

**THE USE AND EFFECT OF GEOGEBRA SOFTWARE IN CALCULUS AT WACHEMO
UNIVERSITY, ETHIOPIA: AN INVESTIGATION**

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Submitted in partial fulfilment of the requirements for the degree

Doctor of Philosophy in Mathematics Education

at the

University of South Africa (UNISA)

Department of Mathematics Education

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Date of submission: August 2021

Dedication

To my wife Diribe Dekeba, my mom Aye Debele and my child Kebeki, who always believe in me.

Declaration

I, TOLA BEKENE BEDADA, hereby declare that the thesis, with the title: **THE USE AND EFFECT OF GEOGEBRA SOFTWARE IN CALCULUS AT WACHEMO UNIVERSITY, ETHIOPIA: AN INVESTIGATION** which I hereby submit for the degree of Doctor of Philosophy in Mathematics Education at the University of South Africa, is my work and has not previously been submitted by me for a degree at this or any other institution.

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I declare that during my study I adhered to the Research Ethics Policy of the University of South Africa, received ethics approval for the duration of my study before the commencement of data gathering, and have not acted outside the approval conditions.

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Abstract

With the rapid growth of technology in the 21st century, traditional teaching and learning methods are considered outdated and not suitable for the active learning processes of the constructivist learning approach. The adjustment of existing methods and the development of new ones to teach and learn calculus with the help of technology is needed. This study aimed to investigate the effect of the use of GeoGebra mathematical software on university students' learning of calculus. It explored the effect of using GeoGebra Mathematical software on students' proficiency in calculus and students' attitudes to using GeoGebra software to learn calculus. To improve the learning process, the study developed a cycle model that posits nine steps. This cycle was implemented in the study. To achieve the goal of the study, a mixed research methodology was employed. In the quantitative part of the study, a quasi-experiment with a pre-test post-test design and questionnaires was used. A case study was used to collect data for the qualitative part of the study. The study was conducted at a university in the Southern Nations and Nationalities region of Ethiopia. The data were collected over four weeks in semester two of the 2020/2021 academic year. The quantitative data were analysed using SPSS version 27 while the qualitative data were coded into themes and analysed using computer software ATLAS.ti 9. The results indicated that the GeoGebra classroom-oriented approach to learning differential calculus with the use of the developed cycle model had a more positive effect on students' conceptual and procedural understanding when compared to students who were taught using a traditional teaching approach. Students in the experimental group showed greater improvement in procedural understanding, with an effect size of $d = 1.2$ and a percentile gain of 49%; in conceptual understanding of differential calculus, however, the students showed only slight improvement with an effect size of $d = 0.02$ and a percentile gain of 2%. Students expressed positive perceptions towards the use of GeoGebra for learning differential calculus. While the GeoGebra oriented learning approach to calculus has the potential to improve proficiency, it remains critically important that it should be designed (cycle model) and aimed to fill a specific gap. Based on the results, additional imperatives for practice and future research are recommended.

Key words: Cycle Model, Calculus, Ethiopia, GeoGebra, Perceptions, Student proficiency

Acknowledgements

I would like to express my sincere gratitude to the following individuals and organisations for their assistance and support in the completion of this study:

- Firstly, I would like to thank God for giving me the good health to complete my study.
- My wife, without whose encouragement and love I would not have finished. Thank you for increasing your burden only to lighten mine. I will love you forever.
- My family, for everyone's love and support during my studies. Mom, I made it with the help of your prayers to God! Long live my mom.
- Professor Masilo France Machaba, my study supervisor, for his inspiration, valuable advice and encouragement, dedication, time and, above all, patience. Thank you for believing in me. Thank you.
- Wachemo University, my employer, thank you for your support and encouragement and, most importantly, for your understanding and patience.
- Wachemo University Department of Mathematics staff, thank you for your endless support of my study.
- University of South Africa (UNISA), thank you very much for giving me this chance to study as a student abroad and for your support and encouragement.
- Lastly, I would like to thank the participants of the study and indirectly involved.

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LIST OF ABBREVIATIONS

No.	Abbreviation	Definition
1	ANOVA	Analysis of Variances
2	CAI	Computer Assisted Instruction
3	CAS	Computer Algebra System
4	CCSS-Maths	Common Core State Standards for Mathematics
5	CK	Content Knowledge
6	DC	Differential Calculus
7	DCAT	Differential Calculus Achievement Tests
8	DGS	Dynamic Geometry system
9	EBSCO	Elton B. Stephens Company
10	EdTech	Educational Technology
11	ICT	Information communication technology
12	IT	Information Technology
13	KR-20	Kuder-Richardson reliability index
14	MASE	Mathematics and science
15	MKO	More knowledgeable than others
16	MOE	Ministry of Education
17	MOSHE	Ministry of science and higher education
18	NCTM	National council of teachers and Mathematics
19	SMASE-Africa	Strengthening of Mathematics and Science Education-Africa
20	SPSS	Statistical Package for Social Sciences
21	STEM	Science, Technology, Engineering, and Mathematics
22	TK	Technology knowledge
23	TPACK	Technological Pedagogical and Content Knowledge
24	TS ² UV	Technology, Self-exploration, social interaction, understanding and visualisation
25	UNISA	University of South Africa
26	WHO	World Health Organization
27	ZPD	Zone of proximal development
28	4IR	Fourth industrial revolution

CHAPTER 1 **INTRODUCTION AND OVERVIEW OF THE STUDY**

1.1. OVERVIEW OF THE STUDY

This study investigates the effect on students' learning of mathematics, specifically calculus, of GeoGebra mathematical software, and students' perception of using GeoGebra. With the dawn of the Fourth Industrial Revolution (4IR) and its ever-increasing global demand on scholars in the 21st century, the integration of technology into higher education has become a non-negotiable imperative. During the ongoing Covid-19 pandemic, countries the world over were forced to conduct online education. Not all countries coped with this transition in an equally effective way, however. In this study, I focused particularly on mathematics instruction at the Wachemo University, located 230 km southwest of Addis Ababa in the Ethiopian town of Hosanna.

This chapter presents the factors contributing to the complexity associated with the learning of mathematics at the tertiary level. In support of the goal of the study, scholars' theories on the integration of technology in mathematics instruction are briefly discussed. They are discussed comprehensively in the literature review presented in Chapter Two. In the 21st century, effective teaching methods take into account recent developments in mathematical software, notably the tool known as GeoGebra. The application of this tool in the teaching and learning of mathematics has proved beneficial globally, from elementary school to higher institution level. In this study, the focus was on the application GeoGebra and its value and effect on conceptualising calculus at the tertiary level.

Following the introduction, the research problem, the purpose, and rationale for the study, and the research questions are included in this chapter.

This thesis consists of six chapters, namely:

- Chapter 1: Introduction and overview of the study
- Chapter 2: Literature review
- Chapter 3: Theoretical framework
- Chapter 4: Research methodology

- Chapter 5: Research findings and analysis of data
- Chapter 6: Discussion, conclusion, and recommendations.

1.2. INTRODUCTION

Society at large continues to be shaped by science and technology in the 21st century. In this regard, each country must reconsider its capacity to remain relevant in the competitive global arena. While some countries are leading the 4IR, several countries are finding this very difficult and require support to reach their desired goals in the future.

The present century has been marked by rapid technological developments; the learning environment has thus undergone irreversible changes, and individuals can learn whatever they choose, as long as they have access to technology that is paired with the skill to use it effectively. Technology has become the foundation of this modern industrial society. Technology-based instruction aims to stimulate students' active participation, purposeful learning and task-oriented activities. The integration of technological aids, specifically in the teaching of mathematics is a move away from teacher-centred instruction towards a learning-centred approach in which the student's conceptualisation of subject matter takes centre stage.

Teaching and learning mathematics, the implementation of information and communications technology (ICT) in the classroom has been slower than expected. Some factors hampering the implementation of new educational technologies are mentioned by researchers (De Witte & Rogge, 2014; Agyei & Voogt, 2010). Agyei and Voogt, (2010) point out that the slow implementation of educational technology may be the result of a lack of teacher professional development.

De Witte and Rogge (2014) argue that the shortage of computers in schools restricts the use of technology. In the same vein, Safdar, Yousuf, Parveen, and Behlol (2011) believe that the financial outlay and resources required by these technologies are responsible for their slow implementation of the integration of technology in education is intended to expedite and enhance the mastering of subject content. So and Kim (2009) argue that the success of integrating technology in education rests on the benefits it holds for the

educator. However, studies on the use of technology in the learning of mathematics have revealed different findings in terms of improvement (or not) in learning. For instance, Biagi and Loi (2013), Goodison (2002) and Hourigan and O'Donoghue (2007) found that the use of technology did not lead to any visible improvements in mathematics learning. In contrast, several scholars have reported gaps in the use of technology in the teaching in the mathematics classroom (Curri, 2012; Miller & Glover, 2007; Novotná & Jančařík, 2018; Tay, Lim, Lim, & Koh, 2012). However, another study revealed that one of the best methods of enhancing student achievements in various mathematical topics, for instance, calculus is the use of technology in the teaching and learning process (Eyyam & Yaratana, 2014).

Calculus has a wide range of applications in disciplines such as economics, engineering, science, business, computer science and information systems (Mendezaba & Tindowen, 2018). As a branch of mathematics, the concepts embedded in calculus are abstract and complex (Gordon, 2004; Sahin, Cavlazoglu, & Zeytuncu, 2015; Zachariades et al., 2007). As such, students need higher-order thinking skills to cope with calculus. Sahin et al. (2015) argue that calculus is often the main reason for the failure of students at the undergraduate level because of the way these students have been trained. In their study, Bressoud, Ghedamsi, Curri (2012), Martinez-Luaces and Törner (2016) found that students' difficulties with calculus emerged between secondary school and tertiary education.

Tall, Smith and Piez (2008, p. 207) argue that "calculus can be taught more by using technology from all fields of mathematics". In a series of research studies demonstrating the power of technology, Tall (1986, 1990, 2003, 2013) found that digital technology enhanced visualisation skills, enable programming language and improved students' understanding of the concepts of calculus. Tall (2019) points out that digital technology enhances the teaching and learning of calculus by allowing students to make fast and accurate numerical calculations, to manipulate symbols and to create dynamic figures that help them to visualise abstract concepts.

Several researchers have demonstrated that most difficulties encountered by students in calculus arise from a poor understanding of function concepts (Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Williams, 1991). To explore these difficulties, educators have resorted in their teaching to instructional multimedia such as graphic software (Robutti, 2010; Lavicza, 2010), computer algebraic systems (Ozguin-Koca, 2010; Mignotte, 1992; Durán, Pérez, & Varona, 2014) or a combination of both (Antohe, 2009; Dikovic, 2009; Hohenwarter & Jones, 2007; Hohenwarter & Lavicza, 2009).

It is known that the concepts involved in calculus include limits and continuity of functions, derivatives, integration, and the fundamental theorem. A function may be constant or a function of a single, two, three or more variables containing dependent and independent variables. Students' first encounter with the concept of functions is in the form of a univariate mathematical relationship where the value of a single independent variable (x) determines the value of a single dependent variable (y). In calculus, this concept is expanded to functions that can have multivariable inputs or outputs (vector-valued functions). The visualisation and conceptualisation of these functions allow the human mind to observe, infer, and predict change and growth (Tall, 2019). At a conference at the University of Agder, Tall (2019) reviewed the evolution and rapid growth of calculus over the past half-century, and the role of digital technology that has helped to make calculus meaningful in a wide range of applications (Tall, 2019, p. 2).

Arango, Gaviria and Valencia (2015) and Nobre et al. (2016) concur that the use of technologies as an alternative and novel way of teaching and learning calculus may support students' understanding of the abstract and complex theoretical ideas that characterise this field of mathematics. On a practical level, interactive technology such as graphing calculators and mathematics software helps students to visualise change and growth through graphical representations (Moses, Wong, Bakar, & Mahmud, 2013; Arslan, Kutluca, & Özpınar, 2011; Liang & Sedig, 2010).

In my seven years of experience in higher education in Ethiopia, I have observed that technology has not been used to support students' performance in calculus, either in or outside the classroom. Ethiopian university students regard calculus as difficult and

conceptually challenging (Abebe & Dirbeba, 2017; Sebsibe, Bekene, & Dorra, 2019). Mathematics is a compulsory subject in all science courses, which meets with resistance, particularly because there are no preparatory bridging courses for students (Semela, 2010). Semela (2010) identified several factors contributing to the challenges, namely students' weak mathematics background as a result of teachers' poor qualifications and a lack of job opportunities ousted the teaching profession, and inadequate pedagogical content knowledge. However, despite this, the interdependence of technology and education has in the past two decades attracted students to the sciences.

Students' regard mathematics as "boring, a burden, scary" (Eng, Han, & Fah, 2011, p. 246). Such attitudes may stem from students being forced to memorise formulae, algorithms, and steps to achieve good scores in tests and examinations. Calculus by its very nature demands step-by-step processes to understand the concepts, definitions and theorems (Matthews, Hoessler, Jonker, & Stockley, 2013). Students have difficulty relating algebraic ideas to graphically represented calculus notions (Tall & Vinner, 1981; Ubuz, 2007).

Calculus teachers mostly make use of traditional methods in their teaching (Lasut, 2015). Most of the time, the traditional method of teaching and learning can be seen as talk and chalk using the traditional paper-pencil approach, while scaffolding was employed within lecturers method-oriented classrooms (Schuetz, Biancarosa, & Goode, 2018). Computational procedures take preference over the true understanding of calculus concepts (Lasut, 2015). Axtell (2006) is concerned that this sequential method of instruction does not help students to understand the basic concepts of calculus. As a result of the lack of true understanding, Studies by Fluck and Dowden (2013) and Nobre et al. (2016) found that many students did not know how to convert calculus concepts to applications in the physical world.

Since 2019, science, technology, engineering, and mathematics (STEM) have become an important focus of the Ethiopian government to improve the teaching and learning of these disciplines, including calculus. As a result, at my University (Wachemo), the promotion of STEM is the focus of one of the community services in the woreda (district)

in the Hadiya zone. In this project, learners are selected from preparatory and secondary school in the woreda to undergo training in STEM subjects at the university during the summer holidays. Unfortunately, this training continues to use traditional methods and does not include new technologies. It is hoped that STEM could change this as it has the potential to encourage all teachers, from elementary to tertiary level, to make use of new technologies, which have as yet not been integrated into mathematics courses in Ethiopia.

Effective teaching programmes in the 21st century are characterised by the integration of technology in education (Pierson, 2001). Gündüz and Odabasi's (2004) study revealed that we can no longer regard the integration of technologies in the learning environment of the classroom as an option; it is an obligation in the information age. This use of technology in the classroom requires thorough planning of how it is to be used to facilitate mathematical understanding (Zho, Pugh, Sheldon, & Byers, 2002).

In this study, I investigated the effect on students' learning of mathematics, and specifically calculus, of GeoGebra mathematical software. GeoGebra was used in this study as it is open-source software, it is simple to use, and anybody can download the software free from the internet. Although there are currently several technologies available to enhance the teaching and learning of mathematics, the choice of the most appropriate technological tool can be difficult (Ruthven, Hennessy, & Brindley, 2004). My decision was guided by the ease of use of GeoGebra and the fact that it is a multi-platform, dynamic mathematical software package designed for students at all levels of education and has a wide range of applications. The program contains both dynamic geometric software (DGS) and computer algebra systems (CAS) (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2008). The software can also be manipulated in various ways in the same window. GeoGebra was designed specifically for educational purposes and has been used in the field of mathematics teaching; it comprises creative and interactive visual application tools that help students to understand complex theoretical mathematical ideas. GeoGebra's display is composed of an algebra window (a window with 2D and 3D graphics), an input bar, an input environment spreadsheet, CAS, statistical abilities and calculus tools.

1.3. RESEARCH PROBLEM

Ethiopia has been part of the programme *Strengthening of Mathematics and Science Education-Africa* (SMASE-Africa) since 2007, a regional sub-section of the Strengthening of Mathematics and Science Education (MASE) programme, where African countries exchange skills and experiences and discuss issues in mathematics and science (MAS) teaching (Ministry of Education (MOE), 2014). The Ethiopian government has also established the Centre for Strengthening Mathematics and Science Education in Ethiopia (CSMASEE) under the auspices of the Federal Ministry of Education (MoE), which is responsible for science and mathematics education. To achieve the goals of SMASE-Africa, the use of technology in mathematics education has become mandatory and non-negotiable, although its implementation has been slow (Ministry of Education (MOE), 2014).

In an additional step to strengthen mathematics and science education, all universities in Ethiopia require students in the technology, social science, business and commerce, and science courses to complete a four-year undergraduate course in mathematics. All the mathematics modules provided include calculus. At Wachemo University where I teach in the College of Natural and Computational Sciences, students are required to take mathematics, partly to reduce their anxiety about the subject.

Instructional aids used by teachers in the classroom, teacher quality and class management all influence students' attitudes towards mathematics (Yilmaza, Altun, & Olkunc, 2010). Teaching methods are also a factor (Papanastasiou, 2000). One subject that is frequently regarded as challenging by most students is mathematics. Nevertheless, mathematics is a basic tool both for all scientific studies and for real life. Whether one is educated or not, mathematics is part of one's everyday life. Mathematics helps students to articulate concepts in the real world; such articulation poses a challenge for many students at university. The use of multimedia technology in the teaching and learning process can simplify this difficulty (Ayub, Sembok, & Luan, 2008). Multimedia learning instruction, defined as using presentations (words) and representations (figures) can

foster learning by combining verbal and visual representations of complex concepts (Mayer, 2009).

Technology is not an end in itself, however; rather, it provides a means for students to engage in deep learning of mathematics (Wassie & Zergaw, 2019). In other words, students will be exposed to a variety of technologies in their education and must thus develop the skills to use them if they are to be successful in their careers and as members of a competitive global economy.

Abstract mathematical concepts make the teaching and learning of mathematics challenging. Teachers must balance the use of multimedia for scaffolding and students clarifying abstract concepts for students (Prieto, Juanena, & Star, 2014). The use of technology, in this case, GeoGebra, in the classroom can help students develop positive attitudes towards using technology in learning; the teacher should thus take care to choose the right software for the particular learning context (Arbain & Shukor, 2015). Beyond its classroom applications, technology plays a major role in the development of educational processes in a country (Gürsul & Keser, 2009). The use of existing and emerging technology packages such as GeoGebra mathematical software, Geometer's Sketchpad (GSP), Math lab, Cabri and Mathematica should be maximised by educational experts. The importance of technology in education as a tool and channel of instruction has been recognised (ten Brummelhuis & Kuiper, 2008); however, students' perceptions of the use of technology have received less attention (Li, 2007).

Several studies have found that day students are not interested in science, specifically mathematics, and are inclined to drop out of science courses. In some circles, there is a mindset that not everybody is good enough to cope in the sciences, and that science courses are made as difficult as possible so that the only the most able students succeed, and the weaker ones tend to fall out (Epstein, 2006, as cited in Gasiewski, Eagan, Garcia, Hurtado, & Chang, 2012, p. 229).

Many students fail to complete mathematics in general science degree courses. Fear of failure and lecturers' pedagogies and instructional methods have been cited as factors in the high attrition rate in mathematics (Bligh, 2000; Booth, 2001; Knight & Wood, 2005;

Novak, Patterson, Gavrin, & Christian, 1999). In his book entitled *What is the use of the lecturer?* Bligh (2000) argues that lectures are an ineffective teaching method that leads to a tendency among students to memorise rather than to develop a conceptual understanding of mathematics. Booth, (2001), Knight and Wood (2005), and Novak et al. (1999) make a similar argument. Scholars such as Handelsman et al. (2004), Hurd (1998), and Williams, Papierno, Makel and Ceci (2004) found that courses at the college level focus more on memorization and less on conceptual understanding. Glasson and Lalik (1993) proposed activities in the classroom that encourage active learning and student participation. Thalheimer (2003) supports this, arguing that learning occurs only when students are cognitively engaged in a process of questions and answers.

In Ethiopia, little research has been done on integrating technology into mathematics teaching at either school or university level, especially in teaching with open access software like GeoGebra. Teaching in Ethiopia is still traditional, and teacher-centred. Thus, this study developed a new model known as the cycle model and implemented it in the study area and investigated its fruit fullness for teaching differential calculus by technology. In the words of Dewey (2016), “if we teach today’s students as we taught yesterday’s, we rob them of tomorrow” (p.1). Thus, in this study, I investigated the effect of mathematical software such as GeoGebra on students’ learning of calculus at Ethiopian universities.

1.4. MOTIVATION FOR THE STUDY

Education and technology play a significant role in bringing about change and transformation in society and human life. Using technology in education has several benefits. Technology can simplify learning (Ianos & Brezeanu, 2020; Julia, Kurnia, Isrokatun, Wulandari, & Aisyah, 2019), encourage students’ active involvement in learning (Schuetz et al., 2018), and enhance heuristic or experiential learning (Shadaan & Eu, 2013).

The importance of STEM subjects and students’ training in these subjects has been endorsed by both the National Council of Teachers of Mathematics (NCTM, 2000) and

the Common Core State Standards for Mathematics (CCSS-Maths, 2010). These standards do not explain how teachers should teach, but rather how to identify the knowledge and skills that students require to create meaningful lessons in a nurturing learning environment (National Governors Association, 2010). In CCSS-Maths (2010), the strategic use of appropriate tools is proposed to identify and express consistent and reliable reasoning (Nadelson, Seifert, & Hendricks, 2015). With this in mind, I selected GeoGebra as an appropriate tool to teach and learn mathematics at the tertiary level. Research has shown that using technology when teaching students is important in increasing students' involvement in STEM. I also believe that using technology to teach mathematics may encourage links with other disciplines such as the social sciences; some scholars have investigated the use of GeoGebra in the teaching of the social sciences (Arini & Dewi, 2019).

As GeoGebra is free software, there is no cost implication for parents or policy makers when students use the program. Furthermore, the use of this software is believed to have a positive impact on student's attitudes, beliefs and perceptions of calculus and provide an alternative approach to learning calculus concepts and solving related problems, whether in algebra or calculus. It is further hoped that teachers will use this study to enhance their students' understanding of the concepts of calculus and even devise interventions based on the one documented in this thesis. The study may also prove significant for students who enjoy learning mathematics in an e-learning (online) environment. In addition, the findings of the study may provide information on how students with different abilities communicate with their peers when engaging in activities in the classroom. Such information is crucial when planning lessons for large classes that include students are of varying abilities. It was hoped that the findings would reveal that the integration of technology is an aid to students learning of mathematics, particularly calculus. The study was also intended to help teachers to redefine their role as facilitators and guide in the learning process.

The introduction of dynamic software and technology in higher education may assist the realisation of the ideal first posited by the Ethiopian Ministry of Education (MoE) and

MoSHE in 2018 to promote STEM-related subjects that have curricula that involve technology. This drive is expressed in the educational reform initiative known as the Ethiopian Educational Road Map for 2018-2030 (Teferra et al., 2018). For instance, starting from 2019, the course *Mathematical Software* was introduced in the first-year BSc course to align university courses with the demands of 21st-century technology (Ministry of Science and Higher Education - Ethiopia, 2020). At present, following revisions of the curriculum, the software programmes *Mathematica* and *MATLAB* are taught; however, it is hoped that the findings of this study will support the inclusion of GeoGebra in future.

The integration of technology can also be regarded as part of teachers' professional development (Trigueros & Lozano, 2012). In the Ethiopian context, the notion that teachers too can benefit from the use of technology (So & Kim, 2009) may feed into MOSHE's views on teachers' professional development. As a lecturer myself, the findings of these and other scholars inspired me to do further research in this area: the purpose of the study was to investigate the effect of the use of GeoGebra software on the learning of mathematics by students at Wachemo University.

1.5. OBJECTIVES OF THE STUDY

This study set out to achieve the following aim and objectives:

1.5.1. Aim

The general aim of the study was to investigate the effect of the use of GeoGebra mathematical software on university students' learning of calculus.

1.5.2. Objectives

- a) To compare the level of proficiency in differential calculus of students in two groups: those taught using GeoGebra (experimental Group 1) and those taught using conventional lecturing (control Group 2).
- b) To compare the level of proficiency in differential calculus pre- and post-intervention in Group 1.
- c) To investigate students' experiences and perceptions of the use of mathematical software (GeoGebra) in learning calculus concepts.

1.6. RESEARCH QUESTIONS

The following research questions were posed in this study:

General research question:

What is the effect on students' proficiency in calculus at a tertiary level in Wachemo University of using GeoGebra Mathematical software?

Specific research questions:

- a) How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group 2)?
- b) How does the level of proficiency in differential calculus compare within the experimental group (Group 1) pre- and post the intervention incorporating the use of GeoGebra?
- c) What are students' experiences and perceptions towards using mathematical software (GeoGebra) when learning calculus concepts?

CHAPTER 2 LITERATURE REVIEW

2.1. INTRODUCTION

The purpose of this literature review is to provide an overview of the theories and research findings of studies conducted in the discipline of mathematics teaching to address the research questions.

This review focused on literature concerned with the following topics:

- The use of technology in education in the 21st century
- Software used in the teaching and learning of mathematics
- GeoGebra software, its components and students' attitudes to using it
- Student's proficiencies in Mathematics education
- The challenges that mathematics poses for students
- Teachers' beliefs about using technology in the classroom.

The focus of the study was on the use of technology in the teaching and learning of mathematics at the university level. In this review, I used a wide range of cross-disciplinary sources including books, journal articles, theses and dissertations, and conference proceedings. In most cases, I used educational databases of e-resources from the University of South Africa (UNISA) (such as EBSCO), Google Scholar, Mendeley and z-library.

2.2. THE USE OF TECHNOLOGY IN EDUCATION IN THE 21ST CENTURY

The 21st century has seen a technological revolution that has had a significant impact on education. The term *technology* can be defined in a variety of ways, depending on the field of its application; literally, it refers to the use of hardware, while metaphorically it can be applied to real-world problem solving (Heinich, Molenda, Russell, & Smaldino, 2002). Huang, Spector and Yang (2019) argue that there are two components to technology: hardware and software. Hardware comprises the tool that embodies the technology, material or physical object, while software comprises the information base underlying the

tool. Some technologies may lack one or both components and may simply take a standard procedure or general-purpose algorithmic approach.

While technology is not a replacement for human intelligence, it certainly reduces the uncertainty in cause-effect relationships involved in achieving the desired outcome. It is a systematic application of knowledge to solve problems (Huang et al., 2019). As such, it has many applications in education and is indispensable in learning and teaching (Pierce & Ball, 2009; ten Brummelhuis & Kuiper, 2008). Technology in education benefits not only researchers but also teachers, governments and funding agencies. Educational technology (EdTech) includes the use of hardware, software, digital content, data and information systems that support and enrich teaching and learning, improving education management and delivery.

In mathematics specifically, technology enables discovery and promotes the discovery method and experimentation. These advantages have encouraged the integration of technology in the mathematics education community and among policy makers (Lavicza, 2008). The extent to which technology is used in mathematics education differs from country to country, however, and even within countries its growth and use may vary greatly from place to place, for example from rural to urban areas. Reasons for this uneven adoption of EdTech include differences in policy initiation and infrastructure expansion. In Southeast Asia (Singapore and Malaysia, for example), the government has led the integration of technology (ICT) in the education system. In Malaysia, various types of technology or dynamic mathematical software such as Geometry's Sketch Pad, Autograph, and the Graphing Calculator have been integrated into secondary school mathematics (Bakar Ayub, & Tarmizi, 2010). Although open software is still new to Malaysia, the internet is widely accessed, even by children. Tapscott (2009) found that children in technologically advanced countries naturally develop technological capabilities, are dependent on technology, regarding it as natural as breathing, and resist teaching that makes use of the old "telling" paradigm (Prensky, 2008). They are inclined to use the Google search function rather than to ask individuals for information (Gibson, 2009).

Because of this rise in the use of technology in education, educators need to integrate technology in their learning and teaching processes and use it as a tool to support the new teaching paradigm (Prensky, 2008). Scholars define the integration of technology or the use of technology in education in a variety of ways; there is no standard definition of the term (O'Dwyer, Russell, & Bebell, 2004). The integration of technology may be viewed either in terms of the use of computers in the teaching process or the presentation of teaching materials (O'Dwyer et al., 2004).

Dockstader (1999) defines the integration of technology in the classroom simply as a way of using computers effectively and efficiently in teaching and learning to enhance student learning. The present study argues that technology should form an integral part of the curriculum, not merely for the sake of integrating it, to support, learning. Most developed countries have adopted technology-based instruction to keep up with the ever-increasing demand for development and progress that characterises the 21st century (Eyyam & Yaratan, 2014; Lasut, 2015).

Students who use technology can discover mathematical concepts, test their emerging mathematical understanding, both procedural and conceptual, experiment and visualise (Olive et al., 2010). However, in my country Ethiopia, the number of students who have the technology for learning in the classroom is very low from my experience of teaching mathematics because of their economic background. The study done in Kenya showed that mathematics teachers lag in adopting new technology, which is directly related to students' experiences of using technology in classroom learning (Mwingirwa & Miheso-O'Connor, 2016). In contrast, the use of GeoGebra affected learners' learning and positively affected the teacher's beliefs regarding teaching and learning even for those teachers found in high-poverty, rural settings where the availability of technological resources is limited (Mthethwa, Bayaga, Bossé, & Williams, 2020). Thus, my government, in which most of the schools are found in rural areas, is ready to integrate technology into the education system (Teferra et al., 2018).

Lacey (2010) argues that a learner-centred classroom and the integration of technology (such as the use of a 3D printer) supports cognitive development, problem-solving, and

active engagement by students in the learning process. Furthermore, modelling and simulation of a range of mental and natural processes become possible when using technology, and computer-based educational environments can provide context and support for meaningful problem-solving activities. This wider view of bridging the zone of proximal development is consistent with Vygotsky's emphasis on human-tool interaction.

Despite this evidence, some studies have challenged the benefits-only view of technology integration in education (Mantiri, 2014). Mantiri's study revealed some disadvantages and challenges associated with educational technology, including copyright issues, the dangers of dehumanised teaching and of breaching privacy and security.

In this regard, Jaffee (1997) lists four valuable pedagogical principles and practices that should occur in the technologically integrated classroom, namely *active learning*, *mediation*, *collaboration*, and *interactivity*.

- *Active learning* involves students' interaction with the subject content in constructing knowledge. Mayer (2009) argues that learning is an enduring change in students' knowledge, attributable to their experience. Learning involves three simultaneous processes, namely acquisition, transformation and evaluation of activities (Bruner, 2006). In most descriptions of learning, the starting point of learning is the interaction of students with their environment (e.g., the scaffolding of students with the help of GeoGebra (please see first steps of cycle model in chapter five Section 5.3), and the integration of new knowledge with existing knowledge. Jaffee (1997) emphasises the imperative of active learning that students must do more than merely receive information. Barak, Lipson and Lerman (2006) found that the use of technology forces students to be engaged, motivated, and focused on activities in the classroom, activities in which they not only learn theoretical concepts but also practise hands-on programming.
- *Mediation*, in Jaffee's (1997) view, is the teacher-student interaction directed at problem-solving, responding to questions and course materials, and the discussion of course-related topics. In Vygotsky's conceptualisation, "mediation"

is defined as the ladder between environmental stimuli and individual students' responses to these stimuli (Vygotsky, 1978b).

- *Collaboration* is the interaction of students with each other, working together through questioning and information sharing.
- *Interactivity*, the last of Jaffee's teaching principles and what he describes as the master concept, refers to knowledge construction through students' interaction with other students, teachers, and resources using technology.

Ruthven (2009) identifies five key structuring features of resources in a technology-oriented :

- *Working environment*: The classroom where students were taught.
- *Resource system*: Collection of mathematical tools and materials in-classroom use.
- *Activity format*: Generic templates for action and interaction which frame the contributions of teachers and students in various types of lesson segments
- *Curriculum script*: Evolving teacher knowledge gained during teacher's own experience of learning and teaching a given topic.
- *Time economy*: This is discussed in detail in the explanation of the framework of the study (see Section 3.4).

The advantages of technology in the mathematics classroom can be affected by students' confidence in using this technology. Galbraith and Haines (1998) found that students' confidence was a factor in the use of technology: those with low levels of confidence in using technology felt disadvantaged while those with high levels of confidence felt self-assured when using technology.

To sum up, the use of technology in teaching and learning in the 21st century cannot be questioned, if it is properly implemented. Applying technology in the classroom requires a more active learning process (Barak et al., 2006; Jaffee, 1997); the engaged student in a classroom environment is a problem solver (Lacey, 2010); technology-oriented classrooms enhance students' learning (Dockstader, 1999; Nobre et al., 2016). The most effective teaching method in the 21st century involves the effective integration of

technology in the classroom (Pierson, 2001). Lastly, to integrate technology in the learning and teaching process, all partners in the education process need to understand the technology (e.g., government, teachers, students, parents, school leadership etc.). The implementation of technology in the classroom must be carefully planned (taking note of the available infrastructure), bearing in mind the criteria for technology implementation (Jaffee, 1997; Ruthven, 2009). It is not the technology itself that facilitates new knowledge and practice; it affords the development of tasks and processes that open new pathways to knowledge (Olive et al., 2010).

2.3. THE USE OF TECHNOLOGY DURING PANDEMIC

The word pandemic comes from the Greek *pandemos*: *demos* mean the population and *pan* mean everyone. So, *pandemos* is a word describing something that affects the world's entire population. The word is generally applied to disease. In 2020 the Corona virus Covid-19 pandemic forced countries around the world to take precautions to stop the disease from spreading. One of these steps was the closure of schools and universities. To continue the teaching and learning process during the Covid-19 pandemic, the World Health Organisation (WHO) urged certain actions; As the pandemic runs its course, many governments have implemented measures limiting the number of people congregating in public places, especially educational institutions (Reimers, Schleicher, Saavedra, & Tuominen, 2020). The Covid-19 pandemic has had a severe impact on higher education, particularly in developing countries such as Ethiopia, as universities closed their premises, without necessarily completely ceasing teaching and learning processes. In general, the Covid-19 pandemic created the largest disruption of education systems in the history of the world (De Giusti, 2020). Education systems globally have been forced to run their teaching and learning on online platforms to reduce physical contact between students and teachers, and thus reduce the spread of the virus. This new teaching pathway is has been enabled by the use of technology (Olive et al., 2010). Thus, the next section discusses the types of software that assists users in the teaching and learning of Mathematics specifically.

2.4. TYPES OF SOFTWARE IN MATHEMATICS INSTRUCTION

In this subsection, I discuss the usefulness of different types of free mathematics software applications, their application in learning and teaching mathematics and their value, particularly in teaching the topic of calculus.

2.4.1. Types of software and their usefulness in teaching mathematics

It is widely acknowledged that students benefit from the teaching of mathematics through technological means (Dossey, McCrone, & Halvorsen, 2016; Heinich et al., 2002; Lavicza et al., 2019; NCTM, 2000; Pierce & Ball, 2009; ten Brummelhuis & Kuiper, 2008).

Inayat and Hamid (2016) focus on the advances of technological tools such as Computer Algebra Systems (CAS) and Dynamic Geometry Systems (DGS), and the combination of the two packages in GeoGebra, in terms of their effectiveness in the teaching and learning of mathematics. They argue that such applications promote more effective learning in a student-centred and dynamic environment. They found that in mathematics, innovation in the teaching and learning process was shaped by modern digital technologies offered by web-based applications. A web image has been used to characterize this new way of teaching mathematics in the digital age. Tall (2019), as discussed in conference proceedings mentioned above, together with several other researchers have shown that the use of computerised technology in Mathematics education has many advantages (Ayub et al., 2008; Ayub, Mukhtar, Luan, & Tarmizi, 2010; ten Brummelhuis & Kuiper, 2008).

Curriculum developers, educators and students all benefit from the advantages of educational technology, not least because students are attracted to this visually entertaining and interactive mode of learning. The introduction of technology in mathematics instruction elevates the level of motivation and affect displayed by students in science-related courses of study. Inayat and Hamid (2016) and Keong, Horani, and Daniel (2005) found that technology-oriented mathematics education enhanced students' understanding of basic concepts. Interactive software can provide an immediate response to students' input, enables interaction and cooperation among students, improves skills,

stimulates active participation and assists in the integration of theory and models (Inayat & Hamid, 2016).

2.4.2. Types of mathematical software suitable for educational purposes

Two categories of mathematical software for educational purposes are prominent (Hohenwarter, Kreis, & Lavicza, 2008; Inayat & Hamid, 2016):

- Computer algebra systems (CAS) software such as Derive, Mathematica, Maple and MuPAD;
- Dynamic geometry software (DGS) such as Geometer's Sketchpad, Cabri Geometry software.

In each category, effective mathematics instruction tools can be found. Both CAS and DGS are essential for higher education, while DGS is also suitable for primary school since it features a mouse-driven user interface and is rich in visualisation.

Some software has been developed for specific applications and others have multi-purpose applications; some are available free of cost and others must be purchased; some are area or country bound while others are globally available (Papp-Varga, 2008). Papp-Varga (2008) observes that Graph is an open-source application suitable for teaching functions, which can be categorised under software with specific packages. Maple can be categorised as software with general packages as it works for almost all fields of mathematics. Some mathematics software such as GeoGebra, has been translated into several languages while others are restricted to one language. Most types of software can be installed on personal computers, notebooks, mobile cell phone devices and laptops.

Some educational software packages for mathematics teaching and learning come at a cost in the market, and many students, teachers and schools cannot afford to buy them. Subsequently, free open-source software, readily available from the internet, is in high demand, especially in developing countries. Apart from interactive software applications, courseware and teaching materials are also available. Given the Ethiopian educational

setting, the focus in this research study is on free open-source mathematical software suitable for teaching and learning calculus at the tertiary level. From the available applications in this category, namely GeoGebra, Wolfram Alpha and Desmos, I chose GeoGebra because it is user-friendly, time-saving, simple to use and easy to manipulate. Any student can download the software onto his or her electronic device at no cost. This freeware is gaining popularity around the world for both educational and research purposes.

Despite the obvious advantages of teaching technology integrated mathematics, many studies have revealed that the integration of technology into mathematics teaching has been slow when compared to the speed at which technology has evolved (Lavicza, 2010; Lavicza et al., 2019). Some teachers are fearful of integrating technology into their classrooms because their skills, knowledge and abilities may be overshadowed by those of their increasingly proficient 21st-century students (Lavicza et al., 2019). The digital age is accompanied by the imperative to conduct technology training for teachers; (Bekene, 2020) regards this as the first phase of the integration of technology into classrooms, as teacher professional development in the implementation of technology into teaching.

Because of the development of GeoGebra, by 2001 the two types of mathematics software mentioned above had increased to three common types, known as DGS, CAS and a combination of the two. However, before 2001 DGS and CAS had not been linked in one program. GeoGebra mathematical software, developed in 2001, integrates the possibilities of both DGS and CAS in one program (Antohe, 2009; Dikovic, 2009; Hohenwarter & Jones, 2007). Kllogjeri and Shyti (2010) argue that GeoGebra software provides bidirectional representations, making it different from software developed previously. For example, GeoGebra makes it possible to write an algebraic equation in one window and the graph of the equation will be displayed in a graphic window. This functionality will be discussed below. In Table 2.1 some common CAS, DGS and CAS+DGS types of software are listed, together with their year of inception.

Table 2.1 Some mathematical software currently available

Software Brand Name	Year of start	Utility
Mathematica*	1988	CAS
Maple*	1985	CAS
MuPAD*	1993	CAS
SAGE	2005	CAS +DGS
Geometry's Sketch pad*	1986-1991	DGS
Autograph	1985	DGS
Graphing Calculator	1925	DGS
GeoGebra	2001	CAS +DGS
MATLAB*	Late 1970	CAS
Derive	1988	CAS
Mathcad*	1985	CSA
Magma*	1993	Arithmetic, number theory
SciLab	1994	CAS
Maxima	1998	CAS
Solver	2002	Differential equation
R	1993	Statistics
Gnuplot	1986	Dynamic software for plotting graphs

(*commercial software)

2.5. GEOGEBRA SOFTWARE IN TEACHING AND LEARNING MATHEMATICS

2.5.1. Characteristics of GeoGebra

Among the multitude of mathematics software programs available in the global market, GeoGebra has gained exceptional popularity as a freely downloadable multi-stage dynamic mathematical software package. It was developed for educational purposes and its use spans all levels, from elementary to university level. It combines the functionality of CAS and DGS in one user friendly application (Hewson, 2009; Hohenwarter et al., 2008; Hohenwarter, Hohenwarter, & Lavicza, 2009). Hewson (2009) points out that GeoGebra software is attractive both in terms of price and of the way it encourages collaboration in learning and teaching. For the end-user in the classroom and at home, there are no licencing concerns and after downloading, it can operate offline. Since its development by Markus Hohenwater in 2001, GeoGebra has built a user community in 190 countries and has been translated into 55 languages (Furner, 2020).

2.5.2. Components of GeoGebra

The components of GeoGebra and their applications have been developed interestingly and appealingly. Ogwel (2009, cited in Akanmu, 2015) listed the elements of GeoGebra as, among others, menus, tools, views, input bar, tool bar, graphics window and algebra window. These components can be seen on the GeoGebra interface at the top and bottom of the screen in Figure 2.1.

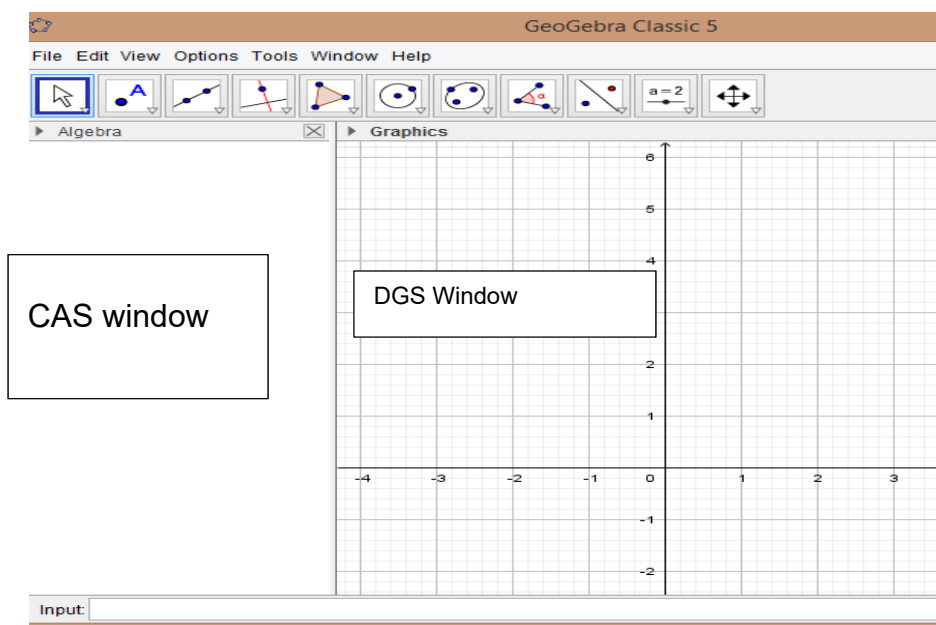


Figure 2.1 Opening screen of GeoGebra

2.5.3. Successful integration of GeoGebra in mathematics instruction

Notwithstanding the speed of technological developments in the 21st century, the majority of teachers do not find its integration in the classroom without difficulties (Ruthven et al., 2004). Teachers are aware that students need the motivation to tackle problems by themselves and to become involved in practical activities. These are the elements of constructivist theory, which is generally regarded as the most effective approach to mathematics teaching and learning. Based on a large body of research, educators are equally aware that the integration of technology in the classroom has the potential to give

rise to motivation, interest and involvement. GeoGebra provides users with considerable opportunity to engage in true constructivist learning (Hohenwarter et al., 2008). The mere introduction of technology into the classroom does not necessarily affect the motivation, interest and involvement of students in mathematics, however. It requires training for educators to master the technology and its uses (Cuban, Kirkpatrick, & Peck, 2001; Ruthven & Hennessy, 2002).

One of the indicators of successful integration of technology in the classroom occurs when students become cognitively involved in the mathematics they learn and do, using technological tools. Karadag and McDougall (2011) investigated the ability of GeoGebra software to provide activities that would involve students cognitively. They found that students created mathematical objects, that they were able to conceptualise ideas and to form relationships among these ideas. They were able to perceive mathematical objects in the physical environment and through social interaction and it became clear that technology was capable of introducing concepts to the working memory where they were systematically processed as integrated knowledge.

From an educator's perspective, GeoGebra is a useful aid when creating mathematics tasks. These tasks include the preparation of teaching and learning materials such as test banks (which may reduce repetition of test items from year to year), module preparation, progress tests and summaries, in both technological and traditionally oriented situations. In the technologically oriented classroom, the use of dynamic software such as GeoGebra accelerates these tasks (Jaffee, 1977; Ruthven, 2009), while allowing best teaching and learning practices to be maintained. GeoGebra can create precise figures that can be manipulated both in the classroom and at home.

The literature reveals that GeoGebra creates an atmosphere conducive to the learning of mathematics, in the sense that it stimulates creative thinking and promotes a problem-solving orientation (Selvy, Ikhsan, Johar, & Saminan, 2020; Žilinskiene & Demirbilek, 2015; Zulnaidi & Zamri, 2017). The software is simple to use, which helps to reduce the teacher's role to that of a knowledgeable guide, while students take on an active role by doing tasks by themselves, only calling for help when they find activities difficult.

Fahlberg-Stojanovska and Stojanovska (2009) found that learning mathematics through technology motivates students to engage in the *process* of searching for solutions at a higher level, not only in finding the *solutions*.

A further advantage of GeoGebra mathematics software is that it helps students to learn calculus by simultaneously displaying the answer of a task in the algebra view window with its visual representation in the dynamic geometry view. This dual-mode of representation facilitates the making of connections and relationships, a prerequisite for high-level mental functioning, which will be discussed in Section 3.2. However, GeoGebra not only unlocks higher-level thinking; it also enables mathematical thinking at all developmental levels. This allows teachers to explore students' potential in mathematics and to unlock their skills (Aydin & Monaghan, 2011).

GeoGebra mathematical software is a cloud-based service and like Office 365 it offers online data processing and self-actualisation of certain actions (Semenikhina, Drushlyak, Bondarenko, Kondratiuk, & Dehtiarova, 2019). It also has great potential in an e-learning environment (Albano & Iacono, 2018; Antohe, 2009; Dikovic, 2009; Gülseçen, Reis, Kabaca, & Kartal, 2010).

There is evidence (Zulnaidi, Oktavika, & Hidayat, 2019) that the use of technology in general, and GeoGebra in particular (Zengin, Furkan, & Kutluca, 2012) enhances students' achievement. Zengin et al. (2012) investigated the effect of GeoGebra software on students' achievement in trigonometry by using a control group that received constructivist instruction and an experimental group whose instruction included the use of GeoGebra software. They found a significant difference in achievement between the experimental group and the control group; those using GeoGebra achieved significantly better than those who were taught without GeoGebra.

GeoGebra has several innovative functions that empower the user when tackling complex tasks. Diković (2009) lists some of the uses of GeoGebra as follows:

- a. GeoGebra is a calculator of graph functions.

GeoGebra enables the student to sketch the graph of a simple linear function not only in one variable, such as $f(x) = 3x + 5$, but also in two variables, such as $f(x, y) = \frac{x^2 + y^2}{x - y}$.

In Figure 2.2 these functions are displayed in a single GeoGebra window.

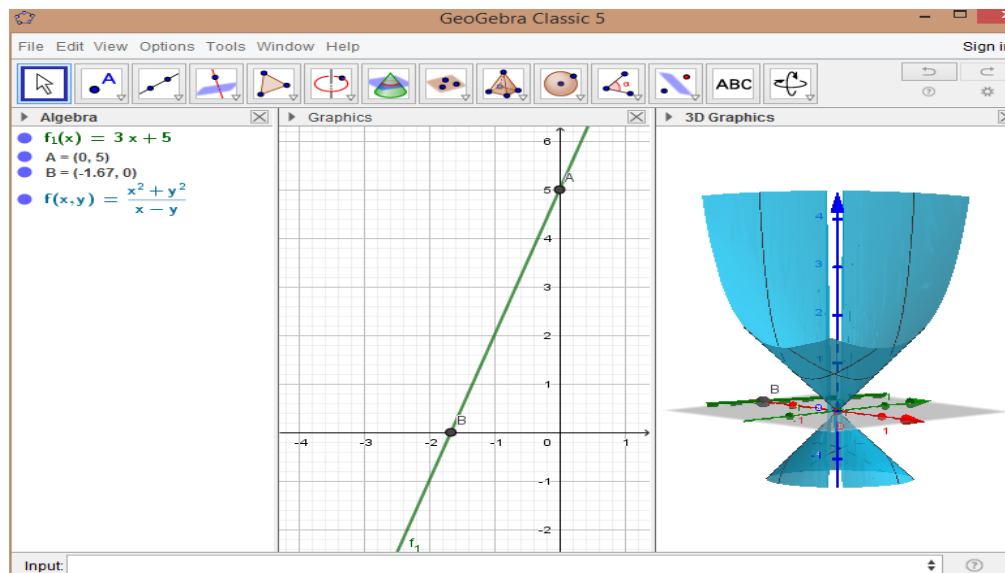


Figure 2.2 The graphs of functions in one and two variables in a GeoGebra window

- b. GeoGebra can be used for investigation projects.

Students can apply GeoGebra to their investigations since it allows experimentation with various representations of a mathematical idea, it is visually rich and it promotes a problem-solving disposition, also termed heuristics (Bruner, 2006). Because of these multiple representations, connections between and among various mathematical ideas can be made. In the case illustrated in Figure 2.2, the CAS window represents the two functions symbolically and the DGS windows represent the graphs of the two functions visually. Students can experiment with these multiple representations by adding, changing, and manipulating input elements at will.

- c. GeoGebra enables original creations.

Students can personalise a graphed function in the GeoGebra interface by changing the language and display elements such as font type, size and colour, the coordinates of axes, the thickness of lines and line styles. GeoGebra simplifies the understanding of mathematical concepts.

A complex and abstract idea, such as the real function $f = x \sin\left(\frac{1}{x}\right) : R/\{0\} \rightarrow R$ approaching zero, once mapped and visualised, may be easier to conceptualise, as illustrated in Figure 2.3 below. When studying the properties of this function near zero, GeoGebra provides a visual conceptualisation of the limits of the function of f at zero.

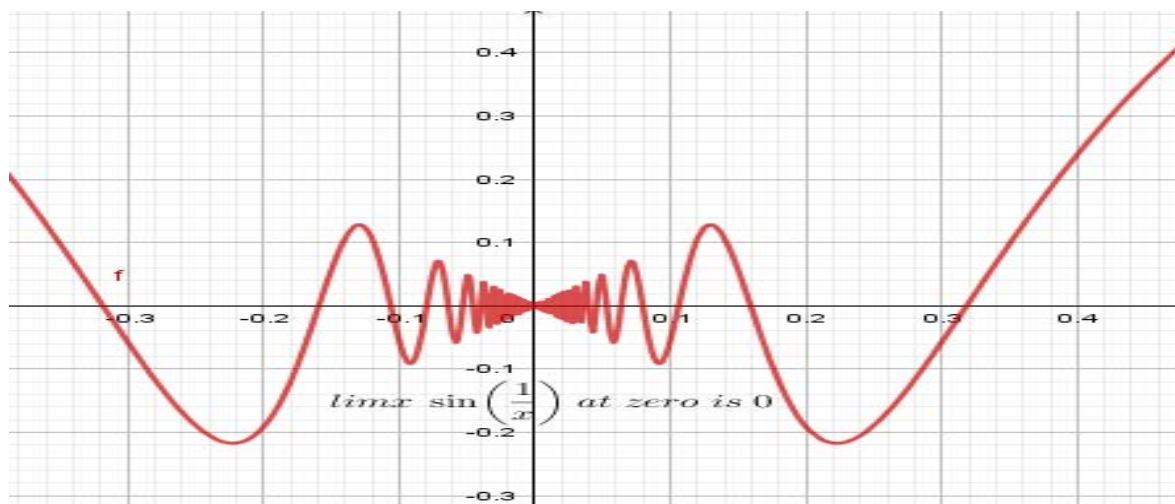


Figure 2.3 The graph of a real function approaching zero.

- d. GeoGebra enables cooperative learning and teaching.

Mathematical problem solving can be approached cooperatively, in pairs, small groups, or by the whole class. It can be used in individual student presentations and in group presentations, as well as when teaching mathematical modelling. Within a pleasant, interactive and friendly environment, students tend to participate actively, cooperating and

collaborating. All this enhances the understanding of the problem and the development of problem-solving strategies (Pasco & Roble, 2020).

- e. GeoGebra enables the generation of mathematical objects.

The GeoGebra software allows students to create new graphs or edit existing ones. It allows the user to easily publish a worksheet as a Web page and in so doing, make online e-learning possible in a virtual classroom. Apart from uploading the activities of students together with their sketched figures onto the GeoGebra platform, students can access activities created by their teacher by simply clicking on the link and using the password to enter the virtual classroom. Diković (2009) found that GeoGebra encouraged not only the students but also their teachers to use the software in their classrooms, whether they were conventional or virtual.

2.6. STUDENTS' PROFICIENCY IN CONCEPTUAL AND PROCEDURAL KNOWLEDGE IN MATHEMATICS EDUCATION

2.6.1. History of mathematics education and mathematical proficiencies

Learning theories influenced the evolution of mathematics education. These theories are can be classified under the two umbrella terms, behaviourism and constructivism. The method of transmission of mathematics knowledge differs in these theories of learning. Behaviourism combines explicit teaching and direct instruction as a method of knowledge transmission, sometimes known as a traditional teaching method, while in constructivism knowledge is constructed when it was imposed or integrated into existing knowledge. This is known as the active teaching method (Hechter, 2020). The procedural-formalist paradigm and the cognitive-cultural paradigm are two paradigms in the history of mathematics education; the procedural-formalist paradigm is built on behaviourist foundations [transmission of knowledge] and cognitive-cultural paradigm is built on the foundation of socio-constructivism [promotes the active role of the student and improvement of conceptual understanding through reflection and shared experiences] (Ellis & Berry III, 2005). Vygotsky believed that knowledge was made within the process

of communication and interaction with others and that scaffolding would lead to the storage of information in the mind and used by the students in the environment (base of the developed cycle model) at a later date (see Section 5.10 of the internalisation and externalisation stage of cycle model and Figure 5.2) (Vygotsky, 1978).

Mathematics is considered to be a difficult subject (Kinnari, 2010). The reviewed literature argues that the transmission of mathematical knowledge in the classroom may be facilitated by the use of technology, as in this study with the use of GeoGebra (see Section 2.6.2.) together with traditional/conventional methods that are “embedded in culture, human experience and social interaction” (Hechter, 2020, p. 5).

Within the classroom, mathematics tasks activities commonly take place while social interaction occurs (see Figure 3.2). Tasks are the basis of students’ learning in the classroom (Stein & Smith, 2011). Tasks that need students to recall step in a monotonous manner lead to one type of student thinking [multiple choice question types]; tasks that require students to think theoretically and that encourage students to make connections lead students to different ways of thinking [working out problem questions]. Students who have difficulty linking the statistical words or calculations with their graphical, tabular or other representations may improve with the help of technology (Ocal, 2017). In general, in mathematics the recognition of students’ starting level (proficiency) by using a pre-test of differential calculus containing the tasks of two types of knowledge (conceptual and procedural) can lay the foundation for successful learning situations in the environment (Kinnari, 2010).

The word proficiency in this study refers to students’ fluency in both types of knowledge (conceptual and procedural understanding) that can be discretely measured, quantified, and stratified using the tasks of differential calculus before and after intervention (see Section 4.11.1) (Ellis & Berry III, 2005; Kilpatrick, 2001). The term mathematical proficiency has been referred to as *mathematical literacy* by Kilpatrick (2001) who posits five strands of mathematical proficiency. These are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. This study

concentrated on the two types of knowledge known as conceptual understanding, and procedural understanding. These are defined as Kilpatrick (2001):

- Conceptual understanding: students' grasp of mathematical concepts, operations, and relations, that is knowledge that students understand (Machaba, 2014).
- Procedural understanding: students' skills that they use to follow mathematical procedures, and whether they use them flexibly, accurately, efficiently and appropriately].

2.6.2. Developing tasks for mathematics proficiency

If mathematics is dealt with in the classroom as a priori knowledge, based on objective reasoning alone, without taking the experiences of students with mathematics or the meaning they make of what they have learned, this can be taken into account by pre and post-test which allows teacher's to identify student's mathematics achievement by discretely measured, quantified, and stratified the delivered pre and post-test (Ellis & Berry III, 2005). Hence, the activities given to the students may be developed depending on the concepts of the two types of understanding – conceptual and procedural understanding.

Conceptual understanding

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas that allows students to reconnect with the designed tasks (Kilpatrick, Swafford, & Findell, 2001). Proficiency in representational activities demands conceptual understanding of the mathematical concepts involved (definition of limits, derivatives, etc.), the operations (of addition, subtraction, division and multiplication) and the relations (the combination of concepts such as the relation between natural exponentials and logarithms [$e^{\ln x} = x$]). It also requires strategic competence to formulate and represent that information. Hence, the conceptual tasks require the ability to recall or connect to previous knowledge. In calculus, we know that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, but if the task for students is

given to evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{5x}$ students need to connect the previous knowledge of basic limits to the given rule for the building of students' prior knowledge (Sumartini & Maryati, 2021). Once students have conceptualised the rule they can simply recall answers. Sumartini and Maryati (2021) suggest two measurements of conceptual understanding, implicit and explicit measures. These measurements of conceptual understanding are *implicit* measures and relate to evaluations where one makes definitive choices, ranks quality, and compares numbers; *explicit* measures, on the other hand relate to definitions and explanations. The factors that hinder the recalling or reconnecting of students' to previous knowledge to new knowledge occur in the classroom and these conditions should be identified by the teachers (Stein & Smith, 2011). Factors that are associated with making connections include:

- Scaffolding of student activities [see the cycle model in figure 3.3].
- Students' own exploration.
- Teachers or capable students modelling high-level performance.
- Teachers providing activities [questioning, comments, and feedback].
- Tasks developed based on students' prior knowledge.
- Teachers making frequent connections in conceptual tasks.
- Sufficient time for exploration.

Procedural understanding

The procedure is the knowledge that shows the order or sequence of actions for comprehensive learning of all the components (Zulnaldi & Zamri, 2017). They elaborated procedural understanding by examples of questions asking students to solve the function equation of $f(x) = x^2 + 1$; to determine the formula of inverse function $f^{-1}(x)$ and to graph the functions. According to the question, students are required to find the formula of an inverse function. In this case, students need to recall the ways how to find inverse functions such as:

- Step one: Letting $y = f(x) = x^2 + 1$
- Step two: Interchange the variables x with y that is $x = y^2 + 1$

-
- Step three: Solve for y variables that is $y = \pm\sqrt{x-1}$
 - Step four: Set $y = f^{-1}(x)$

Here we understand that to arrive at the required formula students need to know these steps or procedures. Thus, in this study procedural tests are tests that require step by step activities to arrive at the answers. Procedural understanding is the knowledge of procedures, when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently (Kilpatrick et al., 2001). These two types of knowledge were also discussed in the framework of this study (please see Section 3.4.1). In general, even if procedural or conceptual tasks/tests are presented for the students' things to be considered in measuring student's cognitive level should be considered.

2.7. STUDENTS' PERCEPTIONS OF GEOGEBRA IN LEARNING MATHEMATICS

In this section, I report on the literature about the perceptions of students about using GeoGebra, particularly when learning differential calculus (DC).

We all have our own unique set of perceptions, attitudes, and behaviours (Aiken, 2002). Attitudes and perceptions are closely related (Pickens, 2005) and in this study, I use the terms interchangeably. The process by which people interpret and organise sensations to produce a meaningful experience of the world is known as perception (Pickens, 2005). Many scholars believe that students' attitudes to mathematics are formed by "social forces" (Singh, Granville, & Dika, 2002, p. 324). One of these forces may be technology, including software such as GeoGebra. If students are not sufficiently prepared to use technology in learning, this may affect the impact of technology-integrated instruction and students' perceptions of their ability to solve complex mathematical problems (Moos & Azevedo, 2009). Singh et al. (2002) argue that negative attitudes of parents and teachers affect students' attitudes towards their own abilities and interests; however, programs such as GeoGebra mathematical software may restore their motivation and improve their perceptions of their own achievements (Doğan & İçel, 2011).

Shadaan and Eu (2013) investigated students' perceptions of GeoGebra in learning circle geometry and found the integration of GeoGebra in their teaching and learning produced

a significant improvement in their level of thinking, creativity, critical reasoning and logical assumptions. Bragg (2007) found that poor societal attitudes towards mathematics leave teachers with the task of motivating students in the classroom. It is the role of the teacher to determine the learning outcomes of students when using technology in teaching and learning mathematics (Smith, 2002).

Leder, Pehkonen and Törner (2002) emphasise that students' beliefs about the social setting within which they learn have a decisive influence on problem-solving behaviour, particularly on the affective aspects of learning (such as emotional reactions to class activities). If students experience positive emotions when using mathematical tools/software, studies have revealed that this has a decisive effect on their studies. Arbain and Shukor's (2015) study, found that students in the experimental group that had been taught using GeoGebra not only had positive perceptions towards learning mathematics but also performed better than students in the control group who had been taught by traditional methods.

2.8. TEACHING AND LEARNING DIFFERENTIAL CALCULUS

2.8.1. Brief overview of studies on differential calculus (DC)

Various *conceptualisations* of *calculus* exist, as follows:

- Calculus is a branch of mathematics that deals with quantities approaching other quantities (Charles-Ogan & Ibibo, 2018).
- Calculus is a branch of mathematics that deals with how a change in one variable is related to changes in other variables (Nobre et al., 2016).

Tall (2009) describes a calculus course of study as the desire to quantify and express:

- How things change (the function concept);
- The rate at which things change (the derivative of functions);
- How they accumulate (the integral of functions); and

-
- The relationship between the two (the fundamental theorem of calculus and the solution of differential equations).

Standard terminology in calculus includes the terms *limits*, *derivatives* and *integrations of functions*, while the main terms in differential calculus are *limits* and *derivatives of functions*. The *big ideas* in calculus are *limits*, *derivatives*, *integrals* and *fundamental theorem*, while the idea of *series* also features in the generalisation of calculus, mathematical analysis (Tall, 2019).

Arango et al. (2015) argue that traditionally explaining differential calculus can be dry and off-putting for students; they believe that the use of technology may render explanations more fruitful. As technology continues to develop at an astonishing pace, teaching and learning calculus becomes more possible and accessible. Technology has migrated from large mainframes to portable desktop computers, calculators, laptops and notebooks, while manual input of data, arithmetic and the subsequent creation of graphs have been replaced by automated calculations and graphs. This makes technology available anywhere, anytime.

A study conducted in Brazil by Nobre et al. (2016) found that calculus (and the way it was taught) was the primary cause of failure among college and university students. Traditionally, students experience calculus as difficult, hard to understand and daunting; innovative methods and approaches are needed to make teaching and learning of calculus more effective (Charles-Ogan & Ibibo, 2018; Lasut, 2015). As early as the end of the last century, Rochowicz (1996) identified calculus as the subject that prevents many students from completing courses in science and engineering. According to his research, the calculus curriculum was outdated (even then) and needed to be revised to align with a technologically oriented educational curriculum.

The rapid growth of technology in the 21st century is ongoing, and studies on the effect of combining technology and calculus instruction have also increased. Tall et al. (2008) identify the dynamic nature of both technology and calculus as the reason for this increased interest in such research. Recognising the importance of calculus as the backbone of many science courses, other scholars (Durán et al., 2014; Lavicza, 2010;

Mignotte, 1992; Ozguin-Koca, 2010; Robutti, 2010) have argued that technology has the potential to simplify complex calculus concepts and is gaining ground as a research interest.

The potential of technology in education to promote constructivist instruction is particularly appealing. Huang et al. (2019) list the characteristics of constructivist learning as follows:

- Instruction is student-centred.
- Learners actively construct internal psychological representations.
- Learning comprises the reorganisation and reconstruction of old knowledge and the meaningful construction of new knowledge.
- Learning is not only individualised, but involves language centred social interaction, communication, and cooperation.
- Learning must be situationally embedded to support meaningful learning.
- The construction of meaning requires appropriate resources.

2.8.2. Learning differential calculus using GeoGebra software

The use of technology in teaching calculus stimulates student participation and motivation by relating subject content and concepts to visualisation and experimentation (Nobre et al., 2016). GeoGebra mathematical software provides significant opportunities for meaningful learning and concept formation in calculus, geometry and algebra at various levels (Tatar, 2013). Ocal (2017) investigated the effect of GeoGebra on applications of derivatives in two calculus classrooms (experimental and control) involving 55 students. Students' conceptual understanding and scores both improved; however, there was no significant difference between the procedural knowledge of the experimental group and the control group. The National Research Council (2001) argues that conceptual and procedural knowledge of mathematics are interrelated components, with the first (conceptual understanding) taking the central position. Procedural fluency can be affected by basic instructional routines and by following steps, algorithms and methods or strategies of calculation and the application of formulae and rules.

In the GeoGebra software-based mathematics classroom, the main task of the teacher is to guide students' work, as the software enables students to explore and discover mathematics concepts by themselves (Preiner, 2008). This idea is consistent with Vygotsky's classical cognitive constructivist theory. Preiner (2008) found that the simple way in which developers of GeoGebra designed the user interface of the software aligns with the characteristics of cognitive constructivism, particularly its visualising and explorative capabilities, its contribution to multimedia environments for learning and the minimisation of cognitive load in learning. Multimedia environments offer new ways of learning and teaching compared to traditional environments (Preiner, 2008).

Akanmu (2015) agrees that technology, well-integrated into mathematics education, enhances students' achievements, "irrespective of gender" (Akanmu, 2015, p. 88). In an analysis of 50 articles published from 1997 to 2014, Cai, Fan and Du (2017) found that male students had more favourable attitudes to technology than females, but these differences were found in small effect sizes.

In summary, studies on the integration of GeoGebra in differential calculus have found positive effects on student performance (Akanmu, 2015; Nobre et al., 2016; Ocal, 2017; Preiner, 2008; Tatar, 2013).

2.8.3. Beliefs about learning and teaching mathematics through technology

Several qualitative studies of teachers' and students' beliefs about mathematics learning with the use of technology, particularly GeoGebra, have been conducted.

Teachers' beliefs about technology-oriented mathematics classrooms

Teachers' perceptions of effective teaching and their cultural beliefs may influence their instructional practices; these beliefs mustn't widen the gap between theory and practice (Purnomo, Suryadi, & Darwis, 2016).

There is no uniform definition of the term *teacher beliefs* in the literature. Ertmer (2006) defines beliefs as suppositions, commitments or ideologies. Variations in teachers' cultural belief systems influence how they view their students, what mathematics should

be learned, and how this should be taught (Tirosh & Graeber, 2003). Galbraith and Haines (1998) view beliefs as a way of imitating a certain set of concepts, while attitudes are an emotional reaction to an object, to beliefs about an object, or behaviour towards the object such as technology. They view emotion as heated or agitated arousal created by some stimulus. In their review of articles, they found that understanding students' attitudes and beliefs about learning is a crucial step in understanding how the mathematics learning environment is affected by the introduction of computers and other technology to the classroom.

Ernest (1989) identified three main components of teachers' mathematical beliefs: the conception of the nature of mathematics as the basis of the philosophy of mathematics; the structures of mathematics teaching; and the process of learning mathematics. The conception of the nature of mathematics is fundamental as it has an impact on the structure of mathematics teaching and the process of learning mathematics (Speer, 2005; Thompson, 1992). Ernest (1989) reasons that the restructuring of teaching cannot take place unless teachers' beliefs about mathematics, its teaching structure and its learning process change. In general, teachers' beliefs are regarded as critical to the restructuring of mathematics education (Cooney & Shealy, 1997; Leder et al., 2002; Thompson, 1992). In particular, teachers' beliefs towards technology in the classroom, their beliefs about the potential of their students and teaching mathematics have a decisive impact on the success or failure of the implementation of technology (Windschitl & Sahl, 2002). Teachers who believe in the potential of instructional technology are catalysts for the transformation of teaching mathematics with technology.

Teachers believe that the integration of GeoGebra in their classrooms is time-saving when preparing worksheets, tests, lecture notes and board work. Prepared work can be stored on a web page or the GeoGebra software; teachers can simply change the variables of the object to create a new set of instructional materials. Interactive lectures can also be created using GeoGebra, and can be uploaded on the internet (Hohenwarter et al., 2008).

Zakaria and Lee (2012) found that teachers were positive perceptions about the use of GeoGebra as far as its features, tools and commands were concerned. In a quantitative survey, these researchers concluded that technology can be used as an alternative method in mathematics instruction. Tatar (2013) used a mixed-methods approach to investigate the effect of technology, in particular GeoGebra, on teacher perceptions and arrived at the same positive conclusion.

Although educational technology is undeniably beneficial and positively perceived by teachers, Pierce and Ball (2009) found that a lack of time, skills and confidence may hinder its implementation in the classroom. They suggest ways for smooth implementation to overcome such barriers.

2.8.4. Students' beliefs about technology-oriented mathematics learning

Leder et al. (2002) explain that students' beliefs about mathematics are "implicitly or explicitly held subjective conceptions" that they believe to be true and "that influence their mathematical learning and problem solving" (p.16). Thompson (1992) states that although the term belief is not clearly defined, it is assumed that the reader knows what is meant in context. In this study, I use the term concerning students' and teachers' perceptions about the use of technology when learning calculus.

Students' attitudes towards mathematics can be affected by technology. Akanmu (2015) found that Nigerian students' attitudes towards mathematics could be linked in a significant way to their knowledge of GeoGebra. Factors that could influence students' attitudes towards the use of GeoGebra include their attitude towards learning mathematics and their knowledge of the technology they will be using to master mathematical concepts (Anthony & Walshaw, 2007; Kele & Sharma, 2014). Anthony and Walshaw (2007) regard students' attitudes towards technology as a central concern when evaluating the impact of technologies on mathematics learning.

Kele and Sharma (2014) found both negative and positive mathematical beliefs among the students in their study and concluded that teachers needed to develop or use new instructional approaches in mathematics instruction to encourage a positive disposition

towards mathematics in all students. Mwei, Wando and Too (2012) noted that the majority of students developed constructivist learning strategies when exposed to computer-assisted instruction (CAI).

Han and Carpenter (2014) define beliefs about mathematics as the cognitive component of attitude, while feelings (emotions) about mathematics comprise the affective component of attitude (Akinsola & Olowojaiye, 2008). Behavioural responses are the observable elements of attitude that students display when dealing with mathematics (Ingram, 2015). Cognitive and affective components of attitude interact with each other and are both important in learning mathematics (Di Martino & Zan, 2007). Student responses to mathematics instruction, i.e. their mathematical behaviour, are the overt expression of the cognitive and affective elements of attitude (Akinsola & Olowojaiye, 2008).

Unsuccessful behavioural attitudes such as negative feelings manifest when students are not confident about mathematics (Di Martino & Zan, 2007). As soon as students observe the importance and value of mathematics in real life, however, they start to engage in learning, gaining confidence and becoming connected (Attard, 2012). Students' beliefs about mathematics influence their achievements, and the cognitive, emotive and behavioural aspects of attitude are intertwined. This holds also for students' cognition, affect and behaviour as far as mathematical software is concerned.

Guiding and scaffolding the effective use of the latest technology in mathematics learning helps students to solve mathematical problems with greater ease (Oldknow, Taylor, & Tetlow, 2010). This holds for the complex mathematical topic of calculus (Ayub et al., 2010), as reflected in the improved performance of students who learned calculus through the aid of technology. Two programs, *Mastering Calculus Computer Courseware* (MACCC) and *SAGE* software were investigated and their effect on the learning of calculus; however no significant difference in student performance was detected (Ayub, Sembok, & Luan, 2008).

Hew and Brush (2007) list some barriers that affect the teaching and learning of mathematics through technology, including a lack of resources, negative attitudes and

beliefs, institutional restrictions, the complexity of the subject and variations in culture, knowledge and skills. These are elaborated on in section 2.7.5 below.

Complex mathematical tasks such as visualising a 3D graph may be difficult for the teacher to demonstrate manually; this is simplified by using technology. Bos (2007) found a better understanding of such concepts among students who used technology than among those not using technology. It may be that such improvements result in altered beliefs about mathematics.

2.8.5. The challenge of improved performance from teaching with GeoGebra software

In recent years, the world has seen rapid growth in technology, including the introduction and design of educational software to support student learning. Up to now, the conventional instructional strategies employed in mathematics teaching in Ethiopia have not improved students' achievement and motivation; in fact, mathematical performance remains poor. The current national assessment confirms that the achievement of students in mathematics is below the expected standard. There is consensus that new pedagogical approaches are necessary to improve science and mathematics instruction in Ethiopia. This was the driving force behind the new Ethiopian educational road map, introduced in 2019 (Teferra et al., 2018). Although the rapid emergence of technological innovations holds great potential, Ethiopian schools are not ready to integrate technology into education. At the tertiary level, the new Ethiopian educational road map resulted in the integration of mathematical software such as MATLAB and Mathematica in the mathematics curriculum. These two packages are not freely available, however; the cost is thus restrictive. This forced me to explore the use of a free alternative at all educational levels, namely GeoGebra software. GeoGebra's mathematical applications are wide-ranging and enable the visualisation and representation of some of the most complex mathematical concepts (Thambi & Eu, 2013).

Glaserfeld (1995) proposes the term radical constructivism as a way of thinking about knowledge and the act of knowing, and formulates the following basic principles:

- Knowledge is not passively received either through the senses or by way of communication.
- Knowledge is actively built up by the cognizing subject.
- The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability.
- Cognition serves the subject's organization of the experiential world, not the discovery of objective ontological reality.

In particular, cognitive constructivists emphasise accurate mental constructions of reality, whereas radical constructivists emphasise constructing a coherent experiential reality. Social constructivists emphasise the construction of an agreed-upon, socially constructed reality in the classroom situation in general. Brau(2020) sees the term constructivism in two camps [radical and social]. According to Brau (2020), “the first form, radical (or cognitive) constructivism, proposes that the process of constructing knowledge is dependent on the individual's subjective interpretation of their active experience. The second form, social constructivism, affirms that human development is socially situated, and that knowledge is constructed through interaction with others”. Vygotsky believed that learners could achieve a much greater level of learning through the help of a More Knowledgeable Other (teacher) (please see Figure 3.1) Brau (2020).

Within the ZPD space, the students are tackling acquiring the knowledge (thinking about knowledge (Glaserfeld's ideas) either with the help of teachers or by themselves. While students are thinking about knowing the need to interact with the tools or subjects, which is the idea coined under Vygotsky's socio-cultural educational theory. To this end, Piaget forwards the stages of cognitive development, starting from born to mortality and is known as Piaget's cognitive theory of constructivism (Piaget, 1959).

Thus, the choice of GeoGebra coincided with my instructional goal of a teacher-student relationship according to the socio-cultural educational theory of Vygotsky (1978) and its later development in modern research termed known as “*post-Vygotskian studies*”

(Daniels, 2001, p. 69). The emphasis in this theory is on the active disposition of the student in both student-teacher and student-student interaction - an aspect that GeoGebra enables. The achievement of students in mathematics can be increased by integrating technology usage in the classroom the gap between potential and actual development of students can be narrowed by the effective scaffolding of knowledge by the teacher.

Young, Reynolds and Walberg (1996) analysed students' achievements about individual and institutional factors. At the individual level, factors that might affect students' achievements include their background, their attitudes towards science, the time they are exposed to instruction, their home environment and parental involvement. Similarly, Singh et al. (2002) argue that students' achievement in mathematics and science is affected by attitude, motivation and academic time, the greatest influence being the time spent on homework. Homework has the advantage that students grapple with ideas on their own but receive feedback from their teacher in the classroom – an effective means of assisting performance and facilitating learning in the Zone of Proximal Development (ZPD)(Tharp, 1993). The zone of proximal development is referred to as the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with MKO. This is discussed further in Sections 3.3 and 3.5. Where there is a lack of feedback from the teacher, student achievement is severely challenged. In my view, academic achievement refers to students' ability and skills and the marks they obtain in a subject. In this study, achievement refers to the effect of GeoGebra in calculus instruction, as reflected in students' pre- and post-intervention test scores.

A prerequisite for teaching and learning mathematics with the help of technology is a working knowledge of the software. Technology can simplify understanding; however, Galbraith and Haines (1998) argue that unfamiliar technology can raise difficulties, even if the tools are powerful. The availability of resources and computers, the awareness of stakeholders, teachers' pedagogical knowledge on integrating GeoGebra in their teaching, student-teacher ratios and the technological fluency of users are some of the requirements for delivering GeoGebra integrated mathematics education effectively in the

classroom (Wassie & Zergaw, 2019). Students' prior experience and their computer skills are further determinants of the effectiveness of computer-assisted instruction (Mwei et al. 2012).

In a developing country such as Ethiopia, the availability of software at an institution may pose a challenge. Tay et al.(2012) identify institutional factors that might hinder the integration of technology in the classroom. These include the institutional context, the departmental ethos, the availability and accessibility of technology facilities, technological expertise in the department, teachers' technological skills and the opportunities for professional teacher development. Eng et al. (2011) found that rural students were less confident when using technology, while urban students' attitude towards mathematics learning through technology was significantly more positive.

Although institutions may face challenges when integrating technology in teaching and learning mathematics, and the process may be slow (Lavicza, 2010), Preiner (2008) reminds us that in this day and age many teachers and students have access to computers and software. The real difficulties are the integration of the software in the teaching and learning and teachers' ability (Ertmer, 2005; Gorder, 2008).

To sum up, once the challenges of employing technology in the mathematics classroom have been overcome, this mode of instruction has many advantages (Hew & Brush, 2007). The Covid-19 world has been forced to use technology, not only for teaching and learning but also in other sectors. Several scholars (Alkhateeb & Al-Duwairi, 2019; Arbain & Shukor, 2015; Doğan & İçel, 2011; Jelatu, 2018; Rohaeti & Bernard, 2018; Saha, Ayub, & Tarmizic, 2010; Thambi & Eu, 2013) believe that GeoGebra mathematical software encourages student achievements more than conventional teaching and traditional methods.

CHAPTER 3 THEORETICAL FRAMEWORK OF THE STUDY

3.1. INTRODUCTION

The theoretical framework of this study is grounded in Vygotsky's theory of learning and extends to the Zone of Proximal Development. The study outlines how communication between students and technology, between students and their peers (among students), and students and teachers (more knowledgeable adults) affect the learning of Mathematics, particularly calculus, with GeoGebra software. Using this theoretical framework, the researcher developed a model to observe how using GeoGebra/Technology in differential calculus relates to Vygotsky's ideas of the Zone of Proximal Development (ZPD) and scaffolding patterns (Wood, Bruner, & Ross, 1976).

3.2. OVERVIEW OF VYGOTSKY'S THEORY OF LEARNING

Social interaction plays a significant role in student learning both at school and in the wider environment. In this learning landscape, learning occurs both at school from the teacher or instructor and other people in the environment (world), that is from the human to the world of the object/technology (Lantolf & Appel, 1994). Learning is a social activity in which the engagement of students in learning takes place through their use of active cognitive and metacognitive knowledge and strategies (Leder, Pehkonen, & Torner, 2002). Metacognition encompasses student's self-regulation, self-determination, self-planning, and self-checking once they have received guidance (Daniels, 2001). In this space, learning can occur in social and cultural scenarios. The culture of a society provides students with the knowledge, as discussed in the literature on mathematics culture, and this affects student achievement. To this end, the sociocultural environment or milieu is the central idea in Vygotsky's theory of learning. His theory of cognitive development is based on a child's ability to learn things socially with the tools at hand (hands, hammers, computers) and to learn the culturally-based signs (language, writing, Mathematics). Morcom (2014) discusses learning, emotion, and motivation, all of which are central and interconnected processes in Vygotsky's sociocultural theory. According to this theory, students' and teachers' perceptions of the learning and teaching culture of

Mathematics can be considered overtly or covertly. The famous scholar Wertsch (1985, p. 123) views Vygotsky's theory as having three core themes. These are a) reliance on genetics, that is, developmental methods; b) the claim that higher mental functions of the individual have their origin in social processes; and c) the claim that mental processes can be understood only as mediated by signs and tools.

In my study, Wertsch's (1985) third theme in particular led me to focus on Vygotsky's theory. Minick's (1987, as cited in Daniel, 2001, p.31) study of Wertsch's ideas on Vygotsky's theory revealed that Vygotsky's thinking moved from a focus on instrumental acts in 1925 to the analytic unit of psychological systems in 1930, and the modification of the descriptive principle from 1933 to 1937, with an emphasis on interactions and actions in individual participation. As cited in Daniels (2001), Vygotsky argues that humans master themselves when actions come from the external symbolic, cultural system, rather than by being conquered by and in them. From my understanding of the literature and Vygotsky's beliefs, learning takes place when an individual receives assistance from educated persons. From an educational point of view, Vygotsky argues that psychology reveals that the human mind is developed through the interactions of subjects with the world as well as the quality of the relationship between students, subject matter and tools (technology) in the classroom (Vygotsky, 1978). In this light, I used students as the subject and GeoGebra Mathematical software as the object to observe the interaction between these phenomena on students learning differential calculus. Modern technology or digital technology enhances human abilities to learn, especially in the subject of calculus, and emphasises the interaction of technology and humans that enables them to increase their capacity to process expressions numerically, manipulate symbolically, create new theories and represent ideas visually (Tall, 2013b). In general, according to Vygotsky, the method reveals the human mind's potential for future development to address the challenges of this century.

Vygotskian theory developed from the Piagetian theory (Piaget, 1959). Since I followed Vygotsky's theory in this study, I do not discuss Piaget's theory in detail, however. In Vygotsky's view, teachers' interaction with students is the most important factor of the learning process (Vygotsky 1984), whereas in Piaget's view the teacher's interaction with

students is a secondary factor. The reason I follow Vygotsky rather than Piaget is that I believe that the teacher must interact with students in the first phase as the students are new to the tools GeoGebra, as social interaction is key to cognitive development, as emphasised by Vygotsky. Lantolf and Appel (1994) regard development as a mediated mental activity. However, in Vygotsky's theory, students and teachers are all active mediators in the process of students' development, that is in teaching and learning.

In the teaching and learning process, teachers' intervention in students' learning is essential. Social constructivism emphasises the quality of importance of teacher-student communication in learning (Gallimore & Tharp, 1988). Social constructivism posits those ideas are constructed by the interaction between teachers and other students; in contrast, cognitive constructivism holds that ideas are created individually. In social constructivist theory, the learning process is cooperative, and knowledge is created not only through the interaction of teacher and student in the environment (psychologically), but also by students themselves (intrapsychologically)(Churcher, Downs, & Tewksbury, 2014).

Lastly, effective learning occurs in a particular place, according to Vygotsky's theory. A place at which 'good learning' occurs is the Zone of Proximal Development (Vygotsky,1978). In response to this notion, another scholar Doolittle (1995) found that not only learning but also cognitive development of students occurs in the zone. Vygotsky (1978) argues that students' intellectual and problem-solving abilities fall into three categories: 1) those that are performed independently (lower level), 2) performed with assistance (higher level), and 3) cannot be performed even with assistance. Those that cannot be performed even with assistance are those found to be beyond the ZPD (outside the concentric circles). This study considers students' intellectual and problem-solving abilities in activities that can be performed with assistance. Assistance, in this case, refers to providing hints and directions, rephrasing questions, modelling, asking the student to restate what has been discussed, or asking what he/she understands or has learned, or demonstrating the task or a portion of it (Bodrova & Leong, 2007; Jones, Rua, & Carter, 1998). All these activities can occur in the classroom in effective teaching and learning processes. In this study, the researcher guided students who were learning calculus with the tools in GeoGebra software.

In general, studies have shown that teachers and students are mediators in the mathematics classroom; adding technology as a tool can lead to the development of a better understanding of mathematical concepts in interactions between individuals (peers), and between students and teachers (MKO). Daniels (2001) found that every meaning in a child's cultural development occurs at the social level (interpersonal) and the individual level (intrapersonal); this supports the arguments for the importance of the zone of proximal development.

3.3. ZONE OF PROXIMAL DEVELOPMENT AND LEARNING MATHEMATICS BY USING TECHNOLOGY

Vygotsky (1896-1934), a Russian psychologist, first introduced the term zone of proximal development (ZPD) in the 1930s. He defined this as the difference between what a student can do without assistance and what he or she can do with assistance (Vygotsky, 1978), and depends greatly on the "more knowledgeable other" (MKO). The MKO is defined as an essential component of the learning process and is a teacher or lecturer who has more knowledge than his or her students (Vygotsky, 1978). The zone of proximal (potential) development (ZPD) is the gap between what a student can do independently and what he or she could potentially do with support (guidance) and assistance (Daniels, 2001). Vygotsky (1984) divided ZPD into two categories of intellect: actual intellect, the distance between the actual developmental level as indicated by independent problem solving ability, and potential intellect, indicated by problem-solving ability with adult guidance or in collaboration with more capable peers. In the present study, the more capable peers were the teachers. Students in the ZPD zone can be successful with instructional guidance (Blake & Pope, 2008).

Literature reveals that the ZPD represents a maturation process. If students are nurtured properly, they will grow. Doolittle (1995) and Warford (2011) argue that social interaction in the learning process is at the heart of ZPD.

Bodrova and Leong (2007) argue that the ZPD is not a static region but rather an active region of learning in which students develop experiences through participation. In this

sensitive region, students learn cultural skills. This study proposed that one of these cultural skills could be mathematical skills. The ZPD is important for the learning and teaching of mathematics because it determines the scope of work to be covered.

Gallimore and Tharp (1990) claim that the ZPD has four stages. In their study they identified these as stage 1: in this stage modifying for transfer, assistance and task performance is applied; stage 2: this is the stage where performance is monitored by self or assisted by self although the learner has not yet automatised the activity; stage 3: this is where the performance is automatised, fossilised and developed; and stage 4: this is the point at which the de-automatisation of performance leads to recursion through the ZPD.

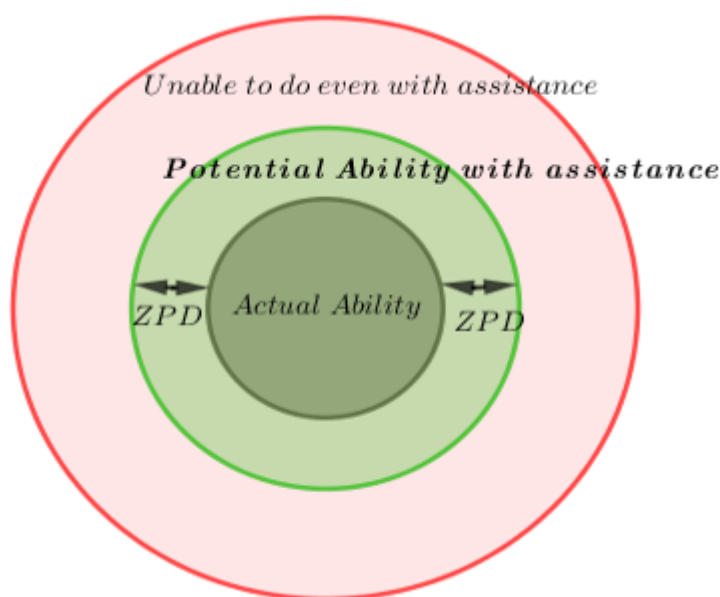


Figure 3.1 The gaps in the Zone of Proximal Development

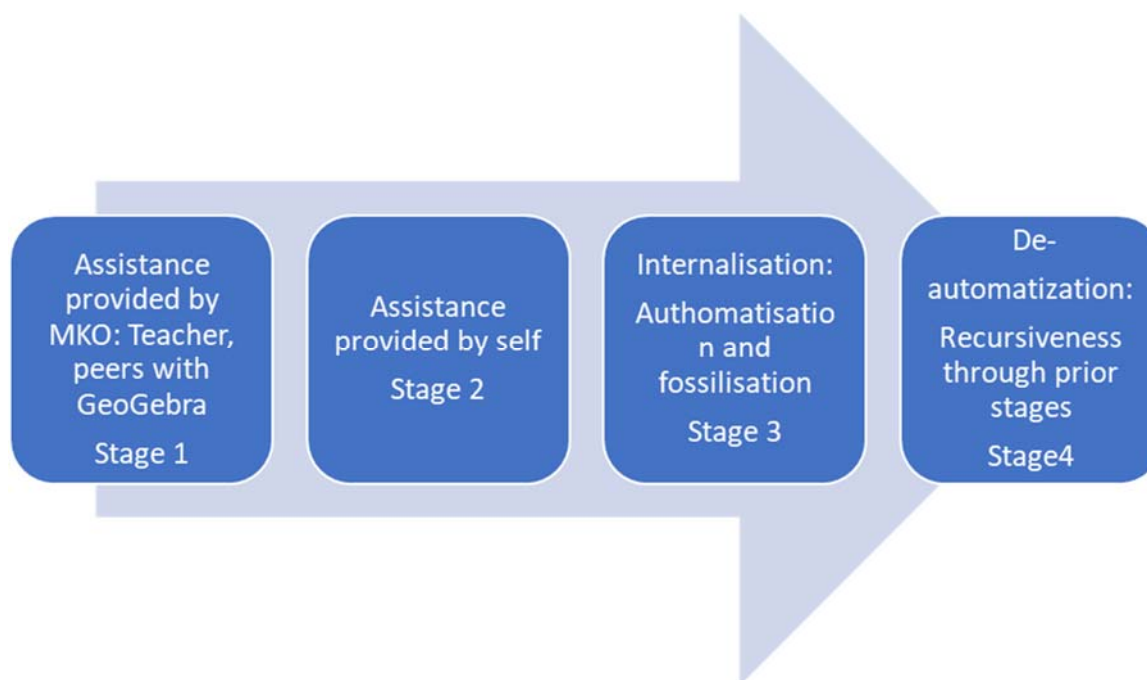


Figure 3.2 Progression through the Zone of Proximal development

Figure 3.1 demonstrates the gaps in concentric circles; these gaps can be narrowed by performing certain activities, depending on the stages indicated in Figure 3.2. These activities should be scaffolded, as discussed in the following section.

3.4. SCAFFOLDING IN TEACHING MATHEMATICS BY GEOGEBRA

The literature review revealed some agreement on the notion of a socio-cultural theory of the mind. In this respect, ZPD is based on Vygotsky's theory, at the heart of which is the notion of scaffolding or guidance. The term scaffolding was first introduced in the context of teaching and learning by Wood et al. (1976, p. 90), who define scaffolding as a form of adult assistance that helps learners achieve a goal that they would not be able to do on their own.

Doing difficult tasks, setting appropriate goals, and guiding students in the classroom are tasks of the teacher in the ZPD. Scaffolding may take several forms, including "increasing engagement, providing alternate learning strategies, resolving learning bottlenecks, and

(paradoxically) taking away support to allow students to master the material” (Lee, 2014, p. 1). Technology can scaffold student learning before, during and after class to provide appropriate assistance to students.

Pea (2004) believes that there are two primary axes to support the processes of learning in the classroom. The first axis depends on students’ needs and the resources that enable them to do more than they would do alone. This axis is social and involves interactive responses. Ruthven (2009) also used resources as classroom practice. The second axis comprises technology and the design of artefacts (Simon, 1996) and focuses on problem-solving.

Theory building and design in education can encompass scaffolding (Quintana et al., 2004); technology supports learning and teaching and has become increasingly important in pedagogical design. For example, to demonstrate GeoGebra in teaching calculus in the classroom, I prepared a lesson plan that was compatible with this software. This led to a dramatic shift from a lecturer centred lesson plan to a technology-oriented lesson plan. This change in pedagogical design may be necessary if this software is to be applied to all chapters of a calculus textbook. Several scholars Quintana (2004) have found that software tools support students by simplifying their learning, and this, in turn, encourages their engagement in learning.

Technology can support multiple methods of studying the same material and can provide visual scaffolds that help students to understand complex concepts. Providing direction to their study and showing students how to do activities can be regarded as scaffolding. Such scaffolding can gradually be withdrawn over some time. Scaffolding should be seen as temporary assisted learning in certain activities that leads to independence; the result will be that students may become self-governing and problem-solvers in their own activities (Lajoie, 2005). If the task is accomplished, then the scaffolding is slowly withdrawn. This dynamic system is recognised by both teacher and student. One of the best technologies to use in the teaching and learning of calculus is the GeoGebra Mathematical software as the software is dynamic. This program provides scaffolding by guiding and assisting students in their learning activities in the classroom. The dynamic

system comprises three ideas that are important in defining the system. These are contingency, fading and responsibility. In my study I used the notion of contingency, which involves modifying and customising the teaching lesson plan according to students' abilities, taking into account the students' calculus syllabus which was designed according to the Ethiopian higher education programme.

Pea (2004) defines the notion of scaffolding by listing questions to ask, such as *What*, *Why* and *How*, when determining which individuals require scaffolding. In the present study' *What* and *Why* questions were used to identify students who needed assistance and *How* questions were used to determine the type of scaffolding, such as guiding, focusing and modelling of activities.

Instructional scaffolding is a mechanism for observing the process by which a student is helped to achieve his or her potential learning in education by a potential teacher (Stone, 1998). Vygotsky believes that with appropriate assistance in the ZPD, students will be able to move from the present zone of proximal development to the actual developmental level in the future (Vygotsky, 1978, p.87). In this case, assisting students by using a given technology within a given period and then stopping the guidance coincides with Vygotsky's ideas; students should master calculus by the use of GeoGebra, with some guidance.

Social interactions play an important role in learning and teaching, for both students and teachers. The social and participatory landscape of teaching and learning in education can be explained by scaffolding and this term is used as a metaphor for educators and researchers in the ZPD (Daniels, 2001). Through social interaction, students learn from each other, as well as from adults, in this case teachers. This is illustrated in the ZPD in Figure 3.2. Students learn first through interactions with their peers [Stage 1 of Figure 3.2] and then on their own by internalisation, finally reaching deep understanding [stage 3 of the same figure]" (Fogarty, 1999, p. 77).

In this view, learning mathematics/calculus through the use of GeoGebra software fits the theory that I followed: I used the technology to teach (assist) the students to investigate the effects this had on students' achievement and understanding. In this study, Vygotsky's

perspective was the most appropriate theory to use to interpret the data. Instrumental mediation allows the researcher to analyse the advantages of technologies (created by human beings) in education (Elizondo-Rami & Hernandez-Solis, 2016). One of the characteristics of humans being is the building of tools, such as GeoGebra software. In principle, this amplifies an intentional activity, whether physical or cognitive. Wertsch (1985) defines Vygotsky's tools in two ways, techniques (artefacts) and psychological aspects (symbols). The tools are mediators or ladders of human activity in an environment for building the concepts of intended activities (see Figure 3.3). Kozulin (2003) argues that both human mediation and symbolic mediation, in which the first enhances the learner's performance (in my case, learning mathematics with GeoGebra), and the second describes changes that occur in a learner's performance (in my case the result of the post-test after students had studied calculus with the aid of GeoGebra). In Vygotsky's conceptualisation, the term 'mediator' is defined as the ladder between an environmental stimulus and an individual response to this stimulus. In this study, this is referred to as the environment and the individual area of the Hypothesised cycle model of teaching mathematics using GeoGebra, as illustrated in Figure 3.3 below. This figure shows the relationship between an environmental stimulus and individual response to the stimulus as well as the ladder between environment stimulus (GeoGebra) and students learning differential calculus.

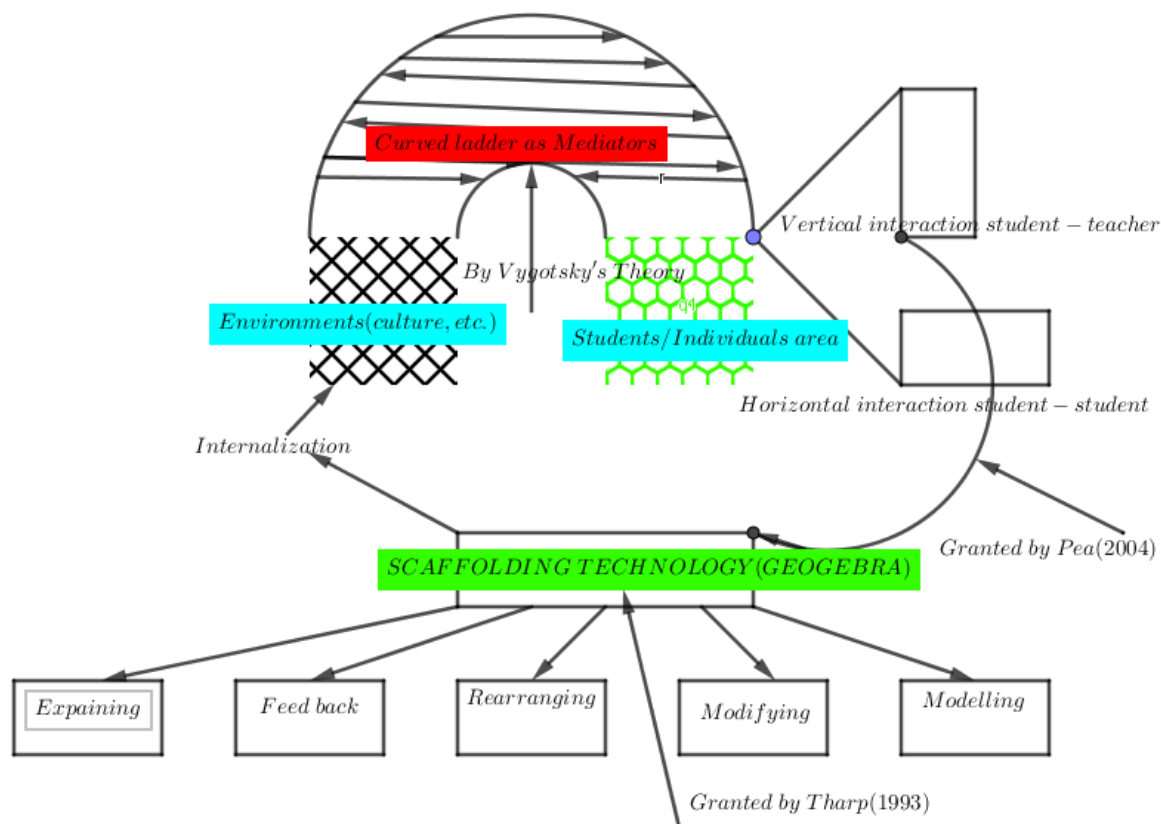


Figure 3.3 The interaction of environment, teachers, and students with technology (IEST). Hypothesised cycle model of teaching mathematics by GeoGebra.

The model was developed from Figures 3.1 and 3.2 and the theoretical framework of the study.

Figure 3.3 indicates five interaction treatments. These are **students-environment**, **teacher-content and tools**, **student-student**, **student-teacher**, **tools, and content**, and **educated individual-environment** (internalised activities).

Within these environments is a variety of objects that may be human (students) or human-created tools (the content of the lesson and technology). Indeed, culture is naturally part of an environment. Students are actors in the learning process and interact with the environment. In Vygotskian theory, this interaction is known as socio-cultural interaction. When human-created objects such as GeoGebra are introduced to the interaction process, as it is indicated in Figure 3.3, interaction may be between student-teacher or

between student-student. In this model, student-teacher interaction is guided (scaffolded) by the technology of GeoGebra. Those students who are guided by more knowledgeable ones (MKO), most of whom will be teachers, will internalise the concepts (self-reflection) in the environment. When the self-reflection that occurs during internalisation is reflected in the environment, for example in the reproduction of culture, externalisation will immediately take place in, for example, the creation of new artefacts made possible by its transformation like that of human growth and understanding of required activities (Vygotsky, 1978b; Vygotsky, Leont'ev, & Luria, 1999). If a well-designed model and activities are implemented in classroom teaching and learning then externalisation will be optimum (Vygotsky et al., 1999). I guided students in such environments to become new teachers, and if these newly qualified teachers apply these activities themselves, the process becomes cyclical. In this study, this cyclical process is referred to as the hypothesised cycle model, as illustrated in Figure 3.3. A learning cycle is not a teaching method; rather, it is a process of teaching and learning (Marek, Gerber, & Cavallo, 1999).

In summary, studies have shown that in Zone of Proximal Development learning can be scaffolded equally by using interactive teaching methods or by using technology. In Vygotsky's theory of learning, students learn a given course (such as calculus) first by interacting with more a knowledgeable person, in this case, the teacher. Gradually, the student internalises the knowledge and engages in activities independently by using tools in the given environment. As students' abilities grow, scaffolding is progressively decreased, an important aspect in ZPD. In other words, where students interact with each other, cooperative learning takes place, and the assistance of the individual occurs during these activities.

For a long time, researchers have tried to understand the steps one takes when solving a problem to comprehend how the mind works and how best to educate the next generation (Singer & Moscovici, 2008). Taking this idea as a starting point, this study posits nine steps in the implementation of the Hypothesised Cyclical model in classroom learning and teaching:

- 1) Identification of area (environment) (laboratory class).
- 2) Identification of individual areas (teacher professional development and student ability, perception). As the researcher was a teacher the MKO occurs, thus teaching and learning can take place. The pre-test was used and to identify students' abilities.
- 3) State objectives of teaching a lesson with GeoGebra (review literature).
- 4) Design teaching materials (lesson plan that is compatible with GeoGebra).
- 5) Implementation of a lesson plan in the classroom (start scaffolding student-student, teacher-student interaction).
- 6) Get feedback from students (responses). This could be in the form of a post-test and interview.
- 7) Evaluation of whether the method had achieved what was intended. Comparison of abilities before and after.
- 8) Internalisation and externalisation. See stage 3 in Figure 3.2.
- 9) Apply in the environment as in Step 1. See Stage 4 in Figure 3.2.

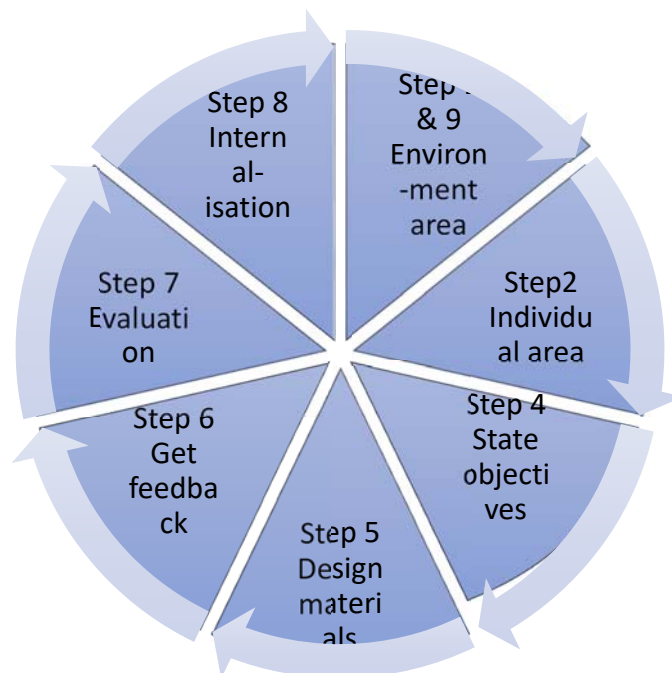


Figure 3.4 Steps in implementation of Hypothesised Cycle model

3.4.1. Framework of the study

In this Section, I included the framework of the study as shown in the Figure3.4 that used to give direction for my study (Akanbi, Amiri, & Fazeldehkordi, 2015). The study employed the GeoGebra mathematical software to investigate the effect of it on students learning differential calculus either by self-exploration or social interaction (vertical and horizontal interaction) that they got because of scaffolding in the zone of proximal development by using Vygotsky's theory. The student development in the zone was investigated both in terms of two types of knowledge known as conceptual and procedural understanding. Conceptual understanding of students is increased with collaborative learning which is the central idea of Vygotsky's theory in education(Hwang, Wu, & Kuo, 2013). The world of conceptual understanding is one of the three mental worlds of mathematics that build on human perceptions and actions by developing mental images (Tall, 2013a). Procedural knowledge is defined as "mental actions or manipulations, including rules, strategies, and algorithms, for completing a task". Conceptual knowledge is defined as "knowledge about facts, [generalizations], and principles"(Baroody, Feil, & Johnson, 2007, p. 123). The difference between conceptual and practical or procedural knowledge is expressed as by Ivic (1991, as cited in Haapasalo & Kadijevich, 2000 p.139):

Piaget made a distinction between 'practical knowledge' (savoir-faire) and 'conceptual knowledge', whereas Vygotsky dealt with three levels of knowledge: 'manifest content' (facts, data, and the like), 'instrumental knowledge' (methods, skills, procedures, etc.), and 'structural knowledge' (knowledge structures with underlying modes of thinking).

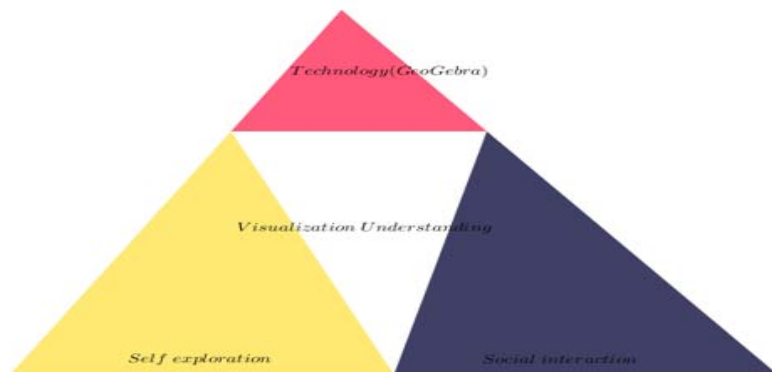


Figure 3.5 Figure drawn using GeoGebra Mathematical software and snipping from GeoGebra window that represents TS^2UV .

The TS^2UV conceptual Framework is explained as follows:

T = Technology. The technology in this study is the dynamic GeoGebra Mathematical software, which is important in scaffolding the gaps in ZPD.

S = Social interaction. At this level, students interact and learn from peers and MKO, specifically teachers. This is the central idea of Vygotsky's theory of learning.

S = Self-exploration. This is the central idea of constructivism. Students learn a given topic actively and project their actual learning level.

U = Understanding. This is the understanding of any mathematical concept included in the study (conceptually and/or procedurally).

V = Visualisation. At this level, the process of learning calculus is visualised by the GeoGebra Mathematical software.

3.5. TEACHING MATHEMATICS IN THE ZONE OF PROXIMAL DEVELOPMENT AND COOPERATIVE LEARNING IN CLASSROOM BY GEOGEBRA

3.5.1. *Teaching mathematics in ZPD*

As discussed in the section above (please see Section 3.3) good learning takes place in the ZPD. Tharp (1993) defines the term teaching as assisting the performance through the ZPD and argues that teaching takes place when assistance is offered at points in the ZPD where performance requires assistance. In this study, the definition of teaching is redefined as the assistance of the performance of students by using GeoGebra in the classroom. Tharp (1993) identifies seven means of assisting performance and facilitating learning in the ZPD, as listed below:

Modelling: Providing behaviour for imitation. Modelling assists the learner by providing information and a remembered image that can serve as a performance standard.

Feedback: The process of providing information on performance. Feedback is essential to improving performance because it allows the performance to be compared to the standard and thus encourages self-correction. Ensuring feedback is the commonest and single most effective form of self-assistance.

Contingency management: Application of the principles of reinforcement and punishment of undesirable behaviour.

Instructing: Requesting specific action. This assists by selecting the correct response and by providing clarity, information and enhancing decision-making. It is most useful when the learner can perform some segments of the task but cannot yet analyse the entire performance or make judgements about what elements to choose.

Questioning: A request for a verbal response that assists by producing a mental operation that the learner cannot or would not produce alone. This interaction assists further by giving the assistor information about the learner's developing understanding.

Cognitive structuring: Explanations. Cognitive structuring assists by providing explanatory and belief structures that organise and justify new learning and perceptions and allow the creation of new or modified schemata.

Task structuring: Chunking, segregating, sequencing or otherwise structuring a task into or from components. It assists learners by modifying the task itself so that the units presented to the students fit into the ZPD when the entire unstructured task is beyond that zone.

In this study, teaching mathematics through ZPD by using GeoGebra Mathematical software was applied to the experimental group. In the GeoGebra oriented classroom all seven identified means of assisting performance and facilitating learning mentioned by Tharp (1993) were implemented in the developed model.

3.5.2. Cooperative learning in ZPD

This study made use of learning activities with guidance from the teacher and discussion between peers and teachers, and as well as between peers themselves. These activities take up a large percentage of the teaching and learning process in the mathematics classroom, and with the aid of GeoGebra Mathematical software, this leads to cooperative learning. In this type of learning, students engage in activities both as a group and as an individual, with the help of the teacher. Cooperative learning is a form of small group teaching and learning in which students work actively in a social setting (Doolittle, 1995). Doolittle (1995) argues that social interaction between teachers and students forms the heart of ZPD; in a social context the ZPD must be regarded as the immersion of students in cooperative activities in a specific social environment.

3.6. TEACHING METHODS IN VYGOTSKY'S THEORY AND HYPOTHESISED CYCLE MODEL

As Vygotsky died before he had fully articulated his ideas there is no clear methodology for the teaching and learning process in the classroom in his theory. Furthermore, this idea itself needs investigation as there is no clarity on this method in his theory. Fani and

Ghaemi (2011) contend that Vygotsky did not discuss any specific methodology for the use of ZPD in teacher education. In their paper, they discuss some factors that hinder the teacher's implementation of ZPD in the classroom. These factors include peers, mentors, contextual constraints, mediators' artefacts, and technology. They regard technology as an important factor when planning activities in the ZPD; one example of technology is GeoGebra, and they point out that technology has proved to be a reliable source of electronic scaffolding and thus a positive change in teacher's professional development (Fani & Ghaemi, 2011). However, in my opinion, there is an implication of teaching methodology in Vygotsky's theory. For example, Palincsar and Brown (1984, as cited Daniels, 2001, p. 110) use the term 'reciprocal teaching method' to cover a combination of modelling, coaching, scaffolding and fading. As discussed in the above Section 3.4, scaffolding is the central idea of Vygotsky's theory and in this dynamic system, the learning and teaching process consists of the four ideas that are important in defining the system. Fading is one of these concepts. Therefore, this theory has indirect references to teaching methods or methodology. For Daniels (2001), the reciprocal teaching approach involves summarising, generating questions, clarifying and predicting the topic in the classroom.

In general, in Vygotsky's theory, the teacher can use a reciprocal teaching method (modelling, coaching, scaffolding, and fading) by integrating technology such as GeoGebra in the classroom to teach mathematics. This is the argument of this study as scaffolding is part of both Vygotsky's theory and the Hypothesised Cycle model developed in this study.

3.6.1. GeoGebra oriented lesson plan teaching in hypothesised cycle model

The main aim of this study is to give special considerations in integrating technological pedagogical and content knowledge (TPACK) in teaching students differential calculus with GeoGebra, a dynamic multi-purpose mathematics software. According to Bekene (Bekene, 2020), GeoGebra oriented lesson is a way of implementing some developed steps or designed teaching-learning(lesson plan) in the classroom. "The designed teaching-learning scenario allows students and teachers to focus on specific mathematics

learning and teaching and to make sense of the mathematics with foreseeable results for the full range of students in the classroom” (Bekene, 2020). On implementation stages of the Hypothesised Cycle model of this study, the teaching material used consists of the topics on differential calculus which can be considered as a GeoGebra oriented lesson plan for the experimental group and traditional oriented lesson plan for control groups (please see Appendix 1.2 and 1.3). It is accepted that planning helps the teachers to organise and systematise the learning and teaching process. Therefore, planning is important for the teaching of students in control manner in the classroom and preparing detailed lesson plans is important, especially for beginner teachers who newly experience explicit instruction, modelling, guided practice, and scaffolding and proficient teachers were found to start their lesson plans with instructional activities included within the developed lesson plan (Allahverdi & Gelzheiser, 2021). The important components of lesson design (lesson plan design tool) sometimes known as task solutions help the communication between the students and teachers around contents (differential calculus), technology/GeoGebra, and pedagogy/developed cycle model during the teaching process. Another scholar entertains the definition of lesson plan within the TPACK frameworks by stating that it is the intersection of the integration of pedagogy knowledge (PK), Technology knowledge (TK), and Content Knowledge (CK) and the teachers may need to develop meta-knowledge of what presuppositions their local theories such as lesson plan (Krauskopf, Zahn, & Hesse, 2015).

To sum up, the developed GeoGebra oriented lesson plan with the use of ideas of TPACK frameworks were implemented by using Hypothesised Cycle model (please see section 5.6).

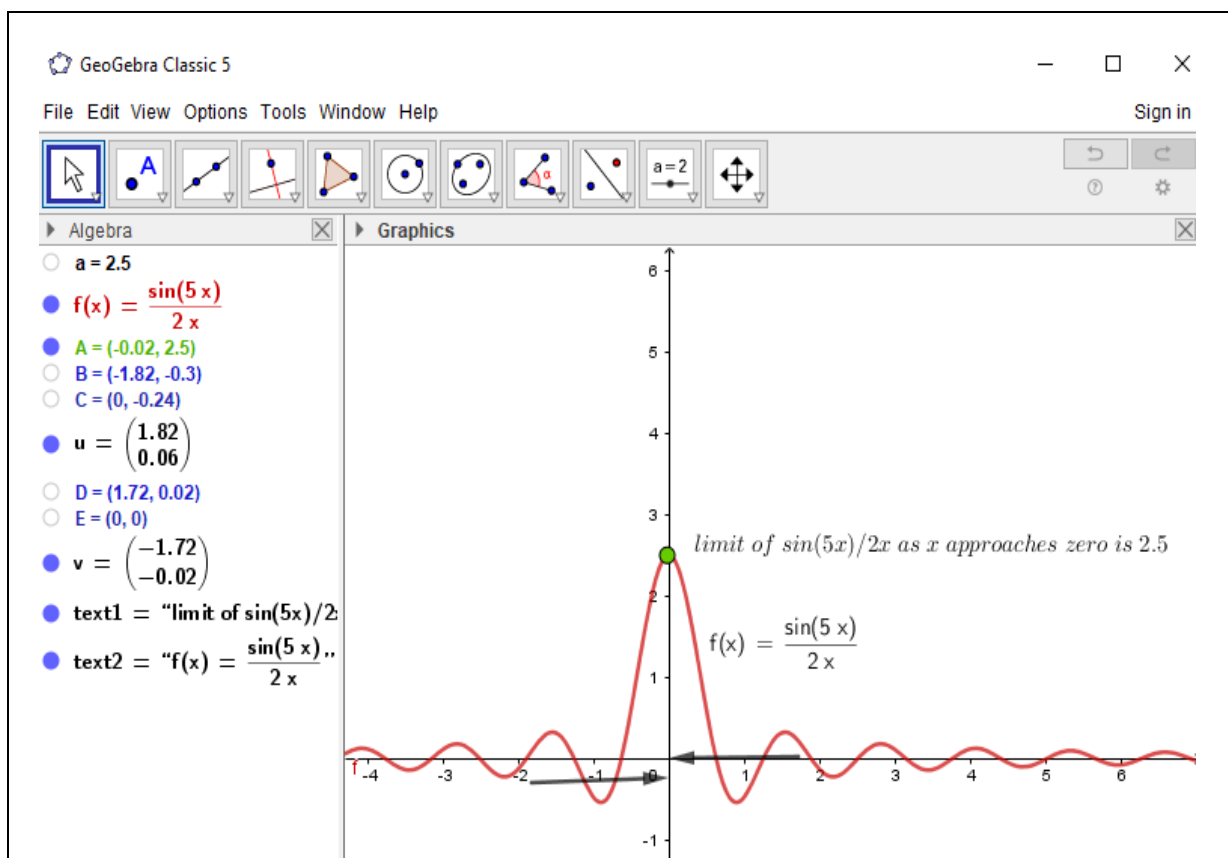
A booklet was developed for teaching differential calculus based on 20 questions, formulated as set out in this section. I took a problem from the question tests to show the contribution of the GeoGebra software for the construction and solution of the problem situation, emphasising the visualization and description of a significant scenario for the learning of mathematics, specifically on differential calculus. This section can be considered as a booklet for the training of teaching calculus. The developed booklet for teaching students by GeoGebra Mathematical software was formulated based on the 20

differential calculus questions used for pre-test and post-test. This booklet was developed by sticking (depending on) the stages of implementation of the cycle model (please see Section 5.7). In each question, the students need to use the activities given within the developed lesson plan drafted from the scaffolding of the cycle model (please see Appendix 1.2). The booklet started from Questions 1 through 20 by showing how to solve and visualize the given question using GeoGebra Mathematical software. This booklet may be best for those who are new to the software [not acquainted with the software].

Question1: What is the value of $\lim_{x \rightarrow 0} \frac{\sin 5x}{2x}$?

- a. $\frac{5}{2}$
- b. $\frac{2}{5}$
- c. 1
- d. $\frac{1}{2}$

For this question, the students need to recall the formula of special limit by using previous knowledge to compute the answer. The teacher may need to inform his/her students by using the GeoGebra applet to find the answer as it was illustrated in the following.

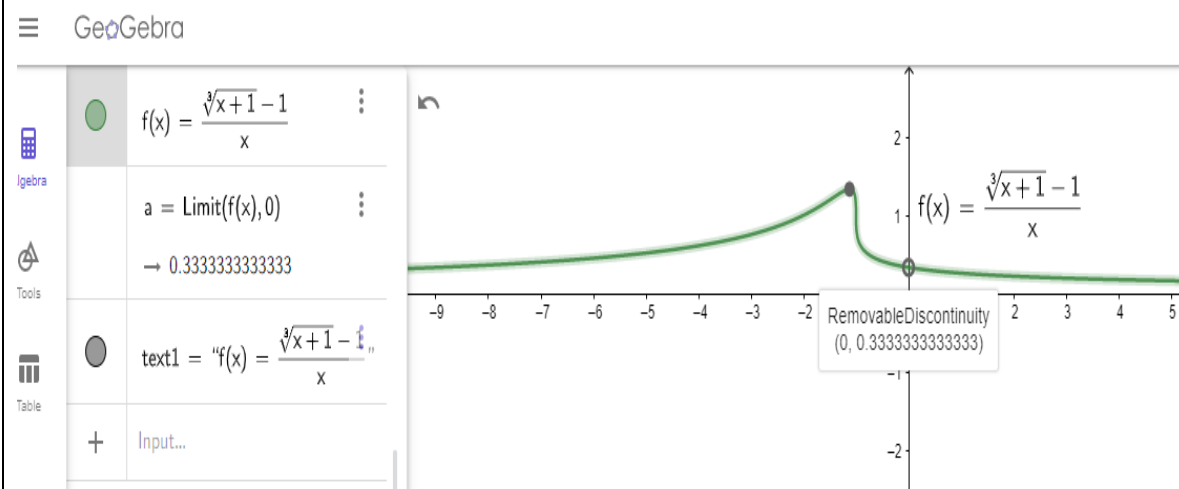


To find out the sketched graph students need to write first the function $f(x) = \frac{\sin(5x)}{2x}$ at the input menu bar of GeoGebra and then hit enter key on the keyboard. After all, students need to write $\text{limit}(f, \text{point}) = \text{limit}(\frac{\sin(5x)}{2x}, 0)$, then hit the enter key. Note that this step holds for all other functions to enter into the GeoGebra menu unless otherwise the function is defined in piecewise form.

Question2: What is the value of $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x+1}-1}{x}$?

- $\frac{1}{6}$
- 3
- 6
- $\frac{1}{3}$
- Does not exist

This question requires some manipulations to arrive at the answer while students are tackling to do it. But, the GeoGebra applet can simply compute the answer and visualize the graph of the function $\frac{\sqrt[3]{x+1}-1}{x}$.



We visualize from the graph that the limit of the function $\frac{\sqrt[3]{x+1}-1}{x}$ at the point zero is $1/3=0.33333$ as the software computes with in the short time. When we click on the graph at which it touches the y-axis, the software informs us the point is removable discontinuity.

Question3: Find the values of a & b such that the diving board function is continuous

$$f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x \leq 3 \\ -2, & x \geq 3 \end{cases}$$

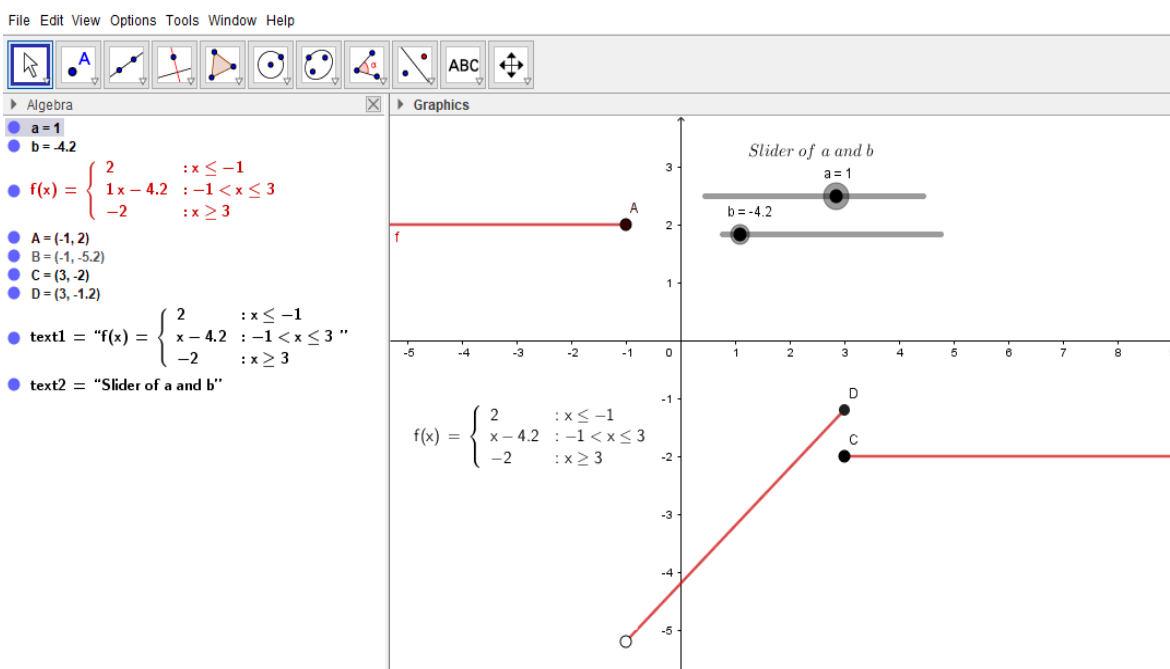
a. $b = 1, a = -1$

b. $a = 1, b = -1$

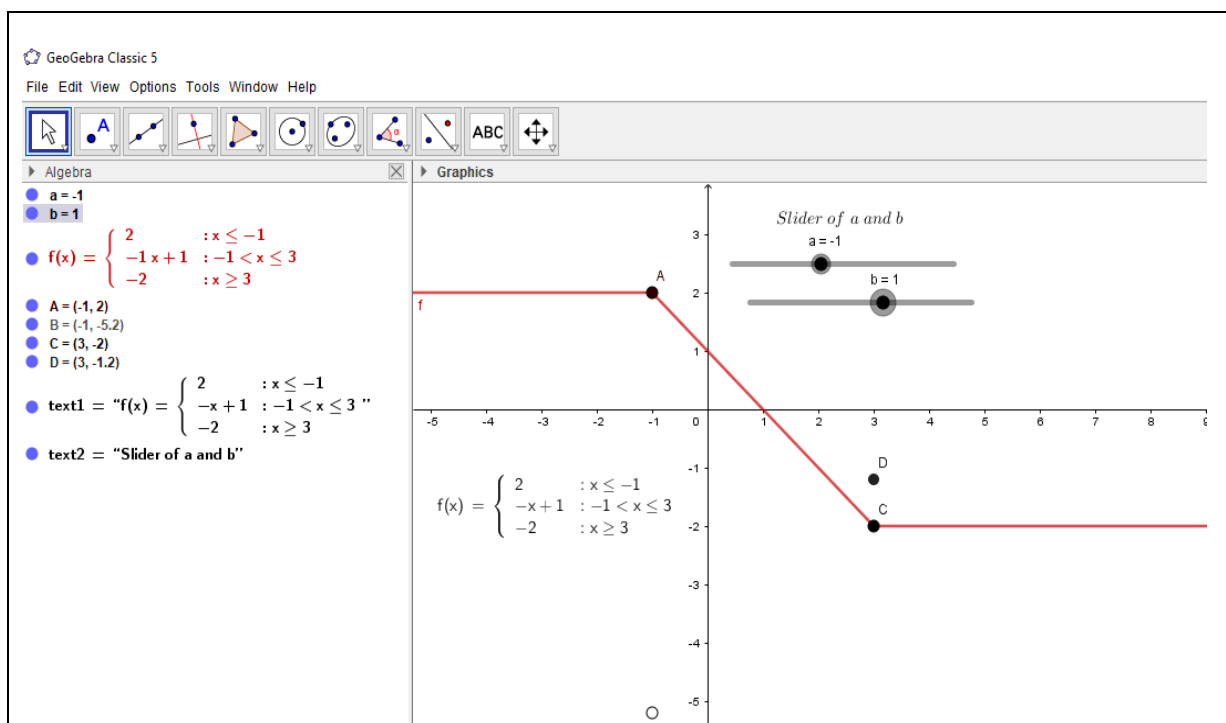
c. $a = 2, b = -2$

This question is the same as question 2 above which needs manipulations to get the answer. But GeoGebra can tell us how to find the constants. To write such like diving board functions students need to write if(conditions, then) in GeoGebra input menu bar

in which the 'condition' stands for each condition of corresponding given function and 'then' stands for the corresponding function. And as the dividing board contains constants we need to have a slider say a and b from Geogebra to indicate the constants are considered as variables. Hence, students need to write as if($x \leq -1, 2, 1 < x \leq 3, ax + b, x \geq 3, -2$).

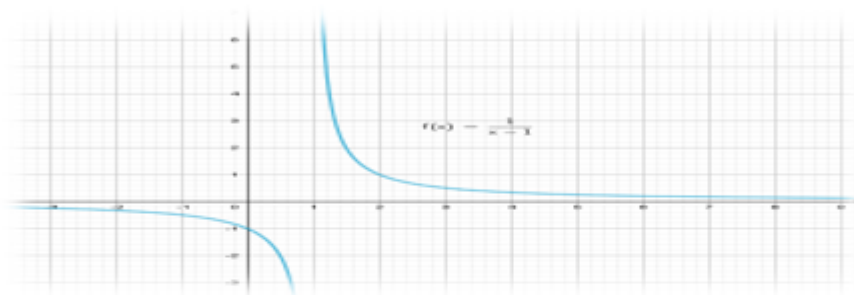


We see that this graph is not continuous as the graphs have a jump by definition of continuous functions when the values of the constants are $a=1$ and $b=-4.2$ that can be read from the slider. But, by moving the two sliders back and front we find the exact values of a and b at which the function is continuous. Whence, the below graph showed this.



This graph showed that the diving board is continuous when the values of $a=-1$ and $b=1$.

Question4: Assuming that the graph of the function $f(x) = \frac{1}{x-1}$ is given by



Which of the following is not true about this graph?

- $f(x)$ is continuous in its domain
- The vertical asymptote the function is line $x = 1$
- $x - axis$ is the horizontal asymptote of the function.
- The value of $\lim_{x \rightarrow 1^-} f(x) = \infty$

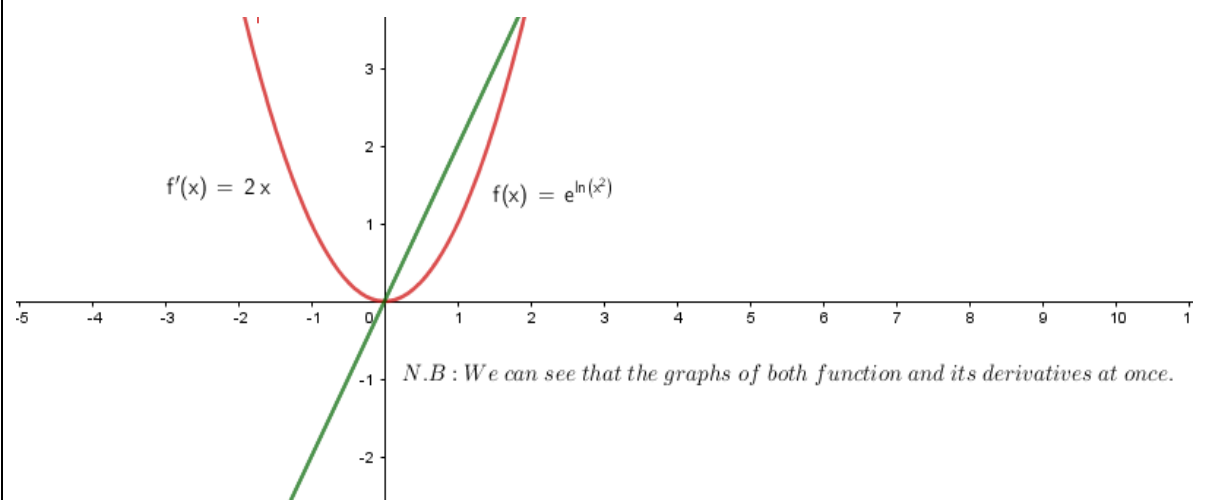
This question can be visualized by students by observing the sketched graph[using GeoGebra mathematical software] by recalling the knowledge students knew(connecting the knowledge they knew without manipulation). The researcher categorizes these questions as conceptual tests.

Question5: Let $f(x) = e^{\ln(x^2)}$ be given function. Which of the following is the derivative of $f(x)$?

- a. $f'(x) = 2x$
- b. $f'(x) = e^{\ln(x^2)}$
- c. $f'(x) = 2$
- d. None of the above

This question more needs conceptual knowledge than procedural knowledge so the researcher categorizes this question as conceptual knowledge. Whatever types of the question(item) is the GeoGebra applet easily compute the derivatives of $f(x) = e^{\ln(x^2)}$.

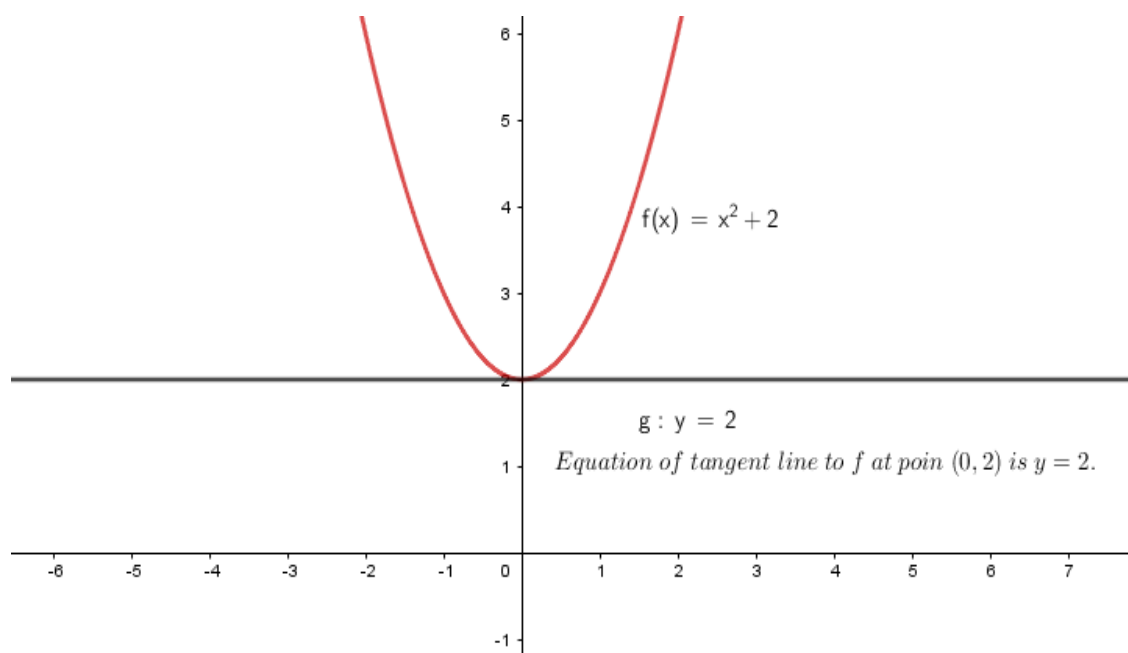
The GeoGebra applet can compute the first derivatives of $f(x) = e^{\ln(x^2)}$ as shown in the following figure.



Question6: Equation of tangent line to the curve $f(x) = x^2 + 2$ that passes through the point $(0,2)$ is:

- a. $y = 2x + 2$
- b. $y = -2x + 2$
- c. $y = 2$
- d. $y = -2$

The researcher categorized this item of the question as procedural knowledge as the students need to compute it or need to manipulate it to arrive at the answer.



To do this students need to write `tangent(point, function)` at the input menu bar of GeoGebra mathematical software. Then immediately the software computes the graphs of the tangent line with its equations. Here we have two options. I) we can first write the function f at the input menu bar of GeoGebra and hit enter key on the keyboard and then write `tangent(point,f)`; II) we can write as it is `tangent(point, function)`.

Question7: Let the composition function $h(x) = f(g(x))$ be given as the differentiable function of x . Which of the following is true about $h(x)$?

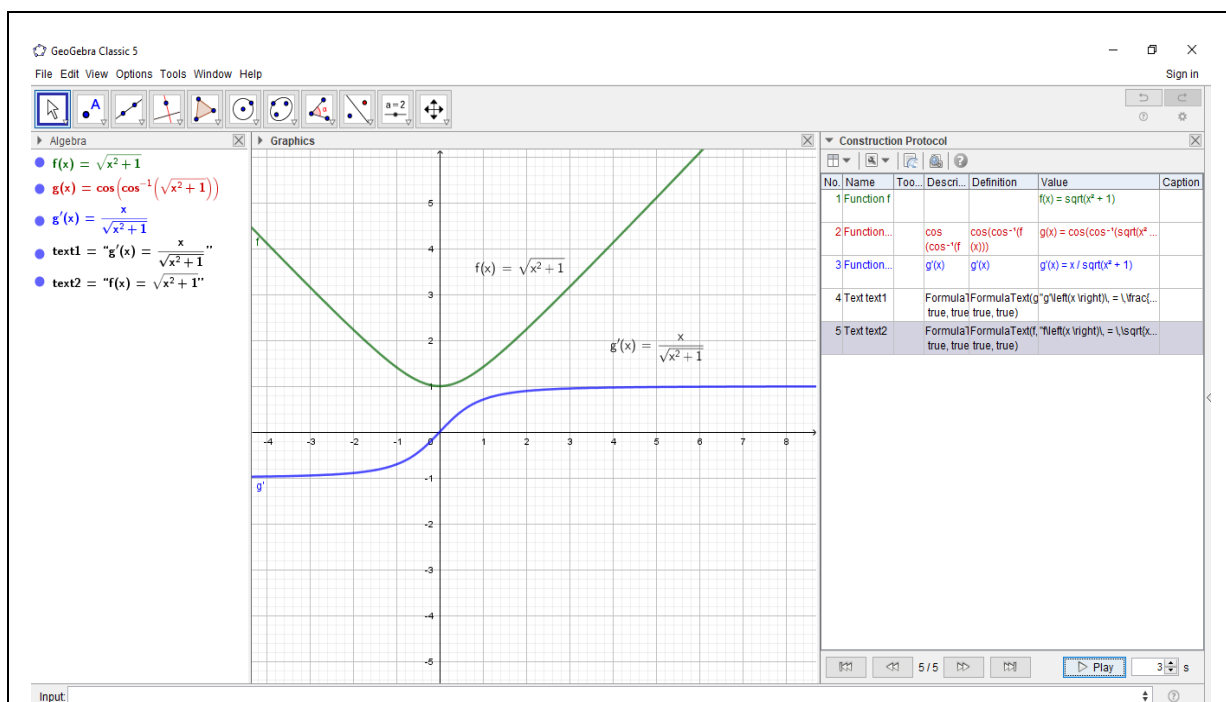
- a. $\frac{d}{dx} h(x) = \frac{d}{dx} f(g(x)) + \frac{f}{dx} (g'(x))$
- b. $h'(x) = f'(g(x)) + g'(x)$
- c. $h'(x) = f'(g(x)) \times g'(x)$

In this question item, the GeoGebra applet cannot enable us to answer the question stated. So, in this case, students need to recall the derivatives of composition functions that they learned earlier.

Question8: The derivative of $g(x) = \cos(\cos^{-1}(\sqrt{x^2 + 1}))$ is

- a. $g'(x) = \sqrt{x^2 + 1}$
- b. $g'(x) = \frac{2x}{\sqrt{x^2 + 1}}$
- c. $g'(x) = \frac{x}{\sqrt{x^2 + 1}}$

For this question, item students need to recall some properties of an inverse function to simplify their life for the manipulation of the item or students may compute the item as it is. So, the item was categorized more or less at procedural test. The GeoGebra mathematical software can compute the first derivatives and draw the graph of them as shown in the following figure.



From the graph, we see that the graph of the derivatives of the function $g(x) = \cos(\cos^{-1}(\sqrt{x^2 + 1}))$ is $g'(x) = \frac{x}{\sqrt{x^2 + 1}}$. We can show students each inscribed item on both algebra and graphics views by hitting play of construction protocol of the graph sketched.

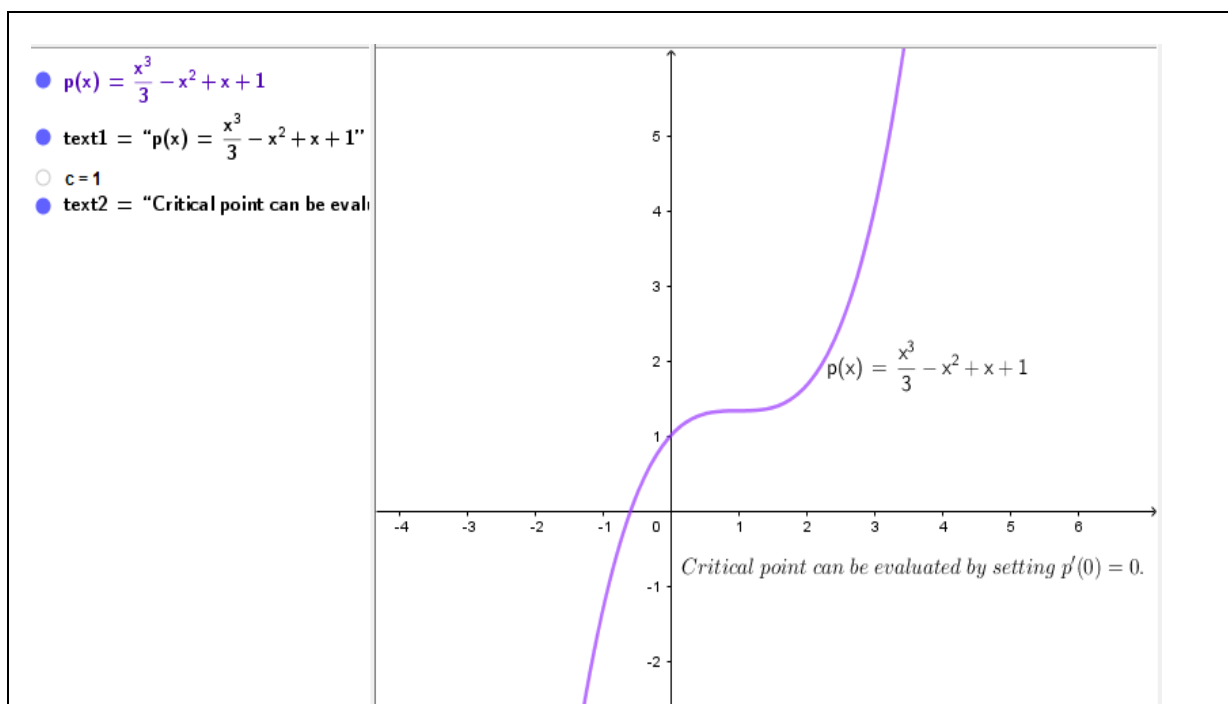
Question9: Which of the following is true about the critical point(s) "c" of the function

$$p(x) = \frac{x^3}{3} - x^2 + x + 1$$

- $c = \pm 1$ is the only critical point.
- $c = 1$ is the only critical point
- $c = -1$ is the only critical point

This question needs to recall both the formula and manipulations to arrive at the answer(s). But the researcher needs to categorize this question/item into the procedural test. The GeoGebra applet can simply solve the critical point of the function

$p(x) = \frac{x^3}{3} - x^2 + x + 1$. We see that the software plots both the graph and its answer.

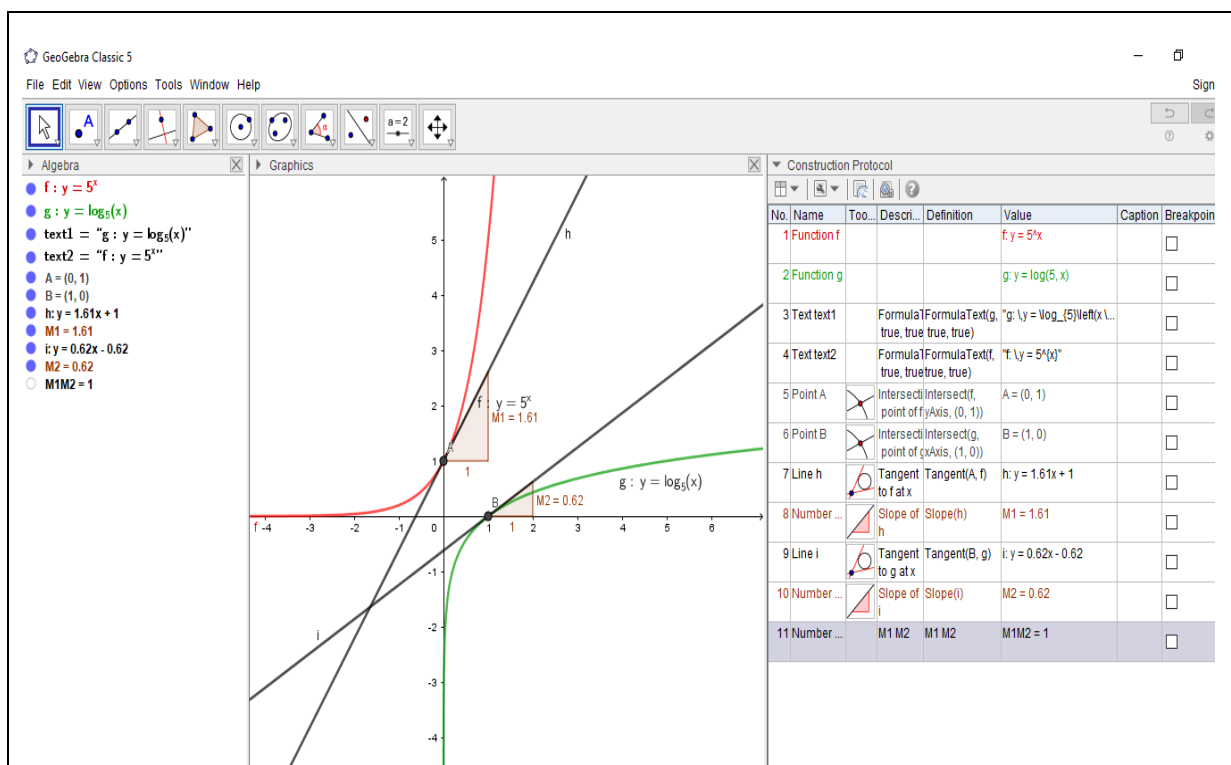


Here students need to write $p'(0)$ in the input bar of GeoGebra to find out the critical point of $p(x)$.

Question10: Let M_1 be the slope of the function $y = 5^x$ at the point $x = 0$ and let M_2 be the slope of the function $y = \log_5 x$ at $x = 1$. Then

- $M_1 = \ln(5)M_2$
- $M_1 = M_2$
- $M_1 = -M_2$
- $M_1M_2 = 1$
- $M_2 = \ln(5)M_1$

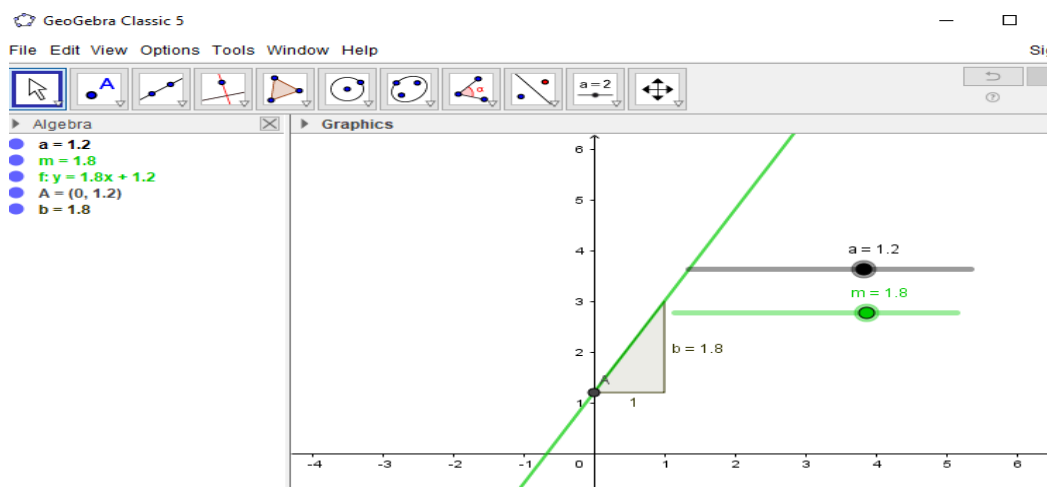
This question can be answered either by conceptual or procedural knowledge. In short, GeoGebra can answer this by simply plotting the graphs of the two functions on the same GeoGebra window. The following figure showed this.



To do this the students need to follow the following a guideline.

- Write both functions on the input menu bar of GeoGebra
- Click on both graphs at the point with the corresponding given point as shown in the figure by selecting the point bar at the top or bottom of the GeoGebra menu as it is arranged by using the setting.
- Write $\text{tangent}(\text{point}, \text{function})$ on input menu bar of GeoGebra. In this case $\text{tangent}(A, f)$ for a tangent line of the graph $f: y = 5^x$ at the point $x=0$ and $\text{tangent}(B, g)$ for a tangent line of the graph $f: y = \log_5 x$ at the point $x=1$.
- Write $\text{slope}(\text{line})$ on the input menu bar of GeoGebra by selecting each tangent line turn by turn. For instance, in this case, I wrote $\text{slope}(h)$ and $\text{slope}(i)$ separately to obtain the sketched slopes.
- Finally, investigate the chooses given to arrive at the answer. GeoGebra can evaluate all mathematical operations. For instance, the software gave us $M1M2=1$ (please see $M1$ and $M2$ from the graph).

Task: Display the variables m and b as a slider in the graphic views. Graph a linear equation $y = m x + b$ which parameters can be modified using sliders. Display the slope of the corresponding line and visualize the y -intercept.

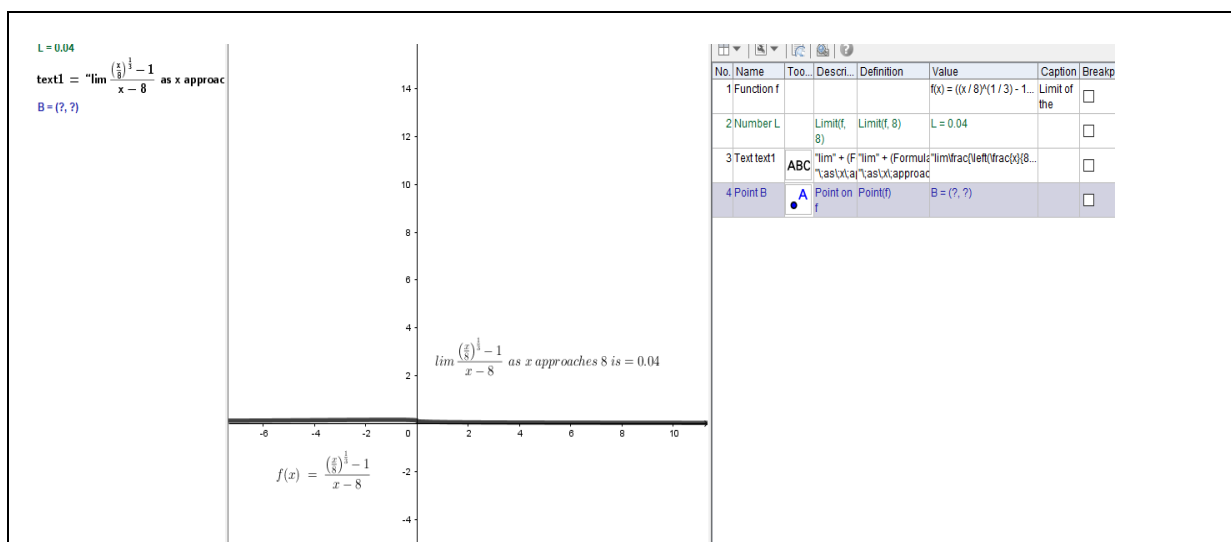


Question11: By using the power rule of derivatives, you that the derivative of $x^{1/3} =$

$\frac{1}{3} x^{-2/3}$ for every $x \neq 0$. Then $\lim_{x \rightarrow 8} \frac{\left(\frac{x}{8}\right)^{1/3} - 1}{x - 8} = \frac{1}{2a}$, where $a =$ _____

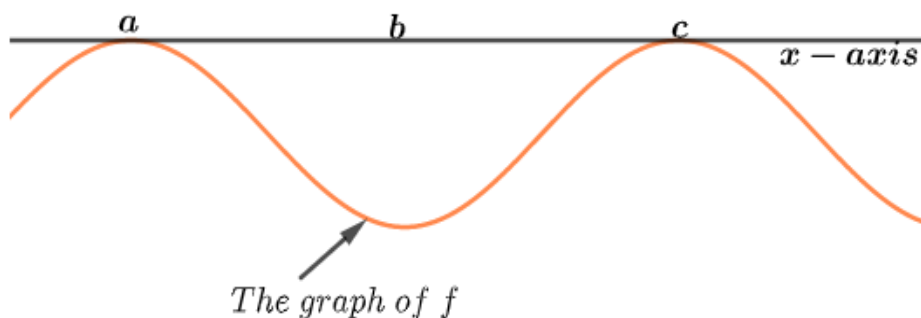
- a) 4
- b) 12
- c) 6
- d) 1

As the question is forced us to use the power rule of finding derivatives of the functions (that need steps), the question was categorized at procedural test. Without using the steps, the GeoGebra can evaluate the limit as shown in the following figure. In addition to evaluating the limit of the given function, the applet enables students to visualize the behavior of the functions at the given point.



The value of a can be calculated by equating $\frac{1}{2a} = 0.04 \Rightarrow a \approx 12$.

Question12: Suppose that the graph of the function f is drawn as in the following figure.

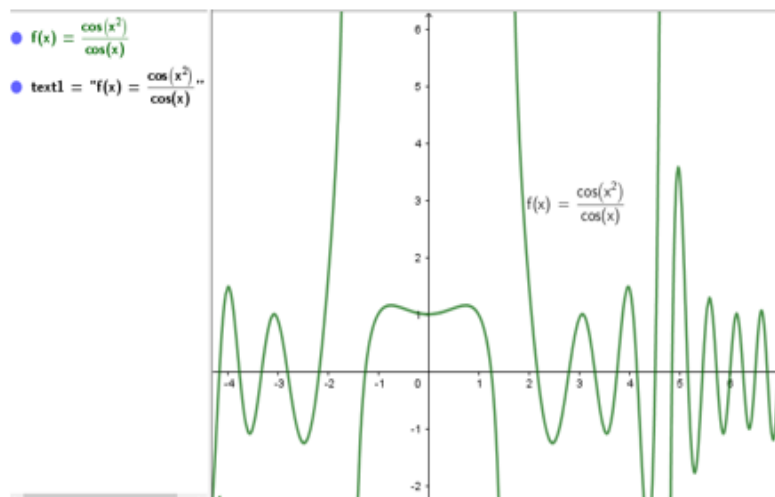


Which of the following is not true?

- The function f is concave upward on interval $[a, c]$
- The function attains the minimum value at point b
- The maximum values of the function occur at the point a and c
- The function has no inflection point.

Students can visualize and run on the chooses to arrive at the answer by recalling the knowledge they do have earlier and thus the item was categorized under conceptual test.

Question13: The first derivative of the function $f(x) = \frac{\cos(x^2)}{\cos x}$ which is indicated in the following is:

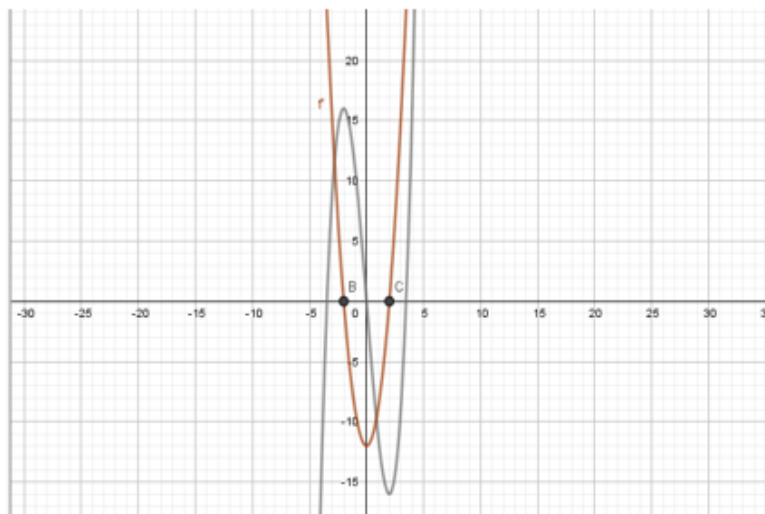


- a) $\frac{\sin(x) \cos(x^2) - 2x \sin(x^2) \cos(x)}{\cos^2 x}$
- b) $\frac{\sin(x) \cos(x^2) + 2x \sin(x^2) \cos(x)}{\cos(x^2)}$
- c) $\frac{\sin(x) \cos(x^2) + 2x \sin(x^2) \cos(x)}{\cos^2 x}$
- d) $\frac{\sin(x) \cos(x^2) - 2x \sin(x^2) \cos(x)}{\cos(x^2)}$

To find the derivatives of the function $f(x) = \frac{\cos(x^2)}{\cos x}$ the students can use a couple of steps to arrive at the answer which requires the recalling of the quotient rule of derivatives. More or less, the question was categorized at the procedural test. But the GeoGebra applet can solve this in a short period by visualizing the answer in graphs and answer in algebra view of GeoGebra window.

Question14: By using the following graph of the function $f(x) = x^3 - 12x$ determine which of the following is true?

- ☒ $f(x) = x^3 - 12x$
- ☒ $f'(x) = 3x^2 - 12$
- ☐ $A = (0, -12)$
- ☒ $B = (-2, 0)$
- ☒ $C = (2, 0)$



- a) The turning point of the derivatives of the function $f(x)$ points A
- b) Between point B and C the function $f(x)$ is increasing
- c) From point B to negative infinite the function $f(x)$ is decreasing
- d) -2 is the only critical point of the function $f(x)$.

Students need to manipulate the behavior of the given function by using bits of knowledge they knew by using a couple of calculations. Note that the GeoGebra can sketch the graphs and the students themselves need to tackle the stated chooses.

Question15: Let $r(t)$ stand for the position of a particle at the time t . Which of the following is false?

- a. $r'(t)$ Represents the velocity of a particle at time t
- b. $r''(t)$ Represents the acceleration of a particle at time t
- c. $r'(t)$ Represents the length of a particle at time t .

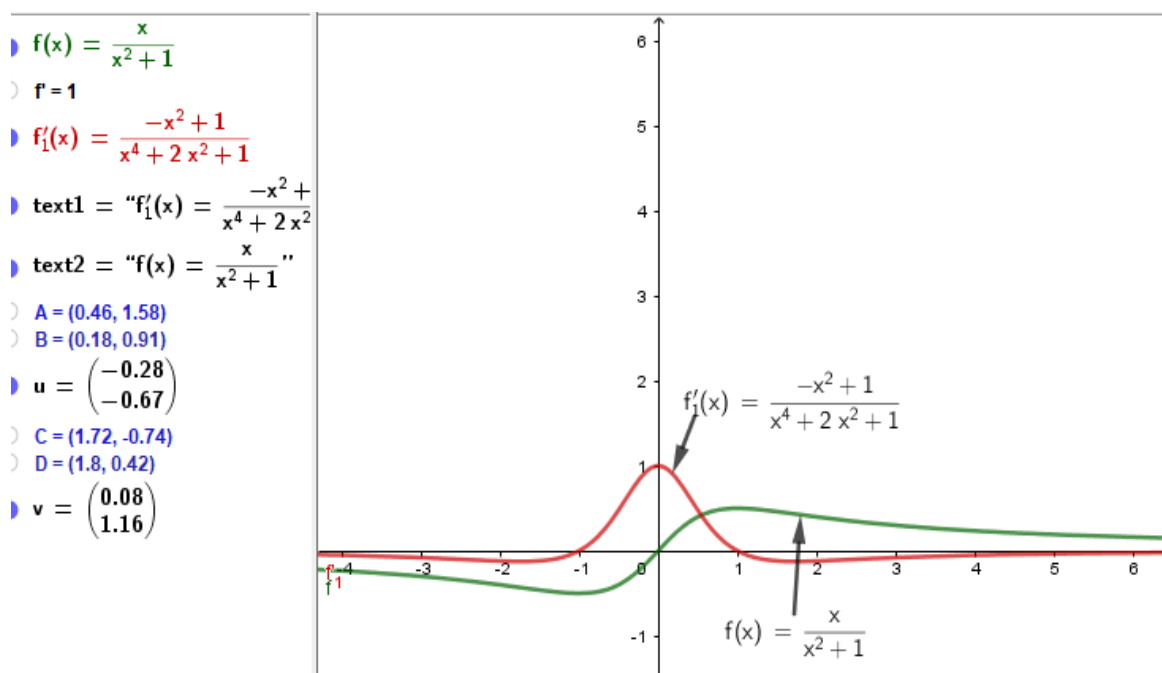
For this question, students need to recall the knowledge they have in their minds.

Question16: What is the first derivative of the function $f(x) = \frac{x}{x^2+1}$ at $x = 0$?

- a) 0
- b) 1
- c) 2
- d) 3

This question can be evaluated by using GeoGebra mathematical software. Task: Graph a polynomial equation $f(x) = \frac{x}{x^2+1}$ with non-parameters. Display the first derivatives of the function and visualize them on the GeoGebra window. Write $f'(0)$ on the input menu bar of GeoGebra.

The answer to the task was given by the following figure.

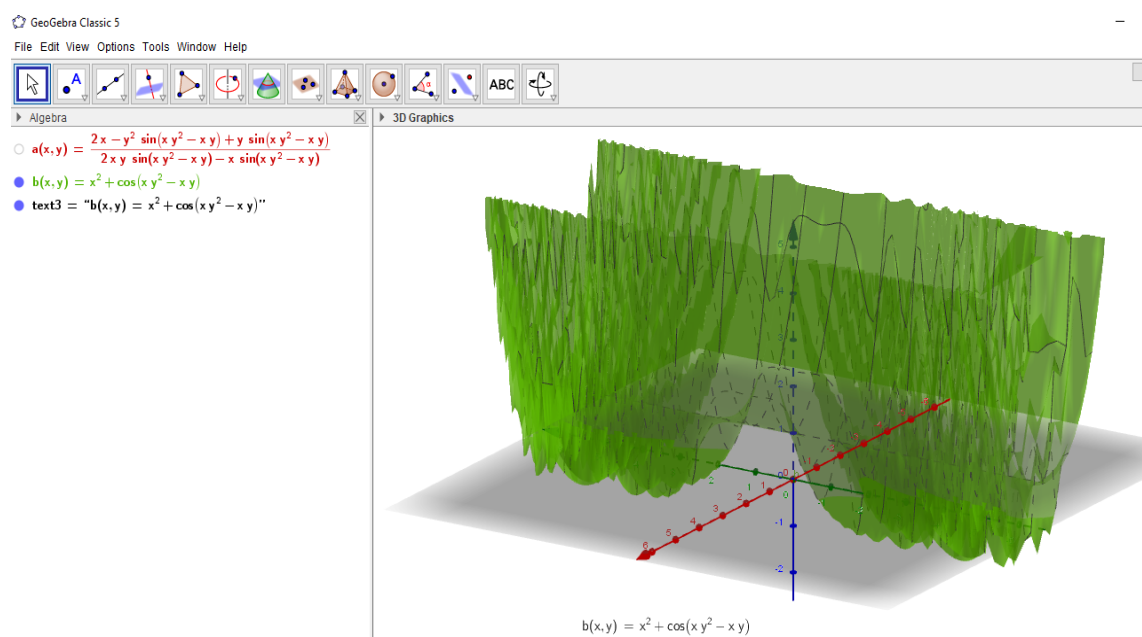


It is trivial that the value of $f'(x) = \frac{-x^2+1}{x^4+2x^2+1}$ at $x=0$ is 1.

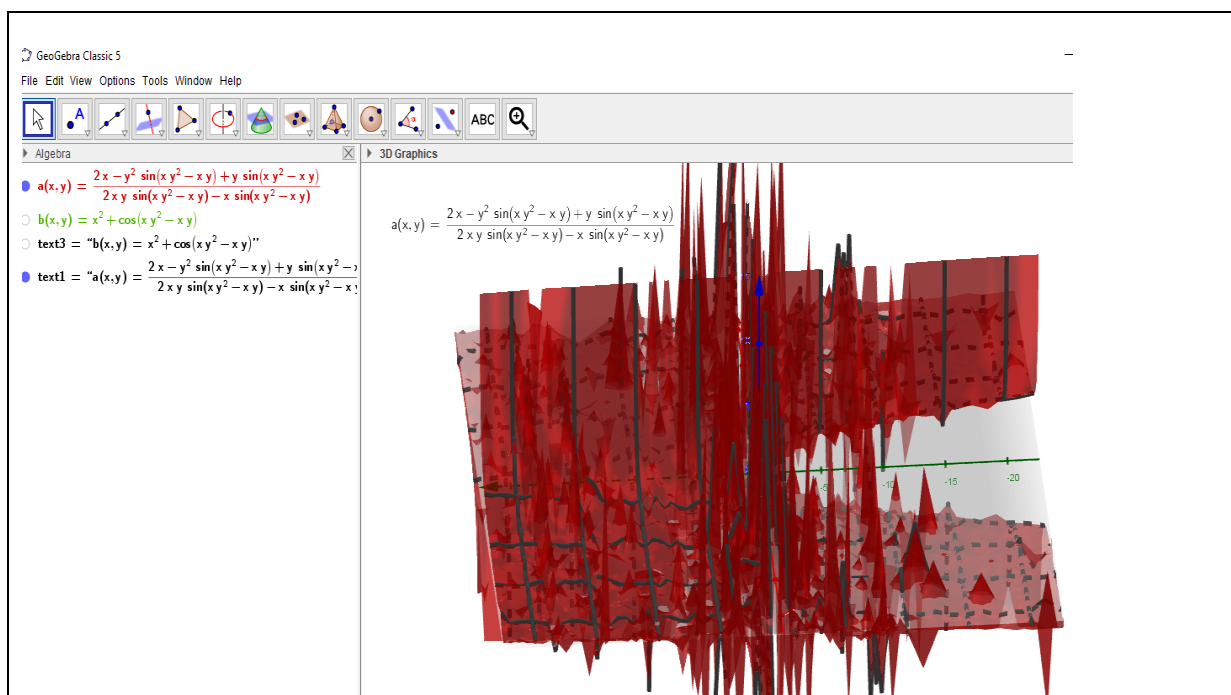
Question17: $\frac{dy}{dx}$ of $x^2 + \cos(xy^2) = xy$ is:

- a. $\frac{dy}{dx} = \frac{2x-y-y^2 \sin(xy^2)}{2xy \sin(xy^2)+x}$
- b. $\frac{dy}{dx} = \frac{2x-y+y^2 \sin(xy^2)}{2xy \sin(xy^2)+x}$
- c. $\frac{dy}{dx} = \frac{2x-y^2 \sin(xy^2)}{2xy \sin(xy^2)+x}$

This question is about finding the implicit derivatives of the curve $x^2 + \cos(xy^2) = xy$. It is very difficult for the students to visualize the nature of the function on the xyz-plane by hand. But the GeoGebra can sketch simply. To do this select 3D graphics and do as usual of inserting the function into the GeoGebra input menu bar. We obtained the following figure. The great advantage of using 3D GeoGebra is just this over the traditional teaching method in which the sketch of such a graph is impossible by hand.



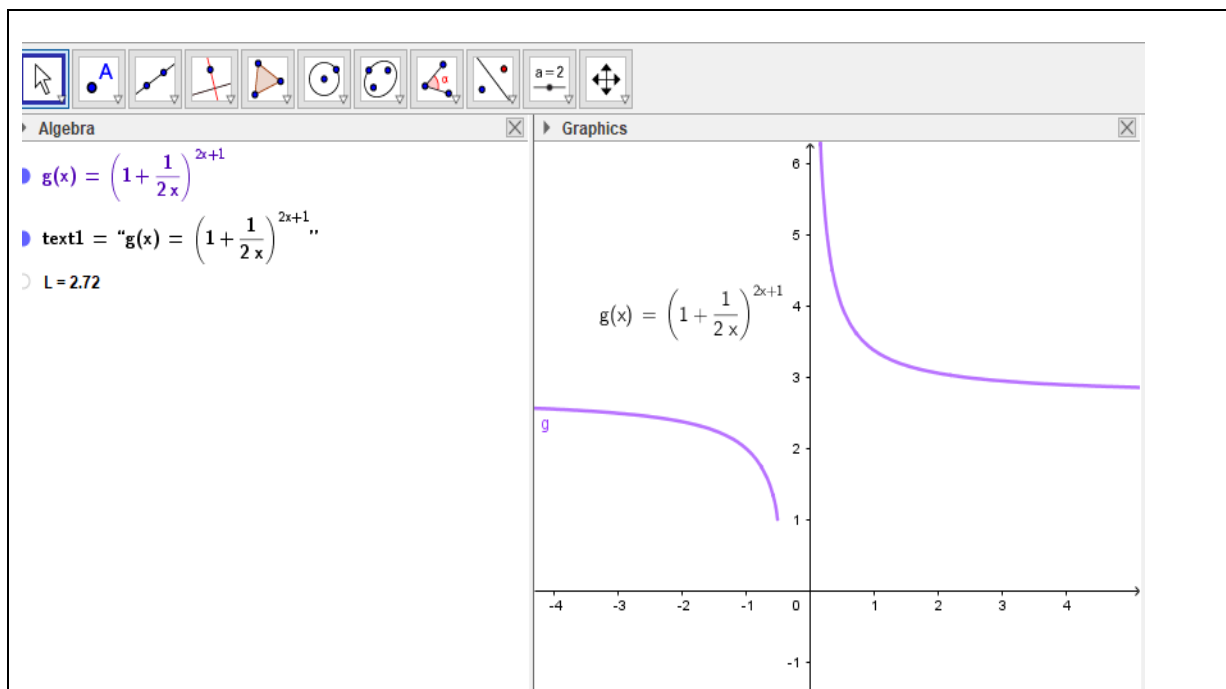
After writing the graph on the input menu bar of GeoGebra, then write implicit derivative(function) and hit enter key on the keyboard. We obtained the following figure.



Question18: Use the fact that $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$. Then $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{2x+1}$ is:

- a. e^2
- b. e^{-4}
- c. e
- d. e^{-1}
- e. None of the above

The question needs some steps to find the required limit of the function $\left(1 + \frac{1}{2x}\right)^{2x+1}$ as the x value tends to be infinite. Task: Sketch the graph of the function $g(x) = \left(1 + \frac{1}{2x}\right)^{2x+1}$ and write $\lim(g, \infty)$. Observe that the GeoGebra applet gave us $L=2.72=e$.



Part 2: Work out the problem.

Show all necessary steps in finding the required answers and write your final answer carefully.

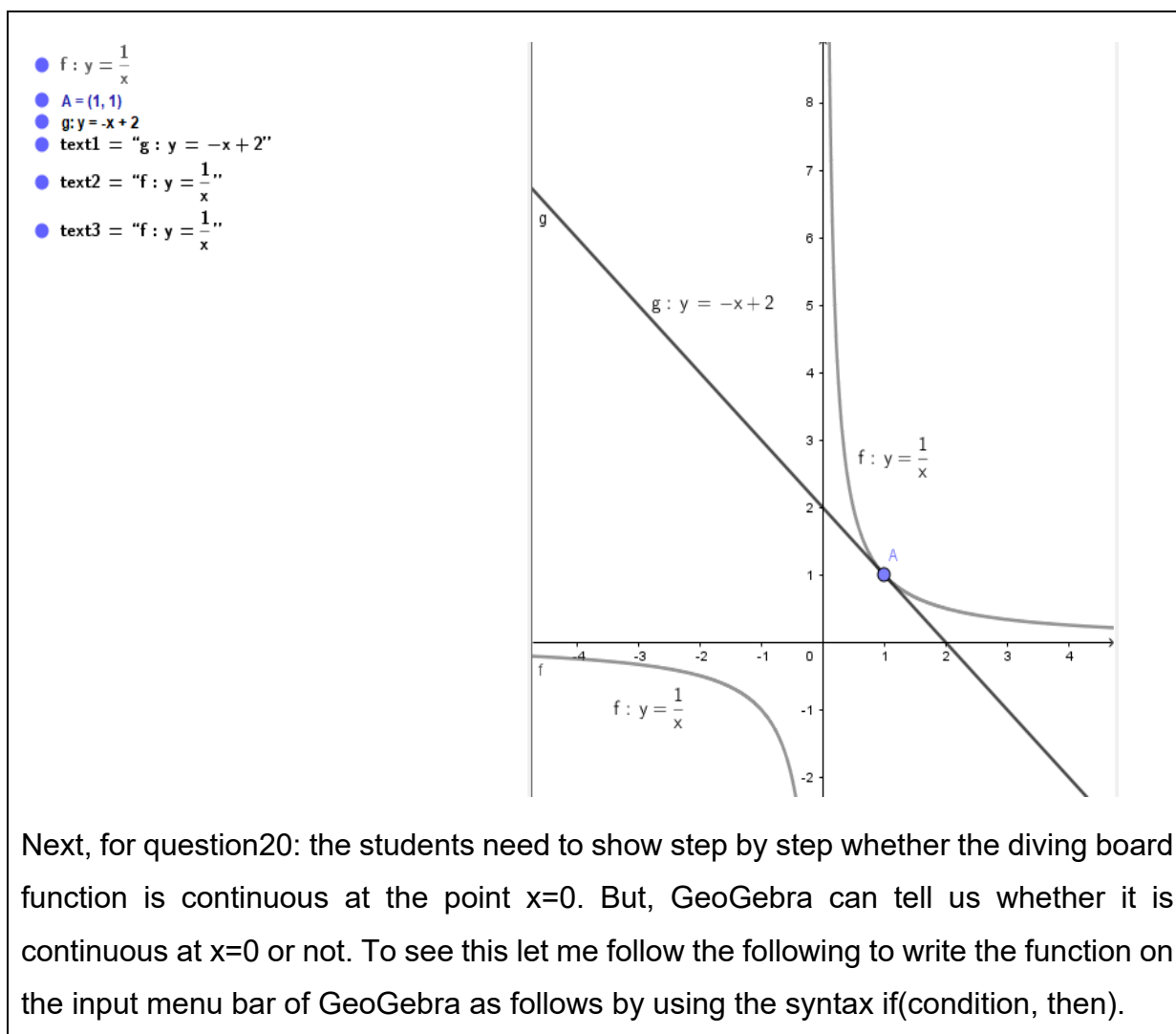
Question19: Find the equation of a tangent line to the function $xy = 1$ at $x = 1$ and sketch the graph of $xy = 1$.

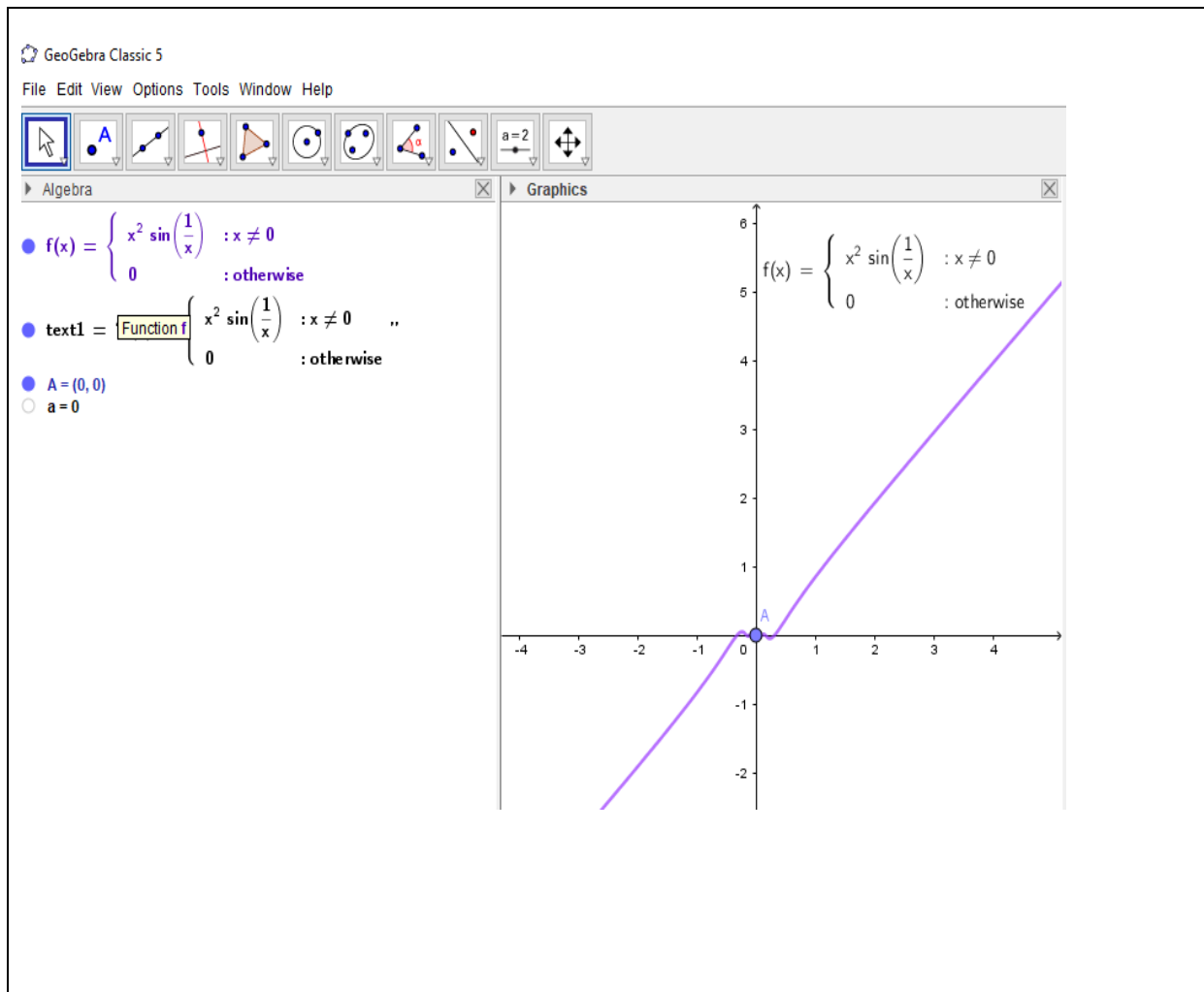
Question20: Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ Is $f(x)$ continuous at $x = 0$? Justify

To sketch the graph of $xy = 1$ and find the equation of the tangent line the curve at $x=1$, we follow the following steps.

- 1) Write $xy=1$ on the input menu bar of GeoGebra
- 2) Click on the curve at $x=1$
- 3) Write $\text{tangent}[\text{line}]$ on the input menu bar of the window
- 4) Finally consider step 3 as the equation of a tangent line

These steps can be visualized by the following figure.





The teacher employed the lesson plan for each activity that guided him in the classroom, as attached in Appendix 1.2.

CHAPTER 4 THE PROPOSED METHODOLOGY

4.1. INTRODUCTION

In this chapter, the research methodology (i.e., the research design) of the study is discussed. This is the strategy or proposed method followed in the study to assist the researcher in addressing the research questions. The methodology comprises aspects such as establishing the data sources (students), data collection techniques and instruments (tests, interviews and questionnaires), sampling methods or techniques (both purposive non-random and random sampling methods in a mixed sampling method), reliability and validity of data, operational terms, the proposed model, and lastly the ethical considerations. In this introduction, the philosophical worldview or paradigm that helped the researcher to choose this specific research design for the study is discussed. As a researcher, I was interested in making a distinction between the term's *method* and *methodology*. In this instance, method refers to the research instruments that were used to collect and analyse the data (Khaldi, 2017), whereas methodology is defined as the design by which the researcher selected appropriate data collection methods and analysis procedures to investigate the specific research problem of the study (Mcmillan & Schumacher, 2014).

4.2. RESEARCH PARADIGM

The term paradigm was defined in the influential book entitled *The Structure of Scientific Revolutions* by Thomas Kuhn in 1962 as a conceptual framework shared by a community of scientists that enables them to model problems and find solutions for community practitioners (Kuhn, 1996,p.155). Guba and Lincoln (1994, p. 107) define a paradigm as “basic beliefs that deal with ultimate principles”; thus a paradigm influences researchers when choosing the research questions and methods of study that will enable them to find solutions to their research questions (Morgan, 2007). Traditionally, there are three common research paradigms: positivist, interpretivism, and critical theory. For example, the interpretive/constructivist paradigm tries understand and interpret what the subject is thinking about the concept (Kivunja & Kuyini, 2017, p. 33). All these paradigms (positivist,

interpretivism, and critical theory) contain opposing ideas that have led to a “paradigm war” (Galvez, Heiberger, & Mcfarland, 2020, p. 613; Maarouf, 2019, p. 1) in terms of the three philosophical dimensions of ontology, epistemology and methodology. As a result, the compromising paradigm known as the pragmatic paradigm has emerged (Teddle & Tashakkori, 2009).

Understanding the most significant differences between the research paradigms and how they approach (ontology, epistemology, and methodology) these three philosophical dimensions helped the researcher to choose the best research paradigm for this study. It is thus to discuss these dimensions. Guba and Lincoln (1994) argue that the philosophical dimensions present three fundamental, interconnected questions:

The ontological question asks, “What are the form and the nature of reality?” Does “objective” reality exist “independent of the researcher”? The objective of this study is to investigate the use and effect on students’ learning of Mathematics of the GeoGebra Mathematical software. By asking this question the researcher hoped to establish a reality somewhere between positivist (quantitative) and constructivist (qualitative) ways of knowing to examine the data in the study from both world views for triangulation purposes.

The epistemological question “What is the nature of the relationships between the knower and what can be known/participant?” is concerned with the acceptable knowledge in the study field (Saunders, Lewis, & Thornhill, 2009). Morgan (2007) defines epistemology as the nature of knowledge and the relationship between researcher and participants in the study. Drawing on ontology to establish a reality between quantitative and qualitative ideology, the researcher’s task was to scaffold students in their learning of calculus with the help of GeoGebra mathematical software, using the hypothesised cycle model of teaching mathematics by GeoGebra (see the theoretical framework of this study Section 3.4). In the case of this scaffolding, and MKO is required (see steps of implementing hypothesised cycle model Figure 3.4). Thus, there is a relationship between the researcher and participants in terms of knowledge. In the epistemological philosophical dimension, reality is represented by objects that are considered to be real, such as computers, trucks and machines (Saunders et al., 2009). Investigating the views of

students on the use of GeoGebra before (Step 2 section 3.4 of Figure 3.4 and after intervention (Step 6 section of the same figure was the task of the researcher in this study.

Methodological questions include, How can the inquirer go about finding out whatever s/he beliefs can be known? The nature of the research question addressed in this study demanded the use of an explanatory methodology, which consisted of the investigation of the cause-and-effect relationships between the variables of the study such as teaching differential calculus with the help of GeoGebra (independent variables) and students' achievements and understanding (dependent variables) in the experimental group, and teaching calculus using conventional methods(independent variables) with their achievement(dependent variables) and hence, the study investigated the relationship between achievements and students' views on using GeoGebra which answered the methodological questions appeared in the study.

Considering these questions and the differences between research paradigms and how each related to the objective of this study, the researcher chose the pragmatic research paradigm for this study. The pragmatic paradigm is based on the researcher's plan to use a methodology that fits the problem to be investigated by the researcher (Teddle & Tashakkori, 2009). In this case, the literature review revealed that mixed-method research (MMR) was appropriate when following the pragmatic research paradigm as it represents a compromise between the positivist and constructivist paradigms (Maarouf, 2019). Thus, a mixed-methods research approach was chosen for this study. Using only one research method (a qualitative or quantitative method) in a study is not always sufficient to obtain viable results, as a researcher strives or provide complete answers that meet the aim or purpose of the study (McMillan & Schumacher, 2014; Offermann, Levina, Schönherr, & Bub, 2009; Zainal, 2007). McMillan and Schumacher (2014) argue that a mixed-methods design is very important when the thinking of individuals or small groups is significantly different from that of the majority.

Table 4.1 Summary of the philosophical dimension of the study and how this is related to the pragmatic research paradigm.

Features and authors	Description
Objective of research	The study investigates the effect on students' learning of mathematics through GeoGebra Mathematical software.
Ontology (nature of reality) (Morgan, 2007)	The researcher aims to find a way of knowing, understanding, mastering (subject matter, and tools) by employing the sociocultural interactions (teacher with students) of Vygotsky's theory while students learn calculus by using GeoGebra Mathematical software.
Epistemology (nature of the knowledge and the researcher/participant relationships) (Morgan, 2007)	The researcher, in this case, the teacher (knower), is more knowledgeable than others (MKO) when teaching calculus with the help (scaffolded) of GeoGebra in experimental groups, and when teaching calculus using conventional methods. Thus, the study comprises the visible relationships between the knower (researcher) and the known (participants of the study, i.e., students) in terms of knowledge that reveals the nature of knowledge. In this case, the researcher's task was to evaluate the learning outcomes and make sense of the constructed meanings of their new experience after gradually ending the intervention.
Axiology (judgements about value (Saunders et al., 2009)	Values play an important role in interpreting results; the researcher adopted both objective (free from bias) and subjective (biased) points of view (Saunders et al., 2009).
Methods	Mixed method research (MMR)
Logic (Creswell, 2009)	Both deductive and inductive (as the study is MMR)
The possibility of causal linkage	There is a causal link in the study as is indicated in the objectives of the study.
The possibility of generalisability	The issue of sample size in the study

Pragmatism holds that truth is what works at the time; it is not based on a dualism between reality independent of the mind and within the mind (J. W. Creswell & Poth, 2018). In keeping with this world view, this study used multiple methods or perspectives to validate quantitative and qualitative instruments by considering information obtained from the reviewed literature (students' perceptions of GeoGebra) because statistics cannot manipulate perceptions obtained from interviews, and to explain quantitative results for better contextualisation in the intervention. Creswell (2013) believes that mixed research methods are suitable for research problems and questions in a study. Before

the researcher administers instruments, he or she needs to explain the statistical results by talking to people; the researcher must determine whether the quantitative and qualitative results match.

Having established the research method to be used in the study in any research, the next question is how to collect data (ways of obtaining data in terms of time available). In MMR, data can be collected sequentially or concurrently to achieve the best understanding of the research problems (J. W. Creswell, 2014). The method allows the researcher to apply two types of research questions (to collect qualitative and quantitative methods), two types of sampling procedures (probability and purposive), two types of data (numerical and textual), two types of data analysis (statistical and thematic) and two types of conclusions (objective and subjective) (Tashakkori & Creswell, 2007). These options and a pragmatic research paradigm encouraged the researcher to adopt this research method (MMR). The philosophical dimension of pragmatism opens doors to the MM researcher to apply multiple methods, different worldviews, different assumptions, different data collection methods and analysis in a study.

4.3. RESEARCH DESIGN AND PROCEDURE

The organisation of the research activities (data collection) to achieve the aim of a study is referred to as the research design (RD). RD comprises the procedures of conducting the research and takes into consideration the 'when' (the time of the intervention), the 'from whom' (the participants) and the conditions under which the data will be obtained. As there may be many types of research design and research questions in a study, the researcher needs to match the research questions with the appropriate research design (McMillan & Schumacher, 2014). The research design in this study is a mixed (quantitative and qualitative) approach method necessitated by the type of research questions (Tashakkori & Creswell, 2007). The researcher used a qualitative approach to determining students' perceptions of learning calculus through technology, in this case, GeoGebra, and used a quantitative approach to investigate the effect on students' learning of differential calculus of GeoGebra software, and the extent to which this software enhanced students' learning of calculus, in terms of both achievement and

understanding of differential calculus. As this study investigated attitudes/perception and other psychological concepts such as behaviours and beliefs, the best methodology was a mixed-methods approach. The study used an explanatory sequential design, which is one example of a mixed approach method, to collect data. I used the quantitative methods first (posed pre and post-test to students) and gave a greater emphasis in addressing the study's purpose, and the qualitative methods (perception of students after intervention employed) followed to help me explain the quantitative results of the study and in this method, qualitative data is enhanced me for an understanding of some aspect of the intervention (J. W. Creswell & Clark, 2018). Therefore, in explanatory sequential design, the researcher first collected quantitative data and then collected qualitative data to help explain or elaborate on the quantitative results (J. W. Creswell, 2014; Maarouf, 2019).

As the study used a mixed research method, it employed a quasi-experimental design on the one hand it had cause and effect relationships between the teaching method and student's scores (McMillan & Schumacher, 2014). A quasi-experimental pre-and post-test and a control group design were adopted for this study. Quasi-experimental research uses non-randomised assignments of the group of the study that are categorised into experimental and control groups (Shadish & Luellen, 2005). Because of chose of this design, the study included 36 and 30 numbers of students in control and experimental groups. A pre-test was administered to the experimental and control group before intervention (statistical control over the groups) for the sake of identifying the abilities of students in the two groups. Where there is no difference between the pre-test scores of two groups, the researcher uses a T-test or an ANOVA. This ensures that the results are real and helps the researcher to manage the initial group difference statistically. In the case of the experimental group in this study, the pre-test helped to categorise students as low or high achievers. The pre-test and the post-test were the same for the two groups in terms of content and the number of questions (McMillan & Schumacher, 2014). The instructional materials for the two groups were also the same (McMillan & Schumacher, 2014).

The research design is illustrated in Table 4.2.

Table 4.2 Research design and procedure

Experimental Group 30 students	Pre-test before intervention	High achievers	Treatment (teaching using GeoGebra)	Post-test intervention after
		Low achievers		
Control Group 36 students	Pre-test before intervention		Conventional teaching (lecturer)	Post-test intervention after

One section of the intervention, the experimental group, was conducted in a computer laboratory over two weeks, while the control group was taught at the same time in an ordinary classroom. The laboratory classroom was arranged so that each student had a computer. The intervention was planned as a set of eight 50-minute lessons, in total about 400 minutes or approximately 7 hours using the dynamic mathematics software GeoGebra (DMSG) as well as conventional methods.

The instructional material (lesson plan) for the experimental group was designed to be delivered using computer-assisted teaching (in this case, GeoGebra software). The same instructional material was used to teach the control group but using traditional (conventional) teaching methods in which the teacher must use the lecturer method, using talk and talk through paper-pen approaches (Please see Appendix 1.1 and 1.2).

The study was made up of four phases. The activities in each phase of the study are depicted in Figure 4.1:

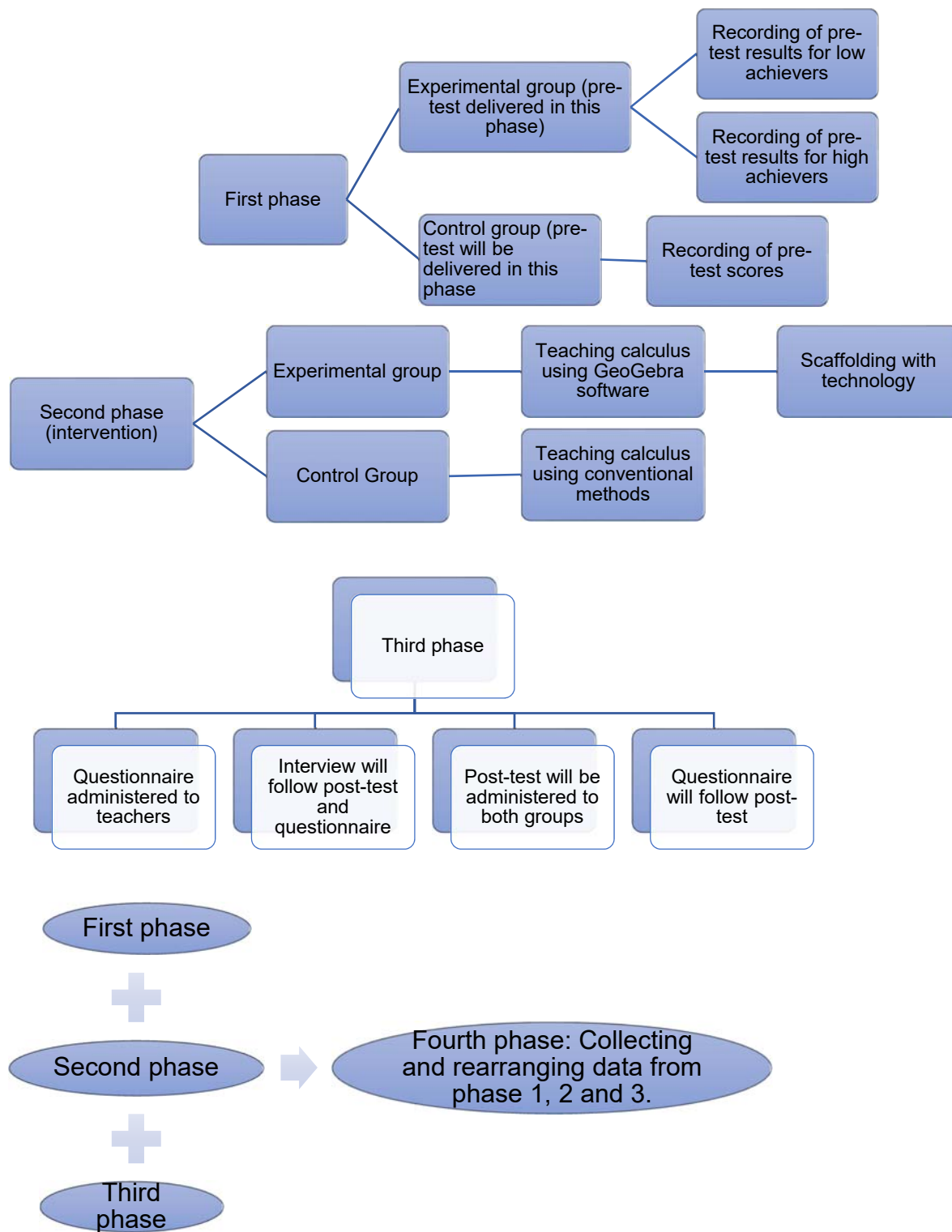


Figure 4.1 Phases of research design and procedure

4.4. DATA SOURCES

The pre-and post-test and closed-ended questionnaires and interviews for students and were the sources of data in this study. This part was discussed more deeply in Sections 5.2 and 5.3 in the steps of the cycle model. The pre-and post-tests were prepared from the instructional material before the intervention took place. These parts again were discussed at steps of the cycle model (please see Section 5.2-5.9).

The items in questionnaires and interviews were based on the research objectives and research questions and the results of the pilot study (please see Section 4.11) were used to check the validity and reliability of the instruments. Questionnaires allow a researcher to obtain data to address the research questions in a study. Richards and Schmidt (2002, p. 438) argue that problems may arise when designing a questionnaire based on research questions; It is the task of the researcher to ensure that a questionnaire has validity, is clearly expressed and that it is scientifically reliable (see Section 4.11.1). Questionnaire items may be closed-ended or open-ended. When using closed-ended questionnaires, data are quantified as they are numerical data, whereas in open-ended questionnaires the data are not numerical but in the form of text.

In general, Mathematics students from the mathematics department at Wachemo University were the sources of data for the study.

4.5. DATA COLLECTION TECHNIQUES

Test, interviews, and questionnaires are data collection instruments that provide data for the researcher to analyse and interpret. The data collection techniques used in this study are elaborated on below.

The researcher constructed 20 multiple-choice items and four problems to be solved for that used for pre-test before the intervention and the post-test after the intervention from differential calculus topics (Mcmillan & Schumacher, 2014). After the pilot study, the researcher narrowed this down to 18 multiple-choices items and two problems, with 20 questions in total. The omitted tests were depending on the criteria of inclusion and

exclusion (please see Section 4.11). Therefore, about 20 tests which are called Differential calculus tests were prepared and ready for the main study. Closed-ended questionnaires were administered to the experimental group after the intervention and students were interviewed. In short, the quantitative data was collected using two instruments, a closed-ended questionnaire and tests, and qualitative data was collected using a focused group interview. The focused group interview was chosen for this study because of the number of the individual chosen by the researcher. There were about five participants of individuals who participated in the interview thus simple to control the data. An interview is a specialized form of communication between people for a specific purpose associated with the research question of the study and whereas a focus group interview is a qualitative technique for data collection by discussions of the participants of the study on a given issue or topic(Dilshad & Latif, 2013).

4.6. ISSUES OF RELIABILITY AND VALIDITY

The reliability and validity of any research study must be assured. In a test or assessment procedure, the two most important psychometric properties are reliability and validity. In psychological and educational testing, reliability is the stability of test scores (scores must be similar on every occasion) whereas validity refers to the accuracy of the inferences or interpretations the researcher makes from the test scores. To assure reliability in this study, this researcher used the test-retest reliability method. The questionnaire and interview were checked by experts to ensure their reliability and validity (see Section 4.11). Some of the factors affecting the internal validity, such as maturation, effects of history, selection, and design contamination of the study, were considered by the researcher while the study was underway. Let us look at how the researcher managed the internal validity of the study:

- ✚ **Maturation:** The difference between the pre-and post-test of differential calculus might be the result of the psychological maturation of the participants rather than differences in the independent variable. Also, differences between experimental and control groups might result from one group changing at a different pace than another (selection-maturation interaction). This is the invisible factor of internal

validity of the study as the duration of the study, the age level, and education level of the study participants are somewhat the same (please see Section 5.2-5.4) of the cycle model (Creswell & Poth, 2018).

- ✚ History: Events during a study might affect one group but not another, leading to differences between groups that are not solely the result of the independent variables. In nonexperimental history, this might refer to events happening (to a group of individuals) beyond the event that the researcher is studying (Creswell & Poth, 2018).
- ✚ Selection: Certain attributes of one group are different from another before the study starts, coinciding with the stages of the cycle model (please see Section 5.2). Hence, differences after treatment are not solely attributable to the independent variable, and thus the researcher selected the participants of the study with the same education level (Teddle & Tashakkori, 2009). This factor was managed by selecting the study participants randomly into experimental and control groups so that the characteristics had the probability of being equally distributed among groups of the study (Creswell & Creswell, 2018).
- ✚ Design contamination: As the study was employed within one university with the participants of the same education level, this factor of internal validity was enabled to be controlled by the researcher. However, the researcher protected students from using the instructional material (GeoGebra oriented lesson plan) that prepared them for teaching purposes to have at their home. The material is used only within the classroom environment.

4.7. SAMPLING TECHNIQUES

In 2019, Ethiopia had 45 public universities. Of these, Addis Ababa University and Haromaya University are first-generation according to the categorisation of Ethiopian universities. Universities are categorised according to the year they were built (from one generation to four generations ago). I chose Wachemo University, a third-generation university purposefully, specifically students studying in the Department of Mathematics. I chose this university because I am a lecturer there and the problem of the study was raised there. Wachemo University is situated in the Southern Nations, Nationalities and

Peoples (SNNP) regional state of Ethiopia and is 230 km from the capital of the country, Addis Ababa.

One group of undergraduate students of mathematics made up the participants of the study. The numbers of these students depend on the capacity of the department and the researcher used a lottery or a simple random sampling method to select an experimental and control group for the study. This was achieved by identifying the section by coding (code number one indicating students who would be included in the study code number two indicating those who would be excluded from the study). In total, 30 and 36 freshman students learning mathematics were included in experimental and control groups. The researcher sampled students by writing the codes 1 or 2 on 60 to 72 pieces of paper. Placing these pieces of paper in a bowl, the researcher asked each student to take a piece of paper from the bowl. This method of including participants in a study is called the fishbowl draw or the lottery method.

4.8. DATA ANALYSIS AND INTERPRETATION

Data collected from the control and experimental groups before, during and after the intervention was analysed using the statistical software SPSS version 27 and ATLAS.ti 9. Depending on the nature of the research questions and the data collected, different statistical techniques were employed. In the case of the quantitative data, the researcher used either a T-test or an ANOVA. The narrative method was used to analyse the qualitative data by using computer software known as ATLAS.ti 9.

As mentioned in Section 4.3, the researcher used a mixed-methods approach. In mixed methods research, several approaches are used, in this case, a combination of qualitative and quantitative approaches. The researcher observed the data from the two approaches separately, analysed and interpreted it. Data were not merged as the study used an explanatory sequential design. Cohen, Manion, and Morrison (2018) argue that the use of different research methods is important for a better understanding of the issues of the study. Answers may be found using either of the approaches and the limitations of one method may be balanced out by the advantages of the other (Creswell, 2009).

In their study, Cohen, Manion, and Morrison (2018) found that the use of data collected using mixed methods enables researchers to generalise to the wider population. The quantitative data in the study were analysed using T-tests and ANOVAs. The qualitative data were analysed using narrative techniques by using computer software known as ATLAS.ti 9 to understand students' views on the tools of the study.

Table 4.3 Summary of data analysis techniques

Mixed-Method Design (Explanatory sequential design)	
The quantitative phase of the study	The qualitative phase of the study
Research question 1: How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group 2)?	Research question 3: What are students' experiences and perceptions about learning calculus using GeoGebra software?
Theoretical Framework Vygotsky's Theory Analysis techniques The difference in means of the two groups, experimental and control groups, was established using both pre-and post-test results.	Theoretical Framework Vygotsky's Theory Analysis techniques The narrative method was used to determine the perceptions of participants about the use of GeoGebra and the existence of this and other software at the university. Frequency distributions
Research question 2: How does the level of proficiency in differential calculus compare within the experimental group (Group 1) pre-and post the intervention incorporating the use of GeoGebra?	

<p>Theoretical Framework</p> <p>Vygotsky's Theory, Zone of Proximal Development, scaffolding</p> <p>Analysis techniques</p> <p>Comparing in experimental group students' achievement in pre-test and post-test.</p> <p>Comparing the mean scores of the experimental and control groups on the post-test.</p>	
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4.9. OPERATIONAL DEFINITION OF TERMS AND VARIABLES

Mathematics: The science of the relationship between numbers, shapes, and quantities. It uses signs, symbols, and proofs, and arithmetic, algebra, and calculus.

Higher education: post-secondary school education usually at the university level.

Calculus: Branch of mathematics that deals with limits, derivatives, and integration of functions.

Attitude: Personal views about using technology to learn mathematics.

Achievement: Success in learning Mathematics by traditional methods and by technology-assisted methods.

Theory of Learning: Students' response to both traditional methods and technology-assisted methods.

Belief: Acceptance of GeoGebra software on learning and teaching Mathematics

Investigate: Carry out a detailed examination of the effect of GeoGebra learning Mathematics.

Students: Individuals enrolled at an institution for higher learning.

GeoGebra: Dynamic mathematical software

Learning: A gaining new knowledge, in this case about calculus.

Scaffolding: Guiding students, in this case with the help of GeoGebra Mathematical software.

Perception: University students' belief about using GeoGebra to learn.

Experiences: The knowledge that students have about the GeoGebra.

4.10. ETHICAL CONSIDERATIONS

4.10.1. Confidentiality

I protected the confidentiality of data obtained from students during the study. This is part of the ethics of research. Ethical standards demand that the privacy of the participants in the study is ensured and that other people's access to information about an individual is controlled. There are two aspects to privacy (Folkman, 2000): 1) A person's freedom to identify the time and circumstances under which information is shared with or withheld from others and 2) the right to decline the information that he or she does not want. All participants in this study took part voluntarily. No names of participants were used, and participants were informed of the findings.

The researcher undertook to keep hard copies of collected data in a locked filing cabinet in an office for future research or academic purpose for not less than five years to ensure their protection. Electronic information was stored on a password-protected computer. The collected data was to be destroyed or deemed invalid after five years.

4.10.2. Informed consent

The researcher asked Wachemo University for permission to conduct a research thesis, should ethical clearance be obtained from the University of South Africa (UNISA) (see Appendix 5.1). The students were also informed that the data would only be used for research purposes.

The literature review revealed that before a researcher uses individuals' existing records consent must be given. Records of students collected for this study would not be released to another researcher for research purposes without the students' consent.

4.10.3. Debriefing

Under the principle of informed consent, I discussed the purpose of the study with the students in the sample. This allowed them to evaluate the procedures to be followed in the study and helped them to make an informed judgement on whether they wanted to participate. A debriefing session was held after the study had been completed . This tool is the form of an interview conducted with each research participant. In this interview, the reasearcher and participant talked about the study.

4.11. PILOT STUDY

A pilot study is a preliminary small-scale study that the researcher conducts to identify issues that could arise in the main study (Crossman, 2020). It allows the researcher to refine the research question, determine the best methods to address it and estimate the time and resources needed for the study. It is also used to test the research instruments to be used in the main study.

One of the instruments in this study was a test, used as both pre-test and post-test. The test was prepared by the researcher from the lesson plan and was checked for content validity by mathematics experts and the researcher's supervisor (Cronbach, 1990). The degree to which a test measures what it purports to measure constitutes its validity when compared to accepted criteria (such as the content of the given textbook and course outline). In short, content validity refers to how well a tool measures what it intends to measure in a test. Pallant (2007) argues that there is no clear-cut indicator of validity; rather, it must be checked by experts. As far as the qualitative data was concerned, the researcher considered dependability, credibility, transferability, and confirmability as trustworthiness criteria to ensure the rigour of the qualitative findings (Anney,2014). The interviews were checked by research experts.

4.11.1. *Validity and reliability of instruments*

Nature of the Instruments

The instruments in this study were: 1) test (comprised of a multiple-choice test and word problem), prepared by the researcher and validated by five university lecturers of mathematics. This was referred to as the differential calculus achievement test (DCAT); 2) an interview; and 3) a questionnaire comprising closed-ended items. The reason the researcher used both multiple-choice questions and a word problem in the test was that multiple-choice questions are no longer regarded as a tool for providing a suitable response (Sharma, 2021; Shute & Rahimi, 2017) because they do not allow a sufficiently accurate assessment of students' knowledge and skills (Whittington & Hunt, 1999). A word problem, on the other hand, allows the teacher to assess what knowledge and skills the student have and which s/he does not (Morgan, 2007). This is closely related to investigating the student's competence in a certain domain. So, the combination of the two types of questions enabled the researcher to judge the competency of the students in the subject of differential calculus during and before the study. The validity and reliability of each instrument are discussed in the following section.

4.11.2. *Validity of instrument*

Thompson (2013) argues that test validation is an ongoing process of gathering evidence to support the inferences made from test scores. Validity cannot be given a numerical index like the reliability coefficient; to ensure content validity in this study, the test, GeoGebra oriented lesson plan and course outline (please see Appendix 1.1, 1.2, and 1.3) were sent to a panel of experts in differential calculus and mathematics education (with a master's degree or PhD in mathematics education). In total, five mathematics experts and my supervisors have participated. Moreover, the table of a specification ensures content validity and interview instruments were checked with a line of research questions by experts.

4.11.3. Reliability of tests and questionnaires

One way of verifying the data collection instruments to be used in the main study was the use of a pilot study. Since the pre-test covered the subject of differential calculus and was prepared by the researcher, he selected students who were familiar with calculus and who would not be participants in the main study. The participants in the pilot study were third-year mathematics students at Wachemo University in the second semester of 2020, who volunteered to participate. They had passed both first- and second-year calculus courses, so they knew about differential calculus. These students would graduate in January 2020 and would not still be at university when the main study was scheduled to begin so there would be no possibility of information contamination between students on the campus. Again, participants in the main study were not on campus when the pilot study was conducted because of COVID-19. During the pilot study, a test and questionnaire for students were administered to 15 participants. This was done to ensure the internal reliability of questionnaires to identify the degree to which the items were cohesive.

To analyse the collected data with SPSS version 27, the researcher coded it for a pilot study. A test contains right or wrong answers (dichotomous data). If students answered correctly, the score was 1; if incorrect, the score was 0. So, *right* = 1 and *wrong* = 0 in the SPSS version 27 database. The pilot study was conducted to check the reliability of the test items that would be used in the main study. Twenty-four differential calculus tests were distributed to 15 students in the pilot study. These tests were divided into 12 procedural tests and 12 conceptual tests depending on the nature of the constructed items (Bergsten, Engelbrecht, & Kågesten, 2017; Jones & Tarr, 2007). A differential calculus test or question is in the form of a statement /item. To achieve reliability of the items, the Cronbach alpha value (α), which is the best indicator of internal reliability, was employed for both categories of tests (Pallant, 2007). To this end, item analysis was conducted to determine the item difficulty level of the differential calculus test of achievement (DCAT). Item analysis is a technique that enables the researcher to accept, reject or adjust items to be included in the main study which is an important tool to increase the effectiveness of the test (Sharma, 2021).

To assess whether the DCAT test score was reliable, Cronbach's alpha value and inter-item correlations were computed to examine the deficit in items (very simple and very difficult that disturbs another test on student's achievement). The pilot study revealed that the Cronbach alpha value for DCAT was 0.716 after deleting four items, two each from both categories of questions (conceptual and procedural understanding). Three of these items were found to have low inter-item correlations with the whole scale, affecting the reliability of the whole test, and one further item was removed by SPSS. as the item had zero variance. Thus, in the main study, student achievement and their understanding of calculus were investigated using 20 DCAT items. I divided the tests into a conceptual and procedural understanding depending on the nature of the constructed tests. Procedural questions were questions that can be obtained by following steps and whereas conceptual questions are a question that can be obtained by remembering only the formula or logic. Each category of understanding (conceptual and procedural) consisted of 10 questions items (please see Appendix 4.1).

Table 4.4 below reflects the data from the pilot study; from DCAT conceptual test categories and the difficulty index of the individual items (P): $p = \frac{n_u + n_l}{N}$, where n_l = lower bound (number of lower achievers), n_u = upper bound (number of higher achieves) and N = Number of students who took an exam(tests).

The number in the upper group(students scored high marks) who answered the item correctly= n_u can be seen by counting four students from below for all questions as the scores arranged in increased see the total part at column twelve of table 4.4 and n_l = number in the lower group(students scored low marks) who answered the items correctly that can be seen by counting four students from above for all questions as the scores arranged in increased order and N is the number of students who tried to answer the items and they are fifteen(15) students in numbers starting from student name MAA to AM. The discrimination index (Di): $DI = \frac{n_u - n_l}{n}$, where n_u and n_l is the number of the examinee in each group (upper(at the bottom of *table 4.4*)) and lower groups(at the top of the same table) respectively and n is the number of examine in each group and computed as $n = 27\% \times N = 0.27 \times 15 = 4.05 \approx 4$, which indicates that we take four

student's scores from above and four items from below of *table 4.4* after ordering the scores of students either in descending or ascending ways, depending on the total score of students on each item, as shown in Table 4.4, that the arrangement is ordered in ascending starting from 4 to 15. Student MAA scores only 4 marks out of 24 questions whereas student AM scored about 15 marks out of 24 questions.

Table 4.4 Difficulty and discrimination index of conceptual DCAT

Student	Q1	Q4	Q5	Q7	Q10	Q12	Q15	Q16	Q20	Q23	Total
MAA	1	0	0	0	0	0	0	0	1	0	4
MYW	0	1	0	0	0	0	0	0	0	0	4
AUC	0	0	0	0	0	0	0	1	0	0	5
BGD	0	0	0	0	0	1	1	1	0	0	6
SAK	0	0	1	0	0	0	0	1	1	0	8
MDJ	1	0	0	1	0	0	1	1	1	0	8
AMS	1	1	1	0	0	1	0	0	1	0	8
CJ	1	1	0	1	0	1	0	1	1	0	9
TKB	1	0	0	1	1	0	1	0	1	0	10
DNL	1	0	1	0	0	0	0	1	1	0	11
SK	1	1	0	1	0	1	0	1	1	0	11
FB	1	1	1	1	1	0	0	1	1	1	12
AMA	1	0	1	1	1	0	0	0	1	0	12
TTD	1	1	1	0	0	1	0	1	1	0	13
AM	1	0	1	1	1	0	0	0	1	1	15
Lower bound	1	1	0	0	0	1	1	2	1	0	
Upper bound	4	2	4	3	3	1	0	2	4	2	
Difficulty index	0.33	0.2	0.267	0.2	0.2	0.133	0.067	0.267	0.333	0.133	
Discrimination index	0.75	0.25	1	0.75	0.75	0	-0.25	0	0.75	0.5	

In the same way, the difficulty index and discrimination index of the procedural test of DCAT was computed and tabulated, as indicated in Table 4.5:

Table 4.5 Difficulty and discrimination index of procedural DCAT

Student	Q2	Q3	Q6	Q8	Q9	Q11	Q14	Q17	Q19	Q22	Total
MAA	0	0	0	0	0	0	0	1	0	0	4
MYW	0	0	0	0	0	1	0	1	0	0	4
AUC	0	0	1	0	1	0	1	0	0	0	5
BGD	0	0	0	0	0	0	0	1	0	0	6
SAK	0	0	0	0	1	0	0	1	1	0	8
MDJ	1	0	0	0	0	0	0	1	1	0	8
AMS	0	0	1	0	0	0	1	0	0	0	8
CJ	1	0	1	0	0	0	0	1	0	0	9
TKB	0	0	0	0	1	1	1	1	0	0	10
DNL	0	1	0	1	0	1	1	1	1	0	11
SK	0	0	0	1	0	1	1	1	1	0	11
FB	0	1	1	0	0	0	0	1	0	0	12
AMA	1	1	1	1	1	1	0	1	0	0	12
TTD	0	1	0	1	0	0	1	1	1	1	13
AM	1	1	1	1	1	1	1	1	0	1	15
Lower bound	0	0	1	0	1	1	1	3	0	0	
Upper bound	2	4	3	3	2	2	2	4	1	2	
Difficulty index	0.13	0.267	0.267	0.2	0.2	0.2	0.2	0.467	0.067	0.133	
Discrimination index	0.5	1	0.5	0.75	0.25	0.25	0.25	0.25	0.25	0.5	

Using the indices reflected in Tables 4.4 and 4.5, the researcher analysed the items of the conceptual and procedural test of DCAT, as summarised in Table 4.6 by using the criteria given in table 4.7 and table 4.8.

Table 4.6 Summary of Item analysis of DCAT

Item analysis			
Tests	Difficulty index (P)	Reliability Coefficient (KR-20)	Remark
Procedural (P) test items	0.2131	0.649	Four items need to be revised (Q9,11,14,19) as all are marginal and difficult items.
Conceptual (C) test items	0.213	0.525	Three items need to be revised (Q12 is a poor and very difficult item; Q16 is average in terms of difficulty and a poor item in terms of differentiation between students, and Q4 is a very difficult and only marginally useful for identifying students); in addition, one item (Q15) is rejected and replaced, as it has a negative discrimination index and is very difficult please see table 4.7 and 4.8)(Sharma, 2021).

4.11.4. Interpretation of difficulty index of conceptual and procedural understanding test/items

I used the following table 4.7 the criteria of categorising the difficulty index of procedural and conceptual items(Sharma, 2021).

Table 4.7 The criteria of categorising the difficulty index of procedural and conceptual items

S. N	Difficulty index	Number of items	Evaluation of item	Recommendation
1	<0.2	3 for Conceptual items(Q12,15,23) and 3 for Procedural items (Q2,19,22)	Most difficulty	Remove and replace
2	0.2-0.39	Seven questions for Conceptual items (Q1,4,5,7,10,16,20) and 7 questions for procedural test items(Q3,6,8,9,11,14,17)	Difficulty	Keep
3	0.4-0.59	0	Moderately difficulty	Keep
4	0.6-0.79	0	Moderately easy	Keep
5	0.8-0.89	0	Easy	Keep
6	>0.9	0	Easiest	Remove

4.11.5. Interpretation of discrimination index of conceptual and procedural understanding test/items

I used the following table 4.8 the criteria of categorising the discrimination index of procedural and conceptual items(Sharma, 2021).

Table 4.8 The criteria of categorising the discrimination index of procedural and conceptual items

S. N	Discrimination index	Number of items	Evaluation of items	Recommendation
1	Negative	1 for conceptual item(Q15) and 0 for procedural item	Worst item	Surely Discard
2	<0.2	1 for conceptual item(Q16) and 0 for procedural item	Marginal item	Revise/discard
3	0.2-0.29	1 for conceptual item(Q4) and five for procedural items(Q9,11,14,17,19)	Moderately discriminating item, fair item	Keep
4	0.3-0.39	0 for conceptual item and	Discriminating item, good item	Keep

5	>0.4	6 for conceptual items(Q1,5,7,10,20,23) and five for procedural items(Q2,3,6,8,22)	Very discriminating item, very good items	Keep
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Next, the validity and reliability of questionnaires were discussed. The closed-ended questionnaire was adopted and rearranged according to the context of the study following research by Bu, Mumba, Henson, and Wright (2013), and the researcher computed its reliability by using a five-point Likert scale starting from *strongly agree* = 1 to *strongly disagree* = 5, with the scales between 1 and 5 coded as *Agree* = 4, *Neutral* = 3 and *Disagree* = 2. As the closed-ended questionnaire was intended for students, the questionnaire was distributed for students and the reliability of the questionnaire obtained from students who participated in the pilot study was computed and Cronbach's alpha was found to be 0.917 for students, implying that the questionnaires were reliable. The questionnaire comprised 14 closed-ended items and five interview questions to investigate the perceptions of students on the use of GeoGebra as an instructional tool.

If the value of Cronbach's alpha of an item is equal to or greater than 0.5, then the item is considered acceptable, implying that it is reasonably reliable (Salvucci, Walter, Conley, Fink, & Saba, 1997; Taber, 2018).

4.12. A GEOGEBRA ORIENTED LESSON PLAN ON THE TOPIC DERIVATIVE OF CALCULUS

Scaffolding students who are learning calculus with the help of GeoGebra Mathematical software is one of the focuses of this study. To use scaffolding in the classroom the teacher (MKO) needs to be prepared. Teaching materials (books, lesson plans from the curriculum), knowledge (of technology, subject matter, and pedagogy) may help teachers to scaffold their students in the classroom. The combination of these three conditions is referred to as TPACK. Ruthven (2012) identifies five features of the structure of classroom practice and discusses how they relate to the use of digital tools and materials in the classroom: working environment (physical context of lessons), resources (collection of mathematical tools and materials in classroom use), activity format (generic templates for action and interaction which frame the contributions of teachers and students to particular

types of lesson segments), curriculum script (involving teacher knowledge gained in the course of her/his own experience of learning and teaching a given topic) and time economy (managing the conversion of time available for classroom activity to didactical time, measured in terms of advances in knowledge). The study was conducted over seven hours of tuition, running over two weeks for both the experimental and conventional methods of teaching and learning differential calculus. The lesson plan was prepared by the researcher from the Ethiopian Harmonized Curriculum on calculus I (please see Appendix 1.2 and 1.3).

The procedure followed in the development of the GeoGebra oriented lesson plan-based learning tools for differential calculus (please see Appendix 1.2 and 1.3 and Section 3.6.1) comprised two phases. Phase 1 was the experts' suggestions that ensured that the study followed the curriculum of the university (please see Section 4.11.2), and the second phase was the pilot study (please see Section 4.11). The two phases were accomplished as discussed in this chapter. PowerPoint and the GeoGebra applet were used when conducting GeoGebra oriented lesson plan-based learning in the classroom.

CHAPTER 5 **DATA ANALYSIS AND INTERPRETATION**

5.1. INTRODUCTION

The purpose of this study was to determine the effect on students' learning of calculus by being taught through GeoGebra Mathematical software at the university level. To achieve this purpose, instructional materials were designed, and the instruments for the study, which included a questionnaire featuring Likert scales and differential achievement tests of both conceptual and procedural understanding, were implemented. The study included one exogenous variable: teacher perceptions of using GeoGebra for teaching calculus at university. As student ability was the same in the experimental group and the control group there was no need to consider the exogenous variables (see Section 5.3.1). The dependent variable was a differential achievement, which was measured using a pre-test and a post-test. A mixed-methods approach was followed to achieve the goals of the study.

The research questions addressed in this study were: (1) How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing methods (control Group 2)? (2) How does the level of proficiency in differential calculus compare in the experimental group pre-and-post the intervention? (3) What are students' experiences and perceptions of using mathematical software (GeoGebra) in learning calculus concepts?

In addressing these research questions, I used a developed cycle model that was the theoretical framework of the study:

There are nine steps to be considered and discussed in this model, starting from section 5.3 as follows:

5.2. BIBLIOGRAPHY INFORMATION OF PARTICIPANTS

The first section of the questionnaire used in the study was used to obtain demographic information such as the gender and age of the participants in the experimental and control groups. Figure 5.1 shows the information for the experimental and control groups of students.

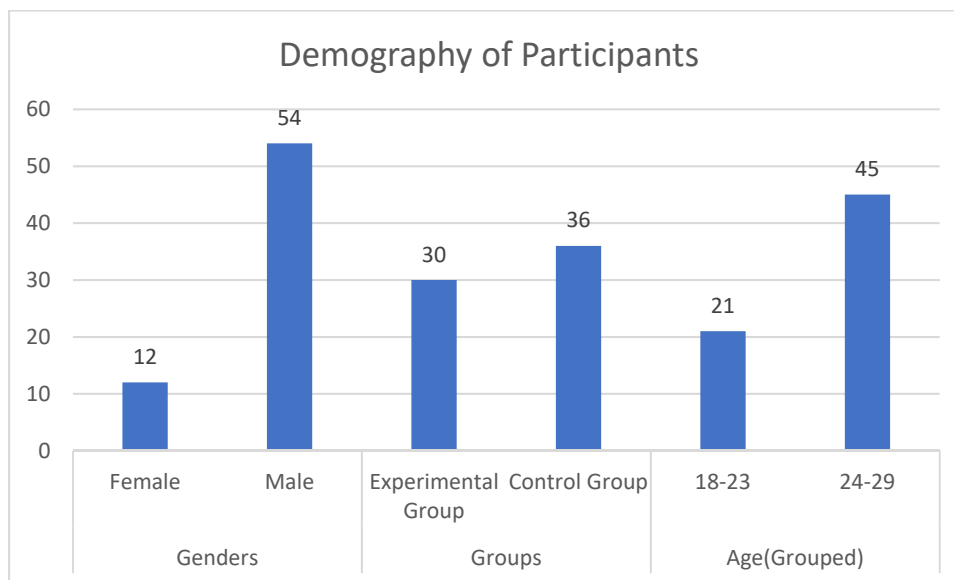


Figure 5.1 Participants' Demographic Information

5.3. STUDY CONTEXT (ENVIRONMENT)

This section provides information on the area (environment) in which the study took place, in this case, a laboratory classroom at the university. Vygotsky's ideas are reflected in the community of practice thinking that addresses the need for continuous professional development and lifelong learning in the environment (Heinze & Procter, 2006). The environment can be viewed from two standpoints: the biological perspective (phylogenesis and fatal development) and the psychological perspective. The 'environment' or 'real world' can be articulated and described only in terms of viable intangible structures by observers (Glaserfeld, 1996). Within the school environment, teaching and learning activities occur, using a variety of reinforcements, such as praise,

rewards, and grades. As this study depends on Vygotsky's theory, I based my view of the environment on psychological perspectives. To initiate articulation of things in the environment, indicated on the left side of the figure below, I first identified the environment.

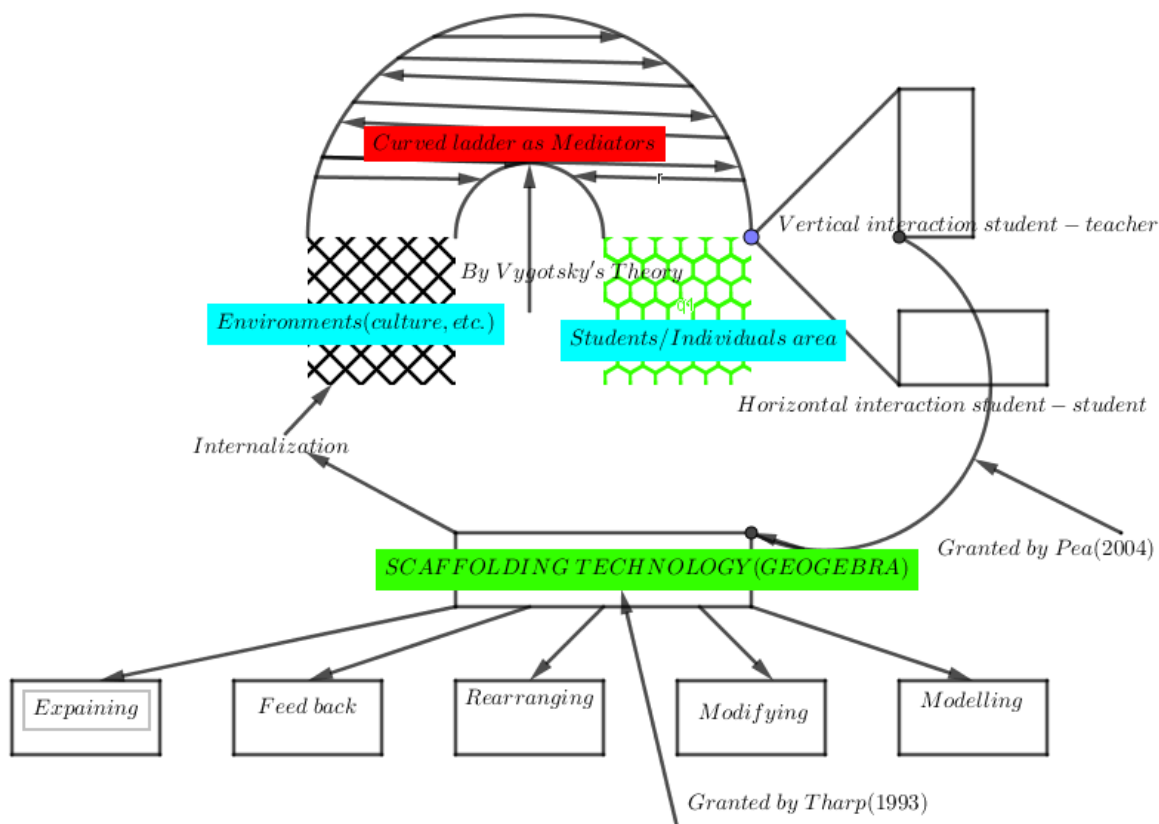


Figure 5.2 The base of the cycle model

The computer laboratory in the Department of Mathematics at Wachemo University was not well organised and not suitable for intervention. Thus, I searched for and found a laboratory before starting the main study. Figure 5.3 provides images of this laboratory at the university. I felt that it was important to determine the study area before commencing with the intended intervention, which is the base of the cycle model.



Figure 5.3 Wachemo University laboratory

5.4. IDENTIFICATION OF INDIVIDUAL ABILITY WITHIN THIS ENVIRONMENT

The next stage of the cycle model was the identification of individual ability. This section considers teacher professional development and student ability. It was important to establish students' abilities before the intervention. I did this by administering a pre-test on proficiency in differential calculus developed by a researcher (see the validity and reliability of the test (see Section 4.11.1). Students' ability or proficiency was analysed by testing two types of understanding: conceptual and procedural understanding. According to the ASSURE model (analysis, state objectives, select instructional materials, utilise materials, require students, and evaluate), this step is regarded as the first step in the analysis, that is identifying students' characteristics on entering the programme (Baran,

2010). The ASSURE model does not take the environment/workplace setting in which the programme would be employed into account (see Figure 5.3 and Section 5.3).

5.4.1. Analysis of group differences in pre-test of differential calculus achievement

Differential calculus is an important part of mathematics because it serves as a basis for more advanced courses in mathematics and engineering at secondary and higher education levels. It has many applications in real life. In this section, students' scores on differential calculus achievement tests (DCAT) were obtained. The test comprised 20 items, 10 items on procedural knowledge and 10 on conceptual knowledge, developed by the researcher and administered at the beginning of the study. This pre-test was used to investigate the initial differences (if any) between the two groups in the study in terms of their performance in a differential calculus achievement test (DCAT) to address the two research questions in the study: (1) How does the level of proficiency in differential calculus in students teach using GeoGebra (experimental) and those taught through conventional lecturing (control) compare? (2) How does the level of proficiency in differential calculus in the experimental group compare pre-and-post the intervention incorporating the use of GeoGebra? Scores obtained from the pre-test were analysed by applying an independent samples T-test, which compares the means of the two groups as shown in Table 5.2. below. To ensure the use of the T-test (Elliott & Woodward, 2007), I computed the normality of pre-test recorded data, as shown in Table 5.1. This showed that the pre-test was normally distributed in both groups in the study as the significance level in both tests was greater than 0.05.

Table 5.1 Test normality of pre-test

	Group	Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	Df	Sig.	Statistic	df	Sig.
Pre-test	Experimental	.119	30	.200	.946	30	.133
	Control	.145	36	.055	.957	36	.172

Table 5.2 Overall descriptive statistics of the two groups' proficiency in differential calculus before the intervention

Differential Calculus achievement (Before Intervention)				
Groups	N	Mean	SD	Std. Error
Experimental	30	27.0000	9.96546	1.81944
Control	36	26.6667	10.82326	1.80388
Total	66	26.8182	10.36400	1.27572

Table 5.2 shows a mean difference of 0.33333 between Group1 ($M=27.000$) and Group 2 ($M=26.6667$). This indicates that the two groups were very similar as the difference was not significant at 0.05 ($p=0.898>0.05$) (see Table 5.3). Students in the two groups had similar academic backgrounds, with each group consisting of both high and low achievers. This is discussed in Section 5.3.2 below.

The uniformity in the results of the two groups was a good starting point for me to be able to deduce whether the effect of the treatment after the intervention had occurred. Hence, if the experimental group scored higher than the control group on the post-test, the researcher could assume that the differences had occurred because of the treatment in the study, by controlling other confounding variables. In this regard, I tried to control all the possible confounding variables such as time allocation for a lesson (see Appendix 1.2 and 1.3), the effect of the teacher (this was controlled by using the researcher as the teacher for both groups), and topics covered (this was controlled by focusing on the curriculum (see Appendix 1.1)). The one-way ANOVA is summarised in Table 5.3 below.

This provided further analysis of the two groups and within the groups (experimental and control).

Table 5.3 Overall One-Way Analysis of Variance Summary table comparing groups' achievement in differential calculus before treatment

Differential Calculus Achievement Test (Before Intervention)					
	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	1.818	1	1.818	.017	.898
Within Groups	6980.000	64	109.063		
Total	6981.818	65			

The results in Table 5.3 show that there was a statistically non-significant difference in pre-test differential calculus achievement ($F (1,64) = 0.017$, $p = 0.898 > 0.05$). The dependent variable in this study was students' proficiency in differential calculus and this may have been influenced by the other variables (groups). Hence, the study investigated the conceptual and procedural understanding of both groups before treatment as a starting point, as tabulated in Table 5.4.

Table 5.4 Overall descriptive statistics of achievement in differential calculus of the two groups (Conceptual and Procedural understanding) before treatment

		Student's proficiency within Groups	
Groups		Pre-test Conceptual	Pre-test Procedural
Experimental	Mean	16.3333	10.6667
	N	30	30
	Std. Deviation	6.55656	5.83292
Control	Mean	13.8889	12.7778
	N	36	36
	Std. Deviation	7.28120	5.90937
Total	Mean	15.0000	11.8182
	N	66	66
	Std. Deviation	7.01646	5.92494

Table 5.5 Students' proficiency by gender before intervention

		Students' proficiency by * Gender	
Gender		Pre-test Procedural	Pre-test Conceptual
Female	Mean	12.9167	12.0833
	N	12	12
	Std. Deviation	7.21688	6.89477
Male	Mean	15.4630	11.7593
	N	54	54
	Std. Deviation	6.95460	5.75911
Total	Mean	15.0000	11.8182
	N	66	66
	Std. Deviation	7.01646	5.92494

Table 5.4 shows that the mean scores of experimental groups 1 on both pre-procedural and pre-conceptual understanding of DCAT were $M=10.6667$ and $M=16.3333$ respectively with a mean difference of 5.6666. This indicates that students in this group had better conceptual understanding than procedural understanding before the intervention. The mean for the control group 2 was $M=12.7778$ and $M=13.8889$ for pre-conceptual and pre-procedural understanding respectively, with a mean difference of 1.1111, indicating that some students in the control group had the same level of procedural and conceptual understanding of differential calculus before the intervention. Table 5.5 shows that both male and female students had a better conceptual understanding of differential calculus than procedural understanding before the intervention. An ANOVA was calculated to determine whether if there was any significant difference between the mean scores of the groups in terms of two types of knowledge. The one-way ANOVA is summarised in Table 5.6 below.

Table 5.6 Overall One-Way Analysis of Variance Summary table comparing groups' proficiency in differential calculus before treatment.

Understanding		Sum of Squares	Df	Mean Square	F	Sig.
Pre-conceptual	testBetween Groups	97.778	1	97.778	2.017	.160
	Within Groups	3102.222	64	48.472		
	Total	3200.000	65			
Pre-test procedural	Between Groups	72.929	1	72.929	2.113	.151
	Within Groups	2208.889	64	34.514		
	Total	2281.818	65			

Table 5.6 indicates that there were statistically non-significant differences in both conceptual and procedural understanding of differential calculus before the treatment, with the values $F(1,64) = 2.017$, $p=0.160 > 0.05$ and $F(1,64) = 2.113$, $p=0.151 > 0.05$ respectively. Next, I was interested in investigating students' abilities within each group in terms of the two types of knowledge involved in understanding differential calculus.

5.4.2. Analysis of students' ability within groups

When dividing students into two groups within the groups, I considered their pre-test score to investigate the GeoGebra treatment effects on diverse achievers. These were divided into two groups, higher achievers and lower achievers, using the pre-test score median of each group. Next, I categorised students into nested groups (below the median of 27.5 (low ability), 16 in number, and above-median of 27.5 as high ability (14 in number) for the experimental group. Of these students, only two of the female students were categorised as high achievers and none were higher achievers in procedural proficiency or conceptual proficiency. However, the sum of the two (procedural proficiency and conceptual proficiency) or one proceed the other (procedural proficiency proceed conceptual proficiency and vice versa) resulted in their categorisation as high achievers (Finn & Metcalfe, 2010; National Research Council, 2001; Rittle-Johnson & Alibali, 1999). Twelve male students were higher achievers but only one male student was a high achiever in procedural proficiency; the others were becoming high achievers, as reflected in the sum of the scores on the two types of proficiency before intervention. Of the 36 students in the control group, 17 were included in the high achiever category as their scores were higher than the median of 25; 19 students were low achievers as their scores fell below the median of 25. Of these students, only three female students were high achievers, and none were high achievers in procedural proficiency or conceptual proficiency; the sum of their scores on the two types of proficiency allowed them to be categorised as high achievers (see Table 5.7). Fourteen males' students and three female students were high achievers in the procedural understanding of calculus; 12male students and two female students were high achievers in procedural understanding.

Table 5.7 Descriptive statistics of students' proficiency by gender before treatment

Proficiency	Student Ability	Genders	Groups	Mean	SD	N
Pre-conceptual.	Low ability	Female	Experimental	6.7	2.9	3
			Control	10.0	4.1	4
			Total	8.6	3.8	7
		Male	Experimental	13.5	4.7	13
			Control	9.7	5.8	15
			Total	11.4	5.6	28
		Total	Experimental	12.2	5.2	16
			Control	9.7	5.4	19
			Total	10.9	5.4	35
	High ability	Female	Experimental	17.5	3.5	2
			Control	20.0	8.7	3
			Total	19.0	6.5	5
		Male	Experimental	21.7	4.4	12
			Control	18.2	6.1	14
			Total	19.8	5.6	26
		Total	Experimental	21.1	4.5	14
			Control	18.5	6.3	17
			Total	19.7	5.6	31
	Total	Female	Experimental	11.0	6.5	5
			Control	14.3	7.9	7
			Total	12.9	7.2	12
		Male	Experimental	17.4	6.1	25
			Control	13.8	7.3	29
			Total	15.5	6.95	54
		Total	Experimental	16.3	6.6	30
			Control	13.9	7.3	36
			Total	15.0	7.0	66
Pre-procedural	Low ability	Female	Experimental	5.0	5.0	3
			Control	11.3	4.8	4
			Total	8.6	5.6	7
		Male	Experimental	7.7	4.8	13
			Control	9.3	4.95	15
			Total	8.6	4.9	28
		Total	Experimental	7.2	4.8	16
			Control	9.7	4.9	19
			Total			

High ability	Female	Total	8.6	4.9	35
		Experimental	15.0	7.1	2
		Control	18.3	5.8	3
	Male	Total	17.0	5.7	5
		Experimental	14.6	3.96	12
		Control	15.7	5.1	14
	Total	Total	15.2	4.6	26
		Experimental	14.6	4.1	14
		Control	16.2	5.2	17
Total	Female	Total	15.5	4.7	31
		Experimental	9.0	7.4	5
		Control	14.3	6.1	7
	Male	Total	12.1	6.9	12
		Experimental	11.0	5.6	25
		Control	12.4	5.9	29
	Total	Total	11.8	5.8	54
		Experimental	10.7	5.8	30
		Control	12.7778	5.9	36
		Total	11.8	5.9	66

N.B pre-conc = pre-test conceptual on pre-test, pre-pro = pre-test-procedural on pre-test

5.4.3. The difference between students' proficiency and students' ability

On admission to both groups of the study, students' ability was the same before intervention they had on differential calculus. Although students' ability before being introduced to differential calculus was very similar (see Table 5.2,3), there were some differences in their proficiency (see Table 5.6,8).

Table 5.8 Overall One-Way Analysis of Variance Summary: students' proficiency in differential calculus compared to their ability before treatment.

Variables		Sum of Squares	df	Mean Square	F	Sig.
Pre-test conceptual	Between Groups	1278.940	1	1278.940	42.608	.000
	Within Groups	1921.060	64	30.017		
	Total	3200.000	65			
Pre-test procedural	Between Groups	785.505	1	785.505	33.597	.000
	Within Groups	1496.313	64	23.380		
	Total	2281.818	65			

Table 5.8 shows whether in terms of their ability, experimental and control group students' procedural and conceptual understanding of differential calculus differed before the treatment. The table shows that there were statistically significant differences in both conceptual and procedural understanding of differential calculus by student ability before the treatment with the values $F(1,64) = 42.6$, $p < 0.5$ and $F(1,64) = 33.6$, $p < 0.5$. To determine the extent of the difference between the two groups in terms of the two proficiencies, I used effect size (ES). For the ANOVA test, the effect size can be calculated by the formula:

$Eta\ squared = \frac{\text{Sum of the squares between groups}}{\text{Total sum of squared}}$ (Cohen et al., 2018). According to the formula, the effect size of the pre-conceptual understanding of the experimental and the control group was computed as:

$$\begin{aligned}
 Eta\ squared &= \frac{\text{Sum of the squares between groups}}{\text{Total sum of squared}} \\
 &= \frac{1278.940}{3200.000} \\
 &= 0.4
 \end{aligned}$$

Eta squared = 0.4 indicates a small effect size; this, in turn, implies that there is a small difference between the two groups (experiment and control) in terms of pre-test conceptual understanding in terms of achievement (Cohen et al., 2018).

The effect size of the pre-test procedural understanding of the experimental and control group was computed as:

$$\begin{aligned} \text{Eta squared} &= \frac{\text{Sum of the squares between groups}}{\text{Total sum of squared}} \\ &= \frac{785.505}{2281.818} \\ &= 0.34 \end{aligned}$$

This indicates that pre-test procedural understanding of students had a small effect size, implying that there were small statistically significant differences in the two groups in pre-test procedural in differential calculus.

The next step according to the cycle model was to develop the aims/objectives of teaching a lesson with GeoGebra applet. This is discussed below.

5.5. STATE OBJECTIVES OF TEACHING A LESSON WITH GEOGEBRA

The experimental group was introduced to the use of the GeoGebra mathematical applet in learning calculus in the classroom during the first session of the intervention. In this step, I familiarised students with this new mathematical software so that they would understand what they would gain from the program and what they would be able to do once the lesson had been completed. I used the review literature on GeoGebra to explain the benefits they would gain from the intervention. I also explained what topics in differential calculus would be covered by the program. In session one I introduced the following keynotes using a PowerPoint presentation:

- GeoGebra emerged in 2001 and was developed by master's student Hohenwarter.
- Over 100 million people have visited www.geogebra.org.
- There are over 10 million active users monthly around the world.
- There are over 1 million interactive materials available.
- It is free for both students and teachers to use for non-commercial educational purposes.

- It is a combination of a computer algebra system and a geometric dynamic system.
- It is multilingual and has been translated into 55 world languages.
- Over 90 countries in the world use GeoGebra for teaching and learning purposes.
- GeoGebra is also used by artists.
- Anyone can download GeoGebra Classic 5 on a computer or smart android phone from www.geogebra.org for mathematical purposes.

5.6. DESIGN TEACHING MATERIALS

To implement an intervention, I designed teaching material or lesson plans (see Appendices 1.1 and 1.2) that were compatible with the use of GeoGebra mathematical software in the classroom. Student abilities and objectives in using the program were identified. The following stage was to design teaching material that was compatible with the use of this technology in a classroom setting.

5.7. IMPLEMENTATION OF LESSON PLAN IN THE CLASSROOM

The researcher used the lesson plan and the teaching materials in the classroom(see Appendix1.1,1.2, and 1.3). These teaching materials were of two types. One set was for the experimental group and was integrated into the GeoGebra applet using PowerPoint presentations. The second was materials designed for a traditional 'chalk and talk' lesson. The topics included in the teaching and learning process for both groups were:

- Definition of derivatives
- Equation of tangent line and slopes of tangent lines
- Notations of derivatives
- Basic derivatives rules
- Examples of derivatives of different functions
- Higher-order derivatives
- Implicit derivatives and equation of a tangent line
- Applications of derivatives

- Extreme values of functions
- Monotone of functions
- Concavity of functions

Some examples of how the GeoGebra applet was integrated into the derivatives of the function.

Find $f'(x)$ for all x where $f(x) = x^2$

$$f'(x) = \lim_{x \rightarrow t} \frac{f(x) - f(t)}{x - t}$$

$$= \lim_{x \rightarrow t} \frac{x^2 - t^2}{x - t} = \lim_{x \rightarrow t} \frac{(x-t)(x+t)}{x-t} = \lim_{x \rightarrow t} x + t$$

Therefore, $f'(x) = 2x$

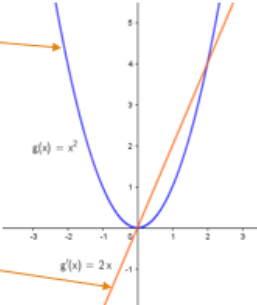


Figure 5.4 The derivatives of the function $f(x) = x^2$ and its visualisation in a graph

Figure 5.4 shows that once the function is inserted into the menu of GeoGebra, the GeoGebra applet immediately sketches a blue graph when one hits the enter key on the keyboard and types f' or $f'(x)$ in the menu. Hitting the enter key again, GeoGebra gives $f'(x) = 2x$ immediately with its graph. The following figure demonstrates how students can find the equation of a tangent line to the curve $x^3 + y^3 = 2xy$ at the point $B = -0.74$. The figure indicates how students go through activities by self-scaffolding while the teacher presents the lessons with the help of PowerPoint.

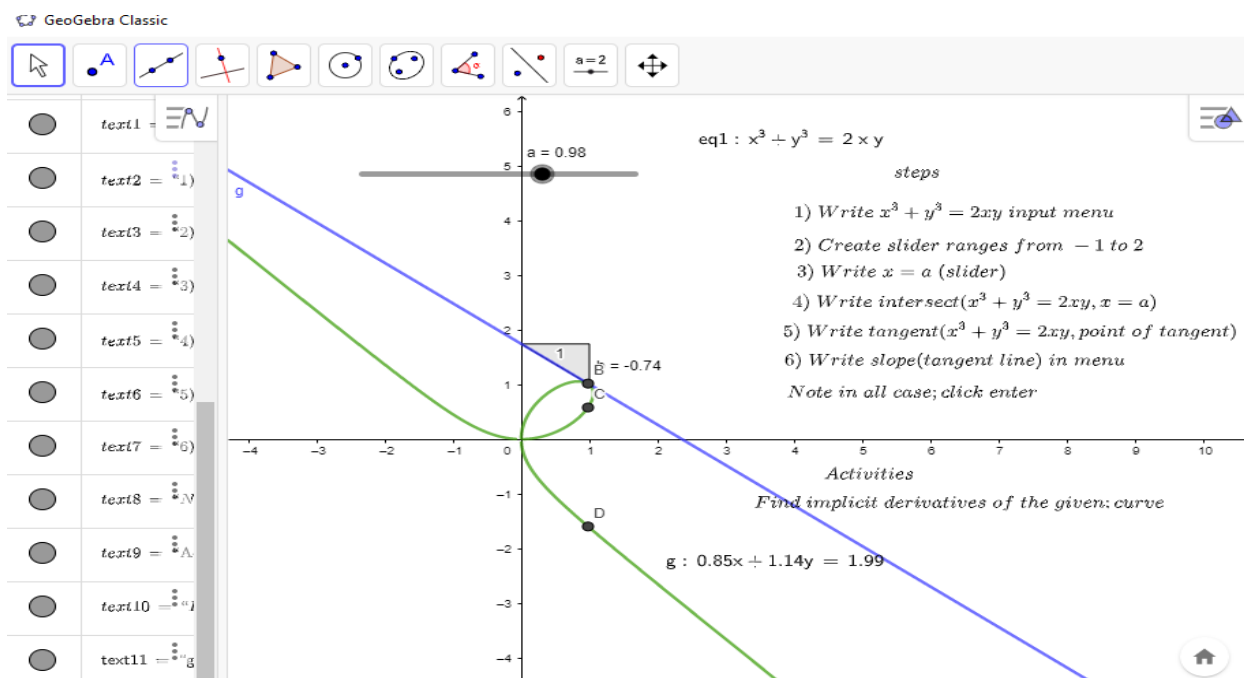


Figure 5.5 The equation of tangent line computed by GeoGebra

By moving the slider backwards and forwards, the student understands that there is an infinite number of equations of a tangent line to the given curve as the curves have an infinite number of points. But the fixed answer of the equation of a tangent line to the curve at the point $B = -0.74$ to the curve $x^3 + y^3 = 2xy$ is $g: 0.85x + 1.14y = 1.99$, as shown in the figure above.

The teacher can ask students to sketch the graph of a given function with the help of GeoGebra, and to fix the correct point on the graph. The teacher poses the questions and students search for the point on the graph by moving the slider backwards and forwards to find the fixed point on the curve. The following question requires students to find a point on the curve $f(x) = x^3 + 2x^2 + 1$ such that the equation of the tangent line to the graph is $y = 7x - 3$ is sketched by GeoGebra applet by moving the slider. The following figures show students who obtained the correct answer (Figure 5.5) and an incorrect answer (Figure 5.6).

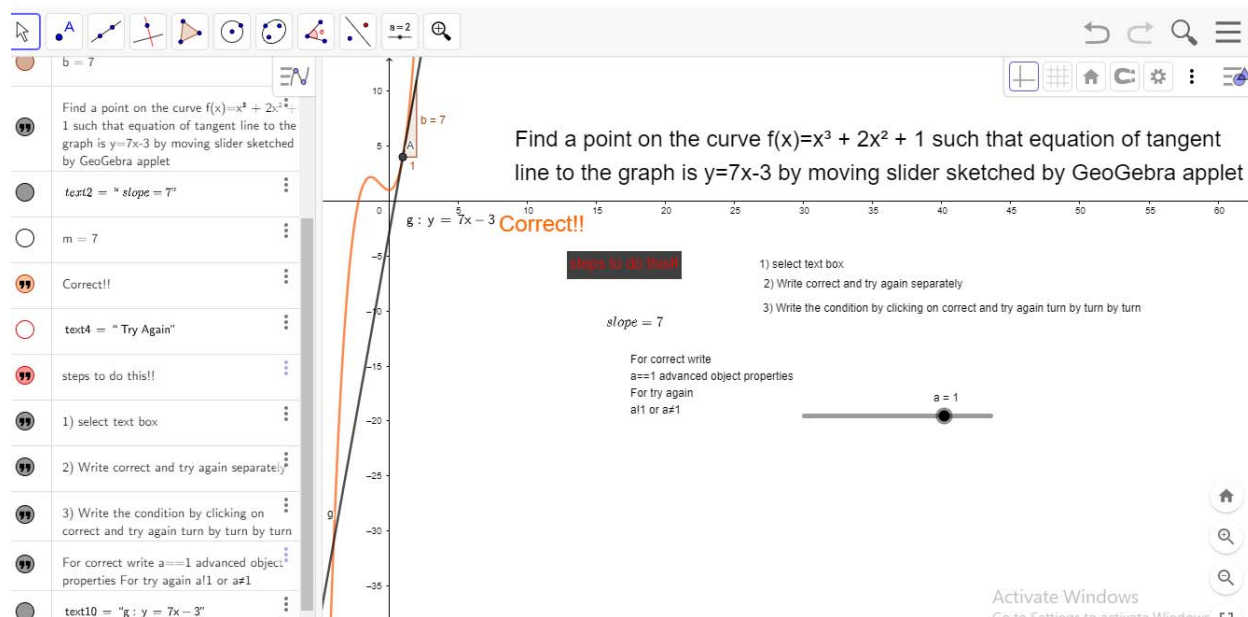


Figure 5.6 The correct answer

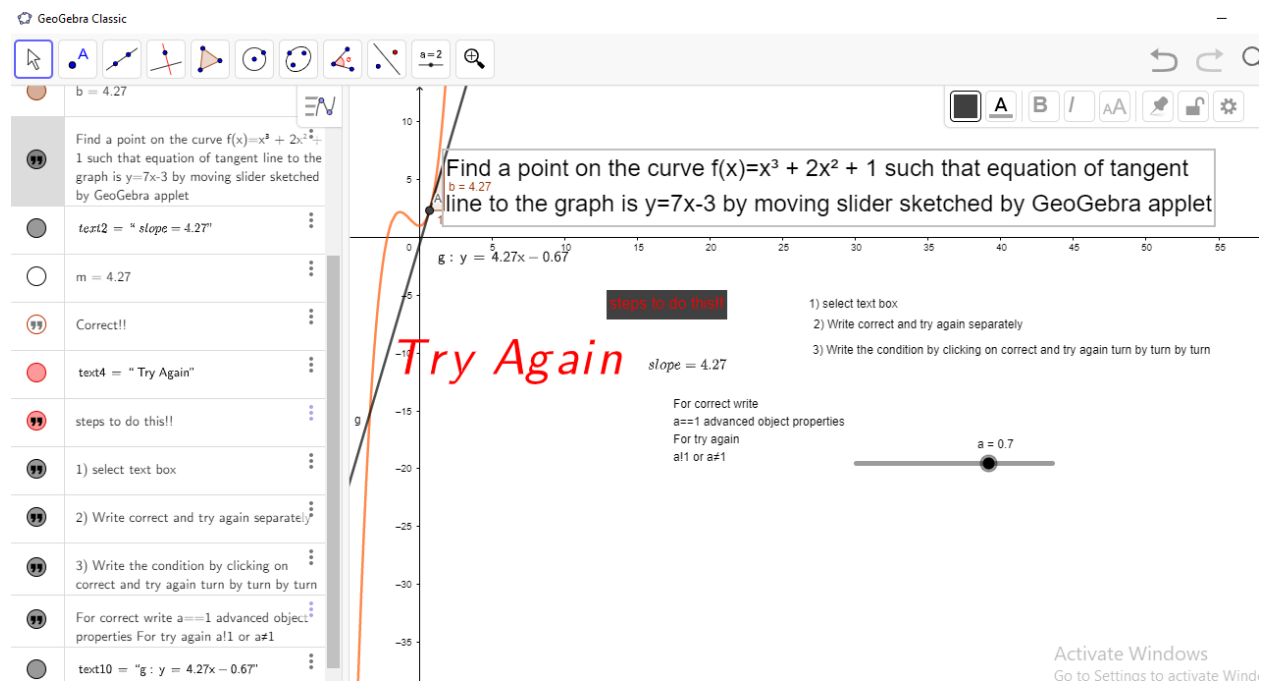


Figure 5.7 An incorrect answer

In cases where students provide incorrect answers, they scaffold themselves or the teacher scaffolds them by using PowerPoint to keep a country's Covid-19 protocol. If

students continue to provide incorrect answers but receive no help them in understanding where they are going might lose interest in learning mathematics (Thambi & Eu, 2013). PowerPoint lesson presentations in the classroom are a wonderful tool for teaching that allows learners to see different images and integrate new information with existing knowledge in both a student and a teacher-directed situation, and in teacher-student interaction, which reflects Vygotsky's theory (Pugsley, 2010). The following figures provide some examples of students scaffolding themselves and of teachers scaffolding students in the classroom.

5.8. FEEDBACK STAGES OF CYCLE MODEL

After the implementation of the teaching and learning of these topics in differential calculus with the help of GeoGebra mathematical software in the experimental group and with the aid of traditional methods in the control group, students' feedback on the activities in the classroom setting was discussed. In this step of the cycle model, I administered the post-test to both groups. I also gave the experimental group a questionnaire designed to elicit students' perceptions of the use of GeoGebra in the learning process.

During the interventions, the activities suggested by Tharp (1993) were used in the classroom setting. These activities included scaffolded feedback on how to find the solution to the problem provided by the teachers. If students are given the correct answer immediately after making a mistake, the correct information will be better remembered. Finn and Metcalfe (2010, p. 1) argue that "scaffolded feedback" builds on retrieval practice by providing "incremental hints" until students are able to find the correct answer themselves. In the present study, about 70% of the students needed scaffolding to manipulate the given activities using GeoGebra in the classroom. Most of the students responded that they did not get time to scaffold themselves. About 70% were more successful, scoring high marks from the scaffolding within the classroom after the treatment (see Figure 5.5). This is discussed in detail in section 5.9. The next step in the cycle model was to investigate the mean gain or loss in students' proficiency or understanding after the intervention.

5.8.1. Analysis of group differences in post-test of differential calculus

After the intervention had been completed, the post-test was administered to both the experimental and the control group. The research questions of the study (1) How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group 2)? and (2) How does the level of proficiency in differential calculus compare within the experimental group (Group 1) pre-and post-intervention incorporating the use of GeoGebra? To address these questions, a post-test was administered to both groups. The recorded post-test scores achieved after the intervention were analysed and are reflected in Table 5.9.

Table 5.9 Overall descriptive statistics for two groups on differential calculus achievement after the treatment

Comparison of Pre-test Scores and Post-test Scores of Groups			
Interventions		Pre-test Scores	Post-test Scores
Experimental (N=30)	Mean	27.0000	41.1667
	Std. Deviation	9.96546	13.81424
Control (N=36)	Mean	26.6667	31.1111
	Std. Deviation	10.82326	11.40871
Total	Mean	26.8182	35.6818
	Std. Deviation	10.36400	13.44179

Table 5.9 shows that the mean score of the experimental Group 1 in the post-test was $M = 41.1667$ and that of the control Group 2 was $M = 31.1111$; the mean difference between the two groups was 10.05556 , indicating that the scores of the two groups were significantly different at 0.05 ($p = 0.002 < 0.05$) after the intervention (see Table 5.11). To determine which gender was responsible for the difference, I computed the overall descriptive statistics for the analysis of gender, as tabulated in Table 5.11.

Table 5.10 Pre-test Scores and Post-test Scores by Gender

Pre-test Scores and Post-test Scores by Gender			
Interventions		Pre-test Scores	Post-test Scores
Female	Mean	25.0000	31.2500
	Std. Deviation	11.07823	9.32372
Male	Mean	27.2222	36.6667
	Std. Deviation	10.26382	14.07527
Total	Mean	26.8182	35.6818
	Std. Deviation	10.36400	13.44179

Table 5.10 shows that both male and female students had benefited from the intervention. Next, I investigated which students' proficiency was causing the differences. For this, an ANOVA was calculated to investigate the difference in students' achievement in both types of knowledge in the post-test of differential calculus. These results are tabulated in Table 5.11.

Table 5.11 Overall One-Way Analysis of Variance Summary table comparing groups on differential calculus achievement after the treatment.

Comparing groups on differential calculus understanding						
Statistic		Sum of Squares	Df	Mean Square	F	Sig.
Pre-test Scores	Between Groups	1.818	1	1.818	.017	.898
	Within Groups	6980.000	64	109.063		
	Total	6981.818	65			
Post-test Scores	Between Groups	1654.596	1	1654.596	10.495	.002
	Within Groups	10089.722	64	157.652		
	Total	11744.318	65			

The results in Table 5.11 show that there was a statistically significant difference in students' achievement in differential calculus post the intervention ($F(1,64) = 10.495, p = 0.002 < 0.05$). There was a statistically significant difference in students' achievement in the pre-test of differential calculus ($F(1,64) = 0.17, p = 0.898 > 0.05$) with effect size (ES) $d = 1$. Thus, it could be argued that the improvement was the result of the treatment. Students' results on the test of conceptual and procedural understanding of differential calculus were analysed.

Research Question1: Table 5.12 shows the level of proficiency in terms of students' conceptual and procedural understanding in differential calculus achievement when taught using GeoGebra (experimental Group 1) and when taught through conventional lecturing (control Group 2).

Table 5.12 Descriptive analysis of student proficiency in conceptual and procedural understanding

Pre-test conceptual-procedural, Post-test conceptual and post-test procedural with Groups					
Groups		Pre-test Conceptual	Pre-test Procedural	Post-test Conceptual	Post-test Procedural
Experimental	Mean	16.3333	10.6667	16.8333	24.3333
	Std. Deviation	6.55656	5.83292	8.14559	7.51244
Control	Mean	13.8889	12.7778	20.0000	11.2500
	Std. Deviation	7.28120	5.90937	9.56183	8.39855
Total	Mean	15.0000	11.8182	18.5606	17.1970
	Std. Deviation	7.01646	5.92494	9.01986	10.30791

Table 5.12 shows that students' conceptual and procedural understanding of the differential calculus material in GeoGebra Mathematical software-assisted learning had improved whereas students' conceptual and procedural understanding of the differential calculus material in the control group indicated improvement only in terms of conceptual understanding. In the case of procedural understanding, nothing had changed, or understanding had diminished slightly. Further statistical tests were required on the post-test data, firstly a normality test on the results obtained from the post-test of differential

calculus proficiency. The reason for carrying out further analysis was to determine whether the data were normally distributed or not, enabling me to choose the types of tests I used (parametric such as a t-test or non-parametric such as a Mann Whitney test) (Elliott & Woodward, 2007). Thus, the descriptive analysis of the normality test of the post-test data was computed and is tabulated in Table 5.13.

Table 5.13 Descriptive analysis of normality test of post-test data

Tests of Normality of post-test							
Groups		Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	Df	Sig.	Statistic	df	Sig.
Post-test	Experimental	.139	30	.144	.924	30	.033
Scores	Control	.128	36	.145	.962	36	.253

Kolmogorov-Smirnov and Shapiro-Wilk tests are designed to determine whether the observed data fit the shape of a normal curve (bell curve) closely. If a test does not reject normality, this suggests that a parametric procedure that assumes normality (e.g. a t-test) can be safely used (Elliott & Woodward, 2007). However, the results in Table 5.13 indicate that the data were normally distributed for the value of $p = 0.144$ and $p = 0.145$ for the experimental and control group in learning differential calculus respectively and were greater than 0.05 in the Kolmogorov-Smirnov test. In contrast, the Shapiro-Wilk test in Table 5.13 indicates that the data in the experimental group were not normally distributed as the p-value was less than 0.05; however, the data for the control group were normally distributed. Thus, further investigation using another test was required. Table 5.14 reflects the post-test data normality test for both types of understanding to determine for which types of proficiency the data were not normally distributed.

Table 5.14 Descriptive analysis of normality test of post-test proficiency data

Tests of Normality							
Post-test	Groups	Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	Df	Sig.	Statistic	df	Sig.
Conceptual	Experimental	.199	30	.004	.909	30	.014
	Control	.144	36	.057	.970	36	.434
Procedural	Experimental	.251	30	.000	.887	30	.004
	Control	.145	36	.055	.923	36	.016

Table 5.14 shows that data from the post-tests of both types of knowledge in the experimental group were not normally distributed; on the other hand, in the control group, these data were normally distributed. Table 5.10 above summarises the descriptive statistics of the post-test of conceptual understanding scores for the experimental group ($n = 30$) and the control group ($n = 36$), $M = 16.8333$ ($SD = 8.14559$) and $M = 20$ ($SD = 9.56183$) respectively. Descriptive statistics of the scores on the post-test of procedural understanding of differential calculus for the experimental group ($n = 30$) and control group ($n = 36$) were reported as $M = 41.1667$ ($SD = 13.81424$) and $M = 11.2500$ ($SD = 8.39855$) respectively. The skewness for participants in the two groups in terms of the scores on the post-test of conceptual understanding was computed as .329 and -.078 respectively, whereas for scores on the post-test of procedural understanding this was reported as .329 and .232, respectively. The kurtosis for participants in the experimental group and the control group in terms of the post-test of conceptual understanding was -1.281 and -.547, whereas for scores on the post-test of procedural understanding this was reported as -1.189 and -.653, respectively. Their scores in terms of post-intervention procedural understanding were slightly positively skewed, which indicated that most participants tended to score lower than the mean score. The result of negative kurtosis meant that their test score distributions for both types of understanding were flatter than the normal distribution, indicating that test scores were spread out rather than grouped. As no data were normally distributed in this study, the non-parametric test used the Mann-Whitney U test (Elliott & Woodward, 2007). The results of the Mann Whitney test on students' achievement on differential calculus are reported in Table 5.15.

Table 5.15 Mann Whitney U test on students' scores in differential calculus

Time	Groups	N	Effect size(r)	Mean Rank	Median	Sum of Ranks	ofZ-value	U	P
Pre-test conceptual	Experimental	30	.19	37.42	15	1122.50	-1.551	422.5	.121
	Control	36		30.24	15	1088.50			
	Total	66							
Pre-test procedural	Experimental	30	.71	30.03	10	901.00	-1.385	436	.166
	Control	36		36.39	15	1310.00			
	Total	66							
Post-test Conceptual	Experimental	30	.72	29.93	17.5	898.00	-1.397	433	.163
	Control	36		36.47	20	1313.00			
	Total	66							
Post-test Procedural	Experimental	30	.83	47.10	20	1413.00	-6.729	132	.000
	Control	36		22.17	10	798.00			
	Total	66							

The Mann-Whitney U test showed that procedural proficiency/understanding was statistically significant in both experimental ($Md = 20$, $n = 30$) and control groups ($Md = 10$, $n = 36$) after the intervention ($U = 132$, $z = -6.729$, $p < 0.05$); students' scores in the post-test of conceptual understanding of differential calculus in the experimental group ($Md = 17.5$, $n = 30$) and control group ($Md = 20$, $n = 36$) did not show any visible significant difference between the two ($U = 433$, $z = -1.397$, $p = 0.163 > 0.05$); pre-intervention procedural understanding of differential calculus of students in the experimental group ($Md = 10$, $n = 30$) and of those in the control group ($Md = 15$, $n = 36$), ($U = 436$, $z = -1.385$, $r = .71$, $p = .166 > 0.05$); conceptual understanding of differential calculus of students in the experimental group ($Md = 15$, $n = 30$) and students in the control group ($Md = 15$, $n = 36$), ($U = 422.5$, $z = -1.551$, $r = .19$, $p = 0.121 > 0.05$) post intervention also showed no visible significant difference. However, using computed effect size (ES), which can be calculated as $r = \frac{z}{\sqrt{N}}$, where N is the total number of participants and z is the z-value computed by SPSS, the groups had small to moderate differences in terms of pre-test conceptual, pre-test procedural and post-test conceptual understanding of differential calculus in both groups (Rice & Harris, 2005). In addition, observation of both types of knowledge in each group revealed that in the experimental group, both differential

calculus proficiency (conceptual) (*median = 15 to median = 17.5*) and procedural (*median = 10 to median = 20*) had increased. In contrast, students' procedural understanding proficiency had diminished in the control group (*median = 15 to median = 10*), whereas the conceptual understanding of differential calculus increased (*median = 15 to median = 20*).

Therefore, for **Research Question 1** that asks How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group 2)? the study found that students who learned differential calculus with the help of GeoGebra scored highly statistically differently, with students improving by 46% (see Table 5.17), and more students made greater progress in procedural understanding (see Table 5.15).

Research Question 2 asks how the level of proficiency in differential calculus compares within the experimental group (Group 1) pre and post the intervention incorporating the use of GeoGebra. The results from students' proficiency variables measured before and after the interventions were used to examine their progression from pre-test to post-test. In this case, the subject was measured twice (before and after the intervention), giving a pair of observations. Thus, the progression of each group from pre-test to post-test on proficiency variables (conceptual and procedural) was analysed by using the paired sample t-test, as all assumptions were met for all variables by the Levene test for equality of variances. The results are reflected in Table 5.16.

Table 5.16 Differences in student proficiency in Experimental group

Student proficiency	Paired Differences of Experimental group							
	Mean gain	SD	Std. Error Mean	95% CI		t	Df	P
				Lower	Upper			
Pre-test-Post-test	-14.2	12.3	2.25	-18.77	-9.57	-6.3	29	.000*
Pre-C-Post-C	-.5	7.92	1.45	-3.46	2.46	-.35	29	.732
Pre-P-Post-P	-30.5	13.5	2.5	-35.5	-25.5	-12.4	29	.000*

Pre-C: Pre-test conceptual *Significant at 0.05 post-C: post-test procedural

Pre-P: Pre-test procedural

Post-P: Post-test procedural

The results of a paired samples t-test, (see Table 5.16) indicate the mean gain in students' proficiency in the two types of knowledge between pre-test and post-test, and in particular the mean gain of conceptual and procedural understanding of DC before and after an intervention. The p-value for the comparison of pre-test and post-test conceptual understanding of differential calculus was $p = 0.732$ and 0 , respectively. Students in the experimental group improved significantly in terms of procedural understanding ($t(29) = -9.36$, $p < 0.05$, $d = -30.5/13.5 = -2.35$) but did not show a visible improvement in terms of conceptual understanding of DC when being taught using GeoGebra ($t(29) = -.35$, $p > 0.05$, $d = -.5/7.92 = -0.06$). In general, students in the experimental group improved their proficiency significantly ($t(29) = -6.3$, $p < 0.05$, $d = -14.2/12.3 = -1.2$). To determine the extent to which the improvement of students occurred after the intervention, I used Cohen's d effect size standard; this is the numerical method of interpreting the strength of a reported correlation, avoiding simply 'binarising' matters. It states the effect size of 0.2 for small, 0.5 for medium and 0.8 and above for large (Cohen et al., 2018; Lakens, 2013; Mills & Gay, 2019). Table 5.17 indicates the interpretation of effect size computed in Table 5.16.

Table 5.17 Computed effect size of pre-test and post-test

Observations	Computed effect size	Percentile gain	Interpretation
Pre-test - post-test	-1.7	46%	Improvement is high as the value is greater than Cohen's d standard 0.8.
Pre-C - Post-C	-0.06	2%	Improvement is low as the value is smaller than Cohen's d standard 0.2.
Pre-P - Post-P	-2.35	49%	Improvement is high as the value is greater than Cohen's d standard 0.8.

The negative value indicates the direction of means and as is indicated in Table 5.16, negative values occurred as the means within post-intervention were subtracted from pre-intervention on each observation. In other words, scores were lowered by the effect of the program used in the study.

5.9. EVALUATION STAGE OF THE CYCLE MODEL

If the average post-test score is higher than the average pre-test score, it makes sense to conclude that the treatment might be responsible for the improvement. The difference between the control group's pre-test and post-test composite violence scores was -4.4444 (26.6667 – 31.1111) while the post-test difference between the experimental and control group was -10.1 (31.1111- 41.6667). The intervention, therefore, boosted the pre-post increase in the aggression score by 44% (-4.444/-10.1).

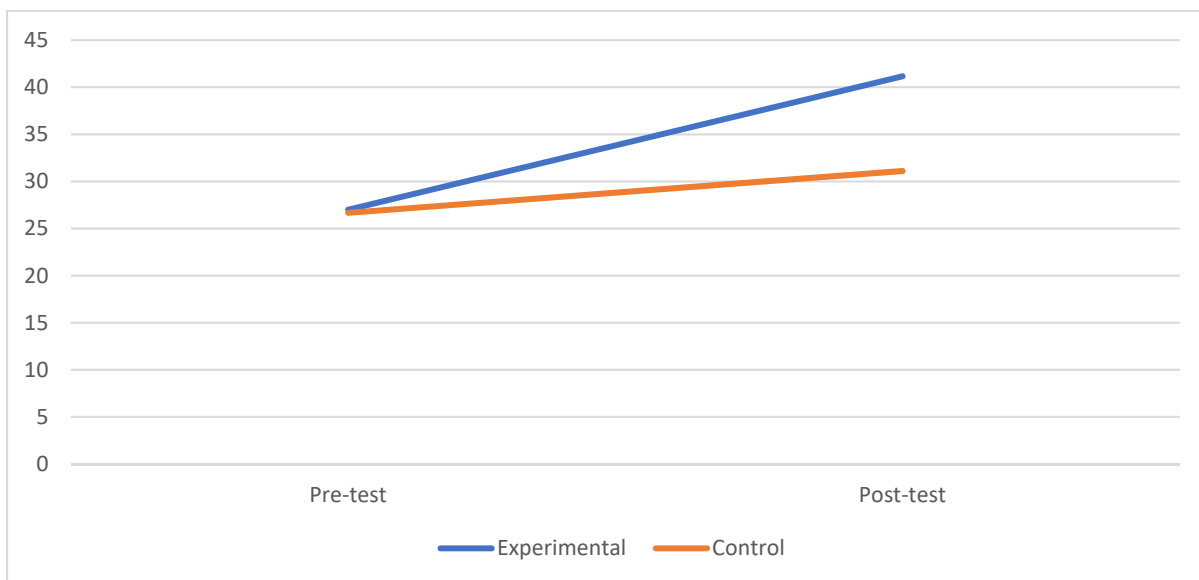


Figure 5.8 Pre-test post-test changes in student proficiency

Figure 5.8 reflects the pre-test post-test changes in proficiency in the experimental and in the control group. Both groups showed an increment in their scores even though they were at the same initial point when the pre-test was administered. However, the treatment group achieved higher scores than the control group in the post-test.

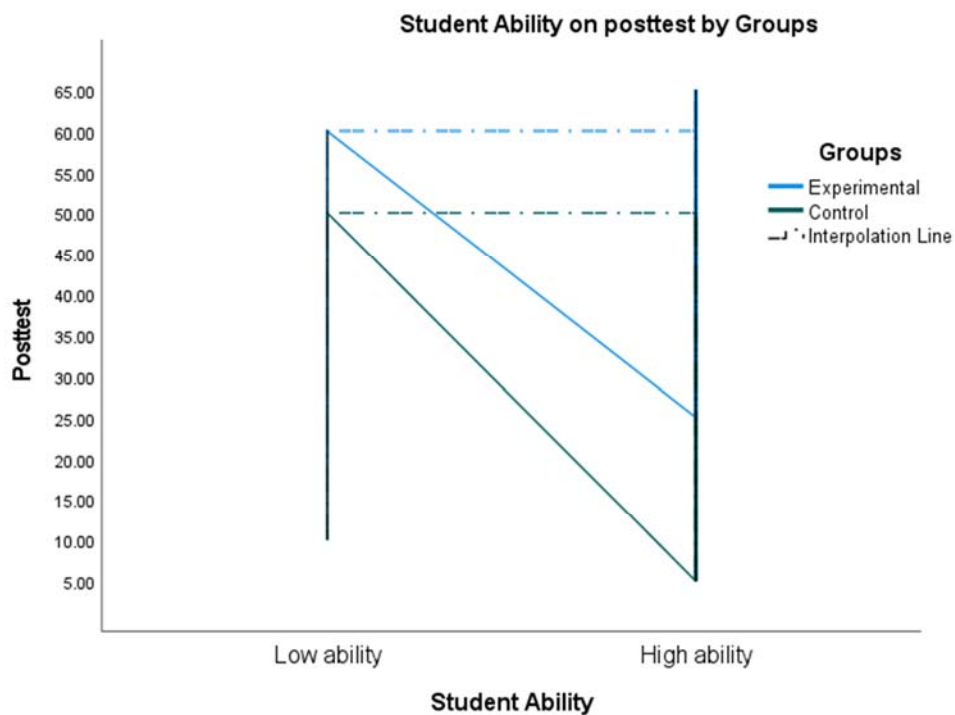


Figure 5.9 Student ability in Experimental and Control groups

Figure 5.9 shows that both high ability and low ability students were advantaged by the treatment, but students in the experimental group scored higher than the control group.

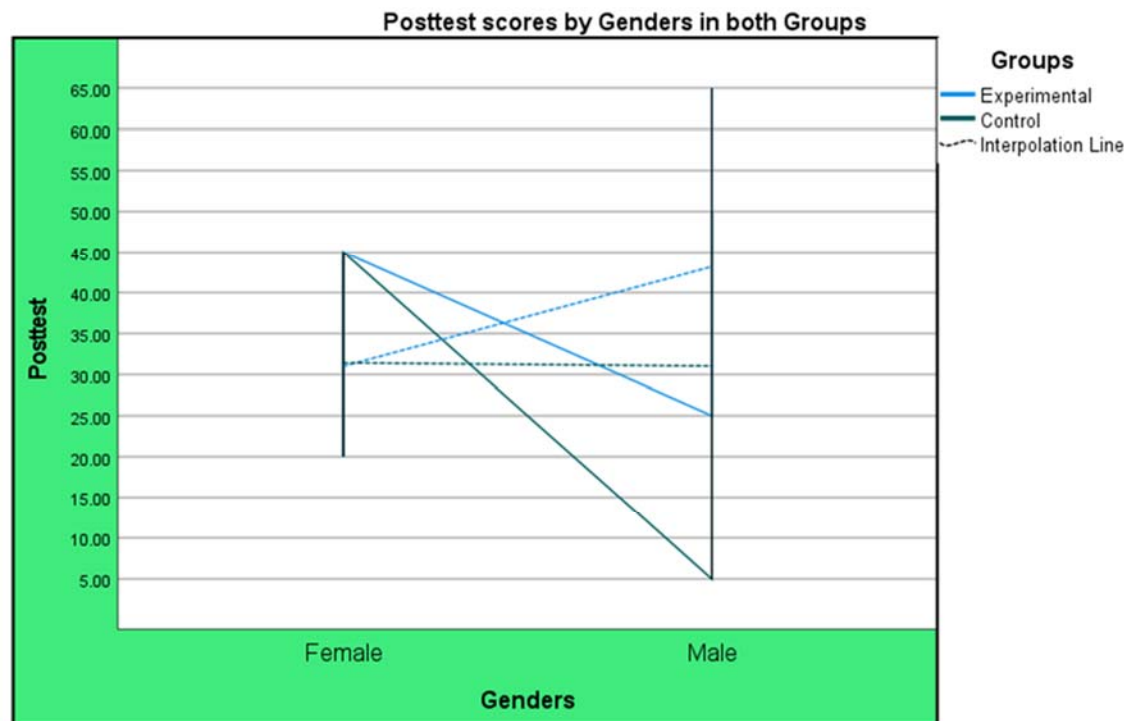


Figure 5.10 Gender difference in scores on post-test in both groups

Figure 5.10 indicates that both female and male students in the experimental group scored higher than students in the control group. These findings are in line with a study that found that female students learning a given course with the help of GeoGebra achieved scores that were superior to those of a control group taught by traditional methods. They also showed greater survival of learning impact, defined by learning output retained in memory as indicated in scores on a post-test (Alabdulaziz, Aldossary, Alyahya, & Althubiti, 2021).

One of the aims of this study was to explore how student participants perceived learning with the aid of GeoGebra after the intervention had been implemented. To this end, in addition to the interview, a questionnaire was distributed to the experimental group after the intervention. The validity of the interview and the reliability and validity of the questionnaire was discussed in Section 4.11.1. Questionnaire items were grouped according to three core themes (perception towards the existence of the technology in the environment (the first step of the cycle model), perception towards scaffolding (the

vertical and horizontal interaction step of the cycle model), and their preference for using technology/GeoGebra (individual perspectives step of the cycle model)). This was called the three-perception scale. Items such as 4.5 are grouped under 'perceptions towards technology', items 1, 2, 3, 7, 8, 9, 10, 11 and 14 are categorised as 'perceptions towards technology in learning, and items such as 6, 12, 13 and 14 are categorised as 'perception towards scaffolding' during the intervention. The results of the analysis of responses are provided in Table 5.18.

Table 5.18 Percentages and means of perceptions scales

Items	Scales					Mean
	SD	DA	N	A	SA	
Preferences in the classroom						
1: At the beginning, I did not like GeoGebra	11(35.5%)	7(22.6%)	5(16.5%)	6(19.4%)	1(3.2%)	2.3 (3.7*)
2: I like GeoGebra because it is dynamic and free for everyone.	6(19.4%)	4(12.9%)	2(6.5%)	7(22.6%)	11(35.5%)	3.433
3: Right now, I'm more open to learning using GeoGebra.	12(38.7%)	7(22.6%)	3(9.7%)	5(16.1%)	3(9.7%)	3.6
7: I think working with GeoGebra is frustrating.	6(19.4%)	9(29.0%)	1(3.2%)	5(16.1%)	9(29.0%)	3.0667 (2.933*)
8: I am comfortable with GeoGebra in learning calculus.	2(6.5%)	-	2(6.5%)	12(38.7%)	14(45.2%)	4.2
9: I do not want to use GeoGebra for my future study.	13(41.9%)	7(22.6%)	3(9.7%)	2(6.5%)	5(16.5%)	2.3 (3.7*)
10: GeoGebra makes calculus more difficult for me.	12(38.7%)	7(22.6%)	3(9.7%)	5(16.1%)	3(9.7%)	2.3333 (3.6667*)
12: The instructional material in learning calculus through GeoGebra is well organised.	2(6.5%)	2(6.6%)	1(3.2%)	11(35.5%)	14(45.2%)	4.1000
15: I achieved better marks after I learned calculus through GeoGebra software.	3(9.7%)	1(3.2%)	2(6.5%)	9(29.0%)	15(48.4%)	4.0667
Overall	2.16	1.42	0.72	2	2.424	3.7
Existence of software	Scales					
4: There is mathematical software for learning calculus in secondary school.	10(32.3%)	2(6.5%)	2(6.5%)	9(29.0%)	7(22.6%)	3.033

5: There is no mathematical software for learning calculus.	3(9.7%)	7(22.6%)	3(9.7%)	10(32.3%)	7(22.6%)	3.3667(2.633*)
Overall	0.42	0.291	0.162	0.613	0.452	2.8
Scaffolding in the classroom	Scales					
6: I need a lot of help when doing new things by using technology like GeoGebra.	3(9.7%)	3(9.7%)	1(9.7%)	9(29.0%)	14(45.2%)	3.9333
12: I get enough time to do the activity on my own in the laboratory classroom.	8(25.8%)	7(22.6%)	4(12.9%)	2(6.5%)	9(29.0%)	2.900
13: I depended on others to do the activity while the program was running in the classroom.	7(22.6%)	8(25.8%)	2(12.9%)	4(12.9%)	9(29.0%)	3.000
14: I achieved better marks after I learned through GeoGebra Mathematical software.	3(9.7%)	1(3.2%)	2(6.5%)	9(29.0%)	15(48.4%)	4.0667
Overall	0.678	0.613	0.42	0.774	0.152	3.5

Note * indicates the reversed mean in positive statements.

Students' perceptions were elicited by a questionnaire consisting of 14 items (nine items for perceptions towards GeoGebra, two items on the existence of the technology and four items on scaffolding by GeoGebra). The questionnaire was distributed to the experimental group only to determine their perceptions based on their experience of using the GeoGebra software. The results of the analysis of the responses to the questionnaire reflect students' perceptions towards GeoGebra for teaching in the classroom (with an overall mean of $M = 3.7$) and perceptions of scaffolding activities (an overall mean of $M = 3.5$) in the classroom. These were positive whereas perceptions towards the existence of technology for the mathematics classroom were negative (with an overall mean of $M = 2.8$). It appeared that students were not familiar with the technology for teaching and learning calculus before the intervention. These students had never used GeoGebra before. This may be why they enjoyed using GeoGebra software for learning as it is a dynamic mathematical software ($M = 3.7$). The study found that the items in the questionnaire that had the highest mean were those which showed that students were comfortable using GeoGebra for learning calculus ($M = 4.2$), indicating that the software increased students' motivation, confidence, and achievement. The lowest mean was item 2.9, responses to which revealed that students did not think that working with GeoGebra was frustrating. Studies have found that technology in the classroom improves not only

student performance and achievement but also student motivation (Harris, Al-Bataineh, & Al-Bataineh, 2016). GeoGebra software can increase students' interest, confidence, and motivation in learning calculus. These findings correspond to those of a study by Arbain and Shukor (2015). The three perception scale was developed by condensing the items in each category/theme; negative statements were reversed and recoded into positive statements (Sadeghiyeh et al., 2019).

Table 5.19 Mean of perception scale.

Perception scales	Gender	Mean	Std. Deviation	N
Preference	Male (M = 3.7111)	3.7111	.54792	30
	Female (M = 3.7111)			
Existence	Male (M = 2.82)	2.8333	.83391	30
	Female (M = 2.9)			
Scaffolding	Male (M = 3.5)	3.4750	1.02837	30
	Female (M = 3.35)			

Table 5.19 shows that there was no difference in means according to gender in the three perception scales measuring perceptions of the use of technology/GeoGebra in classroom learning and teaching of differential calculus.

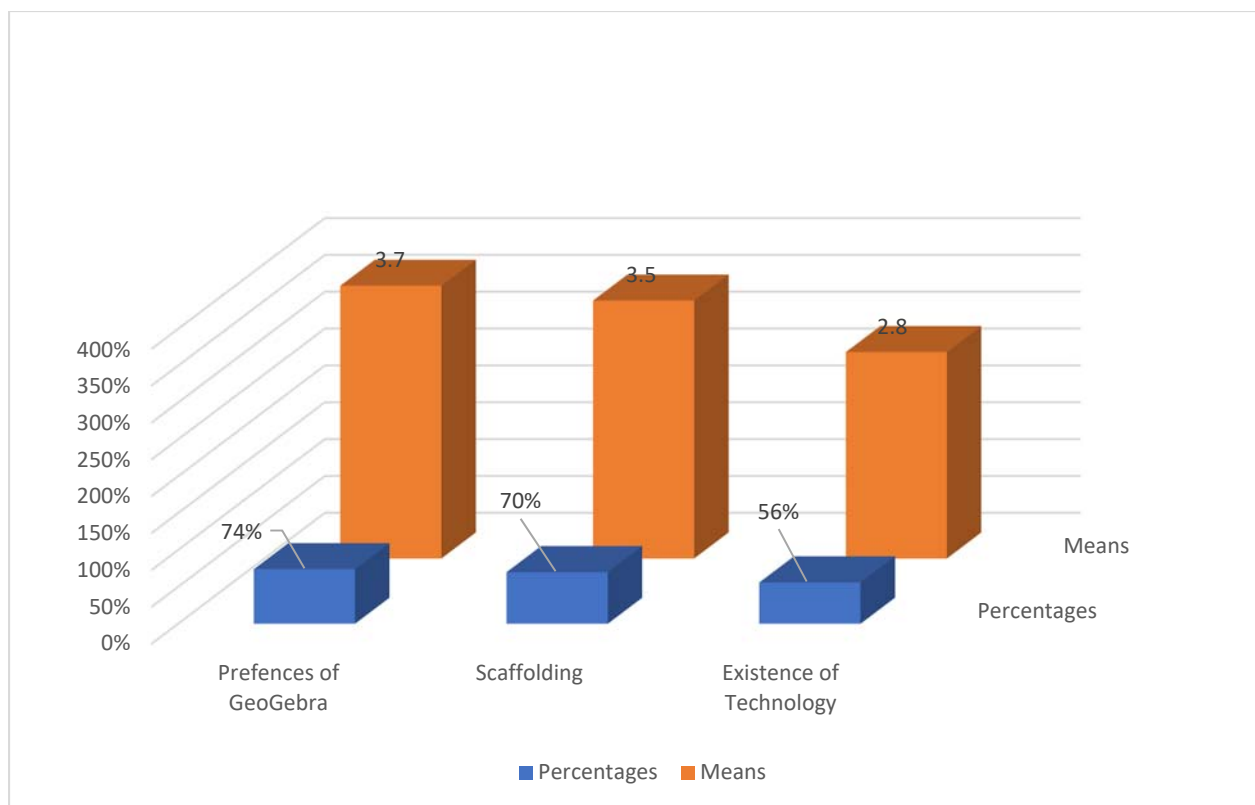


Figure 5.11 Percentages and means of three scales measuring student perceptions towards GeoGebra/technology

Figure 5.11 shows that 74% of students liked GeoGebra, implying positive attitudes towards the use of GeoGebra in the classroom; 70% of students were also interested in scaffolding activities (see Tharp's (1993) activities) that were included in the developed model during the intervention. During the interviews, one student stated that he was "very interested in the program, and it should be expanded in [an] Ethiopia[n] context". He added that he was introduced to technology while at secondary school in a course on IT, but he indicated that there was no technology for teaching and learning calculus at elementary or secondary school. Another four students (56%) disagreed with this, saying that there was mathematical software available for learning calculus at secondary school. A further student (44%) ignored these ideas and can be accepted as neutral (mean approximately ~ 3) (see Table 5.19 above.)

5.10. INTERNALISATION AND EXTERNALISATION STAGES OF CYCLE MODEL

These findings suggest that students in the experimental group gained more advantage from the intervention than the control groups gained from traditional teaching. It was anticipated that students in the experimental group would internalise the GeoGebra mathematical software and externalise their knowledge in the environment with their mentors or students after they had completed their studies at university. Vygotsky's concept of internalisation is a model of learning alienated activities; interconnected dialogic processes (scaffolding) (i.e. decontextualised) in which the individual uses sociocultural practices (teaching and learning of differential calculus with the aid of GeoGebra Mathematical software) through engagement with these interconnections (activities designed by Tharp (1993))(Smith, Dockrell, & Tomlinson, 1997). In Vygotsky's theory, externalisation occurs when learning and teaching process outcomes in sociocultural practices are fossilised in terms of the cognitive proficiency (e.g., conceptual and procedural) of human adults. Behaviour-based proficiency in competencies such as how to approach a task, how the subject's meta strategic understanding has evolved in the course of engagement with the task, and successful search procedures in the form of self-produced state-based feedback, may well constitute a separate layer of competence with a powerful potential role in the growing interaction between subject and environment (Smith et al., 1997).

In general, technological (GeoGebra) aids within the cycle model were provided to the students to increase students' motivation toward learning differential calculus, increasing students' opportunities to operate with mathematical representations of both conceptual and procedural knowledge, making learning more meaningful and enjoyable in the progression of ZPD by a scaffolder (teacher) (please see Figure 3.4), maximising visualizations of the learned topic (differential calculus) by the software and maintaining the students' attention on the lesson to make them ready for applying the situations in the environment stages of the cycle model.

5.11. APPLY IN THE ENVIRONMENT STAGE OF THE CYCLE MODEL.

Those students who had internalised the activities were expected to externalise the activities again in the school environment, which is known as communities of practice. This stage was similar to step1 of the cycle model, but participants were now familiar with the environment and familiar with the activities they had engaged in during the treatment.

5.12. ANALYSIS OF QUALITATIVE DATA

Once the intervention had been completed, selected students from the experimental group were interviewed. Five willing students were selected purposively as this method is unique to qualitative research (Ranney et al., 2015). Directed content analysis, in this case, was recorded and enabled me to conduct data identification and categorisation of the data into major themes (Reeves et al., 2015). These themes were identified/ or determined depending on the types of items that the researcher used. The interviews were audio-recorded on a smartphone and transcribed verbatim. The transcribed student's interviewed were attached as Appendix 3.3. This constituted the data for analysis. The transcripts were uploaded to the ATLAS.ti 9 computer software and coded. This also analysed the data from the five students (Parker, 2021). The responses from the interviewed students were grouped for data analysis purposes and presented according to three themes (see Figure 5.11). Each of these themes related to students' perceptions towards the use of GeoGebra/technology in classroom learning as elicited by the closed-item questionnaire (see Table 5.18) in line with the cycle model steps. The three themes connected to the research question were: perception of GeoGebra in terms of preference, perception of scaffolding in the classroom, and perception of the existence of technology in the classroom. According to Vygotsky's theory of education, the changes in the development of perception (qualitative part) come about in association with the old (in my case before the intervention of the experiment) and the development of new psychological systems (after the intervention of the experiment throughout ZPD) (Zaporozhets, 2002). To this end, I identified students' perception towards GeoGebra mathematical software after they were acquainted with the software during the

intervention, and it was discussed deeply by using ATLAS.ti 9 computer software, as set out in the following paragraphs.

A revealing look at the narrative with the help of ATLAS.ti 9 computer software:

- The initial steps consisted of an initial analysis of the material by listening to the recorded data to better understand it.
- The document was uploaded as an audio file into ATLAS.ti 9 computer software.

Listening to the recorded audio in ATLAS.ti 9 computer software again and again to make text files. These (the narratives obtained from the interviews) were included in the ATLAS.ti 9 computer software.

- Initiate the identification of the basic units of analysis (according to the themes) to provide ways to develop categories/themes to be analysed by ATLAS.ti 9.
- Pick keywords from each interview corresponding to all interview questions (see Figures 5.12, 5.13 and 5.14).
- Coding according to the similarities in text files
- Finally, I used a graphics network for better analysis, and the narratives of all students were presented in boxes as shown in figures 5.12, 5.13 and 5.14.

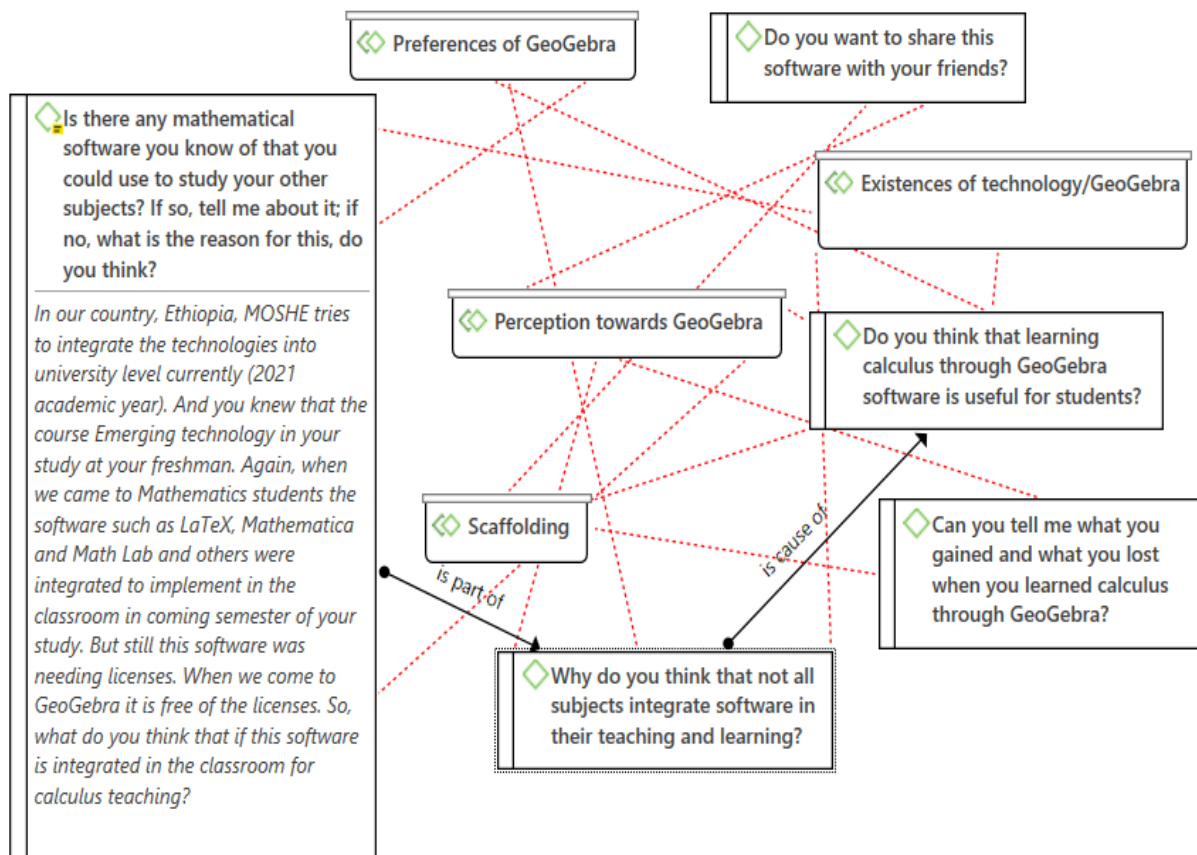


Figure 5.12 Interview themes

Figure 5.12 reflects interview themes grouped into categories and provided in boxes with related ideas. These were:

- a. Perceptions towards GeoGebra
 - i. Why are your beliefs about learning calculus through GeoGebra software important for students? Could this software be helpful to high school students?
 - ii. Why do you think all subjects is not integrated with the mathematical software? Can you tell me what you gain and what you lose when you learn calculus using GeoGebra?
- b. Perceptions towards scaffolding during classroom:
 - i. Can you tell me what you gained and what you lost when you learned calculus through GeoGebra?

- ii. Do you want to share this software with your friends?
- c. Perceptions towards the existence of technology/GeoGebra:
 - i. Is there any mathematical software you know of to assist you in studying your subjects? If so, tell me about it; if not, what do you think is the reason for this?
 - ii. Why do you think mathematical software is not integrated into your subjects?

The views of students expressed during the interview were analysed by grouping them according to three themes as follows:

Theme1 Preferences:

The following figure shows the interaction between interviewers and interviewees as categorised according to the preference themes. Each interviewee interacted with all interview questions and recorded this on his or her smartphone. ATLAS.ti 9 allowed me to code the transcribed words and analyse the data. Figure 5.10 indicates that students' responses demonstrated positive perceptions towards GeoGebra as all words were positive. The selection and decision of what technology to use and how to use it (implying preference) in the classroom increases the user positive perceptions towards integrating ICT/technology in the classroom (Thambi & Eu, 2013).

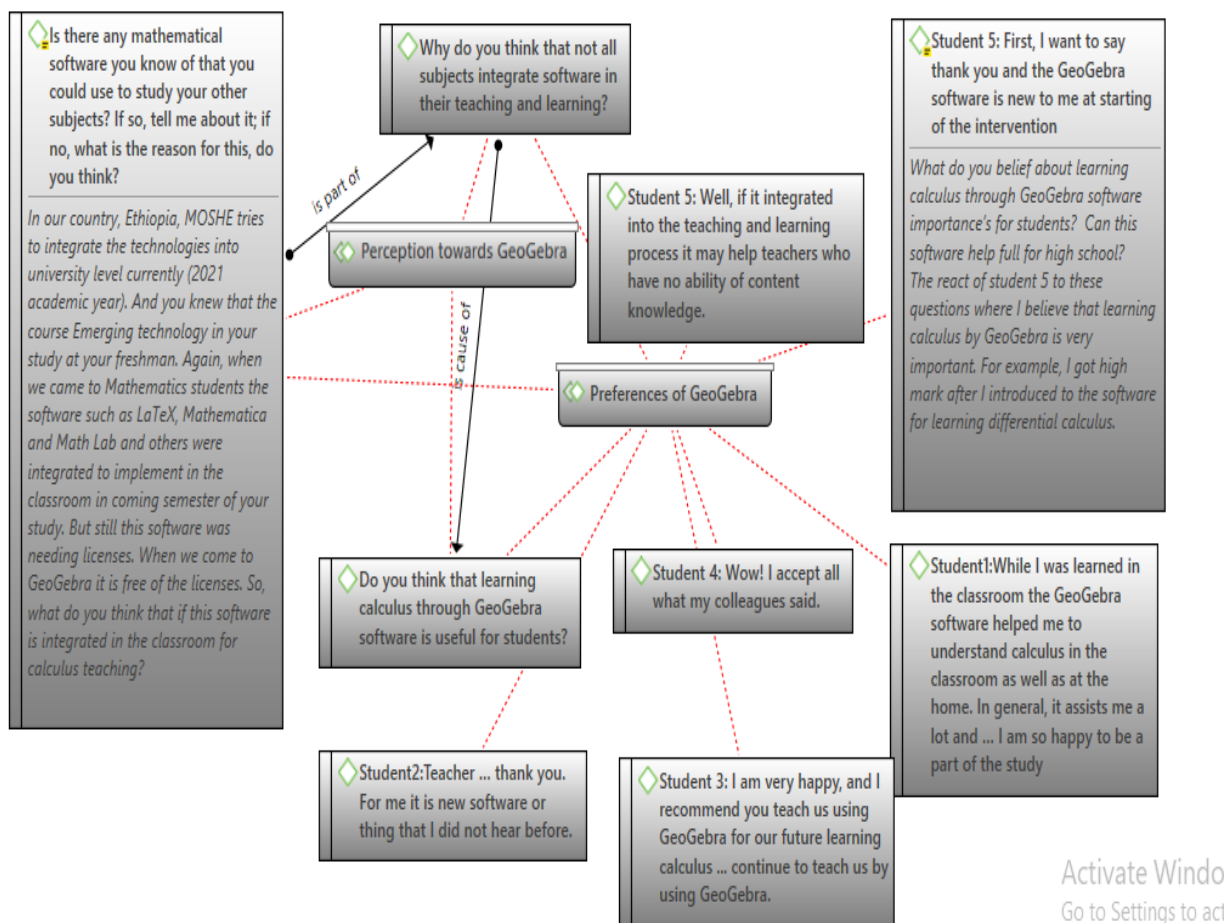


Figure 5.13 Students' responses during interviews on perception about preferences towards GeoGebra.

The responses to the interview questions (narratives of which are provided in each box) indicated that most students liked the GeoGebra oriented lessons using the computer (see the box in Figure 5.13): *They think that it helped them to interact with each other to discuss and share their ideas.* Generally, the worksheets developed by the applet helped them to learn the differential calculus topics. However, the students believed that they were helpless without the guidance of a teacher as they were new to the software despite their interaction with computers. In general, information gained from the interview responses when it had been coded by ATLAS.ti 9 can be summarised as follows: even though students were new to the GeoGebra software, which can be regarded as a

deficiency in computer-guided learning, they were nonetheless encouraged when using it to learn calculus in the classroom as it enhanced their independence and saved time. This suggests that embedding technology in the teaching and learning process is important, but it may be hindered by a lack of resources. The use of GeoGebra provides both teachers and students with a free tool, which is a new method of using technology to visualise calculus, helping students to interact with mathematical concepts individually (self-scaffolding) or in groups (vertical and horizontal interaction of cycle model see figure 3.3), in the classroom, at home or in any suitable place (see Section 5.2).

Lastly, during the interview, one student explained:

I found that GeoGebra gives a good impression of learning calculus. I feared Mathematics especially calculus for my study, but after I have installed the GeoGebra and try out calculus on my own at home, I am very impressed to use it for my future study.

The other four participants agreed with this. But one student wished to add something about the preference for GeoGebra in the classroom, saying,

I have a mobile, and I have a brother with the age of 7 years old, but when I come back home from work, he immediately comes to me and took my mobile to play a game.

This implies that the seven-year-old knew how to manipulate the technology. I, therefore, recommend that instead of showing our children in Ethiopia how to play a game we should introduce them to mathematical software.

Theme 2 Scaffolding:

As indicated in Figure 5.14, the scaffolding theme was covered by two of the interview questions presented to the five selected students. Their responses were recorded and coded and presented on the nodes of the figure as shown below.

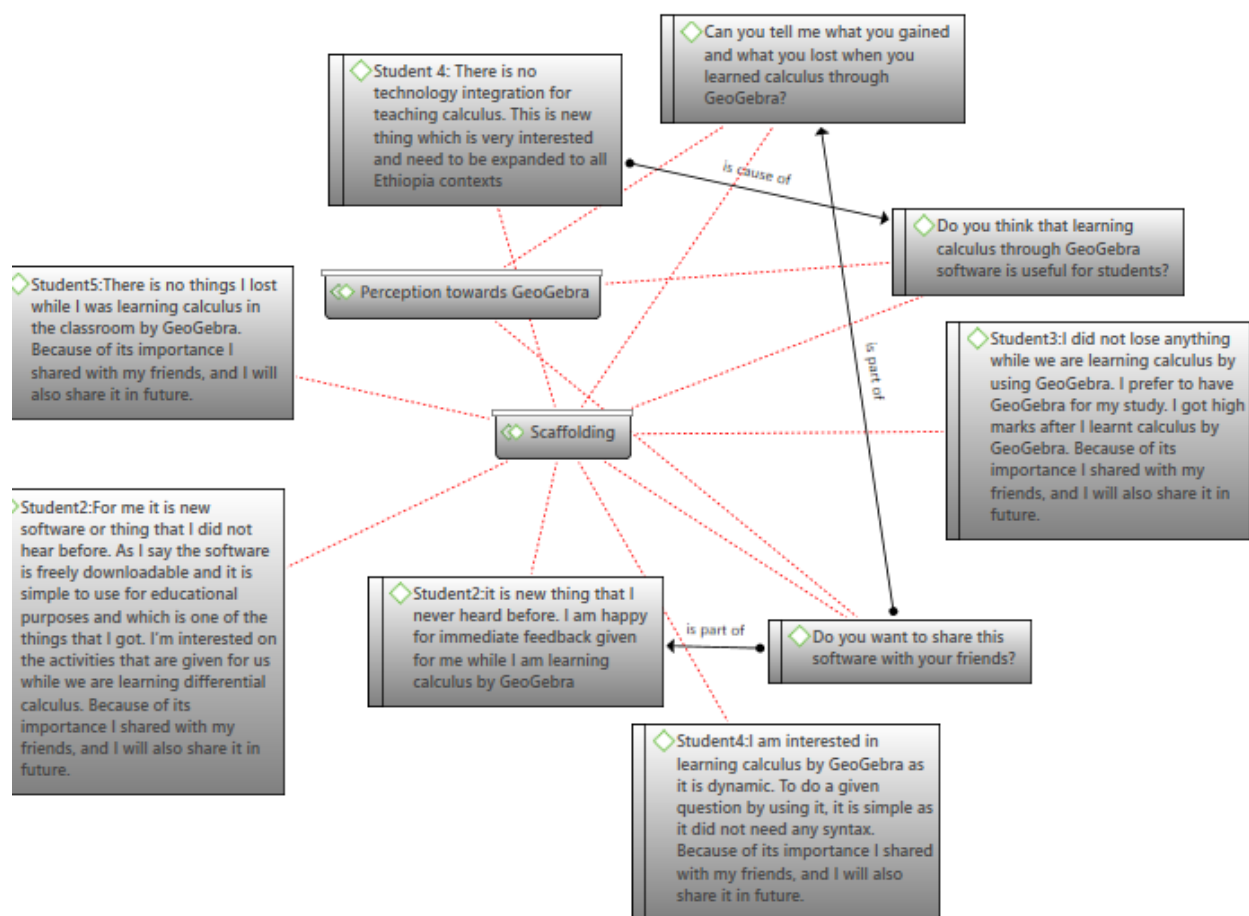


Figure 5.14 Students' perceptions towards scaffolding activities

Figure 5.14 shows the responses as they were coded by ATLAS.ti 9. These indicate that most respondents said that a GeoGebra oriented classroom allowed students to communicate directly with the teacher and other students, and among themselves. This suggests that learning with software could trigger on-task interaction. These interactions increased students' interest in learning mathematics. They were also happy with the immediate feedback given to them while scaffolding was employed in the classroom.

Educational scaffolding is the dynamic intervention the zone of proximal development in a situation-based intervention by the teacher in the learning process of students creates a useful area (e.g. by posing questions, answering, getting feedback etc.) for exploring specific individual and group forms of support that can be captured in the interaction

between teachers and students in the classroom environment (Seberová, Göbelová, Šimik, & Sikorová, 2020).

In summary, in this study students revealed positive perceptions towards scaffolding activities in the classroom. Students benefited from scaffolding in terms of immediate feedback and communication and improved their achievements in differential calculus. This allowed them to discover new mathematical knowledge (procedural and conceptual), supporting the findings of Željka and Trupčević (2017).

Theme 3 Existence:

Two interview questions covered this theme: i) Is there any mathematical software you know of to study your subjects? If so, tell me about some of them; if you do not know of any, why do you think this is so? ii) Why do you think that mathematical software is not integrated into all your subjects? The responses were recorded and uploaded into the ATLAS.ti 9, helping me to code the data as indicated in Figure 5.14.

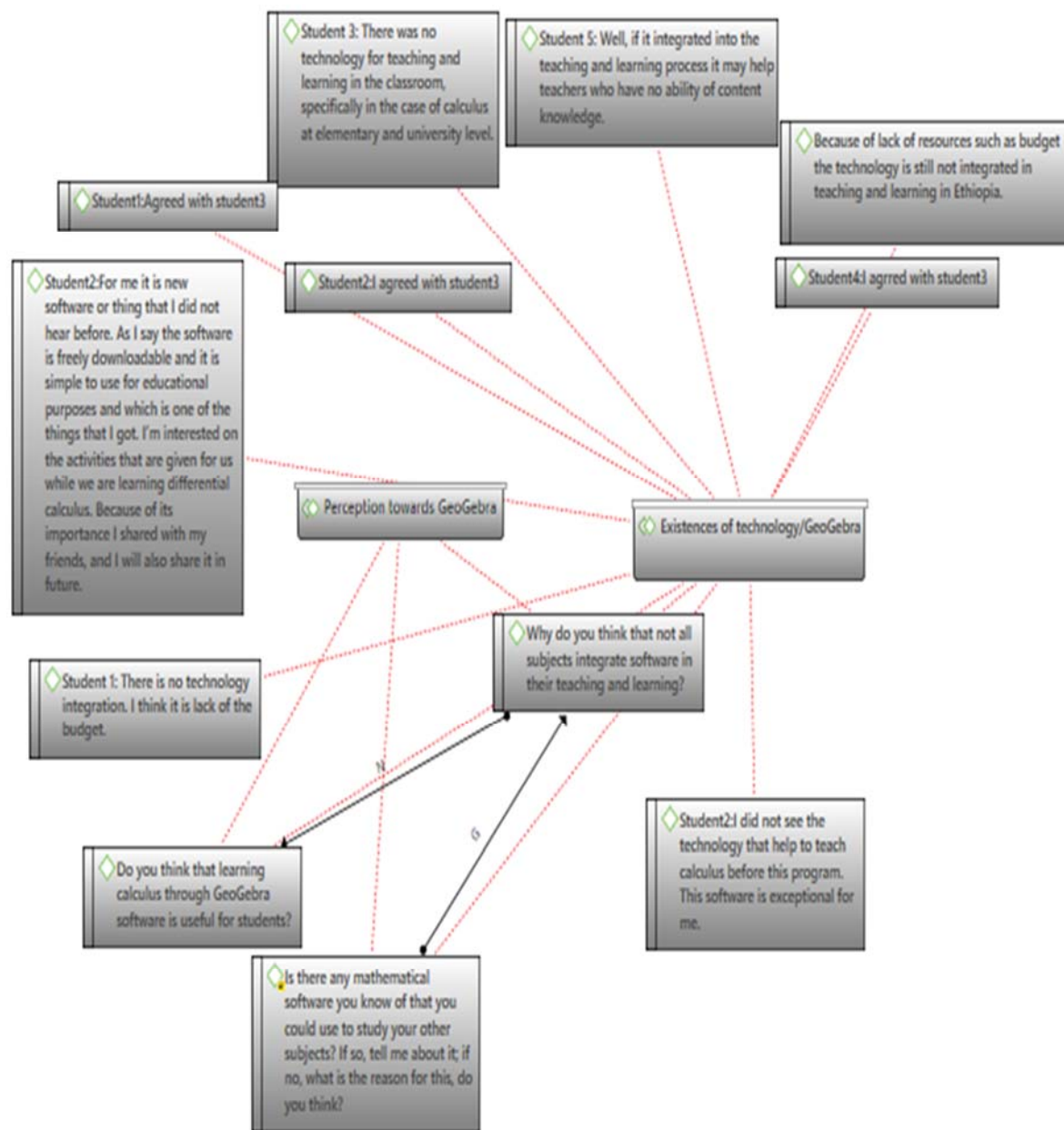


Figure 5.15 Student perceptions about the existence of technology

During the interview, most students expressed the belief that there was no technology for teaching and learning in the classroom, specifically in the case of calculus at the elementary and university levels (see Figure 5.6). Of the five students, only one was

familiar with software known as Photomath; the others were new to technology. Some students said that the integration of technology was hindered by a lack of resources. These findings strengthen those from the perception scale (see Section 5.8) in the quantitative phase of this study. The existence of computers/technology has a significant effect on the use of technology in the classroom (Nikolopoulou & Gialamas, 2013). One student said that MOSHE integrated one course known as Emerging technology, and this had helped him to think about technology integration in the classroom. Another student recommended that the bodies concerned should consider the integration of technology in teaching and learning because he had benefited from the GeoGebra oriented classroom.

Most of the respondents stated that they were familiar with one course, known as IT, at secondary school but there was no software for learning calculus or other courses. They also stated that as the software used in the classroom in this study was freely accessed from the internet, educators should focus their attention on integrating it in the classroom to teach and learn calculus.

To sum up, in this phase of the study the results of the quantitative analysis were discussed, and these were explained by the responses to the qualitative questions in the interview protocol. The results of the qualitative analysis showed a positive perception among students towards the GeoGebra oriented classroom. This is supported in Figure 5.10, which indicates that students' preferences for technology and scaffolding had a mean greater than neutral ($n = 3$). This indicates positive perceptions toward GeoGebra in the classroom, in line with the findings of Arbain and Shukor (2015) and Doğan and İçel (2011). In general, the existence of technology, a preference for technology and scaffolding affects students' perceptions of the use of technology in the classroom. These findings support those of other researchers in the field (Nikolopoulou & Gialamas, 2013; Thambi & Eu, 2013; Željka & Trupčević, 2017).

CHAPTER 6 **CONCLUSION, SUMMARY, AND RECOMMENDATIONS**

This chapter concludes the thesis by providing a summary of the major findings and recommendations. The chapter opens with a brief overview of the research design, followed by a summary of the empirical findings. Finally, further reflections and implications of the study with recommendations are provided.

This chapter is organised under the following headings:

- Research agenda (Section 6.1)
- Summary of major findings (Section 6.2)
- Limitations of the study (Section 6.3)
- Recommendations (Section 6.4)
- Significance of the study (Section 6.5)
- Conclusion (Section 6.6)

6.1. RESEARCH OUTLINE

This research study comprised the following core ideas: higher education, Covid-19, integration of technology, particularly GeoGebra, in the school environment, using GeoGebra for teaching and learning, and differential calculus.

Higher education holds the key to harnessing the full potential of one of the important national resources, human capital. This is important in the generating of new ideas, specifically for the teaching and learning process, but generally for the development of the economy of the country. For a nation to produce professionals who fulfil the demands of the workplace, the first necessity is the preparation of the human resources (the students) by higher education (Alakrasha & Razakb, 2020). Berisha, Mustafa, and Ismail (2018) argue that higher education institutions should focus on the six strategic dimensions of teaching: research, internationalisation, business alliances, size, infrastructure, and recruitment. Changing or reshaping the teaching process to bring it up to international standards is the key strategic focus of higher education and the fourth revolution in education (Ally & Wark, 2020). However, in Ethiopia, the teaching of

differential calculus in particular through traditional classroom-based approaches (face-to-face teaching) may have prevented students from achieving the intended goals (Arango et al., 2015). In the 21st century, such practices in teaching and learning may mean that students not only fail to achieve the required competence but the latched/stopped of the teaching and learning process may occur. The best example of this came with the outbreak of the Covid-19 virus. All schools in Ethiopia and all countries that had not embraced the fourth industrial revolution were locked down. With the current technological advancements, traditional ways of teaching and learning are considered to be old fashioned approaches that do not suit the active learning required by this generation of students. The current and future digital generations demand technologically supported and interactive ways of teaching and learning. The integration of technology in the education system is one of the most important aspects of a technologically supported GeoGebra oriented classroom. This is an interactive way of teaching and learning that meets the demands of the 21st century.

Inayat and Hamid (2016) and Keong, Horani, and Daniel (2005) believe that technology-oriented learning in mathematics education provides many opportunities for students to enhance their understanding of basic concepts. But in Ethiopia, the integration of technology, more specifically GeoGebra Mathematical software, is still new. To this end, this study investigated the use and effect of GeoGebra Mathematical software in learning differential calculus at the tertiary level. Integration of GeoGebra in the teaching of differential calculus is not new. A review of the literature in this field revealed that lessons supported by GeoGebra enhance students' achievement in their studies (Akanmu, 2015; Nobre et al., 2016; Ocal, 2017; Preiner, 2008; Tatar, 2013). In this study therefore the researcher designed a model and implemented it to teach differential calculus with the aid of GeoGebra mathematical software. The model was designed to fit the context and to determine the benefits of this teaching method for students' proficiency. It presents a way that students of different abilities can communicate with their peers during activities in the classroom. Lastly, the study investigated students' perceptions of GeoGebra and the activities in the developed model.

Accordingly, the objectives of the study were: to compare the level of proficiency in differential calculus of students taught using GeoGebra (experimental Group 1) and of students taught through conventional lecturing (control Group 2); to compare the level of proficiency in differential calculus pre-and-post the intervention incorporating the use of GeoGebra in Group 1, and to investigate students' experiences and perceptions towards using mathematical software (GeoGebra) when learning calculus concepts.

To guide the processes of data collection and analysis, the main research objectives were broken down into the following general research questions and specific research questions:

General research question: What is the effect on students' proficiency in calculus at a tertiary level in Wachemo University of using GeoGebra Mathematical software?

Specific research questions:

- a) How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group 2)?
- b) How does the level of proficiency in differential calculus compare within the experimental group (Group 1) pre-and-post the intervention incorporating the use of GeoGebra?
- c) What are students' experiences and perceptions towards using mathematical software (GeoGebra) when learning calculus concepts?

To address these questions, research issues were considered in a critical review of existing literature, presented in Chapter 2. This review covered topics such as the use of technology in education in the 21st century, types of software in teaching and learning mathematics, an introduction to and the importance of GeoGebra software, components (elements) of GeoGebra, students' attitudes to using mathematical software, specifically GeoGebra Mathematical Software, challenges facing students when learning mathematics, and teachers' attitudes to using technology in the classroom. In addition,

the researcher used Vygotsky's theory as the theoretical framework when developing the model for the study.

The study was guided by a pragmatic philosophical paradigm (Teddle & Tashakkori, 2009). Apart from the objectives of a study, the choice of a research methodology and the method of analysis depends on the ontological and epistemological dimensions. In this regard, the research methodology used in this study was a mixed-methods approach, giving priority to quantitative data (explanatory research design) but including qualitative data. This was because of the cause-and-effect nature of the study. The qualitative approach was employed to better understand the implementation process of the teaching and learning of differential calculus with the aid of GeoGebra mathematical software and with the help of a model. It also allowed the researcher to understand students' attitudes to or perceptions of GeoGebra and its activities in the classroom during the intervention.

The study was conducted at Wachemo University, Ethiopia, and two groups of students learning differential calculus were included. These two groups' students were divided into experimental and control groups. The participants numbered 30 ($N=30$) in the experimental group and ($N=36$) in the control group. Hence, the total number of participants in the study was 66 ($N=66$).

A questionnaire, differential calculus achievement tests (DCAT) and interviews were used to collect data for this study. The questionnaire comprised three scales to measure perceptions towards GeoGebra in terms of preferences, scaffolding, and existence. DCAT was developed in such a way that it measured both types of knowledge, that is students' conceptual and procedural understanding. The interview covered the themes of the three-perception scales (see Table 5.19) to form a deep understanding of the developed model and its activities.

To increase the validity of all the instruments and the entire study, various strategies such as triangulation and expert reviews were followed. The content validity of the qualitative instrument was mediated and approved by my supervisor and colleagues. The internal consistency or reliability of the instruments was tested in the analysis of the pilot study data using the Cronbach alpha and Kuder-Richardson (KR-20) reliability index. The

quantitative and qualitative data were collected and analysed sequentially, giving greater emphasis to the quantitative than to the qualitative data.

A statistical package, Statistical Package for Social Sciences (SPSS) version 27.0 was used to analyse the quantitative data, by applying both descriptive and inferential statistics. The statistical tests used were the t-test, ANOVA, and a repeated ANOVA. An alpha level of .05 was used to decide whether the observed differences were statistically significant. The analysis and reporting of the qualitative data were done by developing themes using ATLAS.ti 9 and coding these for analysis. Important ethical issues in human research and the treatment of participants were emphasised.

6.2. SUMMARY AND DISCUSSION OF MAJOR FINDINGS

In this section, a summary of major findings of the study is organised according to the three research objectives, followed by the discussion of the findings, in Sections 6.2.1: validity and reliability of the instruments, 6.2.2: Student proficiency, and 6.2.3: perception scales.

6.2.1. Validity and reliability of the instruments

Twenty-four differential calculus tests were developed and distributed to 15 students in the pilot study. These tests were divided into 12 procedural tests and 12 conceptual tests, depending on the nature of the questions (see Appendix 4.1). Fourteen closed-ended perception items in the questionnaire with five Likert scales starting from strongly agree (5) to strongly disagree, and five open-ended questions for the interview were included in the study. Key findings for the validity and reliability of the study were summarised as:

- The pilot study guided me in retaining, revising or discarding test items (see Section 4.10). Items with an acceptable difficulty index and discrimination index were selected for inclusion in the main study.
- The pilot study revealed that the Cronbach alpha value for DCAT was 0.716 after deleting four items, two from each category of tests. These three items were found to have a low inter-item correlation and one item was removed by SPSS

version 27 as the item had zero variance. Thus, for the actual study, students' achievement and understanding of calculus were investigated using 20 DCAT items.

- The difficult index of procedural test items and conceptual items was 0.2131 and 0.213, respectively. The items included were difficult and helped to identify students for the main study (Sharma, 2021).
- The reliability coefficient of the procedural and conceptual items was 0.649 and 0.525, respectively. Thus, the items were reliable for the main study (Salvucci et al., 1997; Taber, 2018).
- The inter-reliability scale of the closed-ended perception questionnaire was 0.917, which indicates high-reliability items.
- The findings of the pilot study also indicated the content validity and validity of the questionnaire items, both closed and open, as determined by my supervisor and an expert in the field (see Appendices G and H).

6.2.2. Student proficiency

A paired samples t-test presented in Table 5.13 indicates the mean gain in students' proficiency in the two types of knowledge between pre-test and post-test and in particular the mean gain of conceptual and procedural understanding of DC before and after an intervention. The p-value for the comparison of pre-test and post-test conceptual understanding of differential calculus and pre-test and post-test procedural understanding were $p = 0,0.732$ and 0 , respectively. Students in the experimental group improved significantly in terms of procedural understanding ($t(29) 9.36, p < 0.05, d = -13/7.6 = -1.7$) but did not show a visible improvement in terms of conceptual understanding of DC ($t(29) = -35, p > 0.05, d = -.5/7.92 = -0.06$). In general, students in the experimental group improved their proficiency significantly ($t(29) = -6.3, p < 0.05, d = -14.2/12.3 = -1.2$). To determine the extent to which the improvement of students had occurred after the intervention, I used Cohen's d effect size standard. The combination of the two mathematical proficiencies of students in the understanding of differential calculus in the experimental group showed great improvement, with an effect size of $d = 1.7$ and with a

percentile gain of 46%. Students in the experimental group showed great improvement in procedural understanding, with an effect size of $d = 1.2$ and a percentile gain of 49%; in conceptual understanding of differential calculus; however, the students showed only slight improvement with an effect size of $d = 0.02$ and a percentile gain of 2%. These findings indicate that using GeoGebra for teaching DC helped students to improve their procedural understanding more than their conceptual understanding, which is in contrast to the findings of Ocal (2017).

The analysis of post-test data using the Mann-Whitney U test indicated that procedural proficiency/understanding was statistically significantly different in the two groups ($U = 132, z = -6.729, p < 0.05$), whereas student's proficiency in procedural understanding of differential calculus after the intervention ($U = 433, z = -1.397, p = 0.163 > 0.05$), and procedural understanding of differential calculus before the intervention ($U = 422.5, z = -1.551, p = 0.121 > 0.05$) showed no visible significant difference ($436, z = -1.385, p = .166 > 0.05$) between the groups. Using computed effect size (ES), the groups showed small to moderate differences in terms of pre-intervention conceptual, pre-intervention procedural, and post-intervention conceptual understanding of differential calculus, indicating that there was a relationship between the two (Rice & Harris, 2005). In addition, when observing both types of knowledge in each group, the findings revealed that in the experimental group, students' differential calculus proficiency (conceptual: median = 15 to median = 17.5, and procedural: median = 10 to median = 20) had increased as had students' overall scores (Diković, 2009). In the experimental group, procedural understanding of differential calculus had increased more than conceptual understanding as GeoGebra enables students' visualisation. The transformation of procedural to conceptual understanding requires an integral gradual reconstruction of students' perceptions towards the use of GeoGebra, even though the students expressed positive perceptions towards the use of GeoGebra during the study (Attorps, Björk, & Radic, 2011). Therefore, the findings indicated that instruction with GeoGebra had a positive effect on students' scores in both conceptual and procedural understanding of differential calculus, contrary to the findings of Ocal (2017), who reported that GeoGebra did not affect procedural understanding. However, procedural understanding can be considered as the mediator between conceptual understanding and student achievement (Zulnaidi &

Zamri, 2017). In contrast, proficiency in procedural understanding was slightly diminished in the control group (*median = 15 to median = 10*), whereas proficiency in conceptual understanding of differential calculus was increased (*median = 15 to median = 20*). The findings by Handelsman et al. (2004), Hurd (1998) and Williams, Papierno, Makel, and Ceci (2004) revealed that at the college level, courses focused more on memorisation and less on conceptual understanding and computational/procedural understanding of the material.

Finally, this study revealed that students in the experimental group were more advantaged than those in the control group in terms of both types of proficiency and had also developed positive attitudes towards the use of GeoGebra in the classroom when used with the developed cycle model in constructivism approaches. To some extent, COVID-19 tells us something about how the development of the cycle model emerges in terms of technology integration. These findings are in keeping with those of several earlier studies on overall student achievement (Akanmu, 2015; Alkhateeb & Al-Duwairi, 2019; Arbain & Shukor, 2015; Doğan & İçel, 2011; Hutkemri, 2014; Jelatu, 2018; Nobre et al., 2016; Ocal, 2017; Preiner, 2008; Rohaeti & Bernard, 2018; Saha et al., 2010; Tatar, 2013; Thambi & Eu, 2013; Zulnaidi & Zamri, 2017).

6.2.3. Perception scale

The **third research question** in the study, *What are students' experiences and perceptions towards using mathematical software (GeoGebra) in learning calculus concepts?* was addressed by the questionnaire and interview. The items in the questionnaire and the questions asked in the interview were grouped according to three perception scales. These were the preference scale, the scaffolding scale and the existence scale (see Table 5.9). Findings from these scales revealed that students had developed positive perceptions towards using the software GeoGebra in the classroom in terms of the preference scale, and towards the scaffolding activities included in the model during the intervention. Students were neutral on whether technology was integrated into elementary and secondary school mathematics teaching and learning, suggesting that they were neutral about the existence of technology or of using

technology, particularly GeoGebra, at the school level for learning calculus (*mean* ~ 3) (Bretscher, 2014). These findings were consolidated in the interviews conducted with five students. ATLAS.ti 9 was used to categorise responses into three themes and to code these data for analysis purposes (see Table 5.10). The three themes connected to the research question were perceptions of GeoGebra in terms of preference, perceptions of scaffolding in the classroom, and perceptions of the existence of technology in the classroom, all found in the steps of the cycle model (see Tables 5.11, 5.12 and 5.13). In general, the existence of technology, a preference for technology, and scaffolding affected students' perception of the use of technology in the classroom, in line with the findings by Nikolopoulou and Gialamas, (2013), Thambi and Eu (2013) and Željka and Trupčević (2017).

6.3. LIMITATIONS OF THE STUDY

This study was not conducted without some limitations. One possible limitation was that the study included self-reported views. It is difficult to determine whether students answered the questions honestly, providing their genuine feelings towards the three scales of perceptions. Depending on social appeal, students may respond based not on what they feel, but on what they think is socially acceptable. The results obtained from the questionnaire may thus not reflect students' actual feelings.

A second issue that might have affected the data quality in this study was the low level of computer ability of students in the experimental group; they might have failed to benefit fully from the approach, especially during the externalisation stage of the cycle model.

In addition, the smooth implementation of the intervention was affected by electrical outages and the absence of a well-organised mathematics laboratory. This situation affected the study, although I did my best to continue the experiment by changing my schedule. That was managed by arranging classes at the times when the university generator was functioning as a power supply for some purpose, such as to power the cafeteria or library.

6.4. RECOMMENDATIONS

With the current rapid technological advancement, good quality education cannot be achieved without the integration of technology. That is why the Ministry of Education of Ethiopia has planned to implement an Ethiopian educational road map (Teferra et al., 2018). To this end, this road map (2019–2030) integrates technology such as Math Lab, Latex and Mathematica as one course named Mathematical Software for the Mathematics Department. However, not all these technologies are freely accessible from the internet. GeoGebra Mathematical software is an open and freely available access software, however. This study thus recommends that the government integrates GeoGebra mathematical software in teaching differential calculus at the tertiary level. As the findings showed that the study was successful in improving both conceptual and procedural understanding of differential calculus, it is therefore recommended that both mathematics teachers and students be encouraged to use computer-based multimedia instruction. GeoGebra can be regarded as a multimedia tool to provide equal opportunities for students of different abilities (Anyanwu, Ezenwa, & Gambari, 2014).

There are several models of learning being practised by various universities abroad that work for all contexts of learning, such as the ASSURE and the ADDIE model. But in the cycle model used in this study, the duration and type of activities in the classroom depend on the context/environment and the reasons for learning by technology (see Chapter 2, the review of literature), the nature of the students, and the availability of technology and laboratories. These elements were considered in this study and evaluated, and it was decided that the cycle model using GeoGebra was most suitable for implementation in the intervention for the teaching and learning of differential calculus, following Vygotsky's theory of constructivism. The study was based on Vygotsky's ideas and the cycle model that was developed posits nine steps. This nine-step cycle model of learning differential calculus by GeoGebra benefited students. This study has shown the potential of a GeoGebra oriented classroom and the cycle model to benefit a developing country such as Ethiopia: the software is freely downloadable and can be installed on any computer or smartphone and it can be used offline. Developing countries, including my country Ethiopia, could thus use this nine-step cyclical model of implementation of GeoGebra in

their own context as educational software technology is still out of reach for many developing countries. This is of course not the complete story and acquiring and using up-to-date technology has associated costs. The lack of internet access, especially in schools is also a constraint (Bekene, 2020; Mainali & Key, 2012). It is thus recommended that the GeoGebra program is included in mathematics curricula at all stages of education (Alabdulaziz et al., 2021). The study strongly recommended to the Ethiopian Government that the cycle model using technology, more specifically GeoGebra, was the best teaching process for all students at any educational level.

In summary, as the integration of technology in mathematics education cannot replace the teacher, teachers and students need to be equipped with both content knowledge (differential calculus), skills to effectively apply the given technology (for instance GeoGebra), and pedagogy (interactive teaching methods) to facilitate the teaching and learning processes (cycle model) for students' achievement (Koehler & Mishra, 2009).

6.5. SIGNIFICANCE OF THE STUDY

The primary aim of this study was to investigate the use and effect of GeoGebra on students' learning of differential calculus at the tertial level. The study successfully applied the nine-step cyclical model, which demands greater participation by both students and teachers in the classroom. The study has hopefully opened gateways in research to using current, open accessed technology for future researchers interested in this area.

With the rapid growth of technology in the 21st century, introducing GeoGebra to the classroom is undoubtedly a possibility because students and teachers can manipulate a mathematics problem by being at their own home or schools freely. The main advantage of using GeoGebra for teaching and learning differential calculus using the cycle model is the improvement of students' mathematical proficiency.

This model could contribute to the relationship between the two mathematical proficiencies, known as the iterative views of Rittle-Johnson (2017) (see Section 5.3.2.), as indicated in Figure 6.1 below. The two types of knowledge (conceptual and procedural understanding) can be seen as bidirectional, iterative views, which means that

improvements in one type of knowledge lead to improvements in the other (Nordlander, 2021; Rittle-Johnson, 2017).

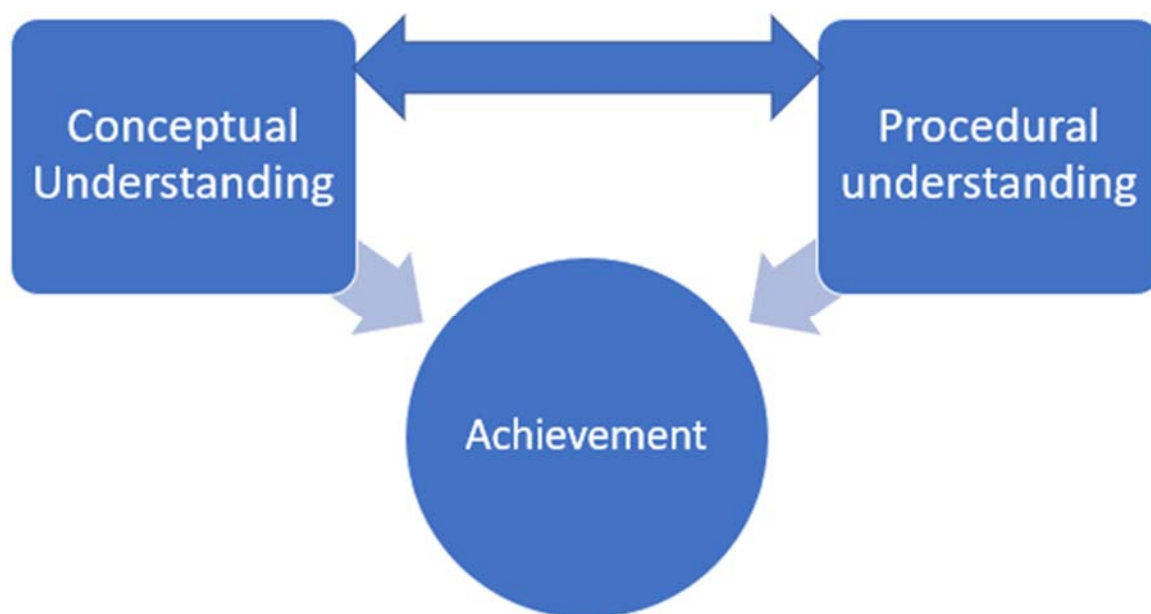


Figure 6.1 A model of conceptual, procedural, and student achievement

Figure 6.1 indicates that both conceptual and procedural understanding lead to achievement in mathematics achievement, and interact with each other (Hoyles & Forman, 1995; Rittle-Johnson, 2017). Rittle-Johnson (2017) proposed *iterative views* of both conceptual and procedural understanding rather than *procedure-first views* and *conceptual-first views* in which the *procedure-first view* posits that students develop procedural understanding before conceptual understanding, while the *conceptual-first view* holds that students develop conceptual understanding first, before procedural understanding. The more students are skilled in both conceptual understanding (a grasp of differential calculus concepts, operations, relations, and focus on domain principles) and procedural understanding (skills obtained during the intervention to carry out procedures flexibly, accurately, efficiently, appropriately and in general a focus on step-by-step activities in discovering the solution to differential calculus problems posed by the teacher), the more they were inclined to score high marks in their studies (see Section 5.16). Mathematics achievement was the strongest predictor in the mathematics creativity score; the creativity score is one of the important 21st-century skills that must be

developed in students because we live in the era of the fourth industrial revolution. This is constantly changing because of technology (Ramdani, Mohamed, & Syam, 2021).

6.6. CONCLUSION

This study shed light on the use and effect of GeoGebra in teaching and learning differential calculus in the Ethiopian context. It was conducted at a time when the Ministry of Science and Higher Education (MOSHE) became aware of the fourth industrial revolution and was making efforts to expand technology integration at every stage of schooling, especially at the university level (PASET Forum, 2019). Learners in the 21st century need technological support in the learning process because of the advancements made in technology for teaching and learning. A GeoGebra-oriented classroom uses one of these technologies that can be implemented in the classroom. Generally, the findings from this study were supported by previous studies discussed in Chapter 5. It developed a new cycle model for the implementation of the technology of GeoGebra in the classroom according to nine steps. Based on the discussion and the findings of the study, the following conclusions can be made.

This study aimed to investigate the effect of GeoGebra software on students' learning differential calculus in terms of two psychologies of knowledge, that is conceptual and procedural understanding. It also investigated students' perceptions towards the use of GeoGebra. The GeoGebra classroom-oriented approach had a more positive effect on the conceptual and procedural understanding of students in learning differential calculus than the traditional teaching approach had on students in the control group. The gap in the zone of proximal development was reduced by using technology/GeoGebra and students were assisted in becoming self-learners after being scaffolded in the internalisation stage of the cycle model (see Section 6.2.2). In the GeoGebra oriented classroom, students benefited more in terms of procedural understanding than conceptual understanding, while in the control group the reverse result was reported.

The improvement in achievement/scores of students can be attributed to the vast learning opportunity they gained from the GeoGebra classroom-oriented approach. One of the

advantages came from the interactivity and supplementary materials. What students found important and attractive during the intervention was scaffolding when explaining the concepts, modelling, rearranging of fixed differential calculus questions on topics discussed in the classroom (see Section 5.4), immediate feedback, discussion forums, and supplementary materials, both online and offline, such as reference books and collections of previous worksheets. Thus, the role of the teacher lay in identifying both environment and student ability, designing, guiding, helping, assisting, facilitating, giving feedback, evaluating, and motivating students to use their learning in the classroom and environment after they had developed their understanding (internalisation) for externalisation. In this regard, Vygotskian theory holds that cognitive development can be described as a process of internalising culturally transmitted knowledge (that can be held by scaffolded) in the cycle model (containing nine steps), in which the exposure to cultural models (cyclical model) stimulates a gradual internal process of knowledge growth (in both conceptual and procedural understanding) in students learning differential calculus with the help of GeoGebra (Nezhnov, Kardanova, Vasilyeva, & Ludlow, 2014; Vygotsky, 1978).

The perceptions of students were found to be positive towards the GeoGebra classroom-oriented approach, as respondents agreed that scaffolding activities offered learning opportunities that were better than those in traditional classrooms. Perception is a part of the process of using technology (Bruce & Hogan, 1998). The study found that 74% of students were satisfied with the preferences of the GeoGebra lesson-oriented course offered in the study while 70% were also interested in scaffolding activities and seeing Tharp's (1993) activities included in the developed model during interventions.

Student respondents felt that the GeoGebra classroom-oriented approach was an interactive, engaging, convenient, and more resourceful approach to logical thinking and discovery. In addition, GeoGebra's classroom-oriented approach allowed students to become familiar with computers and to build some essential skills for their studies. The developed cycle model was evaluated and brought positive changes to students' learning of differential calculus, in terms of both perception and scores. These findings suggest that the cycle model that emanated from the study for learning and teaching could improve

students' procedural and conceptual understanding, as depicted in Figure 6.2. The study satisfied the principles of the fourth educational revolution which are that the teaching and learning process should be reshaped (Ally & Wark, 2020) and consistent with Common Core State Standards that do not recommend traditional teaching and learning approaches (Alabdulaziz et al., 2021). This study thus produced the cycle model for teaching and learning differential calculus using technology (Koehler & Mishra, 2009). Figure 6.2 shows that the improvement of both proficiencies was the result of the scaffolding of student and teacher activities by GeoGebra, such as explaining (defining the problem), feedback, rearranging (as the software is dynamic and the teacher can change a given question as many times as he/she wants to) and modifying and modelling lesson learning within the classroom.

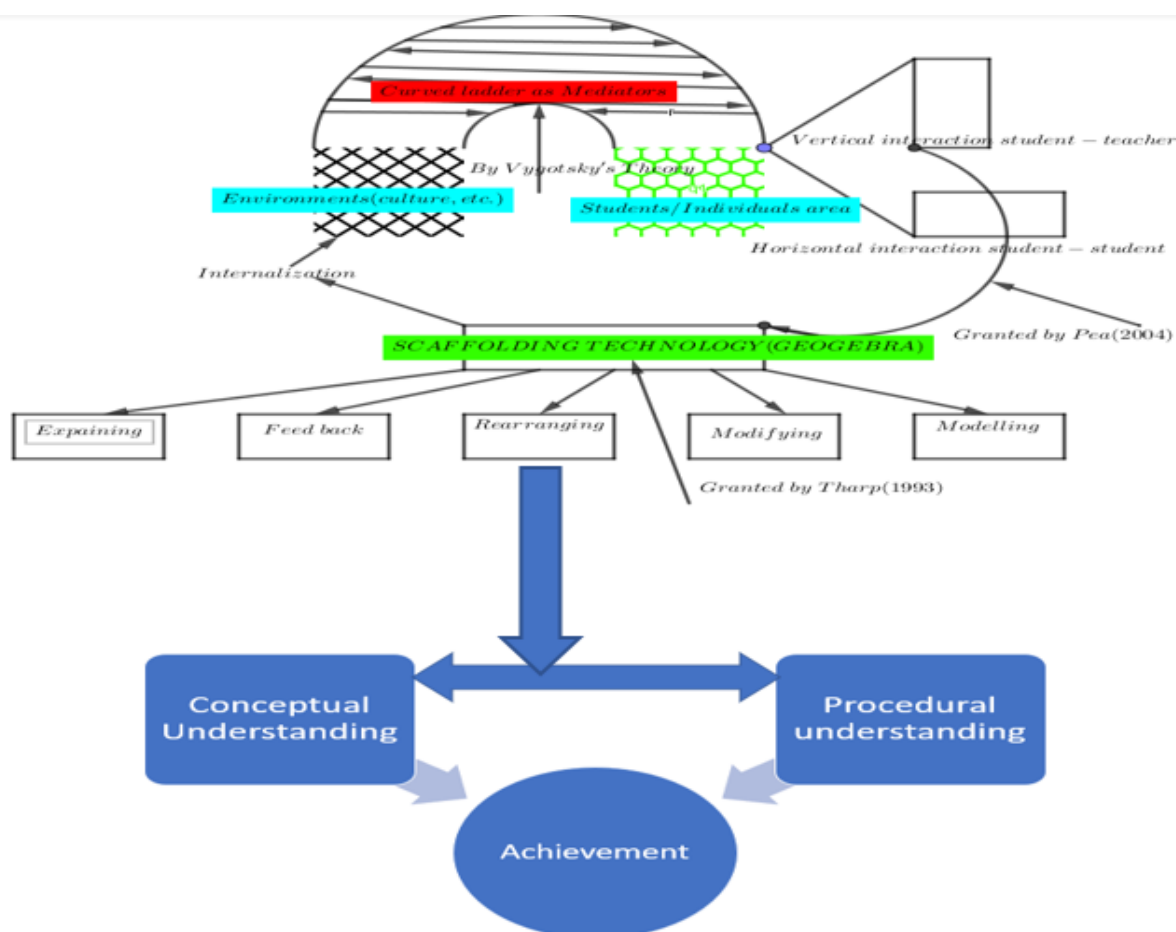


Figure 6.2 The contribution of cycle model to student proficiency

In summary, I strongly believe that the use of GeoGebra had a positive impact on visualisation through self-exploration and social interaction when learning differential calculus, in terms of both scores and perceptions of students (see Section 3.4 and Section 5.18) (Semenikhina et al., 2019). Technology/GeoGebra provides an environment of communication and interaction between students and students, and teachers and students (see cycle model steps) during the process of scaffolding (learning and education) that leads to effective teaching and learning landscapes (Ayub et al., 2008; Ayub et al., 2010; ten Brummelhuis & Kuiper, 2008). This creates positive perceptions among students towards the technology (see Section 5.9) as an educational method and towards the subjects students study, which supports the findings of Alabdulaziz et al. (2021). Achieving conceptual and procedural understanding through combining different concepts can be significantly enhanced by using the digital tools of GeoGebra Mathematical software at the tertiary level, supporting the 21st century generation in the learning environment by employing the developed cycle model, which follows the concept of Koehler and Mishra (2009).

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APPENDICES

1. COURSE OUTLINES AND LESSON PLANS

1.1. COURSE OUTLINES

Calculus I: Course outline						
University						
College/Faculty						
Department of Mathematics						
Department/ programme	BSc in Mathematics					
Module name	Basic functions of one variable					
Module No.	02					
Course Title	Calculus I					
Module Code	Math-M1021					
Course Code	Math1011					
Course EtCTS	7					
Instructor's Information	Name _____ Office No: _____ Phone No _____ Consultation hours _____ E-mail _____					
Study workload (in hours)	Lecture	Tutorial	Lab work	Assessment	Home study	Total study
	64	32	0	8	85	189
Lecture Hours, Days & Rooms						
Tutorial Days & Hours						

Target Group	First-Year Mathematics students			
Semester	II			
Prerequisites	Basic Mathematics for Natural Sciences			
Course objectives: On completion of the course, successful students will be able to: understand the formal definition of limit and continuity, evaluate limits of functions, determine points of discontinuity of functions, apply Intermediate Value Theorem, evaluate derivatives of different types of functions, apply derivatives to solve problems, evaluate integrals of different types of functions, apply integrals to find areas and volumes.				
Course Description: This course introduces the basic concepts of limit, continuity, differentiation, integration, and some of their applications.				
Week	Content	Methodology	Teacher's activity	Students' activities
1	Revision of intuitive definition and general concepts of limit The formal definition of limits and examples Basic limit theorems One-sided limits	Lecturing, Problem-solving, Gapped lecture, Pair work, Group discussions.	Lecturing, Forming and leading group discussions, Asking and answering questions, Preparing homework, Giving feedback.	Listening and taking t notes, Asking and answering questions, Doing homework.
2	Infinite limits, and limit at infinity Continuity The intermediate value theorem (IVT) and its applications	Lecturing, Problem-solving, Gapped Lecture, Pair work	Lecturing, Forming and leading group discussions, Asking and	Listening and taking notes, Asking and

		Group discussions.	answering questions, Preparing homework Giving feedback.	answering questions, Doing homework.
3	<p>Definitions of derivative</p> <p>Tangent and normal lines</p> <p>Properties of derivative</p> <p>Derivatives of different functions (polynomial, rational, trigonometric, exponential, logarithmic and hyperbolic functions)</p> <p>The chain rule and parametric equations</p> <p>Higher-order derivatives</p>	<p>Lecturing,</p> <p>Problem-solving</p> <p>Gapped lecture,</p> <p>Pair work</p> <p>Group discussions.</p>	<p>Lecturing, Forming and leading group discussions, Asking and answering questions, Preparing homework and feedback, Giving assignments.</p>	<p>Listening and taking notes,</p> <p>Asking and answering questions,</p> <p>Doing homework and completing assignments.</p>
4	<p>Implicit differentiation</p> <p>Extreme values of functions</p> <p>Rolle's theorem and the mean value theorem and their applications</p>	<p>Lecturing,</p> <p>Problem-solving,</p> <p>Gapped lecture, pair work,</p> <p>Group discussions.</p>	<p>Lecturing, Forming and leading group discussions, Asking and answering questions, Preparing homework and feedback, Giving Assignments.</p>	<p>Listening and taking notes,</p> <p>Asking and answering questions,</p> <p>Doing homework and completing assignments.</p>

5	<p>Monotonic functions and the first and second derivative test</p> <p>Applications to extreme values and related rates</p> <p>Graph sketching</p> <p>Tangent line approximation and the differentials</p> <p>Indeterminate forms and L'Hôpital's rule</p>	<p>Lecturing,</p> <p>Problem-solving,</p> <p>Gapped lecture,</p> <p>Pair work,</p> <p>Group discussions.</p>	<p>Lecturing,</p> <p>Forming and leading group discussions,</p> <p>Asking and answering questions,</p> <p>Preparing Homework,</p> <p>Giving feedback,</p> <p>Setting assignments,</p> <p>Administering take-home examination.</p>	<p>Listening and taking notes,</p> <p>Asking and answering questions,</p> <p>Completing take home examination.</p>
6	<p>Ant derivatives</p> <p>Indefinite integrals and their properties</p> <p>Partitions, upper sum, lower sum</p> <p>Riemann sums</p> <p>The definite integral</p> <p>fundamental theorem of calculus</p>	<p>Lecturing,</p> <p>Problem-solving,</p> <p>Gapped lecture,</p> <p>Pair work,</p> <p>Group discussions.</p>	<p>Lecturing,</p> <p>Forming and leading group discussions,</p> <p>Asking and answering questions,</p> <p>Giving feedback.</p>	<p>Listening and taking notes,</p> <p>Asking and answering questions.</p>
7	<p>Techniques of integration</p> <p>Application of integration</p>	<p>Lecturing,</p> <p>Problem-solving,</p> <p>Gapped lecture,</p> <p>Pair work</p> <p>Group discussions.</p>	<p>Lecturing,</p> <p>Forming and leading group discussions,</p> <p>Asking and answering questions,</p>	<p>Listening and taking notes,</p> <p>Asking and answering questions.</p>

			Preparing and conducting tests, Giving feedbacks	
8	Revision and a final examination week			
Assessment	<div>1. Quiz (1)5%</div> <div>2. Assignment (1)10%</div> <div>3. Test (1 & 2)20%</div> <div>5. Presentation5%</div> <div>6. Takehome exam.....10%</div> <div>7. Final exam50%</div> <div>Total.....100%</div>			
Course policy	<div>A student must:</div> <div>Attend at least 85% of the classes.</div> <div>Complete all continuous assessments.</div> <div>Take the final examination.</div> <div>Respect all rules and regulations of the university.</div>			
<div>Textbook</div> <div>R. Ellis and D. Gluck, Calculus with Analytic Geometry, 3rd edition.</div> <div>H. Anton, Calculus with Analytic Geometry, 5th edition.</div> <div>References</div> <div>Edwards & Penney, Calculus with analytic geometry, 5th edition.</div> <div>Leithold, Calculus with Analytic Geometry, 3rd edition.</div> <div>Swokowski, Calculus with Analytic Geometry, the Alternate edition.</div> <div>Thomas, Calculus and Analytic Geometry, 9th edition.</div>				

1.2. LESSON PLAN FOR EXPERIMENTAL GROUP

Daily lesson plan (GeoGebra oriented lesson plan) for the experimental group		
Day 1	Students' achievement /goals, behaviours	<p>GOAL 1: Acquaint yourself with derivatives of different functions</p> <p>Behaviours:</p> <p>State the definition of derivatives.</p> <p>Explain the meaning of a tangent line and normal lines and find the equation of a tangent line and normal lines to the curve at a given point.</p> <p>List properties of the derivatives.</p> <p>Find derivatives of different functions (polynomial, rational, trigonometric, exponential, logarithmic, and hyperbolic functions).</p> <p>GOAL 2: Differentiate between the derivatives of different functions and their original functions.</p> <p>Find the derivatives of different functions with the help of GeoGebra.</p> <p>Visualise the difference between the graph of functions and their derivatives.</p>
	Teaching and learning methods	Teaching with GeoGebra, scaffolding using a cyclic model of GeoGebra implementation, question and answer, problem-solving
	Tools, equipment and sources	Textbook, MoSHE (Ministry of Science and Higher Education) approved supplementary books, GeoGebra, computer lab, projector, pre-prepared GeoGebra content published on the website.
	Summary	Attendance by students
Day 2	Students' achievement/goals, behaviours	<p>GOAL 1: Acquaint yourself with chain rules and parametric equations.</p> <p>Behaviours</p> <p>Explain the chain rule.</p> <p>Explain the meaning of parametric equations.</p> <p>Evaluate the derivatives of function involving the chain rule.</p> <p>Differentiate between the equation of a line and parametric equations.</p> <p>GOAL 2: Define visualisation.</p> <p>Familiarise yourself with manipulating the derivative of a function and its graph with the help of GeoGebra.</p> <p>Investigate the graph of any function with its derivatives by simulation in the algebra window and graphic window of the GeoGebra applet.</p>

	Teaching and learning methods	Teaching with GeoGebra, scaffolding using a cyclic model of GeoGebra implementation, question and answer, problem-solving
	Tools and equipment's and Sources	Textbook, MoSHE approved supplementary books, GeoGebra, computer lab, projector, pre-prepared GeoGebra content published on the website.
	Summary	Attendance by students
Day 3	Students' achievement/goals, behaviours	GOAL 1 Acquaint yourself with how to evaluate implicit differentiation.
		Behaviours
		Define implicit differentiation.
		Explain the meaning of extreme values of functions.
		Define Rolle's theorem and the mean value theorem and their applications.
		GOAL 2: Sketch the graph of functions implicitly and explicitly.
		Familiarize yourself with manipulating the derivative of any function (defined both implicitly and explicitly) and its graph with the help of GeoGebra.
		Can you sketch the graph of any function by hand? Can you use this applet?
	Teaching and learning methods	Teaching with GeoGebra, scaffolding using the cyclic model of GeoGebra implementation, question and answer, problem-solving
	Tools and equipment and sources	Textbook, MoSHE approved supplementary books, GeoGebra, computer lab, projector pre-prepared GeoGebra content published on the website.
	Summary	Attendance by students
Day 4	Students' achievement/goals, behaviours	GOAL 1: Acquaint yourself with the application of derivatives.
		Behaviours
		Explain the first and second derivatives test of determining extreme value(s).
		Explain the maximum point and minimum point of a given function and locate them on the graph.
		Define concavity and inflexion of the function.
		GOAL 2: Explore the maximum, minimum and concavity from the given graph.
		Familiarise yourself with the withdrawing of any function with the help of GeoGebra and identify the maximum and minimum value(s).
	Teaching and learning methods	Teaching with GeoGebra, scaffolding by the cyclic model of GeoGebra implementation, question and answer, problem-solving

	Tools and equipment and sources	Textbook, MoSHE approved supplementary books, GeoGebra, computer lab, projector, pre-prepared GeoGebra content published on website
	Summary	Attendance by students

1.3. LESSON PLAN FOR CONTROL GROUP

Daily lesson plan (Conventional lesson plan) for the control group		
Day 1	Students' achievement/ goals, behaviours	<p>GOAL 1: Acquaint yourself with the derivatives of different functions.</p> <p>Behaviours</p> <p>State the definition of derivatives.</p> <p>Explain the meaning of a tangent line and normal lines and find the equation of a tangent line and normal lines to the curve at a given point.</p> <p>List the properties of the derivatives.</p> <p>Find derivatives of different functions (polynomial, rational, trigonometric, exponential, logarithmic, and hyperbolic functions).</p> <p>GOAL2: Show the graph of derivatives of different functions and with their original functions.</p> <p>Work out the derivatives of different functions.</p> <p>Visualise the difference between the graph of functions and their derivatives.</p>
	Teaching and learning methods	Presentation method (lecturing), question and answer, problem-solving
	Tools and equipment and sources	Textbook, MoSHE approved supplementary books, board
	Summary	Attendance by students
Day2	Students' achievement/ goals, behaviours	<p>GOAL 1 Acquaint yourself with chain rules and parametric equations.</p> <p>Behaviours</p> <p>Explain the chain rule.</p> <p>Explain the meaning of parametric equations.</p> <p>Evaluate the derivatives of function involving the chain rule.</p> <p>Differentiate the difference between the equation of the line and parametric equations.</p> <p>GOAL 2: Define visualisation.</p>

		<p>Familiarise yourself with manipulating the derivative of any function and its graph.</p> <p>Investigate the graph of any function with its derivatives.</p>
	Teaching and learning methods	Presentation method (lecturing), question and answer, problem-solving
	Tools and equipment and sources	Textbook, MoSHE approved supplementary books, board
	Summary	Attendance by students
Day 3	Students' achievement/goals, behaviours	<p>GOAL 1: Acquaint yourself with evaluating implicit differentiation.</p> <p>Behaviours</p> <p>define implicit differentiation.</p> <p>Explain the meaning of extreme values of the functions.</p> <p>Define Rolle's theorem and the mean value theorem and their applications.</p> <p>GOAL 2: sketch the graph of functions which is given in terms of implicitly and explicitly.</p> <p>Familiarise yourself with manipulating the derivative of any function (defined both implicitly and explicitly) and its graph.</p> <p>Can you sketch the graph of any function by hand?</p>
	Teaching and learning methods	Presentation method (lecturing), question and answer, problem solving
	Tools and equipment and sources	Textbook, MoSHE approved supplementary books, board
	Summary	Attendance by students
Day 4	Students' achievement/goals, behaviours	<p>Goal1: Acquaint yourself with the application of derivatives.</p> <p>Behaviours</p> <p>Explain the first and second derivatives test of determining extreme value(s).</p> <p>Explain the maximum point and minimum point of a given function and locate them on the graph.</p> <p>Define concavity and inflection of the function.</p>

		GOAL 2: Explore the maximum, minimum and concavity from the given graph. Familiarise yourself with drawing the graph of any function and identify the maximum and minimum value(s) by visualisation if possible.
	Teaching and learning methods	Presentation method (lecturing), question and answer, problem-solving
	Tools and equipment and sources	Textbook, MoSHE approved supplementary books, board
	Summary	Attendance by students

2. CONSENT FORM

CONSENT FORM FOR STUDENTS

Section A

Title of the questionnaire: An investigation of the effect of GeoGebra mathematical software on students' learning of mathematics

Dear Respondent

My name is Bedada T B (Tola Bekene Bedada) and I am currently a PhD student at the University of South Africa (UNISA), doing my thesis in the College of Education in the Department of Mathematics Education under the supervision of Prof. M.F. Machaba.

This questionnaire forms part of my doctoral research for the degree DEd at the University of South Africa entitled: An investigation of the effect of GeoGebra mathematical software (GMS) on students' learning of mathematics (GMS) at Wachemo University. You have been selected by a purposive sampling strategy from a population of 45 public Ethiopian universities. I invite you to take part in this survey.

This study aims to investigate the effect on students' learning of Mathematics of teaching with GeoGebra Mathematical software. The findings of the study may benefit students and their parents, teachers and policymakers.

You are kindly requested to complete the three sections of this survey questionnaire as honestly and frankly as possible and according to your personal views and experience. No foreseeable risks are associated with the completion of the questionnaire, which is for research purposes only. The questionnaire will take approximately 30 minutes to complete.

You are not required to indicate your name or organisation on the questionnaire and your anonymity will be ensured; however, an indication of your age, gender, occupation position and so on will contribute to a more comprehensive analysis. All information obtained from this questionnaire will be used for research purposes only and will remain

confidential. Your participation in this survey is voluntary and you have the right to omit any question if so desired, or to withdraw from answering this survey without penalty at any stage. After the completion of the study, an electronic summary of the findings of the study will be made available to you on request.

Permission to undertake this survey has been granted by the Ministry of Science and Higher Education (MoSHE) on behalf of Wachemo University, Ethiopia, and the Ethics Committee of the College of Education, UNISA. If you have any research-related enquiries, they can be addressed directly to me or my supervisor. My contact details are phone: +251922347244, e-mail: leencaoro@gmail.com or 67119557@mylife.unisa.ac.za and my supervisor can be reached at 0124298582, Department of Mathematics Education, College of Education, UNISA, e-mail: emachamf@unisa.ac.za.

By completing the questionnaire, you imply that you have agreed to participate in this research study. Please return the completed questionnaire to the department secretary before the date indicated on the questionnaire. I would like to express my gratitude for your time and cooperation, beforehand, in completing this questionnaire. This study is purely for academic purposes. Your sincere, honest and timely responses are vital to the success of this study.

There is no “right” or “wrong” answer here; rather, what is required is your opinions.

Section B

GIVING INFORMED CONSENT

This section indicates that you are giving your informed consent to participate in the research:

I confirm that I have read this consent requesting my consent and understand the information provided and do agree to participate in this study. I do understand that my participation is voluntary and I hereby add my signature below as I am over 18 years of age.

Participant's signature _____ Date _____

Section C

Demographic Information

Please, tick in the appropriate boxes by using this "☐" mark

Age: 18-23 ☐ 24-29 ☐ 30-35 ☐ 36-41 ☐

Sex: Female ☐ Male ☐

Educational level: Undergraduate ☐ Postgraduate ☐

3. STUDY INSTRUMENTS

3.1. Student Questionnaire

The items use a five-point Likert scale ranging from strongly disagree (1) to strongly agree (5). Note that 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly agree.

Questions (Items)	1	2	3	4	5
1. At first, I did not like GeoGebra.					
2. I like GeoGebra because it is dynamic mathematical software and free for everyone.					
3. Right now, I'm more open to investigations using GeoGebra.					
4. There is mathematical software for learning calculus in secondary school.					
5. There is mathematical software for learning calculus, but I did not know how to manipulate the software at my institution.					
6. I need a lot of help when doing new things when using technology like GeoGebra.					
7. I think working with GeoGebra is frustrating.					
8. I am comfortable with GeoGebra when learning calculus.					
9. I do not want to use GeoGebra in my future studies.					
10. GeoGebra makes calculus more difficult for me.					
11. The instructional material for learning calculus through GeoGebra is well organised.					
12. I get enough time to do the activity on my own in the laboratory classroom.					
13. I depend on others to do the activity while the programme is running in the laboratory classroom.					
14. I achieved higher marks after I learned calculus through GeoGebra software.					

Thank you in advance.

3.2. Interviews with students

1. Can you tell me what you gained and what you lost when you learned calculus through GeoGebra?
2. Do you think that learning calculus through GeoGebra software is useful for students?
3. Do you want to share this software with your friends?
4. Why do you think that not all subjects integrate software in their teaching and learning?
5. Is there any mathematical software you know of that you could use to study your other subjects? If so, tell me about it; if no, what is the reason for this, do you think?

3.3. Transcribed students' focus group Interviews

3.3.1. Transcription of student's interview into English

Researcher: Can you tell me what you gained and what you lost when you learned calculus through GeoGebra?

Student 1: There are no things that I lost while I was learning differential calculus rather than I increased my motivation towards learning because of the software. I obtained higher marks than before. ... I knew GeoGebra this is also what I got. Because of its importance, I shared it with my friends, and I will also share it in future.

Student 2: For me, it is new software or thing that I did not hear before. As I say the software is freely downloadable and it is simple to use for educational purposes and which is one of the things that I got. I'm interested in the activities that are given to us while we are learning differential calculus. Because of its importance, I shared it with my friends, and I will also share it in future.

Student 3: I did not lose anything while we are learning calculus by using GeoGebra. I prefer to have GeoGebra for my study. I got high marks after I learnt calculus from GeoGebra. Because of its importance, I shared it with my friends, and I will also share it in future.

Student 4: I am interested in learning calculus by GeoGebra as it is dynamic. To do a given question by using it as simple as it did not need any syntax. Because of its importance, I shared it with my friends, and I will also share it in future.

Student 5: There are no things I lost while I was learning calculus in the classroom by GeoGebra. Because of its importance, I shared it with my friends, and I will also share it in future.

Researcher: Do you think that learning calculus through GeoGebra software is useful for students?

Student 1: While I was learned in the classroom the GeoGebra software helped me to understand calculus in the classroom as well as at the home. In general, it assists me a

lot and ... I am so happy to be a part of the study. When it is embedded into the learning calculus it is very essential for me and it saves time for me for the study.

I found that GeoGebra gives a good impression of learning calculus. I feared Mathematics especially calculus for my study, but after I have installed the GeoGebra and try out calculus on my own at home, I am very impressed to use it for my future study.

Student 2: Teacher ... thank you. For me, it is new software or thing that I did not hear before. As I say the software is freely downloadable and it is simple to use for educational purposes. Having the software on own smartphone or laptop it is simple to manipulate a calculus problem--- for example derivatives of the function as we were learnt it. Hence, in my opinion, its assists self-independence learning. Encourage learning.

I found that GeoGebra gives a good impression of learning calculus. I feared Mathematics especially calculus for my study, but after I have installed the GeoGebra and try out calculus on my own at home, I am very impressed to use it for my future study.

Student 3: I am very happy, and I recommend you teach us using GeoGebra for our future learning calculus ... continue to teach us by using GeoGebra.

I found that GeoGebra gives a good impression of learning calculus. I feared Mathematics especially calculus for my study, but after I have installed the GeoGebra and try out calculus on my own at home, I am very impressed to use it for my future study.

I have a mobile, and I have a brother with the age of 7 years old, but when I come back home from work, he immediately comes to me and took my mobile to play a game.

This implies that the seven-year-old knew how to manipulate the technology. I, therefore, recommend that instead of showing our children in Ethiopia how to play a game we should introduce them to mathematical software

Student 4: Wow! I accept all that my colleagues said.

I found that GeoGebra gives a good impression of learning calculus. I feared Mathematics especially calculus for my study, but after I have installed the GeoGebra and try out calculus on my own at home, I am very impressed to use it for my future study.

Student 5: First, I want to say thank you and the GeoGebra software is new to me at starting of the intervention. There is nothing I lost while I was learning calculus in the laboratory classroom. As we are new to the technology if it will be introduced to the teacher at the elementary as well as university-level it is very good for teaching.

Researcher for **student 5:** What do you believe about learning calculus through GeoGebra software importance for students? Can this software help full for high school?

The reaction of **student 5** to these questions where I believe that learning calculus by GeoGebra is very important. For example, I got high marks after I was introduced to the software for learning differential calculus. The activities we were imposed while we learnt differential calculus were interesting. These interesting activities were made because of the well-planned lesson and GeoGebra software. So, I think if it is implemented well at the high school level it is good.

Researcher: Why do you think that not all subjects integrate software in their teaching and learning?

Student 1: I agreed with Student3.

Student 2: I agreed with Student3.

Student 3: First, to me, I prefer technology use such as Smartphones, in addition, the teachers, students (they) may learn by themselves. Because of a lack of resources such as budget the technology is still not integrated into teaching and learning in Ethiopia.

Student 4: I agreed with Student 3.

Student 5: I agreed with Student 3.

Researcher: Do you want to share this software with your friends?

Student 1: It corrected me while I was learnt by myself as well as when I was scaffolded by the teacher in the classroom. Oh ... I did not lose anything during the intervention. I become friends with my teacher as the software increase communication between teachers and students.

Student 2: As I was mentioned in the previous questions it is a new thing that I never heard before. I am happy for the immediate feedback given to me while I am learning calculus by GeoGebra.

Student 3: I have already shared it with my friends and my friends love it.

Student 4: I have already shared it with my friends and high school teacher.

Student 5: I have already shared it with my friend who is a master's student, and I will continue to share it.

Researcher: Is there any mathematical software you know of that you could use to study your other subjects? If so, tell me about it; if no, what is the reason for this, do you think? For these questions, the **researcher** gives clues for all students by stating that:

In our country, Ethiopia, MOSHE tries to integrate the technologies into university-level currently (2021 academic year). And you knew that the course Emerging technology in your study at your freshman. Again, when we came to Mathematics students the software such as LaTeX, Mathematica and Math Lab and others were integrated to implement in the classroom in the coming semester of your study. But still, this software was needing licenses. When we come to GeoGebra it is free of the licenses. So, what do you think that if this software is integrated into the classroom for calculus teaching?

Student 1: There is no technology integration. I think it is a lack of the budget.

Student 2: Still, I did not see the technology that helps to teach calculus before this program. This software is exceptional for me.

Student 5: Well, if it is integrated into the teaching and learning process it may help teachers who have no ability of content knowledge.

... When I was at my secondary school some teachers faced a problem with difficult questions while they tried to teach. So, the technology may help those teachers to simplify difficult maths problems. So, I recommend the concerned bodies start the integration of the technology at the elementary level.

Student 4: There is no technology integration for teaching calculus. This is a new thing that is very interested and needs to be expanded to all Ethiopian contexts. I recommend that the concerned bodies should give attention to integrating technology into the classroom. I am very surprised with the course Emerging technology that we have taken and then I am surprised with the software GeoGebra we heard from you. Thank you. For me, the GeoGebra is important if it is integrated into the teaching of calculus. I understood that the software is important for graphics/visualization purposes and may use as student guides. Finally, the reason why it is not integrated still is because of lack of resources (can be infrastructures may be laboratory, electricity).

Student 3: There was no technology for teaching and learning in the classroom, specifically in the case of calculus at the elementary and university level. I know one mathematics technology that was familiar to me known as Photomath and I saw those other students even teachers at secondary level are not familiar with the technology. I think in my opinion the integration of technology in the classroom was hindered by a lack of resources.

4. CONSENT FORM FOR TEST QUESTIONS

Dear Students

I am a PhD student at the university of south Africa (UNISA) doing my thesis at the College of Education in the Department of Mathematics Education. I am conducting a research study on an investigation of the effect of students' learning mathematics through GeoGebra software at Wachemo University. Therefore, I request your assistance by inviting you to participate in the study by answering the questions below. The insights gained from these Pre-test Questions will provide helpful information, clarify mathematics student-teachers beliefs and help me to accomplish my research. The results would help to improve and develop mathematics teaching and learning at universities and elementary schools. The completion of these pre-test questions will take about 60 minutes. Your participation is voluntary, and you are free to discontinue at any time. As a participant, you have the right to ask for clarification and refuse to answer any questions. All information you provide will be kept strictly confidential and the researcher and the researcher's supervisors are the only ones who will be able to access this information. Your name not be used or associated with the study. There are no risks to you or your privacy if you decide to participate in my study. But if you choose not to participate that is fine. However, your participation and your opinions are important in helping me to obtain answers to my research questions. I would appreciate your taking the time. If you are willing to participate in the research study, please put your signature here.

4.1. QUESTION TYPES

Instructions

Answer all the questions carefully and neatly.

These are multiple-choice questions and questions requiring short answers.

The time allowed to complete these questions is 60 minutes.

	Groups	(Experimental Group or Control Group)
Name:	_____	_____
ID:	_____	_____

Part 1: Choose the best answer and encircle it.

1. What is the value of $\lim_{x \rightarrow 0} \frac{\sin 5x}{2x}$?

a. $\frac{5}{2}$

c. 1

b. $\frac{2}{5}$

d. $\frac{1}{2}$

2. What is the value of $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x+1}-1}{x}$?

a. $\frac{1}{6}$

d. $\frac{1}{3}$

b. 3

e. Does not exist

c. 6

3. Find the values of a & b such that the diving board function

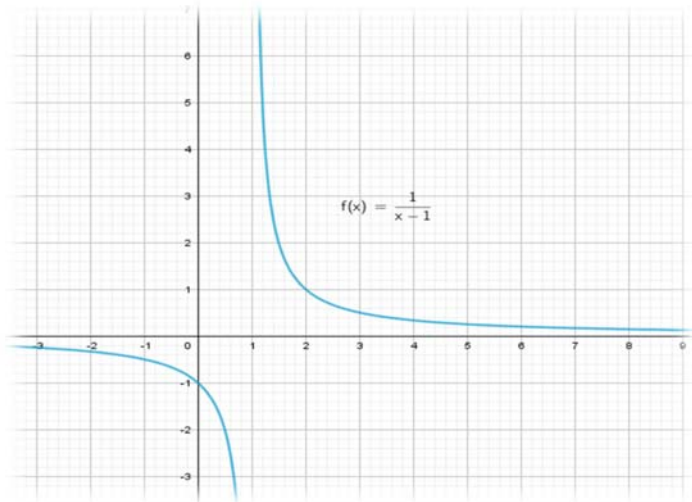
$$f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x \leq 3 \\ -2, & x \geq 3 \end{cases}$$

a. $b = 1, a = -1$

b. $a = 1, b = -1$

c. $a = 2, b = -2$

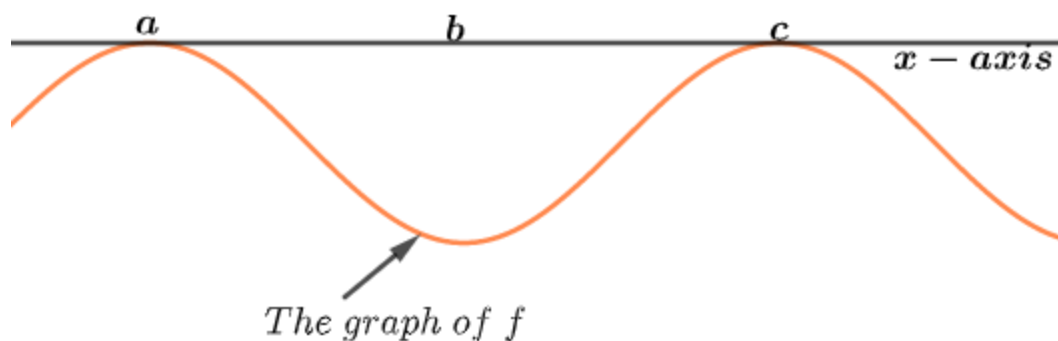
4. Assuming that the graph of the function $f(x) = \frac{1}{x-1}$ is given by



Which of the following is not true about this graph.

- a. $f(x)$ is continuous in its domain
 - b. The vertical asymptote the function is line $x = 1$
 - c. $x - axis$ is the horizontal asymptote of the function.
 - d. The value of $\lim_{x \rightarrow 1^-} f(x) = \infty$
5. Let $f(x) = e^{\ln(x^2)}$ be given function. Which of the following is the derivative of $f(x)$?
- a. $f'(x) = 2x$
 - b. $f'(x) = e^{\ln(x^2)}$
 - c. $f'(x) = 2$
 - d. None of the above
6. Equation of tangent line to the curve $f(x) = x^2 + 2$ that passes through the point (0,2) is:

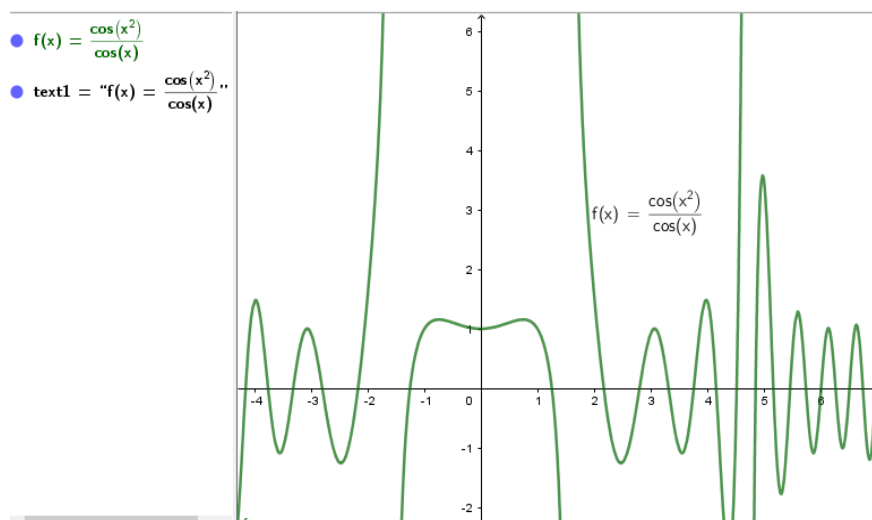
- a. $y = 2x + 2$
 b. $y = -2x + 2$
 c. $y = 2$
 d. $y = -2$
7. Let the composition function $h(x) = f(g(x))$ be given as the differentiable function of x . Which of the following is true about $h(x)$?
- a. $\frac{d}{dx}h(x) = \frac{d}{dx}f(g(x)) + \frac{f}{dx}(g'(x))$
 b. $h'(x) = f'(g(x)) + g'(x)$
 c. $h'(x) = f'(g(x)) \times g'(x)$
8. The derivative of $g(x) = \cos(\cos^{-1}(\sqrt{x^2 + 1}))$ is
- a. $g'(x) = \sqrt{x^2 + 1}$
 b. $g'(x) = \frac{2x}{\sqrt{x^2 + 1}}$
 c. $g'(x) = \frac{x}{\sqrt{x^2 + 1}}$
9. Which of the following is true about the critical point(s) "c" of the function $p(x) = \frac{x^3}{3} - x^2 + x + 1$
- a. $c = \pm 1$ is the only critical point.
 b. $c = 1$ is the only critical point
 c. $c = -1$ is the only critical point
10. Let M_1 be the slope of the function $y = 5^x$ at the point $x = 0$ and let M_2 be the slope of the function $y = \log_5 x$ at $x = 1$. Then
- a. $M_1 = \ln(5)M_2$
 b. $M_1 = M_2$
 c. $M_1 = -M_2$
 d. $M_1M_2 = 1$
 e. $M_2 = \ln(5)M_1$
11. By using the power rule of derivatives, you that the derivative of $x^{1/3} = \frac{1}{3}x^{-2/3}$ for every $x \neq 0$. Then $\lim_{x \rightarrow 8} \frac{\left(\frac{x}{8}\right)^{1/3} - 1}{x - 8} = \frac{1}{2a}$, where $a =$ _____
- a. 4
 b. 8
 c. 6
 d. 12
12. Suppose that the graph of the function f is drawn as in the following figure.



Which of the following is not true?

- a. The function f is concave upward on interval $[a, c]$
- b. The function attains the minimum value at point b
- c. The maximum values of the function occur at the point a and c
- d. The function has no inflection point.

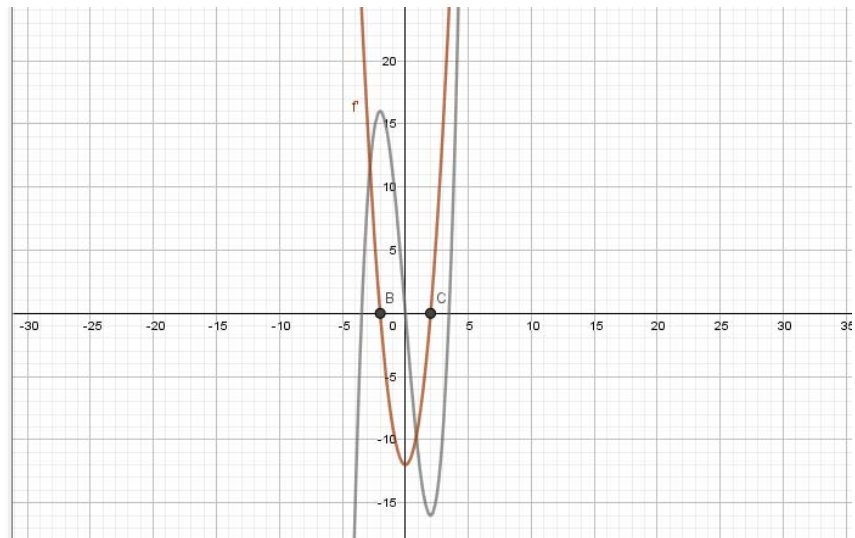
13. The first derivative of the function $f(x) = \frac{\cos(x^2)}{\cos x}$ which is indicated in the following is:



- a. $\frac{\sin(x) \cos(x^2) - 2x \sin(x^2) \cos(x)}{\cos^2 x}$
- b. $\frac{\sin(x) \cos(x^2) + 2x \sin(x^2) \cos(x)}{\cos(x^2)}$
- c. $\frac{\sin(x) \cos(x^2) + 2x \sin(x^2) \cos(x)}{\cos^2 x}$
- d. $\frac{\sin(x) \cos(x^2) - 2x \sin(x^2) \cos(x)}{\cos(x^2)}$

14. By using the following graph of the function $f(x) = x^3 - 12x'$ determine which of the following is true?

- $f(x) = x^3 - 12x$
- $f'(x) = 3x^2 - 12$
- A = (0, -12)
- B = (-2, 0)
- C = (2, 0)



- a. The turning point of the derivatives of the function $f(x)$ points A
 - b. Between point B and C the function $f(x)$ is increasing
 - c. From point B to negative infinite the function $f(x)$ is decreasing
 - d. -2 is the only critical point of the function $f(x)$.
15. Let $r(t)$ stand for the position of a particle at the time t. Which of the following is false?
- a. $r'(t)$ Represents the velocity of a particle at time t
 - b. $r''(t)$ Represents the acceleration of a particle at time t
 - c. $r'(t)$ Represents the length of a particle at time t.
16. What is the first derivative of the function $f(x) = \frac{x}{x^2+1}$ at $x = 0$?
- a. 0
 - b. 1
 - c. 2
 - d. 3
17. $\frac{dy}{dx}$ of $x^2 + \cos(xy^2) = xy$ is:

a. $\frac{dy}{dx} = \frac{2x-y-y^2 \sin(xy^2)}{2xy \sin(xy^2)+x}$

b. $\frac{dy}{dx} = \frac{2x-y+y^2 \sin(xy^2)}{2xy \sin(xy^2)+x}$

c. $\frac{dy}{dx} = \frac{2x-y^2 \sin(xy^2)}{2xy \sin(xy^2)+x}$

18. Use the fact that $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$. Then $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{2x+1}$ is:

a. e^2

b. e^{-4}

c. e

d. e^{-1}

e. None of the above

Part 2: Work out the problem.

Show all necessary steps in finding the required answers and write your final answer carefully.

19. Find the equation of a tangent line to the function $xy = 1$ at $x = 1$ and sketch the graph of $xy = 1$.

20. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ Is $f(x)$ continuous at $x = 0$? Justify

5. ETHICAL CLEARANCE

5.1. ETHICAL CLEARANCE FROM UNIVERSITY OF SOUTH AFRICA



UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/03/11

Ref: **2020/03/11/67119557/10/AM**

Name: Mr TB Bedada

Student No.: 67119557

Dear Mr TB Bedada

Decision: Ethics Approval from
2020/03/11 to 2025/03/11

Researcher(s): Name: Mr TB Bedada

E-mail address: 67119557@mylife.unisa.ac.za

Telephone: +251922347244

Supervisor(s): Name: Prof. M.F. Machaba

E-mail address: emachamf@unisa.ac.za

Telephone: 0124298582

Title of research:

**AN INVESTIGATION OF EFFECT OF STUDENTS' LEARNING MATHEMATICS
THROUGH GEOGEBRA SOFTWARE: THE CASE OF WACHEMO UNIVERSITY,
ETHIOPIA**

Qualification: PhD Mathematics Education

3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.

7. No field work activities may continue after the expiry date **2025/03/11**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

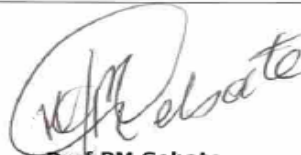
Note:

*The reference number **2020/03/11/67119557/10/AM** should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,



Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za



Prof PM Sebate
ACTING EXECUTIVE DEAN
Sebatpm@unisa.ac.za

5.2. PERMISSION FROM WACHEMO UNIVERSITY

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F.D.R.E Ministry of Science and Higher Education
Wachemo University
College of Natural and Computational Science
Department of mathematics
Bonsara Ethiopia


RefNo: WCU/CNCS/0018/12
Date: 10/01/2020


To: - Department of Mathematics Education, University of South Africa (UNISA)

Subject: - Giving Letter of Consent

Mr.T B Bedada (Tola Bekene Bedada) requested our University to get letter of consent from the participant in his research work. The department of Mathematics in behalf of **WACHEMO-UNIVERSITY** communicated all mathematics teachers in the department of Mathematics to give their permission to participate in the study by explain his research thesis and shown positive response. And the University again permit him to do his thesis on the Mathematics Department students in the Mathematics Laboratory of our department.

Therefore, we would like to inform your institution that Mr. T B Bedada (Tola Bekene Bedada) to conduct his PhD project/thesis in our University as well as has got permission from all mathematics teachers (participant of the research) and students beside the Mathematics department of our university to participate in the study by giving all the necessary data he needs in the study.

With Best Regards

Wubeshet Seyoum Merabo
Head, Department of Mathematics



Phone No: +251-846-555-1910 Fax: +251-846-555-1910 P. O. Box 667

6. TURNITIN REPORT

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THE USE AND EFFECT OF GEOGEBRA SOFTWARE IN CALCULUS AT WACHEMO UNIVERSITY, ETHIOPIA: AN INVESTIGATION

TOLA BEKENE BEDADA

Submitted in partial fulfilment of the requirements for the degree

Doctor of Philosophy in Mathematics Education

at the

University of South Africa (UNISA)

Department of Mathematics Education

Supervisor: Prof. M.F. MACHABA

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7. LETTER FROM EDITOR

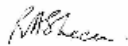
Certificate
Dr Ruth A Scheepers Language Specialist

This is to confirm that I, Ruth Angela Scheepers, edited this thesis for language and style. In doing so, I complied with all professional requirements for editing research theses. The onus is, however, on the student to implement the changes that I have suggested.

Title of thesis:

THE USE AND EFFECT OF GEOGERRA SOFTWARE IN CALCULUS AT WACHEMO UNIVERSITY,
ETHIOPIA: AN INVESTIGATION

Student: TOI A BEKENE BEDADA



Dr RA Scheepers

30 August 2021

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SATI Membership number: 1000028

8. WORK PLAN

This study will be completed according to the following timetable:

Work plan

Steps in the research plan	Deadline for completion
Submission of the proposal	April-July 2019
Design of the research study	April-July 2019
Permission to work in Wachemo University to access data	December 2019
Literature review	March 2019-March 2022
Defining universe and setting up criteria for selection of students. Grouping students into two groups. Designing a lesson plan.	December 2019-October 2020
Administering pre-tests to students to determine which test to use.	October 2020-November 2020
Delivering course to both experimental and control groups.	November 2020-December 2020
Post-test administered to both groups.	December 2020
Interviews and questionnaires were conducted with experimental group students. At the same time, the questionnaire will be given to teachers.	November 2020
Editing of completed post-test, interviews and questionnaires, grouping and coding of data, entering data into a computer program.	January 2021-March 2021
Categorising the draft analysis of qualitative and quantitative data.	March 2021-April 2021
Analysis of data	May 2021-July 2021
Report findings	August 2021-September 2021
Presentation of final research product(s)	January 2022