TEACHERS' PERCEPTIONS OF PROBLEM SOLVING IN MATHEMATICS: A CASE OF MANZINI REGION IN ESWATINI

by

NKOSIKHONA CALVIN NHLABATSI

submitted in accordance with the requirements
for the degree of

MASTERS OF EDUCATION

in the subject

Mathematics Education

at the

UNIVERSITY OF SOUTH AFRICA

SUPERVISOR: DR JOSEPH J DHLAMINI

November 2019

DECLARATION

Name: Nkosikhona Calvin Nhlabatsi

Student number: 48419052

Degree: Master of Education in Mathematics Education

Title: *Teachers' perceptions of problem solving in mathematics: A case of Manzini Region in Eswatini*

I, Nkosikhona Calvin Nhlabatsi, declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

Btsi

10 October 2019

SIGNATURE DATE

DEDICATION

To

my wife Nompumelelo

and our two daughters

Simelokuhle and Unathi

ACKNOWLEDGEMENTS

Firstly, I would like to give thanks to the Almighty God for providing me with the volition and strength to complete this research project. Secondly, I would like to express my sincere gratitude and appreciation to the following people:

Special thanks to my wife, Nompumelelo. I am short of words when considering what you have done for me while I was dealing with the challenges and intricacies of doing this research. Truly, you are the pillar of my strength, and it is to you that this work and research are dedicated.

To my parents Caiphus and Thelma. I thank you for your love, prayers and for your concern and constantly enquiring about the progress of my study, and always wanting to know when I would finish my masters' dissertation.

To my sister Hlehle and my brothers Sabelo, Phathwakahle and Kwenziwe. I thank you for your continued support to my research and giving me the strength to complete the study.

To my research supervisor, Dr Joseph J Dhlamini, whose advice, expertise, recommendations and insightful guidance were largely instrumental towards the completion of this research. I would not have managed to complete this study without your unswerving reassurance.

To the Director of the Ministry of Education and Training in Eswatini. I thank you for granting me permission to collect the study related data from participating schools in the Manzini Region.

To the principals and the School Committees of the participating schools. I sincerely appreciate your allowing me to conduct an empirical investigation in your schools.

To all Form 1 mathematics teachers who participated in the study. I thank you not only for sacrificing your time, but also for your professional assistance that you gave me while doing my research. You have all played a significant role in the success and completion of this study.

ABSTRACT

Problem solving is an innovative instructional practice currently receiving attention and advocacy in mathematics education. It may support learners to develop knowledge to employ ideas in real world situations. It motivates learners to see practical reasons for learning mathematics. Such an instructional efficacy may be unachievable if traditional teaching methods are prevalent in mathematics classrooms. In the latter the teacher talks, demonstrates and solely work out problems to transfer mathematical knowledge to learners. Problem solving is a shared learning experience with teachers and learners working together to find a solution to a problem.

This study investigated, (1) teachers' conceptions of problem solving; (2) teachers' articulations of problem solving, such as constructing problem solving tasks; and, (3) shared experiences and challenges in teaching problem solving. The study aimed to gain insights into teachers' perceptions of problem solving in Form 1 at the Eswatini schools. The study employed a case study research strategy to optimize the production of desirable data from four Form 1 mathematics teachers in four high schools around the Manzini region. Data collection instruments comprised of semi-structured interviews and lesson observations. Lesson observations served to affirm interview data.

Study findings revealed that most participants' conceptions of mathematics problems, and problem solving in mathematics, were at variance with literature definitions. One teacher demonstrated desirable knowledge of what a problem and problem solving in mathematics is. Teachers agreed that using problem solving as a teaching strategy is useful. They agreed that problem solving helps learners develop critical thinking skills and ability to solve problems in real life. The method cannot be implemented properly until the curriculum, textbooks and the assessment system reflect its values. Teacher's lack of problem solving knowledge, large classes and inadequate time provide hindrances. Proper training in mathematical problem solving is needed to full actualize the aspirations of the curriculum regarding problem solving.

KEY WORDS

Mathematical problem

Mathematical word problem

Mathematical problem solving skills

Non-Routine Problem

Problem solving

Problem solving strategy

Routine Problem

Traditional teaching method

ABBREVIATIONS

IGCSE	International General Certificate of Secondary Education
‘O’ Level	Ordinal Level
SGCSE	Swaziland General Certificate of Secondary Education
JC	Junior Certificate
NCTM	National Council of Teachers of Mathematics
SPC	Swaziland Primary Certificate
UNISA	University of South Africa
NCC	National Curriculum Centre
EDSEC	Education Sector
PGCE	Post Graduate Certificate in Education
STD	Secondary teacher’s diploma
BSc	Bachelor of Science
MSc	Master of Science
MOE	Ministry of Education
REC	Research Ethics Committee

TABLE OF CONTENTS

		Pages
	Declaration	ii
	Dedication	iii
	Acknowledgement	iv
	Abstract	v
	Key words	vi
	Abbreviations	vii
	Table of contents	viii
	List of Figures	ix
	List of Tables	x
CHAPTER 1: INTRODUCTION		**1**
1.1	A brief introduction to the study	1
1.2	Contextual background of the study	2
1.3	Problem statement	4
1.4	Aim of the study	5
1.5	Objectives of the study	5
1.6	Research questions	6
1.7	The rationale for the study	6
1.8	The significance of the study	7
1.9	Definitions of key words	7
1.10	Overview of the dissertation	9
CHAPTER 2: LITERATURE REVIEW		**11**
2.1 Introduction		11
2.2 Mathematics problem solving themes in the Eswatini curriculum		11
	2.2.1 Problem solving at the primary school level	12

2.2.2 Problem solving at the secondary school level 12

2.2.3 Assessment objectives for both JC and SGCSE 14

2.3 Definitions of a problem and problem solving in mathematics 15

2.3.1The definition of a problem 15

2.3.2 Definition of the notion of problem solving 18

2.4 Choosing teaching methods to use in a lesson 20

2.4.1 Using problem solving as a teaching strategy 20

2.4.2 Problem solving classroom ecology 23

2.4.3 Problem solving strategies 24

2.4.4 A Conceptual Framework of problem solving instruction 25

2.4.4.1 The classroom culture 25

2.5 Varying perspectives on the goal of problem solving 26

2.6 The role of the teacher in mathematics problem solving instruction 27

2.7 Metacognition and problem solving 27

2.8 Benefits of problem solving in mathematics 28

2.9 Challenges encountered by teachers when problem solving is used as a teaching strategy 29

2.10 Traditional method of teaching mathematics 31

2.11 Teachers' perceptions of a mathematical problem and problem solving 31

2.12 Summary 32

CHAPTER 3: RESEARCH METHODOLOGY 33

3.1Introduction 33

3.2 Research Design 33

3.3 Research Site and Participants 34

3.3.1 The research site 34
3.3.2 The study participants 35
3.3.3 Sampling procedures and techniques 36
3.4 Data Collection Instruments for the Study 38
3.4.1 Naming of instruments 38
3.4.2 Purpose of data collection instruments 38
3.4.3 Development of instruments 38
3.4.3.1 Interview Schedule 39
3.4.3.2 Lesson Observation Schedule 40
3.4.4 Data Collection 41
3.5 Ensuring the trustworthiness of the study 42
3.5.1 Credibility 42
3.5.2 Dependability 43
3.5.3 Confirmability 43
3.6 Trialling the Research Process 44
3.7 Data Analysis 45
3.7.1 Coding 48
3.8 Research Ethics 49
3.8.1 Full disclosure 49
3.8.2 Negotiate access 50
3.8.3 Informed consent 50
3.8.4 Confidentiality and anonymity 50
3.9 Chapter Conclusion 51

CHAPTER 4: DATA PRESENTATION AND ANALYSIS 52

4.1 Introduction 52
4.2 Analysis of data from semi-structured interviews 53
4.2.1 Teachers' profile 54
4.2.2 Teacher's conception of a mathematical problem 56
4.2.2.1 Example of a mathematics problem 57
4.2.3 Meaning of problem solving in mathematics 59

4.2.3.1 Development of effective problem solvers 61
4.2.4 Advantages of using problem solving as a teaching strategy in mathematics 62
4.2.5 Challenges, encountered by teachers when using problem solving as a teaching strategy 64
4.2.6 Teacher training on problem solving 66
4.3 Analysis of teachers' lesson observation 67
4.3.1 Problem posing 70
4.3.2 Problem solving strategies 71
4.3.2.1 Observing SC02 lesson in terms of theme 2 72
4.3.3 Modeling problem solving 75
4.3.4 Limiting teacher input 77
4.3.5 Promoting metacognition 80
4.3.6 Highlighting multiple solutions 81
4.4 Presentation of interview findings in relation to lesson observation findings. 83
4.4.1 Teacher SC02 83
4.4.2 Teacher LL03 83
4.4.3 Tabulating the differencing in the observed lessons of SC02 and LL03 85

4.5 Summary 87

CHAPTER 5: DISCUSSION 88
5.1 Introduction 88
5.2 Achieving the Objectives of the study 88
5.2.1 First objective of the study 89
5.2.2 Second objective of the study 89
5.2.3 Third objective of the study 90
5.2.4 Fourth objective of the study 91
5.2.5 Fifth objective of the study 91
5.3 Discussion of findings 92
5.3.1 Meanings that teachers attach to problem solving in mathematics 92

5.3.2Teacher’s conception of a mathematical problem, the types of problem solving questions and tasks that are posed by mathematics teachers 93
5.3.3 Classroom practices that characterise teachers’ problem solving lesson 94
5.3.4 Challenges teachers encountered when implementing problem solving 96
5.4 Summary 96

CHAPTER 6 SUMMARY OF STUDY, CONCLUSION AND RECOMMENDATIONS 97

6.1 Introduction 97
6.2 Summary of Findings 97
6.2.1 Summary of literature review 97
6.2.2 Summary of research methodology and design 98
6.2.3 Summary of findings of empirical investigation 99
6.3 Answering the Research Questions 100
6.3.1 Research question 1 100
6.3.2 Research question 2 101
6.3.3 Research question 3 101
6.3.4 Research question 4 102
6.3.5 Research question 5 103
6.4 Limitations of Study 104
6.5 Recommendations 104
6.6 Future Research 104

REFERENCES 106

APPENDICES SECTION 117

APPENDIX A: Request letter to the Ministry of Education 117
APPENDIX B: Permission letter from the Ministry of Education 118
APPENDIX C: Request letter to the school principals 119
APPENDIX D: Reply receipt for the school principals 120
APPENDIX E: Consent Form/Request letter from teachers 121

APPENDIX F: Reply receipt for the teachers 125
APPENDIX G: Semi-structured interview schedule 126
APPENDIX H: Lesson observation schedule 128
APPENDIX I: Sample of transcribed teacher interviews 129
APPENDIX J: Language editing certificate 132
APPENDIX K: Turnitin certificate 133
APPENDIX L: Ethics clearance certificate 134

LIST OF FIGURES

Figure	Title	Page
2.1	Conceptual Framework used in the study	26
3.1	The map of Eswatini with four regions	35
4.1	A problem given by Teacher SC02	71
4.2	Groups' responses on the value of *pi*	74
4.3	Learners' solution on the problem	79
4.4	Example of ratio and proportion problem	82

LIST OF TABLES

Table	Title	Page
2.1	Weighting of papers	14
2.2	Teaching and learning strategies	22
3.1	Description of sampled school	36
3.2	Steps followed to analyse the qualitative data	46
3.3	Transcript of a lesson observation	47
4.1	Teachers' profiles	55
4.2	Teacher's conception of a mathematics problem	56
4.3	Teachers' responses on problem examples and frequency of giving word problems	58
4.4	Meaning of problem solving in mathematics	60
4.5	Development of effective problem solvers	62
4.6	Advantages of using problem solving as a teaching strategy in mathematics	63
4.7	Teachers' challenges when using problem solving as a teaching strategy	65
4.8	Teacher's responses on training and support on problem solving	66
4.9	Example of lesson observation schedule used in SC02 and LL03 mathematics lessons	69
4.10	Using theme 2 to observe the lesson of SC02 in terms of Conceptual framework and lesson observation	73
4.11	The approximate value for pi.	74
4.12	Examples of problems LL03 gave to learners during interview and actual lesson	84
4.13	Teachers' observed classroom practices in relation to the themes in Table 4.	85

CHAPTER 1

INTRODUCTION AND BACKGROUND OF THE STUDY

1.1 A BRIEF INTRODUCTION

Problem solving is an advanced instructional practice that is currently receiving massive attention and advocacy in mathematics education (Lee & Kim, 2005). This is a very important and groundbreaking concept that is integral in mathematical education. "Problems and related problem solving strategies play a significant role in life, and are considered an integral component of mathematics" (Ersoy & Guner, 2015: 120). Why should problem solving lessons be part of mathematics? A possible answer may be that problem solving instruction enhances the systematic growth of the learner's ability; enabling them to deal with numerous and varied problem types encountered in everyday life. Dhlamini (2012: 22) acknowledged that problem solving is an essential social skill going beyond academic, social, political and professional boundaries.

In Eswatini[1] there are several educational programmes aimed at elevating the problem solving skills of learners. According to Isaacs (2007: 4), "in 1997, the government of Eswatini largely adopted a national strategic position known as the "National Development Strategy" or the "Vision 2022". This national strategy pronounces predicted growth and first world development in the economic sector, including education. In a technologically developed society, people who are largely critical thinkers, who are able to analyse and apply logical thinking procedures when dealing with complex problem solving issues, are highly sought after (Broody, 1998). To produce learners who are able to function in such a society, teachers are encouraged to generate

1. Eswatini meaning "land of the Swazis" was announced by The King of Swaziland, King Mswati iii as the new name for Swaziland during a celebration marking the monarch's birthday and 50 years of independence from British colonial rule. Although the study was conducted in Eswatini, it was supervised by a South African lecturer. Hence, in some parts of the dissertation the discussions would be influenced by a South African context and educational background. However, this influence only took place in cases where terms would be explained and clarified in the dissertation to find and locate their meaning and equivalence in a South African context.

meaningful learning experiences of the problem solving process, cultivate productive problem solving strategies, and to generate a constructive attitude in learners towards problem solving.

This study was fundamental because incorporating problem solving into mathematics instruction[2] can offer "a learning environment for learners to explore realistic problems and hence invent ways to find solutions to a problem, a skill that could be essential in later life" (Lee, 2007: 3). The current study identified the types of mathematical problems considered by mathematics teachers to be appropriate for learning. Also, teaching, instructional practices as well as possible challenges to using problem solving as a teaching strategy were also explored.

1.2 CONTEXTUAL BACKGROUND OF THE STUDY

Eswatini introduced the International General Certificate of Secondary Education (IGCSE) curriculum at the beginning of 2006, which replaced the Ordinal Level ('O' Level) curriculum. According to The Times of Swaziland (18 February 2008: 1), the IGCSE curriculum was introduced with resistance from teachers. Teachers felt that they were not consulted in the matter nor given the time to become oriented to the IGCSE curriculum, which appeared to be different from the previous 'O' Level. Teachers further pointed out that the 'O' Level curriculum was more teacher-based while IGCSE was learner-based. The success of the new curriculum mainly depends on whether teachers can translate the envisaged educational goals of curriculum developers into educational versions that are meaningful and applicable to their learners. Teachers would have to have first embraced and acknowledged the intended educational transformations that the new curriculum was seemingly attempting to achieve in Eswatini classrooms (see, also, Zanzali, 2003). The IGCSE curriculum ran for three years before it was replaced in 2009 by a localized curriculum, namely, the Swaziland General Certificate of Secondary Education (SGCSE) that was still being implemented at the time of conducting this study.

2. The word *instruction* has been used in this study to refer mainly to the teaching and learning activities taking place in the mathematics classrooms. It was the researcher's view that this word could also be used interchangeably with the word pedagogy. In this study instructional activities that were studied are those relating to the teaching and learning of mathematics in the Form 1 classrooms at Eswatini, mainly from the perspective of teachers.

In addition, various documents were reviewed to find out other educational stakeholders' views on problem solving. These documents consisted of the syllabi, the Eswatini Education Sector Policy (EDSEC)[3] and the prescribed Prism Alive Series[4]. Data sourced from the documents seemed to suggest that mathematics teachers in Eswatini predominantly use teaching methods that can be classified as learner centered, such as problem solving, to develop learner problem solving skills among others. The Eswatini Education and Training Sector Policy (2018: 7), the Swaziland General Certificate of Secondary Education (SGCSE) (2015: 14) syllabus and the prescribed Prism Alive Series (2010: 4), all outline goals referring to problem solving without explaining how this concept might translate into classroom practice and allowing for multiple interpretations in Form 1[5]. For instance, both the SGCSE Syllabus (2015: 4) and Junior Certificate (JC) Syllabus (2015: 4) aim to enable the learner to "apply combinations of mathematical skills and problem solving techniques". However, Desire (2014) has argued that it may not be a given fact that mathematics teachers can implement problem solving effectively into their lessons in a manner that stimulates the intended skills in learners.

The phrase "problem solving" may be perceived as problematic since it suggests multiple meanings in one's understanding of mathematics teaching and learning. Wilson, Fernandez and Hadaway (2015: 1) have argued that some teachers may think largely of the selection and presentation of 'good' problems to learners. Some may have in mind mathematics program goals where the curriculum is structured around problem content. Others may think of program goals that emphasize techniques and strategies of problem solving. Indeed, mathematical problem solving conversations may often combine or blend several of these ideas. Lupahla (2014) noted that "teaching problem solving requires an agreed upon definition of problem solving" (p. 8).

3. Eswatini Education Sector Policy is a document, the Ministry of Education and Training regards as a guide for the operations of the education sector.
4. Prism Alive Series is a set of prescribed mathematics books, book 1, book 2, book 3 and book 4 taught at the entire secondary school level in Eswatini.
5. A Form is an educational stage, class or grouping of learners in Eswatini schools. Form 1 is equivalent to grade 8 in South African context.

1.3 PROBLEM STATEMENT

The last two decades have seen a marked increase in the use of problem solving as one of the main approaches to teaching and learning mathematics (Hung *et al*., 2008; NCTM, 2000; Alreshidi, 2016: 3). It was noted in a mathematics reader of The University of South Africa Unisa (Unisa) (2009: 93) that problem solving supports learners in developing a deeper understanding, employing ideas into real world situations, as well as motivating and showing learners the practical reasons for learning. However, such an understanding cannot occur in mathematics if traditional teaching methods still dominate lessons; where the teacher talks and the only means of transferring mathematical knowledge to learners is by way of demonstrations or worked problems. For instance, the teacher may state that in order to calculate the circumference of a circle, one must multiply the value for $pi(\pi)$ by the diameter of a circle. Given that the diameter of a circle is 10cm and taking $pi(\pi)$ as 3.14, the circumference is 3.14 multiplied by 10cm, which results in 31.4 cm.

Traditionally, learners master the knowledge taught by the teacher through repetitive practice and are then later required to reproduce this knowledge. It was indicated in a mathematics module for Unisa (2011: 13) that the problem with this method of teaching is that the learning of mathematical concepts takes place by rote, meaning that learners can easily forget what is learnt. Hence, rote learning cannot lead to deep understanding but can only produce surface learning. In addition to that, “learners fail to apply what they learn in one context to another” (Unisa, 2011: 13). For example, learners usually face challenges in trigonometry because they cannot recognize the relationship between algebraic equations and trigonometric functions. Furthermore, Nardi and Steward (2003) argue that in using the traditional method of teaching, learners are often led to believe that school mathematics are irrelevant and boring as it calls for the learning of rules in a passive manner without a clear purpose, and that it restricts the chances of working collaboratively, neglecting the needs of the individual.

In the First (2016) and Second (2017) Mathematics Symposiums as well as at teachers' workshop in 2015 and 2016 organized by the Mathematics Department of the Ministry of Education, teachers were encouraged to use problem solving when teaching mathematics. However, no attempt has been made to find out what mathematics teachers think about problem solving. How teachers' conceived meanings differ from those expressed in the educational literature has not been adequately explored, nor what kinds of mathematical problems are considered by mathematics teachers to be suitable for their learners. There is a need to think differently about problem solving, to strive towards meeting the learning needs of 21st century learners in mathematics classrooms. To achieve these educational aspirations, teachers need to fully understand what problem solving is in mathematics.

Teaching mathematical knowledge using problem solving as a teaching strategy was opted in the current study to improve learning and education in mathematics. This was because "mathematical knowledge does not simply exist out there waiting to be discovered, but it should be constructed by learners using the teacher's guidance and the resources in their environment" Unisa (2011: 13).

1.4 AIM OF THE STUDY

The aim of the study was to determine mathematics teachers' perceptions of problem solving in Form 1 at the Eswatini schools.

1.5 OBJECTIVES OF THE STUDY

The following objectives were considered to achieve the aim of the study:

- **1.5.1** To establish teachers' explanations of the meaning of problem solving in selected Form 1 mathematics classrooms;
- **1.5.2** To ascertain teachers' definitions and conceptions of a mathematical problem;
- **1.5.3** To identify the kinds of problem solving questions and tasks that teachers pose during a Form 1 mathematics lesson;

1.5.4 To determine the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom? ; and,

1.5.5 To identify possible challenges, if any, encountered by Form 1 teachers when problem solving is used as a teaching strategy.

1.6 RESEARCH QUESTIONS

The study presented the following research questions:

1.6.1 What meanings do teachers attach to mathematical problem solving?

1.6.2 How do teachers in Form 1 conceive a mathematical problem?

1.6.3 What kind of mathematical problem solving questions, and tasks, do teachers in Form 1 pose?

1.6.4 What are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?

1.6.5 What challenges, if any, do teachers encounter when using problem solving as a teaching strategy to learn mathematics in Form 1?

1.7 THE RATIONALE FOR THE STUDY

Problem solving is an important component of teaching and learning mathematics. Aydogdu and Ayaz (2008) put forth that problem solving allows learners to develop understand and explain the processes used to arrive at solutions, rather than remembering and applying a set of procedures. The current study strived to determine mathematics teachers' perceptions of problem solving in Form 1 at the Eswatini schools. The researcher's conceptual understanding of problem solving was changed significantly after studying a reader entitled, *The impact of the curriculum on effective mathematics classroom practices*, while doing an Honors' Degree. The researcher saw problem solving as a good strategy for teaching mathematics. However, what the researcher used to practice and observing the colleagues' practices, the researcher noted that there was a gap between the researcher's knowledge and the general practice in the teaching of mathematics. Hence this study was commissioned.

1.8 THE SIGNIFICANCE OF THE STUDY

According to the (NCTM, 2003: 3), teaching mathematics using problem solving as a strategy leads to greater levels of understanding on the part of the learner. Hence, the findings of this study may help curriculum developers and teacher professional development facilitators to influence the curriculum of the country to be more educationally productive. An educationally productive curriculum should be one that improves learners' performance and achievement in mathematics where instruction has to be designed to promote mathematical problem solving skills. Every mathematics teacher must be geared towards designing effective instruction that promotes meaningful learning. In addition, the results of the study may be used by mathematics teachers to reflect on their own and gain new conceptions of mathematical problem solving. Teachers may then use these insights to reflect and improve their teaching to ameliorate mathematics teaching and learning.

1.9 DEFINITION OF KEY TERMS

The following are definitions applicable to the study in the way that the study d them to mean:

1.9.1 Mathematical problem

A contextualized, real world situation that does not have an immediate solution at hand. A mathematical problem may require higher order cognitive abilities where the problem solver must think before attempting to solve the problem (Schoenfeld, 1992).

1.9.2 Mathematical word problem

A mathematical task where important background information is given in words instead of presenting it in mathematical notation or symbols.

1.9.3 Problem solving

It is what one does to achieve a given goal with no prior knowledge of the solution method. Krulick and Rudnick (1980) states that an individual should use previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation.

1.9.4 Problem solving strategy

A teaching strategy employed to present educational material from a meaningful, contextualized, customized and real-world approach. A problem-solving approach strives to provide productive educational guidance and meaningful instruction to learners to support and bolster the intended development of their problem solving skills.

1.9.5 Routine problem

A problem that can be solved by the application of an already-known process, computational strategies, making use of formulas and only need a single step.

1.9.6 Non-routine problem

A problem that requires the solution method to be ascertained as a component embedded within the process of solving the problem. It exists when one does not have a clue, or has relatively little information, regarding the solution process, and is unable to see the solution because it is not conspicuous (Mayer & Hegarity, 1996).

1.9.7 Traditional teaching method

A method that takes on the following steps: introduction, development and review, normally used to develop memorizing and assessing the learner's knowledge of the content.

1.9.8 Mathematical problem solving skills

Mental processes which let a learner to tackle a mathematical problem, select the best of many mathematical problem solving techniques for that certain situation, and go through the process of solving the problem.

1.10 OVERVIEW OF THE DISSERTATION

The dissertation is outlined as follows:

Chapter 1: This Chapter presents the theoretical background to the current study. Issues addressed in this Chapter include: a contextual background, the main problem, the aim and related objectives, the key research questions, and the rationale. This Chapter also outlines the significance of the study and presents the definitions of key terms. It further highlights an overview of the entire dissertation.

Chapter 2: This Chapter addresses the literature review, beginning with an overview of problem solving themes in the Eswatini curriculum. Definitions of a problem, problem solving in mathematics and varying perspectives on the goal of problem solving are reviewed in this Chapter. In addition, this Chapter provides a useful discussion on problem solving instruction and how a teacher should facilitate this form of instruction. The benefits of mathematical problem solving and the challenges associated with implementing mathematical problem solving are also discussed in this Chapter. Finally, an analysis of problem solving models is discussed in the last section of the Chapter.

Chapter 3: The research design and its articulation form part of the discussions in this Chapter. The data collection process, participants' selection criteria and characteristics, followed by a discussion of ensuring the trustworthiness of the study are covered. The Chapter concludes with a discussion of the pilot study, data analysis and ethical review procedures.

Chapter 4: Data analysis is explored in this Chapter. In this study data analysis was carried out in three stages: interviews, lesson observations and presentation of interview findings in relation to lesson observation findings, and all these stages are discussed comprehensively bin Chapter 4.

Chapter 5: This Chapter provides a discussion of the qualitative analysis from the semi-structured interviews and lesson observations, the analysis of which generated the findings of the study. This pertinent discussion is comprehensively covered in Chapter 5.

Chapter 6: This Chapter provides a summary of the study. The research questions are revisited and reviewed in line with the results of the study. Limitations are highlighted and avenues for future related studies are considered. The Chapter concludes with recommendations and general conclusions.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The first section of this Chapter provides a review of the literature and sets the stage through a brief reflection on what constitutes problem solving themes within the Eswatini mathematics curriculum. This discussion is linked to mathematics teaching in both primary and secondary school levels. The assessment objectives for both Junior Certificate (JC) and the Swaziland General Certificate of Secondary Education (SGCSE) are also discussed. The second section of this Chapter provides definitions of a mathematics problem, and problem solving in mathematics.

The next section is a brief commentary on the following themes: choosing a teaching method to use in a lesson, problem solving as a teaching strategy, emerging and diverse perspectives on the goal of problem solving in mathematics classrooms, the role of a mathematics teacher when problem solving is used as a teaching strategy and, the perceptions of metacognition and problem solving. The final section of the Chapter discusses the following subjects: research on the benefits of problem solving in mathematics classrooms, possible challenges, if any, encountered by teachers when problem solving is used as a teaching strategy, and problem solving frameworks.

2.2 MATHEMATICS PROBLEM SOLVING THEMES IN THE ESWATINI CURRICULUM

The education system in Eswatini consists of the following hierarchical components:

- Lower Primary (Grade 1 to Grade 4), a period of four years;
- Upper Primary (Grade 5 to Grade 7), a period of three years;
- Junior Secondary/ Junior Certificate (JC) (Form 1 to Form 3), a period of three years; and,

- Senior Secondary (Form 4 to Form 5), a period of two years.

2.2.1 Problem solving at the primary school level

According to the Education and Training Sector Policy (2018), the education and training sector aims "to empower the Eswatini people to analytically integrate, think critically about, and synthesize knowledge and come up with conclusions from complex material; communicate effectively in written, oral, and symbolic form; develop technical, mathematical and quantitative skills needed for analysis, calculation and problem solving" (p.7). Using problem solving as an instructional strategy to teach mathematical knowledge was an innovative initiative that the current study intended to add value to the improvement of education in mathematics

According to The Primary Syllabus (2016), teaching problem solving in the Eswatini mathematics curriculum starts in Grade 3. An expectation at this level is that learners can solve story-related problems linked to situations that inspire learners to make use of the four basic operations: addition, subtraction, multiplication and division. For the lower primary level, when solving problems, mathematical operations require a learner to apply and justify their logical strategy (Primary Syllabus, 2016). The Upper Primary (2016) syllabus further suggests that activities incorporating word problem solving must form a climax in most topics.

2.2.2 Problem solving at the secondary school level

The Junior Secondary, also known as Junior Certificate (JC) Syllabus (2016: 2), functions as an instrument that direct instruction and assessment in the classroom and guide examinations. The syllabus states that learners should have acquired knowledge, understanding and skills during their study of mathematics at the Primary Level (SPC). Therefore, the JC syllabus is packaged to offer a three-year course with an external examination in Form 3. The curriculum content of the syllabus is arranged into topics meant to cover four areas: (i) Numbers, (ii) Shape, Position and Space, (iii) Algebra; and, (iv) Data Handling. These four themes, or learning areas, are dealt with holistically throughout the course.

Overall, the JC syllabus is aimed at encouraging ingenious and instructional approaches that are innovative and oriented to encourage learners to enjoy the course. The JC Mathematics and Additional Mathematics Syllabus (2015) have devoted some part of its curriculum guidelines in addressing the problem-solving approaches. For instance:

> Learners engage in problem solving within contextual situations by communicating, reasoning and connecting to: representing and use numbers in a variety of equivalent situations by contextualized situations; explore, identify, analyse and extend patterns in mathematical and contextual situations; collect, organize and represent data; use and apply geometric properties and relationship to describe the physical world (JC Syllabuses (2015: 6-7).

The course is designed to evaluate learners' knowledge, understanding and ability. As such, it enables learners to progress to higher-level courses in the study of mathematics. The Junior Certificate encompasses topics in ordinary mathematics and in additional mathematics. In Form 3, all learners are provided with mathematical work that is presented at the same level of difficulty, therefore, they are assessed similarly in the identified competencies and skills. The following are two syllabi packages for mathematics in Form 4 and Form 5:

- The SGCSE level mathematics, core component; and,
- The SGCSE level mathematics, extended component.

In Eswatini, learners who have opted to do the core curriculum will eventually sit for an examination that allows them to qualify to be awarded a grade level of C to G only. Candidates who have opted to pursue the extended curriculum will qualify for a grade level of A to E only. Learners in the core curriculum write two papers, while those in the extended curriculum write three papers. Candidates who are expected to achieve a grade level of C or above are assessed on their performance on Paper 3 first to see if they can achieve grade levels A to C. In the case that these learners fall short, they are then assessed on their performance on Papers 1 and 2 (see, SGCSE Syllabus, 2016: 8; see, also, Table 2.1).

Table 2.1: Weighting of Papers

Paper	Weighting Core Curriculum (Papers 1 and 2 only)	Weighting Extended Curriculum (Papers 1, 2 and 3)
1	40%	40%
2	60%	60%
3	Not for Core Curriculum	100%

2.2.3 Assessment objectives for both JC and SGCSE

The SGCSE syllabus (2016: 3) indicates the following skills that should be developed: 1) communication and language skills; 2) independent learning; 3) technological awareness and applications; 4) numeracy competencies; 5) problem solving skills; 6) study and work techniques; 7) skills linked to critical thinking; and, 8) being able to work with others. It is worth noting that the mathematics syllabus shows a curriculum with the expressed goal of developing problem solving skills. Assessment objectives of mathematics in both JC Syllabus (2016: 5) and SGCSE Syllabus (2016: 5) spell out the abilities to be assessed that include a single assessment objective, technique with application. Therefore, learners should be able to:

1. Arrange, analyse and give out information appropriately in text, tables, graphs, and diagrams;
2. Calculate using appropriate approaches;
3. Utilize a calculator;
4. Understand to some extent the measurement systems applicable to real world use and apply them in solution-finding processes;
5. Estimate, approximate and perform to a certain level of accuracy with regard to the context or a related scenario relating to everyday life;
6. Apply mathematical tools to measure and generate drawings and sketches with a certain level of accuracy;

7. Interpret, change, formulate and utilize mathematical statements expressed as words or symbols;
8. Identify and make use of observed spatial relationships in either two-dimensional (2D) or three-dimensional space (3D);
9. Remember, implement and understand mathematics-related knowledge observable in real world or every day context;
10. Make valid and systematically sound conclusions by using the provided data;
11. Identify patterns and arrangements that appear in various situations;
12. Solve problems linked to almost non-structured settings by formulating it into a logical structured that can be systematically resolved;
13. Interpret a problem, choose an appropriate problem solving strategy and implement suitable problem solving techniques to reach the solution stage;
14. Effectively implement and apply a variety of mathematical strategies and problem solving techniques; and,
15. Formulate mathematical tasks, including the anticipated solution of the problems, in a systematic and logically sound format by making use of appropriate mathematical symbols and terminology.

2.3 DEFINITIONS OF A PROBLEM AND PROBLEM SOLVING IN MATHEMATICS

The definitions of a *mathematics problem* and that of *problem solving* are discussed in the next sections.

2.3.1 The definition of a problem

In the past, mathematicians and mathematics education researchers have provided several definitions of a problem and problem solving (for examples, see Gelbal, 1991; Guclu, 2003; Polya, 1962; Schoefeld, 1992). According to Schoenfeld (1992) "a problem could be a situation that one is unfamiliar with, whereby the problem solver is unable to execute its solution" (p. 827). Lupahla (2014) argues that "a problem is a task for which an individual fails to know instantly what to do to get to the answer" (p. 36). Saglam and Dost (2014) noted that, "grooming learners for life and making them well-equipped problem solvers both in business life and in their private lives has an important place in enabling them to acquire analytic thinking skills" (p. 303). In respect to this, mathematical problems provide essential tools to achieve such ends.

McDougal and Takahashi (2014) argue that the task may "not necessarily be a word problem to qualify as a problem. It could be an equation or calculation that learners have not previously learned to solve". Heddens and Speer (1997) argue that problems are comprehended as exercises needing basic computational skills to solve in mathematics courses. The Prism Alive Series (2010) has prescribed in Eswatini that secondary school mathematics should contain both equations or calculations (mathematical exercises) and word problems. Mathematical exercises are normally used for developing computational skills, while word problems are for application purposes. Whether something can be classified as a problem also depends on when the task is given. Early in the year before learners acquire a certain skill the task may be considered a problem. This same task can later become an exercise, since the learner now instantly knows how to find the solution. Mathematics teacher's choice of teaching strategy, therefore, determines whether a task is a problem or not (McDougal & Takahashi, 2014).

Okon (1966) further described a problem as "a theoretical or practical challenge that a learner has to solve independently by his own active research" (p.311). Dostal (2015) argued that "the base of this challenge is normally a systematic and intentionally organized situation, whereby the learner aims to subdue the challenges in conformity with the exact needs and by this he or she attains new knowledge and experience" (p.2). As indicated by the University of South Africa [Unisa] (2011: 219), a mathematical problem is one that meets the following criteria: the learner faces the task, genuinely wants to solve the task, has no usable procedure at hand for solving the task, and puts some effort into solving the task.

Schoenfeld (1988) differentiated between tasks that are exercise-oriented and those that are problems in mathematics. The author asserted that both types of tasks are of great value, but a lot of high school learners in mathematics classrooms participate mainly in exercises and seldom embark on solving mathematical problems. In this context a problem would then be perceived as a task that has no obvious solution method. Pennant (2013) further argued that a gap must exists between where one is and where they will get by embarking on a path towards obtaining a solution. Hence, learners need to think on and play with the problem for some time. They have to try out ideas, make hypotheses, go up 'impasses', and adjust their thinking according to what they have learnt from this, talk to others about the idea and be at ease in taking risks. When learners become confident with this process, they become capable of independently stepping into problems, instead of instantly turning to teachers for the solution.

When analyzing the definitions, one may therefore define a problem as a contextualized, non-routine situation whereby an individual does not know instantly what to do to obtain the solution. The problem being dealt with must be new, thought-provoking and the person confronting the problem must be willing to solve it. Literature generally categorized problems into two approaches: routine problems and non-routine problems. According to Polya (1957), routine problems are made by summing up dissimilar information to already solved problems. In this context, the solution finding process relies only on already known and familiar algorithm processes, rather than inventing and formulating new and non-familiar problem solving techniques. In this regard, Mayer and Hegarity (1996) noted that a routine problem appears when the person who is expected to provide the solution is familiar with the related problem solving process or the way of obtaining the expected and correct answer to the given problem.

Unisa (2011: 109) explained routine problems as those that can be solved by the application of an already known process. They also indicated that many simple mathematical problems fall into this category and are probably best referred to as drill and practice exercises. Furthermore, Lupahla (2014: 36) also referred to routine problems as those that are not hard to interpret and only need a single step. However, solving routine problems plays a significant role in acquiring computational skills (Altun, 2005). According to a mathematics module for Unisa (2011: 236), the problems found in many mathematics textbooks are typical routine problems. Generally, these are designed to afford learners practice in applying specific pieces of knowledge, or practice in applying certain problem solving strategies. Routine problems may require the use of certain computational strategies and making use of formulas.

Jurdak (2005) noted that non-routine problems may require one to rearrange the given information, formulate patterns and classify, while also being able to do proper computations. Furthermore, Mayer and Hegarity (1996) argued that a non-routine task exists when one does not have a clue, or has relatively little information, regarding the solution process, and is unable to see the solution because it is not conspicuous. As implied in a mathematics module for Unisa (2011: 234), non-routine problems are those that require the solution method to be ascertained as a component embedded within the process of solving the problem. Often times, such problems can be solved in multiple ways and may require a sustained effort. Therefore, for learners to come across good mathematics problems, the responsibility rests with the teacher's skill in integrating problems from several sources, not only from prescribed textbooks like the Prism Alive series in the case of Eswatini.

2.3.2 Defining the notion of problem solving

It looks like there are some discrepancies in the ways mathematicians and mathematics educators define a problem and problem solving. Some of these discrepancies may be brought about by a difference of opinion on what constitutes a problem, or by the many ways of presenting well-matched ideas concerning what is crucial in problem solving. For instance, Polya (1962) defined problem solving as "a way around an obstacle, achieving an aim that was not instantly achievable" (p. 2). The National Council of Teachers of Mathematics (NCTM) (2000) concurred with Polya's definitions of problem solving, specifically applying it to mathematics. In its Principles and Standards, problem solving is described as carrying out a task with no prior knowledge of its solution method. As learners acquire skills and knowledge about mathematics, the edification of this type of problem will increase. In order to find a solution, one must consider certain variables that are important, like recognizing the unknown problem-related data that is given and the structure of the problem, and then determine a pathway to a solution.

Kenney (2005, cited in Bain, 2010) describes problem solving as "a process that involves modeling, formulating, transforming, manipulating, inferring and communicating" (p. 7). Bain (2010) further claims that problem solving can be about taking information in the problem and changing it into another medium so that one is able to better identify solution strategy. For instance, a problem solver can translate words into a diagram or an equation. Mathematics educators perceive problem solving in mathematics in a number of ways. For example, Polya, (1945) comprehended problem solving as a heuristic process, while Newell and Simon (1972) perceived it as a program based on logic. Furthermore, a framework for goal-oriented decision making (Schoenfeld, 2011, 1985), a standard (NCTM, 1989), and a model-eliciting activity (Lesh & Zawojewski, 2007).

The way one perceives the purpose of problem solving to be, is affected by each conception of problem solving in mathematics (Schoenfeld, 1992). For instance, "comprehending problem solving as a process is pedagogically and epistemologically not the same as comprehending problem solving as discovery" (Silver, 1985: 1). Krulik and Rudnick (1980: 4) further described problem solving as acquired knowledge, skills and understanding that an individual has previously used to meet the demands of a situation that is unfamiliar. Thus, learners should synthesize the learned information, and utilize it in different and new situations.

Schoenfeld (1985) tendered the view that teaching and understanding mathematics ought to be approached as a problem solving domain. Therefore, beyond defining problem solving in mathematics, Schoenfeld (1985) outlined four competencies a problem solver needs to be successful, namely: 1) heuristics; 2) beliefs; 3) resources; and, 3) control.

> Resources have to do with the tools of mathematics needed by the solver to obtain the desired problem solution and heuristics are the different strategies used to solve a problem. Control is about metacognition and self-regulation whereby the solver meditates on his or her resources and heuristics. Beliefs concerns what preconceived opinion the solver has regarding problem solving in mathematics and mathematics (Schoenfeld, 1985).

The definition of problem solving given by The National Council of Teachers of Mathematics (NCTM) (2000) in its Principles and Standards was considered applicable to the current study.

2.4 CHOOSING TEACHING METHODS TO FACILITATE A LESSON

Even (2005, cited in a mathematics module for Unisa, 2011: 11) claimed that it is a generally understood fact that the actual teaching of mathematics is much more perplexing than the straightforward implementation of the national curriculum. In this way, balancing attention on the learner, the content and the curriculum becomes a challenge for a lot of teachers. Content may be taught by some mathematics teachers at the expense of the other two factors by concentrating on the delivery of textbook content regardless of learners' understanding. Therefore, when choosing a teaching method to use in a lesson, a mathematics module for Unisa (2011: 11) has encouraged teachers to prioritize teaching learners to understand mathematics as well as the relevant curriculum outcomes, over covering the entirety of the content.

2.4.1 Using problem solving as a teaching strategy

Mathematics educators have widely accepted that the principal objective of mathematics instruction should be developing learners' problem solving abilities, and that problem solving must play an integral role in the curriculum of mathematics programmes (Lester, 1994). Mills and Kim (2017: 1) also argued that the skills of problem solving do not necessarily develop naturally. They must be taught in an explicit manner such that they can be moved across multiple contexts and settings. According to Alsawaie (2003), the NCTM calls for programmes of teaching that will make all learners be capable of constructing new knowledge in mathematics through problem solving, solving mathematical problems that may arise and those appearing in other settings, employing and adapting a lot of suitable techniques in finding solutions to problems, and reflecting and monitoring the methods and strategies utilized to work out a mathematics solution to a given problem. The JC mathematics and additional mathematics syllabi (2015) devote some part of their curriculum guidelines to addressing curriculum approaches. For instance:

> Learners engage in problem solving within contextual situations by communicating, reasoning and connecting to: representing and use numbers in a variety of equivalent situations by contextualized situations; explore, identify, analyse and extend patterns in mathematical and contextual situations; collect, organize and represent data; use and apply geometric properties and relationship to describe the physical world (JC Syllabuses, 2015: 6-7).

However, the ways in which problem solving can be incorporated as an integral part of the curriculum remain very unclear (Lester, 1994). Rather than providing teachers with coherent directions to guide problem solving instruction, teachers are often left to their own devices, resulting in a well-intended attempt to teach story problems through very rigid and inflexible ways (Lester, 1994). The syllabi make no suggestions to teachers as to how problem solving should be incorporated into the classroom. That is why the current study strived to determine mathematics teachers' perceptions of problem solving in Form 1 at the Eswatini schools. Problem solving as a teaching strategy is also included by the National Curriculum Centre (NCC), which is the main vehicle for curriculum development in Eswatini, in its teaching and learning strategies. The NCC's main function is to "analyse and understand the Ministry of Education's educational policies as incorporated in official documents such as the Reports of Education Review Commissions, Imbokodvo Manifesto and National Development Plans, as well as to come up with objectives and yield educational programmes that can be used in the school system" (www.ibe.unesco.org/.../Mauritus.pdf). The NCC has provided a set of teaching and learning strategies that can be utilized in various subjects to promote learner participation during a lesson (see, Table 2.2).

Table 2.2: Teaching and learning strategies

♦ Memorization	- Educational visits/ Field trips
♦ Demonstrations	- Problem Solving
♦ Question and Answer	- Debate
♦ Project method	- Individual work
♦ Guest presentations	- Writing method
♦ Reading Aloud	- Showing method
♦ Singing method	- Drama
♦ Discussion	- Demonstration
♦ Experimenting	- Note-taking
♦ Games for Learning	- Lecture
♦ Group Discussion	- Silent Reading
♦ Simulations	- Resource person
♦ Role-playing	- Research using reference material

Hatfield (1978) argued that different approaches exist when problem solving is used as the focus for teaching mathematics. These include: (1) teaching *for* problem solving; (2) teaching *about* problem solving; and, (3) teaching *through* problem solving. According to a mathematics reader of Unisa (2009: 95), in a "traditional" approach to teaching, to get learners to be able to get solutions to complex mathematical problems, you first teach them the skills and concepts they would need (teaching *for* problem solving), then you would teach them the processes used to solve these types of problems (this would be teaching them *about* problem solving).

Teaching about problem solving centers around having learners research and work out problem solving strategies and processes (Ontario Ministry of Education, 2014: 11). After learners have developed their basic knowledge and their problem solving skills, you could then get them to utilize these skills in familiar contexts and beyond; getting learners to use their problem solving skills to acquire something new, be it in mathematics or other fields of study (teaching *through* problem solving).

It was the researcher's view that using problem solving as a teaching strategy was educationally important. Similarly, Mayo, Donnelly, Nash and Schwartz (1993) justified that problem solving strategy as a "teaching strategy where important, contextualized, real world situations are presented and guidance, resources, and instruction to learners are offered as they acquire problem solving skills and content knowledge" (p. 227). Mayo, Donnelly, Nash and Schwartz's definition of problem solving strategy was applicable in the study.

2.4.2 Problem solving classroom ecology

This study is concerned mainly with the processes that teachers can use to teach through problem solving. That is, using problem solving as a technique to help learners to learn other things. Mathematics teachers should make and uphold a suitable learning climate for learners to learn through problem solving. Alsawaie (2003) contended that teachers need to choose rich and suitable problems, organize their use, assess learners' understanding and apply strategies to assist their learners to become problem solvers. According to Donaldson (2010), "teaching through problem solving is an instructional approach where teachers utilize problem solving as the main means to teach mathematical concepts and assist learners synthesize their mathematical knowledge" (p. 5). Having learners develop, broaden, and ameliorate their understanding by finding solutions to problems is the focus of using problem solving as a teaching strategy (Hiebert & Wearne, 2003: 5).

Taplin (2015: 1) agreed that concentrating on teaching topics in mathematics through problem solving contexts is observed when the teacher helps learners to formulate an in-depth understanding of their own interpretation of ideas in mathematics. This could be done by "letting them carry out mathematics such as creating, testing, exploring, verifying and conjecturing" (Lester *et al.*, 1994: 154). According to McDougal and Takahashi (2014: 114), a lesson based on teaching through problem solving begins with the teacher establishing the context and introducing the problem. Learners then try to find the solution to the problem using the problem solving strategies for about ten minutes as the teacher keeps an eye on their progress.

The teacher also takes notes of the approaches used by the learners. The teacher would then model problem solving. "Modeling problem solving consists of the following: demonstrating skills and concepts of mathematics, thinking aloud in order to offer learners insights into the metacognitive aspects of problem solving, and demonstrating a positive attitude and perseverance when confronted with challenges" (Donaldson, 2011: 90) Next, the teacher starts a discussion with the whole class. Just like in a lesson where the problem solving teaching approach is used, learners may be requested by the teacher to share their ideas. However, instead of concluding the lesson here, the teacher goes on to require learners to ponder and compare their various ideas; which ideas are wrong and why, which are correct, which are alike, which are more effective or refined. It is through such discussions during lessons that allow learners to acquire new ideas or procedures in mathematics.

2.4.3 Problem solving strategies

While learners are engaging in self-learning, consisting of single handed tasks and whole class discussions, they use problem solving strategies. Pressley (1995) described strategies as conscious and containable activities executing cognitive objectives. A suitable strategy causes the problem solver to ponder the meaning of both the mathematical equation and the problem sentence (Aydogdu, 2014: 54). According to Posamentier and Krulik (1998: 4120):

> The strategies of problem solving may comprises of working backwards, adopting different viewpoints, discovering a pattern, creating a drawing, solving an easier or analogous problem, considering extreme cases, well-informed guessing and testing (approximation), explaining all possibilities, logical reasoning and coordinating data.

When learners solve problems during self-learning or whole class discussion, they make use of the problem solving strategies in the problem solving mathematics classroom. By making careful moves such as pursuing productive leads and abandoning fruitless paths, the problem solver succeeds to solve the problem (Schoenfeld, 1985). Hatfield, Edwards, Bitter and Morrow (2007) emphasize that, the problem solving strategies help learners make progress in solving more challenging and hard problems.

2.4.4 A CONCEPTUAL FRAMEWORK FOR THE STUDY

According to Lupahla (2014), a conceptual framework is a set of broad theories and ideas obtained from pertinent fields of enquiry which assist a researcher recognize in the right manner the problem being studied, determine appropriate literature and frame their questions. The classroom ecology for problem solving and the strategies for problem solving were adopted from Intaros, Inprasitha and Srisawadi (2014: 4121) as conceptual framework of this study (see, Figure 2.1). Intaros, Inprasitha and Srisawadi (2014: 4121) claimed that this conceptual framework supports the empirical data for the problem solving mathematics classroom, which encourages learners to acquire information by themselves through finding solutions to the problems along each Phase of the lesson. The first Phase consists of posing a problem in the classroom. Learners may be asked the meaning of the problem. Phase 2 is learners' self-learning which consists of learners working either individually, in groups or pairs trying to find solutions to the problem using the problem solving strategies. The third Phase is whole class discussion and comparison. Learners share their ideas while the teacher try to identify those learners who have difficult in understanding the problem and offer suggestions in order to solve the problem. Phase 4 is about summarization through linking learners' mathematical ideas which came out in the classroom.

2.4.4.1 The classroom culture

Gradually generating a classroom culture that encourages a problem solving approach in mathematics is exactly what this study endeavored to achieve, namely, by generating and encouraging a mathematical problem solving culture within the mathematics teachers in Eswatini. This is crucial since self-reliant problem solving skills are important for 21^{st} century life and employment (Pennant, 2013). Pennant (2013) suggested that teachers can encourage learners to develop the skills required to solve problems by way of the classroom culture they make. When questioning and deep thinking are appreciated, mistakes and errors are educationally of value. All learners would participate and their suggestions are appreciated, being stuck is regarded as honorable and learners acquire knowledge from shared discussions with their teacher and peers, such a problem solving culture is attained.

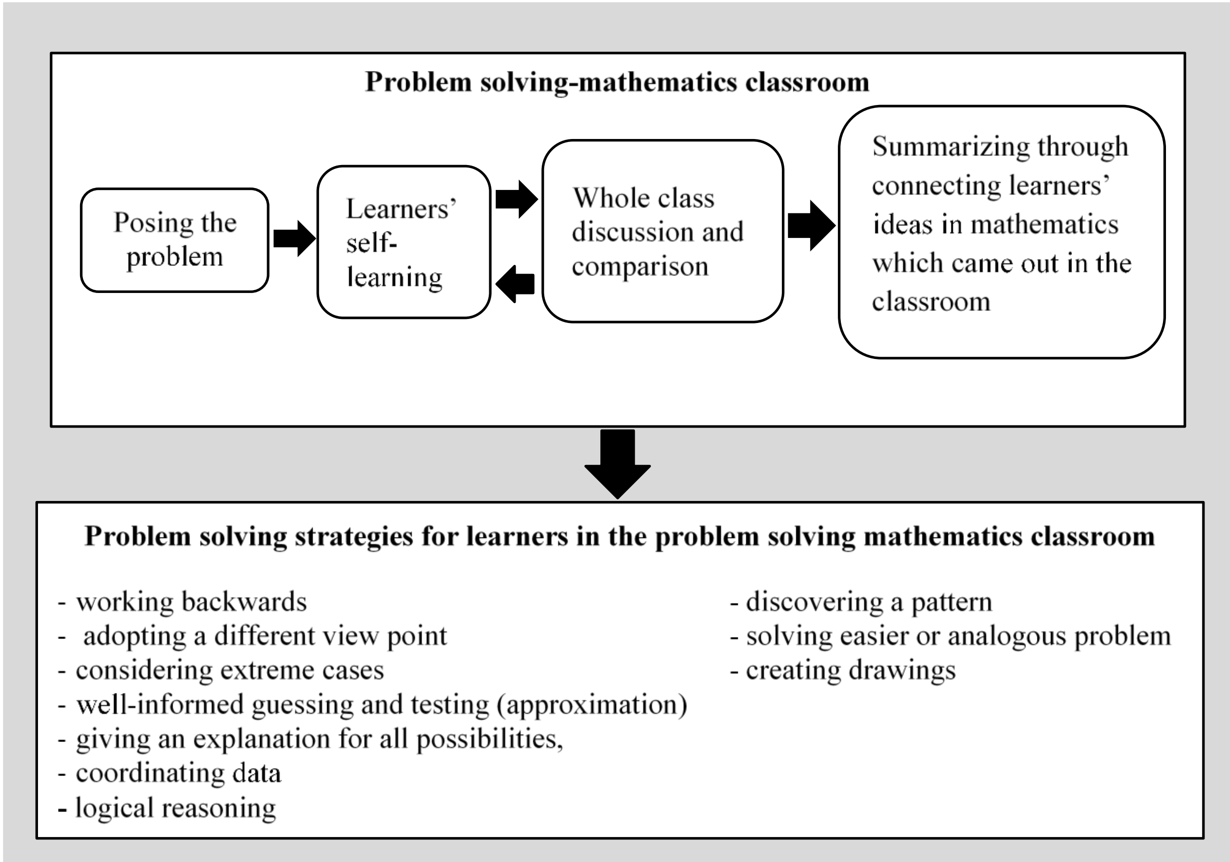

Figure 2.1: A conceptual framework used in the study

2.5 VARYING PERSPECTIVES ON THE GOAL OF PROBLEM SOLVING

For many years, a lot of perspectives concerning problem solving have been formulated by mathematicians and mathematics educators in mathematics education (Yuan, 2016: 9). There are two main perspectives that exist in problem solving instruction: direct instruction and constructivist instruction. According to Lester (2013), the argument over the benefits of these two perspectives has gone on for many years and may impact the way in which mathematics programmes are set up. It may also influence the method of problem solving instruction used in classrooms. Yuan (2016) argued that the main difference between direct versus constructivist instruction is the outcome goal of a problem solving task. Lester (2013) states that if problem solving is intended to be an end result of instruction, then learners should learn about problem solving. In a similar way, if problem solving is the means through which mathematical concepts are learned, then learners should learn through problem solving.

2.6 THE ROLE OF THE TEACHER IN MATHEMATICS PROBLEM SOLVING INSTRUCTION

Matheson (2012: 16) has argued that the role of the teacher from that of an informational front-end loader is changed when mathematics becomes problematic for learners. Thus, teachers are required to know how to give the right information at the right time to learners. Alsawaie (2003) indicated that it is the duty of the teacher to be aware of when learners require help as well as when they can carry on working productively without help. Therefore, it is crucial that learners are given time to research problems. Offering help too early can rob learners of their chance to create discoveries in mathematics. It is also absolutely essential for learners to be aware that a thought-provoking problem is time consuming and that persistence is a crucial element of carrying out mathematics and of the problem solving process.

Sweetland (2016) emphasized that the teacher's function during problem solving is to give hints not answers, provide a heuristic in problem solving, teach different kinds of problem solving strategies, grant learners some time to battle with the problem, select problems requiring some time to find the solution, offer dissimilar cases of problems, give similar problems in various ways and be a role model as the teacher solves the problem. Sweetland (2016) further said that the teacher should provide encouragement and appreciation. That is, encouraging learners to find more than one solution to a problem, to keep trying and to learn by correcting their mistakes as well as appreciate different solutions and strategies. In fact, this role begins before learners start to solve the problem effectively and continues throughout the teaching and learning process (Alsawaie, 2003).

2.7 METACOGNITION AND PROBLEM SOLVING

Research indicates that metacognition is central for all areas of academic achievement (Ward, 2012: 5). Alzahrani (2017: 522) defines metacognition as one's knowledge, monitoring and control of one's own systematic cognitive activity which requires particular metacognitive skills like planning and evaluation. According to Stephan, 2017: 1):

> A learner's ability to carry out metacognition may teach them to self-monitor their learning, the manner in which they pose questions of their thinking while in the process of learning, using information, and what strategies to utilize in order to help them through their thought processes.

Therefore, the extent to which a learner learns can be determined by the teacher's ability to skillfully reply to a learner's feedback in the classroom setting (Stephan, 2017: 1). Donaldson (20011: 24) stated that teachers can promote metacognition by displaying metacognitive behaviour, such as asking metacognitive questions and thinking aloud. The NCTM (2000: 260) argued that it is important that classroom instruction develop learners' metacognitive abilities. Hence, learners must be urged to assess and supervise themselves.

2.8 BENEFITS OF PROBLEM SOLVING IN MATHEMATICS

In professional and every day settings, problem solving is generally viewed as a crucial and significant cognitive articulation (Aksoy *et al*., 2015). Educators propose that problem solving be utilized as a general teaching and learning approach because of its amalgamating purpose in mathematics curricula (Cai, 2003; Cockroft, 1982; National Council of Teachers of Mathematics [NCTM], 1989). It is thought that this approach can assist learners acquire a much deeper and better understanding of mathematics. Since understanding is an internal and unobservable phenomenon which takes place when learners' minds incorporate new information with previous understanding, it cannot be taught directly. Therefore, using problem solving as a teaching strategy is a very strong way of promoting this kind of thinking (Lambdin, 2003).

Yavuz, Karatas, Arslan and Erbay (2015) argued that in the teaching and learning of mathematics lessons, the problem solving process is of value. Since problem solving is a scientific method, it needs reflective thinking, critical thinking, analysis, and creative and synthesized abilities. Ersoy (2016: 79) stated that concentrating on problem solving in lessons builds up the learners' high-level thinking. It is for this reason that learners are able to self-teach in mathematics lessons with the problem solving process. Chauraya and Mhlolo (2008: 73) asserted that the benefits of problem solving include the fact that it is a learner-centered approach where learners investigate, as well as research mathematical ideas by themselves. When learners validate solutions, they develop evaluation and reflection skills on the entire process of solving the problem.

According to Brehmer (2015:12), teaching mathematics problem solving to teachers develops general cognitive skills and encourages learners to learn mathematics. Brehmer (2015) highlighted the importance of developing learners' ability in mathematics problem solving was agreed upon by educators. This is reflected in the national steering documents of many countries (MOE, 2007; NCTM, 2000; Skolverket, 2012), which focus on mathematics problem solving. In addition, through mathematical problem solving, learners are able to learn and develop the practical and logical skills they need to be successful in everyday life (www.kevbotlearning.weekly.com). Chauraya and Mhlolo (2008: 75) concurred that there are rich educational benefits in using problem solving in mathematics instruction.

It seems the benefits are: allowing learners to actively participate, giving an opportunity to apply their mathematical knowledge and skills, furnishing rich experiences for learners to have the pleasure of discovery, learning new mathematical concepts with greater understanding, nurturing positive attitudes towards mathematics, developing thinking, problem solving and cooperative skills, and developing flexibility and creativity.

2.9 CHALLENGES ENCOUNTERED BY TEACHERS WHEN PROBLEM SOLVING IS USED AS A TEACHING STARTEGY

The methods of problem solving often contrast with teaching methods that teachers use most frequently in classroom or traditional lecturing. Kim (2005) described traditional teaching methods as "one that takes on the following steps: introduction, development and review" (p.13). Similarly, Akhter, Akhtar & Abaidullah, 2015: 3) argued that traditional teaching normally develops memorizing and assessing the learner's knowledge of the content. Chauraya and Mhlolo (2008) investigated in-service mathematics teachers on their conception of problem and problem solving in the subject. The authors found that the respondents' conception of a mathematical problem was a task with an explicit solution procedure, or one that required the application of learned and clear skills and procedures. With such conceptions, classroom mathematics problems are likely to remain confined to the drill and application type of task as encountered in most standard mathematics textbooks.

Zanzali (2003) stated that examinations which define how and what should be taught in mathematics influences teachers' perceptions regarding what problem solving is and how it is relevant. The author argued that teachers possess little influence on the mathematics content of the curriculum, and thus, consider modifying and adapting it is out of their control. In addition, Zanzali (2003) claimed that perceptions projected by most teachers are still traditional in nature despite various efforts made by the Curriculum Development Center of the Ministry of Education to transform them. Akhter, Akhtar & Abaidullah, 2015: 3) argued that the shortage of time and the huge amount of material to be learned by learners makes it challenging to always use problem solving in all elements of the teaching process.

Akhter, Akhtar and Abaidullah (2015: 4) claimed this is due to the large number of learners in a class which results in needing more time than usually expected, both in terms of preparation and implementation. Furthermore, "problem solving method is not accommodated properly in the curriculum that relies too much on textbooks and an assessment system overburden with formal examination that reinforce recall skills" (Akhter, Akhtar & Abaidullah, 2015: 4).

2.10 TRADITIONAL METHOD OF TEACHING MATHEMATICS

Various teaching methods like traditional and problem solving have been employed in educational system all over the world. The Platonist, formalists, behaviourists and structuralists models of teaching mathematics belong to the traditional paradigm of teaching (Unisa, 2011: 12). This is because there is a narrow line that divides them in that they all use the transmission principles of teaching mathematics. According to Bonato (2018), the teacher is regarded as the only knowledgeable person and is the only person who can impact appropriate knowledge to learners, who are empty of the universal knowledge. Therefore, in a mathematics class which uses a traditional method, the teacher talk, demonstrations or worked examples are the means of transferring the mathematical knowledge to learners (Unisa, 2011).

Bonato (2018) emphasized that in traditional methods, rules are taught first and then drilled into learners via memorization and solving problems. For instance, a mathematics lesson based on the traditional method could have learners told the rule that the order of multiplication of two numbers does not matter. The teacher could demonstrate some examples on the chalkboard and then learners would work on problems related to that topic. Although the traditional method of teaching mathematics is still adhered to in some schools, it is teacher centered, it lack collaboration and group learning, teacher acts as a mode of knowledge disperser rather than a facilitator and there is more emphasis on examinations rather than understanding of concepts (Nazzal, 2014).

2.11 TEACHERS' PERCEPTIONS OF A MATHEMATICAL PROBLEM AND PROBLEM SOLVING

According to Ekici (2013), the knowledge levels, perceptions and views about problem solving process of teachers are of great importance for them to teach the problems to their learners and use problem solving skills in their life. Chapman, (2013) concurred with Ekici (2013) in that teachers need to hold knowledge of mathematical problem solving for themselves as problem solvers and to assist learners to become better problem solvers. Teachers' knowledge and abilities to reason abstractly, make sense of word problems and progress through problem-solving tasks are critical elements for teachers' mathematical problem-solving teaching success (Yee & Bostic, 2014).

In a study conducted by Kaino and Yaqiang (2004), it was reported that mathematics teachers had an average understanding of problem solving, and both teachers and pupils had low scores in solving a mathematical problem. Andesta (2012) stated that teachers are still likely to have the perception that problem-solving problems are application problems. Lee and Kim (2005) investigated a group of elementary school teacher candidates' perceptions of 'good problems' and found that the majority considered typical routine problems as good and showed strong resistance to some non-routine problems that have atypical characteristics. The same applies to Hiltrimartin (2017) who investigated teachers' perception about problem-solving task and found out that the question procedures are not usual non-routine.

2.11 SUMMARY

The Chapter began with an overview of problem solving themes in the Eswatini mathematics curriculum, problem solving at the primary and secondary school levels and assessment objectives of the SGCSE core and extended components. A description of definitions of a mathematics problem and problem solving in mathematics followed, as well as discussions on choosing a teaching method to use in a lesson, using problem solving as a teaching strategy, varying perspectives on the goal of problem solving, the role of a mathematics teacher when using problem solving as a teaching strategy, and metacognition in problem solving. Finally, the Chapter reflected on the research-based, documented gains linked to problem solving in mathematics classrooms. Also, discussions on possible challenges, if any, encountered by teachers when problem solving is used as a teaching strategy and the traditional method of teaching mathematics formed part of the concluding remarks in this Chapter.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

This study aimed at determining mathematics teachers' perceptions of problem solving in Form 1 at Eswatini schools. The research methodology that was opted to achieve this aim is, thus, of important consideration. This Chapter begins with a discussion of the research design and how it was executed. A comprehensive reflection on the data collection and data analysis processes that characterized the current study is captured in this Chapter. The criteria used to select the study participants and issues of trustworthiness that ensured trustworthiness of the whole research process are also discussed. The Chapter concludes with a brief discussion on the pretesting of data collection tools that occurred and how the ethical issues entrenched in the study were handled.

3.2 RESEARCH DESIGN

Since teaching is nuanced and complex, additional meaning may be established by the scrutiny of a small number of teachers, as compared to a larger sample. This research used a qualitative approach to determine and describe how teachers perceived the notion of problem solving in mathematics. A qualitative research method was used because this type of research method has a lot of important features that corresponded well with the nature of this research study. Rather than producing numerical data to support or refute a clear-cut hypothesis, this study opted to generate factual descriptions of problem solving in mathematics based on personally conveyed knowledge of mathematics teachers.

The research needs of the current study suited a case study strategy. Merriam (1998) defined a qualitative case study as "an intensive, holistic description and analysis of a bounded phenomenon such as a program, an institution, a person, a process, or a social unit" (p. xiii). In line with this view, the usage of case studies allows for a deep, holistic perspective of the research problem, making it easier to describe, comprehend and clarify. A case is a thing, a single entity, a phenomenon of some sort occurring in a bounded context (Merriam, 1998). The case in this study was problem solving.

Yazan (2015) contended that a case can be studied by making use of three types of case studies depending on the aim and purpose of intended research. These can be classified as: (1) a particularistic case study; (2) a heuristic case study; and, (3) a descriptive case study. *Particularistic case studies* try to focus on particular situations, event, program, or phenomenon. It is possible that when a particular event, program, or phenomenon is examined, a more general aspect of the problem may be illuminated. When examining the particular event, program, or phenomenon it may or may not be influenced by the researcher's bias. Yazan (2015) also asserted that:

> *Heuristic case studies* involve illuminating the reader's understanding of phenomenon under study. This can bring about the discovery of new meaning, extend the reader's experience, or confirm what is known (p.148).

Lastly, the *descriptive case study* endeavours to yield a rich, thick description of the phenomenon under study. The descriptive case study describes the influence of people such as differences of opinion of those interviewed. Given this background, the descriptive case study was used in the current study as it was observed that its features would fit with the aim of the study, which was mainly to develop insights into what constituted teachers' perceptions of mathematical problem solving in Eswatini.

3.3 RESEARCH SITE AND PARTICIPANTS

The choice of site and the sampling techniques used for participant selection are discussed in this section.

3.3.1 The research site

The current study was conducted in four schools located in the Manzini Region of Eswatini. Eswatini is bordered by South Africa and Mozambique (see, Figure 3.1).

Figure 3.1: The map of Eswatini with four regions

3.3.2 The study participants

Four teachers who were teaching mathematics in Form 1 participated in the study. The four teachers were drawn from four secondary schools located in the Manzini region (Section 3.3.1; see also, Figure 3.1). Form 1 was chosen since problem solving is a strand in primary school mathematics but not in secondary school mathematics in Eswatini. Learners who are in their first year of secondary school are still fresh on problem solving lessons from primary school. Form 1 teachers need to utilize these competencies coming with learners to cultivate and promote problem solving abilities, which are largely embedded in mathematical topics and tasks encountered at secondary school level. To accomplish this, teachers need to have a solid and mathematically coherent perspective of problem solving.

3.3.3 Sampling procedures and techniques

This section reflects on the sampling procedure and the sampling criteria that informed the selection of the study participants (Section 3.3.3). In this study, maximum variation purposive sampling was used to get hold of the sample. This strategy is also known as "heterogeneous sampling" because it involves the process of selecting participants who have shared similar experiences or events (characteristics) but who may be scattered across a broad spectrum (Liker, Sulaiman & Rukayya, 2016: 3). The Manzini region is geographically large and provides an ideal space to generate desirable variations in teachers' classroom experiences and wide-ranging conceptions in relation to mathematical problem solving. Table 3.1shows a description of schools where teachers were sampled for the study.

Table 3.1: Description of sampled school

School	Description
LL	• Newly establishing mixed sex school, in the rural area, community school
LN	• Well established mixed sex school, in a semi urban area, supported by Government
ST	• Well established single sex school, in the urban area, it's a mission school
SC	• Well established mixed sex school, in the urban area, it's a mission school

Thus, multiple and varied forms of teachers' perceptions of mathematical problem solving were expected to be generated. The maximum variation sampling strategy provides a space for schools that are in seemingly varied educational settings to coexist as all of them offer mathematics as a subject. The sampling of such teachers could provide deeper insights into how problem solving in mathematics is perceived, and also understood among different teachers, in different schools and at different times, therefore reaching the aim of this study.

The researcher recruited participants by moving from one targeted school to the other, requesting mathematics teachers to participate in this research. For a school to fit the description of a target school it needed to host teachers who met the three requirements set out in the criterion to participate:

> (1) Participants must have experience of more than 3years teaching mathematics in secondary school at Eswatini. It was the researcher's view supported by literature that in exploring the different teaching strategies of mathematical problem solving, teacher experience could bring about more plentiful and rich opportunities if they had been teaching for at least a few years. Kini and Podolsky (2016) stated that teaching experience is positively related with learner achievement gains throughout a teacher's career and when the teacher accumulate experience in the same grade level or subject, teacher effectiveness increases at a greater rate;
> (2) Participants may be teaching mathematics, but not have qualifications for this subject in an educational space (there were teachers who possessed other forms of qualifications like Bachelor of Science in Agriculture but could still teach mathematics); and
> (3) Participants must be teachers drawn from schools around the Manzini region at the time the study was conducted.

The researcher asked the teachers to participate voluntarily and provide written consent to ensure that they would not feel pressured or coerced into participating. Four teachers participated in the study interviews, two of which also participated in lesson observations, all located in different schools.

3.4 DATA COLLECTION INSTRUMENTS FOR THE STUDY

To address the research questions of this study, semi-structured interviews and lesson observations were conducted.

3.4.1 Naming of instruments

Semi-structured interviews were conducted. Interviews form an essential component of numerous data sources in qualitative research (Harrell & Bradley, 2009). The interview method is mainly used in qualitative research methodology to enhance the collection of individuals' unique experiences (Harrell & Bradley, 2009; Lupahla, 2014: 87). The methods of data collection in qualitative studies emphasized by Lupahla (2014: 87) are well in line with the goal of this study (Section 1.4). After the semi-structured interviews, lesson observations were carried out as a follow up on two of the interviewed teachers.

3.4.2 Purpose of data collection instruments

Interviews opened a room for teachers to relate their views and understanding on the meanings they held on mathematical problem solving, and their conceptions of mathematical problems, problem solving, the nature of problem solving questions and the tasks they pose to Form 1 mathematics learners. It was anticipated that semi-structured interviews could offer a chance to communicate possible challenges, if any, encountered when problem solving is used as a teaching strategy.

Lesson observation enabled the researcher to observe the verbal actions and physical ways in which teachers were incorporating mathematical problem solving in their teaching and instructional practices. Lesson observation prevented teachers from just claiming incorporating mathematical problem solving in their mathematics classrooms.

3.4.3 Development of instruments

In this section the process of developing the data collection instruments for the study are discussed.

3.4.3.1 Interview Schedule

The interview schedule was adopted from Donaldson (2011) and a few modifications were made to suit the current study. The few changes were context based with respect to Eswatini. For instance, Donaldson simply asked the teachers what comes to their mind when they hear the term problem solving in mathematics. The researcher first stated that in the first mathematics symposium for 2016 as well as at a mathematics teachers' workshop in 2016 and 2015 organized by the Mathematics Department of the Ministry of Education, teachers were encouraged to use problem solving methods in teaching mathematics. It was then that the researcher asked teachers what comes to their mind when they hear the term problem solving in mathematics. On the question on how teachers incorporated problem solving into the mathematics class, The researcher began by stating that the Swaziland Education and Training Sector Policy (2011), the Swaziland General Certificate of Secondary Education (SGCSE) (2015), the Junior Certificate (2016) syllabuses and the prescribed Prism Alive series (2010) for Forms 4 and 5, all outline goals that refer to problem solving. Then the question was posed. The interview schedule from Sophia's (2016) study was proved to be reliable in the study in which it was implemented. It was trialed before the main study and study results proved to be reliable since it was able to facilitate the answering of the research question for the current study.

Even though most of the semi-structured interview questions may be prepared in advance, a dialogue form interaction is often used to make the questioning method more conversational, other spontaneous questions may also arise during the discourse and subsequent probing instances (Lupahla, 2014: 87). The semi-structured interview method was open-ended enough for teachers to share their personal and lived experiences, and instructional expertise they employ while communicating a mathematical problem solving teaching approach in their classrooms. The questions were so much guided that each teacher stayed on the desired topic. Such guided questions included: *What do you think a mathematics problem is?; What comes to your mind when you hear the term problem solving in mathematics*?; and *What do you consider to be good ways to help learners become better problem solvers*? (see, Appendix G).

3.4.3.2 Lesson observation schedule

For triangulation purposes, lesson observations were conducted as a follow-up from semi-structured interviews. Two teachers (SC02 and LL03) participated in the lesson observations. SC02 and LL03 were selected purposively because (1) during the semi-structured interviews SC02 and LL03 indicated that they incorporated problem solving in their mathematics lessons (see, Section 4.3; see, also, Tables 4.3, 4.5 & 4,6); and, (2) during the semi-structured interviews SC02 and LL03 indicated that they used problem solving as a teaching strategy in their mathematics lessons (see, Section 4.3; see, also, Tables 4.5 & 4,6). For this reason, the researcher saw the need to make a comparison between data collected from the semi-structured interviews and lesson observation. Since the semi-structured interviews preceded the lesson observations, the latter served to confirm and verify the former.

According to Statistics Solution (2017), "triangulation necessitates using data sources, multiple methods, theories or observers in order to attain a more complete understanding of the phenomenon being studied" (p. 1). A lesson observation schedule was adopted as is from Donaldson (2011). Donaldson (2011) piloted the observation schedule before their main study and was able to generate results that proved to be reliable. The lesson observation in the Donaldson (2011) study was judged to be reliable or to produce qualitative results that were trustworthy because it was able to answer the research questions.

The practices for problem solving instruction as provided in Appendix H included sub-headings like, (1) teaching problem solving strategies, (2) modeling problem solving; (3) limiting teacher input; (4) promoting metacognition; and, (5) highlighting multiple solutions. Further details of each practice are provided in Appendix H. Otherwise, The lesson observation guide consisted of questions or items such as: *Does the teacher demonstrate problem solving or particular problem solving skills?; Do learners work together to solve problems?; How does the teacher promote metacognition in the classroom?; and, Does the teacher encourage learners to find various ways to solve a particular problem?* (see, Appendix H).

3.4.4 DATA COLLECTION

Prior to collecting data for the study, permission was granted by the Ministry of Education (MOE, The Director) to carry out this research in prospective schools for educational purposes (see, Appendix B). Upon arriving at the schools, the school principal was met with and made aware of the MOE permission. They were then requested to identify their Form 1 mathematics teachers. In the current study probing was advanced to gain more clarity on teachers' responses (Gay, Mills & Airasian, 2011). Each respondent was interviewed individually in a private room after the official tuition time at their school. A schedule was used to ensure commonality and to guide the conversation towards the desired topic. Each interview lasting between 10-15 minutes. Only one interview session was anticipated with each mathematics teacher. Interviews were audio recorded and hand notes were also taken during session with each teacher. Interviewee confidentiality and anonymity were assured using pseudonyms and storing all collected data in a password protected folder on a laptop kept in a locked cupboard.

Two teachers participated in lesson observations during their lesson presentations, thus occurring without interruption to the normal school program. These teachers were purposively selected because they seemed to possess knowledge about problem solving and were occasionally using the problem solving approach in their classrooms to facilitate the teaching and learning of mathematics. This choice was made because it opened opportunities to explore the teachers' perceptions of mathematical problem solving in more depth. In my view, the selection of a teacher with a seemingly subtle knowledge of problem solving could limit the depth at which these issues could be explored in the study. Some of the foci of the lesson observations where the ways in which the teacher dealt with problem solving instructionally, strategies for learner engagement to promote meaningful problem solving dialogue in the classroom, and how the teacher handled learners' problem solving solutions.

Given that schools in Eswatini subscribe to a common syllabus program that advocates for the common pacing and sequencing of topics, it was anticipated that the duration of the mathematics periods would be similar in participating schools. Therefore, the lesson observation schedule was implemented similarly in terms of duration of lessons. Each lesson was audio recorded, noted with hand notes and visually recorded with a camera used to capture crucial inaudible data like items written on the chalkboard. The audio recorder helped me to gather thoughts and ideas more accurately. I arranged and decided with the certain teachers on the day and time in which the lesson observations were to take place. I suggested no specific topic, as it was the teacher's discretion to select a topic to teach in mathematics. The teacher and learners were regarded as participants in the lesson observation while I was a non-participant, just an observer.

Participants reflected on the daily teaching and learning of mathematics in the classroom. In each lesson observation, any questions that were a follow up from what was discussed during the lesson was written down. That pertained questions that has to do with teacher's reasoning behind a certain issue or decision which came to mind due to something that took place during class. I did not focus much on the specific mathematical content of the lesson but took note of mathematics, with the examples and problems posed during the lesson. For instance, one way of using problem solving as a teaching strategy is to stress that a problem can be solved using multiple methods. Giving a complete description of such a case, it was significant that certain problems and solutions that teachers and learners discussed were written down.

3.5 ENSURING THE TRUSTWORTHINESS OF THE STUDY

The study achieved its trustworthiness by paying attention to the following constructs: credibility, conformability and dependability.

3.5.1 Credibility

Credibility refers to the degree to which the study represents the actual meanings of the research participants (Moon, Brewer *et al.*, 2016). In the current study, credibility was enhanced by using a method of triangulation and the fact that both the interviews and lesson observations were conducted by the researcher. Semi-structured interviews and lesson observation were used to "assist[s] the researcher to cross-examine the integrity of participants' responses and it reduce bias" (Anney 2015: 277) as well as to achieve an in-depth understanding of the phenomenon that the study was focusing on. To further enhance the credibility of the study, a summary of the interview was then given to the teachers for confirmation of accuracy to increase credibility and reliability.

3.5.2 Dependability

According to Korstjens and Moser (2018), dependability is the stability of findings over time. In the current study the quality of dependability was established as follows: (1) the research steps were transparently described from the beginning of the research to development and reporting of the findings (see, Sections 3.3 & 3.4); (2) all interviews were conducted by the researcher; (3) all interview questions were asked in the sequence in which they appeared in the interview schedule and using the same words; and, (4) all interviews were conducted after contact time, at school, and for 10 to 15 minutes with all participants. Adherence to these research procedures ensured that the study's results are similar and can be replicated if the study was repeated using the same context, methods and participants.

3.5.3 Confirmability

Confirmability is the degree to which the findings of the research could be confirmed by other researchers (Korstjens & Moser, 2018). To achieve confirmability, triangulation was considered to reduce the effect of investigator bias. Pandey and Patraik (2014) described triangulation as ascertaining the consistency of findings produced by data collection methods that are different. Semi-structured interviews and lesson observation were used to determine teachers' perceptions of problem solving in mathematics. A teacher may claim to be using the problem solving method in an interview yet demonstrate a lesson based in the traditional methods of teaching mathematics when observed during lesson observation.

3.6 TRIALLING THE RESEARCH PROCESS

Crossman (2017) described a trialling or a pilot process as "a preliminary small-scale study that researchers conduct to assist them to decide how best to conduct a large scale research project" (p. 1). In the current study, I conducted a pilot process before the actual research activities were facilitated. According to Teijlingen and Hundley (2017), a pilot or trialling process helps to assess the efficiency and applicability of data collection instruments. The pilot process may also help to anticipate the feasibility of the intended full-scale study, and overall, to evaluate the practicality, applicability and feasibility of the research methodology that informs the envisaged study (Teijlingen & Hundley, 2017). In addition, a pilot study may help to reveal the inherent problems to the study and its methodology.

The items for the semi-structured interviews and the lesson observation guide were pre-tested on a group of four Form 1 mathematics teachers from four different schools. The pilot schools were chosen because they were located closer to the researcher and met the sampling criterion that guided the selection of schools in the main study. In this way the researcher was content that the pilot schools closely resembled the schools that would later participate in the main study. For instance, during the pilot study, one teacher described problem solving as teaching using a scenario whereby the teacher bring real life problems involving numbers and then learners carryout whatever strategy they think of to come up with the solution to a problem.

Another teacher described problem solving as giving a problem to learners where they are to solve that particular problem using mathematical concepts. The results from the pilot study suggested to a certain extent that such descriptions of problem solving were observed to be in line with literature definition (Kenney, 2005, cited in Bain, 2010: 7; Mayo, Donnelly, Nash & Schwartz, 1993; NCTM, 2000).

3.7 DATA ANALYSIS

The study focused on teacher experiences and it was open-ended in nature. The framework analysis and the use of the conceptual framework are two approaches employed to analyse the qualitative data in the study. It was important for the data analysis process to carefully analyse the key themes (of the study) in relation to teacher's responses. According to Gale et al (2013), the framework analysis is an approach that attempts to identify commonalities and differences in qualitative data, before focusing on relationships between different parts of the data, thereby seeking to draw descriptive and/or explanatory conclusions clustered around themes. Central to the approach is the development of a 'thematic framework' specific to the research study. This enables the researcher to label, classify and organise data in relation to main themes, concepts and categories (Ritchie et al 2010). The classroom ecology for problem solving and the strategies for problem solving were adopted from Intaros, Inprasitha and Srisawadi (2014: 4121) as conceptual framework of this study (see, Figure 2.1). The use of a conceptual framework to analyse the qualitative data was to act as a comparison base where the researcher can relate available problem solving concepts with the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom during the study. Generally, Table 3.2 indicates the steps that were considered when analyzing data from both the semi-structured interviews and lesson observations.

Table 3.2: Steps followed to analyse the qualitative data

Sequencing of data analysis (steps)	Method of analysis	How each step was carried out in current study
1	Familiarising yourself with your data	During semi-structured interviews and lesson observations, and while listening to the recorded interviews, notes were made. Responses relating to either a research question or objective were noted. Once the recordings were transcribed, the new interview script was read again to ensure that it was accurate and sense-making.
2	Identifying a thematic framework	An initial list of codes featuring interesting and meaningful data on problem solving instruction was constructed based on conceptual framework. Codes were grouped together into categories and then clearly defined. Different codes were sorted into potential themes and relevant data extracts were collated within the identified themes. Identified themes formed the basis of a thematic framework
3	Indexing	Portions or sections of data that corresponded to a particular theme were identified.
4	Charting	The data was lifted from its original textual context and placed in charts that consisted of the headings and subheadings that were drawn during the thematic framework,
5	Mapping and interpretation	Involved the analysis of the key characteristics as laid out in the charts.

Adapted from Jane Ritchie and Liz Spencer (1980)

Although the whole of each lesson was audio recoded, relevant sections of each recording were transcribed. A table with three columns was made when carrying out the process of transcribing the data. The first column indicated time, the second column was about the description of activities and the third column consisted of notes and coding. Table 3.3 shows an example of a transcript for a lesson observation.

Table 3.3: Transcript of a lesson observation

Teacher code: SC02
Topic: Triangles
Date: 2/11/17
Class: Form 1

Time	Activities	Notes/coding
1150-1200	-Teacher recapped from the previous lesson. -lesson objectives stated	- learners sitting in groups of 6
1200-1245	- Teacher posed the problem.	-Limiting teacher input
	- Learners worked in groups, solving a problem given by teacher	-Limiting teacher input
	-Teacher walked around groups providing guidance, "Note that the diagram is not drawn to scale, so it cannot be solved using a protector".	- Limiting teacher input
	-"Good people am happy that some of you are able to find angle x".	-Promoting metacognition
	-"What is the meaning of the short line across the sides of the upper triangle stand for?"	
	- Learners were asked to display their solution of angle x on the chalkboard	- Limiting teacher input
	-"Why did you add the sizes of the two angles and subtract them from 180˚?" asked the teacher.	-Promoting metacognition
	- Another group representative solved angle y and z on the chalkboard using the concept on vertically opposite angles.	- Limiting teacher input
	-"Is there any other method we can use to find the sizes of angle y and z?"	-highlighting multiple solutions.
	- "We can also solve angle y and z using the ideas of a straight angle and properties of an isosceles" said the teacher.	-highlighting multiple solutions
1245-1250	Teacher summarize through connecting learners' ideas which came out in the lesson.	

Part of the transcribed lesson consisted of quotations from the teacher or exchange of words between the teacher and the learners.

3.7.1 Coding

Relevant sections of the audio recordings of the lessons were coded after transcribing. Cases that involved the teacher engaging in a teaching practice related to problem solving were noted. An initial list of codes on problem solving instruction was constructed based on the conceptual framework of the study, adopted from Intaros, Inprasitha and Srisawadi (2014: 4121) (see, Section 2.4.4). The list consisted of: (1) limiting teacher input; (2) multiple solutions; (3) metacognition and (4) modelling problem solving

During the lesson observation, there were particular teaching practices that were easily identifiable. For example, limiting teacher input. Cases of limiting teacher input included: (a) learners working together to solve a problem during the lesson; and (b) assistance or guidance the teacher offers to learners as they solve a problem. The mere fact that learners sat in groups did not constitute limiting teacher input unless learners actually worked together to solve a problem.

It was sometimes a challenge to code modelling problem solving because the researcher wanted to make a differentiation between a teacher just demonstrating to learners how to find the solution to a problem and actually modelling the problem-solving process. The researcher did not code as modelling problem solving, if a teacher just engaged in "show and tell". If, however, a teacher showed his or her thought process or highlighted certain problem-solving strategies while finding a solution to a problem in front of the class, the researcher coded that as an instance of modelling problem solving.

The researcher coded as metacognition any question the teachers asked that focused to thought processes, knowledge, or decision making either theirs or the learners. In addition, any instance a teacher encouraged learners to monitor their problem solving including checking their work or reflect on the problem or solution, the researcher coded it as metacognition. In addition, the researcher coded as metacognition even when the teacher was observed encouraging learners to check their work or reflect on the solution or problem.

The researcher coded as multiple solutions if the teacher encouraged learners to find various ways to find the solution to a problem, when learners share their solution with one another or even when the teacher discusses connections between different solutions. A list of codes on problem solving was constructed based on literature. The conceptual framework for problem solving instruction (in Figure 2.1) emphasizes that after posing a problem, learners should engage in self-learning or/ and a whole class discussion. While engaging in self-learning or/and a whole class discussion, learners can make use of problem solving strategies. In addition to the codes, the researcher coded as teaching general or specific problem solving strategies if the teacher talked explicitly about problem solving or else the teacher mention or demonstrate general or specific problem solving strategies. The codes were then regarded as the study themes.

The researcher coded each teacher's interview using a similar method, making use of literature and taking note of cases in which the teacher referred to any of the items listed on the interview schedule (see, Appendix I).

3.8 RESEARCH ETHICS

In line with the Unisa research ethics policy, I looked at the following ethical considerations:

3.8.1 Full disclosure

The researcher was upfront and loyal to the teachers in relation to pertinent issues that characterized the current study. For instance, the researcher made it clear that he was not an officer, or an agent sent by the Ministry of Education, but was rather a teacher enrolled at Unisa conducting a study on teachers' perceptions of problem solving in mathematics in Eswatini.

3.8.2 Negotiating access

The researcher applied to the Director of the Ministry of Education to be granted permission to conduct the research activities in schools that participated in the study. Subsequently, the researcher requested permission from the school principals and from the governing body chairpersons of participating schools to gain access to initiate the research. The aim of the study was explained (Section 1.4), and participation was completely voluntary. In addition, the researcher applied for the research ethics clearance certificate from the institutional Research Ethics Committee (REC), and this was granted prior to the commencement of the study. The pilot study also occurred after I have obtained the ethics clearance certificate from REC.

3.8.3 Informed consent

The researcher provided a consenting form to all prospective study participants acknowledging that they understood the research (see, Appendix E).

3.8.4 Confidentiality and anonymity

Confidentiality has to do with researcher's efforts not to reveal the study participant's real identities, while anonymity emphasizes that the real names of study participants are not used when a report is generated from a research process (see, Wiersma & Jurs, 2009: 458). Prior to the commencement of the current study, all prospective participants were given an assurance that their confidentiality, anonymity and privacy would be observed respectfully. The researcher informed the participants that their responses would be aggregated, and that their real names and identifications would not be used. Lastly, I assured study participants that any information that would be deemed as providing a hint or traces to participant's names or identifications would be discarded in the final report. Code names, instead of actual names, were used in all reports of this dissertation; however, the school and teacher names were written on the interview schedule.

3.9 CHAPTER CONCLUSION

This Chapter furnished with an explanation of the study design and the method in line with the research methodology that the current study employed. These research components were systematically streamlined to lead to the eventual responding to the research questions of the study. A detailed account of the data collection instruments (i.e., semi-structured interviews and lesson observations) was provided in terms of their development, purpose and administration (Section 3.4). The identification and selection of study participants was also well documented in this Chapter (Section 3.3). Finally, this Chapter provided an explanation of the data collection and analysis (Section 3.7). The next Chapter will reflect on the actual data collection episode and the resulting analysis that gave rise to the findings of this study.

CHAPTER 4

DATA PRESENTATION AND ANALYSIS

4.1 INTRODUCTION

This Chapter presents the findings from the analysis of data drawn from the semi-structured interviews and lesson observations aimed at answering the following research questions (see, Section 1.6; see, also, Section 3.4):

- What meanings do teachers attach to mathematical problem solving?
- How do teachers in Form 1 conceive a mathematical problem?
- What kind of mathematical problem solving questions, and tasks, do teachers in Form 1 pose?
- What are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?
- What challenges, if any, do teachers encounter when using problem solving as a teaching strategy to learn mathematics in Form 1?

The process of data analysis followed the three phases, namely: (1) the analysis of data from the semi-structured interviews; (2) the analysis of data from the lesson observation; and, for triangulation purposes (3) presentation of the findings of the analysis of semi-structured interviews in relation to the findings of the lesson observations. The first phase of data analysis addressed the research questions 1, 2, 3 and 5 (Section 1.6). In this regard, participants reflected on their experiences and views regarding problem solving as a teaching strategy in mathematics lessons.

The second phase of data analysis addressed research question 4 and included descriptions of teaching practices that the teachers either demonstrated in the observed lessons or described in course of interviews (see, Sections 1.6 & 3.4). Literature reveals that there are mainly five teaching practices that characterize a problem solving lesson, namely: (a) teaching problem solving strategies (Aydogdu, 2014); (b) modelling problem solving (Taplin, 2015); (c) limiting teacher input (Alsawaie, 2003); (d) promoting metacognition (Donaldson, 2011); and, (e) highlighting multiple solutions (Ersoy, 2016). Two teachers participated in lesson observations to follow-up from semi-structured interviews (Section 3.4.3.2). The rationale to choose two teachers, out of the four study participants interviewed, is provided in Section 3.4.3.2. The third phase of data analysis was meant to compare results from the lesson observations and semi-structured interviews as a way of achieving triangulation to optimize the credibility and trustworthiness of the study (see, Section 3.4.4; see, also, Sections 3.5 & 3.5.1).

4.2 ANALYSIS OF DATA FROM SEMI-STRUCTURED INTERVIEWS

Semi-structured interviews were conducted with four teachers to explore their perceptions of problem solving in mathematics (Section 1.6; see, also, Sections 3.4 & 3.4.3.1). Analysis of data from the semi-structured interviews entailed the use of framework analysis (see, Section 3.7). Table 3.2 outlines the steps used as a framework to guide the analysis of data (Section 3.7). These steps were adapted from Jane Ritchie and Liz Spencer (1980) (see, Section 3.7). Five themes were highlighted and discussed in this section and these consisted of: 1) Teacher's conception of a mathematical problem; 2) Meaning of mathematical problem solving; 3) Advantages of using problem solving as a teaching strategy; 4) Challenges, encountered by teachers when using problem solving as a teaching strategy; and, 5) Teacher training on problem solving.

As part of the interviews, respondents were requested to provide information on their teaching qualifications and teaching experiences, for instance, *what are your teaching qualifications and for how long have you been teaching mathematics*? It was the researcher's view that this data would reveal the status teachers' professional qualifications in terms of their suitability to teach mathematical problem solving in Form 1 at the Eswatini secondary schools. Also, it was the researcher's view that teaching experience would be essential in exploring teaching strategies employed by teachers to facilitate mathematical problem solving since two of the interviewed teachers were to be further observed delivering their lessons. (Section 3.3.4). Four teachers participated in the semi-structured interviews (Section 3.3.3).

4.2.1 *Teachers' profiles*

Data in Table 4.1 represents teachers' responses relating to their professional qualifications and teaching experiences. Since the study aimed at finding out teachers' perceptions on problem solving in order to contribute to the improvement of Form 1 learners' mathematical problem solving performance in Eswatini schools, it was important to look into teacher's professional qualifications and teaching experiences. Participants may be teaching mathematics, but not have qualifications for this subject in an educational space (there were teachers who possessed other forms of qualifications like Bachelor of Science in Agriculture but could still teach mathematics). A qualified teacher should aim at learning which strengthens the capacities of learners to act progressively on their own through acquisition of relevant knowledge, useful skills and appropriate attitudes (Bernard, 1999 in Benegusenga *et al* 2017). Participants must also have experience teaching mathematics in secondary school and for more than three years. It was the researcher's view supported by literature that in exploring the different teaching strategies of mathematical problem solving, teacher experience could bring about more plentiful and rich opportunities if they had been teaching for at least a few years (Section 3.3.4). Ball, Thames and Phelps (2008) in Chapman 2015:19) also emphasized that general mathematical ability does not fully account for the knowledge and skills needed for effective mathematics teaching. The quality of a teacher itself is a crucial factor in the teaching and learning of mathematics.

Teachers who took part in the interviews were coded as ST01, SC02, LL03 and LN04 with capital letters standing for a school (see, Section 3.3.4), and the numbers representing the numerical sequencing of the interview sessions with teachers. For example, ST01 referred to a teacher participant in school ST (Table 3.1) who was the first on the interview list. Using this name coding would imply that LN04 was interviewed last.

Table 4.1: Teachers' profiles

Teacher code name	Teacher's qualifications	Teaching experience in years
ST01	Bachelor of Science (BSc) in Mathematics and Chemistry and Post Graduate Certificate in Education (PGCE)	10 -15
SC02	Bachelor's degree (BSc) in Mathematics and Chemistry and Post Graduate Certificate in Education (PGCE)	0 - 5
LL03	Secondary Teachers' Diploma (STD) in Mathematics and Science	20 - 25
LN04	Bachelor's degree (BSc) in Mathematics and Chemistry, Post Graduate Certificate in Education (PGCE) and Master of Science (MSc) in Chemistry	5 - 10

In Eswatini, a person should have at least a secondary teachers' diploma to be considered as a qualified Form 1 teacher. Table 4.1 reveals that all the respondents were qualified teachers with a major in mathematics. Teachers should be aware of a set of teaching and learning strategies that can be utilized in various subjects to promote learner participation during a lesson (see, Table 2.2) which include problem solving.

According to Kini and Podolsky (2016), teaching experience is positively linked with learner achievement gains throughout a teacher's career and when the teacher accumulate experience in the same grade level or subject, teacher effectiveness increases at a greater rate. Of the four teachers in Table 4.1, three had more than five years teaching experience. Hence, they brought plentiful and rich opportunities of their experience on to the study.

4.2.2 Theme 1: Teachers' conceptions of a mathematical problem

Table 4.2 presents teachers' responses of their conception of a mathematics problem. In relation to teachers' responses in Table 4.2 the following question had been asked (see, also, Appendix G):

Question: What do you think a mathematics problem is?

Table 4.2: Teacher's explanation or conception of a mathematics problem

Teacher/ respondents' code name	Teachers' conceptions of a mathematics problem
ST01	One that applies mathematics to real life situations that learners may be familiar with.
SC02	A question or scenario given in mathematics in which there is no obvious way or method to get to the solution. The problem solver has to think before attempting to solve the problem. It requires higher order abilities.
LL03	One that subjects the learner to reaching with proper understanding of the problem at hand in order to classify all conditions given before attempting to solve it. It should not allow the learner to just recall answers without subjecting them to proper thinking.
LN04	One that is worked out using a method so that one gets a solution

Schoenfeld (1992) stated that a problem could be a situation that one is unfamiliar with, whereby the problem solver is unable to execute the solution. It is observed from Table 4.2 that, to a certain extent, Schoenfeld's description of a problem is contrary to ST01's explanation of a problem. Schoenfeld stated that one should not be familiar with the situation while ST01 contends that learners may be familiar with the situation (Schoenfeld, 1992). McDougal and Takahashi (2014) concurred with Schoenfeld in that before learners acquire a certain skill the task may be considered a problem, but the same task can later become an exercise, since the learner now instantly knows how to find the solution.

LN04's definition of a problem referred to a task requiring clear solution procedures and the application of learnt method(s). It seems LN04's conception of a problem is like that of Heddens and Speer (1997). The latter perceived problems as exercises that need basic computational skills to solve in mathematics courses. Whereas, problems are not just confined to mathematics courses. Pennant (2013) defined a problem as one that has no obvious solution method requiring learners to think on, try out ideas, make hypotheses and adjust their thinking according to what they have learnt from this. Lupahla (2014) argued that, “a problem is a task for which an individual fails to know instantly what to do to get to the answer” (p.36).

In line with the preceding discussion, the definitions of a problem given by SC02 and LL03 in Table 4.2 were considered to be in line with literature definitions. The definitions of SC02 and LL03 featured the phrases like “no obvious way or method of getting the answer” and “not allow the learner to just recall answers without subjecting them to proper thinking”. By proper thinking LL03 was referring to engaging in the process of trying out ideas and making hypothesis.

4.2.2.1 Example of a mathematics problem

Table 4.3 provides a list of teacher’s responses to a question that asked them to provide the example(s) of a mathematic problem, as well as a frequency of giving word problems and the use of other sources for non-routine problems. In this regard the questions asked:

Questions: (a) Can you give me an example of a mathematics problem?

(b) How often do you give learners word problems in a particular exercise?

(c) Do you use other sources for non-routine problems?

Table 4.3: Teachers' responses on problem examples and frequency of giving word problems

Teacher code name	(a) Example(s) of a problem	(b)Frequency of giving word problems	(c) Use of other sources for non-routine problems
ST01	Find the height a flag pole when given information about the angle where you are standing and the distance from the pole to where you are standing.	Rarely	Yes, for problems that may not be in the prescribed book or may not be enough in terms of strength and number of questions
SC02	What three-dimensional shape has the highest volume and least surface area?	Very often	Yes
LL03	A girl is *m*-years old, her mother is four years older than twice the girls' age. The sum of their ages is 44 years. How old is each one of them?	Rarely	Yes
LN04	A sequence of numbers where learners can work out the next term (e.g., Find the 20^{th} term and the general way of finding n^{th} term	Rarely	No

Literature has generally categorized problems as routine and non-routine problems (Altun, 2005; Jurdak, 2005; Lupahla, 2014; Mayer & Hegarity, 1996; Polya, 1957; Unisa, 2011: 109). Lupahla (2014: 36) referred to routine problems as those that are not hard to interpret and only needing a single step. Solving routine problems plays a significant role in acquiring computational skills (Altun, 2005). Mayer and Hegarity (1996) argued that a non-routine task exists when one does not have a clue, or has relatively little information, regarding the solution process, and is unable to see the solution because it is not conspicuous.

LN04's example of a problem indicated in Table 4.3 was classified as relating to a routine type of a problem. The analysis of LN04's response in Table 4.3 presents a problem which can be solved by the application of a solution procedure that is already known. The latter is seemingly not difficult to interpret and may only need a single step to get the solution. ST01, SC02 and LL03's examples of a problem are those of a non-routine problem type. In the responses of ST01, SC02 and LL03 the problem solver may have no clue or relatively little information regarding the solution process. It is such problems that can be solved in multiple ways and may require a sustained effort. Sidenvall (2019) argued that the main reason learners experience difficulties in learning mathematics is due to the over emphasis on learning procedures and working with routine problems. Yet, learners' knowledge would improve if more emphasis was placed on non-routine problems.

Table 4.3 also reveals that only SC02 response referred to often giving learners word problems. Word problems may benefit learners in a number of ways. For instance, word problem calls for the learner to "parse the question critically, take into account what is really being asked and then use the proper approach to solve that certain problem" (Mathnasium, 2018; 4). Marsh (2018) concurred with Mathnasium (2018) reiterating that word problems develop logical analysis, boost creative thinking and mental skills.

Also, it was observed from Table 4.3 that ST01, SC02 and LL03 would refer to other mathematics textbooks for non-routine problems in their teaching of mathematics. In order for learners to come across good problems, the responsibility rests with the teacher's skill in integrating problems from several sources, not only from prescribed textbooks like the Prism Alive series in the case of Eswatini.

4.2.3 Theme 2: Meaning of problem solving in mathematics

In the first symposium for 2016 as well as at a teachers' workshop in 2016 and 2015, organized by The Mathematics Department of the Ministry of Education, teachers were encouraged to use problem solving methods in teaching mathematics. To elicit teacher's conception and meaning of mathematical problem solving the researcher posed the following question:

Question: What comes to your mind when you hear the term problem solving in mathematics?

Table 4.4 presents teachers' responses to the posed questions.

Table 4.4: Meaning of problem solving in mathematics

Teacher code name	Meaning of problem solving in mathematics
ST01	a situation that one wants learners to solve, probably a real-life situation where they will use mathematical concepts to attempt.
SC02	giving questions to learners that require them to reason or think before giving the solutions when teaching mathematics. There is no obvious way to get to the solution.
LL03	involves questions in the higher order in bloomy Taxonomy, starting with application, analysis, evaluation and synthesis, where learners are subjected to serious thinking to obtain a solution
LN04	an existing problem in the mathematics field where solutions need to be found

The National Council of Teachers of Mathematics (NCTM) (2000) described problem solving as carrying out a task with no prior knowledge of its solution method. According to Anderson and White (2004), problem solving is "the process of learners examining non-routine questions, making use of a number of strategies to work out unfamiliar problem and improving in processes of reasoning, analyzing, abstracting and generalizing" (p.127). Norton, McRobbie and Cooper (2002: 39) stated that problem solving is an approach in which "teachers consider themselves as guides, listeners and observers instead of authorities and answer givers". Problem solving is a "teaching strategy where important, contextualized, real world situations are presented and guidance, resources, and instruction to learners are offered as they acquire problem solving skills and content knowledge Mayo, Donnelly, Nash and Schwartz (1993: 227).

It may be observed from Table 4.4 that the respondents perceived the meaning of problem solving in mathematics differently from each other as well as from literature definition. The conception of a mathematical problem solving given by ST01, LL03 and LN04's seemed to be at variance with literature definitions (see Anderson & White, 2004; Mayo, Donnelly, Nash & Schwartz, 1993; NCTM (2000): Norton, McRobbie & Cooper, 2002). These authors defined a mathematical problem solving as carrying out a task with no prior knowledge of its solution method. It may be noted that SC02 ' s definition of problem solving featured phrases like "*giving learners a question and* *requiring them to think, without an obvious way to get to the solution*", which was considered to be in line with the literature definition of problem solving (NCTM, 2000). The results in Table 4.4 show that the phrase "problem solving" is seemingly problematic as it may connote and invoke various meanings on the teachers' understanding of mathematics teaching and learning. ST01, LL03 and LN04 thought of problem solving as the selection and presentation of a 'good' problem to learners while SC02 thought of program goals that emphasize techniques and strategies of problem solving.

4.2.3.1 Development of effective problem solvers

The Swaziland Education and Training Sector Policy (2018), the Swaziland General Certificate of Secondary Education (SGCSE) (2015), the Junior Certificate (2016) syllabi and the prescribed Prism Alive series (2010) for Forms 4 and 5, all outline goals that refer to problem solving. Table 4.5 indicates if respondents incorporated problem solving in their mathematics lessons, also, show the ways of helping learners to become better problem solvers. Table 4.5 shows that SC02 and LL03 incorporated problem solving in their mathematics lessons. Teachers' responses were followed up with the lesson observations (Section 4.3) and were then confirmed and verified.

A teacher may claim to be using the problem solving method in an interview yet enacting a lesson that is largely based on traditional methods when observed during lesson. Table 4.5 shows that ST01 and LN04 were not sure if they incorporated problem solving in their mathematics lessons. The latter is in line with the fact that in Eswatini teachers are not guided on how to infuse and implement problem solving into their mathematics instruction. Rather than providing teachers with guidelines to implement problem solving instruction, teachers are often left on their own resulting in a well-intended attempt to teach story problems through very rigid and inflexible ways (Lester, 1994). To generate teachers' responses in Table 4.5 the following questions were asked:

Questions: (a) Do you incorporate problem solving in mathematics lessons?
(b) What do you consider to be good ways of helping learners become better problem solvers?

Table 4.5: Development of effective problem solvers

Teacher code name	Teachers' responses to the question	Ways of helping learners become effective problem solvers
ST01	Not sure	Use problem solving strategy more often
SC02	Yes	train learners to read questions with understanding, helping learners understand the mathematics language;
LL03	Yes	subject them to many word problems that will require them to read with understanding, like mathematic contests or Olympiad questions starting from primary school.
LN04	Not sure	allow learners to explore the different ways of solving a problem

It was observed in Table 4.5 that all four teachers provided good ways of helping learners become good problem solvers which even concurs with literature assertions. Wood (2017) stated that teachers may utilize open-ended questions in their lessons so that learners carefully think and secure their answers as they question each other and discuss possible solutions. "In order for learners to see the value of working smarter in trying new and different strategies and revising their process, teachers need to make a classroom environment in which learners are problem solvers by connecting struggles to strategies" (Mills & Kim, 2017: 2).

4.2.4 Theme 3: Advantages of using problem solving as a teaching strategy in mathematics

Problem solving as a teaching strategy to facilitate learners' active participation is also included by National Curriculum Centre (NCC) in its teaching and learning strategies. Table 4.6 presents teachers' responses with regards to the advantages of using problem solving as a teaching strategy in mathematics. In this regard the question asked to the teachers was:

Q: Do you use problem solving as a teaching strategy in teaching and learning of mathematics? If yes, what do you think are the advantages/ benefits of using problem solving as a teaching strategy?

Table 4.6: Advantages of using problem solving as a teaching strategy in mathematics

Teacher name code	Teachers' responses to the question	Teachers' explanations/ elaboration of their responses
ST01	Not sure	Develops critical thinking
SC02	Yes	It develops learners who are critical thinkers; Helps them to be creative; Helps them enjoy mathematics; Helps learners to be independent thinkers; Helps learners create their own problems.
LL03	Yes	Learners become very independent and are able to solve their own personal problems
LN04	No	N/A

According to Mills and Kim (2017: 1), problem solving skills do not necessarily develop naturally and spontaneously. Alsawaie (2003) stated a need for teaching strategies that cultivate learners' ability to construct new knowledge in mathematics through problem solving.

In Table 4.6, ST01's response was "Not sure" and that of LN04 was a "No", suggesting that the latter was not using or incorporating problem solving as a teaching strategy in their teaching of mathematics. In terms of Milla and Kim (2017), and Alsawaie (2003), teachers' responses of ST01 and LN045 seemed to imply that their learners would hardly acquire mathematical knowledge consisting of deep understanding, and would hardly develop problem solving thinking and cooperative skills, and would also hardly develop flexibility and creativity mathematically.

Also, in Table 4.6 SC02 and LN03 suggested that they were using or incorporating problem solving as a teaching strategy into their mathematics lessons. The latter responses, regarding the benefits of using problem solving as a teaching strategy in mathematics, agree with Ersoy (2016: 79). Ersoy (2016) stated that concentrating on problem solving in lessons cultivates learners' high-level problem solving thinking. In such instructional lessons, learners can learn mathematics on their own. As learners self-teach they "develop evaluation and reflection skills on the entire process of solving the problem" (Chauraya & Mhlolo, 2008: 73). When problem solving is prioritized as a mathematical instructional strategy learners develop practical and logical skills, problem solving and cooperative skills, and develop mathematical flexibility and creativity (Brehmer, 2015: 12).

According to Taplin (2015), individuals can no longer function optimally in society by just knowing the rules to follow to get a correct answer. They also need to have the ability to decide through a process of logical deduction. Taplin (2015) further argues that problem solving should not only be used just as the means to an end but can be developed as a valuable skill and a way of thinking. It was observed in Table 4.6 that learners in SC02 and LL03 stand a chance to learn mathematics meaningfully and experience mathematical learning and problem solving in terms of Taplin (2015).

4.2.5 Theme 4: Challenges encountered by teachers when using problem solving as a teaching strategy

Despite the recent emphasis on problem solving in syllabi and the avowed support of teachers for it, research indicates that several teachers may not use problem solving due to certain challenges (Cavanagh, 2008; 1). Participants in the current study listed challenges they experience when attempting to enact problem solving as a teaching strategy in mathematics classrooms. To generate teachers' responses in this regard the following question was posed:

Q: What challenges, if any, do you encounter when using problem solving as a teaching strategy in your mathematics lesson?

Teachers' responses to the preceding question are captured in Table 4.7.

Table 4.7: Teachers' challenges when using problem solving as a teaching strategy

Teacher name code	
ST01	Lack of problem solving knowledge, inappropriate textbook, inappropriate curriculum
SC02	Language barrier, large number of learners in class
LL03	Large classes, inadequate time
LN04	lack of problem solving knowledge, Inappropriate assessment, large number of learners

Table 4.7 shows that teachers shared almost similar experiences and challenges when using problem solving as a teaching strategy in their mathematics lessons. Teachers' observed challenges in Table 4.7 concur with those highlighted by Akhter, Akhtar and Abaidullah (2015: 4). Akhter, Akhtar and Abaidullah (2015: 4) noted that problem solving cannot be used effectively as a teaching strategy due to the shortage of time, a lot of material to be learnt by learners and the large number of learners in a class, which results in needing more time than usually expected both in terms of preparation and implementation. In addition, Anderson (2014) argued that most teachers avoid using problem solving as a teaching strategy because they are unsure of what they must tell learners to do.

In Table 4.7 ST01 argued that problem solving method does not fit properly with the curriculum, as the curriculum depends a lot on the prescribed textbook, namely, Prism Alive. ST01 proceeded to argue that the assessment system is overloaded with formal examinations that reward recall skills, not encouraging learners' cultivation of problem solving skills. Since there are no clear guidelines on how problem solving should be used as a teaching strategy in the teaching and learning of mathematics, it often depends on teacher's problem solving knowledge to incorporate problem solving into the curriculum.

Teacher LL03 contended that more time is needed to prepare and implement a problem solving instruction or use problem solving as an instructional strategy. This means that teachers are failing to prioritize the understanding mathematics during learning as well as emphasizing the relevant curriculum outcomes, over covering the entirety of the content (see, Section 2.4).

In Table 4.6, ST01 responded as not being sure if problem solving was used as a teaching strategy in their experience of teaching and learning of mathematics. The lack of problem solving knowledge indicated in Table 4.7 may be the reason behind the teacher being unsure and not using problem solving as a teaching strategy in teaching and learning of mathematics.

4.2.6 Theme 5: Teacher training on problem solving

Training teachers on the facilitation of a problem solving instruction should be prioritized. Teacher support needs to be strengthened to mitigate the classroom challenges related to implementing problem solving as an instructional strategy. To source teachers' (respondents') views on this aspect the following question was asked:

Q: Do you think teachers need more training to implement effective problem solving lessons in their mathematics classrooms? If yes, what platforms can be utilized in addressing the lack of adoption of problem solving teaching approach?

Table 4.8 provides teachers' responses regarding training and support received to facilitate effective problem solving instruction in mathematics classrooms.

Table 4.8: Teachers' responses on training and support on problem solving

Teacher name code	Teachers' responses	Teachers' elaboration of their responses
ST01	Yes	workshops, symposiums
SC02	Yes	seminars, symposiums and workshops
LL03	Yes	workshops, symposiums
LN04	Yes	workshops

All respondents in Table 4.8 agreed that there is a need for more teacher training regarding problem solving in mathematics. All respondents responded with a "Yes" to the question asking them if they thought there is a need to train teachers on mathematical problem solving.

In addition, respondents suggested that this could be done during teacher workshops, seminars and symposiums. Anderson (2014) emphasized that the use of appropriate professional learning experiences, which can take the form of regular collaborative meetings between groups of teachers, workshops, conferences, networking between schools as well as school and university partnerships can be utilized in addressing the lack of adoption of problem solving teaching approach.

4.3 ANALYSIS OF TEACHERS' LESSON OBSERVATIONS

The second phase of data analysis was meant to address the fourth research question of the study (Section 1.6), namely:

> *What are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?*

Responding to this question would require one to identify some aspects of teaching practices either, by observing the lessons or by interviewing the concerned teachers. During the interviews, SC02 and LL03 indicated that they incorporated problem solving in lessons and used problem solving as a teaching strategy in their mathematics lessons. For this reason or for providing these kinds of interview responses, SC02 and LL03 were purposefully selected to participate in the subsequent session of data collection involving the lesson observations (see, Section 3.4.3.2).

For instance, during the semi-structured interviews SC02 and LL03 were probed on how to teach learners to become effective problem solvers (see, Section 4.2.3.1; Table 4.5; see, also, Appendix G) the following responses were generated:

SC02: *Learners must be trained to read questions with understanding, helping learners understand the mathematics language*.

LL03: *Learners should be subjected to many word problems requiring them to read with understanding, like mathematics contests or Olympiad questions starting from primary school*.

Hence, both teachers were selected as participants in the subsequent lesson observations. The analysis of data from lesson observations in this study has been done in comparison with the data analysis from the semi-structured interviews to facilitate triangulation of results to strengthen the credibility and trustworthiness of the study (see, Section 3.4.3.2; see, also, Sections 4.4 & 3.5). Analysis of data from the lesson observations also entailed the use of framework analysis and the conceptual framework (see, Table 3.3: Section 3.7). Table 3.2 indicates the steps used as a framework to guide the analysis of data. These steps were adapted from Jane Ritchie and Liz Spencer (1980) (Section 3.7).

The following six lesson themes on problem solving guided the implementation of a lesson observation schedule (Appendix H): (1) observing how the teacher poses a problem; (2) observing the problem solving strategies that the teacher uses during a lesson; (3) observing how a teacher models problem solving during a lesson; (4) observing the extent to which a teacher limits learners' input, if this is done by the teacher, during the lesson; (5) observing whether a teacher promotes or encourages metacognition[5] during a lesson; and, (6) observing how a teacher utilizes the tool of exploring multiple solutions, if indeed the teacher incorporates such a tool into their lessons (see, Section 3.4.3.2).

In addition to these themes the researcher used the lesson observation schedule in Appendix H to facilitate the observing of teachers' problem solving practices during the Form 1 mathematics lesson (see, also, example in Table 4.9). The right column of Table 4.9 provided the space for the researcher to capture the research-related moments during

5. Metacognition is one's knowledge of monitoring and controlling one's own systematic cognitive activities requiring some level of metacognitive skills like planning and evaluation. (Section 2.7).

the lessons of teachers SC02 and LL03. Also, the researcher used the columns of Table 4.9 to capture data that related to the themes mentioned in the preceding section.

Table 4.9: Example of lesson observation schedule used in SC02 and LL03 mathematics lessons

Theme sequencing	Theme and related sub-questions	Researcher's comments/ observations
1	**Teaching general or specific problem solving strategies** o *Does the teacher talk explicitly about problem solving?* o *Does the teacher mention or demonstrate general or specific problem solving strategies?*	
2	***Modelling problem solving*** o *Does the teacher demonstrate problem solving or particular problem solving skills?* o *Does the teacher ever highlight, either implicitly or explicitly, Polya's four phases of problem solving?*	
3	**Limiting teacher input** o *Do learners work together to solve problems?* o *Do learners explain or demonstrate their solutions to classmates?* o *What assistance or guidance does the teacher provide to learners as they work on problems?* o *How does the teacher respond when learners pursue unproductive solution paths or dead ends? How far does the teacher let learners go before intervening?*	
4	**Promoting metacognition** o *How does the teacher promote metacognition in the classroom?* o *Does the teacher ask questions or make comments that encourage learners to be reflective about problem solving? If so, how?* o *Does the teacher model metacognitive behaviour regarding problem solving?*	
5	**Highlighting multiple solutions** o *Does the teacher encourage learners to find various ways to solve a problem?* o *Do learners share their solutions with one another?* o *Does the teacher discuss connections between different solutions?* o *Is there discussion of advantages and disadvantages of particular problem solving strategies?* o *Does the teacher encourage learners to develop more efficient problem solving strategies?*	

Both teachers were observed conducting lessons in a Form 1 class. The duration of SC02 lesson was sixty minutes, and that of LL03 was seventy minutes. The number of learners in SC02 classroom was forty-two and in LL03 classroom was forty. Learners were sitting in groups in both classrooms. There were seven groups of six learners in SC02 classroom and eight groups of five learners in LL03 classroom.

4.3.1 Theme 1: Problem posing

Theme 1 that is used here is based on the first stage of generating a problem solving mathematics classroom, as suggested by the conceptual framework of the study (Figure 2.1; Section 2.4.4). According to McDougal and Takahashi (2014: 114), a lesson based on teaching through problem solving begins with the teacher establishing the context and introducing or posing the problem for the lesson. Teachers must provide adequate information to bring about the background or intent of the problem (Taplin, 2015).

At the time of conducting the lesson observations, SC02 was teaching the topic of triangles to Form 1 learners. One of the lesson objectives of SC02 was, *to teach learners to calculate unknown angles in the triangles*. Teacher SC02 started by recapping or drawing from the previous lesson, which seemed to have been about kinds of triangles and ascertaining learners' prerequisite knowledge about triangles. For instance, learners were asked to state the properties of the different types of triangles. SC02 posed the problem in Figure 4.1.

In this regard, it may be reasonable to say the teacher was able to establish the context and introduced the problem as would be expected in a lesson based on teaching through problem solving indicated in the conceptual framework of the study (see, Section 2.4.4) and also mentioned by McDougal and Takahashi (2014) (see, Section 2.4.2).

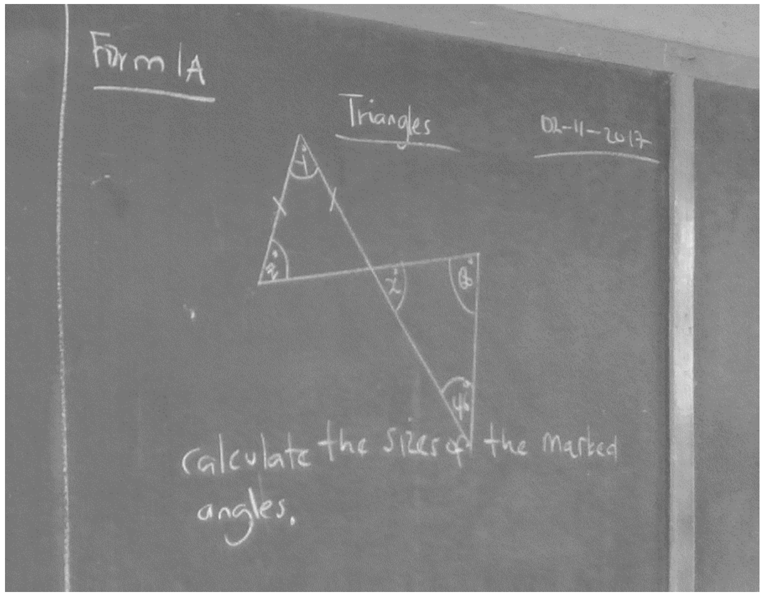

Figure 4.1: A problem given by teacher SC02

Teacher LL03 also taught a Form 1 mathematics class on the topic, *The circumference of a circle*. The learning objective of this learning was: *Learners are expected to be able to calculate the circumference of a circle*. The teacher started the lesson by discussing the value of pi (π), providing the background knowledge of 'pi' and highlighting the importance of learning about 'pi'. In the case of LL03 the lesson was introduced in a traditional teaching approach where learners remained passive and were mere recipients of information from the teacher. In the introductory part of the lesson, no problem was posed to learners as is recommended by the study's conceptual framework. Looking at the way the teachers introduced their lessons, it was observed that LL03 failed to pose a problem to the learners while SC02 was able to pose a problem as would be expected in a lesson based on teaching through problem solving indicated in the conceptual framework of the study (see, Section 2.4.4) and also mentioned by McDougal and Takahashi (2014) (see, Section 2.4.2).

4.3.2 Theme 2: Problem solving strategies

The lesson observation schedule had items that sought to determine if teachers mentioned or demonstrated general or specific problem solving strategies (see, Appendix H; see, also, Table 4.9), posed as a first theme in the lesson observation schedule. The conceptual framework for problem solving instruction (in Figure 2.1) emphasizes that after posing a problem, learners should engage in self-learning or/and a whole class discussion. This is where learners should be allowed to make use of their self-developed problem solving strategies, and in this regard self-learning takes place (Figure 2.1). A teacher then monitors the progress made by learners as well as take note of various problem solving strategies used by learners.

Gleason, Livers and Zelkowski (2015) argued that learners should be permitted to take part in exploration, investigation and/or problem solving activities to acquire a flexible use of mathematics. The conceptual framework of the study suggested the following problem solving strategies, which learners may use during a problem solving instruction: working backwards, adopting different viewpoints, discovering a pattern, creating a drawing, solving an easier or analogous problem, considering extreme cases, well-informed guessing and testing (approximation), explaining that all possibilities, logical reasoning and coordinating data (Figure 2.1).

4.3.2.1 Observing SC02 lesson in terms of Theme 2

The following observations were made during the lesson of SC02. The observations made are demarcated in Table 4.10 in terms of the conceptual framework of the study and the lesson observation schedule in Figure 4.9.

Table 4.10: Using Theme 2 to observe the lesson of SC02 in terms of conceptual framework and lesson observation

Conceptual framework (Figure 2.1)	Lesson observation schedule (Table 4.9)
• **Learners' self-learning** - *Learners worked in groups (solving a problem given by teacher in Figure 4.1)* • **Whole class discussion and comparison** - *Classroom divided into groups of learning in which robust problem solving discussions ensued* • **Summarizing through connecting learners' ideas in mathematics which came out in the classroom** - *Teacher revealed that diagram (Figure 4.1) would not be solved with protector since it was not on scale*	• **Limiting teacher input** - *Learners worked together in groups* - *Teacher provides support by guiding and directing learner discussions* • **Highlighting multiple solutions** - *Learners displayed different points of view during problem solving discussions*

Despite the observations captured in Table 4.10, the following are some of the issues that played out during the problem solving lesson of SC02:

- The teacher gave a problem to learners (see Figure 4.1);
- Learners embarked on logical thinking and argumentation while working in groups of learning; and,
- One learner suggested that the value of *x* (in Figure 4.1) could be obtained by using a protractor.

Learners were observed explaining the problem to themselves and looking for possible entry points to its solution. SC02 moved around monitoring learners' work as they analyse givens, relationships, constrains and goals of the problem. Avcu and Avcu (2010) emphasized that strategies should be used in solving a problem when teaching through problem solving.

After introducing the lesson, teacher LL03 gave learners circular card box and a string (see Figure 4.2). The circular card boxes given in each group were of different sizes in terms of diameter. The teacher requested learners to use a string to find the circumference and measure the diameter of a circular card box to fill in Table 4.11.

Table 4.11: The approximate value for pi.

Object	Circumference (C)	Diameter (d)	$\frac{c}{d}$
Circular card box			

The teacher moved around the different groups, trying to assist learners to do accurate measurements. After obtaining the value of the circumference and diameter, the teacher asked learners to divide the circumference by the diameter. The teacher then stated that the obtained value was an approximate value of pi. A learner from each group was asked to present their work on the chalkboard (see, Figure 4.2).

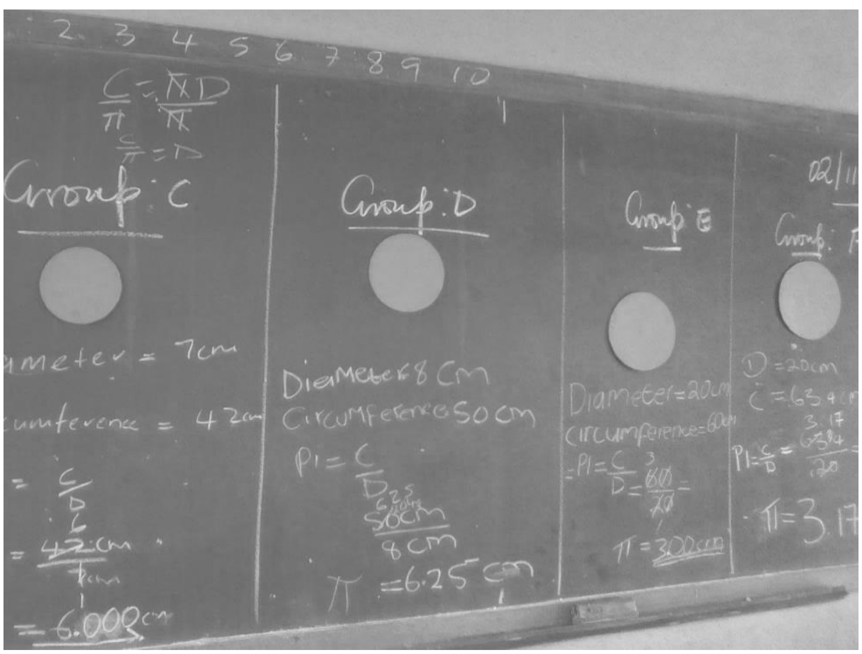

Figure 4.2: Groups' responses on the value of pi

The teacher summarized the activity by stating that the value of the ratio of the circumference C, to the diameter d, is given by $\frac{C}{d} = \pi$. Therefore, the value of π is given as 3.14 when written to two decimal places. The teacher then wrote on the chalkboard, different values of diameters and radiuses of circles as well as the value of pi and requested learners to practice independently to calculate the circumference of the circles using the given formula $C=\pi d$. The teacher moved around marking learners' classwork.

It was observed that this lesson featured characteristics of a traditional method of teaching mathematics. The rate at which the teacher interacted with learners was relatively minimal when compared to SC02. In most cases LL03 dominated the lesson. Hence, learner involvement was kept at a minimal level. Learners were largely passive recipients of mathematical knowledge, which mainly came from the side of the teacher. For instance, LL03 talked for about 15 minutes giving a historical background of *pi*, something that could have been researched by learners on their own, and later presented in class.

LL03 continuously talked, demonstrated or worked out examples as means of transferring mathematical knowledge to learners. For example, after deriving the value for *pi*, the teacher then demonstrated on the chalkboard how to calculate the circumference of a circle given the diameter or radius. Gleason, Livers and Zelkowski (2015: 2) argued that it does not count as problem solving if learners follow a procedure established by the teacher. Instead, learners should be finding out their own problem solving strategies without necessarily knowing that the strategy will lead to the desired result. This is not in line with the conceptual theorem of the study, which advocates an instructional approach of learners' self-learning (Figure 2.1). Teacher LL03 never mentioned or demonstrated general or specific problem solving strategies.

4.3.3 Theme 3: Modelling problem solving

In terms of the observation schedule that was developed for the study this aspect fell in the fourth theme under the sub-question, *Does the teacher model metacognitive behaviour regarding problem solving*? In this regard the teacher would be expected to demonstrate to learners, meta-cognitively, certain problem solving behaviour without being perceived by learners to be overly dominating the lesson. In terms of the conceptual framework a problem solving instruction should foreground aspects of learners' self-learning. Therefore, when judged against this theme the teacher would have to be seen to strike the balance between overly indulging themselves in the lesson and trying to allow the voice of learners to dominate the problem solving instruction.

Hence, some of the items in the lesson observation schedule (Table 4.9) specifically attempted to observe if teachers would take a lead in demonstrating problem solving behaviour or skills, and in addition, if the teachers would make use of Polya's four phases of problem solving. According to Donaldson (2011: 90), modelling a problem solving behaviour consists of the following actions: demonstrating skills and concepts of mathematics, thinking aloud in order to give learners insights into the metacognitive aspects of problem solving, and demonstrating a positive attitude and perseverance when faced with difficulties (Section 2.4.2).

In this regard, SC02 was able to show learners how to interact with relevant mathematical concepts and skills when solving the problem. The teacher (SC02) stated that identifying the two equal sides of the upper triangle gave rise to an idea that the diagram was an isosceles triangle (see, Figure 4.1). Talking through the teacher's thought process in the class was a common feature of SC02's modelling of problem solving. While solving a problem on the chalkboard the teacher would ask questions like, "*What is it that we know about this problem*?"

Solving problems may need patience and perseverance. As learners were solving the problem, SC02 encouraged them not to abandon a solution attempt before it was completed or reaching the final solution stage. In Figure 4.1 some of the learners appeared to be disappointed when told not to measure the angles using a protractor since the diagram was not drawn to scale. The teacher later provided a hint to assist learners to solve the problem. For example, the teacher asked learners to identify and name the upper triangle, and subsequently provide a justification the name given to the triangle.

However, some learners were able to solve the problem without teachers' assistance. In this way the classroom instruction was seen to develop learners' metacognition abilities, which is regarded by NCTM (2000) as very important (see, Section 2.7). SC02 never highlighted Polya's four phases of problem solving in the course of the lesson. Modelling was a major component of the lesson.

Teacher LL04's lesson did not include much of modelling aspects of mathematics. The teaching mainly involved demonstrating relevant facts, rules, skills and processes; monitoring activities in which learners repeat and practice the preceding items as well as correcting errors that occurred. According to Bonato (2018), in traditional methods, rules are taught first and then drilled into learners via memorization and solving problems. Lesson observation in LL04 classroom revealed that the teacher solely and overly demonstrated skills and concepts of mathematics like how to use the formula for calculating the circumference of a circle.

4.3.4 Theme 4: Limiting teacher input

The lesson observation schedule had items to observe the extent to which teachers limit their input, if this is done by the teacher, during the lesson. Under the theme, limiting teacher input, the lesson observation schedule included items like (see, Table 4.9); a) *do learners work together to solve problem*?; b) *do learners explain or demonstrate their solutions to classmates*?; c) *what assistance or guidance does the teacher provide to learners as they work on problems*?; and, d) *how does the teacher respond when learners pursue unproductive solution paths or dead ends? How far does the teacher let learners go before intervening*?

Gleason, Livers and Zelkowski (2015) stated that the Standards for Mathematical Practice, National Governors Association Center for Best Practices, Council of Chief State School Officers (2010) encouraged learners to be active in establishing conjectures, examining the truth for the conjectures, and replying to the conjectures and generating reasoning for others.

In both SC02 and LL03's lessons, learners were observed talking about the mathematics in the lessons. Learners become effective problem solvers when they take the responsibility of solving problems rather than waiting for the teacher to tell them what to do. Alsawaie (2003) indicated that it is the duty of the teacher to be aware of when learners require help as well as when they can continue working productively without help. Lester *et al*. (1994) further emphasized that teachers must know when it is appropriate to intervene and when to step back and let learners make their own way. Therefore, it is important to give learners time to research problems. Limiting teacher input was largely observed when learners work in groups, with a teacher refraining from telling learners too much, permitting learners to struggle and sharing authority for correct answers. In both lessons, learners were sitting in groups of four or five. According to Hastings and Chantrey (2002), it is standard practice for learners to sit around grouped tables-usually with four to six learners in each group. Working in groups allows learners to share ideas, explain their thinking, and help each other to solve problems Donaldson (2011).

Teacher SC02 gave a problem and walked around the class supervising the groups and encouraging them to compare their solutions. The teacher encouraged learners to look to one another and to their own reasoning abilities before looking to the teacher for solutions. In this way, learners were observed sharing ideas, explaining their thinking and helping each other find the solution to the problem. Then, rather than having the teacher demonstrating the solution, each group chose a representative and explained their solution on the chalkboard (see, Figure 4.3). SC02 limited teacher's input in that regard.

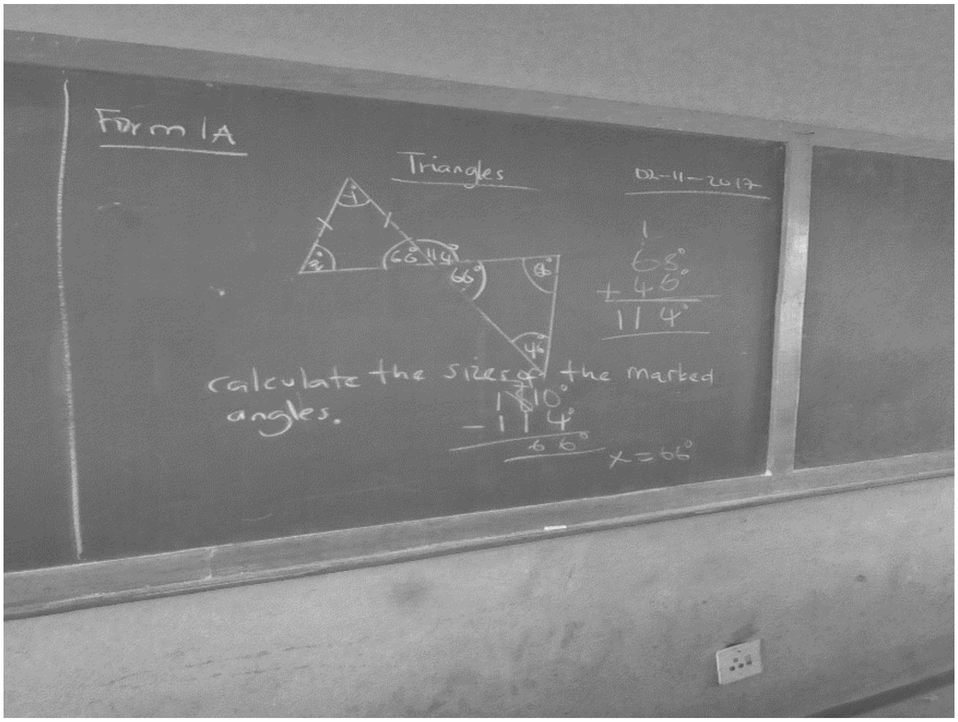

Figure 4.3: Learners' solution on the problem

As learners explained their solutions, one learner from another group asked why angle *z* was equal to 66º. Without hesitating, the presenter correctly answered the question without any assistance from the teacher. By so doing SC02 was able to limit teacher's input. Authority was shared for correct answers with the learners. In SC02's lesson learners engaged in mathematical thinking instead of just following a set of instructions.

Observations turned different in LL03's lesson. After deriving the value of *pi*, LL03 asked learners to calculate independently the circumference of a circle with a diameter of 6*cm*. LL03 then moved around, coaching and marking as learners were solving the problem. The teacher demonstrated the solution of the problem on the chalkboard, gave homework and concluded the lesson. It was observed that there was no teamwork among learners when solving problems on the circumference of a circle. Teamwork only featured when learners were deriving the value of *pi*. There was a lot of memorization of facts starting from the origin of the value of *pi* to following rules or formulae for calculating the circumference of a circle. It was also observed that LL03 offered clear, step by step demonstrations of the activity and provided illustrations of how to use the formula for the circumference of a circle. By so doing, the teacher could not refrain from telling learners too much.

Given these observations it is reasonable to conclude that LL03 failed to limit teacher's input with regards to, refraining from telling learners too much, permitting learners to struggle and sharing authority for correct answers. Teamwork in a problem solving lesson plays a pivotal role in learners as they ask questions, discuss opinions, listen, have the responsibility of what to learn, criticizing constructively and constituting an atmosphere of mathematical learning (The NCTM, 1989).

4.3.5 Theme 5: Promoting metacognition

The lesson observation schedule had items attempting to determine, (a) *how the teacher promotes metacognition in the classroom*?; (b) *does the teacher ask questions or make comments that encourage learners to be reflective about problem solving*?; and, (c) *does the teacher model metacognitive behaviour with regard to problem solving*? As stated earlier on modelling problem solving, when the teachers model problem solving they are modelling metacognition behaviour. The teacher is promoting metacognition in this way. Another way a teacher can use to motivate learners to be aware of their own cognition is assisting learners in thinking about what they do and do not know.

To get learners to think about the knowledge that would help them to solve the problem on triangles, SC02 asked questions during the lesson. For instance, the teacher asked learners about the name of the upper triangle and why it was called by that name. Bjuland (2004) noted that it is inadequate for learners to just complete tasks, but they must be motivated to reflect on their tasks as they engage in self-learning or whole class discussion and comparison (Figure 2.1). Being asked to justify a method of solution promotes reflection. It was observed in SC02 lesson that as learners explained their solutions, one learner from another group asked why angle z was equal to 66^{o}. Without hesitating, the presenter correctly answered the question without assistance from the teacher. In this way learners encouraged each other to be reflective on problem solving. Since metacognition is one's knowledge, monitoring and control of one's own systematic cognitive activity, one element of monitoring is checking for mistakes along the path of solving a problem.

While learners were working in their groups SC02 asked learners if they were checking on their group members' work to avoid careless mistakes. According to Donaldson (2011), "one aspect of monitoring the problem solving process is checking for mistakes along the way when solving a problem" (p. 108).

In LL03 lesson there were few instances of promoting metacognition. Most of the time, the teacher was telling learners what to do, and how to do it. For example, LL03 demonstrated the computing of the circumference of a circle with a diameter of 6*cm* instead of letting the learners solve the problem themselves. It was the researcher's view that learners were adequately equipped to solve the problem on their own since they had already learnt about the relationship between *pi*, circumference and diameter. Izzati and Mahmudi (2018) argued that problem solving is not enough when learners just mimic how to solve problems that they must know. Learners should attempt to put more effort, like modifying the problem in such a way that one is familiar with the way of solving it, solving the problem using multiple solutions.

4.3.6 Theme 6: Highlighting multiple solutions

According to The Standards for Mathematical Practice, National Governors Association Center for Best Practices, Council of Chief State School Officers (2010, cited in Gleason, Livers and Zelkowski, 2015), learners should be motivated to look for multiple problem solving methods and to deal with problems that have multiple solutions based on various assumptions. The lesson observation schedule items consisted of highlighting multiple solutions which may occur, *when teachers stress that there could be more than one way to reach the solution for solving a certain problem*. For example, given that the ratio of males to females in a meeting was 5: 4 learners may be requested to compute the number of either males or females (see, Figure 4.4).

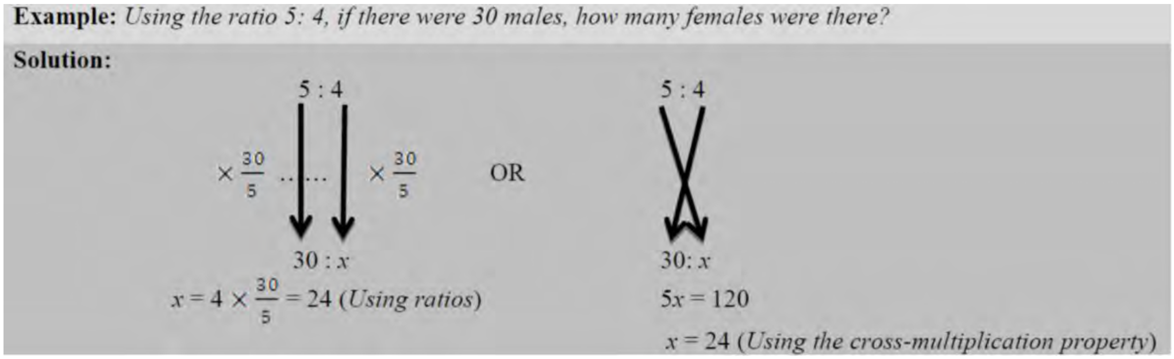

Figure 4.4: Example of ratio and proportion problem

While learners were working out the solutions of the problem given in Figure 4.1, SC02 moved around the seven groups in the class observing learners as they work and checking their progress. Learners in SC02 lesson were observed to be developing more effective problem solving strategies. SC02 also noticed one of the groups using a different, yet correct method to get the size of angle *z*. After the activity the teacher requested a group representative to go to the chalkboard and share their method with the rest of the class. The method was that after obtaining the size of angle *x*, the learners used the principle that vertically opposite angles are equal. Since the upper triangle was an isosceles triangle, angle *z* would also be equal to the angle vertically opposite to angle x. In this way, learners were sharing their solutions with one another. The teacher then discussed the connections between the different solutions. Multiple solutions were a significant part of the lesson. SC02 then encouraged learners to get used to finding various ways to solve a problem.

Comparatively, there were minimal instances of highlighting multiple solutions in LL03's lesson. It was noted that LL03 guided learners with a formula for the value of *pi* during the activity. When they were to apply the value of *pi* to calculate the circumference of a circle, given its diameter, LL03 further demonstrated the method on the chalkboard. Learners imitated the teacher's method when solving similar problems. Such a teaching practice is mainly rooted on the traditional teaching methods where learners become proficient with mathematical concepts through repetitive practice and later required to reproduce the concepts. Bonato (2018) emphasized that in traditional methods, rules are taught first and then drilled into learners via memorization and solving problems.

4.4 PRESENTING INTERVIEW FINDINGS IN RELATION TO THE FINDINGS OF LESSON OBSERVATIONS

Lesson practices of teachers SC02 and LL03 are now compared in the next discussions.

4.4.1 Teacher SC02

Observing the way SC02 conducted the lesson, it reflected a good picture of what was said during the interview. During the interview SC02 described problem solving as giving questions to learners that require them to reason or think before giving the solutions when teaching mathematics. This was also in line with the first stage (posing the problem) of the conceptual framework (of the study) for problem solving instruction (Figure 2.1). Learners were given a non-routine problem in which there was no obvious way to get to the solution.

Learners were indeed given a question that required them to think ahead of the solution (see, Figure 4.1). The question had no obvious way or method to get to the solution; some learners pulled out a protractor attempting to measure the angles. In SC02 lesson, problem solving was incorporated as learners worked together to solve a problem and explaining their methods on the chalkboard to the rest of the class. Hence, various ways to solve the given problem were explored. These teaching practices concur with the conceptual framework of the study for problem solving instruction (see, Section 2.4.4). Also, the observations in SC02 lesson concurred with literature (see, McDougal & Takahashi, 2014: 114; see, also, Section 2.4.2).

4.4.2 Teacher LL03

The lesson presentation of LL03 did not correspond well with what the teacher said during the semi-structured interviews. The teacher described problem solving as a question in the higher order in the Bloom Taxonomy (see, Table 4.4), starting with application, analysis, evaluation and synthesis, where learners are subjected to serious thinking to obtain a solution. The lesson of LL03 posed questions that only required recalling of learnt facts and the usage of formula to obtain a problem solving solution. The types of problems given during the LL03's lesson differed from the ones the teacher stated during the interview (see, Table 4.12).

Table 4.12: Examples of problems LL03 gave to learners during interview and actual lesson

Mathematical problem solving task	
Provided during semi-structured interview	**Observed and given during the lesson**
A girl is m-years old, her mother is four years older than twice the girls' age. The sum of their ages is 44 years. How old is each one of them?	Calculate the circumference of a circle with diameter $12cm$. Take π as 3.14.

The lesson observation problems in Table 4.12 were different in the sense that they had clear solution procedures, requiring application of learned rules or formulas, such as the circumference of a circle is equal to *pi* multiplied by diameter (see, Figure 4.2). This was found to match the characteristics of the common classroom drill exercises and application word problems, which is not what constitutes a mathematics problem, as applied to problem solving (Mhlolo & Chauraya, 2008).

Learners in LL03 lesson were observed sitting in groups, computing the value of *pi*, but that was not enough for problem solving to be considered as having been effectively incorporated into the lesson. Learners in groups should be seen engaging in solving problems using problem solving strategies, finding multiple ways to solve a particular problem and learners explaining or demonstrating their solutions to classmates. Teaching practices that were demonstrated by LL03 during a lesson observation were that of a knowledge dispenser rather than that of a facilitator. For example, the formula for calculating the circumference of a circle was taught first and then learners drilled via memorization and solving problems. Such teaching practices (of LL03) are like those of the traditional method as described by Bonato (2018) (see, also, Section 2.10).

4.4.3 Tabulating the differencing in the observed lessons of SC02 and LL03

Table 4.13 is drawn to highlight differences in the way in which the lessons of SC02 and LL03 were conducted. The differences in Table 4.13 are outlined in terms of the themes that had been identified in Table 4.9.

Table 4.13: Teachers' observed classroom practices in relation to the themes in Table 4.9

Teacher			
SC02: Lesson observation		**LL03: Lesson observation**	
Theme	**Teacher's observed responses to the theme and related sub-questions**	**Theme**	**Teacher's observed responses to the theme and related sub-questions**
1	- The teacher was able to establish the context and pose the problem	**1**	- The teacher failed to pose a problem to the learners
2	- Learners were able to use the problem solving strategies as they embarked on logical thinking and argumentation while working in groups of learning.	**2**	- Learner did not engage in utilizing problem solving strategies but carried out an activity on deriving the value for *pi* in order to calculate the circumference of a circle.
3	- The teacher was able to model metacognitive behaviour regarding problem solving. - Talking through teacher's thought process in the class was a common feature. - Learners were encouraged to be patient and persevere when solving a problem.	**3**	- Lesson did not include much of modelling aspects of mathematics. - The teacher solely and overly demonstrated skills and concepts of mathematics like how to use the formula for calculating the circumference of a circle.
4	-Teacher was able to limit teacher input. -The teacher gave a problem and walked around the class, supervising the groups, encouraging them to compare their solutions. - Learners were encouraged to look to one another and to their own reasoning abilities before looking to the teacher for solutions. -Each group chose a representative and explained their solution on the chalkboard.	**4**	-The teacher failed to limit teacher's input with regards to, refraining from telling learners too much, permitting learners to struggle and sharing authority for correct answers.
5	-Teacher was able to promote metacognition in the classroom. - learners explained their solutions, -learners checked on their group members' work to avoid careless mistakes along the way when solving a problem.	**5**	- Few instances of promoting metacognition. - Most of the time, the teacher was telling learners what to do, and how to do it.
6	- Multiple solutions were a significant part of the lesson. -Teacher encouraged learners to get used to finding various ways to solve a problem.	**6**	-There were minimal instances of highlighting multiple solutions - Learners imitated the teacher's method when solving similar problems

4.5 SUMMARY

This Chapter presented the analysis of data for the current study. Data for the study were collected through semi-structured interviews with four teachers, and subsequent lesson observations with two teachers. The process of data analysis for the study occurred in three phases, namely, (1) analysis of data from the semi-structured interviews; (2) analysis of data from the lesson observations; and, (3) presenting the findings of the semi-structured interviews in relation to the findings of the lesson observations.

Four teachers who were interviewed provided their perceptions about what a mathematics problem and problem solving in mathematics mean. The most common characteristics of a problem included: real life situations, challenge and motivation and word problems, although some fit the characteristics of routine drill exercises and application tasks, which are typical of many mathematics textbooks. Teachers gave varying perceptions of what problem solving in mathematics means.

The data analysis from the lesson observations revealed teachers' classroom practices when using problem solving as a teaching strategy. Teachers' classroom practices were analysed in terms of teaching problem solving strategies, modelling problem solving, limiting teacher input, promoting metacognition, and highlighting multiple solutions. The next Chapter provides an extension of the discussion of the findings of the study.

CHAPTER 5

DISCUSSIONS

5.1 INTRODUCTION

This study aimed to determine mathematics teachers' perceptions of problem solving in Form 1 at the Eswatini schools (Section 1.4). The study set out five objectives to guide the research activities of the study (Section 1.5). To set into motion the planned research activities, the study employed a descriptive case study research strategy to develop insights into teachers' conceptions and practices of mathematical problem solving (Section 3.2).

Four teachers from the Manzini Region in Eswatini participated in the study (see, Section 3.3; see, also, Figure 3.1). These teachers taught mathematics in Form 1 and were selected to participate in the study using purposive sampling techniques (Section 3.3.4). Data collection procedures involved semi-structured interviews with all participants and the conducting of lesson observations with two teachers (see, Sections 3.4.1, 3.4.3.1 & 3.4.3.2). Chapter 5 provides a comprehensive discussion of the findings of the study emanating from the data analysis in Chapter 4.

5.2 ACHIEVING THE OBJECTIVES OF THE STUDY

The study was guided mainly by five objectives (Section 1.5), which tended to spell out the planned research activities that the study anticipated to respond to the research questions (Section 1.6). For instance, the action verbs in the study objectives necessitated a series of research activities, and these had been constructed as follows: *To establish…*; *to ascertain…*, *to identify…*, *to determine…*; and *to identify…*. To act in line with the objectives of the study, the researcher embarked on a series of research activities.

5.2.1 First objective of the study

The first objective of the study was formulated as follows: *To establish teachers' explanations of the meaning of problem solving in selected Form 1 mathematics classrooms*. This exploration was done through a process of interviewing the four teachers who participated in the study (see, Section 4.2.3). Among other things the study asked the respondents to explain the meanings (conceptions) of a mathematical problem. Examples of the interview questions that tended to address this objective were: *what comes to your mind when you hear the term problem solving in mathematics? and what do you think a mathematics problem is?* The respondents perceived the meaning of problem solving in mathematics differently from each other as well as from literature definition. Some respondents defined problem solving as carrying out a task with clear solution procedures requiring a formula. While another respondent's definition of problem solving featured phrases like "*giving learners a question and requiring them to think, without an obvious way to get to the solution*",

Some respondents perceived problems as exercises that need basic computational skills to solve in mathematics courses. Other respondents' definition featured phrases like "no obvious way or method of getting the answer" and "not allow the learner to just recall answers without subjecting them to proper thinking". By proper thinking, the respondent was referring to engaging in the process of trying out ideas and making a hypothesis. Teachers' responses regarding the meanings they hold are well documented in Section 4.2.3 (see, Table 4.2 & Table 4.4).

5.2.2 Second objective of the study

The second objective of the study was formulated as follows: *To ascertain teachers' definitions and conceptions of a mathematical problem*. In an attempt to make sense of teachers' theoretical definitions and conceptions of mathematical problem and problem solving the researcher visited two teachers, SC02 and LL03, to observe their mathematical problem solving lessons (Section 4.3). In line with the second objective of the study the lesson observations were purposefully arranged to make sense (understand) of teachers' conceptions (definitions) of mathematical problem solving, from a practice perspective.

Teachers who participated in the lesson observations had been purposively selected based on their earlier responses that they incorporated problem solving in lessons and used problem solving as a teaching strategy in their mathematics lessons (see, Sections 3.4.3.2 & 4.3). Classroom and lesson exposure helped the researcher gauge if participants' conceptions of mathematics problem and mathematical problem solving were in coherence with participants' actual classroom practices. Section 4.3 of the dissertation provides a comprehensive account of the researcher's exploration in teachers' classrooms, and the summary of this exploration is documented in Section 4.4 (see, also, Table 4.13). It must also be noted that the second objective of the study tended to compliment the fourth research question of the study, which asked, *What are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?*

5.2.3 Third objective of the study

The third objective of the study was formulated as follows: *To identify the kinds of problem solving questions and tasks that teachers pose during a Form 1 mathematics lesson*. In an attempt to respond to the third objective of the study the researcher asked teachers, during the interview, to give examples of a mathematics problem (Section 4.2.2.1) and problem solving task. In addition, lesson observations involving SC02 and LL03 helped to provide a meaningful exposure into teachers' formulations and posing of mathematical problem solving questions and tasks (see, Sections 4.3 & 4.5). It must also be noted that the third objective of the study also tended to compliment the fourth research question of the study, which asked: *what are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?*

5.2.4 Fourth objective of the study

The fourth objective of the study was formulated as follows*: To determine the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom.* The attainment of the fourth objective of the study is documented fully in Sections 4.3 and 4.4, in which teachers SC02 and LL03 are observed in their classrooms while conducting their mathematical problem solving lessons. It must also be noted that the third objective of the study also tended to compliment the fourth research question of the study, which asked*: what are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?* During lesson visits the researcher was also privileged to observe teachers' challenges possibly experienced during a classroom instruction on problem solving (see, Section 4.2.5). In addition, the researcher observed teachers' instructional practices whether these practices supported the development of effective problem solvers or not.

5.2.5 Fifth objective of the study

The fifth objective of the study was formulated as follows: *To identify possible challenges, if any, encountered by Form 1 teachers when problem solving is used as a teaching strategy*. The enactment of this objective played out during the semi-structured interview sessions in which study participants were probed to list some of the instructional challenges experienced during the mathematical problem solving lessons. In this regard the interview question asked, *What challenges, if any, do you encounter when using problem solving as a teaching strategy in your mathematics lesson*? Subsequently, all four teachers responded to the interview question listing classroom challenges such as lack of problem solving knowledge (ST01), large number of learners in the classroom (SC02), inadequate time to effectively enact mathematical problem solving instruction (LL03); inappropriate assessment procedures (LN04), etc. It must also be mentioned that the fifth objective of the study fell within the same space as the fifth research question of the study, which asked, *What challenges, if any, do teachers encounter when using problem solving as a teaching strategy to learn mathematics in Form 1*? (Section 1.6).

5.3 DISCUSSION OF FINDINGS

The following sections of the dissertation provide a comprehensive discussion of the results and findings of the study. In this discussion the researcher makes effort to respond to the research questions (Section 1.6) and objectives (Section 1.5) of the study.

5.3.1 Meanings that teachers attach to problem solving in mathematics

The first research question of the study asked:

What meanings do teachers attach to mathematical problem solving?

Study participants gave responses demonstrating varying perceptions of what problem solving in mathematics means. Some of the teachers' views of what constitutes mathematical problem solving were observed to diverge from the literature's definitions (Section 4.2.3). Some of the teachers had in mind primarily the selection and presentation of "good" or effective problems to learners. That is, their conception of problem solving revolved around giving learners a question, subjecting them to serious thinking, and applying learned and practiced skills and techniques to solve mathematical tasks whose solution procedures are explicit. Of the four interviewed teachers, only one indicated that they knew exactly what problem solving was.

That teacher' s definition of problem solving featured phrases like "giving learners a question and requiring them to think, without an obvious way to get to the solution", which is in line with the literature (The National Council of Teachers of Mathematics (NCTM) (2000). The results from participants proved that the phrase "problem solving" is indeed problematic as it suggests various meanings on the teachers' understanding of mathematics teaching and learning (Section 4.2.3). This could mean that using problem solving as a teaching strategy calls for an agreed upon definition of what problem solving is, which should be clearly stated on policy documents like the Eswatini Education and Training Sector Policy and syllabi.

Problem solving plays a crucial role in mathematics and should have a prominent role in the mathematics education of learners. The Eswatini Education and Training Sector

Policy (2018: 7), The Swaziland General Certificate of Secondary Education (SGCSE) (2015: 14), the Junior Certificate (2016: 4) syllabi, and the prescribed Prism Alive series (2010: 4) for Forms 4 and 5, all outline goals that refer to problem solving. However, knowing how to meaningfully incorporate problem solving into the mathematics curriculum is not necessarily obvious or easy for teachers, as evidenced by the one teacher who was unsure of whether she incorporates problem solving into her teaching method (see, Table 4.5).

Two teachers were actually not sure if they incorporate problem solving into their mathematics lesson. They claimed to lack the problem solving knowledge. Hence, all the teachers strongly recommended a need for more training regarding problem solving in mathematics (see, Table 4.5). The other two teachers considered giving learners an unfamiliar question where they apply different strategies to get to the solution as incorporating problem solving into the mathematics class. This is one of the teaching practices associated with using problem solving as a teaching strategy (see, Section 2.4.2).

5.3.2 Teacher's conceptions of a mathematical problem, the types of problem solving questions and tasks that are posed by mathematics teachers

The second and third research questions of the study asked:

> *How do teachers in Form 1 conceive a mathematical problem?*
> *What kind of mathematical problem solving questions, and tasks, do teachers in Form 1 pose?*

Using problem solving as a teaching strategy may begin with a teacher choosing and posing "good" problems to learners as indicated in the conceptual framework (Section 2.4.4). Grouws (2003) indicates that a good problem is one that is clearly stated, incorporates a mathematical context or a real-world context that has the potential to attract and maintain a learner's interest, and the solution can be obtained with more than one method. Bayazit & Donmez (2015: 827) argued that a situation is considered a problem if it causes cognitive conflicts in the minds of individuals. Schoenfeld (1988) emphasizes that a problem is a task for which the method of solution is not instantly conspicuous, and that requires a longer consideration.

The participants of this study believed that problems should be challenging yet manageable and should engage the learners' interests (Section 4.2.2). For instance, two teachers claimed that a problem should not allow the learner to just recall answers without subjecting him or her to proper thinking, but the problem solver must think before attempting to solve the problem. Problems should require higher-order abilities. Some characteristics of a mathematics problem stated by one teacher were to have no obvious way or method to get to the solution and to relate to real life situations with which learners were familiar (Section 4.2.2). Such types of problems are referred to as non-routine problems. It should be noted that the way some teachers described mathematical problems during the interviews concurs with the educational literature.

It was observed during the lesson observations that one teacher assigned problems which were of a different type to the ones he talked about during the interview (see, Table 4.12). The mathematical problems were that of a task with clear solution procedures, requiring application of learned rules or formulas and having only one solution method. Such types of problem are called routine problems. This was found to match the characteristics of the common classroom drill exercises and application of word problems, which does not constitute a mathematical problem as applied to problem solving (Section 2.3.1). As such, it was not surprising to see that three out of the four teachers indicated that they rarely gave learners word problems in a particular exercise, nor consulted other mathematics textbooks for non-routine word problems in the teaching and learning of mathematics.

5.3.3 Classroom practices that characterize teachers' problem solving lessons

The forth research question of the study asked:

> What are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?

The research findings generally indicate several categories of teachers with different perceptions of problem solving with regards to classroom practices that characterize teachers' problem solving lessons; 1) a teacher who have knowledge about using problem solving as a teaching strategy and make significant effort to implement what is required by the curriculum and to a certain degree become successful in doing so (Section 4.3). For instance, SC02 began the lesson by establishing the context and posing a problem. Talking through teacher's thought process in the class was a common feature of SC02's modelling of problem solving, The teacher encouraged learners to look to one another and to their own reasoning abilities before looking to the teacher for solutions (Section 4.3.4), as well as encouraged learners to get used to finding various ways to solve a particular problem (Section 4.3.6); 2) a teacher claiming to be knowledgeable about problem solving as a teaching strategy but failed to implement teaching practices that characterize problem solving lessons. Traditional teaching practices dominated the lesson. For instance, LL03 started the lesson by discussing the value of $pi(\pi)$, giving the background knowledge on pi and highlighting the importance of learning about pi. Learners were passive recipients of information. The teacher offered clear, step by step demonstrations of the activity and provided illustrations of how to use the formula for the circumference of a circle.

The learners were only left to imitate the teacher's method to solve similar problems. Highlighting multiple solutions in Teacher LL03's class was very minimal (see, Section 4.3.5); 3) teacher not sure if they use problem solving as a teaching strategy. 4) teacher who seemed to have a very little idea about problem solving as a teaching strategy in mathematics. Hence, problem solving is never used as a teaching strategy in the teaching and learning of mathematics (see, Table 4.6). The research findings also indicate that all the teachers saw the value of using problem solving as a teaching strategy, some emphasizing on the development of learners' critical thinking skills and ability to solve problems even in their own life, learners becoming creative and enjoying mathematics (Section 4.2.4). However, the teachers also indicated that they rarely used problem solving as a strategy in the teaching and learning of mathematics due to the following challenges (Section 4.2.5).

5.3.4 Challenges teachers encountered when implementing problem solving

The fifth research question of the study asked:

> *What challenges, if any, do you encounter when using problem solving as a teaching strategy in your mathematics lesson?*

Some challenges exist for Form 1 mathematics teachers when using problem solving as a teaching strategy (Section 4.2.5). Teachers argued that the method cannot be implemented properly until the curriculum, textbooks and the assessment system reflect the value of this approach. Other hindrances included the teacher's lack of problem solving knowledge, large classes and inadequate time to teach. It was also noted that all the respondents felt they required face-to-face training on problem solving in mathematics in order to effectively implement the aspirations of the curriculum in this regard (Section 4.2.6). The teachers even suggested that such training could be done through workshops and in mathematics symposiums (see, Table 4.8).

5.4 SUMMARY

This Chapter offered a discussion of the research findings, with reference to the research questions and study objectives. Most of respondents' views of a mathematics problem and problem solving in mathematics were in contest with the literature definitions except for one teacher. Overall, teachers agreed that using problem solving as a teaching strategy is a useful approach and they agreed that problem solving helps learners develop critical thinking skills and solve problems in their own lives. The method cannot be implemented properly until the curriculum, textbooks and the assessment systems reflect explicitly on the principles and values of mathematical problem solving. There are also issues of teachers lacking problem solving knowledge, large classes and inadequate time to teach (see, Table 4.7). Hence, teacher training for problem solving in mathematics to effectively implement the aspirations of the curriculum regarding problem solving is necessary (see, Section 24.2.6 & Table 4.8). Study participants agreed that teacher training on mathematical problem solving is necessary.

CHAPTER 6

SUMMARY OF THE STUDY, CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

Problem solving is an important component in teaching and learning mathematics, hence the results of the current study could contribute to the improvement of Form 1 learners' mathematical problem solving performance in the Eswatini schools. This Chapter provides a summary of the study, answers to the research questions, and success of the study's aim.

6.2 SUMMARY OF FINDINGS

The research findings provided readers, educators, curriculum developers, teacher professional development providers and mathematics teachers with insight into the current understanding, conception and actualization of effective teaching of problem solving in mathematics. In this section of the Chapter, a summary of the literature review, the research methodology and design and the findings of the empirical investigation are provided.

6.2.1 SUMMARY OF LITERATURE REVIEW

This study aimed at determining teachers' perceptions on problem solving in order to contribute to the improvement of Form 1 learners' mathematical problem solving performance in Eswatini schools. One way this was achieved was by conducting an extensive literature review. The review of literature consisted of components such as problem solving themes in the Eswatini Mathematics Curriculum, definitions of a problem and problem solving in mathematics, choosing a teaching method to use in a lesson, using problem solving as a teaching strategy, problem solving classroom ecology, problem solving strategies and the conceptual framework for problem solving instruction.

Other aspects that were covered were classroom culture, varying perspectives on the goal of problem solving, the role of a teacher in problem solving instruction, the benefits of problem solving in mathematics, metacognition and problem solving, and possible challenges, if any, encountered by teachers when implementing problem solving as well as the traditional method of teaching mathematics..

6.2.2 SUMMARY OF RESEARCH METHODOLOGY AND DESIGN

The current study employed a qualitative research design to explore and describe how teachers perceived the notion of problem solving in mathematics. This research design was chosen for its important features that corresponded well with the nature of this research study. Instead of generating numerical data and supporting or refuting clear-cut hypotheses, this study aimed at producing factual descriptions of problem solving in mathematics based on personal knowledge of mathematics teachers.

The descriptive case study strategy addressed the research needs of this study, which was mainly to establish and examine teachers' perceptions of problem solving in mathematics (Section 3.2). This style was chosen because the major aim of the current study was to develop an understanding of what problem solving in mathematics is, by describing characteristics of problem solving held by mathematics teachers in Eswatini. With respect to this, the case study presented the researcher with the chance to acquire a deep, holistic perspective of the research problem, and made it easier to describe, comprehend and clarify the research problem.

Two forms of data collection tools were used in this study to optimize the richness and credibility of the study's findings: semi-structured interviews and lesson observations (Section 3.4). The interviews schedule and lesson observation guide were pretested in a similar setting as that of the main study as a way to improve the functionality and effectiveness of the items.

6.2.3 SUMMARY OF FINDINGS OF EMPIRICAL INVESTIGATION

The analysis and discussion of findings indicated that most of the respondents' view of problem solving in mathematics was at odds with the literature's definitions. Their conception of problem solving revolved around giving learners a question, subjecting them to serious thinking, and applying learned and practiced skills and techniques to solve mathematical tasks, whose solution procedures were explicit. Of the four teachers interviewed, only two indicated that they knew what problem solving was. In their definitions of problem solving, they mentioned phrases like "giving learners a question, requiring them to think, without an obvious way to get to the solution", which concurred with literature definition of problem solving in mathematics (Section 4.2.3).

One teacher sounded unsure about whether problem solving goals are even stated in The Eswatini Education and Training Sector Policy (2018), The Swaziland General Certificate of Secondary Education (SGCSE) (2015) syllabus, the Junior Certificate (2016) syllabus and the prescribed Prism Alive series (2010). All the respondents appreciated the value of using problem solving as a teaching strategy (Section 4.2.4). Some even went to the extent of mentioning that problem solving develops learners' critical thinking skills and ability to solve problems in their own lives. However, the teachers indicated that they rarely used problem solving as a strategy in Form 1 due to a number of challenges (see, Table 4.7).

One of the two teachers who had their lessons observed was able to use problem solving as a teaching strategy in line with the current study's conceptual framework. There were some teaching practices such as posing a question that had no obvious way or method to get to the solution and having learners sharing ideas. However, the lesson presentation by the other teacher did not match well with what the teacher said during the interview. The lesson observation problems had clear solution procedures requiring application of learnt rules or formulae. This was found to match the characteristics of the common classroom drill exercises and application word problems, which is not what constitutes a mathematics problem as applied to problem solving.

Teachers argued that the method cannot be implemented properly until the curriculum, textbooks and assessment system reflect the value of this approach. Teacher's lack of problem solving knowledge, large classes and inadequate time also are hindrances (see, Table 4.7). It was also noted that all the respondents felt that they required face-to-face training about problem solving in mathematics in order to effectively implement the aspirations of the curriculum with regard to problem solving (Section 4.2.6).

6.3 ANSWERING THE RESEARCH QUESTIONS

6.3.1 Research Question 1: *What meanings do teachers attach to mathematical problem solving?*

In this study the researcher used problem solving largely as a teaching tool and an instructional strategy for mathematics. Hence, problem solving is a teaching method employed to present educational material in a meaningful, contextualized, customized and real world approach. A problem solving approach strives to provide productive educational guidance and meaningful instruction to learners to support and bolster the intended cultivation of problem solving skills in them. The respondents comprehended problem solving in different ways.

For instance, SC02 stated that, in problem solving, there is no obvious way to get to the solution and learners are required to reason or think before giving the solution when teaching mathematics. Teacher LL03 on the other hand mentioned that problem solving in mathematics involves questions in the higher order in the bloomy Taxonomy, starting with application, analysis, evaluation and synthesis, where learners are subjected to serious thinking to obtain a solution. LN04 described problem solving as an existing problem in the mathematics field where solutions need to be found (Section 4.2.3).

6.3.2 Research question 2: *How do teachers in Form 1 conceive a mathematical problem?*

Also discussed in the literature review, Schoenfeld (1992) notes that a problem could be a situation that one is unfamiliar with, whereby the problem solver is unable to execute its solution (Section 2.3.1). The respondents' general definition of a mathematics problem was a situation that should not allow the learner to just recall answers without subjecting him or her to proper thinking, but the problem solver must think before attempting to solve the problem. Problems should require higher-order abilities. Some characteristics of a mathematics problem stated by one teacher were to have no obvious way or method to get to the solution and to relate to real life situations with which learners were familiar (Section 4.2.2). Such a mathematical problem was posed and discussed in one of the two observed lessons. It should be noted that the way some teachers described mathematical problems during the interviews and lesson observation concurs with the educational literature.

6.3.3 Research question 3: *What kind of problem solving questions and tasks, do teachers in Form 1 pose?*

Avcu and Avcu, (2010: 1283) state that problems are generally categorized into routine and non-routine problems. Routine problems may require one to solve them by making use of certain computational skill and making use of formulas. Jurdak (2005) says that non-routine problems may require one to rearrange given information, formulating patterns and classifying while being able to do computational properly. The respondents gave examples of problems such as asking learners to find the height of a flag pole when given information about the angle where you are standing and the distance from the pole to where she was standing (a question based on trigonometry); what three-dimensional shape has the highest volume and least surface area; A girl is m years old, her mother is four years older than twice the girls' age. The sum of their ages is 44 years. How old is each one of them?, as well as find the 20^{th} term and the general way of finding the n^{th} term (see, Table 4.3).

The participants of this study believed that problems should be challenging yet manageable and should engage the learners' interests. Problems should require higher-order abilities. Some characteristics of a mathematics problem stated by one teacher were to have no obvious way or method to get to the solution and to relate to real life situations with which learners were familiar. It was observed during the lesson observations that one teacher assigned problems which were of a different type to the ones he talked about during the interview (see, Table 4.12). The most common characteristics included real life situations, challenge and motivation, and word problems, although some did fit the characteristics of routine drill exercises with clear solution procedures, requiring application of learned rules or formulas and having only one solution method.

6.3.4 Research question 4: *What are the perceived level of knowledge of Form 1 teachers enacting problem solving as a teaching strategy in mathematics in the classroom?*

Regarding the literature review, the JC mathematics and additional mathematics Syllabus (2015: 6-7) has devoted some part of its curriculum guidelines in addressing the curriculum approaches. For instance:

> Learners engage in problem solving within contextual situations by communicating, reasoning and connecting to: representing and use numbers in a variety of equivalent situations by contextualised situations; explore, identify, analyse and extend patterns in mathematical and contextual situations; collect, organize and represent data; use and apply geometric properties and relationship to describe the physical world (JC Syllabuses (2015: 6-7).

Teacher SC02 started the lesson by recapping from the previous lesson and determined prerequisite knowledge about the topic before posing a problem (Section 4.3.1). Modelling problem solving and talking through her thought process in the class was a common feature of the lesson of problem solving (Section 4.3.3). As learners were solving the problem, the teacher encouraged them to avoid the temptation of abandoning an unsuccessful solution attempt completely. Hence, teacher provided a hint so that the learners gained courage in solving the problem. Furthermore, in one of the lesson observations conducted, learners were sitting in groups of four or five observed explaining their solutions to each other.

Teacher SC02 gave a problem and walked around the class, supervising the groups, telling them to also compare their solutions. The teacher encouraged learners to look to one another and to their own reasoning abilities before looking to her for solutions. In this way, learners were observed sharing ideas, explaining their thinking and helping each other to solve the problem (Section 4.3.4). The teacher shared authority for correct answers with the learners since a group representative was chosen to go and explained their solution on the chalkboard. Teacher SC02 was observed asking questions during class in an attempt to get learners to think about the knowledge they had that would help them solve the problem on triangles as well as moving around groups, observing learners working and checking their progress (Section 4.3.5).

6.3.5 Research question 5: *What challenges, if any, do teachers encounter when using problem solving as a teaching strategy to learn mathematics in Form 1?*

There are some challenges which exist for Form 1 mathematics teachers when using problem solving as a teaching strategy. Teachers argued that the method cannot be implemented properly until the curriculum, textbooks and the assessment system reflect the value of this approach. Other hindrances included the teacher's lack of problem solving knowledge, large classes and inadequate time to teach (Section 4.2.5). It was also noted that all the respondents felt they required face-to-face training on problem solving in mathematics in order to effectively implement the aspirations of the curriculum in this regard. One teacher even suggested that such training could be done through workshops and in mathematics symposiums (see, Table 4.8).

6.4 LIMITATIONS OF STUDY

The study was limited to the Manzini region due to financial and time constraints. Since this study was a descriptive case study, generalizations about large populations are not intended, however the small sample of teachers is still a limitation as not every way of using problem solving as a teaching strategy was represented.

6.5 RECOMMENDATIONS

The results of the study indicated that it was not clear for most teachers what a problem is or how to use problem solving as a teaching strategy in mathematics as a way to activate learners' mathematical problem solving skills. It is therefore, the researcher's view that the Ministry of Education should allocate a portion of its educational budget to initiate teacher development programmes where mathematics teachers become largely acquainted with problem solving teaching and possible learning approaches.

The Eswatini Education and Training Sector Policy (2018), The Swaziland General Certificate of Secondary Education (SGCSE) (2015) syllabus, the Junior Certificate (2016) syllabus and the prescribed Prism Alive series (2010), should also clearly indicate an agreed upon definition of problem solving and how it can be used as a teaching strategy in the teaching and learning of mathematics in Form 1 since the respondents perceived problem solving differently.

The current study indicated that using problem solving as a teaching strategy is possible. School mathematics curriculum designers, teacher professional development providers and policy makers could be well informed in terms of the intended mathematical problem solving skills that are needed to be learned by learners and therefore, design supportive curriculum programmes to promote the intended development of learners' problem solving skills in mathematics. The new curriculum should address the educational aspirations of problem solving, rather than advocating traditional teaching approaches and views when transmitting mathematical knowledge. Teachers are still using the traditional methods of teaching mathematics. The description of practices for using problem solving as a teaching strategy described in this study can be helpful to any teacher interested in implementing such a teaching approach.

This study recommends that efforts should be made to support and motivate teachers with regards to problem solving to generate positive teacher identities and instructional attitudes needed to utilize problem solving as a teaching strategy in the teaching and learning of mathematics. This is in respect to providing adequate resources and having an appropriate number of learners in a class. The study showed that all the respondents saw the value of using problem solving as a teaching strategy. It is therefore recommended that problem solving should be clearly integrated in secondary school as is the case at the primary school level. Concerted efforts need to be made to enculturate the perceptions of problem solving even in practicing teachers. The centralization of problem solving in teacher training could be included in such efforts. This will enable the monitoring and supervision of its implementation during teaching practice and micro-teaching sessions. As a result, new teachers are introduced with the culture of problem solving and may adopt problem solving as a teaching strategy. The success of this teaching strategy could be enhanced by the introduction of examination formats that also incorporate testing problem solving abilities in national standardized examination.

6.6 FUTURE RESEARCH

The researcher recommends that further studies be conducted in this area, to provide a more detailed understanding of teachers' perceptions of problem solving not only in Form 1 but in other Forms as well. Such studies could include mathematics teachers' abilities to solve mathematics problems, since teachers are sometimes required to model problem solving when using problem solving as a teaching strategy. Problem solving skills of learners need to be investigated in order to understand the challenges that they encounter when solving mathematics problems. It is also recommended that the assessment process of problem solving be investigated. The researcher recommends that more time be provided for carrying out such a study since the time spent on this study was limited.

REFERENCES

Aksoy, Y, Bayazit, I., & Donmez, M. K. (2015). Prospective primary school teachers' proficiencies in solving real world problems: Approaches, strategies and models. *Eurasia Journal of Mathematics, Science and Technology Education*, 11(4):827-840. [Online] Available from http://wwwejmste.com {accessed 23 April 2016}.

Altun, M. (2005). *Matematik ogretimi*. Bursa: Erkam Publishing.

Alzahrani, K. S. (2017). Metacognition and its role in mathematics learning: An exploration of the perception of teachers and students in a secondary school. *International Electronic Journal of Mathematics Education, 12*(5), 521-537.

Anderson, J. (2014) Implementing problem solving in mathematics classrooms: What support do teachers want? *Journal of Educational and Instructional Studies in the World*, 5(2), 35-40 [Online] Available from https://www.researchgate.net {Accessed 20 July 2019}.

Avcu, S., & Avcu, R. (2010). Pre-service elementary mathematics teachers' use of mathematical problem solving. *Procedia-Social and Behavioural Science, 9*(1), 1282-1286. [Online]. Available from www.sciencedirect.com {Accessed 2 April 2017}.

Aydogdu, M. (2014). A research on geometry problem solving strategies used by elementary mathematics teacher candidates. *Journal of Educational and Instructional Studies in the World,* 4(1), 53-62 [Online]. Available from www.wjeis.org {Accessed 9 June 2017}.

Aydogdu, M & Ayaz, M.F. (2008). The importance of problem solving in mathematics curriculum. *Journal of New World Science Academy*, 3(4), A0095, 538-545[Online] Available from https://www.researchgate.net {accessed 12 July 2019}.

Bal, A. P. (2015). Examination of the mathematical problem solving beliefs and success levels of primary school teachers' candidates through the variables of mathematical success and gender. *Journal of Educational and Instructional Studies in the World* 10(5) 2573- 2575 [online] Available from http://www.estp.com.tr {accessed 20 March 2016}.

Benegusenga, A., Ntawiha, P. & Nzabalirwa, W (2017). Teacher qualification and teacher techniques in nursery schools in Kicukiro District, Rwanda. *Rwanda Journal of Education.* Volume 4 No 1 (2017) University of Rwanda. College of Education.

Bjuland, R. (2004). Student teachers' reflections on their leaning process through collaborative problem solving in geometry. *Educational Studies in Mathematics*. Volume 55, issue 1-3, 199-225 [Online] Available from https://www.researchgate.net {accessed 12 July 2019}.

Bonato, A. (2018). No matter what method is used to teach Mathematics, make it fun. Ryerson University *American Journal of Theoretical and Applied Statistics, 5*(1), 1-4. [Online] Available from https://theconversation.com {accessed 12 April 2019}.

Brehmer, D. (2015*).* Problem solving in mathematics textbooks. (Master's Thesis, Malardalen University, Sweden [Online] Available from http://www.diva-portal.org {accessed 23 April 2016}.

Cai, J., & Lester, F. (2016). What research tells us about teaching mathematics through problem solving. Mathematical Thinking and Learning: An International Journal, 2(2000) 309-340 [online] Available from https://www.researchgate.net/publication/228597905 {accessed 15 April 2018}.

Cavanagh, M. (2008). One secondary teacher's use of problem solving teaching approach. Macquarie University. [Online] Available from www.sciencedirect.com {Accessed 2 April 2017}.

Chapko, M. A., & Buchko, M. (2004). Mathematics instruction for inquiring minds, principal (Reston,Va.) *An International Journal* 8(4), 30-33. [online] Available from https://eric.edu.gov {Accessed 2 April 2017}.

Cohen, L., Manion, L., & Morrison, K. (2007). Research methods in education. 6th edition. Britian: Alejandra Navarro Sada.

Creswell, J. W., & Plano Clark, V. L. (2011). Designing and conducting mixed method research. Thousand Oarks, CA: Sage.

Curtin, M., & Fossey, E. (2007). Appraising the trustworthiness of qualitative studies: Guidelines for occupational therapists. *Australian Occupational Therapy Journal*, 54, 88-94. doi:10.1111/j.1440–1630.2007.00661.x.

D'Ambrosio, B. (2003). Teaching mathematics through problem solving: A historical perspective. In F. K. Lester, & C. I. Randall (Eds.), *Teaching mathematics through problem solving: Prekindergarten to Grade 6,* (pp.37-50). Reston, VA: The National Council of Teachers of Mathematics Inc.

Dhlamini, J. J. (2012). *Investigating the effect of implementing a context-based problem solving instruction on learners' performance*. Unpublished doctoral thesis, University of South Africa, Pretoria. [Online]. Available from https://core.ac.ak {Accessed 8 March 2018}.

Donaldson, S. E. (2011). *Teaching problem solving: Practices of four high school mathematics teachers*. PhD thesis, The University of Georgia, Georgia [Online].Available from http://getd.libs.uga.edu/pdf/donaldson_sarah_e_201105_phd.pdfbySEDONALDSON. {Accessed 25 September 2015}.

Ekici, D. I. (2013). The perceptions and views about problem solving process of pre-service science teachers. *Procedia-Social and Behavioural Science,* 141(2014) 308-312 [Online] Available from www.sciencedirect.com {Accessed 17 September 2019}

El-Masri, M. M. (2017). Terminology 101:Introduction to research sampling. *Internaional Nursing Journal*, 5(2) 506-516 [Online] Available from https://canadian-nurse.com {Accessed 17 September 2017}.

Ersoy, E. (2016). Problem solving and its teaching in mathematics. *The Online Journal of New Horizons in Education,* 6(2), 79-87 [Online] Available from www.tojoined.net {Accessed 2 April 2017}.

Ersoy, E., & Guner, P. (2015). The place of problem solving and mathematics thinking in the mathematics teaching. *The Online Journal of New Horizons in Education 5*(1), [Online] Available from www.tojned.net{Accessed 1 March 2016}.

Florida Department of Education. (2010). Classroom cognitive and meta-cognitive strategies for teachers. Florida: Department of Education.

Gale, N. K., Heath, G., Cameron, E., Rashid, S., & Redwood, S. (2013). Using framework method for the analysis of qualitative data in multi-disciplinary health. *Journal for BMC Medical Research Methodology*, 13(1) [Online] Available from http://www.biomedcentral.com/1471-2288/13/117 {Accessed 1 March 2019}

Gay, L. R., Mills, G. E., & Airasian, P. (2012). Educational research: Competencies for analysis and application. 10th edition. New Jersey: Pearson education

Gay, L. R., Mills, G. E., & Airasian, P. W. (2011). Educational research: Competencies for analysis and application. 10th edition, Pearson: United States.

Gibson, W. J., & Brown, A. (2009). Working with qualitative data. London: Sage.

Goldman, E. (2012). Leadership practices that encourage strategic thinking. *Journal of Strategy and Management, 5*(1), 25-40.

Grouws, D. A. (2003). The teacher's role in teaching mathematics through problem solving. In Fulop, E, Teaching problem solving strategies in mathematics. [Online] Available from http://www.luma.fi/file_download/485 {Accessed 3 October 2015}.

Gunawan, J. (2015). Ensuring trustworthiness in qualitative research. *Belitung Nursing Journal*, 1,10-11. [Online] Available from http://belitungraya.org/BRP/index.php/bnj/ {Accessed 24 August 2017}.

Harrell, M. C., & Bradley, M. A. (2009). *Data collection methods: Semi-structured interviews and focus groups*. National Defense Research Institute. The Rand Corporation: Pittsburgh.

Hastings, N., & Chantrey, K. (2002). Group seating in primary schools: an indefensible strategy, *Paper presented at the annual conference of the British Educational Research Association*. Nottingham: Trent University.

Hatfield, L. L. (1978). Heuristical emphases in the instruction of mathematical problem solving: Rationales and research. In L. L. Hatfield & Bradbard, D., A (Eds.), *Mathematical problem solving: Papers from a research workshop*. Columbus, OH: ERIC Clearing house for science, Mathematics and Environmental Education.

Hatfield, M. M., Edwards, N. T., Bitter, G. G. & Morrow, J. (2007). *Mathematics methods for elementary and middle school teachers,* (6th ed.). USA: Allyn & Bacon.

Heddens, J. W., & Speer, W. R. (1997). *Today's mathematics. Part 1: Concepts and classroom methods.* New Jersey: Prentice-Hall, Inc.

Hiebert, J., & Wearne, D. (2003). Developing understanding through problem solving. In H. L. Schoen (Ed.), *Teaching mathematics through problem solving: Grades 6-12* (pp.3-13). Reston, VA: National Council of Teachers of Mathematics.

Isaacs, S. (2007). ICT in education in Swaziland. Survey of ICT and education in Africa: Swaziland country report. [Online] Available from www.infodev.org {Accessed 2 May 2017}.

Izzati, L. R. & Mahmudi, A. (2018). The influence of metacognition in mathematical problem solving. *Journal of Physica Conference Series*, 1097(1):012107. [Online] Available from https://www.researchgate.net {Accessed 2 July 2019}.

Jeffrey Barnes, Kerri Conrad, Christof Demont-Heinrich, Mary Graziano, Dawn Kowalski, Jamie Neufeld, Jen Zamora, and Mike Palmquist. (1994 - 2012). Generalizability and Transferability. Writing@CSU. Colorado State University. [Online] Available from https://writing.colostate.edu/guides/guide.cfm?guideid=65 {Accessed 28 August 2017}

Jurdak, M. (2005). Contrasting perspectives and performance of high school students on problem solving in real world situated, and school contexts. *Educational Studies in Mathematics,* 63, 283-301.

Kaino, L. M., & Yaqiang, L. (2004). *In-Service teachers' knowledge of mathematical problem solving*. Paper presented at the Southern African Association for Research in Mathematics, Science and Technology Education., Durban.

Kim, K. M. H. (2017). Teaching problem solving: Let students get 'stuck' and 'unstuck'. *Journal of Human Resource and Sustainability Studies 5*(1), 25-40. [Online] Available from https://www.brookings.edu [Accessed 04 March 2018].

Kini, T., & Podolsky, A. (2016) Does teaching experience increase teacher effectiveness? A Review of research. Palo Alto: Learning Policy Institute. [Online] Available from https://learningpolicyinstitute.org [Accessed 04 March 2018].

Krathwohl, D. R. (2009). *Methods of educational and social science research*. 3rd edition. Long Grove, IL: Waveland Press.

Krulik, S., & Rudnick, J. A. (1980). Problem solving: *A handbook for teachers*. Boston, MA: Allyn and Bacon.

Lambdin, D. V. (2003). Benefits of teaching through problem solving. In F. K. Lester (Ed.), Teaching mathematics through problem solving: Prekindergarten - *Grade 6* (pp.3-13). Reston, VA: NCTM.

Lee, E. S. (2007). The effect of alternative solutions on problem solving performance. Taipei Municipal University of Education, Taiwan (R.O.C.).

Lee, J. E., & Kim, K. T. (2005). Elementary school teacher candidates' perceptions of good problems. *IUMPST: The Journal, Vol. 1 (Content Knowledge)*, September 2005. [Online] Available from http://www.k-12prep.math.ttu.edu/journal/ contentknowledge/lee01/article.pdf {Accessed 04 March 2018}

Lester, F. K. (1994). Musings about mathematical problem solving research: 1970-1994. *Journal for Research in Mathematics Education, 25*(6), 660-675.

Liker Etikan, S. A. M. R. S. A., (2016). Comparison of convenience sampling and purposive sampling. *American Journal of Theoretical and Applied Statistics, 5*(1), 1-4.

Lupahla, N. (2014). *Assessing the algebraic problem solving skills of Grade 12 learners in Oshana region, Namibia*. Unpublished Masters' dissertation, University of South Africa, Pretoria.

Maphalala, I (2008). *I brought IGCSE to Swaziland*, The Times of Swaziland, 18 February, p. 1.

Marsh, J. (2018). Why mathematics word problems are important*? Journal of Mathematics Teacher Education,* 4(1) 227-233 [Online]. Available from https://medium.com {Accessed 16 July 2019}.

Matheson, D. (2012). Teaching through problem solving: Bridging the gap between vision and practice. *An International Journal* 5(4), 20-33. [Online]. Available from www.peterliljedahl.com {Accessed 13 June 2017}.

Mathnasium. (2018). Why do I need to learn this? The benefits of word problems in Garner. *The Online Journal of New Horizons in Education* 4(2), 232-240

Mayer, R. E., & Hegarity, M. (1996). The process of understanding mathematical problems. In R. J. Stenderg& T. Ben-Zee (Eds.), *The nature of mathematical thinking* (pp. 29-54). Mahwah, NJ: Lawrence Erlbaum Associates.

Mayo, P., Donnelly, M.B., Nash, P.P., & Schwartz, R.W. (1993). Problem-Based learning. University of South Alabama. *Journal of Human Resource and Sustainability Studies* 4(1) 227-233. [Online] Available from www.southalabama.edu Accessed 2 September 2015}.

McClure, L. (2013). Problem solving and the new curriculum. An International Journal, 5(2) 55-60 [Online] Available from http://nrith.math.org {accessed 15 March 2016}.

McDougal, T., & Takahashi, A. (2014). Teaching mathematics through problem solving. *National Association of independent schools Journal*, *13*(1), 27-83. [Online] Available from http://www.nais.org {accessed 20 March 2016}.

McMillan, J. H., & Schumacher, S. (2010). *Research in Education: Evidence-Based inquiry*.7th edition, Boston: Pearson education.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.

Mhlolo, M. K., & Chauraya, M. (2008). An analysis of in-service mathematics teacher's conception of problem solving in the subject at Midlands State University. *African Journal of Research in SMT Education*, 12(1), 72-83. [Online]. Available from www.researchgate.net {Accessed 27 October 2015}.

Mills, K., & Kim, H. (2017). Teaching problem solving: Let students get 'stucked' and 'unstucked'. *Mathematics Education Research Journal*, 13(1), 27-39.

Mouton, J. (2001). *How to succeed in your masters' and doctoral studies*. Pretoria: Van Schaik.

Nardi, E., & Steward, S. (2003). Is mathematics T.I..R.E.D? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal,* 29(3), 345-366

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Nazzal, N. (2014). Modern vs traditional teaching method. *Procedia-Social and Behavioural Science,* 8(2), 1342-1356.

Norton, S., McRobbie, C., & Cooper, T. (2002). Teachers' responses to an investigative mathematics syllabus: Their goals and practices. *Mathematics Education Research Journal, 14*(1), 37-59.

Pennant, J (2013). Developing a classroom culture that support a problem solving approach to mathematics. *Procedia-Social and Behavioural Science*, 2(6), 203-306.

Polya, G (1962). *Mathematical discovery: On understanding, learning and teaching problem solving.* Creative Education Journal, 3(4), 513-519

Polya, G. (1957). *How to solve it: A new aspect of mathematical method*. London: Penguin Books.

Posamentier, A. S. & Krulik, S. (1998). *Problem-solving strategies for efficient and elegant solutions. A Research for the Mathematics Teacher*. California: Corwin Press.

Punch, K. F. (1998). *Introduction to social research: quantitative and qualitative approaches*. London: Sage.

Pressley, M. (1995). Cognition, teaching and assessment. Harper Collins College Publishers, (p. 28-30 & p. 315-322).[Online] Available from https://books.google.com, {Accessed 2 April 2017}.

Ritchie J, Spencer L, O' Connor W (2010) *Carrying out qualitative analysis*. In Ritchie J, Lewis J (Eds) Qualitative Research Practice: A Guide for Social Science Students and Researchers. Sage, London, 219-262

Saglam, Y., & Dost, S. (2014). Preservice science and mathematics teachers' beliefs about mathematics problem solving. *Procedia-Social and Behavioural Science*, 116, 303-306. [Online] Available from www.sciencedirect.com {Accessed 2 April 2017}.

Schoen, H. L. (2013). (Ed.), *Teaching mathematics through problem solving: Grades 6-12* (pp.129–141). Reston, VA: National Council of Teachers of Mathematics.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In Grouws (Ed), *Handbook of research on mathematics teaching and learning*, pp 334-370. Macmillan. New York.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. San Diego: Academic Press Inc.

Sidenvall, J. (2019) *Problem solving helps pupils to learn mathematics*. Umea University. [Online] Available from https://m.phys.org {Accessed 16 July 2019}

Silver, E. A. (1985). *Teaching and learning mathematical problem solving: Multiple research perspectives.* Hilldale. NJ: Lawrence Erlbaum Associates.

Simon, M. K. (2011). Validity and reliability in qualitative studies. *Qualitative Health Research Journal* 12(2) 279-289. [Online] Available from http://dissertationrecipe.com {accessed 5 October 2016}.

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, Ca: Saga.

Stephan, R. A. (2017). *The interview: Ascertaining teachers' ability to foster student metacognition*. Doctoral Thesis, University of Pittsburgh, Available from http://d-scholarship. pitt.edu. (accessed on 29/03/2018).

Swaziland General Certificate of Secondary Education. (2015). *Mbabane: Examination Council of Swaziland*.

Swaziland. *The Swaziland education and training sector policy*. (2018). Mbabane: Government Printer.

Sweetland, R. (2016). Teachers' role in problem solving. McGill Journal of Education, 51(1), 615-634. [Online] Available from www.homeofbob.com {Accessed 13 June 2017}.

Taplin, M. (2015). *Mathematics through problem solving*. Institute of Sathya Sai Education. [Online] Available from www.mathgoodies.com {Accessed 13 June 2017}.

Tashakkori, A., & Teddlie, C. (2003). *Foundations of mixed methods research: integrating quantitative and qualitative approaches in the social and behavioural science*. London: Sage. The education system at the end of the twentieth century: An overview. [Online] Available from http://www.unesco.org/.../Mauritus.pdf {accessed 23 April 2016}.

University of South Africa. Department of Educational studies. (2011). *Analyse modeling in school mathematics: study guide for HBEDAMU*. Pretoria, South Africa.

University of South Africa. Department of Educational Studies. (2009). *The impact of the curriculum on effective mathematics classroom practice: Reader for HBEDAIQ*. Pretoria, South Africa.

Ward, R. (2012). *Problem solving toward mathematical understanding: Instructional design for students with learning disabilities*. Unpublished Master's dissertation, University of California, San Diego.

White-Clark, R., DiCarlo, M., & Gilchriest, N. (2008). Guide on the side: An instructional approach to meet mathematics standards. *The High School Journal*, 91, 40-44.

Wiersma, W., & Jurs, S. G. (2009). Research methods in education: An introduction, 9th edition. Boston: Pearson education.

Wilson, W. J., Fernandez, M. L., & Hadaway, N. (2015). *Mathematical problem solving*. The University of Georgia. [Online] Available from www.jwilson.coe.uga.edu {Accessed 2 May 2017}.

Wood, I. (2017) Want to develop your pupils' problem solving skills? Here's the solution. *Journal for Research in Mathematics Education*, 26, 4600-6600 [Online] Available from https://www.tes.com {Accessed 17 July 2019}

www.kevbotlearning.weekly.com/.../math 11 May 2015 {accessed 23 April 2016}.

www.statisticssolution.com [Accessed 12 /10/ 2017].

www.worldatlas.com/webimage/.../sz.htm {accessed 15 July 2016}.

Yavuz,G., Karatas, Y. D., Arsbay, H. N. (2015). *Research trends on mathematics problem solving in Turkey*: Masters' Dissertations, University of Turkey. [online] Available from www.sciencedirect.com {accessed 01 April 2016}.

Yazan, B. (2015). *Three approaches to case study methods in education*. Yin, Merrian and Stake. The Qualitative Report, University of Alabama, 20(2), 134-152. [Online] Available from https://nsuworks.nova.edu/tqr/vol20/iss2/12 {Accessed 20 March 2016}.

Yee, S. P., & Bostic, J. D. (2014). Developing a conception of students' mathematical problem solving. USA, University of South Carolina. [Online] Available from www.elsevier.com {Accessed 01 April 2016}.

Yin, R. K. (2003). *Application of case study research*. (2nd edition). Thousand Oaks, Ca: Saga

Yin, R. K. (2003). *Case study: Design and methods* (3rd edition). Thousand Oaks, Ca: Saga

Yuan, S. (2016). The teachers' role in problem solving: A study of elementary mathematics program from teacher's perspectives. Master's thesis, University of Toronto, Toronto. [Online] Available from http://tspace.library.utoronto.ca/../ {accessed 5 October 2016}.

Zanzali, N. A. A. (2003) *Implementing the intended mathematics curriculum: Teachers' belief about the meaning and relevance of problem solving*. Malaysia: Universiti Teknologi

APPENDICES SECTION

APPENDIX A: Request letter to the Ministry of Education and Training

Research title: *Teachers' notion(s) of problem solving in mathematics: A Case of Manzini Region in Eswatini*

PO Box3568
Manzini
Eswatini
M200
25 July 2017

The Director of Education
Teaching Service Commission
P. O. Box 69
Mbabane

Dear sir/ Madam
I, Nkosikhona Nhlabatsi am doing a research under supervision of Dr Joseph Jabulane Dhlamini, a lecturer in the Department of Mathematics Education towards a Masters' qualification in Education (MEd) with specialization in Mathematics Education at the University of South Africa. We are inviting you to participate in a study entitled, ***Teachers' notion(s) of problem solving in mathematics: A case of Manzini Region in Eswatini.*** The proposed study aims to find out how teachers in the Manzini Region perceive the notion of problem solving in mathematics. It shall involve secondary schools mathematics teachers. The benefit of this study is to raise awareness to mathematics teachers about the importance of problem solving in mathematics and its incorporation in the aims, assessment objectives and in the approaches of the mathematics curriculum in order for them to make use of teaching strategies that would enable learners' understanding concepts in context and that this understanding is constructed by the learners themselves as opposed to being transmitted by the teacher.

I propose to carry out my field work by interviewing 4 mathematics teachers and carrying out 2 lesson observations. The only foreseeable risk is that of inconvenience. There will be no reimbursement or any incentives for participation in the research. Teachers who participate in this study will receive a soft copy of the final report on the data analysis on request.

If you have any questions about the study, please feel free to contact the following:

Researcher: Nkosikhona Calvin Nhlabatsi
Contact: (+268) 76135425
Email: cnhlabatsi@yahoo.com / 48419052@mylife.unisa.ac.za

Supervisor: Dr Joseph Jabulane Dhlamini
Tel: +27 12 429 2023
Email: dhlamjj@unisa.ac.za

Thanking you in advance for your support.
Yours Sincerely
Nkosikhona C Nhlabatsi
The Researcher

APPENDIX B: Permission letter from The Ministry of Education and Training

Ministry of Education & Training

Tel: (+268) 2 4042491/5 P. O. Box 39
Fax: (+268) 2 404 3880 Mbabane, SWAZILAND

23rd September, 2016

Attention:
Headteachers:
List of Schools Attached

THROUGH
Manzini Regional Education Officer

Dear Colleague,

RE: REQUEST FOR PERMISSION TO COLLECT DATA FOR UNIVERSITY OF SOUTH AFRICA (UNISA) STUDENT – MR NKOSIKHONA CALVIN NHLABATSI

1. Reference is made to the above mentioned subjects.
2. The Ministry of Education and Training has received a request from Mr. Nkosikhona C. Nhlabatsi, a student at the University of South Africa (UNISA), that in order for him to fulfill his academic requirements at the University of South Africa, he has to collect data (conduct research) and his study or research topic is: *Teachers' Notion of Problem Solving in Mathematics in Swaziland: A case of Manzini Peri-Urban Area.* The population for his study comprises of 80 teachers from the mentioned schools who shall fill the questionnaire and 50 teachers who shall respond to interviews from the mentioned schools. All details concerning the study are stated in the participants' consent form which will have to be signed by all participants before Mr. Nhlabatsi begins his data collection. Please note that parents will have to consent for all the participants below the age of 18 years participating in this study.

3. The Ministry of Education and Training requests your office to assist Mr. Nhlabatsi by allowing him to use the above mentioned schools in the Manzini region as his research sites as well as facilitate him by giving him all the support he needs in his data collection process. Data collection period is one month.

DR. SIBONGILE M. MTSHALI-DLAMINI
DIRECTOR OF EDUCATION AND TRAINING

MINISTRY OF EDUCATION TRAINING 2016 -09- 23 BOX 39 MBABANE SWAZILAND

cc: Regional Education Officer –Manzini
Chief Inspector – Secondary
22 Head Teachers of the above mentioned schools
Dr. Joseph Jabulane Dhlamini

Page 1

APPENDIX C: Letter notifying the school principal/ governing body chairperson

P.O. Box 3568
Manzini
M200
Eswatini
25 July 2017

The School Principal/ SGB Chairperson
……………………………………………
……………………………………………
……………………………………………

Dear Sir/Madam

<u>An educational research</u>

I, Nkosikhona Calvin Nhlabatsi, student number 48 419 052. I'm a Masters of Education student in Mathematics Education at the University of South Africa (UNISA). I would like to kindly notify you that I have been given permission to carry out a research project entitled: *Teachers' notion of problem solving in mathematics: A case of Manzini Region in Eswatini***.** The aim of his study is to find out how teachers in the Manzini Region perceive the notion of problem solving in mathematics.

Through the participation of your mathematics teachers, the study endeavors to raise awareness to mathematics teachers about the importance of problem solving in mathematics and its incorporation in the aims, assessment objectives and in the approaches of the mathematics curriculum in order for them to make use of teaching strategies that would enables learners' understanding concepts in context and that this understanding is constructed by the learners themselves as opposed to being transmitted by the teacher.

I propose to carry out my field work by interviewing 4 mathematics teachers and carrying out 2 classroom observations. The only foreseeable risk is that of inconvenience. There will be no reimbursement or any incentives for participation in the research. Teachers who participate in this study will receive a soft copy of the final report on the data analysis on request. I guarantee that their responses will not be identified with them personally. Their participation in this case study is voluntary. If they decide not to participate, they are free to withdraw their consent and discontinue participation at any time without penalty.

If you have any questions about the study, please feel free to contact the following:

Researcher: Mr Nkosikhona Nhlabatsi (Tel: +268 76135425)
Email:cnhlabatsi@yahoo.com / 48419052@mylife.unisa.ac.za
Supervisor: Dr J.J Dhlamini (Tel: +27 12 429 2023): Email: dhlamjj@unisa.ac.za

Thanking you in advance for your contribution to this research.
Yours Faithfully

Nkosikhona Nhlabatsi
The Researcher

APPENDIX D:

Principal/Deputy Principal's Consent letter

I....Zolile Ndlovu.......................................~~Principal~~/Deputy Principal of Lobamba Lomdzala.......high school agree to allow mathematics teachers to participate in the study " Teachers' notions of problem solving in mathematics: A case of Manzini region in Eswatini" conducted by Nkosikhona Nhlabatsi. I allow the mathematics teachers to participate through interviews and lesson observations.

Principal's signature:..

Date:..........30-07-2017..

LOBAMBA LOMDZALA HIGH SCHOOL
DEPUTY PRINCIPAL
30 JUL 2017
P.O. BOX 244
MAHLANYA H122
TEL 2550 4301
SWAZILAND

APPENDIX E

Consent Form

Date

Teachers' notion of problem solving in mathematics: A Case of Manzini Region in Eswatini".

Dear Mathematics Teacher

I, NkosikhonaNhlabatsi, am doing a research with Dr Joseph Dhlamini, a lecturer in the Department of Mathematics Education towards a Masters in Education (M Ed) at the University of South Africa. You are being asked to participate in a study entitled teachers' notion of problem solving in mathematics: A Case of Manzini Region in Eswatini.

Research Purpose and Description

The study endeavors to raise awareness to mathematics teachers about the importance of problem solving in mathematics and its incorporation in the aims, assessment objectives and in the approaches of the mathematics curriculum in order for them to make use of teaching strategies that would enables learners' understanding concepts in context and that this understanding is constructed by the learners themselves as opposed to being transmitted by the teacher.

By virtue of being a mathematics teacher at secondary and/or high school in Eswatini, you have been selected to participate in this study.

Explanation of Procedures

Data collection will include semi-structured interviews and lesson observation with mathematics teachers. Questions like: what, in your view, is a mathematics problem? And how would you answer the question "what is meant by problem solving in mathematics? will be asked. It shall take about 20–30 minutes to conduct the interviews. An audio recording of the interview and lesson observation will be made to ensure that the teachers'' thoughts and ideas are collected completely.

Confidentiality

Confidentiality will be guarded by the researcher in the following ways: interviews will be conducted in a private room to reduce the chance of the teacher being overheard. The researcher will be the only person who knows the identity of the teacher interviewed. Your answers will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. Participants' anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings.

All data, when not being collected or analysed, will be stored in a password protected folder on the researcher's laptop and /or in a locked cupboard at the researcher's home.

All data will be destroyed (hardcopy will be shredded) while electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme after 5 years.

Benefits
Teachers who participate in this study will receive a soft copy of the final report on the data analysis on request. This report should be helpful in understanding exactly what problem solving in mathematics is? With this information they get the opportunity to use teaching strategies that would enable learners' understanding concepts in context and that this understanding is constructed by the learners themselves as opposed to being transmitted by the teacher.

Risks
Participants in this study will be reflecting on the teaching and learning that occurs daily in their classrooms with regard to problem solving in mathematics. The following risk may be uncounted through sharing of these reflections; knowledge of teachers may result in damaged relations (i.e. mistrust by learners, other teachers or administrators). To reduce this risk, the researcher will make every effort possible to ensure the confidentiality of the individuals.

Refusal/Withdrawal
Refusal to participate in the study will have no effect on any present or future services or benefits that I may be entitled to from the University. Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw at any time without a penalty or giving a reason.

THE FINDINGS/RESULTS OF THE RESEARCH

If you would like to be informed of the final research findings, please contact;

RESEARCHER
Full Name: Nkosikhona Calvin Nhlabatsi
Contact: (+268) 76135425
Email: cnhlabatsi@yahoo.com .

Should you have concerns about the way in which the research has been conducted, you can contact;

SUPERVISOR
Full Name: Dr Joseph Jabulane Dhlamini
Department of Mathematics Education
University of South Africa (UNISA)
AJH Building, Floor 7, Office 13 (7–13)

Tel: +27 12 429 2023
Cell: +27 76 495 0067
Email: dhlamjj@unisa.ac.za

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

Signature:______________________

Nkosikhona Calvin Nhlabatsi

CONSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, _______________________________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interview and lesson observation as part of the data collection tool.

I have received a signed copy of the informed consent agreement.

Participant Name &Surname (please print)______________________________________

__________________________ ______________________________

Participant Signature Date

Researcher's Name & Surname (please print)____

____
Researcher's signature Date

APPENDIX F: Reply receipt for the teachers

CONSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, Queen Mkhwanazi (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interview and lesson observation as part of the data collection tool.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) Queen Mkhwanazi

Participant Signature   Date 30/10/2017

Researcher's Name & Surname (please print) Nkosikhona Nhlalaki

Researcher's signature   Date 30/10/17

APPENDIX G: Semi-structured Interview Schedule

Initial information for the participants:

Thank you for your willingness to participate in this interview regarding teachers' notion of problem solving in mathematics in Swaziland. I am Nkosikhona Nhlabatsi and this research is being conducted for my dissertation in mathematics education at the University of South Africa (UNISA).

Your answers to these questions are very important to me and the study I am conducting. Your confidentiality is assured. Pseudonyms will be used so that participants cannot be identified by location or individual statements. All data that I collect will be stored in password protected folder on my laptop and/or in a locked cupboard at my residence.

With your permission, I would like to make an audio recording of this interview as well as take notes while you speak. This will help me collect your thoughts and ideas more accurately.

(Check the recording device and begin interview)

Reference information

Name:…………………………………………………

School:…………………………………………….…..

Pseudonym:………………………………………….

Experience

0–5 year 5– 10years 10–15 years 15–20years Above 20 years

Teacher's Qualification

Secondary or high school Teaching Diploma Bachelor's degree Honours degree Master's degree or above

Other………..……… ….……..…….. …………………… …..…………… ……………. …….

1.(a) What do you think a mathematics problem is?
(What are the characteristics of a mathematics problem?/ What makes it a good problem?)
(b) Can you give me an example of mathematics problem? (Think of some problems you have used recently in your teaching).

2.(a) How often do you give learners word problems in a particular exercise?
(b) Do you consult other mathematics textbooks for non-routine word problems in the teaching and learning of mathematics?

3. In the first mathematics symposium for 2016 as well as at a mathematics teachers' workshop in 2016 and 2015 organized by the Mathematics Department of the Ministry of Education, teachers were encouraged to use problem solving methods in teaching mathematics.
What comes to your mind when you hear the term problem solving in mathematics?

4. The Swaziland Education and Training Sector Policy (2011), the Swaziland General Certificate of Secondary Education (SGCSE) (2015), the Junior Certificate (2016) syllabuses and the prescribed Prism Alive series (2010) for Forms 4 and 5, all outline goals that refer to problem solving. (a) Can you explain how this might translate into classroom practice? / How do you incorporate problem solving into your mathematics classes?)

5. The National Curriculum for Forms 4 and 5 in the SGCSE syllabus set out several mathematical essential skills that should be incorporated into the curriculum delivery. These essential skills include communication and language skills, numeracy skills, problem solving skills, critical thinking skills, word and study skills, independence learning and working with others. (What do you consider to be good ways to help learners become better problem solvers?)

6. Problem solving as a teaching strategy to facilitate the learner's active participation is also included by National Curriculum Centre (NCC) in its teaching and learning strategies.
(a) Do you use problem solving as a teaching strategy in the teaching and learning of mathematics?
(b) If yes, what do you think are the advantages/benefits of using problem solving as a teaching strategy?
(c) What could be constrains/challenges that may prevent you from implementing problem solving?
7. Do you think more training is useful for mathematics teachers with regard to problem solving in mathematics? If yes, what platforms can be utilized in addressing the lack of adoption of problem solving teaching approach?

APPENDIX H: Lesson Observation Schedule

TEACHER'S CODE NAME: ....
SCHOOL CODE NAME: ....
GRADE LEVEL/ CLASS: ....
DATE OF LESSON: ....
LESSON TOPIC: ....
LESSON OBJECTIVES: ....

Theme sequencing	Theme and sub-questions	Researcher's comments/ observations
1	**Teaching general or specific problem solving strategies** ○ *Does the teacher talk explicitly about problem solving?* ○ *Does the teacher mention or demonstrate general or specific problem solving strategies?*	
2	***Modeling problem solving*** ○ *Does the teacher demonstrate problem solving or particular problem solving skills?* ○ *Does the teacher ever highlight, either implicitly or explicitly, Polya's four phases of problem solving?*	
3	**Limiting teacher input** ○ *Do learners work together to solve problems?* ○ *Do learners explain or demonstrate their solutions to classmates?* ○ *What assistance or guidance does the teacher provide to learners as they work on problems?* ○ *How does the teacher respond when learners pursue unproductive solution paths or dead ends? How far does the teacher let learners go before intervening?*	
4	**Promoting metacognition** ○ *How does the teacher promote metacognition in the classroom?* ○ *Does the teacher ask questions or make comments that encourage learners to be reflective about problem solving? If so, how?* ○ *Does the teacher model metacognitive behavior regarding problem solving?*	
5	**Highlighting multiple solutions** ○ *Does the teacher encourage learners to find various ways to solve a problem?* ○ *Do learners share their solutions with one another?* ○ *Does the teacher discuss connections between different solutions?* ○ *Is there discussion of advantages and disadvantages of particular problem solving strategies?* ○ *Does the teacher encourage learners to develop more efficient problem solving strategies?*	

APPENDIX I: Sample of transcribed teacher interviews

Reference information

Name:… Queen Mkhwanazi……………

School:…Lobamba Lomdzala high ………..

Pseudonym:…LL02… ………………….

Experience

0 – 5 years 5- 10years 10 – 15 years 15 – 20years Above 20 years

Teacher's Qualification

Secondary or high school Teaching Diploma **Bachelor's degree** Honours degree Master's degree or above

Other:………………. …………………… **(BSc) mathematics ………….. …………………**

and Chemistry ………………… ………………
(PGCE)

1.(a) **What do you think a mathematics problem is?**

(What are the characteristics of a mathematics problem?/ What makes it a good problem?)
It is a question or scenario given in mathematics in which there is no obvious way or method to get to the solution. The problem solver has to think before attempting to solve the problem. It requires higher order abilities

(b) Can you give me an example of mathematics problem? (Think of some problems you have used recently in your teaching).
What 3 Dimensional shape that has the highest volume and least surface area?
Here: learner has to think, you can't just answer the question, all the different shape you have and then you compare and contrast until you find the shape. So in this case the answer is a sphere but it is not obvious.

2. (a) How often do you give learners word problems in a particular exercise?
Very often
(b) Do you consult other mathematics textbooks for nonroutine word problems in the teaching and learning of mathematics?
Yes

3. In the first mathematics symposium for 2016 as well as at a mathematics teachers' workshop in 2016 and 2015 organized by The Mathematics Department of the Ministry of Education, teachers were encouraged to use problem solving methods in teaching mathematics.
What comes to your mind when you hear the term problem solving in mathematics?

It whereby you are giving questions to learners that require them to reason or think before giving the solutions when teaching mathematics. There is no obvious way to get to the solution.

4. The Swaziland Education and Training Sector Policy (2011), The Swaziland General Certificate of Secondary Education (SGCSE) (2015), the Junior Certificate (2016) syllabuses and the prescribed Prism Alive series (2010) for Forms 4 and 5, all outline goals that refer to problem solving. (a) Can you explain how this might translate into classroom practice? / How do you incorporate problem solving into your mathematics classes?)
In class the teacher gives scenarios or questions which the learners are unfamiliar with and apply or employ different strategies to get to the solution. I sometimes create questions or copy them from other books or past exam papers and then ask learners to form groups and work together to solve the problem. Sometimes I let them do the question at home and submit the next day.

5. The National Curriculum for Forms 4 and 5 in the SGCSE syllabus set out several mathematical essential skills that should be incorporated into the curriculum delivery. These essential skills include communication and language skills, numeracy skills, problem solving skills, critical thinking skills, word and study skills, independence learning and working with others. (What do you consider to be good ways to help learners become better problem solvers?)
- It's to train learners to read the questions with understand, understanding that mathematics has no one way of solving a problem and that as long as you do the right thing in mathematics/ using the right method, you will always get the answer.
- helping learners understand the mathematics language
- letting learners write down whatever they think is correct
- letting learners do problems independently and believing in themselves

6. Problem solving as a teaching strategy to facilitate the learner's active participation is also included by National Curriculum Centre (NCC) in its teaching and learning strategies. (a) Do you use problem solving as a teaching strategy in the teaching and learning of mathematics?

Yes

(b) If yes, what do you think are the advantages/benefits of using problem solving as a teaching strategy?

- Its creates learners who are critical thinkers
- Helps them to be creative
- Helps them enjoy mathematics
- Helps learners to be independent thinkers
- Helps learners creates their own problems

(c) Can you briefly explain how you use problem solving as a teaching strategy in mathematics?

I sometime use word problems and most of the time diagrams

(d) What could be constrains/challenges that may prevent you from using problem solving as a teaching strategy.

Time- normally we don't have enough time yet the maths syllabus is too long. Time during lesson delivery and in preparation because sometimes when solving a problem to apply a skill they need an hour in class just to answer one question yet you have least time to finish the syllabus for the whole year.

Sometimes word problems it becomes more of an English problem than a mathematics problem so it becomes more difficult to employ because of that, if a learner does not understand English then definitely he /she will not understand the question.

7. Do you think more training is useful for mathematics teachers with regard to problem solving in mathematics? If yes, what platforms can be utilized in addressing the lack of adoption of problem solving teaching approach?

Yes, some teachers tend to enjoy recall questioning to their teaching and learning. More training can even help us deal with the constrains we have indicates earlier. This can be done through teacher seminars, symposiums and workshops.

APPENDIX J: Language editing certificate

Editing Press

Editorial Certificate

This certificate is to affirm that Editing Press Inc., comprising faculty and postgraduates from the Universities of Oxford and Cambridge, has edited, to the best of its members' abilities, the work entitled

TEACHERS' NOTIONS OF PROBLEM SOLVING IN MATHEMATICS: A CASE OF MANZINI DISTRICT IN SWAZILAND

by

NKOSIKHONA CALVIN NHLABATSI

This certificate is issued without prejudice to the author on

27 August 2018

Charles Anderson
Director of Academic Editing
Editing Press Inc.
www.editing.press

APPENDIX K: Turnitin Certificate

Final Nhlabatsi dissertation

ORIGINALITY REPORT

29%	20%	8%	22%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	uir.unisa.ac.za Internet Source	5%
2	getd.libs.uga.edu Internet Source	3%
3	Submitted to University of Newcastle Student Paper	2%
4	Million Chauraya, Michael Kainose Mhlolo. "An analysis of in-service Mathematics teachers' conceptions of problem-solving in the subject at Midlands State University in Zimbabwe", African Journal of Research in Mathematics, Science and Technology Education, 2013 Publication	1%
5	nau.edu Internet Source	1%
6	Submitted to University of South Africa Student Paper	1%
7	Submitted to University of KwaZulu-Natal Student Paper	1%

APPENDIX L: Ethical Clearance Certificate

UNISA | university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2017/08/16

Dear Mr NC Nhlabatsi

Decision: Ethics Approval from 2017/08/16 to 2020/08/16

Ref: **2017/08/16/48419052/20/MC**
Name: Mr NC Nhlabatsi
Student:48419052

Researcher:
Name: Mr NC Nhlabatsi
Email: 48419052@mylife.unisa.ac.za
Telephone: +26876135425

Supervisor:
Name: Dr JJ Dhlamini
Email: dhlamjj@unisa.ac.za
Telephone: 076 495 0067

Title of research:

Teachers' notions of problem solving in mathematics: A case of Manzini district in Swaziland

Qualification: M Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2017/08/16 to 2020/08/16.

The low risk application was reviewed by the Ethics Review Committee on 2017/08/16 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.

Telep

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
hone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date 2020/08/16. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2017/08/16/48419052/20/MC** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Dr M Claassens

CHAIRPERSON: CEDU RERC
mcdtc@netactive.co.za

Prof V McKay

EXECUTIVE DEAN