THE IMPACT OF TECHNOLOGY INTEGRATION IN TEACHING GRADE 11 EUCLIDEAN GEOMETRY BASED ON VAN HIELE'S MODEL

by

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Submitted in accordance with the requirements for the degree of

MASTER OF EDUCATION

in the subject

MATHEMATICS EDUCATION

at the

UNIVERSITY OF SOUTH AFRICA

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October 2021

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The impact of technology integration in teaching grade 11 Euclidean geometry based on van Hiele's model.

I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the dissertation to originality checking software and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at Unisa for another qualification or any other higher education institution.

10 October 2021

SIGNATURE DATE

DEDICATION

To Yahweh be the glory for providing me with the strength to reach this far.

To my son Bismark, and my twins Prince and Princess, for the inspiration they provide me each day.

ACKNOWLEDGEMENTS

I wish to extend my sincere gratitude to my supervisor, Professor Zingiswa Mybert Monica Jojo. What an inspiration she has been in my research journey. Your guidance, including those constructive criticisms, has really shaped this study. Thank you, Professor.

A special thank you to my family for their physical and emotional support.

To the Free State Department of Education, I say thank you for your permission for the use of your learners for this research. A big thank you to the Subject Education Specialists and educators who reviewed the research instruments of this study. To all the Grade 11 learners who took part in this study, I really appreciate your input.

Finally, I wish to acknowledge the timely financial assistance (M&D bursary) provided to me by the Directorate of Student Funding (DSF) of UNISA. A further thank you to the Research Ethics Committee of UNISA for the ethical clearance.

ABSTRACT

This quantitative study reports on the impact of using GeoGebra software to teach Grade 11 geometry through van Hieles' levels theory, merged with some elements of the Technological Pedagogical Content Knowledge framework. Control (n=30) and experimental (n=30) groups were purposefully sampled from two secondary schools in Motheo District of Free State, South Africa. The experimental and control groups were taught using GeoGebra software and traditional teacher-centered instruction, respectively. The two groups wrote pre and post-tests on all six van Hieles' levels. Both groups significantly improved in the post-tests at each van Hieles' level. However, the experimental group's true difference in population means (-39.57; -32.90) outweighed that of the control group (-15.65; -10.15).

The study recommends teaching geometry to learners based on their cognitive abilities instead of their age groups. In addition, the effective use of smartphones in geometry lessons is recommended. Finally, geometry lessons should be planned.

Key terms

GeoGebra software; geometry; technology integration; van Hieles' levels; van Hieles' phases; cognitive levels; technological knowledge; pedagogical knowledge; content knowledge; smartphones.

ACRONYMS

CAPS: Curriculum and Assessment Policy Statements, South Africa

DoBE: Department of Basic Education, South Africa.

FET: Further Education and Training phase, South African.

FSDoE: Free State Department of Education, South Africa.

GET: General Education and Training phase, South Africa.

ICT: Information and Communications Technology

MoE: Ministry of Education, Ghana.

NSC: National Senior Certificate, South Africa.

PDA: Personal Digital Assistant

TPACK/TPCK: Technological Pedagogical Content Knowledge

VHL: van Hieles' levels

VHP: van Hieles' phases

VHLT: van Hiles' level theory

PrV of PrT represents Pre-visualization of pre-test

PrV of PoT: Pre-visualization of post-test

V of PrT: Visualization of pre-test

V of PoT: Visualization of post-test

A of PrT: Analysis of pre-test

A of PoT: Analysis of post-test

Ab of PrT: Abstraction of pre-test

Ab of PoT: Abstraction of post-test

D of PrT: Deduction of pre-test

D of PoT: Deduction of post-test

Rg of PrT: Rigor of pre-test

Rg of PoT: Rigor of post-test

VHL1-6 of PrT: combined Van Hieles' levels 1 to 6 of pre-test

VHL1-6 of PoT: combined Van Hieles' levels 1 to 6 of post-test.

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CHAPTER ONE: INTRODUCTORY ORIENTATION

1.1 Overview of the Study

This is a quantitative study that comprises five incremental chapters. This chapter sets the stage for the entire study by providing an overview, introduction to the study, background, rationale, purpose, and significance of the study. In addition, the research aims and objectives are highlighted together with the limitations and delimitations of the study. The chapter further provides a summary of the reviewed literature, methodology, validity and reliability of tests, the analysis procedure, as well as the ethical considerations. It ends with brief explanations of the key terms used in this research.

1.2 Introduction

According to Rideout, Foehr and Roberts (2010:2-4), in a day, 8-18year old children spend over $7\frac{1}{2}$ hours watching television, playing video games, surfing the web, and listening to music. This implies that the new generation pays more attention to technology-delivered programs. They, therefore, tend to spend most of their time on their mobile phones, laptops, social media, software, etc. This raises the question of whether those technological tools have any place in today's world of teaching and learning mathematics. Should teachers continue teaching learners whose interest in technology grows by the day using the same traditional teaching strategies? These are some of the very intriguing questions that are faced by today’s dynamic educators. Dynamic teachers do not stick to one teaching approach for all topics but prefer to analyze mathematical content before choosing the method or environment that would suit the topic. However, John Dewey, the educationist, once said, "if we teach today's students as we taught yesterday's, we rob them of tomorrow" (Pilgrim, Bledsoe & Reily, 2012:16). It is worth mentioning that sticking to the traditional talk-and-chalk method would mean an endorsement of the same absolutists’ beliefs that seem to undermine the intellectual capabilities of individuals or groups of learners, who have a genuine will and passion to explore the world of mathematics through technology. Absolutists are of the philosophical view that learning takes place by reading well-known books with no consideration for the ability of the learner to construct their concepts through the help of, for instance, technology. In addition, absolutists believe in the teacher being the main provider of information to the learner. Absolutism is a learning philosophy that encourages the behaviorists' approach to teaching and learning, where the teacher is at the center of the learning process as opposed to the learner. Perhaps it is time for teachers to stop bombarding learners with information and give them the necessary tools to construct their geometric concepts, and this is where technology integration comes in handy.

Ironically, the previous centuries might have considered the traditional teaching approach relevant during their time, while information was accessed via newspapers, books, libraries, etc. But today, one only needs to pick their personal digital assistant (PDA) or a mobile phone to access millions of articles and publications which prior to technology exposure, was not so easy to retrieve. Nowadays, most learners use social networks and many other technological tools to enrich their own learning. Interestingly, our learners seem fully aware of the gap between the teaching methods used for learning in the classroom and the reality out there. From personal experience, there are situations where learners, on the blind side of the teacher, would be busily browsing the content the teacher is presenting from their phones during the teaching process. For instance, the teacher might be revising Grade 12 past examination questions through talk-and-chalk, but the learner might be using free educational software like HeyMath to check and provide answers to the teacher. Arguably, there seems to be some great mismatch between the teaching approaches used by educators and the level of technological background high school learners possess. Perhaps, learners secretly use technology during or after lessons to draw teachers' attention to the notion that they probably need educators to bring the lesson to their technological level. It could also be an indication of the reality regarding how students of today learn with technology. Whichever the case may be, this great mismatch arising out of teachers' continuous use of the traditional transmission approach in teaching technologically inclined learners requires urgent redress. Unfortunately, a number of our classrooms have shown little sign of change to match the modern trend. Some educators stick to the theories of learning such as behaviorism, cognitivism, and constructivism which they acquired years ago, to teach today's learners. Although no single learning approach could be considered best, perhaps considering the modern trends in teaching is commendable. The modern trends include the daily exposure of learners to cell phones, laptops, etc., where learners probably prefer watching videos of lesson presentations as against the transmission approach encouraged by proponents of behaviorism. In a related example, a learner might be struggling to use established formulae to calculate the general or missing terms of number patterns. Meanwhile, that learner plays many games on their phones that use diagrams to form patterns. This is where teachers could take advantage of to maximize the learning outcome by dwelling on learners' desire to use technology. However, with the abundance of literature to support technology inclusion, one still wonders why most educators are hesitant to link related educational technology into their lessons. There may be genuine concerns as to why there is limited adoption of technology integration in schools. Some researchers (Agyei, 2013; Akcay, 2017) lament the lack of resources, training of educators on the effective integration of technology, among others. Irrespective of the constraints out there, learners still deserve to be taught in a manner that maximizes their inherent potential, and what better way to do this than through technology integration.

In addition, Xu, David, and Kim (2018:91) contend that the world is quickly shifting from the 3rd industrial revolution where information technology was mainly used to boost production, to the 4th revolution where production, management, governance, and almost all aspects of human lives are managed through connected technology in a digital world. This implies that educators and the schooling system have a responsibility as an agent of change. Schools should be able to produce learners that are capable of meeting the demands of the constantly evolving society. However, by engaging our learners through the integration of relevant educational tools and software, mathematics teachers would all be helping to make the 4th industrial revolution (4IR) a reality and, in turn, make the world a better place. Perhaps the time has come for South Africa to advance the prospects that come with the 4IR, which encompasses technology integration, such as the ability of educational technology to motivate learners to explore geometry on their own. Probably, the 4IR is a call for mathematics educators to find a better alternative to the traditional teaching approach when it comes to teaching geometry. The alternative, from the available literature and prevailing state in which the country finds itself, seems to point to integrating technology into the teaching process, to help learners achieve more in terms of their understanding and application of geometric concepts.

1.3 Background of the Study

The South African education system has gone through a lot of reforms pre-and post-independence. The reforms brought about the Outcomes-Based Education (OBE), the National Curriculum Statements (NCS) of 2007, and currently, the Curriculum and Assessment Policy Statements (CAPS) effective 2012. These reforms were necessitated by the growing demands of society. However, in the aforementioned, Euclidean geometry became an optional Paper 3 component in the high school system from 2007 until the 2012 reforms (DoE, 2003, 2011). Perhaps, the relevance and the practicality of geometry to our everyday lives were more than enough to grant its inclusion into the CAPS. It was the implementation of CAPS that brought back the topic to be compulsory for all high school mathematics learners in the country. This could be attributed to the desire of the country to empower its future leaders with critical thinking skills needed for solving problems that require spatial reasoning skills which Euclidean geometry seeks to do, as implied under the principles and aims of CAPS (DoBE, 2011:4). The inclusion of the topic also hopes to make learners ready for the challenges in the job market as employers usually require the innovative services of employees which Euclidean geometry readily offers through its riders. Perhaps, the compulsory nature of Euclidean geometry would help correct the educational imbalances of the past by providing equal opportunities for further education to all students.

According to CAPS, the Grade 11 Euclidean geometry deals with circle theorems, proofs, and their applications (DoBE, 2011:34; Phillips, Bason & Botha, 2013:207-246). In addition, the topic allows for the acceptance of related axioms from previous grades. One such axiom is that a radius is perpendicular to the tangent at the point of contact, which is useful in solving many geometry problems. Those geometry problems include situations where one needs to prove the tangent-chord theorem or its application. Moreover, geometry currently contributes to 33.3% of the entire Paper 2 questions of both Grades 11 and 12 final examinations (DoBE, 2011:55). This implies that Euclidean geometry constitutes more than the required pass percentage of mathematics nationwide and that a learner could pass Paper 2 with Euclidean geometry alone since the pass mark for the paper is set at 30%.

However, geometry is perceived to be difficult even at the international level (Agyei, 2013). According to researchers (Abdullah & Zakaria, 2013a; Haviger & Vojkůvková, 2014; Mthethwa, 2015), geometry scares some educators and learners. However, recent diagnostic reports (DoBE, 2015:162-164; 2017b:151; 2018b:143-144) indicate that learners have improved in solving routine problems in Euclidean geometry. The challenge is that learners still struggle in interpreting complex geometry problems such as riders. According to the findings, that challenge may be due in part to how subject educators approach the topic in their everyday interactions with students, particularly in the early grades (DoBE, 2017b:171). To corroborate the findings from the diagnostic reports, researchers (Manganyana et al., 2020; Ogundile et al., 2019) assert that it is not only learners who dislike geometry but some teachers do too. The researchers claim teachers who mostly fear geometry are those who were deprived same during their schooling and tertiary years. Learners' geometry performance in rural schools seems worse (Bayaga et al., 2019:33). However, the 2015 diagnostic report showed that learners achieved 34% on geometry questions that required application, and 38% on those that involved analysis (DoBE, 2015). From the above discussion, it seems that Euclidean geometry in South African high schools has not been producing enough critical thinkers, as intended by the curriculum planners. For example, learners have consistently been scoring better on low-cognitive geometry questions but not in the higher order questions (DoBE, 2015; 2017b; 2018b: 2019b). Geometry is mostly associated with the development of learners' cognition required for problem-solving (Bayaga et al., 2019). However, those problem-soving skills required in the job market seem limited in South Africa (Jonck, 2014). Therefore, learners scoring lower marks in geometry during examinations implies that their level of preparedness and their opportunities in securing jobs is low. Available literature (Bhagat & Chang, 2015; Faruk Tutkun & Ozturk, 2013; Gweshe, 2014; Jelatu et al., 2018; Khalil et al., 2019; Kutluca, 2013; Manganyana et al., 2020; Mthethwa, 2015) supports the use of GeoGebra and other technological tools as a corrective measure. What seems limited is the use of GeoGebra software to enhance learners' cognitive abilities in rural settings. However, if the outcome of this study in rural schools proves successful, then educators may be empowered to adopt GeoGebra as a technology integration tool to help improve learners' academic performance.

Furthermore, Euclidean geometry could be taught in a manner that harnesses learners' real-life experiences (Clements & Battista, 1992; Masilo, 2018). Learners experience geometry in many forms daily as they play with football, kites, cylindrical cups in their homes, rectangular and square boxes, etc. This means that whenever teachers select geometry problems such as proving riders, the contents need to be realistic and taken from those experiences that are directly connected to learners' socio-cultural setting of which technology is a vital part. A learner's socio-cultural setting embodies the surroundings, way of life, physical amenities, and others that help mold their total development. Fortunately, the socio-cultural environment of today's learners is surrounded by technology; they opt for video games instead of their traditional ones, some prefer using interactive software on their phones to solve basic arithmetic problems as against using manual computations. However, access to these technological devices would depend on the economic status of learners' families and the level of internet connectivity in the particular locality, which mostly favors the urban areas. These revelations imply that today's classroom instruction, especially around the villages and townships, should be structured around learners' active environment that challenges them to freely visualize and logically analyze given geometry problems.

Due to financial and logistical constraints, it might prove difficult for the South African Department of Education to fix, at once, all challenges on the provision of technological devices to schools. Although not all classrooms are equipped with technological tools, individual schools could start the technology integration drive by investing in teacher development, and the needed technological devices. That notwithstanding, no single teaching approach could be considered superior over the other. However, many researchers (Biccard & Wessels, 2015; Osmanoglu & Dincer, 2018) agree that the traditional ways of teaching do little to develop learners' understanding and construction of their own concepts. The traditional teaching approach is the transmission system that considers learners as mere recipients of information from the presumably all-knowing teacher. Unfortunately, a teacher who lacks knowledge in a certain aspect of a topic implies that the whole class could be heading for failure if this traditional teacher-centered method is used. On the contrary, technology has been and is still an important tool for the cognitive development of learners (Yakymchuk & Kazachenok, 2018). Cognitive development refers to the ability of the learner to process whatever information that they receive by reorganizing, explaining, or making changes to them. The use of the abacus, for instance, helps learners to master numeracy, and the same can be said of using calculators, mobile phones, or the internet to explore, learn, or adapt different mathematical concepts. Consequently, educators may change their traditional teaching approach to include technology if they do not want to continue putting the future of mathematics at risk.

Moreover, despite the lower-income status of most African countries, efforts are being made to embrace the use of technological tools in the teaching and learning of mathematics, precisely, geometry. Ghana, for instance, has through reforms in 2008, 2015, and 2019, made information and communications technology a compulsory subject for all basic and high school students and recommends its integration within "learner-centered classrooms in teacher-centered schools" (MoE, 2018:2). Having a learner-centered classroom implies developing all classroom activities around the learner. In that classroom, it is learners who lead group and individual discussions on projects, investigations, experiments, etc. The teacher would be there to provide supervision without necessarily thinking for the learners. On the contrary, teacher-centered schools are characterized by educators being responsible for the total welfare of the learner to ensure that learners enjoy learning. This includes making improvisations for situations where teaching and learning materials are lacking.

However, since technological devices like phones or computers cannot be improvised to yield the same result as the actual technological devices, efforts need to be made to ensure their availability for learning. Regrettably, this is where most African countries are not doing well as a result of their economic conditions or political will. There seem to be insufficient technological devices in almost all African schools. The African countries are unable to provide enough resources such as laptops and licensed educational software and develop many teachers for effective technology integration (MoE, 2015, 2018, 2019). However, researchers (Mereku & Mereku, 2015) concord that the challenge of insufficient technological resources is not so different in Mauritius, Nigeria, Senegal, and many other African countries. These countries share the same problem as they all have deficits in infrastructure and technological tools. The challenges with technology provision in African schools pose a serious threat to technology integration for the continent.

In South African high schools, there are subjects such as computer applications technology (CAT), and information technology (IT). However, those subjects are optional, and as such do not necessarily serve as technology integration in teaching and learning of other subjects. Technology integration deals with making use of available educational technology to improve teaching and learning. Although technology education, as a subject, is not compulsory in all high schools in the country, the South African Department of Education (DoE) has put in place measures to ensure that some schools are equipped with the required technological resources ranging from laptops, whiteboards, projectors, etc., to the internet connection in schools. Moreover, the provision of these technological gadgets to schools does not only assist mathematics educators in their quest to integrate technology, but it also helps the school as a whole for their day-to-day administrative work. Ramorola (2010) complains that despite most South African schools having some technological tools, their integration into teaching and learning is not encouraging. Perhaps, the failure of teachers to integrate the available technology could be related to teacher belief systems or lack of teacher development, or both. For example, a teacher may not integrate technology in teaching due to the belief that learners only get assessed through the use of pen and paper. On the contrary, a learner who can acquire and understand the basics in the various theorems through the help of technology is in a better position to apply the same in examinations and different aspects of real life.

In an attempt to encourage technology integration, the Free State Department of Education (FSDoE) has been supplying Hey-Math laptops and software to schools and updating them yearly. Those measures have been in place since the inception of CAPS with the sole aim of helping mathematics educators integrate technology into their daily teaching. This initiative is an attempt to make mathematics more interesting for both learners and educators. However, researchers (Schleicher, 2012:44; Drijvers, 2013:15) warn education practitioners of not to get carried away by technology. They argue that technology alone is not enough. In other words, technological tools should not be considered as some magic wands because it takes the skill and careful planning of a teacher to make a technology-integrated lesson useful. This implies that technology per se is not capable of replacing poor teaching practices. For instance, an educator’s failure to build a lesson from the known to the unknown may not yield the intended objectives, and that cannot be blamed on the technology used. This is where educators need to be reminded that teaching is an art and that one needs to conduct a situational analysis, and plan all activities well, and in advance, to be able to reach the lesson's set objectives. Furthermore, to integrate a particular educational tool or software into geometry lessons, educators may consider the relevance of the said technology to the topic as well as the benefits the learner would derive from the lesson. For example, using GeoGebra Graphing Calculator to present geometry would be less useful as compared to using GeoGebra Geometry since the latter has more tools to help navigate through geometric concepts. This is an indication that one cannot just decide to integrate technology without a plan. Likewise, it would be worthless to adopt technology integration just for the fun of it without structuring the lesson. Rather, each result from a technology-enhanced lesson should be immediately linked with learners' daily pen-pencil-paperwork as well as their already established mathematical concepts. Otherwise, learners would be more interested in the fun part of the technology at the expense of learning geometry.

Researchers (Akcay, 2017; Polly, 2014) have emphasized the importance of technology in our teaching. A lot of them have written about this in and outside South Africa. Many studies stress the improvement and positive effect that technology has in the teaching of mathematics. Some make use of GeoGebra software to address challenges in different aspects of geometry such as reflections, congruency, 3D geometry, etc., (Mosese, 2017; Mushipe, 2016; Shadaan & Leong, 2013). Others have also looked at the effects of GeoGebra software in teaching circle geometry in South African schools (Chimuka, 2017; Gweshe, 2014). For instance, Gweshe (2014:24) used the socio-constructivist approach to assess the effect of computer-assisted instruction in geometry. However, that approach did not clearly provide progressive levels to track learners' development in geometry as compared to Van Hieles' levels. Manganyana et al (2020:100) recommended the use of GeoGebra in "deep rural schools". However, their study only focused on the properties of quadrilaterals. Similarly, Bayaga et al (2019:38) used "one week" to provide treatment for both the control and experimental groups in their study in a "high poverty rural" school. Although the results showed significant improvement, a longer time frame would have helped considering the diverse nature of grade 11 circle geometry. Numerous studies on geometry and technology integration exist. Perhaps what is limited are studies on the use of GeoGebra and Van Hieles' levels in teaching circle geometry to Grade 11 learners in a rural setting, where such studies capture all the grade 11 circle theorems. This study, therefore, intended to measure the impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model in two of the rural secondary schools in South Africa.

1.4 Significance / Rationale of the Study

This study intends to inform education practitioners of the need to make use of educational technology in our schools to improve learners' performance in mathematics. It hopes to empower educators to improve practice through the use of interactive modern technologies such as educational software installed on mobile phones and laptops that relate to the environment of learners. The study aimed to make learners more actively involved in the instructional process as teachers make use of GeoGebra software in presenting Euclidean geometry. The study further hoped that policymakers would use the outcomes of the study to address the teaching and learning challenges faced in our schools.

1.5 Literature Review

In this chapter, I only present a brief on a few issues on the literature reviewed in this study. A full account of the literature review is given in Chapter 2.

Literature was reviewed under the theoretical framework and in context. In context, the literature review analyzes the various studies conducted using GeoGebra and other educational software in and outside South Africa. Numerous studies have used GeoGebra software to assess its impact in teaching geometry and many other aspects of mathematics. Firstly, this study begins with a review of the true nature of geometry education in the country. It became evident that the spiral approach is mostly used in teaching high school mathematics of which geometry is included. Consequently, most of the Grade 11 concepts are deeply rooted in the axioms and theories learnt during the General Education and Training (GET) phase. For instance, a Grade 11 learner could best understand the fact that a perpendicular bisector from the center of a circle bisects the chord, only if that learner is abreast with visualization of all the parts of a circle, not forgetting the concept of congruency learnt in Grades 7 to 10. According to the Department of Basic Education (2018b:150), educators should put more emphasis on the basic work because "*The fact that learners are naming angles incorrectly at Grade 12 level indicates that this issue has not been dealt with effectively in earlier* *grades*". Probably, mathematics educators could capitalize on learners' prior geometric knowledge and develop the new geometry content around such knowledge.

This and many more are what this review found very disturbing and needed urgent redress, probably, with the help of GeoGebra integrated lessons. Furthermore, there was enough literature on different available educational software. However, GeoGebra software seems to be the most widely used of them all. Perhaps, it is due to its open-source nature or its friendly user interface in different versions or both. Literature supports the use of GeoGebra in teaching in the sense that most of the studies showed significant improvement in different aspects of mathematics. An example is a study by Mwingirwa and Miheso-O'Connor (2016) which highlights the benefits of using GeoGebra in schools. However, the researchers, just like Chimuka (2017), strongly encourage effective teacher training prior to the use of such educational software. Among the challenges in using educational software, the review pointed out insufficient technological tools in our schools, low levels of teacher development, the beliefs held by some teachers regarding teaching methods, amongst others. Many other researchers (Drijvers, 2013; Drijvers et al., 2015; Jelatu et al., 2018; Seloraji & Eu, 2017; Shadaan & Leong, 2013) have emphasized the usefulness of making technology a part of the mathematics teaching process. Interestingly, the researchers caution against any attempt to replace our rich teaching methods with technology as this has the potential of ruining the creativity that comes along with teaching. In this technology-driven world, perhaps what educators need is a carefully planned lesson that utilizes van Hieles' levels and also takes into account all the pedagogical content knowledge in a technological environment to maximize the learning output.

1.6 Theoretical Framework

The theoretical framework entails a critical overview of the Technological Pedagogical Content Knowledge (TPACK) model, constructivism, and the five levels and phases of geometric thoughts as outlined in the model of van Hieles'. Furthermore, the review assessed the suitability of each of the aforementioned models for this study together with a critique of the possible weaknesses in the models.

Firstly, the van Hieles' model is one of the popular theories in mathematics education that provides direction to teachers on the effective teaching of geometry. This model was developed by husband and wife, Dina van Hiele-Geldof and Pierre van Hiele, as part of their doctoral dissertation at Utrecht University, Netherlands. Their model has five distinct levels and five phases for teaching geometry. The levels comprise visualization, analysis, abstraction, formal deductions, and rigor. However, these levels are not mutually exclusive as Level 2 requires the application of Level 1 knowledge for one to excel, and so on. Moreover, each of the five levels requires the learner to progress through five phases viz. information, guided orientation, explanation, free orientation, and integration. Secondly, the Technological Pedagogical Content Knowledge (TPACK) framework which was designed by Khoehler and Mishra was reviewed (Khoehler & Mishra, 2005). Finally, the epistemology of constructivism, which was seen to be much related to van Hieles' model, especially, as it was manifested during van Hieles' free orientation phase was discussed. All aspects of the aforementioned frameworks and theories have been discussed in detail under Chapter 2 of this study.

1.7 Statement of the Problem

Most 21st-century learners have access to modern technology devices like cell phones, laptops and others. They also get excited and enjoy working with such devices. Moreover, various methods have been used to teach geometry including those that strengthened visualization and deduction from proofs. However, learners' performance in geometry in South African high schools does not look encouraging (Chimuka, 2017). Since 2015, the pass percentages per question in the National Senior Certificate examination in Euclidean geometry range from 28% to 61% (DoBE, 2015, 2017, 2019a, 2020b). Seemingly, learners struggle to interpret given diagrams because of their weak understanding of the basics of geometry. With the desire to improve the performance of learners in Euclidean geometry, educators would need to take advantage of learners' technology advancement and modify their approach to teaching to meet the current societal trends. This study therefore aims to investigate the impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.

1.8 Aim of the study

The study aims to determine the impact of GeoGebra as a technology integration tool in teaching Grade 11 Euclidean geometry based on Van Hieles' model.

1.9 Objectives

The objectives of this study were to:

- Identify the challenges that learners experience in understanding circle geometry.
- Determine how the use of GeoGebra software through each van Hieles' level in the classroom affects learner achievement in the teaching and learning of Euclidean geometry among Grade 11 students.

1.10 Main Research Question

The main research question is:

What is the impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model?

1.11 Research Questions

The following questions will seek to find answers to the main research question:

1) What challenges do Grade 11 learners experience in understanding circle geometry?
2) How does the use of GeoGebra software through each van Hieles' level in the classroom affect learner achievement in the teaching and learning of Euclidean geometry among Grade 11 learners?

1.12 Research Hypothesis

The study was also guided by the following four (4) null and four (4) alternative hypotheses, in that order, where H_0 ~ Null Hypothesis and H_1 ~ Alternative Hypothesis. The findings from the study were used to determine whether to accept or reject these hypotheses.

i. ***H₀*:** There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners.

***H*₁**: There is significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners.

ii. ***H₀*:** There is no significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

H_1: There is significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

iii. ***H_0*:** There is no significant interaction between the control/experimental groups and their pre- and post-tests.

***H_1*:** There is significant interaction between the control/experimental groups and their pre- and post-tests.

iv. ***H_0*:** Using GeoGebra software in classroom does not improve Grade 11 learners' academic achievement across each van Hieles' level in the teaching and learning of Euclidean geometry.

***H_1*:** Using GeoGebra software in classroom improves Grade 11 learners' academic achievement across each van Hieles' level in the teaching and learning of Euclidean geometry.

1.13 Methodology

In this study, I only present a summary of the methodology used in this study. Full discussions on methodology are covered in Chapter 3.

This study followed a quasi-experimental design situated in a positivist paradigm. The design adopts the control group interrupted time-series format which is known for its ability to withstand maturation, history and pre-testing (McMillan & Schumacher, 2014:303). The research used quantitative methods by taking the population from approximately 600 Grade 11 mathematics learners enrolled in Circuit 7 of Motheo District in Free State, South Africa. Out of that population, a sample of 30 participants each, were taken from two secondary schools in the Free State province, making a total of 60 subjects for the control and experimental groups. In addition, a purposive sampling technique was used since this design needed an already existing classroom with technological resources. The data collection instruments were self-designed three pre- and another three post-tests based on the van Hieles' theory. The data were analyzed using descriptive and inferential statistics.

1.14 Validity and Reliability

According to McMillan and Schumacher (2014:189), test validity is a measure of the appropriateness of inferences based on the scores from the test. In other words, a test of higher validity should be capable of measuring what it intends to measure. However, the outcome from a particular test may be valid only for that test but invalid for others, an indication that validity is case-specific. Consequently, factors such as the purpose, population, and the environment used for data collection were carefully considered to ensure content validity. Moreover, researchers (Ayre & Scally, 2014; Lawshe, 1975) seem to agree that a test item has content validity if over 50% of experts consider it relevant to the study. However, Ayre and Scally (2014) suggested a minimum number of experts needed to agree on an item, for it to be considered as either essential or critical. Therefore, this study adopted expert opinion and the Content Validity Ratio (CVR) of Lawshe in determining test content validity. To achieve reliability, a pilot study was conducted on 15 Grade 11 learners from a different secondary school situated in the same locality where the main study was done. Thereafter, the results of those tests were subjected to Pearson product-moment (r) reliability testing for equivalence and internal consistency. Finally, steps were taken to manage other aspects of reliability including stability. All processes that were followed to ensure the reliability and validity of the test items can be found in Chapter 3.

1.15 Data Analysis

The scores from the tests written by the control and experimental groups were coded and analyzed using descriptive and inferential statistical procedures. This analysis was augmented with the International Business Machines Corporation's Statistical Product and Service Solutions (IBM-SPSS), formerly known as Statistical Package for the Social Sciences (SPSS). This software package was chosen due to its friendly user interface and its broad statistical features such as descriptive, bivariate, predictive, geo spatial analysis, among others. In conclusion, this analysis provides the mathematics educator with evidence that supports the need to include technology in the teaching of geometry in South Africa.

1.16 Ethical Considerations

This study commenced after the issuance of an ethical clearance from the ethics committee of UNISA. In addition, permission was sought from the Free State Department of Education (FSDoE), as well as the circuit manager of the schools where the research was conducted. Moreover, permission was requested from the principals of two schools in Free State to use one each of their Grade 11 classes, during after-school hours for a maximum of an hour, for this study. The two schools were further requested by the researcher to grant access to the list of the participants who were

Grade 11 learners, from their school's database for progression purposes only. Assent and consent forms that ensure the protection of learners' privacy, anonymity, the confidentiality of data, etc., were issued out to learners and their parents/guardians that sought their voluntary completion and participation. Each permission letter or assent/consent form addressed the issue of coronavirus (Covid-19), and the measures put in place to avoid exposing learners to the virus.

1.17 Scope and Limitations

Although the study was done for over two years, conducting data collection in only six weeks limited the outcomes due to the short time, especially, since the control group interrupted time design is more suitable for longer periods. Moreover, the sample size of 60 learners for the two groups was still not adequate for generalizations. A larger sample size would be preferable, but time constraints and resources did not allow it. More so, using an already established class of learners makes it difficult to control threats such as attrition, statistical regression, selection, instrumentation, among others (McMillan & Schumacher, 2014:303). There were threats regarding exposure of participants to Covid-19, which restricted the researcher from using a larger class size or encouraging group work. In terms of the scope, the researcher restricted this study to the visualizations, analysis, deductions, and rigor of circle geometry. This did not provide room for more formal proofs and problem-solving in geometry as a whole. Furthermore, the fact that classes were mostly held on weekends led to participants sometimes forgetting learnt concepts as compared to having the study continuously on weekdays. Finally, using tests for data collection is mainly suitable for subjects who can only read and write, implying that any wrong interpretation of the test items would result in unreliable outcomes.

1.18 Definition of key terms

The following key terms, concepts and variables relate to this study:

Technology integration: In this dissertation, the term refers to making use of educational software, mobile phones, laptops, and projectors to facilitate teaching and learning without necessarily replacing the mathematics teacher. It is not about learning ICT as a course in schools.

Constructivism: The mathematical philosophy that knowledge does not exist somewhere waiting to be discovered, but to be acquired by constructing them through the various resources available to learners in their surroundings through intuition, creativity, observations, experiments, and our five senses.

Constructivist teaching approach: The method of teaching where the learner's interest is placed above that of the teacher as the latter serves as a facilitator to enable learners to construct their own mathematical concepts individually or with their peers.

The zone of proximal development (ZPD):

A term in Vygotsky's socio-cultural theory, a constructivist approach, depicting the gap between what each learner is capable of doing without any assistance, and what they can do through the help of their more capable peers or facilitators through structured and unstructured activities.

The traditional teaching method

This is the talk-and-chalk, transmission, teacher-centered, behaviorist, or absolutist way of teaching where knowledge is mainly transferred from the teacher to the learner, where the latter is usually considered as a *tabula rasa* (a clean slate with no innate ideas).

Group A: The experimental group that received the intervention in the form of using GeoGebra software to solve problems.

Group B: The control group that wrote only the tests after being taught through the traditional approach.

X: The intervention received by Group A (using GeoGebra software to explore circle geometry problems).

O_1: A pre-test used for data collection which centers on the visualization and analysis levels of van-Hieles' model.

O_2: This is the second pre-test which assesses learners on Levels 3 and 4, which are abstraction and deduction respectively, of van-Hieles' model.

O_3: The final pre-test which focuses on van-Hieles' rigor level of geometric thoughts.

O_4: This is the first post-test, similar to **O_1**, which assesses van Hieles' Levels 1 and 2.

O_5: The second pre-test, just like **O_2,** dedicated to Levels 3 and 4 of van Hieles' model.

O_6: This is the final post-test, structured just like **O_3**, to assess rigor.

1.19 Chapter Orientation

This chapter sought to present a general orientation as to what the entire study is about. Efforts were made to explain the background to the main research problem. Furthermore, the researcher outlined the aims, objectives, and hypotheses in addition to research ethics, limitations, and delimitations of the study. A summary of the literature review, methodology, as well as key terms related to technology integration, have been explained.

Next is Chapter 2, which presents a detailed review of all relevant literature that is related to this study. The summaries and critiques of previous studies are presented as well as their significance to the current study. The review focused on studies done using different educational software to improve teaching pedagogy, the nature of geometry education in South Africa, and the measures to adopt for successful technology integration. The theoretical framework discusses van Hieles' theory on geometric reasoning, constructivism, and the TPACK model.

Further, Chapter 3 is dedicated to the methodology for the study. This chapter presents a positivist research orientation. It exposes the reader to a quantitative study that uses non-probability sampling techniques to select the sample in a quasi-experimental design. It also gives a vivid account of the entire procedure used for collecting data. Moreover, issues relating to ethics have been addressed together with the measures to ensure the validity and reliability of the study.

In Chapter 4, the researcher organizes and analyzes the collected data and presents the findings. Both descriptive and inferential statistical procedures were adopted for the data analysis through the use of the SPSS software. In addition, the presented findings have been interpreted in this chapter.

Finally, Chapter 5 is dedicated to making conclusions about the study. A summary of the findings, recommendations, and suggestions for further research are outlined here.

CHAPTER TWO: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

This chapter presents relevant literature related to the integration of educational technology in the teaching and learning of geometry. The related studies are discussed under two main categories: the context, and the theoretical framework that underpins this study. The chapter begins with the presentation of the theoretical framework. This is followed by a discussion of the nature of geometry in South African General Education and Training (GET), and Further Education and Training (FET) phases. In addition, the findings from different studies, challenges in teaching and learning of geometry, available educational software, features of GeoGebra software, and learners' performance with GeoGebra are presented. Other aspects that are discussed include the significance of educational technology, and the challenges and control measures for using educational technology when teaching Grade 11 Euclidean geometry. The chapter concludes with a summary of where this study fits into the related literature on the impact of teaching with technology.

2.2 Theoretical Framework

Researchers (Jojo, 2017; Ernest, 1991) opine that teaching geometry in a way that actively engages the learner in a social environment, to make the most of the learning situation, is a more effective approach than the traditional behaviorist paradigm. Woollard (2010) explains the behaviorist paradigm as a teaching and learning approach where learners react to stimulus or information from the teacher which motivates the learner to form a new learning behavior. In behaviorism, learners are considered as blank slates, and so, learners rely mostly on the teacher as their main source of knowledge, with little to no active engagement from the learner. However, some theorists (Piaget, 1967; Ernest, 1991; Dubinsky, 2014; 1984; van Hieles, 1986; Smaldino, Heinich, Molenda & Russel, 2008; Koehler & Mishra, 2005b; Rosenberg & Koehler, 2015) have been used in researching best practices to use in the teaching and learning of geometry. Those theories include Piaget's cognitive development theory, social constructivism, van Hieles' level theory (VHLT) of geometric reasoning, and Smaldino et al.'s (2008) Technological Pedagogical Content Knowledge (TPACK) model. The others include Actions, Processes, Objects, Schemas (APOS) theory by Dubinsky (1984; 2014) and finally, the Analyse Learners; State Objectives; Select

Methods, Media, and Materials; Utilize Media and Materials; Require Learner Participation; Evaluate and Revise (ASSURE) model. In summary, Piaget’s cognitive development theory provides four stages of development, from sensorimotor to formal operational stage, which could help the teaching of geometry in South African schools based on the learner’s cognitive level. However, Piagets’ theory could not be used for this study because it is more general in application with emphasis on the age of learners, but no specific focus on geometry or technology integration, which is contrary to what this study aims to investigate. Similarly, Ernerst’s (1991) social constructivism does not apply to this study although the theory has its own affordances. For instance, social constructivists promote the active peer-to-peer interaction between learners during lessons as a means of ensuring meaningful learning. However, the constructivists’ theory seems to have no specific guidelines for integrating technology into the learning of geometry. Similarly, the APOS model, which was developed from Piagets’ work, is equally not suitable for this study compared to van Hieles’ model. That is because APOS does not necessarily focus on geometry or technology integration unlike VHLT or TPACK respectively. On the contrary, the ASSURE model, just like TPACK, seems to lay out some specific steps to follow in the technology integration process. However, the ASSURE model does not take into consideration the various forms of knowledge, as in TPACK, that the geometry learner or the teacher should possess prior to, and during the learning process.

Consequently, this study adopted van Hiele’s level theory (VHLT) of geometric reasoning, after analyzing the kinds of knowledge in the TPACK model, which were then compared with the nature of mathematics taxonomy used in South African schools. The South African Mathematics taxonomy is a set of cognitive levels that each outlines the formal tasks, tests or examinations should cover. For instance, a geometry test should have 20% of Knowledge (recall of basic geometric facts), 35% of Routine procedures such as proofs of well-known theorems, 30% of Complex procedures such real-world problems, and 15% Problem-solving that deals with high order reasoning. All the identified frameworks for this study (VHLT and TPACK) are in line with the epistemology of constructivism where learners are given the needed support to construct their own concepts. However, the VHLT and TPACK model were selected for several reasons. For instance, the van Hieles’ theory provides incremental levels for developing learners’ geometric understanding, which is supplemented by five transitional phases for teaching and learning. Alternatively, the TPACK model provides an in-depth understanding of the various potentials or knowledge possessed by the teacher and learner before and during the learning process. A blend of the van Hieles' and TPACK is expected to make the learning process more intriguing. A detailed presentation of the two theories is given in the next sections.

2.2.1 Van Hieles' five levels of geometric reasoning

In separate doctoral dissertations at the University of Utrecht in 1957, Pierre Marie van Hiele and his wife Dina van Hiele-Geldof developed what has become the van Hieles' Levels Theory (VHLT) on geometric reasoning. According to Usiskin (1982), the VHLT became popular when the husband took it upon himself to throw more light on the couple's project after the death of Dina, the wife. Subsequently, the publication of VHLT by Freudenthal in 1973, who was a mentor to the husband and wife, helped enhance the popularity of the model. The work done by Freudenthal (1973), Pyshkalo the Soviet (1968), Wirszup (1976), among others, ensured that the VHLT spread across the globe.

The VHLT has two stages comprising five levels and five phases. According to van Hieles' (1986), the five levels of geometric thought processes are visualization, analysis, abstraction/informal deduction, deduction, and rigor as shown in Figure 1.1. Each preceding level seems to overlap into the next level making the levels not mutually exclusive but dependent on each other.

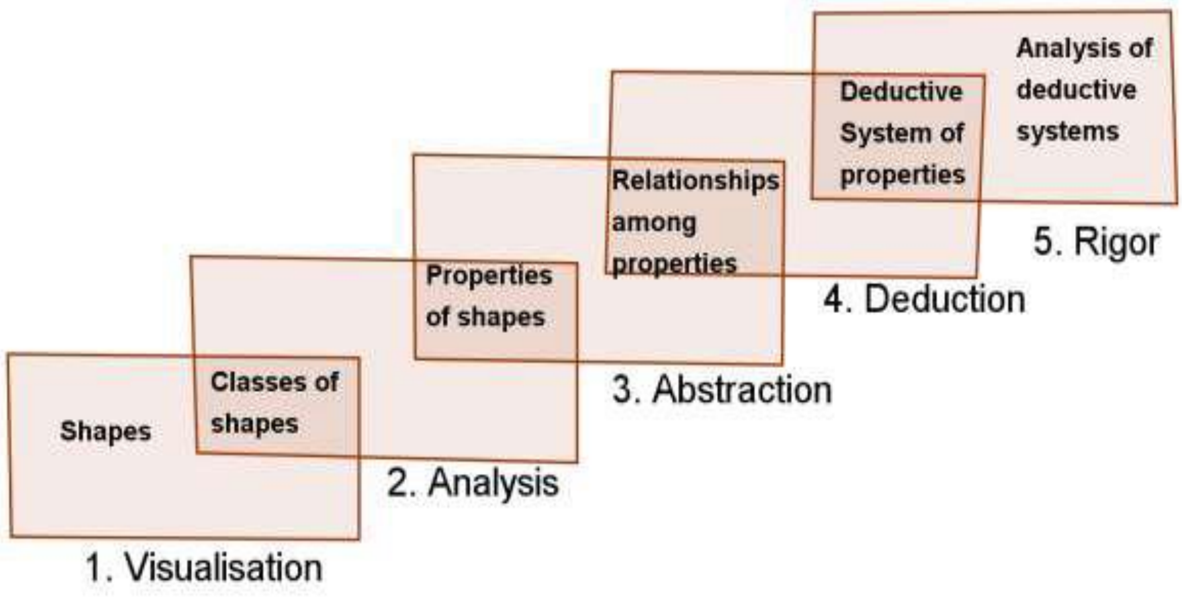


FIGURE 2.1: THE VAN HIELE'S LEVELS (VHL) OF GEOMETRIC THOUGHTS (Adapted from Rezky & Wijaya, 2018)

Besides the five levels, there exist five van Hieles' phases (VHP) of development through which a learner progresses from one level to the next. Those phases, from one to five, in that order are inquiry/information, directed/guided orientation, explanation/explicitation, free orientation, and integration. A detailed presentation of the levels and phases has been presented below.

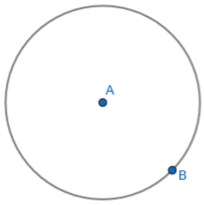
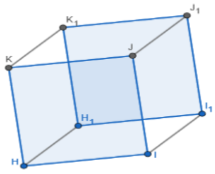
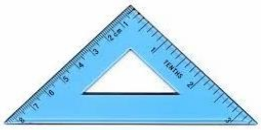
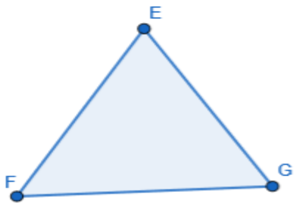
However, it is worth mentioning that the five van Hieles' levels (VHL) originally begin with level 0, which Usiskin (1982) called Level 1, and it continues that way until Level 5 (originally Level 4).

Level 0 (VHL0): Visualization

According to van Hieles (1957), visualization, also known as recognition, is the first and basic level that each learner must acquire when developing their geometric concepts. In simple terms, to visualize means to create a mental picture of an object or a situation. Similarly, van Hieles' visualization level (VHL0) deals with the identification of polygons, parts of a circle, or any geometric shape by their unique appearances or by their look-alike/prototype, without necessarily understanding all the characteristics of the particular shape. For instance, Table 2.1 displays the likelihood that learners would compare some shapes such as a circle to the sun or full moon, a shape which most Grade 11 learners should be familiar with. In effect, visualizing is the skill of relating known shapes to geometric figures. Likewise, learners could be made to easily recognize a semi-circle as the half-moon, the cube as a dice, or parallel lines as railway lines all due to their resemblance.

However, inasmuch as Pegg (1995: 90) agrees to seeing or recognizing geometric figures through their prototypes, the researcher identified other aspects of VHL0 as part of the visualization process. According to the researcher, instead of identifying say, a square, by its properties, learners would rather look at other aspects like the flatness, points, or corners in identifying the shape. At other times, learners may use only one feature, for instance, equal sides, to quickly remember an equilateral triangle. The implication is that at VHL0, learners mostly rely on perception to identify shapes and teachers should use more real objects to nurture learners' ability to visualize.

TABLE 2.13: VISUALIZING GEOMETRIC SHAPES

PROTOTYPE	GEOMETRIC SHAPE	NAME OF GEOMETRIC SHAPE
  Sun Full-moon		Circle
 Half-moon		Semi-circle
 Dice		Cube
 Warning Triangle  Set-square		Triangle

However, it is worth noting that not all geometric shapes or their prototypes could be physically obtained and shown to learners during a lesson. Sometimes the educator would have to improvise a particular geometric shape, such as the sun, to depict a circle.

Unfortunately, there seems to be a challenge amongst Grade 11 learners when it comes to identifying the various parts of a circle. As a result, learners struggle to grasp the various circle theorems. However, recognizing geometric shapes could be enhanced by displaying images of their look-alikes via technological tools like phones and laptops. Hopefully, such a display of shapes using technology could probably deepen and create long-lasting mental images about the various geometric shapes including parts of the circle. That display could also pave the way for any future analysis or deductions of axioms or theorems based on established properties of the geometric shapes.

In conclusion, effective visualization requires the ability to not only identify shapes, but to classify them as well. The learner needs to develop the skill to see that triangles are different from quadrilaterals by their unique physical looks.

Level 1 (VHL1): Analysis, the aspect of geometry

Analysis involves breaking into parts the various features of a shape for a better understanding of geometric concepts. It is at this stage that the properties of a shape are used to distinguish it from other shapes, as opposed to using their prototypes.

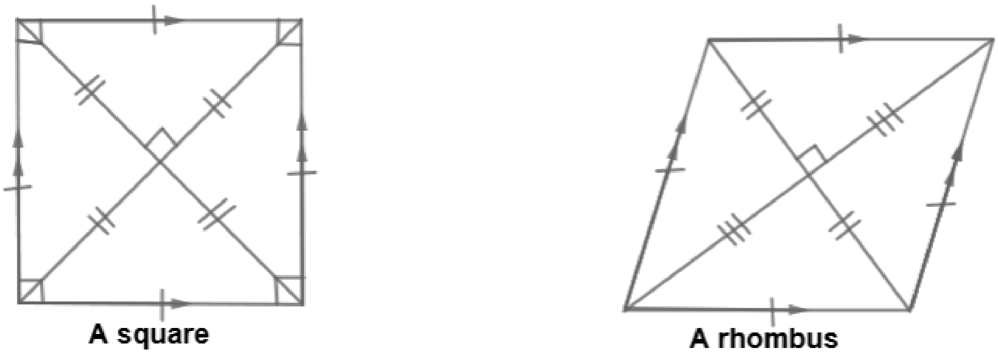


FIGURE 2.2: A RHOMBUS DIFFERS FROM A SQUARE

In Figure 2.2, the Grade 11 geometry learner should be able to use the various properties of the two shapes to distinguish them. Although the two shapes have a lot of similarities, for instance, opposite sides are parallel and equal in the square as well as rhombus, however, a square has four right angles that a rhombus does not have. In addition, the diagonals of a square are equal, and they bisect each other. On the contrary, the diagonals of the rhombus have different lengths although the diagonals bisect each other just like the square. Perhaps, it is these basic and unique characteristics about the various plane figures that made van Hiele-Geldof (1957) classify this level (VHL1) as "*the aspect of geometry".* Moreover, those properties of geometric shapes are so essential that they influence learners' ability or otherwise to deal with high-order geometric proofs. The identification of shapes by their properties could help avoid the situation where a learner confuses a chord with a diameter, a kite with a rhombus, or a trapezium with a parallelogram. In other words, the more a learner understands the basic features of geometric shapes, the better the learner can apply them in advanced geometry problems.

However, with abundant evidence (DoBE, 2011-2019b) pointing to the way Grade 11 and 12 learners struggle with riders in geometry, one could only interpret it to be due to the failure by educators to pay much attention to the features of VHL0 and 1 (Masilo, 2018b; Rezky & Wijaya, 2018). Perhaps, mathematics educators could use technology integration to assist learners to master the particular features of geometric shapes. For example, GeoGebra software allows learners to measure angles and line segments or create their own geometric shapes (Tay & Mensah-Wonkyi, 2018:4). As a result, learners could construct the diagonals or any other part of a square and measure to see if the lengths or angles are equal or not and compare their measures with that of other shapes. In that way, learners would understand more and own their self-developed concepts as they use GeoGebra to analyze the features of various geometric shapes under VHL1.

Level 2 (VHL2): Abstraction/Order, the essence of geometry

This level is about logic, order, and relationships among geometric shapes (Rezky & Wijaya, 2018). At this level, the learner of geometry makes simple and informal deductions without any mathematical system to link one shape or theorem to the other (Masilo, 2018). This means that learners do not understand the mathematical proofs attached to learners' deductions. For example, diagram X in Figure 2.3 depicts the theorem which states that "*the angle at the center of a circle, subtended by an arc or chord is double the angle at the circumference formed by the same arc or chord".* To explain further, if arc AB subtends 130^0 angle at the center as shown in diagram X, half of that 130^0 should form at the circumference since it is the same arc AB that formed both angles.

However, diagram Y in Figure 2.3 shows diameter AD formed by moving point B in diagram X to point D. That move from B to D leads to the formation of 180^0 at center O, implying that the angle, K, at the circumference must be 90^0. But the theorem for diagram Y says that "*angles in a semi-circle are supplementary"* (DoE, 2011). A close observation of the two diagrams would reveal that both theorems are the same since they all revolve around the theorem for diagram X. Moreover, circle theorems such as *"angles in the same segment are equal", "equal chords subtend equal angles at the circumference of the same circle",* etc., could all be traced to the theorem of "*angle at center is double the angle at circumference"*.

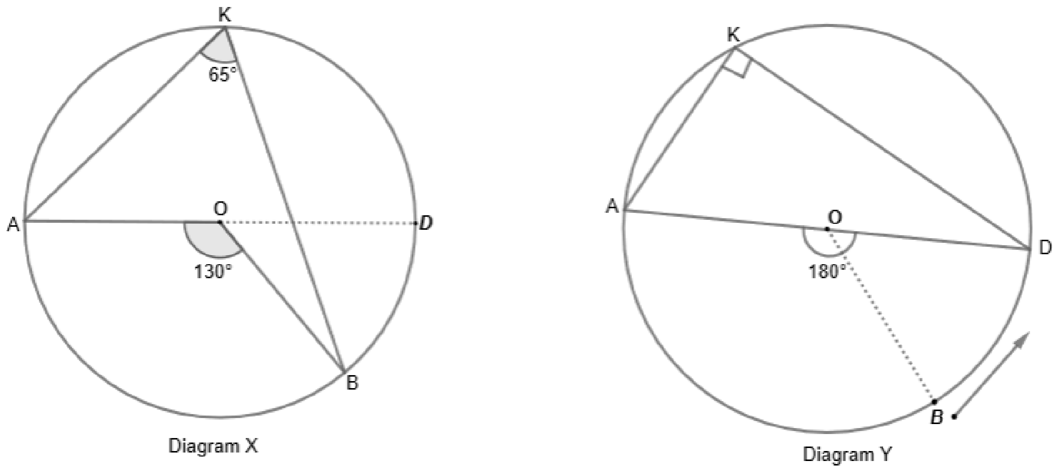


FIGURE 2.3: ANGLE AT CENTRE IS DOUBLE THE ANGLE AT CIRCUMFERENCE

According to van Hiele-Geldof (1957), VHL2 is "*the essence of geometry"* in the sense that VHL2 moves the learner's reasoning from just identifying properties of shapes, to establishing direct relations between the shapes. This abstraction level further draws learners' attention to the importance of order in geometry. Similarly, ordering could help establish that the square, rhombus, and rectangle are all special forms of a parallelogram. This is because a parallelogram is a plane figure with opposite sides parallel and equal, and all the aforementioned shapes share those properties.

Level 3 (VHL3): Deduction, an insight into the theory of geometry

Dina van Hiele-Geldof (1957) considers this deduction level as the one that provides learners with adequate *"insight into the theory of geometry"*. Perhaps, that insight is gained from the numerous formal proofs, making postulates, and the derivation of theorems in this level. For example, the tangent-chord (tan-chord) theorem says:

"An angle formed between a tangent and a chord, at the point of contact, equals the angle formed by the same chord at the circumference of the circle".

To prove this tangent-chord theorem, the learner would have to make many deductions similar to the ones outlined in the four steps below:

Step 1: Construction

Diagram Y in Figure 2.4 is one of the constructions that could be used to prove the tan-chord theorem. Although the construction in diagram Y might look routine to some learners in Grade 11, the details in it span through VHL0-2. For instance, to be able to construct diagram Y means the learner can visually identify and differentiate all the parts of the circle from VHL0 to 1. Moreover, the learner should be conversant with the properties of the individual parts of the circle as well as the relationship existing between some of the parts, which fall under VHL1-2. Examples of such relationships include: a radius is perpendicular to a tangent; the angles that are opposite the equal sides of an isosceles triangle are equal, etc. Once again, the use of technology integration could assist learners to have a better view of the various connections between circle parts in their constructions. The technology, such as GeoGebra, could help improve learners' understanding of concepts as learners themselves predict their outcomes and test their assumptions with accurate measurements from the software.

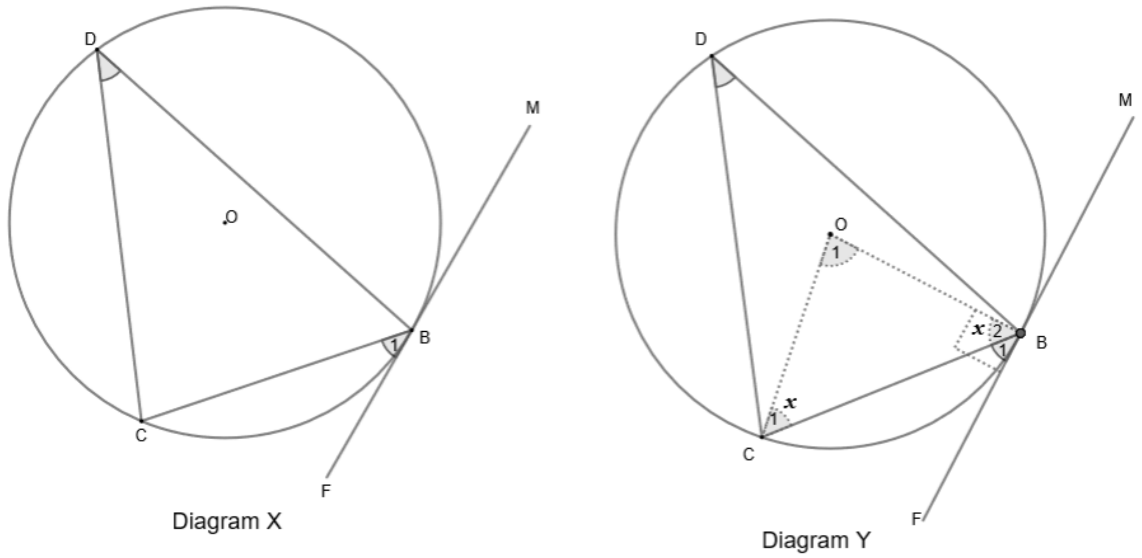


FIGURE 2.4: PROVING THE TANGENT-CHORD THEOREM THROUGH DEDUCTIONS

Step 2: Justifying the value of angle O_1

This is where the actual deductions begin. From Figure 2.4, the Grade 11 learner is expected to use the properties of the isosceles triangle in $\Delta COB,$ to deduce the following:

$In\ \Delta COB,$

$$\hat{C}_1 = \hat{D}_2 = x \quad \ldots\ldots.. \quad angles\ opposite\ equal\ sides\ of\ \Delta$$

$$\therefore \hat{O}_1 = 180° - 2x \quad \ldots\ldots.. \quad angle\ sum\ of\ \Delta$$

Step 3: Relating $\hat{O}_1$ to $\hat{D}$

At this stage, the learner should be remembering the connections learnt from VHL2 to link the two angles. Since $\hat{O}_1$ is an angle at the center and $\hat{D}$ is the angle at the circumference of the same circle, then half of $\hat{O}_1$ is equal to $\hat{D}$. That means, $\hat{D} = 90° - x$. Those are very useful deductions that help learners to build their level of interest in geometry. Those deductions could also help learners to have durable memory of each learnt geometric concept, like the fact that the angle at the center is double the angle at the circumference of a circle, the radius is perpendicular to tangent, or the tangent-chord theorem, and apply them in the next VHL.

Step 4: Relating $\hat{B}_2$ to the radius and tangent

By inference and from the learners' construction of diagram Y above,

$$\hat{B}_1 + \hat{B}_2 = 90° \quad \ldots\ldots \quad radius\ \perp tangent$$

$$\text{But } \hat{B}_2 = x$$

$$\therefore \hat{B}_1 = 90° - x$$

$$\therefore \hat{B}_1 = \hat{D}$$

In conclusion, the above steps attempt to demonstrate the various deductions that the Grade 11 learner could make to prove given geometric theorems or problems. However, the learner at VHL3 is expected to be able to derive and prove each of the main geometric theorems from learners' own constructions. Sometimes, this level (VHL3) demands making proofs through a combination of other theorems. Unlike VHL2, the learners at VHL3 understand the proofs that they make as learners connect, in a more logical manner, all related axioms and other theorems to derive new theorems. From the aforementioned, it probably seems logical to say that the deductions in this level prepare learners for more "*insight"* (van Hiele-Geldof, 1957) into the learning of Grade 11 circle geometry.

Level 4 (VHL4): Rigor, the scientific insight into geometry

It could be said that VHL4 is an advanced VHL3. This is because learners gather all the necessary insight into the theories in geometry from VHL3 and turn them into "*scientific insight into geometry"* at VHL4 (van Hiele-Geldof, 1957). By scientific insight, it denotes making proofs that could withstand the test of time. It also means that the geometric proofs should use all available and well-established geometric theories, axioms, postulates, etc., to arrive at valid conclusions. In addition, the nature of those VHL4 problems is so open-ended that different learners could use different approaches, which are all geometrically acceptable, to arrive at the same conclusion. Furthermore, VHL4 tests learners' understanding of various systems in mathematics through direct or indirect arguments, contra-positive proofs, and sometimes, by non-Euclid means. This is where learners may be required to think deeper as they debate, and come to a consensus on the best option to solve given geometry problems. However, the Curriculum and Assessment Policy Statements (CAPS) of the South African education system refers to such VHL4 questions as riders. Those riders mainly focus on the application of all learnt concepts and theorems of geometry from both General Education and Training (GET) and Further Education and Training (FET) phases.

Unfortunately, most diagnostic reports (DoBE, 2015, 2017, 2019a, 2020a, 2020b) complain year by year of consistent low performance of Grade 12 learners on riders during their National Senior Certificate (NSC) examinations. Interestingly, most of the geometry questions in NSC examinations come from Grade 11 circle geometry. Perhaps if Grade 11 learners were given the necessary attention by schools, and carefully taken through all van Hieles' levels, learners' performance would be better than it is currently. Furthermore, although literature shows the positive impact of using GeoGebra, the outcome of this study, if successful, could guide educators as to how best to use the visuals and the tools for measurements in GeoGebra to maximize learners' performance in geometry.

Besides the five van Hieles' levels, Usiskin (1982) laments the inability of some learners to fit all Van Hieles' levels, hence, the creation of a forced Van Hieles' level (p.44) to address that shortfall. In creating that forced VHL, the researcher first had to remove VHL5 because the specific behaviors listed under VHL were too "*vague"* (p.23)*.* For instance, VHL5 talks about *"logical thinking itself"* as a "*subject matter"* (p.22). However, it is not clear whether that logic refers to axioms or the normal symbolic deductions teachers use in class. As a result, the researcher posited that VHL5 is *"hardly attainable"* in secondary schools because it may not be testable (p.23), hence, its removal. That removal of VHL5 paved the way for the researcher to reorganize the van Hieles' levels which resulted in the forced VHL. Usiskin realized that "*not every geometry question"* (p.29-30) can be classified under the VHL. For instance, questions that involved using a theorem to solve problems of numerical nature were considered by the researcher to be outside the VHL since those questions do not satisfy the listed characteristics of VHL. Moreover, although VHL assumes that a learner can do proofs under level 4, Usiskin argues that there may still be learners who can memorize and write simple proofs with no understanding of the theorem itself (not yet attained VHL4). Such a learner could not be necessarily placed under a particular VHL but in a forced no fit VHL. Those reasons probably accounted for the creation of the six “Modified van Hiele levels” which included the forced no fit VHL, by Usiskin. However, Usiskin (1982:44) maintained that classifying a learner under a no-fit category does not mean that the van Hieles’ theory is flawed, rather, it only shows a learner who demonstrates random geometric capabilities.

On the contrary, Usiskin (1982:45) asserts that increasing the number of tests items or improving the test quality and classifying them appropriately to minimize the number of errors could ensure that all the learners fit all the van Hieles’ levels. For that reason, this study opted to increase test items and improve test item classification instead of creating a no-fit VHL. However, this study agrees with the work of Clements and Battista (1992) who believe that the van Hieles’ levels should begin with a Pre-visualization level before Visualization. The expected behaviors under Pre-visualization have been captured under Section 2.2.5 of this study. Consequently, this study adopted 6 instead of 5 van Hieles’ levels where the first level is Pre-visualization, as explained in Table 2.2.

Furthermore, Usiskin (1982) highlights the need for educators to give as many as 20 or 50 lessons to move a learner from level 1 to 2 or level 2 to 3 respectively. Following the notion that constant practice puts one close to perfection, educators could probably give more tasks that address various aspects of geometry to help enrich learners’ understanding of geometric concepts. This implies that to teach Euclidean geometry successfully requires more practice which goes beyond just literally progressing among some levels.

2.2.2 Features of van Hieles’ level theory

There are five unique properties of van Hieles’ levels. Firstly, and according to Usiskin (1982), the van Hieles’ levels follow a “*fixed sequence”* such that, each learner ought to successfully understand all concepts in VHL1 before progressing to the next level. For example, a learner who struggles to differentiate the properties of different quadrilaterals, or of the parts of a circle would have difficulties drawing connections among the parts of the same circle or the quadrilaterals. Therefore, each learner ought to gain the basics in geometry such as the visuals or properties of shapes, to enable a better understanding of more challenging Euclid problems. In other words, the geometry learner needs to fully understand level *n-1* (previous VHL) before starting level *n* (current VHL). In addition, researchers (Abdullah & Zakaria, 2013b; Chimuka, 2017; Jojo, 2017; Masilo, 2018) assert that advancing from one VHL to the next is mostly influenced, not by the learner's age, but by the quality of instructional materials, methodology, and learning environment within which the lesson takes place. That assertion places a quick reminder to geometry teachers: effective preparation before a geometry lesson cannot be underestimated when success and learner understanding are the objectives.

However, the opposite of moving from level *n-1* to level *n* is possible because according to Usiskin, (1982), a learner at a higher van Hieles' level displays *"adjacency".* This second property *"adjacency",* according to Pierre van Hiele (1958-59), implies that the learner at a higher VHL has already achieved considerable success at the lower levels. Therefore, the learner is capable of turning the skills initially considered "*intrinsic"* at lower VHL to "*extrinsic*" ones at higher levels. Intrinsic skills could be explained as those skills that are applicable within a particular geometric concept or van Hieles' level whilst extrinsic skills are applied outside a particular concept or level. For instance, within van Hieles' Abstraction level, learners should have connecting or ordering skills (*intrinsic*) to be able to identify different ways of using the same theorem, say, the angle at the center is twice the angle at the circumference, in diverse ways to generate converse theorems. However, those connecting, and ordering abilities tend to serve "*extrinsic*" purposes in the next van Hieles' level (Deductions). That is because a learner who is presented with a geometric problem that requires deductions would be able to break the problem down into parts for analysis if the learner understands the connections between different geometric theorems from the previous VHL.

Thirdly, the researcher identified some "*distinctions"* in terms of learners' use of language and the associated mathematical symbols at different levels. For instance, learners at the visualization level would rely on real objects to identify or classify shapes. However, a VHL 4 learner would use mathematical notations for angles, triangles, etc., to make valid proofs.

Moreover, the next property *"separation",* according to Usiskin (1982:15), says that if two people operate at two different VHL, they fail to understand one another. To exemplify, a mathematics educator may be using established axioms and theorems to prove riders in the classroom. Should some learners in the class have no idea what each circle theorem says or cannot even tell the relationship between radius and diameter, the teacher's proof would result in futility as the learners cannot comprehend the arguments being made regarding the particular geometric proof.

Finally, there is the "*attainment"* property. This property carefully outlines the various phases that the geometry learner has to pass through to move from lower to higher VHL. Those phases apply to each van Hieles' level, and each of those five phases has been explained below. To conclude, Usiskin (1982) explains that the nature of van Hieles' level theory (VHLT) demonstrates *"elegance, comprehensiveness, and wide applicability".*

The elegance in VHLT is manifested in how simple the theory is. For example, there are five phases within each VHL, and that learners ought to complete VHL1 to allow their progression to the next VHL, showing elegance in the form and structure of the model. In terms of geometry, VHLT could be described as comprehensive because the theory could be used to teach geometry irrespective of a learner's grade. In addition, VHLT provides clarity on learners' challenges in geometry as well as providing comprehensive levels and phases to use to address those geometric challenges. According to Usiskin (1982), Pierre van Hiele believes in the applicability of VHLT to all mathematical topics including functions, although most researchers prefer geometry to the other topics. Those assertions by Pierre seem to confirm how comprehensive the VHLT is. Moreover, researchers (Masilo, 2018; Rezky & Wijaya, 2018) assert that VHLT is known to be adopted by most mathematics researchers across the globe. That assertion shows the wide applicability of VHLT. Perhaps the theory's wide usage is due to its elegant and comprehensive nature.

2.2.3 Van Hieles' five phases within the levels

According to researchers (Rezky & Wijaya, 2018: Usiskin, 1982), there are five van Hieles' phases (VHP) of "*attainment"* that are attached to each VHL. These phases are inquiry/information, directed/guided orientation, explanation/explicitation, free orientation, and integration.

VHP 1: Inquiry / information

Some researchers (Abdullah & Zakaria, 2013b; Dockendorff & Solar, 2018; Masilo, 2018) believe that effective learning of geometry begins with information sharing between the teacher and learners. However, to share information, according to Hoffer (1981), implies the teacher asking thought-provoking questions to elicit responses that are geared towards the activation of learners' prior knowledge. For example, at VHL4, a teacher may introduce the proof of the theorem, *"angle at center is twice the angle at circumference"* by asking learners the following instruction and questions which I suggest below:

1. *Mention any part of the circle that is also a straight line.*
2. *What is the sum of all the angles in any triangle?*
3. *What is the relationship between a diameter and a radius?*
4. *Describe the isosceles triangle with particular reference to its angles.*
5. *Is there any relationship between the exterior angle of a triangle and its interior angles? If so, what is it?*

Although those self-developed questions above might not be the best thought-provoking questions, those questions could enhance the geometric habits of mind (Bülbül, 2021; Cuoco et al., 1996) required to prove and apply the indicated circle theorem. In effect, the questions posed during van Hieles' inquiry phase (VHP1) have two objectives: the first is to ascertain the level of existing knowledge possessed by learners, and secondly, to prepare learners' minds by giving them a clear path as to what learners should expect in the lesson. Similarly, most of the questions posed above seek to draw learners' attention to the relationships among geometric shapes or parts of a shape that they learnt in VHL3 (Bülbül, 2021). Those relationships, which were previously intrinsic at VHL3, would then serve as prior knowledge to help both the learner and teacher delve into the geometry topic for the day (Yi et al., 2020). To corroborate the work of Hoffer (1981) on the five van Hieles' phases, Crowley (1987), explains that this inquiry phase aims to establish some rapport between the teacher and the learner regarding learners' geometry experience, the direction the study would take, including the kind of learning tools, materials, and pedagogy to use.

VHP 2: Directed/guided Orientation

Having finished preparing the learner’s thoughts at the inquiry phase, it should be expected that the learner’s anxiety on the chosen geometry topic would rise, and that is when the teacher comes in with the direction. According to Masilo (2018), the guided orientation phase could be described as an exploratory or discovery exercise for learners. Perhaps, it is called exploratory due to how educators actively engage individual learners to search for geometric knowledge through the use of drawing and measuring tools, educational software, or by learners’ own constructions. Once again, the educator’s main role is to facilitate learning by giving more room for learners to create and form their own geometric ideas. The educator may use well-prepared learning materials such as geo-board, cut-outs, etc., that could provide support for learners to develop more geometry concepts. Moreover, the teacher could guide learners through technology integration, to learn the functions of the various tools in GeoGebra, and use the tools to construct circles, zoom in and out, draw and measure line segments and angles, etc.

VHP 3: Explanation

According to (Masilo, 2018; Siyepu, 2005; Crowley, 1987) this phase (VHP 3) is for explication. Explication involves the analysis and development of ideas or principles for gaining more knowledge. On the contrary, Jojo (2017) refers to VHP 3 as the explicitation phase. However, explicitation originates from the word explicit, which means to make a clear and detailed statement that leaves no doubt among those listening. Moreover, both explication and explicitation seem to have knowledge acquisition in common. Therefore, this study adopts Usiskin (1982)’s term, explanation, for VHP 3 since the term encapsulates both explication and explicitation. Furthermore, a detailed explanation of concepts during lesson presentation ensures a better understanding and provide direction in the development of knowledge in geometry.

However, the explanations given during knowledge development could not always be coming from the bosom of the teacher. Learners at this phase (VHP3) should be able to exchange geometric thoughts with their peers too. Moreover, learners could teach or learn from their group members, and that is similar to what Vigotsky (1978) describes in his Zone of Proximal Development (ZPD) as learning from *“more capable peers”.* Perhaps, learners feel free to share ideas with their peers due to the same level of language/linguistics used, or due to peers finding the simplest way or method to explain their acquired concepts to other learners.

Furthermore, researchers (Ernest, 1991; Jojo, 2017) believe that learners learn better from each other through active engagements like group work, leading to the acquisition of relevant skills applicable to geometry. In addition, Masilo (2018) is of the view that it is during VHP 3 that learners begin to cement their self-identified relationships about geometric shapes to form axioms. The learners probably develop those geometric connections faster through active interactions in a well-organized environment. Furthermore, the researcher asserts that those axioms are mostly developed as learners express themselves by giving explanations of identified patterns or structures in given geometric shapes. For example, learners can explain why the rhombus, square, and rectangle are all parallelograms, or why a diameter's length is double that of a radius. However, mathematics educators should keep facilitating to ensure that learners'-built concepts could stand the test of time. In addition, the learners should be encouraged to use correct mathematical notations as and when concepts are being developed. Those skills gained from directed orientation are so important for the next van Hieles' phase of learning.

VHP 4: Free Orientation

At this phase, learners are challenged to make use of their own established geometric relationships, postulates, axioms, or theorems to solve open-ended geometry questions (Clements & Battista, 1992). Perhaps the idea of the open-endedness is for learners to come up with as many solutions as possible so that the class could enrich their critical thinking skills as they debate on the different approaches used and agree or disagree on those concepts.

However, the free orientation phase requires some amount of time for learners to apply their knowledge. To achieve this, teachers could probably actualize Jojo (2017)'s approach of "*disrupting the learning environment".* The researcher encourages educators to modify the mathematics learning environment by "*reorganizing the mathematical possibilities in a geometric task"* (Jojo, 2017:258) to suit the skills and needs of particular learners. That means individuals or groups of learners should be supported with techniques, tools, and materials specific to their geometry problem. Those relevant tools and materials may include a display of charts and diagrams connecting different circle theorems, educational software, geoboard, etc. Perhaps, a geometry-friendly environment would motivate learners to think independently or make them persist amidst challenging and complex activities during the free orientation phase.

VHP 5: Integration

Finally, the integration phase (VHP 5) involves making a synthesis to form a general overview of learnt concepts. It is VHP 5 that serves as the bridge between successive van Hieles' levels. However, this phase does not produce new concepts. Rather, learners conduct reviews to connect all previously identified geometric connections and theorems (Masilo, 2018:70). For instance, learners may summarize all properties of angles that are formed at the center of a circle by an arc or chord and make a general remark about them, supported by valid reasons. However, such reviews tend to add to or even replace learners' existing knowledge about geometric shapes. For instance, a learner might previously be familiar with congruent triangles. However, after investigating congruent triangles inside a circle, the learner then realizes that a perpendicular line from a chord that passes through the center of a circle bisects that chord. Finally, after the completion of the integration process, the learner then becomes ready to enter a fresh van Hieles' level, leading to a repeat of the same cycle of phases within each level.

2.2.4 Critiques of van Hieles' Level Theory

Van Hiele's model has been criticized by some researchers. Although the model encourages hierarchical transition among levels, De Villiers (1987) believes that this is not always necessary for formal deductions. This is because some talented learners may skip levels as evident in, for instance, the use of Structure of the Observed Learning Outcome (SOLO) model (Pegg, 2002). By implication, just using levels may not lead to the complete understanding of the desired geometrical concepts. Rather, educators may consider learners' needs as well as the goals and context of the lesson and allow some flexibility among the levels whiles maintaining effective integration of relevant concepts.

In concurrence, some researchers (Masilo, 2018; Abdullah & Zakaria, 2013; Clements & Battista, 1992) posit that the original van Hieles' visualization level (VHL 0) lacks the exposure of learners to the real-world geometric experiences. Perhaps, that was the motivation for the splitting of VHL0 to produce pre-visualization and visualization levels. Moreover, the researchers assert that some learners perceive geometry as abstract concepts that have no direct relation to their real-life experiences. That perception makes it difficult for learners to remember and own geometric concepts. From personal experience, some Grade 11 learners cannot mention more than two of the various parts of a circle. Others too fail to differentiate the names of the basic triangles. Perhaps, learners have these problems due to little or no exposure to visuals. To address the challenge with visualization, Clements and Battista (1992) reworked the original van Hieles' levels to include a Pre-visualization level which precedes the visualization level. This Pre-visualization level has been explained under Section 2.2.5.

Despite the criticisms of van Hiele's theory, available literature shows that some researchers (Masilo, 2018; de Villiers, 2004; Abdullah & Zakaria, 2013), still trust that the VHLT is relevant and gives more direction as far as the teaching and learning of geometry is concerned, hence, the choice of van Hieles's model for this study.

2.2.5 Comparing Van Hieles' Levels (VHL), Bloom's Taxonomy (BT), and South African Mathematics Taxonomy (SAMT)

As a means of addressing the loopholes criticized by some researchers on van Hieles' level theory, this study postulates that the pre-visualization level is critical for cognition in geometry. As a result, Table 2.2 has been developed by the researcher to actualize this new van Hieles' level so that there will be six instead of the five original levels. However, there seems to be a direct connection between the van Hieles' levels (VHL), Bloom's taxonomy, and the mathematics taxonomy currently being used in South African secondary schools. Table 2.2 has been dedicated to the various links existing between the three taxonomies.

Originally, there are six Bloom taxonomy levels. These are Knowledge/Remember (1), Understand (2), Application/Apply (3), Analysis/Analyze (4), Synthesis/Synthesize (5), and Evaluation (6). However, in this study, the researcher opted to swap the last two levels of Bloom's taxonomy. The swapping became necessary so that Bloom's Evaluation and Synthesis could match the action verbs that apply to van Hieles' Deduction and Rigor levels, respectively.

TABLE 14.2: COMPARING THREE TAXONOMIES FOR LEARNING GEOMETRY

(ADAPTED FROM: MASILO, 2018; CLEMENT AND BATTISTA, 1992; ABDULLAH AND ZAKARIA, 2013; VAN HIELES', 1986)

Level	Van Hieles' Level	Bloom's Taxonomy	SA Math Taxonomy	Geometric Reasoning
1	Pre-Visualization **(Pre-V)**	Knowledge/ Remember	Knowledge	• To relate real-world experiences to geometry. • Learners rely on concrete models, real artefacts, pictures, electronic and computer resources to learn about geometry. • To use learners' five senses and perception to identify shapes. • To duplicate, repeat, state, define, memorize basic facts and recall ideas about geometric shapes from long-term memory.
2	Visualization **(V)**	Knowledge and understanding	Knowledge	• Identifying geometric shapes without relying on their properties. • Learning and using appropriate geometric terms/linguistics. • To describe, recognize, locate, select, report, classify, translate, and explain basic geometric concepts
3	Analysis **(A)**	Understand and apply	Routine Procedures	• To distinguish and classify given shapes using their properties. • To sketch, interpret, implement, solve, operate, and make use of geometric concepts in new situations.
4	Abstraction/ Order/ informal deductions **(Abs)**	Analyze	Complex Procedures	• To compare, connect, contrast, differentiate, test, examine, and draw a network of relationships among geometric shapes. • To break geometric ideas into parts and connect them.
5	Deductions **(D)**	Evaluate	Complex Procedures	• To judge, appraise, defend, weigh, value, critique, support, and justify a decision with logic. • To use axiomatic systems to establish/prove geometric theories. • To distinguish geometric statements from their converses using the connection between postulates, terms, definitions, axioms, and other theorems.
6	Rigor **(Rg)**	Synthesize and create.	Problem-solving	• To operate in different mathematical systems including axiomatic and non-Euclidean systems. • To plan, invent, assemble, summarize, review, design, construct, conjecture, formulate, and investigate. • To hypothesize, make sets of rules, and produce original/new concepts.

Similarly, the mathematics taxonomy of South Africa has four levels, viz. Knowledge (1), Routine Procedures (2), Complex Procedures (3), and Application (4) (DoE, 2011). However, this study finds it useful to put both VHL 1 and 2 in the same group as South Africa's Knowledge level, since the two taxonomies focus on the basics of geometry at this stage. Likewise, VHL 3 and 4 have been paired with Routine Procedures because the geometry tasks at this level usually require a few steps to finish.

Moreover, all three frameworks assess basic knowledge at the new VHL 1 and VHL2. However, VHL 2 adds the use of correct terminology in geometry. In addition, some aspects of the taxonomies have been noted to overlap with others. This overlap was influenced by the nature of van Hieles' levels where some concepts operate implicitly and later become explicit at the next VHL.

2.2.6 The Technological Pedagogical Content Knowledge (TPACK/TPCK) Model

Shulman (1986, 1987) developed a model called Pedagogical Content Knowledge (PCK) to address the critical knowledge requirements for teaching and learning different school subjects. However, in this 21st century, learning has evolved to include technology usage in many ways, which the PCK alone could no longer address. Consequently, Koehler and Mishra (2005, 2009) introduced the Technological Pedagogical Content Knowledge (TPCK/TPACK) after the researchers criticized the PCK model. Some of the criticisms were that the model (PCK) is only useful for "*time-starved teachers*" (Koehler & Mishra, 2009:398) who have little or no knowledge of the affordances and demerits of modern educational technology. However, the researchers admit that both PCK and TPACK show the disparity between a person's cognition and actions, as well as the subject matter and its accompanying methodology. Perhaps that disparity is what motivated the framers of TPACK to use the "*learning by design"* approach, which is a discovery learning approach, to enhance the teaching and learning process. However, TPACK features different forms of knowledge. The unique features of the TPACK model have been explained below, and are also shown in Figures 2.5 and 2.6.

Content Knowledge (C/CK): This is the knowledge about the specific subject matter that educators need to teach their learners. For instance, every mathematics learner in Grade 11 in South Africa should be taught exponents, quadratic patterns, functions, Euclidean geometry, and measurement, among others. However, due to the spiral nature of the mathematics curriculum in the country, just knowing the topics is not enough. This is because all grades in the Further Education and Training (FET) study, for example, Euclidean geometry, except that the specific contents differ from one grade to the next. By implication, this means the mathematics educator should be very familiar with the specific aspects of Euclidean geometry such as angle properties on parallel lines, midpoint theorem, etc. Geometry in Grade 11, for example, covers the discovery, proof, and application of circle theorems whilst that of Grade 12 deals with proportionality and similarity theorems. Perhaps, knowledge of such contents would guide the mathematics educator to choose or design more effective teaching pedagogies.

Pedagogical Knowledge (P/PK): Selecting the right teaching approach, managing the classroom effectively, or finding out which assessment strategies suit a chosen content are usually challenging for some educators. However, having effective P/PK requires teachers to find answers to questions like these: What method of teaching would be appropriate for the content and the learners? Should that teaching method involve the use of technology, drill, brainstorming, lecture, or a mix of methods?

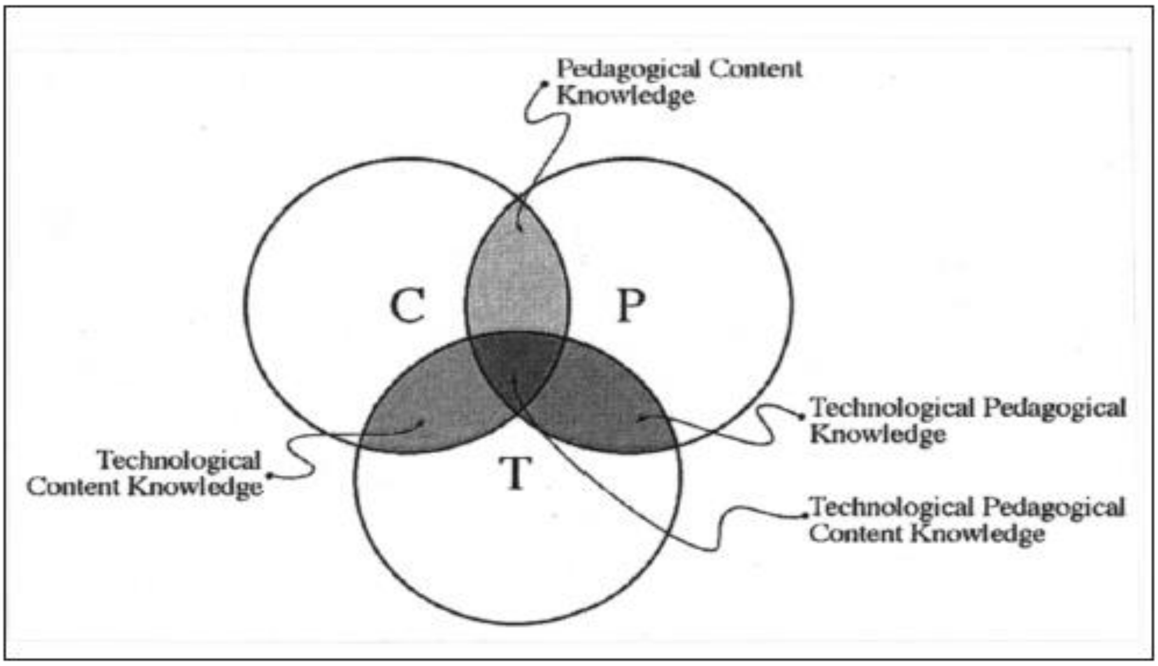


FIGURE 2.5: THE ORIGINAL TPCK FRAMEWORK (SOURCE: KOEHLER AND MISHRA, 2005)

Sometimes, it is important to know what specific teaching practices, procedures, or processes are to be adopted in teaching a particular mathematics topic. It could be that a particular geometry lesson requires group or individual work, a project, or some investigation. Other equally important aspects of PK are the aims and objectives of the lesson. It is necessary to ensure that the aims and objectives are in agreement with the anticipated outcomes, which then influences the kinds of assessment to be used. Hopefully, the answers to the above questions would strengthen the PK of the mathematics teacher. This is because when the mathematics teachers become pedagogically adept, they can use “cognitive, social and developmental learning theories” (Harris, Mishra & Koehler, 2009:397) to design lessons that acknowledge the unique needs of each learner.

Technological Knowledge (T/TK): As educators prepare for the 4th industrial revolution (4IR), the level of teachers’ knowledge regarding basic and advanced teaching tools should be updated regularly (Xu et al., 2018). Similarly, teaching Euclidean geometry requires teachers to abreast themselves with the effective use of relevant technological gadgets and software. It means educators should be conversant with how to draw, drag and paste, copy, save, insert, zoom, etc., using GeoGebra software, for example. It also means that teachers need development on the affordances and weaknesses that come with identified technological tools to maximize learning (Joshi, 2016).

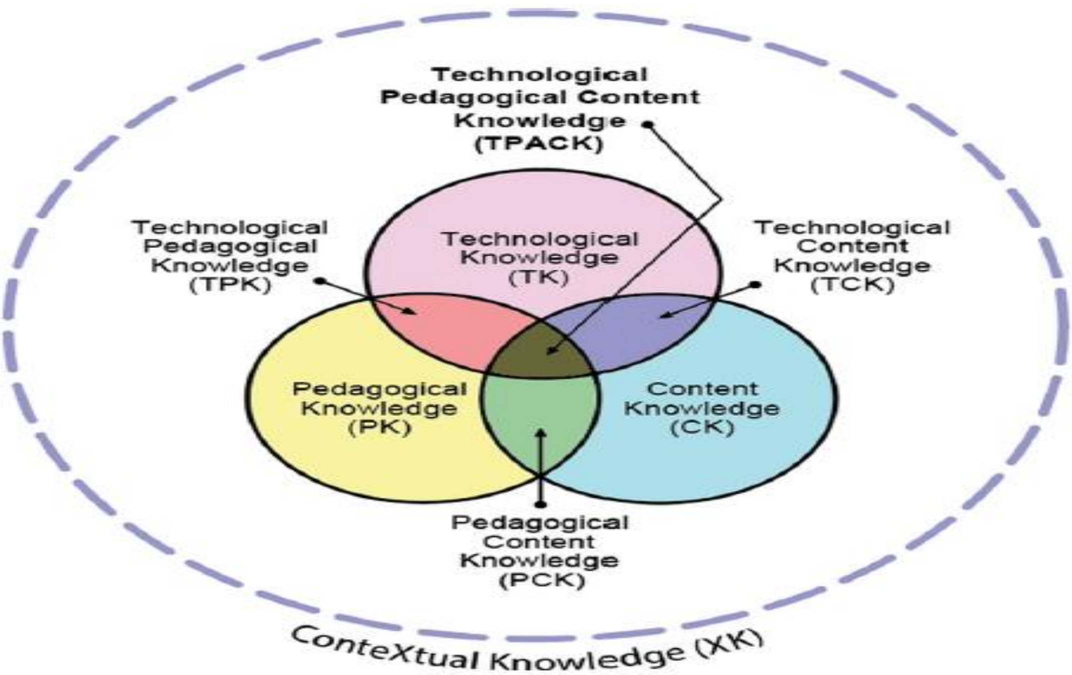


FIGURE 2.6: THE UPDATED TPACK MODEL (SOURCE: MISHRA, 2019)

However, Harris, Koehler and Mishra (2009:395) warn against having "*technocentric*" development in teaching. This refers to putting more effort into learning how the GeoGebra software, for example, operates before considering the exact needs, strengths and weaknesses of the learners involved. On the contrary, the researchers recommend the reverse situation where learners' needs come first. For instance, avoiding *technocentrism* when teaching circle geometry may involve making some considerations such as the following: Grade 11 learners should be developed to acquire deductive skills that would enable them to draw circles, make assumptions, inscribe triangles, measure parts of the lines and angles to verify their assumptions, and make proofs. Thereafter, the teacher tries to find out if the selected technological tool is capable of addressing learners' particular learning needs.

Moreover, the researchers (Koehler & Mishra, 2005; Mishra, 2019) posit that any meaningful integration of technology into teaching should find a way of connecting the elements CK, TK, and PK. Both Figures 2.5 and 2.6 seem to connect the various forms of knowledge that come together to form the TPACK framework. However, the updated model (Figure 2.6) throws more light on the relevance of contextual factors in teaching and learning in general, which could be applied in geometry too. The aspect of contextual factors in TPACK has been further explained in the later part of this section. Epistemologically, the success of using TPACK revolves around the careful interaction between CK, TK, and PK. However, that interaction between the various components of TPACK immediately leads to another set of knowledge, which is the next point of discussion.

Technological Content Knowledge (TCK): The relationship between technological knowledge (TK) and content knowledge (CK) results in the formation of TCK. This is the stage where educators analyze how technology impacts content and vice-versa (Justine, 2017). Unlike the traditional approach where a teacher prepares a geometry lesson separately, TCK would rather consider situations where a mobile phone, GeoGebra, whiteboard, projector, or any technological device or software could be used to maximize the learning output. For example, an educationist may ask, how could learners benefit from the proof of the tangent-chord theorem if learners could use GeoGebra on their phones to do the construction? Or do the displays and visualization of geometric shapes using software improve learners' cognition in geometry in any way? If the answer to any of the above questions is yes, then educators should be encouraged to harness the potential that comes with technology, and integrate the same in a carefully planned, intentionally developed lesson, and an active and constructive learning environment.

Pedagogical Content Knowledge (PCK): When the pros and cons of mathematics content and the related pedagogy are merged, the result is PCK (Koehler & Mishra, 2005a). PCK begins with having an understanding of the mathematics curriculum and making a good selection of a topic like Euclidean geometry, together with all related sub-topics that the mathematics learner should know. Thereafter, the identified content should be matched with suitable teaching strategies, assessment techniques, etc. However, managing the learning environment (Mishra, 2019) through class discipline, availability of learning aids, setting clear objectives for groups and individuals may help to achieve the teacher's outlined objectives in the lesson.

Moreover, the PCK framework fails to acknowledge the role played by technology. Although it could be argued that technology may be considered as part of the teaching pedagogy, the emphasis on its usage under PCK is almost non-existent. Perhaps, this is what motivated the development of TPACK to address that gap by giving technology its rightful place in this 21st century.

Technological Pedagogical Knowledge (TPK): Is it possible for teaching and learning to change because of the introduction of a new technological tool? If so, what sort of knowledge should the mathematics educator have to be able to use the right technologies with appropriate technology? To answer both questions, the teacher should be equipped with developments that enable the teacher to distinguish educational technological tools by their strengths and weaknesses (Joshi, 2016). Hopefully, constant exposure of educators to technology and pedagogy would drive mathematics teachers to change from a traditional teaching approach to be more creative and flexible during their teaching. Flexibility is mentioned here because the technology itself evolves by the minute. Therefore, both learners and teachers should be ready to embrace new techniques and challenges during technology integration (Mhlanga, 2018). However, that does not mean a deviation from the learning objective. It is the pedagogy that gets upgraded because of technology but not by lowering the standards and expectations in geometry. The standard to be maintained includes reasoning with relationships, analyzing, synthesizing, and evaluating concepts based on sound logic. Besides, researchers (Bülbül, 2021; Couco, Goldenberg & Mark, 1996; Sakirudeen & Sanni, 2017) agree that each geometry learner is expected to develop creative, reflective and critical reasoning abilities as part of the habits of mind: standards that cannot be underestimated or lowered in the 4th industrial revolution. Perhaps TPK would help teachers to develop lessons that challenge learners to be inventors, pattern sniffers, or make conjectures and models.

Contextual Knowledge (XK): The particular context within which learning takes place may have some influence on how a learner understands the lesson. It could be that a mathematics teacher plans to integrate technology into the learning process, but the school authorities do not allow learners to bring electronic gadgets to school, or the parents do not want their children to use such devices. At other times, the classroom environment itself could pose a threat to technology integration due to the poor lighting system or non-functioning electrical sockets. Whichever the context may be, its impact on teaching and learning is inevitable. For instance, a school whose learners are allowed regular access to technology usage may find technology integration lessons less challenging than another school with limited or no access to the internet or technological tools.

According to researchers (Kol, 2019; Rosenberg & Koehler, 2015), contextual factors are so many, but they are the "least understood" aspects of the TPACK framework. That misunderstanding makes some educators downplay the role that contexts play in teaching. As a result, some educators fail to analyze the very contexts within which they find themselves before preparing or delivering their lessons. However, Rosenberg and Koehler (2015) assert that the effectiveness of learning activities depends largely on how teachers can reorganize the classroom to suit the particular context of the school and the needs of the learners. Furthermore, the researchers believe that contexts emanate from the interaction between the teacher's characteristics, motivation and beliefs, the learner's characteristics, and the TPACK of both the learner and teacher. Figure 2.7 details the connections between the various contexts in teaching and learning. Originally, the researchers (Porras-Hernandez and Salinas-Amescua, 2013) were the ones who classified contextual factors into "micro, meso, and macro".

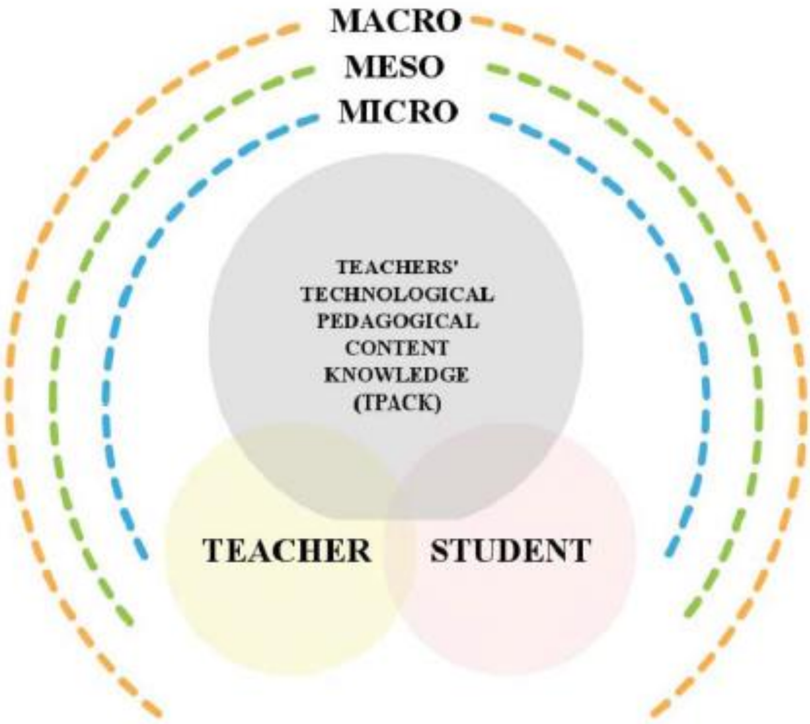


FIGURE 2.7: CONTEXTUAL FACTORS UNDER TPACK (SOURCE: ROSENBERG AND KOEHLER, 2015)

The “micro” level entails the classroom contexts such as the display of learning aids, or the general arrangement of the environment to be used for teaching and learning. Other examples include situations where there is insufficient furniture in the classroom or the unavailability of computers and software needed for technology integration lessons.

However, if the school lacks support staff who are supposed to ensure that copies of documents are made, or who are responsible for the regular maintenance of school computers, then there exist “meso” contextual factors. “Meso” factors could also refer to school management decisions that directly or indirectly affect the integration of technology in mathematics. According to Mishra (2019:77), it is not enough to have knowledge of the Technology, Pedagogy, or Content. Rather, a good understanding of the contextual knowledge plays a key role to a successful geometry lesson. For example, a school may set up a protocol to be followed should a particular teacher want to use the computer laboratory for learning. Such protocols sometimes tend to discourage the integration of technology if different teachers decide to use the computer laboratory at the same time. Therefore, Mishra (2019) would prefer teachers who understand, and are able to manage the context within their schools.

With regards to “macro” factors, they are the external factors that are outside the direct control of the school or the teacher (Rosenberg & Koehler, 2015). For example, a disagreement in the community or during a parents' meeting regarding how a particular school policy is implemented could affect teaching and learning. Moreover, the decisions that are made regarding curriculum development at provincial or national levels could be described as “macro” contextual factors (Rosenberg & Koehler, 2015:189). If there is a lack of support or direction from policymakers regarding technology integration, the teacher could only do little in pushing the technology integration agenda.

That notwithstanding, this study acknowledges the presence of contextual factors during the implementation of van Hieles’ levels. The next section has been dedicated to those contexts that connect TPACK to van Hieles’ model.

2.2.7 Decolonizing VHLT and TPACK: The V-T model

From the above literature, there seems to be a direct connection between van Hieles’ level theory (VHLT) and the Technological Pedagogical Content Knowledge (TPACK) framework. That connection could be decolonized to suit the teaching and learning context of South African Grade 11 geometry, with respect to technology integration.

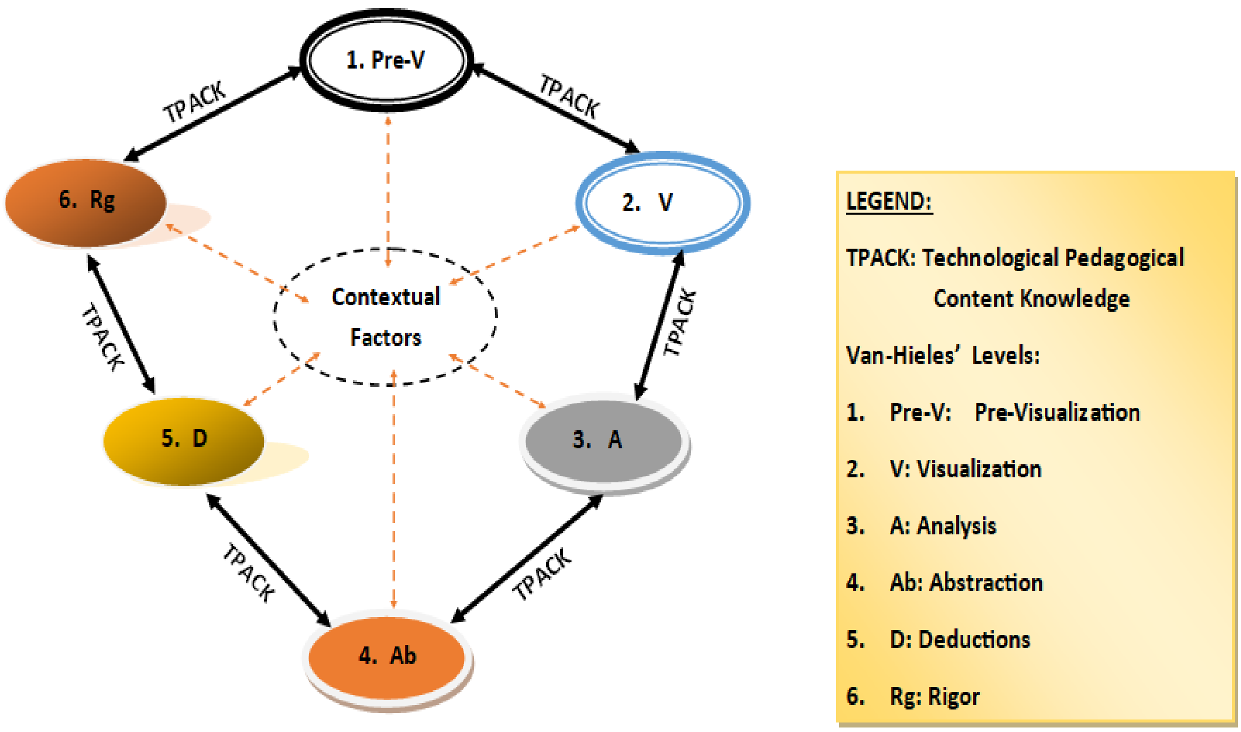


FIGURE 2.8: THE V-T MODEL

The researcher was hopeful that van Hieles' model could be merged with the TPACK framework to achieve the desired outcomes of this study. As a result, I developed the V-T model in Figure 2.8 to help address the possible shortfalls in both VHLT and TPACK models. In the V-T model, the V comes from van-Hieles' theory whilst the T represents the TPACK framework. Those shortfalls have been discussed below.

Firstly, the TPACK model on its own only analyzes the forms of knowledge that apply to specific learning areas. Although the developers of TPACK used "*learning by design"* (Koehler & Mishra, 2005a) to actively engage students when teaching identified topics, perhaps the van Hieles' theory would be more appropriate for teaching geometry in the context of this study. That is due to the ability of van Hieles' model to provide levels of progression through five distinct phases. However, van Hieles' levels also do not necessarily factor in technology integration. Therefore, this V-T model tries to merge technology from TPACK, with the tried and tested VHLT to help make learners gain more from teachers' presentations. In addition, the V-T model adopts six van-Hieles' levels and five van Hieles' phases. The change in levels from five to six is to address the loopholes that were discussed under Sections 2.2.4 and 2.2.5.

Secondly, the typical South African mathematics classroom seems to have a lot of contextual factors that sometimes militate against the effective teaching and learning of geometry. Such factors may include the unavailability of or insufficient textbooks for all learners, unexpected interruptions during lessons due to meetings or workshops, lack of geometric charts in the classroom, among others. Those contextual factors may fall under micro, meso, or macro, as explained by Rosenberg & Koehler (2015). However, this study is of the view that contextual factors exist at every single van Hieles' level, which if not well managed, those factors could derail any success achieved in Euclidean geometry. Therefore, each contextual factor should be well analyzed and addressed as educators move from one VHL to the other.

Finally, the V-T model fully encourages the use of all five van Hieles' phases before progressing to a higher level. However, for effective technology integration, probably what is required is for educators to go through all the available and applicable knowledge forms, as in TPACK, before moving to a new van Hieles' level. Hopefully, applying TPACK in-between van Hieles' levels will provide educators with the required information about the learners, the context, and the concepts learnt or to be learned, or the most useful methodology and assessment techniques, etc. With such information, the teacher may be well prepared to anticipate any possible challenges in the next van Hieles' level and prepare adequately for that.

In conclusion, the V-T model is specifically intended to address the teaching and learning challenges faced when teaching Euclidean geometry in South African high schools. Hopefully, the outcome from this study would determine the effectiveness or otherwise of the V-T model.

2.3 Review in context:

2.3.1 The nature of geometry in South African GET - FET phases

South Africa adopts the spiral approach to curriculum development for most topics in Mathematics education and geometry is one of such. The spiral approach ensures that learning takes place progressively from the less difficult to the advanced ones. It also ensures that each subsequent topic relies on previously learnt content in a gradual, logical, and comprehensive manner. This approach is somehow motivated by constructivism where learners' previous knowledge and learning environment play a crucial role in determining any new content to teach.

For instance, learners are taught the proportionality theorem in Grade 12 (DoE, 2011) but it takes a good understanding of the midpoint theorem from Grade 10 to be able to make meaning from the Grade 12 content. Likewise, the properties of quadrilaterals learnt in Grades 7-9 are applied in Grade 11 when dealing with cyclic quadrilaterals. This makes it imperative to look at successful geometry teaching as a whole although this study focuses on Grade 11 geometry. This study considers geometry teaching to be successful if the teacher's facilitation can motivate the learner to actively construct and own every single geometric concept and apply them in solving real-life problems. However, the level of the learners' understanding of geometry at the General Education and Training (GET) phase seems to affect their further application of geometry in high schools. In Table 2.3, I present the progression of geometry across the GET-FET learning phases in relation to the Grade 11 Euclidean geometry as prescribed in the CAPS document.

TABLE 2.15: PROGRESSION OF EUCLIDEAN GEOMETRY FROM SENIOR PHASE UNTIL GRADE 11

MAIN CONCEPTS IN GRADE 11 (FET) CIRCLE GEOMETRY	ROOT CONCEPTS FROM SENIOR PHASE & GRADE10
GROUP 1: Tangents to circles • Radius is perpendicular to tangent • Tan-chord theorem • 2 tangents from same point outside circle are equal in length	• Drawing and naming parts of a circle from grade 7 • Measuring 90^0, 180^0 in grade 7, and bisection of angles in grade 8 • Construction of angles from grades 8-9 • Properties of isosceles triangles from grade 8 • Investigating sum of angles on straight lines; and angles inside a triangle in grades 8-10
GROUP 2: Circle-center and chords • Angle at center is twice angle at the circumference • Line perpendicular to chord, from center, bisects the chord • Angles in the same segment are equal • Equal chords subtend equal angles in the same circle	• Drawing circles; measuring angles from grade 8 • Constructing perpendiculars and bisecting angles from grade 7 • Measuring angles; drawing and naming parts of a circle in grade 8
GROUP 3: Cyclic quadrilaterals • Opposite angles of cyclic quadrilaterals are supplementary • Exterior angle of cyclic quad equals the interior opposite angle • The 4 vertices of angles in same segment help form a cyclic quad.	• Properties of quadrilaterals in grade 8 - 10 • Relationship between angles on a straight line; exterior angle of triangles from grade 7-10 • Investigating diagonals and angles formed inside quadrilaterals from grade 8-10.

Although Table 2.3 gives the progression of contents in circle geometry, the FET geometry covers content of mathematics topics. Other contents of geometry in FET include using conditions for congruency and similarity to make proofs, the midpoint and proportionality theorems, angle properties on parallel lines, among others. In Grade 11, the aforementioned contents are mostly applied when solving circle geometry problems. For instance, to prove that a perpendicular from a chord to the center of a circle will bisect that chord, congruency has to be fully applied. Similarly, proving that the angle at the center is twice the angle at the circumference relies on the relationship between the exterior angle of a triangle and its interior opposite angles.

However, the inability of learners to understand the basic geometry concepts or even identify the parts of a circle could be traced to many factors. One of such factors, according to the 2019 diagnostic report of DoBE (p.200), could be the passive nature in which some educators teach geometry by not covering "*the basic work"* in "*earlier grades thoroughly"*. As a result, learners seem to have problems with visualizing geometric shapes, and that makes it difficult for them to perform better under higher van Hieles' levels. Perhaps what learners need are real geometric objects, or computer-aided instructions to help them visually identify and classify shapes and their properties.

Most education departments like the Free State, Gauteng, Western Cape, etc., have initiated many developmental programs to encourage teachers to embrace technology integration in their lessons. The Free State department of education, for example, has been collaborating with the University of Free State to develop teachers in short learning programs (SLP) on how to teach online with the technology of which I enrolled as a student. However, educators could not solely rely on computers or software without considering all the pedagogical aspects of teaching. The pedagogy including methods, learning environment, assessment techniques, etc., are all critical and need to be intertwined and connected if learners are to master and create their own geometry concepts. For example, the choice of software should influence the learning approach to adopt, which in turn determines the applicable assessment strategy. In support of the effective use of technology to address learners' understanding of basic geometric concepts, the following quote from Mandell, Sorge and Russell (2002:43) could be useful:

> *"Can teachers teach successfully without using computers? Yes, but it is getting more difficult to avoid them altogether. Can lessons be enhanced with the integration of technology? Most definitely. Will computers ever replace teachers? No. It's been said that any teacher who could be replaced by a computer should be."*

It is not and has never been the objective of technology integration to substitute teachers with software or any electronic devices. However, the nature of the FET geometry content demands lots of visualizations, analysis, and sound deductions that help to arrive at valid conclusions in given problems. All those deductions and applications in geometry may be possible if learners were taught geometry through visuals and assessed after each van Hieles' level of geometric reasoning right from the GET phase. Failure to teach for understanding through each VHL may leave learners confusing the various concepts in geometry at a later stage. Nonetheless, if teachers in the GET phase fail to expose learners to all the correct and basic concepts in geometry, FET teachers tend to struggle fixing learners' errors. However, the challenge of learners not being able to understand and apply their GET geometry at the FET could be addressed with technology (Hamilton, 2015). According to researchers (Mudaly & Uddin, 2016; Mudaly & Budaloo, 2016), through the use of visuals and the interactive nature of GeoGebra, for example, educators could facilitate learners to explore the geometric shapes, their properties and classifications, etc. In addition, learners may learn through the software to develop durable concepts because they did the drawings, measurements, constructions, and proofs all by themselves. In that case, learners tend to own the concepts, and that would boost their morale and also increase their ability to apply the same in different geometry problems.

2.3.2 Some principles and guidelines for teaching with technology

a. Technology should be a slave to pedagogy

Irrespective of the technological device used for teaching a mathematics lesson, it is the teacher who plans and facilitates the lesson. According to Ross (2018:1), technology on its own cannot teach learners geometry. This implies that the selection of a particular technology, the teaching method to use, the arrangement of the learning environment, the learning context, and the kinds of assessment to use should all be well connected. More importantly, researchers (Mudaly & Fletcher, 2019; Nisiyatussani et al., 2018; Sutiarso et al., 2018) believe that technology integration should rely on available and appropriate pedagogy to survive in the learning space. For example, a teacher conducts situational analysis by identifying learners' needs, chooses a content in geometry and selects which learning materials and methods to use. If the chosen content demands a visual aspect, the teacher may go for real objects, drawings, charts, etc. to demonstrate the lesson (Mudaly & Naidoo, 2015). However, if those real objects are difficult to find, the teacher may use a computer or educational software to design those objects and show them to learners. The affordances in GeoGebra software go beyond merely showing pictures of geometric shapes. GeoGebra allows learners to interact with lines, angles, measuring tools, etc., and use those tools to create other shapes. It is evident from the above explanations that the selection of a technological teaching tool should not be the main objective of the learning process. Rather, it should be the pedagogy that influences technology integration.

b. Teach and learn geometry actively

The euphoria that accompanies the use of technological devices sometimes makes teachers forget to follow certain pedagogical principles during the teaching process. For technology integration to improve the learner's performance, the learning has to be active. Active learning implies that the learners do things physically or mentally without just sitting down and listening or watching. In a book on audio-visual teaching methods, Dale (1969:109) presented the "*Cone of Experience"* from the most basic to the abstract level. The researcher asserts that learners easily recall 90% of the things they do physically. However, when learners only say and write things in the class, they remember only 70% of them. Perhaps that is because just saying and writing things are behavioral activities that do not involve much cognitive effort as compared to doing things physically or thinking about geometric concepts. The researcher continues to say that learners remember 50% of what they see or hear, 20% of what they only hear, and just 10% of their readings. That assertion implies that educators ought to actively engage learners when teaching geometry, even if technology is being used. It also means that every van Hieles' level or phase should be demonstrated actively in the classroom. In addition, the visualizations and all other VHL could only be facilitated by the educator whilst the drawing of the shapes, identifying of the properties, constructing, and applying the geometric concepts, etc., should be left to the learners to do. Perhaps, active technology integration lessons could help learners to see the results of their activities as their own. Assuming ownership of learners' own work could motivate them to do more (Woolfolk, 2014), thereby making the learning process a self-rewarding activity.

However, learning geometry actively may be put into two groups: behavioral and cognitive activities. The former refers to allowing a learner to physically take part in the class activities through the answering of questions, writing solutions on the board or in workbooks, marking their peer's work, among others. The latter on the other hand involves situations where learners have to think about ideas, concepts, methods, etc., which are relevant to the given geometric problem. For instance, a learner may be asked to explain how a square differs from a rhombus. In other instances, a learner may be asked to predict the size of an angle before calculating, and later confirming or discarding the initial predictions.

Moreover, educators could engage learners in some activities during technology integration, to build their cognition. Such activities may include "Post-It-Parade" where learners write their ideas, for instance, properties about given geometric shapes, and learners write all and post all the related properties on the board. Post-It-Parade may be useful during the analysis level of van Hieles' model where learners distinguish and classify shapes according to their properties. Furthermore, learners could individually think about given proof-problems, form groups to think about how to make the proofs and share their outcome with the larger class in a "Think-Pair-Share" activity. In addition, learners may use the write a "1-Minute Paper" at the end of each VHL to allow reflection on all concepts learnt a particular level before progressing to the next level.

c. Intentional Learning of Geometry

To teach or learn geometry intentionally, according to Stott (2020), means teachers have to make up their minds on the planned geometry contents to be taught, before and during the lesson. The plan may include the time and the specific technological tools to be used in the lesson. By implication, intentional learning involves situations where the entire geometry lesson is being directed by specific learning outcomes. Such outcomes may include teaching learners to prove why the angle between a tangent and a chord of a circle equals the angle in the alternate segment, by the end of a one-hour lesson.

However, intentional learning of circle geometry may begin with goal setting by the educator to delimit the lesson (Ross, 2018). Setting goals and objectives could help the mathematics teacher and the learner to understand why the topic is important, what learners will achieve after the lesson, what to do in order to achieve the goals, or whether there is a more effective way to learn the content. Secondly, the lesson has to be structured to indicate when the lesson will be introduced, and at what time each activity will take place. Moreover, using VHL provides a structure through which geometry lessons could be delivered. VHL is structured to provide gradual progression from one level to the other, after the completion of all five phases in each VHL (Abdullah & Zakaria, 2013b). Thirdly, intentional learning has to provide learners with time to review their work by pausing and going over their assumptions, calculations, or proofs (Kostiainen et al., 2018; Skuballa et al., 2018). Moreover, learners have to be able to apply their learnt concepts, reflect, or teach others in their class to help learners perfect their developed geometric ideas.

d. Teach and learn geometry meaningfully

According to some researchers (de Sousa, Formiga, Oliveira, Costa & Soares, 2015), learning becomes meaningful when the lesson focuses on sense-making, and the *learner assumes authorship of each of their own knowledge. A technology-integrated* geometry lesson that makes sense is the one that is designed to build and link new concepts with learners' prior learning experiences (Frankel & Mountford, 2021; Kostiainen et al., 2018; Skuballa et al., 2018). Such a lesson challenges learners to explain learnt concepts in their own words and apply them in related geometric riders. In addition, meaningful lessons are structured such that different aspects of the lesson connect. To elaborate more on sense-making, Wolfolk (2014:270) presents three lines similar to the following:

1. *TNUQRVBZPKJLGISCDVBTVNHUCKMRETXYAZKGP*
2. *TANGENT SEGMENT RADIUS CHORD CIRCLE PERPENDICULAR*
3. *RADIUS IS PERPENDICULAR TO TANGENT OF CIRCLE*

From the first three lines above, perhaps Line 3 is the easiest to remember for some reasons. The reason could be that Line 3 seems more organized and the words connect leading to sense-making and retention. In addition, the third line seems more meaningful because the previous knowledge about each word that is stored in the long-term memory is activated in the mind. However, the many words in Line 2 could only be interpreted separately because they do not link to each other as compared to Line 3. As for Line 1, there seems to be no meaning at all.

Similarly, geometry lessons could be developed for sense-making by carefully connecting the related concepts in a structured manner, as in line 3 above. It may not make academic sense to teach learners how to apply the tangent-chord theorem to solve riders when the learner is not even familiar with a tangent, or any other part of the circle. Interestingly, the van Hieles' levels provide that framework for learning geometry on a step-by-step basis (Rezky & Wijaya, 2018). In VHL, learning begins with pre-visualization, then visualization, etc., which helps build a reservoir of the knowledge base for any future development of geometric concepts. For example, learners may be taught to visualize the radius, tangent, circle, etc., before being introduced to the relationship between a radius and a tangent. Unfortunately, teachers sometimes present lessons with little consideration for what learners already know. To make matters worse, some educators teach geometry passively to learners with little to no contribution from the learner. On the contrary, meaningful learning requires the active involvement of the learner at all stages of the learning process (Frankel & Mountford, 2021; Skuballa et al., 2018). That implies making the learner the initiator of ideas, methods, techniques, etc., at every VHL. Perhaps it is only when the learner is totally involved in the knowledge construction process at all VHL that the learner would accept responsibility for each concept developed.

2.3.3 How learners learn geometry

According to Woolfolk (2014:247), each person learns new information, retains or forgets that information, through three main processes: sensory memory, working memory, and long-term memory. Any new concepts that learners learn start from the sensory memory. For instance, the instructions from the teacher, the geometric shapes that learners see, the names of the shapes they hear, the properties of those geometric shapes, etc., all pass through the sensory memory for a maximum of three seconds (Woolfolk, 2014:248). However, the human brain cannot store every single piece of information in it. Some data may be thrown away or kept in the mind due to its relevance to the learner. Moreover, how teachers guide learners to perceive new geometric concepts and pay particular attention to their meanings, usually determines if that new concept is useful to the learner for later usage, and whether such information should be stored in the brain or not. According to Woolfolk (2014:250), all relevant learning concepts from the sensory memory that received enough attention from the learner are then encoded and sent to the working memory for further processing.

Similarly, van Hieles' pre-visualization (VHL1) and visualization (VHL2) stages seem to serve a key role in ensuring that learners use their five senses effectively to understand basic geometric ideas. VHL 1 and 2 provide learners with the opportunity to feel, touch, see, or draw geometric shapes which in turn helps learners to increase their perception of the shapes. However, due to the high volume of information that learners perceive through their sensory memory, selective attention may be required to help learners move from one VHL to the next. Selective attention implies managing the data that enters the sensory register such that certain geometry concepts may be recognized more than others, depending on the given context. For instance, in a diagram where the given circle includes tangents, the learner's attention should be directed towards the tangent-chord theorem, the radius is perpendicular to tangent, etc., instead of, for instance, the angles in the same segment. However, more emphasis should be placed on the learner's prior knowledge since that knowledge stimulates the giving of more attention to the new and related geometry concepts.

Furthermore, some researchers (Ingram, 2014; Webb & Graziano, 2016) assert that a person only pays attention to one cognitive activity at a particular time. By implication, that means teachers should minimize the level of distractions during a geometry lesson. Minimizing distractions may include removing irrelevant content, learning material, or images from the learning environment, and focusing on the very important and related aspects of the topic. In effect, the short duration of data in the sensory memory implies that the teaching and learning process should be well planned by doing, for instance, visualization at a time, and analysis or deductions at other times. Perhaps, it is only when learners can recognize as many as possible Euclidean geometry concepts that the teacher may then introduce multitasking activities such as deductions or rigor. Besides, deductions and rigor are multitasking activities because they fall under higher-order cognitive levels which demand the use of different prior concepts learnt from the preceding VHL.

However, Woolfolk (2014:250) refers to the working memory as the "*workbench"*. That, according to the researcher, is because it is in the working memory that the learner processes the encoded data from the sensory memory and combines that data with the stored information from long-term memory. That newly processed data is then converted into useful information to be stored and reused in solving geometric problems, or for understanding a teacher's presentation. However, Woolfolk (2014: 252) explains that all the sounds, words and verbal information are processed in the phonological loop whiles images and spatial data go through the visuospatial sketchpad. In other words, there are two main channels in the working memory: images and words/texts. In a technology-integrated lesson, the presence of the two channels implies the need to make good use of each medium because each channel accepts different loads of work that cannot be swapped. For instance, if no images or diagrams are used during the lesson, no number of words or texts can fill that space in the visuospatial sketchpad, and vice-versa. According to Stott (2020), that unused space becomes wasted. However, texts and images could be balanced to avoid overloading learners with only one form of data which ends up demotivating them. Moreover, DoBE (2021:207) recommends that to help learners easily recognize and differentiate geometric theorems, teachers should associate statements of geometric theorems with their specific drawn diagrams.

Furthermore, one of the outcomes expected from the geometry learner is to develop critical thinking skills that prepare learners for the ever-evolving job market (DoBE, 2011). What is not certain is whether merely following van Hieles' levels may yield the expected outcome in a technology-integrated lesson. Educators may have to follow some presentation principles within the van Hieles' levels and phases. Stott (2020) discussed such principles to include the pre-training principle where learners are taken through the key items related to the lesson. For instance, each learner should be abreast with the concept of a tangent, radius, chord, etc., as well as their relationships before learning the circle theorems. The second is the signaling principle. According to Mayer (2014), signaling involves highlighting important parts of the lesson through the use of different colors. The signaling principle is very important in geometry teaching because one rider may integrate different theorems at the same time. It then becomes the learner's responsibility to carefully link all related theorems, preferably with colors, to enhance easy recognition. The next principle is coherence. A coherent presentation does not include irrelevant materials, items, or content in the lesson. Coherent lessons direct the learner's attention to the key parts of the lesson in an organized manner. The geometry lesson could avoid ambiguity by making the lesson more coherent, leaving the learner nor room for doubts about the topic, while at the same time giving more room for critical thinking. The final presentation principle is segmentation. Instead of drawing or trying to explain one whole diagram that involves different theorems, educators may cut the diagram into smaller manageable segments. The segmentation principle by Mayer (2014) may be very useful during the deduction or rigor level in van Hieles' theory since those levels mostly combine different circle theorems. Hopefully, the different segments could help learners to connect, synthesize and apply the various theorems, making geometry questions less difficult for the learner. In addition to the above, sometimes learners pay more attention to particular concepts when teachers vary their tone of voice, the pace of delivering the lesson, the verbal and facial expressions used, or even by varying the volume of the voice. Perhaps, when educators follow the discussed theory of multimedia learning and also include the presentation principles in VHL, learners may increase their level of creativity in geometry through critical thinking. Consequently, learners may retain lots of vital geometry concepts in their long-term memory for easy recall when needed.

2.3.4 Challenges in the teaching and learning of Euclidean geometry

Serow and Inglis (2010:10) argue that teaching circle geometry is "often regarded as time-consuming" and learners usually struggle to comprehend and apply geometry's underlying concepts. However, learners in Grade 11 require more time to comprehend the basic concepts of geometry and thereby apply them in solving riders. On the contrary, recent diagnostic reports (DoBE, 2018b; 2017b) indicate that the time given to learners is woefully inadequate, hence, their inability to correctly apply their geometric thoughts. From personal experience in teaching, it is true that time is mostly insufficient for teaching geometry. Perhaps it is not just a matter of insufficient time needed to finish Grade 11 geometry. In my view, the issue of time could be linked to the backlog of work that a learner was expected to cover before reaching Grade 11, which unfortunately was not done at the GET phase (See Table 2.3). For instance, the Grade 11 learner should not be struggling to identify congruency among shapes, angle properties on parallel lines, among others, since those contents are done throughout the senior phase of the South African education system (DoBE, 2012; DoE, 2011).

Perhaps, GET educators are also unable to finish most geometric contents due to time, or they teach the learners in the traditional approach. Whichever the reason might be, the Grade 11 educator should be able to fully prepare the learner on circle geometry (DoE, 2011) before the learner reaches Grade 12 to continue with proportionality and similarity theorems.

Sometimes, to be able to make up for the insufficient time, some educators resort to extra classes to complete the geometry schedule. However, per the CAPS document, the Grade 11 learner is assumed to be able to identify and distinguish different geometric shapes by their properties (DoE, 2011). In addition, Grade 11 learners are usually expected to operate at the abstraction level of VHL. Unfortunately, because educators sometimes want to finish with the work schedule to avoid having any problems with their supervisors, some teachers are unable to give more attention to individual learner needs regarding Euclidean geometry. Moreover, the other unfortunate reality, from my personal experience, is that the mathematics work schedule for schools is mostly designed such that teaching and learning takes place even when learners are scheduled to be writing examinations. Therefore, if teachers are unable to adjust their time to include extra teaching hours, learners may write their tests or exams without knowing much about the topic.

Another challenge, according to Sadiki (2016:18), is that Euclidean geometry in South African schools is taught at a higher van Hieles' level than what learners can attain. This study concords to that assertion. However, the seemingly higher level of the FET geometry could be the result of learners not being exposed to the required primary geometry concepts from the GET phase (DoBE, 2015b). Moreover, such learners with little background knowledge may find it extremely difficult to absorb or apply higher-order geometry concepts. When that happens, the teacher also becomes automatically affected such that s/he cannot proceed to teach the content as laid out in the CAPS document but to go back to cover the basics which learners lack. Should a teacher fail to realize the gap between learners' knowledge in geometry basics, and proceeds to strictly teach per the CAPS document, learning may not take place, which could then lead to the conclusion that geometry content is at a higher van Hieles' level than what learners are expected to be taught.

Furthermore, some researchers (Siyepu, 2005) argue that the geometry contents in the South African GET-FET system are neither sequential nor hierarchical. This study agrees with the researchers because although the spiral curriculum is in use, there seems to be no clear link between the circle geometry that learners already know from GET and what is to be learnt in Grade 11. For instance, although CAPS introduces learners to visualizing a circle from Grade 4 to 6, learners are only introduced to the parts of a circle, for the first time, in Grade 7 (DoBE, 2012; DoE, 2011). In Grade 7, learners learn to describe and name parts of a circle under the geometry of 2-dimensional shapes, and then use a pair of compasses to construct the circle. Thereafter, almost nothing is learnt about the circle until learners reach Grade 11. The break in continuity of circle geometry seems to pose challenges in Grade 11 since most learners remember almost nothing about the terminologies used to describe parts of the circle. According to Alex and Mammen (2012), it is the terminologies in every single field that uniquely distinguishes that field from the rest. Similarly, students of circle geometry should at least be conversant with terms like angles, subtend, radius, arc, diameter, etc. Unfortunately, most Grade 11 teachers who want learners to fully understand circle theorems usually have to reteach those basic concepts, which the CAPS assumes that learners already know from GET (DoE, 2011). On the other hand, if a particular teacher goes ahead to teach those circle theorems without touching the terminologies, the result may not be desirable because learners may confuse the parts of the circle and as a result confuse the entire given question. Perhaps, CAPS should be realigned such that learners could continually make use of basic axioms and postulates that are directly related to the circle right from GET to FET so that learners could apply the same in the Grade 11 geometry of circles.

Furthermore, most of the learners fail to recognize the basic properties of triangles, quadrilaterals, parallel lines, etc. As a result, learners are unable to connect such shapes and their unique or shared properties to the circle theorems properly (Machisi, 2021; Tachie, 2020). For instance, a Grade 11 learner who might want to prove why a given quadrilateral is cyclic, would have to rely on how angles are formed at the center of circles or how to even identify and measure such angles, or what it means to classify a shape as a quadrilateral. However, those classifications of 2D shapes are not directly covered in the Grade 11 geometry, but in GET up to Grade 10 (DoBE, 2012; DoE, 2011). Moreover, the structure of the Grade 11 geometry assumes that learners already know the aforementioned concepts and so, CAPS does not give more room for visualizing or analyzing the various properties of the parts of the circle or other the properties of other related plane figures. It would take a resourceful teacher to bring together all such related concepts (Bora & Ahmed, 2018), but from different sources, to make a learner fully understand a particular concept or theorem. If that fails to happen, the learners lose in terms of understanding leading to more confusion of the theorems.

According to DoBE (2011:4-5), the mathematics curriculum is aimed at developing "active and critical thinking" learners who are capable of serving the needs of the job market. Therefore, any attempt to teach Euclidean geometry in Grade 11 without addressing the accompanying challenges could amount to lowering the standards in teaching the topic. To keep the standard high in geometry requires learners to think outside the box to unravel, for example, which different concepts and principles to apply in solving given problems/riders (Bora & Ahmed, 2018). Perhaps that is why geometry is perceived to be challenging.

To address the above challenges, all stakeholders of educations would have to assist in resolving the identified challenges to produce learners who have the required capabilities to face the spatial and critical thinking needs of the job market.

2.3.5 Available educational technology software

Abacus is one of the oldest technological tools in mathematics education used for computations. However, technology keeps evolving. In 1972, hand-held calculators were introduced (Waits & Demana, 2000). Later, microcomputers, graphing calculators, televisions, the internet, interactive whiteboard, instructional software, etc. were developed (Akcay, 2017). Irrespective of the type of software, Heddens and Speer (2006) explain that educational software may serve one or more of the following functions: exploratory; simulation; games; drill and practice; tutorials; and problem-solving. Some of the available instructional software and web-based tools include GeoGebra, IXL, Khan Academy website, Mathplayground, Mathbits, Math Buffalo State, Touchmathematics, etc. (Akcay, 2017:163-170).

In parallel, attempts have been made by researchers to classify the various educational software. Karadag and Aktumen (2013) explain that Computer Algebra Systems (CAS) is one category of software that combines the capabilities of advanced calculators, Mathematica, Maple, etc., to make mathematics more engaging to learners. However, using CAS requires learners to be well equipped with the basic mathematical concepts and thoughts, else, learners would not benefit from their usage since merely using procedural knowledge to arrive at solutions is usually not enough in mathematics.

Another group of software is the Dynamic Geometry Software (DGS). Examples of DGS include Cabri and Geometer's SketchPad. DGS provides the platform for learners to use specific details to make, test and generalize conjectures that arise out of their interaction with the software. These software are user-friendly by providing users with functions such as lines, circles, points, etc., that users can manipulate to identify numerous patterns embedded in geometric shapes.

However, Karadag and Aktumen (2013) introduced a new term Dynamic and Interactive Mathematics Learning Environment (DIMLE) due to the limitations associated with DGS. The researchers aimed to expand DGS to include other packages whose features are more interactive and suitable for different learning situations. The expanded list of DIMLE includes software such as the Thinker Plots Geometer's SketchPad, Cabri, Fathom, Geocadabra, Desmos and GeoGebra (Karadag & Aktumen, 2013:ii; Martinovic & Karadag, 2011:208). According to the researchers, the key features of DIMLE are their ability to provide dynamic and interactive learning environments for teaching. By dynamism, it means learning will no longer be the same old-fixed-transmission style where learners only receive information from the knowledgeable teacher. Rather, each learner becomes exposed to new technologies and styles of teaching, giving learners the chance to try things out on their own. As learners explore and have insight into geometry, they become confident and capable of making interesting constructions that a teacher might not have taken note of (Martinovic et al., 2014). In addition, DIMLE is interactive because that software provides immediate feedback to learners as they make use of the features of that software through simulations.

According to Karadag and Aktumen (2013:ii), the final class of software is the Intelligent Tutoring Systems (ITS) which make models of educators through robotics and artificial intelligence. Perhaps the ITS could help champion the 4th industrial revolution by making use of the best methodological practices in developing those interactive learning devices for teaching and learning.

Nonetheless, Polly (2014) reveals that although technology integration in teaching mathematics has increased, most educators still prefer to use projectors and document cameras. This could be attributed to the fact that those gadgets are not so difficult to operate as compared to using educational software like Desmos or GeoGebra which requires the user to learn how each tool works. More preference for projectors and cameras could also be attributed to low teacher development in terms of correct selection of technological tools. Perhaps those teachers who have not been developed on the correct usage and merits of different educational tools would not want to risk it. Such teachers may not opt for software or programs they are unfamiliar with but would rather stick to the easy ones.

To sum all up, all those software and tools above confirm one thing: gone are the days when we used to say the pen is mightier than the sword, to wit, educated persons could use their knowledge to solve problems that warriors of the stone-age/pre-colonial era could not probably fix. But of late, the ideas which were previously penned down are now being typed onto computers on daily basis. Others get stored on websites and accessed through smartphones, laptops, etc., for the consumption of the general public. It is that above-mentioned educational software and many others that educators can take advantage of to ensure that learners understand and appreciate the value of each concept they learn in Euclidean geometry.

2.3.6 The GeoGebra software

A study by Akcay (2017:170) shows that at the secondary school level, most educators like using GeoGebra software out of the available educational technology software. GeoGebra is an open-source educational software that can be freely downloaded and is easy to use, which could probably be the reason why it is the favorite of most educators. The software comes in different versions which are useful for handling geometry, functions, etc. Its friendly user interface makes it more appealing and interactive, and that influenced its selection for this study.

Furthermore, GeoGebra, like any other Dynamic and Interactive Mathematics Learning Environment (DIMLE), provides learners the opportunity to learn through visualizations and simulations. Karadag and McDougall (2011) assert that learners enjoy visualizing more than any other form of methodology. In reality, visual learning does not only interest school learners. All adults including teachers like it too. Naturally, learners easily pick useful lessons from their direct contact with artefacts, images, or software programs and make better interpretations than when being told by someone else. Similarly, today's learners view and interact with almost all features of their cellphones, laptops, etc., regularly (Rideout, Foehr & Roberts, 2010). Learners do play their games, surf the web, and many more by the minute, to the extent that some cannot even live without their technological tools for a day, to say the least. As learners interact with their technological tools, they respond to questions, edit, text, and update or create new things on their own. Therefore, it might look strange to some learners to continue receiving mathematical instruction through symbols that they cannot easily picture or visualize. Probably, learners would appreciate our algebraic and geometric symbols after they have been well versed in visuals (Mudaly & Budaloo, 2016). Hopefully, the different versions of GeoGebra may assist learners to continue with their regular explorations, but this time, in the learning of Euclidean geometry.

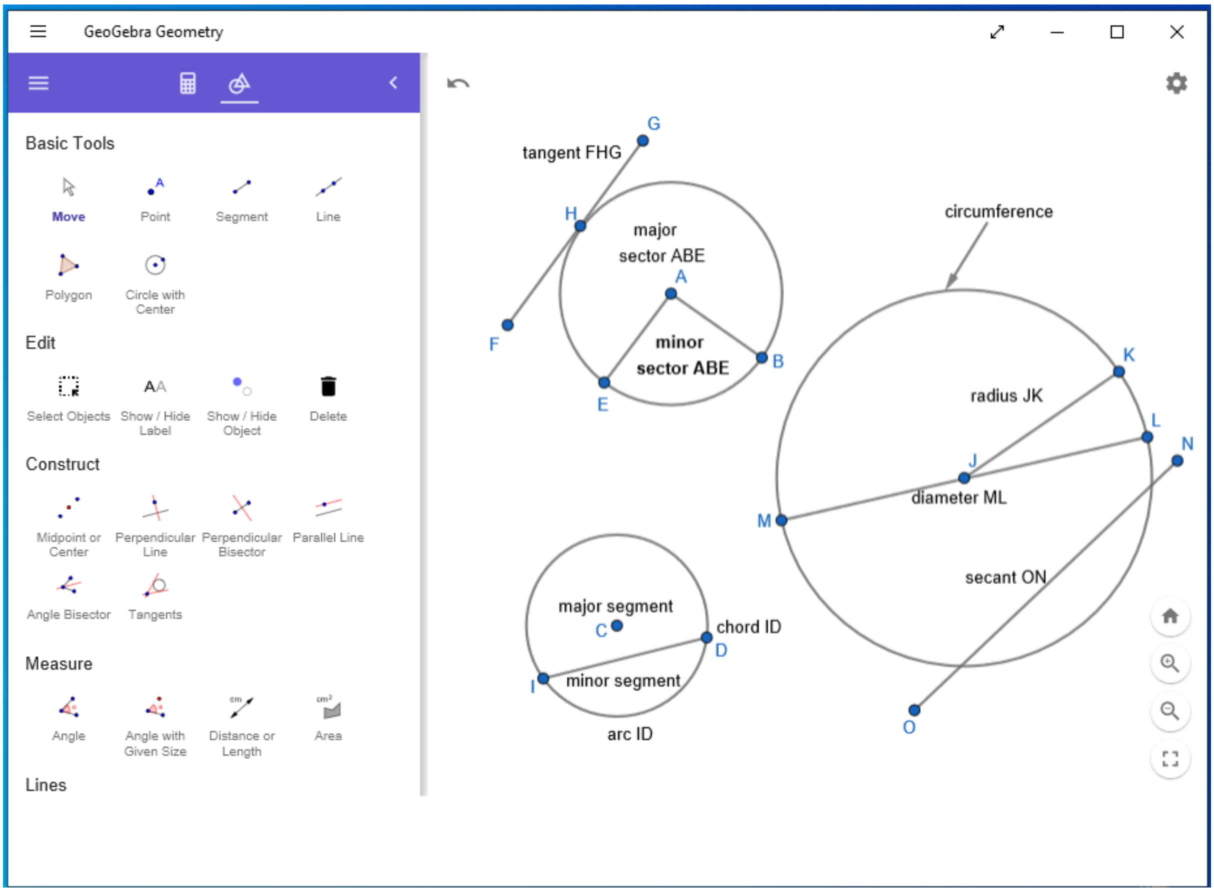


FIGURE 2.9: FEATURES OF GEOGEBRA GEOMETRY SOFTWARE

GeoGebra software comes in different forms. There are GeoGebra Graphing Calculator, GeoGebra Geometry, GeoGebra 3D Calculator, GeoGebra Scientific Calculator, and GeoGebra Classic. Figure 2.1 displays the user interface of GeoGebra Geometry. On the top left corner of the app, there are the main menu, algebra, and the tools tabs. Under the tools box, there are basic tools used to move drawn objects, those for drawing line and line segments, as well as those for creating circles and different polygons. Besides the basic tools, there are other tools for specific functions. Those functions include *Edit, Construct, Measure, Lines, Circles, Polygons, Transform*, and *Media*.

The *Construct* tool, for instance, allows the user to bisect lines and angles, draw tangents, perpendicular and parallel lines, and locate the midpoint of any two points, segment, circle, or conic shape. Similarly, one may use the *Measure* tool to do simulations by determining the sizes of drawn angles, length of line segments, areas, or to preconfigure values for specific angles and draw them. When learners visualize the various tools and start exploring them one after the other, they tend to master the usage of each tool thereby making it easy to engage other learners, to draw and discuss the properties of their drawn shapes (Mudaly & Budaloo, 2016; Velichová, 2011). In addition, the *Edit* tool gives users the chance to add any new ideas or remove unwanted parts, whilst the *Transform* tool enables users to reflect an object, rotate, translate, or dilate from a point.

However, the approach to this study was not to learn how to use GeoGebra, but to maximize the potential the software has in developing the learners' geometric abilities. For that reason, this study only used a particular GeoGebra tool or tab as and when it was considered essential for the topic of the day.

2.3.7 Significance of technology in education

Jojo (2017: 258-259) recommends a classroom environment that challenges learners to "explore, explain, extend and evaluate" given tasks that are easy for learners to make sense of. She challenges educators to adopt learner-centered pedagogy for geometry lessons. Moreover, researchers (Bakir, 2016; George & Sanders, 2017; Mudaly et al., 2015) agree on the importance of using educational technology to improve learners' understanding of different mathematical concepts. They believe technology helps learners to understand, observe, construct, and develop mental images about given problems, which enhances knowledge development in a learner-centered classroom where pre-visualization, visualization, analysis, deductions, and rigor as outlined in van Hiele’s model could be implemented. Furthermore, learning with technology may help develop learners’ self-confidence in handling geometry problems, leading to an increase in retention of memory. That assertion is corroborated by Hamilton (2015: 5) that in a technology integration lesson, “one student’s discovery of a tool, solution or a factoid”, even in a noisy classroom, can easily become a very useful knowledge for the entire class in minutes. That is because learners talk a lot when using technological tools in class. Learners may be seen arguing through peer interactions as they try to make use of, for example, the tools used to draw or measure angles in the GeoGebra software. According to researchers (George & Sanders, 2017; Nel, 2017), as learners share ideas with the guidance of a teacher, they tend to learn faster as compared to always receiving direct instructions from teachers, and this makes learning more interesting, meaningful, and constructive .

2.3.8 GeoGebra and learners’ performance in geometry

Various studies in and outside South Africa have emphasized the impact that educational technology makes on learners’ academic achievements. The GeoGebra software has been identified as one of the tools that engage learners to learn in dynamic ways as opposed to the static nature used in traditional teaching. The use of the software also enhances learners’ ability to observe and make visual representations of geometric shapes, and more importantly, helps learners to explore geometry with little assistance from their educators.

In a recent study, Jelatu, Sariyasa and Ardana (2018) analyzed learners' understanding of geometry concepts using a GeoGebra-Aided REACT strategy in Indonesia. The REACT is a pseudo name for Relating, Experiencing, Applying, Cooperating, and Transferring, which was developed by the Center for Occupational Research and Development in the United States of America (Crawford, 2001). In that study, sixty Grade 8 learners were used in a quasi-experimental design. They used a spatial ability test, prior to the intervention, to put learners in the experimental group into low and high ability sub-groups. They then conducted a test of 5 questions on the experimental group. The control group was taught with the conventional expository teaching approach and, they did not write any pre-test except the same 5-questions test. After using the two-way ANOVA to analyze their outcomes, they reported that the use of GeoGebra-REACT strategy helped learners with the high spatial ability to be more creative whiles the low ability group became very active in class. The control group, however, showed no major output in the written test. This emphasizes the role that educational software plays in teaching and learning. Although the researchers did not work on Euclidean geometry, the outcome of their study is motivating enough to warrant the use of GeoGebra software in teaching mathematics.

Another experimental study was conducted by Seloraji and Eu (2017) in Malaysia where data were collected from year 1 learners aged 5-6 years. The purpose was to determine how GeoGebra software impacts learners' achievement in the learning of geometrical reflections. There were 24 participants of different nationalities comprising 12 boys and girls appease who were randomly sampled and put into control and experimental groups. Each group was made up of three ability sub-groups of extension, core and support groups. A pre-test was written by both groups. However, the intervention was given to the experimental group through the use of GeoGebra on computers to reflect the shapes of buildings. It was followed by the writing of the post-test by both groups. The researchers used the paired sample t-test to analyze the given tests which showed significant improvement from the pre-test to the post-test of the experimental group. The ANOVA also showed differences in terms of gender and ability groupings. However, the outcome revealed that girls perform better than the boys in geometrical reflections using GeoGebra, with the girls recording a mean score of 86.00 as against 68.33 from the boys. Although their study was not on Grade 11 learners, the positive outcome in terms of the control group’s performance could serve as a reference for this study. Interestingly, the exposure to GeoGebra still produced good results despite the different nationalities of the participants. This means the software may work well to help get the desired results irrespective of the geographical background or the kind of learners we have in the classroom.

In another related study, Shadaan and Leon (2013) studied how GeoGebra impacts the teaching of circle geometry through a quasi-experimental study using year 9 students. The researchers administered pre and post-test to the experimental (n=28) and control (n=25) groups. The pre-test showed no significant difference but there was a significant difference in the post-test. The mean difference between the two groups was 4.22, a t-value of 3.989, and a p-value of .000 at p< .05. The questionnaire used also gave “positive feedback” on the use of GeoGebra software. Most participants praised their ability to visualize concepts, think critically, and make logical conclusions after using the software through effective peer-to-peer and teacher interaction under the theoretical framework of Vygotsky’s zone of proximal development (Vygotsky, 1978). That is another confirmation at the international level, of how GeoGebra software impacts the teaching of geometry. The researchers ensured that their study was designed to make learners do more of what is expected under van Hieles’ model. For example, the study encouraged the use of logic to solve problems many problems, which is one of the targets of the current study.

The above studies (Jelatu et al., 2018; Seloraji & Eu, 2017; Shadaan & Leon, 2013) are very relevant to this research because they all used quantitative approaches in determining how educational software (GeoGebra) influences the learning of different aspects of geometry. The research by Shadaan and Leon (2013) which focused on circle geometry, however, failed to indicate exactly what challenges most of these learners face and how to address them. Moreover, some African researchers (Mwingirwa & Miheso-O’Connor, 2016) support the use of GeoGebra for teaching and learning, adding that the software’s effective usage should commence with effective training of the teachers involved. Besides, teacher development is very key in software integration as each software has its own features which the user must be familiar with before using them in class. Hopefully, effective teacher development on the use of educational software will help avoid any embarrassment in class as learners of today are very observant of every action of their teachers. This is because most learners prefer to do what teachers do instead of what they hear from teachers. That means if a teacher provides the wrong direction in terms of software usage, learners will more like adopt the same wrong approach in solving given geometry problems.

In South Africa, a lot of researchers have made some interesting observations regarding the integration of technology into the teaching and learning of Mathematics. Ford and Botha (2010:4) assert that South Africa’s problem with technology integration is more of a negative perception as some school administrators oppose the use of mobile phones during lessons. Once again, it could be that those school administrators have little development on the affordances that come with integrating smart devices into the learning process. The reality, however, is that the same learners use the phones without teachers noticing. According to Mhlanga (2018), it is pointless for institutions to continue to ban smartphones in class without considering the affordances. Instead, schools could adopt, promote, and at the same time regulate smartphone usage with the aim of improving learners' academic performance.

Chimuka (2017) compared the effects of using GeoGebra software in teaching circle geometry to the traditional teacher-centered approach using Van Hiele's theory in Limpopo, South Africa. There were 22 and 25 in the experimental and control groups, respectively. It emerged from the study, after the post-test that there was a significant difference among the achievement of the experimental group. However, a detailed comparison under van Hiele's levels showed no significant difference from Van Hiele's levels 3 to 5 but Levels 1 and 2 had statistically significant differences. Nonetheless, there were sufficient data to confirm that the use of GeoGebra software makes learners more motivated to tackle circle geometry problems. This shows that some learners can still do better without the use of GeoGebra or any other educational software and so teachers need not rely solely on educational technology. Rather, educational technology may be used as supporting tools when teaching. The revelations from Chimuka's study also pose a challenge to educators on the kind of preparation teachers make prior to the integration of educational technology. Since some learners are still able to perform better with or without educational technology (Ross, 2018), educators may have to plan well in advance to cater for the needs of each learner in the class before deciding on whether to use a particular technology or not. In conclusion, teachers' role as curriculum implementers and facilitators during geometry lessons should never be underestimated since the level of facilitation somehow affects learners' outputs directly or indirectly. This study therefore sought to analyze the role played by the integration of GeoGebra software into Euclidean geometry instruction by taking into consideration the various challenges of learners in Euclidean geometry, whilst measuring the effect those challenges have on the mathematics learners in the typical South African high school classroom.

2.3.9 Challenges and control measures when using educational technology

Every intervention comes with its own challenges, and technology integration is no exception. The challenges associated with technology integration range from insufficient resources, poor teacher development, etc., to the beliefs held by stakeholders of education. The discussion commences with some identified challenges, followed by literature that will guide the effective use of technology in schools.

a. Insufficient resources

Insufficient resources usually pose threats to any programme and classroom technology integration cannot be an exception. Mthethwa et al (2020:1) lament the limited use of technology in rural classrooms due to the problem with accessibility, affordability and unstable internet connection. Fortunately for South Africa, the problem of resources is being taken care of as days go by. Data available from the websites (http://www.digitalclassroom.co.za/digitalclassroom/free-state) of the various provinces in the country indicate that each province has some district teacher development centers. Most of those centers were developed in conjunction with Vodacom South Africa and equipped with technological devices for training educators and transmitting learning resources to schools. In addition, Ramorola (2018:8) asserts that about 50.9% of schools in South Africa have access to computers. That percentage seems too small considering the population of the country. However, efforts are being made by the various educational provinces to provide laptops to schools and educators. For example, the Free State province has over the years been supplying Hey-Math schools with two laptops per school. Gauteng's efforts at encouraging technology integration was faced with many challenges including infrastructural deficits (Ford & Botha, 2010:2). However, with the growing number of learners and teachers in the education system, providing schools with fewer laptops could not close the technological gap. Perhaps, if each learner has access to one computer during a technology integration lesson, the facilitator will spend less time giving instructions over and over. However, that is mostly not the case as teachers are sometimes required to let learners share computers, or reschedule the lesson to accommodate all learners, but in separate sessions. Moreover, with the already limited time for teaching mathematics, regrouping learners because of insufficient resources may not only cause time-wasting, but also to loss of focus and the lesson may not become meaningful to the learner.

The issue of insufficient technological resources confirms the problem of technology integration in the country as the available resources are not enough for or directed towards the core objective of schooling which is teaching and learning (). It then serves as a wake-up call to both the government and education professionals to make the most out of technology.

b. Lack of / ineffective teacher development

The challenges associated with technology integration cannot be blamed solely on the lack of resources. Interestingly, Ramorola (2018) argues that although most South African schools have access to computers, only 22.6% of those computers are used for teaching and learning. That assertion raises some teething questions that demand urgent answers. For instance, why are the schools not using the given technological tools for the intended purposes? Could it be that the schools lack the technical capabilities to operate the devices, or the pedagogical beliefs of the teachers do not allow the effective integration of technology? Well, maybe the DoE itself is not providing an adequate supervisory role to ensure that the technological devices given to schools are used for the intended purpose and maintained regularly. Perhaps the latter is the problem because most schools have dysfunctional IT centers with just a few computers in average working condition (Ford & Botha, 2010). My current school and many other schools in the country are examples in this case. Unfortunately, that is what happens when you assign technological tools to schools without providing regular development for the teachers involved (Tachie, 2020:299). This study is of the view that if teachers are well resourced with technological tools and software, and well developed such that they understand how to make the most of educational technology in teaching, there would be minimal room for failure in geometry. This is because there are numerous technological resources over the internet, or in the form of software that teachers, if well developed, could take advantage of to improve the learning of Euclidean geometry.

Unfortunately, seldom do educators get invited for technology integration workshops. When DoE invites educators, most of the workshops are content-based (De Silva, 2015; Tshuma, 2018). Sometimes, even planned workshops get cancelled at the last hour due to financial and logistical constraints. If individual schools were to organize their own inset on technology integration, maybe the level of development would have gone higher. However, the challenges faced in most schools are not so different from that of DoE or the entire country, as some schools do not have enough human capital to take charge of or lead technology-related workshops, seminars, or even lessons.

According to researchers (Ashirbayev et al., 2018; Kriek, 2011), effective teacher development also has to do with knowing which software or technological tool to use for a particular lesson. This is where comprehensive teacher development comes in (Gilakjani, 2013). For instance, it would be out of place to attempt to teach functions with GeoGebra Geometry when GeoGebra Graphing Calculator is available and vice-versa. Usually, deciding which software to use and designing tasks to suit them, go hand-in-hand, and it takes a well-developed teacher to achieve both. Furthermore, teacher development could enhance teachers' planning of mathematical tasks such that each chosen task suits the selected technological tool.

In parallel, Martinovic, McDougall and Karadag (2012:5) argue that well-developed teachers can use different illustrations and comparisons to engage and sustain learners' attention in class. This implies that educators must be conversant with the technical and pedagogical aspects of a selected educational tool or software. On the technical aspect, the GeoGebra Geometry software, for instance, has functions such as basic tools, edit, construct, measure, polygons, transform, media, etc. Each function contains specific tools that could be used for identified tasks. When educators master the use of each tool, and plans lessons accordingly, learners would be in a better position to understand and extend their understanding to solve more geometry problems. It also implies that teacher education programs in tertiary institutions need to start looking at merging technology integration with teaching methodology content so that teachers get used to technology integration before they enter the workspace.

c. Teacher beliefs and negative perception amongst stakeholders

Some parents and even education officials are of the view that using mobile phones or electronic devices in class makes learners lose focus (Dias & Brito, 2021), and for that reason, learners should not be allowed to access these devices whilst in class. No matter the belief of stakeholders on this matter, the reality is in direct contrast. Mudaly (2013) contends that learners rather enjoy lessons that make use of visuals such as GeoGebra "especially if those images create some cognitive conflict with their existing knowledge." This is so because a conflicting visual would drive the learner to find out more, to know the why, when, and how these new images are formed. For that reason, technology integration cannot be relegated because of the perception of poor learner concentration, especially when this problem of concentration could be controlled with effective teacher development.

According to NCTM (1991), mathematics educators perform four major roles. These are goal and task setting, providing stimulus during discussions, providing an active and interactive learning environment, and making effective analyses of learners' progress in given tasks to provide informed remedial. However, all those roles are largely influenced by the nature of belief systems held by each educator (Mthethwa et al., 2020; Mthethwa, 2015). Unfortunately, one of the biggest hindrances to technology integration comes from none other than educators. Some believe that some mathematical concepts are true because the textbook or the teacher says so. Other educators, on the other hand, strongly believe that axioms, theories, and other geometry concepts are true and relevant because they have tested or applied them in real life, and they can attest to that. As one of the teachers who love to investigate most geometry concepts, it sometimes becomes clear to me that most of the topics in the popular textbooks are not so exhaustive. As a result, it requires extra effort from teachers to fill in the missing geometry content by sourcing for more information from the internet, other books, or by making more proofs on their own.

However, when a learner or even a teacher can prove for themselves certain geometry concepts or theorems, they do not only gain self-confidence, it gives some relief, ownership, and form of authority to validate the said ideas which in turn urges them to do more (Nurjanah et al., 2020). On the contrary, if some teachers merely follow what the textbooks say without exploring more on the particular concept, the level of knowledge tends to be narrow and that poses serious risks to learners' conceptual developments. On the other hand, although proofs are nice when done with pen and paper, integrating such proofs with technology could however serve as a boost to help learners visualize and throw more light on each geometry concept. Moreover, the fact that learners are mostly excited about technology usage could be a plus for teachers. Perhaps, educators could capitalize on learners' preference for technological devices to guide and direct learners' quest for new knowledge to geometric proofs.

Finally, irrespective of a teacher's belief systems, Cuoco, Goldenberg and Mark (2010; 1996) identified some habits of mind that mathematics teachers and learners ought to possess. These include being visualizers, the ability to make conjectures, pattern sniffers, experimenters, thinking big and talking less, etc. However, using educational software could help achieve those habits in many ways. The interactive nature of GeoGebra, for instance, helps learners develop the urge to find out more about geometric shapes. The software has simulation properties that teachers could use to let learners develop the attitude to guess possible solutions and conduct an experiment to confirm or reject their predictions. By predicting and testing learners' own assumptions with GeoGebra, the learners develop a love for shapes which makes the work of teachers less difficult, as learners themselves become more capable of extrapolating their knowledge onto other geometric shapes.

d. Measures for successful technology integration

Ross (2018:3) asserts that avoiding excessive use of technology is very important. Rather, educators could creatively weave the geometry content around the individual features of the particular technological tool being used. For example, learners are expected to know how to use the "move" tool in GeoGebra Geometry to drag, enlarge or minimize drawn objects just the same way as using the "line segment" tool to draw specific lengths of lines. By learning the use of each tab or tool in GeoGebra to draw shapes, learners would see technology as a scaffold instead of a solution to all their geometry problems. That is because the learners only use the software to convey their ideas and test them in geometry but not the other way round. Moreover, Mayer (2014) would prefer educators using a set of principles to formulate a well-laid-out plan to guide the successful use of educational software in teaching and learning geometry. For starters, that plan may only focus on the basic features in the particular software such that learners could explore the rest of the software's features on their own.

However, during a technology integration lesson, it is not the technology that yields the results but the teacher's skill in choosing the content, methodology and suitable assessment techniques (Mayer, 2014; Ross, 2018). Researchers (Drijvers, 2013; Mandell, Sorge & Russell, 2002) argue that one of the most important tasks in technology integration is the ability to select, implement, and integrate suitable software. Moreover, the teacher already uses tools like the calculator, chalk, duster, projector, etc. on regular basis. There is no way a lesson involving a calculator would be successful if the teacher himself is not very conversant with all or at least the basic functions on the calculator. Likewise, the GeoGebra or any relevant educational software cannot produce results on their own unless teachers add their skills to their usage (Ross, 2018). In addition, a teacher may decide to let learners work individually or in groups depending on the objectives of the lesson. However, the teacher's role as a facilitator would drive learners to gather and interpret information on their own, which is perhaps the most important asset teachers can ever give to learners. All those facilitator's roles underscore the relevance of the educator's level of development as far as technological tools are concerned.

However, it takes a well-structured lesson plan to be able to ensure success in the technology integration class. The ASSURE model is one of the instructional strategies developed by Heinich, Molenda, Russell and Smaldino in 2002 (Mandell et al., 2002). It begins with *Analyzing Learners* (A) to find out about their needs, strength, and weaknesses to match them with appropriate objectives, content, and materials. This is followed by *Stating Objectives* (S) to specify what each learner would be capable of doing after the instruction. The third and fourth steps are *Selecting Software and Materials* (S) *and Utilizing Software and Materials* (U), respectively. During the utilization stage, the teacher needs to adopt a step-by-step approach since this is the main part of the lesson. A review of the chosen materials would sometimes be necessary and the same applies to the mode of delivery or presentation of the lesson. However, for someone who would like to adopt Van Hiele's model, this would be the time to follow the levels one after the other. The next stage is to *Require Learner Participation* (R). It is at this stage that learners are expected to practice what they have learnt. This could be done in the form of structured activities or worksheets whiles the teacher continues to serve as the facilitator. The final stage has to do with *Evaluating and Revising* (E) the entire instructional process. Factors to consider may include whether the laid down objectives were achieved, the usefulness of the instructional materials and software, and to make reflections on the lesson.

Furthermore, Ramorola (2010:165) proposes that the lesson plan for technology integration needs to have an introduction, development, consolidation, expanded opportunities, and reflection. Similarly, Tatar, Aldemir, and Niess (2018) highlight a four-step guide from the TPACK framework for the planning of technology integration lessons. The first step is the *Examination of Technology Concepts and Documents.* It may involve choosing the topic and setting up the related objectives for the lesson. In

Step 2, the teacher *Determines Learners' Knowledge and Skills* such as the previous and current background knowledge and technological skills of learners. The teacher then decides on the approach to use in delivering the new knowledge. Then comes the identification of the necessary resources needed to realize the laid down objectives. Step 3 is all about selecting which *Technology Product to Apply to Learning.* That also entails outlining what content the product will have, specifying the software, enumerating all skills expected to be used in relation to the anticipated tasks, and making learners use the product. The final step is the *Knowledge and Selection of Intructional Strategies, Assessment method and Evaluation Criteria.* This stage allows teachers to carefully analyze the worth of the technological tool, content, methods used. However, the discussion reveals that these lesson plans do not differ much from the normal lesson plans used for traditional instruction. The difference lies in the kind of tools used to support the learning process.

Moreover, irrespective of the software or technological tools adopted for the lesson, rules and regulations need to be specified before the study. This would help to avoid learners breaking computers or browsing through irrelevant websites during the lesson. However, those dos and don'ts would become more effective if learners themselves set the rules.

2.4 Where this study fits in the literature

After reviewing the literature for this study, there seems to be enough evidence to support the impact that technology usage brings to the learning of Euclidean geometry in South African schools (Bayaga et al., 2019; Bhagat & Chang, 2015; Manganyana et al., 2020; Mthethwa et al., 2020; Mthethwa, 2015; Mudaly & Uddin, 2016). The literature points to the fact that using appropriate educational technology such as GeoGebra for geometry lessons encourages learners to be critical observers, thinkers, and conjecturers (Bülbül, 2021; Cuoco et al., 1996). However, a word of caution was manifested throughout the entire literature warning educators to avoid becoming over-reliant on technology (Ross, 2018; Schleicher, 2012). Rather, educators could enrich their lessons with technology through well-thought-out pedagogy. However, researchers (Koh, 2017; Mayer, 2014) posit that any chosen pedagogy should be the type that engages learners to actively use technology in a meaningful geometry lesson. In addition, the geometry lesson becomes meaningful when learners' prior knowledge and real-life experiences are well integrated into the current lesson (Koh, 2017; Schleicher, 2012), and when learners have a friendly environment to create and share their geometry ideas.

However, although this study was undertaken in a rural setting, the researcher hoped that a careful selection of appropriate pedagogy could ensure effective technology integration. That means the teaching methodology, assessment strategies, the learning environment, the context, etc., should influence the kind of technology to be adopted and not the other way round (Ross, 2018). Consequently, this study hoped to ensure meaningful and active technology integration as suggested by Koh (2017) through the use of van Hieles' levels theory, TPACK, and GeoGebra software. Furthermore, available literature (Ford & Botha, 2010; Kutluca, 2013) encourages the consideration of learners' needs, the learning context, available and appropriate assessment techniques, etc., before settling on, for example, GeoGebra software or van Hieles' model. However, the GeoGebra software seems to provide easy access to learners since it is free to download and use (Kutluca, 2013). The software also provides interaction and simulation that draw learners' interest and curiosity in geometry higher as learners navigate through the various geometry concepts using the many features of GeoGebra (Bayaga et al., 2019). As a complement, van Hieles' theory (VHLT) provides a theoretical basis for teaching Euclidean geometry in a logical and meaningful manner. The literature (Masilo, 2018; Rezky & Wijaya, 2018; Yi et al., 2020) has shown that VHLT combines levels with phases which, in turn allows careful progression of learners from stage to stage. However, instead of the five levels and phases in the original VHLT, this study uses six levels and five phases. What seems limited are studies that combine VHLT and TPACK frameworks through the use of GeoGebra, for effective technology integration. Also limited are geometry studies conducted in rural settings. In addition, only a limited number of studies consider the entire grade 11 geometry as a whole (Bayaga et al., 2019; Chimuka, 2017; Mudaly & Uddin, 2016; Ogbonnaya & Alfred, 2014; Ramorola, 2010). Whilst some studies consider a few aspects of geometry, for instance, quadrilaterals only, other studies are more general (Kutluca, 2013; Masilo, 2018). This study hoped to address the identified loopholes by having a comprehensive look at the technology integration of grade 11 circle geometry in a rural environment through GeoGebra, van Hieles' levels (VHLT), and the Technological Pedagogical Content Knowledge (TPACK) frameworks. With regards to the VHLT, this study hoped to adopt the change in levels from five to six which emanates from the splitting the original visualization level such that there are pre-visualization and visualization levels separately (Clements & Battista, 1992; Masilo, 2018). However, this study acknowledges the knowledge and contexts that surround the learning process after reviewing the TPACK model (Rosenberg & Koehler, 2015). Therefore, the researcher hoped to analyze the various TPACK knowledge types in the con+text of this study before applying the VHLT to integrate GeoGebra into the learning of Euclidean geometry. Hopefully, the merging of TPACK and VHLT to integrate GeoGebra software in the teaching process would improve the required pedagogy by mathematics teachers to enhance learners' performance.

2.5 Chapter Summary

The literature has thrown more light on the need for educators to modify their methodology regarding geometry teaching. The review has shown that geometry is more visual than theoretical. Therefore, learners should be exposed to technological tools that may help learners visualize or create mental pictures of each geometry concept so that, any analysis of the properties of shapes could be backed up by meaningful interpretations from those visuals. Moreover, by using van Hieles' model to integrate GeoGebra into the learning of Euclidean geometry, the learners could make the most of the learning process by constructing as many as possible geometric concepts in a more enjoyable and meaningful manner. Hopefully, technology integration would improve learners' academic performance, and minimize the negative perception that some stakeholders have with regards to technology integration. Perhaps effective technology integration could encourage the DoE and the government to renew their commitment to providing more technological resources to schools so that schools could ready the learners for the numerous prospects of the 4th industrial revolution.

CHAPTER THREE: DESIGN AND METHODOLOGY

3.1 Introduction

This chapter begins with a presentation of the aim, objectives, and research questions that guided this study. Furthermore, the hypothesis, paradigm, design, research procedure, and research methods used in the study are discussed. The methodology section covers the population and sampling techniques, instrumentation, and a detailed process of data collection for the experimental and control groups. Next, I present the data analysis, validity, reliability, ethical issues, and the scope and limitations of the study. The chapter concludes by summarising the entire procedure used to design, collect, and analyze data for this study.

3.2 Aim

This study aimed to determine the impact that the integration of technology has on the teaching of Grade 11 Euclidean geometry in Motheo District of South Africa, based on van Hieles' level theory.

3.3 Objectives

The following three research objectives guided the entire study. The study sought to:

a. Identify the challenges that learners in Motheo District of Free State experience in understanding circle geometry in Grade 11.
b. Determine how the use of GeoGebra software through each van Hieles' level in the classroom affects learner achievement in the teaching and learning of Euclidean geometry among Grade 11 students.

3.4 Main Research Question

The main research question was:

What is the impact of technology integration in teaching Grade 11 Euclidean geometry on Mathematics learners based on Van Hieles' model?

3.5 Research Questions

These questions were used to provide answers to the main research question:

i. What challenges do Grade 11 learners experience in circle geometry?

ii. How does the use of GeoGebra software through each van Hieles' level in the classroom affect learner achievement in the teaching and learning of Euclidean geometry among Grade 11 learners?

To be able to answer Question (b), six underlying issues were considered and analyzed with reference to each of the van Hieles' levels (VHL). Those concerns included ways in which:

(i) The use of GeoGebra software in teaching Grade 11 geometry impacts learners' performance under van Hieles' Pre-visualization (VHL1) level.

(ii) The use of GeoGebra software affects Grade 11 learners' achievement, under the Visualization stage (VHL2) of van Hiele's model of geometric thinking.

(iii) The use of GeoGebra software affects Grade 11 learners' achievement under the Analysis stage (VHL3) of van Hieles' model.

(iv) The use of GeoGebra software affects Grade 11 learners' achievement under the Abstraction/ordering stage (VHL4) of van Hieles' levels.

(v) The use of GeoGebra software impacts Grade 11 learners' achievement during the formal Deductions stage (VHL5) of van Hieles' model.

(vi) The use of GeoGebra software affects Grade 11 learners' achievement under the Rigor level (VHL6) of van Hieles' level theory of geometric reasoning.

3.6 Hypothesis

The study adopted four null and four alternative hypotheses. The findings from the study were used to determine whether to accept or reject these hypotheses.

i. $\boldsymbol{H_0}$**:** There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners.

   $\boldsymbol{H_1}$: There is significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners.

ii. $\boldsymbol{H_0}$**:** There is no significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

H_1: There is significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

iii. ***H_0*:** There is no significant interaction between the control/experimental groups and their pre- and post-tests.

***H_1*:** There is significant interaction between the control/experimental groups and their pre- and post-tests.

iv. ***H_0*:** Using GeoGebra software in classroom does not improve Grade 11 learners' academic achievement across each van Hieles' level in the teaching and learning of Euclidean geometry.

***H_1*:** Using GeoGebra software in classroom improves Grade 11 learners' academic achievement across each van Hieles' level in the teaching and learning of Euclidean geometry.

In the next section, I introduce the research paradigm through which this study was conducted.

3.7 Research Paradigm

This study was situated in the positivist paradigm. According to researchers (Makombe, 2017; Shah & Al-Bargi, 2013)**,** positivism mostly embraces real reality as the ontology through objective epistemology under quantitative approaches. In other words, positivist research should begin with the identification of the reality of existing knowledge (ontology) to lead the knowledge acquisition process through objectivity (epistemology) during the collection, analysis, and interpretation of quantitative data. Similarly, this study considered technology integration as relevant to the teaching and learning process of Mathematics. As a result, the impact of effective technology usage was considered as the ontology of this research in the sense that, almost every aspect of our human lives relies on some form of technological advancement. For instance, from checking daily temperatures at the hospitals to commuting to and from work with automobiles, or recording learners' academic profiles on South African School Administration and Management Systems (SA-SAMS), or using a particular educational software to enhance learning, are all somehow linked to technology usage or its integration. However, there seem to be some questions regarding technology usage. For example, what constitutes effective technology integration? Is it even necessary to include technological tools during classroom instruction? Is technology integration worth the time and resources when some teachers can or have been teaching for years without technology? Those were some of the questions that guided the ontological views of this study. Consequently, this study aimed at identifying the impact which technology plays when used to teach Euclidean geometry at Grade 11 in South African schools.

Moreover, ontology seems to work best when connected to appropriate and related epistemology. Perhaps the reason is that ontological views alone may not necessarily yield an outcome that is proven, tested and supported by research or scientific data. Therefore, this study set out specific objectives to help gather numerical data, which guided the epistemological process. Objectivity was the epistemological stance of this study instead of assuming subjective views used in other paradigms such as the interpretive or constructivist paradigms.

Furthermore, adopting objectivity based on the discussed ontology demanded that the research methodology used in this study be quantitative. That is because this study required data from both experimental and control groups to be collected and analyzed to help accept or reject the identified hypotheses of the study.

The next section of this study explains in detail, the adopted research design.

3.8 Quasi-experimental: Control group interrupted time series design

The design for this study is quasi-experimental that is situated in a positivist paradigm. Specifically, the researcher adopted the control group interrupted time series design. According to McMillan and Schumacher (2014:300), time-series designs are appropriate when the study intends to *"repeatedly"* measure the performance of subjects before and after an intervention as opposed to other designs where single pre- and post-tests are used. However, there are two kinds: the single group and the control group interrupted time series designs (p. 300-302). The single group time-series design only uses one group with many pre-observations, an intervention, and post-observations. Subsequently, the control group time-series design was used in this study, and that design is similar to the single group design barring the addition of a control group to help strengthen the threat posed by history due to the repetition of tests. To give more strength to the design, the researcher used parallel forms of the pre- and post-tests such that the respondents did not have to write the same test twice. A diagram of the research design is displayed in Figure 3.1.

Moreover, this design was selected for several reasons. For example, McMillan and Schumacher (2014:300) explain that time series design is mostly suitable for situations where the dependent variable could be timeously observed in a "*continuous, naturally occurring"* environment as the researcher administers one or more unique interventions during those observations. Similarly, the van Hieles' levels (VHL) being used in this study required the sequential development of geometric concepts from level to level. Therefore, using the time-series design in this study allowed the researcher to make objective observations regarding Grade 11 learners' academic performance (the dependent variable) as the geometry learner progresses along the VHL through technology integration (the independent variable). In addition, the inclusion of a control group to the time series design, as explained earlier, is known to help minimize the threats of history and instrumentation (McMillan & Schumacher, 2014:302), which have been further explained under reliability and validity in subsequent sections. In the next section, I present the research procedure.

3.9 Research procedure

Figure 3.1 provides a summary of the entire research design process used in this study. In that design, and through purposive sampling, an already existing Grade 11 mathematics class from one school was sampled to represent the control group.

Similarly, another existing class from a nearby school was also purposefully selected to form the experimental group for the study. Thereafter, there were three pre-tests O_1, O_3, and O_5, as well as three post-tests O_2, O_4, and O_6, which were written by both groups. However, there were three interventions in-between the pre- and post-tests of the experimental group. The control group, on the other hand, received no intervention except being taught by the researcher through the traditional teaching approach.

In addition, Pre-test 1 and Post-test 1 were designed to assess van Hieles' Pre-visualization, Visualization, and Analysis levels (VHL1-3) whilst Pre-test 2 and post-test 2 were dedicated to Abstraction and Deduction levels (VHL4-5). The final pre- and post-test 3 focused on Rigor (VHL6). The construction of the test items has been detailed in the next section.

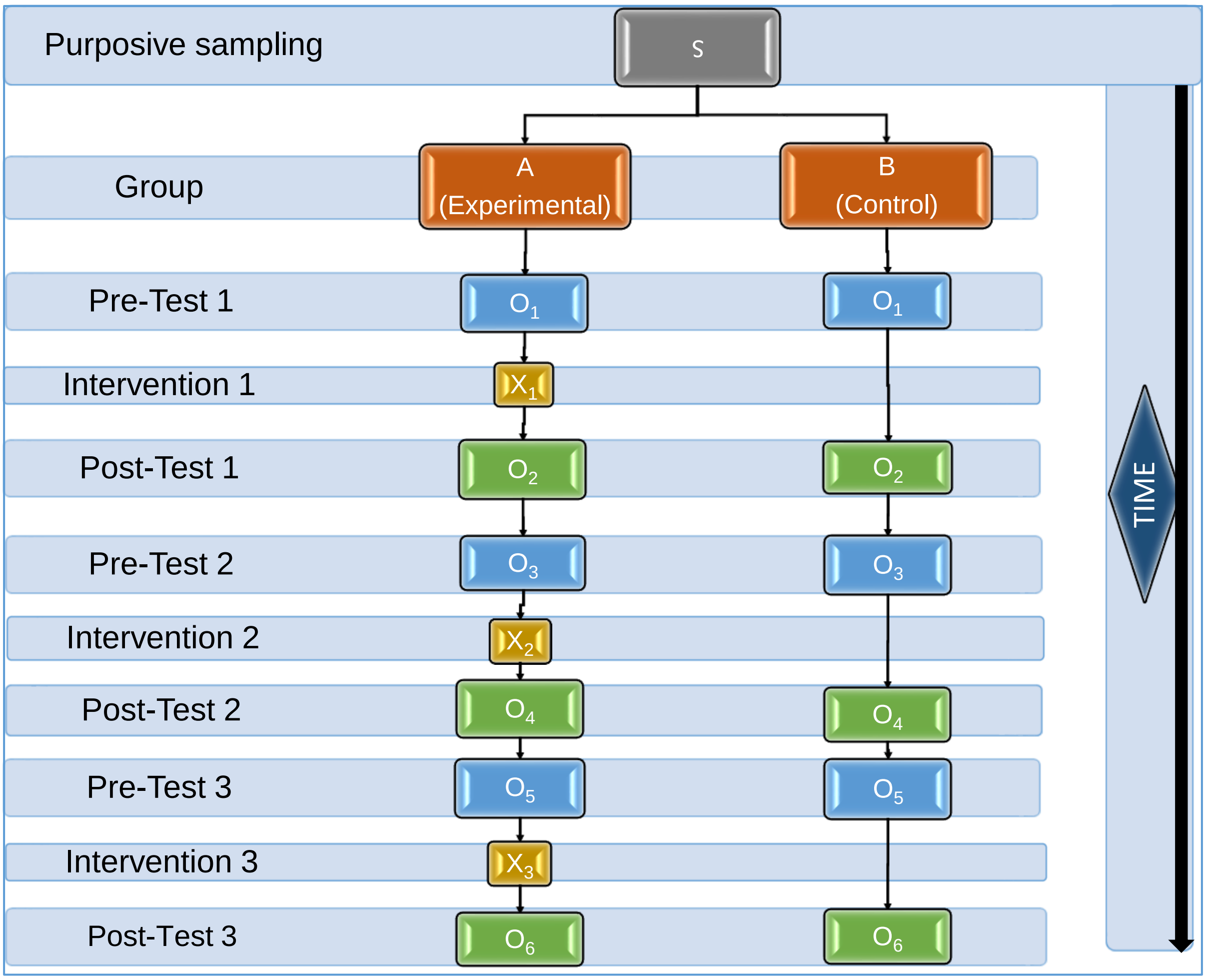


FIGURE 3.2: THE CONTROL GROUP INTERRUPTED TIME-SERIES DESIGN

Furthermore, the three interventions for the experimental group had one thing in common: technology integration. Prior to the testing and interventions, the researcher assisted each learner in the experimental group to download and install GeoGebra, a free software, on learners' mobile phones. In addition, the researcher installed the same software on the computers in the information technology (IT) laboratory of the experimental group, for use by the experimental group during the intervention phase. However, each intervention was unique. For instance, Intervention 1 was administered immediately the respondents submitted their Pre-test 1. That Pre-test 1 focused on Pre-visualization, Visualization, and Analysis of van Hieles' levels. Therefore, the first intervention was structured such that the researcher could use GeoGebra software to correct all learners' mistakes in Pre-test 1 before they write post-test 1. Prior to the correction of learners' errors in Pre-test 1, the researcher used the GeoGebra software to highlight the key concepts associated with each of the first three van Hieles' levels.

Some of those key concepts include the ability to physically and visually identify geometric shapes and their prototypes, sort and classify shapes according to their unique properties.

Similarly, Intervention 2 targeted the skills learners were expected to possess as they solve geometric questions under van Hieles' Abstraction (VHL4) and Deduction (VHL5) levels. This intervention used GeoGebra software to highlight the interconnections between some geometric concepts and theorems, to help provide the required foundation for learners to do good deductions on given geometric problems. However, Intervention 2, just like Intervention 1, was done by the researcher after learners had finished writing their Pre-test 2, waiting to write the Post-test 2.

Finally, Intervention 3 was purposefully structured to assist learners on the various possible ways of solving geometric riders or real-world geometric problems. The emphasis, however, was the need for learners to be calm when faced with questions under Rigor (VHL6). Questions under VHL 6 are sometimes so advanced that learners have to rely on the various forms of knowledge acquired from VHL 1-5 without introducing any new concepts (Masilo, 2018:73), to carefully break the given problem into manageable parts before re-assembling the various solutions to the problem. Moreover, all the interventions were administered at regular intervals, always at the same time in-between a pre-test and the corresponding post-test, to ensure fairness and consistency in the data collection process.

To control contextual factors emanating from the use of two different schools, the researcher sought the assistance of the Principal and the Mathematics teacher of each school to assist with supervision. Their supervisory role included ensuring that the allocated classrooms were well organised and conducive for learning. The computers from both schools were checked in advance to ensure that they were in working condition. Moreover, the assigned times for teaching, learning, and the writing of the tests were the same in both schools. However, it was the days for instruction that were rotated. For example, if school A learners are taught today, school B is taught the next day at same times by the same researcher.

3.10 Methodology

I begin this section with the procedure used for selecting the population and the sample for this study.

3.10.1 Population and sampling techniques

According to McMillan and Schumacher (2014:143) it is important for researchers to clearly define their target population/universe and their sampling frame or survey population. For this study, the target population was Grade 11 Mathematics learners of Motheo District in Free State, South Africa. To understand the level of performance of the target population, I collected data from some teachers in Circuits 6 and 7 of Motheo District regarding their Grade 11 learners' mathematics performance in schools for the period 2018 to 2020. Table 3.1 shows the data from some schools in the said district.

TABLE 3.1: GRADE 11 MATHEMATICS PERFORMANCE OF SOME SCHOOLS IN MOTHEO DISTRICT.

	Circuit	2018 November Exam Pass %	2019 November Exam Pass %	2020 November Exam Pass %
School A	6	53	70	75
School B	6	52	71	95
School C	6	56	34	66
School D	7	67	81	51
School E	7	70	53	61
School F	7	47	33	45

The data from Table 3.1 seems to portray some level of inconsistencies in the performances, coupled with below-average performances in some of the schools. Perhaps Euclidean geometry contributed to the low performance of those schools since geometry weighs a large percentage of 33.3% (50/150 marks) of Mathematics Paper 2 (DoBE, 2011:10), and 16.7% (50/300) of both Paper 1 and 2 put together (DoBE, 2017b, 2018b). This implies that an improvement in learners' geometry understanding would directly affect their performance in examinations.

Predictably, the pattern does not look any different when analyzing the Grade 12 performance of the same schools from 2017 to 2020. Comparing Tables 3.1 and 3.2, School A in 2018 achieved a 53% mathematics pass rate in Grade 11. However, since the same group of learners wrote the final National Senior Certificate examination the following year, the performance of 58.3% in 2019 does not seem surprising considering the group's previous performance (DoBE, 2019b:189). That 5.3% increase in performance could be an indication of hard work in Grade 12 although that hard work could not make much difference. It is the belief of Department of Education (DoBE, 2021:207) that addressing the fundamentals of geometry in earlier grades (including Grade 11) such as drawing and classifying geometric shapes or proving circle theorems could project higher pass percentages in future examinations at the Grade 12 level and beyond.

TABLE 3.2: NSC GRADE 12 MATHEMATICS PERFORMANCE OF SOME CIRCUIT 6 AND 7 SCHOOLS IN MOTHEO DISTRICT

SOURCE: DOBE (2018A-2020A). NSC SCHOOL SUBJECT REPORT.

	Circuit Number	NSC 2018 PASS %	NSC 2019 PASS %	NSC 2020 PASS %
School A	6	59.5	58.3	52
School B	6	66.7	68.4	93.3
School C	6	96.2	96.2	97.5
School D	7	100	68.9	74.2
School E	7	84.3	64.6	45.6
School F	7	40	52.2	41.2

Interestingly, the Grade 11 performance of schools in Circuit 6 and 7 for the same period 2018-2020 seems to permeate through the Grade 12 performance of the district, and to the national level. Table 3.3 indicates how Motheo District fared compared to the provincial and national mathematics pass percentages. Once again, the data in Table 3.3 concurs with that of the two previous tables above, all pointing to the need to reinforce learning of the basic concepts in geometry as repeatedly captured in the various school subject reports used in Table 3.3.

TABLE 3.3: COMPARING MOTHEO DISTRICT'S GRADE 12 MATHEMATICS PERFORMANCE

RETRIEVED FROM: NSC SCHOOL SUBJECT REPORT (2017A; 2018A; 2019A; 2020A)

Year	2017	2018	2019	2020
National NSC Math Pass %	51.9	58	54.6	53.8
Free State NSC Math Pass %	70.6	74.3	68.5	66.3
Motheo District NSC Math Pass %	69.5	78.7	71.2	67.9

In addition, the National Diagnostic Reports (2017b-2020b) keep lamenting the poor performance of learners in Euclidean geometry; a performance that is similar to that of the learners in schools of Motheo District. Consequently, the researcher hoped to find a way of addressing those concerns related to geometry that are contained in those diagnostic reports, which are also pertinent in Motheo District. Hence, the use of Motheo District as this study's target population.

However, the researcher sourced the study's survey population or sampling frame from approximately 600 Grade 11 mathematics learners in Circuits 6 and 7 of Motheo District. Moreover, that sampling frame was targeted due to the varied nature of the mathematics pass percentages of the various schools in the Circuits over the past years. In addition, it was noted that there was a lack of consistency in the performance of learners in most of the identified schools as shown in Tables 3.1 and 3.2 of which geometry is a contributing factor (DoBE, 2011, 2017b, 2018b). Furthermore, the critical role played by geometry in Further Education and Training (FET) cannot be overemphasized. For example, all of the circle geometry content in schools are taught at grade 11 and revised in Grade 12. That makes the Grade 11 geometry a very critical component when analyzing Grade 12 learners' performance. Therefore, this study considered the mathematics performance of schools in Circuits 6 and 7, and accordingly used those schools as the survey population to whom the outcome of this study would be applied, and for onward generalization of the results to Grade 11 learners in Motheo District of the Free State province. Next, I present the sampling procedure.

The sample size for this study was 60 in total representing 10% of the sampling frame. Those 60 participants were made up of 30 learners per group representing the control and experimental groups. Each of the groups was an already existing Grade 11 class. Besides the selection of the two groups, Creswell (2012: 319) seems to agree that a study, such as this one, being quasi-experimental requires pre-determined classes so that the study does not cause much interference in the normal academic programme of the host schools.

However, the researcher used the purposive sampling technique in choosing the sample for this study. According to McMillan and Schumacher (2014:152), using purposive sampling in quantitative studies demands that the researcher selects subjects who satisfy the required *"characteristics"* of the study in order to form a representative sample. With regards to the characteristics of the participants, this study chose participants who were all Grade 11 Mathematics learners from Circuit 6 and 7 of Motheo district in Free State. However, the participants in the experimental and control groups were 30 each, making a total of 60. Considering the sampling frame of 600 learners, that 60 participants represent 10% which lies within the recommended range of 8% to 12%, according to McMillan and Schumacher (2014: 155). Furthermore, the researchers (p.156) recommend a minimum of 15 participants per group for comparison and experimental designs. However, this study's 30 subjects per group are double the suggested benchmark of 15 using the rule of thumb, thereby increasing the probability of getting statistically significant results. Moreover, all the subjects in the experimental group came from a school that had a well-functioning information technology laboratory since the group required technology integration as their intervention. On the contrary, participants from the control group, also 30 in number, did not have to work in any computer laboratory since they received no intervention except being taught by the traditional approach by the researcher.

Interestingly, the control group's average academic performance from their Grade 10 end-of-year reports, was not so different from that of the experimental group. Data from participants' end-of-year (Grade 10) academic reports showed that the average percentage of the control group was 45.3% whilst that of the experimental group was 43.6%, showing only a small margin of 1.7%.

Considering the characteristics of the participants explained in the previous sections, Grade 11 learners from schools C and D in Table 3.1 were purposefully selected by the researcher as the experimental and control groups, respectively. The inclusion of other schools in Tables 3.1 and 3.2 sought to provide a general view of the performance of schools in the identified education circuits. However, the said learners in those two schools (C and D) showed some comparable levels of consistency in their academic performance. Moreover, learners from the control group were from Circuit 7 whilst that of the experimental group were chosen from Circuit 6 of the same community, so as to ensure a balance in the selection, and a fair generalization of the findings of the study.

3.10.2 Instrumentation: Designing of Pre- and Post-tests

This study made use of pen-and-pencil tests (McMillan & Schumacher, 2014:204). Tests were preferred to other instruments such as interviews because this study is quantitative, and the researcher aimed at assessing learners' cognitive abilities under each of the six van Hieles' levels, which is more suitable for tests. In total, the pre-tests were three in number. However, Pre-test 1 (PrT1) and Post-test 1 (PoT1) were not the same questions, but they had a similar structure in terms of the cognitive level for the test items. The choice of similar test items for pre- and post-tests was done because according to McMillan and Schumacher (2014:197), such parallel test items enhance test equivalence.

In addition, PrT1 and PoT1 assessed participants on van Hieles' Pre-visualization (VHL1), Visualization (VHL2), and Analysis (VHL3). However, the test items in VHL1 were 6 whilst that of VHL2 had 16 items. That number of test items for VHL1, although only 6, were developed to allow learners to have a feel of some real geometric objects or their prototypes before progressing to VHL2. As for VHL3, there were 20 test items, making a total of 40 test items for 40 marks in both PrT1 and PoT1 combined. Some of the questions were multiple choice whilst others were the short-answer type. Moreover, the researcher gave each participant an hour to answer the 40-mark tests. That means each learner spent a maximum of 1.5 minutes on each 1-mark question. However, that 1.5 minutes is a little higher than the 1.2 minutes spent on a 1-mark question at the national level in all Mathematics tests and examinations. The time increase from 1.2 to 1.5 minutes was to afford learners more opportunity to analyze the given problems and apply the basic geometry concepts in solving those questions.

Similarly, the researcher designed Pre-test 2 (PrT2) and Post-test 2 (PoT2) to assess the performance of learners under van Hieles' Abstraction (VHL 4) and Deductions (VHL 5) levels. The test items under VHL 4 tested the ability of learners to informally deduce the interconnection between different geometric theorems. In addition, the test items only assessed the same cognitive levels under VHL 4 and 5 respectively, but the questions were not the same. However, VHL 4 sought to provide learners with an insight into established geometric theories through the use of formal deductions to make conjectures and proofs. Moreover, PrT2 and PoT2 had a maximum of 40 marks each although there were only 9 in all. Each question required learners to provide written and valid statements and reasons as answers. Likewise, PrT1 and PoT1, the researcher allocated sixty minutes for PrT2 and PoT2.

Furthermore, the participants wrote their final Pre-test 3 (PrT3) and Post-test 3 (PoT3) which focused only on van Hieles' rigor level (VHL 6). Those tests were also similar in terms of cognition, but the questions differed for the same reasons as the previous tests discussed above. However, those tests (PrT3 and PoT3) assessed learners' cognitive abilities when it comes to using scientific means to make insight into geometric systems. In other words, the given questions under van Hieles' rigor level were structured to challenge learners to use all available theorems, axioms, etc., to break the given open-ended geometry questions down, prove them in parts, and assemble the various conclusions afterwards. However, each test under VHL6 consisted of two questions with a total of 15 marks. Due to the high cognitive level of the tests under VHL6, the respondents were given 30 minutes to answer the 15 marks questions.

3.10.3 The Data Collection Process

Right after ensuring the validity and reliability of the research instruments, the researcher began collecting the data from the two groups as discussed below.

3.10.3(a) Experimental Group

This group was selected from Circuit 6 during the sampling process with a total of 30 participants. The researcher, with the assistance of the Grade 11 mathematics teacher of the host school, conducted Pre-test 1 of 40 marks. Thereafter, marking of the scripts was done by selected mathematics teachers from the same community as the host school. That was followed by a feedback session that included the provision of the designed intervention for this group. After the free installation of GeoGebra software onto learners' phones and the school's computers, each learner was assigned a computer to use during the intervention. The researcher led the class through the pre-visualization stage (VHL1) by showing different electronic models of geometric shapes. Learners were guided to draw as many as possible of such geometric shapes using GeoGebra software.

Subsequently, the researcher drilled learners on van Hieles' visualization level (VHL2) as learners were tasked to assign names to various geometric shapes, draw those shapes, or identify their prototypes. In addition, learners were introduced to the properties of the various geometric shapes through the use of GeoGebra. That was done by making learners draw and measure, for example, the lengths of each side of a square to confirm in the sides are equal or not. Moreover, the researcher urged learners to form small groups behind the computers, to debate and predict the various properties of their own drawn shapes before measuring with the simulation tools in the

GeoGebra software. All those processes were geared towards fixing learners' mistakes that they made during PrT1. This intervention lapsed over two sessions where each session lasted for an hour.

Immediately after Intervention 1, the learners wrote PoT1 during the following session. Thereafter, a similar cycle ensued where the experimental group was exposed to PrT2, followed by Intervention 2, and then PoT2. However, unlike the first intervention, Intervention 2 targeted making the participants develop the ability to establish relationships among theorems and also make formal deductions. For that reason, the researcher used standard past exam papers on geometry, for learners to reproduce those shapes using GeoGebra software, and try to synthesize or make deductions using the measuring tools in the software. However, most of the work under Intervention 2 was done by the learners themselves whilst the researcher only took a facilitator's role. There were many instances where learners actively argued and debated each other in their attempts to find a viable solution to the given geometry problems. This intervention also lasted for two sessions of 1 hour each before learners proceeded to write PoT2.

Finally, the respondents were given PrT3 to write for 30 minutes. Thereafter, the researcher regrouped the learners to make use of the GeoGebra software to find or test solutions to the answers they provided in PrT3. Surprisingly, most learners came up with unique ideas regarding how to prove the given problems in different ways. Thereafter, the researcher administered the final test, PoT3. Moreover, feedback on all the tests was provided to the participants prior to the commencement of the subsequent sessions.

3.10.3(b) Control Group

This group received no intervention. However, the group wrote all the pre- and post-tests in the same manner as their counterparts in the experimental group. To explain further, the control group wrote PrT1 at the same time as the experimental group through the supervision of the principal and the Grade 11 mathematics teacher of that host school. The researcher limited the role of the host schools' mathematics teachers and their principals to that of supervisors only, to ensure that their collaboration (Elabdali, 2021; Laal & Ghodsi, 2012) did not directly influence the outcome of the study. It was only the researcher who led all the teaching and learning processes in both the control and experimental groups. However, the researcher only provided feedback sessions to address learners' mistakes through the traditional talk-and-chalk teaching approach. Thereafter, the learners wrote PoT1. However, the same process used in the first test was used in writing PrT2, followed by a feedback session, and then PoT2. That same approach was also used when learners had to write PrT3 and PoT3. However, both groups were given the same time frame to write, complete and submit all the tests.

3.11 Data Analysis

The analysis for this study was done through descriptive and inferential statistical procedures. The scores of the various groupings, for example, control and experimental groups, or each van Hieles' level were analyzed using the mean and standard deviation of the groups, as well as the mixed factorial analysis of variance (2-way mixed ANOVA) to compare the significance and performance of the subjects in the identified groups. The level of interaction between learners' tests and the groups was further analyzed using a paired samples t-Test at each van Hieles' level. According to MacMillan and Schumacher (2014:327), the 2-way mixed ANOVA is useful for situations where there is a minimum of two independent groupings to be analyzed. Similarly, this study made use of control and experimental groups who each wrote tests that covered six different van Hieles' levels. Therefore, the many van Hieles' levels and the two main groups (control and experimental) made the choice of ANOVA more relevant and suitable for this study.

The researcher started the data analysis process by compartmentalizing and coding the tests. For example, the various scores of respondents were classified and captured under the six van Hieles' levels. In addition, the coding used included Pre-visualization of Pre-test 1 (PrV PrT1), Visualization of Pre-test 1 (V PrT1), Analysis of Pre-test 1 (A PrT1), Abstraction of Pre-test 1 (Ab PrT1), Deductions of Pre-test 1 (D PrT1), and Rigor of Pre-test 1 (R PrT1). Moreover, the researcher used the same approach of coding for Pre- and Post-test 2 and 3. Furthermore, a similar set of codes were adopted for the post-tests, starting from Pre-visualization of Post-test 1 (PrV PoT1), up until Rigor of Post-test 3 (R PoT1). However, the total for each van Hieles' level (VHL) was computed and analyzed for significance against the stated hypotheses of this study. In all, the total scores for each of the pre- and post-tests were coded, captured and analyzed descriptively and inferentially, and presented in the form of tables and graphs.

Finally, all the recorded scores from the tests were captured into the International Business Machines Corporation's Statistical Product and Service Solutions (IBM-SPSS). SPSS software was chosen because it is designed to provide users with descriptive, bivariate, and geo-spatial, among others, analysis tools that have been incorporated into a friendly user interface.

3.12 Validity

McMillan and Schumacher (2014:189) opine that when the score from a particular test can provide the study with a measure of appropriate inferences, that test could be deemed to have content validity. However, the validity of instruments may depend on the particular instrument or the corresponding research design. For instance, the test results from this study may only be valid for this study but not for others. Therefore, the researcher did not only look at whether the *"test is able to measure what it is supposed to measure",* rather, the study took into account the context of this study which covers the purpose, population, and the environment used for data collection to ensure content validity.

In addition, this study focused on geometry, technology, and performance of Grade 11 learners in Motheo District of South Africa. The researcher posits that a valid instrument for this study ought to factor in the methodology and how mathematics tests and examinations are conducted in South Africa. For example, the Curriculum and Assessment Policy Statements prescribes that all mathematics tasks should have 20% knowledge (K), 35% routine procedures (R), 30% complex procedures, and 15% problem-solving. Similarly, the tests used for this study were structured to contain all those cognitive levels which sometimes overlap compared to the van Hieles' levels (VHLs), as shown in Table 2.2 of Chapter 2. That overlapping became more evident due to the restructuring of the original 5 VHLs to form 6 VHLs for this study. Regarding methodology, the traditional teaching method seems to be the most commonly used approach in the classroom despite the technological advancements in education. Therefore, the researcher structured the test items to allow the control group to continue receiving traditional instruction whilst the experimental group took to the use of GeoGebra software. However, all the tests were written under normal examination conditions to match the reality on the ground in terms of mathematics testing in schools, but with technology integration, as a means of ensuring that the tests could assess Grade 11 learners' geometric abilities irrespective of the technology integration. Despite the measures put in place to ensure test validity, the researcher adopted further research techniques to affirm the validity of instruments, as explained below.

Researchers (Ayre & Scally, 2014; Lawshe, 1975) assert that a test item has content validity if over 50% of experts consider it relevant to the study. This study used expert opinion to ensure that the test items were valid. Those experts were 5 in total, and they involved 3 Subject Education Specialists (SES) for Mathematics in Free State Province, together with 2 experienced mathematics educators selected from the schools where the control and experimental groups were hosted. However, Ayre and Scally (2014) further note that for a study like this one that used 5 experts, all 5 experts had to agree on each test item for that item to be considered essential to the study. The number of experts who must agree on each test item was calculated as follows:

The original formula by Lawshe (1975) was $CVR = \frac{n_e - \frac{N}{2}}{\frac{N}{2}}$ where CVR represents the content validity ratio is; n_e is the number of essential test items identified by the experts; *N* is the number of experts on the panel. In addition, the *CVR* values range from -1 to 1 where -1 is "perfect disagreement" and 1 represents "perfect agreement." Moreover, it is only when the *CVR* is above 0 (50%) that the item is considered essential as experts have the option to choose from "essential, useful but not essential, and not necessary" (Ayre & Scally, 2014).

However, in an attempt to verify if Lawshe's (1975) calculations were valid, Ayre and Scally (2014) used the normal approximation to the binomial to calculate *CVR* as shown below:

$CVR = \frac{z\sqrt{N}+1}{N}$, where z represents the normal approximation to the binomial; *N* is the total number of experts on the panel.

This study therefore used the *CVR* formula produced by Ayre and Scally (2014) to arrive at selecting the 5 panel of mathematics experts and having all 5 agree on each item to be considered essential to study. Interestingly, all the 5 experts were asked by the researcher to indicate whether the test item was essential/relevant or not essential/irrelevant following the aims and objectives of this study, the cognitive levels used in the South African mathematics curriculum, and the 6 van Hieles' levels. However, all the experts agreed that the test items were essential, leading to a *CVR* of 1 for each test item. The only criticisms from some of the panels were the wording of a few questions. The researcher immediately paraphrased those statements and resubmitted them to those experts for review, which the panel accepted as relevant before the researcher used those tests for data collection.

3.13 Reliability

As part of measures to ensure that the data collected for this study was reliable, the researcher conducted a pilot study on 15 Grade 11 mathematics learners from a nearby school that is situated in the same community as the control and experimental groups. The same research instrument used in collecting data for this study was used for the pilot study. However, the researcher controlled the threat of pre-testing by ensuring that the intervention received by participants did not necessarily influence participants' performance in the post-tests. That threat of pre-testing was controlled by creating 'parallel forms' of the pre- and post-tests, instead of using the same instrument twice (MacMillan & Schumacher, 2014:197).

TABLE 3.16: TEST OF RELIABILITY USING PEARSON PRODUCT-MOMENT

	VHL 1	VHL 2	VHL 3	VHL 4	VHL 5	VHL 6
n	15	15	15	15	15	15
df (residual)	13	13	13	13	13	13
Pearson's r	.87	.81	.90	.84	.77	.83
Alpha (*a*)	.001	.001	.001	.001	.001	.001
Significance (*p-value*)	.000	.000	.000	.000	.001	.000

However, the researcher analyzed the outcome of the pilot study through the use of Pearson product-moment (*r*) to test for equivalence between the pre- and post-tests (MacMillan & Schumacher, 2014:182). However, since this study used van Hieles' level theory (VHL), the *r* and its level of significance were determined for each pair of pre- and post-test at the various VHL as shown in Table 3.4.

Moreover, the results from those computations in Table 3.4 revealed a positive and strong relationship between participants' pre- and post-test across all van Hieles' levels. For instance, at VHL 3, this pilot study reported $r(13)=.90$ at $p= .000$ where $p< .001$. That result, together with the rest of the data from Table 3.3, indicates a strong and positive relationship between the pre-test and post-test scores at each VHL except VHL5 where $a=p$, but still significant at $p=.05$. In effect, the data provide evidence of equivalence and stability in the research instrument used for this study. A summary of the scores from the pilot study for each VHL is attached under Appendix 1.

3.14 Ethical Considerations

Some specific measures were put in place by the researcher to ensure that all ethical procedures were followed to the later. Those measures have been elaborated below.

3.14.1 Permissions

The researcher sought permission from the University of South Africa by applying for ethical clearance from the Ethics Review Committee of the College of Education. My approval has reference number **2020/06/10/56098324/14/AM.** That clearance paved the way for the commencement of the study. In addition, since the study was being done in Free State, the researcher applied for and received approval from the Free State Department of Education to allow the use of 60 learners (30 each) from two secondary schools and three subject advisors for the data collection process. Furthermore, parents and guardians of respondents were sent consent forms for their approval to allow learners to partake in the study. Finally, each permission letter outlined the particular measures put in place to prevent the spread of coronavirus (Covid-19) during the data collection process.

3.14.2 Consent and Assent

Parents received and signed consent and assent forms to permit their children to partake in this study. However, those forms specified the intent of this study, the possible risks involved, as well as benefits learners and teachers could derive from the outcome of the study. Voluntarily, parents gave their consent for the study. However, the consent and assent forms made provisions to ensure the safety and privacy of all respondents, which have been explained below.

3.14.3 Privacy, Confidentiality and Safety

The researcher assured participants of their safety, which was achieved by ensuring that the studies took place immediately after the normal weekday classes, with active supervision from both the principal and the mathematics teachers in the host schools. In addition, learners' scores from the tests were locked in a safe cabinet for five years before disposing them off. Moreover, the researcher coded the scores of learners for all the tests to avoid tracing the scores to the learners who wrote that test. Furthermore, no learner was required to write their names on the tests. Rather, the researcher assigned unique codes to each learner, which made learners' responses more anonymous.

3.15 Scope and Limitations

Although this study spanned over 2 years, the actual data collection was done in about a month. The scope of this study covered only Grade 11 learners in Motheo District of Free State, South Africa. Specifically, the learners were sampled from only two schools in Circuits 6 and 7 of the said District due to some constraints which have been detailed here. For instance, that short period for collecting data was influenced by the onset of Covid-19, making it difficult to have regular access to respondents. That affected the time series design of the study which mostly requires lots of time.

In addition, this study used a total sample size of 60 learners, a number that could have been increased if not for the time constraints and the Covid-19 restrictions. Another limitation of the study was the use of 2 already existing Grade 11classes for the study. Although the existing classes are mostly suitable for quasi-experimental studies, there seems to be the possibility of a bias in selected classes especially if there are many existing classes of the same Grade 11 learners in the same school, where only 1 class gets selected for either the control or experimental group. According to McMillan and Schumacher (2014:305), the use of pre-existing classes makes it difficult to control threats such as attrition, statistical regression, selection, instrumentation, among others. However, the efforts made by the researcher to control those threats have been explained under the validity and reliability sections of this study.

3.16 Chapter Summary

This chapter highlighted the quasi-experimental design, the procedure and methodology used for the entire study. There was a detailed explanation of the procedure for selecting the 60 respondents as well as how the research instruments were developed by the researcher. The chapter further explained the possible threats that could undermine the validity and reliability of the data collected, and the measures put in place by the researcher to control such threats. In addition, this chapter detailed the intervention that was given to the experimental group, as well as how ethical issues were managed by the researcher. Although the outcome of the study would be explained in the next chapter, the processes that were used for data collection and analysis have all been explained in this chapter.

CHAPTER FOUR: DATA ANALYSIS AND DISCUSSION OF FINDINGS

4.1 Introduction

This chapter presents the analysis and discussion of the findings of the entire study. I commence with the factors of analysis.

4.1.1 Factors of analysis

In this section, the factors of analysis of the data are presented. The analysis was done using the International Business Machines Corporation's Statistical Product and Service Solutions (IBM-SPSS) software. As explained in the previous chapter, a 2-way mixed ANOVA was employed in this study to compare the performance of the subjects within and between groups. The between-groups comparisons (1st factor of analysis) were done between the experimental (E) and control (C) groups as shown in Table 4.1.

TABLE 4.1: WITHIN- AND BETWEEN-GROUPS FACTORS OF ANALYSIS

		Within-subjects Factors													
		1. PrV of PrT	2. PrV of PoT	1. V of PrT	2. V of PoT	1. A of PrT	2. A of PoT	1. Ab of PrT	2. Ab of PoT	1. D of PrT	2. D of PoT	1. Rg of PrT	2. Rg of PoT	1. VHL1-6 of PrT	2. VHL1-6 of PoT
Between-groups Factors	C														
	E														

In Table 4.1, **PrV of PrT** represents Pre-visualization of pre-test; **PrV of PoT**: Pre-visualization of post-test; **V of PrT**: Visualization of pre-test; **V of PoT**: Visualization of post-test; **A of PrT**: Analysis of pre-test: **A of PoT**: Analysis of post-test; **Ab of PrT**: Abstraction of pre-test: **Ab of PoT**: Abstraction of post-test; **D of PrT**: Deduction of pre-test: **D of PoT**: Deduction of post-test; **Rg of PrT**: Rigor of pre-test: **Rg of PoT**: Rigor of post-test; **VHL1-6 of PrT:** Van Hieles' Levels 1 to 6 of pre-test; **VHL1-6 of PoT:** Van Hieles' Levels 1 to 6 of post-test. In all, a label of 1 is pre-test whilst 2 represents post-test.

However, since each group wrote tests on two occasions (pre- and post-tests), the groups' performance over time was the within-subjects variable (2^{nd} factor of analysis). The mean test scores of each group were assigned to the empty cells in Table 4.1 and analyzed accordingly. The final stage of the analysis considered the overall performance (interaction) of the groups in the tests for all van Hieles' levels (VHL1-6). Those between- and within-group factors of analysis were used throughout the analysis of scores in this section of the study. Finally, that analysis led to the acceptance or rejection of the hypotheses, which consequently helped to answer the research questions of this study.

4.1.2 Testing assumptions: Normality, independence, and homogeneity

According to (Lane, 2016:114), analyzing data in a study like this should consider having independent "observations, normality within groups, and homogeneity of variances." For the entire study, the observations in the between-group variable were independent since a respondent could only be in one group at a time. In addition, the normality of scores within groups was explored and it was found that the majority of scores at all van Hieles' levels were normally distributed. Evidence from the Shapiro-Wilk normality test (See Appendix 2) revealed that 13 out of the 28 values were not normally distributed ($p<.05$) (Ahad et al., 2011; Hanusz et al., 2016). However, the fact that the overall sample size remained 60 for each van Hieles' level also assumed the normality to be less prominent in this study.

Regarding the assumption of homogeneity, Gastwirth, Gel, and Miao (2009:1) assert that Levene's (1960) test is very useful when analyzing the "homogeneity of variances". Levene's test could have been ignored for the entire study since the sample size was bigger (N=60). However, to verify if the error variance of the learners' geometry scores in the tests was the same across the two groups (within-groups), Levene's test for equality of variances was adopted. For example, Table 4.2 shows non-significant *p*-values ($p>.05$) to signify the non-violation of Levene's (1960) test under van Hieles' level 1.

TABLE 4.2: LEVENE'S TEST OF EQUALITY OF ERROR VARIANCES UNDER VHL1

		Levene Statistic	df1	df2	Sig.
Total of Pre-Visualisation for Pre-Test 1	Based on Mean	.068	1	58	.795
	Based on Median	.023	1	58	.880
	Based on Median and with adjusted df	.023	1	55.701	.880
	Based on trimmed mean	.104	1	58	.749
Total of Pre-Visualisation for Post-Test 1	Based on Mean	1.671	1	58	.201
	Based on Median	1.330	1	58	.254
	Based on Median and with adjusted df	1.330	1	53.680	.254
	Based on trimmed mean	1.590	1	58	.212

Interestingly, all other van Hieles' levels in this study, except VHL6, recorded non-significant values (see Appendix 3). The higher non-significant differences further imply that the majority of scores in the control and experimental groups had a similar dispersion throughout the study.

Based on the above, it was decided that the assumptions were adequately met to proceed with the analysis process. I began the analysis with van Hieles' pre-visualization level.

4.2 Analysis under van Hieles' Pre-visualization (PrV) level (VHL1)

This section tested the study's hypotheses by analyzing the performance between the control and experimental groups, the total scores from the pre- and post-tests, and the interaction among variables.

4.2.1 Between Subjects Effects under van Hieles' Pre-visualization level

The analysis in this section focused on testing the following null hypothesis:

(i) There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners who are taught with or without GeoGebra software.

The ANOVA results in Table 4.3 show that there is a significant main effect of the groups (F (1, 58) =29.00, p=.000, $n_p{}^2$=.33) on their geometry performance. The direction of the difference is illustrated in Table 4.4, where the control (C) group's average performance for both pre and post-test was 3.42 compared to the 4.33 average score of the experimental (E) groups. The difference in the mean scores of the two groups is 0.91, which is significant in this study at p=.000.

TABLE 4.3: TEST OF BETWEEN-SUBJECTS EFFECTS UNDER VHL1

Measure:Test
Transformed Variable
Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	1801.875	1	1801.875	2072.901	.000	.973
Group	25.208	1	25.208	29.000	.000	.333
Error	50.417	58	.869			

TABLE 4.4: DESCRIPTIVE STATISTICS OF TESTS FOR VAN HIELES' PRE-VISUALIZATION (VHL1)

	Group	Mean ($\bar{x}$)	SD (σ)	N
Total of Pre-visualization for Pre-test	C	3.30	.998	30
	E	3.27	1.081	30
	Total	3.28	1.027	60
Total of Pre-visualization for Post-test	C	3.53	.776	30
	E	5.40	.621	30
	Total	4.47	1.171	60

The reported significant main effect could then be attributed to the impact of technology integration at van Hieles' Pre-visualization level (VHL1). Therefore, this study rejects null hypothesis (i) and reports that there is a significant main effect between the group performances of Grade 11 learners who are taught using GeoGebra software and those taught without it under van Hieles' Level 1.

Comparatively, this finding concurs with the position by Mandell, Sorge and Russell (2002:43) who believe that although educators are capable of presenting their lessons using the traditional behaviorists approach, "*lessons can be enhanced with the integration of technology".* However, the reported main effect size of 33%, although large (n_p^2>.14), shows that learners still have challenges at the pre-visualization stage (VHL1). To confirm the existence of learners' challenges in geometry, the scores of learners in both control and experimental groups were analyzed further. There were 19.7% of the 60 learners who scored a maximum of 2 marks out of 6; 37% scored exactly 3 marks; 45.3% scored from 4 to 6 marks. Having Grade 11 learners who scored less than 50% in the basics of geometry (pre-visualization), portrays the gravity of the challenges learners face in geometry in South African schools. Those learners in Groups C and E struggled in the pre-test to identify geometric shapes by their names, the number of edges, parts of a circle, etc. Perhaps that is why the Department of Basic education constantly emphasizes the need to actively teach learners those basic geometry concepts (DoBE, 2019:200). However, the same two groups performed extremely better after the post-test where all the learners scored 50% and above, except only 2 out of the 60 learners who scored below 3 marks under pre-visualization. Once again, that performance highlights the impact technology plays in geometry if the lessons are well structured to allow learners to enjoy visualizing as many shapes as possible as asserted by Karadag and McDougall (2011). However, the findings support assertions by researchers (Mandell, Sorge and Russell, 2002; 43) that technology cannot and should not replace teachers. This is because the control groups also showed significant improvement although that group received no intervention except the traditional teaching approach.

4.2.2 Within-subjects Effects of Van Hieles' Pre-Visualization Level

This section tests research hypothesis (ii) which states that:

There is no significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

TABLE 4.17: TEST OF WITHIN-SUBJECTS EFFECTS AT VHL1

Measure:Test

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Pre_post	Sphericity Assumed	42.008	1	42.008	60.284	.000	.510
	Greenhouse-Geisser	42.008	1.000	42.008	60.284	.000	.510
	Huynh-Feldt	42.008	1.000	42.008	60.284	.000	.510
	Lower-bound	42.008	1.000	42.008	60.284	.000	.510
Pre_post * Group	Sphericity Assumed	27.075	1	27.075	38.854	.000	.401
	Greenhouse-Geisser	27.075	1.000	27.075	38.854	.000	.401
	Huynh-Feldt	27.075	1.000	27.075	38.854	.000	.401
	Lower-bound	27.075	1.000	27.075	38.854	.000	.401
Error(Pre_post)	Sphericity Assumed	40.417	58	.697			
	Greenhouse-Geisser	40.417	58.000	.697			
	Huynh-Feldt	40.417	58.000	.697			
	Lower-bound	40.417	58.000	.697			

In Table 4.5, there was a significant main effect of a large effect size of the pre- and post-tests on the Grade 11 learners' performance (F (1, 58) = 60.28, p=.000, $n_p{}^2$=.51) ($n_p{}^2$=.510) (Crawford et al., 2014; Lane, 2016). To corroborate that significant effect, Table 4.4 captures the combined pre-test (VHL1) average of 3.28 as against the post-tests' 4.47. Comparatively, those averages provide an improvement of 1.19 mean points from the pre to post-test. As a result, this study rejects null hypothesis (ii) by stating that, there was a significant main effect of the pre- and post-tests on Grade 11 learners' geometry performance.

4.2.3 Interaction among variables

Null hypothesis (iii) of this study states:

> *There is no significant interaction between the control/experimental groups and their pre- and post-tests*.

To verify if the null hypothesis (iii) holds for this study under VHL1, the researcher used Table 4.5 to analyze the interaction between the pre-post tests and the two groups (C and E) (Lane, 2016). Data from Table 4.5 show significant interaction between the pre-post scores of learners under VHL1 and the groups (F (1, 58) =38.85, p=.000, $n_p{}^2$=.40). The data also show a large effect size, where 40.1% of the variance in the dependent variable was accounted for by the independent variable (Richardson, 2011: 142). In Table 4.4, the control (C) and experimental (E) groups' average performance in the post-test were 3.53 and 5.40, with standard deviations .776 and .621, respectively. The difference in the mean scores of the two groups is 1.87, which is significant in this study at p=.000. The study rejects null hypothesis (iii) and states that there was a significant interaction between the control/experimental groups and their pre- and post-tests.

According to Alex and Mammen (2018), building and understanding the terminologies in every field cannot be overemphasized. For example, VHL1 builds learners' appreciation of geometry through exposure to real objects and their associated terminologies. Hopefully, the rejection of that null hypothesis (iii) and the subsequent indication of improved performance under VHL1 by the experimental group would help Grade 11 learners to build enough geometric vocabulary needed for higher van Hieles' levels.

4.2.4 Graphical representation of main effects and interaction

Loftus (1978) asserts that main effects that result from probabilities are mostly interpretable just as crossover interactions. This study has reported a significant interaction (F (1, 58) =38.85, p=.000, $n_p{}^2$=.40) of time and groups. In addition, the graphs in Figure 4.1 also cross over each other. That significance and the crossover confirm the existence of significant interaction between the variables as previously discussed in this study. Besides, the two graphs in Figure 4.1 are far from being parallel, a further indication of an interaction (Lane, 2016; Petty et al., 1996).

The graphical representation of the marginal means in Figure 4.1 affirms the data provided in Table 4.4. In Figure 4.1, the numbers 1 and 2 represent respectively, the pre- and post-tests of van Hieles' Pre-visualization level. Although the two groups (C and E) performed similarly in the pre-test, the post-test shows a significant difference in the mean scores of the two groups as explained earlier in this section. That difference was measured using a paired sample t-Test.

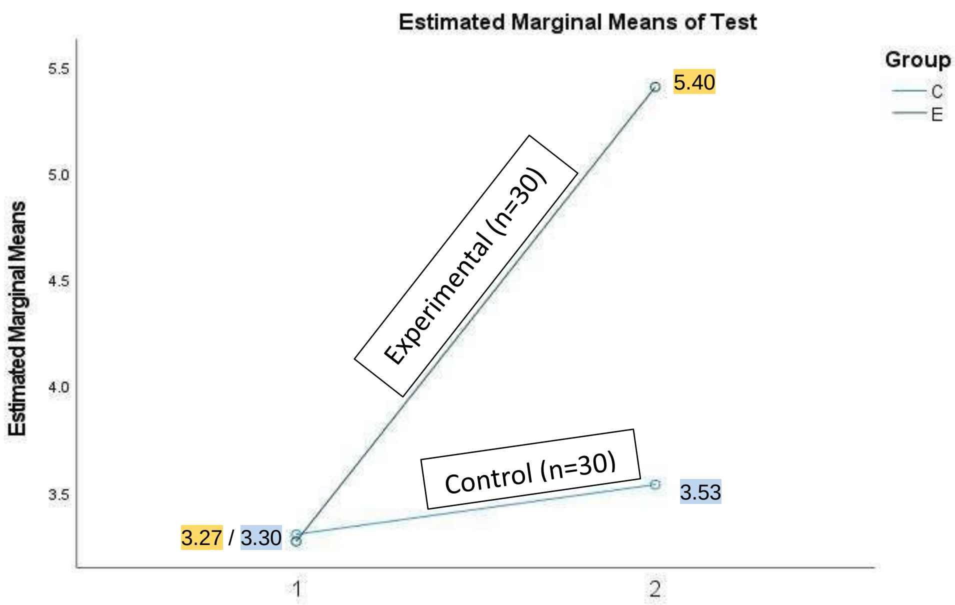


FIGURE 4.1: ESTIMATED MARGINAL MEANS OF TESTS UNDER VAN HIELES' PRE-VISUALIZATION

All the discussed tables above and Figure 4.1 provide the ANOVA data that highlights the significant interactions and main effects which exist between and within the variables. This study therefore accepts research hypothesis (iv) under VHL1 because the available data shows that Grade 11 learners who were taught geometry using GeoGebra software performed significantly higher than their counterparts who received no such intervention. The improvement in performance under VHL1 concurs with findings by researchers (Masilo, 2018; Abdullah & Zakaria, 2013; Clements & Battista, 1992) who consider the pre-visualization level critical to the teaching and learning of geometry.

Next, I present the ANOVA outcomes for van Hieles' Visualization level (VHL2) of this study.

4.3 Analysis under van Hieles' Visualization (V) level (VHL2)

Having reported significant main effects and interaction in Section 4.2, this section (4.3) hoped to find out if a similar or different outcome would emerge. The between-group factors were considered first.

4.3.1 Between Subjects Effects under van Hieles' Visualization level (VHL2)

This section tested the following null hypothesis under VHL2:

(i) There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners who are taught with or without GeoGebra software.

TABLE 4.18: TESTS OF BETWEEN-SUBJECTS EFFECTS UNDER VHL2

Measure:Test
Transformed Variable
Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	8484.008	1	8484.008	1729.706	.000	.968
Group	99.008	1	99.008	20.186	.000	.258
Error	284.483	58	4.905			

TABLE 4.19: DESCRIPTIVE STATISTICS FOR VAN HIELES' VISUALIZATION (VHL2)

	Group	Mean ($\bar{x}$)	SD (σ)	N
Total of Visualization (VHL2) for Pre-test	C	7.20	1.919	30
	E	7.37	2.236	30
	Total	7.28	2.067	60
Total of Visualization (VHL2) for Post-test	C	7.80	1.901	30
	E	11.27	1.874	30
	Total	9.53	2.561	60

According to Table 4.6, there was a significant main effect of the groups, with large effect size, on the Grade 11 learners' scores in Euclidean geometry (*F* (1, 58) =20.12, *p*=.000*,* $n_p{}^2$=.26) (Norouzian & Plonsky, 2018:267). Table 4.7, however, details the pattern of the average scores and spread of data such that Group C's overall average was 7.50. On the other hand, Group E achieved a combined mean of 9.32, making the difference between the two group means 1.32. The experimental group's overall performance in this section was far different and better than that of the control group. This study therefore rejects null hypothesis (i) under van Hieles' visualization level.

By rejecting the null hypothesis, this study agreed with the stance of researchers (Jojo, 2017: 258-259; White 2012; Dogan 2010) who believe in the significant role played by technology integration in the classroom, especially when the geometry lesson is structured to challenge learners to actively explore geometry on their own.

4.3.2 Within-subjects Effects of Van Hieles' Visualization Level

In this section, the analysis focused on investigating if there was no significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

TABLE 4.20: TEST OF WITHIN-SUBJECTS EFFECTS AT VHL2

Measure:Test

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Pre_post	Sphericity Assumed	151.875	1	151.875	50.640	.000	.466
	Greenhouse-Geisser	151.875	1.000	151.875	50.640	.000	.466
	Huynh-Feldt	151.875	1.000	151.875	50.640	.000	.466
	Lower-bound	151.875	1.000	151.875	50.640	.000	.466
Pre_post * Group	Sphericity Assumed	81.675	1	81.675	27.233	.000	.320
	Greenhouse-Geisser	81.675	1.000	81.675	27.233	.000	.320
	Huynh-Feldt	81.675	1.000	81.675	27.233	.000	.320
	Lower-bound	81.675	1.000	81.675	27.233	.000	.320
Error(Pre_post)	Sphericity Assumed	173.950	58	2.999			
	Greenhouse-Geisser	173.950	58.000	2.999			
	Huynh-Feldt	173.950	58.000	2.999			
	Lower-bound	173.950	58.000	2.999			

Table 4.8 reports a significant main effect of the pre-post tests on performance (F (1, 58) = 50.64, p=.000, $n_p{}^2$=.47). That significant effect was accompanied by a large effect size where 46.6% of the Grade 11 learners' scores were accounted for by the independent variable (Richardson, 2011). In addition, Table 4.7 gives the direction of the scores by reporting a difference in means of 2.25 between the overall pre- and post-tests. That difference showed a performance improvement which resulted from the difference in total means of 7.28 and 9.53 for the pre- and post-test respectively. Consequently, this study rejects null hypothesis (ii) and concludes that, for VHL2, there was a significant main effect of the tests on Grade 11 learners' geometry performance.

To produce significant within-pre- and a post-test main effect for both control and experimental groups in this study could be a good start to correct the negative perceptions held by some school administrators. Ford and Botha (2010:4) assert that some schools still disallow the use of technological devices due to the possibility of causing disruptions in class. Interestingly, the literature shows that irrespective of the technology to be used, schools should consider *“when is technology most effective”* (Ross, 2018:2) and whether that technology is *“underpinned by theories and pedagogical principles”* (Ford & Botha, 2010:5), than focusing on the disruptions they may cause. Interestingly, findings from this study support both researchers on the notion that although technology cannot replace educators, but they can replace many *“low-level instructional duties”* (Ross, 2018:2). To be precise, this study has shown that the benefits of technology integration could be harnessed through van Hieles’ model as teachers facilitate geometry lessons in South African schools.

4.3.3 Interaction under VHL2

The third null hypothesis of the study states that:

> *(iii) There is no significant interaction between the control/experimental groups and their pre- and post-tests*.

Table 4.9 provides evidence that shows the existence of significant interaction between the pre-post and the groups, with a large effect size (F (1, 58) =27.23, p=.000, n_p^2=.32). However, the effect size is not as large as it was at the pre-visualization level (VHL1) which recorded n_p^2=.40. Moreover, the descriptive statistics (see Table 4.8) for VHL2 revealed an improved performance for Group E in the post-test (Mean=11.27, SD=1.874) compared to Group C (Mean=7.80, SD=1.901). A similar pattern was observed during the pre-test with Groups C and E having means of 7.20 and 7.37 as against standard deviations of 1.919 and 2.236, respectively.

Graphically (see Figure 4.2), the two groups performed similarly at the pre-test level with a mean difference of .17. However, a paired samples t-test showed that the experimental group performed significantly better in the post-test (mean=11.27, SD=1.87) than they did in the pre-test (mean=7.37, SD=2.24); t(29)=-9.13, p=.000). At the 95% confidence interval, the interval for the experimental group’s true difference in population means was (-4.77: -3.03).

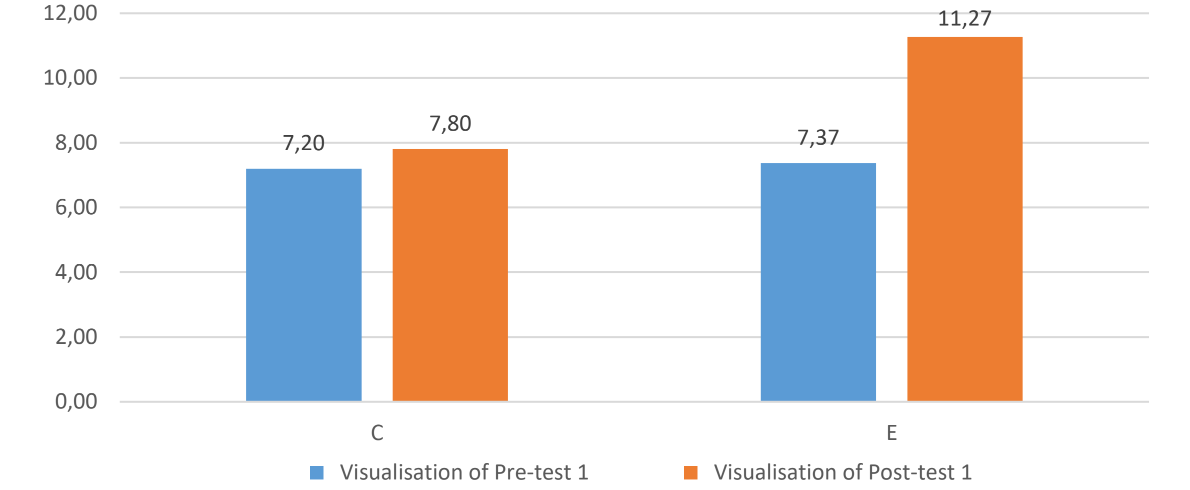


Figure 4.2: Average performance of groups at Visualisation level (VHL2)

On the contrary, although Group C recorded a significant mean difference in the post-test (mean=7.80, SD=1.90) as against (mean=7.20, SD=1.92) in the pre-test, the experimental group performed better after the intervention, as shown on the graph. As a result of the above discussion, this study rejects null hypothesis (iii) under VHL2 at p=.000. That means there was a significant interaction with a large effect size (n_p^2=.32) between the groups and their test scores.

Based on the presented data, this study accepts research hypothesis (iv) under VHL2 because the available data show that Grade 11 learners who were taught geometry using GeoGebra software performed significantly higher than their counterparts who received no such intervention. However, that acceptance of the hypothesis aligns with the study by Chimuka (2017) who also reported significant differences in learners' performance under VHL1 and 2. Moreover, the test items under VHL2 addressed contents such as identifying various circle theorems, properties of quadrilaterals, differentiating triangles, among others. Therefore, for this study to report significant interaction after the intervention, with large effect size, also affirms the impact GeoGebra makes in correcting Grade 11 learners' errors in Euclidean geometry. The next discussion analyses the ANOVA outcomes for van Hieles' Analysis level (VHL3).

4.4 Analysing van Hieles' Analysis level (VHL3)

Since all three assumptions were met, the analysis under VHL3 proceeded as follows:

4.4.1 Between Subjects Effects under van Hieles' Analysis level (VHL3)

This section tested null hypothesis (i) under VHL3:

(i) There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners who are taught with or without GeoGebra software.

TABLE 4.21: TESTS OF BETWEEN-SUBJECTS EFFECTS UNDER VHL3

Measure:Test
Transformed Variable
Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	15732.300	1	15732.300	3666.024	.000	.984
Group	388.800	1	388.800	90.600	.000	.610
Error	248.900	58	4.291			

This study reports a significant main effect of the groups on the Grade 11 learners' scores in Euclidean geometry (F (1, 58) =90.60, p=.000, n_p^2=.61). In addition, the effect size was large such that 61% of the learners' scores were accounted for by the groups in this study.

TABLE 4.22: DESCRIPTIVE STATISTICS FOR VAN HIELES' ANALYSIS LEVEL (VHL3)

	Group	Mean ($\bar{x}$)	SD (σ)	N
Total of Analysis (VHL3) for Pre-test	C	8.53	2.013	30
	E	9.93	1.596	30
	Total	9.23	1.934	60
Total of Analysis (VHL3) for Post-test	C	10.77	2.046	30
	E	16.57	2.750	30
	Total	13.67	3.785	60

Moreover, Table 4.10 explains the direction of the scores where the combined average of Group C was 9.65 compared to 13.25 achieved by Group E. The mean difference between the two groups was 3.6, which contributed to the reported large effect size. Based on the above analysis, the null hypothesis (i) is rejected for van Hieles' Analysis level of this study. For this study to report such a significant main effect underscores the need to develop educators to be able to effectively use appropriate technological tools such as GeoGebra, to improve learners' geometry scores under van Hieles' Abstraction level. Hopefully, an increase in technology integration lessons in schools would address shortfalls in geometry basics under VHL 3 that make learners make assumptions without valid reasons during tests and exams (DoBE, 2019a, 2020a, 2021).

4.4.2 Within-subjects Effects under Van Hieles' Analysis Level

Table 4.11: Tests of Within-Subjects Effects under Van Hieles' Analysis Level

Measure: Test

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Pre_post	Sphericity Assumed	589.633	1	589.633	120.772	0.000	0.676
	Greenhouse-Geisser	589.633	1.000	589.633	120.772	0.000	0.676
	Huynh-Feldt	589.633	1.000	589.633	120.772	0.000	0.676
	Lower-bound	589.633	1.000	589.633	120.772	0.000	0.676
Pre_post * Group	Sphericity Assumed	145.200	1	145.200	29.741	0.000	0.339
	Greenhouse-Geisser	145.200	1.000	145.200	29.741	0.000	0.339
	Huynh-Feldt	145.200	1.000	145.200	29.741	0.000	0.339
	Lower-bound	145.200	1.000	145.200	29.741	0.000	0.339
Error (Pre_post)	Sphericity Assumed	283.167	58	4.882			
	Greenhouse-Geisser	283.167	58.000	4.882			
	Huynh-Feldt	283.167	58.000	4.882			
	Lower-bound	283.167	58.000	4.882			

Table 4.11 helped to determine if there was any significant main effect of pre- and post-tests on the geometry scores of the participants. Data from Table 4.11 indicates the existence of a significant main effect of the pre-post tests on performance (F (1, 58) = 120.77, p=.000, $n_p{}^2$=.68). The effect size was large ($n_p{}^2$>.14) (Richardson, 2011).

To understand the direction of the reported effect, Table 4.10 records the total mean scores of the pre- and post-tests as 9.23 and 13.67, respectively. That shows an increase of 4.44 mean points. As a result, this study has shown sufficient evidence to reject the null hypothesis (ii) under VHL3. The study therefore concludes that there was a significant main effect of the tests on Grade 11 learners' geometry performance under van Hieles' Analysis level (VHL3).

4.4.3 Interaction under van Hieles' Analysis level

Null hypothesis (iii) reads:

> *(iii) There is no significant interaction between the control/experimental groups and their pre- and post-tests*.

Once again, there was evidence that showed significant interaction with large effect size between the pre-post and the groups (F (1, 58) =29.74, p=.000, $n_p{}^2$=.34) (see Table 4.11).

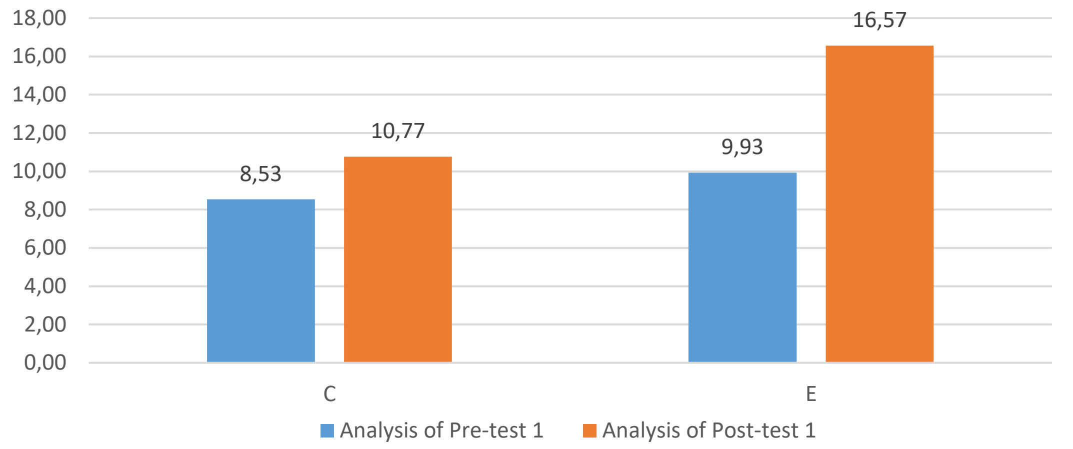


Figure 4.3: Average performance of groups at Analysis level (VHL 3)

In addition, data from Table 4.10, as well as a paired samples t-Test, showed that the experimental group significantly increased their performance from the pre-test (Mean=9.93, SD=1.60) to the post-test (Mean=16.57, SD=2.75); $t(29)= -11.98$, $p=.000$). Similarly, the control group also performed significantly better from their pre-test (Mean=8.53, SD=2.01) to post-test (Mean=10.77, SD=2.05); $t\ (29) = -3.81$, $p=.001$. However, the graph in Figure 4.3 shows that the mean of Group E is better than that of Group C. moreover, the true difference of population the population means for Group E at the 95% confidence interval was better (-7.766; -5.500) than that of Group C (-3.433; -1.034). The study therefore rejects null hypothesis (iii) under van Hieles' Analysis level at p=000. On the contrary, the available evidence presented supports this study in accepting alternate hypothesis (iv) by stating that, the use of GeoGebra software in the classroom improves Grade 11 learners' academic achievement under van Hieles' Level 3.

A similar significant improvement was reported by Sadiki (2016:51) in the experimental Group A, although the researcher failed to specify exactly which van Hieles' level was being analyzed. As a reminder, it is under van Hieles' Analysis level that learners appreciate "*the aspect of geometry*" (van Hiele-Geldof, 1957). In other words, the Grade 11 learner begins to use all prior knowledge from the pre-visualization and visualization levels, to distinguish between two or more geometric shapes based on their properties. Therefore, reporting significant interaction under VHL3 shows how well technology integration influences learners' ability to use a figure's characteristics to see for example, that a right-angled triangle differs from an equilateral triangle just the same ways as a rectangle differs from a square.

Notwithstanding the reported significant main effect between the groups, there were many errors detected in the learners' work from both control and experimental groups. For example, only 45% (27 out of 60) of learners were able to correctly indicate that the diagonals of a parallelogram intersect at right angles in the pre-test. However, there was an improvement such that 63.3% (38 out of 60) got the same property correct in the post-test. Similarly, the intervention assisted 78.3% (47 out of 60) of the learners to recall during the post-test that only one pair of sides of a trapezium are parallel, compared to the 41.7% (25 out of 60) who got that property right during the pre-test. However, the level of improvement between the overall scores of Groups E and C were not significant enough as reported earlier in this study. However, those findings confirm Masilo's (2018:122) conclusions that learners in the experimental group perform better under van Hieles' Analysis level by showing increased ability to analyze geometric shapes from the shapes' properties. Moreover, by combining technology integration with van Hieles' Analysis level, this study has further shown that learners could perform better in geometry when an appropriate teaching methodology is adopted.

In conclusion, this section of the study has shown the existence of significant main effects for both within- and between-groups. The results also showed a significant interaction between the pre-post and group leading to the rejection of all three null hypotheses in favor of the alternate hypothesis of the study. In the end, the experimental group performed significantly better than their cohorts in the control group under van Hieles' Analysis level.

Abstraction, which is classified under van Hieles' Level 4, is the next subject of analysis.

4.5 Analysis of van Hieles' Abstraction level (VHL4)

This part of the analysis dealt with testing three null and alternate hypotheses based on the factors of analysis. The geometric contents covered were the ability of learners to detect one geometric theorem in different ways, and to connect and apply those theorems to solve given circle problems. The between subjects' factor was considered first.

4.5.1 Between Subjects Effects under van Hieles' Abstraction level (VHL4)

Null hypothesis (i) under VHL4 states:

> *(i) There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners who are taught with or without GeoGebra software.*

TABLE 4.12: TESTS OF BETWEEN-SUBJECTS EFFECTS UNDER VHL4

Measure:Test
Transformed Variable
Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	12505.208	1	12505.208	608.875	.000	.913
Group	63.075	1	63.075	3.071	.085	.050
Error	1191.217	58	20.538			

TABLE 233: DESCRIPTIVE STATISTICS FOR VAN HIELES' ABSTRACTION LEVEL (VHL4)

	Group	Mean ($\bar{x}$)	SD (σ)	N
Total of Abstraction (VHL4) for Pre-test	C	7.30	4.75	30
	E	6.73	4.09	30
	Total	7.02	4.40	60
Total of Abstraction (VHL4) for Post-test	C	11.67	2.95	30
	E	15.13	2.80	30
	Total	13.40	3.35	60

Evidence from Table 4.12 showed that there was no significant main effect of the groups on the Grade 11 learners' performance (F (1, 58) =3.07, p=.085, n_p^2=.05). In addition, the total average of Group C was 9.49 as against the 10.93 total average achieved by Group E (see Table 4.13). The control and experimental groups recorded a difference in means of 1.44 which was non-significant in this study. Therefore, this study considers the above data evident enough to accept the null hypothesis (i) under van Hieles' Abstraction level.

The reported non-significant effect in this study breaks the cycle of continuous progression from one van Hieles' level to the next as expected in van Hieles' theory (VHLT) (Crowley, 1978:4). However, those findings only affected the between-group factor. The analysis of the pre- and post-tests presents the exact opposite of the outcome in this section. I, present the within-subject effects under VHL 4.

4.5.2 Within-subjects Effects under Van Hieles' Abstraction Level

TABLE 4.14: TESTS OF WITHIN-SUBJECTS EFFECTS UNDER VAN HIELES' ABSTRACTION LEVEL

Measure: Test

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Pre_post	Sphericity Assumed	1222.408	1	1222.408	166.009	0.000	0.741
	Greenhouse-Geisser	1222.408	1.000	1222.408	166.009	0.000	0.741
	Huynh-Feldt	1222.408	1.000	1222.408	166.009	0.000	0.741
	Lower-bound	1222.408	1.000	1222.408	166.009	0.000	0.741
Pre_post * Group	Sphericity Assumed	122.008	1	122.008	16.569	0.000	0.222
	Greenhouse-Geisser	122.008	1.000	122.008	16.569	0.000	0.222
	Huynh-Feldt	122.008	1.000	122.008	16.569	0.000	0.222
	Lower-bound	122.008	1.000	122.008	16.569	0.000	0.222
Error (Pre_post)	Sphericity Assumed	427.083	58	7.364			
	Greenhouse-Geisser	427.083	58.000	7.364			
	Huynh-Feldt	427.083	58.000	7.364			
	Lower-bound	283.167	58.000	4.882			

Unlike the between-subject effects, the study reports a significant main effect of the pre-post on Grade 11 learners' performance in geometry under the Abstraction level (F (1, 58) = 166.01, p=.000, $n_p{}^2$=.74) (see Table 4.14). Furthermore, the tests accounted for large effect size (74.1%) of the geometry scores (Richardson, 2011). To corroborate that significant large effect, Table 4.13 shows a higher post-test's total average of 13.40 which is almost double that of the pre-test (Mean=7.02). Based on the available data presented, this study rejects null hypothesis (ii) under VHL 4.

Crowley (1987:4) explains that although the levels in VHLT are *"sequential",* the *"advancement"* from one level to the next has more to do with the nature of the topic and the methodology adopted, rather than the learner's age. Hence, this study reports that the nature of the organization of the circle theorems with the GeoGebra software adequately facilitated learners' ability to link circle theorems and to make informal deductions.

To corroborate the reported findings, 63.3% of the pre-test (Question 3 of Pre-test 2) scores correctly linked and applied the angle at center theorem with the angles formed in a semi-circle by a diameter. As an improvement, 88.3% (53 out of 60) of the post-test scores (Question 3 of Post-test2) were able to correctly relate the same angle at center theorem with the angle in a semi-circle. I now present the interaction.

4.5.3 Interaction under van Hieles' Abstraction level

Null hypothesis (iii) reads:

(iii) There is no significant interaction between the control/experimental groups and their pre- and post-tests.

There was evidence (see Table 4.14) that showed a significant interaction of the pre-post and group with a large effect size (F (1, 58) =16.57, p=.000*,* n_p^2=.22). However, the effect size was not as large as the previously discussed van Hieles' levels above since this section only accounted for 22.2% of the geometry scores.

Moreover, a paired samples t-Test indicated a significant increase in the performance of the experimental group in the post-test (Mean=15.13., SD=2.80) compared to the same group's performance during the pre-test (Mean=6.73, SD=4.09); t(29)= -13.04, p=.000). The control group also achieved significant improvement in the post-test (Mean=11.67, SD=2.95) as compared to their pre-test scores (Mean=7.30, SD=4.75); t(29)= -5.80, p=.000). However, by comparing the averages of Groups E (n=30) and C (n=30) on the graph below (Figure 4.4), there is a higher mean mark in favor of Group E. The level of improvement reported in this study coincides with the findings of Masilo (2018), who reported that the deduction skills possessed by most learners in the "comparison group" were low. On the contrary, the researcher identified a huge number of learners who initially had "low informal deduction skills" moving to "average, advanced, and proficient" (p.126) abstraction skills.

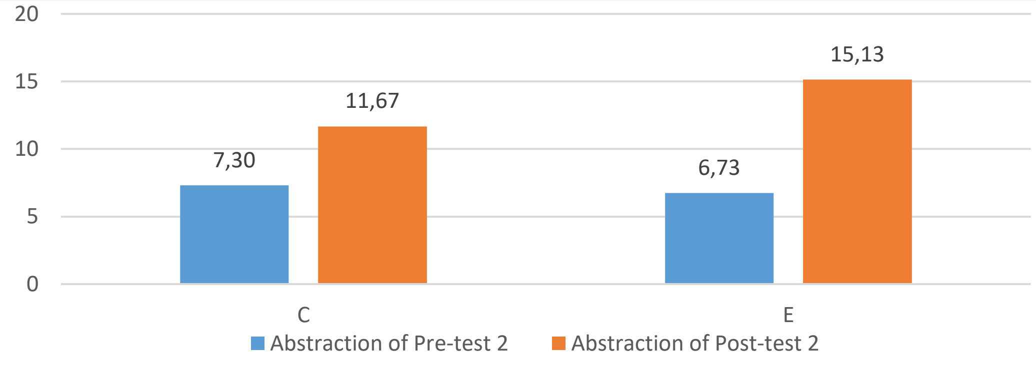


Figure 4.4: Average performance of groups at Abstraction level (VHL 4)

Furthermore, the paired samples t-Test, at the 95% confidence interval, reported the true difference of the population means for Group E (-9.72; -7.08) compared to that of Group C (-5.91; -2.83). As a result of the presented evidence, the study rejects null hypothesis (iii) in favor of alternate hypothesis (iv) under van Hieles' Analysis level at p=000.

The study has shown that technology integration, in the form of using GeoGebra, significantly improves Grade 11 learners' performance in Euclidean geometry under van Hieles' Level 4. The next analysis looks at van Hieles' Deduction level.

4.6 Analysis of van Hieles' Deduction level (VHL5)

Van Hieles' Level 5 in this study, exposed learners to formal deductions and proof in Euclidean geometry. As a result, this section of the study was dedicated to the analysis of the between-groups factors, within-subjects, and interaction of the pre-post and groups. That analysis of data served as evidence for the acceptance or rejection of the study's hypotheses.

4.6.1 Between Subjects Effects under van Hieles' Deduction level

The analysis in this section was guided by the following null hypothesis:

(i) There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners who are taught with or without GeoGebra software.

TABLE 4.15: TESTS OF BETWEEN-SUBJECTS EFFECTS UNDER VAN HIELES' DEDUCTION LEVEL

Measure:Test
Transformed Variable
Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	7145.633	1	7145.633	840.265	.000	.935
Group	282.133	1	282.133	33.176	.000	.364
Error	493.233	58	8.504			

TABLE 4.16: DESCRIPTIVE STATISTICS FOR VAN HIELES' DEDUCTION LEVEL (VHL5)

	Group	Mean ($\bar{x}$)	SD (σ)	N
Total of Deductions (VHL5) for Pre-test	C	4.80	2.295	30
	E	4.83	2.119	30
	Total	4.82	2.190	60
Total of Deductions (VHL5) for Post-test	C	7.57	2.459	30
	E	13.67	2.940	30
	Total	10.62	4.084	60

From Table 4.15, there was a significant main effect of the groups with a large effect size on the Grade 11 learners scores under VHL 5 (F (1, 58) =33.18, p=.000, $n_p{}^2$=.36). In addition, Table 4.16 reveals the direction of the scores where in total, the control group achieved an average of 6.18 whereas the experimental group obtained a mean of 9.25. Therefore, the study rejects null hypothesis (i) at *p*=.000. However, the effect size between the control and experimental groups, under VHL5 (36.4%), was higher than that of the abstraction level (5%). The outcome of this study is a direct opposite of Chimuka's (2017) findings who recorded a non-significant difference between the mean score of the control and experimental group's performances in geometry. However, the average mark of the experimental group was higher than that of the control group in that study (p. 65).

4.6.2 Within-subjects Effects under Van Hieles' Deduction Level

TABLE 4.17: TESTS OF WITHIN-SUBJECTS EFFECTS UNDER VAN HIELES' DEDUCTION LEVEL

Measure: Test

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Pre_post	Sphericity Assumed	1009.200	1	1009.200	271.282	0.000	0.824
	Greenhouse-Geisser	1009.200	1.000	1009.200	271.282	0.000	0.824
	Huynh-Feldt	1009.200	1.000	1009.200	271.282	0.000	0.824
	Lower-bound	1009.200	1.000	1009.200	271.282	0.000	0.824
Pre_post * Group	Sphericity Assumed	276.033	1	276.033	74.200	0.000	0.561
	Greenhouse-Geisser	276.033	1.000	276.033	74.200	0.000	0.561
	Huynh-Feldt	276.033	1.000	276.033	74.200	0.000	0.561
	Lower-bound	276.033	1.000	276.033	74.200	0.000	0.561
Error (Pre_post)	Sphericity Assumed	215.767	58	3.720			
	Greenhouse-Geisser	215.767	58.000	3.720			
	Huynh-Feldt	215.767	58.000	3.720			
	Lower-bound	215.767	58.000	3.720			

Table 4.17 provides data to show a significant main effect with a large effect size of the pre-post on Grade 11 learners' geometry scores (F (1, 58) = 271.28, p=.000, n_p^2=.56). In addition, Table 4.16 records a higher post-test total mean score of 10.62 compared to the 4.82 mean obtained during the pre-test. Consequently, this study rejects null hypothesis (ii) under van Hieles' Deductions level at *p*=.000.

4.6.3 Interaction under van Hieles' Deduction level (VHL5)

The following null hypothesis was tested under VHL 5:

(iii) There is no significant interaction between the control/experimental groups and their pre- and post-tests.

Under van Hieles' Deduction level, this study reports a significant interaction of the pre-post and group (F (1, 58) =16.57, p=.000, n_p^2=.22) (see Table 4.17). The effect size was also large (n_p^2>.14). The direction of the effect is demonstrated in both Table 4.16 and Figure 4.5 where the experimental group scored higher than the control group in both pre- and post-tests. In addition, a paired samples t-Test confirmed that there was a significant increase in the performances of both groups. For example, the post-test performance of Group E was better (Mean=13.67., SD=2.94) compared to the same group's performance in the pre-test (Mean=4.83, SD=2.12); t (29) = -18.89, p=.000). On the other hand, Group C performed significantly better in the post-test (Mean=7.57, SD=2.46) as compared to their pre-test scores (Mean=4.80, SD=2.30); t (29) = -5.25, p=.000).

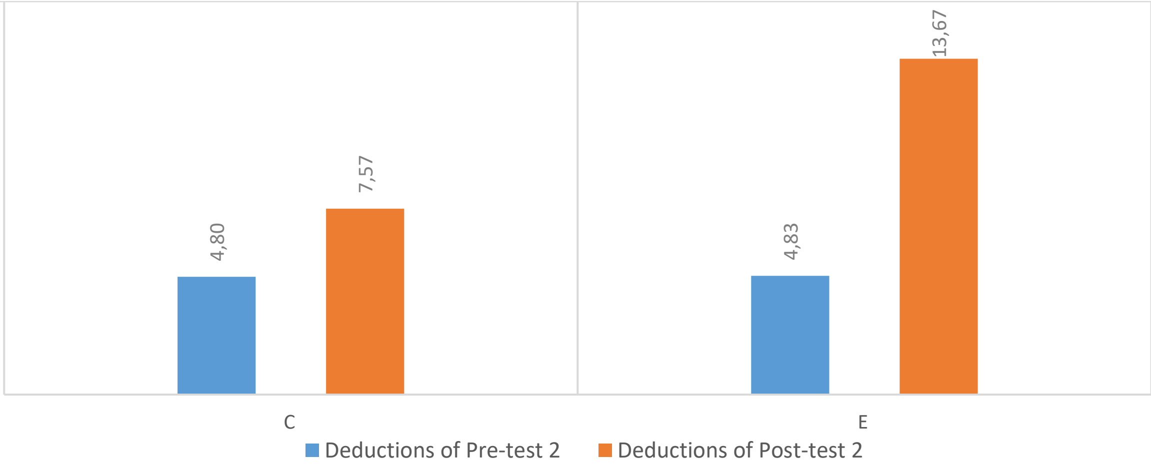


Figure 4.5: Average Performance of Groups at Abstraction Level (VHL 5)

Initially, at the pre-test level, the two groups performed almost the same with a mean difference of .03. However, the post-test results showed the contrary. Although the two groups performed significantly better in the post-test, Group E's mean score after the intervention far outweighs the mean of Group C in the post-test (see Figure 4.5). Based on those findings, this study rejects the null hypothesis (iii). The study concludes that under VHL5, the use of GeoGebra software improves learners' performance in the learning of Grade 11 geometry.

According to researchers (Chua et al., 2017; Machisi, 2021), when learners are being taught using VHLT, the lesson should provide room for individual search for information through trials and explorations. The researchers believe that learners develop creative and critical thinking skills when learners think for themselves. In support, Jojo (2017) urges teachers to present geometry lessons actively and in geometry-friendly environments, unlike what she observed where 12 out of 13 lessons presented by some teachers had passive learners. However, the significant outcome of this study promotes not only van Hieles' theory or technology integration, but also promotes the views of the active and critical thinking researchers above. This is because formal deductions (VHL5) expose learners to independent thinking with minimal supervision from teachers, just as was done in this study during the intervention. Therefore, the outcome from this VHL5 of the study served as a foundation filled with creativity through formal deductions, to prepare learners for the next level of rigor.

4.7 Analysing van Hieles' Rigor level (VHL6)

The rigor level is the last and the highest cognitive level in van Hieles' theory. According to Crowley (1987:3), the rigor level exposes learners to numerous "*axiomatic and non-Euclidean systems".* However, since the high school geometry does not cover *"non-Euclidean systems"* (DoE, 2011), the study limited itself to Euclidean geometry only. This part of the study analyzed how the various factors of analysis (see Section 4.1.1) turned out at VHL6.

4.7.1 Between Subjects Effects under van Hieles' Rigor level

The tested null hypothesis in this section was:

(i) There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners who are taught with or without GeoGebra software.

TABLE 4.18: TESTS OF BETWEEN-SUBJECTS EFFECTS UNDER VAN HIELES' RIGOR LEVEL

Measure:Test
Transformed Variable
Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	38556.675	1	38556.675	884.322	.000	.938
Group	612.008	1	612.008	14.037	.000	.195
Error	2528.817	58	43.600			

Data from Table 4.18 show a significant main effect of groups with large effect size on Grade 11 learners' geometry performance (F (1, 58) =14.04, p=.000, n_p^2=.20). The groups accounted for 19.5% of the geometry scores in the study. Comparatively, this effect size, although large (n_p^2>.14), was still lower than the effect size of most of the other van Hieles' levels discussed above.

TABLE 4.19: DESCRIPTIVE STATISTICS FOR VAN HIELES' RIGOR LEVEL (VHL6)

	Group	Mean ($\bar{x}$)	SD (σ)	N
Total of Rigor (VHL6) for Pre-test	C	2.40	1.754	30
	E	3.60	2.111	30
	Total	3.00	2.017	60
Total of Rigor (VHL6) for Post-test	C	5.10	1.788	30
	E	9.93	3.352	30
	Total	7.52	3.610	60

However, the direction of the scores was captured in Table 4.19. The evidence showed that the combined mean from both pre- and post-test of Group E was higher ($\bar{x} = 6.77$) than that of group C($\bar{x} = 3.75$). The difference between the two averages was 3.02. The presented data provide the basis to reject the null hypothesis (i) under van Hieles' Rigor level of this study at the 95% confidence interval.

4.7.2 Within-subjects Effects under Van Hieles' Rigor Level

This section tested the following null hypothesis:

(ii) There is no significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

According to data from Table 4.20, there was a significant main effect of the pre-post tests on performance (F (1, 58) = 120.62, p=.000, $n_p{}^2$=.68). On the effect size, the tests under VHL 6 accounted for 67.9% of learners' performance in geometry.

Table 4.20: Tests of Within-Subjects Effects under Van Hieles' Rigor Level

Measure: Test

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Pre_post	Sphericity Assumed	612.008	1	612.008	122.620	0.000	0.679
	Greenhouse-Geisser	612.008	1.000	612.008	122.620	0.000	0.679
	Huynh-Feldt	612.008	1.000	612.008	122.620	0.000	0.679
	Lower-bound	612.008	1.000	612.008	122.620	0.000	0.679
Pre_post * Group	Sphericity Assumed	99.008	1	99.008	19.837	0.000	0.255
	Greenhouse-Geisser	99.008	1.000	99.008	19.837	0.000	0.255
	Huynh-Feldt	99.008	1.000	99.008	19.837	0.000	0.255
	Lower-bound	99.008	1.000	99.008	19.837	0.000	0.255
Error (Pre_post)	Sphericity Assumed	289.483	58	4.991			
	Greenhouse-Geisser	289.483	58.000	4.991			
	Huynh-Feldt	289.483	58.000	4.991			
	Lower-bound	289.483	58.000	4.991			

In addition, the reported total mean score from Table 4.19 for the pre- and post-tests were 3.00 and 7.52, respectively. That shows a mean difference of 4.52 in favor of the post-test. The evidence presented was used to reject the null hypothesis (ii) under VHL3.

4.7.3 Interaction under van Hieles' Rigor level

The level of interaction was tested using the following null hypothesis:

> *(iii) There is no significant interaction between the control/experimental groups and their pre- and post-tests*.

Based on data in Table 4.20, the study reports the existence of significant interaction with large effect size between the pre-post and groups (*F* (1, 58) =19.84, *p*=.000, n_p^2=.26). Moreover, using paired samples t-Test (2-tailed) revealed a significant improvement in the performance of Group E in the post-test (Mean=9.93, SD=3.35) compared to the pre-test of the same group (Mean=3.60, SD=2.11); *t*(29)= -8.96, *p*=.000). A similar significant improvement was recorded by Group C also in the post-test (Mean=5.10, SD=1.79) as against their pre-test score (Mean=2.40, SD=1.75); *t* (29) = -6.63, *p*=.000). However, Group E performed better with a comparatively higher mean score as displayed in Figure 4.6.

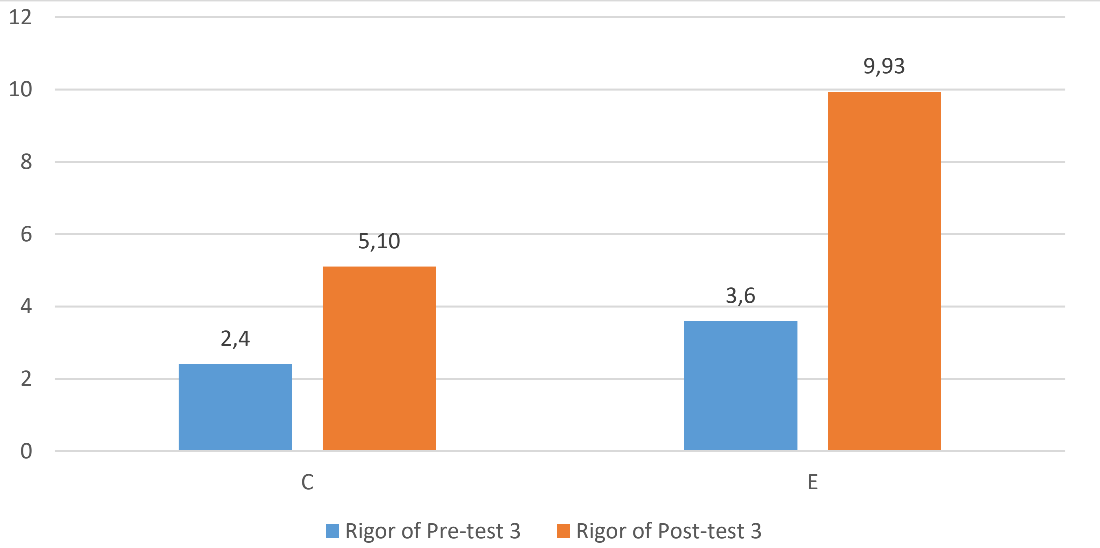


Figure 4.6: Average performance of groups at Rigor level (VHL 6)

Furthermore, the recorded true difference of the population means for Group E at the 95% confidence interval was better (-7.779; -4.888) than Group C's (-3.533; -1.867).

Based on available data, this study, , rejects null hypothesis (iii) under van Hieles' Analysis level at p=000. The study rather accepts alternate hypothesis (iv) by stating that, the use of GeoGebra software improves Grade 11 learners' academic performance under van Hieles' Level 6. Similarly, Masilo (2018:131) recorded "double" the performance of the experimental group in the post-test compared to the group's pre-test scores. However, the control group dropped in performance when comparing their pre- to the post-test. The difference, however, is that this study used GeoGebra to facilitate the teaching through van Hieles' Rigor level. Therefore, the recorded significant improvement could be attributed to the technology used through VHLT.

Finally, the next analysis combines all six van Hieles' levels to assess the impact of technology integration in teaching Grade 11 geometry.

4.8 Analysing all van Hieles' levels (VHL 1 – 6)

Most of the individual van Hieles' levels discussed above have reported similar main effects and interactions. Will the study continue to report similar or different outcomes when all learners' scores from the six van Hieles' levels are combined? This section of the analysis was dedicated to answering that question. The discussion began with the between-group analysis.

4.8.1 Between Subjects Effects of all van Hieles' levels

The following null hypothesis was tested in this section of the study:

(i) There is no significant main effect of the control and experimental groups on the geometry performances of Grade 11 learners who are taught with or without GeoGebra software.

TABLE 4.21: TESTS OF BETWEEN-SUBJECTS EFFECTS OF ALL VAN HIELES' LEVELS

Measure:Test
Transformed Variable
Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	264140.833	1	264140.833	2669.655	.000	.979
Group	5768.533	1	5768.533	58.302	.000	.501
Error	5738.633	58	98.942			

The combined statistic from all 6 van Hieles' levels, as illustrated in Table 4.21, shows a significant main effect of groups on Grade 11 learners' geometry performance (F (1, 58) =58.30, p=.000, $n_p{}^2$=.50). The groups accounted for a little over half (50.1%) of

Grade 11 learners' scores in Euclidean geometry. According to Richardson, (2011), an effect size of this nature (n_p^2>.14) could be classified as large. Consequently, the evidence calls for the rejection of null hypothesis (i) for all combined van Hieles' levels in this study.

TABLE 4.224: DESCRIPTIVE STATISTICS OF ALL VAN HIELES' LEVELS

	Group	Mean ($\bar{x}$)	SD (σ)	N
Overall total of all van Hieles' levels for Pre-test	C	33.53	7.587	30
	E	35.73	8.200	30
	Total	34.63	7.910	60
Overall total of all van Hieles' levels for Post-test	C	46.43	5.649	30
	E	71.97	10.397	30
	Total	59.20	15.316	60

Moreover, Table 4.22 shows that the control group's combined mean ($\bar{x} = 20.57$) from pre- and post-tests was lower than that of the experimental group($\bar{x} = 26.90$). That means the experimental group outperformed the control group by a mean of 6.33. Subsequently, this study rejects null hypothesis (i) under van Hieles' levels 1 to 6 combined. Clearly, this study has emphasized the impact made by integrating GeoGebra software into van Hieles' theory as a learning process. That impact has been evidenced one way or the other in many research findings (Abdullah & Zakaria, 2013a; Chua et al., 2017; Haviger & Vojkůvková, 2014; Machisi, 2021; Ogbonnaya & Alfred, 2014; Sadiki, 2016). This study adds to the existing knowledge on effective technology usage, with a particular merger with van Hieles' level theory.

4.8.2 Within-subjects Effects under Van Hieles' Levels 1 to 6

Null hypothesis (ii) was tested in this section of the study:

(ii) There is no significant main effect of pre- and post-tests on Grade 11 learners' geometry performance.

As indicated in Table 4.23, this study reports a significant main effect of the pre- and post-tests on Grade 11 learners' geometry performance (F (1, 58) = 541.01, p=.000, n_p^2=.90). The effect size was very large such that the tests accounted for 90.3% of the geometry scores, leaving a small room for possible errors from the research instruments.

TABLE 25: TESTS OF WITHIN-SUBJECTS EFFECTS FOR OVERALL TOTAL OF ALL VAN HIELES' LEVELS

Measure: Test

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Pre_post	Sphericity Assumed	18105.633	1	18105.633	541.014	0.000	0.903
	Greenhouse-Geisser	18105.633	1.000	18105.633	541.014	0.000	0.903
	Huynh-Feldt	18105.633	1.000	18105.633	541.014	0.000	0.903
	Lower-bound	18105.633	1.000	18105.633	541.014	0.000	0.903
Pre_post * Group	Sphericity Assumed	4083.333	1	4083.333	122.014	0.000	0.678
	Greenhouse-Geisser	4083.333	1.000	4083.333	122.014	0.000	0.678
	Huynh-Feldt	4083.333	1.000	4083.333	122.014	0.000	0.678
	Lower-bound	4083.333	1.000	4083.333	122.014	0.000	0.678
Error (Pre_post)	Sphericity Assumed	1941.033	58	33.466			
	Greenhouse-Geisser	1941.033	58.000	33.466			
	Huynh-Feldt	1941.033	58.000	33.466			
	Lower-bound	1941.033	58.000	33.466			

In addition, the total average of the pre-test was 34.63. However, the overall total of the post-test differed from the pre-test by an increase in mean of 24.57 to produce 59.20. That significance and the large effect size led to the rejection of null hypothesis (ii) for the entire study.

4.8.3 Interaction for Overall Total of all van Hieles' levels

To test the interaction for the entire study, the following null and alternate hypotheses were adopted:

> *(iii) There is no significant interaction between the control/experimental groups and their pre- and post-tests*.
>
> *(iv) Using GeoGebra software in classroom improves Grade 11 learners' academic achievement across each van Hieles' level in the teaching and learning of Euclidean geometry.*

Available evidence (see Table 4.23) shows that between the pre-post and groups, there was a significant interaction (F (1, 58) =122.01, p=.000, $n_p{}^2$=.68). However, the interaction was not only significant, but it also had a large effect size ($n_p{}^2 > .14$). Moreover, a paired samples t-Test confirmed that significant interaction. The t-Test showed that the experimental group's performance was better after the intervention ($\bar{x}$=71.97, SD=10.4) than their scores in the pre-test ($\bar{x}$=35.73, SD=8.20); t (29) = -22.23, p=.000). On the other hand, the control group also showed significant improvement in the post-test ($\bar{x}$=46.43, SD=5.65) compared to the same group's pre-test ($\bar{x}$=33.53, SD=7.59); t(29)= -9.60, p=.000). The distinction, however, is that Group

E performed better with a higher mean score of 71.97 as against the 46.43 achieved by the control group (see Figure 4.7).

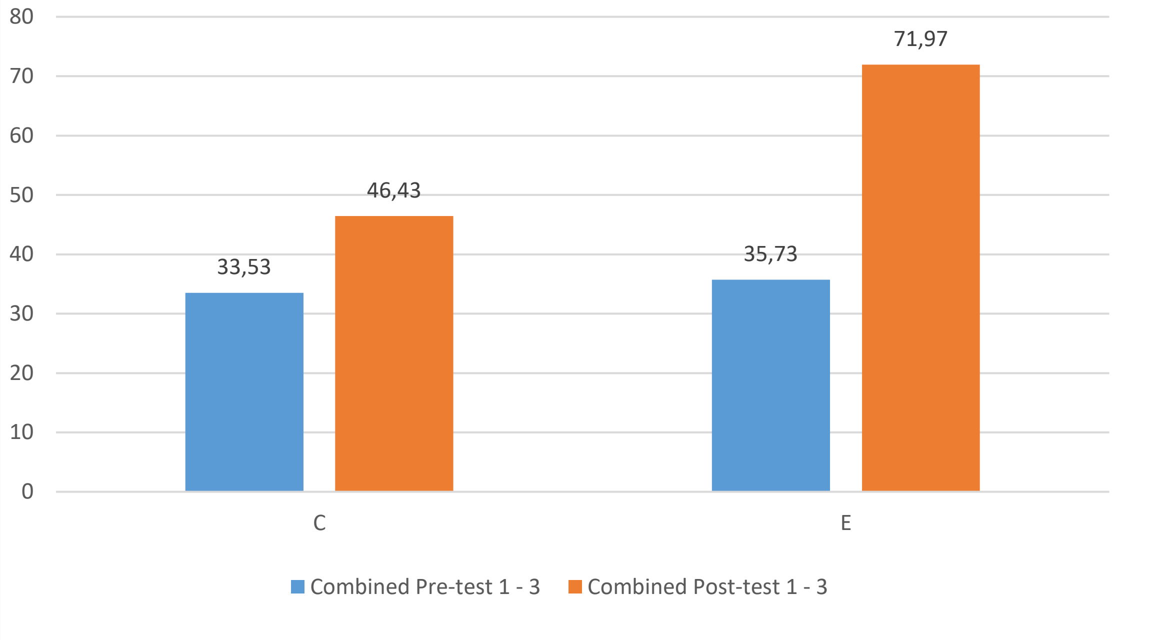


Figure 4.7: Average performance of groups for all van Hieles' levels combined

Meanwhile, at the 95% confidence interval, the true difference of population means for Group E (-39.57; -32.90) was better than that of Group C (-15.65; -10.15). Based on all the presented evidence, the null hypothesis (iii) is rejected for the entire study at the 95% confidence interval. Furthermore, the evidence points to an improvement in Grade 11 learners' performance after using the GeoGebra software to solve geometry problems (see Figure 4.7). As a result, this study accepts an alternate hypothesis (iv).

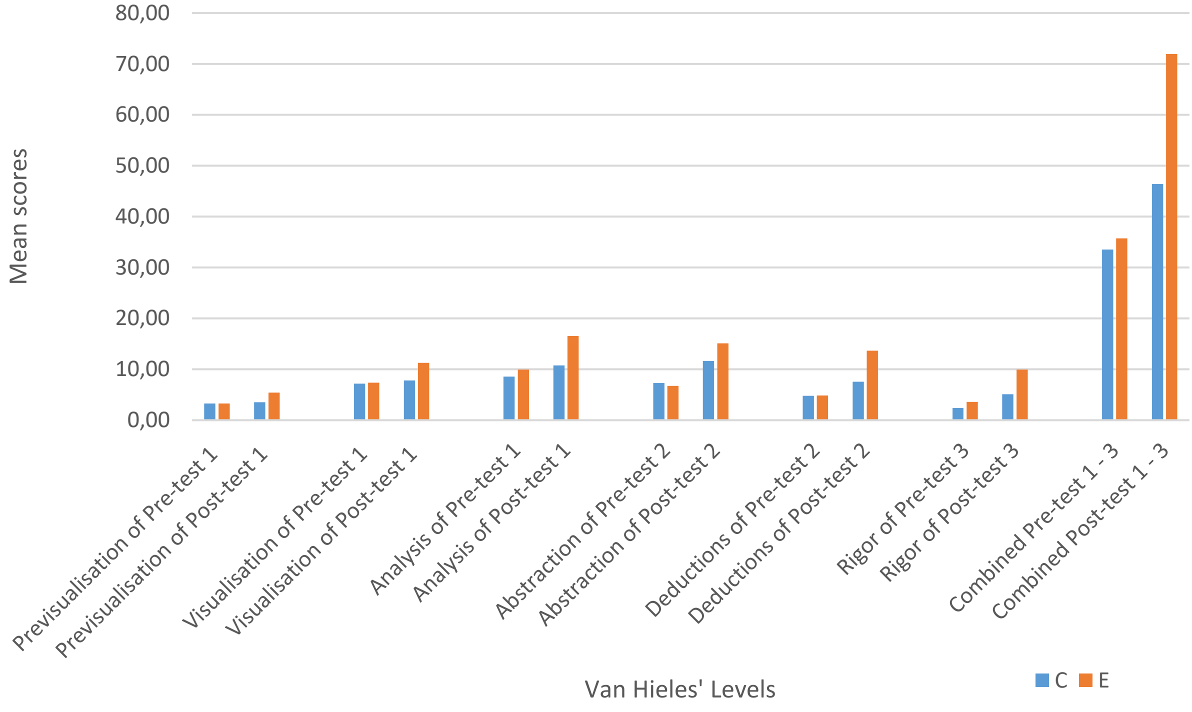


Figure 4.8: Comparing average group performances at each Van Hieles' level

Notwithstanding the fact that the experimental group reported significant main effects and interactions in the study, learners who were excluded from the intervention (Group C) also recorded significant main effects and interactions in most of the van Hieles' levels (see Figure 4.8). That means technology integration alone could not be the only factor that accounted for the improved learners' performance; traditional instruction and contextual factors are still relevant to the teaching process. Perhaps, that is why the TPACK framework emphasizes familiarizing oneself with contextual, technological, and pedagogical knowledge (Mishra, 2019) during the planning of lessons such as a geometry lesson. Moreover, the reported improvements by Group E from one van Hieles' level to another in this study could not have come up by chance. The presented data showed that learners gradually acquired knowledge from pre-visualization through to the rigor level, through careful planning of the geometry lesson at each van Hieles' level and phase. Once again, although technology integration has shown significant improvement across all van Hieles' levels, evidence from this study strongly agrees with Mandell, Sorge and Russell (2002: 43) on the assertion that technology cannot replace teachers and that *"any teacher who could be replaced by a computer should be"* replaced.

In conclusion, the only exception in terms of performance in the study was the Abstraction level (VHL4) which reported no significant main effect of the groups on the Grade 11 learners' performance. However, that same VHL4 showed significant between-tests main effects as well as significant interaction of the tests and groups. Moreover, since most of the van Hieles' levels, including the overall performance, showed significant main effects and interactions with large effect sizes, this study concludes by agreeing with researchers (Yakymchuk & Kazachenok, 2018; Schleicher, 2012; Drijvers, 2013; Masilo, 2018) that the effective use of educational software such as GeoGebra in teaching Euclidean geometry through van Hieles' model, improves Grade 11 learners' performance significantly.

CHAPTER FIVE: SUMMARY, RECOMMENDATIONS AND CONCLUSION

5.1 Introduction

In this chapter, a summary of all previous chapters has been presented. That summary covers the aim, how each research question and hypothesis was addressed, the literature review and the methodology used. The chapter further accounts for the implication, limitations, and possible recommendations of this study.

5.2 Summary of the study

5.2.1 Aim, objectives, and research questions

This study focused on exploring the effects that the use of GeoGebra software through van Hieles' level theory has in the teaching and learning of Grade 11 Euclidean geometry in Motheo District of the Free State.

The objectives of the study were to:

i. Determine ways in which the use of GeoGebra software through each van Hieles' level in the classroom, affects Grade 11 learners' achievement in the teaching and learning of Euclidean geometry.
ii. Identify the challenges that learners experience in understanding circle geometry.

The above objectives led to the formulation of the following research questions:

i. What challenges do Grade 11 learners experience in understanding circle geometry?
ii. How does the use of GeoGebra software through each van Hieles' level in the classroom affect learner achievement in the teaching and learning of Euclidean geometry among Grade 11 learners?

However, the study tested eight hypotheses (see Section 3.6).

5.2.2 Summary of how research questions were addressed

This section summarizes how the three research questions were answered based on the findings from the study.

5.2.2 (a) Research question 1: Challenges in teaching Grade 11 geometry

To identify the challenges that Grade 11 learners face in geometry, the pre- and post-tests were analyzed. The results of those tests confirmed the view held by the Department of Basic Education that learners lack the understanding of the basic geometric concepts (DoBE, 2019 b). This study showed an improvement in the post-tests at van Hieles' level 1 (VHL1) such that there was a significant interaction with a large effect size (F (1, 58) =38.85, p=.000, $n_p{}^2$=.40). However, learners' marks were not so encouraging in the pre-tests. For example, the analysis revealed that at the Pre-visualization level, prior to the intervention, only 19.7% of the 60 learners scored a maximum of 2 marks out of 6; 37% scored exactly 3 marks; 45.3% scored from 4 to 6 marks. Those percentages are very low considering how important the basic geometric concepts are in the entire Grade 11 Euclidean geometry (DoBE, 2015, 2019a, 2020a, 2021; DoE, 2011). Those below-average scores simply mean that learners still have difficulties with recognizing basic geometric shapes in their surroundings. In such situations, progressing from one van Hieles' level to the next becomes a challenge (Masilo, 2018:40).

TABLE 5.1: COMBINED PRE-TEST SCORES OF SUBJECTS IN BOTH GROUPS

Van Hieles' Level	Total Marks	Average	Standard Deviation	N
Pre-visualization	6	3.28	1.027	60
Visualization	14	7.28	2.067	60
Analysis	20	9.23	1.934	60
Abstraction	20	7.02	4.40	60
Deduction	20	4.82	2.190	60
Rigor	15	3.00	2.017	60
Combined VHL1 - 6	**95**	**34.63**	**7.910**	**60**

In addition, this study assessed learners on how to identify basic circle theorems and two-dimensional shapes based on how the shapes looked like, under the Visualization level. Just like the Pre-visualization level, the Visualization level also recorded significant interaction of the pre-post and the groups with large effect size (F (1, 58) =27.23, p=.000, $n_p{}^2$=.32). However, the combined pre-test scores of all learners (control and experimental groups) showed that from the possible 14 marks, the recorded mean mark was 7.28 with a standard deviation of 2.07. that means approximately half of the learners had difficulty with visualizing geometric shapes (see Table 5.1). To be precise, only 45% (27 out of 60) of learners were able to correctly indicate that the diagonals of a parallelogram intersect at right angles in the pre-test at the Analysis level. The same could be said about the Abstraction level.

The average performance of the learners started dropping right after the Analysis level (see Table 5.1). For instance, the average for the Deduction level was only 4.82 whilst that of Rigor level was 3.00. That means some learners could not apply their knowledge from the lower van Hieles' levels at the Deductions and Rigor levels. As a recall from Chapter 4, only 63.3% of learners were able to correctly apply their abstraction skills to relate the angle at center theorem to the angles in a semi-circle. The remaining 36.7% could not connect those 2 circle theorems. Therefore, it came as little surprise when those same learners did not perform well at VHL 5 and 6 since those levels depend on learners' understanding of concepts from VHL 1-4. The inability of learners to apply their cognition from lower VHL in solving higher cognitive problems confirmed one of the unique characteristics of van Hieles' levels: key concepts developed from preceding levels become fundamental concepts in succeeding levels. In other words, if learners did not develop enough knowledge at the preceding level, they cannot apply the same at higher van Hieles' levels.

Next, I summarize how technology integration impacted geometry teaching in this study.

5.2.2 (b) Research question 2: Impact of GeoGebra software on teaching geometry

The impact of using technology to teach Grade 11 geometry through van Hieles' model was categorized into 6 parts, based on the six van Hieles' levels adopted for the study. In each case, the findings considered the main effects between groups, within-subjects main effects, and the interaction of the groups and the tests. In addition, a paired samples t-Test was used to analyze the interaction among variables.

<u>Impact of technology integration under Pre-visualization level</u>

The study reported significant between-subject main effects of the groups (F (1, 58) =29.00, p=.000, n_p^2=.33) with a large effect size on learners' geometry performance. The control group's average performance for both pre and post-test was 3.42

compared to the 4.33 average score of the experimental group. Similarly, there was a significant within-subjects main effect of large effect size on the Grade 11 learners' performance (F (1, 58) = 60.28, p=.000, $n_p{}^2$=.51). A combined pre-test mean score of 3.28 was recorded against the 4.47 mean recorded in the post-test. Those averages provided an improvement of 1.19 mean points from the pre- to the post-test.

On the interaction of the variables, this study reported significant interaction of large effect size between the pre-post scores of learners under VHL1 and the groups (F (1, 58) =38.85, p=.000, $n_p{}^2$=.40). The control and experimental groups' average performance in the post-test were 3.53 and 5.40, with standard deviations .776 and .621 respectively. A significant difference in the mean scores of 1.87 was recorded between the two groups. Under van Hieles' Pre-visualization level, this study rejected null hypothesis (iii) in favor of the alternate hypothesis.

Impact of technology integration under Visualization level

The study showed a significant between-group main effect with large effect size, on the Grade 11 learners' scores in Euclidean geometry (F (1, 58) =20.12, p=.000, $n_p{}^2$=.26). The combined mean of Group C was 7.50 compared to that of Group E's 9.32, making the difference between the two group means 1.32. There also was a reported significant within-subjects main effect of the pre-post tests on performance (F (1, 58) = 50.64, p=.000, $n_p{}^2$=.47). The ratio of the averages of the pre-test to the post-test was 7.28;9.53, showing a difference in means of 2.25. In addition, there was a significant interaction between the pre-post and the groups, with a large effect size (F (1, 58) =27.23, p=.000, $n_p{}^2$=.32).

Furthermore, a paired samples t-test showed that the experimental group performed significantly better in the post-test (mean=11.27, SD=1.87) than they did in the pre-test (mean=7.37, SD=2.24); t(29)=-9.13, p=.000) where the true difference in population means was (-4.77: -3.03). The control group also recorded a significant mean difference in the post-test (mean=7.80, SD=1.90) as against (mean=7.20, SD=1.92) in the pre-test. However, the data showed that the experimental group performed better after the intervention since their mean score was higher. The study therefore accepted research hypothesis (iv) under VHL2 by stating that Grade 11 learners who were taught geometry using GeoGebra software performed significantly higher than those in the control group.

Impact of technology integration under Analysis level

Under van Hieles' level 3, this study reported a significant main effect of the groups on the Grade 11 learners' scores in Euclidean geometry (F (1, 58) =90.60, p=.000, $n_p{}^2$=.61). The effect size was large (61%) such that the combined average score of the control group was 9.65 compared to the 13.25 average achieved by the experimental group. The mean difference between the two groups was 3.6. There was also a significant within-subjects main effect with a large effect size of the pre-post tests on performance (F (1, 58) = 120.77, p=.000, $n_p{}^2$=.68). The direction of the scores was such that the total mean scores of the pre- and post-tests were 9.23 and 13.67 respectively. That shows an increase of 4.44 mean points. With regards to interaction, the study recorded significant interaction with a large effect size between the pre-post and the groups (F (1, 58) =29.74, p=.000, $n_p{}^2$=.34). From a paired samples t-Test, the experimental group performed significantly better by increasing their performance in the pre-test (Mean=9.93, SD=1.60) to the post-test (Mean=16.57, SD=2.75); t(29)= -11.98, p=.000. Similarly, the control group also performed significantly better from their pre-test (Mean=8.53, SD=2.01) to post-test (Mean=10.77, SD=2.05); t (29) = -3.81, p=.001). The reported significant interaction led to the rejection of null hypothesis (iii) in favor of the alternate hypothesis. The study therefore concluded that the use of GeoGebra software in the classroom improves Grade 11 learners' academic achievement under van Hieles' Level 3.

Impact of technology integration under Abstraction level

This aspect of the research question was addressed by starting with the between-group effects. Under the Abstraction level, this study reported a non-significant main effect of the groups on the Grade 11 learners' performance (F (1, 58) =3.07, p=.085, $n_p{}^2$=.05). However, there was a significant between-subjects main effect with a large effect size under van Hieles' Abstraction level (F (1, 58) = 166.01, p=.000, $n_p{}^2$=.74). In addition, there was a significant interaction of the pre-post and group with a large effect size (F (1, 58) =16.57, p=.000, $n_p{}^2$=.22). To further understand the direction of the scores, the data were subjected to the paired samples t-Test. The t-Test showed significant improvement in the post-test scores of the experimental group (Mean=15.13., SD=2.80) compared to the same group's performance during the pre-test (Mean=6.73, SD=4.09); t(29)= -13.04, p=.000. Although the control group also recorded significant improvement in the post-test (Mean=11.67, SD=2.95) as compared to their pre-test scores (Mean=7.30, SD=4.75); $t(29)= -5.80$, $p=.000$), the difference in means favored the experimental group. The presented evidence led to the conclusion that technology integration, in the form of using GeoGebra software, significantly improves Grade 11 learners' performance in Euclidean geometry under van Hieles' Abstraction level. For that reason, the null hypothesis (iii) was rejected.

Impact of technology integration under Deduction level

This part of the study recorded a significant between-group main effect of large effect size on the Grade 11 learners' performance (F (1, 58) =33.18, $p=.000$, $n_p{}^2=.36$). In addition, there was a significant main effect with a large effect size of the pre-post tests on Grade 11 learners' geometry scores (F (1, 58) = 271.28, $p=.000$, $n_p{}^2=.56$). That effect size was large such that, a higher combined mean score of 10.62 was recorded in the post-test compared to the initial 4.82 in the pre-test. This study further showed a significant interaction of the pre-post and group (F (1, 58) =16.57, $p=.000$, $n_p{}^2=.22$). That interaction had a large effect size ($n_p{}^2=.22$). However, the use of a paired-samples t-Test confirmed that there was a significant increase in the performances of both groups, where the post-test of the experimental group improved (Mean=13.67., SD=2.94) over the same group's performance in the pre-test (Mean=4.83, SD=2.12); t (29) = -18.89, $p=.000$). Similarly, the post-test of the control group (Mean=7.57, SD=2.46) was significantly better as compared to their pre-test scores (Mean=4.80, SD=2.30); t (29) = -5.25, $p=.000$). In the post-test, group difference in means favors the experimental group. As a result, the study concluded that the use of GeoGebra software improves learners' performance in the learning of Grade 11 geometry.

Impact of technology integration under Rigor level

At the highest van Hieles' level in this study, there was a significant between-groups main effect with large effect size on Grade 11 learners' geometry performance (F (1, 58) =14.04, $p=.000$, $n_p{}^2=.20$). There was also a reported significant main effect of the pre-post tests on performance (F (1, 58) = 120.62, $p=.000$, $n_p{}^2=.68$). The within-subject effect had a large effect size by accounting for 68% of the scores. With regards to interaction, evidence showed a significant interaction with a large effect size between the pre-post and groups (F (1, 58) =19.84, $p=.000$, $n_p{}^2=.26$). The outputs from a paired samples t-Test (2-tailed) showed significant improvement in the experimental group's performance in the post-test (Mean=9.93, SD=3.35) compared to the pre-test of the same group (Mean=3.60, SD=2.11); *t*(29)= -8.96, *p*=.000). The control group also had significant increase in their post-test scores (Mean=5.10, SD=1.79) compared to the pre-test performance (Mean=2.40, SD=1.75); *t* (29) = -6.63, *p*=.000). However, the true difference in population means showed that the experimental group (-7.779; -4.888) performed comparatively better than their counterparts in the control group (-3.533; -1.867). The data available led to the rejection of null hypothesis (iii) at p=000. Therefore, the use of GeoGebra software improves Grade 11 learners' academic performance under van Hieles' Level 6.

In the next discussion, the overall impact of integrating technology into the learning of geometry is presented.

The overall Impact of technology integration in this study

Aside the individual levels analysis already presented in this chapter, a combination of all the scores of learners indicated a significant between-group main effect of large effect size on Grade 11 learners' geometry performance (*F* (1, 58) =58.30, *p*=.000, n_p^2=.50). The data accounted for approximately half (50.1%) of the Grade 11 learners' scores in geometry. The pre- and post-tests reported significant within-subjects main effect on Grade 11 learners' geometry performance (*F* (1, 58) = 541.01, *p*=.000, n_p^2=.90) with a very large effect size.

Furthermore, there was significant interaction (*F* (1, 58) =122.01, *p*=.000, n_p^2=.68) of the pre-post and the groups, with a large effect size. Further analysis of the data from the t-Test showed that the control group improved significantly in the post-test ($\bar{x}$=46.43, SD=5.65) compared to their pre-test ($\bar{x}$=33.53, SD=7.59); *t*(29)= -9.60, *p*=.000).

However, the experimental group outperformed the control group with a higher mean difference. The former recorded better results after the intervention ($\bar{x}$=71.97, SD=10.4) as against ($\bar{x}$=35.73, SD=8.20); *t* (29) = -22.23, *p*=.000) in the pre-test. Moreover, the true difference in population means for Group E (-39.57; -32.90) was better than that of Group C (-15.65; -10.15). Based on all the analyzed data, this study concludes that GeoGebra software, as a technology integration tool, helps improve Grade 11 learners' geometry performance significantly.

5.2.3 Addressing Grade 11 learners' geometry challenges

In addressing the challenges faced by Grade 11 learners in Euclidean geometry, the study considered the effects of the measures put in place by the researcher during the intervention. Since the intervention focused on Group E, I present how the experimental group fared in the study.

Firstly, learners in the experimental group were made to work actively, individually, and with their peers where necessary at each van Hieles' level. Secondly, the researcher acted as a facilitator in most instances. However, a great deal of the work focused on the ownership of the basic geometric concepts by learners. To achieve that, learners' prior and real-life experiences with geometric shapes were relied upon in the introductory phases of each lesson. The assumption was that, if learners acquire and own enough geometric ideas, they would be able to apply the same at higher van Hieles' levels. In addition, there were situations where learners had to brainstorm over the given question in small groups after visualizing that geometry problem. The researcher only assisted learners where there were challenges.

Thirdly, learners were guided by the researcher to fully complete each van Hieles' phase in the learning process to enable learners' progression to the next level. Learners were given the opportunity and the environment to freely debate, and challenge other learners' solutions based on reasonable geometric statements and terminologies. Occasionally, the facilitator would project a group's work onto the screen for the entire class to comment on it.

Lastly, each lesson was planned with great emphasis on active participation, to ensure that both the educator and the learners stayed focused throughout the learning process. The lesson plans factored in the contextual factors affecting the smooth delivery of each lesson by analyzing the Technological and Pedagogical Content Knowledge (TPACK) of learners and the educator. Those forms of knowledge only influenced the delivery of the lessons at each van Hieles' phase and level by serving as a point of reference. The main structure of each lesson was entirely based on van Hieles' levels.

The adopted strategies seemed to produce desired results as shown in Section 5.2.4 (b) of this study. It emerged from the findings that all van Hieles' levels recorded significant interaction between learners' pre- and post-tests and the groups. The study further showed after the intervention at VHL3 that 63.3% (38 out of 60) of the learners could correctly indicate that a parallelogram's diagonals intersect at right angles compared to the 45% who could do the same task during the pre-tests. Similarly, 78.3% (47 out of 60) of the learners correctly recalled during the post-test that only one pair of sides of a trapezium are parallel, compared to the 41.7% (25 out of 60) who got that property right during the pre-test. After the post-test in VHL 4, there was an improvement such that 88.3% (53 out of 60) of the learners correctly applied the relationship between the angle at center theorem and the angles formed in a semi-circle by a diameter (Question 3 of Post-test 2). That was an improvement because only 63.3% of the pre-test (Question 3 of Pre-test 2) was correctly answered by learners.

5.3 Implications of the study

The findings from this study present some implications for research methodology as well as effective teaching and learning of Euclidean geometry in High Schools. The implications have been segmented into theory, teaching and learning, and research methods. I commence with the theoretical implications.

5.3.1 Implications for theory

The use of van Hieles' level theory (VHLT) and the associated significant improvements in this study go to support the relevance of teaching geometry according to learners' cognitive levels instead of their age. For example, all the six van Hieles' levels used in this study were sequenced such that a learner must complete the expected cognitive activities attached to one level before progressing to the next. More so, the inclusion of the extra level, Pre-visualization (VHL1), allowed learners to familiarize themselves with real objects from their environment to pave way for the visualization and other higher cognitive levels. That VHL 1 also provided the researcher with a rich source of prior knowledge from which to build and develop learners' geometric reasoning for long-term knowledge retention.

In addition, van Hieles' theory and the Technological Pedagogical Content Knowledge (TPACK) frameworks seem to work well when the two theories are effectively structured. Under each van Hieles' level, this study structured its lessons by first assessing the kinds of knowledge that learners possess or lack, prior to the start of each van Hieles level. The significant results obtained at all van Hieles' levels imply that VHLT and TPACK could be maximized for effective teaching of geometry, and to prepare learners for the prospects of the 4th industrial revolution where technology dictates the pace of almost everything.

5.3.2 Implications for teaching and learning

As a teacher of the 21st century, this study has shown that teaching with technology improves learners' cognition in geometry more than it does with the traditional rote learning approach. The user-friendly interface of GeoGebra software used in this study offered the platform for learners to construct many geometric shapes thereby increasing learners' ability to visualize those shapes and interpret their own works. However, that does not in any way suggest the relegation of the traditional teaching approach. On the contrary, there were significant improvements in almost all the van Hieles' levels in the control group, except that the experimental group performed better. Moreover, since Euclidean geometry is considered challenging for some learners, the traditional approach could come in handy to assist struggling learners.

The levels in van Hieles' theory suggest that learners should be taught based on their cognitive abilities. For instance, how do you teach the relationship between two or more theorems to a learner who operates at van Hieles' visualization or even Analysis level? The geometric basics should precede the higher cognitive skills as stipulated in the characteristics of van Hieles' theory (Usiskin, 1982; Masilo, 2018). In other words, teachers should teach geometry from the known to the unknown, and from the basics to the more challenging content. In addition, learners vary in terms of TPACK and van Hieles' levels. Therefore, each learner's abilities should be fully developed by the teacher when progressing through the five phases of VHLT. This is the stage where learners' prior knowledge becomes relevant in the learning process. When teachers identify each learner's prior skills in geometry, the teacher becomes empowered to accurately prepare lessons that suit individual learners, or groups of learners with the same abilities.

Moreover, the study highlighted the importance of using cell phones in schools. For example, besides installing GeoGebra on the school computer of the experimental group, learners who had smartphones were assisted to install the same software too. What transpired was that learners became motivated to use their phones to do as many geometric constructions as possible. That motivation turned into self-regulated learning and the teacher's work became less of a source of knowledge to more of a facilitator. The facilitator occasionally projects learners' work on their phones onto the screen as a means of monitoring learners' usage of those phones. In addition, the researcher engaged with the entire class to set up basic rules for the use of the phones during the geometry lessons. Unfortunately, most high schools do not allow the use of smartphones in schools (Masilo, 2018:194). However, this study has shown that by regulating its usage in class, smartphones can help learners achieve more with the learning of geometry.

One final implication for teaching was the issue of time in teaching geometry. The researcher observed that more time is needed by educators to effectively teach geometry to learners. That could be due to the fact that most learners lack the basic geometric concepts in high schools. Therefore, it serves no good purpose if Grade 11 teachers were to start a new geometry topic by merely assuming that learners have already learnt certain concepts in their lower grades. Rather, educators may adopt baseline tests to ensure that they are familiar with what learners know before introducing new geometry content. However, that requires some amount of time which teachers do not usually have. Perhaps with careful planning, educators would be able to fit in all the critical components of the topic and teach learners at designated times.

5.3.3 Implications for research methodology

Having situated this study in the positivist paradigm, it became imperative that quantitative data be collected for the study. This is because positivism aligns with objectivity. Therefore, a quasi-experimental design was actualized in the study. In addition, the quantitative data collected using the pre- and post-tests ensured objective discourse in the study. Moreover, the objective stance also led to the acceptance or rejection of the null and alternate hypotheses of the study. Furthermore, the use of mixed factorial analysis (2-way mixed ANOVA) was relevant as it allowed a comparison of learners' performance between groups, within groups, and the interaction between the groups and the tests. By implication, the paired samples t-Test used in the study was required to help clarify the interaction among the variables through a comparison of means. In effect, collecting quantitative data in this and future studies is useful in assessing learners' performance in geometry since that methodology provides numerical data that could be easily verified.

5.4 Scope and Limitations

The study only sampled learners from Circuits 6 and 7 of Motheo District in Free State. Therefore, the findings cannot be generalized to cover all Grade 11 learners in the country. This is because the contextual factors in those two circuits may vary from what persists in other parts of South Africa. Consequently, the findings only apply to learners in the Motheo District. Similarly, since only Grade 11 Euclidean geometry was used in this study, the conclusions only apply to geometry in high schools. However, this study posits that the intervention could be adapted to suit similar topics like analytical geometry, trigonometry, or even functions. Moreover, the use of pre-existing classes affected the internal validity of the study because it was difficult for the researcher to claim complete oversight and control during the writing of the tests.

Although quantitative data was collected, a mixed-method approach would have provided more clarity as to why learners are, for example, having a particular challenge in geometry. The design therefore deprived the study of understanding learnings feelings about Euclidean geometry.

In addition, the study was limited in terms of time. The literature revealed that Euclidean geometry should be taught in detail by addressing all related basic concepts. However, the limited time used for the intervention could not allow the researcher to address all the geometry contents that learners missed in their GET phase. To worsen the situation, the covid-19 restrictions at the time of the intervention did not motivate learners to attend classes regularly, thereby impacting the outcome of the study. This is because most learners feared being infected with the virus since the pandemic was still new with no scientifically proven medication at the time of this study’s data collection. Therefore, although all the learners wrote the test, the absenteeism during the intervention phase affected the internal reliability of the results.

5.5 Conclusion

Based on the findings of this study, GeoGebra software integrated into the learning of geometry significantly improved learners' scores across all van Hieles’ levels. The traditional approach also proved useful as the control group showed some improvement in their scores too. However, at all levels, the experimental group performed significantly better than the control group. For example, scores from the tests showed that a learner who scored 25 during the pre-test, scored 60 in the post-test. There were many such instances where the experimental group learners increased their marks significantly in the post-test. The combined scores from all six van Hieles' levels also showed significant interaction (F (1, 58) =122.01, p=.000, n_p^2=.68) of the groups and the pre-post such that the experimental group's true difference in population means (-39.57; -32.90) was outweighed that of the control group (-15.65; -10.15). The results therefore indicate that the intervention worked, and that GeoGebra software improves learners' overall geometry achievement across the van Hieles' levels.

5.6 Recommendations

Findings from the study showed that learners' performance at the Deductions and Rigor declined compared to the previous van Hieles' levels (VHL). However, that was attributed to the inability of learners to acquire enough basic skills at the lower cognitive levels (VHL 1-4)**.** This study therefore recommends that teachers teach from the basics in geometry. For example, a good exposure of learning to real geometric shapes during Pre-visualization serves as a good foundation for Visualization. Similarly, learners who become equipped with visualization skills are more capable of applying identifying shapes by their properties at the Analysis level, and so on. Moreover, analyzing learners' technological or pedagogical content knowledge places the teacher in a better position to plan lessons to effectively address learners' geometric needs. Therefore, this study recommends a merge of van Hieles levels theory with the TPACK framework for effective teaching and learning of Grade 11 Euclidean geometry.

For further research, this study recommends that researchers allocate more time to geometry contents at all van Hieles' levels in their interventions, more especially VHL 1 to 4 because those are basic levels. This is to ensure that subjects have understood and can apply concepts that fall under van Hieles' levels 1 to 4 before attempting levels 5 and 6 questions. The study also recommends to curriculum developers to research into different ways of assessing how technology integration and the combination of van Hieles' theory with other frameworks like TPACK, could improve geometry learning in schools. Finally, a mixed-method approach to future studies of this nature is recommended. This could add some qualitative data that explains the reasons or circumstances surrounding the obtained results of the study.

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APPENDIX A: RELIABILITY TEST WITH PEARSON'S PRODUCT MOMENT

PARTICIPANT NUMBER	VAN HIELES' LEVEL 1 (6 marks)		VAN HIELES' LEVEL 2 (14 marks)		VAN HIELES' LEVEL 3 (20 marks)		VAN HIELES' LEVEL 4 (20 marks)		VAN HIELES' LEVEL 5 (20 marks)		VAN HIELES' LEVEL 6 (15 marks)	
	Pre-Test Score	Post-Test Score	Pre-Test Score	Post-Test Score	Pre-Test Score	Post-Test Score	Pre-Test Score	Post-Test Score	Pre-Test Score	Post-Test Score	Pre-Test Score	Post-Test Score
1	3	3	7	5	12	14	6	8	5	7	1	1
2	2	2	8	9	8	8	9	8	5	6	2	0
3	4	4	11	12	13	12	4	5	7	7	2	4
4	4	3	8	9	15	14	12	13	5	3	3	2
5	4	4	10	9	11	12	9	10	8	8	5	6
6	1	1	9	9	9	8	7	7	4	3	3	4
7	3	3	9	10	7	8	10	8	8	7	5	6
8	4	3	6	6	5	6	7	8	3	4	4	4
9	3	3	9	9	9	9	10	11	8	7	7	6
10	4	5	9	7	10	11	12	12	9	8	9	8
11	2	2	7	8	7	7	7	8	6	7	4	3
12	3	3	7	7	12	14	10	8	9	8	5	3
13	4	5	10	11	11	9	9	10	9	9	5	6
14	2	2	7	8	9	10	8	9	5	7	2	3
15	3	3	9	10	8	9	9	10	8	6	4	3
r	.87		.81		.90		.84		.77		.83	

APPENDIX B: TESTS OF NORMALITY

Tests of Normality

	Groupnum	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
TOTALA_PoT1 Total of Analysis for Post-Test 1	C	.155	30	.065	.961	30	.321
	E	.171	30	.026	.782	30	.000
TOTALA_PrT1 Total of Analysis for Pre-Test 1	C	.229	30	.000	.901	30	.009
	E	.187	30	.009	.883	30	.003
TOTALAb_PoT2 Total of Abstraction for Post-Test 2	C	.153	30	.072	.944	30	.115
	E	.155	30	.064	.923	30	.033
TOTALAb_PrT2 Total of Abstraction for Pre-Test 2	C	.101	30	.200*	.961	30	.328
	E	.178	30	.016	.927	30	.042
TOTALD_PoT2 Total of Deductions for Post-Test 2	C	.130	30	.200*	.967	30	.464
	E	.122	30	.200*	.972	30	.584
TOTALD_PrT2 Total of Deductions for Pre-Test 2	C	.168	30	.030	.948	30	.154
	E	.120	30	.200*	.961	30	.332
TOTALPoT13 Overall Total for Post-Test 1 to 3	C	.103	30	.200*	.982	30	.875
	E	.173	30	.022	.960	30	.310
TOTALPrT13 Overall Total for Pre-Test 1 to 3	C	.105	30	.200*	.961	30	.325
	E	.220	30	.001	.886	30	.004
TOTALPrV_PoT1 Total of Pre-Visualization for Post-Test 1	C	.254	30	.000	.859	30	.001
	E	.300	30	.000	.749	30	.000
TOTALPrV_PrT1 Total of Pre-Visualization for Pre-Test 1	C	.294	30	.000	.831	30	.000
	E	.203	30	.003	.912	30	.016
TOTALR_PoT3 Total of Rigor for Post-Test 3	C	.144	30	.112	.958	30	.268
	E	.225	30	.000	.888	30	.004
TOTALR_PrT3 Total of Rigor for Pre-Test 3	C	.166	30	.034	.921	30	.028
	E	.158	30	.054	.940	30	.092
TOTALV_PoT1 Total of Visualization for Post-Test 1	C	.191	30	.007	.931	30	.052
	E	.210	30	.002	.890	30	.005
TOTALV_PrT1 Total of Visualization for Pre-Test 1	C	.128	30	.200*	.973	30	.636
	E	.196	30	.005	.950	30	.166

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

APPENDIX C: LEVENE'S TEST OF HOMOGENEITY OF VARIANCES

VHL1: Pre-visualization

		Levene Statistic	df1	df2	Sig.
Total of Pre-Visualisation for Pre-Test 1	Based on Mean	.068	1	58	.795
	Based on Median	.023	1	58	.880
	Based on Median and with adjusted df	.023	1	55.701	.880
	Based on trimmed mean	.104	1	58	.749
Total of Pre-Visualisation for Post-Test 1	Based on Mean	1.671	1	58	.201
	Based on Median	1.330	1	58	.254
	Based on Median and with adjusted df	1.330	1	53.680	.254
	Based on trimmed mean	1.590	1	58	.212

VHL 2: Visualization

		Levene Statistic	df1	df2	Sig.
Total of Visualisation for Pre-Test 1	Based on Mean	1.101	1	58	.298
	Based on Median	.914	1	58	.343
	Based on Median and with adjusted df	.914	1	57.108	.343
	Based on trimmed mean	1.084	1	58	.302
Total of Visualisation for Post-Test 1	Based on Mean	.060	1	58	.808
	Based on Median	.040	1	58	.843
	Based on Median and with adjusted df	.040	1	57.946	.843
	Based on trimmed mean	.069	1	58	.793

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group Within Subjects Design: Pre_post

VHL 3: Analysis

		Levene Statistic	df1	df2	Sig.
Total of Analysis for Pre-Test 1	Based on Mean	.963	1	58	.331
	Based on Median	.671	1	58	.416
	Based on Median and with adjusted df	.671	1	52.230	.417
	Based on trimmed mean	.967	1	58	.330
Total of Analysis for Post-Test 1	Based on Mean	.220	1	58	.641
	Based on Median	.080	1	58	.778
	Based on Median and with adjusted df	.080	1	47.102	.778
	Based on trimmed mean	.150	1	58	.700

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group Within Subjects Design: Pre_post

VHL 4: Abstraction

Levene's Test of Equality of Error Variances[a]

		Levene Statistic	df1	df2	Sig.
Total of Abstraction for Pre-Test 2	Based on Mean	1.733	1	58	.193
	Based on Median	1.452	1	58	.233
	Based on Median and with adjusted df	1.452	1	57.971	.233
	Based on trimmed mean	1.758	1	58	.190
Total of Abstraction for Post-Test 2	Based on Mean	.015	1	58	.904
	Based on Median	.000	1	58	1.000
	Based on Median and with adjusted df	.000	1	57.987	1.000
	Based on trimmed mean	.009	1	58	.925

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group Within Subjects Design: Pre_post

VHL 5: Deductions

		Levene Statistic	df1	df2	Sig.
Total of Deductions for Pre-Test 2	Based on Mean	.001	1	58	.980
	Based on Median	.008	1	58	.927
	Based on Median and with adjusted df	.008	1	55.102	.927
	Based on trimmed mean	.000	1	58	.997
Total of Deductions for Post-Test 2	Based on Mean	.848	1	58	.361
	Based on Median	.920	1	58	.341
	Based on Median and with adjusted df	.920	1	57.461	.341
	Based on trimmed mean	.862	1	58	.357

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group Within Subjects Design: Pre_post

VHL 6: Rigor

Levene's Test of Equality of Error Variances[a]

		Levene Statistic	df1	df2	Sig.
Total of Rigor for Pre-Test 3	Based on Mean	.786	1	58	.379
	Based on Median	.632	1	58	.430
	Based on Median and with adjusted df	.632	1	56.391	.430
	Based on trimmed mean	.889	1	58	.350
Total of Rigor for Post-Test 3	Based on Mean	10.312	1	58	.002
	Based on Median	4.909	1	58	.031
	Based on Median and with adjusted df	4.909	1	40.579	.032
	Based on trimmed mean	8.824	1	58	.004

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group Within Subjects Design: Pre_post

APPENDIX D: PRE-TEST 1 (Pre-Visualization, Visualization, and Analysis)

TITLE OF STUDY: The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.

Duration: 60 minutes **Date: ……./……./2020** **Marks: 40**

Name of your school: ____

Tick with X : Female ☐ **Male** ☐

Instructions:

Read the following instructions carefully before answering the questions.

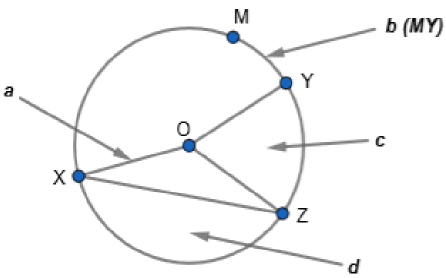
1. This question paper consists of 40 questions.
2. Answer ALL questions.
3. Write your answers **in the spaces provided** on this question paper or by **underlining** the correct answer where applicable.
4. Clearly show ALL calculations, diagrams, etc., which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
8. Diagrams are NOT drawn to scale.
9. Write legibly and present your work neatly.
10. Do all your **rough work** on this paper.

VHL	Q. NO.	MAX MARK	LEARNER MARK	VHL	Q. NO.	MAX MARK	LEARNER MARK
1. Pre-V	1	1		3. A	18	1	
	2	1			19	1	
	3	1			20	1	
	4	1			21	1	
	5	1			24	1	
	6	1			25	1	
2. V	7	1			26	1	
	8	1			28	1	
	9	1			29	1	
	10	1			30	1	
	11	1			31	1	
	12	1			32	1	
	13	1			33	1	
	14	1			34	1	
	15	1			35	1	
	16	1			36	1	
	17	1			37	1	
	22	1			38	1	
	23	1			39	1	
	27	1			40	1	
				TOTAL		40	
				%		100	

<u>In the following table, pictures of some real objects have been shown. Write the geometric names that match those pictures in questions 1 to 3.</u>

QUESTION	PROTOTYPE	NAME OF GEOMETRIC SHAPE
1	 Sun	(1) Pre-V
2	 Dice	(1) Pre-V
3	 Warning triangle	(1) Pre-V

In the next diagram, O is the center of each circle. Write the names of each missing part of the circles from **questions 4 to 6.**

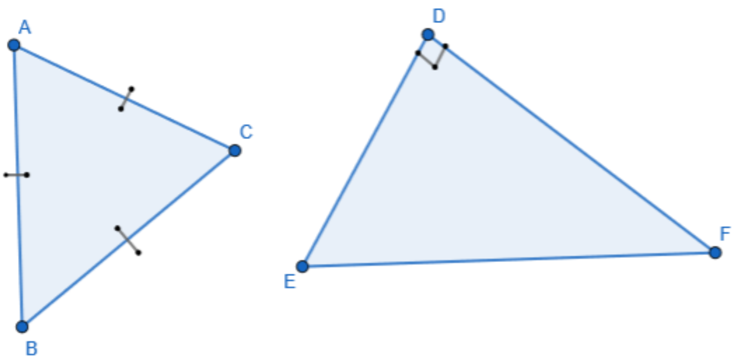


iii. Letter a: .... (1)Pre-V

iv. Letter b: .... (1) Pre-V

v. Letter d: .... (1) Pre-V

<u>Use the triangles below to answer questions 7 and 8:</u>

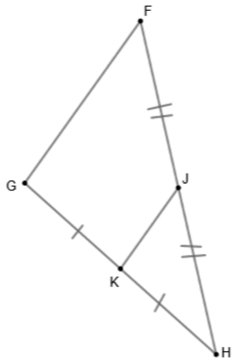


7. Write down the name of triangle ABC:...................................... (1)V

8. What type of triangle is DEF ?

.... (1)V

Consider the triangle below and use it to answer questions 9 and 10:



9. Which of the following statements is true?

a. $JK \perp GH$ **b.** $FG \parallel JK$ **c.** $\frac{GF}{2} = 2KJ$ **d.** If $JK = 3x, then\ FG = 9x$ (1)V

10. Which theorem is the diagram referring to?

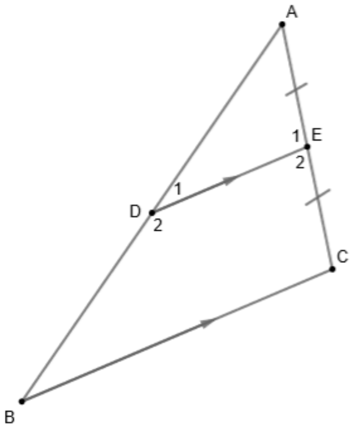
a. tan-chord theorem

b. angle at center is double angle at circumference

c. mid-point theorem

d. perpendicular from center bisects chord [1]V

Use the following triangle to answer questions 11 and 12:



11. Which of the following statements is **NOT** true?

a. $DE = \frac{1}{2}BC$ **b.** $\hat{D}_1 = \hat{B}$ **c.** $BD = DA$

d. $BC = \frac{1}{2}DE$ (1)V

12. Why is $\hat{E}_1 = \hat{C}$?

a. corresponding angles are equal

b. converse mid-point theorem

d. perpendicular from center bisects chord

e. alternate angles are equal [1]V

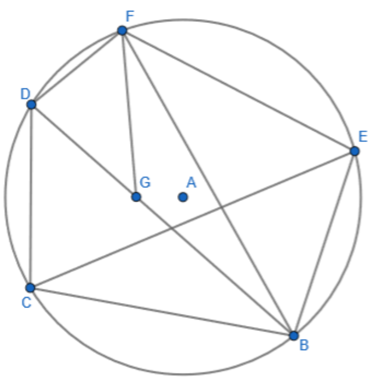
From questions 13 to 16, choose TRUE (T) or FALSE (F) for the given statement:

13. The angle formed between a radius and a tangent equals 90^0. **T / F** ? (1) V

14. Co-interior angles are equal. **T / F** ? (1) V

15. The opposite angles of a cyclic quadrilateral are supplementary. **T / F** ? (1) V

16. A line from circle center which is perpendicular to a chord is parallel to the chord. **T / F** ? (1)V



17. A is the center of the circle below. B, C, D, F, E are points on the circumference.

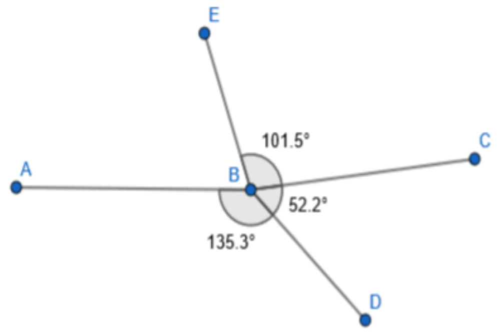
Underline the correct answer.

Which of the following is **not** a cyclic quadrilateral?

a.	ECDF	b.	BDFE
c.	FGBE	d.	DCBF

(1)V

18. Determine the value of $A\hat{B}E$ from the options provided.



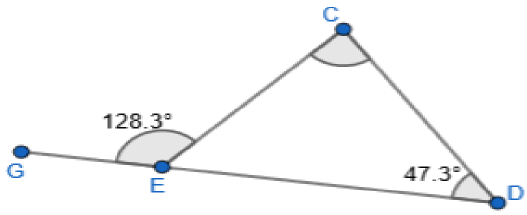
a. 71^0

b. 78.5^0

c. 52.2^0

d. 81.5^0 (1)A

19. Which of the following is the size of angle C ?

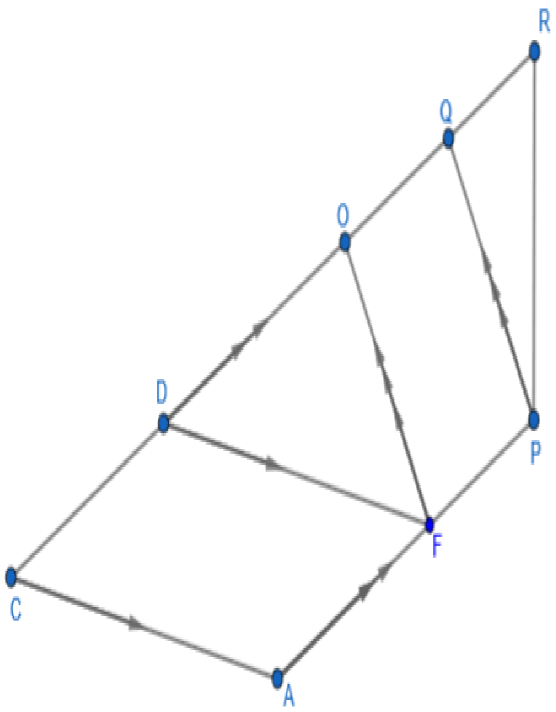


a. 81^0 b. 64^0

c. 51.7^0 d. 47.3^0 (1)A

<u>Use the diagram below to answer questions 20 to 24.</u>

In the diagram below, $CR \parallel AO,\ CA \parallel DF\ \ and\ \ OF \parallel QP$. Answer the questions that follow:



20. How many quadrilaterals can be found in this shape?

a. 2 b. 8

c. 7 d. 3 (1)A

21. Which of the following is/are **not** parallelograms?

a. CQPA b. OQPF

c. DFAC d. ORPF (1)A

22. Which name would be appropriate for DFPR?

a. parallelogram b. square c. trapezium

d. rectangle (1)V

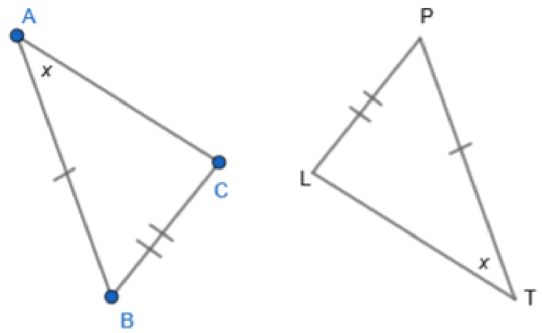
23. How many parallelograms are there?

a. 1 b. 6 c. 2 d. 3 (1) V

24. If angle DFA= 30^0, determine the size of angle C.

a. 30^0 b. 60^0 c. 150^0 d. 15^0 (1) A

Observe the two triangles below and **complete** the statements with an appropriate triangle.



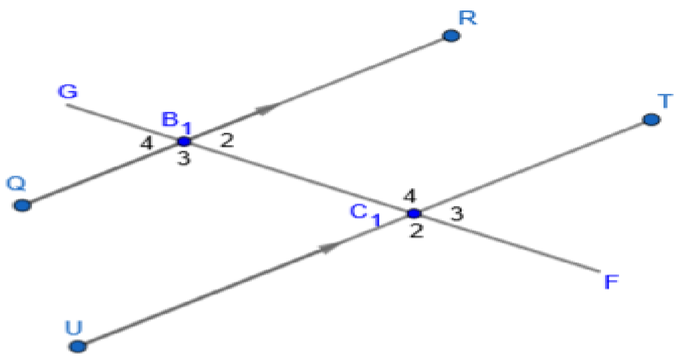
25. $\Delta ABC \equiv \Delta$ ... (1) A

26. Δ ... $\equiv \Delta LPT$ (1)A

27. Which reasoning has been used in question 26 ?

a. SAS b. AAA c. SAA d. SSS (1) V

In the diagram that follows, $QR \parallel UT$ and GF is a transversal. Use the diagram to complete the missing statements and/or reasons in the table below:



QUESTION	STATEMENT	REASON	MARK
28.	$\hat{B}_1 = \hat{C}_4$	...	(1)A
29.	$... = \hat{C}_1$	Alt $< s$ are =	(1)A
30.	$\hat{B}_3 = 180° - \hat{C}_1$	...	(1)A

Complete the following table for the properties of the quadrilaterals given.

If the property **APPLIES** to the shape, tick with ✓.

If the property **DOES NOT APPLY** to the shape, tick with **X.**

Fill **ONLY** the **10** spaces indicated with **question numbers 31 to 40.**

PROPERTY → **2D SHAPE ↓**	**Only 1 pair of sides ∥**	**2 pairs of adjacent sides =**	**Opposite sides are ∦**	**Angle sum = 360^0**	**Diagonals intersect at ⊥**	**Diagonals bisect each other**	**Each angle = 90^0**
Parallelogram	X	X	**31.**	✓	**32.**	✓	X
Rhombus	X	✓	**33.**	✓	✓	✓	**34.**
Rectangle	X	X	✓	✓	**35.**	✓	**36.**
Kite	X	**37.**	X	✓	✓	**38.**	X
Trapezium	**39.**	X	**40.**	✓	X	X	X

1 mark each (A)

TOTAL: 40

APPENDIX E: POST-TEST 1 (Pre-Visualization, Visualization, and Analysis)

TITLE OF STUDY: The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.

Duration: 60 minutes **Date: ……./……./2020** **Marks: 40**

Name of your school: ______________________________________

Tick with X : **Female** ☐ **Male** ☐

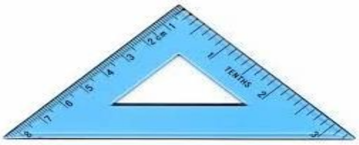
Instructions:

Read the following instructions carefully before answering the questions.

1. This question paper consists of 40 questions.
2. Answer ALL questions.
3. Write your answers **in the spaces provided** on this question paper or by **underlining** the correct answer where applicable.
4. Clearly show ALL calculations, diagrams, etc., which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
8. Diagrams are NOT drawn to scale.
9. Write legibly and present your work neatly.
10. Do all your **rough work** on this paper.

VHL	Q. NO.	MAX MARK	LEARNER MARK	VHL	Q. NO.	MAX MARK	LEARNER MARK
1. Pre-V	1	1		3. A	8	1	
	2	1			9	1	
	3	1			10	1	
	4	1			15	1	
	5	1			16	1	
	6	1			17	1	
2. V	7	1			18	1	
	11	1			19	1	
	12	1			20	1	
	13	1			23	1	
	14	1			29	1	
	21	1			31	1	
	22	1			33	1	
	24	1			34	1	
	25	1			35	1	
	26	1			36	1	
	27	1			37	1	
	28	1			38	1	
	30	1			39	1	
	32	1			40	1	
				TOTAL		40	
				%		100	

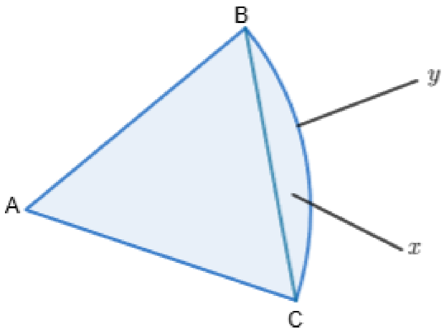
In the following table, pictures of some real objects have been shown. Write the geometric names that match those pictures in questions 1 to 3.

QUESTION	PROTOTYPE	NAME OF GEOMETRIC SHAPE
1	 Full moon	(1) Pre-V
2	 Half-moon	(1) Pre-V
3	 Set-square	(1) Pre-V

Use the following diagram to answer questions 4 and 5

The diagram below is a cut-out of a circle. A is the center of the circle with B and C as points on the circumference.

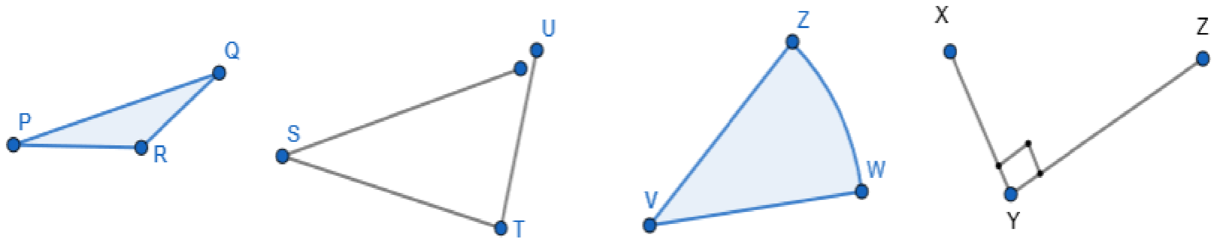
What would be the names of the following part?



4. part x :……………………………………………………………(1) Pre-V

5. part y :……………………………………………………………(1) Pre-V

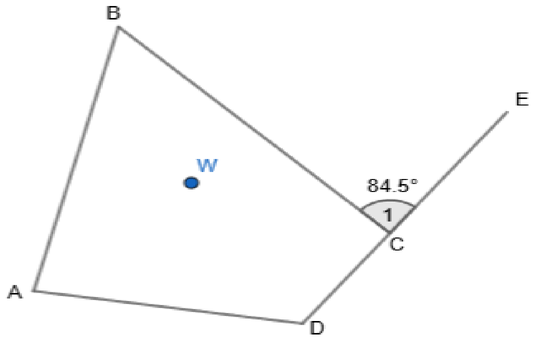
Consider the following shapes and answer the next question:



6. Which of the above shapes is/are triangle(s)? ………………………………………………………………(1) Pre-V

Use the following diagram to answer questions 7 to 10:

In the diagram below, W is the center of a certain circle (hidden). A, B, C and D are points on the circumference of that circle. DCE is a straight line such that $\hat{C}_1 = 84.5°$



7. Which of the following would be the correct value for $\hat{A}$?

a. 15.5^0 b. 95.5^0 c. 90^0 d. 84.5^0 (1)V

8. Which of these reasons apply to the answer in question 7?

a. Exterior angle of cyclic quadrilateral equals interior opposite angle

b. Tan-chord theorem

c. Opposite angles of cyclic quadrilateral add up to 180^0

d. Angles in same segment are equal (1)A

9. If it is further given that $\hat{B} = 35°$, what would be the value of $\hat{D}$?

a. 145^0 b. 55^0 c. 84.5^0 d. 100^0 (1)A

10. Select the reason for the answer in question 9 above.

b. Equal chords subtend equal angles

b. Exterior angle of cyclic quadrilateral equals interior opposite angle

c. Opposite angles of cyclic quadrilateral are supplementary

d. Angles in same segment are equal (1)A

From questions 11 to 14, choose TRUE (T) or FALSE (F) for the each statement:

11. A line from circle ecenter which is perpendicular to a chord bisects the chord. **T / F** ? (1)V

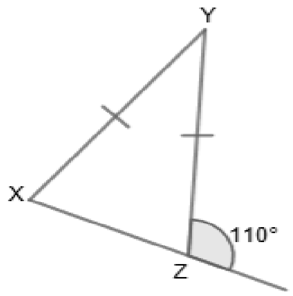
12. The angle between a tangent and a chord equals 90^0. **T / F** ? (1) V

13. Co-interior angles are supplementary. **T / F** ? (1) V

14. The opposite angles of a cyclic quadrilateral are supplementary. **T / F** ? (1) V

Use the following diagram to answer questions 15 and 16:

In the diagram, XY=YZ and the obtuse angle Z=110^0.



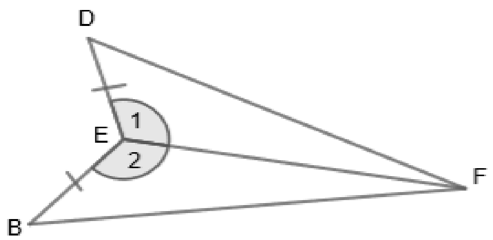
15. Determine the value of $\hat{X}$.

a. 110^0 b. 40^0 c. 70^0 d. 99^0 (1)A

16. What is the value of angle Y?

a. 55^0 b. 40^0 c. 70^0 d. 99^0 (1)A

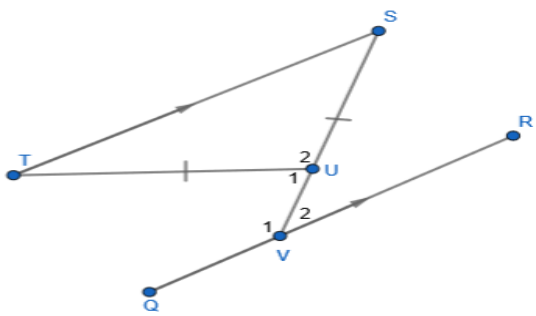
17. The following diagram shows a two-joined triangles with BE=ED. If it is further given that E_1=E_2, which reason could best be used to show that $\Delta DEF \equiv \Delta BEF$?



a. SAA b. RHS

c. AAA d. SSA (1)A

Use the following diagram to answer questions 18 and 19.



$TS \parallel QR$ and SV is a transversal in the diagram on the left. TU joins SV such that $TU = US$.

18. Given that $\hat{U}_1 = 85^0$, calculate the size of $\hat{T}$.

a. 42.5^0 b. 40^0 c. 70.5^0 d. 99^0 (1)A

19. Which of these 2 reasons could be used, one after the other, to show that $\hat{V}_2 = \hat{T}$?

a. vertically opposite angles are equal; alternate angles are equal

b. angles opposite equal sides of triangle are equal; co-interior angles are supplementary

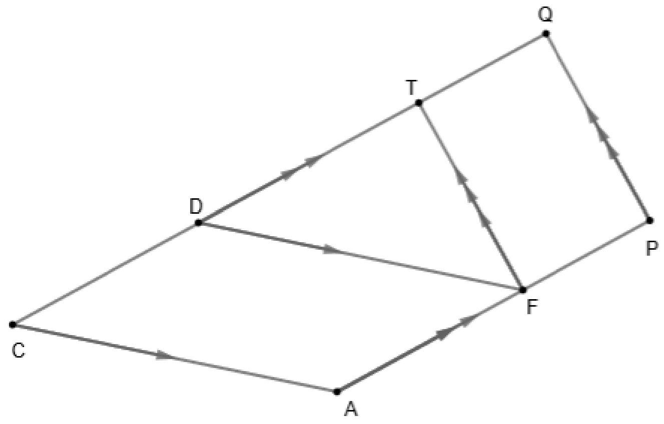
c. alternate angles are equal; angles opposite equal sides of triangle are equal

d. tan-chord theorem; corresponding angles are equal. (1)A

20. A certain triangle KTL, has the following properties: $\hat{K} = x$, $\hat{T} = 4x$ and $\hat{L} = 70$. What would be the name of that triangle?

a. Equilateral b. Isosceles c. Right-angled d. Scalene (1)A

Use the next diagram to answer questions 21 and 22



Given:

$CQ \parallel AP, \quad CA \parallel DF \;\; and \;\; TF \parallel QP$

Answer the questions that follow:

21. Which name would be appropriate for $DQPF$?

a. parallelogram

b. square

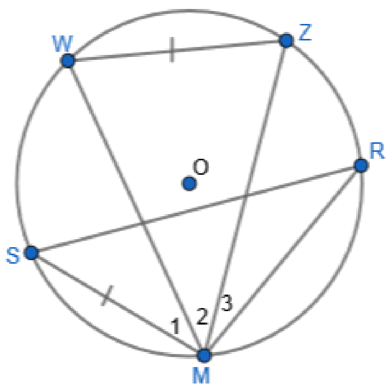
c. trapezium

d. rectangle (1)V

22. How many parallelograms are there?

a. 1 b. 6 c. 2 d. 3 (1) V

Use the diagram below to answer questions 23 and 24:



Circle center O is drawn such that WZ = SM and $\hat{R} = 47^0$.

23. Determine the size of $\widehat{M}_2$

a. 94^0 b. 53^0 c. 47^0 d. 35^0 (1)A

24. Which reason applies to the statement from question 23 ?

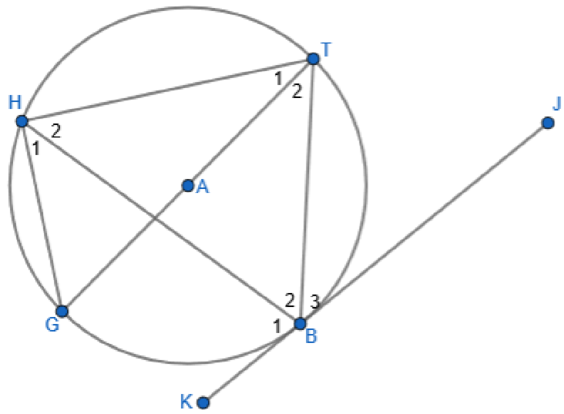
a. angles opposite equal sides of triangle are equal

b. angle at center is double angle at circumference

c. angles in a semi-circle are supplementary

d. equal chords subtend equal angles (1)V

Use the diagram below to answer questions 25 to 28:



The diagram alongside has KBJ as a tangent to the circle at B. The points T, H, G and B lie on the circumference. A is the center.

25. Using the tan-chord theorem, which of the following angles is equal to $\widehat{H}_2$?

a. $\widehat{H}_1$ b. $\widehat{B}_1$ c. $\widehat{T}_2$ d. $\widehat{B}_3$ (1)V

26. What is the size of $G\widehat{H}T$?

a. 90^0 b. 60^0 c. 85^0 d. 45^0 (1)V

27. Which of the following reasons applies to question 26 ?

a. tan-chord theorem

b. angles in same segment

c. angles in semi-circle

d. radius $\perp$ tangent (1) V

28. Give a reason why $\widehat{G} = \widehat{B}_2$.

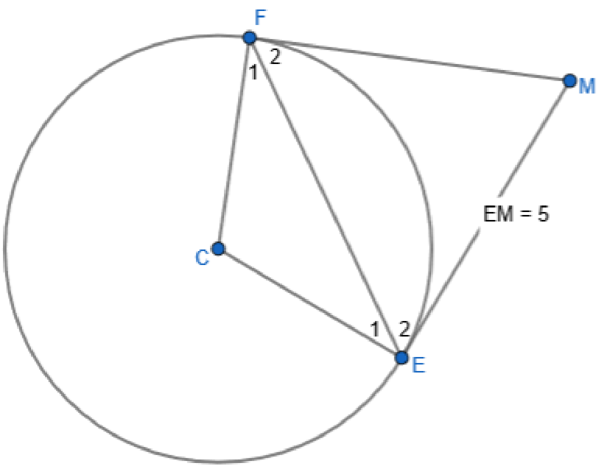
a. tan-chord theorem

b. angles in same segment

c. angles in semi-circle

d. radius $\perp$ tangent (1) V

Use the diagram below to answer questions 29 to 33:



h

In the diagram, FM and EM are tangents to circle center C. Answer the questions that follow:

29. Determine the length of FM if EM=5units.

a. 5units b. 10units c. 15units d. 2.5units (1)A

30. Which of the following reasons applies to the answer in question 29 ?

a. tan-chord theorem b. angles in same segment

c. angles in semi-circle d. radius ⊥ tangent (1) V

31. What is the size of angle MEC ?

a. 74^0 b. 52^0 c. 90^0 d. 60^0 (1)A

32. What reason is true for the answer in question 31 ?

a. radius ⊥ tangent b. tan-chord theorem

c. angles in same segment d. angles in semi-circle (1) V

33. If $FE = EM$, what would be the name of triangle FEM ?

a. Equilateral b. Isosceles c. Right-angled d. Scalene (1)A

Complete the following table for the properties of the quadrilaterals given.

Fill **ONLY** the **7** spaces indicated with **questions 34 to 40.**

If the property **APPLIES** to the shape, tick with ✓.

If the property **DOES NOT APPLY** to the shape, tick with **X.**

PROPERTY→ 2D SHAPE ↓	Diagonals bisect each other	Opposite sides are ⋕	2 pairs of adjacent sides =	Only 1 pair of sides are ∥	Each angle = 90^0	Diagonals intersect at ⊥
Parallelogram	**34.**	**35.**	X	**36.**	X	**37.**
Trapezium	**38.**	X	X	**39.**	X	**40.**

1 mark each (A)

TOTAL: 40

APPENDIX F: PRE-TEST 2 (Abstraction, and Deductions)

TITLE OF STUDY: The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.

Duration: 60 minutes **Date: ..../......./2020** **Marks: 40**

Name of your school: ____

Tick with X : Female ☐ **Male** ☐

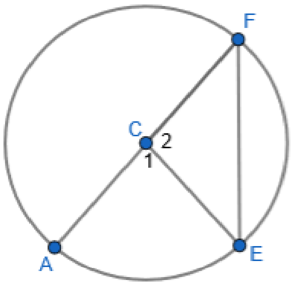
Instructions:

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL questions.
3. Write your answers **in the tables provided** on this question paper by giving clear statements and reasons.
4. Clearly show ALL calculations, diagrams, etc., which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
8. Diagrams are NOT drawn to scale.
9. Write legibly and present your work neatly.
10. Do all your **rough work** on this paper.

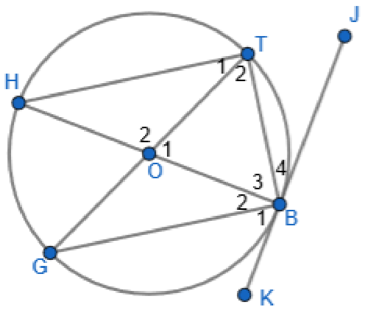
VHL	**QUES. NO.**	**MAX MARKS**	**LEARNER'S MARKS**
4. Ab	**1**	3	
	2	4	
	3	4	
	4	4	
	5	5	
5. D	**6**	5	
	7	5	
	8	5	
	9	5	
TOTAL MARKS		**40**	
MARKS (%)		**100**	

1. C is the center of the circle below. If $\hat{E} = 28^0$, calculate with reasons, the size of $\hat{C}_1$.



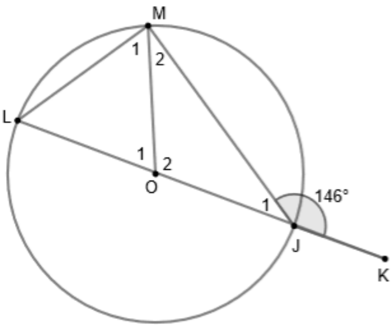
STATEMENT	REASON
	(3) **Ab**

2. In the given circle, O is the center. HB and GT are diameters. KJ is a tangent and $\hat{B}_2 = 40^0$. Determine, with reasons, the value of $\widehat{H}$.



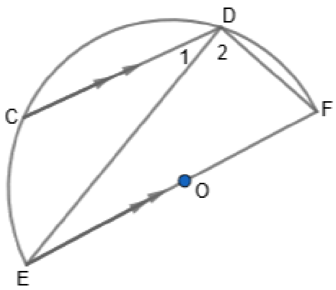
STATEMENT	REASON
	[4]Ab

vi. In the diagram, JL is a diameter of the circle. O is the center and $M\hat{J}K = 146^0$. Calculate the size of $\widehat{M}_1$.



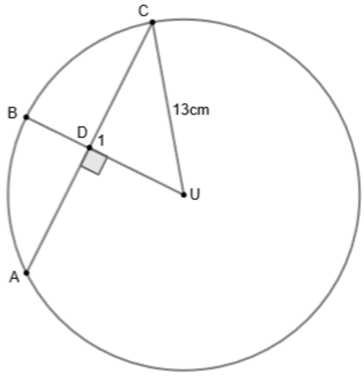
STATEMENT	REASON
	(4) **Ab**

4. The diagram below is a semi-circle cut from a circle with center O. $CD \parallel EF$ and $\hat{F} = 60^0$. Determine $\widehat{D}_1$



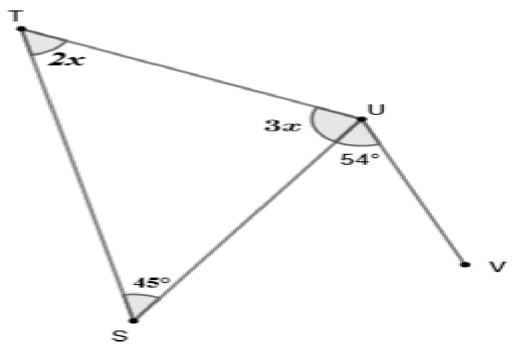
STATEMENT	REASON
	[4]Ab

5. Drawn below is circle center ***U*** with radius 13cm and $A\widehat{D}U = 90^0$. If AC=16cm, calculate the length of BD. Let $DU = x$.



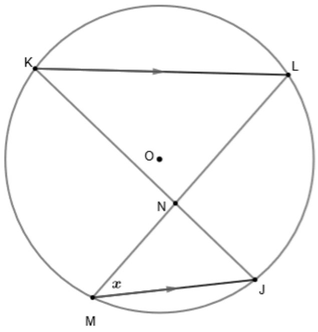
STATEMENT	REASON
	[5] **Ab**

vii. Consider the diagram below. Prove that line UV is a tangent to a circle that passes through points S, U and T.



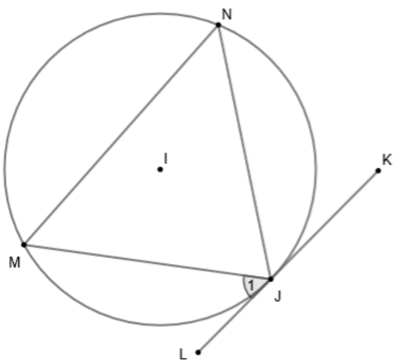
STATEMENT	REASON
	(5) **D**

viii. O is the center of the following circle. KJ and LM intersect at N and $KL \parallel MJ$. If $\widehat{M} = x$, show that $KN = LN$.



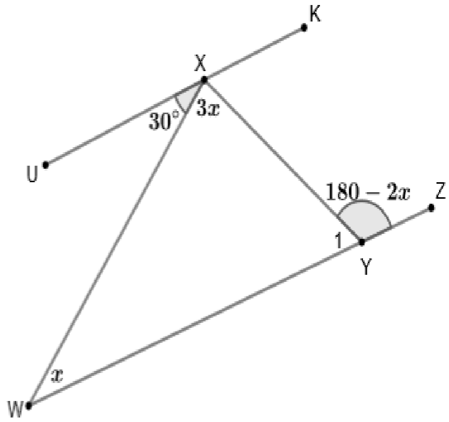
STATEMENT	REASON
	(5) **D**

ix. KJL is a tangent to circle center I. Prove the theorem which states that $\hat{J}_1 = \widehat{N}$.



STATEMENT	REASON
	(5) **D**

x. In the diagram, XWY is a triangle. WZ is a straight line such that angle XYZ=180-2x. UK is another straight line that passes through point X. Show, with reasons, that $UK \parallel WZ$. Hence, determine the size of $\hat{Y}_1$.



STATEMENT	REASON
	(5) **D**

TOTAL: 40

APPENDIX G: POST-TEST 2 (Abstraction, and Deductions)

TITLE OF STUDY: The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hiele's model.

Duration: 60 minutes **Date: ……./……./2020** **Marks: 40**

Name of your school: ____

Tick with X : **Female** ☐ **Male** ☐

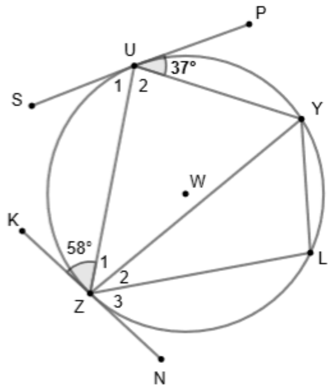
Instructions:

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL questions.
3. Write your answers **in the tables provided** on this question paper by giving clear statements and reasons.
4. Clearly show ALL calculations, diagrams, etc., which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
8. Diagrams are NOT necessarily drawn to scale.
9. Write legibly and present your work neatly.
10. Do all your rough work on this paper.

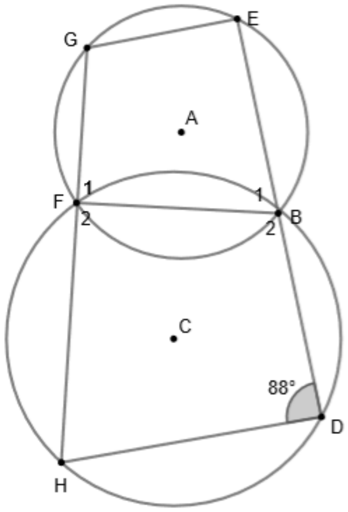
VHL	QUES. NO.	MAX MARKS	LEARNER'S MARKS
4. Ab	**1**	4	
	2	3	
	3	5	
	4	4	
	5	4	
5. D	**6**	5	
	7	5	
	8	5	
	9	5	
TOTAL MARKS		**40**	
MARKS (%)		**100**	

1. SUP and KZN are tangents to circle center W. If $\widehat{U}Y = 37^0$ and $K\hat{Z}U = 58^0$, calculate the value of $\hat{L}$.



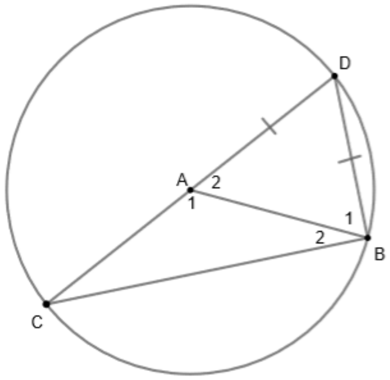
STATEMENT	REASON
	(4) **Ab**

2. In the given diagram, A and O are the centers of the smaller and bigger circles respectively. FB is a common chord and $\widehat{D} = 88^0$. Determine the size of angle E.



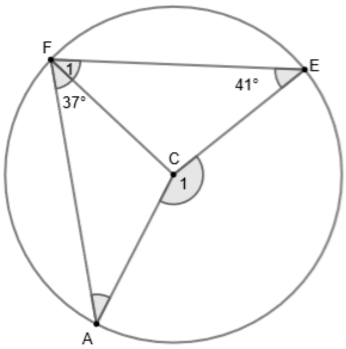
STATEMENT	REASON
	[3]Ab

3. Drawn below is circle center A. CD is the diameter and AD=DB. Determine the value of angle C.



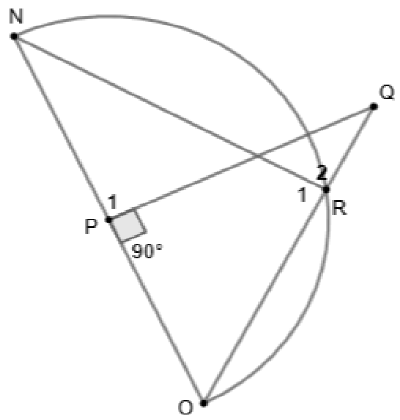
STATEMENT	REASON
	[5]Ab

xi. The diagram below is a circle with center C. Determine $\hat{C}_1$.



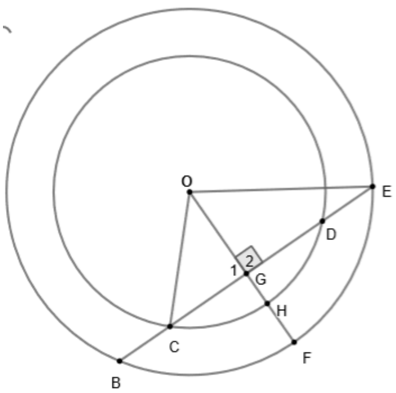
STATEMENT	REASON
	(4) Ab

xii. Drawn below is semi-circle cut from circle center P. N, R and O are points on the circumference. Show that the points $N, P, R\ and\ Q$ are concyclic (lie on the circumference of a circle).



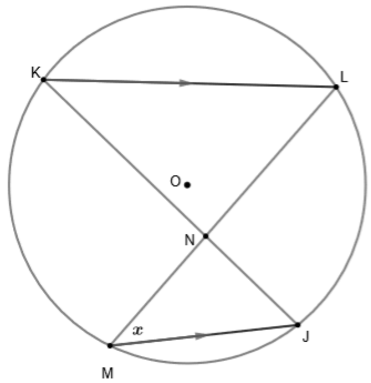
STATEMENT	REASON
	(4) **Ab**

xiii. Consider the 2 concentric (same center) circles drawn below. $OC = 100mm\ \ and\ OE = 170mm$. If $CD = 120mm$, determine the length of BE.



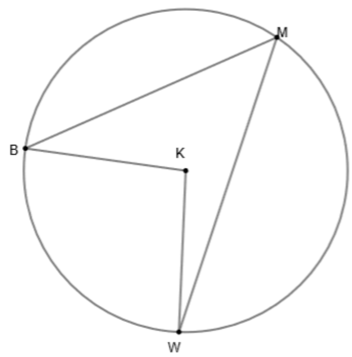
STATEMENT	REASON
	(5) **D**

xiv. O is the center of the following circle. KJ and LM intersect at N and $KL \parallel MJ$. If $\widehat{M} = x$, show that $KN = LN$.



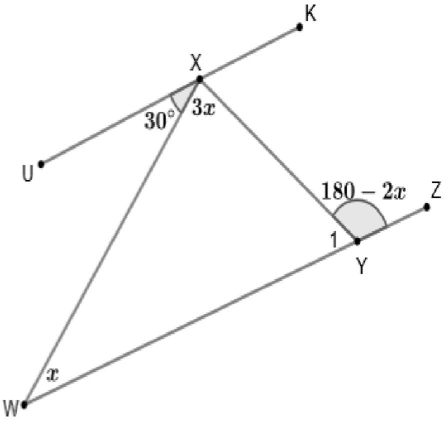
STATEMENT	REASON
	(5) **D**

xv. Consider circle center *K* below. Prove that the angle at the center is double the angle at the circumference.



STATEMENT	REASON
	(5) **D**

xvi. In the diagram, XWY is a triangle. WZ is a straight line such that angle XYZ=180-2x. UK is another straight line that passes through point X. Show, with reasons, that $UK \parallel WZ$. Hence, determine the size of $\hat{Y}_1$.



STATEMENT	REASON
	(5) **D**

TOTAL: 40

APPENDIX H: PRE-TEST 3 (Rigor)

TITLE OF STUDY: The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.

Duration: 30 minutes **Date: ….../….../2020** **Marks: 15**

Name of your school: ____

Tick with X : Female ☐ **Male** ☐

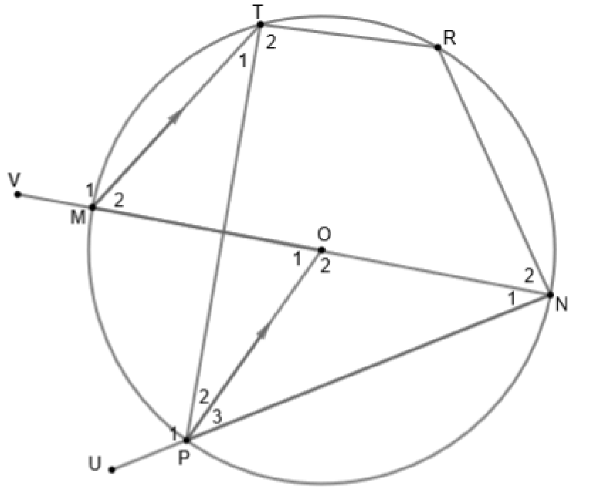
Instructions:

Read the following instructions carefully before answering the questions.

1. This question paper consists of 2 questions.
2. Answer ALL questions.
3. Write your answers **in the spaces/tables provided** on this question paper by giving clear statements and reasons.
4. Clearly show ALL calculations, diagrams, etc., which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
8. Diagrams are NOT drawn to scale.
9. Write legibly and present your work neatly.
10. Do all your **rough work** on this paper.

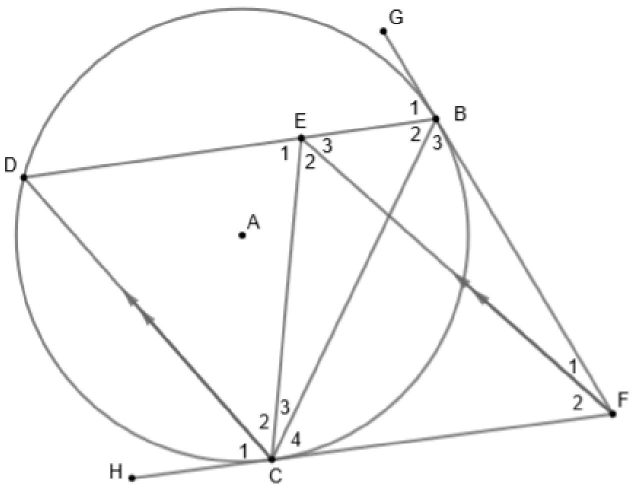
VHL	QUES. NO.	MAX MARKS	LEARNER'S MARKS
6. Rigor (Rg)	**1**	8	
	2	7	
TOTAL MARKS		**15**	
MARKS (%)		**100**	

1. *MD* is the diameter of circle center *O.* If $MT \parallel PO$, *prove that* $\hat{O}_2 = \hat{P}_1$.



STATEMENT	REASON
	[8]Rg

2. Circle center A is drawn below with tangents *HCF and GBF.* Point *E* lies on chord *DB* such that $EF \parallel DC$. Let $\hat{C}_4 = x$. Show that a new circle, center *T,* will pass through the points *E, B, F and C.*



STATEMENT	REASON
	[7] Rg

TOTAL: 15

APPENDIX I: POST-TEST 3 (Rigor)

TITLE OF STUDY: The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.

Duration: 30 minutes **Date: ……./……./2020** **Marks: 15**

Name of your school: ____

Tick with X : Female ☐ **Male** ☐

Instructions:

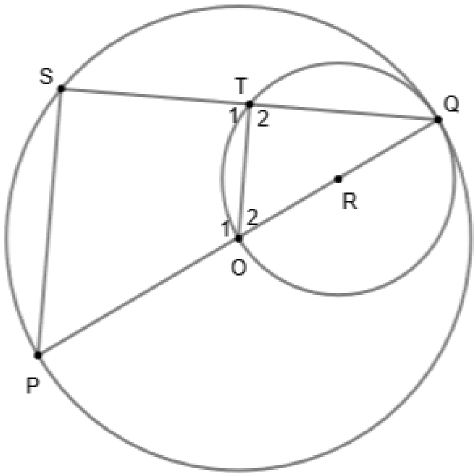
Read the following instructions carefully before answering the questions.

1. This question paper consists of 2 questions.
2. Answer ALL questions.
3. Write your answers **in the tables provided** on this question paper by giving clear statements and reasons.
4. Clearly show ALL calculations, diagrams, etc., which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.

8. Diagrams are NOT drawn to scale.
9. Write legibly and present your work neatly.
10. Do all your **rough work** on this paper.

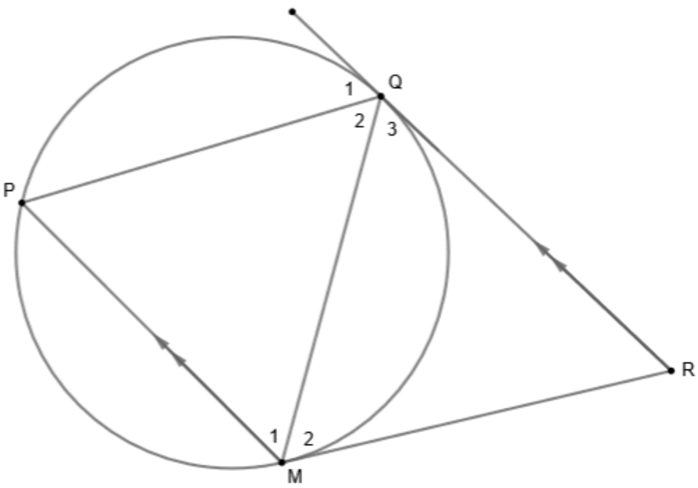
VHL	**QUES. NO.**	**MAX MARKS**	**LEARNER'S MARKS**
6. Rigor	**1**	7	
	2	8	
TOTAL MARKS		**15**	
MARKS (%)		**100**	

1. In the diagram, P*, S,* and *Q* are points on the circumference of circle centre *O.* Similarly, *O, T and Q* lie on circle ercenter R. *PQ and OQ* are the diameter of the bigger and smaller circles respectively. Prove that $OT^2 = \frac{1}{4}PS^2$.



STATEMENT	REASON
	[7]Rg

2. *P, Q,* and *M* are points on the circumference of the given circle. *QR* and *RM* are tangents such that $QR \parallel PM$. Show that $\hat{Q}_2 = \hat{R}$. (Let $\hat{Q}_3 = x$)



STATEMENT	REASON
	[8]Rg

TOTAL: 15

APPENDIX J: UNISA ETHICAL CLEARANCE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/06/10

Ref: **2020/06/10/56098324/14/AM**
Name: Mr A Bediako
Student No.: 56098324

Dear Mr A Bediako

Decision: Ethics Approval from 2020/06/10 to 2023/06/10

Researcher(s): Name: Mr A Bediako
E-mail address: 56098324@mylife.unisa.ac.za
Telephone: +27 74 571 7174

Supervisor(s): Name: Prof ZMM Jojo
E-mail address: jojozmm@unisa.ac.za
Telephone: 012 429 6627

Title of research:

The impact of technology integration in teaching grade 11 Euclidean geometry based on Van Hiele's model.

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/06/10 to 2023/06/10.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2020/06/10 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2023/06/10**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2020/06/10/56098324/14/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof PM Sebate
ACTING EXECUTIVE DEAN
Sebatpm@unisa.ac.za

Approved - decision template – updated 16 Feb 2017

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX K: FREE STATE DEPARTMENT OF EDUCATION'S APPROVAL

Enquiries: MZ Thango
Ref: Research Permission: A Bediako
Tel. 051 404 9257/ 9207 / 082 537 2654
Email: MZ.Thango@fseducation.gov.za

education
Department of Education
FREE STATE PROVINCE

557 Section G
Botshabelo
Bloemfontein
9781

Dear Mr. A. Bediako

APPROVAL TO CONDUCT RESEARCH IN THE FREE STATE DEPARTMENT OF EDUCATION

This letter serves as an acknowledgement of receipt of your request to conduct research in the Free State Department of Education.

Topic: The impact of technology integration in teaching grade 11 Euclidean Geometry based on Van Hieles' model.

1. **List of schools involved:** Senakangwedi Secondary School and Setjabasemaketse Combined School.

2. **Target Population:** Two SES: Mathematics, Motheo District, One SES: Mathematics, Lejweleputswa District and Sixty Grade 11 Mathematics learners at the selected Secondary Schools (thirty learners per school).

3. **Period of research:** From date of signature of this letter until 30 September 2020. Please note that the department does not allow any research to be conducted during the fourth term (quarter) of the academic year. Should you fall behind your schedule by three months to complete your research project in the approved period, you will need to apply for an extension. The researcher is expected to request permission from the school principals to conduct research at schools.

4. The approval is subject to the following conditions:

   4.1 The collection of data should not interfere with the normal tuition time or teaching process.

   4.2 A bound copy of the research document or a CD, should be submitted to the Free State Department of Education, Room 319, 3rd Floor, Old CNA Building, Charlotte Maxeke Street, Bloemfontein.

   4.3 You will be expected, on completion of your research study to make a presentation to the relevant stakeholders in the Department.

   4.4 The ethics documents must be adhered to in the discourse of your study in our department.

5. Please note that costs relating to all the conditions mentioned above are your own responsibility.

Yours sincerely

DR JEM SEKOLANYANE
CHIEF FINANCIAL OFFICER

DATE: 18/08/2020

RESEARCH APPLICATION BY A BEDIAKO, PERMISSION LETTER 13 AUGUST 2020, MOTHEO DISTRICT
Strategic Planning, Research & Policy Directorate Private Bag X20565, Bloemfontein, 9300 - Old CNA Building, Room 319, 3rd Floor, Charlotte Maxeke Street, Bloemfontein
Tel: (051) 404 9257/9207

www.fsdoe.fs.gov.za

APPENDIX L: CONSENT FORM TO PARENTS

Title of research: **The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.**

Dear Parent,

Your child is invited to participate in a study titled "The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hiele's model". I am undertaking this study as part of my masters' research at the University of South Africa. The purpose of the study is to explore the impact of using GeoGebra software in the teaching and learning of Grade 11 circle geometry. The possible benefits of the study are the improvement of the existing knowledge on the impact that the use of educational software has on the teaching of Euclidean geometry. It will also give educators more options in their daily selection of teaching methods. Moreover, your child will be exposed to the various ways of handling geometry problems through technology, thereby helping increase their level of interest in the topic and improve their academic performance. I am asking permission to include your child in this study because he/she is in Grade 11, which the study hopes to focus on to improve their learning approaches. I expect to have a total of 60 children participating in the study including your child.

A total of 30 learners from Senakangwedi Secondary shall form the experimental group, who will be taught using GeoGebra software at Senakangwedi Secondary School, and the other 30 learners from Setjabasemaketse Secondary will be taught using the traditional teaching method at Senakangwedi Secondary School. The academic reports of the learners for the year 2019 would be needed to help in determining learners' progression.

Weekdays, after school, will be used for the research with each session taking a maximum 1 hour per day, from 15:20 to 16:20. **Classes for the control group shall be Mondays and Wednesdays at Setjabasemaketse, and that of the experimental group shall be done on Tuesdays and Thursdays at Senakangwedi, from Monday 24 August 2020 to 25 September 2020.**

If you allow your child to participate, I shall request him/her to voluntarily write 3 pre-tests and 3 post-tests. The tests will focus on Grade 11 geometry only. Each test will take a maximum of 60 minutes to complete. Participants' scores from the test will be immediately communicated to them right after marking to help them correct their mistakes. The tests will take place on weekdays, after school, at Senakangwedi and Setjabasemaketse for the experimental and control groups respectively.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

To avoid exposing learners to COVID-19, arrangements shall be made with principals to ensure strict adherence to all the safety protocols outlined by the Department of Health and the government. This includes daily screening of learners and teachers, sanitizing, wearing masks, and ensuring social distancing. The researcher shall cooperate with the schools and communicate any possible health risks to the principals and parents/guardians immediately for the necessary action. Neither your child nor you will receive any type of payment for participating in this study. The principal and the Grade 11 mathematics teacher of the school shall assist in supervision during the study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be in the study now and change your mind later without any penalty. However, if you do not want your child to participate, an alternative activity will be available which is to continue receiving similar instruction during the normal classroom activities, which does not form part of this research.

In addition to your permission, your child may agree to participate in the study and you, and your child will also be asked to sign the consent/assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely in locked cabinets and the researcher's password protected computer for five years after the study. Thereafter, records will be erased.

If you have questions about this study, please ask me or my study supervisor, Prof Z.M.M Jojo, Department of Mathematics Education, College of Education, University of South Africa. My contact number is 0745717174 and my e-mail is adjeibediako@gmail.com. The e-mail of my supervisor is jojozmm@unisa.ac.za. Permission for the study has already been given by the

Ethics Committee of the College of Education, UNISA, the Principals of Senakangwedi and Setjabasemaketse Secondary Schools, and the Free State Department of Education.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:

Sincerely,

______________________	____________________	____________
Parent/guardian's name (print)	Parent/guardian's signature	Date
<u>BEDIAKO, A.</u>	________________________	____________
Researcher's name (print)	Researcher's signature	Date

APPENDIX M: ASSENT/CONSENT FORM TO PARTICIPANTS

Title of research: **The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hieles' model.**

Date: 05 July 2020

Dear Learner,

I am doing a study on "**The impact of technology integration in teaching Grade 11 Euclidean geometry based on Van Hiele's model"** as part of my studies at the University of South Africa. Your principal and the Free State Department of Education have given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your teachers can use to improve the teaching of Euclidean geometry. This may help you and many other learners of your age in different schools.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents/guardian about this before you decide if you want to be in this study.

This study will require that you attend weekday classes, after school, from Monday **24 August 2020 to 25 September 2020**, at your school. Each session will take a maximum of an hour per day, from **15:20 to 16:20**. A total of 30 learners from Senakangwedi Secondary shall form the experimental group, who will be taught by the researcher using GeoGebra software at Senakangwedi Secondary School, and the other 30 learners from Setjabasemaketse Combined School will be taught by the same researcher using the traditional teaching method at Setjabasemaketse Combined School. **Classes for the control group shall be held on Mondays and Wednesdays at Setjabasemaketse, and that of the experimental group shall be done on Tuesdays and Thursdays at Senakangwedi, from Monday 24 August 2020 to 25 September 2020.**

During the classes, you will be asked to voluntarily write 3 pre-tests and 3 post-tests on circle theorems, proofs, and applications in geometry. Each test will constitute a maximum of 40 marks to be written in 60 minutes. You may decide to write or not write the tests. Your academic reports will be assessed to only determine your progression status in Grade 11. Your information will be kept confidential.

I will write a report on the study, but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you don’t want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

The benefits of this study include adding to the existing knowledge on the impact that the use of educational software has on the teaching of Euclidean geometry. It will also give educators more options in their daily selection of teaching methods. Moreover, you as a learner will be exposed to the various ways of handling geometry problems through technology, thereby helping increase your level of understanding and interest in the topic.

There is that risk of anyone contracting COVID-19. However, measures have been devised to avoid exposing you to the coronavirus. There shall be arrangements with your principal to ensure strict adherence to all the safety protocols outlined by the Department of Health and the government during the study. This includes daily screening of learners and teachers, sanitizing, wearing masks, and ensuring social distancing at all times. The researcher shall cooperate with the schools and communicate any possible health risks to the principals and parents/guardians immediately for the necessary action. Your principal and the Grade 11 mathematics teacher shall assist in supervision during the study.

You will not be reimbursed or receive any incentives for your participation in the research.

If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me on the phone number below. Please, do not sign the return slip for consent/assent until you have all your questions answered and understand what I would like you to do. Ask your questions first and ensure that someone answers those questions. If you have understood the information given, proceed to the return slip below.

Researcher: BEDIAKO, A Phone number: ………………………..

Researcher’s Signature: ………………………………….

APPENDIX N: RETURN SLIP FOR CONSENT/ASSENT

I, ………(Participant's Surname, Initials), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

- I have read (or had explained to me) and understood the study as explained in the information sheet.
- I have had sufficient opportunity to ask questions and am prepared to participate in the study.
- I understand that my participation is voluntary and that I am free to withdraw at any time without penalty.
- I understand the existence of COVID-19 and the measures put in place by the researcher to mitigate the spread of the virus.
- I shall subject myself to daily screening, wearing of mask, observe social distancing and follow all other safety measures as outlined by the government and the Department of Health.
- I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.
- I agree to write the pre and post-tests, and allow the recording and analysis of my scores for research purposes only.
- I have received a signed copy of the informed consent/assent agreement.

Participant's Name & Surname (please print) : ____________________________________

Participant's Signature : ____________________ Date : ____/______/2020

Researcher's Name & Surname: ADJEI BEDIAKO

Researcher's signature: ____________________ Date: ____/______/2020

APPENDIX O: EDITORIAL CERTIFICATE

EDITORIAL

CERTIFICATE

Author: Mr Adjei Bediako

Document title: The impact of technology integration in teaching Grade 11 Euclidean geometry based on van Hiele's model

Date issued: 09/10/2021

This document certifies that the above manuscript was proofread and edited by Prof Gift Mheta (PhD, Linguistics).

The document was edited for proper English language, grammar, punctuation, spelling and overall style. The editor endeavoured to ensure that the author's intended meaning was not altered during the review. All amendments were tracked with the Microsoft Word "Track Changes" feature. Therefore, the authors had the option to reject or accept each change individually.

Kind regards

Prof Gift Mheta (Cell: 073 954 8913)