A FRAMEWORK TOWARDS IMPROVED INSTRUCTION OF PROBABILITY TO GRADE SEVEN STUDENTS: A CASE OF SOUTH AFRICAN SCHOOLS IN MPUMALANGA PROVINCE

by

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DECLARATION

I declare that the above thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the thesis to originality checking software and that it falls within the accepted requirements for originality. I further proclaim that I have not previously submitted this work, or part of it, for examination at Unisa for another qualification or at any other higher education institution.

_____ 03 February 2022

SIGNATURE DATE

DEDICATION

This thesis is dedicated to my parents, Piet Mabuse Bokaba (deceased) and Violet Kgomotso Bokaba, who were my first teachers. They instilled in me their intelligence, energy for life, thirst for knowledge and love. I thank them for their unwavering support, and I am grateful that they taught me the value of my origins.

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ABSTRACT

This empirical phenomenological study explored teachers' conceptual understanding of probability to gain insights into how this understanding enhanced their instructional classroom practice. The study was motivated by the fact that many teachers did not receive training on this topic during their pre-service training, and their students often demonstrate below average performance in this topic. Studies have found that primary school mathematics teachers require specific training to enhance students' understanding of probability. Scholars advocate the implementation of intervention programmes to ensure effective practices in probability and mathematics education in general.

In this study the researcher used purposive sampling to select nine participants. These individuals included five qualified teachers; the snowballing method was then used to select another teacher, two subject advisors and one mathematics coordinator. Participants' experience in the field of mathematics teaching ranged from 10 to 25 years. Open-ended questionnaires, semi-structured interviews, lesson observations and focus group discussions were used to collect data.

An initial needs analysis phase was conducted to determine what intervention strategies could be used to empower primary mathematics teachers and subject advisors. These intervention strategies were implemented following a lesson study model first used in Japan. The data collection instruments were developed from the conceptual tools of Kilpatrick, Swafford and Findell's (2001) model. The items in the open-ended questionnaire were adapted from school textbooks and national and international assessments.

The findings from the first phase of the study revealed that teachers had limited conceptual understanding of probability. The analysis of the data collected during the second phase of the study (the intervention) revealed the following:

- There was an improvement in participants' understanding of probability in relation to teaching, and how teachers' classroom instructional practices could be enhanced.
- Teachers and subject advisors acknowledged that readily available common lesson plans provided by the Department of Education would not have a significant impact on their classroom practices unless they were modified.

- Greater collaboration between teachers and higher education institutions was required to enhance teachers' professional development.
- A platform for subject advisors and teachers to share their experiences and provide support should be created.
- The use of a lesson study process as an accelerant promotes collaborative learning and enhances teachers' instructional practices. In this study, teachers were encouraged to share their practices with others and to implement what they had learnt at school level (Lipscombe, Buckley-Walker & McNamara, 2020).

The study found that the absence of policy on the support and mentoring of teachers has resulted in a lack of confidence when teaching the concept of probability. The unsystematic way in which issues of mentoring and teacher support are dealt with does not serve teachers well. This seems to suggest that mentorship and teacher support programmes require regulation by policy. Likewise, subject advisors need training if they are to provide proper support to mathematics teachers.

These recommendations were translated into specifications to empower teachers and subject advisors. These specifications form the basis of the RIRAD (Review, Identify, Reconsider, Adapt and Develop) framework conceptualised in this study. The framework was conceptualised within the conceptual tools of Kilpatrick et al.'s (2001) model and strengthened by the findings that emerged from data. This framework has a dual integrated purpose, aimed at enriching teachers' conceptual understanding of probability and empowering them to enhance their classroom instructional practices. Secondly, the framework empowers subject advisors to offer appropriate support to teachers. Based on findings from the literature review and empirical inquiry, more recommendations for the improvement of practice were made.

KEYWORDS: framework, instructional practice, probability, grade seven, students, conceptual understanding, professional development, senior phase

Table of Contents

DECLARATION ..... **ii**

DEDICATION ..... **iii**

ACKNOWLEDGEMENTS ..... **iv**

ABSTRACT ..... **v**

LIST OF FIGURES ..... **x**

LIST OF TABLES ..... **xii**

LIST OF ACRONYMS ..... **xiii**

Chapter 1 : Orientation to the Study **1**

1.1 Background to the problem 2
1.2 Statement of the problem 5
1.3 Research questions 7
1.4 Aim of the study 7
1.5 Purpose of the study 8
1.6 Significance and possible contribution of the study 8
1.7 Research methodology 8
1.7.1 Pilot Study 8
1.7.2 Main study 9
1.8 Ethical considerations 10
1.9 Model of data collection process 11
1.10 Limitations of the study 12
1.11 Assumption of the study 12
1.12 Summary of chapters 13

Chapter 2 : Theoretical and conceptual Framework **15**

2.1 Introduction 15
2.2 Five Strands of mathematical proficiency 16
2.2.1 Conceptual Understanding 17
2.2.2 Productive Disposition 23
2.2.3 Procedural Fluency 24
2.2.4 Adaptive Reasoning 26
2.2.5 Strategic Competence 28
2.3 The central idea that frames the study 30
2.4 Practical implications of the theoretical and conceptual framework 33

2.5 Conclusion 37

Chapter 3 : Literature Review 38

3.1 Introduction 38
3.2 Research orientation 38
3.3 South African and international reports 39
3.3.1 TIMSS Reports 39
3.3.2 SACMEQ Report 45
3.4 Indicators of conceptual understanding in relation to probability 54
3.4.1 Definition of probability 56
3.4.2 The Importance of Probability 61
3.4.3 Language of Probability 63
3.5 Practices in teaching probability 66
3.5.1 Teachers' Understanding of Probability 67
3.5.2 Facilitation Processes 70
3.6 Practices in teacher professional development 80
3.7 Gaps identified in the literature 87
3.8 Conclusion 91

Chapter 4 : Methodology 92

4.1 Introduction 92
4.2 Research strategy 94
4.2.1 Research paradigm 94
4.2.2 Research approach and design 97
4.3 Study population 98
4.3.1 Population, sampling and sampling procedures 98
4.3.2 Research Instruments 103
4.4 Data collection procedures 106
4.4.1 Pilot study 106
4.4.2 Main study 108
4.5 Data analysis procedures 119
4.6 Report findings 123
4.7 Ethical considerations 123
4.8 Quality assurance criteria: trustworthiness of data 124
4.9 Conclusion 126

Chapter 5 : Data Presentation and Analysis of Findings 128

5.1 Introduction 128
5.2 Findings: Phase 1 129

5.2.1 Theme 1: Limited conceptual understanding of probability 129
5.2.2 Inferences from the overarching theme: Limited conceptual understanding of probability 159
5.3 Findings: Phase 2 participants' professional development and learning context 161
5.3.1 Theme 2: Strategies to develop a conceptual understanding of probability 161
5.3.2 Theme 3: Augmenting teaching activities in probability 172
5.3.3 Theme 4: Participants' views about the study 184
5.4 Conclusion 191

Chapter 6 : Summary, Discussion and Conclusion 192

6.1 Introduction 192
6.2 Summary of the study 193
6.2.1 Teachers' conceptual understanding of probability 196
6.2.2 Development of scenarios 199
6.2.3 Difficulty in making sense of multiple meanings of modal verbs. (The use of language) 201
6.2.4 Ways of representing probability 204
6.3 Conceptual understandings that impede or enhance the teaching of probability 204
6.4 Areas that require greater attention for effective intervention processes 210
6.5 Limitations and mitigations 210
6.6 Implications of the findings: Enhancing classroom practice 212
6.7 Enhancing teachers' instruction practices 214
6.8 Suggestions for further research 229
6.9 Conclusion 230
6.10 Autobiographical reflection: Experiences in conducting research 231
6.11 Reflexivity: Life is probabilistic in nature 232

References 235

Appendices 263

PARTICIPANTS' ACTIVITIES 279

LIST OF FIGURES

Figure 1.1. Mathematics profile of selected countries participating in TIMSS 2015..... 2
Figure 1.2. Assessment item on probability ……. 4
Figure 1.3. Data collection processes ……..11
Figure 2.1. Five strands of mathematical proficiency 15
Figure 2.2. Interrelatedness of strands of mathematical proficiency..... 16
Figure 2.3. Web of representations 19
Figure 2.4. Probability of landing on different colours 21
Figure 2.5. Language use challenges..... 22
Figure 2.6. Proposed framework for the teaching and learning of mathematics ……..30
Figure 2.7. Potential web of idea illustrating components of conceptual understanding …….32
Figure 3.1. Average achievement of Grade 5 and 9 South African students across assessment years . 41
Figure 3.2. Percentages of students reaching international benchmarks of Mathematics achievement across assessment years..... 43
Figure 3.3. Students' reading and mathematics scores for 2000–2013..... 45
Figure 3.4. Grade 12 average performance in probability..... 49
Figure 3.5. Extracts from Diagnostic report (Students A - C)..... 50
Figure 3.6. The chance of rain in Zedland ……...…52
Figure 3.7. Grade 8 students' performance on probability ……...…53
Figure 3.8. Indicators of conceptual understanding in relation to probability …….55
Figure 3.9. A construct to help define the concept of probability ……....58
Figure 3.10. Student's tree diagram to determine probability ……..70
Figure 3.11. Task design in mathematics education ……74
Figure 3.12. Extract from Grade 7 Mathematics 2020 DBE workbook ……..76
Figure 3.13. Teachers' needs for fuure professional development in mathematics ……..…82
Figure 3.14. Lesson study and pathways of impact: A theoretical model ……...…84
Figure 3.15. The impact of lesson study in different countries ……....85
Figure 3.16. Overview of the methodology process..... 91
Figure 4.1. Schematic representation of methodology..... 93
Figure 4.2. Overview of the main study ……110
Figure 4.3. Lesson study processes ……...115
Figure 4.4. Level 1 and Level 2 of the intervention sessions …….116
Figure 4.5. Intervention processes ……....116
Figure 4.6. Overview of the study …….117
Figure 4.7. Five phases of analysis ……...…121
Figure 4.8. Focus of the report …….124

Figure 5.1. Scenario 1: Views of participants on students' conception of chance 130
Figure 5.2. Scenario 2 Participants' solutions to questions on prediction..... 137
Figure 5.3. Participants' solutions to questions on prediction..... 137
Figure 5.4. Teaching and learning activities: Before and during the lesson..... 141
Figure 5.5. Use of probability words..... 141
Figure 5.6. Representation of fractions on the number line 145
Figure 5.7. Task in the lesson plan..... 145
Figure 5.8. DBE lesson plans: Grade 7 Lesson Plan: Term 4 PROBABILITY: Probability (lesson 2) 151
Figure 5.9. Circular S5 of 2018..... 155
Figure 5.10. Trends observed in each category..... 159
Figure 5.11. PR_1's views on the augmented activities……....179
Figure 6.1. Definition of probability: examples given by participants..... 199
Figure 6.2. Basis for formulating learning objectives 207
Figure 6.3: RIRAD Framework: An integrated reflective dual perspective..... 218
Figure 6.4. Clarifying cores of conceptual understanding in relation to probability..... 221
Figure 6.5. Processes involved in reconstructing the definition of probability …….....223
Figure 6.6. Overview to augment teaching and learning activities …….….225
Figure 6.7. Redesign and development of teaching and learning activities ……...227
Figure 6.8. Further adjustment to activities ……..….228

LIST OF TABLES

Table 1.1 Percentage of students considering different options as correct answers ……4
Table 1.2 South African students' performance in data and chance ……7
Table 2.1 Characterisation of procedural knowledge and conceptual knowledge 25
Table 2.2: How conclusions are derived through reasoning 27
Table 2.3: Indicators of conceptual understanding in the context of probability 36
Table 3.1: SACMEQ mathematics competency levels and their description..... 46
Table 3.2 South African teachers' and learners' competence levels in percentages. 47
Table 3.3: Sample of tasks and specific manifestations 54
Table 3.4: Essential aspects of probability 59
Table 3.5: Purpose and values of modal verbs 65
Table 3.6: Proposed questions that make sense of the language of probability 80
Table 3.7: Analysis of gap found in literature 88
Table 4.1 Participants' demographics 100
Table 4.2: Participants' profile 107
Table 4.3: Schools' profile 111
Table 4.4: Five phases of analysis and their practical implications 121
Table 4.5: Trustworthiness and credibility of the study 126
Table 5.1: Guiding questions on how to develop students' proficiency in general..... 167
Table 5.2: PR10's reflections on the augmented activities 176
Table 6.1: Reflection activities..... 228

LIST OF ACRONYMS

ANA	Annual National Assessment
ATP	Annual Teaching Plan
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
FET	Further Education and Training
GET	General Education and Training
HSRC	Human Sciences Research Council
IEA	International Association for the Evaluation of Educational Achievement
MST	Mathematics, Science and Technology
MTF	Mathematics Teaching and Learning Framework
NCS	National Curriculum Statement
NQT	Not-Qualified-Teachers
OECD	Organisation for Economic Co-operation and Development
PBL	Problem-Based Learning
PCK	Pedagogical Content Knowledge
RIRAD	Review, Identify, Reconsider, Adapt and Develop
RME	Realistic Mathematics Education
RNCS	Revised National Curriculum Statement
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
TIMSS	Trends in International Mathematics and Science Study

FREQUENTLY USED PHRASES

Word	Justification
Strands of mathematical proficiency	Mathematical perspectives that assist in explaining mathematics knowledge acquisition.
Conceptual understanding	The central idea that drives the methodology of the study.
Probability	The phenomenon under investigation.
Framework	The word is used in the Theoretical and Conceptual Framework as well as the RIRAD framework conceptualised in this study.

STUDENTS VERSUS LEARNERS

This study refers to school children as students. Some of the scholarly reports and participants in this study refer to them as learners.

PARTICIPANTS VERSUS TEACHERS

I used the term ‘participant’ when collecting and analysing data. The term ‘teacher’ was used in the recommendations made in this study, significance of the study, participants’ responses and when acknowledging work of other scholars.

CHAPTER 1 : ORIENTATION TO THE STUDY

In this study, I explored a group of primary school teachers' conceptual understanding of probability by investigating how they comprehended the concept in relation to teaching. I conducted a multi-faceted needs analysis by observing lessons, interviewing participants, and analysing how participants interpreted students' solutions to probability questions. I also examined details of the intervention processes followed in teacher professional development.

In order to foster an understanding of what probability entails, the Curriculum and Assessment Policy Statement (CAPS) specifies the concepts and skills that should be addressed in each grade. The skills to learn include defining the concept of probability, determining and listing possible outcomes when conducting experiments (Department of Basic Education (DBE), 2011). The South African curriculum aims to develop, amongst other things, "deep conceptual understandings in order to make sense of mathematics" (DBE, 2011, p. 8). Kilpatrick, Swafford and Findell (2001) define conceptual understanding as an understanding of mathematical ideas that emphasise the connectedness of essential skills. These essential skills are developed through the use of specialised language, how they are applied and represented. Kilpatrick et al.'s (2001) definition of conceptual understanding played an important role in providing a context for this study.

In Chapter 1, I provide an overview of the methodologies followed in tracing and exploring teachers' practices when teaching the concept of probability. In addition, I make some recommendations and suggestions on how such practices could be enhanced. The problem statement, consequent research questions and assumptions are also presented in this chapter. The significance of the study, its scope and limitations are also briefly discussed.

In order to add to the understanding of the phenomenon under investigation, this thesis incorporates ideas from international assessment reports and includes a discussion of good practices in high performing countries. The concluding chapter of the thesis discusses the findings and presents the RIRAD framework that exposits ways of enhancing teachers' instructional practices in the teaching of probability. This framework has emerged from combining various concepts, especially from Kilpatrick et al.'s model, as well as information provided by participants. This introductory chapter concludes with a summary of the remaining chapters of the thesis.

1.1 Background to the problem

Currently one of the most significant discussions globally is the concern over poor student performance in mathematics. Recent developments have heightened the need for more research in the field of mathematics. A report on Trends in International Mathematics and Science Study (TIMSS, 2015) by Reddy et al. (2016) showed the degree to which students had mastered mathematics concepts and skills taught in schools. This report indicates that a) 36 countries participated at Grade 8 level and three participated at Grade 9 level; b) South Africa participated at Grade 9 level in the 2015 and 2019 TIMSS assessments and c) Botswana participated at Grade 8 level in 2003 and at Grade 9 level in 2015. Considering the fact that South African Grade 9 students participated in the Grade 8 assessment suggests that their competency levels in mathematics were low. Figure 1.1 presents performance in mathematics in the selected countries for the year 2015.

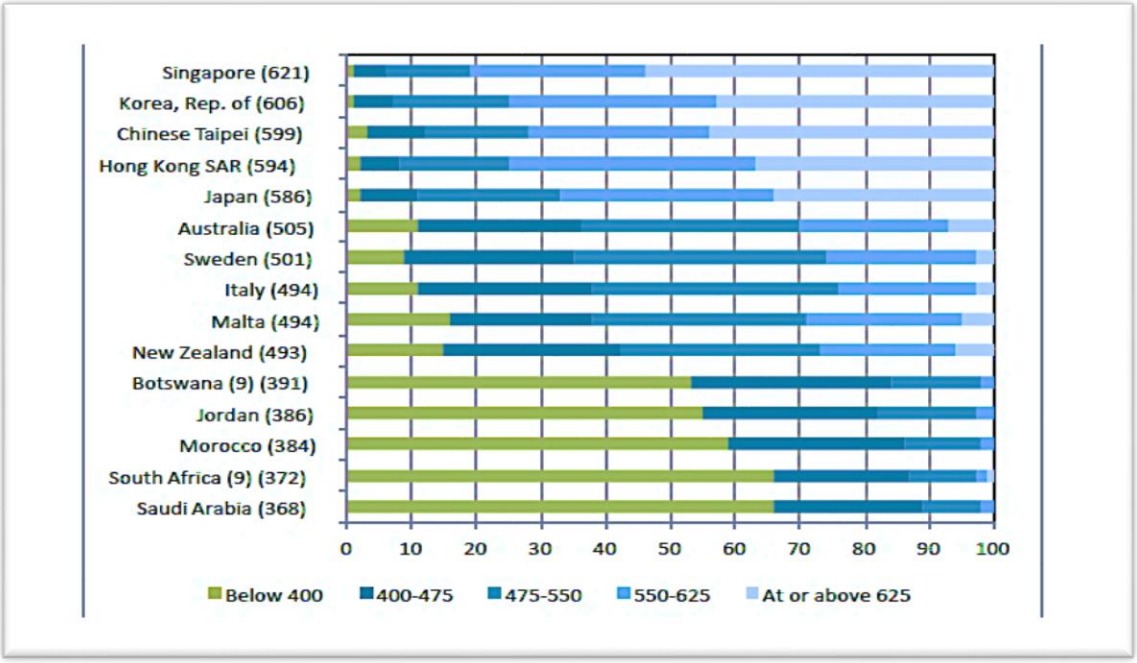


Figure 1.1. Mathematics profile of selected countries participating in TIMSS 2015
Source: Reddy et al., 2016, p. 4

Figure 1.1 indicates that the top five countries were from East Asia - Singapore, Republic of Korea, Chinese Taipei, Hong Kong SAR and Japan. South Africa was rated one of the lowest performing countries with an average performance of 372, far below the centre point scale of 500. In this report, it was also highlighted that "For mathematics, performance in the number

and geometry content areas was close to the overall mean, higher for algebra (by 22 points) and lower for data and chance (by 15 points)" (Reddy et al., 2016, p. 4). It is concerning to see that South African students are still struggling to make meaning of mathematical concepts and are amongst the below average performers in international comparative studies.

In their study, Gómez-Torres, Batanero, Díaz and Contreras (2016) found that a high proportion of participants demonstrated poor combinatorial reasoning and made errors in computing conditional probability and interpreting frequentist probabilities. Batanero, Chernoff, Engel, Lee and Sánchez (2016) suggest a need for teachers to acquire strategies and ways of reasoning to help them make proper decisions in situations related to chance. A study by Danişman and Tanişli (2018) revealed that the pedagogical content knowledge of secondary mathematics teachers about probability was inadequate and recommended that more studies be conducted in this area. In support of this recommendation, Ogbonnaya and Awuah (2019) add that regular, content-specific professional development workshops should be conducted for all teachers, especially when topics are newly introduced, such as probability.

In 2008, a new strategy to track student performance each year in Grades 3, 6 and 9, called the Annual National Assessments (ANA) was introduced by the South African education department. In her study comparing Grade 9 students' performance on the ANA and summative assessment, Bansilal (2017) concludes that poor performance in the ANA was the result of instructions that were not clear, bad timing and overly long tests.

I was employed as a mathematics subject advisor and coordinator for sixteen years. My duties included training teachers in the Revised National Curriculum Statement (RNCS), National Curriculum Statement (NCS) Grades R-9 and NCS Grades R-12. During the monitoring and support of mathematics teachers at schools, I observed that in some cases the concept of probability was not taught; in other instances, it was taught superficially only to complete the syllabus. Some teachers merely taught the skill as it was covered in the final examination. The reason given by teachers for these practices was that they did not have enough time to teach the concept, and it was taught in the last term when students were about to sit their final examinations.

To add to the challenges experienced by students in making sense of questions involving probability, Mosimege et al. (2017), reported that the South African learners performed lower than the average of their international counterparts in Question 69 as depicted in Figure 1.2

M02_12

Question 69 **Cognitive area: Knowing**

A machine has 7 different colours of gumballs. Liv noticed that people bought 306 gumballs and that 23 of them were blue. Which is the best statement that the next gumball will be blue? A. $\frac{1}{7}$ B. $\frac{7}{23}$ C. $\frac{7}{306}$ C. $\frac{23}{23}$

Figure 1.2. Assessment item on probability

Source: Mosimege et al., 2017, p. 81

Table 1.1

Percentage of students considering different options as correct answers

	A	B	C	D	Omitted
South Africa	8,9	22,7	19,5	**42,1**	1,6
Internal average (n =38)	14,5	17,4	15,8	**46,0**	2,3

Source: Mosimege et al. (2017, p. 81)

Mosimege et al. (2017) further highlighted that:

> Learners did not understand that the concept of probability relies on the total number of outcomes, therefore they select B as an option. The word "estimate" versus "probability" may be of concern for some learners. Perhaps learners failed to recognise that the question refers to "blue" only. Language may be an issue here. Due to slow coverage of topics through the year, these final concepts in CAPS are often ignored by the teacher. (p. 81)

Probability is one of the final concepts taught in the fourth term (DBE, 2011). Although we as researchers in most cases focus on students' poor performance, we also acknowledge the efforts made by the Human Sciences Research Council (HSRC) and DBE to develop this educator resource which acts as a supplement to other resources used in schools. The strengths and

weaknesses identified and recommendations made in Mosimege et al.'s (2017) item analysis report would strengthen teachers' instructional practices.

Because classroom instructional practices are integral to effective teaching, this study explored how teachers made sense of their practices when teaching probability. Le Donné, Fraser and Bousquet (2016) point out that good instructional practices are built on three critical components:

Active learning: This strategy includes promoting engagement, working in groups, reflecting on activities, and using technology to foster an interactive environment.
Cognitive activation: This approach refers to students' ability to communicate their thinking processes through creativity and engaging in problem-solving activities.
Teacher-directed instruction: This technique is based on the understanding that teachers must be able to formulate explicit goals, and deliver clear and orderly lessons.

These critical components convey the understanding that instructional practice is a broad term that embodies a multitude of concepts that depend on the context in which they are applied (Ojose, 2011) and their intended goal. To add to the implications of the third critical component, Ni, Zhou, Li & Li (2014) emphasise that the engagement between the teacher and the student is directed by the intentions set. In other words, goals set should serve as a road map to guide the development of teaching activities.

The items in the lesson observation protocol clarify the perspectives from which the concept of instructional practice is viewed. This study supports the view that instructional practices should encourage learning by enabling students to construct knowledge and produce a product that can be measured and demonstrated (Von Oppell & Aldridge, 2021). The following section explains why this study explores teachers' practices in teaching the concept of probability.

1.2 Statement of the problem

The desire to explore teachers' practices in teaching the concept of probability was motivated by the decline in the pass rate in mathematics in South Africa over the years and in the way this subject is taught. As discussed in Section 1.1 above, probability continues to pose a challenge in schools, and very little guidance on how this problem can be resolved has

emerged from international studies. Although extensive research has been conducted on the teaching of probability, no single study exists that adequately covers the teaching of probability in primary schools in a South African context. The focus of research on probability tends to be on secondary schools.

From my experience as a subject advisor and mathematics lecturer, it appears that the terminology of probability is an obstacle. This view is supported by Mosimege et al. (2017) who conclude that "Learners don't know the terminology regarding the probability scale (0% = "will not" and 100% = "it will" and the rest in between" (p. 87).

In a study conducted by Gürbüz and Birgin (2012) in Turkey, it was found that the main reason that probability was not taught effectively was misconceptions related to the concept. Gürbüz and Birgin pointed out that such misconceptions may be caused by linguistic difficulties or by extracting mathematical structures from their practical embodiment. In addition, Groth, Butler and Nelson (2016) recommended that more research was required to investigate the extent to which students become successful probabilistic reasoners if they learn the related vocabulary at an early stage.

Having acknowledged the fact that students' understanding of probability remains problematic, I decided to explore how probability was conceptualised at Grade 7 level. I was particularly interested in this topic because I realise that many people do not understand its relevance to real-life situations. I was also motivated by the fact that probability is a topic that could be taught practically and in a way that raises awareness of the risks and opportunities that arise in life. The aim was to identify techniques and new ways of doing things to revive students' interest and engagement in mathematics (Peters, 2016). An added impetus, according to Mosimege et al. (2017), was that the CAPS curriculum is too packed, and Grade 7-9 teachers struggle to get through the concepts.

To conclude this section, Table 1.2 shows how South African students performed in Data and Chance especially in the constructed-response questions where the Grade 9 students had a 12.5% pass rate compared with the 36,4 % internationally.

Table 1.2

South African students' performance in data and chance

	Constructed-response questions	
	South Africa	International average
Number	33,3	54,1
Algebra	18,3	35,8
Geometry	24,8	44,5
Data and chance	12,5	36,4

Source: Mosimege et al., 2017, p. 12

Thus, the problem statement gave rise to the research questions discussed in Section 1.3

1.3 Research questions

Three research questions guided the study. The first question focused on teachers' understanding of the concept of probability. The second question was designed to explore teachers' conceptual understanding of probability, and how this understanding influenced teaching and learning. The third question was designed to outline the type of support given to teachers to enhance their classroom practice.

Primary research questions

- What are teachers' conceptual understanding of probability?
- What conceptual understandings of teachers impede and/or enhance the teaching of probability?
- How can teachers' conceptual understanding of probability be enhanced pedagogically to improve classroom practice?

1.4 Aim of the study

The aim of this study was to design a framework to enhance teachers' instructional practices in the teaching of probability. This framework also focuses on assisting teachers to think of and suggest strategies that could promote the development of students' conceptual understanding in probability.

1.5 Purpose of the study

This study intended to identify and explore the skills required by Grade 7 mathematics teachers to teach probability effectively. Drawing on the findings from the needs analysis stage and through professional development, this study provides ideas to support teachers in enhancing their instructional practices in the teaching of probability.

1.6 Significance and possible contribution of the study

The findings of the study may contribute to the current literature, which at present is scant, on this topic. Firstly, it may enhance or add to teachers' understanding of the concept of probability. Secondly, it could provide some guidelines on augmenting teaching and learning activities to promote proficiency in the learning of probability. Thirdly, it may highlight some initiatives to improve instructional practice in teaching probability and, lastly, the findings could be used to develop targeted interventions at all levels, aimed at teachers' professional development. Hence, the RIRAD framework presented in this study is all-encompassing and intended to benefit teachers, subject advisors and, to a certain extent, researchers. Above all, it is hoped that the study will make mathematics teachers more aware of how they can create opportunities for students to develop their conceptual understanding of probability. In other words, teachers will develop teaching and learning activities to help students define the concept properly, apply the concept in real-life situations, represent the concept in various ways and use the correct terminology when explaining the concept.

In an effort to guide teachers in creating such opportunities, this study highlights specific issues that would benefit from effective interventions. It also identifies those aspects of teachers' conceptual understanding that impede or enhance the teaching of probability. The last chapter provides an in-depth teacher empowerment framework that describes the pillars of effective instructional practices that relate to the improvement of the teaching of probability in South Africa, but which could be adapted for other countries.

1.7 Research methodology

1.7.1 *Pilot Study*

I collected data from two teachers, two lecturers and one researcher to assess the appropriateness and the validity of the data collection instruments. I ensured that teachers selected for the pilot study shared similar backgrounds with the target teachers of the main

study. The aim of this study was to form a clearer idea of the demands of the research data collection instruments.

1.7.2 *Main study*

This qualitative study was focused mainly on exploring mathematics teachers' conceptual understanding of probability, and the skills they require to improve their instructional practices when teaching probability. Methods used in this study enabled me to obtain an in-depth understanding of opportunities teachers created to develop the concept of probability at Grade 7 level. This study followed a social constructivist model and was based on the assumption that individuals make meaning of the social world through a process of investigating and structuring knowledge (Adem, 2021; Günes, Bati, & Katranci, 2017). The study took an interpretive stance in analysing the situation and the way that the participants make sense of their circumstances. This conception echoes the findings of a study by Maree (2007), who argues that an interpretivist view is based on the hypothesis that the origin of meaning and human behaviour is affected by what is experienced in the social world.

In this study, I went into the field to see how meaning was constructed. Understanding people's experiences in their natural environment reflects what Van Merriënboer and de Bruin (2014) refer to as social construction—the collaborative construction of meaning through interaction with others. I collected data through lesson observation, semi-structured interviews and focus group discussions. Although going into the field has many advantages, such as gaining first-hand information of what transpires, I was mindful of the fact that my presence at the research site might influence the data. However, I felt that it was necessary to go into the field to see how teachers facilitated the learning of probability in the classroom and how they helped students to construct knowledge.

The study used purposive sampling. The sample for the needs analysis phase comprised eight teachers, all of whom had over ten years' experience of teaching mathematics to Grade 7 learners. Owing to the outbreak of COVID-19 and the subsequent lockdown regulations implemented in the country, some participants were unable to participate in the intervention phase of the study. For the intervention phase, I used the snowballing method to sample a second team of participants, which included two of the teachers who had participated during the needs analysis phase. The second team consisted of one more mathematics teacher, two subject advisors and one mathematics coordinator. In total, the sample for the intervention

phase was made up of three teachers, two subject advisors and one mathematics coordinator. It was important that each member of the team had access to a computer and the internet as all communications and activities were conducted online.

As part of the data collection process in the first phase of the study, some of the participants presented two lessons and others only one; individual interviews were conducted immediately after presentation of the lesson(s). In addition, more data were collected during the intervention workshop. These data provided insight into a) how teachers experienced the teaching of probability, b) the nature of this experience, and c) critical matters arising from their understanding and teaching of probability. Categories that emerged were coalesced into sub-themes and these sub-themes were consolidated into an over-arching theme. Confirmability and trustworthiness were addressed by member checking to ensure reliability (Prosek & Gibson, 2021). In the case of member checking, I constantly repeated and confirmed what participants said. In other words, I ensured that the key areas that I highlighted at the end of each discussion were a true reflection of what the participants had said.

1.8 Ethical considerations

I obtained permission to conduct this research study from the University of South Africa, the Mpumalanga Department of Education, subject advisors, the principals and teachers at the sampled schools. In the letters I wrote requesting permission to conduct research at these schools, I provided information on how I would conduct the study, and on the data collection and analysis processes.

I assigned pseudonyms to teachers, their schools and to subject advisors. These pseudonyms comprised the two letters PR and numbers. These letters indicated ‘participant probability’. Probability was the focus of this study. Participants were consulted about the use of pseudonyms and gave their consent (Geiger, 2021). The reason for using these pseudonyms was to ensure the anonymity of participants. Data were collected during school hours, during mathematics periods in Term 4 to avoid any disruption of the proper running of the schools. I had no link to any of the schools selected for this study.

1.9 Model of data collection process

The data collection process was conducted in two phases, namely the needs analysis phase and intervention phase. I visited the schools to discuss the rollout plan of the study with teachers **(Stage 1).** I delivered the open-ended questionnaires (see Appendix C) to school in person and collected them after completion. Teachers were given two weeks to complete the document **(Stage 2)**. I analysed participants' responses **(Stage 3).** I observed lessons and interviewed teachers. Triangulation was achieved by using different techniques to explore the phenomenon and to enrich the data. Interviews were conducted in the last meeting with participants in the first phase of the study (**Stage 4)** (McGinley, Wei, Zhang & Zheng, 2021). In the second phase of data collection (the intervention phase), I conducted an intervention workshop to collect data on how participants made meaning of the strands of mathematical proficiency, and how they developed and used the guidelines to adapt the teaching activities on probability. During this phase, I assumed the responsibilities of both the facilitator and observer.

The last stage involved analysing the data. The findings formed the basis of the proposed framework for teacher empowerment **(Stage 5).** Figure 1.2 is a diagrammatical representation of the data collection process in this study.

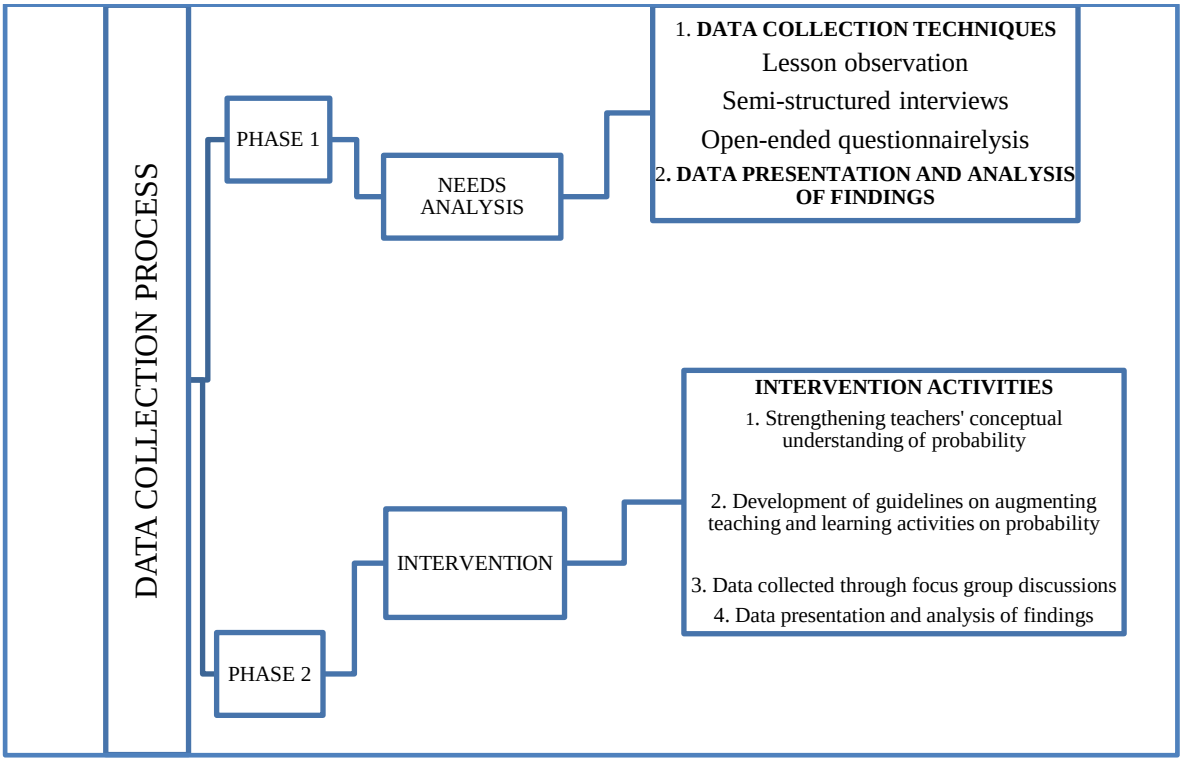


Figure 1.3. Data collection processes

1.10 Limitations of the study

As data were collected from observations and interviews, my role in the data collection process was key. As such, my views, values and beliefs and prior experiences may have affected both processes. In order to mitigate this limitation, I asked a person outside the project to conduct a review of the study and to report back in writing on its strengths and weaknesses (Creswell, 2008). To make sure that the data were accurate, I asked two of the teachers to check the transcripts of the interviews. Also, to minimise costs of travelling, data were collected from only one of four districts and this small sample cannot be taken as representative of all teachers in Mpumalanga province.

The study was focused on the teaching of probability, which makes up a small section, 3%, of the mathematics curriculum. In other words, the study was carried out on a specific topic in a specific context and any application of findings to other contexts should be done with caution as the results are not generalisable. The sample used in the investigation included only six teachers and three subject advisors from the district. Since the study took a qualitative approach, the findings cannot be extrapolated to other contexts.

1.11 Assumption of the study

I embarked on the study with certain claims and assumptions in mind. These assumptions were based on the principles of the paradigm, constructivism, and on the conceptual framework that guided this study. In other words, I could not adopt challenges identified in the literature and turn them into the assumptions of the study because the reality could not be predicted nor described. The following assumption about the identified problem of the conceptual understanding and teaching of probability was made in the study:

Assumption: Grade 7 mathematics teachers' conceptual understanding of probability is guided by how they define probability, use the language of probability, apply probability to real-life situations, and how connections between probability terms and representations probability are made. This assumption is embedded in the paradigm used to investigate the phenomenon, to describe a set of concepts in a subject area and to clarify how knowledge is discovered (Hays & McKibben, 2021).

1.12 Summary of chapters

This thesis consists of six chapters. A brief description of each chapter is provided below:

Chapter 1: Introduction and overview

This chapter has provided the introduction to the study. The problem of the study was presented, and the research questions, significance of the study, and context of the study were explained. The possible limitations of the study were discussed, as were the main assumptions. The chapter concludes with a summary of the remaining chapters.

Chapter 2: Theoretical and conceptual framework

This chapter provides a discussion of the theoretical and conceptual framework on which the study was based. The framework provides a deeper understanding of the model used to guide the processes in this study. Concepts in the framework were applied as mathematical perspectives to assist in explaining mathematics knowledge acquisition and were used when making arguments, analysing the findings and when developing the items for the data collection instruments.

Chapter 3: Review of the literature

This chapter provides information gleaned from a review of scholarly work that is related to various issues and debates concerning the concept of probability, and how it is taught and learned at school level. Because the intention of this study was to design a framework, the discussion in this chapter includes teacher professional development strategies as practised in high performing countries. The review of literature gave me the understanding and insight I needed to understand terms used in probability, teachers' practices in probability and how to refine the research question to keep my research focused and on track.

Chapter 4: Research design and methodology

The chapter comprises a discussion of the qualitative methodological approach that was followed in this study. This includes a description of the paradigm and the philosophical assumptions, the main assumptions in the study and the research design. The study population and sampling procedures are described. This study included a pilot study, which is described in detail in this chapter. The main study comprises the needs analysis and intervention phases. The data collection processes and instruments (lesson observation protocol,

interview schedule and open-ended questionnaire) used in the pilot and the main study are discussed, as is the data analysis.

Chapter 5: Analysis of data and discussion of findings

This chapter presents the evidence from the field and the analysis of findings from these data. It provides a description and interpretation of teachers' views regarding their practices in teaching probability. The data analysis revealed several thematic strands and the findings are presented in an integrated manner according to these strands. This study found that teachers have limited conceptual understanding of probability. However, they were highly motivated and believed that they could design activities that are appropriate to develop students' proficiency in probability.

Chapter 6: Summary and conclusion

In this concluding chapter, the main findings are discussed. The implications of these findings for the teaching and learning of probability are examined. Recommendations and suggestions for further research are provided. The findings of this study are discussed as they relate to other studies in this field. The contributions to knowledge made by this study are discussed, as are the limitations of the study. Finally, this chapter concludes and presents the RIRAD framework which is conceptualised within Kilpatrick et al.'s (2001) model. This framework aims at deepening teachers' conceptual understanding of probability and enhancing their classroom instructional practice.

CHAPTER 2 : THEORETICAL AND CONCEPTUAL FRAMEWORK

2.1 Introduction

The theoretical and conceptual framework provides a deeper understanding of the model used to guide this study. It is a perspective through which to view teachers' conceptual understanding of probability and helps to create a conceptual base from which to enrich teachers' instructional practices (Islam & Cullen, 2021). This framework allows the researcher to look more deeply into the practical implications of teaching probability and provides some guidelines on how to extract and make sense of data.

In this chapter, I detail the conceptualisation and the operationalisation of Kilpatrick et al.'s (2001) strands of mathematical proficiency in the current study. The five strands of mathematical proficiency are explained and discussed in detail and how they provided me with a lens to explore and understand teachers' conceptual understanding of probability. In other words, I explain the way in which I used the strands to assist me in establishing teachers' conceptual understanding of probability and showing how their experience of teaching probability formed the basis of their professional development (Ngulube & Mathipa, 2015).

Kilpatrick et al.'s (2001) strands of mathematical proficiency were espoused as relevant conceptual tools for the current study as they provided the vocabulary to explain how the ideas outlined in the RIRAD framework relate to enhance teachers' classroom practices in developing the concept of probability. Figure 2.1 provides a schematic representation of the strands of mathematical proficiency.

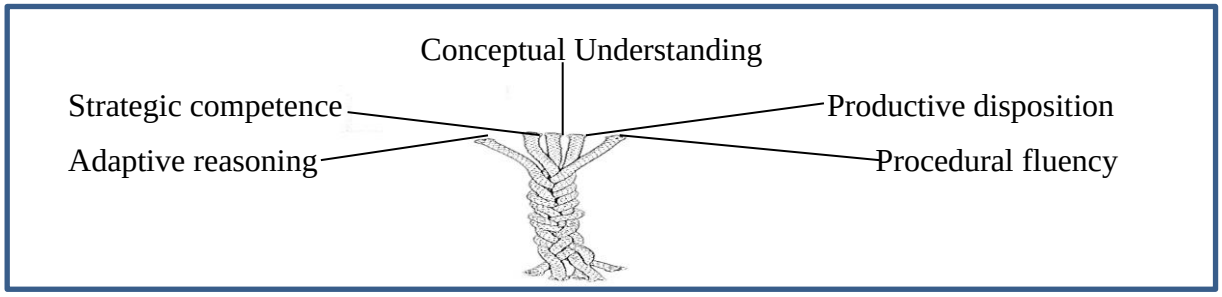


Figure 2.1. Five strands of mathematical proficiency

Source: Kilpatrick et al., 2001, p. 117

What follows is a discussion of the five strands of mathematical proficiency.

2.2 Five strands of mathematical proficiency

In this section I discuss what each strand of mathematical proficiency entails. In the book, *Adding it up: Helping children learn mathematics*, the National Research Council (2001) describes mathematical proficiency as five connected and interwoven strands. To emphasise the interrelatedness or connectedness of these strands, Groves (2012) concludes that mathematical proficiency cannot be achieved by focusing on one strand; each strand forms a subset of the whole and none is independent. Figure 2.2 illustrates the interrelatedness of the strands of mathematical proficiency.

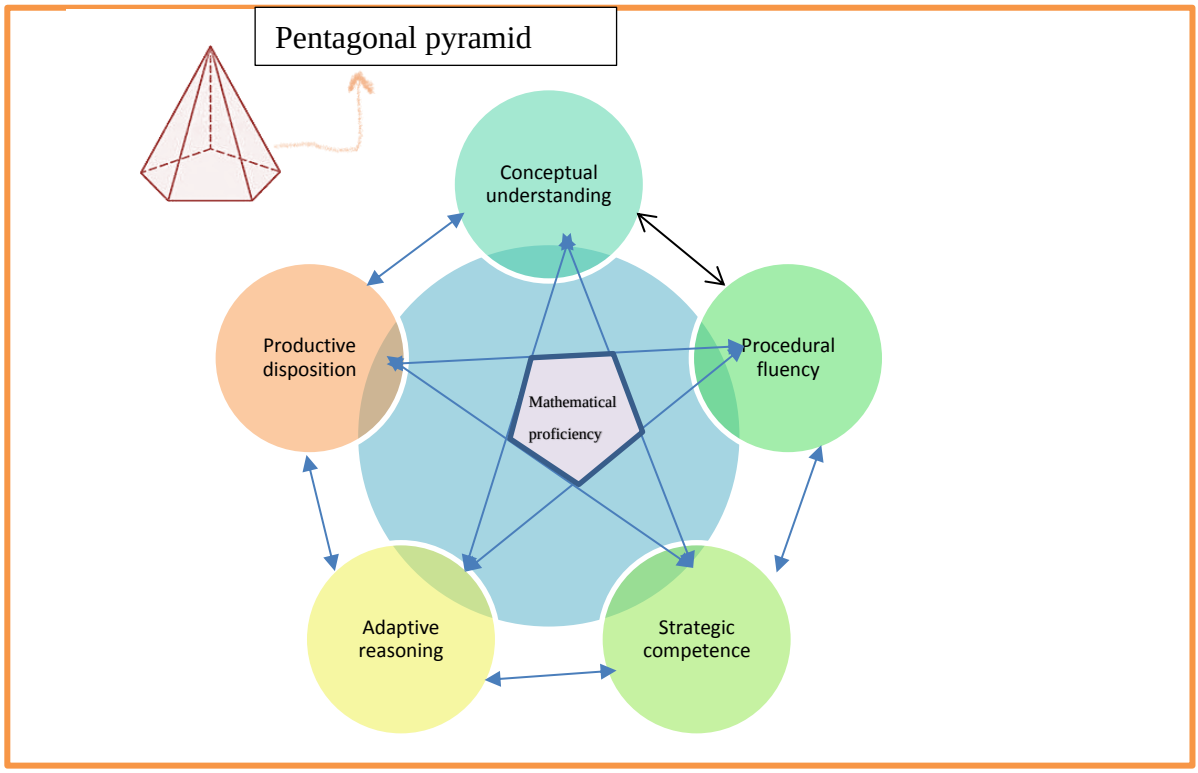


Figure 2.2. Interrelatedness of strands of mathematical proficiency

As indicated in Figure 2.2, the net of a pentagonal pyramid resembles the complex whole of mathematical proficiency. The pentagonal base of the pyramid appears regular. In mathematical terms, this means that all the sides of the pentagon are equal, and all the angles are equal. Analogically, this figure suggests that the strands of mathematical proficiency are equally rated in terms of their contribution towards developing student proficiency in different concepts of mathematics. With this remodelling, if the net is formed, one cannot detach the triangular shape

from the base. One side of each triangle is anchored to the pentagonal base. This diagram shows how the strands are interrelated and connected.

To add to this, one nodal point from each triangle forms one vertex of the pentagonal pyramid. From the perspective of this study, the five strands of mathematical proficiency share common boundaries, the base and the vertex. The way the Figure 2.2 is presented seems to provide evidence that all the strands contribute towards the attainment of mathematical proficiency; they are interrelated and none of them can be developed in isolation. Acknowledging that profound insight is needed to develop students' mathematical proficiency, the Department of Basic Education (DBE, 2011, p. 154) emphasises that "knowing students' level of proficiency in a particular mathematics topic enables the teacher to plan her/his mathematics lessons appropriately and to pitch them at the appropriate level". This view is supported by Ally (2011) who notes that there is a great deal of literature that suggests that strands of mathematical proficiency have a positive impact on the teaching and learning of mathematics.

Mathematical proficiency is achieved by the development of these five strands, which are the main concepts that govern the model proposed for the theoretical and conceptual framework in this study. Although Wilson, Heid, Zbiek and Wilson (2010) argue that a sixth strand that focuses on historical and cultural knowledge is possible, this study is confined to the five strands mentioned above. The argument made by Wilson et al. (2010) is that the sixth strand is likely to lead to deeper understanding of mathematical rules.

The allegorical interpretation of the connectedness of strands of mathematical proficiency could be analogically equated to the function of the legs of a table. Each leg of a table helps to disperse the weight and reinforce the base. Secondly, they set the tone for the design and give the table its unique identity and purpose. In other words, these strands serve as anchors to promote student mathematical proficiency in a teaching and learning situation. In an effort to shed more light on what teaching for proficiency means in the context of the theoretical and conceptual framework used in this study, a brief explanation of what each strand entails is provided in the next section.

2.2.1 *Conceptual Understanding*

Kilpatrick et al. (2001) argue that conceptual understanding refers to the comprehension of mathematical concepts, operations and relations. In this case, it relates to full knowledge and

understanding of the concepts, operations and relations used in mathematics. This definition points to how students see relationships and make connections between mathematical concepts.

Van de Walle, Bay-Williams and Karp (2016, p. 48) define conceptual understanding as "a flexible web of connections and relationships within and between ideas, interpretations and images of mathematical concepts". These definitions embody a multitude of concepts such as basic operations, relationships and terms used in mathematics. Referring to Figure 2.2, my understanding is that exploring conceptual understanding includes understanding the foundational ideas of the topic. Foundational ideas in this context refer to basic mathematical concepts and how teachers engage students to make the concepts more meaningful and sensible. Some of the elements that are vital in developing students' conceptual understanding of probability include the way teachers themselves interpret probability concepts, make meaning of the relative nature of probability, and how they incorporate concepts associated with knowledge of probability (Danişman & Tanişli, 2018).

Kvatinsky and Even (2002) argue that the explanatory framework that teachers use could include analogies, illustrations, and examples to explain the content more clearly and through demonstrations. This view is supported by Van de Walle et al. (2016), who observe that conceptual understanding includes a network of representations and interpretations of concepts created using pictures, manipulatives, tables, graphs and words. In other words, if students are to build connections between ideas, teachers need to provide them with the opportunity to make connections and to see how the same ideas play out across different representations. Figure 2.3 illustrates various ways of representing an idea to assist in building a relational understanding of probability.

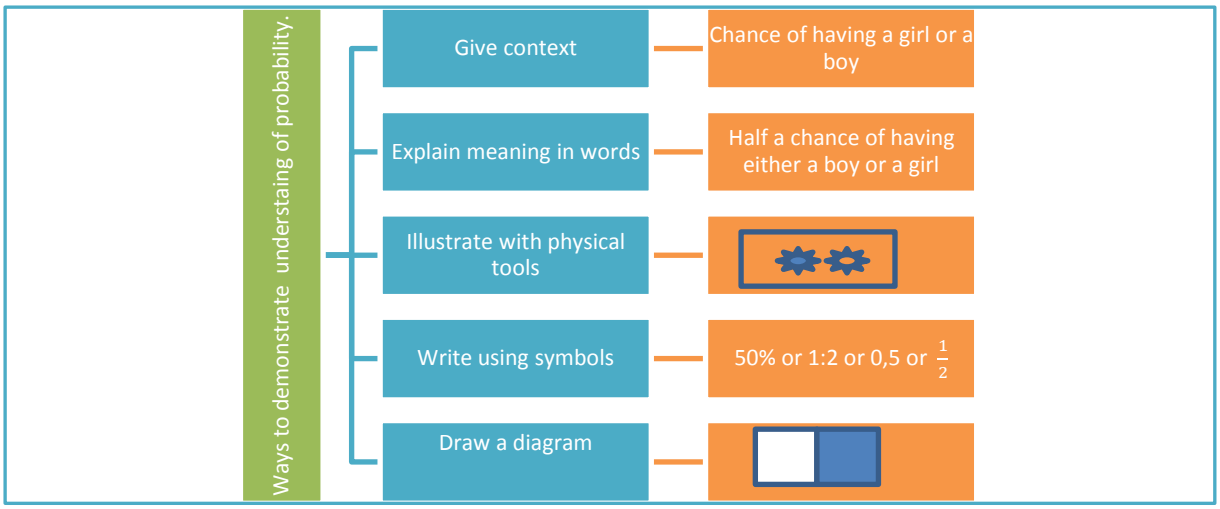


Figure 2.3. Web of representations

Source: Adapted from Van de Walle et al., 2016, p. 45

Multi-representation of an idea in the context of Figure 2.3 include writing problems and translating them into mathematical statements, drawing pictures and applying mathematical concepts in real-life contexts. Teachers also need to ensure that students understand the use of various representations in dealing with problems on probability. Representations include words, tables, tree diagrams, physical manipulatives and graphs. Understanding connections between these representations could extend students' experience of probability. The next section provides a discussion of the understanding of connectedness of concepts.

- *Understanding connections between concepts and procedures*

Kilpatrick et al. (2001) argue that conceptual understanding concerns the ability to see the relationship and make connections between mathematical concepts. To build on an understanding of connectedness, Van den Heuvel-Panhuizen and Drijvers (2014) believe that the level principle must be understood from the perspective that students pass through different levels of understanding. These authors suggest that students could learn through informal context-based activities, creating shortcuts and schematisation and by acquiring the skill of identifying interrelated concepts. In other words, all these strategies are aimed at incorporating basic concepts in the building of a solid foundation for the concept of probability, and at providing arguments to explain why some facts are consequences of others. Other forms of connection refer to how teachers draw on students' prior knowledge to connect formal learning and how they develop strategies to help students acquire appropriate skills to make sense of probability.

- *Application to real-life situations*

In emphasising the importance of drawing from real-life situations, Skovsmose (2016, p. 418) observes that "mathematics education might be located along the horizons of the students' life-worlds". This understanding suggests that any teaching and learning activities should be at students' cognitive level and should be aimed at building awareness of the role of mathematics in real life. Van den Heuvel-Panhuizen and Drijvers (2014) present the Realistic Mathematics Education (RME) theory as a domain-specific instruction theory characterised by the fact that realistic situations serve as a basis for initiating the development of mathematical concepts. My view is that students should start with meaningful real-life problems and attempt to devise useful mathematical representations of the connections between facts. Problems can only be meaningful if they are realistic and if students see the importance of solving such problems. In addition, Alsulami (2016) believes that applying knowledge to real-life situations promotes communication between students and society.

There is a definite need for teachers to be aware of their students' understanding if they are to decide on an appropriate context for learning (Depaepe, Verschaffel, & Kelchtermans, 2013). Examples of contexts that could be used to explore the concept of probability might include but would not be limited to sports, business and weather. If learning is to take place, students need to conceive their own context. It is also important that teachers use their own strategic competence to interrogate students' choice of context. Carpenter, Fennema, Peterson and Carey (1988) also stressed the importance of correlating teachers' knowledge and students' achievement.

- *Understand the language used in mathematics*

The discussion of this strategy is two-fold. It refers to both language across the curriculum (language of teaching and learning) and the language of probability. These languages are both necessary for students to understand and solve problems involving probability. It is also critical to highlight that what amplifies the quasi-equity of the languages is the understanding that there are words unique to mathematical communication which take on unique meaning in mathematical context (Morgan, Craig, Schuette & Wagner, 2014).

To develop proficiency in the language of teaching and learning, Bretuo (2020) mentions that language support centres are established for peer-to-peer teaching through the guidance of the

teacher or the supervisor. In her study, Nene (2017) advised teachers to engage with and listen to students' feedback on the difficulties they experience in their learning, particularly learning related to their real-life experiences. She added that engagement would enable teachers or facilitators to adapt their lessons to students' circumstances. All these views confirm that students need teacher support to improve their language competence (language of learning and teaching) to get to grips with the language used in probability.

I used the following example to articulate the importance of understanding the language as a vehicle of communication to make sense of probability concepts. In this example, students were asked to look at the spinners on the board. They were instructed to use the terminology of probability, words such as "likely", "unlikely" and "certain" to describe the likelihood of the spinner landing on the green colour.

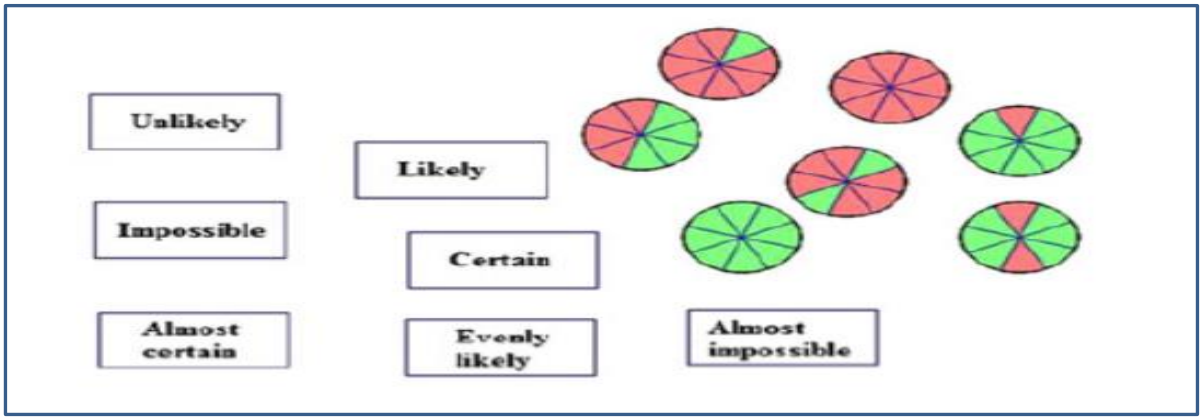


Figure 2.4. Probability of landing on different colours
Source: Groth et al., 2016, p. 105

Groth et al. (2016, p. 106) show how students comprehended the terms and language used in this probability problem when they responded to the question:

> They all agreed upon the all-green spinner. Rebecca explained: "Last time we were here, we learned that certain meant that it was like 100% going to happen. Even if you had a tiny sliver of a different colour, you could still land on it because it is an option." Upon hearing Rebecca's comment, we moved the all-green spinner to the top rung of the probability ladder along with the term certain. We put the fraction $\frac{8}{8}$ and asked if there were any other ways to express the probability. Students' knowledge of equivalent fractions and percentages led them to offer 100%, $\frac{4}{4}$ and 1 as different possibility.

From this extract, I realised that students were able to use the correct language of probability to draw on their prior knowledge and to use different representations to illustrate the likelihood of an event happening. In trying to respond to this question, students drew on their basic knowledge of fractions. This understanding indicates that students had conceptual understanding of fractions, including equivalent fractions. They were also able to connect new ideas of probability to certain and 100%. One other critical point was the fact that Rebecca realised that certainty could also be described in terms of area. Her response also suggests how activities could be sequenced to unpack the concepts of probability. In this case, Rebecca could make choices and apply her understanding in the language of probability through active engagement (Andamon & Tan, 2018).

The following extract provides an example of how language might pose a challenge for students who speak English as a second language.

Your sister has four children. What is the probability that your sister has:

I. Boys only?
II. A boy or a girl or another boy?
III. Two boys and then a girl?
IV. Two boys and a girl?
V. A girl as first born?

Figure 2.5. Language use challenges

Source: Brijlall, 2014, p. 724

Brijlall (2014) highlights the fact that English second language speakers might encounter problems differentiating (ii), (iii) and (iv) and, as such, students might need more clarity on what it means if the word ‘then’ is included or excluded from a question. It is therefore important for teachers to ensure that the way concepts are used conveys the intended meaning. Language plays a crucial role in probability because it is a vehicle through which a message is conveyed. In this study, I look at how Grade 7 mathematics teachers use the language of teaching and learning to explore the concepts of probability. It is also important to observe and make meaning of how teachers use and interpret the concepts of probability in relation to everyday life experiences.

2.2.2 *Productive Disposition*

Productive disposition refers to a habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick et al., 2001, p. 116). To shed more light on this definition, Kilpatrick et al. (2001) argue that students need to believe that mathematics is understandable, that it makes sense, can be learned and used. Seeing sense in mathematics is likely to have an impact in students' attitudes and beliefs and, in turn, to influence their mathematics performance. Previous studies (Andamon & Tan, 2018; Ma & Kishor, 1997; Suh, 2007) support this argument and the belief that there is a relationship between students' attitude towards mathematics and their performance. Andamon and Tan (2018, p. 104) further emphasise that "the higher the attitude towards mathematics, the better the performance in mathematics". In the current study, I examine how teachers persuade students during the lesson presentation to become effective and doers of mathematics

A recommendation made by Lomibao, Luna and Namoco (2016) is that students should be encouraged to write down how they construct their own understanding of concepts, their experiences about mathematics and how they foster a positive attitude towards mathematics. I regard it as a positive move if teachers devise effective ways to address students' attitudes toward mathematics. Although this is to be encouraged, it must be cautiously done as it takes an effort to make mathematics sensible. For example, to develop productive disposition, teachers are expected to give students frequent opportunities to make sense of probability, to recognise the benefits of perseverance and to experience the rewards of sense-making in mathematics.

Le Roux (2020, p. 58) conceptualises seven essential indicators for assessing productive disposition as follows:

> Mathematics as a sense-making endeavour, mathematics as beautiful or useful and worthwhile; beliefs that one can, with appropriate effort, learn mathematics, mathematical habits of mind, mathematical integrity and academic risk-taking, positive goals and motivation and self-efficacy.

This conceptualisation provides some idea of what to look for when assessing students' productive disposition, not overlooking the fact that it takes an effort to form a positive attitude towards mathematics. In the study by Graven (2012), the scholar concludes that most research

focuses on assessing the other four strands (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning) and overlooks the productive disposition strand. This understanding suggests that teachers need to devise strategies to ensure that this strand is also catered for during their lessons. It would help teachers to develop this strand effectively if they could create a learning space for students.

2.2.3 *Procedural Fluency*

According to Kilpatrick et al. (2001, p. 116), procedural fluency refers to the "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". Procedural fluency is also understood as "the process through which mathematics is done where students need to perform mathematical procedures accurately and efficiently" (DBE, 2018a, p. 9). Students need to know when to use a procedure and why they have decided on that procedure. In relation to this study, these definitions encourage teachers to inculcate in students the spirit of striving for accuracy, efficiency and to use methods appropriately when solving problems involving probability.

Through proper guidance, teachers need to facilitate lessons in such a way that students demonstrate the understanding of effective ways of choosing appropriate methods when solving problems. It is also advisable to use appropriate methods when conducting experiments for acquisition of the language of probability (likely, equal chance, certain and uncertain). Through the support of teachers, students need to stick to the facts and limit the influence of personal biases or preconceived ideas. To add to the preceding view, the strategy used in incorporating resources, if well conceptualised, would develop students' efficiency and accuracy in performing basic computations that lead to the solution of problems involving probability. In other words, students would be able to carry out the steps of solving a problem easily and know when a particular procedure should be used. Teachers are therefore expected to support students in refining their informal methods of calculating probability to develop their skills in standard procedures and algorithms.

Table 2.1 provides descriptions of conceptual and procedural knowledge of probability and statistics topics.

Table 2.1

Characterisation of procedural knowledge and conceptual knowledge of probability and statistics topics

Procedural Knowledge	Knowledge of definitions of terms, formulae and procedures for computing measures of central tendency (mean, median and mode), standard deviation for ungrouped and grouped data. Knowledge of notations, representations and algorithms. Knowledge of formulas and procedures for computing probability of different events, knowing when and how to use combinatorial rules and probability rules.
Conceptual knowledge	Understanding of statistical and probability concepts and rules, justifying and interpreting data distribution using different representations. Relating counting principles to probability rules, comparing probabilities of events involving different experiments, understanding the relationships between two probability events, understanding classical definitions and meaning of probability events; the ability to distinguish which events are independent and which are not; understanding when and how to use combinatorial rules and probability rules.

Source: Legesse, Luneta and Ejigu , 2020, p. 3

This information led me to realise that characterisation of both procedural and conceptual knowledge still points to the indicators of procedural and conceptual understanding as proposed by Kilpatrick et al. (2001). A broader perspective was adopted by Hiebert, 1986 (as cited in Joersz, 2017, pp. 1-2), who defines procedural knowledge as having two distinct parts:

> The first part is the formal language of mathematics, or symbol representation system. It includes familiarity with the symbols used to represent mathematical ideas and an awareness of the syntactic rules for writing symbols in an acceptable form. The second portion of procedural knowledge consists of rules, algorithms, or procedures used to solve mathematical tasks. These are described as step-by-step instructions for how to complete a task.

Nance (2018, p. 16) emphasises the importance of balance between conceptual and procedural fluency and indicates that:

> Balance in learning can occur when students are able to use their knowledge flexibly by applying the appropriate methods and reasoning within a problem setting. Students may show balance when they are able to make connections between their conceptual understanding and the algorithms they use.

Rittle-Johnson, Siegler and Alibali (2001) point out that procedural knowledge demonstrates the ability to perform action sequences to solve mathematical problems. Although the knowledge of algorithms, sequences and the use of formulae is emphasised in this case, this understanding calls for teachers' strategic competence to ensure that fluency is promoted as computational procedures need to be efficient, accurate and to result in correct solutions. This conception also suggests that students should use other procedures to check the reasonableness of their answers. In this way, students become skilled in solving problems. In a more recent study, Rittle-Johnson (2019) argues that procedural knowledge and conceptual knowledge are the most fundamental types of knowledge. The balance between conceptual understanding and procedural fluency is also acknowledged by Joersz (2017), who maintains that conceptual understanding improves procedural fluency more than procedural understanding improves conceptual knowledge. Concepts discussed, and arguments made in this section provide a lens to critically explore how teachers promote efficiency, flexibility and accuracy when dealing with problems involving probability.

2.2.4 *Adaptive Reasoning*

Adaptive reasoning refers to the "capacity for logical thought, reflection, explanation and justification" (Kilpatrick et al., 2001, p. 116). This definition relates to how students navigate facts, procedures, concepts and solution methods to see if they fit together in a way that makes sense. Kilpatrick et al. (2001) argue that adaptive reasoning develops when students can think critically and justify their solutions by reasoning. In the context of probability, the likelihood of an event happening could be determined through logical reasoning. To put it another way, students need to provide sound reasons for the way they make sense of relationships amongst quantities in problem situations. Awofala (2017) argues that students become mathematically proficient if they can justify their solutions through logical reasoning about the existing problem. Table 2.2 provides activities to assist students in reaching conclusions through reasoning.

Table 2.2

How conclusions are derived through reasoning

Research phase	Activities	Reflection and discussion
Phase 1: Posing a problem	Esha and Sarah decide to play a die rolling game. They take turns to roll two fair dice and calculate the difference (bigger number minus smaller number) of the numbers shown. If the difference score is 0, 1, or 2, Esha wins, If the score is 3, 4, or 5, Sarah wins. Is this game fair?	Why do you think the game is fair? Or unfair? Explain your thinking.
Phase 2: Playing the game in pairs	In pairs, pre-service teacher participants play the game with 20 trials and record the data.	On the basis of your results, do you think the game is fair? Why, or why not? If you wanted to win this game, which player would you choose to be? Explain your answer. If you played the game 30 more times, would the results be the same as or different from the first game? If they would be different, how?

Source: Dayal and Sharma, 2020, p. 95

In the case of the questions posed in Table 2.2, students need to justify the source of their reasoning. For instance, they should understand what the word “fair” means in the context of the game. In other words, students should realise that it is important that all the players (i) use the same die, (ii) play on the same surface and (iii) are given equal time to play; if not, the game will not be fair. Through the guidance of the teacher, students need to work out what the impact of deciding on such rules is on the game. For development to take place, students are also expected to formulate their own questions as part of the learning process. In the case where the experiments are conducted, students should use the collected data to challenge the misconceptions and wrong assumptions they might have. This would be part of developing the skill of reasoning.

The capacity of students to reflect on their work and adapt as necessary constitutes adaptive reasoning. In such cases it is also advisable for teachers to develop questions that probe every step of students’ learning. These probing questions should be informed by how students answer the questions. As a recommendation, Wong, Kaur, and Tong (2021) encourage teachers to use analytical discussions as a mechanism to probe students’ responses so that they acquire a deeper understanding of the concepts. Teachers need to present lessons in such a way that students are given opportunities to discuss their ideas and make their reasoning clear. On the

other hand, teachers need to develop more similar questions so that students can justify their reasoning in several other related problems on probability. Similarly, teachers should use their reasoning skills to help students justify how they made their choice of strategies.

In developing skill in logical reasoning, teachers could pose questions such as: Which method do you like most and why? Likewise, teachers need to consider their questioning strategies very carefully. For example, after students have responded to a question, teachers could ask probing questions such as: (i) What do you mean by …? (ii) Think about what you did (iii) Analyse your experiences (iv) Apply your learning to your practice (v) What informed your selection of resources? (vi) How would you explain your approach to your classmates? (vii) How could you solve this problem differently? Based on the discussions and illustrations provided in this section, it appears that when students are given a problem to solve, it is important for them to explain the reasoning behind the processes they follow.

2.2.5 *Strategic Competence*

Strategic competence refers to the "ability to formulate, represent and solve mathematical problems" (Kilpatrick et al., 2001, p. 116). This process involves taking a situation and turning it into a solvable problem that can be represented by a mathematical model. Furthermore, strategic competence also refers to the ability to use appropriate mathematical skills to find a solution, interpreting and evaluating the solution in the context of the problem. Teachers need to assist students in designing their own strategies to solve a problem. Strategies could include the use of models, diagrams, paper folding and so on. This study takes the holistic approach to understand the role teachers play in ensuring that students are able to choose strategies appropriately, checking reasonableness of different strategies when solving problems involving probability.

Drawing from the understanding of strategic competence, teachers need to encourage students to find concise and effective procedures to solve problems related to probability. It is important to note that while acknowledging the use of multiple strategies, teachers themselves need to have the skill of evaluating options when choosing the best strategy to solve probability problems. For example, if teaching activities centre around what students have experienced, such activities would build awareness of the role mathematics plays in real-life situations, and it becomes a reality for students.

It is also critical for teachers to use context that seeks to inform and educate people about probability with the intention of influencing their attitude, behaviour and beliefs. For instance, teachers could ask students to investigate the type of vitamins contained in specific ingredients and how these benefit the body. This kind of activity raises students' awareness of health issues. Further investigations could include students' views regarding the use of vegetables instead of fruit to make a smoothie. Findings could be represented using tables, graphs or charts, or be described in words. Apart from raising awareness, these kinds of investigations enable students to use various representations to explain an idea and show the integration or connectedness of the mathematical concepts involved.

Using logical reasoning, teachers should encourage students to use or invent any mathematical representation that they understand and that is appropriate to solving a problem. If students are to develop strategic competence, particularly through the use of investigations and projects, teachers need to analyse students' thinking. Other examples that highlight the complexity of making learning meaningful include how teachers use context, incorporate resources in their lessons, how they should reformulate questions if students experience difficulties in understanding a problem and how they develop cognitive skills that students could use throughout their life. Teachers should also ensure that students are able to formulate their own mathematical problems to assess their understanding of probability.

Drawing on the discussion of strands of mathematical proficiency provided in section 2.2, it is evident that these strands are interrelated and interdependent. However, although I acknowledge that the strands of mathematical proficiency are interwoven, the focus of this study was to explore how teachers' conceptual understanding of probability informed their classroom practices. Figure 2.6 presents the framework that draws on Kilpatrick et al.'s (2001) five strands of Mathematical proficiency. According to DBE (2018a), the dimensions of the framework represent a contextualisation and adaptation of these strands to the South African context (p. 8).

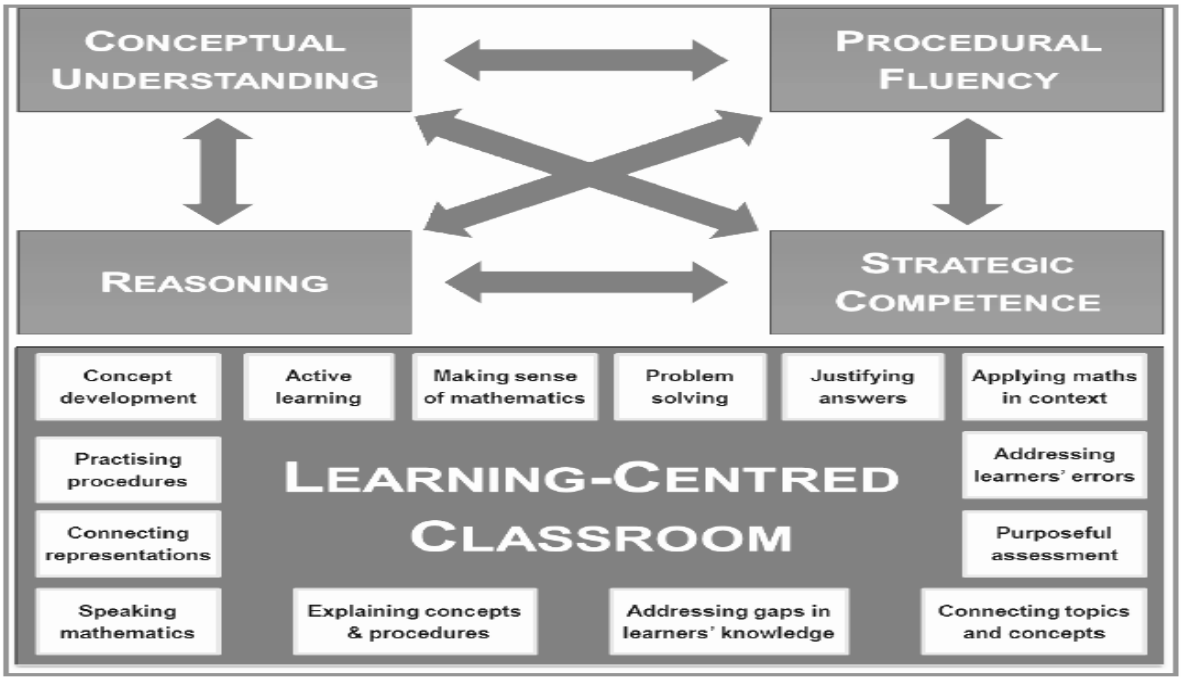


Figure 2.6. Proposed framework for the teaching and learning of mathematics
Source: DBE, 2018a, p. 14

The framework, according to DBE (2018a), is aimed at changing the way mathematics is taught in South Africa and supports the key activities of the reviewed Mathematics, Science and Technology (MST) Education strategy (2019–2030). This framework is based on the premise that "A learning-centred classroom focuses on learning – where the teacher designs learning experiences to help learners learn mathematics, using whatever teaching and learning strategies s/he thinks are most suitable for the specific lesson that will be taught" (DBE, 2018a, p. 19).

It is important to highlight the point that conceptual understanding is premised on the fact that it cannot be described in isolation from the other strands of mathematical proficiency. Groth (2017, p. 104) argues that "[t]he strands of mathematical proficiency develop in tandem rather than in isolation". In other words, one cannot separate the strands as each contributes towards attaining proficiency in probability. The next section presents the central idea that frames the study, and assists in explaining probability knowledge acquisition (Smyth, 2004).

2.3 The central idea that frames the study

The central idea in this context refers to what is integral to this study, what sets the tone for the design to give the study its unique identity and how the research purpose is clarified. This

central idea was derived from earlier studies conducted on how to develop students' mathematical proficiency. Studies by Suh (2007), Ally (2011) and Groth (2017) focused on aspects such as designing activities that promote the strands and designing observation instruments to analyse the effectiveness of traditional and problem-solving teaching strategies, including how teachers use the strands to reflect on their classroom practice.

The framework that guided this study is helpful not only for articulating a clear approach to teaching for proficiency, but also as an analytical lens through which to study the development of teachers' knowledge of designing activities for teaching probability. It aims to explore the teaching and learning practices, to identify teachers' needs in creating a learning-centred environment in the classroom, and students' need to be mathematically proficient, to guide teachers' professional development processes and to provide a foundation for reviewing the mathematics curriculum. In this conceptual model, the strands of mathematical proficiency are central to developing student mathematical proficiency. However, rather than treating these strands as separate anchors of mathematical proficiency, this model emphasises their complex connectedness, suggesting that separating them might be difficult (Gunawan, Nurhayati, & Hendrawan, 2021).

What sets this approach apart is the fact that this study explores how teachers' conceptual understanding of probability frames the plan to enhance their instructional classroom practices. In practical terms, the central idea in this study was establishing how teachers:

- comprehend the concept of probability
- create a learning-centred classroom to develop student conceptual understanding
- use classroom practices that could enhance students' conceptual understanding of probability.

My view is that teachers need to have a better idea of what conceptual understanding means and the implications such understanding has for classroom practice. I chose to focus on this aspect because I believe that conceptual understanding and how it is developed could enhance students' comprehension of probability. The core of my argument is that having a conceptual understanding of probability means more than knowing how to develop it or promote it in the classroom; teachers' intentions may be affected by how they put the idea into practice.

Having a deeper conceptual understanding of probability assists teachers in discovering and articulating practices involved in teacher professional development. This echoes Sands' (2014, p. 10) argument that "conceptual understanding could be considered to be in a fluid state that is constantly evolving". Sands adds that what students might say about the concepts can change the way they use them. I acknowledge the fact that exploring a phenomenon in a research study is a complex activity that draws on how concepts are understood. Figure 2.7 provides a breakdown of what I used as a guide to explore how teachers use their conceptual understanding of probability to frame the plan to enhance their instructional classroom practices.

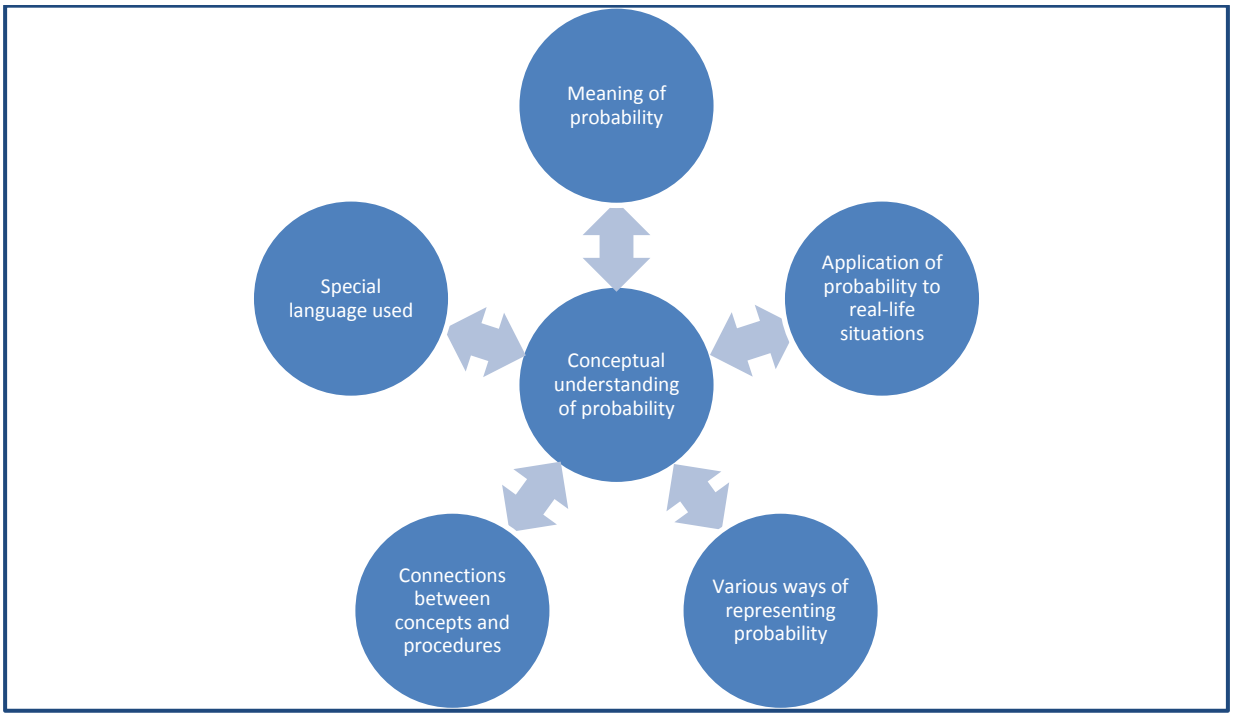


Figure 2.7. Potential web of ideas illustrating components of conceptual understanding
Source: Adapted from Kilpatrick et al., 2001

In framing the plan, I focused on teachers' perspectives and experiences on how they define the concept of probability in real-life situations, use different representations to explain probability concepts, make connections amongst probability concepts and procedures and how they use the specialised language to make sense of probability. The visual representation in Figure 2.7 is useful to this research study in theoretical and practical ways. Theoretically, it shows the components of conceptual understanding. It is practically useful as it provides a consistent language with which to describe how teachers use their conceptual understanding of probability to make sense of the concept of probability. It also articulates an explicit concept to analyse the data in this study.

Although these strands have been adopted as the conceptual basis for exploring teachers' knowledge of probability, conceptual understanding was selected as the central idea to understand how teachers make sense of the probability concept. This central idea points to two issues: firstly, how teachers' conceptual understanding of probability guides the teaching and learning process and secondly, how teachers' conceptual understanding of probability frames their strategies to enhance their instructional classroom practices. In support of this, the DBE (2018a, p. 13) argues that "more emphasis should be placed on conceptual understanding since it is the metaphorical foundation on which all the dimensions build".

2.4 Practical implications of the theoretical and conceptual framework

Having explained the foundations of Kilpatrick et al.'s (2001) model, the question arises: How would the elements of this model come together to facilitate research and a clear understanding of results? The practical implications of the model for this study are presented as follows:

- *Classroom teaching and learning*

What was envisaged for methodological practices, procedures and processes in this study is discussed in terms of the concepts used in Kilpatrick et al.'s (2001) model. As I have argued earlier in this chapter, this model forms an important knowledge base to guide the processes in this study. From a social constructivist point of view, Van den Heuvel-Panhuizen and Drijvers (2014) suggest that students should be treated as active participants in the learning process. To effectively transform a knowledge of probability into content that students can understand clearly, teachers must:

- Have a grasp of what the strands mean to them in relation to what learners need to achieve. This refers to how teachers understand the concept of probability and how they teach it. Van de Walle et al. (2016) argue that an important aspect of knowing is understanding. Thus, a grasp of these terms should provide guidance on teaching and support for learners.
- Ensure that they explore more deeply how to make sense of mathematical terms (that is, attach meaning to them), because problems with language may affect students whose home language or mother tongue is not English.
- Critically analyse the aims and objectives of classroom lessons and use the vocabulary of probability to formulate such objectives. The well-formulated objectives implicitly suggest teaching, learning and assessment activities that promote student proficiency in probability.

Casserly (2016, p. 9) argues that teachers need to ensure that the vocabulary used enhances an understanding of the strands of mathematical proficiency by developing the following skills:

- Developing beliefs: Instruction should support students in developing beliefs that mathematics is sensible, worthwhile, and doable.
- Engaging students in mathematical practices: Instruction should provide opportunities for students to engage in a set of core mathematical practices: (1) making sense of problems and persevering in solving them, (2) reasoning abstractly and quantitatively, (3) constructing viable arguments and critiquing the reasoning of others, (4) modelling with mathematics, (5) using appropriate tools strategically, (6) attending to precision, (7) looking for and making use of structure, and (8) looking for and expressing regularity in repeated reasoning.

The work of Casserly (2016) suggests ways in which instruction could be developed to enhance the understanding of mathematical concepts. These instructional purposes reveal the interrelatedness of the strands of mathematical proficiency and the importance of supporting students in developing beliefs that mathematics is sensible, worthwhile, and doable. To develop productive disposition, this study explores how teachers encourage and motivate students. For example, the teachers could probably use positive remarks to keep them focussed on the task. Further implications of the conceptual model in relation to this study are discussed as follows:

- *Data collection instruments*

Previous research studies (Ally, 2011; Legesse et al., 2020; Suh, 2007) have described how Kilpatrick et al.'s (2001) framework provided guidance in developing teaching and learning activities, observation tools to explore how the various strands are promoted in the classroom and in analysing students' levels of proficiency in understanding mathematics concepts. In this study, the concepts used in Kilpatrick et al.'s (2001) model serve as perspectives to make sense of how teachers engage students to acquire knowledge of probability.

- *Teacher professional development*

In the context of this study, the main question asked is: How could the ideas of Kilpatrick et al. (2001) enhance teachers' practices that support students to develop knowledge, skills and competencies they need to make sense of probability? To answer this question, the study

provides a supportive setting in which teachers could try out different approaches to promote better outcomes for students. The strands and their implications guide the processes in developing intervention activities that teachers need to engage with. This approach takes into account the fact that initial data collected be used to inform targeted intervention. For this study, it means the model is used to provide means for teachers to improve their practice to effectively meet the needs of their students and to shift the focus from simply fulfilling the minimum requirements for teaching to being able to increase its quality.

I argue that the framework used in this study provides a core of conceptual understanding that furnish guidelines to foster deeper understanding of mathematical concepts. However, it was important for me as the researcher to conceptualise how such cores were to be interpreted in order to yield the intended results. Of critical importance is to note that the model discussed in this section serves to guide teachers' practices to a proficiency in probability. The indicators in Table 2.3 provide guidance on how to accurately and reliably interpret participants' conceptual understanding of probability

Table 2.3

Indicators of conceptual understanding in the context of probability

Domains of conceptual understanding	How indicators guided the study	The role of the teacher in relation to the identified indicators. How do teachers help students achieve the following learning outcomes?	Strands of mathematical proficiency reflected
Definition of probability	Are they able to define probability? Are they able to clarify other terms used in the definition? Are they able to reconstruct their definitions? Are they able to provide examples in context and through representations when they define probability?	***Outline*** and grasp the meaning of probability, in terms of the identified indicators	Conceptual understanding
Ways of representing probability and knowing how it could be useful	Do they know what informs their choice of representations, considering advantages and disadvantages? Do they understand the importance of each representation and its usefulness to the purpose? Are they able to connect representations by considering similarities and differences? Do they realise the importance of using different solution methods?	***Illustrate*** ways of representing probability and how it could be useful in learning	Conceptual understanding and procedural fluency
See connections between probability concepts and procedures (organising knowledge in a coherent whole)	Do they know more than isolated facts and methods? Can they connect the new ideas to what they already know?	***Acquire*** skills to identify interrelated concepts Drawing on students' prior knowledge to connect formal learning	Conceptual understanding
Application of probability concepts to real-life situations	Do they understand why probability is important? Are they able to apply their knowledge to real-life scenarios? Do they know the types of contexts in which probability is useful? Are the activities within students' cognitive grasp? Do the activities build awareness of the role of probability? Are students able to define their own context?	***Instil in*** the minds of students the importance of probability in real-life situations	Adaptive reasoning and conceptual understanding
Understand the language used in probability	Are they able to verbalise connections among probability concepts and representations? Is learning related to students' experiences?	***Promote*** the use of special language in mathematical processes involved in solving problems; listen to students' feedback on challenges faced in their learning, particularly learning related to their own experiences	Conceptual understanding

Source: Adapted from Kilpatrick et al., 2001

2.5 Conclusion

This chapter outlined the framework that guided this study. In the chapter, I discussed strands of mathematical proficiency, how they map out and relate to each other. The concepts used in this framework provide guidance on how I could advance my field of practice in terms of assisting teachers to enhance their practices. The model allows for more detailed discussion on the kind of questions to ask when collecting data, the nature of evidence to collect, the methodologies appropriate for collecting data and viable strategies for analysing the data. I also believe that this model can be of great help in terms of making sense of the RIRAD model conceptualised in study.

Although I argued that this framework provided guidance on the methodological processes, I remained mindful of the fact that no single theory covers every aspect of a phenomenon. For this reason, I discussed important studies that enriched my understanding of the model used in this study. The next chapter explores scholarly work relating to and debates surrounding the teaching of probability and teacher professional development.

CHAPTER 3 : LITERATURE REVIEW

3.1 Introduction

This chapter provides some insights into the background and context of the study. It explores scholarly work relating to and debates surrounding the teaching of probability. I have outlined, substantiated, summarised and analysed the past and current state of information on practices in the teaching and learning of probability (Creswell, 2008). I have used the conceptual tools discussed in Chapter 2 as a guide to critique the work of other scholars in relation to the intentions of the study. This chapter is divided into six main sections. The first section focuses on research orientation, the second explores South African and international reports, the third describes indicators of conceptual understanding in relation to probability, followed by teachers' practices in teaching probability, practices in teacher professional development and the last section presents gaps identified in literature. The chapter ends with a conclusion.

3.2 Research orientation

As pointed out in Chapter 1, the main objective of this study was to investigate how teachers' conceptual understanding of probability framed the plan to enhance their instructional classroom practices. Previous studies (Ally, 2011; Browning, Goss, & Smith, 2014; Jojo, 2011; Smith & Freels, 2017; Tatar & Zengin, 2016) have provided findings on how conceptual understanding is developed in fractions, calculus and statistics. Although the concept of probability and how it is taught has been explored, the focus has been mainly on computer-assisted teaching. The use of computers was aimed at (i) addressing misconceptions students have about probability concepts, (ii) teaching probability with the support of statistical software, (iii) restructuring probability tasks, (iv) exploring pedagogical content knowledge (PCK) for teaching probability in middle school (Grades 7-9), (iv) promoting teaching probability through games, and planning probability lessons for students to work in pairs (Brijlal, 2014; Dos Santos Ferreira, Kataoka, & Karrer, 2014; Groth et al., 2016); Gürbüz & Birgin, 2012; Gürbüz, Erdem, & Uluat, 2014; Zaranis & Synodi, 2017).

Sands (2014) emphasises the importance of teaching students to recognise their inconsistency in reasoning instead of trying to stop them developing misconceptions. In their study, Mahmud and Porter (2015) found that university students had difficulty in understanding probability concepts because of lack of exposure to these concepts while at school. A further dimension

was highlighted by Pale (2016), who found that students complained about how teachers rushed through the syllabus. These findings touch on the importance of developing the appropriate skill to effectively and meaningfully teach the concepts in probability. The next section presents South African and international assessment reports of student achievements in mathematics.

3.3 South African and international reports

This section, concerned with local and international assessment reports, presents key findings and controversies in the literature and what national and international assessment reports have highlighted as concerns in the teaching of mathematics. It is important to emphasise that the discussion that follows in the subsequent paragraphs provides facts that will enlighten the reader on how students performed in mathematics globally, which served as part of the motivation for conducting this study. This section reviews two studies, the Trends in International Mathematics and Science Study (TIMSS) and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), as South African students participated in both assessments.

Many studies have been conducted to investigate the cause of poor performance in mathematics, but performance has not yet improved as expected (Alkhateeb, 2021; Machisi & Feza, 2021; Tian & Siegler, 2017). It has become increasingly difficult to ignore this situation, that is why researches are conducted in mathematics to build knowledge about reality. Venkat and Spaull (2015) stress the need to gather information that has a positive impact on pedagogical content knowledge of teachers as a matter of urgency, to guide the intervention processes. To draw specific attention to the strength of the evidence of students' performance in international assessments, the reports are discussed as follows:

3.3.1 *TIMSS Reports*

TIMSS was developed by the International Association for the Evaluation of Educational Achievement (IEA) to compare students' educational achievement across borders (Reddy et al., 2016). Mathematics assessment emphasises two domains: content and cognitive domains (Mullis, Martin, Foy, Kelly, & Fishbein, 2020). Mullis and Martin (2017) describe the two domains as follows:

PART A: CONTENT DOMAIN

- The content domain for Grade 8 covers Numbers, Algebra, Geometry, Data and Chance and Grade 4 focuses on Numbers, Geometric Shapes, Measures and Data display (p. 7).
- Target percentage devoted to content domain in Grade 4 (p. 14) for data is 20%.
- In Grade 8 (p. 18), the target percentage for data and probability is also 20%, which is spread as 15% for data and 5% for probability (p. 21). The questions on probability are based on equally likely outcomes and experimental outcomes like rolling a die.

PART B: COGNITIVE DOMAIN (p. 23)

Domain	**Focus**	**Fourth grade**	**Eighth grade**
Knowing	Mastery of facts and concepts	40%	35%
Applying	Use of facts to solve problems	40%	40%
Reasoning	Dealing with unfamiliar situations, complex contexts and multi-step problems	20%	25%

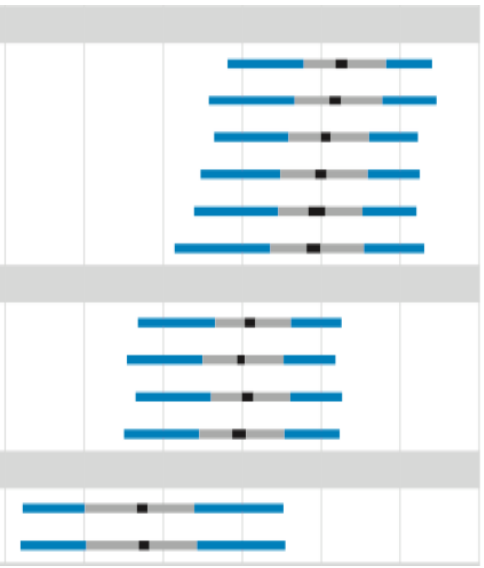
It is clear from this clarification that the assessment items aim to assess how students make sense of the mathematical facts through computations, how they apply such facts and concepts to solve familiar and unfamiliar situations and how they reason or make arguments to solve problems for which the answers are not obvious. It is also evident from this clarification that items on chance are only covered in Grade 8. Because schools participate in these assessments, the question that arises is: To what extent are these cognitive domains promoted in textbooks or resources used by teachers? As a researcher, it would also be important to understand how teachers make sense of these cognitive domains in relation to the aims and objectives of the curriculum. Figure 3.1 reflects the average achievement of Grade 5 and 9 South African learners across assessment years.

Mathematics • Grade 4

IEA TIMSS 2019

Exhibit 1.4: Differences in Average Mathematics Achievement Across Assessment Years

Singapore						
[3] 2019	625 (3.9)	8	20 ▲	26 ▲	31 ▲	35 ▲
[2] 2015	618 (3.8)		12 ▲	18 ▲	23 ▲	27 ▲
[2] 2011	606 (3.2)			6	11	16 ▲
2007	599 (3.8)				5	9
2003	594 (5.6)					4
1995	590 (4.5)					
Slovak Republic						
[2] 2019	510 (3.5)	12 ▲	3	14 ▲		
2015	498 (2.5)		-9	2		
2011	507 (3.7)			11		
2007	496 (4.5)					
South Africa (5)						
2019	374 (3.6)	-2				
2015	376 (3.5)					



Mathematics • Grade 8

IEA TIMSS 2019

Exhibit 3.4: Differences in Average Mathematics Achievement Across Assessment Years

(Continued)

Country	Average Scale Score	Differences Between Years					
		2015	2011	2007	2003	1999	1995
Singapore							
[2] 2019	616 (4.0)	-5	5	23 ▲	10	11	7
[2] 2015	621 (3.2)		10 ▲	28 ▲	16 ▲	17 ▲	12 ▲
[2] 2011	611 (3.8)			18 ▲	6	7	2
2007	593 (3.8)				-13 ▽	-12	-16 ▽
2003	605 (3.6)					1	-3
1999	604 (6.3)						-4
1995	609 (4.0)						
South Africa (9)							
[*] 2019	389 (2.3)	17 ▲	38 ▲				
[*] 2015	372 (4.5)		20 ▲				
[*] 2011	352 (2.5)						

Mathematics Achievement Distribution

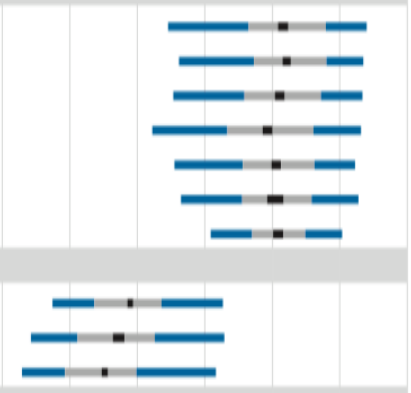


Figure 3.1. Average achievement of Grade 5 and 9 South African students across assessment years

Source: Mullis et al., 2020, pp. 23 & 16

It can be seen from Figure 3.1 that the performance in Grade 5 dropped while Grade 9 improved in subsequent years. The countries ranked in the top five were all from East Asia: Singapore, Hong Kong SAR, Republic of Korea, Chinese Taipei and Japan. South Africa was rated among the lowest performing countries (Reddy et al., 2016; Mullis et al., 2020). South Africa obtained position 38 out of 39 countries, with an average performance of 376 in 2015 (Reddy et al., 2016). It is also evident from Figure 3.1 that South African students achieved far below the international scale centre point of 500.

It is clear from these reports that mathematics poses a challenge and that South African students continue to perform below international and national benchmarks. What is interesting in this data is that South African students are falling behind relative to the curriculum; hence they find TIMSS tests very difficult. This is substantiated by the fact that the Grade 5 learners completed the Grade 4 test and Grade 9 learners, the Grade 8 test. In other words, South African learners are performing one grade behind other countries. More evidence to underscore the ranking of Asian countries is displayed in Figure 3.2.

Mathematics • Grade 8

IEA TIMSS 2019

Exhibit 3.9: Percentages of Students Reaching International Benchmarks of Mathematics Achievement Across Assessment Years

Country	Advanced International Benchmark (625) Percent of Students							High International Benchmark (550) Percent of Students						
	2019	2015	2011	2007	2003	1999	1995	2019	2015	2011	2007	2003	1999	1995
Singapore	51	54	48	40 ▲	44 ▲	42 ▲	40 ▲	79	81	78	70 ▲	77	77	84 ▽
Chinese Taipei	49	44 ▲	49	45	38 ▲	37 ▲		75	72 ▲	73	71 ▲	66 ▲	67 ▲	
Korea, Rep. of	45	43	47	40 ▲	35 ▲	32 ▲	31 ▲	74	75	77 ▽	71 ▲	70 ▲	70 ▲	67 ▲
Japan	37	34	27 ▲	26 ▲	24 ▲	29 ▲	29 ▲	71	67 ▲	61 ▲	61 ▲	62 ▲	66 ▲	67 ▲
Hong Kong SAR	32	37	34	31	31	28	23 ▲	66	75 ▽	71	64	73 ▽	70	65
Russian Federation	16	14	14	8 ▲	6 ▲	12	9 ▲	48	46	47	33 ▲	30 ▲	39 ▲	38 ▲
Israel	15	13	12					40	38	40				
United States	14	10 ▲	7 ▲	6 ▲	7 ▲	7 ▲	4 ▲	38	37	30 ▲	31 ▲	29 ▲	30 ▲	26 ▲
Turkey	12	6 ▲	7 ▲					32	20 ▲	20 ▲				
Australia	11	7 ▲	9	6 ▲	7 ▲		7 ▲	36	30 ▲	29 ▲	24 ▲	29 ▲		33
Hungary	11	12	8 ▲	10	11	13	10	36	37	32 ▲	36	41	43 ▽	40
England	11	10	8	8	5 ▲	6 ▲	6 ▲	35	36	32	35	26 ▲	25 ▲	27 ▲

Mathematics • Grade 4

IEA TIMSS 2019

Exhibit 1.9: Percentages of Students Reaching International Benchmarks of Mathematics Achievement Across Assessment Years

(Continued)

Country	Intermediate International Benchmark (475) Percent of Students						Low International Benchmark (400) Percent of Students					
	2019	2015	2011	2007	2003	1995	2019	2015	2011	2007	2003	1995
Singapore	96	93	94	92 ▲	91 ▲	89 ▲	99	99	99	98 ▲	97 ▲	96 ▲
Hong Kong SAR	96	98 ▽	96	97	94 ▲	87 ▲	100	100	99	100	99	97 ▲
Korea, Rep. of	95	97 ▽	97 ▽			94	99	100	100			99
Chinese Taipei	96	95	93 ▲	92 ▲	92 ▲		100	100	99 ▲	99 ▲	99 ▲	
Japan	95	95	93 ▲	89 ▲	89 ▲	89 ▲	99	99	99	98 ▲	98 ▲	98 ▲
Northern Ireland	85	86	85				96	97	96			
England	83	80	78 ▲	79 ▲	75 ▲	54 ▲	96	96	93 ▲	94 ▲	93 ▲	82 ▲
Russian Federation	91	89	82 ▲	81 ▲	76 ▲		99	98	97 ▲	95 ▲	95 ▲	
Ireland	84	84	77 ▲			73 ▲	97	97	94 ▲			91 ▲
United States	77	79	81 ▽	77	72 ▲	71 ▲	93	95 ▽	96 ▽	95 ▽	93	92

Figure 3.2. Percentages of students reaching international benchmarks of Mathematics achievement across assessment years

Source: Mullis et al., 2020, pp. 41 & 177

Figures 3.1 and 3.2 show that Singapore is one of the top performing countries in international assessments.

On the other hand, the Organisation for Economic Co-operation and Development (OECD) (2020, p. 4) emphasises that:

> Reading is no longer mainly about extracting information; it is about constructing knowledge, thinking critically and making well-founded judgements. Against this backdrop, the findings from this latest PISA round show that fewer than 1 in 10 students in OECD countries was able to distinguish between fact and opinion, based on implicit cues pertaining to the content or source of the information. In fact, only in the four provinces of China, as well as in Canada, Estonia, Finland, Singapore and the United States, did more than one in seven students demonstrate this level of reading proficiency.

In their study Yang, Kaiser, König and Blömeke (2020) found that Singapore, Korea and Japan performed better in problem solving than students in other participating countries. In view of what has been highlighted in these reports, there seems to be a need to explore the practices of performing countries more deeply while not neglecting the contextual advantage of each. Teachers also need the skill to create an environment that promotes meaningful learning. In other words, whilst acknowledging the fact that researchers could source the ideas from performing countries, it is important for teachers to create a learning environment that enables students to construct their own meaning of probability.

The study by Reddy et al. (2016) draws our attention to the fact that students' performance in data and chance is low and only a third of students achieved more than 400 points. The following section provides a short review of the Southern and Eastern Africa Consortium for Monitoring Education Quality (SACMEQ) report.

3.3.2 *SACMEQ Report*

SACMEQ undertook four research projects, namely SACMEQ I, SACMEQ II, SACMEQ III and SACMEQ IV. Makuwa and Maarse (2013, p. 349) note that the aim of SACMEQ is to:

> (a) build the technical capacity of educational planners and researchers in participating Ministries of Education to monitor and evaluate the conditions of schooling and the quality of their own basic education systems; (b) undertake co-operative large-scale cross-national assessments of student learning; and (c) utilise innovative information dissemination approaches and a range of policy-dialogue activities to ensure that SACMEQ research results are widely discussed, debated and understood by all stakeholders and senior decision-makers and subsequently used as the basis for policy and practice.

Figure 3.3 presents students' reading and mathematics scores for 2000–2013. The mathematics score in 2013 was more than the SACMEQ IV average score.

	Learner reading score				Learner mathematics score			
	2000	2007	2013	Diff (2007-2013)	2000	2007	2013	Diff (2007-2013)
1. Mauritius	536	574	597	23	585	623	694	71
2. Kenya	547	543	601	58	563	557	651	94
3. Seychelles	582	575	602	27	554	551	630	79
4. Swaziland	530	549	590	41	517	541	601	68
5. Botswana	521	535	582	47	513	521	598	77
6. South Africa	**492**	**495**	**558**	**63**	**486**	**495**	**587**	**92**
7.Uganda	482	479	554	75	506	482	580	98
8.Zimbabwe	505	508	528	20	**	520	566	46
9.Lesotho	451	468	531	63	447	477	559	82
10.Namibia	449	497	599	102	431	471	558	87
11.Mozambique	517	476	519	43	530	484	558	74
12. ?		537	562	25		490	538	48
13. Zambia	440	434	494	60	435	435	522	87
Tanzania	***546***	***578***			***522***	***553***		
Zanzibar	***478***	***540***			***478***	***486***		
14. Malawi	429	434	494	58	433	447	522	75
SACMEQ	**500**	**507**	**558**	**51**	**500**	**507**	**584**	**77**

Figure 3.3. Students' reading and mathematics scores for 2000–2013

Source: Parliament of the Republic of South Africa, 2016, p. 2

According to DBE, (2017), South Africa's overall performance in SACMEQ IV was 538 in reading and 552 in mathematics. The report also indicates that, for the first time, South Africa achieved above the mean SACMEQ score of 500 in both reading and mathematics. Between 2000 (SACMEQ II) and 2013 (SACMEQ IV), South Africa's achievement was on an upward

trend with greater improvements in mathematics than in reading (language). In addition, the SACMEQ IV report highlighted that the

> improvements can be ascribed to, among others, a) the streamlining and strengthening of the national curriculum between SACMEQ III and SACMEQ IV, b) the focus on monitoring of teaching and learning through the National Strategy for Learner Attainment and c) regular exposure to standardised assessments through the Annual National Assessment (ANA). (DBE, 2017, p. 27)

SACMEQ uses eight levels of achievement to assess student performance. The SACMEQ reading competency levels and their descriptions are summarised in Table 3.1:

Table 3.1

SACMEQ mathematics competency levels and their description

	Level	Descriptor	Competencies
BASIC MATH SKILLS	1	Pre- Numeracy	Applies single step addition and subtraction.
	2	Emergent Numeracy	Applies a two-step addition and subtraction involving carrying.
	3	Basic Numeracy	Translates verbal information into arithmetic operations.
	4	Beginning Numeracy	Translates verbal or graphic information into simple arithmetic problems.
	5	Competent Numeracy	Translates verbal, graphic, or tabular information into an arithmetic form in order to solve a given problem.
ADVANCED MATH SKILLS	6	Mathematically Skilled	Solves multiple-operation problems (using the correct order) involving fractions, ratios, and decimals.
	7	Concrete Problem Solving	Extracts and converts information from tables, charts and other symbolic Presentations in order to identify, and then solve multi-step problems
	8	Abstract Problem Solving	Identifies the nature of an unstated mathematical problem embedded within verbal or graphic information and then translate this into symbolic, algebraic or equation form in order to solve a problem.

Source: DBE, 2017, p. 31

Table 3.2 reflects the percentage of South African teachers and learners who reached various mathematics competency levels.

Table 3.2

South African teachers' and learners' competence levels in percentages

Teachers	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6	Level 7	Level 8
SACMEQ III	0	0	0.2	3.2	9.8	21.8	37.2	27.8
SACMEQ IV	0	0	0	1.4	7.2	23.4	32.4	35.4
Learners	**Level 1**	**Level 2**	**Level 3**	**Level 4**	**Level 5**	**Level 6**	**Level 7**	**Level 8**
SACMEQ III	5.5	34.7	29	15.4	7.1	5.9	1.9	0.6
SACMEQ IV	0.8	14.1	35.1	20.3	14.8	7.7	4.6	2.6

Source: DBE, 2017, pp. 31-32 & 35-36

Table 3.2 raises immediate concern; if there are still teachers who perform at Level 3, how will the learner performance improve? On the other hand, SACMEQ IV study results point to several gains, which are outlined below:

- South Africa registered the highest improvement margins among participating countries in the region.
- The provincial inequality gap narrowed, resulting in eight of the nine provinces scoring more than 500, which was the centre point.
- Although the number of non-numerate learners at Grade 6 level declined, there is still an urgent need to focus on developing reading and numeracy skills.

The report also emphasised that

> At the same time further improvement is possible if the sector: Enables more learners to achieve higher Reading and Mathematics competency levels with a greater focus on learners coping with questions of higher cognitive demand. (DBE 2017, p. 52)

Although South African learners have improved in mathematics, according to the SACMEQ reports, it was recommended that pre-service and in-service training of teachers in pedagogical and subject content knowledge and in the setting of higher cognitive demand questions required improvement. I do align myself with this recommendation because even though students' performance has improved, teachers' and students' competency levels are below 50% (see Tables 3.1 and 3.2). I therefore argue that focusing on the basics in mathematics should be a priority for every teacher. All basic skills are important for any mathematics topic prescribed for higher secondary schools. Hence my study targeted Grade 7, which is the primary school exit level. In the context of this study, mastery of Level 6 (See Table 3.1) is important because students require knowledge of fractions to be proficient in probability.

The evidence provided in sections 3.2 and 3.3 indicates that there are still problems that need to be addressed in the teaching and learning of mathematics. In emphasising the importance of paying attention to these issues, Batanero et al. (2016) endorse the fact that as the research field in probability grows and diversifies, it is important also to focus on how best to account for preconceptions and difficulties in the learning of probability.

Mosimege et al. (2017, p. 81) emphasised that "teachers must recognise the importance of basic probability knowledge from Grade 4 because this topic runs through to Grade 12". In other words, concepts from intermediate phase are fundamentals and as such they gradually build and expand towards an understanding of more complicated ideas in higher levels. The aim was also to highlight that the research focus is mostly on FET rather than primary schools and also that gaps identified might be as a result of the challenges experienced at primary school level. In relation to the challenges experienced by FET students, De Kock (2015) discovered that in most cases, when answering questions on dependent, independent and mutually exclusive events in examinations, students are unlikely to remember the identity or the rules of probability. This finding supports what is illustrated in Figure 3.4.

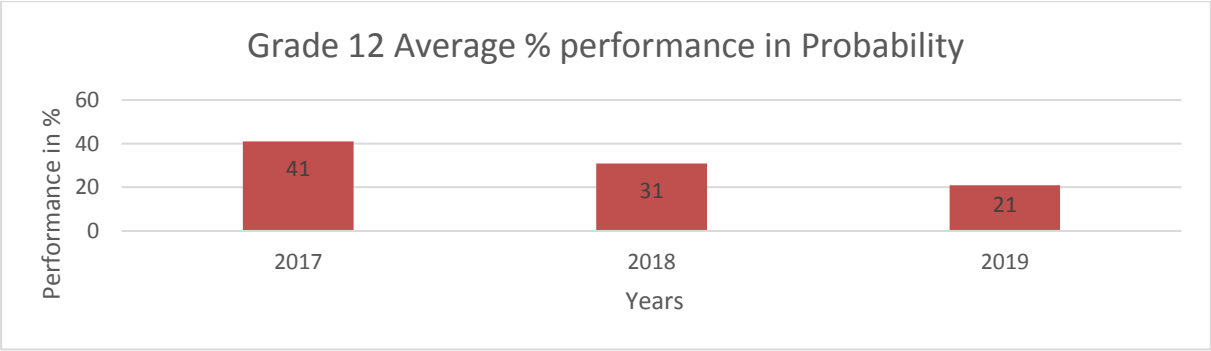


Figure 3.4. Grade 12 average performance in probability

Source: DBE, 2018b; DBE, 2019; DBE, 2020b

A closer examination of Figure 3.4 reveals that the performance of Grade 12 students declined from 2017 to 2019. Several reasons for this performance were highlighted in the DBE (2020b) report: students confused mutually exclusive events with independent events, confusing P (A or B) and P (A and B); very few students were able to draw a tree diagram; the language of probability posed a challenge; and some students attempted to use the formulae [e.g., P(A) × P(B)] without understanding how to apply them. The following suggestions were made to improve this situation (DBE, 2020b, p. 191). I have summarised the suggestions as follows:

- An in-depth explanation of the Venn diagram and formulae should explain formulae in the context of these diagrams.
- The difference between "repetition is allowed and not allowed" in the context of probability should be explained to students.
- An understanding of probability concepts such as mutually exclusive independent and complementary events is essential.
- Teachers should refrain from teaching rules and try to encourage working out the calculations.
- Tree diagrams are a useful tool to visualise compound events and help students to understand the sequence of events.

Considering these suggestions for improvement, I realised that there was a need to reflect on the guidelines provided in CAPS on how to teach probability. These guidelines do not mention the use of sets to explain the concept of probability. It is my view that this should form part of the content for primary schools to serve as the basis for secondary school mathematics. It would

not be easy to apply set theory in secondary school mathematics if students are not familiar with the language of sets and Venn diagrams. The essence of my argument on poor performance in probability is also evident in the diagnostic report by the DBE (2012), which emphasises the point that many Grade 6 students have great difficulty understanding the concept of probability. Challenges faced by students include their inability to differentiate between the concepts of probability and ratio. These difficulties can be observed in the following example extracted from the 2012 ANA paper that is typical of student responses:

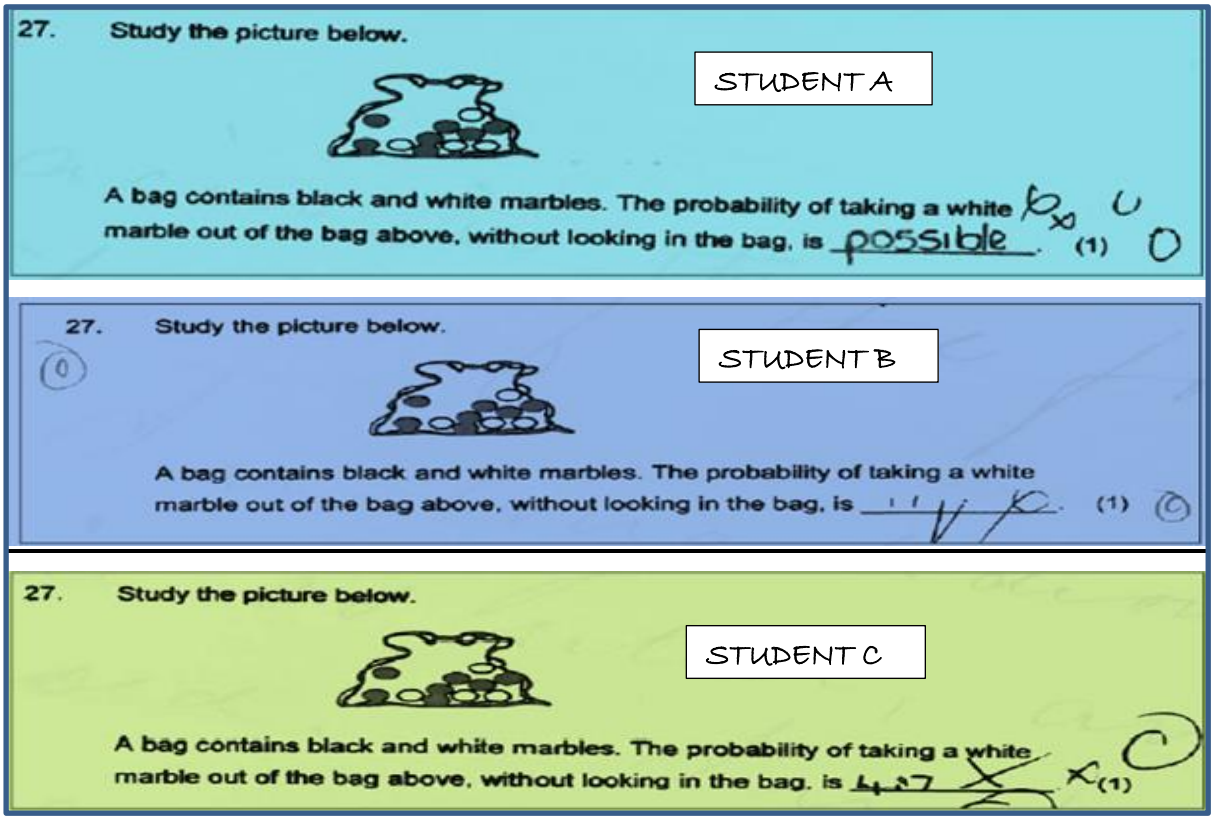


Figure 3.5. Extracts from Diagnostic report (Students A - C)
Source: DBE, 2012, pp. 22-23

Figure 3.5 illustrates the kinds of difficulties experienced by students in probability. It is not clear whether such difficulties may be attributable to the fact that the topic was not taught at all or whether the question was not clear to the students. It may well be that probability was not well conceptualised when taught. Cordova and Tan (2018) advise teachers to conduct diagnostic tests to determine the nature of students' difficulty.

It appears from Figure 3.5 that students did not understand the question. Student B simply added up the marbles and wrote the total number of marbles. Student A's response suggests that the student is familiar with the terms used in probability to describe the degree of chance. In my view, Student B's answer appears to suggest that the student had not been introduced to concepts such as outcomes, sample space and possible outcomes. It might also be that the student uses the terminology inappropriately. On the other hand, one might challenge the marker by indicating that Student A's answer might be correct because it is possible that the event could occur. These responses spell out a need for teachers to facilitate a discussion about the misconceptions identified, focusing on having the students explain their thinking. In the same vein, it could be argued that the question should have been extended to allow students to provide justification for their answers. Student C had an idea of how to express probability as ratio, but this was not captured correctly.

The challenges identified in Figure 3.5 could be effectively addressed if students are given the opportunity to reflect on their responses, critique the reasoning of other students and justify their arguments (Kilpatrick et al., 2001). If students are engaged in discussions that promote logical and critical thinking, students would be able to balance conceptual understanding and procedural fluency (Joersz, 2017; Nance, 2018) when solving problems involving probability. These students failed to understand that they were required to calculate the theoretical probability of picking a white marble from the bag. Another concern is that teachers do not provide constructive feedback from which students could learn from their mistakes. Making a cross does not mean anything to the student.

If Grade 6 students are struggling with simple concepts of probability, as indicated in Figure 3.5, how will they then manipulate simple formulae when calculating probability? The same concepts were dealt with from Grade 4, but these students still had difficulty dealing with the basics of probability in Grade 6. Drawing on students' responses (see Figure 3.5), I argue that more probing questions could have been asked to allow students to articulate their current understanding of a topic, to make connections with other ideas and also to become aware of what they do or do not know.

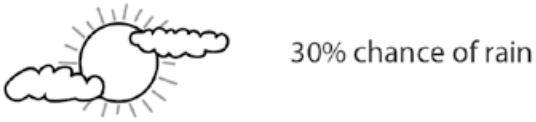
Equally important is the fact that teachers need to reconsider the way feedback is given to students. In other words, the main question to ask at this stage is: Does the feedback given to students promote learning? In response to this question, Molloy, Boud, and Henderson (2020)

encourage teachers to use feedback to promote learning-centred classrooms. They further suggest that students should be engaged in a genuine dialogue that will make feedback a reciprocal process.

It was proposed in the diagnostic report (DBE, 2012) that teachers should set practical and written problems on probability for students to learn the concept from different perspectives. One way would be to do practical activities on probability with beads, counters, and so on, while asking questions related to probability. Most importantly, the same report emphasises the importance of understanding the language of probability.

Additionally, the report by Mosimege et al. (2017) indicates that students do not know that 0% = “will not” and 100% = “it will”. This understanding was informed by the options chosen by students when responding to the question presented in Figure 3.6. It therefore came as a recommendation in this report that Grade 7 students need to know that certain words are equivalent to certain percentages and probability. It was also highlighted in this report that students did not even use the information next to the cloud to help them answer the question.

QUESTION 75 The weather forecast for tomorrow in Zedland is below.



How likely is it that it will rain tomorrow in Zedland?

A. It is certain to rain.

B. It is likely to rain.

C. It is unlikely to rain.

D. It will not rain.

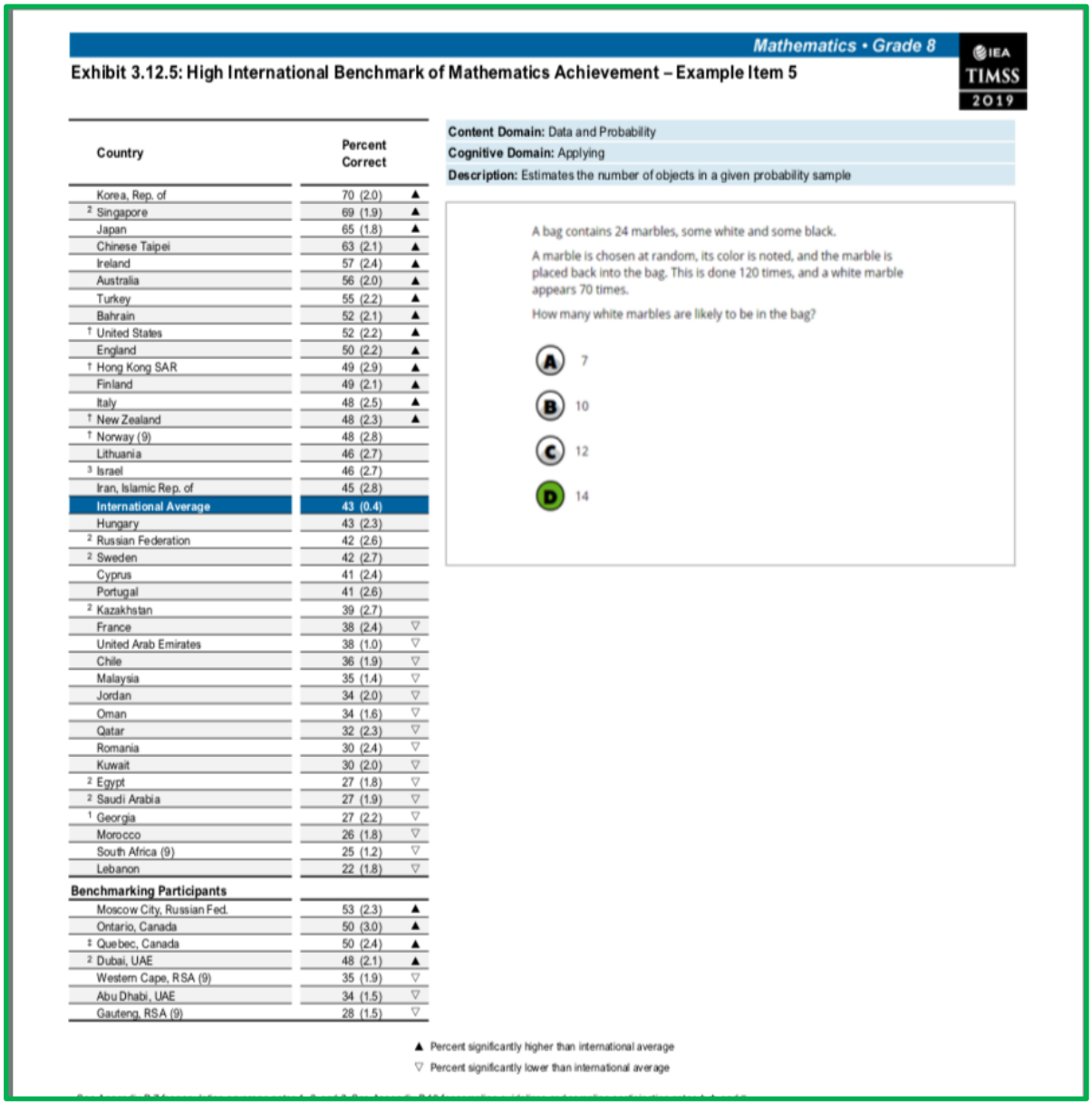
Figure 3.6. The chance of rain in Zedland

Source: Mosimege et al., 2017, p. 87

Implicitly, these findings seem to suggest that gaps identified in secondary schools might be because of the challenges experienced at primary school level. If concepts are not well developed in earlier grades, students will tend to have such misconceptions. Lack of conceptual

understanding of probability could have a negative impact on other strands of mathematical proficiency (Kilpatrick et al., 2001).

Figure 3.7 indicates how students performed on questions on probability in the TIMSS (2019) study.



Mathematics • Grade 8

IEA TIMSS 2019

Exhibit 3.12.5: High International Benchmark of Mathematics Achievement – Example Item 5

Country	Percent Correct	
Korea, Rep. of	70 (2.0)	▲
[2] Singapore	69 (1.9)	▲
Japan	65 (1.8)	▲
Chinese Taipei	63 (2.1)	▲
Ireland	57 (2.4)	▲
Australia	56 (2.0)	▲
Turkey	55 (2.2)	▲
Bahrain	52 (2.1)	▲
† United States	52 (2.2)	▲
England	50 (2.2)	▲
† Hong Kong SAR	49 (2.9)	▲
Finland	49 (2.1)	▲
Italy	48 (2.5)	▲
† New Zealand	48 (2.3)	▲
† Norway (9)	48 (2.8)	
Lithuania	46 (2.7)	
[3] Israel	46 (2.7)	
Iran, Islamic Rep. of	45 (2.8)	
International Average	**43 (0.4)**	
Hungary	43 (2.3)	
[2] Russian Federation	42 (2.6)	
[2] Sweden	42 (2.7)	
Cyprus	41 (2.4)	
Portugal	41 (2.6)	
[2] Kazakhstan	39 (2.7)	
France	38 (2.4)	▽
United Arab Emirates	38 (1.0)	▽
Chile	36 (1.9)	▽
Malaysia	35 (1.4)	▽
Jordan	34 (2.0)	▽
Oman	34 (1.6)	▽
Qatar	32 (2.3)	▽
Romania	30 (2.4)	▽
Kuwait	30 (2.0)	▽
[2] Egypt	27 (1.8)	▽
[2] Saudi Arabia	27 (1.9)	▽
[1] Georgia	27 (2.2)	▽
Morocco	26 (1.8)	▽
South Africa (9)	25 (1.2)	▽
Lebanon	22 (1.8)	▽
Benchmarking Participants		
Moscow City, Russian Fed.	53 (2.3)	▲
Ontario, Canada	50 (3.0)	▲
‡ Quebec, Canada	50 (2.4)	▲
[2] Dubai, UAE	48 (2.1)	▲
Western Cape, RSA (9)	35 (1.9)	▽
Abu Dhabi, UAE	34 (1.5)	▽
Gauteng, RSA (9)	28 (1.5)	▽

▲ Percent significantly higher than international average

▽ Percent significantly lower than international average

Content Domain: Data and Probability

Cognitive Domain: Applying

Description: Estimates the number of objects in a given probability sample

A bag contains 24 marbles, some white and some black.

A marble is chosen at random, its color is noted, and the marble is placed back into the bag. This is done 120 times, and a white marble appears 70 times.

How many white marbles are likely to be in the bag?

A 7

B 10

C 12

D 14

Figure 3.7. Grade 8 students' performance on probability

Source: Mullis et al., 2020, p. 192

South African students' performance in stating the number of marbles that are likely to be in the bag, as illustrated in Figure 3.7, was below 50%. Asian countries were ranked top among

all countries represented. The views of Kilpatrick et al. (2011) regarding the meaning of conceptual understanding are exemplified in the work by Legesse et al. (2020, p. 4), as follows:

Table 3.3

Sample of tasks and specific manifestations

Sample of test tasks	Task specific manifestation	Category
Let Qi, Di, and Pi be respectively the i^{-h} quartile, decile, and percentile of data arranged in ascending order. Establish the relationship between Q2, D5, P50, and median of the data.	It requires knowledge of the definition of probability, determination of sample space and favourable outcome	Conceptual understanding
If a coin is tossed three times, which sequence of events do you think is more likely to occur? (H stands for Heads and T stands for Tails) a) The probability of getting HTH **OR** b) The probability of getting TTH **OR** c) Both (a) and (b) have an equal chance of occurring.	It asks for comparing the probability of events and understanding the concept of "more likely" and the definition of probability.	Conceptual understanding

Source: Extract from Legesse et al., 2020, p. 4

Table 3.3 contains terms such as events, equal chance, more likely, sample space and favourable outcomes to elucidate the concept of probability. The task specific manifestations as described in Table 3.3 signal the need for teachers to clarify new embedded terms in order to develop student conceptual understanding of probability. The examples provide additional evidence that probability enhances an understanding of statistics. In other words, probability is a topic that forms part of the layers of statistics and shows the connectedness of mathematical concepts. The next section describes the indicators of conceptual understanding in relation to probability.

3.4 Indicators of conceptual understanding in relation to probability

Van de Walle et al. (2016, p. 48) describe conceptual understanding as "a flexible web of connections and relationships within and between ideas, interpretations and images of mathematical concepts – a relational understanding". In this study, conceptual understanding is viewed from the perspective of how probability is defined, its importance, the terminology used in elucidating the concept and various ways of representing probability. In extending the ideas of Kilpatrick et al. (2001), Van de Walle et al. (2016) argue that conceptual understanding

includes multiple ways of representing probability. On the other hand, Zulu (2019) highlights that the concept could sometimes be clarified by comparing the concept itself and its contrast. Figure 3.8 attempts to advance the understanding of probability, aiming at providing teachers with practical guidance and an insight into how to describe conceptual understanding in relation to probability.

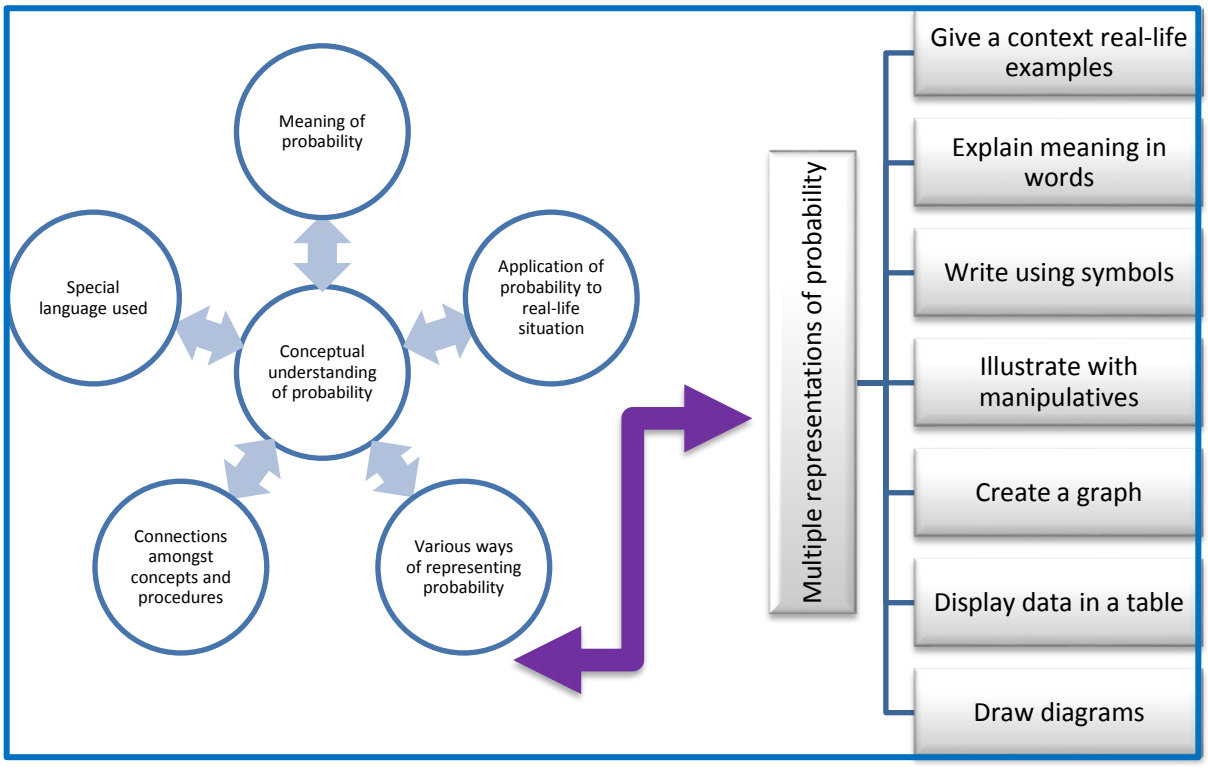


Figure 3.8. Indicators of conceptual understanding in relation to probability

Source: Kilpatrick et al., 2001 and Van de Walle et al., 2016, p. 48

According to this model, teachers develop students' conceptual understanding when they determine what each component means, how each component can be effectively used and how the connection of these representations deepens an understanding of probability. The use of various ways of representing probability is critical if teachers are to make sense of their world. In addition, when these representations make sense to students, they are more likely to use them appropriately to suit the relevant context. For example, at Mokoukou (pseudonym) Primary School, Vuyo sells the following snacks during break: popcorn, Smarties, biscuits and peanuts. A survey shows that students (the customers) have the following preferences:

Popcorn	Smarties	Biscuits	Peanuts
$Two - fifths$	0,01	one sixth	One third

It is important for Vuyo to consider the information provided if he needs to stock a reasonable quantity of each product. Teachers should then give students the opportunity to use any appropriate method to make sense of Vuyo's decision. This could include the use of models, pie graphs, area, length or set models to compare the fractions. In other words, students' conceptual understanding of fractions could be strengthened through the use of different representations.

To enhance the understanding of new concepts, teachers should facilitate learning in such a way that students are given the opportunity to connect previously learnt concepts to give meaning to the new experiences. Teachers' assistance in this sense is critical because previously learnt concepts enhance the understanding of the new concepts. For example, concepts such as fractions, ratios, percentages, types of numbers, and sets are examples of prior knowledge that students need to have to make sense of probability. It is also important for teachers to ensure that students understand how mathematical concepts originate, extend and connect across and within the grade.

A research study by Gravemeijer, Bruin-Muurling, Kraemer and van Stiphout (2016) on fractions found that although students were not able to deal with more complex variations in tasks, they moved swiftly to formal and general understanding of fractions, if taught with understanding. In other words, the grasp of concepts of probability is subject to how meaning is constructed. Additionally, teachers should demonstrate connections between mathematics and other subjects to provide context for learning. For example, in Geography, weather forecasters use fractions and percentages to predict the chance of rain. Bertsekas and Tsitsiklis (2002) indicate that probability is defined in terms of how one discusses the uncertain situations. In the next section, the relative attempts to use the concept of probability to describe an uncertain situation are discussed.

3.4.1 *Definition of probability*

Bhat (2007) relates probability to measuring the possibility of an event taking place. Likewise, Grimmett and Stirzaker (2020) hold the view that probability is the likelihood of occurrences of events. Van de Walle et al. (2016, p. 583) capture the essence of probability in the following definitions: Firstly, "probability is a ratio that compares the desired outcomes to the total possible outcome. Secondly, probability is about how likely an event is". Drawing from these

definitions, the term probability is understood to mean the extent to which an event is likely to occur.

While a variety of definitions of the term have been suggested, in this study the definition of probability is derived from the work of Naidu and Sanford (2017) ,who regard it as a numerical value between 0 and 1 indicating the likelihood for an event occurring. Using numbers to model situations, the number "0" is applied to situations where it is can be said with certainty that an event will not happen; the number "1" on the other hand, indicates that the event will certainly occur. It is helpful to use the numbers between 0 and 1 to describe the degree of uncertainty for certain events. Such events show that probability is relative. The use of fractions is ascribed to understanding the degree of chance**.** In order to explain the degree of chance, terms such as 'unlikely', 'likely' and 'might be' should be used. These underline the fact that certainty cannot be guaranteed. For instance, in many cases journalists often use jargon from probability when they report cases or incidents. This is because one can only report with certainty if the investigations into the matter in question have been concluded.

Furthermore, how teachers connect knowledge of previously learnt mathematical terms to the concept of probability is critical, particularly when dealing with numerical values. Mathematical terms learnt in real-life situations should be used to enhance students' understanding of the concepts of probability, including the use of numerical values. On the question of context, Kazeni and Onwu (2013) point out that the situatedness of learning (learning that is intertwined with environmental, social and cultural factors) and the connection of teaching to students' real-life situations are two emerging ideas that are common to various definitions of context. This statement underlines the need for teachers to create a learning space in such a way that students' understanding of probability manifests in how they experience probability.

To show that the indigenous knowledge system has the potential of being used in making sense of formal learning, Pradhan (2017) acknowledges that knowledge passed down from ancestors outside of the formal schooling context is factual and stands on a strong base for people's work and living. A closely related finding from a study conducted in Malawi by Simamora and Saragih (2019) emphasises the importance of integrating local culture with mathematics to improve students' learning achievements. Ozofor and Onos (2018) also recommend the use of instructional methods that reflect people's cultural background and that link students to

immediate environmental experience. In explaining how traditional knowledge can be integrated in cultural activities, Enock (2013) provided the example of prediction as follows:

> Among the indigenous rainfall prediction indicators that farmers in Zaka use include the density of spider webs in their locality, with a lot of spider webs indicating a very wet season (spiders don't want damp or wet conditions). Also, when spiders close their nests, an early onset of rain is expected because spiders do not like any moisture in their nests. They also look at the circular halo around the moon, known locally as dzivaremvura, to predict the wetness of that particular period of the season. Other indicators include animal and plant behaviour, as well as wild fruit availability and wind direction prior to the rainy season. Although these indicators can be said to be indigenous, they certainly show some level of dynamism and integration of western science which has also tapped into these and use wind direction to predict rainfall patterns. (p. 117)

A foundation on which students could define the concept of probability is represented diagrammatically in Figure 3.9.

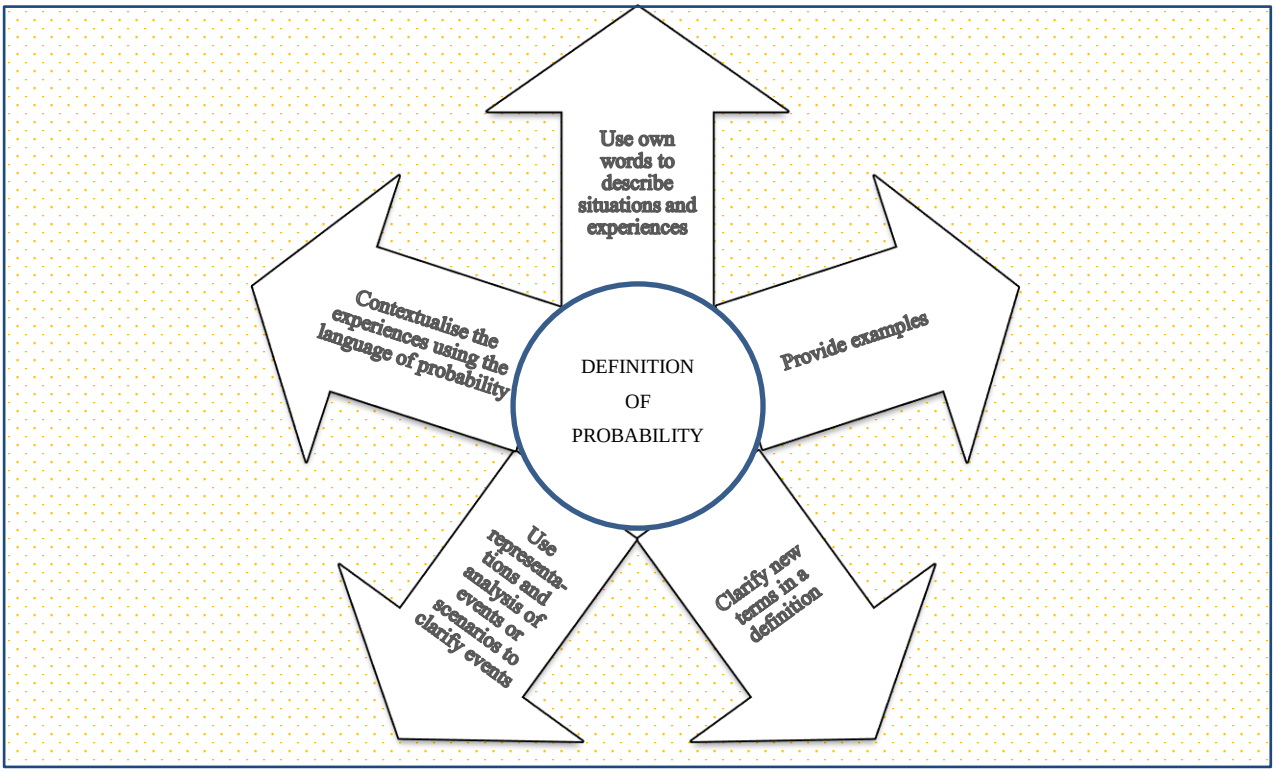


Figure 3.9. A construct to help define the concept of probability

Figure 3.9 presents strategies that teachers could adopt to move students beyond superficial knowledge of probability towards a deeper conceptual understanding of the uncertainty of events. In the same vein, Shaughnessy (1977) indicates that one of the strategies that could develop an understanding of probability is a course that encourages students to build their own models to represent ideas.

What are the essential aspects of probability? In this section, each of these aspects is clarified by discussing how teachers could help students gain deeper understanding of probability. Kvatinsky and Even's (2002) paper offers some insights into what to consider as essential aspects (fundamentals) in understanding the concept of probability. These essential aspects are described in Table 3.4 as follows:

Table 3.4

Essential aspects of probability

- **Essence of probability**

There are two approaches to the essence of probability. The first is the objective approach, which is assigned to experiments or practical activities. The second is the subjective approach, which is interpreted as the extent of the belief that an event will occur, rather than its relative occurrence.

Implications: The following questions might be asked to clarify an understanding of the essence of probability. Do students understand the purpose of doing experiments? How are these experiments related to the theory of probability? From which perspective are teachers viewing probability? Shedding more light on this aspect, are the findings from a study conducted in Egypt by Elbehary (2019) who indicates that teachers promote the development of procedural knowledge which might hinder students' probabilistic reasoning.

- **Strength of probability**

This concerns the way teachers draw on real-life situations when using examples to explain the concept of probability.

Implications: The strength of probability could be explored by investigating how teachers draw from real-life situations to make connections between informal learning and formal learning of probability concepts. This understanding refers to how teachers could use ideas that learners bring into class as tools to construct meaning.

- **Different representations and models**

This essential refers to the explanatory framework that teachers use to look more closely at content. It may include the use of analogies, illustrations, demonstrations and symbols.

Implications: The focus in this case is mainly on how teachers select and use various representations to explore the concept of probability.

- **Alternative ways of approaching probability**

Alternative ways of approaching probability in this context refers to descriptions that suggest ways of teaching probability.

Implications: The understanding is more on how teachers create a learning-centred classroom to help students move swiftly from informal to abstract learning of probability (DBE, 2018a)

- **Basic repertoire**

These are basic mathematical concepts connected to probability, such as percentages, ratio, fractions, including sets.

- **Different forms of knowledge and understanding**

This essential refers to knowledge of the design of instruction that requires an interaction between mathematical understanding and understanding of procedural issues that affect students' learning.

Implications: It is critical for teachers to realise that understanding the concept of probability is as important as processes involved in teaching it.

- **Knowledge about mathematics**

The CAPS: Mathematics for Grades 7-9 (DBE, 2011, p. 8) states that if students are to develop the skills that are essential to succeed in mathematics they should:

Learn how to use the language of mathematics (including symbols and numbers) correctly;

Learn to use the vocabulary of probability, such as 'fair', 'likely', 'certainly', 'probably', 'definitely' and so on;

Learn to listen, communicate, think, reason logically and apply the mathematical knowledge they have gained;

Learn to investigate, analyse, represent and interpret information;

Learn to pose and solve problems.

Source: Adapted from Kvatinsky & Even, 2002

These essential aspects are pillars to gain deeper understanding of probability. Central to making sense of probability, is the importance of its applicability in everyday life situations. The next section describes the importance of probability in real-life scenarios.

3.4.2 *The Importance of Probability*

As part of statistics and data handling in the mathematics curriculum, probability is used all the time and relates to many aspects of life (Mahmud & Porter, 2015). Aspects of life in this regard might include sports, environmental issues, companies, engineering, computer science and weather forecasting. Durand-Guerrier et al. (2016, p. 98) argue that "cognitive mathematical activity is connected to speech activity". In other words, because probability relates to the realities of life, it is important to master a language that conveys such realities. Therefore, the art of attaching meaning to such realities demands an understanding and acknowledgement of the complexities of the language of teaching and learning. Additionally, such realities might be predictable or may respond in a way that is expected, hence the term 'uncertainty'.

The word 'uncertainty' means that one cannot be sure of what might happen at any given time. For instance, because of the outbreak of the Coronavirus, we witnessed the global shutdown of industries, the loss of jobs, home-schooling and so on. People had to adapt quickly and master technology in the face of uncertainty about the disease. This pandemic has taught us that we live in a world where unforeseen events may occur.

We therefore need to be on the alert and learn to analyse situations in order to make some predictions and informed decisions. In this case, together with plans to curb and combat the disease, there is nonetheless increasing concern about the resurgence of the disease, or a second and third wave of the COVID-19 virus. This pandemic provided an opportunity to advance the understanding of the probability concept. The outbreak of the Coronavirus could be one area that teachers might use to explore concepts such as prediction and how to make an informed judgement. This perspective has several important implications for classroom practice. For example, teachers and subject advisors need to consider the importance of reviewing their practice and providing proper guidance that will encourage students to reflect on their experiences and to construct their own understanding of the world they live in.

In the proceedings of the British Congress for Mathematics Education, Borovcnik and Kapadia (2010, p. 42) cited the following as key reasons for the strong role of probability in the mathematics curriculum:

- Misconceptions on probability affect people's decisions in important situations such as in investments, medical tests, assessment etc.
- Probability is essential to understand any inferential procedure of statistics.
- The concepts of risk (not only in financial markets) and reliability are closely related to and dependent on probability.

A useful example to clarify the concept of risk is the erection of a building in an area situated in a windy environment. This building could collapse if the wind force is not considered when designing the building. Yang, Gao, Bai, Li and Tamura (2018) advise that in such cases building arrangements and construction methods should be chosen with wind resistance in mind. In other words, if design and construction methods are not chosen accordingly, the likelihood of wind destroying the building would be very high. Some natural phenomena such as strong winds can be difficult to predict exactly, and thus have the potential to cause disasters. Gazdula and Farr (2020) argue that awareness of risks involved in actions could be raised by a discussion of the Fukushima nuclear disaster. These kinds of events could spark debates that would be understood better if the language of probability is correctly used.

Another practical example of using probability occurs when insurance companies cover inflation for pure investments and risk for death and accidents. One of the most important factors to consider when calculating risks for car insurance, for instance, is the experience of the driver. An experienced driver does not carry the same risk as a novice driver. My view thereof is that the insurance cover for the two categories of drivers would differ. In their study, Mohammed, Ambak, Mosa and Syamsunur (2019) suggest that to reduce the risk and level of accident severity, authorities need to come up with strategies to build awareness to target the audience. In this case, the understanding is that the level of accidents and the rate of fatalities might be reduced.

Through analysis of prevailing situations, people become empowered and will be able to make some predictions or informed decisions. This understanding suggests that the concept of probability should be developed through the interpretation and analysis of experiences in real-life scenarios. Aven and Bouder (2020) concluded in their study that risk is a key concept in

probability and the Coronavirus pandemic could be used to demonstrate an understanding of risk, how to communicate risk and how to handle risk. The analysis of such risks would need relevant probability concepts and models to guide the explication. The following section provides a discussion of how language is used to convey meaning of probability.

3.4.3 *Language of Probability*

When a new topic is introduced in mathematics teaching, the use of both the language of probability and the language across the curriculum is essential. As highlighted by Aven and Bouder (2020), students are taught that different probability representations and the correct use of language can convey information about probability. Students are expected to learn the concepts of probability through the language of learning and teaching. Vukovic and Lesaux (2013) have emphasised that students need language and content-based instruction with a focus on teaching language both as a specialised vocabulary and as a tool for critical thinking. The issue of language in this section is reviewed under two headings, namely, language across the curriculum and the specialised language used in probability.

This section looks at how meaning of probability is communicated and comprehended through the language of teaching and learning. Bozkurt (2017); Kusmaryono, Jupriyanto and Kusumaningsih (2021) argue that learning is fostered through guidance and assistance provided during a child's development. We use a specific language in mathematical communication to model our real-life experiences. To provide appropriate guidance in child development, Khaliliaqdam (2014) believes that the knowledgeable other should be experienced in that specific field. The next section explains how the language of probability is used.

Probability Terms

It is important to understand the unique language of probability to acquire knowledge or develop the skills to interpret real-life experiences. Although Groth et al. (2016) acknowledge that even though students recognise that if one event is possible the other will be impossible, they point out that the ability to assign numerical values is more developed than their use of probability language. With reference to the definition of probability discussed in Section 3.4, it is therefore advisable for students at primary school level to learn and understand the terms used in probability. For example, the terms 'possibility' and 'probability' might sound complex or confusing to them.

Batanero (2015) describes the term ‘random’ from different perspectives. Firstly, from the experimental point of view, where each outcome in a set of events has an equal chance of being selected. Secondly, from the perspective that, although a selection can be made randomly (with no particular order prescribed), they acknowledge that there might be factors that could affect judgement of randomness. Such factors might include fairness in terms of how the rules of the game were developed, the kind of resources used by individual members and the conditions under which experiments are conducted. How players understand a fair game will be based on the nature of the game and their experiences. For the game to be fair, players should ensure that all events have an equal chance of happening.

Another platform where the concept of randomness and fairness could be learnt is when experiments are conducted. Experiments should be designed carefully and repeated several times (Hering, Durell, & Morgan, 2021) to guard against jumping to conclusions without enough evidence. Conducting experiments several times might benefit students in acquiring the language of probability and developing vocabulary to make reasonable arguments. Students would also realise that they can predict the results if an experiment is fairly conducted.

Modal Verbs

Elicer (2019) defines modality as the expression of probability that describes the likelihood of the expected outcome to happen. Zhang (2019) describes modality as having to do with the expression of possibility and necessity. In clarifying the concept of modality, Zhang (2019) explains that modal verbs are auxiliary verbs that add more information about the level of necessity and possibility. Likewise, Sadia and Ghani (2019, p. 144) indicate that the “term modality has been observed as a way to express attitudes of the speaker or writer towards the world”. To offer further analytical description and interpretation of modality, Sadia and Ghani (2019, p. 144) list the following:

I. Modality
    - Deals with presentation of the writer’s/the speaker’s world view and ideologies.
    - Is a comprehensive term to describe propositions of the speaker’s attitudes to a situation.
    - Is considered an important linguistic device to express social roles and relationships between speaker/writer and hearer/reader.

II. Purpose and values of modal auxiliary/verbs are presented in Table 3.5 as follows:

Table 3.5

Purpose and values of modal verbs

FUNCTIONS OF MODAL VERBS			EXAMPLE	VALUES	EXAMPLE
Predictions	Making a prediction that is not definite		Will Would	High value	Must
Obligation	An act that is obligatory Makes formal request or offer		Must Should Can	Median value	Will, Would Shall
Possibility/ability	Expresses the degree of possibility informed by past experience		Could May Might	Low value	May Might Could

Source: Sadia & Ghani, 2019, p.145

Information provided in Table 3.5 suggests that teachers need to ensure that students understand the context in which the modal verbs are used, be it a possibility, obligation or a prediction. Curto (2019) offers an equally important view and indicates that teachers should give students sentences in which the same modal verb is used in different contexts. To achieve this, teachers could draw on students' diverse skills, experiences, interests and beliefs to build on new knowledge. Furthermore, students need to have background information about an event or a situation before they can make any conclusion or judgement (Kvantisky & Even, 2002). According Fong (2020), the use of modals should allow one to understand and explain the implications of the sentence or the event occurring.

Although it could be argued that the notion of modal verbs is relevant only within the English language context, verbs are important for students to learn probability effectively and develop conceptual rigour. On the other hand, small nuances in meaning may cause a person to misunderstand an idea, for example, an erroneous interpretation of the following scenario: "If a child has flu, the probability that she will test positive for Covid-19 is 88%" is often confused with the statement "if a child tests positive on Covid-19, the probability that she has flu is 90%". Although I acknowledge that knowing the meaning of words in everyday language may be different from mathematical meaning, the mathematical situation may require precise language that has been defined to have meaning. The ability to express the idea is what holds the idea in thought.

At the moment I cannot rule out the importance of language in communicating mathematical ideas. According to DBE (2011, p. 11), "The study of probability enables the learner to develop skills and techniques for making informed predictions and describing randomness and uncertainty". For students to be able to describe randomness and uncertainty, the use of language is critical. In their study, Morgan et al. (2014) recommend a substantial and coordinated research effort to answer the following questions:

- What are the linguistic competencies and knowledge required for participation in mathematical practices?
- How do students develop linguistic competence and knowledge in subject-specific learning?
- What knowledge and skills might teachers need and use to support the development of students' linguistic mathematical competence? (p. 851).

These questions suggest that grammar and mathematics vocabulary should not be taught in isolation.

Since the main aim in this study is to design a framework to enhance teachers' instructional practices in the teaching of probability, I have adapted Kilpatrick et al.'s (2001) notion of conceptual understanding to explain what instructional practice entails. Toh and Chiu (2020) suggest that in developing students' conceptual understanding of probability, it is important to use real-world contexts. To put into context, Toh and Chiu propose the implementation of the TIDE model (Tackle students' misconceptions, Introduce probabilistic reasoning, Draw connections, and Encourage problem-solving). These indicators of the TIDE model are embedded in the strands of mathematical proficiency. The next section presents teachers' practices in developing the concept of probability.

3.5 Practices in teaching probability

Wessels (2014) highlights the point that deep understanding of mathematics is reflected in various ways of representing concepts and application of such concepts in different situations.

Le Donné et al. (2016) directed their attention to the fact that active learning, cognitive activation through engaging students in problem-solving activities and clear teacher-directed instruction form the basis of good instructional practices. The same study indicated that evidence has shown that a) students exposed to teacher-directed instruction are somewhat more

likely to solve the easiest mathematics problems; b) teacher-directed strategies seem to be more conducive to solving easier tasks but cannot prepare students for more complex tasks; and c) cognitive activation practices are positively associated with student performance.

An arguable challenge, according to Ojose (2011), is that the approaches used by teachers to teach mathematics may influence how students construct meaning. It may be the case, therefore, that these approaches defining teachers' instructional practices depend on the intended goal and the context. Based on Wessels's (2014) work, the current study provides perspectives on how teachers mentor, control and facilitate the learning process. Practically, these perspectives refer to what teachers know, say and do to create a learning space that allows students to make meaning of probability. Creating a learning space for students means moving beyond the use of formulae to calculate the probability of an event occurring. To provide more insight into the work of other scholars, this section reviews literature focusing on teachers' practices in making sense of probability.

3.5.1 *Teachers' Understanding of Probability*

Probability is a topic in the content area of data handling, as stated on page 10 of the CAPS document (DBE, 2011). The aim of this topic is to develop the skill of making predictions, describing uncertainty and randomness. The focus of this study was an exploration of how teachers' conceptual understanding of probability frames their practices. Content on probability is captured in the CAPS as follows:

> Perform simple experiments where the possible outcomes are equally likely, and list possible outcomes based on the conditions of the activity and also for students to determine the probability of each possible outcome using the definition of probability. (DBE, 2011, p. 36)

Batanero et al. (2016) argue that the Grades 6–8 United States syllabus includes understanding the terminology used to describe types of events and computing probabilities for simple compound events. According to Batanero et al. (2016), the focus is also on understanding the language of probability, as in South Africa. What CAPS also emphasises is the use of more manipulatives when explaining the concept of probability. The clarification notes in Section 3 of the CAPS document suggest that students should be able to do the experiments, list the possible outcomes and use the definition of probability to determine the probability of each possible outcome. There is also experimental language that students need

to construct to explain the relationship between language and thought in the learning of probability. Such language clarifies puzzling aspects of probability with which students typically struggle. To emphasise the importance of effective use of the language in enhancing understanding, Erath, Ingram, Moschkovich, and Prediger (2021) advise researchers to carefully consider conceptualisations of language and language development that match the focus of learning mathematics.

Van de Walle et al. (2016) define probability as a ratio of the desired outcomes and the total possible outcomes. In other words, the concept is defined in relation to the sample space. From this understanding, one may conclude that probability is relative; the degree of likelihood is a continuum between 0 and 1. From my experience as a subject advisor, teachers' classroom activities focussed mainly on the use of probability formulae to determine the number of ways in which the event could happen.

While acknowledging that the use of a formula might be appropriate in solving probability problems, probing questions are also key to developing conceptual understanding (Lanuza et al., 2020). The importance of follow-up questions as a way of encouraging students to develop their understanding is illustrated in an article by Ang and Shahrill (2014), in which they present the results of a study conducted in Malaysia with Grade 10 and 11 students. The aim of the study was to determine the extent of students' misconceptions when learning about probability. Examples of questions asked in the study included the following: If a coin is tossed five times, which of the following ordered sequences of Heads (H) and Tails (T), if any, is most likely to occur?

a) HTHTT
b) THHHH
c) HTHTH
d) Sequences (a) and (c) are equally likely
e) All the above sequences are equally likely

One student responded to the question as follows:
"*Sequences (a) and (c) are equally likely*". The same student justified this answer, saying that *"Tossing a coin is a random event, it is really rare you get four heads in a row"* and added that there ought to be an equal number of heads and tails (Ang & Shahrill, 2014, p. 27). Ang and Shahrill (2014) argue that students' problems may persist in the learning process if they

incorrectly think that samples that correspond to the population distribution are more probable than samples that do not. They recommend that intervention classes should be provided to treat any misconceptions that may still exist.

From my experience as a subject advisor, I have observed that even in the Intermediate phase, particularly in Grade 6, teachers focus more on the use of the formulae when calculating the probability of events. This practice does not develop conceptual understanding in students and it points to the fact that students who experience some difficulties in making sense of probability should first learn more about the meaning of the likelihood of an event (this is a critical point at the heart of understanding probability) before thinking of using the formula. Based on what the CAPS document provides as guidelines for teaching probability in Grade 7, it is also important for teachers to ensure that all the embedded terms in the definition of probability are clarified if students are to grasp the concept.

The content area of numbers, operations and relationships also provides an opportunity to develop understanding of the basic knowledge used in probability. The application of this involves comparing quantities of the same kind (ratio), which makes connections with the other concepts of probability. Well-formulated learning objectives and understanding of basic knowledge would assist teachers in designing tasks that are appropriate in helping students to gain an understanding of probability. Teachers' own conceptual understanding of probability can be gauged by how these learning objectives are formulated and how the tasks are designed. What is also important to note is that the way the objectives are formulated dictates the focus and emphasis of teaching. If the learning objectives are explicit, this would allow teachers to deliver clear and orderly lessons.

Furthermore, Sullivan (2011, p. 39) indicates that "Having students pose questions in their own words allows for explicit articulation of mathematical learning and for the understanding that there may be multiple ways of solving a problem". I find this understanding very useful because it helps to determine the focus of the lesson, and to assess whether or not the teaching activities will enable the teacher to achieve the goal. Moreover, it is important for teachers to realise that because assessment is integral to teaching and learning, they need to ensure that assessment activities guide the presentation processes to achieve the intended goals. When integrating and practising their foundational and reflexive competencies, teachers must decide on effective approaches to facilitate the learning of probability.

3.5.2 *Facilitation Processes*

Processes and practices in the teaching of probability are facilitated by considering several issues: how probability is defined in relation to chance; the correct use of the language of probability; the use of modal verbs; procedures for developing concepts; representations of the probability concept; the incorporation of manipulatives and other teaching resources in the lesson; the promotion of active learning; the explanation and connection of concepts; seating arrangement in the classroom; an understanding of how challenges are addressed; how assessment is conducted; how mathematical representations are connected; how context is used to explain probability concepts and how to instil a belief in students that probability makes sense (Kilpatrick et al., 2001; Van de Walle et al., 2016). All these processes and practices are equally important to create a leaning space for students.

In their study, Mutara and Makonye (2016) analysed students' responses on how they construct a diagram to illustrate their solutions. The following question was asked: What is the probability of getting two 3s if a die is tossed twice? A Grade 10 student used a tree diagram to answer the question as follows:

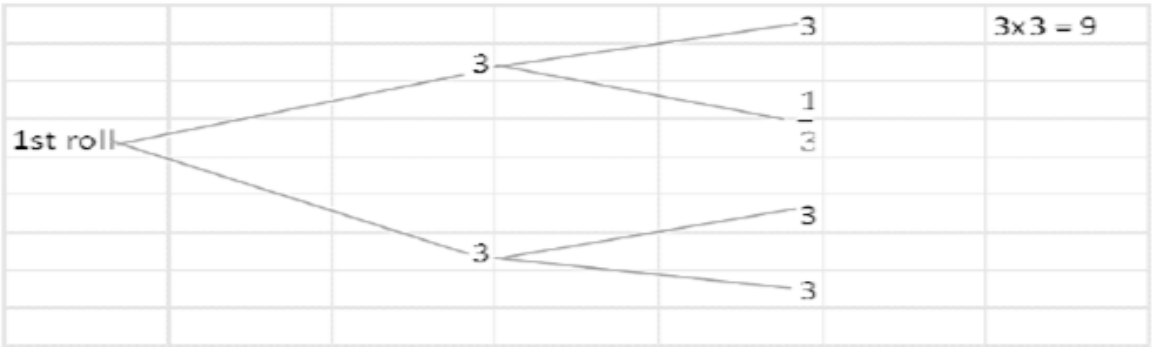


Figure 3.10. Student's tree diagram to determine probability
Source: Mutara & Makonye, 2016, pp. 1-4

According to Mutara and Makonye (2016), most students in this study failed to understand how the tree diagram is used to represent probability events. Also, none of the students used other representations like a two-way table to answer the question. Mutara and Makonye (2016) concluded that the problem might be that students had not practised rolling a die more than twice. They also found that the students believed that three was the only possible outcome and other possibilities were ignored.

The student's answer is not correct because probability of an event happening cannot result in a number more than 1. Evidence from Mutara and Makonye's (2016) study suggests that the student has no idea of what each branch means, or that probability represents the level of uncertainty. The reason for this kind of error is not clear, but it may have something to do with the student not knowing what a tree diagram entails and its purpose. A further implication of this error is that the student may lack understanding of the following probability terms: outcomes, sample space, probability. These errors raise several questions and it would be more interesting to see how the student would have responded if asked to construct the representation for the second roll of the die.

On the other hand, Brodie and Berger (2010) point out that these kinds of errors might be connected to the fact that students are not familiar with the representation used. I tend to agree with this view because students could become excited to work on something that they have seen before or they have interest in. The student's response is a good illustration that more should be done by teachers to make sense of probability. In their study, Batanero et al. (2016) suggest that chance terminology should be introduced as early as possible to help students build upon their intuitions so that they can begin to use sophisticated words such as 'randomness' appropriately.

In the same study, Mutara and Makonye (2016) indicated that students were required to answer the following questions: What is the probability of getting 1 head, 2 heads and 3 heads when a coin is tossed three times? Thulani wrote: a) H, b) HH, c) HHH. Mutara and Makonye (2016) argue that this answer is partially correct in the sense that it shows the required outcomes in the correct sequence, although the tails are missing. On the other hand, they highlight the fact that the student did not cross the threshold and determine the numerical probability that was required for each event.

The evidence presented thus far illustrates the difficulties students have in understanding concepts in probability, representation of probability outcomes and the use of various representations when solving probability problems. The case studies discussed here confirm that teachers should focus on finding strategies that will help students to learn probability concepts in an understandable and meaningful way, particularly at the elementary level. This view is supported by Molina (2014), who advises teachers to avoid taking shortcuts; they should

focus instead on instructional language and developing a deeper understanding of mathematics in the classroom. Molina (2014) stresses that the use of confusing language must be avoided. This includes explanations such as 'the probability of getting a head when tossing a coin is one over two'. This is an abstract statement in that the two numbers are pronounced as whole numbers and will be interpreted as such. In a study by Ersanli (2017), the questionnaire used for the needs analysis revealed that mathematics teachers in elementary education needed to develop their English language skills, especially those they would require in a specialised area such as probability.

Being an effective teacher requires the implementation of creative and innovative facilitation strategies in order to meet the needs of students. One other strategy to facilitate learning is how assessment is integrated into teaching and learning. Aligning assessment with the objectives of the lesson is key because it is through assessment that teachers evaluate students' understanding of the concept. In other words, assessment should be planned and implemented in such a way that it promotes learning in the classroom. To assess teachers' understanding of probability, Smith, Bill, and Raith (2018) argue that one approach is to align the tasks to the goals of the lessons. The importance of the use of a textbook was investigated by Elbehary (2019) from the perspective that it can have a great influence on the process of teaching and learning. Batanero et al. (2016) also regard the analysis of curricular guidelines and curricular materials such as textbooks as important for research.

The present study supports the findings of Elbehary (2019) and Batanero et al. (2016) because what I have observed as a subject advisor is that many teachers take activities directly from textbooks without any modification. The questions then arise: Are the teaching and learning activities taken from textbooks applicable to the aims and objectives stipulated in the curriculum? Are the instructions coherent with the children's current learning levels? These questions suggest that teachers need to interrogate the curriculum to ensure that they understand the content and decide on the appropriate techniques to facilitate the teaching of probability. Cheng, Leong, and Toh (2021) urge teachers to make a careful selection of instructional material and learn to adapt the activities to suit the needs of the classroom. This view suggests that teachers should engage in material review processes.

Despite the fact that the study conducted by Fan, Zhu and Miao (2013) found that the selected textbooks were not good examples of all that were available, they reported that the evidence of

a link between the textbook and students' learning outcomes was weak. Figure 3.12, the extract from the DBE workbook confirms the views of Fan et al. (2013) and contributes additional evidence that suggests that the challenge lies in regulating a swift transition from informal to abstract learning.

On the other hand, in a study conducted by Takahashi (2016), a Japanese textbook was reviewed, and the latest edition included the following aspects:

- Write down the way you thought about doing it using pictures and math sentences
- Look at the math sentence Takumi wrote and explain how he thought about the problem
- Look at the math sentence Yumi wrote on the next page and explain how she thought about the problem
- What is common amongst the three students? (Takahashi, 2016, p. 317)

A reprint of the textbook included activities to support students in developing note-taking skills throughout the grade. According to Takahashi (2016), as early as Grade 2, textbooks included some examples of how to take notes to foster students' mathematical thinking and problem-solving skills. I find this strategy useful in several ways. Firstly, it encourages students to focus and pay attention in class. Secondly, it compels students to play an active role in a learning situation as they will be expected to explain their views. Thirdly, the strategy could assist in developing language skills. Lastly, students can learn from their peers. The strategy of taking notes could also inspire students to be innovative by creating their own pictures (mind-maps) to solve problems.

The kinds of questions highlighted by Takahashi could be useful in developing students' adaptive reasoning by forcing them to justify their responses, which in turn develops conceptual understanding (Kilpatrick et al., 2001). Teachers should also ask follow-up questions to probe students' understanding based on the answers they provide. To provide more advice on how to design teaching and learning activities, Watanabe, Takahashi, and Yoshida, 2008 (as cited in Fujii, 2015, p. 5) presents a model that could be of assistance to many teachers:

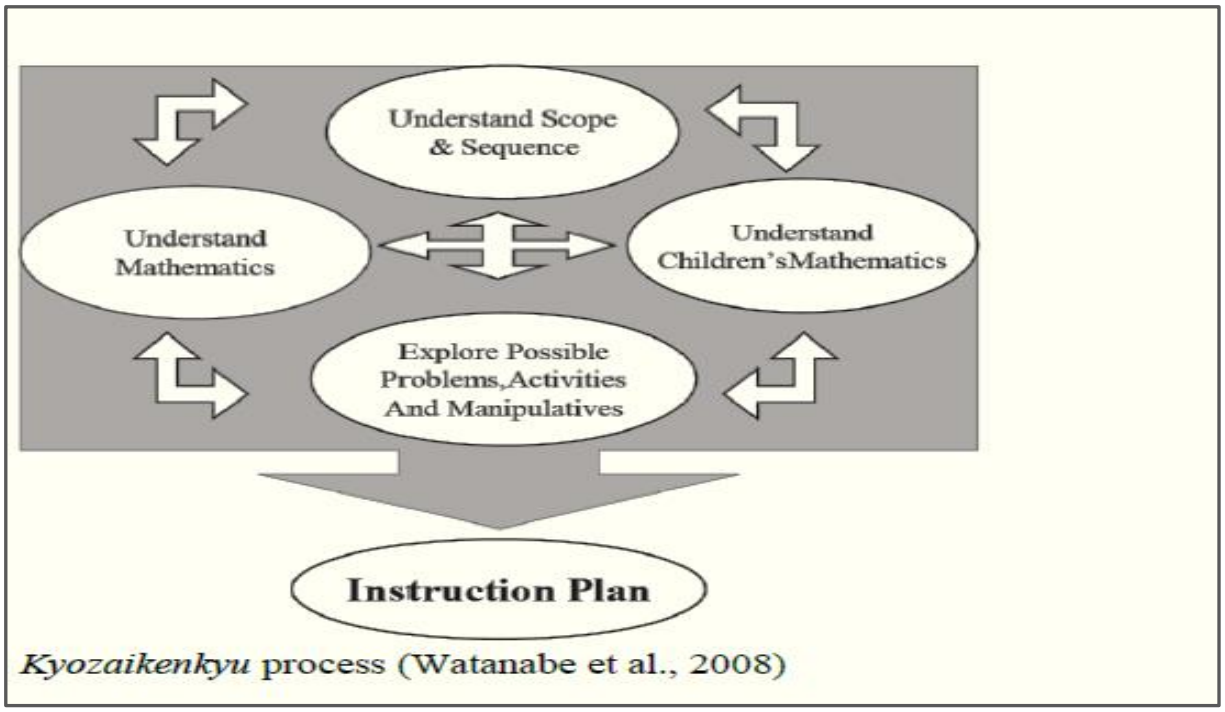


Figure 3.11. Task design in mathematics education

Source: Fujii, 2015, p. 5

Fujii (2015) explains that the *Kyozaikenkyu* process requires teachers to seriously examine their teaching materials and/or tasks from their own and their students' point of view. Van der Walle et al. (2016) also suggest how teaching and learning activities could be sequenced and provide the following guidelines:

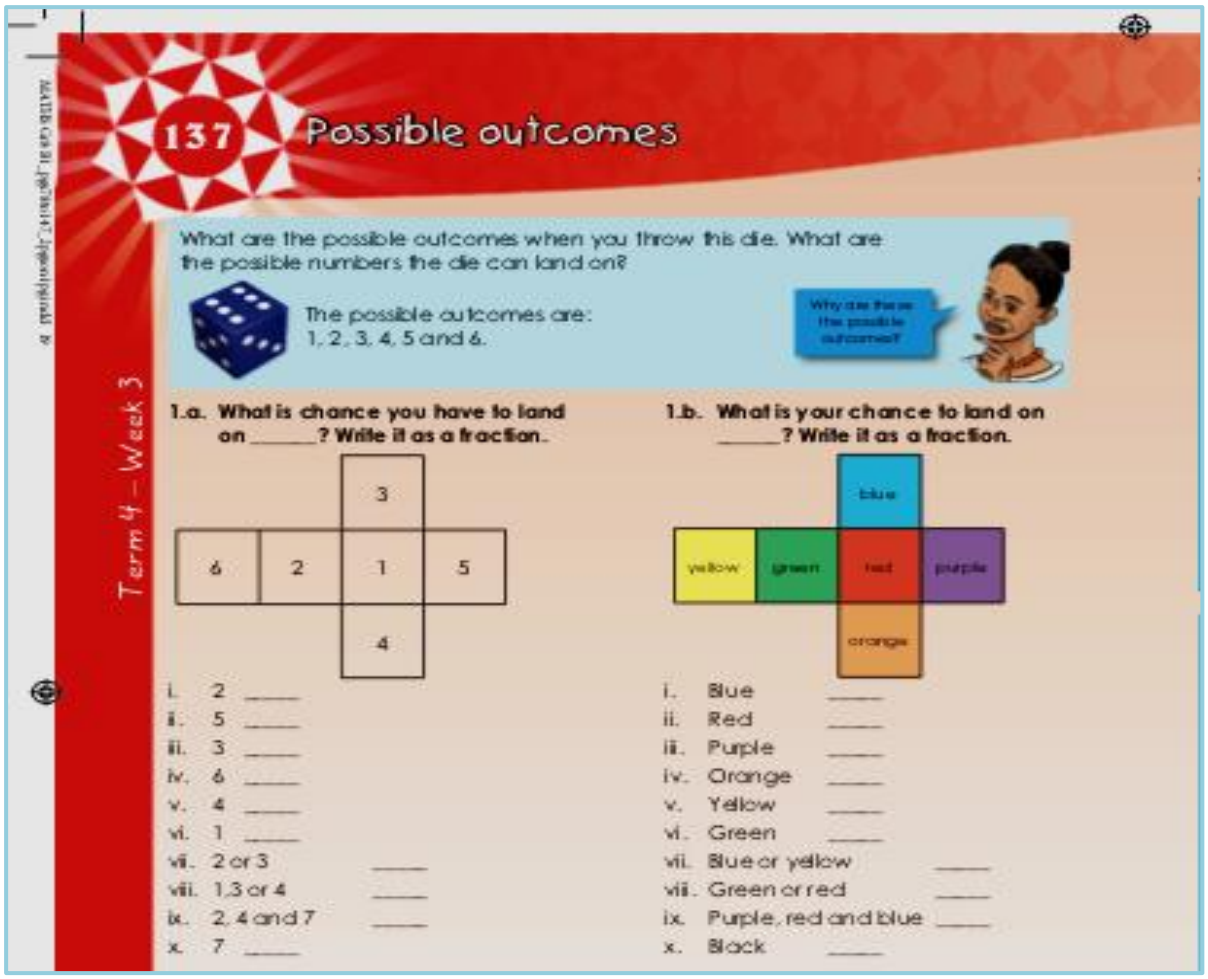
- A good place to begin is with a focus on possible and not possible. Later, impossible, possible and certain.
- Create a table, labelling one column 'Possible', the other 'Impossible'.
- Use games, songs, riddles, debates and record each statement in the appropriate column.
- Ask students to judge various events using certain, impossible or possible.
- For each event, students should justify their choice of how likely they think it is.

This is an opportunity to bring in students' identities and cultures (Van de Walle et al., 2016, pp. 583-584). Using this strategy exposes students to different genres. Although Van de Walle et al. (2016) argue that the sequencing of activities develops conceptual understanding of probability, I believe that these guidelines should not be prescripts because students come from

different backgrounds. In other words, teachers need to make intentional efforts to connect with diverse students.

Language, the primary means of communication, also plays an important role in making sense of complex and abstract thoughts. It is the facilitation strategy that enables students to share ideas, opinions and views during the learning process. Certain verbs are used in probability to indicate the likelihood of an event occurring. Such verbs accompany the base form of another verb and are called modal verbs. The modal verb used most frequently by teachers when formulating teaching and learning activities in probability is the word “will”, which suggests that probability deals with future events only. How teachers use words to explain the concept of probability form part of the strategies used to measure teachers’ conceptual understanding of probability.

Another strategy that promotes meaningful learning is to get students to engage in learning activities that are likely to result in achieving the intended learning outcomes. Figure 3.12 presents some teaching and learning activities that South African teachers use when teaching the concept of probability:



137 Possible outcomes

Term 4 – Week 3

What are the possible outcomes when you throw this die. What are the possible numbers the die can land on?

The possible outcomes are: 1, 2, 3, 4, 5 and 6.

Why are these the possible outcomes?

1.a. What is chance you have to land on _____? Write it as a fraction.

3
6 2 1 5
4

i. 2 ____
ii. 5 ____
iii. 3 ____
iv. 6 ____
v. 4 ____
vi. 1 ____
vii. 2 or 3 ____
viii. 1,3 or 4 ____
ix. 2, 4 and 7 ____
x. 7 ____

1.b. What is your chance to land on _____? Write it as a fraction.

blue
yellow green red purple
orange

i. Blue ____
ii. Red ____
iii. Purple ____
iv. Orange ____
v. Yellow ____
vi. Green ____
vii. Blue or yellow ____
viii. Green or red ____
ix. Purple, red and blue ____
x. Black ____

Figure 3.12. Extract from Grade 7 Mathematics 2020 DBE workbook

Source: DBE, 2020, p. 174

The questions posed in Figure 3.12 indicate an abrupt transition from informal understanding of chance to abstract learning of probability. For instance, the way the two questions are posed confirms my observations that many questions developed by teachers focus on the definition of probability. As such, probability is understood from the perspective that it must be expressed in numerical values. If this is the case, it seems that very little is being done by teachers to develop students' conceptual understanding of probability. In this example, questions are asked but there is no opportunity for students to justify their answers. In addition, the activities do not provide an opportunity for students to show their understanding of chance in relation to probability.

Secondly, sub-questions $(vii - x)$ in both questions, take for granted that students understand the language of sets. The introduction of sets at this level might lead to an understanding of

probability concepts that deal with mutually exclusive, independent and complementary events; this is only covered in the Further Education and Training (FET) band. From my experience as subject advisor, sets do not form part of the South African curriculum in the General Education and Training (GET) phase. In this case, it is no wonder that students regard the language of probability as complex. It would be very difficult for students to answer the identified sub-questions if they do not understand the terms ‘or’ and ‘and’.

If students are to develop a conceptual understanding of probability, it is essential that their learning is contextualised. In support of this view, Van den Heuvel-Panhuizen and Drijvers (2014, pp. 522-523) argue that a) Realistic Mathematics Education (RME) theory is characterised by the fact that realistic situations serve as the basis for initiating the development of mathematics concepts and that b) the theory is underpinned by the following principles:

- *Activity principle*

Students are treated as active participants in the learning process. This encourages students to learn best by doing. For example, in probability, students need to conduct experiments using selected resources. Each student should have a role to play when such experiments are conducted. The roles include facilitating the session, reporting back to class and writing down the outcomes of the experiments. Additionally, students could also share their presentations with other classes or amongst themselves. Being active in the learning process is one of the building blocks towards developing conceptual understanding.

- *Reality principle*

Learning should start from meaningful real-life problems that will later develop into a construct. Students should start from meaningful real-life problems to provide a useful mathematical representation of the connections between facts. Real-life problems can only be meaningful if they are realistic and if students see the importance of working on such problems. For example, if teaching activities centre around what students have experienced, then it becomes reality on their part. The other critical issue to explain reality is that activities should be within the cognitive level of students. The use of context-based problems when building awareness of the role mathematics plays in real-life situations is an attempt to make teaching of mathematics more relevant.

Veenstra (2014) subscribes to a context-based teaching strategy in a study evaluating the impact of real-life activities on the teaching of probability in K-7, the seventh grade in the American school system. It emerged that revision of study units was necessary to promote higher levels of engagement by students and to provide more real-life activities to develop the probability concept. It is also important to highlight that in 2014, Van den Heuvel-Panhuizen and Drijvers published a paper in which they described realistic situations as using problems from the real world as long they were experientially real in the minds of children. Even though feedback on difficulties experienced by students in solving problems on chance was not provided by Mullis et al. (2020), the SACMEQ study provides more information on students' understanding of basic numeracy skills and problem solving.

Data from several sources have revealed the use of contextual problems in the teaching of mathematics and science as one of the most effective strategies in making teaching and learning more meaningful (Johar, Patahuddin, & Widjaja, 2017; Mattarima & Hamdan, 2011; Peters, 2016; Walan, McEwen & Gericke, 2016). Ültay and Usta (2016) explain that context-based problems refer to content that is associated with real-life situations. They believe, as do other scholars such as Harvey and Averill (2012) and Zaranis and Synodi (2017), that to develop a positive attitude towards mathematics, it is important that the subject is taught in a context that students are familiar with in their own lives. In the same vein Malan, Ndlovu and Engelbrecht (2014, p. 13) argue that "it is possible to influence students' learning patterns in a favourable direction, if appropriate problem-based learning (PBL) context problems of interest to learners are selected and integrated into the curriculum".

Valdmann, Rannikmae and Holbrook (2016) regard a context-based teaching approach as a strategy to motivate students in the learning environment; they emphasise that learning becomes possible if teaching is initiated from perspectives that are relevant to students. This implies that if students' experiences are considered, teaching in the classroom becomes real. Johar et al. (2017) advise that the problem posed must be clear and students must also be given enough time to explore different strategies to solve such problems. Jing, Tarmizi, Bakar and Aralas (2017) bring to our attention that diversity in cultural and economic background may affect students' motivational goals. Whilst most of the studies mentioned in this review were conducted with teachers, there is evidence that students in elementary grades experience some challenges in making sense of probability (DBE, 2013). More principles, according to Van den

Heuvel-Panhuizen and Drijvers (2014, pp. 522-523) that underpin the theory of RME are discussed as follows:

- *Level principle*

Students pass through different levels of understanding. These include informal context-based learning situations, creating short-cuts and schematisation, and acquiring the skill to identify interrelated concepts. How students learn, depends on how teachers approach lessons. The approach could include how teachers draw from students' prior knowledge to connect formal and informal learning and how they develop strategies for students to acquire appropriate skills to make sense of mathematical concepts.

- *Intertwinement principle*

This principle promotes integrated learning. This is twofold in the context of this study. It is considered from the perspective that mathematical concepts are interrelated, and how integration with mathematics and other contexts could be practiced.

- *Interactivity principle*

Mathematics is regarded as a social activity. Learning should start from meaningful real-life problems that will later develop into constructs. The interactivity principle is also related to the activity principle in the sense that students may be active by participating in group activities and learning from their peers. In relation to the strands of mathematical proficiency, students can instil a love for mathematics in their peers through the support of their teachers. The RME principles are embedded in the essential aspects outlined in Table 3.4.

On the other hand, Cheng (2016, p. 283) suggests that errors made by students and the misconceptions they have regarding mathematical concepts could be clarified by using the variation theory to "draw upon their personal experiences and discern learning from different perspectives". For example, if teachers want to introduce the term 'impossible' in relation to chance; they could start by discussing events that are possible and later those that are impossible. Table 3.6 sheds some light on how each of the provided statements could be clarified:

Table 3.6

Proposed questions that could help learning probability

Statement	How do you understand each statement? Explain in your own words.	Categorise each statement as certain or impossible and justify your answer.	How are the two statements similar?	How do the two statements differ?
It is impossible for a two-month old baby to walk				
A two-month old baby can				

The questions in Table 3.6, if used effectively, would assist teachers to be more precise in their use of language, and as such students' misunderstandings are more likely to be clarified. In addition, teachers need to involve students in a series of follow-up activities, using multiple strategies to enable students to develop more insight into the event. Strategies should enable students to realise how the two statements differ if categorised using probability terms. The following section presents practices in teacher professional development.

3.6 Practices in teacher professional development

This section reflects on practices in performing countries. Unsatisfactory performance by mathematics students in general calls for all stakeholders to become involved directly or indirectly in improving students' performance in probability. Professional development in the context of this study is generally much more than simply the training of teachers. This study views professional development from the perspective that it should improve teachers' instructional practices and convey vision for change to teachers. In other words, if professional development processes are well-planned and implemented, they could develop teachers' instructional skills (Desimone, 2009).

Valdmann et al. (2016) believe that provision should be made for effective professional development programmes to support teachers to raise their confidence and competence and to cater for their needs. Nel and Luneta (2017) recommend mentoring as an effective intervention strategy to empower teachers, particularly when it is informed by teachers' instructional and content needs. This view highlights the notion that schools could appoint mentors to support

teachers to rethink the teaching and learning taking place in their classes in order to devise appropriate interventions that are customised to the needs of their students. In order to shed light on what effective teacher professional development entails, Darling-Hammond, Hyler and Gardner (2017) describe it as professional learning that impacts positively on students' learning.

Lesson study is one of the models that serves as an accelerant for teacher professional development. Ni Shuilleabhain and Seery (2017) define lesson study as a school-based model of professional development, where teachers come together to plan classroom activities, as well as present and reflect on research lessons in several cycles. Leavy and Hourigan (2016) argue that the purpose of lesson study is to provide an avenue and focus for the discussion of effective practices that bring about improvements in learning outcomes for students.

Early, Maxwell, Ponder, and Pan (2017) describe professional development as a face-face-model in which a group of teachers meet with trained instructors to learn to identify, analyse effective interactions in classrooms and discuss ways to interact intentionally to increase children's learning. In addition, Early et al. (2017, p. 59) describe "my Teaching partner as a one-to-one remote coaching model that provides specific feedback to teachers about emotional climate, organisational structure, and instructional support using standardised coaching". They further explain that the participating teacher, in this case, makes a video, sends it to the coach who then reviews it and posts feedback about interactions with students. On the other hand, Belbase (2020) points out that the psychological beliefs on the use of technologies have strong pedagogical implications for meaningful learning.

Gaible and Burns (2005) point out that professional development includes ongoing workshops, reflections, observations and assessments which aim at improving teachers' practices as professionals. They explain that "professional development takes many forms such as when teachers plan activities together, when a master teacher observes a young teacher and provides feedback, when a team of teachers observes a video lesson, reflects and discusses on the lesson" (p. 16).

According to Magudu (2014), teacher professional development can also be described in terms of a continuum from pre-service training that continues throughout the career of a teacher. She further argues that induction, as part of the continuum of teacher professional development, aims to ensure that teachers receive ample assistance to grow professionally. Because the focus

of Magudu's (2014) study was on the newly qualified teachers, she emphasises that it is important to strengthen the role of the orientation programme (mentoring strategies) to be comprehensive. Further than that, she stresses that structures put in place should be relevant to the context of the study. In her study, she highlighted that mentees needed assistance on how to plan lessons, and that not-qualified-teachers (NQT) lacked some teaching skills, including the ability to interpret the syllabus. O'Hara, Brookmeyer, Pritchard, and Martin (2020) suggest that during professional development, mentors should share and reflect on their practices of supporting teachers.

There is clearly a need to foster teachers' growing interest in improving their practice, particularly in making the teaching of probability real. In support of this argument, Arsaythamby and Zubainur (2014) recommend that teachers' ability to invent context-based activities should be improved. Such activities have been found to be effective and they should be explored further to enhance the teaching of probability. There is also research evidence from a study by Mullis et al. (2020) to indicate that teachers need support for further professional development, as illustrated in Figure 3.13.

Teachers' Needs for Future Professional Development in Mathematics

Students' Results based on Teachers' Reports

Country	Percent of Students by Teacher Indicating a Need for Future Professional Development*						
	Mathematics Content	Mathematics Pedagogy/ Instruction	Mathematics Curriculum	Integrating Technology into Mathematics Instruction	Improving Students' Critical Thinking or Problem Solving Skills	Mathematics Assessment	Addressing Individual Students' Needs
South Africa (9)	77 (2,4)	81 (2,3)	71 (2,6)	88 (2,0)	89 (1,7)	77 (2,0)	86 (2,0)
International Average	**47 (0,6)**	**60 (0,6)**	**49 (0,6)**	**71 (0,6)**	**69 (0,6)**	**57 (0,6)**	**65 (0,6)**
South Africa (5)	70 (3,7)	78 (3,3)	68 (4,1)	83 (3,3)	84 (3,0)	73 (3,8)	83 (3,8)
International Average	**45 (0,5)**	**55 (0,5)**	**44 (0,5)**	**72 (0,5)**	**69 (0,5)**	**54 (0,5)**	**64 (0,5)**

Figure 3.13. Teachers' needs for future professional development in mathematics
Source: Mullis et al., 2020, pp. 400 & 402

It is clear from Figure 3.13 that most teachers need assistance on integrating technology into mathematics instructions and on improving students' critical thinking or problem-solving skills. This is also reflected in the SACMEQ results, where teachers obtained 35,4% and students 2,6% in problem solving activities. Mullis et al.'s (2020) report also indicates that addressing

individual students' needs was the third most commonly reported area of need for teachers. Furthermore, the same report indicates that only 52% of students had experienced high instructional clarity in mathematics lessons. This underlines the importance of teachers ensuring that their instructions are clear, and the practice is guided (Cooper & Scott, 2017).

In efforts to strengthen teacher professional development so that instruction in mathematics and science improves, several studies have been conducted in several countries to analyse student achievement in mathematics (see Section 3.2). Most countries that performed well were in Asia and they used the lesson study model for teacher professional development. In her study, Adler (2017) concluded that students taught by teachers who underwent teacher professional development programmes outperformed students who were taught by teachers who had not taken these courses. Kaur, Tay, Toh, Leong and Lee (2021) found that in Singapore, teachers' content knowledge was upgraded through a master's degree programme (MSC Mathematics for Education), and by holding annual meetings, seminars and conferences for teachers. Teachers were also encouraged to participate in research projects.

Fauziyah and Uchtiawati (2017) describe lesson study as a training model to enhance teachers' practices through collaborative learning. This model is the brainchild of a scholar in Japan. Lesson study "is a translation of the Japanese term '*jugyou kenkyuu '(*授業研究). *Jugyou* means live instruction (a single lesson or many lessons); *kenkyuu* means research or study (C. Lewis, 2016, p. 571). Joubert, Callaghan and Engelbrecht (2020) define the lesson study as a collaborative process for teachers to reflect on their instructional practices and the manner in which learners learn. For Arani (2017, p. 23), lesson study "helps examine pedagogical theories of knowledge about teaching which are constructed through lesson study, especially main aspects of it, kyouzai Kenkyuu, the study of teaching material". These definitions promote adaptation of the lesson study to serve as an accelerant for the establishment of learning communities.

One of the highlights of lesson study, according to Hadfield and Jopling (2016), is that the model yields positive results when integrated into improvement strategies that result in modifications to the instructional process. Figure 3.14 illustrates the several stages in the lesson study process.

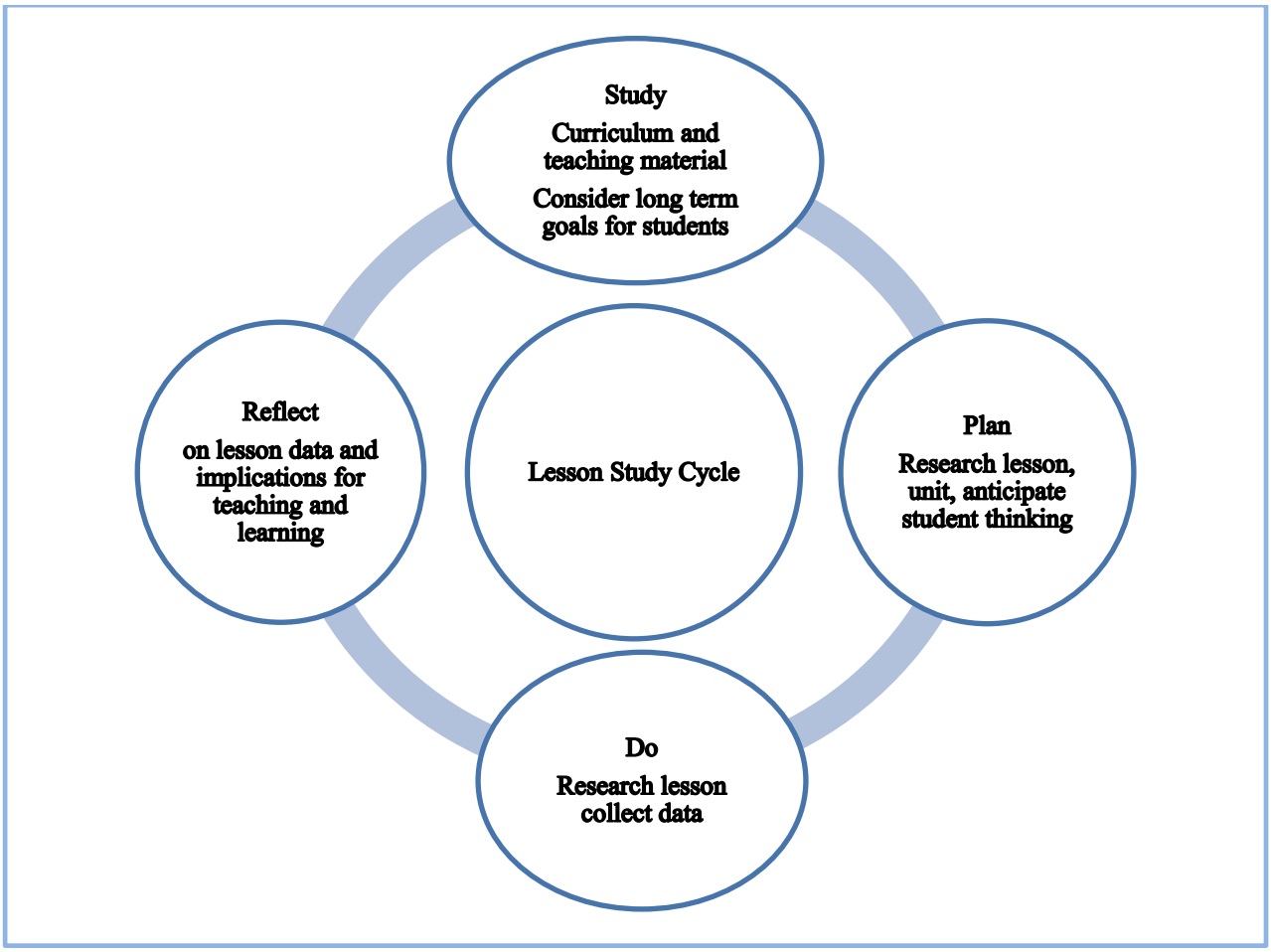


Figure 3.14. Lesson study and pathways of impact: A theoretical model

Source: C. Lewis, 2016, p. 572

Borrowing from the summary by C. Lewis (2016), the lesson study process can be condensed as follows:

- **Study**: Study the needs of the teachers and formulate the research theme.
- **Plan**: Teachers sit together and collaboratively plan the research lesson.
- **Do**: One member from each group presents the lesson.
- **Reflect**: Teachers comment on good practices and discuss areas that need attention.

This Japanese model has aroused increasing attention and its effectiveness has been revealed in many studies (Brown, Taylor & Ponambalum, 2016; Darling-Hammond et al., 2017). The lesson study model has had an impact on improvement strategies that have resulted in modification to the process. To confirm this view, C. Lewis (2016, p. 577) highlights pathways of lesson study impact as follows:

Bruce et al. 2016 Canada	Disposition	Increased ability to look for student thinking, and expanded estimations of what students can do
Fujii 2016 Japan	Knowledge	Expanded knowledge of mathematics student thinking, curriculum and instruction as teachers: discussed the task, anticipated student solutions, planned class discussion, read the standards, and learned from the outside final commentary on the research lesson
Groves et al. 2016 Australia	Knowledge and Dispositions	Saw the value of students sharing and discussing their work at more length. Learned to formulate goals related to students' mathematical *understanding*, not simply observable outcomes. Saw the value of the professional "culture of giving and receiving feedback"
Gu and Gu 2016 China	Knowledge	Feedback from mathematics teaching research specialists to practicing teachers during post-lesson conferences focused on practical knowledge for teaching, particularly behaviour improvement, rather than mathematical or pedagogical knowledge
Huang et al. 2016 China	Instructional materials	Over two trials of a lesson plan, teachers revised the lesson tasks to reflect what was learned from the first teaching
Pang 2016 Korea	Knowledge	Teachers learned to monitor and purposefully select student solution methods to support class discussion, and expanded their lesson goals to emphasise mathematical understandings beyond simply solving the problem correctly
J. Lewis 2016 US	Knowledge, Professional Community	Two educators new to lesson study learn to facilitate it through observation of experienced groups and reflection on their own facilitation process; facilitators learn to negotiate tensions between building professional community and instructional knowledge
Takahashi and McDougal 2016 US	Knowledge, Dispositions, Professional Community	Development of school research theme and school-based structures to support lesson study cycles at multiple grade bands supports increased collaboration, knowledge about mathematics teaching-learning and a disposition to ask "why" in instructional planning and decisions
Warwick et al. 2016 UK	Knowledge, Professional Community	From post-lesson discussions, episodes of teacher learning (i.e., stated learning about mathematics and its teaching-learning) were analysed for discourse features, which included building on each other's ideas, providing evidence, etc. Teachers' learning related to student thinking, lesson structure, etc.

Figure 3.15. The impact of lesson study in different countries

Source: C. Lewis, 2016, p. 577

C. Lewis (2016) regards these reviews as evidence that the lesson study model and its success in improving teaching methods received a great deal of attention globally. Figure 3.15 illustrates how different countries benefited from the lesson study. Reviews of the model indicate that it is possible to focus on any activity as long it contributes to solving the identified problem. C. Lewis (2016) acknowledges that lesson study will remain a challenge when applied in some countries, because of its complex ecology.

I have had experience in using the lesson study method in South Africa, particularly across four districts in Mpumalanga. I spent six weeks in Japan receiving training in the model. Although it is a very useful model to use to develop teachers' planning and implementation of lessons, the process takes some time. All stakeholders in the district where the model is being applied need to be thoroughly informed about the lesson study model. Secondly, teachers are obliged to leave the school early to attend workshops, which compromises their teaching time. I thus strongly endorse C. Lewis' claim that the lesson study method is a complex process that needs proper planning and implementation.

These views correspond to those of Zhang, Shi and Lin (2020) who argue that teacher professional development is a significant barrier that restrains teachers from spending sufficient time in schools. At the same time, all the teachers need to present the research lesson in a real classroom setting. Duez (2018) argues that the lesson study method will continue to be applied globally and recommends that implementers need to understand how much adaptation is needed so that this does not detract from the intentions of the method.

Enderson, Grant and Liu (2018) investigated the connections between coaches' understanding of probability concepts and their interpretation of students' work. They found that coaches' probability solutions to the problems asked lacked accuracy. The study also revealed that a limited understanding of content led to a misinterpretation of students' work. It was apparent that teachers needed to have ongoing and regular opportunities to learn from each other or from subject specialists.

Enderson et al. (2018) suggest possible modifications and recommend that future studies should engage coaches to reflect on their work and discuss this with colleagues before asking students to work on the same problems. They also recommend that coaches listen to students'

explanations in order to offer relevant support to teachers. In support of this viewpoint, Russell et al. (2020, p. 447) view the four key components of pre-lesson planning conversations as:

> (a) the appropriateness of mathematical content in the task for the grade level, (b) the depth at which multiple solution paths for the task were discussed, (c) whether specific questions to advance the conceptual goals of the lesson were identified and discussed in depth, and (d) the depth of discussions about the specific math content and goals of the task.

I therefore anticipated that if these key components of the pre-lesson could be conceptualised, they would help coaches to understand their own strengths and limitations when working with content as well as giving them insight into student work. Based on an understanding of what lesson study entails, researchers could adapt the model to address the gaps found in the literature and the challenges identified in various studies. The following section highlights the research gap from which the aim of this study or the problem statement emanated. Items identified in Table 3.7 serve as key motivating issues that inspired me to pursue the study.

3.7 Gaps identified in the literature

Reservations about how teachers make sense of probability, how they guide and support students to construct the meaning of probability, and how they are developed professionally have been varied. Although the scholars mentioned in this chapter have made efforts to devise strategies to solve some of the identified problems, there are still some gaps to fill if we are to enable teachers to create a learning-centred classroom that will make learning meaningful. An analysis revealed by the literature review is presented in Table 3.7

Table 3.7

Analysis of gaps found in literature

Identified gap	Reasons for the gap	Activity plan: propositions
Instructional practice Teaching and learning activities are not experientially real. Activities are drawn from real-life situations but are not informed by context. Students' ability to assign numerical values is more developed than their use of probability. Ways in which teachers incorporate such tasks from the resource material into their lessons. Integrating technology in mathematics instructions. How to intertwine the language of learning and probability. Poor performance in problem solving activities. Address students' individual needs.	Use activities from scripted curriculum pacing guides. Sequencing of lessons is prescriptive. Not clear how the tasks are aligned to the outcomes of the lesson. Not clear what informs the development of the tasks. Most of the activities in the textbooks are concerned with theoretical probability. Cultural differences Understanding the essence of probability through language. Questioning strategies are one directional. **Sources** Sadia and Ghani (2019) Arsaythamby and Zubainur (2014) Ersanli (2017) Groth et al. (2016) Nene (2017) Curto (2019) Molloy et al. (2020) Zhang et al. (2020) Litke (2020) Kvantisky and Even (2002) Yeo (2021).	Think of activities that students have experienced (at their cognitive level) and build awareness of the role mathematics plays. Think of activities that at the cognitive level of students to make the teaching of probability real. Realise that learning opportunities depend on the types of representations and the teacher's questions when orchestrating the classroom discourse. Use feedback to make students probabilistic reasoners. Train teachers how to incorporate computers into the teaching of mathematics.
Teacher professional development Spending too much time attending professional development workshops outside schools. Lesson study process recommended.	Too many activities involved in developing a research lesson. Context not considered. **Sources** Takahashi (2016)	Think of activities that will have the desired impact on instruction and teacher growth and teachers' tools at identified stages of the lesson plan. Adaptation is needed but should not detract from the intentions of the lesson study; the lesson study processes in this study were based on the prevailing situation so its impact would be situational.

	Ni Shuilleabhain and Seery (2017) Fujii (2016) C. Lewis (2016) Nel and Luneta (2017) O'Hara et al. (2020) Magudu (2014)	Regarded as a year's in-service training project for all the teachers. Recommended that teachers' ability to invent context-based activities should be improved.
Contextual problem	No research conducted in Mpumalanga on probability.	Explore how teachers' conceptual understanding of probability informs their practices.
Design and use of resources Inadequate These resources are incorporated in teachers' lessons to promote conceptual understanding of probability. Many studies reveal that commonly used resources are dice, spinners, coins and cards.	Just use them as resources with which to play games but the reason for choice of these resources is not clear, hence their appropriateness is questionable. **Sources** Darling-Hammond et al. (2017)	Think of effective ways of incorporating the resources. Think of designing and using different resources.
Definition of probability The likelihood of occurrences of events. Numerical value between 0 and 1 to indicate the likelihood of an event occurring. A measure of the chance of the occurrence of events. Probability is a ratio that compares the desired outcomes to the total possible outcome.	Definitions are not derived. **Sources** Grimmett and Stirzaker (2020) Naidu and Sanford, (2017) Bhat (2007) Van de Walle et al. (2016)	Explore the perspectives from which a definition could be derived. How teachers make sense of the definitions.

The choice of context for this study was the result of the following considerations:

- Such an investigation had never been conducted in Mpumalanga;
- The interest I had in the topic because of its applicability to real-life situations;
- The intention to explore the highlighted difficulties and the recommendations made in other studies.

One of the challenges identified in national and international assessments is that teachers need assistance in problem-solving strategies (Mullis et al., 2020; Reddy et al., 2016). Based on this problem, my view was that problem-solving is not an entity, but is laid on a specific foundation. My interest was also on the fundamentals of understanding problem-solving as a strategy for teaching to maximise learning. In other words, I was interested in exploring the initial skills that teachers need to have to understand problem solving.

Yayuk and Husamah (2020) define problem solving as a tool to enrich knowledge and suggest that this enrichment can be achieved by active and innovative learning. This echoes Yayuk and Husamah's (2020) argument that students should be treated as active participants in the learning process. Ulandari, Amry, and Saragih (2019) similarly point out that learning material based on a realistic mathematics education approach is effective in enhancing problem solving skills. On the other hand, Barham (2020, p. 139) found that "pre-service elementary teachers had difficulty applying the essential skills required for success in solving mathematical problems, used limited and incorrect mathematical terminology". Further difficulties were found in a study by Di Leo, Muis, Singh, and Psaradellis (2019), where students did not have the skills to deal with their confusion and frustration that arose during complex problem solving.

The issues emerging from these definitions relate to active participation, innovative learning, integration of cultural practices, the use of realistic teaching and learning activities, integration of problem-solving activities in normal lessons, the importance of addressing confusion and frustration that might arise during problem solving. Gaps identified in the literature (teachers' instructional practices, professional development, design and use of resources and the definition of probability) set the tone for the design of the study and gave the study its unique identity and purpose. Challenges identified, and propositions listed in Table 3.7 seemed to suggest that the way the concept of probability was introduced to students required a serious rethink; if not, it would remain obscure and abstract for students.

Based on the arguments provided in this section, the choice was made to present the methodology chapter as presented in Figure 3.16 in the hope that this would yield more useful results. The figure provides an overview of methodology processes and guidelines used to understand the phenomenon within the limits of critical bounding assumptions. These processes were informed by scholarly reviews, the problem statement and the framework that guided the study.

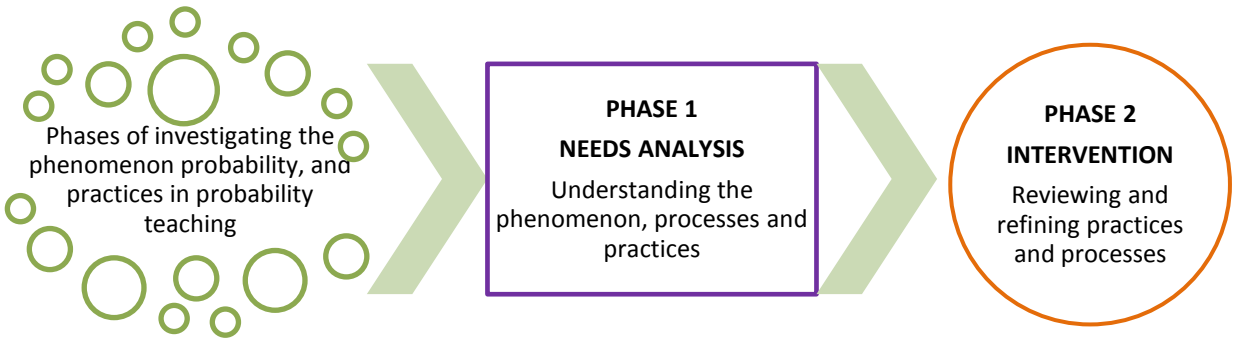


Figure 3.16. Overview of the methodology process

The discussion in this chapter reinforced salient points taken from the literature and extended them to guide the processes of the plan to enhance teachers' instructional classroom practices.

3.8 Conclusion

In this chapter, fundamental issues of the teaching of probability were explored. The chapter provided an overview of studies on student performance, curriculum policies and educational context, research perspectives on mathematics teaching and the teaching of probability. The theoretical and conceptual framework that underpinned the study and the models of teacher professional development were discussed. It is through this discussion and analysis of literature that I understood teachers' practices in teaching the concept of probability.

This chapter also presented the gaps identified in the literature. These gaps served as a guide when refining the focus of the current study. The chapter also provided an overview of challenges experienced in the teaching and learning of probability and discussed some local and international assessment reports on students' performance in mathematics. The notion of conceptual understanding and teachers' instructional practices were also discussed. The next chapter provides a description of the research methodology used in this study to collect and analyse the data.

CHAPTER 4 : METHODOLOGY

4.1 Introduction

In this chapter, a detailed description of the research methods used in exploring teachers' practices in teaching probability is provided. Strauss and Corbin (1998) describe methodology as a specific procedure or techniques used to study and understand social reality. The purpose of this study was to explore how Grade 7 mathematics teachers' conceptual understanding of probability guided their classroom instructional practices. The main question that drove the processes in this study is as follows: How do teachers' conceptual understanding of probability frame the plan to enhance their instructional classroom practices? This purpose had an influence on the type of research design I chose to follow. This chapter includes discussions of the data analysis procedures, ethical considerations and quality assurance criteria. The data collection and analysis plan followed in the study is also explained. A diagrammatical representation of the research procedures followed in collecting data and analysing findings (the methodological choice) is provided in Figure 4.1.

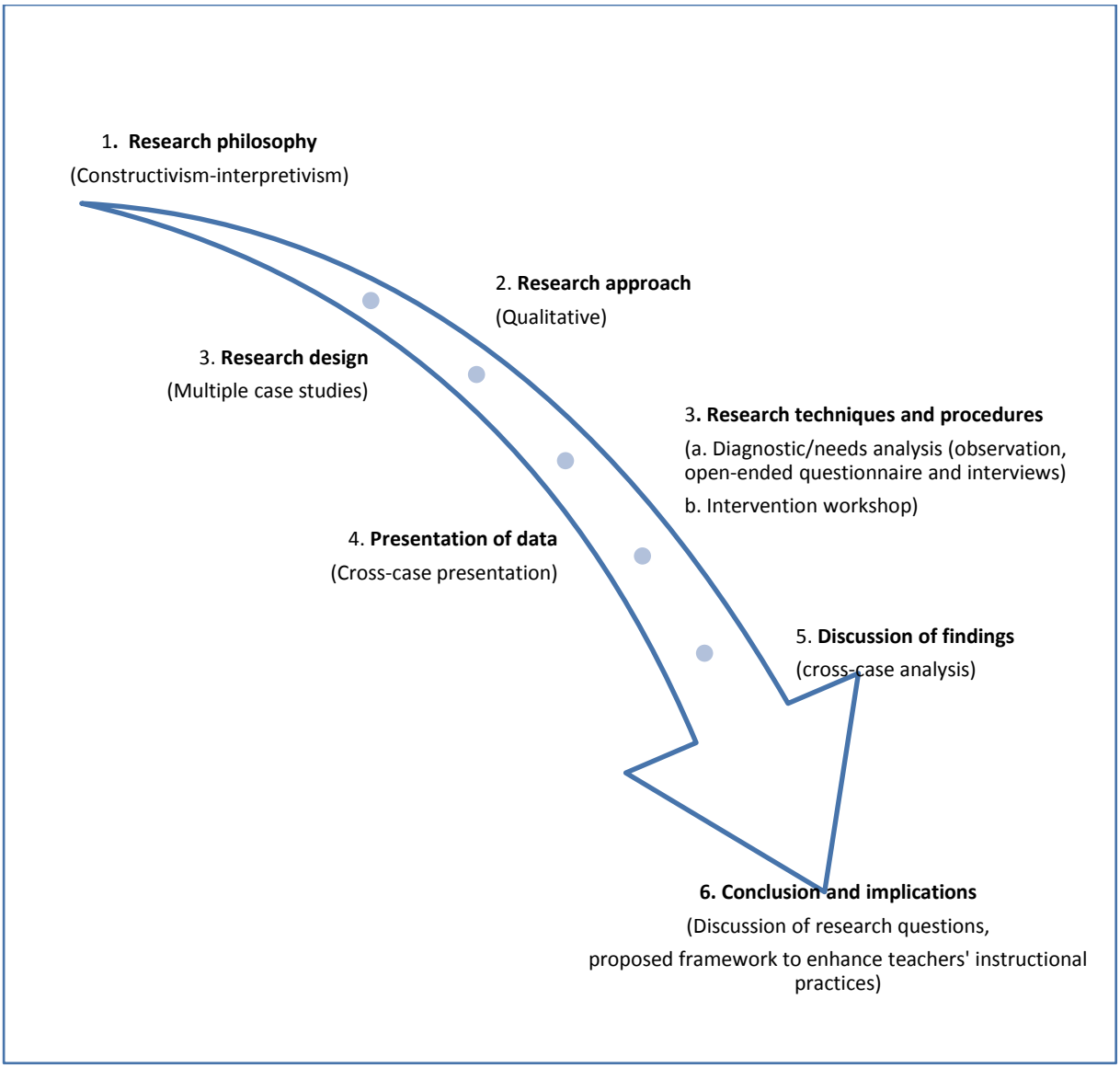


Figure 4.1. Schematic representation of methodology

The first step in the process depicted above served as a foundation of the methodological choice because it outlined beliefs or assumptions that gave rise to my particular world view, which is a fundamental aspect of one's reality. This was important because I regard methodology as a belief that dictates how one should conduct research and how results should be interpreted. An explanation of this understanding is that the philosophical assumptions that a researcher brings to a study affect how the research is approached, as well as the methods and procedures used to collect data on the phenomenon in question.

4.2 Research strategy

4.2.1 *Research paradigm*

This study followed a qualitative paradigm or "a set of assumptions or beliefs about fundamental aspects of reality which gives rise to a particular view" (Maree, 2007, p. 47). Bryman, 2012 (as cited in Du Plooy-Cilliers & Cronje, 2014, p. 19) indicates that "a paradigm describes a cluster of beliefs and dictates which for scientists in a particular discipline influence what should be studied, how research should be done and how results should be interpreted". A broader perspective is adopted by Denzin and Lincoln (2005) who view a paradigm as a notion that represents what one thinks about the world in which one finds oneself. Davies and Fisher (2018) embrace a similar standpoint when describing a paradigm but add that research paradigms are influential in determining research questions and the way data is collected.

Although I acknowledge that it is sometimes difficult to differentiate philosophical assumptions, paradigms and theoretical orientations (Creswell & Poth, 2018), I align myself with the understanding that a research paradigm is a model or pattern of assumptions and beliefs with which to interpret reality (Maree, 2007). I describe the philosophical foundations of this research as follows: Ontology is "the study of the nature and form of reality, epistemology is the method of knowing the nature of reality and methodology is the technique used by the researcher to discover the reality" (Maree, 2007, pp. 53-55). I believe that knowledge construction in mathematics should be built upon the views of teachers' experiences. It is noted that although teachers follow the same curriculum, their teaching experiences cannot be the same. Hence it was necessary, in this study, to acknowledge the uniqueness of each individual case.

Whilst ontology refers to the nature of reality, epistemology refers to how knowledge is acquired (Maree 2007). In this study, I hold an interpretive position that believes in analysing the situation as it reveals itself and how the participants make sense of their experiences. I also understand that students' learning is driven by how each participant interprets his/her own situation. Further, this understanding is associated with the viewpoint of Scotland (2012), who emphasises that:

> Every paradigm is based upon its own ontological and epistemological assumptions. Since all assumptions are conjecture, the philosophical underpinnings of each paradigm

> can never be empirically proven or disproven. Different paradigms inherently contain differing ontological and epistemological views; therefore, they have differing assumptions of reality and knowledge which underpin their particular research approach. This is reflected in their methodology and methods. (p. 12)

I regard each situation as unique because everyone has his/her own beliefs and experiences. The way I view educational knowledge is inevitably linked to the methodology chosen in this study. I relied on the views of the participants and conducted the inquiry in a subjective manner (Creswell, 2008). I also believe that there is a correlation between the ontology, epistemology and methodology used in this study (Sikes, 2004). How one understands the correlation, depends on how the situation is interpreted. In the context of this study, I used the situation to refer to what is happening in a particular place at a particular time. As such, the chances of the situation repeating itself are unlikely.

Krauss (2005, p. 767) points out that "the unique features of qualitative data analysis (for example, the strategic use of data to elicit important themes building toward the development of theory) contribute greatly to the construction of meaning". In their study, Kivunja and Kuyini (2017) conclude that the methodological implications of paradigm choice permeate the research question/s, participants' selection, data collection instruments and collection procedures, as well as data analysis. The choice of methodology drives and informs the research processes.

The basic assumption or the research philosophy that underpinned this study was social constructivism. Van Merriënboer and de Bruin (2014) argue that the most important aspect that influences learning is how meaning and knowledge are constructed through interaction with others. Creswell (2008, p. 50) agrees that "the central perspective of constructivism stresses the importance of participants' view and the environment in which the view is expressed". A qualitative research paradigm "focuses on the social construction of people's ideas and concepts" (Creswell, 2008, p. 54). This study investigated the various ways individual teachers dealt with probability, how they endeavoured to help students understand this concept and appreciate its importance in real-life situations.

Educational knowledge, in my view, is linked to the methodology chosen in this study. As indicated in this section, I used social constructivism as my epistemological stance. For example, having spent time with the participants as an insider and relied on information given

to me by these participants, I realised that knowledge is socially constructed, not discovered. Dialogue with the participants played a major role in gaining a sense of reality. Although I acknowledge that research is value-laden and that biases are present in the study context (Smith & Noble, 2014), I presented participants' views without any omission and I have put aside my bias by focusing on what had value in the research process.The methods of collecting and analysing the data and interpreting the findings to construct meaning were specific to this study.

- *Paradigmatic assumptions*

Regarding my epistemological assumption, this study holds an interpretive position that offers a perspective of analysing the situation. Teaching and learning took place in a classroom context. I observed, interpreted and analysed how participants facilitated the lessons in relation to the learning outcomes formulated. In an interpretivist perspective, meaning is influenced by knowledge of the social world (Maree, 2007). Drawing from what Maree (2007) holds, the reality is that circumstances or external factors influence how a person makes sense of the world. The nature of this study was subjective. I was involved in constructing meaning from situations in the classroom and during the intervention workshop. My teaching philosophy was to enhance proficiency in learning probability and this was grounded in the following aspects:

Teaching approach: This was guided by the Mathematics Curriculum and Assessment Policy Statement (CAPS), teaching strategies, the self-directed learning approach and the student active participation approach.

Use of technology: I used audio recordings, communicated through emails and cellphone, during the research study. I searched information on websites and participated collaboratively on Microsoft Teams. The cellphone was particularly useful when reminding participants about their tasks. I also compensated participants with data to access the internet and be able to participate on Microsoft TEAMS. The importance of using technology is supported by Suh, Gallagher, Capen, and Birkhead (2021) who encourage teachers to adapt video-based Lesson Study to enact their instructional practices and back peers as coaches.

Teaching and learning material: Teachers developed their own teaching and learning material and some activities were adapted to suit the context of their classrooms. This was done during the intervention workshop.

The use of theories: My teaching philosophy was also grounded in active and critical learning. In other words, I encouraged an active and critical approach to learning, rather than rote and uncritical learning of given truths. I also placed value on indigenous knowledge systems (Ahuriri-Driscoll, Lee, & Came, 2021). This included placing a focus on connectivity, motivational learning and social constructivism.

Evaluation of students: The teaching and learning activities were designed following Bloom's taxonomy, cognitive learning categories, constructive feedback and teacher-self-reflection. These critical aspects provided guidance on how teachers identify and understand students' needs to help them realise their potential.

4.2.2 ***Research approach and design***

4.2.2.1 Research approach

I used a qualitative approach in this study. Creswell (2008) describes qualitative research as an inquiry in which the researcher collects data in a subjective manner and analyses findings from themes. Rossman and Rallis (2011) note that an investigation in qualitative research is conducted in a natural setting. Babbie and Mouton (2001) describe it as an approach that provides a detailed report on the understanding of a social problem and acknowledges the fact that the behaviour of a human being cannot be predicted. All these definitions point to the importance of listening to participants' views and providing a detailed report of activities that occur in a natural setting.

In this study, I explored how the teachers created and utilised opportunities in the classroom to develop the concept of probability. I was interested in exploring how teachers' conceptual understanding of probability informed their instructional practice. This necessitated recording their personal feelings, opinions about the teaching of probability, and observing how they used their knowledge of this topic in their classrooms. The overall goal of this qualitative empirical study was to design a framework for teacher empowerment to improve their classroom instruction practices when teaching probability.

4.2.2.2 Research design

McMillan and Schumacher (2010) understand research design from the perspective that it involves investigating a phenomenon under certain conditions and specified period. I chose to use an interpretive case study design to collect and analyse data in this study. The reason for

this choice was to enable me to study the context of each case and to interpret the approaches used by mathematics teachers to teach probability. I was also interested in identifying important aspects that enabled participants to teach the concept in a meaningful way.

Furthermore, a case study permitted me to investigate the rationale for the use of strategies during the presentation of lessons by engaging teachers in individual semi-structured interviews. I was also very interested to learn in these interviews why participants chose to use the identified strategies. In short, I wanted to experience the uniqueness of each case so that I could understand how each participant facilitated lessons based on the nature of the topic under investigation. I provided participants with the opportunity to give their own views and kept a note of all issues that were casually raised.

4.3 Study population

4.3.1 *Population, sampling and sampling procedures*

Creswell (2008) describes a population as a well-defined group of individuals known to have similar characteristics. The population in this study was Grade 7 mathematics teachers in Nkangala district, which comprises 20 circuits. The sample of participants for the first phase of the study from this population included eight teachers from four different senior phase schools (Grades 7—9) in two different circuits. All four schools selected for this study were in rural settings. The characteristics of these teachers are described in the next paragraph.

I employed a purposeful sampling technique to select participants for this study. This is because participants were chosen for the purpose of obtaining relevant information about Grade 7 mathematics teachers' understanding of probability and the teaching of probability. This was a homogenous group of selected teachers who have similar work experience. I decided to target Grade 7 teachers because few studies have been conducted at Grade 7 level (see sections 1.1, 1.2 and 3.2). Most national and international studies have focused on Grades $4, 6, 8$ and the Further Education and Training (FET) phase (Mullis et al., 2020). In the South African context, Grade 7 is in primary schools but is the entry level to the senior phase (Grades7-9). In other school settings Grade 7 forms part of middle school. Thus Grade 7 serves as a link between the Intermediate phase (Grades 4-6) and secondary school. In addition to this, I wanted to explore how teachers transit to close the gap between intermediate and senior phases. My experience

as subject advisor was that some of the teaching strategies used in primary school were not suitable for senior phase students.

In Grade 7, formal and theoretical learning of mathematics is emphasised. The formal learning includes the use of abstract formulae in solving mathematics problems. If teachers are to develop the concept gradually, they need to use strategies that are appropriate to the cognitive level of students. As Grade 7 is the entry level to the senior phase, teachers are expected to be very cautious on how to transit from primary to secondary schools; transition in the context of specific strategies and methods used to teach probability. One of the difficulties I observed when I was the mathematics subject advisor was that when teachers were involved in both the GET and (FET) bands, generally more effort was channelled into FET. This was a reality in schools.

The criteria for the selection of participants were teaching experience, location of school, grades taught and qualifications. Age and gender were not considered. Teachers' experience was a criterion because it plays a major role in enriching teachers' professionalism, content knowledge and pedagogical expertise. The selection criteria were similar to those used by Etikan and Bala (2017), who explain that purposive sampling involves selection of participants who will provide the best information to attain the aim of the study. In other words, I selected participants who had the specific characteristics that were being studied.

My choice of these schools in Mpumalanga was motivated by the fact that they were close to each other, they followed the same curriculum and it was thus convenient for me to conduct the workshops with all the participating teachers at a central venue. Most importantly, it was convenient to use schools that were close to where I lived. However, as a result of the outbreak of COVID-19, there was a need to adapt the methodology and conduct virtual sessions during the intervention workshop (Robinson & Rusznyak, 2020). I was granted permission by the university through the ethics committee to modify the methodology. Adaption in this case included the involvement of the mathematics subject advisors and the mathematics coordinator. These participants were included because they met the selection criteria and had access to the technology resources to go virtual. During the first phase of the study, eight teachers participated to facilitate the development of meaningful themes and interpretations that formed the basis of the intervention workshop. Through snowball sampling, I managed to

involve one more Grade 7 teacher, two senior phase (Grades 7-9) mathematics subject advisors and one mathematics coordinator.

The teachers in the sample were all teaching mathematics at Grade 7 level. When selecting the participating teachers, I considered teaching experience and qualifications. An experienced teacher, in this context, was one who had taught Grade 7 mathematics for more than five years. In this case, participants' teaching experience ranged from 10 to 25 years. The other criterion was that these teachers should have achieved an average pass rate of more than 60% in the end-year examinations over the last three years (2017–2019). The participants were all working in a rural setting. They had all completed a three-year teacher education training programme in the same Department of Education in South Africa. Participants had relatively similar experiences. They would also have undergone similar in-service training. Table 4.1 indicates demographic information of the teachers who participated in this study.

Table 4.1

Participants' demographics

Participant	Gender and age category	Qualifications	Teaching experience	Grades taught, and support provided
PRIC	M 50-55	B.Ed. (Hons) - Maths Edu HED – Major: Mathematics UDES – Majors: Mathematics & Physical Science	**Senior Education Specialist** **Teacher-** 1992 – 2005 **HOD** 2005 – 2014 **Deputy Principal** 2014 – 2016 Middle school: 2016 to date	Support Grades 7-9 mathematics teachers at district level
PRIA	F 50-55	HED - Major Mathematics UDES – Majors: Mathematics & Physical Science B.Ed. (Hons) Curriculum Management	**Senior Education Specialist** 2010 to date	Support Grades 7-9 mathematics teachers at district level

PRIB	M 50-55	HED – Major: Mathematics UDES – Majors: Mathematics & Physical Science B.Ed. (Hons) Mathematics Education	**Mathematics Coordinator** 2012 to date	Support Grade 7-9 mathematics teachers at provincial level
PRD_O	M 45-50	HED – Major: Mathematics UDES – Major: Mathematics & Physical Science B.Ed. (Hons) Mathematics Education	**Senior education specialist** **2013 to date**	**Experience**: Grades 7, 11 and 12 Currently teaching Grades 7 and 11
PR_1	F 50-55	Grade 12 Mathematics UDES - Majors: Mathematics and Physical Science ACE (Mathematics) B.Ed. (Hons. Inclusive Education)	28 years	**Experience:** Grades 7, 8 and 9 HOD Currently teaching Grades 7 and 9
PR_2	M 50-55	Grade 12 mathematics UDES (Majors: Physical Science and Mathematics) ACE (Mathematics) B.Ed. (Management)	24 years	**Experience**: Grades 7; 8 and 9 Currently teaching Grades 7 and 8
PR_3	F 45-50	Grade 12 UDEP BA (Major: Setswana) ACE (incomplete) Mathematics Education)	24 years	**Experience**: Foundation Phase (Grade 3) Grades 7, 8 and 9 Currently teaching Grades 7 and 8
PR_4	F 45-55	Grade 12 Mathematics UDE (Mathematics) ACE (Tech) ABET (Certificate) B.Ed. (Hons Management)	11 years	Experience: Grades 7 and 8. Currently teaching Grade 7.
PR_5	F 45-50	Grade 12 Mathematics ACE (Mathematics) B.Ed. (Hons Management)	10 years	Experience: Grades 7, 8 and 9. Currently teaching Grades 7 and 9.

PR_6	F 40-45	Grade 12 Mathematics Higher Diploma In ABET (Maths and Adult Teaching) Not permanently employed B.A. (Social Work) - Current study	16 years (12 years teaching mathematics)	Experience: Grade 7 Currently teaching Grade 7
PR_7	M 50-55	Grade 12 Mathematics UDEP (Mathematics) ACE (Mathematics and Science)	25 years	Grades 4-12 Currently teaching Grade 7 HOD
PR_8	M 35-40	Grade 12 Mathematics B.Ed. (Senior Phase) NS and Computers	8 years	Grades 7-9 Currently teaching Grades 7 and 9

Participants' experiences ranged from teaching Grades 7–12 and providing subject advisory services for senior phase (Grades 7–9). Table 4.1 indicates that some participants had qualifications at honours level such as in management, for instance, which was more general and not specific to the support of mathematics teachers. However, all participants had experience of teaching mathematics at Grade 7 level and had, on average, more than ten years' teaching experience.

I observed eight Grade 7 mathematics teachers' instructional practices in their natural context in order to investigate their creation and opportunities to develop the concept of probability. Instructional practice in the context of this study refers to strategies that teachers apply to facilitate teaching and to help students develop a deeper understanding of probability. I was guided by indicators of conceptual understanding to explore teachers' instructional practices in unpacking the concept of probability (see section 2.3).

These observations took place during school hours and interviews were conducted during school breaks. During the first phase of the study, each classroom lesson was an hour in duration and interviews 50 minutes. Although teachers planned their own individual lessons using textbooks and resources of their own choice, they all followed the Annual Teaching Plan (ATP) developed by the Mpumalanga Department of Education. The ATP outlines topics, concepts and skills to be addressed and the dates on which each topic should be taught. According to DBE (2011), probability is allocated 4.5 hours per year. This is approximately 2.5 % of the total time allocated to mathematics per year. This might be one reason why teachers do not put much

effort into the topic. All in all, three teachers, two subject advisors and one mathematics coordinator participated in the intervention workshop.

4.3.2 *Research instruments*

Data were collected from the open-ended questionnaire, lesson observation, interviews and a focus group discussion. The use of various tools to collect data allowed for triangulation of the data. Using many data collection techniques is more likely to capture multifaceted aspects of teaching and to increase the depth of the study (Walan et al., 2016). Litke (2020) believes that a lesson observation protocol should be developed in such a way that items are aimed at supporting teachers and serve as a lens for instructional improvement. I support this view because what is observed in the classrooms should form the basis for the intervention workshop. Another critical point is that most of the questions in the interview protocol should emanate from what transpires in the classroom in order to develop a clearer idea of the phenomenon under study.

Although I acknowledge that the data collection instruments had both strengths and weaknesses, their collective use made it possible to address the research questions. The development of the classroom observation protocol was informed by the challenges identified in the open-ended questionnaire. Follow-up questions after the lesson observations formed part of the semi-structured interview. The report drafted from the lesson observation, open-ended questionnaire and interviews provided guidance on the design of the intervention workshop.

The South African curriculum includes tasks such as a) performing simple experiments in which possible outcomes are equally likely; b) listing the possible outcomes based on the conditions of the activity; and c) determining the probability of each possible outcome using definitions of probability. Clarification notes or teaching guidelines linked to the probability concepts and skills are expressed as follows:

> Probability Experiments: In the intermediate phase students did experiments with coins, dice and spinners. In this grade experiments can be done with other objects, like different coloured buttons in a bag, choosing specific cards from a deck of cards, etc.

Example: If you toss a coin, there are two possible outcomes (head or tail). The probability of head is $\frac{1}{2}$ which is equivalent to 50% (since it is one out of two possibilities). (DBE, 2011, p. 73)

4.3.2.1 Open-ended questionnaire

An open-ended questionnaire (see Appendix C) was used to gain understanding of participants' views regarding how probability and the teaching of probability were conceptualised by teachers. Teachers' conceptions of probability and how they developed the concept to make learning meaningful to students was explored. The nature of the questions required teachers to explain how students responded to the questions and how they could assist students in developing conceptual understanding of probability. Questions or items for open-ended questionnaire were generated as follows:

- Adapted from the textbooks
- Taken from the Annual South African diagnostic report
- Adapted from Grade 7 mathematics textbooks
- Developed using the teaching guidelines from CAPS.

The open-ended questionnaire consisted of students' responses and structured questions to gain an understanding of teachers' conceptual understanding of probability. The questions were also structured in such a way that teachers would explain how they dealt with challenges faced by students' understanding of the concept of probability. The responses from the open-ended questionnaire guided the development of the lesson observation protocol.

4.3.2.2 Lesson observation protocol

The second instrument was a lesson observation protocol. This was made up of closed and open-ended questions designed to elicit information on teachers' practices when teaching probability at Grade 7 level. This observation protocol is referred to hereinafter as Appendix A. It was constructed to observe how lessons were taught. In order to address the research questions, I focused on certain important aspects. These included how teachers understood the concept of probability, how the concept was developed, and any difficulties encountered in the teaching and learning of probability. The formulation of questions was guided by teachers' responses analysed from the open-ended questionnaire (See Appendix P7) and the vocabulary

used in the theoretical and conceptual framework. All the participants presented lessons during the first phase of the study (needs analysis stage).

4.2.2.3 Interview schedule

The third instrument was a post-lesson observation interview schedule, referred to hereinafter as Appendix B. Some of the questions in the interview schedule were intended to probe what transpired during the lesson observation and the responses from the open-ended questionnaire. The questions were open-ended to encourage more explanatory responses. I chose the semi-structured type of interview to facilitate follow-up questions on the views of the participants (Leedy & Ormrod, 2001). I used Seidman's (2006) format for conducting phenomenological interviews to construct the items in the interview schedule. The questions were conceptualised and outlined as follows:

Question 1 was based on the participants' conceptual understanding of probability

Question 2 was based on participants' experience in teaching probability and what they considered important in making each lesson interesting and comprehensible to students. This included reflecting on the success of the lessons that had been presented in the first phase of the study

Question 3 focused on external support received to allow teachers to reflect on their teaching methods and

Question 4 focused on the participants' reflections on their participation in the study.

Interview questions were expressed in such a way that they addressed issues that emerged during the lessons presentations by each participant, and elicited information that highlighted common themes.

Audio recording

For the purpose of this research study, I audio-taped interviews and lesson observations during the first phase of the study. This was beneficial to me because I managed to listen to the recordings more than once so that I did not miss any important information. Although DuFon (2002) argues that video recording is essential and provides researchers with more contextual data, I listened to the audio recording repeatedly to ensure that information had been correctly captured. I assumed the responsibility of both the observer and facilitator during this workshop.

4.3.2.4 Focus group discussions

Focus group discussions were held during the intervention workshop sessions. Creswell (2008) defines focus group discussions as a process used to collect data from a group of four to six people. Petty, Thomson and Stew (2012) added that focus groups are useful when the researcher wants to gain a range of views about a particular issue. In this study, the main benefit derived from the focus group discussions were the views that emerged from the interaction between participants. In his work, Maree (2007, p. 91) emphasises the point that:

> an understanding of group dynamics is important for focus group researchers in two respects. First, it can help the researchers identify the conditions that promote interaction and open discussion of participants' views and experiences within groups. Secondly, it can assist the researcher in the analysis of the data through an understanding of what was happening in the group as well as why it might have happened.

In this study, I conducted a focus group discussion during Level 1 and Level 2 of the second phase of the study. Although I was aware that some participants might dominate the discussion or be outspoken, I felt that it was necessary to hold focus group discussions so that participants could share their experiences and learn from each other. During focus group discussions, participants also reflected on their experiences throughout the study. They were asked to comment on what they had learnt about teaching probability, the type of support they had received in the form of in-service training, how they had benefited from the study, challenges they had experienced, and any recommendations they wished to make.

4.3.2.5 Field notes

I took detailed field notes to consolidate the other data gathering methods. The aim was to record critical incidents that would form part of the record of the study.

4.4 Data collection procedures

4.4.1 ***Pilot study***

Prior to the commencement of the main study, a pilot study was conducted. The purpose of this was to test the feasibility of the data collection instruments, that is, to validate the data collection instruments. I piloted the open-ended questionnaire with two teachers, one subject advisor and two university lecturers, none of whom were participants in the main study. The two teachers were teaching in a middle school from a similar socio-economic background to the schools

sampled for the main study, but not in the same circuit. The other participants were two university lecturers and one subject advisor. These participants did not have any link to the participants in the main study. I ensured that there was no "contamination" or no possible interaction between the participants in the pilot study and those in the main study.

The purpose of the pilot study was to evaluate the appropriateness of the instruments, namely the open-ended questionnaire (see Appendix P3). I communicated with the participants of the pilot study telephonically. They were asked to comment on the clarity of the items in the instruments, specifically on how the questions were phrased. I provided the participants with the research questions so that they could determine whether items would elicit the expected responses. The teachers completed the open-ended questionnaire. The subject advisors and lecturers reviewed the items in the questionnaire. Their responses to the open-ended questionnaire and their recommendations guided me in adjusting the items in the lesson observation protocol and the interview schedule.

The pilot study did not include all the research processes involved in the main study; participants in the pilot study only reviewed the open-ended questionnaire. The profiles of the participants in the pilot study are presented in Table 4.2:

Table 4.2

Participants' profile

Participant and age category	Qualifications	Teaching experience	Grades taught
Teacher 1 45-50	Grade 12 Mathematics UDES (Majors: Mathematics and Physical Science) ACE (Mathematics)	20 years teaching mathematics	**Teaching experience** Grades 6-9 Presently teaching Grades 7-9
Teacher 2 50-55	Grade 12 Mathematics UDES (Majors: Mathematics and Physical Science) ACE (Mathematics) B. Ed. (Honours) Mathematics Education	30 years teaching mathematics	**Teaching experience:** Grades 7, 9, 11 and 12 Presently teaching Grades 7, 9 and 12

Subject advisor 45-50	Grade 12 Mathematics UDES (Majors: Mathematics and Physical Science) ACE (Mathematics) B.Ed. (Hons.) Mathematics Education	15 years teaching mathematics and six years of supporting mathematics teachers	**Teaching experience:** Grades 7, 9, 11, 12 Presently supporting Grades 4-9 mathematics teachers
University lecturer 1 50-55	Grade 12 Mathematics B.Sc. (Education) (Mathematics) B.Ed.(Hons. Mathematics Education M.Ed. (Mathematics education)	18 years teaching mathematics and presently having four years' experience lecturing mathematics	**Teaching experience** Grades 6-10 Presently lecturing statistics and financial mathematics module
University lecturer 2 30-40	Grade 12 Mathematics B.Ed. (Honours) Mathematics Education (Education.) M.Ed. (Mathematics Education) Ph.D. (Mathematics Education)	Six years teaching mathematics and four years lecturing mathematics.	**Teaching experience** Grades 7, 9, 11 and 12 Presently lecturing statistics module

4.4.2 *Main study*

The main study was conducted in two phases, Phase 1 and Phase 2. Phase 1 of the study focussed more on the systematic processes of identifying gaps and the needs of the participants. The second phase of data collection (intervention workshop) focused on teacher professional development and learning context through an analysis of teachers' deliberations on the realities of classroom practices. This study comprised three stages: (i) ***needs analysis stage,*** where participants completed the open-ended questionnaire, presented lessons in a classroom setting and participated in the post-lesson interviews; (ii) ***collaborative planning stage,*** where participants shared their experiences and consolidated the development of their teaching and learning activities (tasks) as a team; and (iii) ***post-collaborative planning reflection stage***, where participants reflected on the tasks they had developed and their involvement in the study.

The reason for adapting and using the lesson study process following this procedure was to present opportunities for teachers to reflect and improve their instructional practices. Working

as individuals, teachers were required to identify their weaknesses and strengths, so that they could close any gaps when they shared good practices with other teachers. This was in keeping with my understanding of how individual teachers construct knowledge. Owing to the outbreak of COVID-19, I was forced to adapt the methodology. That is the main reason why some of the teachers who participated in Phase I of the study could not take part during second phase of the study. The stages of the main study are depicted in Figure 4.2

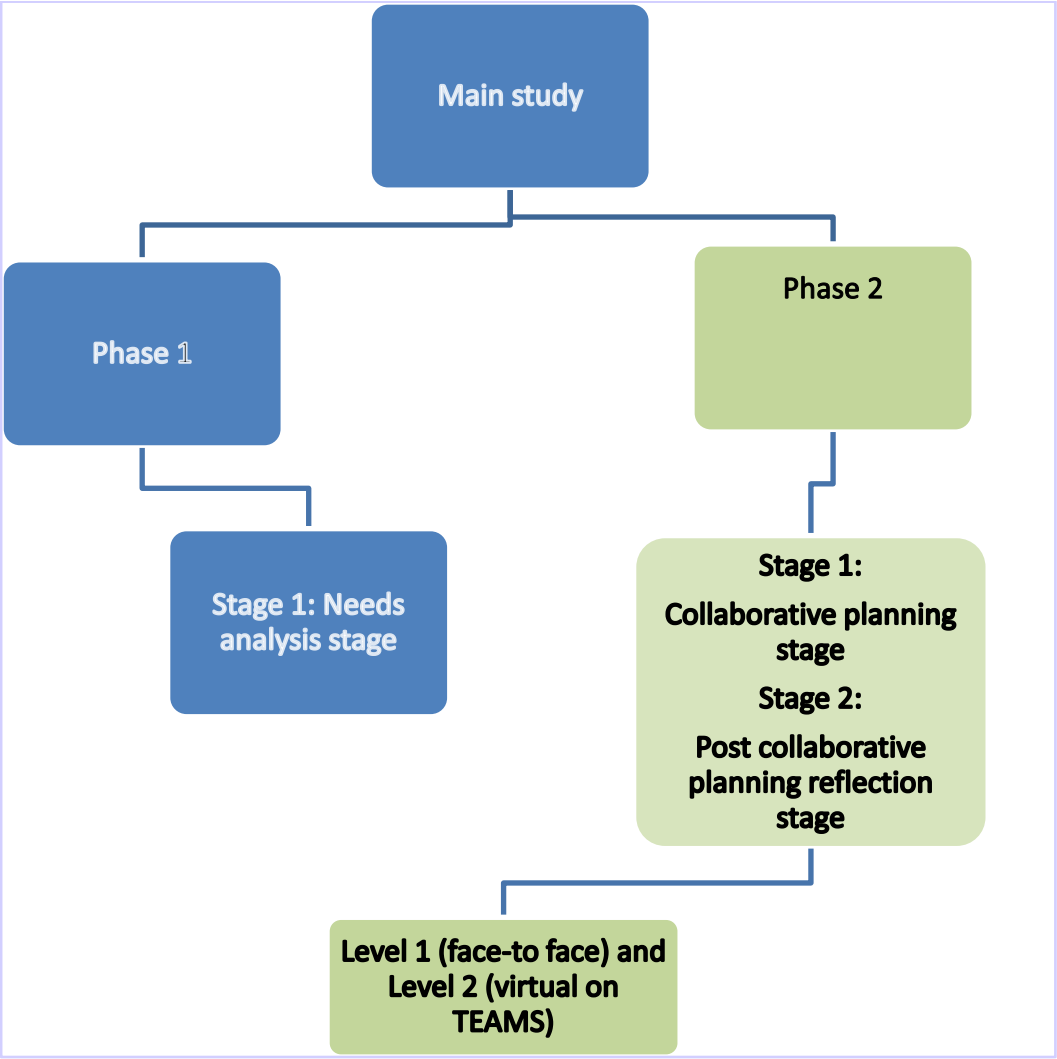


Figure 4.2. Overview of the main study

Source: Adapted from C. Lewis, 2016

The introductory part of this section has presented the context of the main study. It is therefore necessary to detail the activities of phase 1 as follows:

4.4.2.1 Phase 1 of the study

Although the data from the questionnaire drove the analysis process, it is important to highlight that lesson observations and semi-structured interviews formed part of the primary data collection process. Data from each instrument were therefore collapsed, combined and discussed as a single unit of analysis. Questions developed for the interviews were informed by participants' responses to the questionnaire as well as observations made during the lesson presentations. Questions were aimed at probing participants' responses to the open-ended questionnaire and to clarify some of the behaviours observed during the lesson observation.

Stage 1: Needs Analysis

During this stage, I analysed the responses to the open-ended questionnaire (see Appendix P7), the observation notes, recordings and the responses to the interviews. The report on the findings was shared with the participants. I acted as the facilitator in discussing the report from the needs analysis stage. The main aim of this activity was to gain an understanding of participants' preconceptions about probability and the teaching of probability at Grade 7 level; that is, to explore teachers' practices and gain an understanding of their views on the teaching of probability. Through this process I was to determine participants' needs. The following three paragraphs provide an explanation of how data were collected during this stage

- **Open-ended questionnaire**

I visited the participating schools and requested the sampled teachers to go through the document and to complete it. I personally took the open-ended questionnaire to the selected schools and all the expectations were outlined at the time. Teachers were given two weeks to complete the instrument. This was done before the topic on probability was taught by each participant. According to the ATP for the province, all schools are expected to teach the probability topic in Term 4. The findings from the questionnaire (See Appendix P6) informed the focus of observation protocol and interview schedule. Some of the questions posed were aimed at probing challenges faced when responding to the questionnaire.

- **Lesson observation**

I visited two schools per week to conduct the lesson observations and interviews. Table 4.3 provides details of each school

Table 4.3

Schools' profile

School number	Site (Code used for each school)	Participants	Number of lessons presented	Number of students
1	PS_1	PR_1	2	45
2	PS_2	PR_2	2	55
3	PS_3	PR_3	1	52
		PR_4	1	52
		PR_5	1	52
4	PS_4	PR_6	1	48
		PR7	1	46
		PR8	1	45

Four schools participated in the study. One teacher from each school participated in the study except for PS_3 and PS_4 that had three participating teachers. Each teacher from PS_1 and PS_2 presented two lessons on probability and the rest presented one lesson each. In all cases, lessons were one hour in duration. The contexts of the schools are explained in Section 4.3.

All schools involved in the study followed the five-day teaching cycle (Monday to Friday). I spent two successive days (day1 and day 2) at each school. I went through the lesson plan of each participant before the lessons commenced. I observed all the lessons presented. PS_1 and PS_2 spread the teaching activities in two lessons to develop the stipulated concepts and skills on probability. Other participants covered the same concepts and skills in one lesson. The lessons were audio recorded and transcribed afterwards (see an extract of Appendix P5). The actions and words of the teachers were classified into categories identified in the observation schedule. Findings emerging from data were analysed. Areas of focus had been established to address the research questions. As I had assigned codes to the participants in order to manage the data and also to ensure participants' anonymity, I could link aspects of behaviour to participants and to schools.

Limiting my visits allowed lessons to run their course as usual at these schools. Visits were scheduled in accordance with the school timetable to avoid unnecessary disruptions. The observed lessons and the interviews were audiotaped with the consent of the participating teachers. Field notes were kept, enriching the data from the instruments. The kind of questions used in data collection instruments were aimed at eliciting the following information:

- type of instructional practices used by teachers to develop the concept of probability
- teachers' good practices in developing the concept of probability
- teachers' challenges when developing the concept of probability
- how teachers address students' challenges
- how teachers use real-life examples and different representations to teach the concept of probability
- How teachers use the language across the curriculum to unpack the concept of probability

- **Interviews**

In the first phase of the study, eight participants were interviewed. Development of the interview schedule was informed by the challenges identified during the lesson observation. The semi-structured nature of these interviews encouraged participants to provide frank descriptions of how they teach probability in Grade 7 level. Interviews were conducted after the presentation of the lessons. The interview was limited to an hour as far as possible and was conducted during break. Each interview was prefaced by an explanation of its purpose and a clarification of how ethical issues had been observed. If participants had any questions about the study and their role in it, these were discussed before the interview commenced.

During the interviews, I probed teachers on 1) their understanding of the concept of probability; 2) their views on how probability should be taught; 3) their views on the incorporation of real-life problems in developing the concept of probability; 4) what they regarded as challenges in teaching the concept of probability; and 5) what they regarded as important to an understanding of the concept of probability.

The aim of the interview was to triangulate the lesson presentation and open-ended questionnaire data (See Appendices P4(a & b)). During the interviews, participants had the opportunity to attach meaning to their actions in the classroom. In other words, data from classroom observations formed the basis for subsequent semi-structured interviews. These

interviews enabled me to make sure that a clear and accurate record of all communications had been kept. Interviews were recorded and then transcribed for analysis. During the interview, I had the latitude to change the order of questions if this allowed for a deeper investigation into the participants' ideas about their teaching.

4.4.2.2 Phase 2: collaborative planning and reflection stages

This section was designed to address the weaknesses identified in the first phase of the study. To address the identified challenges, the following question was asked: How can teachers' conceptual understanding of probability be enhanced pedagogically to improve classroom practice? The second phase of the study (intervention workshop) focused on teacher professional development and learning context through an analysis of teachers' deliberations on the realities of classroom practices. This phase drew on the processes followed in conducting an intervention workshop. The practices that were given prominence included collaborative planning where teachers came together and developed guidelines on how to augment their teaching activities. In this phase, I provided a forum for discussions, presentations, and reflections on the proposed guidelines.

Stage 1: Collaborative planning

I acted as participant observer in this phase of the study. The sole purpose of this stage was to enrich the teaching activities on probability in a developmental way. I took the participants through the strands of mathematical proficiency as proposed by Kilpatrick et al. (2001) (see Chapter 2 Section 2.3) to give participants an idea of what would be discussed and to expose them to the research articles. To do this activity, I wanted participants to understand that the information they needed could also be accessed from journal articles, not only from textbooks.

Participants were advised not to use the strands of mathematical proficiency like a cooking recipe, but as something to support them. The initial plan was to conduct the second phase of the study, illustrated in Figure 4.3 as follows:

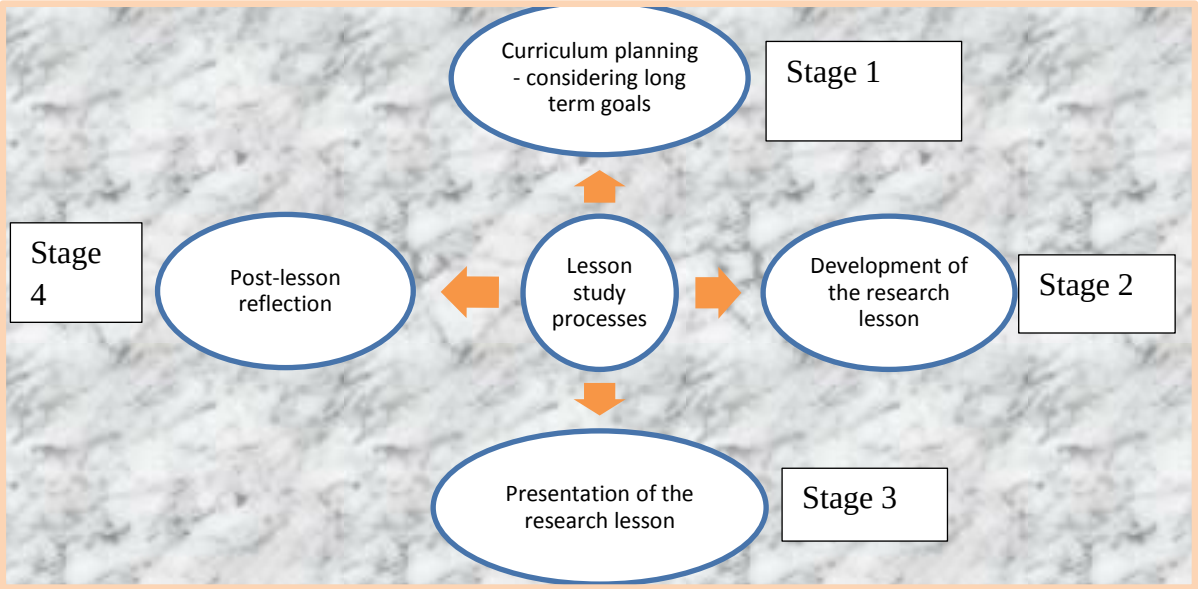


Figure 4.3. Lesson study processes

Source: C. Lewis, 2016, p. 572

It is important to emphasise that this study focused on Stage 1 as outlined in Figure 4.3 because of unforeseen circumstances. The original lesson study processes were adapted to meet the dynamics of the current study. To scale up lesson study, the model in this study includes innovatory activities such as reflecting on students' responses, conducting post-lesson interviews with students, simulating research lessons, teaching in other team members' classrooms and discussing common challenges. These processes encourage teachers to develop interest in student thinking and assisted them to adjust tasks according to what they have learnt in lesson study.

The intervention workshop was conducted on two levels. The first level was focused on the development and reflection of guidelines for augmenting teaching activities. The second level was focused on analysing guidelines developed during the first level, teachers sharing their views regarding developed guidelines as well as reflecting on their participation in the study. The processes of the lesson study were adapted and translated into two levels as follows:

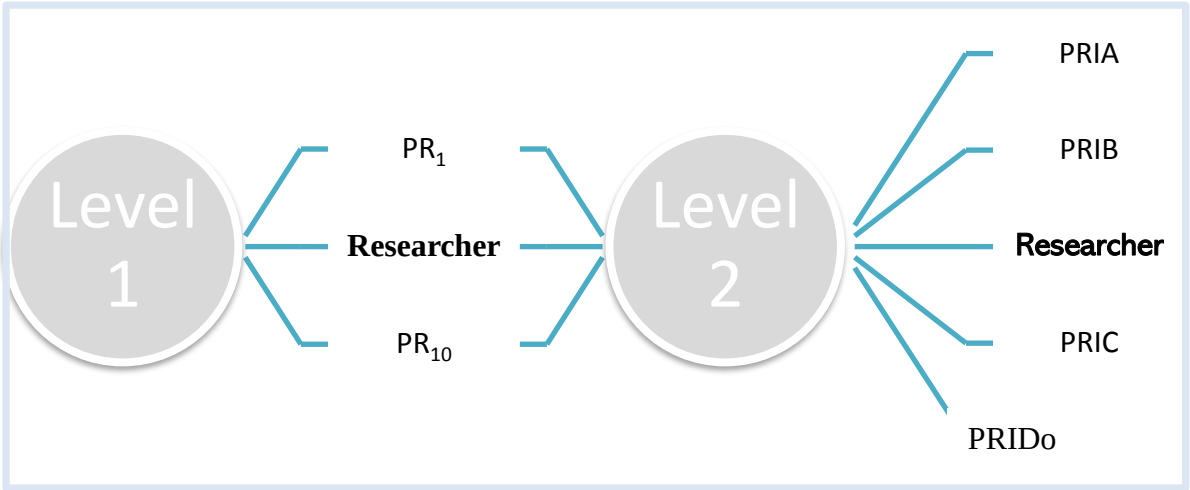


Figure 4.4. Level 1 and Level 2 of the intervention sessions

In the two levels, I assumed the role of both the facilitator and the participant observer. The main role that I played was to plan, guide and manage the processes during the intervention workshop. The activities at each level of the second phase of study are summarised in Figure 4.5 as follows:

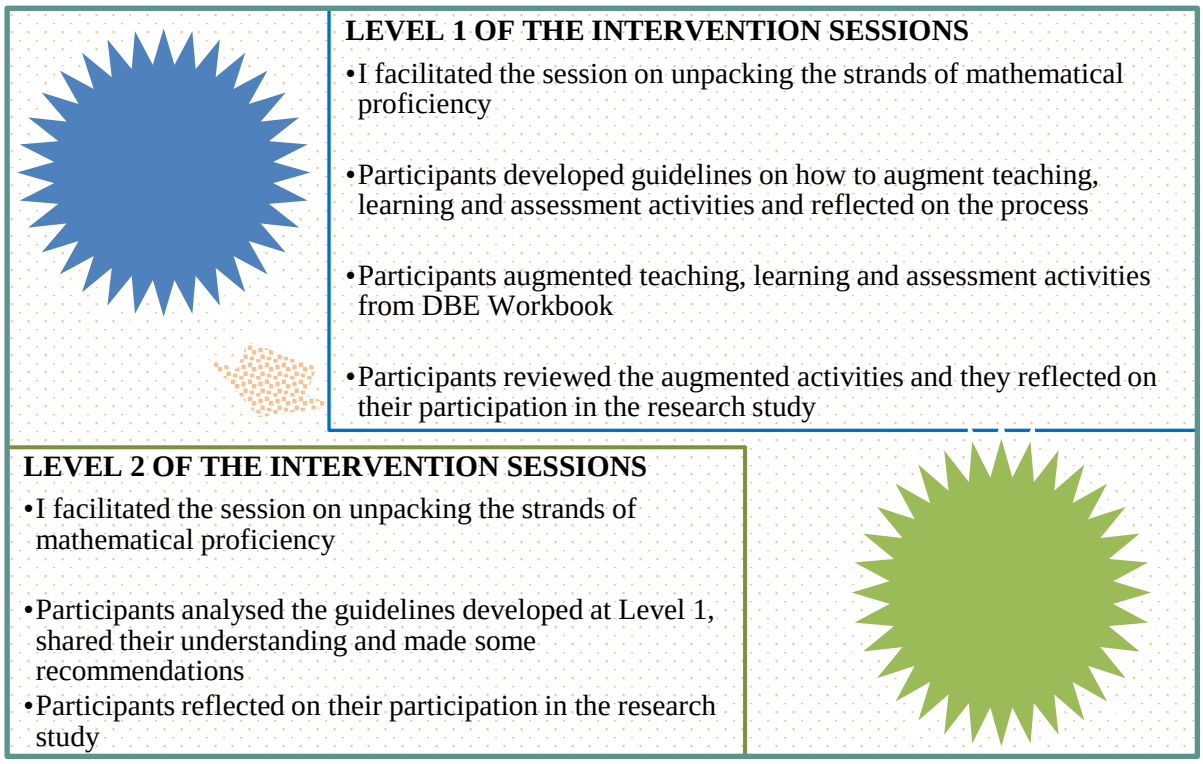


Figure 4.5. Intervention processes

Figure 4.6 presents what constitutes phases of the study

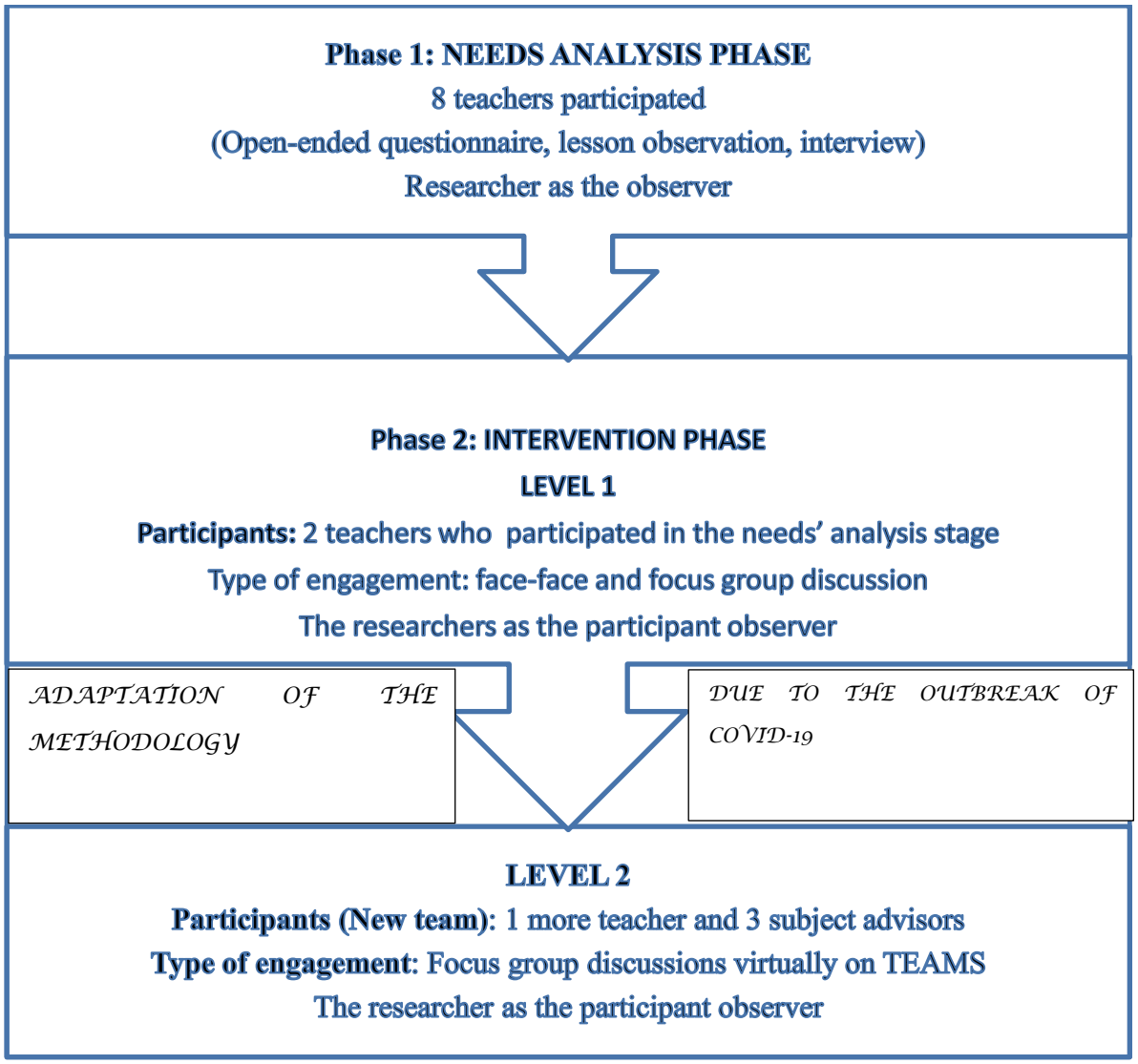


Figure 4.6. Overview of the study

I used the relay method during the intervention workshop. This meant that participants in Level 1 were engaged for a period of 40 weeks (4 hours in 10 days). To carry the activities of the intervention sessions, they were then replaced by an almost similar group of participants to work on the activities developed in Level 1 of the intervention. The group is similar in the sense that they all have experience of teaching mathematics at school level and studied mathematics up to honours level except for one who did B.Ed. (honours) in Curriculum Management. I had to get more participants because I could not make some recommendations based on the findings from two participants. The next section details Level 1 activities for the intervention sessions.

LEVEL 1 INTERVENTION WORKSHOP

Participants at **Level 1** of the intervention sessions developed guidelines for augmenting teaching and learning activities and reflected on their participation in the study. A common venue was used to conduct the intervention workshop. Unfortunately, only two participants attended the workshop. Despite this unexpected circumstance, the two participants succeeded in providing ideas on what might constitute guidelines for augmenting teaching and learning activities on probability. The guidelines were informed by challenges identified during the needs' analysis stage. The participants also used the strands of mathematical proficiency to enrich the guidelines. Activities for teaching probability were designed in such a way that they could be used in a real classroom scenario.

Although the suggestions made by various authors are not explicit as to what should actually be done to assist teachers, I believe it is important for teachers to learn how to design effective teaching and learning activities as a first step to improving their practice. I regard proper development of such activities as critical to effective teaching. This initiative was consistent with the ideas of Valdmann et al. (2016), who indicate that there should be provision for effective professional development programmes that cater for teachers' needs and support them in increasing their confidence and competence. This understanding also mirrors the recommendation of Nel and Luneta (2017), who emphasise that mentoring is an effective intervention practice in professional development programmes, especially when it is informed by the teacher's instructional and content needs.

Of critical importance, especially in a South African school context, participants modified the activities from the DBE workbook, which is used by all the schools at Grade 7 level. The focus was on how to incorporate the workbook activities into their lessons. The participants decided to look at what the workbook had to offer and how this could be used creatively. Creativity, in this context, means that the activities in the workbook were neither deleted nor changed, but modified. It is also very important to highlight that the sole purpose of focusing on workbook activities was because concepts are elucidated using teaching and learning activities.

Workbook activities were adapted to make learning more meaningful and comprehensible for learners (see Appendices P1 and P2). Appendix P1 provides guidelines on how to modify teaching, whereas Appendix P2 presents an excerpt for the intervention workshop activities.

The aim of this exercise was to modify the activities to fit the students' needs in such a way that they would promote conceptual understanding. To ensure that the activities from the workbook were not totally changed, they were adapted from the same core task. In political terms one could say that learning how to adapt activities to address the dynamics of the classroom is the ***key constituency*** of effective teaching. To take the process forward, two participants augmented the activities on probability from the DBE workbook and reflected on the augmented activities (see Appendix P_2). The next section presents activities that were carried out during the second level of the intervention sessions.

LEVEL 2

Having two participants only for the intervention workshop was a challenge and a limitation for this study. This was an unforeseen circumstance because of the outbreak of the Coronavirus. According to Reviglio (2019, p. 152) Serendipity is "the art of discovering new things by observing and learning from encountering unexpected information". In this instance the outbreak of the Coronavirus was an unforeseen circumstance. McCulloch (2021, p.3) highlights that "serendipity should rather be seen as occurring when advantage is found in a chance occurrence". McCulloch further indicates that "when we talk about serendipity, we are referring not to the simple operation of chance but to the confluence of events and occurrence associated with the skill and ability of the observer to make sense of what they are seeing" (p. 3).

Drawing from the definitions of serendipity, and as a result of the outbreak of the Coronavirus, I had to use the snowballing method of sampling to select participants who had access to computers and the internet in order to go virtual. The second team consisted of a further one mathematics teacher, two subject advisors and one mathematics coordinator. In total, the sample for the intervention phase of research was made up of three teachers and three subject advisors. It was important that each member of the team had access to computers and the internet as all communications and activities were conducted online. The subject advisors participated in this study, as part of their roles, to demonstrate how lessons are taught in class. This understanding suggests that although they are subject advisors they are still office-based teachers. This arrangement compelled me to make another application to the ethics committee to adapt the methodology in this study. As highlighted earlier in this section, the intervention processes were adjusted, and Level 2 was conducted as follows:

The intervention processes were adjusted and taken further to involve one more teacher, two mathematics subject advisors and one mathematics coordinator. Participants in Level 2 were asked to analyse the guidelines developed at Leve1 of the intervention (see Appendix P_1). This analysis was guided by participants' understanding of the strands of mathematical proficiency. Participants gave their input as a team. They were then asked to reflect on the guidelines developed at Level 1. At this stage, participants were advised to focus on aspects that would benefit them in terms of enhancing their instructional practice, particularly teaching activities. To take this process forward, participants shared their thoughts and exchanged ideas regarding the developed guidelines. At this point, and as part of teacher professional development, I needed to understand what meaning was attached by each individual participant to the developed guidelines. By asking questions and making comments, I was able to identify more gaps in participants' understanding.

For developmental purposes, I tracked participants' progress during the intervention sessions through inputs in discussions. The post-collaboration planning reflection occurred immediately after reviewing the developed guidelines to ensure that no critical area was overlooked or left unattended. As part of the processes during this stage of the research, participants reflected on their participation in the study; particularly on what they gained from the study, their own challenges and what they thought could be done better. Owing to time constraints, participants could not go further to develop the lesson plans. During this stage, other needs of participants, in terms of developing and presenting teaching and learning activities, were identified. Thus, the whole process of lesson study was cyclical in nature. The next section describes how the collected data were analysed.

4.5 Data analysis procedures

This section explains how the data from the open-ended questionnaire, classroom observations, semi-structured interviews, focus group discussions and field notes were analysed. Maree (2007, p. 99) argues that "qualitative data analysis is usually based on an interpretive philosophy that is aimed at examining meaningful and symbolic content of qualitative data".

I analysed data manually by identifying and describing patterns that allowed me to establish themes (see Appendix P7). Braun and Clarke (2014) argue that thematic qualitative analysis should be the first method learnt by researchers because it provides core research skills,

compelling insights into the real world, and provides experiences and perspectives of participants. In this study, I identified patterns from data which were coalesced into themes. The analysis of data was carried out stage-wise, using a combination of deductive and inductive coding, particularly during the second phase of the study. Categories and themes that were identified through this coding process were developed gradually and carefully. For the purpose of triangulation, categories from both the open-ended questionnaire, lesson observation protocol and the interview schedule were reconciled. The data analysis protocol and the coding system used in this study was guided by the work of Yin (2016) as is illustrated in Figure 4.7.

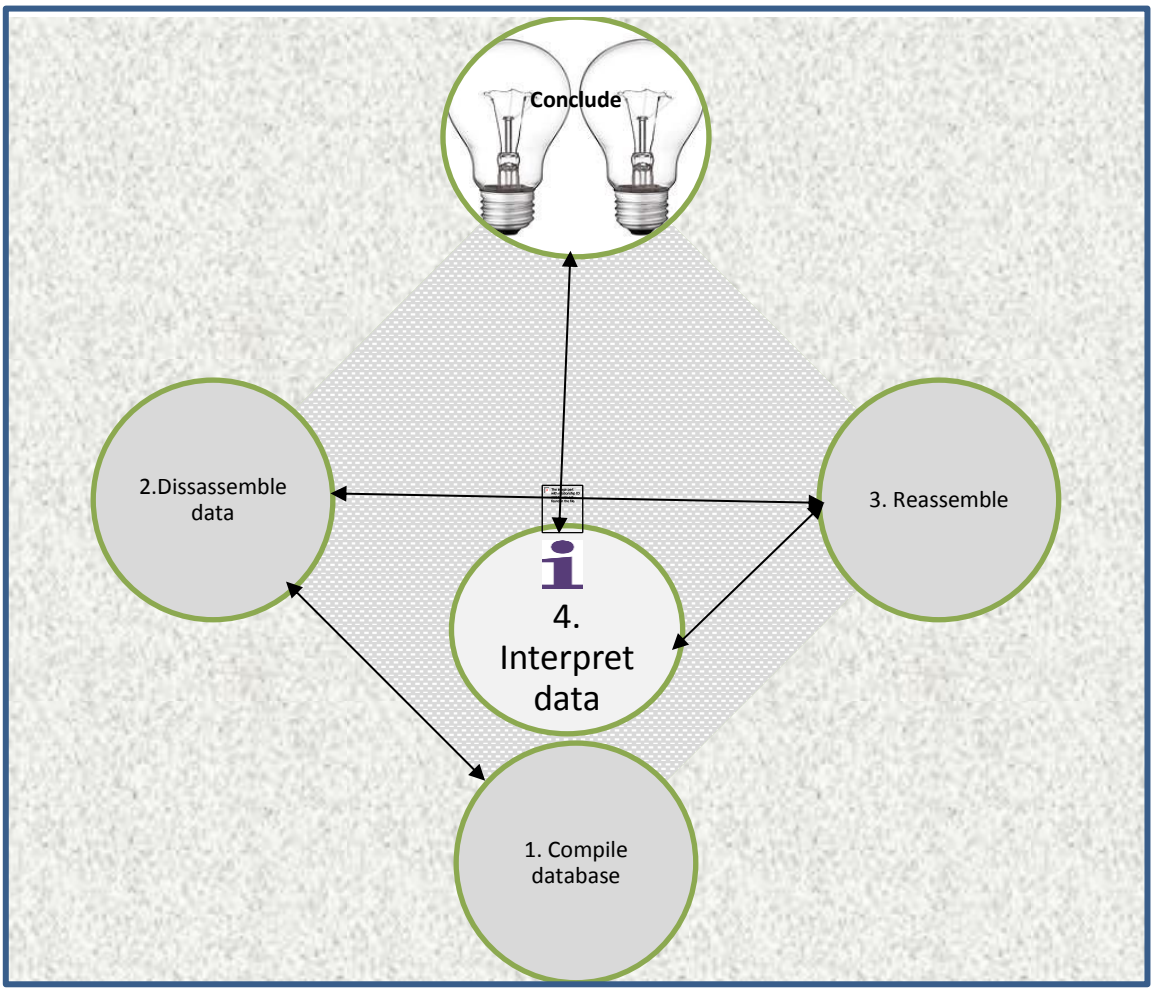


Figure 4.7. Five phases of Analysis

Source: Yin, 2016, p. 186

The phases of data analysis previewed in Figure 4.7 are explained as follows: the two-way arrows imply that you can expect to go back and forth between phases many times. The model

suggests how unlikely it is that analysis will occur in a linear fashion. Table 4.4 provides an explanation of what each of the phases entailed.

Table 4.4

Five phases of analysis and their practical implications

Phases of data analysis	Practical implications
Compile the database	The notes were arranged so as to organise and prioritise data, record and describe participants' experiences verbatim. I read the responses closely several times in order to make sense of the data.
Disassembling procedure	This entailed breaking down the compiled data into smaller fragments or pieces. Codes were assigned to fragments. This procedure was repeated many times with the aim of refining the codes. That is indicated by the double arrow (two-way) between phases.
Reassembling procedure	The disassembled pieces were reorganised, rearranged and/or recombined into fragments called themes. The two-way arrow suggests how the reassembling and disassembling phases are repeated one or more times.
Interpreting data	This phase involved interpretation of the reassembled data. The interpreted data was also reviewed and recompiled in a fresh way.
Conclusion	This is where I drew conclusions from the data.

To put Yin 's (2016) model into context, I used Kilpatrick et al.'s (2001) model to guide the description of the categories and themes that emerged from the data. The following steps present the processes involved in coding data (see Appendix P7).

<u>Step 1:</u> Pre-analysis stage

I read collected data from the open-ended questionnaire, semi-structured interviews and focus group discussions several times to get an understanding of participants' views regarding their experiences. This process was repeated many times with the aim of refining the codes

<u>Step 2:</u> Analysis of data (categorising and coding data)

At this level, I started asking myself the question as to what the findings mean to me as the researcher. I first looked at the general impression of each participant's comments in the scenario, wrote down their words verbatim and described the implication of each. I coded data by looking at the patterns, interpreted findings and ensured that gaps, errors, misconceptions were also identified and highlighted. The disassembled pieces were reorganised, rearranged

and/or recombined into categories and subsequently themes. The second reason for reviewing the initial sub-theme was to cater for overlapping codes.

Step 3: Refining the codes

At this stage I asked myself the question as to what these findings mean to my study. To answer this question, the findings were translated into implementable practice in the next chapter. Conclusions reached from data analysis and recommendations made formed the basis of the intervention workshop and enhanced the RIRAD framework conceptualised within Kilpatrick et al.'s (2001) model. To put the findings emerging from data into context, I used the theoretical and conceptual framework to structure the aspects of data analysis and make the research findings more meaningful.

I used an inductive approach and line-by-line coding of every word of transcribed responses. Codes were developed and modified throughout the coding process. Each segment of data relevant to a research question was coded. Each teacher's documentary sources, lesson observations, post-lesson interviews and post-collaborative planning discussions, were coded and analysed separately during both phases of the study. Cross-case analysis carried out to identify commonalities and differences. However, an inductive approach to data analysis was used when I created new coding labels to identify sub-themes as they emerged during the research process. Open coding was used where new information came to light.

Maguire and Delahunt (2017) argue that themes should be coherent and distinct. They believe that to review the themes when analysing data, the following questions should be asked:

- Do the themes make sense?
- Does the data support the themes?
- If themes overlap, are they really separate themes?
- Are there other themes within the data? (Maguire & Delahunt, 2017, p. 3358)

All these questions provide an overview of the concept of a theme and how it is developed. The next section presents how findings were reported in this study.

4.6 Report findings

The study findings were reported in two phases. Phase 1 reported teachers' understanding of probability and how they facilitated lessons in the classroom settings. Phase 2 reported findings on how participants developed their initial conception of probability. The findings of the needs' analysis stage were infused into the discussions during the focus group engagements. This was done through giving the scenarios to brainstorm the idea and also through describing their experiences of teaching the concept of probability. Explanations and examples of typical teacher and students' responses were provided and compared to findings in the literature. Raw data were used as evidence to demonstrate an analytic point, not simply to display data. Data from all the sources were captured using WORD and presented in tables for purposes of comparison. Figure 4.8 presents an overview of the focus of the report

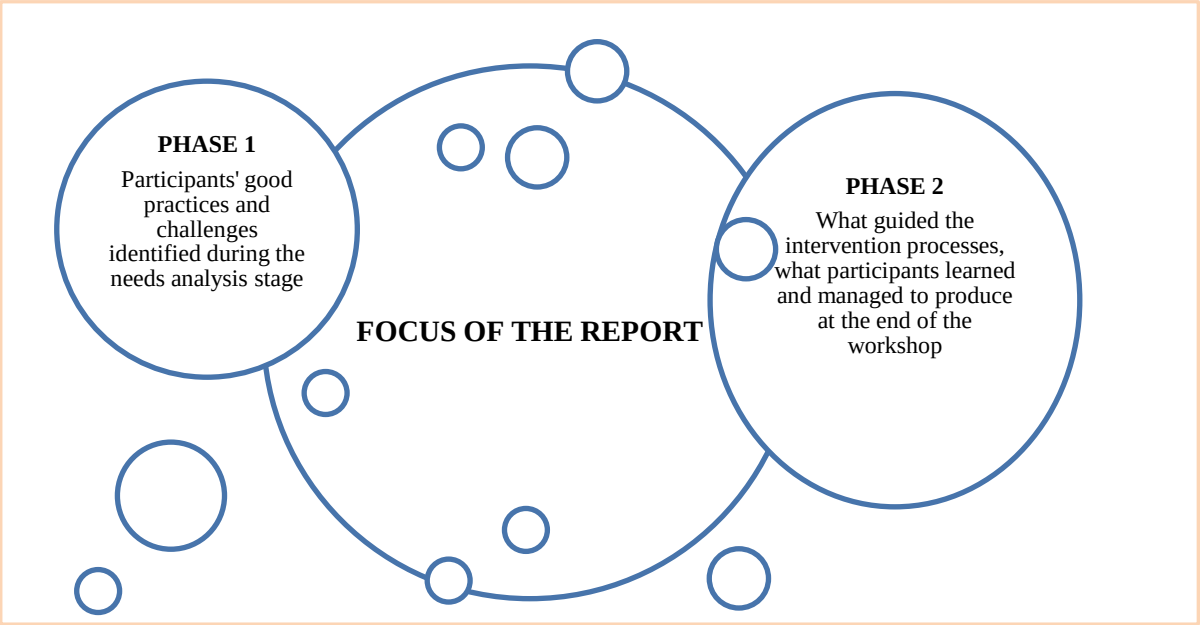


Figure 4.8. Focus of the report

4.7 Ethical considerations

I requested permission to conduct the study from the Mpumalanga Department of Education (Head Office), District Education Office, Circuit offices and schools. Then I received permission from both the Provincial Department of Education and the District Education office for principals to allow me entry to their schools (see the attached permission letters). I then made a preliminary visit to each school to meet the principal. The aim was to request permission to meet with the mathematics teachers to explain the purpose of the study. The permission

letters (see Appendices G-J) reflected the purpose of the research, the role of the researcher, how data would be collected and analysed. In the letters, I assured participants that any information they provided would be confidential In addition, permission to conduct research was obtained from the University of South Africa.

Owing to the outbreak of COVID-19, I was forced to modify the methodological process and to request permission to adapt the methodology. The second ethical approval was granted by University of South Africa. Other ethical considerations included receiving permission from the principals, teachers at selected schools and subject advisors by means of signed consent forms. Gaining access to schools was simplified by the fact that all the people I requested permission from were eager to learn from the study. These applications were submitted after the proposal was accepted and before I could commence with the field work.

Informed assent was obtained from the participants to use audio recordings during lesson observations and interviews. Participation in this study was voluntary, and participants were free to withdraw at any time if they so wished. They were informed that data from the study would become the property of the University of South Africa. These data would be kept by the university for a minimum period of five years. The first phase of data collection was done in 2018 and 2019 in schools. The second phase of data collection was done virtually in 2020. The teachers were informed verbally and in the application letter about the purpose of the study and their roles.

To protect the anonymity of the participants, I assigned pseudonyms to teachers: PR_1-PR_{10} for the first team of participants and PRIA, PRIB, PRIC and $PRID_O$ for the second team. Schools were also assigned pseudonyms. I had no link with any of the schools because I was not attached to the District of Education. I was not in a position of power over teachers and my presence held no threat.

4.8 Quality assurance criteria: trustworthiness of data

The internal validity of this study was assured by spending a double period at least (1 hour) on observations of lessons with each participant. I used an open-ended questionnaire, lesson observations, filed notes, and interviews to develop a comprehensive understanding of how

teachers make sense of the concept of probability. I conducted the post-lesson observation interviews with the intention of probing and confirming what I had observed in the classroom.

As a way of exploring credibility of the results, I checked accuracy and resonance with participants' experiences. Member checks were carried out verbally, particularly during the fieldwork. What I used to do, particularly during the focus group discussions, was to constantly check my understanding of the phenomenon by utilising techniques such as paraphrasing and consolidation for clarification. In other words, member checking during the post-observation interviews and focus group discussions meant that I could verify the accuracy of the data by questioning participants. This platform enabled me to correct any misinterpretation of information.

I used Kilpatrick et al.'s (2001) model to guide the description of the categories emerging from the data. Data were coded according to these categories and themes. Reliability was ensured by using the same observation protocol when observing lessons presented by each teacher; the instrument elicited similar information in both cases. If the information differed, it would have been because of the different teaching approaches used for a specific focus. The open-ended questionnaire was piloted and recommendations (see Appendix P3) made by the reviewers were considered.

The observed lessons and interviews were audiotaped to ensure credibility of the study. Cross-analysis of findings was done per data collection technique. This assisted me in constantly verifying whether the recording had been accurately transcribed. All these processes were conducted at a time that was convenient for participants. The feedback I received informally from teachers in the post-observation interviews and during focus group discussions improved the credibility of data. Table 4.5 shows how trustworthiness and credibility were ensured in the study.

Table 4.5

Trustworthiness and credibility of the study

Strategy	Criterion	Application
Credibility	Prolonged engagement	I conducted the research over two phases and spent extended time with participants to gain better understanding of teachers' classroom practice. I facilitated the activities during the workshops and that gave me an opportunity to probe participants' views and understanding of the concepts. Additionally, I conducted reflection meetings during the second phase of data collection to ascertain if participants' view were correctly captured (Lincoln & Guba, 1985). I piloted the open-ended questionnaire whereby two teachers, two lecturers and one researcher provided guidance in terms of its relevance and clarity (Anney, 2014). The participants in the pilot study provided feedback and helped improve the questionnaire (see Appendix P3).
Dependability	Dense description of methodology	The data collection methods and procedures were comprehensively described. Participants will be informed of the findings and recommendations made in this study.
Confirmability	Audit trail	Interviews were recorded and transcribed and saved on computer in both audio and text format. Reflective notes were kept that indicated decisions made. All versions of the coding process, illustrating the development of the findings, were kept. Participants' responses are included in the study.
Consistency	Repeatability of procedures	All interviews were conducted in the same way, using the same leading questions.
Transferability	Description of the sample	The study population was described in detail. Various characteristics of the participants were described, making it possible for the reader to compare their samples with that of this study The RIRAD framework conceptualised in this study, is applicable to other subjects.

Source: Adapted from Lincoln and Guba, 1985

4.9 Conclusion

This chapter described the sampling criteria, data collection procedures, and data analysis procedures. A discussion of quality criteria and ethical considerations was also provided. Finally, the processes followed in the analysis of the data from an interpretivist perspective were explained. Data collection was achieved by using three instruments, an open-ended questionnaire, lesson observations protocol, interview schedule and field notes gathered during

a focus group discussion. Categories obtained from literature and the field were used to interpret data and analyse findings. Interviews were used to triangulate the data. All processes and procedures followed in conducting this study were described with reference to the research paradigm and assumptions, and the research approach and design. In the next chapter, the results of the study are presented and discussed.

CHAPTER 5 : DATA PRESENTATION AND ANALYSIS OF FINDINGS

5.1 Introduction

In the previous chapter, the process and purpose of a qualitative research design was explained in detail. In this chapter, the findings from this qualitative research study are presented and interpreted in a systematic manner. Presentation and analysis in this chapter is divided into two phases. The first phase presents the findings from data collected during the needs analysis phase of the study, while the second presents findings from data collected during the intervention phase. The second phase drew on the analysis, findings, conclusions and recommendations made in Phase 1 of the study. Findings from the first phase of the study are presented under the overarching theme, limited conceptual understanding of probability. This theme emerged during the process of coding. This overarching theme was conceptualised within Kilpatrick et al.'s (2001) model.

The findings in this study are not presented as per data collection technique, but according to critical discovery. Participants' responses are quoted verbatim. In the study, the data collection techniques were triangulated to provide a comprehensive understanding of the phenomenon under investigation. It was deemed appropriate to present the findings according to issues experienced by teachers in their classroom practice and in their professional development process.

These findings are presented thematically, drawing on the analysis of primary and secondary data. Primary data were collected from lesson observations, semi-structured interviews and focus group discussions. The secondary data comprised questionnaire responses (open-ended questions). In the final section of this chapter, a conclusion and a summary of the main issues is provided. The findings presented in this chapter served to address the following research questions for the study:

PRIMARY QUESTIONS

- What are teachers' conceptual understanding of probability?
- What conceptual understandings of teachers impede and/or enhance the teaching of probability?
- How can teachers' conceptual understanding of probability be enhanced pedagogically to improve their classroom practice?

The next section presents the findings from Phase 1.

5.2 Findings: Phase 1

This section presents and analyses findings of the study from the needs-analysis phase. The main finding at this level revealed that Grade 7 mathematics teachers have a limited conceptual understanding of probability. In this section, I start by presenting the overarching theme that drove the analysis process. Detailed discussion of the broad theme is provided below.

5.2.1 *Theme 1: Limited conceptual understanding of probability*

This theme emerged when data from lesson observations, interviews and the open-ended questionnaire were presented. I have analysed findings (See Appendix P7) from the perspective of how teachers understood the concept of probability, how they promoted learning by using multiple representations, how they connected teaching and learning activities to real-life situations, how they encouraged students to use the language of probability and how they connected related concepts to promote learning. The view that connecting teaching activities to real-life situations is important, is supported by Romberg and Shafer (2020) who argue that if students are to improve their ability to solve mathematical problems, teachers should teach mathematics in context. In this way, students will be able to see the benefit of learning such topics.

The development of the overarching theme arose from discussions of the following sub-themes (limitations that were coalesced into sub-themes): teachers' comprehension of chance, as well as the complexities of second language challenges in relation to the use of modal verbs and the language of probability, including teachers' classroom instructional practices. The next section presents data and analyses findings that emerged from the first phase of the study.

5.2.1.1 Subtheme 1: Teachers' comprehension of chance

The scenario in Figure 5.1 formed part of the items in the open-ended questionnaire completed by participants. The aim was for participants to analyse how student A understood and made meaning of the degree of chance. The scenario was presented to participants as follows:

Scenario 1: The degree of chance

"The weather forecast says there is a 40% chance of rain in our area on Friday."
This statement was made by a Geography teacher in a Grade 7B class. Student A understood the sentence to mean the following:
Student A So, it means only 40% of our area will get rain.
Reflect on students' responses by answering the following questions.
(i) Do you agree with students' understanding? Explain.
(ii) If you do not agree, explain why the understanding is incorrect?

Figure 5.1. Scenario 1: Views of participants on students' conception of chance

To provide a little background, in this scenario the participants were expected to comment extensively on student' interpretation of the provided statement. The aim of the scenario was to test students' understanding of the concept of chance in the context of a weather forecast. Student A interpreted the statement as presented in Figure 5.1. I presented and analysed participants' responses in terms of how many agreed with the student's interpretation, how many disagreed, and the justification they provided in each case(See Appendix P7). In other words, the presentation included how participants made arguments to prove their point. It is also important to highlight the point that the analysis of participants' responses was reinforced by evidence from the lesson presentations and interviews.

What follows is an account of how participants made sense of Student A's interpretation of the weather forecaster's statement in relation to the concept of chance. Keeping in mind the fact that some participants had a limited understanding of the concept of probability, participant **PR5** commented and said:

> *Not agree: The probability of getting rain is even. There is fifty-fifty chance of getting rain, on Friday. There is a chance of rain not a possibility of rain. The student needs to know that the chance of an event happening depends on outcome probabilities of something happening and the probability of the same thing not happening add up to 1.*

It is evident from **PR$_5$**'s response that the student does not see the connection between chance and possibility. This view attests to the findings by Groth et al. (2016) who acknowledge that students' ability to assign numerical values is more developed than their use of probability language. Additionally, there are two issues arising from **PR$_5$**'s response. Firstly, **PR$_5$** understands that it might or might not rain and, secondly, she highlighted the importance of understanding the degree of chance. This view seems to have some important evidence to indicate that instead of interpreting what the student means, the participant clarified the student's response by introducing new terms like: 50-50 chance, chance of rain, not a possibility of rain and complementary events which at this stage might not be relevant or are premature. This is one challenge that usually surfaces when participants try to define mathematical concepts. Instead of explaining the concept they complicate it by introducing new terms which need further clarification. **PR$_5$**'s response suggests the participant had difficulty in explaining the concept of chance.

In the same vein, Participant **PR$_9$** said: *"Not agree: The statement is misunderstood to be referring to 40% of an area instead of 40% chance of raining. The correct statement means 40% chance of raining and 60% chance of not raining"*. A possible explanation for this comment may be the lack of adequate terminology to interpret or maybe to give students a bigger picture of what the statement means so that they can respond appropriately. It is also worth noting that apart from the participant's interpretation of the student's understanding of the statement, **PR$_9$** prefers using percentages to clarify the meaning. However, the use of percentages in this context serves as evidence that the participant also introduces the notion of complementary events. **PR$_9$**'s interpretation suggests that there is a need to assist teachers to make sense of the concept of chance. Some of the issues emerging from this finding contradict the fact that there are words unique to mathematical communication which take on unique meaning in mathematical context (Morgan et al., 2014). It appears that it is difficult for the participant to make a smooth transition from algorithmic calculations to conceptual understanding of numbers.

The assumption that each outcome has a fifty-fifty chance of happening is the most common misconception that continues to resurface when dealing with probability. It seems participants understood it to mean "equally likely events", which means events that have the same theoretical probability of happening. This misconception was identified and highlighted as one

of the issues to be addressed during the intervention workshop. More on this aspect is evidenced in the following discussion:

During an interview, **PR$_5$** indicated that "*50-50 chance means there is a chance of rain and not rain"*. I understood **PR$_5$**'s response to mean that 50-50 chance implies there is a chance for an event to happen and it does not mean it will rain. Although **PR$_5$** clarified her understanding of the statement, she did not say anything about the degree of chance. She reiterated and said: "*There is a chance of rain, not a possibility of rain".* This finding helped me to understand that differentiating between possibility and probability appeared a challenge to some of the participants.

Again, misconceptions arose when **PR$_5$** detached the word chance from 50-50 and explained without emphasising the degree of chance. This finding confirms that such misconceptions may be caused by extracting mathematical structures from their practical embodiment (Gürbüz & Birgin, 2012).

Although **PR$_3$** understood the statement to mean it might or might not rain, she could not explain exactly what 40% chance of rain meant. To confirm this, she said: *"No, because 40% chance of rain means there is a possibility of 40% of rain. The understanding is incorrect because 40% chance of rain means it can rain or not rain on Friday"*. When probed further during the interview about her understanding of probability, **PR$_3$** had difficulty defining the concept. Comparisons of the participants' responses revealed that the term possibility was used differently and could have caused confusion among students. Likewise, it appeared that participants did not understand possibility as 'it might be the case, or it might happen and does not indicate the extent of being probable' as opposed to probability which refers to the likelihood of the event happening. In other words, likelihood is about the extent to which something is likely to happen.

Still on the issue of how the statement was interpreted by student A, one finding was that **PR$_1$** agreed with the student's explanation and commented as follows:

> *I agree with the student because 40 is a part of the whole 100. The student perceived the statement using knowledge of fractions. The student understood that part of their area will receive rain of only 40 out of hundred.*

What this implied was that the participant focused her interpretation on representation and the meaning of percentages rather than on the meaning of chance. When questioned about this response during the interview, she said that "*It is just to calculate the chances of anything to happen or an event to happen".* She clarified the statement by adding, "*probability is about highlighting the possibility of chances to students and that can be calculated using the mathematics formula"*. This answer revealed that the participant's understanding of probability was centred on calculations and mathematical representations. During the interview, a related viewpoint was provided by **PR$_6$** when she explained probability as follows: "*It's like eh … that scale in relation to that probability scale"*. This explanation suggests that her understanding of probability might be explained better with the use of the probability scale. The emphasis on the use of scale to describe probability resonates well with the view by Groth et al. (2016) who point out that the ability to assign numerical values is more developed than their use of probability language.

Examining closely the responses by **PR$_1$** and **PR$_3$**, I concluded that teachers needed support on how to set forth the meaning of probability. This view is supported by the findings of Ndlovu (2014), who emphasises that teachers' classroom mathematical needs could be addressed if they engage with content meaningfully. To attain this, teachers are encouraged to design purposeful activities to develop students' cognitive skills that they will use throughout their lives.

It was interesting to note that in this phase of the study, only one participant seemed to have some idea of what chance entails. The following response from this participant, **PR$_2$**, suggests this:

> *Not agree: The forecast is based on the climatic conditions that leads to rainfall and not the area coverage of the rain. The students' understanding is that 40% of their area will get rain as a matter of certainty which is flawed as the forecast basically says that conditions are 40% probable that their whole area will get rainfall.*

Drawing from this response, it appears that **PR$_2$** has knowledge of the climatic conditions that lead to rainfall, which is mostly dealt with in geography. **PR$_2$**'s response agrees with the suggestion that students need to have background information about an event before they can make any judgement (Kvantisky & Even, 2002).

Overall, there was evidence to suggest that insufficient attention had been paid to unpacking and comprehensively explaining the concept of chance. Relating to this remark, one could assume that participants' understanding of chance was influenced by how the activities in the textbooks are designed. Taken together, the findings pointed not only to how participants explain or define the concept of probability, but also to how they understand chance in relation to probability. It was apparent that they needed assistance in understanding what chance entails in the context of weather. Teachers would need to modify their definition of concepts to encompass other fundamentals of probability, such as the essence, strength, and basic repertoire of probability (Kvantisky & Even, 2002) if they were to form a better understanding of the concept.

In view of the above discussion, which indicates that participants had difficulty in understanding chance in relation to probability, it seemed logical to conclude that there was a definite need for designing teaching activities that would help participants to gain a conceptual understanding of chance. Strategies to enhance a conceptual understanding of chance could include creating scenarios that would help participants understand how to teach probability in context. It is also argued that students' conceptual understanding enables them to reason effectively and draw on a range of strategies with which to engage in mathematics (DBE, 2018a). Taken together, participants' responses provide an important insight into how teachers could be assisted to unpack the concept of probability and how it relates to chance. The next section presents how participants concisely explain the essential nature of probability.

- Definition of probability

The following interview scripts confirmed more challenges that faced participants when it came to what probability entails.

Interviewer: *What is your understanding of the concept, probability?*

PR6*:* *An event taking place.*

Interviewer*:* *What do you mean by an event taking place?*

PR6*:* *Mmm……By eh….an event taking place. I am referring to* ... (pauses for a few seconds) *Should I use the scenario?"*

Interviewer: *It's fine you can!*

PR6*:* *Like…* (5 second pause) *Maybe saying. It's like eh… that scale in relation to probability. Will it be possible for me to teach a cat to maybe*

cook? It is not possible because it is an animal but with relation to eh... something that geographically, the sun will rise from the east. It is a definite thing, that is a certain thing. (I continued the interview to try to get a clearer explanation from **PR6**'s understanding of probability).

Interviewer*:* *In a nutshell, how can you explain the concept of probability?*

PR6*:* *It can be explained as event occurring and certainly that event will take place.*

The participants' definitions of probability illustrated some of the difficulties they had, such as realising that the term embodied a multitude of concepts that made it difficult to define precisely. They gave examples of a range of terminology used in probability. Based on this observation and the way it was a struggle for some of the participants to define the concept of probability, it appeared that this challenge rests upon how words in the language of probability are correlated. The next section discusses how participants describe activities that give students a sort of immersion in what they learn.

- Development of scenarios

Also emerging from data were evidence that some participants had difficulty using procedures appropriately when solving probability problems. This finding helps the researchers to understand the importance of using real life examples in order to develop meaningful learning (Laurens, Batlolona, Batlolona & Leasa, 2017). In this study, participants were asked to provide scenarios that would show how probability was applied in sports. I asked the question as follows: How would you help students design a scenario, so that they could realise the importance of probability in real-life situation?

The aim of the question was to check if the scenarios developed would promote learning that starts from a meaningful real-life situation and that could later develop into a construct. Two of the eight participants did not respond to the question. Those participants who made an attempt to develop scenarios, merely asked simple questions without providing the necessary background. **PR7** drew on his empirical knowledge of the concept of probability and answered the question as follows: *"Team A and B with one given a red card. What is the probability of Team A to get a red card? It will be…."*.

This kind of response raised several questions. When asked during the interview about this response, **PR$_7$** said: "*I was not even aware that I have answered a question like that. Eh! This is really amazing"*. This response reaffirms the observation made whilst I was the subject advisor that the participant seemed to have some knowledge of probability but lacked the proficiency to formulate the scenario.

Responding to the same question**,** **PR$_3$** indicated that: *"In sports, e.g., netball, we play in order to win or lose. Playing is an action. Win or lose are the outcomes".*

Another participant, **PR$_4$** responded to the question as follows:"*Which sport is the most popular?"*

PR$_2$, in sharing his opinion of what the scenario entailed, stressed the fact that

> *In a sporting context where there has to be an outcome, there are only two outcomes which are win or lose. The outcomes are therefore determined by different aspects such as preparation, performance, determination and even pure luck. So, in this case the probability is even or 50-50.*

PR$_5$ contextualised her understanding of scenario saying that:

> *Athletes and coaches use probability to determine the best sports strategies for games and competitions, e.g., a player with a 200-batting average on cricket means he's gotten a base hit two out every 10 bats. A player with 400 batting is more likely to get a hit four out of 10. If a football kicker makes nine out of 15 goal attempts over 40, he has a 60 percent chance of scoring.*

PR$_5$'s response is an example from sports; however, it might be difficult for students to make sense of the information if they do not understand the language used in cricket. Some of the scenarios developed by teachers suggested that participants still needed assistance on how to develop realistic teaching activities. Looking closely at **PR$_3$**'s response, the focus is on understanding the meaning of an outcome instead of developing the scenario. The activity does not promote learning in a real-life situation. **PR$_4$**'s response has nothing to do with probability per se. The scenarios would not develop students' conceptual understanding in probability.

The responses provided by participants might be a result of participants' total reliance on textbooks, workbooks, or readily available lesson plans. In contrast to this finding, Tibebu (2015) emphasises the importance of adapting teaching activities to make learning meaningful for students. This view is supported by Purba and Surya (2020) who suggested that teachers need to encourage students to make connections between knowledge and its application in their lives, through social relations. To support this view, teachers could also encourage students to formulate their own problems and to use mathematical reasoning to solve such problems. Drawing from teachers' responses, it can therefore be assumed that students will not be able to develop their own scenarios if teachers themselves experience some challenges in doing that. This view agrees with Ojose's (2011) finding which indicates that how students construct meaning is influenced by teachers' approaches in facilitating learning.

During the interview, four of the participants indicated that they experienced challenges in dealing with problems on prediction. This is attested to in participants' responses to the question in Figure 5.2.

SCENARIO

- What guidance would you give to learners to find the solution to the following problems? A spinner is divided into seven equal sections numbered 1 to 7. Predict how many times out of 280 spins the spinner is mostly likely to stop at an even number.

Figure 5.2. Scenario 2 Participants' solutions to questions on prediction

Figure 5.3 illustrates participants' responses to the question asked in the scenario:

PR_{10}	**PR_4**	**PR_5**	**PR_3**
Step 1: Write down all the numbers (1-7) 1;2;3;4;5;6;7. Step 2: How many even numbers are here? 3. Step 3: Divide 3 by 280. = 1,07% or 0,010.	The spinner has three even numbers, which are 2; 3 and 4. There are three chances.	2 - 4 - 6 -	Out of 280 spins, we will have 160 even numbers.

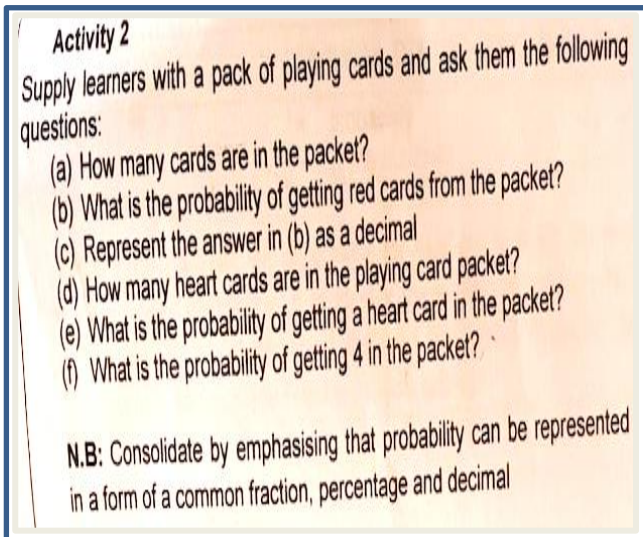
Figure 5.3. Participants' solutions to questions on prediction

None of the participants answered the question correctly. Although **PR_3** did not show how she arrived at the answer, she found it very difficult to explain her reasoning behind the steps in her solution. Equally so, it was not clear how **PR_4, PR_5** and **PR_{10}** introduced even numbers in solving the problem. The reason for this is not clear, but it may have something do with the fact that the even number was mentioned in the scenario. When probed further on their responses, they provided various reasons that reflected an incorrect understanding of the question. **PR_5** said "*I did not even have a clue on where to start. In most cases when we teach prediction in class, we focus only on whether it will rain or not, not on the calculations"*.

Considering **P_{10}**'s response it was clear that the participant did not realise that the answer should be a counting number because the question was "how many times?" Failing to realise that the expected answer is a counting number is a serious challenge. The challenges identified on **PR_3-PR_5** and **PR_{10}**'s responses link with the findings of Danişman and Tanişli (2018) who revealed that secondary mathematics teachers' pedagogical content knowledge is inadequate, and it was further recommended that content-specific professional developmental workshops be conducted specifically for new topics like probability (Ogbonnaya & Awuah, 2019). The challenges identified showed that teachers have limited conceptual understanding of probability. Such challenges might be ascribed to lack of making connections, given any contextual situation. The next section presents the type of teaching activities that were predominantly used by the participants during the lessons.

- The use of formulae to calculate probability

The questions asked in Vignette 0 involved the use of standard algorithms in solving problems on probability. It was observed that questions did not encourage students to invent their own strategies and justify their answers. It seems that the assessment activities developed for students cater only for the cognitive level of routine procedures (Lubis, Widada, Herawaty, Nugroho & Anggoro, 2021). Routine procedures, according to DBE (2011), include performing well-known procedures and direct use of the correct formula. To illustrate how the formula for calculating probability is used, extracts from the lesson plan and students' work are provided below as vignettes:



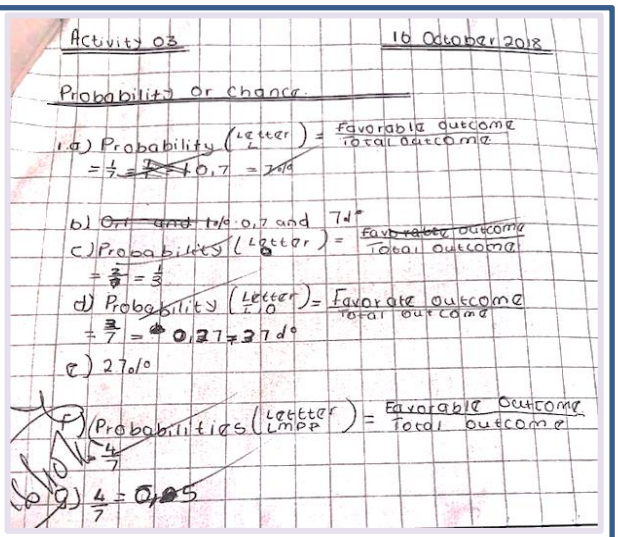
Activity 2

Supply learners with a pack of playing cards and ask them the following questions:

(a) How many cards are in the packet?
(b) What is the probability of getting red cards from the packet?
(c) Represent the answer in (b) as a decimal
(d) How many heart cards are in the playing card packet?
(e) What is the probability of getting a heart card in the packet?
(f) What is the probability of getting 4 in the packet?

N.B: Consolidate by emphasising that probability can be represented in a form of a common fraction, percentage and decimal

Vignette 0 Extract from the common lesson plan



Activity 03 — 16 October 2018

Probability or chance

1.a) Probability (Letter L) = Favorable outcome / Total outcome
= 1/7 = 0,7 = 7%

b) 0,7 and 7%

c) Probability (Letter O) = Favorable outcome / Total outcome
= 2/7 = 1/3

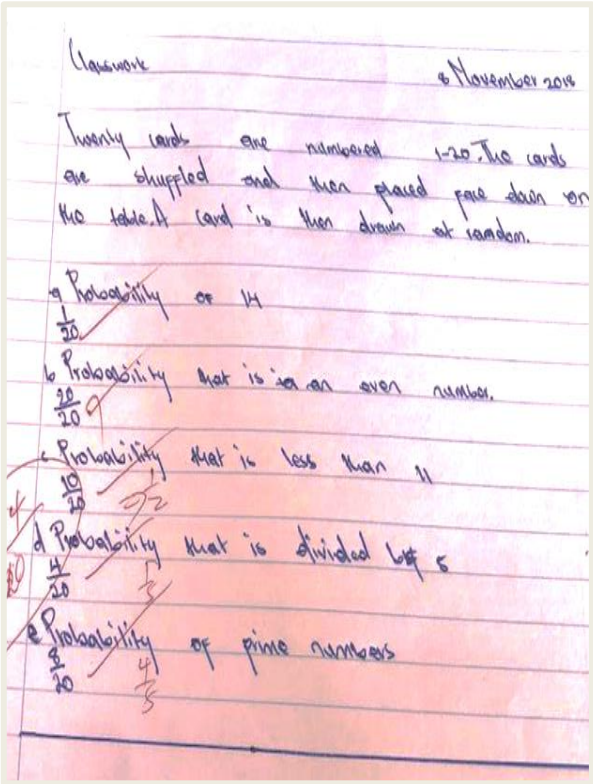
d) Probability (Letter I O) = Favorate outcome / Total outcome
= 3/7 = 0,37 = 37%

e) 27%

f) Probabilities (Letter LMPP) = Favorable Outcome / Total outcome
4/7

g) 4/7 = 0,5

Vignette 1 Student A's Responses



Classwork — 8 November 2018

Twenty cards are numbered 1-20. The cards are shuffled and then placed face down on the table. A card is then drawn at random.

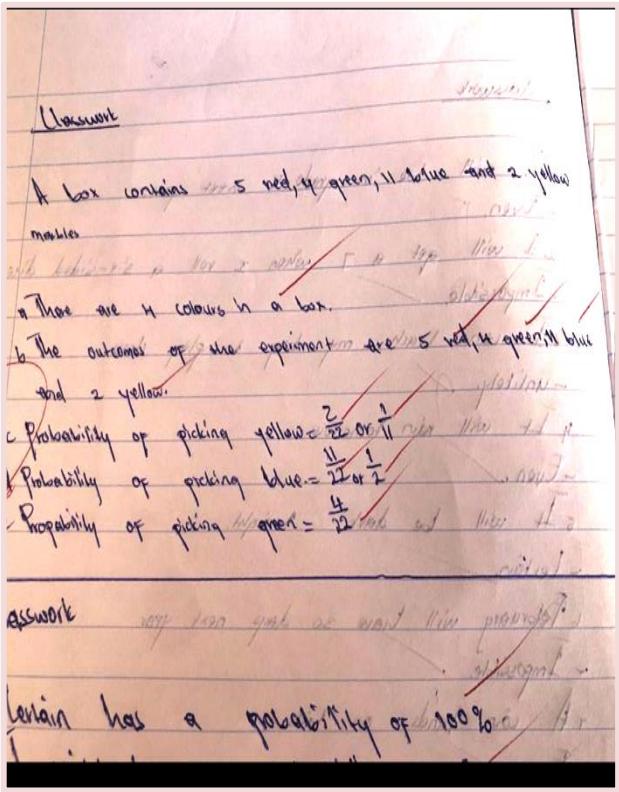
a Probability of 14
1/20

b Probability that is an even number.
20/20

c Probability that is less than 11
10/20

d Probability that is divided by 5
4/20

e Probability of prime numbers
8/20



Classwork

A box contains 5 red, 4 green, 11 blue and 2 yellow marbles

a There are 4 colours in a box.

b The outcomes of the experiment are 5 red, 4 green, 11 blue and 2 yellow.

c Probability of picking yellow = 2/22 or 1/11

d Probability of picking blue = 11/22 or 1/2

e Propability of picking green = 4/22

Classwork

Certain has a probability of 100%

Vignette 3 Student B's Responses

Vignette 1 Student C's Responses

Looking at these vignettes and the extract from the common lesson plan provided by the Provincial Department of Education, activities did not encourage students to:

- Invent their own strategies or methods for solving problems related to probability.
- Describe the procedures used.
- Justify why they used such procedures.
- Use different strategies to solve the same problem.

Likewise, feedback given to students does not engage them in a genuine reciprocal dialogue to promote learning (Molloy et al. 2020). Information gathered in this section formed the basis for developing interventions targeted at preparing teachers to teach probability effectively at a very elementary level. There is a definite need to re-examine the principles pertinent to understanding the concept of probability. Participants could not realise that developing an understanding of probability is not about solving problems taken from the textbooks, but rather about how knowledge is constructed to make learning meaningful (Lombardo & Kantola, 2021).

A bird's eye view of the challenges identified in this phase suggests prioritising the importance of reinforcing ideas gleaned from the literature, extending the ideas that emerged from this study and bridging gaps in knowledge to help teachers to engage in innovative ways. Common themes in participants' responses served as a basis for the intervention workshop and the decision to empower participants. The findings were therefore put into perspective to guide the intervention workshop. The next section discusses the complexity of the second language.

5.2.1.2 Subtheme 2: Complexity of second language issues

In the previous section, participants' understanding of the concept of chance in relation to probability was discussed. But equally important is the fact that understanding chance is not possible without effective communication to convey information. What follows is a discussion on how participants used the language of probability and modal verbs to develop the concept of probability. The complexity of the language was looked at from the perspectives of how participants used the language to extend their knowledge of probability and how they interpreted and clarified situations. The intention was also to look at how they expressed ideas, arguments, opinions when providing direction and conveying information to students. The first difficulty identified under the sub-theme of complexity of second language issues relates to:

The use of modal verbs

This sub-theme relates to how participants used modal verbs to help students understand mathematical situations when teaching probability. I identified and explored modal verbs used most frequently by participants in their lessons and when they conducted assessment. This included investigating how teachers understood what the use of these modal verbs implied to them.

What emerged from this study was that participants appeared to have difficulty in affirming that the meaning of a modal verb changed according to the context of its use and sometimes by how it is used. In line with my views on these issues, the activities shown in Figure 5.4, presented by **PR1**, provided data on the context within which the participants operated. She started her lesson by asking students to respond to the statement according to the instructions for each activity. The first activity was performed at the beginning of the lesson and the second during the presentation of the lesson.

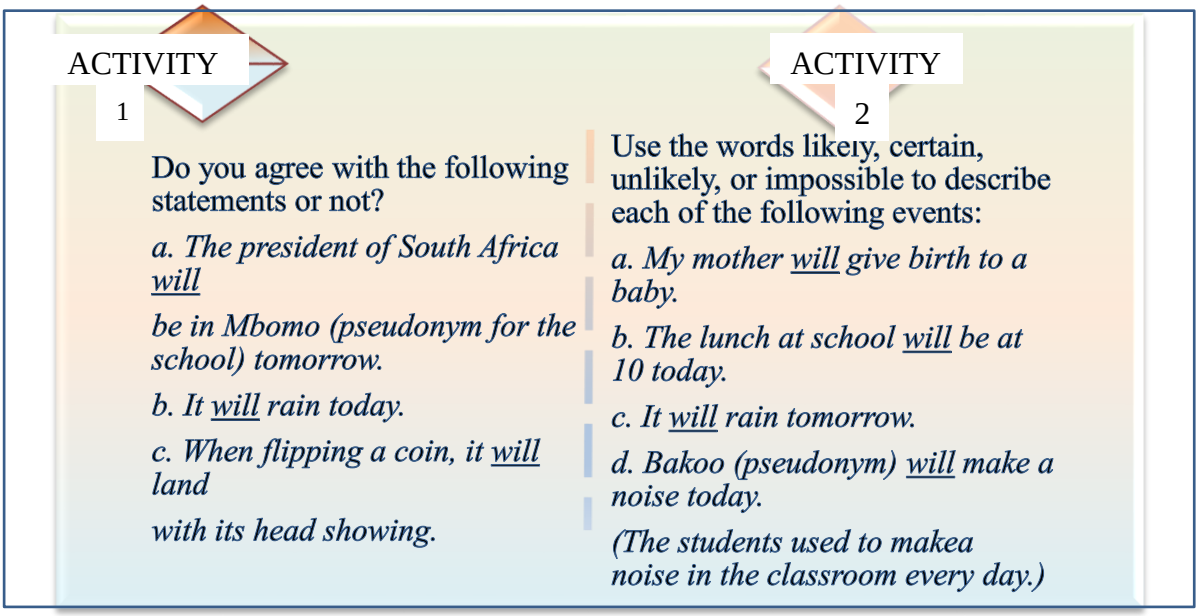


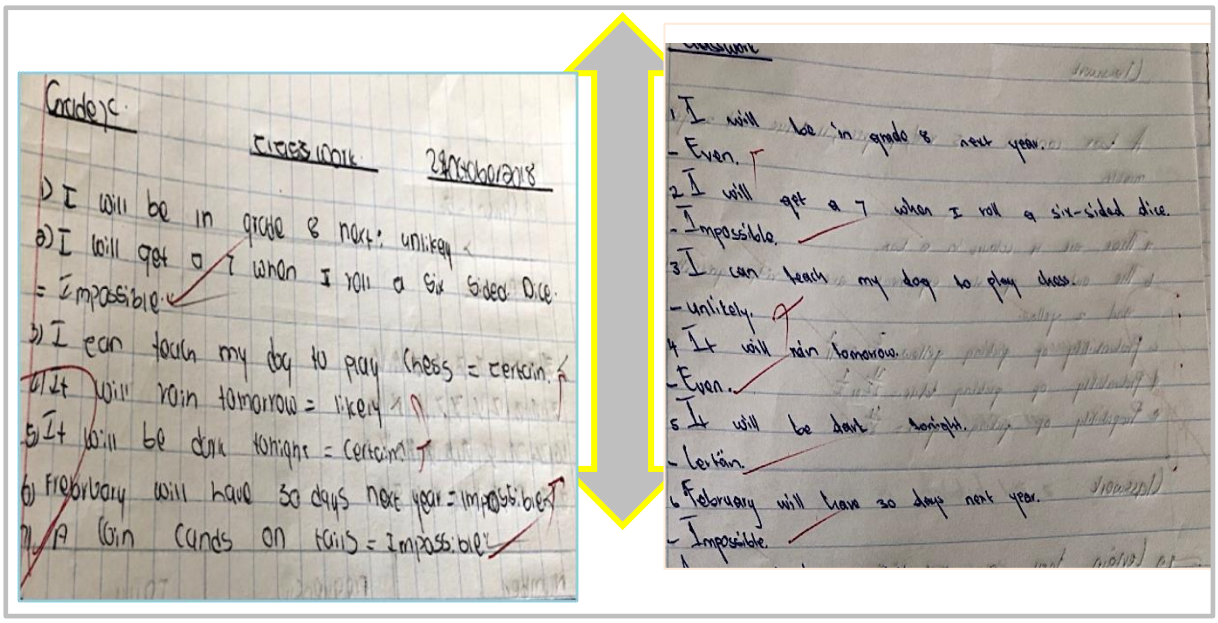
Figure 5.4. Teaching and learning activities: Before and during the lesson

Figure 5.4 reveals that **PR1** used only one modal verb, "will" throughout the presentation of the lesson. Highlighting this observation further, **PR7** wrote the statements on the board, as shown in Figure 5.5, and asked students to describe them using the words *impossible*, *even*, *certain*, *likely* and *unlikely*:

- *I **will** write and talk at the same time.*
- *I **will** pass at the end of the year.*
- *I **will** eat and sleep at the same time.*
- *I **will** be at school and in Johannesburg at the same time.*
- *I **will** be in Johannesburg at the same time.*

Figure 5.5. Use of probability words

It was interesting that all the participants used the modal verb “will” to generate discussions of the concept of probability. It might be because participants associated probability with what will happen rather than what could happen in the future. There was no provision made for students to justify their answers. Students’ responses in Vignettes 4 & 5 serve as clear evidence of this view. A few excerpts from students’ work are provided in Vignettes 4 & 5 as illustration:

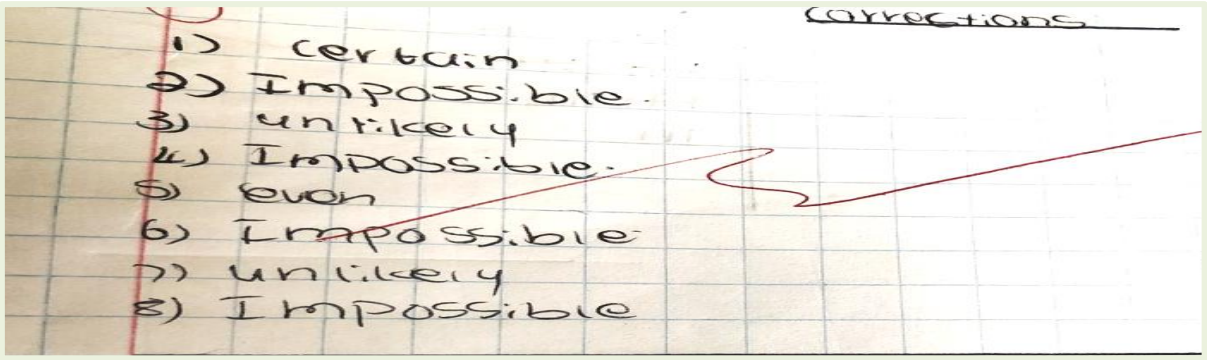


Vignette 2 Student D’s Responses

Vignette 5 Student E’s Responses

The frequent use of the modal verb ‘will’ provides an impression that the probability relates to events that could happen in future only. Equally so, students might experience some difficulties in understanding when to use other modal verbs and for what purpose. In both Vignettes 4 and 5, students’ responses for some of the questions were marked wrong. The reason for this is not made clear, although it might have had something to do with the fact that the teacher argued from his own perspective. It could also relate to the fact that students were not given the opportunity to justify their answers. Because feedback was provided in a way that does not promote learning, it was very difficult to infer from answers if the reasoning was looked at from the perspective of teachers or students. It appears that student D did not understand what it means to “be in Grade 8 next year”. Under normal circumstances, you cannot say this is unlikely if you intend to pass at the end of the year. For the teacher to have proper understanding of the student’s thought processes, she should have immediately opened up a discussion to interrogate the student’s thinking.

Student E described the same event (Question 1) using the word ‘Even’. This response suggests that the student might or might not be in Grade 8 in the following year. If the teacher had provided constructive feedback, and not simply given a tick or cross, this would have made anyone understand why students were marked wrong. It would have been better if the participant had clarified what the statement meant. What follows is an account of how feedback was given by the teacher.



Vignette 3 Memo 1

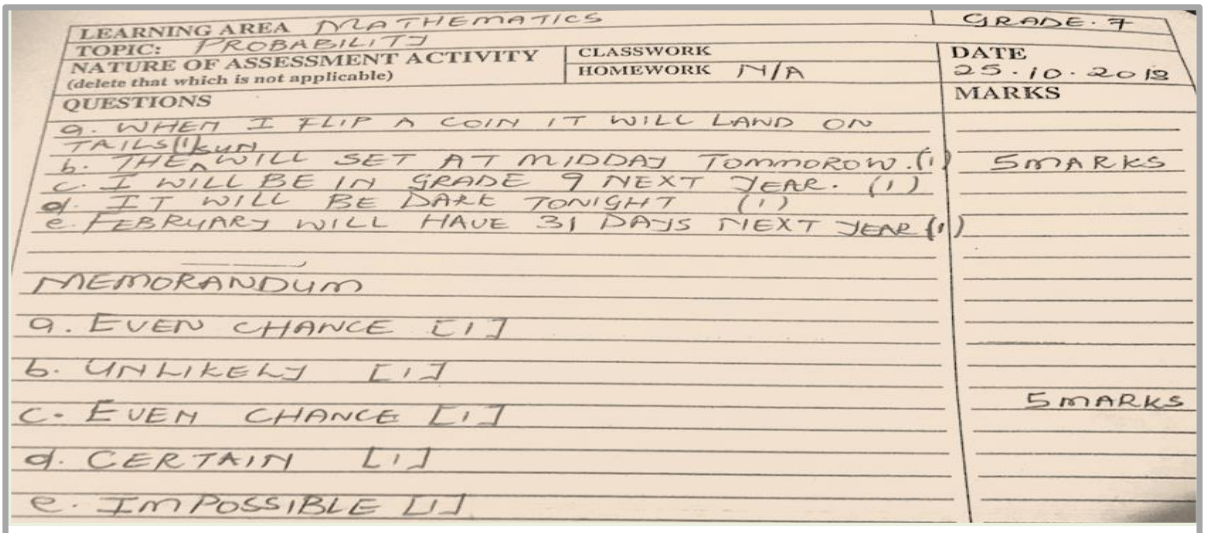
Several issues of concern arise from the feedback given to students’ responses. If this memo serves as the corrections given to the whole class, the main questions to ask here are: Who is certain? Was it the teacher or the students? Could it be a one size fits all answer? Secondly, the teacher was not consistent when marking Questions 5 and 6 (See Vignettes 4 and 5) for the two students. They wrote the same answers, but these were marked differently. This kind of question would elicit a range of responses if context is not considered (Belbase, 2020). Despite displaying the use of probability terms, it is not clear how the modal verb (will) add to different levels to either denote necessity or possibility (Zhang, 2019). It seems possible that this finding is due to the fact that students were not asked to justify their answers. This view is supported by Casserly (2016) who argues that it is important to engage students in mathematical practices that construct viable arguments. The next section covers the second limitation identified under this sub-theme.

The use of the language of probability

Important questions this study raised were: What terminology and background might students need to understand the concept of probability? and: Are teachers using the correct language to express caution, possibility or awareness? To put it another way: Are teachers cautious in coming to conclusions, making judgements, reporting issues, taking decisions, and responding

to students' misconceptions when reporting in meetings? All these questions relate to what the degree of certainty entails.

What emerged from the data was that participants used a subjective approach when introducing the language of probability. The objective approach, according to Kvantisky and Even (2002), is assigned to an experimental event, whereas the subjective approach is characterised by a subjective judgement. Although participants managed to introduce the language of probability, they had some difficulty in providing activities that helped students to make decisions under conditions of uncertainty. The memo below substantiates this:



LEARNING AREA MATHEMATICS GRADE: 7
TOPIC: PROBABILITY
NATURE OF ASSESSMENT ACTIVITY (delete that which is not applicable) CLASSWORK HOMEWORK N/A DATE 25.10.2012
QUESTIONS MARKS

a. WHEN I FLIP A COIN IT WILL LAND ON TAILS (1) RUN
b. THE WILL SET AT MIDDAY TOMMOROW. (1) 5MARKS
c. I WILL BE IN GRADE 9 NEXT YEAR. (1)
d. IT WILL BE DARK TONIGHT (1)
e. FEBRUARY WILL HAVE 31 DAYS NEXT YEAR (1)

MEMORANDUM
a. EVEN CHANCE [1]
b. UNLIKELY [1]
c. EVEN CHANCE [1] 5MARKS
d. CERTAIN [1]
e. IMPOSSIBLE [1]

Vignette 4 Memo 2

The memo to the classwork given to students was developed by the participant and was part of the lesson plan. What is implied is that the participant did not give the students opportunity to make their input on the memo. It therefore suggests that all these answers were reasoned out from the perspective of the teacher. During the presentation of the lesson, there was an argument about why the answer to (b) was *unlikely*. The students raised an argument, indicating that the answer should have been *impossible*. As Proudfoot and Kebritchi (2017) note, this immediately suggests that it is of paramount importance to give students enough opportunity to explore the issue to form a better understanding of the concept. It became indisputable that teacher professional development should be the focal point when considering responses from conversations such as:

As **PR3** was presenting the lesson, she asked students the following question:

PR3: "*The rain comes from the clouds. Is this possible?"* Some of the students said it was possible and some said it was impossible. Without any comment, the teacher wrote *impossible* on the board. Another significant observation was that **PR3** said: *"We are going to do an activity which is called an event.*" The participant did not elaborate further on what kind of an activity an event was. What immediately came to my mind as the researcher was that teachers still experience some challenges in using the probability terms appropriately. A further finding was that after drawing the probability scale as indicated in Figure 5.6, **PR3** asked the following question: "*Who can tell me? The numbers are increasing from zero by how much*?"

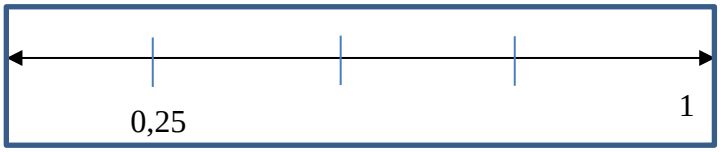


Figure 5.6. Representation of fractions on the number line

Student: *(responding with confidence*): "*The number is increasing by 25."* The participant did not make any comment on the student's response. I found it difficult to understand this, but it might be related to the fact that the participant thought the student meant 25%. The participant's focus was more on the lesson plan than making sense of the student's answer. Figure 5.7 shows the solution the participant was looking at in the lesson plan. It immediately came to my mind that although the student said 25, to him it registered as 25%. This appears as a caution that when presenting lesson, teachers need to focus and fully apply their minds.

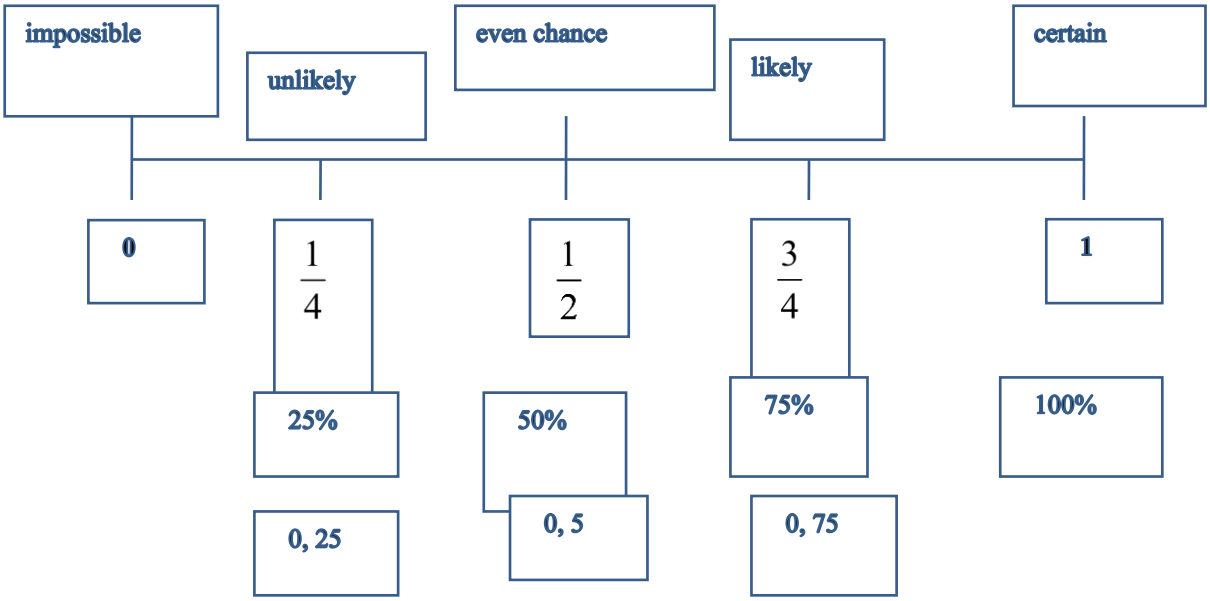


Figure 5.7. Task in the lesson plan

Focusing on the lesson plan might be the reason why the participant could not comment on the student's answer which reads "*the number is increasing by 25"*. Because the student did not get confirmation of whether the answer was correct or not, it became difficult for me to conclude that learning took place because the student's response was not probed. As discussed earlier in this chapter, **PR3** had some difficulty with regards to the connection between probability terms and mathematical concepts that had been taught previously; it appeared that the participant lacked the ability to build previously learnt concepts into the learning of probability. This finding was also noted and analysed in Subtheme 1.

It was somewhat surprising to observe that even though most aspects were not clarified, students did not show any signs of frustration. Examining the data further highlighted another issue: **PR3** continued to use the language of 1 over 2, instead of a half. Naming the fractions in this way could result in students losing the contextual meaning of a fraction. During the interview, I followed up on what I had observed during the lesson as follows:

Interviewer: *Thank you very much for the presentation that you made this morning. I have few questions I would like to ask.*

PR3: *Ok, ma'am.*

Interviewer: *I heard you saying, "We a going to do an activity which is called an event". Am I correct?*

PR3*:* *Ja…* (She looked surprised).

Interviewer: *What did you mean?*

PR3: (She was not sure how to answer the question). *Eh … I meant an experiment with a die.*

Interviewer: *What is the difference between an experiment and an event?*

PR3: *I take that the two terms are the same.*

Quite clearly, responses from some of the participants signified a limited understanding of the concepts used in probability. The interchange above endorses the importance of teachers' attendance at an intensive workshop on how to use the appropriate language to develop the concept of probability. This situation could have been caused by teachers not being exposed to this topic during their pre-service training. In relation to **PR3**'s responses, Khaliliaqdam's (2014) argument that the knowledgeable other should be experienced in that specific field could be taken to mean teachers need to attend intensive workshop to deepen their understanding of

probability. I used participants' written and oral responses to identify the core groupings of conceptual difficulties that participants experienced in using modal verbs and the language of chance. These core categories are captured as follows:

a. "Will" was the commonest modal verb.
b. Questions did not give students the opportunity to justify their answers.
c. Teachers marked answers without providing proper feedback that students could learn from.
d. Students' answers were not interrogated to get an indication of whether they understood what they had written, or whether they had simply guessed the correct answer
e. Answers provided by teachers in the form of a memorandum were not justified; the way feedback was provided was additional evidence that learning had indeed not taken place because understanding modal verbs in relation to context is crucial.
f. Questions led to ambiguity and misinterpretation.

The following section presents the findings on classroom instructional processes and procedures.

5.2.1.3 Subtheme 3: Teachers' classroom instructional practices: processes and procedures

In seeking to investigate and analyse the characteristics of teachers' classroom instructional processes and procedures in lesson presentations, it was useful to consider how teaching, learning and assessment activities were implemented in the classroom. The aim of this study was to provide insight into how participants experience the teaching of probability (Vijaya Kumari, 2014). Turning to the empirical evidence of how instructional processes and procedures unfolded during the study, the next section presents observations of eight participants when teaching probability to Grade 7 classes.

These observations were designed to gain insight into how participants confronted the underlying ideas of probability. The findings drew attention to distinctive categories of instructional processes and procedures. These processes and procedures are discussed in an integrated manner to obtain a comprehensive understanding of how teachers develop the

concept of probability. It is also important to note that the content of all lessons presented in this study was informed by the CAPS document.

Although participants were all guided by the same curriculum policy document, two participants presented two lessons each, and six of them presented only one lesson each. In the case of those who presented two lessons, the first lesson focused on introducing the language of probability and the second on calculating the probability of events occurring in a given situation. Those who presented one lesson covered most of the aspects that were addressed by the other participants in two lessons. This observation therefore raised some concern about how the content of probability is divided into deliverable units for implementation in the classroom. The findings from the eight participants were compared, integrated and categorised. These categories are discussed and summarised in the subsequent sub-sections to validate the challenges identified in each.

This sub-theme is generalised because lessons were presented in more or less the same way by all participants. It was difficult to explain this phenomenon, but it may have been related to the fact that five of the eight teachers (63%) used the common lesson plans provided by the provincial Department of Education. The remaining three reviewed the lesson plans although their modification was limited. The categories emerging from participants' underlying view of probability comprised the following a) Prior knowledge**;** b) Selection and use of teaching resources; (c) Use of workbooks. The presentation of content in these sub-categories was linked to participants' facilitation strategies. How participants drew from students' real-life experiences to build on new knowledge is discussed in the next section.

Students' prior knowledge

This category relates to the following aspects: how teachers drew on students' prior knowledge to build on new knowledge, and how they made connections amongst mathematical concepts. It was clear from all the lessons observed that participants' initial questions were aligned with the students' prior knowledge. They began with the simple version of the task and asked context related questions. The following conversation between participants and students serves to substantiate this view:

PR7:	*In five minutes, I will be in Pretoria*
Student 1:	(Asking the teacher a question) *How will you get there?*
PR7:	*In other words, it won't be easy. Is that what you mean?*
Student 2:	*No, it won't be possible.*
PR7:	*It will be impossible.*

Although the participant tried to engage the students at the beginning of the lesson, it was very difficult to identify the purpose of these questions. It was clear that the participant was trying to draw on students' real-life experiences, but he could not establish students' learning difficulties and misconceptions because of poor questioning techniques. The aim of asking such questions was not made explicit. This is contrary to Darling-Hammond et al.'s (2017) argument that prior knowledge includes understanding of students' thinking, distinct perceptions and knowledge base.

To add to this, the participant did not encourage students to reflect on the questions to make links, thus it was not easy for the participant to detect language patterns or misconceptions students had. It appears the participant did not take much effort to plan these introductory teaching activities. **PR7** also asked students to give them examples of impossible events. When responding to questions the participant advised students to start as follows: "It is impossible to/that …". For example, one student responded, saying, "*It is impossible to turn a table into cat"*. The teacher responded and said, *"Yes you are right, it is impossible to turn a table into a cat"*. This suggests that the teacher did not realise that the student's answer might be confusing if the aim was to describe the events using probability terms like 'impossible', 'certain' and 'possible'.

Just to elaborate on this aspect, if the student says, "It is impossible to turn a table into cat", I can immediately respond and say, "Yes, it is impossible for the event to happen or it is certain that the event won't happen". The student should have said, "I can turn the table into a cat". It would have been clear that the action was impossible. This might be an indication that teachers need to be cautious when giving students the leading statements to answer the question. Alternatively, it might mean that the teachers' way of asking questions needs some reconsideration. Acknowledging the challenges experienced by **PR7** links closely with the view that teachers must be able to deliver lessons with clear objectives (Le Donné et al., 2016).

Although these questions caught students' interest, none of the participants asked questions that required students to identify learnt topics and sub-topics related to probability. It was concerning to note that some of the participants were not even aware that probability was also taught in the Intermediate Phase. Confirming this, **PR2** said during the interview:

> **Interviewer:** *Did you ever go through the content on probability that is prescribed for Grade 6 students?*
>
> **PR2**: *For Grade 6, no*?
>
> **Interviewer**: *The reason I ask is because what you emphasised during the presentation of lessons was dealt with in Grade 6.*

The following quote from the participant is a case in point:

> **PR2**: *Okay… I haven't check, I thought it is the first time, they hear about probability. I was not even aware that they are doing probability in Grade 4.*

Although most of the participants claimed that students did not have a strong foundational background in probability, it was clear that participants had not considered how probability develops from one grade to the other. This finding suggests that teachers need to familiarise themselves with the curriculum policy so that they know what to focus on in each grade. In other words, they need to consider the level at which the same content should be pitched at different grades. In addressing this challenge, Section 3 of the CAPS document provides some clarification notes and teaching guidelines. On the other hand, despite references to previous knowledge, one of the important findings to emerge from this study was that the link between the introductory activity and the lesson development was not clear. In conclusion, extracts from the common lesson plans provided to teachers that support the above findings are captured below:

INTRODUCTORY ACTIVITY

Give learners a die to throw and then ask them to answer the following questions:

- How many possible outcomes can you get when throwing a die?
- What is the probability of getting an even number?
- What is the probability of getting an odd number?
- What is the sum of probability of getting even and odd numbers when you throw the die?

LESSON DEVELOPMENT ACTIVITY

N.B: Consolidate the activity in the introduction by introducing probability scale as shown below:

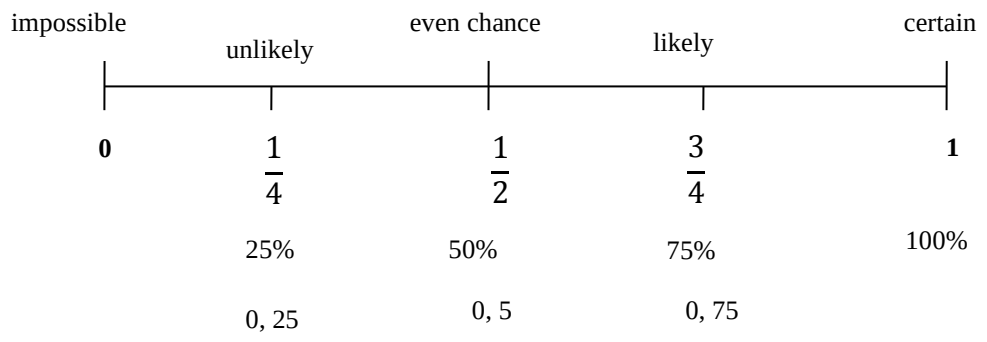


Figure 5.8. DBE lesson plans: Grade 7 Lesson Plan: Term 4 PROBABILITY: Probability (lesson 2: page 2 of 3)

According to Figure 5.8, it was not clear what the statement 'Consolidate the activity in the introduction by introducing probability scale" meant. It was difficult for me to deduce how consolidation could be done, because no instructions were provided for the lesson development activity. This extract underlines that teachers must ensure that their instructions are clear, and activities developed for the introduction are relevant. All these lesson plans were provided as additional resources not as material for publication. The next sub-category discusses how the resources were incorporated into the lessons.

Selection and use of teaching resources

A second issue emerging from data was that participants used only the resources stipulated in the curriculum policy and in the readily available lesson plans. This finding supports the suggestion made earlier that teachers need to be more innovative and creative when adapting material. They need to be flexible and avoid using the lesson plan like a recipe or as a prescript. In certain instances, the resources used by participants bore no relation to students' experiences. For example, a majority of the girls had no idea how to use the resources like a die and cards. In one case where the teacher used cards, students did not have an interest in taking part because they did not understand the rules of the game. Girls especially were reluctant to be part of the game when the experiment was conducted. The following examples and responses from **PR2** illustrate this:

Interviewer: *Ok, today you did an experiment using the cards. As I was observing the lesson, and when some of the students in each group were shuffling the cards, I realised that some students, particularly the girls, did not understand how to play cards and, as such, were not interested in the game.*

PR2: *Ja…* (embarrassed)

Interviewer: *What do you say about this situation?*

PR2 *Actually, I could have started by introducing the cards and explained the rules of playing the cards so that those who don't know the cards could be exposed to them.*

This exchange suggests that the teacher did not make time to introduce the resources to students. **PR2**'s acknowledgement of his approach links closely with Cheng et al.'s (2021) advice that teachers need to make a careful selection of instructional resources. Also implied was the difficulty experienced by the participant in incorporating resources in such a way that they made sense to all students; the purpose of using these resources was not explained. It was difficult to explain this finding, but it may have been related to the way questions are asked in the textbooks and in the workbook that these participants were using. The following conversation supports the view that some teachers did not understand the importance of using teaching aids during lesson presentations:

Interviewer: I saw this morning, in your presentation, you used a die; cards and a coin.

PR2: *Yes.*

Interviewer: Were these the only resources you could use to teach probability?

PR2: *No. I did not have time to make a spinner. I was able to get the ones that I have mentioned and used.*

Interviewer: Apart from the four that you mentioned, are there any other resources that you could have used?

PR2: *No, only those*.

The following interview transcript makes the point more clearly:

Interviewer: Which teaching aids do you prefer to use when teaching probability, and why?

PR$_1$: *Teaching aid?* (It appeared that I had made a mistake by asking such a question.)

Researcher: Yes, the resources.

PR$_1$: (She took her time before she responded.) *I don't know because I am not using the resources.*

Indeed, in the lessons that **PR$_1$** presented, she did not use any resources. This might suggest that the participant did not realise the importance of doing experiments in relation to the theoretical aspect of probability. It is probable that a weak link exists between the purpose of conducting experiments and how the resources are introduced. None of the participants highlighted the fact that as the number of trials keeps increasing, the experimental probability tends towards the theoretical probability. They did not relate the experiments conducted to real-life experiences either. This confirmed the need for the review of teaching strategies to teach probability and the use of realistic examples that make sense to students. To elaborate more on real-life experiences, the cultural perspectives of elders are presented as follows:

African perspective: cultural beliefs and experiences

I included this information to highlight the importance of having a proper background of information before any conclusion can be made, whether in a form of prediction or an indication of the degree of uncertainty. If students do not have information based on cultural practices and experiences, and a question is asked, it would be unlikely that they would answer correctly. In one of our family gatherings, I gained some insight when some elders explained how they understood probability from a cultural perspective. Culture is the dominant discourse that elders use when constructing meaning of common patterns that happen in their environment. They indicated that they acknowledged certain cultural beliefs by observing trends as they happened every year. One elder provided two examples of what this meant. This is a paraphrase of his words:

Sign that there will be heavy rain

According to the elder, this happens almost every year. What he indicated was that through experience, a big bright star like a "burning coal" appears in the western sky in the evening. In a specific position it signals the probability of heavy rain. But if it occupies a position a little

bit higher than normal, then average rainfall can be expected. If there is no sign of this star, then there will be no rainfall. This then gives a signal for them not to plant any mealies as they will experience drought during that year. Equally, if the moon rests in this position, it means that they will experience heavy rain. If the moon rests in this position, (like rotating the object clockwise) it means they might have little or no rain at all. He also indicated that it was a sign that people would become ill. What frightened me was when the elder emphasised that if the moon rests in this position, (like rotating the object anti-clockwise) it was a clear sign that it would be very bad during that year; meaning that they would experience drought and illnesses.

The cow's horn

If the cow has one calf, there will be one mark on the cow's horn. If there are two marks, then it means the cow has two calves. How they benefited from this mark was that no one (not even the cowherd) could steal calves and, secondly, if you bought a cow from someone else you could immediately see whether it was old or still young. If a cow has a mark next to where the horn ends, it means it is old and you should not keep it for too long. If you sell it to other people, they will not benefit from it.

A very cold winter

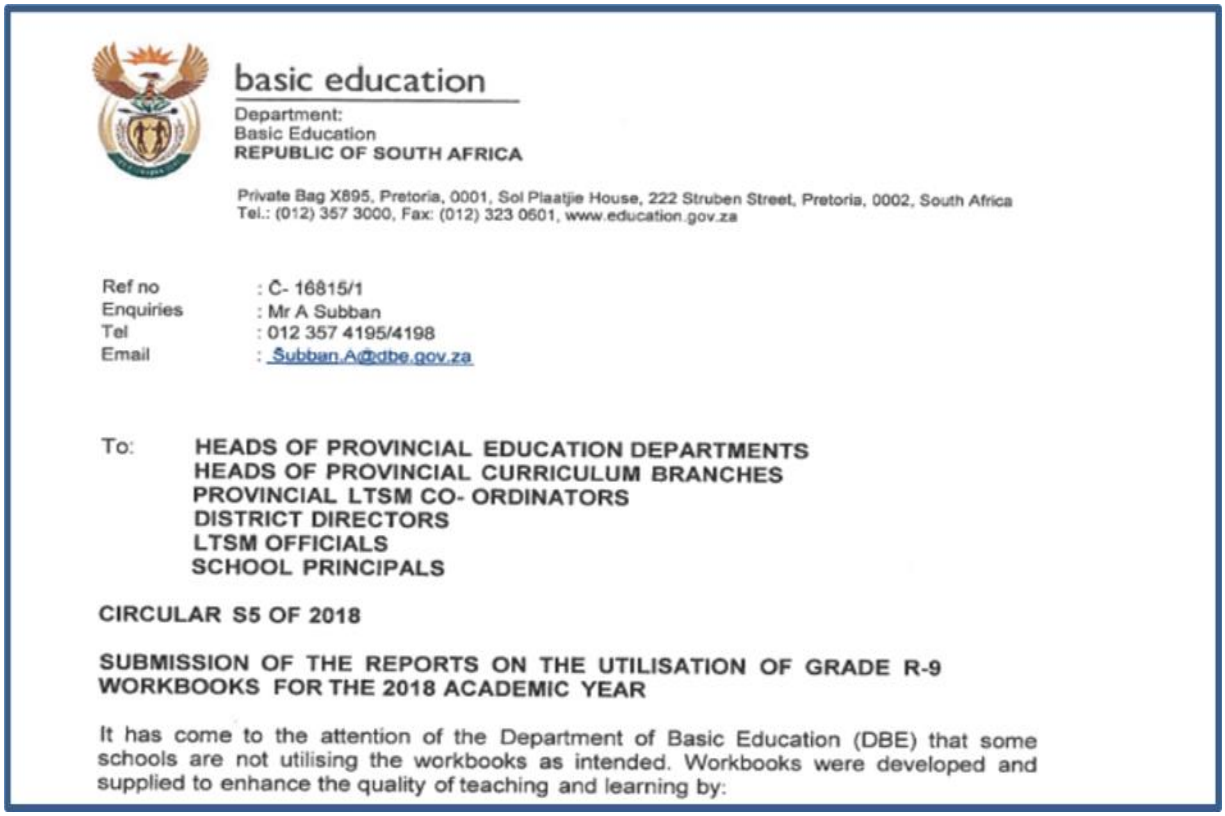
The elder indicated that a cluster of stars (called '*Magala*' in their native language) usually signifies that during that year they will experience very cold weather. According to the elder, the stars usually appear on 25 May. I was surprised because the elder did not mention any external factor that influences natural and cultural processes. The elder believed and emphasised that observing cultural practices would resist man-made interference. He believed that everything is controlled by God and happens naturally.

Despite the apparent uncertainties highlighted above, the elders appeared to have an insight into phenomena that were beyond their educational reasoning. They appeared to understand the concept of certainty through a respect for cultural practices. This suggests that the more we experiment (in this case what the elder experiences as a pattern) and the number of trials (happenings) keeps increasing, the experimental probability tends towards the theoretical probability (expected outcomes).

The elder's perspective concurs with Ozofor and Onos's (2018), Pradhan's (2017), Simamora and Saragih's (2019) views who emphasise that the local culture that links to the students' immediate environment should be integrated into the lessons. Although these are the views of individuals they are, according to the elder, also an indication of strong beliefs about the importance of respecting God and nature. The following sub-category relates to how participants used the DBE workbook as part of their classroom practice.

The use of DBE workbook

Although schools use different textbooks to facilitate learning in the classroom, the DBE recommends that teachers use the workbooks provided to enhance the quality of mathematics teaching. The extract from Circular S5 of 2018, as shown in Figure 5.9, provides for this urge to use the DBE workbook.



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LTSM OFFICIALS
SCHOOL PRINCIPALS

CIRCULAR S5 OF 2018

SUBMISSION OF THE REPORTS ON THE UTILISATION OF GRADE R-9 WORKBOOKS FOR THE 2018 ACADEMIC YEAR

It has come to the attention of the Department of Basic Education (DBE) that some schools are not utilising the workbooks as intended. Workbooks were developed and supplied to enhance the quality of teaching and learning by:

Figure 5.9. Circular S5 of 2018 (DBE, 2018c).

Although participants used the workbooks, following the directive from the DBE, some lamented the use of textbooks, and this was attested to in the following conversation between participant PR_1 and the interviewer:

Interviewer: *In the two lessons that you presented, you did not even once refer students to any activity in the workbook.*

PR1: *Most of the time I use the textbooks because the workbook is supporting us, but there are certain things that I don't understand. I found that they are not clear to the students.*

Interviewer: *Why didn't you consult the teacher's guide?*

PR1: *There is no teacher's guide for the workbook. It is also difficult to gauge students' understanding of the concepts if there are no answers or guidelines given for the problems. Eh… mmm… Maybe I should also indicate that it does not mean that I am not using the workbook totally, I sometimes used it for the classwork or homework.*

Interviewer: *If you describe the use of the workbook in terms of percentages, how would you rate this.*

PR1: *I only use it 40%.*

Interviewer: *What does 40% mean in the context of this discussion?*

PR1: *It means that I am not relying on it as much as I do with the textbook.*

Interviewer: *Did you ever raise your concerns about the use of the workbook in your meetings with the departmental officials?*

PR1: *No, maybe next time.*

The participant's responses pointed to a reluctance to use the workbook. This may have been because there were no teacher guidelines to confirm the answers. Alternatively, it may have had something to do with the way the questions were posed in the workbook. When discussing their experience of the professional development workshop, participants had different views. **PR1** not only highlighted the importance of this workshop, but also voiced concerns about its focus and responded as follows during the interview:

Interviewer: *How did you find the (1+4) lesson plans provided by the provincial government?*

PR1: *We are benefiting from it.*

Interviewer: *The reason I am asking this question is because you have developed your own lesson plan. Why are you not using the other lesson plans?*

PR1: *I found that when I look at the lesson plan, I found that there are certain terms that have been skipped. I wanted to clarify them, and they are very important.*

This discussion revealed that the participant saw the necessity of reviewing the teaching activities to close identified gaps. Even though this was done, the difficulty remained of designing teaching and learning activities in such a way that students would be able to:

- Formulate scenarios that would enable them to design their own teaching resources,
- Devise their own scenarios to introduce the language of probability
- Understand the importance or the aim of conducting experiments in probability lessons.

Participants found it particularly difficult to initiate discussions and respond to students' questions. The challenges identified also relate to how teachers give feedback to students and how they address errors and misconceptions identified. Another challenging area relates to how participants incorporate students' questions and answers into a lesson. Most participants found it challenging to:

- Shift from a traditional teaching approach to the facilitation of learning.
- Manipulate the designed tasks to encourage students to engage in constructive learning on their own as creators of their own knowledge.
- Incorporate students' questions and responses into the lesson to inform teaching.
- Manage the discussions so that they allowed the exploration of ideas.

During the presentation of lessons, most participants could not identify students' learning difficulties or misconceptions because of poor questioning techniques. In addition, some teachers could not mark all their students' work because of the time factor. It was therefore difficult for me to determine whether learning took place or not. While participants appeared to have an idea of which probability concepts and skills to develop, more work needed to be done in terms of questioning techniques to expose students' learning difficulties and misconceptions.

Drawing from participants' responses, it appeared that most teachers had not completed formal training in the teaching of probability. During the interview, participants indicated that they had attended training workshops on probability but usually these comprised only discussions of the

activities developed at provincial level. No amendments were made to these activities. The aim was merely to check the accuracy of the methods and the answers provided.

It appears that development of teaching activities was guided by how the content clarification is explained in the CAPS document. This was confirmed by the fact that all the resources used by participants were prescribed in this document. Teachers were not innovative in creating their own resources. Their teaching approach was characterised by following the common lesson plans to the letter. There was no adaptation of activities, and questions did not raise students' awareness of the importance of probability. It also came to light that the common lesson plans referred teachers to the DBE workbook for homework and classwork exercises. All these challenges point to the importance of providing support to teachers to enhance their classroom practices, particularly on how to design teaching activities that are real to promote conceptual understanding.

These findings revealed that teachers did not have much experience in the augmentation of activities to cater for the diverse contexts of the students. This view alludes to advice by Cheng et al. (2021) who urge teachers to adapt the teaching activities and engage in the material review processes. Additionally, these findings suggest that participants relied solely on the activities provided in the common lesson plans and those in the textbooks or workbook.

It was also not clear why some participants were unable to improvise and incorporate the resources into their lessons. The most common resources used by the participants when teaching the concept of probability included a die, spinners and cards. These are the only resources suggested in the CAPS document. Figure 5.10 summarises the findings discussed in the previous sections.

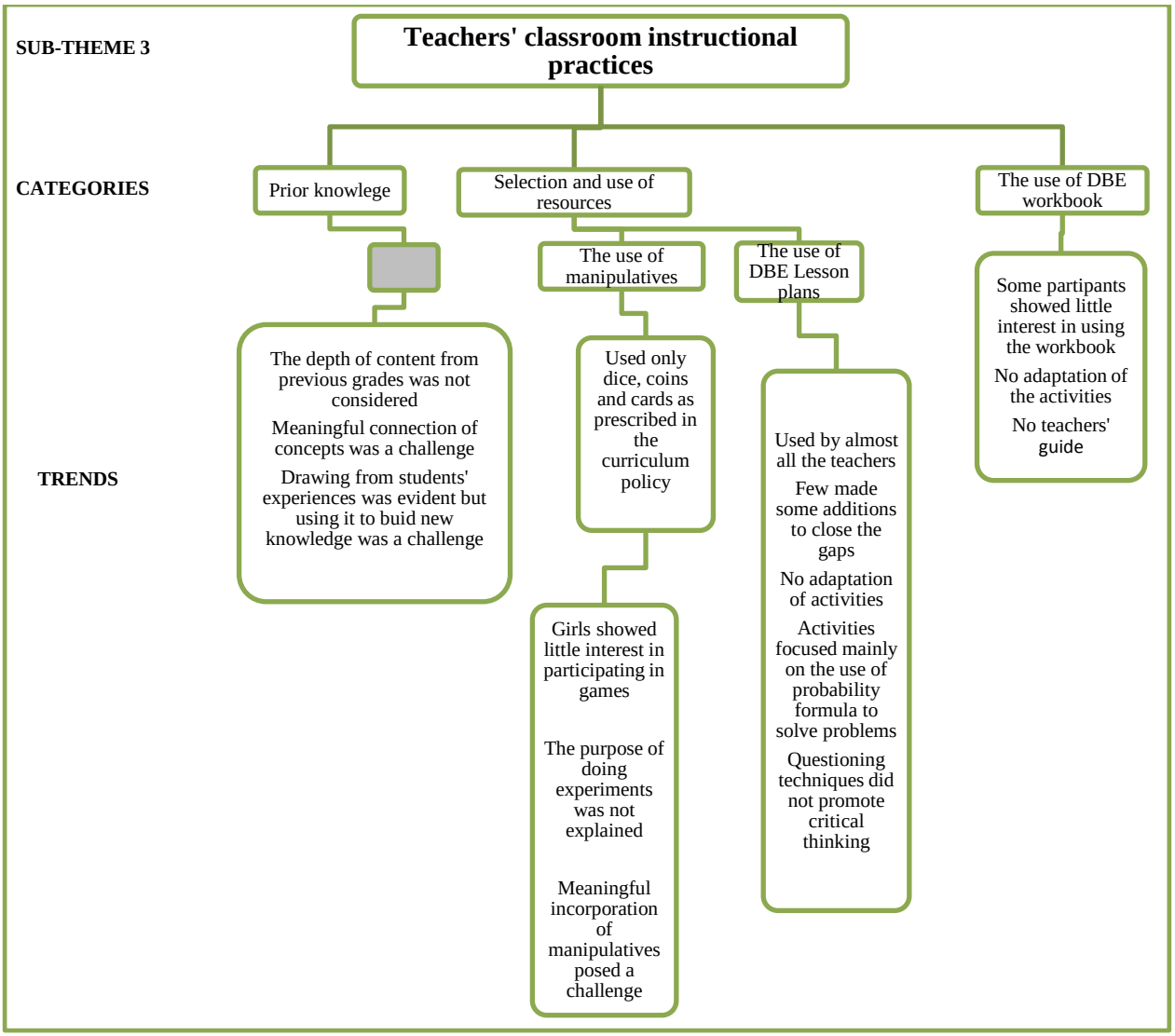


Figure 5.10. Trends observed in each category

5.2.2 ***Inferences from the overarching theme: Limited conceptual understanding of probability***

The main aim of this section is to summarise and provide an overview of the analysis of findings in relation to the overarching theme: Limited conceptual understanding of probability. I explored teachers' experiences of how they developed the concept of probability with the intention of gathering evidence to inform processes (the development of the framework) in enhancing their instructional practice. I analysed participants' written and spoken responses to identify the core indicators of conceptual difficulties they experienced**.** Trends in expressions shared by all participants served as a basis for the intervention workshop and the decision to empower participants. These trends guided the next phase of the study as follows: They

- provided an overview of the findings to place the intervention workshop in the proper context.
- provided a global view or scope for processes and procedures to highlight aspects for teacher professional development.

Such challenges were interpreted, deliberated on and theoretically contextualised in the second phase of the study. A bird's eye view of the challenges identified in this phase suggests prioritising the importance of reinforcing ideas gleaned from the literature, extending the ideas that emerged from this study and bridging gaps in knowledge to assist teachers to engage in an innovative way. The process of connecting the qualitative data to the literature study and the conceptual framework contributed to the design and the propositions of the intervention workshop.

The workshop was intended to enhance participants' strengths and address their difficulties. Specific material was developed for the workshop. All these activities were informed by the challenges identified in the first phase of the study. The structure and the content of the intervention programme were similarly guided. To build on participants' comprehension of chance, evidence from this study suggested that the intervention workshop should be designed to empower teachers to be able to:

- search appropriate information on their own to empower themselves. For example, to find out how forecasters predict the climatic conditions; that is, to have a proper understanding of what informs this prediction;
- make meaning of the essence of probability; to explain the concept of chance;
- engage students in constructive learning to construct their own knowledge;
- develop or adapt teaching and learning activities from the resources;
- develop appropriate activities with clear instructions; to ask questions in such a way that they spark discussion and lead to further questions;
- use various modal verbs to convey varying degrees of certainty;
- understand that one modal verb can be used in different contexts, the meaning of a modal verb changes according to context and understand that modal verbs are used to communicate awareness of opportunities and risk involved.

The next section presents a discussion of the findings from the intervention workshop.

5.3 Findings: Phase 2 participants' professional development and learning context

This section was designed to address the weaknesses identified in the first phase of the study. To address the challenge of limited conceptual understanding of probability, the following question was asked: How can teachers' conceptual understanding of probability be enhanced to improve their classroom practice? The intervention processes unfolded over four stages: Firstly, through the review of journal articles; secondly, through the development and analysis of guidelines for adapting teaching activities; thirdly, through reflecting on the developed guidelines and, lastly, through looking back on teachers' participation in the study.

5.3.1 *Theme 2: Strategies to develop a conceptual understanding of probability*

- **Making sense of the notion of conceptual understanding**

This section discusses how participants made sense of the concept of probability. However, it emerged during deliberations that it was not possible to focus on conceptual understanding in isolation to other strands of mathematical proficiency. This means that although the focus was on conceptual understanding, other strands of mathematical proficiency automatically featured as part of the development to give a global perspective. However, the vocabulary used to analyse the finding was borrowed from the core indicators of conceptual understanding as proposed by Kilpatrick et al. (2001). The findings in this section are quoted verbatim and analysed from the perspective of the participants.

All participants were given two articles to read and summarise according to their own understanding. Participants were expected to highlight the key ideas in each article. One article discussed how teachers developed mathematical proficiency in a classroom setting. The second was on data analysis using the five strands of mathematical proficiency.

On seeking views on how participants made sense of the two articles, **PR_{10}** responded as follows: *The articles assisted me to unpack and thoroughly understand the five strands of mathematical proficiency.* She went on to name all the strands of mathematical proficiency as mentioned in the articles. Her responses included references to the importance of providing examples when explaining the strands of mathematical proficiency. Even though she felt that the understanding of these strands could enhance teaching and learning in the classroom, **PR_{10}** alluded to the fact that "*Drawing a line between the strands of mathematical proficiency is a challenge, since they are related and contribute to each other".* She added that *"The strands*

should be accompanied by practical examples in order to show the difference between the strands and not to confuse them". **PR$_1$** also responded and said: "*The five strands of mathematical proficiency to be taken into account while teaching mathematics".*

Although **PR$_1$** and **PR$_{10}$** managed to touch on key aspects of the articles, I realised that the question (where I had asked participants to read and summarise the article) exposed a serious weakness as it was broadly answered. Furthermore, participants could not articulate the underlying meaning and practical implications of the strands of mathematical proficiency. This finding serves as a hint for researchers to use questions that develop participants' thinking beyond simple recall of knowledge. The preceding view is brought about by the fact that participants lack experience of analysing articles. I regard it as a limitation for having not asked questions that develop thinking and understanding beyond summarising the article. I immediately realised that I needed to review, adapt and ask the questions that lead to exploring new ideas. In other words, even though the focus might be on articles, the questions should be relevant to assist the teachers to develop from the level where they are.

The difficulty participants had in analysing the articles was made clear when **PR$_1$** said*: "From the two articles I read, I learnt that teaching of mathematics is best using qualitative classroom data".* What I have picked up from this response was that the participant did not even understand research terms like 'qualitative'. Although the participant was pleased that she had been able to learn about two new concepts in the articles, the meaning of the two terms was not fully understood. **PR$_1$** reiterated the fact that she had little understanding of the two research approaches and perhaps it would have been better if the two terms were explained using examples. When questioned further about the two research approaches, she said: "*I thought quantitative refers to content coverage and qualitative meant the quality of activities given to students".* It was clear from the response that the two research terms were relatively new to the participant.

I acknowledge the fact that participants managed to explain the strands of mathematical proficiency as described in the two articles. However, they only gave their own opinions grounded on insufficient information on how to analyse an article. I took it for granted that they would be able to identify the main ideas in the articles and explain in detail the meaning of each strand of mathematical proficiency. A possible explanation for participants not being able to demonstrate their ability to come up with clear opinions and conclusions might also relate to

insufficient time given to read and analyse the articles. In view of these findings, I decided to narrow the question and ask participants to explain the notion of conceptual understanding, based on what they had read in the articles.

In response, **PRIA** indicated that she understood conceptual understanding as "*the manner in which students organise knowledge".* She added that conceptual understanding means "*grasping a concept*". When elaborating on her understanding of grasping a concept, she said "*for example, if the student has understood the linear equations from earlier grades, he or she will be able to apply the understanding in trigonometric equations".* This idea was expanded on by **PRIB,** who said "*this is about making connections amongst concepts and they are able to justify on what they did in solving the mathematical problem".* In his explanation of what conceptual understanding entailed, **PRIB** unknowingly touched on adaptive reasoning, another strand of mathematical proficiency, when he emphasised that students should learn to justify the choice of methods used to solve mathematical problems. This observation confirms the understanding that strands of mathematical proficiency are intertwined (see section 2.3).

It was difficult for participants to explain what it meant to say the strands of mathematical proficiency are intertwined. This was attested to by **PRIA** when she said,

> *For me you may say I had a misconception. The way I think things through, you know, I think of myself as someone who is technical, things have to follow a particular order. I was struggling to understand that the strands are intertwined, we don't learn this strand, this strand, this strand and eventually get there, but actually learnt that there is no order. In my head, I wanted conceptual understanding to be the first strand that you actually have. As you are learning other strands, you are eventually gaining conceptual understanding.*

PRIA's understanding appears to propose that that the strands cannot be developed in a particular order or in isolation.

Taken together, the participants' responses suggest that the manner in which the strands of mathematical proficiency are developed is critical. Critical in the sense that, for teachers to fully develop each of the strand of mathematical proficiency, it is necessary for them to reconsider their questioning strategies.

A view shared by **PRIC** and **PRIDo** was that a proper understanding of basic operations was key to conceptualising mathematics. In support of this, **PRIDo** said:

> *I was lost, hei!!!* (laughing). *Colleagues are debating what is conceptual understanding. In my case, I said conceptual understanding can be tested maybe by giving two learners a R10.00 to share amongst themselves. I wanted to see if they understand the concept of division and also the concept of half. Just to understand that if they have R10.00 each one of them will be taking a half of what is given to them.*

The common key issues that emerged from the discussion of participants' comprehension of conceptual understanding was the application of mathematical concepts, the interrelatedness of the strands and the contextual understanding of basic operations. However, realising that little had been said about the other strands, particularly the productive disposition (encouraging students to love and see mathematics as worthwhile), I took it upon myself to further clarify the interrelatedness of the strands. For example, I emphasised the fact that, to develop student disposition, it is important to give them enough time and appropriate resources to make sense of the concept. Moreover, I argued that *"Teachers need to use their strategic competence and conceptual understanding to develop a spirit of curiosity and the love for mathematics*" (DBE 2011, p. 8). One further important observation was that participants explained the notion of conceptual understanding by giving practical mathematical examples.

Before closing the discussion of what the strands of mathematical proficiency entail, **PRIA** asked a question about the issue of productive disposition and she provided the following scenario:

> *Sometimes learners maybe eh…, are seating there and they are really stuck, and you want them continuing solving the problem and you as the teacher can see that they can answer a particular question, for instance, if they are able to answer the question, then they will be able to move and get ahead of solving the problem. So, asking a question you thought will help them answer the problem. My question is: Does is it help in developing the disposition or hamper the development of the disposition?*

This question was immediately redirected to the other participants. A variety of perspectives were expressed. **PRIC** said:

> *In my opinion, by asking that question, it will definitely uplift the morale, motivate them to answer the question. In a mathematics class, learners need to struggle, it will boost not hamper the disposition.*

Similar sentiments were expressed by **PRIB**:

> *Students should be allowed to get stuck. As a teacher you need to question their methodology to get how their understanding is developing, for them to identify what could help them to proceed … it is unfortunate that the challenge that I see is for teachers having pressure to complete the syllabus within a specified particular time frame and as such, it might not be easy to develop the dispositions.*

This confirms what **PRIA** mentioned during the telephonic interview:

> *In my opinion the article is an eye opener for a mathematics teacher such that when one focuses only on test scores and on teaching learners procedures, they are not doing justice to the learners but are simply just complying with finishing the syllabus.*

On a more positive note, **PRIB** referred to the significance of developing the dispositions:

> *But what I think it does in the long run, it will help them to develop a skill of questioning themselves like: students asking themselves questions like: Where did I get stuck? Why did I stuck when solving this problem? They will continuously stuck but let us encourage them to ask questions.*

PRIDo did not seem to be comfortable with asking students leading questions when drawing out the views of students:

> *Most of the time we should not only explore the leading questions, but we should ask ourselves what makes them not to connect into the topic. Because it might be that somewhere in the process, they might lose meaningful concepts that should form transition to what they are doing at that present moment. So, asking them leading questions might help but then you need to investigate as a teacher to say what is it they might have missed into my presentation.*

An implied common view of **PRIDo** was that students should not be assisted in solving problems per se but should be led to understand their mistakes. Likewise, Sands (2014) holds the view that teachers should think of other strategies to assist students but not trying to stop

them developing misconceptions. Students learn best from the misconceptions they have. In principle, participants agreed on the importance of leading questions in developing disposition; however, there should be a reasonable approach or strategy to tackle this issue. What participants suggested was rooted in:

- questioning students' methodology in solving problems;
- reflecting on their own lessons to identify missing links or where students might not have grasped concepts well enough to allow a smooth transition;
- raising students' morale.

To sum up, **PRIA** made a suggestion and said: "*In future it would be important to explore further how to effectively develop productive disposition in probability and I think this could be applicable to any topic in mathematics".* It was very interesting and encouraging to realise that the participant has acquired new knowledge and skills. I was impressed because **PRIA** highlighted the fact that what they had learnt in mathematics through the workshop could also be applicable to other areas.

Participants' inputs on the importance of developing students' productive disposition are broadly in line with Le Roux's (2020) suggestion that mathematics should been seen as a sense making endeavour - useful and worthwhile. The idea of generating questions came from participants. They indicated that to enhance their understanding of strands of mathematical proficiency, they preferred to generate questions that may spark discussions and encourage deeper thinking. Such questions were aimed at guiding them on how to develop the strands of mathematical proficiency. They used the core indicators of mathematical proficiency to generate such questions. Some of the ideas that came from the participants are captured in Table 5.1 as follows:

Table 5.1

Guiding questions on how to develop students' proficiency in general

How to develop conceptual understanding	Procedural fluency	Strategic competency	Adaptive reasoning	Productive disposition
Can students apply what they have learnt? Can they solve routine and non-routine problems? Do they use the correct language when interpreting word problems? What kind of representations do they use to answer questions?	What informs the procedure used to solve the problem? Is the procedure appropriate, well-organised, exact and true? Is it the correct way to complete the task? Can the strategy be adapted to similar but new situations?	Can the students arrive at their own way of solving problems? Are they able to test that their strategies work? Can they formulate their own problems in relation to what they have learnt? Are the students clear about the problem?	Can students explain what they did and how? Are they able to reflect on the processes of solving the problem? Can they share their experiences? Do they consider evidence when solving problems? Can they weigh the options before they make any decisions? Are they able to make an informed judgement or decision?	Do they show an interest in working on the given problem? Are they motivated? Do they persevere? Are they patient? Are they diligent? Do they enjoy the task? Do they all participate?
More ideas provided by participants to explain adaptive reasoning and productive disposition further.			Using reasoning consistently to conclude. Serious thought or consideration, thinking about what they say, analysing their experiences, applying their learning to their practice.	Provide some remarks to develop the disposition. Understand that in the end they are doing something worthwhile and not pointless.

Source: Adapted from Kilpatrick et al. (2001)

These questions are firmly grounded in Kilpatrick et al.'s (2001) strands of mathematical proficiency. The idea of generating questions proves to us that learning occurs in different ways. In other words, acquisition of knowledge or skills could happen through study, experience or being taught.

Acknowledging the importance of asking these kinds of questions **PRID$_0$** said:

> *The questions are very good, and I will therefore suggest that we have a glossary of the highlighted words in order to know exactly what kind of teaching and learning activities are we to develop.*

Sharing the same sentiments, **PRIC** said:

> *To have a proper and conceptual understanding of the underlined words and statements, we need a thorough preparation.*

When probed further to explain what he meant, **PRIC** indicated:

> *The reason why I say this, is because, for example, I have seen that the questions asked in most of the textbooks are not really using the verbs we have identified.*

Table 5.1 is quite revealing in several ways. Firstly, the verbs used could assist the participants to adapt the teaching and learning activities. Secondly, it might provide teachers with some ideas on how to develop the rubric for marking students' tasks and, thirdly, questions might serve as a basis for mathematics subject advisors to develop a lesson observation protocol. All the questions provided by the participants reflected an insight into conceptual understanding of probability, although the process would have been more interesting if they had had the opportunity to include and explain them in a lesson plan.

Reflecting on the articles, **PRIA** said:

> *I remember when I was reading an article, I saw that it has all the levels of Bloom's taxonomy, oh my word! I couldn't believe that one strand can just have all these things, you know, I just … you think that when you run from knowledge to evaluation, then you would have learnt, only to find that all of that talk to one strand and then, you know, the rest support you to sort of achieve that strand. Now I understand what it means when they say the strands are intertwined.*

The manner in which discussions unfolded in trying to make sense of strands of mathematical proficiency, links well with the views of Sands' (2014) argument that conceptual understanding develops gradually and is based on solid mathematical reasoning.

In order to get more clarity, I asked **PRIA** the following question: What do you mean when you say all of that talk to one strand? **PRIA** explained:

> *When I analysed the constructs or indicators of conceptual understanding closely, I realised that for teachers to develop it effectively, it must be supported by other strands.*

To probe **PRIA's** response indirectly, I directed the following question to other participants: What about the development of strategic competence? Do you think it also needs the support of other strands? Although the question was not directly responded to, **PRIDo** said:

> *Eh… I think so because earlier on we discussed and emphasised that the strands are not separable. I also want to say, I understand it better if my focus is to develop conceptual understanding. If I have to focus on strategic competence, I don't know where to start without first ensuring that students have conceptual understanding.*

Other participants just nodded their heads to show that they agreed with what **PRIDo** said. Although they had indicated some challenges posed by developing other strands of mathematical proficiency, it was clear that most participants understood what it means to be mathematically proficient and to say the strands are intertwined. All the participants emphasised that the other four strands of mathematical proficiency are critical to developing productive disposition. All these findings point to the fact that teachers need to devise strategies on how to fully develop each strand.

On the other hand, it was also important to point out that a variety of perspectives were expressed as participants made sense of the articles. This is illustrated in the following:

> *The framework on strands of mathematical proficiency acknowledges the importance of procedural knowledge whilst also emphasising the foundational role of conceptual knowledge.* (**PRIB**)

When questioned further, **PRIB** indicated that

> *Because facts and methods learned with understanding are connected, they are easier to remember and use, and can be reconstructed when forgotten. If students understand a method, they are more likely to remember it correctly.*

PRIB's view seems to suggest that to carry out procedures efficiently, it is important for students to understand the underlying concepts. Equally important, **PRIA** emphasised more about how strands are related:

> *In teaching this topic, it was found that in paying attention to underlying reasons for learners' answers rather than whether the answer is correct or incorrect provides an opportunity to probe learners' understanding on a deeper level, which relates to the strategic competence and adaptive reasoning strands.*

Still on the issue of analysing the articles, responses included references to teachers' classroom practices: *"There is a tendency of wanting students to generate correct responses, rather than eliciting student's underlying reasoning"* ***(PRIDo).*** He further indicated that "*sometimes students give correct answers that do not match their explanations, use contradictory probability language when explaining their responses"*.

Similarly, **PRIA** said

> *In scrutinising students' responses, it was found that in teaching probability, one needs to pay thorough attention to the language of probability where the words need to be explicitly explained during instruction.*

It was significant that this issue of language surfaced during the needs' analysis stage of the study as well. These views also suggested that although participants acknowledged the fact that the strands are intertwined, it would still be necessary to explore the strands further to gain more insight into what each entailed. As pointed out in the literature review, the main aim of this study was to explore how teachers' conceptual understanding of probability framed the plan to enhance their instructional classroom practices. To achieve this aim, all the challenges identified in this chapter would be reflected upon and translated into proposals to guide the processes in enhancing the conceptual understanding of probability.

The participants unanimously confirmed that the discussion was fruitful and that the issue of productive disposition had been clarified with appropriate examples. This was also affirmed by **PRIDo** when he said:

> *Yes, I was impressed by actually everything, especially the last part of getting learners interested which was productive disposition. It is inspiring, you know, to get learner interested in what you do in mathematics. But it comes all the way from how the strands are developed, but at the end of the day, it gets to productive disposition. Much was not said in the article about productive disposition. I think I was more impressed about productive disposition.*

PR_{10} echoed this:

> *Yes, the implementation of the five strands in our classrooms might change the negative picture painted by student performance in mathematics. If all teachers can teach mathematics for conceptual understanding, I am hopeful that learners will definitely achieve high scores.*

PR_1 offered an equally important view:

> *I can advise other educators to read and implement what was discussed because the strands make it easier for us to understand maths proficiency.*

These remarks were evidence that participants learnt from this exercise. Although he was happy about deliberations during the meeting, **PRIB** reiterated the importance of reading more articles on the phenomenon under study and remarked:

> *The article requires one to have read other articles of related topic and this was not part of the brief. The other articles that relate to the same topic should be provided or referred to for reading purposes to have broad context on the topic.*

The implication of the participant's suggestions is that they need to read several articles to intensify their understanding of strands of mathematical proficiency. Although I fully agree with **PRIB'**s sentiments, this needs proper planning, but it might work well in action research or when preparations for a workshop are made well in advance to allow for more engagement with participants.

Participants' responses, comments, suggestions and questions regarding their comprehension of conceptual understanding directed the focus of the subsequent activities in the workshop. This practice aligns well with Visnovska and Cobb's (2019) advice that it is always important to develop the material during the workshop. The advice is further linked to Schoenfeld, Thomas and Barton's (2016) suggestion that it is always critical to address people's perspectives of a phenomenon before engaging in the actual activities of the professional development programme. How teaching and learning activities are developed and implemented plays an important role in how students construct knowledge. The next section discusses processes involved in augmenting teaching activities.

5.3.2 *Theme 3: Augmenting teaching activities in probability*

What emerged from the discussions was that participants had no idea of how to adapt the tasks in the workbook. To guide the processes of developing the guidelines to augment the task in the DBE workbook, I asked the following question: How can we adapt the activities in the workbook in such a way that they promote meaningful learning in the classroom?

Two salient views arose from the discussions: participants mentioned the importance of focusing on current issues, and on context that was accessible to students. In support of the former view, **PR$_{10}$** remarked:

> *Although we don't have sufficient information about the Coronavirus, I think we should focus on the outbreak of Coronavirus, because everybody talks about the disease. The other reason is that it will be very interesting to get students' views about the virus.*

Similar sentiments were expressed by **PR$_{1}$**, who reiterated the importance of focusing on the Coronavirus:

> *What I like most about this topic is the fact that it can be dealt with in many subjects like natural sciences, geography, languages etc.*

When probed on the idea that the topic cuts across several subjects, she elaborated thus:

> *Ja…hey, you know* (laughing) *we are really learning here. You know in languages the teachers could come up with any topic around Coronavirus and students engage in debates or maybe the scenarios whereby students could engage in a dialogue. In addition, you know I am also an English teacher, students could also write a poem about Coronavirus.*

I was so excited so see how guidelines to augment teaching activities in probability emerged from the discussion. These were individual views but were also an indication of creativity and strong beliefs concerning the importance of drawing on real-life experiences. These views also signified the need for each teacher to consider the dynamics of his/her classroom when developing or modifying activities. Significantly, it emerged that the exercise of augmenting activities was key to developing teachers professionally in terms of what content to cover, how to facilitate learning, and the use of realistic examples. It is also important to emphasise that the activities of the workshop sessions would only make sense if driven by the participants' responses.

Although participants emphasised that the use of experiments was important to making learning interesting, they still regarded a die, coin, spinner and cards as the only resources that could be used for experimental activities. This could be partly attributed to the fact that even if they know of many other resources, they lack the skill of incorporating them in their lessons. It may also be an indication of their total reliance on the textbook and the common lesson plans provided to them. My experience as a subject advisor was that in most cases teachers introduce the resources at the beginning of a lesson; most of them had no idea how to incorporate the resources meaningfully in their lessons.

Owing to time constraints and the outbreak of the Coronavirus, participants **PR_1** and **PR_{10}** managed to develop guidelines on how to augment the teaching activities in probability. These guidelines were revisited and reviewed several times to check if what has been written match their intentions. Even though this was regarded as a fruitful exercise by the participants, **PR_1** lamented and said that:

> *It is unfortunate that we don't have enough time due to the outbreak of Coronavirus and we also don't have proper resources like laptops and internet, otherwise we were supposed to work collaboratively and prepare a formal lesson plan together.*

In the same vein, **PR_{10}** observed:

> *It is true because with that lesson we would be able to assess if the augmented activities could promote the development of student conceptual understanding of probability.*

This is an important area that should be explored to assess the impact of the modification of textbook or workbook activities on student performance. **PR_1** and **PR_{10}** emphasised the fact that

they had now begun to see the importance of developing their own lesson plans or augmenting any material provided by senior officials. This view was supported by **PR$_1$**:

> *We really appreciate the lesson plans provided by the provincial department of education; however, we need to realise that they basically serve as a guide in terms of what to focus on. It therefore depends on us as to how we present or implement them in the classroom.*

The acknowledgement by **PR$_{10}$** mirrors the advice by Nene (2017) who indicated that listening to students' feedback would enable teachers to adapt their lessons to students' circumstances. This was an indication that participants needed to be encouraged to adjust teaching activities to address the diverse context of the students. Because the two participants mentioned challenges of not having access to laptops and internet, through snowballing, they referred me to another teacher, two subject advisors and one mathematics coordinator whom I contacted to take the process forward. The second team analysed and reflected on the guidelines developed. The switch-over mirrored a relay in an athletic field whereby one group of participants engaged in a task for a period of time and were then replaced by a similar group to take the process forward.

Commenting on the proposed guidelines, **PRIA** indicated:

> *The proposals are significant in two major respects. Firstly, the strands of mathematical proficiency were incorporated to encourage teachers to create a learning-centred environment, Secondly, the importance of highlighting the purpose of each activity is key when planning for a lesson.*

However, **PRIA** suggested that the terms *probably, definitely* and *possible outcomes* be added to form part of other terms identified in the guidelines. In other words, she realised that there were missing probability terms that she thought should be included as part of the activities in the guidelines. **PRIC** was impressed about the proposed guidelines:

> *The ideal situation was to engage the participants who developed the guidelines on how to augment tasks from the DBE workbook so that we can learn from their experiences. We however understand that the situation is beyond our control.*

The fact that the two teams could not work jointly is a limitation of this study. Further research could be conducted to explore ways to bring teachers and subject advisors together to jointly input on the guidelines.

Although **PR_1** and **PR_{10}** did not seem to be comfortable with the fact that they could not engage in the actual process of developing the lesson plans, they did say that they had an idea of what to consider in developing a lesson plan. It was indeed a difficult moment for participants as lockdown was declared in the country. This was also a limitation for this study because teachers could not apply their minds fully as a team to the process of developing a lesson plan.

As way of making participants aware of the importance of revisiting and reflecting on the guidelines they had developed, I asked them to provide further suggestions. This exercise was also critical in tracking each participant's development, as it was an individual activity. In addition to this, I needed participants to understand that learning is a process, cyclic and reiterative. The aim was to allow participants to provide additional data not covered in the initial phase of developing the guidelines. It was important that participants could revisit and reflect on their work in order to enhance their instructional practices. Table 5.2 is an extract from feedback they provided after reflecting on the activities they developed. Information highlighted in yellow and green are **PR_{10}**'s inputs.

Table 5.2

PR10's reflections on the augmented activities

Strands of mathematical proficiency	The classroom implications. Use a tick to indicate if the activity meets the demands of the strand.		Your own comments (additions, corrections, suggestions etc.)
Adaptive reasoning The capacity to think logically about relationships among concepts and situation Knowledge to justify the conclusions	**Logical reasoning** **The TEACHER will recognise this when the activity encourages students to:**		Proving that something is true of false can be extended to accommodate a Yes/No, correct/incorrect etc True or false option not given on the activity but option such as yes/no is given **Suggestion to replace the dimension** Able to make a choice when given more than one option
	• Explain their thinking and add to the thinking of others.	✓	
	• clarify and justify a conclusion or an idea, understand reasons for underlying procedures and determine the legitimacy of a proposed strategy	✓	
	• Deduce and justify strategies and conclusions reached	✓	
	• Prove that something is true or false		
	• Compare related ideas and explain their choices	✓	
Procedural fluency Knowledge of procedure and knowledge of when and how to use procedures appropriately and skill in performing them flexibly, accurately and efficiently. Can they solve problems correctly according to the rules or formulae	**Accurate application of procedures** **The TEACHER will recognise this when the activity encourages students to:**		I suggest it should be stated on the activities that learners should use their own strategies as an instruction e.g. use your own strategies to calculate probability of.... Again, on describing how strategies will be used, it can also be linked to a question where teacher ask learners on how they managed to calculate probability and relative frequency. It will make it easy to see that learners are being encouraged to use own strategies from the question given. I think this strand can be evaluated by making sure that the instructions/ questions address the dimensions e.g. Use standard algorithm to calculate 123, use own strategy to calculate 123, use a different strategy applied in number 2. above to solve 123...
	• Invent their own strategies, methods for solving problems related to probability.		
	• Describe how they are going to use each strategy in solving probability problems		
	• Use algorithms (written or mental).		
	• Describe the procedures they are using.		
	• Justify why are they using such procedures		
	• Explain their invented strategies.		
	• Use different strategies to solve the same problem.		
	• Give them enough activities for practice to improve accuracy	✓	
	• to have knowledge of when and how to use procedures		

PR10 suggested that *"Prove that something is true or false"* could be replaced by the generic statement "*Make a choice when given more than one option"*. This suggestion makes sense as it would accommodate any kind of questioning strategy. On the issue of students inventing their

own strategies, **PR_{10}** indicated that no activity was included that asked students to invent their own strategy. This sentiment was shared by **PR_1** who said: "*Not clear"*. When probed she said: "*The issue of invented strategies was not catered for in the augmented activities. The extract expresses participants' views on the DBE workbook*". The conversation between the interviewer and the participants continued as follows:

Interviewer: *What is your comment about the activity extracted from the DBE workbook?*

PR_{10}: *The activity might not be beneficial to every student, since they come from different backgrounds. Some learners might not have experienced the game used in the activity.*

To clarify **PR_{10}**'s view further, Jing et al. (2017) caution that diversity in cultural and economic background may affect students' motivational goals.

In response to the question, **PR_1** said: *"The activity does not start from the reality of students. It has only one form of question or types of questions"*. Other views highlighted by the participants included considerations to take into account when using resources and drawing from real-life experiences **(PR_1).** The following conversation shows how **PR_{10}** expressed her views on the issue of context:

Interviewer: *What considerations did you make in analysing the augmented activity?*

PR_{10} *The augmented activity is developed in line with the learners' context which might benefit all the learners since the teacher considered the background and the environment of the learners.*

Interviewer: *If you were tasked with the responsibility of augmenting activities (from any textbook) on probability, how would you approach the process?*

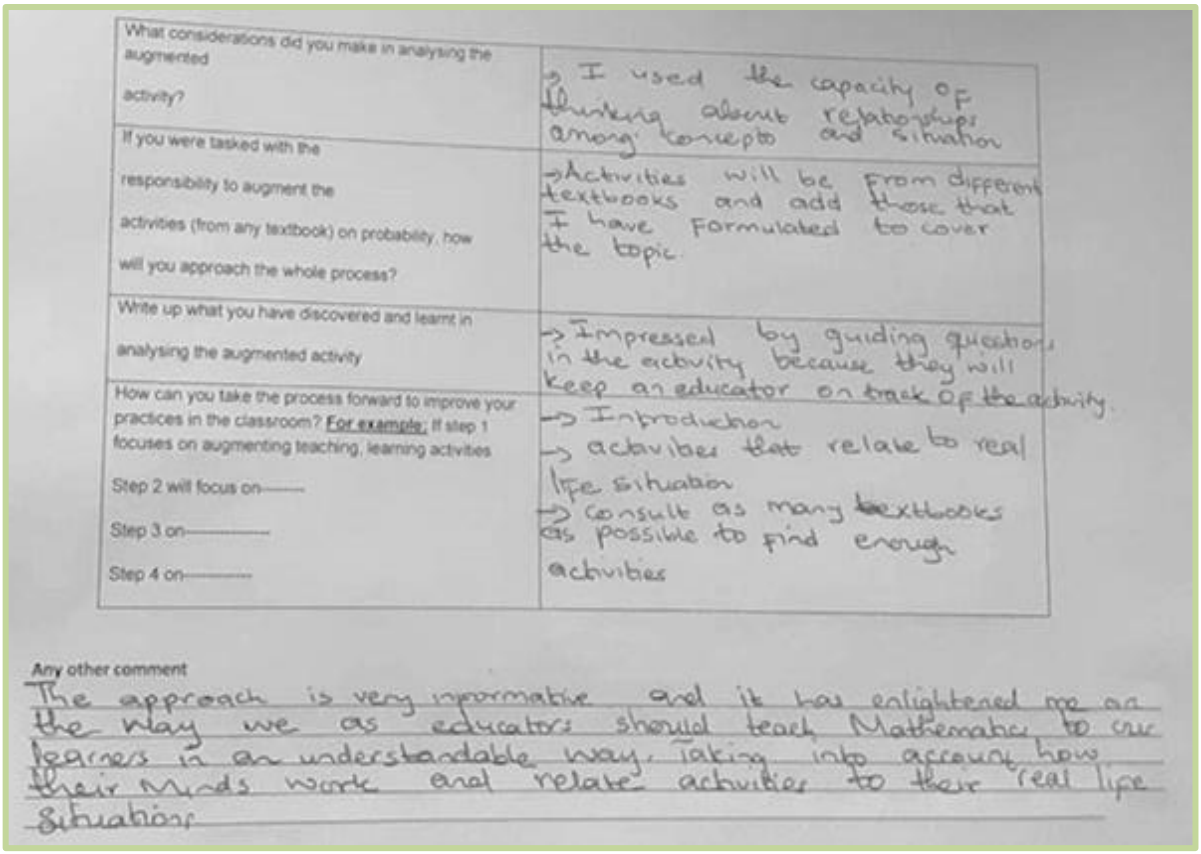
PR_{10}: *First of all, it is important to consider the context and the background of the learners, so as to link what they know with what they will learn. Every activity will have to consider learners' background. Again, the five strands of mathematical proficiency will be incorporated into every activity in order to help develop learners that are proficient in mathematics.*

Implied in this conversation is the importance of considering students' diverse context.

Interviewer: *What is your view on the augmented activities?*

PR$_{10}$: *My experience from analysing the augmented activity is that every activity can be improved to suit one's learning environment. Again, it is possible to incorporate the five strands when teaching a particular topic as the main aim of teaching mathematics. Augmented activities might have a greater impact in learning mathematics than the ready-made activities. If teachers could contribute in augmenting ready-made activities, they might have positive impact in the classroom, rather than using ready-made activities. Augmentation of activities could help teachers to be proactive and be able to use relevant activities in the classroom, since some activities in the textbooks have irrelevant context as far as learners' environment is concerned.*

Implied in **PR$_{10}$**'s response is the acknowledgement that augmented activities could help teachers to be proactive and use their own creativity to adapt teaching activities. Figure 5.11 indicates some of the responses from **PR$_1$**.



What considerations did you make in analysing the augmented activity?	→ I used the capacity of thinking about relationships among concepts and situation
If you were tasked with the responsibility to augment the activities (from any textbook) on probability, how will you approach the whole process?	→ Activities will be from different textbooks and add those that I have formulated to cover the topic
Write up what you have discovered and learnt in analysing the augmented activity	→ Impressed by guiding questions in the activity because they will keep an educator on track of the activity
How can you take the process forward to improve your practices in the classroom? For example: If step 1 focuses on augmenting teaching, learning activities Step 2 will focus on—— Step 3 on—— Step 4 on——	→ Introduction → activities that relate to real life situation → consult as many textbooks as possible to find enough activities

Any other comment

The approach is very informative and it has enlightened me on the way we as educators should teach Mathematics to our learners in an understandable way, taking into account how their minds work and relate activities to their real life situations

Figure 5.11. PR$_1$'s views on the augmented activities

Sharing these sentiments, **PR_1's** comments highlight critical issues such as the importance of consulting as many textbooks as possible when planning activities. She also indicated that knowledge of both content and students was very important. The most significant comment **PR_1** provided was: *"I am impressed by guiding questions in the activity because they will keep the teachers on track of the activity"*. My understanding of this comment was that every teaching activity designed should assist teachers to achieve the objectives of the lesson. What was particularly noticeable during the study was that:

- participants used the language of proficiency to discuss issues pertaining to effective teaching and meaningful learning. This finding was evident when participants showed interest in reading more articles to explore the strands of mathematical proficiency further.
- Participants supported the idea of augmenting teaching activities; this finding has an important implication for DBE officials concerning the monitoring of the implementation of the workbook. It also suggests that monitoring instruments should be revised in terms of what they focus on to support teachers.
- Although participants agreed on the importance of the use of appropriate resources, data showed that the proposed framework should provide more guidelines on how to effectively incorporate resources in the lessons.

PR_1 and **PR_{10}** explained that they felt nervous during the classroom observations but gained confidence as the study progressed, particularly during the interview. They indicated that although they could not develop a lesson plan, they were pleased that they had managed to share ideas and resources. **PR_{10}**, said: "*Collaborative planning allowed us to point out and discuss areas that needed improvement in our teaching"*. These views were confirmed by **PR_1** when reflecting on the augmented task: "*I am impressed about the guiding questions in the activity because they will keep the teachers on track in the activity"*.

It should also be mentioned that participants repeatedly stressed the importance of drawing from real-life situations and the understanding of the strands of mathematical proficiency. Two participants, **PR_1** and **PR_{10}**, argued that the modified activities were good, although the issue of invented strategies was not catered for. On the other hand, I felt that there was a need for an intensive workshop on how to develop lesson plans in the way participants suggested. Another area requiring attention was to help teachers to read and make sense of the articles.

In general, **PR_1** and **PR_{10}** applauded the process of learning how to augment and review the tasks because they benefited in terms of how to make teaching activities relevant. What was also promising and served as evidence that they had really learnt from the analysis of augmenting activities was a remark from **PR_1**: "*I would take the process forward by consulting as many textbooks as possible to have enough activities".* This indication suggests that **PR_1** would not rely on only one textbook or resource. It could also mean that comparing activities from different textbooks would help her to devise appropriate activities. The most interesting finding was that both **PR_1** and **PR_{10}** felt that they were *half cooked or half-baked* because the ultimate aim was for them to develop their own lesson plans, not to rely on the one provided by their superiors.

Analysis of the augmented activities provided an exciting opportunity to advance participants' ability to create a learning-centred classroom. The most striking result to emerge from the discussion was that participants learnt individually how to justify their conclusions or decisions. This is the skill that they need to inculcate in their students. What was also regarded as an achievement was the fact that participants were able to make connections amongst fractions, percentages, ratio, and probability terms. They made suggestions about various contexts which were within students' experiences.

- **More input on the developed guidelines**

This section provides an account of participants' views on the developed guidelines in relation to their conception of teaching for conceptual understanding. I felt that it was important to establish whether teachers and subject advisors had the same understanding of teaching for conceptual understanding. At the same time, it was important for me to understand how subject advisors viewed their role in supporting teachers. This activity was aimed partly at exploring how subject advisors made sense of the guidelines developed during Level 1 of the intervention workshop by:

- Establishing whether participants could draw on what they had learnt in the first session of the workshop;
- Assessing the extent to which participants could implement the suggestions made in the proposed guidelines;
- Advancing participants' knowledge of chance.

When asked to make some comments on how to strengthen the guidelines developed during the first level of the intervention workshop, participants expressed different views. However, the perspectives from which participants viewed the developed guidelines had one thing in common, the idea of using the indicators of conceptual understanding to analyse the activities: **PRIA** said

> *Honestly, I did not know what kind of activity I would have developed after going through the guidelines. The only thing that I could realise is that the task does not cater for strategic competence.*

This echoed what **PR10** said when reflecting on the guidelines they developed. **PRIA** indicated that

> *What stood up for me is the integration of geometry and measurement in these activities. The only thing I could do is to make follow-up activities to the ones already developed.*

From this I deduced that **PRIA** was able to make sense of the guidelines. The fact that she was able to highlight that the task did not cater for strategic competence suggested that she acknowledges that the strands of mathematical proficiency are intertwined and cannot be developed in isolation.

PRIC gave another example to describe the probability of the event happening. He added: *"It is likely that we will get a cure for Coronavirus".*

When asked why the term 'likely' was more appropriate, he reiterated and said: "*It is likely, not certain".* I questioned his response further and asked: *"From whose perspective are you arguing your point? Is it from the teacher's perspective or from the students' perspective?"* Although it was not easy to answer this question, he said: (taking a deep breadth) *"Er … it means we are hopeful that we will get vaccine".*

These words indicate that **PRIC's** responses was the result of the limited information he had on the Coronavirus and the government's plan to provide the vaccine. Although **PRIC** does not say that the vaccine would definitely be available, he raised a question: *Would students answer the question from the same perspective?* From these responses one could deduce the fact that teachers sometimes provide answers to open-ended questions instead of listening to the views

of students. Rather than emphasising what students understand from the background they have, teachers provide their own views. This view was explained further by **PRIA**:

> *I learnt a few things when it comes to probability, the learners' reasoning must concur with the answer they provide. If the reasoning is totally different, then the answer cannot be accepted.*

PRIDo added:

> *More activities to probe students' reasoning should be included because they develop their critical thinking.*

PRIA and **PRIDo**'s points of view suggest that the idea of requiring students to elaborate on their thinking is critical as one way of developing learners' critical thinking. In support of the idea that reasoning is one of the key aspects to making learning meaningful, **PRIDo** noted:

> *I will prefer to give the students probability terms and they provide scenarios and from those scenarios, I will be able to probe their thinking.*

It appeared from these discussions that one of the most suggested approaches to reinforce students' understanding was to develop questions that encourage students to support their arguments through reasoning and justification. As participants were expressing their views, I realised that there was still confusion about how to differentiate between the following terms: probability, possibility and chance. The responses below revealed that participants needed help in making sense of the identified terms. When asked to describe how these three concepts differed, they responded as follows:

> **PRIA:** *For me chance is the day-to-day language that is used. Possibility is broader than that. It can be possible, impossible, certain, chance of happening or no chance of happening. Probability is one chance out of thousand chances, it is a fraction* (laughing) *… once you talk about it, the terminology, you get tongue-tied as well. It sounds that you are saying one and the same thing.*

> **PRIC:** *Possibility and chance to me are almost the same. It is a likelihood of something to happen. Probability is when you express in fraction, like what PRIA has indicated.*

> **PRIDo:** *Probability has to do with sample space … neh… unlike possibility. For example, when I was in town, there is a possibility that I can meet my girlfriend. Possibility happens outside sample space. You need to look at the size of the sample space. Probability means there is a certain sample space. For example, if you are given a sample space of apples, what is the probability of picking an apple of a particular colour? Probability is the chance within the sample space and possibility is the chance out of the sample space.*

Although **PRIA**'s and **PRIC**'s explanations were not clear, it was apparent that **PRIDo (**the Grades 7 and 12 teacher) went further to include sample space. The reason for this was not clear, but it may have had something to do with how the concepts were explained in the textbooks. Another possible explanation for **PRID$_0$'**s response might be related to the fact that some of these concepts are dealt with in FET.

PRID$_0$ confessed:

> *I have been teaching matric for more than six years, neh! But to tell you the truth I have solved complex probability problems and I realise that even myself as a teacher I could not thoroughly clarify this terminology: possibility, chance, probability, unlikely. We all know that we can look at examples and solve problems, but to develop that understanding behind problem solving, the conceptual understanding is lacking. Before we move forward, we are at least clear of what is conceptual understanding.*

Implied in this confession, **PRIDo** understood that conceptual understanding is key to the development of proficiency in probability. When the participants were advised to consult resources for clarification of the identified probability concepts (probability, possibility and chance), **PRIA** said:

> *You cannot define probability without attaching the word chance or possibility. Probability is a measure of chance that a particular event will take place. Because it is a measure, then we can attach numbers to probability. We use possibility and chance to explain what probability is.*

Trying to clarify the concepts, **PRIC** said:

> *You can quantify probability as a number between 0 and 1 and no numerical value is attached to chance and possibility.*

PRIDo agreed with the preceding views but said explicitly:

> *A chance neh is an opportunity of an event in taking place and there is no quantifiable data for possibility and chance.*

PRIB defined probability as the *"chance of events happening within parameters"*. Quite clearly, these responses taken together suggest three important points. Firstly, that the three probability terms are linked. Secondly, that probability is a measure of chance and, lastly, that the degree of possibility can be described in words, fractions or percentages.

PRIB added:

> *Possibility* (pauses), *you are dealing with something that is either possible or impossible.*

This might suggest that if students are to clearly understand the term 'possibility', they need to find scenarios that differentiate between *possible* and *impossible*. The next section presents participants' views about the study.

5.3.3 ***Theme 4: Participants' views about the study***

On the issue of how participants viewed their engagement in the study, their responses referred to (i) their understanding of strands of mathematical proficiency; (ii) suggestions on how strands of mathematical proficiency could be introduced to teachers; and (iii) what could have been done better in the study.

- Strands of mathematical proficiency

The overall impression from the focus group discussion was that they benefited a great deal from the study. Participants' discussions and responses revealed that they did understand what the strands of mathematical proficiency entailed. The following quotations from participants' responses support this view:

PRIB:

> *The whole concept of strand is key to the study. I found it very useful in approaching, you know, learning in the classroom, especially if one is determined to have conceptual understanding of certain terms. The whole connectedness of the strands, If I have to develop someone, I would love to proceed from the conceptual framework of mathematical proficiency to guide my activities.*

To emphasise how useful the strands were in approaching learning in the classroom, **PRIC** said:

> *We need to align tasks to enhance student mathematical proficiency in probability. What I have observed is that we struggle to introduce new topics. We now know how we can introduce new topics, particularly drawing from real-life situations. I think we should now focus on how we introduce different topics at different levels. I think it will assist teachers as well as us as subject advisors and other stakeholders.*

These views also manifested in related responses:

> *We must always be aware how the teaching activities designed are developing the five strands of mathematical proficiency so that learners can be proficient in probability.* **(PRIA)**

This reasoning endorses the importance of incorporating the strands of mathematical proficiency when explaining mathematical concepts. Participants' views were focused on introducing topics in a classroom setting. In view of all that has been mentioned so far, one could suppose that participants' responses emphasised the importance of developing teaching and learning activities that would develop proficiency in probability. The preceding view is supported by Cheng et al. (2021) who indicated effective use of the curriculum material includes skilful selection and modification of teaching and learning activities.

Another significant comment was:

> *We can still use the strands of mathematical proficiency to analyse ourselves in terms of how we understand the notion that the strands are connected.* (**PRIB**)

What I have picked up from **PRIB**'s view is that strands of mathematical proficiency could be adapted to explore different situations. This view echoes the understanding that the core indicators of the strands of mathematical proficiency could be used to analyse how people understand a situation of any kind. It is therefore remarkable that participants proved that the strands of mathematical proficiency are connected and cannot be developed in isolation.

When giving participants the opportunity to provide additional responses on aspects related to the competencies required to introduce the strands of mathematical proficiency to teachers, **PRIDo** responded as follows:

> *I wouldn't give teachers articles to read. Introducing by saying people should read is a very difficult one. I don't know exactly how to do it but introducing by saying people should read it is risky. As I am sitting here, I am thinking to give them the framework [MTF]to read, but still the framework is more than the articles we read, 92 pages. Honestly, I don't know how to approach it, but I am still thinking.*

Although **PRIC** did not respond immediately, he eventually said:

> *Jo, ja nee…that is a difficult one, that one! But especially introducing this one er… remember er…as teachers when we meet someone it is once-off. You had a chance to give us the articles and set the date but normally is one day, no homework etc. In my view, maybe give practical examples of what they are doing in class and after that you bring er… they relate their activities to strands of mathematical activity. Let them link the activity to the strands.*

PRIC recommended that teaching activities should be linked to the strands of mathematical proficiency; however, as to how he would do this was not made clear.

The argument constructed by **PRIB** demonstrates the need for better strategies:

> *In Sotho* bare ke laka leo' **[**In Sotho language, we say I concur]. *Look, if one was to look also on the issue of methodology, ehm… in which we were proceeding from one step to another, you know there was this moment where we had to come up with our understanding of the articles. At the beginning I struggled with the article itself, but as you know, you came up with the PowerPoint presentation of some sort, when I could see the linkages … er, look, it is important that were given the same thing and analyse the articles because it was a new concept, loosely connected.*

To shed more light on what **PRIB** meant, Figure 2.2 (See section 2.2) was presented to them in order to clarify the notion of intertwined strands of mathematical proficiency:

After a lengthy discussion, **PRIB** said:

> *At first it was like strands are different things that are not talking to each other, but then it became clear how they link, and it became a very big lesson for me. To answer your question, I could have given them the MTF* [Mathematics Teaching and Learning Framework] *to read specific sections than giving them the articles.*

The lengthy response below explains how **PRIA** found it difficult to support teachers if they themselves were not capacitated. As to how best to introduce the strands of mathematical proficiency and how they could benefit the teachers, **PRIA** said**:**

> *Just to add, to be honest, I did not know about MTF until you spoke of it and I googled that and I found a draft on the document from the SA Foundation maths website and I downloaded it and I started reading it and then I saw that the framework is actually based on five strands and I started thinking that what could be learnt more is the engagement on a lesson plan because I believe lesson planning and preparation is more important for mathematics learning.*

This response confirmed the limited support these participants receive from their superiors and suggests that further training focused on lesson planning and preparation is required.

PRIA went on:

> *I acknowledged that we were trained on lesson plan and preparation without knowing about MTF and without knowing about strands of mathematical proficiency. Now we need again the training on lesson planning and preparation. We need that as subject advisors to be able to capacitate teachers.*

To clarify what she meant, she added:

> *Sometimes you go to school, you find that the teacher is trained on a particular thing and you are not thoroughly trained in that field, then you are unable to support teachers, hence the whole thing will fall apart. Because now teachers are doing one thing and you are not sure on how to support teachers. But whatever training that happens should cut across for teachers and subject advisors and it makes it easy for the process to be moving. Planning and preparation are important for us.*

PRIA was then asked the following question: *"If you are trained on how to plan and prepare a lesson, does it mean that you can still go to class and teach?"* She responded:

> *Yes, it is true, and it touches on what **PRIC** has said that introducing a topic is very critical because I believe how you introduce the topic would eventually lead to how you attain mathematical proficiency.*

PRIA's view touched on the limited support subject advisors receive from the district and province in relation to classroom practice; she believed that one could only provide relevant support on aspects that she personally had experience on.

In sharing his experiences as a classroom-based teacher, **PRIDo** said:

> *I am a classroom-based teacher, let me talk to what is happening in the classroom* (he laughs). *One thing that needs to be looked at is true is preparation. I agree, but what is lacking is coaching, neh… coaching a teacher maybe ahead of time. You know er… I can prepare but what if what I prepared even myself, I am not sure of, but, yet I have prepared.*

When questioned further on what he meant by coaching, **PRIDo** replied:

> *It is important to link preparation with coaching if you are the HOD or subject advisor. You make sure that you meet with teachers and sit down with them the whole day and cover what should be done in the following week as prescribed in the ATP. Allow the teachers to present how to introduce the lesson. So, teachers will introduce it differently, then amongst us, there will be those who are good, and we suggest perhaps this how I approach it.*

In sharing his experience, **PRIDo** emphasised that there are some teachers with brilliant ideas and in-depth understanding of the content; such teachers would be able to share good practices with fellow teachers. Although this might be one strategy to allay teachers' fears, it is also critical for teachers to creatively modify their practices themselves so that the method of transferring the content benefits the student.

A significant finding emerged from a response by **PRIA** in which an explicit connection between the essence of probability and what it means to assess students' understanding of probability was made. She said:

> *What I have learnt is that probability is subjective and that when you teach students probability, you shouldn't be quick to jump to a memorandum and try to mark the answer correct or incorrect without understanding the context in which students were basing their arguments.*

Significantly, it also emerged that participants were faced with several challenges that impinged on their effective delivery of content. Responses from one participant provide an example:

> *One thing that I also noted, neh…maybe because I am at the exit level, I feel you must also include this in the study that curriculum development of DBE speaks more to time-on-task than real conceptual understanding of mathematics where people are demonstrating that we are doing this model. We teach to assess rather than teach to understand the concept.* **(PRIDo)**

PRIDo described the ideal situation as follows:

> *What I am suggesting is what we are doing in the district, where we meet as subject advisors and teachers every second week. We take one day and cover the whole ATP for two weeks, then we coach each other, and we present. When teachers leave the training by Friday, they know exactly where they are going to start.*

This view is in consistent with Nel and Luneta (2017)'s understanding that mentoring becomes an effective teacher empowerment strategy if informed by teachers' instructional and content needs. These findings revealed that the relationship between district officials and teachers should aim at coming up with strategies that could assist teachers to teach for conceptual understanding. Put differently, what teachers wanted was rooted in efforts that would assist them in designing effective strategies to enable students to engage in meaningful learning. One cannot, however, disregard the point raised about the importance of reflecting on their practices as teachers. The following quotation from participant **PRIDo** substantiates this understanding:

> *But apart from every other thing, neh… on the other hand, it is also important to give ourselves time. If we don't give ourselves time, then we won't be ready for preparation on the kind of activities that need to take place.*

When asked to clarify 'giving yourself time', he said:

> *Giving us time because the curriculum design of the department is heavily overloaded. One of the challenges we have sometimes for conceptual understanding is that even before you introduce the topic, you are already late neh… before you can start and go to practical and demonstrate conceptual understanding, the ATP is against you.*

It must be clearly stated, however, that it does not mean that subject advisors are not helping at all but, as they have other responsibilities, they need to prioritise activities that will enhance teachers' instructional practices. On the other hand, participants provided some practical guidelines, recommendations and specific approaches that could benefit them. They expressed their ideas about how things could have been done differently in the study. The significance of these concerns, for methodological aspects, forced me to pay attention to areas that might have influenced the perspective of participants. Examples were:

PRIA:

> *We took a long break between some of the sessions, maybe what we are saying now, we might not have said if we had done reflection sooner after presentations. Even if it is not a day after, maybe a week after. But I am not saying today's reflection is not good. It is, but we could have said more.*

PRIDo:

> *It was a good thing for me to have a break because I was engaged in a big study, but not disputing the fact that we were supposed to meet earlier. I was really under pressure for exams and all kinds of things, but me myself neh … I would have loved if we in one session we have met face-face and demonstrate practically conceptual understanding of probabilities so that at the end of that session would also take the same activity that we did and be able to apply them into classroom. I think that could have contributed a lot in terms of developing as the teacher in the classroom.*

PRIC:

> *Getting immediate feedback, putting into practice. How do we now practice in the normal classroom situation?*

All these responses pointed to a need for teachers to reflect deeply after every session in order to interpret their experiences. I do acknowledge what the participants said but meeting

frequently would have had financial implications, particularly for me as the researcher. Equally important, the participants suggested that more time could have been devoted to planning a lesson as a team, presenting it, correcting it, teaching it in a classroom setting and conduct post-lesson reflection. These suggestions correlate with the findings of Ni Shuilleabhain and Seery (2017), Leavy and Hourigan (2016) and Early et al., (2017) which recommend that provision be made to reflect on effective practices, interactions in the classroom and effective ways to improve students' performance. I do not dispute that this practice of developing, presenting and reviewing lessons would have been most worthwhile; however, it was necessary to analyse the needs of the participants so that relevant support could be provided. Furthermore, these responses served as basis for framing the plan to enhance teachers' instructional practice.

5.4 Conclusion

In this chapter data from classroom observation, semi-structured interviews, an open-ended questionnaire and focus group discussions were presented. The analysis of findings was guided by the aim, the research questions, and the objectives of the study. Methodological processes and procedures were also discussed. Attention was drawn to the importance of diagnosing the needs of teachers before any intervention is conducted. Issues that emerged from the data in the first phase of the study were pertinent to the development of intervention programmes. Consequently, the process of connecting the qualitative data to the literature study contributed to the design and the propositions for the intervention workshop. Significant details from participants' responses that served as guidelines to conceptualise the proposed framework are discussed in the next chapter.

CHAPTER 6 : SUMMARY, DISCUSSION AND CONCLUSION

6.1 Introduction

This chapter concludes the study and provides the summary and discussion of findings as analysed in the previous chapter. The aim of this study was to design a framework to enhance teachers' instructional practices in the teaching of probability. This study revealed some difficulties that should be acknowledged by the DBE and other external bodies. The overarching challenge identified in this study relates to the limited conceptual understanding of probability demonstrated by participants. Although I acknowledge that there were some challenges with regard to how participants understood the concept of probability, it is important to highlight that there were positive aspects that emerged during the study. This includes, but is not limited to, the ability of participants to:

- Interact with the South African curriculum policy for Grade 7
- Facilitate and engage students during their lessons
- Modify the teaching activities on probability from the DBE workbook
- Assess the students to measure the effectiveness of their teaching
- Use teaching resources, although incorporating them in a lesson was a challenge
- Read and analyse two journal articles
- Reflect, share their views and good practices during the focus group discussions

What motivated me to explore how teachers make sense of probability is two-fold: Firstly, it has some practical implications in the sense that it serves as a tool to compare the likelihood of different events in a world full of uncertainties. Its importance would be evident if it is taught in a way that brings awareness of risks involved or opportunities available. Secondly, many teachers did not receive training on this topic during their pre-service training, and their students often demonstrate below average performance in this topic. This chapter opens with the synopsis of the findings, discussion of findings, the limitations of the study, its contribution to the body of knowledge and suggestions for future studies. The next section presents the summary of the findings.

6.2 Summary of the study

This section discusses the findings from the open-ended questionnaire, semi-structured interviews, lesson observations and intervention workshop. I used the qualitative approach to explore how teachers' conceptual understanding of probability informed their classroom practices. Secondly, the study also explored how teachers' conceptual understanding of probability framed the plan to enhance their instructional practices. The findings in this study are conceptualised within Kilpatrick et al.'s (2001) model. Although I acknowledge that the strands of mathematical proficiency are intertwined, I used conceptual understanding as the central idea to make meaning of the investigation based on findings that emerged. What follows is a discussion of how the findings relate to each question. The purpose of conducting this study was expressed in three key research questions:

RESEARCH QUESTION 1

What are teachers' conceptual understanding of probability?

The purpose of this question was to gain understanding of how participants define probability, represent probability, connect between probability concepts and procedures (with the aim of organising knowledge in a coherent whole), apply probability concepts to real-life situations, and how they use language to communicate information. This study used the indicators of these core essentials from Kilpatrick et al.'s (2001) model to make meaning of the findings.

Findings

In responding to the question, four issues were found as the major difficulties identified in all processes:

- Difficulty in defining probability, particularly in differentiating the terms chance, probability and possibility.
- Difficulty in making sense of multiple meanings of modal verbs.
- Lack of skills to develop scenarios that allow students to explore probability in depth.
- Predominant use of two common representations, namely probability scale and standard algorithms, to get a comprehensive conceptual view of probability.

RESEARCH QUESTION 2

What conceptual understandings of teachers impede and/or enhance the teaching of probability?

The purpose of this question was to explore how teachers' conceptual understanding limits or enhances their effectiveness in teaching probability

Findings

The challenges in the study that hindered the effective teaching of probability emerged during the lesson observations and focus group discussions. In responding to the research questions, several issues were found as major difficulties identified in the study:

- Lack of skills in drawing from real-life situation to build on new knowledge
- Unrealistic teaching activities to develop students' skills in analytical thinking and communication
- Difficulty in incorporating resources into the lessons
- Overuse of standard algorithms
- Limited support for teachers and subject advisors
- Limited exposure to research articles

RESEARCH QUESTION 3:

How can teachers' conceptual understanding of probability be enhanced pedagogically to improve classroom practice?

This question was aimed at exploring strategies that offer teachers opportunities to deepen their conceptual understanding of probability and improve their classroom instructional practice. In other words, the question explored how teachers' conceptual understanding of probability framed the plan to enhance their instructional practices. These plans are expressed as guidelines and explained in the RIRAD framework conceptualised in this study. The guidelines streamline the processes on how teachers' classroom instructional practice could be improved (See Figure 6.3).

Findings

The major finding in this specific research area suggests that there is a need for a re-examination of the way teachers approach the teaching of probability. The RIRAD framework provides a conceptual structure and show how different elements are connected to facilitate ways of deepening conceptual understanding of probability. Because this finding is multi-faceted, answers to the research question are building blocks of the RIRAD framework and are captured as follows:

- Clarifies cores of conceptual understanding in relation to probability as informed by the empirical study. In other words, the framework describes a distinct culture that attempts to capture some of the essential qualities of conceptual understanding of probability.
- Provides guidelines on how to redesign and develop teaching activities.
- Suggests ways to implement the adapted lesson study process as an accelerant for enhancing teacher classroom practice.

In this chapter, I report that probability is a topic that appears complex and difficult for both teachers and learners because of the manner in which it is introduced in schools. In view of this understanding, I concluded that the complex nature of probability requires the following:

- The need to study and understand the complex nature of probability;
- The selection of relevant and various approaches to meet students' learning needs;
- Implementation of teaching activities that are real. For example, activities that build awareness of opportunities available for students and risks involved. Implied in this view is that teachers should know and understand the purpose of teaching probability.

I regard the above identified aspects as critical to provide guidelines to deal with the complexities around the teaching of probability. An overarching theme emerged and according to the evidence from the needs' analysis, teachers had a limited conceptual understanding of probability. This refers to a) teachers' difficulties in comprehending the concept of chance, b) how teachers experience complexities of the second language and challenges faced within guiding interactions in the classroom: teachers' classroom instructional practices.

It was also necessary for me to adapt the methodology in this study due to the outbreak of Coronavirus. It is important at this stage to highlight that participants spent much time learning how to use TEAMS application and were forced to learn it on the spot. This was because all the meetings during the intervention workshop were virtual. One other finding was that the intervention workshop could not be pre-planned because the activities in the workshop manual were based on the challenges identified in the first phase of the study. Findings in this study place teachers' conceptual understanding of probability at the centre. In other words, how teachers make meaning of teaching for conceptual understanding drives planning and instructional practices. Furthermore, the intervention sessions conducted enabled me to identify more teacher needs that linked to what subject advisors can offer. The next section discusses findings of this study.

6.2.1 *Teachers' conceptual understanding of probability*

This sub-theme is discussed in relation to the following cores of conceptual understanding: definition of probability, ways of representing probability, connections between probability concepts and procedures (organising knowledge in a coherent whole), application of probability concepts to real-life situations, connections between probability concepts and procedures and the language used in probability. This theme is discussed from the perspective of how teachers experienced difficulties in:

- Defining probability, particularly in differentiating the terms chance, probability and possibility;
- Making sense of multiple meanings of modal verbs;
- Developing scenarios that allow students to explore probability in depth.

The complexity in defining probability

I sampled teachers based on their qualifications and experience in teaching mathematics. However, in this study it has emerged that even though participants were experienced, there were some challenges in defining the concept of probability. Most of the participants described probability through a system of representations like the probability scale, fractions and percentages. They also defined probability using the sample space. What emerged as critical was how participants experienced some difficulties in differentiating the three terms: chance, probability and possibility.

I observed that some participants used the terms probability, chance and possibility interchangeably. For example, they asked questions such as "What is the probability of / What is the possibility of / What is the chance of …?" These questions meant one and the same thing to some of the participants. My argument at that point was that if the terms are used interchangeably, why are they regarded as distinct in many resources? These concepts were further discussed during the intervention workshop. To build on this aspect, subsequent paragraphs discuss how the participants connected their background knowledge to clarify the difference between the three terms: chance, probability and possibility.

During the intervention workshop, **PRIDo** explicitly said: "*A chance, neh, is an opportunity of an event taking place and there is no quantifiable data for possibility and chance"*. This particular response bears some relation to the views of Naidu and Sanford (2017) who indicate that how the concept is introduced has a bearing on how it is defined. Offering a valuable tip in this regard, **PRIC** said: "*Possibility* (pauses), *you are dealing with something that is either possible or impossible"*. **PRIC's** view seems to suggest that it might be easier to grasp the term ''possible' if examples to describe possibility and impossibility are discussed.

PRIC's view not only points to how a concept could be explained but also to the critical role that the contrast of the word plays. To provide an example, if one wants to define the term possible, the meaning could be clearer if one starts by exploring events that are impossible. The idea is that it might be easy to understand the concept if it is explained in relation to its contrast. This view seems to align well with Marton, 2009 (as cited in Jing et al., 2017), who understand contrast as the awareness of variations in the object of learning. Participants agreed unanimously that, although it was not easy to differentiate the three terms, they acknowledged that it is better to look at them from the perspective that they are interrelated.

The way participants struggled to differentiate the three terms (chance, probability and possibility) suggests that even students in secondary schools might have problems in defining each of the three terms. To justify this view, **PRIDo** admitted: "*I have been teaching matric for more than six years, neh! But to tell you the truth I have solved complex probability problems and I realise that even myself as a teacher I could not thoroughly clarify this terminology: possibility, chance, probability, unlikely"*. These sentiments show that there is a

need to start teaching students the concept at elementary level to help them build upon their intuitions so that they can begin to use sophisticated words such as 'randomness' appropriately (Batanero et al., 2016).

Frequent descriptors in participants' definitions of probability included: chance; numerical value between 0 and 1; possibility; and quantifiable. Although it took the participants some time to clarify their understanding of the terms discussed in this study, the majority of them agreed that probability is quantifiable, but chance is not. This understanding aligns with Naidu and Sanford (2017) who define probability as a numerical value between 0 and 1. Similarly, Van de Walle et al. (2016, p. 583) capture the essence of probability in the following definition: "Probability is a ratio that compares the desired outcomes to the total possible outcome".

Although participants agreed that probability is quantifiable, it took them time to understand that the meaning of the numerical value between 0 and 1 means that probability is relative. On the other hand, I realised that participants experienced some difficulty in gaining deeper understanding of chance because they used abstract numbers to define the concept. That is why it was a challenge for the participants to reconstruct the definitions to clarify aspects.

The word prediction kept on resurfacing as participants were trying to clarify the concept of probability. When I asked **PR5** how she understood the problem on 'prediction', she responded by saying, "*I did not even have a clue on where to start. In most cases when we teach prediction in class, we focus only on whether it will rain or not, not on the calculations."* It was a clear that participants use various terms that they are unable to explain. This is an indication that participants lack effective strategies to develop students' conceptual understanding in probability. This view is reflected in **PRIDo's** response: *"We teach to assess rather than teach to understand the concept".* Patterns, trends, argumentation and deliberations in this section point to the fact that the way teachers defined probability had limitations.

To sum up the discussion, participants understood probability to refer to the likelihood of events occurring and also as the numerical value between 0 and 1 (Grimmett & Stirzaker, 2020; Naidu & Sanford, 2017). Examples in Figure 6.1shws this.

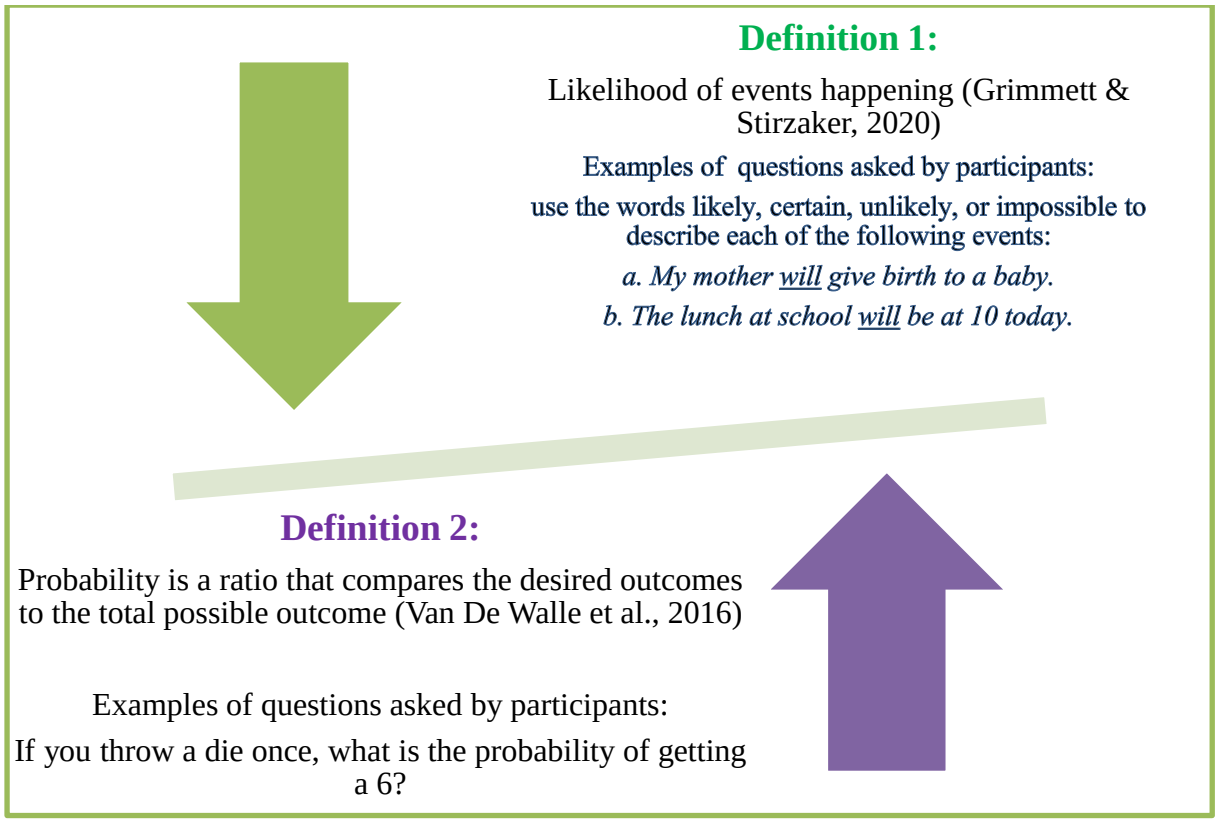


Figure 6.1. Definition of probability: examples given by participants

Participants defined probability from the two perspectives as presented in Figure 6.1. Although the participants acknowledged that their understanding of possibility, chance, and probability was enhanced through the discussions held during the intervention workshop, they believe the terms could be clarified better by developing appropriate teaching activities. The next section presents participants' experiences in developing scenarios.

6.2.2 ***Development of scenarios***

Participants were asked to generate the scenarios that could assist students to realise that probability is used in everyday applications whether we are aware of it or not. The main aim of developing scenarios was to explore how teachers instil in the minds of students the importance of probability in real-life situations. This study found that teachers experience some challenges in developing scenarios. Findings revealed that some teachers preferred to use scenarios when defining probability. To attest to this view, **PR$_6$** responded as follows when asked to define probability during the interview: "*mmm…by eh…. An event taking place. I am referring to...* (pauses) *should I use the scenario?".* Participants were also asked to develop

scenarios in the context of sports to explain the concept of likelihood in real-life situations. **PR$_3$** responded as follows: "*In sports such as netball, we play in order to win or lose. Playing is an action. Win or lose are the outcomes".* **PR$_4$** said: "*Which sport is the most popular?"* Teachers provided examples in the context of sports; however, it appeared that **PR$_3$** wanted to introduce the concept of outcomes to students.

Several challenges could be identified in these examples. Firstly, **PR$_4$** provided information, not a scenario in which students would need to draw on their own experiences to build new knowledge. Secondly, it is a given that the purpose of playing netball is to win. This is a weakness because there was no background information provided that would lead to the conclusion that the purpose of playing netball is to win. Thirdly, it was not clear whether questions would be derived from the provided example or not. Lastly, the scenario does not build awareness of opportunities available for students or risks involved. **PR$_4$'s** response would not assist students in making sense of probability because information provided was insufficient; it would therefore be difficult for students to reflect on their experiences and to construct their own understanding of probability if these kinds of scenarios are provided as an example.

It is clear that teachers faced challenges when capturing the essence of what a scenario entailed. To elaborate, participants often asked questions such as *"What is the probability of a particular event occurring?"* and regarded it as the scenario. It was important for the participants to realise that beliefs, period, interest, situation, place, environment and time references have a bearing on how to make a conclusion or judgement. In other words, the scenarios should provide a better background for students to make a reasonable judgement or conclusion.

What emerged from the data in the present study runs counter to the findings of Proudfoot and Kebritchi (2017, p. 8), who define scenario-based learning as follows:

> *This approach is based on the understanding that in order for a learner to acquire and retain skills and knowledge, the learner must be placed in a scenario where his/her decisions affect, or alter subsequent events leading to new events*.

In view of this, the limited understanding of scenario-based learning had impacted on how students respond to questions.

To address this challenge, I therefore recommend that subject advisors organise in-service training workshops wherein they focus on the following aspects: a) ways of representing probability and how it could be useful in learning, b) development of scenarios that aim at ensuring that students:

- understand the importance of probability;
- are able to apply their knowledge and draw from real-life situations to build on new knowledge;
- know the types of contexts in which probability is useful;
- are engaged in activities within their cognitive grasp;
- engage in activities that build awareness of the importance of probability;
- are able to define their own context;

On the other hand, although participants managed to conduct the experiments, they did not understand their purpose as they lacked the skill to relate them to real-life situations. The next section presents how participants made sense of the modal verbs.

6.2.3 *Difficulty in making sense of multiple meanings of modal verbs. (The use of language)*

A further finding from this study was that teachers used the language of learning and teaching to explain the concept of probability; they used the correct language of probability to explain processes involved when teaching the concept of probability; however, their verbalisation of the connections between probability concepts and representations was not explicit. Participants had difficulty in affirming that the meaning of a modal verb is changed by the context in which is used and often by how we say it.

The example discussed below builds on some aspects that serve as fundamentals (pointers) for the conceptual understanding of probability. Such pointers include how teachers categorise the modal verbs in their core meaning. To elaborate, a sentence such as ‘Orapeleng has been elected to explain to Grade 7 learners how she understands the statement; the probability of contracting Coronavirus’. This statement should spark a discussion to reach an understanding

of the context. The following questions could be posed to guide students how to make meaning of the statement:

- Does it mean that Orapeleng is good at explaining concepts?
- Does it mean Orapeleng has been invited to develop her presentation skills?
- Does it mean Orapeleng can well describe the symptoms of a person who has contracted Coronavirus?

These questions have important implications for developing an understanding of the use of modal verbs and context. They also provide a clear guideline to students when making judgements because they provide context. In the same vein, a statement such as 'The teachers are conducting on-line lessons for Grade 12 students' could mean that lessons are in progress, or it is essential that the lessons should be conducted as planned. Drawing from the meanings attached to the given statement in the on-line classes, it suggests that the use of the modal verb might not be explicit but implied. How the modal verb is used depends on the context and the point of view of the person. Most importantly, do the questions develop any awareness among students? This is another critical area that highlights the purpose of asking such questions.

Although Curto (2019) offers an equally important view by indicating that teachers should give students sentences in which the same modal verb is used in different contexts, a difficulty remains when insufficient information is provided to students to enable them to make reasonable conclusions or judgements. Adding to the views of Curto (2019), Batanero et al. (2016) encourage teachers to acquire strategies and ways of reasoning to help students make proper decisions in situations related to chance. Background information is critical when making informed decisions; without it, questions might be ambiguous or lead to misinterpretation.

What I also picked up during the lesson observations was that students were not given an opportunity to justify their responses. With similar experience, Graven (2012) notes that much of the literature focuses on other strands of mathematical proficiency and neglect productive disposition, which focuses on how to develop reasoning skills. A broader perspective has been adopted by Nance (2018) who emphasises that learning occurs when students are able to use their knowledge reasoning within a problem-solving setting. The evidence presented in this

section suggests that activities should be designed in such a way that the use of modals does not lead to ambiguity or misinterpretation.

Ambiguous context makes the declarative sound vague. For example, the statement ‘I should come home’ is ambiguous in the sense that there is no background information that clarifies the decision made. This study found that the most frequently used modal verb was the word ‘will’ which focuses only on what will happen in future. Other modal verbs were used rarely to convey varying degrees of certainty. This finding runs counter to the findings by Belbase (2020), who indicated that students’ learning is affected by their beliefs that relate to emotional and mental contexts. In other words, the belief that something might happen depends on how modal verbs are used. It can thus be inferred from the above discussions that participants in this study needed some assistance in understanding how modal verbs convey the message.

I have also realised that sometimes it is not easy to make a judgement on the examples of events provided because the core meaning of the modal verb and the context in which it was used has not been made clear. This is illustrated in the following example: “The devastating outbreak of Coronavirus disease in 2020 that *may* have resulted in hundreds of deaths globally *could* have been due to local transmissions”. Teachers should be able to make meaning out of this statement. The main questions that the teachers could ask would be: Do students have sufficient information to enable them to make an informed conclusion or judgement? Do students know or understand what the word ‘could’ in the provided statement expresses? These kinds of questions would make teachers understand that language also relates to how the construction of modal verbs occurs (Fong, 2020).

Groth et al. (2016) highlight the importance of learning vocabulary at an early stage to become successful reasoners. Listening to students’ feedback is another area that has the likelihood of getting to know the context in which the modal verbs are used. The findings in this section may help us to understand that proper use of probability language to express ideas and opinions should always be guided by the context in which the events happen. Two important things emerged from these findings: the use of modal verbs in context and the importance of ensuring that scenarios provided do not lead to ambiguity or misinterpretation. Similarly, understanding context also plays a major role in students’ interpretation of situations, which will ultimately inform the decisions or judgements made.

6.2.4 *Ways of representing probability*

It was a common practice for participants in this study to use language, tables, percentages, fractions and scales to represent probability. However, it was not clear what informed their choice of these representations. It is possible that the participants did not even consider the advantages and disadvantages of using each representation. What I have also observed was that participants did not understand the importance of each representation and its usefulness to the purpose. As to how the representations used were connected, was not clear. Students were not encouraged to build models to represent their ideas on probability (Shaughnessy, 1977). These findings suggest that teachers need to understand the purpose of using each representation and provide reasons on what informed the choice they made.

To answer the research question: What are teachers' conceptual understanding of probability? this study revealed that teachers have limited conceptual understanding of probability. It is limited in the sense that although the cores of conceptual understanding were evident in teachers' practices, there was no evidence of how teachers reconstructed the definition of probability, demonstrated the importance of each representation and role that probability plays in real-life situations (i.e., teaching activities were not developed in such a way that they build awareness of the role of probability). In the same vein, participants experienced some challenges in developing proper scenarios, using modal verbs in context, organising knowledge in a coherent whole, showing connections between methods, procedures, and representations - which contradicts Kilpatrick et.al.'s (2001) theory. The next section presents factors that contribute or impede effective teaching of probability.

6.3 Conceptual understandings that impede or enhance the teaching of probability

This section discusses factors that impede or enhance the teaching of probability. The factors are discussed as a guide or indication of a future course of action. Most of the issues presented in this section relate to how participants facilitated lessons on probability. From my understanding as researcher and subject specialist, learning takes place when students are able to construct knowledge and make sense of their learning (Adem, 2021). If material is provided simply as a recipe for implementation, knowledge cannot be constructed.

- **Prior knowledge**

The success of transiting from informal to formal learning depends on how teachers draw from students' experiences to build on new knowledge. Teachers need to first understand that probability is all around us. During the initial stages of the lesson, it is important for teachers to use the language students know best; language that will easily connect students' prior knowledge to the new knowledge. It might not make sense to the students if teachers start by introducing numerical values right from the beginning of the lesson. Numbers are too abstract to develop conceptual understanding of mathematical concepts.

An informal situation provides a useful background: teachers should be strategically competent to devise ways to identify any misconceptions students have about probability. Understanding such misconceptions should help teachers to plan realistic and appropriate teaching and learning activities. This view is supported by Carpenter et al. (1988) who emphasise that it is important to recognise the knowledge and procedures that students bring to the classroom, and any misconceptions they have about the topic. Problems used during discussions should be experientially real in the mind of the students. Therefore, it becomes critical for teachers to ensure that their questioning strategies spark discussions that promote learning.

Moving from informal to formal learning is a continuum and cannot be achieved overnight. Therefore, to enhance the teaching of probability, teachers need to plan the right questions and ensure that the intention behind such questions is explicit. They should reflect on the questions and make links to help students detect language patterns and misconceptions. One area that was overlooked by almost all the participants in this study is the consideration of how the concept of probability develops amongst the grades. In other words, how probability progresses from grade to grade.

- **The use of resources**

The manner in which resources are incorporated in the teaching and learning of probability is critical. The resources used in lessons should make sense to students, be accessible to students, simplify the complexity of mathematics and model real-life situations. It is the responsibility of teachers to check the relevance of the resources prescribed in textbooks, and either modify these or think of alternatives that will make sense to students if necessary (Cheng et al., 2021).

It is also important for teachers to meaningfully incorporate resources into lessons, and not introduce them in isolation if they are to promote learning.

Most of the textbooks used by teachers at the time of this study contained activities that encouraged students to calculate the probability of events using a formula. In line with this finding, Groth et al. (2016) reiterate that students' ability to assign numerical values is more developed than their use of the language of probability. The views of Groth et al. (2016) suggest that students perform better when using the formula than using the language of probability. I align myself with this view because one of the findings in this study was that participants do not understand the context in which the modal verbs are used, hence most of the activities were more on using standard algorithms in determining the probability for an event occurring.

The importance of selecting material purposefully is explained by Cheng et al. (2021, p. 228) as follows:

> *Using the curriculum materials effectively includes not only being able to recognise and distinguish between high- and low-quality materials. Skilful selection and modification of instructional materials guided by clear goals of the teachers—in this study the characteristics inherent in the teachers' instructional materials—for classroom use are also critical.*

Implied in this quote is that material should be purposefully selected and incorporated to make the lesson more meaningful.

- **The formulation of objectives**

The findings suggest that participants need a thorough knowledge of the curriculum to ensure that learning objectives are well-formulated. Clear learning objectives would assist teachers in ensuring that their tasks and activities are appropriate. These objectives are what teachers want their students to do and what they should be able to do. If teachers are to formulate measurable objectives clearly, they need to know their roles (what they have to do), have knowledge of probability and what they should say (the complexities of language in relation to the use of modal verbs). In support of these views, Smith et al. (2018) argue that one approach to assess teachers' understanding of probability is to align tasks to the goal of a lesson.

Furthermore, Sullivan (2011) indicates that the way goals are formulated justifies the focus of the lesson. In other words, the focus of the lesson should be explicit in teachers' planning and modes of presentation. In this study I observed that the emphasis was more on explaining probability concepts and using the formulae to calculate probability. Very little was done in terms of building awareness of the role of probability in real life situations. The reason for this is not clear, but it may have something to do with the fact that teachers struggle to develop scenarios to use as tools for learning.

In their study, Fan et al. (2013) found that the link between the textbook and students' learning outcome was not clear. To enhance the teaching and learning of probability, Fan et al. (2013) suggest that teachers re-align the activities in the textbook to suit the needs of the classroom. Figure 6.2 depicts the basis for formulating learning objectives:

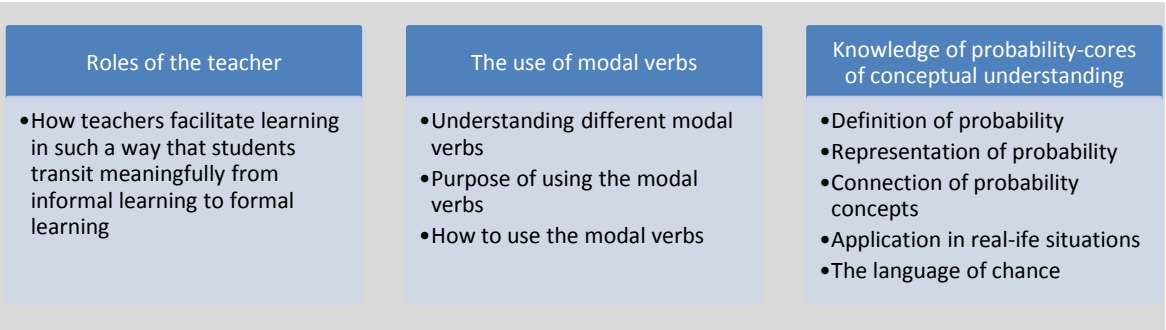


Figure 6.2. Basis for formulating learning objectives

Source: Kilpatrick et al., 2011 and Zhang, 2019

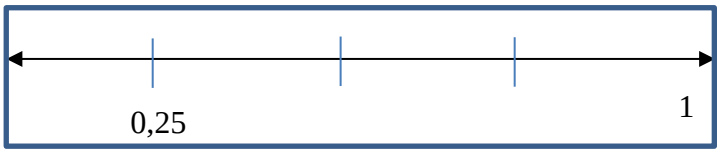
Figure 6.2 is developed from the understanding that teachers should strive to develop activities that promote meaningful learning. The findings in this study suggest that the success of the lesson depends on how teachers understand their roles, use modal verbs and how they make sense of probability. The three key areas in Figure 6.2 help to clarify and organise teaching.

Teaching by negotiation should replace teaching by imposition; students should be actively involved in "learning" probability. Students will also engage in constructive learning on their own, working quietly through set tasks, allowing their minds to sift through the materials they are working with, and consolidating new ideas with existing ideas (Creswell, 2008). Learning is influenced by how meaning and knowledge are constructed (Van Merriënboer & de Bruin,

2014). It is therefore important for all teachers always to keep this question at the back their minds: "How are ideas constructed by students?"

- **Teaching and learning activities**

Probability questions should be drawn from the content of statistics because there is evidence to justify their arguments. Information in graphs, articles, tables and video clips could be discussed through investigations, games, speeches and so on. Although gamifying the lesson could be one strategy to construct knowledge and promote learning, Lanuza et al. (2020) recommend that probing questions should be asked to make meaning of students' responses and activities. A further finding in this study was that after drawing the probability scale as indicated below, **PR3** asked the following question: *"Who can tell me? The numbers are increasing from zero by how much?"* The participant expected students to refer to the diagram below when answering the question.



One student responded with confidence and said: *"The number is increasing by 25"*. This was a concern because the teacher did not make any comment on this response. My argument was that 0,25 is not the same as 25. Although I acknowledge that the teacher used the scale to represent probability, it would have been better for the participant to address the misconception on the spot. This example illustrates how teachers' response to students' answers could impede the teaching of probability.

Paying attention to students' explanations and encouraging arguments amongst students enhance the teaching of probability (Nene, 2017). When assessing students' learning through formative assessment, it is important that teachers avoid asking closed-ended questions. Teaching activities should also be designed in such a way that they develop listening and language skills (DBE, 2011). The content should not be academic only but should relate directly to students' everyday experiences.

- **Content coverage**

The focus on content coverage is one aspect that could impede the teaching and learning of probability. It often means that teachers either teach to finish the syllabus or teach only what is to be assessed. **PRIDo** attests to this**:**

> *One thing that I also noted, neh … maybe because I am at the exit level, I feel you must also include this in the study that curriculum development of DBE speaks more to time-on-task than real conceptual understanding of mathematics where people are demonstrating that we are doing this model. We teach to assess rather than teach to understand the concept.*

If the intention is not to teach for conceptual understanding, then learning loses meaning. Because it is important to assess for learning, teachers are therefore encouraged to use different forms of assessment effectively. For example, investigations and projects require tasks that provide real problem-solving situations, that encourage enquiry and exploration (Ulandari et al., 2019). Stimulating the exploration of concepts also requires teachers to guide students and ask thought-provoking questions. If this is not done, students might be left with no option but to regurgitate facts and memorise rules. Tasks that encourage students to discover rules cannot be answered instantaneously; for instance, if students are given the scenario, they will have to apply their minds and teachers need to probe their responses to establish if the answer is just not a guess. To achieve this, teachers need to adjust teaching activities and review assessment strategies.

In this study, the reviewed literature revealed that the effective teaching of probability was hampered by the use of confusing language and students' lack of exposure to these concepts (Mahmud & Porter, 2015; Molina, 2014). Akin to this, Brodie and Berger (2010) associate this challenge with error of familiarity which means that students are attracted by what is familiar to them.

When planning the intervention workshop, I took certain issues into consideration: a) the need for teachers to realise that modal verbs are accelerants to communicating awareness and risk; b) an understanding that the meaning of a modal verb is changed by the context and often by how we say it. These considerations were informed by how participants developed and used the scenarios in their teaching. The scenarios did not assist students to make better and

informed decisions. Also, participants needed to understand that defining a concept is a process, not an end in itself. Teachers could not realise that the definition of a concept is derived through the following: what students experience and how they experience the situation. This might be the reason why participants did not give students an opportunity to reconstruct the meaning of probability from available evidence. Moreover, the gradual introduction of the language of probability could help students to understand the content better. Introducing several concepts over a short period and without contextual understanding could disadvantage students.

6.4 Areas that require greater attention for effective intervention processes

- The first workshop should focus on unpacking conceptual understanding of probability and strands of mathematical proficiency. Teachers and facilitators should be well-grounded with the phenomenon under investigation.
- Models of teacher professional development should be adapted to address local challenges.
- One-day workshops are not effective. Workshops should be on-going and relevant.
- Use of the lesson study process as an accelerant for teacher professional development could empower teachers as they share good practices.
- Engaging subject advisors and teachers in the same activities allows subject advisors to identify the kind of support they could offer teachers. This initiative needs proper planning.

6.5 Limitations and mitigations

This section presents limitations of the study, however there are areas where I explained options and actions to enhance opportunities and reduce risks that could impact on the study. As in all studies of this nature, this one had its limitations. I acknowledge them because they had an impact on the way findings were analysed. The following limitations were experienced:

- Eight teachers participated in the first phase of the study and only two proceeded to the second phase. Due to the outbreak of Coronavirus, the two that proceeded to the second phase of study could not participate in all the activities developed for the intervention workshop. Although participants managed to develop guidelines on how

to augment teaching activities, data collected from the two participants might not be sufficient to make a conclusion.

- Through snowballing, I managed to recruit the second team. The team reviewed the developed guidelines and made their own recommendations. This was a new team but had similar characteristics to the first team. It might have been more beneficial for the second team if they could have interacted with the first team. Although the method used was like a relay in an athletic sports field, the second team did not experience processes involved in developing the guidelines.
- The intervention workshop was conducted in two levels. Level 1 was conducted over six weeks (96 hours) and the second level over five weeks (80 hours), four hours a day. This period may have been too short to influence the results. Teachers required more time to critically analyse the research articles to get deeper into exploring the notion of conceptual understanding in probability. This finding made me realise the importance of reviewing some of the activities to ensure that the workshop is developmental. Areas like development of lesson plans were not covered during the intervention workshop. The aim was to avoid overloading programmes with too much information.
- A further suggestion to mitigate the challenge of time is that the number of administrative periods teachers have should be reduced or combined with planning periods and other non-instructional time (Zhang et al., 2020). This would make more time for teachers to attend professional development workshops.
- The second team was in a ratio of one teacher to three subject advisors. This was not balanced and might have had some influence on the results of the intervention workshop in particular. However, all in all six teachers, two subject advisors and one mathematics coordinator participated in the study.
- The study was conducted in one out of four districts. This made it impossible to generalise the findings to a larger population.
- A further limitation arose from the fact that during the intervention workshop, the teacher and subject advisors may not have felt free to mention all the things that bothered them because one participant was a teacher and the others were seniors, although not from the same circuit. However, that arrangement benefited all participants, particularly the subject advisors, because they knew exactly what was happening in schools and how they could plan and improve their support to schools.

- The study focused on only one topic and one grade. However, the RIRAD framework might be tested in other subjects.
- Engagement with the research articles was not dealt with in detail due to time constraints. This aspect requires an in-depth workshop.
- The intervention sessions were of most benefit to those participants who had the resources to participate online.
- Teachers were prepared for my visit and this might not have provided the true reflection of what normally transpires in schools. It is possible that this might have affected the results of the observations.
- Because of the time constraints, participants did not have enough time to develop the lesson plans and analytic rubrics.
- Participants only managed to adapt the activities from the DBE workbook.

6.6 Implications of the findings: Enhancing classroom practice

This study was undertaken with the aim of filling a gap in existing research into mathematics education. This study has made the following contributions:

Firstly, since teachers are provided with common lesson plans by the Provincial Department of Education, this study could help them to use the activities in the lesson plans as a guide, and augment them to suit the needs of their own classrooms. It is my view that one of the important initiatives in improving instructional classroom practice is the adaptation of teaching, learning and assessment activities. In this study, guidelines were developed on how teaching and learning activities from various resource materials could be adapted and augmented. The material used by participants in this study was imposed in the form of policies and teachers had no power to change or ignore it; with guidance they could, however, review and modify these activities.

This study designed an exemplar of teaching and learning activities adapted from the DBE workbook. The context was derived from situations that learners are likely to interact with in their daily lives. These include social responsibilities, health and educational issues. There is strong evidence to support the fact that mathematics is useful and can be used to solve problems in our everyday lives.

Secondly, the RIRAD framework conceptualised in this study could make an important contribution to the field of Education in the following ways: It could be applicable and adapted to any subject; guidelines discussed in the framework could be used as a prototype to unpack the new MTF piloted in South African schools. From this framework, teachers would be able to develop analytical rubrics to analyse teaching activities, lesson plans and to reflect on their classroom practice. This framework was conceptualised after extensive engagement with the participants, and it is hoped that it will be useful and continue to provide the system with valuable feedback on its implementation.

Thirdly, it was hoped that the study would add to a growing body of literature on methodology by extending the knowledge of scholars in terms of the processes and procedures to follow in empowering teachers and diagnosing their needs.

Fourthly, the study offers some insights into how teachers could work collaboratively to share their practices. In this area, the suggestions provided may help teachers to improve their content knowledge, facilitation skills and post-lesson conferencing (Ni Shuilleabhain & Seery, 2017). The findings also underline the fact that a teacher is a lifelong researcher. The study has gone some way towards enhancing teachers' understanding of the need to explore the field of mathematics and the theories governing its teaching and learning thoroughly.

Finally, although the study was conducted with a small sample of participants, the findings suggest that provincial officials need to review their approach to supporting teachers in schools. For instance, if teachers manage to augment activities, this requires that officials also modify their monitoring instruments. It is also hoped that the findings of this study will contribute to teachers' understanding of the curriculum and how to develop students' conceptual understanding of probability.

Enhancing the theory

The feedback from participants revealed that the amalgamation of strands of mathematical proficiency can help teachers to reflect on their practices. They even acknowledged the fact that the mathematics teaching and learning framework (MTF) draws on Kilpatrick et al.'s (2001) five strands of mathematical proficiency with four interdependent dimensions that are

adopted as the theoretical and conceptual basis for exploring the teaching of probability in this study.

I reconsidered cores of conceptual understanding and added few aspects to help address the complexity of probability. This includes, amongst other things, the use of modal verbs, adapted lesson study processes and analysis of journal articles. Equally, Stages 1 and 4 of the RIRAD framework provide further details on how the lesson study processes in relation to the teaching of probability could be actualised.

It is hoped that these findings will guide and inform curriculum developers, subject advisors, and authors of textbooks on how to support teachers to enhance their classroom practice. It will also make an important contribution to the scholarly literature on how to augment teaching activities in probability and how to provide relevant support to enhance teachers' classroom instructional practice.

6.7 Enhancing teachers' instruction practices

In addressing the following research question: How can teachers' conceptual understanding of probability be enhanced pedagogically to improve classroom practice? The RIRAD framework is intended to provide support to teachers and subject advisors in the teaching of probability for conceptual understanding. Professional development practices are inculcated through the lesson study that serves as an accelerant for intervention processes. In other words, the proposed framework's purpose is two-fold: to enhance teachers' conceptual understanding of probability and to improve their classroom instructional practice in relation to the teaching of probability. This framework is anchored in four key distinct steps: Review, Identify, Reconsider, Adapt and Develop, hence the name RIRAD framework.

There are several important and useful elements underpinning the framework, namely, an understanding of policy issues, conceptual understanding of probability and teacher professional development practices. In this study, I drew from Kilpatrick et al.'s (2001) model and findings that emerged from data to conceptualise and spell out how the RIRAD framework could influence practices in supporting schools. In this framework, teachers play a leading role, while subject advisors support teachers' efforts. It is hoped that by using this framework, subject advisors will offer systematic and sustained support to Grade 7 mathematics teachers.

All the components of the framework complement each other in the provision of teaching for conceptual understanding. The frame of reference are teachers' views, perceptions, emotions, experiences, knowledge and practices in developing the framework. The suggested components of the framework were developed from the reviewed literature, from my experience as a subject advisor and from the study findings and workshops. The framework is presented in the form of stages which are linked and interrelated.

The order of teaching activities depends on the focus of a lesson; if the teachers are to augment teaching and learning activities, I recommend that they start by following the suggestions (as a trial) provided in the framework. If not, the framework can be adapted to suit the needs of the audience without compromising the essence of the framework. Because teachers are viewed as key people in bringing change, it is important that they are given more support so that they are not focused only on emphasising facts and procedures. In each stage of the RIRAD framework, the objectives have been clarified and the activities spelt out clearly, specifically to give guidance to teachers. The point of departure for conceptualising this framework was the theory of social constructivism (paradigm) and the lesson study processes.

The Japanese lesson study served as a catalyst in the development processes. Borrowing from the summary by C. Lewis (2016), the lesson study process can be condensed as follows:

- **Study**: Study the needs of the teachers and formulate the research theme. It is therefore my understanding that intervention in this area could be looked at from different perspectives. Firstly, it could inform the teachers about students' mathematical problems that have the potential to hinder their performance and learning. Secondly, it could assist subject advisors in identifying challenging areas for teachers and include them in their intervention programmes and, lastly, this could also inform research studies on how to improve teacher practices or community engagement projects.
- **Plan**: Teachers sit together and plan the research lesson collaboratively. In the context of this study, participants developed guidelines on how to augment teaching and learning in probability.
- **Do**: One member from each group presents the lesson. This area was not covered in this study, it came in as a recommendation.

- **Reflect**: Teachers comment on good practices and discuss areas that need attention.

The reflection step was critical because that was where I tracked the development of each participant. Participants were sometimes given homework which they were expected to present in subsequent meetings, first as individuals and then discuss it as a group. In so doing, they were able to reflect on their own practices. Reflection is thus not a stand-alone component of the lesson study; rather, it should be integrated at all the levels of professional development. As evidence that learning occurred during the intervention workshop, participants managed to do the following:

- They developed guidelines on augmenting teaching and learning activities (see Appendix P_1). These guidelines were developed and reviewed during the first level of the intervention workshop. Further input was provided by a second team of participants during the intervention workshop. Based on this, the RIRAD framework provided some guidelines on how teachers could deepen their understanding of probability and how teacher professional development processes could enhance their instructional practice.
- When reflecting on their participation in the study, teachers and subject advisors appreciated the fact that they now understood the strands of mathematical proficiency. This was evident when participants generated questions that provided insight into what each strand entailed.

Figure 6.3 presents a breakdown of the levels and the processes involved in the RIRAD framework. In Stage 1, the focus is on understanding curriculum policies. Stage 2 focuses on making sense of conceptual understanding. Stage 3 reconsiders cores of conceptual understanding and Stage 4 focuses on the processes of augmenting teaching and learning activities. This framework takes the process one step further to include Stage 5. Stage 5 focuses on development of the lesson plan and implementation in a classroom setting. Reflection is integral to all processes involved in refining the adapted activities and the lesson. The five stages complement each other in the enhancement of conceptual understanding and teacher professional development.

I explored the experiences and identities of six teachers, two mathematics subject advisors and one mathematics coordinator over a period of four years. The results obtained are not generalisable because these participants differ with respect to qualifications, experience in teaching mathematics, training, support received from their seniors, and pre-service training. It is therefore important to highlight that while this case study attempted to offer some insights into the complex process of interrogating conceptual understanding in the teaching of probability, the participants developed guidelines on augmenting teaching and learning activities from the DBE workbook. The limited time I had did not allow participants to plan and present the lessons in a classroom setting. The next section presents how the third research question was answered.

Research question 3: How can teachers' conceptual understanding of probability be enhanced pedagogically to strengthen their classroom practice? This question was aimed at exploring strategies that could offer teachers opportunities to deepen their conceptual understanding of probability. These strategies were translated into key performance indicators that indicated concrete actions that teachers should be able to perform to foster deeper understanding of probability. The framework conceptualised in this study was the result of four years' research on how teachers' conceptual understanding of probability frames the plan to enhance their instructional practices. It describes a distinct culture that attempts to capture some of the essential qualities of a conceptual understanding of probability. The findings in this study suggest that there is a need for a re-examination of the way probability is taught. Figure 6.3 presents the RIRAD framework: An integrated reflective dual perspective.

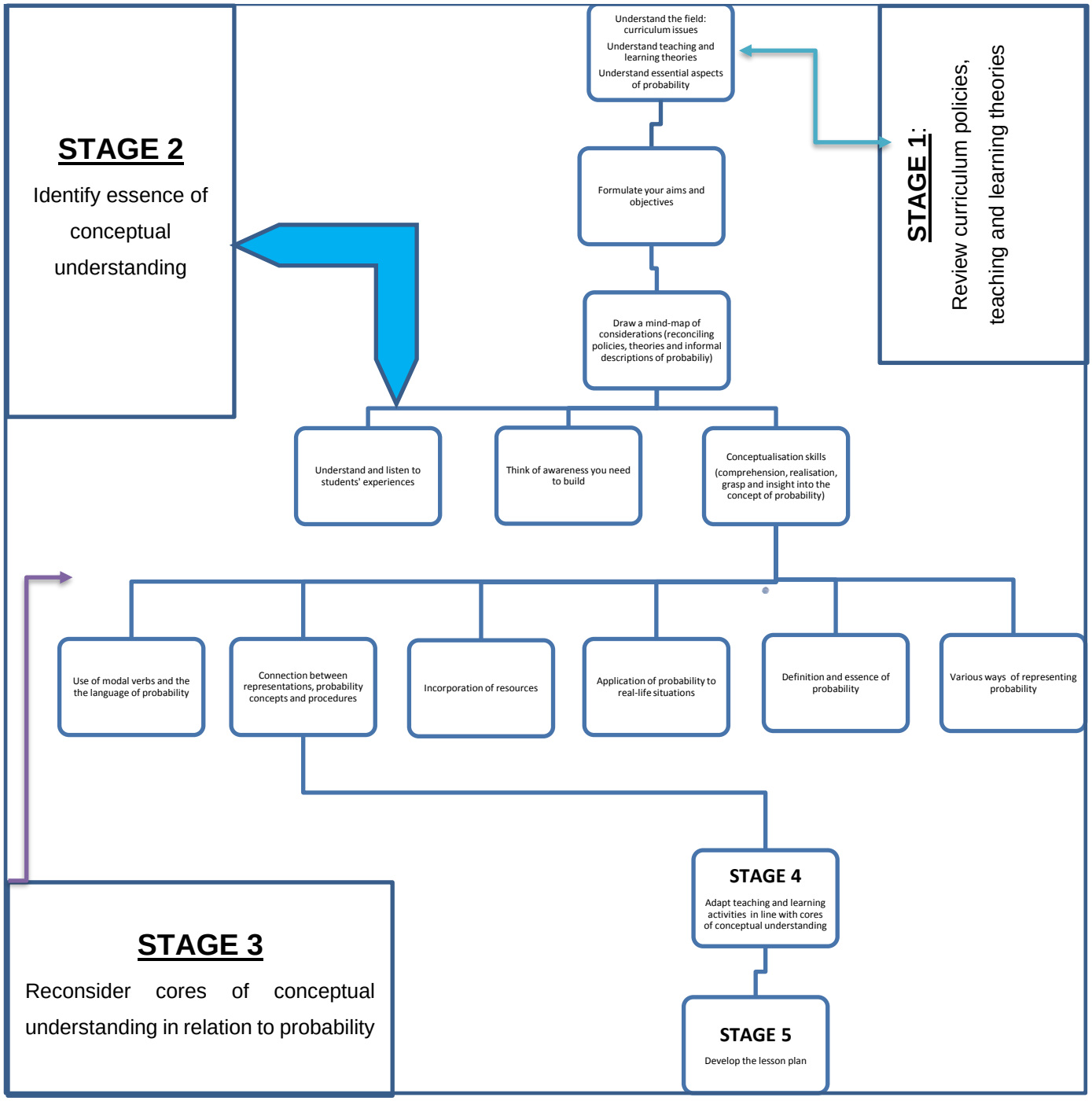


Figure 6.3. RIRAD framework: An integrated reflective dual perspective

Source: Kilpatrick et al., 2001 and findings from the study

This framework is dual in nature in the sense that it looks at what it means to deepen the conceptual understanding of probability, and at the enhancement of teachers' instructional practices. The pillar of this framework is "evaluate and adjust each stage" to make it relevant and significant in developing conceptual understanding. But because this framework was also

based on the views of subject advisors, I structured it to benefit both teachers and subject advisors. It should be of benefit to them both as classroom-based teachers and as office-based teachers. The framework could assist teachers to reflect on their classroom practices and improve their knowledge and skills (Desimone, 2009). While acknowledging that I used some tools from Kilpatrick et al.'s (2001) model and findings that emerged from data to conceptualise the RIRAD framework, the next section presents how I discussed each stage of the framework.

EXPOSITION AND CONCEPTUALISATION OF THE RIRAD FRAMEWORK

STAGE 1: Review curriculum policies, teaching and learning theories

In this stage, teachers need to understand theories of teaching and learning to know exactly how to make the learning of probability meaningful. Two important processes are involved at this stage. Firstly, teachers should explore the notion of conceptional understanding as explained by Kilpatrick et al. (2001). Secondly, they need to gain deeper understanding of what probability and teaching of probability entail. Knowledge of curriculum policies and the essence of probability are critical to effective teaching of probability.

This stage is about making propositions to enhance the conceptual understanding of probability and will lead to the adaptation of instructional practices. Stage 1 was based on the study findings, the literature, theories of teaching and learning. The work of other scholars could help teachers to understand various approaches used in solving problems of probability. These policies, which specify and clarify what content to cover in each grade, together with findings from literature, provide guidance in developing students' conceptual understanding of probability.

STAGE 2: Identify and explain essence of conceptual understanding

In this stage, teachers consider the skills and background that students have, and this information helps them to formulate appropriate themes for investigation. In their study, Cordova and Tan (2018) recommended that teachers need to conduct diagnostic tests of students' mathematics proficiency to help the next teacher determine where to begin. Teachers

could collect data by presenting an initial lesson in the classroom. From this lesson and through formative assessment, they should be able to identify and study the needs of the students. Equally important, teachers need to gather information regarding their own strengths and challenges; they need to reflect on their practice. Students' needs will immediately suggest strategies to teachers that might help students to learn ways of comprehending, realising, grasping and developing insight into the concept of probability. For learning to take place, teaching activities should aim at building awareness of opportunities available for students and risks involved, amongst others.

STAGE 3: Reconsider cores of conceptual understanding in probability

Darling-Hammond et al. (2017) advise that to implement the framework effectively, common obstacles should be anticipated and planned for. These include:

- Inadequate resources, including required curriculum materials;
- Lack of shared vision of what high-quality instruction entails;
- Lack of time for planning and implementing new instructional approaches;
- Conflicting requirements, such as scripted curriculum pacing guides; and
- Lack of adequate foundational knowledge on the part of teachers.

Some insightful ideas to enhance conceptual understanding of probability are presented in Figure 6.4 and expressed in the form of questions. These questions could help teachers towards filling their knowledge gaps and addressing challenges. Additionally, responses to these questions have the potential to articulate teachers' current understanding of probability and prompting teachers' reflective thoughts. The questions are comprehensively elaborated in Figure 6.4 as follows:

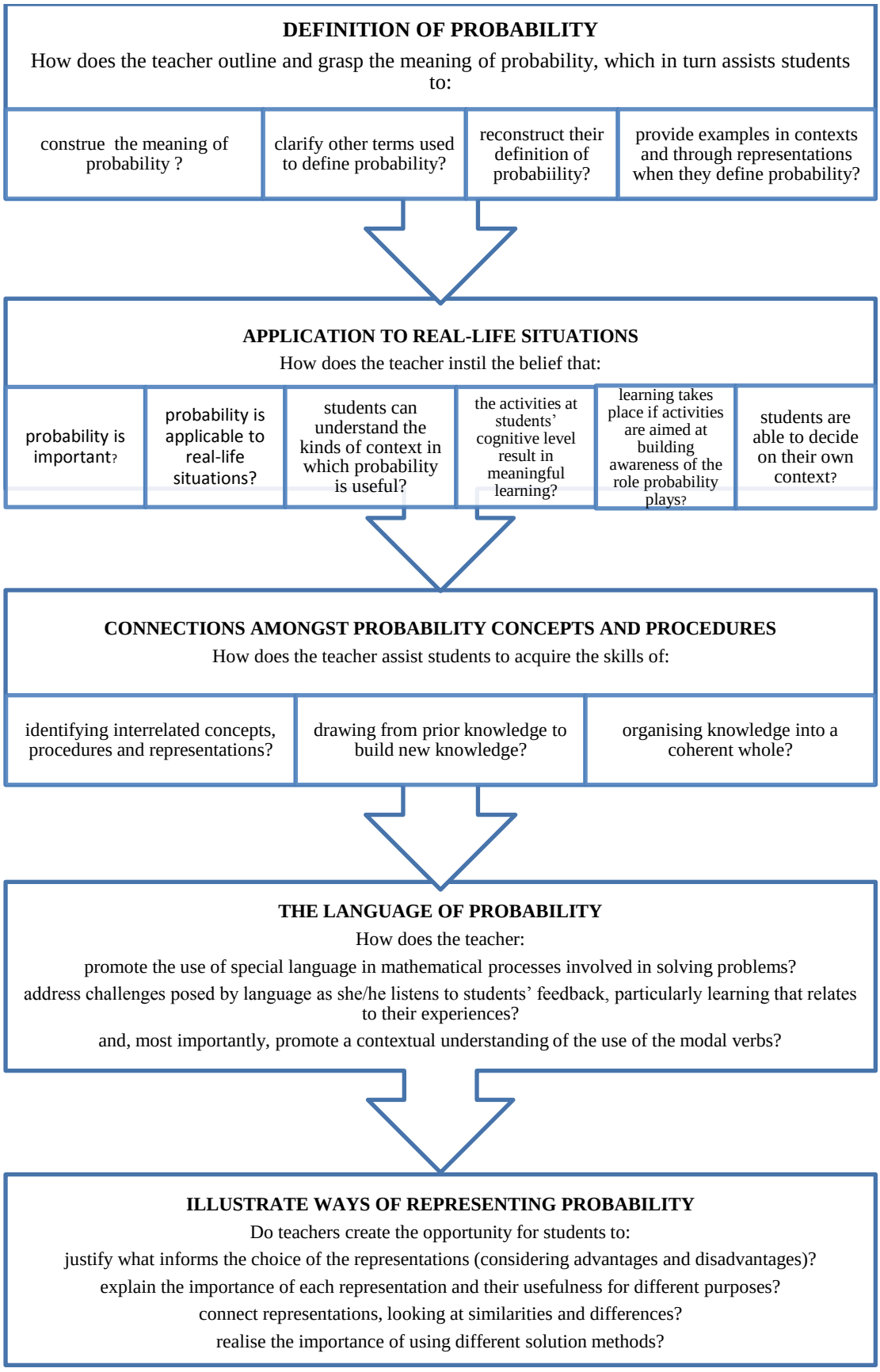


Figure 6.4. Clarifying cores of conceptual understanding in relation to probability

Source: Kilpatrick et al., 2001

Figure 6.4 provides some guidelines on how teachers could use their strategic competence to draw the students deeper into constructing meaning of probability. Figure 6.5 illustrates processes teachers could engage in to reconstruct the definition of probability

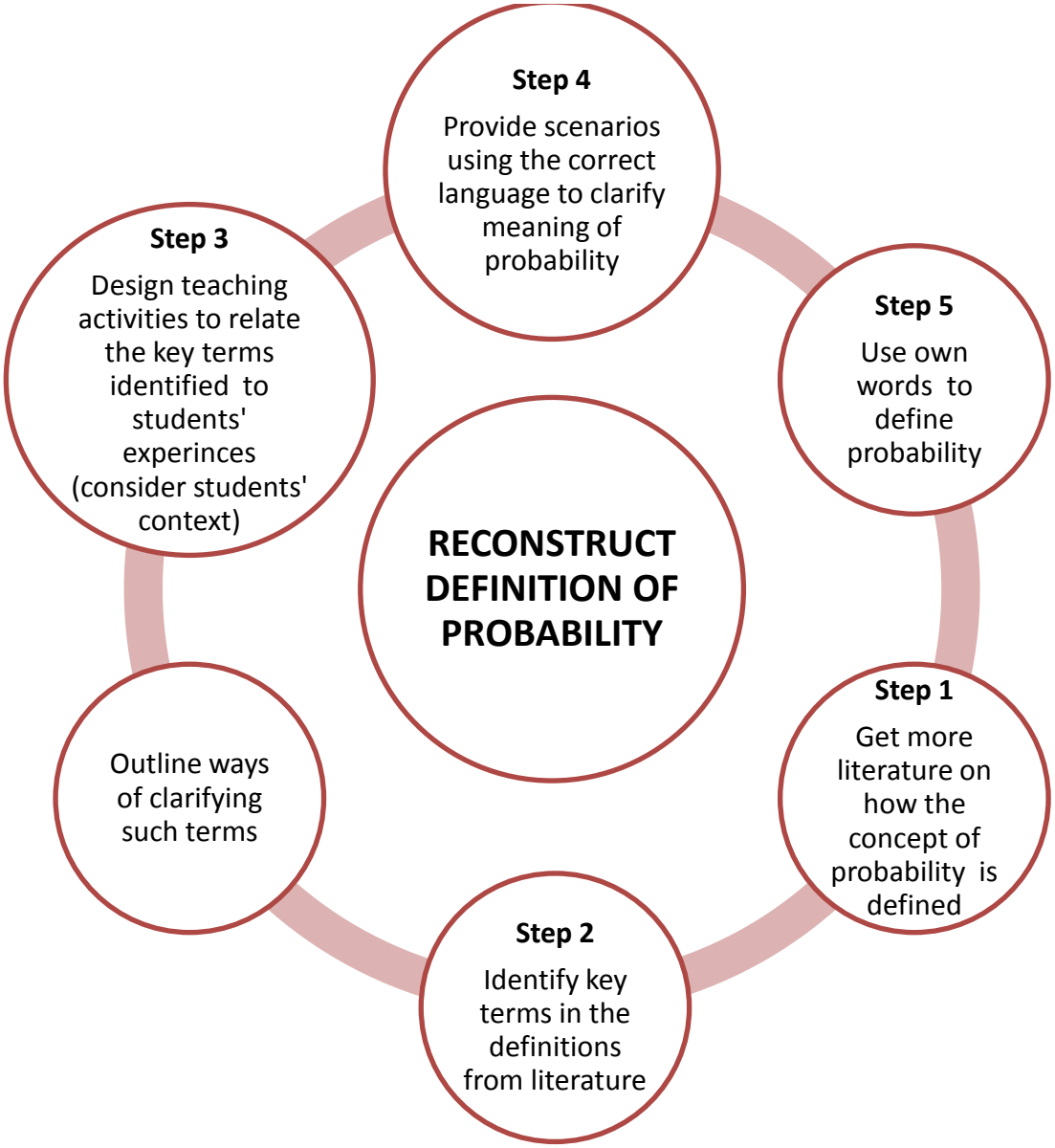


Figure 6.5. Processes involved in reconstructing the definition of probability

Source: Informed by findings in the current study

To fully realise the gains that are desired, teachers could use the ideas in Figure 6.5 to design their own models to explore the cores of conceptual understanding as outlined in Figure 6.4.

STAGE 4: Adaptation and re-design of teaching activities

This is the stage where teachers engage in the process of augmenting teaching and learning activities on probability. Their strategic competence should enable them to use effective strategies and proper resources to redesign these activities. The findings and all the processes involved in this study suggest that augmenting teaching activities helped teachers to acquire a conceptual understanding of probability.

To use an analogy, if the objective of a lesson is a baked cake, then the design of teaching activities is the list of ingredients. The taste of the cake will depend on the method and utensils used. In other words, this will depend on the how the ingredients were mixed, and the steps followed in the baking. In the same way, well-designed teaching activities promote meaningful learning. Figure 6.6 provides an overview of the processes involved in augmenting teaching activities on probability.

Figure 6.6. Overview to augment teaching and learning activities

Source: Takahashi, 2016 and Tibebu, 2015

The next stage was informed by the fact that teachers felt the need to review how lessons were planned, based on what they learnt in this study. Although this forms part of the framework, it could be implemented as part of in-service training by subject advisors. In addition, researchers could test its workability in the community engagement projects.

STAGE 5: Plan, prepare and implement the lesson

In this stage, teachers are encouraged to work collaboratively as a team. All the processes involved are enriched through reflection and adjustment. In order to guide the planning, preparation and facilitation of the lesson, teachers need to be aware that the most important aspect that influences learning is how meaning and knowledge are constructed through interaction with others (Van Merriënboer & de Bruin, 2014). This is aligned to the view of Van den Heuvel-Panhuizen and Drijvers (2014), who argue that students should be treated as active participants in the learning process. In other words, teachers should design activities in such a way that students become engaged in constructing knowledge meaningfully. Because the proposed framework caters for teacher professional development, teachers need to work together and share good practices. Teachers should plan and simulate the lesson before presenting it in a real classroom setting.

- **Simulate the lesson amongst peers**

The lesson could be simulated twice before it is presented in the real setting of the classroom. One of the team members presents the lesson and the others become the observers and write field notes. The subject advisors facilitate the sessions. Peers reflect collaboratively on the simulated lesson. As they will be observing lessons, peers need to note all the challenges and make recommendations. They should understand the principles of constructivism in order to guide the facilitation strategies. They could also videotape the lesson for reflection at a later stage.

- **Present the lesson in the classroom and reflect on the presentation**

At this stage, teachers observe one of their peers presenting the lesson in a real classroom setting. As observers they need to follow the steps as illustrated in Figure 6.7.

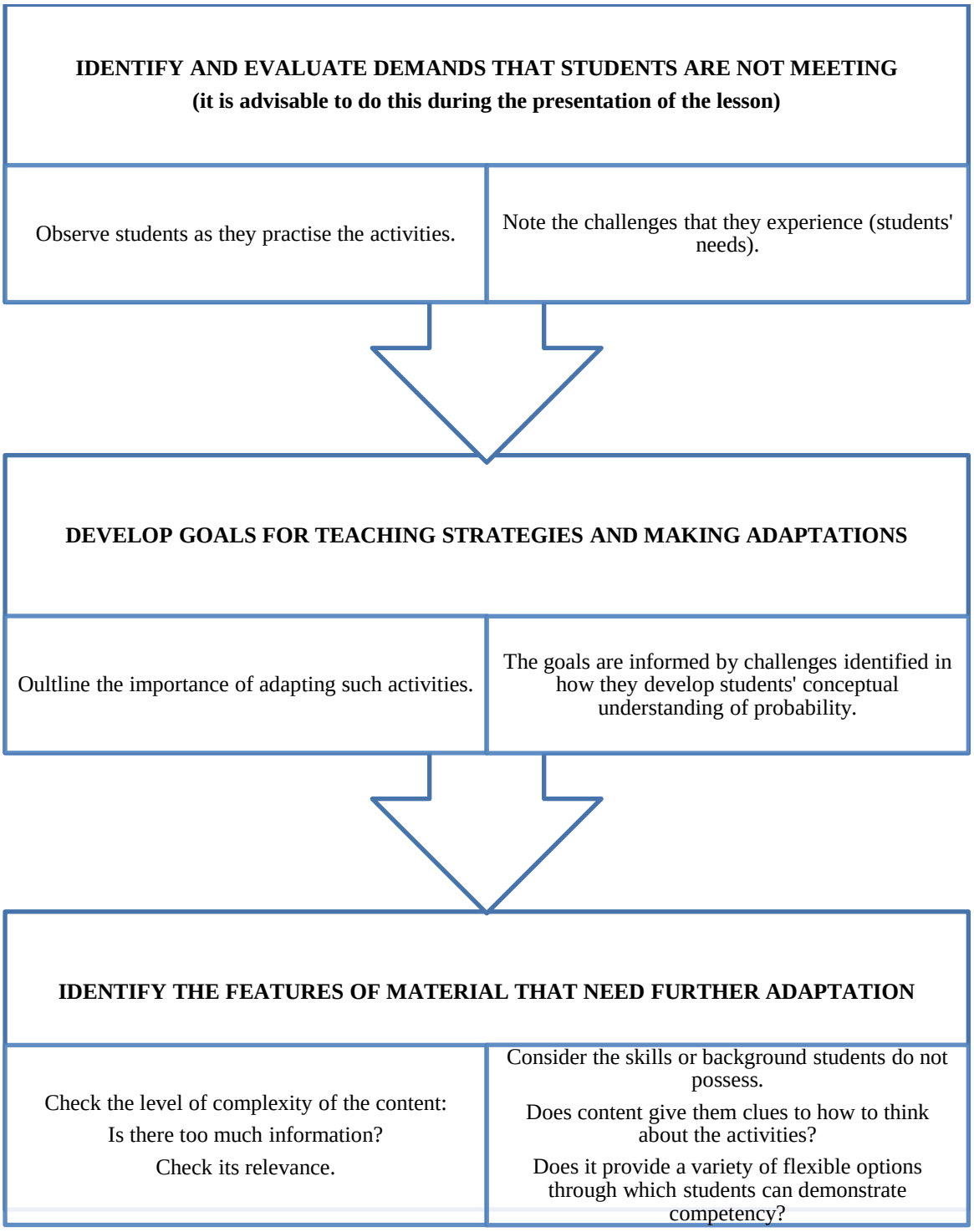


Figure 6.7. Redesign and development of teaching and learning activities

Source: Tibebu, 2015

- **Further redesign and development of teaching and learning activities**

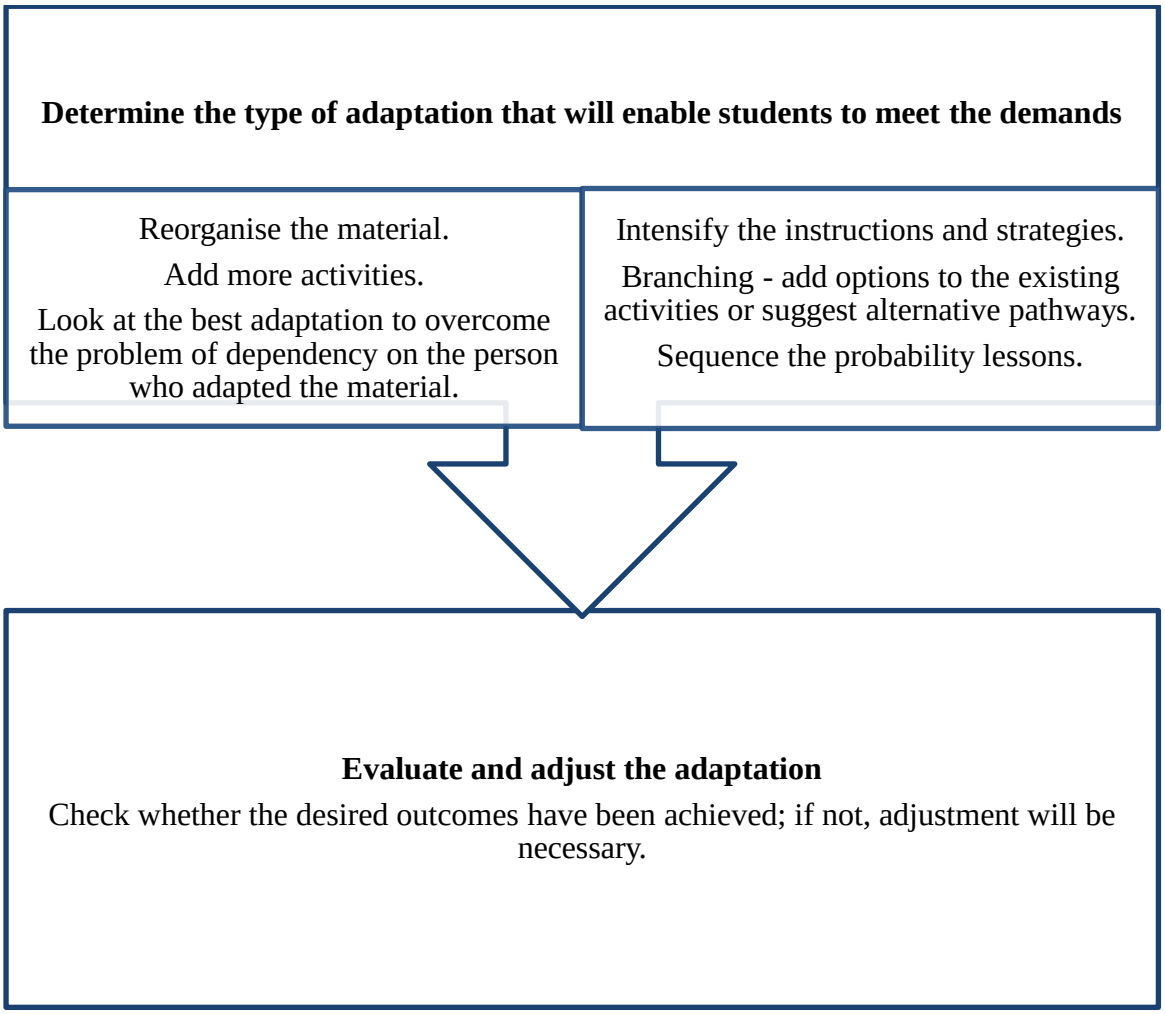


Figure 6.8. Further adjustment to activities

Source: Tibebu, 2015

Teachers, subject advisors and researchers can determine the extent to which the framework makes sense to them. Takahashi (2016) recommends that an investigation be conducted to check whether guidelines provided in the framework have the desired impact on instruction and teacher growth. Teachers could assess if the adapted teaching activities made some impact or would need further modification. It is equally important that teachers take responsibility in developing lessons and acknowledging the importance of doing this exercise.

The following aspects are also critical to enhance conceptual understanding of probability:

- Sequencing of teaching activities cannot be scripted; it depends on how knowledge is constructed and how instructional practices are guided by the learning outcomes.
- The needs of both teachers and subject advisors should be considered to provide more guidance on how the intervention workshop should unfold.
- The nature of engagement between teachers and students or between teachers and subject advisors is informed by the goals of the developed activities (Ni et al. (2014).
- Teachers could explain probability concepts using the contrast of the concept (Zulu, 2019) For example, if the aim of the lesson is to make students understand the term possible, then teachers could start by focusing on examples that are impossible.
- Material for the intervention workshop should be developed during the process, not beforehand, if it is to be valuable (Visnovska & Cobb, 2019).
- If teachers are to facilitate meaningful learning they need to identify students' needs through assessment and follow-up interviews (Warwick, Vrikki, Færøyvik, Dudley, & Vermunt, 2019).

As part of the process of testing the practicability of the framework, teachers could assess how they benefited from the workshop by answering the questions in Table 6.1:

Table 6.1

Reflection activities

1	**Teachers**: Reflect on what transpired in the classroom and make recommendations to enhance your practices. **Subject advisors:** Conduct needs analysis through lesson observations and interviews, develop your own framework on how to support teachers in enhancing their instructional classroom practice.
2	**For both teachers and subject advisors** Write a short presentation of not more than four pages on how you would share your experiences with other teachers (or subject advisors). Use examples to justify your arguments. Your presentation should include highlights, challenges and recommendations.
	PLEASE NOTE The focus of your activities at each stage should be guided by your aims, objectives and your continuous engagement with the students (or teachers).

Reflection activities

- Encourage teachers and subject advisors to know their field.
- Build the capacity, competence, and confidence of teachers (subject advisors) when dealing with the concept of probability.
- Do not encourage focus on student achievement but rather on what goes into enhancing effective instructional practice.
- Encourage listening to student explanations to guide planning and implementation.

I conclude the exposition of the framework by making the following justifications. Augmenting teaching and learning activities is one of the key areas that teachers should master to guide the effective delivery of lessons in the classroom if students are to make sense of the concept of probability. Collaborative planning and reflection on the planned activities will enable teachers to share good practices. It cannot be overstated that purposeful teaching and learning activities will result in meaningful learning. The findings of this study have provided evidence that there are different areas that one can focus on to enhance teachers' and subject advisors' practices. In this study, strategies and mechanisms for augmentation of teaching and learning activities were perceived by all participants to be most effective and efficient. In this section, I described the theory behind the proposed RIRAD framework, provided examples of activities based on the framework, and illustrated methodological contributions that resulted from this work

- **Future practices**

I believe that the framework could function in other countries because it is adaptable to any context, particularly for classroom practice. Although no generalisations can be made, I do believe that this study will inspire teachers and subject advisors to use the proposed framework to improve the teaching of probability to Grade 7 learners. I argue that this framework would be suitable for any subject area. It is hoped that the framework designed in this study will help teachers to adapt and design their own activities.

6.8 Suggestions for further research

It is recommended that the proposed framework be expanded by implementing Stage 5 (development of the lesson plan). This was found to be one of the critical stages to enhance

teachers' instructional practice. Subject advisors could implement this stage in a workshop as part of in-service training. Closely related to this is the suggestion that researchers could also engage teachers at this level in community engagement projects. It is therefore crucial that the DBE identifies and plans for teacher support services that are relevant to teachers' needs, particularly within the limits of available resources. More research is needed to understand how greater support can be provided for mathematics subject advisors.

Further research should be conducted to establish how the framework could contribute to new evidence. In other words, the framework could also be tested in other subjects and disciplines to determine whether its use improved performance. It is my understanding that this framework offers a complete package of planning, teaching, learning and assessment.

Any research study in the field of teacher education should ensure that teachers understand the theories of teaching and learning. Teachers are lifelong researchers and learners. Theories are lenses to guide research studies and implementation to enhance practices. Teachers' planning and implementation should be grounded in the theories. I look forward to refining this framework through a community engagement project, to the point where it is a model of educational excellence.

6.9 Conclusion

This chapter discussed the conclusions drawn from the findings. It also made recommendations based on the findings. One important issue arising from the study findings was that there is still a great deal of work to be done to foster deeper understanding of the concept of probability in South African Grade 7 mathematics classrooms. The study followed a vigorous step-by-step approach to data collection and analysis. The findings from this study identified the challenges that teachers experienced in carrying out their responsibilities and reasons that underlie teachers' limited conceptual understanding of probability.

Major concerns that were flagged that limit teachers' conceptual understanding of probability include, amongst other things, ineffective ways of incorporating resources, challenges in formulating scenarios, inability to restructure the definition of probability, using teaching activities that are sometimes not real, complexities in using the modal verbs, the use of probability terms interchangeably, inability to develop own teaching resources (concrete

objects), inability to highlight what informs representations used in probability and also awareness that teaching activities build in relation to opportunities available or risks involved.

Secondly, the participants' biographical details did not appear to have any link to how participants comprehended probability because, on average, they used the common lesson plans provided by the provincial Department of Education. These findings provided useful insights into how readily available teaching and learning activities could be augmented and implemented. To provide some assistance, this study conceptualised the RIRAD Framework that suggests ways to enhance teachers' conceptual understanding of probability and classroom practice. Suggestions made in the framework highlight various ways in which the teachers could amend and adjust their practices. The RIRAD framework is deemed a useful tool because it addresses the multi-dimensionality of teacher support systems. Another advantage of this framework is that it could be used by any teacher at any school. Additionally, this framework identified several issues that require attention and it is therefore my belief that this framework, if properly implemented, would make teachers realise that total reliance on a prescribed textbook does not benefit them.

Although the study's degree of impact varied, the majority of participants acknowledged that they benefited from this study, particularly when it came to the importance of augmenting the teaching and learning activities in any resource material. They regarded this exercise as a key learning tool. This acknowledgement suggests that teachers might function better if they engage in the process of adapting the activities to suit the needs of their own classrooms. The view of most of the participants was that all the strands of mathematical proficiency were key to effecting planning and teaching for proficiency. It is important to highlight that participants also appreciated sharing good practices during the intervention sessions. They agreed that they had enjoyed and benefitted from taking part in the study. In conclusion, the Department of Basic Education should provide support to facilitate the enhancement of teachers' classroom instructional practices in this regard.

6.10 Autobiographical reflection: Experiences in conducting research

Undertaking this research study was an invaluable learning experience. I gained some understanding of the cyclical nature of research. This can be explained as follows: firstly, I acknowledge that all the research stages are equally important and interdependent. Secondly,

analysis of data is not linear; the researcher needs to assemble, disassemble, and reassemble data and read closely in order to make sense of it (Yin, 2016). Results are interpreted with caution, and I began to understand why journalists use the language of probability in reporting news.

Research is like an essay; one cannot write it in bits and pieces. Most critical research is very broad; one needs a lens to narrow down the focus. I also learnt that any view highlighted in the process needs justification. Evidence is necessary to substantiate any claim made. Ambiguity in research might lead to readers interpreting other scholars' claims incorrectly. As the researcher, one needs to understand the field of study to make a sensible argument. For example, if my focus is on teaching, I need to know about the policies and theories on teaching and learning.

Every topic in research study has its own specialised language; what was most crucial in this study was that terms describing degrees of uncertainty took centre stage in the discussion of the findings. This study taught me that knowledge is not static and new discoveries are constantly being made. Thus, I need to keep on reading. The process of the study sharpened my research skills and provided insights into aspects such as ethical issues, how to analyse the needs of the participants and issues of teacher professional development.

Going forward, our teachers will need to understand how important self-directed learning is to keep up with the pace of professional development. The participants' interest sometimes guides the professional development processes, that is why activities of the workshops cannot be pre-planned.

6.11 Reflexivity: Life is probabilistic in nature

As Robinson and Rusznyak (2020, p. 526) observe,

> … the changing and demanding contextual conditions of Covid-19 have shown us that nothing can be taken for granted, that we cannot build on our 'tried and tested' approaches to teacher education, and that our decisions about teacher preparation need to remain ever-open to new modalities and conditions.

I acknowledge the words of Robinson and Rusznyak (2020) by indicating that whenever the concept probability is taught, teachers need to ensure that they build awareness of risks involved and opportunities available in real life situation. This means that learning should be framed to gain some insight into day-day issues affecting the teachers, students and the nation at large.

In relating this topic to life, I therefore conclude this study by saying:

> *I understand the concept of probability (the degree of chance) to refer to the lens with specialised language through which the realities of life could be understood. The deeper understanding of this lens will enable people to make informed decisions and judgements. The topic, probability, assists us to understand that there is no entitlement or declaration to certainty. Every moment or stage in life is an era of uncertainty. For us to understand and respond accordingly, we need to experience, experiment, reflect, predict, adjudicate situations and be realistic so that we become relevant and significant.*

OMB expressed her views and said:

> *It is undeniable that life is fraught with unknowns. We have no idea what will happen in the next hour, much less what will happen tomorrow. We prepare our lives and our schedules to the best of our abilities as beings, but nobody knows what will happen next or whether things will go as expected, no matter how well we prepare. You already embrace a lot of uncertainty every day, no matter how hard you try to remove doubt and instability from your life. You accept a degree of uncertainty when you get behind the car wheel or when you cross the road when the little man is green, not knowing if a car will ignore the traffic light signs that day. As a student, I can fully attest to this. When Covid-19 hit the world, it was all games until I had to start online classes. I had never done this before. Time management skills and self-discipline were some of the qualities which I needed but didn't possess. There was only one way to cope with this change, Adapting. I had to adapt to this new way of operations or else I was headed for a dead-end in my Grade 11 academic year. Change is all around us, and it is the single factor that has the greatest effect on our lives. Change is inevitable; it will find you, question you, and force you to rethink your life choices. It's how we respond to the change that matters most.*

RIM set forth her opinion and said:

> *Life is full of uncertainty. Due to the scarcity of experts in the civil engineering field, as a student, I was certain that I would get a job after the completion of my studies, of which didn't happen. I got to realise that even the most certain careers may bring about uncertainty when job seeking. When I entered workspace, the allocation of civil engineering work was dependent on the type of current projects in the company, which was another type of uncertainty presented to me as an employee. My interests in the field laid in roads and storm water but the projects at hand were traffic engineering related. This then persuaded me in venturing in the traffic engineering part of civil engineering. As a female engineer and leading some of these projects, when I interacted with other stakeholders, they became reluctant to engage in conversations due to them being uncertain of the quality of work that I could produce as a professional. The transition into adulthood especially in the workplace, brings about an inevitable degree of uncertainty. The best solution to this is being able to adapt and transform to the changes.*

REFERENCES

Adem, S. (2021) Constructivism. In *Postcolonial Constructivism. Global Political Thinkers*. Cham.: Palgrave MacSkovan.

Adler, J. (2017). Mathematics discourse in instruction (MDI): A discursive resource as boundary object across practices. In *Proceedings of the 13th international congress on mathematical education* (pp. 125-143). Cham.: Springer.

Ahuriri-Driscoll, A., Lee, V., & Came, H. (2021). Amplifying indigenous voice and curriculum within the public health academy–the emergence of indigenous sovereign leadership in public health education. *Higher Education Research & Development*, *40*(1), 146-161. https://doi.org/10.1080/07294360.2020.1857343

Alkhateeb, H. M. (2021). Redesigning Developmental Mathematics Education: Implementation and Outcomes. *PRIMUS*, 1-23. DOI: 10.1080/10511970.2021.1872749

Ally, N. (2011). *The promotion of mathematical proficiency in grade 6 mathematics classes from the uMgungundlovu District in KwaZulu-Natal* [Doctoral dissertation, University of KwaZulu-Natal]. http://hdl.handle.net/10413/5791

Alsulami, S. (2016). Toward a constructivist approach in Saudi education. *English Language Teaching*, *9*(12), 104-108. DOI: 10.5539/elt.v9n12p104

Andamon, J., & Tan, D. A. (2018). Conceptual understanding, attitude and performance in mathematics of grade 7 students. *International Journal of Scientific & Technology Research*, *7*(8), 96-105.

Ang, L. H., & Shahrill, M. (2014). Identifying students' specific misconceptions in learning probability. *International Journal of Probability and Statistics*, *3*(2), 23-29.

Anney, V. N. (2014). Ensuring the quality of the findings of qualitative research: Looking at trustworthiness criteria. *Journal of Emerging Trends in Educational Research and Policy Studies (JETERAPS), 5*(2), 272-281.

Arani, M. R. S. (2017). Raising the quality of teaching through Kyouzai Kenkyuu–the study of teaching materials. *International Journal for Lesson and Learning Studies*, *6*(1), 10-26.

Arsaythamby, V., & Zubainur, C. M. (2014). How a realistic mathematics educational approach affect students' activities in primary schools? *Procedia-Social and Behavioral Sciences*, *159*, 309-313.

Aven, T., & Bouder, F. (2020). The COVID-19 pandemic: How can risk science help? *Journal of Risk Research*, *23*(7-8), 849-854. DOI: 10.1080/13669877.2020.1756383

Awofala, A. O. A. (2017). Assessing senior secondary school students' mathematical proficiency as related to gender and performance in mathematics in Nigeria. *International Journal of Research in Education and Science (IJRES), 3*(2), 488-502. DOI: 10.21890/ijres.327908

Babbie, E., & Mouton, J. (2001). *The Practice of social research.* Cape Town: Oxford University Press.

Bansilal, S. (2017). The difficulty level of a national assessment of Grade 9 mathematics: The case of five schools. *South African Journal of Childhood Education 7*(1), a412. https://doi.org/10.4102/sajce. v7i1.412

Barham, A. I. (2020). Investigating the development of pre-service teachers' problem-solving strategies via problem-solving mathematics classes. *European Journal of Educational Research*, *9*(1), 129-141. https://doi.org/10.12973/eujer.9.1.129

Batanero, C. (2015, February). Understanding randomness: Challenges for research and teaching. *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education* (pp. 34-49).

Belbase, S. (2020). Early Undergraduate Emirati Female Students' Beliefs about Learning Mathematics Using Technology. *European Journal of Educational Research*, *9*(3), 1235-1255.

Bertsekas, D. P., & Tsitsiklis, J. N. (2002). *Introduction to probability* (Vol. 1). Belmont, MA: Athena Scientific.

Bhat, B. R. (2007). *Modern probability theory*: *An introductory text book*. New Delhi: New Age International.

Borovcnik, M. G., & Kapadia, R. (2010). Research and developments in probability education internationally. *Proceedings of the British Congress for Mathematics Education*, *30*(1).

Bozkurt, G. (2017). Social constructivism: Does it succeed in reconciling individual cognition with social teaching and learning practices in mathematics? *Journal of Education and Practice*, *8*(3), 210-218.

Braun, V., & Clarke, V. (2014). What can "thematic analysis" offer health and wellbeing researchers? *International Journal of Qualitative Studies on Health and Well-being*, *9*.

Bretuo, P. (2020). Using language to improve learning: Teachers' and students' perspectives on the implementation of bilingual education in Ghana. *Language, Culture and Curriculum*, 1-16.

Brijlall, D. (2014). Exploring the pedagogical content knowledge for teaching probability in middle school: A South African case study. *International Journal of Educational Sciences*, *7*(3), 719-726.

Brodie, K., & Berger, M. (2010) Toward a discursive framework for learner errors in mathematics. *18th annual meeting of the Southern African Association for Research in Mathematics, Science and Technology Education, Durban*.

Brown, C., Taylor, C., & Ponambalum, L. (2016). Using design-based research to improve the lesson study approach to professional development in Camden (London). *London Review of Education*, *14*(2), 4-24.

Browning, C., Goss, J., & Smith, D. (2014, July). Statistical knowledge for teaching: Elementary preservice teachers. *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS)*.

Bruce, C. D., Flynn, T. C., & Bennett, S. (2016). A focus on exploratory tasks in lesson study: The Canadian 'Mathematics for Young Children' project. *ZDM Mathematics Education*, *48*(4), 541-554. doi: 10.1007/s11858-015-0747-7.

Bryman, A.(2012). *Social science research* (4th ed.). New York: Oxford University Press.

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for research in mathematics education*, *19*(5), 385-401.

Casserly, M. (2016). *A Framework for Re-Envisioning Mathematics Instruction for English Language Learners.* Council of the Great City Schools. https://eric.ed.gov/?id=ED580863

Cheng, E. W. (2016). Learning through the Variation Theory: A Case Study. *International Journal of Teaching and Learning in Higher Education*, *28*(2), 283-292. http://www.isetl.org/ijtlhe/

Cheng, L. P., Leong, Y. H., & Toh, W. Y. K. (2021). Singapore Secondary School Mathematics Teachers' Selection and Modification of Instructional Materials for Classroom Use. *Mathematics Instructional Practices in Singapore Secondary Schools* (pp. 205-230). Singapore: Springer.

Cooper, J. T., & Scott, T. M. (2017). The keys to managing instruction and behaviour: Considering high probability practices. *Teacher Education and Special Education*, *40*(2), 102-113.

Cordova, C., & Tan, D. A. (2018). Mathematics proficiency, attitude and performance of Grade 9 students in private high school in Bukidnon, Philippines. *Asian Academic Research Journal of Social Sciences and Humanities*, *5*(2), 103-116.

Creswell, J. W. (2008). *Educational research: Planning, conducting and evaluating quantitative and qualitative research* (3rd ed.). USA: Pearson Prentice Hall.

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches* (International student edition). SAGE.

Curto, J. C. (2019, February). The Use of the Modal Verb Shall in Legal Texts. *Ideas*, *4*(4).

Danişman, Ş., & Tanişli, D. (2018). Examination of mathematics teachers' pedagogical content knowledge of probability. *MOJES: Malaysian Online Journal of Educational Sciences*, *5*(2), 16-34.

Darling-Hammond, L., Hyler, M. E., & Gardner, M. (2017). Effective teacher professional development. *Learning Policy Institute*. https://learningpolicyinstitute.org

Davies, C., & Fisher, M. (2018). Understanding research paradigms. *Journal of the Australasian Rehabilitation Nurses Association*, *21*(3), 21-25.

Dayal, H. C., & Sharma, S. (2020). Investigating Probability Concepts of Secondary Pre-service Teachers in a Game Context. *Australian Journal of Teacher Education, 45*(5). http://dx.doi.org/10.14221/ajte.2020v45n5.6

De Kock, S. M. (2015). *Relating teachers' content knowledge to learner performance in probability* [Doctoral dissertation, University of Pretoria].

Denzin, N. K., & Lincoln, Y. S. (2005). Introduction: The discipline and practice of qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative and quantitative research* (pp. 1-32). Thousand Oaks, California: Sage Publications.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12-25.

Department of Basic Education (DBE). (2011). *Curriculum and Assessment Policy Statement (CAPS) Senior Phase (Grades 4-6) Mathematics.* Retrieved from http://www.education.gov.za

Department of Basic Education (DBE). (2012). *Diagnostic reports: Annual National Assessments.* Retrieved from. http://www.education.gov.za

Department of Basic Education (DBE). (2013). *Annual National Assessment: Diagnostic report and 2014 Framework for improvement.* Pretoria: Government Printers. Retrieved from. http://www.education.gov.za

Department of Basic Education (DBE). (2017). *The SACMEQ 1V Project in South Africa: Study of the Conditions of Schooling and the Quality of Education.* https://www.education.gov.za/LinkClick.aspx?fileticket=tWsFPndM_gI%3d&tabid=92&portalid=0&mid=4366

Department of Basic Education (DBE). (2018a). *Mathematics teaching and learning framework for South Africa. Teaching Mathematics for understanding.* Retrieved from http://www.education.gov.za

Department of Basic Education. (2018b). *National Senior Certificate 2017 Diagnostic Report: Part 1*. Pretoria: DBE.

Department of Basic Education. (DBE). (2018c). *Circular S5 of 2018*. Retrieved from https://www.education.gov.za/Portals/0/Documents/Publications/Circular%20S5%200f%202018.pdf?ver=2018-06-14-133733-950

Department of Basic Education (DBE). (2019). *National Senior Certificate 2018 Diagnostic Report: Part 1*. Pretoria: DBE.

Department of Basic Education (2020a). *Grade 7 Mathematics in English.* https://www.education.gov.za/LinkClick.aspx?fileticket=oERr08gNqiI%3d&tabid=2598&portalid=0&mid=9229

Department of Basic Education (DBE). (2020b). *Report on the 2019 National Senior Certificate 2019 Diagnostic Report: Part 1*. Pretoria: DBE.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*(3), 181-199.

Di Leo, I., Muis, K. R., Singh, C. A., & Psaradellis, C. (2019). Curiosity… Confusion? Frustration! The role and sequencing of emotions during mathematics problem solving. *Contemporary Educational Psychology*, *58*, 121-137.

Dos Santos Ferreira, R., Kataoka, V. Y., & Karrer, M. (2014). Teaching probability with the support of the R statistical software. *Statistics Education Research Journal*, *13*(2).

Duez, E. (2018). Global applications of the Japanese? ‘Lesson Study’ teacher education and training model. *International Dialogues on Education: Past and Present*, *5*(1), 65-73.

DuFon, M. (2002). Video recording in ethnographic SLA research: Some issues of validity in data collection. *Language Learning & Technology*, *6*(1), 40-59.

Du Plooy-Cilliers, F., & Cronje, J. (2014). Research paradigms and traditions. *Research matters*, 18-35.

Durand-Guerrier V., Kazima, M., Libbrecht, P., Ngansop, J. N., Salekhova, L., Tuktamyshov, N., & Winslow, C. (2016) Challenges and Opportunities for Second Language Learners in Undergraduate Mathematics. In R. Barwell et al. (Eds.), *Mathematics Education and Language Diversity* (pp. 85-101). New ICMI Study Series. Cham.: Springer. https://doi.org/10.1007/978-3-319-14511-2_5

Early, D. M., Maxwell, K. L., Ponder, B. D., & Pan, Y. (2017). Improving teacher-child interactions: A randomized controlled trial of Making the Most of Classroom Interactions and My Teaching Partner professional development models. *Early Childhood Research Quarterly*, *38*, 57-70. http://creativecommons.org/licenses/by-nc-nd/4.0/

Elbehary, S. (2019). The necessity of revising primary school content of Probability in Egypt to enhance students’ Probabilistic reasoning. Decision making based on data. *Proceedings of the Satellite conference of the International Association for Statistical Education (IASE)*.

Elicer, R. (2019). The role of decision-making in the legitimation of probability and statistics in Chilean upper secondary school curriculum. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 930-937). Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Enderson, M. C., Grant, M. R., & Liu, Y. (2018). Supporting Mathematics Coaches' Learning of Probability through Professional Development Tasks. *International Journal of Research in Education and Science*, *4*(2), 613-632.

Enock (2013). Indigenous knowledge systems and modern weather forecasting: Exploring the linkages. *Journal of Agriculture and Sustainability*, *2*(2).

Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM–Mathematics Education*, 1-18.

Ersanli, Y. C. (2017). Identifying the English language learning needs of prospective mathematics teachers in elementary education. *New Trends and Issues. Proceedings on Humanities and Social Sciences*. [Online]. *4*(3), pp. 39-45. Available from: https://doi.org/10.18844/prosoc.v4i3.2514 (Original work published October 14, 2017) www.prosoc.eu

Etikan, I., & Bala, K. (2017). Sampling and sampling methods. *Biometrics & Biostatistics International Journal*, *5*(6), 215-217.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *Zdm*, *45*(5), 633-646.

Fauziyah, N., & Uchtiawati, S. (2017). Developing a model of educators' professional training special for remote areas through the implementation of lesson study. *International Education Studies*, *10*(8), 108-115. https://doi.org/10.5539/ies.v10n8p108

Fong, R. (2020). Is knowing the constructions enough to understand modality patterns in English? A response to 'Taming English modals–how a construction grammar approach helps to understand modal verbs' by Sergio Torres–Martínez, English Today, 138, 35(2), 50–57, 2019. *English Today*, 1-7. DOI: https://doi.org/10.1017/S0266078420000425

Fujii, T. (2015). The critical role of task design in lesson study. In *Task design in mathematics education* (pp. 273-286). Springer, Cham.

Fujii, T. (2016). Designing and adapting tasks in lesson planning: a critical process of Lesson Study. *ZDM Mathematics Education*,*48*(4), 411-423. doi: 10.1007/s11858-016-0770-3.

Gaible, E., & Burns, M. (2005). *Using technology to train teachers: Appropriate uses of ICT for teacher professional development in developing countries*. Washington, DC: infoDev/World Bank. Available at: http://www.infodev.org/en/Publication.13.html

Gazdula, J., & Farr, R. (2020). Teaching risk and probability: Building the Monopoly® board game into a probability simulator. *Management Teaching Review*, *5*(2), 133-143.

Geiger, B. B. (2021, January). Performing trustworthiness: The 'credibility work' of prominent sociologists. *Sociology*. DOI:10.1177/0038038520977805

Gómez-Torres, E., Batanero, C., Díaz, C., & Contreras, J. M. (2016). Developing a questionnaire to assess the probability content knowledge of prospective primary school teachers. *Statistics Education Research Journal*, *15*(2), 197-215.

Gravemeijer, K., Bruin-Muurling, G., Kraemer, J. M., & van Stiphout, I. (2016). Shortcomings of mathematics education reform in The Netherlands: A paradigm case? *Mathematical Thinking and Learning*, *18*(1), 25-44. https://doi.org/10.1080/10986065.2016.1107821

Graven, M. (2012). Accessing and assessing young learner's mathematical dispositions. *South African Journal of Childhood Education*, *2*(1), 49-62.

Grimmett, G., & Stirzaker, D. (2020). *Probability and random processes* (4th ed.). Oxford University Press.

Groth, R. E. (2017). Classroom data analysis with the five strands of mathematical proficiency. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *90*(3), 103-109. DOI: 10.1080/00098655.2017.1301155

Groth, R. E., Butler, J., & Nelson, D. (2016). Overcoming challenges in learning probability vocabulary. *Teaching Statistics*, *38*(3), 102-107.

Groves, S. (2012). Developing mathematical proficiency. *Journal of Science and Mathematics Education in Southeast Asia*, *35*(2), 119-145.

Groves, S., Doig, B., Vale, C., & Widjaja, W. (2016). Critical factors in the adaptation and implementation of Japanese lesson study in the Australian context. *ZDM Mathematics Education*, *48*(4), 501-512. doi: 10.1007/s11858-016-0786-8.

Gu, F., & Gu, L. (2016). Characterizing mathematics teaching research mentoring in the context of Chinese lesson study. *ZDM Mathematics Education*, *48*(4), 441-454. doi: 10.1007/s11858-016-0756-1.

Gunawan, I., Nurhayati, L., & Hendrawan, B. (2021). Mathematical proficiency profile of prospective mathematics teacher students. *Journal of Physics: Conference Series*, *1764*(1). DOI:10.1088/1742-6596/1764/1/012109

Günes, G., Bati, K., & Katranci, M. (2017). An examination of the epistemological views and learning styles of pre-service teachers. *International Journal of Progressive Education*, *13*(3), 112-128.

Gürbüz, R., & Birgin, O. (2012). The effect of computer-assisted teaching on remedying misconceptions: The case of the subject "probability". *Computers & Education*, *58*(3), 931-941.

Gürbüz, R., Erdem, E., & Uluat, B. (2014). Reflections from the process of game-based teaching of probability. *Croatian Journal of Education: Hrvatski časopis za odgoj i obrazovanje*, *16*(Special Edition. 3), 109-131.

Hadfield, M., & Jopling, M. (2016). Problematizing lesson study and its impacts: Studying a highly contextualised approach to professional learning. *Teaching and Teacher Education*, *60*, 203-214. http://dx.doi.org/10.1016/j.tate.2016.08.001

Harvey, R., & Averill, R. (2012). A lesson based on the use of contexts: An example of effective practice in secondary school mathematics. *Mathematics Teacher Education and Development*, *14*(1), 41-59.

Hays, D. G., & McKibben, W. B. (2021). Promoting rigorous research: Generalizability and qualitative research. *Journal of Counselling & Development*, *99*(2), 178-188.

Hering, A. S., Durell, L., & Morgan, G. (2021). Illustrating Randomness in Statistics Courses with Spatial Experiments. *The American Statistician*, 343-353. DOI:10.1080/00031305.2020.1871070

Hiebert, J. (1986). Conceptual and procedural knowledge: *The case of mathematics.* Hillsdale, NJ: L. Erlbaum Associates.

Huang, R., Gong Z., & Han, X. (2016). Implementing mathematics teaching that promotes students' understanding through theory driven lesson study. *ZDM Mathematics Education*, *48*(4), 425-439. doi: 10.1007/s11858-015-0743-y.

Islam, S., & Cullen, J. (2021). Food traceability: A generic theoretical framework. *Food Control*, *123*. https://doi.org/10.1016/j.foodcont.2020.107848

Jing, T. J., Tarmizi, R. A., Bakar, K. A., & Aralas, D. (2017). The adoption of variation theory in the classroom: Effect on students' algebraic achievement and motivation to learn. *Electronic Journal of Research in Educational Psychology*, *15*(2), 307-325.

Joersz, J. R. (2017). Changing the way that math is taught: Conceptual versus procedural knowledge. *Learning to Teach*, *5*(1). http://utdr.utoledo.edu/learningtoteach/vol5/iss1/4

Johar, R., Patahuddin, S. M., & Widjaja, W. (2017). Linking pre-service teachers' questioning and students' strategies in solving contextual problems: A case study in Indonesia and the Netherlands. *The Mathematics Enthusiast*, *14*(1), 101-128. http://scholarworks.umt.edu/tme/vol14/iss1/8

Jojo, Z. M. M. (2011). *An APOS exploration of conceptual understanding of the chain rule in calculus by first year engineering students*. [Doctoral thesis, University of Kwa-Zulu Natal].

Joubert, J., Callaghan, R., & Engelbrecht, J. (2020). Lesson study in a blended approach to support isolated teachers in teaching with technology. *ZDM*, *52*(5), 907-925.

Kaur, B., Tay, E. G., Toh, T. L., Leong, Y. H., & Lee, N. H. (2021). A Study of the Enacted School Mathematics Curriculum in Singapore Secondary Schools. In *Mathematics Instructional Practices in Singapore Secondary Schools* (pp. 17-43). Singapore: Springer.

Kazeni, M., & Onwu, G. (2013). Comparative effectiveness of context-based and traditional approaches in teaching genetics: Student views and achievement. *African Journal of Research in Mathematics, Science and Technology Education*, *17*(1-2), 50-62. DOI:10.1080/10288457.2013.826970

Khaliliaqdam, S. (2014). ZPD, scaffolding and basic speech development in EFL context. *Procedia-Social and Behavioral Sciences*, *98*, 891-897. doi.org/10.1016/j.sbspro.2014.03.497

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.

Kivunja, C., & Kuyini, A. B. (2017). Understanding and applying research paradigms in educational contexts. *International Journal of Higher Education*, *6*(5), 26-41.

Krauss, S. E. (2005). Research paradigms and meaning making: A primer. *The qualitative report*, *10*(4), 758-770.

Kusmaryono, I., Jupriyanto, & Kusumaningsih, W. (2021). Construction of students' mathematical knowledge in the zone of proximal development and zone of potential construction. *European Journal of Educational Research*, *10*(1), 341-351. https://doi.org/10.12973/eu-jer.10.1.341

Kvatinsky, T., & Even, R. (2002). Framework for teacher knowledge and understanding about probability. *Proceedings of the Sixth International Conference on Teaching Statistics (CD). Cape Town, South Africa: International Statistical Institute.* https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.136.4097&rep=rep1&type=pdf8

Lanuza, M. H., Mendoza, R. V., Bayan, R. J. R., Elipane, L., Dimaculangan, K. A. C., & Hilario, F. F. A. (2020). A gamification technique through the method of a lesson study in teaching probability. *International Journal of Scientific & Technology Research*, *9*(4), 3734-3740.

Laurens, T., Batlolona, F. A., Batlolona, J. R., & Leasa, M. (2017). How does realistic mathematics education (RME) improve students' mathematics cognitive achievement? *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(2), 569-578.

Leavy, A. M., & Hourigan, M. (2016). Using lesson study to support knowledge development in initial teacher education: Insights from early number classrooms. *Teaching and Teacher Education*, *57*, 161-175. https://doi.org/10.1016/j.tate.2016.04.002

Le Donné, N., Fraser, P. & Bousquet, G. (2016), Teaching Strategies for Instructional Quality: Insights from the TALIS-PISA Link Data. *OECD Education Working Papers*, No. 148, Paris: OECD Publishing. https://doi.org/10.1787/5jln1hlsr0lr-en

Leedy, P. & Ormrod, J. E. (2001). *Practical Research Planning and Design* (7th ed.). London: Merrill Prentice-Hall.

Legesse, M., Luneta, K., & Ejigu, T. (2020). Analyzing the effects of mathematical discourse-based instruction on eleventh-grade students' procedural and conceptual understanding of probability and statistics. *Studies in Educational Evaluation*, *67*, DOI:10.1016/j.stueduc.2020.100918

Le Roux, J. P. (2020). *A case study of secondary school mathematics teachers evaluating their classroom-based assessments for the purpose of teaching for mathematical proficiency.* [Doctoral thesis, University of Stellenbosch].

Lewis, C. (2016). How does lesson study improve mathematics instruction? *ZDM*, *48*(4), 571-580. https://doi.org/10.1007/s11858-016-0792-x

Lewis, J. (2016). Learning to lead, leading to learn: how facilitators learn to lead lesson study. *ZDM Mathematics Education*, *48*(4), 527-540. doi: 10.1007/s11858-015-0753-9.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Sage.

Lipscombe, K., Buckley-Walker, K., & McNamara, P. (2020). Understanding collaborative teacher teams as open systems for professional development. *Professional Development in Education*, *46*(3), 373-390.

Litke, E. (2020). The nature and quality of algebra instruction: Using a content-focused observation tool as a lens for understanding and improving instructional practice. *Cognition and Instruction*, *38*(1), 57-86. https://doi.org/10.1080/07370008.2019.1616740

Lombardo, E., & Kantola, J. (2021). Social constructivism. In *The Routledge Handbook of Gender and EU Politics* (pp. 43-55). Routledge.

Lomibao, L. S., Luna, C. A., & Namoco, R. A. (2016). The influence of mathematical communication on students' mathematics performance and anxiety. *American Journal of Educational Research*, *4*(5), 378-382.

Lubis, A. N. M. T., Widada, W., Herawaty, D., Nugroho, K. U. Z., & Anggoro, A. F. D. (2021). The ability to solve mathematical problems through realistic mathematics learning based on ethnomathematics. *Journal of Physics: Conference Series, 1731*(1), 012050.

Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education*, 26-47.

Machisi, E., & Feza, N. N. (2021). Van Hiele theory-based instruction and Grade 11 students' geometric proof competencies. *Contemporary Mathematics and Science Education*, *2*(1), ep21007. https://doi.org/10.30935/conmaths/9682

Magudu, S. (2014). *Induction experiences of newly qualified primary school teachers in Zimbabwe.* [Doctoral thesis, University of South Africa]. http://uir.unisa.ac.za/handle/10500/18585

Maguire, M., & Delahunt, B. (2017). Doing a thematic analysis: A practical, step-by-step guide for learning and teaching scholars. *All Ireland Journal of Higher Education*, *9*(3).

Mahmud, Z., & Porter, A. (2015). Using Rasch analysis to explore what students learn about probability concept. *Faculty of Engineering and Information Sciences – Papers: Part A. 4502* https://ro.uow.edu.au/eispapers/4502

Makuwa, D. K., & Maarse, J. (2013). The impact of large-scale international assessments: A case study of how the Ministry of Education in Namibia used SACMEQ assessments to improve learning outcomes. *Research in Comparative and International Education*, *8*(3), 349-358.

Malan, S. B., Ndlovu, M. & Engelbrecht, P. (2014). Introducing problem-based learning (PBL) into a foundation programme to develop self-directed learning skills. *South African Journal of Education*, *34*(1),1-16

Maree, K. (2007). *First steps in research*. Pretoria: Van Schaik.

Marton, F. (2009). *Sameness and difference in learning*. Lecture at the Swedish Research Links Symposium on Phenomenography and Variation Theory, The University of Hong Kong, Hong Kong SAR, December 1-3.

Mattarima, K., & Hamdan, A. R. (2011). The teaching constraints of English as a foreign language in Indonesia: The context of school-based curriculum. *Sosiohumanika*, *4*(2).

McCulloch, A. (2021). Serendipity in doctoral education: The importance of chance and the prepared mind in the PhD. *Journal of Further and Higher Education*, 1-14. https://doi.org/10.1080/0309877X.2021.1905157

McGinley, S., Wei, W., Zhang, L., & Zheng, Y. (2021). The state of qualitative research in hospitality: A 5-year review 2014 to 2019. *Cornell Hospitality Quarterly*, *62*(1), 8-20.

McMillan, J. H., & Schumacher, S. (2010). *Research in Education: Evidence-based Inquiry* (7th ed.). USA: Pearson Education, Inc.

Mohammed, A. A., Ambak, K., Mosa, A. M., & Syamsunur, D. (2019). A review of traffic accidents and related practices worldwide. *The Open Transportation Journal*, *13*(1). http://dx.doi.org/10.2174/1874447801913010065

Molina, C. (2014). Teaching mathematics conceptually. *SEDL insights*, *1*(4), 1-8. http://www.sedl.org/insights/1-4/teaching_mathematics_conceptually.pdf

Molloy, E., Boud, D., & Henderson, M. (2020). Developing a learning-centred framework for feedback literacy. *Assessment & Evaluation in Higher Education*, *45*(4), 527-540.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: An overview of research in the field. *ZDM*, *46*(6), 843-853.

Mosimege, M., Beku, U., Juan, A., Hannan, S., Prinsloo, C. H., Harvey, J. C., & Zulu, N. (2017). *TIMSS item diagnostic report: South Africa: Grade 9 Mathematics.* South Africa: Human Sciences Research Council. Retrieved from http://repository.hsrc.ac.za/handle/20.500, 11910, 11448.

Mullis, I. V. S., & Martin, M. O. (Eds.). (2017). *TIMSS 2019 Assessment Frameworks.* Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2019/frameworks/

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International results in Mathematics and Science.* Retrieved from Boston College, TIMSS & PIRLS International Study Center website: https://timssandpirls.bc.edu/timss2019/international-results/

Mutara, L., & Makonye, J. (2016). Learners' use of probability models in answering probability tasks in South Africa. *Promoting understanding of statistics about society, Proceedings of the 13th International Conference of Mathematical Education, July* (pp. 24-31).

Naidu, J. T., & Sanford, J. F. (2017). Teaching Basic Probability in Undergraduate Statistics or Management Science Courses. *American Journal of Business Education, 10*(1), 17-22.

Nance, M. S. (2018). *Building procedural fluency from conceptual understanding in equivalence of fractions: A content analysis of a textbook series.* [Theses and dissertation 7331, Brigham Young University]. https://scholarsarchive.byu.edu/etd/7331

National Research Council. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford & B. Findell (Eds.). *Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education*. Washington, DC: National Academy Press.

Ndlovu, M. (2014). The effectiveness of a teacher professional learning programme: The perceptions and performance of mathematics teachers. *Pythagoras*, *35*(2), a237. http://dx.doi.org/10.4102/pythagoras.v35i2.237

Nel, B., & Luneta, K. (2017). Mentoring as professional development intervention for mathematics teachers: A South African perspective. *Pythagoras*, *38*(1), 1-9. https://doi.org/10.4102/pythagoras.v38i1.343

Nene, J. D. (2017). *Supporting TVET students to improve English language competency: An action research study.* [Doctoral thesis, University of KwaZulu-Natal, Pietermaritzburg].

Ngulube, P., & Mathipa, E. R. (2015). Theoretical and conceptual frameworks in the social and management sciences. In E. R. Mathipa & M. T. Gumbo (Eds.). *Addressing research challenges: Making headway in developing researchers.* Mosala-Masedi Publishers. http://doi:10.13140/RG.2.1.3210.7680

Ni, Y., Zhou, D., Li, X., & Li, Q. (2014). Relations of instructional tasks to teacher–student discourse in mathematics classrooms of Chinese primary schools. *Cognition and Instruction*, *32*(1), 2-43.

Ni Shuilleabhain, A., & Seery, A. (2017). Enacting curriculum reform through lesson study: A case study of mathematics teacher learning. *Professional Development in Education*, *44*(2), 222-236. https://doi.org/10.1080/19415257.2017.1280521

OECD (2020), *PISA 2018 Results (Volume V): Effective policies, successful schools*, Paris: OECD Publishing. https://doi.org/10.1787/ca768d40

Ogbonnaya, U. I., & Awuah, F. K. (2019). Quintile ranking of schools in South Africa and learners' achievement in probability. *Statistics Education Research Journal*, *18*(1), 106-119.

O'Hara, S., Brookmeyer, J., Pritchard, R., & Martin, R. (2020). Mentoring secondary novice teachers to develop the academic language of English language learners. *Journal of Educational Research and Practice*, *10*, 26–-40. https://doi.org/10.5590/JERAP.2020.10.1.02

Ojose, B. (2011). Mathematics literacy: Are we able to put the mathematics we learn into everyday use? *Journal of Mathematics Education*, *4*(1), 89-100.

Ozofor, N. M. & Onos, C. N. (2018). *Effect of ethnomathematics on senior secondary school students' achievement in Ikwuano local government area, Abia State. Researchjournali's Journal of Mathematics, 5*(1). ISSN 2349-5375

Pale, J. W. (2016). Teacher and student based instructions on Probability achievement outcomes and attitudes of secondary school students in Bungoma North, Kenya. *Journal of Education and Practice*, *7*(24), 43-53.

Pang, S. K. (2016). Improving mathematics instruction and supporting teacher learning in Korea through lesson study using five practices. *ZDM Mathematics Education*, *48*(4), 471-483. doi:10.1007/s11858-016-0768-x.

Parliament of the Republic of South Africa. (2016). *Portfolio Committee on Basic Education: Overview and analysis of SAQMEQ 4 study results.* Cape Town: Government Printers.

Peters, B. I. (2016). *Realistic mathematics education and professional development: A case study of the experiences of primary school mathematics teachers in Namibia* [Doctoral dissertation, Stellenbosch: Stellenbosch University]. http://scholar.sun.ac.za/handle/10019.1/98416

Petty, N. J., Thomson, O. P., & Stew, G. (2012). Ready for a paradigm shift? Part 2: Introducing qualitative research methodologies and methods. *Manual therapy*, *17*(5), 378-384.

Pradhan, J. B. (2017). Mathematical ideas in Chundara culture: Unfolding a Nepalese teaching and learning system. In D. Orey & M. Rosa (Eds.). *Ethnomathematics and its diverse approaches for mathematics education* (pp. 125-152). Cham.: Springer.

Prosek, E. A., & Gibson, D. M. (2021). Promoting rigorous research by examining lived experiences: A review of four qualitative traditions. *Journal of Counseling & Development*, *99*(2), 167-177.

Proudfoot, D. E., & Kebritchi, M. (2017). Scenario-based e-learning and stem education: A qualitative study exploring the perspectives of educators. *International Journal of Cognitive Research in Science, Engineering and Education*, *5*(1), 012019.

Purba, G. I. D., & Surya, E. (2020). The improving of mathematical understanding ability and positive attitudes of Unimed FMIPA students by using the Contextual Teaching (CTL) approach. *Journal of Physics Conference Series*, *1462*(1), 012019. DOI:10.1088/1742-6596/1462/1/012019

Reddy, V., Visser, M., Winnaar, L., Arends, F., Juan, A. L., Prinsloo, C., & Isdale, K. (2016). *TIMSS 2015: Highlights of mathematics and science achievement of grade 9 South African learners.* (Commissioned by the Department of Basic Education, December) http://hdl.handle.net/20.500.11910/10673

Reviglio, U. (2019). Serendipity as an emerging design principle of the infosphere: challenges and opportunities. *Ethics and Information Technology*, *21*(2), 151-166.

Rittle-Johnson, B. (2019). Iterative development of conceptual and procedural knowledge in mathematics learning and instruction. In J. Dunlosky & K. A. Rawson (Eds.), *The Cambridge handbook of cognition and education* (pp. 124–147). Cambridge University Press. https://doi.org/10.1017/9781108235631.007

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346-362. https://doi.org/10.1037/0022-0663.93.2.346

Robinson, M., & Rusznyak, L. (2020). Learning to teach without school-based experience: Conundrums and possibilities in a South African context. *Journal of Education for Teaching*, *46*(4), 517-527.

Romberg, T. A., & Shafer, M. C. (2020). Mathematics in Context (MiC)—Preliminary evidence about student outcomes. In S. L. Senk & D. R. Thompson (Eds.), *Standards-Based School Mathematics Curricula* (pp. 225-250). Routledge.

Rossman, G. B., & Rallis, S. F. (2011). *Learning in the field: An introduction to qualitative research* (3rd ed.). Thousand Oaks, California: Sage.

Russell, J. L., Correnti, R., Stein, M. K., Thomas, A., Bill, V., & Speranzo, L. (2020). Mathematics coaching for conceptual understanding: Promising evidence regarding the Tennessee Math Coaching Model. *Educational Evaluation and Policy Analysis*, *42*(3), 439-466.

Sadia, S., & Ghani, M. (2019). Modality in editorials of Pakistani English newspapers: A corpus based study. *International Journal of English Linguistics*, *9*(1), 144-151. https://doi.org/10.5539/ijel.v9n1p144

Sands, D. (2014). Concepts and conceptual understanding: What are we talking about? *New Directions in the Teaching of Physical Sciences*, (*10*), 7-11.

Schoenfeld, A.H., Thomas, M., & Barton, B. (2016). On understanding and improving the teaching of university mathematics. *International Journal of STEM Education*, *3*(1), 1-17. https://doi.org/10.1186/s40594-016-0038-z

Scotland, J. (2012). Exploring the philosophical underpinnings of research: Relating ontology and epistemology to the methodology and methods of the scientific, interpretive, and critical research paradigms. *English language teaching*, *5*(9), 9-16.

Seidman, I. (2006). *Interviewing as qualitative research: A guide for researchers in education and the social sciences*. Teachers College Press.

Shaughnessy, J. M. (1977). Misconceptions of probability: An experiment with a small-group, activity-based, model building approach to introductory probability at the college level. *Educational Studies in Mathematics*, *8*(3), 295-316. https://doi.org/10.1007/BF00385927

Sikes, P. (2004). Methodology, procedures and ethical concerns. In C. Opie (Ed.), *Doing educational research* (pp. 15-33). Thousand Oaks, CA: SAGE Publications.

Simamora, R. E., & Saragih, S. (2019). Improving students' mathematical problem solving ability and self-efficacy through guided discovery learning in local culture context. *International Electronic Journal of Mathematics Education*, *14*(1), 61-72.

Skovsmose, O. (2016). An intentionality interpretation of meaning in mathematics education. *Educ Stud Math*, *92*, 411–424. https://link.springer.com/article/10.1007/s10649-015-9644-9

Smith, J., & Noble, H. (2014). Bias in research. *Evidence-based nursing*, *17*(4), 100-101.

Smith, M., Bill, V., & Raith, M. L. (2018). Promoting a conceptual understanding of mathematics. *Mathematics Teaching in the Middle School*, *24*(1), 36-43.

Smith, T. M., & Freels, A. (2017). Instructional practices that promote conceptual understanding and procedural fluency: Profile of practice brief. *Promoting Student Success in Algebra*. Washington: American Institutes for Research.

Smyth, R. (2004). Exploring the usefulness of a conceptual framework as a research tool: A researcher's reflections. *Issues in Educational Research*, *14*(2), 167-180.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research techniques*. Thousand Oaks, CA: SAGE Publications.

Suh, J. M. (2007). Tying it all together: Classroom practices that promote mathematical proficiency for all students. *Teaching Children Mathematics*, *14*(3), 163-169.

Suh, J., Gallagher, M.A., Capen, L. & Birkhead, S. (2021). Enhancing teachers' noticing around mathematics teaching practices through video-based lesson study with peer coaching. *International Journal for Lesson and Learning Studies*, *10*(2), 150-167. https://doi.org/10.1108/IJLLS-09-2020-0073

Sullivan, P. (2011). *Teaching Mathematics: Using research-informed strategies.* Camberwell, Victoria: Australian Council for Educational Research. https://research.acer.edu.au/cgi/viewcontent.cgi?article=1022&context=aer

Takahashi, A. (2016). Recent trends in Japanese mathematics textbooks for elementary grades: Supporting teachers to teach mathematics through problem solving. *Universal Journal of Educational Research*, *4*(2), 313-319.

Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: maximizing the impact of lesson study. *ZDM Mathematics Education*, *48*(4), 513-526. doi: 10.1007/s11858-015-0752-x.

Tatar, E., & Zengin, Y. (2016). Conceptual understanding of definite integral with Geogebra. *Computers in the Schools*, *33*(2), 120-132.

Tian, J., & Siegler, R. S. (2017). Fractions learning in children with mathematics difficulties. *Journal of Learning Disabilities*, *50*(6), 614-620.

Tibebu, M. (2015). *Secondary school English teachers' Practice of adapting textbooks: The case of Ninth And Tenth Grades English teachers at Bulga General Secondary and Preparatory School.* [Doctoral dissertation, Adama Science and Technology University, Adama, Ethiopia].

Toh, T., & Chiu, Q. (2020). Development of framework with exemplars using real-world context to teach probability. Learning Science and Mathematics, 15(December), 16 to 29. http://myjms.mohe.gov.my/index.php/lsm/article/view/9768

Ulandari, L., Amry, Z., & Saragih, S. (2019). Development of learning materials based on realistic mathematics education approach to improve students' mathematical problem solving ability and self-efficacy. *International Electronic Journal of Mathematics Education*, *14*(2), 375-383.

Ültay, N., & Usta, N. D. (2016). Investigating Prospective Teachers' Ability To Write Context-Based Problems. *Journal of Theory and Practise in Education*, *12*(2), 447–463.

Valdmann, A., Rannikmae, M., & Holbrook, J. (2016). Determining the effectiveness of a CPD programme for enhancing science teachers' self-efficacy towards motivational context-based teaching. *Journal of Baltic Science Education*, *15*(3), 284.

Van de Walle, J. A, Bay-Williams, J. M., & Karp, S. P. (2016). Elementary and middle school mathematics: Teaching developmentally (9th ed.). New York: Pearson Education.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 521-525). London: Springer.

Van Merriënboer J. J. G., de Bruin, A. B. H. (2014). Research paradigms and perspectives on learning. In J. Spector, M. Merrill, J. Elen, & M. Bishop (Eds.). *Handbook of Research on Educational Communications and Technology*. New York, NY: Springer. https://doi.org/10.1007/978-1-4614-3185-5_2

Veenstra, T. (2014). Teaching seventh grade statistics and probability: An inference based approach. *Master of Education Program Theses, 1*. https://digitalcollections.dordt.edu/med_theses/1

Venkat, H. & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development*, *41*, 121-130.
DOI: http://dx.doi.org/10.1016/j.ijedudev.2015.02.002

Vijaya Kumari, S. N. (2014). Constructivist approach to teacher education: An integrative model for reflective teaching. *Journal on Educational Psychology*, *7*(4), 31-40.

Visnovska, J., & Cobb, P. (2019). Supporting shifts in teachers' views of a classroom statistical activity: Problem context in teaching statistics. *Mathematical Thinking and Learning*, *21*(4), 285-305, DOI: 10.1080/10986065.2019.1576003

Von Oppell, M. A., & Aldridge, J. M. (2021). The development and validation of a teacher belief survey for the constructivist classroom. *International Journal of Educational Reform*, *30*(2), 138-162.

Vukovic, R. K., & Lesaux, N. K. (2013). The language of mathematics: Investigating the ways language counts for children's mathematical development. *Journal of Experimental Child Psychology*, *115*(2), 227-244. http://dx.doi.org/10.1016/j.jecp.2013.02.002

Walan, S., McEwen, B., & Gericke, N. (2016). Enhancing primary science: An exploration of teachers' own ideas of solutions to challenges in inquiry- and context-based teaching. *Education, 3-13*, *44*(1), 81-92.

Warwick, P., Vrikki, M., Færøyvik Karlsen, A. M., Dudley, P., & Vermunt, J. (2019). The role of pupil voice as a trigger for teacher learning in Lesson Study professional groups. *Cambridge Journal of Education*, *49*(4), 435-455.

Warwick, P., Vrikki M., Vermunt J. D., Mercer, N., & Halem, N. V.(2016). Connecting observations of student and teacher learning: an examination of dialogic processes in lesson study discussions in mathematics. *ZDM Mathematics Education*, *48*(4), 555-569. doi: 10.1007/s11858-015-0750-z.

Watanabe, T., Takahashi, A., & Yoshida, M. (2008). Kyozaikenkyu: A critical step for conducting effective lesson study and beyond. *Inquiry into Mathematics Teacher Education*, *5*, 131-142.

Wessels, H. (2014). Number sense of final year pre-service primary school teachers. *Pythagoras*, *35*(1), 1-9.

Wilson, P. S., Heid, M. K., Zbiek, R. M., & Wilson, J. W. (2010). *Framework for mathematical proficiency for teaching*. Athens, GA: Center for Proficiency in Teaching Mathematics.

Wong, L. F., Kaur, B., & Tong, C. L. (2021). Meaningful mathematics talk that supports mathematics learning in Singapore secondary schools. In B. Kaur, & Y. H. Leong (Eds.). *Mathematics Instructional Practices in Singapore Secondary Schools*. *Mathematics Education – An Asian Perspective*. Singapore: Springer. https://doi.org/10.1007/978-981-15-8956-0_9

Yang, Q., Gao, R., Bai, F., Li, T., & Tamura, Y. (2018). Damage to buildings and structures due to recent devastating wind hazards in East Asia. *Natural hazards*, *92*(3), 1321-1353.

Yang, X., Kaiser, G., König, J., & Blömeke, S. (2020). Relationship between pre-service mathematics teachers' knowledge, beliefs and instructional practices in China. *ZDM*, 1-14.

Yayuk, E. & Husamah, H. (2020). The difficulties of prospective elementary school teachers in item problem solving for mathematics: Polya's Steps. *Journal for the Education of Gifted Young Scientists*, *8*(1), 361-378. DOI: http://dx.doi.org/10.17478/jegys.665833

Yeo, K. K. J. (2021). Learning opportunities to promote conceptual understanding in Singapore secondary school mathematics instruction. In B. Kaur, & Y. H. Leong (Eds.). *Mathematics instructional practices in Singapore Secondary Schools: Mathematics education - An Asian perspective* (pp. 63-77). Singapore: Springer.

Yin, R. K. (2016). *Qualitative research from start to finish (*2nd ed.). New York: The Guilford Press.

Zaranis, N., & Synodi, E. (2017). A comparative study on the effectiveness of the computer assisted method and the interactionist approach to teaching geometry shapes to young children. *Education and Information Technologies*, *22*(4), 1377-1393.

Zhang, J. (2019). A Semantic Approach to the English Modality. *Journal of Language Teaching and Research*, *10*(4), 879-885. DOI: http://dx.doi.org/10.17507/jltr.1004.28

Zhang, S., Shi, Q., & Lin, E. (2020). Professional development needs, support, and barriers: TALIS US new and veteran teachers' perspectives. *Professional Development in Education*, *46*(3), 440-453. DOI: 10.1080/19415257.2019.1614967

Zulu, S. (2019). *Teachers' use of examples to promote mathematical proficiency in statistics.* [Doctoral thesis, University of the Witwatersrand]. https://hdl.handle.net/10539/29446

APPENDICES

Data collection instruments:

APPENDIX A: Lesson observation protocol

Grade 7 mathematics

Participant code: _____ Site Code: _____

Topic: Probability Concepts and skills: _____

Constructs	Classroom Implications	Observer's comments (Emerged data)	Implications
Focus Area	**Indicators**		
Conceptual understanding Showing deep and thorough conceptual understanding of probability and the teaching of probability	**1. Functional grasp of mathematical ideas** The intention is to understand how teachers: • Interpret students understanding of probability concepts • see connections amongst the probability concepts and different ways of solving probability problems. • comprehend mathematical concepts • Clarify the contextual understanding of the specialised language used in Probability • make connections for similar problems • apply the learnt concepts to new areas. For example, how probability can be applied in different contexts like in sports, entrepreneurship etc. see mathematics as useful, understandable and not arbitrary • solve real life probability problems • Relation to life experiences (teachers' application. • Scenarios, events or stories • use models and multiple representations to explain the probability concepts • identify topics and sub-topics related to probability, and connected them to Probability. **2. Teaching and learning activities** • Do they encourage active participation? • Do they promote integrated learning? • Are they relevant and realistic to promote achievement of the intended goals **3. Explanatory framework** This involves the use of various representations and models such as representations, experiments, tables listening, illustrations, examples, modelling , analysis, analogy, explanations and		

	demonstrations to unpack the concept of probability. **4. <u>Facilitation strategies</u>** How do teachers: • Guide the students to decide on the appropriate context to apply probability in real-life situation • Deal with students' misconceptions and answers • Select and Incorporate teaching and learning resources • Encourage self-directed learning • Use or incorporate students' remarks(questions) to make a mathematical point and decide when to pause for clarification. **5. <u>Questioning strategies</u>** • Types of questions asked • Do the questions provide students opportunities to learn? • Do questions promote curiosity, creativity, participation in students? • Does the teacher probe students' answers • Does the teacher encourage students to justify their answers • Do teachers' questions promote inquiry learning • Does it allow exploration of ideas? **6. <u>Communication approach</u>**: Interactive-authoritative or interactive -dialogic **7. <u>Assessment</u>** strategies • Is assessment integral to teaching and learning		
What to note for further probing			

Overall Comment/Remarks

_____

_____

APPENDIX B: Post-lesson interview schedule

INTERVIEWEE CODE: _____

SITE CODE: _____

DATE:_____

PART A: BIOGRAPHICAL INFORMATION
Gender :_____ Age :_____ Teaching experience :_____ Number of students in each class :_____ Position held :_____ Qualifications :_____ Specific mathematics qualifications:_____ Matric or Grade 12 mathematics : YES/NO
PART B: QUESTIONS RELATED TO CONTENT AND PRACTICE
1. What is your understanding of the concept of probability? 2. Do you find students enjoying learning about probability? Motivate. 3. How do you develop teaching and learning activities for students? What guides you in developing teaching activities? 4. How do you implement such activities in the classroom? 5. Which resources do you prefer to use in teaching probability and why?
PART C: QUESTIONS RELATED TO TEACHER PROFESSIONAL DEVELOPMENT
6. Have you ever attended a training/workshop on the teaching of probability? Elaborate 7. How do DBE and HODs support you as a mathematics teacher? 8. How does CAPS inform your instructional practice?
PART D: QUESTIONS RELATED TO INVOLVEMENT IN THE STUDY

9. If you were to develop a framework for effective classroom practice in the teaching of probability, how will it look like? 10. If you were to teach this topic, again what would you change? 11. What contribution do you like to make in your cluster/circuit? 12. What aspect of your involvement in this study have you found most rewarding? 13. Which area needs some improvement? 14. Is there anything else that you would like to add?
Thank you for your thoughts, time and effort in participating in this research study

APPENDIX C: Open-ended questionnaire-analysis of learners' responses

TEACHER ☐ **CODE** ☐ **SITE CODE**

Thank you for agreeing to participate in this study. The nine scenarios below will give you a picture of how learners interpret and answer probability questions. Please answer the questions below to reflect your views on learners' conception of probability. Your information will be kept strictly confidential.

SCENARIO 1

"The weather forecast says there is a 40% chance of rain in our area on Friday". This statement is made by Geography teacher in the Grade 7B class. Learners understand the sentence to mean the following:

Learner A: So it means only 40% of our area will get rain.

Learner B: It means that it will rain for 40% of the time on Friday and 60% of the time on the other days.

Learner C: It is a promise that it will rain; 40% is a big number.

Learner D: 40% of the forecasters think it will rain, the other 60% think it will be sunny.

Reflect on learners' responses by answering the following questions.

(i) Do you agree with learners' understanding? Explain.

(ii) If you do not agree, explain why the understanding is incorrect.

Learner A

(i)_____

_____

_____

_____

_____

(ii)_____

_____

_____

_____

_____

Learner B

(i)_____

_____

_____

_____

_____

(ii)_____

_____

_____

_____

_____

Learner C

(i)_____

_____

_____

_____

_____

(ii)_____

_____

_____

_____

_____

Learner D

(i)_____

_____

_____

_____

_____

(ii)_____

_____

_____

_____

_____

SCENARIO 2

A tuck-shop at a primary school sells the following: popcorn, toffees, chips and Danone. A survey shows that the learners (tuck-shop customers) have the following preferences:

a) Popcorn:	$\frac{2}{5}$	b)	Toffees:	$\frac{1}{10}$
c) Chips:	$\frac{1}{6}$	d)	Danone:	$\frac{1}{3}$

What advice could you give to the learners to assist the tuck-shop owner in stocking the correct products?

_____

_____

_____

_____

_____

_____

SCENARIO 3

Learners could choose between three different juices, namely mango juice, orange juice and peach juice. They were asked to determine the probability that peach juice would be chosen, given that the probability that mango juice would be chosen is 20% and the probability that orange juice would be chosen is $\frac{3}{7}$. None of the learners answered the question.

i. In your opinion, what made students not to answer the question.

_____

_____

_____

_____

_____

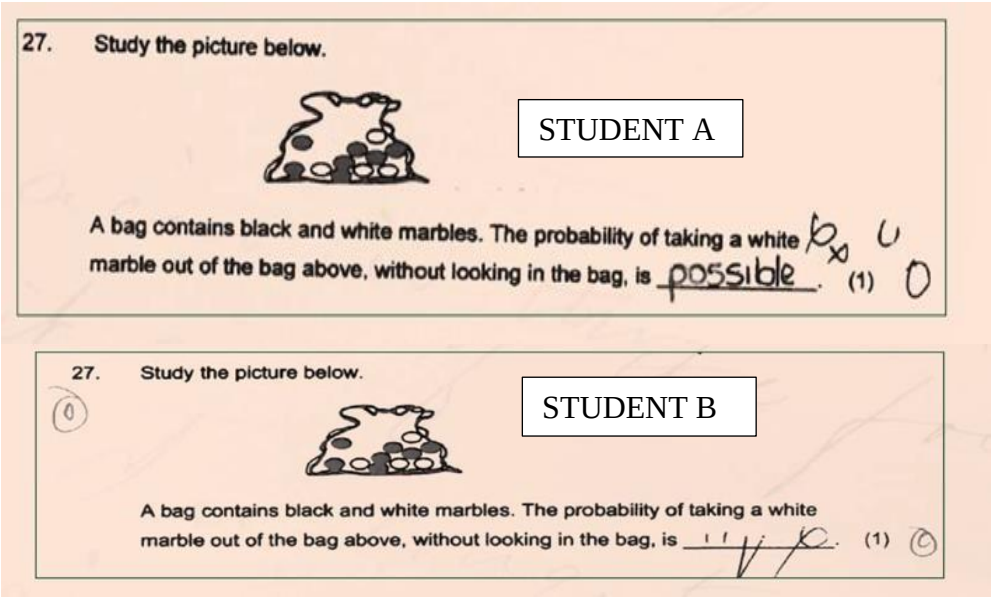
ii. Explain step by step how you would assist learners in answering the question.

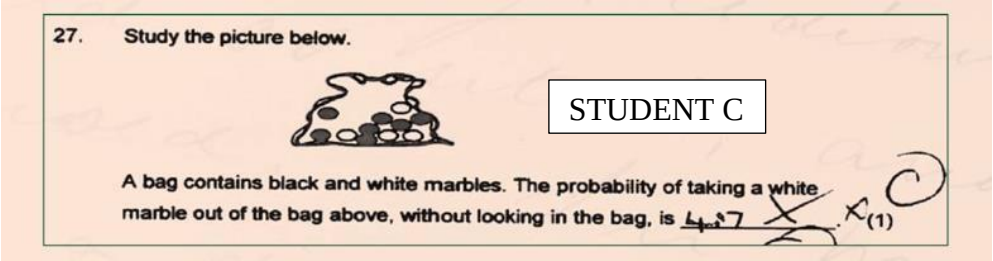
SCENARIO 4

The following question was asked in Grade 7 term test: Morongwa was given the following items to acknowledge her good achievement in Mathematics, 2 pairs of gold earrings; 4 pairs of silver earrings; and 6 pairs of bronze earrings. She put them in a jewellery bag. As she was preparing to attend her party, the power went out. What was the probability that she would grab the silver earrings without looking in the bag? How would you, the teacher, assist learners in answering this question?

SCENARIO 5

A few other misconceptions related to probability can be inferred from the typical learner responses below (extract from *Diagnostic report: annual national assessment 2012* (DBE, 2013, pp. 22 -23])





5.1 Comment on the three learners' answers.

STUDENT A

_____

_____

_____

STUDENT B

_____

_____

STUDENT C

_____

_____

_____

5.2 What concepts do learners in Grade 7 need to know in order to solve this problem?

_____

_____

_____

5.3 How can you assist these learners in answering the question?

_____

_____

SCENARIO 6

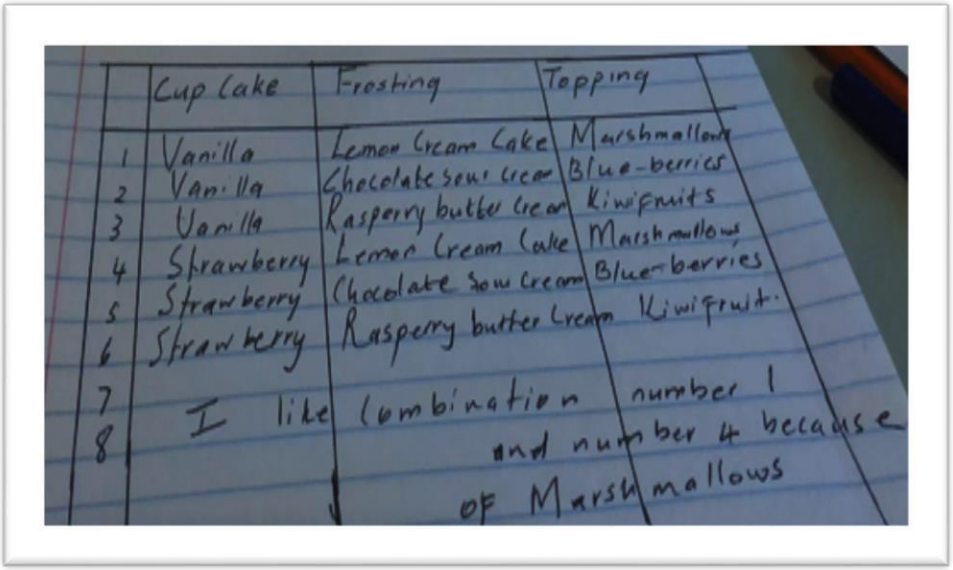
A bakery sells vanilla cupcakes and strawberry cupcakes. It offers a choice of three different types of frosting: lemon cream cheese; chocolate and sour cream; and raspberry buttercream. It also offers three different toppings: marshmallows, blueberries and kiwifruit. Learners were asked to write down the different combinations that would result from choosing a vanilla or a strawberry cupcake, one kind of frosting and one topping. The following diagram represents the scenario:

Cupcake	
Vanilla	Strawberry

Frosting		
Lemon cream cheese	Chocolate and sour cream	Raspberry buttercream

Topping		
Marshmallows	Blueberries	Kiwifruit

Morris, a Grade 7 learner, answered the question using the table below. This is an extract from his journal:



	Cup Cake	Frosting	Topping
1	Vanilla	Lemon Cream Cake	Marshmallows
2	Vanilla	Chocolate sour cream	Blue-berries
3	Vanilla	Rasperry butter Cream	Kiwifruits
4	Strawberry	Lemon Cream Cake	Marshmallows
5	Strawberry	Chocolate Sour Cream	Blue-berries
6	Strawberry	Rasperry butter Cream	Kiwifruit.
7			
8			

I like combination number 1 and number 4 because of Marshmallows

Consider Morris's attempt to find the solution and answer the following questions:

a) What are the advantages and disadvantages of using this approach?

Advantages

_____

_____

_____

Disadvantages

_____

_____

_____

b) What questions would you ask him to clarify his understanding?

_____

_____

c) What advice would you give to him to extend his knowledge of probability?

_____

_____

_____

SCENARIO 7

Learners were asked to provide scenarios that would show how probability could be applied in real-life situations. How would you help learners to recognise the importance of probability in the following contexts?

a) Sports

_____

_____

_____

_____

_____

_____

_____

_____

b) Health

_____

_____

_____

_____

_____

_____

_____

c) Business

SCENARIO 8

What procedure would you suggest to help your learners answer the following question?

Create a spinner that has five sections, labelled MARKS, and has to meet the following requirements.

Probability	Allocation
P(M)	15%
P(A)	$\frac{1}{5}$
P(R)	0,03
P(K)	_____
P(S)	$\frac{1}{2}$

_____

_____

_____

_____

SCENARIO 9

9.1 What guidance would you give to learners to find the solution to the following problems?

A spinner is divided into seven equal sections numbered 1 to 7. Predict how many times out of 280 spins the spinner is mostly likely to stop at an even number.

_____

_____

_____

_____

_____

_____

_____

9.2 In an experiment, Emmerson rolled a six-sided die 54 times. The results are shown in the following table:

Number on the die	1	2	3	4	5
Frequency	6	10	15	9	14

Which number on the die was obtained the expected number of times according to theoretical probability? Explain your answer.

_____

_____

_____

_____

_____

_____

_____

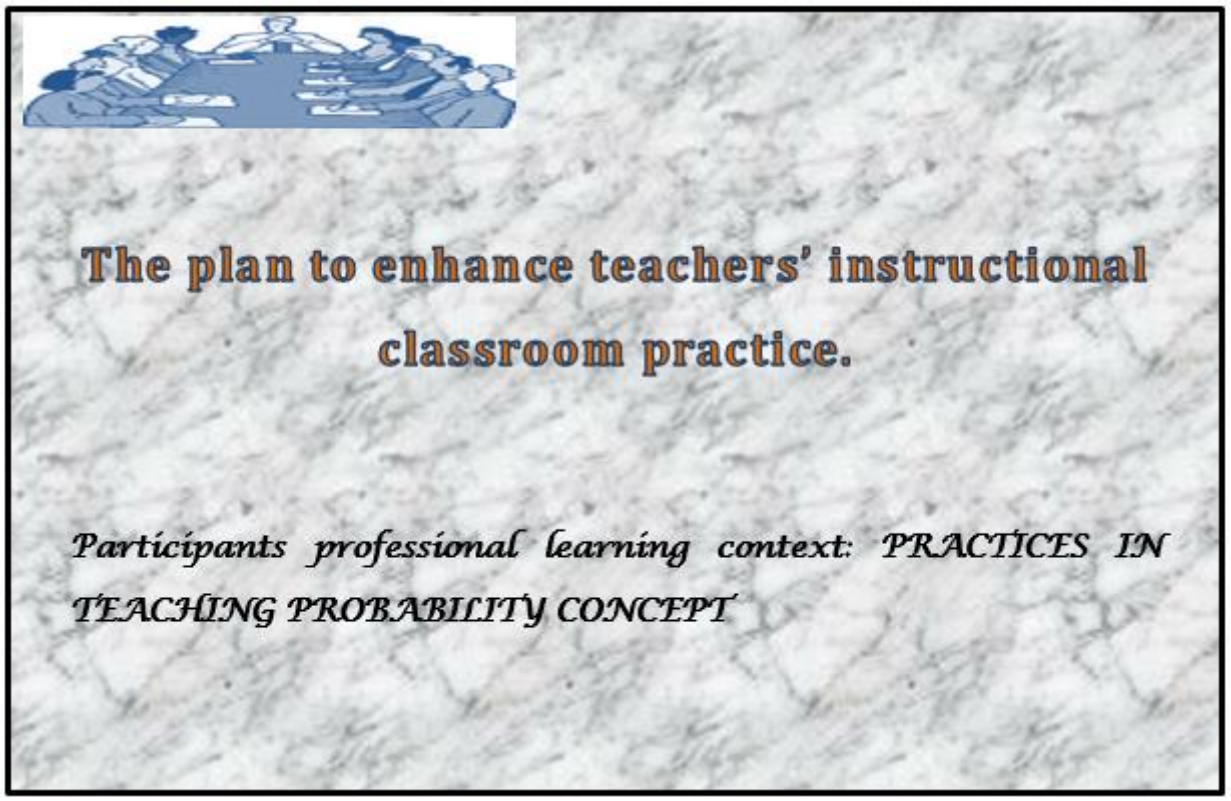
Thank you for your thoughts, time and effort. We appreciate your input.

INTERVENTION WORKSHOP DOCUMENTS

APPENDIX P1: Exemplar: guidelines on how to augment/modify teaching, learning and assessment activities

	KEY COMPONENTS TO CONSIDER IN MODIFYING TEACHING AND LEARNING ACTIVITIES IN THE DBE WORKBOOK		
Copy the activities in the DBE workbook	**Guide questions for each stage of the teaching and learning activities**	**Example of an activity**	**What students need to learn from the activity**
	• What terminology and background might students need to be ready for the task **(Probability concepts discussed in the previous activity and knowledge of resources)** • What questions will you ask to help students access their prior knowledge? • Indicate how you will use the resources in your teaching	**Do the experiments** **(NB: every time you do an experiment you run the random event)** **Step 1:** Let students do the experiment in small groups **Step 2:** The following information could be written on the board as an example "A fair coin has two faces numbered 1 and 2 that are equally likely to show when the coin is flipped. What is the experimental probability of a coin landing on a tail after a total flip of 100? ***(This question could be asked before they do the experiment, to arouse interest. Do you think it will more likely/less likely to be heads or just as likely to be heads as tails? They can then do the experiment to confirm their guess)*** **Step 3**: Let students predict what will then	They will learn: • Terminology used in experimental probability like a trial, even, outcome, experiment and probable outcome,50-50 chance • Mathematical concepts are interconnected • To explain the purpose of doing an experiment • To predict from real life experiences and experiments conducted. • Learn from each other • Practice making arguments with justification • responsibilities by assigning roles. Should all contribute to the task. • To present their work • The key concepts in probability • That the real-life situations can be represented in different contexts • That one can be able to make a conclusion and justification based on practical experience • That mathematical tasks are meaningful, make sense, realistic and not abstract

APPENDIX P2: Excerpt from the Participant Manual(Activities for the intervention workshop)



AIM OF THE INTERVENTION WORKSHOP

The main aim of the intervention workshop is to provide guidelines for modifying teaching, activities to best meet the range of needs of the curriculum and diverse context of the students. The aim will be achieved through:

- Reflecting on your understanding of the strands of mathematical proficiency (***Individual activity***)
- Collaboratively designing guidelines and providing an overview of what informs adaptation of the activities. (***whole group activity***)
- Adapting identified teaching activities from the DBE workbook (***Individual activity)***

The next section outlines the activities that you need to go through to guide the process of engaging with the material and to get the opportunity to participate in a professional learning context.

PARTICIPANTS' ACTIVITIES

At the end of the workshop, you should be able to adapt and review the adapted teaching activities on probability.

DAY 1: HOME ACTIVITY

FOCUS: Conceptualizing the strands of mathematical proficiency

ACTIVITY 1: (INDIVIDUAL ACTIVITY)

Read the two attached documents on teaching for mathematical proficiency and make a summary of your understanding of the following concepts:

- Conceptual understanding, Procedural Fluency, Strategic Competency, Adaptive Reasoning and Productive Disposition

The understanding of strands of mathematical proficiency will serve as the basis for the next activity in the coming workshop.

DAY 2: FACE-FACE MEETING

FOCUS: Conceptualizing the strands of mathematical proficiency

ACTIVITY 2: (WHOLE GROUP ACTIVITY)

Let us quickly reflect on our understanding of the five strands of mathematical proficiency?

DAY 3-9 : FACE_FACE MEETING

ACTIVITY 3: (WHOLE GROUP ACTIVITY)

FOCUS: Developing guidelines on how to adapt teaching and learning activities

Part 1: Based on your understanding of the strands of mathematical proficiency do the following activities:

- Brainstorm around and used the key indicators of conceptual understanding to guide the modification of activities. Just throw in the ideas
- Develop the guidelines on how to augment teaching and learning activities.

APPENDIX P3: The recommendation made on the open-ended questionnaire (Pilot study)

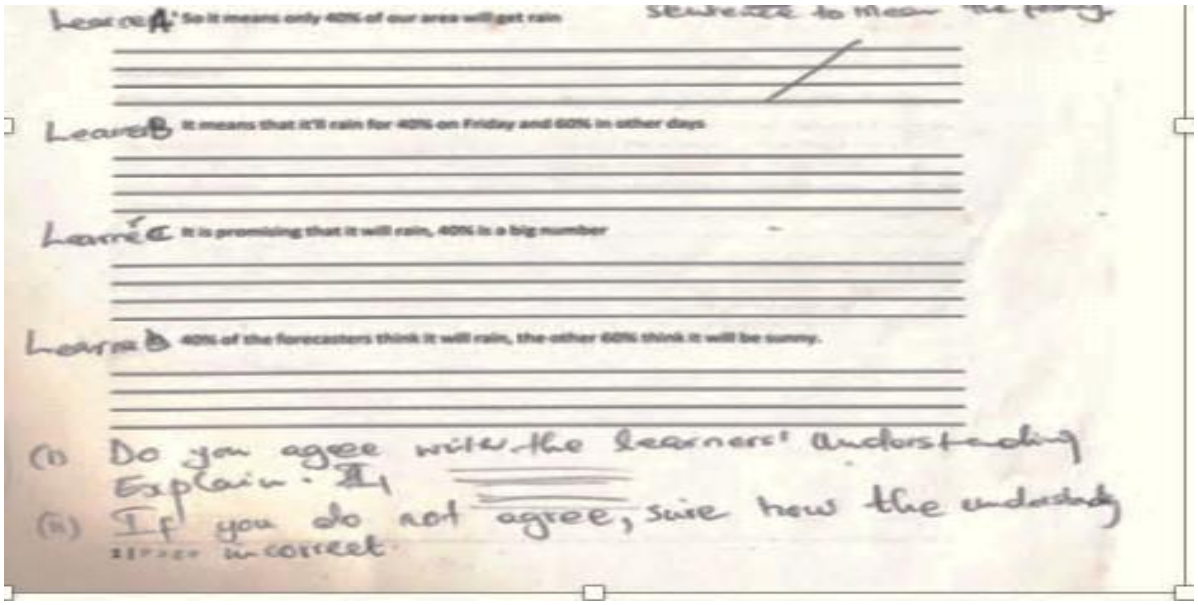
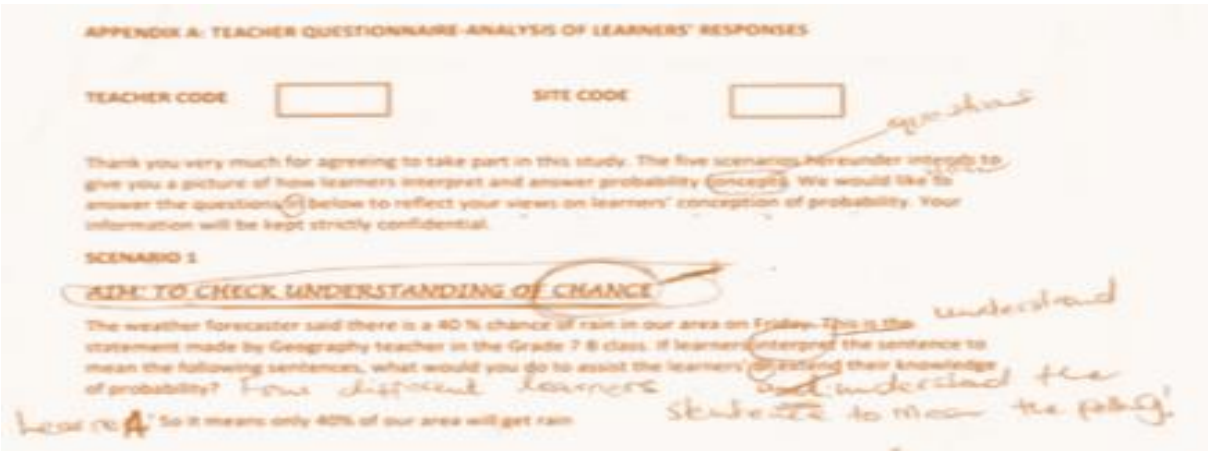


Figure R1: Extract 1 of Feedback from Reviewer1

Reviewer 1 mainly recommended that instead of writing alphabets only, I need to write it as Learner A; Learner B; Learner C and Learner D respectively. For the purpose of this study, I wrote them as Student A-Student D. In addition to that, the reviewer advised that I need also to determine if teachers do or don't agree with the students' understanding. This suggestion has assisted also in determining the degree of teachers' understanding of chance. **Figure R2**: shows more inputs made by Reviewer 1



Review **Figure R1**: Extract 2 of Feedback from Reviewer 1

Reviewer 2 also suggested replacement of other terms as exemplified in Figure R_2

SCENARIO 1

AIM: TO CHECK UNDERSTANDING OF CHANCE

understand

The weather forecaster said there is a 40 % chance of rain in our area on Friday. This is the statement made by Geography teacher in the Grade 7 B class. If learners interpret the sentence to mean the following sentences, what would you do to assist the learners' or extend their knowledge of probability?

and ?

a) So it means only 40% of our area will get rain

b) It means that it'll rain for 40% on Friday and 60% in other days

it will

c) It is promising that it will rain, 40% is a big number

d) 40% of the forecasters think it will rain, the other 60% think it will be sunny.

who is this? Students? viewers?

Figure R_2: Extract 3 of Feedback from Reviewer 2

APPENDIX P4(a)

INTERVIEW SCHEDULE:

PARTICIPANT: PR_6

DATE: 26 OCTOBER 2018

SITE: **PS_4**

Researcher: I just want to get your understanding of Probability? If someone can just ask you a question to say what is your understanding of Probability? What would you say?

PR_6: An event taking place

Researcher: Can you elaborate on your answer?

Interviewer*:* What is your understanding of the concept, probability?

PR_6*: "An event taking place."*

Interviewer*:* What do you mean by an event taking place?

PR_6*: "Mmm……By eh….an event taking place. I am referring to ... (pauses for some few seconds) (Should I use the scenario?)"*

Interviewer: It's fine you can!

PR_6*: "Like… (5 second pause). Maybe saying. It's like eh… that scale in relation to probability. Will it be possible for me to teach a cat to maybe cook? It is not possible because it is an animal but with relation to eh... something that geographically, the sun will rise from the east. It is a definite thing, that is a certain thing."* I continued the interview to try to get a clearer explanation from **PR_6's** understanding of probability.

Interviewer*:* In a nutshell, how can you explain the concept of probability?

PR_6*: "It can be explained as event occurring and certainly that event will take place."*

Interviewer: ok….(paused whilst making sense of the participant's response)

Interviewer: Is there anything that you would like to regarding the topic Probability?

PR_6*:*mmm……Probability looking at it , its easier for one to understand. Personally like…with regards to my own perspective, its easier for me to say I can deal with this probability myself. But somehow measures should be put in place to assist us as educators how maybe to develop us further from what I know to the next person.

Interviewer: ok. Let me give you this scenario then. When we teach concepts or topics, we need to start by making learners realise the importance of the topic. How can you apply probability in real life situation?

Participants: It is more like stressing a point of taking a real-life situation to the classroom. It should be more practical use every day. eh….A daily encounters , we need to take them from maybe the outside world, bringing back to the classroom to a formal situation.

APPENDIX P4(b)

INTERVIEW SCHEDULE:

PARTICIPANT: PR_3

DATE: 23 OCTOBER 2018

SITE: PS_3

Interviewer: Thank you very much for the presentation that you made this morning. I have few questions I would like to ask.

PR_3: *"Ok, ma'am."*

Interviewer: I heard you saying, "We a going to do an activity which is called an event". Am I correct?

PR_3*: Ja…* (She looked surprised).

Interviewer: What did you mean?

PR_3: (She was not sure how to answer the question). "*Eh … I meant an experiment with a die".*

Interviewer: What is the difference between an experiment and an event?

PR_3: "*I take that the two terms are the same."*

Interviewer: ok, what is your understanding of Probability?

PR_3: (*She was silent for some few second)* A chance of something to happen

Interviewer: ok, what else?

PR_3: Again, I can say eh….probability, you are not certain, eh… about what is going to happen, is a 50-50 chance.

Interviewer: Are you saying probability is about certainty and 50-50 Chance?

PR_3: Mmmm……...(*Could not respond*)

Interviewer: ok, what else?

PR_3: (*kept quiet for some seconds and did not respond*)

Interviewer: ok. Do learners enjoy learning this topic ?

PR_3: Ja….

Interviewer: What is your experience?

PR_3: It is just that these learners enjoy when they are having teaching aids or when they are hands-on and involved. And when you come and just do it, or just teach, only the learners who are intelligent can follow in my lesson

Interviewer: Do you have enough teaching aids to use in your classroom? The reason why I am asking is because you had one resource to when demonstrating to learners.

PR3: I do have

Interviewer: Where did you get the one you used during your lesson?

PR3: I have improvised. The cards are from the learners and those coloured things, yes. Improvisation.

Interviewer: Ok. Ehm….What guides you or what informs the development of teaching and learning activities?

PR3: Mmm… come again.

Interviewer: Where did you get the teaching activities that you used in your lesson?

PR 3: Some of them, I got from the textbook and then others from the workbook and others were provided in the (1+4) lesson plans.

APPENDIX P5

LESSON OBSERVATION : LESSON 2

PARTICIPANT: PR_1

DATE: 16 OCTOBER 2018

SITE: PS_1

Teacher: Good Morning class

Learners: Good Morning Mam

Teacher: What is probability?

Learner: A chance for an event to happen

Teacher: (She wrote the following statement on the board: Our school will go on a trip) . How are you going to assess that using the words I taught you yesterday?

Learner: impossible

Teacher: Why do you say impossible because you are not given information about the trip?

Teacher: Today's business

Teacher: What are possible outcomes when you spin a spinner with four colours?(she rephrased the question). How many outcomes will you get?

Learners; (in a chorus form): Four outcomes

Teacher: A boy has five red sweets 5 green sweets and 5 yellow sweets. How many sweets are there? Then we are going to calculate the chances of getting sweets.

Teacher: We are going to have two types of outcomes: favourable outcomes and total outcomes.

Teacher: What is the probability or what is the chance of taking out a red sweet?

Learners: (no response from students)

Teacher: (Rephrasing the question) How many red sweets are in the bag?

Learner: five red sweets.

Teacher: The red five sweets are our favourable outcome because we are focussing on only five. Outcomes out of 15 outcomes.

Teacher: So, when we calculate probability, we are going to say probability equals to = probability of taking out a red sweet P(taking a red sweet)

Teacher: Ask yourself how many red sweets there are, then is your favourable outcome.

Teacher: Probability = $\frac{favourable\ outcome}{Total\ outcomes}$

$= \frac{5\ red\ sweets}{15\ sweets}$

Teacher: We may simplify by looking at the highest common factor = $\frac{1}{3}$

APPENDIX P6

EXTRACT FROM THE TRANSCRIPT OF THE OPEN-ENDED QUESTIONNAIRE

Student responses	**PR_4**	**PR_5**	**PR_3**
Scenario 2	Buy buying same number of products and the one that goes very fast to increase their number	The tuck shop owner must calculate the preference of learners and compare it to his stock. Then calculate the percentages and the one with a good percentage is a good product of popcorn and Danone are preferred.	Let learners give the tuck shop owner the most liked products by them (most preferred). The prices should not be too high.
Scenario 3	According to my opinion the learners were confused by 20% not knowing it can be changed The learner must know how to collect data by using tally mark of frequency table for the product that the tuck shop sell. Asked to check which is the most item that is liked by the most learners	The question seems to be abstract for learners. Learners could not interpret the question and differentiate between percentage given and fraction given Show learners how to calculate the number of different juices. Convery percentages of mangoes to fraction 20%=1/5. Convert fraction of orange to percentages 3/7=40%. Then from the given information, calculate the number of peach juice and covert it to percentages 3/7=40% Learners need to know that in order to calculate probability they need to be able to know and calculate the number of items given.	They are confused, because the probability of the other juice is in percentage and the other is in the form of a fraction. Firstly, learners must understand what possible outcomes are. If we have three juices, the outcomes should be out of three meaning the probability of choosing mango juice is $\frac{1}{3}$ meaning out of three juices there is a possibility of picking 1 juice.
Scenario 4	The learner must know the total number of earing in the bag. The total number per item. Know many silver earrings are in the bag. Probability is written in fraction form using the total number of items divided by the total number of items	Teach learners to write probability in 3 forms of fractions, decimals and percentage. To answer this question, learners need to check number of favourable outcome and divide them by possible outcome, eg collect number of silver earrings(4 pairs) divide by all pairs(12 pairs 4/12=1/3)	1 out of 24 = $\frac{1}{24}$. You add the total of earrings which is 242 pairs = 4 and 4 pairs = 8,6 pairs =12 pairs. We add 4+8+12=24 and there is a possibility of picking one of all the colours out of 24 earrings.

Scenario 5.1				
LEARNER	**PR10**	**PR1**	**PR 2**	**PR 9**
Learner A	The learner could not count the black marbles correctly. He got 6 instead 7. Picking the white marbles is possible yes, but the probability was not calculated.	Learner was unable to relate the diagram with the question.	The learner understands that probability is the chance of something happening. The learner does not understand that the outcomes are determined by the concentration of a particular kind of marble as opposed to the other. In this scenario the outcome can be possible or not possible, hence the probability range from 0 to 100%. The probability of taking a white marble can be 0 or 100% since there are only two marbles black and white.	The learner did not answer the question exactly as asked.
Learner B	The added the white marbles and the black marbles as the total outcomes. However, the probability of taking a white marble was not calculated. The learner only calculated the total outcomes not probability.	The learner concentrated on the diagram and does not understand the word probability.	The learner understands the concept of probability and the outcomes but does not understand how to express the probability on a probability scale. Probability is expressed as percentage, fraction or decimal fraction. The learner cannot express the outcome 'even' as a percentage. Instead of writing probability is 50% or $\frac{1}{2}$ or 0,5 she wrote 11. According to him probability is 11 meaning there is 1 white and 1 black marble.	The learner just added the number of marbles.
Learner C	The learner only showed the ratio of white marbles to black marbles.	The learner has little knowledge of probability but not in detail.	Does not understand probability and its outcomes.	The learner made a ratio of white marbles to black marbles.

	Probability was not calculated.			
Scenario 5.2	1. Addition (to be able to get total outcomes). 2. Fractions (to be able to calculate probability).	Concepts like chances of an event to happen changes go with the total number of white balls from the total number of balls (white and black). Learner must be able to identify the number of times an event will happen e.g. (white)= $\frac{1}{4}$	Learners need to know that for every event there are outcomes. Outcomes may be impossible, unlikely, even, likely or certain/possible. Probability is the measure of these outcomes.	Addition, fraction, percentage and ratio.
Scenario 5.3	Step 1: I would ask the learners to add the outcomes, 4+7=11 Step 2: I would ask them to divide the total number of white marbles by the total of outcomes $\frac{4}{11} = 0.36\ or\ 36\%$	Learners have to count white balls in a bag then count total number of balls in the bag. Then write P(white balls)= $\frac{4}{11}$ balls(white).	Learners should be able to determine the number of outcomes in a given trial. In this scenario there are only two outcomes white or black marbles, so the probability of taking a white marble is $\frac{1}{2}$ P(WM)= $\frac{1}{2}$. Learners should understand that outcomes can be possible or impossible and probability is the measure of the possibility or impossibility of an outcome.	Find the sum of all marbles then divide the number of white marbles by the sum (i.e. $\frac{4}{11}$)
Scenario 6a)	*Advantages* It is advantageous when dealing with two sets of data. *Disadvantage* It is disadvantageous when dealing with more than two sets of data, since a table is a 2 way.	*Advantages* It reflects all types of frost and toppings of the two types of cupcakes. *Disadvantage* Lack of information on the outcomes of the combinations. We cannot answer the questions from looking at the approach.	*Advantages* It is a very simple approach that anyone can use. *Disadvantage* Other options from this kind of approach are excluded.	*Advantages* It reflects all types of frost and toppings of the two types of cupcakes. *Disadvantage* Lack of information on the outcome of the combinations. He

				cannot answer the questions from just looking at the approach.
b)	Is it not possible to have a combination of vanilla, lemon, cream cheese and blueberries?	How many types(outcomes) of cupcakes will be available after writing all combinations?	Did you list all the possible options?	How many types(outcomes) of cupcakes will be available after writing all combinations?
c)	I would an advice of using tree diagram	I will advise him to also use a tree diagram to represent the combinations. It will give him accurate answers or number combinations.	To teach learners a tree diagram so that he can see all branches of all possible outcomes.	I will advise him to use a tree diagram to represent the combinations. It will give him accurate number of combinations.

APPENDIX P7

OPEN-ENDED QUESTIONNAIRE: responses for pr1 and pr2 and processes ofcoding data

SCENARIO 1: focus on Students A-C

Participants' understanding of students' responses

How the statement *"The weather forecast says there is a 40% chance of rain in our area on Friday"* was interpreted by each student and how each participant interpreted the students' responses

			Coding of data PR2	**Coding of data PR1**	**Refining codes**
Participants	**PR1**	**PR2**	**Emerging categories**	**Emerging categories**	**Sub-Categories**
Student A So it means only 40% of our area will get rain.	Agree with the learner because is about interpreting percentages using knowledge of fractions Only part of the area will receive rain for only 40 out of hundred	Not agree: the learners understanding of probability is incorrect since the forecast is based on the climatic conditions that leads to rainfall and not the area coverage of the rain The learners' understanding is that 40% of their area will get rain as a matter certainty which is flawed as the forecast basically says that conditions are 40% probable that their whole area will get rain	Not agree the forecast is based on the climatic conditions that leads to rainfall conditions are 40% probable that their whole area will get rain	Agree interpreting percentages using knowledge of fractions	Climatic conditions that leads to rainfall 40% probable that their whole area will get rain
Student B It means that it will rain for 40% of the time on Friday and 60% of the time on the other days	Not agree: The learner's understanding focus on time instead of rain.	Not agree: the forecast is not based on time-frames of the rainfall but actual conditions that leads to rainfall and is not specified on the times of the day. The learner's understanding is completely incorrect since the forecast is only for Friday and excludes all other days that the learner refers to.	Not agree: the forecast is not based on time-frames of the rainfall but actual conditions that leads to rainfall	Not agree conditions are 40% probable that their whole area will get rain	Conditions that lead to the rainfall
Student C It is a promise that it will rain; 40% is a big number.	Not agree: The learner does not understand percentage. The learner see percentage as a large concept	Not agree: Even if 40% seems like a big probability, 60% chance of it not raining is even a bigger probability if you take into account the learner's understanding The learner's understanding could have been correct if their	Not agree: higher the percentage the more the probability.	Not agree see percentage as a large concept	Higher percentage higher probability

		grasp of the conditions was based on the higher the percentage the more the probability. It is only flawed because the chances of it not raining are bigger than the chances of it actually raining			
Student D 40% of the forecasters think it will rain, the other 60% think it will be sunny.	Not agree: The learner misunderstood the teacher. He has a problem of misinterpretation(relating 40% to forecasters)	Not agree: The weather forecast is a consolidated forecast of all forecasters. It does not enumerate number of forecasters who come with the forecast The learner does not understand what weather forecast is all about. They need to be taught that the weather forecast is based on the climate conditions that leads to a particular outcome	Not agree It does not enumerate number of forecasters who come with the forecast	Not agree (relating 40% to forecasters) Problem of relating 40% to the forecaster than the possibility of rainfall	Challenge of relating 40% to the forecaster than the possibility of rainfall
Refined sub-categories	Conditions that lead to the rainfall Percentage determines probability of rain Challenge of relating percentage to the number of forecasters				

APPENDIX G: Permission letter

LETTER TO REQUEST PERMISSION TO CONDUCT RESEARCH

The HEAD OF DEPARTMENT

Private bag x11341

NELSPRUIT

1200
Republic of South Africa

(ATTENTION: Research UNIT: Contact Person: _____)

Enq: SM Kodisang **e-mail:kodissm@unisa.ac.za**

Subject: Request to conduct a research in schools in your province

Dear Sir

I am currently enrolled for a doctoral degree in Education (Ph.D.) through the University of South Africa. I would like to request your permission to undertake a research study in four primary schools, Nkangala District. My research topic focuses on the **use of context-based problems to develop the concept of probability: A case in selected rural primary schools in, Mpumalanga, South Africa**

The purpose of the study is to explore skills required by Grade 7 Mathematics teachers to develop and integrate context-based problems in teaching probability. Probability is one of the topics that form part of mathematics curriculum. The motivation to study probability is based on poor performance of learners in Mathematics, particularly in GET (General Education and Training) phase. Probability is still a challenge for learners. My assumption is that the incorporation of context-based problems will make teaching effective and result in a meaningful learning.

My study will involve four (one from each school) mathematics teachers and learners in Grade 7. Four schools will be selected from different circuits in Nkangala District. The researcher will use a qualitative approach that aims at full understanding of social problem. The aim of the study is to derive a model that conceptualizes the underlying components of teaching and learning that incorporates context-based problems in the teaching of probability. This will be achieved through the following objectives: Each teacher is expected to interpret and analyse learners' responses on probability questions and present one lesson in his/her own school. Intervention to assist teachers will be conducted during school holidays to avoid disruptions of the proper running of the schools. Reflective interviews will be conducted towards the end of the data collection process, during the holidays or after school.

The purpose is to expand and extend thinking through follow-up questions. The engagement/interaction with teachers will take 12 days spread in two months. Lesson presentations and the interviews will be audiotaped in this study with the consent of the participating teachers.

Data will be collected in Term 1, 2018. All the data collected from each school will be analyzed and a report regarding the study will be written. The information from this study will only be used for academic purposes. In my research report, and in any other academic communication, pseudonyms will be used and no other identifying information will be given. Collected data will be in my or my supervisor's possession and will be locked up for safety and confidentiality purposes.

After completion of the study, the material will be stored at the University according to the policy requirements. This research study is being carried out with the hope that it will contribute to the body of knowledge. The findings will make several contributions to the current literature. First, it will enhance or add substantially to teachers' understanding of probability concept; second, it will provide additional evidence with regards to the effectiveness of using context-based problems to develop probability concept, and lastly the findings could be used to develop targeted interventions, at all levels, aimed at teacher professional development

Kindly be informed of the following conditions of participation in the research study.

1. All participation is voluntary.
2. The school's name will not be revealed in the findings of the research study.
3. All discussions with participants will be treated with confidentiality.
4. The schools can withdraw from the research study at any time.
5. If the school is willing to participate, it will kindly be requested to sign the consent form provided

Yours sincerely

Signature of student

Name of student: Kodisang SM (Supervisor) Prof. N Feza Date

APPENDIX H

PRINCIPAL CONSENT LETTER AND FORM

Enq: SM Kodisang **e-mail: kodissm@unisa.ac.za**

The Principal of----------Primary School

Circuit

Subject: Permission to conduct a Research Project in your school.

Dear Principal,

I am currently enrolled for doctoral degree in education through the University of South Africa. My area of interest is on the use of context-based problems to develop the concept of probability for Grade 7 mathematics learners. My supervisor is Prof NN Feza I have obtained formal approval from the Mpumalanga Department of Education (MDE) to conduct research in four primary schools. I have selected your school as one of the four schools to conduct my study. The targeted research participant is one mathematics Grade 7 teacher from each school. The study intends to explore skills required by Grade 7 Mathematics teachers to develop and integrate context- based problems in teaching probability. The information obtained from the interviews and observations will be treated with the strictest confidentiality and will be used solely for research purposes. Pseudonyms will be used when making reference to the school and participants, whose participation is completely voluntary. You can withdraw from the research study at any time.

The researcher will use a qualitative approach that aims at full understanding of social problem. The aim of the study is to derive a model that conceptualizes the underlying components of teaching and learning that incorporates context-based problems in the teaching of probability. This will be achieved through the following objectives: Each teacher is expected to interpret and analyse learners' responses on probability questions and present one lesson in his/her own school. Intervention to assist teachers will be conducted during school holidays to avoid disruptions of the proper running of the schools. Reflective interviews will be conducted towards the end of the data collection process, during the holidays or after school. The purpose is to expand and extend thinking through follow-up questions. The engagement/interaction with teachers will take 12 days spread in two months. Lesson presentations and the interviews will be audiotaped in this study with the consent of the participating teachers.

The participants will therefore be expected to sign the consent form provided to them. Each selected school will serve as a site for conducting this research. The schools can withdraw from the research study at any time. All the data collected from each school will be analysed and a report regarding the study will be written. This study is carried out under the Supervision of Prof NN Feza at the University of South Africa. This research study is being carried out in the hope that it will contribute to the body of knowledge on how probability concept could be developed in Grade 7 Mathematics learners.

Yours Sincerely,

Signature of student _____

Name of student: _____ (supervisor) _____ Date_____

Contact number of student: 0824665968

E-mail address of student: **sophykodisang@yahoo.com**

APPENDIX I: Teacher consent letter and form

Enquiries: SM Kodisang
kodissm@unisa.ac.za

The Teacher

School:_____

Dear Mr/Mrs/Miss _____

Following consent from the principal regarding the participation of Grade 7 mathematics teacher at the school to conduct my research, I am submitting this letter to formalize the process. I am currently a part time student studying for a doctoral degree in Education through the University of South Africa. My area of interest involves exploring skills required by Grade 7 Mathematics teachers to develop and integrate context-based problems in teaching probability

With all documentation retrieved, the anonymity of the school and yourself will be maintained and will only be considered for the purposes of this research study.

It is with great enthusiasm that I look forward to conducting the research at your school and involving you as a willing research participant. If you give your consent to participate in this research, you will join a group of fellow participants who will also taking part in the investigation. You will be expected to analyse learners' responses on probability questions and present one lesson in your own school. An intervention programme will be conducted during school holidays to avoid disruptions of the proper running of the school. Reflective interviews will be conducted towards the end of data collection process. The purpose is to expand and extend thinking through follow-up questions. The engagement/interaction with teachers will last for 12 days spread in two months. Lesson presentations and the interviews will be audiotaped in this study with the consent of the participating teachers.

The researcher will use a qualitative approach that aims at full understanding of social problem. The aim of the study is to derive a model that conceptualizes the underlying components of teaching and learning that incorporates context-based problems in the teaching of probability..

In this study, I will be a non-participative observer for one probability lessons that will be presented by each teacher. A semi-structured interview will be conducted with the participant after the classroom observation at the place that is convenient to both the participants and the researcher. The lesson observation and the interview will both be audio recorded for a verbatim transcription.

Attached to this letter is a consent form that you need to complete as a way of giving concern to participate in the study.
Kind regards
Kodisang Sophy

This letter serves as a consent form for your participation in this investigation and it shall be retained for safe- keeping by UNISA. If you need clarity on any aspect of the investigation or have any questions in regard to the investigation and your participation, and if you would like to have a copy of the final report of this investigation please contact me at 082 466 5968. You can also contact my supervisor at UNISA, Prof. NN Feza on +27 012-337-6168. If you agree to participate in this investigation, please sign this consent form and pass it over to me.

Consent Statement:

I hereby declare that I have read the above consent form and therefore understand the nature of the project as explained to me. I understand that I have the opportunity to ask questions about the project. I also understand that I have the right to withdraw my consent and thus terminate my participation in the project at any time without any fine. I hereby agreed to participate and be audio-recorded in the research project conducted by Mrs. Sophy Mamanyena Kodisang, entitled **"The use of context-based problems to develop the concept of probability: A case in selected rural primary schools in, Mpumalanga, South Africa"**

Name of Participant: _____ Participant Signature: _____ Date:

Certification Statement:

I, Sophy Mamanyena Kodisang (Researcher), hereby confirm that I have explained to the above named participant the purpose and nature of this research project. I have also mentioned to the participant that he or she is free to ask any possible questions regarding the project and participation. I also certify

to have provided the participant a copy of this signed consent form.

Researcher Signature Date

APPENDIX J: Participant information sheet

Ethics clearance reference number: **2017_CGS/ISTE+012**
Date: 31 July 2020

Title: The use of context-based problems to develop the concept of probability: A case in selected rural primary schools in, Mpumalanga, South Africa

Dear Prospective Participant

My name is Sophy Mamanyena Kodisang and I am doing research with Prof N Feza, the Rector at Walter Sisulu University, Buffalo City Campus. We are inviting you to participate in a study entitled "The use of context-based problems to develop the concept of probability: A case in selected rural primary schools in, Mpumalanga, South Africa "

WHAT IS THE PURPOSE OF THE STUDY?

I am conducting this research to explore skills required by Grade 7 Mathematics teachers to develop and integrate context-based problems in teaching probability.

WHY AM I BEING INVITED TO PARTICIPATE?

I went back to my previous records to search for participants I previously invited to participate in my study and have shown interest. Through e-mails I invited them to participate in the professional development workshop. I indicated that those who have interest to participate in the study should include their cell numbers when they respond. I needed to have more participants to add to the two numbers that participated before. I contacted them individually explaining the study and invited them to participate voluntarily**.** I needed to have four more participants so that each get ample time to share his/her experiences and for me to probe participants' responses to ensure that I capture their views correctly and not misquote them. With this number, I am confident that I will be able to track the development of each participant

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?

The study involves reading and 296nalysing articles; developing/ modifying existing teaching and learning activities and sharing your experiences with other participants on the ZOOM on-line platform. In addition to that, you will be expected to participate on a group focus interview to reflect on your experiences in the study. The discussions will be audio-recorded so that I may be able to listen to our discussions at a later stage, to make sure that I capture your views correctly. The material on the audio recorder will not be reproduced or used anywhere. The questions that will be asked during the focus group discussion will mainly be on your views regarding the participation in the study. All the activities will be done individually as home activities. There will be two ZOOM sessions for participants to share their experiences with regards to analysis of articles and development of teaching and learning activities. The participants will reflect on their participation in the study in the third ZOOM meeting. The first two sessions will last for 1hour 15 minutes each and the last session will be conducted for an hour. The total number of hours will be 3 hours 30 unless if there is a need to review. But it will not be less than the proposed time

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE?

Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw at any time and without giving a reason. Your participation will be kept confidential and your anonymity cannot be guaranteed for the focus group discussions.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The study will develop a framework to teacher empowerment in creating interactive classroom practices to enhance the teaching of probability at grade 7 level. This interactive teacher support model will:

- enable you to understand the quality of classroom instructional practices within Teacher Education. This will be informed by how you make meaning of the strands of mathematical proficiency.

- deepen your conceptual understanding of Probability.
- enable you to develop the analytic instruments to guide the development of teaching and learning activities.
- enable you to come up with the exemplars of teaching and learning activities augmented from the DBE workbook.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?

All the participants will be participating from their convenient houses in an enclosed area. The participants will be compensated for buying data. The on-line discussion might pose a risk of participants identifying each other's participation in the research.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?

You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research. No-one will be able to connect you to the answers you give. Your answers will be given a code number, or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. The language editor and external examiners will have access to data and they will maintain confidentiality by signing a confidentiality agreement. Your answers may be reviewed by people responsible for making sure that research is done properly, including the transcriber, external coder, and members of the Research Ethics Review Committee. Otherwise, records that identify you will be available only to people working on the study, unless you give permission for other people to see the records. I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings

A report of the study may be submitted for publication, but individual participants will not be identifiable in such a report. Please keep in mind that it is sometimes impossible to make an absolute guarantee of confidentiality or anonymity, e.g. when focus groups are used as a data collection method. While every effort will be made by the researcher to ensure that you will not be connected to the information that you share during the focus group, I cannot guarantee that other participants in the focus group will treat information confidentially. I shall, however, encourage all participants to do so. For this reason, I advise you not to disclose personally sensitive information in the focus group.

HOW WILL THE RESEARCHER(S) PROTECT THE SECURITY OF DATA?

Hard copies of your answers will be stored by the researcher for a minimum period of five years in a locked cupboard/filing cabinet at the University of South Africa for future research or academic purposes. The electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. Hard copies will be shredded, and/or electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

Participants will be compensated for data usage.

HAS THE STUDY RECEIVED ETHICS APPROVAL?

This study has received written approval from the Research Ethics Review Committee of the School of Science, Unisa. A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?

If you would like to be informed of the final research findings, please contact SM Kodisang at 0824665968/kodissm@unisa.ac.za. Should you require any further information or want to contact the researcher about any aspect of this study, please use the same contact details. Should you have concerns about the way in which the research has been conducted, you may contact Prof. Nosisi Feza nosisi.piyose@gmail.com. You may also contact the research ethics chairperson of the School of Mathematics Research Ethics Committee, Prof Melusi Khumalo, at Khumalom@unisa.ac.za if you have any ethical concerns.

Thank you for taking time to read this information sheet and for participating in this study.
Thank you.

SM Kodisang

CONSENT TO PARTICIPATE IN THIS STUDY

I, _____ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings. My participation will be kept confidential and my anonymity cannot be guaranteed for the focus group discussions.

I agree to the recording of the of the TEAMS on-line sessions

I have received a signed copy of the informed consent agreement.

Participant Name & Surname…… (Please print)

Participant Signature…….Date……

Researcher's Name & Surname……(please print)

Researcher's signature…… Date……

UNISA university of south africa

UNISA INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION (ISTE)
RESEARCH ETHICS REVIEW COMMITTEE

Date: 3 November 2017

ERC Reference # : 2017_CGS/ISTE+012
Name: Sophy Mamanyena Kodisang
Student #: 4072-482-4

Dear Sophy Mamanyena Kodisang

Decision: Ethics Approval from 6 November 2017 to 6 November 2022

Researcher(s): Name: Sophy Mamanyena Kodisang
Address: PO. Box 21, Skilpadfontein, 0431
E-mail address: 40724824@mylife.unisa.ac.za
Tel: +251 918 760874

Supervisor (s): Name: Prof. NN Feza
E-mail address: nfeza@cut.ac.za
Tel: +27 (051) 5073362

Working title of research:

The use of context-based problems to develop the concept of probability: A case in selected rural primary schools in, Mpumalanga, South Africa

Qualification: PhD (Mathematics Education)

Thank you for the application for research ethics clearance by the Unisa ISTE Ethics Review Committee for the above mentioned research. Ethics approval is granted for **5 years**.

The low-risk application was reviewed by the ISTE Ethics Review Committee on 9 September 2017 in compliance with the Unisa Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the ISTE Research Ethics Committee.

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UNISA university of south africa

UNISA SCHOOL OF SCIENCE ETHICS REVIEW COMMITTEE

17 August 2020

ERC Reference # : 2020/CSET/SOS/049
Name : Ms Sophy Mamanyena Kodisang
Student # : 40724824
Previous ERC Reference # : 2017_CGS/ISTE+012

Dear Ms SM Kodisang

Decision: Ethics Approval from 17 August 2020 to 16 August 2022.

Amended 12 August 2020 to allow online recruitment gathering of data.

Researcher(s): Name: Ms Sophy Mamanyena Kodisang
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Supervisor(s): Name: Prof Nosisi Nellie Feza
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Telephone #: +27515073362

Working title of research:

THE USE OF CONTEXT-BASED PROBLEMS TO DEVELOP THE CONCEPT OF PROBABILITY: A CASE IN SELECTED RURAL PRIMARY SCHOOLS IN, MPUMALANGA, SOUTH AFRICA

Qualification: MSc

Thank you for the application for research ethics clearance by the Unisa School of Science Ethics Review Committee for the above mentioned research. Ethics approval is renewed for a period of two years from 12 August 2020 until **16 August 2022**.

*The **low risk application** was **reviewed** by the School of Science Ethics Review Committee on 17 August 2020 in compliance with the Unisa Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa COVID-19 position statement on research ethics attached.

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education
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Litiko le Temfundvo, Umnyango we Fundo | Departement van Onderwys | Ndzawulo ya Dyondzo

Ms. SM Kodisang
PO BOX 21
SKILPADFONTEIN
0431

RE: APPLICATION TO CONDUCT RESEARCH: Ms. SM KODISANG

Your application to conduct research study was received and is therefore acknowledged. The title of your study reads thus: **"The use of context-based problems to develop the concept of probability: A case of selected rural primary schools in Mpumalanga of South Africa."** I trust that the aims and the objectives of the study will benefit the whole department in particular the curriculum division. Your request is approved subject to you observing the provisions of the departmental research policy which is available in the departmental website. You are also requested to adhere to your University's research ethics as spelt out in your research ethics document.

In terms of the research policy, data or any research activity can only be conducted after school hours as per appointment with affected participants. You are also requested to share your findings with the relevant sections of the department so that we may consider implementing your findings if that will be in the best interest of the department. To this effect, your final approved research report (both soft and hard copy) should be submitted to the department so that your recommendations could be implemented. You may be required to prepare a presentation and present at the department's annual research dialogue.

For more information kindly liaise with the department's research unit @ 013 766 5476 or a.balovi@education.mpu.gov.za.

The department wishes you well in this important project and pledges to give you the necessary support you may need.

MRS M.D.C MHLABANE
HEAD: EDUCATION

17/10/17
DATE

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ENQUIRIES: MR. J.J. MABENA
TEL: 013 947 1710
E-mail: t.thubane @ education.mpu.gov.za

TO : **Ms. KODISANG S.M**
UNIVERSITY OF SOUTH AFRICA

FROM : **MR.J.J. MABENA**
DISTRICT DIRECTOR

DATE : **18 JANUARYN 2018**

SUBJECT : **PERMISSION TO DO RESEARCH**

This matter bears the following reference:

The District is appreciating your initiative to conduct a research programme in Mathematics Grade 7 teachers at **Mmametlhake** and **Nokaneng** Circuits.

You are required to ensure that this research does not interfer with contact time for teaching and learning as per appointment with effected participants.

You are requested to share your findings with the relevant sections in the Department,

We wish you well in this research.

Hoping that you will find this in order.

Mr. J.J. Mabena
District Director

2018/02/02
Date

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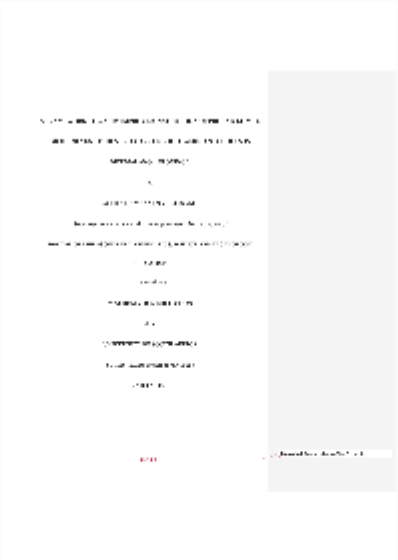

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EDITOR'S LETTER

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Mrs G Hannant
28 Hillcrest Avenue
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13 December 2021

TO WHOM IT MAY CONCERN

I certify that I have edited the PhD thesis

A FRAMEWORK TOWARDS IMPROVED INSTRUCTION OF PROBABILITY TO GRADE SEVEN STUDENTS: A CASE OF SOUTH AFRICAN SCHOOLS IN MPUMALANGA PROVINCE

by

SOPHY MAMANYENA KODISANG

However, the correction of all errors remains the responsibility of the student.

G.C. HANNANT
BA HED