EXPLORATION OF GRADE 8 LEARNERS' MISCONCEPTIONS IN LEARNING SURFACE AREA AND VOLUME OF PRISMS AT A HIGH SCHOOL IN JOHANNESBURG EAST DISTRICT

by

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DECLARATION

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Exact wording of the title of the dissertation as appearing on the electronic copy submitted for examination:

Exploration of Grade 8 learners' misconceptions in the learning of surface area and volume of prisms at a high school in Johannesburg East District

I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the dissertation to originality checking software and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.

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Limitation of the summary:

This study was conducted in order to explore Grade 8 learners' perceptions through misconceptions they display when learning surface area and volume of prisms, which hinder conceptual understanding. A case study was used to identify, analyse and interpret the nature of learner misconceptions following constructivism proponents.

Constructivism is birthed from the philosophical paradigm of known as interpretivism, Adom, Yeboah and Ankrah (2016). The interpretivist approach emphasises the researcher to appreciate differences between people. Constructivism seeks to understand how individuals make sense of their everyday lives in natural settings. In this study, misconceptions in surface area and volume are interpreted from the learners' point of view, as they construct meaning of the concepts through a test and interviews.

Data were collected through a scholastic test on surface area and volume, followed by in-depth interviews over purposefully sampled participants. Data were analysed using the narrative interpretative approach following Battista and Clement (2003), framework of misconceptions in surface area and volume. Battista and clement propound the five types of misconceptions in surface area and volume as: treating 2-D shape as 3-D objects; confusing surface area with volume; assuming that a shape has more than one surface area and volume; counting only the visible faces and incorrectly enumerating cubes in arrays.

Educational implications are drafted in the concluding chapter, with recommendations and suggestions for effective conceptual development as well as minimising misconceptions on curriculum interpretation and implementation under the topic surface area and volume.

The following is a list of key words that were used in this research.

Key Terms to describe the Research Topic:

cognition; concept; conceptualisation; conceptual knowledge; constructivism; generalisation; misconception; perception; relationship; understanding

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I would also like to thank God, our Heavenly Father, for the gift of life and grace to see this vision come to pass.

LIST OF ABBREVIATIONS

ANA	Annual National Assessment
CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
TIMMS	Trends in International Mathematics and Science Study
2-D	Two-dimensional
3-D	Three-dimensional

ABSTRACT

This study was aimed at exploring the misconceptions in learning surface area and volume among Grade 8 learners at a school in the Johannesburg East Education District.

In order to understand how learners conceptualised and perceived surface area and volume, I used the constructs found in constructivism. A qualitative design was employed to explore misconception displayed among Grade 8 learners at a selected school following a case study approach. The approach enabled me to interpret and analyse dynamics found in solitary settings, following ideas from Miles, Huberman and Saldaña, (2018).

The study enabled the researcher to investigate the causes of misconceptions in learning surface area and volume through a scholastic test and post-test interviews. Misconceptions from previous studies were confirmed while some not found in the literature were discovered during the study. Some causes of misconceptions in surface area and volume were discussed, suggestions and recommendations made to minimise misconceptions in learning the discussed concepts.

Most learners could not define surface area or volume, nor would they describe how surface area or volume could be obtained, without using some formula. However, they could calculate such values when given measurements, largely by application of the relevant formula. Where some learners gave correct responses, at times it was not through the proper procedure.

Among the discovered misconceptions was that learners confused volume with mass and surface area with the perimeter of edges around a 3-D object. Learners also considered that when one was not given measurements of a diagram built of unit cubes, then they would not be able to find the surface area and volume. They also thought the same of "irregular" blocks and shapes composed of blocks and spaces.

Regarding prisms of the same volume, learners were not able to derive that longer prisms would have larger surface areas than their shorter counterparts.

In this study, misconceptions in surface area and volume were largely attributed to lack of mastery of basic facts and concepts, among other reasons. Learners lacked conceptual knowledge but displayed some procedural knowledge in most of the test items under-study.

Keywords:

Cognition; concept; conceptualisation; conceptual knowledge; constructivism; generalisation; misconception; perception; relationship; understanding

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1 CHAPTER ONE: OVERVIEW, BACKGROUND AND RATIONALE OF THE STUDY

1.1 INTRODUCTION AND PROBLEM STATEMENT

Having taught mathematics at Senior Phase for more than two decades, I have anguished when seeing learners, especially at Grade 8 level, struggle to understand and distinguish between the volume and surface area of given 3-D figures. In most cases, learners would define the concepts by simply reciting their respective formulae and merely substituting the values into the formula to solve problems.

1.1.1 PROBLEM

What misconceptions do Grade 8 learners at a high school in Johannesburg East district of Gauteng have, regarding the concepts surface area and volume of prisms? In this study, I am investigating misconceptions that hinder learners form understanding the concepts surface volume and area of prisms.

Trends in International Mathematics and Science Study (TIMSS) administered a test at Grade 9 level in 2015. This was followed by an item-by-item Diagnostic Report, which was released in November 2016 revealing a shocking result, concerning South African learners. The percentages of correct responses from South African learners were very low, in comparison with international averages on questions related to surface area and volume.

In 2014, the Department of Basic Education (South Africa) conducted an Annual National Assessment (ANA) in all subjects. The Diagnostic report on Grade 9 mathematics recorded a weakness in understanding surface area, where learners lacked knowledge of the formula to calculate the given polyhedron's surface area (DBE, 2016). This then raises many questions regarding the depth and intensity of coverage under misconceptions, specifically those related to volume and surface area.

In this chapter, I highlight the statement of purpose, significance and benefits of this study. The chapter also contains the location of the topic under study in the current

South African Curriculum Assessment Policy Statement (CAPS). Lastly, I enumerate the research questions, objectives and chapter outline for this research project.

1.2 STATEMENT OF PURPOSE

The purpose of this study was to identify, explore and investigate misconceptions that Grade 8 learners from one school in the Johannesburg East District have, in the learning of the concepts surface area and volume. The investigation should lead to the identification of the causes of the misconceptions as well as strategies that can be employed to reduce barriers in learning. Although data were collected from the learners' perspective, it has been used to help teachers understand why learners develop misconceptions in the concepts under study.

1.2.1 RATIONALE

One of the challenges mathematics teachers face in today's classroom is space and measurement, particularly surface area and volume (Thompson and Preston, 2004). Learners struggle with misconceptions in surface area and volume because due to lack of strong foundation skills and practice with the composing and decomposing of rectangular arrays. The underlying factors faced by learners today revolve around the absence of attention to detail and meaningful attempts of real life application to what measurement represents. The study will help teachers to address the many concerns surrounding learner perplexity and remedy inaccuracies in dealing with surface area and volume.

The rationale for investigating the causes of misconceptions in the learning of surface area and volume is to help teachers and curriculum planners minimise barriers to learning mathematics. Study has shown that learners struggle with measurement and geometry concepts in mathematics. Learners with undiscovered misconceptions develop wrong mathematics. Eventually these learners find mathematics difficult and drop the subject in later school years. The country is in great need of mathematicians and scientists. This need cannot be realised with the current trend of mathematics drop outs, due to poor results, related to misconceptions in learning. It is for this reason that misconceptions need immediate identification and correction.

Misconceptions have affected learner performance at various national and international benchmark assessments at the Grade 9 level over the years. The research will help teachers develop ways for improving learners' understanding of the concept, thereby reducing barriers to learning the topic. When misconceptions are cleared, learners gain confidence in learning the subject at this level (Senior Phase) and choose mathematics in Further Education and Training, resulting in candidates choosing careers in engineering and architecture, which the nation needs for development and growth.

1.3 SIGNIFICANCE OF THE STUDY

1.3.1 Why this Research?

From the teaching experience I have acquired in the past 30 years as a mathematics educator, I have noticed that the concepts surface area and volume had been a nightmare to most learners in the senior phase level. Many learners tend to use the terms interchangeably, an indication of some misconceptions around the learning process. As an educator, I have tried to find meaning and the reasons why learners struggle with the topic. It is for this reason, that I decided to conduct this research.

1.3.2 The Benefits of Doing this Research

I believe that this study's findings will help practising teachers deal with the challenges that learners meet in realising meaningful learning and conceptual acquisition of surface area and volume. Other stakeholders such as subject specialists who advise teachers, curriculum planners, the heads of mathematics departments in schools, and parents or guardians whose children cover surface area and volume at school will also benefit from this study's results. Ultimately, it is of national benefit as it could produce citizens who: understand phenomena; are life-long learners; have self-esteem, and are not frustrated by mathematical concepts.

Dewey (1916) postulates that anything can be taught to anyone at any time provided it is scaled down to the learner's level. This challenges teacher to focus on creating the notion that mathematics is easy and fun.

1.3.3 Location in the Curriculum

Volume and surface area are both sub-concepts of measurement, which are dealt with in the Senior Phase Mathematics Curriculum Assessment Policy Statement (CAPS), (DBE, 2012), in the current South African curriculum. The CAPS document's specific focus at Grade 8 level is the use of appropriate formulae in measuring surface area and the volume of cubes, rectangular prisms and triangular prisms, describing the relationship between surface area and the volume of these 3-D figures, as well as selecting and appropriately converting into suitable units of measurement under the two concepts (surface area and volume) (DBE, 2012).

Between Grade 7–9, the CAPS document promotes the use of nets to create models of platonic solids as well as the derivatives of formulae from nets to calculate surface area and volumes of given solids. Possibly, this follows the assumption that area and volume concepts have been introduced and developed in the lower grades and earlier phases, as measurement is covered as early as the foundation phase. Area and volume are extensively covered in the Intermediate phase, Grade 4–6 (DBE, 2012), using Standard International Units. However, the concepts are also evident in the Foundation Phase from Grade 1–3, using discrete measurements (DBE, 2012).

The clarification notes, or teaching guidelines for teachers, provided for the senior phase in the CAPS curriculum, provide the formula that learners should use for the volume of a cube and rectangular prism (DBE, 2012). The notes further emphasise that the amount of space inside a prism is called its capacity, while the space occupied by the prism is its volume. With respect to the surface area, at senior phase, Grade 7–9 (DBE, 2012), the notes stress that learners will investigate the nets of cubes and rectangular prisms to derive the formulae for calculating their respective surface areas. This eventually results in misconceptions, as learners try to find and construct meaning in the development of volume and surface area concepts.

1.3.4 Theoretical Underpinning and Conceptual Framework

This study is informed by the theories in constructivism. Constructivism is birthed from the philosophical paradigm of known as interpretivism, Adom, Yeboah and Ankrah (2016). The interpretivist approach emphasises the researcher to appreciate differences between people. Constructivism seeks to understand how individuals make sense of their everyday lives in natural settings. In this study, misconceptions in surface area and volume are interpreted from the learners' point of view, as they construct meaning of the concepts through a test and interviews.

Learners use properties of 2-D shapes in nets of 3-D objects as previous knowledge required in deriving a formula for finding surface areas of prisms. Piaget's (1936) theory of cognitive development purports that learners' understanding of new concepts is dependent on the previously acquired knowledge. Learners make constructs of new knowledge by associating it with the old schemata. The finding of meanings of new knowledge associated with the old is linked to Piaget's (1974) theory of constructivist learning, which then guides this study. Learners construct new knowledge on the foundations of their existing knowledge, and all knowledge is constructed rather than perceived through the senses (Von Glasersfeld, 1974). Learners tend to make errors when they try to find their own meaning of new knowledge, which results in misconceptions in learning. According to Piaget (1936), when learners construct the meaning of new knowledge, misconceptions do arise, resulting in many barriers to learning. In this study, I explored misconceptions in surface area and volume guided by Battista's framework (2003). I used the framework discussed in Chapter 2.2 to analyse the misconceptions that Grade 8 learners display.

Van Hiele (1985) places learners in Grade 8 as operating at least at level 2 of the geometric thought scale, characterised by relationships between geometric objects' properties (Van de Walle et al., 2014). Van Hiele (1986) states that at this level students try to appreciate informal deductive arguments about shapes and their properties. Thus, it is common to find misconceptions and incorrect generalisations in learners' mathematical reasoning at this level.

Jaworski (2008) argues that the time allocation for the coverage of a topic in the curriculum is a limitation on learners' conceptualisation of surface area and volume, resulting in teachers eventually, reluctantly, explaining concepts to learners. No remedy to this challenge has yet been given by previous researchers, hence the need to further pursue this topic. I have designed three research questions to guide my study to find the causes of misconceptions in the learning of surface area and volume,.

1.4 THE RESEARCH QUESTIONS

What are Grade 8 learners' (at a high school in Johannesburg East District) perceptions about misconceptions in the learning of surface area and volume of prisms?

1. How do learners in Grade 8 define the concepts of surface area and volume of prisms?
2. How do learners in Grade 8 solve problems involving surface area, volume of prisms and the relationship between them?
3. What misconceptions prevail when learners in Grade 8 deal with problems involving surface area and volume of prisms?

1.5 THE OBJECTIVES OF THE STUDY

The objectives of this study are:

1. To find out how learners at Grade 8 define the surface area and volume of rectangular prisms.
2. To explore how Grade 8 learners solve problems involving surface area and volume of rectangular prisms.
3. To describe evident misconceptions that Grade 8 learners display when solving problems in surface area and volume.

1.6 CHAPTER OUTLINE

This research is structured into five chapters. Chapter 1 outlines the background of my research, the statement of purpose, objectives of the study, research questions and the significance of the study.

Chapter 2 gives a detailed account of previous work completed by other academic writers and researchers covering misconceptions in the learning of surface area and volume. A literature review of misconceptions in the two concepts is also covered here, guided by the constructivism framework, thereby identifying the research gap. Chapter 3 discusses the pedagogy of the methodology followed in this research. Issues pertaining to the research design used, the sample, participant sampling method, research instruments, trustworthiness are covered in this chapter in conjunction with ethical considerations and study's limitations.

In Chapter 4, an analysis and interpretation of the data, collected through the research test and interviews, is given. Identified misconceptions in the conceptualisation of surface area and volume are described and discussed. Research findings are tabulated and represented in graphs, then critically analysed. Chapter 5 discusses the findings of the entire study and after that, concludes the study by giving recommendations for further related study in the field of mathematics.

1.7 CONCLUSION

This chapter highlighted the background of the study, the statement of purpose, rationale, objectives and research questions. The significance of the study, as well as its location in the curriculum have also been outlined.

Some research has been covered on misconceptions occurring as learners interact with surface area and volume. Chapter 2 outlines a great deal of them, as I discuss and review literature related to misconceptions in learning surface area and volume across the board.

2 CHAPTER TWO: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter covers the definitions of surface area, volume and misconception. I also briefly highlight known misconceptions in surface area and volume as given in the literature to date. I will also provide an overview of the causes of misconceptions in surface area and volume. Finally, a reflection on the literature gap, the theoretical framework underpinning this research study, and a list of key words are given.

2.2 LITERATURE REVIEW

2.2.1 What is Surface Area?

Surface area refers to the total area of all 2-D faces on a 3-D shape or solid object, whether flat or curved, which cover its outer surfaces. Area tells us how much material is required to cover a two-dimensional (2-D) shape. The total surface area of a 3-D shape refers to the sum of the areas of all polygons or 2-D figures that enclose the shape. The most fundamental property of surface area is its summation, which is the area of the whole shape, which is the sum of the areas of all the 2-D parts (Vincent, 2005). Surface area can be defined as the number of identical square faces on the outer surface of a shape (Mbedzi and Samson, 2013); it tells us how much material is needed to cover the shape.

The above definitions all explain that total surface area is a measure, quantity or summation of areas of all the 2-D faces (polygons) around a 3-D object (polyhedron), whether flat or curved. Research Question 1 of this study seeks to provide evidence of how Grade 8 learners interact with the above definitions while constructing meaning of surface area and volume.

2.2.2 What is Volume?

Volume is the quantity of three-dimensional space occupied by a liquid, solid or gas, (Sasanguie, Göbel, Moll, Smets & Reynvoet 2013). Mbedzi and Samson (2013) define volume as the total number of wooden cubes making up the prism. It is the space occupied by a substance or enclosed by a surface. Volume tells us how much material is required to fill a three-dimensional shape. In other words, the concept of volume refers to the amount of matter contained in an object (Sáiz, 2003). Voulgaris and Evangelidou (2003) explain that interior volume is the volume defined by the boundary surface of an object and occupied volume is defined in relation to the object's surroundings in space.

The above definitions qualify volume as an amount of some space, a quantitative measure of some space that an object or a three-dimensional figure wholly occupies, encompassing all its features. For example, if we take a coffee mug, its volume would be the space it takes up, which includes the base and the handle. Conversely, the coffee mug capacity would be the space that can be filled by the coffee one puts into it. Hence, one can subtract the thickness of the walls of the mug from its volume to obtain its capacity. This, therefore, shows the difference between the two concepts of volume and capacity.

2.2.3 What are Misconceptions?

Brodie (2010) states that the main element of constructivist research in the past has been work on misconceptions. Learners' misconceptions are generally systematic and consistent resistance to instruction. These misconceptions are arguably meaningful and practically, defensibly correct, from the learner's perspective; hence many researchers prefer referring to them as "alternative conceptions" (Brodie, 2010).

Hadjidemetriou and Williams (2002) describe a misconception as erroneous responses to a question. Smith et al. (1993) state that a misconception is a student's conception that produces a systematic pattern of error, which may be caused by misapplication of a rule, in the form of an over- or under-generalisation, or an alternative conception of a situation. This definition entails that we can identify a misconception by recognising repetitive erroneous patterns in learners' work. This challenges teachers to be on-guard, daily searching for such patterns and planning remediation or intervention programmes to help the learners correct their misconceptions.

Drews (2005) sees a misconception as a part of a faulty cognitive structure of ideas in the learner's schema that causes, lies behind, explains or justifies an error. Learners usually tend to defend the way they perceive things, even when no one can make sense of their suppositions. It becomes the mathematics teacher's role to undo the faulty structures and replace them with real or true concepts that can then equip the learners to explore the world around them.

In this study, I defined errors as organised, insistent and prevalent mistakes characteristic of learners when dealing with surface area and volume. I further distinguish errors from slips, which can easily be corrected when identified. Brodie and Berger (2010) describe errors as systematic, persistent and pervasive patterns of mistakes performed by learners across a range of contexts. Olivier (1992b) states that slips are wrong answers due to processing; they are not systematic but are sporadic and carelessly made by both experts and novices. Slips are easily detected and spontaneously corrected. Spooner (2002) regards misconceptions as indicators of poor understanding, which denote a line of thinking that causes a series of errors. He argues that misconceptions result from an incorrect underlying premise, rather than sporadic and non-systematic errors.

In constructivism, errors are said to result from misconceptions, which are conceptual structures constructed by the learner that make sense in relation to individuals' current knowledge, but which are not aligned with conventional mathematical knowledge (Nesher, 1987; Smith et al., 1993). Thus, one of this study's objectives is to describe evident misconceptions when Grade 8 learners are solving problems in surface area and volume.

2.2.4 Misconceptions in Surface Area and Volume

Battista and Clement (2003) identified the five most reported learner difficulties in comprehending the concept of surface area and volume as:

a) Treating 3-D figures as 2-D ones;
b) Counting visible faces or unit cubes instead of calculating the volume thereof;
c) Enumerating cubes in 3-D arrays incorrectly;
d) Believing that a shape has more than one surface area; and

e) Confusing the concept of volume with that of surface area, including interchangeable use of their formulae.

The above five learner difficulties became the basis of my conceptual framework. I explored these misconceptions and any others that arose among my participants as I conducted this research. I have also further explored the literature to discover what other researchers have to say about these misconceptions.

2.2.4.1 Treating 3-D shapes as 2-D ones

In a study of sixth graders' misconceptions in measurement, (2015) found that learners produced diverse varieties of formulae for volume, such as "volume = length + width + height", a misconception of treating volume as a one-dimensional structure, and "volume = length x width", thereby treating 3-D as 2-D structures. The misconception is shown in the next example in Figure 2.1, extracted from Sisman's (2015) work.

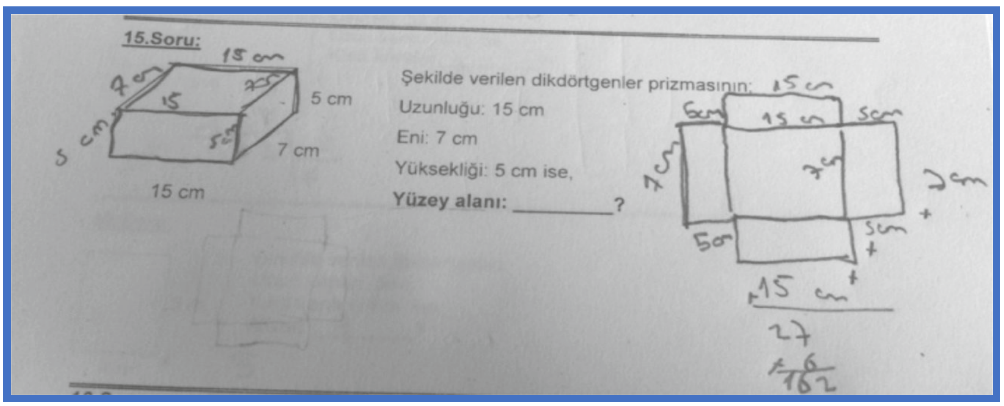


Figure 2.1: An Example of Treating 3-D shapes as 2-D one

Sisman and Aksu's (2015) participants would sometimes merely double count unit cubes for volume, while in other cases, they would simply double the number of counted unit cubes. This misconception had roots in the learners' inappropriate use of geometric figures' properties and, specifically, polyhedra.

The van Hiele (1986) theory indicates that learners in Grade 8 operate at level 2 of geometric thought, which is characterised by providing the relationships between geometric figures' properties. This calls for the use of my intended test and interview to address issues regarding the properties of 3-D shapes. The calculation of the surface area of prisms requires learners to know the properties of the respective prism, such as the number of faces as well as the relationships between them, for instance, that vertically opposite surfaces have equal area. This relationship can be used in deriving the formulae for calculating the surface areas of various solid objects.

2.2.4.2 Counting visible faces or unit cubes instead of calculating the volume thereof

Another misconception highlighted in Sisman and Aksu's (2015) work is that instead of calculating the volumes of given prisms, learners would just count the faces of a unit cube and multiply that number by three with the justification that cuboids have three dimensions. In this instance, learners counted only the visible unit cubes. Sisman and Aksu (2015) attribute this to an indication of area and perimeter confusion, which also includes using the measurement for area or volume units while reporting perimeter, as seen in the extracted illustration in Figure 2.2.

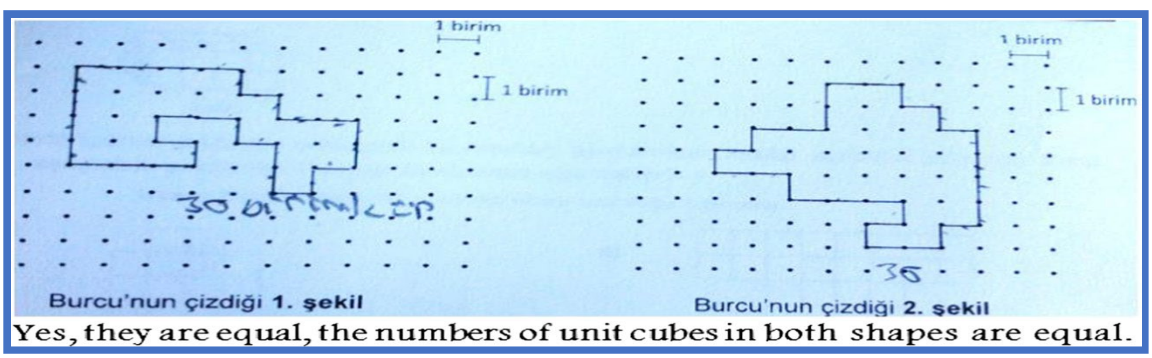


Figure 2.2: An example of counting visible faces or unit cubes instead of calculating the area thereof

2.2.4.3 Enumerating cubes in 3-D arrays incorrectly

Battista and Clements (1996) reiterate that in dealing with 3-D rectangular arrays, students' spatial thinking is related to their enumeration strategies. Battista and Clements' findings suggest that many students cannot count the cubes in a 3-D array because they lack the ability to synchronise separate views of an array before integrating them into one entity.

Given the following cube arrays, for instance, to establish which building block occupies the most space and give reasons, learners would only count the visible cubes in each array and, at times, fail to establish that an equal number of cubes of the same volume can be arranged to obtain different surface areas. Hence, learners would need concrete manipulative models to use, rather than just pictures in diagrams.

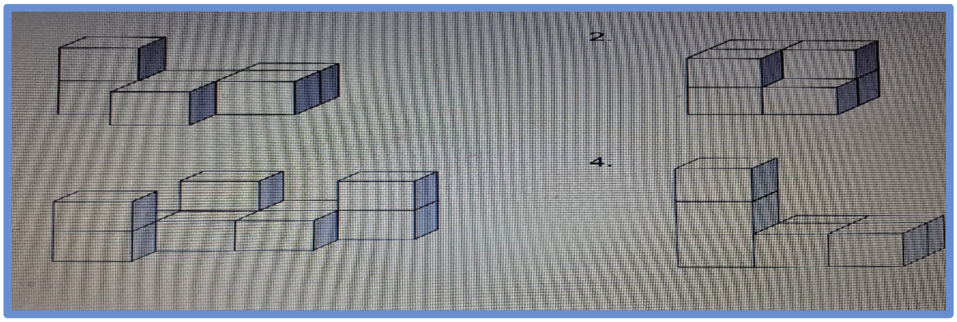


Figure 2.3: Enumerating cubes in 3-D arrays incorrectly

In research conducted by Voulgaris and Angelidou (2003), learners' responses to tasks involving volume of rectangular prisms composed of unit cubes, participants incorrectly counted visible and invisible cubes, resulting in failure to include the correct number of invisible cubes. The learners counted surface area (i.e. squares and not cubes) on the rectangular structures' visible faces, neglecting the 3-D forms (Voulgaris & Angelidou, 2003).

2.2.4.4 Believing that a shape has more than one surface area

In Sisman and Aksu's (2015) work on the concept of surface area, learners were required to establish the surface area and the volume of prisms, given all the dimensions. Their findings indicate that about three-quarters of the participants could not elucidate the association between the expanse of wrapping material needed to cover a box and its surface area. Many learners confused the concept of surface area with that of volume.

2.2.4.5 Confusing the concept of volume with that of surface area, including interchangeable use of their formulae

Fifty percent of the learners in Sisman and Aksu's (2015) work counted the square units in the net of a rectangular prism to find out its volume, which the two researchers highlighted as a clear indication of the confusion of surface area with volume. This is illustrated in Figure 2.4 and 2.5 below, extracted from Sisman's (2015) analysis.

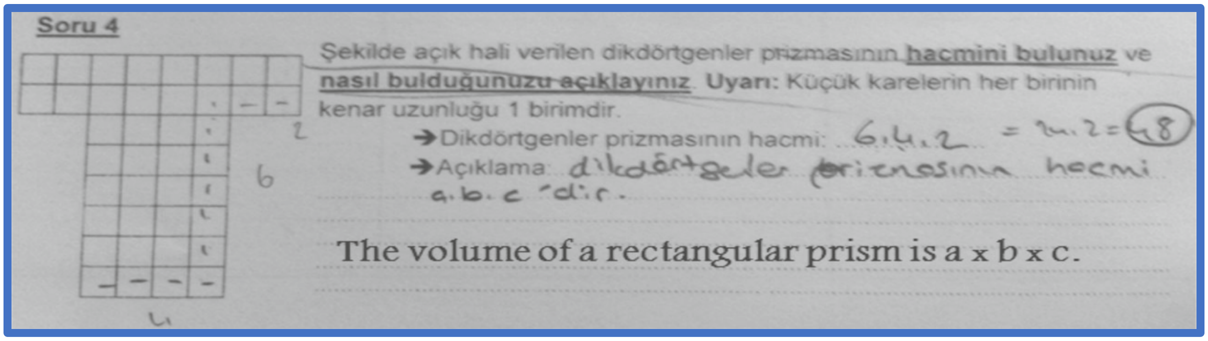


Figure 2.4: Confusing surface area with volume

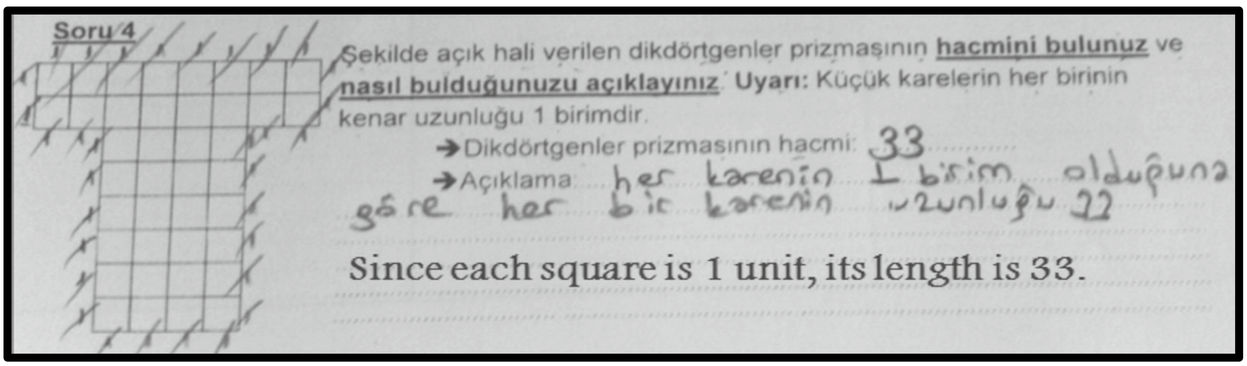


Figure 2.5: Confusing surface area with volume

In a study of measurement misconceptions among Sixth Grade students in Taiwan, Sisman and Aksu (2015) have identified a misconception in the learners' error of using the volume formula to calculate surface area. Sisman and Aksu's (2015) study has immediacy in terms of what I have experienced in South African schools in the past decade of teaching mathematics at the senior phase level. However, their research does not extensively identify possible strategies to rectify the misconceptions, especially as it covered many topics from measuring length, to measuring area and volume.

Tůmová (2017), who studied what influences Irish Grade 6 to 9 learners' success in solving conceptual tasks on area and volume, found the same misconceptions, thereby indicating a global problem. Although the two mentioned studies identify some of the misconceptions involving surface area and volume, they are both silent about what causes these misconceptions, which has become the basis of my study. This study focuses on enhancing understanding of the differences between these two concepts by looking at conceptual definitions rather than merely memorising procedural formulae.

Tůmová (2017) also states that learners often used an entirely different formula, and at times substituted incorrect measurements into formulae. Dorko and Speer (2013) add that misconception in surface area and volume is also evident among prospective teachers. The scenario results in a vicious cycle; with some teachers only possessing procedural knowledge of surface area and volume, taking charge of the mathematics classroom and unconsciously imparting incorrect knowledge to the next generation.

2.3 CAUSES OF MISCONCEPTIONS IN SURFACE AREA AND VOLUME

2.3.1 Insufficient Mastery of Basic Facts

Legutko (2008) claims that misconceptions are caused by the insufficient mastery of basic facts, concepts and skills. It can be derived from this claim that teachers need to ensure that learners have adequate mastery of learnt concepts. Baturo and Nason (1996) found that misconceptions are based solely on students' memory of disconnected rules and formulae. In measuring the surface area and volume of 3-D objects, traditional teaching methods rely on the application of formulae, falling short of opportunities for students to manipulate the objects under study, resulting in an insufficient mastery of basic facts (Hwang et al., 2009). In response to this issue, Hwang et al. (2009) suggest that the development and understanding of volume should emerge through the experience of working with cuboids and assembling and manipulating unit cubes. Extending on this, Battista and Clement (1996) allude to the three phases of acquiring concepts via manipulating spatial objects (concrete experience), brainstorming (imagery concepts) and writing symbolic solutions (abstract representation).

2.3.2 Over-Generalisation of Basic Facts

Misconceptions sometimes arise when learners apply procedures that are appropriate for one situation into a different, although related one. For example, the volume of a pyramid is not calculated using the same formula as the volume of a prism, yet they are both the same concept. This is an example of over-generalisation. Another over-generalisation is seen in the idea that we cannot subtract a bigger number from a smaller one, which is only correct when dealing with natural numbers but not integers

(Brodie, 2010). These misconceptions are prevalent in teaching strategies and pedagogies that emphasise procedures and individualised instructions more than collaborative learning (Brodie, 2010).

In many instances, learners tend to memorise the rules and algorithms to solve problems without the justification of why such procedures are selected. For example, when asked for the meaning of the volume of a rectangular prism, learners would quickly respond as follows:

$$\text{Volume} = L \times W \times H.$$

Problems would soon emerge when, for example, the volume of a triangular prism is required. Over-generalising the above formula results in incorrect answers and frustration for both the learner and the teacher, which manifests following the insufficient mastery of basic facts.

The reason learners respond in such a way is usually due to memorising a routine procedure and the over-generalisation of the formula. For example, a child might argue that the volume of a wedge is obtained by using the formula of length x width x height due to the misuse of this formula for rectangular prisms, when working with triangular prisms.

Learners who suffer from this misconception know how to compute selective volume problems but lack a true understanding of the meaning of volumes. They probably had limited or no examples of the volume concept. Furthermore, they may have been taught to memorise and use the volume formula for cuboids through rote learning in a classroom where the teacher was the only source of information, and learners were passive recipients of learning. This perception regards children as blank slates, the tabula rasa hypothesis (Locke, 1990). Such teaching-learning experiences are strongly criticised by constructivists, who believe that children have to create their own constructs and find their own meaning of the knowledge from their environment. The suggested experiences can only happen when the teacher acts as a facilitator in learning, guiding learners to correct their errors in constructing meaning.

2.3.3 Rote Learning of Calculating Procedures without Understanding

Borko et al. (1992) note that procedural and conceptual knowledge are regarded as essential facets of mathematical comprehension facets. However, there is evidence that procedural knowledge gets more emphasised in most schools and that teachers bestow less time and attention to conceptual knowledge (Porter, 1989). Machaba (2005) denotes this type of reasoning as instrumental understanding, which is demonstrated by learners' ability to use rules without reasoning, not knowing the origin of these rules and formulae. Skemp (1976) describes this scenario as "instrumental understanding", and contrasts it with "relational understanding".

2.3.4 Poor Connections between Existing Knowledge, Resulting in Insufficient Mastery of Basic Facts

Machaba (2016) states that understanding mathematical concepts depends on two facets: the existence of appropriate ideas and the formation of new connections in the learner's schema. Hiebert (1986) defines conceptual knowledge as a connected web of knowledge that is rich in associations, while procedural knowledge is composed of step-by-step instructions that prescribe how to complete tasks. Learners who memorise only the procedural steps in solving problems are most likely to apply these procedures incorrectly in different situations due to a lack of conceptual understanding, hence a major cause of misconceptions in learning mathematics constructs.

Ashlock (1990) finds that students' errors and misconceptions when learning mathematics are considered as rich vehicles for discovering their conceptual assimilation, thought levels and learning difficulties. This complements Vygotsky's (1978) Zone of Proximal Development (ZPD), which teachers should manipulate in helping learners to reach the equilibrium stage of conceptual cognitive development, as propounded in Piaget's Cognitive Theory (1936). In my study, I intend to explore the factors behind learner misconceptions in volume and surface area, investigating how these factors can be channelled into positive tools of learning.

2.4 LITERATURE GAP

The arguments discussed above, covering both identifying and classifying misconceptions in surface area and volume, were taken from studies that investigated the concept of measurement under surface area and volume and included numerous other mathematical concepts. Most of the recorded work to this day is an interpretation of analysed responses on given benchmark tests, which does not reveal and display the challenges leading to misconceptions faced in the learning process. The latter forms a gap in the literature, which this study seeks to cover.

2.5 THEORETICAL FRAMEWORK

This study followed constructivism, a theory which propounds that learners can effectively and efficiently construct or restructure new knowledge or skills based on what they already know or can do. This is their existing prior knowledge, used to make sense of one's outside world (Brodie, 2010; Piaget, 1970; Hatano, 1996; Kramer, 1996; Vygotsky, 1969). In this form of learning, Piaget's three phases of assimilating new knowledge, accommodating it and entering equilibration are realised. Constructivism is informed by the interpretivist paradigm, which postulates that knowledge is multi-faceted and a distinct phenomenon can have compound interpretations. In this study, I used different techniques to help me understand how learners interpret and interact with problems involving surface area and volume.

The theoretical work used to explain the persistence of learner misconceptions in learning is often drawn from constructivism (Brodie, 2010). This is why I used this theory throughout my research, particularly in collecting data, analysing it and finding insights regarding how learners construct the meaning of surface area and volume.

One fundamental principle of Piagetian constructivism, which has direct immediacy with the classroom, is that what people learn is controlled and afforded by what they already know. The other key principle is that there is integrity to learners' thinking, in other words, what learners think, say and do makes sense to them concerning what they already know (Brodie, 2010). The significance of the constructivist paradigm in the mathematics classroom is the implication that teachers need to discover how learners perceive concepts, to help them build relationships between ancient and modern knowledge.

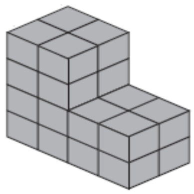
My study was focused on how learners find the meaning of surface area and volume as they interact with and interpret the two mathematics classroom concepts. In social constructivism (often described as interpretivism), the developed subjective meanings lead the researcher to consider a diversity of views instead of narrowing the meanings into a few classes or thoughts (Creswell, 2013). The goal of this research was to rely on the participants' views in a studied situation by addressing the processes of interaction among individuals.

One of the key implications of constructivism is that teachers are "facilitators" of learning; hence they are urged not to "tell" learners mathematics for fear of inhibiting the learners' constructions (Brodie, 2010). Through discussions, questions, arguments and explanations with other learners and the teacher, a learner experiences Piaget's (1936) notions of assimilation, accommodation and equilibration, thereby acquiring new knowledge. The previously discussed list of misconceptions enlisted in Battista's (2003) work on types and sources of misconceptions will be used as the analytical framework for this study.

Table 2.1: Table linking the misconception to the source of misconception

Concept	Definition	Misconception	Source/Cause of misconception
Surface area	The sum of the areas of all the 2-D faces surrounding a 3-D solid shape e.g. a rectangular prism.	L x w x h L x w L + w	Confusion with volume formula. Confusion with the formula for the area of a rectangle. Confusion with part of the perimeter of a rectangle.
Volume	The total amount of space occupied by a 3-D shape. The total amount of space a shape can hold.	L x w x h L + w + h	Supplies the formula instead of defining the concept. Supplies a wrong formula.
Count the blocks	This block is made of 24 unit cubes.	The block is made of only 18 unit cubes.	Counting only the cubes with visible parts. Error in

array perception.



The following flow-diagram tries to link some misconceptions to their sources in the researcher's perspective.

Definition: The surface area of a rectangular prism is the total sum of all the areas of the two-dimensional faces about the prism.

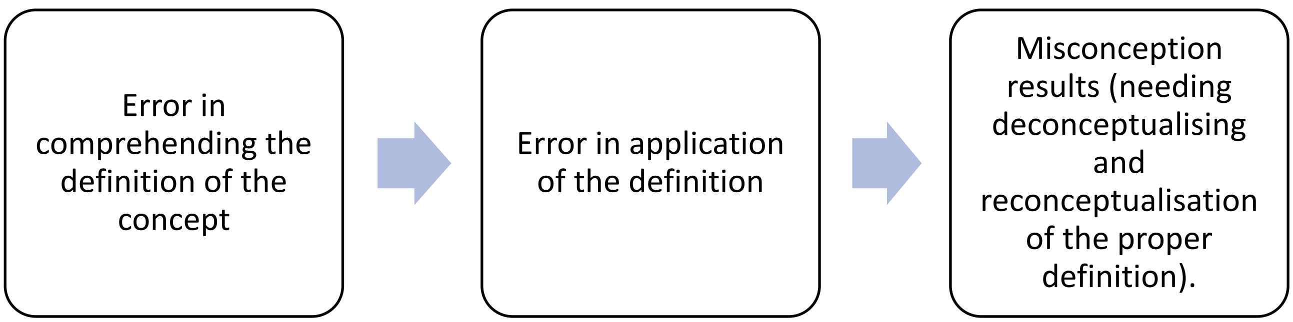


Figure 2.6: Flow diagram linking definition of concept to misconception

2.6 DEFINITIONS OF KEY TERMS, CONCEPTS AND VARIABLES

Misconception: A learner's understanding of a concept that yields to a methodological configuration of error. This could be caused by misappropriation of a rule, an over- or under-generalisation and a surrogate conception of a situation.

Conceptualisation: The grasping of knowledge by individuals; it can be hampered by several environmental factors as well as the learner's experiences.

Cognition: The assimilation and accommodation of newly learnt material which learners can apply in different situations giving several representations.

Concept: A new phenomenon to be learnt.

Generalisation: The transferability and application of learnt concepts to relevant situations of a similar phenomenon.

Constructivism: The theory of education, which underscores that the composition of knowledge is reliant on what the child already knows.

Perception: How a learner views and understands concepts.

Relationship: The link between one phenomenon and another. Learners need to link new knowledge with previously acquired knowledge, learning from known to unknown (Bruner 1956; Skemp, 1986).

Conceptual knowledge: Knowledge of the fundamental composition of a concept. It includes the associations and interrelationship of notions that describe and give meaning to procedures.

Procedural knowledge: Knowing how to identify a problem in its broadest and most routine sense and how to solve it correctly.

Understanding: The attribute and magnitude of connections that an idea has with other existing ones (Van de Walle, 2007).

2.7 CONCLUSION

The reviewed literature indicates that most studies conducted so far have not dealt extensively with misconceptions envisaged during the learning process. This has directed my study to focus on misconceptions from the learners' perspective. The next chapter gives a detailed outline of my research design and methodology in this study.

3 CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

This chapter outlines the research method followed in this study. I discuss the design, the sources of data collection, as well as the techniques used thereof. I will also highlight the sampling techniques, issues of trustworthiness and dependability, the data analysis procedure and ethical considerations, among other aspects.

3.1 RESEARCH DESIGN

The research follows a qualitative research design, which explores how Grade 8 learners comprehend surface area and volume through a test, followed by an interview. This study utilises a case study approach where the phenomenon of the cognitive representation of surface area and volume among Grade 8 learners is examined. The test comprises seven questions on surface area and volume, whose responses give an insight into how learners perceive the two concepts. The interview addresses issues regarding how learners learn these two mathematics concepts. A list of about ten questions related to surface area and volume is set to guide the first interview. A voice recorder is used to record the interviews for future analysis, such as transcribing and coding.

Participants for this study are from a high school in the Johannesburg East District of Education, purposefully chosen as it was within my workplace's neighbourhood. This was a co-educational school with multilingual classrooms and students of mixed socio-economic backgrounds and mixed abilities in mathematics. The research was conducted with a Grade 8 class of 29 learners, where rigorous data collection was done through tests and interviews. I have chosen this class as it was the entry year to secondary school education. This further allowed me to determine whether or not problems in Grade 9 emanate from previous classes' conceptualisation. Furthermore, learners at this level have covered surface area and volume topics for at least a year in this phase. At the Grade 8 level, the current Mathematics CAPS Curriculum requires learners to use formulae for surface area and volume in their calculations. However, the concepts of volume and area are covered in the curriculum from the foundation phase onwards, while surface area is a new progression of an area in the senior phase.

I got permission to conduct this research at the secondary school under study, which is in the neighbourhood of my place of work. I chose this secondary school due to convenience, reducing the occasions when I would leave my own classes to undertake the research. My school’s exit year is Grade 7 level; hence I could not conduct the study at my workplace since the target group was Grade 8. I prepared the test and used a different school as a first pilot of the case study, before administering it at the identified school.

I visited the school to administer the test, having received permission from the district, principal, the subject teacher, the learners, as well as their parents to take part. I could also have requested the respective subject teacher to administer the test for me; then I would mark it. However, for authenticity and reliability, I preferred to conduct this aspect of the research personally to build trust and a good relationship with the participants in preparation for the interviews.

All interested and willing learners wrote a test on surface area and volume. After writing the test, a sample of seven participants was interviewed. This comprised roughly 24% of the class, which allows a trustworthy and dependable comparison of the class performance. The interviews were conducted so that the participants could justify their written responses, giving their individual perceptions of the tested concepts. These interviews were recorded, to avoid missing data when transcribing the participants’ responses, thereby enhancing meaningful data analysis.

I used a case study approach in this research, which emphasises understanding of the dynamics found in solitary settings (Huberman et al., 2018). This was because my data were gathered from a single class at a chosen school where I investigated learners’ understanding of the concepts of volume and surface area, identifying misconceptions by so doing. In this case study, which is sometimes also called idiographic research by Leedy and Ormrod (2019), I used a test and interviews to describe the generated evidence of learners’ experiences in learning surface area and volume at the Grade 8 level. To improve my research's reliability and dependability, I spent a considerable amount of time on site and regularly interacted with the participants to record details about the context surrounding the misconceptions in learning volume and surface area.

The idiographic research (a case study) allowed me to gather information about the physical environment and any historical, economic and social factors that might have had any bearing on the misconceptions in understanding volume and surface area (Leedy et al., 2019).

I triangulated constructivism with the phenomenological theoretical perspective, in which, according to Taylor et al., (2016), the researcher views human behaviour as a product of how people define their world. My task, as Berger and Luckmann (1967) state, was to capture how learners construct their realities. This identifies with symbolic interactionism or social constructionism (constructivism). The theory follows the works of Charles Horton Cooley (1902), John Dewey (1930), Herbert Blumer (1967), Jean Piaget, Vygotsky and many other psychologists. Blumer (1969) stated that symbolic interactionism depends on three critical foundations, which are similar and related to constructivism:

- People act toward things influenced by the meanings these things hold for them;
- Meanings are not innate in objects but are a result of experiences that arise during interactions;
- Social players attach meanings to circumstances through interpretative processes (Taylor et al., 2016).

3.2 RESEARCH METHODOLOGY

A qualitative research approach was employed in this research. I chose qualitative research as it helped me best answer my research questions, which aimed to explore learners' experiences in representing, understanding, and interpreting surface area and volume. A qualitative approach enabled me to collect narrow but rich data from the participants (Braun & Clarke, 2013), contributing to understanding volume and surface area conceptualisation. A qualitative approach was also applicable for this study as it dealt with a phenomenon at a single school and hence was not generalisable but rather gave insights into the teaching and learning of mathematics. It is for this reason, that I opted to use a case study design.

The study was conducted through the collection of data by the following two methods:

- an analysis of the participants' responses to a test on surface area and volume using Battista's (2003) framework, and
- interviews with a sample of seven selected participants from the group that wrote the test.

3.3 THE SAMPLING TECHNIQUES

In this case study, I used a type of purposive sampling known as focal participants, selecting a class of Grade 8 learners from a school located close to my workplace. The school was thus selected as it would reduce travelling expenses since it is within my every day working surroundings. The Grade 8 class was selected as it is the beginning of secondary (high) school, a phase where the concepts of surface area and volume are intensely covered, according to the South African CAPS document of the Senior Phase. It is also a class that I have taught for a long time and consisted of experienced learners encountering conceptualisation problems in surface area and volume. The strategy of selecting focal participants is important when conducting observations in classroom situations where it is not possible to pay attention to every individual (Leedy & Ormrod, 2019).

After administering the test to the whole class of 29 learners and marking it, I employed an extreme case sampling strategy to identify about six participants for interviewing (Leedy et al., 2019). This is where a researcher selects participants with unique or specific characteristics within a case and then conducts in-depth interviews with them.

This sample had participants that yielded the most information on my topic. Since the study required exploration of learners' perceptions of volume and surface area at Grade 8 level, I gathered information from learners during the third quarter of the year, when this topic had been covered nationwide. Interviewees were selected according to individual performance in the test and classified into groups based on the level of understanding of the question. Codes were given for each level of understanding, on a scale of 1 to 4, with the least symbol being for complete comprehension of the problem, without errors, while the highest represents the presence of most misconceptions. The data were organised on a table, comparing participants against cognitive level of competence for each question. At most, two learners were then selected from each group of competences, to constitute the interviewees.

3.4 THE DATA COLLECTION METHOD

3.4.1 Data Collecting Techniques

In this study, two data collection techniques were used: qualitative scholastic tests and in-depth interviews. A task sheet with questions testing learners' understanding of surface area and volume was given to them on completion. It was marked and a sample of six participants selected for interviews.

3.4.2 Data Sources

I used a test and in-depth interviews to collect data. I chose to conduct interviews because, according to Braun and Clarke (2013), these give valuable and comprehensive data about individual practices and viewpoints. I am positive that interviews were an appropriate method for data collection of this nature as they are suitable for smaller samples.

Other methods such as qualitative surveys, although less daunting than interviews, have potentially limited anonymity. Interviews are also useful in reconnoitring insight and perception- and construction-type research questions (Sloan et al., 2010), which characterise this study. Grounded theorists have used interviews to answer influencing factor-type questions, such as the factors that influence people's decisions to regular genetic screening (Michie et al., 1996).

3.5 WRITTEN SCHOLASTIC TEST

The test was composed of six questions with a total of 23 items on understanding and calculating the volume and or surface area of selected prisms, purposively chosen from the scope of work covered at Grade 8 level. The items included definitions, manipulations and calculations. The test items focused on testing both the procedural and conceptual knowledge of surface area and volume. I set the test items, and requested permission to administer the test at a secondary school in the Johannesburg East district of Education as a one hour or one-period test among the Grade 8 learners during the third quarter of the year. I chose a test as it shows both the participants' procedural and conceptual understanding from their own perspective and cognition. The interviews complemented the written responses and drew individual characteristics that could not be deduced from the learners' written work. These characteristics include learners' feelings towards the subject or concepts of surface area and volume. Tests alone would not have yielded results regarding how the learners understood concepts, as understanding is a subjective, individual experience. The interviews help learners to express themselves freely while assisting the researcher in transcribing dependable and reliable data about the participants. Other data collecting techniques, such as focus groups, would not have been best for this study as they are characterised by data distortion due to other participants' influence.

3.6 RESEARCH INSTRUMENT

The test comprised six main questions with an average of four sub-questions each, totalling 23 marks; however, the total mark was not indicated, neither was the mark allocation for each question. This was because the researcher was not interested in quantifying how much the participants attained but rather the quality of responses.

To verify the misconception of confusing the concept of volume with that of surface area

Question 1 had two items on definitions of surface area and volume. This question assessed the participants' understanding of the concepts of surface area and volume by definition. Question 2, similar to Question 1, had two items describing how to find the surface area and volume of prisms or cylinders without using any formula.

To verify the misconception of enumerating cubes in 3-D arrays incorrectly

Question 3 dealt with manipulating arrays of blocks to explain the surface area and volume in five items, one of which required using the formula. This question assessed the use and manipulation of relevant formulae for surface area and volume, including understanding the arrangement of arrays of blocks in a rectangular prism.

To verify the misconception of believing that a shape has more than one surface area

Question 4 entailed six items on manipulation and arrangement of blocks or arrays, describing the surface area and volume without using a formula. This question assessed the conceptualisation of surfaces on a 3-D figure. Question 5 assessed the conceptual knowledge of surface area and volume.

To verify the misconception of counting visible faces or unit cubes instead of calculating the volume thereof

Question 6 was a development and progression from Question 5. This time, manipulation was followed by the use of a formula in determining the surface area and volume. However, one could also establish these without the use of the formula but just by definition. It had five parts requiring manipulation and making at least three entries on the table. The question assessed the manipulation of arrays and arrangement of blocks in unfamiliar figures (problem-solving), expecting participants to conclude that shapes can have the same volume but different surface areas.

To verify the misconception of treating 3-D figures as 2-D ones

Most items in Question 3 described and discussed the existence or non-existence of volume and surface area, according to the learner's perspective. It was application mainly without the use of a formula. The items assessed the visualisation of surface area and volume problems.

To verify the misconception of confusing the concept of volume with that of surface area, including interchangeable use of their formulae

Question 6 introduced the application of surface area to volume relationships. This question had four items that included the use of formulae. The question assessed the application of the volume and surface area concepts to check whether the learners used insight or merely over-generalised the use of the formulae on rectangular prisms. The question required the learners' intuition in rearranging a given cube into rectangular prisms of the same volume. The learners then compared the surface areas of the different rectangular prisms formed. The newly formed prisms all had differing lengths and widths but maintained the same height of one unit. A conclusion was expected to be reached on the relationship between the surface areas of different rectangular prisms of the same volume.

3.7 INTERVIEWS

In-depth interviews were conducted with a sample of six learners from the 29 who had written the test. I believe that this sampling provides a true reflection of the population of learners who completed the task. A voice recorder was used to capture the interviews to gather the audio data for transcription.

The interviews were used to answer questions on what factors influenced the misunderstanding of concepts such as surface area and volume among Grade 8 learners. Qualitative interviewing appeals for flexible research design (Taylor et al., 2016). While I began with general ideas on what to interview my participants about, I was prepared to shift course after the preliminary interviews. The accessibility of in-depth interviews made it easy to collect data from learners at the Grade 8 level. Furthermore, my control over the produced data as the researcher increased the likelihood of generating useful information.

There was continuous movement between a semi-structured and an unstructured interview in the interview conducted on the selected sample of six learners. A few questions were listed for guidance as an interview schedule, just to begin the interviews. Otherwise, questions were based on how the individual learners performed in the test. The participants explained their understanding of the concepts and the methods used in solving problems within the test, in conjunction with challenges encountered while relating with problems of surface area and volume.

In conducting these in-depth interviews, I endeavoured to develop close, trust-based relationships with the participants. Varying interviews from one participant to the other enabled the learners to make authentic disclosures about their thoughts (Gomm, 2008). I used Battista’s (2003) list of the five misconceptions listed in my conceptual framework as the design and constructs to analyse the data collected from the tests and the interviews.

3.8 PILOTING THE RESEARCH-INSTRUMENT (TEST)

Cassandra (2014) identifies and describes four types of qualitative testing methods, direct exploration, monadic testing, sequential monadic testing, and discrete choice testing. I chose to use the direct exploration method over the other three as it is used to measure a sample's expectations, attitudes and interest in a concept. The approach relied on participants giving open-ended, unassisted feedback, which is used as the initial building block for this research.

I administered the first piloting test to 6 learners from a different school to check whether my questions were vague, ambiguous, or unclear. I also asked a colleague to pre-moderate the test for authenticity. My supervisor also advised and scrutinised each item of the test. As I went through the test with my piloting group of learners, I noticed that the initial test was too long for an hour, let alone a 30 minute period; for that reason, I needed to shorten it. Some of the questions were similar, requiring an adjustment to the paper.

As I had not received much information from my first piloting, I decided to conduct a second pilot at a different school. This time I targeted the Grade 8 class. Unfortunately, due to the disruption of schooling as a result of the COVID-19 impact forcing an adjustment to the curriculum, I was then compelled to pilot with the Grade 9 class instead. This was because the Grade 8 class of 2020 had not covered the topic of surface area and volume yet, as it was moved to the fourth quarter. Piloting with the Grade 9 learners had an advantage that at least all questions would be responded to, giving more insight to the researcher.

Like the first pilot, the second piloting helped me understand the way learners interpreted questions. As such, I made further changes to the test, including further reduction of the test items and reducing similarities in questions. The test is attached in Appendix H, while the semi-structured interview schedule is included as Appendix J.

3.9 TRUSTWORTHINESS AND DEPENDABILITY

McMillan and Schumacher (2014) state that qualitative validity refers to the extent of resemblance between clarifications of phenomena and the authenticities of life. This study used audio and visual recorded (written) data to ensure that the research was valid. This was done in the form of voice recordings, participant interviews and the learners' written tests.

I used (participant review) member checking to enhance my study's credibility, where participants were asked to review interview transcripts to double-check the accuracy of data representation (Leedy et al., 2019). In so doing, the agreement became the type of evidence for reliability. McMillan and Schumacher (2014) describe the agreement as the consistency of ratings or observations. Comparisons of the ratings of different interviewees were checked for consistency, thereby testing my collected data's reliability. I believe that by following the above procedure, the data were declared credible, transferable, dependable and confirmable.

To further improve the trustworthiness and credibility of my collected data, I used various strategies. I took steps to reduce identified personal, social, political or philosophical biases that could influence my ability to collect and interpret data, which helped increase reflexivity (Leedy et al., 2019). The triangulation of data collection methods through a test and interviews helped find consistencies or inconsistencies in the data (Leedy et al., 2019).

The test and interviews focused on checking for rigour and trustworthiness of collected data in this study. For the test, the questions specifically addressed learners' understanding of surface area and volume. The test was conducted within the school set-up with both the class teacher and the researcher present. Trustworthiness required that the test be done under strict examination conditions to avoid learners sharing answers and other forms of cheating that would have affected the results of the research.

The interviews were recorded and transcribed, which required at least two people's views. To this effect, I submitted the recorded material together with the transcribed written material to the participants for member checking. The findings were discussed and read by participants; however, pseudonyms were used to identify participants.

3.10 INDUCTIVE AND DEDUCTIVE DATA ANALYSIS PROCEDURES

Twenty-nine participants responded to the six questions of 23-item, one hour test on surface area and volume, designed by the researcher. The researcher analysed the responses presented by all the participants, and code them into four classes. Attention was given to the number of learners who responded correctly, partially correctly, incorrectly, or did not attempt to respond at all for each item to analyse the data. Two participants were randomly selected from each code, and preparations set for interviews. Interviews were conducted one on one with the participants. Each interview took at most an hour.

I employed thematic analysis to analyse the written test and the interview thereof. This was when I looked for themes present in the whole set or sub-set of interviews for making comparisons among different respondents (Gomm, 2008). Thematic analysis is a way of describing what things mean to people. I chose this method over the other two methods of analysing qualitative interviews, which are reports and linguistic analysis, as its main emphasis is in searching for meaning rather than the use of language.

3.11 TIME FRAME

The test and interviews were both administered and conducted in the third quarter of the year, in 2020. The participants wrote the test in July, while the interviews followed in August. These were both conducted after having been granted the permission by the Gauteng Department of Education in late June of the same year. The test was conducted on a day when the class had a free period of an hour. It took the class 45 minutes to complete the test. The interviews were then conducted outside school time, when individual participants had free time. This was on different afternoons before leaving the school premises. The school guest room was used as the venue for the interviews. All interviews were complete over a fortnight.

3.12 ETHICAL CONSIDERATIONS

Creswell (2013) states that a qualitative researcher confronts many moral issues that appear when collecting data, analysing and distributing qualitative reports. Lipson (1994) categorises these moral or ethical issues into informed consent processes, dishonesty or stealthy practices, confidentiality or privacy of participants, sponsors and colleagues, the benefits of research for the participants compared to the risks, and participant requests that go beyond social norms.

I considered Murphy and Dingwall's (2001) Ethical Theory as the four principles guiding ethics issues regarding my participants and data collection (Flick, 2011). These four principles are:

1. Non-maleficence: Implying that as a researcher, I will endeavour to avoid harming participants.
2. Beneficence: A feature demanding that the research about human subjects yields positive and identifiable benefit to them rather than simply being carried out just for the sake of researching.
3. Autonomy or self-determination: A call for researchers to respect participants' values and decisions.
4. Justice: A demand for treating all people equally (Flick, 2011, p. 216).

Following the above-discussed principles, I avoided harming my participants by ensuring that there was informed consent and confidentiality in all issues. The participants were treated with respect, and I also ensured that the study was beneficial to them. This was done by sharing back the findings and requesting the participants' feedback and verification.

3.12.1 Confidentiality

The recorded interviews were not availed to unauthorised people to promote confidentiality. Pseudonyms were used instead of the learners' real names to identify the participants to protect the participants' anonymity. I also assigned numbers or aliases to each individual. However, participants' actual marks were not disclosed for confidentiality purposes, protection from labelling, and emotional safety, especially for those who did not do well in the test.

Taylor et al. (2016) state that qualitative research potentially threatens people's privacy and confidentiality. I reduced this threat, essentially by:

- The use of pseudonyms and exclusion of identifying material in my data;

- Securely preserving stored data; and
- Deleting of audio recordings after transcription and analysing.

The above measures were taken not to expose the participants to harm or interfere with their activities. Once in the field, I established a rapport with the learners and gained trust and openness with them to be accepted as a non-judgmental person. I did not provide any intervention when a learner's responses showed any misconceptions but strived rather to be a good listener and scribe.

3.12.2 Informed Consent

Participation in this study was voluntary, as it was explained that they could choose to leave the study at any point in time. Permission was sought from the individual learners, their parents, the subject class teachers at the school, and the educational provincial and district officials, to conduct the study at the chosen school. I obtained forms from the University of South Africa and had all my participants complete them.

As Creswell (2013) denotes, participants are informed of the following issues:

- Their right to voluntary withdrawal from the research at any time;
- The aim, importance and purpose of the study as well as the methods or techniques to be used in collecting data;
- The fortification of the confidentiality of participants;
- The known hazards accompanying participation in this study;
- The predictable profits accumulated by participants in this study.

I was also granted permission to conduct the study by the university's ethics committee.

3.13 CONCLUSION

I have given an account of the research design and methodology of my study in this chapter. Issues covered include the target population, the sample and sampling techniques, and the research instruments used. I have also addressed how trustworthiness, dependability and ethical issues were accounted for in addition to the data analysis procedures. The next chapter deals with data analysis and interpretation.

4 CHAPTER FOUR: DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

In this chapter, I recorded the actual data collection for this study. Inductive analysis was employed in this research, where I synthesised and made meaning from the data, beginning with specific data, through categories to general patterns in participants' work (McMillan & Schumacher 2014), thereby interpreting the collected data. As the study's purpose was to explore the misconceptions Grade 8 learners at a school in the Johannesburg East Education district had in learning about surface area and volume, a 23-item scholastic test from 6 main questions in this concept was administered. The test was aimed at answering the following research questions:

1. How do learners in Grade 8 define the concepts of surface area and volume of prisms?
2. How do learners in Grade 8 solve problems involving surface area, volume of prisms and the relationship between them?
3. What misconceptions prevail when learners in Grade 8 deal with problems involving surface area and volume of prisms?
4. What are the sources and causes of misconceptions in the process of learning volume and surface area of prisms?

4.2 DATA ANALYSIS PROCESS

4.2.1 Test Response Codes

The responses of the 29 learners who wrote the test were analysed. The participants are identified in alpha-numeric characters, for instance A1, A2 and A3 up to Y3. Learners' responses to the test were categorised into four classes and captured as follows: C–correct responses to the asked question, PC–partly correct responses, I–incorrect responses; NA–not attempted response.

The above categories were used to analyse the learners' responses per item. The test was set such that Question 1 required definitions, Question 2–descriptions, Question 3–stating availability or non-availability of surface area and volume on given objects, Questions 4 and 5 manipulating arrays of cubes and finding surface area as well as volume on structures with or without dimensions, Question 6–relationship between surface area and lengths on cuboids of the same volume. Participants' responses are recorded in Table 4.1. Highlighted participants (grey) indicated unavailability for interviews, due to scepticism and fear of interviews. The categories are coloured (correct-green; partly correct-yellow; incorrect-red, and blue for not attempted responses) for easy identification.

Table 4.1: Summary in manner of participants' responses throughout the test paper

P	**TEST ITEM NUMBERS**																		
	1a	**1b**	**2a**	**2b**	**3a**	**3b**	**3c**	**3d**	**3e**	**4a**	**4b**	**5a**	**5b**	**5c**	**6a**	**6b**	**6c**	**6d**	**6e**
A2	C	C	I	I	I	I	PC	I	C	I	I	PC	I	I	I	I	I	I	I
A3	C	I	I	I	I	C	I	PC	PC	C	C	C	C	C	I	I	NA	NA	NA
B1	PC	C	PC	I	I	NA	I	I	I	C	NA	NA	NA	NA	NA	NA	NA	NA	NA
B2	C	C	I	NA	PC	I	C	I	PC	I	I	I	I	I	NA	NA	NA	NA	NA
B3	PC	I	I	I	I	C	I	I	I	I	C	I	C	C	PC	I	NA	NA	NA
C2	C	C	I	C	PC	I	PC	I	PC	I	I	I	I	I	I	I	I	I	NA
C3	PC	NA	NA	NA	I	PC	I	I	I	C	PC	C	I	I	PC	I	PC	NA	NA
D2	C	PC	I	I	PC	I	PC	I	I	I	I	NA	PC	NA	NA	NA	NA	NA	NA
D3	I	I	PC	NA	NA	NA	NA	I	NA	C	PC	C	C	I	PC	PC	PC	PC	I
E1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	C	C	PC	NA	NA	NA	NA	NA
E2	PC	I	PC	C	NA	I	PC	NA	NA	I	I	NA	NA	NA	NA	NA	NA	NA	NA
E3	PC	PC	NA	NA	I	C	PC	I	NA	C	PC	C	C	C	C	C	C	NA	NA
F1	C	C	I	C	I	I	I	I	PC	I	PC	C	C	PC	C	I	C	I	NA
F2	C	PC	I	I	I	C	PC	I	NA	I	I	I	I	I	I	I	NA	I	NA
G2	PC	C	I	I	I	I	I	I	C	I	I	I	I	I	I	I	I	I	I
G3	PC	PC	I	I	I	I	I	I	I	I	I	C	I	I	I	I	I	I	I
H2	I	PC	I	I	I	I	I	I	C	I	I	NA	PC	NA	I	I	I	NA	NA
H3	PC	I	PC	I	PC	PC	I	PC	I	C	PC	C	C	I	I	I	I	NA	I
I2	C	PC	I	I	I	I	I	I	NA	C	I	I	I	I	I	I	PC	NA	NA
J2	PC	C	PC	C	C	C	PC	I	NA	C	I	NA	NA	NA	NA	NA	NA	NA	NA
L3	PC	NA	NA	NA	NA	NA	NA	NA	NA	C	PC	C	I	I	NA	NA	NA	NA	NA
M3	PC	I	I	I	NA	C	NA	NA	NA	I	I	I	C	NA	NA	NA	NA	NA	NA
N3	PC	I	I	I	I	C	I	I	I	I	I	I	I	I	NA	NA	NA	NA	PC
P3	PC	PC	I	I	I	NA	NA	NA	NA	I	I	I	NA	NA	NA	NA	NA	NA	NA
Q3	PC	I	I	I	I	C	I	I	I	I	I	I	I	C	I	I	I	PC	NA
R3	PC	I	NA	NA	I	PC	NA	PC	NA	I	I	I	I	I	I	I	I	I	I
S3	I	PC	I	NA	NA	NA	NA	NA	NA	I	I	I	I	I	NA	NA	NA	NA	NA
T3	I	I	I	I	NA	NA	NA	NA	NA	C	I	I	I	I	NA	NA	NA	NA	NA
Y3	PC	PC	NA	NA	NA	I	NA	NA	NA	NA	PC	C	NA	I	NA	NA	NA	NA	NA

KEY: P – Participant, C – Correct (green), PC – Partly Correct (yellow), I – Incorrect (red), NA – Not attempted (blue)

Learners willing to be considered for interviews *Learners not willing to be considered for interviews*

The total number of entries or cells in the above table is 551. Green cells total 72 of 551, which indicated that there were only 13% correct responses throughout the test. The yellow cells amount to 72, which is equivalent to 13% partly correct responses. The red cells add up to 237, which accounts for 43% of incorrect responses throughout the entire test. There are 170 blue cells altogether, which means 31% of the test was not attempted, largely due to a lack of knowledge of what the questions require. The above statistics indicate that learners have great trouble responding to surface area and volume problems. Possible misconceptions are shown in Table 4.2 below.

Table 4.2: Potential misconceptions noticed in participants' written test

P	TEST ITEMS WITH EVIDENT POSSIBLE MISCONCEPTIONS PER PARTICIPANT																	
	1a	1b	2a	2b	3a	3b	3c	3d	3e	4a	4b	5a	5b	5c	6a	6b	6c	6d	6e
A2			●	●															
A3			●	●	●														
B1		●	●	●			●	●	●										
B2			●		●			●	●										
B3		●	●	●	●		●	●	●										
C2		●	●					●	●								●	●	
C3					●		●	●	●				●	●					
D2			●	●	●			●	●										
D3								●											
E1																			
E2	●	●	●	●															
E3							●												
F1			●			●	●	●											
F2		●	●	●			●									●	●		
G2			●	●			●									●	●	●	●
G3			●	●	●	●	●	●	●										
H2	●	●	●	●			●	●								●	●	●	
H3	●	●		●			●		●							●			
I2			●	●		●	●	●								●	●		
J2	●		●					●											
L3	●													●	●				
M3																			
N3		●	●	●	●		●	●	●										
P3			●	●															
Q3		●				●	●												
R3					●														
S3			●																
T3	●		●	●															
Y3						●													

Key: P – Participant

On this table, a dot represents a possible potential misconception in the participant's response to the respective test item, as seen from the written response. I called them potential misconceptions as they had not been confirmed during the interviews. The descriptions of each of these identified possible misconceptions are given in the next section.

Table 4.2 above compares item variances of participants' responses. The number of correct responses as well as percentages thereof for each test item is compared against the partly correct, incorrect and not attempted questions. The table depicts which test items were easy and which ones were difficult for the participants. It shows an overview of the frequency of each category of responses per learner throughout the test.

4.2.2 Distribution of Codes on Participants' Responses to the Test

Table 4.2 shows noticed potential misconceptions from the participants' written scripts, using the conceptual framework. I referred to them as potential misconceptions as I had not confirmed them through the interviews yet. The eight participants highlighted in grey had indicated an unwillingness to take part in interviews, as they feared embarrassment in the event that they had not done well. Table 4.2 shows the frequency of potential misconceptions per participant as well as the test items where the assumed misconceptions prevail. The type of misconception is indicated separately in Table 4.3.

In Table 4.3, on the next page, participants' potential misconceptions are grouped according to the conceptual framework. Participants with the same preconceived misconception per question are grouped in the same cell.

A list of other additional discovered misconceptions is provided in Section 4.2.5, together with participants who displayed these characteristics. The discovered misconceptions are also discussed in the next session.

The above noted potential misconceptions are described according to the conceptual framework categories below, in Table 4.3.

Table 4.3: Classification (codes) of evident misconceptions in participants' (learners') work:

TYPE OF MISCONCEPTION FROM THE CONCEPTUAL FRAMEWORK					
Test Item	Treating 3-D figures as 2-D ones	Counting visible faces or unit cubes instead of calculating the volume thereof	Enumerating cubes in 3-D arrays incorrectly	Believing that a shape has more than one surface area / volume	Confusing the concept of volume with that of surface area including interchangeable use of their formulae
1a					Participant T3
1b					Participants B3, N3, Q3, D3, H3
2a	Participant D2				Participant G3, P3, H2, A2, A3, I2, Q3 H3, N3
2b					Participant G3, P3, B1, C2, G2, A3, B3, H3, N3, Q3
3a	Participant C2	Participant H2	Participants A3, B3, M3, B1, G2, A2, N3, R3, C3, G3		
3b	Participant C2	Participant C2, H2, B2, C3, E2, F2, H3	Participant G2, A2		Participant B1, H2, G2
3c	Participant C2	Participant H2	Participant G2, A2	Participant H2	Participant B1
3d	Participant A2,	Participant B1, A2	Participant B1, F1, A2, C2	Participant F1	Participant B1, F1, A2
3e		Participant H3	Participant C3	Participant B1, F1	
4a	Participant A2, H2		Participant G2, H2		
4b	Participant G2, A2, H2	Participant F1	Participant C3, E3, G3, H3, L3, B1, F1, G2, H2	Participant L3, P3, R3, T3	
5a	Participant T3, P3,	Participant S3, Q3, M3	Participant R3, N3		
5b	Participant S3, N3	Participant R3		Participant Q3	Participant G3
5c	Participant R3, H3	Participant G3, D3	Participant Y3	Participant S3, N3	
6	Participant B1, F1, A2		Participant C3, H3, B1, F1, G2, A2, C2		
Total	**19 occurrences = 16%**	**19 occurrences =15%**	**40 occurrences = 33%**	**11 occurrences = 9%**	**33 occurrences = 27%**

Classification of Misconceptions, Evident in Learners' Work, as Found in Literature

Note: before the interviews; using the conceptual framework

Of the one hundred entries in Table 4.3, comparing occurrences and frequencies of potential misconceptions among the five codes, I noticed that the most prevalent misconception during the test period was enumerating cubes in 3-D arrays incorrectly, at 33% of the classified data. The second most frequent misconception is confusing the concept of volume with that of surface area, including interchangeable use of their formulae, seen amongst 27% of the test scripts. The misconception of treating 3-dimensional figures as 2-dimensional ones appears third most frequently, identified in 16% of the written test. The misconception of counting only the visible faces, instead of calculating the requested surface area or volume appears less frequently; at 15%, while that of believing that a shape has more than one surface area and volume is the least prevalent at 9%. The information in Table 4.3 is represented pictorially in Figure 4.1, the bar graph below, to compare the given codes.

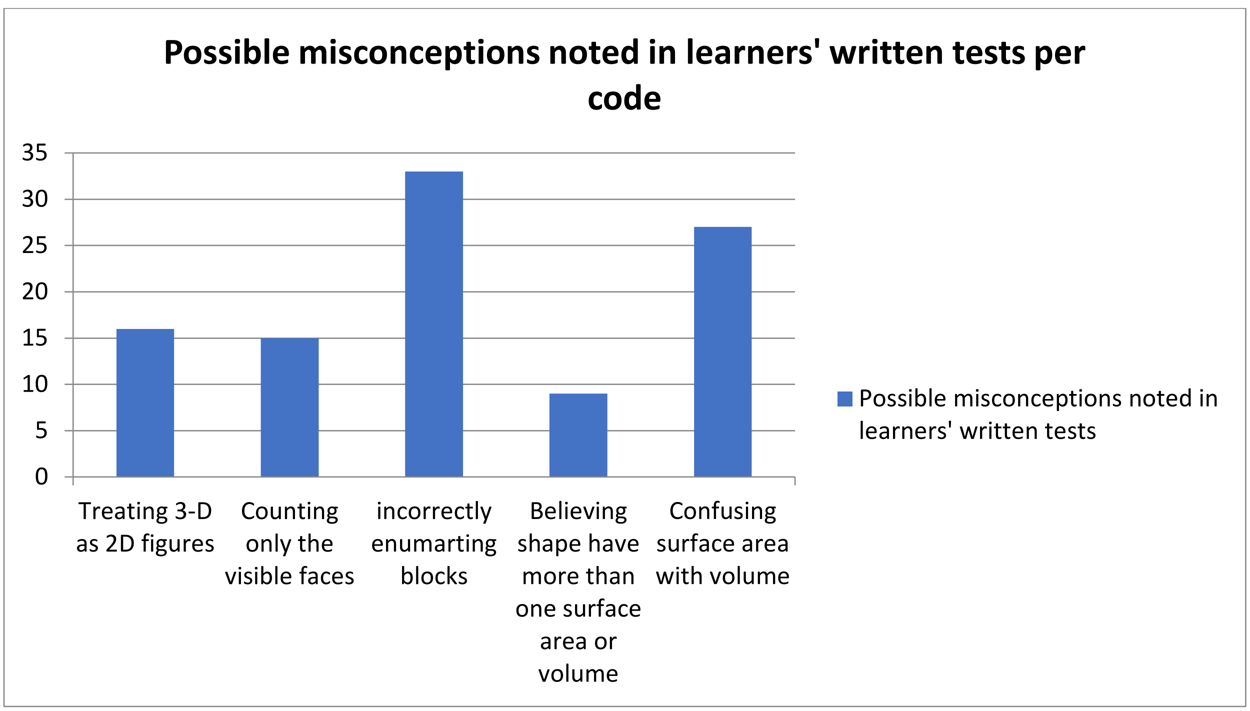


Figure 4.1: Possible misconception codes found in learners' written responses to the test

The occurrences and frequency of potential misconceptions per category and item displayed in Table 4.3 and the graph in Figure 4.1 were used to identify and purposefully select participants to be invited for interviews. Cells with high volumes of participants' misconception were identified, and respective participants purposefully selected, except those who had indicated that they would not be available for interviews.

4.2.3 Selection of Possible Interviewees

Table 4.3 above indicates test items with a high frequency of possible potential misconceptions per category of the conceptual framework. The dat is interpreted in the following manner, for example:

Treating 3-D figures as 2-D figures:

This misconception was mainly noticed in Test item 4b, in the work of participants G2, A2 and H2, and in test item 6 from participants A2, B1 and F1. It appeared less frequently in other test items. It was the third most common misconception in the test, with an occurrence rate of 16%.

Counting visible faces or unit cubes instead of calculating the actual surface area or volume:

This misconception was most observed in Test item 5a from participants Q3, M3 and S3. It is second least among the most common misconceptions in the administered test, at 15% occurrence.

Incorrectly enumerating cubes in 3-D arrays:

This misconception is frequently evident in most test items. In test item 3a, it is manifested by participants A3, G2, A2, N3, B1, C3, M3, G3, R3 and B3. In test item 3d, it is noted among participants A2, C2, B1 and F1. In test item 4b, it is evident from participants C3, E3, G3, H3, L3, G2 and H2. Lastly, in Test item 6, the misconception is noted within participants C3, H3, G2, A2, C2, B1 and F1. This misconception is the most prevalent in the test, at 33% occurrence.

Believing that a shape has more than one surface area or volume:

This misconception is evident in test item 4b from participants L3, P3, T3 and R3. It is the least common misconception in this test at a 9% occurrence.

Confusing the concept of volume with surface area, including interchanging their formulae usage:

This is the second most common misconception in the test, appearing in 27% of the occurrences. It is evidenced the most in four test items. In test item 2a, it is seen in participants G3, H2, A2 and P3. In test item 2b, the misconception is most prevalent in participants G3, C2, G2, P3, A3, B3, H3, N3, Q3 and B1. In test item 3b, we observe it in participants H2, G2 and B1. Lastly, in test item 1b, it is noticed from participants N3, Q3, D3, H3 and B3.

From the above observation, participants purposefully selected for the post-test interviews were the ones with the most possible prevalent misconceptions from Table 4.2. These participants had also indicated a willingness to be considered for post-test interviews. While the possible participants were: A3; C3; G3; H3; N3; Q3, A2, C2, G2 and H2, only seven were then selected for the post-test interviews. Most of the test items (questions) were covered in this interviewee selection, according to the anticipated possible prevalent misconceptions. Participants A2; A3; C3; G3; H3; N3 and Q3 were finally selected and confirmed to participant in the post-test interviews.

Interviews were then conducted to confirm and verify the supposed misconceptions and discover and identify other misconceptions not found in the literature. Other misconceptions found in participants written tests, which were not yet described in the literature, are listed and discussed in Section 4.2.5 below.

4.2.4 Discovered Misconceptions, Evolving (Emanating) from the Participants’ work, before the Interviews

1. Giving the formulae for calculating surface area and volume when asked for definitions

This misconception has been seen in Test item 1 in participants, including H3 and L3, among others. The participants would give the correct respective formula instead of defining the requested concepts. However, the given formula is often for finding surface area or volume of rectangular prisms only and not other types of prisms.

2. Providing the formulae for calculating surface area and volume of prisms instead of describing how to find these without using any formula

This was the most common misconception in Test item 2, identified in work submitted by, for example, participants A3 and B3. The majority of the participants who responded to this item would simply provide the formula. The participants battled to describe how surface area or volume could be found without using any formula.

3. Believing that surface area and volume cannot be found without having the measurements of prisms or cylinders;

This misconception had the highest frequency of all types, and it appeared in many test items such as in Items 2, 3 and 5. In Test item 2, the misconception manifested among participants M3, N3, Q3, S3 and participant T3. In Test item 3, the misconception was prevalent in participant C3's work. In Test items 5b and c, the misconception was evident in participant L3 and C3.

4. Believing that a solid object (3-D shape) composed of blocks and spaces is incomplete, and thus has neither surface area nor volume;

This was the most frequent misconception throughout the test, and it prevailed in Test item 3 as participants' perceptions, especially for item 3d. Participants D3, E3, G3, H3 and Q3 manifested this misconception, to mention but a few. The participants believed that for a shape to consist of volume and surface area, it should be a solid, without open spaces. A shape with some spaces was considered incomplete or not even and sometimes referred to as inconsistent.

5. Assuming that a (3-D shape) solid object with unmarked dimensions has neither surface area nor volume;

The greater part of Test Item 3 had no dimensions, with the exception of Test item 3b. Most participants perceived that the absence of marked dimensions and units disqualified a shape from having surface area and volume. This probably emanated from the desire to use a formula for calculation. This misconception was most common in Item 3e among participants B3, G3 and N3.

4.3 TEST ITEM ANALYSIS AND INTERPRETATION

Each test item was analysed in an attempt to check which questions were contributing to a high frequency of potentially possible misconception. The analysis is tabulated in Table 4.4 as percentages of responses for each response category. The responses per item are analysed after that. For example, the number of participants who had correct responses to Question 1a is 8, out of the total number 29 who wrote the test, giving us a class percentage of 28%.

Table 4.4: Comparison of participants' responses per test item

Test	**Code of responses per test item**			
Item	Correct	Partly correct	Incorrect	Not attempted
1a	8/29 = 28%	16/29 = 55%	4/29 = 14%	1/29 = 3%
1b	7/29 = 24%	9/29 = 32%	10/29 = 34%	3/29 = 10%
2a	0/29 = 0%	5/29 = 17%	18/29 = 62%	6/29 = 21%
2b	4/29 = 14%	0/29 = 0%	16/29 = 55%	9/29 = 31%
3a	1/29 = 3%	4/29 = 14%	15/29 = 52%	9/29 =31%
3b	8/29 = 28%	3/29 = 10%	11/29 = 38%	7/29 = 24%
3c	1/29 = 3%	7/29 = 24%	12/29 = 42%	9/29 = 31%
3d	0/29 = 0%	3/29 = 10%	18/29 = 62%	8/29 = 28%
3e	3/29 = 10%	4/29 = 14%	8/29 = 28%	14/29 = 48%
4a	10/29 = 34%	0/29 = 0%	17/29 = 59%	2/29 = 7%
4b	2/29 = 7%	7/29 = 24%	18/29 = 62%	2/29 = 7%
5a	14/29 = 50%	0/29 = 0%	15/29 = 50%	0/29 = 0%
5b	11/29 = 38%	0/29 = 0%	15/29 = 50%	3/29 = 12%
5c	7/29 = 25%	0/29 = 0%	19/29 =63%	3/29 = 12%
6a	6/29 = 21%	1/29 = 3%	11/29 = 38%	11/29 = 38%
6b	1/29 = 3%	4/29 = 14%	13/29 = 45%	11/29 = 38%
6c	7/29 = 24%	1/29 = 3%	7/29 = 24%	14/29 = 49%
6d	5/29 = 17%	0/29 = 0%	5/29 = 17%	19/29 = 66%
6e	0/29 = 0%	3/29 = 10%	2/29 = 7%	24/29 = 83%

Considering the very low frequency of “C- Correct” responses in many questions on the table, most learners performed poorly in this test. The most problematic areas were the descriptions in Question 2, manipulations and evaluations of Question 3 as well as the relationships in Question 6.

The participants found it easier to define volume than surface area. It is disappointing that a high percentage of learners could not describe how to find the volume or surface area without using any formula in Question 2, despite some having defined the concepts in Question 1. The evidence to be discussed later show that correct definitions had just been provided from memorisation, without much comprehension and mastery of the concepts.

Question 3’s responses indicated that learners could not apply what they learnt in surface area and volume to new situations. The same is seen and said of the high frequency in the “NA-Not attempted” responses in Question 6, where learners were expected to show their application, synthesis and evaluation skills. As shown by the interview, the reason for this could be attributed to inadequate mastery of concepts.

Notice the high percentages of incorrect responses and not-attempted questions from Table 4.4, compared to the very low percentages of correct and partly correct responses per item. This is another indication that learners do not respond well to questions on surface area and volume.

From the responses to Question 1 in Table 4.4, it is evident that most learners are more comfortable with defining the volume (28%, 1a), than the surface area (24%, 1b). The responses for Question 2 reveal that almost all participants had difficulty in describing or explaining how to find the volume (2a) or surface area (2b) without using any formula. Again, a few learners responded positively in describing how to find the surface area, 14%, in comparison with none, (0%) on the volume.

The participants performed dismally in Question 3, where most correct responses lie between 0% and 3 %, except for Question 3b, which had 28% correct responses and 3e, at 10%. It is interesting to note that only Question 3b had dimensions, although in discrete units. As discussed below, the results show that the most prevalent misconception in Question 3 was that the rest of the blocks could not have volume and surface areas because they were considered “incomplete” in the participants’ minds. As discussed below, another misconception prevalent in Question 3 was that where there are no dimensions or measurements, the surface area and volume cannot be obtained. A reasonable number of learners could enumerate arrays of cubes in Item 4a, at 37% but not in Item 4b, where only 7% of the learners obtained correct responses.

Question 5a and 5b also had higher correct response percentages than similar objects in Question 3. It is also important to note that the block or prism in Question 5 is considered complete by most participants, despite not having measurement; hence they could figure out the number of unit cubes used to construct and also the required volume. Conversely, 63% of the participants gave incorrect responses for Question 5c, which required them to find the same structure's surface area.

Misconceptions in Question 6 were expected to be confirmed during the interviews, as most learners either did not attempt responding to the item or had not completed their work due to time constraints. A detailed narrative analysis of each test item is provided below, under the relevant research questions.

The following are written extracts and responses from some of the ten interviewed participants, including extracts of the subsequent interviews. Probing the participants on each question helped the researcher understand each participant’s point of view. Participants A2, A3, C3, G3, H3, N3 and Q3 took part in the interviews.

4.4 INTERPRETATIONS FROM TEST AND INTERVIEWS

4.4.1 How Grade 8 Learners define the Concepts Surface Area and Volume

<u>Question 1a: Define volume</u>

The purpose of this question was to find out how Grade 8 learners defined volume. The most favourable response was that volume is the amount of space occupied by a three-dimensional object. Participants could also have noted that volume measures the amount of space an object or substance takes up. Table 4.4 shows that only eight participants defined volume correctly, which is 28% of the participants who took part in this study. Sixteen other participants, 55% of the participants, partly defined volume.

This leaves the other 17% of the participants who were totally unable to define volume. Considering the 28% that gave correct responses, 72% of the participants under study struggled with the definition of volume.

In response to Question 1a, participant H2, A3, B3, Y3 and R3, that is 5/29; all defined volume as the capacity an object takes up. On another note, participants C3, D3, G3 and Q3 defined volume as mass. Participants H3, L3 and T3 defined volume by citing the formula for finding the volume of prisms. These statistics indicate a total of 12/29 participants with some misconception regarding defining volume. This implies 41% of the participants under study either confused volume with capacity, mass, or provided a formula instead of a definition.

The extract below shows how participant A3 justifies this answer.

Here is how Participant A3 responded:

Researcher: Question 1a, you said volume is the capacity or how much an object takes; do you think volume and capacity are the same?

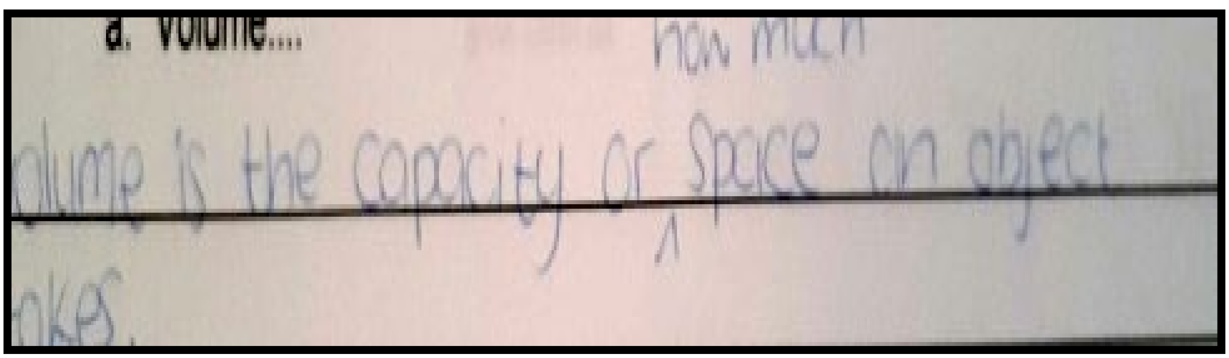


Figure 4.2: Participant A3's written response to test item 1a

Participant A3: No sir, capacity measures liquids

Researcher: And volume?

Participant A3: It is the amount of space an object takes up.

This response sounds like clear memorisation and mere recall of the definition, as the volume initially had been equated to capacity. A similar argument is portrayed in participant C3's work, where volume is erroneously referred to as just matter, and also as can be defined in terms of the formula: length x width x height.

Participant C3 accounts as follows:

Researcher: Please expand on Question 1a, what you meant by volume is how much matter an object takes.

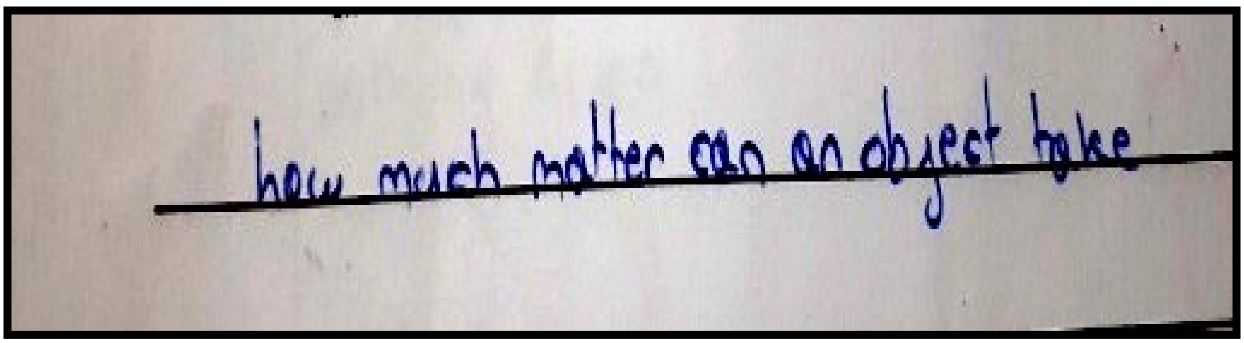


Figure 4.3: Participant C3's written response to test item 1a

Participant C3: I meant to say, I do not really know how to explain it, but kind of like this piece of paper (holding one page of the question paper), how much space it takes, like the space around us. That is what I thought volume was. That's my answer.

Researcher: Explain how that piece of paper can have volume.

Participant C3: We can calculate the volume of this paper by take the length, width and height measurement. It is going to be Length x base x height then we find the volume of this paper.

Researcher: If we consider this piece of paper and the First Aid Kit box behind you, are they similar?

Participant C3: No, they are not.

Researcher: How are they different?

Participant C3: They are different in size, the height and the weight.

Researcher: What can the height of this paper be?

Participant C3: The height can be 0,001 mm, it is very small.

At least, the learner's response seemed to show that the participant knew what volume was, although it was hard to give an explicit definition. However, there was a noted misconception of taking 2-D space as 3-dimensional, as evidenced in the volume of a piece of paper example that the learner gave. Contrary to participants A3 and C3, participant G3 and Q3 defined volume as mass; a clear indication that learners dealt with mathematical concepts having covered minimum or no definition at all, hence the learner finally said, "it is a total something". Interesting to note is the number of learners referring to volume as capacity, also found in participant G3. Learner G3 also thought volume was the air inside an object; an assumption that solid objects could not be linked to volume but only the hollow ones.

Participant G3 responds likewise:

Researcher: Explain what you mean by volume is the total mass of an object in question *1a.*

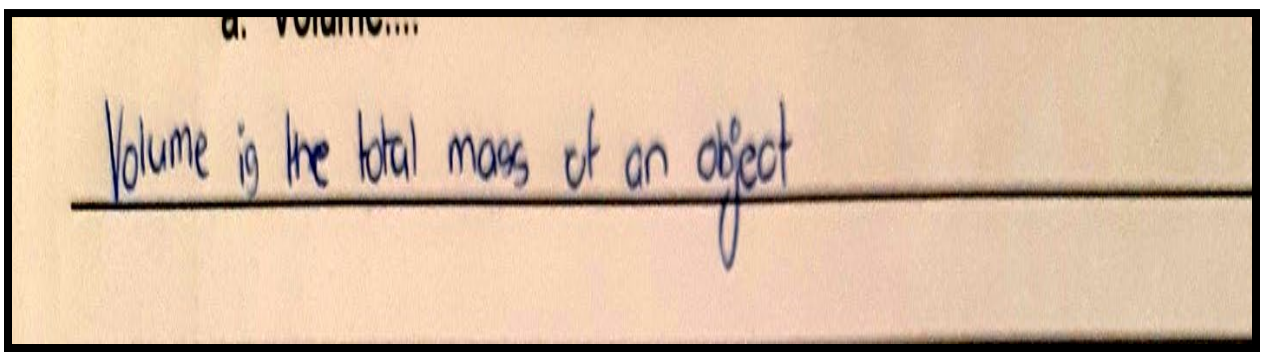


Figure 4.4: Participant G3' written response to test item 1a

Participant G3: I meant that volume is the total capacity, the weight, no that's not it, total capacity, total something.....how much air it has.

Participant H3 displayed misconceptions in this interview as follows:

Researcher: 1a, what do you mean by volume is the total calculation of length and width multiplied by the height of the shape.

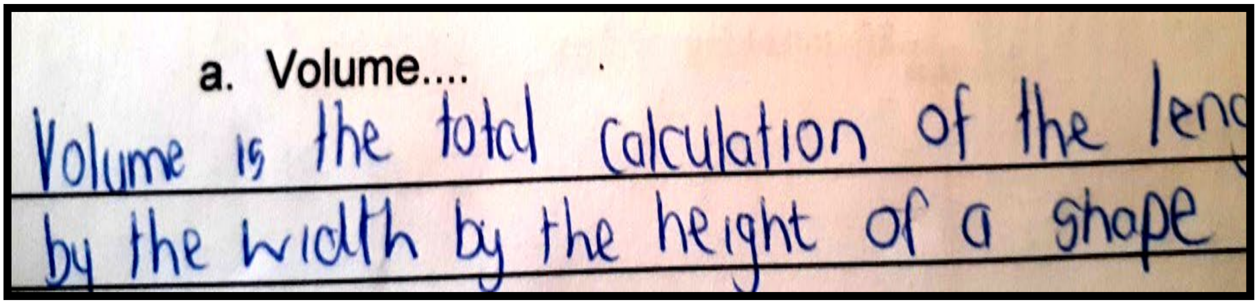


Figure 4.5: Participant H3's written response to test item 1a

Participant H3: I meant its formula

Researcher: Is it possible to define it without using the formula, could there be another way of describing volume

Participant H3: No sir

The learner associated the definition of volume with some formula description. This term of reference was evident in most of the learners' scripts; hence it displays a potential misconception.

Participants L3 and T3 were not available for interviewing, but also displayed similar characteristics as H3 in their work, as shown in the extract below from participant L3, who also defines volume by citing the formula.

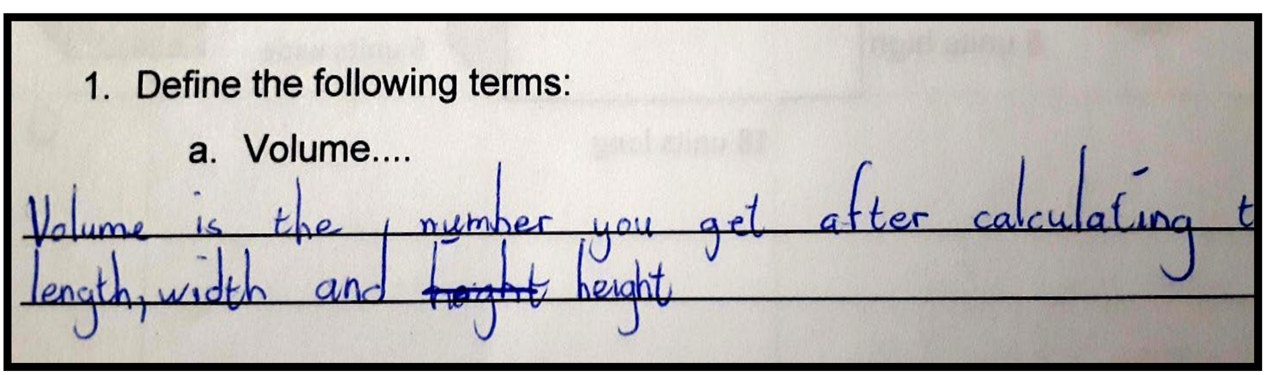


Figure 4.6: Participant L3's written response to test item 1a

The other learners, for example participant N3, could not express any vivid definition of volume. That was proof that some learners lacked the conceptual understanding for interaction with the learnt mathematics. Mathematics seemed to be learnt by rote and formula memorisation.

Lastly, participant Q3 responded in the following manner:

Researcher: Please explain what you meant by volume is the amount of mass in a shape for question 1a.

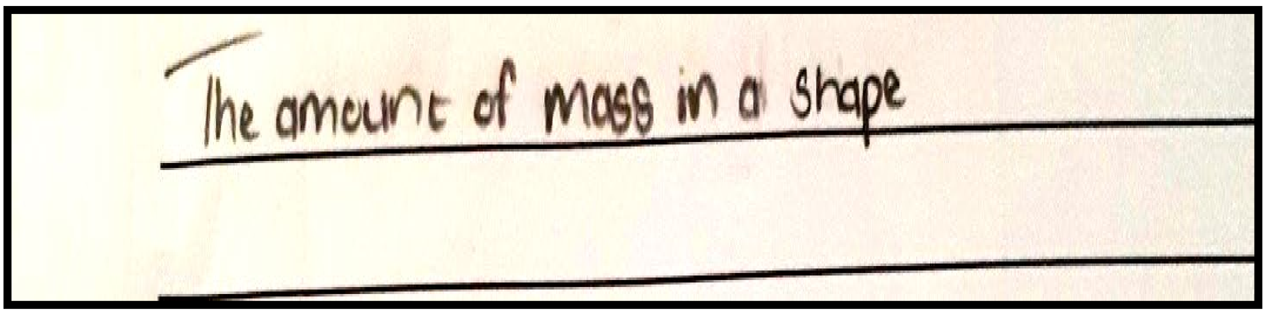


Figure 4.7: Participant Q3's written response to test item 1a

Participant Q3: Let's say we have this box sir, we want to know how much it weighs, like how many things are inside this box, that's what I meant. But I think I mixed it with surface area, because volume is how much space it occupies.

While participant A3 had a somewhat clear understanding of what volume was, the learner initially took volume and capacity to mean the same. It was only after probing that the learner explicitly distinguished between volume and capacity. This was indeed one of the other misconceptions discovered during this study. Participant C3 took volume to be matter, but matter has both mass and volume. There was a clear indication of a misconception by Learner C3, who considered a piece of paper that is 2-Dimensional to consist of volume. Interestingly, the learner gave the height of a piece of paper as 0,001 mm in the motivation of the fact that it should therefore have the calculated volume, using the formula L x W x H. Participant G3 confused mass with volume. This was another misconception discovered in this study, although not yet recorded or found in the existing literature.

Probing Learner G3 revealed another misconception of equating volume to capacity, weight, "how much air", and other unknown quantities the learner refers to as "some things". Participant H3 defined volume by giving the formula for calculating it. Probing the learner reached a conclusion that there was no other way one could define volume without the use of the formula, another misconception. Participant Q3 initially equated volume with mass, then with weight, and later with the number of things inside a box. Finally, the learner gave the correct definition of volume. It was evident that while the learner eventually gave the correct definition, it was by default, probably by memorisation and not understanding of the concept. One can conclude that learners had difficulty defining the concept volume.

<u>Question 1b: Define surface area</u>

Similar to Question 1a, this question aimed at exploring learners' definitions of surface area. The most appropriate response would have been that surface area refers to all the faces' total areas on a 3-D shape or solid. Participants could also have mentioned that surface area gives us how much material would be required to cover all the 2-D faces of a 3-D object. Considering the summary in Table 4.4, seven of 29 participants could precisely define surface area, which means only 24% correct responses. Only 9 of 29 participants could partly define surface area, this is only about 32% of the subjects. With 10% of the participants not attempting the question and 34% giving incorrect responses, it means 44% of the learners could not define surface area.

Responding to Question 1b, participants B3, D3, H3, N3, Q3 and T3 define surface area as volume, with three of these learners even supplying the formula for volume. These responses confirm the fifth misconception in Table 4.3, namely confusing surface area with volume, including interchanging the formulae. Participants D2, E2, F2 and R2 describe the perimeter of 2-D faces around an object instead. This is another misconception that had not been seen in the literature review concerning surface area. In addition to participants E1, C3 and L3 who did not attempt this question, it follows that 13/29 of the participants–45%, had difficulty defining surface area due to some misconception.

The following extracts show how different participants justified their answers.

Participant A3 denotes as follows**:**

Researcher: In question 1b, you indicated that surface area is the total area an object occupies, would you please elaborate here?

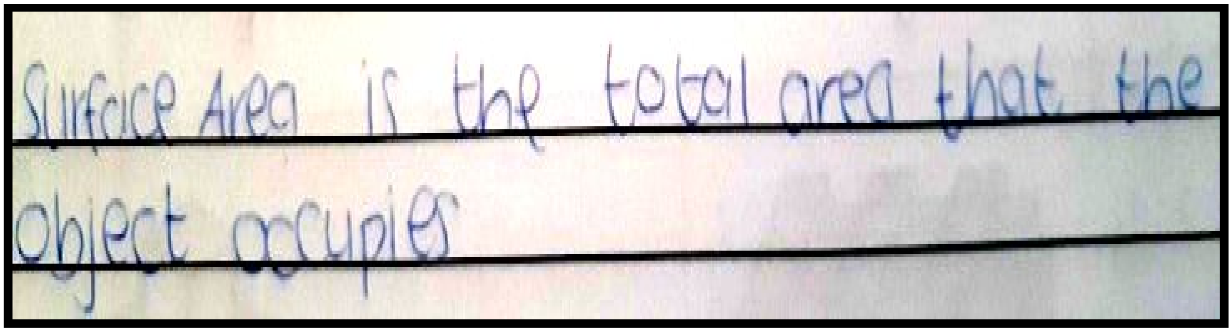


Figure 4.8: Participant A3's written script response to test item 1b

Participant A3: Total surface area is basically the entire area of an object, it's the total area of each side, all the areas added up.

Participant C3 accounted in the following manner:

Researcher: You had not responded to question 1b-2b, what would you want to say about that?

Participant C3: I am not good at remembering definitions, but I am good at Math, just these definitions. I don't think I can even try it.

Participant C3 had no vocabulary to define surface area but could solve problems involving surface area through formula usage. It is most likely that participant C3 memorised the calculation procedure, but without understanding what the concept of surface area entails.

Participant G3 elaborates the above definition with some resemblance to participant A3 but also indicating a misconception as follows:

Participant G3's argument:

Researcher: In Question 1b, what exactly do you mean by surface area being the total area around the object?

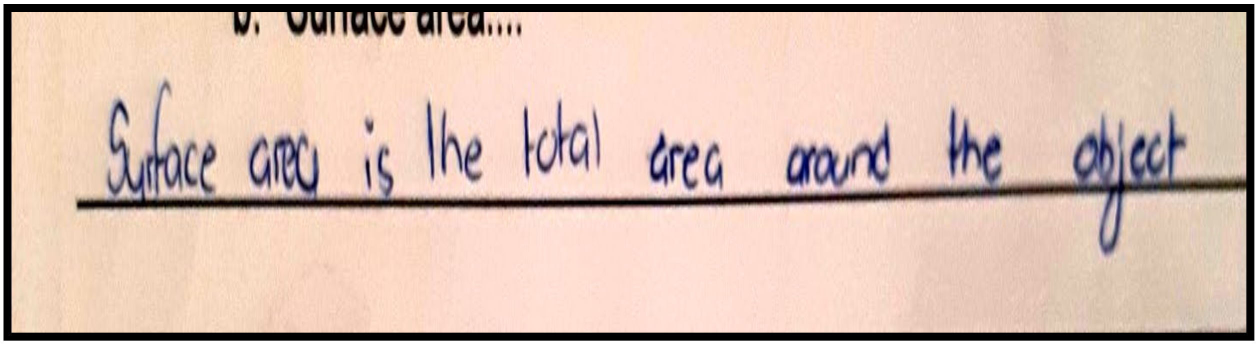


Figure 4.9: Participant G3's written script response to test item 1b

Participant G3: It is the distance around the object.

Researcher: Here is a box, please show me what you refer to as the distance around.

Participant G3: This distance of the surface, of all the sides

Researcher: And for volume, show me which part would be the volume

Participant G3: The components inside; that would make up its weight.

Researcher: How do you define surface area?

Participant G3 initially takes perimeter to be the surface area then compares the volume to mass, similar to the misconceptions discussed in Question 1a above. The following participants all confused surface area with volume in their responses.

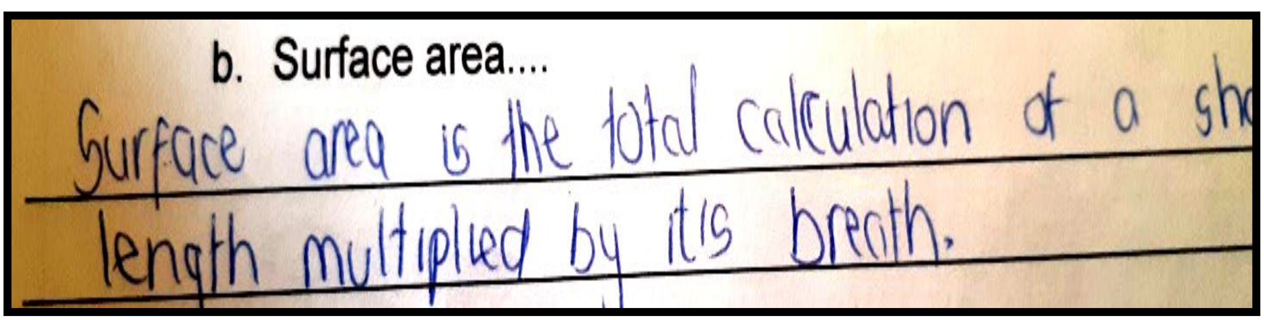


Figure 4.10: Participant H3's written script response to test item 1b

Researcher: How do you define surface area?

Participant H3: Total calculation multiplied by the breadth.

In this case, participant H3 provides the formula for finding the area of a rectangle, displaying the first misconception from the framework in on Table 4.3, where learners treat 3-D objects as 2-D shapes. Furthermore, participant H3 provided a formula when asked to define the surface area, a characteristic displayed by participant C3 as well.

Researcher: In question 1b, what definition can you give for surface area?

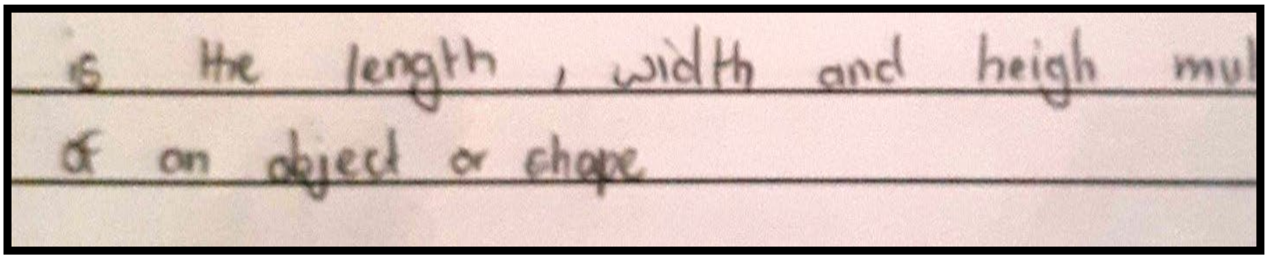


Figure 4.11: Participant N3's written script response to test item 1b

Participant N3: It is the length, width and height multiplied together on an object or shape. Basically I wanted to show how it is calculated.

Researcher: Can we define it without using the formula.

Participant N3: No, first they have to give you the length, the height and the width. I would find the surface area then multiply by 2 to find volume, because volume is much heavier per say.

Participant N3 confused surface area with volume and interchanged the formulae thereof. The participant further described an incorrect way of finding volume, taken from surface area. I noticed a combination of errors here, which all culminate in a misconception.

Researcher: Question 1b on surface area, what really did you mean by amount of space in a shape?

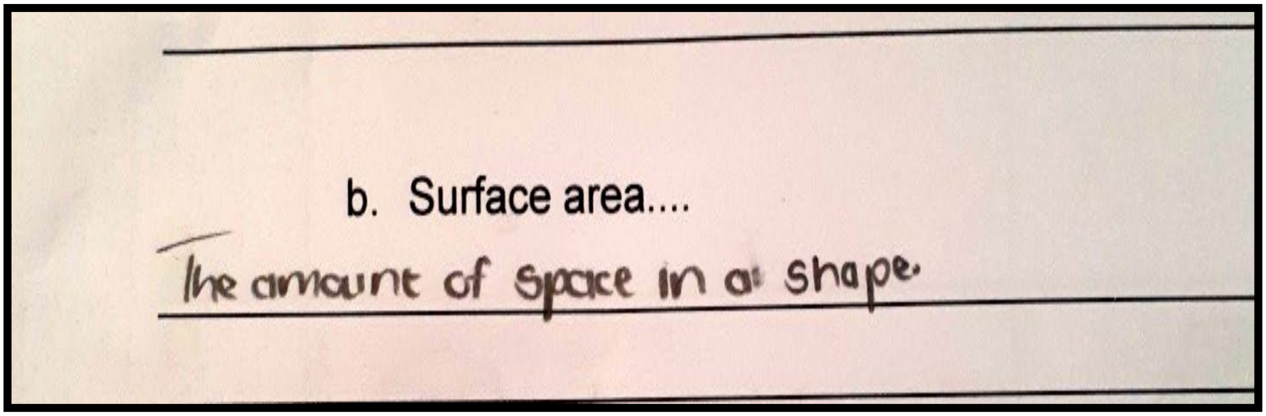


Figure 4.12: Participant Q3's written script response to test item 1b

Participant Q3: I think it is the calculation of the whole shape, like how much is inside. The measurement of the object, yeah, that's what I wanted to say.

Researcher: If you look at this first aid kit here, which one would you say is its volume and which one would be surface area?

Participant Q3: It would be the outside of the object, if you know what I mean.

Researcher: What about the outside?

Participant Q3: Its measurements, I think.

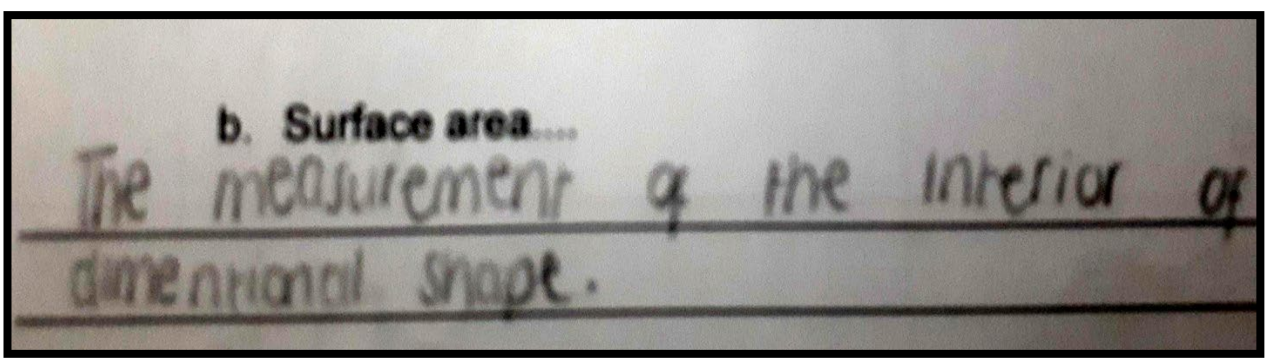


Figure 4.13: Participant T3's written script response to test item 1b

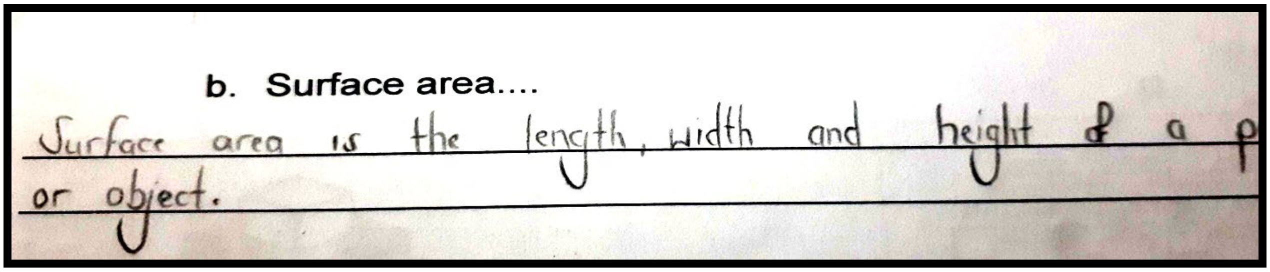


Figure 4.14: Participant B3's written script response to test item 1b

Participant A3 initially included terms used in volume, "occupies" but on probing during the interview, learners seemed to have an idea of what total surface area entails. It was only the vocabulary of expressing oneself and defining the surface area that lacked. Participant C3 could not define surface area. The learner claimed to be very good at mathematics but not so in definitions. When asked to elaborate on what participant G3 meant by the surface area being the total area around an object, the learner mixed the terms with perimeter. Participants H3 defined surface area by giving the formula of finding the area, while participant N3 gave the formula for volume instead. Participant Q3 seemed to refer to volume when asked to define surface area. The learner mentioned some form of calculation, "the inside and the outside of an object".

It was clear from the above responses that the learners had more challenges in defining surface area than in defining volume. In many instances, surface area was confused with volume. This is a misconception in surface area and volume. One notices that regarding definitions, learners' misconceptions fell into the fifth part classified as confusing the concept of volume with that of surface area, including interchangeable use of their formulae.

4.4.2 How Grade 8 Learners Describe the Process of Finding Surface Area and Volume

<u>Question 2a: Describe how you can find the volume of prisms without using a formula.</u>

Related, but not similar to Question 1; Question 2 required learners' descriptions of how to find the volume and surface area of prisms, but without using the formulae. This was aimed at establishing whether learners were comfortable giving descriptions of events or procedures instead of providing a formula regarding volume and surface area. The question responded to the objective where the researcher intended to explore how Grade 8 learners responded to surface area and volume questions.

Table 4.4 shows that none (0%) of the participants gave correct responses to this question. Seventeen per cent (17%) of the participants had partly correct answers while 62% responded incorrectly, with the other 21% not responding to the question at all. These figures indicate a dire lack of vocabulary to describe how the volume of prisms can be found. Eighty-three per cent (83%) of incorrect and non-response is a very high percentage, indicating the learners perceiving difficulty and misconception. It indicates an inability and a lack of skills to describe and explain procedures in mathematics, possibly largely caused by memorisation of formulae.

The most suitable response to this question was that one would count the number of unit cubes that make up the required shape. Other alternatives like displacement of liquids in measuring cylinders followed by converting capacity units to volume were also acceptable, since the question did not allow any use of formula.

Most of the learners, 13/29 participants (45%), responded by using a formula. This was the main misconception in this test item. Four of 29 participants (14%) tried to supply some formula but gave the wrong formulae. These were participants I2, G3, P3 and Q3. It follows that 17/29 participants (59%) used the formula, which was against the question instruction. A few other participants described the 2-D figure-nets formed when 3-D shapes are opened. This confirmed the misconception of treating 3-D objects as 2-D figures.

The following learners' written responses and interview extracts display the challenges participants encountered in their attempt to describe how to find the volume of a prism without using a formula.

In answering Question 2a, participant A3 indicated, as displayed below, that to find volume of a prism, you would multiply the length by breadth by the height.

When asked and probed in the interview, participant A3 said the following:

Researcher: In question 2a, you said that to find volume, you would multiply the length by breadth, times height, can you expand on this?

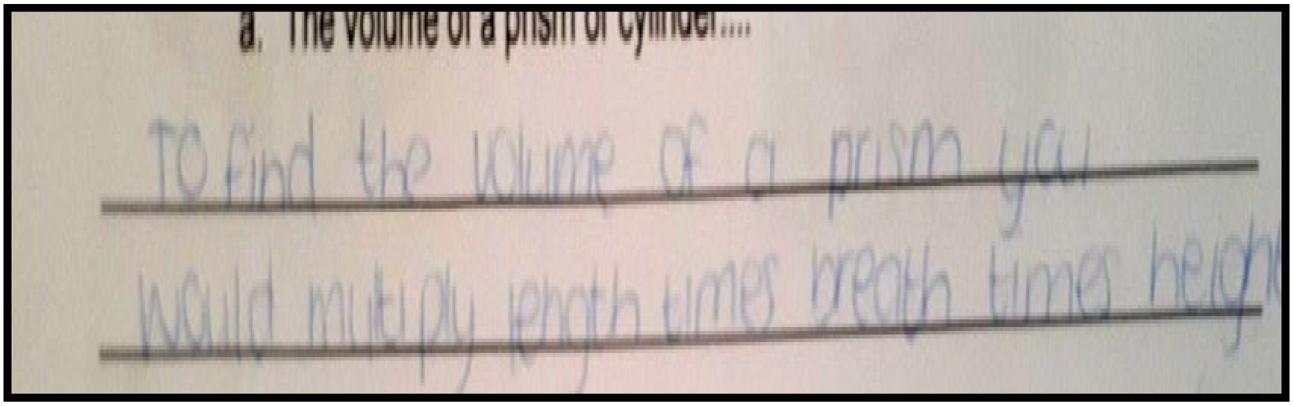


Figure 4.15: Participant A3's written response to test item 2a

Participant A3: It is the formula to find volume sir

Researcher: Alright, and I think our question does not allow us to use the formula, so could there be another way of describing how to find volumes of prisms without the use of a formula?

Participant A3: No, the question says you can't write the formula sir but they didn't say you can't use it in words. I used words to explain the formula. It wasn't specific on how not to use it.

Researcher: Is there another way you can explain, without mentioning the formula?

Participant A3: I am not sure Sir, this is the only way I can explain it.

It appeared though, according to participant A3, that there was no other way of finding the volume of a prism except by the use of the formula "length x breadth x height". This is a misconception of describing a procedure by the quantitative formula. This is another misconception discovered during this study, as it is not among the ones displayed in Table 4.3. Probing participant C3 showed a similar response to participant A3 regarding the use of a formula in an attempt to describe how to find the volume of a prism.

Participant C3 had this to say about the same question:

Researcher: In question 2a. You initially left it blank, what was your reason for that?

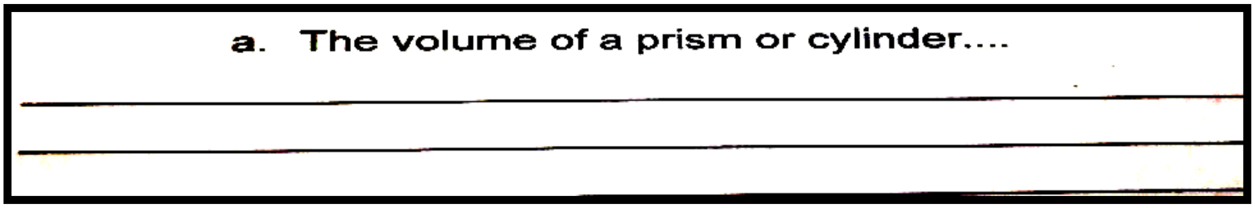

a. The volume of a prism or cylinder....

Figure 4.16: Participant C3's written script response to test item 2a

Participant C3: Actually here I had forgotten the formula for a prism. I don't know how to say it but.....

Researcher: Alright, but here the question instructs you not to use a formula, so there could be another way you can describe how to find surface area.

Participant C3: Let me try, the area of the prism, I think it is height multiply by one area of the triangles of the prism and the base, multiplying those two will give us the volume.

Researcher: When you are multiplying...don't you think you are using a formula?

Participant C3: Yes, I think I used the formula again.

Researcher: Now how can we find the volume without using any formula?

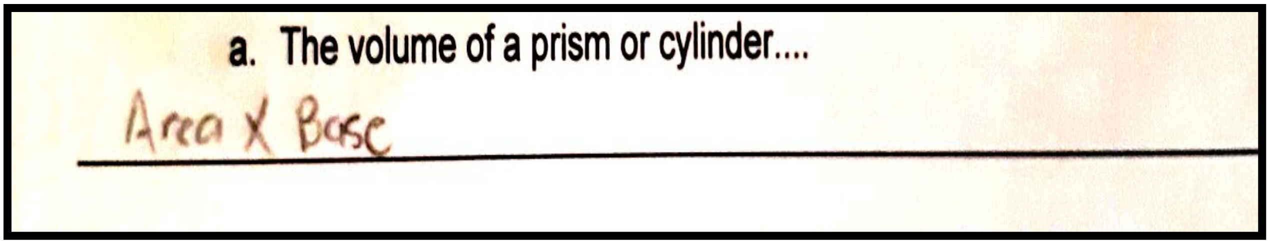
Participant C3: Volume, no we cannot, there is no other way. Maybe I can, but right now, I don't think so.

Participant C3's response to Question 2a was similar to the one given when probed in Question 1a. The learner defined and described how to find a quantity by its formula. This misconception was displayed in participant A3 above as well, in Question 2a.

The above perception from participants A3 and C3 was also observed in participant G3's written work. Although in this case the learner gave an incorrect formula, the fact that a formula was submitted instead of describing how to find volume confirms the misconception of defining and or describing a quantity by its formula.

Participant G3 responded in the following manner:

Researcher: For question 2a, you described how to find the volume of prisms or cylinders by..... (learner interjects)..


a. The volume of a prism or cylinder....

Area x Base

Figure 4.17: Participant G3's written script response to test item 2a

Participant G3: ....Area x base

Researcher: I think that you are using some formula when you say so, when the question restricts us from using any formula, is there another way you can describe how to find the volume?

Participant G3: Let's see, I think you can measure around, measure the surface, measure the sides and multiply by......., no, everything I am thinking of seems to be going back to the formula

Researcher: Do you remember when you learnt about area or volume?

Participant G3: Yes

Researcher: When was that?

Participant G3: I think around grade 6

Researcher: Do you remember the very first activities you did in finding area and volume, before using the formula?

Participant G3: No, they first introduced the formula and then we would take it from there.

Participant G3's response to probing exposes poor pedagogy of concept development. According to participant G3, volume was introduced in terms of the formula, with no definition and description of examples or non-examples of the concept. This has resulted in the learner also defining and describing the quantity volume by its formula. One of the causes of a misconception in the learning of volume and surface area is exposed here, which is rote learning and memorisation of the formula.

The following is an extract from participant H3 in response to Question 2a, which displays the definition and description of quantities by their formula. However, participant H3's written response was contrary to what the learner said during the interview, moving from a partly correct response to a misconception of describing by supplying the formula.

Participant H3 described the process of finding volume of prisms as follows:

Researcher: In question 2a, you said if there are blocks, you would count them, what if there are no blocks: how would you find the volume?

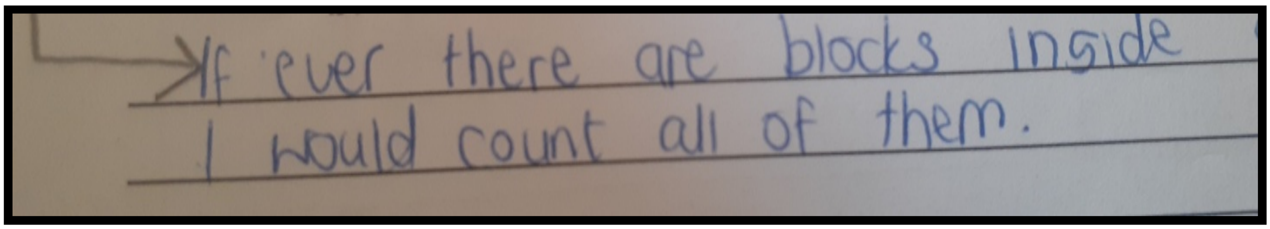


Figure 4.18: Participant H3's written script response to test item 2a

Participant H3: I would measure all the sides.

Researcher: What would you do with the measurements?

Participant H3: ...then I would definitely use the formula to calculate.

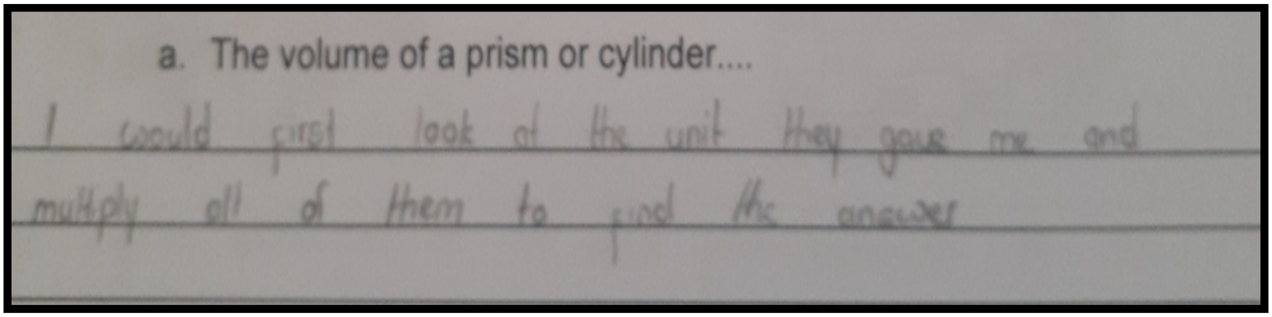
Researcher: ...but the question says you must not use any formula, so in what other way can you find the volume?

Participant H3: There is definitely no other way, sir, that is what I think.

I was left wondering how participant H3's interview response was different from the written one. It is possible that the learner was guided by Question 5 while writing the text but forgot about it during the interview. According to participant H3, one needs measurements or dimensions to calculate the volume of a prism using a formula, besides which there is no other way. That was the same misconception displayed by participants A3, C3 and G3 earlier on. Participant N3 below displayed the same reasoning as the others discussed above.

In response to Question 2a, participant N3 has the following to say:

Researcher: Question 2a, can you find the volume without being given the units and the formula? I see you said you would first look at the given units.


a. The volume of a prism or cylinder....

I would first look at the unit they gave me and multiply all of them to find the answer

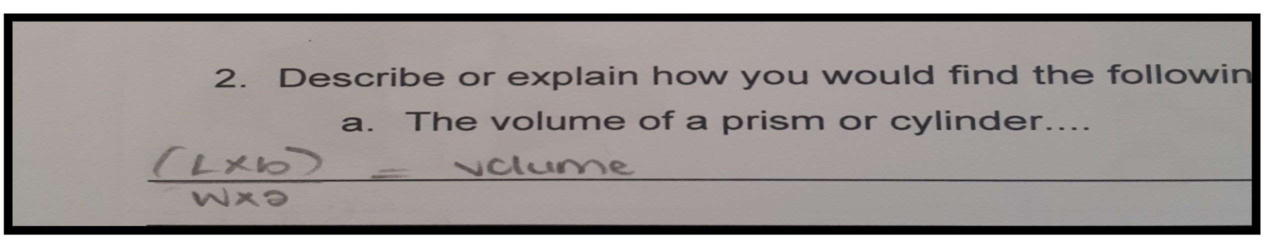
Figure 4.19: Participant N3's written script response to test item 2a

Participant N3: In my opinion, you cannot, because formula is the maths, everything needs a formula, without the formula you can get it by chance, once or twice but not always, it's not guaranteed.

As argued by participant N3, the learner believed that there was no other way of finding the volume of a prism besides the use of a formula. The learner continued to state that everything in mathematics revolved around a formula. This misconception was the most prevalent in this question and throughout the study.

The last participant I discuss in this question is participant Q3, who also responded by writing a wrong formula instead of describing how to find the volume. This is how participant Q3 accounted for the question after probing at the interview:

Researcher: In Question 2a, what do you mean by this answer, for describing how to find volume of a prism?


2. Describe or explain how you would find the followin

a. The volume of a prism or cylinder....

(Lxb) = volume

Wx2

Figure 4.20: Participant Q3's written script response to test item 2a

Participant Q3: I think I did it wrongly, because I used the formula, I should have done the calculation of....., you know the calculation of a rectangle or perimeter. I think I was thinking of area... I gave a wrong formula.

This response exposed three other errors in participant Q3’s perception. Firstly, that volume and area were the same; secondly, that volume was the same as perimeter and thirdly, suggesting that you can find the volume of a rectangle. The identified errors exposed the misconception of treating 2-D figures as 3-D shapes.

The interview proceeded as follows:

Researcher: Yes, I see you have recorded a formula, but the question requested us not to use any formula. Is there another way we can find volume or surface area without the formula?

Participant Q3: Isn’t just the same thing that we must have the length, width and the height?

Researcher: And that means we would be going back to the formula

Participant Q3: I think I need to rephrase sir, there is no way you can find the surface area and the volume, without the formula, you would confuse yourself.

Researcher: Do you remember the first time you learnt about volume or surface area?

Participant Q3: Yes

Researcher: When was that?

Participant Q3: That was in Grade 6

Researcher: And do you remember the first activities that you did under the topic volume?

Participant Q3: No not really, because I didn’t understand what volume was, until I got to grade 8....

Researcher: And the first activities you did then in Grade 8 on volume, do you remember them?

Participant Q3: We did 3-D shapes and then surface area because it was together with volume.

Researcher: So, then, how can we find the volume of a prism, without using any formula?

Participant Q3: Do you want me to use these unit cubes, sir?

Researcher: If you used them, how would you find the volume?

Participant Q3: You would find how many are in the length, width and height then calculate them using the calculation, maybe, length x 2; width x 2 and height x 2 then you add them all up together. Or you say length x width x breadth.

Not only did participant Q3 supply a formula for Question 2a, but also gave several incorrect ones. The learner's perception mostly follows the participants' point of view that one describes how to find the volume of a prism by stating the formula.

A close look at the given responses shows that only participant H3 initially described how to find volume without the use of a formula. The rest of the interviewed participants linked the description of any procedure of finding the volume of prisms with some form of formula, whether in words or algebraic form. Figure 4.4 indicates that none of the participants got this question correct, only 17% were partly correct, but with high misconception prevalence. This suggests that 83% of the participants struggled with describing how to find the volume of prisms. This inability to describe concepts indicates a lack of conceptual knowledge.

In the same manner as in Question 2a, Question 2b was testing the ability to describe how to find the surface area, but without using any formula. The participants were expected to indicate that they would draw the nets of all the prism faces, find the areas of each face and then add up all the areas together for surface area. Table 4.4 shows that only 14% of the participants gave correct responses to this question, while 55% had incorrect answers, and the other 31% could not attempt responding. This statistic reveals that 86% of participants could not describe how to find the volume of prisms.

The low percentage of correct answers confirms difficulty in giving mathematical, conceptual descriptions.

The main misconception noted in this question was the same as in Question 2a, the substitution of a description by a formula. Of the participants, 12/29 (41%) described how to find the surface area of prisms by attempting to supply some form of formula, with a couple of them writing it incorrectly.

<u>Question 2b: Describe how you could find the surface area of prisms without using formulae.</u>

In response to this question, participant A3 has the following to say on describing the procedure of finding surface area without using a formula:

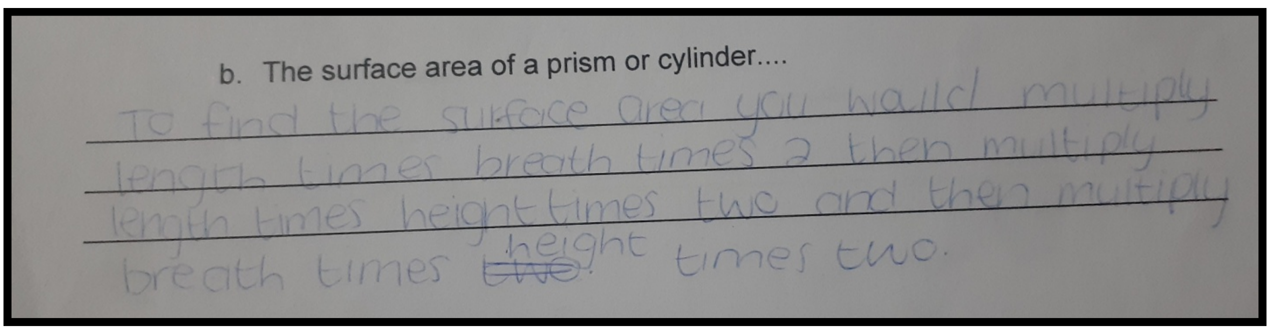


Figure4.21: Participant A3's written response to test item 2b

Researcher: Please describe how you can find the surface area of a prism.

Participant A3: It is length x breadth x 2 + length x height x2 + breadth x height x2, that is the formula.

Researcher: Can you try to describe it without using a formula?

Participant A3: It is not possible. You will need the units and a formula for surface area.

Researcher: What if the units are not indicated?

Participant A3: Do you mean like the picture on question 5, sir?

Researcher: Sort of, how can you find the surface area of a prism without using a formula?

Participant A3: For surface area, we can count the blocks on the top from left to right then....

Researcher: Are we counting blocks?

Participant A3: The units, cubic units, square units....There is something one can count but I do not remember what it is. I think I have forgotten the method, Sir.

Initially, participant A3 provided the formula but after probing, changed position to mention the count of some undisclosed entity. It was clear that the learner interchanged surface area with volume procedures, as seen in the use of units, cubic units and square units in describing the same phenomenon. The fact that the learner emphasised having forgotten material at Grade 8 level, is disheartening as the concepts under discussion were introduced and developed well ahead in the foundation and intermediate phase and early in the senior phase at Grade 7 level. An observer is forced to conclude that the concepts were not covered well enough for the learners to master and comprehend independently to successfully apply the acquired knowledge in different future situations.

Responding to the same question, participant G3 wrote the formula and argued as follows:

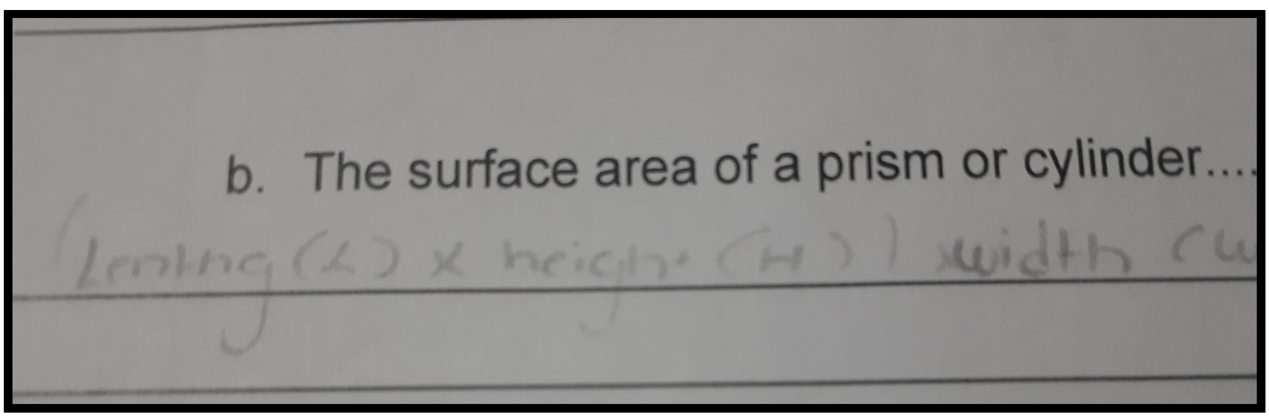


Figure 4.22: Participant G3's written response to Test item 2b

Researcher: Please explain how you would find the surface area of a prism or cylinder, without using a formula.

Participant G3: It would be the length times the height times the width.

Researcher: But I think you have just used a formula, can you explain in another way without the formula?

Participant G3: That is impossible, sir, surface area needs a formula, always.

Participant G3 supplied the formula for finding the volume of a prism rather than describing how to find its surface area. This misconception falls under the '*confusing surface area with volume'* misconception as well as interchanging the formula listed in Table 4.3.

Participants B1 and B3 were not available for an interview but also expressed the same thinking as participant A3 and G3 above. Participants B1 and B3 written responses are given as follows:

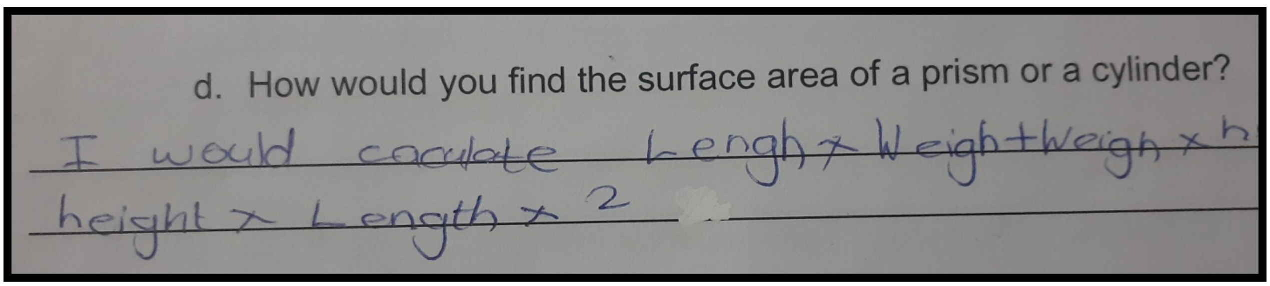


Figure 4.23: Participant B1's written response to Test item 2b

Similarly, participant B3's written response follows the same line of thought as participant A3.

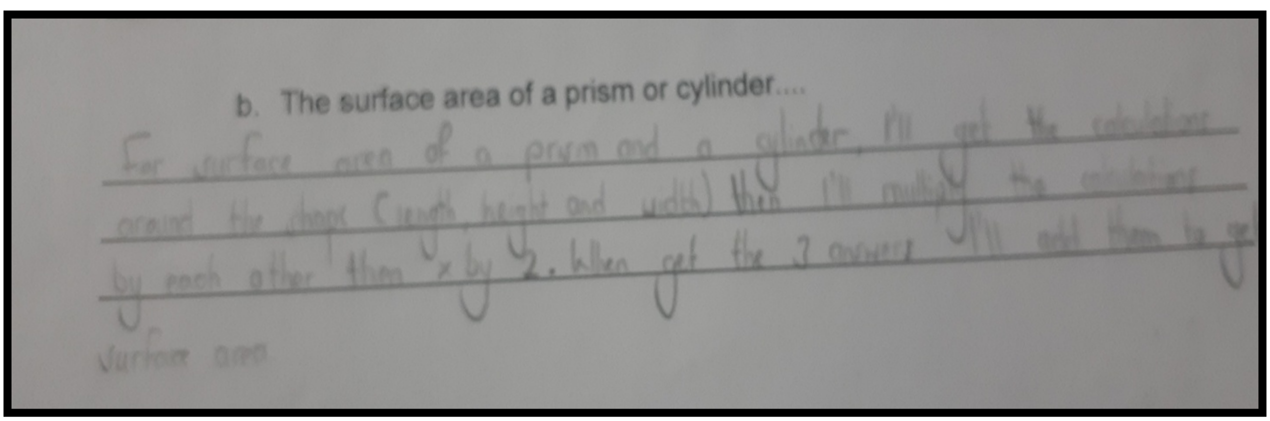
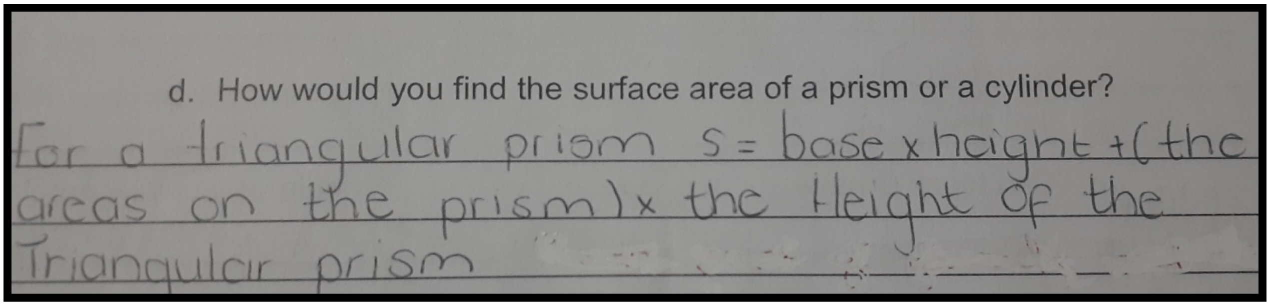


Figure 4.24: Participant B3's written response to Test item 2b

Participant G2 also described the formula for finding the surface area of a triangular prism, but ended up mixing it with formula for volume, as shown in the next figure.

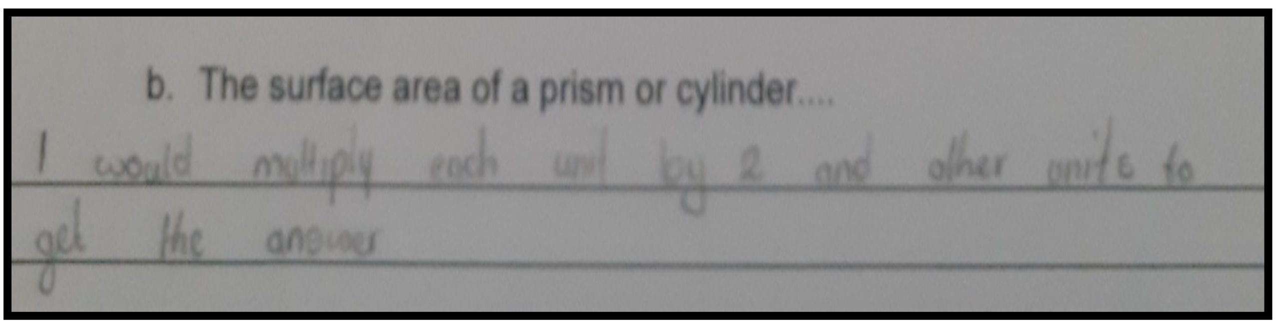

d. How would you find the surface area of a prism or a cylinder?

For a triangular prism S = base x height +(the areas on the prism) x the Height of the Triangular prism

Figure 4.25: Participant G2's written response to Test item 2b

Participant N3 had the following description:

Researcher: Question 2b, surface area, can you explain how to find it without using the formula


b. The surface area of a prism or cylinder....

I would multiply each unit by 2 and other units to get the answer

Figure 4.26: Participant N3's written response to test item 2b

Participant N3: I will take out everything inside, then figure out the units, find the surface area then multiply by 2 to find its volume. Volume is going to be heavier than surface area.

Researcher: Why would you multiply by 2

Participant N3: Because it is 2 times the mass, in my opinion

Researcher: Do you remember when you first covered the topic of surface area, what were the first activities you did?

Participant N3: Yes, in Grade 8. We were given formula and take it from there. No formula, no volume and no surface area.

Participant N3 seemed to concur with participant G3 in assuming that only the formula describes the surface area of a prism. This misconception has already been confirmed.

Having responded to question 2b with a formula, participant Q3 argues as follows:

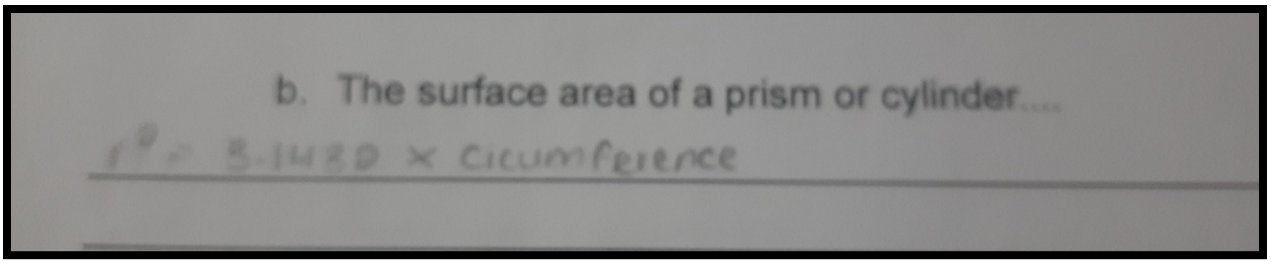


Figure 4.27: Participant Q3's written response to test item 2b

Researcher: How would you find surface area of a prism or cylinder without using the formula?

Participant Q3: There is no way you can find the surface area and the volume without the formula, you would confuse yourself.

Researcher: But since the question requires you to describe without the formula, don't you think there is another way?

Participant Q3: It is all about formula sir. No formula, no surface area. There must be a formula of some sort.

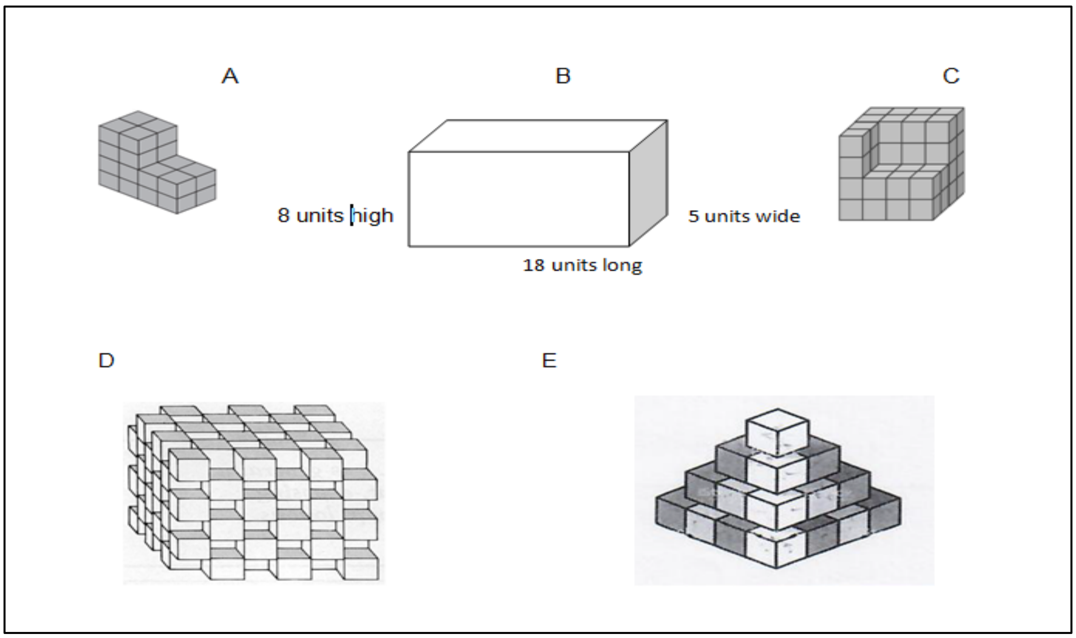
Participant Q3 also displayed the same line of thought and perception as Participant N3 and G3.

All scripts and the interviewed participants associated any description and procedure of finding the surface area with a formula of some sort. That is a gross misconception emanating from the introduction of the concepts using the formula, as indicated in participants C3, G3, N3 and Q3. Figure 4.4 shows that 31% of the participants did not attempt this question, while 55% had signs of misconceptions in responding to it. This gives a result of 86% incorrect responses, as displayed in Figure 4.4. The dominating misconception in this question is that only the formula describes any mathematical phenomenon.

4.4.3 How Grade 8 learners solve problems involving surface area and volume

Question 3: Surface area and volume

> State whether each of the following blocks has surface area and volume. If they do, find the surface area and volume is in each case. If any of these blocks neither has volume nor surface area, provide a reason.



The objective in including this question was to assess learners' conceptual understanding of surface area and volume. This developed from Question 1, which required definitions from first principles as well as Question 2 calling for descriptions. Question 3 was then the application part of the defined and described mathematics. Only Diagram 3b had measurements on the dimensions for computational purposes. The rest of the diagrams had no specified dimensions to test whether learners would appropriately use the unit squares and unit cubes provided at the beginning of the paper. As had been hypothesised, most participants felt that where there were no units, volume and surface area could never be obtained. Figure 4.4 indicates that only Question 3b, which had dimensions, had the highest percentage of correct responses, at 28% correct and 10% partly correct answers; that is to say, about 38% of the participants were in the right direction regarding this question. Question 3b had 38% incorrect responses and the lowest not attempted percentage of 24% in this section.

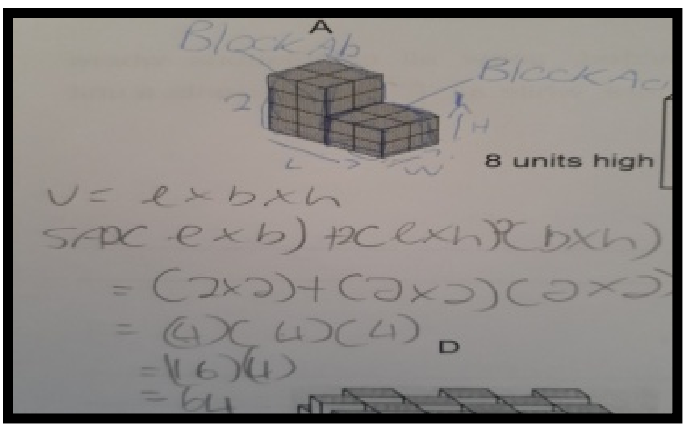
The rest of Question 3 sub-questions displayed an average of 81% incorrect responses and not attempted responses. That was an indication that supports what participants highlighted in previous questions, being comfortable only when given dimensions and using a formula.

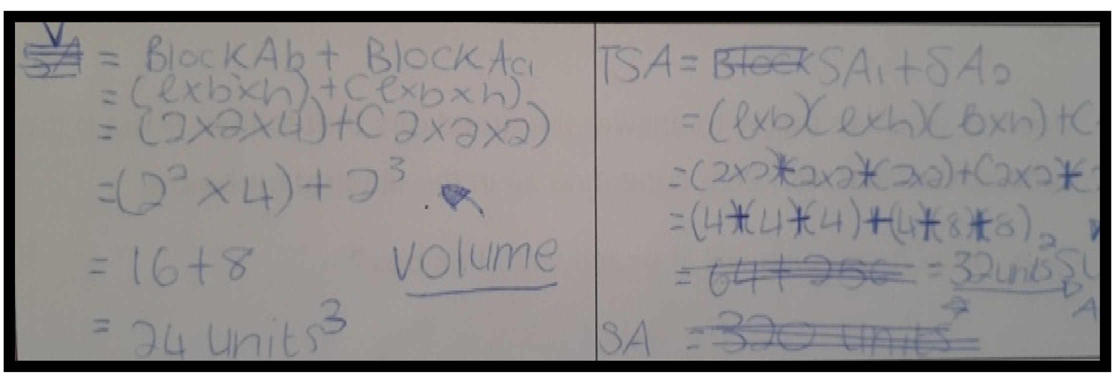
Questions 3a and 3c had 3% correct responses each, while Question 3e had 10% correct responses. The worst attempted question was Question 3d which had no (0%) correct responses, as displayed in Table 4.4. All four of these questions had no dimensions but required participants to use the provided unit cubes and unit squares for finding volume and surface area respectively.

When asked to find the surface area and volume of Diagram 3a, if they existed, 5/29 (17%) of the participants responded positively to the volume but could not get the correct surface area. These were participants A3, B2, C2, D2 and H3. However, they all used some formula, including participants B3, E3 and Q3, who calculated both the surface area and volume incorrectly. Eight other participants (A2, B1, F2, F1, G2, H2, I2 and P3), about 28% of the participants, gave incorrect responses for both surface area and volume without indicating the method used to solve the problem. This gave a total of 13/29 participants (45%) who used a formula to find surface area and volume where a formula would not have been needed. Nine other participants (D3, E1, E2, J2, L3, M3, S3, T3 and Y3), (31%) did not attempt the question. A clear misconception was seen in the work of four participants (14%) (C3, G3, N3 and R3), even before the interviews, who wrote that the shape could not have a volume and surface area because it had no unit measurements.

The most appropriate response would have been that both the volume and surface area of block 3a could be sought by counting blocks and squares respectively. The volume was 24 cubic units, while the surface area was 56 square units. While a few participants got the volume correct, none of them got the surface area right. Most participants were getting a surface area of 64 square units instead. This difference of eight (8) more square units was caused by the inability to arrange cubes in arrays. These learners considered the number of squares on the front and back face of the block to be 16 each, instead of 12. In fact, they did not count the squares but used their established length, breadth and height of four (4) units each and then applied the formula for finding the surface area of a cube, obtaining $4^3 = 64$.

When asked to state whether each of the blocks had surface area and volume, in question 3a, participant A3 wrote the following three different results:





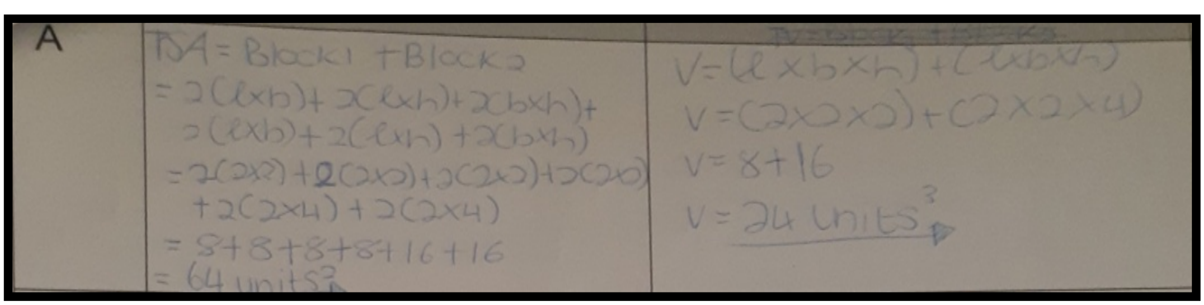


Figure 4.28: Participant A3's three written script responses to Test item 3a

When asked to justify this solution during the interview, it was clear that participant A3 used a formula to get the answer. This is evidenced by the following extract:

Researcher: Can you show me how you obtained the volume and surface area for question 3a.

Participant A3: I used the formula and calculated them all. At first I made a mistake, that is why I asked for another page to redo that section.

While participant A3 cited the use of the formula for surface area, the working indicated some counting. This showed that the learner had some form of conceptual knowledge of surface area, but the early use of the formula came in as a barrier to learning, resulting in a misconception. The formula was used without knowing how it originated.

Researcher: Which formula did you use?

Participant A3: Volume = length x breadth x height.

Researcher: But there is no indication of the length, breadth and height, how did you determine them?

Participant A3: I figured out from the diagram, there are 2 squares here and 4 there (touching edges of the diagram) so that must be the length and height.

In this discussion, participant A3 refers to cubes as squares, thereby treating 3-D objects as 2-D figures, the first class of misconceptions listed on the framework in table 4.3.

Researcher: If you had not used the formula, would you have found the volume and surface area in another way?

Participant A3: What do you mean sir, is there any other way?

Researcher: I want to find out from you.

Participant A3: Maybe just for volume, but I do not think it will give the same answer.

Researcher: How can you find it?

Participant A3: I am not sure. I think it is just by the formula and nothing more.

Researcher: What is volume by the way?

Participant A3: It is the space occupied by a shape.

Researcher: Now when we look at the figure in question 3a, what is occupying this shape?

Participant A3: What do you mean sir? I do not understand.

Researcher: I mean to say, what is the shape composed of? What is taking up its space?

Participant A3: (pointing at the unit cubes making up the shape) It is composed of these squares, sir.

Researcher: By the way, are these squares or something else? Can you define a square then compare with the one you are pointing at?

Participant A3: (thinking) mm....no they are cubes.

Researcher: Ok, and what are these unit cubes taking up?

Participant A3: They are taking up the space of the shapes.

Researcher: If the cubes are taking up the shape space, what one word would you refer them as?

Participant A3: It is volume!

Researcher: And so how can you find the volume of this shape?

Participant A3: I guess by counting these squares, I mean the cubes, but won't it be different, sir, because calculations are different from counting now.

The learner's narrative shifts to misconception type two of the conceptual framework, namely that a shape can have more than one volume or surface area. The researcher then had to probe a little further, for the participant to experience and reach the "Aha – I have found it" conclusion.

Researcher: Let us try it, how many cubes make that block?

Participant A3: (counting) mm... 24 Sir, oh I see, it is the same, but you can't count that way for surface area, you have to multiply.

Researcher: How did you find the surface area?

Participant A3: Since it looks like a cube, it must be l^3, which gives 64.

Researcher: Why do you say so?

Participant A3: Because that is the rule, the formula for surface area of a prism.

Researcher: Is there no other way of finding the surface area, especially since you were not given the dimensions?

Participant A3: There is none sir, only the formula.

Participant A3's response is consistent with the one given on volume above. The learner argues that volume and surface area can be found only by the application of a formula.

Researcher: What is surface area by the way?

Participant A3: It is area of all faces put together.

Researcher: And what is area?

Participant A3: It is space covered or enclosed by a 2-D shape.

Researcher: Which 2-D shapes can you see on figure 3a?

Participant A3: There are squares.

Researcher: How do you find their area?

Participant A3: side x side sir.

Researcher: How do you find the area of one face?

Participant A3: Area of one square times the number of squares in a face.

Researcher: What is the area of 1 square?

Participant A3: 1 x 1, it is 1 square centimetre.

Researcher: But we are not told that they are centimetres, are we?

Participant A3: oh no, it is one unit square.

Researcher: Now how can you find the area of one face without using the formula?

Participant A3: Waal, I get it, by counting all the squares.

Researcher: And how can you find the total surface area of that shape, no formula please?

Participant A3: I think by counting all the squares around.

Researcher: Can you try counting and writing down, then we compare with your calculations

Participant A3: (counts and writes down) Oh, it is 56 square units sir, but when I did with the formula I got 32 for surface area.

Participant A3 got the volume correct, but by default, as the learner applied the formula and inserted measurements that were not given. Luckily the unit squares on each edge corresponded with the assumed dimensions. Participant A3 had procedural knowledge but lacked an understanding of the conceptual knowledge of surface area and volume. The reason for this lack of knowledge could have emanated from the approach in teaching and learning of the concepts – volume and surface area, especially the introduction of the topic by its formula. Misconceptions around this question would then largely be associated with inadequate coverage of the concept.

In response to the same question, participant Q3 wrote the following, which was in the same line of using a formula as participant A3:

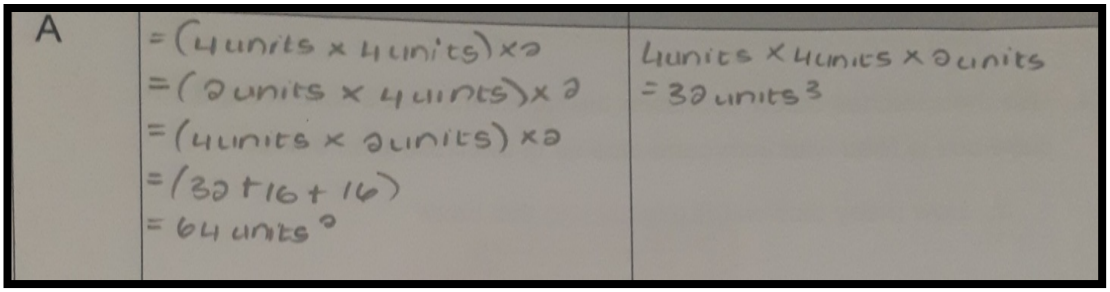


Figure 4.29: Participant Q3's written response

Researcher: I want to find out how you found the surface area of the shape in question 3a.

Participant Q3: I measured the height, length and width then multiplied by 2 to find surface area.

Researcher: Where you trying to use any formula?

Participant Q3: I don't think so. What I know is that you would find how many are in the length, width and height then calculate them using the calculation, maybe length x 2; width x 2 and height x 2 then you add them all up together. Or you say length x width x breadth.

Researcher: I think that is a formula of some sort, what do you say?

Participant Q3: No sir, I never really understand formula, I take long to figure out.

The participant was not even aware that multiplying the length x width x height x 2 for the surface area was actually a use of some formula. Another misconception was noted here, as described in Table 4.3 category 5; confusing volume with surface area – including interchanging their formulae. Here, the participant interchanged the volume formula for the surface area formula, before mixing it up by multiplying by 2.

Researcher: And for volume, how did you find it?

Participant Q3: I measured and calculated the three, there is 4 here (pointing at the edges of the shape), 4 here and 2 there, so I calculated then multiplied by 2 to find the volume.

Researcher: Tell me why you multiplied by 2.

Participant Q3: For surface area, you always square. What am I really saying? I think it will help you find the answer.

Researcher: And for volume?

Participant Q3: It is the way you do it, and for volume you always put a 3 (referring to the cubic unit)

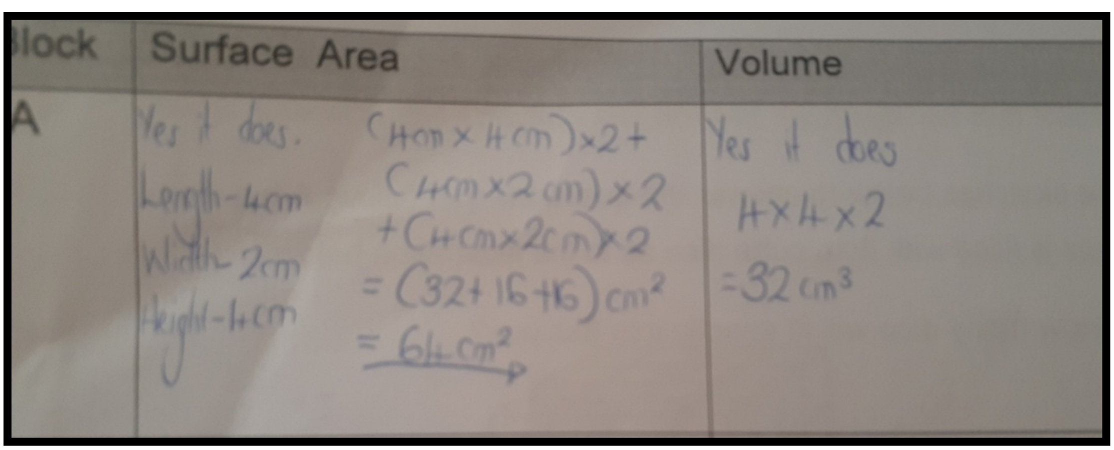
Researcher: Please explain to me this use of 2 in surface area and 3 for volume.

Participant Q3: That is how it is, sir. Either you do that or you have no volume. So after multiplying, volume is 3 and surface area is 2: That's all.

Participant Q3 had some procedural knowledge but lacked conceptual knowledge and understanding of the question. Participant Q3 appears to have learnt by rote memorisation of the procedure for finding the surface area and volume of prisms through the use of the formulae. There is evidence of inadequate mastery of concept as the learner could not explain the existence and use of the cube and square units for volume and surface area respectively.

While participants B3 and E3 were not available for the interview, they both wrote similar responses to participants A3 and Q3, which meant that they probably displayed the same misconceptions. The following two written responses were submitted in support of the above assertion.

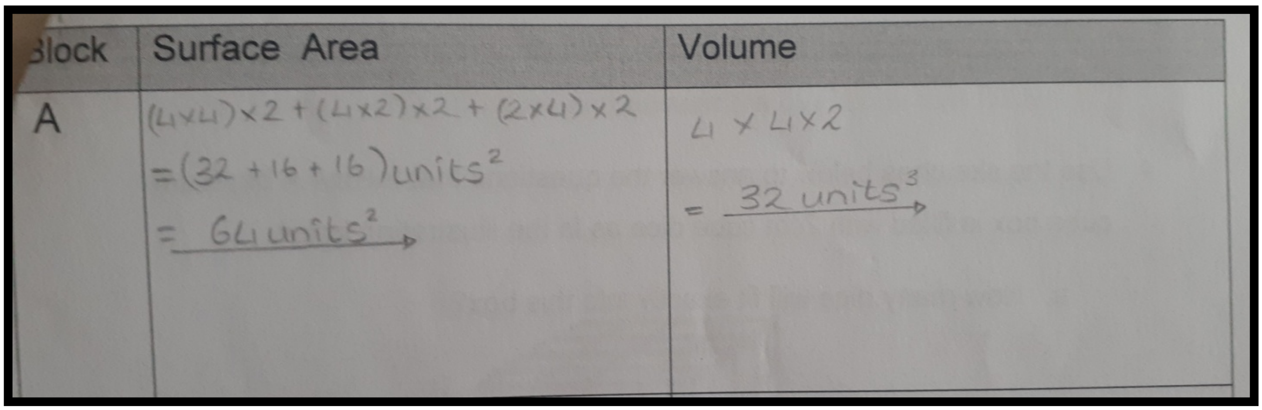
<u>Participant B3 written response to question 3a</u>



Block	Surface Area		Volume
A	Yes it does. Length-4cm Width-2cm Height-4cm	$(4cm \times 4cm) \times 2 + (4cm \times 2cm) \times 2 + (4cm \times 2cm) \times 2$ $= (32 + 16 + 16) cm^2$ $= 64 cm^2$	Yes it does $4 \times 4 \times 2$ $= 32 cm^3$

Figure 4.30: Participant B3's written script response to Test item 3a

Participant E3 written response to question 3a



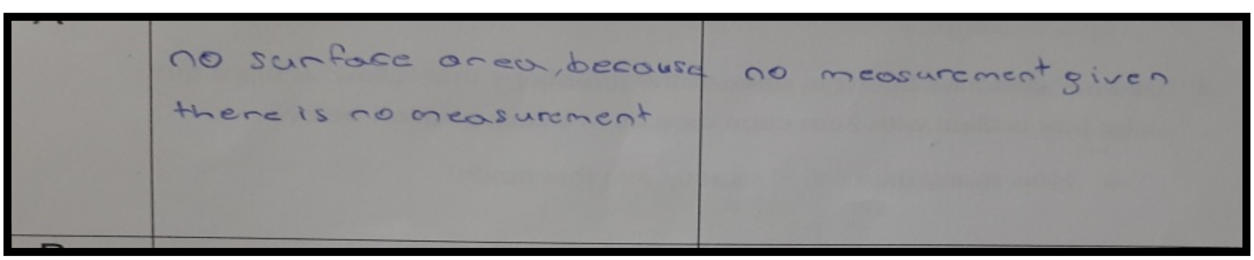
Block	Surface Area	Volume
A	$(4\times4)\times2+(4\times2)\times2+(2\times4)\times2$ $=(32+16+16)$ units^2 $=$ 64 units^2	$4\times4\times2$ $=$ 32 units^3

Figure 4.31: Participant E3's written script response to test item 3a

Although participants B3, E3 and Q3 used the formula for volume, they all got the same incorrect answer due to enumerating arrays in 3-D figures incorrectly, giving them an incorrect height, unlike participant A3 who partitioned the block and got the correct volume.

The other misconception that was the second most prevalent in this question was that lack of measurement, dimensions and units meant the non-existence of surface area and volume. This was evident in participants C3, G3, N3 and R3's written work.

When asked whether shape 3a had volume and surface area, participant C3 wrote the following:



	no surface area, because there is no measurement	no measurement given

Figure 4.32: Participant C3's written script response to test item 3a

In support of this response during the interview, participant C3 said the following:

Researcher: Let's look at question 3a, how can one find the surface area of that shape?

Researcher: Explain why you said this diagram cannot have surface area and volume please?

Participant C3: No measurements are given here sir, so it is obvious, no surface area and no volume.

Researcher: What measurements were you expecting?

Participant C3: The length, breadth and height.

Researcher: If you were given such, how would you have found the volume?

Participant C3: By saying length x breadth x height.

Researcher: And for surface area, how would you have found it.

Participant C3: I would apply the formula again.

Researcher: What is the formula?

Participant C3: It is length x breath + length x height + height x breadth

Researcher: Since the length, breadth and height are not given, how else can you find the surface area?

Participant C3: There cannot be another way, we always need the rule first. Only if there are measurements, like say one cube is 1cm or so, we could count, say this one, plus this one (points to the visible edges), ... so the height would be 4 cm, and this side would be 4 cm the base would be 2 cm, then calculating all to find surface area.

Researcher: What do you call this? (Pointing at the top right corner of the shape)

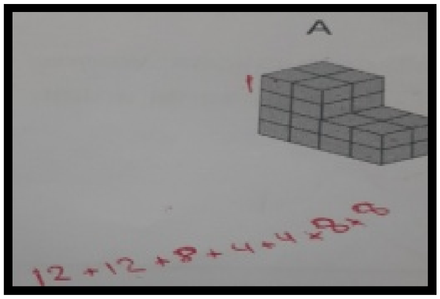


Figure 4.33: Participant C3's written script response to Test item 3a

Participant C3: It is a unit cube

Researcher: What does a unit cube tell you?

Participant C3: I think it shows the volume.

Researcher: And what do you understand by the space enclosed by one of the cube faces.

Participant C3: It is the area of the face of a cube.

Researcher: What is the area of 1 face of this unit cube?

Participant C3: It is 1 square something.

Researcher: How did you get the answer?

Participant C3: I just counted, it is 1 square.

Researcher: What is the volume of one cube?

Participant C3: It is 1 unit cube.

Researcher: How did you find that?

Participant C3: I counted as well.

Researcher: What did you count?

Participant C3: The number of cubes.

Researcher: Now can you tell me how you can find the surface area and volume of this block without using the formula?

Participant C3: For volume we can, but that would be too long, although the simplest method I can use.

Researcher: How would you do that?

Participant C3: By counting all the unit cubes on that shape.

Researcher: Can you count them for us please

Participant C3: This is pretty long (counting) 1, 2, 3.....22, there is 22 of them, even if we can't see some of them.

Learner C3 showed signs of misconception in arranging the cubes in arrays incorrectly. The researcher continued to probe. Another noted misconception was that of counting only the visible cubes.

Researcher: How many can we not see?

Participant C3: Ah....., actually one, only one of them. This behind this one, under this one...16 + 8 = 22 (probes until learner corrected the sum to be 24 instead of 22)

Researcher: So what is the volume of the shape then?

Participant C3: I think it is 24 units

Researcher: Please show me the 24 units

Participant C3: I think 24 unit cubes

Researcher: And surface area, can we find it?

Participant C3: Yes, we can, now I see it.

Researcher: How can we find the surface area?

Participant C3: Since they gave us that this square represents a unit square, we can count them to find the area.

Researcher: Please can you count them now and see.

Participant C3: (counting and writing on the script) 1, 2, 3....40, they are 40.

Researcher: Show us how you got 40

Participant C3: (counts and omits other, the invisible ones. Researcher probes until learner got the correct surface area) it is 48 (omitting the base area of 8 squares), researcher probes until learner realises the omission and eventually gets 56 squares).

We notice that participant C3 initially thought that when there are no measurements, volume and surface area could not be sought. Participant C3 also associated volume and surface area with some rule or formula. The learner had procedural knowledge but inadequate conceptual knowledge, as evidenced by the responses in the above extract.

Another learner who had the same line of thought as participant C3 was participant N3. Participant N3 wrote that the shape could not have volume and surface area because it had no units. Participant N3's written record is seen in the script below:

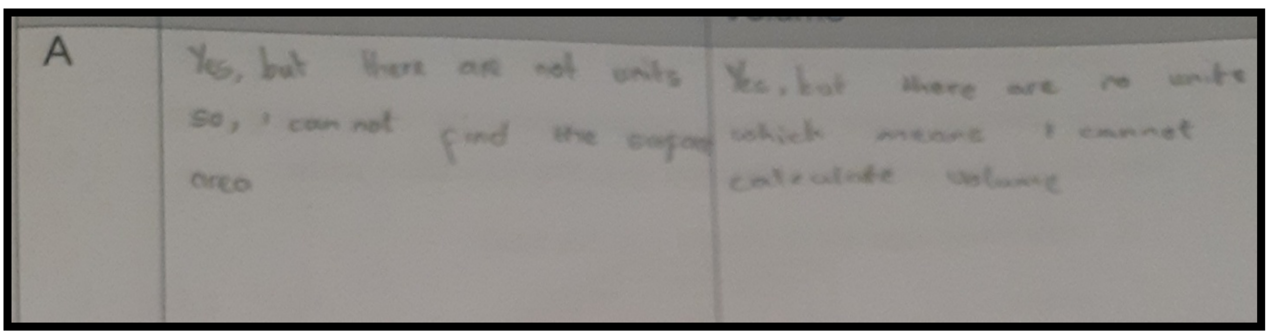


Figure 4.34: Participant N3's written script response to test item 3a

When asked to support this answer during the interview, participant N3 said the following:

Researcher: Question 3a, you wrote yes to having a surface area but then said there are no units cannot find surface area, please verify, which is your conclusion?

Participant N3: No, there are no units, so you cannot find the volume or surface area as well.

Researcher: If you had measurements, how would you have found the surface area?

Participant N3: It would have been easy. For surface area = length x breadth x 2 + length x height x 2 + height x breadth x 2

Researcher: What have you just given me?

Participant N3: It is the formula for finding surface area of any prism.

Researcher: Would you have found the surface area without using the formula?

Participant N3: No, no ways. The formula is very important and it is the first step.

Researcher: How about for volume?

Participant N3: For volume, it would have been length x breadth x height. That is the formula.

Researcher: But then, if I asked you to find the volume and surface area of that figure without using the formulae, how could you do it?

Participant N3: Like I said, the formula is the first to be written, so no measuring units means there is no volume or surface area.

Participant N3 was consistent with the written script that the absence of dimensions negated the surface area and volume's existence. The participant also reiterated that surface area and volume would only be found by applying formulae, calculating using given units.

Another participant who had similar reasoning than participants C3 and N3 was participant R3. However, participant R3 was not available for interviews. Below is a script from participant R3's written response to Question 3a.

<u>Participant R3's written response to question 3a</u>

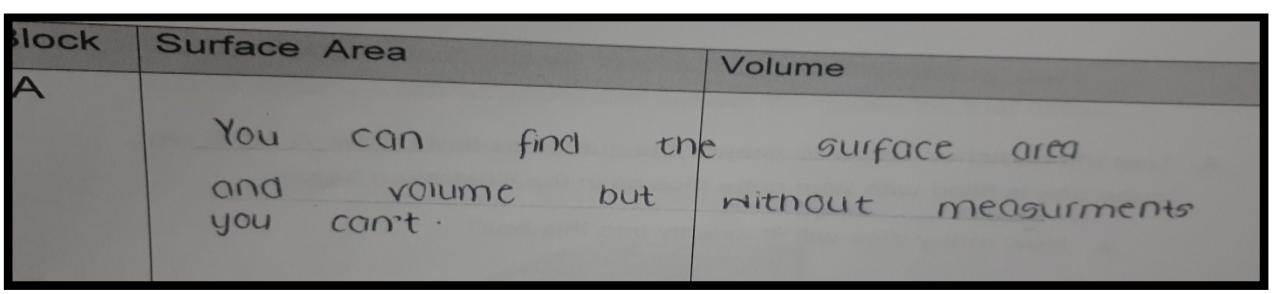


Figure 4.35: Participant R3's written script response to test item 3a

It was evident that the participants were not used to finding surface area and volume of the type of question such as Figure 3a because 83% of them had either incorrect or not attempted answers, as shown in Figure 4.4.

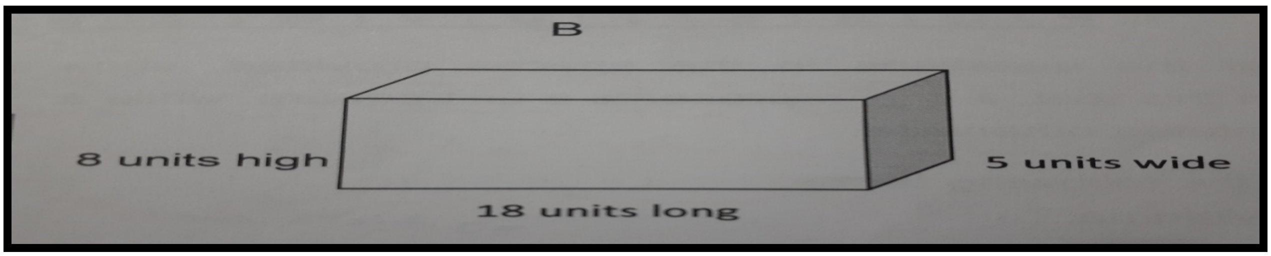
The major misconception in this question was that because the shape was not a complete solid, with a definite length, width and height, its volume and surface area could not be found. A few learners corrected this misconception after probing during the interviews. Some of the participants considered the shape to be incomplete, not level and inconsistent and hence it could not consist of a surface area and volume.

Question 3b involved calculations since it was the only one with marked measurements on all three dimensions. Interestingly, most of the learners got the surface area and volume of this shape correct, despite having difficulties in the rest of the diagrams. This was arguably due to the privileged use of the formulae. The shape was a right rectangular prism. The learners were familiar with this shape as they covered the volume and surface area using formulae in previous grades. All participants who applied the formulae well got the answer right. Evidently, this type of diagram is the most commonly used from the foundation phase, through the intermediate and then the senior phase when dealing with surface area and volume of prisms.

The suitable answer to this question was that since it had three pairs of opposite faces around, the surface area existed and also, as the enclosed shape occupied some space, it had volume. Since there were given dimensions, the participants would then use the relevant formula for surface area and volume to calculate as required, resulting in a surface area of 548 square units against a volume of 720 cubic units.

Of the participants, 7/29 (24%) got both the surface area and volume correct. Four of the 29 (14%) got the volume correct but not the surface area, with one partly correct on the surface area. Another 6/29 (21%) did not attempt the question, while 12/29 (41%) had incorrect responses for this question. A few errors were noticed in participants' tests. Errors noted in the participants' written work were largely around either using a wrong formula or incorrectly applying a chosen formula. Most of the errors were surrounding surface area, where a couple of participants found only the surface area of the visible faces. That error equated to the second misconception of the ones categorised in Table 4.3; counting only the visible units or cubes instead of calculating the entire volume or surface area. This was visible in participants H3, E2, F2, C3 and B2's written scripts.

When asked to show whether Shape 3b had surface area and volume, participant C3 wrote the following:



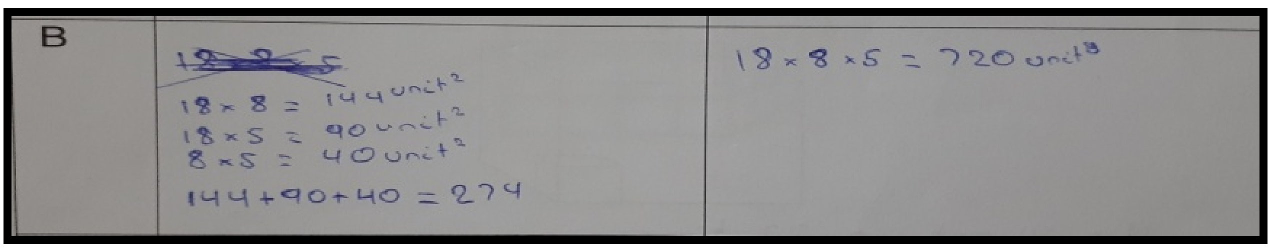


Figure 4.36: Participant C3's written script in response to test item 3b

When asked to motivate this response during the interview, participant C3 responded as shown in the next extract:

Researcher: Looking at question 3B, how did you find the surface area?

Participant C3: I just calculated the surface area for this one (pointing at one visible face of the prism), and this rectangle, and this last one, then I added them together.

Researcher: Look again at the shape and check if there is anything you missed.

Participant C3: No, sir, there is nothing I missed, because this shape has 3 surfaces, it is three dimensional.

Researcher: What name do we give to this shape?

Participant C3: It is a rectangular prism.

Researcher: Can you name any properties of a rectangular prism that you know?

Participant C3: It has 6 faces, 8 vertices and 12 edges.

Researcher: How many faces did you find their area in your answer?

Participant C3: I did 3, sir, now I see I had missed the others.

Researcher: Which ones had you missed?

Participant C3: The hidden one at the back.

Researcher: So are you changing your mind in your answer? If so, what do you want to say is the surface area of this figure?

Participant C3: Yes I am changing, for the back there and the base, oh I forgot to write the back ones (referring to the invisible faces), so the answer should be ... 548.

Researcher: Why do you say so, how did you get the answer?

Participant C3: Because they say for surface area, you have to multiply by 2?

Researcher: Who said so, and why?

Participant C3: That is the formula we were given for calculating surface area, even last year.

<u>Participant Y3's written response to question 3b</u>

Participant C3 also displayed the consequences of concept development through the introduction of the formula by rote learning. Two other participants who had the same written responses, resembling the same line of reasoning as participant C3, were participants F2 and Y3. As these two were not available for interviewing, only their written work is shown below.

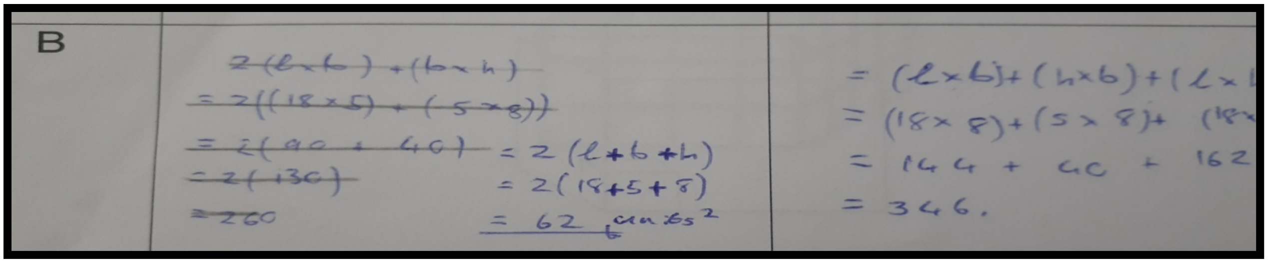


Figure 4.37: Participant Y3's written script in response to Test item 3b

<u>Participant F2's written response to Question 3b</u>

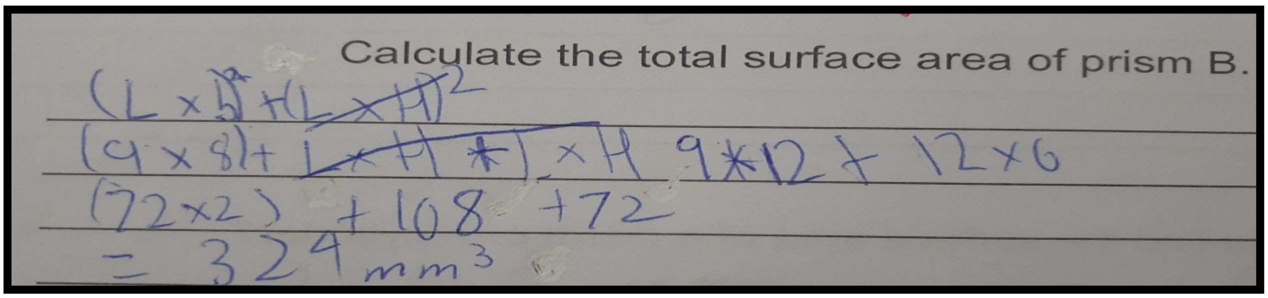


Figure 4.38: Participant F2's written script in response to Test item 3b

It can be noticed from the formula written by Y3 that the participant's thoughts were aligned with C3. Participant F2's calculation also had the same traces of a similar misconception. The following participants indicated an error in selecting and using a wrong formula for calculating the surface area in Question 3b, caused by inadequate mastery of concepts.

Participants A2, G3 and H3 responded to Question 3b's surface area aspect by calculating only the area of one face. This was ascribed to the use of a wrong formula altogether, even though it could have been associated with a lack of mastery of concept as well.

Participant G3 wrote the following as a response to Question 3b:

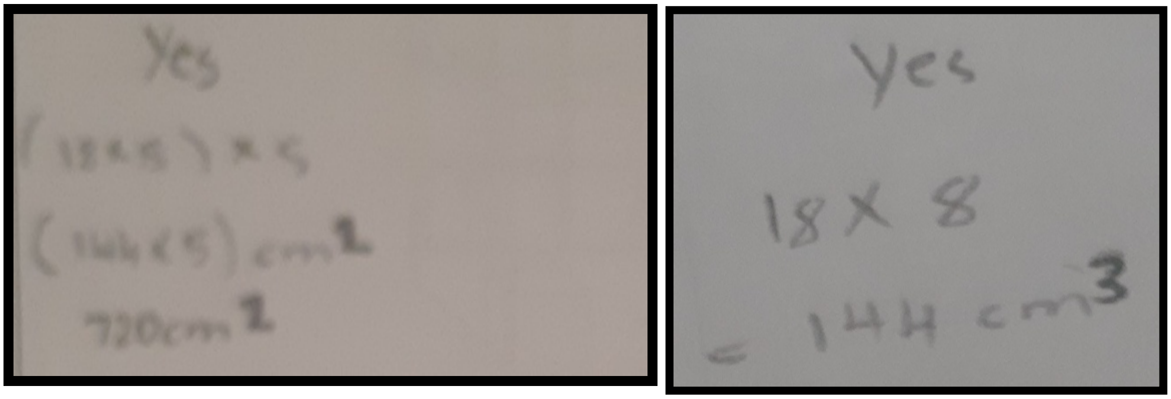


Figure 4.39: Participant G3's written script in response to Test item 3b

Researcher: In 3b, how did you find the surface area and volume?

Participant G3: For surface area, I used the formula, which is height x breadth x width. This one...(showing the length and breadth of one face) x by the other given numeric value (height). 18 x 8 that is base x height, bracket then x 5, the width.

Researcher: How about the volume?

Participant G3: It is length x base, as I remember from grade 6, they said it is side x bottom, so I said 18 x 8 and then with cm to the power of 3.

This discussion showed that the participant had interchanged and confused surface area for volume. In addition to that, participant G3 mixed up the formulae and said one thing but wrote the other. Nevertheless, it was established that the participant had tried to use a poorly grasped formula. This revealed the potential cause of rote learning and memorisation of the formula. The same error was noted in participant H3's written work.

<u>Participant H3's written answer to Question 3b:</u>

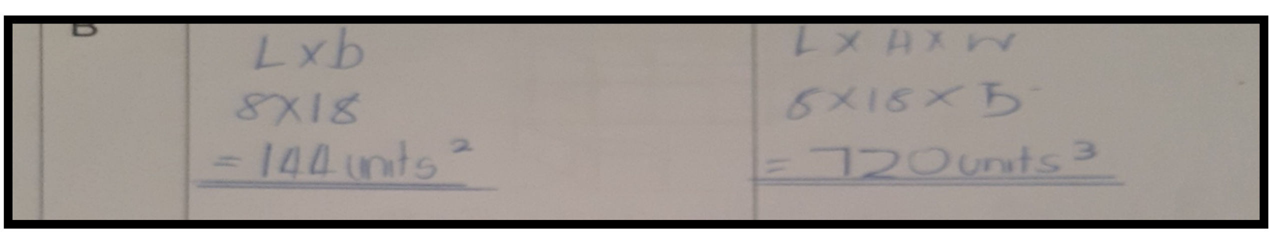


Figure 4.40: Participant H3's written script in response to Test item 3b

Asked to support this answer, participant H3 said the following:

Researcher: 3b for surface area, how did you calculate here?

Participant H3: I said length x breadth. I took this 8 units high and 18 long (pointing to the dimensions of one face) 8 x 18 = 144unit ²

Researcher: When you took length x breadth, how many face areas have you calculated?

Participant H3: It is this one, and it shows us the surface area of the whole shape.

Researcher: What shape is this, by the way?

Participant: It is a rectangular prism.

Researcher: How many faces does it have?

Participant: Let me see, I mean let me count.... they are 6 faces.

Researcher: What does total surface area require?

Participant: It is all areas of the six faces, I see I made a mistake, I should have calculated the other faces as well.

Researcher: And for volume?

Participant H3: I multiplied using the formula...length x breadth x height.

Participant H3 also indicated the other misconception that had not been discussed yet, namely that the area of one 2-D face of a 3-D figure was equivalent to the figure surface area. This could have been the same assumption that participant G3 had when we look at her calculation. Participant A2's written script also showed the same reasoning.

When requested to find the surface area of the rectangular prism in Question 3b, participant A2 wrote 8mm x 8 mm = 64 mm. This is shown in the next script.

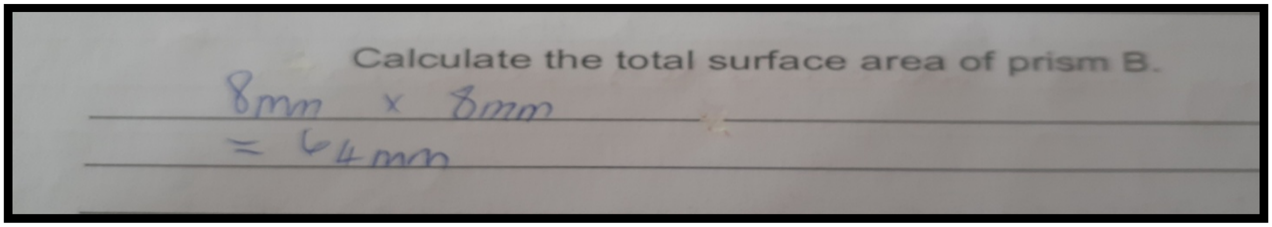


Figure 4.41: Participant A2's written script in response to Test item 3b

When asked to shed more light on this calculation, participant A2 responded as recorded in the following extract.

Researcher: How would you find the volume of shape 3b?

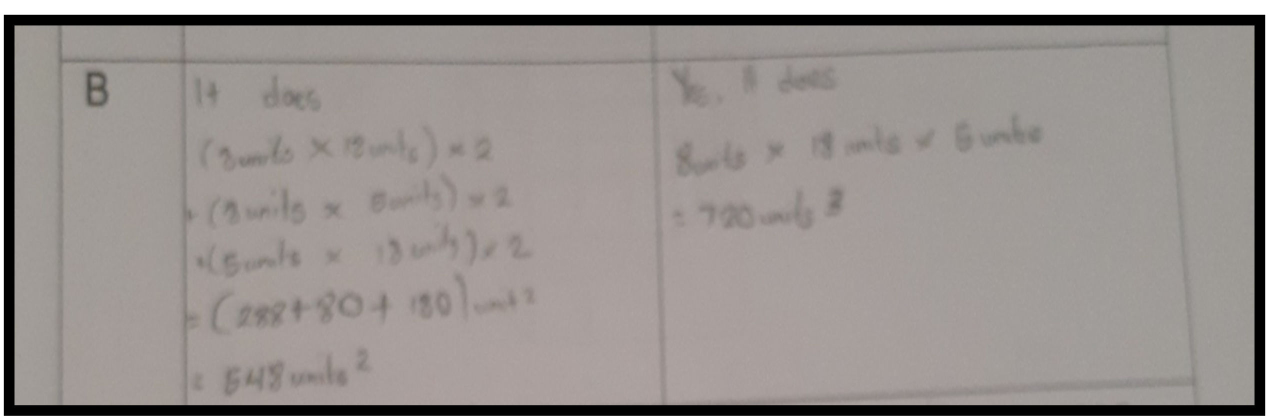
Participant A2: Ah it looks like a rectangular prism because it is not a cube, but, ah I am not sure; you say length x length and then width x width then x height x height

Researcher: How about the surface area?

Participant A2: We say that height divided by width and then the width divided by the length then the length divided by the width

This response indicated that the participant had only guessed the formula and wrote what came to mind first, as what was said did not correlate with what was written. The last two extracts in this question served to indicate that the participants performed well where there were given units, through the correct application of a formula. It was seen and experienced in the work and interview of participants N3 and Q3.

<u>Participant N3's written work for Question 3b</u>



Researcher: Explain how you found the surface area and volume for question 3b.

Participant N3: I said it does have surface area and volume, because they gave us the units and I figured out the volume and surface area.

Researcher: How did you figure out the volume and surface area?

Participant N3: It was easy, sir, I used the formulae that we learnt about calculating surface area and volume.

Researcher: What do these formulae state?

Participant N3: Volume = length x breadth x height and surface area = 2lb + 2lh + 2bh, that's all.

Where there was the use of a formula, learners appeared quite comfortable operating the routine procedure. Still, when one removed the formula, the learners became oblivious, could not even define or describe any procedure for finding the volume or surface area of prisms. The above response indicated memorisation of the formulae. There was no evidence of how the formulae were mastered. However, one could not rule out the rote learning approach. Work from participant Q3 below showed the same method of conceptualisation as in participant N3.

<u>Participant Q3's written script in response to test item 3b</u>

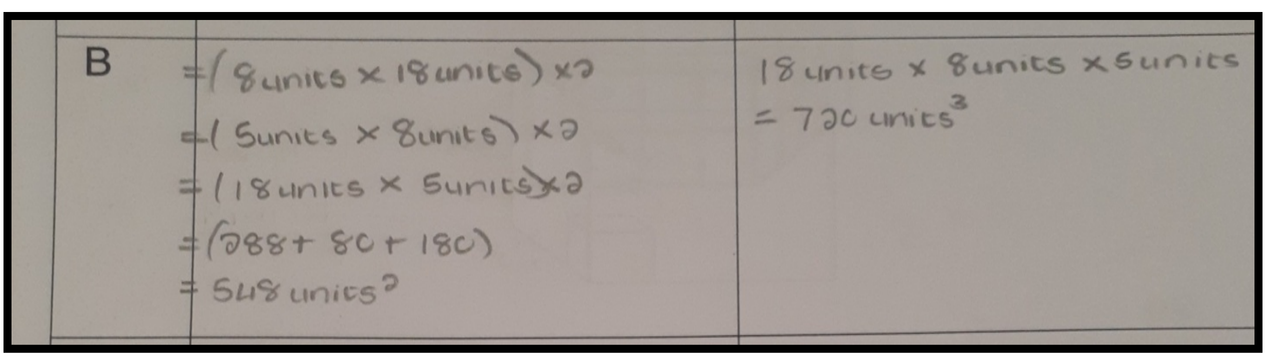


Figure 4.43: Participant Q3's written script in Question 3b

When asked to motivate this answer, participant Q3 responded as evidenced in the following extract*:*

Researcher: In question 3b, how did you find your answers?

Participant Q3: This one was easy, because they gave me all units, it was easy to calculate.

Researcher: How did you calculate?

Participant Q3: Volume = length x breadth x height and surface area = length x breadth x 2 + breadth x height x 2 + height x length x 2

Researcher: And what does all that mean to you?

Participant Q3: That is the formula for calculating volume and the other for calculating surface area.

Researcher: Could you have done it without using the formulae?

Participant Q3: No, no ways. You would just confuse yourself if you do not use a formula. You would also get a different answer.

Participant Q3 was responding in the same manner as the response given in Question 3a that everything in mathematics revolved around a formula. It can be argued that the rest of the other participants who got the correct surface area and volume for Question 3b might have had the same line of thought as participant N3 and Q3. Examples are participants B3, E3 and M3, who, however, were not available for the interviews.

Question 3C

Although this question looks similar to Question 3A, the shape had some narrower parts than the others. The objective was to test learners' conservation of space regarding volume and surface area, thus finding the conceptual understanding of these two concepts.

The acceptable answer to this question was that since the shape had enclosed faces and also occupied some space, it had surface area and volume. The surface area would have been sought by counting the unit squares covering all around the block faces, while the volume was found by counting the unit cubes used to build the block. A surface area of 96 unit squares and 52 unit cubes was the targeted answer. The following is an account of the participants' perceptions of this question.

Table 4.4 indicates that only 3% of the participants got this question correct, with 24% giving partly correct responses, while 42% had incorrect answers, leaving 31% not attempting the question. With only 3% correct responses, 97% of the participants did not do well in this question. It was one of the questions with which the participants were not familiar.

Among the partly correct responses, some participants would get the surface area correct but not the volume, while others would state reasons for the existence of the surface area and volume but fail to provide the respective digits. Most incorrect responses were that the shape neither had surface area nor volume because it was, according to the participants' descriptions, *"not level, inconsistent, irregular, not a full shape, incomplete, not equal, and no given measurements".* This was the major misconception in this question. Most learners felt that as the block was not evenly distributed, it had neither surface area nor volume.

Participants G2, C3 and N3 wrote that the block could not consist of a surface area and volume because the measurements were not given. On the other hand, participants Q3, H2, F1, B1, I2, H3 and G3 advocated that the quantities of surface area and volume could not be obtained due to the shape's irregularity. Similar to Question 3a, the learners expected to find the surface area and volume only on right rectangular prisms.

Asked to find the surface area and volume of Shape 3c if it existed, participant G3 wrote the following script:

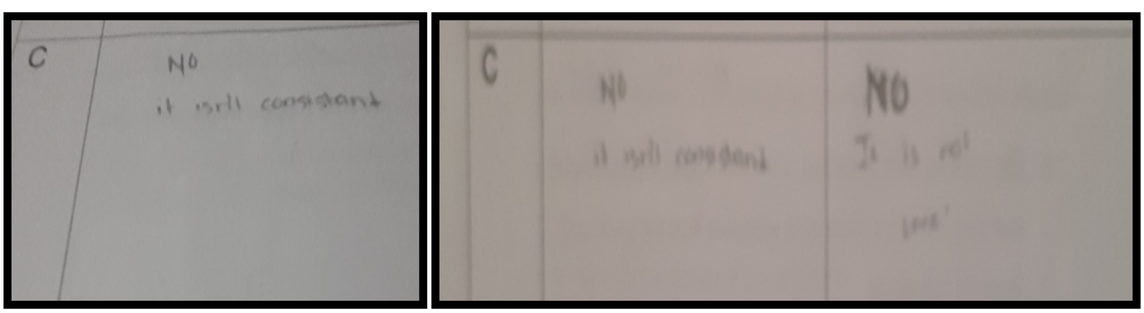


Figure 4.44: Participant G3's written script response to test item 3c

Researcher: Does shape 3c have volume and surface area, if so, how can we find them?

Participant G3: The shape is incomplete and irregular that we cannot work out the volume and surface area.

Researcher: What do you mean by being irregular?

Participant G3: It must have the same length level, same height and same breadth all around, in order to give us surface area and volume.

Researcher: If it was that "regular", how would you have found the surface area and volume?

Participant G3: I would have checked the measurements for length, breadth and height then work out.

Researcher: Which measurements do you see on the shape at the moment?

Participant G3: There are no measurements at all, so you cannot find the volume and surface area.

From this discussion, one can identify the two misconceptions around the participants' reasoning: Firstly, the 'incompleteness/irregularity' supposition and secondly the absence of units of measurement, which are both evident and consistent with the previous questions. Similar reasoning is found in Participant H3's work, who wrote the following answer for the same question:

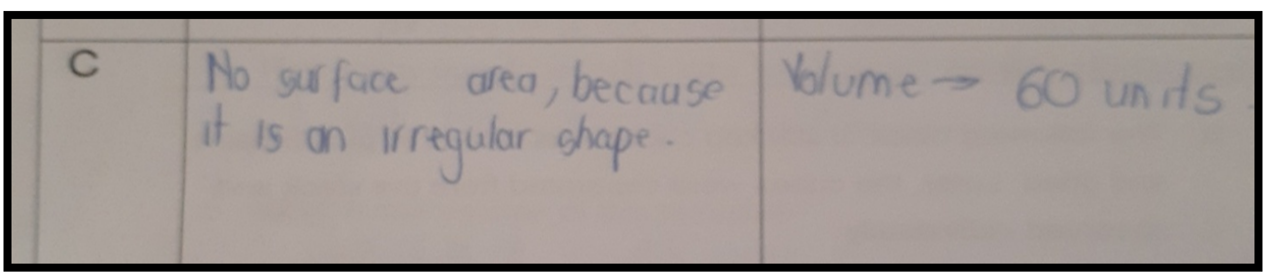


Figure 4.45: Participant H3's written script response to test item 3c

During the interview, participant H3 reasoned as shown in the next extract in support of this solution:

Researcher: For questions 3c take us through how you got volume when you said surface area could not be found.

Participant H3: I think for volume, I just guessed. I don't remember working it out. The shape is irregular.

Researcher: What do you mean by the shape is irregular?

Participant H3: It does not have the same length, breadth and height, very confusing. How can the height keep changing like that!

Researcher: If the shape had the same length, breadth and height, would you have fond the surface area and volume?

Participant H3: Yes, it would have been possible.

Researcher: How would you have found the surface area and volume, if the shape was that "regular"?

Participant H3: I would check for length, breadth and height then multiply.

Researcher: How would you multiply?

Participant H3: Using the formula

Researcher: Try to complete the shape now, make it "regular", then find the surface area and volume.

Participant H3: Mm..... no, it is not possible. There are no units here, can I measure using a ruler?

At this point, it was clear that the participant was going back to the previously discussed scenario, in the same cognition as participant G3 above. Participant N3 also had the same sentiments as participants G3 and H3, shown in the next script and interview extract.

<u>Participant N3 wrote this answer to question 3c</u>

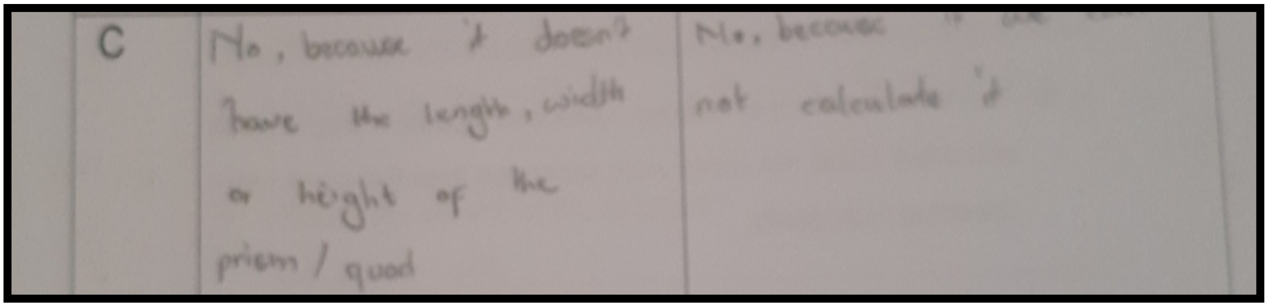


Figure 4.46: Participant N3's written script response to test item 3c

This is how participant N3 supported the written answer during the interview:

Researcher: Explain what you meant for Question 3c, why can you not find the surface area and volume?

Participant N3: I said no surface area and volume because they don't give us the length, width and height. It does not have that height that much. We cannot calculate the volume because there is a space here (pointing at one of the edges), so it is incomplete.

Researcher: What did you expect the shape to look like, in order for you to find the volume and surface area?

Participant N3: It should definitely have been closed here (pointing and the "uneven" face).

Researcher: Please close it and see if we can now find the surface area and volume?

Participant N3: Yes, I can close it, but still we have a problem, there are no measurements.

Researcher: What measurements now?

Participant N3: There is no length, no width and no height. These things must be given.

The same reasoning and argument as participants G3 and H3 above was seen here. All the interviewed learners (100%) in this question believed that the shape was inconsistent, irregular or had gaps; hence no surface area and volume could be found. It took some probing to have some of them revisit and recognise this misconception, while others stuck to their initial suppositions. Figure 4.4 show that 73% of all the participants who took part in the test got the answer to this question wrong, as either incorrect or not attempted. One can safely deduce that the learners had probably not been exposed to this type of question in their learning of surface area and volume. Another question that tested the participants' cognition of surface area and volume was Item 3d, which follows next.

Question 3d

This question was a consolidation of testing the conservation of space through volume and surface area.

Similar to the previous questions in this section, but in this case, there were gaps in between the 3-D block. The aim was to see whether learners could still find the surface area by counting the square units all around the shape and then the volume by counting the cubic units used when building the block. It also focused on how Grade 8 learners would respond to spaces within the shape, as to whether they formed part of volume or not. Figure 4.4 indicates that none of the participants got all the parts correct in this question, with 0% correct responses. Ten per cent (10%) of all participants' responses were partly correct, while 62% of participants had incorrect responses, and another 28% did not attempt responding to this question at all. Combining incorrect responses and not attempted ones showed that 90% of the participants were completely ignorant of how to deal with Question 3d.

The appropriate response to Question 3d was that as the structure was composed of several faces and built of cube blocks, it had surface area and volume. The surface area could have been found by counting all the square units around each face. It was noted that each face had 36 unit squares, giving a total surface area of 216 square units. The volume would have been obtained by counting all the unit cubes used in building the structure, resulting in the answer of 108 cubic units.

Of the partly correct responses, only 3%, one participant (H3) had the correct volume but not surface area. Two other participants, J2 and H2, partly gave reasons for the existence of volume. Participants J2, G2, and C2 also tried using some formula to find the surface area and volume but got stuck in the process. This was the first misconception associated with this question. Among the incorrect responses, 4/29 (14%) of the participants wrote that Figure 3d did not have a surface area and volume because it had a lot of spaces, in addition to not having been given measurements (length, width, height). Ten of the 29 participants' written work (34%) stated that Figure 3d did not have any surface area and volume as it "had spaces, not regular, not consistent lines, not a fixed shape". That was the most prevalent misconception of this question.

When asked to deduce whether Figure 3d had surface area and volume, participant A3 wrote that one could not find the surface area and volume as one could not identify the distinct length, width and height. This is shown in the script below:

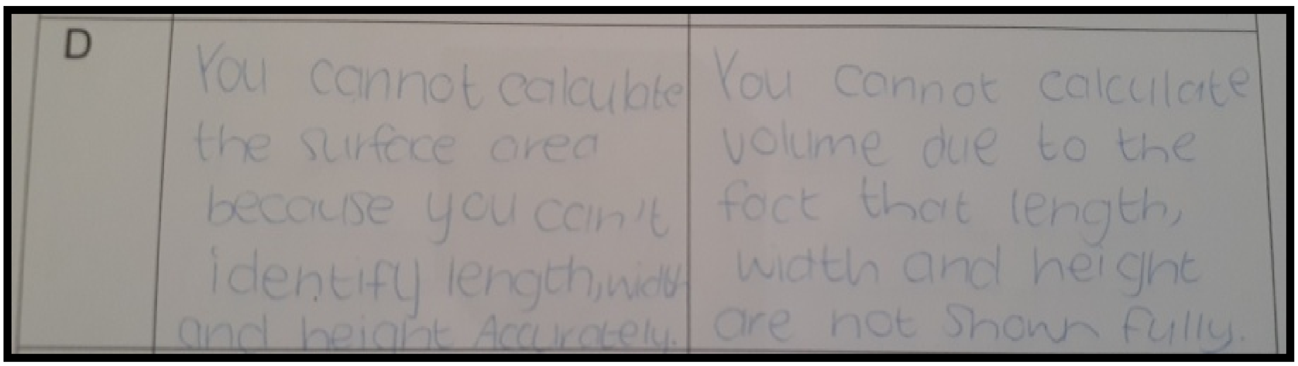


Figure 4.47: Participant A3's written script response to Test item 3d

During the interview, participant A3 supported the written answer, arguing in the manner displayed in the following extract:

Researcher: In question 3d, you said one can't find the surface area and volume, how did you derive that?

Participant A3: I saw that the shape is not a definite shape, there are gaps in between.

Researcher: How does a definite shape look like?

Participant A3: It must have a proper length, width and height, without all these gaps.

Researcher: Assuming that the shape had no gaps, would you find the surface area and volume?

Participant A3: It would also depend on whether the shape has the length, width and height.

Researcher: If the shape had a distinct length, width and height were, how would you have found the surface area and volume?

Participant A3: I would have multiplied length x width x height for volume.

Researcher: And for surface area?

Participant A3: There is a formula for surface area as well, sir.

Researcher: What is it?

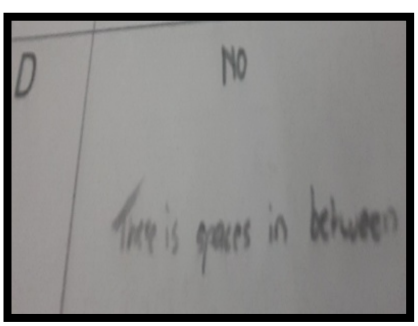
Participant A3: It is 2lw + 2lb +2bw

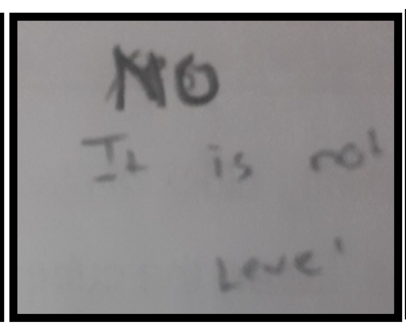
Researcher: Now think of another way of finding the surface area and volume of the shape, besides using the formula, can you?

Participant A3: No sir, no ways, there is no length, width and volume, so you cannot find the surface area and volume in this question.

This participant was adamant that the volume could not be sought as long as there were no measurements. Participant A3's comments in Question 3d were consistent with what was said in Question 2 above about using a formula for any mathematical process. Participant G3 also had the same sentiments as participant A3, as shown in the following discussion.

Responding to whether the figure in Question 3d had surface area and volume, participant G3 wrote that both quantities could not be found as the shape was "irregular". This is evident in the next script.





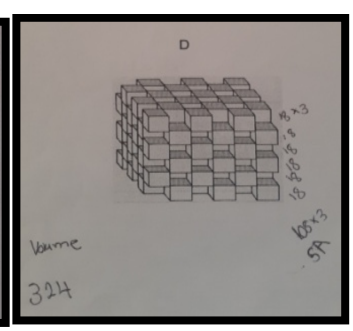


Figure 4.48: Participant G3's written script response to test item 3d

During the post-test interview, participant G3 gave the following explanation in defence of the assertion:

Researcher: How did you solve question 3d?

Participant G3: I said there are spaces in between, so no surface area. For volume, I also said no volume because it is not level.

Researcher: What do you mean by the shape not being level?

Participant G3: There is no proper length, width and height, because of these spaces.

Researcher: When would the shape have had a proper length?

Participant G3: If all the gaps are filled and there length is measured?

Researcher: If you had all that, would the shape have surface area and volume?

Participant G3: Yes, it would have both?

Researcher: How would you determine the surface area and volume in that case?

Participant G3: For surface area, I would use the rule length x width x height and for volume, I would multiply x 3.

It was clear that the learner expected the shape to be solid and have dimensions in specific units to use some formula to calculate the surface area and volume. Participant G3 also confused the formula for finding the volume of a prism with the surface area before giving an incorrect formula for volume. These two misconceptions have been discussed at length in the previous questions. Two other participants who appeared to have the same reasoning and displayed the same misconceptions as participants A3 and G3 for Question 3d were participants B3 and C3. During the interview, participant C3 gave the same response as was given for Question 3a. Participant B3 was not available for the interview. The following written scripts from participant B3 and C3 are evidence of this supposed misconception.

<u>Participant B3 written answer to question 3d</u>

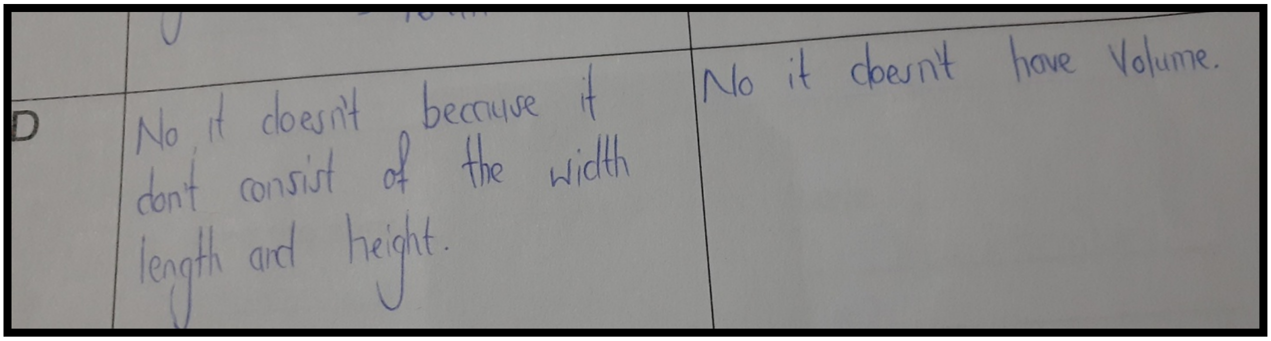


Figure 4.49: Participant B3's written script response to Test item 3d

<u>Participant C3 written response to question 3d</u>

Figure 4.50: Participant C3's written script response to test item 3d

Notice that participant C3 meant that no measurement was given, as to say no length, width and height, the same response given by the other four participants.

The most prevalent misconception in this question was that there were too many spaces in the shape. Hence, according to the participants, neither surface area nor volume could be determined. Most learners continued to regard the shape as "incomplete, irregular and inconsistent" as said in the previous shapes. Disturbingly, volumes of irregular shapes can be found by the use of liquid displacement. If the issue was of irregularity, then other methods of finding the volume and even surface area could also have been used. For example, learners could have suggested covering the shape with a piece of paper or cloth, then finding the cover's surface area. The responses indicated that the participants lacked conceptual comprehension of surface area and volume, which stemmed from the inability to define and describe the concepts in question.

<u>Question 3e</u>

The diagram consisted of cube blocks that were piled to form a pyramid structure. Although this diagram was similar to the ones in part 3a–3d, it was included in the test to determine whether the learners could find its surface area and volume. It was hoped that the learners would not consider the shape incomplete as they did on the previous diagrams, considering that there were no such gaps as those found in Part 3d. Shockingly, the results of the participants' responses in Table 4.4 indicated that 76% of the participants had either not attempted a response or provided incorrect answers for this question. Fourteen per cent of the participants had partly correct answers, while only 10% achieved correct responses.

Question 3e was targeted at enumerating cubes in arrays and testing the conservation of area and volume. The learners were expected to find the volume by counting unit cubes in each layer of square numbers from 16; 9; 4 and 1. The majority of the learners continued to state that the shape was irregular and inconsistent, suggesting that no volume or surface area could be found.

The most appropriate answer for this question was that the shape had six independent faces; hence it definitely had a surface area, obtained by counting the squares around the block, a total of 72 square units. The block was composed of 30 unit cubes, which happened to be its volume, by counting. Four of the 29 (14%) participants (A2, C2, G2, H2) who got partly correct answers for this question did so by describing and justifying that the diagram had surface area and volume. Only one participant (F1) could calculate the volume correctly while slightly missing the surface area. Seven of the 29 participants (24%) of the incorrect responses stated that the shape in Question 3e could not have surface area and volume, citing the same reasons as in previous questions such as "not regular, inconsistent, not proportional and no given measurements". Only one learner, participant Q3, tried to use the formulae for surface area and volume of prism but applied them incorrectly due to the use of wrong values for length, width and height.

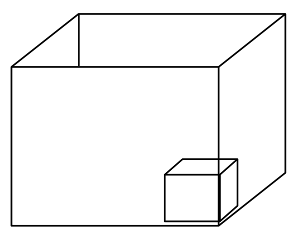
Even this type of a diagram was extremely difficult for the learners to figure out the surface area and volume. The learners did not seem to be able to use what they already learnt in an application in new situations. The only situation that the learners were comfortable with was the one where a definite length, width and height were evident;

then only would the learners find surface area and volume by substituting the dimensions into the formulae. This showed that most participants who took part in this study could not think outside the box but were only one-dimensional-thought dominant. The written scripts and interview extracts for this question have been excluded from this record as they were identical to the previous questions and displayed the same misconceptions.

Question 4a

How many 2 cm cube dice can fit in the given 6 cm cube box?

Use the sketches below to answer the questions that follow: A big 6 cm cube box is filled with 2cm cube dice, as in the illustration below.
How many dice will fit exactly into this box?



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This question was designed to test the learners' manipulation of arrays arranged in a block. It also tested the learners' methods of comparing two cubes of different volumes and giving the relationships between them. Besides comparing how many smaller cubes (dice) would fit on each dimension of the bigger cube (box) and then establishing the number of cubes to fit in the box, learners could also have calculated the two volumes and then find out how many times the larger cube (box) was bigger than the smaller (dice). By arranging blocks in arrays, the learners were expected to have noticed that there were three layers of nine cubes (dice) that could have been fit into the large box, thereby giving us 27 dice altogether.

Table 4.4 shows the distribution of responses by the 29 participants in this study. Thirty four per cent of the participants had correct responses, while 0% was partly correct, 59% incorrect, and 7% not attempted.

Of the incorrect responses, two participants (7%), wrote that the number of small cubes that would fit into the large one was 9. This number actually represents the cube that one could see on one face or a single layer. It seemed these participants (F1 and M3), had only focused on the 2-D squares that they drew on the diagram on one face, thereby treating cubes as 2-dimensional rather than 3-dimensional, a misconception recorded in the first column of Table 4.3.

Another 7% of the participants wrote that the box would hold 18 dice. It seemed though that these participants' (E2 and R3) thoughts might have developed from the previous pairs by considering square faces seen on two faces of the box, probably the side view and the top view. There were other individual responses such as 72, 6, 30, 54, and 24.

Three participants, 10%, wrote that the larger cube (box) would only hold three of the smaller cubes (dice). These three (H2, C2, B2) most probably checked and found that each edge of the box would fit 3-dice-lengths. Lastly, five (17%) of the participants (G2, D2, B3, S3, F2), wrote that the box would fit 12 dice. Only the interviews would help determine out how this answer was derived at by most of these participants, since no calculation steps were indicated. One of these participants (G2) obtained the answer by just multiplying the given dimensions without thinking about the concepts.

When asked how many small cubes (dice) would fit in the given lager cube (box), in Question 4a, participant N3 wrote the answer 54, as shown in the script below.

<u>Participant N3 written response to Question 4a</u>

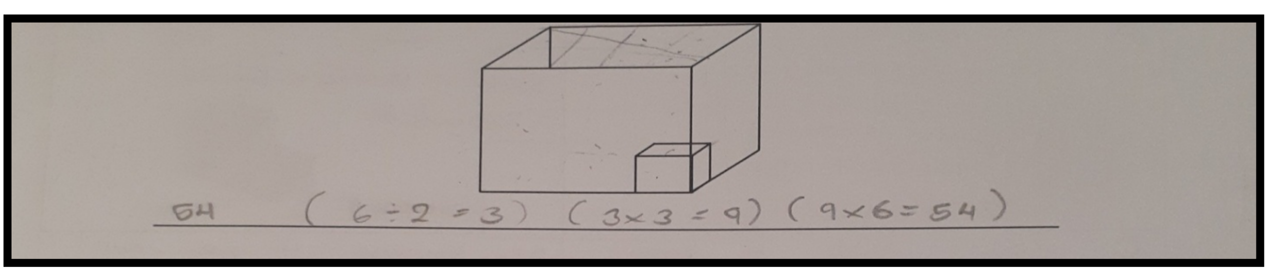


Figure 4.51: Participant N3's written script response to test item 4a

During the post-test interview, participant N3 supported the written response in the following manner:

Researcher: In Question 4a, how did you come to 54 dice filling the box?

Participant N3: I see there we can fit 3 on one side (2-D face), and 3 along the height, that is 9 cubes on one face and there are 6 faces so 9 x 6 = 54.

Researcher: How did you get the 9 by the way?

Participant N3: That is the 3 length x 3 height will give 9 in one face.

Researcher: But you first wrote 6 ÷ 2 = 3, what were you trying to achieve?

Participant N3: I wanted to see how many small cubes can fit on one side of the box.

Researcher: If you had not worked out this way, is there another way you can find the answer?

Participant N3: I think that is the only, the use of the formula.

Researcher: What formula did you use?

Participant N3: To divide and multiply, then multiply again.

Researcher: And that gives you the formula for which quantity?

Participant N3: The number of things fitting in another.

Researcher: You said there are 9 cubes you see on one face of the box. Please show me on the picture how these 9 are arranged.

Participant N3: It is this one and this (counting the 2 cm squares on one face of 6cm square)....7, 8 and this one 9.

Researcher: Please show me the 6 faces that you said there are.

Participant N3: Here I can see 1, 2, 3 but I know there are 6 altogether, I can't show you now.

In counting the squares instead of cubes, participant N3 displayed the misconception of treating 3-D figures as 2-D ones. While participant N3 knew one of the properties of a cube (having 6 equal faces), the participant could not enumerate cubes arranged in arrays, another displayed misconception in the same question. Thirdly, participant N3 displayed the misconception of rendering a formula where not necessary, and used it incorrectly.

Participant G3 also wrote the same answer as participant N3. The evidence is provided below.

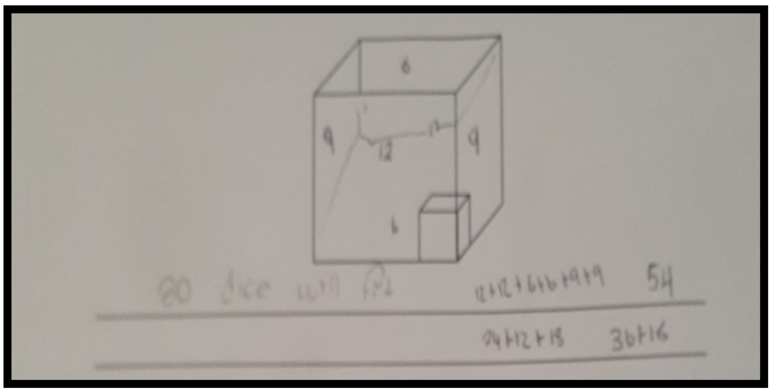


Figure 4.52: Participant G3's written script response to Test item 4a

When asked to support this answer during the interview, participant G3 argued as follows:

Researcher: Please explain how you got your answer for question 4a.

Participant G3: There are 6 here and the back (pointing at the visible edges of the bottom layer), 9 in two sides and 12 on the other 2 so it must be 54. I was wrong the first time when I said 30, it does not make sense.

Researcher: Show me how you get to the result 54 dice.

Participant G3: We have 8 + 8 + 6 + 6 + 9 + 9

Researcher: How did you come up with the 8, 6 and 9s?

Participant G3: Sir, for this side (referring to the front face), we can have 8, and the back. The top and the bottom both have 6, then these two (right and left side elevations) will have 9 each.

Researcher: Please check your addition again.

Participant G3: Oh yes, it is 36 + 16 = 54

Apart from the error in addition, participant G3 seemed to have counted squares for surface area instead of counting blocks for volume, a misconception of interchanging the two concepts as listed in Table 4.3. There were also signs of the Type 3 misconception in enumerating cubes in 3-D arrays incorrectly, as found in participant N3's work. This could have been caused by the insufficient mastery of the definitions of the concepts' volume and surface area.

Another learner who was interviewed in this question was participant Q3. Although participant Q3 had initially not written the same answer as participants N3 and G3, the post-test interview showed that they all had the same line of thinking, regarding treating 3-D objects as 2-D figures.

Participant Q3 wrote that 24 dice would fill the box, as shown in the written script that follows.

<u>Participant Q3's written answer to question 4a:</u>

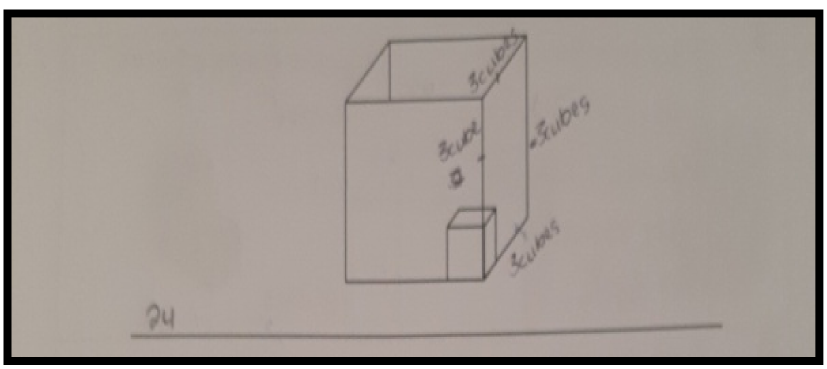


Figure 4.53: Participant Q3's written script response to Test item 4a

Researcher: In Question 4, may you please explain how you got 24 dice filling the given box.

Participant Q3: In a cube sir, there are 6 faces, so I checked on one side, we have 3 here, 3 here and three there (writing on the four edges of one visible face) then I added that together.

Researcher: Show me what you added please.

Participant Q3: I said 12 x 6 ... then 72 - 12 that I already calculated is 60 – 6 = 54

Researcher: So, what is your answer?

Participant Q3: It is 54. That is the correct one. I wrote 24 by mistake, it is 54.

Researcher: Explain why you multiplied the six faces by 12

Participant Q3: Because of these lines on the smaller cube (referring to the edges of the die).

Researcher: And why did you subtract 12 from 72?

Participant Q3: Because I had already used the 12 at first, I had to remove it.

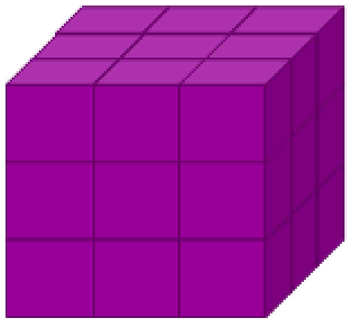
Researcher: Then the last 6 you subtracted?

Participant Q3: Yes, I had also used it before. It had to be subtracted, so that it is not double.

This discussion indicated that participant Q3 had multiple misconceptions. The participant enumerated cubes around the edges instead of by faces, layers, columns or rows. In this case, participant Q3 found the perimeter of one face and then multiplied by six (6) faces of the cube. Some edges were repeated in the process, however, the participant Q3 thought it was only one face's edges that had been repeated, hence subtracted one set of 12 from 72. From the calculated value of 60, participant Q3 then subtracted six (6) faces, eventually settling for 54. It was evident that the learner had interchanged concepts, as the answer was different from the original 24, yet failed to explain the difference.

<u>Question 4b</u>

> A large cube of 27 unit cubes is dipped in paint, dried, and the unit cubes dismantled; establish how many faces of the unit cubes will be painted…0 faces, 1-face, 2-faces, 3-faces, 4-faces?
>
> The following block of stacked cubes was dipped in purple paint and dried. Later, the cubes were separated from the stack and observed individually.



How many cubes do you think will have the following features?

I.	Four faces painted purple	____________________
II.	Three faces painted purple	____________________
III.	Two faces painted purple	____________________
IV.	One face painted purple	____________________
V.	No face painted purple	____________________

This part of the question was largely to test how learners would enumerate cubes in arrays as well as describing the features or properties of enclosed cubes. It was assumed that learners could establish the number of visible faces on the smaller cubes that would be painted if a cube of 27 blocks was dipped in paint and detached. The number of painted faces required for each part represented each small cube's visible faces as seen from the large cube. Table 4.4 indicates that only 2/29 participants gave accurate responses to this question; that is just 7% of all the participants. The other 24% had partly correct answers, while 62% gave incorrect responses, with another 7% not having attempted responses.

The correct responses would have been that 0-cubes had four painted faces; 8 cubes had three painted faces; 12 small cubes had two painted faces; 6 cubes had one painted face each, while 1 cube would have no painted face at all.

Ten of the 29 participants (34%) figured out that none of the small cubes would have four painted faces, and their argument was based on the diagram being 3-D; hence only a maximum of three (3) painted faces would be realised. Five of the 29 participants

(17%) also established that eight (8) cubes by the corners would be painted on three faces each. Only 2/29 participants (7%) identified the number of cubes that would be painted on two faces while 4/29 participants (14%) correctly established the number of cubes painted on one face. Six of the 29 participants (21%) stated the correct number of cubes that would be without any painted face from the stack. The difficulties faced indicated the inability to enumerate cubes in 3-D arrays correctly, which was recorded as a Type 3 misconception in learning volume and surface area, shown in Figure 4.3.

When asked to identify the number of cubes that had 0, 1, 2, 3 or 4 painted faces each, participant G3 wrote the following:

<u>Participant G3 written answer to question 4b</u>

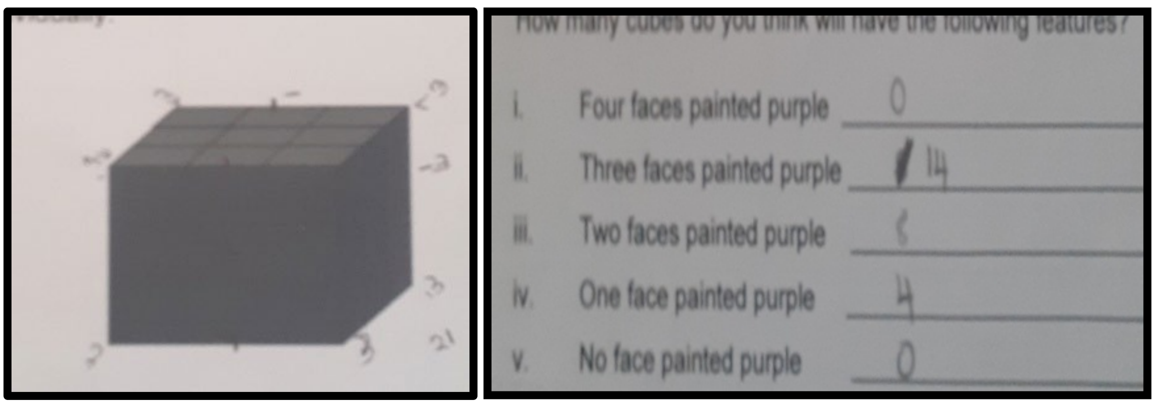


Figure 4.54: Participant G3's written script response to Test item 4b

In support of the above answers, participant G3 argued as follows:

Researcher: Take us through question 4b, how did you determine the number of painted faces on the detached unit cubes.

Participant G3: Part (i) - I said 0 because every cube there has three faces, so none can have 4 painted faces. Part (ii) - I said ... let me see, I was counting the faces of the cubes, this cube 1, 2, 3 for this other cube 1, 2, 3 (counting the edges of the cubes by the corner).

Researcher: The ones with two painted faces?

Participant G3: It is these (counting the middle cube on each edge of the front face x 2 for the back one), then one painted face is this one (pointing at the centre cube of every 9 square face but missing some of the invisible, only added one invisible)

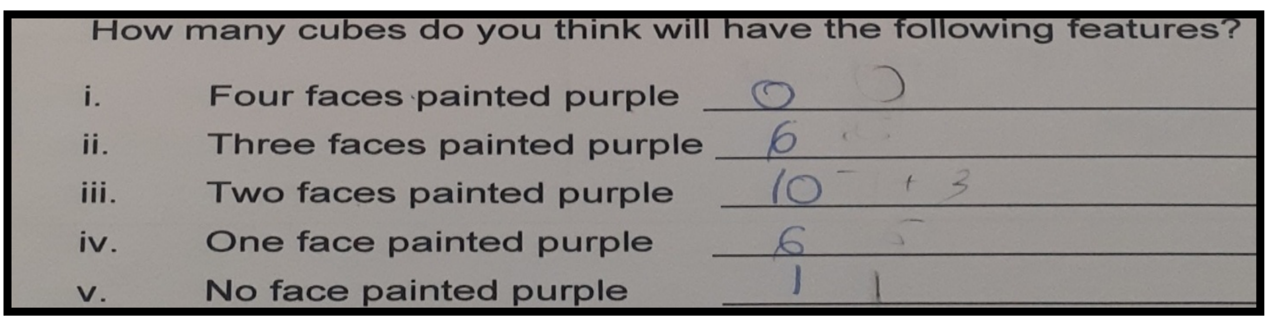
Researcher: How about cubes with no painted face?

Participant G3: I said none because all these cubes are outside and all painted, there is none enclosed underneath, all are painted.

Participant G3 only noted one centre cube on three (3) visible faces and added only one invisible one for part (vi). The learner did not recognise that there is another invisible one directly opposite for every visible face drawn in 3-D isometric and oblique projections. G3 also failed to deduce that there was one enclosed cube in all directions, at the centre of the array, which remains unpainted. Participant G3 displayed the misconceptions of incorrectly enumerating cubes in arrays and treating 3-D objects as 2-D figures. The same reasoning was observed in participant H3's work.

In the same question, participant H3 wrote the following:

<u>Participant H3's written work</u>


How many cubes do you think will have the following features?

i. Four faces painted purple 0

ii. Three faces painted purple 6

iii. Two faces painted purple 10 + 3

iv. One face painted purple 6

v. No face painted purple 1

Figure 4.55: Participant H3's written script response to Test item 4b

Participant H3 motivated the written response as follows:

Researcher: Question 4, how did you tell that 6 cubes will have 3 painted faces?

Participant H3: I imagined pulling all the unit cubes out and taking the corner cubes, give us 6.

Researcher: Can you count them now and verify.

Participant H3: 1, 2,...6, 7, 8.(counting the corners to get 8)

Researcher: What happened, can you explain the difference?

Participant H3: I think when I wrote this, I forget to count the hidden ones.

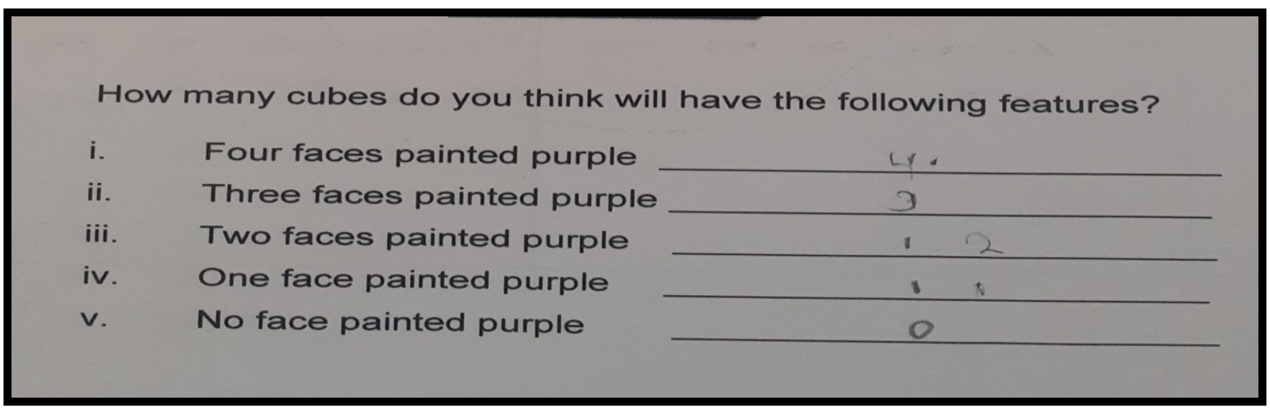
Researcher: Which are these 10 you said will have 2 painted faces?

Participant H3: It is these ones, let me count ...(counts every middle cube's visible face, has some counted twice).

Participant H3 displayed the same misconceptions as participant G3 by taking some 3-D objects as 2-D figures and incorrectly enumerating some cubes in this array, irrespective of having different answers in some parts of this question.

Participant N3 wrote the following response to question 4b:

<u>Participant N3 written response to question 4b</u>


How many cubes do you think will have the following features?

i. Four faces painted purple ____4.____
ii. Three faces painted purple ____3____
iii. Two faces painted purple ____1 2____
iv. One face painted purple ____1 1____
v. No face painted purple ____0____

Figure 4.56: Participant N3's written script response to Test item 4b

Researcher: In part 4b, can you take us through how you figured out these responses

Participant N3: Since the question said 4 faces painted, I started from 4; 3 and I think I made a mistake here, it should have been 4 (counts the visible faces and one invisible), 3 (counting the visible faces) , 2, 1 and 0 (means the whole shape painted). I don't know how I got 1, maybe it was the correct one.

Researcher: But the question requires the number of smaller cubes with 4; 3; 2; 1 or no painted faces, not the number of faces on the larger cube.

Participant N3: I think this face will have 9, the other 9 so it will be 9x6 all 54 will be painted.

Researcher: Read the question statement again, how many cubes does it say build the block in question 4b?

Participant N3: We are given that there are 27 smaller cubes altogether.

Researcher: Now from 27 cubes, how would you get 54 painted ones?

Participant N3: I do not know which formula I used, but it does not make sense.

Researcher: Could you have found out the answers in another way besides thinking of a formula to use?

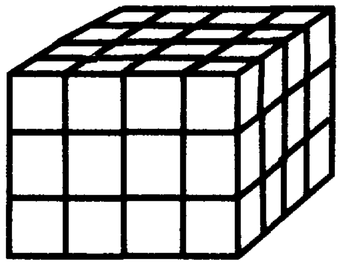
Participant N3: I do not think so. Either you use a formula or you guess, estimate.

It was evident that participant N3 counted the individual unit squares on each face of the larger cube, the surface area, instead of the number of cubes that were painted. The error could have been caused by a language barrier, as the learner seemed not to have understood the question. In each attempt to probe further, participant N3 changed from one supposition to the other. The misconception of treating 3-D objects as 2-D figures also manifested in the responses participant N3 gave, similar to participants G3 and H3.

The participants' responses indicated some difficulty in enumerating cubes in arrays of three-dimensional structures and shapes. This had been listed as a Type 3 misconception in the framework in Table 4.3. The identified misconception could have been a result of insufficient mastery of the concepts of surface area and volume.

Question 5

> Examine a structure filled with cubes without gaps inside, then, find its surface area and volume.
>
> The structure below is entirely filled with cubes, there are no gaps inside. Examine this structure and answer the questions that follow.



a. How many unit cubes will it take to construct the structure above?

____

b. What is the volume of this structure?

____

c. Find the surface area of the structure above.

____

Although this question is similar to Question 3, it was included to verify and prove that most learners were more comfortable dealing with right or uniform prisms than with irregular prisms when finding surface area and volume.

Question 5 required participants to count the number of unit cubes making a block of arrays, representing its volume. The participants were also expected to count the number of unit squares on each of the six faces of the structure, a rectangular prism, to find its surface area.

The correct answers for this question were supposed to be 5a = 48 cubes; 5b = 48 cubic units and 5c = 80 square units.

Table 4.4 indicates that 50% of the participants had correct responses for the number of unit cubes building the block as asked in 5a, with the other 50% giving incorrect answers. However, in 5b, when asked to find the block's volume, only 38% of the participants were correct. There were 50% incorrect and 12% not attempted responses to Question 5b. In Question 5c, which required participants to find the structure's surface area, only 25% of the participants managed to give the correct answer. Sixty-three per cent had incorrect responses, while 12% had not attempted the question. This shows that 75% of the participants could not find the surface area of the block. The learners could not link the number of cubes they counted correctly to the volume of the block. This showed a lack of understanding of the definition of the volume and inability to find the volume without the use of the formula. This indicated the cause of insufficient mastery of the concepts.

Among the incorrect responses in 5a, 2/29 participants (R3, S3) wrote that the block had 36 cubes. There were other answers like 12, 36, 72, 80, 96 and 104. The incorrect responses in question 5b included three participants (T3, N3, S3), who stated that the volume of the block was 36. Other incorrect responses for 5b were 16 cm^3, 96 and 900 $units^3$. Two participants (D3, R3) wrote that the block's surface area in Question 5c was 32 cm^2. Some of the incorrect responses were 12 $units^2$, 72; 24 and 840 $units^2$.

Participants C3 and L3 wrote that the dimensions for length, width and height were necessary to find the surface area and volume in both 5b and 5c. This was the only misconception easily identifiable in Question 5, before the post-test interview.

When asked how many cubes built the block in question 5a, participant A3 wrote that there were 48, as evidenced in the script below.

Participant A3 written response to question 5a

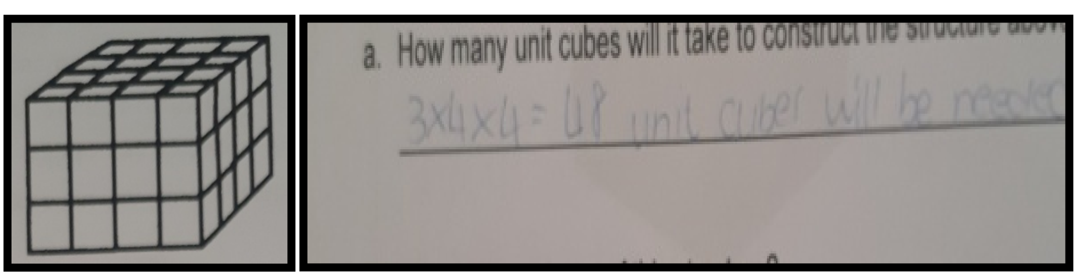


Figure 4.57: Participant A3's written script response to test item 5

Asked to support this answer during the interview, participant A3 gave the following account:

Researcher: In Question 5a, how did you establish that the block is made up of 48 unit cubes?

Participant A3: It is 3 cubes high, 4 (showing the length and the width) this way and 4 that way, so it is 3x4x4 = 48 unit cubes needed.

Researcher: Can you tell why you multiplied?

Participant A3: Because it is the formula for finding the total number inside.

Researcher: Could you not have used another method besides this "formula"?

Participant A3: No, sir that is the only way one can find out.

Although participant A3 mentioned the formula, in this calculation, it appears though the multiplication was done through counting the number of blocks on each dimension rather than writing the formula l x w x h, and substituting the relevant values. Conceptual knowledge seemed to exist here, however, coupled with relating the use of some formula without understanding.

Researcher: And how did you find the volume in question 5b?

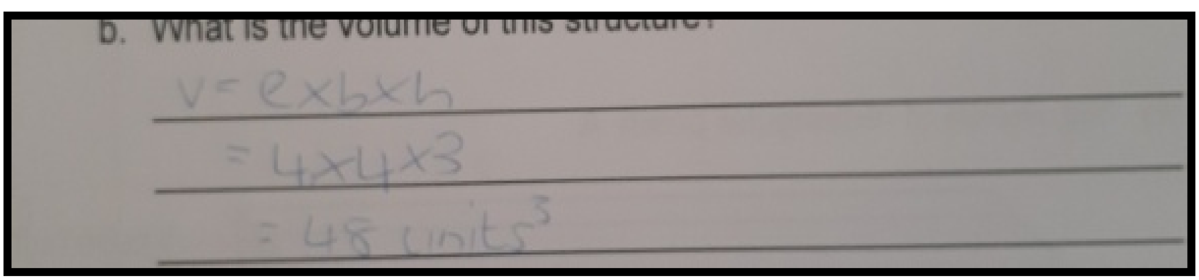


Figure 4.58: Participant A3's written script response to Test item 5b

Participant A3: I used the formula for volume, which is L x W x H so it is 4 x 4 x 3 = 48 units³.

Researcher: Do you think you could have found the volume in any another way?

Participant A3: That is not possible, another formula would give another answer, so another way would mean something else not volume.

Researcher: How about for surface area, explain how you obtained 80 unit²?

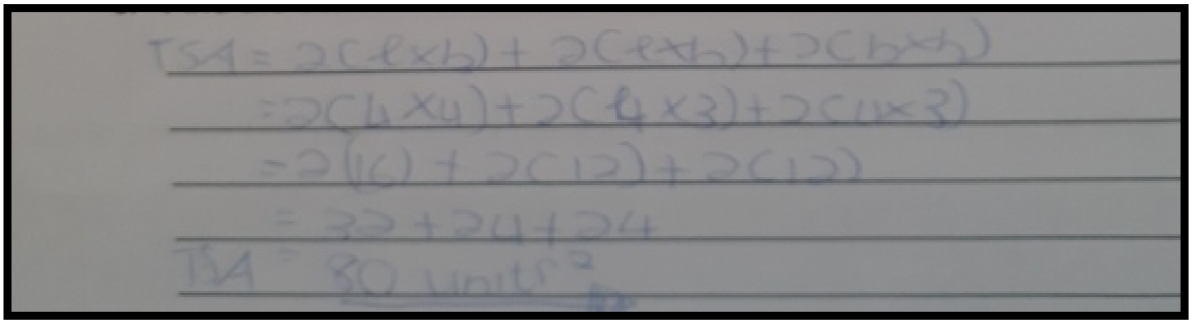


Figure 4.59: Participant A3's written script response to test item 5c

Participant A3: Surface area you say length x breadth x 2 and breadth x height x2 then height x length x2 so it will be 80 unit².

Researcher: Explain why you used that rule.

Participant A3: It is the formula for surface area. That is why I used.

Researcher: Please use another way to find the surface area of that shape.

Participant A3: There cannot be another, as it would give a different answer.

The learner had mastered the use of the formula in finding the surface area and volume. However, there is a misconception that using a different method would result in a different answer. Participant A3 displayed the misconception of defining surface area and volume by their respective formulae.

When asked how many cubes the block in Question 5 had and what its surface area and volume were, participant C3 wrote the following:

<u>Participant C3's written answers to question 5</u>

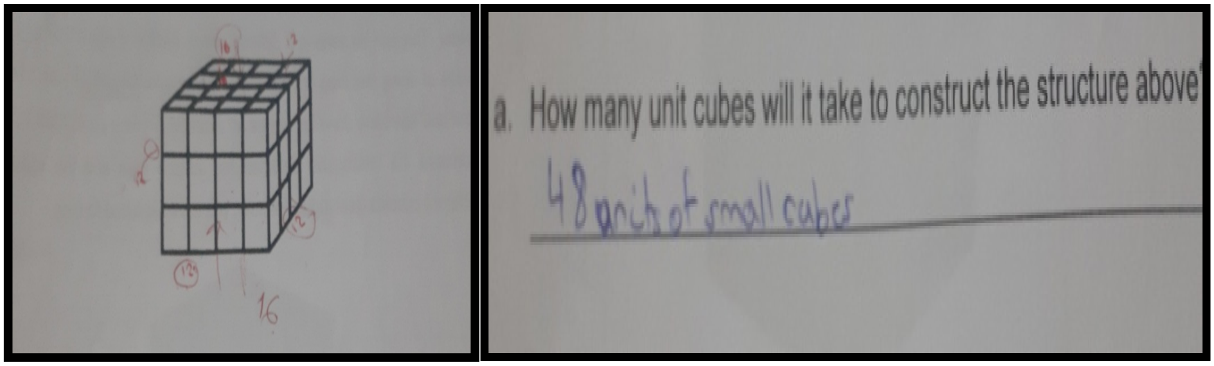


Figure 4.60: Participant C3's written script response to test item 5a

Participant C3 supported the given answers to question 3 in the following manner:

Researcher: Please show me how you obtained 48 unit cubes in Question 5a.

Participant C3: I counted all the cubes, we have 12 here (pointing at the front visible face), 12 behind here (points at the 2nd column) and the last 2 also have 12 each so 12 x 4 = 48.

Participant C3 seemed to be mixing the number of cubes on one face with the number of cubes in one column. This showed that there was a misconception of treating and confusing 2-D figures with 3-D ones. Despite getting the correct answer, participant C3 indicated some misconceptions in this response. It was evident that the correct answer had coincidentally been obtained by using an incorrect method.

Researcher: Looking at question 5b and c, you indicated that there are no dimensions, hence volume and surface area cannot be found, can you want to explain?

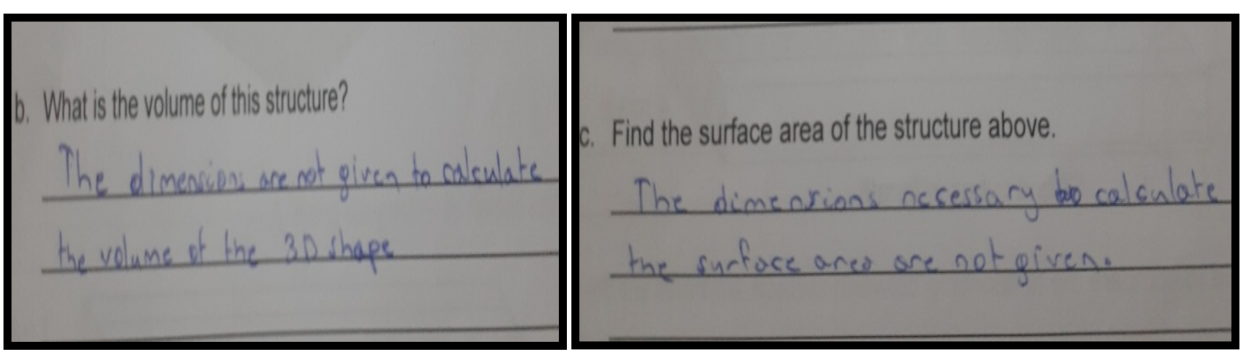


Figure 4.61:Participant C3's written script response to Test item 5b and c

Participant C3: Certainly, to find volume and surface area, you need the dimensions for length, width and height. This diagram does not have such.

Researcher: But since you were asked for those quantities in this question, there might be a way to solve without the dimensions, don't you think so?

Participant C3: But if there is another way, how will you apply the rule?

Researcher: What rule do you mean?

Participant C3: The one that says length x width x height.

Researcher: What is the rule for?

Participant C3: That is the rule for finding the volume of this prism.

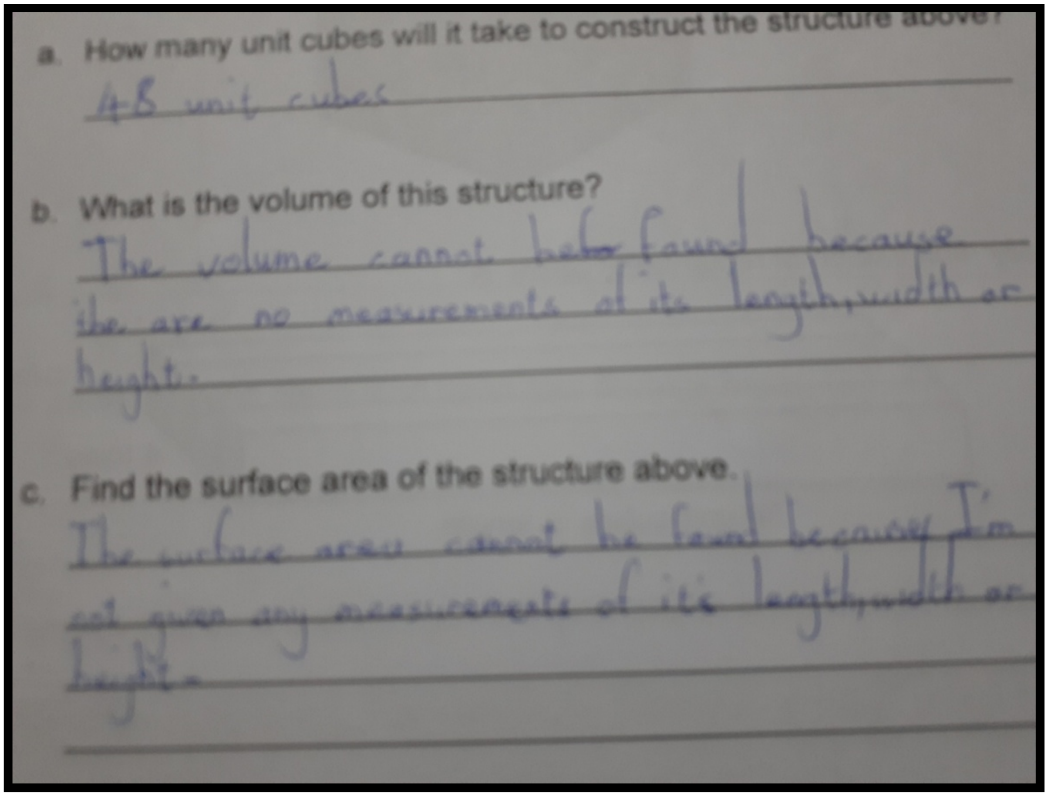
Researcher: And how about the one for surface area?

Participant C3: It should be volume x 2, because surface area is bigger.

Participant C3 also displayed the misconception of defining the quantity of the volume by its formula, just like participant A3. The formula for finding the volume of a prism is well memorised but not the one for surface area. It is most probably because the quantity total surface area had only been covered in the past two years of the participant's schooling curriculum, at Grade 7 and 8, unlike volume, which was covered thoroughly from the intermediate phase and partly in the foundation phase. That probably explains why participant C3 gave an incorrect formula for the surface area of the given block.

Another learner who wrote exactly the same response as participant C3 was participant L3. This, actually, indicates that participant L3 had the same line of thinking as participants A3 and C3, that surface area and volume could only be found when given the dimensions, to use the formulae. However, participant L3 was not available for the post-test interview. Nonetheless, participant L3's written response to Question 5 is indicated here as evidence and affirmation to the above assertion.

Participant L3 written responses to question 5



a. How many unit cubes will it take to construct the structure above?

48 unit cubes

b. What is the volume of this structure?

The volume cannot be found because there are no measurements of its length, width or height.

c. Find the surface area of the structure above.

The surface area cannot be found because I'm not given any measurements of it's length, width or height.

Figure 4.62: Participant L3's written script response to Test item 5

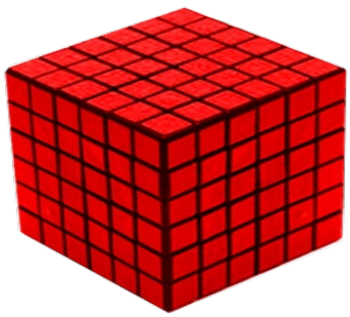
The above responses are similar to those of both participants A3 and C3. The three participants believed that the surface area and volume could only be obtained by the use and application of some formula. They believed that the absence of the dimensions length, width and height meant that surface area and volume could not be obtained since there would not be any measurement digits to substitute in the formulae. It was noted that most participants had difficulty enumerating the cubes arranged in 3-D arrays.

4.4.4 How Grade 8 Learners Describe the Relationship between Surface Area and Dimensions of Prisms of the Same Volume

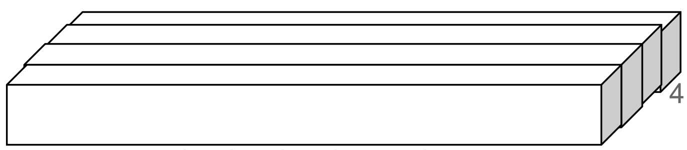
Question 6

Construct prisms of the same volume, find their surface areas then compare the relationships between these different prisms.

A cube made from 216 identical smaller cubes of 1 cm³ each is changed into rectangular prisms of one layer, where no two cubes are placed one above the other. This can be done in 8 different ways. Sketch diagrams of any other three ways of doing this then find the surface area and volume of your sketches. The first one has been done for you as an example. Write a paragraph to describe or compare the relationship between the surface area and the volumes of the different cuboids (rectangular prisms) you have sketched formed.



My first sketch of rectangular prism A



Use this table to compare the relationship between surface area and volume of cuboids.

Shape letter name and dimensions	Surface Area	Volume	Relationship of surface area to volume
A Length – 54 cm Width – 4 cm Height - 1 cm	(1 cm x 54 cm) x 2 + (1cm x 4cm) x 2 + (4cm x 54cm) x 2 = (108+8+432)cm² = 548 cm²	54 cm x 4cm x 1cm =216 cm³	548:216 = 274:108 =137:54
B			
C			
D			

Write your paragraph and conclusion upon your findings here:

____

____

This question dealt with finding the relationship between surface area and volume of prisms of different sizes. In this question, the volume was kept the same. The comparison was between different lengths, resulting in the conclusion that longer prisms have larger surface areas than shorter ones of the same volume.

The participants were required to dismantle a large cube built of 216 small cubes of one cubic unit each. Having dismantled the block, the participants would then construct different prisms with all 216 small cubes. There was only one restriction: No one cube should be on top of the other. It meant that the height of each constructed prism would be one unit. Eight different prisms would have been constructed, with the following dimensions: 1x1x216; 1x2x108; 1x3x73; 1x 4x54; 1x 6x36; 1x8x27; 1x9x24; 1x12x18. The various respective surface area values would then be calculated, together with the volumes. However, it was expected that the learners would realise that the volume remained the same. The participants were also expected to reach the desired conclusion by comparing these prisms dimension against their surface areas.

One example was provided on how to go around the question from 6a up to 6d. It meant that from the eight possible prisms, the participants were only going to construct three from which, together with the example, they would be able to compare and make a conclusion.

Table 4.4 indicates that 6/29 of the learners (17%) of the participants could establish correct dimensions of the alternative constructed prism in part 6a. Three percent (3%) of the participants achieved partly correct answers regarding the dimensions of constructed prisms of the same volume. Thirty-eight per cent (38%) gave incorrect dimensions, while 38% did not attempt assigning the dimensions. Thus, 76% of the participants had no clue of finding dimensions of different prisms that could be constructed from a block of 216 cubes of the same volume.

When asked to find the surface areas of the constructed prisms in Question 6b, only 1/29 participants (3%) had the correct response. Another 14% had partly correct values for the surface area. 45% of the participants had incorrect responses for surface area, while 38% did not attempt finding the respective surface area. This gave a total of 83% of participants who could not find the surface area of their own constructed prism of the same volume.

Part 6c required the participants to find the volumes of their own constructed prisms. It was expected that since the participants were constructing the prisms from 216 congruent cubes, they would deduce that the volumes of the different prisms would be the same (equal). Unfortunately, it seemed to be not so easy to derive, as Table 4.4 indicates that only 7/29 of the participants (24%) managed that. Three per cent (3%) got partly correct responses while on the other hand, 24% had incorrect responses against 49% not having attempted to obtain a result.

Question 6d required the participants to compare the result obtained in 6b with 6c, thereby giving the surface area to volume relationship. Five of 29 of the participants (17%) got this part correct, as shown in Figure 4.4. None of the participants (0%) had a partly-correct response, but 17% were incorrect, while 66% did not attempt to compare the relationship between surface area and volume of different prisms.

Part (e) of Question 6 required the participants to draft a conclusion of their discoveries regarding the relationship between surface area and lengths of different prisms of the same volume. The most suitable response was that longer prisms have larger surface areas than shorter prisms of the same volume. None of the participants reached this conclusion 90% correct responses), as shown in Table 4.4. Only 3/29 participants (10%) had partly correct results in this question, whereas 7% gave incorrect responses compared to 83% who did not attempt to answer the question. That gave us 90% of the participants being unable to come to the desirable conclusion.

Among the incorrect responses in this questions were participants who wrote dimensions with heights above one unit. These participants probably misread or did not understand the limitation stated in the question instructions. Other participants drew different sorts of polyhedra, and wrote the formulae for finding their respective surface areas and volumes. Most participants had incomplete written work. A few participants followed the example up to the calculation of surface area and volume rather than the comparison ratio in column 6d. It was difficult for the participants to get to the point of deducing the required conclusion. This was due to their incomplete work as most learners had constructed only one of the three required prisms, which made it impossible to compare results and eventually make conclusions.

When asked to construct different prisms and find the surface area and volume, and then compare them in ratio form, participant G3 wrote the following:

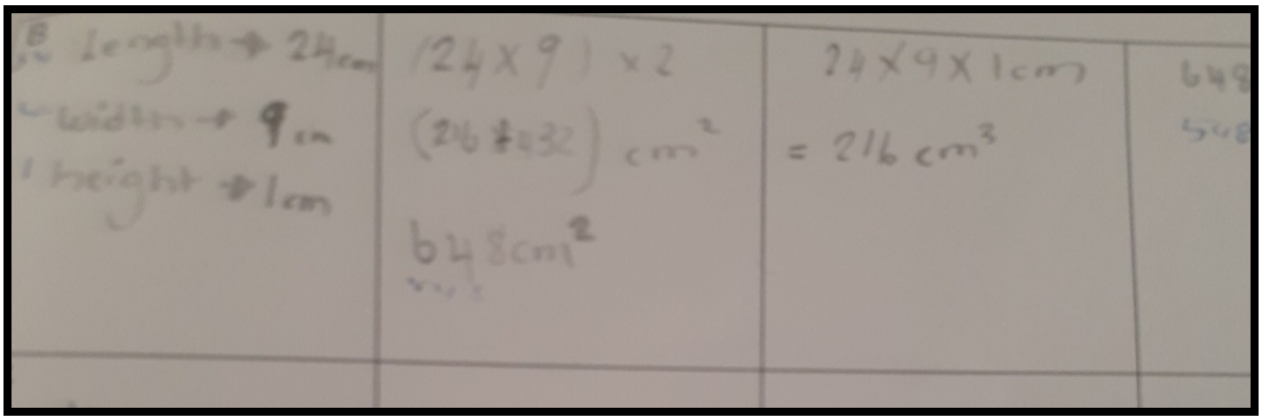


Figure 4.63: Participant G3's written script response to test item 6a-d

When asked to support the written response during the interview, participant G3 reasoned as follows:

Researcher: Let us look at number 6. I see you did not complete the task, what could have been the reason?

Participant G3: I didn't understand really what to do.

Researcher: But you worked out the first one, seems like you knew what to do then. Tell me how you found your dimensions, surface area and volume of your sketch please.

Participant G3: the length is 24, the width is 9 and the height is 1. So for surface area I say 24 x9 x2 that is 216 + 432 = 648cm² and volume is 24 x 9 x1 =216cm³, leaving our ratio as 648:216

In this response, participant G3 obtained the correct volume but the incorrect surface area. The researcher probed to determine the reason behind the incorrectly obtained surface area.

Researcher: You said 24x9x2 that gives us 432. Show us where the 216 that you added came from please.

Participant G3: I think it is from 1x9x24

Researcher: What were you looking for by multiplying 1x9x24?

Participant G3: It is in the formula

Researcher: Which formula is that and what is it for?

Participant G3: Is it not for volume and surface area?

While participant G3 claimed the use of some formulae, it was mixed up, without the real knowledge of which formula to use when. The same was evident in the following supposedly formula for surface area.

Researcher: I wanted you to inform me. So when you added the volume to 432, what were you looking for?

Participant G3: I am sure it gives us the surface area. That is the rule.

Researcher: Ok, here you said (24 x9) x 2, which 2 is this?

Participant G3: This is when you want to get the exponent. You multiply by 2 for surface area.

Researcher: Alright, let us compare the ratios of the relationships between your sketch and the example, what can you say?

Participant G3: One is smaller and the other is larger, mine is larger?

Researcher: What makes it larger, and what is large?

Participant G3: The surface area...

Researcher: In your conclusion, you said the measurements do not change but position of the prism changes, can you explain here?

Participant G3: It still remains the same, but other things change.

Researcher: Which other things change?

Participant G3: The length, width and height, but measurements remain the same.

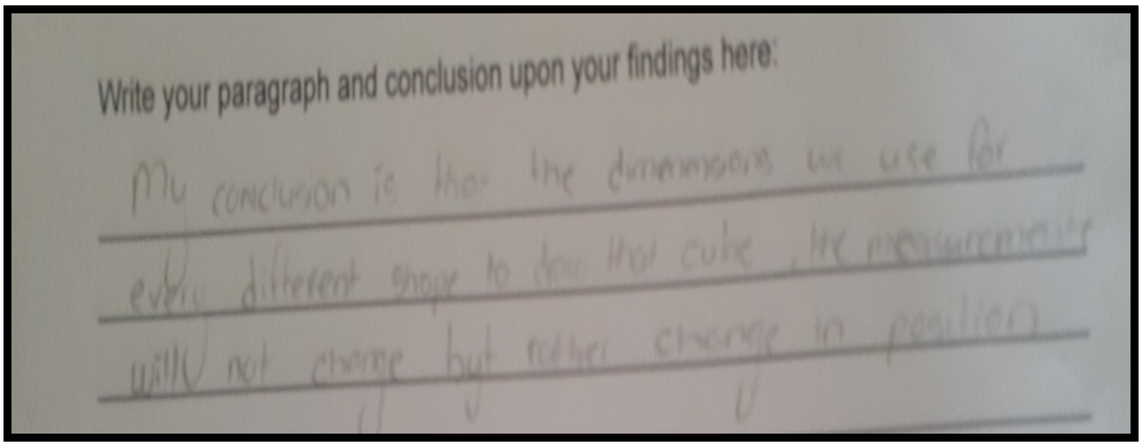


Figure 4.64: Participant G3's written script response to Test item 6e

Participant G3 lacked the knowledge and vocabulary to use for describing the relationships required in this question. That was consistent with a lack of vocabulary used for the definition and description of Question 1 and 2. The participant failed to explain that while changing dimensions or prism measurements changed the surface area, it kept the volume unchanged. The final conclusion that longer prisms have larger surface areas than shorter prisms of the same volume was not reached. The misconception prevalent in participant G3 was confusing surface area with volume, including interchanging formulae and sometimes using the wrong formula altogether. Similar to G3's reasoning was participant H3, whose interpretation follows.

When asked to respond to Question 6, participant H3 wrote the following.

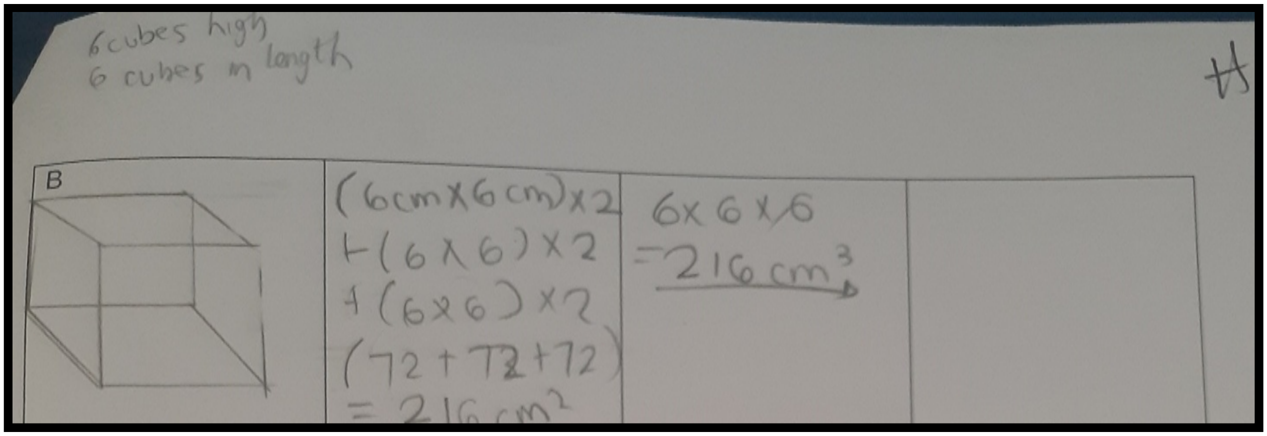


Figure 4.65: Participant H3's written script response to Test item 6a - d

During the interview, participant H3 supported the written answer as follows:

Researcher: Please explain how you obtained the answers for question 6.

Participant H3: I drew another cube in different ways but the surface area to remain the same.

Researcher: Show us how you obtained your dimensions.

Participant H3: I have 6 x6 x6 for my dimensions, so volume is 216cm^3 and the surface area is also 216cm^2.

Participant H3 had not read the instructions clearly, in the same manner participant G3 had not followed to instructions in the same question. This resulted in the incorrect dimensions being supplied and jeopardised the intended result. Participant H3 also seemed to be more formulae oriented.

Researcher: Explain the relationship between the surface area and the dimensions of prisms with the same volume.

Participant H3: I noticed that the volume does not change. It remains the same.

Researcher: And what about the other quantities?

Participant H3: The surface area and the measurements.

Researcher: How do they change, do you notice any pattern?

Participant H3: They change because I changed them. I think it is because length x width x height changes all the time.

Researcher: What does length x width x height provide you and what makes it change all the time?

Participant H3: It is the surface area of these blocks. It keeps changing when volume does not change.

It was evident that participant H3 was confusing the surface area with the volume and using the same formula for both quantities. Participant H3 also failed to describe the relationship between the dimensions of prisms of the same volume with their surface area. These were the same characteristics found in participant G3, in addition to the lack of a proper vocabulary for the descriptions. Similarly, participant H3 did not reach the desired conclusion.

Researcher: Explain why you did not complete writing about the relationship between surface are and volume.

Participant H3: I had no clue of what to do, but I could see that the volume remained the same.

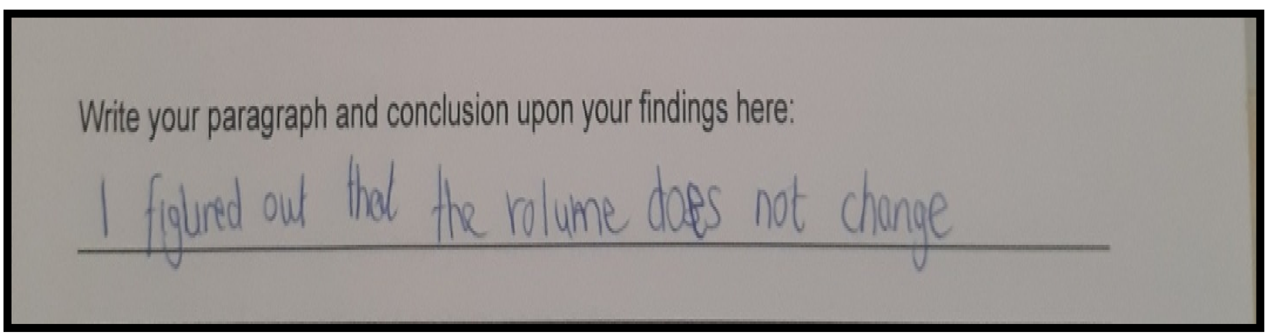


Figure 4.66: Participant H3's written script response to Test item 6e

Participant H3's lack of knowledge to manoeuvre in this question could be ascribed to insufficient mastery of concepts. Another participant who displayed similar misconceptions as participant G3 and H3 was participant N3.

Participant N3 wrote this script in response to Question 6:

Figure 4.67: Participant N3's response to Test item 6 a - d

When asked to motivate this answer during the post-test interview, participant N3 stated the following:

Researcher: Let us look at question 6, please explain why you left the surface area, volume and relationship unanswered?

Participant N3: When I was writing, I couldn't think of a way to find everything but I figured out that in the end there would be the same amount of block, same amount of volume but different structures, maybe different surface areas.

Researcher: How would you know that the surface area would be different?

Participant N3: Because of the different units they give us.

Researcher: How would you find the surface area of your prisms?

Participant N3: By using the formula Lx B x H

Researcher: And how would you find the volume of your structures?

Participant N3: Also by saying Lx Bx H.

Participant N3 used the same formula for both volume and surface area, a misconception discussed in Table 4.3, also noticed in participant G3 and H3. Participant N3's conclusion was similar to the one given by the other two participants discussed in this section.

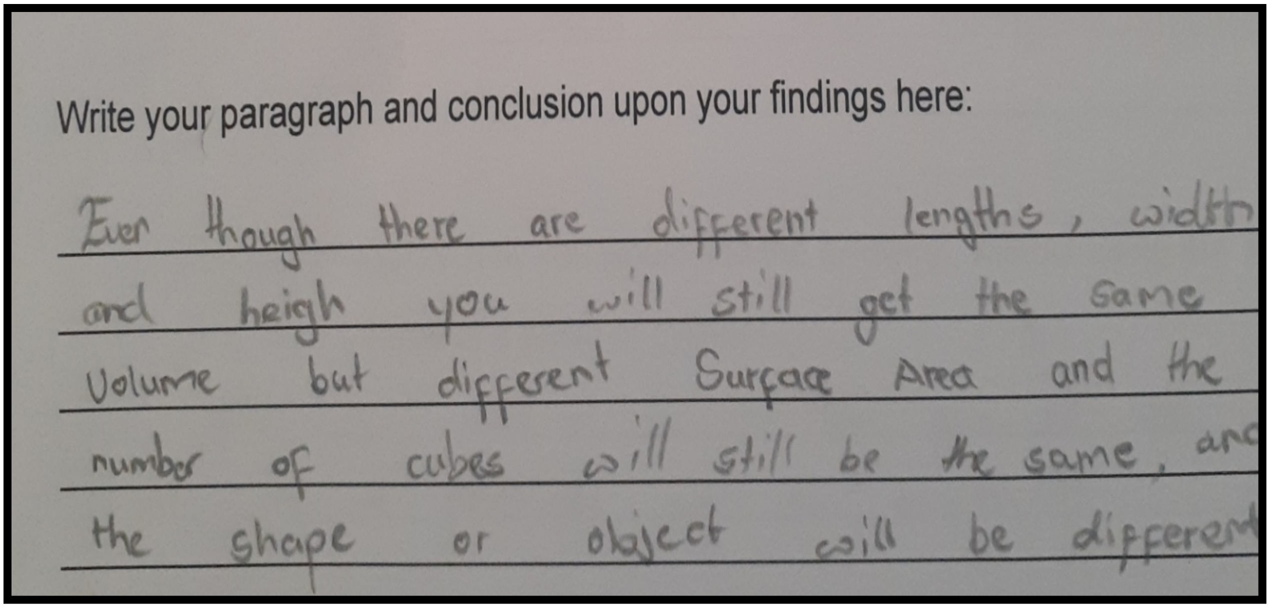


Figure 4.68: Participant N3's response to Test item 6 e

With the correct dimensions, the learners could find the surface area and volume of the constructed prisms. There was, however, a weakness in the comparisons between surface area and volume. Regarding drafting a conclusion following the results of an investigation, the participants could not independently deduce the desired outcome. One could conclude that the participants were not exposed to all the levels of the cognitive domains in mathematics learning. It is only the recall and routine procedures that the learners seemed to excel in; there was a lack of synthesis, application and evaluation in the learner's work. It appeared though learners were not used to solving higher order mathematical problems.

4.4.5 Summary of confirmed misconceptions listed on the conceptual framework

Table 4.5 shows the distribution of misconceptions stated in the conceptual framework as confirmed among the interviewed participants for the entire test results. The interviewed participants who displayed the particular misconception are listed in the table per item per code.

Table 4.5: Distribution of confirmed misconceptions during interviews

	Misconception code			
Test item	Treating 3-D as 2-D figures	Incorrectly enumerating of arrays	Confusing surface area with volume	Counting only the visible faces
1a – b	C3		A3; N3; G3	
2a – b	Q3		A3; G3	
3a – e	H3	C3	G3	C3; G3; H3
4a	G3; N3; Q3	Q3		Q3
4b. (i – v)	G3, H3; N3	G3; H3; N3	N3	G3; H3
5a – c	C3		C3; G3	C3
6.a – e			A3;G3; H3;N3	
Entry total	10	5	12	7
Entry percentage	10/34 30%	5/34 15%	12/34 35%	7/34 20%

The information from Table 4.5 was represented graphically for a pictorial representation in the pie chart depicted in Figure 4.69. Figure 4.69 compares the occurrences of codes and misconceptions found or confirmed during the participants' interviews.

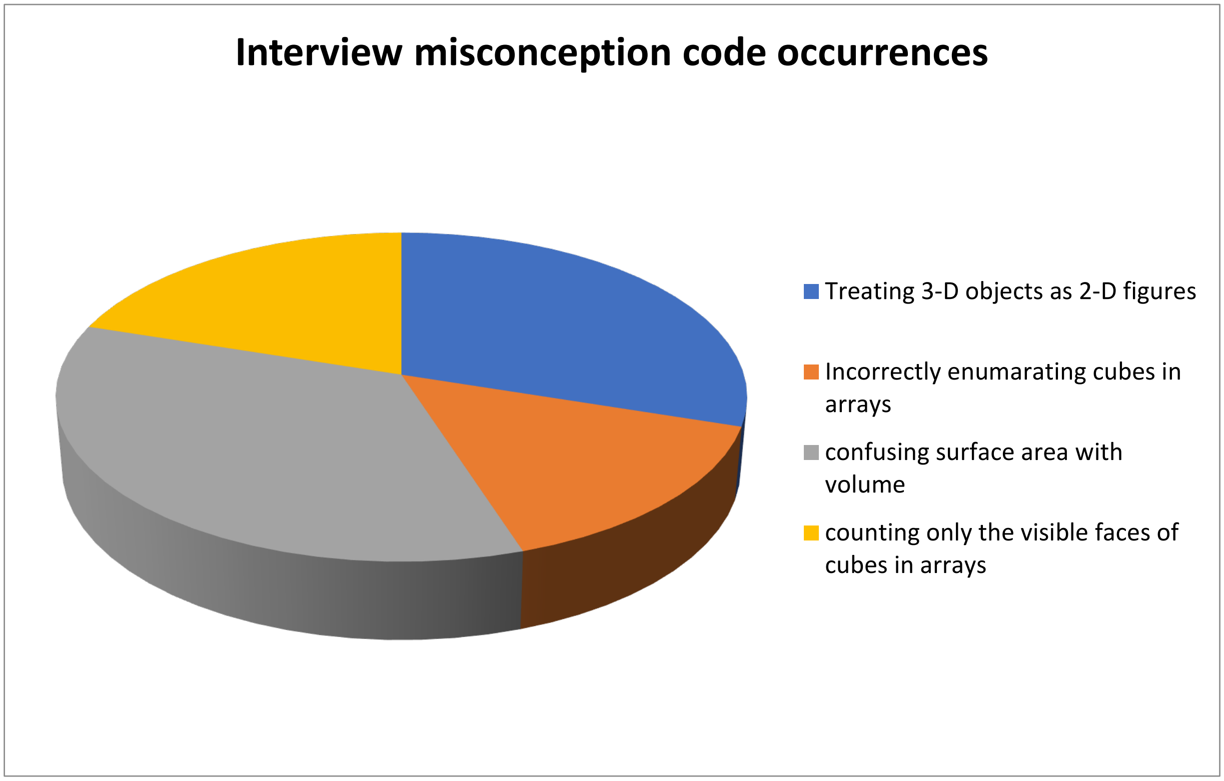


Figure 4.69: Occurrences of codes and misconceptions confirmed during interviews

Figure 4.69 shows that the most prevalent misconception confirmed in this study was confusing surface area with volume. This was followed by the misconception of treating 3-D objects as if they were 2-D figures. The misconception of incorrectly enumerating arrays and that of counting only the visible faces were almost equally likely in this study. The misconception of assuming that a shape has more than one surface area or volume was not confirmed during the interviews, as it does not appear in table 4.5 and figure 4.69 alike.

Apart from confirming the misconceptions listed in my conceptual framework, during the study, I discovered additional misconceptions in the learners' work. These misconceptions were not listed in my conceptual framework under the literature review. These were discussed in Section 4.2.5, now confirmed in Section 4.5. The new misconceptions found and confirmed in this study are shown in Table 4.6.

Table 4.6: Distribution of misconceptions found in the study but not in the conceptual framework

	NEW MISCONCEPTIONS				
Test item	Defining volume by formula	Confusing volume with mass	No units of measurement means no volume or surface area	Irregular shape have no volume or surface area	Shapes with blocks and spaces have nor surface area or volume
1a	A3;C3;G3,H3	A3;C3;G3;Q3			
1b	H3;N3	G3;Q3			
2a	A3;C3;G3;H3;N3;Q3	Q3	A3; G3; N3		
2b	A3;G3;N3;Q3				
3a	A3;Q3		C3;G3;N3		
3c			G3;H3:N3	G3;H3	
3d			A3;G3		C3;G3
5	A3; C3				
6	A3; H3; N3				
Entry total	23	7	11	2	2
Entry percentage	23/45 50%	7/45 16%	11/45 24%	2/45 5%	2/45 5%

The information given in Table 4.6 is shown graphically in a bar graph in Figure 4.70 for visual comparisons of the confirmed discovered misconceptions among the interviewees.

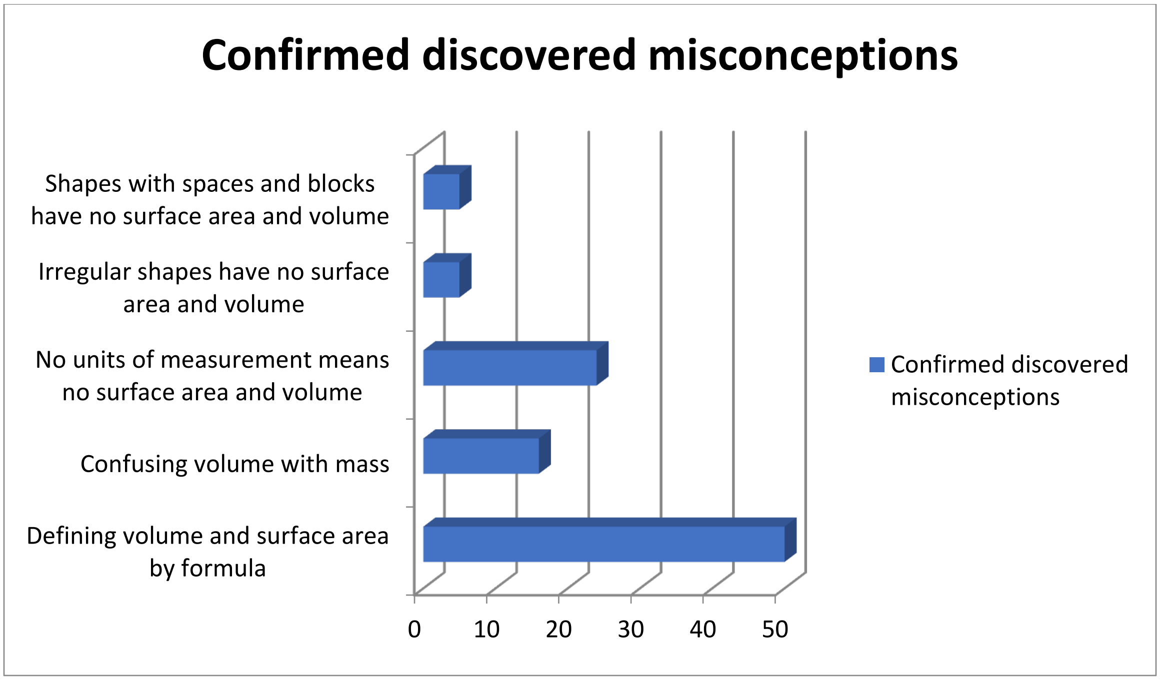


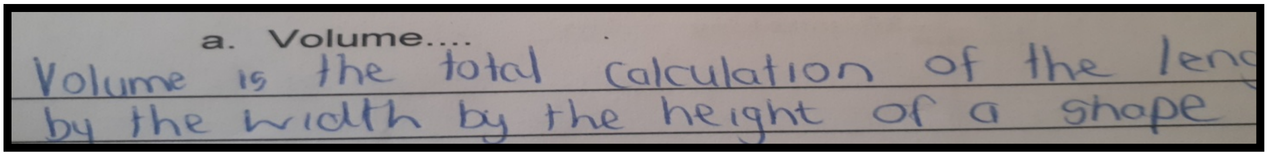
Figure 4.70: Confirmed discovered misconceptions not in literature

Figure 4.70 indicates that in close to 50% of the interviews, participants defined volume and surface area by their formula. This was the most prevalent misconception discovered in this study but not originally stated in the conceptual framework. Just above 20% of the interviews shown on figure 4.70 confirmed that the absence of units of measurement and dimensions meant that surface area and volume could not be found. These misconceptions are dealt with in detail in Chapter 4.5.

4.5 MISCONCEPTIONS FOUND IN LEARNERS' RESPONSES BUT NOT IN THE LITERATURE

More misconceptions about learning surface area and volume were found and confirmed in learners' responses during the interviews. These were discussed in Section 4.2.5.

1. Giving the formulae for calculating surface area and volume when asked for definitions. This was confirmed in 14% of the interviewees, as seen in participant H3's response.



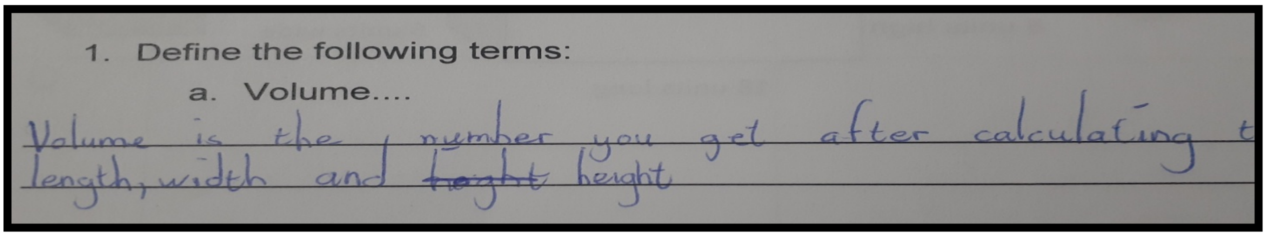
a. Volume....

Volume is the total calculation of the len
by the width by the height of a shape

Figure 4.71: Participant H3's written response.

Participant H3: I meant its formula

It was also found in participant L3's written work.



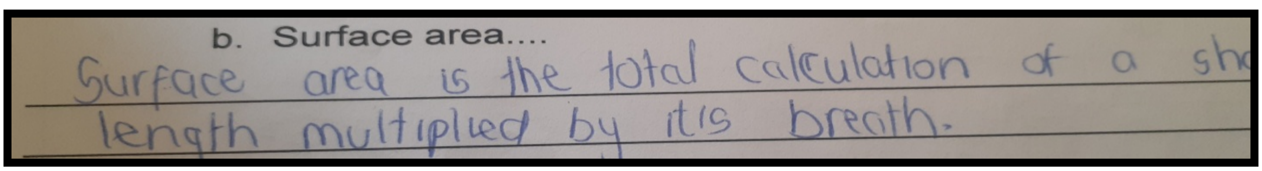
1. Define the following terms:

a. Volume....

Volume is the number you get after calculating t
length, width and ~~height~~ height

Figure 4.72: Participant L3's written response.

2. Providing the formulae for calculating surface area and volume of prisms instead of describing how to find these without using any formula, as noted in 29% of the interviewees, through participants H3 and N3's responses during the interviews.

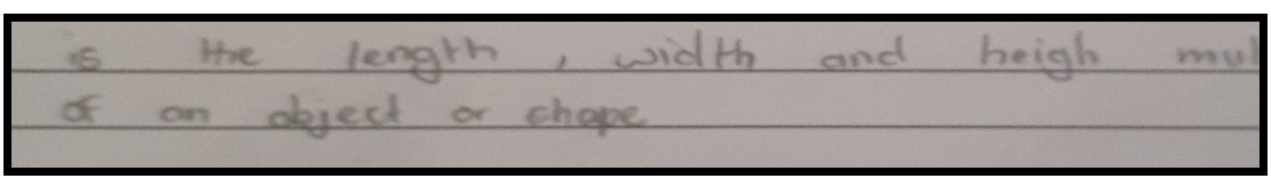


b. Surface area....

Surface area is the total calculation of a sh
length multiplied by it's breath.

Figure 4.73: Written responses, participant H3

Participant H3: Total calculation multiplied by the breadth.



is the length, width and heigh mul
of an object or shape

Figure 4.74: Written response, participant N3

Participant N3: It is the length, width and height multiplied together on an object or shape. Basically I wanted to show how it is calculated.

3. Believing that surface area and volume cannot be found without having the measurements of prisms or cylinders. This was most prevalent in Question 3 as evidenced in 29% of the interviewees, in respect of participants C3 and N3's descriptions during the post-test interviews.

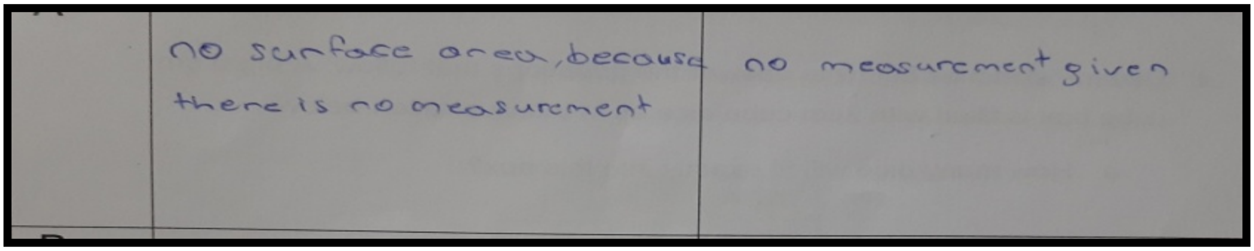


Figure 4.75: Participant C3's comments on no units of measurements

Participant C3: No measurements are given here sir, so it is obvious, no surface area and no volume.

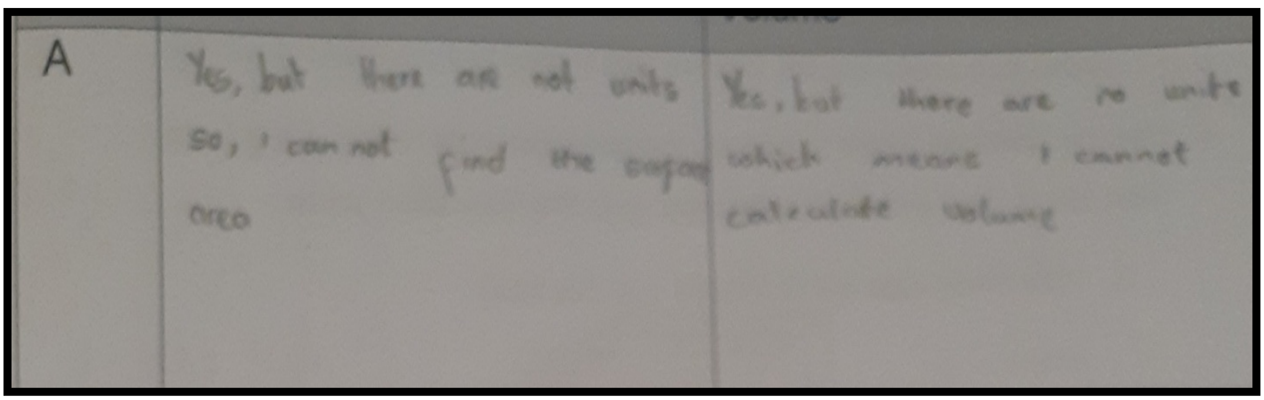


Figure 4.76: Participant N3's comments on no units of measurements

Participant N3: No, there are no units, so you cannot find the volume or surface area as well

4. Believing that a solid object (3-D shape) composed of blocks and spaces is incomplete (in the learner's perception) and thus has neither surface area nor volume: As seen in responses to Question3d, displayed in interviews by participant G3, 14% of the interviewees.

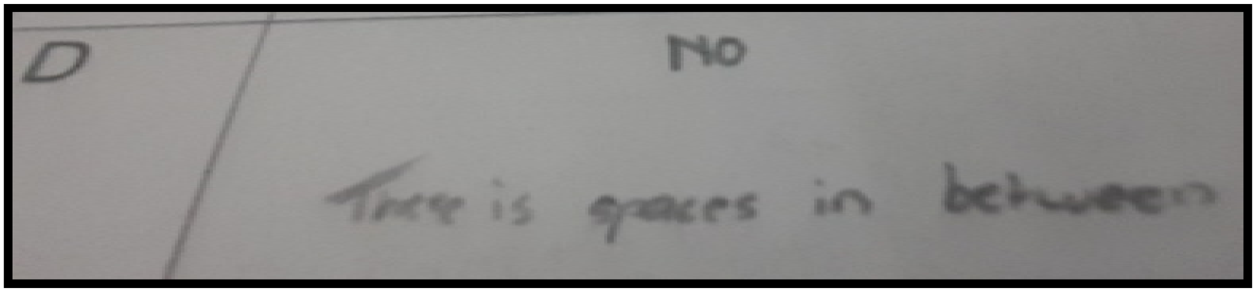
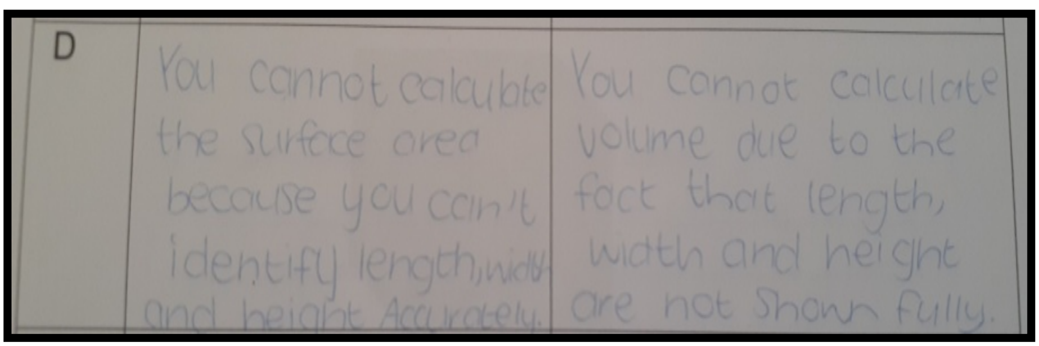


Figure 4.77: Participant G3's comments on incomplete objects

Participant G3: I said there are spaces in between, so no surface area. For volume, I also said no volume because it is not level.

5. Assuming that a solid object (3-D shape) with unmarked dimensions has neither surface area nor volume. This was the commonest response in most Question 3 items, as shown during the interviews with participants A3; C3 and N3. This misconception was seen among 43% of the interviewees.

Participant A3: It must have a proper length, width and height, without all these gaps.



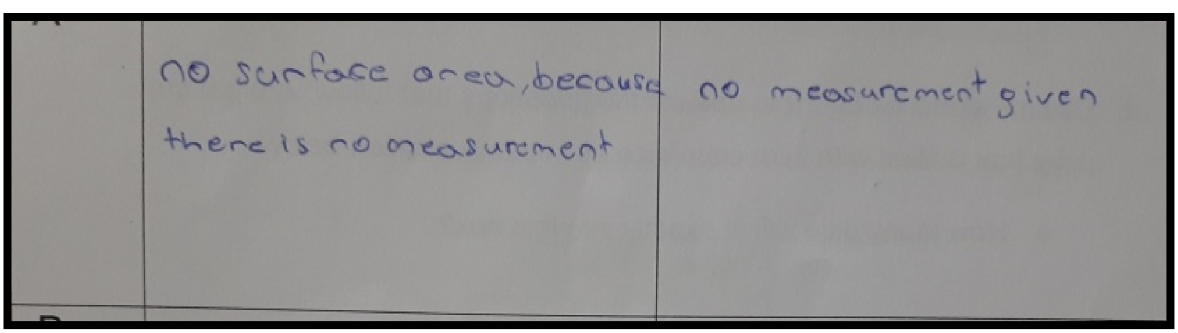


Figure 4.78: Participant A4's comments on incomplete objects

Participant C3: No measurements are given here sir, so it is obvious, no surface area and no volume.

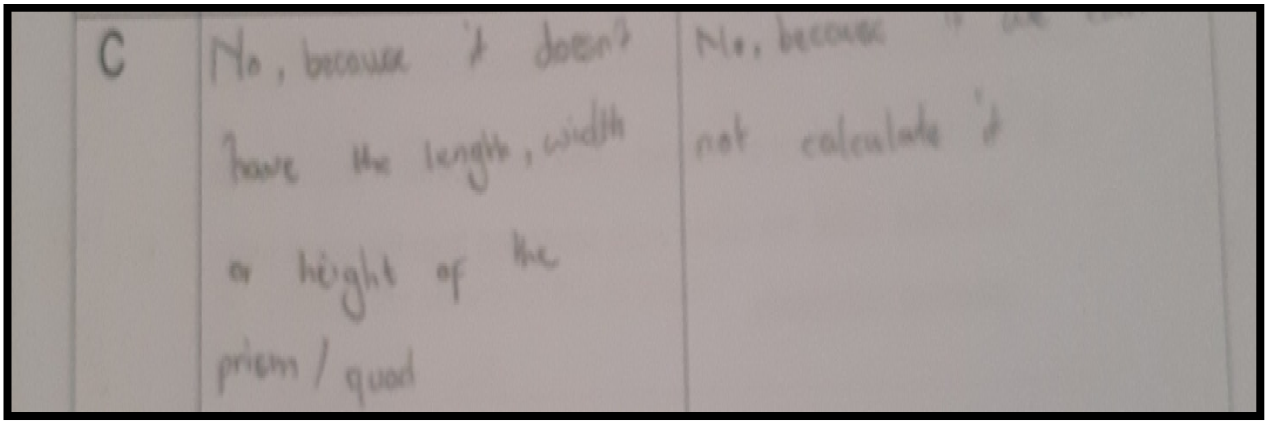


Figure 4.79: Participant N3's comments about incomplete objects

Participant N3: I said no surface area and volume because they don't give us the length, width and height. It does not have that height that much. We cannot calculate the volume because there is a space here (pointing at one of the edges), so it is incomplete.

4.6 CHAPTER SUMMARY

In this chapter, I recorded the actual data that were collected in this study. A scholastic test was administered; thereafter, the responses from 29 participants were compiled into categories which helped the researcher to identify themes and ten participants were invited for interviews. The data from both the test and interviews was analysed and interpreted in this chapter. The main focus was to find resemblances between the literature review, the conceptual framework and the participants' responses. Numerous misconceptions have been identified from the learners' raw data, and meanings were extracted during the interviews through probing. In this chapter, I recorded evidence on how Grade 8 learners defined the concepts of volume and surface area of prisms and how they solved problems involving surface area and volume, including the relationships between them. The chapter also recorded misconceptions in surface area and volume found in learners' work, with a couple of sources and causes exposed in interviews.

The next chapter reports the findings and suggested recommendations for minimising misconceptions in the learning of surface area and volume at the Grade 8 level.

5 CHAPTER FIVE: CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter deliberates on the entire study's findings, as displayed in the previous chapter's sections on data analysis and interpretation. It concludes the study in two parallel ways. First, the chapter shares the study's educational implications for the teacher, curriculum planner and implementer. Lastly, the chapter gives an account of recommendations for further related study in the field of mathematics.

Research aim:

This research was aimed at exploring the misconceptions that learners displayed when dealing with surface area and volume.

The research questions for this study were:

1. How do learners in Grade 8 define the concepts of surface area and volume of prisms?
2. How do learners in Grade 8 solve problems involving surface area, volume of prisms and the relationship between them?
3. What misconceptions prevail when learners in Grade 8 deal with problems involving surface area and volume of prisms?
4. What are the sources and causes of misconceptions in the process of learning about volume and surface area of prisms?

The objectives of this study were:

1. To find out how learners at Grade 8 define the surface area and volume of rectangular prisms.
2. To investigate how Grade 8 learners solve problems involving surface area and volume of rectangular prisms.
3. To describe evident misconceptions Grade 8 learners display when solving problems in surface area and volume.
4. To explore possible causes of misconceptions in learning about surface area and volume at Grade 8.

5. To suggest possible strategies that can help enhance understanding of surface area and volume at Grade 8.

In this chapter, conclusions are reached concerning the aim, the research questions and the listed objectives. The chapter evaluates whether the aim was attained; the research questions answered; and whether the set objectives were achieved.

5.2 SURFACE AREA AND VOLUME CONCEPTUALISATION FROM THE LEARNERS' PERSPECTIVES

Grade 8 learners conceptualised volume better than surface area based on the analysis done in Chapter 4 and drawing from the interpretations of both the learners' tests and interviews, considering the analysis of the collected data. However, the researcher found that learners had inadequate vocabulary to define or describe surface area and volume.

Most prevalently, the learners' understanding of volume or surface area was limited to the application of formulae. By so doing, the participants displayed over-generalisation of facts. Over-generalisation of facts was similar to and confirmation of the proposition by Brodie (2010). This was consistent with Brodie's (2010) study, proposing that that learners had procedural knowledge rather than conceptual knowledge, a proposition also supported by Machaba (2018). The participants' use of the formula when asked to define the surface area or volume proved that they exercised rote learning of calculating procedures without understanding. It was also confirmation of Machaba's (2005) instrumental understanding, demonstrated by learners' ability to use rules and formulae without reasoning. As seen in Questions 3, 5 and 6 of this research instrument, some participants used an entirely inappropriate formula, and at times substituted incorrect measurements into formulae, confirming Tůmová's (2017) findings.

In instances where there was no measurement given for a figure, learners were likely to say there was no volume or surface area because there were no measurement. According to the participants who took part in the study, volume and surface area could be defined in terms of formula, but they seemed not to know where the formulae originated. Some learners attempted to use a formula for those figures which had no measurements, which was correct and praiseworthy, except that they did not understand the origin of the formula. They could not link the developmental process of the concept and the formula. This could be attributed to the misconception of incorrectly enumerating cubes in 3-D arrays.

As evidenced in learners' work, they held that prisms without dimensions had neither volume nor surface area. This was a reflection of Legutko's (2008) claim of insufficient mastery of basic facts, concepts and skills. Needless to say, in the learners' point of view, only right prisms with distinct and uniform dimensions had volume and surface area; otherwise, they were rendered irregular, not level or inconsistent, thereby disqualifying them from having surface area and volume.

Comparing the relationship between surface areas and lengths of prisms of the same volume seemed a far-fetched task for 83% of the participants, as seen in Figure 4.4. The reason for this could be ascribed and attributed to insufficient mastery of basic facts as well.

5.3 HOW LEARNERS DEFINED SURFACE AREA AND VOLUME

With 72% of the participants struggling to define volume precisely and only 28% of the population accurately defining it, the researcher reluctantly accepted that learners generally struggled to give definitions of this concept. It was alarming that most learners at this level lacked the conceptual understanding of the volume concept in order to define it, but rather resorted to giving the formula for finding volume of prisms. In addition to this lack of conceptual understanding, learners also interchanged volume with capacity. 17% of the studied learners confused volume with mass, weight and matter. This confusion of concepts was consistent with Sisman and Aksu's (2015) report.

When it came to the surface area, 17% of the participants could at least attempt to partly define the concept. Some of the learners who attempted to define surface area failed to explain the totality of the areas of faces enclosing a three-dimensional object. In attempting to describe areas of the 2-D shapes around a 3-D solid, learners ended up describing perimeter, thereby treating 3-D shapes as if they were plane shapes, a misconception also stated by Sisman and Aksu (2015). The rest of the participants (83%) accounted for surface area with many misconceptions. Some learners considered the surface area as space inside a shape while others confused it with volume, providing the formula for finding the volume of prisms. It was evident that learners had more difficulty in defining surface area than volume. In most cases, the surface area was mistaken for and interchanged with volume.

Only 17% of the participants attempted describing how to find the volume of prisms without using a formula; while 14% of the learners could accurately describe how to find surface area. Instead of providing the descriptions, the learners who were closer to a reasonable response simply applied the respective formulae. Definitions and descriptions did not seem to be part of the Grade 8 learners' curriculum.

5.4 HOW LEARNERS SOLVE PROBLEMS INVOLVING SURFACE AREA AND VOLUME

When Grade 8 learners in this research were given knowledge recall as well as routine procedural problems involving surface area and volume of prisms, they solved them with ease, provided they were given the distinct length, width and height of the prism. In this case, the learners recalled and applied the relevant formula in solving the problems, as seen in Question 3bs results, as shown in Table 4.4.

In test item 3b of this research, with distinct dimensions and units, 38% of the participants had correct computations and answers, against 62% who incorrectly responded or never attempted, as seen in Figure 4.4. However, even where learners applied the correct formula for finding the surface area of a prism, they failed to explain why they must find the areas of visible faces in isometric drawings and then multiply by two. They just used the formula without understanding it, a misconception concurring with Battista's (2003) findings.

Concerning the square units for surface area and cubic units for volume, learners could not deduce where the symbols came from but claimed it was just part of the formula. Also, when Grade 8 learners were given diagrams without units or dimensions and with differing edge lengths like in test items 3a; 3c and 3e, they faced challenges in finding surface area and volume. At least 78% of the participants had incorrect responses in these three types of problems, as shown in Figure 4.4. In fact, in such questions, the learners thought surface area and volume could not be computed, citing that the shapes were uneven, inconsistent or irregular. In addition, learners could not find surface areas and volumes of prisms that had gaps, such as in Test item 3d, where more than 90% of the participants responded either incorrectly or did not attempt to respond (Figure 4.4). In such problems, learners declared that gaps disqualified the shape from having surface area and volume. This misconception was attributed to poor connections between existing knowledge and new knowledge, consistent with Hiebert's (1986) findings.

In problems involving comparing volumes of different cubes, such as in Test item 4a, only 34% of the participants could manoeuvre well; the rest (66%) faced difficulties. A possible strategy would have been to find the number of arrays of smaller cubes that could be arranged on one layer and enumerate the number of layers. Most learners tried to find how many cubes fitted on each edge, thereby treating volume as one-dimensional length. This misconception confirmed Sisman and Aksu's (2015) work. When 66% of Grade 8 learners could not solve such a problem, it indicated gross deficiency in the learning process.

Problems like test item 4b exposed the learners' inadequacy of visual and spatial conservation, as only 7% of the participants could respond positively. At the Grade 8 level, one would have expected learners to operate well in symbolic or at least in iconic (pictorial) mode, but here most participants appeared to be at the enactive stage (Bruner, 1966), where they would need concrete (physical) objects to respond to this question. This explained that 50% of the interviewees declared they guessed in most parts of the question.

Test item 5 was purposefully included to compare with results from diagrams in items 3a-e, as this particular block was solid, with no gaps and having uniform dimensions. Fifty per cent (50%) of the participants could easily count the number of unit cubes making the block in 5a, even though one would have expected all 8th-grade learners to provide the correct response. While a couple of learners enumerated the blocks by identifying how many cubes formed the base or top layer and then multiplied by the number of layers, some learners simply applied the volume formula, using the number of units along the edges as their dimensions. The participants could not correctly enumerate cubes in arrays, confirming Battista and Clement's (1996) work. Twenty-five per cent (25%) of the interviewees treated 3-D objects as if they were 2-D shapes, where they repeated counting edges of the same block, resulting in incorrect responses. It was disheartening to note that most 8th grade learners could not associate the number of unit cubes in a block with its volume, as in 5b, where only 38% correct responses were achieved, a drop from 50% of 5a, with the same question asked differently. The participants indicated that they could only respond positively if the questions were asked in the same familiar manner. A further drop to 25% of the participants correctly responded to item 5c, requiring a surface area of the block, which proved that surface area was less understood than volume among the participants.

5.5 HOW LEARNERS DESCRIBE THE RELATIONSHIP BETWEEN SURFACE AND VOLUME

Regarding the comparison between the surface area and volume relationship, more than 83% of the learners did not notice that different sized prisms could have the same volume but different surface areas. Only as few as 12% of the interviewees could deduce, after intensive probing, that longer prisms had larger surface areas than shorter ones of the same volume. Figure 4.4 shows that none (0%) of the participants responded correctly to this type of problem (6e). Learners were only comfortable with routine operations of calculating surface area and volume but could not make conclusions using information from computed results. Some learners could get the ratio of surface area to volume correct but failed to explain what it meant, stating that they had no idea what the question required them to do. Only 6% of the participants could notice that different cuboids had the same volume but varying surface areas; nevertheless, they could not deduce the required relationship. This seemed a new concept for the learners, a question they had never seen before.

5.6 MISCONCEPTIONS THAT LEARNERS DISPLAYED WHEN DEALING WITH CONCEPT OF SURFACE AREA AND VOLUME

The grade 8 learner sample understudy displayed the following misconceptions in their tests and interviews. Some of the misconceptions confirm previous studies such as Battista and Clement (2003), while others were new discoveries from this study. I recorded the prevalent misconceptions in two sections. 'Section A' reports confirmed misconceptions evident in learners' work while 'Section B' highlights the ones discovered in this study but had not stated in the conceptual framework.

Section A- Confirmed misconceptions as found in literature and the learners' work:

While the following misconceptions had already been identified in literature, I re-arranged them here, according to the frequency of occurrence in my study:

1. Confusing the concept of volume with that of surface area, including interchanging their formulae

The pie chat on figure 4.69 indicates that this was the most prevalent confirmed misconception among the interviewees. At 35% of the interviewee presence, the misconception was evidenced in test item 1a-b by participants A3; N3 and G3 as shown in table 4.5

Table 4.4 showed that 24% of the participants got correct responses in test item 1b; 32% were partly correct; 34% had incorrect responses while 10% did not attempt defining surface area. Fifty percent of participants in Sisman and Aksu's (2015) study were reported to have displayed the misconception of confusing surface area with volume. The confusion of concepts seen in this section are attributed to insufficient mastery of basic facts and concepts.

Participant N3 defined surface area in test 1b as, *"...the length, width and height multiplied together of an object or shape, how it is calculated..."*

Besides presence in the item 1 definition question, the confusion of surface area with volume was also confirmed during the interviews with participants A3; G3; H3 and N3 in test item 6 of comparing the relationships between surface area and volume, with reference to table 4.5. In this question, table 4.4 indicated that 0% of the participants responded correctly to test item 6e; 10% were partly correct; 7% incorrect and 83% did not attempt answering the question.

2. Treating 3-D figures as 2-D shapes

This misconception was mostly evident and confirmed during interviews with participant G3; H3; N3 and Q3 in test item number 4, indicated on table 4.5. As much as 30% of the interviews proved the misconception of treating 3-dimensional objects as 2-dimensional figures. Shown in table 4.4 was 34% of the participants positively responding to test item 4a when 0% had partly correct responses, leaving 59% incorrect and 7% not attempted tasks. Sisman and Aksu (2015) found that learners produced formulae for volume, such as “volume = length + width + height”, a misconception of treating volume as a one-dimensional structure, and “volume = length x width”, thereby treating 3-D as 2-D figures. The general cause of this misconception, like many others, is insufficient mastery of basic facts and concepts.

In test item 4a, where learners had to find how many small cubes would fit in a larger one, participant N3 drew arrays and said, *“...I see 3 cubes on one side, 3 along the height; that is 9 on one face, therefore for 6 faces 9 x 6 = 54.”*

3. Counting cubes only from the visible faces in an isometric diagram

Figure 4.69 shows that counting visible faces or cubes was almost equally likely to incorrect enumeration of arrays as the sectors are seemingly equal. Table 4.5 shows that this misconception was most prevalent and confirmed in interviews with participants C3; G3 and H3 in test items 3 and 4. As shown in table 4.5, 20% of the interviewees counted only the visible faces of cubes in arrays. Table 4.4 showed that 0% of the participants gave correct responses in test item 3d, with 10% partly correct, 62% incorrect and 28% not attempting to answer the question. The next extract from participant C3 in test item 3b had proof the assumption. Instead of calculating the volumes of given prisms, learners in Sisman and Aksu (2015) just counted the visible faces of a unit cube and multiplied that number by three, with the justification that cuboids have three dimensions. I believe that it is insufficient mastery of basic facts and concepts which leads to this misconception

When asked to explain how surface area was obtained for test item 3b, participant C3 said, “...*I just calculated the surface area for this one, and this rectangle, and this last one, then I added them together.”* In this response, only the visible faces were considered.

4. Enumerating cubes in an array incorrectly.

Incorrect enumeration of cubes in arrays was most prevalent in test item 4b as shown on table 4.5 among participants G3; H3 and N3 interviews. The misconception was evidenced in 15% of the post-test interviews. Considering test item 4b, table 4.4 showed that 7% of the participants positively responding, when 24% had partly correct responses, leaving 62% incorrect and 7% not attempted task. In Voulgaris and Angelidou (2003), participants incorrectly counted visible and invisible cubes, resulting in failure to include the correct number of cubes in arrays. Insufficient mastery of basic facts and concepts could be the major cause of this misconception.

In an attempt to determine the number of painted faces on cubes detached from a block dipped in paint, test item 4b, participant G3 counted edges of the corner cubes rather than faces.

5. Believing that a shape has more than one surface area

None of the interviewees confirmed this misconception during the interviews; hence it was not captured in both table 4.5 list and graph of figure 4.69. However this misconception is largely caused by insufficient mastery of basic facts.

Section B: Misconceptions confirmed in the study but not in the conceptual framework

Here, I listed the other misconceptions about surface area and volume which were discovered in this study. The order is not related to occurrence. Some, while they appear similar to one or two of the ones listed in Section A, I believe that they have unique references. These have been dealt with much in chapter 4.5.

1. Substituting definition or description with formula: Giving the formulae for calculating surface area and or volume, when asked for definitions. Providing the formulae for calculating surface area and or volume of prisms instead of describing how to find surface area or volume without using any formula.

Table 4.6 indicates that this was that 50% of the interviewees displayed this misconception in various questions. The bar graph on figure 4.70 shows how dispersed this misconception was from the others. It was most prevalent that learners would define a concept by its formula in this study.

2. Belief that lack of dimensions (units) equals to no surface area and volume: Believing that surface area and volume cannot be found without having the measurements of prisms or cylinders. Assuming that a (3-D shape) solid object with unmarked units or dimensions has neither surface area nor volume.

Figure 4.70 shows the bar second longest in the comparison of the five discussed misconceptions. Table 4.6 records this misconception appeared in 24% of the interviewed participants. As shown in section 4.5, a lot of learners displayed the misconception of assuming that a shape without dimensions has neither surface area nor volume.

3. Complete-incomplete shape assumption: Believing that a solid object (3-D shape) composed of blocks and spaces is 'incomplete/ not level/ irregular/ inconsistent' (in the learner's perception) and therefore has neither surface area nor volume.

This appeared in 5% of the interviews, as shown in table 4.6. However, as shown in section 4.5 and table 4.3, many participants displayed this misconception in their written test.

4. Invisible surfaces dilemma: Counting only visible unit squares for surface area.

In addition to stating that irregular shapes had no surface area and volume, this was evidenced in 5% of the interviews shown in table 5.2. Similar to the assumption that irregular shapes do not have surface area and volume, this misconception was less evident during interviews but a lot of written scripts shown in section 4.5 proved otherwise.

5. Surface area to perimeter-summation equality and confusing volume with mass: Assuming that surface area is the same as summation of perimeters of each 2-D face on a 3-D object.

Together with confusing volume with mass, this misconception took up 16% of the interviews, as seen in table 4.6. It was very common for learners to call volume and mass the same thing, while surface area was equated to the sum of perimeters of the 2-D faces rather than their areas.

5.7 CAUSES OF MISCONCEPTIONS HELD BY LEARNERS IN RELATION TO THE CONCEPT OF SURFACE AREA AND VOLUME

5.7.1 Insufficient mastery of basic facts, concepts and skills

We have seen from the learners' responses, considering a lack of vocabulary to define concepts or describe processes involved in finding the volume or surface area, that there was insufficient mastery of basic facts. A learner who had grasped a considerable amount and knowledge of a phenomenon should be at liberty to use the relevant jargon to define, describe, classify, compare and contrast different mathematical concepts with others, among other topics. There is no better explanation of why an average 8th grade learner would fail to define, for instance, volume (a concept that begins in the 1st grade curriculum), other than insufficient mastery of basic facts, concept and skills as supported by Hwang et al., (2009).

With adequate content-coverage and concept mastery, a learner develops skills to solve problems in new situations. It was demonstrated in this study, as learners were recorded assuming that a solid shape (object) has neither surface area nor volume because no dimensions or units are given, or where learners conclude that a shape has no volume because it is rendered uneven. There were several strategies that grade 8 learners should have been able to use in describing the procedure for finding volumes of irregular shapes if there had been sufficient mastery of basic facts, the concept and skills. This, for instance could be by the displacement of liquids, then conversion of capacity units to volume. The same can be said of describing how to find surface area of irregular shapes, for instance using a cover over the shape, then finding the area of the cover.

5.7.2 Over-generalisation of basic facts (over-generalisation of the formulae)

In this study, the formulae for finding surface area and volume of prisms was over-generalised, to the extent of being used in place of a definition or description. Learners were given some non-familiar shapes to find their surface area and volume. In the case of blocks in research tool items 3a-e, where opposite faces of the shapes were not congruent, learners would simply find the area of one face then multiply by 2; just because the formula stated so. This misconception confirmed the work by Brodie (2010). Similarly with volume, where two opposite edges were not equal, learner would still use either edge as a dimension for length, width, or height to apply the formula. One notices that the formula had been used where it was not necessary. Where learners could just have counted the unit cubes to obtain the volume, a formula was arbitrarily used due to over-generalisation. No wonder some learners confidently claimed that mathematics was all about formulae; no formula = no mathematics.

5.7.3 Rote learning of calculating (routine) procedures without understanding

A substantial number of learners indicated that the earliest activities they remembered being given in surface area and volume were the formulae. Then they started applying it to given diagrams, which happened to be right prisms. This scenario points to rote learning of routine calculating procedures, where learners were expected to memorise the formula and apply it. This practice was the reason learners could not define volume and surface area; they simply did not have the vocabulary to do so, except for reciting the formula, without understanding with what they were dealing. When rote-learning was employed, learners were deprived of opportunities to discuss and discover the mathematics they were expected to learn—considering that learners had different learning styles and preferred different teaching strategies used to explore all their senses in learning. Rote-learning simply makes the learner a passive recipient in learning; hence it affects the mastery of concepts. Rote learning inculcates procedural knowledge but hinders the declarative knowledge needed for an understanding of the fact, concepts and development of mathematical skills. The result leads to a lack of conditional knowledge; hence we found learners unable to relate learnt concepts to new situations.

5.7.4 Poor connections between existing knowledge and new knowledge, resulting in insufficient mastery of basic facts

If we consider the interview with participant C, the learner inappropriately associated volume with 2-D shapes. The learner found a way to justify that a piece of paper had some kind of height (thickness). A learner who had the correct mastery of the volume concept, would easily have determined that volume was associated with 3-D objects but not plane shapes like the rectangular piece of paper. Rote-learning of the formula resulted in learners' associating the volume with the formula. No wonder learners just made use of the formula where they could simply have counted the unit cubes as defined by Hiebert (1986) and Skemp (1976).

5.8 CONCLUSION

In this qualitative research, following constructivists' theoretical framework, I used a case study approach to explore misconceptions that Grade 8 learners display when they are dealing with surface area and volume problems. I administered a scholastic test with items on the definition, description, routine procedures, problem solving and relationships between surface area and volume questions. Data were collected through testing a class of 29 learners and interviewing ten of them. Using a narrative analysis of the data and Battista and Clement's (2003) work as my conceptual framework, I discussed my participants' misconceptions. The possible sources and or causes of identified misconceptions were also examined, considering the data collected in this study.

The identification of possible causes of misconceptions in surface area and volume from this study led to the suggestion of the following implications and recommendations regarding teaching and learning of the concepts.

5.9. IMPLICATIONS AND RECOMMENDATIONS FOR TEACHING AND LEARNING SURFACE AREA AND VOLUME

Most of the misconceptions in teaching and learning surface area and volume revolve around insufficient mastery of basic facts, concepts and skills caused by rote learning, poor teaching-learning approaches, and improperly planned curricula. This implies that for the misconceptions to be addressed and minimised, high standards of planning, presentation and evaluation is required across the board, from curriculum planning to lesson delivery in the classroom.

The researcher recommends spending time in defining and describing concepts prior to deriving formulae for procedural purposes. Learners should also derive the formulae as opposed to being given such during topic introductions. A lot of examples and non-examples of concepts should be at learners' disposal for proper conceptual development.

Teachers need to desist from employing traditional teaching approaches that emphasise lecturing and indoctrinating information (knowledge) to learners. The 21^{st} century learners are characterised by multiple intelligences, requiring the exploration of different teaching and learning styles in concept formation. Numerous forms of lesson delivery or concept presentation should be applied in methods that prompt inquiry-based learning, to motivate active involvement of the brain. Concepts must be presented and formed in multiple ways, giving multiple examples and non-examples of the concepts. The study revealed that learners were only exposed to finding the surface area and volume of right prisms with uniform dimensions. Participants considered this "complete", but what about the other types of prisms and when there are no dimensions or units, and one cannot use any formula?

To enhance understanding of surface area and volume, learners should move between the five representations or models of concepts as identified by Van de Walle (2004) which are:

1. Manipulative models
2. Pictures
3. Written symbols
4. Oral language
5. Real-world situations

Learners need to be able to define concepts in oral language, explain procedures represented by symbols in a written formula, draw diagrammatic representations of situations and use manipulative models to demonstrate or solve problems constructed from real-life situations. Only when learners can freely move between the above representations, will they master the concepts.

In addition to helping learners move through the five conceptual representation modes, teachers need to follow the six-stage theory of learning mathematics (Dienes, 1960), when presenting learners with concept development activities. The following order of conceptual development steps that the theory suggests, is a good strategy teachers can use.

1. Free play (free experimenting): Familiarisation of the situation or concept confronted through "trial and error" until some regularity begins to emerge.
2. Rules of the game (play by the rules): Use regularities to formulate or invent rules that match the situation (rules can then be used to create a game).
3. Comparison stage/Abstract: Let games be discussed and compared to each other for similarities and differences to confirm the common rule, structure or elements embodied in different games.
4. Diagrammatic representation/Representation stage: Use diagrams, tables, arrows or coordinate systems to link/map the abstraction.
5. Symbolisation stage: Use language to identify and describe properties of the yielded representations and discoveries. The used language can approximate the language conventionally used by mathematicians, though exercising freedom to invent new symbols.
6. Formalisation/Generalisation stage: Establish some order in the maze of descriptions; deduce other properties, realisation of axioms, theorems and proofs.

When we desist from telling or giving learners the formula right at the introduction of a lesson and follow the above six stages in our teaching of any mathematical concept, our learners will truly comprehend the learnt material and then find meaning and interest in the subject well. This strategy would help minimise misconceptions in surface area and volume. We also notice from the theory of learning mathematics, that Dienes (1960) emphasises play and discovery in meaningful learning.

In respect of planning, it is evident that the proposed six-stages of learning mathematics concepts can never become a reality if the curriculum is rigid and assessment focused. Annual teaching plans should allow enough time for proper concept development with the learner in mind. More time should be given for covering specific concepts in-depth at a specific grade level, then there would be better mastery of facts, concepts and skills than when every topic reappears each year, and there is limited time to cover it, resulting in insufficient mastery. In most instances, the vocabulary and language necessary for concept development in mathematics are overshadowed by routine computational procedures, leading to many misconceptions. Learning needs to follow the above discussed steps in concept development to maximise the understanding of surface area and volume concepts.

Some of the causes of learner misconceptions in learning concepts such as surface area and volume are rushing to cover the curriculum, teaching-learning strategies and curriculum structure, though not part of the findings in this study. These three possible causes of misconceptions are discussed below.

Rushed-curriculum coverage

One of the reasons for misconceptions in the teaching-learning of mathematics concepts is that of a "rushed curriculum". To cover the curriculum in the given time, teachers can be indirectly forced to rush through the curriculum and move to the following week's learning material, leaving a lot of learners behind.

The major push factors for rushing curricula coverage could be the compliance to pre-set milestones and work schedules from the school, cluster, circuit, district, provincial or national authorities. In some cases, a topic is hurriedly covered because it was included in some assessment during the planning phase. The assessment date could be due, while learners were behind schedule for one reason or the other. Rushed-curricula are mostly content-oriented rather than being child centred.

Spiral curriculum-structure challenge

In as much as there are numerous advantages of a spiral curriculum, one disadvantage thrust among both learners and teachers is complacency. An educator in the lower grade realises that a concept is not well mastered, but does nothing about it with the view that the learner will catch-up the next year as the same topic will be repeated. This postponed mastery of concepts becomes a barrier to learning the following year as a background of misconceptions, blocking assimilation and accommodation of new knowledge, hindering equilibration.

On the other hand, a learner who learns the same concept year in and year out becomes bored each time the topic is introduced. Some learners tend to become over-confident that they already know the lesson content, resulting in less attentiveness and they are likely to lose-out. This particular learner can also become disruptive and hamper mastery of concepts in the process. I have seen the same content coverage of volume from intermediate phase to senior phase in our current CAPS curriculum in some grade, if the same activities were done, in say four consecutive years, truly the third and fourth years will be wasted. Total surface area is introduced in the curriculum at around Grade 7. If concepts were not well mastered at the initial stages, an attempt to reintroduce the concept at Grades 8 and 9 is met with a negative attitude by the learner, as the connotation would be that they were not paying attention in previous years. De-teaching a wrongly conceived concept is a real tall order for teachers, let alone learners, so is re-teaching a concept, resulting in inadequate mastery of the concept.

Teaching-learning strategies

In constructivists' philosophy, it is argued that a teacher shapes the way the learner grasps and understands concepts, learning does not just happen on its own, as stated in Adom, Yeboah and Ankrah (2016). The learning environment has a great impact on the conceptualisation of new schema. The teaching-learning styles play a pivotal role in the learning of mathematics. This is where teachers' integral roles become critical. Poor lesson preparation, presentation and evaluation all end in poor concept mastery; so does a poorly executed teaching-learning strategy. Learning by only utilising one strategy, approach and type of activity reduces the learner's enthusiasm, resulting in a barrier to concept formation.

Many participants in this study indicated that their first experiences in surface area and volume learning they could remember were being introduced to a formula and then starting manipulating figures after that. This is clearly not a good approach and strategy to teaching concept formation, hence the discovered learners' misconceptions. It is also possible that poor teaching-learning strategies were caused by the lack of knowledge of the curriculum, supported by Baturo and Nason (1996).

5.10. RECOMMENDATIONS FOR FURTHER STUDY

Since the understanding of surface area and volume revolves around the teaching and learning styles, approaches and activities (environment), it could be worthwhile to study or research the study areas listed below. The list is not exhaustive but proposed as a guide to address other factors facilitating misconceptions in the learning surface area and volume.

1. Teachers' approaches to teaching surface area and volume
2. Content presentation in sources of material (books etc.) in teaching surface area and volume
3. Strategies that maximise curriculum implementation of surface area and volume
4. Hidden curriculum practices that foster or hinder understanding of surface area and volume
5. Concept development of surface area and volume
6. Determining how much concept development of topics like surface area and volume is covered in teacher training institutions.
7. Level of teacher support in concept development, e.g. surface area and volume.

5.11. CHAPTER SUMMARY

I began this chapter by highlighting the aim of the study, the objectives and the research questions thereof to determine the degree of coverage through the entire research process. In this chapter, the findings of the research were discussed. This study's findings pointed to a couple of implications for the classroom teacher and curriculum planner, which I also highlighted in this chapter. The discussed implications can go a long way in minimising misconceptions in the learning of surface area and volume. Finally, topic areas on recommendations for further study have been given. It is my hope that covering the recommended areas of study will enrich the mathematical body of knowledge and facilitate maximum understanding of the concept.

6 REFERENCES

Adom, D., Yeboah, A., & Ankrah, A.K. (2016). Constructivism philosophical paradigm: implications for research, teaching and learning. *Global Journal of Arts Humanities and Social Sciences. 4 (10), 1-9- researchgate.net.*

Ashlock, R. B. (1990). Parents can help children learn mathematics. *The Arithmetic Teacher*, *38*(3), 42-46. https://doi.org/10.5951/at.38.3.0042

Battista, M.T. & Clements, D.H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education, 27*(10), 258-292. https://doi.org/10.2307/749365

Battista, M. T. (2003). Levels of Sophistication in Elementary Students Reasoning about Length. *International Group for the Psychology of Mathematics Education*, *2*, 73-80. https://doi.org/10.5951/tcm.13.3.0140

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, *31*(3), 235-268. https://doi.org/10.1007/bf00376322

Berger, P. L., & Luckmann, T. (1967). Aspects sociologiques du pluralisme. *Archives de Sociologie des Religions*, 117-127. https://doi.org/10.3406/assr.1967.2621

Blumer, H. (1969). Fashion: From class differentiation to collective selection. *The Sociological Quarterly*, *10*(3), 275-291. https://doi.org/10.5040/9781847887153.v4-0103

Borko, H., Eisenhart, M., Brown, C., Underhill, R., Jones, D., & Agard, P. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*(3), 194-222. https://doi.org/10.2307/749118

Braun, V & Clarke, V. (2013). *Successful Qualitative Research; a practical guide for beginners.* London: Sage.

Brodie, K. (2010). *Teaching Mathematical Reasoning in Secondary School Classrooms*. New York: Springer.

Bruner, E. M. (1956). Primary Group Experience and the Processes of Acculturation 1. *American Anthropologist*,*58*(4). 605-623. https//doi.org/10.1525/aa.1956.58.4.02a00030

Cassandra, M. (2014, June 23). Types of Qualitative Testing Methods. www.gutcheckconstellation/qualitative-test: http://www.gutcheckconstellation/qualitative-test

Creswell, J. (2013). *Qualitative Inquiry & Research Design: Choosing Among Five Approaches (3rd ed.).* London: Sage.

Dewey, J. (1916). Nationalizing education. *Journal of Education, 84*(16), 425-428. https://doi.org/10.1037/e575312009-034

Dienes, G. J. & Damask, A.C. (1960). Theory of vacancy annealing in impure metals. *Physical Review*, *120*(1), 99. https://doi.org/10.1103/physrev.120.99

Drews, D. (2005). Children’s mathematical errors and misconceptions: perspectives on the teacher’s role. Children errors in mathematics. *Understanding Common Misconceptions in Primary Schools, 2*(12), 14-21.

Dorko, A., & Speer, N. M. (2013). Calculus students’ understanding of volume. *Investigations in Mathematics Learning*, *6*(2), 48-68.https://doi.org/10.1080/24727466.2013.11790332

Dorko, A., Speer, N. (2013). Calculus students', understanding of volume. In S. Brown, G., Karakok, K., Roh, & Oehrtman (Eds.). *Proceedings of the 17th Annual Conference on Research in Undergraduate Mathematics Education (Pp. 190-204).* Denver, CO: University of Northern Colorado.

Education, D. o. (2016). *Annual National Assessment (ANA) Diagnostic Report .* Pretoria: Department of Education .

Flick, U. (2011). Introducing *Research Methodology: A Beginner's Guide to Doing a Research Project.* Thousand Oaks: SAGE.

Gomm, R. (2008*). Social Research Methodology: A Critical Introduction* (Second ed.). New York: Palgrave Macmillan.

Hadjidemetriou, C. & Williams, J. S. (2002). Children's graphical conceptions*. Research in Mathematics Education, 4,* 69-87. https://doi.org/10.1080/14794800008520103

Hatano, G., & Inagaki, K (1996). Young children's recognition of commonalities between animals and plants. *Child Development*, *67*(6), 2823-2840.

Hiebert, J. (1986). A theory of developing competence with written mathematical symbols. *Educational Studies in Mathematics, 19*(3), 333-355. https://doi.org/10.1007/bf00312451

Miles, M.B., Huberman, A.M. & Saldaña (2018). *Qualitative Data Analysis: A methods sourcebook (Fourth ed.).* London: Sage.

Hwang, W.-Y., Su, J.-H., Huang, Y.-M., & Dong, J.-J. (2009). A Study of Multi-Representation of Geometry Problem *Solving. Educational Technology & Society, 12*(3), 229–247. https://doi.org/10.1007/978-3-540-73011-8_44

Jaworski, B. (2008). Building and sustaining enquiry communities in mathematics teaching development. Teachers and didacticians in collaboration. In The Handbook of Mathematics Teacher Education, 3(Brill Sense), 309-330. https://doi.org/10.1163/9789087905491_015

Kramer, R. M. (1996). Divergent realities and convergent disappointments in the hierarchic relation. *Trust in organizations: Frontiers of Theory and Research*, 216-245. https://doi.org/10.4135/9781452243610.n11

Leedy, Paul D. and Ormrod, Jeanne Ellis. (2019). *Practical Research: Planning and Designing* (Twelfth ed.). New York: Pearson.

Legutko, M. (2008). An analysis of students' mathematical errors in the teaching-research process. Handbook for mathematics teaching. *Teacher Experiment. A Tool for Research, 1*(17), 141-152.

Locke, J. (1948). *An essay concerning human understanding*. Kay & Troutman.

Machaba, F. (2005). *Insights and misconceptions that Grade 10 learners in one school in Soshanguve have with the concepts of area and perimeter.* Unpublished BEd. Honours project. Johannesburg: Faculty of Education University of the Witwatersrand.

Machaba, F. (2018). Insights and misconceptions of Grade 10 learners. Pythagoras. *The concepts of area and perimeter, 37*(1), 1-11. https://doi.org/10.4102/pythagoras.v37i1.304

Michie, S., McDonald, V., & Marteau, T. (1996). Understanding responses to predictive genetic testing: a grounded theory approach. *Psychology and Health*, *11*(4), 455-470. https://doi.org/10.1080/08870449608401982

McMillan, J & Schumacher, S. (2014). *Research in Education: Evidence-Based Inquiry* (7th ed.). Edinburgh: Pearson.

Murphy, E and Dingwall, R. (2001). *Qualitative Methods and Health Policy Research.* London: SAGE.

Piaget, J. (1974). Piaget's theory. In *Piaget and his school* (pp. 11-23). Heidelberg.

Porter, M. E. (1989). From competitive advantage to corporate strategy. In *Readings in Strategic Management* (pp. 234-255). Palgrave. Prestage, S. and Perks, P. (2001). Models and Super-Models: Ways of Thinking About Professional Knowledge in Mathematics Teaching. *Research in Mathematics Education, 3(*1), 101-114. https://doi.org/10.1080/14794800008520087

Mbedzi, S. & Samson, D (2013). Investigating surface area and volume of rectangular prisms. *Learning and Teaching Mathematics. 15(1), 3-6. https://hdl.handle.net/10520/EJC146076*

Sáiz, M. (2003). Primary Teachers' Conceptions about the Concept of Volume: The Case of Volume-Measurable Objects. *International Group for the Psychology of Mathematics Education, 4*(7), 95-102.

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number–space mappings: What underlies mathematics achievement?. *Journal of Experimental Child Psychology*, *114*(3), 418-431. https://doi.org/10.1016/j.jecp.2012.10.012

Sisman, T.G. & Aksu, M. (2015). A study on sixth grade students' misconceptions and errors in spatial measurement: Length, area, and volume. *International Journal of Science and Mathematics Education [online], 14*(7), 1293–1319. https://doi.org/10.1007/s10763-015-9642-5

Schmidt, F. L. (2004). *Methods of meta-analysis: Correcting error and bias in research findings*. London: Sage.

Sloan, C. Gough, B & Connor, M. (2010). *Successful Qualitative Research : A practical guide for the beginners.* London: Sage.

Smith, J.P., diSessa, A.A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *Journal of the Learning Sciences, 3*(2), 115–163.http://dx.doi.org/10.1207/s15327809jls03021. https://doi.org/10.1207/s15327809jls0302_1

Taylor, Steven J., Robert Bogman& Marjorie, L. DeVault. (2016). *Introduction to Qualitative Research Methods: A Guidebook and Resource* (4th edition ed.). New York: John Wiley and Sons.

Thompson and Preston. Integrating Measurement across the Curriculum, 2004.

Tůmová, V. (10, Feb 2017). *What influences grade 6 to 9 pupils' success in solving conceptual tasks on area and volume*. Dublin: CERME .

Van Hiele, P. M. (1986*). Structure and insight: A theory of mathematics education*. https://pdfs.semanticscholar.org/f13d/86aea2cda3779d03a32ae11fd86f0dcb0067.pdf

Van de Walle, J. (2007). *Teaching Student-Centred Mathematics: Grades K-3. Education Review.* Boston: Pearson Education.

Van de Walle, J.A., Karp, K.S., & Bay-Williams, J.M. (2014). *Elementary and middle school mathematics: Teaching developmentally* (Eds ed.). Boston, M.A: Pearson Education.

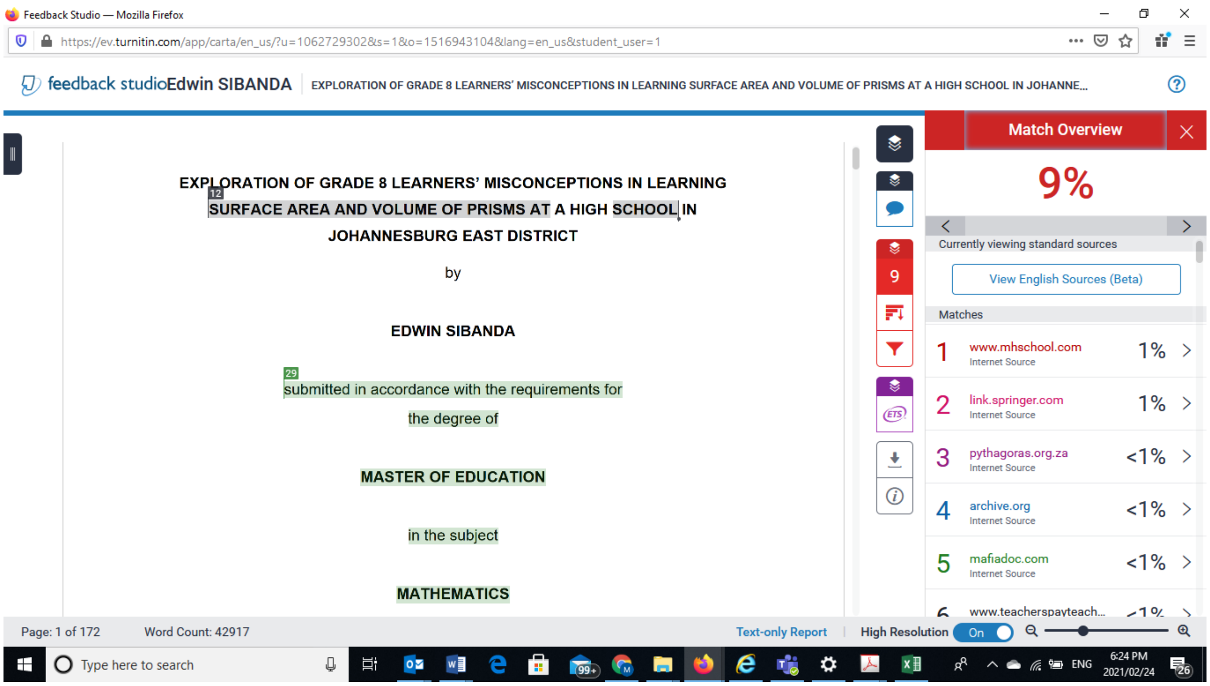
Vincent, J. 2. (2005). Maths World Year 7. AU: Macmillan Education.

Von Glasersfeld, E. (1974). Piaget and the radical constructivist epistemology. *Epistemology and education*, *1*, 24. https://doi.org/10.4324/9780203454206_chapter_1

Voulgaris, S & Evangelidou, A. (2003). *Application of Mathematical concepts across all disciplines: an example. Application of mathematical concepts across disciplines: an example.* (pp. 123-149). Cyprus: Cyprus Ministry of Education.

7 LIST OF APPENDICES

7.1 APPENDIX -A - TURN-IT-IN REPORT



7.2 APPENDIX-B- ETHICAL CLEARANCE LETTER

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/06/10

Ref: **2020/06/10/49813536/15/AM**
Name:Mr E Sibanda
Student No.: 49813536

Dear Mr E Sibanda

Decision: Approved on condition that all the recommendations below are effected on or before 19 June 2020

Researcher(s): Name: Mr E Sibanda
E-mail address:49813536@mylife.unisa.ac.za
Telephone:0833136109

Supervisor(s): Name: Prof. M.F. Machaba
E-mail address:emachamf@unisa.ac.za
Telephone: (012) 429 8582

Title of research:

Exploration of grade 8 learners' misconceptions in the learning of surface area and volume at a high school in Midrand, Gauteng Province.

Qualification:MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research.

Research Ethics Committee Recommendations:

Adjust application in terms of responding to the country's prevailing COVID-19 protocols.

3.7.2 The teacher from the school (who will help with conducting the test) is not indicated here.

3.7.3 • Under (sample) Size, the researcher indicates that 3 x 4 = 12 learners will be purposively selected, while under 'Participant selection' he indicates "about eight". • Provide scientific justification for participant selection.

3.7.4 b) Semi-structured interviews do typically have a list of guiding questions. c)

Especially if learners' faces will be obscured, this data collection instrument does not seem to be needed. What will be the justification of the video recording when participants write the test, shouldn't it be during the teaching and learning? 3.8 Now the sample selected is indicated as ten; ensure consistency. *3.9 While 3.7.4 b) indicates both initial, semi-structured interviews and follow-up unstructured interviews, here it looks like only a single interview? 4.2 Talking to affected learners' subject teacher and principal compromises confidentiality and anonymity.* *5.1 In Gauteng, permission is obtained directly from the provincial office, and not the district office. Typically, researchers are only allowed to conduct research from February until the end of the third quarter (September) of the school year. The sub-research questions indicate that the researcher would like to obtain data "at the end of Grade 8" – this needs to be reconsidered.* *Appendix B 4.7 Only single interview seems to be indicated?* *4.9 If research will take place in the second and third terms, the sub-research questions need to be accordingly adjusted.* *Appendix C falls away; once permission is granted from the provincial office, the district office only needs to be informed.* *Appendix D • "ten participants"? • Will follow up interviews be recorded?* *Appendix E • Follow up interviews not mentioned. • The consent form has to have a part where participants grant permission for audio, the interview and video recording of their work. • Please remove name of school to align with anonymity; you may replace with: "I obtained your contact details from the principal and administrator of your school."* *Appendix H falls away, as it is covered by Appendices E and F.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

7.3 APPENDIX-C-LANGUAGE EDITOR'S CERTIFICATE

Member South African Translators' Institute
www.language-services.online

PO Box 3172
Lyttelton South
0176
22 February 2021

TO WHOM IT MAY CONCERN

The thesis titled "Exploration of Grade 8 learners' misconceptions in learning surface area and volume of prisms at a high school in Johannesburg east district" has been proofread and edited for language by me.

I verify that it is ready for publication or public viewing in respect of language and style, and it has been formatted as per the prescribed style of the journal.

Please note that no view is expressed in respect of the subject-specific technical contents of the document or changes made after the date of this letter.

Kind regards

Anna M de Wet

BA (Afrikaans, English, Classical Languages) (Cum Laude), University of Pretoria.
BA Hons ((Latin) (Cum Laude), University of Pretoria.
BA Hons (Psychology), University of Pretoria.

7.4 APPENDIX- D-GAUTENG DEPARTMENT OF EDUCATION PERMISSION

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**25 June 2020**
Validity of Research Approval:	**04 February 2020 – 30 September 2020** **2019/502**
Name of Researcher:	**Sibanda E**
Address of Researcher:	**510 Newprot Flat**
	Beria Park
	Pretoria
Telephone Number:	**0833136109**
Email address:	**49813536@mylife.unisa.co.za**
Research Topic:	**Exploration of grade 8 learners 'misconceptions in the learning of surface area and volume at a high school in Midrand ,Gauteng Province**
Type of qualification	**Master's in Education**
Number and type of schools:	**1 Secondary School**
District/s/HO	**Johannesburg East**

<u>Re: Approval in Respect of Request to Conduct Research</u>

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

25/06/2020

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of COVID 19 pandemic researchers can ONLY collect data online, telephonically or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate. The approval letter will then indicate the type of arrangements that have been made with the school.***
4. ***The Researchers are advised to make arrangements with the schools via Fax, email or telephonically with the Principal.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 25/06/2020

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

7.5 APPENDIX-E-PERMISSION LETTER TO THE PRINCIPAL

UNISA university of south africa

510 Newport

208 Scheiding Street

Pretoria

0002

Mr B. Moyo
The Principal
Glen Austin High School
38 Hampton Rd Glen Austin
Midrand
1685
Tel: 011 023 7340/1/2
Email: admin@glenaustinhigh.co.za
Admin cell: 065 641 2759

16 May 2020

RE: Request for permission to conduct research at Glen Austin High School.

Title of the research: Title: Exploration of grade 8 learners' misconceptions in the learning of surface area and volume at a high school in Midrand, Gauteng Province.

Dear Mr Moyo

I, Edwin Sibanda, am doing research under the supervision of Professor France Masilo Machaba, a professor in the Department of Mathematics Education towards a Master of Education Degree at the University of South Africa. We have no funding from any organization. We are inviting you to participate in a study entitled, "Exploration of grade 8 learners' misconceptions in the learning of surface area and volume at a high school in Midrand, Gauteng Province."

The aim of the study is to find, identify, describe and explain causes and sources of misconceptions in the learning of volume and surface area at Grade 8 level, from the learners' perspective, in the hope to have an insight on why learners develop misconceptions then establish how these can be minimized.

Your school has been selected because it deals with learners who constitute the target group. Furthermore, your school is closer to my work place, as a result, I will not waste a lot of time travelling from my workplace to the research site, thereby reducing disruptions during learning time.

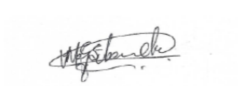
The study will entail a scholastic test to a class of Grade 8 learners, which will be followed by individual interviews to a selected sample of learners not exceeding ten participants. The test period can be videotaped but the interview will definitely be voice recorded. The information gathered from the test results and the transcribed interviews will then be analysed in order to draft conclusions regarding causes and sources of learners' misconceptions around surface area and volume conceptualization.

The benefits of this study are that educators will understand more of how and why learners manifest misconceptions in learning mathematics, especially the concept of surface area and volume. It will also give the participants an opportunity to express their opinions on how they create misconceptions, during the interview sessions. The follow up interviews will help the participants correct some easily identified errors in conceptualization of the covered concept.

Potential risks are emotional stress in the case of learners that would not have done very well in the test. This could harm the individual learners' self esteem, resulting in a negative impact on one's learning experiences and style. However, the potential risks have been planned for as learners' responses will be confidentially and safely stored. If any learners are identified to be emotionally affected by participating in the research, I will refer these to the educational psychologist for therapy and professional guidance thereafter.

There will be no reimbursement or any incentives for participation in the research.
Feedback procedure will entail a participants group discussion of the findings as well as email sent to the school and the department of education officials. If there could be any issues you need to know regarding this study, please feel free to contact me at 0833136109 or email 49813536@mylife.unisa.ac.za. You can also contact my supervisor, Professor France Masilo Machaba at (012) 429 8582 or email emachamf@unisa.ac.za.

Yours sincerely



Edwin Sibanda

7.6 APPENDIX-F-PARENTAL CONSENT

UNISA university of south africa

EXAMPLE OF A LETTER REQUESTING PARENTAL CONSENT FOR MINORS TO PARTICIPATE IN A RESEARCH PROJECT

Dear Parent

Your _______child> is invited to participate in a study entitled___________(add title **exactly** as it appears on your CEDU REC Application Form).

I am undertaking this study as part of my ________________master's research at the University of South Africa. The purpose of the study is _________________ and the possible benefits of the study are the improvement of _______________. I am asking permission to include your child in this study because _______________. I expect to have ___________other children participating in the study.

If you allow your child to participate, I shall request him/her to(delete what is not applicable):

- Take part in a survey(explain procedures, when, where, time to complete survey)
- Take part in an interview(explain procedures, when, where, time to complete survey)
- Take part in a group interview(explain procedures, when, where, time to complete survey)
- Complete a test (explain procedures, when, where, time to complete survey)
- Other(special attention must be given creating and using video recordings).

If you are going to use audio/video recording during the interview/group interview, you must indicate it and ask permission to record the interviews

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study(if, however, there are any risks involved in your study, they should be mentioned here). Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are _________________ (indicate benefits). Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities (or state when, if at an alternative time) with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available (state what the alternative activity will be).

In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password ~~locked –~~d computer in my locked office for five years after the study. Thereafter, records will be erased.

The benefits of this study are ________________________ (indicate realistic benefits)
Potential risks are_____________________________ (if no risk is involved also state it)
There will be no reimbursement or any incentives for participation in the research.

If you have questions about this study please ask me or my study supervisor, Prof/Dr____________(supervisor's name), Department of ____________,College of Education, University of South Africa. My contact number is_____________and my e-mail is_____________.The e-mail of my supervisor is______________________. Permission for the study has already been given by _____________________ (DET/principal/SGB etc.) and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:

Sincerely

______________________________ ____________________________ ________________

Parent/guardian's name (print) Parent/guardian's signature: Date:

Edwin Sibanda

Researcher's name (print) Researcher's signature Date: ________________

7.7 APPENDIX-G-LEARNERS' ASSENT-FORMS

EXAMPLE OF A LETTER REQUESTING ASSENT FROM LEARNERS IN A <u>SECONDARY SCHOOL</u> TO PARTICIPATE IN A RESEARCH PROJECT

Title of your research (**exactly** as it appears on your CEDU REC application form)

Dear ____________ Date

____

I am doing a study on ______________________ as part of my studies at the University of South Africa. Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your _________________ (teachers, counsellors, coaches, etc.) can use to_______________better. This may help you and many other learners of your age in different schools.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

Indicate what the child's participation will entail.This is an example: I would like toask you ___________________(questions/interview you about.../complete a questionnaire about....../involve you in a focus group (a group of 6 or 8 participants). Answering the ______________ (questions/completing the questionnaire/discussion in the focus group will take no longer than)______________________ (indicate the time it will to complete).

I will write a report on the study but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you don't want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

The benefits of this study are __________________________ (indicate realistic benefits)

Potential risks are________________________________ (if no risk is involved also state it)

You will not be reimbursed or receive any incentives for your participation in the research.

If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at ___________________ (insert contact number).Do not sign the form until you have all your questions answered and understand what I would like you to do.

Researcher: ______________________________ Phone number:

____

Do not sign the written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

____________________________ ____

Learner's name (print):Learner'ssignature:Date:

____ ________________________

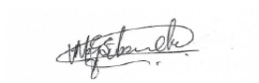
Witness's name (print) Witness's signature Date:

(The witness is over 18 years old and present when signed.)

____________________________ ____________________________

____

Parent/guardian's name (print) Parent/guardian's signature: Date:



Edwin Sibanda ________________________

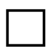
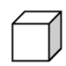
Researcher's name (print) Researcher's signature: Date:

7.8 APPENDIX- H-TEST INSTRUMENT

This became the research instrument after all considerations and corrections had been done.

Grade 8 Surface Area and Volume Test

Learner: A / B / C / D / E / F / G / H / I / J / K / L / M / N / O / P / Q / R / S / T / U / V / W / X / Y / Z

Answer all of the questions in the spaces provided, show all calculations where necessary. In this test, a  represents a unit square, while a  represents a unit cube, unless stated otherwise.

1. Define the following terms:

Volume....

Surface area....

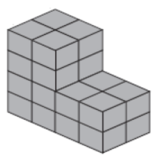
2. Describe or explain how you would find the following, without using any formula:

The volume of a prism or cylinder....

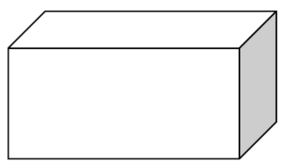
The surface area of a prism or cylinder....

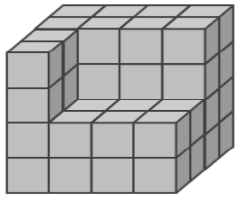
3. State whether each of the following blocks has surface area and volume. If they do, find the surface area and volume is in each case. If any of these blocks neither has volume nor surface area, provide a reason. Use the spaces in the table on the next page for your answers.

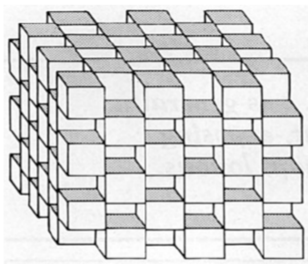
A

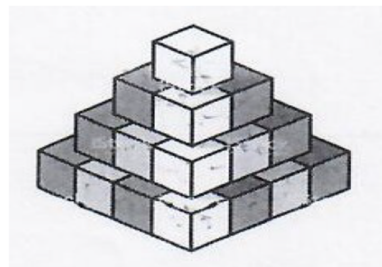


B



C 

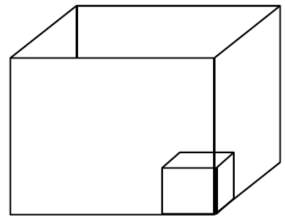
D 

E 

Block	Surface Area	Volume
A		
B		
C		
D		
E		

4. a. Use the sketches below to answer the questions that follow: A big 6 cm cube box is filled with 2cm cube dice as in the illustration below.

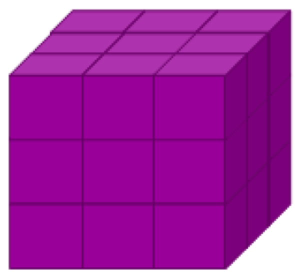
How many dice will fit exactly into this box?



____

____

4. b. The following block of stacked cubes was dipped in purple paint and dried. Later, the cubes were separated from the stack and observed individually.



How many cubes do you think will have the following features?

i. Four faces painted purple ____________________

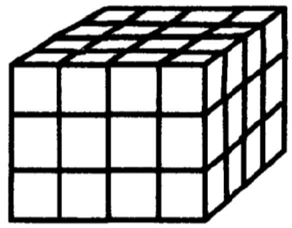
ii. Three faces painted purple ____________________

iii. Two faces painted purple ____________________

iv. One face painted purple ____________________

v. No face painted purple ____________________

5. The structure below is entirely filled with cubes, there are no gaps inside. Examine this structure and answer the questions that follow.



a. How many unit cubes will it take to construct the structure above?

____

b. What is the volume of this structure?

____

____

___

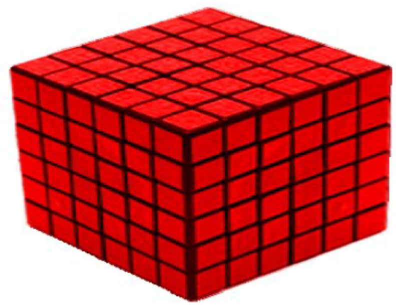
c. Find the surface area of the structure above.

____

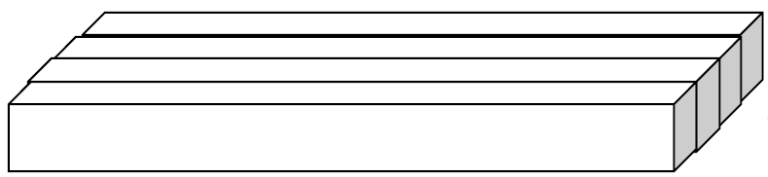
____

___

6. A cube made from 216 identical smaller cubes of 1 cm^3 each is changed into rectangular prisms of one layer, where no two cubes are placed one above the other. This can be done in 8 different ways. Sketch diagrams of any other three ways of doing this then find the surface area and volume of your sketches. The first one has been done for you as an example. Write a paragraph to describe or compare the relationship between the surface area and the volumes of the different cuboids (rectangular prisms) you have sketched formed.



My first sketch of rectangular prism   A



Use this table to compare the relationship between surface area and volume of cuboids.

Shape letter name and dimensions	Surface Area	Volume	Relationship of surface area to volume
A Length – 54 cm Width – 4 cm Height - 1 cm	(1 cm x 54 cm) x 2 + (1cm x 4cm) x 2 + (4cm x 54cm) x 2 = (108+8+432)cm² = 548 cm²	54 cm x 4cm x 1cm =216 cm³	548:216 = 274:108 =137:54
B			

C			
D			

Write your paragraph and conclusion upon your findings here:

____

____

____

____

____

7.9 APPENDIX-I-TEST MEMORANDUM

Define the following terms:

Volume

Volume is the amount of space occupied by a three dimensional object. Volume measures the amount of space an object substance takes up.

Surface area

Surface Area refers to the total area of all the faces on a 3-D shape or solid object. It tells us how much material is required to cover all the 2-D faces of a 3-D object.

2a. How would you find the volume of a prism or a cylinder?

I would find the volume of a cylinder or cube by filling it with 1 cm cubes then count the number of cubes thus used. Alternatively, I could fill it up with water, pour the water into a measuring cylinder, convert the measurement from millilitres to cubic centimetres, thus finding the required volume. When using the formula, I would find the area of the base of the prism or cylinder, then multiply it by the height thereof.

b. How would you find the surface area of a prism or a cylinder?

I would draw the net of all the faces of the prism or cylinder on a flat sheet then find the individual areas of each 2-D shape formed, then add the respective areas together, thus obtaining the total surface area required.

Question 3: Surface area and volumes of the five given blocks

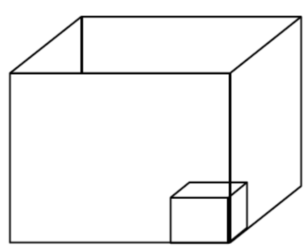
Block	Surface Area	Volume
A	This shape has six faces and we can find the area of each face and add them together, therefore it has surface area. The top and the bottom both have 8 unit squares each. The right side and the left	Since the block is made of unit cubes, it therefor has volume as well. The volume of this block is equal to the amount of the unit cubes used to build it. In this case it is 24 unit cubes, counted 12 unit cubes

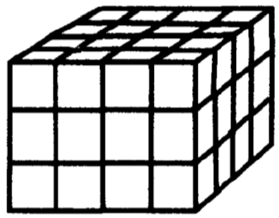
	side both have 8 unit squares as well. The front side and the back both have 12 unit squares each. Hence the total surface area of the block will be eqaul to 4 x 8 unit squares + 2 x 12 unit squares = 32 + 24 unit squares = 56 unit squares.	in the front side (column) and the other 12 behind or 2nd column of cubes.
B	As this is a solid cuboid (rectangular prism) with 3 pairs of equal rectangular faces, we can find the areas of the six faces and add them together; therefore it has surface area. As we are given the dimensions, we can apply the formula too to find the total surface area. In this case it would be = 2 faces of 5 units x 8 units + 2 faces of 5 units x 18 units and 2 faces of 18 units x 8 units which gives a surface area of 80 square units + 180 square units + 288 square units. Therefore the total surface area of this block is 548 square units.	A solid cuboid will also have a volume as it occupies space. Since we are given the dimensions, we can apply the formula for the volume of a rectangular prism. Thus the volume will be 5 units x 8 units x 18 units = 720 cubic units.
C	This shape has six faces and we can find the area of each face and add them together, therefore it has surface area. The top and the bottom both have 16 unit squares each. The right side and the left side both have 16 unit squares as well. The front side and the back both have 16 unit squares each. Hence the total surface area of the block will be eqaul to 6 x 16 unit squares = 96 unit squares.	The block truly has a volume too, as it occupies space, by definition of volume. In this case we count the number of the unit cubes used to make it. The bottom layer is made 16 unit cubes and so is the second one. The top two layers are built of 10 unit cubes each. So the volume of the block is 2 x 16 unit cubes + 2 x 10 unit cubes = 32 unit cubes + 20 unit cubes which is equal to 52 unit cubes.

D	Each of the six faces of the structure is composed of individual unit squares, so it has a surface area, since the areas of each unit square can be obtained then all put together. A surface area of 36 square units per face is realised, giving a total surface area of 36 squares x 6 = 216 square units.	This structure, like any other object, occupies space, so it has volume. The diagram is in the form of a cube but if compressed to close up the spaces it forms a cuboid (rectangular prism). The space occupied by this diagram is equal to the number of used blocks, which is equal to 3 columns x 6 blocks x 6 layers = 108 unit cubes. There are 108 cubes making this structure.
E	The shape has six independent faces, hence it definitely has surface area. Surface Area = (2 x 16) + (4 x 4) + (3 x 4) + (2 x 4) + (1 x 4) which gives 72 square units	The shape occupies space, so it has volume. Volume = (16 + 9 + 4 + 1) which gives 30 cubic units. This by counting all the unit cubes from the bottom layer to the single cube at the top.

Question 4: Use the sketches below to answer the questions that follow: A big 6 cm cube box is filled with 2cm cube dice as in the illustration below.

How many dice will fit exactly into this box?





a. How many unit cubes will it take to construct the structure above?

Since there are three layers of 16 unit cubes each, the total number of unit cubes building the structure will thus be 16 x 3= 48.

b. What is the volume of this structure?

Since the shape is built up of 48 unit cubes, the volume of the structure is 48 cubic units.

c. Find the surface area of the structure above.

There are 4 faces of 12 unit squares and 2 faces of 16 unit squares each, the total surface area 4 x 12 + 2 x 16 = 48 + 32 results in a total of 80 square units.

Question 6: An example of some of the possible sketches with their dimensions

B

1 unit cube

216 Unit cubes

C

2 unit cubes

108 unit cubes

D

Use this table to compare the relationship between surface area and volume of cuboids.

Shape letter name and dimensions	Surface Area	Volume	Relationship of surface area to volume
A 1 unit x 216 units	(216 x 4) + (1 x 2) 866 square units	216 cubic units	866:216 433: 108
B 2 units x 108 units	(2 x 108) + (2 x 216) + (2 x 2) 652 square units	216 cubic units	652:216 163:54
C 3 units x 72 units	(2 x 72) + (2 x 216) + (2 x 3) 582 square units	216 cubic units	582:216 97:36
D 4 units x 54 units	(2 x 54) + (4 x 54 x 2) + (4 x 1 x 2) 548 square units	216 cubic units	548:216 137:54
E 6 units x 36 units	(36 x 1 x 2) + (36 x 6 x 2) + (6 x 1 x 2) 516 square units	216 cubic units	516:216 43:18
F 8 units x 27 units	(27 x 1 x 2) + (27 x 8 x 2) + (8 x 1 x 2) 502 square units	216 cubic units	502:216 151:108
G	(24 x 1 x 2) + (24 x		

9 units x 24 units	9 x 2) + (9 x 1 x 2) 498 square units	216 cubic units	498:216 83:36
H 12 units x 18 units	(18 x 1 x 2) + (18 x 12 x 2) + (12 x 1 x 2) 492 square units	216 cubic units	492:216 41:18

The cubes can be arranged into cuboids in 8 different ways, without placing one above the other. The different surface areas and volumes are recorded in the above table. While the volume remains the same, the surface area of longer cuboids is far greater than the shorter ones. The longer the cuboid, the greater or bigger the sufarce area, even if the volume can still be the same. Longer prisms have larger surface areas than shorter prisms of the same volume. Prisms of the same volume can have different surface areas.

7.10 APPENDIX J: INITIAL INTERVIEW SCHEDULE SAMPLE

Proposed Interview Schedule

Questions for the interview to follow up after marking the test:

Explain what you understand by the terms surface area and volume, then give the differences between them, if any.

Describe how you felt when you were writing the volume and surface area test.

State which questions you found easy and which ones were difficult in the test.

What makes you say these questions were difficult?

What are your perceptions regarding the topics of surface area and volume?

Do you think these topics should be part of the school programme at this grade level, and why?

What are your major strengths and weaknesses regarding surface area and volume?

Describe what you think can be done to help reduce the weaknesses or turn them to strengths?

Regarding question number ________, what do you mean by _____________?

If you were to teach someone about surface area and volume, what and how would you do it?