RELATIONSHIP BETWEEN LEARNERS' PROFICIENCY IN COMMON AND ALGEBRAIC FRACTIONS: A CASE OF GRADE 10 LEARNERS AT A LIMPOPO HIGH SCHOOL

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I declare that the above thesis is my work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references. I further declare that I have not previously submitted this work or part of it, for any degree or examination in any other higher education institution.

SIGNATURE**:**____________________________ DATE **2021**

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DEDICATION

I dedicate this thesis to my husband, Malvern, children, Tanyaradzwa, Tinofara, Atipaishe and Akatendekaishe, my late father Muxabango Ntuli and my mother Gladys Ntuli who have always been my source of inspiration.

ABSTRACT

In Mathematics, the topic of fractions is one of the most difficult topics for both teachers and learners. The primary focus of the study was to examine how pedagogical strategies could be used in the teaching of common fractions on the learner's performance achievement on algebraic fractions. The study used a mixed method action research approach. The data were collected using pre- and post- test instruments, which were complemented by semi-structured interviews. A class of 26 Grade 10 Mathematics learners was purposively chosen including five Mathematics teachers. The results of the study revealed that proficiency in common fractions improves proficiency in algebraic fractions. The study also revealed that learners preferred to use the easiest common denominator (ECD) in addition and subtraction of fractions instead of using the lowest common denominator (LCD). Besides, the current study showed that the use of a calculator was a barrier in learning fractions since learners were reluctant to learn and internalise the concepts of common fractions from the basics. Therefore, the study recommended that the use of a calculator must be discouraged at lower levels until learners have mastered the basics of solving fractions. The study also recommends the teaching of fractions using models in a hierarchical order, starting with the definition until the most difficult concepts.

Keywords: Proficiency; algebraic fractions; performance achievement; relationship; computational skills; misconceptions; errors; learners

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LIST OF ACRONYMS

AR	Adaptive Reasoning
CAPS	Curriculum and Assessment Policy Statement
CU	Conceptual Understanding
DBE	Department of Basic Education
ECD	Easiest common denominator
FET	Further Education and Training
GET	General Education and Training
KWL learner	K-what the learner already know (prior knowledge) W-what the want to know L-what the learner has learnt from the lessons
LCD	Lowest Common Denominator
ME	Misconception Errors
MP	Mathematical Proficiency
MLE	Meaningless Errors
OLS	Ordinary least squares
OOE	Order of Operation Error
PCK	Pedagogical Content Knowledge
PD	Professional Development
PF	Procedural Fluency
SC	Strategic Competence
SME	Silly Mistake Errors
SP	Senior Phase
ZPD	Zone of Proximal Development

CHAPTER 1

INTRODUCTION

1.0. Overview

This study is divided into 6 chapters. The first chapter introduces the study. It comprises the introduction, background to the study, statement of the problem and purpose of the study. Also, Chapter 1 highlights the research questions and objectives that guide the study. In Chapter 2, literature was reviewed and a conclusion was given. Research methodology and design were described in Chapter 3. Chapter 4 comprises the data gathered and analysis of results. The discussion of results was done in Chapter 5 while Chapter 6 comprises the summary, conclusions and policy implementations.

1.1. Introduction and background

There is growing concern over low performance in Mathematics at the matriculant level in secondary schools across South Africa (Mji and Makgato, 2006; Howie, 2003). This problem is increasing at an alarming rate and warrants an in-depth investigation of its root causes (Mji and Makgato, 2006). One area that has received attention from scholars is the lack of understanding and computational skills in dealing with fractions by learners starting at lower levels (Baidoo, 2019, Petit et al., 2015, Mhakure et al., 2014). Evidence shows that learners who struggle with fractions also struggle with understanding basic topics in algebra (Brown and Quinn, 2007). Common fractions and algebra are critically important components of Mathematics education for learners across different fields (Department of Basic Education, 2011). Problems arising from not understanding fractions start from the primary level and continue into adulthood, with reasonable to severe consequences for every day and occupational decision-making (Ross and Bruce, 2009).

It is important to note that the problem with teaching fractions was documented beginning in the 1900s. De Morgan (1910) brings to light two of the major problems with the teaching of fractions which are: first, the concept of a fraction is never clearly defined (Wu, 2001), thus returning to the original definition is very important. Second, more time is needed to allow students to invent their own ways to operate on fractions rather than memorising a procedure (Huinker, 1998). The definition of fractions should be re-visited as a way of making students understand fractions better. According to De Morgan (1910), students having difficulties with fractions should return to the original definition and reason upon the suppositions, neglecting the rules until he or she cognitively establish them by reflecting upon familiar instances.

If the concept of fractions is not mastered at lower grades, say the Senior Phase (SP) which consists of grades 7 to 9, then it follows that students will find it challenging to solve fractions at higher grades, which is Further Education and Training (FET) phase comprising grades 10 to 12, due to the congested nature of the South African curriculum. Lamon (1999) claims that teachers are not well-equipped to teach the concept of fractions, which leads to students' inability to learn fractions proficiently. As a result, teaching and learning fractions are characterised by rote learning (Vinner, 2002).

In 2012, the government of South Africa put in place a teacher professional development programme geared towards improving teachers' pedagogical content knowledge (PCK) in Mathematics. A few studies have been done to evaluate the impact of professional development (PD) in the classroom (Hill et al., 2005). Specifically, studies that use action research to document if the pedagogical methods provided at the workshops contribute to the improvement of learners' ability to solve fractions are scarce. A recent study reports on teacher's concerns about "one-size-fits-all" PD programmes and the limited time allotted to PD is inadequate to grasp the important pedagogical concepts to be developed in this exercise (Nel and Luneta, 2017). This current study seeks to bridge the gap that has been identified in research by conducting action research that documents detailed lesson plans that show systematically how to teach common and algebraic fractions using concrete examples that include tangible objects.

1.2. Statement of the problem

Solving fractions is one of the most problematic areas in Mathematics across all grades and it continues even at SP and FET phases. Research in Mathematics education indicates that fractions are a challenging concept in the curriculum (Brown and Quinn, 2007, Pienaar, 2014, Makonye and Khanyile, 2015). The problems in understanding how to solve fractions continue into adulthood and these problems impact negatively on every day and occupational decision making (Ross and Bruce, 2009, Mhakure et al., 2014). For instance, in baking, fractions are used to tell how much of an ingredient to use. Across the globe, people used to survive by informal trading of commodities like vegetables to manufactured goods.

Therefore, understanding fractions made trading efficient and profitable. Ultimately, to avoid incompetence at the workplace arising from not understanding fractions and algebraic fractions, there is a need to equip learners with problem-solving skills starting from lower levels. Having taught grades 8 to 12, the researcher found that learners struggle with comprehending addition, subtraction, multiplication and division computations when simplifying algebraic fractions.

1.3. Significance of the study

Several groups may find the results of this study useful, inclusive of Mathematics teachers in rural contexts who may not be exposed to strategies that could be effective when teaching the concept of fractions to struggling learners. Many teachers are mentors to teacher candidates during clinical field experience and student teaching. Most programmes for teacher education get feedback on the success of their programmes through working together with in-service teachers who act as mentor teachers for teacher candidates. This study has the potential to help instructors to reflect on the way they model the teaching of fractions in education programmes and if there is a need for improving their practice. In addition, the study could help those who offer Mathematics professional development programmes to plan and deliver effective professional development courses regarding the concept of fractions. Finally, the study can provide pragmatic evidence to policymakers, which could allow them to interrogate their strategies in implementing teaching programmes.

1.4. Research questions

The study sought to investigate the following main research question:

> How does proficiency in common fractions in grade 10 learners at a Limpopo high school affect performance achievement in algebraic fractions?

The specific questions are as follows:

i. What is the relationship between proficiency in common fractions and learners' achievement performance in algebraic fraction?

ii. Which barriers do learners encounter when learning algebraic fractions?

iii. Which pedagogical strategies could be used to improve understanding and problem-solving skills in algebraic fractions?

1.5. Research aim and objectives

1.5.1. Main aim

The main aim of the research was to establish the relationship between proficiency in common fractions and performance achievement in algebraic fractions among grade 10 learners at a Limpopo high school.

The study will address the following specific objectives as a way of trying to achieve the main aim of the study.

1.5.2. Objectives

i. To establish if proficiency in common fractions would lead to an increase in the performance achievement in algebraic fractions.
ii. To determine the barriers and challenges to learners' understanding of fractions.
iii. To examine how pedagogical strategies can be integrated to improve learner proficiency in solving algebraic fractions.

1.6. Literature review

The literature was reviewed so as to get an insight into the challenges of teaching and barriers to learning, what other studies had revealed on the teaching and learning of common and algebraic fractions. By reviewing this literature, the researcher hoped to identify common themes and errors, establish stylized facts about the relationship between common and algebraic fractions, and to come up with effective strategies that could be used to enhance the learning of fractions. A review of related literature was completed on the history of fractions, challenges and interventions in the teaching and learning of common and algebraic fractions and studies that were done on fractions in South Africa and elsewhere. Studies done in South Africa had revealed that common and algebraic fractions are among the most challenging topics in the curriculum (Dhlamini and Kibirige, 2014, Baidoo, 2019, Spaull, 2011, Jooste, 1999). Other studies done elsewhere had also revealed that fractions are considered to be one of the most challenging topics in the Mathematics curriculum (Brown and Quinn, 2007, Hecht et al., 2007, Torbeyns et al., 2015). A detailed account of the literature review is in Chapter 2.

1.7. Theoretical framework

This study was guided by the constructivist theory in trying to explain the errors and misconceptions that learners display when simplifying algebraic fractions (Makonye and Khanyile, 2015). Besides, it was guided by Skemp's notions of understanding algebraic fractions, which is relational and instrumental. The constructivists teaching used in this action research will include hands-on, exploratory and inquiry methods as advocated by Vygotsky (1978) and the NCTM (2014). A constructivist's perspective claims that mathematical concepts are not mastered directly from experience but the ability of learners to learn from experience depends on the quality of the learner's prior knowledge. An adult is unable to engage in abstract reasoning if they lack prior knowledge. In this study, where learners lack the prior knowledge necessary to engage in abstract reasoning when learning algebraic fractions, concrete examples that include tangible objects shall be used during the demonstration. Conceptual understanding of mathematical concepts proceeds primarily from the learner's prior knowledge. This takes us to Skemp's notions of Mathematics understanding, which Skemp referred to as relational and instrumental understanding. Skemp (1976) argues that instrumental understanding is using rules without reasons, which can be regarded as the absence of conceptual understanding. On the other hand, relational understanding is knowing both what to do and why (Skemp, 1976). Therefore, it is paramount to improve learner's prior knowledge so that they can understand both instrumental and relational concepts.

1.8. Research methodology and design

The study used a mixed-method action research approach, which allows the researcher to provide a comprehensive initial assessment of the problem, develop a more solid plan of action and conduct a more rigorous intervention through integrating qualitative and quantitative data sources. An action research design was used in this study to seek new knowledge and understanding on how to improve teaching practices in general and in particular how to resolve significant problems associated with solving common and algebraic fractions in the classroom (McMillan, 2015).

1.9. Data collection

The data used in this study was collected from participants using pre-and post-test and semi-structured interviews. Permission was sought from the Limpopo Department of Basic Education, principals and participants in that order before data collection commenced. Intervention lessons were audio-recorded and all field notes were kept for future use in data analysis.

1.10. Data analysis

Data analysis comprised descriptive statistics like frequencies, percentages averages, standard deviations, range and median to characterise the sample. Descriptive statistics were used to organise and describe the characteristics of data collected (Salkind and Shaw, 2019). This was followed by a comparison of observations from the pre-test and post-test scores. The dependent sample t-test was used to determine whether there was a significant difference in performance resulting from the intervention by comparing the pre-test and post-test results. We used the dependent sample t-test because measurements were taken for the same individual based on the pre-and post-test. For this study, the variable of interest was performance, measured as the scores attained in the pre-and post-test. The difference-in-difference technique were employed to examine the differential impacts of our intervention on performance, while ordinary least squares (OLS) regression analysis was used to determine whether the difference in performance was as a result of the intervention or other factors. The dependent variable was performance and the independent variables were the characteristics of the student. The independent variables were identified from the literature. We hypothesised that the socio-economic variables were less important in explaining the gap between pre-and post-test scores. We also expected variables such as use of calculator and perception of common and algebraic fractions to be important explaining the different between the pre- and post-test. The study controlled for these two time invariant independent variables in the regression models. The researcher also looked for independent ideas bearing in mind that the data collection technique used semi-structured interviews, which are less structured formats that “assume that the individual respondents define the world in unique ways” (Merriam and Tisdell, 2015). Unique ideas were given special attention or treatment. In this respect, semi-structured interviews provided a deeper understanding of participants’ perspectives and produced rich data, which could be analysed and inform practice.

1.11. Limitations of the study

Marshall and Rossman (2014) note that all proposed research projects have limitations. The size of the sample used in the study was small and as a result, the researcher could not make generalisations using the findings of the study since there was no random selection of participants. The researcher used purposeful sampling during data collection, which is viewed as a biased sampling method. The qualitative data collected added depth to the study but did not provide for generalisation.

1.12. Population and sampling

In this study, the sample constituted all grade 10 Mathematics learners at one high school in Seleka representing the population of all grade 10 Mathematics learners in the study area and grade 10 Mathematics teachers around Seleka. In this study, the participants were selected using purposeful sampling. For this study, the researcher purposefully targeted a class of grade 10 learners who were taking Mathematics and grade 10 Mathematics teachers.

1.13. Ethical considerations

This study followed subject approval and informed consent, risks and benefits, confidentiality, anonymity and trust. The rights and values of the participants were handled with caution by the researcher. Participants were not exposed to any risk while participating in the study. Detailed ethical considerations are in Chapter3.

After acquiring ethical clearance from the University of South Africa (UNISA), the researcher sought permission from the District Director of Waterberg District in Limpopo province to carry out research in the circuit. The researcher attached a research proposal and ethical clearance from UNISA. The permission was granted by the district director of the Waterberg district. Letters seeking permission were then dispatched to the principal of the high school selected, learners and teachers who participated in the study. Parents of selected learners were also asked for permission in writing and filling in consent forms. Participants were assured that confidentiality would be maintained by not using their names. Instead, pseudonyms were used in the thesis and will be used in any other publications that the researcher might publish based on this data. The participants would be informed of the findings by the researcher formally and the principal and teacher participants will discuss the findings with the researcher.

1.14. Validation of study instruments

In the current study, the research instruments were reviewed by the researcher's supervisor and two other Mathematics education experts for content validity. Using recommendations from the supervisor and these experts, changes were made to reflect what the pre-and post-test instrument measure were intended to measure. Interpretive validity was used in this study which refers to the accuracy of presenting the inner worlds (DeLuca, 2011) In other words if a test is given it must accurately measure what it is intended to measure. Participants were asked to check if the written scripts (transcribed data) reflected their actual responses, which is referred to as member checking (Merriam and Tisdell, 2015).

1.15. Reliability

To ensure reliability, the researcher conducted a pilot study of the pre-and post-test instrument with a convenience sample of learners who were not to be part of the study. The pilot study helped to determine whether the individuals in the sample were capable of completing the pre-and post-tests and that they could understand the questions (Creswell, 2002). Also, to ensure the reliability of the study, the researcher employed corroboration. The purpose of corroboration is to help researchers increase their understanding of the probability that their findings will be seen as credible or worthy of consideration by others (Stainback and Stainback, 1988). One process involved in corroboration is triangulation. The researcher used a variety of techniques important in mixed methods study. Since this study had multiple data sources, the researcher employed the convergence of multiple data sources where the researcher used information from pre-and post-test data and interviews to study the problem.

1.16. Definition of key terms

Computational skills: Mabbott and Bisanz (2008) defined computational skills as the selection and application of arithmetic operations to calculate solutions to mathematical problems.

Common fractions: These are fractions where the numerator and the denominator must be integers or whole numbers and the denominator cannot be zero. Common fractions are further subdivided into proper and improper fractions. Proper fractions have denominators bigger than the numerator while improper fractions have numerators bigger than the denominators.

Algebraic fractions: An algebraic fraction is a fraction where either the numerator or denominator is represented by letters or both are letters. It is indicated by a quotient of two algebraic expressions, for example, $\frac{x}{4}$ or $\frac{2x}{x+1}$.

Proficiency: Cowan et al. (2011) defined proficiency as an advancement in knowledge or skills. It is a high degree of skill or expertise.

Performance: It is the completion of a task with an application of knowledge, skills and abilities

Pedagogical Content Knowledge (PCK): Oleson and Hora (2014) defined PCK as the overlapping of information about subject knowledge, that is, knowledge of the subject being taught and pedagogic knowledge, which is, knowledge of how to teach.

Rote learning: This is learning by memorisation technique based on repetition and drilling of concepts without reasoning and thinking.

Algorithm: Is a set of rules or formulas for solving a problem, based on conducting a sequence of specific actions.

Misconceptions: These are wrong facts or ideas arising from a learner’s prior experiences.

Algebra: This is a branch of Mathematics where numbers are represented by letters.

Arithmetic: Refers to addition, subtraction, multiplication and division of numbers.

1.17. Chapter division

This study is divided into seven chapters, as briefly discussed.

Chapter 1 provides the orientation to the study with the focus on the introduction, background to the research, statement of the problem, significance of the study, research questions, aims and objectives, definition of terms and chapter division. A summary of literature review, theoretical framework, research methodology and design, data collection and analysis, limitations of the study, population and sampling, ethical considerations, validation of study instruments and reliability, the definition of terms and summary of chapter divisions are also provided.

Chapter 2 contains a review of related literature organised in terms of the main research questions and objectives, literature review on the history of fractions, challenges faced in the learning and teaching of fractions and studies that were done on the learning and teaching of fractions. The chapter concluded by providing a gap in the literature that this study sought to address.

Chapter 3 focuses on the research paradigm which guides the study, research methods and design used in the study, specifically, action research design, methods and procedures. Important detailed discussions in this chapter include population and sampling, data collection and analysis, triangulation of data, measures to ensure validity and reliability and ethical considerations.

Chapter 4 includes data analysis, findings and interpretation of the study.

Chapter 5 contains a discussion of the main research findings.

Chapter 6 contains the summary, conclusion, recommendations, suggestions for further research, policy implementations and reflections on the study.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

This chapter starts with a discussion of the review of the literature. The purpose of this review was to assess the challenges that both learners and teachers face in the teaching and learning of common and algebraic fractions. Also, the review was aimed at understanding the connection between common and algebraic fractions and what makes algebraic fractions more challenging to learners than common fractions. The pedagogic methods used in the solving of fractions and other strategies to simplify the teaching of common and algebraic fractions were also reviewed. In depth, the researcher came up with the following themes from the literature: the context of the research, history of fractions, problems faced in the learning of common and algebraic fractions, the relationship between common fractions and algebraic fractions and when is the right time to teach common fractional concepts. Besides, they included studies conducted in South Africa on fractions, common errors in simplifying common and algebraic fractions, the importance of pedagogical content knowledge in solving common and algebraic fractions and what is fractional proficiency. Furthermore, additional factors contributing to learners' understanding or misunderstanding of fractions were included. The theoretical framework that guides this study is also discussed in this chapter. The chapter ends with a conclusion and knowledge gaps of the research.

2.1.1. Context

As a Mathematics teacher, understanding common fractions is crucial to a learner's success in algebra and other Mathematics topics. Recent studies regarding proficiency in fractions and success in algebra, have been conducted at lower levels (grade 7–9) and higher levels (grade 10–12) including tertiary level (Torbeyns et al., 2015, Khanyile, 2016, Pienaar, 2014). Most studies reviewed in this chapter focused on learning what fractional proficiency is and how the proficiency can be used to understand algebraic fractions (Brown and Quinn, 2007, Thomas, 2010, Preciado, 2016). However, the reviewed literature did not come up with effective strategies on how to improve the understanding of algebraic fractions, hence the reason for embarking on this study.

2.1.2. The history of fractions

A fraction is a number that represents a whole number that has been divided into parts (Schwartzman, 1994). It can either be a common fraction that comprises two parts, the denominator and the numerator or a decimal fraction whose denominator is a power of ten and the numerator is expressed by figures placed to the right of a decimal point, for example, 0.01. The denominator is the bottom number of the fraction, which originated from the Latin word '*denomino*' meaning to name while the top part is the numerator, which also comes from the Latin word '*numerus*' that means a number (Schwartzman, 1994). The denominator tells you how many parts the whole number has been divided into while the numerator informs you how many of those pieces are. The word 'fraction' comes from the Latin word ' *fractio*' which means to break (Schwartzman, 1994). Fractions were invented by Egyptians to organise taxes on land. Fractions were used to represent ratios and division, for example, $\frac{3}{4}$ can mean a ratio of 3: 4 or division 3÷ 4.

From a historic perspective, research has shown that students in lower grades and higher grades harbour a dislike of fractions primarily because they find that fractions were irrelevant in their daily lives (Wu, 1999). In addition, Resnick (1987) argues that using fractions in calculations has no practical utility. The arguments put forward by both Wu (1999) and Resnick (1987) support the historical justifications for students to dislike learning fractions. However, Resnick's notion in 1987 is justified if the learning and understanding of fractions by learners is separated from solving other mathematical problems. The current study seeks to show the usefulness of fractions in most Mathematics topics, especially in algebra. Through this study, Mathematics teachers and learners were exposed to strategies that could make the teaching and learning of fractions easier. Furthermore, this study contributed to the body of literature by focusing on effective ways of teaching algebraic fractions.

Brown and Quinn (2007) claim that when solving rational equations and simplifying rational fractions, it is necessary to apply generalised common fraction concepts. Students with a weak understanding of the fractional concept are likely to find rational algebraic fractions concepts difficult (Thomas, 2010). Further, the author alluded that rational algebraic fractions comprise all the difficulties found in common fractions but in addition, they also include difficulties usually associated with algebra. These challenges include addition and subtraction involving like terms as well as the multiplication and division of algebraic terms (Thomas, 2010). In addition, the other challenge arises from finding the lowest common denominator of algebraic fractions and the operations involved in adding and subtracting these terms in the same way that common fractions are added or subtracted (Brown and Quinn, 2007).

2.1.3. Problems faced in the learning of common and algebraic fractions

A study carried out by Hecht et al. (2007) revealed that learners often have imbalances in procedural knowledge and conceptual knowledge of fractions (either more conceptual and less procedural or vice-versa). The authors revealed that these discrepancies are a result of a lack of prior knowledge of fractions and this affects the understanding of fractions. A study by Torbeyns et al. (2015) revealed that the difficulty experienced by Belgian children in learning fractions stemmed from the important dichotomy of procedural and conceptual knowledge. The authors further reported that learners learnt fractions by rote learning rather than by understanding. Results from a study by Bempeni and Vamvakoussi (2015) documented that learners with a strong conceptual fraction knowledge adopt a deep approach to Mathematics learning and understanding, whereas learners with poor conceptual fraction knowledge adopt a shallow approach to reproduce. Other authors noted that the reason for difficulties in solving questions related to fractions includes over-generalisation of procedures even in situations that are inappropriate (Bempeni and Vamvakoussi, 2015, Torbeyns et al., 2015).

Lortie-Forgues et al. (2015) documented that factors that are not integral to common fractions but instead are determined by cultural values and characteristics of educational systems also contribute to difficulties in learning fractions. The authors further emphasise that these cultural factors are not unique to rational number arithmetic but instead, extend to Mathematics in general. In this study, factors that were not directly related to fractions but can impact the understanding of algebraic fractions were investigated, for instance, learners' background can be a barrier to the learning of fractions. If a learner comes from a poor home where they cannot buy a pizza and a teacher uses it as an example of how to divide fractions, it becomes a fruitless task. The difficulties can be minimised by improving teaching styles that will be highlighted in the intervention section. To come up with better and doable strategies, it is important to consider the relationship between common and algebraic fractions, which will be discussed in the section that follows.

2.1.4. Relationship between success in common fractions and proficiency algebraic fractions

A study done by Brown and Quinn (2007) revealed that there is a relationship between success in algebra and proficiency in common fractions. Usiskin (2007) reported on the significance and justification for doing common fractions in the foundational years of Mathematics. The author further revealed that fractions represent the division and constitute the most common way in which division is represented in algebra. Also, fractions feature in most Mathematical topics of which algebraic fractions is one of them. Hence, this has caused a demand for increasing proficiency in common fractions. The study conducted by Brown and Quinn (2007) on the relationship of algebra and common fractions was not in the context of this current study. The authors looked at how proficiency in common fractions enhances the learning of algebra in general, thus, there is a need to investigate if a learner who has mastered the arithmetic of common fractions will also be proficient in algebraic fractions in particular.

Research has shown that knowledge of fractions at the age of 10 would foretell their algebra knowledge and overall Mathematics performance in high school, above and beyond the effects of general intellectual ability (Siegler et al., 2012). Hence, it is therefore paramount to teach common fractions in a way that allows for conceptual understanding from the primary level. Siegler et al. (2012) further argued that if specific areas of Mathematics can be identified that are most consistently predictive of mathematical proficiency, then we can determine why those types of knowledge are uniquely predictive to improve instruction and learning in those areas. In line with this argument, Brown and Quinn (2007) reiterated that understanding arithmetic should be a prerequisite to understanding the structure of algebra. Therefore, the understanding of common fractions is, important before learners can do algebraic fractions. The current study will elucidate if learners who do not understand the arithmetic of common fractions (i.e., addition, subtraction, multiplication and division of fractions) also struggle with simplification of algebraic fractions.

It is of paramount importance to understand and know arithmetic concepts since algebra is the generalisation of arithmetics and the first experience in symbolic representation of numbers (Wu, 2001). Furthermore, Brown and Quinn (2007) point out that a bridge must be built to close the gap between arithmetic and algebra. According to the authors, the building materials of closing the gap between arithmetic and algebra are conceptual understanding and the ability to perform arithmetic manipulation on whole numbers, decimal fractions and common fractions. For learners to be able to solve rational algebraic equations and simplifying algebraic fractions, learners must apply knowledge of generalised common fractional concepts (Brown and Quinn, 2007). This section has brought to light the relationship between common and algebraic fractions, it is, therefore, necessary to know the right time to introduce fractional concepts to learners. The next section discusses the right time to introduce fractional concepts.

2.1.5. When is the right time to teach common fractional concepts?

Research has revealed that learners must learn fractional concepts from lower grades if they are to succeed in topics related to fractions in the future. Wu (2001) pointed out that if we are to understand why fractions can be considered as having the potential to be the best kind of 'pre-algebra' (fractions should be done before algebra), it is important to consider the nature of algebra and what makes it different from whole number arithmetic (Wu, 2001; Brown and Quinn, 2007). Furthermore, Wu (2001) proposed that fractional concepts be taught at a young age. On the other hand, Brown and Quinn (2007) advocate postponing teacher-taught algorithms for working with fraction operations until high school. From their studies, they discovered that students who choose to use algorithms were unsure of the correct process and the fractional operations in general. According to Brown and Quinn (2007), if students are taught algorithms at a young age, they resort to memorisation rather than internalising and conceptual mastering the algorithm and the process. Therefore, it is critical to equip learners with hands-on skills to solve common fractions at lower grades (7 – 9) so that they can apply these skills in algebraic fractions. In the next section we peruse what happens in South African schools on the teaching and learning of common and algebraic fractions.

2.1.6. Studies done in South Africa on fractions

A study was done by Dhlamini and Kibirige (2014) in the Gauteng province where they examined the addition of fractions, which revealed that the majority of grade 9 learners were able to add fractions if they were numerical. However, the pupils could not perform well when they were algebraic. Furthermore, their findings revealed that when adding fractions, learners produce errors and reveal misconceptions as a result of shortcuts that they perceive as easier methods of adding fractions instead of following the apparent long and burdensome algorithmic correct method of adding fractions.

In a most recent study done by Baidoo (2019), the computation of algebraic fractions was found to be challenging to students, possibly because of the understanding of mathematical concepts like division, variable, equation, perfect squares, exponents, factorisation and rational numbers was required. Inadequacy in the understanding of the above concepts led to difficulties in solving algebraic fractions (Baidoo, 2019). Therefore, it is of paramount importance to research on proficiency in common fractions and understanding algebraic fractions.

Other studies conducted in South Africa have a different opinion on the success of understanding algebraic fractions by pupils. In a study done by Chamane (2016), Mathematics was considered to be a subject for the elite during the apartheid era in South Africa. Black children were discouraged to take Mathematics during apartheid since it was considered as one of the subjects that would create better opportunities for them. Also, Chamane (2016) highlighted that the few blacks who managed to do Mathematics were taught in such a manner that memorisation was the norm of the day rather than teaching for understanding. This could be the reason why most teachers who schooled during the apartheid era and chose Mathematics are arguably ineffective in the teaching of the subject. The focus of the study was on teachers' experiences when they teach fractions, how they teach fractions and why they have particular experiences. According to research, the way a teacher was taught influences how they also teach their learners (Oleson and Hora, 2014). If a teacher was made to believe that this is how a problem is solved without querying why it is done in that way, they will also use the same way to teach their learners. For example, in the teaching of fractions, learners are only told that when dividing fractions, you change the division sign to multiplication and invert the divisor. The reason why it is done that way is never explained to learners. Additionally, Chamane (2016) highlighted that most Mathematics teachers are not keeping abreast with new and more effective ways of teaching Mathematics and fractions, in particular. Further, the studies revealed that even though Mathematics was now made one of the most important subjects in the curriculum in South Africa, the effects of how it was taught in the apartheid-era still shapes how it is still taught today. The dislike of Mathematics and particular topics like fractions by learners still harm Mathematics pass rate in South Africa (Chamane, 2016).

Studies conducted by Hugo et al. (2010) and Spaull (2011) on primary school Mathematics teachers' content and pedagogic knowledge revealed that none of the teachers got 100% for a test on the content that they were teaching. A total of 24% of the participants got less than 50% and on average 47% managed to get test answers correct. Fractions were amongst the concepts in which pupils faced challenges. It is evident from the aforementioned studies that if a teacher is not conversant with fractions, they cannot successfully teach the concept to learners. Consequently, as articulated by many authors, there is concern about the poor background of primary school learners in basic Mathematics (Ubah and Bansilal, 2018, Siegler et al., 2013). This challenge is inherited in high schools as the teachers tend to assume that learners were fully taught the basics of Mathematics at the primary level and just plunge into rules and algorithms.

Another study done by Jooste (1999) has shown that primary school teachers harbour misconceptions about basic operations on fractions and can only work in an externally-driven manner on fractional operations. This could be another reason why learners have a poor background in fractional concepts. Arslan and Altun (2007) also noted that older learners are capable of applying "routine skills or algorithms" but do not have an understanding of the basic concept of fractions. To this effect, Chamane (2016) recommended that teachers must be guided by rationales in their teaching and learning of fractions. Further, Chamane (2016) went on to allude that there is a need for on-going professional development for Mathematics teachers so that they keep abreast with current and innovative teaching approaches, particularly for teaching fractions.

From the above discourse, it is evident that many black South Africans were not given the privilege to learn Mathematics during the apartheid era. Some of these individuals became primary school teachers where they were required to teach all subjects including Mathematics in which they were not taught problem-solving skills and basic mathematical concepts like fractions. Subsequently, the same individuals lacked pedagogical skills to deliver difficult concepts such as fractions as they were never taught those skills. Thus, a closer analysis of the study by Chamane (2016) revealed that these teachers cannot impart problem- solving skills because they are also deficient in the skill. However, this warrants further investigation.

2.1.7. Common errors in simplifying common fractions and algebraic fractions

Table 1 depicts the categories and the common errors in simplifying common and algebraic fractions as reported by Baidoo (2019), Makonye and Kanyile (2015) and Brown and Quinn (2007). The authors categorised the errors differently. However, an in-depth analysis revealed that some of the errors were similar. The most common error identified by the researchers was the lowest common denominator error where learners failed to find the correct LCD (Table 2.1a and 2.1b). Furthermore, Baidoo (2019), Makonye and Kanyile (2015) and Brown and Quinn (2007) mentioned the concept error as another common error where learners showed a lack of understanding on how to simplify fractions. Learners also failed to understand the concept of reducing fractions to their lowest terms by taking out the highest common factor (Table2.1a,2.1b and 2.1c). Brown and Quinn (2007) identified seven categories of errors, which were as follows: like term error, mistaken operation error, equationisation, partial division error, transforming a fraction into a non-fraction, failure to reduce fractions into their lowest terms and concept error after administering a test to students who had been specifically taught rational expressions as also shown in Table 2.1(b). Makonye and Kanyile (2015) came up with eight categories of errors (confusing the factors, no recognition of the common factor, unable to factorise a trinomial, lowest common denominator error, correct answer obtained by using incorrect mathematical rule, dropping the denominator, careless error, failure to take out the common factor) before and after they used their intervention of using probing as a scaffolding technique to overcome Mathematics errors (Table 2.1a). In a study done by Baidoo (2019), four categories of errors were identified as follows: mathematical language error, procedural error, conceptual error and application error after the learners were given a test (Table 1c). Most of the errors identified in Table2.1appeared to be a result of a lack of conceptual understanding of fractions. Therefore, the researcher decided to put the errors from the different authors under categories 1 to 6 (Table 2.1).

These common errors are among some of those that learners make when they simplify fractions. It is reported by some researchers that these types of errors are not dealt with by teachers because they do not realise that learners struggle to cope with the essential concept of reducing fractions to their lowest terms by singling out a common factor amongst other hitches (Laurillard, 2013). Furthermore, Vinner (2002) reports that students apply rote learning when simplifying fractions and rational algebraic fractions. In this study, the researcher will investigate whether learners who have mastered the concept of the LCD still make errors in adding and subtracting algebraic fractions.

2.1.8. Content pedagogical knowledge in teaching the solving of common and algebraic fractions

Mji and Makgato (2006) attributed learners' poor performance in Mathematics to teachers' lack of pedagogical content knowledge (PCK) as a result of teachers being under-qualified or unqualified to teach Mathematics. Research indicates that most teachers lack PCK to teach fractions effectively (Borko et al., 1992, Ma, 1999, Mewborn, 2001). Borko et al. (1992) and Ma (1999) concur that many teachers are unable to generate a word problem for a whole number divided by a fraction, often providing a problem that represents a multiplication situation. This shows the need for effective PCK on the part of the teachers. On another angle, according to Shulman (1986), PCK goes beyond subject matter knowledge. Shulman (1986) further stresses that PCK involves teaching for understanding and knowing what makes some topics easy and others difficult for learners. Therefore, teachers must improve on their PCK. Learners must be made aware that understanding Mathematics is to be able to "do" Mathematics and knowing why certain steps are done (Ball, 2003). Ball (2003) suggested that 'doing Mathematics' is the ability to solve mathematical problems without using procedures but instead by using multiple mathematical methods approach to a mathematical problem.

Jooste (1999) did a study using what the author called *Malati* project ideas. In the study, grade 3 and 4 teachers were helped to develop a classroom culture based on a problem-centred approach of learning through social interaction amongst learners in their activities to make sense of their own construction and that of others in the group. According to Jooste (1999), learning is a social activity in which learners are involved in discussing, inventing, explaining, negotiating, justifying, sharing, reflecting and evaluating. Research has also shown that children who are taught using a grounded curriculum approach that focused mainly on relationships between representations did much better on fraction tasks compared to those who were taught using traditional methods of teaching (Ubah and Bansilal, 2018). The current study will focus on the appropriate PCK to enhance the learning and teaching of algebraic fractions. Table 2.1a – c illustrate the common errors identified by different researchers in simplifying common and algebraic fractions.

Table 2.1a: Common errors identified by different researchers in simplifying common and algebraic fractions.

Category					**Reference**
1	**3**	**4**	**5**	**6**	
a) Confusing the factors $\frac{4x^2+16}{x^2+4}$ $=\frac{4(x^2+4)}{(x+2)(x+2)}$ $=\frac{4(\cancel{x+2})(\cancel{x+2})}{(\cancel{x+2})(\cancel{x+2})}$ $= 4$ b) No recognition of the common factor. $\frac{\cancel{a^2b}-\cancel{ab}}{\cancel{a^3}+a^{\cancel{2}}}\times\frac{\cancel{a^3}-\cancel{a}}{\cancel{a^2b}-2\cancel{ab}+b}$	a) Unable to factorise a trinomial $\frac{3}{x^2+6x+9}-\frac{2}{x^2-9}-\frac{1}{x^2-6x+9}$ $=\frac{3}{(x+3)(x+3)}-\frac{2}{(x+3)(x-3)}-\frac{1}{(x-3)(x+3)}$ b) Lowest common denominator error e.g $\frac{3}{x^2+6x+9}-\frac{2}{x^2-9}-\frac{1}{x^2-6x+9}$ $\frac{3}{(x+3)(x+3)}-\frac{2}{(x+3)(x-3)}-\frac{1}{(x-3)(x+3)}$ LCD $=(x+3)(x-3)$ $=(\cancel{x+3})(x-3)\ \frac{3}{(\cancel{x+3})(x+3)}-$ $(\cancel{x+3})(\cancel{x-3})\frac{2}{(\cancel{x+3})(\cancel{x-3})}$ - $(\cancel{x+3})(\cancel{x-3})\frac{1}{(\cancel{x-3})(\cancel{x+3})}$	a) Correct answer obtained by using incorrect mathematical rule. $\frac{4x^2+16}{x^2+4}$ $\frac{4(x^2+4)}{(x+2)(x+2)}$ $\frac{4(\cancel{x+2})(\cancel{x+2})}{(\cancel{x+2})(\cancel{x+2})}\quad = 4$ b) Dropping the denominator LCD=$x+3$ $\frac{2}{x+3}-\frac{1}{(x+3)^2}$ $=\frac{2}{x+3}-\frac{1}{(x+3)(x+3)}$ $= x+3(2)-+3(1)$ $=2x+6-x+3$ =1x +3 $= x+3$	Careless error $\frac{\cancel{x}}{\cancel{x}}+\frac{1}{x}=\frac{1}{x}$	Cancellation error e.g $\frac{\cancel{a^2}-\cancel{ab}}{\cancel{a^3}+a^{\cancel{2}}}\times\frac{\cancel{a}^3-\cancel{a}}{\cancel{a^2b}-2\cancel{ab}+b}$	Makonye and Kanyile, 2015

Table 2.1b: Common errors identified by different researchers in simplifying common and algebraic fractions.

Category				Reference
2	**3**	**4**	**6**	
(a)Like term error $\frac{2x-6y}{4x+6y} = \frac{2x-4x}{6y+6y}$ b) mistaken operation error $3\frac{1}{x-3} = \frac{3}{x-3} + \frac{1}{x-3}$ c) Equationisation $\frac{1}{2}+3$ learners will write $\frac{1}{2}+3=0$	Partial division error $\frac{3x+2}{x+2} = \begin{array}{r\|l} & 3 \text{ r } 8 \\ \cline{2-2} x+2 & 3x+2 \\ \cline{1-1} & -(3x+6) \\ & 8 \end{array}$ $= 3 + \frac{8}{x+2}$ Instead of the following $\frac{3x+2}{x+2} = \begin{array}{r\|l} & 3 \text{ r } (-4) \\ \cline{2-2} x+2 & 3x+2 \\ \cline{1-1} & -(3x+6) \\ & 8 \\ & -4 \end{array}$ $= 3 - \frac{4}{x+2}$	Transforming a fraction to a non-fraction e.g. $\frac{1}{3}$ becomes 3 or $\frac{2}{x}$ becomes $2x$.	Cancellation error $\frac{2(\cancel{5})}{3(5)}, \frac{8\div4}{12\div4}$ and $\frac{2(3x+2)}{2(x+y)}$ will be the same as $\frac{2}{3}, \frac{8}{12}\ and\ \frac{3x+4}{x+y}$ respectively will not be the same as $\frac{5+3}{7+3}, \frac{5-2}{8-2}$ and $\frac{4a+3}{4b+3} = \frac{5}{7}, \frac{5}{8}$ and $\frac{a}{b}$ respectively	Brown and Quinn, 2007

Table 2.1c: Common errors identified by different researchers in simplifying common and algebraic fractions.

Category				Reference
1	**2**	**3**	**4**	
Mathematical language error	Procedural errors	Conceptual errors	Application error	
$\frac{x \times x}{x} = \frac{x \times 1}{x}$ $= \frac{x \times 1}{1}$ $= x = 1$	$\frac{3x+9}{9+3x^2}$ $= \frac{3(x+3)}{3(3+x^2)}$ $= \frac{x+3}{3+x^2}$ $= \frac{x+3}{(3-x)(3+x)}$ $= \frac{1}{3-x}$	$\frac{x^2-1}{x^2-2x+1}$ $= \frac{x^2\ (1+(-1)}{x^2(-2x+1)}$ $= \frac{1+(-1)}{-3}$ $= \frac{0}{-3}$ $= 0$	$\frac{3x+9}{9+3x^2}$ $= \frac{3(\ x+3\ )}{3\ (3+x^2)}$ $= \frac{x+3}{3+x^2}$ $= 3^{1-1}.x^{1-2}$ $= 3^0.x^{-1}$ $= x^{-1}$ $= \frac{1}{x}$	Baidoo, 2019

2.1.9. What is fractional proficiency?

Teaching fractions by applying rules without a sound understanding of why we are using those rules can be detrimental (Pienner,2014). Usually, teachers teach learners to memorise rules instead of honing for proficiency in fractions. According to Cowan et al. (2011), proficiency is an advancement in knowledge or skills. Therefore, fractional proficiency will mean the ability to perform efficiently the basic operations involving fractions namely the addition, subtraction, multiplication and division of fractions as well as understanding the "whys" of the process (Thomas, 1992). It is not merely procedures with fractions that the learners must master but the concept of what the fraction means (Thomas, 1992). For one to be proficient in fractions, one must have both a procedural and conceptual understanding of fractions. To this effect, the researcher used a model of mathematical proficiency (MP) by Kilpatrick et al. (2001). According to the authors, there are five strands of MP which are conceptual understanding (CU), procedural fluency (PF), strategic competence (SC), adaptive reasoning (AR) and productive disposition (DP). The current study focuses only on two of the strands that are: CU and PF. The reason is that fractions are part of Mathematics, which requires a learner to have both CU and PF to master them successfully. According to Kilpatric et al. (2001), CU and PF are needed for learning Mathematics successfully and connecting the two strands helps learners to remember, use and reconstruct those ideas when needed. Fractions are one such topic that needs a good knowledge of CU and PF to be fractional proficient. For learners to build a strong CU and PF, learners must have a learning experience that provides them with connected ideas (Kilpatrick et al.,2001). Learners must see the connection between related concepts, for example, the relationship between common fractions and algebraic fractions. In short, CU can be seen as knowing why certain procedures are done in Mathematics while PF can be seen as knowing when and how certain procedures are done.

Jooste (1999) reiterated that learners should be allowed to use their intuitive knowledge to solve problems and fractions so that they can develop a sound concept of fractions over a period of time during which each learner's method should be valued and appreciated. In his study, Jooste (1999) intended to determine the outcomes of allowing learners to engage with problems where they have to construct their own fractional concepts involving equal sharing. According to the study, the rationale for fraction introductory materials has its origin based on the fact that grade 1 learners should be able to make sense of fractional problems with frequent exposure to such problems. This is possible if such concepts are regularly revisited and developed through problem-solving and not via demonstrations, rules, recipes or definitions. In other words, formal methods should be deferred for as long as possible (Jooste, 1999). The Curriculum and Assessment Policy Statement (CAPS) curriculum in South Africa only allocates four hours for teaching fractions at grades 8 and 9 respectively. This will not give enough time for learners to develop a sound concept of fractions given the small time period of learning and also considering their poor background of fractions from the primary level. Researchers also alluded that fractions require much attention because they present a hurdle as learners attempt to transfer their understanding of whole numbers to a new but a related class of numbers (Siegler et al., 2012, Chinnappan and Forrester, 2014).

2.1.10. Additional factors contributing to learners' understanding or misunderstanding of fractions

Hecht et al. (2007) mentioned a factor overlooked by many researchers regarding the understanding of fractions. The authors maintained that misunderstanding of fractions is not only mental in nature, but that the way a learner behaves also impacts negatively on a learners' understanding of Mathematics and more specifically fractions. Attitude towards the learning of fractions can contribute to a lack of understanding, thus Thomas (1992) pointed out that if a learner feels that it is a waste of time to follow the long method of finding the LCD when adding and subtracting fractions, he or she will end up using a calculator, which makes them get the answer without any effort. Mhakhure et al. (2014) bring up the point that learners do not like learning fractions since they feel that fractions are irrelevant in their daily lives because historically, fractions were not considered as numbers but rather were just used as a way to compare whole numbers. Learners' backgrounds and cultures also affect learners' understanding of fractions (Preciado, 2016). If a learner comes from a home where there are no shapes and patterns that depict fractions, then, it is a struggle to make them understand that a fraction is a part of a whole. If a teacher gives learners an example of sharing a pizza or a cake and the learner has never seen a cake or a pizza, it does not make sense to him or her.

2.1.11. Interventions in the teaching of fractions

Having looked at factors contributing to learner's misunderstanding in the learning of common and algebraic fractions, it is vital to look at intervention strategies that can be used to alleviate these misunderstandings. A teaching intervention is a programme or set of steps to help kids improve at skills they struggle with (Baker et al., 2002). Makonye and Kanyile (2015) used probing as a teaching intervention to understand the errors that learners made in simplifying algebraic fractions. The authors believed that probing learners on the errors they make helps them resolve the errors on their own. Makonye and Kanyile (2015) used two types of probing which they called funnelling and revoicing. Chen (2013) alluded that this kind of teaching concerns a teaching process with practice consciously guided by thinking, inspiring teachers to teach more effectively. Makonye and Kanyile (2015) further mentioned that the probing method consists of a discussion between the teacher and the learner and encourages reflection and critical thinking about a given task. One important fact about this strategy is that learners and teachers must admit when they find out that their initial position was incorrect (Makonye and Kanyile, 2015). Tirosh (2000) also explains that a class is a learning community in which each member, including the teacher, learns with and from others. The teacher needs to listen to what learners come up with because it helps on ironing out misconceptions and sometimes learners come up with good ways of solving problems.

Collaboration is another strategy that can be used to improve the teaching and learning of fractions (Martin-Stanley and Martin-Stanley, 2007). Collaboration underpins the theory of constructivism. According to the constructivist perspective, there must be a collaboration between learners and also between the teacher and learners for meaningful learning to take place (Vygotsky, 1978). The theory highlights learner-centred learning and learners are responsible for creating knowledge. Van Steenbrugge et al. (2015) mentioned that using collaboration and showing relevancy to real life when teaching and learning fractions can deeply impact learners' abilities to learn abstract concepts. Martin-Stanley and Martin-Stanley (2007) further affirm that teachers must work alongside learners and become partners in forming new knowledge.

The way a teacher questions learners in the classroom has an impact on the learning of Mathematics and can be used as an intervention strategy to improve understanding of difficult concepts in Mathematics like fractions. Ntuli and Godfrey (2018) state that while teacher questioning is an important part of the mathematical discourse, if not implemented correctly, it can affect learner discussions and can be damaging to self-confidence and growth in their mind set. They also claim that most teachers use funnelling questions more than focusing. Ntuli and Godfrey (2018) described funnelling as a type of questioning where the teacher decides on a particular path for the discussion to follow and leads the learner along that path, not allowing learners to make their own connections or build their own understanding of the targeted mathematical concepts. Some learners become discouraged by the way teachers use funnelling prompts; they end up giving up. On the other hand, the focusing technique is described as a pattern of questioning that involves the teacher attending to what the learners are thinking, pressing them to communicate their thoughts clearly and expecting them to reflect on their own thoughts (NCTM, 2014). The topic of fractions in Mathematics needs the teacher to use good questioning techniques such as the focusing technique that can lead to reflection and justification of one's way of thinking. McCarthy et al. (2016) suggested that teachers who can question appropriately and effectively at various levels of Bloom's taxonomy are better able to discern the range of learner's thinking. From the aforementioned discussion on mathematical discourse, it is clear that if teachers use the correct questioning techniques in topics like fractions and algebraic fractions, learners may end up arriving at answers on their own.

While the above interventions can improve learners' conceptual understanding of fractions teachers can also contribute to learners' difficult in constructing fractions. This happens when the teacher is not well versed with all the definitions of fractions (Park et al., 2012). Learners need to understand a fraction as a: part of a whole, ratio, number, operator, set of objects relationship, representation of division and a unit of measurement otherwise without all these definitions learning the concept of fractions becomes an uphill. The explanations are illustrated in Table 2.2. If learners are not exposed to the holistic picture of fractions this limited understanding interferes with understanding more complex algebraic fractions. Park et al. (2012) further reported the dominance of part-whole interpretation in students' and teachers' thinking about fractions and their failure to conceptualize them as an extension of whole numbers.

Table 2.2: Definition of fractions and their examples

	Fraction use (X)		Description/example (Y)
A	As a ratio	1	For every two boys there are four girls
B	As a number	2	$\frac{2}{3}$ is more than 0 but less than 1
C	As an operator	3	$\frac{3}{4}$ means multiplying by 3 and dividing by 4
D	As part of a set of objects	4	$\frac{2}{5}$ of the marbles are black
E	As a relationship	5	Jim earns a half of what his sister earns
F	As representation of division	6	$\frac{5}{8}$ means $5 \div 8 = 0{,}625$
G	As part of a whole	7	One third of a fruit cake
H	As a unit of measurement	8	Two-fifths of a kilogram

2.2. Theoretical framework

This study was guided by the theory of constructivism in trying to explain the errors and misconceptions that learners display when simplifying algebraic fractions and also by Skemp's notions of understanding, which are, instrumental and relational.

Constructivism is an epistemology that states that learning Mathematics entails that learners actively create, interpret and reorganise knowledge in individual ways (Lee, 2009). Constructivism also emphasises on the learners' prior knowledge for learning of new concepts to take place. Relating this to the learning of algebraic fractions, learners should master the concept of common fractions before they can engage with algebraic fractions. Conceptual understanding of mathematical concepts proceeds primarily from learners' prior knowledge. The more connections a learner can make between new concepts and older ones, the easier it is to remember the new concepts being learnt. A constructivist perspective also claims that mathematical concepts are not mastered directly from experience but that the learners' ability to learn from experience depends on the quality of the learner's prior knowledge. According to the social theory of constructivism by Vygotsky (1978), when a learners' prior knowledge is increased to a higher level, it is called the zone of proximal development (ZPD). This is the difference between what a learner can do using his or her prior knowledge without help and what he or she can do with help from a teacher or more experienced peers. The ZPD theory requires that the teacher provides a lot of scaffolding as the level of challenge increases. In learning algebraic fractions, the researcher provided learners with scaffolds and gradually removed them so that they can solve the algebraic fractions independently. Models were used as scaffolds in the simplification of common fractions and then rules and algorithms were later used.

The above discussion brings to light that prior knowledge is essential to build a deep understanding of a concept and through the ZPD theory scaffolds are needed for learners to have a more solid foundation of concepts. Skemp (1976) claimed that there are two types of mathematics understanding that are classified as instrumental and relational understanding. The author further argues that instrumental understanding is using rules without reasons, which can be regarded as the absence of conceptual understanding (Kilpatrick et al., 2001). On the other hand, Skemp (1976) defines relational understanding as knowing both what to do and why. It can further be argued that when mathematical conceptual understanding does not occur as the teacher had planned, a learner resorts to mathematical procedures (Kilpatrick et al., 2001).

Skemp (1976) came up with two situations that are referred to as a mismatch. These two situations are as follows: the author claims that the first mismatch happens when a teacher wants learners to understand relationally but learners' goal is to understand instrumentally. The second mismatch occurs when learners want to understand relationally but the teachers' goal is for them to understand instrumentally. In the first situation, learners do not bother if they understand the concept; all they want is to get to the answer faster with no difficulties. That is the reason why learners tend to use calculators in solving common fractions and other shortcut methods, which get them to the answer easily and faster. However, when it comes to algebraic fractions, the calculator can no longer be used. Relational understanding will be now needed and thus they will get wrong answers. It will be now essential for teachers to use their PCK in addressing such situations to empower learners with mathematical concepts for relational understanding.

In the second situation, Skemp (1976) argues that it is more detrimental when a teacher teaches instrumentally while learners require a relational understanding of Mathematics concepts. This kind of situation can be the source of mathematical misconception (Kanyile, 2016). Learners who have a relational understanding of concepts can tackle any situation because they understand the concept. Looking at algebraic fractions, if learners understand that addition and subtraction of common fractions and the lowest common denominator (LCD) are important, then they can apply the same concept in algebraic fractions.

2.3. Conclusions and knowledge gaps

The topic of fractions is a very important part of the Mathematics curriculum in South Africa and the world over. Fractions feature in most Mathematics topics and impact negatively on Mathematics results if they are not mastered well. This review established that if fractions were not mastered at lower grades, it becomes hectic to master them at high school because of the crowded nature of the curriculum. The literature also revealed the common errors that learners do in the simplification of algebraic fractions and the intervention strategies that can be used to reduce those errors. The need for teachers to continue professional development for teachers to keep abreast with newer and more innovative ways of teaching fractions and Mathematics in general featured in some of the studies, thus there was a need to embark on studies to look for newer and better ways of teaching algebraic fractions. Most studies reviewed in the current literature review were conducted in an urban setup with better infrastructure and technology. It is interesting to investigate the topic in a rural school with small infrastructure and less technology. Studies conducted concentrated on more complex algebraic fractions, which needed the application of other concepts like factorisation. However, none has focused on the arithmetics of simple algebraic fractions. No studies have been conducted to show whether a learner who has mastered the arithmetics of common fractions will fail to simplify simple algebraic fractions before proceeding to the complex problems, hence there is still a need to investigate. Furthermore, most studies used quantitative and qualitative research designs and under correction, no study in South Africa has used the action research design.

CHAPTER 3

RESEARCH METHODOLOGY AND DESIGN

3.0. Introduction

This chapter discusses the research paradigm, research methodology and research design as well as how data were collected and analysed to answer the research questions posed for the study. The main objective of this research study was to identify the barriers and challenges that learners encounter when learning algebraic fractions and to determine the relationship between learners' proficiency in common fractions and their performance in algebraic fractions. The pedagogical strategies that can be used to reduce the barriers and challenges were also investigated.

3.1. Research paradigm

The research was conducted within the pragmatist paradigm. A paradigm is a set of basic beliefs that guides the actions and defines the worldview of the researcher (Lincoln et al., 2011). The philosophical foundations of the pragmatism research paradigm are embedded in the historical contributions of the philosophy of pragmatism (Maxcy, 2003) and embrace a variety of methods. The researcher used a methodology that worked best for the research questions that used both qualitative and quantitative methods in data collection. Hence, the pragmatist paradigm was suitable since it is based on the proposition that researchers should use the methodological approach that works best for the particular research problem that is being investigated (Tashakkori et al., 1998). Pragmatists also favour working with both quantitative and qualitative data because it enables them to understand social reality (Tashakkori et al., 1998). Therefore, this research used pragmatism as a paradigm since it used both quantitative and qualitative data. The fact that pragmatists also accept that there can be single or multiple realities that are open to empirical inquiry made it a more suitable paradigm for this research.

3.2. Research methodology

A research methodology is a philosophical framework within which the research is conducted or the foundation upon which the research is based (Venkatesh et al., 2013). A research methodology should meet the following two criteria: first, the methodology should be the most appropriate to achieve the objectives of the research and second, it should be made possible to replicate the methodology used in other researches of the same nature (Venkatesh et al., 2013). There are three types of research methodologies namely qualitative, quantitative and mixed-methods. Qualitative research focuses on collecting and analysing data in form of words or narratives while quantitative research describes, infers and resolves problems using numbers (Tracy, 2010). The mixed-method methodology combines the best of both qualitative and quantitative methodologies to integrate perspectives and create a rich picture (Wisdom et al., 2012). The nature, purpose and other attributes of the study are used to decide which methodology to use in a particular research. The research methodology for the current study was guided by the research design chosen. This current study used a mixed-method methodology because action research allows both qualitative and quantitative methods to be use in data collection.

3.2.1. Action research design

The nature of the current research renders the study suitable for the action research approach because the researcher was seeking new knowledge and understanding about how to improve teaching practices in general and, in particular, how to resolve significant problems associated with solving fractions in the classroom. Action research is a structured design process in which teachers identify, examine and improve aspects of their teaching (Smith and Sela, 2005). The essence of action research is used by teachers in solving everyday problems in schools to improve student learning and teacher effectiveness. The researcher was interested in improving the teaching and learning of fractions in schools. Mertler (2019) outlines the basic process of action research design with four steps: a) Identifying a problem in one's teaching or students' learning, b) collecting data, c) analysing and interpreting the data and d) developing a plan of action for implementation. Action research design allows the teacher to use qualitative and quantitative methods in data collection and analysis. For instance, a teacher can use qualitative methods such as one-on-one interviews, focus group interviews and observations to collect data that is used to describe what is happening and to understand the effects of some educational intervention (Mertler, 2019). The researcher can also collect quantitative data that require analysis that include statistics, inferential statistics or both (Mertler, 2019). It is important to note that there are two types of action research namely exploratory action research and mixed methods action research approach (Creswell, 2002). This study used mixed methods action research approach to give a voice to participants and ensure that the study findings are grounded in participants' experiences.

In this study, the first phase involved the collection of quantitative descriptive data using a pre-test instrument that comprises both closed and open-ended questions. The second phase of the study allowed the comparison of quantitative data collected in phases one and two. The action plan included the implementation of sequential lesson plans (Appendix 1, LAP1 to LAP5) that used four strategies to teach common fractions. At the end of the second phase, data were collected from learners using a post-test instrument that comprised closed-and open-ended questions (Appendix 2).

The third phase of the study involved triangulation of the results using qualitative data collected through qualitative interviews with teachers and learners. The qualitative data provided complementary findings or filled in the gaps from the information gathered from the post-test instrument. In the current study, the instrument used for quantitative data collection does not probe deeply into participant's opinions (Gall et al., 1996), therefore, interviews with the learners and teachers were used to fill in the potential explanatory gap in the data. Additionally, the fact that some of the research questions asked required to collect information on "what" and "how" questions suggests that a qualitative approach would be useful in establishing a rich data set that would complement data from the pre-and post-test instrument. Yin (2003) notes that in general, qualitative approaches are preferred when "how", "what" or "why" questions are posed when the investigator has little control over the events.

3.2.2. Instrumentation

For the purpose of this study, the researcher designed a pre- and post-test questionnaire (Appendix2, PTQ1 and PTQ2). The pre-test questionnaire was administered by the researcher to the study participants before the implementation of a unit on fractions while the post-test questionnaire was administered after the implementation. The pre-test and post-test questionnaires had three sections: the demographic (identification and student characteristics) section with 18 questions, the disposition and perception section with 8 questions and the problem-solving section with 12 questions (five questions on solving common fractions and seven questions on solving algebraic fractions). The disposition questions were adapted from Cai et al. (2012) who designed a survey instrument to study 9^{th} grade learners' mathematical dispositions. It is important to note that the pre-and post-questionnaire contained similar questions. The only difference was the numerical values used in the questions. For instance, in pre-test, the researcher could use $\frac{5+3}{7+3}$ and in the post-test instrument, the researcher could use $\frac{3+7}{3+5}$. This type of pre-and post-test allowed the researcher to do comparisons that show mastery and growth between pre- and post-test.

This pre-and post-questionnaire carried both open- and closed-ended questions. Each response to the closed-ended questions on disposition was associated with a point value, for example, (Yes=1, NO=0). The closed-ended questions enabled the researcher to describe and compare participants' responses or reactions to how they practice fractions and how they feel about Mathematics (Creswell, 2002). The numeric values that were assigned to the pre-test responses on closed-ended questions allowed the researcher to compute frequency tables and do cross-tabulations. Interview questions were developed based on the gaps found in the literature. Open-ended questions required the learner to demonstrate and comment on the procedure that they took to solve the common and algebraic fractions. Spaces were provided under each open-ended question for participants to write responses, that is, participants were asked to solve fractions providing detailed procedures on how they arrive at the answer. The qualitative data collected through open-ended questions probed deeply and explored the participant's performance, perspectives and experiences with fractions.

After administering the pre-test data instrument, the researcher performed a preliminary analysis of the results and implemented a sequential lesson plan on fractions that used four strategies on Mathematics teaching practices (NCTM, 2014). For a description of the Mathematics practices, see Appendix 1, LAP6. After the implementation of the sequential lesson plan, the researcher administered the post-test data instrument. Subsequently, after writing the post-test, the researcher interviewed participants based on the performance achievement on the post-test. The researcher used time after school to conduct the interviews.

This study also collected and used qualitative interview data from Mathematics teachers of grade 10 learners. Themes from the interview data with teachers were compared with what the research found in the analysis of the themes from interview data with the learners. This type of triangulation of data helped strengthen the study. Detailed procedures, benefits and time requirements of the study were described in the consent forms (Appendix 3).

3.2.3. Study population and sample population

Polit and Beck (2006) describe a population as comprising all possible elements that could be included in a research study. A sample, on the other hand, is a subset taken from a population of interest (Cohen et al., 2007). In this study, the population will constitute all grade 10 learners at one high school in Seleka taking Mathematics and grade 10 Mathematics teachers around Seleka in Limpopo province.

3.2.4. Sampling

In this study, participants were selected using purposeful sampling. Many authors recommend the use of purposeful sampling in research because the sampling procedure is based on the assumption that the investigator wants to discover, understand and gain insight, therefore, must select a sample from which the most information can be learnt (Merriam, 1998). Purposeful sampling was, thus, used to produce quality data directly related to answering the research questions. Patton (2002) noted that the logic and power of purposeful sampling lie in selecting information-rich cases for an in-depth study. Merriam (1998) contends that the criteria of purposeful sampling directly reflect the purpose of the study and guide the identification of information-rich cases. Information-rich cases are those from which one can learn a great deal about issues of central importance to the objective of the inquiry (Patton, 2002). For this study, the researcher purposefully targeted a class of grade 10 learners who were taking Mathematics since grade 10 is a bridge between the SP and FET phases. The sample contains all the 26 participants from the study population and hence becomes a 100% sample since the participants were too few. Most statisticians argue that if a population is less than 100 participants, then use all the subjects. It is important to note that the number of learners doing Mathematics in our schools in the district is between 15 – 30 learners per class in grades 10 – 12. Participants were recruited from one public high school located in Seleka, Limpopo Province in the summer of 2020. In the current study, the sample also comprised 5 teachers around Seleka who were purposeful recruited because they were within the researcher’s circuit. The researcher handed recruiting letters and consent forms to the teachers at their schools in summer 2020 (Appendix 3, PL5). Only teachers of grade 10 learners were purposefully selected for interviews. To be more specific, typical sampling, which is one type of purposeful sampling was used to select participants for interviews. Merriam (1998) notes that a typical sample would be selected because it reflects the average person, situation or instance of phenomenon of interest.

3.2.5. Ethical considerations

Research ethics is concerned with the protection of the rights and interests of research participants (Dowling, 2000). According to Marianna (2011), the researcher is ethically responsible for protecting the rights and welfare of the subjects who participate in a study and this involves issues of physical and mental discomfort, harm and danger. Therefore, the researcher should ensure that the planned research is ethically accountable, that is, it conforms to acceptable norms and values. The researcher obtained an ethics clearance certificate from UNISA (Appendix 3, PL6). Once an ethical clearance certificate was granted, the researcher sought permission from the Department of Education through the circuit manager (Appendix 3, PL3). The researcher then sought permission from the principal of the school, parents and last, participants in that order. This was done since human subjects were used as part of the study. Then, the researcher contacted the participants with recruitment letters (Appendix 3). The recruitment letters were issued together with consent forms (Appendix 3) for both learners and teachers.

During the first phase of the study (pre-test phase), the researcher handed permission letters and consent forms to 26 learners. The researcher instructed the students to take the consent forms and have their parents/guardians sign if they wanted them to participate in the study. Then, the researcher collected the consent forms and allowed only those who were granted permission by their parents/guardians to take the pre-test before the beginning of the unit on fractions.

To maintain researcher and participant confidentiality, the learner responses and scripts for pre-test and post-test and interview script files were kept under lock and key in the cabinet located in the researcher’s office. The researcher was aware of the risks that might take place if the files were made public. For instance, if data about how learners performed were to be released, it could be embarrassing and one’s confidence could be affected to have other learners read about how little they know about fractions. To ensure security, the only person who had access to all the files was the researcher. As a researcher, it was important to ensure that the information provided by participants did not reveal their identity to a third party, as supported by (Kimmel, 2009). Participants were also ensured that they would remain anonymous by not writing their names on the answer script. Data collected was not linked to participants' names; instead analysis codes were used.

After using the pre-and post-test responses and interview files, the researcher will destroy all data files after two years. The researcher was honest with the reporting of procedures and findings and ensured that there was no misrepresentation of data including misrepresentation by omission.

3.3. Validity and reliability

3.3.1. Validity

Content validity is the degree to which a test measures an intended content area or the extent to which a measurement reflects the specific intended domain of content (Carmines and Zeller, 1991, Gay and Airasian, 1996). In this study, the instrument was reviewed by the researcher's supervisor and two other Mathematics education experts for content validity. Using recommendations from the supervisor and experts, changes were made to reflect what the pre-and post-test instruments measure what they were intended to measure.

The researcher used interpretive validity, which refers to the accuracy of presenting the inner worlds (DeLuca, 2011). One method that was used to establish interpretive validity was the use of low inference descriptors in which the actual language of the participants from the verbatim interview scripts was used to describe the phenomena (Johnson, 1997). In this study, member checking in which participants were asked to determine if the written scripts (transcribed data) reflected their actual responses was another way employed to ensure interpretive validity (Merriam, 1998).

3.3.2. Reliability

To ensure reliability, the researcher conducted a pilot study of the pre-and post-test instrument with a convenience sample of learners who were not to be part of the study. The pilot study helps to determine that the individuals in the sample are capable of completing the pre-and post-tests and that they can understand the questions (Creswell, 2008). Also, to ensure the reliability of the study, the researcher employed corroboration. The purpose of corroboration was to help researchers increase their understanding of the probability that their findings will be seen as credible or worthy of consideration by others (Stainback and Stainback, 1988). One process involved in corroboration was triangulation. Denzin (2017) identified several types of triangulation. One type involves the convergence of multiple data sources. The researcher used a variety of techniques important in mixed-methods study. Since this study had multiple data sources, the researcher employed the convergence of multiple data sources where the researcher used information from the pre-and post-test data and interviews to study the problem. The researcher also employed investigator triangulation where the cross-examination of the data was conducted by asking the participants to verify if the written interview data scripts represented their original responses as advised by Denzin (1988). This type of triangulation was important in that it helped to ascertain the consistency and reliability of data collected from participants.

Since the data collected in the third phase was purely qualitative, the researcher ensured credibility by utilising Guba’s Model of Trustworthiness discussed by Edmonson and Irby (2008). Edmonson and Irby (2008) identified four components of trustworthiness: truth value, applicability, consistency and neutrality. From another angle, Johnson (1997) argued that when qualitative researchers speak of research validity, they are usually referring to qualitative research that is plausible, credible, trustworthy and therefore, defensible. These characteristics can be demonstrated by a variety of techniques to ensure trustworthiness. First, internal consistency in the interview questions was exercised through pilot testing the questions with learners and one convenient teacher who was knowledgeable in the area of study. A check on the internal consistency within the interview is one way for establishing trustworthiness in truth value. Second, reflexivity was used and this minimised researcher bias. Reflexivity is a strategy employed in which the researcher reflects on his/her influence in the study and provides opinions when appropriate (Creswell, 2005; Johnson, 1997). The researcher analyses and reflects on her influence in the study to determine whether the responses given by participants were unbiased because of the researcher’s influence. To be more specific, the researcher reflected on the probing questions to determine if participant’s responses and line of thought were not led by the way the researcher phrased the probing questions.

3.4. Data analysis procedures

In this study, the data analysis procedure was conducted in line with the research questions and data collection procedures. The variables that were considered in the analysis are shown in Appendix 2, PTQ1 and PTQ2 and include learner's demographic information such as age, gender and distance from home to school in Part A, learner's disposition section shown in Part B, that is, learners attitudes and perceptions towards fractions, which consist of closed and open-ended questions. Part C of the instrument which is the problem-solving section was also open-ended and the last part on commenting. Coding was done on these open-ended questions to find themes.

Data collection was done in two phases. First, a pre-test was administered and a preliminary data analysis was done during data cleaning. Data cleaning involves running frequencies to identify errors that might have occurred during data capturing and outliers. Data cleaning is the process of removing invalid data points from a data set. Sequential lessons followed on common and algebraic fractions after writing the pre-test (Appendix 1, LAP1 to LAP5). Then, a post-test was administered and contained similar sections to the pre-test except for the time-invariant variables such as demographic characteristics of the learner. Participants were required to answer only sections on which their responses differed from those of the pre-test, that is, the disposition and problem-solving sections. The same process of data cleaning was done on the post-test as above by running frequencies on closed-ended questions and coding the open-ended responses to find themes.

After preliminary data analysis based on the pre-and post-test, the researcher identified gaps and a follow-up was made through interviews. Qualitative data analysis was done by coding, categorising and looking for recurring themes. Caudle (2004) simplifies codes as labels that assign themes to the evaluation data. Saldaña (2009) explains a 'code' as a word or short phrase that symbolically assigns a summative, salient, essence-capturing and or evocative attribute for a portion of language-based or visual data. The assignment of codes as noted by Merriam (1998) is a continual and emergent process throughout the conduct of the study. During the process of coding, the researcher identified categories and subcategories of information about the phenomenon being studied. The researcher wrote analytical memos on the categorised data. Saldaña (2009) reports that even after you have coded a portion of your data and categorised the codes into various lists, analytical memo writing serves as an additional code-and category-generating method. This type of coding was used by the researcher in this study.

Initially, data analysis comprised descriptive statistics like frequencies, percentages averages, standard deviations, range and median to characterise the sample. Descriptive statistics were used to organise and describe the characteristics of a collection of data (Salind, 2008). Then, this was followed by a comparison of observations from the pre-test and post-test scores. The dependent samples t-test was used to determine whether there was a significant difference in performance resulting from the intervention by comparing the pre-test and post-test results. We used the dependent sample t-test because we were comparing performance for the same individual before and after intervention measured using the pre-and post-test respectively. For this study, the variable of interest was a performance, which was measured by the scores attained in the pre-and post-test. Ordinary least squares regression analysis was used to eliminate other factors as reasons for the difference in performance, that is, to determine whether the difference in performance was as a result of the intervention or other factors. The dependent variable was performance and the independent variables were the characteristics of the student. We hypothesised that the socio-economic variables were less important in explaining the gap between pre-and post-test scores. The researcher also looked for independent ideas bearing in mind that the data collection technique used semi-structured interviews that were less structured formats that assumed that the participants defined the world in unique ways (Merriam, 1998). Unique ideas were given special attention. In this respect, semi-structured interviews provided a deeper understanding of participants' perspectives and produced rich data, which could be analysed and inform practice.

3.5. Strengths of the study

The strength of this study lies in the fact that the researcher used different forms of data and that the triangulation of data established the validity and reliability of the study. This action research design captured the best of both quantitative and qualitative data (Creswell, 2008) because the researcher obtained quantitative results from the population in the first and second phases and then refined or elaborated the findings through an in-depth qualitative exploration in the third phase.

3.6. Limitations of the study

Marshall and Rossman (2014) note that all proposed research projects have limitations. The researcher used purposeful sampling during data collection. Purposeful sampling is viewed by some researchers as biased (Morse and Richards, 2002) even though many researchers justify purposeful sampling based on the assumption that the investigator wants to discover, understand and gain insight, therefore, must select a sample from which most of the information can be learnt (Merriam, 1998). The researcher purposely focused on grade 10 learners at one high school only.

The fact that only those who were willing to participate were part of the study was one limitation beyond the researcher’s control. Closely connected to this was the honesty of participants. Some participants might not be honest when responding to interview questions and this could have affected the findings.

Since this study only focused on learners from one high school, it would be important for other studies to continue to explore the same phenomenon using a larger sample size with students from different high schools in South Africa. The qualitative data added depth to this study but did not provide for generalisation.

3.7. Conclusion

This chapter provided a broad description of the research design and methodology. Research instrument procedures used to collect data, sampling, population, validity, reliability and ethical considerations were described in detail. The next chapter focused on data analysis in the study.

CHAPTER 4

ANALYSIS OF RESULTS

4.1. Introduction

This chapter presents the analysis of the results based on the research aim, objectives, questions and theoretical framework. Considering the main objective, the study investigated the connection between proficiency in common fractions and performance achievement in algebraic fractions. The barriers and challenges that learners encountered when solving fractions were also examined together with the pedagogical strategies that could be used to improve learner proficiency in solving algebraic fractions.

The data were collected using pre- and post-test questionnaires and also from semi-structured interviews. Semi-structured interviews were conducted at each stage to add further depth to the answers obtained in the pre- and post-test by asking open-ended questions. An intervention of five lessons was done after the pre-test to close the gap shown in the pre-test. The study aimed to answer the following research questions: i) how does a lack of conceptual understanding in common fractions affect performance achievement in algebraic fractions? ii) which barriers do learners encounter when learning algebraic fractions? and iii) which pedagogical strategies could be used to improve understanding and problem-solving skills in algebraic fractions? To answer the final question, the study relied on both literature reviews on pedagogical strategies that are used in trying to make the teaching and learning of fractions simpler together with the intervention lessons.

4.2. Demographic characteristics of participants

The data collected on learner's characteristics showed great variability and is presented in Table 4.1. The average age of the learners was 16.8 years with a standard deviation of 1.50 years. The minimum age was 15 years while the maximum was 20 years. The results showed that 46% of the learners were males while 54% were females. The average age of the household head was 47.8 years with a standard deviation of 11 years, minimum age of 19 years and maximum age of 70 years. There was great variability in the ages of the household heads. The average distance that learners travel from their homes to school was 2.60 km while the maximum distance was 5 km and the minimum distance was 0,80 km with a range of 4.20 km. The average household size was 6 members with a standard deviation of 2.70 members per family.

Table 4.1: Demographic characteristics of learners and household attributes.

Variable		**Mean**	**Std. Dev**	**Min**	**Max**
Age of learners (years)		16.8	1.50	15	20
Gender of learners (0=F, 1=M)		0.46	0.36	0	1
Age of household head (years)		47.8	11.0	19	70
Distance to school (km)		2.60	1.39	0.80	5
Household size		6	2.70	1	10
Household head	Mother	0.46	0.03	0	1
	Father	0.42	0.03	0	1
	Grandmother	0.04	0.00	0	1
	Sister	0.04	0.00	0	1
	Self	0.04	0.00	0	1
Employment of household head		0.31	0.12	0	1
Level of education					
Siblings	None	0.00	-	0	1
	Below Matric	0.54	0.36	0	1
	Matric	0.04	0.00	0	1
	Certificate	0.19	0.05	0	1
	Diploma	0.12	0.02	0	1
	Degree	0.12	0.02	0	1
Parents	None	0.04	0.00	0	1
	Below Matric	0.46	0.03	0	1
	Matric	0.31	0.12	0	1
	Certificate	0.12	0.02	0	1
	Diploma	0.08	0.01	0	1
	Degree	0.00	-	0	1

Std. Dev – Standard deviation; km – kilometres

Most of the learners (46%) indicated that the household head was their mother while 42% indicated that it was their father, 4% indicated grandmother, another 4% indicated sister and a further 4% indicated that they were the household heads. Considering the employment status of household heads, most learners (69%) reported that the head of the household was not employed and 31% were employed. Considering the level of highest qualifications attained by most of the learner’s parents (46%) or siblings (54%) which was below matric, it was evident that most of them might not have the capacity to help their children or siblings with homework and not to mention fractions.

4.3. Attitudes and perceptions of learners towards the learning of common and algebraic fractions

The learning of fractions is affected by several factors including learners' attitudes towards the topic, teachers' methods of teaching, learners' background and the learning environment (Sarmah and Puri, 2014). In the current study, the researcher investigated learners' attitudes towards the learning of common and algebraic fractions as well as ways to try and improve these attitudes.

Figure 4.1 summarises the learner's responses on their attitudes and perceptions towards fractions before and after an intervention. Before intervention, 57.8% of the learners claimed that they enjoyed learning fractions and this percentage increased to 92.3% after the intervention. Those who claimed that fractions were easy to understand were 38.5% of the learners before intervention and the proportion rose to 92.3% after the intervention. All the learners (100%) indicated that their teacher was good at fractions before and after the intervention. Of the 26 students, 42.3% reported that algebraic fractions were difficult to simplify before intervention and dropped to 30.8% after the intervention. Similar results were observed when learners were asked to compare fractions with food or animals to gauge how they felt about the learning of fractions before and after the intervention. The learners' responses on their dispositions are captured in Appendix 5, LD1. Based on the literature, such examples are used to establish consistency in the learner's answers (Cai et al., 2012). The percentage of learners who used a calculator to simplify fractions before intervention was 92.3%, which dropped to 69.2% after the intervention.

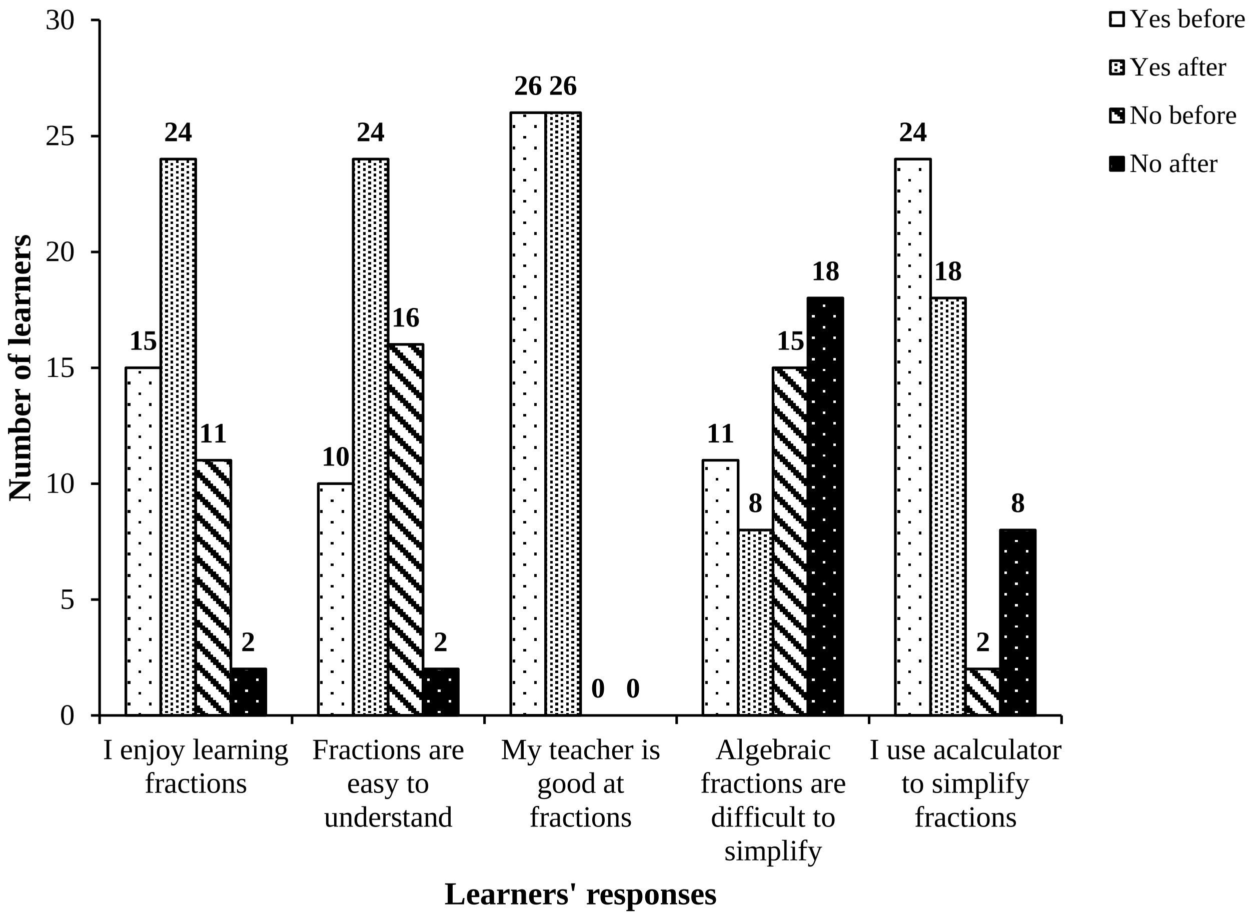

Figure 4.1: Learners’ attitudes and perceptions towards common and algebraic fractions before and after intervention lessons.

To get an insight into how the learners practised fractions, learners were asked to indicate who assisted them with homework. According to Table 4.2, 46% of the learners indicated that they did not get assistance with homework. The study showed that fathers do not assist with homework at all while 7.70% indicated their mothers, 15.4% sister, 11.5% brother and 19.2% got help from friends and uncles.

Table 4.2: Proportion of learners who got assistance with Mathematics homework

Person assisting the learner with homework	**N°. of learners indicating assistance**	
	Count	**Percentage (%)**
None	12	46
Father	0	0
Mother	2	7.70
Brother	3	11.5
Sister	4	15.4
Other (friend, uncles, cousins)	5	19.2

All the learners (100%) reported that their households have electricity meaning this variable cannot be a barrier to learning. Table 4.3 presents the results of the learners' behaviour. Considering the studying behaviour, on average, learners studied for 2 days in a week and 2 h per day with a standard deviation of 0.93 h and 1.10 h, respectively. Learners engaged in household chores for an average of 2 h per day with a standard deviation of 1.50 h. Both the time that learners spent studying and the time they engaged in household chores did not vary that much as indicated by the standard deviation.

Table 4.3: Learners studying behaviour and participation in household chores

Variable	**Mean**	**Std. Dev**	**Min**	**Max**
Studying hours	2	1.10	0	5
Studying days	2	0.93	0	3
Household chores (h)	2	1.50	0	6

Std. Dev – Standard deviation

The results in Table 4.4 revealed the learners' rating of their level of comprehension based on a scale from 0 meaning total lack of understanding of common and algebraic fractions to 5 meaning complete mastery. In categories 0 and 1, there were only 7.70% of the learners before intervention and none indicated those categories after the intervention. Of the 26 learners, 42.3% indicated that they had little understanding of the fractions while 42.3% were in the middle (level 3). No learners indicated levels 4 and 5 before intervention. After intervention, 7.70 % still indicated level 2 and those in level 3 increased from 42.3% to 50%. In level 4, there were now 38.5% and 3.80% in level 5. As for the teacher's understanding of fractions, only 1 learner indicated that the teacher had an average understanding (level 3) while 2 rated the teacher on level 4 and the rest, 88.5% rated the teacher at level 5. After the intervention, 3 learners rated the teacher at level 4 and the rest at level 5.

Table 4.4: Learners rating on the extent to which learners and teachers understand common and algebraic fractions

		Number of learners indicating each scale											
Scale		**0**		**1**		**2**		**3**		**4**		**5**	
		Bef.	*Aft.*	*Bef.*	*Aft.*	*Bef.*	*Aft.*	*Bef.*	*Aft.*	*Bef.*	*Aft.*	*Bef.*	*Aft.*
Learner	Count	1	0	1	0	11	2	11	13	0	10	0	1
	(%)	3.80	0	3.80	0	42.3	7.70	42.3	50.0	0	38.5	0	3.8
Teacher	Count	0	0	0	0	0	0	1	0	2	3	23	23
	(%)	0	0	0	0	0	0	3.80	0	7.70	11.5	88.5	88.5

Bef. – Before
Aft. – After

4.4. Analysis of pre-test results

A pre-test was administered to the participants under supervision so that they could not discuss their responses. The pre-test questionnaire comprises three sections, Part A with eighteen questions, Part B with eight questions and Part C with twelve questions. The duration of the pre-test was 45 minutes, 15 minutes for completing part A and part B (Questionnaire part) and 30 minutes for part C. The pre-test part is attached in Appendix 2, PTQ1. Evidence of learners' work and errors they committed in both pre-test and post-test are found in Appendix 6.

4.4.1. Definition of errors made by the learners

From the pre-test, the researcher identified errors and misconceptions that learners had when solving fractions (Table 4.5). These errors were determined after marking the pre-test as part of the analysis and would also assess the learners' performance after taking the post-test. The errors were put in seven categories of which 3 were pre-determined from literature and 4 emerged from the researcher's findings. The 3 errors from the literature were Lowest Common Denominator Error coded as LCDE, Conceptual Error coded as CE and Simplification Error coded as SE (Baidoo, 2019, Khanyile, 2016, Makonye and Khanyile, 2015, Otten et al., 2008). The researcher identified Silly Mistake Errors[1] coded as SME, Misconception Errors coded as ME, Order of Operation Errors coded as OOE and Meaningless Errors coded as MLE. These errors are described in Table 4.5.

Table 4.5: Description of errors made by learners in the pre-test and post-test

Codes		**Explanation**
LCDE	Lowest Common Denominator Error	The learner makes this error when they did not get the correct LCD or ECD
CE	Conceptual Error	The learner makes this error when they show a lack of a concept
SE	Simplification Error	The learner makes this error when they do not reduce answers to the lowest terms or cannot simplify algebraic expressions in algebraic fractions
SME	Silly Mistake Errors	The learner makes this error when they show correct working but copies something wrongly or add, subtract, multiply and divide in the wrong way.
ME	Misconception Errors	The learner makes this error when they perform wrong rules or operations to a problem
OOE	Order of Operation Errors	The learner makes this error when they perform the wrong order of operation, for example, they add before division or multiplication.
MLE	Meaningless Errors	The learner makes this error when they write anything that comes to mind with no connection with a particular problem.

4.4.2. Pre-test results

QUESTION 1

This question asked the learners to reduce the following fraction to its lowest terms: $\frac{4}{8}$

Eighteen learners (69.2%) could solve the question. Eight learners (30.8%) failed to solve the question, of these, 8.5% did not reduce the fraction to its lowest terms. They left the answer

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[1] Silly Mistake Errors are those that the learner performed because of carelessness and lack of due care and these could be avoided if the student was more careful.

as $\frac{2}{4}$ instead of $\frac{1}{2}$. This can be regarded as an SE error. Two learners (7.70%) did not write the question while 1 learner made an error that could be classified as an MLE error and it is shown in Figure 4.2 below.

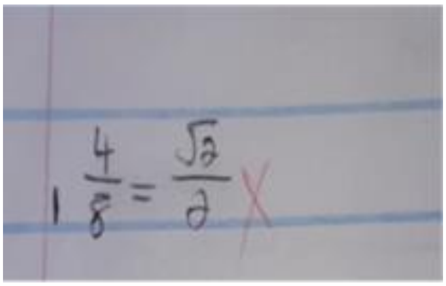


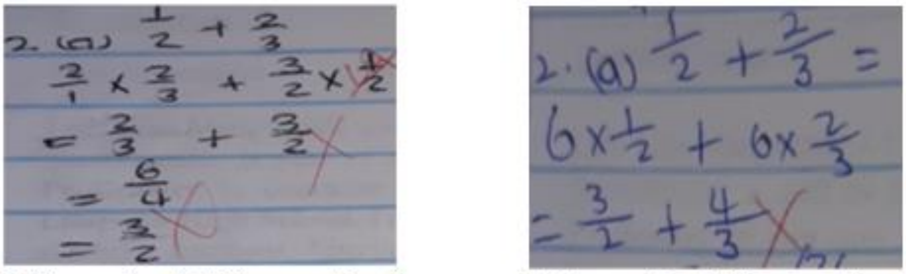
Figure 4.2: Meaningless error made by L22 in question 1

QUESTION 2

These questions asked the learners to calculate different common fractions from (a) to (d) based on addition, subtraction, multiplication and division.

(a) $\frac{1}{2} + \frac{2}{3}$

Twenty-two learners (84.6%) got the question correct and of these 22 learners, ten used the LCD and got all correct steps and correct answers while 8 used the ECD and got the correct answer. The learners simply multiplied the denominators to find the common denominator but got the correct answer. Four learners just wrote the correct answer meaning they might have used a calculator. Four learners (15.4%) got the question wrong. The errors committed by learners in these questions were ME errors, for example, L 17 and a different type of ME error emerged as shown by L 1 in Figure 4.3. L1 multiplied each fraction by the LCD like we do when solving equations with fractions.

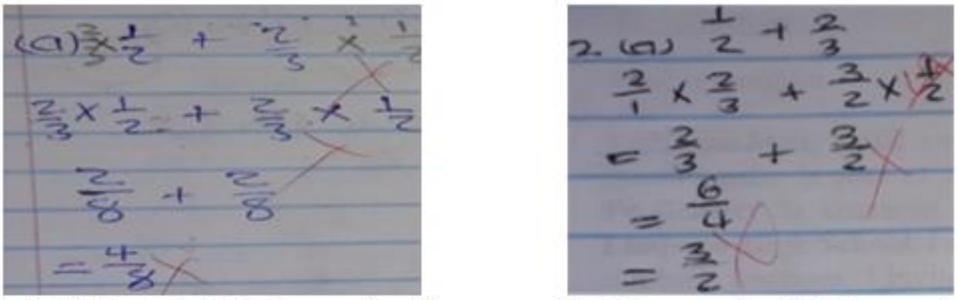


ME error by L17 in question 2a

ME error by L1 in question 2a

Figure 4.3: Misconception errors made by learners in question 2a

Other learners made LCD errors, for example, L 9 and L17shown in Figure 4.4. These learners did not understand that they should multiply each fraction by the denominator of the fraction written as a whole as in $\frac{3}{3} \times \frac{1}{2} + \frac{2}{2} \times \frac{2}{3}$



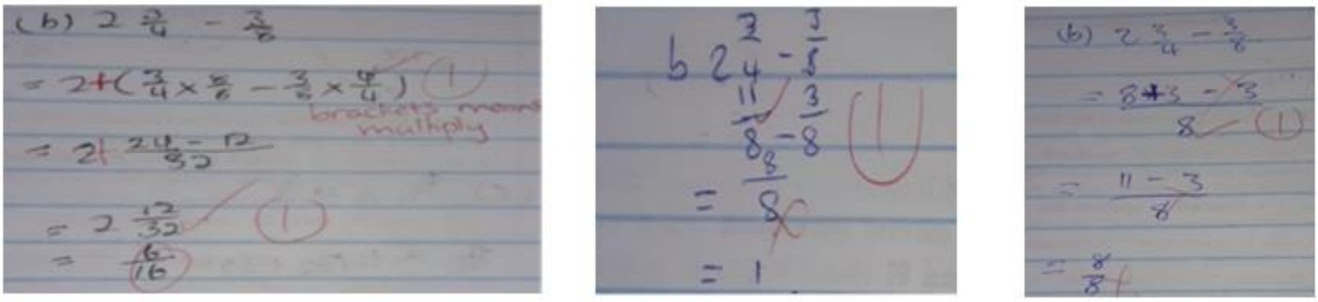
LCDE Error by L9 in question 2a

LCDE error by L17 in question 2a

Figure 4.4: Lowest common denominator errors made by learners in question 2a

(b) $2\frac{3}{4} - \frac{3}{8}$

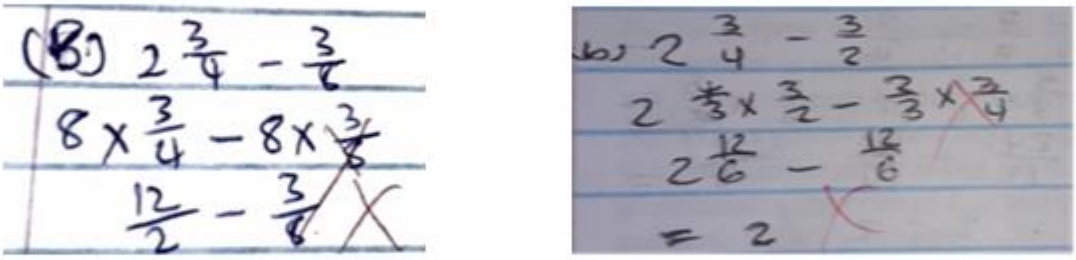
Thirteen learners (50%) got the question correct. Out of these 13, five learners used the LCD and all steps were correct; six learners used the ECD and all steps were correct; two learners changed the mixed number to an improper fraction and then just wrote the correct answer (might have used a calculator). Some learners used the ECD but did not reduce the answer to the lowest terms. The other 13 learners (50%) did not get the question correctly. Some learners did SME errors, for example, L 10 and other learners made CE errors such as L 24 and L 9 as shown respectively in Figure 4.5.



SME error by L10 in question 2b CE error by L24 in question 2b CE error by L9 in question 2b

Figure 4.5: Silly mistake errors and concept errors made by learners in question 2b

These learners knew how to write the fraction as a mixed number and to find the LCD but lacked the conceptual understanding of how to write the fractions under the same denominator. Other learners displayed MLE errors, for example, L 1 and L 17 shown in Figure 4.6.



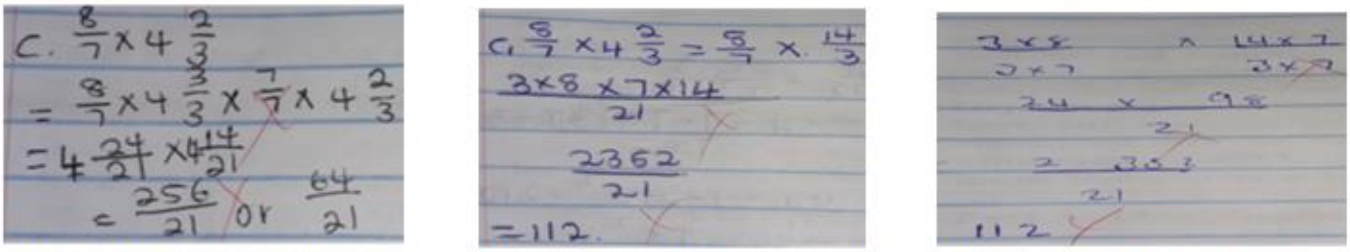
MLE error by L1 in question 2b MLE error by L17 in question 2b

Figure 4.6: Meaningless errors made by learners in question 2b

Consistent with Idris and Narayanan (2011), this shows that these learners lacked the concept of changing a mixed number into improper fractions so that they could subtract the fractions. These authors classified these errors as systematic errors, which may be repeated, systematically constructed or reconstructed over a while due to the grasp of incorrect conceptions of solving a particular problem (Idris and Narayanan, 2011).

(c) $\frac{8}{7} \times 4\frac{2}{3}$

Only four learners (15.4%) got the question correctly while the rest failed (84.6%). All those learners who got the question wrongly showed a lack of conceptual understanding in multiplying fractions. Learners did not understand that when multiplying fractions, they multiply numerators alone and denominators alone. Instead some learners were finding the LCD. They made the following errors: L 17 (CE error), L 12 (ME error), and L 3 (ME error) as shown in Figure 4.7.

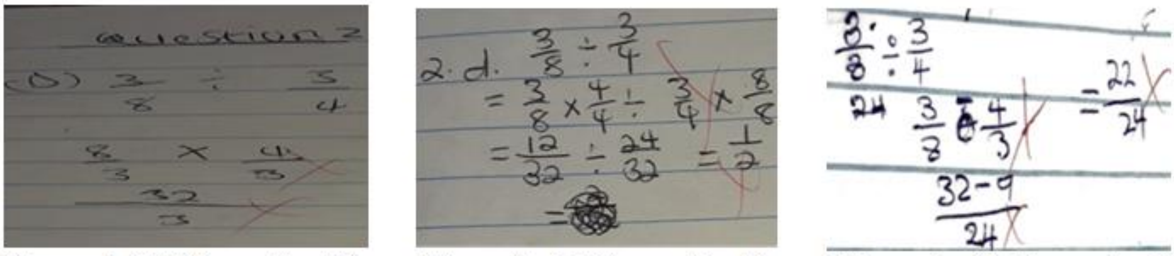


CE error by L17 in question 2c   ME error by L12 in question 2c   ME error by L3 in question 2c

Figure 4.7: Meaningless errors and concept errors made by learners in question 2c

(d) $\frac{3}{8} \div \frac{3}{4}$

Fourteen learners (53.8%) got the question correctly and out of these 14, nine learners managed to do the question showing all the correct steps while 5 wrote correct answers only suggesting that they used a calculator. The other 12 learners (46.2%) did not get the question correctly and out of the 12, one learner did not even attempt to write the question. Ten of the learners showed concept errors as follows: L 10, L 16 while others also showed ME errors as in L 14 and are shown in Figure 4.8.



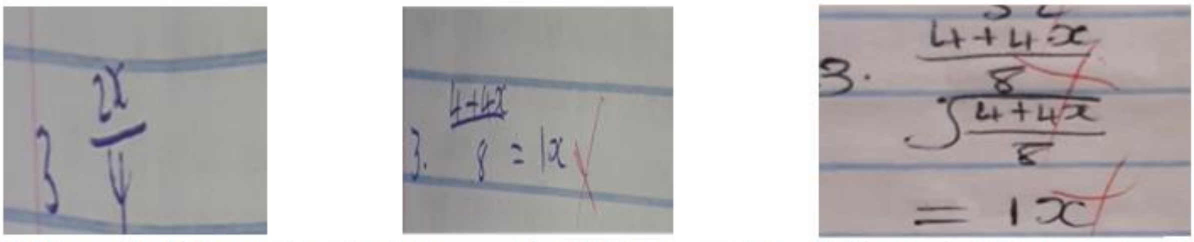
CE error by L10 in question 2d   CE error by L16 in question 2d   ME error by L14 in question 2d

Figure 4.8: Conceptual errors and misconception errors made by learners in question 2d

QUESTION 3

This question asked learners to reduce the following to its lowest terms: $\frac{4+4x}{8}$

In this question, nine learners (34.6%) got the question correct. Some showed all steps but others just wrote the answer. Seventeen learners (65.4%) did not get the question correctly and of these 17, three of the learners did not attempt the question while fourteen showed a lack of conceptual understanding in reducing algebraic fractions to the lowest terms. Examples of the errors they exhibit are shown in the examples in Figure 4.9 which are MLE errors shown by L 1, L 20, L 23.



MLE error by L1 in question 3 MLE error by L20 in question 3 MLE error by L23 in question 3

Figure 4.9: Meaningless errors made by learners in question 3

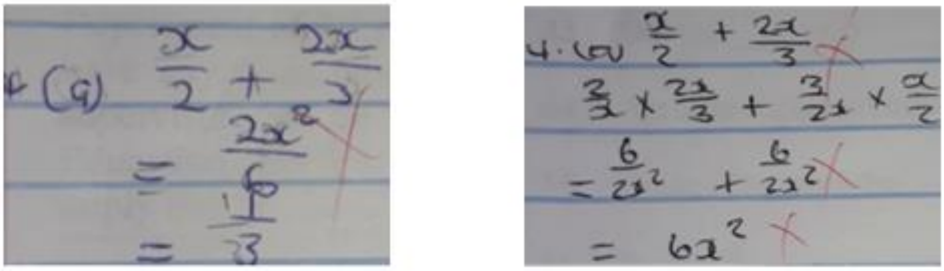
Questions 1 and 3 were testing the same concept. The only difference was that question 3 was an algebraic fraction while question 1 was a common fraction. Comparing the responses to questions 1 and 3, eighteen learners (69.2%) could do question 1 while nine managed (34.6%) to do question 3. Two learners did not write question 1 and 3 did not write question 3. Only one learner failed to do question 1 while 14 failed to do question 3.

QUESTION 4

This question asked learners to simplify different algebraic fractions (a) to (f) based on addition, subtraction, multiplication and division.

(a) $\frac{x}{2}+\frac{2x}{3}$

Twelve (46.2%) learners got this question correctly but 5 of these 12 used the LCD while 7 used the ECD. These same 12 learners also got question 2(a) correctly but those who got 2(a) by just writing the answer failed to get question 4(a) correctly. Fourteen learners (53.8%) failed to do the question and of these, 5 learners partly got the question correctly. They made SME errors which affected their final answer while 9 learners (34.6%) failed to do the question and some made CE errors, for example, L 24 while others made LCD errors as in L 17 error 4 and the errors are shown in Figure 4.10.

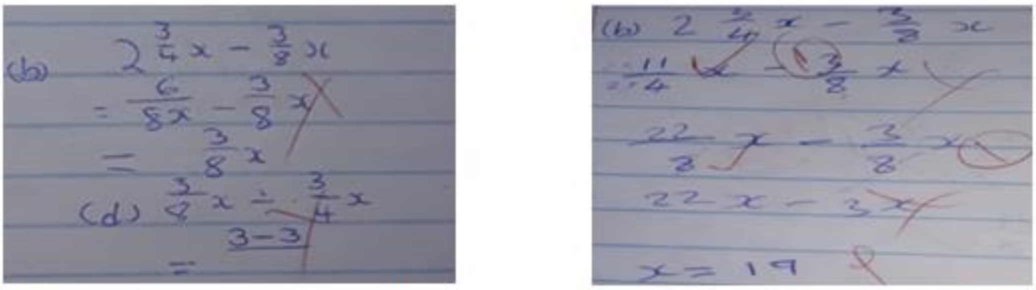


CE error by L24 in question 4a LCDE error by L17 in question 4a

Figure 4.10: Conceptual errors and lowest common denominator errors made by learners in question 4a

(b) $2\frac{3}{4}x - \frac{3}{8}x$

Ten learners (38.5%) got the question correctly of which 7 learners did the question with all steps correctly. Only one did not reduce the answer to the lowest terms and 2 learners dropped the letters in the final answer. Sixteen learners (61.5%) did not get the question correctly and of these, 3 learners did correct steps but made SME errors and failed to get the correct answer. Comparing this question with question 2(b),13 learners (50%) got 2(b) correctly while 10 (38.5%) got 4(b) correctly. Those who used the ECD did not reduce their final answer to the lowest terms which resulted in SE errors. Sixteen learners did not get the question correctly compared to 13 in question 2(b). Examples of errors the learners made in this question are shown in Figure 4.11 below. This error is a conceptual error (CE). Most of the learners (42.3%) who failed to change the mixed number to an improper fraction in question 2(b) also failed to do question 4(b). Those who wrote the answer only in question 2(b) (assumed to have used a calculator) also failed to do 4(b). Some learners also exhibited ME errors, for example, L 4 shown in Figure 4.11.



CE error by L24 in question 4b

ME error by L4 in question 4b

Figure 4.11: Conceptual errors and misconception errors made by learners in question 4b

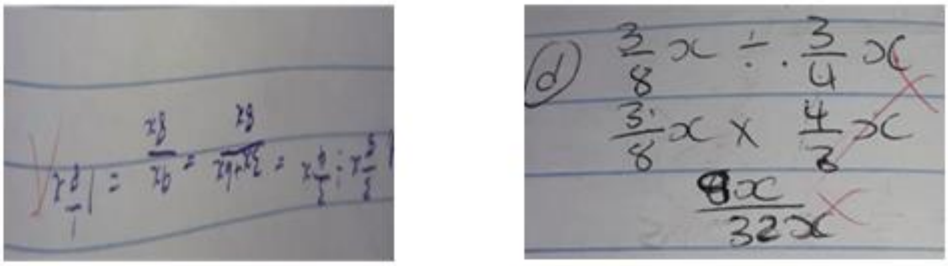
(c) $\frac{8}{7}x \times 4\frac{2}{3}x$

Four learners (15.4%) managed to do the question although some learners did not reduce the answer to the lowest terms which led to SE errors. One learner did not attempt the question. The rest (22 learners, 84.6%) did not get the question correctly. Most of the learners made CE and ME errors as they did in question 2(c).

(d) $\frac{3}{8}x \div \frac{3}{4}x$

No learner managed to do the question; 3 learners did not even attempt the question. Some of the learners made ME errors, for example, L18 and L 20 Figure 4.12. This shows that the learners did not understand that x represents a number and when you invert the divisor, it must also move. Some learners showed a complete lack of conceptual understanding of dividing fractions while others made MLE errors. Examples of these errors are shown by L 2 and L 7 Figure 4.12. Comparing question 2(d) and this question, more learners (9) did well in 2(d) while no learner managed to do question 4(d). The researcher included questions 4(e) and (f) to see which other barriers do learners face when solving algebraic fractions that needed other concepts like factorisation to be applied.

ME error by L18 in question 4d ME error by L20 in question 4d

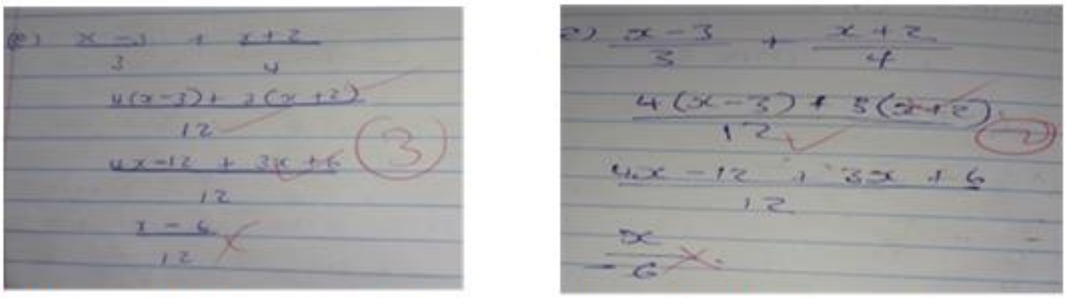


MLE error by L2 in question 4d MLE error by L7 in question 4d

Figure 4.12: Misconception errors and meaningless errors made by learners in question 4d

(e) $\frac{x-3}{3}+\frac{x+2}{4}$

Six learners (23.1%) managed to do the question even though some made SME errors, for example, L 3 shown in Figure 4.13. Four learners managed to find LCD and the second step correctly but showed a lack of conceptual understanding in simplifying algebraic expressions, for example, L 9 Figure 4.13. The rest of the learners (76.9%) did not get the question correctly and out of these, 15 learners did not get any of the steps correctly. Some did not even get the correct LCD, for example, L 15 Figure 4.13. One learner did not even attempt the question. Some managed to get the correct LCD but lacked conceptual understanding of how algebraic fractions are added or subtracted, for example, L 5 Figure 4.13

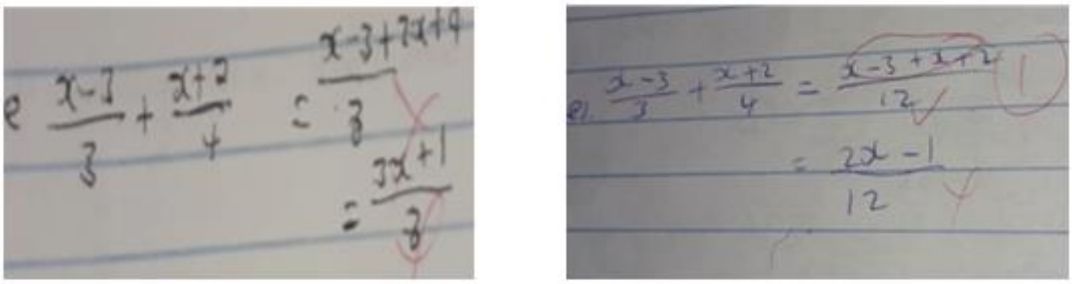


$$\frac{x-3}{3} + \frac{x+2}{4}$$
$$\frac{4(x-3)+3(x+2)}{12}$$
$$\frac{4x-12+3x+6}{12}$$
$$\frac{x-6}{12}$$

$$\frac{x-3}{3} + \frac{x+2}{4}$$
$$\frac{4(x-3)+5(x+2)}{12}$$
$$\frac{4x-12+3x+6}{12}$$
$$\frac{x}{-6}$$

SME error by L3 in question 4e

CE error by L9 in question 4e



$$\frac{x-3}{3} + \frac{x+2}{4} = \frac{x-3+2x+4}{3} = \frac{3x+1}{3}$$

$$\frac{x-3}{3} + \frac{x+2}{4} = \frac{x-3+x+2}{12} = \frac{2x-1}{12}$$

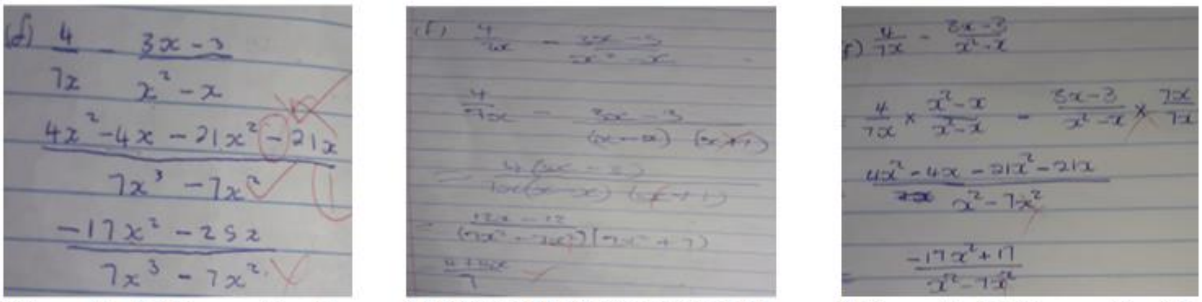
LCDE error by L15 in question 4e

CE error by L5 in question 4e

Figure 4.13: Silly mistake errors, conceptual errors and lowest common denominator errors made by learners in question 4e

(f) $\frac{4}{7x} - \frac{3x-3}{x^2-x}$

No learners got the final answer correctly but 5 learners got some of the steps correctly. This could have been attributed to the fact that they did not simplify the fractions first before they could subtract them. The other reason is that they used the ECD instead of the LCD. Examples of the errors and misconceptions learners showed are shown by L 19 Figure 4.14. When the learner used the ECD, the problem became complicated, hence the learner made SE errors. Four learners did not attempt the question and 17 learners did not get the answer correctly or any step correctly. All the 17 learners made LCDE and ME errors and some of the errors are shown by L 9 and L 18 Figure 4.14. All these learners lacked the concept of how to reduce algebraic fractions to the lowest terms before they found the LCD.



SE error by L19 in question 4f LCDE error by L9 in question 4f ME error by L18 in question 4f

Figure 4.14: Simplification errors, LCDE errors and misconception errors made by learners in question 4f

4.4.3. Comparison of common and algebraic fractions before intervention

Figures 4.15 and 4.16 show the comparison in performance between question 1 (common fraction) and question 3 (algebraic fraction) when reducing fractions to lowest terms and also question 2 (common fractions) and question 4 (algebraic fractions) when performing the four mathematical operations on fractions. Figure 4.15 showed that learners did very well in common fractions (question 1) than in algebraic fractions (question 3).

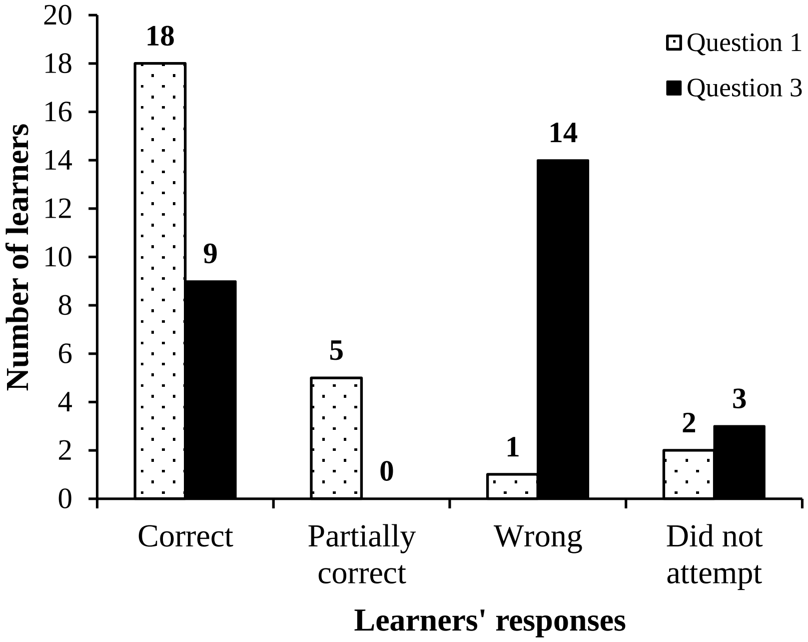


Figure 4.15: Pre-test performance of learners in question that required solving common and algebraic fractions with similar formats.

Figure 4.16(a) compares question 2a and 4a, (b) 2b and 4b, (c) 2c and 4c, (d) 2d and 4d which is shown below.

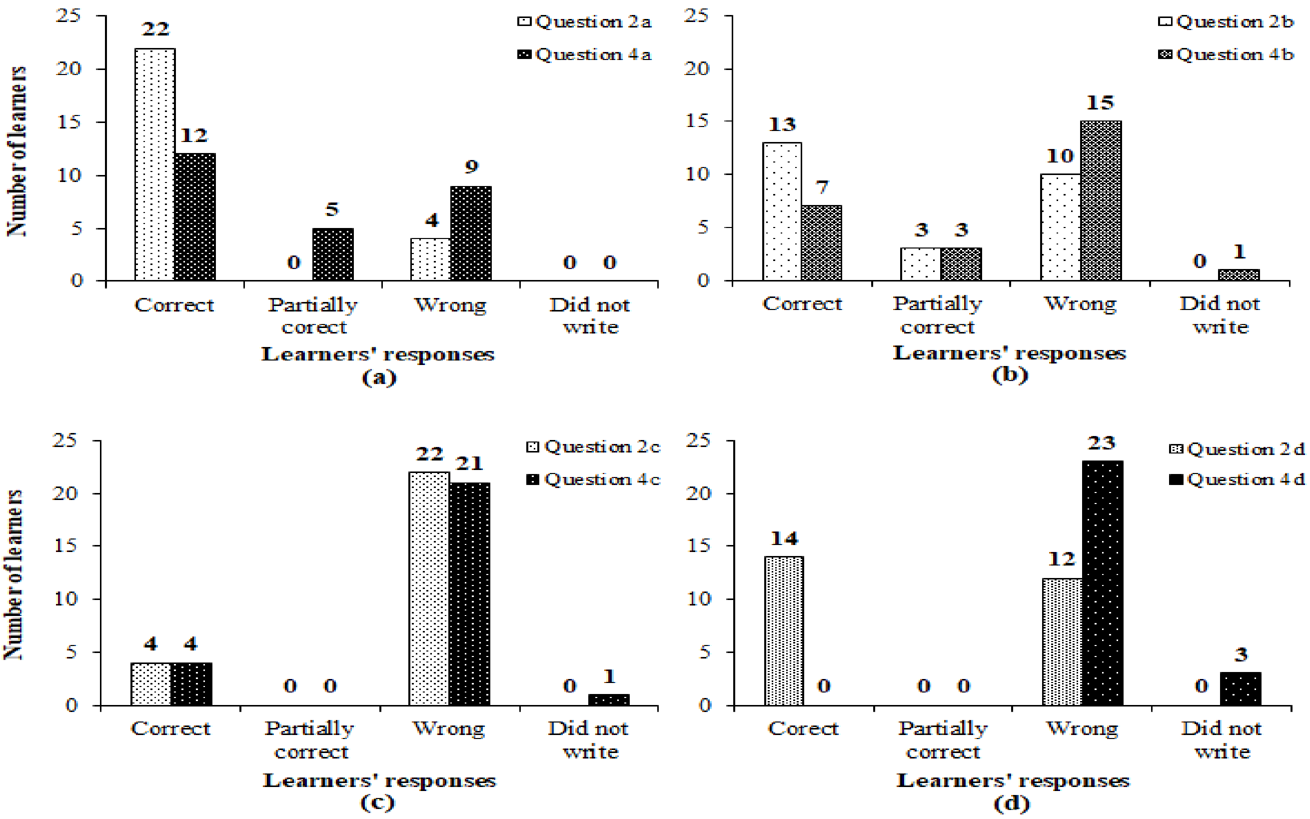


Figure 4.16: Pre-test performance of learners in questions that required solving common and algebraic fractions with similar formats.

(**a**) question 2a format $\frac{1}{2}+\frac{2}{3}$ versus 4a format $\frac{x}{2}+\frac{2x}{3}$

(**b**) question 2b format $2\frac{3}{4}-\frac{3}{8}$ versus 4b format $2\frac{3x}{4}-\frac{3x}{8}$

(**c**) question 2c format $\frac{8}{7}$ x $4\frac{2}{3}$ versus 4c format $\frac{8x}{7}$ x $4\frac{2x}{3}$

(**d**) question 2d format $\frac{3}{8}\div\frac{3}{4}$ versus 4d format $\frac{3x}{8}\div\frac{3x}{4}$

From the bar graphs in Figure 4.16 above, if a learner lacks a conceptual understanding of how to simplify common fractions, they will also struggle with simplifying algebraic fractions. Learners who did well in questions on common fractions also performed well in algebraic fractions, for example, L 7 got questions 1 and 2(a) correct also got 3 and

4acorrectly, question 2b, c, d wrong 4b, c, d wrong. Learner 19 got 2 a, b, c correct and also got 4 a, b, c correctly (Figure 4.17).

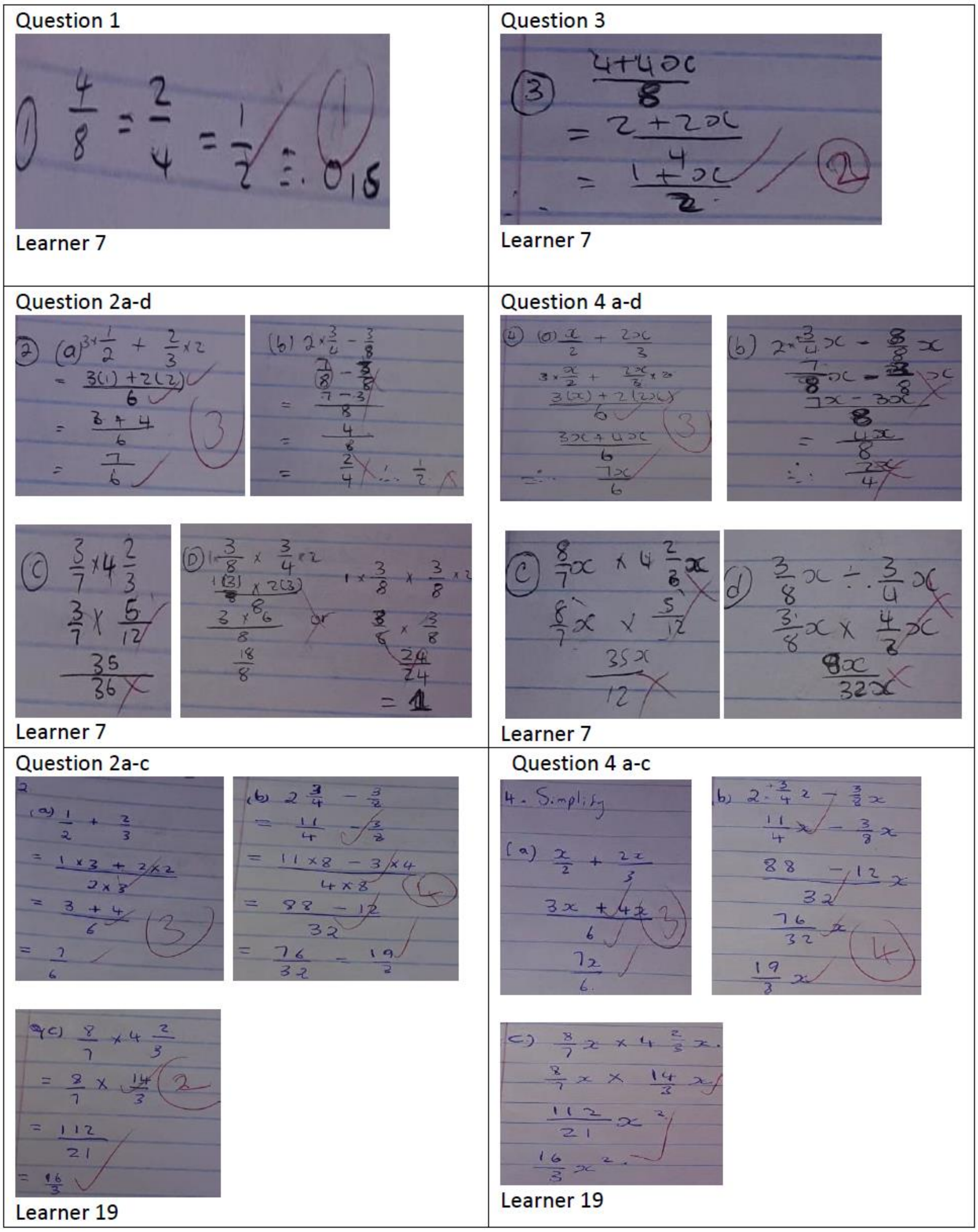


Figure 4.17: Performance of L7 and L19 in common fractions versus their performance in algebraic fractions of similar formats.

4.5. The Intervention

Intervention

The intervention comprised 6 lessons. The first and second lessons introduced some of the concepts of fractions, history and important definitions while the third lesson introduced the idea of reducing fractions to their lowest terms. The multiplication and division of common and algebraic fractions were done in lesson four while lesson five concretised the principles of division. The final lesson was devoted to adding and subtracting common and algebraic fractions. These lesson plans were using four teaching practices that were adopted from NCTM (2014). These practices were: i) establishing goals to focus learning; ii) implement tasks that promote reasoning and problem solving; iii) pose purposeful questions and iv) build procedural fluency from conceptual understanding.

Lesson 1: Introduction to fractions

Learners were asked to give examples of fractions but instead, they gave types of fractions, for example, improper, proper and mixed fractions. After probing and rephrasing the question, they could give examples of fractions. Then, the researcher asked learners to define what a fraction is. All the learners were unable; they claimed that they had forgotten. The researcher then defined a fraction as part of a whole, a ratio, number, operator, a set of objects, a relationship, a representation of division and a unit of measurement. Table 2.2 was used to explain each of the above definitions. For learners to understand the first definition (part of a whole), the researcher demonstrated by dividing a shape into equal parts and cut the parts into separate entities. Then, learners were required to say what fraction of the whole was each entity. As a practical way of internalising the definition, learners were given shapes to divide into groups and shade a part of their shapes. They were then asked what fraction they had to shade. All the groups could do the task. To emphasise that a fraction is a part of the whole, learners were given a packet of snacks that contained 52 small packets to share. They came up with a fraction $\frac{2}{52}$ since each learner got 2 packets out of the whole. At the end of the lesson, learners had a conceptual understanding of the definition of a fraction.

Lesson 2: Definition and history of a fraction

Learners were reminded of the activities they had done in lesson 1 and were then asked to define a fraction. Most of the learners could give the definition. The teacher gave learners the history of fractions that they were first developed in Egypt and were used to divide the land. The word ‘fraction’ came from the Latin word ‘fraction’, which means ‘to break’. Since learners were now having an understanding that a fraction is a part of a whole, the fraction was now defined as a number that represents a whole number that has been divided into two parts. Learners were then asked to name the two parts that make up a fraction. All the learners could name the top number, the numerator and the bottom number, the denominator. Learners did not understand what both the numerator and denominator represent in a fraction. Learners were made aware that the word numerator comes from a Latin word numerous, which means number and it represents the number of things you have while the denominator also comes from the Latin word name and it shows how many pieces the whole has been divided into. The example of the shapes they had divided was given, where each group had one shape and divided it into equal parts. The one shape becomes the numerator and the number of parts they divided the shape into became the denominator.

The teacher also brought a cake to make the lesson more interesting and bring real-life situations to the classroom. The cake was to be shared first amongst 2 learners who were identified by the whole class. Learners managed to realise that the whole cake that was one was the numerator and the denominator was the number of parts into which the cake was to be divided into, which was 2. The number of learners was increased to 4 and they shared the cake again and came up with a fraction $\frac{1}{4}$. Finally, they shared the cake with the whole class and came up with the fraction $\frac{1}{26}$. The numerator 1 (one) did not change; what changed was the denominator. Learners also realised that as the number of pieces increased, the share decreased. In other words, the bigger the denominator, the smaller the fraction. Learners were then given a worksheet in which they were coming up with fractions from sharing.

The activity took the learners 15minutes to complete and the learners marked their work as the teacher discussed the answers with the whole class. The learner who answered was requested to explain how they got their answers and their responses were recorded. The items were as follow:

Item 1

Share 10 marbles to 5 people. What fraction of the whole did each one of them get?

Learner’s response:

Answer :$\frac{2}{10}$

Explanation: I divided 10 by 5 and get 2 which is the number of marbles each one of them would get. Then, I went ahead and divided 2 by 10 since the total number of marbles was 10. So each got 2 parts out of the 10 parts.

From the learners' answer and explanation, it is evident that the learner understood the question.

Item 2

Three friends wanted to share 4 chocolate bars. Each bar is further divided into 6 equal parts. How many bars did each one of them get?

Answer: 8 bars

Explanation: I multiplied 4 by 6 and got 24, then, I divided 24 by 3 and got 8. The teacher probed the learner and ask how could the 3 friends end up getting 8 bars each from only 4 bars. The learner realised that the answer did not make sense but was unable to see the mistake. The teacher probed further and asked why the learner multiplied the four bars by 6. The learner was then able to realise that each bar had 6parts, so for easy sharing, it was wise to break all the bars into 6 so that each one of them would get 8 pieces, not bars. The answer was supposed to be in bars, so the learner now realised that from the 8 pieces, there was only 1bar and 2 pieces. Then, the learner converted the 2pieces to a bar, which would be $\frac{2}{6}$. Thus, the correct answer would be $1\frac{2}{6}$ bars. Another learner's answer was $\frac{4}{3}$ bars.The learner's explanation was as follows:

I divided the number of chocolates, which was 4 (the whole) by the number of people, 3 (the number of pieces the whole should be divided into) and I got $\frac{4}{3}$ bars. This learner showed a conceptual understanding of fractions.

Item 3

You do 7 subjects at school and each subject is allocated a certain number of hours per week as follows: Mathematics $4\frac{1}{2}$ hours, English 4hours, Agriculture 4hours, Setswana 4hours, Physics 4hours, Life Sciences 4hours and Life Orientation 4hours. What fraction of the whole time is allocated to Mathematics?

Answer: $\frac{1}{7}$

Explanation: I divided 1 (one) by 7 because Mathematics is 1 subject and there are 7 subjects in total. This learner did not realise the question was asking for the fraction of time that Mathematics is allocated out of the total time available.

Another learner's answer was $\frac{4.5}{26.5}$

Explanation: I added all the hours allocated to each subject and got 26.5, then, I divided the hours for Mathematics by the total time for all subjects and got $\frac{4.5}{26.5}$.

From this activity, it was clear that some of the learners were now clear about fractions but in all their answers, they did not simplify to the lowest terms. The learners were reminded of the importance of giving answers to the lowest terms. The next lesson deals with this issue. Learners were asked to give the importance of fractions as a way of concluding the lesson. Some indicated that they used fractions in sharing food, allocating time on the time table and dividing things like land. The teacher also added other reasons why we do fractions like in baking, you need to know how much of each ingredient is needed or in trading if you are selling something as a whole when now you want to sell a part, fractions are needed. Learners were given homework to go and ask their parents how inheritance was shared in their families as an outreach effort to show how fractions are useful in daily life.

Lesson 3: Reducing fractions to lowest terms

The answers that learners got in item 2 were used as a point of reference,$1\frac{2}{6}$ and $\frac{4}{3}$. The learners were asked to visualise these fractions and see if they could see if these fractions were equal. Some thought $1\frac{2}{6}$was bigger than $\frac{4}{3}$. The teacher then referred learners back to cake sharing and that the smaller the denominator, the bigger the fraction. It was, therefore, important to reduce the fraction so that:

(a) It becomes easy to see how big a fraction is,

(b) When multiplying or dividing fractions, it becomes easy if fractions are in the lowest terms

The teacher then highlighted that to reduce fractions to the lowest terms, learners found the Highest common factor (HCF) from both the numerator and the denominator of the fraction.

Learners were then asked to reduce the fractions they came up with when they divided the shapes they were given in lesson 1. All the learners were able to do the task. Learners were given another worksheet on reducing common fractions to the lowest terms and all learners managed. They were then given another worksheet with algebraic fractions to reduce to their lowest terms. After completing the activity, each learner marked their work and some learners were asked to give their answers and explain how they got those answers.

Item 1: $\frac{2x+4}{8}$

Answer: $\frac{x+2}{4}$

Explanation: I found the HCF, which is 2, then, I factored out 2 from the numerator. Then, I divided both the numerator and the denominator by 2, that is, .$\frac{2(x+2)}{8} = \frac{x+2}{4}$. In this item, some learners did not get the correct answer; they made ME errors, for example, .$\frac{2(x+4)}{8} = \frac{x+4}{4}$. They showed that they did not understand that when factorising an algebraic expression, you take out the HCF from the 2 terms in the expression in the numerator.

Item 2 :$\frac{3x}{15}$

Answer :$\frac{x}{5}$

Explanation: I found the HCF, which was 3 and divided both the numerator and denominator by 3. All the learners got the correct answer.

Item3: $\frac{6x-14}{8}$

Answer :$\frac{3x \_7}{4}$

Explanation: I looked for the HCF in the numerator, which is 2 and factor it out, that is, $\frac{2(3x-7)}{8}$. Then, I divided both the numerator and the denominator by 2. Most of the learners did the question but a few made an ME error like in item 1.

Item 4: $\frac{6}{9x}$

Answer :$\frac{2}{3x}$

Explanation: I found the HCF which is 3 and divided both the numerator and denominator by the HCF 3. All the learners did well in this item.

Item 5: $\frac{5}{10x-5}$

Answer :$\frac{1}{2x-1}$

Explanation: I got the HCF for both numerator and denominator, which is 5, I then, factored out 5 in the denominator, that is, $\frac{5}{5(2x-1)}$, then I divided both the denominator and the numerator by 5. Some learners committed Order of Operation Error (OOE), that is, $\frac{5}{10x-5}$. They divided each term in the fraction by 5. They did not know that the same way we use BODMAS in common fractions also applies in algebraic fractions, that is, $\frac{5}{10-5}$. You simplify the denominator first before you can divide.

Item 6 :$\frac{x}{x^2}$

Answer :$\frac{1}{x}$

Explanation: I found the HCF, which is x and then divided both the numerator and denominator by$x$. Some learners did not write the item because they could not find the HCF. A clarification was made on how to find the HCF when the fraction is algebraic.

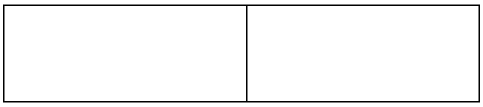
The lesson was concluded by giving learners homework on reducing common and algebraic fractions to the lowest terms.

Lesson 4: Multiplication and division of common and algebraic fractions

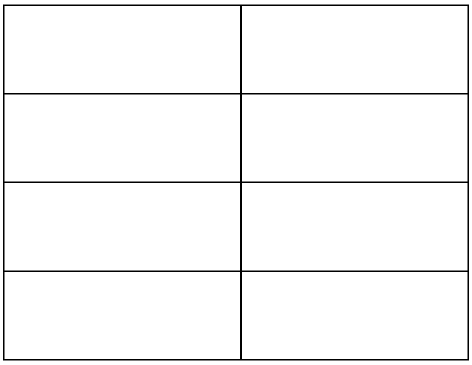
In the introduction, the teacher acknowledged that learners had prior knowledge of the multiplication and division of fractions. Learners were asked to list the steps they followed when multiplying and dividing fractions. Most of the learners could list the steps even though some were blank. Learners were then asked to explain why they did some steps. For example, in multiplication why do we multiply numerators alone and denominators alone? All the learners admitted that it was something they had been doing since primary. This lesson was structured in such a way that learners would know why (conceptual understanding) and how (procedural understanding) they did certain procedures. An area model for multiplying fractions was used to build the conceptual understanding of learners. An example was given where learners were to multiply $\frac{1}{2} \times \frac{1}{4}$

The following instructions were given:

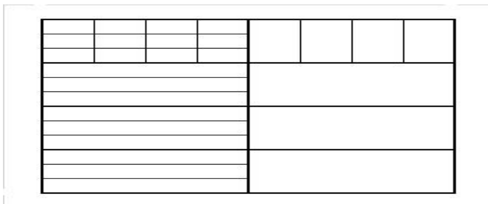
- Draw a rectangle of any measurement
- Divide the rectangle into halves using a vertical line as follows:



Further divide the rectangle into quarters using horizontal lines as follows:



Shade one part of the 2 parts with a different colour and one part of the quarters with another colour as follows:



Count how many parts have both shadings out of the total number of divided parts.

The learners could see that 1 part out of 8 parts contained both shadings which is $\frac{1}{8}$ as a fraction. They were given other fractions to multiply using the model, which is $\frac{2}{3} \times \frac{1}{6}$. They could get the answer $\frac{2}{18}$. The teacher asked the learners if the answer was in the lowest terms to which they were they said no and reduced it to its lowest terms. They were then asked to explain what they could deduct from the two examples. They realised that in the first example to get $\frac{1}{8}$, you can simply multiply the numerators and denominators alone, that is, $\frac{1 \times 1}{2 \times 4}$ similarly , for example, $\frac{2 \times 1}{3 \times 6} = \frac{2}{18}$. Now they could know why we multiplied numerators and denominators alone. To further clarify why we do that, we could also assume that the $\frac{1}{2}$ represented a cake and there were 4 people to share the cake. What fraction of the cake will each one of them get? The learners gave the answer $\frac{1}{2} \div 4$, which directly translated to $\frac{1}{2} \times \frac{1}{4}$.

From earlier work, they mentioned mixed numbers and improper fractions, learners were given the following fractions to multiply and explain how they multiply.

Item: $2\frac{1}{4} \times \frac{1}{2}$

Explanation: I change the mixed number to an improper fraction by multiplying the denominator with the whole number and then add the numerator, that is, $4\times 2 + 1$ = 9, then, the fraction will be $\frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$. The teacher then highlighted that since they now knew how to multiply fractions, it became easy to learn division.

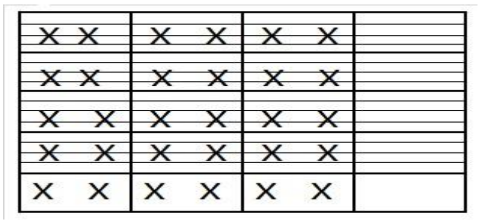
Lesson 5: Division of fractions

Learners were asked again to list the steps they follow when dividing fractions using their prior knowledge. Most of the learners were able to reproduce the steps but lacked the conceptual understanding of why they performed some steps. For easy understanding, the teacher asked 6 learners to share 7 chocolate bars equally. The learners shared the 6 full bars and left one bar. The teacher asked the learners to share that bar as well. They decided to break it into six pieces since it was segmented into six equal pieces. Each one of them ended up having $1\frac{1}{6}$ bars. The teacher then asked learners to change the mixed fraction to an improper fraction which gave them $\frac{7}{6}$. From this, learners were made to realise that sharing represented division, that is, 7 bars divided by 6 people. Division is also the same as multiplying the dividend (the first fraction) by the reciprocal of its divisor (second fraction), for example, $\frac{7}{1} \div \frac{6}{1} = \frac{7}{1} \times \frac{1}{6} = \frac{7}{6}$. Learners were now asked: What is the reciprocal of a number? They were unable to describe it; the teacher then explained that the reciprocal of a number is when you multiply that particular number with it to get 1, for example, the reciprocal of 2 is $\frac{1}{2}$ so that when you multiply 2 by$\frac{1}{2}$,you get 1. In other words, the reciprocal of any number is 1 divided by that number. An area model was also used to emphasise that dividing fractions is the same as multiplying the dividend by the reciprocal of its divisor. The following steps were given to learners for the following example $\frac{3}{4} \div \frac{4}{5}$

- Draw a rectangle of any size
- Divide the rectangle into quarters using vertical lines and into fifths using horizontal lines

- Shade 3 parts of the quarters and 4 parts of the fifths using different shadings.
- Count the number of parts that have been shaded for three quarters and it became the numerator and the number of parts shaded for the four fifths and it became the denominator

The final diagram is shown below



$$\frac{3}{4} \div \frac{4}{5} = \frac{15}{16}$$

Using the rules it will be $\frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} = \frac{15}{16}$

Learners were then given a worksheet on common and fractions to multiply and divide. The teacher checked the learners' answers with the learners and all learners were able to work the problems. Another worksheet on algebraic fractions was given to learners and the work marked in class, learners explained their answers. Learners were picked at random to explain their answers.

Item 1: $\frac{x}{4} \times \frac{2}{3}$

Learner's answer: $\frac{x}{6}$

Explanation: I multiplied the numerators alone and the denominators alone and got $\frac{2x}{12}$, then I went ahead and reduced the fraction to its lowest terms and got $\frac{x}{6}$. The teacher checked if all learners managed to get the item correctly. The majority managed even though some did not reduce to the lowest terms.

Item 2 : $\frac{2}{x} \times \frac{6}{x}$

Learner's answer : $\frac{12}{x^2}$

Explanation: I multiplied the numerators alone and the denominators alone. When the teacher checked with all the learners, they got the item correctly.

Item 3: $\frac{1}{2}x \times \frac{3}{4}x$

Learner's answer :$\frac{3}{8x^2}$

Explanation: I multiply numerators alone and denominators alone, then I got $\frac{3}{8x^2}$. The teacher further asked the learner which numerators and denominators were the learner referring to.

Learner's response: I multiply 1×3 in the numerator and $2x \times 4x$ in the denominator.

Teacher: But the xs are in the numerator not in the denominator.

Learner: I thought if the x is in the middle, that is, $\frac{1}{2}x$, it means it is in the denominator.

This is another misconception that most learners were having about algebraic fractions which affected their solutions and the teacher corrected the misconception.

Item 4 :$\frac{x}{4} \div \frac{2}{3}$

Learner's answer :$\frac{3x}{8}$

Explanation: I changed the division sign to multiplication sign and then swapped the divisor, that is, $\frac{x}{4} \times \frac{3}{2} = \frac{3x}{8}$. All the learners were able to do the item.

Item 5 :$\frac{2}{x} \div \frac{6}{x}$

Learner's answer :$\frac{x}{3}$

Explanation: I changed the division sign to multiplication sign and inverted the divisor, that is, $\frac{2}{x} \times \frac{x}{6} = \frac{2x}{6x}$. I then reduced the fraction to its lowest term and got $\frac{x}{3}$. When the teacher probed the learner further about the answer, it came to light that the learner was unaware that if you divide $x\ by\ x$, you get 1. It was a misconception that some of the learners also had. The correct answer for this item is $\frac{1}{3}$.

Item 6 :$\frac{1}{2}x \div \frac{3}{4}x$

Learner's answer :$\frac{4x^2}{6}$

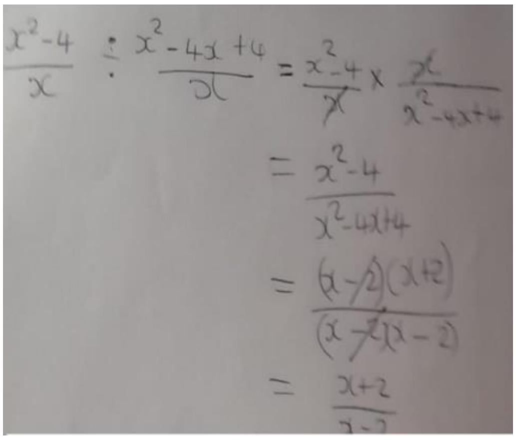
Explanation: I changed the division sign to multiplication sign and then inverted the divisor, that is, $\frac{1}{2}x \times \frac{4}{3}x = \frac{4x^2}{6}$. Most of the learners got this answer wrongly because they did not take the x on 3 as a number such that when they inverted the divisor, it was also supposed to move, that is, $\frac{3}{4}x = \frac{4}{3x}$. The correct answer was supposed to be $\frac{1}{2}x \times \frac{4}{3x} = \frac{2}{3}$. Some learners made the same error they did in item 3 of thinking that the xs were in the denominator, that is, $\frac{1}{2x} \div \frac{3}{4x} = \frac{1}{2x} \times \frac{4x}{3} = \frac{2}{3}$. They got the correct answer but the working was wrong.

Item 7 : $\frac{x^2-4}{x} \div \frac{x^2-4x+4}{x}$

Learner's answer : $\frac{x+2}{x-2}$

Explanation: The learner could not explain so was asked to show the working on the board.

Learner's working:



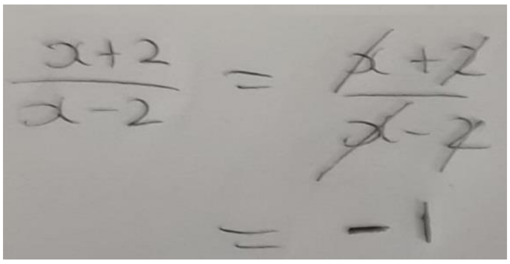
$$\frac{x^2-4}{x} \div \frac{x^2-4x+4}{x} = \frac{x^2-4}{\cancel{x}} \times \frac{\cancel{x}}{x^2-4x+4}$$

$$= \frac{x^2-4}{x^2-4x+4}$$

$$= \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-2)}$$

$$= \frac{x+2}{x-2}$$

Another learner asked why we did not simplify the answer as follows:



$$\frac{x+2}{x-2} = \frac{\cancel{x}+\cancel{2}}{\cancel{x}-\cancel{2}}$$

$$= -1$$

The teacher threw back the question to the whole class and no one could explain. Then, the teacher wrote the following problem on the board and ask the learner who asked to work it out.

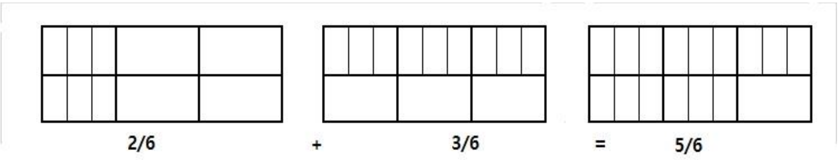
Problem: $\frac{1+2}{1-2}$ the learner wrote the following answer $\frac{3}{-1} = -3$. The teacher then asked why the learner did not just divide 1 by 1 and 2 by -2 as the learner had done with the algebraic fraction. The learners said they followed the correct order of operation. The teacher then emphasised that it also applied in algebraic fractions.

Lesson 6: Adding and subtracting common and algebraic fractions

Models were used to add and subtract the fractions before learners went back to the rules and algorithms. A worksheet with common fractions was handed to learners in groups. The following instructions were given to learners in groups:

- Draw 3 rectangles of the same size, side by side, the teacher demonstrated on the board
- Using the first question (a) $\frac{1}{3} + \frac{1}{2}$ from the worksheet
- Divide the first rectangle into thirds using vertical lines (these divisions come from the denominator of the first fraction) and then shade 1 part of the thirds. The teacher demonstrated on the board
- Divide the second rectangle into halves using horizontal line and shade 1 part of the halves (the division comes from the second fraction). The teacher demonstrated on the board
- Divide the first rectangle into halves and the second into thirds. The teacher demonstrated. This was done so that the 2 rectangles have the same number of parts
- Count how many parts shaded in the first rectangle and write it as a fraction of the total parts in the rectangle
- Count the shaded parts in the second rectangle and write them as a fraction of the total parts in the rectangle
- Divide the third rectangle into the total parts formed in the other 2 rectangles
- Shade the total parts shaded in the 2 rectangles in the third rectangle and write them as a fraction.

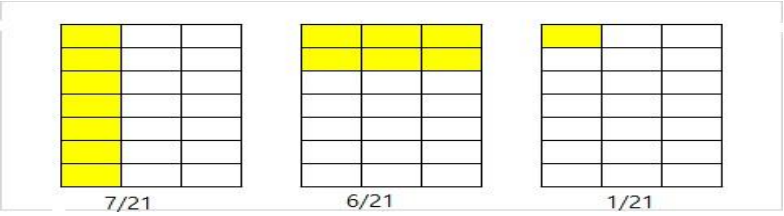
The learners’ final work will look like the following:



A problem on subtraction (b)($\frac{1}{3} - \frac{2}{7}$) was also done in groups while the teacher gave the following instructions:

- Draw 3 rectangles of the same size side by side; the teacher demonstrated on the board
- Divide the first rectangle into thirds using vertical lines (these divisions come from the denominator of the first fraction) and then shade 1 part of the thirds. The teacher demonstrated on the board
- Divide the second rectangle into sevenths using horizontal lines and shade 2 parts of the sevenths (the divisions come from the second fraction). The teacher demonstrated on the board
- Divide the first rectangle into sevenths and the second into thirds. The teacher demonstrated. This was done so that the 2 rectangles have the same number of parts
- Count how many parts shaded in the first rectangle and write it as a fraction of the total parts in the rectangle
- Count the shaded parts in the second rectangle and write them as a fraction of the total parts in the rectangle
- Divide the third rectangle into the number of parts that the other 2 rectangles are divided into
- Shade the number of parts that were shaded in the first rectangle in the third rectangle
- Using an eraser, rub the number of parts that were shaded in the second rectangle from the third rectangle

The learners’ final work will look like the following:



All the groups were able to do the task and were then given the remaining problems to do in their groups using the area models. The remaining problems were as follows:

(c) $\frac{3}{4} + \frac{1}{5}$

(d) $\frac{2}{3} - \frac{1}{4}$

(e) $\frac{1}{4} + \frac{2}{7}$

(f) $\frac{4}{10} - \frac{2}{9}$

All the groups were able to finish the remaining problems well. The teacher then asked learners to explain what conjectures they came up with from the exercise.

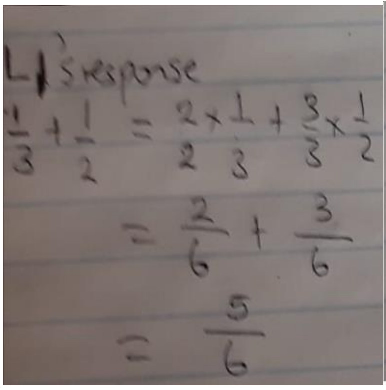
Learners' responses: when you divide the rectangles using the denominators of the fractions, you get the common denominator. Other learners also realised that to get how many parts each rectangle must be divided into; you just multiply the denominators. It was easy for learners to use the models and the learners concluded that if you want to add or subtract fractions, you must put them under the same denominator. Then, you add or subtract the numerators. One learner asked why we only add or subtract the numerators, not the denominators. Learners were asked to respond to the question but no one responded. The teacher answered by giving learners a practical example: if you are given half of a cake and another half of a cake, how much cake will you have altogether? Learners responded and said 1 whole. They were then asked to add $\frac{1}{2} + \frac{1}{2}$ using the other learner's suggestion of adding numerators alone and denominators alone, that is, $\frac{1+1}{2+2} = \frac{2}{4}$ which reduces to $\frac{1}{2}$.

The learners realised that it would not make sense to say half plus half will give them half when it was supposed to be one whole cake. In this activity and the previous ones, models were used as scaffolds and it was now time to remove them. Learners were asked to state the steps they used when adding and subtracting fractions. The learners' responses were as follows:

- find common denominator
- put the two fractions under the same denominator
- then add or subtract.

They were then asked to show their working on the board using one problem from the previous activity which is $\frac{1}{3}+\frac{1}{2}$

The learners’ responses were as follows:

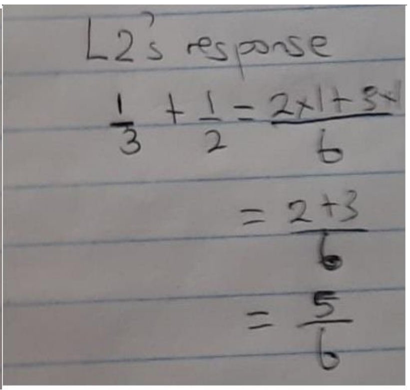


L1's response

$$\frac{1}{3}+\frac{1}{2}=\frac{2}{2}\times\frac{1}{3}+\frac{3}{3}\times\frac{1}{2}$$

$$=\frac{2}{6}+\frac{3}{6}$$

$$=\frac{5}{6}$$



L2's response

$$\frac{1}{3}+\frac{1}{2}=\frac{2\times1+3\times1}{6}$$

$$=\frac{2+3}{6}$$

$$=\frac{5}{6}$$

The teacher commented on the 2 learner’s answers while the first learner got the common denominator by simply multiplying the denominators and it was easy to do it but has its shortfalls which they discovered when dealing with algebraic fractions. This kind of denominator is called the easiest common denominator (ECD). In common fractions, its shortfall might be that you will still need to reduce the answer to its lowest terms. This might be avoided by using the second method where you use the denominator called the lowest common denominator (LCD). How do we find the LCD?

- List down multiples of the 2 denominators
- Find the LCM of the 2 denominators
- The LCM is the LCD.

Let us look at an example where you use an ECD and you still need to reduce the final answer but if you use the LCD, the answer will be already in the simplest form.

$$\frac{1}{5}+\frac{1}{10}$$

Using ECD

$\frac{10}{10}\times\frac{1}{5}+\frac{5}{5}\times\frac{1}{10} \quad =\frac{10}{50}+\frac{5}{50}$

$=\frac{15}{50}$

$=\frac{3}{10}$

or Using the LCD

M_5 = 5,10 15…….. M_{10} = 10, 20,30……

LCM =10 =LCD

$$\frac{2\times 1+1\times 1}{10}=\frac{2+1}{10}$$

$=\frac{3}{10}$

When using the LCD, it might not always be the case that answers are in the lowest terms but they are always simpler to simplify. Learners were given a worksheet on adding and subtracting common fractions using the LCD to consolidate what they had learnt. The work was marked and most learners had improved even though some learners continued to use the ECD as a common denominator. A worksheet on the addition and subtraction of algebraic fractions was administered to learners to see if they could transfer their knowledge of adding and subtracting common fractions to algebraic fractions. The learners' answers were discussed with the teacher and the learners to identify misconceptions and errors learners still had.

Item 1: $\frac{x}{3}+\frac{x}{2}$

Learner's answer: $\frac{5x^2}{6x}$

Explanation: I multiplied $\frac{x}{3}$ by $\frac{2}{2}$ and $\frac{x}{2}$ by $\frac{3}{3}$ and I got $\frac{2x}{6x}+\frac{3x}{6x}=\frac{5x^2}{6x}$

The teacher further probed the learner to explain where the x in the denominator came from. The learner responded that it came from the x in the numerator. The teacher then gave a common fraction to clarify this misconception, that is, $\frac{2}{3}+\frac{2}{5}=\frac{5}{5}\times\frac{2}{3}+\frac{3}{3}\times\frac{2}{5}=\frac{10}{15}+\frac{6}{15}$.

The learner showed the working above and then the teacher asked why she did not multiply by 2 in the denominators of the 2 fractions. She realised her mistake in the preceding question. The learner was also asked how she got $5x^2$; she showed a lack of understanding between adding and multiplying algebraic terms. The teacher explained that when adding, we add coefficients and when multiplying, we add exponents and the misconception was clarified. The learner could then give the correct answer $\frac{5x}{6}$. Most of the learners preferred to use the ECD. However, the teacher did not ask why and waited for the question where the problem became unavoidable to comment.

Item 2 :$\frac{3}{4x}+\frac{x}{5}$

Learner's answer :$\frac{15+4x^2}{20x}$

Explanation: I found the LCD which is $20x$ I then put the fractions under the same common denominator by dividing each denominator into the LCD and multiplying with the numerators as follows :$\frac{3}{4x}+\frac{x}{5}=\frac{5\times 3+4x\times x}{20x}$

$$=\frac{15+4x^2}{20x}$$

Another learner asked why we cannot add $15+4x^2$ to get $19x^2$

Answer: They are not like terms 15 does not have an x^2

Another learner also asked why we cannot divide $4x$ into $20x$

Answer: The correct order of operation must be observed the same way it is done with common fractions, hence reference to Item 7 lesson 4.

Item 3:$\frac{1}{3x}-\frac{2}{7x}$

Learner's answer :$\frac{x}{21x^2}$

Explanation: I made the denominators be the same by multiplying each fraction with the denominator of another fraction, that is, $\frac{7x}{7x}\times\frac{1}{3x}-\frac{3x}{3x}\times\frac{2}{7x}=\frac{7x}{21x^2}-\frac{6x}{21x^2}$

$$=\frac{x}{21x^2}$$

The teacher probed the learner if the answer is in lowest terms until the learner realised that the HCF of the numerator and denominator was x and finally came up with the answer $\frac{1}{21x}$. The teacher now pointed out to learners that one of the shortfalls of using the ECD instead of the LCD is that answers will still need to be reduced to the lowest terms. They can lose a mark in a test for not reducing answers to the lowest terms. Those who used the LCD showed the following working:

$$\frac{7\times1-3\times2}{21x}=\frac{7-6}{21x}$$

$$=\frac{1}{21x}$$

It was short with few hassles. A learner asked how do we find the LCM of algebraic terms?

Answer : for $3x\ and\ 7x$

Method 1 :M_{3X}= $3x, 6x, 9x, 12x, 15x, 18x, 21x, 24x \ldots..$

M_{7X} = $7x, 14x, 21x, \ldots..$

As soon as you reach a common multiple, you stop so the LCM=$21x$ =LCD

Method 2: LCM for 3 and 7 is 21then for the letter take the letter with the highest power, in this case, it is x. Another example can be for $3x^2$and $7x^3$, the LCM will be $21x^3$.

Item 4: $\frac{2}{3x^2}-\frac{1}{4x}$

Learner's answer: $\frac{8x-3x^2}{12x^3}$

Explanation: I multiplied the denominators and got the common denominator of $12x^3$. Then, I divided each denominator into the CD and multiplied by the numerator. The teacher probed further if the answer was in the lowest term then the learner was able to reduce it. Again, it shows that some learners were reluctant to use the LCD even though some used the LCD.

Item 5 :$\frac{x-3}{3}-\frac{x+2}{4}$

Learner's answer :$\frac{x-3}{6}$

Explanation: I found LCD then I put the two fractions under the same denominator.

The teacher probed the learner again if 6 was the LCD for 3 and 4, that is, whether 4 can get into 6. The learner realised that it was incorrect and now gave the correct answer 12. The learner was also asked to explain how she came up with the answer $\frac{x-3}{6}$. She admitted to just writing any answer without any logical explanation. Another learner was asked to take the class through the correct process of solving the fraction. Some learners committed SME errors which they were able to correct from the correct explanation. Learners were encouraged to use brackets when multiplying expressions to avoid making sign errors, for example, $\frac{4(x-3)-3(x+2)}{12}$.

Item 6: $\frac{4}{7x}-\frac{3x-3}{x^2-x}$

Learner's answer: $\frac{-17}{7x}$

Explanation: I factorised the numerator and denominator of the second fraction. The teacher asked why did the learner do that?

Learner's response: I wanted to get the answer.

Teacher: I know that you wanted to get the answer but I want the reason why factorising. All learners did not know why. The teacher then explained that the reason was to reduce the fraction to the lowest terms by taking out the HCF, in this case, which was $x-1$. The learner explained that she took out the HCF from the second fraction and remained with the following fractions $\frac{4}{7x}-\frac{3}{x}$. I then found the LCD, which was $7x$ and put the fractions under the same denominator as follows:

$$\frac{1\times 4-7\times 3}{7x}=\frac{4-21}{7x}$$

$$=\frac{-17}{7x}$$

One learner disagreed with this answer and could give the correct answer:

Learner's answer $\frac{-17x^2+17x}{7x^3-7x^2}$

Explanation: I found the CD by multiplying the denominators and then put all the fractions under the same CD as follows:

$$\frac{4(x^2-x)-7x(3x-3)}{7x(x^2-x)}=\frac{4x^2-4x-21x^2+21x}{7x^3-7x^2}$$

$$=\frac{-17x^2+17x}{7x^3-7x^2}$$

The teacher probed the learner and asked if the numerator and the denominator could not be factorised. The learner managed to factorise and the teacher probed further and asked if there was no HCF between the numerator and the denominator. Again, the learner managed and came up with the following: $\frac{-17x(x-1)}{7x^2(x-1)} = \frac{-17x}{7x^2} = \frac{-17}{7x}$. The learner realised that the answer reduced to the same answer as the first learner got. The teacher now emphasised the importance of reducing fractions before adding or subtracting and also using the LCD instead of the ECD. Some learners after using the ECD failed to even simplify the expression because the expression became complicated. They now appreciated why we must use the LCD. Learners were then given another worksheet to do in groups on algebraic fractions before they could write the post-test.

This intervention was done to address the challenges that were discovered in the pre-test. During the intervention, learners brought to light some errors and misconceptions that were unclear to the researcher after marking the pre-test. The intervention improved learners' proficiency in common fractions as seen in the learners' performance on worksheets given during the lessons. This improved proficiency in algebraic fractions as demonstrated by learners during the lessons.

4.6. Post-test results

The questions answered by the learners are shown in Part C of the research instrument in Appendix 2, PTQ2.As before, the post-test Part C comprised twelve questions which measured the learner's ability to perform different operations on common and algebraic fractions and 30minutes were given to complete the section. The demographic characteristics of the learners reported in Section 4.1 are time-invariant while the use of calculators, attitudes and perceptions of the learners reported in Section 4.3 might have changed because of the intervention.

QUESTION 5

This question required learners to reduce the following fraction to its lowest terms :$\frac{2}{6}$.

As expected, all learners (100%) did very well in this question after the intervention suggesting that the lessons might have had an impact in terms of equipping the learners with the necessary skills and understanding of the concepts.

QUESTION 6

This question asked learners to calculate different common fractions from (a) to (d) on addition, subtraction, multiplication and division.

(a) $\frac{3}{8} + \frac{4}{5}$

The question was done very well with 24 learners (92.3%) getting it correctly but 12 of these used the LCD as the common denominator while the other 12 used the ECD. Two learners (7.7%) did not get it correctly but managed to get the CD. They showed a lack of conceptual understanding in putting the fractions under the same denominator as in, $\frac{3}{8} + \frac{4}{5} = \frac{3+4}{40} = \frac{7}{40}$.

(b) $3\frac{2}{3} - \frac{5}{6}$

In this question, 17 learners (65.4%) got correct answers, 2 used LCD, 12 used ECD but of these 12, 5 did not reduce answers to the lowest terms. Three of these 17 wrote answers only (might have used a calculator). The other learners (34.6%) did not get the question correctly and of these, 3 learners showed some of the steps correctly but made SME errors and hence the final answer was wrong. Six learners knew how to find the ECD but all of them could not change the mixed number into an improper fraction. This is similar to CE errors made by learners in question 2b Figure 4.5.

(c) $\frac{6}{5} \times 5\frac{2}{3}$

Sixteen learners (61.5%) got the question correctly but 3 of these 16 did not reduce answers to the lowest terms and one just wrote the answer. Ten learners (38.5%) did not get the question correctly and 2 of these learners partly got the question correctly but made some SME, which affected their final answer. Eight learners did not get the question correctly. Some of them failed to change the mixed number into an improper fraction like in (b) above. The other learners confused multiplication with the addition of fractions, which is an ME error similar to the one shown in question 2c Figure 4.7.

(d) $\frac{7}{9} \div \frac{2}{5}$

Eighteen learners (69.2%) got the question correctly but 2 just wrote the answer. Eight learners (30.8%) did not get the question correctly but 5 of these learners got some steps correctly as in changing division sign to multiplication sign and inverting the divisor. But, some showed ME errors similar to errors shown in question 2d Figure 4.8. These learners seemed to confuse the addition of fractions with multiplication. Three learners failed to get any correct steps. This is a result of a lack of conceptual understanding of how to divide fractions.

QUESTION 7

This question asked learners to reduce the following fraction to the lowest terms :$\frac{2+2x}{6}$

Sixteen learners (61.5%) got the question correct while 10 learners (38.5%) did not get the question correctly of which one of the learners got some steps correctly but made an SME. Seven learners (26.9%) did not get it correctly and these learners still showed CE similar to those shown in question 3 Figure 4.9. In addition to the CE and MLE errors learners made in this question, they also made OOE errors where they just divided the first term in the numerator by 2 and also the denominator, that is, $\frac{\cancel{2}+2x}{\cancel{6}}$ =$\frac{1+2x}{3}$ instead of factorising the numerator first. Two learners (7.7%) did not even attempt to write the question.

QUESTION 8

This question asked learners to simplify different algebraic fractions from (a) to (f) on addition, subtraction, multiplication and division.

(a) $\frac{3x}{8} + \frac{2x}{5}$

Eighteen learners (69.2%) got the question correctly and of these 18, 3 used LCD while 15 used ECD. The remaining 8 learners (30.8%) did not get the question correctly and of these,6 partly got the question correctly but made some SME, which affected their final answer. Two learners did not get any steps correctly; they made CE errors. Errors made in this question were similar to those made in question 8a Figure4.10.

(b) $3\frac{2}{3}\mathrm{x}-\frac{5}{6}\mathrm{x}$

Seventeen learners (65.4%) got the question correctly. Of these, one used LCD while 13 used ECD. Of these 13, 6 did not reduce their answers to the lowest terms. From the 17, 3 just wrote the answer. Nine learners did not get the question correctly and of these, 3 learners got some steps correctly but made some SME. Six learners did not get any steps correctly and these learners made similar errors as they made in 6(b). They failed to change the mixed number into an improper fraction.

(c) $\frac{6}{5}\mathrm{x}\ \times 5\frac{2}{3}\mathrm{x}$

Eight learners (30.8%) got the answer correctly but one of them did not reduce the answer to the lowest terms. The other 18 learners (69.2%) did not get the question correctly and 6 of these learners committed CE to expanding algebraic expressions. They did not comprehend that when you multiply x by x, you get x^2 instead of x. Ten learners failed to get any steps correctly. From these 10, one learner still committed the ME of not realising that in a fraction like $\frac{6}{5}\mathrm{x}\ \times 5\frac{2}{3}\mathrm{x}$, the x is in the numerator not in the denominator. Another learner made an MLE similar to the ones made in question 6c. Seven of the 10 learners made the same ME errors they made in 4c Figure 4.11 and one learner made a misconception error similar to the one shown in Figure 4.11.

(d) $\frac{7}{9}\mathrm{x}\ \div \frac{2}{5}\mathrm{x}$

Eight learners (30.8%) were able to do the question but only 2 reduced answers to the lowest terms; the rest left the answer as follows: $\frac{35\mathrm{x}}{18\mathrm{x}}$. Eighteen learners (69.2%) failed to do the question and of these, 8 learners were able to do some of the steps but did not invert the x in the numerator of the divisor. They still show an ME. Three other learners also made an ME where they use the concept for addition similar to errors made in question 4d Figure 4.12. Seven learners did not get any steps correctly.

(e) $\frac{2\mathrm{x}-3}{6}+\frac{\mathrm{x}+2}{8}$

Thirteen learners (50%) got the question correctly but 4 used LCD while 9 used ECD of which 5 did not reduce their answers to the lowest terms and 1 committed an SME. The other 50% of the learners did not get the question correctly but 7 of these could find the LCD or the ECD but made a CE in putting fractions under the same denominator. Some made CE errors in simplifying algebraic expressions. Five learners did not get any correct steps. Some made LCD or ECD errors while others made MLE errors. One learner did not attempt the question. All errors made in this question were similar to those exhibited by learners in question 4e.

(f) $\frac{5}{x} - \frac{x-1}{x^2-x}$

Five learners (19.2%) got the question correctly even though 1 (one) did not reduce the answer to the lowest terms and used the ECD. Most of the learners (80.8%) did not get the question correctly but 7 learners partly got the question correctly like factorisation. However, they failed to reduce the fraction to the lowest terms. Other learners used the ECD and failed to simplify the fraction since it became complicated. Twelve learners did not get any correct steps while others did ECD or LCD errors similar to the ones made in question 4f Figure 4.14.

4.7. Comparison of errors and performance before and after intervention

Table 4.6 below shows the frequency of errors that learners showed in pre-test and post-test. According to Table 4.6, learners made some fewer errors in the post-test than in the pre-test. Most of the errors that learners had committed in the pre-test had gone down in the post-test, that is the LCDE, CE, ME and MLE but the SE, SME and OOE had gone up.

Table 4.6: Frequency of errors made by learners in pre-test and post-test

Pre-test								**Post-test**							
Error code								**Error code**							
Question	**LCDE**	**CE**	**SE**	**SME**	**ME**	**OOE**	**MLE**	**Question**	**LCDE**	**CE**	**SE**	**SME**	**ME**	**OOE**	**MLE**
1	N/A	0	5	0	0	0	1	5	N/A	0	0	0	0	0	0
2a	1	1	0	0	0	0	2	6a	0	1	1	0	0	0	0
2b	3	6	1	3	0	0	4	6b	0	4	5	3	0	0	2
2c	N/A	7	0	2	8	0	0	6c	N/A	6	3	0	4	0	2
2d	N/A	10	0	2	5	0	2	6d	N/A	1	0	0	7	0	0
3	N/A	11	0	0	0	1	2	7	N/A	3	0	0	0	3	3
4a	2	4	1	1	3	0	2	8a	0	2	0	7	0	0	0
4b	2	5	1	1	0	0	6	8b	0	1	6	3	0	0	5
4c	N/A	4	0	0	7	0	10	8c	N/A	1	6	3	5	0	3
4d	N/A	4	0	0	8	0	11	8d	N/A	2	7	0	12	0	2
4e	2	7	6	3	2	1	6	8e	1	4	7	2	1	1	2
4f	8	7	2	0	0	0	5	8f	1	10	2	3	0	0	2
Total N°. of errors	**18**	**66**	**16**	**12**	**33**	**2**	**51**		**2**	**35**	**37**	**21**	**29**	**4**	**21**

LCDE – Lowest Common Denominator Error, CE – Conceptual Error, SE – Simplification Error, SME – Silly Mistakes Error, ME –Misconception Error, OOE – Order of Operation Error, MLE – Meaningless Error
N/A – Not applicable

Figure 4.18 shows the comparison in performance in common fraction question versus algebraic fraction, that is, question 5 versus question 7 in reducing fractions to the lowest terms. Question 5 is a common fraction while question 7 is an algebraic fraction. Learners did very well in reducing the common fraction to the lowest terms than the algebraic fraction but there was an improvement from the pre-test results. A similar trend was also observed for question 6 (common fractions) and question 8 (algebraic fractions).

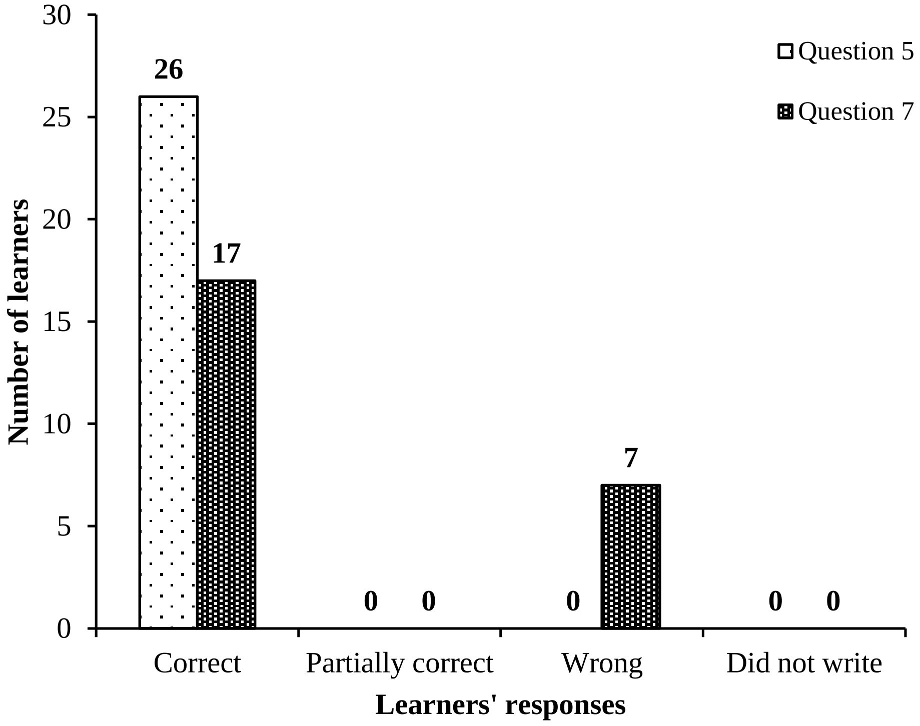


Figure 4.18: Post-test performance of learners in question that required solving common and algebraic fractions with similar formats.
Question 5 format $\frac{2}{6}$ versus 7 format $\frac{2+2x}{6}$

From Figure 4.18, as the learners' proficiency in common fractions improved so did the proficiency in algebraic fractions. Comparing Figures (4.15 and 4.16) with Figure 4.18, there was an improvement in learners' performance in both common and algebraic fractions because of the intervention. Table 4.7 below also summarises the comparison in performance in the pre-test and post-test per question.

Table 4.7: Comparison in performance of pre-test and post-test questions

Pre-test			Post-test		
Question	**Correct response**	**Wrong response**	**Question**	**Correct response**	**Wrong response**
1	18(69.2%)	8(30.8%)	5	26(100%)	0(0%)
2(a)	22(84.6%)	4(15.4%)	6(a)	24(92.3%)	2(7.70%)
2(b)	13(50%)	13(50%)	6(b)	17(65.4%)	9(34.6%)
2(c)	4(15.4%	22(84.6%)	6(c)	16(61.5%)	10(38.5%)
2(d)	14(53.8%)	12(46.2%)	6(d)	18(69.2%)	8(30.8%)
3	9(34.6%)	17(65.4%)	7	16(61.5%)	10(38.5%)
4(a)	12(46.2%)	14(53.8%)	8(a)	18(69.2%)	8(30.8%)
4(b)	10(38.5%)	16(61.5%)	8(b)	17(65.4%)	9(34.6%)
4(c)	4(15.4%)	22(84.6%)	8(c)	8(30.8%)	18(69.2%)
4(d)	0(0%)	26(100%)	8(d)	8(30.8%)	18(69.2%)
4(e)	6(23.1%)	20(76.9%)	8(e)	13(50%)	13(50%)
4(f)	0(0%)	26(100%)	8(f)	5(19.2%)	21(80.8%)
Total(Actual)	112(35.9%)			186(59.6%)	
Total(Expected)	312(100%)			312(100%)	

According to the table above, it was clear that performance in the post-test had improved, for instance, comparing questions that were testing the same concepts in questions 1 and 5, the percentage of correct answers increased from 69.2% in the pre-test to 100% in the post-test. This trend could be seen for all the similar questions. In addition, the total for all correct responses increased from 35.9% in the pre-test to 59.6% in the post-test.

4.8. Interview results

4.8.1. Leaners' responses

The results of the learners' responses are shown in Appendix 7, IR1. The responses came from 5 predetermined questions: questions 1, 2, 3, 4 and question 6 while 4 responses came from questions that emerged from the post-test meaning questions 5a, 5b, 5c and 5d. The researcher chose responses from three categories, that is, 4 learners from low achievers, 4 middle achievers and 4 high achievers.

Most of the learners admitted that they continued to use a calculator or ECD instead of LCD because it was something that they were used to and it was also easy for them. From learners' responses, most learners had challenges with algebraic fractions. Learners also show that reducing fractions to the lowest terms was a challenge for them. The concept of multiplying and dividing algebraic fractions was still a challenge to learners; maybe they needed more practice.

4.8.2. Teachers' responses

Appendix 7, IR2 summarises teachers' responses to interview questions. Each teacher was given a code from T1 to T5. From their responses, teaching experience ranged from five years to ten years and four of the five teachers indicated fractions to be amongst the difficult topics. Two of the teachers did not indicate fractions as a difficult topic but gave a lot of challenges to when learning fractions especially algebraic fractions. Most of the challenges that the five teachers highlighted complemented what the researcher observed in this study and findings from other studies. The teachers were asked to explain how they taught fractions and three of the teachers indicated that they used rules and algorithms where learners were required to just know how they solved fractions. Learners are not allowed to learn through problem-solving and from discovering. Two of the teachers used models to capture learner interest. All the teachers felt that their grade ten learners still lacked the conceptual understanding of common and algebraic fractions even though they did better in common fractions than in algebraic fractions.

4.9. Impact results

In this section, we demonstrated the impact by testing for significant differences before and after interventions and using regression analysis techniques controlling for time-variant factors. All the variables measured in the experiment were time-invariant except for the learners' perceptions and use of calculators before and after the intervention besides the learners' scores.

4.9.1. Test for significant difference in the test scores

Table 8 presents the results of the t-test for the significance difference between the pre-test and post-test scores. The difference of 9.15 marks was significant at the 1% level of significance. This result confirmed that the intervention had a positive and significant impact on the test scores of the learners, which resulted in an increase in performance by 9 marks.

Table 4.8: Test for significant differences between pre-test and post-test results

Variable	Obs.	Mean	Std. Err	Std. Dev	95% Conf. Int.
Posttest	26	20.5	1.47	7.50	17.5 – 23.5
Pretest	26	11.4	1.36	6.95	8.54 – 14.2
Diff	26	9.15***	1.25	6.39	6.57 – 11.7

```
ttestposttest == pretest
mean(diff) = mean (posttest - pretest)
degrees of freedom =      25
t =   7.3096
Ho: mean(diff) = 0
Ha: mean(diff) < 0          Ha: mean(diff) != 0          Ha: mean(diff) > 0
Pr(T < t) = 1.0000        Pr(|T| > |t|) = 0.0000         Pr(T > t) = 0.0000
```

Obs. – Observations
Std. Err – Standards error
Std. Dev – Standard deviation
Conf. Int – Confidence interval

Table 4.9 presents the results of the t-test for significance differences in learner perceptions before and after the intervention. The differences in the perception indexes are significant at the 1% level of significance. Similarly, we concluded that the intervention had a positive and significant impact on the perceptions of the learners. Although smaller in magnitude, the perception index increased by about 0.19 points from 0.65 before the intervention to 0.85 after the intervention.

Table 4.9: Test for significant differences in perceptions about algebraic fraction before and after intervention

Variable	Obs.	Mean	Std. Err	Std. Dev	95% Conf. Int.
Perception before (bpercep index)	26	0.85	0.02	0.11	0.80 – 0.89
Perception after (apercep index)	26	0.65	0.04	0.21	0.57 – 0.74
Diff	26	0.19***	0.05	0.23	0.10 – 0.29

```
ttestapercep == bpercep
Paired t test
mean(diff) = mean(apercep - bpercep)
degrees of freedom =      25
t =   4.1861
Ho: mean(diff) = 0
Ha: mean(diff) < 0          Ha: mean(diff)!= 0          Ha: mean(diff) > 0
Pr(T < t) = 0.9998        Pr(|T| > |t|) = 0.0003         Pr(T > t) = 0.0002
```

Obs. – Observations
Std. Err – Standards error
Std. Dev – Standard deviation

Conf. Int – Confidence interval

Table 4.10 presents the results of the test of significance difference in proportions for the use of calculators before and after the intervention. The difference in proportion was significant at the 5% level of significance. The intervention had a negative and significant impact on calculator use, that is, it resulted in a reduction of 23% in using calculators from 0.69 before to 0.92 after the administration of planned lessons.

Table 4.10: Test for significant differences in calculator use before and after intervention

Variable	**Obs.**	**Mean**	**Std. Err.**	**Z**	**P>|z|**	**95% Conf. Int.**
Use after (b7b_uc)	26	0.92	0.05			0.82 – 1.03
Use before (b7a_uc)	26	0.69	0.09			0.51 – 0.87
Diff	26	0.23**	0.10			0.03 – 0.44
Under Ho:			0.11	2.11	0.04	

```
Two-sample test of proportions
rtest b7b_uc == b7a_uc
diff = prop(b7b_uc) - prop(b7a_uc)
z =   2.1112
Ho: diff = 0
Ha: diff < 0              Ha: diff != 0              Ha: diff > 0
Pr(Z < z) = 0.9826        Pr(|Z| > |z|) = 0.0348        Pr(Z > z) = 0.0174
```

Obs. – Observations
Std. Err – Standards error
Std. Dev – Standard deviation
Conf. Int – Confidence interval

4.9.2. Regression analysis

Table 4.11 shows the determinants of the pre-test scores. The model is significant at 10%. The variables that are significant in this model are the age of the household head, the level of education of the siblings, time devoted to studying and household wealth status. The age of the household head and time devoted to studying were negative and significant at 10% while the sibling's level of education and household wealth were positive and significant at 5%.

Table 4.11: Determinants of pre-test score

Number of Obs.	**25**	
Prob>F	**0.0557**	
R-squared	**0.795**	
Root MSE	**4.771**	
Pretest	**Coef.**	**Std. Err**
Age	1.356	1.474
Gender	-4.711	4.333
Distance to school	-1.391	0.952
Household size	-1.393	0.692
Age of household head	-0.250*	0.192
Employment status of household head	-2.817	3.074
Parent’s education	0.48	1.15
Sibling’s level of education	2.77**	1.04
Time devoted to household chores	-1.404	0.840
Time devoted to studying	-3.231*	1.551
Wealth status (asset index)	15.06**	7.636
Perception before	7.647	6.567
Cons	4.422	25.64

Obs. – Observations
Coef. – Coefficient
Std. Err – Standards error
Cons – Constant
MSE – Mean square error

Table 4.12 shows the impact without the intervention by regressing a dummy variable (0=no intervention, 1 = intervention) on the post-test scores. This result confirmed the difference between the post-test and pre-test in Table 1. The intervention increased the learner marks by 9.15 points. The constant was large and significant at a 1% level of significance.

Table 4.12: Impact of the intervention

Number of Obs.	52	
Prob>F	0.000	
R-squared	0.296	
Root MSE	7.368	
Posttest	**Coef.**	**Std. Err.**
Dummy (0=no intervention, 1 = intervention)	9.154***	2.005
Cons	11.35***	1.418

Obs. – Observations
Coef. – Coefficient
Std. Err – Standards error, Cons – Constant, MSE – Mean square error

Controlling for learners' perception and the use of calculators, the results in Table 4.13 showed that the impact of the intervention increased by a magnitude of 0.26 from 9.15 to 9.41 points.

Table 4.13: Impact of the intervention after controlling for perceptions and use of a calculator

Number of Obs.	52
Prob>F	0.000
R-squared	0.294
Root MSE	7.228

Post-test	**Coef.**	**Std. Err.**
Dummy (0=no intervention, 1 = intervention)	9.407***	2.446
Perceptions	-1.879	6.274
Use of calculator	-0.471	2.713
Cons	13.01**	5.076

Obs. – Observations
Coef. – Coefficient
Std. Err – Standards error
Cons – Constant
MSE – Mean square error

In conclusion the main findings of the study were that: proficiency in common fractions improves proficiency in algebraic fractions, learners preferred to use the easiest common denominator (ECD) in addition and subtraction of fractions instead of using the lowest common denominator (LCD). Besides, the current study showed that the use of a calculator was a barrier in learning fractions since learners were reluctant to learn and internalise the concepts of common fractions from the basics. The next chapter discusses the results and the findings in line with the aim, objectives and questions of the study.

CHAPTER 5

DISCUSSION OF RESULTS AND FINDINGS

5.1. Introduction

This chapter discussed the results in line with the research aim, objectives and research questions. The findings from this study were compared with findings from other studies to see if there were similar trends.

5.2. Discussion

The current study was envisaged to address three questions. The first question enquired on how the lack of conceptual understanding of common fractions affected performance achievement in algebraic fractions. The findings from the study revealed that learners who had not mastered the concepts of common fractions failed to solve common fraction questions and the algebraic fractions of similar formats. On the other hand, learners who had mastered the common fractions concepts could apply the concepts in algebraic fractions of similar formats. The second question attempted to explain the barriers that learners encounter when learning algebraic fractions. This question was addressed when it came to light that the use of a calculator and using ECD affected the learners' ability to solve algebraic fractions questions. Using a calculator and the ECD by the learners acted as barriers given that learners were adamant in solving common fractions from first principles, which could help them in solving algebraic fractions. Lastly, the current study endeavoured to answer which pedagogical strategies could improve understanding and problem-solving skills in algebraic fractions. This study revealed that after applying pedagogical strategies as intervention lessons that are reviewed under the literature review section, learners improved significantly in the post-test results. In the subsequent paragraphs, the findings are discussed.

The demographic characteristics of learners had no direct impact on the outcome variable of interest in this study since they were time-invariant suggesting that the experiment controlled for such factors. The only variables that could bear the low performance in Mathematics and difficult concepts are the learners' perceptions and attitude towards common and algebraic fractions and use of calculators.

The mean age of the learners is consistent with the mean age of learners in grade 10 and 11 in South African rural and urban schools (Spaull, 2015). This could be attributed to the fact that learners repeat grades along the way. The results show that there were more female than male learners which was expected because of the high dropout rate of male students in the country who choose to look for employment instead of continuing with education (Motala et al., 2009). In South Africa, dropout has reached a national crisis. Approximately, 60% of first graders will ultimately drop out rather than complete 12th grade. Likewise, by grade 12, only 52% of the age-appropriate population remains enrolled (Department of Basic Education, 2015). The results also show great variability in the ages of the household heads suggesting that we could find very old and young guardians in the learner sample. Coleman (2018) observed that older people are less likely to assist their children with homework compared to younger parents. In the current study, most of the learners indicated that they did not get assistance with their homework, which might explain why learners did not perform well on challenging topics like fractions. The average walking distance to school for most learners was 2.6 km, which was consistent with most rural schools in South Africa (Simons et al., 2018). Simons et al. (2018) also found that many learners in rural areas in Africa stay away from school, which might have serious implications on educational outcomes as they spend most of their time travelling, arrive at school late and are exhausted.

The result that most households are female-headed was typical of many rural areas in South Africa and the rest of the African continent since men tend to look for greener pastures (Weybright et al., 2017). The indication by the learners that the majority of the household heads were their mothers could be favourable since women tend to care more about success and the educational outcomes of their children in the African context (Shabaya and Konadu-Agyemang, 2004). Weybright et al., (2017) also observed that women spend more money on household activities, education, health and child nutrition than their male counterparts. Most household heads were unemployed, which could be an indication that most of the learners came from very poor backgrounds. This could have an impact on their learning outcomes since they might not have access to educational tools that could facilitate learning of fractions such as calculators, computers and the internet at home (Umugiraneza et al., 2018). Examples of objects that could depict fractions in the classrooms could also be affected by the very fact that these learners' households could not afford some of the items like pizzas, cakes or chocolates which could be used in sharing and division of fractions. This result is consistent with other studies done in rural areas in South Africa (Brijlall et al., 2011) and the region as a whole (Taukeni, 2019).

The level of education of most parents and siblings of learners was below matric, which might imply they were not in a capacity to help those learners with homework and not to mention difficult concepts like algebraic fractions. Another finding was that most learners did not have anyone to help them with homework which can also be the reason why learners fail to practice challenging concepts like fractions. Once they try alone and fail at home, they eventually give up because of frustration.

Scholars reported that frustration can result in negative attitudes towards a certain subject which they perceive to be difficult (Rikhotso, 2015). As a result, learners may end up concentrating on the subject or concepts they enjoy or understand at the expense of the difficult subjects or concepts. This behaviour is observed across grades, schools (rural or urban) and regions (Kim, 2020). The negative perceptions of learners towards Mathematics and Science subjects, in general, have been observed to impact negatively on their attitude and performance not only in developing countries but also in first world countries (Mata et al., 2012). Factors that can influence Mathematics performance are demonstrated by Kupari and Nissinen (2013) when they show that poor performance in Mathematics is a function of cross-factors related to students, teachers and schools. Among the students' factors, attitude is regarded by many researchers as a key contributor to higher or lower performance in Mathematics (Mohamed and Waheed, 2011, Mata et al., 2012, Ngussa and Mbuti, 2017). Attitude refers to a learnt tendency of a person to respond positively or negatively towards an object, situation, concept or another person. Attitudes can change and develop with time and once a positive attitude is formed, it can improve students' learning. In the current study, most learners had negative attitudes and perceptions towards fractions but through the intervention, most learners changed their attitudes.

In line with the arguments above, the amount of time spent studying and performing household chores could potentially affect performance in mathematical and other science subjects. These low levels of study time and erratic behavioural patterns observed in this study are also reported by previous researchers working in other African countries (Darling-Hammond et al., 2020). Coupled with erratic behavioural pattern, student participation in household chores are observed in the literature as the main drivers of poor performance in

Mathematics and Science subjects (Moyana, 1996). Many children suffer from unpredictable home environments such as parents being arrested for always quarrelling due to substance abuse. The presence or availability of parents is crucial since they provide information, learning opportunities, behavioural models and connection to other resources (Rammala, 2009). The absence of such support severely limits these transactional protection processes and results in learners having low self-esteem. According to Saiduddin (2003), educators should create a positive school environment for learners to feel at home at school in such a way that they can openly discuss what prevents them from performing to the required standards. Hence, the involvement of parents would allow the school to seek assistance from relevant authorities to provide the necessary intervention (Rammala, 2009).

To change the learner's perception or attitude, a lot of time and practice is needed so that learners develop the necessary cognitive skills, which enhance their understanding of difficult concepts (Mata et al., 2012). This should start from lower grades and should be ongoing until all learners appreciate the strategies used (Darling-Hammond et al., 2020). In teaching incremental concepts, more time should be invested in early concepts so that students can develop these skills at an early stage. Serdyukov (2017) observed that students who master concepts at lower levels are also likely to excel at higher levels when the concepts become more abstract as they mature and develop cognitive skills.

In this study, learners showed that if they developed a good understanding of common fractions, that is adding, subtracting, multiplying and dividing fractions, it became easy to transfer that knowledge to algebraic fractions especially those that did not require other concepts like factorisation. Learning fractions must follow Bloom's taxonomy of cognitive development (Kliegr et al., 2021, Pickard and Education, 2007). Learners must be given resources and guidelines so as to come up with the definition of what fractions are which must be the first lesson followed by, multiplying, dividing and then adding and subtracting common fractions before they can tackle algebraic fractions. The constructivist perspective advocates for quality prior knowledge and the use of scaffolds in learning (Hmelo-Silver and Eberbach, 2012). The researcher endeavoured to make the learning of common fractions be the prior knowledge for the learning of algebraic fractions and the models were used as the scaffolds which were then removed gradually as the learners were introduced to rules and algorithms. By using the scaffolds, the researcher hoped that the learners would develop the conceptual understanding of fractions before procedural understanding, which Skemp (1976) referred to as relational and instrumental understanding. It was hoped that if learners could understand why they did some procedures before they knew how they did those procedures, then it could make more sense. Chinnappan and Forrester (2014) argue that procedural knowledge supports instrumental understanding while conceptual knowledge supports relational understanding.

The researcher planned the intervention in such a way that learners could bring out the challenges they face in solving fractions, especially algebraic fractions. The researcher also allowed learners to explain their answers and gave them an opportunity to learn from discovering so that they could have an (Aha) experience. According to Van Lange et al. (2013), an aha experience is an emotional response that takes place whenever someone unexpectedly realises or fully comprehends something which baffled them before. Once learners learn from discovery and have the aha experience, they will master the concept and are motivated to explore more. The researcher used the stance that the teacher is not the source of knowledge and everyone is a learner. The researcher borrowed the constructivists perspective that mathematical knowledge does not simply exist out of their waiting to be discovered but it is constructed by learners using resources in their environment. The researcher gave learners resources to construct knowledge on fractions and allowed them to critically question why some procedures were done. Questions on problem-solving were given to develop learners’ thinking skills.

The use of a calculator was another interesting finding. A good number of learners indicated that they used a calculator to solve fractions in the pre-test and also gave a reason that they did so since it was easy and hustle free. This was reported in the interview. In the post-test, the number had gone done and those who continued saw no reason why they could struggle when it could be done so easily. Based on previous empirical accounts, the researcher had the notion that using a calculator to simplify fractions affects the conceptual understanding of learners on difficult concepts like fractions (Vinner, 2014). If learners use a calculator, they find no reason in understanding how and why certain procedures are done. For instance, finding the LCD is important when adding and subtracting fractions. When learners use a calculator to solve common fractions, they do not realise that the tool is limited when it comes to solving algebraic fractions which require deriving from first principles (Mutsvangwa, 2016). Mutsvangwa (2016) investigated to prove that a calculator can improve the conceptual understanding of fractions but agrees that it is important for learners to know how and why procedures are done in cases where the calculator is limited. Zulfa et al. (2020) also reiterated that understanding fractions is vital in other mathematical concepts such as trigonometry and algebraic fractions and in these mathematical concepts, a calculator will not provide the final answer. The conceptual understanding of solving common fractions from first principles is needed for learners to develop a relational understanding when solving algebraic fractions.

Calculators had been introduced prematurely such that to discourage them at a later stage it becomes futile. Thus, the saying ‘it is difficult to teach an old dog new tricks’. Therefore, it is paramount to teach fractions from first principles, starting from lower levels and other concepts that need conceptual understanding before calculators can be introduced. The intervention helped to reduce the number of learners who used calculators in solving common fractions, which improved proficiency in algebraic fractions. Similar sentiments were echoed by (Kellman et al., 2008, Brown and Quinn, 2007, Ford, 1994).

The results from the interviews with the learners suggested less confidence in the learners dealing with both common and algebraic fractions but very high confidence with the teacher. This was a good indication that the teacher’s performance could not have influenced the results of this study. However, previous studies reported that some Mathematics teachers in South Africa show little understanding and appreciation of mathematical concepts such as common and algebraic fractions (Ubah and Bansilal, 2018). These studies recommended in-service teacher development programmes to equip them with the necessary skills and understanding of mathematical concepts (ibid).

The study identified several errors that were made by learners while solving both common and algebraic fractions. The most common errors in the literature were Lowest Common Denominator Error (LCDE), Conceptual Error (CE) and Simplification Error (SE). However, Silly Mistake Errors (SME), Misconception Errors (ME), Order of Operation Errors (OOE) and Meaningless Errors (MLE) were identified by the researcher as part of the analysis. Of these seven errors, the LCDE type is the most common (Makonye and Kanyile, 2015; Mhakure et al., 2014). The researcher realised that learners preferred to use the ECD to avoid the LCD, which they felt was difficult to find. While they managed to solve most of the fractions using ECD, they tended to make other mistakes like (SE). Watanabe (1991) also noted that some learners use shortcuts to solve mathematical problems which result in errors.

Brown and Quinn (2007) noted the same errors when learners did not understand that they cannot take out a factor if they are not multiplying or dividing fractions (Maelasari and Jupri, 2017).

In the current study, learners were not simplifying answers to lowest terms and some failed to simplify algebraic fractions to the lowest terms. Most of the learners were unaware that if you simplify fractions before performing the four mathematics operations, it becomes easier especially when it comes to algebraic fractions. During interviews with learners, it came to light that the simplification error is a result of a misconception that learners have. Learners believed that to reduce a fraction, they divided both the numerator and the denominator by 2. These learners lacked the understanding that what was needed was the HCF of both the numerator and the denominator to reduce a fraction to the lowest terms. Learners also made conceptual errors where they showed a lack of conceptual understanding of how to multiply and divide fractions. The teacher participants also noted the same challenges with their learners while Baidoo (2019) reiterated the same challenges in a study he carried out.

Identified amongst other errors were SME errors that show that learners pay no attention to the arithmetics of fractions or numbers in general and do not check their work before submitting it. As noted in previous studies, this could be a sign of laziness, panic, rushing or lack of care (Ben-Hur, 2006). These same kinds of errors were referred to as careless errors and claimed to contribute to low performance in Mathematics (Kanyile, 2016). Lukhele et al. (1999) termed these as unsystematic errors, which are exhibited without the intention of learners; learners may not repeat such errors and learners can correct them independently. The OOE errors were also shown by learners but the researcher was only able to identify these errors during the intervention and interviews when strategies like probing and rephrasing of questions were done. Otten et al. (2008) classified this kind of simplification error as “no recognition of the common factor”. Misconception errors were also prominent in most learners’ solutions.

The most common misconceptions were in multiplication and division where learners used the LCD like they do in addition and subtraction of fractions. Teachers who were interviewed also reiterated the same challenges. Other studies on fractions also identified these errors (Otten et al., 2008). The other misconceptions came from learners’ lack of knowledge in the simplification of algebraic expressions. Some learners had challenges with factorisation, which impacted their solving of algebraic fractions even if they had mastered the concepts of common fractions. From the researchers' point of view, it is important to learn algebra before doing algebraic fractions to lessen the burden of learning two difficult concepts concurrently. Noted amongst other errors were meaningless errors (MLE) where learners wrote solutions that did not make any sense or did not correlate with the problem. It came to light during the interview that some of these MLE errors were a result of CE errors and ME errors. After much probing and rephrasing of questions, learners revealed their challenges even though some still failed to explain how they came up with their solutions.

Overall, these results suggest that the intervention had an impact on learner's attitudes and perceptions towards fractions. In particular, the study revealed that learners who have mastered the arithmetic of common fractions were more capable when dealing with the arithmetic of algebraic fractions. This evidence is demonstrated by a significant difference in the pre-test and post-test scores using the learner t-test and regression analysis techniques. This result applies in many areas be it Mathematics or in Science in general where concept starts at lower levels and advance in terms of difficulty as the learner progresses to higher levels (Lewis and Smith, 1993). These authors recommended that more time should be invested by teachers to impart the necessary skills at lower levels before moving to more abstract concepts which are also difficult to teach. Other studies established that interventions such as extra lessons funded by parents outside normal school work might help to improve learners' ability to deal with difficult concepts but at an extra cost (Darling-Hammond et al., 2020).

Learners' proficiency in common fractions had considerably improved, which in turn had improved proficiency in algebraic fractions. Even though there was this improvement, the intervention should start from lower levels so that by the time algebraic fractions are introduced, learners are already proficient in common fractions. In the more difficult algebraic fractions questions, the improvement was not all that much pronounced since learners had problems with other concepts like factorisation, which were not part of the intervention and were not dealt with in detail. The interviews helped to bring to light some of the challenges learners had with algebraic fractions. From the learners' work, it was not easy to deduce why they continue to make the same errors but through the interview, they were able to elaborate on their challenges. This chapter discussed the results and finding while the next chapter deals with the summary, conclusions and policy implications.

CHAPTER 6

SUMMARY, CONCLUSIONS AND POLICY IMPLICATIONS

6.1. Introduction

This chapter summarises the major findings of the study on the relationship between proficiency in common fractions and algebraic fractions, the challenges learners face in algebraic fractions and the misconceptions and errors learners do in solving fractions. It also gives detailed recommendations and conclusions, which can be of benefit to the Department of Basic Education (DBE), Mathematics educators and future researchers in the teaching and learning of common and algebraic fractions.

6.2. Summary of the findings

The results of the study revealed that proficiency in common fractions improves proficiency in algebraic fractions. The results of the post-test showed that a learner who did well in questions on common fractions also performed well in algebraic fractions of the same level of difficulty. It also came to light that the higher-order algebraic fractions needed proficiency in other concepts like factorisation, which needed to be taught beforehand and also learners' inability to simplify fractions to the lowest terms made addition, subtraction, multiplication and division of algebraic fractions difficult. The other major finding was the preference of learners to use the ECD in addition and subtraction of fractions instead of using the LCD. This had shortfalls when dealing with algebraic fractions since learners failed to simplify the algebraic fractions. They became complicated in some cases. Noted amongst other findings were the errors and misconceptions that learners do as they solve fractions. For instance, learners tend to use the concept of LCD or ECD in multiplying or dividing fractions and when reducing fractions to the lowest terms learners believe that the fraction should be divisible by 2. Learners also do not treat letters as numbers, which shows that their algebra concepts needed to be improved before doing algebraic fractions.

The use of a calculator proved to be a barrier to the learning of fractions. Learners were reluctant to learn and internalise the concepts of common fractions from the basics. They felt it was a worst of time to go through the rigorous process of finding LCD and show all the steps on how to put the fractions under the same denominator when a calculator can give them the final answer in a second. They did not realise that those basics will be needed when dealing with algebraic fractions. It takes good teaching strategies and resources to overcome this barrier. The intervention was done in such a way that learners changed their perceptions and attitudes towards fractions, which improved their performance in both common and algebraic fractions as shown in the post-test results.

6.2.1. Summary of pre-test results

The researcher did a preliminary analysis of the pre-test results before implementing the intervention. In the analysis, the researcher identified the following problems that needed attention. First, learners had problems in reducing fractions to their lowest terms especially when they are algebraic. Second, learners preferred to use the ECD instead of the LCD and usually fail to reduce the fractions to their lowest terms. When fractions are algebraic and learners use the ECD to simplify, sometimes the problem became complicated hence they failed to simplify. Third, some learners did not have a conceptual understanding of how to solve common fractions. They only had procedural understanding, hence they failed to apply their understanding to algebraic fractions. Last, learners also lacked the conceptual understanding of how to simplify algebraic expressions, which impacted the simplification of algebraic fractions.

6.2.2. Summary of post-test results

From the preliminary analysis of the post-test results, learners had improved in overall performance in the post-test. The average performance in the pre-test was 11 marks while in the post-test it was 20 marks out of 35 marks. The percentage pass rate in the pre-test was 15% while in the post-test, it was 65%. There was also a significant improvement in solving common fractions and a remarkable improvement in solving algebraic fractions. All learners (100%) could reduce common fractions to the lowest terms while 61.5% of the learners reduced the algebraic fraction to the lowest terms. Out of those who failed to reduce the algebraic fractions, some could not factorise the algebraic expressions in the fractions and others were just careless. In questions 2(a) to 2(d), there was a moderate improvement while in the algebraic questions (4a to 4f), there was a remarkable improvement. The errors and misconceptions learners still had now dropped considerably even though SME errors had gone up perhaps because learners were rushing to finish and not paying much attention. The number of learners who just wrote answers (suggesting may have used a calculator) had gone down. The performance of learners in algebraic questions of higher order was still bad even though there was an improvement. Some learners continued to use the ECD instead of the LCD as the researcher had recommended but there was a significant improvement in that regard.

6.2.3. Summary of interview results

Results of the interviews revealed that learners struggle more with algebraic fractions than common fractions. Learners also brought to light the challenge they have with algebraic fractions that letters confuse them such that when dividing fractions, they do not invert a letter in the divisor. They also use the LCD or ECD when multiplying or dividing fractions and cannot reduce fractions to the lowest terms because they think only 2 must be the HCF for one to reduce a fraction to the lowest terms. This can result from systematic errors that were repeated or reconstructed over some time because of the grasp of the incorrect conception of solving a particular problem (Idris, 2011). Learners did not know the difference between the LCD and the ECD and some preferred to use the ECD than the LCD. Teachers also revealed the following challenges learners have with fractions: failure by learners to change mixed fractions to improper fractions and vice-versa, learners cannot find the LCD and learners cannot multiply and divide fractions. The methods that most teachers use when teaching the concept of fractions were more procedural than conceptual understanding as revealed by the teachers' responses in the interview about the methods they use when teaching fractions.

6.2.4. Summary of impact results

There was a significant difference in performance between the pre-test and post-test of 9.15 marks at a 1% level of significance. This confirms that the intervention was effective to some extent. A test for learner perceptions before and after intervention showed a significant difference at a 1% level of significance, which also confirms that the intervention had an impact on how learners view fractions. Now learners had positive perceptions. The proportion of learners who used calculators when solving fractions also dropped as a result of the intervention. There was a significant difference at a 5 % level of significance. The characteristics that determine the pre-test scores were modelled using the regression analysis and the significant variables were the age of the household heads, level of education of siblings, time devoted to studying and household wealth status. The age of household heads and time devoted to studying are negative at a 1% level of significance while siblings' level of education and household wealth status are positive at a 5% level of significance. The other tests done by regressing a dummy variable also showed that the intervention increased the marks of students by 9.15 points. Besides, controlling learners' perceptions and the use of a calculator showed that the intervention increased the marks from 9.15 to 9.41 points.

The results support both empirical and theoretical accounts. Speaking to previous empirical accounts, the results demonstrate proficiency in common fractions has a positive and significant impact on the performance in algebraic fractions, which highlight the need for teachers to spend more time on the former to build conceptual skills. This entails that, in learning abstract Mathematical concepts, learners will actively create, interpret and reorganise knowledge in individual ways. In this sense, the results support the theory on constructivism since learners need prior knowledge (of common fractions) for the learning new concepts (algebraic fractions) to take place. Theoretically, some of the errors observed could be explain by the mismatch that occurs when the goal of the teacher is to equip learners with the necessary skills so that they understand how to solve a mathematical problem in a relational manner but the learners' goal is to understand instrumentally and vice versa. This is also evidenced by the use of calculators to solve both common and algebraic fractions. The use of calculator is also supported throughout the analysis, first when the learners just provide and answer without showing all the working, some learners acknowledged the use of calculators during one-to-one interviews and by the student t-test results of significant differences. Furthermore, both the interview results and t-test agreed on the differences in perceptions before and after intervention.

6.3. Recommendations

The researcher suggests the following recommendations to Mathematics teachers, policymakers and future researchers in algebraic fractions:

1) Discourage the use of a calculator at the primary and GET phases until learners had mastered basic concepts of each topic

2) Teach fractions using models in a hierarchical order starting with the definition until the most difficult concept, which is the addition and subtraction of fractions from lower grades
3) Teach algebra before algebraic fractions
4) Teach learners how to find the LCD before you can introduce them to ECD from lower grades so that when they are doing algebraic fractions, they know which one to use
5) Policymakers to increase the time allocated for the teaching of fractions
6) There is a need for studies to be done on the impact of teacher developmental workshops on the teaching and learning of fractions.
7) A similar research study should be conducted on a larger scale to see if the same findings will be observed.

6.4. Conclusion

In conclusion, it became evident in this study that improving proficiency in common fractions improved proficiency in algebraic fractions of similar format and level of difficulty. This study focused on how proficiency in common fractions can be improved, which would improve proficiency in algebraic fractions in a class of grade 10 learners at a high school in Limpopo province. The challenges learners face in the learning of common and algebraic fractions were investigated. Misconceptions and errors learners made in solving common and algebraic fractions were evidence that mathematical knowledge cannot be transferred directly from the teacher to the learner like pouring something in an empty vessel but learners need to construct fractional knowledge using resources in their environment. If learners are given enough time with excellent teaching strategies, learners' proficiency in fractions can improve. In this study, it became evident that a learner who understands how to find the LCD and why to use the LCD instead of ECD or using a calculator, solves algebraic fractions. This study also allowed learners to learn fractions from a constructivist perspective where they developed the conceptual understanding of why they do some procedures and why those same procedures will not work in other cases. Last but not least, recommendations are given on how to improve the teaching and learning of common and algebraic fractions.

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APPENDIX 1: LESSON PLAN AND MATHEMATICS TEACHING PRACTICES

Sequential Lesson Plan: An Instructional Sequence of five Lessons on Fractions in Grade 10

Developed August 2019

By Lina Mangwende

LAP1

Learning Activity Plan N°: 1 – Introduction to Fractions

Estimated Time: **45 Minutes**

Good Math Practices:

- Establishing goals to focus learning
- Implement Tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Pose purposeful questions

(Adopted from NCTM)

ACHIEVEMENT TARGETS/ OBJECTIVES: **Assessments:**

• **Achievement Target #1:** Learners will define a fraction using knowledge gained from the lesson • **Achievement Target #2:** Learners will appreciate learning fractions and will value that fractions are useful in our everyday life, i.e., in sharing items, allocating time for lessons, dividing a piece of land or sharing an inheritance.	• **Pre-Assessment: Worksheet with items to identify fractions** • **Interim Assessment: worksheet on coming up with fractions from sharing.** • **Post-Assessment: Worksheet on identifying fractions using models.**

SPECIAL PLANNING/PREPARATIONS:

Procedures	**Time**	**Materials**	**Adaptations/Modifications for Learners with Diverse Needs**
Introduction: The teacher introduces the lesson by asking learners to give examples of fractions. The teacher then asks learners to define what is a fraction and explain what they understand about a fraction. The teacher will use probing and “focusing techniques” (NCTM, 2014, Ntuli, 2019) until learners give answers to the teacher's expectation.	10 Min.	Overhead projector	
Build the Knowledge: Learners are given activities where they use models to show that fractions are part of a whole. For instance, they can divide a shape like a circle into halve, a third, a quarter and so on. They can also divide themselves to do certain chores and come up with what fraction of the class does certain chores.	20 Min.	Papers, scissors and coloured pencils	

Apply the Knowledge: Give activities in which they divide the area or can share a cake to 2 people and then continue to increase the number of people and observe what will happen.	15 Min.	Cake	
Assess the knowledge: Learners will be given activities that will make them understand what fractions are.			
Conclude the lesson: The teacher will conclude the lesson by assigning learners to go and investigate what happens to the fraction if the denominator increases.			

Integration of Technology: Using overhead projector or laptops to show videos on models of fractions.

Outreach Efforts: Learners will be required to go and discuss how inheritance is shared in their families and what part each individual gets out of the whole inheritance. They will be required to give feedback to the class.

Reflection:

LAP2

Learning Activity Plan N°. 2 – Simplifying fractions to their lowest terms

Estimated Time: **45 Minutes**

Good math practices:

- Establish mathematical goals to focus teaching
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Pose purposeful questions

ACHIEVEMENT TARGETS/ OBJECTIVES:	Assessments:
• **Achievement Target #1:** Learners will be able to reduce fractions to their lowest terms by dividing both the numerator and the denominator by the highest common factor(HCF). • **Achievement Target #2:** Learners will appreciate learning fractions and value that factorisation is important when reducing algebraic fractions	• **Pre-Assessment: Learners will be given a worksheet with common fractions to reduce to their lowest terms** • **Interim Assessment: Learners will be given a worksheet with algebraic fractions to reduce to their lowest terms.** • **Formative Assessment**: Worksheet with both common fractions and algebraic fractions to reduce

SPECIAL PLANNING/PREPARATIONS:

Procedures	Time	Materials	Adaptations/Modifications for Learners with Diverse Needs
Introduction: The teacher introduces the lesson by asking learners to explain what they understand about HCF. Learners are then asked orally to give the HCF of ordinary numbers and algebraic expressions.	10 Min.		
Build the Knowledge: The teacher will provide a guided instruction on common fractions to write as equivalent fractions. They will then be given a worksheet to reduce common fractions into lowest terms in groups.(collaboration will be used)	20 Min.	Worksheet	
Apply the Knowledge: Learners will be required to now apply the knowledge acquired in the build the knowledge to algebraic fractions. Again, the learners will be required to discuss with peers and the teacher.	15 Min.	Worksheet	
Assess the knowledge: Learners will be given an activity where they reduce both common fractions and algebraic fractions to their lowest terms.			

Conclude the lesson: The lesson will be concluded by assigning learners to go and discuss with peers what they now know and seek clarification on what they still struggle with.			

Integration of Technology:

Outreach Efforts: Learners to go and seek clarification from more experienced peers and share their achievements.

Reflection:

LAP3

Learning Activity Plan N°. 3 – Multiplication and division of common and algebraic fractions

Estimated Time: **45 Minutes**

Good math practices:

- Establish mathematical goals to focus teaching
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Pose purposeful questions

ACHIEVEMENT TARGETS/ OBJECTIVES:	Assessments:
• **Achievement Target #1:** Learners will be able to multiply and divide both common and algebraic fractions • **Achievement Target #2:** Learners will appreciate learning fractions and apply rules after they understand how the rules come about.	• **Pre-Assessment: Worksheet with items on multiplying and dividing common fractions.** • **Interim Assessment: Worksheet with items on multiplying and dividing algebraic fractions.** • **Formative Assessment**: Worksheet with both common and algebraic fractions.

SPECIAL PLANNING/PREPARATIONS:

Procedures	Time	Materials	Adaptations/Modifications for Learners with Diverse Needs
Introduction: The teacher introduces the concept of multiplying fractions first using models of multiplying fractions. The teacher then introduces division by asking learners to share an odd number of chocolate bars (each chocolate bar comprising six divisions) to even-numbered people e.g., 7 chocolate bars to 6 people(bringing the real world to the class)	10 Min.	Chocolate bars	
Build the Knowledge: learners will then be asked to explain how they managed to share the bars equally with the people. Then, the teacher will then show them how their answers lead to the division that is $\frac{7}{6} = 1\frac{1}{6}$. A video of the model of how to multiply and dividing fractions will be shown to learners. The teacher will then introduce rules to the learners. If we are multiplying fractions, multiply numerators first then denominators and when dividing 7 chocolates between 6 people, it is the same as multiplying a number by its reciprocal, i.e., $7\times\frac{1}{6} = \frac{7}{6}$. After this explanation, learners will then be required to use the reciprocal concept in dividing fractions. Learners will then be asked to write the steps followed when multiplying and dividing fractions. Learners are allowed to come up with their rules on multiplying and dividing fractions.	20 Min.		

Apply the Knowledge: Learners will then be required to apply the knowledge acquired above in multiplying and dividing algebraic fractions (building procedural fluency from conceptual understanding)	15 Min.		
Assess the knowledge: Learners will be given an activity on multiplication and division of fractions in which they must demonstrate problem-solving and reasoning skills.			
Conclude the lesson: The teacher will conclude the lesson by asking learners to go and investigate if there are any other ways of multiplying and dividing fractions.			

Integration of Technology: videos showing models on how to multiply and divide fractions.

Outreach Efforts:

Reflection:

LAP4

Learning Activity Plan N°. 4 – Addition and subtraction of common and fractions

Estimated Time: **45 Minutes**

Good math practices:

- Establish mathematical goals to focus teaching
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Pose purposeful questions

ACHIEVEMENT TARGETS/ OBJECTIVES:	Assessments:
• **Achievement Target #1:** Learners will be able to add and subtract fractions. • **Achievement Target #2: Learner**s will appreciate learning fractions and add and subtract fractions they must be able to find the LCD.	• **Pre-Assessment: Learners will be given a worksheet on adding common fractions in groups.** • **Interim Assessment: Learners will then be given a worksheet on subtracting common fractions in groups.** • **Formative Assessment**: Learners will be given individual work on adding and subtracting fractions.

SPECIAL PLANNING/PREPARATIONS:

Procedures	Time	Materials	Adaptations/Modifications for Students with Diverse Needs
Introduction: The teacher will introduce the lesson by using models on how to add and subtract fractions before introducing the idea of rules and algorithms.	10 Min.	Models	
Build the Knowledge: The teacher will provide a guided instruction in which learners will be able to realise the LCD from the models. The models are used as scaffolds. Models will then be removed and learners should discover on their own that the LCD comes from multiplying the denominators of the fractions. Learners will also be probed until they realise that the LCD is the same as finding the LCM (lowest common multiple) of the two denominators.	20 Min.		
Apply the Knowledge: Learners will be required to apply the concepts acquired from models in simplifying fractions. Tasks will be given that promote reasoning and the teacher will pose purposeful questions when learners are stuck.	15 Min.		
Assess the knowledge: Learners will be given an activity on simplifying common fractions using rules.			

Conclude the lesson: Teacher concludes the lesson by asking learners to complete a KWL template			

Integration of Technology: videos showing models on how to add and subtract fractions
Outreach Efforts: Reflection:

LAP5

Learning Activity Plan N°.5 –Adding and subtracting fractions

Estimated Time: **45 Minutes**

Good math practices:

- Establish mathematical goals to focus teaching
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Pose purposeful questions

ACHIEVEMENT TARGETS/ OBJECTIVES: **Assessments:**

• **Achievement Target #1:** Learners will be able to add and subtract algebraic fractions • **Achievement Target #2:** Learners will appreciate learning fractions and that understanding the arithmetic of common fractions will enhance the understanding of algebraic fractions	• **Pre-Assessment: A worksheet on adding algebraic fractions in groups.** • **Interim Assessment: A worksheet on subtracting algebraic fractions in groups.** • **Formative Assessment**: A worksheet on adding and subtracting algebraic fractions.

SPECIAL PLANNING/PREPARATIONS:

Procedures	Time	Materials	Adaptations/Modifications for Learners with Diverse Needs
Introduction: The teacher introduce the lesson by asking learners to list steps on how to add and subtract fractions and complete the KWL chart.	10 Min.		
Build the Knowledge: Learners do activities on simplifying algebraic fractions using rules. The teacher will probe learners until they discover on their own how to find the LCD of algebraic fractions and how to simplify them. The teacher asks questions and allows learners to build knowledge from what they know. Learners are also given a chance to learn from their peers.	20 Min.		
Apply the Knowledge: learners will be given more challenging problems in which they will be required to apply what they have learnt.	15 Min.		
Assess the knowledge: The post-test instrument will be used.			
Conclude the lesson			

Integration of Technology:

Outreach Efforts:

Reflection:

LAP6

Table LAP 6: MATHEMATICS TEACHING PRACTICES

Establish Mathematics goals to focus learning	Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving	Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representation	Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem-solving.
Facilitate meaningful mathematical discourse	Effective teaching of mathematics facilitates discourse among students to build a shared understanding of mathematical ideas by analysing and comparing student approaches and arguments.
Pose purposeful questions	Effective teaching of mathematics uses purposeful questions to assess and advance students] reasoning and sense-making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding	Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skilful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics	Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking	Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Source: NCTM (2014). Principles to Actions: Ensuring Mathematical Success for All (p. 10)

APPENDIX 2: PRE AND POST TEST QUESTIONNAIRE

PTQ1

PRE-TEST QUESTIONNAIRE

Ques no. __________

Introduction

Dear respondent

I am Lina Mangwende, a Master student with the University of South Africa (UNISA). This questionnaire is part of my Master's research entitled, **Relationship between Learners' Proficiency in common and Algebraic fractions: A case of Grade 10 learners at a Limpopo High School.** I am collecting data regarding the learning of common fractions and algebraic fractions. I invite you to take part in this survey. The data collected will be solely to improve the learning and teaching of fractions at grade 10 level. This information is **confidential** and will only be used for this study, which will not refer by name to anyone respondent. The findings of the study may benefit you as it is geared towards improving the learning of algebraic fractions. I will be grateful if you could assist me in filling out this questionnaire in an honest manner as possible. The questionnaire comprises four sections and will take approximately 60 minutes of your time. Besides, participation is voluntary.

You are not required to indicate your name or organisation and your anonymity will be ensured. However, an indication of your age, gender, occupation and position to mention a few will contribute to comprehensive analysis. Permission to undertake this study has been granted by the Limpopo Department of Education and the Ethics Committee of the College of Education, UNISA. If you have any research-related enquiries, they can be addressed to me or my supervisor. My supervisor can be reached at +27124296993, Department of Mathematics, College of Education, UNISA, email: phoshmm@unisa.ac.za. By completing the questionnaire, you imply that you have agreed to take part in this research.

Part A: Identification and student characteristics

☐

A1 Student code __________ ☐

A2 Date _______/_________/________ ☐

A3 Start time ____________________ ☐

A4 Age of the respondent [state number] ☐

A5 Gender of the respondent 0 = Female 1 = Male ☐

A6 Distance from home to school in km [state number] ☐

A7 Household size [state number] ☐

A8 Household head 1 = Father 2 = Mother 3 = Brother 4 = Sister ☐

A9 Age of the household head [state number] ☐

A10 Employment status of the head of the household
0 = Unemployed 1 = Employed ☐

A11 a) Parent's highest level of education 0 = None 1 = Below matric
2 = Matric 3 = Certificate 4 = Diploma 5 = Degree ☐

A12 b) Most educated sibling's level of education 0 = None 1 = Below matric
2 = Matric 3 = Certificate 4 = Diploma 5 = Degree ☐

A13 Who helps you with your homework 0 = None 1 = Father 2 = Mother
3 = Bother 4 = Sister 5 = Other (state ________________) ☐

A14 Does your household have electricity? 0 = No 1 = Yes ☐

A15 How many days of the week (Mon, Tue, Wed, Thur, Fri, Sat, and Sun) do you study mathematics? Indicate in the box by writing M if it is Monday only, MTW if it is Monday, Tuesday and Wednesday or TSS if it is Tuesday, ☐

Saturday, and Sunday.

A16 Amount of time devoted by the student towards household chores (hrs) ☐

A17 Amount of time devoted to studying in a day (hrs) ☐

A18 Does household own (please tick)
- a) Livestock ☐
- b) Car ☐
- c) Bicycle ☐
- d) Tv ☐
- e) Radio ☐

Part B: Attitude and perception towards fractions

				Code
B1	I enjoy learning fractions	0 = No	1 = Yes	☐
B2	Fractions are easy to understand	0 = No	1 = Yes	☐
B3	Please rate the extent to which you understand fractions using a scale from zero to five (0 - 5)			☐
B4	My teacher is good at fractions	0 = No	1 = Yes	☐
B5	Please rate the extent to which your teacher understands fractions using a scale from zero to five (0 - 5)			☐
B6	Algebraic fractions are difficult to simplify	0 = No	1 = Yes	☐
B7	I use a calculator to simplify fractions	0 = No	1 = Yes	☐

We are interested in learning how you think and feel about Mathematics. Please consider the following questions and tell us how you truly feel. There is no right or wrong answer.

B8 a) If fractions were a food, it would be____________ because_______________

a) If fractions were an animal, it would be_______________ because ____________

Part C: Solve the Following Fractions

1. Reduce the following fraction to its lowest terms: $\frac{4}{8}$
2. Calculate:
   (a) $\frac{1}{2}+\frac{2}{3}$
   (b) $2\frac{3}{4}-\frac{3}{8}$
   (c) $\frac{8}{7}\times 4\frac{2}{3}$
   (d) $\frac{3}{8}\times\frac{3}{4}$
3. Reduce the following to its lowest terms: $\frac{4+4x}{8}$
4. Simplify the following:
   (a) $\frac{x}{2}+\frac{2x}{3}$
   (b) $2\frac{3}{4}x-\frac{3}{8}x$
   (c) $\frac{8}{7}x\times 4\frac{2}{3}x$
   (d) $\frac{3}{8}x\div\frac{3}{4}x$
   (e) $\frac{x-3}{3}+\frac{x+2}{4}$
   (f) $\frac{4}{7x}-\frac{3x-3}{x^2-x}$

Part D: Comments

Explain in a few lines what makes the simplification of algebraic fractions difficult.

____

____

____

Finish Time______________________Thank you for your responses they go a long way in assisting to identify problems learners encounter with fractions.

PTQ2

POST-TEST QUESTIONNAIRE

Ques no. ___________

Introduction

Dear respondent

I am Lina Mangwende a Master student at the University of South Africa (UNISA). This questionnaire is part of my Master's research entitled **Relationship between Learners' Proficiency in common and Algebraic fractions: A case of Grade 10 learners at a Limpopo High School.** I am collecting data regarding the learning of common fractions and algebraic fractions. I invite you to take part in this survey. The data collected will be solely to improve the learning and teaching of fractions at grade 10 level. This information is **confidential** and will only be used for this study, which will not make reference by name to anyone respondent. The findings of the study may benefit you as it is geared toward improving the learning of algebraic fractions. I will be grateful if you could assist me in filling out this questionnaire in as honest a manner as possible. The questionnaire comprises four sections and will take approximately 60 minutes of your time and participation is voluntary.

You are not required to indicate your name or organisation and your anonymity will be ensured, however, an indication of your age, gender occupation position e.tc will contribute to comprehensive analysis. Permission to undertake this study has been granted by the Limpopo Department of Education and the Ethics Committee of the College of Education, UNISA. If you have any research-related enquiries, they can be addressed to me or my supervisor. My supervisor can be reached at +27124296993 Department of Mathematics, College of Education, UNISA, email: phoshmm@unisa.ac.za. By completing the questionnaire, you imply that you have agreed to take part in this research.

Part A: Identification and student characteristics

☐

A1 Student code __________ ☐

A2 Date _______/_________/________ ☐

A3 Start time ____________________

A4 Age of the respondent [state number]

A5 Gender of the respondent 0 = Female 1 = Male

A6 Distance from home to school in km [state number]

A7 Household size [state number]

A8 Household head 1 = Father 2 = Mother 3 = Brother 4 = Sister

A9 Age of the household head [state number]

A10 Employment status of the head of the household
0 = Unemployed 1 = Employed

A11 a) Parent's highest level of education 0 = None 1 = Below matric
2 = Matric 3 = Certificate 4 = Diploma 5 = Degree

A12 b) Most educated sibling's level of education 0 = None 1 = Below matric
2 = Matric 3 = Certificate 4 = Diploma 5 = Degree

A13 Who helps you with your homework 0 = None 1 = Father 2 = Mother
3 = Bother 4 = Sister 5 = Other (state ________________)

A14 Does your household have electricity? 0 = No 1 = Yes

A15 How many days of the week (Mon, Tue, Wed, Thur, Fri, Sat, and Sun) do you study mathematics? Indicate in the box by writing M if it is Monday only, MTW if it is Monday, Tuesday and Wednesday or TSS if it is Tuesday, Saturday, and Sunday.

A16 Amount of time devoted by the student towards household chores (hrs) ☐

A17 Amount of time devoted to studying in a day (hrs) ☐

A18 Does household own (please tick)
- a) Livestock ☐
- b) Car ☐
- c) Bicycle ☐
- d) Tv ☐
- e) Radio ☐

Part B: Attitude and perception towards fractions

Code

B1 I enjoy learning fractions 0 = No 1 = Yes ☐

B2 Fractions are easy to understand 0 = No 1 = Yes ☐

B3 Please rate the extent to which you understand fractions using a scale from zero to five (0 - 5) ☐

B4 My teacher is good at fractions 0 = No 1 = Yes ☐

B5 Please rate the extent to which your teacher understands fractions using a scale from zero to five (0 - 5) ☐

B6 Algebraic fractions are difficult to simplify 0 = No 1 = Yes ☐

B7 I use a calculator to simplify fractions 0 = No 1 = Yes ☐

We are interested in learning how you think and feel about Mathematics. Please consider the following questions and tell us how you truly feel. There is no right or wrong answer.

B8 a) If fractions were a food, it would be____________ because_______________

b) If fractions were an animal, it would be_______________ because ____________

Part C: Solve the Following Fractions

5. Reduce the following fraction to its lowest terms: $\frac{2}{6}$
6. Calculate:
   (a) $\frac{3}{8}+\frac{4}{5}$
   (b) $3\frac{2}{3}-\frac{5}{6}$
   (c) $\frac{6}{5}\times 5\frac{2}{3}$
   (d) $\frac{7}{9}\times\frac{2}{5}$
7. Reduce the following to its lowest terms: $\frac{2+2x}{6}$
8. Simplify the following:
   (a) $\frac{3x}{8}+\frac{2x}{5}$
   (b) $3\frac{2}{3}x-\frac{5}{6}x$
   (c) $\frac{6}{5}x\times 5\frac{2}{3}x$

(d) $\frac{7}{9}x \div \frac{2}{5}x$

(e) $\frac{2x-3}{6} + \frac{x+2}{8}$

(f) $\frac{5}{x} - \frac{x-1}{x^2-x}$

Part D: Comments

Explain in a few lines what makes the simplification of algebraic fractions difficult.

____

____

____

Thank you for your responses they go a long way in assisting to identify problems learners encounter with fractions.

Finish time __________________________

APPENDIX 3: PERMISSION LETTERS AND ETHICAL CLEARANCE

PL1

PARENT/GUARDIAN CONSENT FORM

Date: January 2020

Dear parent/ guardian

I am **Lina Mangwende,** your child's Mathematics teacher. I am currently doing a Master of Education (Mathematics Education) degree at the University of South Africa (UNISA) and conducting research titled: **Relationship between Learners' Proficiency in Common and Algebraic Fractions: A Case of Grade 10 Learners at a Limpopo High School.** My proposal has been accepted by the university. I am kindly requesting your permission for your child to participate in my final research project. I hope to investigate whether improving learners' proficiency in common fractions can enhance their understanding of algebraic fractions. Your child was identified as a possible participant in this study.

In general, the teaching interventions I will use will enhance their understanding of algebraic fractions and common fractions. Even though there are no monetary benefits for participating, your child may benefit from the intervention strategies provided in this research project. I know you might be worried about your child's privacy; all data collected from students will remain confidential. I will use a pseudonym (fake name) for your child to protect his/her right to privacy. Information collected from this research will be used to inform the teacher's teaching methods and complete the research project at UNISA that may be shared with other teachers seeking effective teaching strategies that help learners understand common and algebraic fractions.

Note: your child's participation in this research is voluntary. It is your decision to give consent or not for him/her to participate and also note that it will not affect his/her academic performance or the relationship with the school. If you decide to allow your child to participate and later change your mind, you and or your child are free to withdraw consent and participation at any time with no consequence.

Yours faithfully
Lina Mangwende
Cell: 0734517296
Email: ntulilina@gmail.com
Supervisor: Professor M. Phoshoko
Telephone: +27124296993
Email: phoshmm@unisa.ac.za

Return slip

I __________________________________ (Full name of parent/guardian) give permission for my child ______________________________ (Full name of child) to be part of the research study. I have read and understood the contents of the above letter addressed to me.

Signature of parent/legal guardian Date

____

____

PL2

LEARNER CONSENT LETTER

Date: January 2020

Dear learner

Hi, as you know, I am **Lina Mangwende (Ntuli)** your grade 10 Mathematics teacher. I am studying at the University of South Africa (UNISA) to do a Master degree in Mathematics Education. To complete my degree, I am conducting a research project and would like to invite you as a participant. I want to investigate the challenges you face in simplifying common fractions and algebraic fractions. I will also come up with intervention methods to help you understand better. This research may benefit you as it is geared towards improving your Mathematics skills and become more engaged with the learning activities in class. I also hope it will help me and other teachers decide how to better teach common and algebraic fractions.

Your privacy is very important to me. I don’t believe there would be any risks to your education or wellbeing. Any information I collect from this study will remain confidential. I will use pseudonyms (fake names) for you in reporting findings from the study. If you do not feel like participating, you are at liberty to turn down my invitation. It is also fine to agree now and change your mind later. Take your time to make your choice but remember all the other school work that your mates will be doing, you will be required to do it. I will not only use your data in my research project.
If you are willing to participate please sign the return slip below and submit it back to me.

Yours faithfully

Lina Mangwende
Cell: 0734517296
Email: ntulilina@gmail.com
Supervisor: Professor M. Phoshoko
Telephone: +27124296993

Email:phoshmm@unisa.ac.za

Return slip

I ________________________ (Full name of learner) agree to be included in the research study. I also agree that data collected from me can be used in the study as long as my identity is not made public.

Signature of learner participant Date

____

____

Signature of Researcher Date

____

____

PL3

PERMISSION- LETTER TO LIMPOPO DEPARTMENT OF EDUCATION

The Head of Department, Limpopo Department of Education
Private Bag
Polokwane
Date: January 2020

Dear sir/ madam

RE: **REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN SOME OF YOUR SCHOOLS**

The above matter bears reference

1. I **Lina Mangwende**, a Master student with the University of South Africa, hereby request permission to conduct research in some of your schools that will meet the requirements of the sampling technique that will be used in the study.
2. The title of my research study is: **Relationship between Learners' Proficiency in Common and Algebraic Fractions: A Case of Grade 10 Learners at a Limpopo High School**.
3. The study will use an action research design using a case where data will be collected from grade 10 learners at one high school and selected teachers.
4. The ethics policy of the the University of South Africa requires that I get permission from the Department of Education to be able to conduct this research. This policy also stipulates that all participants of this study be protected by keeping their identities and information confidential.
5. The Department of Education will benefit from this study by getting information about strategies on how to teach common and algebraic fractions.

6. When the study has been completed, a copy of the report will be made available to the Department of Education.
7. I have attached a brief research proposal containing all the information required.
8. Hoping for a favourable response to my request.

Yours faithfully

Lina Mangwende
Cell: 0734517296
Email: ntulilina@gmail.com
Supervisor: Professor M. Phoshoko
Telephone: +27124296993
Email: phoshmm@unisa.ac.za

PL4

LETTER OF PERMISSION FROM PRINCIPAL

Date: January 2020

Dear principal

RE: **Requesting permission to conduct research at your school**

I, **Lina Mangwende,** am currently studying with the University of South Africa (UNISA) doing a Master degree in Mathematics Education and wish to request permission to conduct a research on the teaching and learning of fractions with grade 10 learners at your school. The title of my research is: **Relationship between Learners' Proficiency in Common and Algebraic Fractions: A Case of Grade 10 Learners at a Limpopo High School.** I hope to investigate whether improving learners' proficiency in common fractions can enhance their understanding of algebraic fractions. I will also investigate which barriers and challenges the grade 10 learners face in the learning of fractions and try to come up with intervention strategies that could enhance their understanding. I have decided to research with the grade 10 learners since they are the bridge between the GET and FET phases. My research will not interfere with normal lessons since I will carry it during the study after school.

The data to be collected will be solely for the research report and academic purposes. The learners' participation will be confidential and anonymity is guaranteed. Pseudonyms will be used so that there would not be any direct link to the learners. If the data is accidentally lost, even the school name will not be mentioned. The learner's participation is voluntary and refusal to participate will neither lead to prejudice or penalty. Learners may withdraw at any time if they so wish. There will be no financial incentives for the participation of learners in the research. A summary of the findings will be available to you and all the Mathematics teachers at your school once the research has been completed. These findings might also inform future teachers on the challenges and strategies that they can use to improve the learning and teaching of algebraic fractions.

If you are happy for your school to participate, please indicate that you read and understood this information letter by signing the accompanying consent form and return it to me.

Yours faithfully

Lina Mangwende
Cell: 0734517296
Email:ntulilina@gmail.com
Supervisor: Professor M. Phoshoko
Telephone: +27124296993
Email:phoshmm@unisa.ac.za

Cut here---cut here

Return consent slip

I ________________________ (Full name of principal) give you permission to conduct the research in my school.

Signature of principal Date:

___________________ ________________

Signature of Researcher Date:

__________________ ________________

Cut here--

PL5

TEACHER'S CONSENT LETTER

Date: January 2020

Dear Teacher

I, Lina **Mangwende**, invite you to be part of my research which is to be carried out on grade 10 learners as they learn algebraic fractions. I am currently studying towards a Masters degree through the University of South Africa (UNISA). I hope to investigate whether improving learners' proficiency in common fractions can enhance their understanding of algebraic fractions. I will also investigate which barriers and challenges the grade 10 learners face in the learning of fractions and try to come up with intervention strategies that can enhance their understanding. The title of my research is: **Relationship between Learners' Proficiency in Common Fractions and Algebraic Fractions: A Case of Grade 10 Learners at a Limpopo High School.** I have decided to research with the grade 10 learners since they are the bridge between the GET and FET phases. I hope you will also share with me your experiences in the teaching and learning of algebraic fractions.

I am kindly requesting you to volunteer to take part in this research and be aware that you are not forced to participate. Also, note that there are no monetary benefits except that the research report will be made available to you to use the findings and recommendations. I will interview you on the teaching and learning of fractions and record our interview in written form. Your confidentiality is one of my priorities. I will use Pseudonyms to hide your identity and any information collected will be confidential. Taking part in this study is voluntary so you can accept the invitation or you can decline.

Yours faithfully

Lina Mangwende
Cell:0734517296
Email: ntulilina@gmail.com
Supervisor: Professor M. Phoshoko (University of South Africa: Mathematics Education Department)
Telephone: +27124296993
Email: phoshmm@unisa.ac.za

Declaration by participant

I_____________________ (Full name of teacher) hereby confirm that I understand the contents of this document and the nature of the research project. I also understand that I am at liberty to withdraw from the research at any time should I so desire. I agree to be included in the research study and also agree that any data collected from me can be used in your research as long as my identity is not made public.

Teacher's Signature ______________________

Date_________________

Researcher's signature_____________________

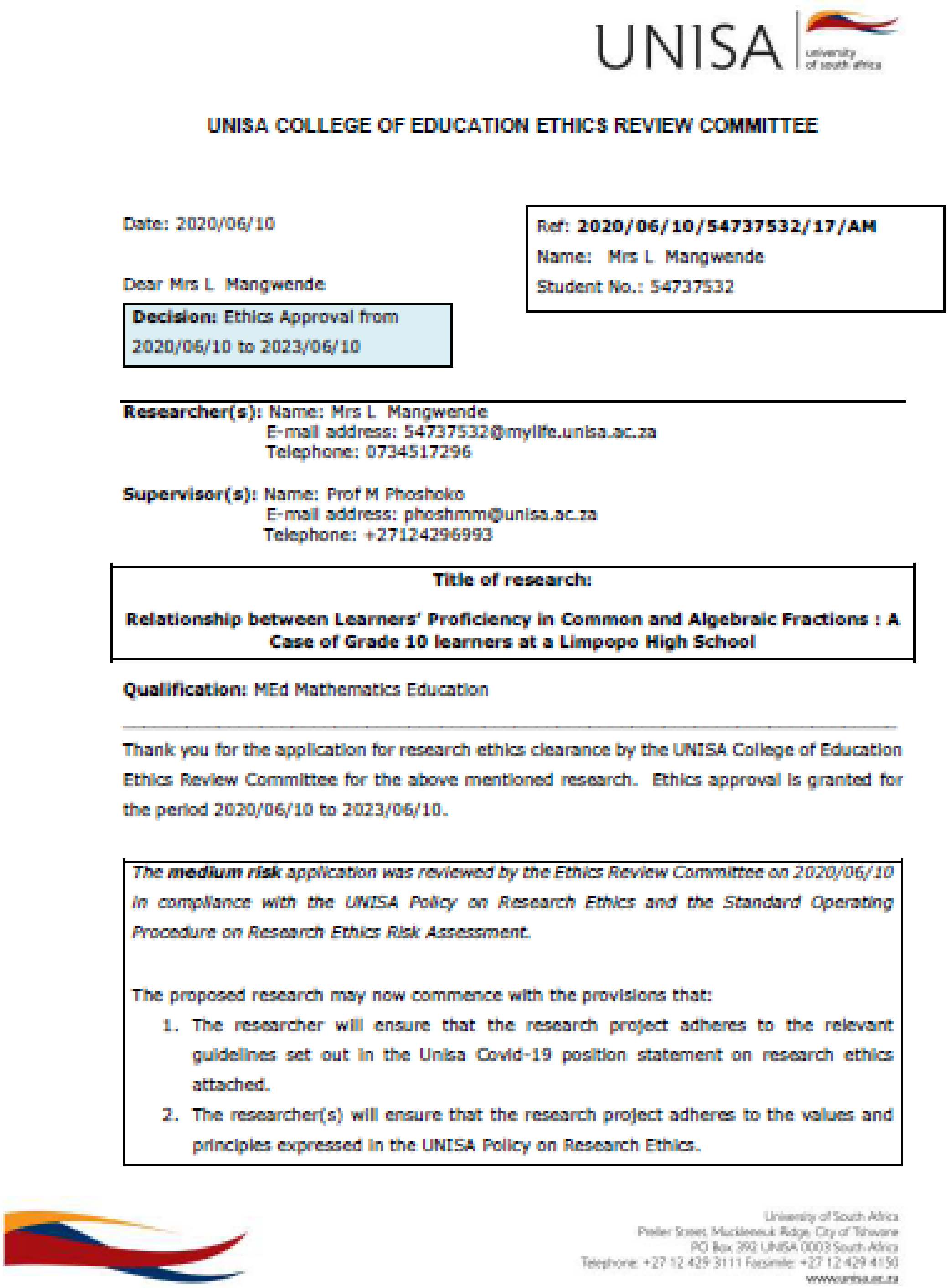
Date__________________

Witness's Signature______________________

Date___________________

PL6

ETHICAL CLEARANCE



UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/06/10

Ref: **2020/06/10/54737532/17/AM**
Name: Mrs L Mangwende
Student No.: 54737532

Dear Mrs L Mangwende

Decision: Ethics Approval from 2020/06/10 to 2023/06/10

Researcher(s): Name: Mrs L Mangwende
E-mail address: 54737532@mylife.unisa.ac.za
Telephone: 0734517296

Supervisor(s): Name: Prof M Phoshoko
E-mail address: phoshmm@unisa.ac.za
Telephone: +27124296993

Title of research:

Relationship between Learners' Proficiency in Common and Algebraic Fractions : A Case of Grade 10 learners at a Limpopo High School

Qualification: MEd Mathematics Education

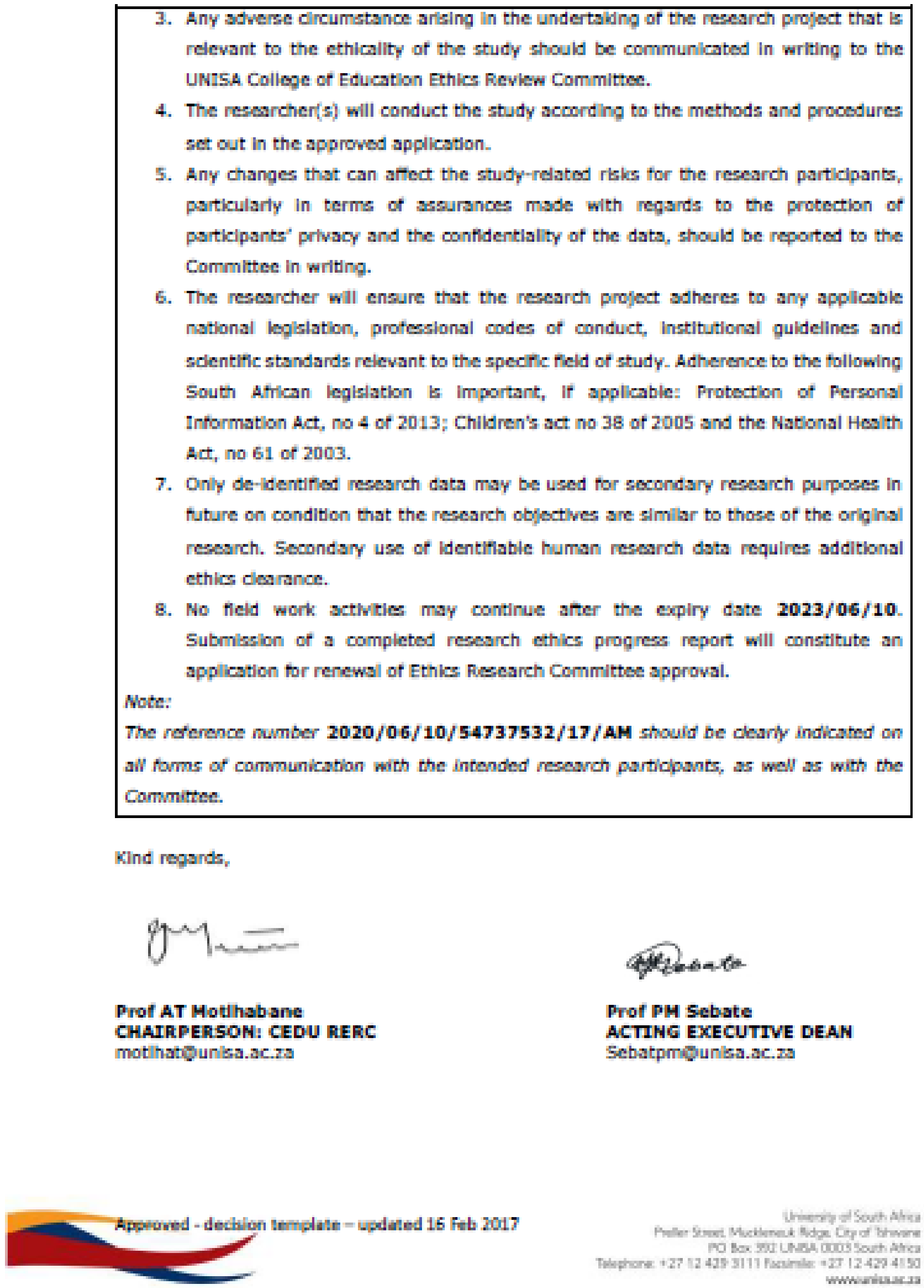
Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/06/10 to 2023/06/10.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2020/06/10 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za



3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2023/06/10**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2020/06/10/54737532/17/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof PM Sebate
ACTING EXECUTIVE DEAN
Sebatpm@unisa.ac.za

Approved - decision template – updated 16 Feb 2017

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APPENDIX 4: INTERVIEW SCHEDULES

IS1

POST – TEST INTERVIEW SCHEDULE FOR LEARNERS

Learners' code: _________

Section A (building good rapport)

1. My name is Lina Mangwende, your Mathematics teacher and thought it would be a good idea to interview you so that I can better understand why you still have challenges in solving algebraic fractions.
2. I would like to ask you some questions about the post-test you wrote to learn more about the challenges you face.
3. I hope to use this information to help me clear out errors and misconceptions you have in simplifying algebraic fractions.
4. The interview should take about 15 minutes. Are you available to respond to some questions at this time?

Section B (main body)

1 Which questions were difficult?

2 Explain how you tried to tackle the questions.

3 Which steps did you fail to do on the questions you mentioned in 1?

4 What do you think contributed to your failure to do the questions.?

5 The questions will come from probing after getting the post-test results.

6 If you were to be given another chance to write a similar test, do you think you can perform better?

IS2

INTERVIEW SCHEDULE FOR TEACHERS Teacher's code: ______________

Section A (building good rapport)

1. My name is Lina Mangwende, a Mathematics teacher at Mazwe High School and would like to interview you about the challenges that your grade 10 learners face in simplifying common and algebraic fractions.
2. I would like to ask you some questions about your challenges and how you solve them.
3. I hope to use this information to help you and other Mathematics educators to help learners who struggle with algebraic fractions.
4. The interview should take about 15 minutes. Are you available to respond to some questions at this time?

Section B (main body)

1. How many years have you been teaching Mathematics?
2. Which topics will you consider to be most difficult for your learners in grade 10?
3. Which challenges and barriers do your learners face in the learning of common fractions and algebraic fractions? Explain and give examples of the errors learners make.
4. How do you teach common fractions and algebraic fractions? Please, elaborate.
5. Do you think your grade10 learners know the concept of common fractions very well? Please, explain by giving details of what they can do and cannot do.

APPENDIX 5: LEARNERS' DISPOSITION

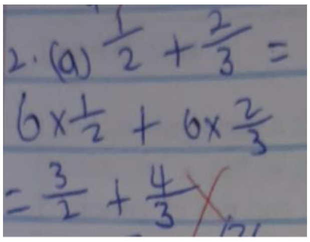
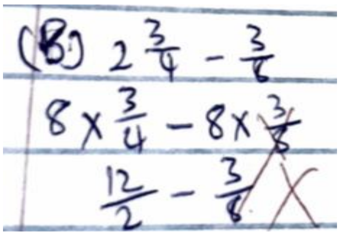
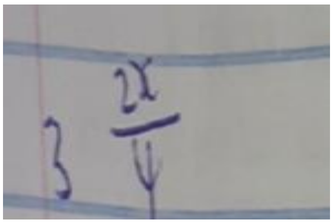
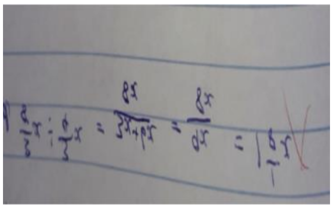
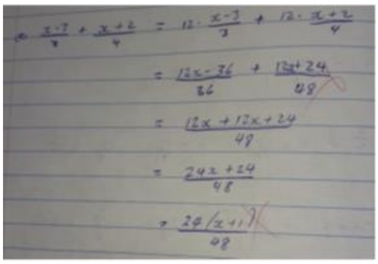
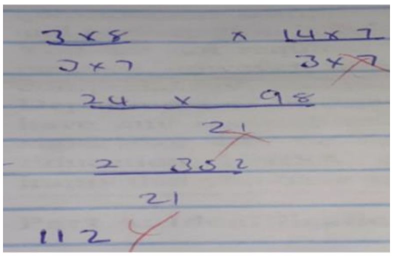
LD1

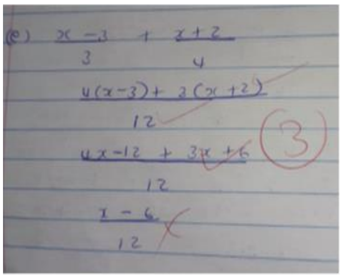
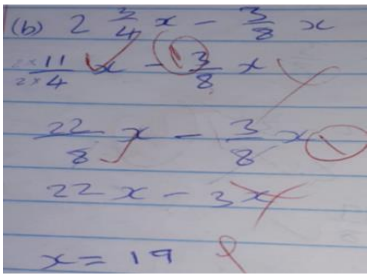
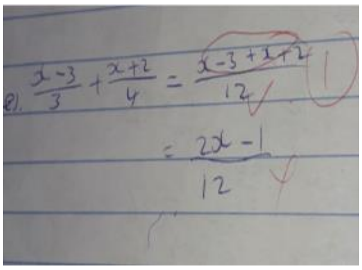
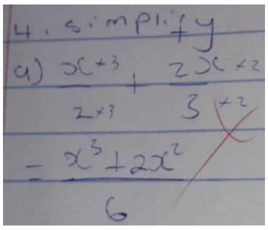
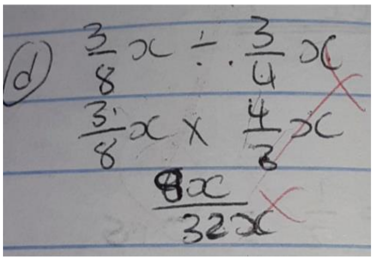
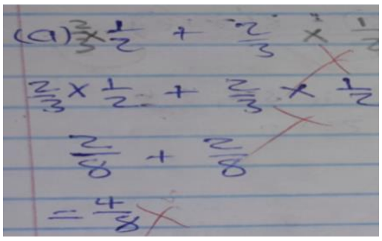
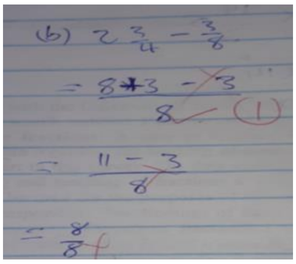
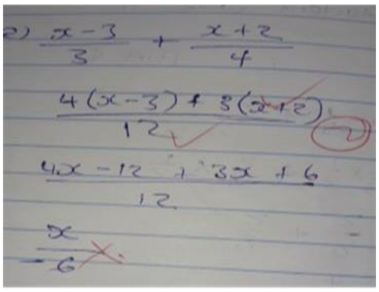
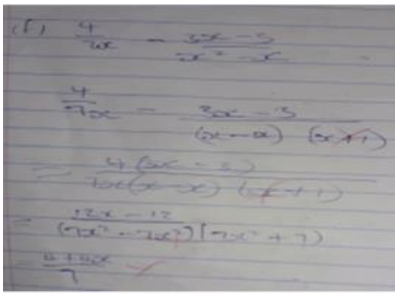
Table LD1: Learners' dispositions about common and algebraic fractions

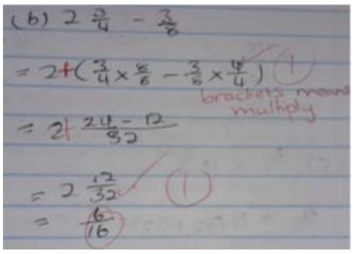
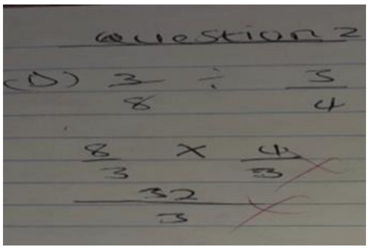
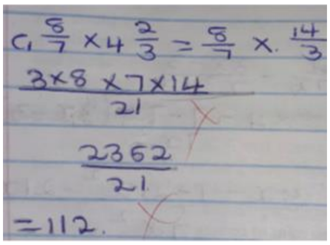
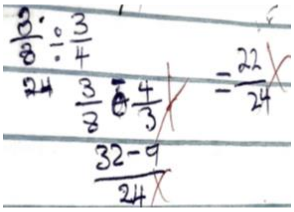
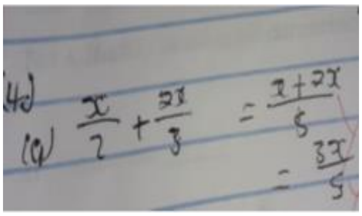
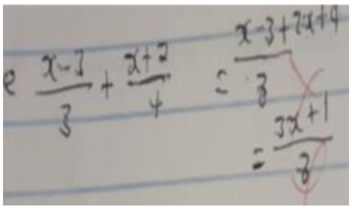
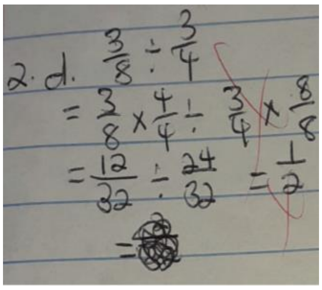
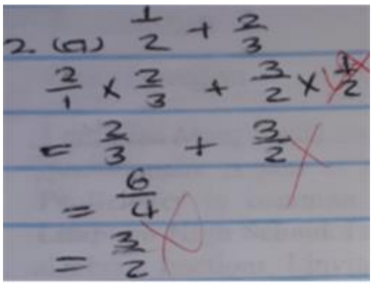
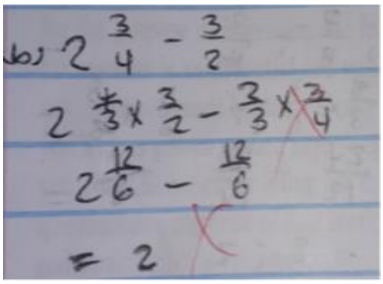
Intervention	**Number of learners who relate fractions to favourite food**	**Number of learners who relate fractions to worst food**	**Number of learners who relate fractions to favourite animals**	**Number of learners who relate fractions to fierce animals**
Before intervention	9	17	9	17
After intervention	22	4	22	4

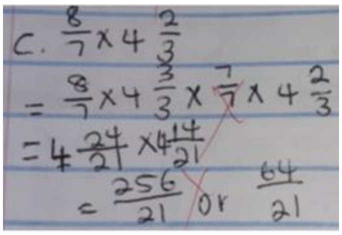
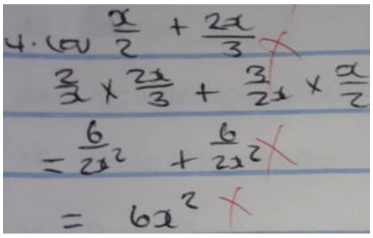
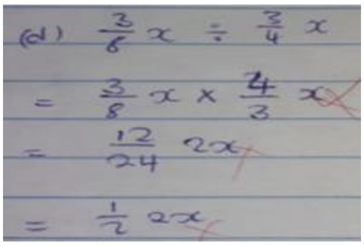
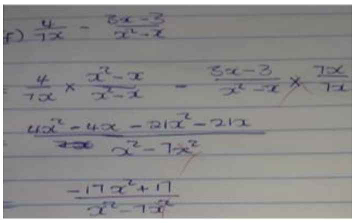
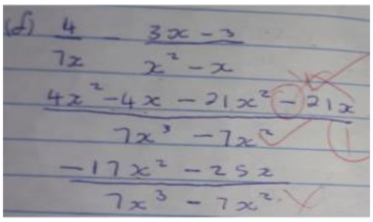
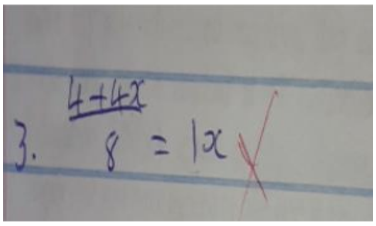
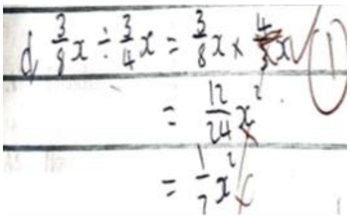
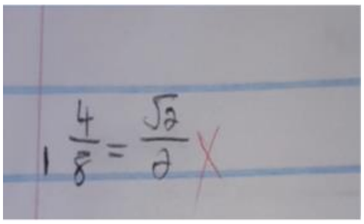
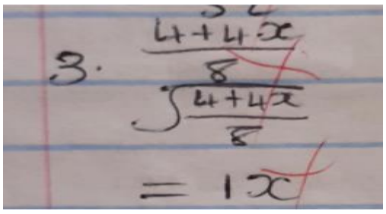
APPENDIX 6: IMAGES OF SCRIPTS SHOWING LEARNERS' WORK

E1

Question 2a  $2.(a)\ \frac{1}{2}+\frac{2}{3}=$ $6\times\frac{1}{2}+6\times\frac{2}{3}$ $=\frac{3}{2}+\frac{4}{3}$ L1 error 1	Question 2b  $(B)\ 2\frac{3}{4}-\frac{3}{8}$ $8\times\frac{3}{4}-8\times\frac{3}{8}$ $\frac{12}{2}-\frac{3}{8}$ L1 error 2	Question 3  $3\ \frac{2x}{4}$ L1 error 3
Question 4d  L2 error 1	Question 4e  L2 error 2	Question 2c  $\frac{3\times8}{7\times7}\times\frac{14\times7}{3\times7}$ $\frac{24\times98}{21}$ $\frac{2352}{21}$ 112 L3 error 1

Question 4e  (e) $\frac{x-3}{3} + \frac{x+2}{4}$ $\frac{4(x-3)+3(x+2)}{12}$ $\frac{4x-12+3x+6}{12}$ ③ $\frac{x-6}{12}$ L3 error 2	Question 4b  (b) $2\frac{3}{4}x - \frac{3}{8}x$ $\frac{2\times11}{2\times4}x - \frac{3}{8}x$ ① $\frac{22}{8}x - \frac{3}{8}x$ $22x - 3x$ $x = 19$ L4 error 1	Question 4e  e). $\frac{x-3}{3} + \frac{x+2}{4} = \frac{x-3+x+2}{12}$ ① $= \frac{2x-1}{12}$ L5 error 1
Question 4a  4. simplify a) $\frac{x+3}{2\times3} + \frac{2x\times2}{3\times2}$ $= \frac{x^3+2x^2}{6}$ L6 error 1	Question 4d  (d) $\frac{3}{8}x \div \frac{3}{4}x$ $\frac{3}{8}x \times \frac{4}{3}x$ $\frac{8x}{32x}$ L7 error 1	Question 2a  (a) $\frac{2}{3}\times\frac{1}{2} + \frac{2}{3}\times\frac{1}{2}$ $\frac{2}{3}\times\frac{1}{2} + \frac{2}{3}\times\frac{1}{2}$ $\frac{2}{8} + \frac{2}{8}$ $= \frac{4}{8}$ L9 error 1
Question 2b  (b) $2\frac{3}{4} - \frac{3}{8}$ $= \frac{8+3-3}{8}$ ① $= \frac{11-3}{8}$ $= \frac{8}{8}$ L9 error 2	Question 4e  2) $\frac{x-3}{3} + \frac{x+2}{4}$ $\frac{4(x-3)+3(x+2)}{12}$ ② $\frac{4x-12+3x+6}{12}$ $\frac{x}{-6}$ L9 error 3	Question 4f  (f) $\frac{4}{7x} - \frac{3x-3}{x^2-x}$ L9 error 4

Question 2b  (b) $2\frac{3}{4} - \frac{3}{8}$ $= 2+(\frac{3}{4}\times\frac{8}{8} - \frac{3}{8}\times\frac{4}{4})$ ① brackets mean multiply $= 2\frac{24-12}{32}$ $= 2\frac{12}{32}$ ① $= \frac{6}{16}$ L10 error 1	Question 2d  Question 2 (d) $\frac{3}{8} \div \frac{3}{4}$ $\frac{8}{3} \times \frac{4}{3}$ $\frac{32}{3}$ L10 error 2	Question 2c  c. $\frac{8}{7}\times 4\frac{2}{3} = \frac{8}{7}\times\frac{14}{3}$ $\frac{3\times8\times7\times14}{21}$ $\frac{2362}{21}$ $=112.$ L12 error 1
Question 2d  $\frac{3}{8}\div\frac{3}{4}$ 24 $\frac{3}{8}$ $\frac{4}{3}$ $=\frac{22}{24}$ $\frac{32-9}{24}$ L14 error 1	Question 4a  (4a) (a) $\frac{x}{2}+\frac{2x}{3} = \frac{x+2x}{5}$ $= \frac{3x}{5}$ L15 error 1	Question 4e  e $\frac{x-3}{3}+\frac{x+2}{4} = \frac{x-3+2x+4}{3}$ $= \frac{3x+1}{3}$ L15 error 2
Question 2d  2. d. $\frac{3}{8}\div\frac{3}{4}$ $=\frac{3}{8}\times\frac{4}{4}\div\frac{3}{4}\times\frac{8}{8}$ $=\frac{12}{32}\div\frac{24}{32}$ $=\frac{1}{2}$ $=$ L16 error 1	Question 2a  2. (a) $\frac{1}{2}+\frac{2}{3}$ $\frac{2}{1}\times\frac{2}{3} + \frac{3}{2}\times\frac{1}{2}$ $=\frac{2}{3} + \frac{3}{2}$ $=\frac{6}{4}$ $=\frac{3}{2}$ L17 error 1	Question 2b  (b) $2\frac{3}{4} - \frac{3}{2}$ $2\,\frac{4}{3}\times\frac{3}{2} - \frac{3}{3}\times\frac{3}{4}$ $2\frac{12}{6} - \frac{12}{6}$ $= 2$ L17 error 2

Question 2c  c. $\frac{8}{7}\times4\frac{2}{3}$ $=\frac{8}{7}\times4\frac{3}{3}\times\frac{7}{7}\times4\frac{2}{3}$ $=4\frac{24}{21}\times4\frac{14}{21}$ $=\frac{256}{21}$ or $\frac{64}{21}$ L17 error 3	Question 4a  4.(a) $\frac{x}{2}+\frac{2x}{3}$ $\frac{3}{x}\times\frac{2x}{3}+\frac{3}{2x}\times\frac{x}{2}$ $=\frac{6}{2x^2}+\frac{6}{2x^2}$ $=6x^2$ L17 error 4	Question 4d  (d) $\frac{3}{8}x\div\frac{3}{4}x$ $=\frac{3}{8}x\times\frac{4}{3}x$ $=\frac{12}{24}2x$ $=\frac{1}{2}2x$ L18 error 1
Question 4f  f) $\frac{4}{7x}-\frac{3x-3}{x^2-x}$ $\frac{4}{7x}\times\frac{x^2-x}{x^2-x}-\frac{3x-3}{x^2-x}\times\frac{7x}{7x}$ $\frac{4x^2-4x-21x^2-21x}{\cancel{7x}\;x^2-7x^2}$ $=\frac{-17x^2+17}{x^2-7x^2}$ L18 error 2	Question 4f  (f) $\frac{4}{7x}-\frac{3x-3}{x^2-x}$ $\frac{4x^2-4x-21x^2-21x}{7x^3-7x^2}$ $\frac{-17x^2-25x}{7x^3-7x^2}$ L19 error 1	Question 3  3. $\frac{4+4x}{8}=1x$ L20 error 1
Question 4d  d. $\frac{3}{8}x\div\frac{3}{4}x=\frac{3}{8}x\times\frac{4}{3}x$ $=\frac{12}{24}x^2$ $=\frac{1}{2}x^2$ L20 error 2	Question 1  $1\frac{4}{8}=\frac{\sqrt{2}}{2}$ L22 error 1	Question 3  3. $\frac{4+4x}{8}$ $\sqrt{\frac{4+4x}{8}}$ $=1x$ L23 error 1

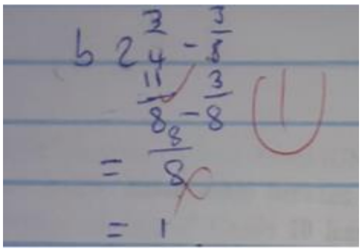
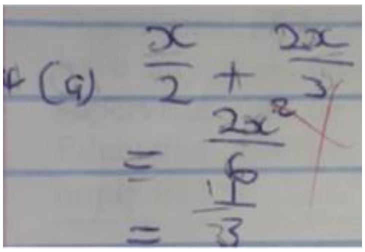
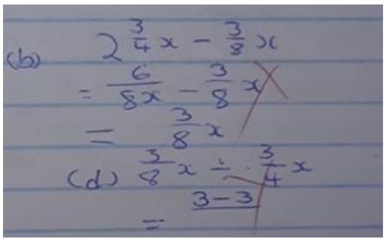
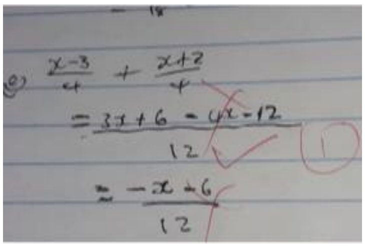
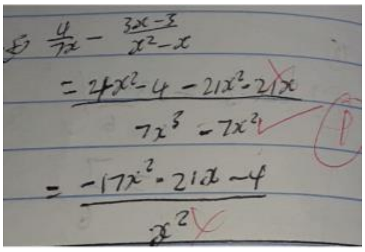
Question 2b  b $2\frac{3}{4}-\frac{3}{8}$ $\frac{11}{8_8}-\frac{3}{8}$ $=\frac{8}{8}$ $=1$ ① L24 error 1	Question 4a  4 (a) $\frac{x}{2}+\frac{2x}{3}$ $=\frac{2x^2}{6}$ $=\frac{1}{3}$ L24 error 2	Question 4b  (b) $2\frac{3}{4}x-\frac{3}{8}x$ $=\frac{6}{8x}-\frac{3}{8}x$ $=\frac{3}{8}x$ (d) $\frac{3}{8}x\div\frac{3}{4}x$ $\underline{3-3}$ $=$ L24 error 3
Question 4e  (e) $\frac{x-3}{4}+\frac{x+2}{4}$ $=\frac{3x+6-4x-12}{12}$ $=\frac{-x+6}{12}$ ① L 25 error 1	Question 4f  (f) $\frac{4}{7x}-\frac{3x-3}{x^2-x}$ $=\frac{4x^2-4-21x^2-21x}{7x^3-7x^2}$ $=\frac{-17x^2-21x-4}{x^2}$ ① L25 error 2	

Figure E1: Errors learners made in the Pre-test

E2

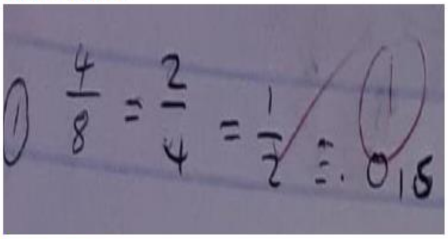
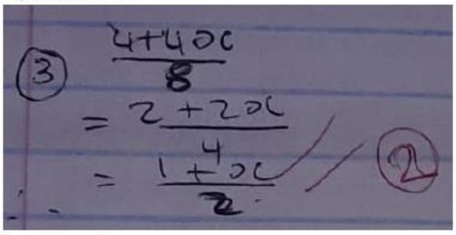
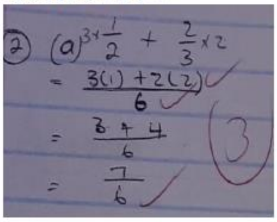
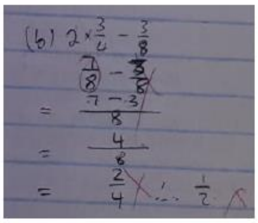
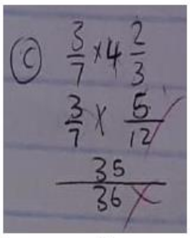
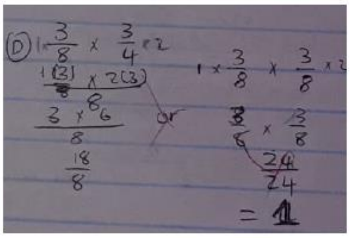
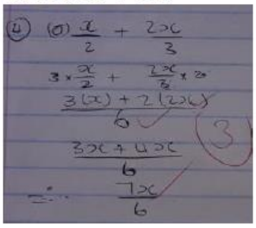
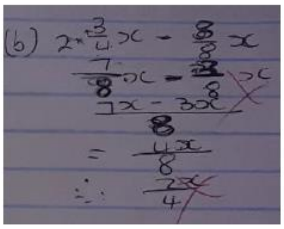
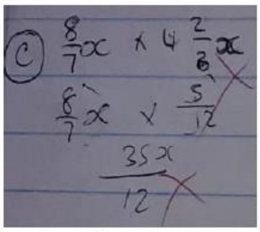
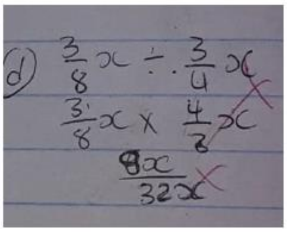
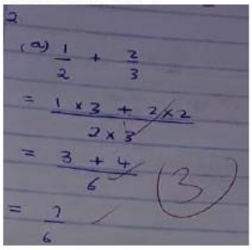
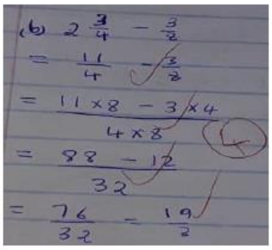
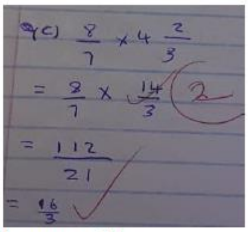
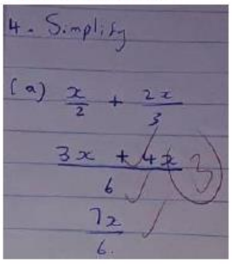
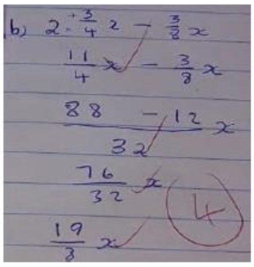
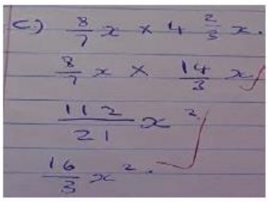
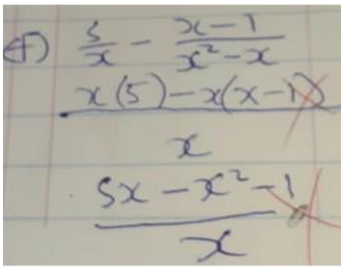
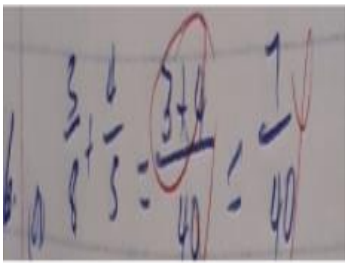
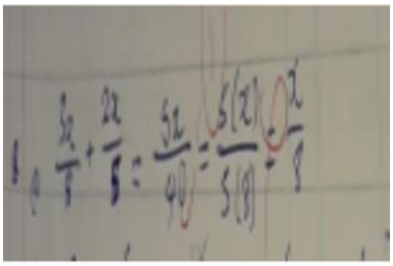
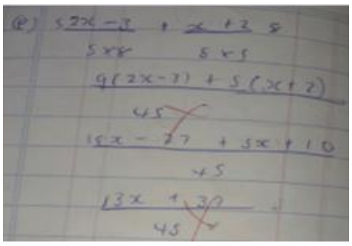
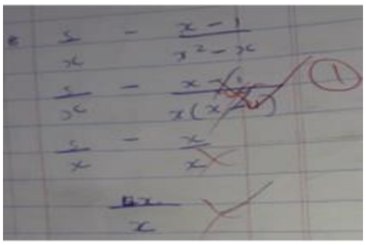
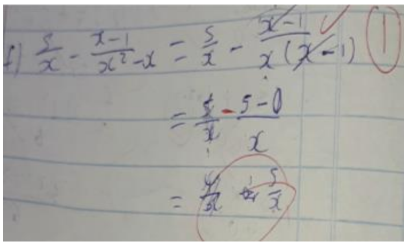
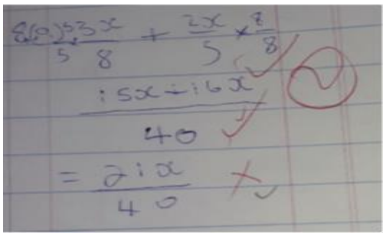
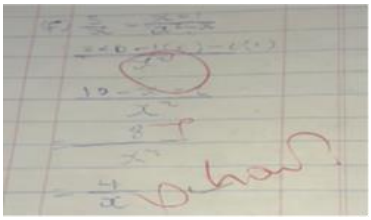
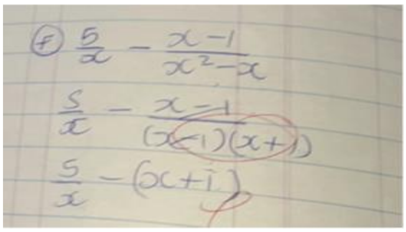
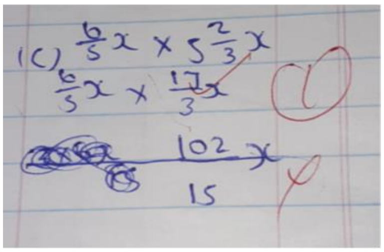
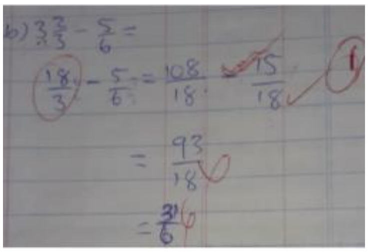
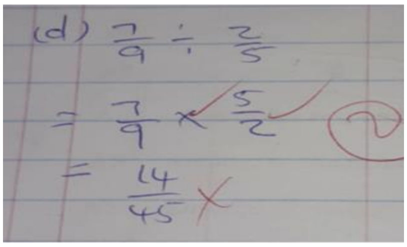
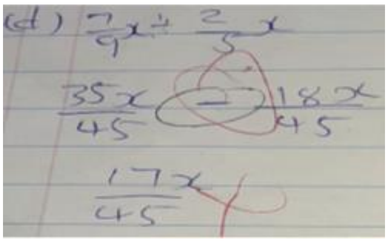
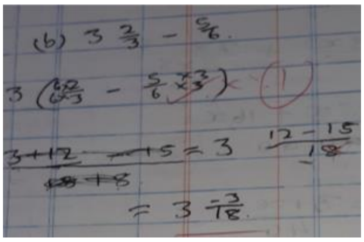
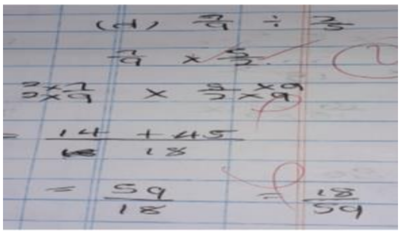
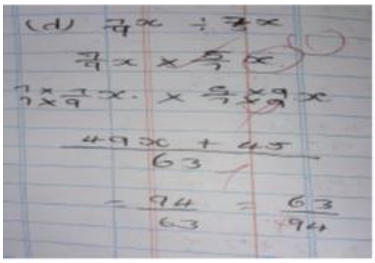
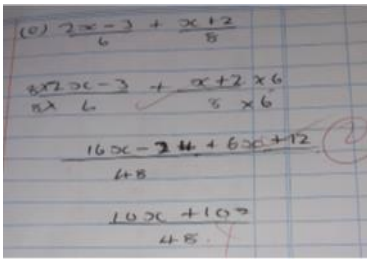
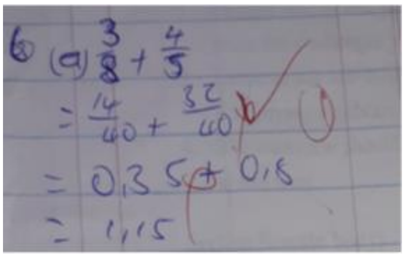
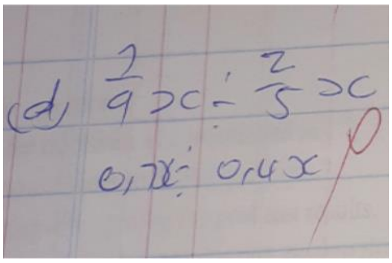
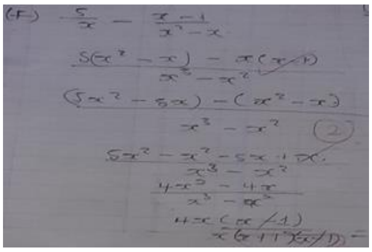
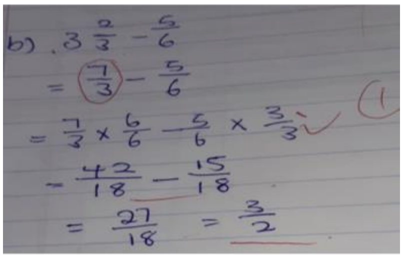
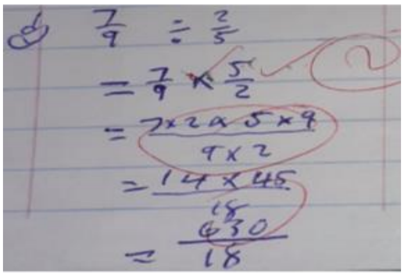
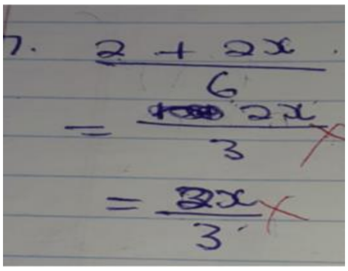
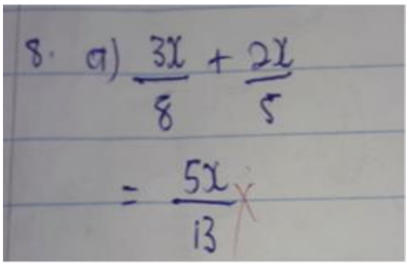
Question 1  ① $\frac{4}{8}=\frac{2}{4}=\frac{1}{2}\therefore 0{,}5$ ① Learner 7	Question 3  ③ $\frac{4+4x}{8}$ $=\frac{2+2x}{4}$ $=\frac{1+x}{2}$ ② Learner 7
Question 2a-d  ② (a) $3\times\frac{1}{2}+\frac{2}{3}\times 2$ $=\frac{3(1)+2(2)}{6}$ $=\frac{3+4}{6}$ $=\frac{7}{6}$ ③  (b) $2\times\frac{3}{4}-\frac{3}{8}$ $\frac{7}{8}-\frac{3}{8}$ $=\frac{7-3}{8}$ $=\frac{4}{8}$ $=\frac{2}{4}\therefore\frac{1}{2}$  (c) $\frac{3}{7}\times4\frac{2}{3}$ $\frac{3}{7}\times\frac{5}{12}$ $\frac{35}{36}$  (d) $1\times\frac{3}{8}\times\frac{3}{4}\times2$ $\frac{1(3)\times2(3)}{8}$ $\frac{3\times6}{8}$ $\frac{18}{8}$ or $1\times\frac{3}{8}\times\frac{3}{8}\times2$ $\frac{3}{6}\times\frac{3}{8}$ $\frac{24}{24}$ $=1$ Learner 7	Question 4 a-d  ④ (a) $\frac{x}{2}+\frac{2x}{3}$ $3\times\frac{x}{2}+\frac{2x}{3}\times2$ $\frac{3(x)+2(2x)}{6}$ $\frac{3x+4x}{6}$ $\therefore\frac{7x}{6}$ ③  (b) $2\times\frac{3}{4}x-\frac{3}{8}x$ $\frac{7}{8}x-\frac{3}{8}x$ $\frac{7x-3x}{8}$ $=\frac{4x}{8}$ $\therefore\frac{2x}{4}$  (c) $\frac{8}{7}x\times4\frac{2}{3}x$ $\frac{8}{7}x\times\frac{5}{12}$ $\frac{35x}{12}$  (d) $\frac{3}{8}x\div\frac{3}{4}x$ $\frac{3}{8}x\times\frac{4}{3}x$ $\frac{8x}{32x}$ Learner 7
Question 2a-c  2 (a) $\frac{1}{2}+\frac{2}{3}$ $=\frac{1\times3+2\times2}{2\times3}$ $=\frac{3+4}{6}$ $=\frac{7}{6}$ ③  (b) $2\frac{3}{4}-\frac{3}{8}$ $=\frac{11}{4}-\frac{3}{8}$ $=\frac{11\times8-3\times4}{4\times8}$ $=\frac{88-12}{32}$ $=\frac{76}{32}=\frac{19}{8}$ ④  (c) $\frac{8}{7}\times4\frac{2}{3}$ $=\frac{8}{7}\times\frac{14}{3}$ $=\frac{112}{21}$ $=\frac{16}{3}$ ② Learner 19	Question 4 a-c  4. Simplify (a) $\frac{x}{2}+\frac{2x}{3}$ $\frac{3x+4x}{6}$ $\frac{7x}{6}$ ③  (b) $2\frac{3}{4}x-\frac{3}{8}x$ $\frac{11}{4}x-\frac{3}{8}x$ $\frac{88-12}{32}x$ $\frac{76}{32}x$ $\frac{19}{8}x$ ④  (c) $\frac{8}{7}x\times4\frac{2}{3}x$ $\frac{8}{7}x\times\frac{14}{3}x$ $\frac{112}{21}x^2$ $\frac{16}{3}x^2$ Learner 19

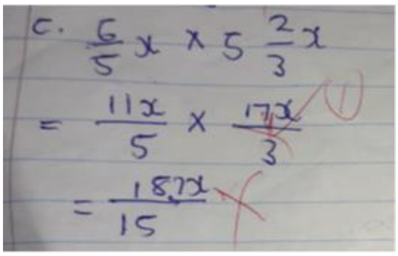
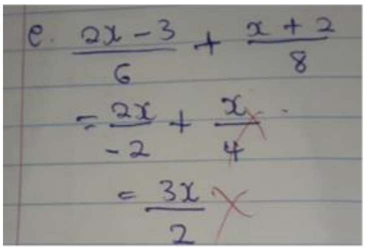
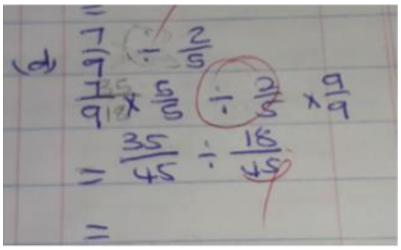
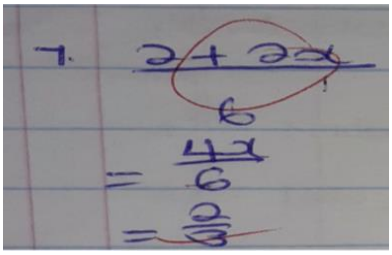
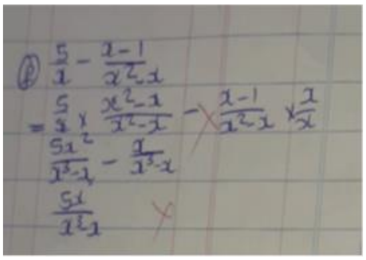
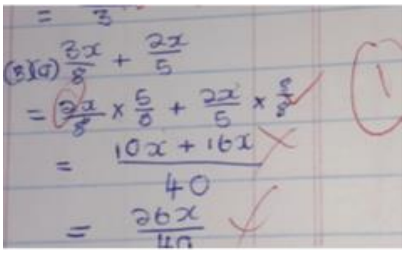
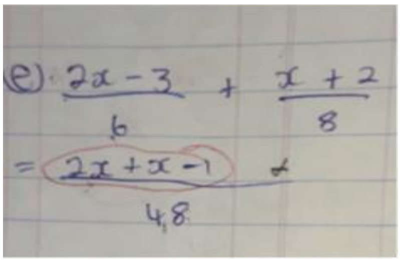
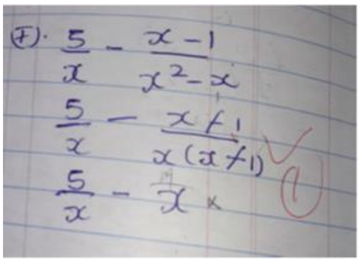
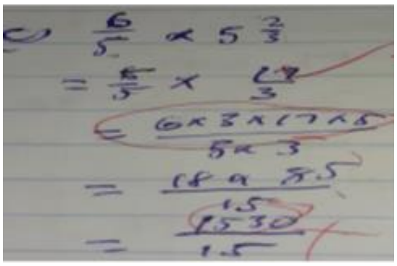
Figure AE2: Comparison of learners' performance in common fractions versus algebraic fractions

E3

Question 8f  L1 error 1	Question 6a  L2 error 1	Question 8a  L2 error 2
Question 8e  L3 error 1	Question 8f  L3 error 2	Question 8f  L5 error 1
Question 8a  L6 error 1	Question 8f  L6 error 2	Question 8f  L7 error 1

Question 8d  L8 error 1	Question 6b  L9 error 1	Question 6d  L9 error 2
Question 8d  L9 error 3	Question 6b  L10 error 1	Question 6d  L10 error 2
Question 8d	Question 8e	Question 6a

 L10 error 3	 L10 error 4	 6 (a) $\frac{3}{8}+\frac{4}{5}$ $=\frac{14}{40}+\frac{32}{40}$ $=0{,}35+0{,}8$ $=1{,}15$ L11 error 1
Question 8d  (d) $\frac{7}{9}x \div \frac{2}{5}x$ $0{,}7x \div 0{,}4x$ L11 error 2	Question 8f  L15 error 1	Question 6b  b) $3\frac{2}{3}-\frac{5}{6}$ $=\frac{7}{3}-\frac{5}{6}$ $=\frac{7}{3}\times\frac{6}{6}-\frac{5}{6}\times\frac{3}{3}$ $-\frac{42}{18}-\frac{15}{18}$ $=\frac{27}{18}=\frac{3}{2}$ L16 error 1
Question 6d  d $\frac{7}{9}\div\frac{2}{5}$ $=\frac{7}{9}\times\frac{5}{2}$ $=\frac{7\times2 \, a \, 5\times9}{9\times2}$ $=\frac{14\times45}{18}$ $=\frac{630}{18}$ L16 error 2	Question 7  7. $\frac{2+2x}{6}$ $=\frac{2x}{3}$ $=\frac{2x}{3}$ L16 error 3	Question 8a  8. a) $\frac{3x}{8}+\frac{2x}{5}$ $=\frac{5x}{13}$ L16 error 4

Question 8c  c. $\frac{6}{5}x \times 5\frac{2}{3}x$ $= \frac{11x}{5} \times \frac{17x}{3}$ ✓ ① $= \frac{187x}{15}$ ✗ L16 error 5	Question 8e  e. $\frac{2x-3}{6} + \frac{x+2}{8}$ $= \frac{2x}{-2} + \frac{x}{4}$ ✗ $= \frac{3x}{2}$ ✗ L16 error 6	Question 6d  = (d) $\frac{7}{9} \div \frac{2}{5}$ $\frac{7}{9} \times \frac{5}{5} \div \frac{2}{5} \times \frac{9}{9}$ $= \frac{35}{45} \div \frac{18}{45}$ = L17 error 1
Question 7  7. $\frac{2+2x}{6}$ $= \frac{4x}{6}$ $= \frac{2}{3}$ L17 error 2	Question 8f  (f) $\frac{5}{x} - \frac{x-1}{x^2-x}$ $= \frac{5}{x} \times \frac{x^2-x}{x^2-x} - \frac{x-1}{x^2-x} \times \frac{x}{x}$ ✗ $\frac{5x^2}{x^3-x} - \frac{x}{x^3-x}$ $\frac{5x}{x^2-x}$ ✗ L17 error 3	Question 8a  = 3 (8)(a) $\frac{3x}{8} + \frac{2x}{5}$ $= \frac{2x}{8} \times \frac{5}{5} + \frac{2x}{5} \times \frac{8}{8}$ ✓ ① $= \frac{10x + 16x}{40}$ ✗ $= \frac{26x}{40}$ ✓ L18 error 1
Question 8e  (e) $\frac{2x-3}{6} + \frac{x+2}{8}$ $= \frac{2x+x-1}{4{,}8}$ L21 error 1	Question 8f  (f). $\frac{5}{x} - \frac{x-1}{x^2-x}$ $\frac{5}{x} - \frac{x \neq 1}{x(x \neq 1)}$ ✓ ① $\frac{5}{x} - \overset{1}{\cancel{x}}$ × L21 error 2	Question 6c  (c) $\frac{6}{5} \times 5\frac{2}{3}$ $= \frac{6}{5} \times \frac{17}{3}$ $= \frac{6 \times 3 \times 17 \times 5}{5 \times 3}$ $= \frac{18 \times 85}{15}$ $= \frac{1530}{15}$ ✗ L25 error 1

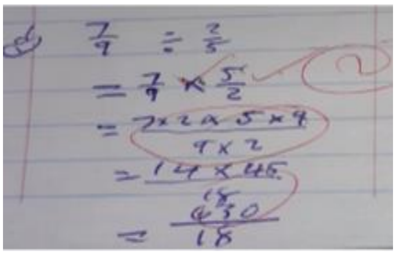
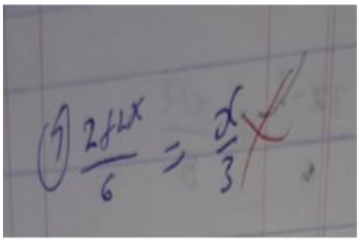
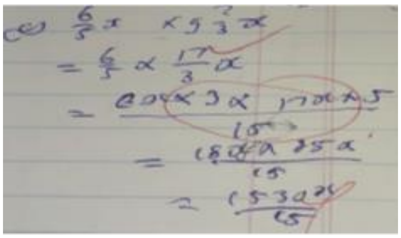
Question 6d  d) $\frac{7}{9} \div \frac{2}{5}$ $= \frac{7}{9} \times \frac{5}{2}$ $= \frac{7\times2 \quad 5\times9}{9\times2}$ $= \frac{14 \times 45}{18}$ $= \frac{630}{18}$ L25 error 2	Question 7  7) $\frac{24x}{6} = \frac{x}{3}$ L25 error 3	Question 8c  L25 error 4

Figure E3: Errors made by learners in the post –test

LANGUAGE EDITING CERTIFICATE

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SACE REGISTERED

11 July 2021

Master thesis*: *RELATIONSHIP BETWEEN LEARNERS' PROFICIENCY IN COMMON AND ALGEBRAIC FRACTIONS: A CASE OF GRADE 10 LEARNERS AT A LIMPOPO HIGH SCHOOL
(LINA MANGWENDE: 54737532)

This serves to confirm that I edited substantively the above document. I returned the document to the author with some tracked changes intended to correct errors and clarify meaning. It was the author's responsibility to attend to these changes.

Yours faithfully

Zano

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APPENDIX 7: INTERVIEW RESPONSES

IR1

Table IR1: Learner’s interview responses

Learner code	1.Difficult questions	2.Method used to tackle difficult questions	3.Steps learner failed to do	4.Reasons for failing to do the steps	5(a)Which denominator did you use and why?	5(b)Why do you use a calculator	5(c) Why did you not reduce your answers to the lowest terms e.g., $\frac{51}{18}$ or $\frac{24x-12}{48}$	5(d)Why do you put fractions under the same denominator when multiplying or dividing fractions	Given another chance will you do better
Lowest achievers									
L1	8a,b,c,d,e,f	I was making the denominator to be the same and change mixed numbers to improper fractions	I failed to find the LCD and how to change a mixed fraction to an improper fraction	I did not quite understand how the LCD is found and also how to multiply and divide fractions	I use the ECD because it is easy to get by multiplying denominators	It is easy to solve fractions with a calculator	I did not get those answers.	I did not do that.	Yes, I have seen the mistakes I was making after the interview
L11	6,7,8	I tried to find a common denominator when adding and	I failed to find the common denominator of algebraic	I don’t just get it and I don’t like fractions	I use the ECD because it’s easy to find but after	It makes the simplificati on of	Two cannot get into the numerator and	I did not do that, I don’t even know how to multiply or	Yes, now I know how to find the LCD and how to multiply and divide fractions after

		subtracting fractions. For multiplication and dividing fractions I used a calculator	fractions and also to reduce algebraic fractions to the lowest terms.		getting it I don’t know what to do	fractions easier.	denominator of the fraction $\frac{51}{18}$	divide fractions.	the interview.
L13	6d,78a,d,e,f	For addition and subtraction of fractions, I cross multiply. I had forgotten the rules for multiplying and dividing fractions.	I failed to find the common denominator and to reduce fractions to their lowest terms.	I do not know how to find LCD and how to divide fractions.	I use ECD because it is easy to find.	It simplifies fractions easily.	I did not get those answers, I used a calculator	I did not do that, I used a calculator.	Yes, I now know how to find the LCD.
L16	6b,c,7,8a-f	To get the common denominator I multiply the denominators.	I failed to get the LCD.	Letters confuse me I don’t know what to do with them.	I use the ECD because it is the one I understand	I do not use a calculator	I did not get those answers.	I was thinking it is the same as adding and subtracting fractions. I had forgotten the steps on how to multiply or divide fractions.	Yes, I now know how to find the LCD and how to change a mixed number into an improper fraction.
Middle achievers									
L22	8c	I find the LCD and swap the denominators.	I did not reduce it	The letters in the problem confused me when they are mixed with numbers.	I use the ECD because the LCD still confuses me.	Calculator makes simplification easier and faster.	I did not get that answer; I used a calculator.	I did not do that	Yes, there are now things I understand
L25	7,6c,d 8,c,d,f	I find LCD and then solve the	I failed to reduce fractions to the	Letters that are used in question	I use the ECD because I do	I do not use a	Two can’t get into the	I thought the rules we use in	Yes, because I realised my mistakes.

		fraction	lowest terms.	8 simplified fractions difficult.	not understand how to find LCD.	calculator.	fraction $\frac{51}{18}$	adding and subtracting fractions also apply in multiplication and division.	
L3	7,6c,d,8d,e,f	When adding and subtracting fractions I tried to find the LCD, for division and multiplication I put both fractions under the same denominator.	I failed to reduce fractions to the lowest terms and to find the common denominator of algebraic fractions especially with letters in the denominator.	The fractions with letters in the denominator made it difficult to find the common denominator.	I use the ECD because it is easy and faster to get.	I do not use a calculator.	I simplified my answers.	I thought it is the same as cross multiplication.	Yes, because I realised my mistakes. I now also understand how to multiply and divide fractions after the interview.
L18	8f	When I add and subtract fractions I make the denominators of the fractions the same.	I failed to find the common denominator of the fraction.	I failed to factorise the expressions to reduce them to the lowest terms.	I use the ECD because I don’t know the difference between ECD and LCD.	I do not use a calculator.	Two can’t get into the numerator and denominator for the algebraic fraction I don’t know how to reduce it.	I did not do that.	Yes, because now I know how to factorise algebraic expressions and how to find LCD.
Top achievers									
L7	8e,f	When I add and subtract fractions	I failed to factorise the	The letters in the denominator	I use ECD I don’t know the	I do not use a	Two can’t divide into	I did not do that, I know how to	Yes, I am now well informed on how to

		I just multiply the denominators of the fractions to get the common denominator.	algebraic expressions so that I could find the common denominator.	confuse me.	difference with LCD.	calculator.	51 and 18 without leaving a remainder.	multiply and divide fractions.	reduce fractions and the advantage of using LCD
L15	No difficult question	N/A	N/A	N/A	I use ECD because I do not understand how to find LCD	I do not use a calculator	I simplified where I didn't I forgot	I know how to multiply and divide fractions.	Yes, I am now perfect at solving fractions and now know the advantage of using LCD.
L19	8f	I multiplied the denominators to get the common denominator.	I failed to simplify the answer I got.	When you multiply denominators of algebraic fractions where there are letters in the denominator it the denominator becomes complicated.	I use ECD because I did not know the difference with the LCD.	I do not use a calculator.	The common fractions I reduced but for algebraic I did not realise they needed to be reduced.	I know how to multiply and divide fractions.	No, because when there are letters in the denominator I get confused I need more time to practice.
L23	No difficult question.	N/A	N/A	N/A	I use the ECD because it is easy to find than the LCD.	I do not use a calculator.	I reduced the answers to the lowest terms	I know how to multiply and divide fractions.	Yes, I now know the advantage of using the LCD.

IR2

Table IR2: Showing teacher's interview responses

Teacher's code	Teaching experience	Difficult topics	Challenges and barriers faced by learners in learning fractions	Methods used by teachers to teach common and algebraic fractions	Do grade 10 learners know the concept of common fractions very well
T1	4years	- Measurement - Trigonometry	- Learners do not know the difference between the denominator and numerator. - In algebraic fractions where factorisation should be done learners fail to factorise. - It then simplifies algebraic fractions difficult since expressions become complicated.	- Use rules and algorithms - Defines the terms numerator and denominator to learners - Teach factorisation beforehand	The teacher was not certain but indicated that learners do well in addition and subtraction. Learners have problems with division.
T2	10 years	- Algebraic expressions - Algebraic fractions - Inequalities - Euclidean Geometry	- Learners cannot determine the LCD when adding and subtracting fractions. Instead, they just multiply the denominators e.g., - $\frac{2}{3}-\frac{3}{4}=\frac{2}{3\times4}-\frac{3}{4\times3}$ $=\frac{2}{12}-\frac{3}{12}$ $=-\frac{1}{12}$ - Learners do not use the minus sign in subtracting algebraic fractions, they do not know it affects all the terms in the expression e.g., $\frac{x-3}{3}-\frac{x+2}{4}$	- Introduce learners to rules and algorithms. - Teach learners how to find LCD and using equivalent fractions in adding and subtracting fractions.	Learners can simplify fractions when they are of a lower order, as they increase in difficulty they fail to solve. They can also simplify fractions to their lowest terms if they are numerical but when they are algebraic they fail.

			$=\frac{4}{4}\times\frac{x-3}{3}-\frac{3}{3}\times\frac{x+2}{4}$ $=\frac{4x-12}{12}-\frac{3x+6}{12}$ $=\frac{4x-12-3x+6}{12}$ $=\frac{x-6}{12}$		
T3	6years	• Euclidean Geometry • Fractions	• Learners cannot simplify fractions to the lowest terms. • Learners also struggle with division, factorisation and addition of fractions. E.g., $\frac{2}{18}+\frac{6}{18}=\frac{8}{18}$ learners add both the numerators and denominators. They also do not simplify answers to the simplest form. • Learners do not understand the concept of LCD and LCM e.g., $\frac{x-1}{x^2-1}+\frac{1}{x+2}$ $=\frac{(x-1)(x+2)+1(x^2\ -1)}{(x^2-1)(x+2)}$ $=\frac{x^2+x-2+x^2-1}{x^3+2x^2-2}$ Learners can no longer know how to simplify further.	I use models to explain the concept of fractions. I will then introduce learners to rules on how to solve fractions e.g., flipping the denominator when dividing fractions.	No, they don't know the whole concept. Some learners still apply rules incorrectly e.g., when multiplying fractions, they find the LCD like they do in adding and subtracting fractions. Expressing fractions to the lowest terms is still a problem. They also struggle with factorisation which is important in solving algebraic fractions

T5	10 years	Fractions especially multiplication and division.	• Learners struggle with changing mixed numbers to improper fractions e.g1 $\frac{2}{3}\times 3\frac{1}{5}=\frac{6}{3}\times\frac{9}{5}$ • They also have challenges with the addition and subtraction of fractions, they add numerators together and denominators together. E.g., $\frac{1}{3}+\frac{1}{4}=\frac{2}{7}$	I use the model approach since it makes learners be interested in learning fractions. I then emphasize the rules.	No, they don't, they still struggle with finding the LCD and simplification of algebraic fractions.
T4	15 years	Euclidean and surds(Exponential laws)	• Learners struggle with the arithmetic of fractions and whole numbers • Lack knowledge of equivalent fractions • Converting mixed numbers to improper fractions and vice-versa. Some of the common errors learners do are as follows:$2+\frac{1}{2}=\frac{3}{2}$	I start with finding LCD of common fractions. Then I emphasise that they only add or subtract fractions when denominators are the same. I deal with proper fractions before mixed numbers and improper fractions.	No, this is the learners' greatest weakness. Most can only work with proper fractions involving the same denominator. To determine the LCD is a problem for most as they don't know the difference between LCM and HCF.