A Study on the Existence of a Low Idiosyncratic Volatility Premium on the Cross-section of Share Returns on the JSE

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DECLARATION

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ABSTRACT

As one of the renowned anomalies in modern investment theory, the low idiosyncratic volatility anomaly may be the most bewildering and captivating of them all. The anomaly defies the traditional asset pricing theories of modern portfolio theory, which state the fundamental principle that high-risk portfolios are compensated for with higher expected returns. This study determined if the low idiosyncratic volatility premium is present on the cross-section of share returns of the JSE. 12-, 36- and 60-month volatility estimation periods were used in this study to determine if this has any significant effect on share returns. A relevant 26-year sample period from January 1994 to December 2019 was employed. In examining the CAPM OLS regression results utilising the 60-month idiosyncratic volatility estimation period, statistically significant evidence was found to support the alternative hypothesis of a low idiosyncratic volatility anomaly on the cross-sectional returns on the JSE. These findings are supported by a statistically significant alpha for five of the six portfolios examined and clearly indicate the superior performance of the low volatility portfolio in contrast to the high idiosyncratic volatility portfolios. These findings of the 60-month CAPM regression analysis provide clear evidence of a low idiosyncratic volatility anomaly and reject the null hypothesis that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 60-month volatility estimation period.

ABSTRACT: AFRIKAANS

As een van die bekendste anomalieë in moderne beleggingsteorie, is die lae idiosinkratiese gestadigheidsanomalie moontlik die mees verbysterende en boeiende anomalie van almal. Hierdie besondere anomalie bied ŉ uitdaging aan die tradisionele bateprysingsteorie van moderne portefeuljeteorie, die grondbeginsel waarvolgens daar vir hoërisiko-portefeuljes vergoed word deur hoër verwagte opbrengste. Die doel van hierdie studie is om te bepaal of die lae idiosinkratiese gestadigheidspremie aanwesig is by die deursnee-aandeleopbrengste op die JSE. In hierdie studie, is gestadigheidsramingstydperke van 12, 36 en 60 maande gebruik om te bepaal of dit enige beduidende uitwerking op aandeleopbrengste het. ŉ Relevante steekproeftydperk van 26 jaar van Januarie 1994 tot Desember 2019 is gebruik. Deur ondersoek van regressieresultate van die kapitaalbateprysingsmodel (KBPM) kleinste-kwadratemetode aan die hand van ŉ idiosinkratiese gestadigheidsramingstydperk van 60 maande is statisties-beduidende bewyse gevind om die alternatiewe hipotese van ŉ lae idiosinkratiese gestadigheidsanomalie in die deursnee-opbrengste op die JSE te ondersteun. Hierdie bevindings word ondersteun deur ŉ statisties-beduidende alfa vir vyf van die ses portefeuljes wat ondersoek is en dit dui duidelik op die superieure prestasie van die laegestadigheidsportefeulje in kontras met die hoë idiosinkratiese gestadigheidsportefeuljes. Die bevindings van die KBPM-regressie-analise van 60 maande voorsien duidelike bewyse van ŉ lae idiosinkratiese gestadigheidsanomalie en verwerp die nulhipotese dat daar nie statisties-beduidende bewyse is ten gunste van ŉ lae idiosinkratiese gestadigheidsanomalie in die deursnee-aandeleopbrengste op die JSE nie nadat gestadigheid geraam is aan die hand van ŉ gestadigheidsramingstydperk van 60 maande.

ABSTRACT: SESOTHO

E le e nngwe ya diphoso tse tummeng kgopolong ya sejwale-jwale ya matsete, bothata bo tlase ba ho hloka botsitso e ka ba ntho e makatsang le e hohelang ka ho fetisisa. Phoso e ikgethileng ha e latele dikgopolo tsa ditheko tsa thekiso ya thepa ya sejwale-jwale, e hlalosang molao-theo wa hore dipotefoliyo tse kotsing e kgolo di lefellwa bakeng sa dikgutliso tse phahameng tse lebelletsweng. Phuputso ena e ne e ikemiseditse ho fumana hore na tefo e tlase ya botsitso e teng dikarolong tse sa tshwaneng tsa dikgutliso tsa dikabelo ho JSE. Phuputsong ena ho sebedisitswe dinako tsa tekanyetso ya ho hloka botsitso ya dikgwedi tse 12, 36 le tse 60 ho fumana hore na sena se na le phello e kgolo ho dikgutliso tsa dikabelo. Nako ya sampole e loketseng ya dilemo tse 26 ho tloha ka Pherekgong 1994 ho isa ho Tshitwe 2019 e ile ya sebediswa. Ha ho hlahlojwa sephetho sa tekanyo ya CAPM OLS ho sebediswa nako ya dikgakanyo tsa ho hloka botsitso ha dikgwedi tse 60, ho fumanwe bopaki ba bohlokwa ho tshehetsa mohopolo o mong wa phokotso dikgutlisong tsa dikarolo tse fapaneng ho JSE. Diphumano tsena di tsheheditswe ke qaleho ya dipalo bakeng sa dipotefoliyo tse hlano ho tse tsheletseng tse hlahlobilweng mme di bontsha tshebetso e phahameng ya potefolio e tlase ya ho hloka botsitso ho fapana le dipotefoliyo tse phahameng tsa ho hloka botsitso. Diphumano tsena tsa tlhahlobo ya tekanyo ya CAPM ya dikgwedi tse 60 di fana ka bopaki bo hlakileng ba phokotso e sa tlwaelehang ya ho hloka botsitso le ho hanyetsa kgopolo-taba ya hore ha ho na bopaki ba dipalo-palo bo tshehetsang boemo bo tlase ba ho hloka botsitso bo sa tlwaelehang dikarolong tse sa tshwaneng tsa dikabelo ho JSE kamora ho lekanyetsa ho hloka botsitso ho sebedisang nako ya dikgakanyo tsa ho hloka botsitso ya dikgwedi tse 60.

KEY TERMS

Volatility, Idiosyncratic Risk, Systematic Risk, Quintile, Winsorisation

KEY TERMS: AFRIKAANS

Gestadigheid, Idiosinkratiese risiko, Sistematiese risiko, Kwintiel, Winsorisering

KEY TERMS: SESOTHO

Ho hloka botsitso, Kotsi ya letsete, Kotsi ya mebaraka, Dihlopha tse hlano tse arohaneng tse ka ajwang ho latela tekanyetso, Phetolo ya dipalo-palo ka ho fokotsa ditekanyetso tse feteletseng

ABBREVIATIONS

Abbreviation	Description
AIV	Abnormal idiosyncratic volatility
BRICS	Brazil, Russia, India, China, South Africa
BTM	Book-to-market
CAPM	Capital asset pricing model
CIV	Common idiosyncratic volatility
CVaR	Conditional value at risk
EMD	Empirical mode decomposition
fBm	Fractional Brownian motion
HML	Value effect (high minus low)
IMFs	Intrinsic mode fluctuations
IVOL	Idiosyncratic volatility variable
JB	Jarque-Bera
JSE	Johannesburg Stock Exchange
MPT	Modern portfolio theory
NBER	National Bureau of Economic Research
Q1	Low volatility quintile portfolio
Q5	High volatility quintile portfolio
S&P 500	Standard and Poor 500 Index
SMB	Size effect (small minus big)
US	United States of America
VaR	Value at risk
ZAR	South African rand

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CHAPTER 1

INTRODUCTION

1.1 Background

One of the trending topics in the field of corporate finance and investment is the low volatility anomaly. Noteworthy studies by Ang, Hodrick, Xing and Zhang (2006), Blitz and Van Vliet (2007), Baker, Bradley and Wurgler (2011) and Xiong, Idzorek and Ibbotson (2014) have assessed the validity of the topic by providing contrasting opinions and theories behind the rationale of anomaly on a global scale. The influential studies by Ang *et al.* (2006), Blitz and Van Vliet (2007) and Baker *et al.* (2011) have inspired a plethora of contemporary literature on the low volatility anomaly.

Understanding the low volatility anomaly requires a fundamental understanding of overall total and systematic volatility. Idiosyncratic volatility, which is also known as unsystematic or firm-specific volatility, is the risk prevalent to an individual asset with no correlation to market risk. Systematic volatility, which is also known as undiversifiable or market risk, is the risk inherent to the entire market or market segment. Total market volatility consists of systematic risk and idiosyncratic risk, in which idiosyncratic risk constitutes the largest component and accounts for the majority of variation in the risk of an individual asset over time.

The results of Ang *et al.* (2006), Baker and Wurgler (2015) and Bhootra and Hur (2015) suggest that low-risk portfolios which have low idiosyncratic volatilities yield significantly higher realised returns than high-risk portfolios with high idiosyncratic volatilities. Blitz and Van Vliet (2007) and Baker *et al.* (2011) conducted their studies on a systematic risk factor (beta) and found evidence that stocks with historically low systematic volatility are associated with superior Sharpe ratios and a statistically significant alpha. These results contradict the basic financial principle that high-risk portfolios are compensated for with higher expected returns, as investors demand a premium in order to hold riskier securities. This contradiction of modern portfolio theory (MPT) is illustrated further by Xu and Malkiel (2003) who state that the presence of a low idiosyncratic volatility anomaly challenges conventional equilibrium asset pricing theory, which affirms that the expected return on an asset is positively correlated to its systematic risk (beta).

This argument is further elaborated by Ang, Hodrick, Xing and Zang (2009), who provide a methodological analysis of how stochastic volatility is priced in the cross-section of expected share returns. They set out to estimate the price of risk for aggregate market volatility by determining if the market return is a systematic factor, arbitrage pricing theory or factor model. If found to be true, aggregate market volatility should be priced in the cross-section of stock prices.

Supporting the argument posed by Malkiel and Xu (2006), Ang *et al.* (2009) found statistically significant evidence that deviations from aggregate volatility have a negative relationship with the price of risk. These findings suggest that assets with positive exposures to aggregate volatility pay off in times when market returns are low. This implies that assets with exorbitant sensitivities to fluctuations in aggregate volatility earn significantly reduced returns. These results found by Ang *et al.* (2009) are consistent with and present in numerous asset pricing studies which estimate the price of risk using time-series and a cross-section of derivative options on an aggregate market index and market portfolio.

The second related objective of the study by Ang *et al.* (2009) was to examine the patterns in cross-sectional expected returns of portfolios formed by categorising stocks by their idiosyncratic volatilities. They measured volatility relative to standard models of systematic risk. Standard asset pricing models such as the capital asset pricing model (CAPM) and the Fama and French 3-factor model assume that idiosyncratic volatility is not priced in the cross-section of average returns. Ang *et al.* (2009) found statistically significant evidence in favour of the presence of a low idiosyncratic volatility anomaly after calculating idiosyncratic volatility relative to the Fama and French (1993) model.

The literature review of this study covered the array of opposing and complementary theories and findings on the low volatility anomaly.

The first distinction investigated in the literature examined in this study was between historical idiosyncratic and systematic volatility measurement methods. The low volatility anomaly was first identified in 1972 when Fischer Black published his study “Capital market equilibrium with restricted borrowing", later that year inspired Haugen and Heins (1972) to draft a working paper entitled "Risk and the rate of return on financial assets: Some old wine in new bottles". The original study of the low volatility anomaly considered total volatility on share returns, which can be reviewed in later studies by Dutt and Humphery-Jenner (2013) and Xiong *et al*. (2014). Ang *et al*. (2006, 2009) and Baker and Wurgler (2015) examined the presence of a low idiosyncratic volatility in share returns. They attempted to price idiosyncratic volatility as an additional risk factor in an asset pricing model to provide a more accurate measure of the asset's expected return. Finally, Blitz and Van Vliet (2007) and Baker *et al*. (2011) conducted their studies of the low volatility anomaly using the systematic risk factor (beta) in an attempt to challenge the theory of market efficiency and outperform the results of Clarke, De Silva and Thorley (2006), who found the minimum variance portfolio to be an effective investment strategy to achieve comparable or higher average returns at an approximately 25% reduction in risk.

The second distinction examined in existing literature in this study was the contrast between volatility and tail risk as a primary risk measure of the low volatility anomaly. It is a well-documented fact by Dennis and Strickland (2004) that volatility is stochastic in nature and appears to be negatively correlated to stock price returns. This indicates that volatility appears to be higher after steady negative returns and significantly lower after a series of positive returns. Due to this asymmetry in volatility, an alternative measure of tail risk is measured to determine if the idiosyncratic risk anomaly believed to be a measure of volatility may in fact be a result of downward tail risk.

The next area investigated was to determine if the presence of the low idiosyncratic volatility anomaly exists within emerging economies such as BRICS. It has been proven by Blitz and Van Vliet (2007) and Xiong *et al*. (2014) that the low returns of high volatility share returns are present on a global scale across numerous exchanges. This investigation determined if the low idiosyncratic volatility anomaly is present on the Johannesburg Stock Exchange (JSE) and if idiosyncratic risk is priced in an asset pricing model. The researcher felt that by testing the presence of idiosyncratic inaccuracies of a mis-specified factor model, the results may provide insight into new aggregate volatility risk factor models which could be used to correctly price assets on the JSE and potentially on a global scale.

The final distinction investigated in the study was to compare the volatility effect with size and value factors as per the Fama and French 3-factor model derived by Fama and French (1992). Future research studies may build on this study by including analysis of the volatility effect by introducing the Fama and French 5-factor model, which introduces profitability and an investment factor variable to the model.

1.2 Problem Statement

The implication of the low idiosyncratic volatility anomaly existing on the cross-section of JSE share returns is that there is an uninhibited contradiction of modern portfolio theory. In MPT, as stated by Markowitz (1952), investors require a premium for taking on additional risk. This implies that the low volatility anomaly is in direct contradiction of MPT, which could have a severe impact on the way investors view risk when deciding on which stocks to include in their risk-adjusted portfolio formation process.

A plethora of contemporary literature has identified the presence of a low volatility anomaly, primarily with a developed market as the sample exchange. As a result, this study determines if low idiosyncratic volatility stocks generate higher returns on the JSE, and whether idiosyncratic risk is taken into account in an asset pricing model.

Building on the work by Markowitz (1952), Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1996) introduced the CAPM, which today is the widely accepted and renowned method to determine the theoretically acceptable required rate of return of an individual asset. Several studies by Prat (1967), Friend and Blume (1970) and Black, Jensen and Scholes (1972) suggest that share returns do not perform as predicted by the CAPM, with the general consensus of low-risk portfolios yielding significantly better returns compared to their high-risk counterparts. Campbell, Lettau, Malkiel and Xu (2001) provide possible explanations on this theory which premise on theory of imperfect diversification in investor's portfolio selection. This would result in investors demanding compensation for the inability to completely diversify volatility away (Malkiel and Xu, 2002 and Jones and Rhodes-Kropft, 2003). The findings by Campbell *et al*. (2001) support the notion of increased idiosyncratic volatility over time relative to market volatility, with idiosyncratic volatility accounting for the greatest share of total volatility. Goyal and Santa-Clara (2003) agree, and demonstrate that idiosyncratic risk has significant forecasting ability in predicting excess market returns. These overall findings would significantly affect investors' share portfolio selection as historically low idiosyncratic volatility stocks would be more desirable from an investment viewpoint and expected return calculation methods would require a new factor to price for idiosyncratic risk.

If the presence of the low volatility anomaly were discovered on the JSE, the aim of the study was to conduct further tests through regression analysis to test for size and value effects on the idiosyncratic premium. The introduction of the Fama and French 3-factor model as a volatility measurement method serves as a tool to address the shortfalls of CAPM. Furthermore, the limited research conducted at a developing and emerging market level provides valuable insight with the inclusion of the Fama and French 3-factor model.

Finally, as a result of limited literature investigating the idiosyncratic volatility premium with respect to multiple idiosyncratic volatility estimation periods, the impact of volatility estimation and time was analysed by introducing 12-, 36- and 60-month volatility calculation periods. The introduction of multiple volatility estimation periods determined the effect of time on volatility estimation and its significance in amplifying or eliminating the idiosyncratic volatility premium on the JSE.

1.3 Research Objectives

1.3.1 Primary Research Objective

To determine if the low idiosyncratic volatility premium is present on the cross-section of share returns of the JSE.

1.3.2 Secondary Research Objectives

In addition to determining whether there is a low idiosyncratic volatility premium present on the cross-section of share returns on the JSE, the study determined:

- The various implications a low idiosyncratic volatility premium has on MPT;
- If an idiosyncratic volatility factor can correctly account for a share's expected return;
- If size, value or momentum effects could provide significant justification of the presence of a low idiosyncratic premium on the cross-section of share returns;
- If the window period applied to estimating idiosyncratic volatility impacts the low volatility premium;
- If certain industries are more susceptible to the effects of a low volatility premium on the cross-section of industry related share returns;
- If JSE stocks with low idiosyncratic risk continue to remain low into the future, or if they revert to higher levels of risk over a period of time;
- If an alternative tail risk metric such as value at risk (VaR) or conditional value at risk (CVaR) could potentially provide significant explanatory power in the cross-sectional variations in JSE share returns.

1.4 Hypotheses

H_0: There is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 12-month volatility estimation period.

H_1: There is statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 12-month volatility estimation period.

H_0: There is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 36-month volatility estimation period.

H_1: There is statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 36-month volatility estimation period.

H_0: There is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 60-month volatility estimation period.

H_1: There is statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 60-month volatility estimation period.

1.5 Importance and Benefits of the Study

1.5.1 Benefits of the Study to Theory

When examining prior literature, it is evident that the vast majority of studies conducted, such as those by Ang *et al*. (2006, 2009), Baker *et al*. (2011), Xiong *et al*. (2014) and Hou and Loh (2016), were conducted primarily on a US and European sample dataset. After conducting extensive analysis of the geographical locations of the literature, few comprehensive studies on the low idiosyncratic volatility anomaly were identified on a South African sample exchange, with fewer published at an international level. Studies by Page, Britten and Auret (2016) and Dutt and Humphery-Jenner (2013) form part of the sparse literature examining the low idiosyncratic and systematic volatility anomaly on a South African market exchange. These studies incorporate the Fama and French 3-factor model as a measurement method of volatility, but may be enhanced by introducing the Fama and French 5-factor model in conjunction with multiple volatility measurement periods. Studies by Xiong *et al*. (2014) and Blitz and Van Vliet (2007) took a global perspective in analysing the low volatility anomaly by using index funds such as the Morningstar's open-end equity mutual fund and the FTSE World Development Index. However, these studies targeted primarily developed markets in developed economies. Studies on the low volatility anomaly conducted within the BRICS association are compared to the South African sample results found in this study and to developed economies such as Europe and the US.

This study aimed at determining whether the low idiosyncratic volatility anomaly discussed does hold in the financial sphere of South Africa. If this anomaly proves to hold, it will be a useful inclusion in current South African financial and economic literature.

1.5.2 Benefits of the Study to Practitioners

Asset pricing models which are widely utilised are known to be modelled on necessary, albeit unrealistic, assumptions. CAPM is a prime example, as the model is formulated on an unrealistic world, failing to consider many real-world complexities. As stated by Brealey, Myers and Allen (2014), CAPM postulates that the return on an asset is distinctly relational to its market beta. This implies that idiosyncratic risk is not a determining factor in calculating a share's expected return.

This study may facilitate investor strategy by attempting to identify a statistically significant idiosyncratic volatility factor, which may provide a supplementary measure to accurately account for a share's expected return. In this regard, it may provide a contrary view to what MPT states.

1.6 Outline of Chapters

The chapter outline for the study is as follows:

1.6.1 Chapter 1: Introduction

The introduction chapter provides the background of the study with a definition of and historical reference to the low volatility anomaly. Furthermore, the research objectives, hypotheses and benefits are discussed in detail.

1.6.2 Chapter 2: Literature Review

In chapter 2, a literature review by relevant authors on the topic of the low-risk anomaly is conducted. The primary literature cited in this study follows the findings of Ang *et al*. (2006) and Baker *et al*. (2011). Further literature from additional authors is included to supplement the findings of the two main sources as well as to provide contrasting opinions and findings on the topic of the low idiosyncratic volatility anomaly.

1.6.3 Chapter 3: Methodology

The methodology chapter of the study comprises the research methodology and data requirements for the empirical analysis. This chapter highlights the data requirements as well as the assumptions and procedures followed in the sample selection process.

1.6.4 Chapter 4: Results

In the results chapter the results of the empirical analysis conducted in this study are analysed and presented. Exploratory data analysis is conducted to summarise the data's primary characteristics. Next, the performance of the quintile portfolios is examined, with reference to the extreme and average expected losses for the worst 5%, 1% and 0.01% of returns. Additionally, the cumulative return series for each quintile portfolio are estimated over the sample period. Finally, the results from the OLS regression analysis for each quintile are examined.

1.6.5 Chapter 5: Conclusion and Recommendations

The conclusion of the study contains the summary of the findings, and the various limitations and further areas of study.

CHAPTER 2

LITERATURE REVIEW

2.1 Literature Review Overview

This chapter deals with the significant literature on idiosyncratic risk and the presence of a low volatility anomaly on the cross-section of share returns. The chapter is organised as follows:

Section 2.2 provides a detailed overview of the theory of the study. The theoretical literature advances key aspects of the idiosyncratic volatility anomaly and the effect of a mis-specified risk factor in the market. Section 2.3 provides detailed evidence of the empirical literature of the study. The empirical literature highlights the practical studies conducted on the volatility anomaly and alternative theories to the existence of a volatility premium. Furthermore, the empirical literature focuses on the contrasting empirical analysis conducted between idiosyncratic, systematic and total volatility anomaly studies. Section 2.4 is an analysis of the empirical literature vis-à-vis the theory of the study. These studies highlight the empirical literature which is the framework for this study. The framework literature noted in this study entails the empirical studies examined within South Africa and internationally which postulate possible reasoning for the presence of an anomaly and highlight a variety of potential explanations.

2.2 Theoretical Literature

The low idiosyncratic volatility anomaly refers to the global phenomenon in which shares with previously low idiosyncratic risk characteristics yield above-average returns in contrast to shares with high idiosyncratic risk characteristics. The anomaly was first discovered and reported on in the early 1970s in a working paper by Haugen and Heins (1972) and later published in a new study by Haugen and Heins (1975). Subsequent to the publication, the low volatility anomaly has been considered to be one of the greatest anomalies of CAPM.

CAPM was popularised by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1996), who individually expanded on the preceding work by Markowitz on diversification and the theorem of MPT. CAPM remains widely used today as a method to determine the theoretically acceptable required rate of return of an individual asset.

In direct contrast to an idiosyncratic volatility anomaly, CAPM states that the return of an asset should exclusively be a linear function of the asset's beta, thereby proving idiosyncratic risk to be an irrelevant factor in asset pricing. CAPM assumes investors will hold a combination of the market portfolio and a risk-free asset; this may be an unrealistic assumption as financial models such as CAPM, which are formulated on "frictionless markets" and strong market efficiency, do not accurately reflect the real world. Investors often do not have perfect information and cannot hold the market portfolio. The basis of the assumption that CAPM is unrealistic is heavily based on the Merton Portfolio Problem, an established dilemma to continuous-time finance which questions how an investor should allot their wealth between equity and the risk-free asset. If the Merton Portfolio Problem is correct, these assumptions assume that idiosyncratic risk should be represented as a determining factor in estimating the share price return.

Idiosyncratic volatility, which may also be referred to as unsystematic or firm-specific volatility, is the risk prevalent to an individual asset with no correlation to market risk. Total market risk consists of systematic risk and idiosyncratic risk, in which idiosyncratic risk constitutes the largest component and accounts for the vast majority of variation in the risk of an individual asset over time (Xu & Malkiel, 2003).

Due to the poor performance of asset pricing models in calculating the expected return on share prices, Black (1972), Dennis and Strickland (2004), Ang *et al.* (2006) and Blitz and Van Vliet (2007) searched for an alternative risk measure which could correctly predict a share's expected return. Despite the assumptions behind CAPM and MPT, from a theoretical perspective idiosyncratic risk may be an important factor in pricing assets when allowing for a degree of imperfect market portfolio selection. Xu and Malkiel (2003) demonstrate this point by arguing that the "effective supply" of shares that investors are able to trade in, which are used to price individual securities, could be significantly different from the total "published" supply of shares that investors can examine. This theory suggests that the market portfolio which investors use to price securities is inefficient, as is the imperfect market portfolio investors hold due to a variety of tax and liquidity constraints. This results in an imperfect market portfolio, with part of the idiosyncratic risk which cannot be diversified away. A correctly specified asset pricing model that can correctly account for these incongruencies may more closely represent the market portfolio.

2.3 Empirical Literature

Historical changes and fluctuations in market, industry and firm-specific volatility were investigated by Campbell *et al.* (2001). Their study provides a breakdown of volatility that is discretional to the assessment of company and industry level betas or the covariance to the market. Campbell *et al.* furthered the study of Schwert (1989), who discovered market volatility to have no significant trend and to remain fairly stable over the sample period of 1926 to 1997. The findings of Campbell *et al.* confirmed the findings of Schwert, but did find firm-specific idiosyncratic variance to display a large significant positive trend, with little correlation to the surge in quantity of publicly traded companies over the sample period. Empirical analysis investigating the movements of historical volatility at a systematic and idiosyncratic volatility level have provided significant evidence of the anomaly, as analysed below.

2.3.1 Systematic Volatility Anomaly Evidence

Blitz and Van Vliet (2007) challenge efficient market theory by stating that a simple investment strategy can generate superior average returns at a considerably lower rate of risk. They constructed decile portfolios based on historical CAPM model betas. The findings of the study document a definitive volatility effect, with low-risk shares returning significantly higher risk-adjusted returns to the market portfolio. In an attempt to unravel the volatility outcome from alternative outcomes, Blitz and Van Vliet further examined the volatility effect by controlling for the effects of size, value and momentum using Fama and French regression models and applying a double sorting methodology. They found the volatility effect to be separate from size, value and momentum effects, and tantamount in significance.

Baker *et al.* (2011) observe that despite the risk measure of volatility or beta, on all securities whether large or small capitalisation shares, low-risk securities regularly exceeded the returns of high-risk securities over the sample period.

Further findings by Baker *et al*. (2011) are as follows:

1. High beta stocks generated superior total returns during bull markets and depressed total returns during long drawn bear markets on a CAPM market-adjusted basis. However, the low beta anomaly was identified during both bull and bear markets.
2. Monthly transaction costs and rebalancing were found to be higher for high volatility composed portfolios as opposed to low volatility composed portfolios.
3. Low-risk securities were found to be genuinely less risky with smoother return patterns, offering the protection they advertised.

2.3.2 Idiosyncratic Volatility Anomaly Evidence

The work of Ang *et al*. (2006, 2009) proved to be a key piece of literature in the analysis and methodology of this study. As it is one of the more significant studies on the low idiosyncratic volatility anomaly in the past decade, the study was critically analysed. Ang *et al*. (2006) found statistically significant evidence of the presence of the low idiosyncratic volatility anomaly on the cross-section of US share returns.

To measure the idiosyncratic volatilities of US share returns, Ang *et al*. (2006) conducted Fama and French (1993) regression tests as opposed to using CAPM due to the failure of CAPM to accurately analyse cross-sectional returns. Value-weighted portfolios were constructed by categorising idiosyncratic volatilities into quintiles according to their preceding 12-month idiosyncratic return volatilities. They found the average returns in the lowest volatility quintile (Q1) to be significantly greater than the average returns of the highest volatility quintile (Q5). Baker *et al*. (2011) concur. The findings by Ang *et al*. (2006) go against the common belief that higher degrees of risk are compensated for with higher levels of return. However, their study could not explain the findings of aggregate idiosyncratic risk levels inherent to the investor or the effect of alternative asset pricing models.

In order to test the robustness of their findings, Ang *et al.* (2006) tested the presence of a low idiosyncratic volatility anomaly in US share returns after controlling for size, book-to-market (BTM), liquidity, volume and momentum effects. When controlling for size, they formed quintile portfolios classified according to size of market capitalisation. The size-based quintiles were categorised by their prevailing 12-month idiosyncratic volatility. The findings suggest that for all size-categorised quintiles, the quintiles with the highest idiosyncratic volatility (Q5) still had a significantly lower alpha. Furthermore, small stocks exhibited the most pronounced effects of the low idiosyncratic volatility anomaly. Sorting quintiles based on BTM found the value effect to be heavily concentrated among small stocks. Potential explanations for this may be idiosyncratic volatility portfolios being concentrated principally in growth shares, with lower average returns than those of value shares. The results after controlling for BTM effects by Ang *et al.* (2006) indicate that the highest idiosyncratic volatilities still exhibited very low alphas after Fama and French regression tests. Based on the study by Pastor and Stambaugh (2003), Ang *et al.* (2006) used historical liquidity betas to proxy for liquidity. After controlling for the liquidity effect, they found liquidity ineffective in mitigating the low average returns of high idiosyncratic share returns, with Q5 still yielding a significantly low alpha. Jegadeesh and Titman (1993) investigated the possibility of a momentum effect driving the low returns attributed to high volatility shares. Ang *et al.* (2006) studied whether the effects of past winner shares, which naturally would have a high idiosyncratic volatility, explained the low returns exhibited by high volatility shares. After rebalancing for share returns over the previous month, they found that there was no effect by removing the very low alpha exhibited by Q5 share returns. Furthermore, they concluded that an overrepresentation of loser shares could be prevalent in the high idiosyncratic volatility quintile, as there was no evidence supporting the theory of a momentum effect in justifying the low idiosyncratic volatility anomaly.

2.3.3 Total Volatility Anomaly Evidence

Dutt and Humphery-Jenner (2013) investigated the relationship between operating performance, share returns and share return volatility, specifically two key issues. Firstly, they determined whether a low volatility anomaly exists outside of the US, particularly in emerging markets. Secondly, if a low volatility anomaly was present in emerging markets, they analysed relationship between volatility returns and operating performance as a potential driver of the effect. They divided the sample exchanges into three distinct emerging market categories: (1) emerging Asia, (2) emerging EMEA and (3) Latin America, and calculated a moving average share return variance and turnover for the prior 500 days. Their study found low volatility portfolios outperformed high volatility portfolios across the three emerging market categories examined. Furthermore, a potential explanation they offered for the low volatility effect was low volatility firms exhibiting higher operating performance.

2.3.4 Behavioural Finance

A large portion of the study by Baker *et al*. (2011) adopted a behavioural finance perspective as they cautioned that trends may be difficult to analyse and interpret using rational theories of asset pricing models. The large group of academics who dispute the findings of CAPM find that beta may be an unsuitable measure of risk, with unrealistic assumptions. However, most newly developed models provide little to no improvement in explaining why high beta stocks are less risky.

The behavioural theories of Baker *et al*. (2011) are classified into two hypotheses:

1. The irrational conduct of investors; and
2. Benchmarking as a limit to arbitrage.

Irrational theory is based on three biases:

- Preference for lotteries: This theory states that investors are risk averse and steer clear of highly volatile securities and the potential losses that follow. This theory may be manipulated when the outcome probabilities change. This indicates that behaviour is essentially linked to positive skewness, which implies that investor behaviour is determined by large positive payoffs rather than volatility.
- Representativeness: This simply means that one group of investors may overpay for volatile stocks by overlooking the high base rates at which small speculative

investments fail, whereas another group of investors may analyse the data and avoid high risk stocks if they are unable to separate potential winners from losers.

- Overconfidence: The process of valuing securities includes forecasting in which overconfident investors are more likely to disagree and demand higher risk securities.

Benchmarking as a limit to arbitrage asks two important questions:

- Why do experienced institutional investors not take advantage of the low volatility, high return anomaly?
- Why do institutional investors not overweigh the low volatility quintile?

To answer the first question, Baker *et al.* (2011) found that institutions do not short high volatility securities as these securities tend to be small stocks which are expensive to trade in large volumes. Investment managers are generally obligated to achieve returns in excess of the market model. This may force them to include riskier securities, which may increase the demand for high idiosyncratic risk securities.

In an attempt to explain the anomaly, the authors tested using "the process of elimination findings" of Ang *et al.* (2009). As in Brennan and Li (2008), it was found that the idiosyncratic component of the Standard and Poor 500 (S&P 500) had a negative payoff, indicating the presence of the low volatility anomaly. The second test was based on the theory that, as the practice of benchmarking has increased, the low volatility anomaly should become more extreme. Findings indicated the forecast to be directionally true, dependent on the sample.

Baker *et al.* (2011) concluded that the low volatility anomaly may be capitalised on by holding securities with homogeneous long-term returns. This suggests that although irrational investors overpay for higher degrees of risk, investment managers are often not rewarded enough to exploit such mispricing. Stockholders who intend to capitalise on returns with respect to risk may exploit mispricing opportunities with the low volatility premium present, as long as the majority of investors remain with standard benchmarks.

2.3.5 Limits of Arbitrage

Gu, Kang and Xu (2018) observed the relationship between pricing of idiosyncratic volatility and the limits of arbitrage, specifically the negative return premium for high idiosyncratic volatility shares present in the China stock market. As stated in Lam and Wei (2011), whenever a mispriced share becomes available, rational investors become aware of the arbitrage opportunity and trade accordingly until the market price converges with the fundamental value of the share. Despite these opportunities, limits of arbitrage prevent these opportunities from being relatively risk free. These limits may include trading, information uncertainty and transaction costs. Influencing the work of Gu *et al.* (2018), De Long, Shleifer, Summers and Waldmann (1990) investigated if noise traders produce an arbitrage risk by significantly influencing share prices to disperse from underlying values for a prolonged period of time. Gu *et al.* (2018) sampled equity from January 2002 to December 2012, traded on the Shanghai Stock Exchange and the Shenzhen Stock Exchange. They found a significant negative relationship between idiosyncratic volatility and share returns, thus supporting the theory of a low volatility anomaly present in emerging markets. The risk-adjusted return difference between the highest volatility portfolio and the lowest volatility portfolio for the equally weighted and value-weighted portfolio resulted in a difference of -1.09% and -1.76%, respectively. Lastly, the negative idiosyncratic volatility premium was found to be significantly present in shares with high limits of arbitrage and robust to a five-factor risk adjustment.

2.3.6 Market Mispricing

Li and Sullivan (2011) attempted to gain deeper insight into plausible explanations for the existence of a low volatility anomaly in US share returns. The premise of the study was to examine whether market mispricing or compensation for elevated levels of market risk could be attributed to such an anomaly. The primary research objective of the study was to determine if systematic risk factors are the fundamental cause of the low volatility anomaly, or alternatively if mispriced shares which may be related to irrational behaviour exhibited by investors are a factor in this unsolved mystery. Over a 46-year period (1962-2008), it was found that market mispricing best characterises the relation between low volatility shares and future excess share returns. This indicates that the premium placed on low volatility portfolios cannot be viewed as compensation for factor risk.

A related study by Black (1972) provides an early theoretical hypothesis for the cause of the low volatility anomaly with respect to mispricing. Black found that agent mispricing arising from borrowing restrictions such as margin requirements may be the fundamental cause of low volatility stocks outperforming their high-risk counterparts when measured according to their market betas.

Further support for the theory of mispricing, as discussed by Lakonishok, Shleifer and Vishny (1994) and Daniel, Titman and Wie (1996) suggest that higher returns are strongly related to market mispricing. These findings indicate that factors such as BTM may be a result of investors placing abnormal expectations of earnings growth rate on low BTM firms. These abnormal expectations may be attributable to a disproportionate optimism of investors in extrapolating future share returns for traditionally well-performing firms.

2.3.7 Market Frictions, Information Risk and Option Pricing

When observing US shares, in their second study, Ang *et al*. (2009) ruled out possibilities based on market frictions (anything preventing the ease of trade), information sharing and option pricing. These were ruled out even after considering the effects of transaction costs, characterising the severity of market frictions and the volume of private information that is used in trading activity.

Yang, Zhang and Zhang (2019) suggest a price-based measure of information volatility referred to as abnormal idiosyncratic volatility (AIV). The measure encapsulates information asymmetry encountered by uninformed investors. They were inspired by Easley, Hvidkjaer and O'Hara (2002), who developed a microstructure model to determine the probability of informed trading and later Easley, Lopez de Prado and O'Hara (2015) developed a new procedure to estimate the volume-synchronised probability of informed trading. Yang *et al*. (2019) determined the measure of AIV by measuring the variation in idiosyncratic volatility between non-earnings announcement periods and pre-earnings announcement periods. The study utilised quarterly and annual earnings announcements for shares listed on the NYSE, AMEX and Nasdaq over a sample period from 1972-1975. They investigated the relationship between corporate insiders and AIV. They found a positive relationship between abnormal insider trading and AIV during pre-earnings announcement periods. Secondly, they investigated if information risk captured by AIV is priced, and they found that high AIV firms have a positive association with high future share returns. Blitz, Huisman, Swinkels and Van Vliet (2019) analysed an attention-grabbing hypothesis which investigated the potential of media attention as being a factor explaining the volatility effect. As information data becomes regularly available, further analysis of the impact of information sharing and media attention is possible. Blitz *et al*. (2019) leveraged the availability of the information data and utilised premium newswires and press releases through a sample period from January 2000 – December 2018. The findings of the study were that there was no standalone media attention effect in global equity markets. Furthermore, shares which had high media attention had low volatility portfolio alphas which were significantly higher than high volatility portfolios. In contrast, shares which had high volatility had alphas of low and high media attention that were statistically indistinguishable. These findings resulted in Blitz *et al*. (2019) rejecting the attention-grabbing hypothesis and finding no evidence to support the notion of media attention as a factor in explaining the volatility premium.

2.3.8 Leverage, Institutional Ownership and Increased Firm Focus

Johnson (2004) had an alternative explanation for this anomaly. In his study, he touched on the relationship between leverage and the anomaly, stating that the idiosyncratic volatility effect is present because of the idiosyncratic volatility's interaction with leverage – "equity is a call option on a firm's underlying assets". There is, however, no explanation that can fully account for this anomaly. In order to attempt to justify the rationale of the existence of a low volatility anomaly, it may be beneficial to understand the determinants of volatility.

Numerous authors have referenced the findings of volatility asymmetry within their research. Campbell *et al*. (2001) found volatility to be stochastic in nature and furthermore, to be negatively correlated with equity returns. This theorem has significant implications for asset pricing, especially in the event of under-diversification. This increase in idiosyncratic volatility of share returns over time may be attributed to several aspects. Firstly, according to Black and Scholes (1972) and Christie (1982), a surge in leverage could amplify the effect of volatility on share returns. Secondly, institutional ownership of stocks has risen over time with institutions trading more than retail investors and displaying herding-like behaviour. Lastly, increases in idiosyncratic volatility may be a result of deviations in firm focus.

Considering the volatility asymmetry effect, aspects of volatility had to be studied to determine the effect on stock returns. Dennis and Strickland (2004) attempted to explain what determining factors may be present in explaining the nature of idiosyncratic volatility in share returns. The findings suggest that idiosyncratic volatility may be positively related to heightened institutional ownership, leverage and increased firm focus.

The findings of Dennis and Strickland (2004) support those of Campbell *et al*. (2001). The findings suggest that idiosyncratic volatility has increased over the past 20 years, with a positive association with increased institutional ownership, increased firm focus and leverage. Noteworthy results of the study suggest that a standard deviation change in institutional ownership results in a 75% rise in idiosyncratic volatility. Furthermore, when conditioning on return, a decrease in idiosyncratic volatility is seen subsequent to positive and negative share returns.

These results are important to note as these factors which affect idiosyncratic volatility may be explanatory factors surrounding the theory of a low idiosyncratic volatility anomaly on share returns.

2.3.9 Rational Expectations and Analysts' Forecasts

Xu and Malkiel (2003) demonstrated that both sales growth and analyst forecasts of long-term growth are positively correlated with idiosyncratic share returns volatility. Lakonishok *et al*. (1994) contend that when formulating prospects of firm future growth, stockholders are predisposed to excessively extrapolate from historical performance. This results in firms with historical high growth rates (often complemented by high idiosyncratic volatility) having a higher probability of exhibiting negative earnings shocks in the future. Lakonishok *et al.* (1994) attribute this investor extrapolation bias to the tendency of high idiosyncratic volatility firms yielding negative future earnings. Jiang, Xu and Yao (2009) examined the findings of Lakonishok *et al.* (1994) and Chan, Karceski and Lakonishok (2003) by measuring investors' expectation of future growth by leveraging forecasts of long-term earnings growth, provided by various brokerage firm analysts. The methodology of Jiang *et al.* (2009) succeeds that of La Porta (1996) who found analyst forecasts of long-term growth to be subject to extrapolation bias. Jiang *et al.* (2009) found inconclusive evidence that idiosyncratic volatility may be attributable to excessive extrapolation of firm growth; instead, they found evidence supporting the notion of idiosyncratic volatility as a result of corporate information disclosure.

Diether, Malloy and Scherbina (2002) found evidence that shares with elevated variability in analysts' forecasts yield significantly lower returns compared to similar shares. The findings are more evident in small and historically poor performing shares with a 12-month "look-back" period. These findings are consistent with their hypothesis that prices reflect investor optimism primarily when the stockholders with the lowest forecasted valuations do not engage in trading. By contrast, they also found evidence which is inconsistent with the view that variability in analysts' forecasts serve as alternatives for volatility.

2.3.10 Liquidity

Another potential explanation that was put forward by Jiang *et al.* (2009) is liquidity. They argued that the negative relation between the share returns and idiosyncratic volatility is a result of an absence of liquidity. The argument of liquidity explaining the cross-sectional fluctuations in expected share returns may be a result of investors requiring higher expected returns on shares which have higher sensitivities to aggregate liquidity. This may be seen in Pastor and Stambaugh (2003), who refer to the long-term capital management crisis. The assumption by Pastor and Stambaugh relies on the theory that any investor who exercises any form of leverage lending faces overall wealth depletion in the event of liquidation, due to solvency or margin calls, and must therefore liquidate some of the portfolio assets to raise capital. In the event that the investor holds assets with higher permutations to liquidity, the probability of liquidation is higher when market-wide liquidity is low. Furthermore, Pastor and Stambaugh state that according to standard asset pricing theory, expected share returns are cross-sectionally associated with the share returns correlation to independent variables which have an immense effect on stockholders' fortune. They investigated if market-wide liquidity is a suitable factor as a priced state variable to fluctuations in expected share returns. They observed expected share returns to be significantly related cross-sectionally to the return sensitivities of share returns to the fluctuations in aggregate liquidity. This finding indicates that shares which have higher sensitivities to aggregate liquidity have significantly higher expected returns even after testing for robustness by accounting for exposures to size, value, market return and momentum.

2.3.11 Volume and Momentum

Lee and Swaminathan (2000) studied the importance of trading volume in predicting the cross-sectional returns for a variety of price momentum portfolios. This included documenting the relation between prior returns and historical trading volume in predicting future returns over the intermediate and long-term horizon. The authors demonstrated this relation between trading volume and future returns by presenting findings of historical trading volume and its paramount importance between momentum and value strategies. They found that firms with high historical turnover ratios exhibit numerous "glamour" characteristics and yield significantly lower future returns complemented by a higher probability of consistent negative earnings surprises in the future. In contrast, firms with low historical turnover ratios were found to exhibit a variety of "value" characteristics and yield significantly higher future returns with reliably more positive increases in earnings in the future. Furthermore, past trading volume predicted the magnitude and persistence of price momentums. Momentum effects reversed in the subsequent 5-year period with high-volume winners experiencing sharper reversals in comparison to low-volume loser portfolios.

Jegadeesh and Titman (1993) found no significant evidence of price reversals through the preliminary three years following portfolio formation. They did find evidence which suggests that between years 3 and 5, an initial winner portfolio significantly underperforms to initial loser portfolios. These findings contradict the assumption that price momentum is a market under-reaction as the evidence suggests that the price momentum is a proportion of the initial momentum gain characterised as an over-reaction.

Charteris, Rwishema and Chidede (2018) examined if the 3- and 5-factor model findings of Chen, Novy-Marx and Zhang (2011) and Fama and French (2015) can explain momentum effects on the JSE. In order to determine if findings were consistent with international evidence by Chen *et al.* (2011) and Ammann, Odoni and Oesch (2012), they determined whether there is a strong correlation between profitability and investment in order to clarify if these findings may provide insight into the momentum effects exhibited by share returns. Charteris *et al.* conducted the analysis by determining if asset pricing models which include profitability and investment as risk factors have any significant correlation to share returns. The results were that neither CAPM, the Fama and French (1993) 3-factor model nor the Carhart 4-factor model (1997) could explain the momentum phenomenon experienced.

In contrast, the 3-factor model of Chen *et al.* (2011) and the 5-factor model of Fama and French (2015) provided strong evidence in support of pricing errors which are drastically lower than those of traditional asset pricing models. Furthermore, investigation of the factor weightings revealed a significant positive relationship between share returns and profitability, notably with winner (loser) shares positively correlated to shares yielding solid (weak) earnings. Finally, the investment factor loading, as was the profitability, was discovered to be positively correlated to past returns, with winner (loser) shares positively correlated to companies with cautious (aggressive) investment approaches.

2.3.12 Volatility as a Measure of Risk

Xiong *et al.* (2014) raised the point of whether volatility is in general a relevant measure of risk. When testing their results, a low volatility anomaly was discovered where the highest volatility quintile had the lowest risk-adjusted return measured by the Sharpe ratio. The main objective of their study was to test if volatility itself is an accurate measure of risk, in turn, refuting the low volatility anomaly as an anomaly of risk. According to Xiong *et al.* (2014), the geometric mean for Q5 and Q1 is identical, suggesting that volatility is not compensated for in returns on a risk-adjusted basis. When looking at tail risk, Xiong *et al.* (2014) found that funds with higher tail risks resulted in higher expected returns – which was found to be consistent with an economy where agents demand a higher premium to compensate for higher risk. This led them to conclude that tail risk is a more accurate measure of risk than volatility. A possible reason for the low volatility anomaly existing in markets globally may therefore be that volatility is not an accurate or appropriate measure of the risk. Xiong *et al.* concluded that excess conditional value at risk, which is a left tail measure, provides the most accurate assessment of risk as opposed to the conventionally used volatility measure. Blitz, David and Pang (2013) examine the risk- return relationship in non-developed markets. The study finds varying evidence to theoretical models such as the CAPM in which findings of a persistent volatility effect seem to strengthen over time. The paper poses this to be a result of increased entrusted asset management in the emerging markets analysed.

2.3.13 Intraday and Interday Distribution of Share Returns

Balaban, Ozgen and Karidis (2018) analysed the intraday and interday distribution of share returns in conjunction with their asymmetric time-varying volatility, on the Bourse Istanbul emerging market. They determined the asymmetric time-varying volatility based on Threshold Generalised Autoregressive Conditional Heteroscedasticity-in-Mean [TGARCH(1,1)-M], which simultaneously determines (1) weak-form informational market efficiency, (2) total systematic risk-return relationship and (3) volatility asymmetry and persistence. Balaban *et al.* (2018) found strong evidence of a positive return effect for second trading sessions on Thursdays and Fridays. Volatility was discovered to be the highest on Mondays with systematic risk priced in for the majority of companies analysed. Finally, Balaban *et al.* (2018) found no conclusive evidence of asymmetry in the volatility of firms examined over the sample period; however, they did find that financial companies had significantly higher levels of systematic risk than industrial companies analysed in the study.

2.3.14 Agency Problems

Titman, Wie and Xie (2004) document that firms which engage in sizeable investments often experience dramatic changes in their business fundamentals, with elevated uncertainty about future cash flows. As a result, firms with higher capital expectations are predisposed to have lower average returns in the future. They attribute this relation between high capital expectations and low average returns to agency problems and the over-investment tendency of empire-building managers. Roger and Schatt (2016) provide further evidence of agency issues affecting idiosyncratic risk in the pricing of shares in their study. They additionally identify numerous quantities of listed companies around the globe which are controlled by under-diversified family block holders who necessitate private reimbursements to compensate the added volatility borne.

2.3.15 High-frequency Financial Data and Empirical Mode Decomposition

Nava, Di Matteo and Aste (2016) conducted an empirical mode decomposition (EMD) to break down intra-day financial time-series into trends and a limited set of oscillations called intrinsic mode fluctuations (IMFs). They found that the volatility of intra-day share indices calculated at various time periods showed a significant difference in behaviour expected from fractional Brownian motion (fBm). The authors applied EMD to fBm in order to decode the power law scaling between the variance and period of IMFs. They analysed 22 share indices inclusive of developed market indices and emerging market indices (inclusive of the JSE), over a period of 6 months, with prices recorded every 30 seconds.

The results of Nava *et al.* (2016) were that fBm tracks the scaling law of EMD, which relates to the linear logarithm of the variance and period of IMFs. When applied to the share indices, the EMD displays different scaling laws which can deviate significantly from Brownian motion and fBm behaviour. They further concluded that EMD of high frequency financial data results in a larger number of IMFs as expected from Brownian motion. These share indices findings result in a curvature that defies the linearity in the log-log relation between IMF variance and the period found in fBm, indicating an anomalous scaling due to complex structures in financial data.

2.3.16 Seasonality as an Explanation for the Volatility Anomaly

Seif, Docherty and Shamsuddin (2017) conducted an empirical analysis to determine if seasonal anomalies exist in emerging markets in contrast to the extensive research conducted on developed markets. They examined the efficiency of emerging economies in contrast to developed economies by testing the presence of five seasonal anomalies (month-to-year, other January, day-of-the-week, holiday and week 44). They state that emerging economies have vast characteristic differences in contrast to developed markets, which could potentially result in skewed assumptions. Commonly known differences between these markets are characterised by emerging economies having low liquidity and market capitalisation levels while exhibiting higher volatility. The study included nine advanced emerging economies based on the FTSE's country classification, with South Africa included in the sample. Seif *et al*. (2017) found no significant evidence to support the notion of a January effect in emerging markets but supported the concept of month-to-year, day-of-the-week and holiday effect. These findings may provide insight into potential drivers which may result in the presence of a volatility anomaly.

2.3.17 The MAX Effect

Bali, Cakici and Whitelaw (2011) investigated the preference of investors for assets with lottery-like payoffs and the resulting under-diversification. The empirical analysis determined the significance of very high positive returns in the cross-sectional pricing of share returns in which they identified a potential influencing factor to explain the negative relationship between the maximum daily return over the previous month and successive monthly returns. This deterministic factor resulting from shares which exhibit extreme maximum daily returns in the prior month to exhibiting low monthly returns in subsequent months is referred to as the MAX effect. The dataset included in the study was based on NYSE, Amex and Nasdaq financial and non-financial companies from the Centre for Research in Security Prices (CRSP) for the period from January 1926 to December 2005.

Bali *et al*. (2011) found a significant negative correlation between the maximum daily return over the previous month and successive monthly returns. Furthermore, the significant and negative MAX effect maintains even after controlling for various factors of BTM, skewness, size, momentum, illiquidity and short-term reversals.

Adding to the findings of Bali *et al*. (2011), Wu, Chimezie, Nartea and Zhang (2019) examined the cross-sectional relationship between expected share returns and the maximum daily return, and idiosyncratic volatility for the five largest emerging African stock markets from 2001 - 2015. Similar to Bali *et al*. (2011), Wu *et al*. (2019) define the MAX variable as a share's maximum daily return in the prior month. Furthermore, the study introduced IVOL following Ang *et al*. (2006, 2009) as an additional measure, and MAX (5) as a final variable to test for robustness. Focusing on the results of the study, Wu *et al*. (2019) report the descriptive statistics of MAX and IVOL. The results were that an average MAX value of 0.1069 with South Africa yielded the highest average MAX value of 1.000. Furthermore, they found MAX and IVOL to be highly positively correlated with a correlation coefficient of 0.9834. The final result of the study was a statistically significant negative relationship between MAX and succeeding share returns, unvarying to the tests controlling for size, BTM, beta, momentum, short-term reversals, illiquidity, skewness and IVOL. A significant negative IVOL effect was also found, which dematerialises when controlling for MAX. The study found evidence supporting the notion of a MAX effect as the true effect in explaining a volatility anomaly, with the IVOL variable effect to be nothing more than a proxy for the MAX effect.

2.3.18 Imperfect Diversification

In an attempt to describe potential reasons for the low idiosyncratic volatility anomaly, Campbell *et al*. (2001) identify potential explanations for imperfect diversification. Firstly, the majority of investors who have large holdings in individual shares fail to diversify in accordance with financial theory, or the investor's portfolio may be restricted by corporate compensation policies. Secondly, in line with MPT, some investors attempt to diversify their portfolio by holding 20-30 shares in order to completely eliminate all idiosyncratic risk. Campbell *et al*. found the adequacy of this assumption to depend on the level of idiosyncratic risk constituting the portfolio. Ingersoll (1987) and Shleifer and Vishney (1997) found that larger pricing errors increase in probability when idiosyncratic volatility is high. These idiosyncratic volatility risks directly affect arbitragers who trade mispriced shares as mis-specified idiosyncratic risks that may be the result of the pricing error rather than market volatility.

2.3.19 Tail Risk as an Alternative Measure

Xiong *et al*. (2014) extended the analysis of the low volatility anomaly by determining if volatility is not compensated for in the equity fund universe as an alternative risk metric such as tail risk. They chose equity funds rather than individual shares, as systematic risk is more relevant to investors' portfolios. The three-risk metric analysed in their study was volatility and the two tail risk metrics were skewness and excess conditional value at risk (ECVaR). Skewness is a measure of the data's asymmetry around a sample mean and ECVaR is a left tail risk measure. These were utilised as the two alternative risk metrics of the study. Xiong *et al*. found the highest volatility quintile (Q5) to have a significantly lower Sharpe ratio than the less risky quintile portfolios. In contrast, they found the highest tail risk quintile (Q5) to exhibit the highest returns and best Sharpe ratio of all quintile portfolios. These results provide evidence that volatility is not compensated for in the equity fund universe as opposed to tail risk.

In testing the robustness of the results, Xiong et al. (2014) incorporated alpha as a measure of performance evaluation. The risk metrics were measured after controlling for size, value, fund beta and momentum in line with the Carhart (1997) 4-factor model and the Fama and French (1993) 3-factor model. They found 13 of the 16 alphas to be negative and none to be significantly positive when measuring volatility, in contrast to all 16 of the alphas to be positive with half significant at a 5% confidence level. These findings are consistent with Xiong et al.'s view that volatility is not compensated for as a risk metric for equity funds, while tail risk is a statistically significant and robust measure.

Slim, Dahmene and Boughrara (2019) investigated the information rooted in the variance risk premium and implied volatility index for developing and developed markets. They conducted their analysis by integrating the relative variance premium into GARCH models, which improve the one-day-ahead VaR forecast for developed markets. The results were significantly accurate for financially distressed markets and alternative measures of the variance risk premium. However, the superior performance of these models did not eliminate the extent of implied volatility as a risk mitigation metric.

2.3.20 Panel Data Methods as an Alternative Measure Technique

Page and Auret (2019) considered a variety of promoted investment methods which have been examined in literature as means to identify potential explanatory factors for the cross-sectional variation in share returns. The factors they analysed were size, value, momentum, low beta, currency risk, liquidity and idiosyncratic volatility. In stark contrast to the majority of literature on the variety of investment methods which conduct their portfolio estimation procedure and sorting methods through Fama-Macbeth regression analysis, Page and Auret applied panel data methods on a share-by-share basis. The motive for utilising these methods was to investigate multiple investment methods in a multivariate parametric framework.

Page and Auret (2019) found significant evidence supporting the investment methods of size, value and momentum risk factors on the cross-sectional variation in share returns on the JSE. Conversely they found no significant evidence to support the investment methods of low beta, currency risk and idiosyncratic volatility risk factors on the cross-sectional variation in share returns on the JSE.

2.3.21 Interest Rate Exposure

Driessen, Kuiper, Nazliben and Beilo (2019) identified a contemporary factor which could potentially explain the presence of a low idiosyncratic volatility premium. They were inspired by the work of Falkenstein (1994), who revealed anomalous findings on NYSE share returns, which yielded negatively correlated variance figures, and questioned the ability of the Fama and French 3-factor model to correctly price share returns. As a result of numerous works such as that by Ang *et al.* (2006) and Baker and Wurgler (2011) failing to identify a significant explanatory factor, Driessen *et al.* (2019) focused on estimating the interest rate risk premium as a potential explanatory factor for the low volatility anomaly. They constructed value-weighted portfolios based on the prevailing 60 days of lagged returns which included NYSE, AMEX, NYSE MRK and NASDAQ share returns. The shares were subsequently divided into 10 risk-based portfolios, with portfolio 10 consisting of the highest volatility and portfolio 1 consisting of the lowest volatility shares. The tests were run over a sample period from 1968-2014. Driessen *et al.* found two main results. Firstly, a substantial portion of low volatility shares outperformed high volatility shares as a result of differences in the interest rate risk exposure in addition to a high interest rate risk premium identified in the equity market. This finding was illustrated in the study in that low volatility portfolios had significant exposure to interest rates. The second key finding of the study involves analysis of the interest rate premiums in the equity market in contrast to the bond market. By estimating the interest rate risk premium for the cross-section of equity portfolios, Driessen *et al.* found a high compensation factor of 0.91% per month. In contrast, by estimating the interest rate risk premium in the bond market, they found a compensation factor of 0.17%. These findings are easily understood as interest rate risk generating a 0.17% excess return for the bond market in contrast to a 0.91% excess return generated by the equity market. As a result, they concluded that interest rate risk is priced differently between markets and asset classes and low volatility portfolios have a strong implicit exposure to bonds, resulting in increased returns, whereas high volatility portfolios have an implicit exposure to short bonds, resulting in poor performance. Finally, they concluded that the interest rate risk exposure, which is priced differently among bond and equity markets, also reduces the variance of the unexplained returns and explains a significant portion of 80% of the low volatility effect.

2.3.22 Size Effect

Ciliberti, Sérié, Simon, Lempérière and Bouchaud (2019) examined one of the most popular and renowned market effects, the size premium. This effect has been analysed within the low volatility anomaly spectrum by Ang *et al.* (2006), Van Dijk (2011) and Schwert (1983). The analysis performed by these authors resulted in comprehensive evidence of a size effect, in which small market capitalisation shares are significantly undervalued and outperform large market capitalisation shares. The significance of a size effect proved to be a factor in explaining excess alpha experienced by the low volatility shares examined in the respective studies. Ciliberti *et al*. (2019) provide a unique take on analysing the size effect by measuring size according to dollar turnover rather than market capitalisation. This unique perspective found evidence to support a size effect presence and highlighted the importance of size in estimating the excess returns of share portfolios. Cho (2019) developed variant conditional asset pricing tests on the size effect to determine if the size effect is present during high volatility periods. The analysis was conducted across UK and US markets from July 1963 to September 2018. The results were that size factor earns a significant risk premium during uncertain macroeconomic periods. These results are interpreted as the size premium existing only during high volatility periods in contrast to recessionary periods, which may prove to be an explanatory factor for the low volatility premium as these portfolios may be more heavily weighted towards small market capitalisation shares.

2.3.23 Common Idiosyncratic Volatility and Household Labour Income Risk

Herskovic, Kelly, Lustig and Van Nieuwerburgh (2016) analysed the behaviour of idiosyncratic volatility on share prices. They postulated that idiosyncratic volatility of US firms was strongly correlated to one another, referred to as common idiosyncratic volatility (CIV). Furthermore they stated that CIV was positively associated with multiple measures of household labour income, which was investigated by determining if deviations in CIV were priced in the cross-section of share returns. They examined the degree of co-movement of idiosyncratic volatility of more than 20 000 shares listed on the CRSP over a sample period of 1926-2010. The primary measure of idiosyncratic volatility was calculated as the sum of residuals of firm fundamental volatilities, i.e. firm-level cash flows and sales growth Herskovic *et al*. found a strong factor structure of a firm's idiosyncratic volatilities and, additionally, deviations in CIV to be priced in the cross-section of share prices. The findings of the study are consistent with an incomplete market's heterogeneous agent model, as an increase in firm-level idiosyncratic volatility increases the average household's marginal utility. Finally, shares with the lowest CIV beta quintile were found to earn annual average returns of approximately 5.4% higher when compared to the highest CIV beta quintile.

2.3.24 Factors affecting Stock Price Returns

When Ang *et al*. (2009) observed the high idiosyncratic volatility effect in America, they had several explanations for why the effect is present, and pointed out several factors that can affect stock returns:

2.3.24.1 Private information

Easley and O'Hara (2004) state that expected share returns vary by the amount of private information that is embedded within the trades of those shares. Shares comprising more private information generate higher levels of expected share returns. Easley *et al*. (2002) determined the degree of private information encompassing the trading activity of shares. They found that shares which encompass higher degrees of private information have significantly higher expected share returns compared to shares associated with low levels of private information. Brogaard, Hendershort and Rierdan (2016) state that information risk is multifaceted, which prevents quantity-based measures from accurately capturing information risk in all its aspects. This results in an informed trading equilibrium to require both price and quantity. These findings may potentially be a result of low idiosyncratic volatility shares whose trades contain excessively high levels of private information as opposed to high idiosyncratic volatility shares whose trades contain low levels of private information.

2.3.24.2 Transaction costs

Lesmond, Ogden and Trzcinka (1999) observed the effect of transaction costs on returns and demonstrated that these costs are closely related to spread and commission estimates. Furthermore, Ang *et al*. (2009) found the volatility effect to occur frequently in shares with the highest transaction costs and in instances where arbitrage opportunities are difficult to find.

2.3.24.3 Analyst coverage effect

The analyst coverage effect determines the correlation between the number of analysts tracking shares and the expected returns associated with those shares. Hou and Moskowitz (2005) investigated if stockholders derive value from the timely dissemination of information. They proposed that shares monitored by fewer analysts will have higher expected share returns than shares monitored by more analysts. If low volatility shares have low analyst coverage, these shares may require higher expected returns to compensate investors for the slow dissemination of news. Ang *et al*. (2009) concluded that controlling the level of analyst coverage leads to misaligned results as the data of highly covered firms positively reflects larger firms. They also found size of firm to have a positive relationship to analyst coverage, therefore indicating a potential size effect driving the anomalous results as opposed to an analyst coverage effect.

2.3.24.4 Institutional ownership

According to Kumar (2007), shares which have low levels of institutional ownership are monitored less by analysts in contrast to shares which exhibit high levels of institutional investment. Furthermore, these shares tend to be smaller and less liquid with dramatically slower response times to news announcements. Ang *et al*. (2009) hypothesise that shares with low idiosyncratic volatility could be shares with low levels of institutional ownership, resulting in them generating high average returns.

2.3.24.5 Delay Price of Stock

Utilising the delay measure developed by Hou and Moskowitz (2005), it was found that the majority of severely delayed firms require larger return premiums. These stocks could be low idiosyncratic volatility stocks with high returns due to the slow response to new information.

2.3.24.6 Skewness

When observing the cumulative prospect theory preferences of Tversky and Kahneman (1992), objective probabilities are transformed by investors that overweight the tails of the probability distribution curve. Positively skewed shares will end up being overpriced, thus earning negative average returns - this may explain, as per Barberis and Huang (2008), why stocks with high idiosyncratic volatility have low returns. Ang *et al*. (2009) discovered that when considering the cross-sectional regression results, Barberis and Huang's findings were true when tested on their data. The more positively skewed the individual returns are, the lower the expected returns to follow.

When Ang *et al.* (2009) sorted quintile portfolios of stocks based on volatility, it was found that the delay effect and the analyst coverage effect played the greatest role in closing the gap in the difference between high and low idiosyncratic volatility. This resulted in these two factors potentially explaining the volatility effect.

2.4 Theoretical Framework

2.4.1 South African Low Volatility Anomaly Studies

Oladele and Bradfield (2016, 2018) and Page *et al.* (2016) conducted studies analysing the presence of a low volatility anomaly within the South African equity market. Pukthuanthong-Le and Visaltanachoti (2009) and Sehgal and Garg (2016) conducted studies on idiosyncratic volatility and the low volatility anomaly, respectively, across a range of emerging market economies, including South Africa.

Oladele and Bradfield (2016) analysed a variety of volatility portfolio construction methodologies for various business sectors in the South African equity market. They aimed to assess the performance of low volatility constructed portfolios utilising seven low volatility estimation techniques relative to the market capitalisation weighted indices using the JSE and FTSE sectors. The seven techniques analysed were the equally weighted low beta portfolio, the naïve equally weighted portfolio, the minimum variance portfolio, the low volatility single index model, the equal risk contribution portfolio, the naïve risk-parity portfolio and the maximum diversification portfolio. The process of analysing the low volatility anomaly was influenced by the work of Leclerc, L'Her, Mouakhar and Savaria (2013), who studied industry-based weighted schemes in the US equity market.

The study by Oladele and Bradfield (2016) analysed nine FTSE/JSE sectors from January 2003 to December 2013. Low volatility portfolios were rebalanced monthly and assumed to incur a 25 bp transaction cost. The industrial betas were estimated using the standard ordinary least squares estimate of the previous 36 months. They found that all sector-based low volatility portfolios outperformed the ALSI at significantly lower risk levels. Additionally, they found low volatility portfolios to be more likely to recover from extreme losses at a faster rate in contrast to the ALSI.

Oladele and Bradfield (2018) conducted a similar study to their 2016 study but analysed the performance of low volatility portfolios constructed from shares as opposed to JSE and FTSE sectors. The low volatility shares were compared to the local market capitalisation-weighted benchmark indices using JSE stocks. A similar 36-month rolling window to estimate the covariance matrix was utilised as a means to back-test the low volatility portfolios. The seven low volatility estimation techniques in their previous study were also used.

The dataset utilised in the study contained weekly total returns of stocks listed on the JSE over a sample period of January 2003 to March 2016. The results were that all the low volatility portfolios outperformed the ALSI over the sample period. Furthermore, the equally weighted, equal-risk contribution and naïve risk parity portfolios' risk-adjusted returns tracked each other consistently over the sample period. Finally, the equal-weight low beta and the maximum diversification portfolio yielded the lowest risk and drawdown of all the low volatility portfolio estimation techniques.

Sehgal and Garg (2016) analysed the cross-sectional volatility of emerging markets. The analysis focused on a systematic volatility factor as well as an unsystematic volatility factor, in determining if either risk measure has any success in identifying a low volatility premium on the cross-section of emerging market share returns. The emerging markets analysed in the study were Brazil, Russia, India, Indonesia, China, South Korea and South Africa. Focusing on the South African component of the study, the sample period analysed was August 1995 to December 2011, evaluating 238 shares listed on the FTSE/JSE All Share.

The results were that high systematic volatility portfolios tended to exhibit low returns in Brazil, South Korea and Russia, with a statistically significant negative risk premium. In contrast, no significant risk premiums were present for China and India. South Africa exhibited the most puzzling result in the study, yielding a significantly positive risk premium. This can be further interpreted as high systematic volatility shares exhibiting a positive risk premium in the South African market, finding no evidence of a low systematic volatility anomaly. Furthermore, CAPM was found to be a poor descriptor of returns on all emerging market systematic risk-sorted portfolios, whereas the Fama and French 3-factor model was able to justify the returns of all emerging market systematic volatility portfolios, excluding the South African market.

In contrast, high returns were associated with high unsystematic volatility constructed portfolios in all emerging markets, excluding China. This result provides further evidence of no volatility anomaly present on the cross-sectional returns in the South African equity market. CAPM, similar to the systematic volatility measure, was unable to provide a suitable explanation for unsystematic volatility constructed portfolio returns. The Fama and French 3-factor model improved on CAPM but also failed to explain the returns on low unsystematic volatility sorted portfolios in South Africa.

Finally, Pukthuanthong-Le and Visaltanachoti (2009) conducted an empirical analysis to determine if idiosyncratic risk should be priced in an asset pricing model. There is an abundance of empirical evidence demonstrating the inconsistencies of CAPM in correctly pricing share returns. As CAPM assumes diversification to fully eliminate idiosyncratic risk, Pukthuanthong-Le and Visaltanachoti conducted exponential generalised autoregressive conditional heteroscedasticity (EGARCH) tests to determine the conditional idiosyncratic volatility of individual shares across 36 countries (including South Africa) from 1973 to 2007.

The results were that idiosyncratic volatility was positively related to expected returns across all 36 sample countries. These results provide further evidence that idiosyncratic risk may be a sufficient additional risk factor to correctly price share returns in the South African market.

2.4.2 The Anomaly in International Markets

Ang *et al*. (2009) investigated the anomalous relationship they previously observed in 2006 between lagged idiosyncratic volatility and future average cross-sectional returns across a broad range of developed markets. They determined if a low volatility anomaly persisted at a global scale by examining the results across 23 developed markets. They found strong evidence of a negative spread between stocks with high and low idiosyncratic volatilities in international, developed markets. This contribution corroborates their previous observation of the presence of a low idiosyncratic volatility anomaly and negative spread in returns between high and low idiosyncratic volatility shares in the US. Furthermore, Ang *et al*. (2009) observed the idiosyncratic volatility effect to be much larger in the US in contrast to other large developed countries. This observation could be due to the US consisting of shares with a significantly wider dispersion of idiosyncratic volatility. Finally, they concluded that in recent history, low idiosyncratic volatility shares produced significantly higher returns in contrast to high idiosyncratic volatility shares and simultaneously appear in different world regions.

Zaremba (2016) examined the correlation between country-level and stock-level low volatility risk by investigating the cross-sectional share returns of 78 national stock markets. Three hypotheses were tested: firstly, whether volatility is a valid determinant of the cross-sectional variation in country indices return, secondly, if risk effects are equally distributed across size and value classes of countries and lastly, whether it is possible to improve cross-national value and size strategies with additional factors of risk. Zaremba found country-level returns to be positively related to variance, VaR and idiosyncratic volatility. Contrary to the findings, a large portion of the results were explained by cross-national value, size and momentum factors. In determining if risk effects were equally distributed across risk metrics, idiosyncratic risk effects were determined to exhibit the strongest risk-return relationship with systematic risk (market beta) non-existent. This finding indicates the potential of idiosyncratic risk as a superior risk factor in determining the risk-return relationship between shares. The final contribution of Zaremba is that sorting on VaR may significantly improve the performance of size and value strategies at country level.

In summary the low idiosyncratic volatility anomaly refers to the global phenomenon in which shares with previously low idiosyncratic risk characteristics yield above-average returns in contrast to shares with high idiosyncratic risk characteristics. The anomaly is in direct contrast to the CAPM, which states that the return of an asset should exclusively be a linear function of the asset's beta, thereby proving idiosyncratic risk to be an irrelevant factor in asset pricing. The literature examined identifies a plethora of potential explanatory factors which may explain the rationale behind the evidence of the anomaly. Despite examining a variety of explanatory factors, this study focuses on two key pieces of literature by Ang *et al.* (2006) and Xiong *et al.* (2014) which form the structure of this study and research methodology. The first key piece of literature examined in this study is by Ang *et al.* (2006) who found statistically significant evidence of the presence of the low idiosyncratic volatility anomaly on the cross-section of US share returns. Ang *et al.* (2006) conducted Fama and French (1993) regression to accurately analyse cross-sectional returns. Furthermore, Value-weighted portfolios were constructed by categorising idiosyncratic volatilities into quintiles according to their preceding 12-month idiosyncratic return volatilities. Ang *et al.* (2006) found the average returns in the lowest volatility quintile (Q1) to be significantly greater than the average returns of the highest volatility quintile (Q5). The second notable piece of literature examined by Xiong *et al.* (2014) raised the point of whether volatility is in general a relevant measure of risk. The main objective of their study was to test if volatility itself is an accurate measure of risk, in turn, refuting the low volatility anomaly as an anomaly of risk. When looking at tail risk, Xiong *et al.* (2014) found that funds with higher tail risks resulted in higher expected returns – which was found to be consistent with an economy where agents demand a higher premium to compensate for higher risk. This led them to conclude that tail risk is a more accurate measure of risk than volatility. Xiong *et al.* concluded that excess conditional value at risk, which is a left tail measure, provides the most accurate assessment of risk as opposed to the conventionally used volatility measure. Finally, numerous studies have been conducted on a South African perspective by Oladele and Bradfield (2016, 2018) and Page *et al.* (2016). The results of the studies find that idiosyncratic volatility was positively related to expected returns. These results provide further evidence that idiosyncratic risk may be a sufficient additional risk factor to correctly price share returns in the South African market.

CHAPTER 3

METHODOLOGY

3.1 Methodology Overview

In this chapter the methods and research design used in the study are explained. Section 3.2 provides a detailed overview of the empirical analysis and methodology. Section 3.3 addresses the sampling and empirical implementation process; an outline, motive and progressive course of action are presented which constitutes the structure of the analysis. Section 3.4 deals with the empirical analysis procedure; the procedure followed and numerous studies conducted in order to determine if low idiosyncratic volatility shares are rewarded with a premium are highlighted. The idiosyncratic volatility estimation process of measuring volatility according to two periods of time is specified in section 3.5. Finally, the expectations of the empirical analysis and of the study are set out in section 3.6.

3.2 Breakdown of Empirical Analysis

3.2.1 Research Type

To acquire familiarity with the global phenomenon of the low idiosyncratic volatility premium and attain innovative insight into the rationale of the anomaly, a quantitative empirical analysis was conducted. Quantitative research was utilised in this study as a means to generate numerical data which can be transformed into usable statistics. Kothari (1990) elaborates that a quantitative empirical analysis is a fundamental form of data-based research to provide observable conclusions of an experiment. This form of research requires verifiable data from a reputable source and methodical analysis to investigate the results to substantiate the hypothesis.

3.2.2 Data Type

Quantitative empirical studies may classify data into data types using a hierarchy of measurement. Berman-Brown and Saunders (2008) and Dancey and Reidy (2008) illustrate this process of numerical measurement to classify data types. This current study at the highest level utilised secondary numerical data of monthly individual share prices for all companies listed on the JSE. The secondary data was sourced utilising INET BFA-provided monthly share prices of all JSE-listed shares. INET BFA was the selected data source provider as the data feed company is highly reputable with stringent requirements for data quality. Furthermore, the university has a collaborative effort with the data feed provider, which allows all students access to high-quality South African financial data that is easily accessible and readily available from the dedicated library. The time frame for gathering the relevant data to conduct the empirical analysis was a total of one work week. The dedicated library required documentation on the data requirements as well as the data sample time frame and data was provided in an efficient manner. The secondary data from INET BFA provided by the UNISA library constituted three different datasets. The first dataset included the monthly share prices for all listed companies on the JSE from January 1994 to December 2019. The second dataset included the monthly asset and liability figures from all JSE-listed companies from January 1994 to December 2019. The third dataset included the market capitalisation rates for all JSE-listed companies from January 1994 to December 2019. The full sample population of JSE-listed shares was tested to gain insight into the summary statistics of the data. The summary statistics included tests of normality, skewness and kurtosis, which were conducted using E-Views statistical software. This software was utilised as the primary statistical software package to run exploratory data analysis and test normality as the software is accessible to students and has an easy-to-use interface and reporting layer. The E-Views platform also proves to be a reliable statistical package, which is used by numerous academics and institutions around the world. The data may further be quantified at source as discrete random data as the prices of shares listed have a finite number of possible values.

The data was cleaned and statistical outliers were removed by eliminating shares which were part of the bottom 5% in market capitalisation. Shares which had traded for less than 12 months, as a result of shares having an IPO or returning from a trading suspension less than 12 months before in study A, were eliminated. Shares which had traded for a period less than 36 months, as a result of shares having an IPO or returning from a trading suspension less than 36 months before in study B, were eliminated. Finally, shares which had traded for a period less than 60 months, as a result of shares having an IPO or returning from a trading suspension less than 60 months before in study C, were eliminated. Furthermore, shares which had over 100 zero daily trades calculated over the previous year were eliminated. Shares were categorised into quintile portfolios based on their prevailing 12-, 36- and 60-month historical returns. The purpose of including a 12-month volatility estimation period was to replicate the period analysed by Ang *et al.* (2006) and Diether *et al.* (2002). Ang *et al.* (2006) constructed value-weighted portfolios calculated from the idiosyncratic risk of daily data over 12 months of returns which ended one month prior to the formation date. The inclusion of a 36-month volatility estimation period drew insight from the studies of Oladele and Bradfield (2016, 2018) who conducted their studies on a low volatility anomaly across the JSE/FTSE. The second estimation technique was introduced in order to determine the effect of time on volatility estimation. A larger look-back period provided a reference to determine which estimation procedure was more accurate and provided contrasting results and theories on the low idiosyncratic premium. Finally, the inclusion of a 60-month volatility estimation period was to replicate the estimation period utilised by Xiong *et al.* (2014). The final estimation technique followed the premise of the 36-month study, as introducing a longer volatility look-back period could provide contrasting results and significant findings. The categorised data resulting from the process was classified as ranked categorical data, as each quintile portfolio was subject to quantitative empirical analysis on each category. The primary intention to exclude these shares was to ensure an accurate measurement of the shares' idiosyncratic volatility. Subsequent to the data cleansing process, statistical analysis to calculate historical monthly share returns over the sample period was conducted. The resulting transformed data was categorised as random continuous data, due to the infinite possible values.

3.2.3 Exploratory Data Analysis

Utilising Tukey's exploratory data analysis (EDA) approach (1977) thorough testing was conducted to summarise the data's primary characteristics before formal modelling and hypothesis testing were conducted. This process included tests of normality, skewness and kurtosis to ensure that the data was of an acceptable standard and to remove any statistical outliers which could skew the results.

3.2.4 Sampling Techniques

In order to generate an accurate representation of the South African equity market, a non-probabilistic sampling technique was utilised in this study. The sample exchange analysed has been predetermined as an accurate representation of the population of the South African equity market. In line with Saunders, Lewis and Thornhill (1997), the non-probabilistic sampling technique can be categorised further as a quota sampling technique. A quota sampling technique is a predefined category which has the same properties as the target population.

3.3 Empirical Implementation

In order to determine the cross-sectional relation between the low idiosyncratic volatility and average return of JSE-listed securities, the methodology of the study made use of the following assumptions:

The sample period analysed was from January 1994 to December 2019, making use of a relevant and accurate 26-year sample period. The motive for using this period was based on the observation by the National Bureau of Economic Research (NBER, n.d.) that there were approximately 11 business cycles from 1949 to 2009, with an average length of 69 months or approximately 6 years. Campbell *et al.* (2001) found that idiosyncratic volatility displays counter-cycle behaviour at different stages of the business cycle. A 26-year sample period was therefore selected as it covers approximately four business cycle periods according to the research by the NBER. This assimilates regular behaviour of idiosyncratic volatility over the sample period with no bias towards specific stages of the business cycle and economic events. The NBER identifies turning points in the economic business cycle when the committee reaches consensus that a turning point has occurred (Chauvet & Piger, 2003). Due to the subjective nature of the economic business cycle calculation process, two primary criticisms have emerged. Firstly, due to the NBER's economic business cycle calculation methodology representing the consensus of individuals, providing contrasting techniques in identifying turning points, the data methodology is neither transparent nor reproducible. The second criticism relates to the NBER business cycle peaks and troughs, which are calculated subsequent to the current economic climate to avoid misrepresentation. This deferred methodology provides an inaccurate view of leading indicators. To corroborate the validity of the findings by the NBER, empirical analysis was conducted in this current study to determine the number and duration of business cycles in South Africa. Composite business cycle indicators (BCIs) are utilised to determine the leading, lagging and coincidence measures of future, prior and current economic conditions, respectively. The BCIs are a broad-based measure of economic conditions, created by the Confidence Board, to forecast changes in the direction of the overall economy of a country. Data to conduct the BCI analysis was sourced from the JSE and had a historical sample period from January 1960 to December 2018.

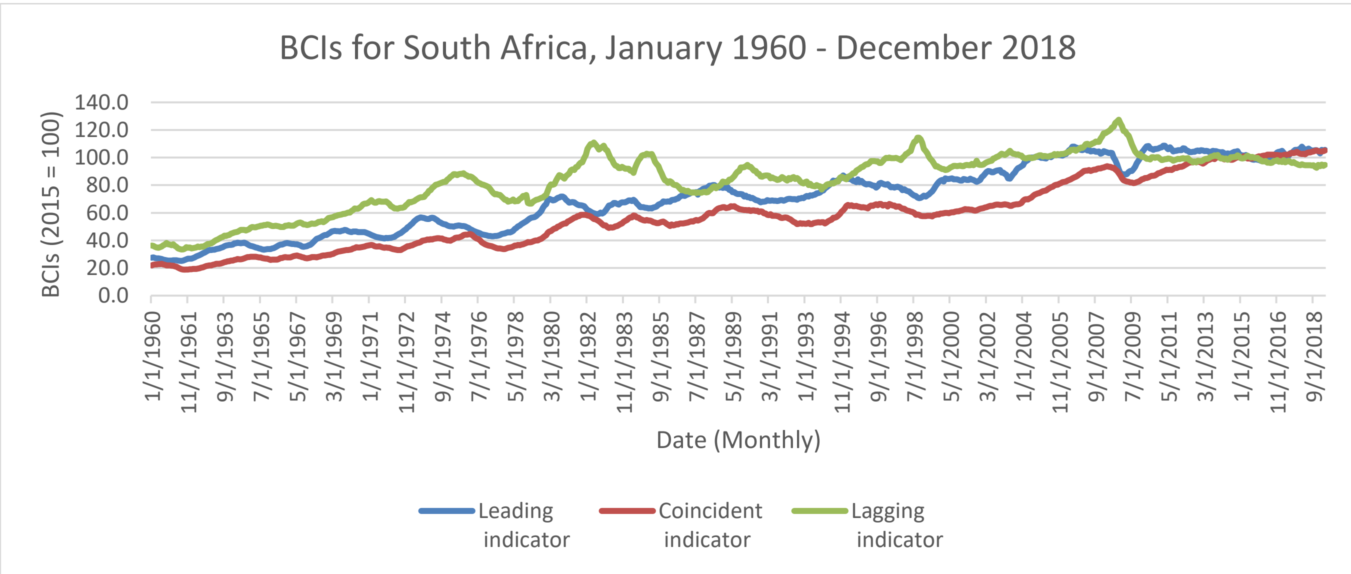

Figure 3.1: BCIs for South Africa from January 1960 to December 2018 Composite Business Cycle Indicators – July 2019.

The leading BCI, which provides foresight into future economic conditions of South Africa, is a defective measure of the actual economic direction the South African economy follows. This may be a result of RMB/BER business confidence indices, which have been highly volatile over the past 10 years. As a result, the South African economic business cycle was examined by means of the lagging and coincidence BCIs, illustrated in Figure 3.2 and Figure 3.3, respectively.

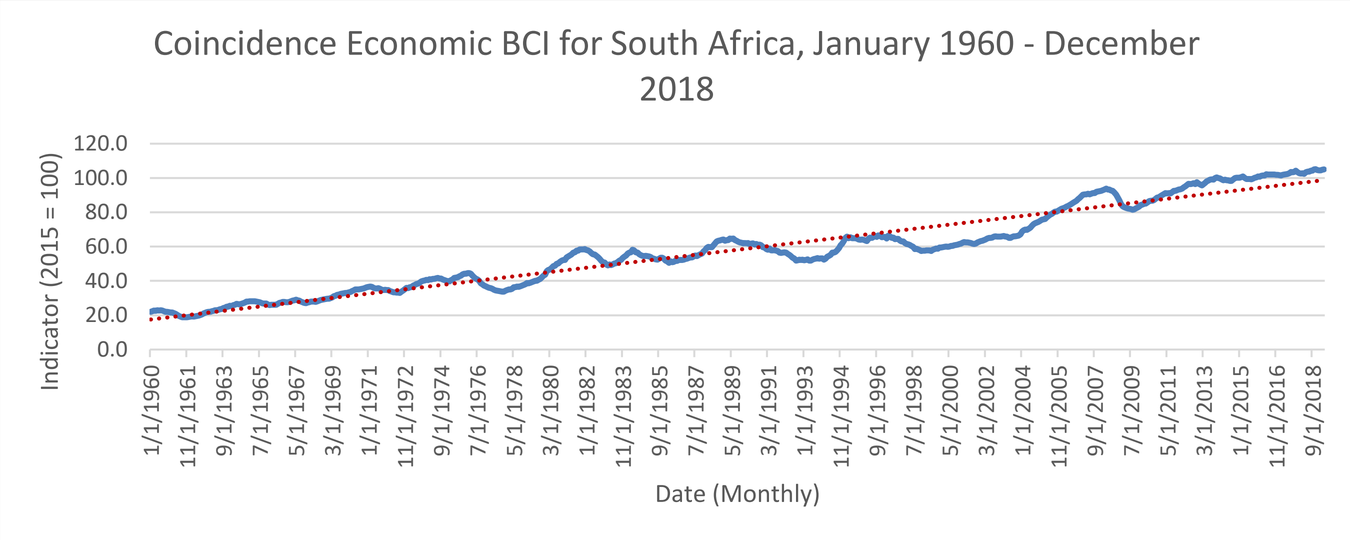

Figure 3.2: Coincidence BCI for South Africa from January 1960 to December 2018. Data sourced from: Composite Business Cycle Indicators – July 2019.

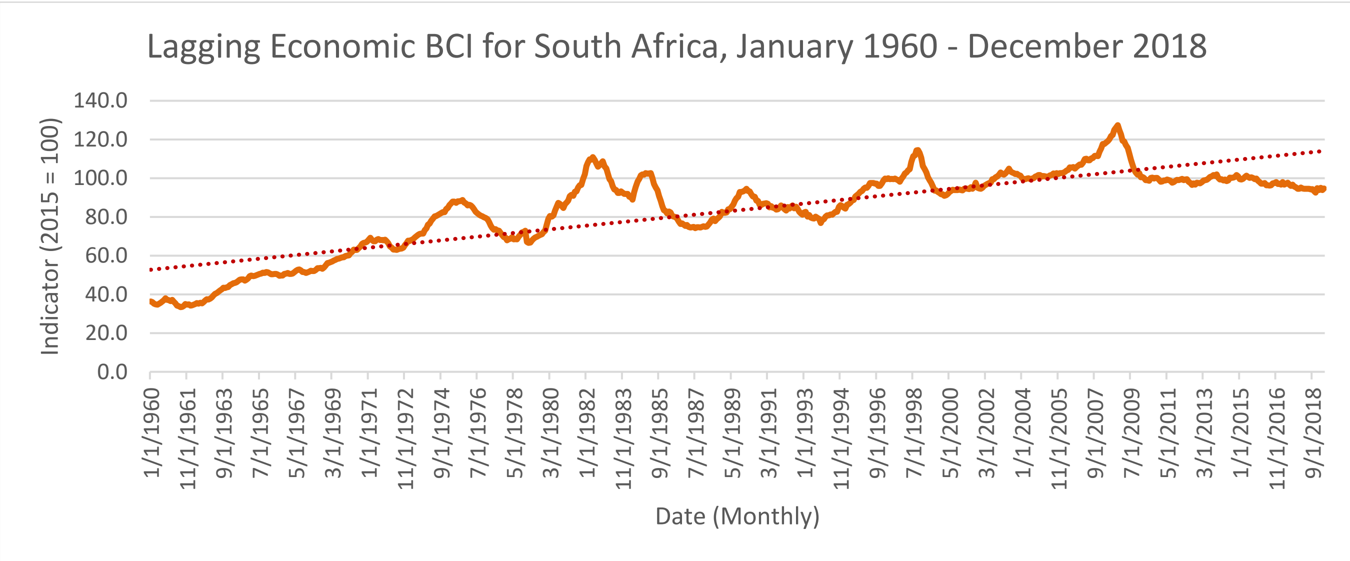

Figure 3.3: Lagging BCI for South Africa from January 1960 to December 2018. Data sourced from: Composite Business Cycle Indicators – July 2019.

The results from Figure 3.2 and Figure 3.3 indicate that the South African economy experienced a prolonged economic expansionary period from 1960 to 1975, with a short recession and depression phase from 1975 to 1979, completing the first economic business cycle in 1980. The next observable economic cycles occurred in 1980 to 1989, 1989 to late 1995, 1996 to 2002, late 2002 to 2010 and finally 2010 to date. We are currently observing a prolonged downward trend in the South African economy post-2010. The overall findings suggest that over the 58-year sample period, South Africa has observed 6 business cycles with an average length of approximately 117 months. These results are significantly higher than the US study conducted by the NBER, but may be a result of skewed data due to the persistent upward trend from 1960 to 1975 observed in the South African economy.

The process of determining volatility was conducted using three estimation periods. Study A estimated volatility utilising 12-month prior data. The purpose of using a 12-month estimation period was to replicate the study of Ang *et al*. (2006) and Diether *et al*. (2002). Ang *et al*. constructed value-weighted portfolios calculated from the idiosyncratic risk of daily data over 12 months of returns which ended one month prior to the formation date. Study B estimated volatility utilising 36-month prior data. The second estimation technique was introduced in order to determine the effect of time on volatility estimation. A larger look-back period provided a reference to determine which estimation procedure was more accurate and provided contrasting results and theories on the low idiosyncratic premium. The rationale for introducing a 36-month volatility estimation period was to replicate the studies of Oladele and Bradfield (2016, 2018) who studied a low volatility anomaly across the JSE/FTSE. Finally, Study C estimated volatility utilising 60-month prior data. The purpose of using this period was to replicate the estimation period conducted by Xiong *et al*. (2014).

Importantly, the data included shares that had been delisted across the sample period in order to eliminate survivorship bias. Furthermore, the returns were adjusted to account for unbundling, dividends, consolidations and stock-splits. Finally, shares which were part of the bottom 5% in market capitalisation and shares which traded for periods less than 12 months in Study A, 36 months in Study B and 60 months in Study C, which had over 100 zero daily trades calculated over the previous year, were excluded to ensure accurate measurement of the shares' idiosyncratic volatility. The JSE was selected as the sample exchange as it incorporates the majority of the South African equity market given the constraints. The share returns used in this study comprised secondary monthly share price data sourced using INET BFA-provided data. The market portfolio used in the study is represented by the JSE All Share Index (J203) and the risk-free rate is proxied by the 90-day treasury bill rate.

Shares were categorised by their preceding 12-, 36- and 60-month volatility rates into quintile portfolios, with Q5 constituting high idiosyncratic volatility shares and Q1 constituting low idiosyncratic volatility shares. Quintile portfolios are rebalanced annually in order to reduce transaction costs in line with common investment strategy. Returns of each quintile portfolio were measured using geometric buy and hold returns, and the risk-adjusted returns were computed using Sharpe and Treynor ratios.

Idiosyncratic volatility was measured according to the Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin CAPM (1996) and the Fama and French 3-factor model (1993). The two methods, which are commonly utilised around the world to calculate the expected return on an asset, are expressed in this study as the return of share *i*, denoted as $E(R_i)$. The idiosyncratic volatility of share *i* is calculated as the standard deviation of the sum of residuals (ε_i) after calculating the expected return using CAPM and the Fama and French 3-factor model methodology, respectively, using the preceding 12-, 36- and 60-month share returns.

3.3.1 Capital Asset Pricing Model

The use of CAPM as a measure of expected return (Malkiel & Xu, 2002) calculates the return of share *i* that is relational to the movement in the market (β_i). This is simply expressed as the mispricing of share *i*.

$$\mathrm{E}(R_i) = R_f + \beta_i \left[(R_m) - R_f\right] \tag{1}$$

$$\mathrm{E}(R_i) = \alpha_i + \beta_i \,(R_m) \tag{2}$$

$$R_i = \alpha_i + \beta_i \,(R_m) + \varepsilon_i \tag{3}$$

Where:

β_i = sensitivity of share return *i* to market variations

R_m = market risk premium represented by the JSE All Share (J203) in excess of the risk-free rate represented by the three-month treasury bill rate

α_i = Jensen's alpha, a measure of the excess returns earned by a portfolio compared to the returns measured by CAPM

Equation (1) is the widely known CAPM equation developed by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1996), which may also be represented as Equation (2) as a measure of expected return of share i. Equation (3) illustrates the calculation method to calculate the actual return of share i. Equation (1) and Equation (3) may be rewritten as Equation (4) in which:

$[E(R_i) - R_f]$ = the expected excess return of share *i*

α = the intercept of the estimated regression line

$\beta_i\,[E(R_i) - R_f]$ = excess return on the market premium

ε_i = random error component

Equation (3) illustrates the measurement process to determine the final Equation (7) to calculate the idiosyncratic volatility of share *i* where $\sqrt{\sigma^2}$ denotes the idiosyncratic volatility of share i.

$$\mathrm{E}\left(R_i - R_f\right) = \alpha_i + \beta_i\,[\mathrm{E}(R_m) - R_f)] + \varepsilon_i \tag{4}$$

Substituting Equation (3) into Equation (2):

$$\mathrm{E}(R_i) - R_i = \varepsilon_i \tag{5}$$

The equation to calculate the variance of share returns is expressed as:

$$\sigma^2 = \Sigma\,\frac{(\mathrm{E}(R_i) - R_i)^2}{n} \tag{6}$$

This can be rewritten as:

$$\sigma^2 = \Sigma \frac{(\varepsilon_i)^2}{n} \quad (7)$$

3.3.2 *Fama and French 3-factor Model*

As discovered by Ang *et al*. (2006), CAPM failed to accurately explain the cross-sectional returns of individual shares. This finding prompted the use of the Fama and French 3-factor model due to its universality in financial applications. The primary benefit of the Fama and French model is the inclusion of a size factor (SMB) and a value factor (HML) to the existing market factor (MRK) model. The market factor is calculated as the value-weighted return in excess of the market portfolio over the three-month treasury bill rate. The size factor is measured as the difference between the smallest one-third and largest one-third of JSE share returns according to market capitalisation. Finally, the value factor is measured as the difference between the largest one-third and smallest one-third of JSE share returns according to BTM ratios. The expected return measured by the Fama and French model is calculated using the regression Equation (8) below:

$$\mathrm{E}(R_i) = \alpha_i + \beta_{mi}\,(\mathrm{MRK}) + \beta_{si}\,(\mathrm{SMB}) + \beta_{vi}\,(\mathrm{HML}) \quad (8)$$

The measurement process to calculate the idiosyncratic volatility of share *i* using the Fama and French model was calculated as the standard deviation of the residual returns (ε_i) after estimating the expected returns by means of Equation (8), using the historical 12-, 36- and 60-month share returns. The factor sensitivities β_{mi} , β_{si} and β_{vi} represent the factor sensitivities for market, size and value effects. To remove any bias towards the small firm effect and value premium which may be present in the CAPM regression results, the size and value factor sensitivities were introduced as part of the Fama and French model. The residuals from the regression results served as the asset pricing factor for the empirical analysis.

Karp and Van Vuuren (2017) analysed a sample of 46 JSE stocks from 2010 to 2015 and found the Fama and French 3-factor model to provide limited explanatory power in estimating expected share returns on the JSE. They do, however, indicate that the model clearly outperformed CAPM consistently over the sample period.

3.4 Empirical Analysis Procedure

The analysis in this study followed the perspective of Ang *et al*. (2006) and Blitz and Van Vliet (2007) who examined the low volatility anomaly by estimating volatility using the Fama and French 3-factor model. The primary benefit of using the Fama and French model is that the model is a better proxy for the return-generating process. This theoretically should result in a better measure of volatility as the factors in the model account for size and value effects. The size and value effects can be explained as shares in the low volatility quintile having significantly positive coefficients for each factor (SMB and HML). This indicates that shares in the low volatility quintile have positive factor loadings towards shares with high BTM ratios (HML) and small firm shares (SMB).

This study set out to determine if there is a low idiosyncratic volatility anomaly present in the cross-section of returns on the JSE by analysing the idiosyncratic volatility measurement procedure, estimated on a univariate and multivariate basis, according to three volatility estimation periods. In layman's terms, three "look-back" periods were used in the volatility estimation procedure. These periods were 12 months (Study A), 36 months (Study B) and 60 months (Study C). All studies calculated volatility measured according to CAPM and the Fama and French 3-factor model.

Shares were winsorised according to a 90% winsorisation as this allowed data below the 5th percentile to be set to the 5th percentile and data above the 95th percentile to be set to the 95th percentile. This transformation of data was done in order to remove and limit extreme values which may have skewed the results. The winsorisation process was conducted using the DescTools library imported into R-Studio (Hastings, Mosteller, Tukey & Winsor, 1947). In line with Tukey (1977), tests of normality, skewness and kurtosis were conducted using SAS analytical software and tools. Once winsorisation had taken place, idiosyncratic volatility was computed according to CAPM and Fama and French 3-factor model procedures discussed in sections 3.3.1 and 3.3.2. The winsorised shares were thereafter sorted into quintiles according to their respective idiosyncratic risk calculated using the 12-, 36- and 60-month volatility estimation methods.

Each quintile portfolio was then analysed by determining the respective mean returns, standard deviations and return per unit of risk measures. Quintile returns were calculated using an equally weighted index and return per unit of risk was calculated using the Sharpe ratio and Treynor ratio. The Sharpe ratio is simply calculated as the quintile's mean return over the standard deviation of each quintile; the Treynor ratio calculates the mean return of each quintile over the quintile's beta. These two measures provide significant evidence of the premium at given levels of risk.

Cumulative returns series for each idiosyncratic volatility estimation procedure were conducted by determining the growth rate of ZAR 1.00 invested in each quintile as well as the market portfolio and differential portfolio. The differential portfolio was calculated as an investor taking a long position in the low idiosyncratic volatility quintile (Q1) while taking a simultaneous short position in the high idiosyncratic volatility portfolio (Q5). This enabled the degree of co-movement between each quintile to be examined. Portfolios are rebalanced yearly to form part of an active trading strategy with lower transaction costs.

Finally, time-series attribution regressions were applied to the quintile portfolios in order to determine whether the idiosyncratic risk (low volatility premium) was present across portfolios, after controlling for market, size and value factor sensitivities.

3.5 Idiosyncratic Volatility Estimation Methods

The use of three estimation periods was based on the realisation that alternative literature on the low volatility anomaly has examined the topic using different volatility estimation periods but rarely on the same sample in a single study. Ang *et al.* (2006) used a 12-month volatility estimation period and Xiong *et al.* (2014) used a longer 60-month volatility estimation period.

Although each study covered a different volatility estimation period, both studies found evidence supporting the low volatility anomaly. However, in the study by Xiong *et al.* (2014) the average returns for quintile 1 (low volatility portfolio) on US equity funds were found to be significantly lower than those of quintile 5 (high volatility portfolio). These results are in favour of the efficient market hypothesis and require further validation. A theory for this finding may be relevant to the volatility estimation period of 60 months as used by the authors. In order to examine the true effects of the volatility estimation period on the low idiosyncratic volatility anomaly, the three formation periods were used in this current study.

3.6 Empirical Analysis Expectations

The primary expectation of this study tracks the various alternative hypotheses stated in the introduction to this study. In line with the findings of Ang *et al.* (2006, 2009), the expectation was that statistically significant evidence would be found in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE at 12-, 36- and 60-month volatility estimation periods. Secondly, idiosyncratic risk was expected to have significant power in explaining the cross-sectional variation in JSE share returns. Thirdly, low volatility portfolios were expected to outperform high volatility portfolios consistently. Fourthly, a larger look-back window period was expected to yield significantly improved results in identifying a low idiosyncratic volatility anomaly. In the fifth place, JSE stocks with low idiosyncratic risk were expected to revert to higher levels of risk over time. Lastly, the alternative risk metric (ECVaR) was expected to provide significant explanatory power in the cross-sectional variations in JSE share returns as an improved measure of risk.

CHAPTER 4

RESULTS

4.1 Results Overview

The results of the empirical analysis conducted in this study are presented in this chapter in four key sections. Section 4.2 details Tukey's exploratory data analysis (EDA) approach (1977), which performs thorough testing to summarise the data's primary characteristics, including tests of normality, skewness and kurtosis. The performance of the various quintile portfolios is examined in section 4.3 by comparing the risk-adjusted returns and volatility of each quintile portfolio relative to the 12-, 36- and 60-month volatility estimation periods. The results of the extreme and average expected loss for the worst 5%, 1% and 0.01% of returns are examined in section 4.4 utilising the VaR and CVaR tail loss metrics. In section 4.5 the cumulative return series is examined for ZAR1.00 invested in each quintile portfolio utilising the 12-, 36- and 60-month volatility estimation periods over the sample period. Lastly, the results from the OLS regression analysis for each quintile at the 12-, 36- and 60-month idiosyncratic volatility estimation periods are presented in section 4.6.

4.2 Exploratory Data Analysis

The EDA approach was used to examine the various descriptive statistics and tests for the monthly returns of the five quintile portfolios examined, for the three volatility estimation periods in this study. The returns of the J203 were also analysed to provide a benchmark to evaluate and compare to the five quintile portfolio descriptive statistics.

4.2.1 12-month Descriptive Statistics

Table 4.1: 12-month descriptive statistics for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
Mean	0.010111	0.001888	0.003452	0.002691	0.001669	0.000705
Median	0.010394	0.001584	0.005368	0.002785	0.002066	-0.003220
Maximum	0.140329	0.074717	0.108849	0.172881	0.189148	0.228424
Minimum	-0.292997	-0.172515	-0.265749	-0.241166	-0.190204	-0.293320
Std. Dev.	0.050885	0.027017	0.039182	0.043214	0.042227	0.065114
Skewness	-0.676658	-0.897634	-1.197007	-0.133001	0.111380	-0.359853
Kurtosis	6.806703	8.226569	10.03511	7.243638	5.736647	6.338467
Jarque-Bera	212.1919	397.0202	717.9134	235.0299	98.00517	151.6233
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	3.154595	0.589058	1.077066	0.839744	0.520706	0.219911
Sum Sq. Dev.	0.805281	0.227006	0.477461	0.580785	0.554546	1.318585
Observations	312	312	312	312	312	312

The results in Table 4.1 provide key insights into the descriptive statistics of the 12-month volatility estimation period returns. Skewness, which measures the asymmetry in the statistical distribution, indicates that Q1 and Q5 are positively skewed, with the remaining quintile portfolios and the J203 negatively skewed. The rule of thumb in interpreting the skewness of data is that any data falling between -0.5 and 0.5 is fairly symmetrical, data between -1 and -0.5 (0.5 and 1) is moderately skewed and data less than -1 (greater than 1) is highly skewed. Utilising this rule of thumb, the data indicates that Q3, Q4 and Q5 are fairly symmetrical, Q1 and the J203 are moderately symmetrical and Q2 is negatively highly skewed. The skewness test also found five of the six datasets to be negatively skewed, with only Q4 showing a positive skewness value. The low and high volatility quintile portfolios each derive a negative skewness result. The 12-month descriptive statistics are further supplemented in Appendix Figure 1, which illustrates the skewness of each quintile portfolio according to a histogram of normality plots.

The kurtosis of the data provides insight into the frequency of statistical outliers in the dataset, resulting in heaviness of the left and right tail measures relative to the normal distribution. The 12-month descriptive statistics in Table 4.1 present all six datasets with kurtosis values which are leptokurtic, with values greater than 3. This results in all six datasets having a longer distribution with fatter tails, as a result of a high frequency in outlier values. The evidence can be observed in Appendix Figure 1, which plots the normal distribution histogram of all six datasets for the 12-month volatility estimation period.

The Jarque-Bera (JB) test for the 12-month volatility estimation period finds all six datasets to have a significantly high JB value, which is significant at a 99% confidence interval. The lowest JB value of 98.00 is exhibited by Q4 with a value substantially lower than the opposing datasets. The result of a p-value less than 0.05 with a high JB value leads to a rejection of the null hypothesis, which states that residuals are normally distributed, and an acceptance of the alternative hypothesis, which states that residuals are not normally distributed.

Table 4.2: 12-month correlation matrix for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
J203	1.000000	0.647002	0.728936	0.660412	0.595741	0.438810
Q1	0.647002	1.000000	0.872254	0.824880	0.711216	0.545554
Q2	0.728936	0.872254	1.000000	0.872426	0.778719	0.585051
Q3	0.660412	0.824880	0.872426	1.000000	0.843392	0.564750
Q4	0.595741	0.711216	0.778719	0.843392	1.000000	0.616300
Q5	0.438810	0.545554	0.585051	0.564750	0.616300	1.000000

The 12-month correlation results in Table 4.2 provide compelling insight into the relationship between each quintile portfolio. The relationship of each quintile portfolio falls within a correlation range of 0.4388 and 0.8723. These figures indicate a moderate to strong positive linear relationship between each quintile portfolio. The weakest relationship documented is between Q5 and the J203, indicating a relatively weak positive relationship between the two datasets.

Table 4.3: 12-month covariance matrix for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
J203	0.002581	0.000887	0.001449	0.001448	0.001276	0.001449
Q1	0.000887	0.000728	0.000920	0.000960	0.000809	0.000957
Q2	0.001449	0.000920	0.001530	0.001472	0.001284	0.001488
Q3	0.001448	0.000960	0.001472	0.001861	0.001534	0.001584
Q4	0.001276	0.000809	0.001284	0.001534	0.001777	0.001689
Q5	0.001449	0.000957	0.001488	0.001584	0.001689	0.004226

The results in Table 4.3 illustrate the covariance relationship between the 12-month volatility estimation quintile portfolios. The results indicate that all quintile portfolios and the J203 have positive covariance values, which is interpreted as all datasets having a positive tendency to increase or decrease together.

4.2.2 36-month Descriptive Statistics

Table 4.4: 36-month descriptive statistics for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
Mean	0.009811	0.009995	0.010712	0.008079	0.007226	0.003994
Median	0.010394	0.010616	0.011145	0.007162	0.006821	0.005358
Maximum	0.140329	0.088582	0.142426	0.139126	0.157057	0.220415
Minimum	-0.292997	-0.166637	-0.223536	-0.214725	-0.202802	-0.296186
Std. Dev.	0.051542	0.029859	0.040044	0.040269	0.042488	0.063581
Skewness	-0.670464	-0.772996	-0.492605	-0.413302	-0.041640	-0.544500
Kurtosis	6.691417	7.023577	7.076133	6.759612	6.157660	6.511511
Jarque-Bera	192.8081	232.2408	219.8187	185.2244	124.7219	168.9579
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	2.943295	2.998538	3.213591	2.423754	2.167874	1.198103
Sum Sq. Dev.	0.794326	0.266583	0.479449	0.484864	0.539762	1.208730
Observations	300	300	300	300	300	300

The results in Table 4.4 provide core insight into the descriptive statistics of the 36-month volatility estimation period returns. In analysing the skewness of the datasets, it is evident that all six datasets are negatively skewed. Q2, Q3 and Q4 emerge as fairly symmetrical, whereas Q1, Q5 and the J203 are moderately symmetrical. The 36-month descriptive statistics are further supplemented in Appendix Figure 2, which illustrates the skewness of each quintile portfolio according to a histogram of normality plots.

Analysing the kurtosis of the 36-month descriptive statistics in Table 4.4 shows that all six datasets have kurtosis values which are leptokurtic, with values greater than 3. These results are interpreted as all six datasets having a longer distribution, with fatter tails as a result of a high frequency in outliers.

The JB test for the 36-month volatility estimation period reveals that all six datasets have a significantly high JB value, which is significant at a 99% confidence interval. The 36-month volatility estimation period shows that Q5 has a JB value of 124.72, which is the lowest JB value in comparison to the opposing datasets. Furthermore, the result of a p-value less than 0.05 with a high JB value leads to rejection of the null hypothesis. These results are consistent with the 12-month descriptive statistics tests reported on in section 4.2.1.

Table 4.5: 36-month correlation matrix for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
J203	1.000000	0.678492	0.716741	0.681993	0.596205	0.419745
Q1	0.678492	1.000000	0.872280	0.778003	0.630461	0.487980
Q2	0.716741	0.872280	1.000000	0.857520	0.693651	0.548133
Q3	0.681993	0.778003	0.857520	1.000000	0.764428	0.527712
Q4	0.596205	0.630461	0.693651	0.764428	1.000000	0.627861
Q5	0.419745	0.487980	0.548133	0.527712	0.627861	1.000000

The 36-month correlation results in Table 4.5 provide compelling insight into the relationship between each quintile portfolio. The relationship of each quintile portfolio falls within a correlation range of 0.4197 and 0.8723. These figures indicate a moderate to strong positive linear relationship between each quintile portfolio. The weakest relationship documented is between Q5 and the J203, indicating a relatively weak positive relationship between the high volatility portfolio and the market portfolio. This finding is comparable to the 12-month correlation matrix, which also found that the weakest relationship was between Q5 and the J203 portfolio.

Table 4.6: 36-month covariance matrix for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
J203	0.002648	0.001041	0.001474	0.001411	0.001301	0.001371
Q1	0.001041	0.000889	0.001039	0.000932	0.000797	0.000923
Q2	0.001474	0.001039	0.001598	0.001378	0.001176	0.001391
Q3	0.001411	0.000932	0.001378	0.001616	0.001304	0.001347
Q4	0.001301	0.000797	0.001176	0.001304	0.001799	0.001690
Q5	0.001371	0.000923	0.001391	0.001347	0.001690	0.004029

The results in Table 4.6 illustrate the covariance relationship between the 36-month volatility estimation quintile portfolios. The results indicate that all quintile portfolios and the J203 have positive covariance values, which means that all datasets have a positive tendency to increase or decrease together.

4.2.3 60-month Descriptive Statistics

Table 4.7: 60-month descriptive statistics for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
Mean	0.009960	0.010023	0.009235	0.008406	0.007828	0.005617
Median	0.009094	0.009273	0.008470	0.009739	0.005053	0.001968
Maximum	0.140329	0.092979	0.111266	0.151473	0.143821	0.161086
Minimum	-0.292997	-0.211709	-0.231717	-0.203198	-0.211022	-0.230597
Std. Dev.	0.052274	0.035627	0.040271	0.040873	0.044080	0.054061
Skewness	-0.645726	-0.961265	-0.677803	-0.620049	-0.302406	-0.044346
Kurtosis	6.611243	8.268096	7.219643	6.390551	5.863474	4.115929
Jarque-Bera	169.1526	361.6630	225.8951	149.8873	98.50070	14.41137
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000742
Sum	2.748935	2.766483	2.548769	2.319962	2.160446	1.550251
Sum Sq. Dev.	0.751454	0.349049	0.445980	0.459424	0.534341	0.803718
Observations	276	276	276	276	276	276

The results from Table 4.7 provide valuable information on the descriptive statistics of the 60-month volatility estimation period returns. After examining the skewness of the datasets, a common result emerges with all six datasets generating negative skewness values as in the 12- and 36-month descriptive tests. Q4 and Q5 are the only portfolios which are fairly symmetrical and slightly negatively skewed. The remaining portfolios are all moderately symmetrical, with

Q1 generating the largest negative skewed result of -0.96. The 60-month descriptive statistics are further supplemented in Appendix Figure 3, which illustrates the skewness of each quintile portfolio according to a histogram of normality plots.

On examining the kurtosis of the 60-month descriptive statistics in Table 4.7, it can be seen that all six datasets are identified to be leptokurtic, with values greater than 3. These results mean that all six datasets have a longer distribution with fatter tails as a result of a high frequency in outliers, as in the 12- and 36-month analyses.

The JB test for the 60-month volatility estimation period reveals that five of the six datasets have a significantly high JB value, which is significant at a 99% confidence interval. In the 60-month volatility estimation period, Q5 has a JB value of 14.41, which is substantially lower than the opposing datasets. The result of a p-value less than 0.05 with a high JB value for portfolios 1-4 and the J203 leads to a rejection of the null hypothesis, whereas a significant and low JB value for Q5 leads to an acceptance of the null hypothesis for the high volatility dataset. This indicates that the residuals for the dataset Q5 are normally distributed.

Table 4.8: 60-month correlation matrix for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
J203	1.000000	0.717943	0.775113	0.717634	0.643040	0.463830
Q1	0.717943	1.000000	0.885515	0.752160	0.600113	0.511045
Q2	0.775113	0.885515	1.000000	0.820250	0.687774	0.547997
Q3	0.717634	0.752160	0.820250	1.000000	0.757581	0.630212
Q4	0.643040	0.600113	0.687774	0.757581	1.000000	0.657855
Q5	0.463830	0.511045	0.547997	0.630212	0.657855	1.000000

The 60-month correlation results in Table 4.8 provide compelling insight into the relationship between each quintile portfolio. The relationship of each quintile portfolio falls within a correlation range of 0.4638 to 0.8855. These figures indicate a moderate to strong positive linear relationship between each quintile portfolio. The weakest relationship documented is between Q5 and the J203, indicating a relatively weak positive relationship between the high volatility portfolio and the market portfolio. This finding is comparable to the 12- and 36-month correlation matrix results.

Table 4.9: 60-month covariance matrix for five quintile portfolios and J203 from January 1994 – December 2019

	J203	Q1	Q2	Q3	Q4	Q5
J203	0.002723	0.001332	0.001626	0.001528	0.001476	0.001306
Q1	0.001332	0.001265	0.001266	0.001091	0.000939	0.000981
Q2	0.001626	0.001266	0.001616	0.001345	0.001216	0.001189
Q3	0.001528	0.001091	0.001345	0.001665	0.001360	0.001388
Q4	0.001476	0.000939	0.001216	0.001360	0.001936	0.001562
Q5	0.001306	0.000981	0.001189	0.001388	0.001562	0.002912

The results in Table 4.9 illustrate the covariance relationship between the 60-month volatility estimation quintile portfolios. The results indicate that all quintile portfolios and the J203 have positive covariance values; as all datasets therefore have a positive tendency to increase or decrease together.

4.3 Quintile Portfolio Performance

In this section the performance of the five idiosyncratic volatility quintile portfolios is examined according to the 12-, 36- and 60-month volatility estimation periods.

4.3.1 12-month Volatility Quintile Portfolio Key Metrics

Table 4.10: Monthly excess returns data of 12-month volatility estimation period over January 1994 – December 2019

Portfolio	Average Excess Monthly Return	Max Excess Monthly Return	Min Excess Monthly Return	Standard Deviation	Sharpe Ratio	Treynor Ratio	Beta	Intercept	Observations
Q5 (high)	0.0705%	22.8424%	-29.3320%	0.06511	0.01082	0.00126	0.56151	-0.00497	312
Q4	0.1669%	18.9148%	-19.0204%	0.04223	0.03952	0.00338	0.49437	-0.00333	312
Q3	0.2691%	17.2881%	-24.1166%	0.04321	0.06228	0.00480	0.56085	-0.00298	312
Q2	0.3452%	10.8849%	-26.5749%	0.03918	0.08810	0.00615	0.56129	-0.00222	312
Q1 (low)	0.1888%	7.4717%	-17.2515%	0.02702	0.06988	0.00550	0.34352	-0.00159	312
Differential	0.1183%	27.7233%	-17.2304%	0.05523	0.02142	-0.00543	-0.21799	0.00339	312
J203	0.2598%	13.0294%	-30.7826%	0.05113	0.05080	0.00259	1.00359	-0.00755	312

The results in Table 4.10 present the following key findings:

With a 12-month volatility estimation period, Q5 yielded the lowest average monthly excess return of 0.0705% over the sample period. In contrast, Q2and Q3 generated the highest average monthly excess returns of 0.3452% and 0.2691%, respectively. The results are not significant at any appropriate confidence interval, which highlights the need for the 36- and 60-month analysis period. The differential portfolio, which is an investment portfolio comprising a long position in the low idiosyncratic volatility portfolio (Q1) while simultaneously holding a short position in the high idiosyncratic volatility portfolio (Q5), generated a positive excess return over the sample period. This finding provides evidence, although insignificant, that the low idiosyncratic volatility portfolio outperformed the high volatility portfolio over the sample period.

As investors are risk averse and primarily concerned about their return per unit of risk, it is vital to measure the performance of each quintile according to its corresponding risk level. The two risk-adjusted measures performed in this study were the Sharpe and Treynor ratios. Risk aversion implies that an investor will accept a lower return (as measured by their portfolio risk premium) in exchange for an adequate reduction in the standard deviation (Bodie, Kane & Marcus, 2013). Firstly, the standard deviation results presented in Table 4.10 reveal that the low idiosyncratic volatility portfolio yielded the lowest levels of risk over the sample period. In contrast, the high idiosyncratic volatility portfolio generated the highest level of risk over the sample period. On examining the Sharpe ratio (reward-to-volatility), it is apparent that the lowest idiosyncratic risk portfolio had a reward per unit of volatility of 0.06988. In spite of this, Q2 yielded the largest risk return ratio of 0.08810, proving to be the best-performing portfolio over the sample period. The opposing portfolio, Q5, generated the lowest risk return result of all the portfolios. On examining the second risk-adjusted return model of the study, i.e. the Treynor ratio, the results are inconclusive as all the portfolios exhibit similar values with no significant preference for any specific risky portfolio.

In conclusion, the results, although insignificant at all appropriate confidence intervals, indicate a clear presence of the low idiosyncratic risk anomaly on the cross-sectional returns on the JSE when the risk-adjusted excess returns are examined. As a result of the insignificant findings, further analysis was required at a 36- and 60-month volatility estimation period.

4.3.2 36-month Volatility Quintile Portfolio Key Metrics

Table 4.11: Monthly excess returns data of 36-month volatility estimation period over January 1995– December 2019

Portfolio	Average Excess Monthly Return	Max Excess Monthly Return	Min Excess Monthly Return	Standard Deviation	Sharpe Ratio	Treynor Ratio	Beta	Intercept	Observations
Q5 (high)	-0.3473%	20.9274%	-30.3796%	0.06367	-0.05455	-0.00666	0.52153	-0.00859	300
Q4	-0.0241%	14.5916%	-21.7631%	0.04277	-0.00563	-0.00049	0.49521	-0.00510	300
Q3	0.0612%	13.2329%	-22.9554%	0.04055	0.01510	0.00114	0.53658	-0.00465	300
Q2	0.3245%	13.5628%	-23.8364%	0.04038	0.08036	0.00579	0.56059	-0.00225	300
Q1 (low)	0.2528%	8.2772%	-18.1465%	0.03013	0.08390	0.00637	0.39681	-0.00136	300
Differential	0.5933% *	28.1777%	-17.3490%	0.05557	0.10677	-0.04769	-0.12441	0.00715	300
J203	0.2344%	13.0294%	-30.7826%	0.05180	0.04525	0.00234	1.00374	-0.00750	300

The 36-month volatility excess returns results in Table 4.11 present the following key findings:

The differential portfolio, yielding an average monthly excess return of 0.5933% at a 90% significance level, is clearly the best-performing quintile over the sample period, significantly outperforming the other portfolios. The differential portfolio's supreme performance can be appreciated by recognising the strong performance of the low idiosyncratic volatility portfolios relative to the negative average excess returns generated by the high idiosyncratic volatility portfolio. Similar to the 12-month study, the differential portfolio produced the highest maximum return over the sample period as well as the lowest negative minimum excess return. The results indicate that with a sharp negative loss of -30.38, Q5 was the hardest affected portfolio during the 2008 financial crisis, only performing better than the market by a mere 0.40%.

After analysing the risk and risk-adjusted returns of Table 4.11, the low volatility portfolio was found to generate the lowest levels of risk over the sample period, with a standard deviation of 0.0301. In contrast, the high idiosyncratic volatility portfolio generated the highest standard deviation of 0.0064. Next, in analysing the risk return models presented for the 36-month volatility estimation period, it can be observed that the differential portfolio, Q1 and Q2 all generated substantially better risk-adjusted returns compared to their higher risk counterparts. The low volatility portfolio generated a Sharpe ratio of 0.0839, which indicates a strong positive return per unit of risk. In contrast, the high volatility portfolio generated a negative Sharpe ratio of -0.0546, clearly providing evidence of a negative return ratio per unit of risk. Finally, on examining the Treynor ratios of the corresponding portfolios, it is evident that Q1-Q3 resulted in positive Treynor values, whereas Q4 and Q5 yielded negative risk return ratios. This supports the findings presented by the Sharpe ratio.

In conclusion, the results (although insignificant for five of the six datasets at appropriate confidence intervals) indicate a clear presence of the low idiosyncratic risk anomaly on examining the risk-adjusted excess returns with a 36-month volatility estimation period. The findings provide further supporting evidence of a low idiosyncratic volatility anomaly in the cross-sectional share returns on the JSE. However, further analysis was required with the 60-month volatility estimation period, in an attempt to provide significant evidence supporting the anomaly.

4.3.3 60-month Volatility Quintile Portfolio Key Metrics

Table 4.12: Monthly excess returns data of 60-month volatility estimation period over January 1997 – December 2019

Portfolio	Average Excess Monthly Return	Max Excess Monthly Return	Min Excess Monthly Return	Standard Deviation	Sharpe Ratio	Treynor Ratio	Beta	Intercept	Observations
Q5 (high)	0.1003%	16.3096%	-28.4847%	0.06014	0.01668	0.00204	0.49085	-0.00389	276
Q4	0.6953% **	17.6791%	-21.2127%	0.04659	0.14923	0.01452	0.47900	0.00218	276
Q3	0.6889% ***	13.5902%	-18.5793%	0.03796	0.18149	0.01383	0.49799	0.00193	276
Q2	1.0647% ***	17.4571%	-22.6156%	0.04074	0.26134	0.01917	0.55538	0.00512	276
Q1 (low)	1.0286% ***	9.3122%	-18.7609%	0.03315	0.31028	0.02393	0.42976	0.00601	276
Differential	0.9640% ***	24.5138%	-17.8860%	0.05152	0.18711	-0.15282	-0.06308	0.01027	276
J203	0.2812%	13.0294%	-30.7826%	0.05252	0.05354	0.00280	1.00377	-0.00719	276

The 60-month volatility excess returns results in Table 4.12 present the following key findings:

Firstly, on examining the average monthly excess return over the sample period, Q2 and Q1 yielded the highest monthly excess returns of 1.065% and 1.029%, respectively. The worst performer over the sample period was once again Q5, yielding a mere monthly excess return of 0.1003%. The differential portfolio generated a positive return over the sample period in addition to yielding the highest maximum excess return and the lowest minimum return. This finding provides further significant evidence that the low idiosyncratic volatility portfolio outperformed the high volatility portfolio over the sample period.

Next, moving on to the risk and risk-adjusted results in Table 4.12, it can be observed that the low idiosyncratic volatility portfolio generated the lowest levels of risk over the sample period with a standard deviation of 0.033. In contrast, the high idiosyncratic volatility portfolio generated the highest level of risk over the sample period, yielding a standard deviation of 0.060. On examining the Sharpe ratio (reward-to-volatility), it is apparent that the lowest idiosyncratic risk portfolio had a reward per unit of volatility of 0.3103, proving to be the best risk-adjusted performing portfolio over the sample period. The opposing portfolio, Q5, generated the lowest risk return result of all the portfolios with a reward-to-volatility ratio of 0.0167. After examining the second risk-adjusted return model of the study, the Treynor ratio, it is evident that Q1 generated the highest Treynor value of 0.0239, whereas Q5 yielded a mere 0.0020. This supports the findings of the Sharpe ratio that Q1 provided the highest level of return per unit of risk over the sample period.

In conclusion, the results provide significant evidence of the presence of a low idiosyncratic volatility on the cross-sectional returns on the JSE anomaly at a 99% confidence interval.

4.4 Extreme and Average Expected Losses Metrics

The 12-, 36- and 60-month extreme and average expected losses return metrics for the five idiosyncratic volatility quintile portfolios and the differential portfolio are tabulated in Tables 4.13- 4.15. Following the analysis conducted by Xiong *et al*. (2014), this study investigated the effects of a left tail risk measure to quantify and measure the statistical riskiness of the five volatility quintiles. VaR, a statistical measure which indicates the maximum loss expected for the worst 5%, 1% and 0.01% of returns, is a commonly used metric in financial intermediaries, despite a number of drawbacks. These drawbacks include issues with the metric as it provides no indication of the extent of loss related to the tail of the probability distribution out of the confidence level. Secondly, VaR is not additive, which implies that VaR figures for individual securities do not add up to the VaR of the overall portfolio (Risk.net, 2020b). To mitigate the flaws of VaR, a secondary measure is introduced, namely ECVaR or expected shortfall (ES). ECVaR is a risk metric which is more sensitive to the tail of distribution of returns on a portfolio. This is computed by averaging all the return values in the distribution that are worse than VaR at the given confidence level (Risk.net, 2020a).

4.4.1 12-month VaR and CVaR Results

Table 13: Extreme and average losses returns utilising 12-month volatility estimation period over January 1994 – December 2019

	Q5	Q4	Q3	Q2	Q1	Differential
VaR(95)	-9.3040%	-5.2655%	-5.1789%	-4.1962%	-2.9181%	-10.1745%
VaR(99)	-15.7148%	-9.1043%	-8.5658%	-9.6900%	-4.8990%	-14.8663%
VaR(99.9)	-28.5710%	-17.5375%	-22.6338%	-25.0920%	-15.7686%	-17.2304%
CVaR(95)	-14.9574%	-8.0810%	-8.4062%	-8.1786%	-5.0675%	-12.7735%
Cvar(99)	-30.8330%	-15.6216%	-16.7765%	-19.0160%	-11.2363%	-19.9101%
Cvar(99.9)	-91.5739%	-56.2099%	-72.5441%	-80.4231%	-50.5403%	-55.2257%
St.Dev	6.5013%	4.2008%	4.2953%	3.8943%	2.6792%	5.5229%

Table 4.13 reports the VaR and CVaR results for the five quintile portfolios examined over the 12-month volatility estimation period.

As expected, Q5 generated the highest negative VaR (95), VaR (99) and VaR (99.9) returns for the 12-month volatility estimation period. These results indicate that there is 95%, 99% and 99.9% confidence that Q5 will not lose more than -9.30%, -15.71% and -28.57%, respectively. Q5 also generated the highest standard deviation of 6.50%, which results from the portfolio construction process described in the methodology of the study. The CVaR results for Q5 reveal that the portfolio has the highest negative CVaR values for all confidence levels. The resulting -14.95%, -30.83% and -91.57% mean that Q5 will lose an average of these figures at the respective 95%, 99% and 99.9% confidence levels.

Q1 generated the lowest negative VaR values of -2.91%, -4.90% and -15.77% for the respective 95%, 99% and 99.9% confidence levels. These results indicate that there is 95%, 99% and 99.9% confidence that Q1 will not lose more than -2.91%, -4.90% and -15.77%, respectively. The corresponding standard deviation of Q1 is the lowest variation in returns of

2.68%. The CVaR results for Q1 suggest that after accounting for the worst 5%, 1% and 0.01% of cases, Q1 will lose an average of -5.07%, -11.24% and -50.54%, respectively.

Finally, some interesting findings from the results are that the differential portfolio performed no better than the high volatility quintiles with significantly high negative VaR and CVaR returns. Furthermore, Q2 generated the second highest negative VaR and CVaR returns, which is a surprising result in light of Q2 being the second-lowest volatility portfolio. These sharp negative returns could be an indication of the poor estimation performance of volatility and the significance of an alternative risk metric which could be an explanatory factor for the presence of a low volatility anomaly.

4.4.2 36-month VaR and CVaR Results

Table 4.14: Extreme and average losses returns utilising 36-month volatility estimation period over January 1995 – December 2019

	Q5	Q4	Q3	Q2	Q1	Differential
VaR(95)	-8.9088%	-5.2482%	-4.7436%	-4.5041%	-3.3189%	-7.9191%
VaR(99)	-23.0620%	-11.6747%	-10.1859%	-10.3573%	-7.7178%	-15.6590%
VaR(99.9)	-29.6186%	-20.2802%	-21.4725%	-22.3536%	-16.6637%	-17.3490%
CVaR(95)	-14.6785%	-8.6319%	-8.2062%	-7.5491%	-5.8148%	-11.4156%
Cvar(99)	-27.4330%	-15.4621%	-15.3334%	-15.3312%	-11.6419%	-16.7316%
Cvar(99.9)	-98.7285%	-67.6008%	-71.5751%	-74.5119%	-55.5455%	-57.8301%
St.Dev	6.3581%	4.2488%	4.0269%	4.0044%	2.9859%	5.5509%

Table 4.14 reports the VaR and CVaR results for the five quintile portfolios examined over the 36-month volatility estimation period.

In conjunction with the 12-month VaR and CVaR results reported in Table 4.13, Q5 generated the highest negative VaR (95), VaR (99) and VaR (99.9) returns for the 36-month volatility estimation period. These results indicate that there is 95%, 99% and 99.9% confidence that Q5 will not lose more than -8.91%, -23.06% and -29.62%, respectively. Q5 also generated the highest standard deviation of 6.36%, which results from the portfolio construction process described in the methodology of the study. The CVaR results for Q5 show that the portfolio had the highest negative CVaR values for all confidence levels. The resulting CVaR (95), CVaR (99) and CVaR (99.9) are interpreted as Q5 losing an average of -14.68%, -27.43% and -99.73%, respectively, in the worst 5%, 1% and 0.01% of cases.

Q1 generated the lowest negative VaR values of -3.32%, -7.72% and -16.66% for the respective 95%, 99% and 99.9% confidence levels. These results indicate that there is 95%, 99% and 99.9% confidence that Q1 will not lose more than -3.32%, -7.72% and -16.66% of returns, respectively. The corresponding standard deviation of Q1 is the lowest variation in returns of 2.99%. The CVaR results for Q1 suggest that after accounting for the worst 5%, 1% and 0.01% of cases, Q1 will lose an average of -5.81%, -11.64% and -55.54%, respectively.

Finally, some interesting findings from the 36-month results are that the differential portfolio performed no better than the high volatility quintiles with significantly high negative VaR and CVaR returns, resulting in the second-highest negative VaR and CVaR returns. Furthermore, Q2, Q3 and Q4 had very similar VaR and CVaR returns, rendering the three portfolios inaccurate as no statistical inferences can be concluded with absolute certainty. The results highlight the reduced losses expected for the low idiosyncratic volatility portfolio. This provides supporting evidence of the low risk associated with Q1 as all three risk measures illustrate the depressed variability in returns and limited left side tail risk.

4.4.3 60-month VaR and CVaR Results

Table 4.15: Extreme and average losses returns utilising 60-month volatility estimation period over January 1997– December 2019

	Q5	Q4	Q3	Q2	Q1	Differential
VaR(95)	-7.6532%	-5.6447%	-5.6028%	-4.7051%	-3.9837%	-8.0165%
VaR(99)	-11.5123%	-13.5240%	-8.7797%	-9.8187%	-8.3384%	-11.5458%
VaR(99.9)	-23.0597%	-21.1022%	-20.3198%	-23.1717%	-21.1709%	-12.9560%
CVaR(95)	-10.8965%	-9.4757%	-8.7492%	-7.9250%	-7.3009%	-10.1150%
Cvar(99)	-17.0645%	-17.5813%	-16.7923%	-16.4038%	-15.2815%	-13.1214%
Cvar(99.9)	-83.5498%	-76.4573%	-73.6225%	-83.9555%	-76.7062%	-46.9419%
St.Dev	5.4061%	4.4080%	4.0873%	4.0271%	3.5627%	4.7152%

Table 4.15 reports the VaR and CVaR results for the five quintile portfolios examined over the 60-month volatility estimation period.

The 60-month volatility returns series provide interesting results for Q5 over the sample period. As illustrated in Table 4.15, Q5 did not generate the highest negative VaR returns consistently over the sample period for the various confidence levels. Firstly, the VaR (95) returns show that Q5 had the largest downside risk of all the risk portfolios besides the differential portfolio, which yielded a moderately higher negative return of -8.0165%. The VaR (95) value of -7.65% was considerably lower compared to Q1-Q4. Secondly, the VaR (99) confidence level returns show that Q5 generated a lower maximum expected loss than Q4 with a difference of 2.01%. The third VaR confidence level result is that Q5 generated a lower maximum expected loss than Q2 with a difference of 0.01%. As this resulting value is marginal, no accurate statistical inferences can be deduced. The CVaR results for Q5 are that the portfolio displays a similar trend to the VaR results, with CVaR (95) generating the largest downside risk with a maximum negative return of -10.90% and Q4 and Q2 generating higher negative downside risk returns for the CVaR (99) and CVaR (99.9) confidence levels, respectively. These results are inconsistent regarding the maximum expected downside risk for the high volatility portfolio.

Q1 generated the lowest negative VaR (95) and VaR (99) return values with maximum expected left tail risk losses of -3.98% and -8.34%, respectively. These results indicate that there is 95% and 99% confidence that Q1 will not lose more than a maximum of -3.98% and -8.34% of returns, respectively. The corresponding standard deviation of Q1 is the lowest variation in returns of 3.56%. Next, the CVaR results for Q1 suggest that after accounting for the worst 5% and 1%, Q1 will lose an average of -7.30% and -15.28%, respectively. It is important to note that the differential portfolio generated a lower average expected loss value of -13.12 for CVaR (99), which is the lowest CVaR (99) return for the 60-month volatility series. Finally, the CVaR (99.9) shows that Q3 and Q4 generated lower average expected losses than Q1, which contradicts the notion of a low volatility portfolio generating lower average expected losses. These findings may provide insight into the poor performance of standard deviation as a measure of risk, where an alternative downside risk metric may be more accurate at measuring the riskiness of a portfolio.

In conclusion, although the volatility of a portfolio of shares is a common metric for estimating the level of riskiness associated with the portfolio, an alternative left tail risk measure that can accurately predict the downside risk of a portfolio may be a better measure and provide more accurate results. The results from Tables 4.13- 4.15 show that CVaR and VaR provide inconclusive results which do not follow the expected results for the volatility portfolios. As a result, further analysis of the presence of a volatility anomaly needed to be made according to portfolios categorised under each share's underlying CVaR to determine if the anomaly holds true under a different risk measurement methodology.

4.5 Cumulative Returns

4.5.1 12-month Cumulative Returns

The 12-month cumulative returns model examined the growth rate of an initial investment of ZAR1.00 in each of the quintile portfolios constructed from January 1994 to December 2019.

Figure 4.1 illustrates the cumulative returns series for each quintile portfolio, using a 12-month volatility estimation period from January 1994 to December 2019. Initially, all six portfolios appear to have similar cumulative returns for a ZAR1.00 investment. However, after July 1996 it is evident that Q5 (high idiosyncratic volatility) commenced a superior and steady short-term outperformance over the other quintiles. Moving towards the turn of the millennium, the steady growth of Q5 came to an abrupt halt, followed by a sharp decline for Q5 from June 1998 to April 2003. This indicates that the high idiosyncratic volatility portfolio endured the largest negative effects of the 1999 crisis. Following April 2000, Q1-Q4 experienced a sharp rise in growth rates, and the high idiosyncratic volatility portfolio (Q5) recovered in late 2002 and began a rapid upward growth trend. Over this early millennium period, the J203 failed to generate any superior growth, as all five contrasting portfolios outperformed the market. Regarding the sharpest decline in portfolio returns over the sample period, the October 2007 to August 2009 period – synonymous with the financial crisis – had a significant negative impact on all the quintile portfolios, resulting in extreme losses and poor performance. The portfolios which experienced the highest negative returns over the period were Q2 and Q5, which yielded maximum extreme losses of -26.57% and -29.33%, respectively. The portfolio which exhibited the lowest negative returns over the 2008 global financial crisis was the low volatility portfolio (Q1) with a minimum return over the period of -17.25%, outperforming the market's maximum extreme loss by 13.53%.

On examining the cumulative returns after the financial crisis of 2008, it can be seen that the high volatility quintile (Q5) realised positive growth, which trended very closely to Q4, Q3 and Q1. In contrast, Q2 outperformed the other portfolios, realising superior returns and lower levels of volatility over the remaining sample period. The final observable economic event which resulted in losses to each portfolio is the apparent downward trend in all the portfolios' cumulative returns around October – December 2015. As a result of S&P and Fitch's decision to downgrade South Africa's economic outlook to "Junk", all portfolios experienced a downward trend in growth, in which the high volatility portfolio and Q2 suffered the largest losses.

The final cumulative returns of Q2 realised in December 2019 of ZAR23.65 indicate the impressive performance of the portfolio over the entire sample period. The final appreciation value of the low idiosyncratic volatility portfolio grew to ZAR16.58, whereas the high idiosyncratic volatility portfolio slumped to a low ZAR6.61. This subsequently resulted in Q5 remaining poor in performance over the entire sample period due to radical swings in volatility. The results of the 12-month estimation period provide evidence in support of the existence of the low idiosyncratic volatility anomaly on the JSE.

When breaking down the cumulative returns into compound annual growth figures, it is revealed that if an investor initially invested ZAR1.00 in the highest idiosyncratic risk portfolio, and continually rebalanced the portfolio on an annual basis (so that Q5 maintained the stocks with the highest idiosyncratic risk), the investor would have achieved capital gains to the value of ZAR6.61, resulting in a compound annual growth rate of return of 6.86%. In comparison, an investor investing ZAR1.00 in the lowest idiosyncratic volatility portfolio would have achieved capital gains to the value of ZAR16.58. The resulting compound annual growth rate of return is equivalent to 11.14%. This shows that Q5 significantly underperformed in contrast to Q1 on a cumulative returns basis, suggesting evidence of a low idiosyncratic risk anomaly on the cross-sectional returns on the JSE. This further indicates that the JSE is not efficient according to MPT and the findings of Markowitz. The evidence highlighted provides supporting proof of the results and findings of Baker *et al*. (2011) and Ang *et al*. (2006, 2009).

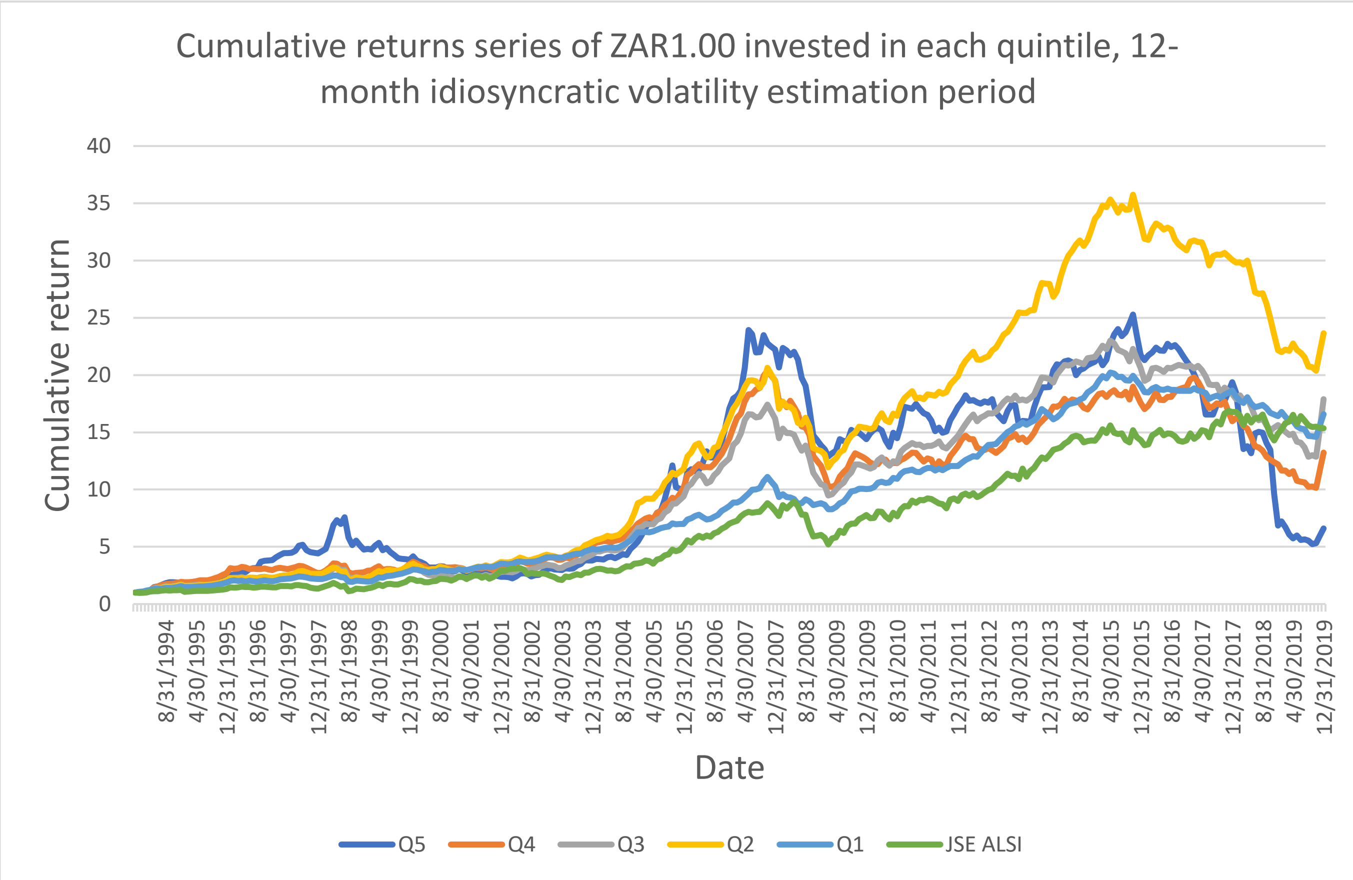

Figure 4.1: 12-month cumulative returns series of ZAR1.00 invested in each 12-month idiosyncratic volatility estimation portfolio from January 1994 – December 2019

4.5.2 36-month Cumulative Returns

Figure 4.2 illustrates the cumulative returns series for each quintile portfolio, using a 36-month volatility estimation period over the period January 1995 – December 2019.

Similar to the 12-month returns series, five of the six portfolios appear to have similar cumulative returns for a ZAR1.00 investment, up to the early 2000s. The high volatility portfolio commenced with rapid short-term growth after July 1996, which took a sharp turn in June 1998, as the turn of the millennium approached and the internet boom of the 1990s went bust. This shows that the high idiosyncratic volatility portfolio suffered the largest negative effects of the 1999 crisis.

Following the effects of the internet boom and bust, all the portfolios experienced a sharp rise in growth rates from early 2000 to 2007. Regarding the sharpest decline in portfolio returns over the sample period, the 2008 financial crisis had a significant negative impact on all the quintile portfolios, resulting in extreme losses and poor performance. The portfolios which experienced the highest negative returns over the period were Q2 and Q5, which yielded maximum extreme losses of -23.83% and -30.38%, respectively. The portfolio which exhibited the lowest negative returns over the 2008 global financial crisis was the low volatility portfolio (Q1) with a minimum return over the period of -18.15%, outperforming the market's maximum extreme loss by 12.64%.

On examining the cumulative returns after the financial crisis of 2008, the high volatility quintile (Q5) is shown to perform very differently from the 12-month study. Figure 4.2 illustrates the poor performance of the high idiosyncratic volatility portfolio for the remaining period of the study, which declined rapidly to a final low of ZAR1.45, eliminating all of the realised growth over the preceding 25 years. In contrast, Q1 and Q2 outperformed the other portfolios, realising superior returns and lower levels of volatility over the remaining sample period. The final cumulative returns of Q1 and Q2 realised in December 2019 of ZAR15.74 and ZAR14.74, respectively, indicate the impressive performance of the low volatility portfolios. The results of the 36-month estimation period provide further evidence in support of the existence of the low idiosyncratic volatility anomaly on the JSE as Q1 and Q2 outperformed the remining portfolios over most of the sample period.

Breaking down the cumulative returns into compound annual growth figures shows that if an investor initially invested ZAR1.00 in the highest idiosyncratic risk portfolio, and continually rebalanced the portfolio on an annual basis (so that Q5 maintained the stocks with the highest idiosyncratic risk), the investor would have achieved capital gains to the value of ZAR1.45, resulting in a compound annual growth rate of return of -3.20%. In comparison, an investor investing ZAR1.00 in the lowest idiosyncratic volatility portfolio would have achieved capital gains to the value of ZAR15.74. The resulting compound annual growth rate of return is equivalent to 11.45%. When interpreting these results, it is shown that Q5 significantly underperformed in contrast to Q1 on a cumulative returns basis.

These findings provide evidence of a low idiosyncratic risk anomaly on the cross-sectional returns on the JSE.

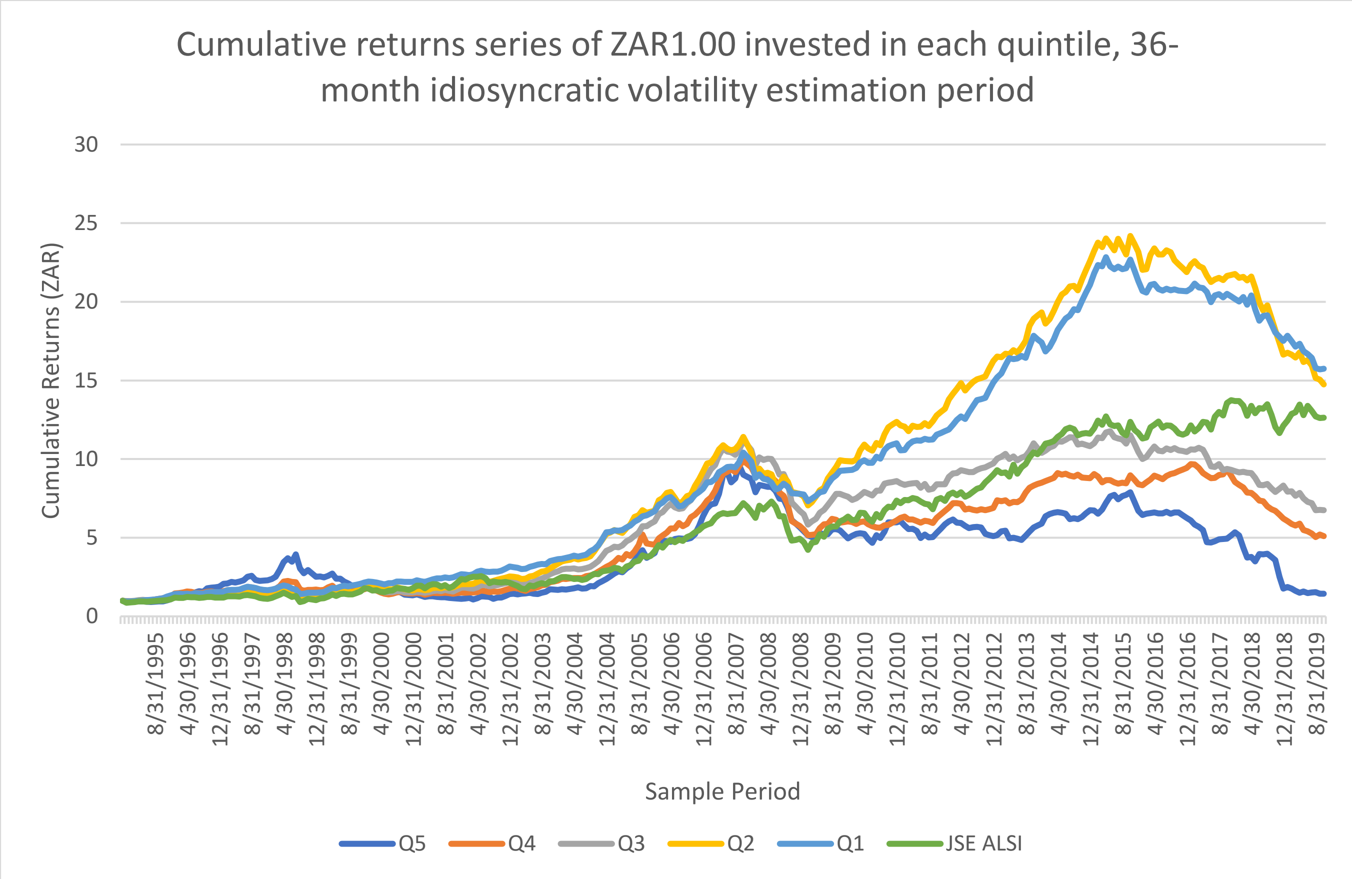

Figure 4.2: 36-month cumulative returns series of ZAR1.00 invested in each 12-month idiosyncratic volatility estimation portfolio from January 1994 – December 2019.

4.5.3 60-month Cumulative Returns

Figure 4.3 illustrates the cumulative returns series for each quintile portfolio, using a 60-month volatility estimation period over the period January 1997 – December 2019.

Similar to the 12-month returns series, five of the six portfolios appear to have similar cumulative returns for a ZAR1.00 investment, up to the early 2000s. The high volatility portfolio commenced with rapid short-term growth after July 1996, which took a sharp turn in June 1998, as the turn of the millennium approached and the internet boom of the 1990s went bust. This indicates that the high idiosyncratic volatility portfolio suffered the largest negative effects of the 1999 crisis.

Following the effects of the internet boom and bust, Q1-Q4 and the J203 portfolio experienced a sharp rise in growth rates from early 2000 to 2007, whereas Q5 experienced moderate to slow growth in cumulative returns over the period. Regarding the sharpest decline in portfolio returns over the sample period, the 2008 financial crisis had a significant negative impact on all the quintile portfolios, resulting in extreme losses and poor performance. These results are consistent with the 12- and 36-month returns series, as no portfolio could effectively immunise against the severe losses experienced in any estimation period. The portfolios which experienced the highest negative returns over the period were Q2 and Q5, which yielded maximum extreme losses of -22.62% and -28.48%, respectively. The portfolios which exhibited the lowest negative returns over the 2008 global financial crisis were the low volatility portfolio (Q1) and Q3 with a minimum return over the period of -18.76% and -18.58%, respectively.

On examining the cumulative returns after the financial crisis of 2008, the high volatility quintile (Q5) is shown to perform similarly to the 36-month study. Figure 4.3 illustrates the poor performance of the high idiosyncratic volatility portfolio for the remaining period of the study, which declined rapidly to a final low of ZAR3.19, eliminating almost all of the realised growth over the preceding 23 years. In contrast, Q1 outperformed the other portfolios, realising superior returns and lower levels of volatility over the remaining sample period. The final cumulative return of Q1 realised in December 2019 of ZAR13.34 indicates the impressive performance of the low volatility portfolio. The results of the 60-month estimation period provide further evidence in support of the existence of the low idiosyncratic volatility anomaly on the JSE as Q1 significantly outperformed the remaining portfolios over most of the sample period.

Breaking down the cumulative returns into compound annual growth figures shows that if an investor initially invested ZAR1.00 in the highest idiosyncratic risk portfolio, and continually rebalanced the portfolio on an annual basis (so that Q5 maintained stocks with the highest idiosyncratic risk), the investor would have achieved capital gains to the value of ZAR1.45, resulting in a compound annual growth rate of return of 3.50%. In comparison, an investor investing ZAR1.00 in the lowest idiosyncratic volatility portfolio would have achieved capital gains to the value of ZAR13.34. The resulting compound annual growth rate of return is equivalent to 11.63%. When interpreting these results, it is shown that Q5 significantly underperformed in contrast to Q1 on a cumulative returns basis.

These findings provide clear evidence of a low idiosyncratic risk anomaly on the cross-sectional returns on the JSE.

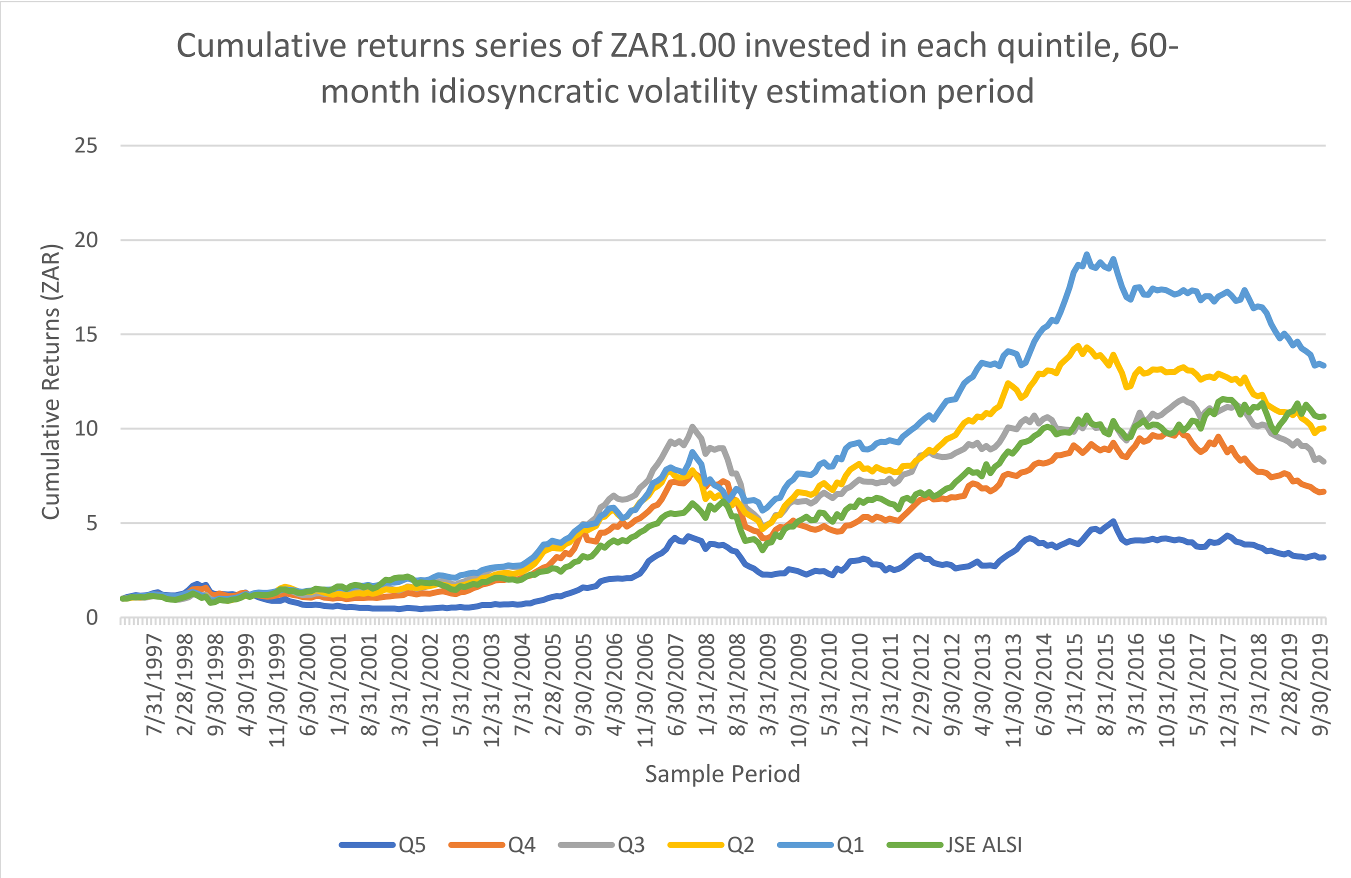

Figure 4.3: 60-month cumulative returns series of ZAR1.00 invested in each 12-month idiosyncratic volatility estimation portfolio from January 1994 – December 2019

4.6 OLS Regression Analysis

The regression outputs for the data collected are displayed in Tables 4.16- 4.21 below, as an extract of the regression output results displayed in Appendices B and C. The regression tests were based on the two idiosyncratic volatility measurement methods, i.e. CAPM and the Fama and French 3-factor model, to ensure robust results.

For the CAPM regression analysis, the dependent variable, which is the excess monthly return portfolios classified according to their idiosyncratic risk (variable Y), was regressed according to its corresponding independent variable (variable X), which is the market risk premium (RMRF) over the sample period.

The Fama and French 3-factor regression analysis constituted the dependent variable, i.e. the excess monthly return portfolios classified according to their idiosyncratic risk (variable Y), and the corresponding independent variables (X_1, X_2, X_3), i.e. the market risk premium (MRK), size effect (SMB) and the value effect (HML), over the sample period.

The 12-month volatility estimation period regression analysis was conducted first, in order to make statistical inferences regarding the short-term estimation procedure. Subsequently the 36- and 60-month volatility estimation regression analyses were conducted for each idiosyncratic volatility measurement method to test if volatility estimation time horizons had any effect on the regression results. Next, the Fama and French 12-, 36- and 60-month volatility estimation period regression analyses were conducted to determine if the size or value effect had any significant impact on the results of the CAPM regression analysis, thus providing a robust set of results.

In the analysis several critical factors expressed in the regression output data were examined in order to make statistical assumptions about the data. The intercept value expresses the alpha value (α), which in layman's terms indicates the return not explained by the independent variables (market returns for the CAPM regression analysis and the market, size or value effects for the Fama and French regression analysis). Testing the significance of the intercept and modelling it over 12 months expressed the annual excess return over and above the risk-free rate (90-day T-bill) after adjusting for these independent risk variables. The next critical factor examined was the portfolios beta to the market (βm). Testing the significance of beta indicated the level of market risk each quintile portfolio was exposed to. Next, the R-squared and adjusted R-squared values were examined for the CAPM and Fama and French regression results, respectively. The R-squared value, which is the percentage of variance in the volatility portfolios that can be explained by the independent variable, was a suitable measure for the CAPM regression results, as only one independent variable (RMRF) was examined. For the Fama and French regression results, the adjusted R-squared value was used, as this indicator, which also compares the value of the volatility portfolio to the independent variables, provided improved descriptive power as more independent variables (MRK, SMB and HML) were included in the regression analysis. Finally, the size and value coefficients were examined to determine if any significance to the results could be explained by these factors.

Testing these critical factors across the portfolios would provide evidence in favour of or against the low idiosyncratic volatility anomaly.

4.6.1 CAPM OLS Regression Analysis

The CAPM regression analysis was modelled according to Equation (4) below:

$$\mathrm{E}\left(R_i - R_f\right) = \alpha_i + \beta_i \left[\mathrm{E}(R_m) - R_f)\right] + \varepsilon_i \tag{4}$$

Where:

$\left(R_{j,t} - rf\right)$ = return on portfolio *j* at time *t* less the risk-free rate modelled by the 90-day T-bill rate

α (alpha) = excess return on portfolio *j* which is not explained by the market

β_j (beta) = degree of market risk portfolio *j* is exposed to

$(R_m - rf)$(market risk premium) = expected return of the market less the risk-free rate

ε_t = error term

4.6.1.1 12-month volatility estimation period

Table 4.16: CAPM OLS regression results for 12-month volatility estimated excess monthly return portfolios against J203

Portfolio	Intercept (α)	βm	R-Squared	Standard Error	Annual Excess Return	Cost of Equity (Annual)
Q5 (high)	-0.0007	0.5599 ***	0.19327	0.05858	-0.899%	10.7613%
Q4	0.0004	0.4944 ***	0.35834	0.03388	0.462%	10.5571%
Q3	0.0012	0.5610 ***	0.44060	0.03237	1.481%	10.7647%
Q2	0.0020	0.5607 ***	0.53537	0.02675	2.395%	10.7637%
Q1 (low)	0.0010	0.3438 ***	0.42323	0.02055	1.194%	10.0877%
Differential	0.0017	-0.2161 ***	0.04002	0.05420	2.093%	8.3427%

The results of Table 4.16 are adapted from the regression results reported in Appendix B Figure 4.

The results of the high idiosyncratic volatility portfolio (Q5) show that the intercept yielded an alpha value of -0.0007, which was not significant at any acceptable confidence level. Multiplying the intercept by 12 provides an annual excess return figure of -0.8993% which, although not significant, indicates the poor performance of the high volatility portfolio. As a result of the insignificant findings of the intercept value for Q5, the conclusion is that Q5 did not make a significant market risk-adjusted return over and above the risk-free rate. The beta for Q5 of 0.5599 has a corresponding p-value exceptionally close to 0, making the resulting value statistically significant at a 99% confidence level. The conclusion is that Q5 appears to be theoretically less volatile than the market, which is an interesting finding due to the perception of extreme volatility surrounding the high volatility portfolio. The R-squared value of 0.19327 is interpreted as 19.33% of change in Q5 being a direct result of and explained by the market. The standard error indicates the average distance the observed values fall from the regression line. Q5 generated a standard error of 5.85%, yielding the largest variance in datapoints. A benefit of the standard error measure is the advantage of utilising the standard error of regression to assess the precision of the predictions. A general rule of thumb is that approximately 95% of observations should fall within +/- 2* standard error of the regression line. This results in approximately 95% of the datapoints falling between the regression line and +/- 11.72% of Q5, which is well over the 5% standard.

The low idiosyncratic volatility portfolio (Q1) was found to yield an intercept value of 0.0017, which is not statistically significant at any acceptable confidence level. Modelling the alpha of Q1 over 12 months provides an annual excess return figure of 1.1941%, which is substantially higher than Q5 but lower than Q2 and Q3. It is interpreted as Q1 not providing a statistically significant market risk-adjusted return in excess of the risk-free rate. The beta for Q1 of 0.3438 is the lowest positive beta of all the volatility portfolios examined and is statistically significant at a 99% confidence level. This result of a significant low beta for Q1 indicates that the low volatility portfolio is theoretically less volatile than all other quintile portfolios as well as the market. The R-squared value of 0.42323 indicates a change of 42.32% in Q1 occurring due to market changes. The standard error results for Q1 are substantially lower than those of the alternative portfolios. The standard error for Q1 of 2.055% means that approximately 95% of the datapoints fall between the regression line and +/- 4.11% of Q1, which is below the 5% standard and can be regarded as Q1 exhibiting a precise set of results.

The differential portfolio, constructed taking a long position in Q1 and a simultaneous short position in Q5, yielded an alpha of 0.0017, which is not statistically significant at any acceptable confidence level. Modelling the alpha of the differential portfolio over 12 months provides an annual excess return figure of 2.093%. This figure indicates the potential benefit and strength of the low idiosyncratic volatility portfolio in contrast to the high volatility portfolio. As the results are not significant at any appropriate confidence level, the annual excess returns of the differential portfolio do not provide a statistically significant market risk-adjusted return in excess of the risk-free rate. The beta of the differential portfolio of -0.2161 is interpreted as the differential portfolio moving in the opposite direction to the market and is statistically significant at a 99% confidence level. The R-squared value of 0.04 indicates a change of 4% in the differential portfolio due to market changes. The differential portfolio yields a standard error of 5.42%, which accounts for approximately 95% of the datapoints falling between the regression line and +/- 10.84% of the differential portfolio.

The remaining portfolios, which are all insignificant at any acceptable confidence level, reveal that Q2 had the highest annual excess return of 2.3948%, and all portfolios generated a statistically significant beta which ranged around 0.35-0.56.

In conclusion, the OLS regression analysis conducted using a 12-month idiosyncratic volatility estimation period found no statistically significant evidence to support the alternative hypothesis of a low idiosyncratic volatility anomaly on the cross-sectional returns of the JSE. The findings do point towards low volatility portfolios generating higher returns in contrast to higher risk portfolios, but as the intercept results are insignificant at all acceptable confidence levels, the 12-month CAPM regression analysis fails to reject the null hypothesis that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 12-month volatility estimation period. As a result, further regression analysis with longer volatility estimation periods was required.

4.6.1.2 36-month volatility estimation period

Table 4.17: CAPM OLS regression results for 36-month volatility estimated excess monthly return portfolios against J203

Portfolio	Intercept (α)	βm	R-Squared	Standard Error	Annual Excess Return	Cost of Equity (Annual)
Q5 (high)	-0.0047	0.5198 ***	0.17879	0.05780	-5.630%	10.4224%
Q4	-0.0014	0.4961 ***	0.36088	0.03425	-1.684%	10.3557%
Q3	-0.0006	0.5368 ***	0.47006	0.02957	-0.775%	10.4702%
Q2	0.0019	0.5613 ***	0.51844	0.02807	2.315%	10.5392%
Q1 (low)	0.0016	0.3972 ***	0.46610	0.02205	1.917%	10.0775%
Differential	0.0062 *	-0.1223 **	0.01300	0.05530	7.440%	8.6164%

The results of Table 4.17 are adapted from the regression results reported in Appendix B Figure 5.

Examining the results of the high idiosyncratic volatility portfolio (Q5) shows that the intercept yielded an alpha value of -0.0047 which was not significant at any acceptable confidence level.

Multiplying the intercept by 12 provides an annual excess return figure of -5.63% which, although not significant, indicates the poor performance of the high volatility portfolio. Q5 therefore did not make a significant market risk-adjusted return over and above the risk-free rate. The beta for Q5 of 0.5198 has a corresponding p-value exceptionally close to 0, making the resulting value statistically significant at a 99% confidence level. Q5 therefore appears to be theoretically less volatile than the market, which was observed in the 12-month CAPM regression analysis. This is an interesting finding due to the perception of extreme volatility surrounding the high volatility portfolio. The R-squared value of 0.17879 is interpreted as 17.87% of change in Q5 being a direct result of and explained by the market. The standard error indicates the average distance the observed values fall from the regression line. Q5 generated a standard error of 5.78%, yielding the largest variance in datapoints. This results in approximately 95% of the datapoints falling between the regression line and +/- 11.56% of Q5, which is well over the 5% standard.

After examining the low idiosyncratic volatility portfolio (Q1) for the 36-month volatility estimation period, it can be seen to yield an intercept value of 0.0016, which is not statistically significant at any acceptable confidence level, as observed in the 12-month CAPM study. Modelling the alpha of Q1 over 12 months provides an annual excess return figure of 1.917%, which is substantially higher than that of Q5 but lower than that of Q2. This is interpreted as Q1 not providing a statistically significant market risk-adjusted return in excess of the risk-free rate. The beta for Q1 of 0.3972 is the lowest positive beta of all the volatility portfolios examined and is statistically significant at a 99% confidence level. This indicates that the low volatility portfolio is theoretically less volatile than all other quintile portfolios as well as the market. The R-squared value of 0.4661 indicates a change of 46.61% in Q1 occurring due to market changes. The standard error results for Q1 are substantially lower than the alternative portfolios. The results of the standard error for Q1 of 2.205% are that approximately 95% of the datapoints fall between the regression line and +/- 4.41% of Q1, which is below the 5% standard. This can be regarded as Q1 exhibiting a precise set of results.

The differential portfolio yielded an alpha of 0.0062, which is statistically significant at a 90% confidence level. Modelling the alpha of the differential portfolio over 12 months provides an annual excess return figure of 7.440%. This indicates the potential benefit and strength of the low idiosyncratic volatility portfolio in contrast to the high volatility portfolio. As the results are significant at a 90% confidence level, the annual excess returns of the differential portfolio are interpreted as providing a statistically significant market risk-adjusted return in excess of the risk-free rate. The beta of the differential portfolio of -0.1233 shows that the differential portfolio moved in the opposite direction to the market, as seen in the 12-month CAPM regression results and is statistically significant at a 99% confidence level. The R-squared value of 0.013 indicates a change of 1.3% in the differential portfolio due to market changes. The differential portfolio yielded a standard error of 5.53%, which accounts for approximately 95% of the datapoints falling between the regression line and +/- 11.06% of the differential portfolio.

The remaining portfolios, which are all insignificant at any acceptable confidence level, show that Q2 had the largest annual excess return of 2.315%, as seen in the 12-month CAPM regression. With respect to the portfolio betas, all remaining portfolios generated a statistically significant beta ranging around 0.496-0.561.

In conclusion, the OLS regression analysis conducted using a 36-month idiosyncratic volatility estimation period found no statistically significant evidence to support the alternative hypothesis of a low idiosyncratic volatility anomaly on the cross-sectional returns of the JSE. The findings do reveal that the differential portfolio generated a statistically significant alpha, which indicates the superior performance of the low volatility portfolio in contrast to the high idiosyncratic volatility portfolios. As the intercept results for both Q1 and Q5 are insignificant at all acceptable confidence levels, the 36-month CAPM regression analysis fails to reject the null hypothesis that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 36-month volatility estimation period. As a result, further regression analysis with the 60-month volatility estimation period was required.

4.6.1.3 60-month volatility estimation period

Table 4.18: CAPM OLS regression results for 60-month volatility estimated excess monthly return portfolios against J203

Portfolio	Intercept (α)	βm	R-Squared	Standard Error	Annual Excess Return	Cost of Equity (Annual)
Q5 (high)	-0.0004	0.4922 ***	0.18474	0.05440	-0.457%	10.2381%
Q4	0.0056 **	0.4791 ***	0.29174	0.03928	6.727%	10.1942%
Q3	0.0055 ***	0.4964 ***	0.47191	0.02763	6.591%	10.2525%
Q2	0.0091 ***	0.5541 ***	0.51028	0.02856	10.907%	10.4471%
Q1 (low)	0.0091 ***	0.4286 ***	0.46119	0.02438	10.897%	10.0237%
Differential	0.0098 ***	-0.0655	0.00445	0.05150	11.789%	8.3563%

The results of Table 4.18 are adapted from the regression results reported in Appendix B Figure 6.

Examining the results of the high idiosyncratic volatility portfolio (Q5) shows that the intercept yielded an alpha value of -0.0004, which was not significant at any acceptable confidence level. Multiplying the intercept by 12 provides an annual excess return figure of -0.457%. Based on the insignificant findings of the intercept value for Q5, it appears that Q5 did not make a significant market risk-adjusted return over and above the risk-free rate. The beta for Q5 of 0.4922 has a corresponding p-value exceptionally close to 0, making the resulting value statistically significant at a 99% confidence level. Q5 therefore appears to be theoretically less volatile than the market, which was observed in the 12- and 36-month CAPM regression analyses. The R-squared value of 0.18474 is interpreted as 18.47% of change in Q5 being a direct result of and explained by the market. The standard error for Q5 indicates the average distance the observed datapoints fall from the regression line. Q5 generated a standard error of 5.44%, yielding the largest variance in datapoints of all six portfolios. This results in approximately 95% of the datapoints falling between the regression line and +/- 10.88% of Q5, which is well over the 5% standard.

After examining the low idiosyncratic volatility portfolio (Q1) for the 60-month volatility estimation period, it can be seen to yield an intercept value of 0.0091, which is statistically significant at a 99% confidence level. This result provides the first statistically significant alpha for the CAPM regression analysis for the low volatility portfolio. Modelling the alpha of Q1 over 12 months provides an annual excess return figure of 10.897%. This is substantially higher than that of the high idiosyncratic volatility portfolio and is interpreted as Q1 providing a statistically significant market risk-adjusted return in excess of the risk-free rate and high-risk portfolios. The beta for Q1 of 0.4286 is the lowest positive beta of all the volatility portfolios examined and is statistically significant at a 99% confidence level. This indicates that the low volatility portfolio is theoretically less volatile than all other quintile portfolios as well as the market. This result is in line with the 12- and 36-month CAPM regression results. The R-squared value of 0.4612 indicates a change of 46.12% in Q1 occurring due to market changes. The standard error results for Q1 are substantially lower than the alternative portfolios. The standard error for Q1 of 2.438% shows that approximately 95% of the datapoints fall between the regression line and +/- 4.88% of Q1, which is below the 5% standard, and can be regarded as Q1 exhibiting a precise set of results.

The differential portfolio yielded an alpha of 0.0098, which is statistically significant at a 99% confidence level. Modelling the alpha of the differential portfolio over 12 months provides an annual excess return figure of 11.789%. This indicates the potential benefit and strength of the low idiosyncratic volatility portfolio in contrast to the high volatility portfolio. As the results are significant at a 99% confidence level, the annual excess returns of the differential portfolio are interpreted as providing a statistically significant market risk-adjusted return in excess of the risk-free rate. The beta of the differential portfolio of -0.0655 shows that the differential portfolio moved in the opposite direction to the market, as seen in the 12-month CAPM regression results and is statistically significant at a 99% confidence level. The R-squared value of 0.0045 indicates a change of 0.45% in the differential portfolio due to market changes. The differential portfolio yielded a standard error of 5.15%, which accounts for approximately 95% of the datapoints falling between the regression line and +/- 10.30% of the differential portfolio.

The remaining portfolios, except for Q4 which is only significant at a 95% confidence level, are all significant at a 99% confidence level. The results illustrate clear dominance in the performance of the low idiosyncratic volatility portfolios as Q1 and Q2 significantly outperformed the remaining higher risk portfolios of Q3, Q4 and Q5. Q2 provided the highest annual excess return of the five risk portfolios, at 10.91%.

In conclusion, the CAPM OLS regression analysis conducted using a 60-month idiosyncratic volatility estimation period found statistically significant evidence to support the alternative hypothesis of a low idiosyncratic volatility anomaly on the cross-sectional returns on the JSE. These findings are supported by a statistically significant alpha for five of the six portfolios examined and clearly indicate the superior performance of the low volatility portfolio in contrast to that of the high idiosyncratic volatility portfolios. These findings of the 60-month CAPM regression analysis provide clear evidence of a low idiosyncratic volatility anomaly and reject the null hypothesis that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 60-month volatility estimation period. To confirm these findings accurately and to eliminate any potential bias as a result of size or value effects, the Fama and French regression analysis was conducted.

4.6.2 Fama and French OLS Regression Analysis

The Fama and French regression analysis was modelled according to Equation (8) below:

$$\mathrm{E}(R_i) = \alpha_i + \beta_{mi}\,(\mathrm{MRK}) + \beta_{si}\,(\mathrm{SMB}) + \beta_{vi}\,(\mathrm{HML}) \tag{8}$$

4.6.2.1 12-month volatility estimation period

Table 4.19: Fama and French OLS regression results for 12-month volatility estimated excess monthly return portfolios against J203

Portfolio	Intercept (α)	β_{MRK}	β_{SMB}	β_{HML}	Adjusted R-Squared	Standard Error	Annual Excess Return	Cost of Equity (Annual)
Q5 (high)	-0.0004	0.5709 ***	0.0559 *	-0.08013	0.1962	0.0584	-0.5099%	10.3717%
Q4	0.0016	0.4961 ***	0.0037	-0.06735	0.3531	0.0340	1.9553%	9.0635%
Q3	0.0008	0.5632 ***	0.0131	0.00593	0.4364	0.0324	0.9870%	11.2588%
Q2	0.0008	0.5593 ***	-0.0030	0.06250	0.5318	0.0268	0.9961%	12.1624%
Q1 (low)	0.0004	0.3433 ***	-0.0001	0.02817	0.4180	0.0206	0.5291%	10.7525%
Differential	0.0009	-0.2275 ***	-0.0560 **	0.10830	0.0464	0.0539	1.0390%	8.9731%

The results of Table 4.19 are adapted from the regression results reported in Appendix C Figure 7.

The results of the high idiosyncratic volatility portfolio (Q5) show that the intercept yielded an alpha value of -0.0004, which is not significant at any acceptable confidence level. Multiplying the intercept by 12 provides an annual excess return figure of -0.5099%. Based on the insignificant findings of the intercept value for Q5, Q5 did not make a significant risk-adjusted return over and above the risk-free rate. This follows the result of the CAPM 12-month regression analysis. The market risk premium beta (β_{MRK}) of 0.5709 for Q5 has a corresponding p-value exceptionally close to 0, making the resulting value statistically significant at a 99% confidence level. Q5 therefore appears to be theoretically less volatile than the market and has a significant positive risk exposure to the market. The size beta (β_{SMB}) of 0.0559 for Q5 is statistically significant at a 90% confidence level. This indicates that the performance of a large portion of the risky shares which form Q5 may be a result of the small firm risk factor. Next, the firm-value beta (β_{HML}) of -0.08013 is not significant at any acceptable confidence level. This shows that Q5 had no significant exposure to the firm-value risk factor. The adjusted R-squared value of 0.1962 is interpreted as 19.62% of change in Q5 being a direct result of and explained by the three independent variables included in the regression analysis. As the MRK and SMB factors are the only statistically significant risk factors for Q5, these two risk factors have the most significant influence over the value of Q5.

The standard error for Q5 indicates the average distance the observed datapoints fall from the regression line. Q5 generated a standard error of 5.84%, yielding the largest variance in datapoints of all six portfolios. This results in approximately 95% of the datapoints falling between the regression line and +/- 11.68% of Q5, which is well over the 5% standard.

The low idiosyncratic volatility portfolio (Q1) for the 12-month Fama and French regression analysis yielded an intercept value of 0.0004, which is not statistically significant at any acceptable confidence level. Modelling the alpha of Q1 over 12 months provides an annual excess return figure of 0.5291%, which is higher than that of Q5 but lower than the remaining portfolios. This means that Q1 did not yield a statistically significant risk-adjusted return in excess of the risk-free rate. The market risk premium beta (β_{MRK}) of 0.3433 for Q1 is the lowest positive beta of all the volatility portfolios examined and is statistically significant at a 99% confidence level. This indicates that the low volatility portfolio is theoretically less volatile than all other quintile portfolios as well as the market. The size beta (β_{SMB}) of -0.0001 for Q1 is not significant at any acceptable confidence level. This indicates that the alpha experienced by Q1 had no significant direct stimulus from the small firm risk factor. Next, the firm-value beta (β_{HML}) of -0.0282 is not significant at any acceptable confidence level. This shows that Q1 had no significant exposure to the firm-value risk factor. The adjusted R-squared value of 0.4180 indicates a change of 41.80% in Q1 occurring as a result of the three independent variables included in the regression analysis. The standard error results for Q1 are lower than those of the alternative portfolios. The standard error for Q1 of 2.06% reveals that approximately 95% of the datapoints fall between the regression line and +/- 4.12% of Q1, which is below the 5% standard, and can be regarded as Q1 exhibiting a precise set of results.

The differential portfolio, constructed taking a long position in Q1 and a simultaneous short position in Q5, yielded an alpha of 0.0009, which is not statistically significant at any acceptable confidence level. Modelling the alpha of the differential portfolio over 12 months provides an annual excess return figure of 1.039%. This indicates the potential benefit and strength of the low idiosyncratic volatility portfolio in contrast to the high volatility portfolio. As the results are not significant at any appropriate confidence level, the annual excess returns of the differential portfolio are interpreted as not providing a statistically significant risk-adjusted return in excess of the risk-free rate. The market risk premium beta (β_{MRK}) of -0.2275 for the differential portfolio shows that the differential portfolio moved in the opposite direction to the market and is statistically significant at a 99% confidence level. The size beta (β_{SMB}) of -0.056 for the differential portfolio is statistically significant at a 95% confidence level. This indicates that the alpha experienced by the differential portfolio had a significant direct influence from the small firm risk factor, which may potentially explain the superior returns experienced and might not be as a result of a low volatility anomaly. Next, the firm-value beta (β_{HML}) of 0.1083 is not significant at any acceptable confidence level, but it is the only risk factor to which the differential portfolio is positively related. This shows that the differential portfolio had no significant positive exposure to the firm-value risk factor. The adjusted R-squared value of 0.0464 indicates a change of 4.64% in the differential portfolio due to changes in the three independent variable risk factors. The differential portfolio yielded a standard error of 5.39%, which accounts for approximately 95% of the datapoints falling between the regression line and +/- 10.78% of the differential portfolio.

The remaining portfolios are all insignificant at any conventional confidence level. However, the results illustrate a clear market risk premium effect as all portfolios have a statistically significant market-risk premium beta.

The results do not show that low volatility portfolios tend to outperform high volatility portfolios at any significant level. The 12-month Fama and French regression analysis therefore fails to reject the null hypothesis as there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 12-month volatility estimation period.

4.6.2.2 36-month volatility estimation period

Table 4.20: Fama and French OLS regression results for 36-month volatility estimated excess monthly return portfolios against J203

Portfolio	Intercept (α)	β_{MRK}	β_{SMB}	β_{HML}	Adjusted R-Squared	Standard Error	Annual Excess Return	Cost of Equity (Annual)
Q5 (high)	-0.0062	0.5291 ***	0.0529 *	0.01777	0.1801	0.0577	-7.4321%	12.2245%
Q4	-0.0012	0.5006 ***	0.0218	-0.03092	0.3583	0.0343	-1.4889%	10.1604%
Q3	-0.0041	0.5353 ***	0.0074	0.16297 *	0.4704	0.0295	-4.9664%	14.6614%
Q2	-0.0001	0.5585 ***	-0.0053	0.10647	0.5162	0.0281	-0.1625%	13.0168%
Q1 (low)	0.0002	0.3941 ***	-0.0096	0.07812	0.4646	0.0220	0.2468%	11.7474%
Differential	0.0063	-0.1348 **	-0.0625 **	0.06419	0.0214	0.0550	7.5015%	8.5790%

The results of Table 4.20 are adapted from the regression results reported in Appendix C Figure 8.

After examining the results of the high idiosyncratic volatility portfolio (Q5), it can be seen that the intercept yielded an alpha value of -0.0062, which is not significant at any acceptable confidence level. Multiplying the intercept by 12 provides an annual excess return figure of -7.4321%. Based on the insignificant findings of the intercept value for Q5, it can be concluded that Q5 did not make a significant risk-adjusted return over and above the risk-free rate, which follows the result of the CAPM 36-month regression analysis and Fama and French 12-month regression analysis. The market risk premium beta (β_{MRK}) of 0.5291 for Q5 has a corresponding p-value exceptionally close to 0, making the resulting value statistically significant at a 99% confidence level. This shows that Q5 appears to be theoretically less volatile than the market and has significant positive risk exposure to the market. The size beta (β_{SMB}) of 0.0529 for Q5 is statistically significant at a 90% confidence level. This indicates that the performance of a large portion of the risky shares which form Q5 may be a result of the small firm risk factor. Next, the firm-value beta (β_{HML}) of 0.0178 is not significant at any acceptable confidence level. This is interpreted as Q5 having no significant exposure to the firm-value risk factor. The adjusted R-squared value of 0.1801 is interpreted as 18.01% of change in Q5 being a direct result of and explained by the three independent variables included in the regression analysis. The standard error for Q5 indicates the average distance the observed datapoints fall from the regression line. Q5 generated a standard error of 5.77%, yielding the largest variance in datapoints of all six portfolios. This results in approximately 95% of the datapoints falling between the regression line and +/- 11.54% of Q5, which is well over the 5% standard.

On examining the low idiosyncratic volatility portfolio (Q1) for the 36-month Fama and French regression analysis, it was found to yield an intercept value of 0.0002, which is not statistically significant at any acceptable confidence level. Modelling the alpha of Q1 over 12 months provides an annual excess return figure of 0.2468%. This is the only positive return figure of the remaining volatility portfolios (excluding the differential portfolio), and is interpreted as Q1 not providing a statistically significant risk-adjusted return in excess of the risk-free rate. The market risk premium beta (β_{MRK}) of 0.3941 for Q1 is the lowest positive beta of all the volatility portfolios examined and is statistically significant at a 99% confidence level. This indicates that the low volatility portfolio is theoretically less volatile than all other quintile portfolios as well as the market. The size beta (β_{SMB}) of -0.0096 for Q1 is not significant at any acceptable confidence level. This indicates that the alpha experienced by Q1 had no significant direct stimulus from the small firm risk factor. The firm-value beta (β_{HML}) of 0.0781 is not significant at any acceptable confidence level. This is interpreted as Q1 having no significant exposure to the firm-value risk factor. The adjusted R-squared value of 0.4646 indicates a change of 46.46% in Q1 occurring as a result of the three independent variables included in the regression analysis. The standard error results for Q1 are lower than those of the alternative portfolios. The standard error for Q1 of 2.20% shows that approximately 95% of the datapoints fall between the regression line and +/- 4.40% of Q1, which is below the 5% standard, and can be regarded as Q1 exhibiting a precise set of results.

The differential portfolio for the 36-month volatility regression analysis yielded an alpha of 0.0063, which is not statistically significant at any acceptable confidence level. Modelling the alpha of the differential portfolio over 12 months provides an annual excess return figure of 7.50%. This indicates the potential benefit and strength of the low idiosyncratic volatility portfolio in contrast to the high volatility portfolio. As the results are not significant at any appropriate confidence level, the annual excess returns of the differential portfolio are interpreted as not providing a statistically significant risk-adjusted return in excess of the risk-free rate. The market risk premium beta (β_{MRK}) of -0.1348 for the differential portfolio shows that the differential portfolio moved in the opposite direction to the market and is statistically significant at a 95% confidence level. The size beta (β_{SMB}) of -0.0625 for the differential portfolio is statistically significant at a 95% confidence level. This indicates that the alpha experienced by the differential portfolio had a significant direct influence from the small firm risk factor, which may potentially explain the superior returns experienced and might not be as a result of a low volatility anomaly. Next, the firm-value beta (β_{HML}) of 0.06419 is not significant at any acceptable confidence level, but it is the only risk factor to which the differential portfolio is positively related. This result is observed in the 12-month Fama and French regression analysis and provides further evidence of an insignificant positive firm-value beta, which indicates that the differential portfolio had no significant positive exposure to the firm-value risk factor. The adjusted R-squared value of 0.0214 indicates a change of 2.14% in the differential portfolio due to changes in the three independent variable risk factors. The differential portfolio yielded a standard error of 5.50%, which accounts for approximately 95% of the datapoints falling between the regression line and +/- 11.00% of the differential portfolio.

After analysing the remaining portfolios, it was found that Q3 exhibited the first observable significant firm-value beta (β_{HML}) of 0.1640, which is significant at a 90% confidence level. This finding is interpreted as Q3 having significant positive exposure to the firm-value risk factor, which may be an explanatory factor for the excess returns of Q3.

The Fama and French 36-month regression analysis found no evidence to support the alternative hypothesis that there is a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE, after estimating volatility utilising a 36-month volatility estimation period. Further analysis was required utilising the Fama and French 60-month volatility regression analysis to determine if a low volatility anomaly is present on the JSE.

4.6.2.3 60-month volatility estimation period

Table 4.21: Fama and French OLS regression results for 60-month volatility estimated excess monthly return portfolios against J203

Portfolio	Intercept (α)	β_{MRK}	β_{SMB}	β_{HML}	Adjusted R-Squared	Standard Error	Annual Excess Return	Cost of Equity (Annual)
Q5 (high)	0.0010	0.5015 ***	0.0373	-0.1060	0.1828	0.0544	1.1742%	8.6066%
Q4	0.0029	0.4812 ***	0.0201	0.1080	0.2883	0.0393	3.5246%	13.3960%
Q3	-0.0011	0.4922 ***	0.0070	0.3125 ***	0.4892	0.0271	-1.3152%	18.1591%
Q2	0.0048 *	0.5481 ***	-0.0100	0.2205 **	0.5161	0.0283	5.7213%	15.6324%
Q1 (low)	0.0054 **	0.4220 ***	-0.0158	0.1959 **	0.4712	0.0241	6.4739%	14.4468%
Differential	0.0052	-0.0812	-0.0533 **	0.2818 *	0.0211	0.0510	6.2400%	13.9050%

The results of Table 4.21 are adapted from the regression results reported in Appendix C Figure 9.

Examining the results of the high idiosyncratic volatility portfolio (Q5) shows that the intercept yielded an alpha value of 0.0010, which is not significant at any acceptable confidence level. Multiplying the intercept by 12 provides an annual excess return figure of 1.1742%. Based on the insignificant findings of the intercept value for Q5, it can be concluded that Q5 did not make a significant risk-adjusted return over and above the risk-free rate, which is the same observed result in all the regression tests conducted. The market-risk premium beta (β_{MRK}) for Q5 of 0.5015 has a corresponding p-value exceptionally close to 0, making the resulting value statistically significant at a 99% confidence level. This indicates that Q5 appears to be theoretically less volatile than the market and has significant positive risk exposure to the market. The size beta (β_{SMB}) 0.0373 for Q5 of is not statistically significant at any conventional confidence level. This indicates that the high idiosyncratic volatility portfolio may be heavily weighted towards large companies with high market capitalisation rates. Next, the firm-value beta (β_{HML}) of -0.1060 is not significant at any acceptable confidence level. This is interpreted as Q5 having no significant exposure to the firm-value risk factor. The adjusted R-squared value of 0.1828 is interpreted as 18.28% of change in Q5 being a direct result of and explained by the three independent variables included in the regression analysis. The standard error for

Q5 indicates the average distance the observed datapoints fall from the regression line. Q5 generated a standard error of 5.44%, yielding the largest variance in datapoints of all six portfolios. This results in approximately 95% of the datapoints falling between the regression line and +/- 10.88% of Q5, which is well over the 5% standard.

Examining the low idiosyncratic volatility portfolio (Q1) for the 60-month Fama and French regression analysis shows that the low volatility portfolio yielded an intercept value of 0.0054, which is statistically significant at a 95% confidence level. Modelling the alpha of Q1 over 12 months provides an annual excess return figure of 6.4739%. This is interpreted as Q1 providing a statistically significant risk-adjusted return in excess of the risk-free rate. Furthermore, this provides conclusive evidence that although the SMB and HML value risk factors were introduced, a statistically significant alpha remained for the low volatility portfolio, highlighting the presence of a low volatility anomaly which is not a result of the risk factors included. The market risk premium beta (β_{MRK}) for Q1 of 0.4220 is the lowest positive beta of all the volatility portfolios examined and is statistically significant at a 99% confidence level. This indicates that the low volatility portfolio is theoretically less volatile than all other quintile portfolios as well as the market. The size beta (β_{SMB}) of -0.0158 for Q1 is not significant at any acceptable confidence level. This indicates that the alpha experienced by Q1 had no significant direct stimulus from the small firm risk factor. The firm-value beta (β_{HML}) of 0.1958 is statistically significant at a 95% confidence level. This shows that Q1 had significant exposure to the firm-value risk factor. The adjusted R-squared value of 0.4712 indicates a change of 47.12% in Q1 occurring as a result of the three independent variables included in the regression analysis. Q1 had a lower standard error result than the alternative portfolios. The standard error of 2.41% for Q1 shows that approximately 95% of the datapoints fall between the regression line and +/- 4.82% of Q1, which is below the 5% standard, and can be regarded as Q1 exhibiting a precise set of results. Overall, these results for Q1 provide statistically significant evidence supporting the findings of the 60-month CAPM regression analysis, namely that the low idiosyncratic volatility portfolio outperformed the high volatility portfolios, indicating the presence of a low volatility anomaly on the cross-sectional returns.

The differential portfolio for the 60-month Fama and French volatility regression analysis yielded an alpha of 0.0052, which is not statistically significant at any acceptable confidence level. Modelling the alpha of the differential portfolio over 12 months provides an annual excess return figure of 6.24%. This figure, although lower than the return yielded by Q1, indicates the potential benefit and strength of the low idiosyncratic volatility portfolio in contrast to the high volatility portfolio. As the results are not significant at any appropriate confidence level, the annual excess returns of the differential portfolio are interpreted as not providing a statistically significant risk-adjusted return in excess of the risk-free rate. The market risk premium beta (β_{MRK}) of -0.0812 for the differential portfolio indicates that the differential portfolio moved in the opposite direction to the market and is not statistically significant at any acceptable confidence level. The size beta (β_{SMB}) of -0.0533 for the differential portfolio is statistically significant at a 95% confidence level. This indicates that the alpha experienced by the differential portfolio had a significant direct influence from the small firm risk factor. Next, the firm-value beta (β_{HML}) of 0.2818 is significant at a 90% confidence level. This result was observed for the first time in all the regression analyses conducted and shows that the differential portfolio had significant positive exposure to the firm-value risk factor when utilising a 60-month volatility estimation period. The adjusted R-squared value of 0.0211 indicates a change of 2.11% in the differential portfolio due to changes in the three independent variable risk factors. The differential portfolio yielded a standard error of 5.10%, which accounts for approximately 95% of the datapoints falling between the regression line and +/- 10.20% of the differential portfolio.

After analysing the remaining portfolios, Q1-Q3 were found to exhibit positive and significant factor loadings to the firm-value beta (β_{HML}). This shows that the low volatility portfolios (Q1-Q3) had significant positive exposure to the firm-value risk factor, which may be an explanatory factor for the excess returns of the low risk portfolios when utilising a 60-month volatility estimation period. Furthermore, only Q1 and Q2 have a positive and significant alpha for the 60-month Fama and French regression analysis. These findings indicate a statistically significant set of results, which highlight excess returns on the low volatility portfolios after controlling for market, size and value effects.

The results of the Fama and French 60-month regression analysis provide statistically significant evidence to support the alternative hypothesis that there is a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE, after estimating volatility utilising a 60-month volatility estimation period. These findings are in line with the CAPM 60-month regression results which provided significant evidence of a low idiosyncratic volatility anomaly and a robust set of results indicating that when utilising a 60-month volatility estimation period, there is a low idiosyncratic volatility premium on the cross-sectional returns on the JSE. These clear and robust results permit rejection of the null hypothesis.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Summary of Findings

The primary research objective of this study was to determine if a low idiosyncratic volatility premium is present on the cross-section of share returns of the JSE. A 26-year sample period from January 1994 to December 2019 was employed. Furthermore, three key volatility estimation periods were analysed to ensure that results were robust and to replicate the study methodologies of Ang *et al*. (2006), Diether *et al*. (2002), Oladele and Bradfield (2016, 2018) and Xiong *et al*. (2014), as these authors all conducted their analyses across either a 12-, 36- or 60-month volatility estimation period. The risk-weighted portfolios were classified into quintile portfolios based on their individual shares' prevailing 12-, 36- and 60-month historical return volatilities. Q1 held the prevailing period's lowest volatility share returns and Q5 the highest. Once the risk-based portfolios were formed, tests on the quality of the data were conducted utilising Tukey's EDA approach (1977). These descriptive tests provided insight into the distribution of datapoints, skewness of data and potential levels of kurtosis. Following the descriptive analysis, a series of empirical tests and regression analyses were conducted to determine the performance and risk-reward relationship each quintile produced over the sample period. The regression analyses were conducted utilising CAPM as a measure of volatility, and the size and value risk factors associated with the Fama and French 3-factor model were introduced. The regression results provided insight into the extent of an abnormal alpha as well as potential implications that market, size and value effects could have for the return distributions. Finally, the empirical analyses examined the effects of an alternative risk metric on the volatility quintiles to provide insight into and identify potential explanatory factors for the effects of a low idiosyncratic volatility anomaly.

The analysis commenced by determining the key metrics and statistical performance results for the 12-month idiosyncratic volatility period. Q2 yielded the highest average monthly excess return of 0.3452% over the sample period, whereas the low idiosyncratic volatility anomaly generated a mediocre 0.1888% excess return. In contrast, the high idiosyncratic volatility portfolio performed poorly over the sample period and generated the lowest excess return of

0.0705%. After analysing the risk metrics for the 12-month excess returns, portfolio standard deviation and the two risk-adjusted measures performed, the Sharpe and Treynor ratios were calculated. The standard deviation results show that the low idiosyncratic volatility portfolio yielded the lowest levels of risk over the sample period. In contrast, the high idiosyncratic volatility portfolio generated the highest level of risk over the sample period. On examining the Sharpe ratio (reward-to-volatility), it was apparent that the lowest idiosyncratic risk portfolio had a reward per unit of volatility of 0.06988. In spite of this, Q2 yielded the highest risk-return ratio of 0.08810, proving to be the best-performing portfolio over the sample period. The opposing portfolio, Q5, generated the lowest risk-return ratio of all the portfolios. The Treynor ratio results were inconclusive as all the portfolios exhibited similar values, with no significant preference for any specific risky portfolio. In conclusion, the 12-month results, although insignificant at all appropriate confidence intervals, indicate the presence of a low idiosyncratic risk anomaly on the cross-sectional returns on the JSE, when examining the risk-adjusted excess returns.

Next, the 12-month VaR and CVaR results were examined. Q5 generated the highest negative VaR returns for the 12-month volatility estimation period for all confidence levels examined. The CVaR results for Q5 were that the portfolio exhibited the highest negative CVaR values for all confidence levels, which is interpreted as Q5 losing an average -14.96% to -91.57% of portfolio returns at the respective confidence levels. In contrast, Q1 generated the lowest negative VaR values for the respective confidence levels, indicating that there is a 95%, 99% and 99.9% confidence that Q1 will not lose more than -2.91%, -4.90% and -15.77%, respectively. The CVaR results for Q1 suggest that after accounting for the worst 5%, 1% and 0.01% of cases, Q1 will lose an average of -5.07%, -11.24% and -50.54%, respectively. These sharp negative returns could be an indication of the poor estimation performance of volatility and the significance of an alternative risk metric which could be an explanatory factor for the presence of a low volatility anomaly.

Progressing to the 12-month cumulative returns results, it was found that if an investor initially invested ZAR1.00 in the highest idiosyncratic risk portfolio, and continually rebalanced the portfolio on an annual basis, the investor would achieve capital gains to the value of ZAR6.61, resulting in a compound annual growth rate of return of 6.86%. In comparison, an investor investing ZAR1.00 in the lowest idiosyncratic volatility portfolio would achieve capital gains to the value of ZAR16.58. The resulting compound annual growth rate of return is equivalent to 11.14%. Q5 significantly underperformed in contrast to Q1 on a cumulative returns basis. This suggests evidence of a low idiosyncratic risk anomaly on the cross-sectional returns on the JSE, which further indicates that the JSE is not efficient according to MPT and the findings of Markowitz.

The CAPM OLS regression results utilising a 12-month idiosyncratic volatility estimation period revealed no statistically significant evidence to support the alternative hypothesis of a low idiosyncratic volatility anomaly on the cross-sectional returns of the JSE. The findings do point towards low volatility portfolios generating higher returns than higher risk portfolios, but as the intercept results are insignificant at all acceptable confidence levels, the 12-month CAPM regression analysis fails to reject the null hypothesis that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 12-month volatility estimation period. The Fama and French OLS regression results did not show that low volatility portfolios tend to outperform high volatility portfolios at any significant level. The 12-month Fama and French regression analysis therefore failed to reject the null hypothesis as there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 12-month volatility estimation period.

Moving on to the 36-month volatility analysis, the key metrics and statistical performance results reveal that the differential portfolio yielded the best average monthly excess return of 0.5933% at a 90% significance level. The result of the differential portfolio's supreme performance can be appreciated by recognising the strong performance of the low idiosyncratic volatility portfolios relative to the negative average excess returns generated by the high idiosyncratic volatility portfolios. On analysing the risk and risk-adjusted returns, it was found that the low volatility portfolio generated the lowest levels of risk over the sample period, with a standard deviation of 0.0301, whereas the high idiosyncratic volatility portfolio generated the highest standard deviation of 0.0064. Next, on analysing the risk-return models of Sharpe and Treynor, it was found that Q1 and Q2 generated substantially better risk-adjusted returns than their higher risk counterparts. In conclusion, the results (although insignificant for five out of six datasets at appropriate confidence intervals) indicate the presence of a low idiosyncratic risk anomaly, when examining the risk-adjusted excess returns with a 36-month volatility estimation period.

Next the 36-month VaR and CVaR results were examined. Q5 generated the highest negative VaR returns for the 36-month volatility estimation period for all confidence levels. Q5 also generated the highest standard deviation of 6.36%, which results from the portfolio construction process described in the methodology of the study. The CVaR results for Q5 show that the portfolio exhibited the highest negative CVaR values for all confidence levels. Q1 generated the lowest negative VaR values for all the respective confidence levels. The corresponding standard deviation of Q1 was the lowest in returns of 2.99%. The CVaR results for Q1 suggest that after accounting for the worst 5%, 1% and 0.01% of cases, Q1 will lose an average of -5.81%, -11.64% and -55.54%, respectively. The results highlight the reduced losses expected for the low idiosyncratic volatility portfolio. This provides supporting evidence of the low risk associated with Q1 as all three risk measures illustrate the depressed variability in returns and limited left side tail risk.

Progressing to the 36-month cumulative returns results, it was found that if an investor initially invested ZAR1.00 in the highest idiosyncratic risk portfolio, and continually rebalanced the portfolio on an annual basis, the investor would have achieved capital gains to the value of ZAR1.45, resulting in a compound annual growth rate of return of -3.20%. In comparison, an investor investing ZAR1.00 in the lowest idiosyncratic volatility portfolio would have achieved capital gains to the value of ZAR15.74. The resulting compound annual growth rate of return is equivalent to 11.45%. Q5 therefore significantly underperformed in contrast to Q1 on a cumulative returns basis. This suggests evidence of a low idiosyncratic risk anomaly on the cross-sectional returns on the JSE.

Examining the CAPM OLS regression results utilising a 36-month idiosyncratic volatility estimation period revealed no statistically significant evidence to support the alternative hypothesis of a low idiosyncratic volatility anomaly on the cross-sectional returns of the JSE. The findings did establish that the differential portfolio generated a statistically significant alpha, which indicates the superior performance of the low volatility portfolio in contrast to the high idiosyncratic volatility portfolios. As the intercept results for both Q1 and Q5 were insignificant at all acceptable confidence levels, the 36-month CAPM regression analysis failed to reject the null hypothesis that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 36-month volatility estimation period. The Fama and French OLS regression results show no evidence to support the alternative hypothesis that there is a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE, after estimating volatility utilising a 36-month volatility estimation period.

Finally, after examining the 60-month volatility estimation period, it was found that the average excess returns for Q2 and Q1 yielded the highest monthly excess returns of 1.065% and 1.029%, respectively. The worst performer over the sample period was Q5, which yielded a monthly excess return of only 0.1003%. The differential portfolio generated a positive return over the sample period in addition to yielding the highest maximum excess return and the lowest minimum return over the period. The risk and risk-adjusted results indicate that the low idiosyncratic volatility portfolio generated the lowest levels of risk over the sample period with a standard deviation of 0.033. In contrast, the high idiosyncratic volatility portfolio generated the highest level of risk over the sample period yielding a standard deviation of 0.060. After examining the Sharpe and Treynor ratios, it is apparent that the lowest idiosyncratic risk portfolio had a reward to risk ratio of 0.3103 and 0.0239, respectively, proving to be the best risk-adjusted performing portfolio over the sample period. The opposing portfolio, Q5, generated the lowest risk-return result of all the portfolios, with Sharpe and Treynor ratios of a mere 0.0167 and 0.0020, respectively. In conclusion, the results provide significant evidence of the presence of a low idiosyncratic volatility on the cross-sectional returns on the JSE anomaly at a 99% confidence interval.

The 60-month VaR and CVaR results conflicted with the 12- and 36-month VaR and CVaR results in numerous areas. It can be concluded that an alternative left tail risk measure that can accurately predict the downside risk of a portfolio may be a better measure and provide more accurate results in comparison to the conventional use of volatility. The CVaR and VaR provided inconclusive results which did not follow the expected results for the volatility portfolios. As a result, further analysis of the presence of a volatility anomaly will need to be done according to portfolios categorised based on each share’s underlying CVaR to determine if the anomaly holds true under a different risk measurement methodology.

Progressing to the 60-month cumulative returns results, it was found that if an investor initially invested ZAR1.00 in the highest idiosyncratic risk portfolio, and continually rebalanced the portfolio on an annual basis, the investor would have achieved capital gains to the value of ZAR1.45, resulting in a compound annual growth rate of return of 3.50%. In comparison, an investor investing ZAR1.00 in the lowest idiosyncratic volatility portfolio would achieve capital gains to the value of ZAR13.34. The resulting compound annual growth rate of return is equivalent to 11.63%. Q5 therefore significantly underperformed Q1 on a cumulative returns basis. These findings suggest clear evidence of a low idiosyncratic risk anomaly on the cross-sectional returns on the JSE.

Examining the CAPM OLS regression results utilising a 60-month idiosyncratic volatility estimation period produced statistically significant evidence to support the alternative hypothesis of a low idiosyncratic volatility anomaly on the cross-sectional returns on the JSE.

These findings are supported by a statistically significant alpha for five of the six portfolios examined and clearly indicate the superior performance of the low volatility portfolio in contrast to the high idiosyncratic volatility portfolios. These findings provide clear evidence of a low idiosyncratic volatility anomaly and reject the null hypothesis that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 60-month volatility estimation period. The Fama and French OLS regression results provided evidence to support the alternative hypothesis that there is a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE, after estimating volatility utilising a 60-month volatility estimation period. These findings are in line with the CAPM 60-month regression results which provided significant evidence of a low idiosyncratic volatility anomaly and a robust set of results. When utilising a 60-month volatility estimation period, a low idiosyncratic volatility premium can be found on the cross-sectional returns on the JSE. These clear and robust results permit the null hypothesis to be rejected.

5.2 Contribution of the Study

This study contributes to literature in several ways.

1. This study demonstrates the presence of a low idiosyncratic volatility anomaly on the cross-section of returns of all JSE-listed firms over a 24-year sample period. Shares which have low idiosyncratic volatilities tend to exhibit significantly higher risk-adjusted returns than the market portfolio.
2. The inclusion of size and value effects in the Fama and French regression analysis provides a robust set of results, removing any bias towards alternative risk factors contributing to the effects demonstrated.
3. The study followed a 12-, 36- and 60-month idiosyncratic volatility estimation period to provide further insight into the effects of short- and intermediate-term volatility estimation on expected future returns.

4. The study investigated if a low systematic volatility (market beta) effect is also present on the cross-section of returns of all shares listed on the JSE over the sample period.
5. The study provides a variety of potential explanations for the anomalous relationship between low idiosyncratic volatility share returns and their high idiosyncratic counterparts.

The study complements the work of Ang *et al*. (2006) by providing a new estimation period of 60 months in addition to the fundamental structure of Ang *et al*. The purpose of this addition was to test the research objective of whether the window period applied to estimating idiosyncratic volatility impacts the low volatility premium. The study finds clear significant evidence to support the notion of a relationship between volatility estimation and idiosyncratic volatility estimation.

Diether *et al*. (2002) found that shares with elevated variability in analysts' forecasts yield significantly lower returns than similar shares. The findings of their study were more evident in small and historically poor-performing shares with a 12-month look-back period. The results of the study described in this thesis may provide a supplementary view to the findings of Diether *et al*. (2002), as the results may not only be limited to small shares, once 36- and 60-month estimation periods are introduced.

Finally, this study complements the work of Xiong *et al*. (2014) by leveraging the 60-month volatility estimation period they utilised and conducting a low volatility anomaly study on an emerging market. Xiong *et al*. focused primarily on a global perspective in analysing the low volatility anomaly by using index funds such as the Morningstar's open-end equity mutual fund. As a result, this study supports the findings of Xiong *et al*. from a South African perspective.

5.3 Limitations of the Study

A small sample size was used, as the JSE has on average listed 389 companies for the past 10 years, with an all-time high of 485 listed companies in September 2002 (Ceicdata, n.d., South Africa …). In contrast, studies by Campbell *et al.* (2001) and Ang *et al.* (2006) dealt with the effects of a low volatility anomaly across the NYSE, AMEX and Nasdaq, which have held approximately 2 800, 1 700 and 3 300 listings over the past 10 years (Ceicdata, n.d., United States …). The significant difference in size between the exchanges in the US and South Africa can indicate that the JSE is relatively illiquid in contrast to more developed market exchanges. Furthermore, availability of data to compute the Fama and French 3-factor model as a method to measure volatility and conduct time-series attribution regression tests may limit the computation of the measure in the study. Lastly, the available results are only relevant to a South African sample and are not representative of an African perspective.

5.4 Further Areas of Study

The subject matter of the low volatility anomaly has been around since it was first identified in 1972, when Fischer Black published his study "Capital market equilibrium with restricted borrowing", later that year inspired Haugen and Heins (1972) to draft a working paper on risk and the rate of return on financial assets. The topic of the anomaly has gained momentum in recent years with numerous authors investigating a variety of factors in an attempt to explain the rationale for the existence of the anomaly. The research which has been conducted focuses primarily on developed markets with the US as the primary market. As this study examined the South African equity market, in particular the JSE, there is limited noteworthy literature to analyse and to use to compare the findings of this study. While this study examined the presence of an idiosyncratic volatility anomaly after controlling for market, size and value effects, further areas of study could include additional analysis conducted by introducing the Fama and French 5 factor model to potentially assess whether, after controlling for the additional risk factors, the volatility premium remains on the cross-section of JSE share returns. Furthermore, the primary measure of risk could be studied to determine different metrics of risk and the resulting effect on the volatility premium. Risk metrics such as skewness and

ECVaR (a left tail risk measure) could be applied to determine if a tail risk measure would be more accurate at estimating expected share returns.

5.5 Conclusion

The three null hypotheses stated in section 1.4 will now be dealt with.

The study finds the first null hypothesis, that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 12-month volatility estimation period, to be accepted at all acceptable confidence levels. An insignificant finding of the presence of a low idiosyncratic volatility anomaly was identified in the 12-month results, as the second lowest risk portfolio outperformed all corresponding risk portfolios, and the low risk portfolio generated the smallest VaR measure. These findings encouraged the further testing and examination of a 36-month volatility estimation period.

The second null hypothesis, that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 36-month volatility estimation period, is accepted at all confidence levels. Comparable to the 12-month volatility estimation period, it was found that the highest volatility portfolio significantly underperformed in contrast to the low volatility portfolio on a cumulative returns basis.

The third null hypothesis, that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns on the JSE after estimating volatility utilising a 60-month volatility estimation period, is rejected at all confidence levels. These findings are supported by a statistically significant alpha for five of the six portfolios examined and clearly indicate the superior performance of the low volatility portfolio in contrast to the high idiosyncratic volatility portfolios. These findings of the 60-month CAPM regressions analysis provide clear evidence of a low idiosyncratic volatility anomaly and the null hypothesis is therefore rejected.

In summary, a low idiosyncratic volatility anomaly was found to be present on the cross-section of share returns of the JSE. Furthermore, when comparing the most accurate idiosyncratic volatility estimation period, it can be stated that the longer time horizon (60 months) produced the most accurate results as opposed to the 12- and 36-month volatility estimation periods. Although the low idiosyncratic risk anomaly was discovered for the 12- and 36-month volatility estimation horizons, the results from the 60-month volatility estimation period were found to be in line with previous literature by Xiong *et al*. (2014) and to be statistically significant at an appropriate confidence level. Ang *et al*. (2006) and Xiong *et al*. (2014) suggest that a larger time horizon will result in more accurate results, which was identified to be true in this study, providing evidence in line with these noteworthy studies. Overall, this study rejects the 60-month null hypothesis which states that there is no statistically significant evidence in favour of a low idiosyncratic volatility anomaly on the cross-section of share returns of the JSE after estimating volatility utilising a 60-month volatility estimation period.

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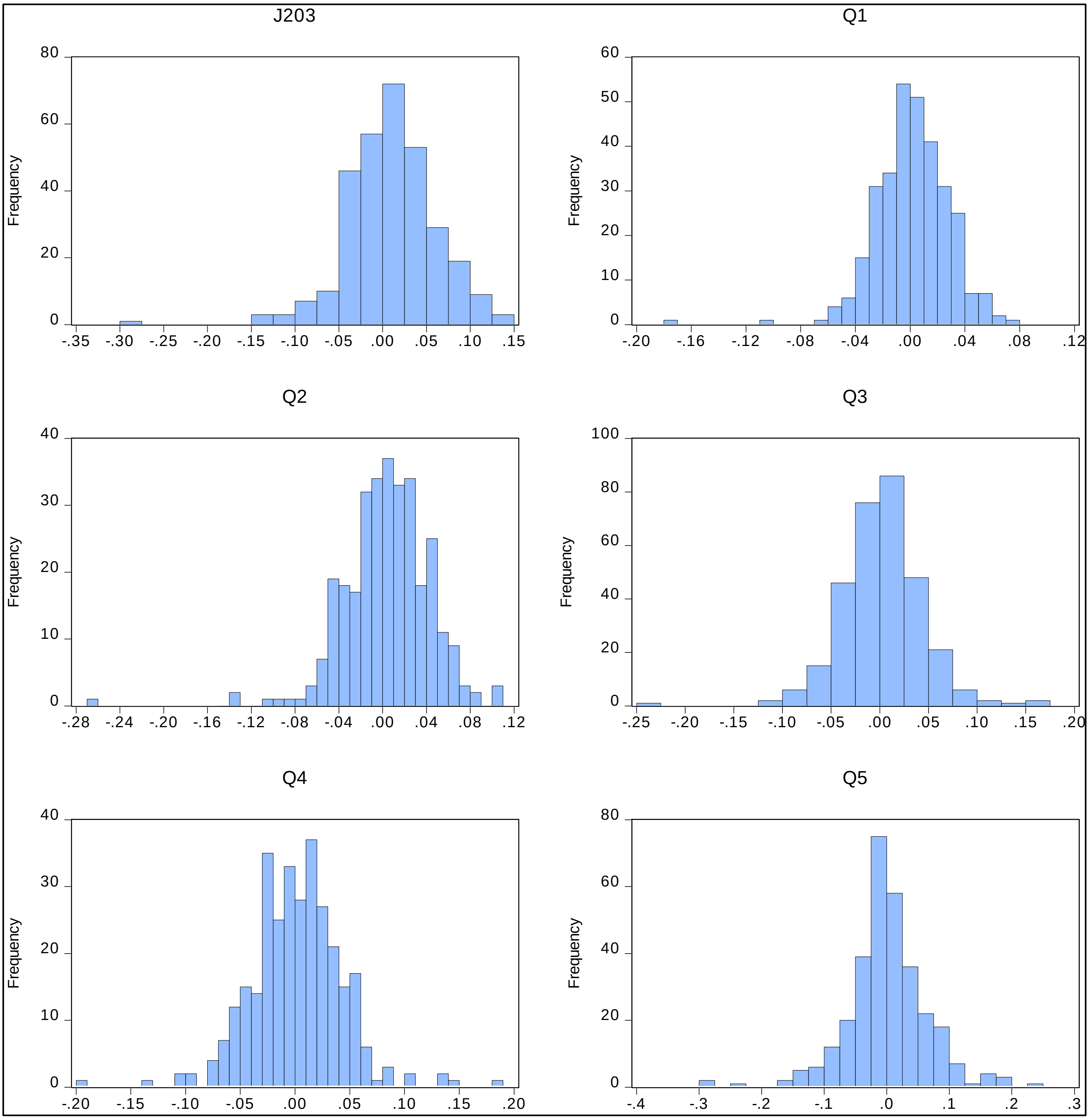
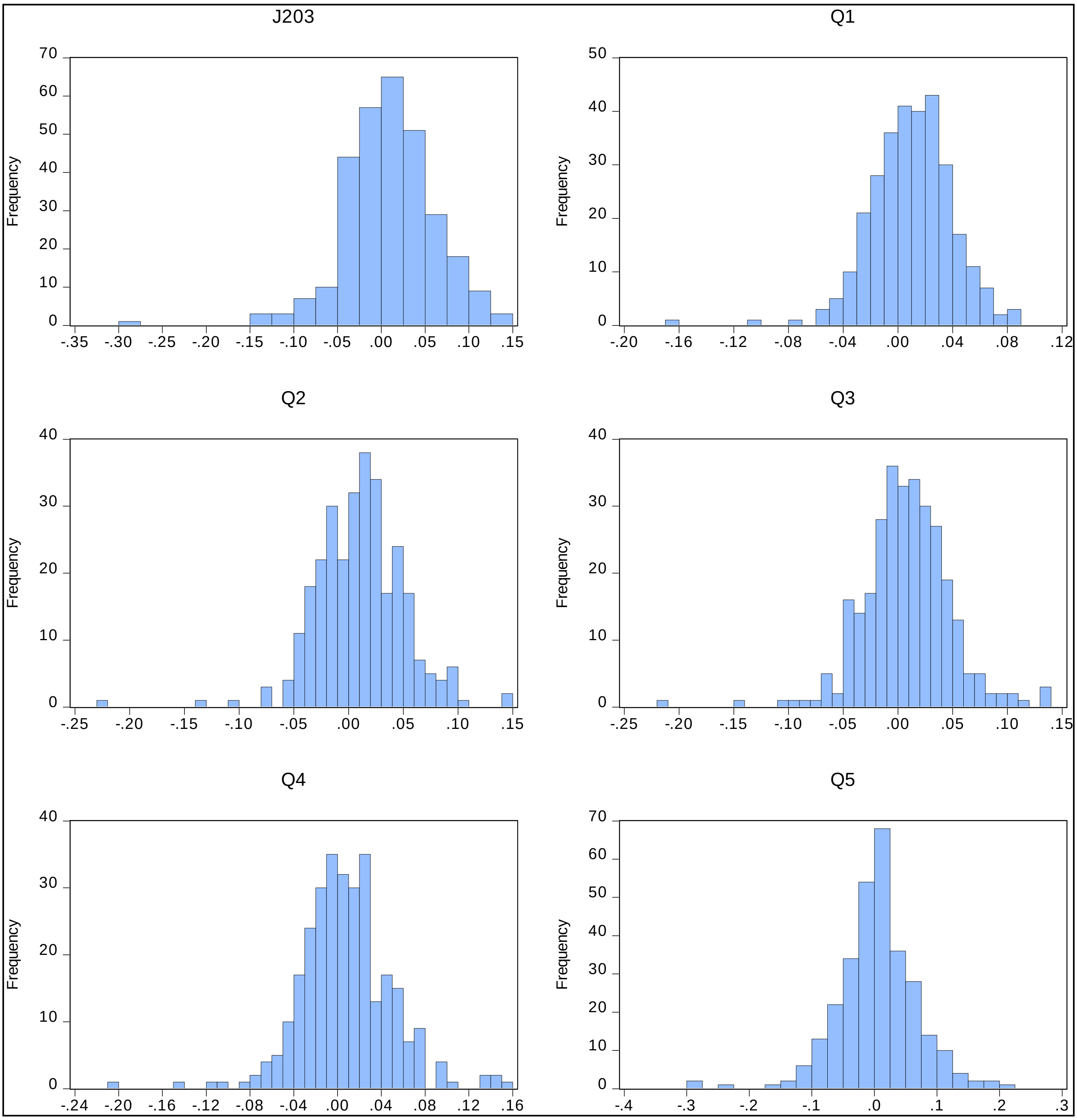
APPENDICES

Appendix A: Distribution Return Series

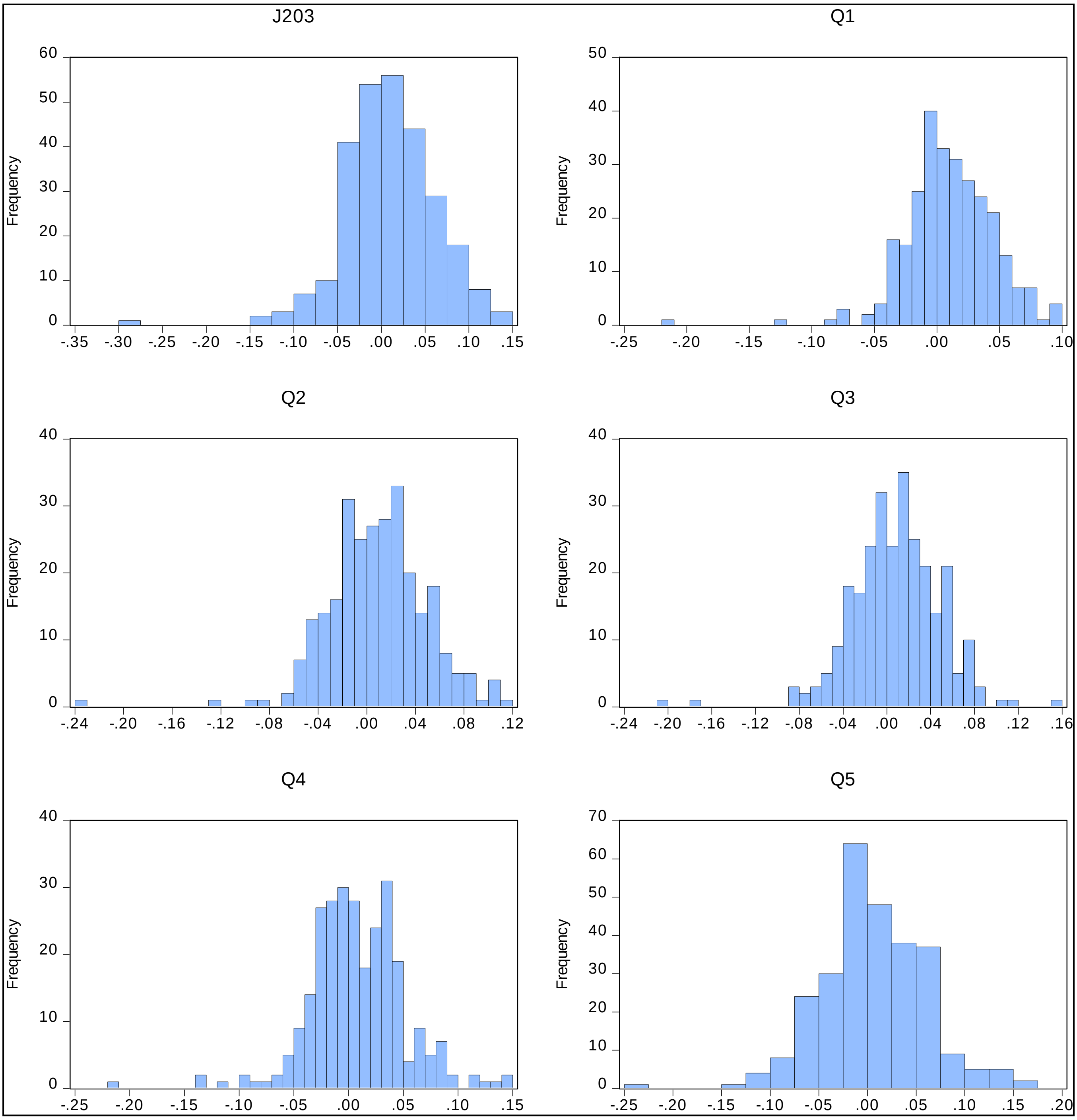
Appendix Figure 1: 12-month distribution returns graphed for five quintile portfolios and J203 from January 1994 – December 2019



Appendix Figure 2: 36-month distribution returns graphed for five quintile portfolios and J203 from January 1994 – December 2019



Appendix Figure 3: 60-month distribution returns graphed for five quintile portfolios and J203 from January 1994 – December 2019



Appendix B: CAPM OLS Regression Results

Appendix Figure 4: CAPM OLS regression results of relationship between individual idiosyncratic volatility quintiles' excess returns and J203 following 12-month volatility estimation period over January 1994 – December 2019

Quintile 5 : High Volatility	
Regression Statistics	
Multiple R	0.439622934
R Square	0.193268324
Adjusted R Square	0.190665964
Standard Error	0.058578433
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.254840681	0.254840681	74.26655251	3.54986E-16
Residual	310	1.063744155	0.003431433		
Total	311	1.318584836			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.000749446	0.003320641	-0.225693001	0.821588922	-0.007283292	0.005784401	-0.007283292	0.005784401
RMRF	0.559873872	0.064967113	8.61780439	3.54986E-16	0.432041597	0.687706148	0.432041597	0.687706148

Quintile 4	
Regression Statistics	
Multiple R	0.598615003
R Square	0.358339921
Adjusted R Square	0.35627005
Standard Error	0.033879766
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.19871592	0.19871592	173.1218434	1.01996E-31
Residual	310	0.35582994	0.001147839		
Total	311	0.554545861			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.00038473	0.001920546	0.20032318	0.841359188	-0.003394224	0.004163684	-0.003394224	0.004163684
RMRF	0.494392818	0.03757476	13.15757741	1.01996E-31	0.420458994	0.568326642	0.420458994	0.568326642

Quintile 3	
Regression Statistics	
Multiple R	0.663774079
R Square	0.440596027
Adjusted R Square	0.438791498
Standard Error	0.032373492
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.255891482	0.255891482	244.1612416	5.36536E-41
Residual	310	0.324893332	0.001048043		
Total	311	0.580784814			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001234205	0.001835159	0.672532745	0.501745574	-0.002376739	0.004845149	-0.002376739	0.004845149
RMRF	0.561026967	0.03590421	15.62565972	5.36536E-41	0.490380194	0.631673739	0.490380194	0.631673739

Quintile 2	
Regression Statistics	
Multiple R	0.731687923
R Square	0.535367216
Adjusted R Square	0.533868401
Standard Error	0.026751182
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.255616911	0.255616911	357.1935575	1.55744E-53
Residual	310	0.221843986	0.000715626		
Total	311	0.477460897			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001995635	0.001516447	1.315993791	0.189148422	-0.000988196	0.004979465	-0.000988196	0.004979465
RMRF	0.560725896	0.02966872	18.89956501	1.55744E-53	0.50234836	0.619103431	0.50234836	0.619103431

Quintile 1: Low Volatility

Regression Statistics	
Multiple R	0.650563982
R Square	0.423233495
Adjusted R Square	0.421372958
Standard Error	0.020551212
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.096076369	0.096076369	227.4792003	6.24809E-39
Residual	310	0.130929221	0.000422352		
Total	311	0.227005591			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000995061	0.001164989	0.854138105	0.393688024	-0.001297224	0.003287346	-0.001297224	0.003287346
RMRF	0.343766944	0.022792568	15.08241361	6.24809E-39	0.29891924	0.388614649	0.29891924	0.388614649

Differential Portfolio (Q1-Q5)

Regression Statistics	
Multiple R	0.200061108
R Square	0.040024447
Adjusted R Square	0.036927752
Standard Error	0.054200019
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.037968701	0.037968701	12.92489019	0.000377101
Residual	310	0.910669028	0.002937642		
Total	311	0.948637728			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001744507	0.003072442	0.567791608	0.570587466	-0.004300971	0.007789984	-0.004300971	0.007789984
RMRF	-0.216106928	0.060111181	-3.595120331	0.000377101	-0.334384448	-0.09782941	-0.334384448	-0.097829408

Appendix Figure 5: CAPM OLS regression results of relationship between individual idiosyncratic volatility quintiles' excess returns and J203 following 36-month volatility estimation period over January 1995 – December 2019

Quintile 5 : High Volatility	
Regression Statistics	
Multiple R	0.422832611
R Square	0.178787417
Adjusted R Square	0.17603167
Standard Error	0.057797348
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.216727212	0.216727212	64.87802461	1.93094E-14
Residual	298	0.995478969	0.003340533		
Total	299	1.212206181			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.004691683	0.003340358	-1.404545054	0.161198039	-0.011265362	0.001881996	-0.011265362	0.001881996
RMRF	0.519779703	0.064531314	8.054689604	1.93094E-14	0.392784884	0.646774521	0.392784884	0.646774521

Quintile 4	
Regression Statistics	
Multiple R	0.600730307
R Square	0.360876901
Adjusted R Square	0.358732193
Standard Error	0.03425076
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.197392783	0.197392783	168.2638553	8.22912E-31
Residual	298	0.349588147	0.001173115		
Total	299	0.546980931			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.001403501	0.001979499	-0.709018424	0.478867356	-0.005299069	0.002492067	-0.005299069	0.002492067
RMRF	0.496053171	0.038241315	12.97165584	8.22912E-31	0.420795927	0.571310414	0.420795927	0.571310414

Quintile 3	
Regression Statistics	
Multiple R	0.685606358
R Square	0.470056077
Adjusted R Square	0.468277742
Standard Error	0.029569613
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.23111456	0.23111456	264.3236485	5.45799E-43
Residual	298	0.260559883	0.000874362		
Total	299	0.491674443			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.00064597	0.001708955	-0.377990939	0.705706425	-0.00400912	0.00271718	-0.00400912	0.00271718
RMRF	0.536755184	0.033014767	16.25803335	5.45799E-43	0.471783559	0.60172681	0.471783559	0.60172681

Quintile 2	
Regression Statistics	
Multiple R	0.720028545
R Square	0.518441105
Adjusted R Square	0.516825136
Standard Error	0.02806767
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.252742932	0.252742932	320.8235812	3.31547E-49
Residual	298	0.234762649	0.000787794		
Total	299	0.48750558			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001929266	0.001622152	1.189325474	0.235258386	-0.001263058	0.00512159	-0.001263058	0.00512159
RMRF	0.561309127	0.031337833	17.91154882	3.31547E-49	0.499637634	0.622980619	0.499637634	0.622980619

Quintile 1: Low Volatility	
Regression Statistics	
Multiple R	0.682713382
R Square	0.466097562
Adjusted R Square	0.464305943
Standard Error	0.02205358
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.126528806	0.126528806	260.1544097	1.66115E-42
Residual	298	0.144935403	0.00048636		
Total	299	0.271464209			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001597201	0.001274571	1.253128032	0.211141464	-0.0009111	0.004105502	-0.0009111	0.004105502
RMRF	0.397152499	0.024623042	16.12930283	1.66115E-42	0.348695423	0.445609574	0.348695423	0.445609574

Differential Portfolio (Q1-Q5)

Regression Statistics	
Multiple R	0.114018483
R Square	0.013000215
Adjusted R Square	0.009688135
Standard Error	0.05530231
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.012004284	0.012004284	3.925090969	0.048489584
Residual	298	0.911386972	0.003058346		
Total	299	0.923391256			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.006220134	0.003196159	1.94612774	0.052579159	-6.97682E-05	0.012510035	-6.97682E-05	0.012510035
RMRF	-0.122329368	0.061745579	-1.981184234	0.048489584	-0.243841983	-0.00081675	-0.243841983	-0.000816754

Appendix Figure 6: CAPM OLS regression results of relationship between individual idiosyncratic volatility quintiles' excess returns and J203 following 60-month volatility estimation period over January 1997 – December 2019

Quintile 5 : High Volatility	
Regression Statistics	
Multiple R	0.429811426
R Square	0.184737862
Adjusted R Square	0.181762453
Standard Error	0.054404432
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.183771329	0.183771329	62.08821912	7.77014E-14
Residual	274	0.810996754	0.002959842		
Total	275	0.994768083			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.000381031	0.00327947	-0.116186697	0.907589689	-0.00683719	0.006075129	-0.00683719	0.006075129
RMRF	0.492168824	0.062461081	7.879607802	7.77014E-14	0.369204215	0.615133433	0.369204215	0.615133433

Quintile 4	
Regression Statistics	
Multiple R	0.540130235
R Square	0.291740671
Adjusted R Square	0.289155783
Standard Error	0.039283356
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.174169621	0.174169621	112.8639477	2.64713E-22
Residual	274	0.422831889	0.001543182		
Total	275	0.597001511			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.005605422	0.002367979	2.367175102	0.018619609	0.000943676	0.010267167	0.000943676	0.010267167
RMRF	0.479138894	0.045100755	10.62374452	2.64713E-22	0.390350859	0.567926929	0.390350859	0.567926929

Quintile 3	
Regression Statistics	
Multiple R	0.686953818
R Square	0.471905548
Adjusted R Square	0.469978196
Standard Error	0.027633765
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.186970965	0.186970965	244.8465793	7.16947E-40
Residual	274	0.209233245	0.000763625		
Total	275	0.396204209			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.005492784	0.001665748	3.297487373	0.001104382	0.002213493	0.008772076	0.002213493	0.008772076
RMRF	0.496434901	0.031725998	15.64757423	7.16947E-40	0.433977211	0.558892592	0.433977211	0.558892592

Quintile 2	
Regression Statistics	
Multiple R	0.714341026
R Square	0.510283101
Adjusted R Square	0.508495813
Standard Error	0.028561908
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.232911589	0.232911589	285.5069328	2.2374E-44
Residual	274	0.223524433	0.000815783		
Total	275	0.456436022			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.009088884	0.001721696	5.279028552	2.64598E-07	0.00569945	0.012478318	0.00569945	0.012478318
RMRF	0.554077827	0.032791587	16.8969504	2.2374E-44	0.489522353	0.618633302	0.489522353	0.618633302

Quintile 1: Low Volatility	
Regression Statistics	
Multiple R	0.679107007
R Square	0.461186326
Adjusted R Square	0.459219853
Standard Error	0.024378893
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.139385098	0.139385098	234.5245855	1.13763E-38
Residual	274	0.162846538	0.00059433		
Total	275	0.302231637			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.009080795	0.001469546	6.179318393	2.32202E-09	0.006187758	0.011973832	0.006187758	0.011973832
RMRF	0.42863082	0.027989118	15.31419556	1.13763E-38	0.373529774	0.483731866	0.373529774	0.483731866

Differential Portfolio (Q1-Q5)

Regression Statistics	
Multiple R	0.066733423
R Square	0.00445335
Adjusted R Square	0.000819968
Standard Error	0.051496835
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.0032504	0.0032504	1.225676196	0.26922054
Residual	274	0.726627173	0.002651924		
Total	275	0.729877573			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.00982383	0.003104201	3.164688556	0.001727486	0.003712714	0.015934945	0.003712714	0.015934945
RMRF	-0.065455122	0.059122904	-1.107102613	0.26922054	-0.181847996	0.050937753	-0.181847996	0.050937753

Appendix C: Fama and French OLS Regression Results

Appendix Figure 7: Fama and French OLS regression results of relationship between individual idiosyncratic volatility quintiles' excess returns and J203 following 12-month volatility estimation period over January 1994 – December 2019

Quintile 5 : High Volatility	
Regression Statistics	
Multiple R	0.45159568
R Square	0.203938658
Adjusted R Square	0.196184814
Standard Error	0.058378368
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.268910422	0.089636807	26.30161916	3.57401E-15
Residual	308	1.049674413	0.003408034		
Total	311	1.318584836			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.000424914	0.004946985	-0.085893507	0.931606882	-0.010159076	0.009309248	-0.010159076	0.009309248
MRK	0.570864308	0.064983096	8.784812435	1.1138E-16	0.44299733	0.698731286	0.44299733	0.698731286
SMB	0.055891376	0.028673078	1.949263208	0.052171465	-0.000528525	0.112311277	-0.000528525	0.112311277
HML	-0.080128054	0.181420158	-0.441671172	0.659037097	-0.437107775	0.276851667	-0.437107775	0.276851667

Quintile 4	
Regression Statistics	
Multiple R	0.599444365
R Square	0.359333547
Adjusted R Square	0.353093289
Standard Error	0.03396326
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.199266931	0.06642231	57.58312666	1.40871E-29
Residual	308	0.355278929	0.001153503		
Total	311	0.554545861			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001629396	0.002878048	0.566146261	0.571706617	-0.004033727	0.007292519	-0.004033727	0.007292519
MRK	0.496061024	0.037805746	13.12131278	1.51251E-31	0.421670808	0.57045124	0.421670808	0.57045124
SMB	0.003708917	0.016681371	0.222338888	0.824197442	-0.029114949	0.036532784	-0.029114949	0.036532784
HML	-0.067345276	0.105546287	-0.638063902	0.523906183	-0.275028284	0.140337732	-0.275028284	0.140337732

Quintile 3	
Regression Statistics	
Multiple R	0.664698423
R Square	0.441823994
Adjusted R Square	0.436387215
Standard Error	0.032442764
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.256604666	0.085534889	81.26575887	9.42787E-39
Residual	308	0.324180148	0.001052533		
Total	311	0.580784814			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.00082254	0.002749201	0.299192507	0.76499495	-0.004587051	0.006232132	-0.004587051	0.006232132
MRK	0.563231729	0.036113227	15.59627243	8.0001E-41	0.492171876	0.634291582	0.492171876	0.634291582
SMB	0.013116521	0.015934565	0.823148964	0.411060159	-0.01823786	0.044470901	-0.01823786	0.044470901
HML	0.005934492	0.100821103	0.058861607	0.953100509	-0.192450789	0.204319774	-0.192450789	0.204319774

Quintile 2	
Regression Statistics	
Multiple R	0.73234521
R Square	0.536329507
Adjusted R Square	0.531813236
Standard Error	0.02681009
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.256076367	0.085358789	118.7549423	4.06864E-51
Residual	308	0.221384529	0.000718781		
Total	311	0.477460897			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000830122	0.002271888	0.365388583	0.715072159	-0.003640263	0.005300507	-0.003640263	0.005300507
MRK	0.559259514	0.029843292	18.73987348	7.99336E-53	0.500536987	0.61798204	0.500536987	0.61798204
SMB	-0.002974899	0.013168025	-0.225918368	0.821414754	-0.028885569	0.022935772	-0.028885569	0.022935772
HML	0.062495039	0.083316664	0.750090515	0.453772781	-0.101446827	0.226436905	-0.101446827	0.226436905

Quintile 1: Low Volatility	
Regression Statistics	
Multiple R	0.650844056
R Square	0.423597985
Adjusted R Square	0.417983679
Standard Error	0.020611313
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.096159111	0.032053037	75.44975889	1.30127E-36
Residual	308	0.13084648	0.000424826		
Total	311	0.227005591			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000440925	0.001746603	0.252447283	0.800863743	-0.002995859	0.00387771	-0.002995859	0.00387771
MRK	0.343329275	0.022943206	14.9643109	1.98802E-38	0.298184019	0.38847453	0.298184019	0.38847453
SMB	-6.40496E-05	0.010123438	-0.006326863	0.994956023	-0.019983898	0.019855799	-0.019983898	0.019855799
HML	0.028174595	0.064052968	0.439863999	0.660344102	-0.097862175	0.154211364	-0.097862175	0.154211364

Differential Portfolio (Q1-Q5)	
Regression Statistics	
Multiple R	0.235831186
R Square	0.055616348
Adjusted R Square	0.046417806
Standard Error	0.053932316
Observations	312

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.052759766	0.017586589	6.046213384	0.000518592
Residual	308	0.895877962	0.002908695		
Total	311	0.948637728			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000865839	0.004570226	0.189452142	0.849863247	-0.008126976	0.009858655	-0.008126976	0.009858655
MRK	-0.227535033	0.060034033	-3.790100756	0.00018114	-0.34566376	-0.10940631	-0.34566376	-0.109406307
SMB	-0.055955426	0.026489358	-2.112373771	0.035459053	-0.10807843	-0.00383242	-0.10807843	-0.003832421
HML	0.108302649	0.167603338	0.646184319	0.518641167	-0.221489769	0.438095066	-0.221489769	0.438095066

Appendix Figure 8: Fama and French OLS regression results of relationship between individual idiosyncratic volatility quintiles' excess returns and J203 following 36-month volatility estimation period over January 1995 – December 2019

Quintile 5 : High Volatility	
Regression Statistics	
Multiple R	0.433922842
R Square	0.188289033
Adjusted R Square	0.180062233
Standard Error	0.057655813
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.22824513	0.07608171	22.88727395	2.36406E-13
Residual	296	0.983961051	0.003324193		
Total	299	1.212206181			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.006193425	0.005074919	-1.220398672	0.223285122	-0.01618092	0.003794071	-0.01618092	0.003794071
MRK	0.529112512	0.064642442	8.185218514	8.16608E-15	0.401895493	0.656329531	0.401895493	0.656329531
SMB	0.052936904	0.028439832	1.861364836	0.063683822	-0.003032991	0.108906799	-0.003032991	0.108906799
HML	0.017768025	0.182926403	0.097132096	0.922687227	-0.342233098	0.377769148	-0.342233098	0.377769148

Quintile 4	
Regression Statistics	
Multiple R	0.603940902
R Square	0.364744613
Adjusted R Square	0.358306214
Standard Error	0.034262134
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.199508348	0.066502783	56.65144438	5.73524E-29
Residual	296	0.347472583	0.001173894		
Total	299	0.546980931			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.00124075	0.003015786	-0.41141864	0.681063315	-0.007175849	0.004694348	-0.007175849	0.004694348
MRK	0.500561196	0.038413959	13.03071087	5.49899E-31	0.424962112	0.57616028	0.424962112	0.57616028
SMB	0.02184635	0.016900453	1.292648772	0.197140668	-0.011413923	0.055106623	-0.011413923	0.055106623
HML	-0.0309175	0.108704547	-0.284417727	0.776289144	-0.244849216	0.183014215	-0.244849216	0.183014215

Quintile 3	
Regression Statistics	
Multiple R	0.689718154
R Square	0.475711132
Adjusted R Square	0.470397394
Standard Error	0.029510616
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.233895006	0.077965002	89.52475365	2.98818E-41
Residual	296	0.257779437	0.000870876		
Total	299	0.491674443			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.004138637	0.002597552	-1.593283323	0.112163889	-0.009250648	0.000973374	-0.009250648	0.000973374
MRK	0.535301374	0.03308666	16.1787671	1.27761E-42	0.470186471	0.600416277	0.470186471	0.600416277
SMB	0.007378569	0.014556676	0.506885588	0.612612487	-0.021269125	0.036026264	-0.021269125	0.036026264
HML	0.162973139	0.093629257	1.740621945	0.082789022	-0.021290243	0.347236521	-0.021290243	0.347236521

Quintile 2	
Regression Statistics	
Multiple R	0.721860481
R Square	0.521082555
Adjusted R Square	0.516228662
Standard Error	0.02808499
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.254030653	0.084676884	107.3535308	4.75387E-47
Residual	296	0.233474927	0.000788767		
Total	299	0.48750558			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.000135388	0.002472067	-0.054766918	0.956361103	-0.005000443	0.004729668	-0.005000443	0.004729668
MRK	0.558521931	0.031488279	17.737455	1.84461E-48	0.49655266	0.620491201	0.49655266	0.620491201
SMB	-0.005272318	0.013853458	-0.380577787	0.703789538	-0.032536073	0.021991436	-0.032536073	0.021991436
HML	0.106465908	0.089106126	1.19482141	0.23311329	-0.068895905	0.281827721	-0.068895905	0.281827721

Quintile 1: Low Volatility	
Regression Statistics	
Multiple R	0.685530598
R Square	0.469952201
Adjusted R Square	0.464580095
Standard Error	0.022047937
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.127575202	0.042525067	87.48006753	1.49613E-40
Residual	296	0.143889006	0.000486112		
Total	299	0.271464209			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000205658	0.00194068	0.105972058	0.915676302	-0.003613622	0.004024937	-0.003613622	0.004024937
MRK	0.394063005	0.024719666	15.94127517	9.88392E-42	0.345414436	0.442711574	0.345414436	0.442711574
SMB	-0.009605238	0.010875566	-0.883194309	0.377848026	-0.03100847	0.011797993	-0.03100847	0.011797993
HML	0.078117114	0.069952179	1.116721664	0.265019007	-0.059549525	0.215783752	-0.059549525	0.215783752

Differential Portfolio (Q1-Q5)

Regression Statistics	
Multiple R	0.176737287
R Square	0.031236069
Adjusted R Square	0.021417515
Standard Error	0.054973831
Observations	300

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.028843113	0.009614371	3.181331042	0.024317633
Residual	296	0.894548144	0.003022122		
Total	299	0.923391256			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.006251249	0.004838849	1.291887729	0.197403788	-0.003271657	0.015774155	-0.003271657	0.015774155
MRK	-0.134813035	0.061635462	-2.187264111	0.029505498	-0.256112286	-0.01351378	-0.256112286	-0.013513784
SMB	-0.062517229	0.027116894	-2.305471649	0.021831511	-0.115883567	-0.00915089	-0.115883567	-0.009150892
HML	0.064188611	0.174417196	0.368017676	0.713123084	-0.279066302	0.407443524	-0.279066302	0.407443524

Appendix Figure 9: Fama and French OLS regression results of relationship between individual idiosyncratic volatility quintiles' excess returns and J203 following 60-month volatility estimation period over January 1997 – December 2019

Quintile 5 : High Volatility

Regression Statistics	
Multiple R	0.43789594
R Square	0.191752854
Adjusted R Square	0.182838364
Standard Error	0.054368651
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.190749619	0.063583206	21.51024252	1.56434E-12
Residual	272	0.804018464	0.00295595		
Total	275	0.994768083			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000978537	0.004999924	0.195710386	0.84498298	-0.008864933	0.010822007	-0.008864933	0.010822007
MRK	0.501512593	0.062718721	7.996218475	3.69713E-14	0.378036752	0.624988434	0.378036752	0.624988434
SMB	0.037279759	0.026980454	1.381732072	0.168187788	-0.015837305	0.090396823	-0.015837305	0.090396823
HML	-0.106049035	0.179408067	-0.591105167	0.554940838	-0.459253975	0.247155904	-0.459253975	0.247155904

Quintile 4	
Regression Statistics	
Multiple R	0.544101972
R Square	0.296046956
Adjusted R Square	0.288282768
Standard Error	0.039307472
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.17674048	0.058913493	38.12980262	1.33587E-20
Residual	272	0.420261031	0.001545077		
Total	275	0.597001511			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.002937192	0.003614847	0.812535423	0.417195049	-0.004179445	0.010053828	-0.004179445	0.010053828
MRK	0.481205963	0.045344408	10.6122448	3.04996E-22	0.391935344	0.570476581	0.391935344	0.570476581
SMB	0.020127065	0.019506341	1.031821648	0.303072253	-0.018275534	0.058529663	-0.018275534	0.058529663
HML	0.107967734	0.129708524	0.832387348	0.405920568	-0.14739253	0.363327998	-0.14739253	0.363327998

Quintile 3	
Regression Statistics	
Multiple R	0.703366829
R Square	0.494724896
Adjusted R Square	0.489152009
Standard Error	0.027129328
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.196012086	0.065337362	88.77353518	4.45662E-40
Residual	272	0.200192123	0.000736		
Total	275	0.396204209			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.001095997	0.002494904	-0.439294164	0.660797188	-0.006007775	0.003815781	-0.006007775	0.003815781
MRK	0.492195533	0.031295916	15.72714877	4.39716E-40	0.430582516	0.553808549	0.430582516	0.553808549
SMB	0.006981292	0.013462935	0.518556472	0.604491661	-0.019523509	0.033486093	-0.019523509	0.033486093
HML	0.312474343	0.089522551	3.490453971	0.000562398	0.136229162	0.488719524	0.136229162	0.488719524

Quintile 2	
Regression Statistics	
Multiple R	0.72203179
R Square	0.521329905
Adjusted R Square	0.516050456
Standard Error	0.028341553
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.237953748	0.079317916	98.74701857	2.91984E-43
Residual	272	0.218482274	0.000803244		
Total	275	0.456436022			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.004767725	0.002606385	1.829248388	0.068457105	-0.000363527	0.009898977	-0.000363527	0.009898977
MRK	0.548116187	0.032694318	16.76487596	8.20263E-44	0.483750104	0.612482271	0.483750104	0.612482271
SMB	-0.009966401	0.014064502	-0.708620975	0.47916658	-0.037655521	0.017722719	-0.037655521	0.017722719
HML	0.220547784	0.093522705	2.358227176	0.019070344	0.036427405	0.404668164	0.036427405	0.404668164

Quintile 1: Low Volatility	
Regression Statistics	
Multiple R	0.690621355
R Square	0.476957856
Adjusted R Square	0.471189009
Standard Error	0.024107593
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.144151753	0.048050584	82.67819224	4.81256E-38
Residual	272	0.158079883	0.000581176		
Total	275	0.302231637			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.005394925	0.002217015	2.433417696	0.015601445	0.001030233	0.009759616	0.001030233	0.009759616
MRK	0.421958571	0.027810096	15.17285577	4.28638E-38	0.367208172	0.476708969	0.367208172	0.476708969
SMB	-0.015757296	0.011963398	-1.317125431	0.188905094	-0.039309923	0.007795331	-0.039309923	0.007795331
HML	0.195913677	0.079551296	2.462733948	0.014408166	0.039299144	0.35252821	0.039299144	0.35252821

Differential Portfolio (Q1-Q5)

Regression Statistics	
Multiple R	0.178334756
R Square	0.031803285
Adjusted R Square	0.021124645
Standard Error	0.050970907
Observations	276

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.023212505	0.007737502	2.978214883	0.031968203
Residual	272	0.706665068	0.002598033		
Total	275	0.729877573			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.005199964	0.004687456	1.109335902	0.268265134	-0.004028343	0.01442827	-0.004028343	0.01442827
MRK	-0.081163678	0.058799142	-1.380354797	0.168610777	-0.196922952	0.034595596	-0.196922952	0.034595596
SMB	-0.053318177	0.025294323	-2.107910823	0.035953619	-0.103115714	-0.00352064	-0.103115714	-0.00352064
HML	0.281791817	0.16819604	1.675377235	0.095009401	-0.049339739	0.612923373	-0.049339739	0.612923373