Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry

By

Khomotso Welcome Kgopane

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Supervisor: Professor Zingiswa MM Jojo

ABSTRACT

This study aimed to explore learning difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in Geometry. To achieve that aim, the study focused on achieving three objectives, namely to: identify learning difficulties experienced by the Grade 9 learners in learning 3D shapes; determine learning processes that enhance the students' cognitive understanding of 3D shapes in Grade 9 Mathematics; and suggest how the 3D shapes learning difficulties in Grade 9 can be minimised.

Furthermore, this study was underpinned by van Hieles' (1984) Levels of Geometrical Thinking. The study adopted a longitudinal mixed-methods research design in which sixty grade 9 learners Lydenburg Circuit, Mpumalanga Province, in South Africa and five grade 9 Mathematics teachers from two schools in the Lydenburg Circuit, Mpumalanga Province, in South Africa participated. Qualitative data were collected through semi-structured interviews with the five grade 9 Mathematics teachers. Quantitative data consisted of learners' scores in the two tests which were administered before and after the intervention activities. The tests were structured according to van Hieles' (1984) Levels of Geometrical Thinking. The first was administered before the intervention activities while the other was administered after the intervention activities.

The study revealed several difficulties that learners face in learning 3D shapes and can be classified as those that relate to the teacher, the learner and the learner's environment. In the findings, difficulties relating to the learner include leaners' poor foundations in arithmetic and learners' inherent challenges with 3D questions that required the abstraction of 3D shapes. Secondly, the teacher's lack of knowledge and inexperience in teaching 3D shapes and unclear instructions were some of the reasons for the difficulties that learners faced in learning 3D shapes. Furthermore, inadequate learner support outside the school and large class sizes which made it difficult to individualise learner support to specific learners' needs were some of the environmental factors which were revealed in this study. The study also found that learners failed to understand questions posed in a language other than the learner's home language. Language difficulties were also reflected in learners' inability to understand instructions that were given in learners' first additional language, English. The structure of questions was also identified as another difficulty, especially where questions are structured in such a way that follow-up questions lead to learners being penalised more than once for the error in earlier questions. Learners' lack of understanding in other Mathematics areas such as change of subject of the formula, algebra and simple arithmetic also led to learners failing questions on 3D shapes in which those skills and knowledge were required. Finally, abstraction was another difficulty common in many learners. Learners failed to work out 3D shapes questions where shapes were not given and learners were required to use their knowledge of the 3D shapes in question to work out answers to activities.

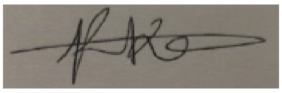
The implications of this study include the need for teachers to comprehensively plan lessons on 3D shapes, taking into account learners' diversity and the general learning environment. For policy-makers, there is a need to facilitate a tripartite learner support system, which empowers the learner to take charge of his/her learning and development, encourage parents/guardians to provide conducive out-of-school learning environments and help schools to provide the necessary stimulus to learners' urge for development. These implications should be viewed in light of some of the limitations of this study, including the small sample size and potential for cultural biases in the study due to limitations relating to time and resources.

Keywords: Learning difficulties; 3D shapes; van Hieles' theory; 3D shapes learning processes; Explore; Learning Difficulties; Grade 9 Learners; Understand 3-D Shapes

DECLARATION

I declare that this Research Report is my own work. It is submitted in partial fulfillment of the degree of Master in Education, Mathematics Education University of South Africa. It has not been submitted for any degree or examination at any other university.

Khomotso Welcome Kgopane

Signature : 

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Most importantly, I am greateful to my family’s unequivocal and loving support. To my wife, Mrs SP Kgopane and the children (Kezia, Botlhale, Khumo and Bravery) thank you for enduring this long process with me. My accomplishments and success are dedicated to you.

DEDICATION

With the schools under lockdown for more than five months, it has not been easy psychologically and physically to complete this study through the Covid-19 pandemic. However, I have always lived with the end product of this study in me and that kept me going.

Therefore, I dedicate this study to myself as a launch to further studies in Mathematics Education.

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ABBREVIATIONS AND ACRONYMS

Abbreviation/Acronym	**Description**
ANA	Annual National Assessment
ANOVA	One-way analysis of variance
CAPS	Curriculum and Assessment Policy Statements
CEDU	College of Education
COVID-19	Coronavirus Disease 2019 – a coronavirus is a type of virus that causes the disease known as COVID-19
DBE	Department of Basic Education
DoBE	Department of Basic Education
EBL	Inquiry-Based Learning
GBPS	Geometry Based Problem-Solving
MKO	More Knowledgeable One
TIMSS	Trends in International Mathematics and Science Study
UNISA	University of South Africa
ZPD	Zone of Proximity Development
1D	One Dimensional
2D	Two Dimensional
3D	Three Dimensional

CHAPTER 1 BACKGROUND AND INTRODUCTION OF THE STUDY

1.1 Overview of the study

This chapter provides the background and introduces the study. It outlines the background of the exploration of challenges experienced by Grade 9 Mathematics learners in understanding 3D shapes in geometry in the classroom. The research aims and objectives are outlined. The literature brief together with the research methodology is introduced, as well as illuminating the manner in which primary data was collected during the progression of the study.

The theoretical underpinnings of the study, introducing van Hieles' theory of geometric thinking, Vygotsky's constructivist learning theory, Fischbein's theory of figural concepts for the learner's understanding of the intrinsic figural nature of geometric objects are all briefly introduced and later explained in detail in Chapter Two. Gerde's complementary methods in learning geometric thinking using traditional and material culture are also integrated into the theoretical literature brief. The rationale and ethical considerations of the study are explained. The chapter concludes with an outline of the chapters developed in making the full research report.

1.2. Introduction

The problem which prompted this study relates to learners' challenges in learning 3D shapes. According to Reddy, Visser, Winnaar, Arenda, Juan, Prinsloo & Isdale (2016), and Mouton, Louw & Strydom 2013:8, South Africa is a country with many problems in its education system, in particular, concerning the teaching and learning of Mathematics. Reddy et al., (2016) further assert that learners in South Africa are advised to continue doing pure Mathematics and or join the mathematical literacy stream when they register for Grade 10. This research corroborates that, depending on the final year Mathematics percentage attained by each learner in various schools in South Africa, a learner with poor performance results in Grade 9 Mathematics is typically advised to do mathematical literacy in Grade 10. This specific decision, however, is mostly determined and guided by the mathematics percentage pass of the student in Grade 9 (Reddy et al. 2016). Consequently, this research opines that such advice and the choices thereof, are not typically considerate of the learner's future aspirations and advancements later in their career choices.

Ironically, the 2016 Trends in International Mathematics and Science Study (TIMSS) at the Grade 9 level indicate that the national average score for South Africa is 372 points for Mathematics and 358 points for science (TIMS 2016; Reddy 2016). Those assessments placed South Africa in the 38th position out of 39 countries for Mathematics and in the last position for science performance. In addition, in TIMSS-Numeracy for Grade 5 Mathematics, South Africa attained 376 points that placed the country at 47th position out of 48 countries although the scale difference with 46th and 45th was not statistically significant (TIMS 2017). This implies that the projection towards improvements in later Grades, and in particular, Grade 9 Mathematics, is with low pass rate expectations.

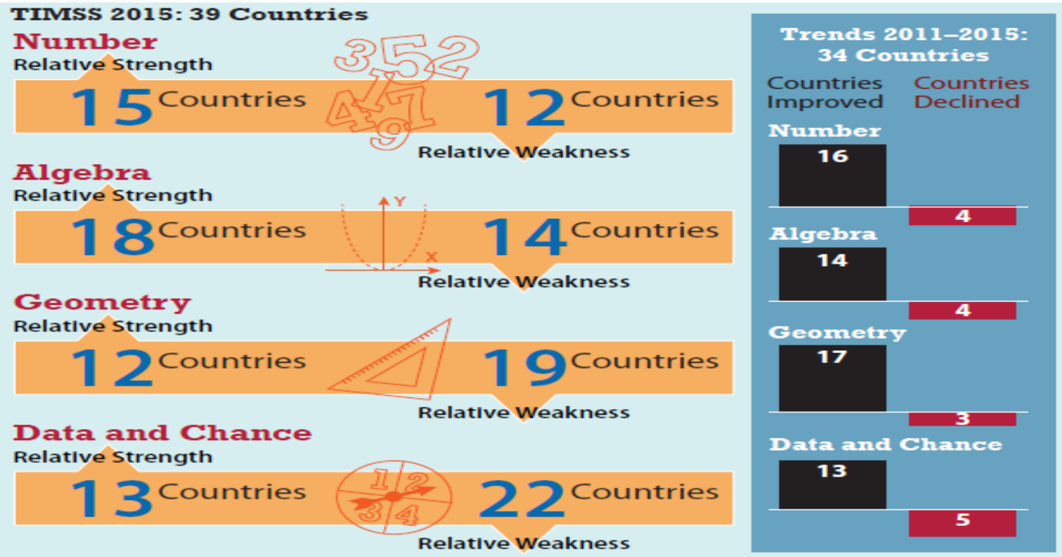


Figure 1.1: Grade 8 Mathematics Achievements by Content Domain TIMSS 2015

Figure 1.1 shows TIMS (2015) achievement results through four content domains namely; Number, Algebra, Geometry, and Data and Chance. The empirical evidence reveals that most countries demonstrate different strengths and weaknesses in one or more of the domains. Specific to this study, it can be observed that relative to TIMSS 2015's achievement per content domain in geometry, 19 out of the 39 countries that participated were relatively weak and 12 relatively strong in the domain. Figure 2 below further illuminates the international benchmarks that are employed in depicting the performance of Grade 8 learners (TIMSS 2015).

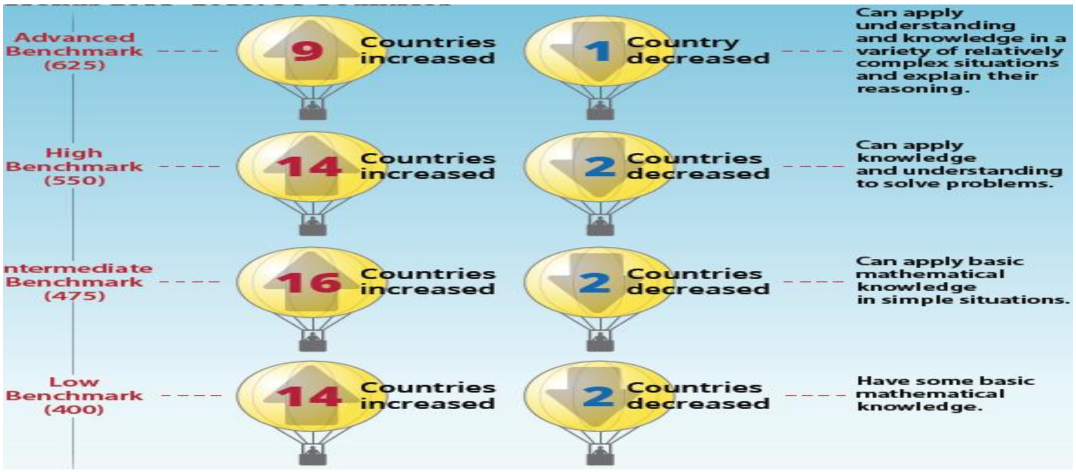


Figure 1.2: Trends in TIMSS International Benchmarks 2015 Grade 8 Learners

The figure shows that the advanced benchmark scores for learners at or above 625 can apply understanding and knowledge in a variety of relative complex situations and be able to explicate their reasoning to problem-solving (TIMSS 2015). The international benchmarks depict the high benchmark score as indicating that learners can apply knowledge and understanding to solve problems, but not sufficiently be able to solve complex problems and explain them (Bowie, Davis, Nxumalo, Pleass & Raju 2014:30; TIMSS 2015). The TIMSS international benchmarking score of 375 is deemed as an intermediate benchmark with learners applying mathematical knowledge in solving simple problems (TIMSS 2015).

Figure 2 reveals a low benchmark score of 400, to which both of South Africa's Grades 5 and 9 participating in the TIMSS international benchmarks reveals that the two groups attained lower than the 400 low benchmark score. The implication of this performance, according to the TIMSS standards, is that countries in the 400 and below low benchmark scores have some basic mathematical knowledge with which, however, they generally fail to apply basic mathematical knowledge in simple situations (TIMSS 2015). The TIMSS international benchmarks further attest that the same group of countries in the low benchmark score (400) also cannot apply knowledge and understanding to solve problems (TIMSS 2015).

Currently, not many learners in Grade 9 achieve satisfactory results that are acceptable for them to pursue Mathematics beyond Grade 9. There is also a growing inference that Grade 9 learners have difficulty in recognising basic geometric shapes and cannot correctly define basic geometrical shapes in their Curriculum and Assessment Policy Statements (CAPS) Mathematics curriculum (Department of Basic Education 2015:69). The following is the international benchmark distribution of South Africa's Grade 9 performances reflected in TIMSS 2015:

Table 1.1: Summary of South African Performance in 2015 TIMSS

Summary of South African Performance on 2015 TIMSS	
International Benchmark	**Grade 9**
	Mathematics
Advanced (>625)	1%
High (550-625)	3%
Intermediate (475-550)	10%

Source: (TIMSS, 2017)

The proportion of learners who obtained Intermediate, High or Advanced international benchmark levels is worryingly low at 14% for South Africa as reflected in Table 1. It is expected that at the advanced level, international benchmark level learners reason and apply knowledge skills in a variety of problem situations such as fractions, percentages, proportions, geometry, averages, expected values, solution of linear equations and generalisations (DoBE 2012). Comparably, about 54% of Singaporean Grade 8 Mathematics learners achieved an advanced level, a very high value compared to 1% of Grade 9s of South Africa (TIMSS 2016). This may sound a call for more training in the classrooms that lack geometry competency for South African students to be capable in mathematical reasoning and problem-solving skills in order to compete with their counterparts internationally. The rationale for this research, furthermore, focuses on the understanding and appreciation of 3D shapes learning for Grade 9 learners. Such understanding may assist in improving the understanding of patterns, precision, and the beauty of the natural and cultural forms of the world as the learners interpret it.

In addition, there is a realisation that South African learners might lack an adequate understanding of Mathematics (Annenberg 2017; TIMSS 2016). Reddy et al. (2016) assert that there still exists limited knowledge to fulfilling the desired pass-rate; subsequently Mathematics learning difficulties remain prevalent in South African classrooms. Roux (2013) acknowledges that frameworks on learning approaches adopted in class settings are not consistent, and as a result present a variety of challenges towards effective learning outcomes for learners. Consequently, the teaching and learning of sections like geometry suffer due to inconsistencies in practical approaches to the subject matter within the classroom.

The challenges relating to the teaching and learning of 3D shapes has been identified by prior researchers. In one such study, Luneta (2014) focused on investigating the student teachers’ conceptual understanding of shapes. The study revealed that while Grade 12 learners are expected to operate at levels 3 and 4 of the Van Hieles’ levels, the majority of the participants in the study were operating at Level 1, the level of the learners they will be teaching when they complete the course. As a result, the study concluded that such knowledge deficiency feeds into the learners who, such teachers will have to teach after graduation, unless there is relearning of the basic geometry concepts. Though the study focused on conceptual knowledge in shapes in general, part of the study focused on the student teachers' knowledge of 3D shapes, which was also found to be deficient.

Oluseyi conducted another study to investigate the effect of using animated computer 3-D figures illustration (ACTDFI) in the learning of polyhedron in geometry. In the study, the commented that it was worrisome that South African learners experienced serious conceptual learning difficulties in 3D shapes. Citing the results of the Annual National Assessments (ANA), Mntunjani, Adendorff and Siyepu (2018) concluded that Senior Phase learners in South Africa faced significant learning difficulties in Mathemaics, generally, and 3D shapes in particular. This study, therefore, explores the learning difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in geometry.

1.3 Background of the Study

In South Africa's education system, Curriculum and Assessment Policy Statements (CAPS) stipulates that one of the main content areas to be covered in the Grade 9 Mathematics curriculum is Space and Shape, Geometry. Geometry learning area has a content weighting of 30%, a high weighting second only to Patterns, Functions, and Algebra (Bowie, Davis, Nxumalo, Pleass, & Raju 2014:33). The weightings are reflected in Table 1.2.

Table 1.2: Grade 7, 8 and 9 Mathematics weighting content areas

WEIGHTING OF CONTENT AREAS			
Content Area	**Grade 7**	**Grade 8**	**Grade 9**
Number, Operations and Relations	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling	10%	10%	10%
	100%	**100%**	**100%**

Adaptated from the Department of Basic Education (2013:15)

Consequently, questions set in the School-Based Assessment and Final Exams, ought to weigh the percentages of the content area as stipulated in the curriculum policy (Department of Basic Education 2013). Given the significant weighting of the space and shape topic in relation to other topics, Bowie et al. (2014) were concerned that Grade 9 learners continue to experience learning difficulties with 3D shapes, which falls under Space and Shape in the CAPS curriculum. Moreover, learners experience challenges in recognising basic geometric shapes and cannot correctly describe basic geometrical shapes as well as the correct properties of shapes in 3D shapes in line with observations by Van de Walle, Karp and Bay-Williams (2013). This study and aligns with the literature that challenges the impact of the understanding of 3D shapes geometry on Grade 9 learners' overall achievement results in the mathematics subject. Hence the study has explored the learning difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in geometry. This is to ensure that 3D shapes geometry learning can be improved to aid learners in attaining better learning outcomes that would be reflected in improved pass rates in overall Mathematics performances of Grade 9 mathematics learners in South Africa.

1.4 Literature review

In this chapter, only a summary of the literature reviewed is presented. Chapter two presents the full detailed literature review of the research. This summary illuminates the importance of learning 3D shapes geometry and how it plays an important role in understanding how different substances behave and the recognition that we live in a three-dimensional world explore and use every minute of every day. Findings on external and local learning challenges are briefly introduced, converging literature guides the study to depicting theoretical constructs explained and identified in theories of learning as explained in the next section.

1.5. The importance of learning 3D shapes in geometry

This section provides for a brief discussion on the importance of the study concerning the importance of learning 3D shapes in geometry.

1.5.1 The world is built of shape and space and geometry is its Mathematics

Annenberg (2017) presents a rationale that geometry is a key part of Mathematics learning and has significance in facilitating the understanding that the world is built of shape and space, and geometry is its Mathematics. The researcher further points out an observation on the class learning experience that students have trouble with abstraction if they lack sufficient experience with more concrete materials and activities during informal geometry learning. Annenberg (2017) concludes this rationale for an investigation into 3D shapes in geometry by alluding to the importance of the visual skill in geometry that can moreover be used as a doorway to students' success in Mathematics. This, therefore, implies that knowledge gained from geometry can be applied to solve problems from other fields more easily for learners in their 3D shapes geometric problem statements in real-world applications. For now, this specific study, therefore, propositions and projects that 3D shapes geometry for Grade 9 Mathematics learners truly becomes a key part of Mathematics learning that takes import on the understanding that the world and its problems for human needs are built of shape and space.

1.5.2 3 D shape's important role in understanding different shapes' substances behaviour in real-life applications

The learning of 3D shapes plays an important role in understanding how different shapes' substances behave when applied in real life. It is therefore important to recognise that we live in an obviously three-dimensional world that we walk through, explore and use in our daily lives.

The study of 3D shapes in geometry improves the understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms (CAPS

2017). Examples that can be illuminated in the Senior Phase Curriculum and Assessment Policy Statements (CAPS) point to learners' heightened ability to draw and construct a wide range of geometric figures and solids by use of appropriate geometric instruments. The Senior Phase CAPS further points to the development of an appreciation for the use of constructions to investigate the properties of geometric figures and solids and attaining and comprehending how to arrive at developing a clear and more precise description and classification categories of geometric figures and solids. In doing so the Senior Phase CAPS projects that learners will attain a heightened ability to solve a variety of geometric problems, drawing on known properties of geometric figures and solids. For example, learners use paper models to construct shapes, combine shapes to form new shapes and decompose composite shapes into simpler ones to aid in exploring shapes and their properties. Learners perform classic constructions with straight edges and compasses as well as with appropriate computer software. They appreciate the presence of geometry in nature and human-built structures and through those geometric applications, they appreciate the role of geometry in life.

As a result, and in agreement with Roux (2013) and van de Walle, Karp and Bay-Williams (2013) it is important that learners learn both a vocabulary with which they can talk about the space that we occupy and the properties that this space or spaces possess as distinct or familiar attributes both in Mathematics and the real world. In the example given above, learners should be familiar with identifying the base of a 3D shape. In order to name and apply the properties of the shape Annenberg (2017) further asserts that informal 3D geometry learning has an equity component that contributes to 3D shapes' understanding and problem-solving. The equity component refers to informal learning, applied in this context. It forms as a pervasive on-going phenomenon of learning 3D shapes properties and angles through participation or learning through knowledge creation.

Informal learning may have influence as part of a conceptual approach to addressing 3D geometry learning challenges. This is because learners' learning processes coupled with a sufficient background experienced through some practice tends to establish improved learning leverage with a fair shot at understanding formal 3D shapes geometry when it occurs in class learning. For example, Van de Walle et al.'s (2013) findings reveal that typically learners practice through informal learning by means of group participation. This literature may imply the use of visual observation and learning through attempting to recognise better shape objects on 3D shapes' patterns, surfaces, edges, and vertices.

For this study, the above literature and empirical findings make it imperative to explore and understand how best learners improve on overcoming the learning challenges towards 3D shapes' geometry and ultimately, attaining the ability to solve 3D shapes' geometrical problems. This entails exploring learning challenges and concepts that aim to aid learners familiarise themselves with specific 3D shapes and their properties. This involves learning and defining familiarities and distinctions of shape properties in predicting and calculating property value variations in shape object properties.

1.6 Problem Statement

The problem which prompted this study relates to learners' challenges in learning 3D shapes. 3D geometry shapes' learning continues to be a challenge for South African's Grade 9 learners as it affects their ability to understand, explain and solve geometry 3D shapes related problems. Geometry is one of the key curriculum components in Grade 9 Mathematics class teaching (Bernstein, McCarthy, & Oliphant, 2013a). In the grade 9 CAPS curriculum, Geometry occupies about 30% of the mathematics content required to be learnt in Grade 9. South African Grade 9 learners persistently perform poorly in Mathematics against top world statistical rankings of pass rates. That is despite efforts to upscale the teaching skills, including recruitment of more experienced teachers from outside South Africa, modernising technological access and resources to learners in line with new teaching and learning resources adopted in the teaching curriculum. Further to that, a report from the latest Annual National Assessment (ANA, 2014) revealed Grade 9learners getting 11% in Mathematics performance, and one of the subject areas of poor performance is geometry on 2D and 3D shapes learning. Further to that, a summary of Mathematics average percentage marks for Grade 9 learners in 2012 to 2014, according to the last Annual National Assessment report reveals performances in geometry to be 13%, 14% and 11% respectively. This is while the percentage of learners achieving 50% or more in Mathematics is poignantly low (2%:2012; 2%:2013; and 3% in 2014) (ANA, 2014).

This study, therefore, proposed to explore the learning challenges experienced by Grade 9 learners in comprehending 3D shapes in geometry. The study, therefore, proposes to find solutions to the persistent learners' learning difficulties in 3D shapes. The study seeks to achieve that by identifying learning difficulties experienced by the Grade 9 learners in learning 3D shapes and experiment on the potential learning processes that can enhance the learners' understanding of 3D shapes in Grade 9 Mathematics. From that, the study will then propose how the learning difficulties in Grade 9 3D shapes can be minimised.

1.7 Aim of the study

To explore learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry.

1.7.1 Specific Research Objectives

The following research objectives were proposed for this specific study:

- to identify learning difficulties experienced by the Grade 9 learners in learning 3D shapes;
- to determine learning processes that enhance the learners' understanding of 3D shapes in Grade 9 Mathematics; and
- to propose how the learning difficulties in Grade 9 3D shapes can be minimised.

1.7.2 Main Research question

What are the learning difficulties experienced by Grade 9 learners in learning 3D shapes and how can those learning difficulties be minimised?

1.8 Research Sub-questions

The following research questions are proposed for this study:

- What are the learning difficulties experienced by Grade 9 learners in learning 3D shapes?
- What are the learning processes that enhance the students' cognitive understanding of 3D shapes in Grade 9 Mathematics?
- How can the difficulties in learning Grade 9 3D shapes be minimised?

1.9 Rationale of the Study

This research finds it prudent to identify the leading causes of Grade 9 learners' poor performance in 3D shapes learning. The study postulates that there is certainly a need for continuous analysis of content-related factors that relate to the mathematical concepts which children learn, and how these concepts are disseminated to them. Mithalal (2015) suggests that one of the ways in which mathematical proof can be taught is to assist learners in resolving 3D shapes and space challenges.

For Mathematics educational curriculum design and pedagogy, the research finds it prudent to identify the leading causes of Grade 9 learners' poor performance in 3D shapes geometry learning. The study postulates that there is certainly a need for continuous analysis of content-related factors that relate to the mathematical concepts which children learn and how these concepts are disseminated to them. Mithalal (2015) suggests that one of the ways in which mathematical proof can be taught is to assist learners in resolving 3D shapes and space challenges. The study postulates and determines that understanding of the learning difficulties encountered by the Grade 9 learners in the learning of 3D shapes provides an opportunity to contribute towards ‘shifting learners out of the bottom end of the performance spectrum in Mathematics learning processes for improved problem-solving individual outcomes. By using rich insights, the study taps into basic concepts relevant to the understanding of Grade 9 prescribed geometric shapes.

For the government, the research may provide rich insights into challenges with the current Mathematics learning the curriculum in South African schools in an effort to influence Mathematics development policy in schools for better comprehension of mathematical problem solving for senior phase and higher mathematical problemsolving abilities with improved accuracy and efficiency.

This study also has significance to research in education, in general. It provides a South African perspective of empirical evidence on education research in general, and 3D learning in particular.

1.10 Overview of the Research Methodology

The overview of the research methodology undertaken in this study is explained briefly. In this section, the research paradigm, research design, and approach, sampling strategy, data collection and method of analysis are introduced.

1.10.1 Research paradigm

Mixed research paradigms are an emerging philosophy with the increasingly novel phenomenon of realistic depictions (Saunders, Lewis, & Thornhill 2016; Creswell 2013; Bernard & Bernard 2012; and Trochim 2012). In addition to pragmatism, other philosophical perspectives include; critical realism, transformative-emancipation and dialectical pluralism. However, pragmatism seems to be the popular perspective underpinning mixed methods research studies and is the focus of the research philosophical underpinning (Saunders et al. 2016). This research adopted pragmatism as its research paradigm. Pragmatism is a deconstructive paradigm that supports the use of mixed methods in research (Saunders et al. 2016; Creswell 2013). Pragmatism is known to “side-step” the argumentative disputes of truth and reality” (Feilzer 2010:8). Instead, it focuses on ‘what works’ as the truth regarding the research questions under investigation (Tashakkori & Teddlie 2013:713).

Pragmatism is used because it binds the meaning, cognition and action in the understanding of 3D shapes.

1.10.2 Research design

The quantitative part of the enquiry made use of a descriptive research design (Saunders et al. 2016; Creswell 2013). This was used in establishing the exact steps the research process took in collecting, gathering, and analysing its quantified survey responses using the statistical programme for social science (SPSS) in interpreting and explaining the survey findings. For the qualitative part of the study, an exploratory research design was used. This allowed the researcher to ask deep-probing questions to elicit information on issues concerning 3D shapes in geometry learning in the classroom setting.

1.10.3 Research Approach

The research uses a mixed research approach that will be concurrent in data collection and analysis. Gathered and analysed data is subsequently integrated and congruencies of the findings illuminated (Young & Hren 2015). As such, this will take the form of concurrent exploratory research where both qualitative and quantitative enquiries are administered on the two schools selected for the population of the enquiry. This also entails that the research will use concurrent data collection and analysis. The researcher projects that the design enhances the validity of the research findings and brings about congruency with multiple data sources such as interviews, surveys, and observations for the study (Saunders et al. 2016; Creswell 2013).

1.10.4 Sampling Techniques and Sample

The study made use of the mixed sampling approach incorporating a stratified simple random sampling for the quantitative data sample selection of survey participants. The qualitative part of the study used a convenient sampling technique to identify and expertly select the most informed and knowledgeable participants for the planned interviews. Moreover, the mixed research's concurrent sampling techniques are preferred to ensure there is equal representation of Grade 9 learners from each of the two schools selected for the study out of the three in the circuit. The two concurrently implemented sampling techniques facilitated for different stakeholder views to enable as much data as possible to be gathered by ensuring that stakeholders with knowledge and information pertaining to the study were purposively selected for primary evidence collection (Creswell 2013).

1.10.5 Data Collection instruments

The data collection instrument took the form of face-to-face interviews, secondary data sources and survey questions that will be used to gather secondary and primary evidence for the study. As such, the research uses three key elements in its instrument design aligned with the research objectives of the study. Semi-structured and open-ended questions will be used on educators, while the Grade 9 mathematics learners will be subjected to a survey in the form of a structured pre and -post intervention tests followed by open-ended question interviews.

1.10.6 Data analysis techniques

The research's qualitative data gathered was analysed using the six stages of conceptual thematic analysis as stipulated by Braun and Clarke (2016). This technique incorporates data familiarisation, code generation, sorting codes into themes, defining and naming themes, and report production which provides enough evidence from themes (Bernard & Bernard 2012).

For quantitative data gathered, the researcher made use of descriptive statistics, where graphs and tables will be used to depict and interpret the quantified data from learners (Saunders et al. 2016). SPSS is used to convert research data variables gathered into statistical frequencies and distributions of gathered data queries and variable correlation coefficients, and their degree of freedom from each other as observed and existing in the study.

Document analysis was used in the study, namely the Grade 9 timetable material, currently used Grade 9 Mathematics learners' textbooks, lesson plans, mark sheets, work schedules, exercises on 3D geometry, and tests. The documents are downloadable from the DBE websites. In addition, the researcher is a "non-participant observer" in the 3D Mathematics class. Classroom observations were conducted to ascertain the performance of the Grade 9 learners in 3D shapes geometry's problem-solving.

Chapter three gives a detailed description / discussion on the research method and explains the above in more detail.

1.11 Research Ethical Considerations

In keeping with any research instrument used on research subjects, there are ethical issues the researcher needs to consider when conducting interviews. With that in mind, first, an application was sent to the College of Education (CEDU) ethics committee. In addition, permission to conduct the study in the selected schools was applied for from the Department of Education in Mpumalanga. The research ethical adherence ensured the reduction of the risk of unanticipated harm, protection of the information of the interviewee, reduction of the risk of exploitation and the efficient informing of interviewees about the nature of the study. The other ethical issues that were adhered to include:

1.11.1 Obtaining Approval to Conduct Study

The researcher ensured that an approval letter from the two schools and or district education office is obtained by way of getting permission to use the schools for the primary research.

1.11.2 Informed Consent

The researcher made certain that the research participants were given consent forms accompanying the letter inviting them to take part in the study.

1.11.3 Confidentiality and Anonymity

Coded numbers were used to identify the 15 Grade 9 Mathematics learners, from the two schools. For purposes of the protection of personal and institutional information, the researcher pledged that all the collected primary information would be safeguarded and published only with the full consent of the subjects/institution.

1.11.4 No Harm on Participants

The researcher pledged that the subjects would be protected from any form of emotional or physical harm owing to their participation in the research. The researcher accomplished this task by securing approval from the relative authorities to conduct the research with learners as respondents.

1.12 Reliability and Validity

Reliability is established in the mixed research approach by provision for the triangulation of primary data between methods at data analysis. The observation and confirmation of data convergence, corroboration, and correspondence in survey data and text data from interviews is best validated by results whose data is sourced using different methods. It also allowed for learning from different perspectives on the subjects relevant to the research problem - in this context, both teachers and learners.

1.13 Delimitations

According to Leedy and Ormrod (2015), Delimitations are choices made by the researcher which should be mentioned. They describe the boundaries that the researcher set for the study. Learning difficulties and learning processes which enhance learners' learning are very broad. In this study, the focus is on 3D shapes' learning difficulties and on the learning processes which can be used to enhance learners' understanding of 3D shapes. In addition, learning difficulties in 3D shapes are very common across South Africa. This study only focuses on learning difficulties which were prevalent the two schools selected in Mpumalanga.

1.14 Organisation of Chapters

Chapter One

Chapter one introduces the study and its contextual background. This chapter has introduced the research focusing on 3D shapes and space (geometry) learning challenges from a global and narrowed to a South African context. The research objectives, questions, and significance of the study are discussed.

Chapter Two

The second chapter presents literature arguments pertaining to 3D geometry learning challenges, opportunities, and constructs influencing student learning abilities. The current thinking will further explore theory on cognitive social learning skills as well as aspects of cultural, social and environmental influences towards student learning processes in geometry at the senior phase level.

Chapter Three

Chapter three focuses on explaining the research methodology and design as the blueprint of the research process. This will explain the research strategy selected, the target population, sample size, the research instruments, and aspects of data collection and analysis. Ethical considerations are explicated in detail.

Chapter Four

Chapter four presents the fieldwork results. This will be done in line with the established research objectives. After the results are presented, the next stage will be the analysis thereof in keeping with the research questions.

Chapter Five

Chapter five focuses on establishing any research gaps and areas for further study. The chapter also provides conclusions for the study in line with analysed findings and concludes with recommendations on how best Grade 9 3D shapes and space learning can be improved from the setting of the study.

1.15 Conclusion

The introduction and research context pertaining to 3D shapes learning for Grade 9 Mathematics has been outlined. The research background, factoring in findings from the TIMSS Mathematics scores for Grade 9 learners have set the pace for the study’s significance towards learning approaches that may improve the scores that Grade 9 mathematics learners currently averagely achieve at a national level. The aim of the study pertaining to 3D geometry learning challenges has been explicated. The research main objective has been outlined. The subsequent research questions have been outlined as well.

The next chapter introduces and discusses current and past literature pertaining to mathematics learning for students and narrowing on the 3D geometry learning dynamics. The empirical findings on the success of van Hieles’ theory on geometric thinking in similar settings are discussed. The literature review will also illuminate on the South African learning environment, the current curriculum on the Grade 9 mathematics learning model and using both the variation theory to learning Mathematics and van Hieles’ theory of geometric thinking. The chapter concludes with a theoretical concept establishing the study’s subsequent analysis of gathered empirical evidence.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

The preceding chapter introduced the research study and gave the rationale for it and its sub-objectives. The current chapter presents the literature reviewed on learning difficulties in Grade 9 mathematics 3D shapes geometry classroom instructional learning processes.

In this chapter, the concept of learning, the learning difficulties in the classroom and learning difficulties in Mathematics are discussed. 3D shapes are introduced, and explained with regard to geometric thinking, while geometry and learning of 3D shapes in the South African context are also discussed in detail. The chapter further presents a contextual depiction of complementary traditional methods of aiding mathematics learners to better comprehend 3D shapes learning in the classroom. Three theoretical concepts and models are introduced and explained in detail in modeling a conceptual approach in exploring the dynamics and contextual realities of learning difficulties for Grade 9 3D shapes learning in geometry for South African mathematics learners. The chapter then concludes by consolidating this discussion and introduces the next chapter.

2.2 Discussion of important concepts

Before proceeding to literature relevant to this study, it is important to discuss important concepts. To this end, the terms learning, and learning difficulties are defined and discussed briefly. Also, this subsection provides an overview of geometry and locates 3D shapes in geometry as a field of study. The subsection also highlights the importance of learning geometry and ends with a discussion of Gerde's traditional methodology in complementing geometric thinking.

2.2.1 Learning

The research theme focuses on an exploration of the learning difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in geometric classroom learning. It, therefore, is critical to understand learning as a critical component for this study. Lachman (2010:477) defines learning as a moderately sustainable or permanent change in behaviour or action that is an outcome of a practical or experimental process of understanding new phenomena. This conception of learning means that learning is aimed at modeling the behaviour of the learner. Secondly, Lachman defines learning as practical so that the learner gains knowledge through experience. This view is supported by Pear (2016:32) who views the science of learning as overlapping with behaviour analysis; but not essentially identical to behaviour analysis. This is to acknowledge that there is a need to understand the learner's behaviour, pre- and post-learning to judge the effectiveness of the learning process. Nonetheless, Webb and Roberts (2017) add that learning is a form of progressive comprehension of new things and responding to the learned phenomena's interactions or problems in a manner that exhibits an understanding of the phenomena in addressing or responding to the learned aspect or concept. Critical here is that learning is progressive. This means that it needs to be divided into smaller units which are progressively added to the process as building blocks of the learner's knowledge.

For the purpose of this study, learning is defined as a process which results in a moderately sustainable or permanent change in behaviour or action and involves progressive comprehension of new things and responding to the learned phenomena's interactions or problems in a manner that exhibits an understanding of the phenomena in addressing or responding to the learned aspect or concept.

2.2.2 Learning difficulties

Webber and Roberts (2017) assert that learning difficulties are inhibiting factors that negatively affect the learning process in an individual or group of people. Learning difficulties are therefore experiences, factors and aspects of environmental and behavioural characteristics that act as hindrances to the learning process (Learning Disabilities Association 2019; Webber & Roberts 2017). This calls for a careful analysis of the learning environment and context on the part of the educator. As the bearer of knowledge to be disseminated, the educator should anticipate these hindering factors and devise ways of mitigating them.

Typically, a suitable learning environment and context that would assist in minimising the learning difficulty of geometry, includes one that fosters learner participation, in which learners feel free to try and solve problems without fear of being reprimanded and one that mirrors learners' everyday family and social environments. According to (Pear 2016:33), it is virtually impossible to succeed in using learning to influence behaviour if learning difficulties exist in the process. Existence of learning disabilities means that almost nothing is constructively (practically or experimentally) learned in a manner that brings about positive change to the individual or group's behaviour and actions relative to the learned phenomenon. For Grade 9 learners in learning 3-D shapes, difficulties usually include the inability to recognise differences and similarities in 3-D shapes, understanding location and symmetry of shapes, and failure to solve problems using visualisation and spatial reasoning.

In a classroom learning setting, learning difficulties are challenges in acquiring knowledge and skills at the same normal level expected of those in the same age groups, and especially because of mental disability or cognitive disorder (Learning Disabilities Association 2019; Webber & Roberts 2017). As such, a learning difficulty is a condition that can cause an individual to experience problems in a learning context. For educators, understanding the nature of learning difficulties that exist in an individual or group can help to inform decisions such as teaching styles, teaching aids and the overall nature of teaching delivery. In the context of 3-D shapes, this may necessitate planning of additional sessions with groups or individuals with similar learning difficulties. For example, learners experiencing symmetry difficulties may be placed in a group different from those having difficulties in solving problems using spatial reasoning. This allows the educator to tailor-make instructions to each group to maximise their ability to understand learned material in a differentiated way.

2.2.3 Geometry: an overview

Bassarear (2012) asserts that Geometry involves the relationships and properties of shapes. This conception is supported by Risi (2015:1) who defines geometry as “shape, size, a relative position of figures, and the properties of space.” Another detailed definition is given by Soanes and Stevenson (2009) who define geometry as the branch of mathematics that deals with solids, surfaces, lines, points, angles, properties, measurements and relationships appropriate to them and their positions in space. This implies that shapes, including 3D shapes, points and sizes, individual and relative sizes, emerge naturally as the central features of Geometry. There are several categories of Geometry including Euclidean, Non-Euclidean, Elliptic, Spherical, Hyperbolic, and Non-Archimedean geometry among others (Risi, 2015). Other forms of geometry are coordinate and analytical geometry. Euclidean refers to a mathematical system attributed to Alexandrian Greek mathematician Euclid. Euclid's method consists of axioms and deduction of theorems. Non-Euclidean can be defined as any geometry that is not the same as Euclidean geometry. In addition, Elliptic geometry refers to a geometry that does not have parallel lines. Spherical geometry can be defined as the geometry of the two-dimensional surface. Hyperbolic geometry can be defined as a non-Euclidean geometry that rejects the validity of Euclid's fifth, the “parallel,” postulate. In Non-Archimedean geometry the properties are significantly different from Euclidean geometry. Coordinate Geometry (or the analytic geometry) describe the link between geometry and algebra through graphs involving curves and lines (Risi , 2015).

This study focuses on Euclidean geometry. The most common classification of geometrical shapes is based on dimensions, which is a measure of the size of an object (Kotzé 2007). The following table summarises classes of Geometry based on dimensions:

Table 2.1: Classes of Geometry based on dimensions

Dimensions class	Shape, size, relative position and the properties of …
Zero-dimensional	… a point
One-dimensional	… a line
Two-dimensional	… plane shapes (e.g., rectangle and triangle)
Three-dimensional	… cuboids, cylinder, pyramids and spheres
Four- /other-dimensional	… tesseract and hypersphere

Source: Blacklock (2018)

The South African CAPS prescribes that Grade 9 learners must investigate the properties of geometric 1-D, 2-D and 3-D shapes and solve geometric problems involving unknown sides, angles, and volume (DoBE 2012). The focus of this study is on 3-D shapes, such as Tetrahedron, Hexahedron (cube), Octahedron, Dodecahedron, Icosahedrons, Area and Volume.

2.2.4 The importance of learning geometry and 3D shapes

The study of Geometry is seen as a foundation for learning Science and Technology (World Economic Forum 2014). Furthermore, Chambers (2008) asserts that the relationship between geometry and everyday human activities creates an interest in the learning of geometry. Indeed, in the construction, design and architecture industries geometry is applied in the drawing of shapes, angles and lines (Blacklock, 2018). Moreover, in architectural engineering, geometry is observed to be applied in angle measurements and architects are expected to know the perimeter and area of shapes to create building products (Risi 2015). Most specifically, the Pythagorean

Theorem is used by architectures in design and measurements of building structures.

In addition, Usiskin (2002) observes that there is one specific reason why geometry is important to teach. The author found that geometry connects Mathematics with the real world and enables ideas from other areas of Mathematics to be pictured. In the real world, for example, most objects that we work with are in 3-D shapes. It would, therefore, be interesting that those objects like rectangular wood blocks would be used in the Grade 9 classroom to develop the geometric language in their descriptions. In this vein, Euclidean Geometry is important in adding the practical side of Mathematics. This would form a bridge between everyday Mathematics languages with the more complex concepts in the field. French (2014) supports this argument by advancing three reasons why Euclidean Geometry should be included in learning and teaching:

- Geometry learning extends spatial awareness. Spatial awareness means the use of geometric motions to create symmetric figures, for instance, paper folding which mirrors as reflections, and also determining congruence;
- Geometry learning improves the development of the skill of reasoning. Geometry teaches learners skills in logic, deductive reasoning analytical and problem-solving reasoning; and
- It prepares learners for challenges and stimulates the problems they are likely to face in the real world.

Therefore, learners may get a deeper perspective of the world through the study of 3D shapes as part of Euclidean Geometry. Also, it improves their reasoning capacity and relates well to other branches of Mathematics. This settles the question of whether Euclidean Geometry should be learnt in the first place and moves the debate to how it should be learnt, including challenges encountered in that process.

2.2.5 Gerdes' traditional methodology in complementing geometric thinking

Gerdes (2014:62) postulates a traditional and cultural complementary methodology towards understanding and comprehending what he termed "hidden moments" in geometry thinking. The author further developed a complementary methodology that aids a person to uncover hidden moments in traditional and material culture concerning geometric thinking. These are characterised as follows: geometric forms and patterns of traditional objects such as baskets, mats, pots, houses, and fish trap. Gerdes (1999) observed that the forms of those objects are ever arbitrary but generally represent practical advantages while typically being the only possible or optimal solution of a production problem.

The traditional forms reflect accumulated experience and wisdom that constitute not only physical and biological knowledge about the material used but also the mathematical knowledge applied in designing and developing the object materials. This entails understanding properties and relations of circles, angles, rectangles, squares, regular pentagons, and hexagons, cones, and cylinders among more geometry related shapes (Webb a& Roberts 2017:192; Gerdes 1999:32).

The discussion here presents critical elements of instructional learning and knowledge on the Mathematics teacher that can be exploited with modern instructional teaching methods to deepen and broaden the cognitive load in 3D shapes learning. Cognitive load refers to the used amount of working memory resources in learners during a learning activity (Sweller 1988). For example, Gerdes' complementary methodology to understanding 3D shapes can be illustrated by children playing with their hands and arranging themselves in a manner that shows shapes and their properties. Such an arrangement also shows 2D shapes as a foundational mathematical base for understanding 3D shapes. This is shown in Figure 2.1.

Figure 2.1: Traditional forms of learning geometry shapes during playtime (Picture credit: Khomotso Kgopane 2019)

According to Piaget (1971), children understand more geometric concepts as they grow and they need to work with the shapes physically for them to have a deeper understanding. This is supported by Clement, Swaminathon, Hannibal & Saram (1999) who state that children's understanding of shapes emanates from a combination of their mental abilities and internalised actions through physical activities. One such way of allowing learners to physically experience shapes is shown in Figure 2.1 above. Figure 2.1 is formed by placing learners in groups of six. Three learners should be standing while the other three should be sitting on chairs or other slightly raised objects or kneeling. The learners in the middle should stretch their arms to form the sides of two blue triangles for those standing, and two black triangles for those seated and the upper side of the yellow trapezium. The learners standing on either side should raise one arm above the middle learner such that their arms meet at the centre of the middle learner. This way, the raised arms complete the black triangle whose base has been formed by the middle learner. Again, those learners on either side of the one standing in the middle should lower their arms towards the two learners sitting on either side of the learner sitting in the middle. Those learners’ lowered arms add two sides to the yellow trapezium and one side to each of the blue triangles. These learners standing on either side of the one standing in the middle also start two new triangles on the bottom edges of the trapezium. The learner sitting in the middle stretches both arms towards the other two learners sitting on ether sides, completing the yellow trapezium and one more side to each on the triangles formed at the edges of the trapezium. The learners sitting on either side of the learner sitting in the middle put their shoulder behind the learner sitting in the middle while raising their arms towards those of the learner standing in the middle. These sitting learners complete both the blue triangles and the black triangles on the bottom edges of the trapezium. Six 2D shapes are be formed at the end of the exercise: 5 triangles and 1 trapezium.

This exercise is one of the illustrations developed by the researcher as one of the methods in which learners can physically experience 2D shapes, triangles and quadrilaterals, and their properties in a traditional way. From this kind of exercise, properties such as the number of sides and the number of angles can be easily understood. This can, therefore, help learners to easily transition to learning 3D shapes where those 2D shapes will be seen as base shapes and faces of 3D shapes. Figure 2.1, therefore can be adopted by teachers in Grade 9 to teach Euclidean Geometry.

2.4 Theoretical underpinnings

The previous section has reviewed literature on the various concepts relating to learning. What is notable in that section is that learning should follow a defined process. That brings the question on what learning processes can enhance 3D learning. This subsection reviews literature on the theories on Geometry. More specifically, it discusses the Vygotsky Constructivist Learning Paradigm and the van Hieles’ Levels of Geometric Thinking theory. The aim of that review is to choose the theory which can be used to provide lenses on understanding 3D shapes.

2.4.1 Vygotsky Constructivist Learning Theory

Vygotsky (1978) believes that learning could lead to the development of learners' critical thinking skills. Critical thinking skills refer to consciously thinking about a problem or an issue to rationally decide what needs to be done or what to believe (Davis & Kazlauskas 2004). The Constructivist theory is concerned with the unity and interdependence of learning and development. For example, learning 3D shapes should be seen and used as a foundation of developing learners' understanding of everyday 3D shapes around them and in their future professional endeavours. Typically associated with the traditional teaching practices that are still prevalent in classroom learning, the theory argues that learners construct their understanding and knowledge of the world through experiencing things and reflecting on those experiences (Sherman, Richardson & Yard 2013). In other words, when learners encounter something new, they reconcile the new experience with previous ideas and experiences perhaps changing what they believe or possibly discarding the new information as extraneous or irrelevant. For example, in Euclidean Geometry learners learn properties of triangles in lower grades. They then need to build on and extend that knowledge to understand the concepts behind the sum of angles in a rectangle, for example. That knowledge is then used for them to learn the behaviour of 3D shapes. In any case, the theory propounds those human beings are active creators of their knowledge. Therefore, learners "must ask questions, explore and assess" what they already know about a phenomenon (Askew 2013:36). The diagram below illustrates the theory. For example, learners in Grade 9 need to explore the relationship between 2D shapes such as squares and 3D shapes such as cuboids.

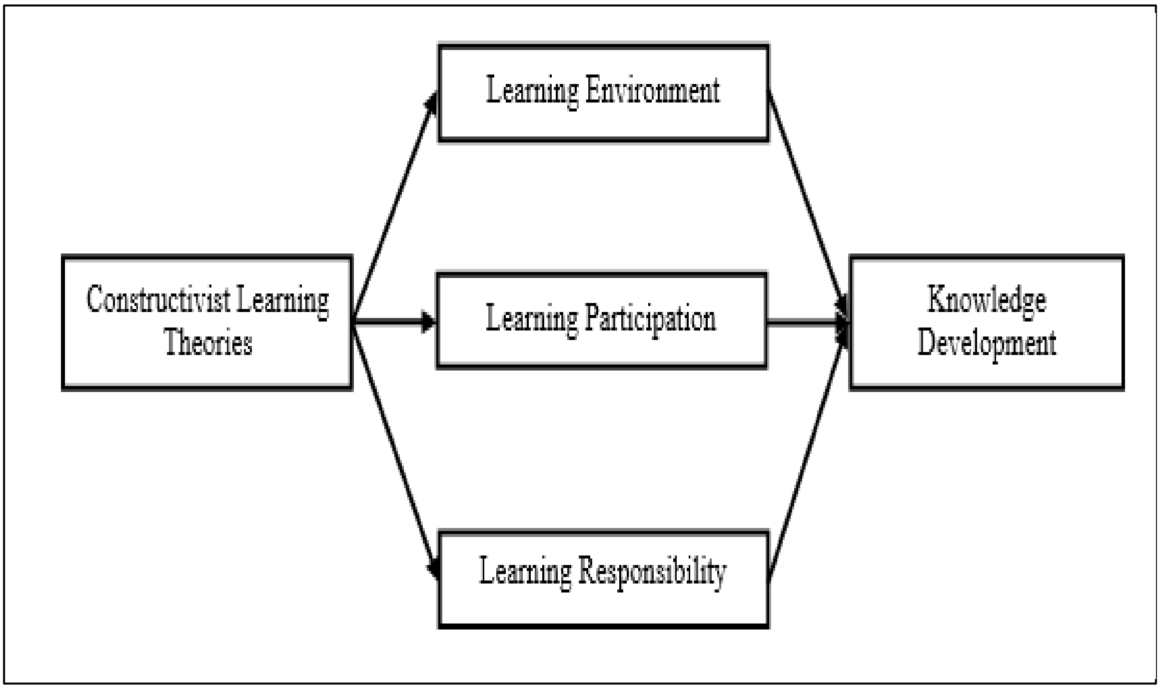


Figure 2.2: Constructivist Learning Theory (Source: Nugroho and Wulandari, 2017:413)

The figure shows that knowledge development is a function of three factors:

Learning environment – there is an emphasis on the theory of creating an environment favourable for learning. Fouze and Amit (2018) argue that such an environment is one that closely mirrors and integrates the learners' ethnocultural values in the learning process. This ensures that learning takes place in the learner's social context so that it is linked to pre-existing knowledge that the learner has already acquired outside of school or from prior learning levels. An environment suitable for the learning of geometry, for example, should encourage learners to experience shapes physically through demonstration and experiment rather than thinking about shapes abstractly.

Learning participation – Nugroho & Wulandari (2017) argue that learning can only be meaningful to the learner if done within a learning community. The community consists of educator, learners of higher level, lower level and level at par with the learner in terms of understanding of the learning concepts. In such a community, learners have the opportunity to participate in the learning process. Participation can be in the form of integrating learning to the learner's environment, solving real-life problems. Also, learning can involve teaching others of lesser cognitive ability or assuming the role of the adult leader (Nugroho & Wulandari 2017). Therefore, learning requires the learner to participate in such activities. For example, in a geometry lesson, a learner can be provided with a 3D shape like a lunch box in the shape of a rectangular prism. He can then be asked to examine its properties such as several faces and vertexes and list them. This way, learning of 3-D shapes becomes practical.

Learning responsibility – the last factor that affects learning is learning responsibility. According to Nugroho & Wulandari (2017:413), this involves learners being given control "over their learning activities to meet the goal of learning and to help them better understand their roles as learners in the learning process." In the context of learning 3D shapes, this may involve giving learners responsibilities of finding 3D objects and bring them into the classroom. Also, learners can be required to demonstrate to other learners the similarities the objects have to 3D shapes found in textbooks.

In addition to the three factors above, Vygotsky also speaks about the more knowledgeable other (MKO) and speaks to the Zone of Proximity Development (ZPD). Each is discussed below:

Zone of Proximity Development – the ZPD refers to "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978:86). Figure 2-3 illustrates the ZPD. In the context of teaching 3D shapes to Grade 9 learners, this means that they can use their knowledge from earlier grades to perform tasks such as calculating the surface area of 3D shapes such as cuboids. However, such learners will need their educator or the MKO to explain the relationship between the surface area and volume in 3D shapes.

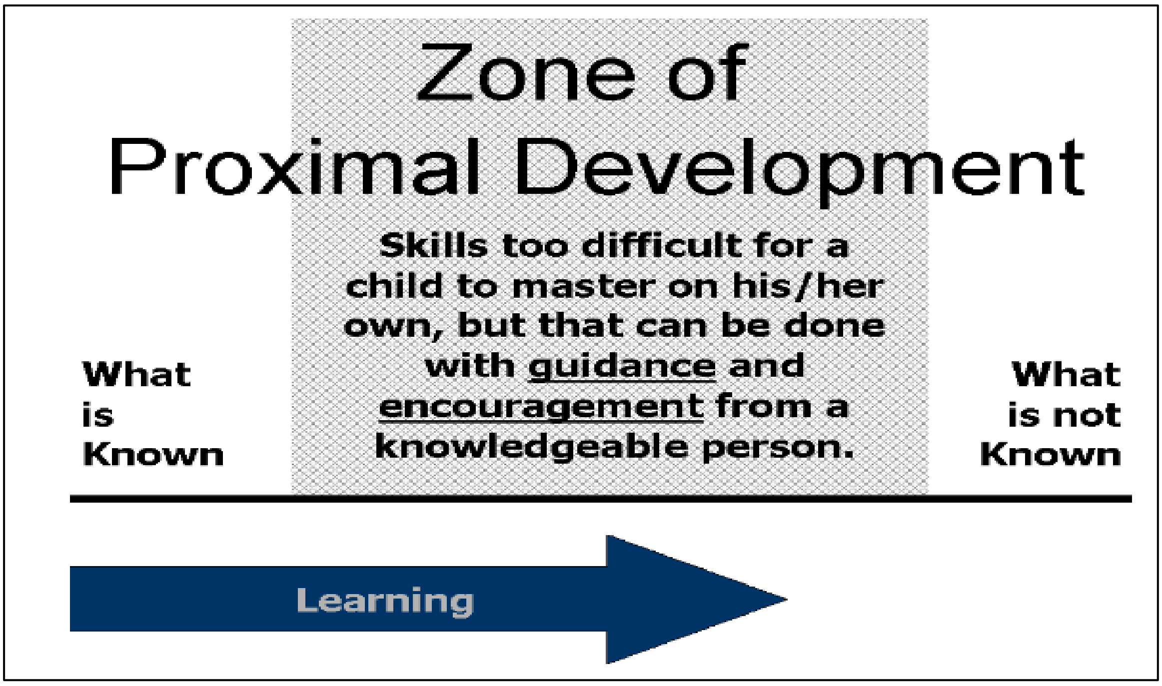


Figure 2.3: Zone of Proximal Development (Source: McLeod 2019: Online)

More knowledgeable one – the MKO refers to "someone who has a better understanding or operates at a higher ability level than the learner for a particular task, process or concept" (McLeod 2019: **Online**). In teaching 3D shapes to Grade 9 learners, the educator fits naturally into the MKO category. However, the MKO can be a learners' peers who may have already acquired this knowledge elsewhere and hence have more knowledge or experience to assist others. This stresses the role of the educator in identifying those MKOs so that they can assist others and internalise their understanding of the process.

2.4.2 Constructivist theory: Fundamental belief

According to Gray (1995:1), constructivists believe that "learning occurs as learners are actively involved in a process of meaning and knowledge construction rather than passively receiving information". This emphasises the idea that learners are the creators of meaning and knowledge and should be viewed as such by educators. The role of educators should, therefore, be confined to the facilitation of this meaning creation process. In the context of teaching 3D shapes, the educators' facilitation can help to develop learners who can think critically and learners who are motivated to learn independently.

2.4.3 A critique of the constructivist theory

There are several proponents of the constructivist theory. For example, Fredrick (2014) supports the theory and observes that this learning approach is closely related to a developing trend of teaching practices in which educators are typically passive and facilitate the generation of new knowledge while retaining their instructional roles. In the context of teaching 3D shapes, the teacher can simply highlight the relationship between these shapes and their 2D counterparts. Thereafter, learners can be required to work out tasks such as calculation of volume, independently. Fredrick (2014) adds that the constructivist theory has implications of significantly diminishing the role of the educator in the learning environment. This could be the situation in teaching 3D shapes since constructivists believe that learners construct their knowledge. Moreover, Orbell, Hodgkins & Sheeran (1997) argue that the constructivist teacher provides tools such as geometry based problem-solving (GBPS) and enquiry-based learning (EBL) activities to learners. Geometry based problem-solving refers to working out problems involving geometry such as the calculation of angles and perimeters of shapes. Hutchings (2017:25) defines inquiry-based learning activities as “an environment in which learning is driven by a process of enquiry owned by the learner.” This means that learning can begin by a scenario, fieldwork or independent research, in which learners familiarise themselves with the learning content. Fieldwork for learners in Grade 9 intended to teach 3D shapes may involve moving around the school or the surrounding area, identifying 3D shapes.

With GBPS and EBL learners would be expected to formulate and test their ideas, draw conclusions and inferences, and pool and convey their knowledge in a collaborative learning environment. For example, learners in Grade 9 can be placed in two groups: one with a solid cylindrical bar and the other with a cylindrical material such as a one formed from folded paper. The group with a solid bar should identify the 2D shapes visible and these would be the two circles on either end. The group with folded paper may be required to cut it straight from one end to the other and identify the shape that is formed. Thereafter, learners may then have to work in their groups and try to identify similarities between 2D shapes identified and the rectangle formed from the cut paper and original 3D shapes. The findings can then be reported to the class. This way, learners will be testing their knowledge of 2D shapes and explore their relationship with 3D shapes. They might also be able to draw conclusions based on their findings in the group exercises.

Moreover, while arguing that this is typically a traditional learning approach; Orbell et al. (1997) add that simulated experience must refer to something that is personally experienced or lived through by a learner. The example just given above is a perfect example of a simulated experience.

On the other hand, Vygotsky's critics such as Zimmerman & Schunk (2013) and Sherman, Richardson and Yard (2013) converge that the process of learning is largely influenced by observations and experiment rather than experience. Those authors further contend that observations establish and shorten the learning curve by providing the learner with an opportunity to visualise a demonstrated or simulated problem-solving technique. The learner should, therefore, be more focused on experimentation in solving similar problems while the teacher is seen to merely shape what new information, adequately or inadequately generated by the independent learner, is relevant for new knowledge. In the example above, the tasks of identifying the shapes in the solid cylindrical bar and the cutting of the open cylinder should be done by the teacher while leaners are observing. The teacher may then give learners similar independent tasks to do on their own. Zimmerman & Schunk (2013) and Sherman, Richardson & Yard (2013) argue that this makes learning faster than having learners figuring out these tasks on their own.

Another, significant criticism of the theory is its failure to acknowledge that the previous knowledge of learners and experience on a learning concept are not essentially at the same level (Fredrick 2014). For example, not all learners clearly understand the link between, for instance, a square and a cube. Because of this, constructivist teaching methods necessitate a different grading system to the traditional ones because the learner's efforts must always be strongly taken account of regardless of whether they have reached the correct conclusions or not (Dubinsky & McDonald, 2015; Fredrick, 2014). Such a grading system may be practical in small classes like those of former Model C schools. However, the administrative burden may prove impractical in class sizes in most South African schools.

2.4.4 The Van Hieles Levels of Geometric Thinking

One of the most dominant theories on the learning of geometry is the Van Hieles' Levels of Geometric Thinking (Kotzé 2007). According to Van Hieles' theory, the development of learners' geometric thinking is considered in terms of the increasing level of thinking sophistication as learners move from one level to the other (van Hiele, 1984). These levels are hierarchical and can predict future learners' mastery of geometry (van de Walle et al. 2013; Mason 2012). The model consists of five levels of Geometry understanding:

- Level 1: Visualisation
- Level 2: Analysis
- Level 3: Informal deduction
- Level 4: Deduction
- Level 5: Rigour

Originally, the levels began at Level 0 and were modified in a study in the United States to begin at Level 1. It is these latter levels that are discussed briefly in this study.

Level 1: Visualisation

The base stage of van Hieles' (1984) geometric thinking requires learners to use visual perception and nonverbal thinking in learning Geometry. According to Vojkuvkova (2012:72) learners at this stage recognise "geometric figures by their shape as "a whole" and compare the figures with their prototypes or everyday things (“it looks like a door”), categorise them (“it is / it is not a…”). The following figure shows the material that can be used to teach at this level.

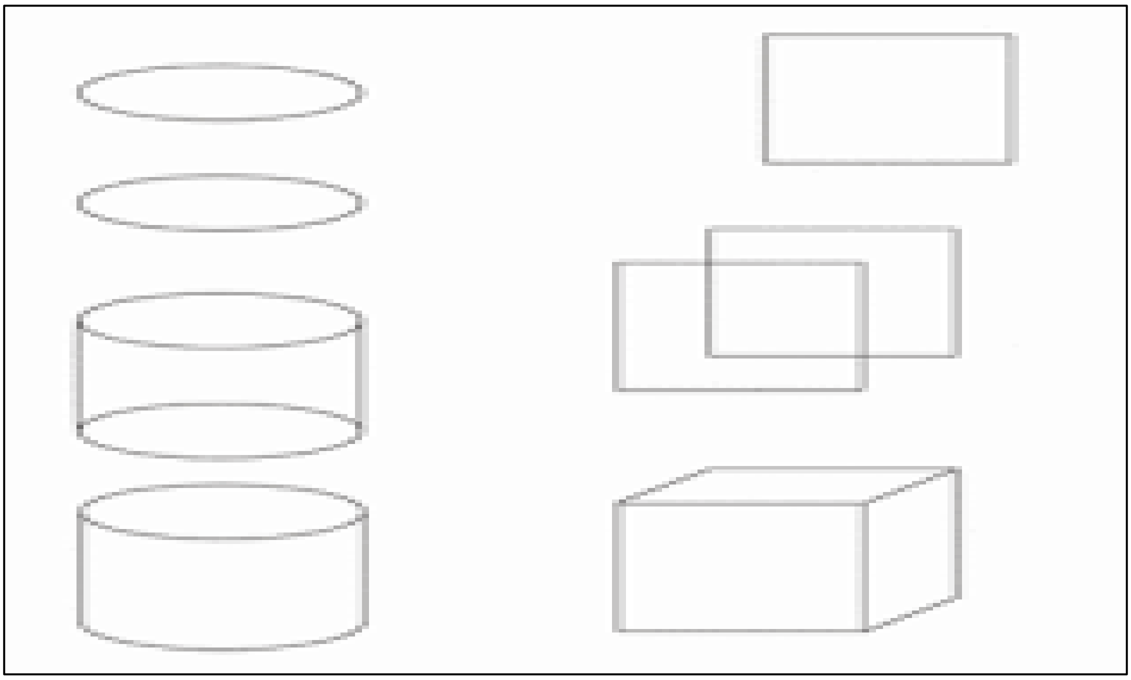


Figure 2.4: Visualising 3D shapes (Source: Education.com, n.d., Online)

Learners should be able to identify the given shapes. There are two circles, two cylinders, three rectangles and one rectangular prism. Moreover, Van de Walle, Karp and Bay-Williams (2013) explain that understanding at this stage includes visualising base shapes. A base shape refers to the surface on which a solid object stands or the shape making up the sides or top of a solid 3D shape. In the figure above, there are two circles while the three rectangles are the base shapes of the rectangular prism. Visualisation is defined as comprehension or seeing initial objects in the learners’ minds (Van de Walle et al. 2013). Therefore, mastery of this level means that learners are able to simply identify the shapes given.

Level 2: Analysis Stage

According to Vojkuvkova (2012), learners start to analyse and name properties of geometric figures at the second level. Van de Walle et al. (2013) further explain that at the analysis stage, learners begin to analyse objects that were only visually perceived at Level 1. They begin to identify Geometric figures' parts and relations among these parts. Learners focus on the properties of these objects. For example, Van de Walle et al. (2013) state that there are many different kinds of quadrilaterals but all have several things in common: all of them have four sides, are coplanar, have two diagonals, and the sum of their four interior angles equals 360 degrees. Referring back to Figure 2.5, Grade 9 learners who operate at this stage are therefore expected to be able to identify features such as the rectangular prism's number of vertices, faces and edges.

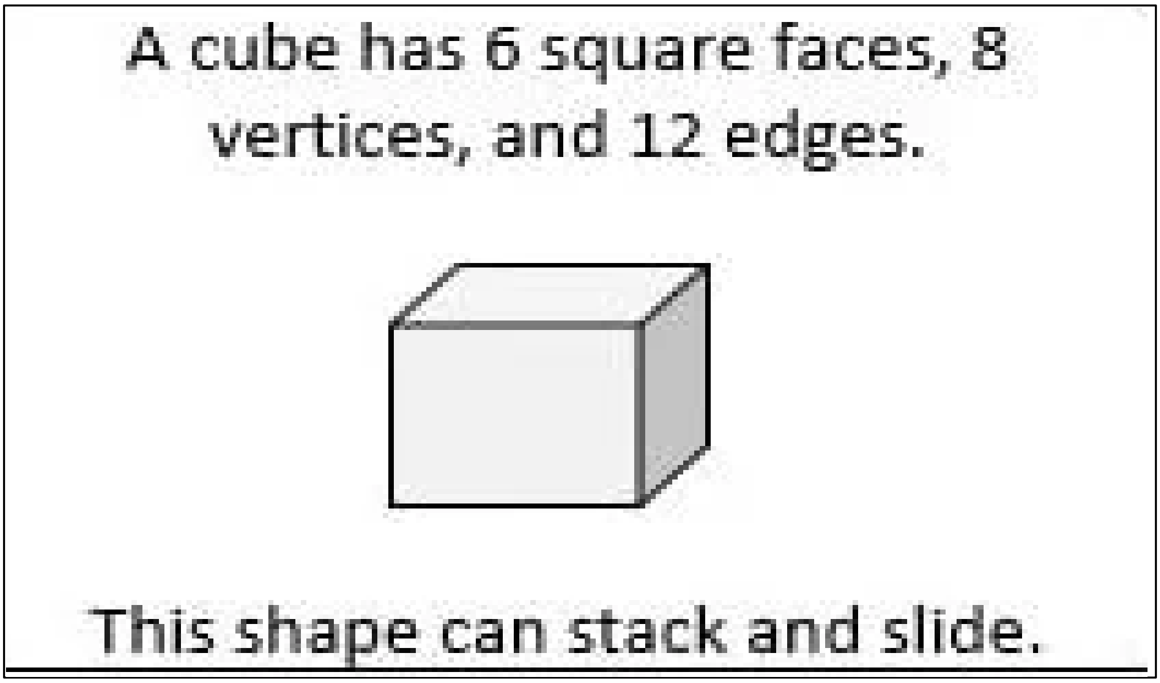


Figure 2.5: Properties of a cube (Source: Elkins, Math and Reading, 2020: Online)

Sarama et al. (2011) assert that the understanding of 3D shapes with common base formulas helps learners better in line with van Hieles' analysis level of geometry thinking. In the context of learning 3D shapes, this means that learners must have sufficient knowledge of 2D shapes which are the base shapes of 3D shapes. This knowledge is brought forward from earlier grades. In addition, Cangelosj (2017) opines that the need for prior understanding of 2D shapes as a foundation of 3D shapes prompts learners to revise and write clear definitions and properties such as sides, angles and diagonals. This way, learners can understand the link between 2D and 3D shapes.

Also, Sarama et al. (2011) note that in solid geometry the analysis stage is where learners begin seeing the properties associated with the different shapes or configurations. For instance, learners at this stage may now see a cube as a shape with 6 equal faces with opposite faces that are parallel and 12 edges and adjacent angles right angles and having equal opposite faces as well as having the diagonals intersect in their middle (Sarama et al. 2011).

Furthermore, Jones (2012) states that at this stage, it is not assumed that learners will be seeking logical relationships between properties such as knowing that a parallelepiped is solid with parallel opposite faces. Parallelepiped refers to a three-dimensional figure formed by six parallelograms (Sawyer & Reiter 2011). In addition, Jones (2012) further points out that it cannot also be assumed that learners may think about a cube as a special type of Parallelepiped. For that reason, learners may identify shapes and solids based on the wholeness of the properties. In other words, relationships between shapes and configurations remain merely on the list of properties they have (Jones 2012). This implies that learners in Grade 9 should only be able to distinguish 3D shapes in terms of basic properties such as the number of faces.

van Hieles' theory postulates therefore that at analysis if a learner were asked to describe a shape or solid, the description would be based on the object's properties (van Hiele 1984). At the same time, if a learner were asked to reproduce a shape or solid based on the list of properties, they would be capable of doing so (Fouze & Amit, 2018). The learner at this stage can recognise the interrelation between figures and their properties. In the context of Grade 9 learners, if learners can identify the properties of a parallelepiped shape, it would be easier for them to deduce that a cube is a special kind of Parallelepiped.

Level 3: Informal Deduction Stage

Informal deduction is known as the third level of geometric thinking (De Wet 2010). Some researchers name this level the abstract/relation level (Battist 1999; Cabral 2004). This is because the level often requires learners to deal with ideas about shapes rather than concrete objects. In this stage, learners are required to reason logically (Clements 2014). Küchemann & Hoyles (2016:64) further explain that the informal deduction stage is achieved when a learner can operate with the relationships of 3D shapes and can apply congruence of geometric figures to prove certain properties of a total geometric configuration of which congruent figures are a part. In the context of teaching Grade 9 learners 3D shapes, this may mean that learners can deduce that the reason why the volume of a cube is one side cube or side x side x side is that the length, height and width are all the same.

At this level, more attention is given to relations among properties of 3D shapes (Clements 2014). Herein, according to the relationship between properties of objects learners attempt to group these properties into subgroups (Cangelosj, 2017). For example, learners should be able to identify similarities between a cube and a rectangular prism. Those two shapes can be placed in one group different from a cylinder and a sphere. Learners try to find out the properties needed to describe the bases of the shapes in the various groupings. The intention would be to categorise properties which are equivalent in a certain situation (De Wet 2010). For example, learners need to categorise properties equivalent for a cube and a rectangular prism such as the number of faces and the number of vertices. The mathematical relationships between properties are the main focus in this stage. Thus, the relationships and or differences between a cube and a rectangular prism would be important at this stage. Understanding and finding these relationships is a kind of informal deduction process. In this stage, the learner would be able to produce proofs and deductions. That is where using tools like *Cabri 3D* as a dynamic geometry software play very important roles to allow learners to experience shapes, produce proofs and make deductions.

Level 4: Formal Deduction

Formal deduction is the fourth level of van Hieles' theory of geometric thinking (Zimmerman & Schunk 2013). At this level, learners start to construct rather than just memorise proofs (Jones 2012). They can find differences between the same proofs (De Wet 2010). The goal of Level 4 is discovering the relations among properties of the bases by the learners (Taylor 2018). At Level 4, those relations are used to deduce theorems about base elements based on laws of deductive logic (Taylor 2018). The main purpose of level 4 is the organisation of the statements about relations from level 2 and 3 into deductive proofs (De Wet 2010). For example, learners at this stage should be able to make the following deduction given the cube in Figure 2.6 below:

- **Known Fact 1** – a cube is a three dimensional solid with 6 square faces
- **Known Fact 2** – the edges form a line segment where 2 faces meet
- **Known Fact 3** – the vertex is where 3 edges meet
- **Deduction** – if I know the length of one of the sides of a cube, such as a size L, I can determine the volume and surface areas

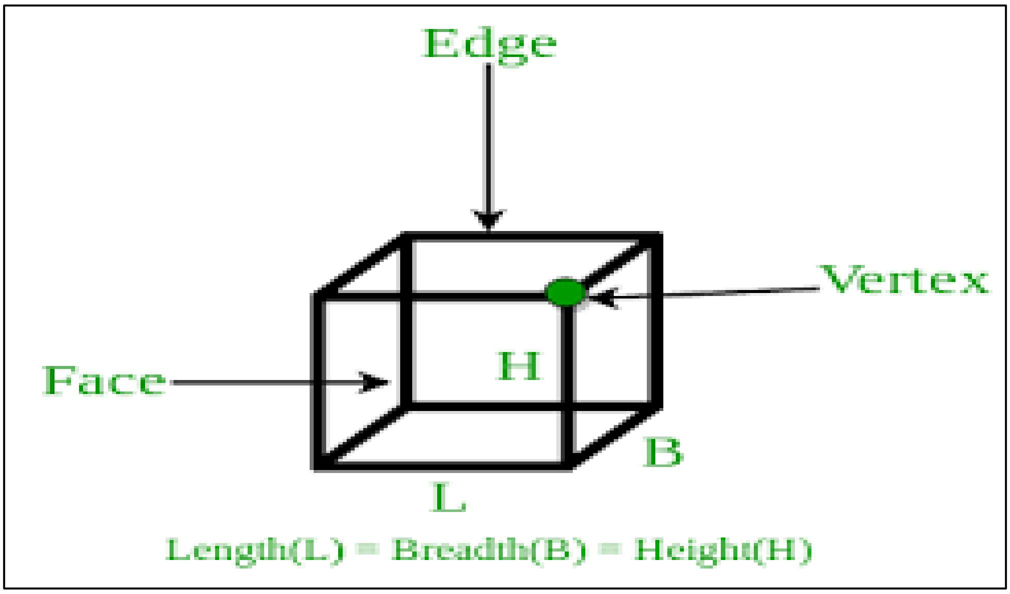


Figure 2.6: Deduction based on a cube (Source: Elkins et al. 2020: Online)

Mason (2012) adds that another point in this stage is that learners begin to become aware, understand and identify the differences between contrapositive, converse and a theorem. Contrapositive and inverse are defined in terms of hypotheses and conclusion. Using the deduction example given in Figure 2.6 about a cube, the following table shows what is meant by the contrapositive and converse statements.

Table 2.2: Contrapositive and converse statements

Statement	**A cube is a 3D shape whose volume and surface area can be calculated when the size of one of its edges is given**
Converse	The shape is not a cube even if it has 6 faces if one cannot calculate the area of its surface area and its volume when given the size of only one edge
Contrapositive	If one cannot calculate the area and surface area of a 6-sided 3D shape when given the sizes on only one edge, then the shape is not a cube

The converse statement exchanges the conclusion and the hypothesis while the contrapositive statement negates both the hypothesis and the conclusion. A theorem, on the other hand, is a “mathematical claim which has been proved to be true” (Math Vault, n.d.: **Online**). The figure below shows how the Pythagoras Theorem can be used in 3D shapes. The theorem states that in a right-angled triangle, the square of the hypotenuse is the same as the sum of the square of the other two sides.

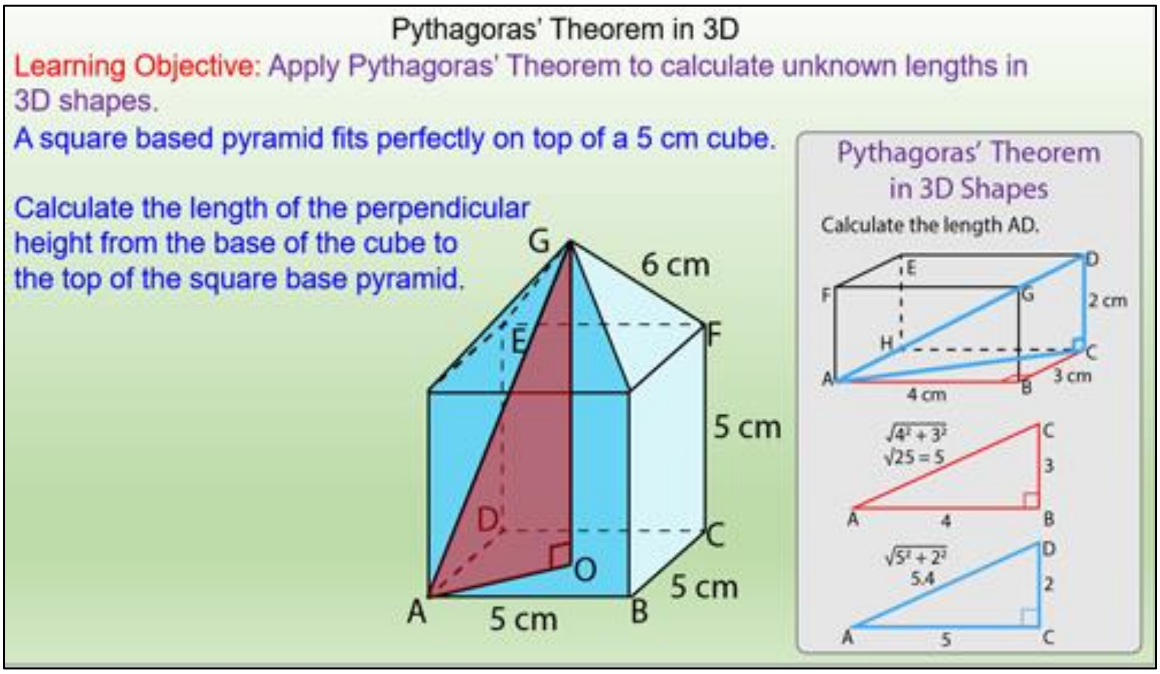


Figure 2.7: Applying Pythagoras' Theorem in 3D shapes (Source: Mr Mathematics 2017)

Learners operating at van Hieles' fourth level should be able to perform the calculations shown in Figure 2.7.

Level 5: Rigor

Sarama et al. (2011) state that the fifth level of van Hieles' theory is called Rigour. At this level, learners should be able to hyper-analyse the deductive proofs from level 4. Learners are looking to find the relationships between proofs. For example, at this level, the questions of "are the proofs consistent with each other?", "how strong is the relationship described in the proof?" and "how do they compare with other proofs?" would be asked (Mason 2012). The level of Rigor involves a deep questioning of all of the assumptions that have come before (Smart 2008).

Mason (2012) and Battista (2009) explain that this type of questioning also involves a comparison to other mathematical systems of similar qualities. In the context of teaching Euclidean Geometry, this may involve comparing the use of the Pythagoras' Theorem in 2D shapes and the use in 3D shapes as shown in Figure

2.7. Mason (2012) states that most learners who are able to do these comparisons, become eloquent in geometry and they can adequately apply the theorems in different axiomatic geometric systems. Therefore, this level is usually found in the work of professional mathematicians and mathematics research learners who research other areas of the geometry.

Jones' (2012) review of the van Hiele theory showed that most of the high school learners are taught at level 3 and 4. This was also observed by van Hiele (1984) who found that most of the learners had difficulty in learning geometry at level 3 and 4 because they had not understood geometry at level 2. As a result, such learners were not able to advance to grasping level 5. This may also relate with the South African learning process where learners were found to have difficulties in comprehending the properties of 3D shapes such as several faces and the number of vertexes, in level 3 (DoBE 2018). As has already been noted, this is one of the reasons why learners at Grade 9 cannot be expected to have a grasp of geometry operations at the fifth stage. In this study, the assessment ends at level 4 as prescribed by the DoBE (2012).

2.4.5 Critiques of the van Hiele theory

van Hieles' theory is an important theory for designing and delivering phase-based learning curricula, as is the case in the South African context. This ensures that learners gain competence as they raise the ladder of learning progression (Luneta 2014). In addition, the theory relates to age-based developmental phases proposed by Piaget (Franzoi 2006) by acknowledging that geometrical development stages are based on experience through different phases of learning (van Hiele 1984). According to Kotzé (2007:22), "these phases may be recursive and are not necessarily achieved in a linear Piagetian progression". A linear Piagetian progression refers to the idea that learners follow a stage of development in which their knowledge and capacity to acquire new knowledge increases with age (Fleming 2018). However, there are several criticisms of the theory.

Firstly, Kotzé (2007) argues that the theory does not explain what teachers should do in a case where learners are in the same classroom but are at different levels of understanding. A further criticism levelled against the theory is its perceived inability to fully aid learners in a figurative understanding of the intrinsic figural nature of a geometry object. An intrinsic figural nature, in this context, is defined as a spatial sensory representation subject to a figural law such as closure (Fischbein 2012; Küchemann & Hoyles 2016). According to Chapman (2019), closure refers to a situation where the brain completes an incomplete object based on prior knowledge. This can be illustrated in the figure below.

Figure 2.8: Closure (Source: WLF, n.d.)

In the image in Figure 2.8 large portions of the panda are missing. However, the brain has no problem constituting the idea of a panda. Even though it is clear that the image shows a 3D geometrical shape, there are no prescribed instructions of teaching this in van Hieles' theory.

Despite the criticisms discussed above, van Hieles' theory remains commonly used in teaching geometry due to the absence of more concrete theories to discredit it.

This study was therefore underpinned by the van Hieles' theory. The next step is to review the literature on Geometry learning difficulties.

2.5 Difficulties in Learning Geometry

The previous section has presented a review of literature on the theories which can be used to structure learning Geometry in general and 3D shapes particular. For example, Van Hieles' Theory of Geometric Thinking shows the stages through which learners should progress when learning geometry, including 3D shapes. The implication of the theory is that if learners find challenges at any one of the stages, it becomes almost impossible to progress to the next level. Once that happens, then learners will have difficulties in learning 3D shapes at higher levels. In addition, learning difficulties can be viewed from Vygotsky's Constructivist Learning Theory. From this perspective, if learners find no link between the 3D shapes they learn and the 3D shapes in their everyday lives, then they will face learning difficulties.

There are several documented learning difficulties in learning Euclidean Geometry. However, there was limited literature found which related specifically to the difficulties encountered in learning 3D shapes in particular. Some of the related studies are discussed in this section. It should be noted from the onset that the following discussion is not in any way intended to be exhaustive. It, however, indicates some of the main challenges faced in a Mathematics classroom.

2.5.1 Teachers' knowledge of 3D shapes

The general requirement in all of the stages of the van Hieles' theory is that there should be an adult leader, an educator in this case in the learning process whose role is to design learning programmes and facilitate the learning process. This means that this leader should be competent and have superior basic knowledge of the learned content than the learners. A study by Luneta (2014) of a cohort of 128 first-year learner-teachers registered for a foundation phase programme at a university in South Africa made some worrying revelations. The study found that the majority of the learner-teachers were operating at Levels 1 of the van Hieles' levels as opposed to Levels 3 and 4 that would be expected of Grade 12 learners entering the university. Luneta (2014) concluded that this deficiency was one of the reasons why learning geometry and 3D shapes, in particular, becomes challenging for learners. Learners usually have adult leaders who do not possess superior knowledge of the concepts to be learned as would be expected in van Hieles’ theory. It can be argued that the study sample was too small to allow generalisations at the national level. However, the study points to potential problems. The findings of this study also support Adolphus (2011) who found that the foundation of most Nigerian mathematics teachers in geometry was poor. Adolphus’ study is discussed further below. In the context of 3D shapes, teachers are expected to visualise such shapes at level 1, identify properties such as faces and vertices at level two, make formal deductions as discussed under level 4 and ultimately be able to prove and evaluate theorems relating to 3D shapes as would be expected at van Hieles’ fifth level.

2.5.2 Failure to identify and integrate Geometry learning in everyday lives

Another challenge that emerges from the literature is the failure to integrate Geometry learning to everyday life as advocated by the constructivists. Utami & Pramudya (2014) carried out another study to determine the difficulties encountered by junior high school learners in creative thinking skills level in resolving rectangular conceptual problems. The study found that though some learners could perform basic operations of the concepts tested, they failed to identify similar objects in their everyday environment. As a result, such learners had difficulties in carrying out the required analysis at van Hieles’ Level 2.

2.5.3 Language of instruction

According to Nugroho & Wulandari (2017), language plays an important role in the teaching and learning of new concepts. In the context of this study, language could help to identify ethno-cultural objects for which learners have pre-existing knowledge and are well acquainted with. Ethno-cultural, in this context, relates to or denotes a person’s ethnic and cultural origin (Oxford Advanced Learner's Dictionary 2015). Kotzé (2007) points out that at early stages of the learning process such as a Grade 9 level, the role of the school is to provide a scientific organisation of knowledge that learners would have already acquired in their ethno-cultural environments. According to McLaughlin (1995), this reorganisation should be done in the same language that the knowledge was initially acquired, lest some of the knowledge is lost in the translation process. In the South African context this is a real challenge given that most learners are taught in their second or even third language.

2.5.4 Poor foundation and inadequate learning support

A further study of geometry learning difficulties was carried out by Adolphus (2011). The study focused on a sample of three hundred learners and thirty teachers drawn from ten secondary schools in Nigeria's Rivers State. The study revealed three critical learning difficulties in geometry learning. The first finding has already been noted above, namely, that the foundation of most mathematics teachers in geometry was poor. Another critical finding was that the learners had a poor foundation in mathematics in general. This takes us back to the hierarchical nature of the van Hieles' theory. If learners' foundation was found to be poor, it would be difficult to teach those learners higher-level concepts. The third finding was that the teaching and learning environment was not conducive for both the teachers and the learners. The study revealed that the classroom environment was congested, making it difficult for the teacher to provide individualised attention needed. The importance of the learning environment has already been discussed in the context of the Constructivist Theory. Adolphus (2011) further explains that even though the school environment could be improved, it would be difficult for learners to reach their full potential without a corresponding improvement in the home and general social environments.

2.5.5 Learners' perception of Geometry

The social environment can both be a facilitator and an inhibitor of Geometry learning, like many other concepts (Forgasz & Rivera 2001). As an inhibitor, the social environment can build negative perceptions about the subject of learners. A mixed design approach study by Gezahegn (2007) focusing on the various school stakeholders in Ethiopia found that the perception of Mathematics was one of the reasons why the subject was being failed in schools. The author found that Mathematics was generally viewed as a difficult subject across the studied stakeholders some of whom indicated that they had inherited that negative perception from their relatives from childhood. Therefore, when such negative perception is passed on to learners, it is difficult to convince the learners that they can be successful in mathematics and geometry.

Another negative impact of this perception is that learners enter formal education with little pre-existing Mathematics knowledge since it is not widely discussed in the learner's ethnocultural settings. According to Nugroho & Wulandari (2017), this ultimately reduces the chances of the learner to succeed unless special interventions are used to deal with the perception. In the context of teaching geometry, this may mean that learners are not able to identify 3D shapes and find it difficult to identify differences and similarities in 3D shapes such as the number of faces and the number of vertices of a cube and a rectangular prism.

2.6 Learning processes that enhance the learners' cognitive understanding of 3D shapes

Theories on geometry learning have already been discussed in Section 2.2. The most common recommendation when it comes to learning Geometry (including 3D shapes) is van Hieles' theory (for example Adolphus 2011; Nugroho & Wulandari 2017; Kotzé 2007). More specifically, Nugroho & Wulandari (2017) explain that the theory's popular use can be attributed to its perfect link with phase-based education curriculums in most countries such as South Africa. To this end, it is recommended that the theory is not only applied at class level but as early as the curriculum development phase (Gezahegn 2007). This ensures that there is a coordinated approach in developing learners throughout their learning cycles. Moreover, it is easier to incorporate ethnocultural values at the curriculum development level than waiting until the learning and teaching phase since it is most probable that teaching and learning material might not have been adapted to incorporate these ethnocultural values such as the type of 3D shapes learners have in their everyday environments (Fouze & Amit 2018). Due to the wide use of the van Hieles' theory, this study used the theory to provide lenses of understanding learning difficulties and processes that can enhance learners' cognitive understanding of 3D shapes. The theory is also supported by two notable large scale studies in the USA (Fuys, Geddes & Tischler 1988; Usiskin 1982).

2.7 Ways of mitigating 3D shapes learning difficulties

There was no literature found specifically on mitigating 3D shapes learning difficulties. However, there is plenty of literature on mitigating mathematics learning in general. Some of this literature is reviewed here. Again, the following discussion indicates some of the strategies that can be used rather than an exhaustive list.

2.7.1 The use of knowledgeable teachers

As has been mentioned in this discussion, one of the challenges faced in learning Mathematics is deficiencies in teachers' competencies of basic numeracy (see Section 2.3 above). Several authors have been advocating for the proper development of Mathematics teachers so that they master the concepts they intend to teach before they go into the classroom (for example Fouze & Amit 2018; Nugroho & Wulandari 2017). Leone, Wilson & Mulcahy (2010), add that teachers' competencies should extend beyond just content mastery to delivery competencies so that teachers can develop context-specific learning programmes that suit learners' specific needs.

2.7.2 Use of graduated instructional sequencing to teach abstract concepts

In addition, Leone et al. (2010) explain that teachers need to think critically about topic and concept sequencing when planning lessons and learning programmes. The authors recommend a hierarchical structure in which learners are taught easier concepts first while progressing to more challenging ones. This supports the methodology espoused by van Hieles'' theory as discussed above, in which concepts are grouped into levels.

2.7.3 Provision of adequate environmental support

The review above has revealed that one of the challenges in Mathematics teaching and learning relates to the environment that is not supportive of the learning process. More specifically Adolphus (2011) cited crowded classrooms as a key challenge since they deny learners the individualised attention they need to succeed. Gezahegn (2007) found that some classrooms were as big as 70 to 80 learners per class. Such classes present additional challenges of learnercontrol in addition to the lack of individualised attention already noted. Therefore, one way of dealing with this challenge is reducing class sizes. This is particularly in the context of teaching 3D shapes since it has already been found that most learners lack pre-existing knowledge of the subject due to the generally negative perception of the subject. Small class sizes may allow the teacher to bridge this knowledge gap easily by providing individualised attention to learners. This should be accompanied by additional learning support such as adequate textbooks and learning aids (Leone, Wilson, & Mulcahy, 2010). Smaller classes also allow the teacher to develop inclusive learning support for learners with natural learning barriers such as those with Dyscalculia (Kadosh & Cohen 2016).

2.8 Teaching and learning of the 3D shapes in the 21st century

Teaching and learning of 3D shapes, just like other aspects of life, should evolve to adapt to changes in environmental factors. In this respect, technology has had a significant impact on how 3D shapes can be learned. One such development is the use of powerful computer-based geometry and visualisation software packages to learn 3D shapes. Ismail & Rahman (2017) studied the impact of one such software, Geogebra, on learners' geometric thinking of in learning 2D and 3D shapes. The authors used van Hieles' levels of geometric thinking to administer pre-test and post-test of learners' geometrical thinking in learning 2D and 3D shapes. The results of the study showed that there was a significant improvement in visualisation and informal deduction for both 2D and 3D geometry. These findings mean that teachers now have better technological tools to teach 3D shapes in addition to the traditional concrete objects.

2.9 Spatial thinking in 3D shapes teaching and learning

According to (Kovaˇcevi´c 2019:2), spatial reasoning can be defined as "the process of forming ideas through the spatial relationship between objects." In the context of 3D geometry, this refers to mental activity in which learners create and manipulate spatial images to solve theoretical and practical problems. For example, instead of thinking about a number line as a line drawn on the board or in the book with numbers written on them, learners utilising spatial reasoning can think of a series of 3D blocks arranged in a sequential form. The position and distance of one block from the others can help learners to think of a number line in concrete terms rather than as an abstract idea. The ability of learners to imagine and visualise a spinning dice can also help them better understand the characteristics of a cube than showing learners an image of a cube in a book. To form an image and visualise a spinning dice, learners make use of spatial thinking. In both of the two cases, it would be more beneficial to the learners if the teacher uses concrete objects to teach 3D shapes than using images in learning materials such as books.

2.10 Conclusion

The literature review has shown that though there are several theories on learning of Geometry, there is no consensus on which one should be used. However, van Hieles' theory emerges from this debate as the commonly used theory in this respect despite having some criticisms levelled against it. The theory is supported by results from large scale studies in the USA by Fuys, Geddes & Tischler (1988) and Usiskin 1982). The literature on the difficulties of learning geometry has also been sufficiently reviewed. Difficulties identified include those relating to the teacher such inadequate mastery of basic geometric concepts expected at levels 4 and 5 of van Hieles' theory, environmental factors and ethnocultural factors such as general negative perception towards Mathematics and geometry in particular. Some of the processes for enhancing understanding of Geometry and 3D shapes have also been discussed together with ways of mitigating Geometry learning challenges. One such way of mitigating these challenges is through the use of geometrical computer software packages. Some aspects of spatial reasoning have also been discussed in the context of teaching and learning of geometry. The next chapter discusses the research methodology that was developed to answer research questions in this study.

CHAPTER 3 Research Design and Methodology

3.1 Introduction

The preceding chapter presented the literature related to learning difficulties experienced by grade 9 learners in understanding 3D shapes in geometry. This included theories pertaining to mathematics learning with Hieles' theory of geometry thinking identified as the underlying theoretical concept for the current study. This chapter discusses the research methodology developed in order to collect, analyse and interpret data for this study in order to answer research questions. The first section provides a discussion of the research paradigm chosen in this study followed by a discussion on research methods. The discussion moves on to issues relating to population and sampling strategy. The chapter also presents a detailed discussion of the research instruments developed in order to collect data. There is also a section focusing on the various aspects of data analysis and interpretation. The final two sections present research quality, validity and reliability and ethical issues that arose from conducting this study, respectively.

3.2 Research Paradigm

In ascertaining the appropriate research strategy for this study, it was imperative to establish the guiding research philosophy. According to Creswell (2013), research philosophy establishes assumptions consisting of a stance toward the nature of reality (ontology), how the researcher knows what he knows (epistemology), and the role of values in the research (axiology), the research language (rhetoric), and the methods used in the research process (methodology). There are two main kinds of research philosophies commonly used in scientific research such as the current study, that is, positivism and interpretivism. These will be explained in the ensuing subsections and a justification of the chosen approach is given thereafter.

3.2.1 Positivist research paradigm

According to Saunders, Lewis & Thornhill (2015), the positivist research paradigm is of the belief that the reality of what is obtaining in a phenomenon can only be accurate and concluded on when the phenomenon's influencing constructs are subjected to numeric significances. Moreover, Cooper & Schindler (2011) assert that the usage of measurable and gauging techniques in assertions of what is happening in a research phenomenon is found to enhance the interpretation of what is considered the objective reality of occurrences. Gauging techniques refer to the use of numerical measures (Sekaran 2016). Also, the objective reality of occurrences refers to factual reality that can be proved from existing facts (Ritchie 1892) such as the fact that the sun rises from the east and falls in the west.

Saunders, Lewis, & Thornhill (2015) further note that belief is grounded on the determination that only what is deduced as the real issues with statistical relevance can be relied on in relation to the need for accurate projections of what is, and what can be done. For the purpose of this study, this means that the learning difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in geometry could only be objectively understood if such challenges can be numerically measured. This is partly true since challenges can be assessed through pre-, during and post-intervention testing. Test scores at those stages could be numerically recorded. However, these were not enough due to some of the challenges identified in Chapter 2, such as learners' perceptions in Geometry which were inherently subjective and could not be reliably reduced to numbers. Even if reducing those challenges to numbers was possible, such numbers are unlikely to be of any use on their own without further qualitative enquiry. The answer to the latter problem lies in the interpretivist research paradigm, discussed in the next section.

3.2.2 Interpretivist Research Paradigm

According to Creswell (2013), an interpretivist believes that the reality of the world cannot only be seen through the lens of measurable metrics to be concluded and relied on. Furthermore, Blumberg, Cooper and Schindler (2016) explain that the truth about reality, in the context of research, cannot be separated from the human participants in the research since their worldviews embody the meaning and truth about a phenomenon. Therefore, according to those authors, truth about a phenomenon cannot be presumed complete if it lacks the qualitative voices of research participants. Those voices cannot be objectively captured numerically. Because of these reasons, this study seeks to tap into the riches of interpretivism at some point in order to qualitatively appreciate the true nature of the difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in geometry. This choice was partly guided by Thanh and Thanh (2015) who observed that an interpretive paradigm allows for different people and different groups to have their assertions and viewpoints integrated into the analysis. Therefore, the interpretivist paradigm permits for the multiplicity of perspectives which are valuable in facilitating the gaining of in-depth and insightful information about challenges of learning 3D shapes. Figure 3.1 summarises the underlying principles of the positivism and interpretivism research paradigms.

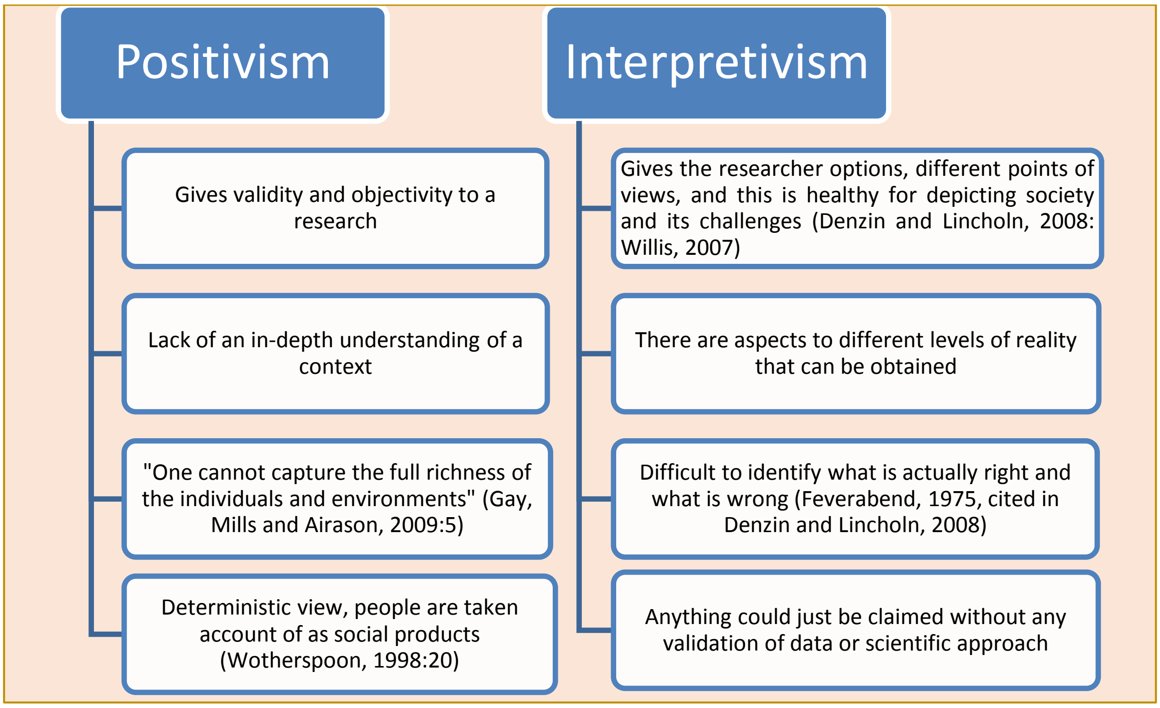


Figure 2.9: A Comparison of Positivism and Interpretivist Research Paradigm

Source: Own illustration based on Denzin & Lincholn (2008); Willis (2007); Gay, Mills & Airason (2009:5); Wotherspoon (1998:20)

In order to benefit from the advantages of each paradigm, this study used the principles of both in a mixed method research design. Mixed method research design is discussed below in section 3.4. In as much as there is need to measure learners' scores in 3D exercises (Quantitative Paradigm), there was also need for a deeper analysis of why learners attain such scores in the first place (Quantitative Paradigm). In other words, there was need for interpretation of those scores in order to appreciate the challenges that learners face when learning 3D shapes.

3.3 Research Methodology

Research methodology refers to a blueprint that explains the systematic and logical steps followed in coming up with empirical evidence for the study (Barbie & Mouton 2011). In the context of this research, the methodology involved the logical steps that were followed in order to find answers to research questions posed in Chapter 1. According to Saunders, Lewis & Thornhill (2015), there are three research methodologies that are typically used in scientific research, namely, qualitative, quantitative and mixed method research methodologies. Each of these is discussed in the following sub-sections.

3.3.1 Qualitative Research Methodology

A qualitative research design considers that in-depth deep probing of primary evidence is of paramount importance to understanding the phenomenon in the study (Cooper & Schindler 2011). In addition, Creswell (2009:4) defines the qualitative research methodology as a "means for exploring and understanding the meanings individuals and groups ascribe to a social and human problem". In the context of this study, this implies that grade 9 leaners should be allowed to explain what they see as the challenges they face in learning 3D shapes. Also, qualitative research fitted well with the interpretivist research paradigm, already chosen above. However, relying on qualitative research methodology alone was insufficient for the purpose of this study. This is because a qualitative study would not identify the level of mastery that learners would have achieved in understanding 3D shapes. Such mastery would only be identified by analysing learners' marks, which are quantitative in nature. This is the reason the quantitative research approach was also used. The quantitative research approach is discussed in the following section.

3.3.2 Quantitative Research Approach

A quantitative research approach is a research approach in which assertions about a research phenomenon are quantified numerically (Hair, Celsi, Money, Samouel & Page, 2016). In addition, Blumberg, Cooper & Schindler (2016) argue that a quantitative research method is designed with structured and close-ended enquiries whose assertions are subjected to a measuring instrument or device with numerical variables. In the context of this study, learners' marks and learner's responses to the numerical parts of 3D shapes problems were relevant in appreciating the challenges that Grade 9 learners faced in learning 3D shapes. However, such quantification of the challenge was not sufficient since there was need for qualitative probing of the underlying reasons which is something that could not be apparent from simply analysing the scores. Therefore, this study adopted a mixed method research approach rather than an outright quantitative or qualitative research approach. The mixed method research approach is discussed in section 3.3.3.

3.3.3 Mixed methods research approach

According to Saunders et al. (2015:152), a mixed method "is the general term used when both quantitative and qualitative data collection techniques and analysis procedures are used in a research design." In other words, a mixed method research approach combines both qualitative and quantitative research approaches. In a mixed method research approach the quantitative and qualitative data collection techniques and analysis procedures can be used "either at the same time (parallel) or one after the other (sequential) but does not combine them" Saunders et al. (2015:152). For the purpose of this study, the parallel mixed methods were used. While qualitative data was gathered through interviews and observations, quantitative data was also being gathered through pre- and post-intervention activities. The concurrent collection and analysis of data saved time for this study. It also ensured that qualitative data such as explanations was gathered in close proximity to learners' activities thereby making it easier to appreciate the Grade 9 learners' challenges in learning 3D shapes.

3.4 Population and Sampling Strategy

3.4.1 Target Population

A target population is a collection of all possible cases upon which a study is intended to be based (Hair et al. 2016). For this study, the target population was made up of all Grade 9 learners in the Lydenburg Circuit of Mpumalanga province. Again, there were 3 schools in the Lydenburg Circuit. There were 960 grade learners and 15 mathematics educators in the Lydenburg circuit in Mpumalanga province. However, carrying out a study of this size was not possible due to the limited time within which this study was supposed to be carried out. Even if there was sufficient time to carry out the study on all Grade 9 learners and teachers in Lydenburg Circuit, financial and human resources would still constraint the size of the study. Because of those constraints, this study was carried out on a sample. The sample and sampling strategy is discussed in sections 3.5.2 and 3.5.3.

3.4.2 Sample Size

A sample is a set consisting of representative cases selected from a target population upon which a study is carried out (Kothari 2015). Basing a study on a sample helps to mitigate the time and resource constraints stated in the previous subsection. The sample for this study was drawn in two stages. First, two schools were conveniently chosen from three schools in the Lydenburg circuit. From these the following sample was drawn. The following table shows the sample for this study.

Table 2.3: Sample frame and sampling technique

	Grade 9 Maths learners		Grade 9 Maths teachers	
Schools	**Target Population size**	**Sample / Sample %**	**Target Population size**	**School total / Sample %**
School 1	334	60 (18%)	4	2 (50%)
School 2	460	60 (13%)	6	3 (50%)
Total	794	120 (15%)	10	5(50%)
Sample selection technique		Convenient sampling	Convenient sampling	

The schools were chosen for convenience, mainly their accessibility to the researcher. School 1 had 334 Grade 9 learners and School 2 had 460 with an average size of 55 learners per class. Since the study employed statistical techniques to analyse data, the sample size needed to comply with statistical rules. One such rule is the Central Limit Theory which prescribes that a sample size for statistical purposes should be at least 30 (Islam, 2018). Mathematically, this means that sample size is represented by $N \geq 30$. In this case, N refers to the number of subjects in the sample.

3.4.3 Sampling Technique

According to Saunders, Lewis & Thornhill (2015), there are two main kinds of sampling, that is, probability and non-probability sampling. In a probability sampling strategy, there is an equal chance that any of the research elements may contribute to primary evidence collection (Keller 2015). On the other hand, a non-probability sampling strategy determines that there is no equal and known chance that any member of the target population can contribute to empirical evidence gathering (Creswell 2013). This study adopted probability sampling to select Grade 9 learners and non-probability convenient sampling technique to select Grade 9 teachers. Each of these techniques is briefly discussed below.

1. Simple random sampling

In this sampling technique, every individual in the population has an equal and independent chance of being chosen for the study (Saunders et al. 2015). In this study, learners were selected at random from 794 grade 9s from the selected schools. This helped to deal with researcher bias which would have arisen if a non-probability sampling method was used.

2. Judgemental sampling technique

The judgemental sampling technique uses an informant and knowledgeable approach in its criteria to selecting the most fitting interviewees in a qualitative research method (Saunders et al. 2015). In this study, the Grade 9 teachers who were interviewed were selected based on several factors including their perceived experience and the convenience with which they could be contacted in order to participate in the interviews. Figure 3.2 illustrates sampling techniques for the purpose of this study.

Figure 3.2: Sampling Techniques

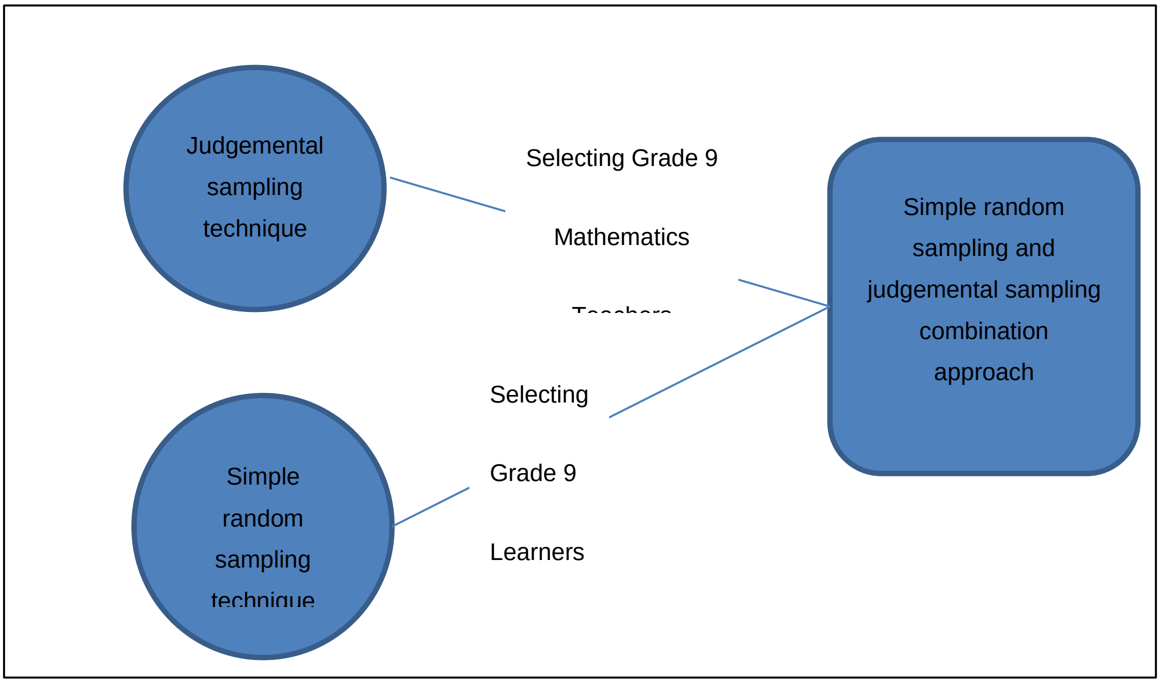


Figure 3.2: An integrated stratified simple random sample and judgemental sampling technique (Source: Author's illustration)

3.5 Instrumentation and data collection techniques

According to Saunders et al. (2015), it is critical to choose research information gathering instruments carefully to adequately answer the research objectives. This study made use of the interview protocol together with test score schedules.

3.5.1 Interview protocol

The interview section of the instrument consisted of structured and unstructured questions. Unstructured questions are open-ended to allow for additional questions to be asked where necessary during the interview process (Creswell 2013). One-on-one semi-structured interviews were conducted with selected teachers through both Zoom, in order to comply with the current COVID-19 pandemic guidelines, and face-to-face, where teachers did not have access to reliable network. Responses were transcribed after the interview. The interview protocol consisted of three broad questions to structure the discussion. Those questions were drawn directly from the sub-questions of the study as presented in Chapter 1. Additional questions were probed as required during the interview process. The interview protocol is included in Annexure F.

3.5.3 Geometry Assessment Exercise

The other data collection instrument used consists of exercises based on learners' understanding of 3D shapes. Two sets of exercises were administered:

1. Pre-intervention exercise

The pre-intervention exercise was in the form of a 3D shapes test given to learners at the beginning of the intervention process. The pre-intervention exercise was aimed at diagnosing learners' knowledge about 3D shapes. The study was organised into five sections according to van Hieles' theory discussed in Chapter 2. Learners' scores in this activity were recorded and were used in quantitative analysis. Based on the results in this pre-intervention test, intervention programmes were developed based on the best practices discussed in Chapter 2. The specific intervention depended on learners' identified challenges from responses.

2. Intervention exercise

Data were also collected as part of the intervention process. The intervention activities and exercises are included in a Google Drive created specifically for this study. It can be found on:
https://drive.google.com/drive/folders/15IvLxNpPz5OjyIYbWadJw4fB3Jto6lJi

3. Post-intervention exercise

At the end of the intervention the learners were required to take a post-intervention test. Like the other exercises based on learners' understanding of 3D shapes, this activity was organised into five sections according to van Hieles'' theory. Learners' scores in this activity were also recorded and were used in quantitative analysis.

3.5.4 Data collection process

As indicated already, the schools from which data was collected were randomly chosen from the Lydenburg Circuit in Mpumalanga province. Each of the three schools in the Lydenburg Circuit was assigned a unique number from which two schools were chosen. The Lydenburg Circuit was chosen because it was convenient and accessible to the researcher it being in the researcher's home area.

Once the schools were chosen, a sample of 30 Grade 9 learners from each school was randomly selected. In addition, two mathematics educators from School 1 and three educators from School 2 were purposely selected for the interviews.

Each leaner was assigned a code with a sequence from 1 to the last number in each class. Initially, 60 learners were supposed to be selected from each school. However, due to the Coronavirus 2019 (COVID-19) pandemic that number was reduced to only 30 learners per school to ensure that the tests were done in compliance with the health guidelines set by the Department of Health. School 1 had 334 Grade 9 learners, while School 2 had 460 Grade 9 learners. A list of all the Grade 9 learners was obtained from the schools' respective administrators, with permission from the principal as discussed in Chapter 3. Thereafter, random numbers were generated. Learners in School 1 were assigned numbers 1 to 334 while those in School 2 were assigned numbers 1 to 460, based on class lists. Class lists were formed with the selected learners, one class in each school. Those two classes were then used as representative samples in each of the two schools.

3.5.5 Administration of tests

The pre-and post-intervention tests were printed together with instructions on how they were supposed to be completed. It should be noted as well that in both schools the study was conducted after the learners had covered the topics on 3D shapes. That was particularly necessary for the pre-intervention activity since it was supposed to test learners' existing knowledge rather than teaching them new things. The tests had space for each learner to add their assigned study numbers. A sample of the activity is included in Appendix 1. The tests were organised into levels based on van Hieles’ theory as discussed in Chapter 2 and Table 4.1 presents a summary of the tests. To ensure consistency, the weighting of each level was the same in both the pre-and post-intervention tests. In the test table “points” referred to the score that the student got after attempting the test.

Table 2.4: Summary of tests

Van Hieles' Levels	Question Number	Points	Weight
Level 1	Question 1: 1.1, 1.2, 1.3, 1.4 and 1.5	5	13%
Level 2	Question 2: All Questions	20	50%
Level 3	Question 3: 3.1, 3.2 and 3.5	6	15%
Level 4	Question 3: 3.3 and 3.4	4	10%
Level 5	Question 4	5	13%
Total Points		**40**	**100%**

3.5.6 Administration of interviews

As indicated in Section 4, the interviews were conducted with two Grade 9 Mathematics teachers from School 1 and three Grade 9 Mathematics teachers from School 2. The teachers were interviewed individually in face-to-face interviews soon after the pre-intervention tests. It should be noted that those interviews, despite being face to face, complied with the health guidelines relating to social distancing and the use of masks to cover the mouth and nose. The teachers were asked questions as presented in the interview guide (See Annexure F). Though Chapter 3 explained how formal interviews would take place with the teachers, it is important to note that informal interviews were also conducted with learners during the intervention process. Since such interviews were informal, there were no pre-prepared questions for the interviews as was the case with formal interviews.

3.5.7 Details of the intervention

3.5.7.1 Interventions Sessions 1 and 2: Understanding 3D shapes

The intervention was done in seven sessions - a session for each day. Sessions 1 and 2 were spent on understanding the nature of 3 D shapes and had the following objectives:

After learning these two sessions, learners should be able to:

- Define polygon, regular polygon and polyhedrons
- Identify different types of polyhedrons
- Name different types of platonic solids
- Find the number of faces, vertices and edges of a given polyhedron
- Use EULER's formula to remember the characteristics of polyhedrons.

The slides that were used in those two sessions are included in Google Drive (https://drive.google.com/drive/folders/15IvLxNpPz5OjyIYbWadJw4fB3Jto6lJi) with the file names *Khomotso Platonic Solids Intervention 1* and the same slides were used in both School 1 and School 2. Several real examples were also used to illustrate the shapes as shown in Figure 4.1 below.

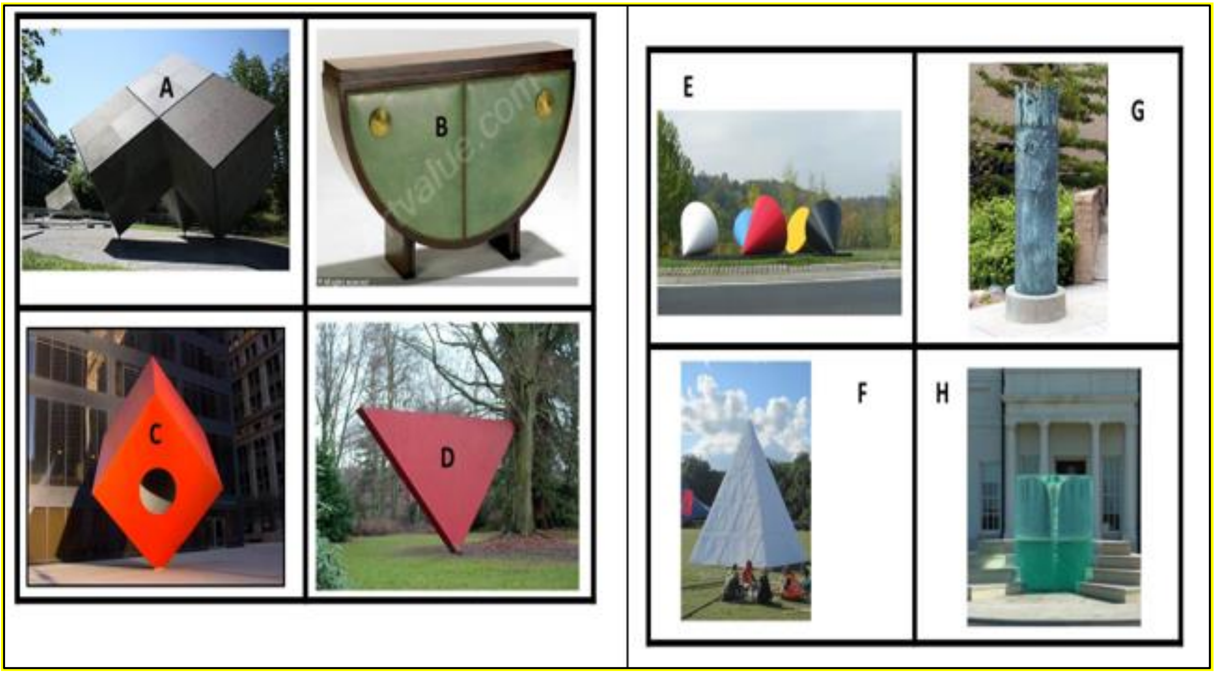


Figure 2.10: Real examples of 3D shapes

(Source: https://www.mixedattainmentmaths.com/uploads/2/3/7/7/23776169/surface_area_and_volume_lesson.ppt)

Further details of how each of the shapes were explained to learners in intervention one and two are included in Appendix 1 while the slides used are included in Google Drive [https://drive.google.com/drive/folders/15IvLxNpPz5OjyIYbWadJw4fB3Jto6lJi], with the file name, *Surface_area_and_volume_Intervention 2.*

3.5.7.2 Interventions Sessions 3 to 7: Surface Area and Volume

In Sessions 3 to 7, the focus was on teaching learners how to calculate the surface area and volume of each of the shapes as stipulated in the Mathematics Senior Phase Curriculum and Assessment Policy Statements (DBE 2012). The sessions were broken down into smaller segments since teachers were still busy with completing the syllabus for the 2020 year which they could not do in time after the COVID-19 lockdown disturbances. Each session had a duration of 30 minutes. The following table shows what was learnt in each of the five sessions.

Table 2.5: Structure of intervention

Day	Learning content
Day 1	The 5 Platonic Solids
Day 2	Area and Volume of Prisms and Cylinders
Day 3	Area and Volume of Pyramids and Cones
Day 4	Surface Area of Prisms and Cylinders
Day 5	Surface Area of Pyramids and Cones

3.6 Data Analysis and Interpretation

According to Cohen, Manion and Morrison (2007:461), data analysis "involves organising, accounting for and explaining the data … making sense of the data regarding the participants' definitions of the situation, noting patterns, themes, categories, and regularities." In this study data analysis refers to the activities carried to convert data into meaningful information that could be used to answer research questions. Both qualitative data from interviews and quantitative data on learners' scores needed to be analysed. Data analysis techniques used in each case are discussed in the following sections.

3.6.1 Thematic analysis

Qualitative data was analysed through thematic analysis. According to Kothari (2015), thematic analysis is a data analysis technique in which data analysis is based on the identified themes that emerge from the collected data. Data reduction is done through coding and classification to categories in order to identify themes that emerge. In this study such themes relate to the various challenges that teachers had experienced while teaching 3D shapes.

3.6.2 Rationale of pre-intervention test

CAPS prescribes that grade 7 learners describe, sort and compare the polyhedral in terms of shape and number of faces, number of vertices and number of edges. In Grade 8 they describe, name and compare the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges. In Grade 9 learners revise properties and definitions of the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges (DBE, 2012). Therefore, learners in Grade 9 are assumed to have a clear understanding of platonic solids and should be able to respond to Questions 1 and 2 in the pre-intervention test.

Also, CAPS prescribes that grade 7 learners should be able to use appropriate formulae to calculate the surface area, volume and capacity of cubes and rectangular prisms. In Grade 8 they use appropriate formulae to calculate the surface area, volume and capacity of cubes, rectangular prisms and triangular prisms. In Grade 9, learners use appropriate formulae and conversions between SI units to solve problems and calculate the surface area, volume and capacity of cubes, rectangular prisms, triangular prisms and cylinders (DBE, 2012). Therefore, learners were expected to answer Questions 3 and 4 that dealt with the pre-intervention test. The details of the test are contained in appendices.

3.6.3 Statistical analysis

Statistical data analysis refers to the use of statistical tools to summarise, organise and interpret data (Keller 2015). In this study, mean and standard deviation were used to analyse the differences between pre-, during- and post-intervention test scores. Mean is a measure of the average score found by dividing the sum of total scores by the number of observations. The following formula was used (Keller 2015):

$$Mean\ (\bar{x}) = \left(\sum_{i=1}^{N} x_i\right) \div N$$

Where:

N = Sample size;

Xi = Test score for learner i

Also the standard deviation defined by Blumberg, Cooper & Schindler (2016) as a measure of dispersion that measures the extent to which observations differ from the mean score. The formula used in this study is as follows:

$$Standard\ deviation\ (\sigma) = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

Where:

N = Sample size;

x_i = Test score for learner i

$\bar{x}$= Mean

Both mean and standard deviations were calculated for each activity, namely, the pre-intervention exercise and post-intervention exercises. Results were then compared to assess learners' challenges and progress in learning 3D shapes. The results were presented in tables and diagrams, as appropriate.

One-way analysis of variance (ANOVA) was also carried out to analyse the differences between in learners' mean scores between schools to determine if there were any statistically significant differences that would make pooling of learners' scores meaningless. In addition, a one-way ANOVA repeated measure was conducted to look at the differences in van Hieles' level of geometric thinking among learners in the pre-intervention exercise and post-intervention exercises. An ANOVA test is used to determine if there are any statistically significant differences between the means of two or more groups. The groups can be independent or be the same group measured at two occasions (Keller 2015). In this study, the mean scores per school in the pre-intervention exercise and post-intervention exercise were repeated measures for the same group. On the other hand, testing for the differences between mean scores in the pre-intervention exercise and post-intervention exercises for each of the two schools involved a test of differences between two independent samples.

3.6.4 Content analysis

According to Leedy and Ormrod (2015), content analysis refers to a detailed and systematic examination of the contents of a particular body of material for the purpose of identifying patterns, themes, or biases within that material. Content analysis was used in this study to analyse learners' scripts for both pre- and post-intervention exercises. That was done to account for the nature of errors and learning difficulties of learners from the tests.

3.7 Validity and Reliability

This section explains the manner in which the research attained its validity and reliability of the data collection instruments as well as the analysis of the research findings. Since this study adopted a mixed method research approach, aspects of validity and reliability were relevant for the quantitative part of the study. Reliability, transferability, dependability and conformability were relevant for the qualitative part of the study. These aspects of research quality are discussed in the following sub-sections.

3.7.1 Validity

Research validity refers to “the extent to which a concept is accurately measured in a quantitative study” (Heale & Twycross 2015:66). There are three forms of validity, namely, content validity, construct validity and criterion validity (Heale & Twycross, 2015). According to Taherdoost (2017), content reliability refers to the extent to which a research instrument accurately measures all aspects of a construct. In the context of this study, content validity refers to the extent to which the activities developed accurately measure all aspects of learners’ mastery of each of the 5 levels of van Hieles’ theory in order to understand the challenges that learners encounter at each of these levels. To ensure the validity of test scores, activities were drawn directly from approved Grade 9 learning material. In addition, the tests and intervention programmes were conducted as part of learners’ day-to-day learning programmes.

Another aspect of validity relevant to this study was construct validity. Construct validity measures the degree to which a research instrument or tool measures the intended construct (Chandrupatla 2016). It is the extent to which an instrument measures a characteristic that cannot be directly observed but is assumed to exist based on patterns in people’s behaviour (such a characteristic is a construct). Motivation, creativity, racial prejudice, happiness – all of these is constructs, in that none of them can be directly observed and measured. When researchers ask questions, present tasks, or observe behaviours as a way of assessing an underlying construct, they should obtain some kind of evidence that their approach does, in fact, measure the construct in question.

In this study, leaners' challenges in learning 3D shapes represent the construct that needed to be measured. Again, the use of activities drawn from approved material meant that all the aspects of 3D shapes relevant for grade learners were incorporated into the exercises.

Lastly, criterion validity refers to "the extent to which a research instrument is related to other instruments that measure the same variables" (Heale & Twycross 2015:66). The activities in this study were compared to those in earlier studies such as Fuys, Geddes, & Tischler (1988), Elkins et al. (2020) and van Hiele (1984). This ensured that activities were similar to those used in previous related studies so as to make results of this study comparable to those other studies.

3.7.2 Reliability

Heale and Twycross (2015) posit that reliability relates to the consistency of a measure. The aspect of reliability relevant to this study was homogeneity. According to Hair et al. (2016), homogeneity refers to the extent to which all the items on a scale measure one construct - 3D shapes in this context. The suitability of activities on 3D shapes was assessed against learning outcomes in the grade 9 curriculum.

3.7.3 Credibility

Given (2012) defines credibility as being certain that the findings are true. In this study, credibility was ensured by purposely selecting teachers who were perceived to have the best possible knowledge on teaching geometry to Grade 9 learners. The choice of those teachers was based on their experience on Grade 9 Mathematics teaching, among other factors.

3.7.4 Transferability

Transferability can be defined as the extent to which research findings can be applied to other contexts similar to the context in which a study was contacted

(Sinkovics, Penz & Ghauri, 2018). In the context of this study, transferability refers to the extent to which the interview protocol would yield results applicable to other similar studies. Given (2012) states that to ensure that findings are transferable to different settings the data must provide rich and detailed explanations of the data collected so that other researchers can transfer it to other contexts. In this study, most of the responses from interviewees were reported as quotations to prevent distortions of meaning. This was followed by the analysis of the meanings of such quotations.

3.7.5 Data dependability

According to Kothari (2015), for data to be dependable it must show consistency and replicability if similar conditions were provided. However, this can prove to be difficult since the study being detailed involves the teachers' perspective which is subject to change over time. However, areas of potential similarities and differences in the participants' responses were identified and reported to aid future researchers who may wish to base their studies on the current study.

3.7.6 Conformability

Conformability relates to the extent to which the respondents shape the findings of a study and not the researcher bias or interests (Cooper & Schindler 2011). This implies that the results should not be about what the researcher thinks about the research phenomenon but those of the participants. The use of open-ended questions was intended to achieve conformability since interviewees were given ample leeway of expressing their opinions based on experience on what they felt were the challenges that grade 9 learners faced in learning 3D shapes.

3.8 Research Ethics

This section discusses the ethical considerations relevant for this study. The research instruments used in this study raised ethical questions. The adequacy of the ethical safeguards discussed here were reviewed and approved by the UNISA

College of Education (CEDU) ethics committee together with the relevant school at which this study was carried.

3.8.1 Obtaining informed consent

According to Kothari (2015), where research involves human participants, the researcher needs to obtain consent in order to collect data. In this research study, ethics compliance forms and the approval letter were used to obtain a consent declaration from the research participants before commencing the data gathering activity. In the case of Grade 9 learners, consent was obtained from the school management, on behalf of learners. Because the study involved minors, the learners' parents were also sent a consent letter so that they consented to their children taking part in the study. For educators, they were also supposed to sign a consent form and adhere to any professional ethics which govern the teaching profession.

3.8.2 Ensuring confidentiality and anonymity

Another relevant ethical issue relating to this study was participants' confidentiality and anonymity. Confidentiality refers to a condition in which the researcher knows the identity of a research subject, but takes steps to protect that identity from being discovered by others (Blumberg, Cooper, & Schindler, 2016). On the other hand, anonymity means that there is no way for anyone to personally identify participants in the study (Cooper & Schindler, 2011). According to Hair et al. (2016), confidentiality and anonymity can be achieved through not sharing raw data with third parties and ensuring that no personal identification data is collected and stored together with research data. In this study, no personal data was collected about participants. Interviewees are referred to as Teacher 1, Teacher 2 and so on instead of using their real identities. In addition, learners were instructed not to write their names or any identification information on the tasks they completed. The data collected was kept securely at the researcher's residence and will be kept there for a period of 5 years after the submission of this study, in compliance with UNISA guidelines.

3.8.3 Ensuring there is no harm to participants

According to Kothari (2015), research participants should always be protected from harm. In this study, participants were not subjected to any physical or mental discomfort. Interviews were conducted via Skype so as to adhere to the social distancing provisions currently enforced by the government.

3.8.4 Voluntary participation and right to withdraw

Even if participants give their consent to take part in a research, they still have the right to do so voluntarily. According to Kothari (2015), participants reserve their right to voluntarily withdraw from the study. In this study, the researcher explained to participants when they were invited to participate in the study that they were doing so voluntarily and they could withdraw at any time if they wish to. This applied to both teachers and learners.

3.8.5 Dealing with research bias

In this study, the researcher administered the tests and interviewed the learners informally during intervention. In addition, the researcher interviewed educators. All these presented opportunities for researcher's biases which would influence results. To deal with research biases several methods were used. For example, the researcher coded learners' scripts and removed any names on such scripts to ensure that they were marked anonymously. In addition, the use of multiple sources of data (learners and educators) provided an internal triangulation which reduced researcher's biases. My supervisor also reviewed my work to provide an independent assurance of the quality of the research outcomes.

3.9 Conclusion

The chapter presented the manner in which the research process gathered its primary information from the selected schools in the study. The chapter has discussed the rationale and nature of a mixed methods research design owing to its leverage on using inductive reasoning in explaining the phenomenon. Again, for grade 9 learners' 3D shapes learning challenges the use of an inductive collection of large data amounts, from the interviews and assessment outcomes was projected to bring the opportunity to expose issues that may not have been anticipated from the research issue. The data collection instruments have been explicated inclusive of the demographic information criteria, interviews and interview questions and assessments that were done on each of the judgementally selected participants to the study. Data analysis procedures and process, and aspects of ethical considerations, as well as the credibility, trustworthiness and reliability considerations of the findings, have also been discussed. The next chapter illuminates on the results of the research findings and their analysis.

CHAPTER 4 Results, Analysis and Interpretation

4.1 Introduction

This chapter presents and discusses the results of the study. The discussion is organised around the themes that were found after the data reduction process through coding, categorising and then thematic identification. The first section discusses the characteristics of the sample of both teachers and learners. That is followed by a discussion of results relating to each research objective before rounding up the chapter in the conclusion. It is important at this stage to remind the reader that this study aimed to explore learning difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in Geometry. The research objectives were to:

- identify learning difficulties experienced by the Grade 9 learners in learning 3D shapes
- determine learning processes that enhance the learners' cognitive understanding of 3D shapes in the Grade 9 mathematics
- suggest how the learning difficulties in Grade 9 3D shapes can be minimised.

The data presented for analysis was collected using three sets of instruments administered at two schools as discussed in Chapter 3, namely the interview guide and pre-and post-intervention tasks. The pre-intervention activity was administered to diagnose the learning difficulties experienced by Grade 9 learners in learning 3D shapes. Thereafter, Grade 9 teachers were interviewed to elicit their views on the learning difficulties experienced by grade 9 learners in learning 3D shapes. In addition, the interviews asked Grade 9 teachers about the learning processes that enhance the learners' cognitive understanding of 3D shapes in grade 9 mathematics. Analysis of interview data helped to devise intervention strategies after which the post-intervention activity was administered to assess the effectiveness of the intervention strategies implemented. The details of the data collection process are discussed next.

4.6 Results of the empirical investigation

This section discusses the details of both the interviews and the tests conducted. However, before that, the characteristics of the sample selected are discussed next.

4.7 Sample

In this section, the researcher presents a description of the sampled learners and educators in both schools

4.7.1 Learners

Having discussed how the data was collected and how the interventions were conducted, this section presents the description of the characteristics of the sample that eventually formed part of the study. The sample from School 1 consisted of 12 boys and 18 girls while the School 2 subjects consisted of 9 boys and 21 girls. Overall, the sample from the two schools was made up of 21 boys and 39 girls in total when both schools are combined. That means there was 35% boys and 65% of girls in the combined sample. According to Mpumalanga's DBE (2020), there are 960 learners in grade 9 in the Lydenburg Circuit, of which 67% are girls. Therefore, the sample of 35% boys and 65% of girls was representative of the gender demographics of the population. However, as already noted, the final sample was lower than the initially planned 60 learners per school. The following table summarises the final sample.

Table 2.6: Sample analysis

School	Gender	Sample	Percentage of total
School 1	Boys	12	20%
	Girls	18	30%
School 1	Boys	9	15%
	Girls	21	35%
Total		**60**	**100%**

4.7.2 Educators

In terms of teachers, the sample was relatively small compared to the Grade 9 Mathematics teachers in Lydenburg Circuit and consisted of two teachers from School 1 and three teachers from School 2. There were about 13 Grade 9 Mathematics teachers in the circuit, meaning that a total of 38% was selected (five teachers in total). However, the teachers selected were experienced, with a mean age and tenure of 30.50 years (SD = 5.44 years) and 5.50 years (SD = 3.03 years), respectively. All the teachers were purposely selected so that only those teachers with at least a degree in education were selected. The teachers could, therefore, be trusted to provide valid insights into the challenges facing Grade 9 learners in learning 3D shapes. The next section presents the results of the pre-intervention test, interviews and the post-intervention test.

4.8 Results from the pre-intervention test

In the following section, the researcher presents a discussion on the rationale for the pre-intervention test, namely, the test of homogeneity of schools, learning difficulties that emerged from the pre-intervention test, and learning difficulties of learners from School 1 and School 2.

4.8.1 Test of homogeneity of schools

Before the analysis of results from each of the two activities was done, the schools were tested for homogeneity, using Levene's Test for Equality of Variances and t-test for Equality of Means. That was done to determine whether the scores from the schools could be pooled together and analysed as one sample. Table 4.4 shows the results of the test.

Table 2.7: Test of homogeneity of schools' pre-intervention

	Levene's Test for Equality of Variances			t-test for Equality of Means			
	Equal variances …	F	Sig.	t	df	Sig.	Mean Difference
Pre-intervention test	… assumed	1.47	.23	2.19	58	.033	1.83
	…not assumed			2.19	54.03	.03	1.83

The Levene's Test for Equality of Variances was calculated to assess if the variance of scores within each school was equal or not (homogeneity within the schools). The above results indicate that the variances within each of the two schools were not significantly different because the p-value (p = 0.23) is greater than 0.05. Equality of variances could therefore be assumed.

The next step was to calculate the t-test for Equality of Means. That aimed to assess whether learners' mean scores in the pre-intervention test from School 1 were significantly equal to those of learners from School 2. Results of the test above shows that there were statistically significant differences between the mean scores of learners from the two schools (t (58) = 2.19, p = 0.03). With a p-value of less than 0.05, the results show that there were statistically significant differences between the mean scores from the two schools.

The final step on the test of homogeneity was to assess the practical significance of the differences observed in the t-test analysis. According to Keller (2015), it is possible for differences to be statistically significant but have little or no significance in a practical sense. The test of practical significance of differences was done using Cohen's d value. As discussed in Chapter 3, differences are assumed to be practically significant if the d value is greater than 0.20. The d value for this study was 0.57 which was way above the 0.20 cut-off criteria. It can be concluded that the differences in the mean scores in the pre-intervention test between learners in each of the two schools were statistically and practically significant. Therefore, the analysis of the results of the pre-intervention test was done on a school-by-school basis.

4.8.2 3D learning difficulties emerging from pre-intervention test

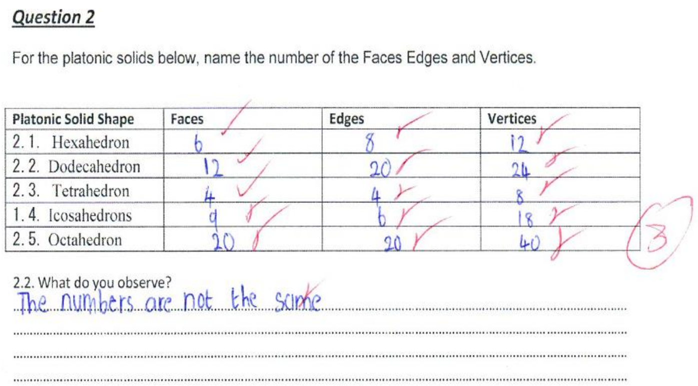
The learning difficulties that emerged from the pre-intervention test were analysed qualitatively. The first step of the analysis was to code the data into three broad areas as shown in Table 4.3. Those broad categories were chosen based on Webber and Roberts (2017) who noted that learning difficulties could be seen as relating to the teacher, the learning environment and the learner.

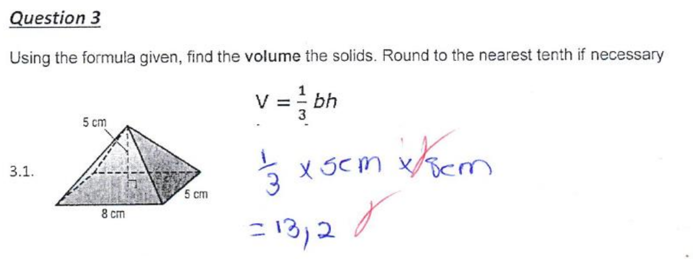
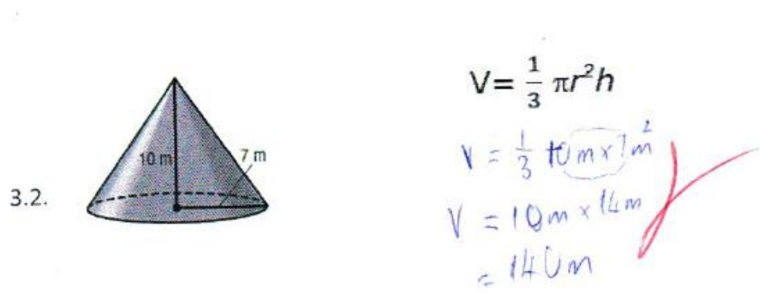
Table 2.8: Data coding: initial categorisation

Excerpt from the initial categorisation			
Category	Learning difficulty relating to…	Description	Code
1	… the teacher	This category consisted of 3D learning difficulties that directly result from the action/inaction by the teacher	TTR
2	… the learning environment	This category consisted of 3D learning difficulties that were caused by the learning environment	TLT
3	… to learners	This category consisted of 3D learning difficulties that resulted from the learner	TLR

Once broad categories were decided upon, subcategories were identified as they emerged from the data. Table 4.6 shows an excerpt from the data categorisation and coding process.

Table 2.9: Data coding: subsequent coding

<table>
<tr><td colspan="5">Excerpt from subsequent coding</td></tr>
<tr><th>Code</th><th>Challenges</th><th>Description</th><th>Sample Learner’s Work</th><th>Learner code</th></tr>
<tr><td>TTR-INS</td><td>Unclear instructions in the test</td><td>Learners struggled with activities where instructions were not clear.</td><td>Question 2.2 is vague and the learner’s response, despite being different from what the teacher was looking for, was factually correct.

<u>Question 2</u>
For the platonic solids below, name the number of the Faces Edges and Vertices.
<table>
<tr><th>Platonic Solid Shape</th><th>Faces</th><th>Edges</th><th>Vertices</th></tr>
<tr><td>2. 1. Hexahedron</td><td>6</td><td>8</td><td>12</td></tr>
<tr><td>2. 2. Dodecahedron</td><td>12</td><td>20</td><td>24</td></tr>
<tr><td>2. 3. Tetrahedron</td><td>4</td><td>4</td><td>8</td></tr>
<tr><td>1. 4. Icosahedrons</td><td>9</td><td>6</td><td>18</td></tr>
<tr><td>2. 5. Octahedron</td><td>20</td><td>20</td><td>40</td></tr>
</table>
3
2.2. What do you observe?
The numbers are not the same</td><td>KG27</td></tr>
</table>

TLR-ART	3D learning difficulties due to arithmetic challenges	Poor arithmetic mastery means that learners could not get the final answer correct even if they got some of the parts of the questions correctly.	Using the leaner's numbers, the answer is 13.33 and not 13.2 	KG28
TLR - ALG	3D learning difficulties due to Algebraic challenges	The challenge here arose from learners' failure to master the algebraic parts of 3D.	$7m^2$ instead of $(7m)^2$ 	KG41

Note: The impact of the learner's environment was not evident because it was difficult to assess from the learners' work. In the end, the following sub-themes emerged from analysing learners' responses to pre-intervention.

4.8.2 Learning difficulties from School 1

In the next section, learning difficulties identified in learners' responses from school 1 are presented.

4.8.2.1 Instructions and question clarity

One of the challenges that learners faced in 3D shapes related to instructions and the clarity (or lack thereof) of questions. That was evident in Questions 1 and Question 2. In question one, the question was given as follows:

Space and Shape: 3D

Total Points

Question 1(a)

Name the platonic solids by choosing from the given below

Tetrahedron, Hexahedron (cube), Octahedron, Dodecahedron and Icosahedrons

Figure 2.11: Question 1(a) on Space and 3D Shapes

While the instruction here was very clear, it was the options given that were not clear to learners. Words such as tetrahedron and hexahedron are rare particularly at the Grade 9 level. As a result, learners struggled to identify the shapes. Learners' struggles were evident from the fact that while only 10% of learners from School 1 were able to identify the dodecahedron (a 3D shape with 12 faces, 20 corners and 30 edges), over 80% of learners could identify the Hexahedron or a cube. The dodecahedron is shown below:

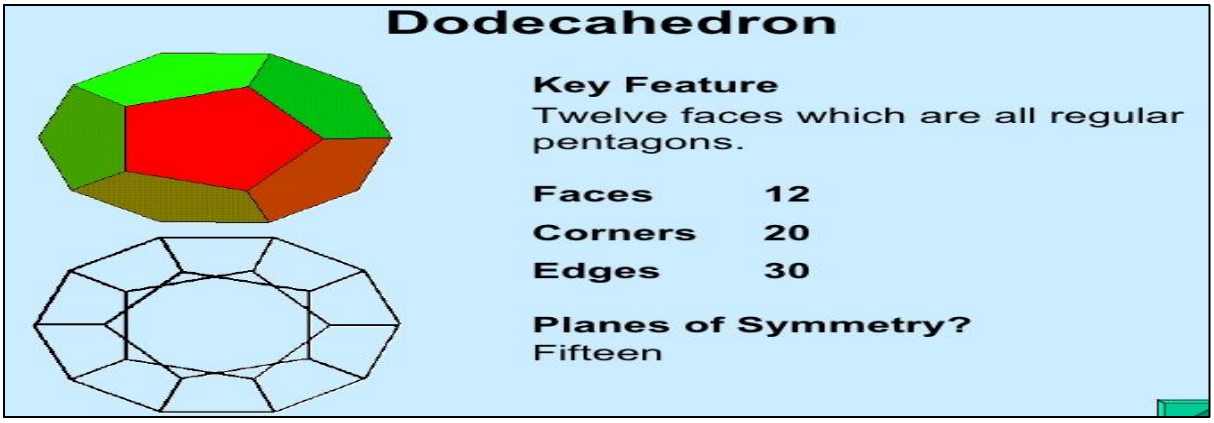


Figure 2.12: Sample platonic shape: Decahedron

(Source: Fairuz 2011)

During the intervention, the researcher asked learners from School 1 the question, "*Why was it difficult to identify the dodecahedron and easier to identify the hexahedron?"* Responses from the learners can be summarised in LKG4's response when he said,

LKG4: '*I could not recall a dodecahedron from my earlier lessons, but for a hexahedron, the inclusion of the word 'cube' gave me a clue and that made it easier to identify the hexahedron.*'

Almost all the learners indicated that though they could not recall, from their earlier lessons, what a hexahedron was, the inclusion of the word 'cube' gave them a clue and that made it easier to identify the hexahedron. When the researcher asked a follow-up question of the shape learners started to name during the test, again the majority indicated that they had started by naming the hexahedron because it was easier for them. These findings are important since they indicate that, where possible, educators should use terms that are familiar to learners when referring to 3D shapes.

Another instructional challenge that learners faced related to the level of instructional clarity. That was evident in Question 2.2. Question 2 is reproduced in Table 4.7:

Table 2.10: Questions 1 and 2

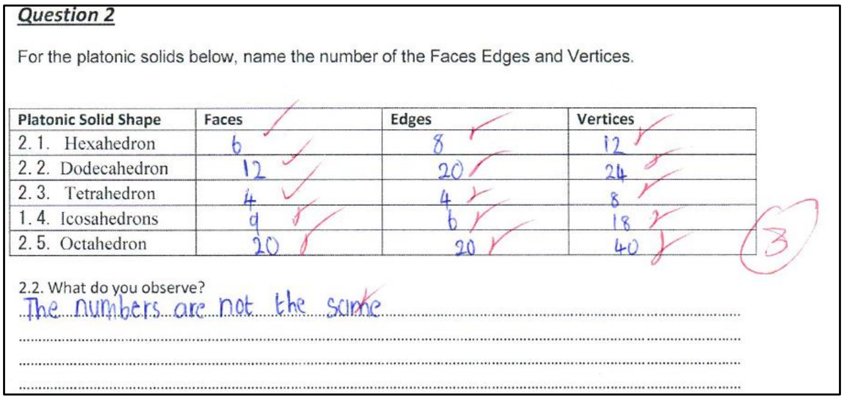
For the platonic solids below, identify the number of the Faces, Edges and Vertices

Platonic Solid Name	**Vertices**	**Edges**	**Faces**
Tetrahedron			
cube			
Octahedron			
Dodecahedron			
Icosahedrons			

2.2. What do you observe?

…………………………………………………………………………………………………

Question 2.2 simply asks, "What do you observe?" However, that was vague and the researcher could not anticipate some of the responses that learners gave. The response that the researcher wanted was that the tetrahedron has the same number of faces and vertices, the tetrahedron has the same number of faces as the vertices of the hexahedron; and that the tetrahedron and the hexahedron have an equal number of edges. Learners were also supposed to observe that the dodecahedron has the same number of faces and vertices as the icosahedron; and that the dodecahedron and the icosahedron have an equal number of edges. However, there were several responses that learners gave, which were factually correct but were not addressing what the researcher was looking for. A sample of these responses was as follows:


Question 2

For the platonic solids below, name the number of the Faces Edges and Vertices.

Platonic Solid Shape	Faces	Edges	Vertices
2. 1. Hexahedron	6	8	12
2. 2. Dodecahedron	12	20	24
2. 3. Tetrahedron	4	4	8
1. 4. Icosahedrons	9	6	18
2. 5. Octahedron	20	20	40

3

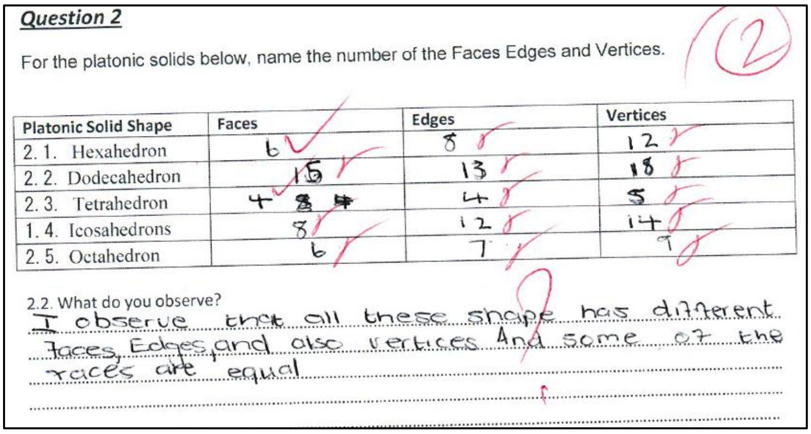
2.2. What do you observe?

The numbers are not the same

Extract 1: Learner KG27's response

The learner's response here was factually correct, the numbers (the student's answers) are not the same and that is a fact coming from the learner's observation. This presented a dilemma to the educator on whether to credit the learner or not. This can be clear from Learner KG19's response

Learner KG19


Question 2

For the platonic solids below, name the number of the Faces Edges and Vertices.

2

Platonic Solid Shape	Faces	Edges	Vertices
2. 1. Hexahedron	6	8	12
2. 2. Dodecahedron	15	13	18
2. 3. Tetrahedron	4 ~~8~~ ~~4~~	4	5
1. 4. Icosahedrons	8	12	14
2. 5. Octahedron	6	7	9

2.2. What do you observe?

I observe that all these shape has different faces, Edges and also vertices And some of the faces are equal

Extract 2: Learner KG19's response

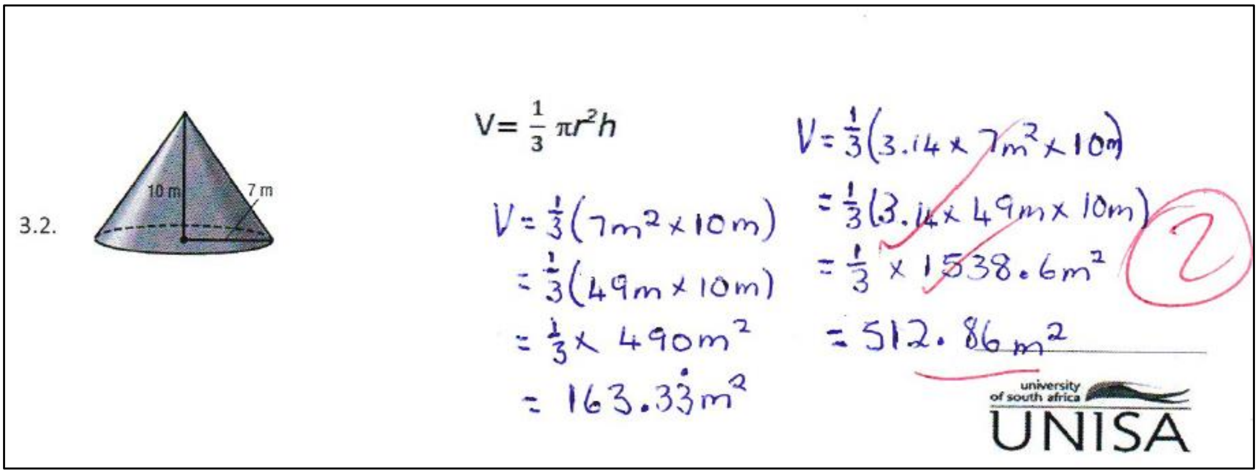
Again, the learner's response for question 2.2 above was factually correct as that referred to the learner's answer. However, that did not address what the researcher wanted, as explained earlier. The majority of learners' responses were similar to the two sampled above with some minor variations. What those responses show is that instructions need to be as clear as possible if teachers want to reduce 3D learning difficulties. The findings here underlie the importance of clarity of instructions in learners' understanding of 3D shapes and support the findings by Nugroho & Wulandari (2017) who explained that achieving a higher level of thought and mastery of 3D shapes, instructions need to be designed carefully and clearly.

4.8.2.2 Structuring of follow-up questions

Learners from School 1 also struggled with follow-up questions. As already indicated above, Question 1 was difficult for the majority of the learners due to the use of words that learners do not generally find in their everyday lives. Because Question 2 was based on Question 1, Question 2 was equally challenging to learners. The implication was that learners were penalised twice for the same concepts. Firstly, learners were penalised for failing to identify the shapes. Secondly, they were also penalised for failing to give the properties of the shapes. Therefore, learners who failed Question 1 generally lost points again in Question 2. On hindsight, the researcher learned that follow-up questions should be asked in such a way that they allow learners to gain credit even where they might have failed the previous question upon which the follow-up question is based.

4.8.2.3 Arithmetic challenges

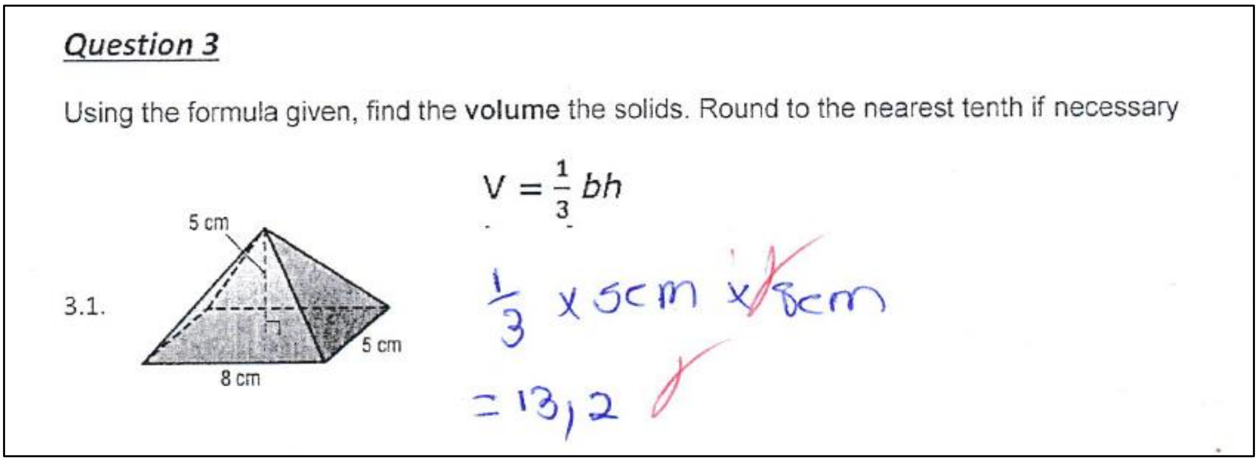
Some learners also failed questions on 3D shapes because they could not perform basic arithmetic. The first arithmetic error made related to rounding off. For example, the following was a typical error that the majority of the learners made.



Extract 3: Learner KG01's response

While the learner's solution was correct (512.8666667), the learner was instructed to round off the answer to two decimal places. As can be seen from the snapshot above, the learner's solution was 512.86 instead of 512.87. Even though the error related to general arithmetic, it ended up costing the learner some points in a 3D shapes exercise.

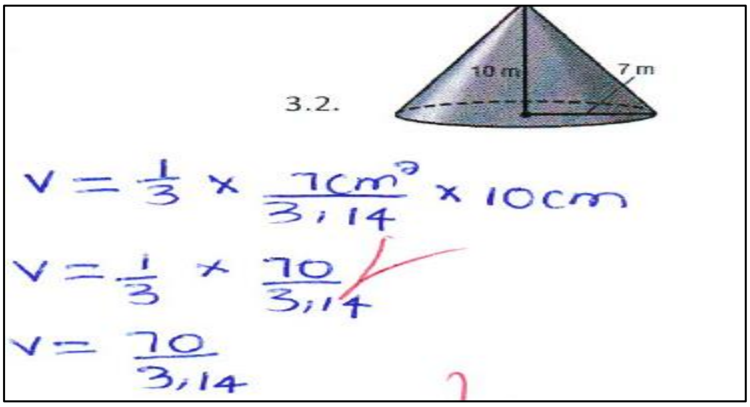
Another example of where learners lost points due to poor foundation in basic arithmetic was in Question 3.1. The question is reproduced below:



Extract 4: Learner KG15's response

Though the learner's solution was incorrect since the learner did not calculate the base of the solid first (b), the final answer shows arithmetic deficiencies on the part the learner above. The solution was supposed to be 13.3. The 13.2 above shows that the learner failed to perform basic operations - division to be specific. When the researcher asked the learner and 15 other learners who had made similar mistakes, they indicated that they had carried out the required operations manually, since they had no calculators because their parents could not afford one. The school had no additional calculators for such learners. This concurs with Adolphus (2011) who asserts that challenges that arise from lack of adequate resources are part of inadequacies of learning support offered to learners.

Arithmetic deficiencies were also evident from learners' failure to perform basic calculations involving exponential in Question 3.2, 3.3 and 3.5. A sample of learners' answers from each of the four questions is reproduced below. Those were typical solutions for learners who made similar mistakes.

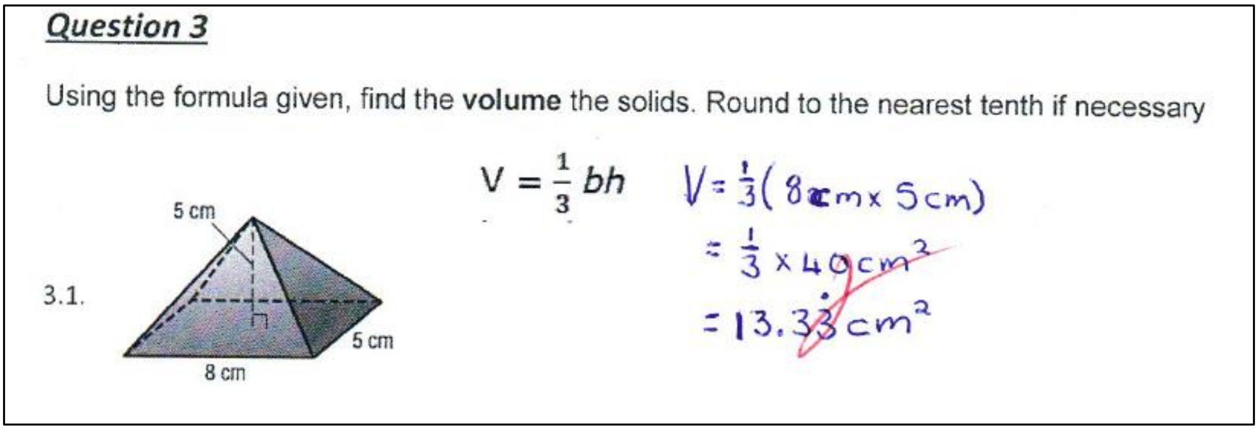


Extract 5: Learner KG10's response

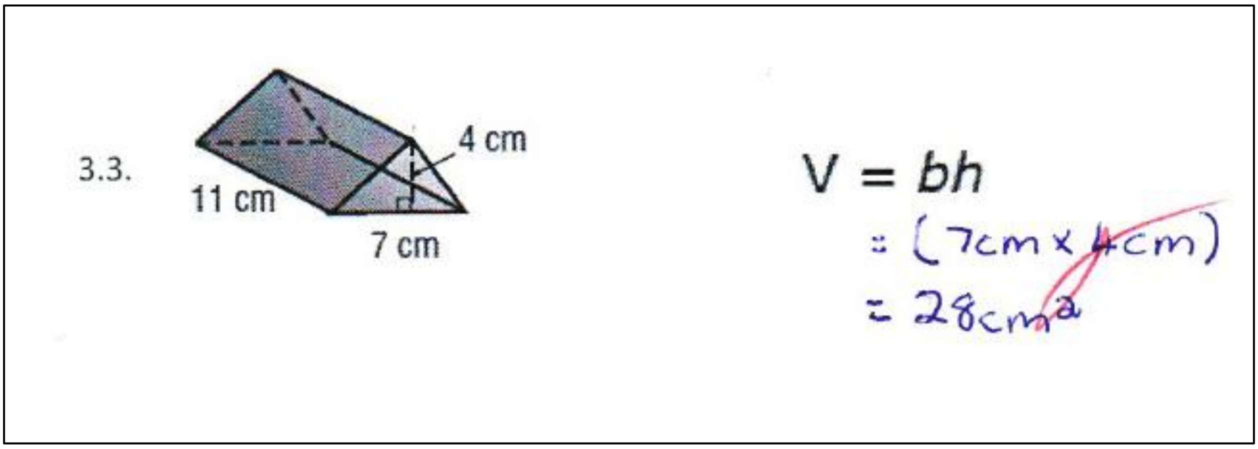
In the sample solution above, the learner should have written $(7\text{cm})^2$ instead of 7cm^2. As a result, the learner failed the question and ended up multiplying 7cm^2 by 10 cm.

4.8.2.4 Leaners' confusion over formula

A further challenge is about learners' use of the formula given on the formulae sheet. That was particularly evident in Questions 3.1 and 3.3. The two questions are reproduced in Figure 8.9 and Figure 4.10.



Extract 6: Learner KG21's response



Extract 7: Learner KG25's response

As can be seen from the two questions, both have the letter 'b' in the formulae given. In either case, the 'b' referred to the base area of the solids. However, only 5 of the learners in School 1 gave correct responses. The two solutions quoted above were typical of most learners' responses. In these responses, learners consistently took 'b' as referring to base length and not base area. When the researcher asked a follow-up question during intervention when revising the pre-intervention test on why learners thought 'b' referred to base length and not base area, most learners, like LKG25 in School 1 indicated that they were surprised their solutions were incorrect. On further enquiry regarding why they were surprised the majority of the learners (17) indicated that they thought the 'b' in the two formulae were similar to the 'b' in the formula of the area of a triangle (½bh). The observations from these mistakes were used as part of the preparations of the post-intervention. The researcher, during the intervention process, stressed the differences between the 'b' in the two formulae in Questions 3.1 and 3.3 and the one in the formula for the area of a triangle. Again, this is in line with the propositions by Sarama et al. (2011) who posits that failure to understand formulae used in 3D shapes may lead to learners failing 3D tasks, especially those in which the formulae will be needed.

4.8.2.5 Failure to reconstruct a shape given dimensions

Learners also found it difficult to work out the solution to 3D questions that had no diagrams drawn. That was seen in questions Question 3.4 and Question 4. Question 3.4 is reproduced below:

Table 2.11: Question 3

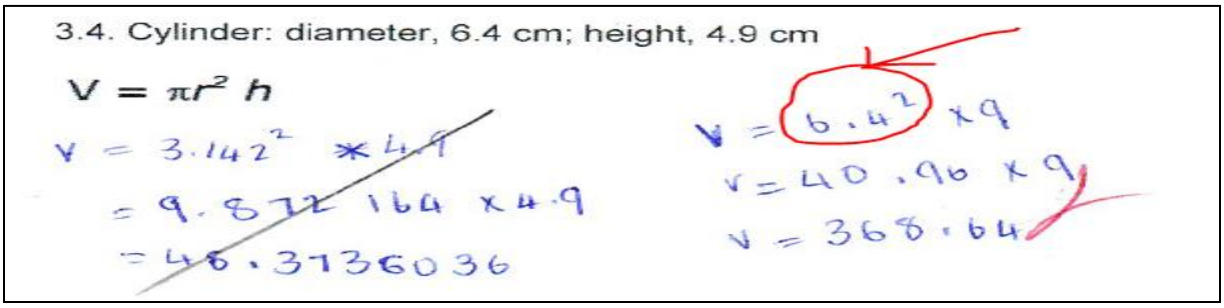
Question 3

Using the formula given, find the **volume** the solids. Round to the nearest tenth if necessary

3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm

$$V = \pi r^2 h$$

The difficulty identified in Question 3.4 was that learners failed to realise that the question information had a diameter of 6.4 cm and as a result, LKG26 for example used that as a radius for r square. The following were typical responses by learners:



3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm

$V = \pi r^2 h$

$\cancel{V = 3.142^2 * 4.9}$

$\cancel{= 9.872164 \times 4.9}$

$\cancel{= 48.3736036}$

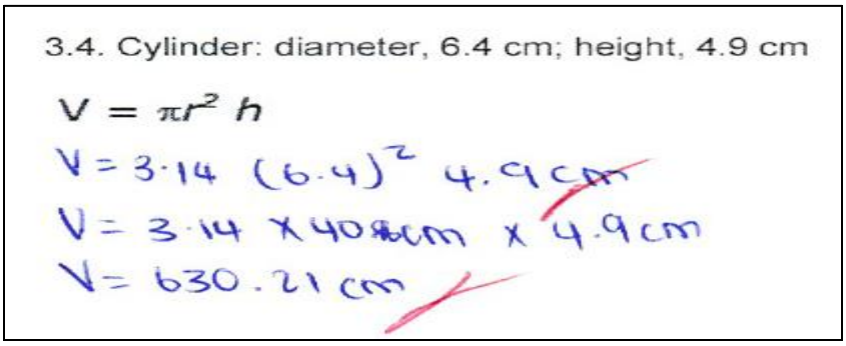
$V = (6.4^2) \times 9$

$V = 40.96 \times 9$

$V = 368.64$

Extract 8: Learner KG26's response

Furthermore, this learner was even careless, instead of recording h as 4,9, she/he wrote 9 and in the response displayed in figure 4.12 it can be noticed that the value of pi was left out. During interviews, this learner had tried to put *pi* but realised that it was not the one that was squared.



3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm

$V = \pi r^2 h$

$V = 3.14\ (6.4)^2\ 4.9cm$

$V = 3.14 \times 40$[illegible]$cm \times 4.9cm$

$V = 630.21\ cm$

Extract 9: Learner KG18's response

In both, the scripts above, LKG18, just like LKG26 used 6.4 cm instead of 3.2 cm. The researcher asked why learners in School 1 had used the diameter as a radius, during the intervention process, when revising the pre-intervention test, one of the learners (KG28) asked for the differences between a radius and a diameter. The researcher explained the difference and demonstrated using the following diagram during the intervention sessions.

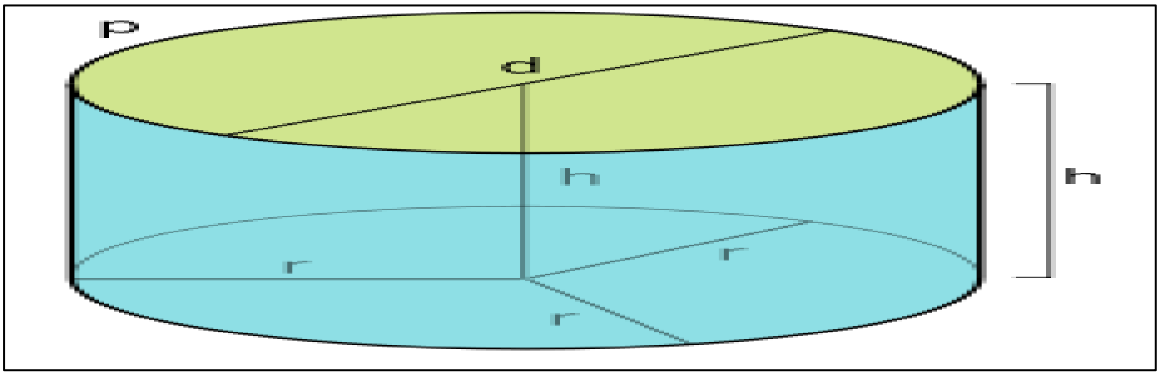
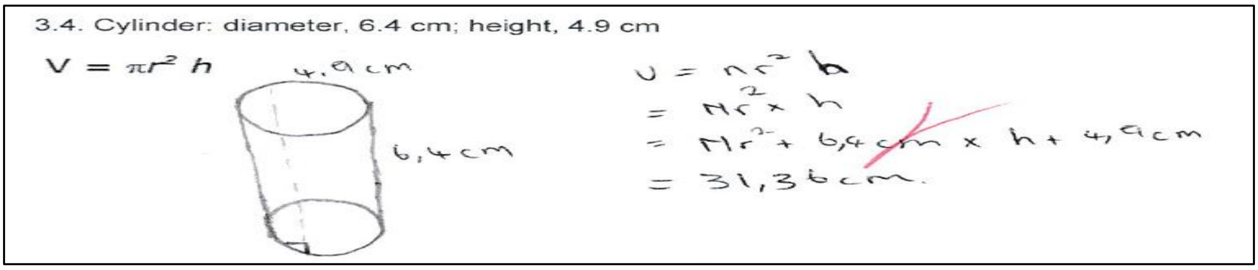


Figure 2.13: The cylinder

In addition, the relationship between the diameter and the radius was also explained (r = d/2). In Question 3.4, the radius was supposed to be 3.2 cm (6.4 cm/2).

Learner LKG15 when asked why he/she had used the diameter as a radius suggested that the question should have had the diagram of the cylinder rather than giving numbers only. The learner indicated that it was difficult to understand the shape they were working on when only dimensions were given. Six other learners in School 1 concurred with that learner who indicated that Question 3.2 could be easier if the question contained the diagram of a cylinder. The following solution justifies the learners' concerns:



Extract 10: Learner KG15's response

As can be seen, Learner KG15 was able to draw the cylinder but failed to label the dimensions correctly. Instead of labelling the diameter as 6.4 cm, the learner used the diameter as a height. Secondly, it is not clear from the solution above which part of the diagram is labelled as a diameter or radius. The researcher asked Learner

KG15 to explain the part of the diagram to which the 4.9 cm belonged. The learner pointed to the circumference rather than the height of the diagram.

Reflecting on the learners' suggestions above, it was clear that such learners had not yet passed Level 3 (Informal Deduction Stage) of the van Hieles' Levels of Geometrical Thinking. According to Clements (2014), Level 3 often requires learners to deal with ideas about shapes rather than concrete objects. Therefore, the learners cited above failed the test at that abstraction stage wherein learners are required to reconstruct shapes when they are given dimensions. According to Sarama et al. (2011), failure to reconstruct shapes when given dimensions is one of the key challenges that mathematics learners face. In addition, Leone et al. (2010) explain that learners who lack skills at van Hieles' Informal deduction level usually struggle to reconstruct figures.

The challenge of reconstructing the shape was also evident in Question 4 which is reconstructed below:

Table 2.12: Question 4

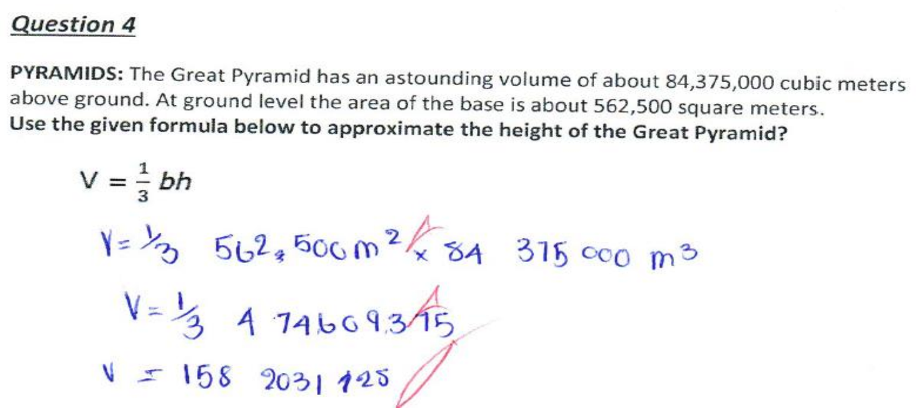
Question 4

PYRAMIDS: The Great Pyramid has an astounding volume of about 84,375,000 cubic meters above ground. At ground level the area of the base is about 562,500 square meters.
Use the given formula below to approximate the height of the Great Pyramid?

$$V = \frac{1}{3} bh$$

In Question 4, learners could not come up with correct responses. During the revision of the pre-intervention test, the researcher asked learners the difficulty they encountered with Question 4. Four learners replied to the question and the following is what they said:

Table 2.13: Leaners' responses to Question 4

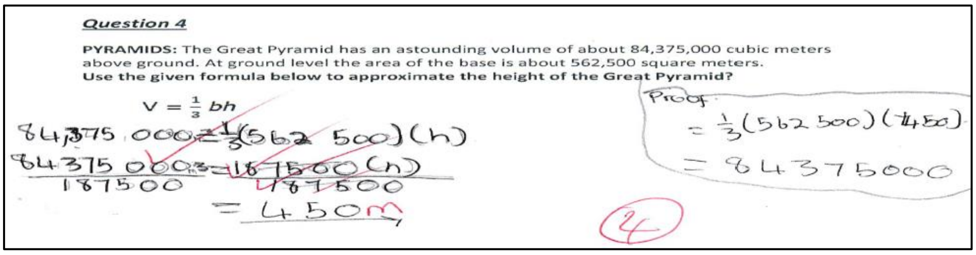
Learner	Response
Learner KG17	*I thought 'b' represented the length of the base. Since that was not given, I could not figure out how to get it. As a result, I left the question blank.* Leaner KG17 did not answer Question Four
Learner KG06	*I did not know how a pyramid looks like. Therefore, all I did was to guess and that is why I simply multiplied the two numbers and multiply the answer by 1/3.* The learner had answered the question as follows:  **Question 4** **PYRAMIDS:** The Great Pyramid has an astounding volume of about 84,375,000 cubic meters above ground. At ground level the area of the base is about 562,500 square meters. **Use the given formula below to approximate the height of the Great Pyramid?** $V = \frac{1}{3}bh$ V= 1/3 562,500 m² × 84 375 000 m³ V = 1/3 4 746 093 75 V = 158 2031 125
Learner KG28	*I could not figure out the shape of a pyramid and did not know how to progress from the given numbers and formula. Again, I misread the question and thought it required volume and not height.* Learner KG28 left the question blank as well.
KG29	*I hate story questions in Mathematics. I did not even bother reading the whole story.* Again, learner KG29 left the question blank.

Difficulties highlighted by Learner KG17 have already been discussed above as involving learners confusing formulae. Again, the difficulty noted by Learner KG22 has been discussed already as involving learners' deficiencies with Level 4 of van Hieles' Theory of Geometrical Thinking. However, learners KG22 and KG28 presented additional learning difficulty, namely the failure to read instructions carefully. Of the 30 learners in School 1, 13 of them had calculations similar to Learner KG06 which used the given values to calculate volume, suggesting the learners failed to read the instructions carefully.

Learner KG29 had a further challenge - not understanding English in general. The researcher conducted a background check with the learner's English teacher who indicated that Learner KG29 struggled with his/her English Language in general. Therefore, the struggles faced by Learner KG29 concur with the views of Nugroho & Wulandari (2017) who noted that language plays an important role in the teaching and learning of new concepts and that if learners struggled with the language used for instruction in a given subject, that leaner may as well be struggling in other subjects. As already indicated in Chapter 2, both McLaughlin (1995) and Kotzé (2007) stressed that the language of instruction is very important for learners of 3D shapes since it determines the extent to which the learner can integrate learning with his/her ethnocultural environments and values.

4.8.2.6 Challenges arising from algebraic deficiencies

Learners also did not respond adequately to questions on 3D shapes due to their poor grasp of simple algebraic operations. That was evident in Question 4 again. Question 4 has already been reproduced above and required learners to calculate the height of the Great Pyramid having given learners the volume and base area of the shape. Since the formula that was given was for the volume of the pyramid, learners needed to make 'h' the subject of the formula before proceeding to calculate the height of the pyramid. Only one learner out of the 30 learners in School 1 managed to get the correct answer and the excerpt of that learner's response was as follows:



Question 4

PYRAMIDS: The Great Pyramid has an astounding volume of about 84,375,000 cubic meters above ground. At ground level the area of the base is about 562,500 square meters.
Use the given formula below to approximate the height of the Great Pyramid?

$V = \frac{1}{3}bh$

$84\,375\,000 = \frac{1}{3}(562\,500)(h)$

$\frac{84\,375\,000}{187\,500} = \frac{187\,500(h)}{187\,500}$

$= 450m$

Proof:

$= \frac{1}{3}(562\,500)(450)$

$= 84375000$

4

Extract 11: Learner KG04's response

Even though Learner KG04 started by making h the subject of the formula, the final solution was correct because the learner was able to perform basic algebra. A further strength in the learner cited above is the knowledge that the learner needed to prove her answer to verify the correctness of his/her calculations.

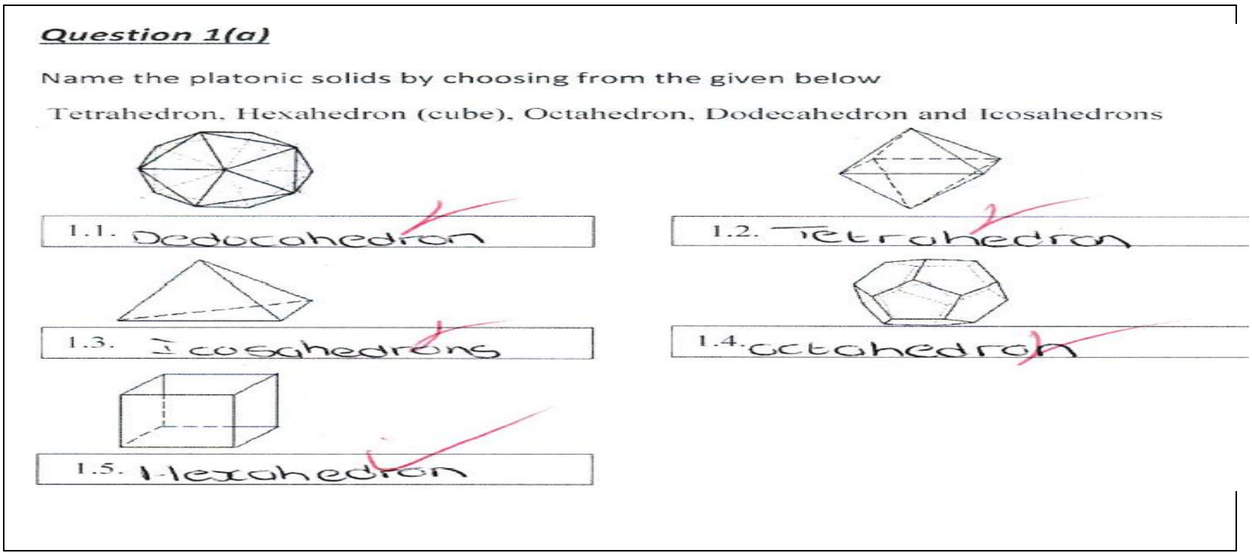
4.8.5 Learning difficulties from School 2

The learning difficulties that were found in School 2 were similar to those observed in School 1. The learning difficulties are discussed below.

4.8.5.1 Failure to understand unfamiliar 3D vocabulary

The problem of unfamiliar terms being the reasons for failing 3D questions identified in School 1 was also found in School 2. That was evident in Question 1(a) as shown below:

Table 2.14: Question 1



<u>**Question 1(a)**</u>

Name the platonic solids by choosing from the given below

Tetrahedron, Hexahedron (cube), Octahedron, Dodecahedron and Icosahedrons

1.1. Dedocahedron

1.2. Tetrahedron

1.3. Icosahedrons

1.4. octahedron

1.5. Hexahedron

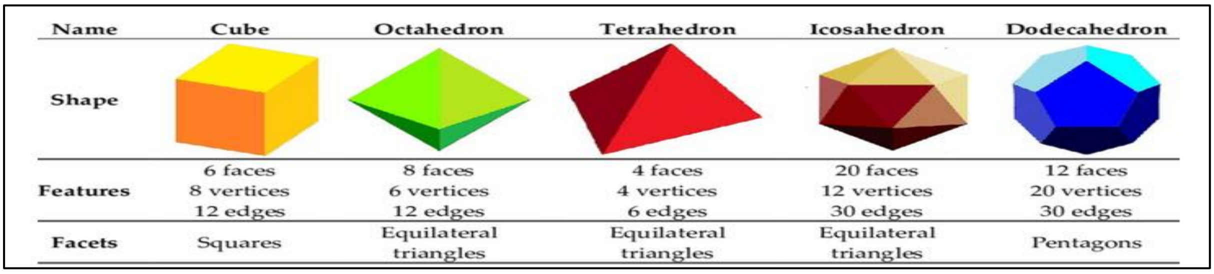
Despite using the word Hexahedron, the name cube, in brackets, made it easier for the learner above to get the Hexahedron correct. Of the 30 learners who wrote the pre-intervention test in School 1, only eight learners got a score of at least three points, which was the passing score in this question. The average score for Question 1 for all 30 learners was 1.5 out of 5 points. The response quoted above represents the typical solutions in Question 1 for learners who failed that question. During the intervention phase, the eight learners who got at least three points were asked how they got such points. Their responses are given below:

Table 2.15: Leaners' interview responses to Question 1

Leaner	Score	Response
Leaner KG33	3	*I remembered the answers from our earlier class. However, the Hexahedron was obvious because of the word cube. We used that word often in other grades.*
Leaner KG37	3	*To be honest, I got the other two shapes (Dodecahedron and Tetrahedron) correct by guesswork. The cube for me was an easier one.*
Leaner KG39	3	*I knew a cube, so the Hexahedron was straightforward for me. We once discussed with the teacher that tri means three so I guessed the Tetrahedron based on that. As for the dodecahedron, that was pure guesswork.*
Leaner KG40	3	*It was pure guesswork since these shapes were difficult for me when we did them in class*
Leaner KG41	3	*I just guessed, sir!*
Leaner KG45	3	*Honestly, I only knew the cube and the rest were unfamiliar to me.*
Leaner KG56	3	*The cube was fine for me, but the rest was guesswork.*

Leaner KG57	3	*I only knew the Hexahedron. I simply guessed the rest*

The common theme in all the answers above is that the Hexahedron was easier for learners due to the inclusion of a more familiar word, cube. Though there are no alternative names for the other platonic solids, educators can make it easier for learners to understand those 3D shapes by relating them to more familiar terms. For example, the revelation by Learner KG39 that she had used her knowledge of the word ‘tri’ to guess correctly. The tetrahedron shape shows other ways of making the shapes’ terminology more accessible to learners. These findings are in line with prior theory, particularly Kotzé (2007) and Nugroho & Wulandari (2017) who explained that language of instruction is usually one of the challenges learners face in schools, especially in cases where instruction is given in an additional language. The following diagram Figure 4.18 was used during the intervention in both School 1 and 2 to link those platonic solids to 2D shapes and that greatly improved learners’ understanding as shall be seen in the analysis of the post-intervention test.



Name	Cube	Octahedron	Tetrahedron	Icosahedron	Dodecahedron
Shape					
Features	6 faces 8 vertices 12 edges	8 faces 6 vertices 12 edges	4 faces 4 vertices 6 edges	20 faces 12 vertices 30 edges	12 faces 20 vertices 30 edges
Facets	Squares	Equilateral triangles	Equilateral triangles	Equilateral triangles	Pentagons

Figure 2.18: Properties of platonic solids

Source: Besson, (2020)

Jones (2000), for example, mentions the linking of learners’ development of spatial awareness and their ability to visualise, to their developing knowledge and understanding of shapes, and the ability to use geometrical properties and theorems. This implies that the use of colour in identifying properties of platonic solids is necessary for the learners’ development of knowledge and understanding of their properties. Learners, for example, after the use of Figure 4.18 to explain the different properties made sounds like, '*Oh Yesss, now I get it'*, '*Mhhh, I can now see the difference!' 'Tjoooo!!!'*

4.8.2.7 Instructional and structure of questions

Analysis of responses for Question 2 revealed three learning difficulties that learners faced. Firstly, because the majority of learners provided inadequate responses, or at least guessed the solutions for Question 1, those same learners were automatically penalised for Question 2 which was based on the learners' solutions in Question 1.

Secondly, Question 2 (b) posed the same challenges it did in School 1, namely that learners ended up giving factually correct solutions which were different from what the researcher wanted. As noted in the previous section, the question was ambiguous. The question simply asked: *What do you observe?* On hindsight, the researcher realised that the question was ambiguous and that was not repeated in the post-intervention test. Typical solutions for learners in School 1 for Question 2 (b) are given below.

Table 2.16: Instructional and structure of questions challenges

<table>
<tr>
<td>Learner KG37</td>
<td>

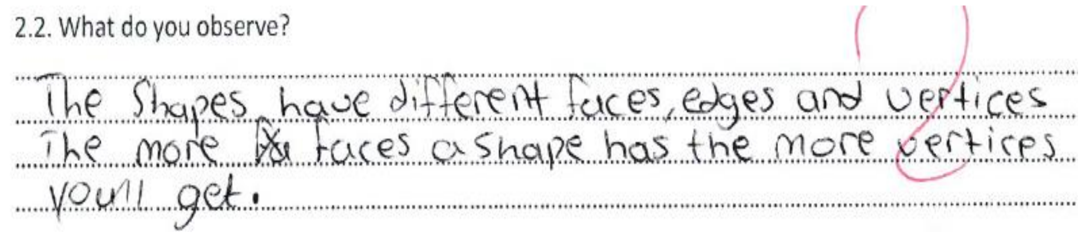
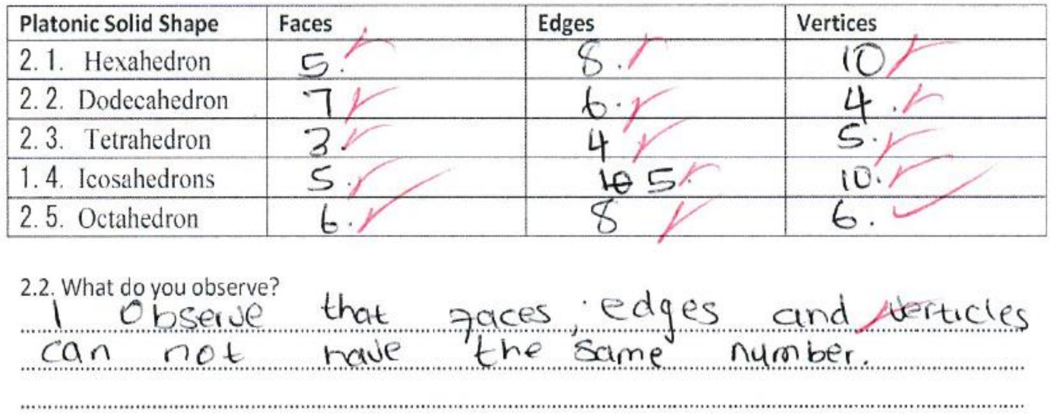
Platonic Solid Shape	Faces	Edges	Vertices
2.1. Hexahedron	6	8	12
2.2. Dodecahedron	~~4~~ 22	10	20
2.3. Tetrahedron	4	5	5
1.4. Icosahedrons	12	14	24
2.5. Octahedron	5	6	12

2.2. What do you observe?

That the vertices are mostly with the highest number than the edges.
</td>
<td rowspan="2">Both Learners KG37 and KG40's responses were factually correct based on the learner's responses to Question 2.1. However, this was not addressing the question that the researcher intended to as noted above.</td>
</tr>
<tr>
<td>Learner KG40</td>
<td>

2.2. What do you observe?

I have observed that all the ~~face~~ shaper are not equal in ~~size~~ facer, edger and vertices they all ~~do~~ dipper to one another.
</td>
</tr>
</table>

<table>
<tr>
<td>Learner KG48</td>
<td>
2.2. What do you observe?
The Shapes have different faces, edges and vertices The more ~~Fa~~ faces a Shape has the more vertices you'll get.</td>
<td>Again, Learner KG48’s observation was factually correct, but still not addressing the question the researcher intended to ask.</td>
</tr>
<tr>
<td>Learner KG56</td>
<td>
<table>
<tr><th>Platonic Solid Shape</th><th>Faces</th><th>Edges</th><th>Vertices</th></tr>
<tr><td>2. 1. Hexahedron</td><td>5.</td><td>8.</td><td>10</td></tr>
<tr><td>2. 2. Dodecahedron</td><td>7</td><td>6.</td><td>4.</td></tr>
<tr><td>2. 3. Tetrahedron</td><td>3.</td><td>4</td><td>5.</td></tr>
<tr><td>1. 4. Icosahedrons</td><td>5.</td><td>~~10~~ 5</td><td>10.</td></tr>
<tr><td>2. 5. Octahedron</td><td>6.</td><td>8</td><td>6.</td></tr>
</table>
2.2. What do you observe?
I Observe that faces, edges and verticles can not have the same number.</td>
<td>Based on the learner’s solutions and mathematical theory, this solution was factually correct, but not addressing the question the researcher intended to ask.</td>
</tr>
</table>

Still, in Question 2, some responses showed that learners had not understood the question requirement well. Some learners in that category went on to give solutions that were unrelated to the question. Unlike learners who gave factually current answers indicated above, learners in the current category gave factually incorrect solutions. The table below shows such learners' responses.

Table 2.17: General 3D shapes deficiencies

Learner KG39	2.2. What do you observe? 3D shapes = how to differenciate between faces, Edges and Verticles and how to calculate them.	The learner is explaining what he/she has learned not what he/she has observed.
Learner KG52	2.2. What do you observe? I observe to make sure that all Faces are equal and Same, Then to Find out that all edges are all same's. And Vertices are not the same with the Faces and Edges	This learner's solution is mixed up and it shows that the learner was not sure of what he/she wanted to write about.

Learner KG54	2.2. What do you observe? The faces, edges and vertices depends on what the prisms are made up of.	There is language confusion here. It is not clear what “they are made of” is referring to. As it is, it might be that the learner is referring to the other properties (faces, edges, vertices etc.) as what makes up the shape. However, one would interpret this as referring to the material that makes up the shapes if they have a physical existence. Either way, the learner’s response is factually incorrect.
Learner KG44	2.2. What do you observe? I observe to count the faces and Edges vertices and how the 3D shapes are drawed to show how IES is	Learners KG44 and KG45 referred to what they had learnt and not observations.

Learner KG45	2.2. What do you observe? We observe the difference between the faces, Edges and Vertices for the piantonic solid shape.	

During the intervention phase, the researcher interviewed the learners who had got the solutions to Question 2.2 incorrectly and asked: Why did you give the solutions you gave for this question? Responses to that question are presented in the following table.

Table 2.18: Learners' informal interview responses

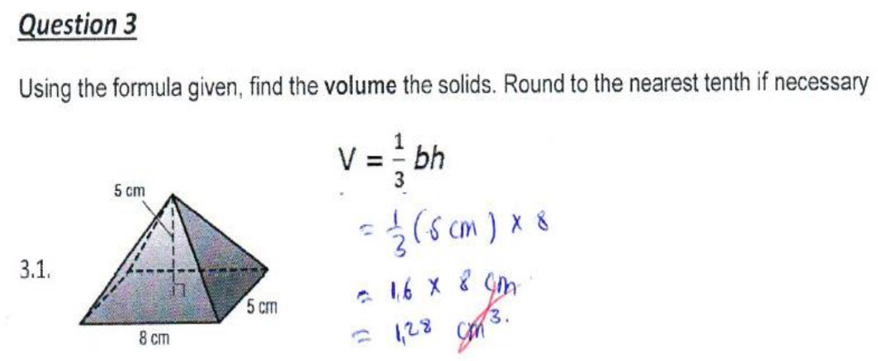
Learner KG39	*I don't think I understood what you wanted sir, I thought what we had learned were our observations.*
Learner KG52	*Honestly, I did not expect a question that required stories in Mathematics. English is difficult sir!* (Whole class laughs)
Learner KG54	*I wasn't sure, your question was not clear and maybe I misunderstood what you meant by "observe."*
Learner KG44	*I thought you wanted us to write about what we would have learnt, sir.*
Learner KG45	*I thought that was the correct answer based on your question. Please explain what was required.*

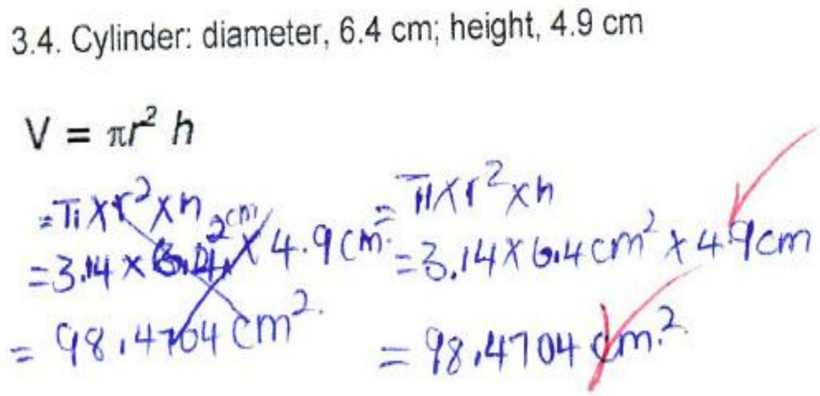
One theme that emerged from the responses above, namely, that learners struggled to understand the language used in the question, English. The language of instruction was identified by Nugroho and Wulandari (2017) as one of the reasons learners find geometry challenging. Also, McLaughlin (1995) points out that learning geometry can be easier for learners if they are taught in their first language, at least early in their school lives. There is no evidence in the discussion above whether the learners could have given correct responses if the questions were set in the learners' vernacular language. However, remarks such as those made by Learner KG52, that "English is difficult!" indicate that using vernacular could have mitigated the challenges identified above.

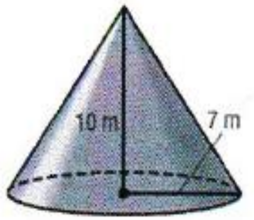
4.8.2.8 Arithmetic challenges

Deficiencies in basic arithmetic calculations have already been identified above as one of the reasons learners failed questions on 3D shapes. Similar challenges were also identified in School 2 and those are detailed in this section.

Table 2.19: Arithmetic challenges

Learner KG57	 ***Question 3*** Using the formula given, find the **volume** the solids. Round to the nearest tenth if necessary 3.1. $V = \frac{1}{3}bh$ $= \frac{1}{3}(5\ cm) \times 8$ $= 1{,}6 \times 8\ cm$ $= 1{,}28\ cm^3$	Learner took ‘b’ to mean base length/width rather than the base area of the shape. The researcher asked the learner during interviews why that was the case and the learner replied, *I had thought since this shape looks similar to a triangle, then ‘b’ is the same as the base in the formula of a triangle.*

Learner KG65	 3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm $V = \pi r^2 h$ $= \pi \times r^2 \times h$ $= 3.14 \times 6.4^2 \times 4.9cm$ $= 98.4704\ cm^2$. $= \pi \times r^2 \times h$ $= 3.14 \times 6.4 cm^2 \times 4.9 cm$ $= 98.4704\ cm^2$	Learner KG65 used the diameter as a radius. In addition, the learner used $6cm^2$ instead of $(6cm)^2$. Upon being asked by the researcher during intervention why that was the case, the leaner remarked On using the diameter as radius, the learner said, "*To be honest, I did not know the difference between the radius and a diameter and I just thought I needed to write something*". On why the learner failed to square 6.4 cm, the learner remarked, "It was my mistake, sir. I realise now that I should have answered the question differently."

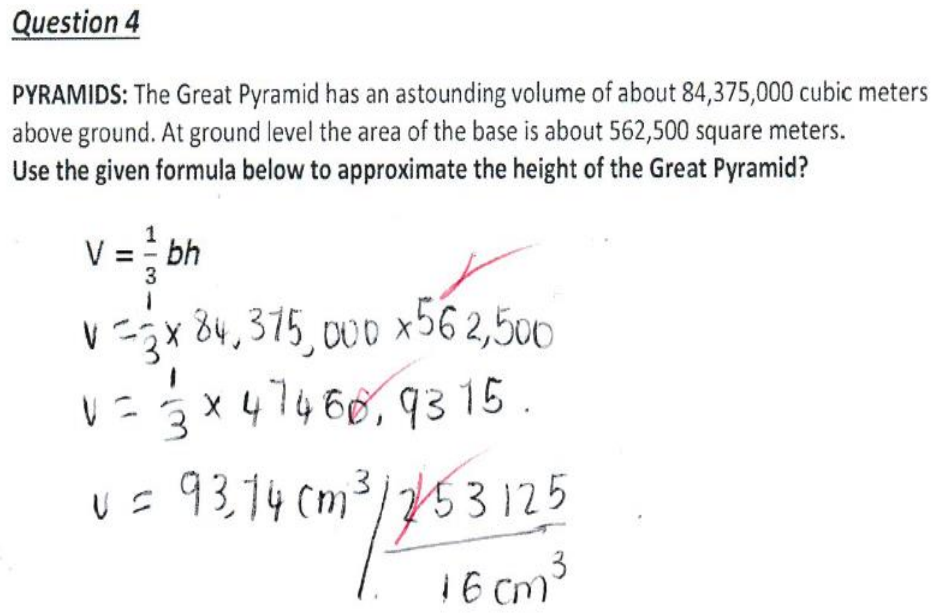
Learner KG65	3.2.  $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}(3{,}14 \times 10)$ $= \frac{1}{3}(31.4)$ $= 1{,}04\ cm^2$	The learner left out the radius in the calculation. In addition, none of the numbers is squared despite that the formula includes a square, meaning that it was supposed to be included somewhere in the learner's answer. The researcher asked the learner why the answer did not include any square despite it being given in the formula and why the learner left out the radius. The learner replied, *I was rushing and did not read the question properly. I guess I could have got the question correct if I did.*

The responses indicated above represent the common mistakes made by learners in School 2. What is clear from these responses is that learners were making arithmetic mistakes. From confusing 'b' in the given formula in Question 3.1 with that in the formula for the area of the triangle, failing to understand the difference between a radius and a diameter to leaving out the square given in the formula. As already said, those mistakes were also found in School 1. These challenges can be explained by a lack of proper foundation in arithmetic as explained by Adolphus (2011).

4.8.2.9 Reconstruction of shapes from given dimensions

Another 3D learning difficulty that was found in School 2 was that learners could not construct 3D shapes from given dimensions. Such difficulties were found in answers to Question 3.4 and Question 4. The samples of typical mistakes in those two questions are shown in Table 4.17 below.

Table 2.20: Reconstruction of shapes from given dimensions

Learner KG34	 ***Question 4*** **PYRAMIDS:** The Great Pyramid has an astounding volume of about 84,375,000 cubic meters above ground. At ground level the area of the base is about 562,500 square meters. **Use the given formula below to approximate the height of the Great Pyramid?** $V = \frac{1}{3}bh$ $V = \frac{1}{3} \times 84,375,000 \times 562,500$ $V = \frac{1}{3} \times 47460,9315.$ $V = 9374\ cm^3 / \frac{253125}{16\ cm^3}$	The learner indicated he/she simply substituted the numbers given, including volume even though the question was asking for height. Upon being asked by the researcher the reasons for the calculation, Learner KG34 said she was not sure what to do. She indicated that perhaps if the question included the shape and clearly labelled dimensions, it would have been easier.

Learner KG56	***Question 4*** **PYRAMIDS:** The Great Pyramid has an astounding volume of about 84,375,000 cubic meters above ground. At ground level the area of the base is about 562,500 square meters. **Use the given formula below to approximate the height of the Great Pyramid?** $V = \frac{1}{3}bh$ x height $V = \frac{1}{3}bh$ 84,375,000 x 562,500 $V = 4.7$	Learner KG56 explained that he was clueless about how to proceed. However, when asked to use the same formula on the same question, but now with a shape drawn on the board, during the intervention, the learner was able to calculate the height.

Learner KG37	3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm $V = \pi r^2 h$ $= \pi \times 6.4^2 cm \times 4.9 cm$ $= 3{,}14 \times 6.4^2 \times 4.9$ $= 630.21\ cm^2$ Nearest tenths $= 630.20\ cm^2$.	In this solution the learner used the diameter as a radius which is an indicator of arithmetic deficiencies as explained above. However, the learner's calculations were arithmetically correct. The learner indicated that it was difficult to understand the dimensions without the shape being given.

Learner KG53	3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm $V = \pi r^2 h$ $V = \underline{6{,}4\text{ cm}} + \underline{4{,}9\text{ cm}}$ $= \underline{11{,}3\text{ cm}}$ is the volume	Learners KG53, KG60 and KG57 all indicated that the question could have been easier if there was a shape given. All three learners simply substituted the values given into the formula instead of calculating height.

Learner KG60	3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm $V = \pi r^2 h$ $v = 6.4cm^2 \times 4,9cm^2$ $v = 983.4496$. $v = 02,18 cm^3$	
Learner KG57	***Question 4*** **PYRAMIDS:** The Great Pyramid has an astounding volume of about 84,375,000 cubic meters above ground. At ground level the area of the base is about 562,500 square meters. **Use the given formula below to approximate the height of the Great Pyramid?** $V = \frac{1}{3} bh$ $= \frac{1}{3}(562,500) \times 84,375,000$ $= 1,875.00 \times 84,375,000$ $=$ $= \frac{1}{3}(562,500) + (84,375,000$ $= 1,875000 + 84,375,000$ $= 86,625,00$	

A common theme in the discussion above is that the learners failed the two questions (Question 3.5 and Question 4) because no shapes were given. The examples given above were typical of the responses by the majority of the learners in School 2. The next section analyses the interview data. As already indicated with regard to School 1, learners who fail to work out solutions when no shapes are given show deficiency in Level 3 of van Hieles' theory. Given that the deficiencies above were reflected in the work submitted by 22 learners in School 1, it can be concluded the learners at School 3 were struggling with the informal deduction stage of van Hieles' theory as well.

4.9 Results from interviews

In the following section the researcher presents the results of the interviews with teachers. The section begins with a discussion of the process that was followed to prepare the interview data before discussing the specific learning difficulties that emerged from the interviews. The section closes by discussing the ways that teachers thought could be used to reduce learners' learning difficulties in 3D shapes.

4.9.1 Data preparation

Once the interviews had been conducted with teachers in both School 1 and 2, analysis followed. It should be noted from the onset that the teachers were assigned codes as follows:

Table 2.21: Data preparation

School	Teacher Number	Code
School 1	Teacher 1	Tr.1.1
	Teacher 2	Tr.1.2
School 2	Teacher 1	Tr.2.1
	Teacher 2	Tr.2.2
	Teacher 3	Tr.2.3

The learning difficulties that emerged from the interviews were analysed qualitatively. The first step of the analysis was to code the data into three broad areas as shown in Table 4.3. Those broad categories were chosen based on Webber& Roberts (2017) who posit that that learning difficulties can be analysed as relating to the teacher, the learning environment and the learner.

Table 2.22: Excerpt from the initial categorisation

Excerpt from the initial categorisation			
Group	**Category** Learning difficulties relating to…	**Description**	**Code**
1	… the teacher	This category consisted of 3D learning difficulties that directly result from the action/inaction by the teacher	Int.TTR
2	… the learning environment	This category consisted of 3D learning difficulties that were caused by the learning environment	Int.TLT
3	… to learners	This category consisted of 3D learning difficulties that resulted from the learners	Int.TLR
4	Learning processes that enhance the learners' cognitive understanding of 3D shapes in the grade 9 mathematics	In this category, teachers provided advice on the learning processes that they believed would enhance learners' cognitive understanding of 3D shapes in grade 9 mathematics	Int.LPL
5	Suggest how the learning difficulties in grade 9 3D shapes can be	The teachers suggested recommendations on how to deal with learning difficulties that learners face in learning 3D shapes	Int.RCM

	minimised		

Once the broad categories were decided upon, subcategories were identified as they emerged from the data. Table 4.20shows an excerpt from the data categorisation and coding process.

Table 2.23: Excerpt from the subsequent coding

Excerpt from subsequent coding				
Group	**Category** Learning difficulties relating to…	**Description**	**Code**	**Sample response**
1	… the teacher	Teacher's knowledge of 3D shapes	Int.TTR.1	*I have had to work hard to convince myself that I could teach 3D shapes and that I attribute to my early experience with the topic. In school, I never enjoyed and could not grasp 3D shapes. That* deprived *me of the necessary foundation. The lecturer I met in university did not help either as he assumed that all of us were at the same level and did have the foundation in the topic. That affected the first few years of my teaching career and I remember rushing through the topic. I'm sure there are many like me* (Tr.1.1).
2	… the learning	Adequacy of learning	Int.TLT.1	*To do well in mathematics, and 3D shapes, in particular, learners require rounded up support both here at school and*

	environment	support		*home. However, most of the time this is not forthcoming. Oftentimes you give learners homework and when they come the following day, they tell you that they had no one to help them. This is a reflection, in my opinion, of our perception as a society towards Mathematics. We don't like numbers* (Tr.2.3)
		Learners' mastery of the language of instruction	Int.TLT.2	*Before we look at content-related learning difficulties, you must appreciate that our language of instruction is one of the challenges. I often see this in my class. If you orally give instructions to the learners in the class, they do well. However, if you simply give them a task in writing, the majority of them fail to understand what is required* (Tr.1.2)
3	… to learners	Learner's intellectual capability	Int.TLR.1	*You should remember that learners have different intellectual capabilities. Some are good at Mathematics and some in other areas. Remember EQ and IQ* [Emotional intelligence and Intelligence quotient, respectively]*? It is our role to identify what learners are good at and try to support them, rather than making them feel bad that they are not good in Mathematics. We should accept that not all our learners will be good in*

				Mathematics and 3D shapes in particular.
4	Learning processes that enhance the learners' cognitive understanding of 3D shapes in the grade 9 mathematics	Experiential learning	Int.LPL.1	*I've realised over time that experiential learning works better for topics such as 3D. Before going to the formal content in learners' prescribed textbooks, I usually use practical examples that are close to the learners' environment. For example, I use shapes such as houses, juice containers and lunch boxes among others. Don't make the mistakes of using shapes that may be imaginary to learners such as the shape of the earth etc. These are abstract ideas in learners' minds – learners need concrete objects to work with* (Tr.2.2)
		Adequate planning	Int.LPL.2	*Planning is very important in Mathematics and 3D shapes in particular. It is not only about knowing what one is going to teach about, it goes beyond that and includes putting together resources for the class, reviewing previous classes on the topic, if any, and deciding how that can be used in the current class* (Tr.2.2)

5	Suggest how the learning difficulties in grade 9 3Dshapes can be minimised	Teacher development	Int.RCM.1	*The concept of 3D shapes remains the same. However, the learners we have in class have changed significantly over time. Some of these no longer know traditional examples such as our traditional baskets and drums used in our traditional ceremonies. This requires teachers to understand these changes and adjust their lesson delivery methods, particularly as it relates to examples used in class. Post-qualifying development sessions that specifically address these dynamics may be useful* (Tr.2.1)
		Peer coaching	Int.RCM.2	*I feel that some of our colleagues who are doing well should share with some of us who are struggling in teaching 3D shapes. The DBE* [Department of Basic Education) *can facilitate this process given that they are in a better position to know who does better among us* (Tr.1.1)
		Reduction in the leaner-teacher ratio	Int.RCM.3	*In as much we have other challenges, the teacher-learner ratio in some of our classes is very high and that needs to be reduced as well. Educators can not realistically offer*

				personalised support to 40 learners in 3 or four classes. That's 120 learners per teacher! (Tr.1.2)
		Developing holistic learner support systems	Int.RCM.4	*Developing a learner should be the responsibility of three parties, namely, the learner who should be empowered to take the change of his/her development, the parent/guardian who should provide a conducive out-of-school environment and the schools which should also provide the necessary stimulus to learners' urge for development. If any of these parties fail to do their part, it is difficult to develop learners in any subject, let alone in mathematics* (Tr.2.1)
		Proper sequencing of lesson delivery	Int.RCM.5	*Sometimes it is important to make sure that as teachers we sequence our classes properly and show linkages to the different parts of the over knowledge body we deliver to learners. For example, you can't teach 3D shapes before you're convinced that learners have grasped 2D shapes and general arithmetic in general. However, sometimes that is what we do because of the pressure we get from facilitators, provincial education departments who seem to focus more on*

				quantity rather than the quality of what we teach (Tr.2.3)

4.9.2 Learning difficulties

Teachers were asked the question, *"What are the learning difficulties experienced by Grade 9 learners in learning 3D shapes?"* This section discusses the responses of teachers. As the coding above showed, the analysis will be done for all five teachers from the two schools.

4.9.2.1 Language of instruction

The first learning difficulty that emerged from the interviews was the language of instruction. Teachers Tr.1.2 and Tr.2.1 were of the idea that some learners fail because they are being taught in a language that they do not fully understand. Tr.1.2 indicated that:

Before we look at content-related learning difficulties, you must appreciate that our language of instruction is one of the challenges. I often see this in my class. If you orally give instructions to the learners in the class, they do well. However, if you simply give them a task, the majority of them fail to understand what the question meant.

Teacher Tr.2.1 shared the same sentiments and noted:

I feel that the language in which we teach our learners matter. That is particularly evident in cases where we give learners written instructions. If a learner fails to understand the instructions because they are written in their second or even third language, it is unrealistic to expect such a learner to eventually do well in 3D tasks.

Teachers Tr.1.2 and Tr.2.1's views support those by Nugroho & Wulandari (2017) who identified language of instruction as one of the reasons why learners find geometry a challenge. In addition, McLaughlin (1995) points out that learning geometry can be easier for learners if they are taught in their first language, at least early in their school lives. Tr.2.1 further explained:

I have observed over the years that teaching learners in their home language make it easier for them to understand topics such as 3D shapes. However, the challenges that we face is that our schools are diverse and teachers can't help every learner in their home language, though I think that should be the case in an ideal situation.

4.9.2.2 Teachers' knowledge and experience

Another learning difficulty that was found from the interview data was the teacher's knowledge and experience in teaching 3D shapes. Tr.1.1 put it as follows:

I have had to work hard to convince myself that I could teach 3D shapes and that I attribute to my early experience with the topic. In school, I never enjoyed and could not grasp 3D shapes. That [deprived] *me of the necessary foundation. The lecturer I met in university did not help either as he assumed that all of us were at the same level and did have the foundation in the topic. That affected the first few years of my teaching career and I remember rushing through the topic. I'm sure there are many like me.*

Teacher Tr.2.3 added:

Experience also counts on how we deliver our lessons. The more experienced we become, the more confident we become in teaching topics such as 3D shapes. Without that confidence, it is not sometime, possible to teach properly and that may contribute to our learners' poor performance in 3D shapes.

It is clear from this response that sometimes teachers can transfer their fears of the topic to learners. More importantly, the response points to the enduring nature of challenges that people have with regard to 3D shapes. Lastly, it is evident from the response that the teacher's experience is an important factor in teaching geometry. To understand that, one needs to imagine the type of learners the teacher in question produced when he was rushing through the topics. When Tr.1.1 was asked a follow-up question, "*You said you would sometimes rush through 3D shapes topics, how then did your learners perform in the topic?*" The teacher admitted in the follow-up question that his “learners were never good at geometry despite them passing Mathematics, overall”. Tr.1.1 further explained, “*I could only do the basics such as the number of sides, identifying vertices and didn’t do much beyond that. That changed with time after discussing with the colleague who recommended that I observe some of his classes*.”

Lack of teachers’ experience was investigated by Luneta (2014) who found that first-year student-teachers registered for a foundation phase programme at a university in South Africa were operating at Level 1 of van Hieles’ levels as opposed to Levels 3 and 4 that would be expected of Grade 12 learners. Another study by Adolphus (2011) also confirmed teachers’ shallow knowledge in Geometry, revealing that most teachers in Nigeria were operating at Level 1 of van Hieles’ model. It is therefore not surprising that our education system produces teachers such as Tr.1.1 who are not confident at some topics such as 3D shapes. As a result, such inexperience and lack of foundation end up being reflected in learners’ performance as Tr.1.1 explained.

4.9.2.3 Lack of learner support outside the formal school

From the data, another theme was evident, namely that of linking everyday activities with mathematics content. Teachers Tr.1.2 and Tr.2.3 expressed frustration that learners were not getting adequate support beyond what teachers offer at school. *Shapes like the platonic solids can be abstract in learners’ minds – learners need concrete objects to work with.*

The above response provides an important insight in teaching 3D shapes, namely, that teachers need to use familiar objects rather than those that are abstract ideas in the learners’ minds. As discussed in Chapter 2, several authors support the use of concrete objects in teaching shapes, and 3D in particular (for example, De Wet, 2010; Battista, 1999 & Cabral, 2004). Tr.2.2 added:

Leaners learn better if they relate to the subject, you’re teaching them. One way of doing that, in the case of 3D shapes, is to use objects that are near to learners’ everyday lives such as their books, their classrooms and similar objects.

4.9.2.4 Comprehensive class preparation

In addition, comprehensive class preparation also emerged from the interviews as an important part of mitigating learners' challenges in understanding 3D shapes. Teachers Tr.1.2, Tr.2.2, Tr.2.3 pointed out the need for planning. For example, teacher Tr.2.2 had the following to say:

Planning is very important in Mathematics and 3D shapes in particular. It is not only about knowing what one is going to teach about, it goes beyond that and includes putting together resources for the class, reviewing previous classes on the topic, if any, and deciding how that can be used in the current class.

Planning is also supported by Leone et al. (2010) as an integral part of an effective programme of teaching 3D shapes. In this study, planning was particularly important to ensure that the intervention sessions covered all of van Hieles' levels. Based on the advice by the Learning Disabilities Association (2019) and Webber & Roberts (2017), planning was also necessary to provide additional support to learners who had learning difficulties.

4.9.4 Ways to reduce learning difficulties

The third and final question of the interview was "How can the difficulties in learning grade 9 3D shapes be minimised?" This section presents the themes relating to that question as they emerged from the interview data.

4.9.4.1 Teacher development

One of the ways that teachers indicated would help to mitigate the challenges that learners face in learning 3D shapes in grade 9, were continuing teacher development. Teacher Tr.2.1 noted that teaching methods have changed significantly since they trained as teachers and indicated she would appreciate periodic training to learn new methods and technologies that can help her to deliver her lessons more effectively. Teacher Tr.2.1 noted:

The concept of 3D shapes remains the same. However, the learners we have in class have changed significantly over time. Some learners no longer know or relate to traditional examples such as our traditional baskets and drums used in our traditional ceremonies. This requires teachers to understand these changes and adjust their lesson delivery methods, particularly as it relates to examples used in class. Post-qualifying development sessions that specifically address these dynamics may be useful.

The response by Tr.2.1 adds a new perspective to the content-specific intervention advocated by Fouze & Amit (2018) and Nugroho & Wulandari (2017) that can improve teachers' mastery of 3D shapes. Teacher Tr.1.2 adds:

Knowledge evolves as society change. Teachers, likewise, need to change with that. Teachers, as part of those who bear societal knowledge, need to constantly develop themselves. For example, there are now several applications that enhance the visualisation of 3D shapes and it is our responsibilities as teachers to learn about them and point learners in that direction. Learners are, in any case, more tech-savvy than most of us, so they just need our facilitation and they will do the rest for themselves.

What the teachers above are saying is that continuing professional development can enhance teacher's delivery of lessons on many topics, including 3D shapes. The responses imply that it is possible for teachers to have sufficient competencies in 3D shapes but fail to deliver that effectively to learners. In other words, the teachers above are explaining the need to keep teachers informed of the existence of a generational gap that exists between teachers' knowledge and what they consider as appropriate examples, and learners' knowledge and experiences. Leone et al. (2010) support that the form of teacher development extends beyond the mastery of subject content to the delivery competencies so that teachers develop context-specific learning programmes to suit learners' specific needs and experiences.

4.9.4.2 Peer coaching and mentoring

Moreover, some teachers explained that there should be programmes of peer coaching and mentoring. For example, Tr.1.1 noted

I feel that some of our colleagues who are doing well should share with some of us who are struggling in teaching 3D shapes. The DBE [Department of Basic Education) *can facilitate this process given that they are in a better position to know who does better among us* (Tr.1.1)

What Tr.1.1 is saying is that teachers who were doing well in teaching 3D shapes should be allowed to assist those who may be struggling. Allowing those successful teachers to impart their knowledge and experience to their peers would be helpful for teacher development. Also, Tr.1.1 explained that such a programme requires the facilitation of school management and DBE officials.

Teacher Tr.2.3 added:

While the district and provincial DBE officials can provide resources and training of school-based coaches and mentors, local schools can help to facilitate the process and explain to teachers the benefits of such programmes.

It is clear that if done properly, those programmes are likely to develop teachers' content and delivery competencies. Teacher Tr.1.2 also emphasised the need for teachers to connect to learn from each other. That teacher noted:

Many teachers are doing well, not only in our schools or South Africa in general but also from around the world. The majority of these share their ideas through blogs, social networks and their writing. We can look up to such teachers for inspiration, and ask them for guidance where we can.

4.9.4.3 Reduction in the learner-teacher ratio

Interviewee Tr.2.1 expressed concern on the impact that high learner-teacher ratios in their school had on learners' understanding of Mathematics in general, and 3D shapes in particular. Teacher Tr.2.1 explained that large classes meant that teachers could not provide the necessary individualised learner support that is required to assist learners to be proficient in 3D shapes. The teacher explained:

The government has tried its best to provide learning material such as textbooks and other learning aids. However, the ratio of learners to the teacher is still very high. As a result, you are forced to adopt a one-size-fits-all approach to lesson delivery, which excludes some learners, especially those who might have learning difficulties in numeric tasks.

Teacher Tr.1.2 also added:

In as much we have other challenges, the teacher-learner ratio in some of our classes is very high and that needs to be reduced as well. Educators cannot realistically offer personalised support to 40 learners in 3 or four classes. That's 120 learners per teacher!

These conversations concur with studies by Gezahegn (2007) and Adolphus (2011), who identified the classroom sizes feature as a challenge in mathematics teaching. In addition, Kadosh and Cohen (2016) explain that smaller classes also allow the teacher to develop inclusive learning support for learners with natural learning barriers such as those with Dyscalculia. This is clear from Tr2.1's explanations displayed in Table 4.1.

4.9.4.4 Adopting a tripartite approach to learner development

A further way to mitigate learners' 3D shapes challenges that was suggested by teacher Tr.2.1 was adopting a tripartite approach to learner development. According to teacher Tr.2.1, the tripartite approach requires the school, the learner and the learners' parent/guardian to work together to support learners' progress in 3D shapes. That teacher had the following to say:

Developing a learner should be the responsibility of three parties, namely, the learner who should be empowered to take charge of his/her development, the parent/guardian who should provide a conducive out-of-school environment and the schools which should also provide the necessary stimulus to the learners' urge for development. If any of these parties fail to do their part, it is difficult to develop learners in any subject, let alone in mathematics.

The importance of a supportive environment has already been tabled in preceding discussions. It was also emphasised by Vygotsky's (1978), Forgasz & Rivera (2001) and Gezahegn (2007).

4.9.4.5 Sequencing lesson delivery

Another theme that emerged from the interviews about the question was that teachers need to sequence lessons properly to enable learners to understand and appreciate the connections between the different parts of the syllabus, including Geometry. For example, teacher Tr.2.3 said:

Sometimes it is important to make sure that as teachers we sequence our classes properly and show linkages to the different parts of the knowledge body we deliver to learners. For example, you can't teach 3D shapes before you're convinced that learners have grasped 2D shapes and general arithmetic in general. However, sometimes that is what we do because of the pressure we get from facilitators, provincial education *departments who seem to focus more on quantity rather than the quality of what we teach.*

Teacher Tr.2.3's response shows the importance of linking knowledge which is in sync with the idea of building on learners' foundation when delivering new knowledge as explained by Webber & Roberts (2017). The next section analyses results from the post-intervention tests.

4.10 Analysis of post-intervention tests

The intervention activities discussed in Section 4.5 take into account the results of the pre-intention tests (Section 4.8). Also considers the learning processes that enhanced learners' cognitive understanding of 3D shapes. Finally, the intervention activities also took into account the ways suggested by teachers on how to mitigate 3D learning difficulties (Section 4.9). After the intervention, the researcher administered a post-intervention test. The results of that test are presented and discussed in this section. The analysis is aimed at evaluating the effectiveness of the interventions that were done in both schools. This will be done by comparing, statistically, the learners' scores in the pre-and post-intervention tests. Mean, standard deviations and t-tests are used to analyse the effectiveness of intervention done in the two schools.

4.10.1 Effectiveness of intervention in School 1

Descriptive statistics

Mean and standard deviations were calculated to assess learner performance according to each of van Hieles' Levels of geometry understanding. Table 4.21 shows the results of the analysis.

Table 2.24: Descriptive statistics

van Hieles' Level	Possible Mark	Mean	Std. Deviation
Pre_Level_1	5	2.23	.77
Pre_Level_2	20	3.30	2.17
Pre_Level_3	6	1.00	1.39
Pre_Level_4	4	.33	.76
Pre_Level_5	5	.30	1.02
Pre_total	**40**	**7.16**	**3.66**
Post_Level_1	5	3.82	1.33
Post_Level_2	20	14.93	5.71
Post_Level_3	6	2.27	2.13
Post_Level_4	4	1.52	1.55
Post_Level_5	5	1.10	1.94
Post_total	**40**	**23.63**	**8.75**

Both the pre-and post-intervention tests had a possible mark of 40 and the possible marks at each of van Hieles' Levels of Geometric Thinking are shown above. From the table above it can be seen that mean marks for the pre-intervention test were all below

50% of the possible marks. On average learners got an average of 7.16 out of the possible 40 (or 18%) in the pre-intervention test. That means the average learner failed (at each level and overall) according to the Department of Basic Education's (DBE) (2012) assessment criteria which require a learner to achieve at least 30% to record an Elementary Achievement. The standard deviations were also very low relative to the possible mark. That meant that the learners' scores were relatively similar and were around the failing marks. In School 1, the highest mark was 17 points (43% of the total points of 40) and only 3 learners achieved percentage points of 30% or above. Even more concerning was the fact that only 5 learners got at least one mark in questions that fall under van Hieles' Level 4 and only 2 learners got a mark on those questions that fell under Hieles' Level 5.

Post-intervention, there were some improvements in the mean scores. Learner scores in van Hieles' Levels 1 and 2 were all above 40% of the possible marks: Level 1 (Mean = 3.82, SD = 1.33) and Level 2 (Mean = 14.93, SD = 5.71). That means that learners in School 1 were able to visualise (Level 1) and analyse (Level 2) 3D shapes after the intervention. These findings support observations such as those by Leone et al. (2010) who advocated for the use of graduated instructional sequencing to teach abstract concepts which was done in this study as part of the intervention process.

Despite the improvement in marks, overall, post-intervention learners still struggled to grasp the concepts assessed in higher levels (Levels 3 to 5). On average learners got lower than 50% of the possible marks at these levels. Generally, learners achieved Elementary Achievement in Levels 3 and 4. The mean marks were 38% apiece of the possible marks with standard deviations of 2.13 (Level 3) and 1.55 (Level 4). Therefore, learners achieved a pass in this level compared to the failing mean mark recorded pre-intervention. Unfortunately, learners still failed to pass Level 5, achieving a mean score of 1.10 out of 5 and a standard deviation of 1.94. That mean score was only 22% of the possible mark, which reflected those learners did not meet the pass criteria, according to the DBE (2012).

Comparison of mean scores

In this section, the differences between the mean scores in the pre-and post-intervention test are compared. If the learning processes applied in School 1 were successful, the mean scores in pre-intervention should be less than mean scores in post-intervention. The statistic used for that purpose is the Paired Samples t-test and the results of the analysis are shown in Table 4.22.

Table 2.25: Effectiveness of learning intervention

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Post - Pre	14.84	8.93	1.17	12.50	17.20	12.66	57	p < .001

From the table above, it can be seen that the mean difference between post- and pre-intervention is 14.84. That means that on average, learners got higher marks in the post-intervention test than in the pre-intervention test (mean score in post *minus* mean score in pre-intervention is positive). The difference is statistically significant ($p < 0.05$). It can be concluded, therefore, that the learning processes adopted at intervention were successful in School 1 since they resulted from a statistically significant difference in mean scores pre-and post-intervention. In addition, it can be said with a 95% confidence that the mean difference in scores was between 12.50 and 17.20 or 31% and 43% of the total marks of 40.

4.10.2 Effectiveness of learning process: School 2

Again, mean and standard deviations were calculated to assess learner performance per each of van Hieles' Levels of Geometric Thinking. Table 4.23 shows the results of the analysis.

Table 2.26: Descriptive statistics

	Possible Mark	Mean	Std. Deviation
Pre_Level_1	5	2.10	1.03
Pre_Level_2	20	2.47	1.98
Pre_Level_3	6	.63	.93
Pre_Level_4	4	.13	.51
Pre_Level_5	5	0	0
Pre_total	40	5.33	2.77
Post_Level_1	5	3.47	1.22
Post_Level_2	20	11.37	6.11
Post_Level_3	6	2.73	1.56
Post_Level_4	4	2.50	.89
Pre_Level_5	5	0.43	1.19
Post_total	**40**	**17.42**	**8.52**

Similar to School 1, the pre-and post-intervention tests had a possible mark of 40 with the possible marks at each of van Hieles' Levels of Geometric Thinking shown above. From Table 4.23, it can be seen that mean marks for the pre-intervention test were all below 50% of the possible marks. On average learners got an average of 5.33 out of the possible 40 (or 13%) in the pre-intervention test. Except for the mean score in Level 1 which was 42% of the possible marks, the average learner failed to achieve the lowest pass level as defined above for Elementary Achievement. The standard deviations were also very low relative to the possible mark as in School 1. That meant that the learner scores were relatively similar and were around the failing mean marks. More importantly, all learners in School 2 failed questions in Level 5. Overall, learners in this school failed the pre-intervention test by achieving a mean score of only 13% of the possible 40.

In the post-intervention test, there were improvements in learner scores. Learner scores in van Hieles' Levels 1, 2 and 4 were all above 40% of the possible marks: Level 1 (Mean = 3.47, SD = 1.22), Level 2 (Mean = 11.37, SD = 6.11) and Level 4 (Mean = 2.50, SD = .89). That means that learners in School 1 were able to visualise (Level 1), analyse (Level 2) and, perform formal deduction on 3D shapes (Level 4) after the intervention. The mean score for School 2 in the post-intervention test was 17.42 (SD = 8.52). That was 44% of the possible 40 marks which means that learners in this school, on average, achieved Moderate Achievement according to the DBE's (2012) rating criteria. Also, as in School 1, learners in School 2 struggled with questions in Level 5 by scoring a mean score of 0.43 (SD = 1.19), which was only 9% of the possible 5 marks. The results in School 2 also raised questions on the presumed hierarchical nature of the van Hieles' Levels (for example, van Hiele, 1984; Mason, 2012 & Walle, Karp and Bay-Williams, 2013), since despite failing Level 3, learners in School 2 went on to get a pass mean mark in Level 4. Table 4.22 shows a comparison of mean scores between mean scores in pre-and post-intervention tests.

Table 2.27: Paired Means t-test

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Post – Pre	12.08	7.15	1.30	9.41	14.75	9.26	29	P< 0.001

For School 2, the mean differences between the scores in post- and pre-intervention are 12.08. This means that on average learners got higher marks post-intervention than during pre-intervention (mean score in post *minus* pre-intervention is positive). The differences are statistically significant ($p < 0.05$). Again, it can be concluded that the learning processes adopted during the intervention process were successful in School 2 since they resulted in a statistically significant difference in mean scores pre-and post-intervention. In addition, it can be said with a 95% confidence that the mean difference in scores was between 9.41 and 14.75 or 24% and 37%. Overall, learners in School 2 performed worse than those in School 1.

4.10.3 School differential factors

The final step of the analysis was to determine whether the mean scores of learners from both School 1 and School 2 were the same after the intervention. Since both schools received a similar intervention, it would be expected that the mean scores were not significantly different. Significant differences in mean scores would point to the existence of context-specific factors that would cause the differential ineffectiveness of similar intervention procedures. The independent t-test was performed for this purpose.

Table 2.28: Test of homogeneity of schools' post-intervention

	Levene's Test for Equality of Variances			t-test for Equality of Means			
	Equal variances …	F	Sig.	t	df	Sig.	Mean Difference
Post-intervention test	… assumed	.001	.98	2.79	58	.007	6.22
	…not assumed			2.79	57.96	.007	6.22

Based on Levene's Test for Equality of Variances, Table 4.25 shows that the variances of scores between the two schools were statistically insignificant ($p < 0.98$). It can be assumed that the variances between scores in post-intervention in School 1 and School 2 were equal. The results also show that the post-intervention scores were statistically and significantly different between School 1 and School 2 ($t (58) = 2.79$, $p = 0.007$). With a p-value of less than 0.05, the results show that there were statistically significant differences between the mean scores from the two schools. It, however, is possible for differences to be statistically significant while there is no significance from a practical point of view. To test the practical significance of the observed differences, Cohen's d value was calculated. Differences are said to be practically significant if the d value is at least 0.20 (Keller 2015).

The d value for this study was 0.72 which indicates that the observed differences in mean scores between the two schools were practically significant since the d value is greater than the 0.20 cut off as discussed above. It can be concluded that though similar interventions were done in both schools the scores for learners from School 1 were better than those for learners from School 2. That implies that other factors could account for the differences. These could be differences in foundational knowledge (Adolphus 2011), as can be seen from lower marks for learners in School 2 in pre-intervention as perhaps pertaining to differences in learners' perception of Geometry (Forgasz & Rivera 2001) or differences in teachers' knowledge of 3D shapes (Luneta 2014).

4.6 Conclusion

This chapter has presented, discussed and interpreted the results of the study. The discussion began by outlining the data collection process that was followed, the administration of tests and how the interviews were conducted. Of particular note is how the data collection process was affected by the ongoing pandemic as discussed. The chapter then moves on to provide details of the intervention that was done in both School 1 School 2. The results of the empirical investigation are then discussed in the next three sub-sections, namely sections on results from the pre-intervention test, interview with teachers and results from the post-intervention tests. The next section concludes the discussion in this chapter and provides recommendations to educators and other interested stakeholders on how to mitigate Grade 9 learners' learning difficulties in 3D shapes.

CHAPTER 5 Conclusions and Recommendations

5.1 Introduction

This is the final chapter of the study and presents conclusions together with recommendations to educators and policy-makers on how to mitigate Grade 9 learning difficulties in 3D shapes. The chapter goes on to give an overview of how the research questions and aims which were stated in Chapter 1 have been addressed and achieved. The chapter also draws conclusions, gives highlights of limitations, and provides conclusions and recommendations directed to the practitioners and researchers in the teaching and learning of 3D shapes.

5.2 Summary of the study

The summary in this section covers and gives an overview of the whole study. In Chapter 1, the aim of the study was stated as: to explore learning difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in Geometry. To unpack that aim, the study focused on achieving three objectives, namely:

- to identify learning difficulties experienced by the grade 9 learners in learning 3D shapes,
- to determine learning processes that enhance the students' cognitive understanding of 3D shapes in Grade 9 mathematics, and
- to suggest how the learning difficulties in Grade 9 3D shapes can be minimised.

These objectives were achieved by, firstly, reviewing literature relating to the topic and, secondly, collecting, analysing and interpreting primary data relevant to answering the research questions. The next sub-section discusses the summary of the literature reviewed.

5.2.1 Summary of literature review

The literature review was approached with three questions in mind:

1. What are the learning difficulties experienced by Grade 9 learners in learning 3D shapes?
2. What are the learning processes that enhance the students' cognitive understanding of 3D shapes in grade 9 mathematics?
3. How can the difficulties in learning 3D shapes in Grade 9 be minimised?

Before concluding on the literature on each of those questions, it is important to first conclude on the literature on the theories dealing with learning difficulties. The first such theory that underpinned this study was Vygotsky's (1978) Constructivist Learning Theory which views learning as a means to learner development. The theory asserts that learning difficulties arise from the way learners experience three interrelated factors: the learning environment, the level of learner participation in learning activities and those tasked with the learners' learning. This theory was preferred because of its emphasis on learning although it has been discredited by scholars such as Zimmerman and Schunk (2013) who argue for observations and experiment rather than experience.

As a result, another theory had to be used for this study culminating in the focus on van Hieles' (1984) Levels of Geometric Thinking. 3D shapes are part of grade 9 geometry and thus a theory that describes geometrical learning was found suitable for this study. Moreover, the theory divided geometry learning into five levels, namely Visualisation, Analysis, Informal deduction, Deduction and Rigor based on the learner's learning progression. Those levels were discussed in detail in Chapter 2, together with examples relevant to the current study. Given the theory's advocacy of designing and delivering phase-based learning curricula (Luneta 2014), it fits into the South African context where learning is divided into phases. The theory was, however, criticised for its lack of prescription of what the teacher should do at each level (Kotzé 2007). Also, its perceived inability to fully aid learners in a figurative understanding of the intrinsic figural nature of a geometry object closure was found to be problematic (Fischbein 2012; Küchemann and Hoyles, 2016). Those criticisms withstanding, this study adopted the van Hieles' (1984) Levels of Geometric Thinking to develop and design intervention activities and learning instructions. The remainder of this subsection presents a summary of the answers found in the literature for each of the research questions stated above.

1. **Summary of literature on learning difficulties experienced by learners in learning 3D shapes**

The teacher's knowledge of 3D shapes or lack thereof was identified by Luneta (2014) and Adolphus (2011) as one of the reasons why learners struggle to learn 3D shapes. Teacher knowledge was found to influence both the educator's confidence to deliver 3D learning activities together with the support that the educator gives to learners. In addition, Utami & Pramudya (2014) explained that learner difficulties in 3D learning also arises from learners' failure to identify and integrate Geometry learning in everyday lives. Because of that, Utami & Pramudya (2014) observed that such learners usually have difficulties in carrying out the required analysis at van Hieles' Levels 2 to 5.

Language of instruction was also cited by Nugroho & Wulandari (2017) as a barrier to learning 3D shapes. More specifically, those authors found that where a language different from the learner's home language was used, it is difficult for learners to connect learning of 3D shapes and the ethnocultural objects for which learners have pre-existing knowledge and are well acquainted. As a result, learners find it difficult to learn 3D shapes. It also emerged from McLaughlin's (1995) observations that learners find it difficult to reorganise pre-existing knowledge into scientific form if such reorganisation is done in a language that is different from the language in which the knowledge would have been acquired. As a result, learning becomes difficult for learners.

Moreover, Adolphus (2011) identified a lack of appropriate foundation in other areas of mathematics as a barrier to learning geometry, including 3D shapes. Furthermore, Adolphus's (2011) study revealed that since learners should progress through van Hieles' levels, any deficiency in lower levels will be carried over to higher subsequent levels. In the same study, Adolphus (2011) stressed the importance of a learning environment and learning support. If those are inadequate, for instance, if learners do not have adequate learning materials such as textbooks, then they find it difficult to learn.

Finally, learners' perception about Geometry was also identified by Forgasz & Rivera (2001) and Gezahegn (2007) as a barrier to learning geometry, 3D shapes included. Specifically, the authors noted that Mathematics was generally viewed as a difficult subject across the studied stakeholders, with some acknowledgement that they had inherited that negative perception from their relatives from childhood. That stresses the importance of the cultural environment of learners in determining learners' perceptions and ultimately their learning abilities. Nugroho & Wulandari (2017) also identified learners' inherited perceptions about Mathematics in general and advocated for the use of special interventions to mitigate the negative impacts of such perceptions.

2. Summary of literature on the learning processes that enhanced the learner's cognitive understanding of 3D shapes in grade 9 mathematics

The literature found was not prescriptive on the learning processes that can enhance learners' cognitive understanding of 3D shapes. However, the structuring of learning processes around van Hieles' theory was one of the main recommendations that emerged from the literature owing to the theory's link to phase-based education curriculum in many countries (for example Adolphus, 2011; Nugroho & Wulandari 2017; Kotzé 2007). Gezahegn (2007) advised policy-makers to consider van Hieles' theory as early as curriculum development to ensure a coordinated approach in developing learners throughout their learning cycles. In addition, Fouze & Amit (2018) advise that educators should be mindful of the learners' ethnocultural values when teaching Mathematics and 3D shapes and incorporate those into learning programmes.

3. Summary of literature on how the difficulties in learning 3D shapes to grade 9 learners can be minimised

Several authors have advocated for the use of knowledgeable teachers (for example Fouze & Amit, 2018; Nugroho & Wulandari, 2017). Leone et al. (2010) also suggested that teachers' competencies should extend beyond just content mastery to delivery competencies so that teachers can develop context-specific learning programmes that suit learners' specific needs.

Another necessary intervention identified by Leone et al. (2010) was the use of graduated instructional sequencing to teach abstract concepts. That involves teachers thinking critically about topics and concepts sequencing when planning lessons and learning programmes so that such topics and concepts are hierarchically sequenced as proposed by van Hieles' theory.

Provision of adequate environmental support was also identified by Adolphus (2011) as important in mitigating learning difficulties. Environmental factors such as class size influence the ability of the teacher to give individualised attention to learners. Where learners have natural learning barriers classes should be relatively smaller than in situations where learners have non-natural barriers to learning. Also in line with the development in learning technology, other supports are emerging, including the use of 3D software.

5.2.2 Summary of empirical investigation

Even though research questions were used to collect data, the nature of the data collected influenced data analysis. In this section, the researcher concludes the empirical investigation. The study involved the collection and analysis of data from sixty learners (Thirty from each school) and educators (two teachers from School 1 and three teachers from School 2) from two schools in Mpumalanga. The data collected was a mixture of quantitative and qualitative data. Qualitative data was collected through semi-structured interviews with educators and informal interviews with learners. Quantitative data consisted of learners' scores from pre-and post-intervention tests. The learning difficulties which were identified from the study are discussed next.

5.2.2.1 Findings in this study

Several learning difficulties emerged from the empirical investigation. These difficulties can be classified into three, namely those that related to the educators, those that related to the learner and those that related to the learning environment.

5.2.2.2 Learning difficulties relating to the teacher

Though all care was taken to prepare the learning materials used in both the intervention and learner testing activities, there were some deficiencies with some of those materials. As a result, these ended up affecting learners' understanding of 3D shapes negatively. Firstly, the study showed that learners find it difficult to master 3D shapes if the instructions given and questions asked were not clear. This was evident in some questions which, on hindsight, the researcher realised were vague. These findings supported the findings by Nugroho & Wulandari (2017) who explained that for learners to achieve a higher level of thought and mastery of 3D shapes, instructions need to be designed carefully and clearly.

In addition, teachers' inexperience in teaching 3D shapes and their poor knowledge of 3D shapes presented learning barriers to learners' understanding of 3D shapes. Again, these findings concur with studies by Luneta (2014) and Adolphus (2011) who both found that teachers' inadequate knowledge of 3D shapes and their inexperience negatively influenced learners' mastery of 3D shapes. This difficulty was particularly apparent at the higher levels of van Hieles' model wherein some educators had difficulty delivering higher-level 3D shapes lessons to learners.

Another challenge revealed by this study was the fact that learners sometimes fail to fully master 3D shapes due to inappropriate ways in which questions were structured, particularly follow-up questions. What was evident from the findings is that questions that are not structured adequately resulted in double penalisation of learners where an incorrect response in one question leads directly to a penalty in another question irrespective of the learner's effort. Those findings were not evident in the literature and represent new insight arising from this study.

5.2.2.3 Learning difficulties relating to the learner

The first learner-specific learning difficulty identified was learners' failure to understand unfamiliar 3D vocabulary. The use of unfamiliar terms such as Dodecahedron and Tetrahedron when teaching 3D shapes needs careful attention as these presented challenges to learners in both schools from which the sample of the current study was drawn.

Moreover, given the close connection between learning 3D shapes and other areas of Geometry and Mathematics in general, learners who lacked knowledge in other aspects of Geometry and Mathematics in general found it difficult to master 3D shapes. One key example was learners' lack of knowledge in Arithmetic. There was clear evidence that the majority of the learners struggled with simple arithmetic concepts such as addition, subtraction and clearing of brackets. As a result, such learners failed to master questions which required computations in 3D shapes. These findings concurred with Adolphus (2011).

Another challenge that was closely related to learners' lack of mastery of mathematical principles was learners' confusion over the formula of the area of 3D shapes. Learners mixed up formulae such as those of cylinders and triangles owing to the similar letter being used in such formula. These challenges can be explained by a lack of proper foundation in arithmetic as explained by Adolphus (2011).

Furthermore, the data revealed that learners experienced challenges relating to inadequate mathematical knowledge. This was evident in learners' difficulties with 3D shapes questions which required simple algebraic knowledge. Questions that required skills such as changing the subject of the formula, finding the missing number in an equation were among those on which the learners performed the least. This could be linked to the learners' poor foundation as already indicated above.

Furthermore, abstraction was one of the major issues with learners' understanding of 3D shapes. This was evident in cases where learners failed to reconstruct a shape from given dimensions. Though these findings were not raised in any of the reviewed studies, they reflect that learners struggled with the informal deduction stage of van Hieles' theory. That stage requires learners to draw on their knowledge of the properties of 3D shapes to respond to questions on 3D shapes (De Wet, 2010; Battista, 1999; Cabral, 2004).

Finally, the language barrier emerged as a barrier in this study as was the case in studies by Kotzé (2007), McLaughlin (1995) and Nugroho & Wulandari (2017). There were clear cases where it was evident that learners had failed because they could not understand instructions. The same issue was also raised by educators during semi-structured interviews. They singled out the language of instruction as one of the key barriers for learners who seek to fully master 3D shapes.

5.2.2.4 Learning difficulties relating to the learning environment

One environmental factor which emerged from the study pertains to lack of learner support outside the formal school. Some teachers expressed frustrations that they were left to teach 3D shapes alone with parents and guardians doing little to support learners. Issues such as parents and guardians' failure to assist learners with homework were raised by several teachers. Though none of these parents and guardians was interviewed, some of the teachers suggested that such parents and guardians' perceptions on arithmetic in general can be a major contributing factor of inadequate learner support.

5.2.3 Ways of reducing learning difficulties

From the results in Chapter 4, majority of the learners were found to be operating at Levels 1 and 2 of Van Hieles' Levels of Geometrical Thinking in the pre-intervention tests in both schools. After the intervention, there were notable improvements in learners' understanding of Geometry with a significant number of students showing understanding of questions at Level 3. This was a notable improvement in learners' knowledge and understanding, even though students still faced challenges with questions at Levels 4 and 5. Therefore, the observation to these results is that the intervention methods used were partly successful in mitigating learners' challenges in 3D shapes. The possible intervention activities were drawn from both literature and interviews with teachers and are discussed in this section.

The teachers who were interviewed in this study suggested experiential learning as a learning process that could mitigate learners' challenges in learning 3D shapes. They advised that teachers need to use familiar and objects when teaching abstract ideas such as perimeter and number of vertices. These suggestions are backed up by literature on 3D by De Wet (2010), Battista (1999) and Cabral (2004). In addition, teachers also stressed the need for comprehensive class preparation when it comes to teaching 3D shapes. That proposition is supported by Leone et al. (2010) who identified adequate planning as an integral part of an effective programme for teaching 3D shapes. In this study, planning was done to ensure that the intervention sessions covered all of van Hieles' levels.

Furthermore, based on advice by the Learning Disabilities Association (2019) and Webber & Roberts (2017), planning was done to identify and provide additional support to learners who had learning difficulties. The results from the post-intervention test showed substantial improvement in learners' scores, showing the effectiveness of the suggested learning processes.

5.3 Recommendations

The following recommendations are made based on the results of this study.

- It is recommended that educators engage in continuous teacher development which should encompass both subject-specific content and new developments in teaching and learning. Currently this pertains to changes within the learners' environments such as new technological developments.
- Teacher peer coaching should be facilitated through formalised programmes led by the Department of Basic Education or it can be informal within the school capacity development set up.
- Policymakers should strive to reduce class sizes to reduce the learner-teacher ratio. This would facilitate individualised learner support by the teacher.
- It is recommended that policymakers capacitate parents/guardians to ensure that there is continuous learner support within and outside the school.
- Teachers are urged to take time to plan their classes and sequence learning programmes to enable learners to understand and appreciate the connections between the different parts of the syllabus, including Geometry and 3D shapes.
- It is recommended that policymakers should seriously consider the need to teach learners in their vernacular languages.

5.4 Limitations of the study

It should be noted that all the objectives set out for this study were reasonably achieved. However, this study was not without some limitations. Firstly, the sample of 60 learners and five teachers used in this study was very small compared to the population which consisted of 960 Grade 9 learners and 13 teachers. One of the reasons for settling for a small sample was limitations in resources and time. Future studies can build on the findings of this study by expanding the sample size.

The coronavirus (COVID-19) pandemic caused two limitations to this study. Firstly, it meant that the time for implementing intervention activities was limited to seven sessions of one hour each since teachers wanted to catch up on learning time that had been lost during the pandemic-induced lockdowns. A period of at least two weeks would have been preferable. However, all care was taken to ensure that most of the planned intervention activities were done.

The cultural environment has already been highlighted several times now as an important aspect of 3D learning. Given the cultural diversity of the two schools on which the study was conducted, it was not possible to factor in all the possible cultural factors in planning the intervention activities, particularly given the time limits already noted. Future research may need to take into account cultural factors when planning intervention activities and compare results from such studies to those of this study. In addition, despite all efforts being taken to ensure that the researcher remained as objective as possible some of the researcher's cultural biases may probably have influenced data collection and analysis, particularly qualitative data from semi-structured interviews with teachers.

Issues such as teacher development, peer coaching and mentoring emerged from the semi-structured interviews with the teachers. These claims were not statistically tested in this study to assess their significance in mitigating 3D learning problems. Future research can build on this study by conducting quantitative studies which statistically test those claims.

5.5 Conclusion

This chapter has presented the conclusions and recommendations of the study. The summaries were grouped into those that arose from literature and those that emerged from the empirical investigation. The chapter has also provided some recommendations to teachers, education policy-makers and other stakeholders such as parents and guardians. Some of the recommendations included the need for continuous teacher development and reduction in the learner-teacher ratio. Finally, this chapter has also presented some limitations of the study such as the potential for cultural biases in data collection and interpretation, sample size and limitations in time to implement intervention activities due to the coronavirus.

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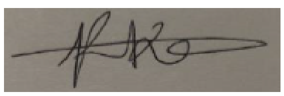
ANNEXURES

Annexure A: Declaration of authenticity research reports

I declare that this Research Report is my own work. It is submitted in partial fulfillment of the degree of Master in Education, Mathematics Education University of South Africa. It has not been submitted for any degree or examination at any other university.

Khomotso Welcome Kgopane

Signature : 

20 July 2021

Annexure B: Language editing certificate

Qondani Mtshane Rwigema

M Ed – Applied Linguistics; M Com – Business Management

EDITING CERTIFICATE

I hereby confirm that I have proof-read, formatted, and

edited the style, layout, references and language of the dissertation for the

MASTER OF EDUCATION (MATHEMATICS EDUCATION) DEGREE

to be submitted to

College of Education, UNISA

By

Khomotso Welcome Kgopane

Entitled

Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry

(188 pages, 39 986 words)

Note: The edited work described here may not be identical to that submitted. The author, at his sole discretion, has the prerogative to accept, delete, or change amendments made by the editor before submission.

Signed:

Date: 19 July 2021

Professional EDITORS Guild

Qondani Mtshane Rwigema
Associate Member

Membership number: RWI001
Membership year: March 2021 to February 2022

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Annexure C: Permission request letter

983, Meester Street, Hotjie, Mashishing, 1123| C: 0712149786, Email:tyetrib@gmail.com

The Circuit Manager
Department of Education
Mashishing Circuit
LYDENBURG
1120

27 Aug. 20

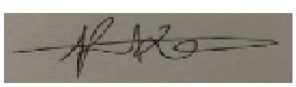
Dear Sir

PERMISSION TO CONDUCT RESEARCH AT MASHISHING CIRCUIT

I, Khomotso W Kgopane am doing research under the supervision of Professor ZMM JoJo under the Department of Mathematics Education towards a MEd at the University of South Africa. We are inviting you to participate in a study entitled Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry. The aim of the study is to explore teachers' experiences in the learning and understanding of 3D shapes by the learners. Your department has been selected because there are teachers specialized in teaching Mathematics at Grade 9 level. The study will entail face to face interview, survey questions, observations and document analysis of the Grade 9 mathematics teacher and learners in the learning difficulties of grade 9 learners.

The benefits of this study are that teachers' experiences will thus strive to improve learners' performance in the Learning of 3D Shapes thereby laying a good foundation in mathematics. This study will be of significant importance to those who guide educational policies and those who contribute to the preparation of the mathematics curriculum. Results and recommendations from this study will shed light in educational reforms. Curriculum planners will offer appropriate in-service training to teachers as well as to conduct workshops which will enable Grade 9 mathematics teachers to discuss challenges they face at various schools and come up with solutions. There are no potential risks involved. There will be no reimbursement or any incentives for participating in the research. If you would like to be informed of the final research findings, kindly contact Khomotso Kgopane on 0712149786 or email 39380408@mylife.unisa.ac.za or tyetrib@gmail.com. Should you have any concerns about the way in which the research has been conducted, you may contact Professor ZMM JoJo on 0734882211 /01242966273964 or email jojozmm@unisa.ac.za

Yours faithfully



Khomotso W Kgopane

Annexure D: Permission letter

Kedibone Jeneva <geraldmokoena@gmail.com>
to me

Aug 28, 2020, 8:35 AM

Morning Mr Kgopane K.W, hope you are well.

In response to your letter dated the 27 August 2020, Mashishing circuit office would like to grant you permission to conduct a research towards your Med under the Department of Mathematics with the University of South Africa at the schools that you have selected in the circuit.

Yours Faithfully

Mokwena T.G
Acting Circuit Manager
Mashishing Circuit Office
083 9731 291

Annexure E: Pre- and post-intervention tests

Annexure E1: Pre-intervention test

Dear Grade 9 Mathematics Learner   Code: KG____

You are invited to participate in a study entitled "Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry". The benefits of this study are that the experiences will thus strive to improve learners' performance in the Learning of 3D Shapes thereby laying a good foundation in mathematics. This study will be of significant importance to those who guide educational policies and those who contribute to the preparation of the mathematics curriculum.

Grade 9 Mathematics Pre-Exercise

Space and Shape: 3D

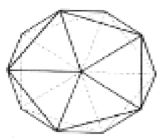
Total Points

40

Question 1(a)

Name the platonic solids by choosing from the given below

Tetrahedron, Hexahedron (cube), Octahedron, Dodecahedron and Icosahedrons



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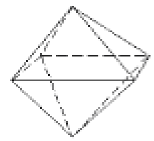

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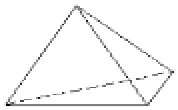

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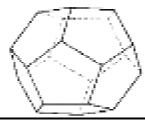

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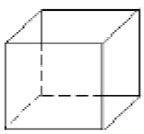

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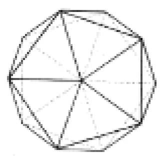
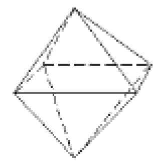
(5 Points)

Name the platonic solids by choosing from the given below

Tetrahedron, Hexahedron (cube), Octahedron, Dodecahedron and Icosahedrons





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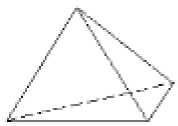

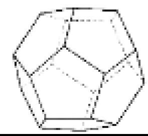

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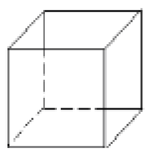

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(5 Points)

Question 2

For the platonic solids below, name the number of the Faces Edges and Vertices.

Platonic Solid Shape	**Faces**	**Edges**	**Vertices**
2.1. Hexahedron			
2.2. Dodecahedron			
2.3. Tetrahedron			

1.4. Icosahedrons			
2.5. Octahedron			

2.2. What do you observe?

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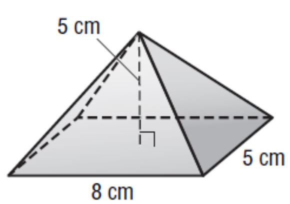
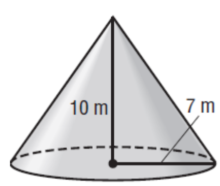
(20 Points)

Question 3

Using the formula given, find the **volume** the solids. Round to the nearest tenth if necessary

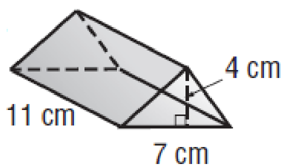
$$V = \frac{1}{3} bh$$

3.1. 



$$V = \frac{1}{3} \pi r^2 h$$

3.2.

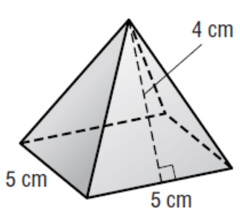
3.3. 

$V = bh$

3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm

$V = \pi r^2 h$

3.5. Find the **surface area** the solid by using the given formula. Round to the nearest tenth if necessary



Surface area of square pyramid = $b^2 + 2bs$

Where *b* is the length of the base and ***s*** is the slant height.

(10 Points)

Question 4

PYRAMIDS: The Great Pyramid has an astounding volume of about 84,375,000 cubic meters above ground. At ground level the area of the base is about 562,500 square meters.

Use the given formula below to approximate the height of the Great Pyramid?

$$V = \frac{1}{3} bh$$

(5 Points)

@@@@@@@@@@@@@@@@@@ THE END @@@@@@@@@@@@@@@@@

Annexure E2: Post-intervention test

Dear Grade 9 Mathematics Learner Code: KG______

You are invited to participate in a study entitled "Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry". The benefits of this study are that the experiences will thus strive to improve learners' performance in the Learning of 3D Shapes thereby laying a good foundation in mathematics. This study will be of significant importance to those who guide educational policies and those who contribute to the preparation of the mathematics curriculum.

Grade 9 Mathematics Post Exercise

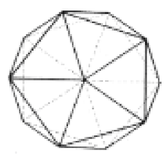
40

Total points

Question 1

Name the platonic solids and the name of the shapes that forms it. Example Tetrahedron,

Three equilateral triangles,



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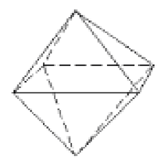
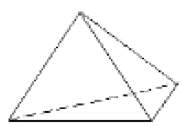
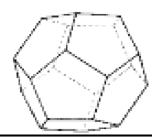


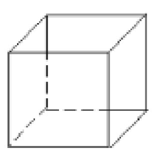
1 2



1.3.



1 4

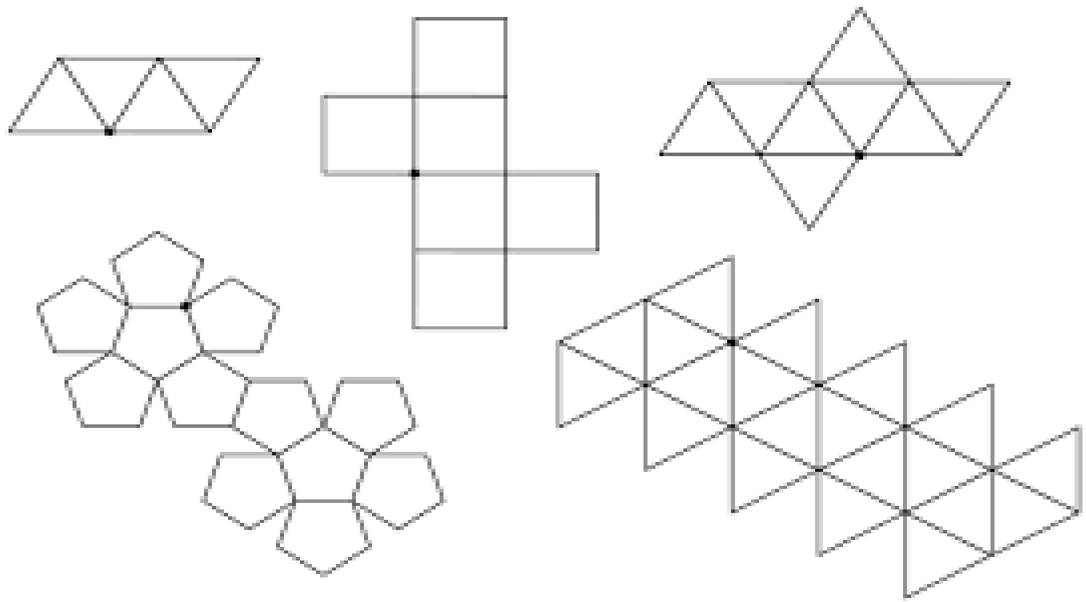


1.5.

(5 Points)

Question 2

Identify the solid. Name **the number** and **shapes of the faces** (F), **edges€ and vertices (V).**

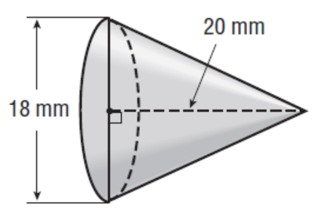


(20 Points)

Question 3 *Use the given formulae for your calculations.*

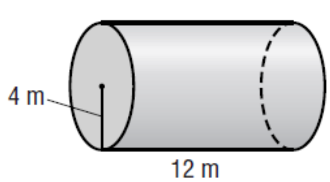
Find the **volume of** the solids below. Round to the nearest tenth if necessary

3.1.

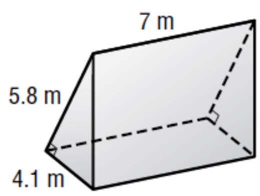


$V = \frac{1}{3}\pi r^2 h$

3.2.



$V = \pi r^2 h$

3.3. 

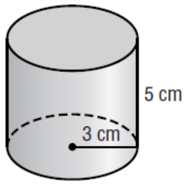
$V = bh$

3.4. Triangular $V = bh$ prism: base of triangle, 8 m; altitude, 8 m; height of prism, 6 m

3.5. Find the **surface area** the solid. Round to the nearest tenth if necessary

Surface = 2π radius X height

$S = 2\pi rh + 2\pi r^2$



(10 Points)

Question 4

ALGEBRA Use the formula below to find the height of a cylinder with a diameter of 5 meters, and a volume of 49.1 cubic meters..

Volume = πr^2 X height

V = $\pi r^2 h$

(5 Points)

@@

I wish to appreciate your participation in this study. This will make a huge impact in the learning and understanding 3D Shapes in grade 9 Mathematics Geometry.

Thank you

Annexure F: Interview protocol

- What are the learning difficulties experienced by Grade 9 learners in learning 3D shapes?
- What are the learning processes that enhance the students' cognitive understanding of 3D shapes in Grade 9 Mathematics?
- How can the difficulties in learning Grade 9 3D shapes be minimised?

Annexure G: Sample assessed tests

Dear Grade 9 Mathematics Learner **Code: KG 023**

You are invited to participate in a study entitled "Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry". The benefits of this study are that the experiences will thus strive to improve learners' performance in the Learning of 3D Shapes thereby laying a good foundation in mathematics. This study will be of significant importance to those who guide educational policies and those who contribute to the preparation of the mathematics curriculum.

Grade 9 Mathematics Pre Exercise

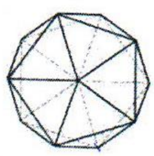
Space and Shape: 3D

Total Points 5 / 40

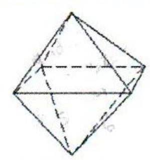
Question 1(a)

Name the platonic solids by choosing from the given below

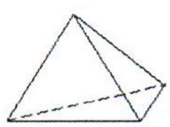
Tetrahedron, Hexahedron (cube), Octahedron, Dodecahedron and Icosahedrons



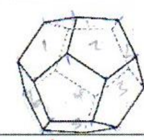
1.1. ~~Icosahedrons~~ Dodecahedron



1.2. ~~Dodecahedron~~ Icosahedrons

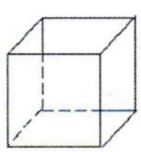


1.3. Tetrahedron



1.4. Octahedron

2



1.5. Hexahedron

(5 Points)

3

Question 2

For the platonic solids below, name the number of the Faces Edges and Vertices.

Platonic Solid Shape	Faces	Edges	Vertices
2. 1. Hexahedron	6	8	11
2. 2. Dodecahedron	~~10~~ 10	~~8~~ 11	~~6~~ 21
2. 3. Tetrahedron	4	5	6
1. 4. Icosahedrons	~~8~~ 6	6	10
2. 5. Octahedron	8	14	17

2.2. What do you observe?

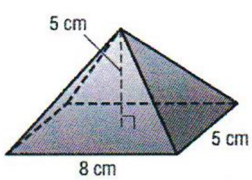
I ~~lear~~ observed to know many kinds of shapes and how many faces, Edges, and vertices they have.

(20 Points)

Question 3

Using the formula given, find the **volume** the solids. Round to the nearest tenth if necessary

3.1.



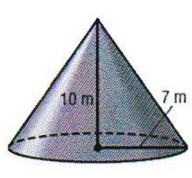
$V = \frac{1}{3}bh$

$= \frac{1}{3}(13cm + 5cm)$

$= \frac{1}{3}(18cm)$

$= 6cm$

3.2.



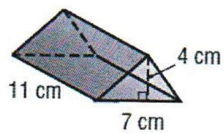
$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(3.14)(r^2)10m$ $\quad \frac{1}{3}(3.141 + 10m)$

$= \frac{1}{3}(31.42)$ $\quad r^2 = \frac{1}{3}(3.142)$

$r = 4.38$

3.3.



$V = bh$

$= \cancel{11\times}(18cm + 4cm)$

$= \cancel{44cm}\ 22\ cm$

3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm

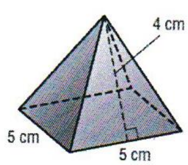
$V = \pi r^2 h$

$= (3.141 + r^2 + 4.9cm)$

$r^2 = \sqrt{8.042}$

$= 2.836$

3.5. Find the **surface area** the solid by using the given formula. Round to the nearest tenth if necessary



Surface area of square pyramid $= b^2 + 2bs$

Where b is the length of the base and s is the slant height.

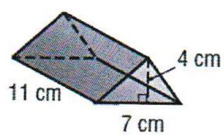
Surface area of Square pyramid $= b^2 + 2bs$

$= 5cm^2 + 5cm + 5cm + 4cm + 4cm$

$= 25cm + 18cm$

$= 43cm$

3.3.



$V = bh$

$= \cancel{11\times}(18cm + 4cm)$

$= \cancel{44cm}\ 22\ cm$

3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm

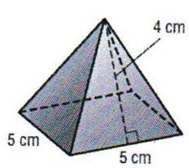
$V = \pi r^2 h$

$= (3.141 + r^2 + 4.9cm)$

$r^2 = \sqrt{8.041}$

$= 2.836$

3.5. Find the **surface area** the solid by using the given formula. Round to the nearest tenth if necessary



Surface area of square pyramid $= b^2 + 2bs$

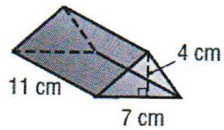
Where b is the length of the base and s is the slant height.

Surface area of Square pyramid $= b^2 + 2bs$

$= 5cm^2 + 5cm + 5cm + 4cm + 4cm$

$= 25cm + 18cm$

$= 43cm$

3.3. 

$V = bh$

$= \cancel{11\times}\ (18cm + 4cm)$

$= \cancel{44cm}\ 22\ \cancel{cm}$

3.4. Cylinder: diameter, 6.4 cm; height, 4.9 cm

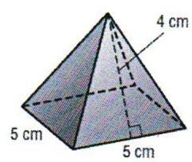
$V = \pi r^2 h$

$=(3.141 + r^2 + 4.9cm)$

$r^2 = \sqrt{8.04}$

$= 2.836$

3.5. Find the **surface area** the solid by using the given formula. Round to the nearest tenth if necessary



Surface area of square pyramid $= b^2 + 2bs$

Where b is the length of the base and s is the slant height.

Surface area of Square pyramid $= b^2 + 2bs$

$= 5cm^2 + 5cm + 5cm + 4cm + 4cm$

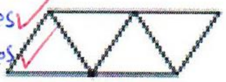
$= 25cm + 18cm$

$= 43cm$

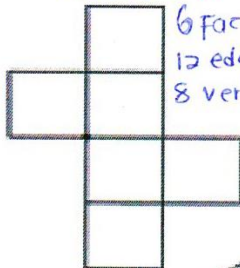
Question 2

Identify the solid. Name **the number** and **shapes of the faces** (F), **edges€ and vertices (V).**

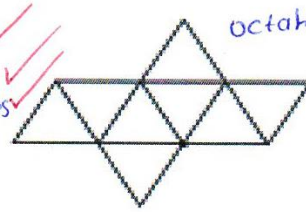
Tetrahedron
4 Faces
6 edges
4 vertices



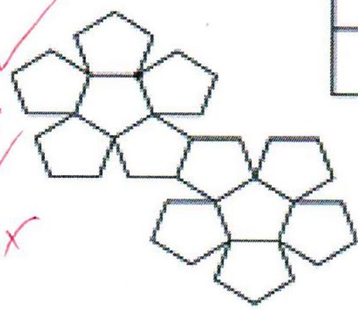
Hexahedron
6 Faces
12 edges
8 vertices



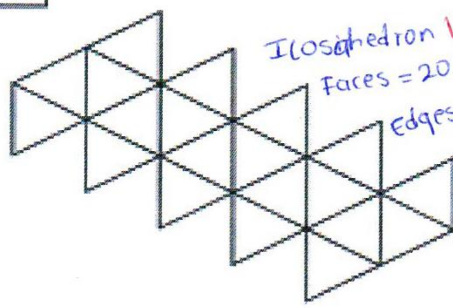
octahedron
8 Faces
12 edges
veterses ~~6~~



dodecahedron
Faces = 20
Edges = 30
~~versise~~
Vertices = 12



Icosahedron
Faces = 20
Edges = 30
verterses = 12



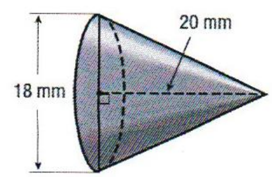
17

(20 Points)

Question 3 *Use the given formulae for your calculations.*

Find the **volume of** the solids below. Round to the nearest tenth if necessary

3.1.



$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \cdot 3.14 \times 2^2 \times 20m$

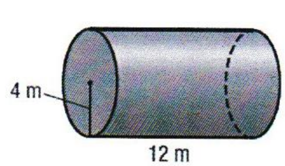
$= \frac{1}{3} (12.56 \times 20m$

$= \frac{1}{3} (251.2)$

$= 83.73$

$= 83.7 m^3$

3.2.



$V = \pi r^2 h$

$= 3.14 \times 4^2 \times 12m$

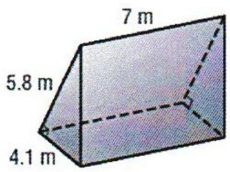
$= 50.24 \times 12m$

$= 602.88m$

$= 602.9m^3$

2

3.3.



$V = bh$

$= \frac{1}{3}(5.8m \times 4.1m)7m$

$= \frac{1}{3}(23,78)7m$

$= 55.48$

$= 55.5 m^3$

3.4. Triangular prism: base of triangle, 8 m; altitude, 8 m; height of prism, 6 m

$V = bh$

$= \frac{1}{3}(8m \times 8m)$

$= \frac{1}{3}(64)$

$= 21.33 m$

$= 21.3 m^3$

$v = bh$

$= \cancel{1\ 5 \times 8}\ 8cm \times 8cm$

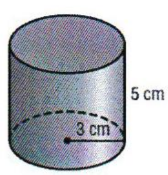
$= 64 cm^3 \cancel{\times 8cm}$

$\cancel{= 512 cm^3}$

3.5. Find the **surface area** the solid. Round to the nearest tenth if necessary

Surface = 2π **radius X height**

$S = 2\pi rh + 2\pi r^2$



Surface $= (2)(3.14 \times 4cm) \times 5cm$

$= 25.12 cm \times 5cm$

$= 125.6cm + 6.28 \times 4^2$

$= 226.08 cm^2$

(10 Points)

Question 4

ALGEBRA Use the formula below to find the height of a cylinder with a diameter of 5 meters, and a volume of 49.1 cubic meters..

Volume = πr^2 X height

$V = \pi r^2 h$

$49.1 = 3.14 \times 4^2 \times h$

$\frac{49.1}{50.24} = \frac{50.24 \times h}{50.24}$

$= 0.977$

(5 Points)

@@

I wish to appreciate your participation in this study. This will make a huge impact in the learning and understanding 3D Shapes in grade 9 Mathematics Geometry.

Thank you

Annexure H: Turnitin receipt

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Annexure I: Supervisor consent

CONSENT FOR SUBMISSION

ME Meyer, Elmarie 15 Jul

To You and Jojo, Zingiswa

Dear Mr Kgopane

With reference to previous correspondence, I have pleasure in informing you that your supervisor has consented to your submitting the dissertation for examination with the following approved title:

Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry

For purposes of examination, please submit the electronic copy of your dissertation in .pdf format to the following link: https://mylifeunisaac.sharepoint.com/sites/StudentSubmissions. Please note that you will have to wait for access on the above link before submitting, and that access is not immediate, it will only take place during office hours.

Go to the following link

Annexure H: Ethical Clearance

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date 14 August 2019

ERC Reference : **2019/08/14/39380408/25/MC**
Name: Mr KW Kgopane
Student no: 39380408

Dear Mr Kgopane,

Decision: Approved

Researcher(s): Name: Mr KW Kgopane

Supervisor(s): Name: Prof ZMM Jojo

Working title of research:

Exploring learning difficulties experienced by Grade 9 mathematics learners in understanding 3D shapes in Geometry.

Qualification: MEd Mathematics Education

Research Ethics Committee Recommendations:

None

Yours sincerely,

Name of the Chair: Prof AT Motlhabane
E-mail: motlhat@unisa.ac.za
Tel: (012) 429-2840