EXPLORING THE GRADE 11 MATHEMATICS LEARNERS' EXPERIENCES AND CHALLENGES IN SOLVING PROBABILITY PROBLEMS IN SELECTED SOSHANGUVE SCHOOLS

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22 November 2019

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DEDICATION

This dissertation is dedicated to my wonderful mother Va-Zebra for instilling in me the value of education and to my wonderful daughters Tanya and Thembelihle who are the pillars of my strength and the source of my inspiration.

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ABSTRACT

This study explored the challenges experienced by Grade 11 mathematics learners in solving probability problems in selected Soshanguve Township Schools in the Gauteng Province of South Africa. The aim of the study was to explore the experiences of Grade 11 learners in analysing, solving and communicating problems relating to probability in order to recommend strategies that can be implemented to resolve such problems. The mixed methods design was adopted for the study following a sequential explanatory research design. The targeted population in this study comprised all Grade 11 mathematics learners and their respective teachers in the Tshwane West District, in the Gauteng province. In this District, 58 schools offer mathematics at Grade 11 with 3676 learners and 20 Mathematics teachers at this Grade level. A total of 380 mathematics learners and their 5 teachers were selected through a purposive sampling method taking a critical case sampling technique. The critical case sample selected was the last three bottom schools who performed poorly at the end of Grade 12 year mathematics examinations for the past three years in the district.

Data collection instruments consisted of a diagnostic test, semi-structured interviews, and a lesson observation guide. Quantitative data from the diagnostic test, which constituted learners' test responses, was then analysed using descriptive statistics. Data were presented in the form of frequency tables and categorised as test responses that are *correct*, *incorrect*, *incomplete* and *blank responses*. Themes and patterns were used to analyse data from semi-structured interviews and lesson observations. The study revealed various challenges experienced by Grade 11 learners in solving probability problems. Most learners (98%) scored below 50% when solving the probability problems involving Venn diagrams, mutually exclusive events, and dependent events. The findings show that learners lacked procedural and conceptual knowledge to solve probability problems. Semi-structured interviews and lesson observations revealed that learners seemed not to have an interest in learning the probability topic. Learners appeared to have inconsistent ideas with probability reasoning and lacked understanding of related probability vocabulary.

The findings of this study may carry important implications for teachers, learners, and curriculum developers in assisting learners to address their challenges, constraints and difficulties in learning the mathematics content in probability. Knowledge of learners' mathematical challenges in Grade 11 may help teachers to generate effective teaching strategies for probability. The researcher recommends that in the learning of Grade 11 probability content the basic concepts should be understood and probability rules should be emphasized.

KEY TERMS

Conceptual knowledge

Diagnostic test

Errors

Experiences

Mathematics learner

Mathematics performance

Misconceptions

Poor performance

Probability performance

Procedural knowledge

Scaffolding

Socio-constructivism

Solving problem

Zone of Proximal Development

LIST OF ABBREVIATIONS

ACE	Advanced Certificate in Education
ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DoE	Department of Education
FET	Further Education and Training
GDE	Gauteng Department of Education
HoD	Head of Department
LTSM	Learning and teaching support materials
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
QUAL	Qualitative
QUAN	Quantitative
SCK	Subject Content Knowledge
TIMSS	Trends in International Mathematics and Science Study
ZPD	Zone of Proximal Development

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CHAPTER ONE

ORIENTATION TO THE STUDY

1.1 INTRODUCTION

This chapter provides a general overview of the study, which has attempted to explore the experiences of Grade 11 mathematics learners and the challenges they face when solving probability problems. The ongoing poor performance of mathematics learners is a concerning subject for policy makers in South Africa. Various research has discussed issues pertaining to mathematics learners' performance in general, and a minimal number of scientific studies addressing the topic of probability have documented these misconceptions (Mutara, 2015; Paul & Hlanganipai, 2014), pedagogical content knowledge useful for teaching probability (Brijlall, 2014), the instruction and apprehension of data handling (Adu & Gasa, 2014) and the role that the language plays in probability (Paul & Hlanganipai, 2014). The current study took place in the Soshanguve township[1] schools, in the Gauteng province of South Africa. This chapter provides, among other things, the contextual background of the study, the rationale undergirding the study, its problem statement, aim and objectives, research questions, and research methodology, along with the design of the study and the organization of its chapters.

1.2 CONTEXTUAL BACKGROUND AND RATIONALE OF THE STUDY

Enu, Agyman and Nkum (2015) assert that mathematics education provides the foundational touchstone upon which learners acquire scientific and technological knowledge that is central in their lives. The development of mathematical skills and knowledge are the goals of the mathematics education curriculum (Department of Basic Education [DBE], 2011). Procedural and conceptual knowledge, reasoning and strategic competences are the main knowledge domain underpinning the mathematics curriculum (Kilpatrick, Swafford & Findell, 2001). Mathematics education and learners' mathematical performance are the main ingredients in the South African technological economic development (Carnevale, 2005). Yet it is concerning

1. A township represents a place in South Africa where black Africans are largely located (Wilkinson, 1998). The apartheid government racially demarcated residential areas to restrict the number of black Africans living within urban areas. Townships are poorly serviced, with informal structures, backyards and widespread poverty (Cocks, Alexander, Mogano & Vetter, 2016). In this study, research participants came from disadvantaged schools located in township.

that The Trends in International Mathematics and Science Study (TIMSS) (2015) noted that learners' mathematical performance is below international and national benchmark standards (Reddy, Visser, Winnaar, Arends, Juan, Prinsloo & Isdale, 2016). It is thus concerning that the data collected in TIMMS (2015) revealed that South Africa performed relatively poor in mathematics when compared to other participating countries (Visser, Juan & Feza, 2015).

Spaull's (2013: 3) reports on Annual National Assessments (ANA)[2] conducted on Grade 9 learners in South Africa confirmed that most mathematics learners were underperforming relative to the mathematics curriculum. Likewise, the South African quality assurance body (Umalusi) generated a disappointing result of Grade 12 mathematics learners' National Senior Certificate (NSC) examinations. Most learners struggle to perform optimally in Grade 12 mathematics and those who are considered to have performed well later struggle at the university level (DBE, 2018). The Grade 12 results of 2014 released by Umalusi documented a decline in mathematics pass rate by 5.6% from 59.1% in 2013. A further decline in 2015 mathematics pass rate by 4.4% from 53.5% in 2014 was documented. Despite there being a documented increase of 3.6% in 2016 from 31.9% in 2015, there is a general low pass rate in the results of Grade 12 learners. It is of great concern to note that the Grade 12 class of 2017 registered an overall end of year mathematics pass of 51.9%, which was below expectations of 100%.

Most of the research has revealed several factors influencing learners' mathematical performance in South Africa (for examples, see, Howie, 2003; Khuzwayo, 2005; Mji, & Makgato, 2006; Graven, 2014; Visser, Juan & Feza, 2015). Visser *et al*. (2015) and Graven (2014) posited that these factors may be classified as those relating to home environment and others relating to school context. Spaull (2015) noted that the relationship between wealth and education models reveals that the poorer learners in South Africa were the worst performing academically. Morkel (2014, cited in Sibanda, 2015) claimed that the fact that the topics of Euclidean Geometry and Probability were included in the high school mathematics curriculum was one key reason for the decline in learner performance. Given this background, the South

2. Annual National Assessments (ANAs) are national standardized literacy and numeracy assessment tests administered to Grades 1-6 and Grade 9 to assess learners' performance relative to others (Van der Berg, 2015). ANAs provide rich information to parents, teachers and bureaucrats such as levels of understanding which forms the basis of accountability (Van Wyk, 2015). In this study, ANA results were used to determine the trends of learners' mathematics performance in public schools.

African educational system has embarked on the process of addressing these challenges, with changes to the mathematics curriculum forming part of this process (DBE, 2011).

Probability was only included in the Further Education and Training (FET)[3] phase in 2006 as part of mathematics curriculum at the Grade 10 level (Makwakwa & Mogari, 2011). According to Mutodi and Ngirande (2014), in the past, probability was regarded as an additional topic for enrichment purposes targeting more 'able' learners in mathematics. Bennie (1998) stated that learners who enrolled for paper 3 would encounter probability related questions in mathematics competitions. Teachers who attended the same schooling system gained new exposure to the probability concepts in 2006. At that time teachers had not been taught the mathematical concepts of probability during their teacher training programs (Makwakwa & Mogari, 2011).

The introduction of probability into the Curriculum and Assessment Policy Statement (CAPS) documents for mathematics was challenging, as teachers and learners lacked the requisite learning and teaching support material (LTSM) (Mutara, 2015). Teachers' lack of pedagogical content knowledge in probability led to documented poor performances by learners in mathematics (Brijlall, 2014; Paul & Hlanganipai, 2014). Mutara (2015) noted that such a scenario presented difficulties for learners and teachers. This led teachers to ask questions such as the following:

- What are learners' abilities in the treatment of the concept of probability?
- How do learners interpret situations requiring the knowledge of probability?
- What are learners' experiences in solving probability problems?
- What is the cause of learners' challenges when dealing with probability?

Given this background, the current study explored the experiences and challenges of Grade 11 learners and challenges they possibly encountered when solving probability problems.

1.3 PROBABILITY CONTENT AND LEARNING

According to Batanero, Chernoff, Engel, Lee and Sánchez (2016), probability is a branch of mathematics, which has seen its introduction in many high school mathematics curricula.

3. Further Education and Training (FET) band is one of the structures of school education in South Africa that comprises of Grades10-12 (Podems, 2017). Mathematics in FET has 10 content areas that include probability, which contributes towards the acquisition of specific mathematical skills (DBE, 2011).

Probability uses the principles of addition and multiplication to do pure digit arithmetic during the calculation process (Shao, 2015). Probability knowledge may assist learners to cultivate skills and strategies to be able to predict and describe the phenomena of statistical randomness and related uncertainty (DBE, 2011). Therefore, probability may engage learners in identifying and solving mathematical problems to arrive at meaningful decisions by making use of critical and creative thinking.

Halpern (2013) defined critical thinking as one’s constructive manipulation of cognitive skills and abilities to optimize prospects of arriving at the desirable solution. The skills needed to think critically vary and include problem-solving, observation, analysis, reflection, interpretation, evaluation, decision making, explanation and inference (Binkley, Erstad, Herman, Raizen, Ripley, Miller-Ricci & Rumble, 2012). DBE (2011) stated that learners should be advised and encourage to understand and apply concepts relevant to probability. In the CAPS mathematics curriculum (DBE, 2011), learners in Grade 10-12 should be able to:

- compare the relative frequency of an experimental outcome with the theoretical probability of the outcome;
- use Venn diagrams as an aid in solving probability problems;
- deal effectively with mutually exclusive events: $P(AandB) = P(A) + P(B)$ and complementary events;
- use the identity for any two events A and B: $P(AorB) = P(A) + P(B) - P(AandB)$ Dependent and independent events: ($P(AandB) = P(A) * P(B)$)
- use Venn diagrams or contingency tables and tree diagrams as aids in solving probability problems (where events are not necessarily independent);
- facilitate the generalization of the fundamental counting principle; and,
- solve probability problems using the fundamental counting principle.

While learners attempt to solve probability problems, they tend to have experiences that may constrain their meaningful comprehension of mathematical concepts in the topic of probability (Paul & Hlanganipai, 2014). From these experiences, learners are faced with challenges in studying probability. It is the researcher’s view that scientific investigations should be conducted on how learners acquire knowledge of probability in particular.

The DBE (2017) National Senior Certificate diagnostic report analysis shows an average percentage performance per sub-question for Paper 1. The topic of probability, which is at the heart of this study, has seen many learners performing poorly. In the end-of-the-year 2016 Grade 12 analysis, the average percentage performance was Q11.1(95%); Q11.2(35%); Q11.3(68%) and Q12(2%). In this analysis, questions Q11.1 and Q11.3 were answered correctly while questions Q11.2; Q11.3.2 and Q12 were poorly answered. According to DBE (2017), learners could not calculate probabilities, solve probability problems and generally lacked critical thinking skills to answer probability questions. Therefore, based on these observations it was necessary to conduct research in Grade 11 mathematics classrooms focusing on analysing learners' challenges in probability. This study has thus explored the challenges Grade 11 learners experience in solving probability problems.

1.3.1 EXPLORING THE LINK BETWEEN PROBABILITY KNOWLEDGE AND PROBLEM-SOLVING ABILITY

The topic of probability is important to include in the mathematics curriculum due to its observed relatedness to the every day livelihood of learners (De Kock, 2015). Batanero *et al.* (2016) posited that learning about probability topics requires learners to develop enquiring minds and to understand the existence of chance situations relating to everyday life. Probability provides learners with problem solving abilities that are helpful to make productive choices and decisions in real life contexts. Gage (2012) asserted that probability language is fundamental in everyday discourse and is being used with increasing frequency. Examples of words relating to probability used in everyday life include *randomly*, *probably*, *likely* and *sometimes*.

The interesting learning activities related to probability have been shown to exert a positive effect on learners' thinking skills, conceptual and procedural understanding (Koparan & Yilmaz, 2015). Probability is important because of the instrumental role it plays in other disciplines, such as the need for basic probabilistic knowledge in many professions (Liu, Guo, Fu & Liu, 2015). Konold (1995) stated that people utilize different heuristics to judge probability and are prone to misunderstand certain concepts. There is a need for learners to engage mathematics through the use of probability concepts to help them identify their learning obstacles. This study also investigated learners’ effective reasoning patterns when solving probability problems (see, Section 5.2).

1.4 RATIONALE FOR THE STUDY

The rationale for conducting this study was to gain insights on learners' poor performance in mathematics. To achieve this, the study zoomed into the topic of probability in Grade 11. Perpetuating poor performance is a cause of concern in South Africa. Thus, the researcher investigated the experiences and challenges of learners in probability to enhance mathematics teaching and learning. This study identified the factors that possibly contribute to the poor performance of mathematics learners in township schools in the topic of probability. This study also revealed the possible causes of failure among learners in the topic of probability. Furthermore, this study identified effective learning approaches that may contribute to the comprehension of probability concepts. The rationale of this study was to improve mathematics achievement results and zooming into probability learning was one such strategy. If this research explains why some learners' performance is poor, it is in the effort to create opportunities for these learners to excel further in their studies.

1.5 THE PROBLEM STATEMENT OF THE STUDY

Probability is a difficult mathematics topic for learners in the secondary school level (Rubin, 2019; Gage & Spiegelhalter, 2016). In public schools, poor performance in the topic of probability is a matter of great concern (DBE 2017; Ogbonnaya & Awuah, 2019). The South Africa mathematics curriculum included the weighting of Grade 11 probability content of 20 ± 3% at the end of year examination in paper 1 (Department of Basic Education [DBE], 2011). Poor performance in probability negatively influences the average pass rate of Grade 10-12 learners such that, when Grade 12 learners perform poorly in mathematics, at the end-of-the-year examination they will be unable to successfully pursue the careers of their choice (DBE, 2017). The DBE's (2017) diagnostic report documented a disappointing probability performance in the analysis of the end-of-the-year examinations. In addition, DBE (2017) revealed that South African secondary school learners possessed limited knowledge and skills in probability. The researcher believed that the study of the causes of learners' challenges experienced when solving probability problems may inform mathematics teachers better about learners' attitudes toward the mathematics curriculum and their relative level of achievement.

Prior to conducting the current study, the researcher observed that most mathematics learners in the school at which the researcher was teaching struggled when solving Grade 11 probability questions. This information was obtained from mathematics teachers and through the researcher's involvement in conducting the analysis of school's examination results. A similar result emerged when the examination results of many schools were analysed at the provincial level. The researcher participated in the mathematics moderation committee at the provincial level. The analysis of 2016 and 2017 Grade 12 results highlighted learners' poor performance in probability relative to other topics. These observations thus motivated the researcher to investigate how the instruction of probability in Grade 11 mathematics classrooms was implemented.

Paul and Hlanganipai (2014) reported that some of the researchers have suggested that learners come into mathematics classrooms with varied misconceptions. Brijlall (2014) reported that learners' ongoing struggles with probability were owing to their teachers' poor knowledge base, extending to their pedagogical content knowledge. The researcher was of the opinion that there is little knowledge of what causes learners' poor performance in probability, and several studies have revealed that all elementary levels in probability have counterintuitive results, as probabilistic reasoning is found to be different from logical or causal reasoning (Batanero, Gea, Díaz & Cañadas, 2014). Probability is different from other branches of mathematics in this respect, as counterintuitive results are encountered when dealing with high order cognitive levels during probability problem-solving. This understanding might help us explain the observed existence of erroneous intuitions and continued learning challenges in high school mathematics classrooms (Kissane & Kemp, 2014).

Probability draws from everyday life and questions in examinations are set involving contextual word-problems. Probability concepts in Grade 11 require logical thinking as well as problem-solving skills (Karatas & Baki, 2017). This topic emphasizes the need for teachers to establish for their learners the tangible factors connecting mathematics with the real world. However, in mathematics classrooms, contextual word-problems may often be perceived by some learners as barriers. Research on contextual word-problems shows that learners perform poorly on contextual word-problems, as they are perceived to be difficult (Magidi, 2015). Contrary to other branches of mathematics, probability lacks the reversibility of random experiments, which may in turn pose challenges for learners. According to Batanero *et al*. (2016), probability utilises language and terminology that is demanding and is not always identical to the notation that is common in other areas of mathematics. The pacing of learning probability in Grade 11 may be a factor that may contribute to poor performance in probability.

According to the DBE (2018) assessment guideline, the instruction of probability is allocated two curricular weeks. The researcher observed that most teachers and learners tend to pace against the syllabus for completion; thus, this study sought to investigate how probability can be taught considering the time constraints and other factors affecting the instruction process.

1.6 THE AIM OF THE STUDY

This study explored learners' academic experiences and challenges, if any, in the process of learning the topic of probability concepts in Grade 11 classrooms.

1.7 OBJECTIVES OF THE STUDY

The objective of this study was to study learners' problem-solving methods in the topic of probability in Grade 11.

1.8 RESEARCH QUESTIONS OF THE STUDY

This study consisted of one primary (main) research question and four subsequent secondary research questions, which were posed as follows:

1.8.1 The primary research question

1.8.1.1 What are learners' experiences and challenges, if any, when solving probability problems in Grade 11?

1.8.2 The secondary research questions

The following secondary research questions were posed:

1.8.2.1 What challenges, if any, do learners experience when solving probability problems in Grade 11?

1.8.2.2 How do learners reason when solving probability problems in Grade 11?

1.8.2.3 How do learners interpret probability situations in Grade 11?

1.8.2.4 What are the causes of those experiences these learners have in probability?

1.9 THE SIGNIFICANCE OF THE STUDY

The importance of this research lies in its aim to address learners' low performance in the topic of mathematics. This study carries the potential to improve mathematics teachers' awareness of effective teaching methods and difficulties learners face when studying probability. Mathematics teachers are likely aware of the benefits of using effective learning methods, such as socio-constructivist teaching and learning practices. McGraner, VanDerHeyden and Holdheider (2011) asserted that the ability of teachers to improve learners' performance in mathematics is predicated on the implementation of a strategy to improve the quality of mathematics instruction. More importantly, this study may inform mathematics learners of their weaknesses and strengths in learning probability (Jang, 2009). This study highlighted challenges faced, and errors committed by learners while studying probability. Halim, Finkenstaedt-Quinn, Olsen, Gere and Shultz (2018) posited that learners' achievement is widely affected by the distinct issues of error and misconception.

1.10 THE RESEARCH METHODOLOGY

This study employed a mixed-methods research approach, using a *sequential explanatory research design*, in particular, to respond to its research questions (Section 4.4). The rationale of this research design was to extrapolate from both quantitative and qualitative data in examining a single study problem to describe and analyze learners' experiences in solving probability problems overall (Greene, 2007; Venkatesh, Brown & Sullivan, 2016). The research paradigm used in this study was pragmatism, as it mixes both positivist and interpretive paradigms in a single study (Morgan, 2014). Using this research methodology, the researcher was less restricted in carrying out the current research, as the selected paradigm and the research design allow the researcher to choose from among different types of research approaches (Johnson & Onwuegbuzie, 2006). The quantitative data that was collected from the test complements the qualitative data collected from semi-structured interviews and classroom observations to gain a deeper understanding of learners' experiences. Chapter 4 then provides an in-depth reflection on the research methodology of the study.

1.11 THE THEORETICAL FRAMING OF THE STUDY

This research used the socio-constructivist theory as a framework for the study (see, Vygotsky, 1978; see, also, Chapter 3). The theory highlights the significance of the social environment in the process of learning. According to Vygotsky (1978), learners construct knowledge in their interactions with others. In the current study, the socio-constructivist theory was used as a lens to explain and gain deeper insights on the analysed data with respect to learners' experiences and challenges in the topic of probability in Grade 11. According to Vygotsky (1979), teachers should help learners to construct knowledge and facilitate knowledge development. The study also payed attention to the role teachers played in facilitating the learning of their Grade 11 learners.

1.12 OPERATIONAL DEFINITIONS FOR THE STUDY

Creswell's (2014) operational definitions will be the definitions that this study will adopt, adapt or instrumentalize to generate working knowledge. The next working terms are explained for the study:

1.12.1 Challenge

A challenge is something considered to be new and difficult requiring great effort and determination (Collins English Dictionary, 2015). Afolabi (2018) defined the word challenge as a situation in which someone is confronted with something requiring excessive mental strength to be performed correctly. The word "challenge" is conceptualised in this study as an obstacle and difficulties experienced by learners possibly hindering the teaching and learning of probability concepts. It may include problems faced by teachers and learners alike in probability classrooms.

Our definition of the word challenge in this study draws on three main semantic sources. The first source stems from epistemological reasoning, which speaks to the challenges that might arise from the nature of probability. Secondly, the psychological dimension of *challenge* refers to the challenges that learners face stemming from difficulties, errors and misconceptions in learning probability concept, which in turn arise out of learners' personal development, state of prior knowledge and skills of understanding. Lastly, the pedagogical dimension of the concept of *challenge* relates to means through which learners acquire content, the resources they use and the methods of instruction they employ.

- *Conceptual knowledge challenges* are conceptualised as the challenges that may arise from the process of learning knowledge of concepts.
- *Procedural challenges* in this study refer to challenges that might arise in the process of executing a problem. When dealing with problem solving tasks participants were expected to demonstrate skills and abilities to solve probability problem such include, facts, skills and algorithms and methods.

- *Prior knowledge challenges* are conceptualised as learner's previous understanding of probability concepts challenges; such include the challenges regarding the background knowledge or experiences learners bring in a new learning situation.
- *Tacit knowledge challenge* refers to a learner's challenges that may arise in the process of describing his experiences in solving probability problems. In this study, participants were expected to explain and justify the reasons for using some identified algorithms, steps and formula used in their process of solving probability problems.
- *Factual knowledge challenge* is conceptualised as the challenges of knowing basic elements and terminology of probability that may arise during learning of probability concepts. In this study, participants were expected to give formal definitions of probability related terms.

1.12.2 Experience

According to the Canadian Senior Dictionary (1979), the phrase '*to experience*' refers to living through something: to act, to do, to respect, to suffer the consequences of, to feel, or to internalize something. The verb *experience* has been used in this study interchangeably with its synonyms *face* and *encounter*. In this study, the word experience refers to an academic event in the learning process that brings about either positive or negative educational outcomes.

1.12.3 Learners

Collins' senior dictionary (1979) defines learner as an individual who studies a particular subject. South African Council for Educators Act (2000) views a learner as a pupil attaining education at any General Education and Training band, Further Education and Training band and or adult learning centre. In this study, the word participant was used to describe those who participated in this study. The main demographic of participants in the current study were mathematics learners in Grade 11. Hence, the word learner in this study refers to Grade 11 learners who were taking mathematics as a subject at the time of conducting the study. Participating learners were aged between 15 and 19 years of age. Learners in this group were found to be vulnerable, as they were in their adolescent developmental stage and frustrated by complex topics such as probability.

1.12.4 Probability

It seems the notion of probability has been interpreted differently in several times (Hacking, 2006). Laplace cited in Jones (2006: 22) defines probability as "a fraction whose numerator is the number of favourable cases and whose denominator is the number of all cases possible". Also, Batanero *et al*. (2016: 112) defined probability as the convergence of frequencies for an event, after many identical trials of random experiments have been completed. Batanero and Diaz (2016) perceive probability as the degree of an individual's belief, based on personal judgment and information about experiences relevant to a given outcome.

1.12.5 Poor performance

According to the Department of Education (DoE) (2005), learners pass secondary school mathematics when they pass with a minimum of 30% (Level 2: Elementary achievement). The final 30% pass requirement in mathematics is composed of a 25% school-based assessment and 75% from examinations (DBE, 2011). The learner must achieve this 30% in mathematics to receive promotions in Grades 10 and 11 and to be certified in Grade 12. In this study, the word *poor performance* refers to a mathematics test score below 50% for a given age group and in keeping with their cognitive skills. A mark lower than 50% does not allow learners to gain university entrance in courses requiring mathematics as a prerequisite.

1.12.6 Diagnostic test

A diagnostic test is a form of assessment that informs teachers of their learners' problem areas as well as inform learners on their strength and weakness in the topic under consideration (Nichols, 1994). In this study, a diagnostic test is a mathematical tool used to assess conceptual and procedural knowledge, mathematics skills, and cognitive skills Grade 11 learners have in solving probability problems to understand their weaknesses and strengths. The researcher used the tool to evaluate learners' competences. The identified problems may in this sense hinder their learners' performance (DBE, 2018). This study employed a diagnostic test, not as a promotional tool but to diagnose learners' mathematical problems and challenges.

1.13 ORGANISATION OF CHAPTERS IN THE FINAL DISSERTATION

This report or dissertation has been outlined in the following manner:

Chapter One briefly describes the background of the study and reflects on the following topics: the problem statement, the aim, the research questions, and the significance of the study, outlining its research methodology and research design, and through providing a brief explanation of the concepts and terms used in the text.

Chapter Two provides a display of the reviewed literature related to the topic of probability in secondary school mathematics. The review of literature also covers the subject of the performance of mathematics learners. Topics such as the resources used in teaching probability and problem solving are explored.

Chapter Three mainly introduces the theoretical framework underpinning this study.

Chapter Four provides a comprehensive discussion relating to the research design and the research methods of the study. The population and study sample, data collection tools, the developments of instruments and data analysis strategies are all discussed in this study. The process through which instruments are developed and the procedures pertaining to data collection are also outlined. The chapter also discusses the measures of the validity of data for the study and reliability associated with instruments used to collect data. The chapter concludes with a reflection on the ethical issues inherent in the study.

Chapter Five discusses the data analysis process and the outcome thereof. The discussions in this chapter cover the interpretation of the study's results.

Chapter Six comprises the conclusion of the study and addresses the following topics: the conclusions, recommendations, and limitations of the study and related reflections.

1.14 CONCLUSION

This chapter was able to set the tone for the anticipated study aimed at exploring learners' experiences and the potential challenges they must overcome when approaching the topic of probability in Grade 11. The background of the study and the problem statement have been discussed in this chapter. The research objectives and research questions were outlined, and a brief description of the methodology has also been provided. The next chapter explores several studies that are deemed to have a bearing on the current study.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents the relevant literature reviewed to ascertain the depth of related research in the field of the current study. This chapter describes the notion of a literature review, discusses the purposes of reviewing the literature on research and explains the roles of primary and secondary literature reviews in general. The chapter includes the following discussions: (a) reflections on the notion of probability; (b) the learning difficulties associated with the topic of probability in mathematics classrooms; (c) intuitions in the research on probability and related characteristics; (d) possible misconceptions and errors associated with the study of probability; (e) learners' understanding of conceptual and procedural knowledge; and, (f) teachers' related pedagogical content knowledge.

2.2 UNPACKING THE NOTION OF REVIEWING LITERATURE IN RESEARCH

A literature review provides an organised way for researchers to identify, evaluate and synthesize documented work produced by other scholars and researchers (Fink, 2019). It is a "systematic search of accredited sources and resources" (Hart, 2018: 3). The literature review involves identifying relevant studies and sources to the researcher's topic. Machi and McEvoy (2016: 3) define the literature review as a written argument that promotes a thesis' (or a dissertation's) position by constructing an argument using authentic previous research. The review of literature helps the researcher identify points of consensus and incongruity in the existing research. Booth, Sutton and Papaioannou (2016: 10) argued for the benefits of the literature review process in highlighting what the knowns and unknowns pertaining to past research. Boell and Cecez-Kecmanovic (2015) gave additional reasons for reviewing past literature, including showing what studies have been conducted in the field and what questions are still outstanding, which in turn allow for a new perspective and conceptual understanding to emerge.

In the current study, the literature review helped to compare and contrast existing research on knowledge relating to learners' experiences in probability. The researcher integrated prior scholars' work in probability and analysed acquired information to influence the research procedures in the current study. The researcher managed to define probability, identify errors and misconceptions, pinpoint probability knowledge relevant to Grade 10-12 learners, and develop appropriate terminology linked to probability. The literature review was helpful in combining international and local knowledge promoting better understanding of challenges and experiences faced by learners in solving probability problems. This study also used the literature review to gain knowledge on a new line of inquiry into learners' perspectives regarding the study of probability, and the process of understanding and integrating conceptual and procedural knowledge, providing further insight into the existing body of research on probability knowledge. Lastly, a literature review was used to relate the findings of this study to previous knowledge drawing similarities and difference of the study findings.

2.2.1 Primary sources of literature

A Primary source is an original work undertaken by researchers (Bloomberg & Volpe, 2018). Salkind (2010: 1092) argued that primary sources consist of first-hand or original information. In this study, the researcher used primary sources to access original knowledge linked to learners' academic experiences and challenges in probability. Some of the primary sources used in this study were learners' classwork books, class tests and assignment scripts to identify the problem of the research study. These sources assisted the researcher to understand learners' abilities in solving probability problems and were used as evidence of learners experiences in learning probability.

2.2.2 Secondary sources of literature

A secondary source of the literature review may be derived from the research that has been conducted on topics addressed by authors, who actively interpret the works of others (Bloomberg & Volpe, 2018). Galvan and Galvan (2017: 24) considered secondary sources as information detailing the research methodologies used in the study and extensive reflection on the study findings. In the current study, secondary sources extended to related book chapters and reviews, journal articles and conference papers. These sources of literature played an important role in gathering important information on learning probability such as learning strategies that may be used in learning Venn diagram, mutually exclusive events and conditional probability. The knowledge acquired in the literature assisted the researcher to explore and understand the nature of learners' academic experiences and challenges in mathematics learning.

During the process of the literature review, initial ideas and understandings of probability teaching and learning were refined and customized to the research aim, objectives and research questions of this study.

2.3 REFLECTIONS ON THE NOTION OF PROBABILITY

Probability is defined from two perspectives; namely, a statistical and epistemic view of probability (Batanero, Chernoff, Engel, Lee & Sánchez, 2016). Chatfield, (2018) defined statistics as a scientific method using data to facilitate the process of decision making in the face of uncertainty. Statistics cuts across measurements and scientific analyses involving exploration, understanding and decision making (De Smith, 2018). Batanero *et al*. (2016) explained the statistics that is implicated in probability such as utilizing the rules of mathematics to influence and regulate the process of randomness. These authors believe that probabilities are determined from the information that is gathered from surveys and experiments. A quantitative notion of probability refers to probabilities that are expressed by means of assigning hypothesis values ranging within the interval 0-1 (Galavotti, 2015). According to Batanero Chernoff, Engel, Lee and Sánchez (2016), an epistemic side of probability is one’s ability to understand concepts and the extent to which they know or believe they know how events unfold. These authors explain the term epistemic as referring to the type of knowledge and the degree of the validity of knowledge. The preceding view of probability is related to uncertainty and is characteristic of human knowledge, in which reasoning is supported by inferential justification. The term epistemic means representing knowledge (Brukner & Zeilinger, 2001).

Epistemic probability depends on the information an individual will share in the process of assigning probabilities. Epistemic probabilities make room for the use of inferences by using appropriate data and information to make logical inductions and deductions (Phillips, 2012). The preceding discussion on probability lies at the roots of the various interpretations of probability (Batanero, 2015; Batanero & Diaz, 2007; Borovcnik & Kapadia, 2014; Chernoff & Russell, 2012). Melchers and Beck (2018) considered probability as the numerical measure of the likelihood that an event will happen relative to a set of alternative events that do not occur. The interpretation of probability stemming from this two-fold perspective includes the intuitive, classical, frequentist and subjective view of probability. In this study, the notion of probability helped the researcher to design some data analysis criteria that encompassed various ways of calculating and interpreting probability situations.

Given this historical background of probability, it is important to understand the origin of the language used in probability because it provides the background on the relevant different approaches to the concept of probability. For example, in this study, learners could determine the probability of an event without using calculations or formulas. In this regard, different views would be considered and accepted in the learning process and solving probability problems.

2.3.1 Intuitive view of probability

Intuitive ideas about probability developed for a variety of cultures and are associated with societal promoting fair play game, such as betting in gambling (Batanero & Diaz, 2007). Batanero and Borovncik (2016) defined intuitive ideas as those associated with a finite mind forming a component of socialisation with the environment of people who live in it. In the real world, individuals engage in discussion relating to the informal chance of many events, such as the soccer team winning a game, getting married one day or going to swim. Games of chance may be used in decision making and in making predictions. Mathematics facilitates the calculation of theoretical probabilities of events (Soyarslan, 2014). Probability reasoning is also important in decision making, though it is difficult to teach and learn. In this sense, learners' difficulties in probability arise from intuitively based misconceptions (Wright, Watson & Fitzallen, 2019).

The study of probability requires learners to be guided during instruction so as to develop conceptual understanding. It is important for mathematics teachers to use effective teaching strategies to guide struggling learners towards developing intuitive probability choices. Kustos (2014) identified a positive connection between teaching strategies and intuitive ideas. According to the author, teaching strategies may influence learners to develop intuitive insights which promote conceptual understanding. Mathematical skills and graphical representation as teaching strategies have been found to develop learners' understanding of sets concepts (Soyarslan, 2014). Batanero *et al*. (2016) asserted that teachers may use learners' intuitive ideas to enhance their understanding. Researchers, philosophers and psychologists have made various contributions to our understanding of the concept and its definitions. Chernoff and Sriraman (2014) defined intuition as a way of knowing where an individual comes from. Volz and Zander (2014) proposed a common definition of intuition referring to an experience-based process emanating from one’s inclination to formulate hypothesis and not considering the underlying cognitive processes.

Bear and Rand (2016) defined intuition as a cognitive process and preferences that come to mind quickly and effortless. Learners’ solutions that come immediately to mind during problem-solving tend to have influence over their conceptual understanding. Batanero, Del Mar López-Martín and Gea (2018) noted that an incorrect solution reached by learners may be attributed to their lack of immediate relevant conceptual knowledge. According to these scholars, individuals fall into decision-making without understanding how the knowledge was acquired. In this study, this event of knowing something without conscious reasoning may lead to errors, misconceptions and inconsistency with probability reasoning (Hamming, 2018). In addition, intuitive views of probability were assessed in a diagnostic test, interviews and lesson observation specifically on how learners interacted with Venn diagrams, mutually exclusive and independent events. Such included, questions that evoked the learners critical thinking and decisions making in the process of problem solving.

2.3.2 Classical meaning of probability

Laplace (1995: 6) classically interpreted probability as the “ratio of the number of favourable cases to that of all possible cases”. For instance, Galavotti (2015) revealed that learners consider all outcomes of a dice play to be equally probable. The classical meaning of probability was linked to the game of chance, including the throwing of a dice. Batanero, Chernoff, Engel, Lee and Sánchez (2016) posited that the formalisation of the classical approach to probability was rooted in the notion that all possible single outcomes are equiprobable in fair games. Kissane and Kemp (2013) viewed the classical approach as the formal definition of probability in terms of the sample space of equally likely events. According to Batanero and Diaz (2016), the classical approach employs the principles of algebra, based on a strict set of axioms. This study explores how learners deal with sample space events in the process of solving probability problems as well as their understanding of probability definition and formula.

2.3.3 Frequentist theory of probability

The frequentist approach to probability is a limit of relative frequencies (Batanero & Boroncvik, 2016). In the frequentist approach, Konold (1991) defined probability as the limit to which the relative frequency of occurrence of an event tends in an infinite number of trials. Renyi (1992, cited in Chernoff & Sriraman, 2014) defined probability as the hypothetical number toward which the relative frequency tends when a random experiment is repeated infinitely many times. Repeated experiments under the same condition are tools used to predict the probability of events. In the event that experiments cannot be undertaken under the same set of fixed conditions, the frequentist approach perspective will not be suitable (Batanero & Sanchez, 2005). During teaching and learning, learners with limited exposure to the frequentist perspective may find the concepts of probability challenging to comprehend. For instance, learners may struggle to comprehend and solve problems emanating from real-life situations. In applying the frequentist definition, it is worth noting that learners may encounter challenges in trying to solve concrete problems, in keeping with the realization that the occurrence of events are affected and influenced by external and internal variables (Eichler & Vogel, 2014). In this study, the researcher used some questions drawn from real-life situations to explore probability knowledge understanding and application of what was learnt.

2.3.4 Subjective approach to probability

Using the subjective approach Batanero and Kapadia (2014) defined probability as being influenced by personalized judgements relating to one's nature of knowledge. These scholars argued that probability is viewed as an evaluation of situations that reside in one's personal mind. Ramsey (2016: 27) defined probability as the measure of belief in the truth of a statement. The decisions made by learners and related conclusions and explanations may be based on their judgement of the likelihood of uncertain events (Kahneman & Tversky, 1972) In this approach, the current knowledge influences an individual's judgement and decision-making. Krawitz & Schukajlow (2018) noted that prior knowledge reduces the likelihood of errors and false starts. Prior knowledge is important to facilitate the development of routines in problem-solving. In this study, learners had to activate prior probability knowledge at the very beginning of the problem solving process.

Krawitz & Schukajlow (2018) argued that a connection between prior knowledge and the acquisition of new knowledge promotes the understanding of concepts. This statement in turn implies that the beliefs a learner brings to a classroom may have an impact on how they learn new knowledge. Vygotsky (1978) acknowledged that the beliefs learners have, not on procedures, but relating to the meaning and interpretation of probability influence their teaching practices in the classroom. In their pedagogical approach, teachers must have an idea of what learners know first before endeavouring to build on their existing knowledge. According to Vygotsky (1978), teachers can facilitate deep knowledge construction by encouraging learners to speak to their peers about their experiences and ideas. This study subjective approach was used to explore the learners' prior knowledge in Venn diagrams, mutually exclusive event and independent events.

2.3.5 Axiomatic meaning of probability

The axiomatic approach defines probability as a function that attributes a real number to the subsets of a sample (Chernoff & Sriraman, 2014). The probabilities may not be negative, and when they are added they should sum up to 1. The probabilities must likewise fulfill the additive property for the two mutually exclusive events. It is important to note that the axiomatic approach is inadequately treated at secondary school level. In this study, the axiomatic approach to probability was used to explore different way learners used to calculate probabilities of events, particularly the use of addition and product rule in solving mutually exclusive and independent events.

2.3.6 Theoretical approach to probability

A theoretical approach is an aspect of probability obtained by the fraction of the outcomes favourable to the event in the sample space (Prodromou, 2012). Borovcnik, Bentz and Kapadia (1991: 41) pointed to theoretical probability as an a priori approach to probability. In this regard, when determining the probabilities of events, the calculation of probabilities is permitted first and followed by trials. Kolmogorov (1933, 1950) defined the sample space using the notion derived from the set theory: *a sample space is a set(s) of all possible outcomes of a random experiment*. The probability is defined from a set (A) containing subsets of the sample space, which is retained under a numerable union in the interval of real numbers [0;1]. Batanero *et al*. (2016) elaborated on Kolmogorov's theoretical definition stating three axioms, from which the properties of probabilities as well as related theories are inferred (see, Figure 2.1).

The interpretation of the preceding axioms of this theoretical definition has an influence over learners' understanding of probability concepts. For example, Anggara, Priatna and Juandi (2018) administered a diagnostic test to 55 participants to investigate learners' difficulties in comprehending and understanding probability concepts. The study questions were constructed to highlight the difficulties of test respondents in relation to comprehension of certain concepts in probability, definitions and terms, union and intersections, and independent and dependent events (Anggara *et al*., 2018). The study analysed the results using their identified cognitive ability levels. The findings showed that learners had many difficulties in using the theoretical probabilities related to Venn diagrams, struggled to explain the sample space, and in putting together the event spaces (Anggara *et al*., 2018). Also, the study found that learners incorrectly explained the Venn diagram as a result of errors in the members of each set (Anggara *et al*., 2018).

1. $0 \leq P(A) \leq 1$, *for every* $A \in A$;

2. $P(S) = 1$; and,

3. For a finite sample space S and incompatible or disjoint events A and B, that is, $A \cap B = \emptyset$, it holds that $P(A \cup B) = P(A) + P(B)$.

Figure: 2. 1: Three axioms for probabilities properties and theories

2.3.7 An experimental approach

Hawkins and Kapadia (2009) referred to the experimental probability of an event as the probability estimated from relative frequencies used to determine the likelihood of an event to occur. According to Batanero (2016), people may relate the frequencies of the events in their every day living when assessing the probabilities of certain events based on the ease of recalling their frequencies. Chaput, Girard and Henry (2011) asserted that experimental probability should be taught in conjunction with theoretical probability to facilitate the comprehension of probability concepts and for learners to apply it in real-life situations.

Today’s school curriculum expects learners to employ and understand the theoretical and experimental approaches to probability. For example, the CAPS mathematics curriculum focuses on “comparing the relative frequencies of an experimental outcome with the theoretical probabilities of the outcome” (DBE, 2011: 14). Prodromou (2012) explored how pre-service teachers tended to reason about experimental probability and theoretical probability. The outcomes of 20 trials of tossing a coin and 50 trials of rolling a die were documented, in the effort to incorporate theoretical probability and experimental probability perspectives into the task. The study found that most teachers could distinguish the theoretical from experimental probabilities (Prodromou, 2012). However, preservice teachers could not coordinate those two different approaches to probability because of their different roles (Prodromou, 2012).

In line with these definitions of probabilities, this study conceptualises probability as a branch of mathematics which employs the principles of addition and multiplication to determine the likelihood or chance of certain events to happen.

2.4 PROBABILITY REASONING

Falk and Konold (1992) define probabilistic reasoning as a mode that involves judgment and decision-making under uncertainty (see also, Batanero, 2016; Borovcnik & Kapadia, 2014). Rubel (2007) described probability reasoning as a mode of reasoning in which learners provide answers and justifications for their responses. The author's findings highlight the importance of asking learners more probability questions and reporting learners' justifications along with their answers. Probability reasoning was found to be important owing to its relevance in everyday life as a mental process unto itself. Qualitative methods for data gathering such as clinical interviews were mostly used to elicit narratives which reveal experiences of self and others, and to probe participants' reasoning (Groleau, Young & Kirmayer, 2006). Research shows that learners engage in probability reasoning using various strategies to make judgements without using fractions or numerical probabilities (e.g., Jones *et al*., 1997; Prodromou, 2013, 2016). Prodromou (2016) captured Grade 12 learners' reasoning about conditional probability when engaged in an activity involving the representation of the problem situation with two-way tables and/ or three-dimensional diagrams.

Prodromou (2016) found that the analyses and representations of problem situations with two-way tables had a strong influence on the types of reasoning that learners used. The tabular representations triggered correct reasoning and contributed to the enhancement of learners' procedural and conceptual understanding. Furthermore, it was found that learners who used two-way tables were able to identify the sample space and the calculated conditional probability of an event. In their study, learners managed to understand the intersection concept when dealing with conditional probability problems. Prodromou (2013) conducted a similar study to attempt to gain insights into the thought processes of learners aged 17, when learning to engage in the task of statistical inference. Prodromou's (2013) study aimed to investigate the evolution of learners' thinking with attention to the use of statistical concepts in decision making. The results were consistent with Batanero and Sanchez (2005) that learners relied primarily on mathematical thinking for their reasoning. It was observed that incorrect mathematical thinking compromised learners' ability to construct meanings about statistics and probability (Prodromou, 2013).

In this study, probability reasoning represented deferent forms that are interconnected and was thought as developing gradually as learners interact with probability situations. Figure 2.2 outlines the example of the indicators of probability reasoning in solving probability problems.

Figure 2.2: Example of probability reasoning construct in solving probability problem

CONSTRUCT	PROBABILITY REASONING INDICATORS	LACK OF PROBABILITY REASONING INDICATORS
Venn diagram	Recognise the language of sets Use the knowledge of intersecting sets Determine the probability of single events	the learner is naïve and wrongly use probability language of the set. Distracted and or mislead by the Venn diagram information
Sample space	Identify the sample space and list the outcomes	Mistakenly did not add a 7 (number of learners do not play both sports) List an incomplete set of outcomes

In this study, the contextualisation of probability reasoning entails how and why learners organise and re-organise their ideas during their interaction with Venn diagrams, mutually exclusive events and independent events. Thus, probability reasoning was used to describe learner's thinking in response to the probability situation, namely, a situation involving sample space and the probability of events. This study focused on how Grade 11 learners solved probability problems involving addition and multiplication rule and the drawing of Venn diagrams. In this study, learners were judged to have acquired probability reasoning if they could make meaning of Venn diagrams by solving for the unknown value and reflect meaningfully on their solutions. Furthermore, learners were expected to use the knowledge of fraction, apply the algebraic calculation to determine the probability of events. It was thought that learners who demonstrated lack of probability thinking would find it difficult to understand the demands of the question, consequently could complete the Venn diagram incompletely. Also, the lack of probability could possibly render the learners incapable of using appropriate rules when calculating mutually exclusive events and independent events. In this study, probability reasoning indicators were explored in the process of problem solving. This study explored the ways learners think during learning the concepts of probability as well as its influence on problem solving abilities.

2.5 THE IMPORTANCE OF PROBABILITY

Probability literacy is largely advocated by curriculum designers and developers of mathematics curriculums in many countries and at different educational levels (Batanero, 2013). In the South African education system, probability literacy starts from the primary education level and extends to Grade 12. Probability concepts enable citizens to overcome their deterministic thinking as well as acknowledge the realities of fundamental change in everyday life. The rationale for learning probability is that it equips learners with the reasoning skills and problem-solving skills that may be used in solving everyday life problems and work-related issues where chance presents (Batanero *et al*., 2016). For learners who have acquired probability, knowledge can raise the level of understanding at which the learner in question interprets what he or she sees in real-life situations.

2.6 CHALLENGES RELATING TO TEACHING AND LEARNING MATHEMATICS

This section reviews existing literature on learners' difficulties experienced when learning about probability concepts. In addition, the nature of learners' difficulties is reviewed. Learners are constantly judging events and involved in decision making. Although learners experience uncertainty, they make judgments and are constantly engaged in processes of making life related decisions. However, the understanding of probability concepts is not easy (Spaull & Kotze, 2015; Kahneman & Tversky, 1972). Research shows that probability concepts are difficult to learn in mathematics (for examples, see, Batanero *et al*., 2016, 2018; Chernoff & Sriraman, 2014; Lee, Park & Kim, 2016). The difficulties facing learners of probability may be linked to its heuristic nature and learners' incorrect theoretical knowledge and misconceptions (Konold *et al*., 1993). Another reason attributed to the difficulty in teaching and learning probability could be linked to learners' difficulties in probabilistic reasoning (Munisamy & Doraisamy, 1998).

Studies have also linked learners' difficulties to learn probability to learners' incorrectly relating of every day knowledge to scientific knowledge (Gürbüz & Birgin, 2012). Learners' negative attitudes towards mathematics and poor performance have been identified as linked to difficulties in learning about probability (Bulut, 2001). Also, the lack of teaching resources may pose various challenges to learning about probability (Fast, 1997). Anggrani and Kusrini (2018) showed that the challenges experienced by learners in solving probability problems poses problems for their ability to understand proportional reasoning and resolve misconceptions in probability. Learners' challenges to learn probability motivated the researcher to explore the types of errors and misconceptions in probability. An error is an incorrect response to a question caused by the insufficient mastery of basic facts, concepts and skills (Sisman & Aksu, 2016). Wilensky (1995) reported that learners use rote formulas to formal exercises without understanding the meaning of the underlying concepts.

2.6.1 Errors and misconceptions in mathematics

Misconception refers to forms of knowledge displayed by learners when incorrectly engaging in problem-solving situation intuitively, thus arriving at incorrect problem-solving outcomes appearing to them as being rational when in actual fact they are derived from inaccurate meanings (Ojose, 2015). Feldman, Cho, Ong, Gulwani, Popovic and Andersen (2018) viewed errors as the inappropriate application of procedures and definitions, while Khuzwayo (2005) stated that misconceptions often surprise teachers and learners and can be difficult to eradicate. In South Africa, research has shown that teachers and learners experience difficulties in the teaching and learning of probability (see, Section 2.9). To understand the background of this problem, the researcher has reviewed the literature on learners' errors and misconceptions when learning the concepts of probability. Hayati and Setyaningrum (2019) investigated the Grade 9 learner misconceptions in mathematics concepts and the possible causes of misconceptions. The study employed a multiple-choice test to identify learners' misconceptions using Certainty of Response Index (CRI) technique. The study revealed that learners had varied misconceptions in working with mathematical problems. Also, results suggested that learners must learn mathematics towards understanding the concepts, not only memorising the concepts.

Aliustaoğlu, Tuna, and Biber (2018) study explored the misconception exhibited by Grade 6 mathematics learners when dealing with fractions. the study employed a fractional information test which consisted of open-ended questions. The data of the study were analysed using the content analysis method where data was organised in frequency/percentage tables. The results showed that learner had misconceptions in terms of parts-whole relation fractions, representation of fractions on a number line and employing algorithms when dealing with fractions. These authors suggest taking into account the different meanings of fractions. Again similar to these results, in the study of Trivena, Ningsih, & Jupri (2017); Tompkins, Guo & Justice (2013) and Kim (2011). It was seen that learners encountered challenges in understanding variables due to text comprehension errors. The relative success of learners engaged in text comprehension pertains to their ability to construct coherent mental representations (Kim, 2011). Learners with text comprehension and language skills, such as vocabulary and syntactic knowledge, can process the meanings of words and phrases (Tompkins, Guo & Justice, 2013). Conceptual errors are incorrect responses arising from difficulties with probability (DBE, 2018). This error arises when learners experience challenges from working with probability systems. Procedural errors are incorrect responses resulting from the faulty use and application of probability formulae and rules (Batanero & Chernoff, 2018). Arithmetic errors arise out of calculations, with errors often emerging from calculation mistakes which do not engage effectively with the problem task.

Daroczy, Wolska, Meurers, and Nuerk (2015) investigated the factors that influence learners understanding of word problems. These authors reported that word problem difficulty was influenced by linguistical factors and numerical factors. Resnick, Jordan, Hansen, Rajan, Rodrigues, Siegler and Fuchs (2016) conducted a study on the arithmetic errors and misconceptions exhibited by Grade 8 secondary school learners on fractions. The latter found that learners experience misconceptions in terms of part-whole relations in fractions. Secondary school learners demonstrated inadequate skills and knowledge to represent fractions on the number line and comparing fractions and computing-related fractional operations.

Ang and Sharill (2014) tested learners' persistence when learning probability in secondary schools. The study asked learners to determine probabilities and assign the qualitative word using the probability scale (imposable, less likely, more likely and certain). Subsequent interview questions sought to understand the nature of learners' misconceptions. The findings showed that the prevalence of participants' responses with misconceptions was high (Ang & Sharill, 2014). Misconceptions and errors may hinder learners' understanding of the probability concepts. Batanero (2016) found that participants lacked comprehension of the laws of probability and form misconceptions. The author argued that learners bring their informal experiences in the classroom in such a way that may be incorrect and inapplicable to probability laws. Learners' misconceptions and errors in probability may reveal learners' conceptions, ways of thinking, and learning difficulties. Research has suggested some common ways that are perceived to impede an individual's understanding of probability concepts, including lack of representativeness, equiprobability bias and beliefs and human control (Batanero *et al*., 2018; Batanero & Borovcnik 2016).

Hay (2014) investigated the complexities of procedures used when learners are engaged in learning conditional probability. The author explored learners' understandings, arguing that learners can construct probability understanding when teachers use a hierarchy of dialogue questions from descriptive to abstract. Hay (2014) explored learners' understanding of probability through proportional reasoning. For example, the probability of obtaining a red chair in the classroom is 10 out of 45, or 0.2222. In probability, this ratio can be expressed as a percentage and/or decimal to illustrate if the probability of an event is great or small. Hay (2014) hypothesized that the effective instruction of probability requires teachers to explore a variety of teaching methods including those deriving from a linguistic orientation and proportional reasoning perspective. The findings suggested that teaching strategies relying on linguistic orientation and a proportional reasoning orientation enhance learners' understanding of probability (Hay, 2014). The researcher inherently thought that teachers should revise the proportionality reasoning to enhance learners' understanding of the mathematical principles of a numerator and a denominator. In this study, the literature on errors and misconceptions discussed enhanced the exploration of academic challenges that emanated from learners' tacit knowledge, factual knowledge and prior conceptions in solving probability problems and their influence in probability problem solving abilities.

2.6.2 Representativeness misconceptions

Misconceptions associated with representativeness are viewed as learners' behaviour to think incorrectly that samples corresponding to the population distribution are more probable than samples that do not (Ang & Shahrill, 2014). Learners should be assisted in studying the composition of the sample in relation to the outcome space (Prodromou, 2016). Learners' understanding of the role of sample space helps them to monitor the composition of the sample space (Chernoff & Sriraman, 2014). Batanero and Borovcnik (2016) stipulated that people rely on biases and, consequently, make incorrect decisions. For example, in tossing a coin, learners with misconceptions may think that a series of coin tosses having approximately equal numbers of heads and tails is more probable than a series with more tails than heads (Batanero & Borovcnik, 2016). The study found that misconceptions of this kind have an influence on the nature of learners' knowledge acquisition and achievement in tests or examinations.

Understanding learners' misconceptions and errors is important too. Sarwadi and Shahrill, (2014) investigated learners' mathematical errors and misconceptions manifested by repeating Grade 11 learners when engaging in probability tasks. The study revealed that learners entertained various misconceptions which negatively contributed to their achievement rates in the test (Sarwadi & Shahrill, 2014). The study results also showed that some learners were not aware of their misconceptions. The authors further revealed that the sources of the misconceptions related to inattentiveness on the part of teachers. Learners may connect misconceptions with newly learnt knowledge, thereby learning erroneous procedures. Hirsch and O'Donnell (2001) designed a test instrument identifying learners with misconceptions of representativeness. Learners completed a multiple-choice test to determine if learners possessed basic knowledge of probability required to answer multiple-choice questions. The test questions were presented in a format allowing learners to justify their responses (Hirsch & O'Donnell, 2001). Study results indicated that learners entertained various misconceptions in probability (Hirsch & O'Donnell, 2001). Test respondents were given several probability questions such as:

> If a fair coin is tossed six times, which of the following ordered sequences of heads(H) and tail (T), if any, is least likely to occur? HTHTHT, TTHHTH, HHHHHT, and HTHTHH.

Figure 2.3: Example of a test item from Hirsch and O'Donnell (2001) study

According to Hirsch and O'Donnell (2001), the results of learners who chose HHHHTT suggested that learners hold a misconception of representativeness believing that the results of repeatedly tossing a coin must be a random mixture of heads and tails. Emmons, Lees, and Kelemen (2018) stated that misconceptions may be difficult to eliminate during classroom instruction due to their nature. In Fishbein and Schnarch (1997) study participants were asked to determine the likeliness of two combinations to occur in a single trial. In this case, a specific combination where all six numbers were consecutive (e.g., 1, 2, 3, 4, 5, 6) or a specific combination in which all six numbers were in any order (22, 6, 10, 36, 18, 9). The findings showed that participants expected the combination with no sequence to be more likely to occur (Fishbein & Schnarch, 1997). The reason for participants' responses was that the numbers contained non-consecutive number combinations consisting of a larger class than the one containing six consecutive number. Shaughnessy (1992) asserted that people tend to estimate the probabilities of an event by considering how well the event represents the aspect of the parent population.

2.6.2.1 Simple and compound events

Another misconception that may arise out of representativeness is distinguishing between simple and compound events (Batanero *et al*., 2016). For example, people often have the tendency to think that when two dice are rolled at the same, getting two sixes carries the same likelihood as obtaining a five and a six (Lecoutre & Durand, 1988). The events of rolling a five and a six is a compound event because there are two outcomes of obtaining this out of 36 possible outcomes of rolling a dice twice. That is, a roll of a five on one dice and six on the other, or a six on the fifth die and a five on the other. A roll of a six and another six is a simple event because there is only the one instant of rolling two sixes out of the 36 possible outcomes of rolling two dice.

2.6.2.2 Equiprobability bias

If the outcome of an experiment has the same chance of occurring then they are equiprobable, meaning they are equally likely to occur (Gauvrit & Morsanyi, 2014). Heyvaert, Deleye, Saenen, Van Dooren and Onghena (2018) studies the ability of high school learners to solve probability problems. The authors used a questionnaire to determine the types of misconceptions of 168 high school learners. The study participants were supposed to examine the results that were more likely to occur when five faces of a fair die painted black and one face painted white was rolled six times. The data gathered showed that learners gave at least one answer that was linked to the equiprobability bias; that is, the possibilities of the painted face was equal to that of the white one. The study asserted that the participants who relied on equiprobability bias focused on the unpredictability and uncertainty of events (Heyvaert *et al*., 2018). The study concluded that all possible outcomes are equally likely because there is no way in which one can certainly tell what the outcome of one specific event can be (Heyvaert *et al*., 2018).

A further illustration of this inappropriate thinking is evident in Carpenter, Corbitt, Kepner, Lindquist and Reys (1981) involving two coins. Carpenter *et al*. (1981) investigated 13-year-olds and 17-year-olds' attitudes toward predicting the probability of obtaining a head and a tail when two coins are tossed. Three concepts on probability were explored; namely, the probability of an event, the probability of independent events and the probability of compound events. Carpenter *et al*. (1981) found that 60% of 13-year-olds and 69% of 17-year-olds gave correct answers of ½ when a two coin was tossed. When earners were asked about the probability of obtaining two heads, they gave the answer of ½. The results revealed that learners were inconsistent with the probability thinking leading to the misconception of equiprobability bias (Carpenter *et al*., 1981).

2.7 PROBABILITY LANGUAGE

The term language is defined as the 'words, pronunciation and the methods of combining words used and understood by a community' (MWD, 2013). According to Roberts (2016), the mathematical language of probability includes language, symbols and language prepositions among other factors. The definition emphasizes the significance of developing proper vocabulary to learn the mathematics of probabilities. Probability makes use of language and terminology that may be challenging to learners (Batanero *et al*., 2016). For instance, probability in mathematics incorporates Greek and capital letters, which is not always similar to the notation used in other mathematics topics and areas. There are many unique terms and concepts, and learning probability is as much about the learning of concepts as it is about language. Terms such as frequency, mutually-exclusive events, sample space and many others need to be explicitly taught and explained to learners.

Research shows that mathematics language that relating to probability hinder learners' achievement and conceptual understanding in general (for examples, see, Ellis & Shintani, 2014; Groth, Butler & Nelson, 2016; Meaney, Trinick & Fairhall, 2012; Nacarato & Grando, 2014). Trinick and Fairhall (2012) noted that probability language can either promote or constrain learners' ability to learn probability. Setati, Molefe and Langa (2008) found that most South African learners are underachieving mathematically as a result of lack of proficiency in English. Nacarato *et al*. (2014) argued that learners' difficulties in solving probability word problems relates to their inability to use suitable language to interpret the problem. Arum, Kusmayadi, & Pramudya, (2018) argued that mathematics learners' responses to the probability problems may not represent their thoughts.

Batanero *et al*. (2016) examined the meaning learners attach to the word *frequency*. The authors developed questions with the word *frequency* to negotiate the probabilistic meaning of the word. During the discussion, a written list explaining the meanings of words was initiated and examples were given (Batanero *et al*., 2016). The study found that the understanding of the word *frequency* facilitated the probability of understanding. The study also found that probability language leads to mistaken interpretations when not mathematically mastered (Batanero *et al*., 2016). Learners who are deficient in the mathematical language of probability may fail to determine the probability of events, leading to inconsistency with probability reasoning. Nacarato and Grando (2014) investigated the development of language relating to probability and related thinking patterns depicted by learners who were 10-12 years old engaging in probability problem-solving. A historical qualitative approach was used to develop a task for investigating language development and learners completed the task in groups. The study highlighted learners' lack of understanding in relation to the language used, such as 'probable' and 'improbable' (Nacarato & Grando, 2014). Learners described an event that cannot occur as *less probable,* a phrase that implies *it can still occur* (Nacarato & Grando, 2014). The study also found that learners relate words to both the school environment and to the broader social context (Nacarato & Grando, 2014). The study concluded that learners misunderstand the meaning of probability terms.

Language difficulties may persist when studying advanced ideas in probability (Watson, 2011). Groth, Butler and Nelson (2016) developed an instrument to enhance learners' learning of qualitative probability vocabulary. Although these authors found that learners hold misunderstandings relating to probability vocabulary, the results of individual interviews revealed that teaching probability language before numerical probabilities help to develop probability language (Groth, Butler & Nelson, 2016). Probability language may confuse learners and reinforce their misconceptions and errors. Nacarato and Grando (2014) highlighted that the inadequate verbal ability of an individual to describe probability problems derives from a probability language barrier. Tarr (2002) investigated learners' conceptions of the word *chance* in probability and found that learners often misuse this language. Learners may use the expression *50-50 chance* to indicate the presence of uncertainty rather than a specific measurement of chance (Nacarato & Grando, 2014).

Learners could find it difficult to comprehend the usage of terms associated with the learning of probabilities. Ellis and Shintani (2014) observed a perpetuation of learners' poor performance in probability until the probability words are understood in a mathematical sense. Learners use everyday terminology and understanding to facilitate a classroom dialogue in the topic of probability and realise the different meanings that is carried by these words. Learners may develop a negative attitude towards the topic due to the difficulties they have encountered (Paul & Hlanganipai, 2014).

2.8 FACTORS INFLUENCING MATHEMATICS PROBLEM SOLVING ABILITY

The questions of *how learners learn, how they are taught* and *how concepts are understood* have inspired the researcher to reflect on the notions of *conceptual* and *procedural knowledge, factual, tacit and prior knowledge* in mathematics. Though conceptual and procedural knowledge cannot always be separated, it is important to differentiate them. Piaget (1977) distinguished between conceptual understanding and successful actions a learner takes to solve the problem. Scheffler (1965) used prepositions, *knowing that* and *how to,* to distinguish between conceptual knowledge and procedural knowledge. Scheffler (1965) defined *knowing that* as knowledge of propositions or facts. *Knowing how* refers to the knowledge of procedures used to execute a process; that is, the possession of a set of skills, competencies and techniques (Scheffler, 1965). This distinction helped the researcher to understand learners' failures and successes during learning. Conceptual and procedural knowledge may not be treated separately, though they do overlap (Hiebert & Lefevre, 1986).

2.8.1 Conceptual knowledge

Conceptual knowledge refers to knowledge of concepts, which are abstract such as general knowledge about everyday concepts and their interrelations (Canobi, 2009; Carey, 2009; Rittle-Johnson, Rittle-Johnson, Siegler & Alibali, 2001; Rittle-Johnson, Schneider & Star, 2015). Conceptual knowledge is defined as a connected web of knowledge: a network in which linking relationships are as prominent as the discrete pieces of information (Hiebert & Lefevre, 1986). These definitions emphasise not only what is known (knowledge of concepts), but the way individuals come to know the concepts (e.g., deeply and with rich connections). Conceptual knowledge can refer to the explicit or implicit meaning that some conceptual knowledge may not be explained in words. Learners may use this knowledge to understand principles in a domain and to memorise and make decisions and predictions (Machery, 2010). Conceptual understanding facilitates understanding and the interpretation of concepts and their interrelationships.

Mathematical competence rests on the development of conceptual knowledge, which is often fragmented and needs to be integrated during learning (Hiebert & Lefevre, 1986). The teaching and learning of mathematics is informed by teaching strategies that support conceptual understanding in mathematics (Rittle-Johnson *et al*., 2016). Teaching conceptual understanding starts with problem posing requiring learners to reason flexibly. Rittle-Johnson *et al*. (2016) evaluated the effects of instruction on mathematics concepts and procedures to second-grade learners engaging in mathematical equivalence problem-solving activity. One such group of learners received conceptual instruction on mathematical equivalence such as,

$$4 + 4 = 8 \; and \; 10 = 3 + \boxed{}$$

The experimental group was taught using the instruction concept of equivalence and the control group was taught using traditional ways of teaching fractions (Rittle-Johnson *et al*., 2016). A retention test was used to measure both the experimental and control groups conceptual and procedural knowledge. The study found that learners in the conceptual instruction condition improved from post-test to retention test. The findings suggested that these learners developed concept understanding. Similarly, Powell's (2012) found that repeated conceptual instruction on the equal sign improved learners' development in understanding the concept. Powell (2012) also found that teaching for conceptual understanding helped learners to connect learnt knowledge to their pre-learning or prior knowledge, thus helping learners to expand their prior knowledge and connect it to novel contexts (NCTM, 2000).

Siegler and Lortie-Forgues (2015) investigated learners' conceptual knowledge of fractions. Pre-serve teachers participated in the study. Dealing with fractions promotes the sense-making of fractional symbols and operations with rational quantities. Participants were expected to anticipate without computing if the solution to inequality would be larger or smaller than the larger fraction (Siegler & Lortie-Forgues, 2015). The study found that participants inappropriately used procedures imported from the addition of fractions to the multiplication of arithmetic fractions (Siegler & Lortie-Forgues, 2015). The results suggested that participants weakly understood the numerical magnitude generated by multiplication and division. The study highlighted the importance of conceptual understanding in mathematics teaching and learning (Siegler & Lortie-Forgues, 2015). Learners who have conceptual knowledge develop critical thinking skills and become problem-solving strategists. Learners who lack conceptual understanding often mindlessly apply procedures to solve novel problems (DeCaro, 2016).

Numerous tasks have been used to evaluate conceptual knowledge. Some of the procedures evaluated the correctness of an example or procedure to providing definitions and explanations of concepts (Crooks & Alibali, 2014). The rationale for the methods of assessment is that learners do not know the procedures for solving tasks, and must rely on their knowledge of relevant concepts to generate methods for solving problems (Siegler & Crowley, 1994). Kilpatrick *et al*. (2001: 118) indicated that a learner develops conceptual knowledge in mathematics and later uses this conceptual knowledge to think and methods of solving problems. The acquisition of conceptual knowledge in scientific domains is the central aim of school instruction because this knowledge helps individuals make inferences and explain complex phenomena. Conceptual understanding and procedural knowledge must be acquired by learners in probability. Probability learning requires learners to use skills and apply problem-solving procedures. Learners must understand the applicability of procedures to differing situations.

2.8.2 Procedural knowledge

Procedural knowledge helps one to apply actions sequentially when solving a problem (Rittle-Johnson, Fyfe & Loehr, 2016). This knowledge is comprised of the formal language, or symbol representation system of mathematics (Hiebert & Lefevre, 1986). It consists of rules and procedural skills for completing mathematical tasks. Star and Seifert (2002) viewed this knowledge as a little toolbox that contains facts, skills, algorithms and methods. Procedural knowledge is specific to the type of a problem and may not be generalizable. Learners who possess this knowledge will be able to detect syntax errors in a mathematical statement. Learning mathematical concepts calls for a varying series of step by step actions. Procedural skill consists of various steps or actions undertaken to accomplish the task or a goal (Hiebert & Lefevre, 1986; Haapasalo, 2003). Possession of this knowledge enables one to execute problem solutions in linear sequential steps such as using a bracket of division, multiplication, addition/ subtraction and exponents (BODMAS) rule.

The BODMAS must be followed when solving problems such as algebraic problems. The process starts with brackets followed by division and multiplication and so on. Baroody and Johnson (2007) indicated that one must be able to justify the reason for using a specific skill. Such processes are important to learners to reinforce their memory recall of methods and skills problem-solving. Teaching by procedural instruction may lead to memorisation and rote learning with little understanding. Teaching definitions, symbols and isolated skills in an expository manner without first focusing on building deep, connected meanings to consolidate conceptual knowledge leads to a lack of comprehension of the underlying concept (Skemp, 2012). Hasan, Bagayoko and Kelley (1999) believed that procedural instruction limits opportunities to explore problems. If this method is not properly used it may impair learning and not equip the learner with necessary skills in mathematics, such as problem-solving skills (Sarwadi & Shahrill, 2014).

2.8.3 Prior knowledge

Prior knowledge has been defined in terms such as prior conceptions, misconception and naïve understanding (Krawitz & Schukajlow, 2018). The common aspects of these definitions are that learner's prior knowledge is important in the creation of baseline knowledge for learning a task (Chareka, 2017). It is important to explore the experience and knowledge a learner brings in a new learning environment. In this regard, the relationship of new ideas and to those ideas already known has a strong impact on the learning process. Mogboh and Okeke (2019) also define prior knowledge as the learner's actual knowledge that is available before learning new knowledge.

Mathematics curriculum emphasises that prior knowledge may be important for the design and implementation of effective learning activities for learners (NCTM, 2000). Prior knowledge is considered to be an important predictor of problem solving performance (Krawitz & Schukajlow, 2018). Furthermore, learning theorist has argued that learners' prior knowledge when used with constructivist approaches to learning enhance understanding of the domain-specific knowledge (Vygotsky, 1975). Bringula, Basa, Dela Cruz, & Rodrigo (2016) investigated the effects of prior knowledge in the mathematics of learners on learner-interface interactions in a learning-by-teaching intelligent tutoring system. The study revealed that prior knowledge in mathematics influenced tutoring learning. These authors revealed that prior knowledge had positive effects on the number of quizzes conducted. It was also shown that learners did not had sufficient prior knowledge consequently lead to poor performance. For learners with insufficient probability, prior knowledge may experience conceptual and procedural knowledge challenges in the learning process.

In addition, Matsuda, *et al*. (2013) investigated the effects of prior knowledge on learning by teaching using interactive technologies. The results showed that prior knowledge of learners and their teachers hinder learning through a negative transfer of knowledge. This was supported by the study of Fyfe & Rittle-Johnson (2016). It was found out that learners with higher prior knowledge in mathematics had higher feedback on the concepts learnt. This study sought to explore the background knowledge in probability in learning new knowledge on mutually exclusive, Venn diagrams and independent events concepts.

2.8.3 Tacit knowledge

Abdullah & Vimalanandan (2017) defined tacit knowledge as cognitive and experiential knowledge that a learner uses to explain the processes of solving problems. Thus, tacit knowledge is embedded in the problem solving process (Yang, Wang, Zhu, & Qu, 2017). As such, it is used by learners to analyse, generalise, select the appropriate problem solving strategies and to trigger mathematics prior knowledge to solve anew problem situation (Yang *et al.,* 2017). This knowledge type may be communicated and made explicit through learner involvement in the learning process. Tacit is important in mathematics because it identifies a learner's conceptual understanding and know-how of subject matter. A study by Abdullah *et al.* (2015) claims that tacit knowledge is enhanced during peer sharing, conferencing with mentors and self-reflection activities. The current study observed learners' tacit knowledge in the learning process and made explicit. Thus the observations will be focusing on how these learners communicate probability as well as their experiences in solving probability task. This was supported by a study by Wang, Zhu, & Qu (2017) who found that tacit knowledge may develop better when learners are encouraged to communicate to each other in the process of mathematics problem solving. For learners with ineffective tacit knowledge may find themselves struggling to solve probability problems.

2.8.4 Factual knowledge

Learners' learning encompasses the application of factual knowledge in the problem solving process. Factual knowledge in probability involves the basic knowledge learner posses, such include terminology of probability. In this study, learners' factual knowledge indicators marked the lower cognitive level questions where a learner was expected to define terms. Learners need to be allowed to try out the meanings of terms in varying contexts (Johansson & Gustafsson, 2016). For a learner with strong factual knowledge, may have the skill to use the probability concept, understand and interpret probability situations (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017). Furthermore, factual knowledge may enable learners to sort probability information leading to the development of basic knowledge. For instance, Esposito & Bauer (2017) investigated the factors that affect learners to generate new factual content knowledge from the interaction of novel facts presented through separate lessons in the classroom. The results showed that self-generated new factual knowledge had a strong effect on comprehension and mathematics academic achievement. these results are in agreement with Cragg *et al*. (2017). Theses authors found a strong positive relation between factual knowledge and achievement in mathematics. When combining the literature of different types of knowledge forms and their influence in mathematics, this study sought to explore this domain-specific knowledge in the process of solving probability problems.

2.9 GUIDELINES AND FRAMEWORK TO TEACHING PROBABILITY

CAPS curriculum for mathematics provides guidelines and/or a framework to teach probability at the school level (DBE, 2011). Instruction on probability involves asking questions, using exploratory data analysis and collecting and interpreting results (see, DBE, 2011; see, also, Appendix L). According to Batanero *et al*. (2018), probability is taught by using a problem-solving approach. Learners may investigate probability problems and conduct their own experiments. Pratt and Ainley (2014) emphasised that teaching probability from an experimental approach leads to a deeper understanding of probability. Pratt and Ainley (2014) asserted that probability must be taught to model real-world phenomena using technology to enhance learners' comprehension of probability. Nilsson (2009) noted that the experimental method engages learners in knowledge construction and is motivational. Nilsson (2009) argued that learners' thinking is developed when engaging with experiments, such as tossing a coin using an urn. The guidelines and framework were used to determine the effective learning methods used by learners in probability classroom.

2.9.1 The learning environments in mathematics

Research shows that learners' roles and actions depend on the teacher's presentation. Grosser (2007) mentions that teachers play many roles in teaching and contribute towards effective learning environments. This author further stated that learners must be awarded opportunities to investigate in the classroom and do meaningful mathematics in a safe and positive space. The teacher must provide conducive learning environments which support learners' investigations (Suurtamm, Quigley & Lazarus, 2015). Nickerson (1999) posited that learning environments ought to be places in which investigations allowing learners to support their ideas and thoughts can occur. Bruce (2007) emphasized that discussion is important in mathematics classrooms, and teachers must create a classroom environment to discuss ideas and develop their understanding. Learners must be co-constructors of knowledge. Suurtamm *et al.* (2015) suggested that teachers should allow learners to ask questions and justify the work they have generated and communicate their ideas to one other. The way the learning environment is organised may have positive effects on learning based on understanding and the ability to apply problem solving skills. Furthermore, conducive learning environment enables learners to achieve a higher order cognitive level of learning. In that way, learners may construct their knowledge so that problem solving abilities may be enhanced effectively and meaningfully.

2.10 LEARNERS' ATTITUDE TOWARDS PROBABILITY

Attitudes are personal psychological constructs that are thought to include emotions, behaviour and cognitive expressions (Hart, 1989; Akinsola & Olowojaiye, 2008). A learner's attitude is determined by his behavioural actions, achievement in the learning of the subject. Freeman cited in Baruah and Gogoi (2017: 166) defined attitude as a dispositional readiness to respond to certain situations, persons or objects in a consistent manner that has been learned and has become one's typical mode of response. To Ifamuyiwa and Akinsola, (2008) attitude comprises learner's feelings, prejudice, preconceived notion of mathematics, ideas, fears, and threats about mathematics learning. Thus, Zan and Di Martino (2007) view an attitude towards mathematics as a positive and negative emotional disposition towards teaching and learning of the subject. Studies have demonstrated that positive attitude facilitates learning while negative attitude hinders the learning of mathematics (Yara, 2009). In this regard, individual attitude towards mathematics is strong factors that affect learners' mathematics achievement (Mohamed & Waheed, 2011; Ogembo, Otanga, & Yaki, 2015).

Some of research findings in mathematics education have shown that learner negative attitude towards mathematics in both primary and secondary often leads to them loose interest and or dislike mathematics, which leads to poor performance (Mata, Monteiro & Peixoto, 2012; Mutodi & Ngirande, 2014). For example, the study carried out by Mohd and Mahmood (2011) revealed that learners who perform well in mathematics tend to like the subject, are encouraged to solve mathematics problems contrary to low performing learners. Moreover, Yara (2009) and Olorunfemi, Olawumi and Adu, (2018) unanimously agreed that learners with declining mathematics achievement are associated with a negative attitude. Additionally, Akinsola and Olowojaiye (2008) believes that a learner's successful experience may make him develop a positive attitude towards learning mathematics. This also supported by Zakaria and Nordin, (2008) in that, learners who fear mathematics perform poorly in mathematics examinations.

According to Zakaria and Syamaun (2017), there various reasons that make learner find it difficult to accept mathematics as an interesting and useful subject; (1) mathematics is a boring subject, (2) mathematics is difficult, (3) mathematics is a subject that requires precision, (4) there are people who think I cannot do mathematics because I am a woman, and (6) mathematics has no relation with my daily life. Similarly, Mohamed and Waheed (2011) categorised the factors that may influence learners' attitude in mathematics to be composed of three factors. Firstly, the factors associated with learners himself includes mathematics achievement, anxiety self-efficacy and self-concept, motivation and experiences in learning a subject. Secondly, the author identified factors associated with teaching material and teaching methods. Lastly, factors related to the home environment and society were identified. This is evident in Kyriacou and Goulding, (2006), work that learners often opt out of mathematics because they find it as difficult, boring and of little or no use to them. For example, Zakaria and Syamaun (2017) conducted a study on the relationship between learners' achievement and attitude towards mathematics among high school learners. These authors found that learning methods that involve learner participation encourage learners to have a positive attitude towards the subject.

When learners are confronted with learning difficulties they develop negative attitude towards the topic. Negative attitudes held toward mathematics derives from a combination of the emotions associated with mathematics (Kiwanuka, *et al*., 2017). Attitudes toward learning are demonstrated directly through learners' responses and/ or reactions to situations. Learners' demonstrated characteristics of negative learning attitudes may extend to not completing tasks, avoiding challenges and being satisfied with just writing for completion (Calder & Campbell, 2016). Such learners frequently demonstrate low self-esteem and low efficacy.

2.11 THE INFLUENCE OF MATHEMTICS LANGUAGE IN UNDERSTANDING MATHEMATICAL CONCEPTS

The term language is defined as ''the words, their pronunciation, and the methods of combining the word, how they are used and understood by a community'' (Merriam-Webster, 2004: 688). Riccomini, Smith, Hughes and Fries (2015) noted that mathematical language involves the ability to use words to explain, justify and communicate mathematically. Mathematical knowledge is language-based (Purpura, Logan, Hassinger-Das & Napoli, 2017). Teaching and learning the language of mathematics enhances mathematical proficiency (Seethaler, Fuchs, Star & Bryant, 2011). Mathematical proficiency promotes the understanding of mathematics, fluent computation, the application of concepts to solve problems, along with reasoning and communication within mathematics (National Research Council, 2001). The knowledge of mathematics vocabulary influences learners' mathematical development and performance (Van der Walt, 2009).

Sinay and Nahornick (2016) argued that mathematics language involves the reading of mathematics text, teacher speech, small group discourses and learners' understanding of mathematical terms and symbols. Learning mathematics in English as a second language has an impact on language development in mathematics. Language is critical for cognitive development, providing the concepts for thinking and the means for expressing ideas and asking questions (Vygotsky, 1989). Language skills enable learners to develop mathematical knowledge. There are several content-specific language concepts and terms for probability (e.g., *and* or *both*) that learners need to master before calculating the probability of events. Sepeng and Webb (2012) stated that linguistic abilities affect performance in probability. Batanero and Borovcnik (2016) noted that learners' challenges in probability are associated to an insufficient level of basic vocabulary knowledge in mathematics. According to Sepeng and Webb (2012), teachers utilizes the knowledge of mathematics incorrectly, carelessly and in a confusing manner. Sepeng and Kunene (2015) argued that learners make several mistakes when they face inconsistent language problems.

The effective vocabulary instruction model may not ultimately constitute a cut and dry model. Its implementation, however, is recommended to promote the mastering of mathematics vocabulary. According to Marzano (2014), the first step is to comprehend learners' challenges that they encounter with vocabulary. Upon diagnosing learners' difficulties that are associated with vocabulary teachers may start to respond to addressing the instructional gaps to enhance its effectiveness (Monroe & Orme, 2002). The second step is to restate the mathematics vocabulary in one's own words, through constructing pictures and diagrams. The effective instruction of vocabulary requires a constant revisiting of vocabulary merging with learners' prior knowledge. Bay-Williams and Livers (2009) supported learners' exposure to mathematics vocabulary and provided appropriate academic support.

2.12 TECHNOLOGY AND LEARNING RESOURCES

The introduction of technology in the education system has led to various learning educational resources to support probability education. According to the National Council of Teachers of Mathematics (2011), teachers and learners have regular access to technologies that support and advance mathematical sense-making, reasoning, problem-solving and communication. The common resources used in the teaching and learning of probability include physical devices such as dice, coins and marbles (Batanero *et al*., 2016). Digital technologies used in learning include the use of laptops, tablets, calculators, interactive whiteboards, and online resource (Tamim, Bernard, Borokhovski, Abrami & Schmid, 2011). According to Nilsson (2014), technology supports learners to develop probability reasoning and thinking. Miron and Ravid (2015) revealed a positive connection between digital technology and learner participation rates. The use of technology through group work for academic purposes is favoured by learners (Miron & Ravid, 2015).

Technology promotes communication between learners and teachers. Ogedebe (2012) found a connection between digital technology and academic performance. Teachers and learners were found to be highly motivated to use technology to render learning more enjoyable, interesting and effective (Silviyanti, 2015). Physical devices may be used to perform experiments where small sample sizes are involved. Learners may examine the object in question to compute the probability of the occurrence of an event. Two or more objects can be used together and/ or simultaneously to explore compound events and conditional probabilities. Thompson (2013) stated that two by two tables and tree diagrams may be utilized in helping with enumerating sample spaces. Leinwand (2014: 89) agreed that calculators should be used strategically to help with reasoning and problem-solving. Katsberg and Leatham (2005) established that increasing learners' access to graphic calculators and encouraging learners to make use of them tended to enhance learners' scholastic performance and promote learning comprehension.

In a study involving 12 learners in a Grade 12 class Harskamp, Suhre and Van Streun (1998) established that learners who enjoyed greater access and exposure to graphing calculators largely acquired problem-solving skills and strategies compared to learners with minimal or no exposure to graphic calculators. Parrot and Leong (2018) studied the effect of using a graphic calculator on learners' problem-solving success in linear problems and their attitude towards problem-solving in mathematics. The study employed a quasi-experimental approach involving a non-equivalent control and treatment group design consisting of a pre-test and post-test treatments. At the end of the study, a significant difference was established between the experimental and control groups signalling that participants with continued exposure to graphic calculators outperformed those with minimal or non-exposure to graphic calculators in areas of mathematical problem-solving (Parrot & Leong, 2018). In addition, the results of a survey study also demonstrated that learners who continuously used graphic calculators tended to have a positive attitude toward mathematical problem-solving (Parrot & Leong, 2018). In a related study Rich (1991) established that learners with better exposure to graphic calculators tended to possess effective problem-solving skills and strategies than those with limited access to graphing calculators.

2.13 Problem solving

Mathematics as an important subject in both primary and secondary school education has a role of equipping learners with well-established knowledge, skills and values. In this way having abilities to doing mathematics operations, capable of solving problems, interpret non-routine problems and possess a meaningful understanding of mathematical operations are prerequisite to good formal reasoning (Rahman & Ahmar, 2016). As such, those things are generally related to mathematics taught at schools and are prerequisite to concrete mathematics reasoning which is the goal of mathematics education. In this regard, mathematics learning should be centred on problem solving activities to enhance the facilitation of conceptual and procedural knowledge. However, several studies have found that learners encounter difficulties in understanding mathematics concepts (Scherer, Beswick, DeBlois, & Opitz, 2017).

Several studies suggest that most learners have weakness in acquiring the mathematics reasoning ability and this have an impact on their abilities in solving problems (Sonnleitner, Keller, Martin, & Brunner, 2013). Vygotsky suggested that children are born with foundations of thinking and in a later stage develop higher order thinking skills and abilities to engage in solving problems and reasoning. Thus, meaningful mathematics learning enables learners to be skilful in analysing and be some good problem solvers. As such mathematics curriculum and mathematics teachers ought to prepare their learners to have problem solving skills. The skill of problem solving is a primary point needed by learners to realise the importance of mathematics in daily life (Rahman & Ahmar, 2016). Therefore, problem solving is a critical skill needed to be installed in learners and must be owned by learners in learning mathematics (Pardimin & Widodo, 2016).

2.13.1 Process of mathematics problem solving

Michalewicz and Fogel, (2004) defined a problem as a situation in which there is a difference between fact and will. As such, a problem calls for a learner to apply mathematical skills, knowledge and values to his potential to reduce the gap between a fact and will. In this view, a problem may be regarded as a non-routine problem. According to Hoosain (2003), a non-routine problem has no common procedure and algorithm to solve. In this study, a learner, who possesses the problem-solving skills utilise his reasoning skills to find information to get to the solution. Problem solving is a complex mental process, involving visualisation, imagination, abstraction, and association of information (Rahman & Ahmar, 2016). Problem solving skills are acquired when the learners participate in the learning process by contributing problems, developing possible solutions for the problems and evaluating the results of the solution (Dhlamini & Mogari, 2011). As such, problem solving can help learners to increase and develop their abilities in the aspect of the application, analysis, synthesis, and evaluation in probability learning processes. In mathematics education, problem solving models have been developed based on the assumption that problems are unique. One of the examples of problem solving models was that put forward by Polya. Polya's (1945) problem solving model consists of 4 steps: namely (1) understanding problems, (2) planning the steps in solving the problems, (3) implementing the strategies to solve the problems, and (4) look back at the completed solution.

In step understanding the problem, the problem must be well understood and believed. In this way, the problem must be read over and over, pose a question on what the reader knows, does not know and the conditions of the problems. In this step, Polya argues that learners must restate the problem in the way he understands it. In the step plan, a learner is anticipated to look for a way to resolve the problems. In so doing, planning enables a solver to know what to do with unknown and known information. Implementing a plan means that each step designed in the plan is carefully examined and outlined to solve the problem at hand. The last stage envisages re-examining the answer to the problems if they are correct. For a learner who inefficiently uses this method may plunge into calculation errors and procedural errors.

2.13.2 Problem-solving skills

According to Polya (1945), in a problem-solving setting the problem solver should respond to a question, and in this case the problem solver is presumably not knowing the desirable problem-solving path to the solution. Lithner (2015) emphasized that in a problem-solving activity the problem solver embarks on a problem-solving process without knowing in advance what the solution to the problem would finally be. The definitions emphasise the routine and not the routine rules of the problem-solving process. These observations may imply that meaningful learning of mathematics may lie largely on learning how to solve problems and learning mathematics through problem-solving (Baki & Karatas, 2013; DBE, 2016). Problem-solving is a higher-order cognitive process requiring optimal comprehension of the problem and articulation of problem-solving skills (Loehr, 2014). To be a successful problem solver, a learner needs to understand the problem and break it into smaller units. According to Baki and Karatas (2017), the problem-solving setting gives learners the chance to analyse their thoughts and share and compare different ideas. Dhlamini (2012) stated that performance demonstrates learners' problem-solving abilities. The instruction and apprehension of probability should promote problem-solving skills to improve probability performance.

2.14 COGNITIVE LEVELS THAT ARE LINKED TO ASSESSMENT

A taxonomy is a measurement tool in teaching and learning which divides the cognitive domain into different categories (Du Plooy & Long, 2013). DBE (2011) specified the four cognitive level used to guide mathematics assessment, such include Knowledge (25%), routine procedures (45%), complex procedures (20%) and problem-solving (10%). The cognitive levels are ordered from *simple* to *complex*, and from *concrete* to *abstract*. The taxonomy symbolizes the cumulative hierarchy of cognitive skills, each simpler skill is a prerequisite for the next, even more complex one. The researcher used the CAPS mathematics assessment guideline to assess probability at different cognitive levels (DBE, 2011). These cognitive levels were used to develop an achievement test to measure learners' performance in probability. The CAPS mathematics describe the first cognitive level as *knowledge* (DBE, 2011). The descriptors to be demonstrated by learners were straight recall, ability to select an appropriate formula in the formula sheet and the use of mathematical facts. Du Plooy and Long (2013) described this cognitive level as consisting of knowledge aspects that learners have stored into their memories.

The second cognitive level is *routine procedures*. In this level CAPS stipulated that learners perform well-known procedures, perform calculations using a sequence of actions, and use the correct formula (DBE, 2011). TIMSS (2015) specified that routine procedures constitute facts, concepts and procedures. The problems should be familiar to learners. The third level is known as *complex procedures*. The way in which the term used signals or encourages the use of higher-order reasoning, performing complex procedures and drawing connections between different representations. The fourth level is described as *problem-solving*, whereby learners use higher-order reasoning in order to solve non-routine problems. This stage is viewed as constituting a no routine problem, which has no common procedure and algorithm to solve, requiring thought to find a piece of useful information to get the solution (Rahman & Ahmar, 2016). In the current research, cognitive levels motivated and assisted the researcher to study learners' understanding of probability. In this way, the researcher assessed the processes, skills, and strategies used by a learner in solving Venn diagrams, mutually exclusive and independent events problems.

2.14.1 Assessment of cognitive skills

In this study, assessment of cognitive skills was important because it promotes understanding, conceptual development and to solve probability problems (Malik & Cajkler, 2018). In this study, diagnostic cognitive assessment aimed to provide the abilities of learners in each questions item. In this regard, a cognitive skills assessment enabled the researcher to design a diagnostic test to facilitate learners understanding of Venn diagrams, mutually exclusive events and independent events. For example, the Venn diagram questions demanded learners to apply knowledge of sets (level 1 skill). Applying knowledge included reading towards understanding and applying the knowledge of intersections of sets. Furthermore, in this question learners were expected to validate results obtained demonstrating problem-solving skills.

2.15 CHAPTER SUMMARY

This chapter covered several discussions on the topic of probability in the mathematics curriculum. Most of the discussions in this chapter reflected largely on popularized conceptualizations that the topic of probability is difficult to teach and not easy to learn at school level. The chapter also discussed mathematical errors and misconceptions that are popularly encountered by learners when learning the topic of probability in mathematics classrooms. Other sections addressed the notions of *conceptual knowledge* and *procedural knowledge* in learning mathematics and how these conceptual constructs may be effectively utilized by teachers to promote deep understanding of mathematical concepts when teaching probability. Although the study focused on learning probability, the literature addressing pertinent issues of, (1) teachers' base knowledge in mathematics; and, (2) teachers' influence in promoting productive learning of probability, were also explored and reviewed in chapter 2. The next chapter discusses the theoretical framework of this study.

CHAPTER THREE

THEORETICAL FRAMEWORK

3.1 INTRODUCTION

This chapter focuses on the development of the theory that was used to frame the current study. After providing the contextual setting of the study, the problem statement, the posing of research questions and reviewing the literature related to the study, it is important to determine the theory that will be used to make sense of the analysed data and to make specific and general conclusions. This study focused on exploring learners' experiences in solving probability problems. The theoretical framework provides a lens to account for the learning of certain concepts in probability. Furthermore, this chapter provides a theoretical basis to answer the research questions of the study. In the next sections the researcher discusses the themes that arose from the theory chosen.

3.2 UNPACKING THE NOTION OF THEORIES

The concept of this theory is widely used in the field of educational settings. Creswell (2002: 15) posits that the term theory is used differently and including theories used to describe and explain phenomena. According to Kerlinger (cited in, Creswell, 2014: 86), a theory is:

> ... a set of interrelated constructs (variables), definitions, and propositions that presents a systematic view of a phenomenon by specifying relations among variables, with the purpose of explaining natural phenomena.

Tracey and Marrow (2012) defined educational theories as a well-documented explanation for what transpires when the teaching and learning processes are unfolding. In my view, educational theories seek to explain the experiences learners have that may influence the goal of their education. The theories of education emanating from different schools of educational psychology undergone many phases of change. According to Aubrey and Riley (2018), some theories have been disputed and some have gone through the process of change advancement. These scholars further postulated that the three main educational and learning theories are behaviourism, constructivism, and humanism.

Wallace (2015: 32) defined behaviourism as a school of thought that views learning as a change in behaviour that can be predicted, measured, and controlled. Constructivist school of thought contests the claim that learning as an activity will take place when learners engaged actively in the process of constructing new knowledge in relation to their prior knowledge (Wallace, 2015). The humanist school of thought argues that teaching and learning should focus on the learners' needs, personal emotional growth (Aubrey & Riley, 2018). In this study, a theory was used to guide, to explain, to describe and to predict the challenges learners' experienced when solving Grade 11 probability problems. More importantly, a theory influences the way the researcher interprets semi-structured interviews and lesson observations results.

3.2.1 The role of theory in educational research

Trace and Morrow (2012) states that the knowledge of theories is important to the educational researcher and teaching practices. Significantly, the role of theories must be recognized and understood because theories help in explaining the reasons why learners experience challenges and to promote the enhancement of a suitable instructional setting. For instance, Trace and Marrow (2012) argue that teachers who theorise that a learners' reading problems is caused by auditory considerations can create a conducive environment for that particular learner. In this regard, the theory applied will be used in the inquiry to explain and predict the behaviour and practices of reading for understanding. Teachers who employ theories in their practices are suggested to develop effective instructional and assessment skills tailored to respond to particular pedagogic contexts.

Trace and Morrow (2012) reported that researchers use theories in their hypotheses and in their discussions. These authors argue that linking theories and research is important, as it defines high quality and scientifically based research. According to Creswell (2002), researchers use theories to provide the frameworks to explain why they expect phenomenon understudy to happen as well as why they believe that this phenomenon has happened in their studies. Creswell (2002) noted that research that is theoretically linked to other research allows for a more substantial contribution toward extending the knowledge base than that which is not linked. In addition, Trace and Morrow (2012) claim that theories enable the researcher to generate and evaluate variables. Creswell (2002) emphasized that theories explain and predict the relationships between independent and dependent variables.

3.2.2 The importance of theories in teaching

According to Trace and Morrow (2012), teachers who understand the set of theories from which instructional strategies stem, may select teaching approaches that best suit the topics and/ or learner needs. Teachers' understanding of educational theories provide them with reasons for choosing teaching and learning strategies they use in different content. In recognition of their understanding of these theories, the teachers' educational practices can radiate, and their range of teaching strategies may expand (Trace & Morrow, 2012). Behaviourism, cognitivist and constructivism constitute the main three theories popularly utilized to generate teaching and learning environments (Goldie, 2016). The current study is framed on the socio-constructivist theory to identify teaching strategies, learner performance and challenges experienced in the topic of probability to selected Grade 11 learners in the Tshwane West district schools.

3.3 THE IMPORTANCE OF THEORIES IN LEARNING

Many theories may be used to promote the effective teaching and meaningful learning of the topic of probability in Grade 11 mathematics. Some of these theories include behaviourism and cognitivism learning theories. Behaviourism is a theoretical perspective of learning, which conceptualizes the learning process as changing one's behavioural patterns (Skinner, 1979). The philosophy underpinning behaviourism is that the outcomes of learning are an observable change in behaviour through a response to stimuli. (Tracey & Morrow, 2012). In the heart of behaviourists, learning is viewed as a change in the rate, frequency of occurrence, or form of behaviour or response which occurs primarily as a function of environmental factors (Schunk, 2012).

The behaviourist's theory stipulates that learning involves the formation of associations between stimuli and responses. In this view, learning is explained in terms of the observable phenomenon. Firstly, the behaviour is viewed as the result of a person's response to stimuli. Secondly, the stimuli can be manipulated to strengthen or reduce an individual's behaviour. Behaviourism theory of learning is interested in the effect of reinforcement, practice, and external motivation on a network of associations and learnt behaviours (Fosnot, 2013). Skinner (1953) stated that the reinforcement can be positive or negative, but it will lead to change in a person's behaviour. In the current study the researcher has discussed the three prominent behavioural theories: Classical Conditioning Theory, created by Ivan Pavlov, Operant

Conditioning put forward by B. F Skinner and Connectionism established by Edward Thorndike.

Pavlov's Classical Conditioning Theory stipulates that learning is observed through the repeated pairing of unconditional and conditional stimuli. While Thorndike's theory of Connectionism acknowledges that stimuli that follow behaviour have an effect on learning. Skinner's Operant Conditioning explains that people operate on their environments based on the antecedents and consequences of their behaviour. In Skinners' (1953) view, learning need not be measured by internal events such as thoughts, beliefs and feeling but rather by the observable environmental events. Behaviourism in educational settings places the responsibility for learning directly on the shoulders of teachers (Jones & Brader-Araje, 2002).

In contrast to behaviourism, cognitive theories emphasise the acquisition of knowledge and skills. The formation of mental structures and the processing of information and beliefs in order to produce learning in individuals is a key aspect of cognitivism (Schunk, 2012). Cognitive theorists view learning as an internal mental phenomenon deduced from what people say and do. The cognitive theories emphasise the importance of mental processes in learning. Although some strands in cognitive theories differ and disagree on the learning processes, they all put the emphasis on the cognitive development and deep understanding of concepts. Bandura's (1986) social learning theory disputes the behaviourist central assumptions that stimuli, responses, and consequences were adequate to explain learning. Bandura's social cognitive theory states that people learn by observing others through direct experience or by observing the behaviour of others (Schuck, 2012). This implies that learning occurs when people interact in the environment where they live. As Bandura (1977) puts it, people use modelling, observation, and imitation as tools to learn from one another. The emphasis of learning from one another gave rise to a social cognitive theory such include socio-constructivism. Social cognitivists believe that learning is a product of social connection and engagement with fellow learners, teachers and one's interaction with real and material world (Wertsch, 1997), a notion shared by Vygotsky's sociocultural theory known as socio-constructivism.

Amineh and Asl (2015) described constructivism as the assimilation of both behaviourist and cognitive ideas. These scholars argue that change in behaviour is not sufficient enough to understand how learners construct knowledge. The constructivism standpoint is that learning is the process of knowledge construction: that is how learners make sense of their experiences

(Caffarella & Merriam, 1999). These scholars found that teachers who employ a constructivist approach turned to develop learners' level of understanding. Furthermore, Caffarella and Meriam (1999) posit that the constructivism theory of learning shows that the understanding of concepts taught leads to higher-level thinking.

3.4 THE ORIGIN OF CONSTRUCTIVISM

The notion of constructivism is linked to Lev Vygotsky, John Dewey, and Jean Piaget. The main two strands of constructivist perspectives are Piaget's constructivist perspective and Vygotsky's socio-constructivist perspective. Dewey (1987), Piaget (1977) and Vygotsky (1978) discarded the behaviourism perspective and made advancement to constructivism as they argued that traditional education plays a major role, in so far as that knowledge was transmitted from the teacher to the learner, shifting the individual autonomy (Dewey, 2013). Dewey (1997) proposed that the educational curriculum must prepare a learner for future challenges. Dewey argued that a learner must strive in an environment where he can be able to interact with the knowledge at hand and must take part in his or her own learning.

The term constructivism is also derived from Piaget's (1977) constructivist view of teaching and learning, which relates to the active construction of meaning, which was later extended by Vygotsky to include social learning theory. Vygotsky (1978) argues that the process of learning is affected and influenced by the society people find themselves in. These proponents of constructivism propose that behaviour and cognition are essential ingredients to facilitate the learning process. Kanselaar (2002) stated that constructivist approach and socio-constructivist approach are the two theories of constructivism that indicate that learners' conceptions of knowledge result from their search for meaning, and that during this process learners formulate and make own understanding and interpretations of their experiences (Amineh & Asl, 2015).

3.5 VYGOTSKY'S SOCIO-CONSTRUCTIVISM THEORY

Socio-constructivism is a philosophy of learning which hinges on the assumption that learners formulate their understanding during their interaction with others and their real world (Mvududu & Thiel-Burgess, 2012). This theory assumes that (a) human beings create meaning through their experiences within the society they live, and (b) language facilitates the construction of knowledge. Socio-constructivism views language (spoken and written) as a socially communicative act and a medium through which the meaning-making searches in the process of interacting with others proceed. Roth (1999) argued that individuals develop knowledge as they interact with their surroundings and adults before knowledge becomes internalised. According to Kim (2001), reality, knowledge, and learning are all influenced by culture.

Kukla (2013) stated that social constructivism offers the perspective that reality does not exist independent of human actions; instead, it is constructed through human interaction with the physical world. The other assumption of social constructivism is knowledge: it is socially and culturally constructed (Kim, 2001). In this regard, social constructivism stipulates that learning takes place when individuals become deeply engaged in the construction of knowledge and the search for meaning (McMahon, 1997). As Vygotsky (1978) puts it, effective teaching and learning instruction is the one that leads to mental development. The socio-constructivist theory helps to understand the unfolding of the learning process in social contexts and assist the teacher to generate productive and active learning situations in the classroom through learners' interactions and communications with their peers (Len, 2018).

Vygotsky (1978) posited that the learning process should be learner driven, that is, a learner must independently discover and transform new knowledge to demonstrate conceptual and procedural understanding. Vygotsky's socio-constructivist theory is used in this study to gain insights on how psychological, social and cultural variables may impact on the process of learning the topic of probability in the mathematics curriculum. Further, socio-constructivist theory acknowledges that a well-designed task from socio-constructivism lessons may create productive learning moments for collaborative work resulting in the enhancement of learners' problem-solving skills, thinking skills and academic performance. This section focuses on the discussion of Vygotsky's socio-constructivist theory and its basic concepts. Vygotsky's socio-constructivist theory states that a learner may engage actively in knowledge construction when receiving support from their peers, teachers and/ or other experts. In this regard, this section discusses Vygotsky's (1978) concept of the Zone of Proximal Development (ZPD) and scaffolding.

3.5.1 Assumptions of Vygotsky's socio-constructivist theory

According to Vygotsky, learners' mental development occurs in two levels: learners can solve a problem independently and may develop when guided by their teacher and receiving support from their peers with higher educational capabilities (Yan-bin, 2009). Vygotsky explained the difference between these two levels as the Zone of Proximal Development (ZPD). Olivares, de

Mello, Adesope, Rolim, Gaševic and Hundhausen (2019) posit that learners who find themselves inside this zone can be helped to develop their skills and strategies through interacting with abler peers. Scaffolding in this sense is a facet of ZPD which teachers may use facilitate teaching and learning activities and knowledge movement from learners' prior knowledge to new knowledge construction. Scaffolding helps teachers develop guiding and leading questions that facilitate problem solving abilities in learners (Lau, Adams, Ambler, Anderson, Andrews, Atherton, Atkinson, Shiffrin, Spence, Spence & Atkinson, 2018).

The theory of the Zone of Proximal Development (ZPD) and scaffolding are crucial in the instruction of probability because they may support problem solving in probability to learners who face difficulties. Learners should receive assistance in order to comprehend the demands relating to probability problems and make sense of their experiences. Moreover, such a strategy is useful for constructivist teachers who use these teaching and learning strategies to promote problem solving abilities in learners. Similarly, Vygotsky (1978) mentioned that actively participating learners in the process of knowledge construction will gain more from ZPD and scaffolding techniques. Lau *et al*. (2018) states that learners who are encouraged to actively participate in solving problems in the effort to enhance their own consciousness perform better than those who may not.

ZPD suggests that individuals find themselves implicated in the process of developing mastery of a practice or understanding a topic as they participate in problem solving activities, such as collaboration. A learning situation that involves a more knowledgeable peer either directly or indirectly has a positive influence on a learner's construction of knowledge (Olivares, *et al*., 2019). Among the other reasons to adopt Vygotsky's socio-constructivist is what Emihovich and Lima (1995) suggested: socio-constructivists pays more attention on active inputs of learners to promote the growth of consciousness.

In this study, socio-constructivism learning theory has been used to pay attention to the significance of learning actively to participate meaningfully in the development of thinking. Study background has demonstrated that learners are generally underperforming in mathematics (see, DBE, 2016). In this regard, socio-constructivism in this study has been used as a lens through which the researcher may explore and analyse data collected from a learner diagnostic test, interviews and lesson observations. However, there are some theories that could have been used to explain learning discourses such as Piaget’s developmental learning theory and Bandura’s social learning theory. Tracey and Morrow (2012) noted that theories are closely linked to learners’ behaviour and practices. The researcher needs in this respect to develop a critical praxis to better understand theories and how they can be correlated with learner performance and teaching and learning practices.

Piaget (1977) asserted that learning takes place when a learner is actively involved in knowledge construction. Despite the fact that such an understanding places an implicit expectation on learners in the construction of knowledge, the learning process suggests that learners as individuals have their own worldviews. In addition to Piaget’s notion of constructivism, Vygotsky’s socio-constructivist approach asserts that knowledge and learning are mediated by society. In this view, the processes of knowledge construction occur as the learner participates in various forms of social interaction. Vygotsky has alluded that learning takes place in stages, which is through the Zone of Proximal Development (ZPD). Vygotsky’s (1978) ZPD points to the idea that learning should be guided by abler learners and or teachers. In this view, a learner who solves a given problem under adult/teacher guidance or in collaboration with more capable peers does it better than when alone. The construction of knowledge occurs when learners use a tool (such as an abacus or calculators) and signs (language, mathematics formulae) which are socially constructed (Vygotsky 1978).

According to Vygotsky (1978), the socio-constructivist theory may help one to understand and justify the place and potential of learner-centred approaches to teaching and learning. This study aimed to explore the learner's understanding of probability concepts. Vygotsky’s theory stresses the interaction of interpersonal factors at the social and cultural-historical levels, and individual factors, are essential to promote the development of humans (Schunk, 2012). Socio-constructivism may be understood in terms of socialisation, which is a process governing the acquisition of skills and knowledge. The learning theory enables individual learners to participate in his or her group or society. These strategies may stimulate developmental processes and foster cognitive growth. In this study, it was significant to explore how learners construct knowledge in the topic of probability. Moreover, the study has used a socio-constructivist perspective to explore learners' understanding of probability concepts.

Vygotsky (1978) stresses the role of language and culture in cognitive development, arguing that language serves to mediate higher-order thinking. He posits that the role of language is the foundation for an individual's ecology and a tool to facilitate conceptual growth. Learners use language to communicate their actions during learning, hence it is a tool that causes a fundamental change in mental functions. The functions of spoken language include signalling, be it social, individual, communicative, intellectual, nominative and indicative (Wertsch, 1985). This implies that learners may develop a meaningful understanding of probability if they can unpack the language used in probability concepts. For instance, language is used to explain one's thoughts to others. Vygotsky argues that the path between objects and thought is mediated by other people through the use of signs or the symbols of language. Simpson and Cole (2015) also emphasize that teachers should be cognisant of the role that the language plays in setting up the classroom norms. Research in education has shown that language influences how learners make meaning of mathematics and that linguistic ability can also predict one's gains in mathematics (Vukovic & Lesaux, 2013). In the current study, the role of language will be used as a lens to explore how learners use probability language to solve problems. The researcher conducted a lesson observed in the topic of probability to determine how learners define relevant terms and solve probability tasks.

3.5.2 ZPD and scaffolding and their influence on learning

Vygotsky (1978) stated that learning is a continuous process stemming from the current intellectual level to a higher level. This movement occurs in the zone of proximal development (ZPD). ZPD is defined as the range of tasks that a learner can perform with the help and guidance of others but cannot yet perform independently (Vygotsky 1978). Bruner refers to this educational guidance and support as 'scaffolding'. Through scaffolding learners may attain educational levels that are higher than those that they may achieve on their own. The ZPD consists of two levels, namely: (1) the level of actual development; and, (2) the level of potential development. The first level refers to the level that the learner has already reached (Vygotsky, 1978). The actual development marks the level at which the learner is capable of solving problems independently (Vygotsky, 1978). He further states that the level of actual development marks the upper limit of tasks that one can be performed with the help of a more competent individual.

ZPD by contrast is the area in which the most sensitive instruction with the help of an adult/ peer should occur (Vygotsky,1978). In this regard, learners are given appropriate challenges that enable them to strive to achieve just beyond what they are currently capable of achieving (Davis, Sridharan, Koepke, Singh & Boiko, 2018). In a classroom situation, ZPD may enable the less competent learner to develop the desirable higher mental skills through scaffolding. Vygotsky points out that through social interaction, learners' cognitive development occurs and what they can achieve when assisted by their peers or teachers becomes indicative of their relative mental development. He maintains that scaffolding within the ZPD awakens and provides an avenue for intellectual development. Scaffolding in this sense is the supporting mechanism which helps learners to cultivate problem-solving skills that are usable in terms of their ZPD (Vygotsky, 1987). Wood, Bruner and Ross (1976) define scaffolding as the collection of items in a task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence.

In the process of scaffolding, the teacher gives a learner a task which is above the learners' current ability, but within his or her capacity for a while, and only intervenes in the educational process when the learner demonstrates that they are experiencing difficulty (Wood, *et al*., 1976). In this regard, scaffolding provided by a teacher paves the way for learners' acquisition of vital problem-solving skills and critical thinking. Schwieter (2010) similarly found that scaffolding writing techniques and feedback debriefing sessions within the ZPD effectively develop writing skills. Thus, within the classroom, scaffolding presupposes that a teacher is continuously attending to learners' thought processes in order to access their individual ZPD. More importantly, the roles of scaffolding may present learners with opportunities ranging from participating and practising extended discourse to negotiating, as both learners and the teacher collaboratively work through extended discourses to make sense of an idea or co-construct meaning (Allahyar & Nazari, 2012).

Burrows, Macdonald and Wright (2013: 21) posit that socio-constructivist learning positions the learner as the active constructor of their knowledge rather than the teachers as the unilateral controller. Schunk (2012) prescribes social constructivist teaching approaches to include reciprocal teaching, peer collaboration, cognitive apprenticeships, problem-based instruction, web quests, anchored instruction, and other methods that involve learning with others. From this perspective, the learner builds their knowledge through understanding what their teacher presents. Amineh and Asl's (2015) claim that the theory of socio-constructivism encourages learners to develop their own version of the truth influenced by his or her background, culture or knowledge of the world. The importance of learners' social interaction with potential peers and other knowledgeable members of society is key to socio-constructivism. The socio-constructivist viewpoint is that learners tend to develop their thinking abilities through interaction. Furthermore, Amineh and Asl (2015) agreed with the position that a learner's background can help to shape the knowledge that he or she discovers, creates and attains in their learning process. Contrary to rote learning in the past, socio-constructivism describes the learner as active participants in the process of constructing new knowledge from prior knowledge (O'Donnell, Reeve & Smith, 2011). Subsequently and through this process, learners' knowledge and constructs become legitimate and eventually internalised.

According to Amineh and Asl (2015), teachers can be viewed as facilitators of a socio-constructivism approach. Teachers should then assist learners to comprehend their instructional content. The role of the teacher is to negotiate and provide guidance by sharing social experiences. In sum, educational reform encourages learners to assume an active role while they are learning. Therefore, Vygotskian' socio-constructivism helps us understand important ways in which knowledge construction proceeds in a probability classroom. In the process of construction, learners need to explain their ideas to one another as well as to discuss disagreements. Allahyar and Nazari (2012) maintained that socio-constructivism classroom activities are in fact classroom discourse governed by interaction patterns either between teachers and learners or among learners. Positioning one's thinking in relation to that of another person leads to deeper cognitive processes (Scardamalia & Bereiter, 2014), including critical thinking. Such processes including collaborative learning encourage learners to cooperate in the solution of complex problems. Furthermore, Scardamalia and Bereiter (2014) posited that interactions, such as those achieved through collaborative learning processes, are thought to provide mechanisms for enhancing high-order thinking as well as learning performance.

3.6 RATIONALE FOR THE CHOICE OF A THEORETICAL FRAMEWORK

This study found value in using some of the principles of the socio-constructivist approach to explore knowledge construction, understanding and the performance of learners within the context of Grade 11 probability. Socio-constructivism pays more attention to the nature of knowledge and how learners acquire, construct and instrumentalize knowledge. This study utilized aspects of socio-constructivist learning approach that include ZPD and scaffolding, along with collaboration learning to assess probability learning. The researcher assessed probability learning in terms of its products and outcomes. The methods that have been identified for assessing learning products include:

- *Testing*: the diagnostic test was administered to learners to measure the cognitive aspect of learning.
- *Direct observation*: I observed instances of learning styles during probability learning. Also, I will observe the teaching approaches if they meet the learners' needs as well as frequencies of learner participation.
- *Oral responses*: the researcher observed the fact that learners interact in probability classes, noting learning approaches from the literature review, such as comments, along with verbal questions asked.
- Written responses were analysed in accordance with a test using techniques taken from the socio-constructivism literature review.
- *Interviews*: in this study, the researcher interviewed learners seeking clarity on the outcomes of the diagnostic test.

The socio-constructivist perspective has provided criteria for exploring the experiences of learners in solving probability tasks. Learners' communicative behaviour has an impact on teaching-learning as it creates an educational space for learners to converse in the classroom (Hardre, Davis, & Sullivan, 2008). The researcher observed how learners unpack probability terms and formulas. The socio-constructivist perspective provided clear guidelines on teaching and learning strategies that may be used especially with learners with difficulty in probability. The emphasis was placed on cognitive development in this context, in which learners construct and re-construct probability knowledge through the social process. In addition, the socio-constructivist theory of learning provided guidelines on aiding knowledge, context, belief, and attitudes that learners bring with them into probability classrooms. Such interviews enabled the researcher to seek clarity on strategies used and the possible reasons for which they were employed. Does the teacher identify a knowledge gap before teaching the new content? The researcher had to seek clarity on the reason why most learners performed poorly in the written test.

The processes of scaffolding were used to check if the teachers considered learners' prior knowledge in relation to the topic of probability before the new content was taught. In this view, scaffolding assists with the process of determining whether teachers understood the position of learners before the new topic could be taught. Scaffolding processes assisted in understanding how well learners who are struggling with probability were supported. The strategies aligned with the socio-constructivism theory of learning can be used in teaching probability, such as in the prediction, observation and explanation of learning discourses, all of which are also incorporated in CAPS. The researcher used these strategies to investigate the learners' initial ideas in probability. In this study, the researcher observed probability lessons using the schedule designed from the socio-constructivist theory of learning. This arrangement was in line with the fact that probability learning is based on real-life adaptive problem solving, which takes place in a social manner through shared experience and discussion with others (Kim, 2001; McMahon, 1997). Hence, the researcher observed how efficiently learners solved probability problems in groups and in the context of class discussions. The methodological issues will be discussed in the chapter that follows.

3.7 CONCLUSION

This chapter reflected on the theories of teaching and learning and introduced the theoretical framework underpinning the current study. The role of theories in education and research, and the importance of theories in teaching was discussed to have a general overview of how theories are used and come to influence teaching and learning. The discussions in this chapter showed how socio-constructivism emphasizes the significant role of ZPD in learning (Vygotsky, 1978). The chapter highlighted the importance of the roles of language and the society in facilitating the construction of knowledge in an education setting. Thus, the knowledge constructed by learners is influenced by the environment and prior knowledge held by learners (Hartle, Baviskar & Smith, 2012). The core of the socio-constructivist philosophy is that knowledge is not forced on to learners, but rather gained over time through real experiences (Piaget & Inhelder, 1969).

CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 INTRODUCTION

The aim of this study was to explore learners' experiences and challenges if they happened to possess any, when approaching the topic of probability in Grade 11 classrooms in selected Soshanguve township schools (Section 1.4) In the previous chapter the theoretical framework of the study was discussed; namely, socio-constructivism (Section 3.1.1). This chapter presents the research methodology of the study and the following information is also presented: the research design of the study, the study population and sampling techniques, its data collection instruments and data analysis procedures. Other discussions in the chapter cover issues relating to the pilot study, validity, reliability, and ethical considerations.

4.2 MOTIVATION FOR THE STUDY

In 2012, the South African education system introduced the Curriculum Assessment Policy Statements (CAPS). The introduction of CAPS in schools led to the re-introduction of the topics of Euclidean Geometry and probability in the mainstream curriculum of mathematics in the Further and Education Training (FET) phase. Prior to the introduction of CAPS in 2012, many township schools were experiencing educational challenges such as overcrowded classrooms and shortages in learning and teaching support materials (Dhlamini, 2012). Most mathematics teachers were experiencing difficulties in their instruction on the subject of probability, which some of them were encountering for the first time (Makwakwa & Mogari, 2012). Research shows that most teachers consider probability to be the most instructional and conceptually problematic topic to teach in mathematics (see, Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya, Dhlamini & Makwakwa, 2011). In addition, it is documented that a majority of poor performing learners in mathematics are located in township schools (Dhlamini, 2012).

4.3 RESEARCH PARADIGMS

According to Kivunja and Kuyini (2017), a paradigm is a conceptual lens through which the researcher examines the methodological aspects of their study and how their data will subsequently be analysed. Johnson and Christensen (2012: 31) define a paradigm as a way of conducting research, and the authors postulated that qualitative, quantitative and mixed methods comprise the three major paradigms in educational research. Boeren (2018) postulated that positivism, interpretation and pragmatism are the three forms of paradigms employed in social scientific research. According to Wahyuni (2012), positivism is associated with quantitative research approaches that deal with numerical data. Numerical data is the data that is measured and expressed as digits and subjected to statistical analysis (Boeren, 2018). Despite the use of quantitative methods in this study, this paradigm can not be used alone because experiences in probability may possibly be understood through the conducting of semi-structured interviews and lesson observations.

Interpretive epistemologies deal with the narrative commonly known as qualitative approaches, which extends to interviews given and lesson observations (Wahyuni, 2012). Thus, explaining the learners' experiences in probability is insufficient if we are to understand the learning processes when learners engage in solving probability problems and the root causes of the challenges. Pragmatism is a perspective that mixes different paradigms in a single study to minimize the weakness of any one paradigm. Creswell and Poth (2016) emphasises that pragmatic researchers focus on actions, situations and the consequences of participating in a study. In the current study employing pragmatism, the paradigm enabled the mixing of positivism and interpretive paradigms (Section 4.2). In this study, research question sought to explore the conceptual and procedural challenges, factual, tact and factual challenges learners experienced in the process of solving probability problems (see, Section 1.8). In that way, pragmatism enabled the researcher to identify the actual experiences and challenges that arose in the process of learning probability and provided some recommendations on how to address the identified challenges.

The research employed the pragmatism paradigm because this study sought to explore learners' cognitive aspect in relation to learning the topic of probability in Grade 11. A diagnostic test generated quantitative results and was further coupled with narrative data from qualitative results from semi-structured interviews and lesson observations. These data collection episodes facilitated the inquiry of, and understanding of learners' cognitive challenges experienced in solving probability problems in Grade 11.

4.4 THE RESEARCH DESIGN OF THE CURRENT STUDY

Creswell (2014) defined research designs as an area of inquiry within qualitative, quantitative and mixed methods approaches that provide specific direction for procedures in research designs. This study employed a mixed-methods research approach involving the collection, analysis and mixing of both quantitative and qualitative data within a single study to acquire a comprehensive understanding of the research problem (Venkatesh, *et al*., 2016). A mixed methods research approach made the study fall within the pragmatism paradigm (Section 4.2). The mixed methods approach was used to gain deeper insight into the challenges that learners seem to experience when solving probability problems at the Grade 11 level (Section 5.6).

There are four types of mixed methods research designs (Creswell, 2014). Triangulation research design is viewed as the merging of qualitative and quantitative data to gain more understanding of the research problem. The embedded research design refers to the use of either the qualitative or the quantitative data to answer the research questions within a largely quantitative or qualitative study. Explanatory use the qualitative data to facilitate the explaining or elaboration of quantitative results. Exploratory research design collects quantitative data to test and explain a relationship found in qualitative data. The specific mixed methods research design that this study uses is a *sequential explanatory research design*. The rationale for employing this research design was to incorporate data from the test into the data stemming from the semi-structured interviews and observations. Creswell and Poth (2016) contends that the sequential explanatory research design first collects and analyses numerical data. The rationale for this process, according to Creswell, is to inform the researcher of the qualities of the participants to be explored in depth through qualitative methods. Venkatesh *et al*. (2016) emphasises that the sequential explanatory research design allows for a more robust exploration of the phenomena under investigation.

The sequential explanatory research design facilitates the collection and analysis of data in two distinct phases. A quantitative phase was followed by the qualitative data collection and analysis phases (Venkatesh, Brown & Bala, 2013). This mixing would then occur when the initial quantitative results inform the collection of secondary qualitative data (Creswell, 2014). In the first phase, a diagnostic test was administered to learners to identify learners' challenges that they presumably experienced when solving probability problems (Section 4.6.2). The second phase constituted the narrative data used to understand the difficulties learners experienced. Qualitative research helped to understand the unique experiences of individuals from their perspective in society. This approach produces results that are not achieved through statistical procedures or means of quantification (Cohen, Manion & Morrison, 2011). Creswell & Plano Clark (2011) states that qualitative research is linked to the social constructivism paradigm supporting the socially constructed nature of reality. In the qualitative phase, a case study research approach was used to collect data using semi-structured interviews. Lesson observations were conducted after analysing data collected from the diagnostic. The open-ended questions for the interviews were developed after this analysis was conducted of the learners' test responses.

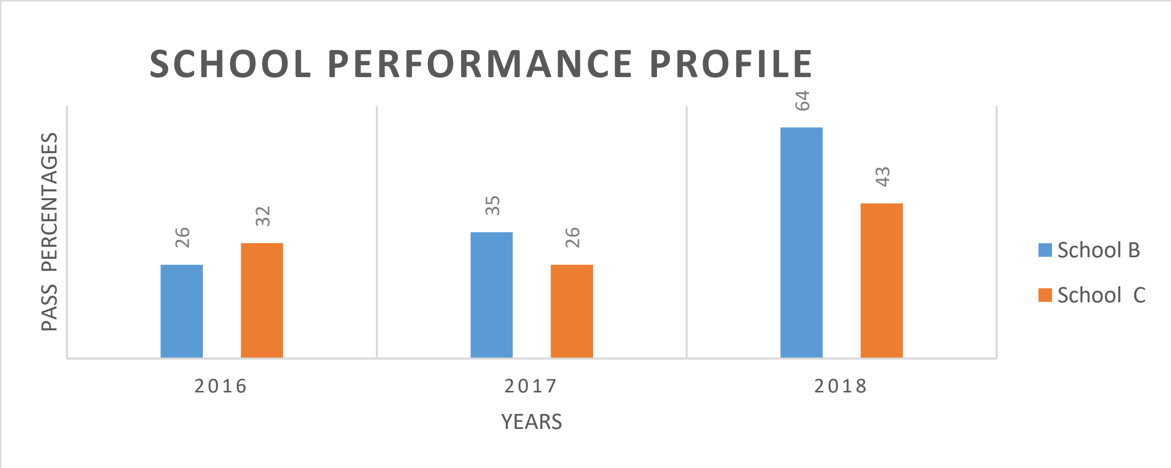
4.5 POPULATION OF THE STUDY AND SAMPLING PROCEDURES

Hartas (2015) defines population as "a group of individuals that have common characteristics that are of interest to the researcher" (p. 67). The targeted population in this study comprised all Grade 11 mathematics learners and their respective teachers in the Tshwane West District, in the Gauteng province of South Africa. In the Tshwane West District, 58 schools offered mathematics in Grade 11. These schools constituted 3676 Grade 11 learners and 70 mathematics teachers. This information was collected from Tshwane West District through the mathematics subject specialist.

4.5.1 Schools' profiles

Schools in the study were coded School A, B and C, where the Capital letter A represented the numerical order of the school that was visited first. Mathematics' end-of-year results in School B was 26% in 2016, 35% in 2017 and, in 2018, the school obtained 63,9%. The pass rate of end of year results in School C was 32% in 2016, 26% in 2017 and 42,6% in 2018 (see, Table 4.1). Although these participating schools showed an increase in the performance over three years, the schools were considered not to be performing to a satisfactory level. Teachers' information regarding their qualifications and professional experience was also obtained. Of the five teachers who participated, four of them had a three-year diploma and an advanced certificate in education (ACE). In a pilot study group, the final teacher was revealed had a four-year degree qualification and relative teaching experience.

Table 4. 1: Schools mathematics performance for three consecutive years



Learners' information relating to their respective socioeconomic status was also obtained. Learners from all participating schools came from informal settlements in Soshanguve Township area. The analysis of learners' data showed that most of them came from disadvantaged socioeconomic backgrounds (see, Section 1.2). The researcher collected this information required from the school first before school study-related visits were pursued. Such information incorporates the number of learners enrolled in Grade 11 mathematics and the location of mathematics classrooms (see, Appendix K).

4.5.2 The sample of the study

The study sample was drawn from three schools in the Tshwane West District in the Gauteng province who almost represented the population of the study (Section 4.4). The sample of the study consisted of 380 learners who were taking mathematics as a subject in Grade 11. One school formed a pilot study and consisted of 78 learners and their teacher (see, Section 4.6). In school B, 189 learners participated in the study and had on average 33 learners per class. These learners from six classes were taught by two teachers who both had more than 10 years of teaching experience, and who had been exposed to different education systems. In school C, 113 learners were included in the study sample together with two teachers. This sample had four classes with a class size of 30 learners. The rationale for including teachers was that the teachers' methods for instruction exert an influence on their learners' performance (see, Suurtamm, Quigley & Lazarus, 2015; see, also, Section 2.8).

4.5.3 Sampling procedures

Sampling techniques are methods used for selecting individuals from which information will be collected (Singh & Masuku, 2014). Prior to collecting data in the pilot and main studies, data was collected to determine the suitability for participating in the study (see, Appendix K; see, also, Section 4.7). The allocation of schools to either a pilot study or a main study was based on the analysis of the performance of the schools over a three-year period (Section 4.4.1). Grade 12 mathematics end of year results for 2016 and 2017 were compared and checked for any declination in their results. Thereafter, 2017 end of year results were compared against 2018 and checked for any deviations. All poor-performing schools had their results compared over these three-year periods and arranged in numerical order of performance decline. Schools in this district with a higher rate of their performances being declined were coded from the lowest to highest performance decline as School A, School B and School C. The category School A means that the school is the lowest in a list of schools with greater percentage decline in end of year mathematics results. School B entails that it is number two in the list and so on.

Participating schools shared similar features as discussed in the background. Participating schools evinced similar features in terms of perpetuating poor performance in Grade 12 mathematics results, inadequate learning and teaching support materials (LTSM), using English as a medium of instruction. The English language plays a key role in the formal instruction of concepts relating to probability (Sepeng & Kunene 2015).

A purposive sampling method was used to target learners with perpetuating poor performance in mathematics end of year examinations. Critical sampling case technique ensured that the sampled learners represented the case of learners with perpetuating poor performance. The reason for the chosen technique was its capability to select a sample whose findings would explain poor performance in probability to all Grade 11 mathematics learners. This sampling method helped to acquire information-rich cases for the study (Palinkas, Hurwitz, Green, Wisdom, Duan & Hoagwood, 2015). Patton, (2015) stated that this method is used for an in-depth exploration of the problem. The author also argued that purposive sampling has the power to select a rich sample, which, when studied in-depth, may provide rich information that is related to the topic of research. The researcher worked with learners who had done the topic of probability in Grade 10. Participants were purposefully selected by researchers in accordance with their performance status in mathematics to answer the research questions

(Section 4.5.1). The diagnostic test constituted the assessment initiative that was administered to the three schools that participated in this study.

The researcher decided on the choice of participants in keeping with the qualities they possessed that were believed to be revealing of the situation under analysis. Schools B and C were interviewed. In school B, 10 learners were purposively selected for the semi-structured interviews to ensure that 10 learners were representative of the two classes. In school C, 20 learners were purposively selected to ensure that these learners were representative of the seven classes. Lesson observations were conducted to Schools B and C in the classrooms with all learners during the two weeks of teaching and learning of probability. During lesson observations, the researcher targeted the participants that had performed poorly in the diagnostic test as well as those participants who provided rich interview data.

4.6 THE PROCESS OF INSTRUMENTATION FOR THE STUDY

The term instrumentation does not only focus on the naming of the data collection instruments but also refers to the entire process of developing instruments and paying attention to technical issues such as reliability, validity and trustworthiness. According to Cohen *et al*. (2011: 377), "instrumentation enables the researcher to gather useful and usable data so as to carry out the practical work of the study". Given this description, it is important to address the instrumentation process sufficiently to collect data that is validated by the study.

4.6.1 Naming the data collection instruments

Data for the study was collected through a diagnostic test, semi-structured interviews, and lesson observations (see, Appendices H, I & J). Interviews are methods of collecting information through oral quizzes using a set of predetermined and pre-planned questions in accordance with the needs and design of the study. Cohen *et al*. (2011: 409) explained interviewing as "interchangeable views between two and or more people on issues of concern". Shneiderman and Plaisant (2005) listed three advantages for the interview methods of data collection:

- direct contact with the users often leads to specific, constructive suggestions;
- they are good at obtaining detailed information; and,
- few participants are needed to gather rich and detailed data.

According to Cohen *et al.* (2011), observation offers the researcher the opportunity to gather live data from naturally occurring social situations.

4.6.2 The purpose of data collection instruments

The data collection instruments may be classified as, (1) those that serve to collect quantitative data; and, (2) those that serve to collect qualitative data.

4.6.2.1 The diagnostic test

The diagnostic test was administered in the pilot study and the main study. The study administered the diagnostic test to identify challenges that learners were experiencing when studying the topic of probability in Grade 11 (see, Appendix H). The test determined learners' cognitive knowledge and skills in the process of solving probability problems. In addition, the test was used to assess procedural knowledge, problem-solving skills and the cognitive skills of learners. The purpose of piloting the test was to ensure that the test satisfied the assessment standards as stipulated in CAPS. Also, it was piloted to check for the clarity of questions and was revised as necessary. This instrument was also used to gather information on the basic understanding of probability concepts.

4.6.2.2 Semi-structured interviews

According to Dhlamini and Mogari (2011), interviews enabled the researcher to enter participants' minds rather than taking only their written responses when dealing with problem-solving tasks into consideration. In this study, semi-structured interviews further investigated learners' challenges observed when analysing their test responses. In this way, semi-structured interviews were mainly conducted to understand learners' experiences in solving probability problems (see, Appendix J). In addition, the purpose of the semi-structured interviews was to understand the underlying thinking of learners when solving mathematics problems on the subject of probability. Semi-structured interviews targeted mainly learners who had performed poorly in the diagnostic test, in order to determine the difficulties these learners experienced in the test. In this study, the researcher interviewed 20 learners from schools A and B for 15 minutes allocated to each interview session.

Semi-structured interviews likewise investigated the ways in which learners perceived, expressed and answered the probability questions in the test. The semi-structured interviews helped the researcher understand how learners responded to questions on probability concepts. Semi-structured interviews were used to gain clarity on questions arising from the analysis of test results. For instance, when learners wrote that they were not taught probability in Grade 10, the researcher probed respondents further for clarification. Also, issues regarding formulae and unclear steps used in solving problems and the reason why one did not attempt to answer the questions were investigated. Extract of the responses by L16 shows misuse of the probability product rule in a question that required the use of mutually-exclusive events. Overall, the semi-structured interviews served the following purpose:

- to clarify an incorrect solution from the test;
- to provide explanations of the observations from the lessons' visitations;
- to seek to clarify the ways in which learners tended to perceive, express and answer probability questions; and,
- to probe learners' reasoning behind their solving of probability problems.

4.6.2.3 Lesson observations

The purpose of lesson observations was to observe ways in which learners solve probability problems and how they engaged or were engaging during a probability lesson in Grade 11. The researcher was interested to observe the classroom interactions between learners in relation to the outcome of their semi-structured interviews. The researcher observed how learners unpacked probability concepts such as how they used certain terms of probability; how they used the language of teaching and learning during the lesson; and, how learners participated during their given lessons. In this regard, the lesson observation instrument would be used to observe how learners constructed mathematical knowledge in relation to the topic of probability in Grade 11, and how they demonstrated their line of reasoning as well as their manifestation of problem solving (see, Appendix I). The lesson observation schedule instrument collected data on how learners interpreted, analysed, solved and calculated the probability problems. The researcher observed five lessons from School B and four lessons from School C. Each observation session lasted for 45 minutes.

The lesson observations were also used to observe teachers. The lesson observation schedule was used to study teacher's instructional methods, to observe how they presented their probability lessons and how they interacted with learners during their teaching to facilitate learning. Observing teachers' lessons was also intended to provide a means for verifying some of the learners' test and interview responses. For example, the interview responses that the teacher provided did not give the learners adequate time to exchange ideas in the classroom.

4.6.3 Development of data collection instruments

All instruments were developed to assist the researcher to collect data for the study to explore the following issues: (1) to identify challenges faced by Grade 11 learners in the topic of probability, and, (2) to study and explore certain aspects of teaching practices thought to have the potential to exert an instructional bearing on the apprehension of the topic of probability in Grade 11.

4.6.3.1 Construction of the questions of the diagnostic test

The researcher developed questions for the diagnostic test using standardized past examination question papers for 2015 and 2016 that covered the topic of probability in Grade 10. Standardized past examination question papers were the end of year examinations set and moderated by the Gauteng Department of Education (GDE). The question papers satisfied the assessment standards as stipulated in the South African Curriculum and Assessment Policy Statement (CAPS, 2012). It is compulsory that all public schools in the Gauteng Province administer these common end of year examinations.

The reason for using past examination question papers was that they are set provincially and moderated by the GDE, which is a statutory body regulating scholastic issues at the school level. In the design of this instrument, the researcher was guided by the CAPS documents of mathematics in Grade 11. This document prescribes that when developing a test in relation to the topic of probability, the learner would be assessed in the following manner: (1) the learner must demonstrate the knowledge of facts in relation to the concept of probability; (2) the learner must be able to solve routine procedures and be able to solve complex procedures and reasoning (see, Appendix L). The CAPS document prescribes that the test must assess competencies such as, (1) the ability to conduct probability calculations; (2) the ability to analyse and interpret Venn diagrams; (3) the ability to work with dependent and independent events; and, (4) the ability to interpret and analyse mutually exclusive events (see, Appendix H). The test items in Figure 4.1 provide an example of those competencies that informed this study.

In Figure 4.1, the test items require a test respondent to analyse a Venn diagram. The Venn diagram in Figure 4.1 shows two events drawn from a real-world context including two sample spaces. Learners were asked to calculate the value of x. A routine procedure was required to create a linear equation. In this case, learners were expected to use their mathematical knowledge and skills to the unknown. In sub-question 1.1.2, a learner (test respondent) would be required to apply a routine procedure in "(a)" and a complex procedure in "(b)" to calculate the probabilities. Learners would also be expected to use the complementary rule when calculating the probability of learners not playing hockey or soccer. In addition, learners would be expected to demonstrate that they understand the distinction between the probability terms embedded in the test questions, such as the words '*or*' and '*only*'.

4.6.3.2 Construction of the items of the semi-structured interview schedule

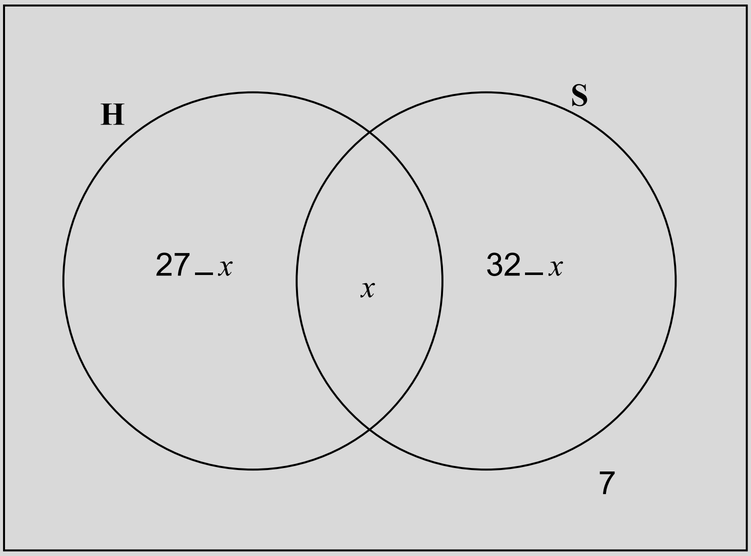
Open-ended questions pertaining to semi-structured interviews were developed after the analysis of the learners' written responses in a diagnostic test (see, Section 5.2). Incorrect test responses were targeted to explore the reasoning and thought processes learners applied when responding to test questions. During a test analysis, the researcher identified key issues and categorised them into themes. The test analysis results of each question item were coded into themes. These themes were then subsequently used to develop interview questions. For instance, the analysis of question 1.1 item was coded as a language in probability, conceptual understanding and procedural knowledge. In this view, the questions asked to reveal their thought processes when answering certain questions. Secondly, what difficulties do you experience when attempting to unpack this probability word problem?

Figure 4. 1: A sample of test items used in the current study

In a certain class of 42 boys:

- *27 play hockey (H)*
- *32 play soccer (S)*
- *7 do not play hockey or soccer*
- *An unknown number (x) play both hockey and soccer*

The information is represented in the Venn diagram below.



1.1.1 Calculate the value of x. *(2)*
1.1.2 If a boy from the class is chosen at random, calculate the probability that he:

(a) *Does not play hockey or soccer* *(1)*
(b) *Plays only soccer* *(2)*

Semi-structured interview questions were used to obtain insight into the responses of errors and misconceptions, and into the inconsistency of probability reasoning and difficulties learners experienced when solving probability problems. In addition, learners were asked to give explanations for the solutions they provided in test questions, which process was thought to be useful in providing the researcher with an understanding of the possible challenges learners experienced when solving probability items. The process assisted in developing some questions which were used to understand the learner's perceptions of probability. Given the purpose of the semi-structured interviews in Section 4.5.2, the following are some of the questions the researcher used in semi-structured interviews (Appendix J).

- *Explain the thought processes which determined your answer to this question?*
- *Why would you say you managed to get this question correct?*
- *You seem to struggle with probability rules and definitions— would you agree?*
- *Why did you use an independent event rule in these questions rather than using mutually exclusive events?*

During the process of analysing the test, the researcher raised issues in need of additional attention and which were informative of the situation under study. Each interview question was posed in a way that generated feedback addressing certain common themes. In relation to learners' observed test responses, the following questions are some of the questions asked to learners during an interview session:

- *When I went through your tests I realised that they are incorrectly answered. What factors might have posed a problem to your ability to understand this?*
- *When I went through your script, I noticed that you struggled with the interpretation of the Venn diagram? What can be a challenge in understanding Venn diagram concepts?*
- *What was your thought process when you answered these questions that way?*
- *Why did you answer this way?*
- *I realise that you struggle with probability calculations. What is your explanation on what you have written?*
- *Explain to me why you did not attempt to answer this question (why blank responses)*
- *It seems like you didn't understand the requirements of the question. Do you agree? What are your challenges to understanding this question?*
- *Tell me what you were thinking when you answered this question?*
- *Explain to me why you answered this question this way?*
- *Why did you use this formula?*
- *I see that you made some calculations. Explain to me what this might mean?*
- *When you have a problem in probability, who helps you?*

4.6.3.3 Construction of the lesson observation schedule

According to Cohen, Manion and Morrison (2018), a non-participant observer adopts a passive and non-intrusive role and uses a lesson observation schedule to collect data. The study adapted and modified Sepeng's (2010) lesson observation instrument for data collection (see, Appendix I). In this regard, the researcher had already pre-established categories from the interview analysis to observe. For instance, the categories which emanated from the interviews included (1) learning and teaching support material; (2) misconceptions and errors; and (3) conceptual knowledge and procedural knowledge (see, Section 2.5). The lesson observation schedule addressed the next issues:

- Observing classroom practices including: the instruction and learning of probability, the classroom arrangement of learners' teaching approach, resources used, and the process of dealing with the correct and incorrect responses of learners;
- Observing learners' productive skills such as defining probability terms, describing events, solving problems, applying and relating probability concepts to real-life situations, being able to independently accomplish tasks on probability concepts, and making problem-solving decisions etc.;
- Observing evocative skills, such as observing if learners can ask related question, and if they able to interpret new information on a given concept;
- Observing evaluative skills, such as observing if learners are able to: (1) evaluate their own work on the concept learnt; (2) identify the error committed; (3) use alternative ways to solve probability problems; (4) discuss the pros and cons of a particular issue by using a specific method to solve a problem; etc.; and,
- Observing reflective skills such as observing if learners can: (1) reflect on errors and misconceptions; and, (2) reflect on decision making in solving a particular problem.

The lesson observation schedule was designed in a way that provided some room for the observer to fill during their teaching and learning on the topic of probability in Grade 11. A five-point scale rating was used to score the observed events (see, Appendix I). The five-point rating scale stipulated by Sepeng (2010) was, *excellent*, *good*, *average*, *needs more attention*, and *not applicable* to the lesson. A rating scale coded excellent means five points and good represented four points and so on.

4.6.4 Addressing issues of scientific rigour for the study

According to Maryam (2016), the data collection instruments had to pass the tests of validity and reliability before they could be used in the study. This section addresses issues of scientific rigour that relate to the current study from two perspectives; namely, those of quantitative and qualitative research methods.

4.6.4.1 Quantitative data collection instruments: The diagnostic test

In this case, reliability and validity are discussed as scientific constructs inherently embedded in the research method of the quantitative segment of the study.

4.6.4.1.1 Reliability of the test

The reliability of a measuring instrument will be achieved when it consistently, and without bias, produces the same results on repeated trials. Additionally, a reliable instrument will measure the construct it is supposed to measure (Sekaran, 2003). Consistency is achieved if the same test produces similar results each time it is administered on a similar sample overtime. Cohen *et al*. (2011) stated that a reliable test administered on a similar group of participants, in a similar context, must yield similar results. Oluwatayo (2012) reiterated that reliability refers to the repeatability of the results in quantitative research. A similar diagnostic test was administered to a pilot study group and thereafter to the main study. The diagnostic test achieved test-retest reliability as it produced the same results with repeated assessments on the same individuals who possed the same characteristics.

4.6.4.1.2 Ensuring the validity of the test results

Validity is demonstrated if an instrument measures what it is purported to measure (Cohen *et al*., 2011). An instrument that can measure what it purports to measure will generate valid data. Face validity, content validity and criterion validity are the forms of validity that were addressed in the current study, and these scientific constructs were linked to the diagnostic test (Section 4.5.2.1). Face validity refers to the way in which the test was presented to the learners. The researcher deemed it necessary to judge if the assessment of learners' performance in probability through the test was worth pursuing. This judgment was achieved by piloting the test (Section 4.6.1).

In the study content, validity established that the test fairly and comprehensively covered all the domains or items that it purported to cover (Carmines & Zeller, 1997, cited in Cohen *et al*., 2011: 188). Content validity was improved by using CAPS documents for mathematics Grades 10-12 (DBE, 2011). The past examination question papers were used to set a test that covered the cognitive levels as stipulated by the Grade 10 mathematics curriculum (DBE, 2011). The content validity ensured that the test addressed the topic of probability adequately. The other type of validity important to this study is criterion validity, which endeavoured to relate the results of one particular instrument to another external criterion (Cohen *et al*., 2011: 189).

As noted earlier, the Grade 10 syllabus was administered to Grade 11 learners using past examination question papers to construct the test (Section 4.5.3.1). All questions were allocated marks according to the cognitive levels. The cognitive levels stipulated by the CAPS mathematics constitute knowledge routine, complex, and problem-solving (DBE, 2011). The test was moderated by the following expert mathematics educators using a test moderation form (see, Appendix O): (1) the Tshwane West District subject specialist (2) the head of the department (HOD) for high school mathematics from a Tshwane North District; and, (3) an experienced mathematics teacher from the researcher's school.

The educators commented on various aspects of the diagnostic test. Some of the comments concluded that: (1) the duration of the test should be an hour; (2) the use of the probability formulae sheet during test should not be permitted; (3) the repetition of questions must be watched for, and (4) the appropriateness of content knowledge covered in the test in relation to the grade level to which this test was pitched. For instance, in point (2) some of the experts raised a concern that final mathematics examinations do not provide learners with the probability formulae and that this practice would not be appropriate in the test. The educators argued that learners are expected to recall the formulae, hence it is necessary to test their knowledge of it rather than provide it as an aid. The educators also observed that the test had additional questions demanding knowledge recall. CAPS mathematics assessment guideline was used by the educators to moderate the test.

4.6.4.1.3 Achieving rigour in the qualitative component of the study

Trustworthiness is a criterion in qualitative research that ensures the rigour of the qualitative findings. The vectors/ values that may be used to ensure trustworthiness on an inquiry in qualitative research are credibility, dependability, transferability and confirmability (Guba, 1981, cited in Anney, 2014). Dependability was employed to both interviews and the observation schedule in this study to ensure that the study findings were consistent and repeatable. The credibility of the interviews was achieved to ensure consistency in this study. A pilot study was conducted before the commencement of the main study to examine the level of bias in the interview process, the interview questions and to trial run the observation process (Johnson & Christensen, 2012). The interview questions were asked to all respondents following the sequence of the interview schedule. Averagely, the times of the interviews ranged between 15 and 25 minutes and were conducted after school hours. Thus, piloting of all interview questions facilitated consistency in data elicitation across respondents (Dhlamini, 2012). The interview data was continuously checked if it addressed the research questions, thus increasing credibility (see, Section 1.5).

The interview results and the analysis were taken back to the respondents for evaluation to avoid misreporting. In this regard, a member would be given a chance to check the correctness and representativeness of the data that they supplied to ensure the credibility of the results and maximizing the conformability aspect of the study. Anney (2014) posited that member checking eliminates researcher bias during data analysis and interpretation. The audio recording device was used when conducting the interviews. Later the recording device was played out repeatedly to facilitate the process of transcribing the recorded data for analysis and interpretation. This, in turn, increased the dependability of the study's findings (Anney, 2014). Later in the study, the researcher made use of an observation schedule to observe the mathematics lessons that were conducted by teachers in participating schools. The lesson observation tool had earlier been examined by the study supervisors to ensure the dependability. Also, the researcher used the observation instrument repeatedly and with similar participants to ensure its consistency. In this regard, the pilot study created an opportunity to achieve this. The findings of the lesson observations produced similar interpretations in both schools. The outcomes of the lesson observations were also checked against the results of the semi-structured interviews, which yielded correlated findings.

Finally, the trustworthiness of the semi-structured interviews and lesson observations raise questions as to whether the tools and questions provided would generate the desired responses (Cohen *et al*., 2011). In this case, convergent validity was established by comparing the interview data with the lesson observations data. In the pilot study, the data from semi-structured interviews was comparable to that of the lesson observations. For instance, the analysis of the interviews data revealed that learners had a negative attitude towards the topic of probability while during a lesson observation it was noted that a relative number of learners tended not to do most of their classwork and homework activities.

4.7 THE PILOT STUDY

A pilot study is a small study to pre-test the research protocols, data collection instruments, sampling strategies and other research techniques in preparation for the main study (Hazzi & Maldaon, 2015). It is a small-scale trial run of all the research procedures that are anticipated to be implemented in the main study. Also, the reasons for pilot testing the data collection instruments were to check and verify the usage of certain words and terms in the data collection instruments. In addition, this process entailed the refinement of certain items in the scales and revising the research plan and the data collection process (Hazzi & Maldaon, 2015). The pilot study aims to determine the degree to which questions were clear and to identify problem areas that need attention. The pilot study was conducted in a purposively selected school. The pilot school had been observed to generate a poor end-of-year mathematics results over three-years in Grade 12. Generally, the socioeconomic conditions characterising the location of the pilot school tended to be comparable to those that defined the schools in the main study (see, Section 4.4.3). The sample in the pilot study consisted of 78 learners and one teacher.

4.7.1 Piloting the diagnostic test

In the case of a diagnostic test, the pilot testing was played out to evaluate the utility of the instrument or test. Upon administering a diagnostic test in the pilot process learners' scripts were marked using a memorandum to ensure consistency. Learners' obtained percentages in each question item were computed and categorized using Didis and Erbas' (2015) categories of written responses (see, Section 4.6.4). The pilot study assisted in identifying test items that were not clear or were deemed to be ambiguous in terms of language. For example, questions item 1.2 was phrased as, "*A bag contains 3 blue ball and X yellow balls*. *Determine the sample space*". The phrasing of the question was not clear to learners (*Determine the sample space*) and was changed to, "*Write down the total number of balls in the bag*". This question intended to determine the learners' conceptual knowledge on sample space.

In addition, piloting the test was used to check if the test would give information on the causes of challenges learners experienced in solving probability as informed in the literature review. The pilot study results revealed that the 45 minutes, which was the time allocated for writing the test was not enough. The researcher observed that learners could not finish writing the test in the stipulated time. These learners asked for more minutes to be added to the duration of the test. Most of the learners submitted their written responses after an hour had elapsed. It was therefore adjusted to the duration of an hour.

The quantitative results from the pilot study suggested that the test was reliable and valid. The test results revealed that learners had varied challenges in solving probability problems. Furthermore, the learner's written frequencies suggested that these learners experienced difficulties in solving probability problems. It was noted that the participants had misunderstandings, errors, and were inconsistent with probability reasoning. From these results, it was possible to conclude that the test was reliable and valid. The research questions had been answered, that is, *what challenges do Grade 11 learners' experience in solving* *probability problems*? Based on the results of the pilot study the researcher anticipated that playing out the research process in the main study would yield a similar and comparable outcome in each parallel point of data collection.

4.7.2 Piloting the semi-structured interview guide

The semi-structured interview schedule was piloted to identify items that could have been unclear or ambiguous (Appendix J). The interview schedule consisted mainly of open-ended questions covering pertinent matters of conceptual understanding, procedural knowledge and probability reasoning relating to the topic of probability in Grade 11 mathematics classrooms. The pilot process motivated the researcher to reviewed certain aspects of the interview questions, for instance, the usage of the language and phrases or wording in constructing the interview items. Also, the relevance of interview items in terms of the teaching and learning contexts in participating school was also taken seriously into account, for instance, the interview question had to be formulated in a manner that would be representative of teaching and learning experiences in the pilot school. In this regard, the researcher would have sufficiently studied the profile of the pilot school prior to conducting the study.

Initially, the first question purported to elicit information on challenges that learners encountered in probability. The wording was not clear and ambiguous to the participants. According to Hershkowitz (2011), better responses may be achieved when the researcher builds a good rapport with study participants. The researcher began with social interaction before the interviews. This prior social engagement helped to generate a free social atmosphere allowing interview respondents to interact with ease with the researcher. The researcher used probing questions to elicit further in-depth information on challenges experienced. Also, the piloting process revealed the interview items that would need to be probed further. Finally, the pilot study gave an idea of what would be the optimal duration of each interview session.

4.7.3 Piloting the lesson observation guide

A lesson observation guide was piloted to evaluate the relevance of the instrument as a suitable data collection tool (Appendix J).

4.8 DATA COLLECTION PROCEDURES

The process of collecting data entailed two phases. The researcher told the interview respondents that the process would be audio recorded. The rationale of audio recording the semi-structured interviews was to achieve accuracy in documenting the respondents' voices or responses, and to allow meaningful access and interaction with the interview data at a later stage to facilitate an accurate data analysis process. During an interview process the researcher used a digital voice recorder as well as a back-up recording device ensuring that all aspects of interview data were captured. All the interviews took place between 14h30-16h00 at the schools. In the end each of the interview respondent was given a copy of an interview schedule for future reference. Also, all study participants had been earlier furnished with the contact details of the research in case they would have research related enquiries in future.

4.8.1 Using the pseudonyms for data collection purposes

The pilot study school and two schools in the main study were coded A, B and C, respectively (Sections 4.5.1 & 4.5.2). The allocated codes had been crafted and arranged alphabetically to inform the process of visiting the participating schools. For instance, School C implied that the school would be visited thirdly to conduct the lesson observations and semi-structured interviews.

4.8.2 Phase 1 of data collection

The first phase of the study consisted of collecting quantitative data for which purposes a diagnostic test was administered to learners in their classrooms after school hours. Subsequent to a brief introduction and explaining the purpose of the study, the research process was initiated. Copies of informed consent were distributed to all study participants and a diagnostic test was administered.

4.8.3 Phase 2 of data collection

The second phase of data collection built upon the first phase, with the two phases being connected in the intermediate stage of the study (see, Venkatesh, *et al*., 2016; see, also, Section 5.3). The goal of the qualitative data collection phase was to further explore learners' written responses, to derive richer insights into their experiences in studying the topic of probability at the Grade 11 level. In this phase, semi-structured interviews and lesson observations were conducted after analysing data collected from the diagnostic test (Section 4.7.1). The semi-structured interviews were conducted three days after the diagnostic test had been administered to learners to avoid retention loss (Didis & Erbas, 2015). All interviews were audio-recorded, and back-up was used to avoid the loss of data.

4.9 DATA ANALYSIS PROCEDURES

Data from the diagnostic test was analysed using quantitative methods and data from semi-structured interviews and lesson observations were analysed using qualitative methods. To analyse data, the researcher adopted Didis and Erbas' (2015) coding categories to obtain the view of learner performance for each question item (see, Section 5.3). The coding categories of learners' written responses per question item take the form of *correct*, *incorrect*, *blank* or *incomplete* responses. The rationale for this process was to have an overall description of learners' solutions (Didis & Erbas, 2015). Learners' written responses were analysed to identify their incorrect mathematical procedures which the researcher assumed could be linked to their poor performance in the topic of probability in Grade 11.

4.9.1 Data analysis in the pilot study

The strategies used at this stage for data analysis resembled those employed in the main study (Section 4.8.2). The purpose of analyzing pilot study data was to verify if Grade 11 learners experienced difficulties in learning probability. The researcher anticipated that the results of the main study would resemble those obtained in the pilot study given that the two study sites were comparable.

4.9.2 Data analysis in the main study

Data analysis procedures in the main study resembled those in the pilot study (Section 4.8.2). However, the purpose of data analysis in the main study was to answer the research questions of the study (Section 1.5).

4.9.2.1 Analysis of quantitative data

The quantitative data that was collected through a diagnostic test was examined and organized into frequencies. Didis and Erbas' (2015) categories of the frequencies of learners' written responses were coded to facilitate the analysis of the test data (Section 1.14). Learners' written responses were coded into categories of *correct, incorrect, blank* and *incomplete* responses. According to Didis and Erbas (2015), incomplete responses are learners' written responses that are not fully completed. The incomplete responses included the written responses that followed the mathematically correct processes and the solution would have been partially achieved. A response would be categorised as being blank in cases in which a learner failed completely to provide a response. In the data analysis, the percentages of the learners' responses that were categorized as *correct, incorrect, blank* and *incomplete* were computed (Section 5.1.1)

Computing percentages assisted the researcher to acquire the background and understanding of the causes of learners' performance and the extent of challenges they experienced when solving probability problems in Grade 11.

4.9.2.2 Analysis of qualitative data

At this stage, data from the semi-structured interviews and lesson observations were analysed concurrently with the gathering of data, making interpretations and writing reports (Creswell, 2009).

4.9.2.2.1 Analysis of data from the semi-structured interviews

The process of data analysis started by coding the learners as L1, L2, L3, etc., with the letter "L" standing for learner and the number next to the letter corresponding to the numerical sequence in which the interview sessions took place (Appendix M). For instance, L11 referred to the learner who was eleventh in the interview list. The analysis of the data from the semi-structured interviews commenced during the data collection stage and continued after the transcription of all interviews data. The emerging themes for the interviews were organised and summarised under similar subheadings to facilitate the analysis (see, Appendix P).

4.9.2.2.2 Analysis of data from the lesson observations

The sessions of the lesson observations scheduled to take place in schools were identified as O1, O2, O3, etc., with "O" standing for observation and the numerical number representing the numerical order of the observations, in terms of sequencing school visitations. For instance, O7 referred to the lesson observation that took place in the seventh position sequentially (Appendix N). Two lesson observations visits were conducted in each class per school and during the times that had been decided prior by the researcher and the teacher. Data from the lesson observations were transcribed and classified according to similar patterns. The transcribed data was then analysed into similar themes (Section 5.3.2). Data from each lesson observation was grouped under sub-headings that related to questions asked in the test and interviews (Section 5.3.3).

4.10 ETHICAL CONSIDERATIONS

Ethics is a system of moral principles, which may be "divided into three interrelated domains: (1) ethics within the research community; (2) ethical responsibilities towards research participants; and, (3) ethics related to the value of educational research for policy-making and practice, and for the quality of education" (Tangen, 2014). Ethical principles such as informed consent, confidentiality, respecting participants and transparency were considered in this study.

The institutional Research Ethical Committee approved and issued the ethics clearance certificate endorsing the conducting of research in prospective schools (Appendix A). Upon obtaining the ethics clearance certificate, the researcher was permitted by the GDE office to conduct the research in selected schools from the Gauteng Department of Education through the Tshwane West District (Appendix B). Informed consent from the teachers was obtained before conducting research in schools (Appendix C). Upon receiving permissions from school heads to conduct research the researcher issued letters of informed consent to prospective study participants. The letters given to learners had a separate tear-off in which the recipient would indicate their participation status in the study (Appendices F & G). In addition, the informed consent letters had a section that sought the approval parents/ guardian in terms of whether they would allow their children to participate in the study or not (Appendix E). Most of the learners who had been earmarked to participate in the study were less than 18years old, the age that rendered them as minors (Appendix E).

The value of upholding the principle of anonymity is to strive to protect the identities of informants for the study (Augustyn, 2014). Learners' real names and other identifying characteristics would not be disclosed. Learners were issued with the answer scripts that did not bear their names on them. The principle of confidentiality entailed not disclosing participants' information that might identify them (Augustyn, 2014). The researcher was able to take appropriate measures to protect the study data and related records. These study related documents were kept away from public access and those that existed in soft copies formats were stored in the researcher's computer using a protected code. The data was saved using codes in a locked software system.

4.11 CONCLUSION

In this chapter, the researcher has discussed the research paradigm, mixed methods approach, instruments and strategies for gathering data, along with the research process used in this study. The population of the study and the sample procedures as well as the development and purpose of the data collection instruments used in this study such as the diagnostic test, semi-structured interviews and lesson observations were outlined. The issue of trustworthiness and rigour of the data collection instruments were also addressed (Section 4.6.4). A discussion of a pilot study and the ethical consideration, and how the permission was acquired from the university, the DBE, the school principals, and the mathematics teachers and their learners, was also outlined in this chapter. In the chapter that follows the researcher presents the analysis of data and the results of the study.

CHAPTER FIVE

DATA ANALYSIS AND DISCUSSION

5.1 INTRODUCTION

This chapter presents the analysis and the discussion of the data collected from a diagnostic test, semi-structured interviews, and lesson observations. At the time of conducting this study the perennial state of learners' poor performance in mathematics was an abiding worry in the South African education landscape. At the time of this study the topic of probability had been a newly introduced topic in the CAPS mathematics curriculum, yet it was perceived as one of the difficult branches of mathematics for secondary school learners to understand. The analysis of quantitative data addressed the following research question:

- What challenges do learners experience when solving probability problems?
- How do learners reason when solving probability problems?
- How do learners interpret probability situations?

The analysis of qualitative data addressed the following research questions:

- What are the learner experiences in solving probability problems?
- What are the causes of those experiences these learners have in probability?

To find answers to the preceding questions the analysis of learners' written responses and to conduct semi-structured interviews with some learners to discuss their written responses. Lesson observations were conducted to confirm what had emerged in the analysis of interview data. A mixed-methods sequential explanatory design was found suitable, in so far as it combined both the quantitative and qualitative processes in this study (Creswell & Plano Clark, 2011). Identifying the challenges in learning probability will help educators to develop an understanding of how learners learn probability concepts and specific concepts which clash with probability knowledge and develop teaching strategies which promote the effective construction of knowledge. It will also assist learners to understand their difficulties in learning probability. The study findings have implications for future studies designed to address these challenges in probability.

Marshall and Rossman (2014) described data analysis as the process of bringing order, structure, and meaning to the mass of collected data. The purpose of the data analysis in this study was to make sense of the challenges learners experienced in solving probability problems and interpreting observed learners' experiences. Data analysis has been subdivided into two distinct phases: the first phase is that of quantitative analysis and the second that of qualitative analysis. The first phase of the research results was presented as an analysis of the quantitative data obtained from the diagnostic tests. The second phase presented was the analysis of the qualitative data collected from interviews and observations.

5.2 ANALYSIS OF QUANTITATIVE DATA

Quantitative data analysis answered the research questions 1 and 2 (see, Sections 1.8.2.1 & 1.8.2.2). Quantitative analysis strategies were used to analyse the data collected from the diagnostic test using descriptive methods. Descriptive statistic measures calculated from the test score are the mean, median, mode and range of the data. Frequencies on the data are organised using Didis and Erbas' (2015) categories of written responses. Didis and Erbas' (2015) data analysis model was employed to analyse the performance of learners in each question item. Learners' written responses for each question item were coded as *correct*, *incorrect*, *blank* or *incomplete* responses (see, Table 5.). Incorrect responses were those that were computationally incorrect, including all responses that followed wrong definitions, formula, procedures and were mathematically incorrect. Incomplete responses were those probability calculations and representations that were not totally complete. Non-attempted items were treated as blank responses. Thereafter, the percentage of coding was calculated. The rationale was to obtain a descriptive view of learners' understanding of probability concepts per question item.

5.2.1 Descriptive analysis of data from the pilot study

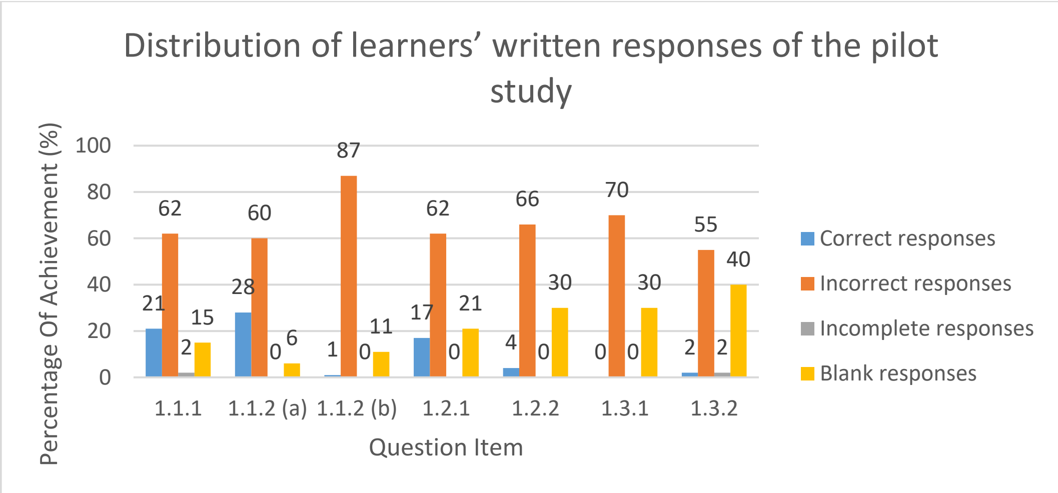
The participants in the pilot study were drawn from the population of the study (see, Section 4.7). Mainly, the pilot study was conducted to trial run the data collection instruments before they could be used in the main study (Hazzi & Maldaon, 2015). Forty-seven out of 78 Grade 11 learners wrote the diagnostic test in the pilot study. The scripts were numbered L1 to L47 to maintain the anonymity of learners. The. researcher marked the written responses using a memorandum and calculated percentage scores for each question item. The percentages calculated were recorded and organised into a frequency table categorised as *correct*, *incorrect*, *incomplete* and *blank* responses (see, Tables 5.1 & 5.2). The descriptive statistics were calculated using the excel program. Measures of the central tendency were computed to summarise the data for the test variable. The measures of dispersion were computed to understand the variability of the scores for the test variable.

The mean was 11%, which was calculated from 47 learners, the mode was 11%, the median and the standard deviations were 14% and 7%, respectively. The mean value shows that most learners performed poorly in the test. The differences in the values of the mean and the median revealed that the test score was symmetrical. The percentage scores of the test takers ranged from 0% to 63% in which latter score (63%) was considered to be high. The standard deviation of the data concerning the test questions is 7%, showing that there was a spread of data around the mean of 11%. The results showed that learners performed poorly in the test, due to the limited context available to learners to interpret the concept. The results also support the study of Batanero et al. (2016) and Ang & Shahrill (2014) that understanding probability was not easy. The poor performance suggested that the participants experienced challenges in the probability concepts. The results warranted proceeded with the main study.

Question 1 consisted of 3 sub-items where learners were asked to interpret the Venn diagram, calculate the probabilities and define mutually exclusive events and use the rule to calculate the probabilities of mutually exclusive events. The items in question 1 are about 1.1.1 (Venn diagram), 1.1.2, 1.2.1 and 1.2.2, which involved the calculating of probabilities. The sub-questions 1.3.1 and 13.2 addressed the concepts of mutually-exclusive events. Table 5.1 presents the responses to these items using Didis and Erbas' (2015) categories of written response analysis. Table 5.1 shows that the pilot study group that performed poorly in solving the Venn diagram, probability calculations, sample space, probability definition, mutually exclusive events, and complementary rule. The highest percentage was 13% and the lowest was found to be 0% in question 1 items.

Table 5.1 shows that 21% of the learners in the pilot study group responded correctly to the question item of the Venn diagram. These learners computed correctly the elements that belong to an intersection of playing both hockey and soccer. As Corter and Zahner (2007) suggested that learners should be able to use a Venn diagram to represent information in a way that facilitates proportional logic and probability problem-solving skills. The Venn diagram facilitated visual representations, which helped to augment thinking and reasoning skills.

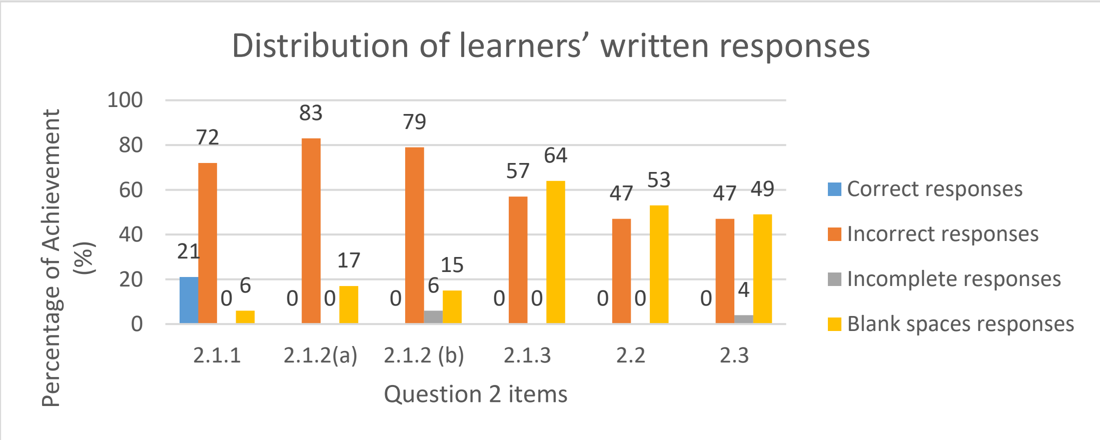
Table 5. 1: Distribution of learners' written responses in the pilot study



However, 62% of the learners in the pilot study group gave incorrect responses. These learners could not interpret the Venn diagram information correctly. The Venn diagram illustrated two events that can occur jointly: an outcome in the circle H can also be an outcome belonging to the circle S. The intersection area marked x represented outcomes that were at the occurrence of both H *and* S. Of these learners who gave incorrect responses, 2% of them had incomplete responses and 15% gave blank responses. It appears that this group of learners lacked understanding of the Venn diagram representations concept. It suggests that learners lacked conceptual knowledge on the relationships between sets; that is, union and intersections. Such include conceptual knowledge that the region inside the curve represents elements that belong to both sets, whereas the outside region represents the elements that are excluded from sets. Table 5.2 provides further information on the pilot study group performance status in question 2, with the highest percentage score being 21% and the lowest 0%.

Table 5.2 demonstrates that a majority of learners performed poorly in question 2 items, indicating that they poorly understood the concepts of probability. This question was designed to identify challenges of the learners' comprehension of the concepts of Venn diagram representations, the aspects of probability calculations, the background relating to union and intersection probability, and mutually exclusive and inclusive events. The low percentage of correct responses indicated some evidence for the use of probability principles and appropriate conceptual and procedural knowledge on probability principles.

Table 5. 2: Distribution of learners' written responses in the pilot study to questions 2 items



A high percentage (72%) of incorrect responses suggests a lack of understanding relating to Venn diagram representations. Based on Table 5.2, it is evident that respondents could not demonstrate conceptual knowledge. Features relating to deficiency in learners′ interpretation of probability concepts included the misunderstanding of probability formulae, rules and the use of irrelevant information (see, Table 5.7). In this question, incomplete responses relating to the percentages recorded 0% of learners performed a mathematically correct procedure. It suggested that this group of learners did not attempt to demonstrate calculation and/ or break down questions into smaller parts to facilitate probability problem-solving skills. It is evident that 6% of learners who left blank spaces were lacking an understanding of the probability concepts and were deficient in their probability reasoning. These learners experienced various challenges in solving probability problems.

5.3 DATA ANALYSIS IN THE MAIN STUDY

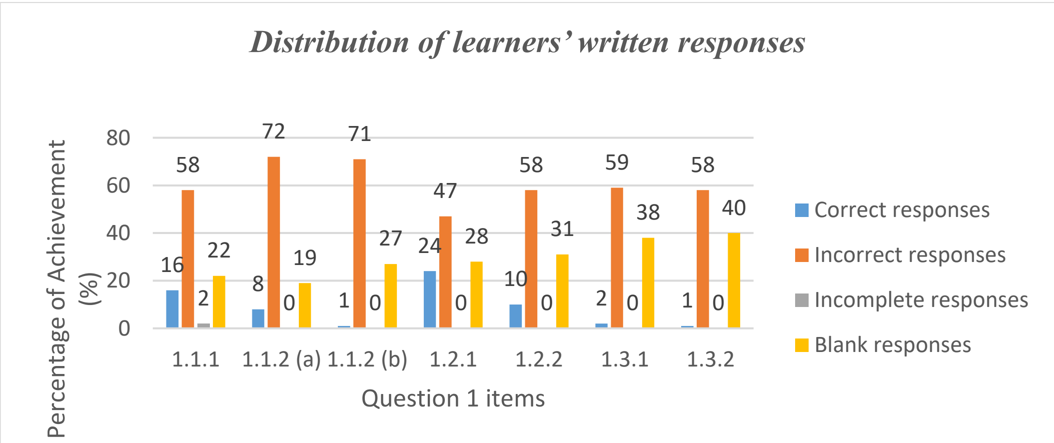
Of the two schools, 302 learners participated in the main study by writing a test. Learners' written responses were analysed item by item. The researcher marked the written responses using a memorandum and calculated the percentage scores of each question item. A frequency table depicting percentages of the scores was used to investigate the levels of the correctness of the solutions (Section 5.1.1). The highest achievement among the learners was *56%*, while the lowest achievement was *0%*. The mean percentage achievement was *12%, median = 11%, mode = 11%* and *standard deviation = 10%*. The differences between the mean and median were calculated depicting the data to be positively skewed.

The percentage scores in the test ranged from 0% to 56%, which is very high. The standard deviation of the data concerning the test questions is 10%, which shows that there is a wide spread of data around the mean of 12%. This indicates that the learners performed similarly, and that the mode value of the test percentage scores of 11% was achieved. The implication is that most learners performed below the mean value of 12%. The mean percentage indicates an average performance level, which is far below the mean of 50%, as stipulated by DBE (2018) assessments guideline. The results revealed that learners performed poorly in the test, suggesting multiple challenges in solving probability problems.

5.3.1 Learner responses to question 1 item

Table 5.3 shows the general performance status of learners in question 1 items.

Table 5. 3: Distribution of learners' written responses in the main study to questions 1 items



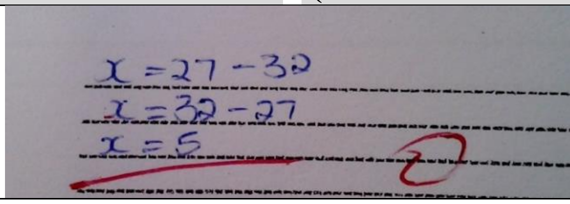
In question 1.1, learners were expected to analyse the Venn diagram and solve for the unknown value. The Venn diagram concept shows two events drawn from the real-world context that included two sample spaces. In these questions, learners were asked to calculate the value of x

A routine procedure was required to create a linear equation and solve for the unknown. Of the 380 learners who participated by writing the test, 16% of the learners applied the probability principles correctly. Participants calculated the number of learners who played both sporting codes correctly, in which context the appropriate quantification information is evident but was incorrectly and incompletely used. Table 5.3 shows that few respondents were able to unpack the terminology of probability that was embedded in the test question. Such terminology included the words '*only*', '*both*' and '*or*' (Appendix H).

A higher percentage (58%) of the learners gave an incorrect response when interpreting probability situations. Paul and Hlanganipai (2014) stated that probability terminology may contribute to a deficiency in conceptual and procedural understanding promoting poor performance. Furthermore, the authors argued that the language of probability affects their understanding of concepts, because the language of probability in schools may be different from that of everyday usage. The percentage scores of 2% and 22% of the learners were achieved by learners whose responses were incorrect and blank responses respectively.

This suggests that most learners would be unable interpret the information in the Venn diagram. For instance, L2 did not include a '7' as the number of elements outside the two circles. It suggests a lack of computational and procedural skills in computing and solving probability word problems. These low percentage scores achieved suggests that learners had various challenges in understanding the terminology used in Venn diagrams. Such included, inconsistency with probability reasoning, a lack of strategic competence, and probability definitions. The results agreed with Hasan *et al.* (1999) and Sarwadi *et al*. (2014) that learners often fail to apply the procedural knowledge todifferent situation.

Table 5. 4: Extract from written responses given by learners to question item 1.1

Question 1.1.1	**Calculate the value of x (Learners' solutions)**
Extract of incorrect responses by L1	

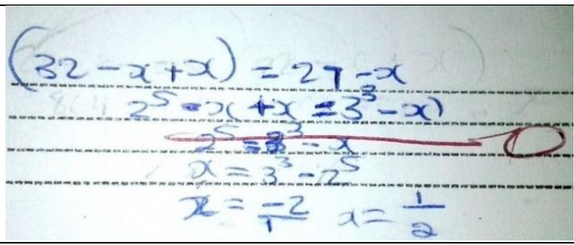
Extract of incorrect response L2	

Table 5.4 shows learners' extracts of the written responses of the question 1 item. Instead of computing $27 - x + x + 32 - 4 + 7 = 42$ and solving for x, L1 and L2 did not consider the total number of learners in the class. In this regard, it suggests a lack of conceptual knowledge of sample space. Based on this extract, there is evidence of random guessing, and an absence of procedural knowledge on solving for the unknown variable. Ellis and Shintani (2014) asserted that learners often struggle with the mathematical concepts associated with probability vocabulary. Such vocabulary includes the use of probability vocabulary requiring additional and/or multiplication rules. It further suggests that L1 and L2 did not know what to do when they encountered the words '*and*', '*both*' and '*or*' in probability word problems, suggesting an absence of conceptual knowledge. Therefore, learners experienced challenges in completing the prerequisite concepts relating to their probability word problems.

In question 1.2, the demands of the questions were subdivided into a routine procedure in, (a) complex procedure, and in (b) where learners were expected to compute the probabilities of single and compound events. Learners were expected to use the complementary rule when calculating the probability of learners not playing hockey or soccer, for example, $P(not\ A) = 1 - P(A)$. The data in Table 5.4 indicates that 92% of the learners answered the question incorrectly. This is a concern, as learners were expected to calculate the probabilities using the given information. It was expected that learners would use their prior knowledge to execute probability calculations. The following is an extract of incorrect and incomplete responses given by L3 and L4 respectively.

Table 5. 5: Extract from written responses by learners to question 1.1.2

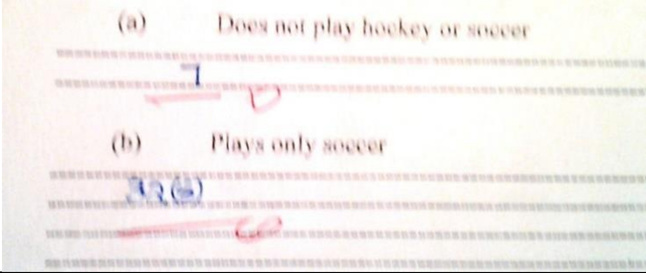
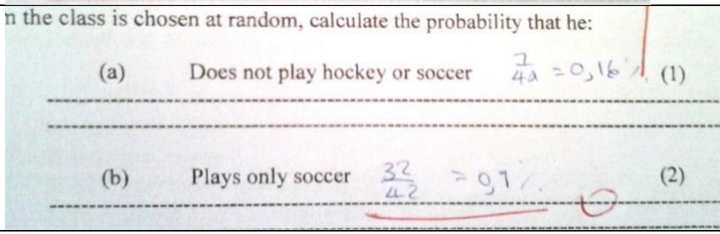
Extract of incorrect responses by L3	
Extract of incorrect and incomplete responses by L4	

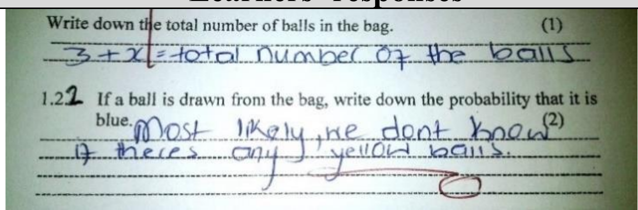
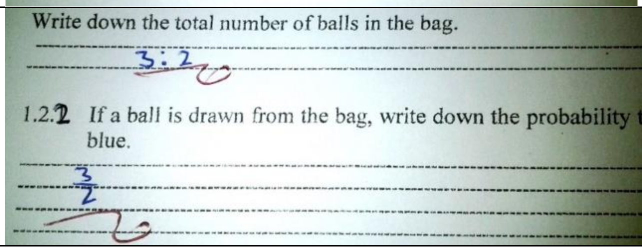
Table 5.5 shows that L3 plugged in numbers in contexts in which calculations of probability were needed. The written responses of L3 showed a lack of conceptual knowledge relating to the probability definitions, probability scale and an absence of knowledge on fractions. The extract demonstrates learners' misunderstanding of the probability vocabulary. L3's solutions lie outside probability boundaries, thus being inconsistent with probability reasoning. Although L4 demonstrated the understanding of knowledge on the numerator and denominator in probability calculations, the learner partially answered this question item correctly. The extract reveals that L4 understands probability definitions as expected. The second part of the questions was incorrectly answered.

The written responses suggest that L4 lacks understanding of the probability terminology and was inconsistent with probability reasoning. It suggests that such misunderstanding results from challenges the learner had with probability. In question 1.2, learners were expected to demonstrate their knowledge in relation to probability definitions in the first part. The questions were designed to identify the challenges of learners' comprehension of probability calculations and their use of probability definitions, sample space and outcomes. The question focused on learners' comprehension of probability vocabulary in interpreting the results involving probability calculations and sample space. Data in Table 5.3 shows that most of the learners performed poorly in this question with 24% managing to answer the question correctly. This suggests that learners experienced challenges in their comprehension of sample space, outcomes and probability calculations. High percentages of 47% and 50% in items 1.2.1 and 1.2.2, respectively, reveal that most learners responded incorrectly. It is noted that 28% and

31% of the participants gave blank responses in items 1.2.1 and 1.2.2, respectively. These observations suggest a lack of comprehension of probability concepts. It also suggests a negative attitude towards learning about probability as indicated by a high percentage of blank responses. Table 5.6 shows answers given by L5 and L6.

When considering L6's response, it is noted that the respondent understood the sample space concept in question item 1.2.1. It seems learners understood the principles of algebra. However, in 1.2.2 it is noted that there was an absence of understanding probability rules and definitions. The responses given by L134 were all incorrect. These written responses suggest that the respondent lacked proportional reasoning, which is relevant to developing learners' understanding and application of fractions. Boyer, Levine and Huttenlocher (2008) argued that learners' difficulties with proportional reasoning have had an ongoing negative influence on learners' ability to reason mathematically. Furthermore, Tso (2012) attributes learners' difficulties with sample space concepts to their inadequate knowledge of handling fractions. The responses of these learners reveal an improper fraction as a solution to this question.

Table 5. 6: Extracts from written responses given by L5 and L6 in questions 1.2.1 and 1.2.2

Question	Learners' responses
Extracts of incorrect incomplete responses by L62	 Write down the total number of balls in the bag. (1) 3+x = total number of the balls 1.2.2 If a ball is drawn from the bag, write down the probability that it is blue. (2) Most likely, we dont know if theres any yellow balls.
Extract of incorrect responses by L134	 Write down the total number of balls in the bag. 3:2 1.2.2 If a ball is drawn from the bag, write down the probability blue. $\frac{3}{2}$

Question 1.3 intended to gauge if learners could recall and use a definition of mutually-exclusive events. The concept of mutually exclusive events pertains to logic and the theory of probability. Events are mutually exclusive when the occurrence of one of the events rules out the possibility of the occurrence of the other events of concern (Zwiers & Kelly, 1986). Learners were required to comprehend the concept of a complimentary event and that the

probability of an event may be found by means of checking for its complimentary event. Item 1.3.2 required learners to use the definition of mutually exclusive events to calculate the probabilities. Respondents were expected to demonstrate comprehension of probability rules when solving the question. The cognitive ability being measured was at the level of knowledge, and the straight recall of mutually exclusive events. Learners needed to build on this question, such that a complex procedure is used to calculate the probabilities using a defined formula, which can include $(P\ (not\ A)) = 1 - P(A)$.

The analysis of this question showed that learners demonstrated challenges in solving probability concepts on mutually exclusive events. This is evident from the low percentages of 2% and 1% of the participants, who correctly answered the questions 1.3.1 and 1.3.2 respectively. Table 5.2.1.4 shows learners' written responses. The table shows that 38% and 40% of the learners gave blank responses to items 1.3.1 and 1.3.2 respectively, this was worrying indeed. Most learners opted not to respond or answer the question on mutually exclusive events and the complementary rule. It suggests that a majority of learners did not have conceptual knowledge of the concepts of mutually exclusive events and the use of the complementary rule. The extracts of learners who demonstrated poor comprehension of concepts relating to probability is demonstrated in Table 5.7.

Table 5. 7: Extracts from written responses given by L5 and L6 in questions 1.3.1 and 1.3.1

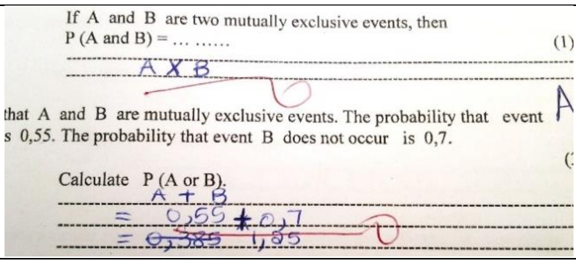
Extract of incorrect responses by L16	 If A and B are two mutually exclusive events, then P (A and B) = … …… (1) A X B that A and B are mutually exclusive events. The probability that event A s 0,55. The probability that event B does not occur is 0,7. Calculate P (A or B). A + B = 0,55 + 0,7 = ~~0,385~~ 1,25

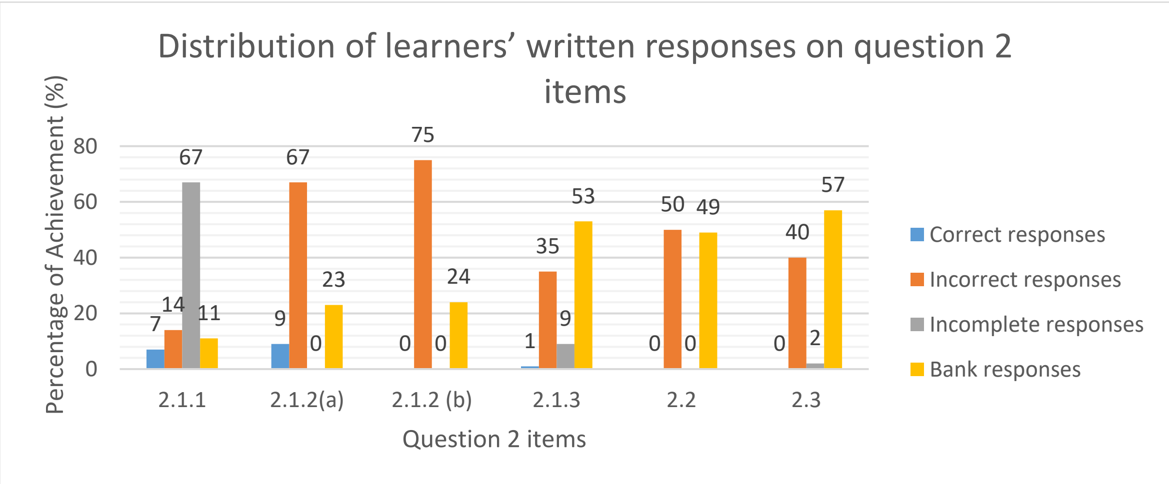
Table 5.7 shows extracts from learners' written responses to question 1.3.1. There is evidence of the use of irrelevant information in the solution given. L16 gave a definition of mutually-exclusive events by multiplying two events. The extract shows that the learner had to add together the given data to calculate the probability of compound events. Noddings, Gilbert-MacMillan and Lutz (1980) stated that learners respond to probability questions by falling into a number-crunching mode, plugging quantities into a computational formula or procedure

without an internal representation of the problem. This suggests that L16 was guessing at random in disregarding the presented information. The results agreed with the assertion of Chareka (2017) that learners hold misconception or naïve theories in their thinking and understanding in mathematics.

5.3.2 Learner responses to question 2 items

In this question, the incidents were associated to real-world context. In this instance learners had been requested to draw a Venn diagram showing the relationship between two events. Learners were asked to execute probability related computations associated with the Venn diagram. This item demanded the attainment of a certain cognitive level relating to complex procedures. Learners needed to demonstrate problem-solving skills to break down the questions into parts. Item 2.1.2 required learners to perform routine procedures to solve the problem. Learners were also supposed to demonstrate a deep understanding of probability formulas. Furthermore, question 2.1.3 involved the understanding of mutually-exclusive events, hence it relied on the knowledge level. Lastly, the item required the respondents to justify the answer obtained employing probability reasoning.

Table 5. 8: Distribution of learners' written responses on question 2 items

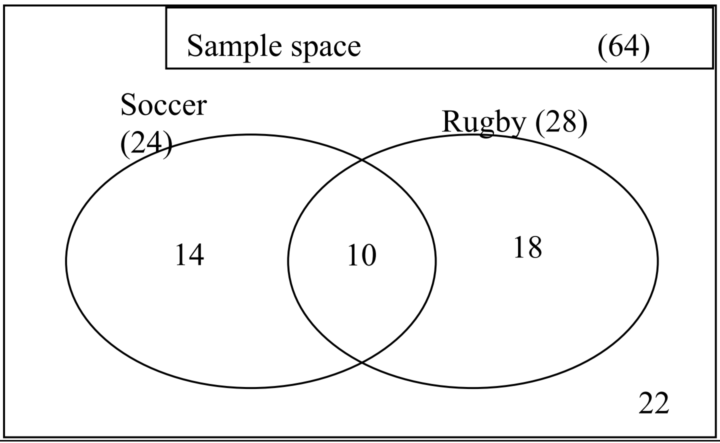


Overall, the quantitative analysis of question 2 items in Table 5.8 shows that most learners were unable to provide answers to the questions that related to completing the Venn diagram, mutually exclusive events and probability rules and definitions. The table shows that 7% of learners got item 2.1.1 correct. It reveals that the Venn diagram completion was a challenge to many learners. The percentage score of 67% of the learners provided incomplete responses and 14% gave incorrect responses on item 2.1.1. This suggests that most learners were lacking probability knowledge, reasoning and thinking skills. It is evident from the extract of the responses of L30, L142 and L161 that most learners experienced various challenges when solving probability problems. In some instances, learners interchangeably used mutually-exclusive and independent identities to determine the probability of there being mutually inclusive events. It is interesting to note that question items 2.1.2 to 2.3 were more poorly performed by most learners than the other questions in this study (see, Table 5.8).

The preceding extracts show of L142 and L30 showed the incomplete set up of their Venn diagrams and procedures. It suggests that the L142 did not know the correct procedure to determine the probabilities of two events. Furthermore, the table depicts that 11%, 23% and 24% of the learners gave blank responses to question items 2.1.1, 2.1.1 and 2.1.2, respectively. Learners were supposed to work from the centre on the two circles outwards towards a possible solution. These preceding extracts of learners L30, L61 and L142 show that learners did not understand the probability language. It is evident that these learners had minimal or limited comprehension of the term intersection and lacked skills on what to do when circles overlap each other. Furthermore, it suggests a lack of probability language coupled with an inconsistency regarding probability reasoning led the respondents to perform poorly.

Table 5. 9: Extracts of responses of learners who performed poorly in questions 2.1

Question
2.1 At a certain school there are 64 boys in Grade 10. Their sport preferences are indicated below:
• 24 boys play soccer
• 28 boys play rugby
• 10 boys play both soccer and rugby
• 22 boys do not play soccer or rugby
2.1.1 Represent the information above in a Venn diagram. (5) **solution**



2.1.2 Calculate the probability that a Grade 10 boy at the school, selected at random, plays:

(a) Soccer and rugby (1)

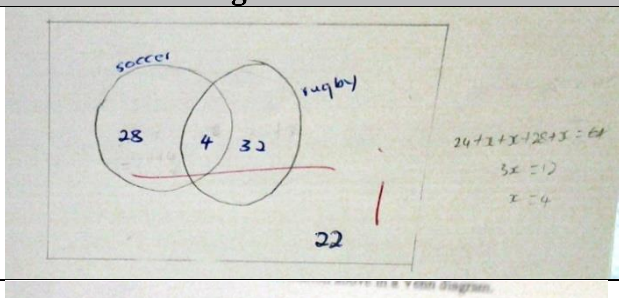
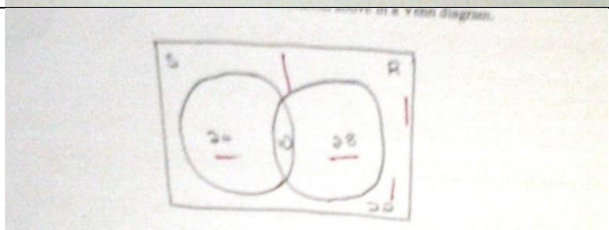
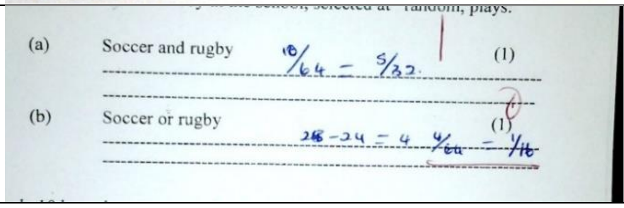
P(Soccer and Rugby) = $\frac{10}{64} = \frac{5}{32}$ = 0,15625 =15,63%

(b) Soccer or rugby (1)

P(Soccer or Rugby) = $\frac{14+10+18}{64} = \frac{42}{64} = \frac{21}{32}$ = 0,65625 = 65,63%

OR

P(Soccer or Rugby) =1- $\frac{22}{64} = \frac{21}{32}$

responses to question on Venn diagram	
Extracts of incomplete responses by L30 on 2.1.1	
Extracts of incomplete responses by L142 on 2.1.1	
Extracts of incorrect and incomplete responses by L61 on 2.1.2	

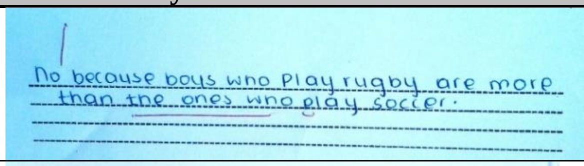
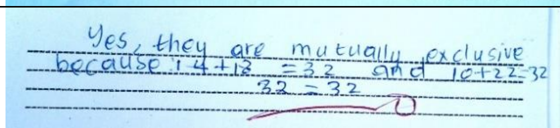
5.3.3 Learners’ written responses to question 2.1.3 item

Item 2.1.3 is a question to which participants frequently performed poorly. It is observed that 1% of the respondents managed to give a correct answer to the concept of mutually exclusive events. Of the 35% of the learners who gave incorrect responses, 9% gave incomplete responses and 53% gave blank responses related to mutually exclusive events.

The preceding extract shows that L2 and L67 were unable to give the correct definition of mutually exclusive events. It suggested that incorrect responses given by L67 coupled with L62 were triggered by insufficient comprehension of the probability identities and rules. Although L2 partially answered the question correctly, the learner was not able to justify why the two events are mutually exclusive.

Incorrect reasoning seemed to indicate that there was a shortage of adequate conceptual understanding of the notion of mutually exclusive. It was observed that some learners used a probability multiplication rule to solve a mutually exclusive events problem.

Table 5. 10: Responses of poor performing learners to question 2.1.3

Question	
2.1.3 Are the events a Grade 10 boy plays soccer at the school and a Grade 10 boy plays rugby at the school, mutually exclusive? Justify your answer. (2) solution No Some boys play both soccer and rugby . **OR** No P(S and R) $\neq$ 0	
. **responses to question on mutually exclusive**	
Extracts of incomplete responses by L2 on 2.1.3	 No because boys who Play rugby are more than the ones who play soccer.
Extracts of incorrect responses by L67 on 2.1.3	 Yes, they are mutually exclusive because 14+18 = 32 and 10+22 = 32 32 = 32 0

5.3.4 Learners' written responses to question 2.2 item

Question item 2.2 calls for an understanding of an experimental approach to determining the probabilities. The questions demanded probability reasoning skills, computational skills and procedural skills in solving the probability problem. The participants were expected to understand the probability language before they determined the probabilities of the events. In this question, learners were expected to understand the words ‘excluding ‘and ‘more than’ to calculate the probabilities. It also calls for learners' general mathematics skills to solve the problem. Out of 302 learners, 50% gave incorrect responses and 49% of the learners gave blank responses on a concept related to probability frequencies. The data in Table 5.8 shows that all test respondents gave incorrect responses to the question related to finding the probability of events. Table 5.11 gives a summative information of learners’ responses.

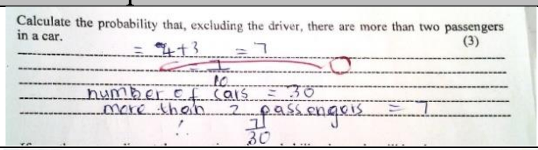
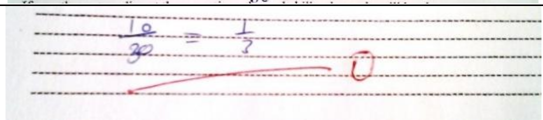
The extracts in Table 5.11 of the L2 coupled with L67’s responses reveal that, (1) learners lacked prior knowledge on a probability calculation, and, (2) learners were inconsistent with probability reasoning to solve probabilistic questions. It appears that the respondents were not able to define precisely the random experiment and the probability of an event in the concept of probability. Learners could not demonstrate understanding of the mathematical meaning relating to the phrase ‘*more than*’. Learners were also confronted with language difficulties in this question item.

Table 5. 11: Learners’ written responses to question 2.2

<table>
<tr><th>Question</th></tr>
<tr><td>One morning Samuel conducted a survey in his residential area to establish how many passengers, excluding the driver, travel in a car. The results are shown in the next table:</td></tr>
<tr><td>
<table>
<tr><td>Number of passengers, excluding the driver</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Number of cars</td><td>7</td><td>11</td><td>6</td><td>5</td><td>1</td></tr>
</table>
</td></tr>
<tr><td>Calculate the probability that, excluding the driver, there are more than two passengers in a car. (3)

Solution
P(more than 2 passengers per car)</td></tr>
</table>

$$= \frac{5+1}{7+11+6+5+1} = \frac{6}{30} = \frac{1}{5} = 0,2 = 20\%$$

Learners' written responses	
Extracts of incorrect responses by L67 on 2.2	
Extracts of incorrect responses by L2 on 2.2	

Hay (2014) asserted that learners often struggle with the mathematical concepts associated with the language used in probability. The written response of L67 shows number plugging and guessing. It implies a lack of willingness in learning the topic of probability (see, Section 5.3). Table 5.12 shows the extracts of learners' responses to item 2.3.

Table 5. 12: Learners' responses to question 2.3

Question
If you throw two dice at the same time, the probability that a six will be shown on one of the dice is $\frac{10}{36}$ and the probability that a six will be shown on both the dice is $\frac{1}{36}$. What is the probability that a six will NOT show on either of the dice when you throw two dice at the same time? (3)

The results in Table 5.12 show that the participants obtained 0.0% for correct responses and 57% blank responses. It suggests that L2, coupled with L22's written responses were unable to determine the probability of two events happening at the same time. These learners demonstrated a weak understanding of independent events and the procedures on how to calculate their probabilities. Written solutions show that the learners did not know when two probabilities should be added or multiplied, in a given situation. Participants did not manage to use the probability rule of a complementary rule to calculate the probability of not a six $P\ (NOT\ A) = 1 - P(A)$. The higher percentage of blank responses may suggest that the abstract and formal nature of probability often cause learners not to be interested to learn the concepts of probability (Borovcnik, 2012). This is evident in this study where blank responses have higher percentages (see, Table 5.3).

Learners were observed to be careless and were providing incomplete solutions when solving probability related tasks in the test. Additionally, these responses suggest a weak understanding of computational methods and procedural skills required for successful solving of probability problems. The next segment of data analysis provides a comprehensive discussion on the semi-structured interviews and lesson observations. Table 5.12.1 shows some of the themes that emerged from the analysis of the learners' work of the diagnostics test.

Table 5. 12. 1: Summary of the themes that emanated from diagnostic results analysis

Themes	Sub-themes and indicators
Conceptual understanding	fail to define terms, solve problems, identify outcomes and graphical representations Interpretation and understanding of the context Misunderstanding of the probability laws and violation of probability laws Misconceptions and errors
Procedural understanding	Lack of computational skills leading to computational errors Problem solving skills
Probability reasoning	Lack of intuition Failed to interpret numerical answers Manipulation of algebraic situations (e.g. 1.2.1 $solution\ is\ 3 + x$
Problem solving	Ineffective problem solving strategies/skills (listing outcomes, completing the Venn diagram, stating the equation) Lack of knowledge on fractions (determining the sample space, meaning of a denominator in a sample space)
Cognitive abilities	Learners' cognitive level was low. Questions of higher order level were poorly answered if not left blank Lack of cognitive skills

5.4 QUALITATIVE DATA ANALYSIS AND DISCUSSION

Analysis of qualitative data answered research questions 3 and 4 (see, Sections 1.8.2.3 & 1.8.2.4). Cohen *et al*. (2011) asserted that qualitative data analyses involve organisation, accounting for and explaining the data. Qualitative data from semi-structured interviews and lesson observations were collected to obtain more background and understanding of the diagnostic test results. Through semi-structured interviews with the learners, the researcher purported to investigate the ways in which learners perceived, expressed and answered the probability problems. The qualitative presentation and data analysis methods used in this study employed patterns and themes to construct a framework of communicating what the data revealed. The interview guide with a list of questions was prepared and all questions followed an open format. Open-ended questions enabled the learners to fully explain, in their own words, their perceptions and experiences encountered in the learning of probability. The lesson observation guide was used to gather live data.

5.4.1 Semi-structured interviews

Some aspects of data collection activities in the interviews entailed a process of note-taking and using audio recording devices. Verbatim transcription of the interviews was done, and an excerpt is included in the qualitative analysis. In some instances, the interviews were translated into English in the case where a participant responded in a vernacular (local) language. The interview questions addressed the themes that arose from the analysis of the diagnostic test (see, Appendix J).

5.4.1.1 Analysis of the semi-structured interviews

The analysis of the semi-structured interviews followed a content analysis process.

5.4.1.1.1 Content analysis process

The content analysis was used to analyze the interviews (see, Appendix P). The content analysis assisted in sorting and summarizing the informational content of the data by item and by common characteristics within the data (McIntosh & Morse, 2015). The researcher transcribed and sorted the participants' responses to the questions. During this process, important words and phrases were highlighted noting commonalities in each response. Thereafter, the commonalities were sorted in categories and coded. Coding is simply attaching a name and or a label to the text that contains an idea (Cohen *et al*., 2011: 559). The meaning in the context was extracted using open coding, language, description of pattern and trends in data. Emerging themes were also analysed, and the meaning was drawn from these themes. In the process of analysing and discussing the qualitative data, research objectives, study aims, and research questions were given preferences (Sections 1.6, 1.7 & 18). Learners were asked the following questions written in italics:

Challenges in probability: *What do you think can be a challenge for the poor performance in probability and why?*

Learner support in probability: *What kind of support do you receive from your teachers and able peers?*

Representativeness misconception: *What kind of difficulties do you experience when dealing with the Venn diagrams representations?*

Computational methods: *What methods do you find easy to use when solving probability problems?*

Procedural knowledge: *Which procedures did you employ in solving a word problem and why?*

Conceptual knowledge: *What probability concepts did you find difficult to solve?*

Probability concepts: *Which probability concepts do you find challenging to grasp and why?*

Curriculum issues: *What do you think about the time allocated to probability for learning?*

A few excerpts of the interviews used as examples are presented in the following section, where the researcher is **R** and the learner (respondent) is **L**.

Researcher (R): *What was your reasoning when you answered the question on the interpretation of the Venn diagram?*

> **Learner 1 response (L1):** I think I don't understand the basic concepts in Venn diagram interpretations. I know how to draw the circles but I fail to calculate the elements in each circle and intersections.

R: *What formula did you use to answer the questions on mutually exclusive events and why?*

L49: I actually took the data from the three circles and add them all together. I am not sure how I got the answer, it's probably that I don't understand the Venn diagram concepts. I don’t remember the formula for calculating mutually exclusive events. My Grade 10 teacher did not teach us the topic of probability.

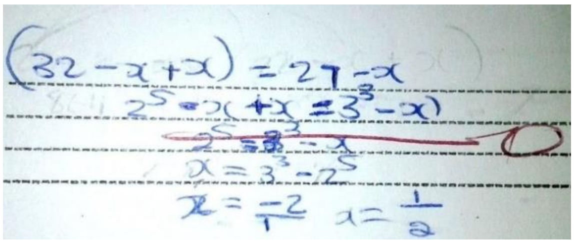
5.4.1.1.1.1 Analysis of item 1.1

R: *When I analysed your script on item 1.1, I realised you struggle to get the correct solution. Can you please explain to me your thought process in solving probability word problems?*

L1: I don’t understand the population of words and terminology used in defining sets. I mean that I don't understand the probability terminology. My teacher told me that when we see these probability terms we should use addition or and multiplications rules to get to the solution. Hence, I multiplied here (pointing on the script). I am interested in mathematical calculations, not probability because it is difficult for me.

L2: This question was difficult for me to answer. Since I didn't understand the interpretation of the Venn diagram, I had to guess by taking a 7 from the data (pointing on answer sheet where it read ‘7 do not play hockey…’). I cannot remember the formula for calculating probability events.3

Figure 5. 1: Extract from written responses given by learners to question item 1.1



R: *this is what you wrote as a solution towards Venn diagram question (*preceding extract*), I see $(32 - x + x) = 27 - x$. You further wrote 2^5 and 3^3as part of your solution right? How were you solving this linear equation? The fourth line you again transpose exponents and look at the bases that they are not the same, explain your calculations here. Your solution of $x - 2 = \frac{1}{2}$, how did you reach this stage?*

L1: Sir, I was looking for the value of x. I expanded the bracket and collected like terms so that I can divide and get the value of x. When we did exponents you said that we should express such numbers to prime factors, hence I did that and subtracted $3 - 5$ which gave me -2. I reached this answer (*learner pointing at a fraction*), it gave me a fraction because it is a negative exponent.

The preceding excerpts show that L1 had challenges with computational methods relevant to solving probability problems. It could be noted from the excerpts that these learners were still trying to understand the given problem information, hence suggesting questionable prior knowledge of probability. Also, learners were still trying to classify the problem when they tried to compute the actual numerical answer. L2 could not understand the probability terminology used in this question and therefore could not interpret the information given on the Venn diagram. Paul and Hlanganipai (2014) stated that learners often face difficulties when dealing with sample space. It is evident in this excerpt that learners had a weak understanding of sets and set notation when interpreting the Venn diagram. A study by Wright et al., (2019) had also shown that learners difficulties in Venn diagram were attributed to intuitive based misconceptions.

R: *I see that you wrote a number instead of a fraction in this question. Do you remember the formula for calculating probability and or definition of probability*?

L3: I cannot remember the formula well. I last did probability last year in Grade 10, but I did not understand it. When I used my calculator I got this answer (*pointing on the written responses where the fraction was outside probability boundaries*).

This excerpt of L3 indicates that the learner did not grasp the concepts of probability definition, probability boundaries and related terms. L3 could not recall any knowledge on how to calculate the probabilities. It shows inconsistency with probability reasoning and thinking skills. Grade 10 probability (see, Annual Assessment Plan for Grades 10-12 in Appendix Q) provides learners with basic skills and reasoning for Grade 11 and12 probability concepts.

R*: When I went through your script I noticed that you managed to answer this item 1.2.1 correct. Can you tell me how you managed to get this question correct?*

L62: I read carefully and understood the question requirements. It was easy to answer anywhere. I mastered most of the formulae in my previous grade.

R*: Now this question was answered incorrectly. What were your difficulties did you experience when you answered this question and why?*

L62: I see that mathematically I can add 3 balls and unknown ones since I don't know the value of. I have challenges in understanding the requirements of the questions because of words used. And I don't have sound skills to deal with sets and intersections, for example, I have a problem in understanding the sample space. In this question, we are not given the total number of balls here (*pointing on the questions*).

The preceding excerpt shows that L62 did not calculate the probability of the questions. It can be noted that learners struggle to identify the sample space in this question, which did not define itself with totals. The incorrect response can be linked to the misconception of the distinction between compound and simple events. The results were consistency with Cragg's *et al*. (2017) assertion that factual knowledge has a strong effect on comprehension and mathematical achievement.

5.4.1.1.1.2 Analysis of Items 1.3.1 and 1.3.2

R: *I see that you struggle to answer this question demanding knowledge you learnt in Grade 10. What difficulties did you experience in solving the probability problems and why*?

L16: I don't know what I was writing. I think I remembered that the word ′and′ means that I should multiply the data given and the word ′or′ means that I should add the data given. These words sometimes confuse me, and I don't understand the terminology used in this question. But Sir, probability language is difficult to understand.

In a discussion with L16 relating to their solutions to questions 1.3.1 and 1.3.2, it came out that L16 lacked essential aspects of prior knowledge relating to the definition of probability and rules. It is evident that L16 did not understand knowledge on Grade 10 probability, which is supposed to form the basis of prior knowledge to Grade 11 probability concepts. The responses further revealed that learners lacked problem-solving skills. According to DBE (2017), problem-solving is at the heart of mathematics. Sepeng and Madzorera (2014) stated that knowledge of vocabulary influences proficiency in word problem-solving. Probability vocabulary, problem-solving skills are some of the challenges identified in this conversation.

5.4.1.1.1.3 Analysis of Item 2.1.1 and 2.1.2

Learners had to draw a Venn diagram using information that was given to them. The questions required learners to employ knowledge of sets. Also, problem-solving skills was one of the requirements for a learner to solve a complex procedure. The question demanded the computational methods and procedural methods as skills needed to be employed towards a possible solution. Computational methods required were the use of known formulae to solve a problem. Thus, the formula-based problem solving was employed when the learner wrote down an explicit formula, then substituted quantities and solve the problems.

Furthermore, it was considered graphical if the participant used an external visual device to solve the problem such as the Venn diagram. It was possible for the learners to carry out calculations without reference to any general formula. Learners were expected to calculate the probabilities without indicating any rationale for that procedure used. This practice may make it easier for the learners to check for needed information, to break down the problem solution into subparts, and/or to make visual associations to relevant formulas.

R: *I see that you struggled with answering this question. Can you explain to me what your thought process was when you answered this question*?

> **L30:** the data given on the question confused me. I don't understand how to complete the circles when given such information. This data is too many for these circles (pointing on the written response). I had to guess about where to put the numbers. I did not use formulas because I could not remember the correct formula to use.

The views of L30 were shared by L142 who added (see next conversation):

> **L142:** We didn't do probability thoroughly last year (*meaning in Grade 10*). Our teacher explained the definition to us towards writing the final examination. I think we did not give the topic of probability time to understand the concepts.

The preceding conversations indicate that teaching and learning of probability were not given enough teaching time in lower grades. It suggests that these learners possessed varying misconceptions such include, representativeness misconception and equiprobability misconceptions in this study. It is evident that L30 lacked conceptual knowledge on sets and sample space, intersections and probability terms and rules. The topic of probability is given two weeks of teaching and learning time in the FET phase (see, Appendix L).

5.4.1.1.1.4 Analysis of Item 2.2 and 2.3

R: *When I went through your written solutions on these two questions, I realise that you did some calculations, can you explain to me what you did and why?*

> **L61:** I did not understand the data given on the table. I had to plug in number in order to finish the test.

R: *What are your difficulties in solving probability problems and why do you experience these problems?*

> **L61:** I don't like the topic of probability. I want to work with mathematical calculations such as algebra. Methods of solving probabilities are difficulty. The words used also make the question tough for me. I don't have a textbook to practice with at home, maybe I was going to do better.

The views of L 61 were shared by L30 who added (see the next comment).

> **L30:** Sir, I don't like probability learning because it's full of words and terminology which is difficult to understand. The textbooks we use do not explain thoroughly on the definitions and how to calculate probabilities. I find some probability concepts difficult to learn. Venn diagrams and mutually exclusive events are giving me hard times to understand. I don't know what to do when I encounter questions involving addition and multiplication rules.

R: *what do you think about two weeks teaching and learning time allocated to the topic of probability?*

> L30: The time allocated to learn the concept of probability is too little for us to grasp the content. Maybe if they can say at least three weeks it might give me time to learn, not learning in a hurry like this. The workload on probability is too much, yet I am expected to understand the concepts within the two weeks allocated time.

It seems from the preceding conversations that learners lacked literacy in probability, reasoning skills and problems solving skills. Furthermore, interviews demonstrate the misconception of equiprobability. The interviewees thought that the event of drawing more than two passengers was equally likely to other events. The discussion further reveals the attitude towards teaching and learning of probability emerged from these discussions. This is evident in L61's interview response on merely pugging numbers to get the solution and leaving blank spaces. This interview response shows a carelessness in so far as the interviewee got the solution incorrect in the test but managed to answer them correctly in the interview. Furthermore, some of the responses highlighted that the time allocated to the topic of probability was too little for the learners to learners towards mastery of the topic.

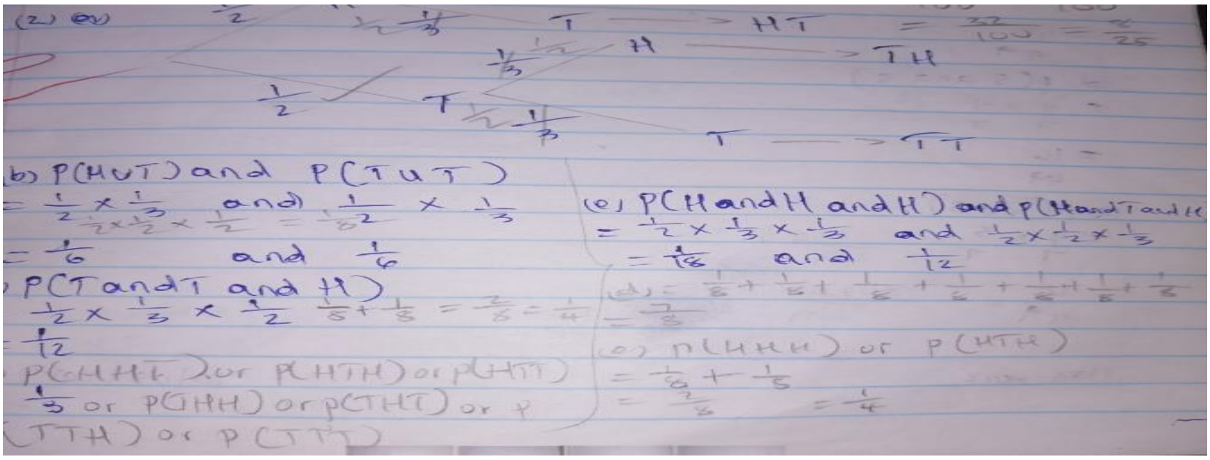
5.4.2 LESSON OBSERVATIONS

The rationale of the observation was to confirm what learners had said in the interviews and what emerged from the test analysis. The data collected from the test and semi-structured interviews were not enough to understand the challenges learners faced. Sepeng’s (2010) lesson observation schedule was used to collect data (see, Appendix I). The lesson observations were in a period of two weeks to School A and School B following the schedule that had been mutually crafted by the researcher and the teachers. This was in accordance with the Annual Teaching Plan (ATP) that stipulated that probability should be allocated two weeks of teaching and learning time (see, Appendix L). In school B, all six classes and their two teachers were observed by the researcher during the two weeks’ period. Similarly, in School C, all four classes and their two teachers were also observed (see, Section 4.5.3). Although teachers were not the focus of this study, the researcher observed how the teachers taught probability concepts to their learners.

The researcher observed the questioning techniques implemented to help learners to develop desirable conceptual and procedural understanding of probability knowledge. Also, the researcher observed how able learners assisted the less able learners to develop knowledge from one level to the other. Vygotsky's (1979, 1978) theory of socio-constructivist and its facets that include the zone of proximal development and scaffolding were used to analyse the descriptions of the learners and their teachers. It was important to observe the participating teachers to explore if some of the challenges faced by learners were contributed by their teachers. The researcher also observed learners in both schools while learning the topic of probability. according to Vygotsky (1978), an individual's mental functioning is derived from participating in social life. During the process of observation, the researcher uses an observation guide to note the learners' participation. Such included, how they ask questions and how they interact with able peers. The following observations were made about productive skills, evocative skills, evaluative skills, and reflective skills:

5.4.2.1 Learner observations on productive skills

Table 5.13.1: observation from the learner's workbook



- Learners were able to read and write notes on the concept taught, however, these learners appeared not to be capable of solving problems provided in the exercises (see, Table 5.13.1). In some cases, the learning strategies used could not facilitate learner knowledge development. The results agreed with Daroczy *et al*. (2015) and Tompkins

(2013) that conceptual understanding is influenced by language/vocabulary encountered in mathematics learning of word problems;

- Learners could not use their knowledge of and experience in the concept in formulating their own responses. Learners found it difficult to accomplish work given on the concept independently;
- Learners were observed not to be able to define and describe learnt terms encountered when dealing with the concept of probability, such include *likely, random and events*. Also, it was observed that these learners were not able to follow steps in solving exercises based on the concept taught. Language of probability was observed to be an obstacle in understanding the concept taught; and,
- The connections between probability and real-life situations were emphasised in the exercises, however, learners were incapable of dealing with problems in real life and abstract context using the concepts. Additionally, learners' ways of making decisions in problem-solving were not enhanced. Learners answers were inconsistence with probability reasoning. The results agreed with Batanero *et al.* (2018) that learners who lack relevant conceptual knowledge often achieve incorrect solutions in the process of solving a problem.

5.4.2.2 Learner observations on evocative skills

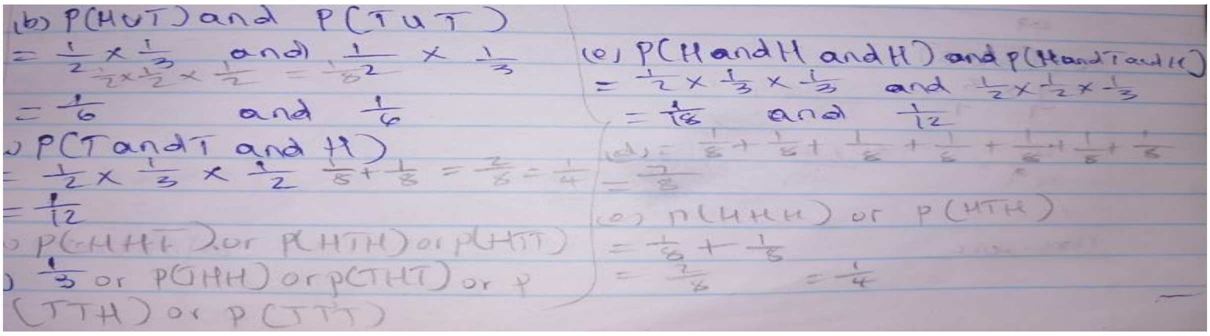
Learners involvement and participation was observed and categorized as evocative skills. Learners were observed to participate and involve in the learning process, however, most of the learners from both schools that participated remained silent and appeared to look and listen. A small percentage of about 5% of the learners were involved in disruptive behaviour often were way out of the probability content. Given that the teachers used questions and answer method in teaching the concepts of probability, the researcher observed that:

- Learners could not ask questions for clarification or to consolidate their understanding of the concepts. Some of the challenges observed were that learners were puzzled by certain areas of the concepts; and,
- Learners were not able to interpret new probability concepts. Some of the challenges observed emanated from the teaching strategies used by a teacher that did not provide learner opportunities to develop probability knowledge. The researcher observed that

the teacher offered no intervention to assist learners who were silent during the learning process.

5.4.2.3 Learner observations on evaluative skills

Table 5.13. 2: Observation from learners' workbook



Learners were observed not capable of evaluating their own work on the concept. For example, some of the learners' solutions constituted an improper fraction. A study by Aliustaoğlu et al., (2018) also showed that learners often have difficulties in handling fraction due to lack of algorithmic skills.   Some of the solutions observed were prone to errors and misconceptions and learners could not identify these errors and misconceptions when dealing with the concepts (see, Table 5.13.2). It was so disappointing to observe that learners could not identify the correct ways of solving probability problems. For example, learners could not use a correct formula for mutually exclusive events and independent events. These formulae were observed to be used haphazardly and used interchangeably. In most cases, learners lacked conceptual and procedural knowledge of probability concepts. This is reinforced by the opinion of Hayati and Setyaningrum (2019) which stated that learners enter a classroom with errors in their cognitive structure and continue to the understanding that has been understood without looking at the new knowledge.

5.4.2.4 Learner observations on reflective skills

The researcher observed that learners were constantly engulfed in a world of errors and misconceptions. There were varied common errors and misconceptions observed in the process about learning of probability, namely, representativeness, equiprobability bias and belief and human control. In most cases, the learner could not justify with reasons why they had used a certain procedure to answer questions. In other words, learners were not capable of reflecting on the decision they had made in solving certain probability problems. Based on the lesson observations it was found that teaching strategies mostly employed question and answer methods. The researcher observed that most of the teaching and learning was dominated by the teachers. Learners were passive recipients of knowledge and information and were not given the opportunity to interact. The data obtained from the lesson observations revealed that the learners appeared to not have been given help by the teachers to unpack the probability terms and language.

The researcher observed that most of the learning strategies used did not facilitate the construction of probability knowledge. According to Vygotsky' (1978), socio-constructivism learning theory, learners tend to understand the concepts taught when they are actively involved in teaching and learning. Furthermore, Vygotsky (1978) stated that interaction with able peers facilitates understanding of concepts taught. Based on the lesson observations, it was found that learners were not provided with the opportunities to discuss in pairs and/ or in groups to facilitate coaching by able peers. Teachers were observed not following the Zone of Proximal Development in teaching probability concepts. Vygotsky (1978) claimed that ZPD confirms what the learners know and whether they can resolve problems beyond their actual development level if they are provided with guidance by someone more capable/ able peers. The researcher further observed that learners were not assisted on those tasks which were too difficult for an individual to master. For example, where the learners lacked knowledge of sets and set notation, the probability lesson was concluded by asking the learner to do their homework and research further. The results disagreed with Abdullah and Vimalanandan (2017) and Yang *et al.* (2017) that learners' tacit knowledge is developed when they learn in groups.

As Vygotsky's (1978) socio-constructive learning theory states that learners construct new knowledge using their current knowledge. It was expected of the teachers to tap learners' prior knowledge before embarking on the Grade 11 concepts. It was an unfortunate situation where some learners were not able to connect between Grade 10 probability knowledge and Grade 11 concepts. In this study, the researcher also observed that Grade 11 learners had inadequate prior knowledge of probability concepts. The results agreed with Matsuda *et al.* (2013); Mogboh *et al.* (2019) that prior knowledge of learners and their teachers hinder learning through a negative transfer of knowledge. It was noted that learners could not give a definition of mutually exclusive and inclusive events. Teachers and their learners were observed not to recap on previously learnt knowledge to facilitate understanding of new knowledge. Learners were observed to not participate in the learning processes. During the probability lesson, Grade 11 learners were very quiet, while some did not appear to be paying attention to the lesson. Further to the point, learners were not motivated to learn probability concepts.

Vygotsky (1978) claims that during knowledge construction, learners negotiate, ask questions and try hard to find the answers themselves. It was surprising that Grade 11 learners did not do their homework. It suggests, again, that they were not motivated to learn the topic of probability. Furthermore, the researcher observed that learners gave blank responses to most of the classwork activities. The Grade 11 learners did not ask questions in the probability classroom. Based on this lesson observation, it was found that there is a lack of sense of ownership in learners for their work and commitment to their learning. Interviews revealed that teachers used textbooks and smart boards in teaching probability. However, there was no usage of smart boards as a teaching aid in teaching probability. Two teachers complained about the incompatibility of using smart boards to teach probability here. Additionally, during lesson observation, it was found that learners were sharing mathematics textbooks and calculators. This suggests inadequate learning and teaching supporting resources. Learners were observed to experience the following challenges during probability lessons: learners were not able to solve probability problems, use or define the probability definitions and rules and lacked probability knowledge, reasoning skills and thinking skills.

Additionally, it was observed that learners had inadequate problem-solving skills and mathematical skills to deal with probability problems. Probability reasoning is a mode of reasoning that refers to judgments and decision making under uncertainty and is relevant to real-life (Batanero, *et al.*, 2016). During the lesson on probability, learners were given classwork from a textbook. The teacher demonstrated the lesson objectives by giving out worked examples. It was observed that learners could not make decisions and judgments on the outcome of the probabilities. The researcher observed that some of the written responses with solutions greater than the probability scale. Probability reasoning is thinking a learner uses that allow for the exploration and evaluation of different possible outcomes in situations of uncertainty. Borovcnik (2011) described probabilistic thinking as being a structure of thinking that is characterised by scenarios that allow one to explore reality.

Probability thinking is activated when learners are presented with real-world situations where one uses analogies that have similar characteristics features to the situation (Batanero, *et al*., 2016). It was observed that learners experience difficulties to transfer learnt concepts in one context to the other context (Cobb, 2007). For example, learners could not transfer thinking learnt when dealing with sample space to a question on whether forecast posed by their teacher. Nevertheless, some examples demonstrated by the teachers were presented with incorrect solutions. Often, teaching approaches in probability classrooms degenerate to a list of formulas and routine applications. Borovcnik (2011) observed that a mathematical approach has resulted in many learners unable to gain access to probabilistic ideas. The results of the lesson observation confirmed the results of the diagnostic test and a semi-structured interview.

5.5 CHAPTER SUMMARY

This chapter has drawn on the data collected from a diagnostic test, semi-structured interviews and lesson observations to study the performance of learners and their challenges in solving probability problems. The data was analysed vigorously in relation to the research objectives and the research questions. The analysis and discussion were conducted in two phases: the quantitative phase and qualitative phase. A diagnostic test data analysis and discussion were done first using statistical methods. Namely, data were categorised in frequencies. The learners' responses were categorised as correct, incorrect, incomplete and blank responses to give an overview of learners understanding of probability concepts. The analysis in Section 5.1 and Section 5.2 shows that learners performed poorly in solving probability problems suggesting challenges in learning the topic of probability.

Section 5.3.1 data from learners’ semi-structured interviews were analysed and discussed. Twenty learners from two schools were sampled for interviews. The analysis of learners’ interviews shows that they experienced varied challenges in solving probability problems. Lesson observation was guided by an observational schedule. The lesson observations were made about learners’ productive skills, evocative skills, evaluative skills, and reflective skills. The analysis in Section 5.3.2 on lesson observations shows that learners experienced challenges in processes of learning probability. Also, it was observed that teachers’ practices contributed to obstacles faced by learners in comprehending the concepts of probability. The quantitative results found in this study confirmed that learners performed poorly and experienced difficulties in solving probability problems.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

The aim of the study was to explore learners' experiences and challenges, if any, when learning the topic of probability in Grade 11 (Section 1.6). This study was motivated by the observed perpetuation of learners' poor performance in the topic of probability in Grade 11 mathematics. Zooming into probability was a way to understand the experiences of learners in learning probability, which at the time the study was conducted was a relatively new topic in the Grade 11 mathematics curriculum. This study sought to identify the causes of difficulties in learning these probability concepts at the Grade 11 level. A summary of these research results is outlined first in this chapter.

6.2 SUMMARY OF THE RESULTS

A diagnostic test, semi-structured interviews and lesson observation instruments were to study Grade 11 learners during lessons on probability in three selected schools. A sample of 380 Grade 11 learners coming from township schools were studied (see, Section 4.5.2). Research in mathematics education has shown that learners from township schools perform poorly in mathematics in general (see, Dhlamini, 2012; see, also, Section 1.2). The results from the study preceding this study has also shown that Grade 12 mathematics learners are under-performing relative to the curriculum (Makwakwa, & Mogari, 2011). This assertion was confirmed by the diagnostic test results. Plainly, the participants performed poorly. The highest achievement among the learners was *56%*, while the lowest achievement was *0%*. The mean percentage achievement was *12%, median = 11%, mode = 11%* and *standard deviation = 10* (see, Section 5.3). The study found that Grade 11 learners could not solve Grade 10 probability problems, suggesting varying challenges. This finding is aligned with the study conducted by Groth, Butler and Nelson (2016), which found that the learners tend to struggle to understand the terminology used in probability. Similarly, the Anggraini and Kusrini (2018) found that learners experience challenges in answering questions related to proportional reasoning because they lack knowledge of numerical fractions.

The results of the semi-structured interviews and lesson observations conducted to selected learners show that learners experienced challenges in understanding probability language, lacked conceptual and procedural knowledge and their reasoning was inconsistent with probability reasoning (see, Section 2.10). The findings of this study further revealed that the causes of the challenges included teacher's content knowledge and pedagogical knowledge, the nature of probability, mathematical skills possessed by individuals and problem-solving skills. The results of this study confirm the results of other studies (Prodromou, 2013; Batanero *et al*., 2018, 2016; Chernoff & Sriraman, 2014; Lee, Park & Kim, 2016; Gürbüz & Birgin, 2012; Anggrani & Kusrini 2018; Sisman & Aksu, 2016; Khuzwayo 2005; Ang & Sharill 2014; Hay, 2014; Sarwadi & Shahrill, 2014).

6.3 OVERVIEW OF THE STUDY

This study was motivated by what was observed to be a trend in Grade 12 mathematics. Since the inception of CAPS in 2012 learners performed poorly in Grade 12 mathematics. The review of literature related to this study suggested a link between probability teaching, learning and achievement (Section 2.3). Chapter 2 discussed issues relating to, (a) the notion of probability, (b) the difficulties of probability, (c) probability intuition and their characteristics, (d) misconceptions and errors in probability, (e) learners' understanding of conceptual and procedural knowledge, and (f) teachers' pedagogical content knowledge. The teaching and learning of probability were used to develop the theoretical framework of the study (Section 2.6). The discussions in Chapter 4 included the research methodology, which involved the sequential explanatory mixed-methods approach (Section 4.4). The instruments used to collect data were discussed and thereafter data analysis and discussion were discussed (see, Sections 4.6 & 4.9).

The data collection for the study was conducted in two phases: the quantitative phase and the qualitative phase. Quantitative phase constituted a collection of numerical data using a diagnostic test. A diagnostic test consisted of examination type questions on the topic of probability. In the second phase, a qualitative data collection was conducted to learners using semi-structured interviews and lesson observations. Quantitative data were analysed using the Didis and Erbas' (2014) categories of learners' responses. Descriptive statistics techniques were employed to analyse the numerical data. Qualitative data analysis followed a content analysis process. The final chapter reflects on the rationale for and the research design of the study. The limitation of this study and the implication of the framework used in this study are outlined. Finally, the recommendation for further research is suggested.

6.4 THE MAIN FINDINGS OF THE STUDY AND CONCLUSIONS

Analysis of data collected through a diagnostic test, semi-structured interviews and lesson observation reveals learners' difficulties when solving probability problems in Grade 11. The questions aimed to highlight learners' experiences and their understanding of the concept of probability and related terms, definitions, theories, background, union and intersection, mutually exclusive events, and dependent and independent events. The results of the diagnostic test revealed that mathematics learners performed poorly in solving probability problems (Section 5.2 & 5.3). The researcher observed that learners had challenges in comprehending the concepts of probability. During the learning process, it was determined in this study that learners attempted to use their own intuitive views of probability and informal procedural strategies (Section 5.4.2). The findings indicate that learners experienced challenges in dealing with probability in various ways.

Similar results have been observed in many studies (e.g., Batanero *et al.*, 2016, Wilensky, 1995). These studies also revealed, among other things, that learners' difficulties with the ability to apply meaning to sample space, probability definitions and rules and the Venn diagrams are apparent. Furthermore, this study revealed learners' challenges that stem from psychological reason, that is, the interpretation and representation of the Venn diagram and misinterpretation of probability language and terminology (Section 5.3). In this study, some of the difficulties that contribute to learners' poor performance included inadequate prior knowledge, probability terminology, representativeness of probability, learner readiness to learn probability concepts, inadequate teaching practice and learning resources, the difference between simple and compound events, sample space and randomness (Section 5.3.1.1.1.1).

The findings gathered from interviews indicated that probability was not given enough teaching and learning time in the syllabus (Section 5.3.1.1.1.1). It was found that learners were not taught some of the concepts of probability in previous grade levels including Grade 11 (Section 5.3.1.1.1.3). Some learners revealed that they did not like this topic and lacked prior knowledge of the concepts of probability (Section 5.3). Moreover, it was found that teachers had no awareness of or knowledge of methods for teaching probability, learners' difficulties in probability and/ or the possible reasons for their difficulties (Section 5.3.1.1.1.4). The interview results also indicated that learners' readiness for probability and their base of knowledge is an important issue to consider when teaching towards understanding (Section 5.3).

The lesson observations revealed that teachers were not familiar with selecting appropriate teaching and learning methods to facilitate productive mathematics lessons. Teachers were not effective when teaching probability concepts to learners (Section 5.4.2). The findings showed that teachers frequently used direct teaching as the main method for imparting these concepts, thus their methods impacted on the learners’ capacity for conceptual understanding (Section 5.4.2.2). Although these teachers sometimes gave emphasis to learners’ centre approach with question and answer methods, teachers were active during the lesson while learners were passive recipients of the information (Section 5.4.2.2). In this view, this teaching and learning strategy was inadequate to present the concepts to learners effectively. Learners who have been exposed to this method turn to rely on memorising the formulae and display minimal grasp and mastery of underlying concepts (Cragg *et al.*, 2017).

The results in this study have shown that some teachers did not teach probability in Grade 10, and this suggests inadequate content knowledge (Section 5.4.2.3). According to Paul and Hlanganipai (2014), some teachers have not encountered probability in their formal learning and training. The findings reveal that the lack of probability content contributes towards a lack of conceptual understanding on the part of learners (Section 5.4.2.4). This situation was considered as one of the reasons for the causes of challenges in learning probability. Based on the lesson observations, it shows that classroom culture contributes to a lack of conceptual understanding and knowledge (Section 5.4.2.4). The lesson observations revealed that socio-constructivism teaching and learning theory was not used in probability lessons (Section 5.4.2.4). These findings suggested a conflict between learners’ knowledge that is gained in real world interactions and the formal knowledge of probability that is learnt at school (Section 5.4.2.4).

Shaughnessy (1992) noted that some intuitions can be useful when learning formal probability. The classroom culture should allow discussion of learners’ experiences so that learners have an opportunity to discuss these conflicts and to assess the validity of the claims of different class members. The study shows that the inability to handle rational numbers and ratios result in learners’ poor performance in the topic of probability. In this study, it was observed that learners lacked proportional reasoning and probability thinking skills, which attribute to learners' failure to comprehend the concepts of probability (Section 5.3). According to Shaughnessy (1992), the learners' weaker rational number concept contributes to the failure to work effectively with ratios, common fractions, decimals and percentages to express probabilities. Also, the results agreed with Aliustaoğlu *et al.* (2018) that that learner had misconceptions in terms of parts-whole relation fractions, representation of fractions on a number line and employing algorithms when dealing with fractions.

The language and terminology used in probability present numerous learning challenges. This study revealed that learners have problems distinguishing between every day and probability terminology and language (Section 5.3.2). In this study, learners misinterpreted the terms used in probability (Section 5.3.4). For example, the word '*likely*' was associated with '*possible*' (Section 5.3.1.1.1.1 & 5.4.2). A learner would misinterpret the likely event as one that could occur. Furthermore, this study pointed out that probability language barriers contribute toward inappropriate use of probability definitions and terminology (Section 5.3.4). It was found that misunderstandings of probability language and terms resulted in weak conceptual understandings and conceptual knowledge on part of learners (Section 5.3.2.1).

Attitude towards the topic of probability has been shown as a challenge to learners comprehending the concept of probability. According to Papaieronymou (2009), a negative attitude can be a barrier to the learning of probability. Based on the interview and lesson observation results in this study, learners developed a dislike of the topic of probability (Section 5.3.1.1.1.4). The finding also shows that learners developed limiting constructions as a direct result of how they have already been taught probability (Section *5.4.2.1*). The study showed that learners make decisions based on their past experiences which are counterintuitive and easily misunderstood (Section 5.3.1.1.1.1). Batanero *et al*. (2016) attribute learners' failure to comprehend the concept of probability to their informal knowledge, which conflicts with formal probability meaning. These intuitions are helpful to the teachers as they form the informal ideas that learners bring in probability learning (Batanero *et al*., 2016). The findings also showed that poor understanding may be due to a lack of experience with the mathematical laws of probability (Section 5.3.1.1.1.2).

According to the diagnostic test results, the conceptual understanding is a challenge to learners and prohibits them from meaningfully learning and understanding the probability concepts (Section 5.3). Learners lacked probability reasoning and thinking skills. Garfield and Ahlgren

(1988) attribute learners’ challenges in meaningfully learning the topic of probability to lack of probability reasoning in solving probability problems. The findings reveal that learners often make errors and apply the wrong formula when given keywords. For example, errors occur when learners are required to engage with the additive and multiplicative rules. In this regard, it was observed that most learner participants gave solutions and views which were inconsistent with probability reasoning.

6.5 THE RESEARCHER’S VOICE FRAMED BY THE THEORETICAL FRAMEWORK OF THE STUDY

The results of this study provided insight and understanding of the teaching and learning of probability. The analysis of the quantitative data suggested that learners need to be taught set notation and representations before they are introduced to probability definitions and rules (Garfield & Ahlgren, 1988). These findings showed the researcher that sound prior knowledge is key in a learner’s understanding of the new concepts (Section 5.3; 5.3.1.1.1.1 & 5.3.1.1.1.2). More importantly, the concepts of probability should be gradually introduced before difficulty concepts are introduced. The results suggest that teachers should use socio-constructivist teaching and learning strategies towards an effective understanding of probability knowledge. As noted in the previous chapters, learners should be actively involved in teaching and learning rather than passive as observed (Section *5.4.2.4*).

The researcher observed that learners come to probability classrooms with informal and formal probability knowledge. These learners held perceptions about probability explain why learners experience difficulties in probability. These results are in agreement with the study of Ang and Shahrill (2014), which found that learners have appropriate quantitative information of probability concepts, but it may be incomplete or incorrectly used. In this study, the researcher noticed that when learners are given time to actively construct their knowledge through able peer assistance they tend to comprehend the concepts of probability (Section *5.4.2.4*). For instance, peer coaching may help learners to do away with misconceptions about probability and improve in the use of probability language. It may also assist teachers to consider prior knowledge about probability. Most of the teachers did not use the ZPD and scaffolding strategies to enhance understanding, otherwise performance would be improved if effectively employed.

During teaching and learning, the language of probability should be unpacked and agreed upon by both teachers and their learners. I observed that learners and their teachers do not thoroughly explain the terminology used in probability, hence they experience difficulties in dealing with the concepts. The reasons for the misunderstandings of these concepts were observed to be due to a lack of teacher probability content knowledge and pedagogical content knowledge (Section 5.4.2.4). Otherwise, the researcher noted that these types of knowledge and skills may produce better comprehension of probability concepts which yields better performance results.

The major findings of this study that have implications for teaching and learning of probability are herein summarized. These are, (1) learners come to class with strongly held misconceptions and errors in probability (Section 2.6.1), (2) learners have theories and intuitions that are inconsistent with probability (Section 2.3.1), (3) inadequate conceptual and procedural knowledge in probability, (Section 2.8), and, (4) lack of problem-solving skills (Section 2.18).

6.6 LIMITATIONS OF THE STUDY

The following limitations emanating from this study have been identified and acknowledged:

- The sample of this study was too small, focusing on only three schools. The findings of this study cannot be generalised.
- This study was confined to Grade 11 learners only.
- This study zoomed onto the topic of probability rather than focusing on broader mathematics to improve mathematics results.

6.7 RECOMMENDATIONS

Findings of this study imply the need for great caution in teaching and learning of probability in secondary schools. In this study, the researcher suggests that the learning process should pay attention to the pedagogical, psychological and epistemological nature of probability (Section 2.6). Also, the researcher recommends an investigation into the instruction methods to facilitate conceptual and procedural understanding of probability concepts. Teachers should be supported and gain greater exposure in the terminology that is used in the topic of probability in Grade 11. As matters stand now, the challenges encountered by Grade 11 mathematics learners when solving probability problems require more time from both the learner and the teacher than is allocated.

6.8 CONCLUSION

This study explored the experiences of Grade 11 mathematics learners and their challenges in solving probability problems. This study was conducted in three secondary schools in Tshwane West District in Gauteng. A sequential explanatory mixed-method design enabled the mixing of both quantitative and qualitative methods in a single study. A diagnostic test, semi-structured interviews and lesson observations were used to collect and analyse data. The data generated from this study revealed that learners perform poorly in solving probability problems, suggesting varying challenges in learning the topic. Upon analysing the diagnostic test, it was observed that learners' challenges were largely associated with linguistic challenges, insufficient comprehension of texts and lack of conceptual and procedural understandings of probability concepts. It further revealed varying experiences learners had in comprehending probability concepts, and such included the lack of prior knowledge and negative attitude to the topic. The finding contributes to the improvement of probability results which influence mathematics performance in general.

The results of this study have provided insights and understanding into the difficulties, conceptions and challenges learners experience with probability, particularly with participating schools. From this research study, the authors were able to identify learners with misconceptions on probability and to familiarize the observer with learners' challenges when learning probability. Initiatives should be made to address learners' problems when learning the topic of probability. This study aimed at exploring the Grade 11 learners' performance and their challenges in solving probability problems. This study was motivated by the perpetuation of poor learner performances in mathematics. Zooming into probability was a way to explore the performance status of Grade 11 learners and their challenges in solving probability problems.

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APPENDIX A: Ethics Clearance Certificate from Unisa

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2018/04/18

Ref: 2018/04/18/32152302/24/MC
Name: Mr T Zhou
Student: 32152302

Dear Mr Zhou

Decision: Ethics Approval from 2018/04/18 to 2021/04/18

Researcher(s): Name: Mr T Zhou
E-mail address: teetine2002@gmail.com
Telephone: +27 84 737 0498

Supervisor(s): Name: Dr S Makgakga
E-mail address: makgasw@unisa.ac.za
Telephone: +27 12 429 4293

Title of research:

Challenges experienced by Grade 11 mathematics learners in Probability in selected Soshanguve schools

Qualification: M Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2018/04/18 to 2021/04/18.

*The **low risk** application was reviewed by the Ethics Review Committee on 2018/04/18 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
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Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date **2021/04/18**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2018/04/18/32152302/24/MC** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

M Claassens

Dr M Claassens
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APPENDIX B: Permission letter from the Gauteng Department of Education

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	20 April 2018
Validity of Research Approval:	05 February 2018 – 28 September 2018 2018/32
Name of Researcher:	Zhou T
Address of Researcher:	5366 Soshanguve East
	Block vv
	Soshanguve 0152
Telephone Number:	084 7370498
Email address:	Teetine2002@gmail.com
Research Topic:	Exploring the challenges experienced by Grade 11 mathematics learners in Probability in selected Soshanguve Schools
Type of Degree:	Master of Education
Number and type of schools:	Two Secondary Schools
District/s/HO	Tshwane West

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

23|04|2018

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Website:

1. The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.
4. A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
5. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
6. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
7. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
8. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
9. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
10. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
11. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
12. On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
13. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
14. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Ms Faith Tshabalala
CES: Education Research and Knowledge Management

DATE: ....23/04/2018.............................

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX C: Request for permission to conduct research at Tshwane West District schools

Mr Tinevimbo Zhou
5366 Soshanguve East
Soshanguve
Pretoria
0152

Request for permission to conduct research at Tshwane west District schools

9 May 2018
The Principal/SGB
Wallmansthal High School
084 737 0498
teetine2002@gmail.com

Dear ___________________
I, Tinevimbo Zhou I am doing research under the supervision of Dr T.P Makgakga a lecturer in the Department of Mathematics Education towards a Master of Education at the University of South Africa. We are inviting you to participate in a study entitled Challenges experienced by Grade 11 mathematics learners in Probability in selected Soshanguve schools. The purpose of the study is to investigate the challenges learners' experience in solving probability problems. The aim of the study was to contribute to the improvement of learners' performance in mathematics. Hence treating the topic of probability is just one way to attain the aim. Your school has been selected because of one of the schools which offer mathematics in the Tshwane west district. Above all, the GDE diagnostic analysis has identified your being amongst school which have perpetuating poor mathematics results in Grade 12.

The study will entail administering of a diagnostic learner test in the topic of probability, observing teaching-learning of probability and interviewing learners. The results of the research will be disseminated to the Department of Education and other stakeholders. The stakeholders include the policymakers, study participants, parents and Grade 11 mathematics teachers. The study may contribute towards identifying challenges relating to teaching and learning of Grade 11 probability. The study may inform mathematics teachers on the challenges learners experience when solving probability tasks and the nature of problems that can be associated with teaching and learning of this concept. McGraner, VanDerHeyden, & Holdheide (2011) assert that central to raising learners' performance in mathematics is improving the quality of mathematics teaching and learning. It is anticipated that the study will assist mathematics teachers in employing effective probability teaching and learning approaches. This study might assist in-service teacher training programmes to influence in designing the professional development of teachers on content-focused instruction which has tremendous effects on learners' achievement.

More importantly, learners may be aware of the difficulties associated with learning of probability concepts and such may include, advised on the conceptions and per-conceptions that some learners may bring in probability classroom. Potential risks are that learners may result in undesired changes in thought process and emotion. Stress and feeling of guilt or embarrassment may arise simply from thinking and talking about challenges one experience. Furthermore, there might be an invasion of privacy where classroom observation is undertaken. There will be no reimbursement or any incentives for participation in the research. Feedback procedure will entail and made available in the form of a research copy upon request.

Yours sincerely___________________________

APPENDIX D: Participants' information sheet

Mr Tinevimbo Zhou
5366 Soshanguve East
Soshanguve
Pretoria
0152

9 May 2018

Dear Teacher/Learner
My name is Tinevimbo Zhou I am doing research under the supervision of Dr T.P Makgakga a lecturer in the Department of Mathematics Education towards a Master of Education at the University of South Africa. We are inviting you to participate in a study entitled: ***EXPLORING THE GRADE 11 MATHEMATICS LEARNERS EXPERIENCES AND CHALLENGES IN SOLVING PROBABILITY PROBLEMS IN SELECTED SOSHANGUVE SCHOOLS***

WHAT IS THE PURPOSE OF THE STUDY?
The purpose of the study is to investigate the challenges learners' experience in solving probability problems.

WHY AM I BEING INVITED TO PARTICIPATE?
You are invited because you are taking mathematics as a major subject, hence your inputs may assist in improving teaching and learning of probability.
I obtained your contact details from the school principal. The study invites the subject teachers and the learners he/she is teaching.

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?
You are kindly asked to answer some probability questions and provide information on issues pertaining to teaching-learning of probability.

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE?
Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent/assent form. You are free to withdraw at any time and without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?
The benefits of this study are that research on factors contributing to performance in probability benefits teaching-learning. As a participant, you may be informed of the difficulties experienced in solving probabilistic problems.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?
Potential risks are that learners may result in undesired changes in thought process and emotion. Stress and feeling of guilt or embarrassment may arise simply from thinking and talking about challenges one experience. Furthermore, there might be an invasion of privacy where classroom observation is undertaking.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?
You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research. Your name will not be recorded anywhere and no one will be able to connect you to the

answers you give. Your answers will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. The anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings. A report of the study may be submitted for publication, but individual participants will not be identifiable in such a report.

HOW WILL THE RESEARCHER(S) PROTECT THE SECURITY OF DATA?
Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet at UNISA library for future research or academic purposes; electronic information will be stored on a password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. Hard copies will be shredded and/or electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme.

HAS THE STUDY RECEIVED ETHICS APPROVAL?
This study has received written approval from the Research Ethics Review Committee of the UNISA. A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?
If you would like to be informed of the final research findings, please contact Mr Zhou on 084 xxx or email teetine2002@gmail.com. The findings are accessible for 3 months.
Should you require any further information or want to contact the researcher about any aspect of this study, please contact Dr T.P Makgakga on makgasw@unisa.ac.za.

Should you have concerns about the way in which the research has been conducted, you may contact CED Unisa.

Thank you for taking the time to read this information sheet and for participating in this study.

Thank you.

____
Tinevimbo Zhou

APPENDIX E: Letter requesting parental consent for learners to participate in a research study

Mr Tinevimbo Zhou
5366 Soshanguve East
Soshanguve
Pretoria
0152

Dear Parent

Your son/daughter/child is invited to participate in a study entitled: **EXPLORING THE GRADE 11 MATHEMATICS LEARNERS EXPERIENCES AND CHALLENGES IN SOLVING PROBABILITY PROBLEMS IN SELECTED SOSHANGUVE SCHOOLS.** I am undertaking this study as part of my master's research at the University of South Africa. The purpose of the study is to contribute to the improvement of learners' performance in mathematics. Hence treating the topic of probability is just one way to attain the aim and the possible benefits of the study are the improvement of probability performance. I am asking permission to include your child in this study because improving probability performance has a positive influence on the overall mathematics results. I expect to have 200 other children participating in the study.

If you allow your child to participate, I shall request him/her to:

- Take part in an interview
- The interviews will be conducted by the research to individuals at the school for a duration not longer than 10 minutes. The selected individuals will be interviewed after school contact times. Individual interviews will audio recorded by the researcher, I, therefore, ask for permission to audio record these interviews.
- Complete a test
- The learners' test will consist of Grade 10 probabilistic concepts out of 45 marks. Learners will take a test under examination conditions under the supervision of their subject teachers during their mathematics period.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only. There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are the improvements in mathematics performance. Neither your child nor you will receive any type of payment for participating in this study. Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities during their mathematics period with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available after school hours with the researcher. In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password-locked computer in my locked office for five years after the study. Thereafter, records will be erased.

The benefits of this study are that mathematics learners will be made aware of the difficulties experienced in learning the concepts of probability. The results of the research will be disseminated to the Department of Education and other stakeholders. The stakeholders include the policymakers, study participants, parents and Grade 11 mathematics teachers. The study may contribute towards identifying challenges relating to teaching-learning of Grade 11 probability. The study may inform mathematics teachers on the challenges learners experience when solving probability tasks and the nature of problems that can be associated with teaching and learning of this concept. McGraner *et al.* (2011) assert that central to raising learners' performance in mathematics is improving the quality of mathematics teaching and learning. It is anticipated that the study will assist mathematics teachers in employing effective probability teaching and learning approaches. It might assist in-service teacher training programmes to influence in designing the professional development of teachers on content-focused instruction which has tremendous effects on learners' achievement.

Potential risks are that learners may result in undesired changes in thought process and emotion. Stress and feeling of guilt or embarrassment may arise simply from thinking and talking about challenges one experience. Furthermore, there might be an invasion of privacy where classroom observation is undertaking. There will be no reimbursement or any incentives for participation in the research. If you have questions about this study please ask me or my study supervisor, Dr T.P Makgakga Department of Mathematics, College of Education, University of South Africa. My contact number is 084 737 0498 and my e-mail is teetine2002@gmail.com. The e-mail from my supervisor is makgasw@unisa.ac.za. Permission for the study has already been given by the principal and School Governing Board and the Ethics Committee of the College of Education, UNISA. You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child: ______________________________

Sincerely

_____________________________ _____________________________

Parent/guardian's name (print) Parent/guardian's signature: Date:

____

Tinevimbo Zhou T. Zhou. 28 March 2018

Researcher's name (print) Researcher's signature Date:

APPENDIX F: Letter requesting learners' assent to participate in a research study

Mr Tinevimbo Zhou
5366 Soshanguve East
Soshanguve
Pretoria
0152

Dear Learner

Date 9 May 2018

I am doing a study on probability as part of my studies at the University of South Africa. Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your teachers can use to improve the mathematics results better. This may help you and many other learners of your age in different schools. This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

I would like to ask you questions about concepts and teaching-learning of probability. Answering the test questions will take no longer than 45 minutes and interviews will not be longer than 10 minutes.
I will write a report on the study but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you don't want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

The benefits of this study are that mathematics learners will be made aware of the difficulties experienced in learning the concepts of probability. The results of the research will be disseminated to the Department of Education and other stakeholders. The stakeholders include the policymakers, study participants, parents and Grade 11 mathematics teachers. The study may contribute towards identifying challenges relating to teaching-learning of Grade 11 probability. The study may inform mathematics teachers on the challenges learners experience when solving probability tasks and the nature of problems that can be associated with teaching and learning of this concept. McGraner *et al*. (2011) assert that central to raising learners' performance in mathematics is improving the quality of mathematics teaching and learning. It is anticipated that the study will assist mathematics teachers in employing effective probability teaching and learning approaches. It might assist in-service teacher training programmes to influence in designing the professional development of teachers on content-focused instruction which has tremendous effects on learners' achievement.

Potential risks are that learners may result in undesired changes in thought process and emotion. Stress and feeling of guilt or embarrassment may arise simply from thinking and talking about challenges one experience. Furthermore, there might be an invasion of privacy where classroom observation is undertaking. You will not be reimbursed or receive any incentives for your participation in the research. If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at __________________. Do not sign the form until you have all your questions answered and understand what I would like you to do.

Researcher: Mr Zhou
Phone number: 084 737 0498

Do not sign the written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

WRITTEN ASSENT
I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

________________________ ____________________ ____________________
Learner's name (print): Learner's signature: Date:

________________________ ______________________ ____________________
Witness's name (print) Witness's signature Date:

(The witness is over 18 years old and present when signed.)

_______________ ______________________ ___________________
Parent/guardian's name (print) Parent/guardian's signature: Date:

Tinevimbo T. Zhou 28 March 2018
Researcher's name (print) Researcher's signature: Date:

APPENDIX G: Consent/ assent letter to teacher/ learner (including return slip)

I, __________________ confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree with the recording of the test, interviews and observations.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) ____________________________________

______________________________ ____________________________________

Participant Signature Date

Researcher's Name & Surname (please print) Tinevimbo Zhou

T. ZHOU . _28 March 2018

Researcher's signature Date

APPENDIX H: Diagnostic test

SUBJECT: Mathematics

GRADE: 10

TOPIC COVERED: Probability

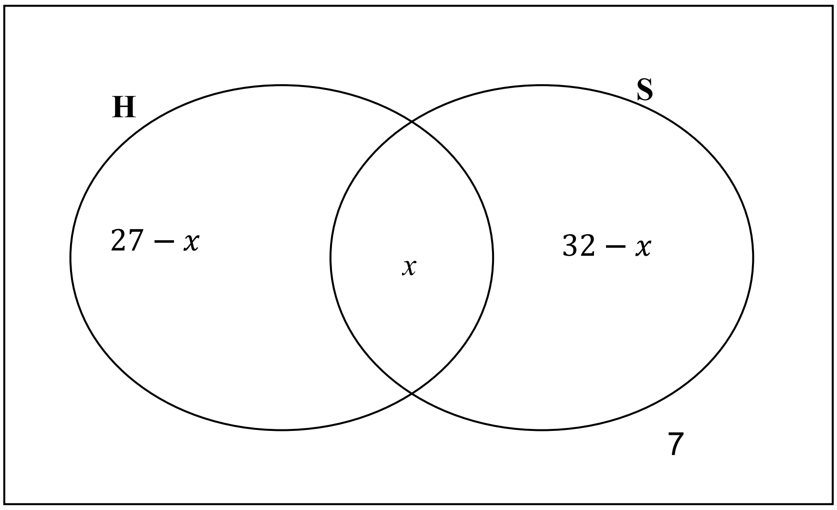
INSTRUCTION: Attempt all questions

QUESTION 1

1.1 In a certain class of 42 boys:

- 27 play hockey (H)
- 32 play soccer (S)
- 7 do not play hockey or soccer
- An unknown number (x) play both hockey and soccer

The information is represented in the Venn diagram below.



1.1.1 Calculate the value of x. (2)

1.1.2 If a boy from the class is chosen at random, calculate the probability that he:

(c) Does not play hockey or soccer (1)

(d) Plays only soccer (2)

1.2 A bag contains 3 blue balls and x yellow balls.

Write down the total number of balls in the bag. (1)

1.2.1 If a ball is drawn from the bag, write down the probability that it is blue. (2)

1.3 1.3.1 Complete the following statement:

If $A\ and\ B$ are two mutually exclusive events, then
$P(A\ and\ B)\ =\ \ldots\ \ldots\ldots$ (1)

1.3.2 Given that A and B are mutually exclusive events. The probability that event A occurs is 0,55. The probability that event B does not occur is 0,7.

Calculate $P(A\ or\ B)$ (3)

[12]

QUESTION 2

2.1 At a certain school there are 64 boys in Grade 10. Their sport preferences are indicated below:

- 24 boys play soccer
- 28 boys play rugby
- 10 boys play both soccer and rugby
- 22 boys do not play soccer or rugby

2.1.1 Represent the information above in a Venn diagram. (5)

2.1.2 Calculate the probability that a Grade 10 boy at the school, selected at random, plays:

(c) Soccer and rugby (1)

(d) Soccer or rugby (1)

2.1.3 Are the events a Grade 10 boy plays soccer at the school and a Grade 10 boy plays rugby at the school, mutually exclusive? Justify your answer. (2)

2.2 One morning Samuel conducted a survey in his residential area to establish how many passengers, excluding the driver, travel in a car. The results are shown in the table below:

Number of passengers, excluding the driver	0	1	2	3	4
Number of cars	7	11	6	5	1

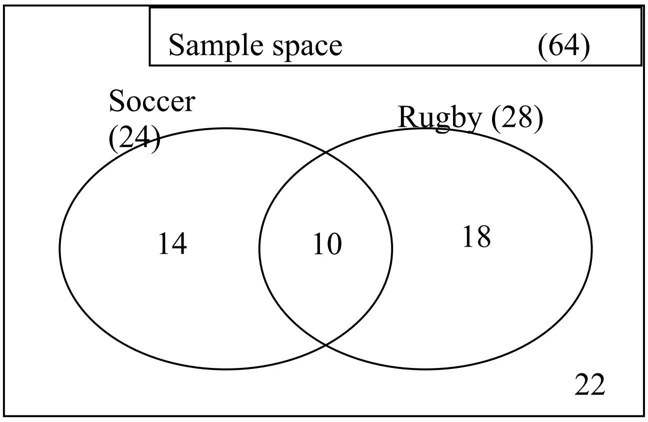
Calculate the probability that, excluding the driver, there are more than two passengers in a car. (3)

2.3 If you throw two dice at the same time, the probability that a six will be shown on one of the dice is $\frac{10}{36}$ and the probability that a six will be shown on both the dice, $\frac{1}{36}$ is. What is the probability that a six will NOT show on either of the dice when you throw two dice at the same time? (3)

[15]

Memorandum

1.1.1	$27 - x + x + 32 - x + 7 = 42$ $-x = 42 - 66$ $x = 24$	**Equation (1)** **Answer (1)**
1.1.2 a	$\boldsymbol{P\ (does\ not\ play\ hockey\ or\ soccer) =}$ $\frac{7}{42} = \frac{1}{6}$	**Answer (1)**
1.1.2 b	$\boldsymbol{P\ (soccer\ only) =}\ \frac{8}{42} = \frac{4}{21}$ **Or** $1 - \frac{3 + 24 + 7}{42} = \frac{8}{42}$	**Answer (2)**
1.2.1	$x + 3$	**Answer (1)**
1.2.2	$\boldsymbol{P\ (blue) =}\ \frac{3}{x + 3}$	**Answer (1)**
1.3.1	$\boldsymbol{P\ (\ A\ and\ B) = 0}$	**Answer (1)**
1.3.2	**P (B)=** $1 - P(B^1)$ $= 1 - 0{,}7$ $= 0{,}3$	**P(B) (1)** **Subs (1)** **Answer (1)**

2.1.1		**✓diagram shape** **✓14 in correct position** **✓10 in correct position** **✓18 in correct position** **✓22 in correct position** **(5)**
2.1.2 (a)	P(Soccer and Rugby) = $\frac{10}{64} = \frac{5}{32}$ = 0,15625 =15,63%	**✓answer (in any form)** **(1)**

2.1.2 (b)	P(Soccer or Rugby) = $\frac{14+10+18}{64}=\frac{42}{64}=\frac{21}{32}=$ 0,65625 = 65,63% **OR** P(Soccer or Rugby) =1- $\frac{22}{64}=\frac{21}{32}$	✓**answer (in any form) (1)**
2.1.3	No Some boys play both soccer and rugby . **OR** No P(S and R) ≠ 0	**No** **Reason** **(2)** **No** **Reason(2)**
2.2	P (more than 2 passengers per car) $=\frac{5+1}{7+11+6+5+1}$ $=\frac{6}{30}$ $=\frac{1}{5}=0{,}2=20\%$	✓ **numerator 6** ✓ **denominator** ✓ **answer (accept** $\frac{6}{30}$ or $\frac{1}{5}$ **or 0,2 or 20%)** **(3)**
2.3	P(not getting a six) = $1-\left(\frac{10}{36}+\frac{1}{36}\right)$ $=\frac{25}{36}$ **OR** $=0{,}69$	✓ $1-(\frac{10}{36}+\frac{1}{36})$ ✓ $(\frac{10}{36}+\frac{1}{36})$ ✓ $\frac{25}{36}$ **(3)** **[15]**

APPENDIX I: Lesson observation schedule

DEMOGRAPHIC DETAILS

1. Name of School: ____

2. Physical Address of School: ____

____

3. Postal Address of School: ____

4. Tel: _______ ___________ Fax: _____________________________

5. Name of Principal: ______________________

Male		Female	

6. Name of Teacher: __________

Male		Female	

7. Grade Observed:________ 8. Number of Learners: __________

OBSERVING CLASSROOM PRACTICE

1. How does teaching and learning of Mathematics occur? (Please list e.g. *whole class*)
   (i) _____________________________ (ii) ________________________

   (ii) _____________________________ (iv) ________________________

2. How is the classroom arranged? (*Furniture*)

____

- What methodology / approach is being used?

  ____

- Which resources are used?

  ____

- How does the teacher deal with correct or incorrect responses?

  ____

OBSERVING TEACHING AND LEARNING IN THE CLASSROOM

____

The PEER system underlies the lessons in a classroom situation. It might not be possible to incorporate all of them in a particular lesson but each lesson will contain some aspects of this system. Please tick (□) your rating.

A	PRODUCTIVE SKILLS	Excellent	Good	Average	Needs More Attention	Not Applicable to the lesson
1.	Learners are able to do reading on the concept being taught.					
2.	Learners write notes on the concept taught.					
3.	Learners are able to solve problems given as exercises.					
4.	Learners are able to relate and apply the concept in real-life problems.					

5.	Learners are able to use their knowledge of and experience in the concept in formulating their own responses.					
6.	Learners are able to accomplish work given on the concept independently					
7.	Learners are able to define and describe learned terms encountered when dealing with the concept.					
8.	Learners are able to follow the steps in solving exercises based on the concept.					
9.	Learners competently use technology (calculators) in areas where it is required in the concept.					
10.	Learners are able to deal with problems in real and abstract context using the concept.					
11.	Learners' ways of making decisions in problem solving are enhanced.					
B	EVOCATIVE SKILLS	Excellent	Good	Average	Needs More Attention	Not Applicable to the lesson
1.	Learners ask questions for clarification.					
2.	Learners ask questions to consolidate their understanding of the concept					

3.	Learners are puzzled by certain areas of the concept and hence very inquisitive.					
4.	Learners are able to interpret new information on the concept.					
5.	Learners ask critical questions to ensure that the methods used are appropriate.					
6.	Learners use their referencing skills to acquire a better understanding of the concept.					
C	EVALUATIVE SKILLS					
1.	Learners are able to do self-assessment tasks in the concept learned.					
2.	Learners are capable of evaluating their own work on the concept.					
3.	Learners are able to evaluate procedures followed in problem solving in the concept.					
4.	Learners are able to identify errors committed when dealing with the concept.					
5.	Learners are able to discuss the pros and cons of using specific methods to solve problems.					
6.	Learners are able to identify incorrect ways of solving problems.					
7.	Learners have alternative ways to solve problems based on the concept.					

D	REFLECTIVEIVE SKILLS	Excellent	Good	Average	Needs More Attention	Not Applicable to the lesson
1.	Learners are constantly engulfed in the world of “exploration in errors.”					
2.	Learners reflect on errors committed to solving problems and work towards eliminating those errors.					
3.	Learners are able to respond to questions testing their comprehension of the learned concept.					
4.	Learners are able to select and use appropriate methods in solving problems.					
5.	Learners are able to hypothesize in problem solving.					
6.	Learners can reflect on the decision they made in solving a particular problem.					

EXAMPLES OF ERRORS CORRECTED

Please provide examples of errors corrected when dealing with the topic being evaluated.

___

___

___

APPENDIX J: The schedule for semi-structured interviews schedule with learners

1. Do you like learning probability? Why?

2. Do you experience any challenges with the learning of probability? (Learners were asked to explain how they responded to the questions).

3. What are those challenges?

4. What aspect(s) of probability do you find difficult to learn?

5. Which aspects of probability do you think you can solve?

6. Which area(s) of probability do you experience problems to learn?

7. What is the cause(s) of the challenges that you are experiencing with the learning of probability, do you think?

8. Why do you experience challenges with the aspect(s) of probability that you have mentioned in question 4?

9. What do you think can be done to alleviate the mentioned challenges?

APPENDIX K: Schools' profiles

1.2.1 School code (to be provided by the researcher): _________.

1.2.2 What is the status of the school (choose one answer): (public; private).

1.2.3 In which area is your school located (choose one answer): (township; town or city; rural).

1.2.4 What is the total number of learners currently doing mathematics in grade 11: _______________

Facilities	YES	NO
Science laboratory		
School library		
Computer laboratory		

1.2.6 If your school has the above-mentioned facilities, are they functional? (YES/ NO):

1.2.6.1 Science laboratory: ____________________;
1.2.6.2 School library: ___________________;
1.2.6.3 Science laboratory: ___________________.

1.2.7 If possible, please provide the school's grade 12 mathematics end-of-the-year pass rates (in percentages) in the following years:
1.2.7.1 2018_______________;
1.2.7.2 2017 _______________;
1.2.7.3 2016 _______________;

APPENDIX L: Annual Teaching Plan (ATP)

ANNUAL TEACHING PLAN: SUMMARY

Grade 10		Grade 11		Grade 12	
	No. of weeks		**No. of weeks**		**No. of weeks**
Algebraic expressions Exponents Number patterns Equations and inequalities Trigonometry	3 2 1 2 3	Algebraic expressions Equations and inequalities Number patterns Analytical geometry	3 3 2 3	Patterns, sequences and series Functions and inverse functions Exponential and logarithmic functions Finance, growth and decay Trigonometry compound angles	3 3 1 2 2
Functions Trigonometric functions Euclidean Geometry EXAMS	4 1 3 2	Functions Trigonometry (reduction formulae, graphs, equations) EXAMS	4 4 2	Trigonometry 2D and 3D Polynomial functions Differential calculus Analytical geometry TESTS /EXAMS	2 1 3 2 2
Analytical geometry Finance, growth and decay Statistics Trigonometry Euclidean geometry Measurement	2 2 2 2 1 1	Measurement Euclidean Geometry Trigonometry (sine, area, cosine rules) Probability Finance, growth and decay	1 3 2 2 2	Geometry Statistics (regression and correlation) Counting and Probability Revision TESTS/EXAMS	2 2 2 1 2
Probability Revision EXAMS	2 3 3	Statistics Revision EXAMS	3 3 3	Revision EXAMS	3 5

The detail which follows includes examples and numerical references to the Overview

GRADE 10: TERM 4

No of Weeks	Topic	Curriculum statement	Clarification
2	**Probability**	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events A and B in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ A and B are mutually exclusive if $P(A \text{ and } B) = 0$ A and B are complementary if they are mutually exclusive and $P(A) + P(B) = 1$. Then $P(B) = P(\text{ not}(A)) = 1 - P(A)$.	**Comment:** It generally takes a very large number of trials before the relative frequency of a coin falling heads up when tossed approaches 0,5. **Example**: A study was done to test how effective three different drugs, A, B and C were in relieving headache pain. Over the period covered by the study, 80 patients were given the chance to use all three drugs. The following results were obtained: 40 reported relief from drug A 35 reported relief from drug B 40 reported relief from drug C 21 reported relief from both drugs A and C 18 reported relief from drugs B and C 68 reported relief from at least one of the drugs 7 reported relief from all three drugs. 1. Record this information in a Venn diagram. (C) 2. How many of the subjects got relief from none of the drugs? (K) 3. How many subjects got relief from drugs A and B but not C? (R) 4. What is the probability that a randomly chosen subject got relief from at least two of the drugs? (R)

The topic of probability is allocated two weeks of teaching time as per Annual Term Plan (DBE, 2011). The instruction time of Mathematics per week is 4,5hours. Given these guidelines, the researcher conducted two observations per class in both schools.

APPENDIX M: Example of a sequencing list for learners' interviews

Class: Grade 11 B	Number of learners: 33

School: B	Subject: Mathematics

Learner Code[4]	D1[5]	D2	D3	D4	D5	D6	D7	D8	D9
1. LB1[6]									
2. LB2									
3. LB3									
4. LB5									
5. LB6									
6. LB7									
7. LB8									
8. LB9									
9. LB9									
10. LB10									

4 Learners were given code name for identification.
5 The code D1 denotes Day 1, D2 means Day 2 and D3 implies Day 3 and so on. D1 is the day a learner was purposefully selected for interviews.
6 The code LB1 was used to denote learners in School B (Section 4.8.2.2.1). the learners in School C were coded LC69 and so on. Learners were sequenced in this way so as not to report the data analysis haphazardly. Moreover, it was easy to identify the learners and the school for follow up interviews.

APPENDIX N: Sequencing list for conducting lesson observations in schools

	07H45-8H30	8H30-9H15	9H15-10H00	10H00-10H45	10H45-11H30	12H00-12H45	12H45-13H30	13H30-14H15
Monday	BO1		C2			C03		
Tuesday		B04	B05			C05		
Wednesday		C06		B07				
Thursday				C08	C08			
Friday								

The schools were coded A, B, and C for identification when analysing the results (Section 4.8.2.2.2). The code B01 was used to denoted School B, O1 denotes the first observation, BO2 means School B, and the second observation, BO3 implies School B and the third observation visits and so on. The code C04 implies the school C with O4 denoting the fourth observation in the numerical order of lesson observations. The researcher used this system of identification so as to be able to link the sources of a particular school.

APPENDIX O: Moderation tool to evaluate the diagnostic test

DISTRICT	**Tshwane west D15**	
SUBJECT	**Mathematics**	
GRADE	**11**	
NAME OF SCHOOL		
NAME OF RESEARCHER		
NAME OF EDUCATOR (S)		
NAME OF MODERATOR		
DATE		
DESCRIPTION OF TASKS/ACTIVITY MODERATING:		

MODERATION			
	YES	**NO**	**REMARKS**
Subject, e.g. Mathematics			
Grade, e.g. Grade 10 or Grade 11 or Grade 12			
Is a **programme of assessmen**t included?			
Does the task correspond with the **programme of assessment**			
Evidence of PRE-MODERATION			
Evidence of SCHOOL MODERATION			
COMMENTS (Compulsory)			

MODERATION	**YES**	**NO**	**REMARKS**
Are the instructions to candidates clearly specified and unambiguous?			
Is the mark allocation indicated?			
Are questions ordered from easy to difficult, e.g. Level 1 to Level 4 (**different** cognitive levels - **MLT Taxonomy)?**			
Mark totals indicated correctly per question?			
Is the assessment activity complete with grid, memorandum, formula sheets and diagram sheets?			
Are tasks dated?			
Is the name of the task indicated?			
COMMENTS ABOUT THE QUALITY OF THE TASK (Compulsory)			

EDUCATOR **SIGNATURE** **DATE**

HOD/ SUBJECT SPECIALIS **SIGNATURE** **DATE**

APPENDIX P: Categories and sub-categories to code the data from semi-structured interviews

Categories	Descriptions
Text comprehension	Understanding of the information contained in the text of the problem. Misunderstanding of the question's demands
Conceptual	Errors and misconceptions involving concept and definitions of probability. Misinterpretation of terms and incorrect formulas
Procedural	Faulty procedures when calculating probabilities Incorrect steps are taken to achieve a goal Procedural errors involving mutually exclusive events, experimental events and complimentary events Arithmetic errors
language	Difficulties in dealing with the language of probability, language of symbols, language of prepositions

APPENDIX Q: Annual Assessment Plan for Grades 10-12

6. PROBABILITY					
10.6.1	a) Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome. (b) Venn diagrams as an aid to solving probability problems. (c) Mutually exclusive events and complementary events. (d) The identity for any two events A and B: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	11.6.1	(a) Dependent and independent events. (b) Venn diagrams or contingency tables and tree diagrams as aids to solving probability problems (where events are not necessarily independent).	12.6.1	(a) Generalise the fundamental counting principle. (b) Probability problems using the fundamental counting principle and other techniques.