demonstrate how effective teaching and learning can be conducted to ensure that learners gain conceptual understanding. By employing the guidelines in mathematics lessons, learners can link prior and new knowledge and be in a position to make connections. They enable learners to gain rich knowledge. The guidelines were designed mainly to promote teaching for conceptual understanding and de-emphasise traditional teaching approaches. The guidelines were developed in line with the views of the constructivists as they target on learning by doing and learning through actions. According to the constructivists, instead of supplying learners with knowledge, they should actively construct their own knowledge through connecting new information with existing information (Kurtes, Larina, & Ozyumenko, 2017; Anagun, 2018; Clark, 2018). The constructivists believe in learners owning their understanding which is a learner-centred approach. The main theme of the constructivists is that learners have the duty to construct their own knowledge with new ideas relying on current and/or past knowledge.

The contribution of this study harmonises with contemporary teaching and learning debates that targets bringing conceptual understanding in mathematics. According to Mashingaidze (2017), there is a continuous call to make teaching and learning more relevant, productive and driven by national goals and challenges to contribute to the national and economic development of the country. The researcher pursued to contribute more knowledge to add it to existing body of knowledge. Adopting the guidelines, together with the other recommendations from this research study can improve the teaching and learning of mathematics and hence improve learner performance.

The following subsection explains how the guidelines can be executed.

5.4.2 Implementation of the guidelines

One of the challenges that the mathematics Curriculum Assessment Policy Statements (CAPS documents) has is lack of guidelines on how concepts should be developed. Therefore, the implementation of this study's guidelines could start with revision of policy documents for grade 11 mathematics. Mathematics education specialists are recommended to conduct workshops with educators for mathematics in the Further Education and Training (FET) phase to equip teachers with strategies of teaching for conceptual understanding. The Department of Education (DBE) should organise workshops to empower teachers with the guidelines. The DBE should fund the implementation of the programme and also provide permission for conduction of the workshops. Mathematics educators need to attend and participate in the workshops. From the workshops, teachers implement the guidelines and review the performance of the learners. This new approach enables learners take an active role in building their own knowledge and thereafter become self-reliant.

5.5 Limitations of the study

The sample for this study was conveniently and purposively selected. The participants of this study were all from one school. Owing to time and financial constraints, the scope of the study could not be extended to other schools. However, triangulating instruments as well as the research designs helped in overcoming this shortfall. The results of this study provide evidence of learners' enhanced conceptual understanding and the findings can be a starting point in introducing teaching and learning approaches that are capable of making changes in the history of mathematics teaching.

5.6 Summary of the study

The study explored the effects of concept-based instruction on the teaching and learning of mathematics. The researcher's main problem was learners' poor performance in mathematics. The reason for not performing well was attributed to lack of conceptual understanding. The study revealed that concept-based instruction has many positive influences on attitudes, beliefs, motivation and construction of knowledge. The findings from this study suggest that concept-based instruction has much more valuable properties than the traditional teaching and learning approaches.

The study was organised into five chapters. The first chapter described the problem statement and its setting, motivation, background of the offset of the study, purpose and significance of the study. Research questions, aims and objectives of the study were also highlighted in the first chapter.

Chapter two outlined the literature review, which entailed examining the research conducted by other researchers in issues related to the study problem. The theoretical framework that underpins this study was acknowledged in this chapter. The research gap was also highlighted in this chapter.

Chapter three described the methodology that was adopted by this study. The research design that was employed by the study was also outlined in this chapter. Descriptions of phases of the research, population, sample, and sampling methods, study participants and research instruments were conferred in the same chapter. Reliability, validity, ethical considerations, trustworthiness and limitations were also discussed in the same chapter.

Chapter four had data presentation and discussions of the findings. The chapter presented data collected through tests, questionnaires and semi-structured interviews. A discussion was presented under each of the four research questions that guided this study.

The roundup of the study was conducted in chapter five. The chapter presented a summary of the findings, recommendations and contribution of the study. The chapter also offered limitations of the study, conclusion and suggestions for further study.

5.7 Suggestions for further study

Based on the findings of the study and literature review, the researcher recommends the following areas for further study:

- Cross sectional study on impact of concept-based teaching and learning of mathematics in Mopani District government secondary schools.
- Investigation into factors that lead learners to shift from pure mathematics to mathematical literacy at the FET phase.

5.8 Conclusion of the study

This study was motivated by poor performance in mathematics and it aimed at finding a teaching and learning approach that could help learners build conceptual understanding and hence improve performance. Data were collected through pre-test, post-test, questionnaire and interviews and from all these data sources. The findings confirmed that concept-based instruction has favourable effects on the teaching and learning of mathematics.

Concept-based instruction was established to be a teaching and learning approach capable of building knowledge and allowing effective understanding of concepts, knowledge and skills in learners. The teaching and learning approach was ratified to have positive gains in learner performance and more advantages over the traditional approaches. Linking and connecting information was achieved through concept-based instruction. The approach brought meaning of aspects and made learners to appreciate the beauty of mathematics. The approach allowed learners to think critically and permitted higher order thinking. The study also provides evidence that points to reform from traditional teaching and learning approaches to approaches that targets concept formation and building.

The study developed guidelines of how mathematics educators can use concept-based instruction to promote teaching for conceptual understanding. The guidelines are meant to assist learners to take active roles in their learning and be in a position to link and connect acquired knowledge. Recommendations based on the findings were proffered to assist educators and stakeholders in bringing meaning to the teaching and learning of mathematics. The study concludes that learning should not focus on getting correct answers but on how answers should be attained.

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APPENDIX A: ETHICS CLEARANCE CERTIFICATE

NISA university of south africa

Unisa COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2018/11/14

Ref: 2018/11/14/48809969/57/MC
Name: Mrs M Ncube
Student: 48809969

Dear Mrs Ncube

Decision: Ethics Approval from 2018/11/14 to 2023/11/14

Researcher(s): Name: Mrs M Ncube
E-mail address: mildretncube@ymail.com
Telephone: +27 73 713 4810

Supervisor(s): Name: Prof MG Ngoepe
E-mail address: ngoepmg@unisa.ac.za
Telephone: +27 12 429 8375

Title of research:

Exploring the effects of concept based instruction in the teaching and learning of mathematics

Qualification: D. Ed in Mathematics Education

Thank you for the application for research ethics clearance by the Unisa College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2018/11/14 to 2023/11/14.

The medium risk application was reviewed by the Ethics Review Committee on 2018/11/14 in compliance with the Unisa Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the Unisa Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 IJNISA 0003 South Africa Telephone: +27 1 2 429 31 I l Facsimile: +27 1 2 429 50 www.unisa.ac.za

2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the Unisa College of Education Ethics Review Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date 2023/11/14. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number 2018/11/14/48809969/ 57/ MC should be clearly indicated on all forms of communication with the intended research participants, as well as with the

Committee,

Kind regards,

Prof MT Gumbo

CHAIRPERSON: CEDU RERC Gumbomt@unisa.ac.za approved - decision template — updated 16 Feb 2017

Prof V McKay
EXECUTIVE DEAN
Mckayvi@unisa.ac.za

University of South Africa Prelier Street. Muckleneuk Ridge. City of Tshwane PO Box 392 Unisa 0003 South Africa Telephone: +27 429 31 Facsimile: +27 1 2 429 41 50 www.unisa.ac.za

APPENDIX B: QUESTIONNAIRE

The questionnaire consists of 4 Sections with 15 items altogether. Some of the items are just to be marked by an X in the box in front of the statement. The other group of items needs to be answered by rating and lastly there are some questions requiring your opinion whereby you have to fill in the blank spaces provided.

SECTION A

1. Fill in the blank spaces

1.1 PARTICIPANT CODE: ....

1.2 AGE: ....

2. Indicate your answer with a √

GENDER: Male..... Female......

SECTION B: Questions about the topic, algebraic functions.

3.1 Indicate your answer with a √

3.1.1 Did you enjoy the topic algebraic functions? Yes...... No......

3.1.2 Was your teacher explaining concepts clearly? Yes..... No......

3.1.3 Did your teacher ask questions during the lessons? Yes.... No....

3.1.4 Were you given a chance to explain and describe mathematical phenomenon? Yes.... No....

3.1.5 Were you free to ask questions? Yes..... No.....

3.1.6 Was it easy to understand the topic? Yes..... No....

3.2 Briefly explain your challenges or your acknowledgements on the topic.

....
....
....
....

SECTION C: General questions.

Choose the applicable answer from these: False, often, partly false, don't know, partly true and true then indicate your choice in the given box using the given abbreviations.

F- False

PF- Partly false

D- Don't know

PT- Partly true

T- True

4.1 Mathematics concepts are difficult to understand. ☐

4.2 One can learn Mathematics better by memorizing facts and procedures ☐

4.3 The important thing in Mathematics is to get correct answers ☐

4.4 It is the teacher's duty to always show me the method to be used to solve Mathematics problems. ☐

SECTION D: Express your feelings about the way you learn Mathematics and give suggestions on what can be done to help you achieve your goals in Mathematics.

....
....
....
....

THANK YOU FOR YOUR PARTICIPATION

APPENDIX C: TEST

GRADE 11 MATHEMATICS TEST-ALGEBRAIC FUNCTIONS

LEARNER CODE: ....

DATE: ....

INSTRUCTIONS

1. Time: 1 hour.

2. Answer all questions.

3. Show all working where necessary in the spaces provided.

4. The use of an electronic calculator is allowed.

Consider the graphs defined by $f(x) = \frac{-2}{x+1} - 3$, $g(x) = \frac{-1}{4}(x + 6)(x - 4)$ and $h(x) = -2x - 5$

1. Draw *f*, *g* and *h* on the same system of axes. Clearly label all asymptotes and intercepts with the axes as well as the coordinates of any turning points.

(10)

2. Write down the range and the domain of $f(x)$, $g(x)$ (and $h(x)$).

....

....

....

....

....

....(6)

3. For which value(s) of x is $f(x) = h(x)$

....

....(2)

4. For which value(s) of x *is* $\frac{-1}{4}(x + 6)(x + 4) > 0$?

....

....(2)

5. For which values of x is $(x).h(x) < 0$?

....

....

....(3)

6. If T(1; 4) a point on function f is reflected in the axis of symmetry with positive gradient, determine the coordinates of its image.

....

....

....(5)

7. Express g in the form $a(x-p)^2 + q$. Hence or otherwise write down the equation of the axis of symmetry and the coordinates of the turning point of g.

....

....

....(5)

8. RT is a line parallel to the y-axis, with R on $f(x)$ and T on $h(x)$.

8.1 Determine a simplified expression for the length of RT.

....

....

....

....(2)

8.2 Hence find the maximum length of RT.

....

....

....

....(4)

9. If *f*(x) is translated 2 units to the right and 1 unit up, state the new equation in the form

y =..................... .... (2)

10. If *h*(*x*) is reflected in the y-axis, state the equation of the reflected graph in the form

y = ...(1)

11. If *g*(*x*) is translated 3units to the left and 2 units down, determine the equation of the

translated graph in the form y = $ax^2 + bx + c$.

....

....

....(2)

12. Determine *f*((*h*(-2)).

....

....

....

....(3)

13. Determine the average gradient of *f*(*x*) between:

13.1 $x = -7$ and $x = -2$

13.2 x = -5 and x = -2

13.3 x = -3 and x = -2

....

....

....

....(3x3)

14. Considering the gradients that you found in 13.3, briefly explain the gradient that best represents the gradient at x = -2

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………(2)

15. Solve for x to two decimal places, g(x) = h(x)

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

…………………………………………………………………………………………...(6)

TOTAL: 50 MARKS

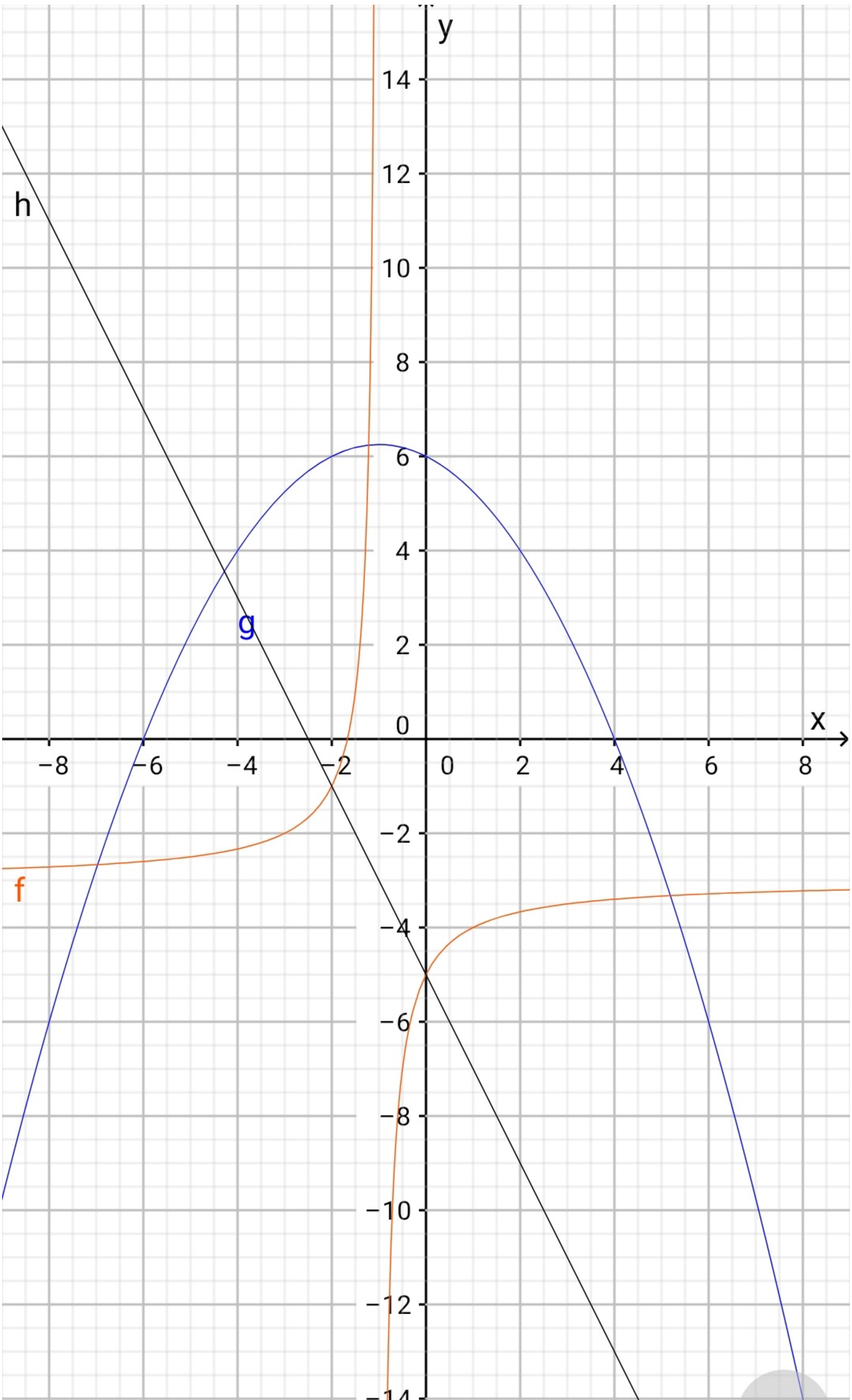
APPENDIX D: MARKING GUIDELINES
MEMORANDUM FOR PRE AND POST TEST

1. See diagram on page 2 and mark as follows:

√ for correct straight line h.

√for shape of g, √correct turning point of g, √√ correct intercepts with axes of g.

√correct shape of f, √√ for asymptotes of f, √√ for correct intercepts with axes.



y
x
h
g
f
14
12
10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
−14
−8
−6
−4
−2
0
2
4
6
8

2. For f , domain: $x \in \mathcal{R}$, $x \neq -1$√

Range: $y \in \mathcal{R}$, $y \neq -3$√

For g, domain: $x \in \mathcal{R}$√

Range: $y \leq \frac{4ac-b^2}{4a}$

$$y \leq \frac{4\left(\frac{-1}{4}\right)(6)-\left(\frac{-1}{2}\right)^2}{4\left(\frac{-1}{4}\right)}$$

$y \leq \frac{25}{4}$.√

For h, domain: domain: $x \in \mathcal{R}$. √

Range: $y \in \mathcal{R}$.√

3. $x = 0$√ or $x = -2$√

4. $-6 < x < 4$√√

5. $x < -2,5$√√ or $x > 0$√

6. Equation of axis of symmetry with positive gradient is:

$y = x + c$, substitute (-1; -3)

$-3 = -1 + c$√

$c = -2$

$y = x - 2$√√, then the image is given by:

$y = 1-2$ and $x = -4 + 2$

∴ $x = -2$√; $y = -1$√

7. $\frac{-1}{4}(x + 6)(x - 4) = \frac{-1}{4}x^2 - \frac{1}{2}x + 6$√

$= \frac{-1}{4}(x^2 + 2x) + 6$

$= \frac{-1}{4}(x + 1)^2 + \frac{25}{4}$√

Axis of symmetry is: $x = 1$√ and turning point $(-1; \frac{25}{4})$√√

8.1 RT = $-\frac{1}{4}(x + 6)(x - 4) - (-2x - 5)$√

RT = $\frac{-1}{4}(x^2 + 2x - 24)$ + 2 x + 5

RT = $\frac{-x^2}{4}$ -$\frac{x}{2}$ + 6 + 2 x + 5

RT = $\frac{-x^2}{4}$ + $\frac{3x}{2}$ + 11√

8.2 RT_{max} = $\frac{4(\frac{-1}{4})(11)-(\frac{3}{2})^2}{4(\frac{-1}{4})}$√√

RT_{max} = $\frac{53}{4}$√√

9. y = $\frac{-2}{(x-2)+1}$ -3+1

y = $\frac{-2}{x-1}$ – 2√√

10. y = -2(-x) – 5

y = 2x – 5√

11. y = $\frac{-1}{4}$ ((x + 3) + 6)((x +3)- 4) – 2√

y = $\frac{-1}{4}$(x + 9)(x - 1) – 2

y = $\frac{-x^2}{4}$ – 2x + $\frac{1}{4}$√

12. h(-3) = -2(-3) – 5

= 1√

f(1) = $\frac{-2}{1+1}$ – 3√

= -4√

13. $\frac{f(-3)-f(-2)}{(-3)-(-2)} = \frac{\frac{-2}{(-3)+1}-3-[\frac{-2}{(-2)+1}-3]}{(-1)}$√√

= 1√

TOTAL: 50 MARKS

APPENDIX E: INTERVIEW GUIDE

1. Performance questions

Referring to the learner's solution in the post-test:

How did you come up with this answer?

Why did you use that procedure?

How did you make the decision?

Can you work it out differently this time?

2. Unexpected questions, example:

Why is the midpoint formula applicable in finding the equation for the axis of symmetry?

3. Twist questions:

If you are given $f(x) = 3^{-x} + 2$, what will be new equation after reflecting $f(x)$ in the y-axis?

4. Construction task:

Given: $p(x) = \frac{-3}{x+1} - 2$ and $q(x) = \frac{3}{x-1} - 2$, find out the transformation that is performed to move *p* to *q*.

5. Give an example task:

Referring to question 8.2 where you were asked to find the maximum length, give another example of a situation where you can apply the method that you used to answer that question and the reason why it is applicable?

6. Reflection:

In question 8.2, you were asked to find the maximum point. Where do you think you can be asked to determine the minimum length between two points on different functions?

7. Attitudes and feelings:

Is Mathematics easy or difficult for you? What makes it easy or difficult for you?

What do you prefer between coming up with your own solutions or being given the solutions by your teacher?

After learning algebraic functions for the second time did you find any changes as far as conceptual understanding is concerned?

What do you think led to the changes that you observed?

How was the teaching and learning approach used in teaching algebraic functions?

What are your suggestions on the teaching and learning of Mathematics?

Are there any additional comments that you would like to make?

APPENDIX F: INTERVIEW TRANSCRIPTION FOR ONE PARTICIPANT
R represents the interviewer (researcher) and L represents the interviewee (Learner 5)

R: Based on your solutions for Question 4 in the two tests. At first you did not get the domains and ranges for all the functions then this time you got all of them correct. What was the problem in the first test?

P: In the first test I did not know what the question was referring to.

R: You mean to say you did not know what range and domain meant?

L: Yes I didn't.

R: So can you now briefly explain the two terms.

L: Domain is the set of all possible values of the independent variable whilst range is for dependent variable.

R: So if you are given any other function, are you able to identify the domain the range.

L: Yes, I now know where to focus my attention on to identify them.

R: Well, what are you saying?

L: I mean to say that if it is a parabola I check its turning point because it gives me the minimum or maximum y values and if it is a hyperbola I know I should exclude the asymptotes.

R: That's great. How about if it is an exponential?

L: I use the asymptote as a boundary for the range.

R: Then comes Question 15 which requested you to find the values of x satisfying $f(x) =g(x)$, in the first test you did not answer the question whilst in the second test you did the question well. Now can you briefly explain what the question actually wants you to find.

L: To my understanding that question says $f(x) = g(x)$, so it means that I must substitute and solve. That is what I did and got the answers.

R: So tell me, is Mathematics a difficult or easy subject to you?

L: Mathematics is difficult for me mam.

R: Why is difficult for you in Mathematics?

L: The subject is boring because a lot of homework and exercises of things that I don’t know are given.

R: Do you want the teacher to give you’re the answers or finding solutions for yourself?

L: It’s better to get answers from the teacher because I hate doing corrections all the time.

L: After learning algebraic functions again, were the any changes that you realised?

L: Yes, I realised that varying class activities can reduce boredom in class and I was also finding some answers for myself.

R: In general, how were the lessons on algebraic functions?

L: They were better as we were given time to think and find friends to explain what we did not understand.

R: Can you give suggestions on what can be done to improve on the teaching and learning of Mathematics.

L: Teachers must help us to get correct answers.

R: Our session ends here. Thank you for participating.

APPENDIX G: LESSON PLANS

LESSON PLANS AND EVALUATIONS FOR THE INTERVENTION

LESSON PLAN TEMPLATE: 1
EDUCATOR: RESEARCHER

SUBJECT: MATHEMATICS	GRADE: 11	TOPIC: ALGEBRAIC FUNCTIONS	DURATION: 1 HOUR

CORE CONTENT: Linear function

LINK WITH PRIOR KNOWLEDGE OF: Grade 10 functions

OBJECTIVES: By the end of the lesson learners should be able to: define and identify: linear function, gradient, y-intercept, increasing or decreasing function, intercepts with axes, domain and range; use function notation

SKILLS: drawing, interpretation, demonstrating, describing, analyzing, communicating, interpreting, problem posing.

Teacher activities	Learner activities	Teaching methods	Resources	Assessment strategy
-Teacher asks learners to define and give examples of functions. Teacher stresses function notation. -Teacher asks learners to determine characteristics of the linear function. -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks	-Learners define and give examples of functions -Learners give the general equation of a linear function and explain what each part of the equation represents. Learners find gradient, y-intercept and also intercepts with the axes. Learners sketch linear functions and answer questions related to the sketch. Learners identify the domain and the range. -Learners given a chance to ask questions. - Learners present solutions of given problems on the board giving	Discussion, Question and answer,	Worksheets, Graph sheets, Pencils, rulers	

	explanations to support their solutions.			

Evaluation: All objectives were covered. Learners did not have problems with function notation. They could identify if a straight line was increasing or decreasing. The sketching of the graph did not give participants problems as they just determined the x and y intercepts and joined the points. Determining the gradient was not a problem as well as finding the y-intercept. However, some of the participants could not interpret questions in which the equation of the straight line was not given in standard form. They could not identify the gradient and the y-intercept.

LESSON PLAN TEMPLATE: 2 EDUCATOR: RESEARCHER

SUBJECT: MATHEMATICS	**GRADE: 11**	**TOPIC: ALGEBRAIC FUNCTIONS**	**DURATION: 2 HOURS**

CORE CONTENT: The hyperbola

LINK WITH PRIOR KNOWLEDGE OF: Grade 10 functions

OBJECTIVES: By the end of the lesson learners should be able to: define and identify hyperbolic function, axes of symmetry, asymptotes, domain and range of hyperbolic functions; sketch hyperbolic graphs.

SKILLS: drawing, demonstrating, describing, analysing, communicating, interpreting, problem posing.

Teacher activities	**Learner activities**	**Teaching methods**	**Resources**	**Assessment strategy**
-Teacher describes the properties of the hyperbola of the form$\frac{a}{x-p}$ + q. - Teacher asks learners to sketch given hyperbolic functions. -Teacher asks learners to describe and explain an asymptote. -Teacher gives learners a chance to identify the domain and the range. -Teacher guides learners to determine axes of symmetry. -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks.	-Learners sketch given functions. -Learners in groups identify axes of symmetry, asymptotes, domain and range for given functions. -Learners given a chance to ask questions. -Learners determine points of intersection of axes of symmetry with the hyperbola. - Learners present solutions of given problems on the board giving explanations to support their solutions.	Discussion, demonstration, Question and answer,	Worksheets, Graph sheets, Pencils, rulers	Group work, pair work and Individual classwork

Evaluation: All objectives were successfully achieved. Participants could draw the hyperbolic graphs though some forgot to draw the other part of the hyperbola especially if that part was not crossing any of the two axes. Determining domain and range was at first a problem as learners did not really understand what an asymptote was. It was noted that participants had misconceptions about an asymptote. They were able to determine and draw it but did not know what it actually represented. The educator then then guided the participants to describe and explain the asymptote. Participants were given tasks to perform that involved substituting functions' asymptote values. The realization that the answers that they were getting were undefined helped them to understand what an asymptote is. This then helped them to determine the range and the domain of a hyperbola. They could find axes of symmetry and interpret them. Reflecting a point on the graph along the axis of symmetry at first gave some of the participants problems of identifying the position of the image. On intersections of axes of symmetry and graphs some had difficulties in solving their created equations which was caused by poor computational background.

LESSON PLAN TEMPLATE: 3
EDUCATORS: RESEARCHER

SUBJECT: MATHEMATICS	GRADE: 11	TOPIC: ALGEBRAIC FUNCTIONS	DURATION: 2 HOURS

CORE CONTENT: The exponential function

LINK WITH PRIOR KNOWLEDGE OF: Grade 10 functions

OBJECTIVES: By the end of the lesson learners should be able to: define and identify exponential functions, asymptotes, and range of exponential functions; sketch exponential graphs.

SKILLS: drawing, demonstrating, describing, analysing, communicating, interpreting, problem posing.

Teacher activities	Learner activities	Teaching methods	Resources	Assessment strategy
-Teacher introduces the exponential of the form y=ab^{x-p}+ q and its properties. -Teacher asks learners to determine increasing and decreasing functions. Teacher asks learners to sketch exponential graphs. -Teacher gives learners tasks to be discussed in groups. -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks	-Learners sketch given functions individually. -Learners in groups identify asymptotes, and range for given functions. -Learners discuss given questions in groups. -Learners given a chance to ask questions. - Learners present solutions of given problems on the board giving explanations to support their solutions.	Discussion, demonstration Question and answer,	Worksheets, Graph sheets, Pencils, rulers	Individual classwork, group work

Evaluation: All objectives were covered. Learners were able to sketch exponential graphs without problems using the table with -1, 0 and 1 as the input values. Identifying the asymptotes and range after sketching the graph was also done without hustles. However just given the equation of a function without the graph saw some learners failing to identify the range.

LESSON PLAN TEMPLATE: 4 EDUCATORS: RESEARCHER

SUBJECT: MATHEMATICS	GRADE: 11	TOPIC: ALGEBRAIC FUNCTIONS	DURATION: 2 HOURS

CORE CONTENT: The parabola (quadratic function) of the form y = $a(x-p)^2$ + q

LINK WITH PRIOR KNOWLEDGE OF: Completing the square

OBJECTIVES: By the end of the lesson learners should be able to: define and identify quadratic functions, x and y intercepts, axis of symmetry, turning points, domain and range of given functions; sketch the parabola.

SKILLS: calculating, interpreting, drawing, demonstrating, describing, analysing, communicating, interpreting, problem posing.

Teacher activities	Learner activities	Teaching methods	Resources	Assessment strategy
-Teacher describes a quadratic function of the form y = $a(x-p)^2$+q -Teacher guided learners to sketch the parabolic function. -Teacher asks learner to apply method of completing the square to given quadratic functions. Teacher asks learners to determine axis of symmetry and range for given functions. -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks	-Learners individually determine shapes and turning points for given functions -Learners individually sketch given functions. -Learners in groups identify axis of symmetry, domain and range for given functions. -Learners given a chance to ask questions. - Learners present solutions of given problems on the board giving explanations to support their solutions.	Discussion, Question and answer,	Worksheets, Graph sheets, Pencils, rulers	Group work, individual work

Evaluation: The sketching of the graph was not a problem. Most of the learners at first struggled with the method of completing the square. However, they finally discussed and assisted each other and ended up understanding conceptually what was actually going on. Writing down turning points, axis of symmetry and the range was not a problem after managing to express the function in the right turning point form.

LESSON PLAN TEMPLATE: 5
EDUCATOR: RESEARCHER

SUBJECT: MATHEMATICS	**GRADE: 11**	**TOPIC: ALGEBRAIC FUNCTIONS**	**DURATION: 2 HOURS**

CORE CONTENT: Parabola (quadratic function) of the form y = ax^2 + bx + c

LINK WITH PRIOR KNOWLEDGE OF: Grade 10 functions

OBJECTIVES: By the end of the lesson learners should be able to: define and identify: intercepts with the axes, axis of symmetry, turning points, domain and range of given functions; determine turning points and sketch the parabola.

SKILLS: interpreting, calculating, drawing, demonstrating, describing, analysing, communicating, interpreting, problem posing.

Teacher activities	**Learner activities**	**Teaching methods**	**Resources**	**Assessment strategy**
-Teacher explains how coordinates of turning points are got. -Teacher then asks learners to sketch given quadratic graphs in groups. -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks	-Learners individually find turning points for given functions Learners find intercepts with the axes for given functions. -Learners in groups identify axis of symmetry, domain and range for given functions. -Learners given a chance to ask questions. - Learners present solutions of given problems on the board giving explanations to support their solutions.	Discussion, Question and answer,	Worksheets, Graph sheets, Pencils, rulers	Individual and group classwork

Evaluation: Set objectives were achieved. Finding intercepts with axes did not give learners problems though few had problems here and there with solving quadratic equations. Learners could determine turning points and also sketch given functions without difficulties. Some of the learners easily appreciated the use of the midpoint formula to find the x-coordinate of the turning point as well as the axis of symmetry.

LESSON PLAN TEMPLATE: 6
EDUCATOR: RESEARCHER

SUBJECT: MATHEMATICS	GRADE: 11	TOPIC: ALGEBRAIC FUNCTIONS	DURATION: 2 HOURS

CORE CONTENT: Determining the equation of a parabola given turning points, intercepts and/or any other point.

LINK WITH PRIOR KNOWLEDGE OF: Substitution and solving equations

OBJECTIVES: By the end of the lesson learners should be able to determine equation of a parabola given: a turning point and any other point, the x-intercepts and any other point as well as the y-intercept and any other two points.

SKILLS: interpreting, calculating, drawing, demonstrating, describing, analysing, communicating, interpreting, problem posing.

Teacher activities	Learner activities	Teaching methods	Resources	Assessment strategy
-Teacher guides learners to determine an equation given a turning point and any other point. -Teacher gives and explain the other case where x-intercepts and another point are given and has to be expressed in the form [a$(x - x_1)(x - x_2)$] -Teacher gives the last case where the y intercept and any other two points are given. -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks	-Learners find equations representing each given case in groups then individually. -Learners given a chance to ask questions. - Learners present solutions of given problems on the board giving explanations to support their solutions.	Discussion, Question and answer,	Worksheets, Graph sheets, Pencils, rulers	

Evaluation: Given the turning point and any other point, learners could substitute though some after substitution had computation problems as a result failed to get the correct value of *a*. After finding *a*, some of the learners again had problems in simplifying the expression. In the other case where x-intercepts were given, some learners still the same problem of failing to simplify after correct substitution. Otherwise they could easily identify the approach to be used and substitute properly. However, most of the participants had application problems. This was seen when the graphs were drawn and the information given was not straight forward for example when a straight line was together with the parabola and was to be used to find either one of the x-intercepts of the additional point. It was difficult for them to link how the straight line could be used to determine points on the parabola.

LESSON PLAN TEMPLATE: 7 EDUCATORS: RESEARCHER

SUBJECT: MATHEMATICS	GRADE: 11	TOPIC: ALGEBRAIC FUNCTIONS	DURATION: 2 HOURS

CORE CONTENT: Translation and reflection

LINK WITH PRIOR KNOWLEDGE OF: Grade 11 functions

OBJECTIVES: By the end of the lesson learners should be able to: shift a graph to the left or right, upwards or downwards and identify its effects to the equation; reflect a graph in the x-axis or y-axis and well as identifying the effects to the equation of the function.

SKILLS: demonstrating, describing, analysing, communicating, interpreting, problem posing.

Teacher activities	Learner activities	Teaching methods	Resources	Assessment strategy
-Teacher guides learners to identify the effects on the equation of shifting the graph upwards or downwards. -Teacher demonstrates reflecting objects in the y and x-axis and ask learners to identify and generalize the effects on signs of x and y-values. -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks	-Learners write new equations after a given vertical or horizontal shift. -Learners in pairs also determine the shift that has occurred given an equation and the new one after the shift. -Learners in groups give new equations after a given reflection or the reflection that has occurred given the original and the new equation. -Learners given a chance to ask questions. - Learners present solutions of given problems on the board giving explanations to support their solutions.	Discussion, Question and answer,	Worksheets, Graph sheets, Pencils, rulers	Group work, pair work and individual classwork

Evaluation: Objectives were successfully achieved. Learners could easily generalize vertical and horizontal shifts as well as reflections in both axes. The practical part of shifting or reflecting a given graph helped learners to grasp the concepts. They also managed to determine new equations after shifting or reflection. Describing the type of reflection that would have occurred gave some of the learners a bit of stress. However, in their groups they explained and understood each other.

LESSON PLAN TEMPLATE: 8
EDUCATORS: RESEARCHER

SUBJECT: MATHEMATICS	GRADE: 11	TOPIC: ALGEBRAIC FUNCTIONS	DURATION: 2 HOURS

CORE CONTENT: Graph interpretation

LINK WITH PRIOR KNOWLEDGE OF: Grade 11 functions

OBJECTIVES: By the end of the lesson learners should be able to: find required lengths, identify when graph is negative or positive, determine average gradient; interpret and make deductions from graphs.

SKILLS: calculating, drawing, demonstrating, describing, analysing, communicating, interpreting, problem posing.

Teacher activities	Learner activities	Teaching methods	Resources	Assessment strategy
-Teacher asks learners to determine stated lengths. -Teacher explains maximum or minimum length between two graphs. -Teacher asks learners to calculate average gradient between two points. -Teacher explains where graph is positive and where it is negative -Teacher gives learners a chance to ask questions on the learnt aspects. -Teacher allocates time for illustrations of individual tasks	-Learners calculate required lengths individually. Learners individually calculate average gradient. -Learners determine an expression for the length between two graphs and the actual length. -Learners in groups interpret and make deductions from given graphs. -Learners given a chance to ask questions. - Learners present solutions of given problems on the board giving explanations to support their solutions.	Discussion, Question and answer,	Worksheets, Graph sheets, Pencils, rulers	Individual classwork, group work.

Evaluation: The interpretations required learners to be given more time to grasp the ideas. Given graphs, some of the participants got confused by questions that involved more than one graph. They could not link the graphs easily. They had problems in realizing the relationship between the two graphs. This was noticed when they were asked to find the either the expression for length between the two graphs or the points of intersection for the two graphs.

Again when asked to identify the parts of the graph where the function was greater than zero or greater than the other function some of the participants struggled. They were given time to discuss in groups and it helped most of them as at first, they were stuck but later on they could answer the questions though they took time to do so. They also asked the educator for clarifications on problems involving inequalities.

APPENDIX H: LETTER TO THE DEPARTMENT OF EDUCATION

Giyani High School

Private Bag X9597

Giyani

0826

15 May 2018

The Circuit Manager

Man'ombe Circuit- Mopani 2 District

Mopani District

Dear Sir

RE: PERMISSION TO CONDUCT A STUDY AT GIYANI HIGH SCHOOL

I am a doctorate student specializing in Mathematics Education with Unisa. My dissertation supervisor is Professor Ngoepe M. G. I am requesting to conduct a research at Giyani High school on Grade 11 learners.

The title of my study is: EXPLORING THE EFFECTS OF CONCEPT BASED INSTRUCTION ON THE TEACHING AND LEARNING OF ALGEBRAIC FUNCTIONS IN MOPANI DISTRICT, LIMPOPO PROVINCE, SOUTH AFRICA. The study aims to investigate the effects of concept based instruction teaching and learning approach in a way trying to away with traditional teaching approaches that that ex[pose learners to passive learning environments. The intention is to move learners towards deeper conceptual understanding and hence improve performance in Mathematics.

Learners will be taught using the concept-based teaching and learning approach. They will write two tests and complete a questionnaire. Furthermore, I will interview some of the participants depending on their responses in the tests. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks. There will be no reimbursement or any incentives for participation in the research.

For more information concerning this request you can contact me at 073 713 4810 or at mildretncube@ymail.com or contact my supervisor Prof Ngoepe M. G at 012 429 8375 or at ngoepmg@unisa.ac.za

Yours faithfully,

Mildret Ncube

APPENDIX I: RESPONSE FROM THE CIRCUIT MANAGER

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

MOPANI DISTRICT – MAN'OMBE CIRCUIT

Enquiries : Monkwe K.W
Tel. No : 015 812 0637

Date: 25 May 2018

The Principal
Giyani High School
Private Bag X9597
GIYANI
0826

ACKNOWLEDGEMENT OF RECEIPT OF THE RECOMMENDATION FOR NCUBE M. TO CONDUCT RESEARCH AT YOUR SCHOOL

1. The above matter bears reference.
2. The recommendation for Ncube Mildret to conduct research at your school is highly acknowledged.
3. The research programme should be conducted after school hours to avoid the interference with the academic programme. However, neither the student nor the School Manager indicated the date when the research will be conducted.
4. We hope you will assist the student with whatever professional information she might need. Furthermore, we greatly invest in your support in this regard.

....
CHABALALA M.E: CIRCUIT MANAGER

DEPARTMENT OF EDUCATION
MAN'OMBE CIRCUIT
PRIVATEBAG X 9654 GIYANI 0826
25 MAY 2018
TEL: 015-812 0637/4421
CIRCUIT DIRECTOR
CHABALALA M.E.
LIMPOPO PROVINCE

DEPARTMENT OF EDUCATION

MOPANI DISTRICT, Man'ombe Circuit, Private Bag X 9654 GIYANI, 0826

Tel 015 812 0637 Fax No. 015 812 4421 or 015 812 1689

The heartland of Southern Africa – development is about people

APPENDIX J: LETTER TO THE PRINCIPAL

Giyani High School

Private Bag X9597

Giyani

0826

15 May 2018

The Principal

Giyani High School

Dear Sir,

RE: PERMISSION TO CONDUCT A STUDY AT GIYANI HIGH SCHOOL

I am a Doctorate student specializing in Mathematics Education with Unisa. My dissertation supervisor is Professor Ngoepe M. G. I am requesting to conduct a research at Giyani High school on Grade 11 learners.

The title of my study is: EXPLORING THE EFFECTS OF CONCEPT BASED INSTRUCTION ON THE TEACHING AND LEARNING OF ALGEBRAIC FUNCTIONS IN MOPANI DISTRICT, LIMPOPO PROVINCE, SOUTH AFRICA. The study aims to investigate the effects of concept based instruction teaching and learning approach in a way trying to away with traditional teaching approaches that expose learners to passive learning environments. The intention is to move learners towards deeper conceptual understanding and hence improve performance in Mathematics.

I intend to teach using the concept based instruction approach and administer questionnaires and tests to Grade 11 learners at your school. Furthermore, I will interview some of the participants depending on their responses in the tests. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks. There will be no reimbursement or any incentive for participating in the research.

For more information concerning this request you can contact me at 073 713 4810 or at mildretncube@ymail.com or contact my supervisor Prof Ngoepe M. G at 012 429 8375 or at ngoepmg@unisa.ac.za

Yours faithfully,

Mildret Ncube

APPENDIX K: RESPONSE FROM PRINCIPAL

GIYANI HIGH SCHOOL

ABORE AD LUCEM

GIYANI HIGH SCHOOL

Private Bag X9597
GIYANI
0826
☎(015) 812-3259
FAX (015) 812 2011
giyanihigh@gmail.com

Ref. No: 9 164 10 164
Enquiries: Khosa M.E (081 040 2432)

21 May 2018

The Circuit Director
Man'ombe Circuit
Private Bag X9597
Giyani
0826

Dear Sir

RECOMMENDATION TO ALLOW UNISA STUDENT (MILDRET NCUBE) TO CONDUCT A STUDY AT GIYANI HIGH SCHOOL.

This letter serves as a recommendation for permission to conduct a study at the above mentioned school. The student requesting the permission is one of the SGB employees at our school, hence I am recommending that she be granted the permission.

Attached to this letter are the application letters to School Manager, Circuit Director and District Director as well as proof for her registration with UNISA as student.

I hope you will find the matter in order.

Yours Faithfully

Khosa M.E (School Manager)

DEPARTMENT OF EDUCATION
GIYANI HIGH SCHOOL
PRINCIPAL
EMIS No.: 916410164
21 MAY 2018
PRIVATE BAG X 9597
TEL:0158123259 FAX:0158122011
LIMPOPO PROVINCE

APPENDIX L: CONSENT LETTER

UNISA | college of education

LETTER TO THE PARENT/GUARDIAN

DATE:

Dear parent/guardian

RE: <u>A REQUEST FOR YOUR CHILD'S PARTICIPATION IN RESEARCH STUDY</u>

I am a Doctorate student specializing in Mathematics Education with Unisa. My dissertation supervisor is Professor Ngoepe M. G. I am requesting to conduct a research at Giyani High school on Grade 11 learners.

The title of my study is: EXPLORING THE EFFECTS OF CONCEPT BASED INSTRUCTION ON THE TEACHING AND LEARNING OF ALGEBRAIC FUNCTIONS IN MOPANI DISTRICT, LIMPOPO PROVINCE, SOUTH AFRICA. The study aims to investigate the effects of concept based instruction teaching and learning approach in a way trying to away with traditional teaching approaches that that ex[pose learners to passive learning environments. The intention is to move learners towards deeper conceptual understanding and hence improve performance in Mathematics.

I intend teaching around 40 Grade 11 learners using concept-based instruction teaching and llearning approach. The learners will write two tests and complete a questionnaire. Some of the learners will be interviewed, depending on their solutions in the tests. The lessons will commence after school hours on Monday to Thursday for four weeks. Each lesson will be for an hour. I am therefore asking for your permission to allow your child to be one of the participants in this study. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research.

Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks. There will be no reimbursement or any incentives for participating in the research.

There is attached form at the back of this letter for you to indicate your decision to allow your child to take part in the study. May you please complete it and return it to me at your earliest convenience.

For more information concerning this request you can call me at 0737134810 or email me at mildretncube@ymail.com or contact my supervisor Prof Ngoepe M. G at 012 429 8375 or at ngoepmg@unisa.ac.za

Your cooperation will be greatly appreciated.

Yours faithfully,

Mildret Ncube

PARENT/GUARDIAN CONSENT FORM

Please fill in the reply slip on granting permission to your child to participate in the study.

I...have read and understood the conditions of the study.

My child .... can/cannot take part in the study. (Delete the inapplicable).

Parent/guardian's signature... Date……………………........

APPENDIX M: LETTER TO THE LEARNER

DATE:

Dear Learner,

I am a Doctorate student specializing in Mathematics Education with Unisa. My dissertation supervisor is Professor Ngoepe M. G. I am requesting to conduct a research at Giyani High school on Grade 11 learners.

The title of my study is: EXPLORING THE EFFECTS OF CONCEPT BASED INSTRUCTION ON THE TEACHING AND LEARNING OF ALGEBRAIC FUNCTIONS IN MOPANI DISTRICT, LIMPOPO PROVINCE, SOUTH AFRICA. The study aims to investigate the effects of concept based instruction teaching and learning approach in a way trying to away with traditional teaching approaches that that ex[pose learners to passive learning environments. The intention is to move learners towards deeper conceptual understanding and hence improve performance in Mathematics.

I invite you to participate in my research study. You will complete a questionnaire and write two short tests then, depending on your responses in the tests can be interviewed on how you would have come up with your answers. The study will be conducted for three weeks, Monday to Thursday, one hour per day. Be assured that you your participation in the study will have no bearing on your grades or evaluation in the subject. The right to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. No one will be able to connect to the information that you will provide in the study. Participation is voluntary and if you decide to withdraw, you can do so at any time without penalty. Participation in the study has no foreseeable risks. There will be no reimbursement or any incentive for participating in the research.

There is an attached form for you to indicate your decision to take part in the study. Discuss your involvement in the study with your parents then complete the form and return it to me at your earliest convenience. A letter has also been sent to your parents to indicate their decision concerning your participation.

For more information concerning this request you can call me at 073 713 4810 or email me at mildretncube@ymail.com or contact my supervisor Prof Ngoepe M. G at 012 429 8375 or at ngoepmg@unisa.ac.za

Your participation will be greatly appreciated.

Yours faithfully,

Mildret Ncube

LEARNER'S ASSENT FORM

Please fill in this form to indicate your decision to participate in the mentioned study.

I...have read and understood the conditions for the study. I accept/do not accept to participate in the study. (Delete the inapplicable).

Learner's signature.. Date……………

APPENDIX N: TURNITIN REPORT

Turnitin Originality Report
DISSERTATION WITH ALL CHAPTERS by Mildret Ncube
From Revision 1 (CEDU M&D Students)

- Processed on 17-Jan-2022 22:51 SAST
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APPENDIX O: LANGUAGE EDITING CERTITICATE

EDITING AND PROOFREADING CERTIFICATE

7542 Galangal Street

Lotus Gardens

Pretoria

0008

21 December 2021

TO WHOM IT MAY CONCERN

This certificate serves to confirm that I have language edited M Ncube's thesis entitled, **"EXPLORING THE EFFECTS OF CONCEPT-BASED INSTRUCTION IN THE TEACHING AND LEARNING OF MATHEMATICS: A CASE OF ALGEBRAIC EXPRESSIONS."**

I found the work easy and intriguing to read. Much of my editing basically dealt with obstructionist technical aspects of language, which could have otherwise compromised smooth reading as well as the sense of the information being conveyed. I hope that the work will be found to be of an acceptable standard. I am a member of Professional Editors' Guild.

Hereunder are my contact details:

Jack Chokwe (Mr)

Contact numbers: 072 214 5489

jackchokwe@gmail.com

Professional EDITORS Guild

Jack Chokwe
Associate Member

Membership number: CHO001
Membership year: March 2021 to February 2022

076 471 6881
012 429 3327
jackchokwe@gmail.com
www.academicproeditor.com

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