USING *UBUNTU* VALUES TO ENHANCE DISRUPTIVE PEDAGOGIES FOR EFFECTIVE AND MEANINGFUL TEACHING OF GRADE 12 EUCLIDEAN GEOMETRY

By

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DECLARATION

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I declare that the above thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the thesis to originality checking software and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at Unisa for another qualification or at any other higher education institution.

10th June 2021

SIGNATURE	DATE

DEDICATION

I dedicate this thesis to my wife, Verna, family, siblings, children, Tanaka, Taonaishe Lameck and Tatenda, as well as my mother, Lucia Chidziva, who was in a wheelchair after suffering a stroke when I was compiling it.

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ABSTRACT

Using a convergent mixed-methods parallel research design, this study explored how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and

meaningful teaching of Grade 12 Euclidean geometry. It employed the social interdependence theory [SIT], *Ubuntu* theory and the Van Hieles' theory of geometric thinking as the theoretical lens. Quantitative data were collected using non-equivalent pre-test-post-test comparison group, while qualitative data were collected using classroom observation, semi-structured interviews, and document review. A sample of two Grade 12 mathematics teachers from two different rural schools of Cacadu Circuit Management Centre, Chris Hani West District of the Eastern Cape Province, South Africa, was purposively selected from a population of 20 Grade 12 mathematics teachers. Each of the two participants taught two groups of learners of less than 20 learners each from their school. One of the groups for each teacher was a control group and the other was an experimental group. Each control group was taught using traditional instructional strategies while the experimental group was taught using the *Ubuntu* values infused in the Dynamic Pairs Model. The Dynamic Pairs Model consisted of learners sitting in pairs whose composition changed every day. The quantitative data from the pre-test and the post-test were analysed using the independent samples t-test from the statistical package SPSS while qualitative data were thematically analysed. Results of the study indicate that:

- Learners from each experimental group produced detailed and accurate solutions as they solved problems in Euclidean geometry as compared to their peers from the respective control group,
- Overwhelming active participation was witnessed from learners from experimental groups as compared to their peers from control groups,
- For each participant's groups, there was a statistical difference between the test scores of the learners from the experimental group and their peers from the

control group in the post-test. The statistical difference was in favour of the experimental group for each participant.

- The geometric thinking levels of learners from the experimental groups were enhanced.

The study recommends that mathematics teachers must be encouraged to incorporate *Ubuntu* values in the teaching of Euclidean geometry and other topics especially from lower grades, to use physical manipulatives in geometry teaching and to use Van Hiele's theory in geometry teaching.

KEY CONCEPTS: *Ubuntu*, disruptive pedagogies, effective and meaningful teaching, Euclidean geometry, traditional instructional strategies, mathematics teachers

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ACRONYMS

APOS-Action, Process, Object and Schema

CAPS: Curriculum and Assessment Policy Statement

CDE: Centre for Development Enterprise

CK: Content Knowledge

DoBE: Department of Basic Education

FET: Further Education and Training

HSRC: Human Sciences Research Council

IK: Indigenous Knowledge

IKS: Indigenous Knowledge Systems

INSET: In-Service Training

MKT: Mathematical Knowledge for Teaching

NCTM: National Council of Teachers of Mathematics

NEEDU: National Education Evaluation and Development Unit

NSC: National Senior Certificate

OBE: Outcomes-based Education

PCK: Pedagogical Content Knowledge

SACMEQ: Southern and East African Consortium for Monitoring Educational Quality

SCK: Specialised Content Knowledge

CHAPTER 1: INTRODUCTION TO THE STUDY

1.1.Overview of the study

This study is divided into seven chapters in which the first chapter presents the overview of the study, introduction, background of the problem, synopsis of literature review and synopsis of the theoretical framework. The first chapter also presents the problem statement, research questions, research aim and objectives, significance of the study, synopsis of the research methodology, trustworthiness of the study, ethical considerations, definition of terms and conclusion. The second chapter is dedicated to review of literature related to the study. The third chapter presents the theoretical framework that underpinned the study. The fourth chapter presents the research methodology used in the study. The fifth chapter presents the pilot study. Data presentation, findings and analysis are presented in the sixth chapter. The seventh chapter presents the discussion of the findings while the eighth chapter consists of conclusions and recommendations.

1.2.Introduction

Since the attainment of democracy in South Africa, the improvement in the teaching and learning of mathematics in the country has been marginal. In 2011, South Africa was one of the poorest performing countries in international assessments administered to Grade 9 learners as compared to other countries (Human Sciences Research Council [HSRC], 2011). Some researchers (Kazima, Pillay & Adler, 2008; Sibuyi, 2012; Hugo, Wedekind & Wilson, 2010; Bolowana, 2014; Venkat & Spaull, 2014) associate the poor performance in mathematics with various factors including teachers' knowledge of classroom practice. According to the Centre for Development Enterprise [CDE] (2011), South Africa's quality of teachers was one of the poorest in the developing world and this had negative effects on learners’ academic performance. According to Goe (2007), teacher quality includes teachers’ academic and professional qualifications in conjunction with some of the following personal characteristics:

i. Thorough subject matter knowledge,
ii. Ability to create a conducive classroom environment,
iii. Ability to motivate students to achieve higher grades in their learning,
iv. Commitment to assist learners to achieve higher academic levels and,
v. Willingness to work diligently with all types of students inclusive of those with special needs.

In this study, teacher quality was defined in conjunction with Goe's (2007) propositions. In addition, the study also considered teachers’ attendance to mathematics In-Service Training [INSET] seminars and professional development workshops as attributes that could contribute to mathematics teacher quality as well. Attendance to those seminars and workshops should add value to the teachers’ academic and professional qualifications.

Furthermore, CDE (2014) reports that 54% of South African Grade 9 mathematics teachers never trained formally to teach the subject. This information displayed some of the reasons why mathematics had poor performance in the country. According to the CDE report, learners’ performance was not impressive because some of the teachers did not have pedagogical skills in teaching mathematics content. According to Parliament of the Republic of South Africa (2016), the performance of South African Grade 6 learners in mathematics in Southern and East African Consortium for Monitoring Educational Quality [SACMEQ] assessments, was the poorest. Countries with weaker economies such as Swaziland, Kenya and Seychelles performed even better than South Africa. The legislators in South Africa were worried that their country was trailing behind countries with smaller economies. The analysis was done. on results obtained from SACMEQ II (2000), SACMEQ III (2007) and SACMEQ IV (2013). Table 1.1 below displays the SACMEQ results of 14 countries in order of performance.

Table 1. 1: Trends in achievement levels of Grade 6 learners in SACMEQ countries

	Learner mathematics score			
Year	**2000**	**2007**	**2013**	**Difference 2007-2013**
1. Mauritius	585	623	694	71
2. Kenya	563	557	651	94
3. Seychelles	554	551	630	79
4. Swaziland	517	541	601	68
5. Botswana	513	521	598	77
6. South Africa	**486**	**495**	**587**	**92**
7. Uganda	506	482	580	98
8. Zimbabwe		520	566	46
9. Lesotho	447	477	559	82
10. Namibia	431	471	558	87
11. Mozambique	530	484	558	74
12. Zambia	435	435	522	87
13. Tanzania	522	553		
14. Zanzibar	478	486		
15. Malawi	433	447	522	75
SACMEQ	500	507	584	77

Source: Parliament of the Republic of South Africa. Parliamentary Portfolio Committee on Basic Education (2016)

Some researchers (Graven, 2013; Reddy, Zuze, Visser, Winnar, Juan, Prinsloo & Rodgers, 2015) argue that the learners' poor performance in mathematics is caused by their lack of basic skills in the subject. Perhaps the lack of strong mathematical background of the learners in lower grades contributes to their poor performance in the Further Education and Training [FET] Phase. Therefore, there is a strong need for building a solid mathematical base in lower grades to enable conceptual understanding of mathematical concepts in the subsequent grades.

According to Alex and Mammen (2014), Grade 10 learners lack basic skills in geometric thinking with respect to Van Hieles' model of geometric thinking. They found that most of the Grade 10 learners in the study were on Level 0. According to Vojkuvkova (2012), Level 0 is a level in which a learner can only visualise or recognise geometric shapes and lacks skills such as analysis, abstraction, deductions and rigour. The poor geometric thinking level of the learners implies that they are promoted to Grade 11 under-prepared. In addition, Makhubele (2014) asserts that Grade 11 learners commit a lot of errors and show a lot of misconceptions in geometry. The majority of the learners have visualisation and analysis skills only and lack other skills inherent in Van Hieles' model. The implications for research include concerted efforts to solve difficulties encountered by learners and build a firm background in geometry. Therefore, there is a need for teachers to be innovative and resourceful to enhance conceptual understanding in geometry.

1.3.Background of the problem

Reports from the Department of Basic Education [DoBE] in South Africa for the years 2016, 2017, 2018 and 2019 bear evidence that learners' performance in geometry in the National Senior Certificate [NSC] Examinations was poor (DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020). In this study Department of Basic Education [DoBE] refers to the South African Department of Basic Education. The average percentage scores of questions in Euclidean geometry in NSC examinations in 2016, 2017, 2018, 2019 and 2020 are displayed in Table 1.2 below.

Table 1. 2: Average percentage scores of learners in Euclidean geometry in Paper 2

Year	Average percentage score in Euclidean geometry	Total Marks on Euclidean in the Paper out of 150 marks
2016	45.67	41
2017	38.75	49
2018	47.00	48
2019	45.67	48
2020	46.67	49

Calculated from the sources: DoBE (2017), DoBE (2018), DoBE (2019), DoBE (2020) and DoBE (2021)

The information displayed in Table 1.2 shows that Euclidean geometry was generally difficult for the learners because the average scores in the topic were less than 50%. In South Africa, a minimum of 50% pass mark in mathematics was a requirement to enrol in universities for degrees in critical skills disciplines such as engineering, medicine, finance and economics. In addition, South Africa as a country faced shortage of skilled and degreed personnel in enginering courses. On national news in April 2021, it was reported that the South African government resorted to hiring degreed engineering experts from Cuba to address the problems of water shortages in the country and there was a national outcry about this decision. According to Tshwane University of Technology (2021), to qualify for studying for a degree in engineering courses such as architectural engineering among others, a student must have a minimum of 50% pass mark in mathematics in National Senior Certificate examinations. Generally, engineering courses require students to have a strong background in geometry since a lot of engineering drawings are done. Table 1.2 shows that the average learners' performances in geometry were below the 50% pass requirement for university entry into critical skills disciplines at universities. For the years 2016, 2017, 2018 and 2019 questions from Euclidean geometry occupied almost one-third of the second paper of mathematics in the National Senior Certificate [NSC] examinations. This implies that teachers and learners needed to exert more effort on the topic. It is envisaged that an improvement in the understanding of geometry may serve to boost the overall performance of learners in mathematics.

The poor performance in mathematics topics such as geometry in national examinations is generally attributable to teachers' lack of mathematical knowledge for teaching, shortage of teachers who are competent to teach mathematics and lack of resources. According to Bansilal, Brijlall and Mkhwanazi (2014), a large number of mathematics teachers have inadequate pedagogical content knowledge [PCK] and mathematical content knowledge. In addition, Gardiner (2008) points out that the majority of teachers in deep rural provinces such as the Eastern Cape did not possess adequate qualifications in teaching mathematics. According to Van der Sandt (2007), pre-service teachers were not adequately trained to reach expected levels of geometric thinking. Moreover, DoBE (2009) reports that South African universities were producing inadequately trained mathematics teachers hence the educational outcomes were mediocre. Also, Taylor (2011) asserts that principals and teachers demonstrate ignorance about the importance of content knowledge in teaching, hence they are devoid of intrinsic motivation. From my experience, a teacher without the relevant subject knowledge is not confident to interact with learners in that subject. He or she always tries to avoid mingling with learners. A teacher who has adequate subject content knowledge, is keen to prepare for lessons, and has the interest to study to gain more pedagogical knowledge in teaching his or her subject of specialisation. Also, the teacher is prepared to share with other teachers and with learners confidently.

Several studies have been conducted on teaching strategies in geometry (Cooper, 2011; Küchemann, 2007; The Royal Society, 2001; Achera, Beleçina & Garvida, 2015; Bikic', Marčic' & Pikula, 2017; Selden & Selden, 2007), while other researchers have conducted studies on teaching geometric thinking, understanding and reasoning (Connolly, 2010; Moss, Hawes, Naqvi & Caswell, 2015; Mabotja, 2017; Mahmood, Khan & Iqbal, 2017; Vojkuvkova, 2012; Panaoura & Gagatsis, 2009). Other researchers (Chiphambo, 2011; Furner & Worrell, 2017) emphasise the importance of the use of geometric manipulatives in the classroom. Such manipulatives include blocks, puzzles, rods, geoboards, cubes and many others. According to these researchers, when learners are exposed to manipulatives, they develop a more positive mathematical disposition.

Research has shown that there were a lot of difficulties and misconceptions encountered in the study of geometry in schools (Chigonga, 2016; Özerem, 2012; Adolphus, 2011; Evbuomwan, 2013). According to Mbugua, Kibet, Muthaa and Nkonke (2012), resources were unequally distributed, and learners from low socio-economic status were exposed to limited or poor quality of resources in democratic South Africa. The researchers further argue that the poor academic achievement of some of the learners may be associated with poor socio-economic backgrounds and limited resources in their schools. On the contrary, CDE (2014) mentions that a variety of resources have some influence on the fruitful teaching and learning of mathematics. A study by Taylor (2011) revealed that time was poorly managed in schools by both teachers and learners. In addition, Mammali (2015) suggests the causes of poor performance in geometry are:

i. The influence of cultural backgrounds whereby learners are influenced by the attitude, beliefs and values of their family members and the society,
ii. The inability of learners to see the importance of geometry in their present and future lives,
iii. Poor mathematics background from primary school level, and
iv. Learners' lack of proficiency in the language of geometry.

Some reasons provided by learners in my former mathematics classes are associated with a lack of mathematics relevance in their real-life environments. It is therefore a challenge paused to mathematics teachers to use real-life examples in mathematics lessons to make learners see the value of mathematics. Other learners would express that they come from families without anyone who was brilliant in mathematics before them and that poor performance in the subject ran through their family lineages. Perhaps such beliefs demotivated learners and they were the root cause of negative attitudes towards mathematics.

Santos (2014, p.92) argues that colonialism resulted in denigration and decimation of indigenous knowledge referred to as 'epistemicide'. Some researchers (Lebeloane, 2017; Mudaly, 2018; Oelofsen, 2015; Santos, 2014) assert that decolonisation in the school system is very important. Decolonisation improves the quality of education by promoting the use of indigenous knowledge systems which allow the African indigenous societies to be creative and inventive. Khupe (2014, p.43) defines IK as knowledge "belonging to or originating in an area or naturally living, growing or produced in an area" and Hewson and Ogunniyi (2011, p.680) refers to IKS as "knowledge systems that existed in numerous parts of the non-Western world before the advent of colonialism…”. The researchers (Lebeloane, 2017; Mudaly, 2018; Oelofsen, 2015; Santos, 2014) argue that the curricula in countries such as South Africa had been Eurocentric and needed to be decolonised. Lebeloane (2017) and Mheta, Lungu and Govender (2018) suggest that crafting the curriculum around *Ubuntu* is the most effective and best method to decolonise the South African education system. According to Letseka (2011), an education system based on *Ubuntu* promotes imagination, deliberation, responsibility, humanity and cooperation among learners together with their teachers. Letseka (2011) further explains that *Ubuntu* supports the use of indigenous knowledge which is an aspect important for promoting inclusivity, equality and social justice. According to DoBE (2011), schools should value indigenous knowledge systems by acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution. Mudaly (2018) explains that indigenous knowledge in mathematics is practical, necessary for living and uses familiar contexts whereas colonial knowledge is factual and uses unfamiliar contexts.

Research (Khupe, 2014; Jojo, 2015) has also shown that using indigenous knowledge in teaching mathematics yields good results. According to these authors, indigenous knowledge helps learners to easily understand and also serves as prior knowledge for the learners, which is very crucial in the classroom. Some researchers (Seepe, 2004; Makgoba, 1996; Higgs & Van Wyk, 2007) support the establishment of African education systems based on African values and philosophy. In such systems, indigenous knowledge systems are promoted. Frere (1970) argues that local Brazilian villagers freed themselves from being oppressed socially, economically and politically with the use of indigenous knowledge. Real education should lead to the elimination of psychological bondage. According to Msila (2009), indigenous knowledge systems promote equality and inclusivity but most profoundly they enhance understanding and participation. In my experience as a mathematics teacher, I have seen many teachers being eager to work with only brilliant learners and neglecting the academically disadvantaged. Some learners sometimes get more materials for learning because teachers naturally like them or because of their fastness in internalising content. Perhaps if all teachers embraced *Ubuntu* in their practice all learners would be attended to according to their abilities and needs.

Teaching in rural secondary schools where English is rarely spoken may also pose a challenge to the teaching of mathematics. In staffrooms teachers always complained about how learners' understanding was affected by their inability to learn other subjects in English. According to Setati (2009), language is important in the study of mathematics because many learners who are taught mathematics in a second language such as English or Afrikaans fail to understand the subject matter fully. Setati (2009) further suggests the use of code-switching to assist learners to understand mathematics. Perhaps if teachers resort to code-switching conceptual understanding in topics such as Euclidean geometry would be enhanced.

According to Taylor (2008), reform in mathematics teaching is difficult to attain due to complacency inherent in some principals and teachers. In addition, CDE (2013) explains that the attitudes of principals and teachers influence their willingness to reform and improve in the teaching and learning of mathematics. It is also evident that some teachers do not see the value of attending workshops to improve their teaching (Mbugua et al*.,* 2012). Usually, attending workshops enhances the content and pedagogical knowledge of teachers in mathematics, especially Euclidean geometry. Some teachers never trained to teach Euclidean geometry which was an optional topic before CAPS was introduced. Taylor (2011) mentions that some incompetent teachers are appointed to teach the subject because of their affiliations with trade unions. This phenomenon compromises the quality of teaching mathematics. According to Taylor (2011), much time is wasted in South African schools because teachers frequently attend union meetings or funerals. Furthermore, teachers and learners decide to come late to school and truancy is rampant. This clearly shows that some teachers lack respect, dignity, compassion for learners and that there is no rapport among teachers and learners. The findings show that *Ubuntu*, though it is espoused in the Constitution of The Republic of South Africa and school curriculum documents, is not practised in its fullest abundance to ensure conducive environments are established in schools. *Ubuntu* refers to the values such as the well-being of the society, consensus, agreement, compassion, empathy, sympathy, respect, human dignity, forgiveness and healing (Mokgoro, 1999; Skelton, 2002; Swanson 2007) and human interconnectedness through inter-dependence, inclusivity and inter-subjectivity (Bolden, 2014). I believe that if these values are present in a mathematics classroom, effective teaching and learning would be achieved.

The global conditions in education are rapidly changing and they require new approaches in pedagogy to be embraced. A lot of technological advancement has pervaded the society and this demands the incorporation of digital technology into classrooms to suit the current flow of events. Also, learners' performance in geometry in South African schools has been poor over the years. This would need the implementation of disruptive pedagogies to match the world's technological advancement. According to Manokore and McRae (2020), disruptive pedagogies are unconventional ways used in teaching to enhance student engagement and learning. In other words, teaching disruptively involves reaching students in new and innovative ways to engage them. In addition, Bassendowski, Mackey and Pamla (2014) explain that disruptive pedagogies also involve the implementation of digital technologies in classrooms. Elsbree (2002) further explains disruptive pedagogies as possessing the following tenets:

i. **Unknowability of teaching:** Teachers do not monopolise knowledge generation but also learn from others including learners,
ii. **The relational nature of power**: Teachers must seek to empower learners
iii. **The fluidity of understanding**: Teachers are open to listen to different opinions from learners and are open to discussion, and
iv. **The importance of seeking change**: Teachers believe the current way of teaching is anachronistic or old-fashioned and there is necessity of change to suit the present world.

In my view, if it would be best to incorporate Ubuntu values in teaching mathematics, and explore how they impact on disruptive pedagogies in the teaching of Euclidean geometry.

Researchers (Muzvidziwa & Muzvidziwa, 2012) argue that schools that implement *Ubuntu* philosophy in their curriculum perform better academically especially if discipline is promoted. According to Jojo (2018), *Ubuntu* allows the transformation of the learning environment and enhances mathematics teaching. These studies on the impact of *Ubuntu* philosophy in the school environment have influenced me to explore its impact on teaching Euclidean geometry. I strongly felt that it was of importance to deal with some aspects of *Ubuntu* to enhance mathematics teaching. This study therefore explored how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry. It specifically focuses on collective work and collaboration, which are integral tenets of *Ubuntu*.

1.4.Synopsis of literature review

In recent years, *Ubuntu* philosophy has pervaded various spheres of life in the African continent and beyond. Education as an ongoing life process has also to a greater extent been influenced by this African philosophy. According to Muzvidziwa and Muzvidziwa (2012), *Ubuntu* is recognised as an African philosophy linking the individual to the collective through brotherly and sisterly relationships. The philosophy makes a significant contribution to indigenous ways of knowing and foregrounds humanness, caring, sharing, respect, solidarity and many other principles. Some researchers have defined *Ubuntu* in single words as humaneness (Mdluli, 1987; Tutu, 2008) and personhood (Letseka, 2013) while others have defined the philosophy in lengthy phrases such as “a person is a person through other people” (Jolley, 2011; Van Nierkerk, 2013; Tutu, 2008; Chitumba, 2013), an African philosophy of humanity and community (Skelton, 2002) and many other explanations. In a mathematics classroom, learners depend on their teacher and vice-versa and they should share ideas among themselves on how to come up with solutions. Meanwhile, diagnostic reports indicate that most learners have challenges in Euclidean geometry. It would be a good idea to encourage learners to work together to achieve better results in that section of mathematics. Perhaps in the mathematics classroom, the teachers need to realise that some individual learners would not experience challenges in Euclidean geometry if they received relevant assistance. Effective mathematics teachers would encourage group interactions to assist learners with difficulties. Through group interactions, learners with difficulties learn from other learners. Mathematics teachers with *Ubuntu* values would have compassion for learners with learning difficulties and would invent strategies to enhance learners’ conceptual understanding of Euclidean geometry.

According to Van Nierkerk (2013), *Ubuntu* embraces African cultural aspects of generosity, hospitality, friendliness, care for others, forgiveness, consensus, compassion, positive group interaction and communal solidarity. For example, the teachers need to care and try to create group interactions to enhance the understanding of various geometric concepts. This can be done by creating small groups or teams in the classroom. Jolley (2011) further expresses that *Ubuntu* is a versatile tool to strengthen a community, to communicate through community support, dignity and identity achieved through mutualism, empathy, generosity and community commitment. The creation of a community of learners who strive to learn and understand mathematics is a necessary environment that mathematics teachers should master for better management of their classrooms. It is with this understanding that Nussbaum (2003) explains that *Ubuntu* is the capacity in the African culture to express compassion, reciprocity, dignity, harmony and humanity in the interest of building and maintaining the community. Reciprocity is the act of mutual exchange of ideas and resources. In a mathematics classroom, reciprocity is made possible by encouraging learners to exchange their ideas on problem solution and share content material and resources mutually. Egocentricity is discouraged, and every learner is encouraged to share all ideas and information he or she got through personal studies. Working collectively would facilitate a mutual exchange of resources and knowledge. Dignity in a mathematics classroom refers to the right of respect each learner has in a mathematics classroom despite levels of giftedness, socio-economic background, gender, race and any other socio-economic and religious disposition.

Ubuntu philosophy addresses people's interconnectedness (Mbigi, 1997) and interdependence (Letseka, 2013). Similarly, Regine (2009) opines that the spirit of *Ubuntu* leads to cooperative and collaborative work environments. The community is encouraged to participate, share and support all team members. Regine (2009) further observes that people work in community groups to be more productive. Perhaps schools in the Eastern Cape need mathematics teachers who apply *Ubuntu*-related values because of the poor socio-economic backgrounds of many learners. Considering the challenges faced in Grade 12 Euclidean geometry, the principles mentioned above can be fruitful since many learners may not understand anything on their own, but they need teamwork to assist their learning. Nussbaum (2003) presents important statements: “Your pain is my pain”, “My wealth is your wealth” and “Your salvation is my salvation” which demonstrate that mutual interdependence is very crucial in this life. In *Ubuntu* terms, teachers should be saying to learners, ‘My understanding of mathematics will be your understanding’ and they should be inculcating in learners the spirit of teamwork more than individual effort.

Many learners generally regard mathematics as a difficult subject. Therefore, it requires teachers who can implement strategies to eliminate barriers in its teaching and learning. Perhaps if learners engage in collaborative learning and make use of communal interdependence through collective work good results would be achieved. Communal interdependence and collective work would allow disadvantaged learners to share their ideas freely with other learners and get easily corrected thus eliminating misconceptions and errors. I also believe that if teachers interact and share ideas, they will be able to quickly formulate effective strategies on how to impart knowledge to learners. Metz and Gaie (2010) assert that, according to *Ubuntu* principles, everyone is a potential member of the ideal family based on friendly and loving relationships; such a philosophy destroys barriers among learners and teachers. In my view, if each learner gets requisite attention in Euclidean geometry, better results can be achieved.

According to Swanson (2007), *Ubuntu*'s role is to focus on human relations taking into consideration moral and spiritual consciousness of what it means to be human and to be in a relationship with another. The author goes on to explain that the strength of the community is a result of community support based on mutualism, empathy, generosity and community commitment. The explanation by Swanson (2007) is crucial considering the difficulties learners encounter in Euclidean geometry because if learners do not share ideas in solving problems, there would be poor performance in the topic. In the classroom, learners who do not share their ideas with others always stick to one method of solving a problem. If they encounter difficulties on the way, they are bound to fail. If these learners are encouraged to embrace the spirit of sharing with other learners, more difficulties may be eliminated thereby enhancing their abilities in solving mathematical problems. Researchers (Sofroniou & Poulos, 2016) assert that collective work such as teamwork or group work enables learners to develop critical thinking, analytical and communication skills, appreciation and respect for other views, techniques and problem-solving methods. In my experience as a mathematics teacher, I have seen many learners who are not gifted excel because of sharing knowledge with other learners in discussion groups. Some exercises in Euclidean geometry can be easily solved by learners in groups. Those learners who are quick to understand how to solve exercises and problems can easily explain to slower classmates. According to some studies (Bernero, 2000; Kocak, Bozan & Isik, 2009), working in groups with members sharing ideas among themselves assists in learning concepts and problem-solving strategies; it improves self-confidence and is instrumental in overcoming learners' fear of committing mistakes. Considering the poor socio-economic backgrounds of the majority of learners in the Eastern Cape, teaching strategies that enable learners to interact and share among themselves are fruitful because fear is dispelled, and the teacher can easily diagnose learners' problems and address them expeditiously. According to my experience and observation in teaching Euclidean geometry, the majority of learners require a lot of support from the teacher and from other learners who internalise procedures and theorems faster. The teacher might have to be patient with learners who take time to understand and has to promote cooperative and collaborative learning among the learners.

Principles of *Ubuntu* are also enshrined in the South African mathematics curriculum. According to DBE (2011, p.4), the role of the mathematics curriculum is:

i. Equipping learners regardless of their socio-economic background, race, gender, physical ability or intellectual ability, with knowledge, skills and values necessary for self-fulfilment and meaningful participation in society as citizens of a free country.
ii. Social transformation wherein ensuring that the educational imbalances of the past are redressed, and the educational opportunities are provided for all sections of the population.
iii. Active and critical learning encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths.
iv. Human rights, inclusivity, environment and social justice as in the Constitution of the Republic of South Africa.
v. Sensitivity to poverty, inequality, race, gender, language, age, disability and other factors.

From the CAPS document, it is clear the Department of Basic Education (DoBE) aspires to ensure that no learner is discriminated against in post-apartheid South Africa. On the ground, the application of such a notion might differ but the education policy is clear. On national television, the nation has watched news reporting about shortages of textbooks in schools especially of mathematics and how DoBE has tried to address the shortage of resources in schools. DoBE spells out inclusivity in mathematics classrooms which encourages teachers to expose all learners to the same conditions despite disabilities, socio-economic backgrounds and many other factors. Inclusivity would ensure that all learners in class receive equal attention even in the teaching and learning of geometry which people consider to be meant for only gifted learners. Perhaps if teachers should be compassionate to learners and could discourage the perpetuation of injustices of the apartheid in the classroom. The South African government's position gives confidence to the disadvantaged learners coming from impoverished communities where competence in mathematics is associated with abnormality. According to Jojo, (2018), when teachers have compassion for the learners, they help to close the gap in learners' pre-requisite background mathematics. In simple terms, compassion helps teachers to assist learners whose mathematics background is poor and help them gradually to learn new concepts through their guidance.

Some researchers (Alexander & Van Wyk, 2014; Dumas, 2003) argue that if teachers promote positive interdependence, learners take a task set before them as a collective responsibility which in turn improves academic achievement. In addition, other researchers (Adams, 2013; Slavin, 1991; Siegel, 2005; Gardner, 1999) argue that all forms of cooperative learning if promoted in classrooms, students' self-esteem and motivation are achieved giving rise to better academic performance. Perhaps if positive interdependence and cooperative learning are promoted in Euclidean geometry, classrooms learners' understanding can be enhanced. Positive interdependence ensures that the teacher and learner realise that they need each other, and progress will not be achieved when they do not work cooperatively and collaboratively. Positive interdependence and cooperative learning are part of *Ubuntu* because they promote collective execution of tasks and success in any given task is attributed to collective hard work. Effective mathematics teaching comprises teachers' efforts in understanding learners' misconceptions and learning difficulties. According to Sibuyi (2012), effective teachers should be able to identify learners' learning difficulties and find ways of eliminating them. Through *Ubuntu,* a teacher approaches each learner with compassion and helps eliminate all learning barriers. An effective teacher would ensure that each learner has resources. He or she also ensures that explanations are effective. In addition, an effective teacher would ensure that classroom interactions are conducive for the sharing of ideas in methods such as group work and classroom discussions and many others.

Another aspect of *Ubuntu* is discipline in schools. Ikoya (2009) argues that there are two major functions of discipline in schools that are:

i. to ensure the safety of students and staff, and
ii. to create a conducive environment for learning.

Safety in schools is important and so is maintenance of discipline in the classroom as disruptive behaviour disturbs effective teaching and learning. Disruptive behaviour in mathematics classrooms usually disadvantages the majority of learners and debilitates the teacher's potential of being an effective facilitator of learning. According to Muzvidziwa and Muzvidziwa (2013), effective school discipline contributes to keeping good order, and also contributes to good academic achievement. A school with discipline and a conducive classroom environment enables conceptual understanding of learners, and also enables teachers to pedagogically prepare themselves to foster fructiferous classroom interactions. There is a multitude of aspects of *Ubuntu* in the lives of learners in any school. However, this study focused on how teamwork, group work and collaboration in a mathematics classroom enhance the teaching of Euclidean geometry.

1.5. Synopsis of the theoretical framework

This study was guided by Ubuntu philosophy, social interdependence theory [SIT] according to Johnson and Johnson (2007) and the Van Hiele theory of geometric thinking as explained by Vojkuvkova (2012).

1.5.1. Brief summary of the social interdependence theory

According to Johnson and Johnson (1989), social interdependence exists when the accomplishment of an individual's goals is affected by the actions of others. The social interdependence theory forms the base to cooperative learning and is also informed by the principles of *Ubuntu* which explain that "a person is a person through other people" (Swanson, 2007; Samkange & Samkange, 1980; Tutu, 1999; Nussbaum, 2003). It means a person is a product of communal and collective endeavours and for someone to reach a certain stage in life, many people have influenced him or her directly or indirectly. No one exists in a vacuum. According to researchers (Chachine, 2008; Adonis, 2008; Letseka, 2012; Mokgoro, 1998), *Ubuntu* and social interdependence are inseparable. According to the above researchers, *Ubuntu* foregrounds the social interdependence of community members involved. In addition, Mbaya (2011) and Msengana (2006) assert that *Ubuntu* involves collectivism, social interpersonal relations and human interconnectedness characterised by the spirit of interdependence and mutual trust. Muzvidziwa and Muzvidziwa (2013) use a Shona idiom "*chara chimwe hachitswanyi inda*" translated to "one finger does not kill lice". This idiom explains how difficult it is to accomplish one's vision and goals without working with others to achieve them. This shows that *Ubuntu* entails the social interdependence of people in a particular community.

The social interdependence theory was considered relevant for the study because it explored how *Ubuntu* philosophy could enhance the teaching of Euclidean geometry in a Grade 12 classroom. The attributes of *Ubuntu* the study focused on were care, compassion, collective work and collaboration. Perhaps collective endeavour for learners in a Euclidean geometry classroom to solving problems given in any task would improve learners' performance. Driven by compassion, the teacher could be able to look for the best resources and prepare adequately to cater for all types of learners. For effective teaching and learning to occur in a geometry classroom, the teacher may encourage gifted learners to assist their classmates who find the topic difficult. Usually, teachers sacrifice their time to conduct extra tuition with learners who are disadvantaged and ensure that those who are gifted are promoted to explain to others. Also, learners may be encouraged to share and allow others to evaluate their work in an open forum.

1.5.2. Brief summary of the Van Hiele's theory

The Van Hiele's theory was also used in the study in conjunction with the SIT. Various researchers (Riizo, 2016; Alex & Mammen, 2016; Khembo, 2011; Abdullah & Zakariah, 2013) have found that the use of the Van Hiele's framework improves conceptual understanding of geometry in learners and incidentally, performance in the topic improves. Van Hiele's theory categorises learners in five hierarchical levels of geometric thinking and phases of teaching geometry. The five levels of geometric presented are:

i. Level 0: Visualisation

At this level, learners use visual perception and non-verbal thinking. Learners are able to recognise geometric figures by shape and compare them with other prototypes. The learners can also categorise them.

ii. Level 1: Analysis (Description)

Learners can analyse and describe properties of geometric shapes and cannot engage in geometric proof.

iii. Level 2: Abstraction (Informal deduction)

Learners can identify and establish relationships between properties and geometric figures and can create meaningful definitions. They are also able to use sketches, grid paper, draw logical maps and diagrams. Also, the learners can make simple arguments to justify their reasoning.

iv. Level 3: Formal deduction

Learners can conduct deductive geometric proofs and are able to distinguish between necessary and sufficient conditions. They are also able to identify which properties are implied by others. Learners understand the role of definitions, theorems, axioms and proofs.

v. Level 4: Rigour

At this level, learners can do all types of proofs. They can also differentiate Euclidean geometry from non-Euclidean geometry. They are also able to describe the effect of adding or removing an axiom on a given geometric system.

The following are five phases of teaching geometry according to the Van Hieles' theory:

i. **Information**: the teacher discusses with learners and he identifies prior knowledge to the topic.

ii. **Guided or directed orientation**: learners are actively involved in solving tasks given by the teacher. Learners are also allowed to explore specific concepts.

iii. **Explication**: learners verbalise what they learnt in their own words. The teacher introduces more concepts and more mathematical terms.

iv. **Free orientation**: learners solve complex exercises and tasks by applying all the relations they learnt in class. The teacher ensures that each learner has a chance to discover their own methods of solution to complete the tasks.

v. **Integration**: learners can summarise all the content learnt in the classroom and make an overview of all concepts.

An effective and compassionate teacher perhaps would need to study the characteristics of his or her learners according to their levels of geometric thought. This might assist in selecting the best didactic methods on how to teach the content effectively. A teacher driven by *Ubuntu* perhaps would also select exercises for learners to do in class according to their levels of geometric thought and creates opportunities to allow learners to move from one level to the next level. Teachers' knowledge of the Van Hiele's theory perhaps would assist them to look for the best resources such as videos and charts for learners in Level 0 to enhance their visual skills and so on. In other words, the zeal to foster conceptual understanding would motivate teachers to look for resources for all learners of different abilities. Perhaps knowledge of the levels of teaching would assist the teacher in how to deliver lessons according to the abilities of learners. The Van Hiele's model assisted in assessing learners' performance during the study.

1.6. Problem statement

Learners' poor performance in geometry had been a cause for concern to teachers and the Department of Basic Education in South Africa. According to Department of Basic Education (2011), Euclidean geometry alone occupied about 30% of the Grade 12 mathematics paper 2 examination whereas analytical geometry occupied about 25% of the same paper. Thus, students' poor performance in this section contributed to their failure of the whole subject. Researchers found that there existed several challenges in the teaching and learning of geometry in South African schools. Perhaps the dismal performance was attributed to various factors such as shortage of resources such as textbooks among others, material resources, mismanagement of time, cultural factors, attitudes, values and beliefs and shortage of competent mathematics teachers. According to Bansilal et al. (2014), some mathematics teachers lacked mathematics content knowledge as well as the skills in classroom practice required to teach mathematics topics including geometry. In addition, Taylor (2011) argues that some incompetent teachers were appointed because of trade unions' influences while others lacked discipline in discharging their duties and were not worried about the future of learners. This might be the reason why learners would display a negative performance towards topics such as Euclidean geometry. Hence this study explored how *Ubuntu* philosophy could enhance meaningful teaching of Euclidean geometry in the Grade 12 classroom.

1.7. Research Questions

The study attempted to answer the following research question:

How can *Ubuntu* values enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry?

The following sub-questions were answered:

i. How can the teachers' PCK be enhanced for effective and meaningful teaching of Grade 12 Euclidean geometry?

ii. What difficulties are experienced by mathematics teachers in the teaching of Grade 12 Euclidean geometry?

iii. What aspects of *Ubuntu* philosophy should teachers apply for the enhancement of the teaching of Euclidean geometry in Grade 12?

iv. To what extent do collective work and collaboration impact the teaching of Grade 12 Euclidean geometry?

v. How can *Ubuntu* philosophy be used to transform the teaching of Grade 12 Euclidean geometry?

1.8. Research aim and objectives

The main aim of the study was:

To explore how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.

The specific objectives of this study were to:

i. Explore how the teachers' PCK can be enhanced for effective and meaningful teaching of Grade 12 Euclidean geometry.

ii. Identify the difficulties experienced by mathematics teachers in the teaching of Grade 12 Euclidean geometry.

iii. Explore which aspects of *Ubuntu* can teachers expose their learners to enhance the teaching and learning of Grade 12 Euclidean geometry.

iv. Investigate the impact of collective work and collaboration on the teaching of Grade 12 Euclidean geometry.

v. Design a model based on *Ubuntu* philosophy that can be used to transform the teaching of Grade 12 Euclidean geometry.

1.9. Significance of the study

According to research (Dhlamini, 2012; Alex & Mammen, 2014; Kotze, 2007; Makhubele, 2014), the performance of learners in geometry has been poor because of various factors ranging from poor quality of teachers, language problems, learners' poor mathematical backgrounds including many others to non-availability of resources. The poor performance of learners in Euclidean geometry motivated this study to be conducted in a bid to explore innovative ways of teaching the topic to enhance learners' conceptual understanding. This study explored how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry. The findings may provide evidence to policy makers and curriculum advisors of the need to incorporate Ubuntu values in the teaching of Euclidean geometry. It was also envisaged that the findings and recommendations may guide mathematics teachers towards choosing instructional strategies that best assist learners to improve their performance in Euclidean geometry. Once the performance in the topic has improved, learners would be motivated to put more effort in mathematics since they would have acquired skills in studying one of the most difficult topics in the subject. Furthermore, the recommendations made in this study would assist in minimising barriers in the teaching and learning of Euclidean geometry and other problematic mathematics topics.

1.10. Synopsis of research methodology

In this section I only present a summary of the methodologies used in this study. A full discussion on the topic is outlined in chapter 4 of this study.

1.10.1. Summary of research paradigm, approach and design

This study was underpinned by the pragmatic paradigm using the mixed methods approach. According to Terrell (2012), pragmatism is a belief that reality is realised through experience and observation. In addition, Creswell (2014) explains that the pragmatic paradigm is concerned with applications and solutions to problems. The paradigm was suitable for the study because it allowed the use of quantitative and qualitative methods which use multiple techniques of data collection. This paradigm allowed the researcher to use a combination of methods of choice without being limited. The purpose of this study was to explore how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.

The mixed methods approach was used in this study. This approach focuses on collecting, analysing and interpreting data using quantitative and qualitative methods in a single study. The study was anchored on the assertion and belief that using mixed methods enhances the validity and reliability of any research study. Creswell (2014) explains that using multiple methods such as mixed methods is appropriate for any research study because one single method cannot give the entire picture of a problem due to the existence of multiple realities. In addition, researchers (Creswell, 2014; Leedy & Ormrod, 2015; Cameron, 2011) argue that the mixed methods approach helps to get comprehensive data than using a single approach. Using mixed methods assists in eliminating the weaknesses of using a single research method. Complementary information is obtained by using both quantitative and qualitative methods. In other words, information that the qualitative method fails to get is obtained by using the quantitative method or vice versa. The mixed methods approach was suitable for this study because I needed to have a deeper understanding of why Euclidean geometry was difficult for both teachers and learners and how the use of *Ubuntu* philosophy would enhance its teaching and learning. Using mixed methods also helps in eliminating or reducing researcher bias. In addition, researchers (Chiphambo, 2017; Riizo, 2016) also used the mixed methods approach in the teaching and learning of geometry.

The research design used in this study was the convergent parallel mixed methods design. It allowed both quantitative and qualitative data to be collected simultaneously during the study. Data were collected from the participants using the preliminary semi-structured interview (refer to Annexure J), final semi-structured interview (Annexture K), classroom observation (refer to Annexure L) and document review (refer to Annexure M), pre-test (refer to Annexure N) and post-test (refer to Annexure O). The pre-test and post-test were written by learners of the purposively sampled participants. Creswell (2014) argues that using both quantitative and qualitative methods simultaneously offsets the weaknesses of using a single data collection method. The other reasons why I used this design were:

i. I wanted to collect the data in one week to avoid prolonging the time of data collection. Prolonged time of data collection would disrupt the normal running of schools.

ii. Collecting both sets of data simultaneously saved time because, according to the Grade 12 national mathematics pacesetter, Euclidean geometry was done in three weeks. I wanted to collect data on one section of the topic, namely, Circle geometry.

iii. I felt that both quantitative and qualitative data were equal sources of information.

iv. I was able to see what was happening at the same time.

According to Creswell (2014), the convergent parallel design allows the researcher to collect both quantitative and qualitative data, analyse separately and compare the results if they confirm or disconfirm each other. In addition, Irwin, Pannbacker and Lass (2014) explain that the convergent parallel design converges both quantitative and qualitative data to provide a comprehensive analysis of the problem.

1.10.2. Summary of population and sample

The population was 20 Grade 12 mathematics teachers from Cacadu Circuit Management Centre in Chris Hani West District, Eastern Cape Province. The reason for selecting a district in the Eastern Cape Province was that DoBE (2016) argues that although there has been some appreciable improvement in mathematics in Eastern Cape, it continued to be one of the poorest performing provinces in South Africa. The participants in this study were two Grade 12 mathematics teachers from two different rural schools who were purposively selected. Data were collected through classroom observation, semi-structured interviews, document review, pre-test and post-test to learners. Merriam (2009) explains that purposive sampling allows the researcher to discover, understand and gain sight. Purposive sampling enables the researcher to select a sample in which relevant information will be obtained. I believed that the sample of two mathematics teachers would give me more information in the study.

Each of the two participants taught two mathematics groups of learners of less than 20 Grade 12 learners each from their school. One of the groups for each participant was a control group and the other was an experimental group.

1.10.3. Summary of data analysis

The qualitative data collected from classroom observations and interviews were categorised and coded for the identification of themes analysed using thematic analysis. Data were transcribed from field notes, video-recorded lessons and audio-recorded interviews. Segments of information were identified to create categories. This involved coding raw data recognising important facts in the data sets. The interpretation of codes developed included comparing theme frequencies and checking theme concurrences and graphically establishing some relationships. The themes created were reviewed guided by research questions to extract more meanings and patterns. All the important facts from the themes were used to produce the final report. Data collected through quantitative means were presented using graphs and tables. The SPSS statistical package was used to analyse the statistical data. The independent samples t-test was used to compare the test scores of learners from the experimental and control group obtained from the pre-test and the post-test for each participant. Thereafter, both quantitative data and qualitative data were integrated.

1.11. Trustworthiness of the study

To ensure trustworthiness, the study was carried out for one week during which I interacted with the participants. In the study, credibility was ensured by physically observing the lessons conducted by the participants. The researcher video-recorded lessons during classroom observation to minimise chances not capturing correctly on what really occurred in the classroom. In addition, field notes were taken during interviews, classroom observation and document review and cross-checking with participants to correct, validate and authenticate the transcribed data. To establish dependability the study, the researcher ensured that the participants did not share information. This was achieved by not disclosing to the participants that they were involved in the same study. The 70km distance between the two schools of the participants was minimise contamination of research instruments.

Confirmability was ensured by safely keeping the data collected which was used for interpretation. This would allow independent critical readers whom I engaged to evaluate the methods used to collect the data. To ensure transferability, the researcher did not make substantive generalisations. The researcher conducted the study when no events, tests and disturbances occurred in the schools to minimise the effects of history. Data was collected inside one week to avoid maturation. The interview questions were formulated in line with the research aims, research objectives and research questions. Experienced researchers and lecturers from universities were requested to critique and evaluate the interview questions and classroom observation schedule. Contribution from curriculum advisors was also sought to evaluate the classroom observation schedule because they used similar instruments to monitor teachers' progress.

The pre-test and post-test were evaluated by experienced lecturers and researchers as well. A pilot study was conducted with the interview questions, pre-test and post-test in a place far away from the intended selected site of the study. Adjustments to these data collection instruments were made following the findings in the pilot study. Also, the data collected in the main study from semi-structured interviews, classroom observation, document review, pre-test and post-test were triangulated to increase the validity of the findings.

1.12. Ethical considerations

Before engaging in the study, the researcher obtained permission from the University of South Africa Research Ethics Committee. Thereafter, written permission was acquired from the Eastern Cape Department of Basic Education (refer to Annexure A and Annexure B) as well as consent from the parents/guardians of the ex-learners who took part in the study. Furthermore, the anonymity of participants was guaranteed by assigning pseudonyms to them. Informed consent forms were developed, and participants signed them before they participated in the study. The teachers and learners participating in the study were assured that they would be allowed to withdraw from the study at any time if they would feel so. Confidentiality was guaranteed to all participants. The researcher clearly explained to the participants the purpose of the research and its benefits of the study and promised to return to them after its completion.

1.13. Definition of key terms

***Ubuntu*:** humanity towards other human beings

Decolonisation: the process of eliminating the effects of colonialism or apartheid especially in the sphere of education

Indigenous knowledge: knowledge specific to a particular group of people and this knowledge articulated and transmitted in the local language

Van Hieles' theory: is a theory by Dina van Hiele-Geldof and Pierre van Hiele which describes how geometry is taught and learnt composed of levels of geometric thinking and phases of teaching

Social interdependence theory: It emphasises that interdependence exists when the accomplishment of an individual's goals is affected by the actions of others.

Positive interdependence: it is a situation when a group of people engages in cooperation in all social endeavours.

Manipulatives: physical objects used by teachers in classrooms to enable learners to learn hands on

1.14. Conclusion

This chapter begins by clearly explaining the focus and purpose of the study. The chapter further describes problems encountered in the teaching of Euclidean geometry, some insights of Ubuntu philosophy, literature related to the study, the theoretical framework, the research methodology and ethical considerations during the study. The next chapter reviews literature on *Ubuntu* in relation to the teaching and learning of Euclidean geometry.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

This chapter presents literature related to the teaching and learning of geometry in secondary schools. It includes literature on *Ubuntu* philosophy in education and pedagogical and socio-economic factors impacting the study of geometry. The chapter further discusses challenges faced in the teaching and learning of geometry as well as how they can be resolved.

2.2. The teaching of geometry in secondary schools in South Africa

According to Chambers (2008), geometry is strongly related to day-to-day human activities. It does not only appear in textbooks, but it is part of the life of all people although most of the time we are not conscious about its application. Therefore, it would be a wise idea for mathematics teachers to use concrete examples in their instruction to make learners understand conceptually. Usiskin (2002) mentions the following as reasons why it is important to teach geometry:

i. It makes mathematics connect with the real world,
ii. It makes it easy for ideas from other areas of mathematics to be pictured,
iii. Spatial awareness of the learners is expanded, extended and enhanced, and
iv. Reasoning skills are developed in learners.

In addition, Suydam (1985, p.41) states that the goals of teaching geometry are to:

i. Develop logical thinking abilities,
ii. Develop spatial intuition about the real world,
iii. Impart the knowledge needed to study more mathematics; and
iv. Teach reading and teaching mathematical arguments.

Perhaps if teachers take into consideration the goals and reasons by Usiskin (2002) and Suydam (1985), their classroom improvement in mathematics performance might be achieved.

The National Council of Teachers of Mathematics [NCTM] (2000, p.41) states that “geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment”. According to research (Muschla & Muschla, 2000; NCTM, 2000) geometry assists learners to be able to develop an understanding of other mathematical concepts and to link with other mathematical disciplines. According to NCTM (2000), learners’ insight into the nature and beauty of mathematics is enhanced because of the ideas such as symmetry and generalisations prevalent in geometry.

According to Mabotja (2017), the South African education curriculum has experienced numerous reforms such as Outcome-Based Education [OBE], Curriculum 2005 and Curriculum Assessment and Policy Statement [CAPS]. In these reforms, Euclidean geometry has been a victim because, in 2008, it was removed from the mainstream mathematics curriculum and was only assessed in an optional paper (Paper 3). Van Putten, Howie and Stols (2010) argue that the exclusion of Euclidean geometry from the mathematics curriculum deprived learners of reasoning skills. In addition, learners who took engineering courses at tertiary level encountered problems because of not having a background in Euclidean geometry (Siyepu & Mtonjeni, 2014). The exclusion of Euclidean geometry in 2008 meant that the teachers had to stop teaching the content which resulted in most of the learners developing disinterest and negative attitudes towards the topic. Mathematics teachers during that period ceased to interact with Euclidean geometry content. The researchers further explain that workshops and seminars in Euclidean geometry subsequently stopped which resulted in new teachers not interacting with the topic only to meet it for the first time when CAPS was introduced.

Bowie (2009) argues that one of the reasons for making Euclidean geometry optional before CAPS was teachers' non-familiarity and negative attitude towards the topic. In addition, Mogari (2004) asserts that learners' poor performance in geometry is a result of abstract and disjointed teaching by some mathematics teachers. This implies that learners are made to believe the topic has no real-life application and they end up resorting to rote memorisation to pass the topic in examinations.

The table below shows the topic overview of Euclidean geometry in South Africa from Grades 10 to 12.

Table 2. 1: Topic overview adapted from CAPS document (DoBE, 2011)

Grade 10	**Grade 11**	**Grade 12**
a) Revise basic results established in lower grade. b) Investigate line segments joining the midpoint of two sides of a triangle. c) Properties of special quadrilaterals	a) Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles. b) Solve circle geometry problems, proving reasons for statements when required. c) Prove riders.	a) Revise earlier work on the necessary and sufficient conditions for polygons to be similar. b) Prove (accepting results established in earlier grades): that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and mid-point Theorem as a special case of this theorem) that equiangular triangles are similar; that triangles with sides in proportion are similar; the Pythagorean Theorem by similar triangles; and riders.

2.3. Suggested strategies from research that enhance geometry teaching

According to Mammali (2015), if learners work in groups cooperatively or collaboratively, their thinking abilities are improved and enhanced. Furthermore, the learners are motivated to work hard to improve their performance in geometry. By working in groups sharing ideas learners achieve more personal, social and psychological skills. Some researchers (Erdem, 1993; Bulut, 2009) have shown that learners taught using cooperative methods achieve better academically than those instructed using traditional strategies. In addition, Guyton (1991) asserts that cooperative group instruction can enhance learners' performance in geometry and consequently learners' self-esteem. Furthermore, Makhubele (2014) suggests the use of the Harkness discussion method whereby 12 learners sit around a table (Tingley, 2002). The learners engage in collaborative discussion sharing ideas in geometry while the teacher facilitates. According to Adams (2013), group processing is promoted, and it allows each group to make reflections on the usefulness of members. The activity of making reflections helps in planning future tasks and assignments. This implies that fostering the spirit of working together assists learners to achieve better results than working individually. This study would assist teachers with *Ubuntu* strategies to adopt for the enhancement of the teaching of Euclidean geometry. *Ubuntu*-geometry combination enables teachers to incorporate *Ubuntu* attributes such as sharing, collaborative and cooperative learning, care, solidarity and among others in geometry instruction. From experience, traditional methods of instruction in which teachers recite theorems have proved to be futile perhaps incorporating *Ubuntu* philosophy might yield better results.

Various researchers (Jones, 2002; Duatepe, 2004; Bankov, 2013) argue that learners understand geometry better if visualisation of geometrical shapes and objects is included in the classroom. In addition, other researchers (Tay, 2003; Guzman, 2008; Mammali, 2015) suggest that if learners touch and see objects, their learning of geometry becomes more imaginative and successful. Visualisation of geometrical objects enhances learners' conceptual and procedural understanding. Visualisation is further enhanced by the use of manipulatives in geometry classrooms especially in primary schools. NCTM (2000) explains that the use of manipulatives should be all-pervasive to all levels and topics in mathematics because conceptual understanding and reasoning are enhanced. Some researchers (Chiphambo, 2017; Bussi & Frank, 2015; Morgan & Sack, 2011; Gürbüz, 2010; Suh & Moyer, 2007) have argued that learners instructed using physical manipulatives understand geometry conceptually better than those who receive instruction without using them. Those researchers further assert that the learners exposed to the use of manipulatives in geometry improve academically. Therefore, this study looked at how strategies linked to *Ubuntu* enhanced the teaching of Euclidean geometry.

2.4. Highlights of some studies conducted in geometry using the Van Hieles's theory

In a survey conducted in the Eastern Cape Province of South Africa by Alex and Mammen (2014) on Grade 10 learners, a sample of 191 Grade 10 learners from 5 different senior secondary schools were selected with the purpose of studying the development of their geometric thinking. The findings of the study show that the participants improved their geometric thinking by moving from Level 2 to Level 3 of the Van Hiele's model. A similar study on grade 10 learners was conducted on the study of congruent triangles by Riizo (2016) in South Africa at Gauteng high schools. The study used mixed methods to collect data using pre-test and post-test matching control group design and classroom observation. Findings showed that the scores of the experimental group where the Van Hiele's model was used improved as compared to the control group which was taught using the traditional methods. The study recommended the Van Hiele's model to be made the instructional model in Gauteng mathematics classrooms. Perhaps using the Van Hiele's model may yield better results in the study of Euclidean geometry in all provinces in South Africa including Eastern Cape which is one of the poor-performing provinces in mathematics.

Researchers on the study of geometry have put Eastern Cape on the spotlight. In a study by Alex and Mammen (2014) a sample of 359 grade 10 learners was selected from a certain district of Eastern Cape, South Africa. 195 learners in the experimental group were instructed geometry using the Van Hiele's framework while 164 learners in the control group received instruction using traditional methods. Pre-test and post-test were administered to both groups. Findings indicate that most of the learners in both experimental and control groups were at Level 0. These learners could not identify simple shapes at the beginning of the study. Only one learner in each group could operate at Level 3 of the Van Hiele's model which indicates that the rest of the learners were not ready to conduct Euclidean geometric proofs. Findings also show that the mean scores of the experimental group were statistically significant than those of the control group. Further findings show that more learners in the experimental group were at Level 2 at the end of the study as compared to the control group. The findings imply that using the Van Hieles' framework in teaching yields positive results as more learners attain higher levels of geometric thinking. Perhaps when teachers incorporate ideas from the Van Hiele's model lessons would be dynamic and would allow learners to attain conceptual understanding. The findings in this study serves as a motivation to study more on alternative strategies to enhance the teaching of geometry especially streamlined to improve geometric thinking.

Khembo (2011) conducted a study on sixth-grade mathematics teachers on their geometric understanding based on the Van Hiele's geometric thinking model. The findings suggested that teachers who participated in the study operated at a lower level of the Van Hiele's model than anticipated. The findings tend to suggest that teachers should be trained to operate at all levels of the Van Hiele's model to assist learners to move to higher levels. If teacher could operate at all levels, it would facilitate them to teach learners to attain geometric thinking abilities.

In Ghana, a study by Armah and Kissi (2019) was conducted on grade 11 mathematics tutors at a college of education to explore the extent to which they facilitated the teaching and learning of geometry at Van Hiele's levels 1, 2, 3 and 4. The Van Hieles' theory was used as a guideline to produce the observation protocol to observe the tutors. The results indicate that the tutors exhibited conceptual understanding in facilitating levels 1 and 2 but fell short of levels 3 and 4. The study further showed that the tutors used rote learning methods which made the lessons less inspiring. The researchers recommended that the Van Hiele's theory should be integrated into teacher education programmes and tutors should design hands-on activities for pre-service teachers.

Chimuka (2017) conducted a quasi-experimental study with grade 11 learners in Nzhelele East Circuit, Vembe District in South Africa. The experimental group comprised of 22 learners and the control group was composed of 25 learners. The experimental group was taught geometry using GeoGebra software whilst the control group was taught using traditional instructional strategies. Using Action, Process, Object and Schema [APOS] theory and Van Hiele's levels of geometric thinking it was found that those learners using GeoGebra performed better in Level 1 and Level 2 (visualisation and analysis) but there were no significant differences in Levels 3, 4 and 5. Also the learners instructed with GeoGebra were found to be more motivated to learn geometry as compared to those learners instructed using traditional methods. This implies that the use of technology should be incorporated to enhance the teaching of geometry and improve the learners' Van Hiele's levels of geometric thinking. In addition, Abdullah and Zakaria (2013) conducted a quasi-experimental the study on 94 students and 2 teachers with the purpose of testing the effectiveness of Van Hiele's phases of learning geometry using the Geometer's sketchpad (GSP). The study took 6 weeks on the experimental group and the control group. Before the study both groups were given Van Hiele's Geometry Test [VHGT] to ascertain their initial levels of geometric thinking. At the end of the study another VHGT test was administered to the learners. Results found using t-test showed that the experimental group achieved better than the control group demonstration the importance of Van Hiele's theory in the classroom. This study would assist teachers on selecting activities that promote geometrical thinking and reasoning and how to improvise resources such as technology and visual aids to enhance conceptual understanding of geometry learners.

A study by Solaiman, Magno and Aman (2012) was conducted in Lanao del Sur, Philipines on 409 third-year high school students in public schools with the purpose of the study was to evaluate the students' Van Hiele's levels of geometric thought and conceptual understanding. Data were collected using mixed methods. Results indicate that 312 students belonged to Level 0 (pre-cognition), 93 belonged to Level 1 (visualisation), 4 belonged to Level 2 (analysis). None of the students belonged to higher levels such as formal deduction and rigour. Recommendations made were:

i. In-service training should be conducted to capacitate teachers and enhance their teaching in geometry,
ii. Qualified teachers should be employed to replace the teachers who did not fully specialize in teaching mathematics; and
iii. Visual aids should be used in the classroom to enhance the geometric thinking of most of the learners. The target should be that most of the learners should attain Level 1 (visualisation) and assist these learners to progress to level 2.

Various studies indicate that both mathematics teachers and learners operate in lower levels of geometric thinking. In addition, the studies indicate challenges faced by both in-service and pre-service mathematics teachers in using the Van Hieles' model in classroom instruction of geometry. Alex and Mammen (2014) suggest that workshops should be organised to capacitate teachers in the use of Van Hiele's theory in the classroom. Those workshops should also cover aspects such as assessment strategies in geometry and preparation of appropriate instructional material. This study interrogated the use of the combination of the use *Ubuntu* and the Van Hieles' theory by mathematics to enhance the teaching of geometry.

2.5. Mathematical knowledge for teaching geometry

Euclidean geometry is one of the topics in the South African Grades 10 to 12 curricula of which the bulk of the content is done in Grade 11. According to Lenhart (2010), not much research has been done in the mathematical knowledge for teaching [MKT] and pedagogical content knowledge [PCK] of teaching geometry, but can be adapted from studies of other topics. Some researchers (Rowan & Ball, 2005; Hill, Schilling & Ball,

2004) have argued that MKT plays a pivotal role in academic achievements in mathematics in schools. According to Ball, Thames and Phelps (2008), all teachers should possess MKT to be able to teach mathematics effectively. This implies that deliberate efforts should be made by mathematics teachers to acquire this knowledge. Ball et al. (2008) further explain that MKT comprises two domains, namely, PCK and content knowledge [CK]. Ahtee and Johnson (2006) assert that lack of CK can lead to teaching difficulties. According to Özden (2008), teachers who possess degree qualifications in mathematics are better prepared to teach mathematics content without difficulties. Perhaps mathematics teachers should have strong Euclidean geometry courses in their preparation to enhance their CK in the topic. On the contrary, Talbert-Johnson (2006) argues that possession of mathematical qualifications is not the only standard for CK but if teachers can attend more INSET programmes their CK is enhanced.

According to Shushua, Kulm and Zhonghe (2004), MKT goes beyond knowledge of mathematics content. It is knowledge that a teacher uses in the classroom which surpasses the knowledge of formal mathematics. Other researchers (Adler & Davis, 2006; Hill et al., 2008) assert that MKT is simply knowledge which a teacher requires to teach in the classroom. According to Ball, Hill and Bass (2005), CK comprises common content knowledge [CCK], specialised content knowledge [SCK] and horizon content knowledge [HCK]. CCK is mathematical knowledge used in other professions apart from mathematics teaching. SCK is mathematical knowledge specific to teachers only and not found in other professions. SCK is the knowledge that a teacher requires to teach mathematics effectively. This implies that mathematics teachers must develop SCK in Euclidean geometry in their training and INSET programmes to enable them to become effective teachers. According to Hill et al. (2004), SCK enables mathematics teachers to explain the content effectively, do presentations, conduct assessments on learners' understanding and select adequate content and curriculum materials. In addition, Ball et al. (2008) assert that a special type called horizon content knowledge exists which is awareness of the relationship of mathematical topics in the mathematics curriculum. It shows teachers' skills to make flow charts and concept maps to link concepts and content across the mathematics curriculum. It implies that an effective teacher should know how Euclidean geometry is taught from lower grades until Grade 12. The teachers are required to know what content is handled per grade and how it should be handled as per CAPS guidelines, teachers' guides and examination guidelines. Figure 2.1 summarises the concept of MKT as conceptualised by Ball et al. (2008).

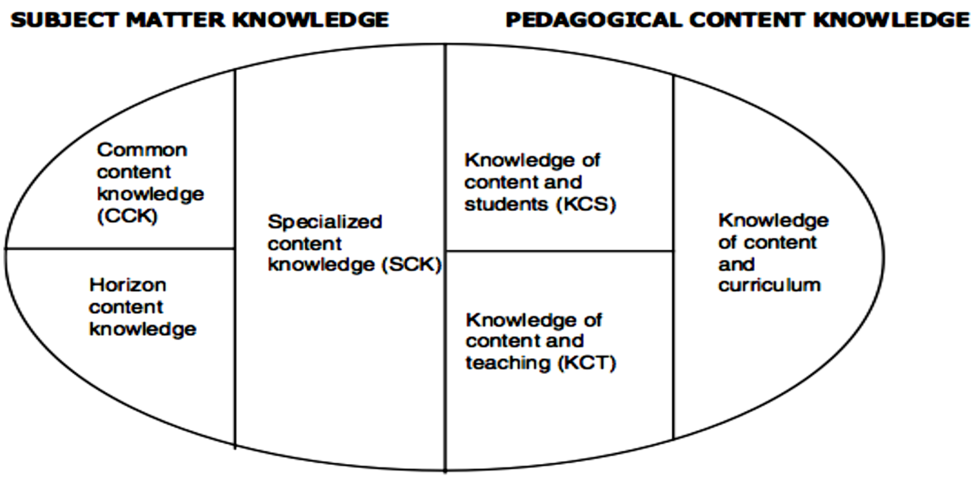


Figure 2. 1: Domains of Mathematical Knowledge for Teaching (Ball *et al.*, 2008:403).

According to Carpenter, Fennema, Peterson and Carey (1988) PCK comprises aspects such as procedural and conceptual knowledge of a topic, knowledge of misconceptions by learners, techniques of conducting assessment of learners and knowledge of instructional strategies, among other things. Shulman (1986:9) refers to

PCK as “the ways of representing and formulating the subject making it comprehensible to others”. This also implies that teachers should possess techniques of simplifying complex aspects of content for all learners to grasp it. Ball et al. (2008) assert that PCK comprises knowledge of content and students [KCS], knowledge of content and teaching [KCT] and knowledge of curriculum and content [KCC]. According to researchers (De Jong, 2010; Schneider & Plasman, 2011) PCK develops with more classroom practice and experience.

Manizude (2006) developed a model of PCK for geometry and measurement comprising:

i. Knowledge of specific difficulties and misconceptions,
ii. Knowledge of useful representations of the content,
iii. Knowledge of developmental levels of the learners,
iv. Knowledge of connections; and
v. Knowledge of understanding appropriateness of learners’ proofs, justifications and mathematical discourse.

This implies that Euclidean geometry teachers who have learners at heart should strive to detect learners’ learning difficulties and misconceptions. This study would assist in encouraging teachers to find strategies to detect problems, errors and misconceptions from learners with the support of diagnostic reports and other curriculum documents.

Loughran, Mulhall and Berry (2004) suggest that for effective teaching, the use of concrete and real-life examples, analogies and demonstrations should be encouraged among teachers. According to Williams, Eames, Hume and Lockley (2012), it is a long process for teachers to acquire PCK. Effective teachers are not born with PCK and they need to learn from others to acquire skills and new knowledge needed to become professional teachers who are experts in the field. This implies that mathematics teachers should interact through INSET programmes such as workshops and seminars to share experiences on how to present Euclidean geometry content. This study focused on *Ubuntu* strategies that promote teacher-teacher interactions which can assist in improving teachers' pedagogical skills in geometry and how to assess learners using Van Hieles' theory of geometric thinking.

2.6. Summary of some problems faced in geometry classrooms

There are various problems faced in the teaching and learning of geometry. These include problems faced by both teachers and learners.

2.6.1. Problems faced by teachers in the teaching of geometry

According to Vidermanova and Vallo (2015), in a study on Slovak mathematics teachers, it was found that some teachers are convinced that there are no real-life everyday problems in mathematics textbooks which poses a challenge to assist in conceptual understanding among learners. In addition, Ngirishi (2015) suggests that using real-life objects and examples is effective for learners' understanding. Perhaps teachers should be innovative to come up with real-life examples in geometry to foster conceptual understanding. The chief cause of poor results in schools is teachers' weak content knowledge and pedagogical skills (Van der Berg, 2015), and they are demotivated to teach geometry because they have a poor foundation in the topic (Adolphus, 2011). In addition, Jones (2000, p.110) laments that "teachers are expected to teach geometry when they are likely to have done little geometry themselves since they were in secondary school…" The researcher further argues that novice teachers have limited PCK for geometry and therefore they should be made to attend professional development programmes. This is an indication that some of the mathematics teachers lack preparedness to impart content in geometry to learners. As an attempt to deal with the poor foundation of the teachers, Ali, Bhagawati and Sammah (2014) argue that teachers should be trained through seminars and workshops on how to teach geometry effectively.

Mammali (2015) argues that the lack of teachers' confidence contributes to learners' thinking that geometry is difficult. Hence, the learners start developing a negative attitude towards geometry. According to Jones (2000), mathematics teachers should be well conversant with the content they have to impart. They need to equip themselves with profound knowledge of geometry content before they deliver it to learners. Dlamini (2012) conducted a study in KwaZulu-Natal on Grade 12 teachers with ten or more years of classroom experience. The study was based on Bloom's taxonomy and Van Hiele's theory of understanding geometry. A sample of 10 teachers was selected. Results indicate that only one teacher achieved level 4 of understanding, 3 of them demonstrated some misconceptions while the other 6 demonstrated poor understanding of the tasks on Euclidean geometry given. The study further shows that many of the teachers did not possess subject matter knowledge [SMK]. Dlamini (2012) further recommends personal professional development by teachers through academic studies and interaction and networking of teachers to share experiences in Euclidean geometry. Sharing is an attribute of *Ubuntu* which can improve teachers' SMK pedagogical skills in geometry. Another recommendation made is that the Department of Basic Education should find specialists to develop material to train pre-service and in-service teachers in Euclidean geometry. In another study conducted by Kotze (2007), findings show that teachers over-estimated their mathematics capabilities. The study confirmed that SMK is a good predictor of student achievement. Teachers emphasised mastery of basic calculations more than the application of procedures. Teachers could not identify geometric shapes in alternative positions. This was due to teachers' exposure to rote learning. A synthesis from two-dimension to three-dimension was needed.

Learners usually develop negative attitudes towards mathematics because there is a lack of connection with reality (Tella, 2007) and these negative attitudes may be the source of misconceptions and errors (Makhubele, 2014). According to Nkwe (1995), teachers' negative attitudes are transferred to learners. Howson (2000) argues that a lot of mathematics teachers have a negative attitude towards Euclidean geometry. This implies that this negative attitude is easily transferred to the learners.

According to Luneta (2013), mathematics teachers are the main cause and source of learners' misconceptions and errors because of the following reasons:

i. Teachers use teacher-centred instructional strategies during lessons which impedes learners' understanding,
ii. Teachers lack knowledge of subject content. They fail to explain confidently to learners because they do not have content knowledge. In addition, they lack skills to explain explicitly to the learners which evokes learners' dissatisfaction and lack of learners' confidence in their teachers; and
iii. Some of the teachers use the wrong techniques in asking questions in the classroom. They ask ambiguous, unclear and difficult questions which make learners commit a lot of errors.

A study by Van Putten et al. (2010) found that students expressed their confusion and frustration in mathematics because teachers appeared not to have dominion of content knowledge and they also showed negative attitudes. Jones (2000) argues that for effective teaching of geometry, the mathematics teacher should master the content very well. In a study conducted by Van der Sandt and Niewoudt (2003), a lot of Grade 7 mathematics teachers lacked content knowledge in geometry.

A two-year study by Van der Sandt (2007) was conducted in South Africa on Grade 7 geometry and was underpinned by Van Hieles' theory. A total of 224 pre-service mathematics teachers, 18 experienced mathematics teachers, and 127 learners were selected for the study. It was found that both experienced and pre-service teachers failed to reach the anticipated level of geometric thinking. All the teachers failed to acquire level 3 and level 4 of the Van Hieles' model demonstrating the ineffectiveness of pre-service training in geometry. In this study, it was argued that if a mathematics teacher lacks higher levels of geometric thought it would be a source and cause of poor achievement in geometry by learners. The study by Sandt (2007) recommended the creation of long-term teacher education programmes that allow teachers to acquire strong mathematical content knowledge. Jones (2000) argues that novice mathematics teachers lack subject matter knowledge and PCK to enable them to teach geometry. The researcher suggests that universities should put in place strong teacher training courses to empower mathematics teachers in teaching geometry at all levels. In addition, in a study by Schoenfield (1988), it was found that teachers tend to teach the way they were taught. They commence learning how to teach from their experiences as learners in the teacher training programmes. When they start to conduct lessons as teachers, they would have been shaped by their experiences as students in their teacher training programmes. According to Schoenfield (1988), teacher training programmes play a vital role in shaping and preparing teachers for future classroom practice. This implies that the exercise of training teachers should be a very serious endeavour because weak training would mean poor classroom performance by the teachers because of poor teacher preparation background.

According to researchers (Mogari, Kriek, Stols & Ogbonnaya, 2009; Mogari, 2004) a well-designed teacher training development programme is required in South Africa to develop competent and effective mathematics teachers. This implies that the quality of teacher qualification depends upon the quality of the teacher training programme. Mogari (2014) argues that recruitment into teacher training programmes should be strict to ensure that the pre-service teachers being recruited possess high-level capability, aptitude and commitment to assist learners.

In another study by Luneta (2014), a group of 128 first-year students at a university level in a foundation phase teacher training programme was selected. The findings show that many of the university students involved in the study were operating at level 1 of Van Hieles' levels of geometric thinking. The teacher trainees lacked basic knowledge of geometry. This implies that the study of geometry begins to suffer from early primary school level because of the incompetence of teachers recruited. The findings of the study imply that the teacher training programmes from the foundation level should be rigorous to produce competent teachers who can operate on all levels of the Van Hieles' model including the acquisition of requisite pedagogy to teach geometry.

2.6.2. Problems faced by learners in the learning of geometry

According to diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020; DoBE, 2021) learners have displayed challenges in answering examination questions in Euclidean geometry in Grade 12. In summary, the challenges faced include:

i. Work with angles between parallel lines,
ii. Correct naming of angles and use of different forms of naming angles,
iii. Basic geometry from previous grades,
iv. Use of theorems and their converses,
v. Working with similarity and proportionality, and
vi. Work with cyclic quadrilaterals and many others.

According to King (2002), it is common practice that primary school teachers spend little time teaching geometry. Hence learners have challenges in conceptually understanding geometry from early primary school. They in turn find difficulties in coping with the topic in higher grades because of poor primary school background. Perhaps if INSET programmes and workshops are conducted rigorously for all primary school teachers, learners' performance and attitude towards geometry would improve. De Villiers (2010) argues that at the primary school level, formal geometry curriculum is not strong. This statement implies that the curriculum does not foster the acquisition of critical geometric thinking at foundation education levels. Hence the learners would find it difficult to develop critical thinking and reasoning to enable them to cope with the demands of high school geometry. De Villiers (2010) further argues that teachers and authors of textbooks appear not to have clear knowledge of Van Hiele's theory and its importance. In this study, strategies to capacitate teachers with information on how to use Van Hieles' theory would be provided so that geometry teaching would be enhanced.

A study was conducted by Feza and Webb (2005) in the Eastern Cape, South Africa with 30 Grade 7 learners from poor socio-economic backgrounds where the majority of learners are Xhosa speaking. The study was based on Van Hieles' levels of geometric thinking. It was found that the learners obtained very low levels. The major cause found for this finding was that learners lacked second-language proficiency. The learners must learn the language of instruction first as a second language then use that language to learn mathematics. Setati (2009) argues that learners cannot handle mathematics content in English as a second language. According to Mammali (2015), the use of English in classrooms populated by black learners is a challenge. The researcher raises the following points:

i. The learners have problems communicating in English. These problems are compounded when the learners are compelled to learn the mathematical language as well; and
ii. The use of English as a second language is one of the reasons why mathematics is poorly performed in black schools.

This implies that mathematics teachers should be able to operate in both the vernacular language of the learners and the language of instruction. This allows the teachers to do code-switching when learners fail to understand. Code-switching might enhance learners' conceptual understanding.

According to researchers (Mammen & Alex, 2016; Clements & Battista, 1992; Siyepu, 2005), high school learners in South Africa are not ready for formal geometric proofs because of weak primary school background in geometry. Learners find it difficult understanding plane geometry language, identifying shapes and classifying them together with the description of properties of shapes. In addition, researchers (Guzman, 2008; Mammali, 2015) argue that some teachers use teacher-centred instructional strategies in classrooms by making learners memorise axioms and theorems. For example, most learners know that the sum of angles of a triangle is equal to 180^0. They might not know where that comes from if they have never done the proof practically. This leads to rote learning in classrooms. The use of visual aids perhaps might enhance learners' conceptual understanding. In the cited example, teachers may draw a straight line and cut all the three vertices of the triangle. They then place the vertices on the line drawn. They will discover that the three angles on the said vertices fit on the straight line which they know form an angle of 180º.

A quantitative descriptive survey was conducted by Sunzuma, Masocha and Zezekwa (2013) in Bindura, Zimbabwe on Ordinary level students (equivalent to Grade 10 and 11 in South Africa). A stratified sampling technique was done on three high schools and a questionnaire of 15 closed questions was administered to the students. The findings indicated that most of the learners did not like solving problems in geometry. The results showed that most of the students showed appreciation of the use of geometry real-life situations and its importance in their future careers. However, their negative attitude towards the topic did not change. Wu (2016) argues that another cause of negative attitude in learners emanates from the discord between the content taught in middle school and high school geometry. High school geometry does not have a close relationship with the rest of the school curriculum from foundation to middle school. This discord might be one of the causes why learners develop negatives attitudes towards the topic because it appears difficult to them.

In another dimension, researchers (Stuart, 2000; Reynolds & Walberg, 1992) argue that negative peer pressure is also a cause for poor performance in geometry. The researchers indicate that some learners are mocked and discouraged by other learners for concentrating on geometry. This distracts their efforts in focusing on their studies and end up failing to understand the topic. Negative peer attitudes impact negatively on the performance of learners because learners become demotivated and lose focus on putting effort into geometry.

Mammali (2015) argues that although the South African education system advocates for learner-centred approaches, the mathematics curriculum is examination-driven. The researcher further explains that, in practice, mathematics teachers are compelled to teach learners to pass examinations at the expense of conceptual understanding. Hence teachers end up teaching only the material which they deem relevant for learners to pass examinations. From experience, teaching for conceptual understanding might be time-consuming. Therefore, teachers may not be willing to apply it because when learners fail the teachers are made to account by the Department of Basic Education. Pressure to teach to finish the syllabus and drill learners to pass examinations would be an option for most teachers to avoid accounting for poor results. This might be a challenge for Euclidean geometry which has a lot of concepts, theorems and axioms in which learners are required to understand them to solve exercises. According to Mammali (2015), there is a shortage of teaching and learning material in schools.

According to Mello and Martins (2015), some of the reasons for poor performance in geometry are:

i. Generally, geometry is conceptually difficult for some learners.
ii. Geometry is not an interactive subject such as science and liberal arts that motivate learners to be active in class. A lot of memorisation of rules and theorems is involved. Learners if they are passive just become demotivated to learn the subject.
iii. Learners might have past negative experiences of geometry learning from previous grades. Such experiences ignite negative attitudes in some of the learners.

iv. There might exist poor relations between geometry teachers and learners which distract learners' focus on learning. According to general life experiences, a learner who has personality clashes with a teacher inherently dislikes everything the teacher has to offer. The learner's concentration is affected, and this has a negative effect on the learning of geometry.

The reasons outlined above call for the conscious attention of the teachers. This implies that teachers should always reflect on their interactions with learners by being exemplary in behaviour and exercise equity and justice in geometry classrooms. Perhaps all teachers of geometry should take responsibility for effectively teaching the topic, from elementary grades to higher grades. They should be innovative to make geometry interactive and interesting for all learners including the academically disadvantaged. The literature reviewed looked at problems faced by both teachers and learners in geometry in some parts of South Africa and other countries. This study explored the challenges faced in the teaching and learning of Euclidean geometry specifically in Chris Hani West District in the Eastern Cape Province, South Africa.

2.7. Manifestation and role of *Ubuntu* in the society

According to Msengana (2006), *Ubuntu* is a collective personality and collective morality hence the motto "I am because we are". The author continues to underscore that people need to eliminate the slave mentality and start to think of themselves as royalty. Moral values and norms are regulated by the community at large and any deviant behaviour is disciplined by the community. *Ubuntu* comprises values that are associated with the welfare and well-being of the society such as reconciliation, compassion, consensus, agreement, human dignity, forgiveness and healing among others (Tutu, 1995; Mokgoro, 1999). Researchers (Mbigi, 1995; Skelton, 2002;

Swanson, 2007) mention that after the advent of democracy in South Africa in 1994 leaders felt that *Ubuntu* should be extended to nation-building, transformation and reconstruction.

Other researchers (Tutu, 2008; Mbigi, 1997; Jolley, 2011; Van Nierkerk, 2013; Samkange & Samkange, 1980; Swanson, 2007) have tried to explain their views on the essence of *Ubuntu* as an Afrocentric philosophy which describes how African communities live. According to researchers (Van Nierkerk, 2013; Oppenheim, 2012; Tutu, 2008), "*Ubuntu*" is a Nguni term which emanates from the phrase "*umuntu ngumuntu ngabantu"* and in *SeSotho "motho ke motho ka batho"* both translated as "a person is a person through other people". Oppenheim (2012) and Broodryk (2006) explain that *Ubuntu* exists in many languages and cultures of sub-Saharan Africa, but the sole aim is to guide humanity. *Ubuntu* philosophy focuses more on human relations and communal interdependence (Swanson, 2007; Msengana, 2006; Muzvidziwa & Muzvidziwa, 2012; Chitumba, 2013; Letseka, 2013; Mnyaka & Motlhabi, 2005). Odari (2020) explains that *Ubuntu* enables each person to understand the pains and problems of other people and put effort to restore happiness in them. In addition, some researchers (Fox, 2010; Nussbaum, 2003; Ndondo & Mhlanga, 2014; Mbigi, 1995) explain that *Ubuntu* comprises respect for others, kindness, compassion, peace, reciprocity, sacrifice, value of dignity, love and that development of the human being should take precedence to political and economic factors. Muzvidziwa and Muzvidziwa (2012) express that *Ubuntu* philosophy, in general, is centralised on the advancement of the community ahead of the individual, discourages individualism and promotes brotherhood and sisterhood among the people. According to Chukwuere (2020), *Ubuntu* promotes a caring spirit for one another regardless of age, gender, race, status, background and among others. This view is an antidote to discriminatory practices which occurred during the colonial and apartheid era. In addition, Mnyaka and Motlhabi (2005) argue that people should not conceal their responsibility in a group effort, but everyone is required to participate as an individual in the spirit of *Ubuntu*. Also, according to Monaheng (2017), *Ubuntu* is manifested in the active participation of individuals in a community for the well-being and benefit of that community.

Ubuntu in modern society is an amalgam of different cultures ranging from African traditional cultures, Christianity, Islamic culture, Western culture and many others. According to Chitumba (2013), *Ubuntu* is not static but dynamic and is susceptible to mutations taking various meanings depending on the social context of any community and it has faced some threats throughout history. Msengana (2006) and Swanson (2007) argue that industrialisation, globalisation and urbanisation have threatened *Ubuntu*, as people mixed with others from different cultures. In addition, Msengana (2006) explains that fathers, the breadwinners, were separated from their families leading to disintegration of homes and this industrialisation brought self-sufficiency. Some community members began to be individualistic and egoistic because of industrialisation. According to Ndondo and Mhlanga (2014), societies that emphasise individualism in their structures normally would produce members who are egoistic and have no feelings to help fellow members of the community. In my experience as a parent and a teacher I have noted that values, morals and attitudes are learnt from home as a child grows. If a child is brought up in a violent home or brought up in a family which is stingy, the same characteristics will tend to surface in the character of the child in the future. Most of the characteristics of *Ubuntu* are acquired as someone interacts with family members around and the community at large. Research (Jolley,

2010) shows that industrialisation and urbanisation deplete *Ubuntu*, disregard human desires and trample on the sensitivities of human beings.

Ubuntu manifests itself in education systems, management systems and many other facets of life where human beings interact with one another. According to Chilisa (2012), *Ubuntu* entails dependence and interdependence of the school and the community. The school and the community should not exist as separate entities. In addition, Sibanda (2019) suggests that schools should be focal points and beacons of community building, community development and collective learning. This implies that there should be cordial relations between the school and the community for development to be realised. The demise of the community development emanates from the divorce and discord between the school and the community. The relationship created between the school and the community enables parents to become responsible for the education of their children and ensures that parents also monitor their children's progress in academic issues. Sibanda (2019) further suggests that the community should use schools as resource centres to champion community development in addition to academic and scholastic issues. Furthermore, Woolfolk (2010) argues that through interaction, more knowledgeable members of the community influence the learning of others through these cooperative dialogues and cultural interactions. In the spirit of *Ubuntu*, it would be prudent for every teacher to introspect, evaluate his or her capabilities and make a judgment about his or her preparedness to teach Euclidean geometry. *Ubuntu* enables the mathematics teacher to want to give his best to the community. *Ubuntu* is required for fostering unity among teachers and villages so that they collectively work together for the success of learners and the development of the community. In this study, *Ubuntu* is interrogated within the classroom parameters and how it can be used to enhance the teaching of geometry.

2.8. *Ubuntu* philosophy in the South African education system

According to researchers (Lebeloane, 2017; Mignolo, 2007; Odora-Hoppers & Richards, 2011; Smith, 1999; Chilisa, 2012; Le Grange, 2015; Msila, 2017; Ngũgĩ wa Thiong'o, 2016), the education systems of African nations such as South Africa should be decolonised for progress to be realised. Those researchers further argue that if the curricula are crafted around an Afrocentric philosophy such as *Ubuntu,* more meaningful development in the education system can occur. In addition, Maris (2020) argues that *Ubuntu* plays the pivotal role of authenticating the African identity that was suppressed and lost during years of apartheid and colonialism. Ubuntu facilitates the transformation of learners' minds through discovery of self-image and identity. The goal for education is "to free the minds of the oppressed in order to destroy social classes and create one human consciousness within the society" (Mahaye, 2018, p.17). According to Chilisa (2012) and Fanon (2008), one of the goals of decolonising the curriculum would be to emancipate people from the inferiority complex brought by the colonial education system which only prepared Africans to be subservient to Europeans. *Ubuntu* instills positive self-image and self-esteem among African learners who were once marginalised during the oppressive the colonial system. In summary, the application of *Ubuntu* values assist in motivating learners from disadvantaged African communities to realise their potential to succeed in life and face the challenges in education and the future.

According to Odora-Hoppers & Richards (2011), during colonisation the settlers captured physical assets and after that colonised the African minds through education, economics, law and science and demonised African indigenous knowledge systems. Smith (1999) argues that processes such as deconstruction and reconstruction whereby inaccurate distorted information is replaced with accurate information, selfdetermination and social justice, use of indigenous languages, internalisation of indigenous experiences and critique should be instituted to decolonise the curriculum.

Apartheid adversely affected the social fabric in South Africa. According to Jolley (2011), *Ubuntu* was eroded during the apartheid era when black people were segregated against, marginalised, ill-treated and dehumanised. As a result of adopting apartheid, sanctions were imposed on South Africa by the international community for violating human rights. Tutu (2008) asserts that dehumanisation of other people is tantamount to dehumanising oneself. The author further vividly describes dehumanisation this way: “when I dehumanise you, I inexorably dehumanise myself”. Researchers (Tutu, 2008; Broodryk, 2006) express that *Ubuntu* is a panacea for unity, reconciliation, love and empathy to others. In the South African context, *Ubuntu* enables people to bury memories of atrocities of apartheid and build a new strong and united nation. It is in this context that mathematics is one of the curricula whose background is founded on those atrocities.

Rousseau (1996) argues that there is need to educate for citizenship from a tender age especially in emerging democracies. According to Ndofirepi and Mathebula (2011), in South Africa, the State is a political parent for children between 7 and 18 years entrusted with the responsibility of fostering good citizenship which former President Mbeki referred to as “new patriotism”. Children are taught to be loyal to values enshrined in the Constitution of the Republic of South Africa. Many constitutions from various countries contain in them values such as those stated in the Constitution of South Africa.

According to researchers (Msengana, 2006; Mbigi, 1997; Mpofu, 2002; Samkange & Samkange, 1980), education in the African traditional system is a communal endeavour. Mbigi (1997) argues that the process of learning is a shared responsibility of teachers, learners, mentors, society and the family unit. Msengana (2006) explains that in the traditional African system, the education of boys and girls was facilitated through initiation schools where principles of *Ubuntu* such as hospitality, spirituality and collective life were taught. In addition, Mpofu (2002) mentions that the education of children in the olden days took place in every sphere of social life ranging from play to storytelling by grandparents and many others. According to Mbigi and Maree (1995), games played a pivotal role in the education of the child whereby many skills were acquired. The researchers explain that modern-day learners lack a lot of skills because their backgrounds are devoid of play. Play helps children in identity formation. Perhaps the education authorities should conduct community campaigns to encourage parents to go back to old ways of story-telling and encourage play at home.

Some researchers (Makgoba, 1996; Seepe, 2004; Higgs & Van Wyk, 2007) advocate for the establishment of an education system based on African values and philosophy. In such a system, indigenous knowledge systems are promoted. Frere (1970) argues that local Brazilian villagers emancipated themselves from social, economic and political oppression and subjugation using indigenous knowledge. Real education should lead to the elimination of psychological bondage. According to Msila (2009), indigenous knowledge systems promote equality and inclusivity but most profoundly they enhance understanding and participation. In addition, Letseka (2011) explains that *Ubuntu* supports the use of indigenous knowledge which is an aspect important for promoting inclusivity, equality and social justice. Furthermore, Letseka (2011) asserts that an education system based on *Ubuntu* promotes imagination, deliberation, responsibility, humanity and cooperation among learners together with their teachers. All these are attributes that are required for learners to understand geometric concepts.

In 2018, the deviant behaviour by learners in South African schools in which some teachers and were stabbed to death was a sign of lack of the spirit of *Ubuntu*. Researchers (Letseka, 2011; Msila, 2008) emphasise that *Ubuntu* manifests itself in schools through disciplined learners, teachers, principals and other stakeholders to achieve efficacious results. Chukwuere (2020) laments that the rampant absenteeism by teachers in schools is a sign of indiscipline that has eroded the gains of freedom. This abseentism requires the implementation of *Ubuntu* to eliminate it. Jansen (2018) argues that the presence of teachers in schools renders emotional, intellectual, and mental support to learners that enhances the teaching and learning process. According to Msila (2008), collective efforts should be exerted by all stakeholders such as parents, civic organisations, school management team [SMT] and teachers to inculcate a sense of responsibility and discipline in the learners. The researcher further suggests that discipline should be fostered among learners and all malpractices should be eradicated in all South African schools.

Chitumba (2013) explains that *Ubuntu* is a moral theory that helps inculcate correct values and norms in students and graduates. However, some researchers (Nzimande, 1988; Khoza, 2006; Sono, 1994) argue that *Ubuntu* should not be applied in schools uncritically. The argument is its enforcement with its traditional value system leads to the discrimination of people according to age, gender and class. Sono (1994) asserts that enforcement of *Ubuntu* uncritically gives rise to the uplifting of societal well-being at the expense of individual well-being. According to Khoza (2006), the uncritical enforcement of *Ubuntu* causes forced conformity whereby people who renege are marginalised and disempowered. However, Louw (2005) argues that the laws and policies that have been enacted have abolished the use of forced conformity such as corporal punishment to protect the rights of learners in schools. These policies have recognised the input of learners in the school system thereby enhancing the societal well-being of learners. The Department of Basic Education [DoBE] (2010) states that the South African Bill of Rights provides for the protection of human rights for teachers and learners so that they operate in a safe environment devoid of violence, fear and discrimination. The document stipulates that the responsibility of the school entails:

i. Fostering that rights and responsibilities are inseparable,
ii. Human rights should be upheld,
iii. Administration of punishment should be educative and not punitive,
iv. Inclusivity is emphasised, and
v. Interaction, hard work and participation.

According to DoBE (2010), schools should incorporate collaborative construction of knowledge embracing indigenous knowledge systems and methodology which allow participation and cooperation. The interdependence of all stakeholders in the school system is also emphasised.

However, Lebeloane (2017) laments that little has been done to decolonise the South African curricula in public schools as they still perpetuate to prepare African learners to lead Western lives. This might be true for Geometry since many textbooks just present it as a series of axioms and theorems without any reference to the African structures in the learners' far and immediate environment. The researcher further argues that although the Constitution of South Africa, various curricula such as the National Curriculum Statement [NCS], Revised NCS and CAPS, South African Schools Act [SASA] advocate for equity, inequality is still prevalent in the education system. An example cited by Lebeloane (2017) and Chilisa (2012) is the sacralisation of English Language and Afrikaans in schools populated by black children who speak local languages such as isiZulu, XiTsonga and many others. According to Hammond (1985), sacralisation refers to the investment of sacredness, superiority and transcendence on an object. During apartheid, those who spoke Afrikaans and English were highly revered and given preference over those conversant only in indigenous African languages. With the advent of democracy, the Constitution of the Republic of South Africa placed all languages in the country at the same level. According to Lebeloane (2017) and Chilisa (2012) if local languages are also used with equal status to English and Afrikaans, confidence among learners is boosted and psychological pathology of inferiority complex is eradicated. According to Mudaly (2018) the mathematics curriculum can be decolonised through use of language whereby teachers strategise to make use of code-switching in mathematics classrooms to enhance conceptual and procedural understanding among learners. This implies that even teachers could use code-switching to deliver lessons in Euclidean geometry as well to enable full participation of learners.

2.9. *Ubuntu* in the teaching of geometry

From experience, people think of *Ubuntu* as a set of values that have to do with life at home only and do not see its applicability in school settings. In other words, when someone talks about *Ubuntu,* it is only about respect for other people especially adults. Even in school contexts, most of the time it is limited to discipline whereas it is all-pervasive in the school system. The implementation of *Ubuntu* in the school environment promotes cordial relationships of teachers and learners, discipline, mutual respect and trust, hard work and harmony. According to Shockley (2011), for conducive environments to be achieved in classrooms, teachers should display values such as caring, respect and some of the values displayed in Figure 2.2 below.

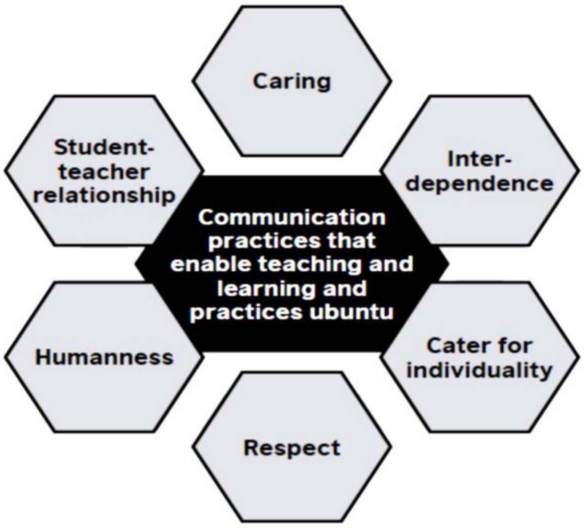


Figure 2. 2: Teacher' role: *Ubuntu* values in the classroom (Source: Adapted from Shockley, 2011).

According to Shockey (2011) the communication practices that enable teaching and learning can be enhanced by inter-dependence, catering for individuals, respect, humanness, caring and others. Inter-dependence refers to the connection between learners whereby one learner's needs can be satisfied by another learner's resources and these learners transfer the resources both ways. It means both learners need each other to satisfy their needs. In summary, it means that when one learner provides resources to another, the one whose needs have been satisfied reciprocates by offering whatever resources he or she might be having. No one between them is stingy to offer whatever he or she possesses, and their relationship is symbiotic, not parasitic. More specifically, when teachers provide individual attention and ensure that learners understand Geometry, it would be most humane as this will improve their performance in Geometry. Since *Ubuntu* pervades all spheres in schools, mathematics teachers should be the protagonists in promoting it when teaching Euclidean geometry in their classrooms. The British Council (2019) encourages teachers to have knowledge of their identity, beliefs and cultural practices, promote change by implementing *Ubuntu* values and respect learners' traditions, culture, history and views of life.

2.9.1. The role of teachers in the implementation of *Ubuntu* in geometry classroom

According to diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020), learners displayed a lot of challenges in national examinations ranging from naming angles correctly, using appropriate theorems to effect geometrical proof, among others. In addition, those diagnostic reports suggest that teachers must encourage learners to gather all the information from any given question before answering. They also encourage teachers to explain to the learners the difference between a theorem and its converse inclusive of conditions applicable for each one. From experience, the suggestions presented require patient, committed and compassionate teachers to execute because the work is arduous and time-consuming. Teachers are also advised to effectively use diagnostic reports to improve the quality of assessments. This exercise obviously would require long-suffering teachers who are full of love, compassion and care for learners to sacrifice their time and effort.

According to Van Breda (2006) a teacher plays a "loco-parentis" role. This a role in which a teacher plays the role of a parent in the school environment. In addition, Jackson (2013) suggests that teachers should strive to establish jointly with learners' relationships that extended beyond classroom interactions. This implies that teachers' influence is needed even when learners are at home to assist them when they study individually. This approach is important in the school system because it enables learners to have parental support from both home and school taking into consideration that some of the learners come from child-headed families. Eliastam (2015) further expresses that, “My child is your child”, which is an important *Ubuntu* view in education. This view gives mathematics teachers a huge responsibility on their shoulders of encouraging and influencing learners to work extra hard as they would do to their biological children. It is common knowledge that every parent desires and dreams to see his or her child becoming an engineer, doctor or any other profession which inherently requires mathematics to study for it. This implies that teachers with *Ubuntu* would go an extra mile to sacrifice and work hard to ensure learners understand the complex content of Euclidean geometry conceptually.

Reformed-based mathematics teaching encourages teachers to use learner-centred approaches in the classroom. According to Jojo (2018) *Ubuntu* enables mathematics teachers to use strategies that promote learner interactions. The researcher suggests that geometry teachers should empathise with learners and fit into their shoes to be able to understand how they perceive geometry and to be acquainted with their ideas, desires and actions. This implies that mathematics teachers should not just teach Euclidean geometry for the sake of finishing the syllabus, but they should invent strategies that promote conceptual understanding for every learner inclusive of those learners with learning difficulties. Jojo (2018) further suggests that gifted learners should be allowed to assist fellow learners with difficulties and that teachers should prepare themselves to work with a diversity of learners. Mathematics teachers are encouraged to find the best ways to make concepts comprehensible to every learner which is in tandem with the CAPS document which promotes inclusivity.

According to Lefa (2015), teachers should encourage learners to engage in cooperative learning through sharing and interaction. In addition, Mahaye (2018) suggests that schools must encourage learners to work cooperatively through sharing and collective work in the classroom. The approach should be inclusive, and it should ensure that all learners benefit. According to Mammali (2015) if learners work in groups cooperatively or collaboratively, their thinking abilities are improved and enhanced. Furthermore, the learners are motivated to work hard to improve their performance in geometry. By working in groups and sharing ideas, learners achieve more personal, social and psychological skills. Some researchers (Erdem, 1993; Bulut, 2009) have shown that learners taught using cooperative methods achieve academically better than those instructed using traditional strategies. In addition, Guyton (1991) asserts that cooperative group instruction can enhance learners' performance in geometry inclusive of enhancement of learners' self-esteem. Perhaps if teachers prepare well on engaging learners in collective and collaborative learning, in Euclidean geometry some difficulties displayed by learners would be dispelled. Furthermore, Makhubele (2014) suggests the use of another collective learning strategy called the Harkness discussion method whereby 12 learners sit around a table in an oval shape (Tingley 2002). The learners engage in the collaborative discussion sharing ideas in geometry while the teacher facilitates. According to Adams (2013) group processing is promoted which allows each group to make reflections on the usefulness of members which helps in planning future tasks and assignments.

Some researchers (Makhubele 2014; Masilo 2018) assert that inquiry-centred instruction of geometry is an effective method that promotes learners' understanding and acquisition of problem-solving skills. Inquiry-based instruction and learning use cooperative learning, exploration, sharing discoveries and guided learning by a facilitator to apply knowledge to solve problems. Results from those studies indicate that better academic performance is displayed by learners. The researchers further recommend active involvement of learners in studying Euclidean geometry and the learners must conduct geometric proofs and solution of problems themselves. This implies that fostering the spirit of working together assists learners to achieve better results than working individually. Among other aspects, this study explored instructional strategies used by mathematics teachers which enabled learners to work together and share ideas to enhance conceptual understanding in Euclidean geometry.

2.9.2. The place of indigenous knowledge in Euclidean geometry classrooms

From experience, learners tend to understand teachers who explain content using examples of objects they visualise within their immediate environment. According to Jojo (2018), effective teachers assist learners to construct meaning from geometric experiences using familiar contexts. This implies that Euclidean geometry should be taught by teachers who use real-life examples for learners to see its relevance in their lives. Letseka (2011) and Khupe (2014) explain that teaching practice based on *Ubuntu* should make use of indigenous knowledge [IK] and indigenous knowledge systems [IKS]. Van Wyk (2002) expresses that for anything to be indigenous, it should naturally exist or be found in any place. This brings to light the importance of adapting instructional strategies to the cultural contexts for learners to appreciate the utility of the Euclidean geometry learnt in the classroom.

The importance of integrating indigenous knowledge in classrooms is decolonising learners' minds and curriculum (Katonga, 2017; Breidlid & Botha, 2015; Dei & Simmons, 2009; Ngara, 2012), teaching relevant knowledge to learners (Keane & Malcom, 2003), helping learners to familiarise with their cultures and traditional practices and restoring the role of parents, elders and the community in educating learners (Mawere, 2015; Khupe, 2014). In addition, King and Schielmann (2004, p.7) express that, "Our cultures and our knowledge must be included in the curricula for indigenous children and youth at all levels of education where everything is based on culture other than ours: our cultures have a rich reservoir of knowledge." Perhaps geometry teachers should be encouraged to explain content using traditional and cultural examples. According to Malloy (2009), teachers need to know the culture and lives of their learners inclusive of their experiences for relevant learning of mathematics to take place. This means that teachers should possess cultural competence. Cultural competence refers to a plethora of aspects that include teachers' awareness and knowledge of their own culture, the role that this culture plays in their lives, teachers' knowledge of learners' cultures and ability to communicate and interact with people from diverse cultures (Ladson-Billings, 2001). This implies that African children need to be exposed to examples and artefacts from their immediate villages, not from European or American cosmopolitan environments which confuse them because they have never been there.

Some researchers (Kaino, 2011; Zengeya-Masuku, Kushure, Zengeya & Bhukuvhani, 2013; Zinyeka, 2014) explain that IK enables learners to understand content conceptually because localised materials and local language are used (King & Schielmann, 2004). In simple terms, it would be fruitless to explain geometry to learners with models of Boeing 767 not indigenous to Eastern Cape but perhaps using examples and models of round huts in which they are used to staying. Chachine (2013) explains that it is of primordial importance to integrate cultural activities which resonate with the cultural backgrounds of the learners and use cultural and indigenous artefacts in classroom activities. This makes learners appreciative of their cultures' relationship with mathematics and make sense of what they learn. Many cultural geometric features such as round huts exist in the Chris Hani West District. According to Chachine (2013), learners would be motivated to see how their culture is embedded in mathematical concepts and to know that their ancestors also used to think mathematically. They would also appreciate that mathematics was practised in their own environment other than Europe. This implies that teachers should take advantage of the natural and cultural artefacts and features to teach geometry practically to enhance understanding. Demonstrating how ancestors used to construct huts using geometry would perhaps ignite interest in the topic among learners.

According to Khupe (2014), learners' theoretical knowledge is strengthened by engaging them in hands-on activities. The researchers further explain that learners enjoy learning from what they perceive from their immediate environment and context. This implies that embracing outdoor or practical activities in geometry enables learners to understand better than forcing them to memorise axioms and theorems. In addition, Jojo (2015) suggests that learners should be allowed to construct objects themselves to enhance conceptual understanding. In another study, Jojo (2018) asserts that *Ubuntu* enables teachers from under-resourced rural schools such as those in the Eastern Cape to improvise and use familiar contexts for a better conceptual understanding of geometry by learners. According to Mosimege and Lebeta (2000), teachers are required to be creative and innovative to make mathematics to be interesting to learners. This suggests that teachers should improvise materials for teaching and learning geometry and desist from forcing learners to do activities found in the textbooks. It is therefore important for learners to see the connection of the geometry they learn in the classroom with their daily experiences.

In support of the use of IKS, some researchers (Chiphambo, 2017; Bussi & Frank, 2015; Morgan & Sack, 2011; Gürbüz, 2010; Suh & Moyer, 2007) suggest the use of manipulatives for geometry instruction because learners taught using physical manipulatives understand geometry conceptually better than those who receive instruction without using them. Physical manipulatives are made using familiar materials for learners. The researchers further assert that the learners exposed to the use of manipulatives in geometry improve academically because they use the skill of visualisation to understand concepts. Various researchers (Blanco, 2001; Jones, 2002; Duatepe, 2004; Bankov, 2013) argue that learners understand geometry better if visualisation of geometrical shapes and objects is included in the classroom. In addition, other researchers (Tay, 2003; Guzman, 2008; Mammali, 2015) assert that if learners touch and see objects their learning of geometry becomes more imaginative and successful. Visualisation of geometrical objects enhances learners' conceptual and procedural understanding. Visualisation is further enhanced by the use of manipulatives in geometry classrooms. NCTM (2000) explains that the use of manipulatives should be all-pervasive to all levels and topics in mathematics because conceptual understanding and reasoning are enhanced. During the study, it was important to interrogate the use of IKS, cultural artefacts, manipulatives and geometrical features native to the Chris Hani West District for the enhancement of the teaching of geometry in Grade 12.

2.9.3. Language and the teaching of geometry

Language plays a pivotal role in the teaching of geometry. According to Adler (2000), language is an important cultural resource to enable effective and efficient communication in mathematics classrooms. In addition, De Villiers and Njisane (1987) suggest that the poor performance displayed by learners is caused by problems in language and lack of knowledge of basic terminology in geometry. This suggests that teachers need to be tactful in using language when they conduct geometry lessons. Furthermore, Letseka (2012) asserts that language is an important cultural aspect because human beings use it to dream, desire and acquire consciousness. In addition, Mampe, Friederici, Christophe and Wermke (2009) assert that the mother tongue is important because babies start learning it in their mothers' wombs. Furthermore, Panthi and Belbase (2017) argue that the best way of making meaning of mathematical concepts is in one's native language or mother tongue. Learners in rural Chris Hani West District are predominantly Xhosa speaking. The assertions by the researchers (Letseka, 2012; Mampe et al., 2009; Panthi & Belbase, 2017) suggest that mathematics teachers should find strategies to deal with language issues in the teaching of Euclidean geometry in the Chris Hani West District considering that the majority of learners are not proficient in English which is the language of instruction.

Language is inseparable from IKS because African children think in their mother tongue and then try to express themselves in the language of instruction (Barnhardt & Kawagley, 2005) and children understand scientific concepts easily in their native language (Nakusera, 2004). According to Jojo (2015), learners should be allowed to use any language in mathematics classrooms to express their ideas and make necessary generalisations. This is in tandem with The Constitution of the Republic of South Africa which espouses *Ubuntu* principles of democracy, social justice and equity and gives equal importance to all eleven languages. The advent of democracy eradicated marginalisation of learners because of language.

Atebe and Schafer (2010) assert that proficiency in the language of instruction is crucial because once learners become proficient it would be easy to understand geometry concepts. I have noticed that learners in rural Chris Hani West District are not well exposed to English. This therefore requires all teachers to have the social responsibility to teach learners to be proficient in the language of instruction. Mntunjani (2017) argues that learners do not automatically become proficient in mathematical language, but patient and caring teachers would need to gradually teach them. The argument by Mntunjani (2017) suggests that teachers need to exercise patience, care and flexibility in language use when teaching geometry because there are a lot of concepts involved. This would not hamper the progress of learners.

According to Ngirishi (2015), mathematics teachers have the responsibility to develop learners' understanding in geometry. Furthermore, Mntunjani (2017) suggests that teachers should use resources in conjunction with language to teach mathematical language to enhance problem-solving skills. The contributions by the researchers imply that geometry teachers in matters of language use, should be resourceful, compassionate, caring and committed for the benefit of learners. This study explored how teachers in the Chris Hani West used language in the teaching of Euclidean geometry.

2.10. Conclusion

The purpose of this chapter was to situate the current study within the confines and context of current literature available in geometry education and *Ubuntu* philosophy. This chapter discussed geometry teaching in South Africa, teacher knowledge including MKT in geometry, problems faced in the teaching and learning of geometry, aspects of *Ubuntu* philosophy, how colonisation destroyed the social and educational landscape which *Ubuntu* had established. Literature on *Ubuntu* in the South African education system and the influence of *Ubuntu* philosophy on the teaching of geometry were discussed. The literature reviewed focused more on socio-economic, political and some educational contexts of humane existence. However, there is lack of literature available focusing on *Ubuntu* philosophy in the teaching of geometry. The closest linked to this study is by Jojo (2018) conducted on Grade 8 geometry in Mount Ayliff, Alfred Nzo West District in Eastern Cape, South Africa. This qualitative study focused on the influence of *Ubuntu* philosophy in the teaching of geometry in Grade 8. This study sought to add to the body of knowledge on the use of *Ubuntu* philosophy in geometry. It was envisaged that the study would instil a positive attitude of teachers towards TPD programmes and would recommend some of *Ubuntu* strategies for teachers in the teaching of Euclidean geometry. This study focused on how *Ubuntu* philosophy enhances the meaningful teaching of Euclidean geometry in the Chris Hani West District. The next chapter presents the theoretical framework of the study.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1. Introduction

This chapter presents a discussion on the theoretical framework that underpinned this study. The study explored how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry. Aspects such as the manifestation of *Ubuntu*, the social interdependence theory and the Van Hieles' theory of geometric thinking, are discussed in this chapter. Also, included in this chapter is the Dynamic Pairs Model, a model which was used in teaching Euclidean geometry during the study

3.2. Attributes of *Ubuntu* that are cardinal in the classroom

According to Broodryk (2002), *Ubuntu* is a comprehensive African philosophy that has a strong base in attributes such as humanness, caring, sharing, compassion, respect and sympathy among others. In addition, Letseka (2012) asserts that *Ubuntu* and communal interdependence are inseparable and if the values of *Ubuntu* are embraced, fruitful results are realised in the school system. Furthermore, Venter (2004, p.151) explains that communalism "is an awareness of interdependence of individuals which uplift the importance of duty to social group more than the individual". This implies that *Ubuntu* and communalism advocate the success of the social group more than personal and individual advancement.

One of the important attributes of *Ubuntu* is sharing. Dlomo (1991) states that, in the traditional African context, the sharing of materials is a common practice. In other words, an African social group is associated with the exchange of ideas and materials to ensure that no individual suffers because of the unavailability of resources. Some researchers (Samkange & Samkange, 1980; Lukhele, 1980; Mpofu, 2002) confirm that the sharing of resources and ideas was important in the development of the traditional African community. Other researchers (Mbigi & Maree, 1995; Khoza, 1994; Tutu, 1995) assert that *Ubuntu* is linked with the spirit of brotherhood and sisterhood inclusive of collective consciousness. This study focused on *Ubuntu* values such as compassion, sharing, caring, collective work and togetherness and many others and how they enhance the teaching of Euclidean geometry. Also, the study considered the importance of the Van Hieles' theory in enhancing the study of geometry.

3.3. Social interdependency theory

Johnson and Johnson (1989) outline the differences between social interdependence, social dependence and social independence. According to the researchers, social interdependence exists when the accomplishments of an individual's goals are affected by the actions of others. Social dependence exists when the achievement of Person A is determined by the input of Person B but the reverse is not true. Social independence is a phenomenon that exists when no one is affected by the actions of other people. Another interesting aspect explained by the researchers is social helplessness which exists when no one influences the achievement of goals. In the case of geometry understanding, learners depend and draw on teachers' content knowledge together with how the teacher presents geometric concepts. However, the learner needs to master the connection of geometric concepts and how they are developed such that he/she becomes independent. Indeed, a learner deprived of basic understanding of geometric concepts undergoes social helplessness. Social interdependence refers to a scenario whereby the actions of a group of people affect the achievement of the goals of each member of the group. Thus, in the case of a group of learners in a geometry classroom, understanding the meaning of geometry concepts affects their achievement in mathematics in general. The social interdependence [SIT] theory was selected as one of the theories to underpin this study because:

i. It is closely linked to *Ubuntu* philosophy. It can guide how the sharing of knowledge can be advanced for meaningful understanding of geometry; and

ii. It was used as a foundational theory on studies on cooperative learning which yielded positive results. Various studies (Muniz & Walmsely, 2003; Musingafi & Rugonye, 2014; Adams, 2013; Johnson, Johnson & Smith, 2007; Huddy, 2012) showed that learners' academic performance improved when cooperative learning was applied in the classroom.

According McCluskey and Lephalala (2010), the core of *Ubuntu* recognises the value system which acknowledges people as social and co-dependent beings. In addition, Louw (2003) asserts that *Ubuntu* promotes communalism and interdependence. The principles of *Ubuntu* explain that "a person is a person through other people" (Swanson, 2007; Samkange & Samkange, 1980; Tutu, 1999; Nussbaum, 2003). There is no way a person would exist without being influenced by the existence of others. Goduka (2000:70) states that "all human beings are connected not only by ties of kinship but also by the bond of reciprocity rooted in the interweaving and interdependence of all humanity". Various researchers (Chachine, 2008; Adonis, 2008; Letseka, 2012 & Mokgoro, 1998; Mbaya, 2011 & Msengana, 2006; Mbaya, 2011; Msengana, 2006) assert that *Ubuntu* and social interdependence are closely intertwined such that the existence of *Ubuntu* inevitably signifies the existence of social interdependence and vice versa. This implies that *Ubuntu* revolves around the spirit of interdependence, communalism and mutual trust. This spirit of *Ubuntu* is characterised by social interdependence in any social endeavour such as education in which human beings interact for the social good and success.

According to researchers (Johnson & Johnson, 1989; Jalilifar, 2010; Johnson, 2003; Castle, 2014; Johnson & Johnson, 2008), two types of social interdependence exist, namely, positive interdependence (cooperation) and negative interdependence (competition). The first phenomenon which is positive interdependence, occurs when individuals in a social setting perceive that their goals can be reached only on the condition that people work together promoting each other to achieve goals. In a mathematics classroom, the teacher may occasionally assign group activities in which learners investigate some properties in circle geometry before the teaching or instruction on any theorem. In that exercise, learners would work collaboratively and hence acquire the geometric skills together. The second phenomenon, negative interdependence exists when people engage in competition and obstruct others to achieve their goals. Consequently, their success is dependent upon the failure of others. This could be the case when learners are given different geometric riders to do individually in a stipulated time; no one learns from the other, rather, they compete. The third phenomenon is no interdependence whereby individuals would perceive that they can achieve their goals regardless of the failure or success of others. Negative interdependence and no independence are antithesis to the spirit of *Ubuntu*. *Ubuntu* is intertwined with positive interdependence where people cooperate and work together with a positive spirit.

According to researchers (Slavin, 1991; Walmsely & Muniz, 2003; Johnson & Johnson, 2013), SIT is a theory that forms the base of cooperative learning. The researchers assert that cooperative learning results in higher academic achievement, high levels of productivity and success, better communication skills, successful social and academic group interactions, high levels of self-esteem among learners and promotes acceptance of others. In the teaching and learning of Euclidean geometry, perhaps if teachers possess the knowledge of the social interdependence theory inclusive of *Ubuntu* principles classroom management and organisation would be enhanced. Perhaps mathematics teachers would encourage learners to work together assisting one another to understand and use theorems correctly to solve exercises. This implies that failure to foster cooperation and sharing would diminish opportunities for academic improvement in Euclidean geometry among learners. Positive interdependence promotes effective teacher-learner interaction and learner-learner interaction. This enables teachers and learners to focus on conceptual understanding of a topic only and also to focus on passing a topic. Figure 3.1 below illustrates elements of the social interdependence theory.

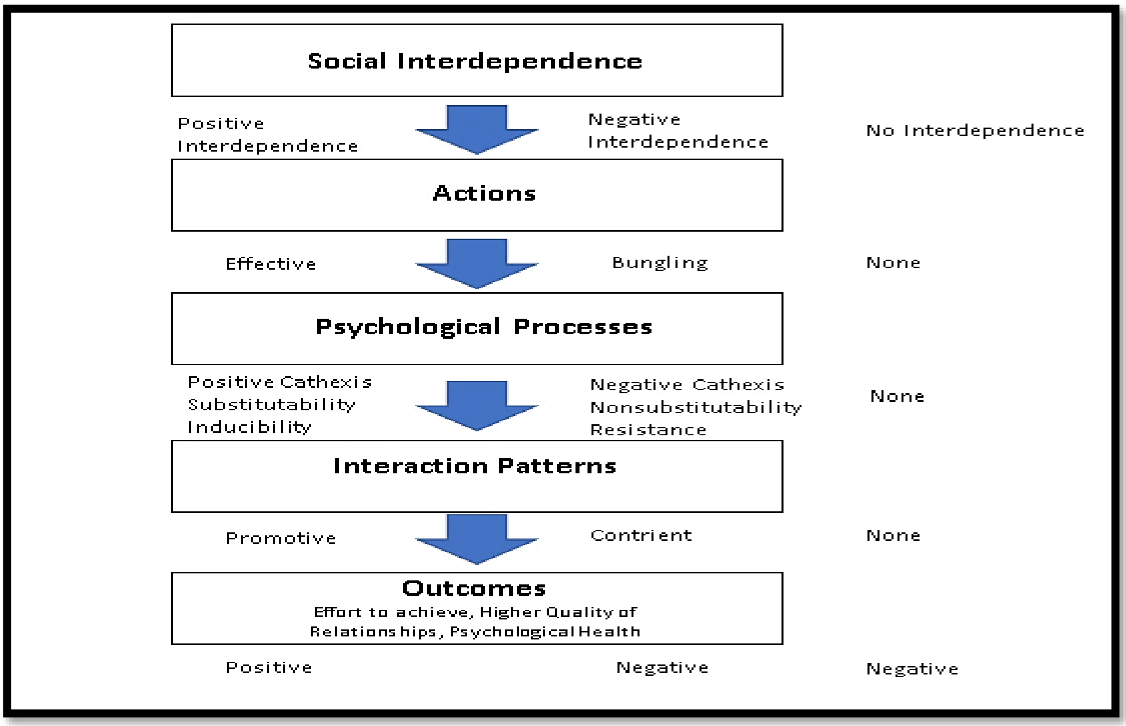


Figure 3. 1: Overview of the Social interdependence theory (Johnson & Johnson, 2008, p.11).

Some researchers (Deutsch, 1949 & Stanne, Johnson & Johnson, 1999) explain the psychological processes which emanate from interdependence. The three processes are:

i. **Substitutability**: the degree to which actions of one person, substitute actions of another person,

ii. **Cathexis:** when a person invests his or her psychological energy in objects outside oneself such as the family, friends and work, and

iii. **Inducibility**: ones' openness to influence others or to be influenced by others.

This implies that both teachers and learners would be encouraged to sacrifice their efforts and time to assist some learners who face challenges in class. All learners are encouraged to share and allow others to evaluate their work in an open forum.

One of the aspects involved in positive interdependence is promotive interaction. It occurs when people encourage one another and facilitate each other's efforts to achieve the group's goals. This encouragement is vital in a geometry lesson especially for meaningful understanding of a concept. Related to negative interdependence is oppositional interaction where people obstruct the development of others. Occasionally, a mathematics teacher would just present a concept and then move on to the next one with those learners who seem to understand. In this way, oppositional interactions may occur when learners who would have failed to understand the concept being explained decide not to listen or sleep while the teacher continues to impart the lesson or make disruptive noise. Another scenario would be when the teacher decides to ignore learners' questions. Oppositional interactions are destructive and militate against the values of *Ubuntu*. The indicators of the existence of promotive interaction are characterised as follows:

i. Individuals behave and act in truthful and trustworthy ways,

ii. Individuals share, and exchange required resources and materials and processing information more efficiently and effectively,

iii. Individuals provide efficient and effective help and assistance to groupmates, being motivated to strive for mutual benefit,

iv. Individuals advocate for all group members to put more effort to achieve mutual goals, and

v. Feedback is availed to group members to improve their individual performance of assignments and other forms of assessments.

According to researchers (McKay, 1995; Quan-Baffour & Romm, 2015), the aspect of social independence that is closely intertwined with *Ubuntu* is positive interdependence. This suggests that *Ubuntu*-inspired teachers inherently value positive interdependence and positive interactions within school environments. The researchers further explain that, in positive interdependence, members of any group depend on each other for success to be achieved. The interests of the group take precedence over individual advancement. Sharing of resources among group members is paramount. In addition, researchers (Sibanda, 2019; Kamwangamalu, 1999; Molefe, 2017) assert that *Ubuntu* is also inseparable from communalism. In communalism, there is positive interaction of group members characterised by the subordination of individual interests to the interest of the whole group. Some researchers (Laal, 2013; Johnson & Johnson, 1989; Johnson, Johnson & Holubec, 1998) further explain that positive interdependence comprises various aspects such as:

i. **Positive goal interdependence** whereby a certain group identifies group goals and individual goals and puts plans to achieve those goals,

ii. **Positive role interdependence** which is a situation in which each group member works towards achieving a specific unique goal,

iii. **Positive environmental interdependence** whereby each group is allocated a designated area to work,

iv. **Positive resource interdependence** whereby resources are shared and each group member receives relevant resources and materials to complete a task,

v. **Positive reward interdependence** whereby each group is given a reward for achieving a goal,

vi. **Positive identity interdependence** whereby each member of a group is identifiable through a motto, name, flag or song,

vii. **Positive fantasy interdependence** which is a situation whereby members dream and imagine what it would be like when the group fails. This motivates group members to collaborate effectively to avoid failure and work towards success,

viii. **Positive outside enemy interdependence** whereby groups are established so that they compete among themselves. Therefore, each group organises itself to collaborate with group members valuing interdependence among themselves so that they supersede other groups and,

ix. **Positive task interdependence**, whereby labour is divided among group members, each one to accomplish the work allocated to her or him.

In a geometry classroom, *Ubuntu* entails positive interdependence among learners together with their teacher to achieve success. Learners would be encouraged to share resources and materials, share vision and dreams, share knowledge, work collaboratively to complete tasks, motivate one another in a brotherly manner and share the working space amicably. Perhaps mathematics teachers in a certain locality may decide to work collectively sharing everything in geometry for the success of every school around. In summary, *Ubuntu* enables the eradication of individualism and ensures that values such as sharing, compassion, solidarity, respect, empathy and among others pervade the geometry classroom. The focus of this study was to explore if *Ubuntu* enhances the meaningful teaching of Euclidean geometry in Grade 12.

3.4. The Van Hieles' theory

According to researchers (Patkin & Barkai, 2014; Vojkuvkova, 2012), the Van Hieles' theory originated from doctoral dissertations of a Dutch couple called Dina van Hiele-Geldof and Piere M van Hiele who were educators. The two University of Utrecht scholars in their separate studies indicated that they noticed difficulties that their students encountered. The theory comprises levels of geometric thinking which are hierarchical. Various researchers (Murray, 1996; Abdullah & Zakaria; De Villiers & Njisane, 1987; Crowley, 1987; Groth, 2005; Holmes, 1995; Usiskin, 1982) have validated the existence of this hierarchy suggested by the Van Hiele theory. Van Hiele (1999) suggests the existence of levels of geometric thinking which each learner has to pass through. The levels are as follows:

i. Level 0: Visualisation or recognition
ii. Level 1: Analysis
iii. Level 2: Ordering or informal deductive
iv. Level 3: Deduction or formal deductive
v. Level 4: Rigour

According to the theory, no learner will breach this hierarchical structure through bypassing some levels to higher levels. However, Burger and Shaughnessy (1986) argue that the levels suggested by the Van Hiele couple are not discrete. The researchers assert that learners transition between levels. Oscillation occurs between the levels during the transition period. Some researchers (Clements & Battista, 1992;

Armah & Kissi, 2019; Solaiman, Magno & Aman, 2017; Abdullah & Zakaria, 2013) confirm the existence of the same levels as in the original Van Hiele model but differ in numbering the levels from Level 1 up to Level 5. The levels are as follows:

i. Level 1: Visualisation or recognition
ii. Level 2: Analysis
iii. Level 3: Ordering or informal deductive
iv. Level 4: Deduction or formal deductive
v. Level 5: Rigour

However, Clements and Battista (1992) suggest the existence of Level 0 which is the pre-recognition level. This level indicates that a learner would not have achieved Level 1 (visualisation). This implies that a lot needs to be done by teachers to assist learners to be able to achieve the ability to visualise.

Van Hiele (1999) argues that the model is different from Piaget's theory. It assumes that movement from one thinking level to another does not depend on biological maturity or age but the teaching and learning experiences. It is through interactions among learners and their teachers in the classrooms that promote movement up the hierarchy. Perhaps teachers in their planning should promote more meaningful and fruitful interactions among learners to enable the development of geometrical thinking of each learner in totality from the lowest level to the highest level. The Van Hiele model is composed of levels of geometric thinking and phases of teaching geometry. According to Crowley (1987), students must go through this hierarchy; no learner jumps any one of the levels to higher levels, and appropriate instruction must be given to each level to ensure effective learning occurs. Teachers have to adapt instructions to each level. If the instructions are given at a higher level of students' abilities, the students would have difficulties in following the thought processes. One of the characteristics of the Van Hiele theory is that people at different levels of geometric thought do not understand each other. Vojkuvkova (2012) suggests this phenomenon highlighted in the Van Hiele theory is one of the reasons why learners fail geometry because teachers and learners normally operate at different levels of geometric thought. Teachers normally operate at higher levels than their learners and they do not understand each other's language hence the existence of poor performance in geometry. It is important to note that there are linguistic symbols attached to each Van Hiele level. There is also a network of relationships at each level that connects those symbols. Symbols include some experiences which the teacher associates with any symbol used in the classroom. Also, it is interesting to note that things deemed to be correct at one level are not necessarily correct at another level.

Van Hiele (1986) suggests that cognitive progress in learners can be enhanced and accelerated by the way the teacher conducts instruction in the classroom. Learners' progress from one level to the next does not depend on age or biological maturity but on the teacher's pedagogical skills to conduct instruction. The Van Hiele theory therefore postulates guidelines on the phases of teaching, namely: information, guided or directed orientation, explication, free orientation and integration. Researchers (Crowley, 1987; Serow, 2008) summarise the phases of geometry instruction as follows:

i. **Information**: In this phase, the teacher uses probing questions to learners engaging them to think critically. The question and answer approach is prevalent during this phase.

ii. **Guided orientation**: In this phase, teachers assist learners to learn geometry through exploration. Activities are planned in which aids are provided to promote learners to explore.

iii. **Explanation**: Learners explain what they observe through activities in which they are involved. In this phase, learners construct their knowledge through their previous experiences. In addition, learners can explain their perspectives about the geometry structure they observe.

iv. **Free orientation**: This is a stage when learners demonstrate their problem-solving skills by solving complex tasks that involve many steps and the tasks that can be solved in a variety of methods.

v. **Integration**: Learners review and summarise the geometry learnt and make an overview of all objects inclusive of all relationships involved.

The phases mentioned above give the geometry teacher clues on how to introduce lessons or discussion, the types of exercises and activities to engage learners with, how to encourage language development and the general classroom management during the instruction of geometry. Also, the knowledge of the phases of instruction enables the geometry teacher to plan teaching in a way that learners move up the Van Hieles' levels of geometric thinking. This implies that a teacher plays a pivotal role in promoting geometrical thinking and reasoning for learners' understanding of mathematics. Moreover, it opens the eyes to those responsible for teachers' training and development to come up with thorough methods of developing teachers to prepare them for future assignments.

3.5. Relationship between *Ubuntu*, SIT and Van Hieles' theory

According to Alexander (2009), caring teachers are required to embrace a diversity of learners in their classrooms. This implies that the pastoral role and caring of the teacher should characterise the work of the teacher to achieve efficacious educational outcomes. In summary, geometry teachers inspired by *Ubuntu* strive to know and practice all aspects of geometry such as the content, didactics and theories revolving around geometry learning. In the teachers' minds should be thoughts of how to enhance learners' achievement. The teacher sacrifices his time, resources and energy for the learner to perform better in geometry. Figure 3.2 illustrates a fusion of *Ubuntu*, social interdependence and Van Hieles' theory.

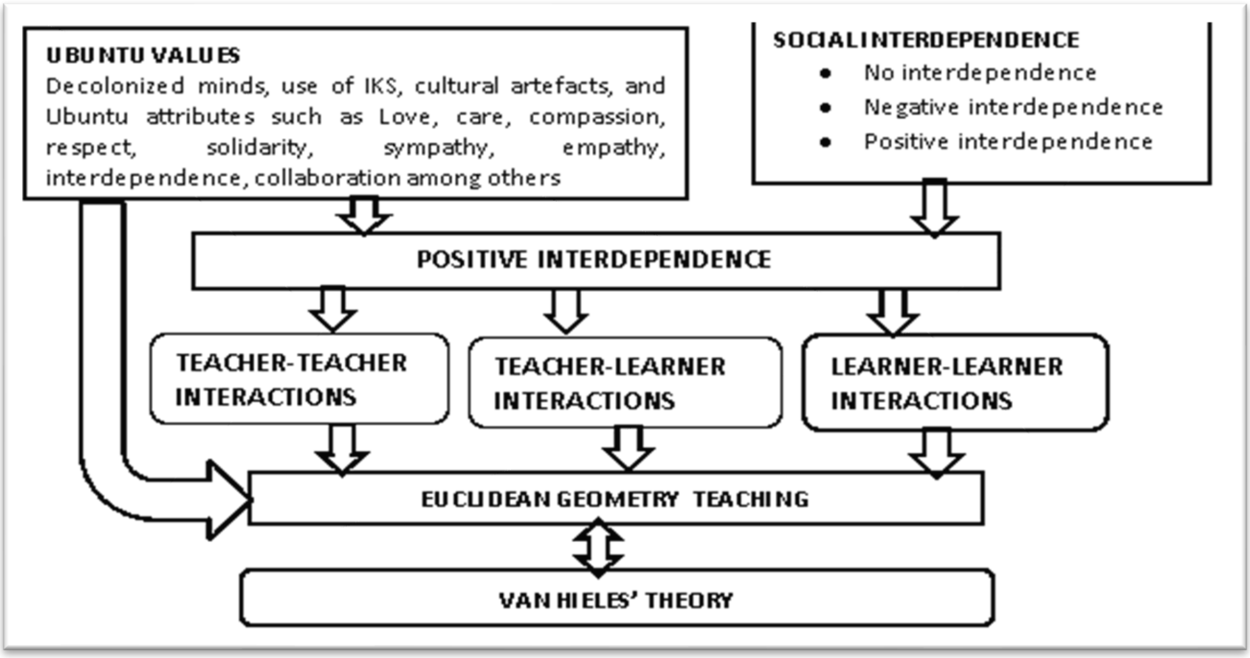


Figure 3. 2: Theoretical framework: The fusion of *Ubuntu*, SIT and Van Hieles' theory.

The positive interdependence of mathematics teachers influences teacher-teacher interactions in which the teachers share ideas and resources of geometry with other teachers at workshops and in professional learning communities. According to Mammali (2015), professional interactions among teachers in professional learning communities, seminars and workshops improve teachers' MKT and PCK in Euclidean geometry. In those workshops teachers who are knowledgeable with Euclidean geometry share the knowledge they possess of Euclidean geometry including the Van Hieles' theory with other mathematics teachers. With that knowledge empowerment, mathematics teachers gain confidence such that in teacher-learner interactions, teachers are compassionate and enabled to play the role of a facilitator. The teacher assists learners with the provision and procurement of resources and also delivers the Euclidean geometry content using learner-centred approaches which enable learners to participate actively in class and give them opportunities to discover for themselves. In that way, teachers may assist learners to move up geometric thinking levels through coordinating well-planned activities following the phases of instruction. In addition, during each lesson the teacher uses the Van Hiele's five phases of learning to assist in develop learners' levels of geometric thinking. The five phases of learning promote learners to interact together and share knowledge. Learners may also be encouraged to interact in geometry classrooms by sharing resources and ideas amongst themselves in solving Euclidean geometry problems. Consequently, the learners would adopt a practice of working as a collective and assist each other and ensure that each of them achieves a desirable result.

3.6. The Dynamic pairs model for teaching Euclidean geometry

The term "Dynamic Pairs Model" is a term coined by researcher to name the model which was used in the teaching of Euclidean geometry to learners in the experimental groups from the two selected schools. The model acknowledged that the effective and meaningful teaching of Euclidean geometry involves a combination of aspects such as social interdependence, *Ubuntu* values and the Van Hiele's theory. The model also recognised that those three aspects added value to the effective and meaningful teaching of Euclidean geometry. The first attributes the teacher needs to possess are PCK, CK and MKT to ensure that every learner is attended to enabling conceptual understanding of Euclidean geometry. In the study, *Ubuntu,* social interdependence and aspects of Van Hiele's theory served to support teachers' PCK, CK, MKT and other pedagogical skills in the teaching of the topic. In this study, PCK, CK and MKT in geometry includes knowledge of how to apply Van Hiele's theory in teaching. The Dynamic Pairs Model was used to close gaps in Euclidean geometry teaching. With the use of *Ubuntu*, the teacher reached out to all learners and enabled them to be active and feel motivated because they realised that their teacher displayed patience compassion, care, empathy and other values. The teacher played the role of facilitator. The following steps were considered in the application of the dynamic pair model:

i. The combination of *Ubuntu*, SIT and Van Hiele's theory plays an important role in the meaningful teaching of Euclidean geometry.

ii. Learners sat in pairs during lessons. The composition of each pair changes composition daily. This means that each learner sat with a new partner different from previous days. In symbolic terms, Learner X shared with Learner A on the first day, Learner B on the second day, Learner C on the third day and so on. In this study, those pairs of learners that changed composition daily were referred to as dynamic pairs. Sitting in dynamic pairs enabled each learner to share with as many learners as possible. This encouraged collective work among learners.

iii. The learners in pairs were given practical activities with steps to follow. Teachers provided mathematical instruments for practical activities, manipulatives such as ropes and pegs, models and artefacts. Learners were encouraged to formulate the theorems after discoveries from practical activities. In a problem-solving lesson, learners shared ideas and came up with solutions among themselves. The teacher visited each pair to assist if difficulties were encountered.

iv. During networking of the dynamic pairs, there was inter-pair collective work whereby each dynamic pair interacted and shared ideas with other pairs on how they came up with solutions or how they carried out the practical activity. The teacher monitored the whole process ensuring that no disruptions occurred. This stage served as a reflective session whereby learners share alternative methods of getting solutions to activities given.

v. The teacher applied the Van Hiele's phases of teaching in combination with *Ubuntu* attributes such as collective work and sharing among others. These attributes manifested in attention to learners' difficulties, allowing gifted learners to interact with other learners to explain how to solve problems, organising extra tuition with learners and inclusivity. The teacher encouraged learners to share during study periods.

vi. The teacher promoted teacher-learner and learner-learner sharing extending beyond the classroom in the context of sharing ideas on solving exercises in Euclidean geometry. This shows the teacher's innovativeness on how to make learners continue to share after school. Figure 3.3 summarises the teaching model.

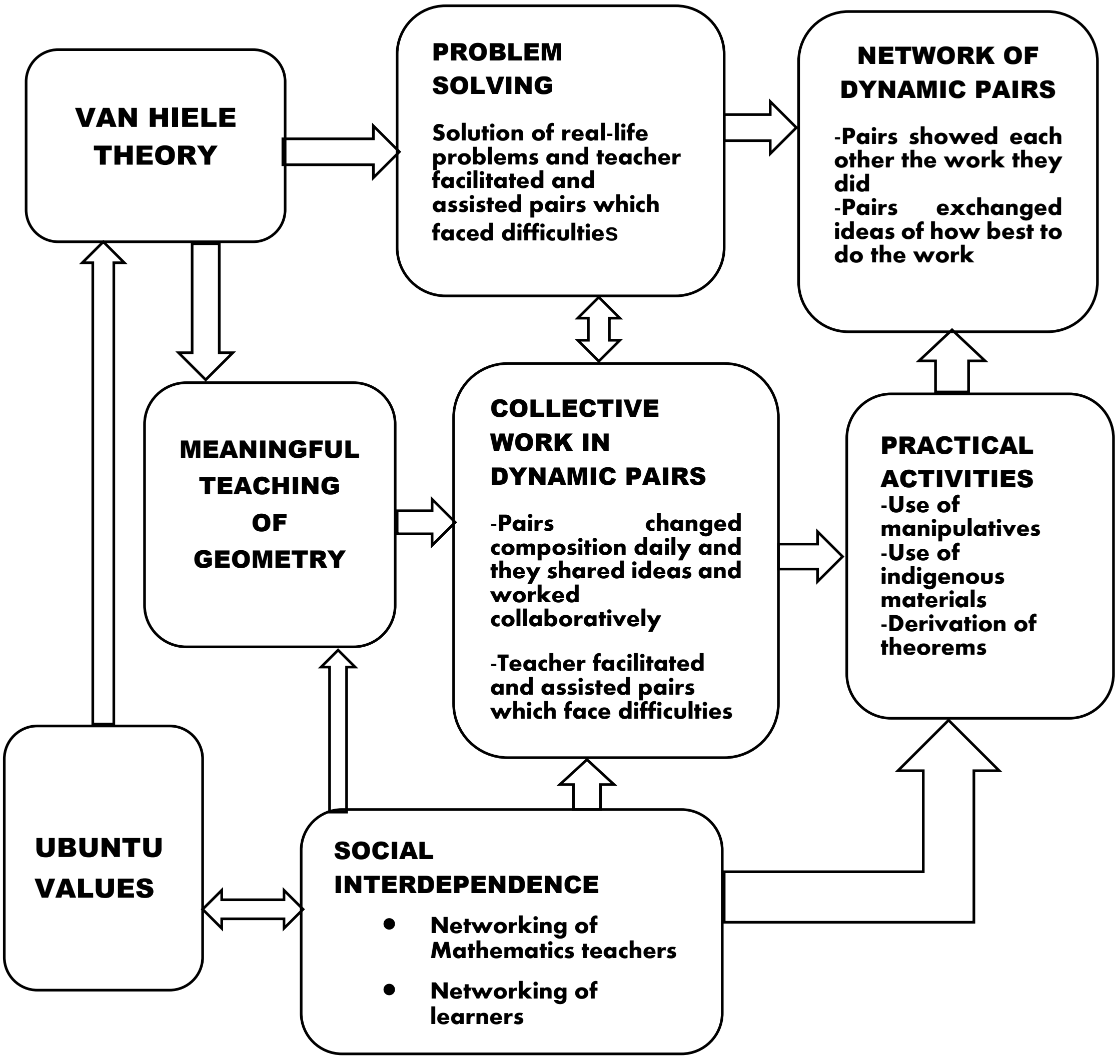


Figure 3. 3: The Dynamic Pairs Model for teaching Euclidean geometry.

To implement the Dynamics Pairs Model, in each lesson each partcicpant used Van Hiele's phases of teaching as follows:

- **Phase 1 (Information):** Each teacher used probing questions to solicit learners' prior knowledge in geometry.
- **Phase 2 (Guided orientation):** The teacher used practical activities which allowed learners to explore for themselves. Aids and models were used in which learners to reach conclusions about relationships between angles.

- **Phase 3 (Explanation):** In their inter-pair and intra-pair discussions, learners explained what they observed in the practical activities.
- **Phase 4 (Free orientation):** Learners solved problems by sharing ideas on how to apply theorems. In addition, learners determined sizes of angles using a series of steps and made simple proofs.
- **Phase 5 (Integration):** At the end of each lesson, learners discussed what they learnt during the lesson. They also had the opportunity to highlight sections in which they had problems in understanding.

3.5. Conclusion

In this chapter, an explanation was given on *Ubuntu*, the social interdependence theory and the Van Hieles' theory. In addition, the chapter explained the relationship among the theories. Furthermore, a proposed teaching model for geometry was presented. The next chapter presents the research design and methodology.

CHAPTER 4: RESEARCH METHODOLOGY

4.1. Introduction

This chapter describes the research methodology with a detailed explanation of the research paradigm, research approach and research design. An explanation is given why each aspect was selected for the research process. Aspects such as sampling techniques, ethical issues, data collection methods, data analysis techniques are also discussed.

The study attempted to answer the following research question:

How can Ubuntu values enhance disruptive pedagogies for effective and meaningful teaching of grade 12 Euclidean geometry?

The following sub-questions were answered:

i. How can the teachers' PCK be enhanced for effective and meaningful teaching of grade 12 Euclidean geometry?

ii. What difficulties are experienced by mathematics teachers in the teaching of grade 12 Euclidean geometry?

iii. What aspects of Ubuntu philosophy should teachers apply for the enhancement of the teaching of Euclidean geometry in grade 12?

iv. To what extent do collective work and collaboration impact on the teaching of grade 12 Euclidean geometry?

v. How can Ubuntu philosophy be used to transform the teaching of grade 12 Euclidean geometry?

4.2. Context of the study

The study was conducted during the period when the pandemic of COVID-19 was prevalent in South Africa. The COVID-19 regulations prescribed wearing of face masks always, practice of social distancing, prohibition of social gatherings of more than 50 people, use of sanitisers and thorough washing of hands with approved detergents. The classrooms in the schools selected could only accommodate a maximum of 20 learners as per regulations from the Department of Basic Education. Therefore, the two schools selected possessed two mathematics groups of learners of less than 20 each in compliance with the specifications and regulations provided by the Department of Basic Education in efforts to reduce the spread of the ravaging virus. The learners' desks were arranged in a manner that the distance between them was two metres. These measures were established to protect teachers and learners from contracting the COVID-19 virus and control the spread of the disease.

4.3. Research paradigm

According to researchers (Mertens, 2010; Creswell, 2014), a paradigm is a worldview. It is a general philosophical disposition or orientation that a researcher has about the world around him or her. It is also a set of beliefs that often guide the action of a researcher based on past research experiences. Sanders, Lewis and Thornhill (2016) assert that a research paradigm is a set of broad terms that refer to a viewpoint in the development and nature of knowledge. In addition, Morgan (2007) explains that a paradigm is a system of beliefs that a researcher has with the capacity to influence the research methods to be used and how data collected can be interpreted. Furthermore, other researchers (Cameron, 2011; Hall, 2012) assert that a paradigm deals with the way researchers perceive the world and that it directs how researchers think and conduct the research process. This study considered pragmatism as the underlying worldview. According to Terrel (2012), pragmatism believes that reality is realised through observation and experience. In this study, the pragmatic paradigm was used because it allowed the researcher to explore a practical and experiential process in mathematics education through the teaching and learning of Euclidean geometry in a district in the Eastern Cape. The study explored how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.

4.4. Research approach

The mixed method research approach was used in this study in which both quantitative and qualitative data were collected. According to Denzin and Lincoln (2011), the qualitative research approach studies phenomena in their natural setting and it also allows people to bring meaning to the data collected. Moreover, Bassey (2001) suggests that using qualitative methods in a study helps to focus more on various experiences and complex issues involved in social settings such as schools. In school settings, there are many issues involved ranging from classroom management, curriculum, school discipline, learners' attitudes, parental involvement and teacher knowledge among others. The use of some qualitative methods in this study was appropriate because geometry is studied under school settings in which some of the aspects mentioned above are involved. In this study, the use of qualitative methods enabled the researcher to generate detailed accounts of how the mathematics teachers taught Euclidean geometry. Furthermore, the qualitative approach helped the researcher to make a deep analysis of the views of teachers in teaching Euclidean geometry.

Quantitative research methods were also used in this study. According to Aliaga and Gunderson (2003), the quantitative approach is a systematic study of a phenomenon that can be observed using numerical or statistical means. Creswell (2014) further explains that it is an approach that allows the use of experiments, surveys and pre-determined instruments in which numerical and statistical data are collected. Data collected using quantitative methods are analysed using statistical procedures obtaining an unbiased generalisation of any phenomenon or any given population. The researcher assumed that this approach would enhance the credibility of the study.

Various researchers (Leedy & Ormrod, 2015; Sweeney, 2016; Cameron, 2011; Hall, 2012) explain that the mixed methods approach allows triangulation, complementarity and completeness of data to take place. According to Lacey and Luff (2009), triangulation enables data to be collected and analysed from different sources to obtain a complete and clear understanding of the phenomenon under study. In addition, Leedy and Ormrod (2015) assert that triangulation enables the data collected from quantitative and qualitative methods to be compared thereby finding similarities between them. Data collected from one method is used to confirm and support data collected from using the other method. By combining mixed methods, weaknesses of one method are eliminated giving validity to the findings. This study used the experimental method, interviews and classroom observation.

According to Greene, Carcelli and Graham (1989), complementarity seeks to elaborate and clarify the findings from one method with the findings of the other method. Complementarity was achieved by using one method to clarify, explain and enhance the findings from the other method. In this study, quantitative data collected were complemented by data collected from interviews and classroom observation. Moreover, one method may miss obtaining some information therefore the other method compensates for that by collecting the data which were missed by the other method. Weaknesses of the qualitative method were compensated by the strengths of the quantitative method and vice versa.

The aspect of completeness is achieved when the approach addresses the research problem. In this case, completeness was achieved by using both quantitative and qualitative methods to collect and analyse the data collected. The researcher analysed data collected from interviews and classroom observation using thematic analysis and used statistical operations to analyse the quantitative data. The researcher felt that using both quantitative and qualitative methods would allow comprehensive data of the phenomenon to be collected, analysed and interpreted to allow the attainment of a comprehensive conclusion. The researcher used both methods to collect data exhaustively and to answer the research questions.

4.5. Research design

This study used the convergent parallel design to collect the data. According to Sweeney (2016) and Creswell (2014), this design is characterised by simultaneous independent data collection and analysis using both quantitative and qualitative approaches. The findings were merged to provide a clear and complete understanding of the phenomenon being studied. In this design, both quantitative and qualitative methods had equal value and weight. Figure 4.1 conceptualises the convergent parallel design.

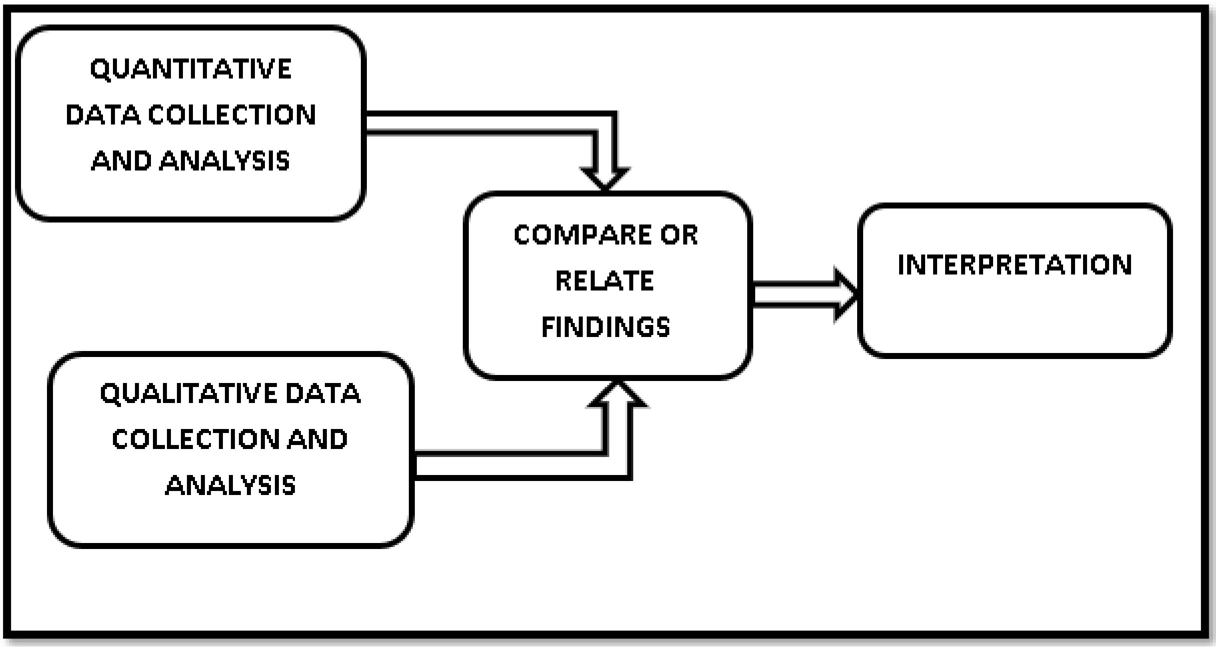


Figure 4. 1: Convergent parallel design (Sweeney, 2016, p.230)

Data were collected using both quantitative and qualitative methods during the same time-point. The findings were integrated during the phase of data analysis. The convergent parallel design was used because:

i. The researcher felt both quantitative and qualitative methods were of equal importance,
ii. Complementary data could be gathered, and
iii. The researcher collected the data in a short space of time therefore both data sets needed to be collected speedily.

The quantitative data were collected using a non-equivalent pre-test-post-test comparison group using a quasi-experimental design. According to McMillan and Schumacher (2010), this design is normally used in educational contexts because it is often impossible to randomly assign subjects. The researcher used already established groups of subjects, gave pre-test, administered intervention to one group and gave a post-test. Using already established groups helped to avoid disruptions in the schools' timetables and the movement of learners between the two schools. Chimuka (2017) used a similar design to collect quantitative data between two different groups from two different schools.

4.6. Research site and population

The research site of this study was the Cacadu Circuit Management Centre [CMC] of the Chris Hani West District in the Eastern Cape Province, South Africa. The term Circuit Management Centre [CMC] refers to a sub-district of the Chris Hani West District. The Chris Hani West District comprised three CMCs, namely, Komani CMC, Cacadu CMC and Cradock CMC and had 84 secondary schools with Grade 12 excluding two prison secondary schools during the time this study was conducted. Table 4.1 below shows the number of schools per sub-district in the Chris Hani West District.

Table 4. 1: Distribution of secondary schools in Chris Hani West District

Circuit Management Centre [CMC]	**Number of schools with Grade 12 classes**
Cacadu	26
Cradock	16
Komani	42
Total number of schools in Chris Hani West	84

The afore-mentioned district was the place where the researcher worked as a lead mathematics teacher and interventionist with a private company in teaching mathematics to Grade 12 learners. The population of the study was 20 Grade 12 mathematics teachers from 20 different secondary schools in Cacadu CMC of the

Chris Hani West District in the Eastern Cape Province, South Africa who were actively involved in intervention programmes. Those intervention programmes were conducted during weekends and school holidays in autumn, winter and spring over three years in which the researcher worked as a lead mathematics teacher in the Chris Hani West District. The population of 20 mathematics teachers was obtained from 20 different secondary schools in Cacadu CMC because the researcher had experience in working with those schools in intervention programmes. Furthermore, the population of 20 mathematics teachers also was reached after consultation with mathematics subject advisors that those 20 teachers were the ones very cooperative in teacher development workshops.

During the time of the study, Cacadu CMC was predominantly rural with all secondary schools situated in the deep rural Eastern Cape. Rural Eastern Cape was deemed to be inhabited by people of poor socio-economic backgrounds where schools were under-staffed, under-resourced, lacking access to information and the place was very remote. The researcher felt that using Cacadu CMC as the site of the study would give a clear picture of how Euclidean geometry and mathematics in general was taught in rural settings where there were teacher shortages, lack of resources such as relevant textbooks, lack of access to the internet and poor connectivity and where schools were enrolling learners without much exposure to the outside world. Furthermore, the researcher felt that the study would assist rural schools since most of the learners needed more attention, compassion, care, support and assistance due to the poor socio-economic backgrounds of the learners.

4.7. Sampling and sampling methods

According to researchers (Babbie, Mouton, Vorster & Prozesky, 2009; McMillan & Schumacher, 2010), a sample comprises a subset of an entire population and this subset is representative of that population under study during a research process. The purposive sampling technique was used in this study. According to Macmillan and Schumacher (2010), purposive sampling is a technique that enables the researcher to select participants from a population that has certain characteristics. The researcher was able to select participants who were deemed to fully represent the population. In addition, some researchers (Burns & Grove, 2007; Kumar, 2014; De Vaus, 2002) assert that purposive sampling allows the researcher to seek participants who provide relevant information and would be willing to share it. Using purposive sampling, the researcher made a conscious decision about which participants to include to collect data from. In this study, two Grade 12 mathematics teachers were purposively selected from two different secondary schools from a population of 20 mathematics teachers from 20 different secondary schools in Cacadu CMC of Chris Hani West District. The population was composed of 20 mathematics teachers who were cooperative and active in teaching in Grade 12 intervention programmes which were spearheaded by the researcher's company for three years. The researcher worked with the two teachers because:

i. of their willingness to participate in the study and their preparedness to share information freely,
ii. The researcher felt that those teachers were reliable sources of information and there was no possibility of them withdrawing from the study,
iii. The two teachers had similar teaching qualifications,
iv. Both teachers showed similar commitment to teaching mathematics, and

v. The two teachers were teaching in similar rural school environments.

Each teacher taught two groups of mathematics in his or her school. One of the groups was a control group whilst the other was an experimental group. In each school, each group comprised less than 20 learners. In summary, School A comprised two Grade 12 mathematics groups, one being the control group and the other being the experimental group, and School B comprised both the experimental group and control group as well. Table 4.2 below shows the distribution of learners in School A and School B. For Teacher B the number of learners in the two groups were different because some learners were affected by COVID-19. The numbers of learners in Teacher B's groups were supposed to be 19 in the experimental group and 19 in the control group.

Table 4. 2: Distribution of learners in experimental and control groups per teacher

Teacher	**Group**	**Number of learners per group**	**Total number of learners per teacher**
Teacher A	Experimental group	18	36
	Control group	18	
Teacher B	Experimental group	16	34
	Control group	18	
	Total number of groups involved in the study= 4		

All four mathematics group wrote a pre-test. An independent samples t-test was conducted using the pre-test scores of the learners to analyse achievement. The independent samples t-test was conducted to examine if the achievements in pre-test for both groups were not statistically different.

4.8. Data collection process

The convergent parallel design was used to collect both qualitative and quantitative data simultaneously. The instruments used for data collection were semi-structured interviews with teachers, pre-test and post-test for learners, document analysis and classroom observation schedule. Figure 4.2 summarises the sources of data in this study.

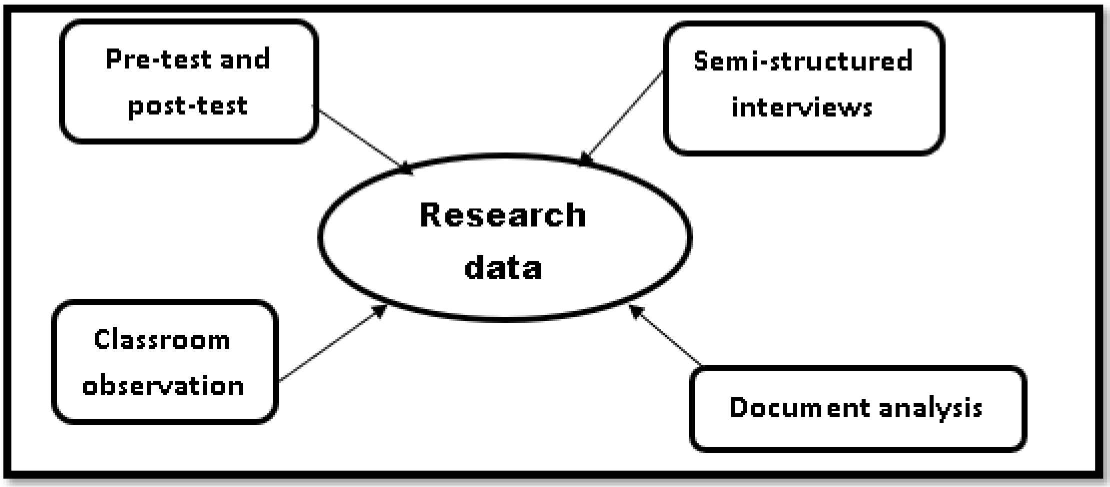


Figure 4. 2: Sources of data in the study.

4.8.1. Pre-test and post-test for learners

The pre-test and post-test were used to collect quantitative data on the performance of the learners in Euclidean geometry. In the two tests, the learners were required to apply the theorems on Euclidean geometry with the pre-test focusing on what learners learnt in Grade 11. During the pre-test and post-test, learners were subjected to the same conditions whereby invigilation was conducted to ensure integrity was achieved and maintained. The pre-test was written at the beginning of the study and the post-test was administered at the end of the study. The researcher administered the tests with the help of the respective teachers of both groups. The two groups wrote the pre-test the same day and same time in each school. This was done to avoid contamination which would affect the validity of the pre-test when the learners would intermingle before others wrote the pre-test. The pre-test aimed to:

i. understand learners’ prior knowledge on Euclidean geometry,
ii. capture information about learners’ problems they had in the topic, and
iii. capture learners’ individual Van Hieles’ levels of geometric thinking.

The objectives of the post-test were to:

i. explore how much learners understood the content on Euclidean geometry,
ii. capture information on what aspects learners did not understand that would need further attention in future lessons by teachers, and
iii. capture learners’ Van Hieles’ levels after the intervention period.

Both groups in each school wrote the post-test at the same time upon arrangement with the school authorities. Also, both schools wrote the post-test the same day.

4.8.2. Semi-structured interviews

According to Cohen and Cabtree (2006), a semi-structured interview is an inquiry method that uses pre-determined open questions which provoke discussion. The interviewer in his or her interaction with the interviewee has the freedom to explore themes and follow up on responses given. In addition, Tuckman (1999) explains that a semi-structured interview is designed with a planned set of questions that enable the interviewer to explore more on the topic under study by interacting with the participants. The study used semi-structured interviews with teachers to collect data. Various researchers (Makamure, 2016; Chiphambo, 2018; Mammali, 2015; Ijeh, 2012; Park & Chen, 2012) also made use of semi-structured interviews in similar educational research studies.

Two semi-structured interviews were conducted with each mathematics teacher involved in the study. The two interviews were namely the preliminary semi-structured interview at the beginning of the study and the final semi-structured interview at the end of the study. The interviews were conducted during the teachers’ free periods in the afternoon upon prior arrangements to avoid disruptions to school activities and lessons. The interviews were conducted in each of the teachers’ offices where no other people were present to ensure the teachers were free to give correct and honest information. The preliminary semi-structured interviews were conducted a day before the pre-tests were written by both groups while the final semi-structured interviews were conducted a day after post-tests were administered. The duration of each interview was one hour. According to Hossain (2011), semi-structured interviews are emergent hence the interview should not strictly adhere to questions on the interview schedule. During the interviews, the questions were adjusted, refined and reorganised in tandem with the nature and responses of each interviewee’s responses. The interviews enabled the researcher to obtain information from each participant through face-to-face interactions.

The preliminary semi-structured interview was conducted to:

i. capture demographic information of each teacher,
ii. obtain information about the teachers’ experiences in teaching geometry,
iii. obtain information about teachers’ awareness of problems faced in Euclidean geometry,
iv. obtain information about teachers’ experience on using cooperative and collaborative instructional strategies, and
v. obtain information about teachers’ awareness of *Ubuntu* principles that could be used in the teaching of geometry.

The final semi-structured interview was conducted to:

i. capture information about actual problems teachers faced in teaching Euclidean geometry,
ii. obtain information about teachers' views on the use of *Ubuntu* aspects of collective work and care in a Euclidean geometry classroom, and
iii. obtain information about teachers' awareness about how *Ubuntu* can be implemented in a classroom.

4.8.3. Document review

According to Bowen (2009), document review is a qualitative process that enables the researcher to interpret and analyse documents. At this stage, learners' workbooks, textbooks, teachers' diaries and lesson plans, as well as worksheets, were examined just after each lesson. During this exercise, teachers' portfolios were examined looking for information about how the teachers reflected on lessons taught taking into consideration learners' misconceptions and learning difficulties. The purpose of document review was to confirm the amount and quality of work covered in class and the depth of the content covered in tandem with the curriculum requirements. Document review also involved the quality of teachers' marking of learners' work. Furthermore, document review enabled triangulation with data collected from semi-structured interviews, classroom observation and pre-test and post-test. Field notes were taken down from those documents.

4.8.4. Classroom observation

According to Cohen, Manion and Morrison (2000), observations enable data to be collected on the spot as events happen. Although classroom observations are time-consuming, they allow the observer to obtain first-hand information as an eyewitness.

During the study, the researcher was a participant observer. Mcmillan and Schummacher (2010) highlight that participant observation helps the observer to get participants' views and perceptions of events by checking their actions, feelings, thoughts and beliefs. Through observation, data were gathered about how the mathematics teachers conducted their lessons when teaching Euclidean geometry. Classroom observations were conducted to help the researcher to obtain information on the behaviour of the participants, how the participants interacted with their environments and to examine the non-verbal expression of emotions and feelings as well as time consumed in activities. In addition, classroom observation enabled the researcher to have a deep understanding of the context in which Euclidean geometry was studied and how the use of *Ubuntu* impacted the teaching of the topic.

The researcher made field notes during each lesson to gather information on what occurred in the classrooms. Video recording of the lessons was conducted by the researcher to revisit them during the phase of data analysis. The videos recorded were revisited to cross-check and verify information on field notes. The aspects focused on during observation were:

i. teacher's level of preparedness for the lesson observed,
ii. classroom interactions focusing on *Ubuntu* aspects manifested during each lesson,
iii. teacher's attention to individual learners and groups of learners,
iv. teacher's awareness of learners' difficulties, and
v. teacher's skills to promote collective work in class.

Just before classroom observations were conducted, a pre-observation meeting with each teacher was done. The purpose of the meeting was to explain the modalities of the classroom observation procedures and expectations and also to create rapport with the mathematics teachers. During classroom observations, the control groups in each of the two schools were taught using traditional methods. For the experimental group in each school, the dynamic pairs model, an *Ubuntu*-based model was used during instruction. The researcher discussed with the two mathematics teachers how to implement the model to the experimental group in each school. The planning of all lessons of the experimental groups was done jointly with the researcher to ensure that all provisions of the dynamic pairs model were adhered to. The teachers were given the freedom to plan lessons not using the proposed dynamic pair model but using the traditional approach. Video-recorded classroom observations were conducted on both teachers' lesson presentations and field notes were taken for use in data analysis. Each teacher was observed six times, that is three times on the control group and three times on the experimental group.

The treatment to the experimental group consisted of the following interventions:

i. Learners sat in pairs during lessons. The pairs changed composition every day. This means that each day each learner sat with another learner whom he or she had never sat with before. The teacher encouraged learners to be honest to choose a partner as prescribed. For example, Learner 1 sat with Learner 2, Learner 3 and Learner 4 on Day 1, Day 2 and Day 3, respectively. The main aim was to ensure learners intermingled as much as possible and not according to friendships. The idea was to encourage sharing among learners but with different types of people each day. The researcher proposed this to remove barriers to learning which might be created by learners sharing with the same people every day. The researcher thoroughly trained the teacher of the experimental group on how to use the dynamic pair model. A clear explanation was given to the learners why this exercise was important. The learners were expected to share ideas about the given exercises. In this study, the researcher refers to these pairs, which changed composition every day as “dynamic pairs”. Sitting in dynamic pairs enabled each learner to share with as many other learners as possible.

ii. The teacher taught all theorems by using practical activities such as using models, artefacts, manipulatives and measurements among others in which each dynamic pair participated. Learners were encouraged to formulate the theorems after discoveries from practical activities. Teachers were encouraged to avail mathematical instruments for practical activities such as ropes and pegs, for example, to construct circles on the school grounds.

iii. Explanations by the teacher were always accompanied by real-life indigenous examples such as huts and many other indigenous geometric features indigenous to the Chris Hani West District.

iv. During problem-solving, the learners shared in pairs. They were also allowed to interact with other pairs.

v. The teacher incorporated collective work and sharing in teaching the experimental group. These attributes manifested when the teacher allowed gifted learners to interact with other learners to explain how to solve problems, organising extra tuition with learners and inclusivity. The teacher encouraged learners to share during study periods. Learners in the control group did not share during the lessons and sat individually.

vi. The teacher promoted teacher-learner and learner-learner sharing extending beyond the classroom in the context of sharing ideas on solving exercises in Euclidean geometry. Making learners continue to share after school demonstrates the teacher’s innovativeness.

The control group received instruction using the traditional method of talk and chalk whereby the teacher gave and explained theorems. According to Tambara (2015), the traditional method of instruction is characterised by having the teacher directing all learning with learners only assigned the duty of memorising facts, rules, algorithms, procedures and definitions. During the study, the teacher in teaching the control group did most of the work on the board for the learners. In this study, teachers provided the same worksheets to their learners. The only difference was the instructional strategies used to teach both groups.

4.9. Data analysis

Qualitative data collected from classroom observations and semi-structured interviews were categorised and coded for identification of themes and then analysed using thematic analysis. Data collected were transcribed from field notes, video-recorded lessons and interviews. Segments of information were identified to create categories. This involved coding raw data recognising important facts in the data sets. The interpretation of codes developed included comparing theme frequencies and checking theme concurrences and graphically establishing some relationships. The themes created were reviewed with reference to answering research questions to extract more meanings and patterns. The themes were drafted in relation to the use of Ubuntu values, SIT and the use of the Van Hiele'theory in classroom interactions. All the important facts from the themes were used to produce the final report.

The data collected through quantitative means were analysed using SPSS package. The data were analysed using independent samples t-test to make a comparison of the achievement between the experimental group and the control group. According to McMillan and Schumacher (2010), this test is conducted to determine if there is a statistically significant difference in the dependent variable between two different populations of subjects. Analysis of data collected from pre-test and post-test examined the statistically significant difference in achievement between a group taught using *Ubuntu* values and the one taught using traditional methods. The Levene's test of equality of variances was used for examine the homogeneity of variances of the test scores between the control group and experimental group for each participant. Descriptive statistical methods such as measures of central tendency, measures of dispersion, graphs and tables were also used. The independent samples t-test was also used to examine learners' test scores in the post-test with respect to Van Hiele's levels of geometric thinking. Afterwards, both quantitative data and qualitative data were integrated.

4.10. Ethical issues

An ethical clearance certificate was obtained from the University of South Africa before the study commenced. Permission from the Eastern Cape Department of Basic Education (refer to Annexure A) was then sought. The researcher went on to seek permission from principals of the schools where the two purposively selected teachers were based. The researcher also requested the consent of parents and learners to conduct the research. The learners who received consent letters took them to their guardians for them to sign even though they were not directly involved in the study. I held meetings with the participants to request them to sign consent forms. During the meetings, I discussed with the participants how classroom observation would be conducted and explained the expectations. An agreement was reached with each participant to conduct observation during times prescribed on normal school timetable and the period when learners would not be writing examinations. Voluntary participation of the mathematics teachers was requested with each one of them being guaranteed privacy, anonymity, and confidentiality. Pseudonyms for schools, teachers and learners were used to ensure that their identities remained anonymous.

4.11. Validity and reliability of the study

According to Mcmillan and Schumacher (2010, p.104), validity means "the degree to which scientific explanations of phenomena match reality". In simple terms, it means the truthfulness of findings and conclusions. Bloor and Wood (2006) refer to reliability as the degree to which results of a study do not change when data are collected and analysed by different researchers. To ensure validity was achieved, triangulation of data collected from the pre-test, post-test, semi-structured interviews, and classroom observation was conducted. The researcher engaged in member checking in which participants were requested to verify if data were correctly captured. Participants cross-checked if the transcribed data were correct and in tandem with the responses they provided. Some experienced lecturers and researchers from universities examined the interview schedules, pre-test and post-test and classroom observation protocol.

Preliminary semi-structured interviews were conducted on the same day to avoid contamination. This arrangement avoided participants to ideas on how to answer interview questions. During interviews, after each question, the interviewer summarised the responses given by the participants and verified with the participants as to whether the responses were captured correctly. To enhance reliability and validity of the semi-structured interviews, the questions asked followed the same sequence and had the same wording for each mathematics teacher. This exercise was carried out to ensure that any differences in the teachers' responses were differences between the two teachers themselves, not the results of the way the questions were articulated.

The effects of attrition were resolved by using the purposive sampling of participants. Purposive sampling allows the researcher to select willing and committed participants. Those selected teachers were ready to sacrifice their time to assist in the study. Purposive sampling also helped in dealing with the effects of maturation because teachers with commitment and willingness were selected. The pre-test and post-test were administered to both the control group and the experimental group on the same day in both schools selected and were professionally invigilated under examination conditions. The pre-test and post-test instruments were verified and corrected by two lecturers from universities who were experienced researchers.

Video recording of lessons was conducted to ensure reliability was achieved in lessons. This process assisted in cross-checking and verification at the stage of coding and allowed the researcher to conduct future references. Researcher bias was eliminated by reference to videos and making each participant check the correctness of field notes recorded by the researcher. All processes and methods of data analysis were vividly described and documented to ensure that other researchers could conduct an audit trail. Furthermore, the interview schedules did not contain leading questions.

4.12. Pilot study

According to Hassan, Schattner and Mazza (2006), a pilot study allows testing research protocols and instruments used in data collection. Creswell (2014) asserts that a pilot study is a feasibility study conducted in preparation of the main study to examine the workability of concepts and ideas behind the study. In addition, Hazzi and Maldaon (2015, p.52) emphasise that, “The overarching purpose of pilot studies is to determine feasibility of a main study prior to it being conducted”. The pilot study assisted in testing the appropriateness, validity and reliability of the research instruments, in testing if the administration of the research instruments was feasible, in improving the design of the research instruments and methodology for the administration of the main study and checking the timing for the administration of the instruments.

In this study, the pilot study was conducted with two mathematics teachers from two different schools who were not part of the population under study and were working in a different Circuit Management Centre [CMC] to Cacadu CMC of Chris Hani West District. The mathematics teachers who were used in the pilot study were from outside the Chris Hani West District. The reason for conducting the pilot study in a different CMC of the Chris Hani West District was to alleviate contamination. The mathematics teachers who were used in the pilot study were from outside the Chris Hani West District. The mathematics teachers used in the pilot study were allowed to air their views about the pilot study. They were requested to critique the questions used in semi-structured interviews as well as the pre-test and post-test. Each mathematics teacher involved in the pilot study provided a group of 20 learners to write the pre-test in Euclidean geometry. This process was done to enable the researcher to evaluate if the time allocation was appropriate.

4.13. Conclusion

This chapter describes the research paradigm, design and methodology in detail. The chapter describes the research site, population, sampling and data collection process. Furthermore, the chapter explains details on the validity, reliability and ethical issues. Data presentation and analysis from this study are focused on in the next chapter.

CHAPTER 5: PILOT STUDY

5.1. Introduction

The previous chapter presented the research methodology on data collection, data presentation and data analysis. This current chapter presents a report on the pilot study. This was a feasibility study before the commencement of the main study to explore the use of Unbuntu to enhance disruptive pedagogies in the effective teaching and learning of Grade 12 Euclidean geometry. In summary, this chapter covers a number of sections including research methodology of the pilot study, sample of the pilot study, summary of findings of the pilot study, summary of adjustments which were made after the pilot study, benefits from the pilot study, and challenges faced during the pilot study.

5.2. Research methodology of the pilot study

The pilot study was conducted within the pragmatic paradigm where mixed methods were used. Both quantitative and qualitative methods were used. Furthermore, the pilot study used the convergent parallel mixed methods design. The reason for using this design was to avoid prolonging the period of data collection because this study was done under COVID-19 conditions. The convergent parallel design allowed both quantitative and qualitative data to be collected simultaneously during the study hence reducing the effects of disruptions in schools' academic programmes as well as allowing health protocols related to COVID-19 to be adhered to effectively.

Data were collected using semi-structured interviews, classroom observation, document analysis, pre-test and post-test. The semi-structured interviews were the preliminary and final semi-structured interviews on each participating mathematics teacher involved in the pilot study. All interview questions were examined by experienced researchers and lecturers from universities. Subject and curriculum advisors were asked to assist in the evaluation of the classroom observation schedule because they used similar instruments to monitor teachers' progress. In addition, the pre-test and post-test were evaluated by experienced lecturers and researchers.

5.3. Sample of the pilot study

The pilot study was conducted on two mathematics from two different schools in Chris Hani West District which were purposively sampled. The two schools purposively sampled were from a Circuit Management Centre [CMC] different from Cacadu CMC where the main study took place. The reason for conducting the pilot study in a different CMC was to avoid contamination when the main study would be carried out. The distance between the two schools used in the pilot study was 40km. In addition, those two schools were in rural Chris Hani West District with geographical, socio-economic, organisational and cultural conditions resembling the conditions found in the two participating schools in the main study.

5.4. Summary of findings from the pilot study

During the pilot study, it emerged that the hard lockdown caused by the COVID-19 pandemic had forced learners to lose three months of classroom time. Initially, the researcher had planned to collect data for two weeks. After consultations with principals from the two different schools involved in the pilot study, the researcher had to adjust and plan for geometry content to be covered in one week. Instead of conducting the study on every aspect of Euclidean geometry, the content was adjusted to cover some aspects of circle geometry, namely, content on centre theorems, tangent theorems, chord theorems and cyclic quadrilaterals.

The Dynamic Pairs Model which was meant to make learners sit in pairs in close contact was adjusted to include social distancing protocols to avoid the risk of the spread of the COVID-19 virus. The researcher saw the need to make learners sitting in pairs to be two metres away from each other as well as using sanitisers whenever they would share any items in the classroom. On conducting the pilot study in the two selected schools, the researcher found out that COVID-19 conditions permitted groups of 20 learners or less in each classroom. The concern of having overcrowded classrooms was eliminated.

5.4.1. Findings from semi-structured interviews in the pilot study

During the semi-structured interviews with the two mathematics teachers, conditions were modified to include social distancing and rooms with enough ventilation. It also emerged that when asked about the challenges they faced in terms of availability of resources, the two mathematics teachers mentioned that learners did not have mathematical sets to use in geometric constructions. They both explained that learners were allocated these resources during the previous year but lost them in due course possibly because of them not seeing their importance as geometric constructions were not part of the content in the study of geometry.

When asked about the challenges faced in the teaching of geometry, Teacher X expressed,

"*Our learners have negative attitudes towards Euclidean geometry. Also, this content was not included in the previous syllabus and us teachers spent some years without teaching it".*

On the same question Teacher Y responded, *"Some of us teachers have problems in handling the content in Euclidean geometry hence sometimes we avoid teaching it thoroughly. Most of the time this content is taught superficially without delving deeper to the levels prescribed by the curriculum documents."*

I have used pseudonyms X and Y for the participants in the pilot study. Some of the conversations held with them after the lessons were observed are presented in the results below:

Results of the interview with Teacher X
Researcher: In the teaching of Euclidean geometry have you ever used collective learning strategies such as group work and teamwork among others? Teacher X: *I have practiced this very little. Normally I do classroom involving all the learners. In very few occasions, I do group work. I always avoid group work because it consumes a lot of time whereas the annual teaching plan [ATP] allocates time that is not enough to accommodate time-consuming teaching methods.* Researcher: Have you ever used principles of *Ubuntu* in your mathematics classroom before? If so, which ones do you think have been dominant? Teacher X: *Yes, I think I have done it though not confident to say I did it best. I have encouraged learners to share resources and knowledge. Only that the time allocated for each lesson is very little to allow such practices.* Researcher: What attributes of *Ubuntu* do you think can be used effectively in a classroom? Do you think these principles can enhance the teaching of Euclidean geometry? Teacher X: *I think I can allow learners to sit in groups sharing when doing classwork. Since Euclidean geometry is difficult for the majority of learners, they will be able to assist one another in solving problems.*

The following is the discussion with Teacher Y

Results of the interview with Teacher Y
Researcher: In the teaching of Euclidean geometry have you ever used collective learning strategies such as group work and teamwork among others? Teacher Y*: I tried to use group work, but it was very difficult to use because my classes are too large and always overcrowded. It was difficult to manage the process.* Researcher: Have you ever used principles of *Ubuntu* in your mathematics classroom before? If so, which ones do you think have been dominant? Teacher Y: *I have encouraged learners to share knowledge in study periods and weekends when they study at home. I always encourage gifted learners to assist the learners who have challenges.*

Researcher: What attributes of *Ubuntu* do you think can be used effectively in a classroom? Do you think these principles can enhance the teaching of Euclidean geometry?
Teacher Y: *I rate sharing and love as paramount attributes. When love is shown to anyone, self-confidence, self-worth and motivation are boosted.*

Both participating mathematics teachers displayed comfort and confidence in answering interview questions asked. In addition, they showed that they understood the questions very well. No adjustments were made to the interview questions.

5.4.2. Summary of findings from classroom observations in the pilot study

Initially, the researcher had planned to collect data in two weeks but due to COVID-19 and requests from participating schools' principals and the Chris Hani District' curriculum officials, the period was reduced to one week. The experimental groups were taught using geometrical constructions. Also, during the pilot study, it emerged that learners in schools were not able to use mathematical instruments in geometric constructions. Furthermore, the researcher found that the learners did not have mathematical instruments to use in the classroom because they had lost them in Grade 11 due to non-usage of them. The researcher provided each learner involved in the pilot study with a mathematical set to enable the study to go on.

5.4.2.1. Summary of findings from classroom observation on Teacher X

The control group was instructed using the traditional method in which the lecture method was predominant. The experimental group was instructed using the *Ubuntu*-based model whereby learners worked in pairs. Teacher X used the Dynamic Pairs model which incorporated some practical activities. The learners from the experimental group were taught using mathematical sets and physical manipulatives in the practical activities. In their pairs, the learners constructed diagrams and made cutting according to specifications to determine relationships between angles. The learners in the experimental group encountered difficulties in using mathematical instruments in making geometrical constructions. This finding confirmed what Teacher X had said during the preliminary interviews that the learners did not have a background in geometrical constructions. The activity of conducting geometrical constructions was time-consuming and few activities were done owing to learners' non-familiarity with the activities and also their inability to use mathematical instruments. The researcher found that Teacher X was not comfortable and well-versed in teaching geometry using geometric constructions due to them not having taught the geometric constructions before.

Learners from the control group received instruction using the traditional method dominated by the lecture method and individual work for learners. Learners were not free to share knowledge and information. Gifted learners in the control group were the ones who dominated participation in classroom discussions. The less gifted were restricted because there was no room for sharing in small groups. On the contrary, the learners in the experimental group discussed in pairs sharing knowledge and ideas. The learners from the experimental group participated more than those from the control group. Perhaps it was because of them being offered the chance to share knowledge and ideas. From both groups, all learners asked for extra tuition and individual learners would be seen calling the teacher to make the request. This was an indication of learners' positive attitude towards the topic contrary to Teacher X's assertion that learners had negative attitudes towards geometry.

5.4.2.2.Summary of findings from classroom observation on Teacher Y

The experimental group was instructed using the dynamic pairs model which embraced the *Ubuntu* attributes as well as some practical activities in which learners used manipulatives to determine relationships of angles. On the contrary, Teacher Y taught the control group using the lecture method in which learners engaged in individual work while the teacher was the source of information. Occasionally Teacher Y would engage the control group in classroom discussions which were dominated by gifted learners who occasionally would solve exercises on the chalkboard. The majority of learners from the experimental group were unable to use mathematical instruments in constructing diagrams. Also, the learners did not possess the instruments because they lost them during the previous grades. The researcher had to provide each learner with an instrument for geometrical construction. Teacher Y confessed that he had never taught geometrical constructions before and would need a workshop for capacitation on that aspect. The exercise of geometrical construction consumed a lot of time since learners were encountering this content for the first time.

During the lessons, learners from the experimental group shared knowledge in pairs and solicited assistance from other pairs which had more knowledge, or which had different approaches. A striking finding was that learners from both groups displayed zeal and enthusiasm in geometry which was contrary to Teacher Y's claims in the preliminary interview that learners had negative attitudes towards the topic. In the control group, learners who faced difficulties would always raise their hands and would try to write in their workbooks. The teacher displayed high command of content knowledge in teaching Euclidean geometry. During instruction to both groups, Teacher Y explained the theorems with exactitude giving different scenarios to achieve conceptual understanding among learners. All the learners attended all the lessons during the course the study.

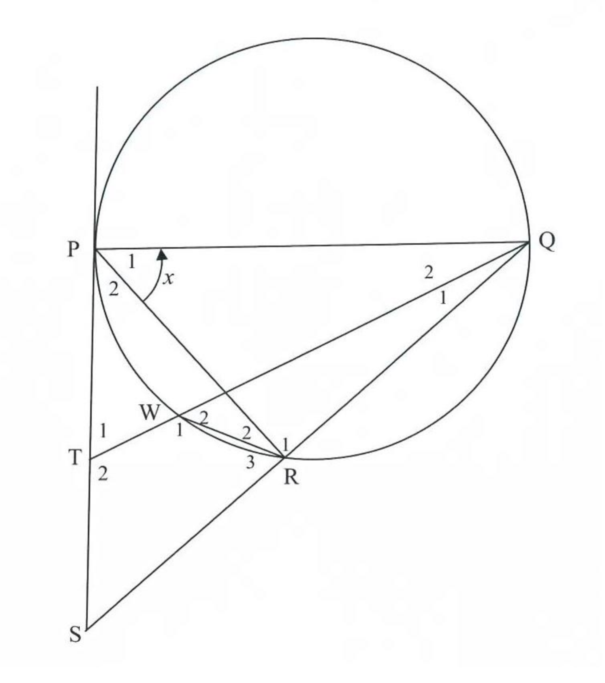
5.5. Adjustments made after findings from the pilot study

Some adjustments and modifications were done on the pre-test. When the pre-test was administered, it was timed to last for 30 minutes. However, less than half of the learners failed to complete the pre-test within the prescribed time of 30 minutes. All the learners completed writing the pre-test in 40 minutes. Therefore, the researcher adjusted the duration of the pre-test in the main study to 40 minutes. Also, the content of the pre-test was not changed.

Some adjustments were also done on the post-test after the duration of the pilot study was changed upon requests from principals of the participating schools. Question 2 of the post-test was originally including similarity as follows:

Original Question 2 in post-test

In the figure below, PQ is the diameter of the circle, PWRQ. SP is a tangent to the circle at P. Let $\hat{P}_1$=X



1.1 Why is $P\hat{R}Q=90^{\circ}$ [1]

1.2 Prove that $\hat{P}_1=S$ [3]

1.3 Prove that SRWT is a cyclic quadrilateral [3]

1.4 Prove that $\Delta QWR /// \Delta QST$ [3]

1.5 If QW=5cm, TW=3cm, QR=4cm and WR=2cm, calculate the length of

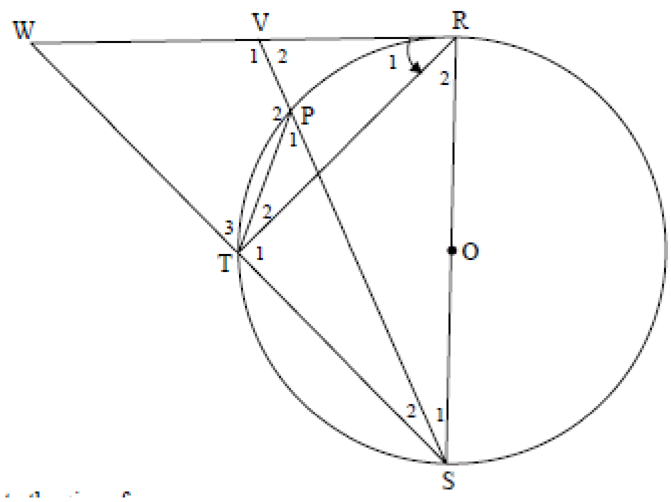
1.5.1 TS [2]

1.5.2 SR [2]

This question needed the application of theorems on similarity on the last two sub-questions. Content on similarity in the main study was removed due to advice from principals and Chris Hani Department of Education on time constraints. Instead, the new question presented below was formulated to replace it.

The New Question 2 in post-test after adjustments

In the diagram below RS is the diameter of the circle with centre O. Chord ST is produced to W. Chord SP produced meets tangent RW at V. $\hat{R}_1=50^{\circ}$



1.1 Calculate, giving reasons, the size of

a) $W\hat{R}S$ (2)

b) $\hat{T}1$ (2)

c) $\hat{W}$ (2)

d) $\hat{P}1$ (3)

2.2 Prove that WVPT is a cyclic quadrilateral (3)

Adjustments to the post-test were done to remove content on similarity which could not be done in one week of teaching Euclidean geometry. In addition, in one week, it was impossible to embark on content on complex geometric proofs.

Adjustments and modifications were also made to classroom observations. The curriculum in geometry from previous grades did not delve deeper into geometric constructions which meant learners involved in the pilot study had no background in geometric constructions. Since the learners from both schools struggled to do geometrical constructions using mathematical sets, the idea of using already made models in the main study was conceived. Owing to the request by the two principals of the two participating schools in the study and Department of Education officials to reduce the duration of the pilot study to one week, the duration of the main study was also changed to one week. The content to be covered in the main study was restricted to only circle geometry, namely, centre theorems, tangent theorems, chord theorems and cyclic quadrilaterals. In the main study, each participating teacher would be observed three times teaching the experimental group as well as another three times teaching the control group. There were no adjustments made on the preliminary and final semi-structured interviews. The participating teachers involved in the pilot study were comfortable in answering the interview questions.

5.6. Some benefits of the pilot study

The following benefits were obtained during the pilot study:

i. The pilot study assisted the researcher to familiarise with prevalent conditions in schools in the Chris Hani West District and gain some insight into what to expect in participating schools involved in the main study.
ii. The researcher gained some insight into the duration of the main study, the times to schedule semi-structured interviews, classroom observation and pre-tests and post-test.
iii. The pilot study enabled the researcher on what elements to modify, adjust or remove from the research instruments.
iv. The preliminary data collection and analysis enabled the researcher to gain some experience necessary and useful for the main study.

5.7. Some of the challenges faced during the pilot study

The pilot study was conducted after a three-month hard COVID-19 lockdown which caused the loss of much classroom time for learners throughout South Africa. There was a need to adjust the duration of the pilot study to one week to avoid further disruption in schools as requested by respective principals. The participating mathematics teachers were uneasy during interviews and classroom observations because they were not exposed to such activities. The researcher had to re-assure the teachers that the reports from the interviews and classroom observations were confidential and anonymity was guaranteed as agreed before. This gave the researcher some green light to orient the participating teachers in the main study. In addition, the main study was adjusted to one week because of findings from the pilot study.

5.8 Conclusion

This chapter examined the pilot study conducted on the use of Ubuntu values in the teaching of Euclidean geometry. It was conducted with two purposively sampled mathematics teachers from two secondary schools in predominantly rural Chris Hani West District of Eastern Cape Province, South Africa. Furthermore, the chapter the discussed findings findings, strengths, challenges and adjustments made relating to conducting a pilot study. The next chapter presents data presentation and analysis of findings.

CHAPTER 6: DATA PRESENTATION AND ANALYSIS

6.1 Introduction

The previous chapter presented the proceedings of the pilot study. This chapter presents presentation and analysis of data collected from two participants who taught lessons in Euclidean geometry. The main aim of the study was to explore how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry. The data collection techniques used in the study were pre-test and post-test, classroom observation, semi-structured interviews and document analysis. In addition, the findings on quantitative data took into account Van Hiele's levels of geometric thinking. This process was conducted to answer the following main research question:

How can *Ubuntu* values enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry?

In order to respond to the main research question, the following sub-questions were asked:

i. How can the teachers' PCK be enhanced for effective and meaningful teaching of Grade 12 Euclidean geometry?
ii. What difficulties are experienced by mathematics teachers in the teaching of Grade 12 Euclidean geometry?
iii. What aspects of *Ubuntu* philosophy should teachers apply for the enhancement of the teaching of Euclidean geometry in Grade 12?
iv. To what extent do collective work and collaboration impact the teaching of Grade 12 Euclidean geometry?

v. How can *Ubuntu* philosophy be used to transform the teaching of Grade 12 Euclidean geometry?

6.2 Demographic profiles of the two mathematics teachers

The two participants in the study, Teacher A [TA] and Teacher B [TB] had experience in teaching in rural settings with under-resourced schools with learners coming from poor socio-economic backgrounds. Table 5.1 shows the demographic profiles of the two mathematics teachers who were involved in the study.

Table 6. 1: Demographic profiles of the two mathematics teachers

Aspect	Teacher A (TA)	Teacher B (TB)
1. Educational qualifications	• Diploma in Education (specialising in Mathematics and Physical Sciences) • Bachelor of Science in Mathematics and Physics	• Bachelor of Education (Further Education and Training) in Mathematics Education • Bachelor of Education (Honours) in Mathematics Education
2. Gender	Male	Male
3. Number of years of teaching Grade 12 mathematics	12	5
4. Subjects trained to teach	Mathematics and Physical Science	Mathematics
5. Number of workshops attended in Euclidean geometry	8	5
6. Other grades taught	Grades 10 and 11	Grades 10 and 11
7. Location of school	Rural	Rural
8. Name of school	School A	School B

The distance between School A and School B was 70km. The information exhibited in Table 6.1 above indicates that the two mathematics teachers in the study had a strong mathematical background and mathematical content knowledge because of their qualifications in mathematics education. Because of teaching at two different secondary schools, the two participants had different exposure to two different organisational cultures, school management systems, socio-economic climate, mathematics instructional strategies and involvement in professional learning communities. During the preliminary interviews conducted, both teachers disclosed that they were trained to teach the CAPS curriculum, and both had experience in attending teacher development workshops in Euclidean geometry.

6.3. Findings from the pre-test in Euclidean geometry to learners

In summary, the pre-test examined the concepts on understanding and application of theorems. The pre-test was administered during the same day as the preliminary interviews. The pre-test, presented in Annexure O, served as a diagnostic test as well as the instrument for the selection of the experimental and control groups.

6.3.1 Findings on pre-test on Teacher A’s learners from School A

The marking of the pre-test was conducted immediately after its administration to ensure that the selection of experimental and control groups was conducted before classroom observation. After marking the pre-test, some challenges were diagnosed.

In the pre-test, TA’s learners displayed the following challenges:

i. 50% of the learners had forgotten the theorem about the relationship between the angle subtended at the centre by an arc or chord with the angle subtended by the same arc or chord at the circumference of a circle.

ii. The theorem of the relationship between the tangent and radius was a problem for all the learners.

iii. 75% of the learners failed to state correct acceptable reasons in steps displayed in their calculations.

iv. 25% of the learners had forgotten the relationships of angles between parallel lines.

v. None of the learners could solve the problem on the cyclic quadrilateral. Furthermore, the learners could not apply the converse theorems of a cyclic quadrilateral.

vi. 67% of the learners found it difficult to identify the size of the angle subtended by the diameter.

In School A, Group A had 18 learners and Group B also had 18 same learners. The total of marks of the pre-test was 27 and the time allocated was 40 minutes. Table 6.2 displays the marks obtained by TA's learners in the pre-test in percentages.

Table 6. 2: Test scores obtained by TA's learners in percentages

Group A (Marks in %)	Group B (Marks in %)
4	15
0	0
4	0
0	4
7	0
11	7
22	7
7	26
11	4
4	15
4	4
19	4
4	4
11	19

4	4
0	11
15	7
33	30
Mean test score=8.89	Mean test score=8.94

An independent samples t-test was conducted to compare the two groups and present descriptive statistics. The following tables were presented from the test.

Table 6. 3: Group statistics of the TA's learners in the pre-test

	Group	Number of learners who wrote the pre-test	Mean	Std. Deviation	Std. Error Mean
Mark	Group A	18	8.89	8.724	2.056
	Group B	18	8.94	8.781	2.070

Table 6. 4: Independent samples t-test for school A pre-test marks

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Mark	Equal variances assumed	.026	.873	-.019	34	.985	-.056	2.918	-5.985	5.874
	Equal variances not assumed			-.019	33.999	.985	-.056	2.918	-5.985	5.874

The homogeneity of variances between the test scores of the two groups was tested using Levene's test of equality of variances.

H_0: There is no significant difference between the variances of the test scores obtained in the pre-test by Group A and Group B

$H_{1:}$ There is significant difference between the variances of the test scores obtained in the pre-test by Group A and Group B.

The p-value was 0.873 a value which was greater than 0.05. Which meant that the null hypothesis was accepted. Therefore, there was homogeneity of variances between the test scores of TA's two groups. This implied that the two groups did not have significant difference in academic performance in Euclidean geometry.

Also, Group A had a mean of 8.89 and a standard deviation of 8.724, while group B obtained a mean of 8.94 and a standard deviation of 8.781. To examine the test score difference between the two groups, the independent samples t-test was conducted. The following null hypothesis was tested at a 95% confidence interval: The null hypothesis was designated H_0 and the alternative hypothesis was designated $H_{1.}$

H_0: There is no significant difference in test scores between Group A and Group B of School A.

H_1: There is a significant difference in test scores between Group A and Group B of School A.

The p-value obtained from the t-test was 0.985 which was greater than 0.05 less than the significance level which was used in the t-test. This means, there was no statistically significant difference between the test scores of Group A and Group B of School A. This implies that any of the two groups could be made either the experimental or the control group. Since there was no statistical difference in test scores among the learners between Group A and Group B, the researcher chose any of the two groups to be the experimental group and the other automatically became the control group. In the study, Group A was selected as the experimental group and Group B was selected as the control group. Therefore, in the subsequent discussions on School A for Teacher A in the study, Group A was referred to as the Experimental group and Group B was referred to as the Control group.

6.3.2. Findings on pre-test on Teacher B's learners from School B

The marking of the pre-test was conducted immediately after its administration to ensure that the selection of experimental and control groups was conducted before classroom observation. After marking the pre-test of the learners from School B, some challenges were observed.

Teacher B's learners displayed the following difficulties:

i. The theorem of the relationship between the tangent and radius was a problem for all the learners.
ii. 50% of the learners failed to state correct acceptable reasons.
iii. 53% of the learners had forgotten the relationships of angles between parallel lines.
iv. None of the learners could solve the problem on the cyclic quadrilateral. Furthermore, the learners could not apply the converse theorems of a cyclic quadrilateral.
vii. 82.3% of the learners found it difficult to identify the size of the angle subtended by the diameter.

In School B, the total number of learners in Group A was 18 and Group B had 16 learners. The total of marks of the pre-test was 27 and the time allocated was 40 minutes. Table 6.5 displays the marks obtained by the learners in the pre-test in percentages.

Table 6. 5: Test scores obtained by learners in School B in percentages

Group A (Control) Marks in %	**Group B Marks in %**
52	41
44	48
44	41
70	33
70	52
11	15
11	15
15	22
30	48
26	44
4	4
81	19
15	19
30	11
15	70
44	26
15	
15	
Mean test score=32.89	Mean test score=31.75

Table 6. 6: Group statistics of test scores of TB's learners in the pre-test

	Group	Number of learners who wrote the pre-test	Mean	Std. Deviation	Std. Error Mean
Mark	Group A	18	32.89	23.341	5.502
	Group B	16	31.75	18.146	4.536

Table 6. 7: Independent samples t-test for school B pre-test marks

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Mark	Equal variances assumed	1.222	.277	.157	32	.876	1.139	7.238	-13.605	15.882
	Equal variances not assumed			.160	31.482	.874	1.139	7.131	-13.395	15.673

The homogeneity of variances between the test scores of the two groups was tested using Levene's test of equality of variances.

H_0: There is no significant difference between the variances of the test scores obtained in the pre-test by Group A and Group B

H_1: There is significant difference between the variances of the test scores obtained in the pre-test by Group A and Group B.

The p-value was 0.277 a value which was greater than 0.05. Which meant that the null hypothesis was accepted. Therefore, there was homogeneity of variances between the test scores of TB's two groups. This implied that the two groups did not have significant difference in academic performance in Euclidean geometry.

Also, Group A had a mean of 32.89 and a standard deviation of 23.341, while group B obtained a mean of 31.75 and a standard deviation of 18.146. To examine the test score difference between the two groups, the independent samples t-test was conducted. The following null hypothesis was tested at a 95% confidence interval: The null hypothesis was designated H_0 and the alternative hypothesis was designated H_1.

H_0: There is no significant difference in test scores between Group A and Group B of School B.

H_1: There is a significant difference in test scores between Group A and Group B of School B.

The p-value obtained in the t-test was 0.876 which was greater than 0.05 less than the significance level which was used in the t-test. This means, there was no statistically significant difference in test scores between Group A and Group B of School B. This implies that any of the two groups could be made either the experimental or the control group. Since there was no statistical difference in test scores among the learners between Group A and Group B, the researcher chose any of the two groups to be the experimental group and the other automatically became the control group. In the study, Group A was selected as the control group and Group B was selected as the experimental group. Therefore, in the subsequent discussions on School A for Teacher A in the study, Group A was referred to as the Control group and Group B was referred to as the Experimental group.

6.4. Findings from the preliminary semi-structured interviews

The preliminary semi-structured interviews were conducted the same day when the pre-test was administered when each of the two mathematics teachers was free. The interview schedule is in Annexure K.

6.4.1 Findings from the preliminary semi-structured interview with Teacher A

Teacher A was a holder of a Diploma in Education and Bachelor of Science degree with mathematics and physics as subjects of specialisation. In addition, TA expressed that he received training as a mathematics teacher and had been teaching the subject for 12 years. During the period when CAPS was being introduced, the teacher received training and attended workshops in teaching the topic Euclidean geometry. When asked about challenges encountered in the teaching of geometry, TA summarised the problems encountered in Euclidean geometry by saying,

"Learners lack the understanding of theorems and how to use them to solve problems, lack the ability to express theorems in words and stating acceptable reasons when solving geometry problems, they have a negative attitude towards the topic hence their passiveness in geometry lessons, they do not have problem-solving skills hence this impacted their performance in Euclidean geometry, they lack spatial knowledge and lack skills on visualisation, analysis and deductive reasoning in the solution of problems and lack geometry language proficiency as well as proficiency in English ".

TA was confident with the content knowledge he had in Euclidean geometry. However, he expressed,

"*I need more workshops on how to teach the topic effectively. Attendance to workshops sharpens the teachers pedagogical and didactic skills because a teacher is updated on how to teach a given section of content effectively using modern instructional strategies"*.

Furthermore, the teacher revealed that he always had a thorough study on the topic and dedicated time to engage in research in the topic consulting various sources such as textbooks, examination guidelines, diagnostic reports and journal articles authored by experts.

Teacher A when asked if he had ever used collective learning strategies such as group work and teamwork in teaching Euclidean geometry, the teacher expressed that group work and teamwork and other collaborative and cooperative learning strategies were time-consuming although he believed they were effective. Table 6.8 further displays the discussion held between TA and the researcher [R]:

Table 6. 8: Summary of preliminary semi-structured interview of TA

Interview questions and responses with TA	**Initial codes**
R: In the teaching of Euclidean geometry, have you ever used collective learning strategies such as group work and teamwork among others?	
TA: Yes, I have used group work before but not in normal daily lessons but revision classes towards examinations. Group work for me is time-consuming. We have limited time to accomplish work on the annual teaching plan [ATP].	-Used group work -Group work time-consuming. -Limited time to accomplish work
R: Have you ever used the principles of *Ubuntu* in your mathematics classroom before? If so, which ones do you think have been dominant?	
TA: Yes, I commit myself to teaching learners extra lessons. These learners come from poor families. My heart pricks when they fail because they go back to their families with empty hands. I want them to pass and go to universities and study to enhance their opportunities to break the cycle of poverty. I have encouraged learners in my classes to share whatever useful material they get.	-Commitment to doing extra tuition -Learners come from poor families -My heart pricks -Break the cycle of poverty -Encourage my learners to share
R: What attributes of *Ubuntu* do you think can be used effectively in a classroom? Do you think these principles can enhance the teaching of Euclidean geometry?	
TA: Sharing, love and patience are very crucial. When learners are shown love and sympathy they are motivated to work hard because they see that people regard them as important. I think learners who have learning difficulties, through sharing with others, will overcome their challenges. Euclidean geometry is complicated for learners in general and it needs cooperative learning strategies to enhance understanding such as learners working in groups during study time.	-Sharing, love and patience very crucial -Love and sympathy -Learners are motivated to work hard -Learning difficulties -Euclidean geometry is complicated -Cooperative learning -Working in groups

The teacher demonstrated knowledge of *Ubuntu* principles by mentioning that *Ubuntu* involves teachers and learners sharing resources, knowledge and ideas as well as promoting the success of every learner for community development to be realised. In addition, Teacher A expressed the belief that the application of *Ubuntu* principles would be effective in the teaching of geometry because gifted learners and learners with difficulties would have the opportunity to cross-pollinate ideas hence improving the overall performance in the topic.

6.4.2 Findings from the preliminary semi-structured interview with Teacher B

Teacher B was free to produce copies of qualifications and expressed willingness to study for a Masters' degree in mathematics education. The teacher was a holder of Bachelor of Education and Bachelor of Education (Honours) degrees both with specilisation in mathematics teaching. In addition, TB confirmed that he attended five CAPS training workshops in teaching Euclidean geometry.

TB summarised the problems encountered in Euclidean geometry by saying: *"Learners have problems with visualisation and analysis of geometric figures, lack problem-solving skills in geometry, do not have the ability to express theorems in words and stating acceptable reasons when solving geometry problems and they negative attitude towards the topic hence their passiveness in geometry lessons."*

Teacher B expressed that he did not have challenges in Euclidean geometry which he indicated was his favourite topic. The teacher went on to reveal that he is always selected to teach in mathematics camps by the District Department of Education because of his deep content knowledge in the topic. In addition, the teacher yearned to be exposed to more content and pedagogical knowledge by attending more seminars and workshops.

During the discussion, the teacher showed that he engaged in a thorough study of the topic in preparation for lessons. TB expressed,

"*I hate mediocrity when it comes to performance. I need to prepare thoroughly to ensure that I deliver the best for the learners."*

The teacher also revealed that he engages in consultation of all policy documents to ensure that the best is imparted to the learners. Additional information from the teacher showed that Teacher B always organised extra tuition classes during weekends to assist learners who struggled in mathematics. Table 6.9 further displays the discussion held between TB and the researcher [R]

Table 6. 9: Summary of preliminary semi-structured interview of TB

Interview questions and responses with TB	Initial codes
R: In the teaching of Euclidean geometry, have you ever used collective learning strategies such as group work and teamwork among others? *TB: Yes, I have used group work in teaching Euclidean geometry. The only challenge I had was that the classrooms were overcrowded with too many groups to manage. Also, group work is time-consuming.*	-Used group work -Overcrowded classrooms -Group work is time-consuming
R: Have you ever used the principles of *Ubuntu* in your mathematics classroom before? If so, which ones do you think have been dominant? *TB: I think I used Ubuntu principles before, but I doubt whether I did it perfectly. What I know is I empathise with learners so much. In this community these learners do not have any community members available who can assist them in mathematics except myself. My failure to assist them is like a death sentence to the whole community.*	-Used *Ubuntu* principles before -Empathise with learners -No community members to assist them -Failure to assist like a death sentence to the community
R: What attributes of *Ubuntu* do you think can be used effectively in a classroom? Do you think these principles can enhance the teaching of Euclidean geometry? *TB: The attributes I know are love for learners, commitment, patience, sharing and empathy. I think these principles will enhance learners' confidence and motivation. When learners see someone feeling for them their self-esteem increases thereby motivating them to put more effort into their studies*	-Love for learners -Commitment -patience -Sharing -Empathy -Learners see someone feeling for them -Their self-esteem increases

	-motivating them to put more effort

6.5 Findings from classroom observation

Each teacher taught the control group immediately after teaching the experimental group. The control group was instructed using the traditional method characterised by limited classroom interactions whereby the teacher was the centre of interactions, whereas the experimental group was instructed using the dynamic pairs model which allowed unlimited interaction patterns.

6.5.1 Classroom observation on Teacher A

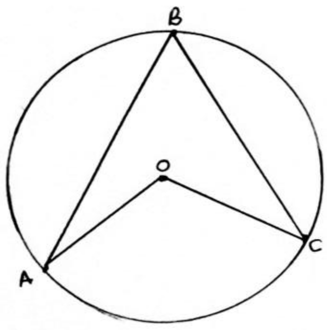
The tables below summarise what transpired in the classrooms. Those tables captured all the proceedings in Teacher A's classroom during instruction. They are a detailed version of the classroom observation schedule (Annexure L).

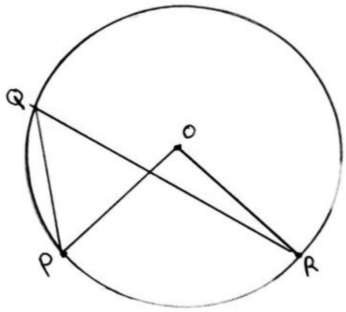
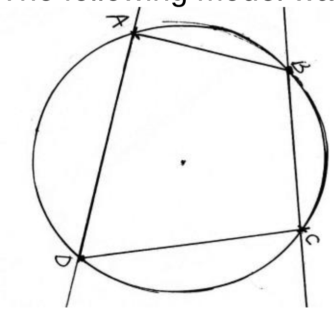
6.5.1.1 Tables summarising classroom observation on Teacher A [TA]

Table 6.10 summarises the proceedings of the first lesson by TA with the experimental group.

Table 6. 10: Lesson 1 of the experimental group for TA

Description of lesson proceedings	**Initial codes**
Topic: Centre theorems, Cyclic quadrilaterals **<u>Introduction </u>(7minutes)** • A verbal revision of theorems was done. • TA used oral probing questions to check on learners' prior knowledge to learners seated in pairs. • Learners responded in chorus answers. • Learners could recite the theorem on the relationship between angle subtended by a chord or arc at the centre and the one subtended by the same chord or arc at the circumference.	-verbal revision of theorems -used oral probing questions –check on learners' prior knowledge -learners seated in pairs -chorus answers -could recite the theorems they could remember -relationship between angle subtended by arc or chord

- Examples were done on the board. - Learners also could recite the theorem on the relationship of opposite angles in a cyclic quadrilateral. - The learners forgot the relationship between interior angle and opposite exterior angle. - TA made use of vernacular language in explanations to make learners understand.	-at the centre -at the circumference -examples were done on the board -recite the theorem -opposite angles in a cyclic quadrilateral -learners forgot the relationship between interior angle and opposite exterior angle -use of vernacular language in explanations
<u>Lesson activities</u> - TA allowed learners to use vernacular language in discussions. - TA explained to learners that they were going to do a practical activity to prove the theorems they had studied before. - Learners were made to sit in pairs observing social distancing protocols. - TA provided already made models of diagrams based on the two theorems revised in the introduction. - The learners in pairs were requested to trace angle ABC on a separate white bond paper Using a pair of scissors, they cut an angle equal to $A\hat{B}C$. The next step was placing the cut angle on $A\hat{O}C$. TA: *How many times does ABC go into AOC*? Learners: *Twice*  -TA emphasised the use of the **skill of visualisation** to establish relationships between angles -Another scenario of the theorem was demonstrated practically. TA: Do the same exercise on the following model	-use of vernacular language in discussions -explained to learners –practical activity -prove theorems -learners made to sit in pairs -observing social distancing -provided already made diagrams -learners in pairs -use of the skill of visualisation to establish relationships

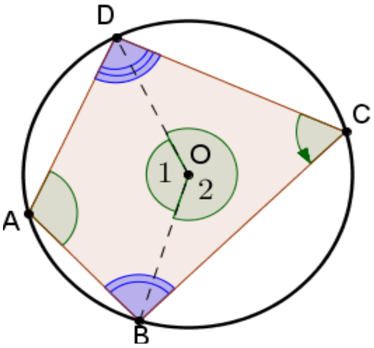
 • Learners in pairs did the cutting and tracing. • TA assisted some pairs which were slower to understand the exercise. Some pairs assisted the neighbouring pairs on how to do the practical activity. • A certain learner exclaimed in isiXhosa after the discovery: *"Titshala, kutheni ungasifundisanga ngoluhlobo kwi-ibanga 11?" meaning "Teacher why didn't you teach us like this in Grade 11?"* • The learner demonstrated approval of the **use of the models in the practical activity** in class. • All pairs now could do the activity and recite the theorem. • All learners participated in class by doing practical activities. **Activity 2: Cyclic quadrilateral** The following model was given to each pair:  Learners were asked to identify opposite angles • TA requested learners to write down opposite angles • In learners in pairs wrote $\hat{A}$ and $\hat{C}$ $\hat{B}$ and $\hat{D}$ TA: *In your pairs trace the quadrilateral ABCD and cut the traced model like what you did in the previous activity.* *Draw a straight line and place opposite angles. What do you observe?*	-learners in pairs -cutting and tracing -assisted some pairs -some pairs assisted neighbouring pairs -practical activity -A certain learner exclaimed in isiXhosa -Approval of the use of models -Pairs could do the activity -All learners participated -In pairs learners wrote -did the work in pairs -Interacted sharing ideas -Pairs with difficulties -Promoted inter-pair interactions -Opposite angles

• Learners did the work in pairs. Neighbouring pairs interacted sharing ideas. TA assisted the pairs with difficulties and promoted inter-pair interactions. • Learners observed that the opposite angles formed an angle in a straight line. • Learners used the skill of visualisation to identify the relationship between angles • The teacher explained the reason why opposite angles were supplementary. • Learners were asked to place A on the exterior angle at C and B on the exterior angle at D. • Learners' participation was overwhelming. Even learners who were habitually passive in other topics were very active in cutting models. Sharing in dynamic pairs assisted some learners who needed assistance. Learners understood the theorem: **An interior angle of a cyclic quadrilateral is equal to the opposite exterior angle of the same quadrilateral.** • Other centre theorems were just revised theoretically because of time constraints • The practical activity was time-consuming • Converse theorems were explained theoretically TA gave the learners the converses of the two theorems which are important to demonstrate that a quadrilateral is cyclic. Throughout the lesson, the teacher was composed and demonstrated patience with learners who were slow in understanding. TA dedicated himself to repeating explanations to learners. Facial expression and body language confirmed that TA enjoyed helping the learners. **<u>Written work</u>** The teacher provided a hand-out with theorems and acceptable reasons. Learners were given an activity to do in pairs. The activity was based on calculating sizes of angles using theorems studied. The time was very limited. Learners could not finish the written activity in class and were requested to do finish in their pairs during one-hour study time from 14h00-15h00.	-Skill of visualisation -Learners' participation was overwhelming -Habitually passive were very active -Cutting models -Sharing in dynamic pairs -Time constraints -Practical activity was time-consuming -Teacher demonstrated patience. -Facial expression and body language showed the teacher enjoyed helping learners. -Acceptable reasons - do in pairs -Time was very limited -Learners shared in pairs at study time

Table 6.11 summarises Lesson 1 by TA with the control group.

Table 6. 11: Lesson 1 of control group for TA

Description of lesson proceedings	**Initial codes**
Topic: Centre theorems and cyclic quadrilaterals **<u>Introduction</u> (10 minutes)** • TA asked oral probing questions about the centre circle theorems the learners remembered. • TA used prior knowledge of learners to introduce the lesson. • Chorus answers were presented verbally by the learners. The chorus answers inhibited critical thinking. • Teacher emphasised that learners should learn to state the theorems correctly in full and memorise the theorems. • TA asked for the theorems associated with a cyclic quadrilateral. The learners failed to state the theorem of the relationship between an interior angle and its opposite exterior angle correctly. • Learners had forgotten the theorem TA: Today we are going to revisit the centre theorems and theorems of the cyclic quadrilateral.	-Asked oral probing questions -Centre theorems -Prior knowledge of learners -Chorus answers -Chorus answers inhibited critical thinking skills -State theorems correctly -Memorise the theorems -Cyclic quadrilateral -Failed to state the theorem -Learners had forgotten the theorem
<u>Lesson activities</u> • TA made sketches on the board to explain centre theorems. • Learners were asked oral probing questions. • Examples were put on the board and the teacher asked specific learners to go and do the work on the board. • The theorem: "*Angle subtended by a chord or arc at the centre of a circle is twice the angle subtended by the same arc or chord at the circumference"* was explained using three different scenarios. TA explained the theorem using the lecture method. Learners did not contribute anything in the derivation of the theorem. They just received the information • TA went on to draw sketches and explained to the whole class the following relationships: -*Tangent is perpendicular to the radius of a circle at the point of tangency*	-Used more of teacher-centred approach to teach -Oral probing questions -Examples were put on the board -Asked specific learners -Lecture method -Learners did not contribute in the derivation of the theorem -TA explained to the learners reciting the relationships to them.

-Line drawn from the centre perpendicular to a chord bisects that chord *-Line drawn from the centre to the midpoint of a chord is perpendicular to that chord* • TA explained to the learners reciting the relationships to them. • Learners' participation was very minimal. • The following sketch of a cyclic quadrilateral was drawn with some angles. The teacher used coloured chalks to designate related angles and enhance the skill of visualisation.  The teacher wrote the following relationships on the board: $\hat{A} + \hat{C} = 180°$ $\hat{B} + \hat{D} = 180°$	-Learners' participation was very minimal -Cyclic quadrilateral -Used coloured chalks -Enhance skill of visualisation
Point to note: The relationship between the interior angles with their opposite exterior angles was not highlighted • An example was put on the board. Learners were asked to get sizes of some angles individually. Sharing of ideas was not allowed among learners. Individual work was encouraged.	-An example was put on the board Sharing of ideas was not allowed -Individual work was encouraged. -Volunteer among learners
• After 10 minutes, a volunteer among learners was allowed to explain on the board. • Gifted learners dominated class participation. • Learners were not free to share knowledge and ideas. • Learners with difficulties and the shy ones failed to participate.	-Gifted learners dominated participation -Learners were not free to share knowledge and ideas -Learners with difficulties and the shy ones failed to participate
• The volunteer did a good job by getting the right answers and stating acceptable reasons with the correct wording. Volunteer learner was one of the courageous and gifted learners.	-Acceptable reasons -Courageous and gifted learners

• **Activity 15 minutes** • An activity was given where learners were to work individually. Exercises were composed of centre theorems and theorems on the cyclic quadrilateral. • TA moved around assisting learners with difficulties. • Some learners had problems in stating acceptable reasons in their working. TA emphasised the need to state correct reasons. • Learners were provided with handouts of acceptable reasons. • Learners could not finish the activity and were advised to finish at study time. All learners requested extra tuition during study time. • Learners showed a positive attitude to request extra tuition • Throughout the lesson, the teacher was very composed and demonstrated patience with learners who were slow in understanding. TA dedicated himself in repeating explanations to learners. Facial expression and body language confirmed that TA enjoyed helping the learners.	-Learners were to work individually -Centre theorems -Cyclic quadrilateral -TA moved around assisting learners -Problems in stating acceptable reasons -Provided with handouts -Acceptable reasons -Learners could not finish the activity -Learners requested extra tuition -Learners showed a positive attitude -Teacher demonstrated patience. -Facial expression and body language showed the teacher enjoyed helping learners

The second lesson on the experimental group is summarised in Table 6.12.

Table 6. 12: Lesson 2 of Experimental group for TA

Description of lesson proceedings	**Initial codes**
Topic: Chord theorems and tangent theorems **Introduction (10 minutes)** • Learners worked in pairs different from Lesson 1 • Collective revision of homework was done and sharing of knowledge and ideas was done by the whole class on one of the activities given as homework. • Volunteer learners wrote their work on the board. • Learners corrected all their mistakes made on the homework.	-Learners worked in pairs -Collective revision of homework -Sharing knowledge and ideas -volunteer learners

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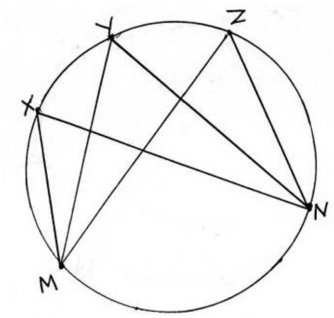
- TA asked about other theorems which had not been looked at. Learners responded in chorus: "Angles subtended by the same arc on the circle are equal".
- Chorus answers inhibited critical thinking.

TA: *Today will look at chord and tangent theorems*
<u>Lesson Activities</u>

- TA allowed learners to use vernacular language in discussions.
- TA gave learners the following model. TA improvised material for learning by using already made models.
- All learners participated in the practical activity.
- All learners were involved in the activity suggesting procedures to be taken.



- TA instructed learners to trace any of the angles subtended by arc MN on the circle. Learners traced and cut the angle equal to the one traced in their pairs.
- TA: *What do we do to the traced model cut?*
- Learner: *We place it on the other remaining subtended angles.*
- In their pairs, learners followed the proposed procedure. They found out that the cut model fitted well on the other angles which made learners agree that all the three subtended angles are equal.
- TA: *In your pairs draw two or more angles subtended on the circle by arc MN on the same side of $\hat{X}$, $\hat{Y}$ and $\hat{Z}$. Continue placing the same cut model.*
- In pairs, learners discovered that all angles subtended on the circle by MN were equal.
- TA: *What is the relationship of angles subtended on a circle by equal chords or arcs?*
Learners: *Those angles are equal.*

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-Learners corrected their mistakes
-Chorus answers inhibited critical thinking

-Use of vernacular language
-TA improvised
-already made models

-All learners participated
-All learners were involved in the activity

-The practical activity allowed learners to discover for themselves

-Sharing in pairs helped learners to discover the relationship between angles.

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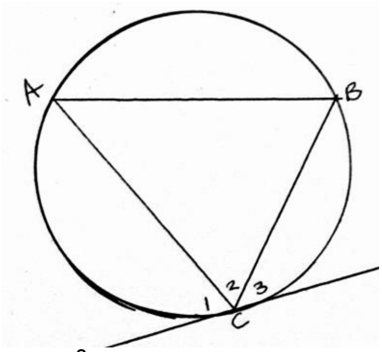
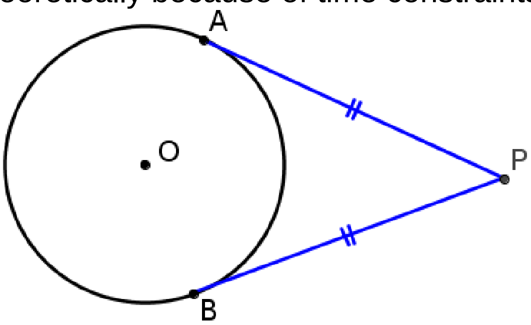
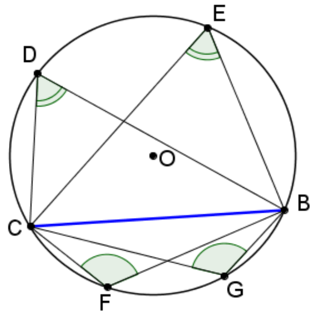
Learners were given the following model to prove the tan-chord theorem.  Learners traced the Â subtended on the circle and placed the cut piece on the angel on angle $\hat{C}_3$. They found that it fitted well proving the equality of the angles. They also cut B and placed it on $\hat{C}_1$ • Skill of visualisation was enhanced. Learners visualised the relationships through the practical activity. • Time constraints compelled the theoretical explanation of the other theorems. • The teacher explained the following theorem theoretically because of time constraints.  ***Tangents drawn from an external point to a circle are equal.*** • Sharing was promoted. Written work on the worksheet was done in pairs. Pairs discussed and shared ideas on how to solve the exercises • All learners were fully engaged. The teacher went around the class explaining to pairs. Also, the teachers emphasised the importance of stating acceptable reasons correctly. • TA stressed the point that learners should analyse diagrams together with the information given in each question before answering questions. The learners then should analyse which theorems would assist in getting solutions. • Homework was given. TA implored learners to share ideas and knowledge when doing homework at study time and even at home.	-Skill of visualisation -Learners visualised -Time constraints -Sharing was promoted -Pairs discussed and shared ideas -All learners were fully engaged -Importance of stating acceptable reasons -Learners should analyse diagrams -Learners should analyse which theorems -to share ideas and knowledge -Study time -ready to help any time -feel for one another

• TA told learners that he was ready to help any time. Requested learners to feel for one another. When one learner fails it should sadden everyone. He emphasised that success should be shared among themselves and not belong to only one person. • The teacher was very patient in explaining to learners who struggled in understanding.	-success should be shared -Compassion and sympathy were encouraged -Teacher demonstrated patience to learners

The proceedings of Lesson 2 by TA are summarised in Table 6.13.

Table 6. 13: Lesson 2 of Control group for TA

Description of lesson Proceedings	Initial codes
<u>Topic:</u> Chord theorems and tangent theorems **<u>Introduction</u>** • TA revised homework collectively with the whole class on the board. The whole class shared ideas and knowledge. • Volunteer learners wrote their work on the board. Gifted learners were very active in explaining to the whole class on the board. Gifted learners dominated participation • Learners corrected their mistakes made in the homework during the class discussion. **<u>Lesson activities</u>** • TA drew the following diagram on the board to explain the theorem of angles in the same segment or subtended by the same arc or chord. • The teacher used the lecture method as the dominant instructional strategy.  • TA used oral probing questions to get answers from learners.	 -Revised homework collectively -Whole class shared ideas and knowledge -Volunteer learners -Gifted learners were very active -Gifted learners dominated participation- Learners corrected their mistakes -Lecture method as an instructional strategy -Used oral probing questions

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- TA: *Can you identify the chord or arc subtending angles in this diagram?*
Learner: *Arc BC or chord BC*
TA: *Name the angles which are equal.*
Learners in chorus: **D̂=Ê and F̂=Ĝ**

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Point to note: TA did not go on to explain why D̂ and F̂ or Ê and Ĝ and other pairs were not equal

- Examples were given and some volunteer gifted learners went to the board to explain.

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<td>-Examples were given
-Gifted learners</td>
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- The tan-chord theorem was also done on the board with relevant examples worked on the board by the teacher.

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<td>-Tan-chord theorem
-Relevant examples worked</td>
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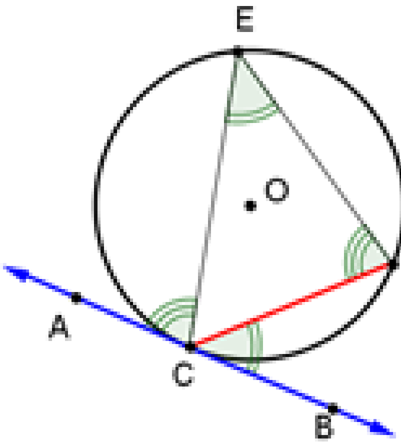
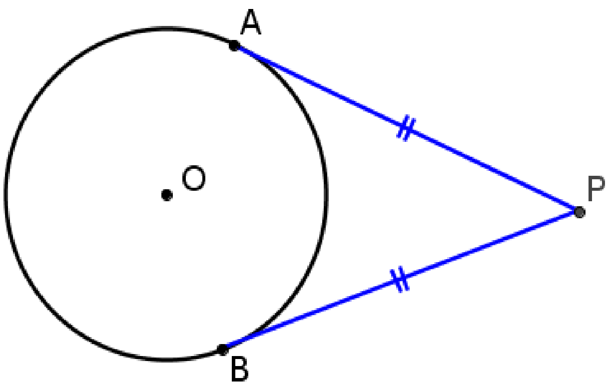
- The following diagram to explain the tan-chord theorem was drawn on the board with coloured chalk to designate equal angles.
- The coloured chalk was used to enhance visualisation.



Angle between tangent and chord is equal to *the inscribed angles which the chord subtends in alternate segment.*

- TA went on to explain the following theorem



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<td>-Coloured chalk to enhance visualisation</td>
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Tangents drawn from an external point to a circle are equal.

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<td>-Sharing of ideas was discouraged</td>
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• Written work was given to learners to do individually. Sharing of ideas was discouraged. • Worksheets were provided. • TA taught learners strategies of problem-solving in geometry. • TA stressed the point that learners should analyse diagrams together with the information given in each question before answering questions. The learners then should analyse which theorems would assist in getting solutions. • TA went round the classroom assisting some of the learners with difficulties. • Some of the learners resorted to waiting for the teacher to come to their desks because they did not know how to solve the exercises. • Only gifted learners volunteered to assist in getting solutions on the board. The less gifted learners displayed difficulties and were only waiting for the teacher to explain. • Homework was given. TA encouraged learners to share ideas when doing homework at study time and even at home. • All the learners showed concern about their performance on the topic and requested the teacher to conduct extra tuition at study time. They showed a positive attitude to learn the topic. • The teacher was patient with learners who were slow in understanding. He would dedicate time to explain more to learners. Showed no frustration when learners continued to commit errors.	-Strategies of problem solving -analyse diagrams -before answering questions -Assisting some of the learners with difficulties - they did not know how to solve the exercises -Only gifted learners volunteered -Less gifted learners displayed difficulties -Encouraged learners to share ideas -Learners showed concern -Extra tuition -Positive attitude -Teacher demonstrated patience to learners

Table 6.14 summarises the key aspects of what happened in Lesson 3 of the experimental group taught by TA.

Table 6. 14: Lesson 3 of Experimental group for TA

Description of lesson proceedings	**Initial codes**
<u>Topic:</u> Exercises on circle geometry **<u>Introduction (10 minutes)</u>** • The Dynamic pairs model was followed. New pairs different from the two previous lessons were formed. • Learners in pairs were given an activity to write down the theorems studied in the two previous lessons. Learners shared knowledge in pairs.	-The Dynamic pairs model was followed -New pairs -Learners in pairs -Shared knowledge in pairs

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- Full participation was experienced. All the pairs were busy.
- Each pair presented theorems of choice verbally in class.
- Homework was revised quickly.

</td><td>-Full participation was experienced</td></tr>
<tr><td>

<u>Lesson proceedings</u>

- TA allowed learners to use vernacular language in discussions.

</td><td>-Use of vernacular language</td></tr>
<tr><td>

- Worksheets with exercises were given to learners to work in their pairs with inter-pair discussions and sharing was allowed.

</td><td>-Work in pairs
-Inter-pair discussions and sharing were allowed</td></tr>
<tr><td>

- TA encouraged learners to limit the amount of noise.

</td><td>-Limit the amount of noise</td></tr>
<tr><td>

- TA emphasised the importance of underlining key words linked to theorems studied and analyse diagrams given making use of information available.

</td><td>-Importance of underlining key words
-Analyse diagrams</td></tr>
<tr><td>

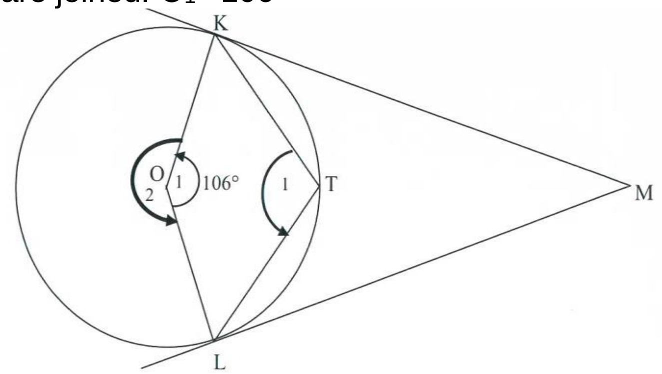
- Pairs worked sharing ideas. Neighbouring pairs compared their solutions correcting one another.

</td><td>-Worked sharing ideas</td></tr>
<tr><td>

<u>The following exercise was one of the activities</u>

- The exercise given was relevant and consistent with CAPS

In the diagram below, O is the centre. KM and LM are tangents to the circle at K and L, respectively. T is a point on the circumference of the circle. KT and LT are joined. $\hat{O}_1 = 106^\circ$



- Calculate, with reasons, the size of $\hat{T}_1$
- Prove that quadrilateral OKML is a kite
- Prove that quadrilateral OKML is a cyclic quadrilateral
- Calculate with reasons the size of $\hat{M}$

Other activities where other theorems were applicable were included as well.

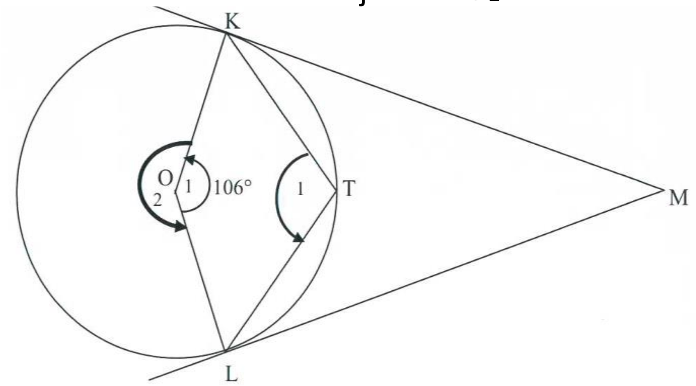
</td><td></td></tr>
<tr><td>The teacher was very patient with learners who were slow in understanding. He would dedicate time to</td><td>-Teacher demonstrated patience to learners</td></tr>
</table>

explain more to learners. Showed no frustration when learners continued to commit errors. **<u>Problems observed</u>** • Learners had problems remembering the properties of a kite. • Learners had problems in applying the converse theorem to prove that OKML is a cyclic quadrilateral. • In other exercises, the use of tan-chord theorems was a challenge. **<u>Learner participation</u>** Learners' participation was excellent as learners assisted their partners in remembering theorems.	-Problems in remembering properties of a kite -Learners had problems in applying the converse theorem -Cyclic quadrilateral -Use of the tan-chord theorem was a challenge -Learners' participation was excellent

The summary of the final lesson of the control group on circle geometry by TA is presented in Table 6.15.

Table 6. 15: Lesson 3 of Control group by TA

Description of lesson proceedings	**Initial codes**
<u>Topic:</u> Exercises on circle geometry **<u>Introduction</u>** • Revision of theorems was collectively done by the whole class on the board. • The teacher put an example on the board which two volunteer gifted learners who dominated participation explained to the whole class. • Homework was revised collectively in class. **<u>Lesson activities</u>** • Worksheets with exercises were given to learners to work individually. Sharing was not encouraged. • TA explained how to do problem-solving. TA emphasised the importance of underlining keywords linked to theorems studied and analyse diagrams given making use of information available. **<u>The following exercise was one of the activities</u>** -The exercise was relevant and consistent CAPS requirements.	-Revision of theorems was collectively done - Volunteer gifted learners -Dominated participation -Homework was revised collectively in class -To learners to work individually -Sharing was not encouraged -Explained how to do problem-solving -Importance of underlining keywords. -Analyse diagrams -Relevant and consistent syllabus

In the diagram below, O is the centre. KM and LM are tangents to the circle at K and L, respectively. T is a point on the circumference of the circle. KT and LT are joined. $\hat{O}_1 = 106^\circ$  I. Calculate, with reasons, the size of $\hat{T}_1$ II. Prove that quadrilateral OKML is a kite III. Prove that quadrilateral OKML is a cyclic quadrilateral IV. Calculate with reasons the size of $\hat{M}$ **Problems observed** • Learners had problems in remembering the properties of a kite. • Learners had problems in applying the converse theorem to prove that OKML is a cyclic quadrilateral In other exercises, the use of tan-chord theorems was a challenge. • Some of the learners could not establish that KM ⊥ OK and LM ⊥ OL to assist in proving that OKML is a cyclic quadrilateral. **Learner participation** Some learners waited for the teacher only to explain to them. Learners' participation was poor.	-problems in remembering properties of kites -problems in applying converse theorem -use of tan-chord theorem was a challenge -Learners' participation was poor

6.5.1.2.Summary of classroom observation on Teacher A

Teacher A taught the experimental group using the dynamic pairs model which embraced the *Ubuntu* attributes as well as some practical activities in which learners used manipulatives to determine relationships of angles. The instruction of the control group was dominated by the lecture method and individual work as an assessment strategy. Learners in the control group had very few occasions in which they would share knowledge especially when TA would engage them in class discussions in the solution of either exercises from the homework or classwork. The classroom discussions in the control group were predominantly dominated by gifted learners who occasionally would solve exercises on the chalkboard. On the contrary, the learners in the experimental group discussed in pairs sharing knowledge and ideas. Learners' participation in classroom activities in the experimental group was better than the participation of learners in the control group. Table 6.16 is a summary of classroom activities during the first lesson of TA on both groups. Table 6.17 presents percentages of the number of learners involved in each activity.

Table 6. 16: Summary of classroom activities in TA's Lesson 1

	Total number of learners in group	**Number of learners involved activity**			
		Participation	**Writing in workbook**	**Answering questions correctly**	**Stating acceptable reasons correctly**
Experimental group	18	18	18	16	15
Control group	18	6	8	4	5

Table 6. 17: Percentage of the number of learners involved in an activity

	Participation	**Writing in workbook**	**Answering questions correctly**	**Stating acceptable reasons correctly**
Experimental group	100	100	89	83
Control group	33	44	22	28

Figure 6.1 illustrates the percentages of the number of learners involved in an activity during TA's first lesson.

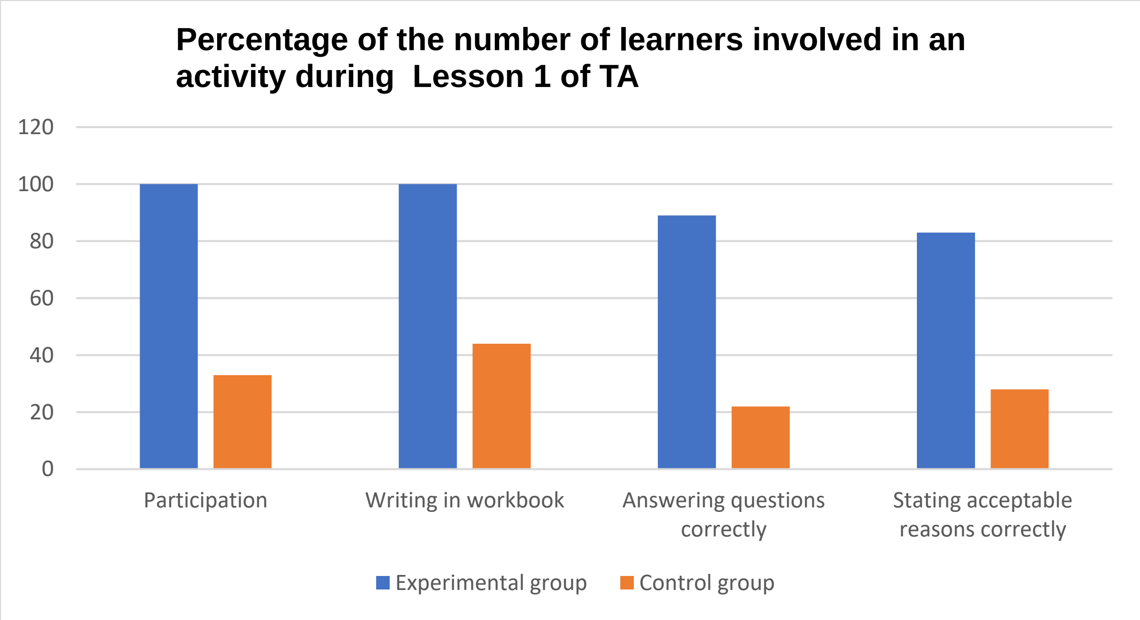


Figure 6. 1: Percentage of the number of learners involved in an activity during TA's Lesson 1

Table 6.18 is a summary of classroom activities during the first lesson of TA on both groups. In addition, Table 6.19 presents percentages of the number of learners involved in each activity.

Table 6. 18: Summary of classroom activities in TA's Lesson 2

	Total number of learners in group	**Number of learners involved activity**			
		Participation	**Writing in workbook**	**Answering questions in workbook correctly**	**Stating acceptable reasons correctly**
Experimental group	18	18	18	14	14
Control group	18	7	9	6	6

Table 6. 19: Percentage of the number of learners involved in an activity in TA's Lesson 2

	Participation	Writing in workbook	Answering questions correctly	Stating acceptable reasons correctly
Experimental group	100	100	78	78
Control group	33	50	33	33

Figure 6.2 illustrates the percentages of the number of learners involved in activities during TA's Lesson 2.

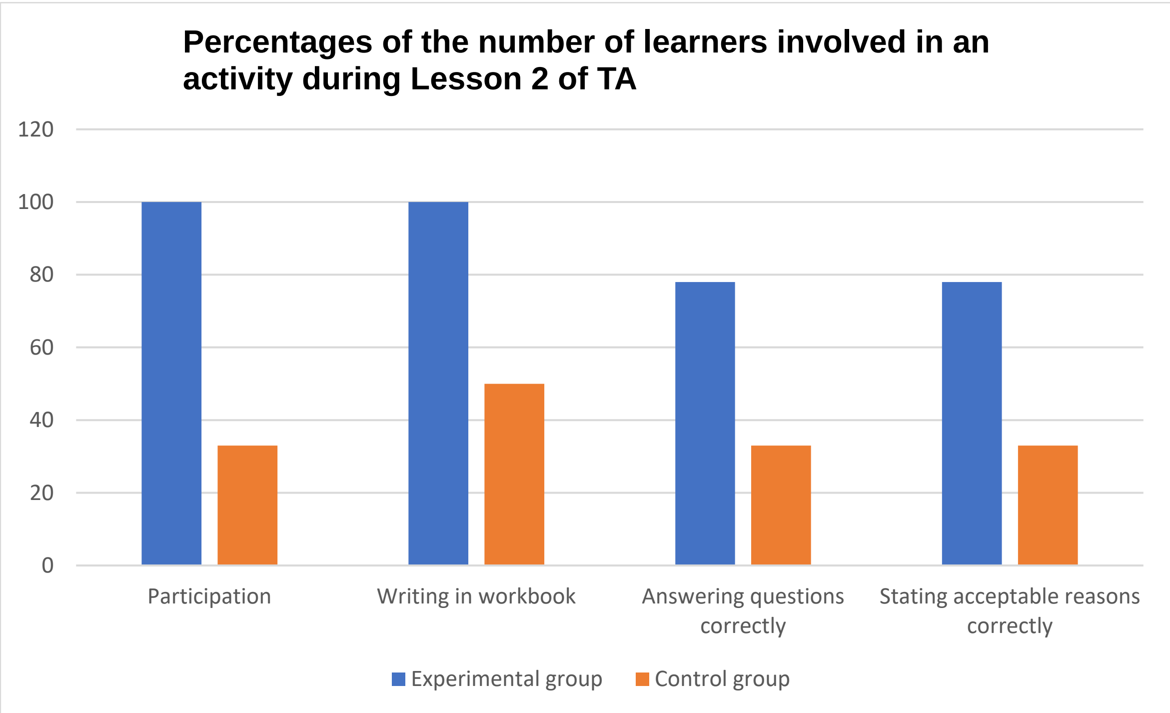


Figure 6. 2: Percentages of the number of learners involved in an activity during Lesson 2 of TA.

The summary of activities that occurred during Lesson 3 in both classes of TA is tabulated in Table 6.20 and Table 6.21.

Table 6. 20: Summary of classroom activities in TA's Lesson 3

	Total number of learners in group	**Number of learners involved activity**			
		Participation	**Writing in the workbook**	**Answering questions in the workbook correctly**	**Stating acceptable reasons correctly**
Experimental group	18	18	18	16	16
Control group	18	9	11	9	9

Table 6. 21: Percentage of the number of learners involved in an activity in TA's Lesson 3

	Participation	**Writing in the workbook**	**Answering questions correctly**	**Stating acceptable reasons correctly**
Experimental group	100	100	89	89
Control group	50	61	50	50

Figure 6.3 displays the percentages of the number of learners who were involved in key activities namely participation, writing in the workbook, answering questions correctly and stating acceptable reasons.

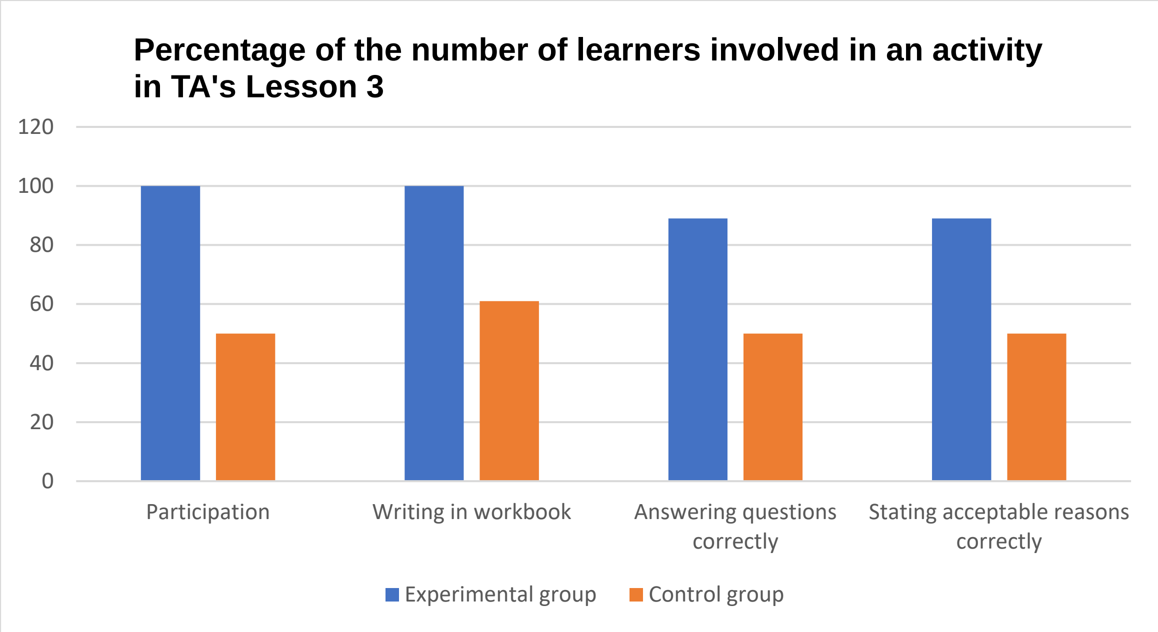


Figure 6. 3: Percentage of the number of learners involved in an activity in TA's Lesson 3.

During the lessons, the pairs in the experimental group would solicit assistance from other pairs nearby. During the preliminary interview, TA mentioned that one of the causes of learners' poor performance in geometry was their negative attitude towards the topic. It was amazing to find out that both groups of learners displayed great enthusiasm and zeal to learn. The expectation was that learners would display a negative attitude as mentioned by the TA. Learners from both groups would always ask for extra tuition. In the control group, learners who faced difficulties would always raise their hands and try to write in their workbooks. The teacher displayed a high command of content knowledge of teaching Euclidean geometry. During instruction to both groups, TA explained the theorems with exactitude giving different scenarios to achieve conceptual understanding among learners. All the learners attended all the lessons during the course the study.

6.5.2 Classroom observation for Teacher B

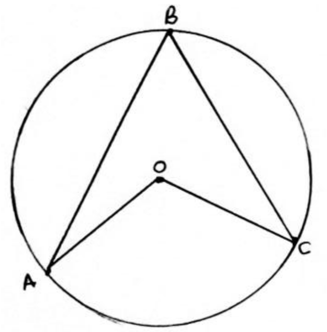
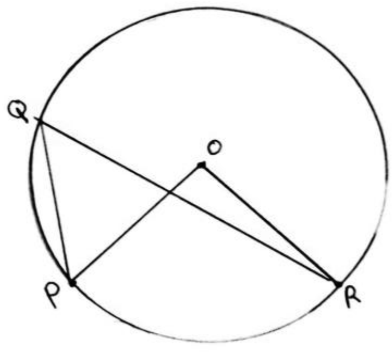
Teacher B taught both groups using two different instructional strategies with the control group being instructed using the traditional method just like how Teacher A did while the experimental group was instructed using the dynamic pairs model. The tables below summarise how instruction occurred in Teacher B's classroom.

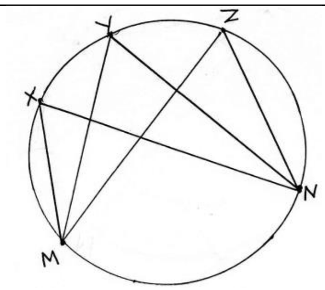
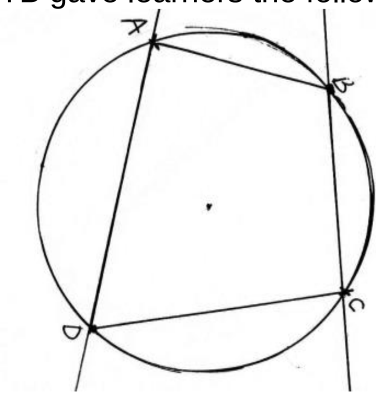
6.5.2.1.Tables summarising classroom observation on Teacher B [TB]

Those tables captured all the proceedings in Teacher B's classroom during instruction. They are a detailed version of the classroom observation schedule (Annexure L).

Table 6. 22: Lesson 1 of Experimental group for TB

Description of lesson proceedings	Initial codes
Topic: Centre theorems, chord theorems and cyclic quadrilateral theorems **<u>Introduction</u> (5 minutes)** • Revision of theorems was done using oral probing questions on the board with the teacher making sketches. Probing questions were asked. • Learners recited some of the theorems in chorus. However, the teacher tried to name specific learners to answer. • Learners also could recite the theorem on the relationship of opposite angles in a cyclic quadrilateral but had forgotten the other theorem. **<u>Lesson proceedings</u>** • TB allowed learners to use vernacular language in discussions. • TB explained to learners that they were going to do a practical activity in pairs to prove some of the key theorems practically. • TB followed the Dynamic pairs model and COVID-19 protocols. Learners were made to sit in pairs observing social distancing protocols. • Learners were provided with already made models of diagrams based on the centre theorems, cyclic quadrilaterals and angles subtended by the same arc revised in the introduction	-Revision of theorems -Using oral probing questions -Learners' prior knowledge in both groups -Learners recited theorems in chorus -Theorem on the relationship of opposite angles -had forgotten the other theorem -Use of vernacular language -practical activity in pairs -Dynamic pairs model -COVID-19 protocols -Social distancing

• TB used vernacular language to put across some ideas and to emphasise when learners would show the misunderstood. He would always ask, *“Simunye?”* to mean, *“Are we together?”*. In other instances, would ask, *“Sizwana?”* to mean, *“Do we understand each other?”* • The learners in pairs were requested to trace $A\hat{B}C$ on a separate white bond paper. Using a pair of scissors, they cut an angle equal to $A\hat{B}C$. The next step was placing the cut angle on $A\hat{O}C$. TA: *How many times does $A\hat{B}C$ go into $A\hat{O}C$*? Learners: *Twice*  • The skill of visualisation was enhanced. • The teacher assisted some pairs who were slower to understand the exercise. Some pairs assisted the neighbouring pairs in the spirit of *Ubuntu* on how to do the practical activity. • TB advised learners to prove the same theorem using the following model in their spare time.  **Activity 2: Angles in the same segment or subtended by the same chord or arc** TB gave the learners the following model: TB: *Which theorem is associated with this diagram?* Learners: *Angles subtended by the same arc are equal* TB: *Which arc subtends angles X, Y and Z?* Learners: *Arc MN*	-Provided with already made models of diagrams -centre theorems -cyclic quadrilaterals -Used vernacular language -Learners in pairs -Skill of visualisation was enhanced -Some pairs assisted neighbouring pairs

	
• Learners were instructed to trace angle Y. They cut the angle traced and placed it on angles X and Z. • In pairs, they discovered that the three angles subtended by arc MN were equal. Learners used the skill of visualisation to establish the relationship of the angles. • Learner's participation in demonstration of the theorem was promoted. • TB told learners to draw two or more angles subtended at the circle by arc MN. They placed the cut traced model. They observed the same relationship holding.	-In pairs they discovered -Used skill of visualisation -Relationship of angles -Learners' participation
Activity 3: Cyclic quadrilateral TB gave learners the following model to each pair 	
• Learners were asked the name of the geometric figure they saw. They responded that it was a cyclic quadrilateral. The skill of visualisation was enhanced. • They were required to write down in pairs the two theorems associated with a cyclic quadrilateral and identify the opposite angles. • TB instructed learners to trace ABCD and cut a model. The next step they were instructed to draw a straight line and place opposite angles. They were supposed to state the observations they made. Learners shared ideas in pairs and interacted with the teacher assisting. • Learners observed that the opposite angles formed an angle in a straight line.	-Cyclic quadrilateral -skill of visualisation -write down in pairs -Cyclic quadrilateral -identify opposite angles -opposite angles -state observations the made -in pairs -Sharing ideas

• TB explained the reason why opposite angles were supplementary. • Learners were asked to place A on the exterior angle at C and B on the exterior angle at D. Learners understood the theorem: An interior angle of a cyclic quadrilateral is equal to the opposite exterior angle of the same quadrilateral. Other centre theorems were just revised theoretically because of time constraints. **<u>Activity</u>** Learners were given an activity to do in pairs to solve exercises in pairs. Time lapsed before they could finish the second exercise because the practical activity was time-consuming. TB encouraged the learners to finish the exercise in pairs sharing ideas during one-hour study time from 14h00-15h00. -Learners' participation was overwhelming. Even learners who were habitually passive in other topics were lively in cutting models. Sharing ideas in dynamic pairs assisted some learners who needed assistance. -Throughout the lesson, the teacher was very composed and demonstrated patience on learners who were slow in understanding. TB dedicated himself to repeating explanations to learners. Facial expressions and body language confirmed that TB enjoyed helping the learners.	-teacher assisting -observed -opposite angles -Explained -opposite angles -Learners understood -Cyclic quadrilateral -revised theoretically -time constraints -to do in pairs to solve -time lapsed -in pairs sharing ideas -Learners' participation -habitually passive -Sharing ideas -Teacher demonstrated patience -Facial expression showed the teacher enjoyed helping learners

The proceedings of Lesson 1 of TB's control group are summarised in Table 6.23.

Table 6. 23: Lesson 1 of Control group for TB

Description of lesson proceedings	**Initial codes**
Topic: Centre theorems, chord theorems and cyclic quadrilateral theorems **<u>Introduction</u> (15 minutes)** • TB asked oral probing questions on learners' prior knowledge on the centre circle theorems, chord theorems and cyclic quadrilateral theorems the learners remembered. • Learners' responses were chorus answers. Chorus answers inhibited conceptual understanding of learners.	 -Oral probing questions -Learners' prior knowledge -Chorus answers -Chorus answers inhibited learners' conceptual understanding

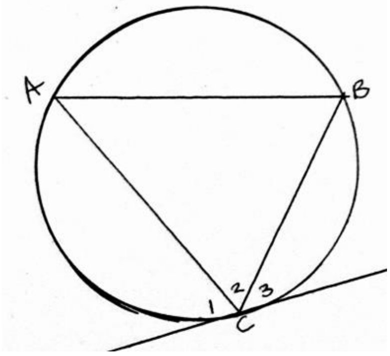
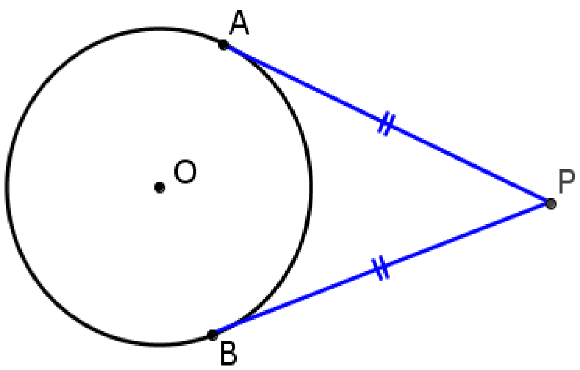
• The majority of theorems were mentioned except the following: ***-Angles subtended by equal chords are equal or equal arcs or chords subtend equal angles*** • TB stressed the importance of stating acceptable reasons in problem-solving. **<u>Lesson activities</u>** • TB put examples on the board and explained those examples using probing questions. • TB used vernacular language to put across some ideas and emphasise when learners would show the misunderstood. He would always ask, *"Simunye?"* to mean, *"Are we together?"*. In other instances, would ask, *"Sizwana?"* to mean, *"Do we understand each other?"* • There was a class discussion and solution of examples. Gifted learners went to the board to solve some of the examples. Only gifted learners dominated participation. • TB would always call the whole class to evaluate solutions presented on the board. • The lecture method was the dominant instructional strategy. • The following theorems were explained in brief theoretically on the board using the lecture method. TB explained as follows: ***-Angle subtended by a chord or arc at the centre of a circle is twice the angle subtended by the same arc or chord at the circumference.*** ***-Tangent is perpendicular to the radius of a circle at the point of tangency.*** ***-Line drawn from the centre perpendicular to a chord bisects that chord.*** ***-Line drawn from the centre to the midpoint of a chord is perpendicular to that chord.*** ***-Opposite angles of a cyclic quadrilateral are equal.*** ***-An interior angle of a cyclic quadrilateral is equal to the opposite exterior angle.*** **<u>Activity</u>** • An activity was given where learners were to work individually. Sharing ideas was not encouraged.	-acceptable reasons -Used examples -used vernacular language -foster learners' understanding -class discussion -examples -Gifted learners -Gifted learners dominated participation -Lecture method -Lecture method -Work individually -Sharing ideas was not encouraged -centre theorems -Assisted learners -Learning difficulties

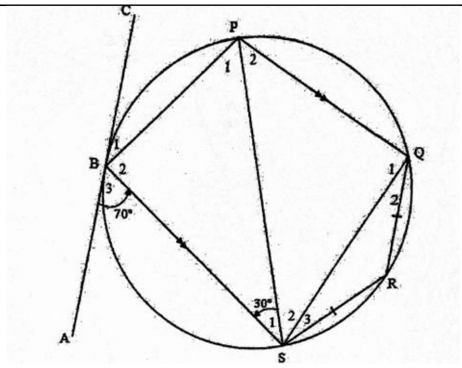
The exercises were composed of centre theorems and theorems on the cyclic quadrilateral. • TB assisted learners with learning difficulties by explaining to them how to apply appropriate theorems to questions. • Problems in stating correct acceptable reasons manifested. TB reiterated the need to state correct acceptable reasons. • TB gave out a hand-out with theorems and acceptable reasons. • Gifted learners assisted in working on the board. Learners' participation was dominated by gifted learners. • All learners requested extra tuition after school and on weekends to enable them to understand the topic better. They showed a positive attitude towards the topic They expressed that since the topic is allocated one-third of the total marks in Paper 2 examination there was a need to put more effort and engage in more practice. • Throughout the lesson, the teacher was composed and demonstrated patience with learners who were slow in understanding. TB dedicated himself in repeating explanations to learners. Facial expression and body language confirmed that TB enjoyed helping the learners.	-Problems in stating acceptable reasons -acceptable reasons. -gifted learners assisted -Learners' participation -dominated by gifted learners. Learners' attitudes -All learners requested extra tuition -They displayed a positive attitude towards the topic. -Teacher demonstrated patience with learners -Facial expression and body language confirmed the teacher enjoyed helping learners

Table 6.24 is a summary of the proceedings of Lesson 2 of TB's experimental group.

Table 6. 24: Lesson 2 of Experimental group for TB

Description of lesson proceedings	**Initial codes**
<u>Topic:</u> Tangent theorems **<u>Introduction (10 minutes)</u>** • Learners sat in pairs different from Lesson 1. The dynamic pairs model was followed. • A joint revision was done by the whole class on one of the activities given as homework. There was sharing of ideas and knowledge by the whole class. • Volunteer learners went to the board to do the work on the board • Learners corrected each other's mistakes made on the homework.	-Sat in pairs -The dynamic pairs model was followed -Joint revision Sharing of ideas -Volunteer learners -corrected each other's mistakes

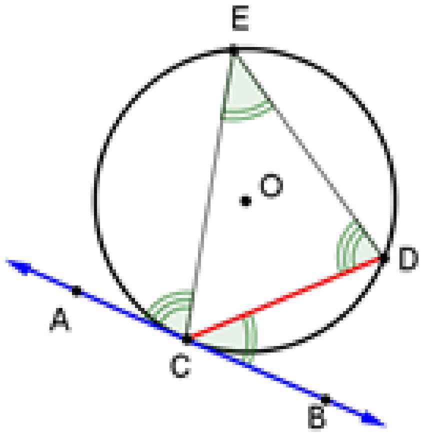
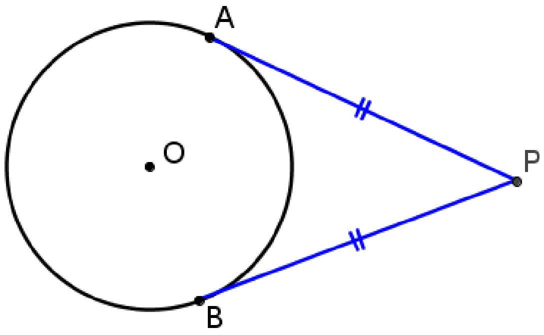
TB told the class that new theorems would be learnt that day **<u>Lesson activities</u>** • TB allowed learners to use vernacular language in discussions.	-Use of vernacular language
TB gave learners the following model  • Learners traced the Â subtended on the circle and placed the cut the traced angle. • TB instructed learners to fit Â on $\hat{C}_1$, $\hat{C}_2$and $\hat{C}_3$. They also cut $\hat{B}$ and placed it on $\hat{C}_1$. Learners saw the relationship between angles. The skill of visualisation was enhanced.	-The skill of visualisation
The theorem was given: **Angle between tangent and chord is equal to** ***the inscribed angles which the chord subtends in alternate segment*** The following theorem was explained verbally due to time constraints. The practical activity was time-consuming. The use of vernacular language was promoted to enhance understanding.	-Time constraints -The practical activity was time-consuming -Use of vernacular language
 *Tangents drawn from an external point to a circle are equal* **<u>Classwork</u>** • Written work on worksheets was given to learners seated in pairs. Sharing of ideas and knowledge was done in pairs and pairs interacted. The learners in pairs solved the following: **ABC is tangent to circle BPQRS at B, PQ ll BS, QR=RS, $\hat{S}_1 = 30^\circ$ and $\hat{B}_3 = 70^\circ$**	- Seated in pairs -Sharing of ideas and knowledge

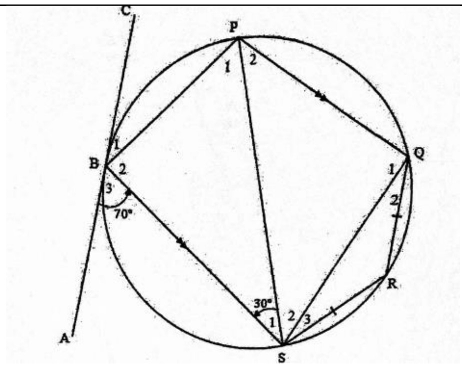
 **Calculate with reasons** I. $\hat{B}_1$ I. $\hat{P}_2$ II. $\hat{R}$ III. $\hat{Q}_2$ • The exercise selected was relevant and consistent with CAPS. • Sharing of ideas was done in pairs. Also, pairs were allowed to interact. Pairs discussed and shared ideas on how to solve the exercises. • Learners' participation was overwhelming. All learners were fully engaged. The teacher went around the class explaining to pairs. Also, the teachers emphasised the importance of stating acceptable reasons correctly. • TB taught learners problem-solving in Euclidean geometry. TB stressed the point that learners should analyse diagrams together with the information given in each question before answering questions. The learners then should analyse which theorems would assist in getting solutions. • Homework was given. TB encouraged learners to share ideas when doing homework at study time and even at home. More work was given to the learners to practice.	-Sharing of ideas -done in pairs -shared ideas -Learners' participation -Acceptable reasons -Taught learners problem solving -Analyse diagrams -Encouraged learners to share ideas

The second lesson for TB's control group is summarised in Table 6.25.

Table 6. 25: Lesson 2 of Control group for TB

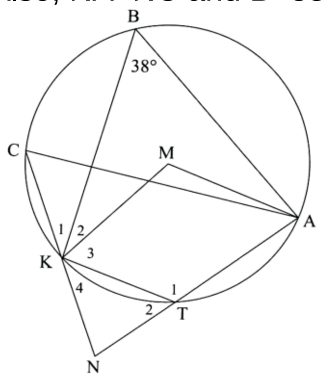
Description of lesson proceedings	Initial codes
Introduction (15 minutes) • TB conducted a collective revision of the homework on the board with the whole class. Homework was used to introduce the lesson.	-Conducted collective revision -Homework was used to introduce the lesson

• Gifted learners did the work on the board. The gifted learners were active in explaining to the whole class on the board. They dominated class participation. • Learners corrected their mistakes made in the homework during the class discussion. The learners shared ideas on how to solve similar exercises.	-Gifted learners -dominated participation -Learners corrected their mistakes -Shared ideas
Lesson activities TB drew the following diagram with coloured chalks on the board to explain the tan-chord theorem. The same colours were used to designate equal angles. This enhanced the skill of visualisation.  **Angle between tangent and chord is equal to *the inscribed angles which the chord subtends in alternate segment*** • TA went on to explain the following theorem  ***Tangents drawn from an external point to a circle are equal.***	-Coloured chalks -Skill of visualisation
• Written work was given to learners to do individually. Sharing ideas was not encouraged. Worksheets were provided. **ABC is tangent to circle BPQRS at B, PQ ll BS, QR=RS, $\hat{S}_1 = 30^\circ$ and $\hat{B}_3 = 70^\circ$**	-Given to learners to do individually -Sharing of ideas was not encouraged

 **Calculate with reasons** i. $\hat{B}_1$ ii. $\hat{P}_2$ iii. $\hat{R}$ iv. $\hat{Q}_2$	
• The written work exercise was relevant and consistent with CAPS. • TB explained to learners how to do problem-solving in geometry lessons.	-Relevant and consistent with CAPS -Problem solving
• TB taught learners problem-solving and stressed the point that learners should analyse diagrams together with the information given in each question before answering questions. The learners then should analyse which theorems would assist in getting solutions.	-taught learners problem solving -Analyse diagrams
• TB went around the classroom assisting learners with difficulties.	-Assisting learners with difficulties
• Some of the learners just waited for the teacher to come to their desks because they did not know how to solve the exercises.	
• Gifted learners volunteered to assist in getting solutions on the board. The less gifted learners displayed difficulties and were only waiting for the teacher to explain.	-Gifted learners volunteered -Less gifted learners displayed difficulties
• Learners requested extra tuition. They displayed zeal and a positive attitude on the topic. More work was given to the learners as homework. TB encouraged learners to share ideas when doing homework at study time and even at home.	-Learners requested extra tuition -Displayed zeal and a positive attitude -Encouraged learners to share ideas

The final lessons taught by TB are summarised in Tables 6.26 and Table 6.27 below.

Table 6. 26: Lesson 3 of Experimental group for TB

Description of lesson proceedings	Initial codes
<u>Topic:</u> Exercises on circle geometry **<u>Introduction (10 minutes)</u>**	
• Learners sat in new pairs different from the two previous lessons. The dynamic pairs model was followed.	-Learners sat in new pairs -The Dynamic pairs model was followed
• Collective revision of homework was done by the whole class. The use of vernacular language was predominant in the discussion.	-Collective revision of homework -Use of vernacular language
• A volunteer learner presented a solution on the board and the class evaluated the work.	-Volunteer learner
• Learners' participation was excellent.	-Learners' participation
<u>Lesson activities</u>	
• Worksheets with exercises were given to learners to work in their pairs sharing ideas with inter-pair discussions allowed. Learners used vernacular language in their discussions. Neighbouring pairs compared their solutions correcting one another.	-Sharing ideas -Inter-pair discussions -Used vernacular language
• TB used classroom management skills to encourage learners to limit the amount of noise.	-Used classroom management skills
• TB taught learners problem-solving in geometry.	-Taught learners problem solving
• TB emphasised the importance of underlining keywords linked to theorems studied and analyse diagrams given making use of information available. .	-Underlining keywords -Analyse diagrams
<u>The following exercise was one of the activities</u> In the diagram, M is the centre of the circle A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also, NA=NC and $\hat{B}=38^\circ$.  o Calculate with reasons ▪ $K\hat{M}A$	

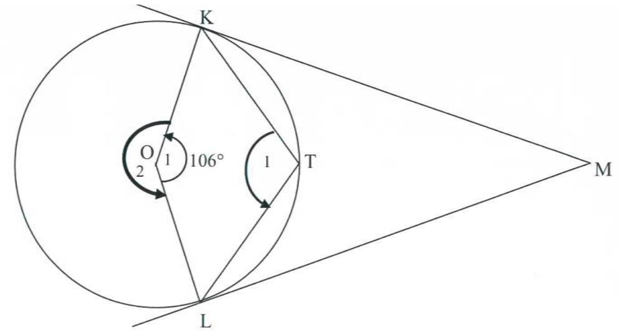
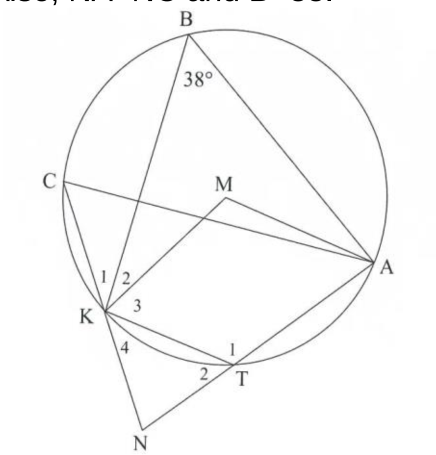
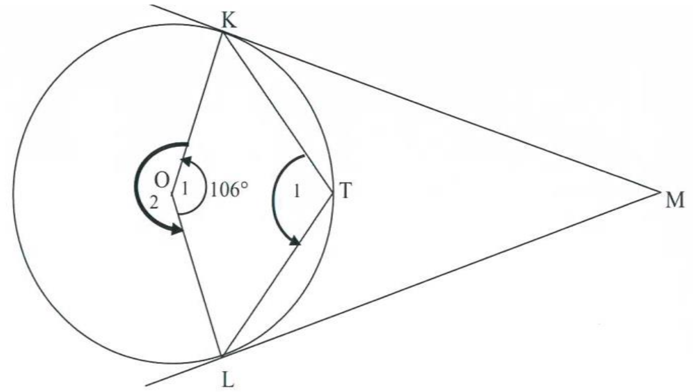
▪ $\hat{T}_2$ ▪ $\hat{C}$ ▪ $\hat{K}_4$ o Show that NK=NT 1.3 Prove that AMKN is a cyclic quadrilateral. 2. In the diagram below, O is the centre. KM and LM are tangents to the circle at K and L, respectively. T is a point on the circumference of the circle. KT and LT are joined. $\hat{O}_1 = 106^\circ$  • Calculate, with reasons, the size of $\hat{T}_1$. • Prove that quadrilateral OKML is a kite. • Prove that quadrilateral OKML is a cyclic quadrilateral. • Calculate with reasons the size of $\hat{M}$. Other activities where other theorems were applicable were included as well. • The selection of questions exercise was relevant and consistent with CAPS. • The teacher enjoyed explaining to learners who were struggling. Facial expression and body language displayed by the teacher showed that he was not frustrated by learners' continued misunderstanding. The teacher would look for simpler ways to explain to each pair that displayed difficulties in understanding. **Problems observed** • Learners had problems remembering the properties of a kite. • Learners had problems in applying converse theorem on cyclic quadrilaterals • In other exercises the use of tan-chord theorems was a challenge **Learner participation** Learners' participation was excellent as learners assisted their partners in remembering theorems.	-Relevant and consistent with CAPS -Teacher demonstrated patience and commitment in helping learners who displayed difficulties -Problems in remembering properties of a kite -Problems in using converse theorem -tan-chord theorem was a challenge -Learners' participation was excellent

Table 6.27 below summarises the proceedings during TB's third lesson with the control group.

Table 6. 27: Lesson 3 of Control group for TB

<table>
<tr><th>Description of lesson proceedings</th><th>Initial codes</th></tr>
<tr><td><u>Topic:</u> Exercises on circle geometry
<u>Introduction</u>
• Collective revision of theorems was done by the whole class on the board.</td><td>-Collective revision of theorems was done by the whole class</td></tr>
<tr><td>• TB used examples to introduce the lesson. TB put an example on the board which two volunteer learners explained to the whole class. Learners shared ideas as a group. Gifted learners dominated with participation.</td><td>-Used examples
-Volunteer learners
-Shared ideas
-Gifted learners dominated with participation</td></tr>
<tr><td><u>Lesson activities</u>
• Individual work on worksheets was given to learners.
• Sharing of ideas was not encouraged.</td><td>-Individual work was given.
-Sharing of ideas was not encouraged</td></tr>
<tr><td>• TB taught learners problem-solving in geometry. TB emphasised the importance of underlining keywords linked to theorems studied and analyse diagrams given making use of information available.
<u>The following exercise was one of the activities</u>
In the diagram, M is the centre of the circle A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also, NA=NC and $\hat{B}$=38°.


Calculate with reasons
1.1.1 K$\hat{M}$A</td><td>-Taught learners problem solving
-Underlining keywords
-Analyse diagrams
-Linked to theorems</td></tr>
</table>

1.1.2 $\hat{T}_2$ 1.1.3 $\hat{C}$ 1.1.4 $\hat{K}_4$ 1.2 Show that NK=NT 1.3 Prove that AMKN is a cyclic quadrilateral 2. In the diagram below, O is the centre. KM and LM are tangents to the circle at K and L, respectively. T is a point on the circumference of the circle. KT and LT are joined. $\hat{O}_1 = 106^\circ$  V. Calculate, with reasons, the size of $\hat{T}_1$. VI. Prove that quadrilateral OKML is a kite. VII. Prove that quadrilateral OKML is a cyclic quadrilateral. VIII. Calculate with reasons the size of $\hat{M}$.	
-The exercise was relevant and consistent CAPS requirements.	-Relevant and consistent CAPS
-The teacher enjoyed explaining to learners who were struggling. Facial expression and body language displayed by the teacher showed that he was not frustrated by learners' continued misunderstanding. The teacher would look for simpler ways to explain to each pair that displayed difficulties in understanding.	-Teacher demonstrated patience for learners with difficulties
<u>Problems observed.</u> • Learners had problems in remembering the properties of a kite. • Learners had problems in applying the converse theorem on cyclic quadrilaterals	-Problems in remembering properties of a kite -Problem in applying the converse theorem
In other exercises, the use of tan-chord theorems was a challenge. • Some of the learners could not establish that KM ⊥ OK and LM ⊥ OL to assist in proving that OKML is a cyclic quadrilateral. **<u>Learner participation</u>** Some learners waited for the teacher only to explain to them. Learners' participation was poor.	-Learners' participation was poor

6.5.2.2 Summary of classroom observation on Teacher B

The experimental group was instructed using the *Ubuntu*-based model whereby learners worked in pairs. Teacher B used the Dynamic Pairs model which incorporated some practical activities. Learners were engaged in using physical manipulatives in the practical activities. In their pairs, the learners used models which they cut according to specifications to determine relationships between angles. The control group was instructed using the traditional methods which were dominated by the lecture method and individual work as an assessment strategy just like what transpired in School A in which Teacher A was at the helm. Activities for Lesson 1 to both groups by Teacher B are summarised in the tables and figure below. Table 6.28 displays the summary of classroom activities during TB's first lesson.

Table 6. 28: Summary of classroom activities in TB's Lesson 1

	Total number of learners in group	**Number of learners involved activity**			
		Participation	**Writing in the workbook**	**Answering questions in the workbook correctly**	**Stating acceptable reasons correctly**
Experimental group	16	16	16	14	12
Control group	18	9	10	9	9

Table 6.29 further illustrates the percentages of the number of learners involved in activities during TB's first lesson.

Table 6. 29: Percentages of the number of learners involved in an activity in TB's Lesson 1

	Participation	Writing in the workbook	Answering questions correctly	Stating acceptable reasons correctly
Experimental group	100	100	88	75
Control group	50	56	50	50

In Figure 6.4 below, the percentages of the number of learners who were involved in classroom activities are illustrated.

The following tables, Table 6.30 and Table 6.31 describe what happened in Lesson

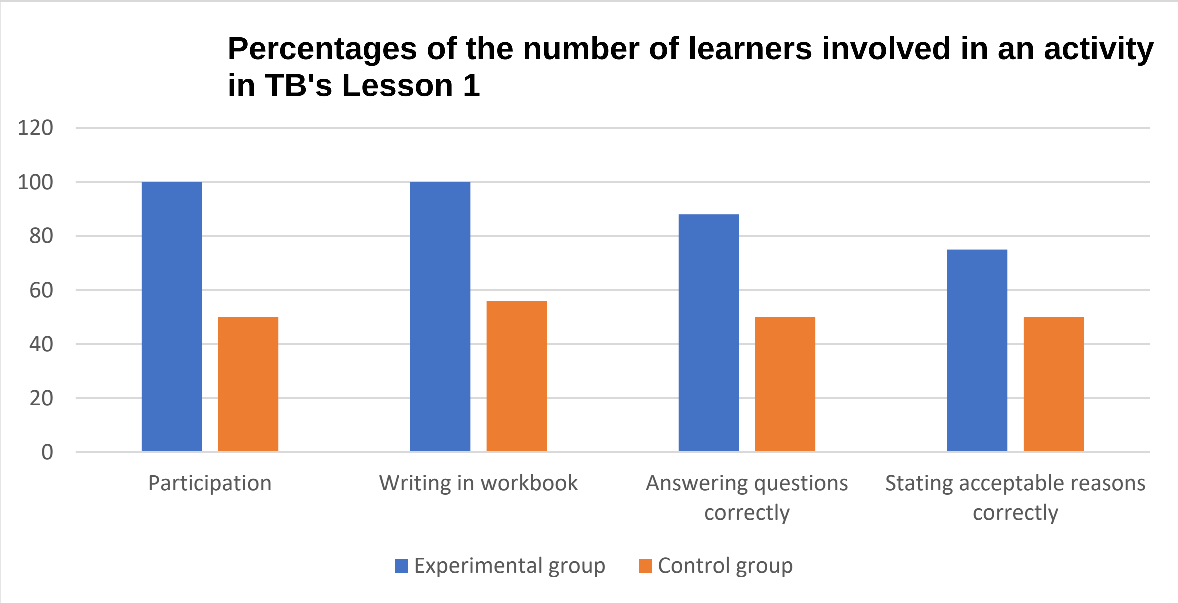


Figure 6. 4: Percentage of the number of learners involved in an activity in TB's Lesson 1

2 for both groups for Teacher B.

Table 6. 30: Summary of classroom activities in TB's Lesson 2

	Total number of learners in group	Number of learners involved activity			
		Participation	Writing in the workbook	Answering questions in the workbook correctly	Stating acceptable reasons correctly
Experimental group	16	16	16	14	14
Control group	18	8	12	9	9

Table 6. 31: Percentages of the number of learners involved in an activity in TB's Lesson 2

	Participation	Writing in the workbook	Answering questions correctly	Stating acceptable reasons correctly
Experimental group	100	100	88	88
Control group	44	67	50	50

The percentages of the number of learners who were engaged in different activities during TB's second lesson are displayed in Figure 6.5.

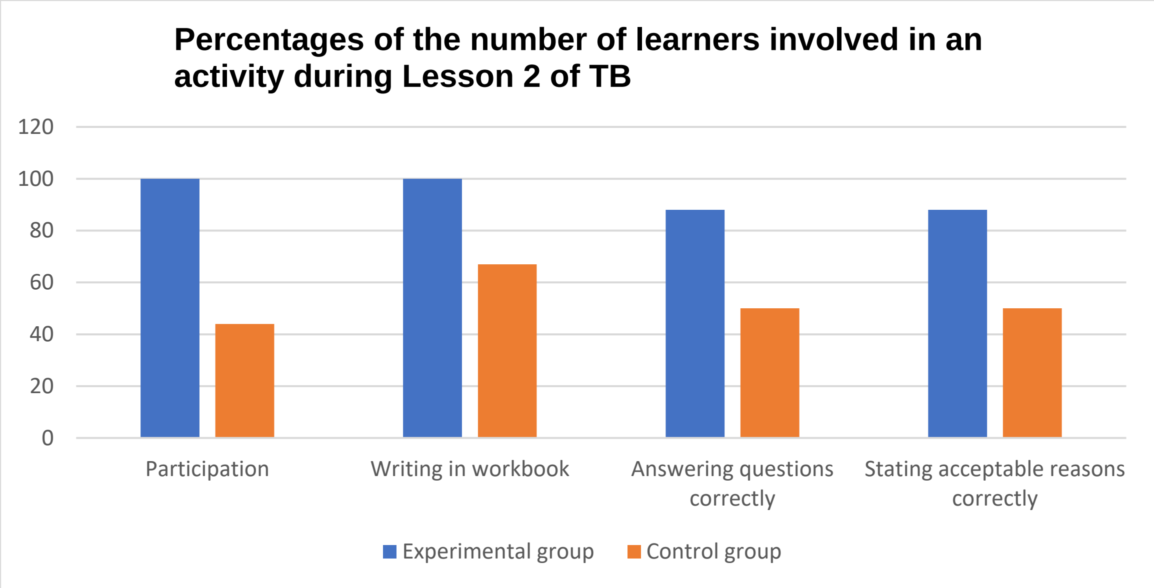


Figure 6. 5: Percentages of the number of learners involved in an activity during Lesson 2 of TB.

Table 6.32 and Table 6.33 and Figure 6.6 summarise the dynamics of classroom activities of Lesson 3 of Teacher B on both the experimental and control groups.

Table 6. 32: Summary of classroom activities in TB's Lesson 3

	Total number of learners in group	Number of learners involved activity			
		Participation	Writing in the workbook	Answering questions in the workbook correctly	Stating acceptable reasons correctly
Experimental group	16	16	16	14	12
Control group	18	7	13	12	11

Table 6. 33: Percentages of the number of learners involved in an activity in TB's Lesson 3

	Participation	Writing in the workbook	Answering questions correctly	Stating acceptable reasons correctly
Experimental group	100	100	88	75
Control group	44	72	67	61

Figure 6.6 further illustrates the percentages of the number of learners who were involved in different activities during the third lesson of TB.

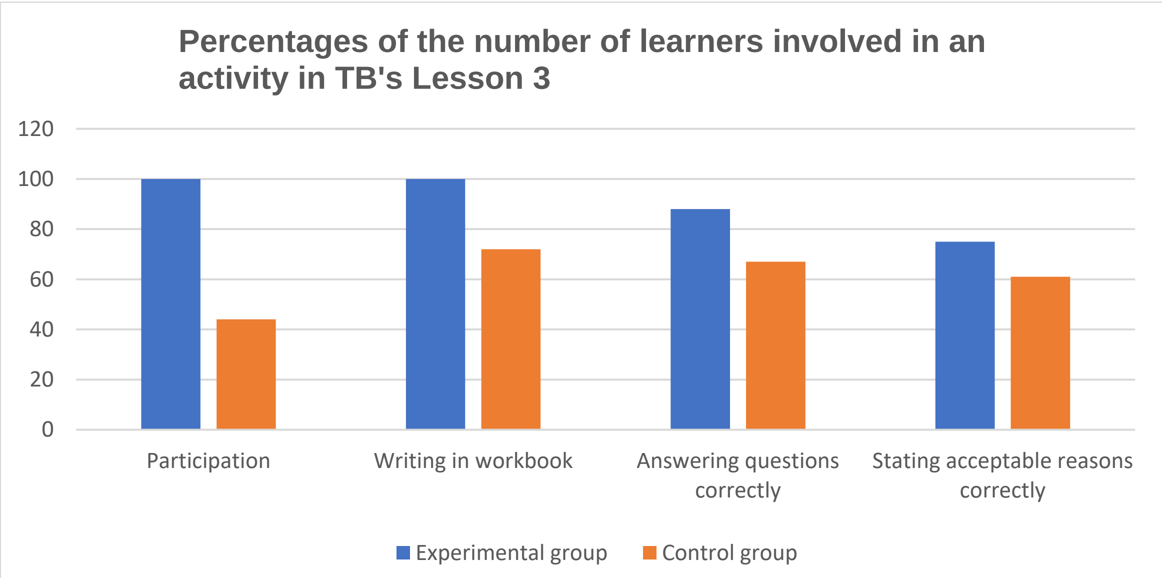


Figure 6. 6: Percentages of the number of learners involved in an activity in TB's Lesson 3

Learners in the control group had very few occasions in which they would share knowledge especially when TB would engage them in class discussions in the solution of either exercises from homework or classwork. Gifted learners in the control group were the ones who contributed much in classroom discussions. They took turns to go and solve exercises on the chalkboard each one explaining all steps used to obtain the solutions. On the contrary, the learners in the experimental group discussed in pairs sharing knowledge and ideas.

During the lessons, the pairs in the experimental group would solicit assistance from other pairs nearby. During the preliminary interview, TB mentioned that one of the causes of learners' poor performance in geometry was their negative attitude towards the topic. It was amazing to find out that both groups of learners displayed great enthusiasm and zeal to learn. The expectation was that learners would display a negative attitude as mentioned by the TB. Learners from both groups would always ask for extra tuition. In the control group, learners who faced difficulties would always raise their hands and would try to write in their workbooks. All the learners attended all the lessons during the course the study.

6.6 Findings from document analysis on both teachers

Document review involved examination of teachers' lesson plans, learners' workbooks as well as teachers' diaries. The lesson plans were examined using requirements for a good lesson plan by Cicek (2013) which include:

i. Statement of the topic,
ii. Written aims and objectives,
iii. Assumed learners' prior knowledge,
iv. Activities and time frames for the activities,
v. Resources to be used in the lesson, and
vi. Assessment activities

In addition, the exercise involved examination of the use of policy documents as well as other resources important in the instruction of geometry. Annexure M was used for document review.

6.6.1 Findings from document review for Teacher A

Table 6.34 summarises the findings from Teacher A's documents.

Table 6. 34: Summary on document review on Teacher A

ITEM	REMARKS	Codes
Did every learner write the prescribed exercises in his/her workbook?	-Every learner had a workbook for both groups. -All learners in the experimental group had work written in their workbooks. -Some learners in the control group failed to write exercises due to lack of understanding.	-All learners had workbooks. -All learnes in the experimental group wrote exercises. -Some learners in the control group did not write.
Does the teacher possess a lesson plan? Is the lesson plan relevant and detailed showing the use of *Ubuntu* values?	-Every lesson had its own lesson plan. -Lesson plans for the experimental group incorporated *Ubuntu* values.	-Lesson plans were available. -Lesson plans of the experimental group displayed *Ubuntu* attributes.
What common errors, misconceptions were observed in the learners' workbooks?	-Wrong reasons were stated in some of the learners' workbooks. -Errors in applying theorems such as tan-chord theorem, converse theorems especially on cyclic quadrilaterals. -Errors were made in analysing questions and diagrams.	-Learners wrote wrong reasons. -Errors in applying theorems and their converses -Errors in analysis of questions and diagrams
Did the teacher's marking indicate that these errors and misconceptions were observed?	The teacher used a red pen in marking. The teacher indicated errors and misconceptions by underlining them and wrote comments for each.	-Marking done using a red pen -Use of red pen to indicate errors -Comments were written.

Which aspects in the topic were learners comfortable with according to the information obtained from written work?	Learners were comfortable in calculating sizes of angles using centre theorems Geometrical proofs were a challenge to the learners.	-Learners were comfortable on centre theorems -Learners not able to do geometric proofs.
Was the written work in line with CAPS requirements?	The work was consistent with the CAPS requirements.	-Work was consistent with syllabus.
Was the written work relevant to channel learners to answer examination questions?	The teacher gave learners questions in line with those which normally appear in national examinations.	-Written work in line with examination standards
Any other aspects observed?	The teacher had handouts for learners with questions for activities to do and acceptable reasons. The lesson plans indicated that the CAPS document, Annual Teaching Plan [ATP] and examination guidelines were consulted.	-Hand-outs given to learners -Lesson plans done using curriculum documents

Teacher A had well-prepared lesson plans in place. The lesson plans reflected the teacher's use of examination guidelines, annual teaching plan [ATP], CAPS document, availability of handouts, as well as reference to diagnostic reports. The lesson plans also reflected that the teacher consulted various textbooks and sources to equip himself for lesson delivery. *Ubuntu* values were incorporated in Teacher A's lesson plans for the experimental group indicating the teacher's cooperativeness with the researcher. These findings indicate that Teacher A was prepared to teach the topic. The teacher's commitment was further reflected in his efforts to mark every workbook and making comments on each written activity. The marking was detailed. Learners' workbooks were regularly marked and corrected with learners writing corrections. Knowledge of learners' learning difficulties was enhanced as the teacher marked the learners' work thoroughly.

6.6.2 Document review on Teacher B

Table 6.35 below summarises the review of Teacher B's documents.

Table 6. 35: Summary of the review of Teacher B's documents

ITEM	REMARKS	Codes
Did every learner write the prescribed exercises in his/her workbook?	All the learners from the experimental group wrote exercises in their write respective workbooks. Some learners from the control group did not write their exercises.	-All learners in the experimental group wrote the exercises given. -Some learners in the control group did not write.
Does the teacher possess a lesson plan? Is the lesson plan relevant and detailed showing the use of *Ubuntu* values?	The teacher prepared a lesson plan for each lesson taught. Lesson plans for the experimental group incorporated *Ubuntu* values.	-Lesson plans were available. -Experimental group lesson plans with *Ubuntu* attributes
What common errors, misconceptions were observed in the learners' workbooks?	Wrongly stated reasons in some of the learners' workbooks Errors in applying theorems such as tan-chord theorem, converse theorems especially on cyclic quadrilaterals Errors were made in analysing questions and diagrams.	-Learners wrote wrong reasons -Errors in applying theorems and their converses -Errors in the analysis of questions and diagrams
Did the teacher's marking indicate that these errors and misconceptions were observed?	The teacher used a red pen in marking and indicated learners' errors and made relevant comments on each error committed.	-Marking done using a red pen -Use of a red pen to indicate errors -Comments were written
In which aspects of the topic were learners comfortable with according to the information obtained from written work?	Learners were comfortable in calculating sizes of angles using centre theorems. They were not comfortable with geometrical proofs.	-Learners were comfortable with centre theorems. -Learners were not able to do geometric proofs.
Was the written work in line with CAPS requirements?	The work was in line with the CAPS requirements.	-Work was consistent with CAPS.

Was the written work relevant to channel learners to answer examination questions?	Past exam questions were given to learners.	-Written work was in line with examination questions.
Any other aspects observed?	The teacher had handouts for learners with questions for activities to do and acceptable reasons. The lesson plans indicated that the CAPS document, Annual Teaching Plan [ATP] and examination guidelines were consulted.	-Hand-outs were given to learners. -Lesson plans done using curriculum documents

The lesson plans were well-structured showing all didactic phases that took place in lessons. Teacher B's lesson plans reflected that a thorough consultation of policy documents such as the CAPS document, examination guidelines and diagnostic reports was done. The teacher indicated the textbooks and material used to prepare for the lesson. In addition, the lesson plans for the experimental group were crafted in tandem with proposals of the *Ubuntu*-based dynamic pairs model. The teacher marked learners' workbooks and reflected teachers' comments on work done by the learners especially on the statement of acceptable reasons in problem resolution.

6.7 Findings from final semi-structured interviews

The final semi-structured interview was held at the end of data collection during the teachers' free time in their schools lasting one hour each.

6.7.1 Findings from the final semi-structured interview with TA

Table 6.36 below displays the final semi-structured interview with TA.

Table 6. 36: Final semi-structured interview with TA

Questions and responses	Initial codes

R: During the study, what challenges did you encounter in teaching the topic? If so, how best do you think you addressed these challenges? TA: *Learners had challenges in recalling all theorems they studied in Grade 11. Also, some of the learners showed that they did not like the topic when I introduced it the first day. I motivated them by telling them the importance of the topic. In the experimental group, it was easier to motivate the learners because the activity-based Ubuntu model involved all learners, and the practical activity assisted learners to understand the theorems.*	-Challenges in recalling Grade 11 content -Learners initially showed that they did not like the topic - It was easier to motivate learners in the experimental group. -The practical activity assisted learners to understand.
R: What common difficulties, misconceptions and errors did you identify among the learners? Among them which ones did you manage to address? TA: *Learners had problems in analysing questions. They wanted to answer questions without analysing questions, diagrams and keywords in the questions. Also, learners stated reasons which were not acceptable. In both classes, I taught learners how to extract information from questions by underlining keywords linked to theorems they had studied. I taught them to write all relationships they could get before attempting to answer questions. I also gave learners a hand-out with acceptable reasons.*	-The learners had problems in analysing questions -Stated reasons which were not acceptable -How to extract information -Underling keywords -Acceptable reasons
R: Did you have enough resources to teach the topic effectively? TA: *Textbooks and past exam papers were there but we needed a variety of updated material to use. Learners did not have mathematical sets to use for the practical activities hence I had to construct models and made copies for them to use.*	-Textbooks and past exam papers were there -Learners did not have mathematical sets -Construct models
R: What new things did you learn through your interaction with the researcher? TA: *I learnt innovative ways of teaching geometry practically which involved all learners even those who are habitually passive in mathematics*	-Innovative ways of teaching geometry -Involved all learners -Habitually passive
R: Are you convinced that *Ubuntu* values can assist in teaching Euclidean geometry effectively? TA: *Yes, the Ubuntu-based activity assisted learners to share knowledge.*	-Assisted learners to share knowledge
R: Do you think learners benefitted during the study? TA: *I believe learners with learning difficulties benefitted a lot because they were all catered for. Even gifted learners had their understanding of theorems enhanced.*	-Learners with learning difficulties benefitted -Gifted learners

R: In your own opinion, do you think you will consider using *Ubuntu* values in your Euclidean geometry classroom again? TA: *Yes, I will consider this. The only aspect I will differ with is the practical part. I will start it in Grade 11 when learners are exposed more to the theorems. I would implement the model there.* R: How do you think *Ubuntu* values can be implemented in a geometry classroom? TA: *During this study, I learnt that if learners sit in pairs or small groups, they learn faster. Learners can be allowed to share ideas in problem-solving.*	-Practical part I will start in Grade 11 -If they sit in pairs and small groups, they learn faster. -Share ideas in problem-solving
R: Do you think it was easy to use the *Ubuntu* model suggested by the researcher? TA: *The first few minutes during the first lesson it was not easy because learners were not used to doing practical activities in mathematics. It was like a cultural shock to them. I could see some learners struggling to use a pair of scissors. However, as the lessons progressed, learners got used to the activities. I think as I said before, these types of activities should start in even Grade 10 or Grade 11 not in Grade 12.* R: What aspects do you suggest adjustments should be made? TA: *I will implement this starting from Grade 11. In Grade 12 it will be repetition and I do practical activities for Grade 12 theorems on proportionality and similarity. Time constraints did not allow all theorems to be proven. However, learners benefitted a lot and even gifted learners admitted they now understood the theorems better than before.*	-Learners were not used to practical activities -Cultural shock -Learners struggling to use a pair of scissors -Will implement this starting from Grade 11 -Time constraints did not allow -Learners benefitted

6.7.2 Findings from the final semi-structured interview with TB

Table 6.37 displays the final semi-structured interview with TB.

Table 6. 37: Final semi-structured interview with TB

Questions and responses	Initial codes
R: During the study, what challenges did you encounter in teaching the topic? If so, how best do you think you addressed these challenges? TB*: The only challenge was that the practical activities were time-consuming.*	-Practical activities were time-consuming.

R: What common difficulties, misconceptions and errors did you identify among the learners? Among them, which ones did you manage to address? TB: *Learners had confusion in applying theorems in problem-solving. They had a tendency of regarding opposite angles of cyclic quadrilaterals as equal. They also confused centre theorems, for example, they would always take the angle subtended by a chord or arc at the centre of a circle as half of the angle subtended by the same arc or chord at the circumference. The experimental group found it easier to understand the theorems because we did the practical activity concerning those theorems. The other group I had to explain and emphasise more to them these relationships by doing more examples with them.*	-Learners had confusion in applying theorems. -The experimental group found it easier to understand. -Doing more examples
R: Did you have enough resources to teach the topic effectively? TB: *The school being rural has no resources. Even mathematics textbooks are in short supply. Mathematical sets are not there. That is why in the practical activity I had to improvise by making models to use in the lessons to ensure time would be saved.*	-School being rural has no resources -Textbooks are in short supply -Had to improvise
R: What new things did you learn through your interaction with the researcher? TB: *I have learnt how to bring theory into practice in geometry. I knew the theorems theoretically but never imagined that experiments can be conducted in this manner and now I understand why mathematics is a science subject.*	-Bring theory into practice -Mathematics is a science subject
R: Are you convinced that *Ubuntu* values can assist in teaching Euclidean geometry effectively? TB: *Yes, I am convinced. I have witnessed learners being free to discover and share information gained from the practical activity. I also was surprised to see learners who had never participated in mathematics since Grade 10 being very active. It shows the Ubuntu-based activities motivated them to act in this manner. Normally these learners would always be passive.*	-Yes I am convinced -Learners being free to discover -Share information -Practical activity -Surprised to see learners who had never participated -*Ubuntu*-based activities motivated -Normally these learners would always be passive
R: Do you think learners benefitted during the study? TB: *I believe learners benefitted a lot by acquiring visualisation skills. Also, learners were able to link theory with practice.*	-Learners benefitted by acquiring visualisation skills. -Able to link theory with practice
R: In your own opinion, do you think you will consider using *Ubuntu* values in your Euclidean geometry classroom again?	

TB: *Yes, I will surely consider using Ubuntu values because I saw many learners working cooperatively sharing ideas. I saw all learners in the experimental group having something to write as compared to those in the control group. Even habitually passive learners in the experimental group participated a lot in the activities.*	-Will surely consider using *Ubuntu* values -Working cooperatively -Sharing ideas -Experimental group having something to write as compared to those in the control group. -Even habitually passive learners participated.
R: How do you think *Ubuntu* values can be implemented in a geometry classroom? TB: *I think in each geometry class the teacher should use indigenous materials as well as allowing learners to discuss in small groups.*	-Indigenous materials -Discuss in small groups
R: Do you think it was easy to use the *Ubuntu* model suggested by the researcher? What aspects do you suggest adjustments should be made? TB: *During the first lesson, it was not easy because learners were resistant to sit with new partners and share. They were used to share with their close friends. I had to use experience and identify learners who were friends not to sit in pairs but assigned them new partners. Learners adapted to the exercise quickly and lessons proceeded. In addition, the practical activities consumed a lot of time, but it was worth it because learners understood the theorems better.*	-During the first lesson it was not easy -Learners were resistant to sit with new partners and share -Used to share with their close friends - Friends not to sit in pairs but assigned them new partners. -Practical activities consumed a lot of time.

6.8 Findings obtained from the post-test

The post-test was administered to all the learners from School A and School B who were taught by Teacher A and Teacher B, respectively during the study. The following are results obtained from the study. The results also took into consideration comparing the groups using Van Hieles' levels of geometric thinking. The levels in summary are:

i. Level 0: Visualisation

ii. Level 1: Analysis (Description)

iii. Level 2: Abstraction (Informal deduction)

iv. Level 3: Formal deduction

v. Level 4: Rigour

Item analysis on the post-test was done using Van Hieles' levels of geometric thinking. The post-test questions were categorised according to Van Hieles' levels. The post-test is in Annexure P. Table 6.38 below summarises the categorisation of the questions. The raw marks obtained by learners per question were converted to percentages of the total marks per question.

Table 6. 38: Categorisation of post-test questions into Van Hieles' levels

Van Hieles' Level	**0**	**1**	**2**	**3**	**4**	**Total marks**
Questions	1.2.1 [1] 1.2.2 [2] 1.2.3 [2] 2.1(a) Reason [1] 2.1(b) Reason [1] 2.1(d) [2]	1.1.1 [1] 1.1.2 [1] 1.2.4 [2] 2.1(c) [2] 3.1 [2] 3.2 [2]	2.1(a) Answer [1] 2.1(b) Answer [1] 2.1(c) [1]	2.2 [3]	3.3 [2] 3.4 [4]	
Total raw marks	9	10	3	3	5	30

6.8.1.Results from the post-test written by Teacher A's learners

Table 6.39 below displays the test scores obtained by TA's learners from both the experimental and control groups.

Table 6. 39: Test scores obtained by TA's learners from the post-test.

Marks of 18 learners of the Experimental group in percentages	**Marks of 18 learners of the Control group in percentages**

63	3
33	20
43	50
43	33
70	20
47	23
30	23
43	33
40	30
40	63
27	47
27	33
27	10
27	70
60	37
73	7
67	43
77	60

Tables 6.40 and Table 6.41 display item analysis of marks obtained by Teacher A's learners based on Van Hieles' levels of geometric thinking.

Table 6. 40: Group statistics at school A post-test marks

	Post-test	Number of learners	Mean	Std. Deviation	Std. Error Mean
Mark	Control group	18	33.61	19.165	4.517
	Experimental group	18	46.50	17.416	4.105

Table 6. 41: Independent samples t-test for School A post-test marks

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper

Mark	Equal variances assumed	.011	.917	-2.112	34	.042	-12.889	6.104	-25.293	-.484
	Equal variances not assumed			-2.112	33.693	.042	-12.889	6.104	-25.298	-.480

The Control group had a mean of 33.61 and a standard deviation of 19.165, and the Experimental group obtained a mean of 46.50 and a standard deviation of 17.416. To examine the difference in achievements between the two groups, the independent samples t-test was conducted. The following null hypothesis was tested at a 95% confidence interval: The null hypothesis was designated H_0 and the alternative hypothesis was designated H_1.

H_0: There is no statistically significant difference in test scores between the experimental group and control group.

H_1: There is a statistically significant difference in test scores between the experimental group and the control group.

The p-value obtained in the t-test was 0.042 which was less than 0.05 less than the significance level which was used in the t-test. This meant the rejection of H_0 and the acceptance of the alternative hypothesis H_1. Therefore, there was a statistically significant difference between the test scores of the experimental group and the control group. In summary, this implies that there was a statistically significant difference between the test scores of learners taught using *Ubuntu*-based dynamic pairs model and those taught using the traditional method of instruction. These results further suggest that teaching Euclidean geometry using *Ubuntu* values was effective.

Item analysis on the post-test was done using Van Hieles' levels of geometric thinking. The post-test questions were categorised according to Van Hieles' levels as explained in Table 6.38 which summarises the categorisation of the questions. The raw marks obtained by learners in each group per level were converted to percentages of the total marks of the respective level.

Table 6. 42: Group statistics at School A item analysis

Group Statistics					
	Treatment	**Number of learners**	**Mean**	**Std. Deviation**	**Std. Error Mean**
Level 0	Control	18	51.28	21.483	5.064
	Experimental	18	63.00	21.379	5.039
Level 1	Control	18	35.00	26.178	6.170
	Experimental	18	48.33	23.577	5.557
Level 2	Control	18	29.56	34.118	8.042
	Experimental	18	49.94	30.924	7.289
Level 3	Control	18	11.11	28.029	6.607
	Experimental	18	26.00	35.452	8.356
Level 4	Control	18	16.67	24.971	5.886
	Experimental	18	22.22	24.628	5.805

Table 6. 43: Independent samples t-test for School A item analysis

		Levene's Test for Equality of Variances		**t-test for Equality of Means**						
		F	**Sig.**	**T**	**Df**	**Sig. (2-tailed)**	**Mean Difference**	**Std. Error Difference**	**95% Confidence Interval of the Difference Lower**	**Upper**
Level 0	Equal variances assumed	.042	.839	-1.641	34	.110	-11.722	7.144	-26.240	2.795
	Equal variances not assumed			-1.641	33.999	.110	-11.722	7.144	-26.240	2.795

Level 1	Equal variances assumed	.867	.358	-1.606	34	.118	-13.333	8.304	-30.209	3.542
	Equal variances not assumed			-1.606	33.634	.118	-13.333	8.304	-30.216	3.549
Level 2	Equal variances assumed	.000	.986	-1.879	34	.069	-20.389	10.853	-42.446	1.668
	Equal variances not assumed			-1.879	33.677	.069	-20.389	10.853	-42.453	1.676
Level 3	Equal variances assumed	5.187	.029	-1.398	34	.171	-14.889	10.652	-36.537	6.759
	Equal variances not assumed			-1.398	32.282	.172	-14.889	10.652	-36.579	6.802
Level 4	Equal variances assumed	.157	.694	-.672	34	.506	-5.556	8.267	-22.355	11.244
	Equal variances not assumed			-.672	33.994	.506	-5.556	8.267	-22.355	11.244

Hypothesis testing was done using achievements according to Van Hieles' levels.

<u>Visualisation</u>

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in visualisation.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in visualisation.

The p-value is greater than 0.05 and H_0 is accepted. This implies that the dynamic pairs model did not have a significant impact on learners' visualisation.

Analysis

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in analysis.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in analysis.

The p-value is greater than 0.05 and H_0 is accepted. This implies that the dynamic pairs model did not have a significant impact on learners' skill of analysis.

Informal deduction

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in the informal deduction.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in informal deduction.

The p-value is greater than 0.05 and H_0 is accepted. This implies that the dynamic pairs model did not have a significant impact on learners' skill of informal deduction.

Formal deduction level

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in formal deduction.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in formal deduction.

The p-value is greater than 0.05 and H_0 is accepted. This implies that the dynamic pairs model did not have a significant impact on learners' skill of formal deduction.

Rigour level

H_0: There is no significant difference in test scores between the experimental group and the control group in rigour.

H_1: There is a significant difference in test scores between the experimental group and the control group in rigour.

The p-value is greater than 0.05 and H_0 is accepted. This implies that the dynamic pairs model did not have a significant impact on learners' rigour.

Looking at the statistical results obtained, the implementation of the *Ubuntu*-based model by Teacher A only had an impact on the overall performance of learners in the post-test but achievement in Van Hieles' levels of geometric thought did not differ significantly.

6.8.2 Results from the post-test written by Teacher B's learners

Table 6.44 below displays the marks obtained by TB's learners in the post-test.

Table 6. 44: Test scores obtained by TB's learners from the post-test.

Marks of the experimental group in percentages For 16 learners	Marks of the control group in percentages for 18 learners
90	37
83	23
73	43
93	27
73	63
73	43
80	27
77	50
73	50
80	57
90	23
57	33
70	23
70	33

93	37
40	60
	40
	50

Table 6.45 summarises descriptive statistics for TB's learners in the post-test scores.

Table 6. 45: Group statistics at School B post-test marks

	Post-test	Number of learners	Mean	Std. Deviation	Std. Error Mean
Mark	Control group	18	39.94	12.964	3.056
	Experimental group	16	75.94	13.704	3.426

Table 6. 46: Independent samples t-test for school B post-test marks

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Mark	Equal variances assumed	.109	.744	-7.867	32	.000	-35.993	4.575	-45.312	-26.674
	Equal variances not assumed			-7.841	31.030	.000	-35.993	4.591	-45.355	-26.631

The Control group had a mean of 39.94 and a standard deviation of 12.964, and the Experimental group obtained a mean of 75.94 and a standard deviation of 13.704. To examine the difference in achievements between the two groups, the independent samples t-test was conducted. The following null hypothesis was tested at a 95%

confidence interval: The null hypothesis was designated H_0 and the alternative hypothesis was designated H_1.

H_0: There is no statistically significant difference in test scores between the experimental group and control groups.

H_1: There is a statistically significant difference in test scores between the experimental group and the control group.

The p-value obtained in the t-test was 0.000 which was less than 0.05 less than the significance level which was used in the t-test. This meant the rejection of H_0 and the acceptance of the alternative hypothesis H_1. Therefore, there was a statistically significant difference between the test scores of the experimental group and the control group. In summary, this implies that for Teacher B's learners there was a statistically significant difference between the test scores of learners taught using *Ubuntu*-based dynamic pairs model and those taught using the traditional method of instruction. Table 6.45 and Table 6.46 display item analysis based on Van Hieles levels of geometric thinking.

Item analysis on the post-test was done using Van Hieles' levels of geometric thinking. The post-test questions were categorised according to Van Hieles' levels as explained in Table 6.38 which summarises the categorisation of the questions. The raw marks obtained by learners in each group per level were converted to percentages of the total marks of the respective level.

Table 6. 47: Group statistics at School B item analysis

Group Statistics					
	Treatment	Number of learners	Mean	Std. Deviation	Std. Error Mean

Level 0	Control group	18	56.22	15.713	3.704
	Experimental group	16	79.25	32.921	8.230
Level 1	Control group	18	43.33	22.492	5.301
	Experimental group	16	81.88	13.276	3.319
Level 2	Control group	18	50.06	20.882	4.922
	Experimental group	16	71.06	46.859	11.715
Level 3	Control group	18	12.83	16.554	3.902
	Experimental group	16	47.88	38.505	9.626
Level 4	Control group	18	16.67	21.963	5.177
	Experimental group	16	67.50	29.098	7.274

Table 6. 48: Independent samples t-test for School B item analysis

		Levene's Test for Equality of Variances		**t-test for Equality of Means**						
									95% Confidence Interval of the Difference	
		F	**Sig.**	**T**	**Df**	**Sig. (2-tailed)**	**Mean Difference**	**Std. Error Difference**	**Lower**	**Upper**
Level 0	Equal variances assumed	3.343	.077	-2.651	32	.012	-23.028	8.687	-40.722	-5.333
	Equal variances not assumed			-2.552	20.932	.019	-23.028	9.025	-41.800	-4.255
Level 1	Equal variances assumed	7.392	.010	-5.984	32	.000	-38.542	6.441	-51.661	-25.423
	Equal variances not assumed			-6.162	28.054	.000	-38.542	6.255	-51.353	-25.731
Level 2	Equal variances assumed	.020	.889	-1.722	32	.095	-21.007	12.201	-45.859	3.845

	Equal variances not assumed			-1.653	20.207	.114	-21.007	12.707	-47.495	5.482
Level 3	Equal variances assumed	17.401	.000	-3.518	32	.001	-35.042	9.962	-55.333	-14.750
	Equal variances not assumed			-3.374	19.861	.003	-35.042	10.387	-56.718	-13.365
Level 4	Equal variances assumed	.549	.464	-5.789	32	.000	-50.833	8.781	-68.720	-32.947
	Equal variances not assumed			-5.694	27.758	.000	-50.833	8.928	-69.129	-32.537

Hypothesis testing was done using achievements according to Van Hieles' levels.

<u>Visualisation</u>

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in visualisation.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in visualisation.

The p-value is 0.012 which is less than 0.05. H_0 is rejected and H_1 is accepted. This implies that the *Ubuntu*-based dynamic pairs model had an impact on learners' visualisation.

<u>Analysis</u>

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in analysis.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in analysis.

The p-value is 0.000 which is less than 0.05. H_0 is rejected and H_1 is accepted. This implies that the *Ubuntu*-based dynamic pairs model had an impact on learners' skill of analysis.

<u>Informal deduction</u>

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in informal deduction

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in informal deduction.

The p-value is 0.095 which is greater than 0.05. H_0 is accepted. This implies that the *Ubuntu*-based dynamic pairs model had no impact on learners' informal deduction.

<u>Formal deduction level</u>

$H_{0:}$ There is no significant difference in test scores between the experimental group and the control group in formal deduction.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in formal deduction.

The p-value is 0.001 which is less than 0.05. H_0 is rejected and H_1 is accepted. This implies that the *Ubuntu*-based dynamic pairs model had an impact on learners' formal deduction.

<u>Rigour level</u>

$H_{0:}$ There is no significant difference in achievements between the experimental group and the control group in rigour.

$H_{1:}$ There is a significant difference in test scores between the experimental group and the control group in rigour.

The p-value is 0.000 which is less than 0.05. H_0 is rejected and H_1 is accepted. This implies that the *Ubuntu*-based dynamic pairs model had an impact on learners' rigour.

Analysis of the statistical results obtained yielded that the implementation of the *Ubuntu*-based model by Teacher B had an impact on the overall performance of learners in the post-test as well as on achievements in Van Hieles' levels of geometric thought except in the skill of informal deduction where there was a significant difference.

6. 9 Problems displayed by learners in the post-test

Learners from both schools displayed similar problems despite that Teacher A's learners scored lower marks in the post-test as compared to those of Teacher B. Teacher A's learners from both the control and experimental showed some improvement. Both groups from teacher A showed improvement in performance as compared to how they performed in the pre-test. Although after the teaching of the three lessons per group, the following challenges were displayed in the post-test:

i. Learners still had challenges with tangent theorems.
ii. Learners had challenges in using converse theorems of cyclic quadrilaterals.
iii. Learners had challenges in stating acceptable reasons correctly.
iv. Learners had challenges in applying a series of theorems in solving a geometric problem.
v. Learners had challenges in conducting geometric proofs.
vi. Learners generally operated on Level 0 and Level 1 of the Van Hieles' levels of geometric thinking as displayed in Table 6.19 and Table 6.23.

The challenges mentioned above were in line with those displayed in the pre-test and those observed in classroom observation. Also, the challenges were not as bad as in the pre-test. However, some learners from Teacher B's experimental group did the geometric proofs brilliantly although generally challenges were displayed on this aspect. Furthermore, learners in the study generally operated on lower levels (Level 0 and Level 1) of geometric thinking. Some gifted learners displayed higher levels of thinking. Some challenges displayed by learners on the use of theorems confirm the contents of diagnostic reports although some learners showed that they never had those challenges.

6.10 Summary of themes derived from qualitative data

Manual coding of the qualitative data was done from the transcribed data from semi-structured interviews, classroom observation and document analysis. From the codes obtained from the semi-structured interviews, classroom observation and document analysis, categories and themes were determined. Table 6.49 summarises the codes, categories and emerging themes derived from all the qualitative data collected during the study.

Table 6. 49: Summary of codes, categories and emerging themes

Summary of codes	Categories	Emerging themes
-Some learners showed learning difficulties in geometry -Learners lack understanding of theorems -Inability to express theorems in words -Inability to state correct acceptable reasons -Lack of problem-solving skills -Lack of spatial knowledge -Lack of visual skills and deductive reasoning skills	1. Learners' lack of geometric thinking and problem-solving skills 2. Learners' lack of geometry language proficiency 3. Learners' forgetfulness and poor background in geometry 4. Learners' negative attitude towards geometry	**Theme 1** Difficulties encountered in the teaching and learning of Euclidean geometry

-Learners' forgetfulness of theorems -Inability to recite theorems correctly. -Problems in stating acceptable reasons -Learners' forgetfulness of what was learnt -Learners' forgetfulness of properties of geometric figures -Problems in applying the converse theorem -Problems in working with the tan-chord theorem -Learners stated wrong reasons in problem-solving -Learners' errors in applying theorems and their converses -Problems in analysing questions and diagrams -Learners initially had a negative attitude -Learners' negative attitude -During the first lesson, learners were resistant to share -Learners were comfortable in working with centre theorems -Learners lack geometry language proficiency as well as proficiency in English		
-Attendance to workshops to sharpen pedagogical skills -Use of revision as an instructional strategy -Oral probing as an instructional strategy -Prior knowledge -Examples as an instructional strategy -Chorus answers -Effects of chorus answers	1. *Ubuntu* values in TA's PCK in geometry 2. *Ubuntu* values in TB's PCK in geometry 3. *Ubuntu* aspects that lacked in the two participants' teaching practice	**Theme 2** *Ubuntu* values in use of PCK and MKT in teaching geometry

-Oral probing as an instructional strategy -Use of examples an instructional strategy -Lecture method as an Instructional strategy -Collective revision of homework -Teacher taught learner techniques in problem-solving -Teacher's knowledge of the curriculum -Lesson plans were available -Lesson plans of the experimental group reflected *Ubuntu* attributes -Lesson plans detailed with relevant information -Teacher used a red pen -Teacher indicated learners' errors -Relevant comments made -Hand-outs given to learners -Some learners did not write the work given. -Evidence of use of curriculum documents on lesson plans		
-No community members to assist learners -Failure to assist like a death sentence to the community -Sharing of ideas and knowledge -Willingness to share -Sharing encouraged -Sharing encouraged -Collective revision done -No sharing of ideas and knowledge -Collective revision of homework	1. The impact of sharing knowledge and other resources. 2. Other *Ubuntu* attributes important in learning geometry	**<u>Theme 3</u>** *Ubuntu* values important in teaching Euclidean geometry

-Learners' full participation -Gifted learners dominated participation. -Sharing was not encouraged. -Learners were active. -Working cooperatively -Sharing ideas -Experimental group having something to write as compared to those in the control group -Even habitually passive learners participated. -Hand-outs were given to learners. -Empathy -Compassion -Love -Teacher demonstrated patience to learners -Facial expression and body language showed teacher enjoyed assisting		
-Written work in workbooks -Written work in line with examination standards --Learners in experimental group understood the content faster.	1. Influence of *Ubuntu* values on TA's learners' academic performance 2. Influence of *Ubuntu* values on TB's learners' academic performance	Theme 4 The influence of *Ubuntu* values on learners' academic performance

6. 11 Conclusion

This chapter comprehensively presented, interpreted and analysed qualitative data gathered from semi-structured interviews, classroom observation and document analysis. Themes were generated from the findings and discussion followed. Furthermore, quantitative data from the pre-test and post-test were summarised using tables, graphs and figures. In addition, the numeric data were analysed using hypothesis testing. The independent samples t-test was the statistic that was used to conduct the hypothesis testing. Both quantitative and qualitative results were presented and analysed separately and then integrated at the end. The analysis of classroom observation either confirmed or refuted the findings of the data collected from semi-structured interviews. Despite those dynamics, the overall findings indicate that the use of *Ubuntu* values in teaching Euclidean geometry is effective. The *Ubuntu* values promote effective classroom interaction patterns giving rise to more learners participating apart from gifted learners only. Chapter 7 presents a comprehensive discussion of the finding of the study.

CHAPTER 7: DISCUSSION OF FINDINGS

7.1 Introduction

This chapter discusses the findings of the main study and how the two mathematics teachers delivered their lessons to the control and experimental groups. In addition, findings from semi-structured interviews, pre-tests and post-tests, classroom observations and document review are examined. The components of the theoretical framework namely *Ubuntu* values, social interdependence theory [SIT] and Van Hieles' theory became the lens through which discussion of the findings were facilitated. The discussion begins by highlighting the research questions that this study sought to respond to.

Those research questions were:

How can *Ubuntu* values enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry?

To respond to the main research question, the following sub-questions were asked:

i. How can the teachers' PCK be enhanced for effective and meaningful teaching of Grade 12 Euclidean geometry?
ii. What difficulties are experienced by mathematics teachers in the teaching of Grade 12 Euclidean geometry?
iii. What aspects of *Ubuntu* philosophy should teachers apply for the enhancement of the teaching of Euclidean geometry in Grade 12?
iv. To what extent do collective work and collaboration impact the teaching of Grade 12 Euclidean geometry?
v. How can *Ubuntu* philosophy be used to transform the teaching of Grade 12 Euclidean geometry?

7.2 Themes for the discussion of the findings

In Chapter 6, themes that emerged from the presentation and analysis of the data collected were tabulated. The following themes were obtained by using thematic analysis of qualitative data and statistical analysis of the quantitative data:

i. Difficulties encountered in the teaching of Euclidean geometry,

ii. *Ubuntu* values in use of PCK and MKT in teaching Euclidean geometry,
iii. *Ubuntu* values important in the teaching of Euclidean geometry, and
iv. The influence of *Ubuntu* values on learners’ academic performance.

The themes mentioned above are discussed at length from the collected data using components of the theoretical framework and literature reviewed in the study.

7.3 Difficulties encountered in the teaching of Euclidean geometry

During the preliminary semi-structured interviews, the two participants argued that one of the major challenges they faced was the shortage of resources to use in delivering lessons. The shortage of resources ranged from the shortage of relevant textbooks to the shortage of mathematical instruments to use in geometrical constructions. The participants used their expertise to improvise by using already made models in delivering the lessons during the study. The following are some of the other difficulties encountered in the teaching of Euclidean geometry during the study.

7.3.1 Learners’ lack of geometric thinking and problem-solving skills

The learners’ lack of geometric thinking and problem-solving skills for TA and TB are presented in the next section.

7.3.1.1 Learners of TA’s lack of geometric thinking and problem-solving skills

During the preliminary semi-structured interview, TA expressed that one of the common challenges encountered by learners was a lack of visualisation skills. In circle geometry, visualisation consisted of identifying angles that were related, angles not related and features of any geometric figure. During classroom observation, most of TA’s learners failed to identify related angles using theorems studied which confirmed his claim that learners lacked visualisation skills. Learners from the experimental group encountered lesser visualisation challenges than those from the control group because of their engagement in practical activities in which they managed to handle geometric figures and visualise for themselves.

The learners from the control group could not easily identify the interior and opposite exterior angles of a cyclic quadrilateral. The failure by learners from the control group to identify and establish relationships among angles in the solution of exercises can be associated with the absence of practical activities in that group. During the practical activities, learners in pairs shared ideas in making cuttings and compared sizes of angles and were subsequently required to establish relationships between the angles. The failure of learners from the control group to visualise important features of geometric figures concurs with Solaiman, Magno and Aman (2012) who argue that most learners operate at pre-recognition level of geometric thought, which is a lower level to visualisation according to Clement and Battista (1992). The learners from the experimental group might have been assisted to visualise features of figures and relationships by engaging in inter-pair and intra-pair interactions and engagement in practical activities.

It was also observed that TA's learners from both groups failed to solve some problems due to lack of knowledge of properties of basic geometric figures especially working with kites in Lesson 3. This confirms TA's argument during the preliminary interview that learners lacked problem-solving skills in geometry as well as the ability to use theorems and their converses in solving exercises. The question in Lesson 3 which needed the use of properties of a kite was poorly answered by most of the learners. The poor performance by learners in that question is confirmed by some researchers (Mammen & Alex, 2016; Siyepu, 2005; Mammali, 2015) and diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020) which reveals that learners lack basic geometry skills from previous grades. According to the curriculum, when this study was conducted, properties of basic geometric figures were part of the content studied at the primary school level and lower secondary school grades in South African schools.

It was also during classroom observation that most of the learners displayed impetuous tendencies in answering questions. They answered questions without engaging in critical analysis of the questions, diagrams given and key statements in the questions to assist in obtaining relevant theorems to use in problem-solving. In addition, learners had challenges in stating acceptable reasons in the solution of problems. Although learners encountered challenges in problem-solving, they showed interest in understanding the techniques used in solving problems. They even requested extra tuition time and more work for practice from their teacher. In response to learners' requests, TA organised to teach the learners some techniques in problem-solving in geometry during the study time. Most of the learners showed a lot of interest and enthusiasm to learn the topic despite encountering challenges. This might have been caused by the conducive classroom interactions and the teacher's ability to establish friendly relations with the learners. In common practice, when learners discover that their teacher is friendly and supportive, their perspectives change towards a subject. The enthusiasm demonstrated by TA's learners from both groups to learn problem-solving techniques contradicts Sunzuma *et al.* (2013) who argue that learners dislike problem-solving in geometry. In the pre-test, most of the learners failed to prove that the given quadrilateral was a cyclic quadrilateral and they also failed to conduct some other trivial proofs. This finding concurs with assertions by researchers (Mammen & Alex, 2016; Clements & Battista, 1992; Siyepu, 2005), that high school learners in South Africa are not ready for formal geometric proofs because of weak background in geometry.

7.3.1.2 Learners of TB's lack of geometric thinking and problem-solving skills

During the preliminary semi-structured interview, TB mentioned that one of the common challenges that learners faced was using visual skills. Failure by learners from both groups to identify and establish relationships between angles during lessons validates TB's claim that indeed learners had problems of visualisation. However, engagement in practical activities and sharing of ideas and knowledge among learners in the experimental group might have assisted in eliminating their problems in visualisation. Those learners from the control group took time to identify and establish the relationships between angles due to lack of assistance from peers and lack of practical activities which required them to manipulate and handle models.

The learners from the control group had challenges in working with the tan-chord theorem. It was also difficult for many of them to quickly identify the angles related through the tan-chord theorem. On the contrary, learners from the experimental group encountered fewer challenges in understanding the theorem. The use of practical activities and sharing by learners might have assisted learners from the experimental group to understand the tan-chord theorem with ease. However, learners from both of TB's groups could not easily identify the interior and opposite exterior angles of a cyclic quadrilateral. Also, TB's learners failed to visualise and identify related angles using the theorems studied. These findings suggest that those learners had problems in visualisation which is the first level of Van Hieles' theory of geometric thinking.

During the preliminary semi-structured interview, TB argued that learners lacked problem-solving skills. In class, some learners were very quick to answer questions without engaging in deep and critical analysis of the questions and diagrams given. The lack of problem-solving skills by learners expressed by TB is confirmed by diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020) which argue that learners lack problem-solving skills as well as the ability to apply theorems in solving exercises. Learners from both groups displayed great enthusiasm in learning problem-solving techniques. Most of the learners paid serious attention to the teacher's explanations. This finding contradicts Sunzuma *et al.* (2013) who argue that learners dislike problem-solving in geometry. The failure by all learners to prove that the quadrilateral in the pre-test is cyclic was consistent with the assertions by researchers (Mammen & Alex, 2016; Clements & Battista, 1992; Siyepu, 2005), that high school learners in South Africa are not ready for formal geometric proofs because of weak background in geometry.

7.3.2 Learners' lack of geometry language proficiency

Learners' lack of geometric communication and language proficiency is discussed in the next sections.

7.3.2.1 Learners of TA's lack of geometry language proficiency

TA mentioned that one of the causes of learners' poor performance in geometry was their lack of proficiency in geometry language and terminology. The teacher's argument concurs with De Villiers and Njisane (1987) who express that learners' poor performance is also attributable to problems in language and lack of knowledge of basic terminology in geometry. TA's concerns about learners' lack of geometry language proficiency were a result of their challenges in proficiency in English. English was the language of instruction in the schools where this study was conducted. According to Atebe and Schafer (2010), proficiency in the language of instruction is crucial because once learners become proficient, it would be easy to understand geometry concepts. The problem in English language proficiency is also echoed by Mammali (2015) who argues that learners have problems communicating in English and that the use of English as a second language is one of the reasons why mathematics is poorly performed in black schools.

It would require teachers to be patient to teach learners English in conjunction with geometry content. Most rural schools in Cacadu CMC were populated with learners who were more proficient in isiXhosa than in English, the language of instruction. The teacher's practice of accommodating learners to communicate in isiXhosa during lessons is supported by (Barnhardt & Kawagley, 2005; Panthi & Belbase 2017) who assert that children think in their mother tongue and then try to express themselves in the language of instruction. The granting of freedom to learners to use isiXhosa their mother tongue was a gesture to empower them in understanding the geometry content conceptually.

Mathematics was examinable in English in the rural schools of Cacadu CMC where learners faced difficulties in the usage of that language. The teacher's practice of resorting to code-switching was a prudent and potent idea as suggested by Setati (2009) to assist learners to understand. During the study, TA tried to address language problems by allowing learners to use vernacular language in class discussions although the presentation of solutions to problems was done in English. Accommodating isiXhosa during lessons might have motivated learners to participate and contribute meaningfully during classroom activities. The teacher exercised a lot of patience and care during lesson delivery by gradually teaching mathematical language to the learners. The practice of gradually teaching mathematical language in geometry is supported by Mntunjani (2017) who suggests that teachers need to exercise patience, care and flexibility in language use when teaching geometry because there are a lot of concepts involved.

7.3.2.2 Learners of TB's' lack of geometry language proficiency

Similar to TA, one of the causes of learners' poor performance in geometry identified by TB was their lack of proficiency in geometry language and terminology. All of TB's learners also came from isiXhosa speaking backgrounds and their proficiency in English was poor. TB's concern about his learners' lack of geometry language proficiency was a result of their challenges in proficiency in English which was the language of instruction. TB's concern is also echoed by Mammali (2015) who argues that the use of English as a second language is one of the reasons why mathematics is poorly performed in black schools. TB's learners faced the challenge of having to learn to express themselves in English first and thereafter learn geometry language in English. The teacher's argument was consistent with De Villiers and Njisane (1987) who express that learners' poor performance is also attributable to problems in language and lack of knowledge of basic terminology in geometry.

During the study, TB granted his learners freedom to use vernacular language in class discussions although the presentation of solutions to problems was required in English. The teacher was patient in teaching learners how to express themselves in English and to answer geometry questions using correct terminology. The patience displayed by TB is supported by Mntunjani (2017) who suggests that teachers need to exercise patience, care and flexibility in language use when teaching geometry because there are a lot of concepts involved. Accommodating learners to be free in using their home language motivated them to participate and contribute meaningfully during class activities. The reason for exercising patience was that all mathematics examinations were in English. Because of love for learners, TB resorted to code-switching and gradually taught learners how to present their solutions in English. The practice of code-switching is supported by Setati (2009) who asserts that it assists learners to understand what teachers deliver in the classroom.

7.3.3 Learners' forgetfulness and poor background in geometry

The next sections discuss learners' forgetfulness and poor background in geometry.

7.3.3.1 Learners of TA's forgetfulness and poor geometry background

During the preliminary semi-structured interview, TA indicated that all the geometry content was covered in Grade 11. Furthermore, TA explained that all the basic theorems were learnt in previous grades including trivial proofs. However, the teacher revealed that complex geometrical proofs were not done. The expectation was that the generality of TA's learners would be able to answer some basic questions from Grade 11 geometry with ease. On the contrary, the poor learners' scores from the pre-test which was composed of basic Grade 11 geometry contradict the teacher's claim that the learners had basic geometry knowledge (see scores in Table 6.2). Some of the learners obtained zeros as scores. The poor scores obtained by TA's learners in the pre-test may be associated with their poor geometry background or their forgetfulness of the content learnt in previous grades. The poor scores may also be associated with the characteristics of teachers who taught those learners in other previous grades.

The following difficulties were diagnosed from the pre-test scripts of TA's learners:

i. Learners' non-remembrance of properties of geometric figures such as kites and cyclic quadrilaterals,
ii. Learners' failure to apply the theorem of the relationship between the tangent and radius was a problem for all the learners,
iii. Most of the learners failed to state correct acceptable reasons,

iv. Learners had forgotten the relationships of angles between parallel lines,

v. Failure to solve the problem on the cyclic quadrilateral,

vi. Failure to apply the converse theorems of a cyclic quadrilateral, and

vii. Most of the learners found it difficult to identify the size of the angle subtended by the diameter.

The challenges observed from the pre-test concur with those listed in diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020) and mentioned by some researchers (Mammen & Alex, 2016; Siyepu, 2005; Mammali, 2015) which indicate that TA's learners' poor performance in geometry was a common phenomenon countrywide.

During classroom observation, some of TA's learners from both groups failed to apply basic primary school geometry concepts. Also, some learners could not apply theorems learnt in Grade 11 correctly. Generally, learners from both groups could not solve circle geometry questions from Grade 11 although those from the experimental group presented better solutions as compared to the learners from the control group by the end of the study. The challenges of learners in solving problems from previous grades concur with De Villiers (2010) that one of the causes of learners' poor performance in geometry was their poor background in geometry.

7.3.3.2 Learners of TB's forgetfulness and poor geometry background

Findings from the pre-test, learners' workbooks and classroom observation reveal that TB's learners had poor geometry background. It was also observed that the challenges that were encountered by TB's learners were the same as those encountered by TA's learners. The difficulties outlined encountered by learners during the study were consistent with findings from diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020). Those reports mention that learners displayed challenges in answering examination questions in Euclidean geometry in Grade 12 which included lack of basic geometry from previous grades, difficulties in the use of theorems and their converses and problems in working with cyclic quadrilaterals and many others. This suggests that teachers should always work using policy documents to get hints and tips on how to approach the topic.

Furthermore, the scores obtained by TB's learners in the pre-test were poor (see Table 6.5). Generally, all of TB's learners involved in the study had forgotten the content from previous grades inclusive of Grade 11 geometry. However, some of the learners still remembered the mid-point theorem from Grade 10. During the preliminary semi-structured interview, TB indicated that the geometry content was covered well in Grade 11. The teacher explained that the poor performance by learners in the pre-test was very discouraging considering that those learners used to perform well in the previous grade. The challenges displayed by the learners indicate that learners either had forgotten Grade 11 content or that their background in Euclidean geometry was poor.

The learners' difficulties in using converse theorems of cyclic quadrilateral were also prevalent in both of TB's groups involved in the study. The information revealed by TB in the preliminary semi-structured interview suggests that learners had forgotten the topic completely and the background was not strong. The challenges diagnosed from the pre-test were consistent with those indicated in diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020). The difficulties found in TB's learners were not peculiar to his school only but were the same challenges experienced in schools countrywide. The poor test scores by learners in the pre-test is an indication that they had forgotten the content from previous grades. During lessons, some of the learners in control groups failed to apply basic primary school geometry concepts. This finding concurs with some researchers (De Villiers, 2010; Feza & Webb, 2005) who argue that learners' primary school background is poor. The poor level of performance by TB's learners suggests that the arguments from research needed serious consideration.

7.3.4 Learners' negative attitude towards Euclidean geometry

Learners' negative attitude towards Euclidean geometry is discussed in the next sections.

7.3.4.1 Learners of TA's negative attitude towards Euclidean geometry

During the preliminary semi-structured interview, TA mentioned that one of the causes of failure in Euclidean geometry by learners could be associated with their negative attitude towards the topic. It was amazing to witness great enthusiasm in learning Euclidean geometry among TA's learners from both the control group and experimental group. Some learners from the control group always called the teacher seeking assistance and clarification on some difficult parts of the problems given. Also, those learners seated in pairs from the experimental group did not tire in asking for help from the teacher and other neighbouring pairs. All learners from both groups requested TA to avail himself at study time for extra tuition and consultation. The enthusiasm by those learners can be an indication of a positive attitude towards the topic refuting the teacher's assumptions. This behaviour by TA's learners also contradicts some researchers (Mello & Martins, 2015; Howson, 2000) who argue that learners display a negative attitude towards geometry.

Results from the pre-test of TA's learners were generally poor and this may be linked to learners' poor geometry background. However, TA indicated that he tried his best to teach those learners all the geometry content including the riders thoroughly in

Grade 11. The active classroom interactions by learners from the experimental group and the zeal to learn by learners from the control group were evidence that all TA's learners were not deterred by their past experiences in geometry. Tella (2007) associates this negative attitude with the discord between middle school geometry and high school geometry. In this case, TA's learners despite having scored poorly in the pre-test and displaying poor geometry background still showed that they were curious to learn.

During lessons with both groups, TA's facial expression and body language showed that he enjoyed assisting learners. Every learner was free to seek help from the teacher without fear. The teacher was always available for each learner in both classes. To the learners from the control group, TA moved from one learner to the other explaining and assisting each one of them to understand procedures on how to answer questions. TA also approached all learners seated in pairs to assist and enabled them to exchange ideas on how to answer questions. The classroom behaviour of working together in harmony displayed by TA and the learners can be associated with the existence of cordial teacher-learner relations. The cordial relations between TA and his learners was an indication of the existence of social interdependence which researchers (Slavin, 1991; Walmsely & Muniz, 2003; Johnson & Johnson, 2013) argue that it results in higher academic achievement, high levels of productivity and success, better communication skills, successful social and academic group interactions, high levels of self-esteem among learners. Learners' attendance to study sessions and discussion groups after school can be associated with their motivation and positive attitude to study geometry. The cooperation among learners and their teacher is supported by researchers (Tutu, 2008; Broodryk, 2006) who argue that *Ubuntu* is a panacea for unity, reconciliation, love and empathy to others. The teacher’s availability to assist learners every time could be the source of motivation for those learners. In addition, the loss of classroom time due to the COVID-19 pandemic and impending examinations might have alerted learners to pay more attention to their studies. Hence, they would need to prepare for examinations thoroughly.

7.3.4.2 Learners of TB’s negative attitude towards Euclidean geometry

In a similar fashion to TA’s argument, TB also argued that one of the causes of failure in Euclidean geometry by learners could be associated with a negative attitude towards the topic. During classroom observation, learners from the experimental group actively participated in the practical activities. They shared pairs of scissors and models and discussed how to do the activities. In Lesson 3, which was predominantly a written activity lesson, learners from the experimental group shared ideas in pairs on how to solve exercises. They also compared their solutions with other neighbouring pairs. The cooperation among learners and their teacher is recommended by McCluskey and Lephalala (2010) who argue that the core of *Ubuntu* values recognise and acknowledge people as socially interdependent beings. Learners from the control individually called the teacher seeking assistance in problem-solving. Those learners who had challenges in understanding requested TB to allow the exercise to be done on the chalkboard for everyone to understand.

During the study, it was amazing to observe the quest for knowledge by the TB’s learners. This contradicted expectations that those learners would display negative attitudes as expressed by their teacher. The learners’ enthusiasm could be a result of TB’s willingness to assist them. The zeal from the learners during lessons also contradicts some researchers (Mello & Martins, 2015; Howson, 2000) who argue that learners display a negative attitude towards geometry. The learners from both groups had the freedom to communicate with TB about their needs. TB availed himself for extra lessons at study time to support the learners. The freedom by learners to seek assistance from their teacher and the teachers' willingness to attend the requested remedial lessons and extra classes was an indication of amicable relations between TB and his learners.

7.4 *Ubuntu* values in teachers' PCK in teaching Euclidean geometry

This section chronicles and discusses each of the two participants' PCK in geometry through the lens of *Ubuntu* philosophy.

7.4.1. *Ubuntu* values in TA's PCK in geometry

During the preliminary semi-structured interview TA said,

"*I need more workshops on how to teach the topic effectively. Attendance to workshops sharpens the teachers pedagogical and didactic skills because a teacher is updated on how to teach a particular section of content effectively using modern instructional strategies"*.

This expression by TA can be interpreted to be his desire to enhance his pedagogical skills in the teaching of geometry. TA expressed that he was not able to teach geometrical constructions effectively. This could have been a result of the non-existence of content on geometrical constructions throughout the mathematics curriculum from lower grades up to Grade 12 in the South African curriculum. TA further expressed that he desired to attend more workshops and seminars to be capacitated with more knowledge in teaching geometry. The desire by TA to attend workshops and seminars is recommended by Ali, Bhagawati and Sammah (2014) who argue that teachers should be trained through seminars and workshops on how to teach geometry effectively.

The desire to equip himself with pedagogical and didactic skills in geometry teaching indicates that TA was concerned with learners' future and academic achievements. It was a pure display of compassion and empathy for learners as well as a commitment to the teaching profession. According to Van der Berg (2015), the chief cause of poor results in schools is teachers' weak content knowledge and pedagogical skills. TA's quest for knowledge sought to strengthen his PCK and MKT in geometry teaching and make a difference in learners' lives. *Ubuntu* was manifested when teachers realised their inadequacies and thought of offering their best to the learners and the community.

Willingness to attend workshops and seminars by TA was a sign of *Ubuntu* because the teacher implicitly expressed his humility and respect for other teachers' capabilities to facilitate in INSET programmes. This is consistent with the assertion by Mammali (2015) who expresses that professional interactions among teachers in professional learning communities, seminars and workshops improve teachers' MKT and PCK in Euclidean geometry. The willingness to interact with other teachers in workshops and seminars is supported by some researchers (McKay, 1995; Quan-Baffour & Romm, 2015) who advocate for the promotion of social interdependence which is an aspect closely intertwined with *Ubuntu*. This suggests that *Ubuntu*-inspired teachers inherently value positive interdependence and positive interactions within school environments. In general, Teachers without *Ubuntu* naturally would always think of themselves as the best and fail to acknowledge that other people might have the knowledge necessary for them to obtain. In addition, the absence of *Ubuntu* makes some teachers fail to network with others for the sake of sharing ideas and knowledge because of pride and personal ego. During workshops, teachers who are knowledgeable with Euclidean geometry share the knowledge they possess of Euclidean geometry including the Van Hieles' theory with other mathematics teachers.

This is encouraged by Sethunga *et al.* (2016) who suggest that teachers should work collaboratively with other teachers and network in professional learning communities to acquire the skills they lack. TA's PCK might have been enriched during the period of acquisition of academic and professional qualifications. Attendance to workshops and seminars in geometry teaching might also have contributed to TA's acquisition of some PCK in geometry.

During the preliminary semi-structured interviews, TA was asked if he had ever used collective learning strategies such as group work and teamwork before. TA responded:
"Yes, I have used group work before but not in normal daily lessons but revision classes towards examinations. Group work for me is time-consuming. We have limited time to accomplish work on the annual teaching plan [ATP]."
TA indicated that he had used collective learning strategies before only during revision sessions with learners towards examination periods. Furthermore, during the final interviews, TA was asked if he would consider using *Ubuntu* values in their Euclidean geometry classrooms again. TA responded:
"*Yes, I will consider this. The only aspect I will differ with is that the practical part. I will start it in grade 11 when learners are exposed more to the theorems. I would implement the model there."*
The response by TA indicates that he was convinced that incorporation of *Ubuntu* values in geometry classrooms was crucial and he would consider incorporating them in future. Even during classroom observation with the control group, TA occasionally engaged learners in classroom discussions where the learners shared knowledge and ideas on how to solve some exercises with the whole class.

TA occasionally requested some of the gifted learners to explain their solutions and ideas to the whole class on the chalkboard. During classroom observation, TA delivered three lessons to each of the two groups. In compliance with the agreements made during the study, TA used the Dynamic Pairs Model to teach the experimental group only. The control group was taught predominantly using the lecture method from Lesson 1 to Lesson 3. The traditional instructional strategies for teaching the control group during the study were not only limited to the lecture method. TA could have used other instructional strategies such as group work, demonstration and class discussion among others. The predominant use of the lecture method as an instructional strategy with the control group might imply that TA had limited PCK in geometry. The use of the lecture method by TA to the control group contradicted recommendations by Jojo (2018) that teachers should use instructional strategies that promote learner interactions. TA could have used other instructional strategies or a hybrid of instructional strategies to enable learner-learner interactions and learners' conceptual understanding in teaching the control group. The poor test scores from the pre-test where some learners got zeros demonstrate that TA's learners had a poor background in geometry (see Table 6.2). Also, some of the difficulties observed in the pre-test indicate that the learners had challenges in basic geometry such as work involving parallel lines. TA could have revised with the learners some basic aspects of geometry learnt from Grade 8 up to Grade 11. Neglect of that practice had ripple effects in the subsequent lessons in which the application of those concepts of Grade 8 to Grade 11 was required.

The teacher's worry about accomplishing the geometry content prescribed by the ATP within the specified time frame overtook his patience to teach basic geometry aspects from the previous grades. The possession of patience could have restrained the teacher from continuing without addressing challenges diagnosed in the pre-test. By not addressing the challenges diagnosed from previous grades, TA manifested that he only had theoretical knowledge of *Ubuntu* values because he failed to exercise patience in addressing problems from Grade 8 to Grade 11. The teacher could have struck the balance between the need to accomplish the prescribed work in time and learners' conceptual understanding.

During all the three lessons with each group, TA used only two phases of geometry teaching according to Van Hieles' theory. The two were Phase 1 [Information] and Phase 2 [Guided orientation]. Phase 1 entails the use of probing questions to enable learners to think critically while Phase 2 consists of learners being engaged to explore through activities. With the experimental group, the teacher used practical activities which enabled learners to establish relationships between angles. On the contrary, TA just dictated theorems to learners in the control group. The other three phases of instruction namely explanation, free orientation and integration and were not reached. Explanation is a stage in which learners explain their own words the activities they are engaged in. There was no activity in both groups in which learners were required to explain in their own words. In a nutshell, all the last three phases of geometry teaching were not reached. The class activities only revolved around the first two phases of geometry teaching. This implies that the workshops and seminars attended by TA did not cater for Van Hieles' theory of geometric thinking. TA needed more workshops to enable him to teach geometry embracing recommendations from Van Hieles' theory.

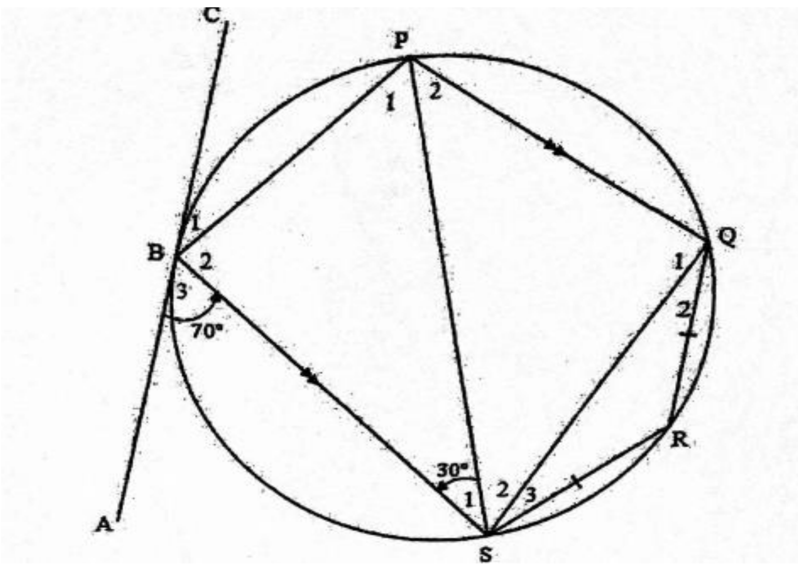
During the classroom observation, TA diagnosed learners' prior knowledge in geometry using probing questions in all three lessons with each of the two groups. In that way, the teacher was able to identify learners' learning difficulties, using oral probing and diagnostic questioning. In addition, TA marked the learners' workbooks indicating where learners had problems. The practice of diagnosing learners' difficulties is recommended by Sibuyi (2012) who argues that effective teachers should be able to identify learners' learning difficulties and find ways of eliminating them. In this regard, it can be said that TA occasionally used effective didactic and pedagogical strategies to enable learners to manifest their learning difficulties. This concurs with Manizude (2006) who asserts that some of the attributes of PCK in geometry are knowledge of specific difficulties and misconceptions, knowledge of useful representations of the content and knowledge of developmental levels of the learners which both teachers manifested in their classroom practice. However, TA could not determine the sources of the learners' learning difficulties as well as determining learners' preconceptions in the topic, especially the misconceptions. TA demonstrated inadequate PCK for knowledge of learners' preconceptions. The teacher could have addressed learners' misconceptions especially if he had dealt with them from Grade 8 content.

Furthermore, TA conducted extra tuition sessions with both the experimental group and the control group in which he addressed the learning difficulties of the learners. The groups were composed of learners of diverse abilities. This was evidence of care for learners by TA, an *Ubuntu* attribute in application. The care and solidarity with learners by the teacher is supported by Alexander (2009) who argues that caring teachers are required to embrace a diversity of learners in their classrooms. TA saw the need for continual interaction with learners after school. During the extra tuition sessions and study time, learners from the experimental groups were encouraged to sit in pairs and discuss how to solve exercises given for homework. The use of collective work as a strategy is recommended by Sofroniou and Poulos (2016) who assert that the use of collective learning strategies allows learners to develop critical thinking, analytical and communication skills and problem-solving problem techniques.

During class activities, TA stressed the importance of analysing questions and diagrams. He explained the need to underline key phrases and properties linked to theorems studied in given activity statements. Some learners from the experimental group quickly understood the technique. The speed of understanding the technique could have been enhanced by sharing of ideas. For example, in Lesson 2 the following activity was given to learners.

Activity

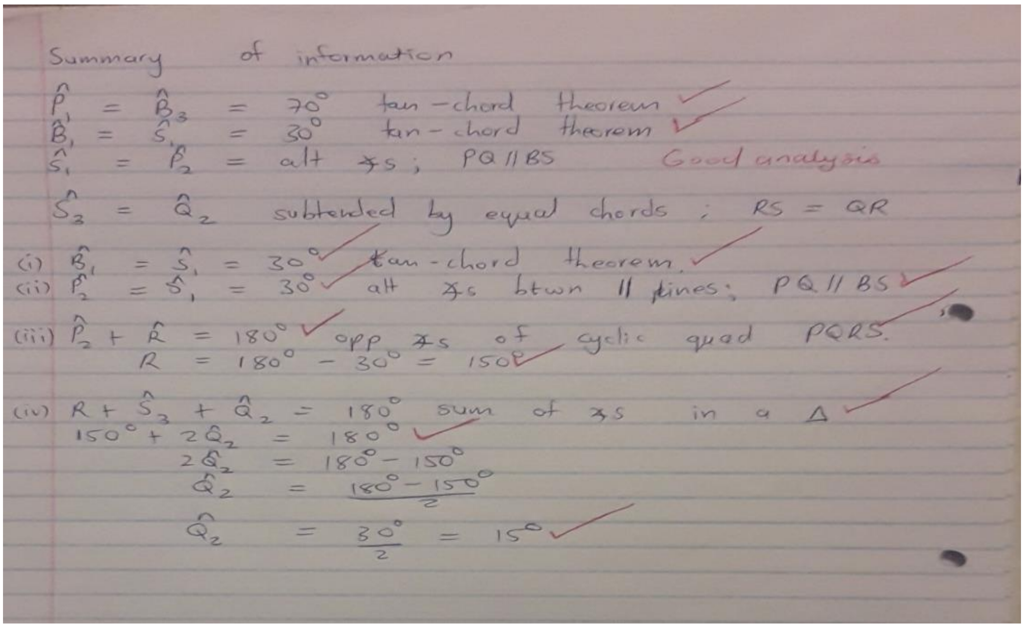
ABC is tangent to circle BPQRS at B, PQ ∥ BS, QR=RS, $\hat{S}_1 = 30^\circ$ and $\hat{B}_3 = 70^\circ$



Calculate with reasons

i. $\hat{B}_1$
ii. $\hat{P}_2$
iii. $\hat{R}$
iv. $\hat{Q}_2$

Fig 7.1 illustrates a sample solution to the activity given in Lesson 2. The solution was provided by a learner from the experimental group.



Summary of information

$\hat{P}_1 = \hat{B}_3 = 70^\circ$ tan-chord theorem

$\hat{B}_1 = \hat{S}_1 = 30^\circ$ tan-chord theorem

$\hat{S}_1 = \hat{P}_2$ = alt ∠s ; PQ // BS Good analysis

$\hat{S}_3 = \hat{Q}_2$ subtended by equal chords ; RS = QR

(i) $\hat{B}_1 = \hat{S}_1 = 30^\circ$ tan-chord theorem.

(ii) $\hat{P}_2 = \hat{S}_1 = 30^\circ$ alt ∠s btwn // lines ; PQ // BS

(iii) $\hat{P}_2 + \hat{R} = 180^\circ$ opp ∠s of cyclic quad PQRS.

$R = 180^\circ - 30^\circ = 150^\circ$

(iv) $R + \hat{S}_3 + \hat{Q}_2 = 180^\circ$ sum of ∠s in a Δ

$150^\circ + 2\hat{Q}_2 = 180^\circ$

$2\hat{Q}_2 = 180^\circ - 150^\circ$

$\hat{Q}_2 = \frac{180^\circ - 150^\circ}{2}$

$\hat{Q}_2 = \frac{30^\circ}{2} = 15^\circ$

Figure 7. 1: Sample solution to an activity given in Lesson 2

From Figure 7.1, it is observable that the learner first analysed the diagram and the question and subsequently wrote summary notes of analysis before answering the questions. On marking, TA commented that the summary of the analysis was good. Most of the learners from the experimental group produced solutions resembling the one presented in Fig 7.1. A few learners from the control group presented a similar quality of solution as compared to the learners from the experimental group. During Lesson 3, similar quality of presentation of solution displayed in Figure 7.1 continued to manifest in activities although this quality was predominant in the experimental group. The quality of solution resembling the one presented in Figure 7.1 can be associated with TA's ability to teach problem-solving techniques to the learners. It can be concluded that TA had some PCK in teaching problem-solving techniques. For learners to produce such quality of solutions it would require patience because this quality of solution presented cannot just be accomplished overnight. It can be deduced that TA possessed some great degree of patience in teaching problem-solving.

It was observed that the workbooks of learners from the experimental group had a lot of written work as compared to those from the control group. In addition, the learners from the experimental group solved a lot of problems correctly as compared to learners from the control group. This finding concurs with the argument by researchers (Bernero, 2000; Kocak, Bozan & Isik, 2009) that working in groups with members sharing ideas among themselves assists in learning concepts and problem-solving strategies. The quality of solutions by learners from the experimental group shows that they understood the concepts and problem-solving techniques.

The models which were cut to compare sizes of angles during lessons with the experimental group were used by TA to follow the guidelines of the Dynamic Pairs Mode to make concepts in Euclidean geometry content to be comprehensible to learners. It can be an indication that TA was also convinced by the usefulness of manipulatives in teaching geometry. The use of manipulatives by TA in the experimental group is supported by various researchers (Chiphambo, 2017; Busi & Frank, 2015; Gürbüz, 2010; Suh & Moyer, 2017) who argue that learners who use manipulatives have their conceptual understanding of geometry enhanced. The use of manipulatives to find effective ways of making learners understand by whatever means can be said to be a manifestation of *Ubuntu* attributes such as empathy and compassion for learners. Figure 7.2 shows the pair of scissors and models of geometric figures in circle geometry which learners made some cuttings to compare relationships between angles.

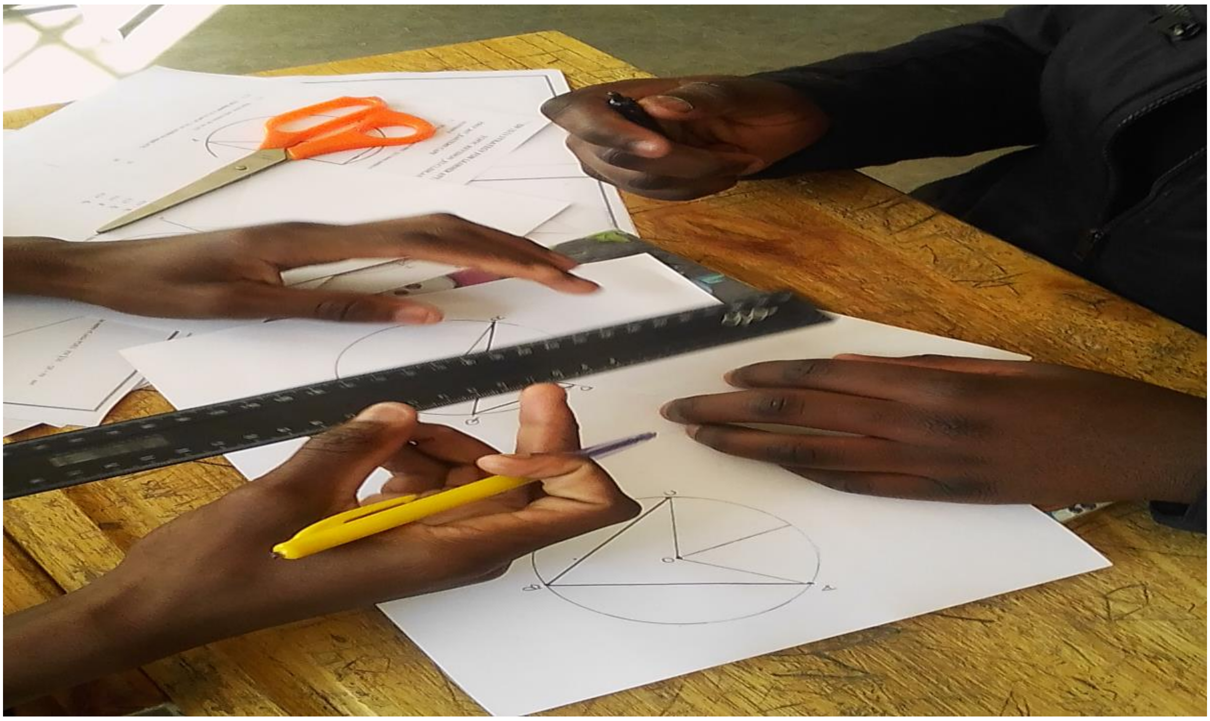


Figure 7. 2: A pair of learners sharing pair of scissors and some models

The years of experience and attendance to workshops might have enabled TA to be familiar and conversant with the mathematics curriculum and its successive changes over the years. In addition, attendance to workshops and other INSET programmes might have exposed TA to opportunities to acquire knowledge of the curriculum. For the three lessons delivered to each group, TA had well detailed and organised lesson plans which included activities consistent with the dictates of the examination guidelines. The lesson plans reflected that curriculum documents such as the CAPS, examination guidelines and diagnostic reports among others had been consulted. The detailed planning by TA showed that the teacher had thorough preparation. The thorough preparation of instructional material by the teacher shows he had respect for his learners. With this *Ubuntu* attribute, TA used it to give his learners activities consistent with examination-type questions. It was an endeavour to expose learners to examination-type questions in preparation for the actual examinations. TA can be regarded as a teacher who possessed specialised content knowledge [SCK] in mathematics. This concurs with researchers (Hill et al., 2004) who assert that SCK enables mathematics teachers to explain the content effectively, do presentations, conduct assessment on learners' understanding and select adequate content from the curriculum. Also, TA during preliminary semi-structured interviews mentioned a list of difficulties encountered by learners which were in line with diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019 & DoBE, 2020) and some researchers (Mammen & Alex, 2016; Siyepu, 2005; Mammali, 2015). The knowledge of the contents of diagnostic reports and examination guidelines as well as other related curriculum documents by TA is evidence of his possession of PCK in geometry. It is also a clear demonstration of applied *Ubuntu* attributes that TA cared about the academic lives of learners.

7.4.2 *Ubuntu* values in TB's PCK in geometry

During the preliminary semi-structured interview TB revealed that the Chris Hani West District Department of Basic Education always selected him to teach in mathematics camps designed for Grade 12 mathematics revision. He explained that he was selected because of his deep content knowledge in geometry despite his few years of teaching experience. Although he possessed deeper content knowledge in geometry, TB expressed the desire to be exposed to more content and pedagogical and didactic knowledge by attending seminars and workshops. The teacher further revealed that he desired to be exposed to teaching geometrical constructions even though they were not in the CAPS geometry syllabus. During the preliminary interview, TB expressed:

"*I hate mediocrity when it comes to performance. I need to prepare thoroughly to ensure that I deliver the best for the learners.*"

The statement by TB reveals that he regarded teachers' mediocre preparation and lack of commitment as one of the causes of poor learners' performance in geometry.

It is a revelation of the sense of responsibility, accountability and obligation of TB to the learners and the community. TB's desire to acquire more pedagogical knowledge in teaching geometry is recommended by Ali, Bhagawati and Sammah (2014) who argue that teachers should be trained through seminars and workshops on how to teach geometry effectively. TB's willingness to attend workshops was an indication that TB valued the contribution of other members of the professional learning community to his didactic and pedagogical endeavours with his learners. The willingness to listen to other people's presentations and ideas shows that TB believed in positive interdependence.

The networking of teachers and cross pollination of ideas was paramount in geometry teaching according to TB's perspective. TB's conviction in professional development workshops is supported by Dlamini (2012) asserts that networking with other teachers enhances teachers' professional development in geometry. TB's beliefs in teacher development workshops and seminars are further supported by Woolfolk (2010) who argues that through interaction more knowledgeable members of the community influence the learning of others through these cooperative dialogues and cultural interactions. By acknowledging other teachers' contributions to his PCK and MKT development, TB demonstrated respect for other teachers. TB showed that he believed in sharing teaching experiences with others.

During the preliminary semi-structured interviews, when asked if he had ever used collective learning strategies such as group work and teamwork before, TB responded: *"Yes, I have used group work in teaching Euclidean geometry. The only challenge I had was that the classrooms were overcrowded with too many groups to manage. Also, group work is time-consuming."*

The response by TB indicates he appreciated the use of collective learning strategies. The only hindrances to his effective teaching were the over-crowdedness in classrooms and the amount of time consumed in implementing the strategy. At the end of the study when TB was asked if he would consider using the *Ubuntu* values in future, he responded:

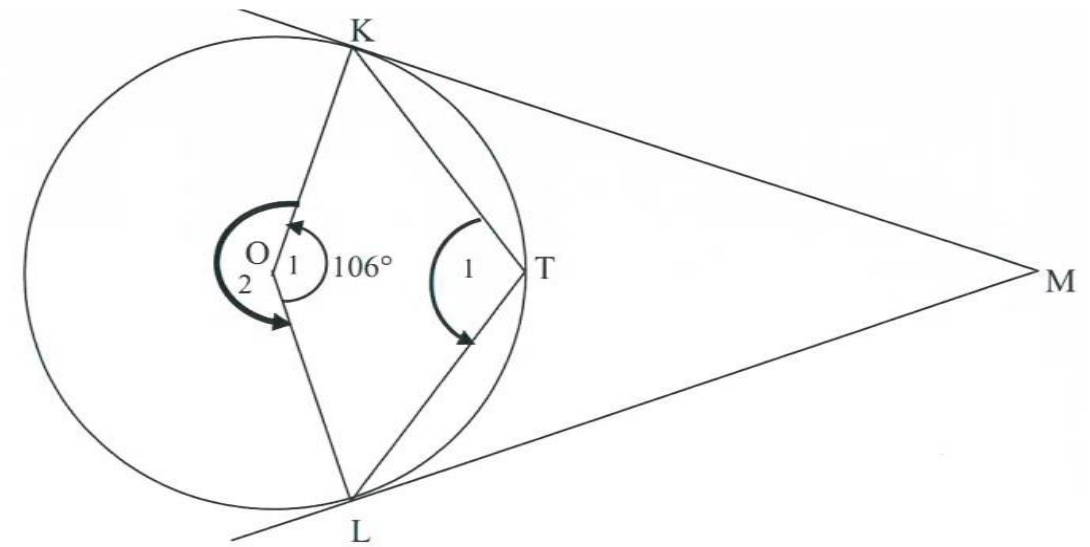
"Yes, I will definitely consider using Ubuntu values because I saw many learners working cooperatively sharing ideas. I saw all learners in the experimental group having something to write as compared to those in the control group. Even habitually passive learners in the experimental group participated a lot in the activities."

The response by TB shows that he was impressed by the amount and quality of work produced by learners from the experimental group. In addition, TB was impressed by the full and active participation of habitually passive learners in classroom activities. The practical work where manipulatives were used could have motivated learners to participate actively. The use of manipulatives is supported by some researchers (Chipambo, 2011; Furner & Worrell, 2017) who assert that learners who are exposed to manipulatives develop a more positive mathematical disposition. Under normal circumstances, difficult topics for learners such as geometry discourage learners' full participation. During lessons, TB was amazed by the amount and quality of work produced by learners from the experimental groups in their workbooks was impressive as compared to the work from control groups. The use Dynamic Pairs Model may be associated with the impressive work by the learners from the experimental group. The phenomenon of the impressive work by learners from the experimental group concurs with some researchers (Erdem, 1993; Bulut, 2002) who argue that cooperative learning strategies improve learners' academic achievements. Furthermore, the collective work among learners from the experimental group is supported by some researchers (Bernero, 2000; Kocak, Bozan & Isik, 2009) who argue that working in groups assists in the learning of concepts and problem-solving strategies and improving self-confidence to overcome the fear of committing mistakes. TB taught his learners how to solve geometry problems of all kinds. He underscored the need to analyse the questions highlighting key phrases and associate them with theorems studied. The teacher taught learners to extract important information from the given questions and diagrams.

For example, the following activity was given to learners of both groups to solve.

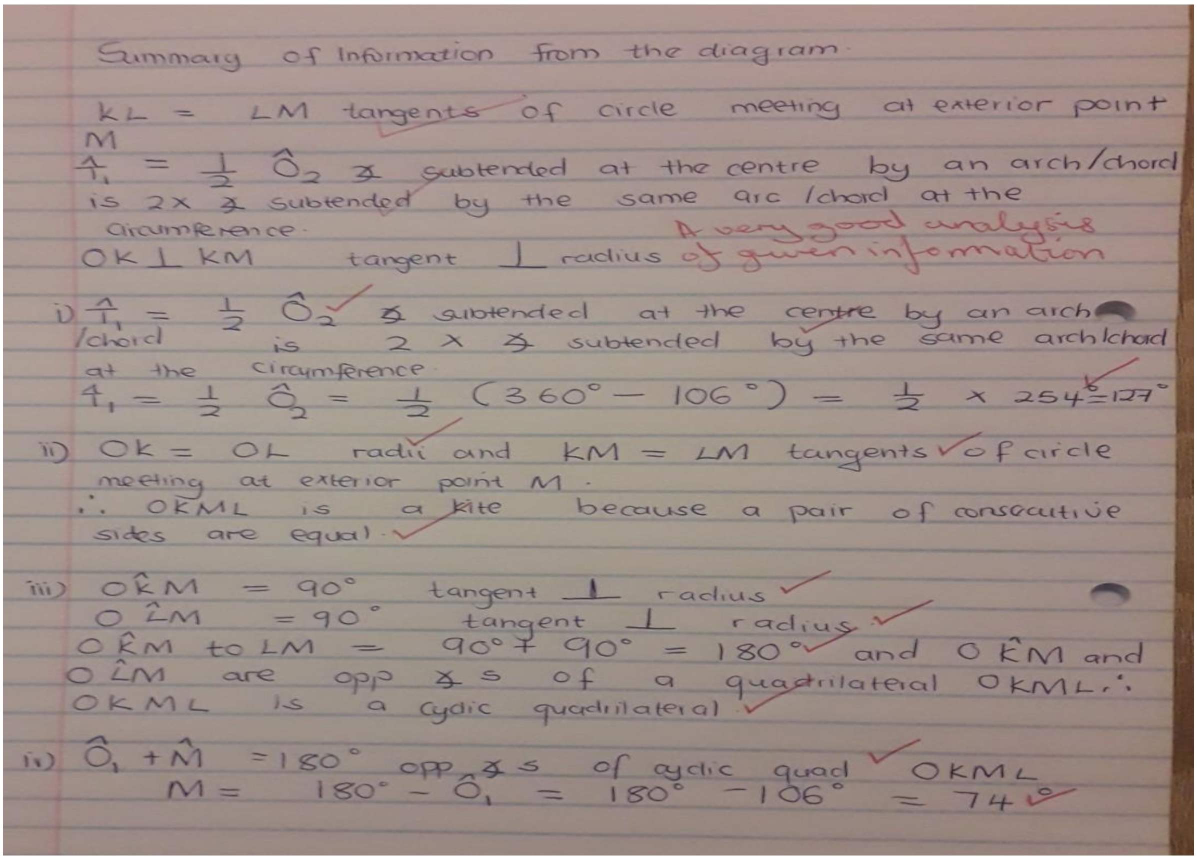
<u>Activity</u>

In the diagram below, O is the centre. KM and LM are tangents to the circle at K and L, respectively. T is a point on the circumference of the circle. KT and LT are joined. $\hat{O}_1 = 106^\circ$



i. Calculate, with reasons, the size of $\hat{T}_1$.
ii. Prove that quadrilateral OKML is a kite.
iii. Prove that quadrilateral OKML is a cyclic quadrilateral.
iv. Calculate with reasons the size of $\hat{M}$.

Figure 6.4 is a sample solution to the activity given above.



Summary of Information from the diagram.

KL = LM tangents of circle meeting at exterior point M

$\hat{T}_1 = \frac{1}{2}\hat{O}_2$ ∡ subtended at the centre by an arch/chord is 2x ∡ subtended by the same arc /chord at the circumference.

OK ⊥ KM tangent ⊥ radius

A very good analysis of given information

i) $\hat{T}_1 = \frac{1}{2}\hat{O}_2$ ∡ subtended at the centre by an arch /chord is 2 x ∡ subtended by the same arch/chord at the circumference.

$\hat{T}_1 = \frac{1}{2}\hat{O}_2 = \frac{1}{2}(360° - 106°) = \frac{1}{2} \times 254° = 127°$

ii) OK = OL radii and KM = LM tangents of circle meeting at exterior point M.

∴ OKML is a kite because a pair of consecutive sides are equal.

iii) $O\hat{K}M = 90°$ tangent ⊥ radius

$O\hat{L}M = 90°$ tangent ⊥ radius

$O\hat{K}M$ to LM = $90° + 90° = 180°$ and $O\hat{K}M$ and $O\hat{L}M$ are opp ∡s of a quadrilateral OKML. ∴ OKML is a cyclic quadrilateral

iv) $\hat{O}_1 + \hat{M} = 180°$ opp ∡s of cyclic quad OKML

$M = 180° - \hat{O}_1 = 180° - 106° = 74°$

Figure 7. 3: Sample solution to activity in Lesson 3 from TB's learner

The incorporation of *Ubuntu* values such as sharing in the experimental group during lessons may be associated with the improved test scores from the post-test and excellent participation of learners. There was a significant difference between the test scores of the experimental group and the control group in the post-test. The test scores of the control group were lower than those of the experimental group. These findings are supported by some researchers (Guzman, 2008; Mammali, 2015) who argue that the use of teacher-centred instructional strategies in classrooms cause of poor performance by learners. Figure 7.4 illustrates TB with learners from the control group during Lesson 1.

Figure 7. 4: TB delivering Lesson 1 to the control group

During lessons, TB was able to detect learners' learning difficulties by monitoring and analysing learners' responses in classwork. However, the teacher could not determine learners' preconceptions. It can be noted that TB had inadequate PCK with respect to knowledge of learners' preconceptions. The pedagogical and didactic effectiveness of TB was manifested in addressing the difficulties encountered by each learner. This practice is recommended by Sibuyi (2012) who argues that effective teachers should be able to identify learners' learning difficulties and find ways of eliminating them. In addition, TB's knowledge of learners' learning difficulties indicates that TB had some PCK in geometry which was in line with Manizude (2006) who assert that some of the attributes of PCK in geometry are knowledge of specific difficulties and misconceptions, knowledge of useful representations of the content and knowledge of developmental levels of the learners which both teachers manifested in their classroom practice. The knowledge of presenting content using manipulatives, class discussions and many other strategies shows that the teacher was not just teaching to satisfy the dictates of the timetable and the curriculum. It was an indication that they were concerned about the future of learners.

It was observed that TB did not use manipulatives during lessons with the control group. This can be associated with his non-preference of their use in class. It could be that TB lacked PCK to use manipulatives in geometry. Also, it can be said that TB showed less care to learners in the control group. The teacher's patience to teach learners problem-solving techniques was also a manifestation of care for learners' future education where techniques in geometry would be required. With the control group, TB used predominantly the lecture method as an instructional strategy. He could have used a variety of instructional strategies for traditional methods. The sole use of one instructional strategy can be deemed as TB's lack of more effective instructional strategies in teaching geometry.

TB used probing questions to solicit answers from learners. The teacher tried his best to discourage chorus answers by targeting questions to specific learners who would have raised their hands. The questions asked were relevant to the content and TB's explanations which ensued showed that the two had deeper content knowledge and PCK. The effective use of the Dynamic Pairs Model with the experimental group is recommended by Sofroniou & Poulos (2016) who assert that the use of collective learning strategies allows learners to develop critical thinking, analytical and communication skills and problem-solving problem techniques.

The academic background of TB reveals that he did Grade 12 when Euclidean geometry was not part of the curriculum from Grade 10 to Grade 12. The year he wrote his Grade 12 examinations, Euclidean geometry content was only done by learners who wrote Paper 3 in Grade 12 mathematics. Mathematics Paper 3 was an optional mathematics paper from the year 2008 to the year 2013. The paper consisted of topics the following content:

i. Probability and counting principles,
ii. Recursive sequences, and
iii. Euclidean geometry.

According to his certificates, TB never wrote Mathematics Paper 3 which reveals that the content knowledge he possessed was acquired at the university. The teacher's fewer years of experience suggested that he did not have enough PCK to teach Euclidean geometry effectively. However, during classroom observation, TB demonstrated that he had stronger content knowledge. In addition, the effective use of relevant examples during lesson delivery reveals that TB possessed some PCK in the use of instructional strategies. Attendance of workshops and seminars could also have enhanced his PCK development in the teaching of geometry.

During data collection preparations, it was revealed that TB received training on CAPS curriculum through workshops organised by the Eastern Cape Department of Basic Education. TB's desire and willingness to attend more workshops was a demonstration of his desire to acquire more knowledge of the contents of the revised annual teaching plans. Furthermore, TB made efforts to expose learners to types of questions consistent with examination guidelines and the curriculum. The teacher's effort to acquire as much information in teaching the topic is evidence that he was committed to his work. It also reveals that he cared for the learners' academic future. The acquisition of good passes in mathematics in Grade 12 gave learners opportunities to select lucrative careers in future. TB's efforts reveal that he wished his learners well in future. TB's quest for knowledge of the curriculum and students is recommended by

Ball *et al.* (2008) who argue that knowledge of curriculum and students [KCS] is manifestation of some PCK. In this case, it can be said that TB had some PCK in the knowledge of the curriculum.

Also, TB during preliminary semi-structured interviews mentioned a list of difficulties encountered by learners which concurred with some researchers (Mammen & Alex, 2016; Siyepu, 2005; Mammali, 2015) and diagnostic reports (DoBE, 2016; DoBE, 2017; DoBE, 2018; DoBE, 2019; DoBE, 2020) and TB's knowledge of the contents of diagnostic reports and examination guidelines as well as other related curriculum documents is a clear demonstration that he cared about the academic lives of learners. This is one of the important *Ubuntu* attributes important for application in mathematics teaching. TB's attendance of workshops concurs with the views by Talbert-Johnson (2006) who asserts that MKT and PCK acquisition are enhanced by participation in workshops.

The well-detailed and organised lesson plans and choice of relevant activities indicate that TB knew the mathematics curriculum in depth. The lesson plans reflected lesson objectives, anticipated learners' prior knowledge, sample questions to ask learners, the activities to be done in class, time allocated per activity, examples to be done with learners and section for evaluation of each lesson. Also, the lesson plans implicitly reflected that curriculum documents such as the CAPS, examination guidelines and diagnostic reports among others were consulted. This was revealed during classroom observations when TB selected activities in tandem with examination guidelines. During lessons, TB gave learners questions and problems with standards of examination type questions. It can be said that TB had some SCK in selecting activities consistent with curriculum requirements. This concurs with Hill et al. (2004) who assert that SCK enables mathematics teachers to explain the content effectively, do presentations, conduct assessments on learners’ understanding and select adequate content and curriculum materials.

During lessons, TB used the same Van Hiele’s phases of geometry teaching used by TA but further used the third phase [Explanation]. In addition, TB worked smoothly with learners from the experimental group using practical activities to enable learners to understand relationships between angles. There were occasions in which TB allowed learners from the experimental group to explain their solutions in pairs. Volunteer learners from the control group explained their solutions to the whole class. It can be said that TB was effective in using the first two phases of geometry teaching. In using Van Hiele’s Phase 3, it can be said that TB was moderately effective in teaching learners allowing them to explain procedures taken among themselves. The other two phases of instruction, namely, free orientation and integration, were not reached. For example, there was no activity in both groups in which learners were required to explain in their own words. In a nutshell, the last two phases of geometry teaching were not reached. The class activities only revolved around the first three phases of geometry teaching. This suggests that the workshops and seminars attended by TB were just content workshops that did not adequately cater for the methodology in teaching geometry. Perhaps, this could be associated with the fact that there were very few or no tasks that related to phase 4 and phase 5 of Van Hiele’s phases of geometry teaching even in the final examination papers. This suggests that TA needed more workshops to enable him to teach geometry embracing recommendations from Van Hieles’ theory.

7.4.3 *Ubuntu* aspects that lacked in the two participants’ teaching practice

The study revealed that the two participants had strong content knowledge of Euclidean geometry because of their attitude towards personal study and academic and professional advancement. Although the two participants demonstrated some *Ubuntu* attributes in their teaching practice, it can be noted that both failed to take advantage of the rich indigenous knowledge in the rural Chris Hani West District. The prevalent features of the rural villages found in Chris Hani West were round huts. In their teaching, the two participants could have used real-life examples which include features such as round huts to explain the idea of a subtended angle. Figure 7.6 shows a picture of Xhosa huts situated closer to one of the two schools involved in the study.

Figure 7. 5: Picture of a group of huts situated closer to one of the schools

In our discussion at the end of the study, the two participants were eager to be empowered with knowledge of the types of real-life examples which were relevant in teaching Euclidean geometry. I explained to them that the use of indigenous examples was the basis of applying *Ubuntu* values in a classroom as advocated by Letseka (2011) and Khupe (2014) who argue that teaching practice based on *Ubuntu* should make use of indigenous knowledge and indigenous knowledge systems.

We discussed how to use a roof of a hut to explain the concept of “subtended angle”. As I shared ideas with each one of the participants, the following diagram was conceived from the cross-section of a hut. Figure 7.6 illustrates a cross-section of a hut which a teacher can use as a real-life example of where geometry is applicable.

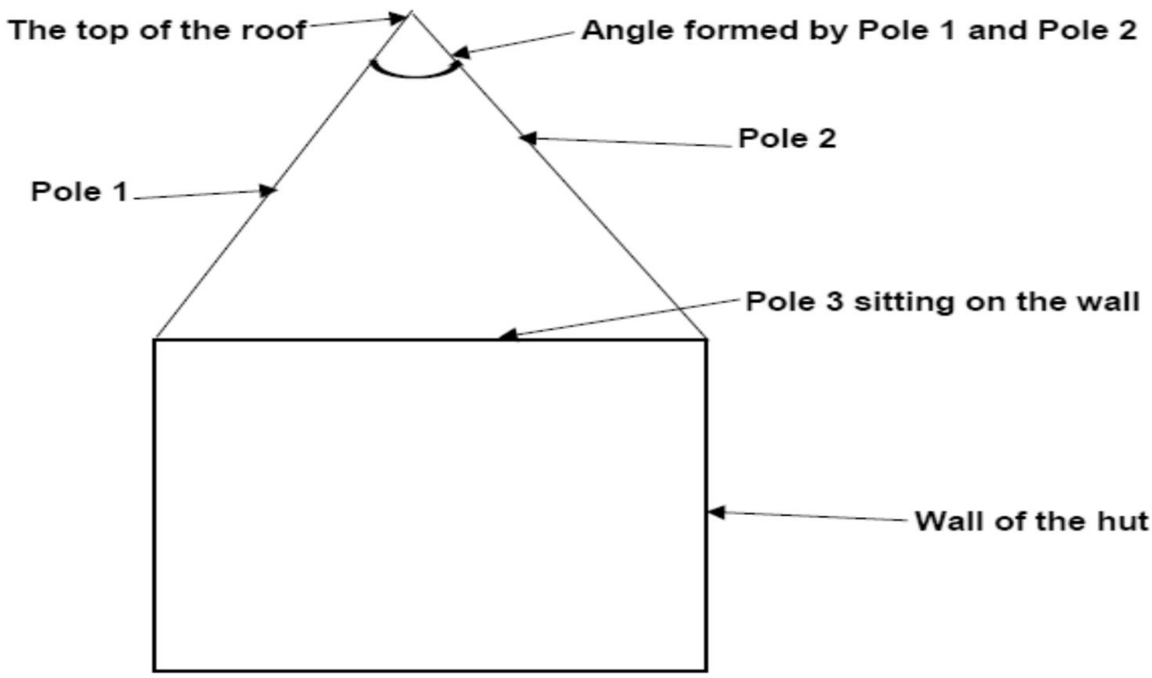


Figure 7.6: Cross-section of a hut

Pole 1 and Pole 2 which form part of a hut meet at the top of the roof forming an angle. The two poles, Pole 1 and Pole 2 each sit at one of the end points of Pole 3 which sits on the wall. The angle between Pole 1 and Pole 2 is said to be subtended by Pole 3 because the rays of the angle, Pole 1 and Pole 2 pass through the extreme points of Pole 3. An example such as this one could have been used to explain important geometry concepts using indigenous knowledge. Incorporating indigenous knowledge such as this example in classrooms decolonises learners' minds and curriculum according to researchers (Katonga, 2017; Breidlid & Botha, 2015; Dei & Simmons, 2009; Ngara, 2012). Recognising learners' indigenous cultures shows respect to the learners as they learn to appreciate that the original architects of those huts were their ancestors who applied geometry. Several real-life examples could be invented by the teacher to enable learners to conceptually understand Euclidean geometry. Each of the two participants requested a workshop in geometry where indigenous knowledge would be incorporated into the instructional strategies to enhance learners' conceptual understanding. Also, I promised to organise a workshop on using technology such as using GeoGebra applets in geometry lessons to enhance visualisation of learners. GeoGebra is a computer software that allows interactive learning whereby aspects such geometry, functions and statistics among others can be taught using simulations.

7.5 *Ubuntu* values in the teaching of Euclidean geometry

This section discusses *Ubuntu* values in each participant's teaching practice.

7.5.1 The impact of sharing of knowledge and resources

The impact of sharing knowledge and resources in each participant's classroom is discussed in this section.

7.5.1.1 The impact of sharing in TA's classroom

During the study, the learners in experimental groups engaged in intra-pair and inter-pair interactions sharing resources such as models, pairs of scissors and mathematical instruments. The learners also shared ideas in conducting comparisons of angles, establishing relationships among the angles and thereafter explained in their own words the relationships found. Also, learners shared knowledge and ideas in solving exercises and problems in pairs. In a nutshell, in all activities, learners in experimental groups were required to sit in pairs and discuss the best methods of either doing the practical activities or doing the written work. On the contrary, learners from control groups did not enjoy the same classroom interactions experienced by those learners from the experimental group because they were taught using restrictive teacher-centred instructional strategies. The learners from the control group engaged in individual work and did not have the opportunity of cross-pollinating ideas with their peers. Figure 7.7 illustrates a pair of learners in TA’s experimental group sharing ideas on how to solve problems but maintaining social distance due to COVID-19 regulations. Both learners wore face masks.

Figure 7. 7: Example of TA’s learners sharing in pairs

Figure 7.8 shows from the control group doing individual work without sharing

Figure 7. 8: TA's learners in the control group

However, there were some instances in which TA would allow whole class discussions among learners in control groups. During those classroom discussions, gifted learners dominated participation. They always offered their solutions including explanations to the whole class on the chalkboard. Figure 6.10 illustrates one of the gifted learners from TA's control group solving a problem on the chalkboard.

Figure 7.9: A gifted learner solving a problem on the chalkboard

TA when asked new things he learnt through the interaction with the researcher he expressed the following:

"*I learnt innovative ways of teaching geometry practically which involved all learners even those who are habitually passive in mathematics."*

Furthermore, TA expressed conviction that learners benefitted from sharing knowledge and resources. He confirmed that he was overwhelmed by the level of participation of learners from the experimental group because he witnessed habitually passive learners deeply absorbed in activities. The active participation of the learners from the experimental group is supported by Monaheng (2017) who asserts that *Ubuntu* is demonstrated by the active participation of community members for the well-being of that community. Table 6.17, Table 6.19 and Table 6.21 illustrate levels of participation of learners in percentages of learners from both experimental and control groups from Lesson 1 to Lesson 3. In Lesson 1, the percentage of participation of learners in classroom activities was 100% from the experimental group as compared to 33% from the control group. In Lesson 2, it was 100% as compared to 33% and lastly, in Lesson 3, it was 100% as compared to 50% from the control group, respectively. The tables show that the level of participation of learners from the experimental group was higher than that of the learners from the control group.

The full participation of learners from the experimental group can be associated with the use of practical activities in which learners were made to share ideas and knowledge. It can also be associated with the inter-pair and intra-pair interactions among learners in which they shared ideas in the solution of problems. Intra-pair and inter-pair discussions among learners in experimental groups also enabled learners with learning difficulties and misconceptions to solicit and obtain assistance from their peers instead of relying on the teacher only. Habitually passive learners were motivated to participate because of the existence of freedom to express themselves in pairs unlike explaining to the whole class. The interactions by learners from the experimental group is supported by Shockley (2011) asserts that the values of *Ubuntu* promote communication and interaction among learners. In addition, the motivation of learners to participate actively in the classrooms was the result of the use of *Ubuntu* attributes which researchers (Adams, 2013; Slavin, 1991; Siegel, 2005; Gardner, 1999) assert that all forms of cooperative learning improve learners' self-esteem and motivation. In this study, this was revealed when the learners' participation in experimental groups was enhanced by sharing, caring, cordial teacher-learner relationships, respect and catering for individuality. Learners shared ideas on how to cut models and compare relationships of angles practically as well as ideas on how to solve problems given by the teacher.

On the contrary, learners from the control group could not share with their peers because of the sitting arrangement whereby each learner sat alone without sharing. Learners only shared ideas of solutions of problems occasionally when TA occasionally allowed some gifted learners to explain their solutions to their peers on the chalkboard. The less gifted learners did not have confidence in presenting their solutions to the whole class. There were also some instances where no learner would volunteer to present solutions on the chalkboard. In those instances, TA would do the explanations to the whole class. However, some learners were shy to show that they had problems in understanding the teacher's explanations. The teacher failed to identify those learners.

At the beginning of the study in each school, there was no statistical difference between the test scores from the pre-test of the experimental group and the control group. The results from the post-test indicate that the mean test score obtained by learners from the experimental group was 46.50% as compared to 33.61% from the control group (see Table 5.39). From the table, 33.3% of learners scored 50% and above from the experimental group as compared to 22.2% of learners from the control group. There was a statistical difference between the test scores of the experimental group and the control group. The mean scores further indicate that the test scores from the experimental group were higher than the test scores from the control group. Also, it can be deduced that the Dynamic Pairs Model which allowed more classroom interactions assisted learners from the experimental group to obtain higher scores as compared to learners from the control group. The sharing of resources by learners from TB's experimental group in the geometry classroom is supported by some researchers (Samkange & Samkange, 1980; Lukhele, 1990 & Mpofu, 2002) who argue that the sharing of resources and ideas is an *Ubuntu* principle important for development. The higher test scores obtained by learners from the experimental group is evidence that learners' academic performance in the topic had improved more than that of learners from the control group.

7.5.1.2 The impact of sharing in TB's classroom

During the study, the learners in the experimental group engaged in intra-pair and inter-pair interactions sharing knowledge, ideas and resources. Sharing strengthened a sense of belonging and membership among learners from the experimental group. On the contrary, learners from the control groups did not enjoy the same classroom interactions experienced by those in experimental groups because they were taught using restrictive instructional strategies such as the lecture method. Figure 7.10 shows TB's learners from the control group during one of the lessons.

Figure 7. 10: TB's control group during one of the lessons

However, in all activities learners in the experimental group sat in pairs discussing the best methods of either doing the practical activities or doing the written work. The learners from the control group engaged in individual work and did not have the opportunity of cross-pollinating ideas with their peers. However, there were some instances in which TB allowed the whole class discussions among learners in control groups. During those classroom discussions, gifted learners dominated participation. They offered to go to the chalkboard to present solutions explaining to the whole class.

TB when asked if he was convinced that *Ubuntu* values could assist in teaching Euclidean geometry effectively. He responded,

"*Yes, I am convinced. I have witnessed learners being free to discover and share information gained from the practical activity. I also was surprised to see learners who had never participated in mathematics since Grade 10 being very active. It shows the Ubuntu-based activities motivated them to act in this manner. Normally these learners would always be passive.*"

TB's expression suggests that he was impressed by the full participation of learners from the experimental group. It was amazing to observe how habitually passive learners were engaged in class activities. The active participation of the learners from the experimental group is supported by Monaheng (2017) who asserts that *Ubuntu* is demonstrated by the active participation of community members for the well-being of that community. Table 6.29, Table 6.31 and Table 6.33 illustrate levels of participation of learners in terms of percentages from both experimental and control groups from Lesson 1 to Lesson 3 in percentages. In Lesson 1, the percentage of participation of learners in classroom activities was 100% from the experimental group as compared to 50% from the control group. In Lesson 2. it was 100% as compared to 44% and lastly, in Lesson 3, it was 100% as compared to 44% from the control group, respectively.

The information from classroom observation indicates that the participation levels of learners from the experimental group were higher than those of learners from the control group. The full participation by learners from the experimental group was enhanced by sharing, caring, cordial teacher-learner relationships, respect and catering for individuality. Intra-pair and inter-pair discussions among learners from the experimental group enabled learners with learning difficulties and misconceptions to solicit and obtain assistance from their peers instead of relying on the teacher only.

At the beginning of the study, there was no statistical difference between the test scores of the experimental group and the control group in the pre-test. At the end of the study, the post-test was administered to both groups. The post-test scores indicate that the mean scores of the experimental group and the control group were 75.94% and 39.94% respectively (see Table 6.44). The mean test score obtained by learners from the experimental group was higher than the mean score obtained by learners from the control group. In addition, the percentage of the number of learners from the experimental group who achieved scores of 50% and above was 93.75% as compared to 33.33% from the control group. Incidentally, the independent samples t-test indicates that there was a statistical difference between the test scores of the experimental group and the control group.

The higher test scores obtained by learners from the experimental group as compared to those of the learners from the control group can be associated with the use of instructional strategies which enabled learners to share ideas in doing classroom activities and in problem-solving. This phenomenon suggests that when learners have the freedom to share and interact more in the classroom, they perform better academically than those learners whose classroom interactions are restricted. The sharing of resources in the geometry is supported by some researchers (Samkange & Samkange, 1980; Lukhele, 1990; Mpofu, 2002) who argue that the sharing of resources and ideas is an *Ubuntu* principle important for development. Learners from the experimental group participated impressively because a conducive environment was created for them to share their ideas, material resources for classroom activities, convictions, conceptions and misconceptions.

In addition, Shockley (2011) asserts that values of *Ubuntu* promote communication and interaction among learners. The motivation of learners to participate actively in the classrooms was the result of the use of *Ubuntu* attributes which researchers (Adams, 2013; Slavin, 1991; Siegel, 2005; Gardner, 1999) assert that all forms of cooperative learning improve learners' self-esteem and motivation. In this study, this was revealed when the learners' participation in experimental groups was enhanced by sharing, caring, cordial teacher-learner relationships, respect and catering for individuality. Learners from the experimental group sat in pairs to share physical manipulatives which they used to establish relationships between angles as well as ideas to solve problems given in geometry.

7.5.2 Other *Ubuntu* attributes in teaching geometry

Other *Ubuntu* attributes in teaching geometry of each of the two participants are discussed in this section.

7.5.2.1 Other Ubuntu attributes in TA's geometry classroom

One striking aspect of TA's teaching practice was his show of sympathy to learners, members of the community and other citizens known and unknown to him. At the beginning of each lesson with each group, TA encouraged learners to get into a minute of silent prayer in honour and remembrance of citizens who succumbed to the effects of the COVID-19 pandemic. Among TA's learners, some lost close relatives due to this devastating pandemic. The learners appreciated the empathy displayed by their teacher. Most of them always verbalised their gratitude about TA's gesture which enhanced emotional connections with them. The teacher taught the learners *Ubuntu* by modelling it in his actions. A sense of belonging pervaded learners from both groups taught by TA.

Respect was also one of the *Ubuntu* attributes which manifested in TA's teaching practice. TA showed great respect for his learners by coming to class well prepared in terms of content delivery. The teacher had great command of the subject content and showed that he had an in-depth study of the content. The teacher's thorough preparation of the content and didactics in geometry showed that he took his learners seriously and respected them. In general, teachers who lack respect for learners are characterised by lack of preparation. In addition, during all lessons with the control group, TA moved from one learner to another offering help. With the learners from the experimental group, TA approached each pair assisting them in problem-solving. During the greater part of his lessons, TA paid attention to all learners who needed attention. He did not ignore learners' calls for help which was a sign of respect to those learners.

During lessons with the experimental group, TA gave learners already made models of circles drawn for them to make cuttings. The exercise of doing those cuttings was meant to enable learners to establish relationships between angles. This was a way to enable learners to understand some of the circle theorems. By using already made models, TA aimed to save time since he considered that the learners did not have a background in making geometrical constructions. Giving learners models of already drawn circles can be interpreted as TA's under-estimation of learners' capabilities of making such geometrical constructions. Under-estimation of the learners' capabilities can be linked to lack of respect. The teacher could have patiently taught the learners such trivial geometrical constructions to show them that he believed in their capabilities even if it was their first time. Allowing learners to conduct the geometrical constructions on their own would be indicative of the teacher’s confidence in them.

TA being cognisant of learners’ inability to express themselves fluently in English, allowed them to discuss class activities in isiXhosa, their vernacular language. The learners were given the chance to think and discuss in class using their mother tongue and thereafter present solutions in English. This was a demonstration of care and respect for learners’ culture. TA’s practice of allowing learners to freely express themselves in vernacular language is supported by Barnhardt and Kawagley (2005) who assert that children think in their mother tongue and then try to express themselves in the language of instruction. In addition, TA’s practice is further supported by Nakusera (2004) who argues that learners understand scientific concepts easily in their native language.

The teacher’s adjustment to allow learners to communicate in vernacular language was consistent with Panthi and Belbase (2017) who emphasise the use of mother tongue in learning mathematical concepts. The main aim was to make learners understand the mathematical concepts fully since their proficiency in English was limited. TA was very patient and explained content in simplified English and gradually introducing more concepts in English. The use of mother tongue in intra-pair and inter-pair discussions in the experimental group created the opportunity for learners to express their ideas hence the overwhelming participation. On the contrary, TA did much of the talking in teaching the control group by predominantly using English and occasionally engaged in code-switching. Learners from the control group had limited time to engage in discussion using their mother tongue. The predominant use of English in the control group might have limited learners’ participation because of their lack of proficiency in English. The researcher felt that TA allowed the over-use of the vernacular which might cause learners to fail to present solutions in English when answering questions in the forthcoming examinations. There was a need to strike a balance on the use of English and isiXhosa to avert those challenges.

During Lesson 3 with both groups, TA was quick to suggest to learners how to solve some of the problems. Some of the learners did not have a chance to present their own solutions in line with their thinking. This gesture can be interpreted as lack of respect for learners' thinking. Also, TA failed to attend to some learners who raised their hands as an indication that they needed help. It might be because of time constraints or TA's vision was compromised. However, this gesture can be interpreted as ignoring learners. Ignoring learners intentionally can be an indication of disrespect. Despite TA's failure to attend to some of the learners, respect characterised TA's classroom practice. To show respect for his learners, TA did the following:

i. Expeditiously giving learners feedback to their answers by marking and giving constructive comments. The teacher provided systematic feedback and corrections to the learners,
ii. Commenting positively to learners' answers,
iii. Acknowledging each learner's contribution as valuable,
iv. Questioning learners on how they got solutions. This showed learners that the teacher had confidence in their abilities, and
v. Expressing that he had high expectations for each learner. There was no undermining of learners' capabilities. Each learner was treated to have great potential.

When TA was asked during the preliminary semi-structured interview if he ever used principles of *Ubuntu* in his mathematics classroom, he said:

"Yes, I commit myself to teaching learners in extra lessons. These learners come from poor families. My heart pricks when they fail because they go back to their families with empty hands. I want them to pass and go to universities and study to enhance their opportunities to break the cycle of poverty. I have encouraged learners in my classes to share whatever useful material they get."

When TA responded his eyes were teary when he spoke the third sentence. Shedding tears by the teacher may be associated with his empathy, compassion and love for learners. This was evidence that TA did not teach only for gaining income but also for gaining a better outcome which was uplifting the learners' lives from poverty to success. The teary eyes also showed that TA's heart was really pricked by learners' poor socio-economic background. The teacher's sensitivity to learners' poor socio-economic background is supported by DoBE (2011) which suggests that *Ubuntu* should be reflected in the curriculum by promoting sensitivity to poverty, inequality, race, gender, language, age, disability and other factors.

The effective interactions of learners from the experimental group might be a result of learners' discovery that TA had empathy for them. Learners seemed to be motivated by the teacher's demonstration of compassion and empathy towards them. No matter how much pedagogy TA displayed, the learners were more attracted by his care, compassion and empathy for them. During classroom activities with the experimental group, TA moved from one pair of learners to the other checking their work and explaining how to go about each activity. In the same vein, with the control group, TA moved from one learner to the other. He spent more time with those learners who did not understand. The teacher's display of empathy to learners is supported by Jojo (2018) who suggests that geometry teachers should empathise with learners to be able to understand how they perceive geometry and to be acquainted with their ideas, desires and actions. During the lessons, TA also demonstrated patience by dedicating time to engage in repeated explanations to learners. The teacher also engaged in extra classes to help learners understand the procedures of conducting activities. The patience exercised assisted TA to modify his instructional strategies to enable learners to solicit help from him and fellow learners. Learners who encountered challenges had opportunities to ask fellow learners a variety of questions and thereafter present their work in different ways.

The teacher's patience and demonstration of compassion and empathy towards learners can be associated with their motivation to work hard. TA's behaviour of playing the "loco parentis" role (Van Breda, 2006) might have created confidence among learners. In addition, the parental roles played by TA is supported by Eliastam (2015) with the dictum, *"My child is your child",* which is an important *Ubuntu* view in education. It might be that this behaviour encouraged and influenced learners to work extra hard because the teacher treated them as his biological children. The learners might have seen TA's sacrifice, love, commitment and patience which made them desire to work extra hard to please them. TA's dedication to putting a lot of effort to change the lives of learners in their poor communities is a recommended demonstration of *Ubuntu* as explained by DoBE (2011) that *Ubuntu* seeks social transformation wherein ensuring that the educational imbalances of the past are redressed, and the educational opportunities are provided for all sections of the population. TA's learners came from historically impoverished families from the days of apartheid. The teacher motivated by *Ubuntu*, sought to address that by exerting concerted efforts in teaching those learners. TA went an extra mile by sacrificing his time to conduct extra classes after school to ensure learners understood the complex content of Euclidean geometry conceptually. The teacher showed commitment to his work by investing his extra time to assist learners. TA always departed from the workplace late after conducting extra tuition and lesson preparation.

During the study, the learners from the experimental group who received instruction using *Ubuntu*-based Dynamic Pairs Model obtained higher test scores in the post-test in Euclidean geometry than their peers from the control groups which were taught using traditional methods. This phenomenon confirms the assertion by some researchers (Slavin, 1991; Walmsely & Muniz, 2003; Johnson & Johnson, 2013) that cooperative learning results in higher academic achievement, high levels of productivity and success, better communication skills, successful social and academic group interactions. Learners managed to share ideas using vernacular language thereby enhancing communication among themselves. The interdependence among learners enabled them to share knowledge, ideas and resources. This assisted some learners who encountered problems to gain knowledge from others.

During the study, TA also encouraged learners to meet after school to engage in discussions to assist one another. This is a demonstration of the promotion of interdependence, communalism and solidarity which are attributes of *Ubuntu*. The teacher's promotion of such practices concurs with Jackson (2013) who suggests that teachers should strive to establish learners' relationships that extend beyond classroom interactions. Those interactions assisted learners to network and share with other learners from other groups as well as other learners from schools around the community.

During Lesson 1 with the experimental group, one of TA's learners exclaimed in isiXhosa:

"Mufundisi, kutheni ungatitshanga ngoluhlobo kwi-ibanga 11?"

The statement translated into English says:

"Teacher why didn't you teach us like this in Grade 11?"

This reaction suggests that the learner enjoyed the use of the practical activity and manipulatives in the classroom. The learner asked a thought-provoking question to make the teacher consider implementing the use of manipulatives and practical activities in lower grades. This learner represented most of his peers in the classroom who displayed enjoyment in the practical activity. According to the learner, had the teacher used the instructional strategy which incorporated the use of practical activities and manipulatives in previous grades, the learner's background would have been firm in geometry. In addition, the statement by the learner suggests that teachers should invent more interesting and innovative ways of teaching geometry and making the topic more enjoyable. The joy that pervaded in the lessons of TA's experimental group suggests that teachers should invent more strategies to make learners have a positive attitude towards geometry. TA testified that the practical activities and manipulatives used motivated even habitually passive learners to participate fully in the classroom interactions. *Ubuntu* was manifested in the lessons with the experimental group when TB made learners derive joy from the classroom activities.

7.5.2.2 Other Ubuntu attributes in TB's geometry classroom

During the study, TB's classroom practice was pervaded by some aspects of respect in all his lessons with both the experimental group and the control group. During the preliminary semi-structured interview, TB expressed the following statement:

"*I hate mediocrity when it comes to performance. I need to prepare thoroughly to ensure that I deliver the best for the learners."*

The statement above was validated when TB went to class well prepared in terms of content. His explanations were in line with the dictates of the CAPS document and examination guidelines. He also showed that he consulted enough geometry literature beyond Grade 12. He ensured that the wording of theorems was accurate in tandem with the way books geometry presented them. Also, he was able to explain the content in various ways to ensure that conceptual understanding was achieved. The expression by TB during the semi-structured interview was supported by his classroom practice of going to conduct lessons well prepared. Thorough preparations for lessons were an indication of respect for learners by TB.

During lessons, TB visited each learner from the control group offering help in problem-solving. Also, the teacher approached learners from the experimental group who were seated in pairs to assist them in solving problems. In addition, TB demonstrated respect by not ignoring learners' calls for assistance. He tried his best to help each learner who indicated that he or she had a need for help. Giving attention to all learners without ignoring their calls for help was a sign of respect by TB. The teacher acted as a role model for the learners on how to respect others.

The granting of freedom to communicate in isiXhosa among learners by TB shows that he cared for the learners and respected their cultural values. The teacher realised that learners were not proficient in English and decided to allow them to discuss in isiXhosa their vernacular language. The teacher's flexibility in allowing learners to communicate in their vernacular language was consistent with Panthi and Belbase (2017) who emphasise the importance of using the mother tongue in learning mathematical concepts. TB's goal was to ensure that all learners understand the mathematical concepts fully since their proficiency in English was limited. TB also resorted to explaining concepts in simple English and gradually introduced more complex concepts and procedures. The overwhelming participation of the learners in classroom activities can also be associated with TB's recognition of the learners' vernacular language which might have motivated them to be active in class.

During all lessons with both groups, TB asked probing questions to enable learners to express their thinking. He would say,

"W*hy don't you find the value of angle A first*?"

He was not quick to provide answers but allowed the learners to present solutions in line with their thinking. This can be interpreted as respect for learners' thinking. However, in some instances, TB failed to attend to some learners who needed help. Failure to attend to learners who called for help can be interpreted as ignoring learners which is associated with disrespect for learners. Also, in Lesson 3, TB did not provide feedback to learners' solutions to problems. Not providing feedback can be regarded as lack of respect for learners. A teacher with respect would naturally regard learners' work as deserving recognition through marking and communicating with them about their performance. Despite some of the negative aspects, TB's classrooms were predominantly characterised by respect. The following indications of respect characterised TB classroom practice:

i. In verbal interactions, the teacher commented positively to learners' responses,
ii. Acknowledged each learner's contribution as valuable,
iii. Questioned learners on how they got solutions. This showed learners that the teacher had confidence in their abilities, and

iv. Expressed that he had high expectations for each learner. There was no undermining of learners' capabilities. Each learner was treated to have great potential.

When TB was asked during the preliminary semi-structured interview if he ever used principles of *Ubuntu* in his mathematics classroom, he said:

"I think I used Ubuntu principles before, but I doubt whether I did it perfectly. What I know is I empathise with learners so much. In this community, these learners do not have any community members available who can assist them in mathematics except myself. My failure to assist them is like a death sentence to the whole community."

The first sentence of the response by TB shows that the teacher was humble. He did not boast of being a champion of displaying *Ubuntu* but expressed that he had once implemented them but needed others to evaluate his practice. Also, at the commencement of Lesson 2 with the control group, TB relayed information to the group that one of the learners from Grade 9 was in hospital after contracting COVID-19. He encouraged all the learners to remember him in prayers. TB demonstrated the empathy and compassion he once verbalised during the preliminary semi-structured interview. The teacher's gesture was fatherly and was a practical demonstration of sympathy towards learners.

TB further expressed that not being empathetic is tantamount to declaring a death sentence to the learners. In addition, TB noted that learners generally did not have support from the community in learning mathematics. Therefore, he was obliged to avail himself to redeem learners from the cycle of poverty. By being sensitive to poverty and language TB acted in line with DoBE's (2011) suggestion that *Ubuntu* should be reflected in the curriculum by promoting sensitivity to poverty, inequality, race, gender, language, age, disability and other factors. The teacher dedicated his efforts to teach learners even beyond school hours playing the “loco parentis” role (Van Breda, 2006) which might have ignited confidence among learners. In addition, the parental role by TB is supported by Eliastam (2015) with the dictum, *“My child is your child”,* which is an important *Ubuntu* view in education. This gesture by TB might be regarded as the source of encouragement and influence for learners to work extra hard because the teachers treated them as their biological children. By being empathetic, TB motivated learners to work extra hard because they noted that the teacher recognised their value.

The sacrifice and hard work by TB enabled learners to understand the complex content of Euclidean geometry conceptually. The experimental group obtained better test scores as compared to the test scores obtained by the control group in the post-test. The better performance by the experimental group confirms the assertion by some researchers (Slavin, 1991; Walmsely & Muniz, 2003; Johnson & Johnson, 2013) that cooperative learning results in higher academic achievement, high levels of productivity and success, better communication skills, successful social and academic group interactions. Learners managed to share ideas using the vernacular language thereby enhancing communication among themselves. The removal of language barriers assisted some learners who encountered problems to gain knowledge from others.

During the final semi-structured interview, TB was asked if he would consider using *Ubuntu* values in teaching Euclidean geometry again. TB responded:

“Yes, I will definitely consider using Ubuntu values because I saw many learners working cooperatively sharing ideas. I saw all learners in the experimental group

having something to write as compared to those in the control group. Even habitually passive learners in the experimental group participated a lot in the activities."

TB was impressed by the level of learners' participation in the experimental group. It can be deduced that practical activities motivated the habitually passive learners from the experimental group. The practical activities in which manipulatives in the form of models of circles and angles which learners cut were an attempt to reach all learners of different levels of understanding. Mosimege and Lebeta (2000) support the creativity and innovativeness of using manipulatives in the classroom.by They argue that creativity in teaching motivates learners who cosequently find mathematics interesting. Learners in the experimental group were visibly comfortable and happy because they understood the relationships between angles through practical activities. Aspects of geometry that seemed complex for them were made clear because they touched and manipulated objects which enabled them to understand. *Ubuntu* was manifested when habitually passive learners began to interact and share with others. The use of manipulatives shows consideration on the part of the teacher towards learners who fail to understand concepts theoretically. Patience was manifested by TB in taking his time to use manipulatives instead of rushing to explain concepts and theorems theoretically in a bid to accomplish the dictates of the ATP.

7.6 *Ubuntu* values' influence on learners' academic performance

In this case, the learners' academic performance was analysed using the results of the pre-test and post-test, Van Hieles' levels of geometric thinking and learners' written work from the workbooks. Comparisons were made between each participant's experimental group and control group using the criteria mentioned above.

7.6.1 Influence of *Ubuntu* values on TA's learners' academic performance

The pre-test results indicate that TA's two groups selected for the study had test scores that were not statistically different although they were numerically different. The test score means were 8.89 and 8.94, respectively. This meant that any group could be selected as an experimental group or a control group. In this case, the group which had a mean score of 8.89 was selected to be the experimental group while the remaining one was selected to be the control group. The test score means of the two groups were far below the test score of 50%. Generally, a 50% pass mark in a mathematics examination was the minimum requirement for eligibility to study for a degree which comprised predominantly mathematical modules at South African universities. The poor test scores suggest that TA's learners had a poor background in Euclidean geometry.

During the study, learners from the experimental group were exposed to *Ubuntu* values using the Dynamic Pairs Model. The model promoted inter-pair and intra-pair interactions whereby learners shared ideas on problem-solving in geometry as well as ideas to do activities. The learners were empowered in how to support one another in Euclidean geometry using *Ubuntu* values such as solidarity, care, patience and respect among others. The good quality of solutions to problems presented by learners from the experimental group can be associated with the use of *Ubuntu* values. For example, TA was patient to teach learners how to extract and analyse information from diagrams and statements of any problem. The learners thereafter used that information to identify appropriate theorems to use in solving the problems. TA further revealed that some of his learners disclosed to him that they engaged in communication through social media (WhatsApp) to discuss solutions to problems.

The post-test results indicate that the test scores between the two groups were statistically different. The statistical difference may be a result of the use of *Ubuntu* values in teaching the experimental group. Also, obtained from Lesson 1, Lesson 2 and Lesson 3 was that 89%, 78% and 89%, respectively, of learners answered questions given as classwork correctly. This is compared to 22%, 33% and 50%, respectively, of learners from the control group who could present correct solutions on the same exercises. The results from classroom observation suggest that learners from the experimental group could present better solutions than those learners from the control group. The results from the post-tests, classroom observation and workbooks from learners indicate that learners taught using *Ubuntu* values perform better academically than those from the control group taught using traditional methods. The finding concurs with researchers (Adams, 2013; Slavin, 1991; Siegel, 2005; Gardner, 1999) who argue that all forms of cooperative learning if promoted in classrooms, students' self-esteem and motivation are achieved giving rise to better academic performance. The positive interdependence of learners from the experimental group through sharing and other values of *Ubuntu* can be associated with better results of the experimental group than the control group.

However, the post-test results reveal that there were no statistical differences between test scores of the experimental group and the control group with respect to Van Hieles' geometric thinking levels. This suggests that although *Ubuntu* values were effective in improving the overall performance of the experimental group, the geometric thinking skills between the experimental group and the control group were the same. This further suggests that *Ubuntu* values used to teach TA's learners from the experimental group had little impact on their geometric thinking capabilities.

7.6.2 Influence of *Ubuntu* values on TB's learners' academic performance

The results of the pre-test indicate that TB's two groups selected for the study had test scores that were not statistically different although they were numerically different. The means of test scores were 32.89 and 31.75 respectively which meant that any of the two groups could be selected as an experimental group or control group. In this case, the group which had a mean score of 31.75 was selected to be the experimental group while the remaining one was selected to be the control group. The test score means of the two groups were far below the test score of 50%. Generally, a 50% pass mark in a mathematics examination was the minimum requirement for eligibility to study for a degree which comprised predominantly mathematical modules at South African universities. The poor test scores suggest that TB's learners had a poor background in Euclidean geometry.

During the study, learners from the experimental group were exposed to *Ubuntu* values using the Dynamic Pairs Model. The model promoted inter-pair and intra-pair interactions whereby learners shared ideas on problem-solving in geometry as well as ideas to do activities. The learners were empowered in how to support one another in Euclidean geometry using *Ubuntu* values such as solidarity, care, patience and respect among others. The good quality of solutions to problems presented by learners from the experimental group can be associated with the use of *Ubuntu* values. For example, TB was patient to teach learners how to extract and analyse information from diagrams and statements of any problem. The learners thereafter used that information to identify appropriate theorems to use in solving the problems. TB further revealed that his learners also would come during evenings to study together. This means TB's learners had more time to share ideas after school.

The post-test mean test scores for the control group and the experimental group were 39.94 and 75.94, respectively. On conducting the independent samples t-test it emerged that the test scores between the two groups were statistically different. The statistical difference can be linked to the use of *Ubuntu* values in teaching the experimental group. Also, obtained from Lesson 1, Lesson 2 and Lesson 3 was that 88%, 88% and 88%, respectively, of learners answered questions given as classwork correctly. This is compared to 50%, 50% and 67%, respectively, of learners from the control group who could present correct solutions on the same exercises. The results from classroom observation suggest that learners from the experimental group could present better solutions than those learners from the control group. The results from the post-tests, classroom observation and workbooks from learners indicate that learners taught using *Ubuntu* values perform better academically than those from the control group taught using traditional methods. This finding concurs with researchers (Adams, 2013; Slavin, 1991; Siegel, 2005; Gardner, 1999) who argue that all forms of cooperative learning if promoted in classrooms students' self-esteem and motivation are achieved giving rise to better academic performance. The positive interdependence of learners from the experimental group through sharing and other values of *Ubuntu* can be associated with better results of the experimental group than the control group.

On analysis of the post-test results, it emerged that there were statistical differences between test scores of the experimental group and the control group with respect to the Van Hieles' geometric thinking levels, namely, visualisation, informal deduction, formal deduction and rigour (see Table 6.42). However, there was no statistical difference between test scores with respect to Van Hieles' level of analysis. This phenomenon suggests that *Ubuntu* values were effective in improving the overall performance of the experimental group as well as the geometric thinking skills of TB's learners. Furthermore, this suggests that *Ubuntu* values used to teach TB's learners from the experimental group had some impact in improving their geometric thinking capabilities. The improvement of geometric thinking levels of learners from the experimental group more than those from the control group can be associated with the use of physical manipulatives which according to some researchers (Chiphambo, 2017; Bussi & Frank, 2015; Morgan & Sack, 2011; Gürbüz, 2010; Suh & Moyer, 2007), enable learners to understand geometry conceptually better than those who receive instruction without using them.

7.7 Summary of the chapter

This chapter presented a discussion of data collected in this study, which was an exploration of how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry. The findings obtained during the study indicate that the use of *Ubuntu* values in geometry classrooms promotes learners' conceptual understanding and in turn enhances learners' problem-solving skills. In the next chapter, conclusions and recommendations for this study are presented including recommendations for further studies.

CHAPTER 8: CONCLUSION AND RECOMMENDATIONS

8.1 Introduction

This chapter presents the findings, conclusions and recommendations of the study based on the data collected and analysed. It also provides an overview of the research aims and objectives as well as research questions. The chapter further highlights the limitations of the study, and provides detailed conclusions and recommendations directed to the Department of Basic Education. The conclusions and recommendations are deemed to be important to researchers in the teaching and learning of geometry as well.

8.2 Synopsis of the study

This section presents an overview of the study. Chapter 1 presented the introduction, background, research problem, research questions and aims as well as the scope of the study. This study attempted to answer the following research question:

How can *Ubuntu* values enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry?

The following sub-questions were answered:

How can the teachers' PCK be enhanced for effective and meaningful teaching of Grade 12 Euclidean geometry?

i. What difficulties are experienced by mathematics teachers in the teaching of Grade 12 Euclidean geometry?
ii. What aspects of *Ubuntu* philosophy should teachers apply for the enhancement of the teaching of Euclidean geometry in Grade 12?
iii. To what extent do collective work and collaboration impact the teaching of Grade 12 Euclidean geometry?
iv. How can *Ubuntu* philosophy be used to transform the teaching of Grade 12 Euclidean geometry?

To answer the research questions, aims and objectives were set. The literature reviewed addressed the research questions presented. Chapter 2 focused on the scope and breadth of *Ubuntu* values in the South African education system and the teaching and learning of geometry, MKT and PCK in geometry and challenges faced in the teaching of geometry. Chapter 3 was dedicated to the theoretical framework whereby the link between the social interdependence theory, *Ubuntu* philosophy and

Van Hieles' theory were discussed. Chapter 4 focused on the research methodology used to collect data, data analysis and all modalities of the study. Chapter 5 focused on data presentation, analysis and discussion while Chapter 6 focused on the discussion of the findings.

The study used the convergent parallel design, a design that falls under the mixed methods approach. Data were collected using the pre-test and post-test on learners using a quasi-experimental design. Two mathematics teachers from two different schools in the Chris Hani West District of the Eastern Cape Province of South Africa were purposively selected to participate in the study. Data were collected from the two participants using preliminary and final semi-structured interviews, pre-test and post-test to their learners, classroom observation and document analysis. Each of the two participants taught an experimental group and a control group. Each experimental group received instruction in Euclidean geometry using the proposed *Ubuntu*-based Dynamic Pairs Model where learners shared in dynamic groups whilst the control groups were exposed to traditional methods of instruction. Quantitative data were analysed using the SPSS package as well as descriptive statistics while the qualitative data were analysed thematically.

8.3 Summary of findings from the study

In this section, the findings based on the main themes identified are presented under the following headings:

i. Enhancement of teachers' PCK for effective and meaningful teaching of Grade 12 Euclidean geometry,
ii. Difficulties experienced by mathematics teachers in the teaching of Euclidean geometry,

iii. *Ubuntu* aspects that teachers should apply in teaching Euclidean geometry,
iv. Impact of collective work and collaboration in teaching Euclidean geometry, and
v. *Ubuntu* philosophy in the transformation of teaching Grade 12 geometry.

8.3.1 Enhancement of teachers' PCK enhancement for effective and meaningful teaching of Grade 12 Euclidean geometry

During interviews, the two participants mentioned that attendance of workshops was of paramount importance in enhancing their pedagogical and didactic skills in teaching geometry. They showed a great desire to acquire more PCK and MKT in the teaching of geometry because, during the 2008-2013 period, they missed training on Euclidean geometry since it was not included in the mainstream syllabus. During the years 2008-2013, Euclidean geometry was an optional topic in the Grade 12 national examinations and was examinable in Mathematics Paper 3 accompanied by other topics such as probabilities, counting principles, statistics and recursive sequences and series. In addition, the participants' desire to attend workshops and seminars on geometry teaching is recommended by Ali, Bhagawati and Sammah (2014) who argue that teachers should be trained through seminars and workshops on how to teach geometry effectively. Furthermore, Woolfolk's (2010) argument that the interaction of teachers with more knowledgeable members of the community influences their choice of instructional strategy supports the participants' desire to attend teacher development workshops and seminars.

During teacher development workshops and seminars, teachers who are knowledgeable with Euclidean geometry share their pedagogical knowledge in teaching Euclidean geometry. Aspects such as Van Hieles' theory and use of physical manipulatives can be with other mathematics teachers. According to Mammali (2015), professional interactions among teachers in professional learning communities, seminars and workshops improve teachers' MKT and PCK in Euclidean geometry. It was also observed that the two participants had degree qualifications in mathematics education. The attainment of higher academic and professional qualifications might have enriched their PCK and MKT.

8.3.2 Difficulties experienced by mathematics teachers in the teaching of Euclidean geometry

From the pre-test, post-test, classroom observations, document analysis and semi-structured interviews, the following difficulties in teaching Euclidean geometry were observed:

i. Learners' lack of geometric thinking and problem-solving skills,
ii. **Learners' lack of geometry language proficiency**: It was observed that the learners involved in the study had problems in communicating proficiently in English. The predominantly rural settings of Chris Hani West where isiXhosa was the vernacular language made it difficult for learners to communicate in English proficiently. The two participants had to exercise patience by giving learners the freedom to communicate in isiXhosa during lessons. The learners were gradually introduced to geometry terms in English,
iii. **Learners' forgetfulness and poor background in geometry**: Generally, learners had forgotten the geometry content from Grade 11 and other previous grades such as work with parallel lines, properties of basic geometric figures and the application of theorems together with their converse theorems in problem-solving, and
iv. **Teachers' assumptions of learners' negative attitude towards Euclidean geometry**: The two participants had assumed that learners had a negative attitude towards geometry. However, the enthusiasm displayed by learners

from both control and experimental groups refuted those assumptions and arguments by some researchers (Matins & Mello, 2015; Howson, 2000) that learners display a negative attitude towards geometry.

8.3.3 *Ubuntu* aspects that teachers should apply in teaching Euclidean geometry

During the study, learners from experimental groups shared resources such as models, pairs of scissors and mathematical instruments. Also, those learners shared knowledge, ideas and best methods of solving problems. The sharing of resources by learners from experimental groups in the geometry classroom is supported by some researchers (Samkange & Samkange, 1980; Lukhele, 1990; Mpofu, 2002) who argue that the sharing of resources and ideas is an *Ubuntu* principle important for development. On the contrary, learners from control groups did not enjoy the same classroom interactions experienced by those learners from the experimental groups because they were taught using restrictive teacher-centred instructional strategies. The learners from the control group engaged in individual work and did not have the opportunity of cross-pollinating ideas with their peers.

It was also observed that learners from the experimental groups engaged in active and full participation in classroom activities in which even habitually passive learners were also motivated to participate. The active and full participation by learners from the experimental group was enhanced by sharing, caring, cordial teacher-learner relationships, respect and catering for individuality. Intra-pair and inter-pair discussions among learners from the experimental group enabled learners with learning difficulties and misconceptions to solicit and obtain assistance from their peers instead of relying on the teacher only. The active participation of the learners from the experimental group is supported by Monaheng (2017) who asserts that *Ubuntu* is demonstrated by the active participation of community members for the well-being of that community. The full participation of learners from the experimental group can be associated with the use of practical activities in which learners were made to share ideas and knowledge.

During classroom observation, the two participants showed sympathy to learners who lost their beloved ones due to the devastating COVID-19 pandemic. Learners expressed their appreciation towards their teachers' gestures of sympathy and humanity to people experiencing hard times. This created a sense of belonging among the learners. It was also observed that the participants were patient to teach their learners in English in which they lacked proficiency. Those learners came from predominantly Xhosa-speaking communities in deep rural Chris Hani West. The teachers had to teach both English and geometry language in English with appreciable patience. The teacher's patience and demonstration of compassion and empathy towards learners can be associated with their motivation to work hard. The participants played the "loco parentis" role (Mclean,1990) which might have created confidence among learners. In addition, the parental roles played by the participants is supported by Eliastam (2015) with the dictum, *"My child is your child",* which is an important *Ubuntu* view in education.

The classrooms of both participants were pervaded by respect. The participants always went to class after having thoroughly prepared in terms of content knowledge and didactics in geometry. This shows that the participants respected the learners. The teacher had great command of the subject content and showed that he had an in-depth study of the content. To show respect for their learners, the participants engaged in the following activities:

i. Expeditiously provided learners feedback to their answers by marking and giving constructive comments. The teacher provided systematic feedback and corrections to the learners,
ii. Commented positively to learners' answers,
iii. Acknowledged the value of each learner's contribution,
iv. Recognised learners' capabilities,
v. Questioned and made follow-ups on learners' solutions. This showed learners that the teacher had confidence in their abilities and,
vi. Expressed high expectations for each learner. There was no undermining of learners' capabilities. Each learner was treated as having great potential.

Also notable was the use of indigenous knowledge systems in enriching the teaching of Euclidean geometry. Eastern Cape being a rural province has indigenous features such as round huts in each homestead. Those features could be used to teach Euclidean geometry concepts such as the subtended angle, triangles, line segments among others. During the study, it was observed that the two participants did not take advantage of the indigenous knowledge to enable learners to appreciate the relevance of Euclidean geometry in their real lives.

In addition, the two participants acknowledged other teachers' contributions to the development of their PCK and MKT. The two participants recognised that when other teachers shared their experiences in workshops and seminars, their PCK and MKT would be enhanced. This suggests that the two participants had great respect for other teachers. The sharing of experiences through workshops is supported by Sethunga et al. (2016) who suggest that teachers should work collaboratively with other teachers and network in professional learning communities to acquire the skills they lack.

8.3.4 Impact of collective work and collaboration in teaching Euclidean geometry

The two participants were impressed by the active and full participation of all learners from the experimental groups in classroom activities. It was interesting to observe habitually passive learners being very active in doing practical activities given to them by their teachers. Also, it was observed that learners from the experimental groups produced high-quality solutions in problem-solving as compared to their counterparts from the control groups. Furthermore, the quantity of written work in the workbooks of learners from experimental groups was more than the written work produced by their peers from respective control groups.

The learners' test scores from the post-test indicate that there were statistical differences between the test scores of learners from the experimental groups as compared to the test scores obtained by learners from the respective control groups. It can be concluded that the use of *Ubuntu* values enabled learners from the experimental groups to obtain better test scores than their peers from the control groups. However, there were no significant statistical differences in test scores between TA's learners from the experimental group and control group with respect to Van Hieles' levels of geometric thought. This phenomenon suggests that the *Ubuntu*-based model for teaching geometry had no impact on TA's learners' levels of geometric thinking. On the contrary, there was a statistical difference between test scores of TB's learners from the experimental group and those from the control group with respect to Van Hieles' levels of geometric thinking except in the Level 2 (Informal deduction/Abstraction level), where there was no statistical difference. This implies that the *Ubuntu*-based Dynamic Pairs Model had some impact on the levels of geometric thinking on TB's learners.

8.3.5 *Ubuntu* philosophy in the transformation of teaching Grade 12 Euclidean geometry

During the study, it was observed that the use of the Dynamic Pairs Model in teaching Euclidean geometry transformed the dynamics of classroom interactions among learners from the experimental groups. The learners from the experimental groups actively and fully participated in all classroom activities. It was also observed that habitually passive learners were seen being very active in doing practical activities. Furthermore, transformation was experienced among learners from experimental groups as they produced high-quality solutions in problem-solving. There was evidence that those learners did not impetuously answer the geometry problems given without critically analysing questions and diagrams. In addition, it can be concluded that the use of *Ubuntu* values enabled learners from experimental groups to obtain higher test scores in the post-test as compared to their peers from respective control groups. The proposed Dynamic Pairs Model is illustrated in Figure 3.3.

8.4 Some educational implications of this study

The findings from this study imply that:

i. The use of *Ubuntu* values in teaching geometry enhances learners' conceptual understanding. This phenomenon was noted when learners who were taught using *Ubuntu* values were able to solve more geometry problems as compared to their peers who were instructed using traditional methods. Also, noted from the post-test was that the test scores of learners from the experimental groups were significantly different from the test scores of their respective peers.

ii. The use of *Ubuntu* values motivates learners' full participation in classroom activities. It was observed that when learners feel that their teachers and peers exercise care, empathy, collective support, solidarity and other *Ubuntu* values, they are energised to participate fully in all classroom activities.

iii. The use of indigenous examples should not be undermined. Because this study was conducted in Eastern Cape, it was noted that the use of indigenous artefacts such as round huts and many others enables learners to realise the significance of studying geometry. In addition, indigenous examples motivate learners because they feel honoured when they find out that their ancestors' ideas of building huts and other indigenous artefacts are recognised in the education and academic fraternity.

iv. The use of vernacular language should not be undermined because it enables learners to express themselves freely. It was observed that when teachers exercise patience in teaching learners in English, the learners become active and their self-esteem is elevated. It is of primordial importance to note that the two participants in this study taught learners both English and geometry language gradually because their learners had a poor background in English. Patience as an *Ubuntu* value enabled the fruitful academic development of learners in the learning of geometry.

v. Teachers should be equipped with knowledge of how to incorporate Van Hieles' theory of geometric thinking in their teaching. It was observed that learners had challenges in analysing geometric figures and information related to them prior to problem-solving.

8.5 Limitations of this study

This study was conducted during Level 2 of COVID-19 lockdown in South Africa where a lot of restrictions were put in place in schools. Social distancing, wearing of face masks and use of sanitisers were practiced during that period. Learners could not sit very close to each other for effective sharing as envisaged by the *Ubuntu*-based dynamic pairs model but could share from at least two metres apart from each other. The prevalence of COVID-19 caused the researcher to collect data in a short space of time allocated by the Department of Basic Education. The pandemic had caused schools to be closed for about three months and much teaching and learning time was lost. In some schools, there were intermittent closures due to cases of outbreaks of COVID-19. It was prudent for the researcher to collect data speedily in a short space of time. The other limitation was organising the lesson observations in schools whereby teachers and learners were psychologically affected by the existence of the pandemic. It was a long process to get permission from the Chris Hani West District Department of Basic Education due to the thorough and rigorous scrutiny and screening process of individuals visiting schools.

Another challenge was the process of organising with principals to modify their schools' timetables to suit the study. However, principals understood the situation and pleaded for the study to be conducted in a short time. Other principals and teachers in Chris Hani District, were apprehensive to engage in the study due to the fear of being put under the spotlight of the Department of Basic Education. Furthermore, the two participants who consented had different years of teaching experience. Research (De Jong, 2010; Schneider & Plasman, 2011) confirms that PCK improves with the number of years of classroom experience. This implies that the two participants in the study had different PCK. The other limitation was that the COVID-19 pandemic compelled schools to have small groups of learners, hence smaller samples were used which impacted immensely on the generalisability of the findings. Each group of learners used in the study had a maximum of 20 learners.

8.6 Conclusions

This study focused on exploring how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry. The findings indicate that *Ubuntu* values had an impact on the teaching of Euclidean geometry in the two schools of Chris Hani West District of the Eastern Cape Province of South Africa.

8.7 Recommendations

The following recommendations are presented by the researcher:

8.7.1 Recommendations to the Department of Basic Education

i. Teachers should be encouraged to incorporate *Ubuntu* values in the teaching of Euclidean geometry.

ii. To effectively deliver lessons in Euclidean geometry, teachers are encouraged to incorporate ideas from the Dynamic Pairs Model which promotes the sharing of knowledge in Euclidean geometry among learners.

iii. During lessons in geometry, teachers should consider using manipulatives and indigenous knowledge in imparting geometry content to learners.

iv. The use of *Ubuntu* values and physical manipulatives should commence in lower grades to strengthen learners' background in geometry.

v. Apart from using manipulatives, teachers should create more activities that enhance the skill of visualisation such as using computers in teaching geometry. In addition, the Department of Basic Education should provide information and communication technology [ICT] tools that enable learners to visualise and reason geometrically.

vi. Teacher development programmes should be organised to capacitate teachers in teaching Euclidean geometry, especially on how to tailor lessons that enhance the acquisition of Van Hieles' levels of geometric thinking.

vii. The use of vernacular language should be incorporated in teaching to enhance conceptual understanding among learners.

viii. Teachers should be cognisant of the fact that learners still have language problems and adjust their instruction to accommodate code-switching.

ix. Teachers should combine Ubuntu values with other theories that are directly related to the teaching of mathematics topics in the classrooms.

8.7.2 Recommendations for further research

i. Comprehensive large-scale and longitudinal research studies should be conducted on larger samples on the use of *Ubuntu* values in the teaching and learning of geometry.

ii. Studies on the use of *Ubuntu* values should be conducted in other districts of the Eastern Cape as well as other provinces in South Africa.

iii. Joint studies among teachers and researchers should be conducted on how to use *Ubuntu* values in teaching geometry as well as other topics across the mathematics curriculum in South Africa.

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ANNEXURES

Annexure A: Permission letter to department of basic education

UNISA university of south africa

The District Director

Chris Hani West District Department of Basic Education

Private Bag X7053

Queenstown 5320

Dear District Director

Re: Request for permission to conduct research in some secondary schools in Chris Hani West District: Myself

I, **Justine Chidziva,** am doing research under supervision of Professor ZMM Jojo in the Department of Mathematics Education towards a PhD in Mathematics Education at the University of South Africa. I hereby request permission to contact research in some of your secondary schools in Chris Hani West District.

The title of my research is: **Using *Ubuntu* values to enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.** In this context, "disruptive pedagogies" refers to new ways of approaching Euclidean geometry to promote learners' understanding deviating from the use of common traditional teacher-centred approaches. It means going an extra mile to encourage teachers to teach the topic effectively using unusual strategies.

The aim of the study is to explore how the use of *Ubuntu* values can enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry. In this study, I have selected Chris Hani District because I am working with a company which conducts intervention programmes there. I have been working with your schools since the year 2017. The study will use mixed methods where data will be collected using convergent parallel design from selected schools. In the study two mathematics from two different schools will be selected to participate. The teachers will be interviewed, and their lessons will be observed, and video recorded. Their learners will be requested to write pre-test and post-test in Euclidean geometry and the learners' workbooks will be viewed for analysis.

Before conducting this research, I am required by the ethics policy of the University of South Africa to get permission first from the Department of Basic Education. This ethics policy also requires that all participants of this study be protected by keeping their identities anonymous and the information confidential. The Eastern Cape Department of Basic Education will benefit from this study by getting information about how to teach Euclidean geometry effectively using *Ubuntu* philosophy. Research and diagnostic reports have shown that this topic is one of the most difficult topics for both teachers and learners in South African secondary schools inclusive of Eastern Cape secondary schools.

When the study has been completed, a copy of the report will be made available to the Eastern Cape Department of Basic Education. I have attached a brief research proposal containing all the information required.

Hoping for a favourable response to my request.

Yours faithfully

Mr Justine Chidziva

Cell: 0787006917 Email address: justgoneka@gmail.com

Supervisor: Professor ZMM Jojo (University of South Africa: Mathematics Education Department)

Telephone: +27124296627 email: jojozmm@unisa.ac.za

Annexure B: Permission letter from Chris Hani West Department of Basic Education

Province of the
EASTERN CAPE
EDUCATION

CHRIS HANI WEST DISTRICT :OFFICE OF THE DISTRICT DIRECTOR
HOMESTEAD SITE, 2 LIMPOPO DRIVE LAURIE DASHWOOD PARK QUEENSTOWN 5320. Private Bag X7053
QUEENSTOWN, 5320 REPUBLIC OF SOUTH AFRICA, Website: www.ecdoe.gov.za

TO	:	**MR JUSTINE CHIDZIVA**
		PRINCIPALS OF THE FOLLOWING SCHOOLS –
		(PUMLANI, NTSONKOTHA, KWAKOMANI
		TAMBEKILE, NKWANCA ,MASIKANYISE)
FROM	:	**THE OFFICE OF THE DIRECTOR**
		CHRIS HANI WEST EDUCATION DISTRICT
SUBJECTS	:	**APPROVAL TO CONDUCT RESEARCH**

This letter serves to approve your request to conduct research with the title: **Using Ubuntu values to enhance disruptive pedagogies for effective and meaningful teaching of grade 12 Euclidean geometry** for your PhD studies in secondary schools in Chris Hani West District. You are requested to make arrangements with principals and educators of the respective schools with the provision that normal teaching time will not be affected. You are also requested to adhere to the provisions of your University's research ethics when conducting the research. After approval of your research by your University, our District will request that you share your findings or copy of the report.

Yours in Quality Education

NRW DE BRUYN
DISTRICT DIRECTOR: CHWED

22/07/2020
DATE

building blocks for growth

PGDP EASTERN CAPE 2004 – 2014

Ikamva eliqaqambileyo!

Annexure C: Permission letter to principals

UNISA university of south africa

Dear Principal

Re: Request for permission to conduct research at your school: Myself

I, Justine Chidziva, am doing research under supervision of Professor ZMM Jojo in the Department of Mathematics Education towards a PhD in Mathematics Education at the University of South Africa. I hereby request permission to contact research at your school. The title of my research is: **Using *Ubuntu* values to enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.** In this context "disruptive pedagogies" refers to new ways of approaching Euclidean geometry to promote learners' understanding deviating from the use of common traditional teacher-centred approaches. It means going an extra mile to encourage teachers to teach the topic effectively using unusual strategies.

The aim of the study is to explore if *Ubuntu* philosophy can enhance the meaningful teaching of Euclidean geometry. In this study I have selected your school because I have been working it for 3 years now with a company which conducts intervention programmes. The study will use mixed methods where data will be collected using convergent parallel design from selected schools.

In the study two mathematics from two different schools will be selected to participate and your mathematics teacher will be one of the participants. The two teachers will be interviewed, and their lessons will be observed and video recorded. Their learners will be requested to write pre-test and post-test in Euclidean geometry and the learners' workbooks will be viewed for analysis.

Before conducting this research, I am required by the ethics policy of the University of South Africa to get permission first from the Eastern Cape Department of Basic Education. I have already been granted permission to conduct the study at any school in Chris Hani West District by the Eastern Cape Department of Basic Education. The university ethics policy also requires that all participants of this study be protected by keeping their identities anonymous and the information confidential. Your school will benefit from this study by getting information about how to teach Euclidean geometry effectively using *Ubuntu* philosophy. Research and diagnostic reports have shown that this topic is one of the most difficult topics for both teachers and learners in South African secondary schools inclusive of Eastern Cape secondary schools.

When the study has been completed, a copy of the report will be made available to you. I have attached the permission letter from the Eastern Cape Department of Basic Education and a brief research proposal containing all the information required.

Hoping for a favourable response to my request.

Yours faithfully

Mr Justine Chidziva

Cell: 0787006917 Email address: justgoneka@gmail.com

Supervisor: Professor ZMM Jojo (University of South Africa: Mathematics Education Department)

Telephone: +27124296627 email: jojozmm@unisa.ac.za

Annexure D: Consent/assent letters from principals

(RETURN SLIP)

I, ________________________________, (Principals' name), confirm that the person asking my consent to conduct research in my school has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to allow the researcher to conduct the research in my school.

I understand that the participation of my school is voluntary and that the teacher and learners are free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my school's participation will be kept confidential unless otherwise specified.

I agree to the recording of the ____________________________(insert specific data collection method).

I have received a signed copy of the informed consent agreement.

Principal's Name & Surname (please print)______________________________

Principal's Signature_________ Date :____________

Researcher's Name & Surname (please print) ____________________________

Researcher's signature______________ Date_____________

Annexure E: Letter to the participating mathematics teachers

Date:

Dear Teacher

I, **Justine Chidziva**, am conducting a research in teaching Euclidean geometry in Grade 12. I am doing research under the supervision of Professor ZMM Jojo, a professor in the Department of Mathematics Education at the University of South Africa. I am studying towards PhD in Mathematics Education at the above-mentioned university. I hereby invite you to take part in this study, which is meant to enhance the teaching of Euclidean geometry.

The title of my research is: **Using *Ubuntu* values to enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.** In this context “disruptive pedagogies” refers to new ways of approaching Euclidean geometry to promote learners’ understanding deviating from the use of common traditional teacher-centred approaches. It means going an extra mile to encourage teachers to teach the topic effectively using unusual strategies.

The purpose of the study is to explore if *Ubuntu* philosophy can enhance the meaningful teaching of Euclidean geometry. In this study two mathematics from two different schools will be selected to participate. I have selected you as one of the two participants because of your experience and track record of teaching mathematics. Also, I believe you will be able to provide important information which I will require in this study. I therefore kindly request you to volunteer to participate in this study. I want to understand that you are not forced to take part in this research. During the study you will be allowed to withdraw your participation at any time if you wish to.

During the study, the researcher will conduct semi-structured interviews with you at the beginning and at the end of the study. Your lessons will on Euclidean geometry will be observed by the researcher in company of a co-researcher. Pre-test and post-test will be given to your learners to write. The learners will use pseudonyms to ensure their privacy and confidentiality. Their names will not be required to appear on the scripts of the tests written. Learners' workbooks, lesson plans and other teaching document will be looked at during the study. Your lessons will be video recorded for future use by the researcher. Also, field notes and critiques will be written about each lesson.

The videos recorded will be kept safe in a password protected computer. Also, hard copies of your answers and field notes written by the researcher and co-researcher will be stored by the researcher for a period of five years in a locked cupboard for future research or academic purposes; electronic information will be stored on a password protected computer. After five years, hard copies will be shred and electronic copies will be permanently deleted from the hard drive of the computer using a relevant software program.

I guarantee that you will not be harmed or affected in any way. If you will need to ask clarity seeking questions the researcher will be free to answer the questions. When the research report will be presented your name and your school will not appear, but a pseudonym will be used to ensure that your privacy and confidentiality are observed. In addition, the research report will be made available to you. This study has received written approval from the Research Ethics Review Committee of the University of

South Africa. A copy of the approval letter can be obtained from the researcher if you so wish.

If you would like to be informed of the final research findings, please contact the researcher and the supervisor. The findings of this study will be available immediately after the University of South Africa approves it. Should you require any further information or want to contact the researcher about any aspect of this study. Should you have concerns about the way in which the research has been conducted, you may contact the researcher and the supervisor of the researcher.

Thank you for taking time to read this information sheet and for participating in this study.

Yours faithfully

Justine Chidziva

Cell: 0787006917 Email: justgoneka@gmail.com

Supervisor: Professor ZMM Jojo (University of South Africa: Mathematics Education Department)

Telephone: +27124296627   email: jojozmm@unisa.ac.za

Annexure F: Consent/assent by teachers to participate in the study

(RETURN SLIP)

I, ________________________________(participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the ____________________________(insert specific data collection method).

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) ______________________________

Participant Signature ________________ Date___________________

Researcher’s Name & Surname (please print_________________________________

Researcher’s signature__________ Date:__________________

Annexure G: Letter requesting parental consent for minors to participate in a research project

Dear Parent/guardian

Your child is invited to participate in a study entitled: **Using *Ubuntu* values to enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.** In this context "disruptive pedagogies" refers to new ways of approaching Euclidean geometry to promote learners' understanding deviating from the use of common traditional teacher-centred approaches. It means going an extra mile to encourage teachers to teach the topic effectively using unusual strategies.

I am undertaking this study as part of my doctoral research at the University of South Africa. I am a PhD student majoring in Mathematics Education at the above-mentioned university. The purpose of the study is to explore how *Ubuntu* philosophy can enhance the meaningful teaching of Euclidean geometry. The possible benefits of the study are the improvement of the teaching and learning of one the difficult topics for learners. I am asking permission to include your child in this study because I will be working with your child's mathematics teacher in this study. I expect to have all other children in the class of your to participate in the study.

In the study two mathematics from two different schools will be selected to participate. I would like to inform you that the mathematics teacher of your child has been selected to participate in the study. The two teachers will be interviewed, their lessons will be observed and video recorded, their learners will be requested to write pre-test and post-test in Euclidean geometry and the learners' workbooks will be viewed for analysis.

If you allow your child to participate, I shall request him or her to:

- write pre-test and post-test and,
- attend video-recorded lessons in Euclidean geometry which will be conducted by the respective teacher.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study. However, the possible benefits to education are that your learners' understanding of Euclidean geometry will be enhanced and also your child will learn more about *Ubuntu* which will be beneficial to the family. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, your child will be learning the topic in a class which will not be involved in the study under another competent teacher. In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

There are no potential risks are involved. There will be no reimbursement or any incentives for participation in the research. If you have questions about this study please ask me or my study supervisor, Prof ZMM Jojo, Department of Mathematics Education, College of Education, University of South Africa. My contact number is 0787006917 and my e-mail is justgoneka@gmail.com . The e-mail of my supervisor is jojozmm@unisa.ac.za. Permission for the study has already been given by the Superintendent General of the Eastern Cape Department of Basic Education and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Yours faithfully

Mr Justine Chidziva

Cell: 0787006917 Email address: justgoneka@gmail.com

Supervisor: Professor ZMM Jojo (University of South Africa: Mathematics Education Department)

Telephone: +27124296627 email: jojozmm@unisa.ac.za

Annexure H: Parent/guardian's consent/ assent of child's participation in this study

(RETURN SLIP)

I, ________________________________ (parent/guardian), confirm that the person asking my consent to allow my child to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that the participation of my child is voluntary and that my child is free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my child's participation will be kept confidential unless otherwise specified.

I agree to the recording of the _________________________(insert specific data collection method).

I have received a signed copy of the informed consent agreement.

Parent/Guardian Name & Surname (please print)______________________________

Parent/Guardian Signature ___________ Date:_____________

Researcher's Name & Surname ____

Researcher's signature:_________________________________Date:_____________

Annexure I: Letter requesting assent from learners in a secondary school to participate in a research project

Dear Learner Date ________________

I am doing a study on Euclidean geometry as part of my studies at the University of South Africa. The topic of the research is: **Using *Ubuntu* values to enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry.** In this context "disruptive pedagogies" refers to new ways of approaching Euclidean geometry to promote learners' understanding deviating from the use of common traditional teacher-centred approaches. It means going an extra mile to encourage teachers to teach the topic effectively using unusual strategies.

Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your teachers can use to teach Euclidean geometry effectively. This may help you and many other learners of your age in different schools.

In the study two mathematics from two different schools will be selected to participate. I would like to inform you that your mathematics teacher has been selected to participate in the study as one of the two participants. The two teachers will be interviewed, and their lessons will be observed, and video recorded. The learners will be requested to write pre-test and post-test in Euclidean geometry and the learners' workbooks will be viewed for analysis.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

If you are willing to participate, you will be involved in this research during your usual mathematics lessons for 2 weeks. During the two weeks of the research, two teachers (myself and another teacher) will observe your teacher delivering lessons in Euclidean geometry and taking note of learners' participation. Just before lessons are observed, you will be requested to write a pre-test. At the end of the research, you will also be requested to write a post-test. In both pre-test and post-test you will not write your name on the answer scripts but only pseudonyms. You will be requested to avail your workbooks for review by the researcher, and also the lessons will be video recorded. The videos from the lessons observed will be kept safe by the researcher in a password-protected computer. Your identity will not be disclosed and you are guaranteed privacy and confidentiality. No potential risks or harm are anticipated during the study. Your participation in this study will be voluntary and you will be allowed to withdraw at any given time. When I finish this study, I will give your class a short talk about some of the helpful and interesting things I found out. It will not cost you or your parents anything.

If you decide to take part, please sign the form overleaf. If you have any other questions about this, you can talk to me or you can have your parent or another adult call me. Do not sign the form until you have all your questions answered and understand what I would like you to do and until your parents have granted you permission to do so.

I will write a report on the study, but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you do not want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

The benefits of this study are:

7 the teachers will improve the way they teach Euclidean geometry for learners to understand, and

8 learners will improve in solving problems in Euclidean geometry.

There are no risks anticipated in this research

You will not be reimbursed or receive any incentives for your participation in the research.

If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at 0787006917. Do not sign the form until you have all your questions answered and understand what I would like you to do.

Researcher: Mr J Chidziva Phone number: 0787006917

Do not sign the written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

--

-------------------------tear off here

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

_______________________ _______________ __________

Learner's name(print) Learner's signature: Date

_____________________ _____________________

Witness's name (print) Witness' signature

Date:____________________

(The witness is over 18 years old and present when signed.)

__________________________ __________________________

Parent/guardian's name (print) Parent/guardian's signature: Date:

____

____________________________ ________________________

Researcher's name (print) Researcher's signature:

Date___________________

Annexure J: Preliminary semi-structured interview with the teachers

1. What qualifications do you have?

____

____

____

____

2. Did you train to teach mathematics?

____

____

3. How many years have you been teaching mathematics?

____

____

4. At college or university did you study any modules in Euclidean geometry?

____

____

5. If you taught mathematics before CAPS was introduced, did you manage to teach Euclidean geometry?

____

____

6. Did you attend any content workshops or seminars in Euclidean geometry?

____

____

____

7. In the teaching of Euclidean geometry what challenges have you encountered?

How do you rate yourself in terms of content knowledge in Euclidean geometry? Do you think you need more assistance or not?

8. What challenges have you observed in the topic among learners over the years?

9. As a teacher have you ever consulted Department of Basic Education diagnostic reports every year?

10. In the teaching of Euclidean geometry have you ever used collective learning strategies such as group work and teamwork among others?

11. Have you ever used principles of *Ubuntu* in your mathematics classroom before? If so, which ones do you think have been dominant?

12. What attributes of *Ubuntu* do you think can be used effectively in a classroom? Do you think these principles can enhance the teaching of Euclidean geometry?

Annexure K: Final semi-structured interview with the teachers

1. During the study what challenges did you encounter in teaching the topic? If so, how best do you think you addressed these challenges?

   ____
   ____
   ____
   ____

2. What common difficulties, misconceptions and errors did you identify among the learners? Among them which ones did you manage to address?

   ____
   ____
   ____
   ____

3. Did you have enough resources to teach the topic effectively?

   ____
   ____
   ____

4. What new things did you learn through your interaction with the researcher?

   ____
   ____
   ____

5. Are you convinced that *Ubuntu* values can assist in teaching Euclidean geometry effectively?

   ____
   ____

6. Do you think learners benefitted during the study?

7. In your own opinion, do you think you will consider using *Ubuntu* values in your Euclidean geometry classroom again?

8. How do you think *Ubuntu* values can be implemented in a geometry classroom?

9. Do you think it was easy to use the *Ubuntu* model suggested by the researcher? What aspects do you suggest adjustments should be made?

Annexure L: Classroom observation schedule

ITEM	REMARKS
1. Is the teaching and learning environment conducive for learning? What items could be added to improve the conditions if any?	
2. Are there sufficient resources that promote effective teaching of Euclidean geometry?	
3. Is the teacher confident in the teaching of the topic? Does he/she show that he/she has dominion over the content?	
4. Are the instructional strategies used by the teacher promoting learner participation?	
5. Is there effective learner-learner interaction in the classroom?	
6. What learning difficulties are learners encountering? How does the teacher intervene in trying to address these difficulties?	

7. Are the assessment strategies used by the teacher effective?	
8. Is the teacher aware of Van Hiele levels of geometric thinking during teaching? How do these levels manifest in the classroom?	
9. What aspects and values of *Ubuntu* are visible in the classroom? Do these aspects enhance the teaching of the topic?	
10. Do learners enjoy learning using these *Ubuntu* values implemented by the teacher?	

Annexure M: Document analysis (learners' workbooks and tests)

ITEM	REMARKS
1. Did every learner write the prescribed exercises in his/her workbook?	
2. Does the teacher possess a lesson plan? Is the lesson plan relevant and detailed showing the use of *Ubuntu* values?	
3. What common errors, misconceptions were observed in the learners' workbooks?	
4. Did the teacher's marking indicate that these errors and misconceptions were observed?	
5. Which aspects in the topic were learners comfortable with according to the information obtained from written work?	
6. Was the written work in line with CAPS requirements?	
7. Was the written work relevant to channel learners to answer examination questions?	

8. Any other aspects observed?	

Annexure N: Pre-test on Euclidean geometry

TOTAL MARKS: 27 **DURATION: 40 Minutes**

INSTRUCTIONS

1. Do **NOT WRITE YOUR NAME** on your answer script. Use the candidate number given to you
2. Answer **ALL** the two questions in this paper
3. Diagrams are **NOT** necessarily drawn to scale
4. Number correctly according to the numbering system in this question paper
5. Clearly show **ALL** calculations, diagrams et cetera that you have used in determining your Answers

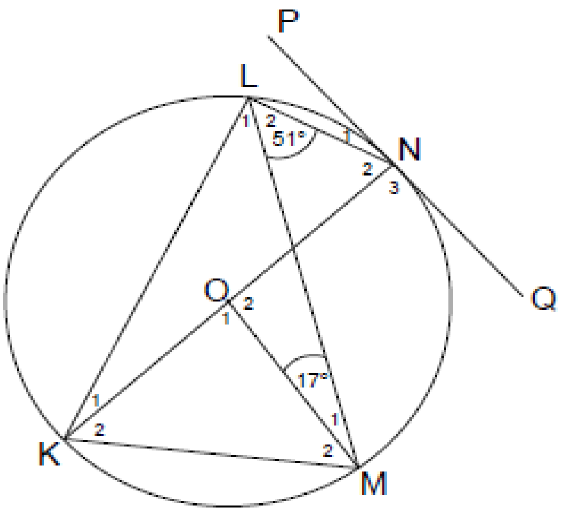
QUESTION 1

1.1. Complete the following statements

1.1.1 In a circle angle subtended by a chord at the centre of the circle is………………………………………………………….. (1)

1.1.2 Opposite angles of a cyclic quadrilateral are…………………………………………………………..(1)

1.2 In the diagram below O is the Centre of the circle KLNM. $\hat{M}_1=17°$ and $\hat{L}_2=51°$. PNQ is a tangent to the circle at N.



Calculate, giving reasons, the size of

1.2.1 $\hat{L}_1$ (2)

1.2.2 $\hat{O}_1$ (2)

1.1.3 $\hat{M}_2$ (2)

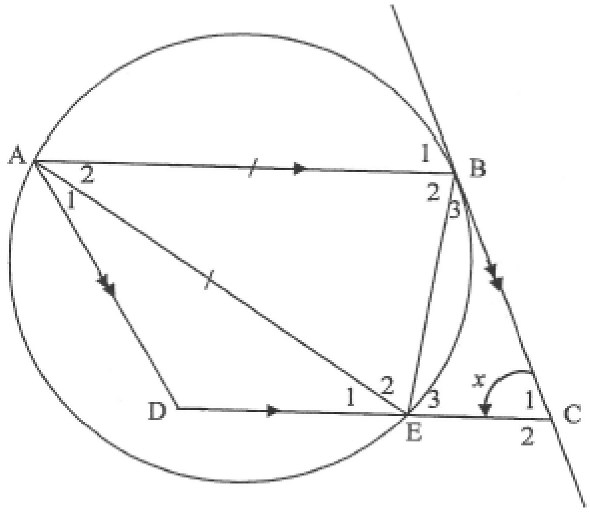
1.1.4 $\hat{N}_2$ (2)

1.1.5 $\hat{N}_1$ (2)

QUESTION 2

In the diagram below the circle passes through A, B and E. ABCD is a parallelogram.

BC is a tangent to the circle at B. AE=AB. $\hat{C}_1$= x.



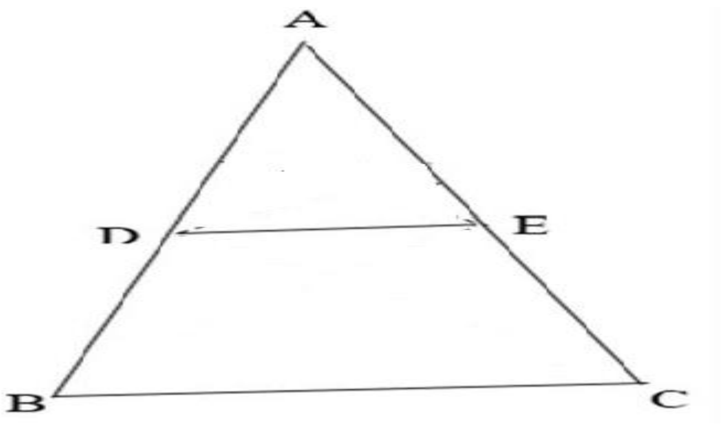
2.1 Give a reason why $\hat{B}_1$=X (1)

2.2 Name with reasons, FOUR other angles equal to X (8)

2.3 Prove that ABED is a cyclic quadrilateral (3)

QUESTION 3

In the diagram below D is the mid-point of AB, AD=DB, DE **II** BC and AE=7cm



Calculate the length of AC (3)

Annexure O: Post-test in Euclidean geometry

TOTAL MARKS: 30 DURATION: 40 Minutes

INSTRUCTIONS

6. Do **NOT WRITE YOUR NAME** on your answer script. Use the candidate number given to you
7. Answer **ALL** the two questions in this paper
8. Diagrams are **NOT** necessarily drawn to scale
9. Number correctly according to the numbering system in this question paper
10. Clearly show **ALL** calculations, diagrams et cetera that you have used in determining your Answers

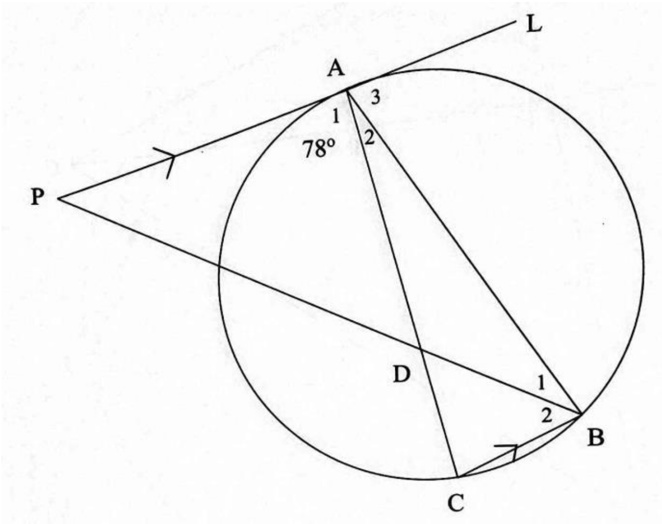
QUESTION 1

1.1 Complete the following statements

1.1.1. Tangents of a circle drawn from a common exterior point……………………………………………………………. (1)

1.1.2. The angle between a tangent to a circle and a chord drawn from the point of contact is………………………………………………… (1)

1.2 In the diagram, PAL is a tangent to the circle at A. C and B are points on the circle such that CB II PAL. AB, AC and PB are drawn. PB intersects AC at D. $\hat{A}_1$=78.



Determine the sizes of each of the following angles with reasons

1.2.1. $\hat{C}$ [1]

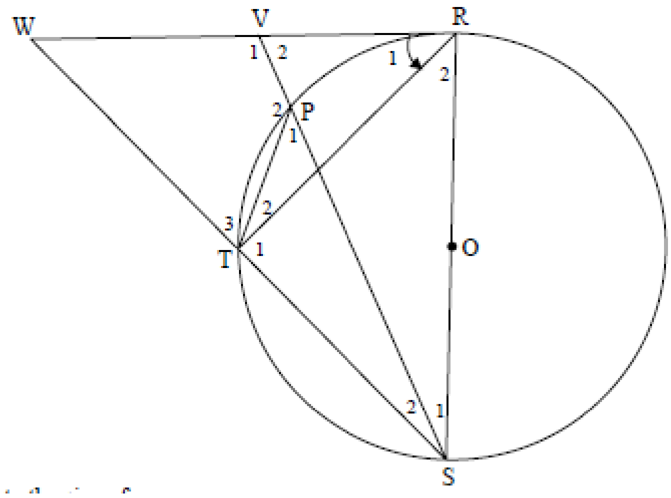
1.2.2. $A\hat{B}C$ [2]

1.2.3. $\hat{A}_3$ [2]

1.2.4. $\hat{A}_2$ [2]

QUESTION 2

In the diagram below RS is a diameter of the circle with centre O. Chord ST is produced to W. Chord SP produced meets tangent RW at V. $\hat{R}_1 = 50°$



8.5 Calculate, giving reasons, the size of

e) $W\hat{R}S$ (2)

f) $\hat{T}1$ (2)

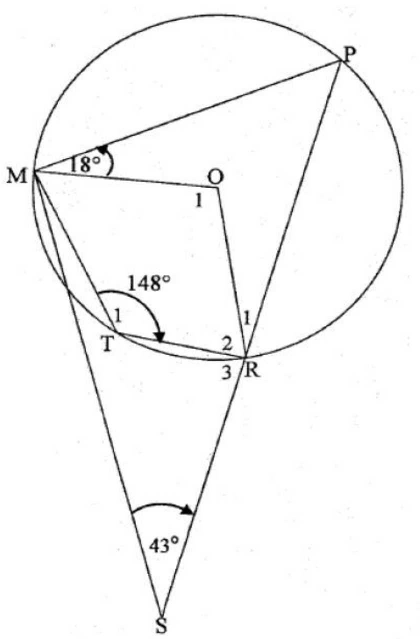
g) $\hat{W}$ (2)

h) $\hat{P}1$ (3)

2.2 Prove that WVPT is a cyclic quadrilateral (3)

QUESTION 3

In the diagram below, P, M, T and R, are points on a circle having centre O. PR produced meets MS at S. Radii OM and OR and the chords MT and TR are drawn. $\hat{T}1=148^\circ$, $P\hat{M}O=18^\circ$ and $\hat{S}=43^\circ$.



Calculate with reasons

3.1 $\hat{P}$ (2)

3.2 $\hat{O}_1$ (2)

3.3 $O\hat{M}S$ (3)

3.4 $\hat{R}_3$, if it is given that $T\hat{M}S = 6^\circ$ (2)

Annexure P: Ethical clearance certificate

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/05/13

Dear Mr J CHIDZIVA

Decision: Ethics Approval from 2020/05/13 to 2025/05/13

Ref: **2020/05/13/46591656/23/AM**
Name: Mr J CHIDZIVA
Student No.: 46591656

Researcher(s): Name: Mr J CHIDZIVA
E-mail address: 46591656@mylife.unisa.ac.za
Telephone: 0787006917

Supervisor(s): Name: Prof ZMM JOJO
E-mail address: jojozmm@unisa.ac.za
Telephone: +27124296627

Title of research:

Using Ubuntu values to enhance disruptive pedagogies for effective and meaningful teaching of grade 12 Euclidean geometry

Qualification: PhD Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/05/13 to 2025/05/13.

*The **low risk** application was reviewed by the Ethics Review Committee on 2020/05/13 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2025/05/13**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2020/05/13/46591656/23/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof PM Sebate
ACTING EXECUTIVE DEAN
Sebatpm@unisa.ac.za

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www.unisa.ac.za

Annexure Q: Editorial Cerificate

EDITORIAL
CERTIFICATE

Author: Mr Justine Chidziva

Document title: Using *Ubuntu* values to enhance disruptive pedagogies for effective and meaningful teaching of Grade 12 Euclidean geometry

Date issued: 10/06/2021

This document certifies that the above manuscript was proofread and edited by Prof Gift Mheta (PhD, Linguistics).

The document was edited for proper English language, grammar, punctuation, spelling and overall style. The editor endeavoured to ensure that the author's intended meaning was not altered during the review. All amendments were tracked with the Microsoft Word "Track Changes" feature. Therefore, the authors had the option to reject or accept each change individually.

Kind regards

Prof Gift Mheta (Cell: 073 954 8913)