Preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-old children

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Submitted in accordance with the requirements for the degree of

MASTER OF EDUCATION

in the subject

EDUCATIONAL PSYCHOLOGY

at the

UNIVERSITY OF SOUTH AFRICA

NOVEMBER 2021

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I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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Mokoena.

________________________ 16 November 2021

SIGNATURE DATE

DEDICATION

I dedicate this dissertation to my late beloved mother Susan Chakonza. May your soul continue to rest in perfect peace.

ACKNOWLEDGEMENTS

I would firstly like to thank the Lord almighty, for blessing me with abundant strength, perseverance, and willpower throughout.

"*Now to Him who is able to do far more abundantly than all that we ask or think, according to the power at work within us"* Ephesians 3 vs 20.

- A very special thanks to my late mother for always ensuring that I get the best education. You always went out of your way to get me the best schools. You always wanted the best for me, I dedicate this degree to you. Rest in perfect peace.
- I am grateful to my family, especially my husband, who stood by me and kept on supporting me when I felt weary.
- Particular thanks to my father, for teaching me the value of education, reading to me under our house tree back then, my sisters and brother for supporting my dream.
- Many heartfelt thanks to a colleague, turned friend, turned mother, Megan Helsby, for being there with me from the beginning, and most importantly for believing in me. Thank you!
- More gratitude to my supervisors, Dr Donna Hannaway and Dr Veronica Mapitso Kgabo. Your prompt and constructive feedback, coupled with your unwavering support and encouragement kept me going. I will forever be grateful.
- To my colleagues, for believing in me and always cheering me on. You are the best!
- Many thanks to the five preschool teachers who offered their time and opened their classroom doors for me: May God bless you.
- To my editor, many thanks for patiently going through all my work with a fine toothcomb.
- Lastly, to everyone who supported me in any way in this journey, I send my heartfelt gratitude.

ABSTRACT

The purpose of this empirical study was to explore the facilitation of emergent mathematics by preschool teachers of three-to-four-year-old children. Literature shows that there is a correlation between early mathematics exposure and later performance. The study aimed to establish how emergent mathematics was being facilitated from the grassroots level. A qualitative approach within an interpretative paradigm was embraced. Purposeful sampling was employed to select five preschool teachers from the Tembisa residential area. A multiple case-study design was employed and data was generated through semi-structured interviews, observations and document analysis.

Thematic analysis was employed to analyse the collected data. The categories that emerged resulted in the formulation of six themes parallel with the findings of the study, namely facilitation of emergent mathematics activities and instructional practices; teachers’ Pedagogical Content Knowledge (PCK) of mathematics; understanding the curriculum; resources for the facilitation of emergent mathematics activities; and planning and beliefs about emergent mathematics

The study concluded that preschool teachers lack PCK for facilitating emergent mathematics activities, and although positive attitudes towards mathematics were evident, this did not manifest in practice.

Based on these findings, recommendations were made at government, institutional and individual levels. Firstly, it was recommended that the Department of Basic Education should create awareness about the existence of the National Curriculum Framework and the implementation thereof. Secondly, teacher training institutions should provide training that is directly linked to appropriate mathematics content. Lastly, teachers should be provided with opportunities for professional development to enhance their PCK and relevant knowledge and skills to facilitate emergent mathematics.

OPSOMMING

Die doelwit van hierdie empiriese studie was ‘n ondersoek na die fasilitering van ontluikende wiskunde aan drie- tot vierjarige kinders deur voorskoolse onderwysers. Literatuur toon ‘n korrelasie tussen vroeë wiskunde-blootstelling en latere prestasie. Die studie was daarop gemik om te bepaal hoe ontluikende wiskunde vanaf die voedsoolvlak gefasiliteer word. ‘n Kwalitatiewe benadering binne ‘n interpretatiewe paradigma is gevolg. Doelgerigte steekproefneming is gebruik om vyf voorskoolse onderwysers uit die residensiële-area van Tembisa te kies. ‘n Veelvoudige gevallestudie-ontwerp is gebruik en data is deur semi-gestruktureerde onderhoude, waarnemings en dokumentanalises gegenereer.

Tematiese analise is gebruik om die versamelde data te analiseer. Die kategorieë wat ontstaan het, het gelei na die formulering van ses temas parallel tot die bevindings van die studie, naamlik die fasilitering van ontluikende wiskunde-aktiwiteite en instruksionele praktyke; onderwysers se Pedagogiese Inhoudkennis (PIK) van wiskunde; begrip van die kurrikulum; hulpbronne vir die fasilitering van ontluikende wiskunde-aktiwiteite; en beplanning en opvattings oor ontluikende wiskunde.

Die studie se gevolgtrekking was dat PIK om ontluikende wiskunde-aktiwiteite te fasiliteer by voorskoolse onderwysers ontbreek, en alhoewel positiewe houdings teenoor wiskunde sigbaar was, het dit nie in die praktyk gemanifesteer nie.

Gebaseer op hierdie bevindings is aanbevelings op regerings-, institusionele en individuele vlakke gemaak. Eerstens is aanbeveel dat die Departement van Basiese Onderwys bewustheid moet skep oor die bestaan van die Nasionale Kurrikulumraamwerk en die implementering daarvan. Tweedens moet onderwysopleidingsinstellings opleiding voorsien wat direk gekoppel is aan gepaste wiskunde-inhoud. Laastens moet geleenthede vir professionele ontwikkeling aan onderwysers verskaf word om hul PIK en toepaslike kennis en vaardighede te verbeter ten einde ontluikende wiskunde te fasiliteer.

OKUCASHUNIWE

Inhloso yalolu cwaningo olususelwa ekubukeni nasekulinganisweni kwezimo bekuwukuhlola ukulekelelwa kwesifundo sezibalo esisathuthuka ngothisha basenkulisa bezingane ezineminyaka emithathu kuya kwemine ubudala. Imibhalo ikhombisa ukuthi kunokuhlobana phakathi kokuvezwa kwesifundo sezibalo sakuqala nokusebenza kamuva. Ucwaningo kuhloswe ngalo ukuthola ukuthi isifundo sezibalo esisafufusa sasisizwa kanjani kusukela emazingeni aphansi. Indlela yokuqoqa nokuhlaziya imininingwane engeyona eyamanani ngaphakathi kokuphathelene nokuqonda umhlaba njengoba kuvela kokuhlangenwe nakho okuzimele kwabantu abathile kwamukelwa. Isampuli enenjongo yasetshenziswa ukukhetha othisha abahlanu basenkulisa endaweni yokuhlala yaseTembisa. Kwasetshenziswa umklamo wokukhethwa amacala amaningi aboshelwe ukuthuthukisa ukuqonda okujule futhi imininingwane yenziwa ngezingxoxo ezihleleke kancane, ukuhlolwa kanye nokuhlaziywa kwemibhalo.

Indlela yokuhlaziya nokuqoqa imininingwane engeyona eyamanani yasetshenziswa ukuhlaziya imininingwane eqoqiwe. Izigaba ezavela zaholela ekwakhiweni kwezindikimba eziyisithupha ezihambisana nokutholwe ocwaningweni, okungukuthi ukulekelelwa kwemisebenzi yesifundo sezibalo esisathuthuka kanye nezindlela zokufundisa; uhlobo lolwazi olwenza othisha besifundo sezibalo babe ngothisha bezibalo kunokuba ngosozibalo (PCK); ukuqonda izifundo nokuqukethwe kwezemfundo okufundiswa esikoleni; izinsiza kusebenza zokwenza lula imisebenzi esanda kuvela yesifundo sezibalo; nokuhlela nezinkolelo mayelana nesifundo sezibalo esisathuthuka.

Ucwaningo luphethe ngokuthi othisha basenkulisa baswele i-PCK yokwenza lula imisebenzi esisathuthuka yesifundo sezibalo, futhi yize izimo zengqondo ezinhle ngesifundo sezibalo zazibonakala, lokhu akuzange kubonakale ngokwenza.

Ngokuya ngalokhu okutholakele, kwenziwa iziphakamiso emazingeni kahulumeni, ezikhungweni kanye nakumuntu ngamunye. Okokuqala, kuphakanyiswe ukuthi uMnyango Wezemfundo Eyisisekelo kumele uqwashise ngobukhona boHlaka lukaZwelonke Lwezifundo nokuqukethwe kwezemfundo okufundiswa esikoleni kanye nokusetshenziswa kwalo. Okwesibili, izikhungo zokuqeqesha othisha kufanele zinikeze ukuqeqeshwa okuxhumene ngqo nokuqukethwe okufanele kwesifundo

sezibalo. Okokugcina, othisha kufanele banikezwe amathuba okuthuthuka kobungcweti ukuze bathuthukise i-PCK yabo kanye nolwazi olufanele namakhono okwenza lula isifundo sezibalo esisathuthuka.

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LIST OF ABBREVIATIONS

CAPS	Curriculum Assessment Policy
CK	Content Knowledge
DA	Document Analysis
ECA	Early Childhood Australia
ECCE	Early Childhood Care and Education
ECD	Early Childhood Development
ECEC	Early childhood education and care
EDTPSETA	Education, Training and Development Practices Sector Education and Training Authority
ELDA	Early Learning Developmental Areas
ELDAS	Early Learning Development Assessment Standards
FEEL	Fostering effective early learning
FP	Foundation phase
HEIs	Higher Education Institutions
KZN	KwaZulu-Natal
NAEYC	National Association of Educators of Young Children
NCF	National Curriculum Framework
NEEDU	National Education Evaluation and Development Unit
NELDS	National Early Learning Development Standards
NGO	Non-governmental organisation
NPO	Non-profit organisation
NQF	National Qualification Framework
PCK	Pedagogical Content Knowledge
PIECCE	Project for Inclusive Early Childhood Care and Education
PIK	Pedagogiese Inhoudkennis
SANCP	South African Numeracy Chair Project
TVET	Technical and Vocational Education and Training
ZPD	Zone of proximal development

CHAPTER 1: INTRODUCTION AND BACKGROUND

"*Teachers make little ones' count":* anonymous

1.1 INTRODUCTION

"I need to experience an environment where I can find patterns, make connections, recognise relationships, work with numbers, sort out objects, match and classify things. This helps me to think, solve problems and ask questions" (Department of Basic Education, 2015:51). This is but one of many voices of young children, speaking to their teachers about wanting an environment where they can learn mathematics playfully. Regrettably, for most children around the world, there are limited chances to acquire and experience emergent mathematics activities and develop skills in an adequate manner (Aunio, Mononen, Ragpot and Tormanen, 2016).

Facilitating emergent mathematics activities, although not a new concept, remains a critical factor in preparing preschool children for advanced operations in higher grades. According to McCray and Chen (2012), mathematical learning opportunities which are thought-provoking, age-appropriate and adaptive to the diverse needs of young children are needed. McCray and Chen (2012) added that three-to-four-year-old children require assistance to notice mathematical interactions in everyday life and to start representing information about patterns, shapes, space and numbers. It is from this line of thinking that Wager and Parks (2016) iterated that the task of the preschool teacher is then to extend children's encounters with mathematics, in addition to organising a rich environment that offers prospects to explore new as well as familiar ideas.

Emergent mathematics forms an integral foundation for young children and conceptual development in preparation for later learning. According to English and Mulligan (2013), there is an increasing acknowledgement in policy and research that emergent mathematics is highly critical for children's development. Chin and Zakaria (2015) added that comprehension of simple mathematics ideas is critical to prepare young children for more complex mathematics operations. This study aims to investigate preschool teachers' facilitation of emergent mathematics activities, since early understanding of mathematics concepts is important for future mathematics learning.

1.2 BACKGROUND

Most would agree that mathematics education in South Africa is worrying (Spaull & Kotze, 2015). Several findings have acknowledged that South African mathematics' teachers do not have subject or instructional knowledge. This has resulted in children having learning deficits during the early preschool years, which have led to their not performing optimally when they reach higher grades (Graven, 2016; Taylor & Taylor, 2013).

Numerous studies have been done in South Africa to get to the root cause of the learner underperformance in mathematics (Bolowana, 2014; Hugo, Jack, Wedekind & Wilson, 2010), NEEDU, 2013, Venkat & Spaull, 2015). These findings endorse that one of the supporting causes to inadequate learner performance is the teachers' insufficient content knowledge. Preschool teachers are expected to prepare the young children for higher-order operations in mathematics, however, they can only do that effectively if they have thorough subject knowledge of emergent mathematics, knowledge of the curriculum and adequate skills to facilitate emergent mathematics acquisition.

According to global studies, South Africa has always had the worst achievements in mathematics hence the government has accepted that there is a crisis requiring urgent intervention (Spaull, 2013). Spaull (2013) added that the poor quality of our teachers, especially in the formative years, has led to poor quality in mathematics results. Green (2011) emphasised that teachers lack the mathematical knowledge required to facilitate mathematics activities. This means preschool teachers should be in possession of the mathematical content knowledge to facilitate emergent mathematics activities effectively, and must have adequate skills to plan and implement activities that will prepare young children to be cognitively ready for primary school mathematics and improve their achievements.

South Africa is not the only country with a problem with mathematics education and teacher expertise in the subject. In the field of mathematics, America has its share of issues. According to a statement released by the National Association of Educators of Young Children (NAEYC, 2010), if success in improving Americans' mathematics skills is to continue, significantly more attention must be paid to early mathematics experiences. Only 66% of children in Washington State, 46% of children from lowincome households, and 43% of Hispanic children are ready for kindergarten math, according to Harris and Petersen (2017). Considering this, Ginsburg, Hyson and Wood (2014) explained that mathematics is now a top priority in the U.S. education system. Ginsberg, Hyson and Wood (2014) further mentioned that in the past, mathematics education has been implemented too little and too late, not beginning in earnest until children were in the elementary grades, and often with ineffective curriculum and teaching methods.

It is this trend in poor mathematics performance and facilitation globally and locally which motivated this study. Because studies have established a link between early mathematical exposure and later performance, the goal of this study was to determine how mathematics is supported from the ground up (Watts, Duncan, Siegler & Davis-Klean, 2014). For this purpose, this study explored the facilitation of emergent mathematics by preschool teachers, who are teaching children in the age groups of three to four years old.

1.3 RATIONALE OF THE STUDY

My role as a trainer of preschool teachers for 8 years gave me insight into the training curriculum and the course content of Early Childhood Development (ECD Level 4 SAQA ID 58761). Knowing the preschool teacher training course content made me question how preschool teachers implemented and facilitated the content knowledge (CK) in the classroom, especially mathematics CK since the teacher training focuses mainly on pedagogical knowledge. Furthermore, the lack of a technical curriculum with day-by-day instructions for preschool teachers to serve as a guideline made me want to investigate what and how they would facilitate the mathematics CK after completion of their studies.

Emergent mathematics skills are critical for children’s mathematics learning, especially in the early years when emergent mathematics competency is required for mathematical learning in higher grades (Aunio et al., 2016). My interest was further sparked by the observation that children enter primary school without having acquired the right mathematical foundation during their years at a preschool. According to researchers, children’s early mathematics achievement serves as a foundation for future academic success (Watts, Duncan, Clements & Sarama, 2017). This compelled me to investigate how emergent mathematics activities are implemented during preschool years, to prepare young children for higher mathematics conceptualisation. It is posited that when preschool teachers facilitate activities that enhance mathematics conceptualisation at an early age, the overall mathematics results in South Africa would improve.

1.4 STATEMENT OF THE PROBLEM

This study sought to explore the preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-olds.

1.5 THE RESEARCH QUESTIONS

The primary research question was:

How do preschool teachers facilitate emergent mathematics activities with three-to-four -year-old children?

The secondary research questions were:

- In what ways do preschool teachers facilitate emergent mathematics activities in early childhood?
- What knowledge and skills are needed to facilitate emergent mathematics activities?
- What curriculum guidelines are used in emergent mathematics and how are they implemented in the facilitation of emergent mathematics in early childhood?
- How do preschool teachers explain the importance of emergent mathematics?

1.6 THE AIM AND OBJECTIVES OF THE STUDY

The study aimed to investigate how preschool teachers of three-to-four-year-olds facilitate and implement emergent mathematical activities in the early years of development; and further to:

- Explore the ways in which preschool teachers facilitate emergent mathematics activities in early childhood.
- Investigate the knowledge and skills needed by preschool teachers to facilitate emergent mathematics activities.

- Investigate what curriculum guidelines are used in emergent mathematics and how they are implemented in the facilitation of emergent mathematics activities.
- Explore and find out how preschool teachers perceive and explain the importance of emergent mathematics.

1.7 THEORETICAL FRAMEWORK

The study reviewed several aspects of the constructivist theoretical perspective derived from the Jean Piaget's theory of cognitive development (1896−1980) and Lev Vygotsky's sociocultural theory (1962), as well as Shulman's theory of pedagogical content knowledge (PCK) as these are relevant to my study. The constructivist theory explains how children come to know what they know, in other words, how children learn mathematics concepts. Shulman emphasised the teachers' role in facilitating mathematics activities. These theories are discussed below and in detail in Chapter 2.

1.7.1 Pedagogical Content Knowledge (PCK)

Shulman (1986, 1987) presented a new method of thinking about teacher CK, claiming that teachers' knowledge base consists of at least these seven types, namely, content knowledge, pedagogical knowledge, curricular knowledge, learner knowledge, context knowledge, educational aims, purposes, and values; and PCK. Empirical research on teachers' expertise, especially in the disciplines of science and mathematics, have given rise to this concept. PCK has sparked the most interest among these categories, since it symbolises the kind of teaching knowledge that distinguishes teachers from content experts. Therefore, in the field of education, teachers' subject-matter expertise, or CK, and their PCK, have received special attention (Kleickmann, Richter, Kunter, Elsner, Besser, Krauss & Baumert, 2013).

In the domain of mathematics, CK and PCK have been shown to affect teachers' instructional practice as well as children's learning (Baumart, Kunter, Blum, Brummer, Voss, Jordan and Tsai, 2010). This means that preschool teachers should know how and what to teach the children. The 'how' is the pedagogy which is attained through professional development and training and the 'what' is the content that is provided by the state, in the form of curriculum documents. This means that preschool teachers should have a thorough subject knowledge of mathematics, accompanied by theoretical knowledge of child development knowledge and the curriculum.

Findings from a study by Baumert, Kunter, Blum, Bummer, Voss, Jordan, Klusman, Krauss, Neubrand and Tsai (2010) showed that teachers' CK of mathematics remains passive in the classroom unless it is accompanied by a rich repertoire of mathematical knowledge and skills related directly to the curriculum, instruction and student learning. Thus, having knowledge of general mathematics on its own is not enough, but what is important is knowledge of mathematics which is directly linked to the curriculum; in other words, subject CK is of utmost importance.

Similar to Shulman, Grossman (1990) proposed a model for PCK that involves the transformation of other knowledge domains such as subject-matter knowledge, general pedagogical knowledge, and knowledge of context. According to Grossman's (1990) ideas, teachers' PCK includes four dimensions, namely, conceptions for purposes of teaching subject matter, knowledge of students' understandings, curricular knowledge and knowledge of instructional strategies. Conceptions for purposes of teaching subject matter are hierarchically more important than other PCK components. This study will align with conceptualisations of PCK to provide a lens through which to analyse the data.

1.7.2 Piaget's Theory of Cognitive Development

Piaget brought attention to his theory on the development of the individual learner who constructs knowledge through interaction with the concrete environment (cognitive constructivism), according to Mulaudzi (2016). According to Gordon and Browne (2014) and Naude and Meier (2014), Piaget's theory suggests that when learners manipulate physical objects in the classroom, they construct their own knowledge (e.g. building puzzles), to learn the relationships between the concepts of shapes and colour when connecting the different puzzle pieces. As they construct knowledge, they develop an understanding of mathematical concepts.

Mensah and Somuah (2014) posited that the teacher's function in the constructivist approach is to act as a facilitator who offers an appropriate learning environment and helps the children to formulate mathematical ideas, discover concepts, and think independently, rather than to transfer knowledge. Piaget's theory also suggests, according to Naude and Meier (2014), that each learner creates his or her knowledge about a mathematical idea by actively engaging with the environment and manipulating physical things to make sense of how the concept works. Piaget's theory is valuable for preschool teachers because it informs them of the fact that children construct their knowledge of mathematics by manipulating concrete objects and interacting with the environment. This understanding guides them to plan an environment that allows children to learn mathematics by manipulating concrete objects.

1.7.3 Lev Vygotsky’s Sociocultural Theory on Learning Mathematics

Vygotsky believed that an individual’s development is heavily dependent on their social and cultural context. To understand a child’s cognitive development, Vygotsky stated that we must analyse the social practices from which a child’s thought processes are produced (Papalia & Feldman, 2011). Vygotsky (1978) stated that cognitive growth occurs first on a social level, followed by individual growth. He saw learning as a continuous progression from one’s current intellectual level to a higher level that more nearly resembles the learner’s potential. As a result of social engagement, this migration happens in the zone of proximal development (ZPD).

The ZPD is defined by Vygotsky (1978:86) as “the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined by problem-solving under adult supervision or in collaboration with more able peers”. Woolfolk (2010) also described it as a dynamically evolving space in which children and teachers connect and exchange knowledges. Lastly, the ZPD was also defined by Papalia and Feldman (2011) as the gap between what children are already capable of and what they are not yet ready to accomplish on their own. With the right instruction, children undertaking activities in this zone can almost, but not quite, complete the activity on their own. Success is possible, and learning responsibility eventually moves from the adult to the child.

Papalia and Feldman (2011) further attested to how Vygotsky stressed that it is critical for children to explore the environment which in turn leads to enhanced thought processes. This implies that preschool teachers should provide support and help for the learners, then slowly decrease the support, so that children can learn independently. This progression further spells out the bigger role preschool teachers have in the development of emergent mathematical skills in children; hence, they should guide children towards understanding concepts by asking probing questions that will assist them to think logically, and lead them to form mathematical ideas and concepts (Naude & Meier, 2014). In my view, interaction is crucial in the learning and teaching process. Preschool teachers should interact with learners, providing support and guidance when facilitating emergent mathematics activities to enhance conceptualisation.

1.8 LITERATURE REVIEW

This study aimed to investigate preschool teachers' facilitation of emergent mathematics. Sources that were relevant to the topic were reviewed. These included journals, research reports and scholarly books.

1.8.1 Facilitating Emergent Mathematical Activities

The teacher's role in facilitating emergent mathematics and enhancing children's conceptualisation with the area is important. According to Cohrssen, Church, Ishimine and Tayler (2013), teachers need to have sound PCK and clear learning outcomes to support and extend children's emerging understanding of mathematical concepts.

Increasingly, there is evidence of ways to meaningfully engross young children in learning and developing mathematics concepts. The success of such efforts relies on the pedagogical approach, the quality of teaching and the connection of the instruction to young children's curiosity as posited by Snow and Pizzolongo (2014).

In view of this, this study explored preschool teachers' use of play and integrating emergent mathematics activities into the curriculum as a means of facilitating mathematical development for young children.

1.8.2 A Play-Based Mathematical Approach

The National Early Learning Development Standards (NELDS) is a curriculum-related policy initiative focusing primarily on the learning needs of children from birth to four years (Department of Basic Education (DBE), 2009). This was a curriculum initiative by the DBE to address the gap at policy level as well as the developmental and learning needs of children from birth to four years. This was more recently replaced by the National Curriculum Framework (NCF), and the Early Learning Development Assessment Standards (ELDAS) embedded in the NCF (DBE, 2015b).

This South African NCF for Children from Birth to Four serves as a curriculum guideline for preschool teachers and has an array of activities for preschool children that can stimulate cognitive thinking and development of emergent mathematical skills in a meaningful and enjoyable way through a play-based and integrated curriculum. The NCF has set six ELDAS, namely, "well-being, identity and belonging, communication, exploring mathematics, creativity and knowledge and understanding of the world" (DBE, 2015b:8).

Exploring mathematics comprises the following content areas:

- Children show awareness of and are responsive to number and counting;
- Children sort, classify, make comparisons, and solve problems; and
- Children explore shape, space, and measurement.

For preschool teachers to be able to facilitate meaningful mathematical activities for young children, they should familiarise themselves with the mathematical content that children will learn as well as the curriculum requirements. Mathematics is an integral part of subjects across the curriculum. Bennison (2015) insists that if all teachers could recognise and use the mathematical learning opportunities that exist in the subjects they teach, then children's mathematics competences along with learning in each subject would improve.

Mathematical concepts are discovered in the classroom during playful activities. Naude and Meier (2014) were of the opinion that learning in a play-based classroom implies that young children are exposed to mathematics opportunities on a regular basis during the day and in the classroom. Preschool teachers should create environments that allow for emergent mathematics learning to take place playfully. Play should be used as a way of facilitating emergent mathematics development.

Preschool teachers have the task of creating a playful environment that enables children to play and discover mathematics concepts. A related question raised by Clements, Sarama, Swaminathan, Weber and Trawick-Smith (2018) is whether preschool teachers can or should take steps to facilitate children's mathematics learning as they play. Different schools of thought hold that teachers have an active role in facilitating emergent mathematics conceptualisation during play. Weisberg, Hirsh-Pasek, Golinkoff, Kittredge and Klahr (2016) suggested that guided free play, in which the teacher questions, gives hints, or poses problems, is more likely to contribute to academic outcomes in classrooms than autonomous free play alone.

Furthermore, in a play-based classroom, teachers support play through guided instruction, materials, and peer interactions. Kotsopoulos, Makosz, Zambrzycka and McCathy (2015) suggested that preschool teachers should create centres with specific learning objectives that allow children to play in multiple ways based on their motivation and selection. This facilitates learning through educational play with direct instruction. Similarly, Cohrssen et al. (2013) added that children have a higher chance of succeeding if they grasp basic mathematical concepts before entering formal education, and these skills can be developed in play-based activities, where teachers actively engage in mathematical talk. In view of this, preschool teachers should create playful mathematical activities and interact with the children during the play activities to facilitate mathematical conceptualisation by probing, asking questions and guiding the children.

1.8.3 An Integrated Approach to Facilitate Emergent Mathematics

Free play alone is not sufficient to promote young children’s mathematical development (Ginsburg & Ertle, 2008; Van Oers, 2010). Mathematics concepts need to be integrated throughout the daily programme. Bjorklund and Ahlskog-Bjorkman (2017) stated that in early childhood education, theme work that integrates several knowledge areas is seen to be appropriate for developing young children’s knowledge and skills. This means preschool teachers can facilitate emergent mathematical development by integrating mathematical concepts in all learning areas and throughout the daily programme.

Baroody, Clements and Sarama (2019) reiterated that a purposefully and thoroughly executed academic approach is congruent with developing emergent mathematical skills with other learning areas. Baroody et al (2019) added that emergent mathematics concepts can be integrated in music and art activities. Similarly, Greenberg (2012) also believed that knowledge of emergent mathematical concepts can lead teachers to being more creative and intentional about facilitating these ideas daily in the classroom.

Preschool teachers should carefully plan their activities to integrate mathematics activities during the daily programme. For example, register time is an important routine activity for mathematics, as mentioned by Naude and Meier (2014). During register time, children in the class can count whether all group members are present. This implies that preschool teachers can integrate and facilitate mathematical development throughout the daily programme and within all learning areas to enhance conceptualisation.

1.9 DEFINITION OF KEY TERMS

1.9.1 Facilitating Emergent Mathematics

Facilitating, according to the Oxford Dictionary Online (2015) means; “to make an action or process easy or easier”. Davin and van Staden (2004) defined emergent mathematics as active hands-on experiences founded in concrete experiences that children have daily.as. In this study, facilitating emergent mathematics refers to the way preschool teachers present, enable and make it easier for young children to develop emergent mathematics concepts through concrete and daily life experiences.

1.9.2 Preschool Teacher

A preschool teacher in the South African context is sometimes referred to as an Early Childhood Practitioner (UNICEF, 2006 cited in Smit, 2015:6). According to the Oxford Learner’s Dictionaries (2021), preschool is defined as “a school for children between the ages of about two and five” relating to the time before a child is old enough to go to formal school. The word teacher is defined as a person who teaches especially in a school (Smit, 2015). In this study, the term preschool teacher means a teacher with at least an ECD NQF Level 4 qualification working with children from three to four years of age in a preschool.

1.9.3 Pedagogical Content Knowledge (PCK)

Pedagogical content knowledge is a notion that has been commonly used in teacher knowledge literature to represent teachers’ professional knowledge (Fernandez 2014). PCK, on the other hand, is defined by Newsome, Taylor, Carlson, Gardner, Wilson, and Stuhlsatz (2019) as the way subject matter is changed for teaching, which occurs when educators interpret the subject matter and find different ways to represent and make it accessible to children based on the context of the material, learning theme, or field of study. In this study, PCK is used to refer to preschool teachers' knowledge of how to facilitate emergent mathematical activities for children between the ages of three and four years of age, including teaching methods and classroom management.

1.9.4 Mathematics Content Knowledge

According to Shulman (1986), knowing a subject requires more than knowing its phenomena and concepts. Knowing mathematics includes mathematical definitions, concepts, algorithms and procedures. According to Van de Walle, Karp and Bay-Williams (2013), mathematics CK is concerned with what the children know about the concepts, rules and procedures of doing mathematics. In this study, mathematics CK refers to the preschool teachers' knowledge of emergent mathematics content areas as specified by the NCF (DBE, 2015a).

1.10 RESEARCH METHODOLOGY

Almalki (2016) described research methodology as the approaches or methods that researchers use in implementing any form of study or investigation as they base their work on the description, explanation and prediction of phenomena. Similarly, Mafuwane (2011) added that the research methodology explains how an inquiry or investigation is undertaken. This section describes and justifies the methods used to conduct the study. The design, method, data collection and analysis are briefly outlined below. They are covered in more depth in Chapter 3.

1.10.1 Research Design

The research design is defined by Seabi (2016:81) as "a plan of how one intends to accomplish a particular task, and in research this plan provides a structure that informs the researchers as to which theories, methods and instruments the study will be based on". This study was located within the interpretive paradigm and followed a qualitative approach using a multiple case-study design.

1.10.2 Research Approach

Denzin and Lincoln (2011) described an interpretivist paradigm as one that endeavours to comprehend a phenomenon being studied by scrutinising the meanings generated by the participants in the study. In addition, Maree (2014) stated that the interpretivist paradigm is based on naturalistic data collection methods such as interviews and observations which are the methods used by the researcher to gather data. I interpreted the information collected from the preschool teachers by means of semi-structured interviews, observations and document analysis.

I employed a qualitative research approach to understand the facilitation of emergent mathematics activities by preschool teachers of three-to-four-year-old children. Manwa (2014) described a qualitative design as a method which attempts to understand and interpret what exists at present in the form of conditions, practices, trend effects, attitudes and beliefs as they are perceived by the actions. In addition, Johnson and Christensen (2020) stated that qualitative research relies primarily on non-numerical data. The qualitative method was best suited for this study as the I wanted to understand and interpret the facilitation of emergent mathematics activities with three-to-four-year-old children and also because it allowed for and presented a body of in-depth, rich and descriptive data about the experiences and views of preschool teachers regarding the facilitation of emergent mathematics (Creswell, 2012).

This study was also embedded in a multiple case-study research design. According to Yin (2014:16), “a case study is an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life contexts”. McMillan and Schumacher (2014) added that with a multiple case study, more than one example or setting is used. A multiple case-study design was chosen to form a “detailed, holistic and contextualised account of the dynamics underlying the group processes” (Maree, 2010:75). Making use of multiple case-study research helped put together a good description and a clear picture of how different preschool teachers facilitate emergent mathematics activities to enhance conceptualisation.

1.10.3 Population and Sampling

1.10.3.1 Population

Population refers to a group of individuals from which a sample is drawn and to which results can be generalised (McMillan & Schumacher, 2014). In this study, the population comprised of preschool teachers in Tembisa, a disadvantaged township in Gauteng Province.

1.10.3.2 The sample

A purposeful sampling method was used in this study to identify participants who could provide valuable and relevant data for the topic under study. Creswell, Ebersohn, Eloff, Ferreira, Ivankova, Jansen, Nieuwenhuis, Pietersen, Plano Clark and Van der Westhuizen (2013) stated that purposive sampling means that participants are selected because of some defining characteristics that make them the holders of the data needed for the study. In addition, McMillan and Schumacher (2014) added that purposeful sampling is a type of sampling that allows the selection of small groups or individuals who are likely to be knowledgeable and informative about the topic under research.

This study employed purposeful sampling to select five preschool teachers from four different preschools in Tembisa, who were teaching three-to-four-year-olds, according to “specific criterion that allowed the researcher to obtain an understanding of the research problem” (Strydom & Delport, 2011:392). Purposeful sampling was employed in this study because participants were intentionally selected to provide relevant and meaningful information to the study. The sample was selected based on the following criteria:

- The preschool teacher should have an accredited ECD qualification from an accredited institution.
- The preschool teacher should have at least a one year’s teaching experience with three-to-four-year-olds.
- The preschool should be situated in the township of Tembisa.

1.10.4 Data Collection Methods

Data was collected from the participants using observations, semi-structured interviews and document analysis. These methods are briefly outlined below.

1.10.4.1 Observations

I used observations as a data collection method to account for the facilitation of emergent mathematics activities by preschool teachers. McMillan and Schumacher (2014) defined observation as a way for the researcher to see and hear what occurs naturally at the research site. Maree (2016) noted that observations allow the researcher to gain an insider's view of the group dynamics and behaviours of the participants. Kumar (2011) recommended the use of observations especially where full and accurate information cannot be obtained by questioning because participants are either uncooperative or do not really understand the questions. This method was employed because it made it possible to watch preschool teachers as they facilitated emergent mathematics activities. As a result, the classroom observations enabled me to closely document the facilitation of emergent mathematics activities of all five participants without interfering in the classroom activities.

1.10.4.2 Semi-structured interviews

According to Nieuwenhuis (2016a), a qualitative interview occurs when the researcher asks one or more participants certain open questions, these are then followed by further probing and clarification, then the answers are recorded. In the same vein, Maree (2016) stated that the aim of an interview is to see the world through the eyes of the participant, which becomes invaluable source of information if acquired appropriately. McMillan and Schumacher (2014) highlight that in a structured interview, topics are selected in advance, but the researcher decides the sequence and wording of the questions during the interview. I applied the semi-structured interview approach to ask the preschool teachers questions and where needed, I probed and requested further clarification to my questions regarding the facilitation of emergent mathematics.

I employed the use of semi-structured interviews in this study to obtain comprehensive and comparable data on how preschool teachers facilitated emergent mathematics activities. I was motivated by Yin (2013) who stated that interviews are one of the most important sources of evidence in a case study.

1.10.4.3 Document analysis

Rule and John (2011:67) stated that "document analysis is a useful place to start data collection in a case study, particularly if the research design includes other methods, such as interviews and/or observations". Yin (2014) advised that researchers should engage with documents as a data source because they provide scientific and valuable information from which considerable detail can be elicited. Like other analytical methods in qualitative research, document analysis requires that data be examined and interpreted to elicit meaning, gain understanding and develop empirical knowledge (Corbin & Strauss, 2008). In this research, document analysis was used as a complementary procedure and for triangulation purposes. The advantage of this method is that documents are easily accessible and it is less time-consuming to gather the data.

I analysed the curriculum guidelines for children birth to four years, namely, the NCF (DBE, 2015a), as well as the participants' planning documents including lesson plans and year plans and the children's workbooks. The policy document was easy to access on the internet and the preschool teachers provided their lesson plans. The document analysis enabled me to gain insight into the theoretical and practical framework of emergent mathematics activities for young children. It enabled me to have knowledge of what the policy document stated and assisted me in comparing what it says and what was really happening on the ground, regarding the facilitation and implementation of emergent mathematics activities.

1.10.5 Data Collection Instruments

Research instruments are the measurement tools used by researchers for the data collection on topics of interest from the participants (Bastos, Duquia, Gonzalez-Chica, Messa & Bonamingo, 2014). I employed the use of observation checklists, journal keeping (field notes) interview guides and document analysis checklists to collect the data.

1.10.2.1 Field notes

Schwandt (2007) describes field notes as evidence to gain meaning and an understanding of the culture, social situation, or phenomenon being studied. In this study, I kept a research diary as part of the data collection process. Gambold (2010) claimed that field notes can be used to describe a physical space, the mannerisms of people and the duration of events. I made use of field notes to record the preschool teachers' attitudes, behaviour, understanding and facilitation of teaching emergent mathematics.

I also designed and employed an observation checklist for recording data collected from general observations of the classroom layout, resources, classroom management as well as activities facilitated. The purpose of this instrument was to make the observations more focused and structured.

1.10.2.2 Semi-structured interviews

Semi-structured interview guides were used for this study. The interview guides consisted of predetermined questions to ask participants during the interviews. Through semi-structured interviews, one can examine attitudes, interests, feelings, concerns, and values that may not be obvious through observations (Gay, Mills & Airasian, 2011:388). The five interviews with the preschool teachers lasted between 30 to 45 minutes each, and each one was recorded.

1.10.2.3 Document analysis

According to Petty, Thompson and Stew (2012), documents include written work, drawings and pictures. Document analysis formed part of this study and served to triangulate the evidence from different sources, according to Steyn (2014). The NCF (DBE, 2015a) was the primary document consulted in the study. Document analysis was carried out to establish and analyse the curriculum materials used by the teachers.

1.11 DATA ANALYSIS

The study adopted a thematic analysis strategy to analyse the collected data. Qualitative data analysis is primarily an inductive process of organising data into categories and identifying patterns and relationships among categories, according to McMillan and Schumacher (2014). Similarly, Maree (2014) noted that researchers within the interpretive paradigm prefer an inductive data analysis strategy due to the potential of the interpretive approach to assist in identifying multiple realities that might be present in the data. Lastly, Walker, Cooke and McAlister (2008) stated that data analysis is a creative and logical process of gathering and arranging data into themes and subthemes so that the analytical scheme becomes obvious. Immediately after data collection, I organised the data into categories to identify patterns and relationships from the data collected.

1.12 ETHICAL CONSIDERATIONS

Stangor (2014) noted that the concern about research participants is only one aspect of ethics in research. Strydom (2011) emphasised that it is important that the researcher follows and abides by ethical considerations throughout the research process. Ethical guidelines serve as a standard and a basis upon which researchers should evaluate their conduct during their research process. Standards should be internalised into the personality of researchers (de Vos et al., 2011).

Ethical measure that were taken in this research included:

- The preschool principals were asked in writing for permission to conduct research at their respective preschools.
- Consent letters were given to the participants, asking their consent to participate in the study and informed them about the procedures and purpose of the study. Similarly, consent letters were given to the participants before the research process began.
- Participants were informed of their right to withdraw from the research process at any time, should they wish to do so. The procedures that the researcher would follow were clearly communicated. Participants also assured that their privacy will be respected, and confidentiality will be ensured.
- The participating teachers were given pseudonyms to ensure their anonymity.

1.13 CHAPTER OUTLINE

Chapter 1: Introduction and background

This chapter gives the contextual background and sets the scene for the whole research study, with specific reference to the rationale, problem statement, aims and objectives of the study. Primary and secondary research questions are highlighted. The theoretical framework underpinning this study and the literature review are introduced. Lastly, the research methodology is presented as well as the ethical issues guiding this study.

Chapter 2: Theoretical framework and Literature review

This chapter explores the literature surrounding preschool teachers' facilitation of emergent mathematics activities with 3-4-year-old children. The literature helped me to gain insight into my phenomenon based on previous scholars' voices and views on the facilitation of emergent mathematics activities. I also present the constructivist theory from the lenses of Piaget's cognitive theory and Lev Vygotsky's sociocultural theory. Lastly, the PCK theory from the lenses of Shulman and Grossman is also presented and discussed.

Chapter 3: Research design and methodology

This chapter outlines the qualitative research methods for gathering the data. These methods comprise semi-structured interviews, observations and document analysis. Discussions about the choice of certain research methods over others to answer the research questions of the study are presented. Lastly, ethical considerations guiding this study are established and presented.

Chapter 4: Findings and discussion of findings

In this chapter, the results of the qualitative study are presented, analysed and interpreted. The data from the case studies is analysed and presented according to codes and categories. The themes and sub themes are generated from the categorised data.

Chapter 5: A summary, limitations, contributions, conclusions and recommendations

This chapter presented a comparison between the empirical results of this study to relevant concepts as well as theories from the literature review. The research questions are addressed and answered. Findings, recommendations, ideas for further research and final thoughts are given.

1.14 CHAPTER SUMMARY

This chapter explored the background of the study. A problem statement was highlighted, and research questions were articulated. The aims and objectives of the study were outlined. A brief theoretical framework and literature review was outlined. Lastly, a brief research methodology was outlined. The next chapter focuses on the theoretical outline and a review of the literature.

CHAPTER 2: A REVIEW OF THE LITERATURE ON THE FACILITATION OF EMERGENT MATHEMATICS

2.1 INTRODUCTION

The previous chapter gave an overview of what this study is about, which included the background and introduction to the study, as well as the research question, research methodology and concept clarification. This chapter analyses the different learning and teaching components pertaining to emergent mathematics, the development of emergent mathematics in young children and the facilitation of mathematics in the early years. The knowledge and skills required for facilitating emergent mathematics as prescribed by the National Curriculum Framework (NCF), literature and the curriculum guidelines that inform the facilitation of emergent mathematics are also analysed. Lastly, the influence of preschool teachers' beliefs and attitudes towards the facilitation of mathematics activities will be reviewed.

This chapter also discusses constructivism and PCK as the theoretical frameworks which provide insight into how preschool teachers facilitate emergent mathematical skills. Piaget's theory of cognitive development, Vygotsky's sociocultural theory, Shulman and Grossman's theories of PCK and other related literature that speak to the development and facilitation of emergent mathematics are reviewed.

As a researcher, I acknowledge that there is a huge gap in literature when it comes to the development and facilitation of emergent mathematics for children between three and four years old. This missing element was the reason for my motivation to conduct research on this topic. Most of what is found in literature about mathematics education focuses on Grade R and lower primary grades and less on preschool.

2.2 THE DEVELOPMENT OF EMERGENT MATHEMATICS

Children are born ready to learn, which is inclusive of mathematical concepts (Hyde and Spelke 2011). It is widely agreed that young children can learn mathematical concepts at a very early age. Harris, Petersen and Smither-Wulsin (2017) stated that infants begin to learn mathematics before they can sit up. When they play in other aspects of their everyday lives, they notice disparities in quantity, compare the shape and size of items, and use early mathematical concepts. Similarly, according to Peucker and WeiBhaupt (2013), research on mathematical development which was conducted when children were only a few days old revealed a significant capacity to work with numbers almost from birth. Peucker and Weißhaupt (2013) went on to state that young children can compare quantities, observe changes in quantities, and are accurate in estimating small set sizes and can estimate with increasing accuracy in larger set sizes. This view is supported by Dehaene-Lambertz and Spelke (2015) who stated that in both newborns and adults, similar brain areas respond to numbers, implying that young children are born ready to grasp mathematical ideas.

Naude and Meier (2014) also affirmed that children gain experience in mathematics from the day that they are born through daily experiences in their homes and communities. When they learn to arrange a table at home for family, they learn to count and understand that each person needs a plate, a knife, and a fork; through this, children construct or build their knowledge of mathematics. The implication of this is that preschool teachers should be aware that young children can develop mathematical concepts very early in their lives and should start to facilitate activities that stimulate emergent mathematics development as early as possible.

Young children engage with mathematics to make sense of the world. Bjorklund (2018) defined mathematising as the process of young children using mathematics to make sense of their surroundings. In the same vein, Garvis and Nislev (2017) were of the notion that mathematics should be presented as a social activity and that it should be relevant to the child's everyday life and that of their family. Furthermore, Van Oers (2016) believed that children participating in playful activities will use mathematical understandings and concepts as part of that play. Considering the above, preschool teachers should create opportunities that allow young children to experience mathematics that is relevant to their daily lives and lifeworld.

In South Africa, the informing policy document on children's development and subsequently the development of emergent mathematical skills is the South African National Curriculum Framework for Children from Birth to Four (NCF) which is prescribed by the DBE (2015a). The NCF (DBE, 2015a: 51) refers to emergent mathematics as the exploration of mathematics and defines it as follows:

- "It is about children developing an understanding of how to solve problems, how to reason and how to use mathematical concepts in their environment.

- Children will use their bodies, minds, and senses to explore their world. There must be opportunities set out for them to explore.
- When they do this, they can develop their knowledge, skills, and attitudes.
- They form ideas and test these out. They also learn to refine these ideas as they interact with their peers and adults.
- Mathematical concepts develop as children investigate and communicate their ideas about numbers, counting, shape, space and measures".

The above policy document suggests that children should learn mathematics concepts through exploration and discovery; that the activities facilitated should encourage problem-solving, using mathematical concepts in the environment and should allow for interaction between children and peers. Likewise, Siraj, Mehush, Howard, Hewett-Neilsen, Kingston, Rosnay, Duursma, Feng and Luu (2018) concur that children should engage in suitable, cognitively challenging activities and that the environment should enable practices that encourage relationships with children. This implies that preschool teachers should take into cognisance the fact that they should create the appropriate environments that enable children to solve problems in their environments and facilitate age-appropriate challenging activities as informed by the NCF.

Preschool teachers must be aware of the mathematics content areas so that they can design activities that stimulate the development of all areas. This view is supported by Jang (2013) and Zhang (2015) who agreed that preschool teachers' PCK, content knowledge (CK) and teaching ability are of great importance in ensuring children learn effectively and achieve success. The next section looks at young children's development of emergent mathematics skills regarding the mathematics content areas of number sense, shape and space, measurement, sorting and classifying. These particular mathematics content areas are analysed since they are the ones prescribed for the preschool teachers by the NCF.

2.2.1 Number Sense and Counting.

There is not much literature on how young children of three-to-four years of age develop mathematical concepts, more so the development of number sense. More focus is placed on language development than on emergent mathematics. Clements and Sarama (2016) explained that even well-regarded programmes for young children tend to give more attention to language and social development than to mathematics concepts.

As emerging mathematicians, children need to acquire a deep understanding of numbers. Bosman (2014) maintained that an understanding of number or number sense is the initial building block for all content areas in mathematics. Equally, Clements, Baroody and Sarama (2014) reiterated that basic number and arithmetic concepts and skills are an important foundation for understanding and acquiring higher-level understanding of other aspects of school mathematics.

Young children need to interact with numbers so that they can develop number sense in an enjoyable, playful and flexible manner. There is a suggestion that children need to interact with numbers flexibly and conceptually. Boaler (2015) stated that in a research study conducted with children aged 7 to 13 on the development of number sense, it was found out that there is a crucial difference between the low and high achievers. High-achieving children solved questions by what is known as number sense; i.e., they interacted with the numbers flexibly and conceptually. The low-achieving children used no number sense and seemed to believe their role was to recall and use a standard method. The researchers concluded that the difference between the high- and low-achieving children was not that the low-achieving children knew less mathematics, but that they were interacting with mathematics differently. Instead of approaching mathematics with flexibility and number sense, they seemed to cling to formal procedures they had learned, following them slavishly. This was probably because they had been set on a wrong pathway from an early age of trying to memorise methods and number facts instead of interacting with numbers flexibly. Although the study was conducted on children between ages 7 and 13, it is relevant for this study in that it informs preschool teachers that children should be set on the right pathway at an early age. The children’s environment should be set up in a way that children interact with numbers playfully and in a flexible and enjoyable way so that they do not view working with numbers as a set of rigid rules.

The South African Numeracy Chair Project (SANCP) suggested that the development of number sense begins in preschool where learners develop a feel for numbers and enjoyment for working with them as they count verbally, count objects, add, as well as take away small numbers (Graven, 2016). Furthermore, according to Clements et al.

(2014), in emergent mathematics, the focus initially should be on working with small collections of objects, one to three items, and progressively moving to large collections of objects. They further affirmed that language, including number words and quantitative terms, such as 'more', are critical for the development of verbal-based number concepts and skills. When children are fluent and comfortable in using and manipulating numbers, they demonstrate the ability to work with numbers in various situations, including problem-solving. This provides them with the opportunity to revisit and venture deeper into various mathematical problems and uses in different contexts (Mukwambo, Ngcoza & Ramasike, 2018).

It is generally agreed that children need an environment that will allow the development of number sense in a playful manner. Teachers are therefore tasked with the responsibility to expose young children to situations and tasks that will enforce the development of number sense and flexible thinking.

2.2.2 Shape and Space

Space and shape are content areas of mathematics that young children should know before they enter formal schooling. Space and shape are also referred to as geometry (Olkun & Sari, 2016). The Spatial Reasoning Study Group (2015) were of the view that geometry and spatial thinking are significant in themselves, but they also serve as a necessary conceptual foundation for understanding other topics in mathematics and other subject areas. Despite this, Dagli and Halat (2016) maintained it is neglected in favour of numbers in early childhood. Limited research has been conducted in primary schools in early mathematics learning about the teaching of geometry (Lelinge & Svensson, 2020). Similarly, Clements, Sarama, Swaminathan, Weber and Trawick-Smith (2018) also agreed that geometry tends to be ignored in the early childhood curriculum, and even if embraced, it is not dealt with in ways suggested by research.

The NCF (DBE, 2015a:56) explains that young children should:

- "Show an interest in shapes by using them, talking about them and taking note of similarities and difference.
- Use familiar objects to make patterns. and
- Begin to use mathematical names for solid 3D shapes and flat 2D shapes".

An understanding of shape follows a definite order, starting with physical manipulation of shapes (Piaget & Inhelder, 1967). Van Hiele (1999) studied how children's comprehension of shapes develops through time and established five stages, the first three of which relate to young children. They are listed in the table below:

Table 2.1: The first three Van Hiele levels of spatial understanding

Level 1 (Visualisation and recognition)	Children are able to see that shapes are different, for example, a square and a triangle, but cannot explain why they are different.
Level 2 (Analysis)	Children are able to name the properties that make up shapes.
Level 3 (Abstraction)	Children can classify shapes according to their properties, for example, a square is a type of rectangle.

Wilmont and Schafer (2015) were of the view that if one argues along these developmental lines above, children should therefore have many opportunities to manipulate shapes, talk about them, classify them, learn vocabulary related to shapes, and construct with shapes in order to consolidate the learning that they bring to school and further extend their understanding and knowledge of shapes.

Children also need to develop spatial skills in preschool as it has been found to link to mathematics achievement in later years. Spatial reasoning is all about how we understand the way things move in relation to the space around us. Resnick, Harris, Logan and Lowrie (2020) referred to spatial reasoning as a set of diverse skills that involve mental manipulation of two-dimensional and three-dimensional relations between and within objects. Several studies have found a positive relationship between spatial ability and mathematics performance from early childhood to adulthood (Mix, Cheng, Hambrick, Levine, Young, Ping & Konstantopoulos 2016; Verdine, Golinkoff, Hirsh-Pasek & Newcombe 2017).

It is critical that preschool teachers create opportunities for young children to interact with shapes in their environment to develop a deeper understanding of the concept of space and shape. Piaget (1952) complemented this notion by stating that children's geometric understanding develops with age; and that for children to generate ideas about space and shapes, they need to interact physically with their environment and concrete objects.

In light of the above, preschool teachers should provide 2D and 3D objects and create opportunities for children to explore different objects so that they can describe shapes, talk about similarities and differences in shapes and use mathematical language to describe shapes. They need to provide concrete apparatus that allows children to describe the position of objects in space and relation to others. It is, therefore, important that geometry should be taught effectively so that young children can understand the concepts of space and shape.

2.2.3 Measurement

Apart from developing shape and spatial awareness, young children need to develop the skill of measuring. Young children develop knowledge of the concept of measurement through their daily activities in a playful and exploratory manner. In the same vein, Kotsopolous et al. (2015) agreed that to improve development of the concept of measurement, a free exploration approach in the context of a play-based learning environment is more effective in teaching length measurement than guided instruction and teacher-centred learning. This implies that preschool teachers should plan the environment in a way that children will develop the skill of measurement while playing in a well-planned environment that allows for discovery and exploration.

A study was conducted in Japan in which a mathematical education programme was implemented for five-to-six-year-old children; they were taught how to measure length and area carefully using paper clips as non-standardised units (Nakawa & Matsuo, 2019). They measured assorted items by connecting clips, for instance, measuring the edge of the desk, the mat and the height of the teacher. Results showed that children continued to understand direct comparison in all activities, but it was difficult to establish understanding of measurement by using non-standardised units. Nakawa and Matsuo (2019) added that play-based activities influenced the children’s understanding in both positive and negative ways. This suggests that play-based activities are effective, but should be meaningful, and mathematical activities should not be superficial. Although this study was done with children between five and six years of age, the use of clips as a play-based manipulative as a vehicle to teach the mathematical content area of measurement can also be implemented with children between ages three-to-four years.

The implication of the study above and subsequent suggestions is that preschool teachers should plan and facilitate age-appropriate measuring activities which are hands-on and allow children to experience measurement in fun ways. Preschool teachers can use a variety of non-standardised measuring tools in a playful but meaningful manner with young children, to enhance the development of the mathematical concept of measurement.

2.2.4 Sorting, Classifying, and Making Comparisons.

Classification is a fundamental pre-number learning concept for children to learn about the world around them. Classification is the skill of sorting or grouping items by similar characteristics, such as colours, shape, or sizes. Children naturally classify toys by type and sort objects according to various attributes. Children between the ages of three and four years enjoy sorting and classifying objects usually by one characteristic, colour, shape, or size, (Byington, Kim, Nazarechuk & Weigel, 2016). Classifying and sorting can be done with or without using numbers, such as separating children or objects into distinct groups; for instance, the colour of their t-shirts, or their hair colour. Children need to learn how to sort and classify before they are able to move on to work that involves numbers; because they need to know what they are counting before they are able to actually count them (Reys, Lindquist, Lambdin, Smith, Rogers, Falle, Frid, & Bennett, 2012)

According to Clark (2019), children should be exploring and classifying concepts in preschool before they begin their primary schooling. This means that preschool teachers should expose young learners to informal activities where they encounter sorting activities using everyday objects that are familiar to them. For example, a cluster of animals can be grouped based on their colour or type of animal. You can have young children classify anything, including blocks, leaves, plates, or toy cars. Once they have classified items, children can compare items further to learn more specific similarities and differences between items, both within and between matched groups (Raney, 2015). The skill of classifying forms the basis for data handling at a later stage; so preschool teachers should provide young children with sorting activities using everyday objects in a playful manner.

2.3 SOUTH AFRICAN CURRICULUM GUIDELINES FOR FACILITATING EMERGENT MATHEMATICS

The definition of curriculum is broad and at times seems to be veiled in obscurity. Qassimi and Wade (2021:2) defined the term curriculum as “a summary of the goals and objectives that a school or specific class must achieve by the end of the school year”. On the other hand, Su (2012:85) defined a curriculum as “all the experiences that a child has in school and this consists of all subjects and the lessons the teacher has to offer, it also includes objectives, teaching content, teaching strategies, assessment methods and other components of learning and teaching in classrooms”. Both definitions point out that a curriculum is a guiding policy which informs teachers on the planning and facilitation of activities.

In South Africa, the NCF is the guiding policy document for early childhood education (DBE, 2015). This is the current curriculum guidance in South Africa, for children between the ages of birth to four years and is relevant for this study. The NCF guides those developing programmes and working with babies, toddlers and young children to facilitate high-quality experiences in a variety of programmes and settings such as ECD centres, homes, neighbourhoods and institutions where children in the early years are found, by providing information on child development and activities that enhance development at each stage of growth.

There are six ELDAS in the NCF which are:

1. Well-being
2. Identity and belonging
3. Communication
4. Exploring Mathematics
5. Creativity
6. Knowledge and understanding of the world.

For the sake of this study, focus will be on the fourth ELDA, which is about exploring mathematics. Exploring Mathematics entails children developing an understanding of how to solve problems, how to reason and how to use mathematical concepts in their environment. Mathematical concepts develop as children investigate and communicate their ideas about numbers, counting, shape, space, and measures. (DBE, 2015b). Figure 2.1 below outlines the mathematics content areas within exploring mathematics:

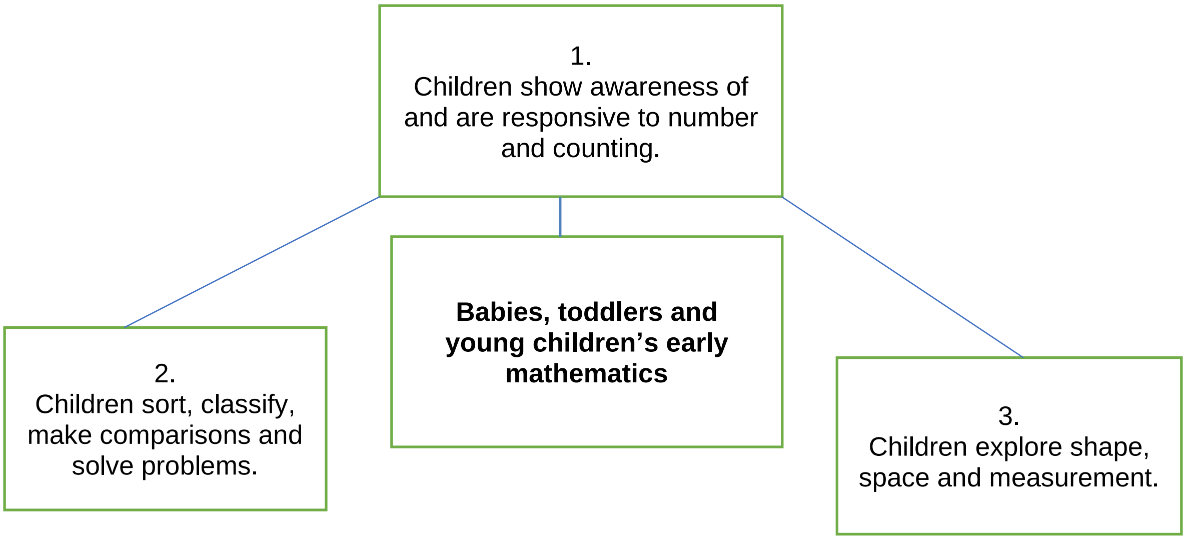

Figure 2.1: Exploring mathematics content areas

Source: (DBE, 2015b:51)

In light of the above, as prescribed by the NCF (DBE, 2015a), preschool teachers should create opportunities in the children's environment that allow for exploration and discovery learning of the concepts of space and shape, measurement, sorting, classifying, problem-solving and number sense.

According to the NCF (DBE, 2015a:52), these are the points to consider when promoting the development of emergent mathematics:

- "What kind of opportunities do the indoor and outdoor environments provide for children to explore, learn and practise their emerging mathematical understandings?
- How do children's own activities, drawings, play, experiments, show their competence in solving problems, thinking logically and making decisions?
- Are there sufficient experiences for children to explore real-life problems, to make patterns, to count, match and measure?

- “How do adults support children who use means of communication other than spoken language to indicate their mathematical ideas?
- “How are early mathematical experiences integrated with other learning opportunities?”

Table 2.1 shows the aims of the developmental area ‘exploring mathematics’ for young children. It also highlights what mathematical concepts young children develop at certain stages of development. Furthermore, it provides guidance to preschool teachers on which activities to facilitate for the children to enhance the development of mathematical concepts.

Table 2.2 shows examples of activities summarised, adapted from the NCF, for preschool teachers to offer while working with babies and toddlers to explore emergent mathematics (DBE, 2015a:52)

Table 2.2: Examples of activities for 3-4-year-olds

Mathematical concepts	Examples of activities for 3- to 4-year-olds
Children show awareness of numbers and counting	• Repeat the counting words children use to show them how counting helps us to find “how many?” • Talk about things that are grouped, for example, toys, clothes, shoes. • Ask children to tell you about the marks and symbols they create on paper and sand. • Use mathematical language as you communicate with toddlers for example, “I would like to give you another toy, then you have two”. • Encourage children to use number language, for example, one, two and so on. • Let children attach number labels to items for example, they can put the number one on an item that belongs to them. • Play counting games and let children count in different situations. • Model and encourage children to ask questions where mathematical language is used for example, how many sweets do you think are in this bottle? • Design activities where children have to find matching items. • Design activities where children have to find items that are different from what they have.
Children sort, classify, and solve problems	• Encourage interactions during activities such as storytelling and rhymes. • Ask questions to solve number problems, for example, Sindisiwe only had 1 shoe. What should she do? • Ask children to talk about how they solved a problem for example, I would like to know how you got up the ladder of the slide. Do you want to teach me how to do it? • Use pictures and objects as stories and rhymes are told. • Encourage children to sort out objects according to their preferences (likes and dislikes), size, shape and colour. Ask them to count on their own. Model counting behaviours to help them. • Use role-play and stories to demonstrate language such as few, less, more than, same as. • Have a vocabulary of around 2000 words. • Show a keen interest in how children are solving problems and answers they come up with.
Children explore shape, space and measurement	• Talk to children about the patterns they see around them for example, patterns on clothing, in nature and on buildings. • Draw children’s attention to the differences in shapes, for example, a biscuit, a pillow. • Allow children to sort out a few objects of different sizes and shapes. • Tell stories to motivate children to use shapes to make their creations. • Encourage children to talk about their interest in shapes. Provide examples of why things are similar or different.

The table contains a summary of the guidelines within the emergent mathematics content areas which preschool teachers should familiarise themselves with to facilitate emergent mathematics activities for young children. The implication of the NCF is that preschool teachers should understand the different stages of emergent mathematical development in young children. With such understanding, teachers will be in a position to design activities that allow children to develop emergent mathematics concepts and also design the environment to allow for exploration and discovery. It implies that children should discover and develop these mathematical concepts in an exploratory or playful kind of way.

2.4 FACILITATING EMERGENT MATHEMATICS ACTIVITIES

For preschool teachers to facilitate emergent mathematics activities effectively, in a manner that enhances mathematics conceptualisation, they should know how children learn best and how emergent mathematics activities can be implemented and incorporated in the daily programme. Various scholars are of the opinion that teachers' importance in any topic, as well as how they portray mathematics, has an impact on children's attitudes about mathematics at all stages of education (Baki & Bektas Baki 2016; Karakuş, Akman & Ergene 2018). This is why it is important to know how mathematics concepts are taught to children in preschool.

Preschool teachers are the chief vehicles that facilitate and enable young learners to grasp mathematical concepts. For them to be able to facilitate emergent mathematics activities and conceptualisation, they need to know the best ways through which young learners learn optimally. The next sections address the different pedagogies that preschool teachers can employ when facilitating emergent mathematics activities.

2.4.1 Facilitating Emergent Mathematics Conceptualisation through a Play-based Learning Approach.

Young children enjoy playing and it is through playing that most of their mathematics concepts are developed. According to Pyle (2018), play is the most developmentally appropriate mode of learning in early childhood, and children learn best through play; it is thus an integral part of early childhood education.

For preschool teachers to understand what it means to facilitate emergent mathematics through a play-based approach, they first need to understand the meaning of play. The definition of play is broad and is an area of debate in research. Play can be defined as activities that "are fun, voluntary, flexible, involve active engagement, have no extrinsic goals, involve active engagement of the child, and often have an element of make-believe" (Weisberg, Hirsh-Pasek & Golinkoff, 2013:105). On the other hand, play-based learning is, essentially, to learn while at play (Danniels & Pyle 2018). The exact definition of play continues to be vague in research, including what activities can be categorised as play (Wallerstedt & Pramling 2014). According to Pyle and Danniels (2017), play-based learning is different from the broader concept of play. Learning is not necessary for an activity to be perceived as play but remains fundamental to the definition of play-based learning.

It is widely agreed that mathematical concepts are developed and mastered through playful experiences. According to Byington et al. (2016), children learn to make predictions, solve problems, think, reason, and establish connections to the world because of their participation in mathematics experiences. Mathematics education for early childhood should contain processes that ensure children's active involvement and learning by doing/experiencing. Similarly, Aksoy (2020) believed that mathematics activities that are limited to paper and pencil, prohibit children from getting real-life experiences and divert them from concrete experiences. Furthermore, Taylor and Harris (2014) added that play can allow young children autonomy, flexibility and motivation to discover the world around them; for example, a children's playground can provide spontaneous opportunities for an early understanding of mathematical concepts such as height, speed, direction, shape, space and comparison.

Within studies that have examined the benefits of play-based learning, two different types of play have been the primary focus: free play and guided play. Catalano (2018) stated that free play is based on respecting children's freedom. On the other hand, Fisher, Hirsh-Pasek, Newcombe and Golinkoff (2013) defined guided play as play that has some level of teacher guidance or involvement. The following sections address these two different types of play in detail.

2.4.1.1 Child-initiated play (free play)

Holt, Lee, Miller and Spence (2015) described free play as play that is child-directed, self-driven, intrinsically motivated and pleasurable. Supporters of the free play approach defend the child's right to choose and the child's innate ability to acquire knowledge through self-directed discovery, child-centred and child-initiated learning activities (Stipek, 2013).

It is widely agreed that free play forms a critical part in stimulating and enhancing the development of emergent mathematical concepts. Presser, Clements, Ginsburg and Ertle (2015) were of the view that free play activities help to promote and build early mathematical skills. Likewise, according to Andrews (2015), child-led play improves their grasp of complex ideas like spatial relations and movement. Fredriksen (2012) asserted that children construct knowledge about the world and experience problem-solving skills during free play. Ivrendi (2016) also observed children's play during free play and reported that play with peers significantly improved children's number sense and self-regulatory skills. Such results suggest that there are many benefits to allowing time for free play as a teaching strategy and to allow for mathematics concepts to take place in early childhood education. Preschool teachers should plan for free play activities in their daily programmes, to enhance and stimulate the development of mathematics concepts in young children. The next section looks at teacher-guided play.

2.4.1.2 Teacher-guided play

Another form of play for young children is called teacher-guided play. According to Nakawa (2020), guided play is a type of learning that combines the child-directed aspect of free play with a focus on learning goals and adult mentoring, with two elements which are child autonomy and adult guidance. Supporters of this approach argue that in guided play an adult initiates the learning process, constrains the learning goals and keeps the focus on these goals as the child guides his own discovery (Weisberg et al, 2013).

Teacher-guided play has a crucial role in the development of emergent mathematics concepts in young children. According to Moss, Bruce and Bobbis (2016), preschool children learn mathematics through play and their teachers are not passive in their efforts to facilitate that learning. In agreement, Marcus, Perry, Dockett and MacDonald (2016) reiterated that the ability of young children to perceive, create, and use mathematical ideas to make sense of their world has been thoroughly established through observation of their play. In the same vein, Nakawa (2020) stated that in mathematics learning, young children can become familiar with mathematical concepts in their environment while they are playing, but in the classroom setting they must also acquire new concepts that they have not seen before, as some mathematical concepts cannot be understood without adult interaction.

Important aspects of early childhood education, like assessing children's competencies and enhancing mathematics conceptual understanding can be implemented during teacher-guided play. According to Rinaldi (2013) and Van Hoorn, Nourot, Scales and Alward (2015), play is widely accepted as an enabling context whereby the breadth and depth of children's true abilities can be observed. Likewise, Weisberg et al (2013) reiterates that the child's development can be enhanced and even accelerated through guided adult activities, through giving commentary while children are playing, co-playing alongside children, encouraging children to explore materials in new ways and asking children open-ended questions (Weisberg et al., 2013).

Although teacher-guided play provides a valuable context for preschool teachers to respond authentically to children's mathematical curiosities, preschool teachers might question their own abilities regarding their mathematical guidance. According to Cohrssen (2015), preschool teachers' understanding of the necessity of interpreting mathematical encounters, as well as their reported lack of confidence and mathematical CK, may limit their ability to respond to these playful interactions. Cohrssen (2015) added that Early Childhood Australia recognises the difficulty some educators experience in responding to mathematical concepts young children engage with, acknowledging that many may feel uncomfortable having conversations with children that enable them to assess their mathematical thinking during play. Taking this into consideration, there is a need to support educators to deal with the complexity of mathematical concepts children are naturally exploring, and to actively engage in questioning and dialogue to elicit children's reasoning to inform future directions for learning (Cohrssen, 2015).

As presented above, both types of play – free play and guided play – are vital in the development of mathematical concepts and learning. Preschool teachers should use both types in the daily programme to facilitate the development of emergent mathematics with young children. Play, however, is not an exclusive pedagogy for emergent mathematics and the following section presents an integrated approach to facilitating mathematics.

2.4.2 An Integrated Approach to Facilitating Emergent Mathematics

The method of instruction in preschool settings in South Africa is informal, and of a playful nature. Preschool teachers use themes and a daily programme as a methodology enabling them to integrate different learning areas in the process. Bjorklund and Ahlskog-Bjorkman (2017) confirmed that theme work that integrates different learning areas is considered suitable for developing young children's knowledge and skills in early childhood education.

Preschool teachers should carefully plan their activities to integrate mathematics activities during the daily programme. Baroody et al. (2019) were of the view that a strategically and carefully implemented academic approach is compatible with developing emergent mathematical skills with other learning areas. In addition, they stated that mathematics and music, art or literacy instruction all involve patterning which can comport with emergent mathematics. However, to facilitate an effective integrated programme, preschool teachers have to be familiar with emergent mathematics concepts. Greenberg (2012) argued that being aware of early mathematical concepts helps teachers to be more thoughtful and intentional about integrating these concepts in everyday experiences and interaction with young children.

Preschool teachers should carefully plan their activities to integrate mathematics activities during the daily programme. For example, register time is an important routine activity for mathematics as mentioned by Naude and Meier (2014). During register time, children in the class can count whether all group members are present. This implies that emergent mathematics is not developed in isolation. Preschool teachers can integrate and facilitate mathematical development throughout the daily programme and within all learning areas to enhance conceptualisation.

2.4.3 Activities for Facilitating Emergent Mathematics Conceptualisation

There is a variety of emergent mathematical activities that are fun and playful that preschool teachers can implement in their daily programme. According to Kühne, O'Caroll, Comrie and Hickman (2013), teachers should provide learning opportunities and objects that incite children's interest to play and to make sense of mathematics concepts. They further explained that preschool teachers should include children in activities that involve numbers, patterns, measuring and problem-solving. Kühne et al. (2013:10) suggested the following activities for facilitating emergent mathematics conceptualisation:

- **Activities that stimulate curiosity and exploration**

"Provide opportunities for children to explore mathematics concepts by connecting mathematical ideas to their play and routines and activities they most enjoy".

- **Activities that allow for learning through doing**

"Include activities that involve numbers during daily tasks; for instance let them help with counting items when shopping, weighing ingredients for cooking and sorting and matching the washing".

- **Activities that include talking, listening and questioning**

"Provide abundant opportunities for children to talk about what they have done and to explain why they did it."

- **Playing**

"Join in and share activities with children as they play, talk to them and think aloud. When you participate in children's play, talk about what you are doing. For instance, I filled three cups with water…".

- **Stories and books.**

"Create an environment rich in storytelling. Look for opportunities in shared storybook reading to explore and explain new mathematical concepts".

- **Songs rhymes and movement**

"Use number rhymes and songs that focus on counting and calculating. Ask children to clap a musical pattern".

- **Problem-solving**

"Encourage children to experience an activity as one of shared exploration where a problem is solved together, rather than one where the adult holds the right answer".

- **Repeating**

"Re-tell stories to provide opportunities for children to practise using their new mathematical vocabulary".

All these activities provide a strong foundation in children's understanding of mathematical concepts. Byington, Kim, Nazarechuk and Weigel (2016) confirmed that children begin to develop emergent mathematics skills at a very young age through their daily playful activities. Byington et al. (2016) further affirmed that preschool children can develop an understanding of numbers, spatial concepts (shapes, measurement) and the ability to sort, classify and solve problems.

In South Africa, the guiding policy document for ECD is the NCF. According to the NCF (DBE, 2015) preschool teachers should create opportunities indoors and outdoors that allow children to explore, learn and practise their emerging mathematical understanding (Table 2.1 above). Preschool teachers can use the guidelines as prescribed by the NCF to design and facilitate emergent mathematics development. Providing a stimulating environment that promotes practical activities and interesting resources (Brock, Dodds, Jarvis & Olusoga 2013) may lead preschool teachers to question what materials they need to structure in the environment. The next section discusses factors for preschool teachers to consider when planning for resources to facilitate emergent mathematics activities.

2.4.4 Resources for Facilitating Emergent Mathematics Development

Piaget's theory of development emphasises the individual learner who constructs knowledge through interaction with the concrete environment, which is cognitive constructivism. According to Gordon and Browne (2014) and Naude and Meier (2014), Piaget's theory suggests that learners manipulate physical objects in the classroom to construct their knowledge. For example, children build puzzles to learn the relationship between the concepts of shape and colour. When connecting the different puzzle pieces as they construct knowledge, children additionally develop an understanding of mathematical concepts.

Moreover, Hill, Blazar and Lynch (2015) stated that in addition to teacher mathematical knowledge, resources are a key determinant of children's mathematical achievement. Likewise, Jones and Tiller (2017:18) stated that "a little creativity and enthusiasm are as effective as cutting-edge tools for teaching mathematics to young children, physical mathematics teaching aids can engage children's minds in valuable ways that result in high retention of the information". This suggests that access to resources is critical to provide first-class mathematical instruction (Peltier, Peltier, Werthen & Heuer, 2020).

While manipulatives provide individuals with sensory experiences, according to West (2018), they do not in themselves impart mathematical understanding; and appropriate discussion and teaching is needed to make the links to mathematics explicit. West (2018) further asserted that manipulatives alone do not give educational experiences unless they are used in conjunction with adequate teaching to promote rich and integrated learning. In agreement, Reys, Lindquist, Lambdin, Smith, Rogers, Cooke and Bennettet (2016:45) affirmed that "effective preschool teachers use manipulatives to provide concrete experiences that help children make sense of mathematics and build their mathematical thinking." Thus, preschool teachers should provide concrete manipulatives so that children can explore and discover mathematical concepts.

Resources are important in the learning and development of concepts, but they need not be expensive. According to Jones and Tiller (2017), teachers can provide great mathematics instruction using waste materials, thus they do not need to acquire pricey, commercially created manipulatives. Ideally, everyday objects found around an average classroom or household can be used as convenient and effective mathematical manipulatives. Jones and Tiller (2017) suggested blocks, sticks, toothpicks, containers and various other counting objects as easily accessible and cheap resources to use. In view of this, preschool teachers can use everyday objects or recyclable waste material in their emergent mathematics instruction.

Preschool teachers should be aware of the resources that enhance the development of emergent mathematics activities. Naude and Meier (2014) suggested number posters, board games, different measuring instruments, such as scales, spoons, measuring cups, tapes and a birthday calendar. They further recommended the use of three-dimensional objects like counters, washing line and all kinds of waste materials to enhance the development of emergent mathematics concepts. Gordon and Browne (2014) also suggested that resources should be age- and developmentally appropriate, durable and safe.

The implication of this is that preschool teachers should use concrete manipulatives to enhance emergent mathematics conceptualisation and should understand that appropriate discussion and links to concepts is vital. Apart from selecting appropriate resources, it is also critical that the environment is conducive and enhances the development of emergent mathematics concepts.

2.4.5 A Conducive Environment that Facilitates Emergent Mathematical Conceptualisation

There are a few empirical studies in mathematics education addressing the environment as a factor for learning. The environment in an early childhood set up plays a very important part in the development of concepts for young children. Children's play is influenced by their environment. The environment in which children learn by means of play, consists of both the indoor and outdoor spaces (Rogers, 2013). It is advised by Andiema, Kemboi and M'mbonne (2013) that preschool teachers should use playtime optimally to structure playful activities for children and be careful not to use that time for personal administration such as work-related teacher responsibilities. Learning through play and children's play requires teachers to plan a playful learning environment that invites young children to explore, examine, question, predict, test, investigate, use trial-and-error and manipulate (Brooker, Blaise & Edwards, 2014).

Preschool teachers should structure the environment in a way that allows children to play, both indoors and outdoors and construct knowledge and understanding through play. According to Canning (2010:93), "children's play is influenced by their environment". Similarly, Van Heerden (2012) stated that the learning environment consist not only of physical features but also the atmosphere which can influence children's play significantly. Gordon and Browne (2014) suggested setting up the indoor and outdoor environment to develop problem-solving skills and enhance cognitive development. Tucker (2010) stated that for mathematics learning to happen, the following should be considered in setting up the physical environment:

- appropriate resources to inspire and motivate learners;
- enough space to enable interaction among peers, and a natural flow from one activity to the other; and
- ample time to explore the opportunities that the materials present.

Preschool teachers should be in a position to plan the environment in a manner that facilitates mathematical development. Both indoor and outdoor areas should be structured systemically and equipped with educational manipulatives that stimulate mathematical thinking and the development of mathematical concepts. Preschool teachers should therefore create an environment that encourages emergent mathematics conceptualisation. There are also other factors that can influence the development of emergent mathematics concepts, as discussed in the following section.

2.5 FACTORS INFLUENCING THE DEVELOPMENT OF EMERGENT MATHEMATICS

Guidelines for facilitating emergent mathematics conceptualisation in early childhood have been discussed above. Factors that influence the development of emergent mathematics development are early childhood educators' training programmes and curriculum guidance in early childhood education. Other influences like early childhood educators' beliefs and attitudes towards emergent mathematics are discussed below. There is little research that examines the importance of preschool children's mathematical knowledge development and teachers' beliefs about how to teach mathematics to young children.

2.5.1 Preschool Teacher Training and Qualifications

The challenges in preschool teacher training programmes are manifold. Morrow (2007:28) in his response to the challenges in South African education maintained that the "remedy is going to have to be professional". Further abroad, according to a statement by the NAEYC and National Council for Teachers of Mathematics (NCTM) (2010), the focus should be directed to preschool teacher training, teacher training curriculum developers and training institutions, that should train teachers of young children on teaching mathematical concepts and pedagogy that are directly linked to their professional role. Courses or in-service training should be designed to help teachers develop a deep understanding of the mathematics they will teach and the habits of mind of a mathematical thinker (NAEYC and NCTM, 2010).

Improved teacher and practitioner qualifications have been linked to better child outcomes in studies conducted in a variety of nations and circumstances (Biersteker & Picken, 2013). Similarly, Ebrahim, Okwany and Berry (2019) stated that quality Early Childhood Care and Education (ECCE) is heavily reliant on properly qualified teachers who have an in-depth understanding of what they are doing, why they are doing it, and who reflect on how to continually improve their practice and adapt to challenges. Also, a study by the Organisation for Economic Co-operation and Development (OECD) (2017) indicated that improved teacher training and qualification levels can enhance the quality of interaction and pedagogy in early childhood education and care. Barnet (2016) stressed that better educated preschool teachers have more knowledge and skills to facilitate activities. This makes them better teachers because they expose children to a wide range of vocabulary, are better at planning and implementing lessons and are better problem-solvers and spend more time facilitating rich learning activities for children.

The quality of teacher education in South Africa is worrying (Botha, 2012). In South Africa, most trainers who are training preschool teachers do not have a bachelor's degree, and only have a teaching certificate. Thus, preschool teachers' knowledge of facilitating and implementing emergent mathematics might be influenced by their training and education or lack thereof. Relevant, age-appropriate learning through play is an important aspect of stimulating learning in the birth-to-four-year age group. Consequently, it is imperative that South Africa has a well-trained workforce of preschool teachers who can meet the needs of children (Harrison, 2020). However, according to Ebrahim (2018) the development of a high-quality workforce for the birth-to-four-year age group is complex and inconsistent. The training of the preschool teachers lies in different sectors (education, health, social services) and in different institutions (non-governmental organisations (NGOs), Higher Education Institutions (HEIs), Technical and Vocational Education and Training (TVET) institutions and private providers). Furthermore, the desktop study conducted by Harrison (2020) revealed that teacher training has been pegged largely as vocational and occupational, with little academic involvement from HEIs beyond a foundation phase degree (FP) that may have a module that addresses the basics of ECD. The overlaps between the sectors are a cause for concern and little exists in terms of harmonising early childhood teacher education, specifically for birth to four years. In view of this, there is a need for standardised quality training of preschool teachers. Preschool teachers need to undergo quality training which will in turn enable them to facilitate quality mathematics activities.

According to the recent policy on minimum requirements for programmes leading to qualifications in higher education for ECD educators (DHET, 2017:24), the minimum entry level qualification is a higher certificate in ECCE. This qualification will qualify graduates as educators, although they should complete a diploma or a Bachelor of Education degree to become professionally qualified. However, to date, NGOs have dominated private provision of the Education, Training and Development Practices Sector Education and Training Authority (EDTPSETA) NQF Levels and offer five qualifications in ECD (Mbalathi; Mthembu & Diga, 2016). This has in turn, raised some concerns as to the quality and provision of the training. The Project for Inclusive Early Childhood Care and Education (PIECCE) report (Harrison, 2017) established that, despite the development of qualifications, there were issues of quality with regard to standards of training and roll-out.

In addition, according to Kagan and Gomez (2014), the training that preschool teachers are receiving is not directly linked to the content that they will teach young children. This influences the amount and quality of knowledge and skills they impart to young children. In addition, there is little in the way of requirements that mandate that the teachers must undergo professional preparation and development that includes content specific to early childhood mathematics (Kagan & Gomez, 2014).

Presently, there is a move towards professionalisation of the early childhood education sector with particular emphasis on the birth-to-four age group through a standardised programme framework for teacher qualifications (DHET, 2017). The impact of this shift cannot be measured yet since it is still work-in-progress, and although it is about early childhood education in general and does not directly address the issue of emergent mathematics conceptualisation, it does give an insight into how different stakeholders are working tirelessly to improve the early childhood sector.

Considering the discussion above, it would seem that there are huge challenges and gaps in preschool teacher training and qualification. The implication of this for this study is that the current practising preschool teachers might not be well-trained and qualified, and might lack the relevant and appropriate skills, knowledge and attitude to facilitate and implement emergent mathematics for young children because of the disconnect in the ECCE sector in South Africa.

2.5.2 Preschool Teachers' Beliefs and Attitudes Towards Emergent Mathematics

Given the importance of emergent mathematics development in children and its implications for future learning, it is critical to discuss the relations between preschool teachers' beliefs and the impact on teaching practice. Numerous researchers promote the argument that teachers' beliefs are linked with their instructional practices (Archambault, Janosz & Chauinard, 2012; Hu, Fan, Yang & Neitzel, 2017; Šapkova 2014).

It has been confirmed that there is a relationship between teachers' beliefs and their facilitation of emergent mathematics activities. Aslan (2013) conducted a study in which he compared prospective and in-service preschool teachers' mathematics anxiety and beliefs about mathematics for young children. He found that in-service preschool teachers manifested more anxiety than prospective preschool teachers. In his study of 31 Head Start teachers, Geist (2015) investigated their attitudes toward mathematics and the effect thereof on how and what they taught in their classes. The findings suggested that there was a relationship between mathematics anxiety and negative attitudes towards mathematics. Moreover, the study illustrated that these feelings affected teachers' curriculum planning choices as well as their ability to involve young children in mathematics activities. Drawing from this line of thought would imply that preschool teachers' beliefs will directly affect how they implement and facilitate emergent mathematics activities in the classroom.

One of the most critical conditions for the preschool teacher to be able to facilitate mathematics activities effectively and to be happy with their job is that their attitude towards the teaching profession and, in relation to this study, their attitude towards mathematics is positive (Miller, Ramirez & Murdock, 2017). A positive attitude in the teaching profession is summarised by Kirkic and

Celinkaya (2020) as being interested in teaching, taking care of the teaching environment and loving and appreciating the children.

In contrast, other research has suggested that there may be a disconnect between teachers' beliefs and their classroom practice (Francis, 2015). Teachers might believe in the importance of mathematics learning in early childhood, yet their teaching practices might reveal the opposite. In addition, Boonen, Van Damme and Onghena (2014) stated that the possible disconnect between teachers' beliefs and classroom practices in mathematics may be reflected in mathematics-related practices in preschool classrooms, including the amount of time, types of practices, and the frequency and duration of time devoted to instructional methods with mathematics content.

Therefore, in view of the above argument, whether preschool teachers' beliefs affect the way they practice and facilitate emergent mathematics activities is subject to debate. However, it is important to note that preschool teachers' beliefs and attitudes towards emergent mathematics do not always reflect how they implement and facilitate emergent mathematics activities in the classroom. The next section discusses the theoretical framework that informed this study.

2.6 THEORETICAL FRAMEWORK

According to McMillan and Schumacher (2010), research should be done to develop a general conceptual framework or theoretical orientation. This justifies the subjects, variables and design of the research, and provides a basis for the interpretation of research results since it is seen in the light of a particular theory. Should a study draw on a theoretical framework, this framework needs discussion and clarification as to its relevance to the research (Basit, 2010).

The study reviewed several aspects of the constructivist theoretical perspective derived from the work of Jean Piaget (1896−1980), Lev Vygotsky (1962), as well as the theory of PCK according to Shulman (1986) and Grossman (1990) as points of departure. These theories emphasise teachers and children's active participation in the facilitation and learning of emergent mathematics, and the teachers' PCK of teaching mathematics.

2.6.1 Constructivism

This study draws from constructivism and specifically, Jean Piaget's theory of cognitive development and Lev Vygotsky's sociocultural theory. Constructivism is entrenched in the cognitive institute of psychology and the theories of Piaget, and Vygotsky (Von Glasersfeld, 1995). There is no universal definition of constructivism but, according to Bada and Olusegun (2015), constructivism represents one of the big ideas in education. Fosnot (2013) stated that constructivism is a theory about knowledge and learning: it describes both what "knowing" is and how one "comes to know". Its implications for how teachers teach and learn to teach are enormous. Some consider constructivism as a theory of learning. Christie (2005) pointed out that constructivism is a learning theory in which learning is both an active process and a personal representation of the world. Knowledge is constructed from experience and modified through different experiences.

Mvududu and Thiel-Burgess (2012) stated that constructivism is widely publicised as an approach to probe for children's level of understanding and to show that understanding can increase and change to a higher level of thinking. This means constructivism refers to the how of learning and thinking. Constructivism is suitable for facilitating and learning emergent mathematics since it allows for young children to create their own meaning and connects it to daily life. Constructivism describes the process of knowledge as an active rather than a passive process (Major & Mangoepe, 2012). It is when the preschool teachers create an environment and experiences that allow for exploration and discovery, that young children can construct their own meaning and learning. This means that preschool teachers have to facilitate activities that allow young children to be actively involved to discover and learn emergent mathematical concepts through exploration and play.

There are two significant theories in the facilitation and learning of emergent mathematics that are important for this study, which are Piaget's cognitive constructivism theory and Vygotsky's social constructivism theory. Though both theorists studied the development of young children, their focus areas were not the same. The differences between these two theorists is in "how" the children construct their knowledge. Bhattacharjee (2015) stated that Vygotsky and Piaget's theories are often contrasted to each other in terms of cognitive constructivism (Piaget), and social constructivism (Vygotsky). Where Piaget believed that a child can construct their thoughts and knowledge of the world from what they already know, Vygotsky believed a child first learns from the social environment and then individualises what was learned. Piaget and Inhelder (1969) suggested that discovery is the most important and fundamental basis of learning. While Vygotsky (1978) believed that Piaget's emphasis focused too much on internal processes of individuals, he considered cognitive development primarily as a function of external factors such as cultural, historical and social interaction rather than individual construction. The two schools of thought are further clarified in the following sections.

2.6.2 Piaget's Theory of Cognitive Development

In the last century, Jean Piaget proposed one of the most well-known theories regarding cognitive development in children. Piaget proposed four cognitive developmental stages for children, including sensorimotor, preoperational, concrete operational, and the formal operational stages of development (Moreno, 2010). During the process of moving through one stage to the next, children's cognitive ability change qualitatively (Sigelman & Rider, 2012).

Based on cognitive development, Piaget divides the cognitive development of children into four stages: sensorimotor that occurs from birth to about 2 years of age; preoperational that occurs at the age of about 2 years to about 7 years; concrete operations that occur at the age of about 7 years to 12 years; and the last stage of formal operations that occur at the age of about 12 years until adulthood (Alhaddad, 2012).

For the sake of this study, the focus is on the preoperational stage. During this stage, children require concrete situations to process ideas (Nurnazarovna & Obidjonovna, 2020). Furthermore, children at this stage are egocentric and have difficulty considering the views of others. They are at a symbolic stage and can imagine and picture objects without being in front of them (Beckley, 2018); however, they do not have the ability to think logically (Ciccarelli & Whith, 2012).

2.6.2.1 The implications of the cognitive theory for teaching and learning of emergent mathematics

Preschool teachers should be aware of the cognitive developmental stages of young children, and what they entail, to be able to facilitate age-appropriate mathematics activities. According to Nuroso and Siswanto (2018), one key point in cognitive development is that teachers should pay attention to the cognitive developmental stage of learning in children, meaning that if children are at the operational stage, teachers should not use abstract concepts in learning. Nuroso and Siswanto (2018) further explained that this is because the success of teaching and learning is influenced by the match between the subject matter and the level of cognitive development of children.

Young children at the preoperational stage are learning to think and love to play; therefore, preschool teachers should be cognisant of that when planning and facilitating mathematics activities. In agreement, Yuriansa and Kurniawati (2021) stated that learning mathematics for early childhood not only teaches children to count but also makes children think. Yuriansa and Kurniawati (2021) further suggested that preschool teachers should provide more opportunities for children to play and learn in an appropriate way, through exploration and real-life experiences that are meaningful.

Despite the effectiveness of the cognitive theory and its use world-wide, researchers have highlighted some weaknesses of this learning theory. Beckley (2018) explained that Piaget’s theory does not acknowledge social and cultural factors or the differences between individual children. This could lead to problems in practice, where preschool teachers may not allow children to attempt activities which could be part of the next step as they may believe the children might not be ready. Similarly, Cacioppo and Freberg (2013) confirmed that Piaget underestimated the effects of interaction influence among peers on cognitive development. Piaget did not consider that children living with their families are influenced by their families. The implications of this for the teaching and learning of mathematics are that preschool teachers should not limit children who show maturity and abilities to do higher-order operations, because every child is different and there are other factors that can enhance cognitive development of children, like social and cultural influences.

2.6.3 Social Constructivism Theory

This study of the facilitation of emergent mathematics activities is also informed by the theory of sociocultural learning by Lev Vygotsky. Vygotsky's work was based on Jean Piaget's idea of a child as an active learner (Verenikina, 2010) though he identified the influence of the greater sociocultural context (Adam, 2017). Vygotsky stated that the human mind is constructed through a subject's interactions with the world and is an attribute of the relationship between subject and object (Verenikina, 2010). A key concept of the sociocultural theory is what is known as the ZPD. According to Vygotsky, good learning occurs in the ZPD where he distinguishes between the children's actual and potential levels of development.

2.6.3.1 Vygotsky's "Zone of Proximal Development"

Vygotsky's theory utilises the concept of the ZPD. The ZPD is the distance between the child's current level of actual development in independent problem-solving activities and the potential level of development that the child can achieve in solving problems under the guidance of a teacher or adult, or a more competent peer (Abtahi, Graven & Lerman, 2017; Eun, 2019; Silalahi, 2019). Likewise, Armstrong (2015) agreed that the ZPD is a sphere in which learning conditions can be optimised through the identification of competencies that the learner could develop only with the right assistance. With reference to emergent mathematics learning and the ZPD, Hembold (2014:32) highlighted that "children performing mathematical tasks in this zone can almost, but not quite, perform the task on their own". Therefore, Papalia and Feldman (2011) suggested that parents, teachers or others should support the child to perform a task until the child can do the task without adult assistance.

2.6.3.2 Implications of the ZPD on teaching and learning

It is clear that help or guidance is essential in facilitating activities and imparting knowledge in child development. Preschool teachers need to enhance and stimulate development of emergent mathematics concepts by playing an active role in the children's learning. According to Kusmaryono, Jupriyanto and Kusumaningsih (2021), the teacher's role is to facilitate learning, and help children to solve problems when needed by the learner. Darlington-Hammond, Flook, Cook-Harvey, Barron and Osher (2020), further asserted that, to make progress to a higher level of development, children in learning need to be assisted, guided and directed so that they can solve problems independently.

A further implication of this concept, according to Armstrong (2015), is that the teacher introduces information and resources appropriate to the learner's current developmental level; then, the learner continues to refine their conceptual landscape through reciprocal learner-teacher and learner-peer interactions. Armstrong (2015) further stated that, though Vygotsky emphasised peer collaboration, he was partial to learner-teacher interaction, which exposes the learner to an expert's conceptual process, thrusting them beyond their current level of development until they become more autonomous in their own understanding of the subject.

To a large extent, Vygotsky and Piaget share a commonality, which is the teacher's involvement and role in the facilitation and acquisition of mathematical concepts as being important in the young learner's mathematical development. Both theories are thus important since this study explores and seeks insight into how preschool teachers facilitate and implement emergent mathematics activities for three-to-four-year-old children. The following section addresses the constructivist learning environment.

2.6.4 The Constructivist Learning Environment

The learning environment plays a crucial role in supporting the facilitation of emergent mathematics in the early years. Tam (2000) outlined that constructivist learning environments have certain basic characteristics such as:

- knowledge will be shared between educators and learners;.
- teachers and learners will share authority;.
- the teacher's role is one of a facilitator or guide; and.
- learning groups will consist of small numbers of heterogeneous learners.

Bada et al. (2015) illustrated in the form of a table, below, what a constructivist class entails and contrasts it with a traditional class. A constructivist environment allows the children to construct their own mathematical meanings and use manipulatives in a playful manner.

Table 2.3: The differences between a traditional and a constructivist classroom.

Traditional Classroom	Constructivist Classroom
The curriculum begins with the parts of the whole. Emphasises basic skills.	The curriculum emphasises big concepts, beginning with the whole and expanding to include the parts.
Strict adherence to a fixed curriculum is highly valued.	The pursuit of learner questions and interest is valued.
Materials are primarily textbooks and workbooks.	Materials include primary sources of materials and manipulative materials.
Learning is based on repetition.	Learning is interactive, building on what the learner already knows.
Teachers' disseminate information to learners and learners are recipients of knowledge.	Teachers have a dialogue with learners, helping learners to construct their knowledge.
Teachers' role is directive, rooted in authority.	The teacher's role is interactive, rooted in negotiation.

(Adapted from Bada et al. 2015).

Bada et al.'s interpretation of the constructivist classroom is that the preschool teacher should create an atmosphere that allows for inquiry, manipulation, and exploration. The young child discovers emergent mathematical concepts by interacting with the environment and making discoveries in the vicinity. This means that teachers should create environments that allow children to discover emergent mathematical concepts through play.

The constructivist theory implies that preschool teachers should expose young children to activities that enable them to learn concepts through manipulation and exploration of concrete objects. Furthermore, it suggests that preschool teachers should be actively involved in children's play by engaging them with questions that allow for thinking, reasoning and problem-solving. This means that teachers should provide concrete objects which children can play with to develop mathematical concepts. Over and above provision for a conducive environment that supports the facilitation of emergent mathematics, a critical resource is the teacher herself. The knowledge and skills in emergent mathematics that a teacher possesses are viewed in light of Shulman's PCK which is clarified further below.

2.6.5 Pedagogical Content Knowledge (PCK)

The term PCK was introduced into the discourse of teacher education in Shulman's 1985 presidential address to the American Educational Research Association. It was defined as "a second kind of CK which goes beyond knowledge of subject matter per se to the dimension of subject-matter knowledge for teaching" (Shulman, 1986:9). It is "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986: 9). Shulman (1987) maintained that knowing the subject matter is not enough to teach it. Teachers need to possess PCK as well. According to Shulman (1987), PCK depends on a teacher's subject-matter knowledge, knowledge of pedagogy and on how the teacher transforms this knowledge into various forms that enable children in different learning environments to understand the subject matter. In the same vein, An, Kulm and Wu (2004) stated that PCK has three components, namely:

- knowledge of content;
- knowledge of curriculum; and
- knowledge of teaching.

Curricular knowledge is the knowledge of programmes designed for the teaching of particular subjects and topics at a given level and knowledge of the available variety of instructional materials that relate to the teaching and learning of those programmes. In South Africa, the NCF currently provides the basis of that curricular knowledge. Preschool teachers need to access the curriculum documents so that they can attain the knowledge of the curriculum and facilitate emergent mathematics activities according to the curriculum guidelines.

Pedagogical Content Knowledge is the knowledge of the forms and strategies teachers use to guide their learners into a meaningful understanding of the concepts of the subject they teach. Woolfolk (2010) argued that good teachers use pedagogical knowledge and skills to help learners understand abstract concepts. In view of this, preschool teachers should be aware of the effective teaching methodologies for facilitating emergent mathematics, to maximise conceptualisation which are illustrated in relation to PCK in Table 2.4 below.

Table 2.4: Shulman's theory in relation to the facilitation of emergent mathematics by preschool teachers

Pedagogical Content Knowledge (PCK)	**Facilitation of emergent mathematics**
Knowledge of curriculum	• Knowledge of and how to implement the NCF • Awareness of topics/themes to be taught and the order in which they will be taught throughout the year.
Knowledge of content	• Knowledge of the emergent mathematics content areas (concepts) as prescribed by the NCF. (i.e. number, counting, comparison, classification, shape, space, measurement, sorting, classifying, and problem-solving)
Knowledge of pedagogy (methodology/instruction)	• Knowledge of how to facilitate emergent mathematics activities (i.e. through play, teacher-guided activities, and integration)

The implication of PCK for the facilitation of emergent mathematics is that preschool teachers should know the curriculum, possess knowledge of the emergent mathematics content areas as well as the knowledge of effective methodology and the skill to deliver activities in a way that enhances mathematics conceptualisation.

Shulman (1986) noted that when teachers' PCK is weak, then it is hard to address misconceptions and errors derived by the students. In New Zealand, a report from the Education Review Office claimed that 23% of the teachers had low PCK (Mayne, 2019). The findings showed the importance of PCK among teachers, since a lack of PCK will influence teachers' effectiveness in managing the teaching and learning process within the classroom environment (Mayne, 2019). Preschool teachers should have thorough PCK to be effective in facilitating emergent mathematical activities for young children. The next section discusses PCK through the lens of Grossman, another theorist who expanded on the same theory.

2.6.6 Grossman's Pedagogical Content Knowledge

Grossman (1990) proposed another variation for the classification of teacher knowledge. He added “context knowledge” as another variation of teacher knowledge in addition to Shulman's PCK. Grossman (1990) was the first to systematise the components of the knowledge base of teachers proposed by Shulman and characterised the concept of PCK in his model of teacher knowledge (Figure 2.2). According to Grossman (1990), there are four interacting components that form the knowledge base for teaching which are:

1. general pedagogical knowledge,
2. subject-matter knowledge,
3. the PCK, and
4. knowledge of context.

In this model, the PCK occupies a central position and is seen as the transformation of pedagogical knowledge, taking the context and content into account. Each of the components in this model encompasses other areas of the knowledge base. Of these, the PCK is one that interacts with all the others. Extracting from the model in his view of PCK (Figure 2.2), Grossman re-examined Shulman's idea of the knowledge base for teachers, including the PCK, the knowledge of the curriculum and the knowledge of the purpose for teaching a specific content. Thus, in Grossman's model, the three knowledges that make up the PCK are guided by a conception of the purposes of teachers for teaching such content. Thus, the formal and practical character of PCK is explained, since the knowledge and beliefs of teachers are present in the model. Preschool teachers need to know the context of the children they are teaching in their classrooms to relate the mathematics activities to the children's lifeworld, and also to provide the necessary support.

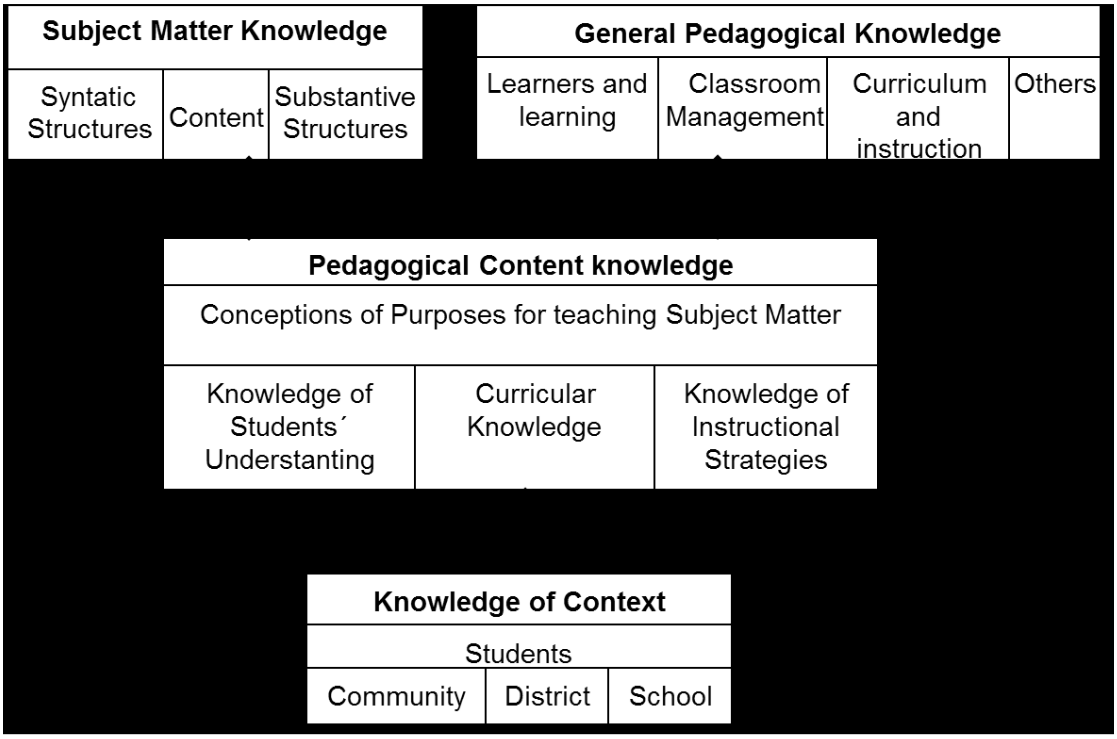

Figure 2.2: Model of Teacher Knowledge

Source: (Grossman, 1990:5).

Grossman's model implies that preschool teachers should have knowledge of the children's background, so that they plan and facilitate emergent mathematics activities that are purposeful and contextualised for the children.

The different theories discussed above shaped and informed this study regarding the facilitation of emergent mathematics activities by preschool teachers who are teaching three-to-four-year-old children. The facilitation of emergent mathematics activities if seen through the lens of Piaget's cognitive theory implies that preschool teachers should provide opportunities for children to construct their own knowledge of mathematics concepts through play and exploration. He also suggested that preschool teachers should take cognisance of the developmental level of the children in their classrooms when planning and facilitating emergent mathematics activities. Lastly, Piaget suggested that concrete manipulatives play a large role in enabling children to construct mathematical ideas and knowledge. On the other hand, Vygotsky focused more on the role of the teacher in the facilitation of emergent mathematics activities. This school of thought implies that preschool teachers have a bigger role in ensuring that children understand mathematics concepts by scaffolding and ensuring that the bridge between what children know and what they are capable of knowing is filled.

Shulman focused on PCK and maintained that teachers should possess knowledge of content, curriculum and teaching methodology. This implies that preschool teachers facilitating emergent mathematics activities should ensure that they have sound knowledge of mathematics as a subject as well as the content areas, knowledge of the mathematics curriculum and the facilitation thereof. Lastly, Grossman's school of thought seems to suggest that apart from having knowledge of content, curriculum and teaching methodology, preschool teachers should also have knowledge of the contextual background of the children in their classrooms. This would allow preschool teachers to plan and facilitate activities that are related to the children's lifeworld, to enable them to understand mathematics concepts better.

2.7 CHAPTER SUMMARY

This chapter built an argument for facilitation of emergent mathematics of preschool teachers by providing relevant findings from literature. The development of emergent mathematical concepts and curriculum guidelines in emergent mathematics was discussed. Furthermore, the facilitation of emergent mathematics conceptualisation was reviewed and factors that influence the facilitation of emergent mathematics were discussed. Lastly, the chapter provided an overview of the theoretical framework from which this study stems, namely the constructivist theory and PCK theory. The following chapter explains the methodology used for conducting this study. Data collection methods and the instruments used are highlighted.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

The previous chapter gave an overview of the relevant literature on the phenomenon of emergent mathematics teaching and the theoretical framework which provided a lens through which to interpret this study. This chapter explains the research design and methodology used in this study to explore the facilitation of emergent mathematics by preschool teachers.

The study aimed to investigate how preschool teachers facilitate emergent mathematical activities; the knowledge and skills they possess; the curriculum which informs the implementation of mathematics activities; and their concepts and beliefs about the implementation of mathematics activities to young children between the ages of three to four years old. To achieve this, three data collection methods were employed, namely, semi-structured interviews, observations and document analysis. The instruments used were interview guides, checklists, a journal for field notes as well as a document checklist for several documents including the NCF, children's work and documents used for planning by preschool teachers. In the following subsections, the research paradigm, research approach and the data collection methods are explained. Lastly, ethical issues are discussed.

3.2 RESEARCH DESIGN

A research design, according to Trochim (2006), is the structure of research that encompasses all components of a research project together, whereas McMillan and Schumacher (2010) viewed it as a plan for selecting research sites, participants and data collection procedures to address the research questions. Mailwane (2016) stated that a good research design must exhibit the significant principles and milestones which hold the research paradigm, research approach and type of research.

This study was located within the interpretive paradigm and followed a qualitative approach using a multiple case study design. The following sections discuss the research paradigm, research approach and type used in this study.

3.2.1 Research Paradigm

I adopted an interpretive paradigm as it provided a greater understanding of the facilitation of emergent mathematics activities by preschool teachers. Interpretivism advocates believe that the truth is made up of a collection of realities gleaned from historical, local, and non-generalisable events (Guest, Namey & Mitchell 2013). Denzin and Lincoln (2011) described an interpretivist paradigm as one that attempts to understand a phenomenon being studied by analysing the meanings generated by the participants in a study. I interpreted the information collected from the preschool teachers by means of semi-structured interviews, observations and document analysis.

Interpretivist researchers carry out their studies in natural settings to achieve the best possible understanding (Creswell, 2007). This study was carried out in the classrooms where preschool teachers were in their natural settings. I completed the research in a "naturally occurring context and not in experimental situation(s)" (Creswell, 2010:51).

3.2.2 Research Approach

This study employed a qualitative approach. According to Martella, Nelson, Morgan and Marchand-Martella (2013), a qualitative approach is a flexible systematic approach in understanding qualities of a phenomenon within a research context. Qualitative research tends to be concerned with the way individual people view the world and the way they experience events. In addition, Tsai (2013) argued that with the qualitative approach, the researcher aims to acquire personal meanings of individuals. Therefore, the qualitative approach was appropriate in my study as I was concerned with the facilitation of emergent mathematics activities by preschool teachers.

Furthermore, according to Maxwell (2013), the frankness and flexibility of the qualitative approach allows the researcher to pursue new discoveries and establish relationships with the participants. Qualitative research designs investigate behaviour as it occurs in the natural settings from the participants' perspective; hence, there is no manipulation of experience (Babbie, 2010; Babbie & Mouton, 2011). The data consists of words in the form of verbal descriptions, rather than numbers (McMillan & Schumacher, 2010). A qualitative research design suited the current study because thick descriptions of data within a naturalistic inquiry related to the facilitation of emergent mathematics activities by preschool teachers were elicited. The qualitative approach was more relevant in my study as the rationale emanates from a lack of research about the facilitation of emergent mathematics activities by preschool teachers.

Lastly, of particular importance to qualitative designs are personal interest and curiosity as a source of the topic (DeVos, Delport, Fouché & Strydom, 2011). I was driven to investigate and explore the facilitation of emergent mathematics by preschool teachers, because of my association with preschool teachers, which led to my curiosity to explore this topic further.

3.2.3 Multiple Case Study

The current study employed a multiple case study design by investigating five preschool teachers to collect rich data on the facilitation of emergent mathematics activities. Multiple case studies are used when the researcher needs multiple sources of evidence or triangulation to provide thick descriptions (Babbie & Mouton, 2011). Baxter and Jack (2008:550) asserted that “if a study contains more than a single case, a multiple case study is required”.

My choice was motivated by Vissak (2010), who stated that using four or more people promotes trustworthiness, whereas fewer than four participants weakens and undermines it. I investigated five cases in this study to increase the trustworthiness of this study.

Yin (2013) emphasised that the results are strengthened by multiple cases since the patterns are replicated, strengthening the authenticity of the conclusions. Using the multiple case study design enabled me to identify similarities and differences among the five participants. In agreement, Creswell (2013) adds that the intention behind a multiple case study is to enable researchers to replicate findings across cases, explore differences within and between cases and draw comparisons from the results. The research questions are highlighted in the following section.

3.3 RESEARCH QUESTIONS

The purpose of this chapter was to answer the following research questions:

3.3.1 Primary Research Question

The primary research question that guided this study was:

How do preschool teachers facilitate emergent mathematics activities for three-to-four-year-old children to enhance mathematical conceptualisation?

3.3.2 Secondary Research Questions

- What knowledge and skills are needed to facilitate emergent mathematics activities?
- What curriculum guidelines are used in emergent mathematics and how are they implemented in the facilitation of emergent mathematics in early childhood?
- How do preschool teachers explain the importance of emergent mathematics?

The questions intended to bring to the surface the facilitation of emergent mathematics activities by preschool teachers of three-to-four-year-old children.

3.4 RESEARCH METHODS

In this section, the selection of participants and the procedures and data collection methods are discussed. Lastly, the process of data analysis is discussed.

Data was collected from the selected sample, which consisted of five preschool teachers in Tembisa, using observations, semi-structured interviews and analysis of documents. The instruments used for collecting data included observation checklists (Appendix I), an interview guide (Appendix J), a journal for field notes and a document analysis checklist (Appendix K).

3.4.1 Selection of Participants

McMillan and Schumacher (2014) referred to a population as a group of individuals or events from which a sample is selected and from which conclusions can be drawn and generalised. DeVos et al. (2011) stated that the term “population” sets boundaries on the study elements, which are in principle considered as individuals or items in the world who have specific characteristics. In this study, the population was preschool teachers at schools in Tembisa, Gauteng. The sample consisted of five preschool teachers who had a qualification in ECD, were teaching three-to-four-year-old children and had at least one year experience in the field.

3.4.1.1 Purposeful sampling

My study used purposeful sampling to select participants, as this sampling method is most appropriate for qualitative case studies (Johnson, 2014). In addition, Oppong (2013) elaborated that in purposeful sampling the participants should show that they have a thorough comprehension of the issue at hand. It is recommended that the research question should be well framed and specify the kind of participants that should be included in the research study (Guest et al., 2013). In my study, only preschool teachers teaching 3-4-year-old children were purposively selected to participate. These preschool teachers were directly engaged in the facilitation of activities of 3-4-year-old children, and they provided knowledgeable and informative information about the facilitation of emergent mathematics. This eliminated the element of generalised information that might not be useful to the study (McMillan & Schumacher, 2014). In essence, these teachers were chosen because of their qualifications, setting, and knowledge of ECD.

Additionally, Creswell (2013) stated that selection decisions need to consider the accessibility of data, data collection methods and the relationship with the participants to be involved. In my study, participants were selected from Tembisa, and I easily established relationships with the preschool teachers as my current work entails training preschool teachers in Tembisa.

3.4.1.2. The study sites

Maxwell (2013) maintained that it is crucial in the field of research for researchers to understand the context within which the people behave and the influences of their behaviour on the context. Creswell (2013) argued that participants should be involved in a study that is relevant to where the research is conducted. Therefore, I sampled participants in preschools and this setting is where the research was conducted. For my study, the participants and the research sites were in preschools situated in Tembisa. The next part of this study discusses the data collection procedures and collection methods.

3.4.2 Data Collection Methods

Flick (2014) emphasised that the qualitative researcher needs to make the correct choice of relevant research data collection methods that yield good results. In my study, I used multiple data collection research methods to maintain credibility of the results (Creswell, 2013). I ensured triangulation using semi-structured interviews, classroom observations and document analysis as recommended by Creswell (2013). Pacho (2015) pointed out that triangulation for collecting data is useful when one needs to corroborate information from one data collection method with that obtained from another. I was guided by this question in collecting the data “How do preschool teachers facilitate emergent mathematics activities for three-to-four-year-old children to enhance mathematics conceptualisation?”. The data I collected through the various research methods aimed to address the research questions. The qualitative data was interpreted and presented without bias and generalisation.

Implementing various qualitative data collection methods helped to record the preschool teachers’ perceptions, views and opinions, thoughts and teaching approaches to offer a descriptive and contextually rich indication of why they acted in certain ways and what their feelings were about these actions (Mukherji & Albon 2010).

3.4.2.1 Observations

I planned classroom observations to gain first-hand information about the facilitation of emergent mathematics activities in the three-to-four-year-old children’s classroom environment. Classroom observations are recommended as a method in which a researcher works in the field to gather information about a phenomenon (Martella et al, 2013). In my study, the three-to-four-year-old children’s classrooms in preschools were the sites where information was collected about the facilitation of emergent mathematics activities in a natural setting. Johnson (2014) defined observations in qualitative research as watching human actions with the purpose of understanding the reasons for those actions and their influence in real-life situations. In this study, the behavioural patterns were the ways in which emergent mathematics activities were facilitated in the preschool classroom.

I employed this method because it allowed for watching natural events as they took place. I was a non-participant-observer, watching from the side lines without being involved in the activities. I sat at the back of the classroom observing and documenting events as they occurred naturally. This enabled me to closely document activities without interfering, as opposed to being a participant-observer, engaging in activities thereby disturbing me from collecting rich data. It also enabled me to assess the preschool teachers' understanding of how emergent mathematics is facilitated. Bertram and Christiansen (2015) state that structured observations can only be successful if the researcher clearly knows what he/she is looking for in a classroom situation. In my study, I was exploring the strategies preschool teachers used in facilitating emergent mathematics activities, their knowledge and attitudes as well as utilisation of resources. I observed the following:

- Classroom layout
- Resources
- Classroom activities
- Teacher-child interaction
- Child to child interaction
- Teacher-child ratio

I employed the use of an observation checklist (see Appendix I) as an instrument for recording data collected from general observations of the classroom layout, resources and classroom management upon arriving at each sample classroom. The checklist enabled me to have an insight into the preschool teachers understanding of setting up a mathematical environment that will enable the facilitation of emergent mathematics activities. The same checklist was used for all the sample teachers. I chose checklists because they are an easy way of collecting a large volume of data in a short period, as it only entails ticking in the appropriate box of set out criteria.

Schwandt (2007) describes field notes as evidence to gain meaning and an understanding of the culture, social situation, or phenomenon being studied. I therefore, kept a research diary as part of the data collection process. Gambold (2010) claims that field notes could be used to describe a physical space, the mannerisms of people and the duration of events. Cohen, Manion and Morrison (2005:146) state that "field notes can either be written *in situ* or away from the site or the study field". From the beginning, I made field notes when I arranged meetings with the preschool teachers. During the observations, I made use of field notes to describe the teachers' attitudes, behaviours, understanding and facilitation of emergent mathematics activities.

3.4.2.2 Semi-structured interviews

I employed the use of semi-structured interviews to gain insight into preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-old children. According to Babbie (2014), a qualitative interview is an interaction between an interviewer and interviewee, where the interviewer has a plan that is inquiry-based in relation to the research problem. Thus, the plan of enquiry was directed towards finding out how emergent mathematics activities were being facilitated in the preschool classroom of three-to-four-year-old children. Semi-structured interviews aimed at better understanding individual experiences and discovering trends in the field of early childhood education (Bertram & Christiansen, 2015).

The qualitative nature of semi-structured interviews enabled me to obtain information from the participants through face-to-face engagement. Flick (2014) states that qualitative research considers that participants have different viewpoints and practices because of their different contextual life experiences. Qualitative research suggests that the participants may respond differently to research questions. I was aware that preschool teachers' perceptions, attitudes and facilitation of emergent mathematics activities might not be the same.

DeVos (2011) states that the guided interview is ideal for obtaining comprehensive and comparable data. In an interview guide, topics are selected in advance, but the researcher decides the sequence and wording of the questions during the interview (Schumacher & McMillan, 2014). For this study, interview questions were purposely selected before the interview, questions which helped to gain insight into what preschool teachers conceive of emergent mathematics, their attitudes, and skills. I chose this interview type as it allows for flexibility, as the interviewer decides the sequence and the wording of the questions as the situation leads.

I designed interview guides (see Appendix J) for the participants and selected my questions and the sequence thereof in advance. All the questions were the same for all the participants. The five interviews with the preschool teachers lasted between 30 and 45 minutes each. The interviews were audio-recorded, while I made field notes and were then transcribed verbatim. Permission was sought from the participants to audio-record the interviews and they were conducted without disrupting the daily activities.

3.4.2.3 Document analysis

Document analysis is a non-interactive qualitative data collection method with little or no interaction between the participant and the researcher (McMillan & Schumacher, 2014). To explore and gain insight on how preschool teachers facilitate emergent mathematics I employed the use of document analysis, as documents are a useful and valuable source of evidence and provides background information on the topic (McMillan & Schumacher 2010). In agreement, Yin (2014) advises the researcher to engage with documents as a data source because they provide scientific and valuable information from which a lot of detail can be elicited. Documents such as the NCF and teachers' planning documents provided the background information to my research topic.

Researchers, Rule and John (2011:67) state that "document analysis is a useful place to start data collection in a case study, particularly if the research design includes other methods, such as interviews and/or observations". In the view of

McMillan and Schumacher (2010), documents are printed or written records of past plans and events. For each preschool visited, I requested the preschool teachers to provide me with lesson plans, year plans, curriculum documents and children's workbooks to determine the extent to which they interacted with the policy documents and how this influenced their facilitation of emergent mathematical activities.

To gain further insight into the research, I collected all the relevant official and unofficial documents such as:

- Policy documents (curriculum guidelines): The curriculum guidelines would to an extent indicate whether preschool teachers follow the policy. The documents would also indicate whether the preschool teachers' facilitation of mathematics activities were in line with the policies.
- Worksheets: Worksheets could reveal the type of mathematical activities facilitated in the programme.
- Daily programme: This would give insight into the types of activities offered in the programme and their alignment to emergent mathematics, and whether there was a specific time allocated for mathematics activities, and the frequency thereof.
- Lesson planning: This would give me an insight as to how emergent mathematics activities were facilitated, the resources used and mathematical outcomes set for the children.

These documents were analysed and integrated with the evidence obtained through the interviews from the preschool teachers. To enable me to analyse these documents in a structured and meaningful way, I employed the use of a document analysis checklist (see Appendix K). The document analysis enabled me to gain insight into the theoretical and practical framework of mathematical activities for young children. The intention was to determine the effectiveness of these documents in implementing and facilitating emergent mathematics activities and to see if the preschool teachers aligned their activities with the NCF. The analysis of the documents was also done to confirm the evidence sought from the lesson observations and interviews. The advantages of using the document analysis method are that documents are easily accessible and are less time-consuming.

The following section discusses how the collected data from the semi-structured interviews, observations and document analysis document analysis, were analysed and interpreted.

3.4.3 Data Analysis

Data analysis entails organising, analysing, and interpreting data, to make sense of the information obtained through the fieldwork (McMillan & Schumacher, 2010). In addition, Lambert (2013), raises a similar view by arguing that data analysis involves breaking down and making connection of the collected data. It is all about making sense and deriving meaning from the collected data. Similarly, Ncube (2016) adds that data collected is converted into useful information that will provide value to the intended end user. This brings order, meaning and structure to the volumes of the collected data.

I adopted and utilised the process of thematic analysis in this study to understand the facilitation of emergent mathematics by preschool teachers. I used an inductive approach to identify similarities and differences that were identified in the various sources of data collection to answer the research problem. Lichtman (2014), highlights the advantage of the inductive approach as it allows identification of central ideas that support themes related to the research problem under investigation. Bertram and Christiansen (2015) state that the inductive approach encourages the researcher to analyse data by looking at topics that emerge and code them. In other words, the inductive approach allows the research findings from the explicit themes to emerge.

Once data had been collected from the preschool teachers, I transcribed the audio recordings of the semi-structured interviews and then reduced the data to smaller, more meaningful categories (Yin, 2014). I read the transcripts several times, sorted, and organised the notes according to research questions to identify important themes that emerged. I identified the themes from the emerging patterns. I employed content analysis to analyse data and interpret the emergent themes. I took advice from Martella et al. (2013) to include my own thoughts, feelings and experiences throughout the investigation. This entailed using my own creativity in interpreting the participants' stories and the field notes focusing on the research question.

I used verbatim quotations from the participants' interview transcripts (Bertram & Christiansen, 2015; Patton, 2015:). The integrated narratives of the participants were written under each theme to produce a readable and descriptive narrative account of the facilitation of emergent mathematics activities. After the data was categorised into smaller parts, I coded it and merged it into themes and subthemes**.** Identifying the themes and subthemes enabled me to link the relevant literature to the collected data (Patton 2002) and subsequent findings. The themes and subthemes were presented by making use of narrative logic (Mukherji & Albon, 2010). Furthermore, in line with the guidelines of Mukerji and Albon (2010), I discussed each case study as one would tell a story with one concept logically flowing into another.

The next section discusses the elements of trustworthiness, which are credibility, transferability, dependability and confirmability.

3.5 ADDRESSING TRUSTWORTHINESS

Trustworthiness is the extent to which the results of the study are judged to be trustworthy and reasonable (McMillan & Schumacher 2010). I ensured that trustworthiness in conducting the study was achieved in an ethical manner and used data triangulation as advised by Tsai (2013). Semi-structured interviews, structured observations and document analysis were used to ensure that the data collected was valid for the problem investigated. To further ensure trustworthiness and accuracy of the data, a peer-debriefing was conducted (Babbie & Mouton, 2011). I sought assistance of an expert who is a colleague, in the field of early childhood education and training at the institution where she works to review the content of the instruments (observation and interview guides) to enhance validity.

3.5.1 Credibility

Woodside (2010) believes credibility as a process of using multiple realities to scrutinise a process adequately. To ensure data is credible, I engaged participants to confirm the truthfulness and correctness of data collected (Birt, Scott, Cavers, Campbell & Walter 2016). After I had analysed and interpreted the data, I sent back the recorded data to the participants to make comments, suggestions and changes were possible. To further increase credibility, triangulation of data collection methods was employed. I collected data through semi-structured interviews, observations and document analysis.

3.5.2 Transferability

Anney (2014) states that a well-descriptive account of the research study provides sufficient detail for readers to understand the context and situation the phenomenon surrounds. Furthermore, Mqulwana (2010) argues that transferability refers to the degree to which the results of qualitative research can be generalised or transferred to other contexts or settings. Lastly, Creswell (2014) adds that transferability refers to the extent to which the findings in qualitative research can be shifted to similar settings. In this study, I achieved transferability through providing contextual information about the research process, as well as giving detailed descriptions on selection of participants and research sites and providing background information.

According to Creswell and Miller (2000:129) “thick descriptions enables readers to make decisions about the applicability of the findings to other settings or similar contexts.” Hence my study gave detailed and thorough descriptions of the research process to enable transferability.

3.5.3 Dependability

Dependability is about obtaining similar findings with similar participants in the same setting if the inquiry were to be repeated by other independent researchers (Babbie & Mouton 2011). To ensure dependability, I documented each step of the research design by using a journal to keep track of events and to establish categories and emerging themes. I also took advice from Anney (2014) who suggests that dependability can be established by employing the use of an audit trail, stepwise replication or triangulation. With that in mind, I collected data using multiple techniques namely, observations, semi-structured interviews and document analysis.

Maree (2016) states that if a researcher can produce the same findings with participants that are alike in the same research context, then he/she has achieved dependability. The facilitation of emergent mathematics by preschool teachers teaching 3-4-year-old children was observed under similar conditions and context, i.e., the site was the classrooms of 3-4-year-old children and all participants were preschool teachers from Tembisa with at least one year's teaching experience. This means a duplication of the study with similar participants in the same context would produce the same results.

3.5.4 Confirmability

Confirmability is the degree to which the findings are the product of the focus of the investigation and not the biases of the researcher (Babbie & Mouton, 2011). I established confirmability through a reflexive journal and triangulation. In my journal, I carefully separated my thoughts from the events that were occurring, to eliminate bias. As advised by Elo, Kanste, Kaariainen, Polkki, Kyngas and Utriainen (2014), I further increased confirmability by ensuring that accurate data was collected and provided information obtained directly from preschool teachers who were the participants in my study. The following section addresses ethical issues, namely, voluntary participation, informed consent, confidentiality and anonymity and lastly, protection from harm.

3.6 ETHICAL CONSIDERATIONS

As a researcher, I considered ethical practices by protecting participants' human dignity in the field of research. It is emphasised by Johnson (2014) that the researcher should use research ethics when conducting research using human beings. Some of these principles include informed consent, freedom to withdraw, confidentiality, anonymity and privacy (Johnson 2014; Yin (2014).

After obtaining ethical clearance from the University of South Africa to conduct my research (Appendix A), I began the process of collecting data. I wrote letters to the respective principals of the preschools (Appendix B), requesting permission to conduct my study in their school and received consent. I then wrote consent letters to the preschool teachers, requesting them to be participants in my study (Appendix C). Lastly, I then wrote information letters to parents (Appendix D), to inform them of the intended study and to let them know that their children were not direct participants in my research study. The following principles guided the process of confirming ethical conduct during the research study:

3.6.1 Voluntary Participation

Many scholars agree that research should be done out of free will. To confirm this statement, McMillan and Schumacher (2014:130) declared that “voluntary participation means that participants cannot be compelled, coerced or required to participate”. Likewise, Israel (2015) added that participants need to first understand and then agree willingly to the nature of the research, including their role within it. Preschool teachers were informed about the research procedures and assured that participation was voluntary and that they could withdraw at any time during the course of the study, with no negative consequences (Strydom, 2011).

3.6.2 Informed Consent

According to Leedy and Ormrod (2015), research with human beings requires informed consent. The participants who participated in the study were informed and consulted during every step of the research process and consent was obtained before the study commenced. By employing this strategy, a good partnership between the participants and the researcher was ensured.

3.6.3 Confidentiality and Anonymity

McMillan and Schumacher (2014) suggested that no one should gain access to the participants’ identity and/or data. Similarly, Cohen et al. (2005) perceived confidentiality as protecting a participant’s identity. The participants were given the assurance that data accumulated during observations and interviews would stay anonymous and confidential (Leedy & Ormrod, 2015). I used pseudonyms to protect the identities of the participants and to ensure anonymity. Therefore, none of the participants’ names were divulged in any of the data.

3.6.4 Protection from Harm

Babbie (2010) pointed out that harm includes both emotional and psychological discomfort that may be caused to participants. The participants were protected from any form of harm, physical or emotional, by asking questions that were carefully formulated and well-thought out, avoiding posing sensitive and private questions to the participants.

3.7 CHAPTER SUMMARY

This chapter described the research methodology, referred to the research approach, the design and the data collection techniques. It also explored how ethical considerations were observed. The following chapter looks at how the data collected was analysed.

CHAPTER 4: DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

The purpose of this chapter is to present an analysis and discussion of the findings on the study that investigated preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-olds. The first section presents the findings of the preschool teachers' facilitation of emergent mathematics from the lesson observations, followed by interviews and then the document analysis. Thereafter, a summary on the findings from the three phases of data collection is presented and lastly findings are compared to existing literature. The data collection and analysis of this study was guided by the main research question of how preschool teachers facilitate emergent mathematics activities for three-to-four-year-old children to enhance mathematics conceptualisation (See 3.3).

4.2 DATA ANALYSIS PROCEDURES

The data gathering followed the procedure of a multiple case study design (see 3.2.2). Information was gathered from preschool teachers teaching three-to-four-year-olds. The aim was to establish how each one understood and facilitated emergent mathematics activities in their classrooms. I also researched their perceptions and views about emergent mathematics.

Five participants at four different schools were observed. Observations, semi-structured interviews and document analysis were conducted to gather relevant data.

The primary research question and the secondary questions were addressed through thematic analysis by means of specific themes that were identified through the data analysis. The themes were established through studying the relationship between the theoretical framework of the study and the data collected. The case studies are written in a narrative style, to explain each case study as descriptively as possible.

The section also includes photographs taken during the observations. The photographs illustrated the materials, the classroom environment and the activities facilitated by the preschool teachers to show a clear picture of the classroom environment and the resources used by preschool teachers to facilitate emergent mathematics. Any pictures of children's faces have been covered to prevent any identification.

4.2.1 Description of Research Sites

4.2.1.1 Preschool 1 (PS1)

Teacher A taught at Preschool 1, a privately owned ECD centre. Preschool 1 was situated in a residential area in Tembisa. The total number of children in Preschool 1 was 24, and all the age groups were accommodated in one big garage. There were two teachers at this school, and one of them, (participant A) had an ECD Level 4 qualification. The preschool was registered with the Department of Social Development as an Non-Profit Organisation (NPO).

The two teachers divided the children according to age groups, so one teacher taught the age group birth to three years and the other one (Teacher A) taught three- to six-year-olds.

4.2.1.2 Preschool 2 (PS2)

Preschool 2 was a privately owned ECD centre situated in the township of Tembisa. The principal (Teacher B) was the only qualified teacher in the school. The preschool had a total of 40 children and three teachers. The facility was a big garage turned into a classroom, and each age group and teachers had their own space in the big room. The different age groups were birth to two years, two to four years and five to six years. All the children shared the big room and used different spaces allocated to them, specific to their age groups.

4.2.1.3 Preschool 3 (PS3)

Preschool 3 was also situated in Tembisa and was a privately owned ECD centre, registered as an NPO. The building was a big garage, which accommodated 26 children. The different age groups all shared the one garage turned into a classroom. The children were divided into two different age groups, namely, birth to two years and two to four years. There were two teachers.

Participant C was from Preschool 3 and was responsible for the two-to-four-year-olds.

4.2.1.4 Preschool 4 (PS4)

Preschool 4 was situated between Tembisa and Midrand. It was a registered NPO funded by different organisations. It was quite a big school which accommodated children from three years of age to Grade 7. The preschool side had six classes, two classes each per grade of 000 (three-to-four-year-olds), Grade 00 (four-to five-year-olds) and Grade R (five-to-six-year-olds). Each class had two teachers, the main teacher and an assistant teacher. Each class had 18 children. Participants D and E were from Preschool 4.

4.2.2 Description of Participants

Purposeful (also known as purposive) sampling was used to select five preschool teachers, according to a specific sampling criterion. The criteria were that each of the preschool teachers should have a qualification in ECD and that they should at least have one year of teaching experience in an ECD centre in Tembisa. Lastly, the preschool teachers had to be teaching the three-to four-year-old age group. To maintain anonymity and confidentiality of the participants, their names were not disclosed, and they were referred to as Teacher A, B, C, D and E (see Table 4.1). The collected data was organised and identified by these pseudonyms to decontextualise it (Nieuwenhuis, 2007).

In this section, the data is discussed according to each participant's information to highlight the codes, together with a detailed description of the data that was collected (Table 4.2 and Table 4.3). Thereafter, the observations and interviews, including references made to document analysis where applicable, of each participant in this study are outlined. Table 4.1 provides the profiles of the five participants in this study. Thereafter, Table 4.2 and Table 4.3 show the codes of the various participants and school, and data collection methods that were applied to the sections that present the findings.

Table 4.1: Profiles of participants

Teachers	Code	Age	Race/ Home language	Experience	Highest school qualification attained	ECD qualification
Teacher A	TA	55 years	Black African Zulu	6 years	Grade 9/ Form 2	ECD Level 4
Teacher B	TB	48 years	Black African Zulu	2 years	Grade 11	ECD Level 4
Teacher C	TC	52 years	Black African Xhosa	11 years	Grade 11	ECD Level 4
Teacher D	TD	40 years	Black African Shona	5 years	Grade 12	Studying towards a B.Ed. ECD
Teacher E	TE	49 years	Black African Zulu	10 years	Grade 12	Bachelors' degree in ECD

Table 4.2 codes the participants and schools according to their pseudonyms. In the analysis, the codes are used to refer to each individual participant.

Table 4.2: Coding of participants and schools

Participant	Code
Teacher A	TA
Teacher B	TB
Teacher C	TC
Teacher D	TD
Teacher E	TE
Preschool 1, 2, 3 and 4	PS1, PS2, PS3, PS4

Table 4.3: Coding of data type

Data type	Code
Observations	Obs.
Semi-structured interviews	SSI
Document analysis	DA

4.3 DESCRIPTION OF FINDINGS OF EACH CASE

This study comprised five individual cases. In this section, each case is presented by an analysis of the observations, interviews and documents, resulting from the data generation, in relation to the literature reviewed.

4.3.1 Teacher A (TA)

The data from observations and interviews with TA is presented in the following section. Documents presented are also analysed.

4.3.1.1 Observation of TA

TA taught at PS1. TA taught a class of 12 children aged between three and six years old. During observations, it was noticed that TA's classroom had no activity areas to engage children in play-based activities (Figure 4.1).

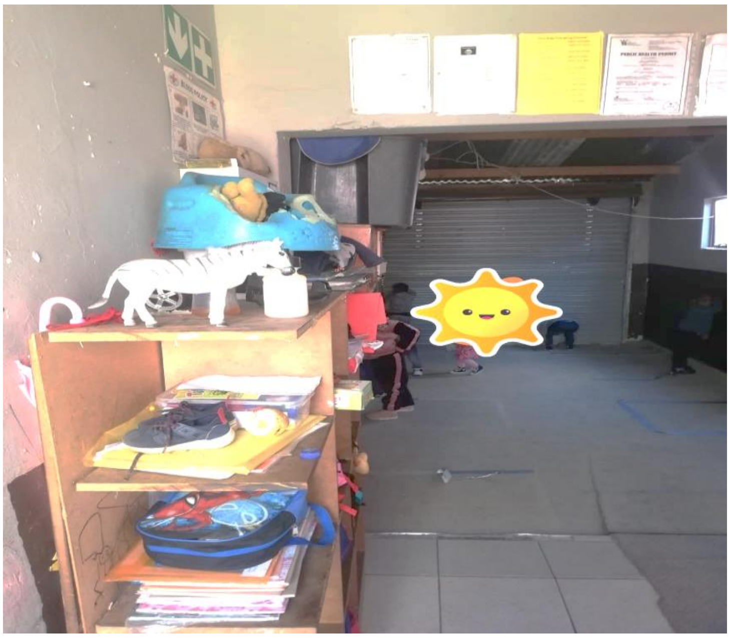

Figure 4.1: TA's classroom layout

A shelf with books and a table were the only items in the large room. This is in contrast with literature which suggests that a well-planned classroom consists of several activity areas (see 2.4.6.) and that for mathematics to happen the environment must be mathematical (see 2.4.6). The only other mathematical evidence in the classroom noticeable on the wall was a number chart and a daily programme (Figure 4.2 below)

Figure 4.2: A number chart and daily programme displayed on the wall.

- **Resources**

There were no visible resources around the classroom, except an abacus displayed on a table and bottle tops (Obs TA). This is contrary to literature (see 2.4.4) which emphasises the importance of resources and that well-planned resources are the basis of high-quality mathematical instruction.

TA presented a teacher-guided activity within the theme “Under the ground”, and the lesson type was indicated as a story (see hand-typed lesson plan, Appendix E).

TA looked confident in her abilities during her presentation. She did all the talking while the children listened attentively, except in one instance where she asked a question. It would seem that TA’s notion of guided play was different from what literature suggests. The role of the teacher, according to Weisberg et al. (2013) during guided activities is to play alongside the children, encourage children to explore and ask open-ended questions. (See 2.4.1.3). TA's presentation was not playful; she did most of the talking while the children sat and listened.

It was evident that TA lacked insight and creativity to create a conducive environment for emergent mathematics to take place. Literature (See 2.4.6) states that a well set-up environment enables children to explore and develop mathematics concepts in a playful way which is antithetical to the practice observed. The implications of this are that preschool teachers who do not create opportunities for or set up activity areas in their classrooms to develop emergent numeracy through play-based activities create obstacles that hamper the development of emergent mathematical concepts.

4.3.1.2 Interview with TA

The interview with TA took place after the observation and lesson presentation. During the interview, I asked TA if the objectives of the lesson were achieved. TA indicated that she was satisfied that the objectives were achieved. TA defined the content area she had aimed to develop in her activity as follows:

> *It was about colours, numbers and different vegetables.* (SSI)

From the response, it shows that TA did not fully understand the term "content areas" and similarly was uncertain about a well-formulated learning outcome.

The instructional approaches TA employed in her classroom were explained as follows:

> *There is no specific instruction. I teach mathematics throughout the day. When they sing, they can sing number rhymes and when they are playing with blocks, they are learning mathematics.* (SSI).

In addition, TA confirmed that there was no specific time allocated for mathematics:

> *I integrate mathematics in all the activities daily and on Fridays, I set aside time for a maths lesson.* (SSI)

TA believed her role in enhancing emergent mathematical understanding during play was to observe and ensure that the children did not harm each other.

> *When the children are playing, I supervise and watch them so that they do not harm each other.* (SSI)

The intention of this question was to find out if preschool teachers were aware of their role during activities. Contrary to her response, literature states that the role of the teacher during children's play is to observe and question, in order to enhance and accelerate understanding of mathematics concepts (See 2.4.1.3).

- **Curriculum guidelines**

TA indicated that she did not follow any curriculum guidelines in her facilitation of emergent mathematics and that she taught from what she learned from workshops that she had attended. Literature states that the use of a curriculum helps preschool teachers in planning and knowing the content to teach. (See 2.3). A curriculum guideline articulates what should be taught and learned before children enter formal school and assists preschool teachers with preparation of activities which are age-appropriate.

> *I have heard of the NCF, but I do not really know what it is about. I teach my children what I have learned from the workshops that I have attended. It would be good to have the curriculum, which will guide us.* (SSI)

Based on TA's response, the emergent mathematics activities planned and facilitated in her classroom were not linked to any specific developmental areas as informed by the NCF (See 2.3). The lack of a curriculum guidance for TA would imply that she lacked proper instructional guidance when it came to planning and presenting emergent mathematics activities.

- **Beliefs and attitude towards emergent mathematics**

TA believed that she could present more meaningful mathematical activities if given the opportunity to learn more. Her response suggests that she felt there was a lot to learn, and she was open to learning to enhance her skills and confidence in presenting emergent mathematics activities.

> *It feels that I must go back to school, I must learn more, I want to know more so that I can teach children the right things.* (SSI)

She further felt that preschool teachers were positive about children's mathematics, but they need more knowledge and skills on how to facilitate it in the right way.

> *They must go back to school or training on how to present emergent mathematics activities in the right way so that they can be more confident. (*SSI*)*

4.3.1.3 Document analysis (TA)

- **National Curriculum Framework**

I requested TA to present me with the NCF or any curriculum guidelines that informed her teaching. My aim was to find out if TA had the NCF, how she understood it and how it informed her facilitation of emergent mathematics activities. The response from TA was that she did not have the curriculum document. The implication of this is that TA might not be facilitating mathematics activities according to the content areas and with clear outcomes as stated in the NCF and might not be aware of activities that enhance emergent mathematics conceptualisation as stated in the NCF (See 2.4.4).

- **Planning documents**

I requested TA to provide me with her daily and weekly lesson plan. The rationale for this was to determine what and how emergent mathematics activities were being facilitated in the preschool programme. TA provided the lesson plan (Appendix E) of the activity she had just presented as she did not have a weekly plan. This implies that TA might be planning on a daily basis, not a weekly basis that might leave her with limited time to gather appropriate resources that are meaningful and engaging.

The lesson plan provided by TA did not correlate with the lesson she presented. The lesson plan indicated that the activity was a number story, which was different from what was observed. This could mean that the teacher decided to change the planned activity and presented another one, or it could mean misinterpreting the written lesson plan.

- **Children's workbooks/sheets**

I asked to see the children's workbooks to see what emergent mathematics activities children engaged in. TA responded that the children took their activity papers home every day to show their parents. There was no evidence of children's work on the wall. (Obs; TA). The workbooks would have given me an indication of the mathematics activities she presented and the mathematical content areas thereof. They could also have indicated the age-appropriateness of the emergent mathematics activities children engaged in.

4.3.2 Teacher B (TB)

The data from observations and interviews with TB are presented in this section. Curriculum documents are also presented and discussed.

4.3.2.1 Observation of Teacher B

- **Facilitation of emergent mathematics**

TB taught at PS2. I observed TB's classroom and outdoor environment, and a mathematics activity to make sense of how she facilitated emergent mathematics activities and how she set up her environment and resources to encourage the development of mathematical concepts. TB taught a class of 15 children between the ages of two and four years old. The classroom was a big and long garage with enough space to move around. There was no evidence of activity areas in the classroom. (See Figure 4.3).

Figure 4.3: TB's classroom layout

Again, the evidence is in contrast with literature which suggests that a well-planned classroom consists of several activity areas (See 2.4.6). The only evidence of numbers in the classroom was on the rules (Figure 4.4)

Figure 4.4: The rules on the wall in TB's classroom

As shown in the picture above, the evidence shows rules. The numbers on the rules do not reflect the right font for preschool numbers.

- **Resources**

There were no visible resources around the classroom except for bottle tops which were placed on the table as depicted in Figure 4.5 below.

Figure 4.5: Bottle tops used for counting in TB's classroom.

When asked to show some of her resources for teaching mathematics, TB responded that she used puzzles and bottle tops to encourage emergent mathematics learning. However she could not present the puzzles, stating that she could not find them where she had packed them. This is not in algnment with what literature suggests regarding learning resources, since resources are an important predictor of children's mathematical achievement in addition to teacher mathematical knowledge. (See 2.4.5). Physical teaching aids can engage children's minds, and this means that access to resources is important to provide high-quality mathematical instruction.

Teacher B presented a teacher-guided activity on mathematics concepts. When asked for a lesson plan, she indicated that she did not have one. The absence of a lesson plan suggested lack of planning by TB.

TB seemed confident as she presented her activity. TB started her activity with a song, then she asked the children to count the bottle tops and to name the colours of the bottle tops. In conclusion to her lesson, TB then asked the children to stick the bottle tops on the matching numbers on the wall. (Obs TB). The activity was short, and the children sat down and listened all the time as the teacher spoke. Evidence suggests TB is not a playful teacher as she presented the activity in a very formal way. This is contrary to literature which states that engagement in playful mathematical experiences helps children develop their abilities in making predictions, solving problems, thinking, reasoning and making connections to the world. (See 2.4.1), TB did not showcase playful learning for emergent mathematics. It also negated the fact that the role of the teacher during play is not a passive one but an active one, to engage in play with the children and ask prompting questions (See 2.4.1.3).

4.3.2.2 Interview with TB

The interview with TB took place after the observation and activity presentation. Teacher B's responses in the post-activity interview and document analysis demonstrated how she facilitated and implemented emergent mathematics in the classroom. TB defined the mathematics content areas she had just presented as numbers and shapes. From the response, it was evident that TB had some understanding of the mathematics content areas.

When asked about the outcomes for the activity and whether they were achieved, TB responded:

> *The outcomes were counting and matching, and the children were able to count the bottle tops and match them with the correct numbers. (SSI)*

The instructional approaches TB employed in her classroom included:

> *I make sure I teach through playing all the time. I just give the children puzzles and bottle tops. (SSI)*

TB indicated that she presented a teacher-guided mathematics activity once a week and for the rest of the time the children learn maths concepts through integration.

- **Curriculum guidelines**

TB indicated that she has heard about a curriculum when she attended a workshop but did not have it. She further stated that she did not even understand what it was. The lack of a curriculum guidance for TB would imply that she lacks proper instructional guidance when it comes to planning and presenting emergent mathematics activities.

When asked what informed her teaching, she indicated that:

> *I use a file that I received from the workshops that I attend regularly. (SSI)*

When asked about which other mathematics content areas she knew she mentioned that she knew about measurement and algebra.

> *Mmmm, I am not sure, I think algebra and measurement. (SSI)*

The response displayed doubt, implying that TB did not have adequate knowledge of the mathematics content areas. The lack of teacher CK suggested that TB might not present opportunities for children to develop other mathematical content areas if she had no full understanding of the content areas herself.

- **Beliefs and attitude towards emergent mathematics**

TB indicated that she loved mathematics and felt happy and comfortable presenting mathematics activities. In responding to whether she believed young children could learn mathematics concepts she responded:

> *Yes, young children can learn mathematics especially at this young age, as all the learning takes place while they are playing.* (SSI)

In addition, TB believed that most preschool teachers loved mathematics and were presenting mathematics activities in their classrooms.

4.3.2.3 Document analysis (TB)

- **National Curriculum Framework**

I asked for the NCF or any curriculum guidelines that informed TB's teaching and she indicated that she did not have any. This is in contradiction with literature which suggests that a curriculum should inform teachers on what activities and resources to plan for so that they can present and facilitate age-appropriate activities. (See 2.3). The NCF serves as a guideline to inform preschool teachers on which activities to present in order to enhance emergent mathematics development. It also clearly states the different mathematics content areas. The implication of this is that TB was presenting mathematical activities which were not informed by the curriculum, and this might lead to children not developing mathematics concepts in all content areas.

- **Planning documents**

I requested TB to provide me with her weekly lesson plan or any planning documentation and she indicated that she did not have any. The rationale for this was to determine what and how emergent mathematics activities were being facilitated in the preschool programme. The lack of any planning documentation suggested poor planning. It also suggested that the mathematics content area that the children are meant to learn in the activity was not clear and that there were no clear outcomes set for the children to achieve by the end of the activity.

- **Children's workbooks/sheets**

I asked for the children's workbooks. The aim was to see what emergent mathematics activities children engage in. TB indicated that they did not have any workbooks since they had just opened recently because of lockdown. There was no evidence of children's work around the classroom (Obs)

The unavailability of the planning documents, children's work and daily programme implies poor planning. This might suggest that TB just went with the flow and taught whatever she felt like, without any form of structure or guidance.

4.3.3 Teacher C (TC)

The data from observations and interviews with TC are presented in this section. Curriculum documents are also presented and discussed.

4.3.3.1 Observation of teacher C

TC taught at PS3. TC taught a group of two-to-four-year-olds in a large classroom. There were 15 children in her classroom. The garage turned classroom was small with very little space to move around. On the walls, there were posters for days of the week, addition and numbers as evident in Figure 4.6 and Figure 4.7. There was no daily programme displayed. (Obs).

Figure 4.6: TC classroom layout

Figure 4.7: A poster displaying the days of the week and concept of addition

Figure 4.8: Number cards on the wall

On the same wall was a colourful assortment of numbers and number symbols from 1 to 10. (See Figure 4.8 above). This confirms what literature states in that the environment should be mathematical for mathematics learning to happen. (See 2.4.6). However, there were no activity areas in the classroom to engage the children in mathematical exploration and discovery (Obs TC). which contradicts what was stated in Chapter 2 (see 2.4.6) that a well-planned classroom should consist of several activity areas.

- **Resources**

When asked to show the resources she uses for mathematics activities, TC brought a few blocks and one set of a puzzle with missing pieces as shown in Figure 4.9:

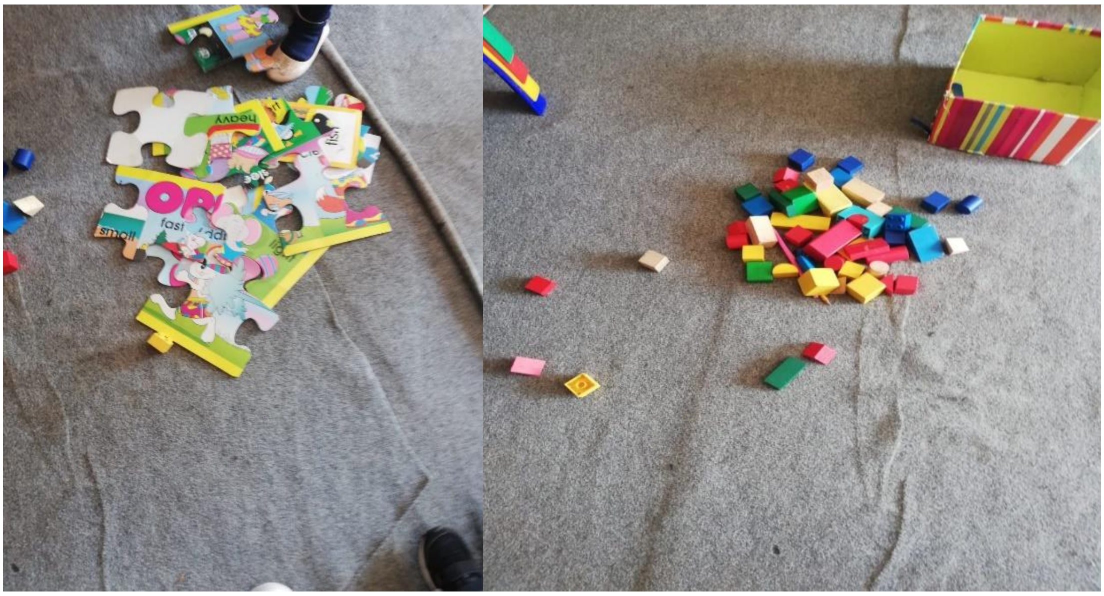

Figure 4.9 : Puzzles and blocks for use in the classroom

Given that literature suggests that children should be provided with many objects to manipulate and create mathematical concepts (See 2.6.2), the lack of resources that were evident can hinder the development of mathematical concepts.

TC presented a teacher-guided activity. TC did not have a lesson plan available when presenting her activity. She introduced her activity by singing the song; “five green leaves” with the children while standing up. After this, all the children faced the wall to recite the numbers on the wall. TC used the number chart as her resource for the activity. TC was very confident as she recited the numbers together with the children. TC pointed to each number using a stick and asked the children to say what number it was and the colour of the number. In conclusion, TC asked the children to clap from number 1 up to number 10.

It was evident that the children were distracted and did not enjoy the activity. The children did not have any manipulatives to play with during the activity. The activity was not presented in a playful way and the teacher was in control all the time. This evidence negates with literature which suggests that teachers and learners should share authority in a constructivist learning environment and that the teacher should take the role of a guide or facilitator (See 2.6.4). Literature further suggests that children learn mathematical concepts best through hands-on activities and in a playful way (See 2.4).

4.3.3.2 Interview with TC

The interview with TC took place after the observation and lesson presentation. TC's response demonstrated how she facilitated and implemented emergent mathematics in her classroom. During the interview, the researcher asked TC for a lesson plan, and she said she did not have one.

TC indicated that she did not have a lesson plan but her intended outcomes for the lesson were that the children should know the primary colours and learn to count. When asked whether she had achieved the outcome TC responded as follows:

> *I think so, the children were able to count at the end of the lesson. (SSI)*

It is evident from TC's response that she knew what she intended the children to learn during the activity even though she did not show evidence of planning.

When asked about which content areas she had presented in her lesson, she responded that it was counting and colours.

> *Mmmm I am not sure, I think it was numbers and colours.* (SSI)

It can be concluded from TC's response that she was not sure about mathematical content areas.

When asked about the instructional approaches she employed in her classroom she responded as follows:

> *Each day in the morning after prayer, they start counting from 1 up to 10, and the alphabet. Throughout the day they also learn maths. Even when going to the toilet, they count the number of steps they are taking. I also make them write numbers in the workbooks every day.* (SSI)

From TC's response, it was evident that she had some understanding of play and integration. TC believed that her role during play was to play with the children.

- **Curriculum guidelines**

When asked about the curriculum guidelines she used, TC indicated that she did not have any and she had only heard a little information about the NCF on TV. From her responses, it was evident that she did not understand what a curriculum guide was.

- **Beliefs and attitudes towards emergent mathematics**

When asked about how she felt when planning and presenting emergent mathematics activities, she responded:

> *I really feel good because I love mathematics a lot.* (SSI)

TC believed that children could learn mathematical concepts even from birth.

When asked about other teachers' perception on emergent mathematics she responded:

> *I think other teachers are not sure of whether young children can learn mathematics or not, so they have a negative attitude and think that children will learn maths concepts in Grade 1.* (SSI)

4.3.3.3 Document analysis (TC)

- **National Curriculum Framework**

I asked for the NCF or any curriculum guidelines that informed TC's teaching, and she indicated that she did not have any. This is in contradiction with literature that suggests that a curriculum should inform teachers on what activities and resources to plan for, so that they can present and facilitate age-appropriate activities. (See 2.3). The NCF serves as a guideline to inform preschool teachers on which activities to present to enhance emergent mathematics development. It also clearly states the different mathematics content areas. The implication of this is that TC was presenting mathematical activities which were not informed by the curriculum, and this might lead to children not developing mathematics concepts in all content areas.

- **Planning documents**

I requested TC to provide me with her weekly lesson plan or any planning documentation and she indicated that she did not have any. The rationale for this was to determine what and how emergent mathematics activities were being facilitated in the preschool programme. The lack of any planning documentation suggested poor planning and poor practice.

- **Children's workbooks/sheets**

The analysis of the workbooks on Figure 4.10 below revealed that children's activities consisted of one content area which is numbers. The children either coloured numbers, traced numbers or made number collages.

Figure 4.10: Children's workbooks/sheets

The evidence shows that there was more emphasis on numbers than other mathematics content areas. This is in contrast with literature that suggest that preschool teachers should include children in activities that involve numbers, patterns, measuring and problem-solving (See 2.4.4). A careful analysis of the workbooks further revealed that the teacher marked the children's workbooks and added comments. This implies that in TC's classroom there was more emphasis on writing and workbooks than playful learning, which confirmed what was said during the semi-structured interview (see 4.3.3.1). This is in contradiction with Piaget's theory of development which strongly states that young children should learn from their environment by using concrete manipulatives in more fun and playful ways. (See 2.6.2).

4.3.4 Teacher D (TD)

The data from observations and interviews with TD are presented in this section. Curriculum documents are presented and analysed.

4.3.4.1 Observation of Teacher D

- **Facilitation of emergent mathematics**

TD taught three-to four-year-olds at PS4. TD's classroom was very spacious. On the one side were children's tables and chairs and on the other was a big mat where children sat for ring presentations. The tables had a divider to divide the children to observe social distancing (See Figure 4.11). There were 18 children in this classroom and TD had an assistant teacher.

Figure 4.11: TD's classroom layout

There was a theme table at one end of the classroom displaying the theme of the week, which was ‘watch these grow’, a birthday chart, children’s worksheets displayed on the wall as well as a pictorial daily programme as seen in Figure 4.12 below:

Figure 4.12: Birthday chart, theme table and pictorial daily programme

TD’s classroom environment was mathematically rich and allowed for mathematics learning to happen, which is confirmation of what literature suggests. (See 2.4.6).

TD presented a teacher-guided activity on numbers, as indicated in her lesson plan, which she handed over to me for my perusal (see Appendix F). She started her activity with a discussion on weather. TD then sang the song, ‘1,2 buckle my shoe’ with the children.

TD then gave each child a set of five blocks (See Figure 4.13) and led them into an activity on counting. Each child had a set of five blocks and was engaged.

Figure 4.13: Blocks for counting

The children seemed to be having fun counting their blocks using one-to-one correspondence and following the teacher's instructions. TD concluded her lesson by singing the song 'Two little blue birds' with the children. The evidence from this activity confirmed that preschool teachers should provide learning activities and materials that stimulate children's interest to play and to make sense of mathematics concepts. (See 2.4.4). TD's presentation aligned with findings from literature in that one-to-one correspondence should be the focal point in teaching children at preschool level and is one of the most fundamental components of the concept of number (See 2.2.1). However, her focus was only on numbers and TD did not use the opportunity to integrate other mathematics content areas in her lesson. This contrasts with the suggestion that preschool teachers should include children in activities that involve numbers, patterns, measuring and problem-solving (See 2.4.4). TD focused only on counting, but she could have brought in patterns, measurement and even sorting and classifying using the blocks.

4.3.4.2 Interview with Teacher D

The interview with TD took place after the lesson presentation. Teacher D's responses in the interview and document analysis demonstrated how she facilitated and implemented emergent mathematics in the classroom.

When asked if the outcomes for the activity were met, TD replied yes.

> *Yes, the outcomes were achieved. At the end of the lesson, the children could count from 1 up to 5 and they developed eye-hand coordination at the same time.*

It was evident from TD's response that she knew exactly what she wanted to achieve in her lesson as stated on her lesson plan (See lesson plan Appendix F).

TD indicated that the content areas she had aimed to teach in her activity were numbers. Asked on her understanding of other content areas she responded:

> *Mm, I know shape and geometry.* (SSI)

The response indicated that TD did not have full understanding of the other mathematics content areas. On some days, TD might be presenting activities related to other mathematics content areas, but it might not be intentional as she did not have full understanding of the other content areas. This was also evidence that TD lacked knowledge of the curriculum. This counteracts the idea that PCK is one of the most important knowledge bases that the teachers should possess to teach effectively where teachers should have knowledge of the content they teach and the methodology (pedagogy) of how to present the content effectively. (See 2.6.5).

TD indicated that the instructional approaches she employed in her classroom were integration and playful methodologies.

TD's responses matched what was in her weekly lesson plan (see Appendix G). It is evident that TD had knowledge of the instructional approaches (pedagogical knowledge) for emergent mathematics and the early childhood programme in general.

TD said that her role during mathematics activities was to observe and play with the children.

Regarding how often she taught mathematics in a week, TD indicated that she integrated mathematics every day in her classroom and twice a week she taught a teacher-guided mathematics lesson. This was supported by the weekly plan provided which showed mathematics activities being presented twice a week.

- **Resources**

TD indicated that she used blocks, puzzles and yoghurt containers to present emergent mathematics activities. She further indicated that the resources were, however, packed away because of Covid-19. Although she stated that she used resources, she could not name more than two. Literature suggests the use of a variety of everyday objects in facilitating and explaining emergent mathematics concepts (see 2.4.4)

- **Curriculum guidelines**

TD indicated that she did not have a curriculum guideline and that she used the daily programme and the lesson plan and read up on the internet on topics regarding her theme of the week. TD did not know much about the NCF; she had just read about it briefly in one of her university books.

> *The principal gives us the topics of the year in the year-plan, and we do research for information to teach on the topic.* (SSI)

TD's response implies that she was not informed by the curriculum in her facilitation of emergent mathematics activities. This contrasts with literature which views curriculum knowledge as very important for both planning and presentation of activities. Curriculum knowledge is the knowledge of programmes designed for the teaching of particular subjects and topics at a given level and knowledge of the available variety of instructional materials that relate to the teaching and learning of those programmes.

- **Beliefs and attitudes towards emergent mathematics**

When asked about how she felt when planning and presenting emergent mathematics activities, TD said that it made her happy especially when the children discovered new things. TD also believed that young children cannot learn much mathematics and they get bored when teachers continuously speak about numbers.

When asked about other teachers' perceptions on emergent mathematics TD responded:

> *I think they love and enjoy teaching mathematics and they have seen that numbers are important, as well as rote counting*. *They should go back to school to get better understanding for teaching emergent mathematics*. (SSI)

4.3.4.3 Document analysis (TD)

- **National Curriculum Framework**

I asked for the NCF or any curriculum guidelines that informed TD's teaching. The rationale for this was to establish which curriculum guidelines were being employed and whether the guidelines were being followed. TD indicated that she did not have any. Yet again, this contradicts the suggestion that a curriculum should inform teachers on what activities and resources to plan for, so that they can present and facilitate age-appropriate activities (See 2.3). The NCF serves as a guideline to inform preschool teachers on which activities to present in order to enhance emergent mathematics development. It also clearly states the different mathematics content areas. The implication of this is that TD was presenting mathematical activities which were not informed by the curriculum, and this might lead to children not developing mathematics concepts in all content areas. It also suggests that she was not implementing emergent mathematics according to the content prescribed by the NCF.

- **Planning documents**

I asked TD if she had any other planning documents apart from the daily lesson plan she had handed me earlier to establish how emergent mathematics activities were being facilitated; whether there was evidence of planning; which resources were used when facilitating emergent mathematics; and what approaches she was going to use. I wanted to find out whether the lessons catered for the development of emergent mathematics concepts for individual children, for groups and not only for the entire class.

TD also presented a copy of the weekly plan (see Appendix G). From the planning documents, it was evident that TD followed her schedule for planning. I established that there was evidence of thorough planning based on the documents provided. The documents displayed that most of the activities were adult-guided and a few activities were child-initiated; however, the evidence indicated that emergent mathematics was being integrated in the daily activities. There was no evidence of group activities, but since the analysis was done on a weekly plan, this did not mean that the teacher did not present group activities in her facilitation of emergent mathematics activities. The weekly plan provided also showed evidence of integration of mathematics concepts in other learning areas.

TD also presented her daily programme (see Appendix H). The daily programme showed evidence of a well-structured and balanced programme. In relation to mathematics, the evidence showed that there was a daily slot for catch up in numbers, shapes and colours, showing that there was time spent daily on numbers, shapes and colours. This indicated that TD put emphasis on the two content areas of numbers and shapes, even though the NCF prescribes other content areas. The implication of this is that children in TD's class might excel in the two content areas that are emphaised daily, but they might show deficiencies in the othert content areas like patterns, measurement and data handling.

- **Children's workbooks/sheets**

I requested the children's workbooks. The aim was to establish what kind of emergent mathematics activities were children doing in their workbooks and the frequency thereof. The evidence suggests that the children used the workbooks mostly for colouring and drawing.

4.3.5 Teacher E (TE)

The data from observations and interview with TE are presented in this section. Documents are presented and analysed.

4.3.5.1 Observation of Teacher E

- **Facilitation of emergent mathematics**

TE taught three-to four-year-olds at the same school as TD at PS4. TE had 18 children in her classroom and an assistant teacher. TE’s classroom was spacious and allowed for easy movement of children (See Figure 4.14)

Figure 4.14: TE’s classroom layout

TE’s classroom was mathematically rich. On the walls were a number chart, birthday chart, a clock, and in one corner was a maths area and in another area was a shape area as indicated in Figure 4.15 below:

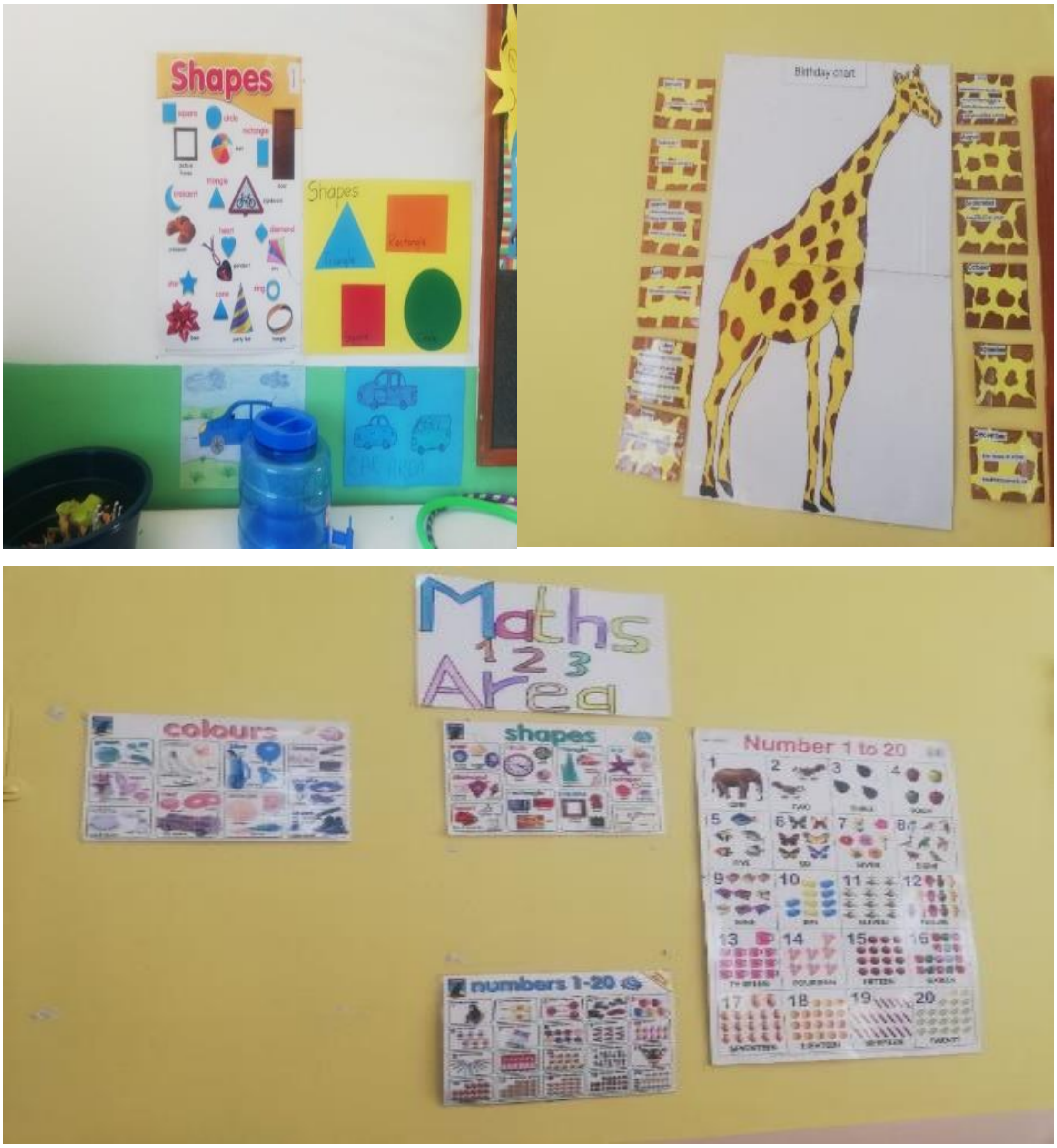

Figure 4.15: Shape area, birthday chart, and maths area

In support of this, literature suggests that the classroom environment for young children should be mathematically rich to enable children to develop mathematical concepts and that the classroom should have an array of displays on the walls, which are colourful and inviting (See 2.4.6). Observations from TE's classroom showed that the young children were exposed to mathematics and could explore and discover mathematical concepts as they played around the classroom.

Like TD, TE also presented a teacher-guided activity on the theme "Watch these grow" since they were in the same school and planned the activities together.

TE introduced her lesson by asking the children to rote count numbers 1 up to 10, then counting their fingers from 1 up to 10 and then backwards from 10 to 1. TE then handed each child a paper plate, clipper, a small container and ten bean seeds (See Figure 4.16). TE instructed the children to each pick the seeds using the clipper, one at a time and put them in the container, while counting them.

Figure 4.16: Resources for the activity: paper plate, bean seeds, small container and clipper

TE called on one child to count while everyone listened and rewarded the child for counting correctly by placing a star on the child's forehead. (Obs).

At the conclusion of her lesson, TE and the children sang the song, 'Two little dickie birds'. She then gave the children worksheets to colour the seeds (See Figure 4.17).

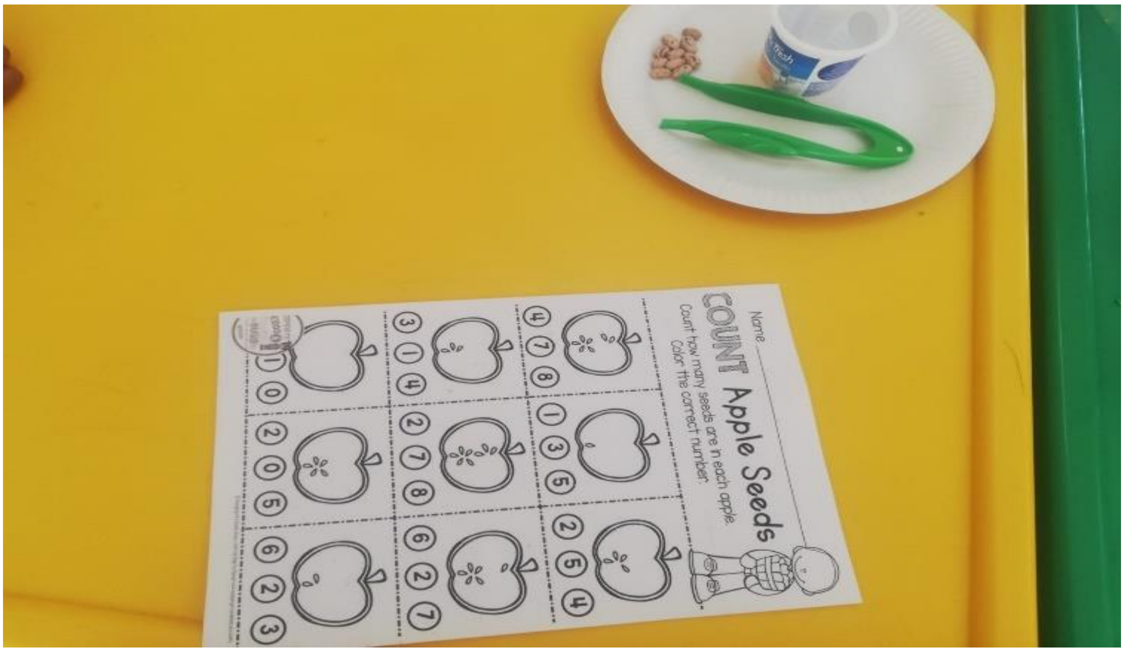

Figure 4.17: Worksheets to colour in

4.3.5.2 Interview with teacher E

The interview with TE took place after the lesson presentation. Teacher E's responses in the interview and document analysis demonstrated how she facilitated and implemented emergent mathematics in the classroom.

When asked if the outcome for the activity were met, TE responded:

> *Yes, the children were able to count forwards and backwards and they were able to count the bean seeds. (SSI)*

It was evident from TE's response that she knew exactly what she wanted to achieve in her lesson as stated in her lesson plan. TE indicated that the content areas she had aimed to teach in her activity were numbers and counting. Asked on her understanding of other content areas she responded:

> *I know geometry, measurement, patterns and data handling. (SSI)*

The response is evident that TE has full understanding of the other mathematics content areas. This is also evidence that TE had knowledge of the curriculum. This confirms the literature that recommends that teachers should possess knowledge of the content that they are teaching children (See 2.6.6)

When asked about the instructional approaches she employed in her classroom she responded as follows:

> *I integrate mathematics activities in the daily programme through play, and I also ensure that, once a week, I present a teacher-guided mathematics activity like the one you have just observed.* (SSI)

TE's response matched what was in her weekly lesson plan. It is evident that TE has knowledge of the instructional approaches (pedagogical knowledge) for emergent mathematics and the early childhood programme in general.

TE indicated that her role during emergent mathematics activities was to play with the children and observe them. Regarding how often she taught mathematics in a week, TE indicated that she integrated mathematics every day in her classroom and every Thursday she taught a teacher-guided mathematics lesson.

- **Resources for emergent mathematics activities**

I requested to see the resources TE used for mathematics activities. The rationale for this was to establish which resources she used in her classroom to enhance emergent mathematics conceptualisation, and whether the resources were appropriate for stimulating emergent mathematics development. TE responded:

> *I am using different things, I have puzzles, number boards, coloured bears for sorting, shape boards for matching and bottle tops for counting. (SSI)*

Figure 4.18: Bottle tops, shape board, small bears of different colours for sorting, puzzle and number board for teaching mathematics

TE’s response and evidence showed that she was aware of resources that provided concrete experiences to help children make sense of mathematics and build their mathematical thinking which is in line with literature (See 2.4.5).

- **Curriculum guidelines**

When asked about the curriculum guidelines she employed in her classroom to facilitate emergent mathematics activities, she responded:

> *I use the topics given by our principal to research on my lessons. Sometimes I use the Curriculum Assessment Policy Document (CAPS) papers to just check the information and I also enquire from other teachers.*

Asked on whether she has heard about the NCF, her response was:

> *Yes, but I do not know about it.*

The evidence suggests that TE used the CAPS curriculum at times. However, the CAPS curriculum is officially for children from Grade R to Matric.

- **Beliefs and attitudes towards emergent mathematics**

When asked about how she felt when planning and presenting emergent mathematics activities, TE said that it made her feel good because she loved mathematics. TE also believed that young children can learn mathematical concepts easily. When asked about other teachers' perceptions on emergent mathematics, TE responded:

> *They have a positive perception. My colleagues love teaching mathematics.*

4.3.5.3 Document analysis (TE)

- **National Curriculum Framework**

I asked for the NCF or any curriculum guidelines that informed TE's teaching. The rationale for this was to establish which curriculum guidelines were being employed and whether the guidelines were being followed. TE indicated that she did not have any but at times she used CAPS papers. When asked for the CAPS papers, she did not have any at the time of the interview. This does not align with literature which suggests that the curriculum should inform teachers on what to teach, and includes objectives, teaching content, strategies and assessment methods (See 2.3). It is the document that should direct every teacher and inform them on what to teach and how to teach. The implication of this is that TE was presenting mathematical activities without clear objectives and strategies. It also implies that TE did not have the prescribed teaching content for mathematics, which was appropriate for her particular age group.

- **Planning documents**

I asked TE if she had any other planning documents apart from the weekly lesson plan (See Appendix G) she handed me earlier. The rationale for requesting the planning documents was to establish how emergent mathematics activities were being facilitated; whether there was evidence of planning; which resources were used when facilitating emergent mathematics; and what approaches the teacher was going to use. I wanted to find out whether the lessons catered for individual children, for groups and not only for the class as a whole. TE had no daily lesson plans and she used the weekly plan in the facilitation of activities.

TE also presented a copy of the daily programme which she shared with TE (see Appendix 8). Similar to TD, it was evident that TE followed her schedule for planning. Evidence suggested adequate planning. The daily programme also showed evidence of structure and balance in the facilitation and presentation of activties. However, like TD, it would seem that the emphasis in relation to emergent mathematics activities was placed on numbers and shapes, resulting in neglect of the other content areas.

- **Children’s workbooks/sheets**

I asked for the children’s workbooks. The aim was to establish what kind of emergent mathematics activities children were doing in their workbooks and the frequency thereof. TE used worksheets for consolidation after an activity.

Figure 4.19: Worksheet

4.4 DATA INTERPRETATION

The process of thematic analysis was employed to understand the facilitation of emergent mathematics by preschool teachers. Thematic analysis was used because it is an appropriate and powerful method to use when seeking to understand a set of experiences, thoughts and behaviours across a set of data (Braun & Clarke, 2012). I also worked inductively, and I was informed by my research questions to generate themes.

After transcribing the interviews, I read and re-read the text to familiarise myself with the data. In formulation of themes based on the codes and organising the themes, I attached meaning and significance to the data. In interpreting and analysing data, I searched for emerging patterns, associations, concepts and explanations in the data to construct new meaning and understanding (Nieuwenhuis, 2016b). As the data were analysed, I began to see patterns and themes; and as the coding layout continued to develop, I obtained inter-rater reliability by requesting one of my colleagues to read one of my interview transcripts to test my codes. I discussed and reconciled any discrepancies that resulted from the independent review by my colleague. This was done with each of the transcripts. Such differences led to additional exploration of data.

Exploration of such differences in which further clarification was needed, helped to refine how I stated my findings including the ensuing analysis and recommendations (Bloomberg & Volpe, 2016).

A table is presented below to summarise and analyse each of the case studies, which are presented according to the codes, categories and themes from the generated data. The codes are highlighted according to patterns that emerged, namely, teacher-guided activity, scaffolding, play, classroom space, PCK, curriculum and manipulatives.

The categories that emerged resulted in the following six themes: Facilitation of emergent mathematics activities and instructional practices, teachers' PCK of mathematics, understanding the curriculum, resources for the facilitation of emergent mathematics activities, planning, perception and beliefs about emergent mathematics. The views on the facilitation of emergent mathematics activities that came to the fore in the themes are used as a starting point for the discussion in Chapter 5.

Table 4.4: Synopsis of codes, categories and themes according to data generated

Themes	Categories	Codes				
		TA	**TB**	**TC**	**TD**	**TE**
Facilitation of emergent mathematics and instructional approaches	Teacher-guided activity Scaffolding Play Space Teacher's role	TA presented a teacher-guided activity but said children learn mathematics incidentally throughout the day. Children played on their own most of the time with no toys. Teacher not involved in play Limited space in the classroom to play freely.	TB presented a teacher-guided activity. Children played alone most of the time with very few toys. Limited space in the classroom. Children played on their own.	TC presented a teacher-guided activity. Believes children learn mathematics while playing. TC facilitates emergent mathematics by integrating it into the daily programme and making children write in the workbooks daily. Limited space in the classroom. Children played on their own.	TD presented a teacher-guided activity on counting. TD had a lesson plan. She asked questions which were intended to give clues to her main questions (scaffolding). She used blocks for counting. Large space and children played freely.	TE presented a teacher-guided activity. She interacts with them and asks open-ended questions. TE facilitates emergent mathematics activities through playing and integration into the daily activities. Large space and children played freely with a variety of toys.
Teachers' PCK of mathematics	Pedagogical content knowledge	Knew little about the content areas of emergent mathematics. She presented an activity on data handling and referred	TB showed limited PCK in her lesson. She defined shapes, classifying and sorting but in a generalised way.	TC showed little knowledge of mathematics content areas and what they entail.	TD had a better idea of PCK. "Measurement is about measuring objects using hands or wool".	TE had a better knowledge of content areas.

Themes	Categories	Codes				
		TA	**TB**	**TC**	**TD**	**TE**
		to it as a counting activity.				
Understanding the curriculum (NCF)	The National Curriculum Framework Lesson planning	TA has heard of the curriculum but knew so little about it. Suggested that it would be good to have it. TA lacks proper and clear instructional guidelines in her teaching.	TB uses a file from the workshop she attended. She has heard about the NCF but does not understand what it is. Lacks proper guidelines and structure in her teaching.	TC has heard about the NCF on TV but knows so little about it. "It is the chart that I am using most of the time and the blocks".	TD has heard about the NCF and thinks it is also a curriculum. She does not have it, she uses the daily programme, lesson plans and the internet. "The principal gives us topics of the year and we do research on the topics". TD's lesson is well-structured and shows evidence of planning.	TE has heard about the NCF but does not know much about it. She uses the CAPS papers to just check the information and enquires from other teachers. The principal also gives her the topics of the year and she does research with colleagues. TE's facilitation of emergent mathematics is well-structured and lesson plan has clear outcomes.

Themes	Categories	Codes				
		TA	**TB**	**TC**	**TD**	**TE**
Resources for the facilitation of emergent mathematics	Manipulatives in a constructivist classroom	TA has very few resources in the classroom. There were only bottle tops, an abacus and shape boards.	TB only presented bottle tops when asked to show resources.	TC had a few blocks and a puzzle which did not have all the pieces.	TD said she uses blocks, bottle tops, flash cards and objects of different shapes but could not present them as they had packed them away due to Covid - 19	TE had a very well-resourced classroom. She had puzzles, coloured bears, shape boards, a clock, bottle tops, and number boards.
Planning for emergent mathematics activities	Planning documents Lesson presentation	TA had a lesson plan but she did not present her lesson according to the plan. She seemed that she did not know what the lesson was about. There was little evidence of planning.	TB had no lesson plan. The lesson was not clearly structured and used only bottle tops.	TC had no lesson plan. There was no evidence of planning and she did not have resources for the activity.	TD had a lesson plan. She presented her activity according to the lesson plan. It showed she had planned her lesson thoroughly and resources were on children's table before the lesson.	TE had a lesson plan. She presented an activity on counting even though her lesson plan indicated that it was a sorting activity. She had resources prepared beforehand and showed evidence of planning.
Preschool teachers' perception on facilitating emergent mathematics activities	Beliefs and attitudes towards emergent mathematics Beliefs of children's mathematics learning	TA had a positive perception about emergent mathematics. Said presenting mathematics activities makes her feel like she must go back to school and learn more. "I	Positive perception. "I feel so happy to present maths activities because I love maths, and I believe young children can learn	Positive perception. "I feel really good when presenting mathematics activities, and I believe young children can learn	Positive perception about facilitating emergent mathematics activities. "It makes me so happy to present mathematics activities, however I	Positive perception. "I feel good when teaching mathematics and I believe that it is easy for children to learn concepts at a young age".

Themes	Categories	Codes				
		TA	TB	TC	TD	TE
		believe young children can learn”	maths concepts at a young age”.	mathematics very well”.	do not think children can learn so much at this young age”.	

4.5 SUMMARY OF EMPIRICAL FINDINGS IN RELATION TO THEORETICAL FRAMEWORKS

The aims of the study (see 1. 6) were to investigate how preschool teachers of three-to-four-year-olds facilitate and implement emergent mathematical activities to enhance mathematics conceptualisation in the early years of development. This section summarises the findings in relation to the theoretical framing of the study, and in relation to the research aims.

Under the first aim, which explored how preschool teachers facilitate emergent mathematics to enhance the conceptualisation of emergent mathematics activities, data confirmed that the preschool teachers facilitated mathematical activities in very formal ways. This is in contradiction with the theoretical framing from this study, as Piaget suggested that children should construct their mathematical knowledge by interacting with the concrete environment (see 2.6.2). Furthermore, the suggestion that preschool teachers should provide physical objects for children to manipulate and construct mathematical understandings through play did not come to the fore in the data. Preschool teachers in this study did not have the different areas that would allow children to develop mathematical concepts through play and had few resources in their classrooms. This refutes Piaget’s notion that the teachers’ role is that of a facilitator, and should offer an appropriate learning environment and concrete manipulatives that helps formulate emergent mathematical ideas. In addition, Vygotsky’s framing (see 2.6.3.1) suggests that teachers should engage with children during play activities to enhance conceptualisation of emergent mathematics activities. However, empirical evidence is in contradiction with this stance since teachers in this study did not involve themselves in children’s play.

The second aim of this study was to investigate the knowledge and skills needed to facilitate emergent mathematics activities and empirical findings revealed that preschool teachers lacked PCK to facilitate effective emergent mathematical activities with young children. Theories regarding PCK (see 2.6.5) state that preschool teachers should have knowledge of the curriculum, knowledge of the mathematical content and knowledge of effective teaching methodology. Preschool teachers from this study lacked the mathematics CK which is essential for planning and facilitating emergent mathematics activities. Data revealed that teachers facilitated activities using very formal methods which proved that the skill and knowledge for facilitating emergent mathematics activities using the playful learning methodology were lacking.

The third aim of this study was to investigate and find out what curriculum guidelines were used and how they were used in facilitating emergent mathematics activities and the fourth and final aim of this study was to find out how the preschool teachers perceived and explained the importance of emergent mathematics. These are summarised in relation to literature in Chapter 5.

4.6 CHAPTER SUMMARY

Five case studies of preschool teachers were discussed and presented in relation to their understanding and implementation of emergent mathematics. The aim of the chapter was to present the research findings according to a case study design and to identify broad categories and themes that emerged. The data collection techniques employed provided enough data to present the participants' experiences, opinions and knowledge of the facilitation of emergent mathematics. A summary of empirical findings in relation to theoretical framings was given.

In Chapter 5, the research questions are answered and the findings and recommendations of this study are presented.

CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In Chapter 4, I presented an analysis of the empirical data according to the themes that emerged during coding in order to provide a deeper insight into the preschool teachers' facilitation of emergent mathematics activities. A summary of the empirical findings in relation to theoretical framings was presented. This chapter presents a comparison between the empirical results of this study to relevant concepts as well as theories from the literature review provided in Chapter 2. The research questions, as indicated in Chapter 1, are addressed, and answered. Findings, recommendations, ideas for further research and final thoughts are given. The chapter begins with a synopsis of the previous chapters followed by the findings from literature and conclusions from the empirical study.

5.2 SUMMARY OF DISSERTATION

A brief overview is provided of the preceding four chapters of this study, highlighting key areas that were of significance to this research.

Chapter 1: This chapter gave the contextual background and set the scene for the whole research study, with specific reference to the rationale, problem statement as well as aims and objectives of the study. Primary and secondary research questions were highlighted. The theoretical framework underpinning this study, as well as the literature review were introduced. Lastly, the research methodology was presented as well as the ethical issues guiding this study.

Chapter 2: This chapter explored the literature surrounding preschool teachers' facilitation of emergent mathematics activities with 3-4-year-old children. The literature helped me to gain insight into my study based on the previous scholars' voices and views on the facilitation of emergent mathematics activities. I also presented the constructivist theory from the lenses of Piaget's cognitive theory and Vygotsky's sociocultural theory. Lastly, the PCK theory from the lenses of Shulman and Grossman was also presented and discussed.

Chapter 3: This chapter outlined the qualitative research methods for gathering my data. These methods comprised semi-structured interviews, observations and document analysis. I also established and presented ethical considerations guiding this study.

Chapter 4: This chapter provided an explanation of how the data was organised and the procedure for the identification of themes and categories in order to make sense and give meaning to data was also presented. Emerging themes, which cut across the five cases, provided answers on the facilitation of emergent mathematics activities.

The results of this study were compared to existing literature from Chapter 2. Table 5.1 shows the relationship between literature, theory and the research findings according to the themes and subthemes that emerged from the study.

Table 5.1: Comparing results in existing knowledge

Themes and subthemes	Result	Literature	Theory	Interpretation and findings
<u>Theme 1:</u> **Facilitation of emergent mathematics activities**	The lessons presented during all the observations were not playful and were very formal. All activities were structured, and teacher-directed. One teacher had evidence of workbooks which children wrote in everyday as a way of learning mathematical concepts.	According to Naude and Meier (2014), children learn best through hands-on activities, where they work concretely to make sense of mathematics.	Constructivism describes the process of knowledge as an active rather than a passive process (Major & Mangoepe, 2012). This means that children construct mathematical knowledge and ideas by actively engaging with the environment.	The findings in this study did not align with literature or the theories framing this study in finding that preschool teachers focused more on teacher-guided formal instruction in facilitating emergent mathematics activities, and not on creating more opportunities for mathematics concepts to develop through hands-on activities where children explore and discover for themselves.
Subtheme 1.1 ***Facilitating emergent mathematics through play activities***	Despite preschool teachers explaining that children learn mathematics through play, there was no evidence of that during the observations. The children played on their own and there was no evidence of the different play areas like fantasy, maths, book or	Moss, Bruce and Bobbis (2016) stated that preschool children learn mathematics through play and their teachers should not be passive in their efforts to facilitate that learning. Preschool teachers need to observe and question children during play. Children's	Vygotsky (1978) stated that teachers have an active role to play during children's play. In the same vein, Vygotsky stated that the teacher should be involved in children's play and encourage learning of concepts and problem-solving by supporting and guiding the	Preschool teachers from this study believed that children can learn through play, but that the teachers' role was only to observe and make sure they were safe. This differs from both literature and theory which suggest that the preschool teacher's role is to question and

Themes and subthemes	Result	Literature	Theory	Interpretation and findings
	science area except in one classroom. The preschool teachers believe that their role during play is to observe and ensure children do not harm each other.	development can be enhanced and even accelerated through giving commentary while children are playing, co-playing alongside others, encouraging children to explore materials in new ways and asking children open-ended questions (Weisberg et al., 2013).	children until they could perform a task without adult assistance. Preschool teachers should provide children with support and guidance to understand mathematical concepts (Vygotsky, 1962).	"scaffold" learning and guide them towards understanding mathematical concepts.
Subtheme 1.2 Facilitating emergent mathematics activities through integration.	Only two of the preschool teachers understood and knew that emergent mathematics activities should be integrated into the daily programme implying that most teachers were not doing this.	Greenberg (2012) maintained that being aware of early mathematical concepts helps teachers to be more thoughtful and intentional about using these concepts in everyday experiences and interaction with infants and toddlers.	Bada et al. (2015) suggested that a constructivist environment is that which integrates mathematics activities throughout the day through playful activities.	Findings reveal that most preschool teachers in this study did not integrate mathematics activities in the daily programme, even though they had a vague understanding of what it was and what it entailed. This is contrary to literature which strongly suggests that mathematical concepts should be integrated throughout the day.
Subtheme 1.3 Resources for emergent	Only one of the teachers had some resources suitable for emergent mathematics learning	Preschool teachers should use manipulatives to give concrete experiences that assist children	Children construct knowledge of concepts through interaction with the concrete environment	Preschool teachers in this study were not able to provide resources that they use to

Themes and subthemes	Result	Literature	Theory	Interpretation and findings
mathematics development	and exploration. The other had some pieces of a puzzle. The preschool teachers did not have in their classrooms, resources to enable children to develop emergent mathematics concepts.	to make sense of mathematics and construct their mathematical thinking (Reys et al., 2016).	(Piaget, 1952). In the same vein, children develop mathematical understanding by exploring different objects in their environment.	enable the development of emergent mathematics activities. This finding is in contradiction with literature which suggests that concrete manipulatives are important and that they help children make sense of mathematics concepts.
***Theme 2:* Preschool teachers' PCK of emergent mathematics**	Preschool teachers displayed very limited PCK. All activities presented were structured. A few knew the mathematics content areas but none of them were in possession of a curriculum document.	According to Mayne (2019), knowledge of PCK influences teacher effectiveness in carrying out the teaching and learning process within the classroom environment.	Preschool teachers should possess knowledge of content, knowledge of curriculum and knowledge of teaching methodology in order to facilitate emergent mathematics effectively (Shulman, 1986).	The findings revealed that preschool teachers lacked PCK to facilitate effective emergent mathematics activities. This evidence does not align with literature which suggests the importance of PCK in facilitating meaningful and effective emergent mathematics activities.
***Theme 3:* Curriculum guidelines**	None of teachers were in possession of a curriculum guideline for preschool.	The NCF gives guidance to those working with babies, toddlers and young children (DBE, 2015).	Su (2012) maintained that a curriculum should guide and inform teachers in terms of material, techniques and all aspects of classroom learning and teaching.	Findings do not align with literature in that preschool teachers in this study are not being informed and guided by the curriculum in their facilitation

Themes and subthemes	Result	Literature	Theory	Interpretation and findings
				of emergent mathematics activities.
Theme 4: ***Perceptions about facilitating emergent mathematics activities***	The preschool teachers felt confident about facilitating mathematics activities even though this did not manifest in practice. Most of them felt that they needed to gain more knowledge so that they can do even better on facilitating mathematics activities. One teacher felt that children would get bored if they were presented with emergent mathematics activities at a young age.	Preschool teachers' attitudes towards mathematics influence how they facilitate mathematics in their classrooms (Lee & Ginsburg, 2009). However, sometimes there is a disconnect between beliefs and practice (Francis, 2015).	Boonen et al. (2014) stated that the possible disconnect between teachers' beliefs and classroom practices in mathematics may be reflected in math-related practices in preschool classrooms, including the amount of time, types of practices, and the frequency and duration of time devoted to instructional methods with mathematics content.	Evidence from this study showed alignment with some schools of thought from literature in the sense that there was a disconnect between belief and practice. The preschool teachers in this study had positive attitudes about mathematics in early childhood but the positive attitude did not culminate in practice. The teachers did not devote much time to emergent mathematics activities and did not use teachable moments to emphasis mathematical concepts during play activities.

Themes and subthemes	Result	Literature	Theory	Interpretation and findings
***<u>Theme 5:</u>* Factors influencing the facilitation of emergent mathematics**	Although all the preschool teachers were qualified, observation evidence shows lack of content and methodology knowledge (PCK) of facilitating emergent mathematics activities.	Botha (2012) stated that the quality of teacher education in South Africa is of concern. According to Kagan and Gomez, (2014), the training that preschool teachers received was not directly linked to the content that they would teach young children.	Shulman's PCK suggests that teachers should receive preservice training in knowledge of content, knowledge of curriculum and knowledge of teaching methodology.	Evidence shows similarity with literature in that the quality of teacher education is a concern, and that preschool teachers should receive training that is linked to the content they will teach children. Teacher training qualifications and training programmes should prepare teachers to effectively impart understanding in the context of the needs of the children.

The above table provides a comparison between the research results and the literature review, as well as the theory framing this study as portrayed through the identified themes and subthemes. An in-depth analysis of the relationship between the data, the literature and he theoretical frameworks is discussed below.

5.2.1 Formal lessons

The facilitation of emergent mathematics activities by the preschool teachers proved to be very formal and lacking in play and hands-on activities. Coherence among the preschool teachers about the facilitation of emergent mathematics activities was observed in this study. The preschool teachers believed that children learn best through play; however this contradicted with how they presented their mathematics activities. This finding does not align with the literature (see 2.4.1) which suggests that children learn emergent mathematics concepts through playful and hands-on activities.

5.2.2 Integrated Approach

All the preschool teachers shared the same sentiment that emergent mathematics activities can be facilitated by being integrated into the programme throughout the day. Even though the preschool teachers shared these sentiments, it was not evident in their practice. This finding coincides and agrees with literature on integration of mathematics concepts in other learning areas (see 2.4.2). In their facilitation of emergent mathematics activities, most of the preschool teachers in this study did not have the necessary resources for facilitating emergent mathematics concepts. This does not align with literature (see 2.4.4) which strongly recommends the use of resources to enhance mathematics conceptualisation.

5.2.3 The Role of the Teacher

Literature suggests that the role of the preschool teacher is to scaffold and question children during play to encourage the children to make meaning of the mathematics experiences (see 2.4.1.3). It was evident in this study that, during free play, the preschool teachers believed their role was only to ensure the safety of the children.

5.2.4 Teacher Training

The study revealed that there is a need for improved teacher training in South Africa. Even though all the preschool teachers in this study were qualified, evidence showed lack of PCK, in that they had little mathematical CK and minimal knowledge about the facilitation of mathematical concepts through the playful methodology which is one of the appropriate methods for the facilitation of early mathematics in early childhood (see 2.6.5).

5.2.5 Lack of Pedagogical Content Knowledge

The preschool teachers in this study displayed lack of PCK during the observations and in-depth interviews. This finding is does not align with the literature (see 2.6.5) which highlights the importance of PCK in planning for and facilitating emergent mathematics activities. None of the preschool teachers were in possession of the curriculum document for ECD. However, a curriculum should inform the preschool teachers on what mathematics content they should facilitate and how to facilitate these mathematics activities (see 2.3).

5.2.6 Beliefs and Attitudes

This study also revealed that the preschool teachers were positive about facilitating emergent mathematics activities and that they believed that there was a need for learning more skills that would enhance their knowledge and confidence in doing so. This finding is subject to debate in literature. On the one hand, some scholars believe that the preschool teachers' beliefs and attitudes determine how they facilitate emergent mathematics activities. On the other hand, some scholars are of the opinion that there can be a disconnect between the preschool teachers' beliefs and practice in the classroom (see 2.5.2). Evidence from the study showed that even though the preschool teachers had a positive attitude about mathematics, their practice revealed the opposite.

5.3 RESEARCH CONCLUSIONS

This section presents the research conclusions which provide answers to the research questions outlined in Chapter 1 (see 1.5 and 3.3). The secondary questions are answered first, followed by the primary question.

5.3.1 Secondary Research Question 1

What knowledge and skills are needed to facilitate emergent mathematics activities?

The preschool teachers had minimal knowledge of PCK to facilitate emergent mathematics activities effectively. It impacted the way they facilitated and presented emergent mathematics activities in their classrooms. This was exacerbated by the fact that they did not have curriculum documents to guide them in their facilitation of emergent mathematics activities which further resulted in the activities lacking direction and outcomes.

Most of them lacked content and instructional knowledge and skills to facilitate and implement emergent mathematics activities to enhance conceptualisation of mathematics concepts. During the observations, most preschool teachers presented numbers and counting activities and could not explain what content area they had presented. Evidence also suggests that they did not have adequate knowledge on effective instruction of emergent mathematics. It was established during the semi-structured interviews that the preschool teachers' training and qualifications influenced their understanding and implementation differently. Two of the preschool teachers who indicated that they were pursuing their formal studies showed basic understanding of emergent mathematics content areas and implementation thereof. The preschool teachers who had non-formal qualifications and were not pursuing their studies showed little understanding and knowledge of the mathematics content areas. The little that they knew, they had learned from the workshops that they had attended. They admitted that they needed to acquire more knowledge and skills in order to facilitate emergent mathematics activities more effectively. This study also revealed that teacher training programmes should prepare preschool teachers to effectively impart understanding and use the appropriate methodology.

The impact of teacher training on facilitating emergent mathematics activities is further discussed below.

5.3.1.1 Teacher training impacting their knowledge and skills to facilitate emergent mathematics activities

Three of the preschool teachers admitted that they needed to gain more knowledge and skills to facilitate emergent mathematics activities effectively. The preschool teachers noted that they had not received any training on mathematics content areas and implementation thereof. Their background information indicated that they had a basic entry Level 4 qualification in ECD. This provides evidence that teacher training in the early years is fragmented and their skills need to be developed. Programmes are lacking and often inefficient and further, emergent mathematics does not receive the attention it requires.

5.3.2 Secondary Research Question 2

What curriculum guidelines are used in emergent mathematics and how are they implemented in the facilitation of emergent mathematics?

All the preschool teachers had heard about the NCF but none of them had seen it. The indication is that the NCF was not being used as an instructional and content guideline when facilitating emergent mathematics activities. They all understood that it was some form of a curriculum. Two of the preschool teachers from the same school use the internet, CAPS and a year-plan provided by the principal. Three preschool teachers referred to and provided lesson plans as a form of guiding document that they used in planning and facilitation of emergent mathematics activities.

Two of the preschool teachers from the same school submitted clearly structured lesson plans with clear outcomes. These two received support from the principal and also collaborated with other teachers when planning. Their planning emanated from the yearly plan of themes provided by the principal. The other two had no curriculum documents or guidance in their implementation of emergent mathematics activities. One of them said she referred to the daily programme as a guiding document. The activities presented by these three had no structure and there were no outcomes set for the children. One teacher had a lesson plan she had received from one of the workshops she had attended. The lesson plan had no clear outcomes or structure. This evidence highlights the lack of planning and the benefits thereof to facilitate emergent mathematics and enhance mathematics conceptualisation in young children.

5.3.3 Secondary Research Question 3

How do preschool teachers explain the importance of emergent mathematics?

The preschool teachers believed that mathematics is very important and that young children can learn mathematical concepts. It was evident from the semi-structured interviews that the preschool teachers had a positive attitude towards the facilitation of emergent mathematics activities, and they believed that it was important that children should learn mathematics concepts. The preschool teachers felt that many teachers in South Africa have positive perceptions about the facilitation of emergent mathematics, but they do not have adequate skills and knowledge to do implement it. This indicates that teachers perceive emergent mathematics as important, but due to the policy-practice void, the facilitation and implementation requires attention.

5.3.4 Primary Research Question

How do preschool teachers facilitate emergent mathematics activities for three-to-four-year-old children to enhance mathematics conceptualisation?

Another finding regarding the facilitation of emergent mathematics activities was that although preschool teachers had knowledge that emergent mathematics activities can be facilitated through integration, they lacked the skills and knowledge of how to implement them. In other activities facilitated during the observations, the preschool teachers did not integrate mathematics with other learning areas like Language and Life skills.

To address this question, the discussion is divided into the following subsections respectively, play, integration, resources and classroom layout.

5.3.4.1 Playful learning

It was established through the research results that the preschool teachers did not fully understand the concept of learning through play. All the activities presented were formal and guided activities in which children sat down the whole time and listened to the teacher. During the interviews, the preschool teachers mentioned that they were aware that children learn best through play, yet the activities observed showed very formal ways of facilitating emergent mathematics activities. During free play, it was observed that three of the teachers let children play on their own with few or no toys. Most of the teachers believed that their role during play was to observe and ensure that children were safe. Two of the teachers from the same school asked the children some questions while they played. Evidence on the ground revealed that they did not understood what learning through play entails and how to facilitate the learning of emergent mathematics activities through play.

5.3.4.2 Integration

During the semi-structured interviews, it was revealed that the preschool teachers were aware of the concept of facilitating emergent mathematics activities through integration. However, the observations conducted throughout the learning day did not show evidence of integrating mathematical concepts into other learning areas.

5.3.4.3 Resources

Most preschool teachers did not use resources when facilitating emergent mathematics activities. Most teachers did not have resources for facilitating emergent mathematics activities citing Covid-19 restrictions on sharing items. One of the teachers had adequate resources suitable for facilitating and conceptualisation of mathematical concepts. The semi-structured interviews indicated that the preschool teachers had a vague idea of the relevant resources needed to facilitate emergent mathematics activities.

As a result, children were limited in their understanding of the mathematical concepts that were being taught because they were not afforded the opportunity to explore and discover through appropriate manipulatives.

5.3.4.4 Classroom layout

Three of the five preschool teachers had limited classroom space to allow for exploration and discovery of emergent mathematics concepts. The lack of space resulted in the classrooms not having the different areas needed in a preschool classroom. Two of the teachers from the same school, however, had large classrooms which had different areas for the children to explore.

Given the above four aspects, it is clear that the preschool teachers from this study were ill-prepared and ill-equipped to facilitate emergent mathematics activities. Teachers did not understand the playful learning methodology in facilitating emergent mathematics activities to enhance conceptualisation. All the activities presented were teacher-guided and structured formally, which is not appropriate for pre-primary school children. It was evident from the observations showing children sitting down in all the preschool teachers' classrooms and the teachers talking in a lecture fashion. Even though the preschool teachers understood the concept of integrating emergent mathematics activities in other learning areas, they did not practise it in their classrooms. Furthermore, most of the preschool teachers did not use resources when facilitating emergent mathematics activities. Lastly, three of the preschool teachers had little space for children to explore and develop emergent mathematics concepts. There were no activity areas to allow children to discover and explore mathematics concepts.

5.4 LIMITATIONS OF THE STUDY

This sample consisted of only five participants and four preschools and therefore results cannot be generalised to the wider population. Also, due to Covid-19, it was difficult to go into the schools and observe the activities for many hours. Lastly the data collection process only included one semi-structured interview per participant, due to time constraints and Covid-19 restrictions.

5.4.1 Addressing the Limitations

Although the sample consisted of only five preschool teachers, these participants were chosen from four different preschools, and I was able to address the trustworthiness of this study.

The initial plan was to observe the classroom activities for five days each but I ended up only observing one day at each school. However, I observed a full day in each classroom which gave insight on how emergent mathematics activities were facilitated.

Due to time constraints and Covid-19 restrictions, I only conducted one semi-structured interview per preschool teacher., However, by means of document analysis and field notes from the observations, these setbacks were addressed, and I was able to collect information-rich data.

5.5 RECOMMENDATIONS

This study focused on gaining a deeper insight into the facilitation of emergent mathematics activities by preschool teachers of three-to-four-year-old children. The findings of this study are significant since they reveal that preschool teachers in South Africa are not supported in facilitating emergent mathematics activities. They have not accessed substantial guidelines nor the curriculum to guide them; knowledge and skills are underdeveloped; and resources and space is limited. This in turn does not lead to enhancing mathematics conceptualisation of preschool children. The findings and recommendations are made below:

Finding 1: There is a need for quality teacher training programmes

In this study, preschool teachers in this study felt that they needed more training and guidance on the implementation and facilitation of emergent mathematics activities.

Recommendation 1:

The DBE should provide ongoing support and professional development on how preschool teachers can implement a playful methodology in the facilitation of emergent mathematics activities. This should include the importance of having manipulatives in the classroom to enhance mathematics conceptualisation and use the classroom to ensure it allows for exploration and discovery.

Finding 2: Preschool teachers lack PCK to facilitate and implement emergent mathematics activities.

Recommendation 2:

Preschool teachers should receive training that is linked to the mathematical content that they will teach children. Teacher training programmes should be designed to include PCK and CK training.

The DBE should ensure that preschool teachers receive ongoing professional development support in the form of workshops. This will empower preschool teachers with knowledge on how and what to teach when it comes to emergent mathematics.

Finding 3: The NCF needs to be easily accessible to preschool teachers and there should be training on how to implement it in the classroom.

Recommendation 3:

Currently, the NCF is available for preschool teachers, but it is not clear to preschool teachers how to access or interpret the curriculum document. The DBE needs to create more awareness of the existence of the NCF and the implementation thereof.

Finding 4: Preschools need to be adequately resourced.

Recommendation 4:

The DBE should support township preschools with resources and the training on how to use the resources thereof.

Finding 5: Preschool teachers have a positive attitude towards facilitating emergent mathematics activities and they believe that there is a need for more knowledge and skills on implementing and facilitating emergent mathematics activities.

Recommendation 5:

The government should award bursaries to preschool teachers who wish to further their studies to acquire more knowledge and skills.

5.6 AVENUES FOR FURTHER RESEARCH

While the findings of this study provided important insight into the facilitation of emergent mathematics activities by preschool teachers, the following gaps which point at several avenues for further research have also been identified.

Suggestion 1

A study on how teacher training programmes could ensure that preschool teachers are properly trained in the content and teaching practice of emergent mathematics.

This study focused only on preschool teachers’ facilitation of emergent mathematics activities. Additional studies could be conducted on the teacher training programmes and their effectiveness in preparing preschool teachers to facilitate emergent mathematics activities for young children.

Suggestion 2

A study on the effectiveness of the preschool curriculum (NCF) in facilitation of emergent mathematics activities.

Although the NCF was one of the documents analysed in this study, there is a need for further research on the effectiveness of the document in guiding preschool teachers to facilitate emergent mathematics activities.

Suggestion 3

This study focused on preschool teachers from Tembisa only. Further studies in other areas and provinces can provide better insight into the context-specific elements that require attention in the facilitation of emergent mathematics activities.

5.7 CONCLUDING REMARKS

Preschool teachers have limited knowledge regarding the facilitation of emergent mathematics activities. It seems they have a vague understanding of what emergent mathematics content areas are, and do not use the curriculum guidelines in facilitating emergent mathematics activities. This highlights that guidance in South Africa concerning the facilitation of emergent mathematics activities and enhancing mathematics conceptualisation is limited.

On a personal note, this study has been an expedition of learning and discovery for me. The different views from the scholars in literature has left me with new and deeper insight and knowledge. The data collection process brought me closer to the reality of what is really happening on the ground. Ultimately, this has made me realise how much work still needs to be done to enhance the facilitation of emergent mathematics activities and other areas as well.

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APPENDICES

APPENDIX A: ETHICAL CLEARANCE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/02/19

Ref: **2020/02/19/49714724/13/AM**
Name: Mrs R Mokoena
Student No.: 49714724

Dear Mrs R Mokoena

Decision: Ethics Approval from 2020/02/19 to 2023/02/19

Researcher(s): Name: Mrs R Mokoena
E-mail address: 49714724@mylife.unisa.ac.za
Telephone: 078 608 4909

Supervisor(s): Name: Dr Donna Hannaway
E-mail address: hannad@unisa.ac.za
Telephone: 012 429 4778

Name: Dr M. V Kgabo
E-mail address: ekgabomv@unisa.ac.za
Telephone: 012 429 6415

Title of research:

Preschool teachers' facilitation of emergent mathematics activities with three-to-four-year old children.

Qualification: MEd Early Childhood Development

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/02/19 to 2023/02/19.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2020/02/19 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX B: PERMISSION LETTER TO PRINCIPAL

College of Education
P.O. Box 392
UNISA
0003
Pretoria

27.07.2020

Name:
Department: Principal
Cell:

PERMISSION TO CONDUCT RESEARCH

Dear Mrs

I, Rosemary Mokoena am undertaking a study under the supervision of Dr Donna Hannaway and Dr Veronica Kgabo, senior lecturers in the department of Early Childhood Education, towards a degree at the University of South Africa. I am requesting permission to conduct the research in your school and the participants will be one of the teachers in your school. The study is entitled; **Preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-old children.**

The purpose of this academic study is to investigate the knowledge, skills and attitudes preschool teachers have, towards enhancing numeracy development to young children, with a view to make recommendations for improvement of the ECD emergent mathematics programmes. Your school has been selected because the research is only being done in Tembisa, where your school is, and it was selected randomly without following any other criterion rather than being a preschool in Tembisa.

The study will entail interviewing and observing teachers during activities, in order to find out what their knowledge, skills and attitudes are, in facilitating emergent mathematics activities. The study is beneficial in the sense that it will highlight the knowledge, skills, attitudes or lack of thereof, of preschool teachers in the facilitation of emergent mathematics activities.

Each interview will range between 30–45 minutes long and will be done in five consecutive days. Observations will be done for five days during classroom activities. The interviews and observations will be recorded. Teaching time and school activities will not be disrupted. The researcher will adhere to the confidentiality and anonymity of the school's name and participants all the time.

The findings will highlight what the teachers know, and how they feel towards promoting and facilitating emergent mathematics activities, and thereby offer recommendations for the enhancement of mathematics conceptualisation in the early years.

The study has no foreseeable potential risks associated as there are non-vulnerable adult participants and non-sensitive information involved. There will be no reimbursement or any incentives for participation in the research. Participants also reserve the right to withdraw from the research project at any time with no penalties of any nature or victimisation.

Findings of the final dissertation will be provided to the participants.

Thanking you in anticipation.

Yours sincerely

ROSEMARY MOKOENA

Rosemary Mokoena

Cell: 078 608 4909

49714724@mylife. unisa. ac. za

Rose. chakonza@gmail. com

Dr Donna Hannaway 072 435 2782

Dr Kgabo 083 277 4942

APPENDIX C: CONSENT LETTER FOR PRESCHOOL TEACHERS

College of Education

P. O Box 392

UNISA

003

Pretoria

27. 07. 2020

Title**: Preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-old children.**

DEAR PROSPECTIVE PARTICIPANT

My name is Rosemary Mokoena, and I am doing research under the supervision of Dr Donna Hannaway and Dr Veronica Kgabo, senior lecturers in the department of Early Childhood Education, towards a Masters' degree in Psychology of Education at the University of South Africa. We are inviting you to participate in a study entitled; Preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-old children**.**

PURPOSE OF THE STUDY

The study is expected to collect information that could help improve mathematics conceptualisation in the early years of development. It could also enhance emergent mathematics development by highlighting the knowledge, skills and attitudes preschool teachers have towards early numeracy. It also aims to contribute to the improvement of policy and practice regarding Early Childhood Development, and emergent mathematics. Lastly, the study aims to contribute to the improvement of ECD teacher training curricula and mathematics results in general.

REASON FOR INVITATION TO PARTICIPATE

You are invited to participate because you are a qualified preschool teacher and have more than one- year experience in the field of ECD. I obtained your contact details from the principal. A total of five participants will take part in this study.

THE ROLE OF THE PARTICIPANT IN THIS STUDY

The study involves observations and interviews during activities. Each participant will be observed throughout the day for a period of about five days, while she engages in activities with the children. Furthermore, every participant will be interviewed for approximately 30–45 minutes.

THE RIGHT TO WITHDRAW FROM THE STUDY

Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw at any time without giving a reason.

POTENTIAL BENEFITS OF PARTICIPATING IN THIS STUDY

This study is expected to collect important information that could contribute to the facilitation of emergent mathematics activities. The study could also contribute to the improvement of training curricular and mathematics results in general.

POTENTIAL CONSEQUENCES OR RISKS FOR PARTICIPATING IN THIS STUDY

There are no risks or negative consequences associated with participating in this study. Questions asked during interviews are not of a sensitive nature and neither do they cause discomfort.

CONFIDENTIALITY

You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research. Your name will not be recorded anywhere and no one will be able to connect you to the answers you give. Your answers will be given a code -number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. Your answers may be reviewed only by people responsible for making sure that research is done properly, in this case, members of the review committee. Otherwise, records that identify you will be available only to people working on the study, unless you give permission for other people to see the records.

Data collected from this study may also be used for other purposes such as research report and journal articles, however, in such instances, participant privacy will be protected, and individual participants will not be identifiable.

PROTECTION AND SECURITY OF DATA

Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard for future research or academic purposes, electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. If necessary hard copies of the information will be shredded and electronic copies will be permanently deleted from the hard drive of the computer by using a relevant software programme.

PAYMENT OR INCENTIVE FOR PARTICIPATING IN THIS STUDY

There will be no payment or incentive for participating in this study.

ETHICAL APPROVAL

This study has received written approval from the Research Ethics Review Committee of the, Unisa. A copy of the approval letter can be obtained from the researcher if you so wish. Permission will also be obtained from the principal of your school.

RESEARCH RESULTS/ FINDINGS

If you would like to be informed of the final research findings, please contact Rosemary Mokoena on 078 608 4909. The findings are accessible for five years.

Should you require any further information or want to contact the researcher about any aspect of this study, please contact the researcher on the following details:

Rosemary Mokoena

Cell: 078 608 4909

49714724@mylife. unisa. ac. za

Rose. chakonza@gmail. com

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

ROSEMARY MOKOENA

CONSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, ………………………………………………………………………………………… confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read and understood the study as explained in the information sheet, and the researcher has also taken time to explain the nature of the study to me. I have had enough opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interviews.

I have received a copy of the informed consent agreement.

Participant Name & Surname (please print).

……………………………………………………………………………..

……………………………………………………………

…………………………………………………………………

Participant Signature Date

Researcher's name & Surname (Please print).

……………………………………………………………………………….

…………………………………………………………………

……………………………………………………………………………..

Researcher’s signature Date

APPENDIX D: INFORMATION LETTER TO THE PARENTS

UNISA university of south africa

College of education

P. O. Box 392

UNISA

0003

Pretoria

27. 07. 2020

Dear Parent

I am undertaking a study entitled; **Preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-old children,** in your child's classroom.

I am undertaking this study as part of my master's research at the University of South Africa. The purpose of the study is to explore and find out the knowledge, skills and attitudes preschool teachers have in facilitating emergent mathematics activities, and the possible benefits of this study are the improvement of the implementation of mathematics activities in the early years of development.

I wish to inform you that your child will not be directly involved in this study, as the focus of the research is the teachers. Your child will not take part in any interviews, survey, or questionnaires.

There are no foreseeable risks to your child in this study, and your child will receive no benefits or incentives from this study, however the possible benefits of education are the improvement of ECD curricular in order to enhance emergent mathematics conceptualisation in the early years.

The study will take place during regular classroom activities, with the prior approval of the school and your child's teacher. The information gathered from the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

For any questions regarding this study please ask me or my supervisor:

Supervisors' details:	**Researcher details:**
Dr Donna Hannaway and Dr Veronica Kgabo	Rosemary Mokoena
Department of Early Childhood Education	Cell: 07806084909
College of Education	49714724@mylife. unisa. ac. za
University of South Africa	
E-mail: hannad@unisa. ac. za	
ekgabomv@unisa. ac. za	

Permission for the study has already been given by the principal and the Ethics Committee of the College of Education, UNISA.

Your signature below indicates that you have read the information provided above and you are aware of the nature of the study. You are also aware that the focus of the study is the classroom teacher and that your child is not an active participant of this study.

Name of child:

Sincerely

Parent/guardian's name (Print). Parent/guardian's signature Date

Researcher's name (Print). Researcher's signature Date

APPENDIX E: LESSON PLAN

LESSON PLAN Weekly Theme: Under the ground **Date:** 27 August 2020

Type of Lesson: STORY **Age:** 3–6 years

Outcomes: The children will:

Develop home language skills by listening to stories and responding to simple questions.

Equipment:

Pictures of beetroot, potato, onion, sweet potatoes, carrots. Number cards. Days of the week cards

Introduction: Put the day flash cards on a board in the sequence of Monday to Friday. Read the names with the learners pointing to each word as you read it. Leave a space next to the word so you can add the number and the picture of the vegetable as you tell the story.

Content: Early one morning a very tiny egg popped open and a very small but very hungry caterpillar crawled out of the egg. He looked around for something to eat and found that he was on a leaf. Monday, he ate through one potato, Tuesday he ate through 2 onions, Wednesday he ate through 3 carrots, Thursday he ate through 4 potatoes. Friday, he ate through 5 beetroots.

Conclusion:

Ask leading questions about the circle life of a caterpillar.

What was on the leaf? What popped out of the egg? What happened to the caterpillar when it ate all the food? Why did he make a cocoon?

Reflection:

APPENDIX F: LESSON PLAN

Date: 24 August 2020

Group: 000 **Type of Lesson:** Mathematics **Age:** 3 to 4 years

CONTENT CONCEPTS SKILLS	The children will… • Develop eye-hand coordination when they move the objects from one side to another • Know how to count from 1 up to 5
LANGUAGE	Learn knew words like; buckle, shoes, sticks, knock and hen
MATHEMATICS	Counting numbers 1, 2, 3, 4, 5
LIFE SKILLS	
ACTIVITY	*RESOURCES*
Preparation: The teacher puts five objects (blocks) on each child's table.	Blocks and tables
Introduction: The teacher will lead the rhyme to warm the learners up. The learners follow the actions 1, 2, buckle my shoes, 3, 4, knock at the door, 5, 6 pick-up sticks, 7, 8 lay them straight, 9, 10 a big fat hen.	Rhyme
Body/Main section: The teacher demonstrates the counting of objects by picking one block and tell the children to pick one as well. The teacher counts together with children, and the children follow after her. The teacher picks up the second block and ask the learner to do the same until to the 5^{th} block which makes 5 blocks. The teacher will tell the learners that we have 5 blocks, and the learners will say the same.	Blocks
Conclusion: The teacher leads the rhyme 2 little blue birds sitting on the wall, one named Peter one named Paul, fly away Peter, fly away Paul.	*Children will go and sit down and relax.*
Reflection:	

APPENDIX G: GRADE 000 WEEKLY PLAN

Grade 000

TEACHER: CLASS: THEME: Watch these grow WEEK: 3 TERM: 3 DATE: 24-27 August 2020

	Monday	Tuesday	Wednesday	Thursday	Friday
7:30	Breakfast *Toilet and*	Breakfast *Toilet and hands*	Breakfast *Toilet and hands*	Breakfast *Toilet and hands*	Breakfast *Toilet and hands*
8.00	Teacher-directed activities. Supervised play (inside)	Teacher-directed activities. Supervised play (inside)	Teacher-directed activities. Assembly	Teacher-directed activities. Supervised play (inside)	Teacher-directed activities. Supervised play (inside)
8:10-0840am	**Morning ring -Bible** **Bible story:** **Ruth 2; 1-9** **Apparatus:** **Bible story book** **Discussion:** **Intro:** **Tell the learners to name the plants they know.** **Main:** Plants Many plants grow now in spring. Plants develop flowers. Plants prepare to bear fruits. Plants have trunk, roots and branches. When it is spring it will develop green leaves. **End Rhyme** Old Mcdonald had a farm.	**Morning ring -Bible** **Bible story:** **Ruth 2: 1-9** **Apparatus:** **Pictures in the book** **Discussion:** **Intro:** **Clap hands once and twice** **Main:**Trees There are different types of trees. Palm tree Fruit tree Vegetable tree Palm trees are very special because we use them during Easter. **End Rhyme** Old Mcdonald had a farm	**Morning ring -Bible** **Bible story: Ruth 2; 9** **Summarise the story in Zulu** **Apparatus:** **Puppet** **Discussion:** **Intro:** **Rhyme** **I was a little flower.** **Main:** Flowers We have different types of flowers and they smell differently Roses Lillies Daisies and sunflowers Sunflowers are poisonous and some are not. We plant seeds to grow plants. **End Rhyme** Sikha maorange	**Morning ring -Bible** **Bible story:** **Mathew 13; 31-35** **Apparatus:** **Bible story book** **Discussion/Maths/Perception** **Intro: Rhyme** **1,2 buckle my shoes** **Main:** Counting Put 5 objects on the table Learners will be counting blocks one by one putting them in the box **Learners will be singing a rhyme.** **End** 1 man went to war and went to war amingo	**Morning ring -Bible** **Bible story: Assembly** **Parable of the sower.** **Apparatus:** **Seeds** **Discussion/Maths/Perception** **Intro:** **Show the learners flash cards with numbers ask them to tell the number they see on the card.** **Main:** Mathematics -Counting and sorting -Children count the seeds to find out how many seeds are inside the tray and sort them according to colours using clippers **End Rhyme** Sing the song 2 little dickie birds
0840-0900am	**CATCH UP** **Intro:** Clap hands, stomp feet, turn around then sit down **Main Activity:** Colours Learners will go out and stand in a square shape, they will be pointing at someone who will be at the corner to make 4 corners. **End rhyme:** **We are marching in the light of God**	**CATCH UP** **Intro:** Put objects on the table **Main Activity**: Counting Learners will be counting blocks on the table and put them in the box one by one They will be counting 5 objects **End rhyme:** **1,2,3,4,5 Once I catch a fish alive**	**CATCH UP** **Intro:** **Song;** Bengili blom elincane **Main Activity**: Square Learners will fold a square paper to a box **End rhyme:** **This is the way we make our box x3**	**CATCH UP** **Intro:** **Song;** We are marching in the light of God **Main Activity**: Square Learners will be matching to make a squareLearners will count the 4 sides of a square while standing in a square shape **End rhyme;** I am a little square	**CATCH UP** **Intro:** **Rhyme;** 1, 2 buckle my shoes **Main Activity:** Numbers Learners will be singing and doing actions. The teacher will lead with actions **End song:** I was a little flower

APPENDIX H: GRADE 000 DAILY PROGRAMME

TIME	PROGRAMME
7:10–7:40am	Arrive/Breakfast/Bathroom routine
7:40–8:10am	Inside play
08:10–08:40am	1st ring-Morning ring/Discussion (Include 1st language)
08:40–0900am	Catch up-Numbers/shapes/colours
09:00–10:00am	Snack/ Outside play
10:00–1050am	Creative activity
10:50–11:20am	2nd ring-Movement
11:20–11:50am	3rd ring-Story time
11:50–12:20pm	Lunch
12:20–12:35pm	Bathroom
12:35–1:35pm	Sleep time
1:35–2:00pm	Bathroom/Snack/Temperature check and departure/Bus

APPENDIX I: OBSERVATION CHECKLIST

Observation guide/checklist of the ECD environment

Available/meets the ELDAS according to the NCF	Yes	No	Comment
Indoor area			
Outdoor area			
Maths centre			
Teacher-child ratio			
Rich Mathematical environment			

Observation checklist for the availability of emergent mathematics resources

Adequate	**Yes**	**No**	**Comment**
Number line			
Number posters			
Birthday chart			
Counters			
Weather chart			
Games such as puzzles, board games etc			
Different measuring instruments such as scales, spoons, measuring cups, tapes etc.			
Blocks and other building/construction materials (2D and 3D)			
Waste material, bottle tops, egg holders, bottles, paper etc.			
Balancing scale			
Story books with numbers			
Number rhymes charts			

Observation checklist for teaching methodology of emergent mathematics concepts

Presentation of mathematics activities	**Yes**	**No**	**Comment**
Mathematics activities are presented in a playful manner			
Principles of teaching concepts followed, that is; start with body kinaesthetic, 3D objects then 2D activities			
Availability of a lesson plan with clear outcomes and sequence of activity			
Evidence of planning			
Maths concepts integrated in other learning activities			
Encourages children to solve problems on their own			
Resources are age-appropriate			

Observation checklist for mathematics integration in the daily programme and the teacher's role

Mathematics content areas	**Yes**	**No**	**Comment**
1. **Numbers, Operations and Relationships**			
Practitioner: Reads number stories to the children			
Sings number rhymes with the children			
Play games in which children can point to parts of their body and count them			
Asks children number questions			
Uses Maths terminology/ number language e. g. few, less, more,			
Provides opportunities for children to count real objects			
Asks children to estimate by playing guessing games e. g. how many?			
Encourages children to count in different situations			
2**. Patterns, Functions and Algebra**			
Talks to children about patterns in the environment			

Mathematics content areas	Yes	No	Comment
Encourages children to talk about patterns they see in clothing, nature, books and buildings			
Creates opportunities for children to copy patterns			
Creates opportunities for children to extend patterns using everyday objects/ their bodies			
Creates opportunities for children to make their own patterns			
3. Space and Shape			
Talks about the things in the environment and mention shape and size			
Provides children with objects of different shapes and sizes to hold and play with			
Joins children to build puzzles			
Creates opportunities for children to put objects into correct shaped openings			
Encourages children to talk about signs and symbols around them			
Name and talk about the shape of items in books and magazines			
Creates opportunities for children to describe positions e. g. in front, behind, on top, under			
4. Measurement			
Talks to children about time, e. g. yesterday, today and tomorrow			
Tell stories about events in the child's past			
Creates opportunities for children to measure length, width using hands and feet (their body)			
Creates opportunities for children to measure objects around them using informal instruments			
Uses terminology like, smaller, bigger, taller			
5. Data Handling			
Creates opportunities for children to categorise items according to characteristics, e. g. length, shape, colour			

Mathematics content areas	Yes	No	Comment
Point out similarities between objects			
Encourage children to play matching games			
Creates opportunities for children to talk about similarities and differences between themselves, and in pictures			

APPENDIX J: INTERVIEW GUIDE FOR PRESCHOOL TEACHERS

THE TEMBISA ECD CENTRES INTERVIEW GUIDE FOR PRESCHOOL TEACHERS ON THE FACILITATION OF EMERGENT MATHEMATICS ACTIVITIES.

Preschool teacher qualifications

1. How long have you been a preschool teacher?
2. What are your qualifications?
3. Is there inhouse professional development in the school?

Probe: Do you participate?

Classroom organisation

1. How many children are in your classroom?
2. How does the teacher-child ratio affect your teaching?
3. To what extent is your principal supportive of your programme?
4. How do you arrange your classroom in a way that encourages mathematics development??

Availability of resources that promotes emergent mathematics development

1. Which resources do you use when facilitating emergent mathematics activities?
2. How do you use the resources to promote emergent mathematics development?

Teaching Methods used in ECD

1. How do you present emergent mathematics activities?
2. How do you feel about learning through play?

ECD curriculum guidelines

1. Which curriculum guidelines do you use for facilitating mathematics activities?
2. Do you have a copy of the NCF document, and how does it influence the way you present emergent mathematics activities?
3. What kind of support do you receive from principal/ Department of Social Development in implementing the curriculum guidelines?
4. Do you use lesson plans in your preparation and how does it affect your presentation of activities?
5. Do you have a copy of your lesson plan?
6. What support do you get in terms of lesson planning?

Preschool teachers' knowledge of the emergent mathematics content and activities.

1. What is your understanding of emergent mathematics?
2. What is your understanding of mathematics content areas?

Preschool teacher's attitude towards developing numeracy concepts in young children

1. How often do you present mathematics activities?
2. What are your thoughts on young children learning mathematics?
3. Do you believe that young children can learn mathematics?
4. How do you feel when presenting mathematics activities?

APPENDIX K: DOCUMENT ANALYSIS CHECKLIST

Type of document	Checklist	Yes/No
Lesson plan	• Is there a lesson plan available? • Does the lesson plan provided show clear outcomes? • Does the lesson plan clearly states the resources to be used? • Is the planned lesson age-appropriate? • Is mathematics integrated/imbedded in the lesson plan?	
Weekly plan	• Is there a weekly plan available? • How frequent is mathematics facilitated in a week? • Does the weekly plan show evidence of mathematics integration in other learning areas?	
Daily programme	• Is a daily programme available? • Is the daily programme well-structured and balanced? • Does it show evidence of mathematics facilitation?	
Curriculum documents	• Is there a curriculum document available? • Is there evidence that the teachers are using the document/s as instructional and content guides? • Does the curriculum document give clear instructional guides for preschool teachers to follow? • Does the curriculum document give clear content and day-by-day instructions and schedules for teachers?	
Workbooks	• Are the children's workbooks available? • Is there evidence that mathematics activities are being facilitated? • Are the activities age-appropriate? • Which content areas are being facilitated?	
General comments		

APPENDIX L: TURNITIN REPORT

Preschool teachers' facilitation of emergent mathematics activities with three-to-four-year-old children

ORIGINALITY REPORT

6%	6%	0%	0%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	repository.up.ac.za Internet Source	6%

APPENDIX M: CONFIRMATION OF PROFESSIONAL EDITING

Blue Diamonds Professional Editing Services (Pty) Ltd

Polishing your brilliance

Email: jacquibaumgardt@gmail.com

Website: www.jaybe9.wixsite.com/bluediamondsediting

1 November 2021

Declaration of professional edit

PRESCHOOL TEACHERS' FACILITATION OF EMERGENT MATHEMATICS ACTIVITIES WITH THREE-TO-FOUR-YEAR-OLD CHILDREN

By

Rosemary Mokoena

I declare that I have edited and proofread this thesis. My involvement was restricted to language usage and spelling, completeness and consistency and referencing style. I did no structural re-writing of the content.

I am qualified to have done such editing, being in possession of a Bachelor's degree with a major in English, having taught English to matriculation, and having a Certificate in Copy Editing from the University of Cape Town. I have edited more than 300 Masters and Doctoral theses, as well as articles, books and reports.

As the copy editor, I am not responsible for detecting, or removing, passages in the document that closely resemble other texts and could thus be viewed as plagiarism. I am not accountable for any changes made to this document by the author or any other party subsequent to the date of this declaration.

Sincerely,

JBaumgardt

Dr J Baumgardt
UNISA: D. Ed. Education Management
University of Cape Town: Certificate in Copy Editing
University of Cape Town: Certificate in Corporate Coaching

Professional EDITORS Guild

Jacqui Baumgardt
Full Member

Membership number: BAU001
Membership year: March 2021 to February 2022

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