EXPLORING NINTH GRADERS' REASONING SKILLS IN PROVING CONGRUENT TRIANGLES IN ETHUSINI CIRCUIT, KWAZULU-NATAL PROVINCE

By

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ABSTRACT

Euclidean Geometry is a challenging topic for most of the learners in the secondary schools. A qualitative case study explores the reasoning skills of ninth graders in the proving of congruent triangles in their natural environment. A class of thirty-two learners was conveniently selected to participate in the classroom observations. Two groups of six learners each were purposefully selected from the same class of thirty-two learners to participate in focus group interviews. The teaching documents were analysed. The Van Hiele's levels of geometric thinking were used to reflect on the reasoning skills of the learners. The findings show that the majority of the learners operated at level 2 of Van Hiele's geometric thinking. The use of visual aids in the teaching of geometry is important. About 30% of the learners were still operating at level 1 of Van Hiele theory. The analysed books showed that investigation help learners to discover the intended knowledge on their own. Learners need quality experience in order to move from a lower to a higher level of Van Hiele's geometry thinking levels. The study brings about unique findings which may not be generalised. The results can only provide an insight into the reasoning skills of ninth graders in proving of congruent triangles. I recommend that future researchers should focus on proving of congruent triangles with a bigger sample of learners from different environmental settings.

Key words: reasoning skills; congruent triangles; Van Hiele theory; deductive reasoning skills; Euclidean geometry

DECLARATION

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I declare that the above dissertation/thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I have not submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.

16 SEPTEMBER 2020

SIGNATURE DATE

Table of Contents

List of Abbreviations

AAS - Angle -Angle –Angle

ANA -Annual National Assessment

ASA - Angle –Side-Angle

CAPS – Curriculum and Assessment Policy Statement

DBE – Department of Basic Education

FET – Further Education and Training

GET - General Education and Training

NSC – National Senior Certificate

RHS – Right- Hypotenuse- Side

SAS – Side- Angle- Side

SSS – Side- Side- Side

2-D – 2 dimensional shapes

List of Appendices

List of Figures

List of Tables

CHAPTER 1

ORIENTATION OF THE STUDY

1.1. Introduction and Background

I have discovered through many years of my teaching experience that most of the grade nine learners are failing Mathematics dismally. Generally passing Mathematics is a pre-requisite for a grade 9 learner's promotion to grade 10. Those who fail Mathematics are either progressed to the next grade on condition that they have repeated the phase once or because they have passed the age limit. Geometry takes about one-third of the marks in the tests. Geometry poses challenges to Grade 9 learners, especially where they are required to prove congruence in triangles. The majority of learners tend to miss communication of congruence in triangles. The last Annual National Assessment (ANA) analysis shows that Grade 9 learners have problems in making deductions about congruent triangles (Department of Basic Education (DBE) 2015). DBE (2015) in its results analysis noted that Grade 9 learners in 2014 displayed weakness in congruency and similarity deductions; angle relationship in parallel lines; terminology and definitions in geometry. Mathematics results of Grade 9 learners in 2013 and 2014 showed that topics in Geometry were difficult for them to answer. Mathematics ANA tests at Grade 9 in 2013 and 2014 assessed learner skills involving properties of angles and 2-D shapes, angle relationships involving parallel lines, and interior angles of a triangle. They were also required to use the congruency axioms to make deductions. The other skills assessed involved proving whether triangles are congruent or similar and make required deductions. Learners were also found to be struggling with geometrical language, terminology and reasoning skills (DBE, 2014). From 2015 the ANA tests were no longer administered until the time of this research proposal.

In addition to ANA results, the same streams of learners of 2013 and 2014 when traced to the time they wrote their Grade 12 examinations were found struggling with same problems.. The streams wrote their Grade 12 examinations in 2016 and in 2017 respectively. The National Senior Certificate (NSC) examinations diagnostic reports showed that these learners did not do well in Euclidean Geometry questions. The 2017 and 2018 reports showed that learners lost marks for incorrect or incomplete reasons for naming angles incorrectly and also could not provide correct reasons for proof questions (DBE, 2017 and DBE, 2018). Both reports say that the fact that learners were naming angles incorrectly at Grade 12 level indicated that this issue was not dealt with effectively in the earlier grades. Failure to answer easy questions like naming angles of triangle at Grade 12 level shows lack of understanding geometry in the lower grades.

Sadiki (2016) takes critical and creative thinking as important tools in learning geometry concepts. Hunt (2008) states that writing a deductive argument was a difficult activity for most of the learners. National Council of Teachers of Mathematics (NCTM) (2000) viewed explanation and justification of reasoning as expectations of any productive mathematical classroom. The importance of reasoning logically cannot be ruled out in any geometric lesson.

However, research suggests that many pre-university educators typically ignore both the importance and the role of proof and reasoning in the classroom (Duturi, 2013). Mukucha (2010) found that most learners lacked conceptual understanding and reasoning skills. Deductive reasoning is one of the important skills which Hunt (2008) says that helps learners to understand the world around them. When proving congruency, the learners are required to know the facts about triangles in order to make conclusions. Alex and Mammen (2014) noted in their study that the Grade 10 learners were not ready for the geometry included in the Curriculum and Assessment Policy Statement (CAPS). The aim of this research is to explore difficulties of learners' experiences in communicating deductive reasoning in the proving of congruent triangles.

Jones (2009) says that many researchers have agreed that the majority of learners do not see the need for logical reasoning even after exposed to such skills. At Grade 9, the learners are expected to justify their answers in Geometry (DBE, 2015). According to the current curriculum, CAPS document, the learning of congruency of triangles is emphasised to the ninth graders. The concept of congruency is developed in the seventh to eighth grade learners where they are expected to identify whether pairs of triangles are congruent and in Grade 9, the four axioms for congruency are introduced (DBE, 2014). The axioms for proving congruent triangles include side - side - side (SSS); side - angle- side (SAS); angle – angle - side (AAS) and right angle – hypotenuse - side (RHS). Geometry is one of the components in the curriculum that influence the way learners develop their reasoning capacity. I have experienced that learners are facing difficulties in communicating reasoning skills in the proving of the congruent triangles. This challenge affects learners' performance as they progress to Grades 10 to12 Euclidean geometry.

1.2. The Rationale for the Study

Hession, Pres-Jennings and Kennedy (2016) noted that geometry is a challenging topic to teach effectively. Chimuka (2017) says that it is evident that the traditional methods of teaching geometry are not achieving the desired results. The effect of poor performance in mathematics at the lower grades has affected the NSC examination results. Success in proving congruent triangles prepares Grade 9 learners for the Euclidean geometry in the Further Education and Training band (FET). Jones, Fujuta and Miyazaki (2013) affirm that proving of congruent triangles is taken to be an important concept in the school geometry. Wang, Wang et al. (2018) say that from the perspective of development of geometry content, congruent triangles reasoning and proof is the beginning of formal mathematical reasoning and proof. Exploring the reasoning skills of the ninth graders helps to establish the learners' geometric levels according to Van Hiele's theory. When learners improve their reasoning skills in proving congruent triangles, they are likely to generally improve their Mathematics performance.

1.3. Van Hiele's Levels of Geometric Thinking

In this research, the Van Hiele theory (1957) of geometrical thinking forms the theoretical framework as it guides on how learners learn geometrical concepts. The researches related to proving of congruent triangles are explained. The researcher will also discuss deductive reasoning skills and communication of deductive reasoning.

Geary, Boykin, Embretso, Reyna, Siegler, Rerch and Graban (2008) noted that Van Hiele's theory provides a generally valid description of the development of learners' geometric reasoning. They developed a pedagogical theory of teaching and learning of Geometry which describes how learners think at different levels. Crowley (1987) describes Van Hiele's theory as a model consisting of five levels of understanding. The levels are Visualisation, Analysis, Informal deduction, Deduction and Rigor. Levels are developmental from the lowest to the highest. They are not dependent on biological age, but on their instructional development. A learner may be in the visual stage of geometrical development when he or she is expected to be in the deduction level.

Level 0 (Visualisation) is when learners start to recognise figures by appearance. It is the stage, where learners are able to name, compare, sort and or describe geometric figures. Learners can be found sorting out shapes as triangles, squares or rectangles. The next level is Analysis (level 1) where they begin to distinguish the properties of figures. Learners are able to analyse figures using their properties and or the relationship between the properties. For example, squares are recognised as having all equal sides and angles. Learners can also use the knowledge of properties of a shape to solve geometric problems. At the Informal deduction level, which is level 2, learners can establish interrelationships of shapes. Shapes or figures can be related among themselves like a square is a rectangle because it has all the properties of a rectangle. Properties of a shape or figure can be related between themselves like in a parallelogram where opposite sides are parallel and opposite sides are bound to be equal. Learners are able to define figures and can make informal arguments. Logical reasoning is developed at this stage when learners start to use the "if … then" thinking. Learners start to make meaningful geometrical reasoning at the deduction level (Level 3). This is the time when they can be able to use axioms to make conclusions. At this level, learners display their geometrical reasoning skills where different ways of solving a problem are endeavoured. Learners experience a sense of achievement as there is no absolute answer to a problem. Learners explore a number of possible avenues to solve a geometric problem. The last level, level 4 (Rigor) is when learners can conceptualise geometry abstractly. The learner can work in different systems of axioms. They can apply a number of axiomatic skills in non-Euclidean geometry. Learners can also compare different systems of axioms. According to Van Hiele's theory rigor is the least developed level since most of the high school geometry emphasise the first three levels. Way (2011) says that Van Hiele concentrated on the three levels that cover the normal period of schooling.

However, Hannibal and March (1999) conducted individual clinical interviews of children aged 3 to 6, emphasising identification and descriptions of shapes and reasons for identifications. They noted that young children initially form schemas on the basis of feature analysis of visual matching to distinguish shapes. The learners were also capable of recognising components and simple properties of familiar shapes.

De Villiers (1987) developed six "geometric thought categories" from Van Hiele's geometric thinking levels. These are:

Identification and representation of figure types (level 1)

Knowing and communication of vocabulary (level 2)

Verbal description of properties of figures (level 2)

Hierarchical classification (level 3)

Single step deduction (level 3)

Multiple steps deduction (level 4)

Levels	Van Hiele's Levels	De Villiers' Geometric Thought Categories
1	Visualisation	Identification and representation of figure types
2	Analysis	Knowing and communication of vocabulary
2	Analysis	Verbal description of properties of figures
3	Informal Deduction	Hierarchical classification
3	Informal Deduction	Single step deduction
4	Deduction	Multiple steps deduction

Table 1.1 Van Hiele's Levels and De Villiers' Geometric Thought Categories

The descriptors suggested by De Villiers, not only provide detailed information about how researchers identify learners' levels of geometric thinking but more importantly shed a light on ways thinking has been communicated through reasoning and the language skills that learners need to develop at each Van Hiele level (Wang, 2016). The geometric thought categories tend to confirm that Van Hiele's levels are connected and hierarchical. For example, learners at level 2 should be able to recognise and represent a figure before they can use the proper language.

Mason (1999) describes a learner's progress according to Van Hiele's Levels of geometric thinking as a product of learning that is organised into phases. In the first phase, information phase, the educator establishes the learners' prior knowledge about a topic before they are introduced to the new content. This phase is followed by a guided orientation phase which allows learners to explore figures. Learners are given an opportunity to manipulate shapes in order to extract mathematical knowledge embedded in the objects. The teacher makes sure that learners explore specific concepts**.** The explicitation phase is when learners describe what they have learned about a topic in their own vocabulary. In the free orientation phase, learners apply the relationships they are learning to solve problems and investigate more open-ended tasks. At the integration phase, learners summarise and integrate what they have learned, developing a new network of objects and relations. The phases are sequential in the sense that each step is prior knowledge for the next phase.

Usiskin (1982) says that studies show that many learners' reasoning is at different levels, or is at intermediate levels. This appears to be in contradiction to the Van Hiele's theory. Usiskin tends to differ with Van Hiele when he says that learners grasp different concepts at different rates depending on how they have been exposed to that concept. For example, learners can operate at the same level on triangles while they can reason at different levels when doing properties of quadrilaterals. Usiskin (1982) says that researchers have found that many children at Visualisation level do not reason in a completely holistic fashion as Van Hiele indicated. He went on to say that learners may focus on a single attribute, such as the equal sides of a square or the roundness of a circle.

Van Hiele (1986) says that level 3 is a transitional stage between informal and formal geometry. Geometry knowledge at this level is constructed by short chains of reasoning about properties of a figure and class inclusions.

1.4. What is Deductive Reasoning?

Deductive reasoning or logical reasoning is the process of demonstrating that if certain statements are accepted as true, then other statements can be shown to follow from them (Serra, 1997). He went on to say, "A logical argument consists of a set of premises and a conclusion" (Serra 1997:687). A conclusion in a geometrical argument is reasonable only when it follows a certain kind of logic. Duturi (2013) maitains that in Mathematics, although there are many ways of solving the same problem, reasoning and logic matters a lot in order to establish facts.

Deductive reasoning is also called proof. Stefanowicz (2014) explains that a proof is a sequence of logical statements, one implying another, which gives an explanation of why a given statement is true. It is reasoning from proven facts using logically valid steps to arrive at a conclusion (Hunt 2008). Usiskin (1982) says that premises used in deductive reasoning are in many ways the most important part of the entire process of deductive reasoning. Usiskin confirms that when premises are not correct it becomes difficult for the conclusion to be valid. A proof is often used to verify that a conjecture is true. Proofs usually help mathematicians to answer the question "why".

Pulley (2010) investigated the effects of assessing reasoning skills on learners' understanding of proof. Pulley found that exposure to reasoning in different formats provided opportunities for classroom discussion. Learners tend to reason logically when exposed to class discussions where they air their views. Data from post-test revealed an increase in learners who were involved in the intervention and ability to identify flawed logical reasoning or incorrect geometric content. Kosze (2011) found that learners improved their use of deductive reasoning in all aspects of mathematical proving after being trained to think logically.

1.5. Research Related to Congruency

Van de Walle, Karp & Bay- Williams (2014: 426) say, “As students begin to think about properties of geometric objects without focusing on one particular object (shape) they are able to develop relationships between these properties”. They aver that with much practice in “the if- then reasoning”, learners can classify shapes using only a minimum set of defining characteristics. Observations go beyond properties themselves and begin to focus on logical arguments about the properties.

Dokai (2014) has shown that the elusive SAS theorem of congruence of triangles can be proved analytically. This research has given the world of Mathematics an entirely new and more uniform procedure for providing the congruence theorem of triangles using the cross section of a double cone. Dokai’s research asserts that the method of superposition be relegated to the primary school while the secondary and the higher levels of education should ensure and emphasize that the analytical method of proof be adopted when proving congruence.

Sears and Chavez (2016) undertook a descriptive study examining students’ performance on a proof task about corresponding parts of congruent triangles. They have noted that proof should not be taught as a topic, but as a way to communicate mathematical concepts.

Stefanowicz (2014) says that direct proof is probably the easiest approach to establish the theorem as it does not require knowledge of any special techniques. He noted that the importance of not missing out any steps as this may lead to a gap in reasoning. At times, learners are tempted to leave out information that turns to be irrelevant. Wilson (2011) says that some facts seem visually obvious to learners and they often see no need to go beyond their observations in proving it to be true. He found out that when learners are asked to prove if two triangles are congruent, they may simply mark the corresponding sides that they see as congruent as part of their proof approach, even if they are not given this information. They may then state the appropriate reason such as SAS, but they arrived at this conclusion by mere observation.

1.6. The Problem Statement

Euclidean Geometry is one of the core concepts in Mathematics Paper 2 in the FET Band in South Africa. The concept geometry constitutes more than one-third of the marks in Mathematics Paper 2. It is important that learners prepare for the Euclidean Geometry while in the lower grades. One of the mathematical competencies that learners must acquire is reasoning. Grade 9 learners tended to struggle with proving of congruent triangles as depicted in the last ANA report of 2014 (DBE, 2015). The report identified areas where learners were facing challenges in displaying their reasoning skills. I traced the same streams which were in Grade 9 in 2013 and 2014 and found that they struggled in the same concept at Grade 12 Paper 2 examination. The National Senior Certificate examination diagnostic reports of 2017 and 2018 revealed that the Grade 12 learners failed to name angles correctly and that was an indication that this issue has not been dealt with effectively in earlier grades (DBE, 2017 and DBE, 2018). Congruence of triangles tends to be a foundation for Euclidean geometry in the FET Band. The learners could not provide reasons for their answers in proving theorems and their converses.

During my many years of teaching both Senior Phase and FET Band learners has shown that there exists a challenge in the development of reasoning skills. The ninth graders also face challenges in communicating reasoning skills in proving congruent triangles. Aldolphus (2011) says that researches have shown that difficulty in teaching and learning of Mathematics, geometry in particular, has resulted in learners' failure in Mathematics.

1.7. Research Questions

In order to understand the problem statement mentioned above, the study looked at the following main research question:

What are the challenges faced by ninth graders in communicating reasoning skills in the proving of congruent triangles?

The following sub-questions were pursued in order to address the main research question:

1. How do ninth graders use properties of 2-dimensional shapes in proving congruency of triangles?
2. How do the ninth graders use congruence axioms to make deductions?
3. How do the ninth graders communicate their reasoning skills in the proving of congruent triangles?

1.8. Purpose, aims and objectives of the study

The study seeks to explore how the Grade 9 learners used the properties of 2-D shapes and the congruent axioms to communicate their reasoning skills in the proving of congruent triangles.

1.8.1 Research Purpose

The purpose of this study was to explore the reasoning skills of ninth graders in the proving of congruent triangles. In order to achieve this, a case study of a Grade 9 class was observed proving congruent triangles in their natural environment.

1.8.2 Research Aim

In order to realise solutions to the above mentioned problem statement and research questions the aims of the investigation are as follows:

- To expose the reasoning skills of ninth graders in the proving congruent triangles.
- To assess the learners' knowledge of the properties of 2-D shapes in the proving of congruent triangles.
- To establish learners' understanding of congruence axioms in the proving of congruent triangles.

1.8.3 Research Objectives

This study is intended to achieve the following objectives:

- To use properties of 2-dimensional shapes in the proving of congruent triangles
- To use the congruency axioms to prove congruent triangles.

- To determine how Grade 9 learners communicate their reasoning skills in the proving of congruent triangles.

1.9. Research Methodology and Design

1.9.1 Research Approach

In this study, a qualitative research design was adopted where a Mathematics class was studied as a case study. Maree (2012) says that qualitative research focuses on describing and understanding phenomena within naturally occurring context with the intention of developing an understanding of the meanings imparted by the respondents. Knowledge obtained from respondents in this manner is mainly reliable as this information from a primary source. McMillan and Schumacher (2010) say that the goal of qualitative research is to understand participants from their own point of view and said it in their own voice. In qualitative research design, emphasis is on the quality and depth of information (Maree, 2012). A case study design was used where the researcher observed participants in their conventional class.

1.9.2 Population and Sampling

The current research was a case study where a Grade 9 class of 32 learners was conveniently selected from six classes in one of the schools in Ethusini circuit in KwaZulu-Natal Province. The class was considered for study because it's small number of learners. Twelve learners were purposefully selected from the class to participate in focus group interviews. Maree and Pietersen (2010) define purposeful sampling as selection of participants because of some defining characteristics that make them holders of the information needed or the study. In this research, a site was selected where learners were involved in proving congruent triangles.

1.10. Instrumentation and Data Collection Techniques

I observed the selected Grade 9 class being taught congruence of triangles. I observed five lessons for one week. I observed the lessons as the teacher delivered them according to the school timetable. Maree (2012) defines observation in research as a systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them. The main focus of observation was how learners communicated reasoning skills in proving of congruent triangles. Queiros, Faria and Almeida (2017) define observation as a way of collecting data simultaneously with the occurrence of the event, without interfering with the occurrence of the event. I played the role of an observer as a participant where I did not influence the educator in the way he presented his lessons. I contracted a camera man who also took audio and videotapes for me. He also signed a consent form to take part in the study. I clearly explained to the participants the purpose of observation. I informed the participants that technology like videotapes and or audiotapes were to be used to collect data. I observed the ninth graders communicating reasoning skills in the proving of congruence of triangles.

I analysed the documents that the teacher used to prepare and plan his lessons. I used document analysis tool to evaluate the influence the documents had on the proving of congruent triangles. The researcher was interested in how participants communicated reasoning skills in the written exercises. To consolidate what learners displayed during observation, the researcher purposively selected two groups of six learners each to participate in focus group learners. Queiros et al. (2017) say that focus groups can provide a broader range of information and they offer the opportunity to seek clarification, if there are topics that need further clarification. I probed them where necessary to clarify their thinking. Although focus groups are believed to be hard to manage, in this case it was manageable as the number was reasonably small. Three one hour- sessions were conducted in focus group interviews. The three sessions were meant to analyse learners' knowledge of identifying 2-D figures, the properties of 2- dimensional shapes and their reasoning skills in proving congruence of triangles. The focus group interviews were audio taped and then transcribed by the researcher.

1.10.1. Data Analysis and Interpretation

Anderson (2010) says that analysis is not left until the end. In the process of collecting data the researcher was going on analysing every response and recorded it. There are a number of ways in which this can be done. I made rich descriptions of the collected data at each stage of data collection (See attached Appendix K and L). When I studied the literature and the theoretical framework, I found that there were concepts which appear more often than others. Concepts like deductive reasoning, proof and congruence dominated the literature. I therefore decided to take these concepts as themes in this research. The data was divided into themes so that the collected data became easy to compare and contrast. The data was coded into categories using the first four levels of Van Hiel's theory. The four levels are Visualization, Analysis, Informal deduction and Deduction. The last level, rigor was purposefully left out as it is above the understanding of the secondary school learners.

1.11. Definition of Key Terms

Here are some of the key words which are constantly used in this research. Their functional meanings are given below.

Geometric reasoning – a way of reasoning used to explore and analyse shape and space.

Deductive reasoning - the process of demonstrating that if certain statements are accepted as true, then other statements can be shown to follow from them (Serra 1997).

Congruency**-** when two polygons are exactly equal in size and shape

1.12. Preliminary Chapter Outline

Chapter 1: Introduction and Overview

This chapter introduces the research problem, justifies the study and gives a brief rationale of the study. It also explains the purpose of the study.

Chapter 2: Literature Review

The theory underlining the study is explained in detail. The research related to proving of congruent triangles and the communication of deductive reasoning are also discussed.

Chapter 3: Research Design and Data Collection

In this chapter, I describe the research design and the methodology to be followed.

Chapter 4: Results and Discussion

This chapter presents the research data and then discusses the findings of the study.

Chapter 5: Conclusions, Recommendations and Limitations of the Study

The study results and are summarised. I make recommendations based on the findings of the study.

1.13. Conclusion

Grade 9 learners face difficulties in the proving of congruent triangles. Proving of congruent triangles helps learners to reason logically. The proving of congruency influences students' reasoning capacity leading to the determination of their performance in Mathematics generally. This research seeks to investigate Grade 9 learners' reasoning skills in the proving of congruent triangles through an in-depth study of students' learning in their natural learning environment. The findings were presented in a descriptive manner. The highlighted difficulties will help inform a Mathematics teacher to find ways of handling such problems.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

The literature review is meant to explore ninth graders' reasoning skills in the proving of congruent triangles. Proving of congruent triangles stands to be an important skill in the preparation of Grade 9 learners for the Euclidean geometry in the Further Education and Training (FET) band. I will discuss learners' reasoning skills as they are vital in the proving of congruent triangles. Deductive reasoning skills stand to be inevitably important in the proving of congruent triangles. I will also unveil gaps that exist in the research one on the proving of congruent triangles. In this research, the Van Hiele's theory of geometrical thinking forms the theoretical framework as it guides on how students learn geometrical concepts. Siew, Chong and Abdullah (2013) believe that the theoretical approaches that are concerned with the development of the geometrical thinking of learners are important areas of pedagogical concern.

2.1 Literature Review

The literature review is an investigation of published journals, reports, research papers and publication in the field of Mathematics Education of the department of Basic Education and other countries (Chimuka, 2017). The identification of gaps that exist on learners' reasoning skills in the proving of congruent triangles in the current study will be explored.

2.1.1 Reasoning Skills

Reasoning and sense making are simultaneously the purpose for learning mathematics and the most effective means of learning it (NCTM, 2000). Clarke (2013) understands reasoning as argumentation, or as thinking used to draw conclusions from available evidence. Mujiasiah, Wahuya, Kartono and Mariani (2018) refer mathematical reasoning to the ability to formulate a given mathematical problem to explain and justify solutions or arguments. In our present secondary schools, teachers tend to be racing against time as they will be preparing learners for scheduled common tasks. Teachers resort to lecture method in order to finish the syllabus at the expense of learners constructing their own knowledge in Mathematics. Primasatya and Jatmiko (2018) found that students need to be trained not only to receive information, but also need to criticize various things related to that information. In our current Mathematics classes, learners tend to be more of passive recipients than constructors of knowledge. They find no reason to argue with the teacher who seems to be an authority of the subject content. This affects the way learners develop their own reasoning skills.

Marchis (2012) points out that geometry has an important place in the school curricula. Studying geometry provides many foundation skills and helps to build the thinking skills of logic, deductive reasoning, analytical reasoning and problem solving (Serra 1997). Lehrer and Chazan (2009) noted that within Mathematics, geometry is particularly well placed for helping people develop ways of thinking. They say that Geometry is an ideal intellectual territory within which to perform experiments, develop visual based reasoning styles, learn to search for invariants and use these and other reasoning styles to spawn constructive arguments. Primastya and Jatmiko (2018) argue that learners need to be trained not only to receive information but also need to criticise various things related to that information. One major purpose of any geometry course is to improve the ability of learners to reason logically (Hunt 2008). Hunt affirms that learners are introduced to the use of deductive reasoning to explain why patterns are true. The teaching methods play an important role to suggest what the learner will be doing in the lesson.

In Japan, Onda, Hirai, Penny, Indurkya and Kaneko (2017) found that learners begin to practice explaining their thoughts to others in primary school, but most learners do not practice generating logical proofs based on concrete reasoning until junior high school. Ahmady and Ruhi (2016) take reasoning skills, including making conjectures and developing deductive argument as important tools for establishing effective learning. Learners need hands on exercise where they defend their own reasoning in proofs. Most of the learners lack the ability to justify their answers verbally and it is worse when they are asked to put their reasoning in writing.

Gunhan (2014) identified geometric reasoning as having three levels called correct reasoning, flawed reasoning and poor reasoning. Correct reasoning is when the learner has a good geometric reasoning level such that the learner is correct in doing the estimation, correct in finding the relationship or is also correct in finding the solution. Flawed reasoning is when the learner has a level of geometric reasoning that is medium, the learner is true in the estimation, correct in finding the relationship but less correct in finding the solution. Poor reasoning skills occur when the learner has a lack of geometric reasoning level, the learner is wrong in doing the estimation, true or less correct in finding the relationship and is wrong in finding the solution. The learners' display of their reasoning skills should be used as a teaching point in the next lesson. Teachers should concentrate on following learners' reasoning skills. Teachers who focus on the average learner disadvantage the high flier and the struggling learner.

Wang, Wang and An (2018) assert that reasoning is one of the mathematical key competencies for K to 12 school learners nationally and internationally. Johnny, Abdullah, Atan, Abu and Mokhtar (2015) take reasoning skills as fundamental aspects in mathematics. Duturi (2013) also says that when learning Mathematics, reasoning and logic matters most in order to establish facts. Learners can solve the same problem in different ways and what matters most is their reasoning skills applied on the task. Bhat (2016) views reasoning as an important aspect which assists learners in gaining true knowledge because knowledge is basic on logic and rational. It helps learners in decision making, problem solving, cause relationships and make inductive and deductive generalisations. Mathematical reasoning, besides providing learners with conditions to engage in proving, allows them to go beyond the routine use of procedures towards learning concepts, properties, and procedures, as being logical, interrelated and coherent aspects of mathematics (Mata-Pereira and Pedro da Ponte, 2017). Geometric reasoning is reasoning from proven facts using logically valid steps to arrive at a conclusion (Hunt 2008).

Aydin and Halat (2009) compared geometric reasoning stages of learners from two categories. They used the Van Hiele Geometry Test designed to collect data. The results show that learners taking logic or proof based courses attain higher reasoning stages than learners taking other college level mathematics courses such as Calculus. The results also show that there is a correlation between Van Hiele levels and proof writing.

Magajna (2013) found that learners consider proving to be difficult and demanding. It is very important for every teacher to be able to determine the geometrical reasoning level of his/her learners. Miyazaki, Fujita and Jones (2017) confirm that learners at the secondary school level and beyond experience difficulties in understanding proof in mathematics in general, and in geometry in particular. Their studies were with learners about 14years of age as this was considered to be the likely time when they could just be starting to learn to construct proofs in geometry. It is very important to know when the learners start struggling with proofs. Their findings are in harmony with what is experienced currently in the secondary schools. Grade 8 or 9 learners really struggle to make proofs. This is the time they start to develop their geometric reasoning skills when they are exposed to proving of congruent triangles. Lim (1992) says that knowing learners' geometric levels helps the teacher to be able to select and use activities appropriate to learners at their level of thinking.

Wang et al. (2018) say that geometry reasoning plays an important role on developing learners' mathematical reasoning ability. They asseverated that the cognitive level of eighth graders is the beginning of formal operations stage. Geometric reasoning and proof development in this stage lays solid foundation for their future geometry learning and mathematical reasoning and proof development. Noraini (2009) says that traditional approaches in learning geometry emphasise more on how much the learners can remember and less on how well the learners can think and reason. Despite that most of the teachers are aware of the ineffectiveness of the traditional methods of teaching that hinder development of reasoning skills, they still resort to them for reasons best known to them. Gunhan (2014:14) say, "When it comes to geometrical concepts learners ought to be presented problems that allow them to use different reasoning skills". The teachers should reveal to the learners that there is no one way of geometry reasoning that is absolute right. The teacher is expected to adopt teaching methods which enhance the development of reasoning skills and proof competence. Teachers should not have all the correct answers as is experienced in today's lessons.

Pulley (2010) investigated the effects of assessing reasoning skills on learners' understanding of proof. Pulley found that exposure to reasoning in different formats provided opportunities for classroom discussion. Learners tend to reason logically when exposed to class discussions where they say out their views. Data from post-test revealed an increase in learners who were involved in the intervention had an ability to identify flawed logical reasoning or incorrect geometric content. The current study agrees with what Pulley discovered that learners need to be exposed to activities that will enhance their reasoning skills in geometry. However, there is need to establish the learners' reasoning skills before they are exposed to different formats for classroom discussion.

Kosze (2011) found that learners improved their use of deductive reasoning in all aspects of mathematical proving after being trained to think logically.

Edwards (1997) proposed five types of reasoning activities that are usually observed before the territory of proof. He identified these activities as noticing and constructing the pattern, describing the pattern, conjecturing, inductive reasoning and deductive reasoning. In young learners, the first reasoning skill is identifying and constructing patterns. This is followed by learners trying to describe patterns by putting rules into words. Some learners may describe the patterns verbally or by way of drawing pictures or diagrams. Some may formally describe the patterns using mathematical notations. Once the learners are able to describe patterns then they can make conjectures. This leads to the next level that is inductive reasoning. Again learners use examples to justify the truth. Learners need a smooth transition to move to deductive reasoning.

2.1.2 Deductive Reasoning

"Deductive reasoning or logical reasoning is the process of demonstrating that if certain statements are accepted as true, then other statements can be shown to follow from them" (Serra 1997: 680). He says that a logical argument consists of a set of premises and a conclusion. A conclusion in a geometrical argument is reasonable only when it follows a certain kind of logic.

Deductive reasoning is also called proof. Heinze and Kwak (2002) say that proof is one of the main methods for developing deductive reasoning ability and promoting the understanding of mathematics. Proof is when true facts lead to a valid conclusion. Usiskin (1982) says that the premises used in deductive reasoning are in many ways the most important part of the entire process of deductive reasoning. He avers that when the premises are not correct it becomes difficult for the conclusion to be valid. The concept of proof is central to meaningful learning, but is hard for students to learn (Stylianides 2011). In Mathematics, a proof is often used to verify that a conjecture is true. Proofs usually help mathematicians to answer the question "why". Hunt (2008) went on to say that deductive reasoning is based on premises and if the premises are true then the reasoning will be valid. Proving is a core mathematical activity, and it has gradually been accepted that justification and proving should be central to mathematical learning at all school levels (Komatsu, 2016). Deductive reasoning is unique in that it is the process of inferring conclusions from known information (called premises) based on formal logical rules, where conclusions are necessarily derived from the given information and there is no need to validate them by experiments (Ayalon and Even 2006).

Sadiki (2016) says that Geometry concepts entail deductive reasoning of proofs and representative diagrams. Jones (2009) found that providing a meaningful experience of deductive reasoning for learners at the school level appears to be difficult. Jones says that a range of research has documented that even after a considerable teaching input many students fail to see a need for logical reasoning such as explanation, argument, verification and proof. At Grade 9, the learners are expected to solve problems where they are required to justify the answers (DBE, 2015). According to the current curriculum, CAPS document, the learning of congruency of triangles is emphasised to the ninth graders. The current study aims to find ways to develop deductive reasoning skills through the proving of congruent triangles in the lower secondary education grades. The concept of congruency is developed in Grades 7 and 8 where the learners are expected to identify whether pairs of triangles are congruent and in Grade 9, the four axioms for congruency are introduced (DBE, 2014). The axioms are side, side, side (SSS); side, angle, side (SAS); angle, angle, side (AAS) and right angle, hypotenuse, side (RHS). Geometry is one of the components in the curriculum that influence the way learners develop their reasoning capacity.

Komatsu (2016) carried out a case study involving a pair of fifth graders and a pair of ninth graders where he was checking the notion of increasing content by deductive guessing which is useful in examining learner processes of generalisation of conjectures. He used a framework of Lakatos which describes learner processes of proofs and refutations. He found that learners could establish truth of a given statement and also generated mathematical knowledge. I agree with Komatsu’s way of establishing true statements about proofs, however there is need to find various ways in which they display their understanding of proof.

The way textbooks are structured has an impact on how the learners are exposed to the concept of proof. Fujita and Jones (2016) researched on the influence of Japanese textbooks for learners learning proof in geometry. They found that the aspect of proof is over-emphasised in the textbooks thereby shadowing them from seeing the rationale for grasping arguments based on empirical evidence while learning to write proofs on geometry. Teachers are supposed to use the textbooks sparingly, in order to encourage reasoning among the geometry learners.

Wang, Wang and An (2018) say that the proving of congruent triangle is the beginning of the rigorous deductive reasoning in the learning of geometry. Congruent triangles also lay a solid foundation for further geometric proof.

Jojo (2015) says that language is an essential tool in communication and perhaps Geometry stresses the use of language more than any other part in mathematics. Communication in mathematics is recognised as an important aspect of mathematics learning and it includes sharing and explaining ideas orally and writing (NCTM, 2000). As learners move towards deductive reasoning in the context of mathematics, the language of reasoning may be the same, but it is the nature of the evidence that is different. The language of mathematics is often critical in learner ability to explain and justify their reasoning. Learners who do not have the necessary mathematical vocabulary are limited, in verbalising their reasoning and being able to justify their reasoning. Powell, Stevens and Hughes (2018) found that teachers were using informal mathematics language to make the content more accessible for middle school learners. The teacher feels a sense of achievement when he/she is able to reach out to the attention and level of every learner. However, when it comes to assessment the students are expected to express themselves in formal mathematics language. Preparing learners to be successful in Mathematics requires teaching learners to recognise and use proper mathematics language to communicate mathematically (Powell et al. 2018). Learners develop a greater understanding of developing proofs based on deductive reasoning if they are given the opportunity to engage in argumentation and conjecturing as part of the proving.

Poon and Leung (2013) administered both a geometry test and a logic test to junior secondary students of between 13 and 14 years in Hong Kong. Their findings suggest that poor performance in deductive proof is highly related to the lack of good logical reasoning. Mariotti, Bartolini, Busi, Boero, Ferri, and Garuti (1997) assert that successful proof construction is dependent on continuity of reasoning or “cognitive unity”.

Brown et al. (2003) say that deductive geometrical reasoning can be more widely interpreted to also include deriving a specific value of a variable using both known theorems and known properties of shapes.

2.1.3 Congruent Triangles

Congruence is an important mathematical idea for human to understand the structure of their environment (Otalora, 2016). It is not a mini task to teach congruent triangles effectively. Wang, Wang et al. (2018) say that from the perspective of development of geometry content, congruent triangles reasoning and proof is the beginning of formal mathematical reasoning and proof. They assert that learners start to use formal language that contains “therefore” or “because” to prove congruent triangles. Alex and Mammen (2016) say that the proving of congruent triangles is the beginning of the rigorous deductive reasoning proof in learning geometry. It is typical material for cultivating learners’ geometric reasoning and proof. Brown, Evans, Hunt, Mclntosh, Pender and Ramagge (2011) say that congruence is an essential part of the early logical foundation of Euclid’s geometry and remains so in our present school courses. In Grade 9 learners are expected to prove congruent triangles in preparation of Euclidean geometry in the Further Education Band (FET), Grade 10 to 12. When learners fail to grasp the concept of proof of congruent triangles, they are likely to face challenges when they deal with Euclid’s geometry in Grade 10.

Patkin and Plaksin (2011) say that in the learning of the concept “congruency”, learners grasp that congruent triangles are triangles that can be placed one on top of the other such that they match at every point. If two triangles are congruent to each other then there are six equalities: the 3 sides are respectively equal and the 3 angles are respectively equal. Sadiki (2016) also says that when two triangles are equal in all respects, they are said to be congruent. Their angles are equal, their sides are equal and they can be placed exactly on top of each other and fit perfectly well. Their areas are also equal. When triangles have been proven congruent using three pairs of equal sides then then remaining three matching pairs are automatically equal. This is not true for three pairs of equal angles their remaining three pairs are not necessarily equal. This only applies in the case of congruent triangles (AAMT-Top drawer teacher 2013). They declare that congruent triangles are the result of combinations of 3 different transformations; translation, reflection and rotation. When shapes are translated, reflected or rotated, their shape and sizes do not change. Brown et al.

(2011) explain that if two triangles are congruent, such movement can always be done by a sequence of translations, rotations and reflections. They say that one will reflect the first figure in axis if it has the opposite parity to the second figure, then rotate the first figure until it fits exactly on top of the second.

Mironychev (2018) highlighted the following elements that must be considered for triangles to be congruent:

If two triangles satisfy Side Side Angle (SSA) conditions included angles are obtuse and the third angles are both acute, then such triangles are congruent.

1. If two triangles satisfy SSA conditions, included angles are acute, and the third angles are also both acute, then such triangles are congruent.
2. If two triangles satisfy SSA conditions, included angles are acute and third angles are both obtuse, then such triangles are congruent,

He pointed out the other theorems about congruent triangles which Atanasian, Butuzov and Kadomzev (2015) say that are regularly studied in geometry courses.

These theorems are:

Triangles are congruent if they have congruent two sides and medians to the third side or have two sides and congruent altitudes or have congruent one side, an altitude and a bisector.

The Curriculum and Assessment Policy Statement (CAPS), Mathematics, Grades 7-9 outlines how the concept of congruence of triangles is development in the Senior Phase. The concept is developed in the Senior Phase. In Grade 7, learners are expected to recognise and describe congruent figures by comparing their shape and size (DBE, 2012). The learners are to explore congruency with any 2-dimensional figures. Learners should recognise that two or more figures are congruent if they have all corresponding angles and sides equal (DBE, 2012). In Grade 8, learners continue to identify and describe the properties of congruent shapes. The curriculum challenges the subject teacher to design and prepare activities that are different in depth but are of the same concept for Grade 7 and Grade 8 learners. When the learners get to Grade 9, they are expected to investigate and establish the minimum conditions for congruent triangles (DBE, 2012). Brown et al. (2011) say that each congruence test will be justified by finding out what is a minimal amount of information in terms of side lengths and angle sizes needed to construct a triangle that is unique up to congruence, meaning that any two such triangles are congruent. At this stage, learners are expected to do constructions that will serve as a useful context for exploring and establishing the minimum conditions for two triangles to be congruent (DBE, 2012). The CAPS document outlines the conditions for two triangles to be congruent in the Senior Phase Band. DBE (2012) says that learners in Grade 9 should explore and establish that triangles are congruent when three corresponding sides are equal (SSS) when two corresponding sides and the included angle are equal (SAS); when two corresponding angles and a corresponding side are equal (AAS) and when right –angle, hypotenuse and one other corresponding side are equal (RHS). Grade 9 learners are expected to discover the minimum conditions for two or more triangles to be congruent. It was of interest to determine the extent to which learners in the current study were able to enact these expectations as per the CAPS document.

Cirillo, Todd and Obrycki (2015) noticed through their past experiences as learners, teachers and as observers of teachers that the teaching of triangle congruence tends to be done by decree. They observed that teachers tell their learners which triangle congruence criteria are valid and they have learners use them as postulates in proofs. The invalid criterion for triangle congruence, the Side-Side Angle (SSA) was quickly discussed with the provision of a counter example (Cirillo et al. (2015). The learners are likely to forget the concepts that they are told as compared to the knowledge which they experienced.

Poon and Leung (2013) found that one of the fundamental learning difficulties in geometry is rooted in learners' weaknesses in the understanding of the definitions and properties of mathematical concepts. These difficulties might be caused by the lack of seamless connection between primary mathematics and secondary mathematics curriculum. Miyazaki, Fujita and Jones (2017) studied the 14 year old learners constructing proofs in geometry. They found that it is the stage when learners start learning simple proofs and it is the time they find proofs difficult to learn.

Hession, Pres-Jennings and Kennedy (2016) acknowledged that Congruent Triangles is a challenging topic to teach effectively.

2.1.4 Communication in Geometry

Eubuomwan (2013) says that teachers must ensure that learners are given the opportunity to communicate in such a way that they develop vocabulary that is not only written but is also mental and pictorial. Every Mathematics topic has its own language appropriate to a particular age group. Jones (2002:126) says, "Encouraging learners to discuss problems in geometry, articulate their ideas and develop clearly structured arguments that support their intuitions can lead to enhanced communication skills and the recognition of the importance of proof". If the learners are exposed to an opportunity where they can discuss their ideas and thoughts, they are likely to understand the intended knowledge. This will also help the teacher to evaluate what the learners are capable of and will be able to plan the future lessons. Wheatley (2001) says that the process of verbalising mathematical concepts is not only important to help the teacher to understand what the child is thinking, but it also encourages the learner to think more deeply about mathematics. Wheatley says that on standardised tests, written explanation will rarely be required, but presenting their solution will help learners to be able to solve new problems whether or not an explanation is required. Learners should be in a position to support their mathematical arguments.

Group work in geometric proofs helps learners to correct each other's misconceptions and false assumptions (Pawlikowski, 2014). His knowledge of small group strategy helped him to realise the importance of teacher facilitation to achieve planned goals. Teachers should set clear objectives to help learners stay focused on the lesson goals.

Van Hiele indicated that the teacher and the students have a problem in communicating because they are on different levels. Eubuomwan (2013) says that one of the reasons for communication breakdown is the difference in the language used for different levels. He goes on to say that each level has its own set of language, symbols and its own network of relationships connecting the symbols. Usiskin (1982) noted that Van Hiele believed that language skills are particularly critical for creating and linking new ideas to past experiences and prior knowledge.

2.1.5 Research Related to Congruency

Van de Walle, Karp and Bay- Williams (2013: 426) say, “As learners begin to think about properties of geometric objects without focusing on one particular shape they are able to develop relationships between these properties”. They also said that with much practice in “the if- then reasoning”, learners can classify shapes using only a minimum set of defining characteristics. Observations go beyond properties themselves and begin to focus on logical arguments about the properties.

Sears and Chavez (2016) had a descriptive study examining learners’ performance on a proof task about corresponding parts of congruent triangles. They have noted that proof should not be taught as a topic, but as a way to communicate mathematical concepts. At times, learners are tempted to leave out information that turns to be irrelevant. Wilson (2011) says that some facts seem visually obvious to learners and they often see no need to go beyond their observations in proving it to be true. He goes on to say that at times learners just mark the corresponding sides that they see as congruent as part of their proof approach, when asked to show congruence in triangles. They may then state the appropriate reason such as SAS, but they arrived at this conclusion by mere observation.

Winer and Battista (2018) investigated learners’ proof reasoning as they moved from verbal planning to written proof. From the clinical interviews conducted, most of the proofs students wrote were not rigorous enough to stand up to scrutiny. They were many gaps in their written proofs’ logical and or axiomatic structure. They suggest that teachers and researchers should evaluate learners’ proofs using both their verbal explanations and their written proofs. Mujiasih et al. (2018) discovered that when the geometric reasoning in solving geometric problems has grown well, it is expected that students are able to write their ideas to be communicative for the reader. Although Mujiasih et al. (2018) were studying the growing of reasoning skills in the learning of Analytic Geometry the same knowledge is also applicable to the study of reasoning skills in the proving of congruent triangles. The learners’ communication skills should not be underrated in the proving of congruent triangles.

Jones, Miyazaki and Fujita (2015) developed a web-based learning support system that is designed for lower secondary school students who are just starting to tackle congruency based proofs in Geometry. The learning system helps learners to solve geometric problems by dragging sides, angles and triangles to one screen cell. The system is motivating as it translates figural elements to their symbolic form. When learners select a congruency condition, the system automatically provides feedback. This is a scaffolding exercise which offers an opportunity for students to learn geometric proofs in a way that is different from the textbook fashion (Jones et al. (2015). The web-based system provides a foundation for learners' development of geometric reasoning skills in proving of congruent triangles. However, the web-based system does not establish the learners' reasoning level in the proving of congruent triangles.

Siew, Chong and Abdullah (2013) say that learning geometry using tangram was perceived as enjoyable to unleash their thinking and creativity. Although their study indicated that effective learning takes place when learners are hands-on the objects of the study, the development of reasoning skills is lacking. Kosze (2011) found that learners improved their use of deductive reasoning in all aspects of mathematical proving after being trained to think logically. The author is of the idea that learners need training in reasoning skills in order to perform well in deductive inferences. Alex and Mammen (2014) carried out research in which they investigated whether the Grade 10 learners in South Africa were ready for formal proof in Euclidean geometry. The findings of the study lead to the importance on the delivery of instruction that is appropriate to learners' level of thinking. They also found that junior secondary school geometry curriculum implementers are not adequately preparing the learners to face the challenges in the senior secondary school.

Onda et al. (2017) developed a system called DELTA that supports the learners' use of backward chaining to prove the congruence of two triangles. DELTA is designed as an interactive learning environment and supports the use of backward chaining by proving hints and a function to automatically check the proofs inputted by the learners. They evaluated the efficacy of DELTA with 36 learners in the second grade of junior high school in Japan. They found that the experimental group performed better that the control group who were not exposed to the backward chaining.

The study of geometry contributes to helping learners develop the skills of visualisation, critical thinking intuition, perspective, problem solving, conjecturing, deductive reasoning, logical arguments and proof (Jones 2002). NCTM (2000) says that geometry enhances the reasoning and proving skills of learners; learners learn the relations among geometric shapes and their characteristics.

One of the most recent studies shows that teachers should provide specific examples or graphics at the introduction of proving congruent triangles lesson (Wang et al.; 2018). They suggest that teachers should guide learners to explore and discover the concepts' connotations and relationship with the other concepts through group cooperation. In most cases, examples provide a basis and guide for proving congruent triangles. Group discussion tends to be helpful for the learners as they practice to prove congruent triangles.

2.2 Theoretical Framework

Shonad, Kusmayadi and Riyadi (2017) acknowledge that one of the most important research conducted about the geometry thinking is the Van Hiele Geometry Thinking Level. Sadiki (2016) views Van Hiele's theory as a learning mode that takes into cognisance how learners progress in geometric cognitive thinking. Watan and Sugiman (2018) see the Van Hiele theory as a theory that can be used as an instructional teacher in the learning process and at the same time it can be used to assess the ability of learners. Jaime and Gutierrez (1995:592) say, "The Van Hiele model of mathematical reasoning has become a proved descriptor of the progress of students' reasoning in geometry and is a valid framework for the design of teaching sequence in school geometry". The Van Hiele's theory proves to be a theory that is suitable to establish the learners' reasoning skills level in the proving of congruent triangles.

Although the theory is many decades old, I have seen it to be a worthy theoretical framework for this study. The Van Hiele theory is one kind of theory where learners know how to apply what they have learned in a new situation (Van Hiele, 1959). Jojo (2017) used the Van Hiele levels of thought as her lens through which her geometry teaching study was viewed. Howse and Howse (2014) also used Van Hiele's theory of geometric thought and phase of learning as their framework for effective learning. Misnanti and Mahmudi (2018) say that it is crucial to describe the characteristics of the learners' geometry skills based on the levels of Van Hiele's thinking development in the geometry learning so that the teacher can provide the appropriate treatment to improve the geometry skill of the student of their thinking level.

2.2.1 Van Hiele's Levels of Geometric Thinking

The Van Hiele couple developed a pedagogical theory of teaching and learning of geometry in 1957. Way (2011) say that the Van Hiele theory puts forward a hierarchy of levels of thinking spanning the ages about five years through to academic adults. The theory is described by Crowley (1987) as a model which consists of five levels of understanding. The levels are Visualisation (level 1), Analysis (level 2), Informal deduction (level 3), Deduction (level 4) and Rigor (level 5). If assisted by appropriate instructional experiences, the model asserts that the learner moves sequentially from the basic level (visualisation), where space is simply observed to the highest level (rigor) which is concerned with formal abstract aspects of deduction. Alex and Mammen (2016) say that it is the quality and nature of the experience in the teaching and learning program that influences a genuine advancement from a lower to a higher level.

The Van Hiele levels were originally defined from level 0 to 4 (Armah, Cofie and Okpoti, 2018). Most of the researchers have renamed the levels from 1 to 5. This has allowed for a sixth level, pre-recognition level to be assigned level 0 (Mason 1998). Mason say that pre-recognition is the stage where learners can be able to differentiate triangles from quadrilaterals but fail to see the difference between a square and a rectangle. Alex and Mammen (2016) noted that the majority of the learners could not recognise common shapes in non- standard positions.

In the current study, I have numbered the Van Hieles' levels from 1 to 5. Level 1 (Visualisation) is the basic level where learners are aware of space only as something that exists around them. Geometric concepts are viewed as total entities rather than as having components. Clements, Sarama and Swaminathan (1997) conducted individual clinical interviews of children aged 3 to 6, emphasising identification and descriptions of shapes and reasons for identifications. They found that young children initially form schemas on the basis of feature analysis of visual matching to distinguish shapes. The learners were also capable of recognising components and simple properties of familiar shapes.

The next level is level 2 (Analysis) where learners begin to discern the characteristics of figures. The shapes are recognised as having parts and are recognised by their parts. After using several examples, students can make generalisations. For example, after learning that opposite angles of a parallelogram are equal, they can generalise that for all parallelograms.

It is until level 3 (Informal deduction) that learners can establish interrelationships of properties both within figures (in a triangle where all sides are equal result in all angles of the same triangle being equal) and among figures (a square is a rectangle because it has all the properties of a rectangle). At this level, definitions are meaningful and informal arguments can be followed and given.

The significance of deduction as a way of establishing geometric reasoning within an axiomatic system is understood at level 4 (Deduction). The learner can construct proofs, the possibility of developing a proof in more than one way is seen, the interaction of necessary and sufficient conditions is understood and the distinction between a statement and its converse is made.

The last level, level 5 (Rigor) is when geometry is seen in the abstract. The learner can work in a variety of axiomatic systems, that is, non-Euclidean geometry and different systems can be compared. According to Van Hiele this is the least developed level since most of the high school geometry is at level 4.

The geometrical thinking levels are sequential and hierarchical. There is an assumption that most teachers think that learners operate at the same level in a Geometry class. Most of the teachers' lesson plans are prepared with an average learner in mind. Van Hiele observed that two persons who are reasoning at different levels will not understand each other (Van Hiele 1984). The situation becomes worse when the classes are over populated. The educator tends to struggle to pull learners of different geometrical thinking levels together in one lesson. Evbuomwan (2013) noted that it is through the discord of the hierarchical nature of Van Hiele's levels within the teacher and the learner operating at different levels that account for much of the difficulties which learners have in the process of learning geometry. I t is the duty of each geometry teacher to determine the learners' reasoning level according to Van Hiele's theory and tries to teach them at their level of operation in order to realise meaningful learning.

De Villiers (1987) developed six geometric thought categories from Van Hiele's geometric thinking levels. The geometric thought categories are intertwined. For example at level 2, one learner may only be able to use vocabulary about isosceles triangles while another learner is able to describe the properties of an isosceles triangle. Within the same level, learners may operate at different sub-levels.

Usiskin (1982) noted that most of the learners do not achieve the deductive level even after successfully completing a proof-oriented high school geometry course, probably because material is learnt by rote, as the Van Hieles claimed. He goes on to say that many high school learners are still at level 2 or even level 1 when they are expected to be at level 3, ready to proceed to level 4. This appears to be similar to the present high school students in South Africa. There is need to develop deductive reasoning as from the lower levels of high school geometry.

Mason (1999) describes a learner's progress according to Van Hieles' levels of thought as a result of instruction that is organised into five phases of learning. The phases are: Information-the teacher identifies what learners already know about a topic and the learners become oriented to the new topic. Guided Orientation - the learners explore the objects of instruction in carefully structured tasks such as folding, measuring or construction. The teacher ensures that learners explore specific concept**.** Explicitation- is when learners describe what they have learned about the topic in their own words. Free orientation- learners apply the relationships they are learning to solve problems investigate more open-ended tasks. At integration, learners summarise and integrate what they have learned, developing a new network of objects and relations. A learner cannot achieve one level of understanding without having mastered all the previous levels. Mostafa, Javad and Reza (2017) say that the importance of learning action between learners and teacher is emphasised within phases of instruction of Van Hiele model. Teachers must organise geometry teaching methods which facilitate growth of learners from the current thinking level to the next thinking level. Way (2011) says that a deliberate instruction is needed to move children through several levels of geometric understanding and reasoning skills.

Usiskin (1982) says that studies found that many children reason at multiple levels, or intermediate levels, which appears to be in contradiction to the Van Hiele theory. He further says that learners also advance through the levels at different rates for different concepts, depending on their exposure to the subject. For example, learners can reason at one level on triangles while they can reason at another level when doing properties of quadrilaterals. Usiskin (1982) points that some researchers have found that many learners at Visualisation level do not reason in a completely holistic fashion as Van Hiele indicated but may focus on a single attribute, such as the equal sides of a square or the roundness of a circle. Although Van Hiele's theory received criticism and modification from other researchers, it still stands as a valid and worthy theory to base on when developing geometric reasoning skills in learners. Alex and Mammen (2017) recommend that educators who facilitate geometry learning in grade 10 need to familiarise themselves with the Van Hiele levels in order to achieve effectiveness in the teaching and learning interface of geometry concepts.

Al-ebons (2016) says that one of the theories that helps greatly and effectively in teaching geometry is Van Hiele's theory which attracted scholars and educationists' attention the world over. It is considered one of the most important models in teaching geometry and geometric concepts. Khoh acknowledges that today's teachers have an advantage of having Van Hiele's theory of geometry learning to help them understand their pupils' difficulties. Although the Van Hiele theory is found to be important by many mathematics educators and researchers, my worry is whether our South African teachers are aware of this theory. If ever they are aware of the theory, how many of our practising educators implement the ideas?

2.3 Conclusion

In the field of geometry, the best and most well-defined model for learner levels of thinking is based on Van Hiele's model (Abdullah and Zakaria, 2013). Although Van Hiele's theory is accepted as one of the most appealing theories for the teaching of geometry, proving of congruent triangles at grade 9 remains a challenge in our South African classrooms. Something that a lot of our lesson plans are missing is an understanding of the Van Hiele levels and how it plays into understanding geometry concepts (Geometry 2017). UK Essays (2018) says that teachers should provide teaching that is appropriate to the level of children's thinking.

CHAPTER 3

RESEARCH METHODOLOGY AND DESIGN

3.1. Introduction

In this chapter, I am going to justify the research philosophy and how the study is carried out. I will present the research design and its justification, research population and sample, data collection instruments and techniques, trustworthiness issues and then ethical considerations.

3.2. Research Philosophy

This study adopts the research philosophy of interpretivism. Pulla and Carter (2018) assert that if a researcher is interested in choosing to interpret a certain human behaviour or wishes to study the interactions or social relationships and build an in-depth understanding of people's lives, a research method that would be suitable in this case would be the interpretivist research paradigm. Adom, Yeboah and Ankrah (2016) assert that the constructivism or interpretivist philosophical paradigm is associated with the qualitative research approach because it seeks to understand the phenomenon under study from the experiences or angles of the participants using different data collecting agents. They say that the researcher engages himself in activities where he experiences it himself or sees others experiencing it.

Knowledge is believed to be socially constructed and therefore becomes subjective to the context of the participants. The core idea of interpretivism is to work with subjective meanings already there in the social world (Goldkuhl, 2012). Kuyinja and Kuyini (2017) are of the view that there is an assumption that the researchers and their subjects are engaged in interactive processes in which they intermingle, dialogue, question, listen, read, write, and record research data. Pulla and Carter (2018) view interpretations as varying from individual to individual and seem to suggest or attest that all truth seems to become relative therefore suggesting that interpretivist approach be a better way of capturing behavior phenomena. In the exploration of the reasoning skills of ninth graders in the proving of congruent triangles, I chose to carry out a case study where the participants are deeply understood in their unique social context.

Pham (2018) outlined the advantages of interpretivist research as the researchers cannot only describe objects, human or events but also deeply understand them in social context.

According to Lincoln and Guba (1995) and Morgan (2007), research conducted under the interpretivist paradigm usually exhibits the following characteristics:

- The admission that the social world cannot be understood from a standpoint of an individual.
- The belief that realities are multiple and socially constructed.
- The acceptance that there is inevitable interaction between the researcher and his/her research participants.
- The acceptance that context is vital for knowledge.
- The belief that knowledge is created by the findings can be value laden and the values need to be made explicit.
- The need to understand the individual rather than universal laws.
- The belief that causes and effect are mutually interdependent.
- The belief that contextual factors need to be taken into consideration in a systematic pursuit of understanding.

In other words, interpretivissts believe that realities depend on the context in which the social interactions occur. Individual truth is accepted on particular context. Context plays a vital role in understanding a particular phenomenon. Interpretivism negates the idea of absolute truth but believes that an outcome depends on context.

3.3. Research Design

In order to address the study problem logically, I have decided to adopt a case study research design. Yazan (2015) says that a case study is one of the most frequently used qualitative research methodologies. Lacono, Brown and Holtham (2009) define a case study as a research strategy which focuses on understanding a phenomenon within its natural setting. They say that case studies are the preferred research strategy when the phenomenon cannot be divorced from its context, and the focus is on contemporary events and the experiences of the actors are important. It again addresses why participants behave in a certain way. In this research, a case study research design helped me solicit learners' real approach and understanding in their proving of congruent triangles. I observed participants in their conventional class.

Nath (2005) asserts that a case study provides a very detailed picture of an individual, an organisation a particular program, a school or other entity. In a case study, instruments like classroom observation, interviews, protocols and audiotapes are used to understand a phenomenon. One of the advantages of a case study research design is that you can focus on specific and interesting cases (https://explorable.com/casestudy-research design). The focus of this study allowed me to use a case study as my research design since the rich descriptions of the study enabled me to explore the learners' reasoning skills in the proving of congruent triangles. Data was collected in the natural setting of the participants. Nath (2005) asseverates that for the research design to be effective it should be guided by research questions and well supported by theory, learning theory, organisational theory or social theory with a detailed review of the literature.

Heale and Twycross (2017) cite the volume of data as one of the disadvantages of a case study because it is difficult to organize and analyse and its integration strategies need to be carefully thought through. They argue that there is a temptation to veer away from the research focus. I asked a co-researcher in this study to assist with the collection of data. We assigned each other roles so as to make sure that all the data is captured accordingly. We used both audiotape and videotape so that we could capture all the relevant details.

3.3.1 Population

This study population includes all the Grade 9 learners in the six Grade 9 classes from the selected school in Ethusini Circuit in KwaZulu-Natal province.

3.3.2 Sampling

I have conveniently chosen to involve my neighbouring secondary school to participate in this study because it is cost effective. A Grade 9 class of 32 learners was conveniently chosen from six classes. The six classes exhibited similar characteristics like that they were all doing Mathematics, were all from the same school and were all doing congruency. Dornyei (2007) defines convenience sampling as a type of nonprobability or nonrandom sampling where members of the target population that meet certain practical criteria, such as easy accessibility, geographical proximity, availability at a given time, or the willingness to participate are included for the purpose of the study. Etikan, Musa and Alkassim (2016) regarded convenience samples as 'accidental samples' because elements may be selected in the sample simply as they just happen to be situated, spatially or administratively, near to where the researcher is conducting the data collection. In this case, the class was chosen because of its small number of learners as compared to the other classes. Patton (1990) says that qualitative inquiry typically focuses in depth on relatively small samples selected.

Two focus groups of six members each were selected to give in-depth information about proving of congruent triangles. The learners were expected to display their reasoning skills in proving congruent triangles. A purposive sampling was done to choose those who participated in the focus group interview. The high fliers were considered for sampling but also considering the gender equity. The learners who dominated in the class discussions were considered for focus group interviews. Focus groups typically involves bringing together people of similar background and experiences to participate in a group interview about major program issues that affect them (Patton 1990). I decided to have two groups to participate in separate focus group learners in order to compare the data.

3.4. Data Collection Instruments

To answer the research questions and establish the reasoning skills of Grade 9 learners in the proving of congruent triangles, three instruments were used. The instruments were classroom observation, focus group interviews and document analysis. The instruments helped to unveil the reasoning skills of learners in the proving of congruent triangles. Learners were observed learning proving of congruent triangles in their conventional class. I played the part of a participant observer in the lessons. At the end of the observation sessions, learners' exercise books were analysed. The learners' daily work on proving of congruent triangles was also checked. I interviewed two focus groups of six learners each.

3.4.1. Classroom Observation

Maree (2012) defines observation in research as a systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them. Queiros, Faria and Almeida (2017) posit that at observation is a way of collecting data simultaneously with the occurrence of the event, without interfering with the occurrence of the event. Mack, Woodsong, MacQueen, Guest and Namely (2005) define participant observation as an appropriate method for collecting data on naturally occurring behaviours in participants' own contexts. Kawulick (2005) defines participant observation as the process enabling researchers to learn about the activities of the people under study in the natural setting through observing and participating in those activities. In participant observation the researcher assumes two different roles at the same time. It then suggests that the research has to be more careful on how he/she collects the data. I assumed the duty of a passive participant observer. Owen (2014) says that passive participant exists when the researcher is physically at the location where observations are being made but he acts as a pure observer. The researcher has the option of place to interact if he/she chooses. I used recording audiotape and videotape to capture the data. I also relied on my memory to record the data after the observation session. I utilized an assistant to record notes for me during the observation.

During classroom observation it is important to observe not only what people are doing, by interpreting their body language and gestures, but also what people are not doing (Spradley, 2016).

Lacono et al. 2009) cite lack of objectivity as the researcher is not an independent observer, but a participant, and the phenomenon being observed is the subject of research. I played the role of an observer as a participant where I did not influence the educator in the way he presented his lessons. I clearly explained to the participants the purpose of the observation. I informed the participants that technology like videotapes and or audiotapes will be used to collect data. I observed the ninth graders communicating reasoning skills in the proving of congruence of triangles. I immersed myself in the daily activities of the people being studied.

I used an observation protocol to record what transpired in each observed lesson (see Appendix H). Shyyan, Christensen, Mitchell and Ceylan (2018) encouraged observers to rely on handwritten notes during a classroom observation. I filled in the pre-observation data which included the physical arrangement of the class, the lesson objectives and the intended outcomes prior to the observation. I recorded what the teacher and the learners were doing, their interaction and how the learning aids were utilised. I took detailed notes in real time as I observed the classroom activities. Immediately after each lesson I made a quick reflection of what happened in the lesson. I recorded the summary of the overall approach to the classroom instruction, the procedure of the activity and how the learners responded.

3.4.2 Focus Group Interviews

Focus group interview is a qualitative research method which has surfaced in educational research to explore learner perceptions, attitudes, beliefs and experiences (Billups, Johnson and Walles University, 2012). Focus group interviews are typically based on homogeneous groups (Patton, 1990). In this study, I consolidated the data collected through participant observation and document analysis with focus group interviews. Queiros et al. (2017) aver that focus groups can provide a broader range of information and they offer the opportunity to seek clarification, if there are topics that need further clarification. Nyumba, Wilson, Derrick and Mukherjee (2017) assert that focus group interview is frequently used as a qualitative approach to gain an in-depth understanding of social issues. For the focus group interview, I prepared semi-structured questions. I probed learners where necessary to clarify their thinking. Although focus groups are believed to be hard to manage, in this case it was manageable as the number of participants was reasonably small. Bolderston (2012) asserts that with semi-structured interview format, the agenda is relatively set but the interviewer is free to follow the respondent's train thought to explore tangential that may arise. I had the advantage of rephrasing the questions and how to ask them. Three one hour sessions were conducted in focus group interviews. The prepared questions were more of a guide to the exploring of learners' knowledge of properties of 2-D shapes, congruence axioms and their reasoning skills in proving congruence of triangles (see Appendix I).

I made sure that the participants were in a relaxed atmosphere. I frequently visited the site myself in order to build a positive relationship with participants in order to gain their confidence. In these focus group interviews, I asked questions to the learners while my colleague took notes and audiotaped the discussions.

I engaged a photographer to take both photographs and videos during the focus group interview. The audio and video tapes were transcribed by the researcher. She also signed the consent form for her to assist with taking of photographs.

3.4.3 Document Analysis

Document analysis is another instrument I used to determine learners' reasoning skills on the proving of congruent triangles. The method was used as a tool to provide details that informants may have forgotten and they can track change and development (Triad 3, 2016). Therefore document analysis was used to complement classroom observation and focus group interviews as it is incomplete by itself. In other words document analysis was used to support and strengthen the research. Triad 3 (2016) says that document analysis is an efficient and effective way of gathering data because documents are manageable and practical resources. In this study, we have chosen document analysis as a method that is cost efficient.

Bowen (2009) defines document analysis as a systematic procedure for reviewing or evaluating documents both printed and electronic material.

Triad 3 identifies types of primary documents as public records, personal documents and physical evidence. In this research endeavour, worked on the learners' written exercises on the proving of congruent triangles and also checked the educator's lesson plans and notes. I analysed the textbooks that they were using to do the exercises.

I designed a tool to use to analyse the documents used by the teacher to prepare and teach his congruence lessons (see Appendix J). I planned to analyse the teacher's lesson plans and the books he consulted in the teaching of congruence. Unfortunately the teacher did not have any lesson plans for the teaching of congruent triangles to the Grade 9 learners. The teacher used the CAPS curriculum, two textbooks and a workbook from the Department of Basic Education. The three textbooks were Mathematics Today Grade 9 Leaner's book, Platinum Mathematics Grade 9 Learner's book and Mathematics in English Book 1 Grade 9 Term 1 and 2. Textbooks have their own advantages and disadvantages depending on educational purpose, the learners' needs and the availability of time for a course (Mironychev, 2018). The workbook was provided by the Department of Basic Education. I have tried to minimise researcher bias by treating the documents as respondents that provide me with relevant information (Triad 3, 2016).

The textbooks used were authentic documents recommended by the DBE to be used in the secondary schools. The author's individual bias in each textbook was treated as a weakness of the textbook on the topic of congruent triangles.

3.5. Credibility and Trustworthiness

Gunawan (2015) postulates that a study is trustworthy if and only if the reader of the research reports or judges it to be so. Therefore the trustworthiness criteria should be pragmatic choices for researchers concerned about the acceptability and usefulness of their research for a variety of stakeholders (Lorelli, Nowell, Norris, White and Moules, 2017). In this section of research, I explained how I endeavoured to make the study findings to be credible, transferable, confirmable and dependable. Kline (2011) defines trustworthiness as the conceptual soundness and standard of credibility with which research is judged in the qualitative paradigm.

MaCnee and McCabe (2008) in Anney (2014) define credibility, as the confidence that can be placed on the truth of a research. Lorelli et al. 2017) say that for data to be credible it should be analysed by more than one person. I explained a few credible strategies such as prolonged engagement, persistent observation, triangulation and member checks. The presence of an observer influences behavior of participants. I frequented the research site in order to build trust and confidence. This helped me to gain insight into the context of the study. I also promised participants that I was not going to judge their behaviour or performance in class. I spent much time with the participants in order to be able to understand their behaviour.

To ensure trustworthiness, the role of triangulation must be emphasised (Gunwan, 2015). He says that triangulation reduces the effect of investigator bias. There are three major triangulation techniques which can be used to validate data. The techniques are data triangulation, methodological triangulation and triangulation that uses multiple researchers to investigate the same problem. I used data triangulation where different instruments in data collection are involved. Anney (2014) says that a researcher can use different sources of data or research instruments to enhance the quality of the data from different sources. I also asked peer researchers to review the collected data. In this research, data collected through observation and focus group interviews was compared and contrasted.

I double checked with the participants for accuracy during data collection and data analysis. I endeavoured to curb researcher bias by doing member checks in the process of analysing and interpreting the findings. In addition I allowed the respondents to read through the data they provided and to give the feedback. McMillan and Schumacher (2010) assert that the researcher should ask the participants to review researcher's synthesis of research findings with the participants for accuracy of representation. I constantly checked what participants meant in each statement.

The research findings should be transferable. Bitch (2005) in Anney (2014) defines transferability as the degree to which the results of a qualitative research can be transferred to other contexts with other respondents. I provided detailed description of each research stage, methods used and context in which the research was conducted. Anney (2014) says that the researcher should elucidate all the research process from data collection, context of the study and production of the final report. I kept every detail of the observation notes, interview audiotape records and written test scripts. This will vindicate me from all biased thought since evidence of study findings are based on collected data, context and observations made. Anney (2014) says that the audit trail also establishes confirmability of the study.

Confirmability is the degree of neutrality in the research findings (Anney, 2014). In this case, I provided detailed record which shows every step of data analysis that was made in order to support findings.

Lastly the issue of dependability should be established in a qualitative research (Anney 2014). Dependability refers to the extent to which another researcher could replicate the study. This is possible when the context in which the study was carried out is well explained. I have made certain that the audit trail is available. This will allow another qualitative researcher to repeat the same research under similar conditions and can still obtain similar study findings.

3.6 Data analysis and interpretation

Bogdan and Biklen (1982) define data analysis in qualitative research as systematically searching and arranging the interview transcripts, observation notes or other non-textual materials that the researcher accumulated to increase the understanding of the phenomenon. Anderson (2010) says that analysis is not left until the end. In the process of collecting data I analysed every response and recorded it. Glaser and Laudel (2013) say that data analysis that moves from texts to theoretical explanations assumes that not all that is said in a text is relevant to a specific research question.

I commenced data analysis by reading the raw data many times so that I became familiar to the collected information. Wong (2008) says that analysing qualitative data entails reading a large amount of transcripts looking for similarities and differences. In the process, I looked for the basic patterns in the data. I checked whether the collected information addressed the research questions. Gibson and O'Connor (2003) opine that qualitative analysis is more concerned with meaning where valid information helps to answer research questions.

I separated and organised data into themes. Themes are defined as common trends or ideas that appear repeatedly throughout the data (https://www.cdc.gov/healthyyouth/evaluation/pdf/). At this stage, coding starts. Coding is the process of identifying and labeling themes within the data that correspond with the evaluation questions I wanted to answer. Gibson and O'Connor (2003) acknowledge that formal systems for the analysis of qualitative data have been developed in order to help researchers get at the meaning of their data more easily. I grouped similar information together in categories. I organized the data into themes. The themes were checked whether they were related to each other.

In case studies, there are deviations that can be recognized from the pattern of the data. I found actors that could explain the deviating data. I checked the corresponding findings from other researches that tended to match the emerging patterns.

Case studies take a more holistic approach to qualitative research. In this single case study, I explored participants' experiences of complex phenomena in a single setting.

I viewed each theme that arose during the coding process and identified similarities and differences in responses from participants with differing characteristics.

3.7 Ethical Considerations

I sought permission from KwaZulu-Natal (KZN) Department of Basic Education to carry out research among the Grade 9 learners in the Ethusini Circuit in Umlazi district. Since participants were minors, their parents were given an informed consent form to sign to show that they agree to let their children to participate in the research without being forced or coerced (McMillan and Schumacher 2010). The participants were given the assent form to fill in before they participated in the study.

I fully disclosed to the participants the purpose of the research and its intended beneficiaries. I made it clear to the participants that participation in this research is voluntary. The participants were promised that their responses to the research were not associated with their names at all. Their responses were confidential and private. The researcher kept the recorded data separate from the research. It is only the researcher who knows their responses and is responsible enough as not to expose them to the public through any other way that would make them identified.

Research ethics deals primarily with the interaction between researchers and the people they study (Mack, Woodsong, MacQueen, Guest and Namely, 2005). They assert that agreed-upon standards for research ethics help ensure that researchers explicitly consider the needs and concerns of the people they are studying, that appropriate oversight for the conduct of research takes place, and that a basis for trust is established between researchers and study participants. To ensure that this study meets the requirements for ethical consideration, I sought ethical clearance certificate from the University of South Africa, sought permission from all relevant authorities and also sought permission from individual participants. In this research, I prioritised the security and respect of my study participants.

3.7.1 Ethical Clearance by the University

This research sought ethical clearance certificate from the University of South Africa before embarking on the collection of data.

3.7.2 Informed Consent

I sought permission from Kwazulu-Natal Department of Education to carry out research among the Grade 9 learners in Ethusini circuit in Umlazi district. Since participants are minors, their parents were given informed consent form to sign to show that they agreed to let their children to participate in the research without being forced or coerced (McMillan and Schumacher 2010). The participants also signed informed consent before engaging them in the research.

I fully disclosed to the participants the purpose of the research and its intended beneficiaries. I clearly communicated to the participants the length of time I will be with them, what I expected from them and promised them that there is no risk involved in participating in this study. I made it clear to the participants that participation in the research is on voluntary. Those who were going to be involved in the interviews could decide to opt out if they were no longer interested.

Informed consent forms for responsible authorities such as the Department of Basic Education, parents and participants were designed. Participant consent forms were also designed (See attached appendices, A, B and C).

3.7.3 Assent Form

The prospect participants in this current research are minors aged below 18 years. They were invited to sign an assent form in order to participant in the research. The purpose of the research was fully explained to the learners before they made their own decision to participate in the research. A learner was allowed to choose to participate in the lesson observation and not in the group interviews. Therefore the learner is expected to sign for each data collection instrument used.

3.7.4 Minimise the risk of harm

The research took place in a normal educational environment which is already risk free. The study did not change the setup or bring in something new to the learning environment. I brought the recording devices which the participants were already informed of before signing their informed consent forms.

3.7.5 Anonymity and confidentiality

The participants were promised that their responses in the research will not be associated to their names at all. Their responses are confidential and private. The researcher will keep the recorded data under a password locked laptop. It is only the researcher who knows their responses and should be responsible enough as not to expose them to the public through any other way that will make them identified. When analysing collected data, I used pseudonyms in order to hide the identity of the participants.

3.8 Limitation and delimitation of the study

The research results are from a school in Ethusini circuit of KwaZulu-Natal Province who were in an urban setting. A class of 32 learners was used to determine the reasoning skills of the Grade 9 learners in the proving of congruent triangles. The focus of this research is to explore the ninth graders' reasoning skills in proving congruent of triangles. Results in this research reflect an in-depth study of a definite setting which cannot be generalised to the performance of the Grade 9 learners in Mathematics.

3.9 Conclusion

This study sought to explore the reasoning skills of Grade 9 students in the proving of congruent triangles. A case study research design has been adopted where participant observation, document analysis and focused group interviews were conducted. Ethical permission was also sought from the University, Department of Basic Education, parents of participants and participants themselves.

CHAPTER 4

PRESENTATION OF RESULTS AND DISCUSSION OF FINDINGS

4.1 Introduction

The instruments used to collect data were discussed in depth in Chapter 3. The three instruments used were the classroom observation, focus group interviews and documents analysis. These methods were used to solicit knowledge about learners' reasoning skills in the proving of congruent triangles. The three methods were seeking to answer the same questions. In this section of the study, I will present the findings of each method used separately. I will then discuss all the findings of the research under the same sub-heading. The collected data was manually coded and interpreted in light of the literature review and the Van Hiele theoretical framework. The recorded data was transcribed into texts as Transcript 1 and Transcript 2 (see appendices K and L). The data was coded into categories using the first four levels of Van Hiele's theory. The four levels are Visualization, Analysis, Informal deduction and Deduction (Van Hiele 1986). Basically, the Senior Phase learners in Grade 9 are expected to be operating at level 3 of Van Hiele's levels. Level 5 of Van Hiele's theory was purposefully left out in this part of research as it is outside the scope of secondary school mathematics (see literature, chapter 2). These categories have been adopted in order to answer the research questions found in chapter 1. The main research question raised was: What are the challenges faced by ninth graders in communicating reasoning skills in the proving of congruent triangles? This research question was further clarified by the following sub-questions:

How do ninth graders use properties of 2-dimensional shapes in proving congruency of triangles?

How do the ninth graders use congruence axioms to make deductions?

How do the ninth graders communicate their reasoning skills in the proving of congruent triangles?

4.2 Presentation of Results

4.2.1 Context

The participants involved a Grade 9 class of 32 learners taught by an experienced and qualified male educator. The educator was 17 years of experience in the teaching of Mathematics to the senior phase band learners by the time of observation. The classroom observations were done from 16 to 20 September 2019. The learners remained in their classrooms while educators took turns to come and teach different subjects as they appear in the timetable. The focus group participants were purposefully chosen from the same class which participated in the classroom observation. Two groups of six participants each were chosen after the classroom observations. I was informed by the competencies of the learners in expressing themselves during the classroom observations. The focus group interviews were carried out one week after the classroom observations. The interview sessions came at the same time of the day for three days for each group. The discussions were carried out in a different room with a different setting from the learners' usual classroom. The focus group interviews were done from 7 to 14 October 2019. The focus group interviews were followed by document analysis. The educator used two textbooks, Mathematics Today Grade 9 and Platinum Mathematics Grade 9 plus the Grade 9 mathematics workbook from the Department of Basic Education. He also consulted the Mathematics CAPS curriculum document.

The observed class was made of mixed ability grouping (see Table 4.1).The learners' ages ranged from 14years to 17 years old. About 84% of the learners were between 14 and 15 years old (see Table 4.2). This is the age group Van Hiele suggested to be at the Informal deduction level (Miyazaki, Fujita and Jones, 2017). The lessons were taught by a professionally qualified and experienced teacher. The teacher prepared worksheets for the learners from the workbook.

Gender	Number	Percentage

Girls	14	43.8%
Boys	18	56.2%
Total	32	100%

Table 4. 1 Distribution of Respondents by gender

Age	Number	Percentage
15	6	18.8%
16	21	65.6%
17	4	12.5%
18	1	3.1%
Total	32	100%

Table 4. 2 Distribution of respondents by age

[Table 4.1 and 4.2 adapted from Adetunji, 2017]

4.2.2 Classroom Observations

I observed five consecutive one hour lessons in Mathematics while learners were taught proving of congruent triangles. The lessons were observed from 16-20 September 2019. The teacher used mainly the talk and chalk method. I used the observation schedule to record the lessons. All five lessons were recorded and transcribed afterwards. I took photographs of the teacher's chalkboard illustrations and learners' written work after every lesson observed. I was among the learners as a participant observer. The educator used the ready-made lesson plans from the Department of Basic Education. (Appendix H). The learners were referred to as Learner 1, Learner 2 and so on in each lesson.

Below is an extract of Transcript 1 of the Classroom Observation, lesson 2.

The educator started the lesson by referring the learners to the previous day lesson.
The learners explained in their own understanding what they know about the rule SSS in proving congruent triangles.
Learner1: *SSS refers to triangles with equal sides.*
Another learner called out from the class saying that *the triangles have 3 sides*.
Learner 2: *The 2 triangles have corresponding sides*.
Learner 3: *SSS means that the 2 or more triangles have their corresponding sides equal and also their corresponding angles equal too*. The learners agreed with this answer.
The educator proceeded to explain congruence in another set of triangles.
The educator emphasized the importance of an included angle. The *axiom SAS means the angle is an included angle*.
The class noted that the angle is always an included angle not just any angle on the triangles.
The class discussed the importance of the order of naming corresponding sides and corresponding angles in the congruent triangles.
Teacher: *The order is not an alphabetical order of the angles of the triangle but the order of the sides of the congruent triangles.*
While discussing the rule SAS, learner 1 raised a question as to whether it was wrong to have SSA as a rule for congruence in triangles. The question drew the attention of many learners.
Learner 4: *Is there anything wrong in having the reason SSA as for* $\Delta ABC \equiv \Delta DEF$.
There was an argument when learners 2 and 3 agreed with her while a few disagreed.

The teacher explained (figure 4.2) saying that the learners should check whether the corresponding sides and corresponding angles of the triangles were equal before making a conclusion.
The majority of the learners seemed not aware of the correct thing to do.
The teacher explained that *the SSA or ASS does not hold as two different triangles can be drawn from such a situation*.

This discussion made the educator to rub the written example so that SAS will hold.

Lesson 1

The teacher introduced the lesson by asking learners to name 2 dimensional shapes they know. The learners named the shapes as *square, parallelogram, rectangle, kite and a rectangular prism* (see Appendix K). Learner 1 in lesson 1 explained that *a rectangular prism is a solid shape with three dimensions.* The educator agreed with what the learner said and added that a rectangular prism was 3-D shape. The teacher shifted his focus from discussing the properties 2-D shapes in general to types of triangles. Learners managed to give the types of triangles and their properties. The learners mentioned *the scalene, the isosceles, the equilateral and the right angled-triangle* (see Appendix K).

The teacher explained to the learners that triangles are named according to their characteristics/ properties. *For example here, an equilateral triangle is named so because all its sides are equal. How about its angles? Also the angles are equal. What is the size of each angle? Each angle of an equilateral triangle is 60° because the sum of a triangle is 180°. Today we want to look at congruent triangles. Do we all know congruent triangles and if so, can you describe them? Congruent triangles are triangles that are the same in all respect* (Appendix K).

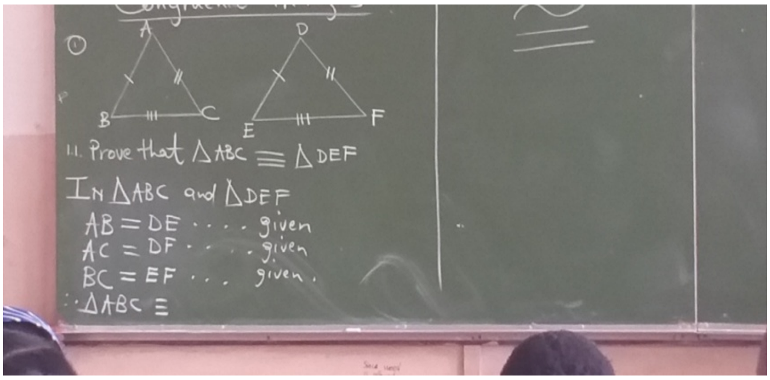


Figure 4. 1 Photography of the SSS rule on the chalkboard

The teacher worked out an example on the chalkboard as shown on Figure 4.1. The teacher explained each step taken to prove congruent triangles. He indicated on the diagram the corresponding sides which are congruent. The teacher tended to dominate in the lesson discussion. It appeared as though the learners grasped the concept of proving congruent triangles.

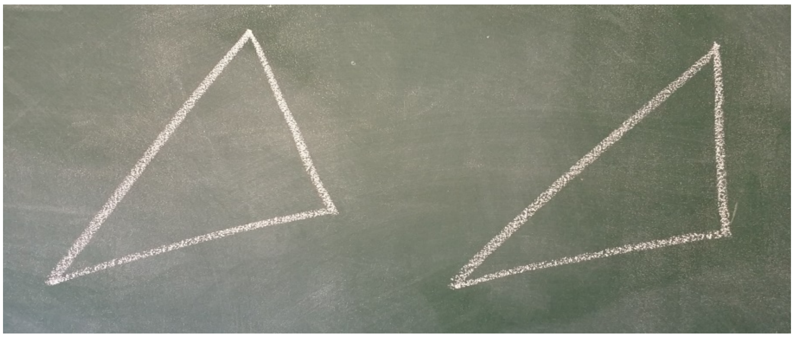


Figure 4.2 Photography of a sketch diagram of congruent triangles

Learner 2 in lesson 1 drew two triangles on the chalkboard (see figure 4.2 above). There were no symbols or measurements on the diagrams to show that the corresponding sides and corresponding angles of the triangles were equal. The teacher accepted the diagrams as correct illustrations of congruent triangles. There was a need for the teacher to point to the learners the importance of showing equal corresponding sides and equal corresponding angles. Accepting the diagrams as they were in figure 4.2 tends to create a misconception in learners that figures can be concluded to be congruent without showing dimensions. The teacher linked the types of triangles to the proving of congruent triangles. The teacher emphasized that *two or more shapes can be congruent if they are the same in shape and size.* The teacher demonstrated how to prove congruent triangles using SSS *rule* (see Figure 4.1).The educator highlighted the importance of checking whether all the sides and angles are equal. He explained that *it is very important to give reasons for every decision made about corresponding sides or corresponding angles.* The teacher explained that the minimum conditions for proving congruent triangles were only three reasons. The teacher emphasized that the moment three reasons were established the remaining three facts become automatically congruent. The learners were not given any kind of work to do as homework. There was not enough time for the teacher to summarise the lesson. The teacher ended the lesson abruptly.

Lesson 2

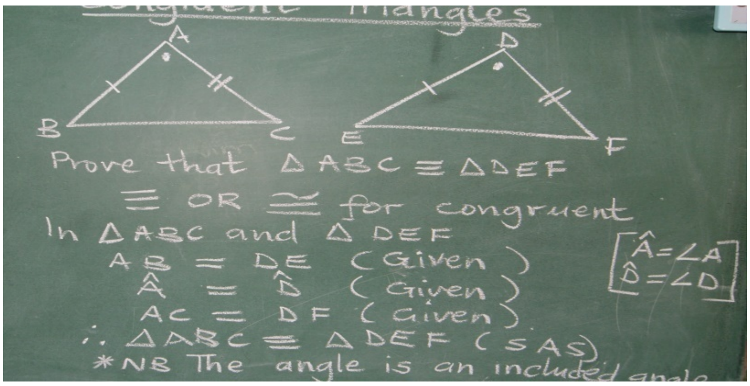


Figure 4. 3 Photography of the SAS on the chalkboard

On day 2, the learners started by revising the previous day's work which was the SSS axiom. Two of the learners were absent on day 2. The teacher was quick to recognise that two boys were absent that day. Most of the learners struggled to explain what SSS rule meant when proving congruent triangles. Learner 1in lesson 2 explained that *SSS meant that three sides of the concerned triangles are equal.* Less than half of the learners (about 40%) tended to agree with learner 1. Learner 2 added to what learner 1 said when he said that *SSS also meant equal sides*. Learner 3 said that *SSS meant corresponding sides*. The educator probed the learners to explain more on the condition SSS. Learner 4 explained that *SSS meant that three pairs of corresponding sides of the triangles are equal.* All the learners agreed with the answer given.

The teacher introduced the SAS rule where he emphasized the importance of an included angle (Figure 4.3). The rule was discussed in class. When the class was making a conclusion of the reason for congruence in figure 2that was when the class discovered that their answer was wrong. According to the explanation and diagram on the chalkboard, the result was giving the reason SSA. The teacher pointed out that the conclusion was wrong. He expressed that learners should *always remember that SAS rule applies only when the angle is an included angle.* The learner raised the issue why SSA or ASS cannot be regarded as reason for congruence in triangles. The class argued at length about the correct way of explaining congruence in figure 4.2. The teacher explained that *the diagram and the stated reasons were correct but the corresponding sides and angles were not equal.* The class agreed that *the congruent triangles were now ΔBAC and ΔEDF when following the corresponding sides and corresponding angles of the triangles.*

The class discarded the conclusion in figure 4.3. This rendered the conclusion in figure 4.3 to be wrong. The teacher explained that *SSA or ASS was/were not reasons for congruence in triangles as they do not always give congruent triangles. There are possibilities of having different triangles*. The learners appeared not to understand the teacher's explanation. There was need for the teacher to use the investigation approach to the proving congruent triangles as suggested by the Mathematics CAPS curriculum (DBE, 2011). The teacher could have employed technological applications to prove congruent triangles. The teacher tended to dominate in the discussion in the lesson. He explained the concept more than once but still the learners showed that they did not grasp the logic. The lesson seemed to be too abstract for the majority of learners as they tended not to grasp the concept.

Lesson 3

The teacher summarised the previous lessons with the learners by saying, "In the previous lessons, we talked about proving congruent triangles using the SSS and the SAS rules. Is there anyone who can tell us what you still remember about proving congruent triangles? The learners were ready to answer the question as they indicated by raising their hands to participate (21out of 32 learners). Learner 1 of lesson 3 pointed out that the 'angle' in SAS rule should always be an included angle. Learner 2 said that SSA or ASS fails to be a rule for congruence as they do not always result in congruent triangles (Appendix K).

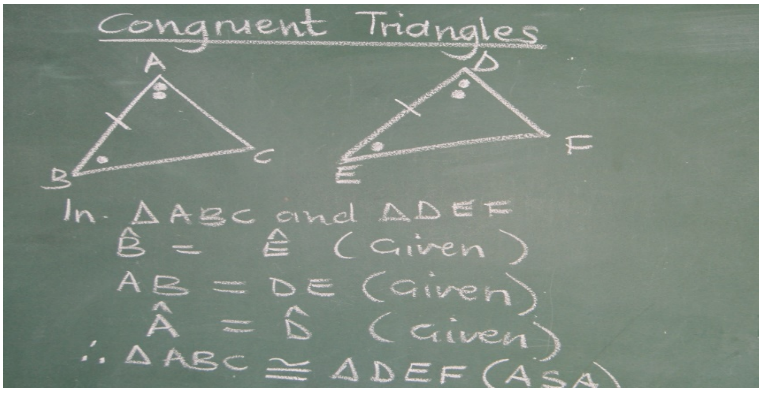


Figure 4. 2 Photography of the ASA rule on the chalkboard

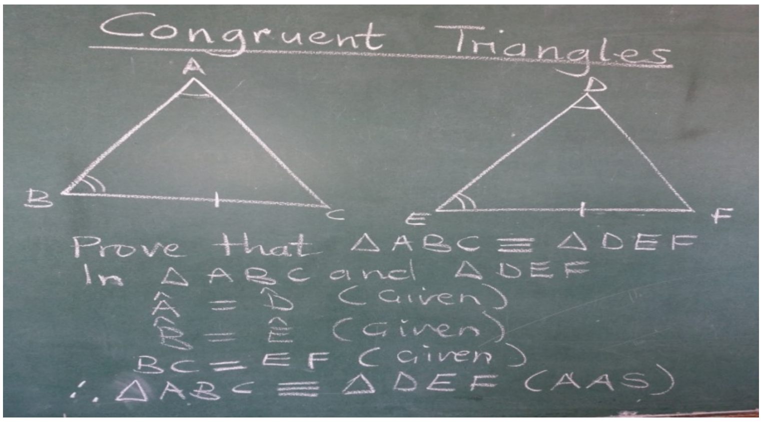


Figure 4. 3 Photography of the AAS rule on the chalkboard

In this lesson, the class discussed the rules SAS and AAS. The learners compared the ASA rule which they referred to as rule number 3 and the AAS rule which they referred to as rule number 4. The educator explained that *rule number 3 and 4 are similar in that they both have one side and 2 angles. The difference is that in ASA, the angle is an included angle and in AAS the side is not included.* The educator drew diagrams on the chalkboard to compare ASA rule and AAS rule

(Figures 4.4and Figure 4.5). The teacher explained the importance of indicating the sides and angles on a triangle by using symbols. Learner 4 of lesson 3 raised a question, *why AAS is accepted as a rule for congruent triangles when we discarded the SSA in the previous lesson?* (Appendix K). The argument was that if 'side' in AAS is not an included side it must not work as explained in the similar case in the previous lesson where' angle' was not an included angle. He led the discussion when learners were giving chorus answers. There appeared to be a disorder when learners were allowed to talk without being given a chance to do so. Unfortunately there was not a convincing explanation from both the educator and the learners who kept on saying that these are different situations. The learners were left without the clear answer to a valid question. There was need for the teacher to refer the learners to research.

Learners were given worksheets to answer questions. They were asked to state with reasons why or why not pairs of triangles were congruent. The learners produced different answers. Some of the learners failed to write anything on their worksheets. Some of the learners' responses are shown in figure 4.6 to figure 4.8.

Five out of 32 learners had their answers as displayed in figure 4.6 while the majority, 17 out 32 had their answers as in fig 4.7. Six out of 32 had their answers as in figure 4.8. Learners appeared to be eager to explain their answers to the class as feedback. Learners were asked to explain their answers in figures 4.6 to 4.8.

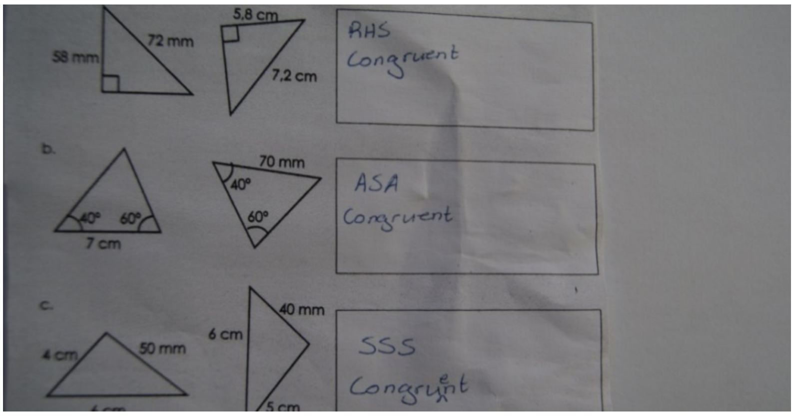


Figure 4. 4 Photography of Learner 1's Answers to Lesson 3 Activity

Learners gave different reasons to the same answer that they indicated as in figure 4.6. In question (a), Learner 1 explained, *I converted millimeter to centimeters and I found out that the measures are the same.* Learner 2 answered saying*, I noted that the triangles are right-angled triangles so their reason was RHS and they concluded that the triangles are congruent. In question (b),* Learner 3 found that *the corresponding sides and corresponding angles of the triangles were equal leading to the triangles to be congruent****.***

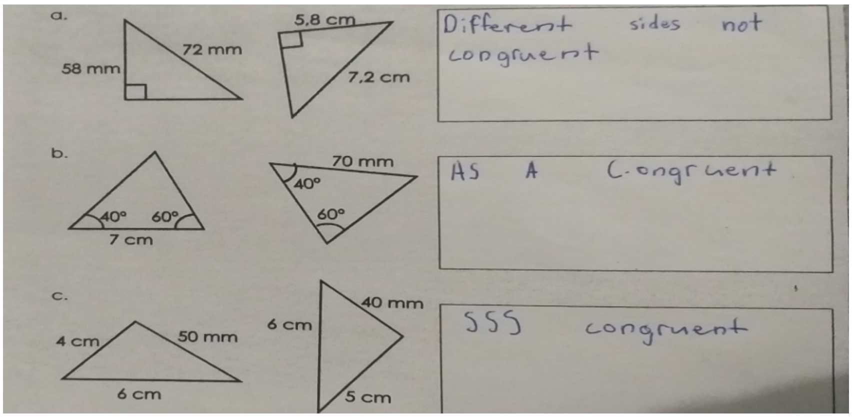


Figure 4. 5 Photography of Learner 2's Answer to Lesson 3 Activity

However, learners whose answers were like in figure 4.6 (b) missed the educator's emphasised point of corresponding sides and corresponding angles. Question (c) seemed to be easy for most of the learners as almost all the learners who answered managed to get it right (see figure 4.6 to figure 4.8).

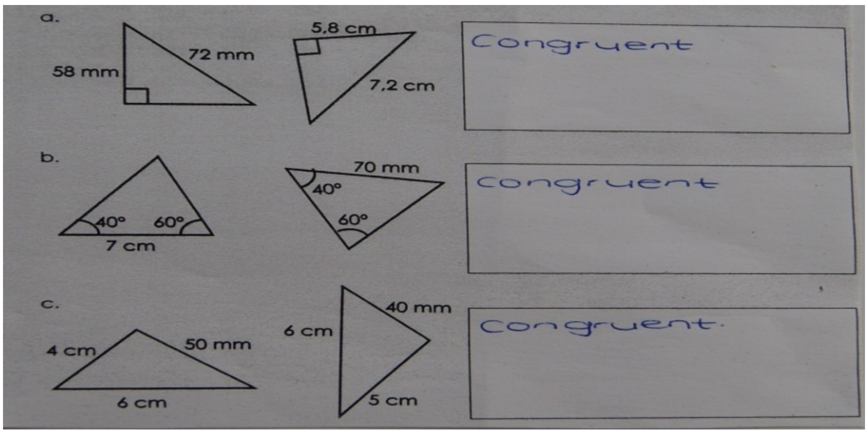


Figure 4. 6 Photography of Learner 3's Answer to Lesson Activity

The learners whose answers were as on figure 4.8 failed to interpret the instruction that they should give a reason for their decisions. The learners provided a variety of reasons for the answers shown on figure 4.8. Learner 4 indicated that *I forgot to write down the reasons.* Learner 5 said he *did not get the instructions correctly*. Learner 6 said that *it was one and the same thing since the most important thing was to indicate that two triangles were congruent and the reason was obvious and there is no need of writing it down* (see Appendix K). Learner 6 indicated that she needed help and the teacher went to assist her with answers. Learner 4 of lesson 3 exclaimed, *Sir! It is obvious that the two triangles are equal and there is no reason for writing that down.*

Lesson 3 appeared to be a busy lesson as compared to the previous two lessons. Most of the learners were involved in answering questions. There seemed to be less interference and most of the learners focused on activity. About a quarter of the learners (eight out of 32) did not write anything on their worksheets. They indicated that they do not know how to answer the questions and were waiting for their colleagues to assist them.

The teacher ended the lesson by summarising the main points. He stated that *proving of congruent triangles is about the relationship between figures. Every step we take should be supported by a reason. We need three facts to prove that triangles are congruent. So far, the rules we have learned include, SSS; ASA; AAS and SAS.* The lesson ended when some learners were still writing their work. He thanked the class for participating.

It appeared that the learners who did not write anything on their worksheets did not receive any kind of help. There was need for the teacher to encourage learners to assist each other.

Lesson 4

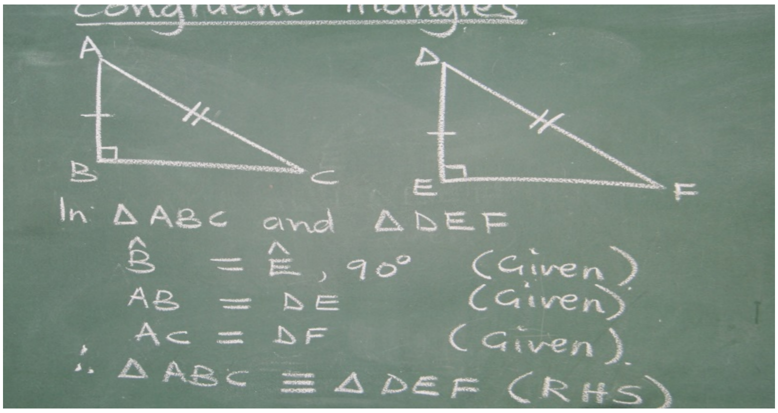


Figure 4. 7 Photography of the RHS rule on the chalkboard

On day 4, the class looked at the properties of a right-angled triangle. Learners outlined the properties of a right-angled triangle as: *the presence of a right angle, the hypotenuse, the sum of the interior angles of a triangle is 180° and the sum of the other 2 non-right angles is 90º.* Learner 1 of lesson 4saidthat a *right-angled triangle reminds her of the theorem of Pythagoras.* The educator invited the other learners to give more information about the theorem of Pythagoras. Learner 2 answered saying that *Pythagoras is used to solve an unknown side of a right-angled triangle.*

The teacher linked the properties of a right-angled triangle to the RHS rule by way of demonstrating on the chalk board (see Figure4.9). It appeared that most of the learners found the RHS rule simple to grasp. Learner 3 of lesson 4 pointed that he felt that the educator did not show when the right angle is an included angle. In the discussion, learner 4 of lesson 4 asked whether *AAA rule is a reason for proving congruence in triangles.* The question came as a result of learners indicating that given one angle on a right-angled triangle we can easily get the size of the angle using the supplementary concept. Learner 5 of this lesson argued that *if two triangles have their corresponding angles equal, they are also congruent* (see Appendix K). The learner went on to explain that the sum of the interior angles of a triangle is supplementary. Learner 2argued that if three pairs of corresponding sides result in corresponding angles equal then when three pairs of corresponding angles are congruent, this also should result in corresponding pairs of sides congruent. She went on to say that if we start with three pairs of corresponding angles equal then the corresponding sides would also be equal. The educator explained that congruence in triangles cannot be determined with angles only.

The learners were given homework to write down notes on the proving of congruent triangles. The learners were to summarise the rules of proving congruent triangles. Figure 4.10 shows what one of the learners wrote as their summary of the congruent axioms.

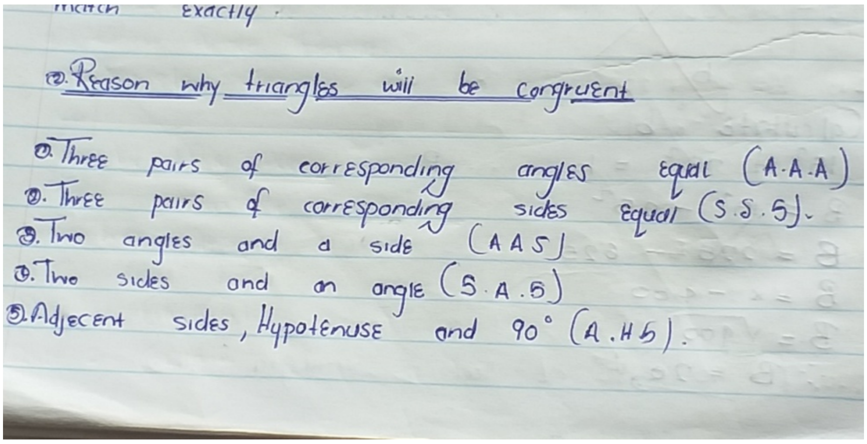


Figure 4. 8 Photography of example of one learner's personal notes on proving congruent triangles

Lesson 5

The teacher started the lesson by revising the congruent cases with the class. As I was checking the learners' homework, I found that three of the learners in the class wrote their notes as shown on figure 4.10. Bulletin number (a) is wrong and was supposed to be corrected. The teacher did not check all the learners' work. Learner 1 of lesson 5 indicated that she got the notes from her friends. There was need for the teacher to emphasise the use of the textbook when making their own notes. There were high chances that that the learners did not consult their textbooks when they were doing this homework. The same summary of congruent rules requested by the teacher was in their textbooks.

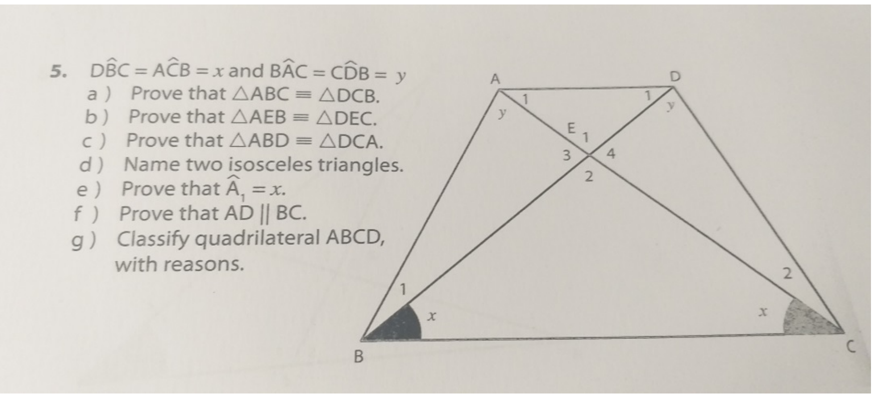


Figure 4. 9 Photography of a question on proving Congruent Triangles (Campbell et al., 2013:134)

In lesson 5, the main task of the day was to prove congruent triangles in figure 4.11. The diagram contains information and symbols about the triangles. The learners were expected to be able to interpret the questions accurately before they attempt to answer the questions. Learners were allowed to work in pairs. They were asked to clearly show their working by drawing diagrams where possible. There was no choice for the learners to start with question 5 (b) because answers to 5(a) unlocks the reasons to do question 5(b). Question 5 is one of the problems which enhance the development of reasoning skills in learners. Question 5 on figure 4.10 was taken from the book “Platinum Mathematics Grade 9 Learner’s book” (Campbell et al. 2013:134).

The learners answered the question in figure 4.11 in different ways. Three answer scripts were cited as examples of what some of the learners were able to do in an attempt to answer the questions. These learners separated the triangles to be proved from the main diagram. They labeled all the given dimensions. They were able to identify common sides in the triangles to be proved congruent as indicated in the diagrams below.

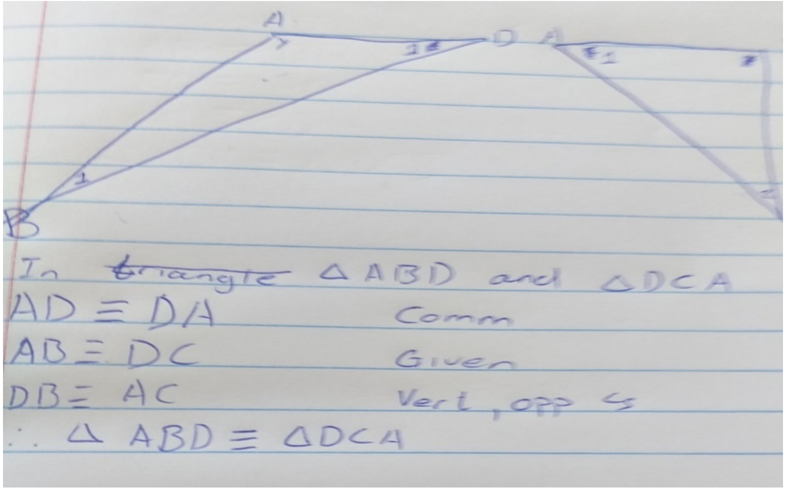


Figure 4. 10 Photography of Learner 1's Answer on Proving Congruent Triangles

In figure 4.12, learner 1 answered question 5 (c) in an interesting way, he separated the triangles and was able to identify a common side. The second point he gives as '*Given*' is actually *proved* in 5 (a). He got lost on the next reason when he stated his reason as *'vertically opposite angles'* when he was talking of sides. The same reason, '*vertically opposite angles'* was the reason 5 (b) on the same question. He might have consulted with Learner 3 on lesson 5doing 5(b) and got that reason. He might have mixed up his reasons for the 2 questions. The learner used congruent sign for equal sign as shown in figure 4.12. The learner could have confused congruency sign for equal sign. He was quick and confident to present his work to the educator for checking.

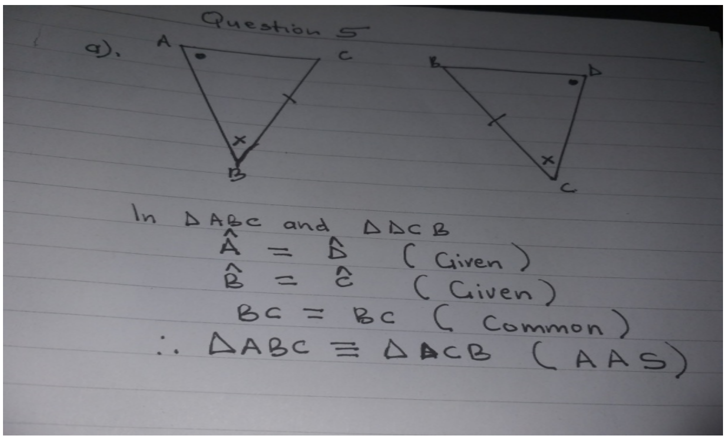


Figure 4. 11 Photography of Learner 2's Answer on Proving Congruent Triangles

Learner 2 showed a smart kind of working that was clear and to the point (Figure 4.13). When learner 2 was asked how he worked out his problem, he explained slowly and accurately. He explained his working in this way: *I separated the triangles to be proved congruent. I made sure that I maintained the same measures of each given angle and given side. I found that side BC was common in the two triangles*. The learner asked whether the reasons of the proved question can be used to answer the next question.

The educator stopped the class and issued the following instructions*. Excuse me class, When you have proved congruence in the first set of triangles, the facts are still valid when you are solving the next question. Use what you have proved to help you answer the next.*

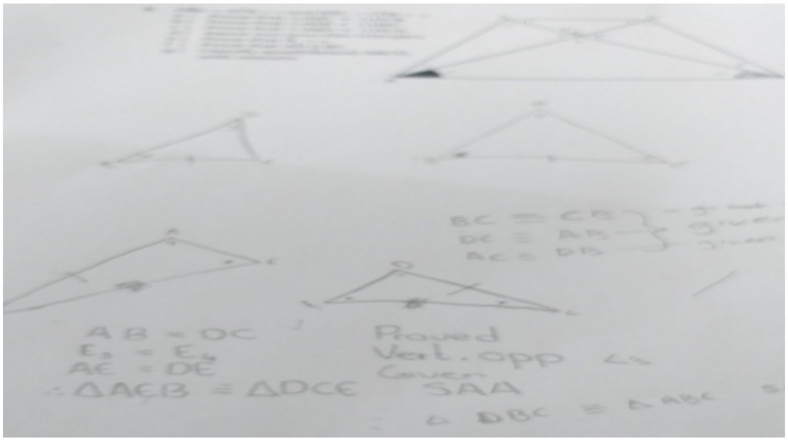


Figure 4. 12 Photograph of Learner 3's Answers on proving Congruent Triangles

Learner 3 managed to use what she proved as a reason for the next question (figure 4.14). Her answer to the first question was not clear until she explained to me what she was doing. Her explanation and what she wrote were different. She explained, *In question 5 (a) I identified 2 pairs of given corresponding angles and a common side. However, I did not write down that but other facts.* She claimed that the other facts were coming from matching equal sides from the diagram. She answered question 5(b) correctly. She managed to pick facts from the previous answer as reason for proving the next question. Unfortunately the learners did not finish the other questions.

The majority of the learners appeared to be struggling to interpret the diagram. They took long to answer the questions. They were constantly asking each other what they were supposed to do in the exercise. The relationship of triangles, corresponding angles and corresponding angles in the Figure 4.10 was not easily picked by the majority of the learners. The learners were requested to complete the remaining portion of the question as homework. The teacher said, *We do not have* *enough time to finish this question together today. When we come next week we will first look at this question before we start a new topic.*

There was need to pursue the remaining questions of figure 4.10 with the learners as they tended to present a challenge on their reasoning skills. There was a need for learners to be able to see the relationship between the triangles proved congruent. I noticed that question 5(d) and 5(g) required learners to show their knowledge of properties of shapes. Question 5(f)) is multi-concept which required learners to be able to follow the relationship between proved triangles, recognise similar shapes and be able to identify the converse of alternate on parallel lines. In Question 5 (e) teaches learners the concept of similar triangles. All these questions required learners to make more than one step to the conclusion.

4.2.3 What did not happen in the Lessons?

The learners were not given an opportunity to rediscover the rules for proving congruent triangles for themselves. The classroom observation and the focus group interviews revealed a gap in the proving of congruent triangles. There was need for a practical activity to help learners to investigate the minimum facts needed to prove congruent triangles. This was evidenced by the learners' questions about why SSA and or ASS fail to be conditions for proving congruent triangles.

The learners were not given chance to discuss in pairs or in small groups what they think or know about certain concepts in the lessons taught. The written work and homes were not revised fully. Little time was given to learners to think for themselves. Although the learners could state reasons why triangles are congruent but they did not comprehend what that meant (Appendix F).

4.2.4 Focus Group Interviews

The focus group interviews were done one week after the classroom observations**.** There were two groups of six participants each involved in the focus group discussions. The focus group interviews took place from 30 September to 7 October 2019. The first group met on Mondays, Wednesdays and Fridays while the second group met on Tuesday Thursday and Monday at 14h00. The groups discussed the same semi-structured questions on the proving of congruent triangles (see Appendix I).

The purpose and reason for having a focus group interview was explained to the two groups. The necessary preparations were done in advance with the help of the subject teacher. The focus group interview was used in this research to complement the participant observation. The participants filled in assent forms for each session they attended. The participants were provided with pens, pencils and paper to use during the interview. The participants were promised that the information shared in the focus group interviews would only be used for the purposes of the research. The learners were promised that their identity will not be exposed to the public in any way.

Below is an extract of Transcript 2 of Focus Group Interviews, Day 2.

Facilitator: *There are situations where facts do not add up to make congruence in triangles. Can you identify such situations?*

Learner 2: *Triangles are not congruent if some of the sides are not equal.*

Learner 3: *I have got a question. Do we have situation when triangles have corresponding sides but one pair of corresponding angles not equal?*

Learner 5*: As long as all the corresponding sides of a triangle are equal then all the other angles are also equal. I wanted to say that triangles are not equal if we have only corresponding angles equal.*

Learner 6: *When the triangles are ASS.*

Facilitator*: Why do you say that? Can someone explain what ASS is?*

Learner 5: *ASS or SSA is not a reason for congruent triangles because the triangles are not congruent. We have done this together in class.*

Learner 2: *I think when you draw the triangles they are not the same.*

Facilitator*: What other challenges are involved in the proving of congruent triangles?*

Learner 5*: I think there is nothing*

Facilitator: *Is there anything else you would like to say about why you like or dislike the proving of congruent triangles?*

Learner 6: *Proving of congruent triangles is very interesting but wants someone who reads every day. I think I can improve my Maths through studying.*

Learner 2*: I understand when I am discussing with someone than when I am doing alone. If we can have a group test we will all pass.*

Day 1

I introduced myself and my colleague who was taking notes as well as recording the discussions. Learners were informed that they were audio taped and or videotaped.

The discussion started by checking their knowledge of properties of quadrilaterals and triangles. In both focus groups, most of the learners admitted that they had forgotten information learned in Grade 8. In the first group, five out of six (83%) and in the second group four out of six (67%) agreed that they had forgotten most of the information about properties of different shapes. There were some important issues like the relationship between a square and a rectangle. In the second focus group, learners argued that *a square is not a rectangle and also that a square is not a rhombus*. Both groups managed to outline a few properties of shapes. Figure 4.14 below was drawn on the chalkboard for learners to discuss. The concept of the diagonals dividing the quadrilateral was introduced. The majority of the learners (in both groups) managed to identify triangles formed by drawing a diagonal across a quadrilateral as congruent or not congruent triangles.

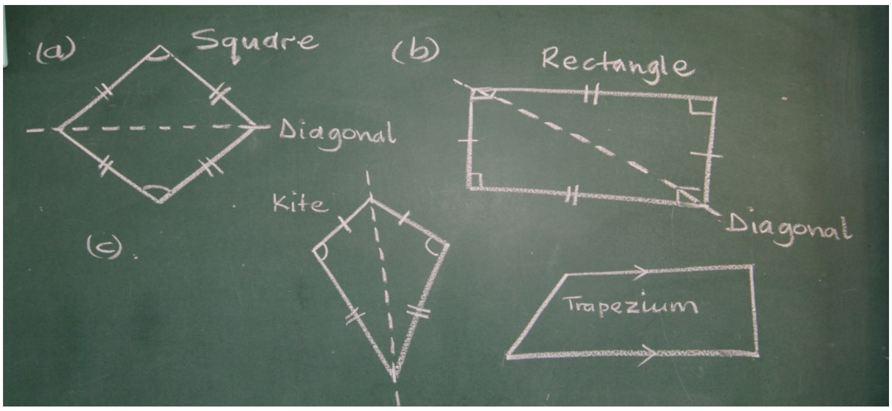


Figure 4. 13 Photograph of Quadrilaterals drawn on the chalkboard

The first question asked whether the learners know the 2-D shapes. All the learners from both groups randomly called out the 2-D shapes as *Pentagon; hexagon; square; rectangle; triangle; octagon; rectangular prism; rhombus; cylinder; circle triangle; pyramid and parallelogram.* The learners then discussed the shapes. Learner1in focus group discussion 1 answered saying, *Ok that is simple sir a cylinder is not a 2 dimensional shape; Learner 2 in the same group added saying, Sir these shapes are confusing 2-D or 3-D shapes?* Learner4 *in focus group discussion 2 tried to explain the difference between 2-D and 3-D shapes when he said a 2-D shape has 2 like a side or directions on the shape like a square. Learner 1in focus group discussion 2defines a 2-D shape* as a *plain shape while a 3-D shape has volume and area* The learners were aware of the shapes and some seemed to have forgotten the properties of these shapes.

The learners were further asked to group the shapes according to their properties. Learner 3 in focus group discussion 1 stated that *there are shapes we call quadrilaterals like a square and a rectangle. Learner 2 from the same group described a quadrilateral as a four sided figure. Learner 1 in focus group discussion 2 concurred with* Learner 1 in focus group discussion1 about quadrilaterals and she added that *there are also shapes called polygons. Learner 1 and 3 both in focus group discussion 2 agreed that a cube was a polygon because it has many sides. Learner 1 and 3 of focus group discussion 2 learners made their conclusion from the definition of a polygon.*

The learners pointed out the different kind of triangles. They identified the triangles *as scalene, isosceles, equilateral, right-angled, acute-angled and obtuse-angled* (see Appendix L).They accurately defined some of the triangles. Learner 1in focus group discussion 1 says, *a scalene is a triangle with all sides different*. Learner 2 in focus group discussion 1 says, *an isosceles is a triangle with two equal sides.* Learner 2 in focus group discussion 2 says, *a right-angled triangle is a triangle with a right angle and a hypotenuse.* The learners tend to define triangles according to their sides. This could be the result of how they were exposed to the different types of triangles. Learners tend to ignore to define triangles according to angles. However, Learner 5 in focus group discussion 1 pointed out an acute-angled triangle and Learner 4 in the same group was excited to draw the triangle on the chalkboard (see Appendix L)

Day 2

In the next discussion, learners expressed their attitude towards the proving of congruent triangles. About 83% in Group 1 and 50% in Group 2 indicated that proving congruent triangles was difficult for them. They all (100%) in both groups agreed that proving congruent triangles help them improve their reasoning skills.

Three out of six in focus group discussion 1 raised a concern as to why there was need to prove what they termed ‘obvious’ facts. In group 2; three out of five of the learners raised a concern about proving congruent triangles. The learners’ argument was that *given triangles are clearly labeled that everyone can see that the corresponding sides and corresponding angles are equal.* The learners’ main problem was to write down the reasons for congruent triangles which they regarded as ‘obvious’.

The learners could explain what congruent triangles are. Learner 2 in focus group discussion 2 explained the congruent triangles as *the triangles should be of the same size and same shape.* I asked the learners to explain how they know that the shape and size of the triangles were the same. Learner 2 in focus group discussion 2 said that *the size of the triangle you check the corresponding sides. The angles I think you can measure. I’m not sure.* In addition to this information, Learner 5 in focus group discussion 1 explained that *in the facts that we check to consider congruent triangles, there must be at least one pair of corresponding sides equal.* Learner 4 in focus group discussion 2 also pointed out this important fact about congruent triangles when she said that *there must be one pair of corresponding sides equal* when considering congruence in triangles. Learner 1 in focus group discussion I gave a difference between similar and congruent triangles when he responded saying, *No they are not. Similar triangles are the same yeah and congruent triangles are like the same size and shape.* Learner 4 in focus group discussion 2 also pointed out this important fact about congruent triangles when she said that *there must be one pair of corresponding sides equal.*

Almost all the learners in both focus group discussions were able to identify the five rules of proving congruent triangles. Learners 2; 3; 4; 5 and 6 in the focus group discussion 1 managed to say one rule each (see Appendix L). In the second focus group discussion Learners 1;2;3;4 and 5 identified the rules for congruent triangles. Learner 1in focus group discussion 2 asked a question saying, *I do not understand why AAS works when ASS does not work for congruent triangles? Learner 4in focus group discussion 2 responded by saying, I think we do not need to argue with* *the law. It is law there is no reason.* The response of Learner 4 in focus group discussion 2 shows that mathematical knowledge is rigid and should not be challenged.

Although the learners could explain what congruent triangles are, they had a challenge to give reasons why triangles were congruent. Checking carefully the properties of each triangle and following the corresponding angles or corresponding sides was a challenge to most learners. In the process of identifying congruent triangles some of the learners mentioned that AAA (Figure 4.15) and SSA are possible reasons for triangles to be congruent. Most of the learners confused the rules for proving congruent triangles. Learner 1 in focus group discussion 1 drew the diagram on Figure 4.15 showing that they are an example of congruent triangles. Learner 1 in focus group discussion 1 forgot that Learner 5 in their group pointed out that there must be one pair of corresponding sides in congruent triangles. The learner was corrected by the group.

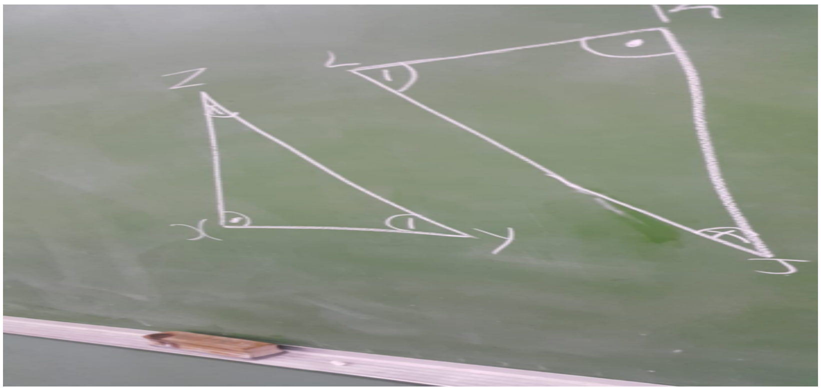


Figure 4. 14 Photography of Similar Triangles on the Chalkboard

The misconception of SSA or ASS taken as a rule for congruence recurred again in the focus group discussions. In focus group discussion 1, learners raised the ASS issue. Learner 6in focus group discussion1explained that ASS rule does not satisfy congruence in triangles as we *can have a small triangle and big triangle which we can draw.* Learner 6 went on to explain that *I don't know how to explain it but there will be two triangles from the same triangle.* In other words, the learner was trying to explain the concept of ambiguity in the triangles.

Day 3

On day 3, the learners started by revising a few questions about the axioms of congruent triangles. Two out of six of the learners in group 1 pointed out that they *try to cram the process of proving* *congruent triangles but the questions kept on changing every time*. Learner 1 in focus group discussion 1 said, *there are many difficult questions in the tests which are difficult. Test questions are different in the tests than in class work. This is the reason why I hate Mathematics.* Learner 2 in focus group discussion 1 said, p*roving congruent triangles is difficult for me. I do not like it. Hey, it's tough for me, there are too many new words involved.* Learner 1 in focus Group discussion 2 indicated, *I try to memorise the steps for proving congruent triangles, at times it works and at times it is difficult as the questions are completely different from what I know*. Learner 2 in group 2 indicated that *questions given during class activity tend to be simple as compared to homework and test items*. In other words, these learners tend to agree that proving of congruent triangles is difficult for them. However, the majority of learners in both focus group discussions acknowledged that they do not have time to practice their work outside lesson time. Learner 2 in focus group discussion 2 indicated that he understood proving of congruent triangles better when he discussed the work with a friend and suggested if it were possible to be allowed to write group tests. He believed he would perform well in group tests.

On the other hand, Learner 6 in focus group discussion 1 said, *Myself, I like Mathematics especially congruent triangles because I know how to solve the problem.* Learner 3 in focus group discussion 1 also said that she liked Mathematics especially the challenging questions on proving congruence in triangles. He felt excited when doing Mathematics.

Questions in figure 4.16 to figure 4.18 were drawn on the chalkboard to check whether learners were able to relate properties of 2-D shapes to the proving of congruent triangles. The learners struggled to explain their mathematical knowledge because of limited formal vocabulary to explain properties of shapes.

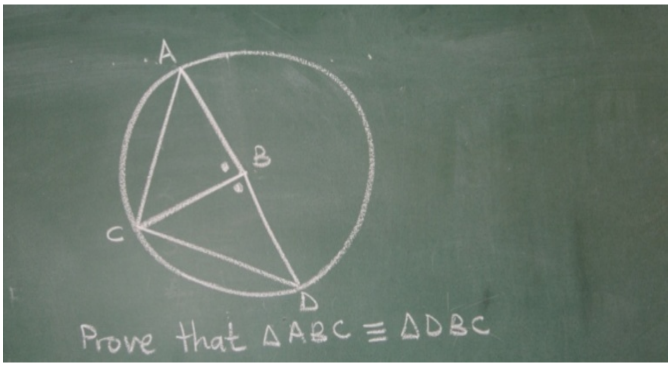


Figure 4. 15 Photography of Triangles on a Circle drawn on the chalkboard

Learner 3 in focus group discussion 1 indicated that she can only show on the diagram than to explain the relationship of angles and or sides. She says, *this angle (pointing at ∠ABC) is equal to this angle (pointing at ∠DBC)* on figure 4.16. There is important terminology for the learners to know in order to function mathematically. For example, learner 5in focus group discussion 1 said, *I can't see any relationship between the sides and angles of the triangles on figure 4.16.This is a difficult and new diagram sir.* In figure 4.16., group 1 learners struggled to pick up the idea of radii in the diagram. The learners constantly shouted that *the triangles in the diagram are not related at all.* In focus group discussion 2, Learner 1 quickly pointed out the radii reason of having AB=BD= BC in the diagram and most of the learners were able to give the reasons why ΔABC and ΔDEF were congruent.

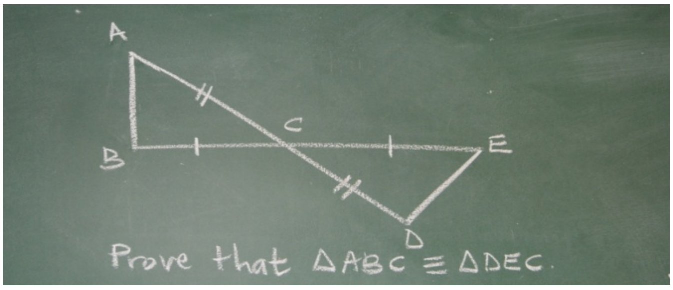


Figure 4. 16 Photography of Intersecting lines drawn on the chalkboard

In figure 4.17 learners in both groups agreed that the two triangles were congruent without identifying the third reason for their congruence. They were quick to say that *it is clear that these triangles are congruent by mere looking at the diagram.* The concept of a common side in the diagram in figure 4.17 was easily noted by the majority of learners from both groups.

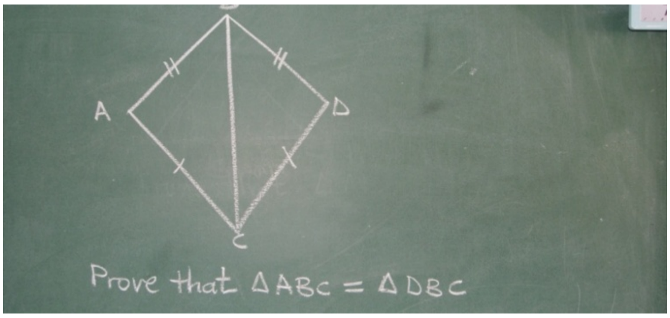


Figure 4. 17 Photograph of a Kite with a diagonal drawn on the chalkboard

In figures 4.18 learners were asked to identify corresponding sides or angles on the given diagram. Learner 3 in group 2 said, *I can see AB = BD and AC=DC and the reason is that they are given.*

Learner4in focus group discussion 2said,*∠A= ∠D and the reason is give.* I invited the learners to discuss the congruence in the triangles. Both groups agreed that BC is a common side between the two triangles.

The learners were engaged in discussing the possible situations where triangles were not congruent. Learner 4 in focus group discussion1 said, I think when Angle-Angle-Angle is the only reason for considering triangles to be congruent. We need one side to prove that 2 or more triangles are congruent. Learner 3 in focus group discussion 1 added saying, *If triangles have their corresponding sides in proportion, then they are said to be similar triangles*. In other words, the learners acknowledge the concept of similarity in triangles and that the same triangles are not congruent. The learners in focus group discussion 2 picked on the concept of ASS or SSA. Learner 5 in focus group discussion 2 said that *ASS or SSA is not a reason for congruent triangles because the triangles are not congruent. We have done this together in class.* Learner 2 in the same group added that he thought that when one draws triangles showing ASS rule, the triangles may not be the same size.

4.2.5 Document Analysis

4.2.5.1 Introduction

Document analysis was one of the instruments used to collect data in this research. I looked at the books the educator used to prepare and plan his five lessons. The educator consulted two textbooks plus a revision book. The books are Mathematics Today Grade 9 Learners' Book, Platinum Mathematics Grade 9 Learners' Book and Mathematics in English Book 1 for Term 1 and 2. He was also guided by the CAPS document.

There are important topics/concepts that work as the base or prior knowledge for the learners proving congruent triangles. The learners must be aware of the properties and definitions of triangles. Learners need to know equilateral triangles, isosceles triangles and right angle-triangles as a prerequisite for proving congruent triangles. The knowledge of the properties and definitions of quadrilaterals enhances the proving of congruent triangles. Learners are expected to have knowledge of properties of parallelogram, rectangle, square, rhombus, trapezium and kite.

Congruent triangles may be treated as an isolated concept in the curriculum. It comes in between concepts under the main topic, Geometry of 2-D shapes. It is again difficult to talk about congruent triangles without similarity concept.

I checked whether the textbooks and CAPS document were aligned to Van Hiele's levels of geometrical thinking on the concept of proving congruence in triangles. The CAPS document is the main source of all the lesson plans that any teacher can have in the teaching of Mathematics. The document outlines the concepts that prepare the learners for the learning of congruent triangles and it also gives a suggestion on how the educators can handle the concept of congruence. The CAPS document pictures the proving of congruent triangles as a spiral concept which starts in Grade 7 and proceeds to Grade 9. This means that whenever the learners miss important facts prior to the proving of congruence, then they were likely to experience challenges in Grade 10.

4.2.5.2 Mathematics CAPS document Senior Phase Grade 7-9

The Mathematics CAPS document outlines the concept of congruent triangles from Grade 7 to Grade 9 clearly. At grade 7 the learners are expected to *recognise and describe similar and congruent figures by comparing shape and size* (DBE, 2011). At Grade 8 level, *learners are expected to identify and describe the properties of congruent shapes*. *Learners should recognise that 2 or more figures are congruent if they are equal in all respect that is, corresponding angles and corresponding sides are equal* (DBE, 2011).

The CAPS document suggests that Grade 9 learners should explore the minimum conditions for 2 triangles to be congruent through investigation. Construction helps the learners to understand the minimum number of facts needed to prove congruency. Construction of geometric figures has been allocated a period of 9 hours in the curriculum and it comes before 2-D shapes. This is fairly enough time for learners to be able to learn how to construct different geometrical shapes. Construction also helps learners to investigate the properties of triangles. The teacher may adopt the construction of congruent triangles

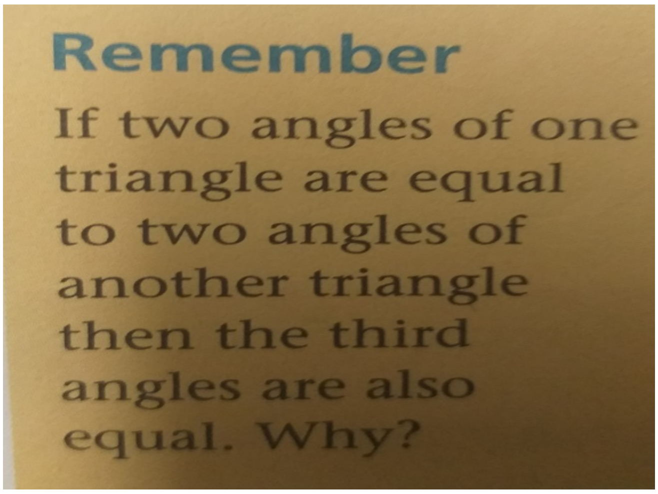
The method of transformation may be used to establish that triangles are congruent. The literature says that if 2 figures are congruent, such that movement can always be done by a sequence of translations, rotations and reflections reflect the first figure in any axis if it has the opposite parity to the second, then translate any point of the first figure to the matching point of the second figure, then rotate the first figure until it fits exactly on top of the second (Hunt 2008).This may require the use of modern technology in proving congruent triangles. Chimuka (2017) found that geoGebra software was effective in the teaching of proof in the high school.

4.2.5.3 Mathematics Today Grade 9 Learner's book

In the textbook, Mathematics today Grade 9, the concepts congruent triangles and similar triangles are under the same unit. The book states that two triangles are congruent if they are the same in all respect (Groenewald et al. 2013). The writers went on to say that the corresponding sides and angles of congruent triangles are equal. The explanation of how you check whether triangles are congruent is also given in the textbook. In the same chapter, reference is given to Grade 7 and 8 concepts which act as prior knowledge to the learning of proving congruent triangles. The data on the concept of congruent triangles appeared to be arranged in a spiral approach. Groenewald et al. (2013) stated the minimum conditions for two triangles to be congruent as follows:

1. SSS
2. SAS
3. ASA
4. AAS
5. RHS

The conditions for congruent triangles were followed by worked examples for the learners to follow.


Remember

If two angles of one triangle are equal to two angles of another triangle then the third angles are also equal. Why?

Figure 4. 18 Photography of a Reminder from the Textbook on supplementary Angles of a Triangle (Groenewald, Otto and Westhuizen (2012: 137)

The above statement serves as a reminder to the learners about the sum of the interior angles of a triangle. It is one of the properties of a triangle which learners should know as they prepare to prove congruent triangles.

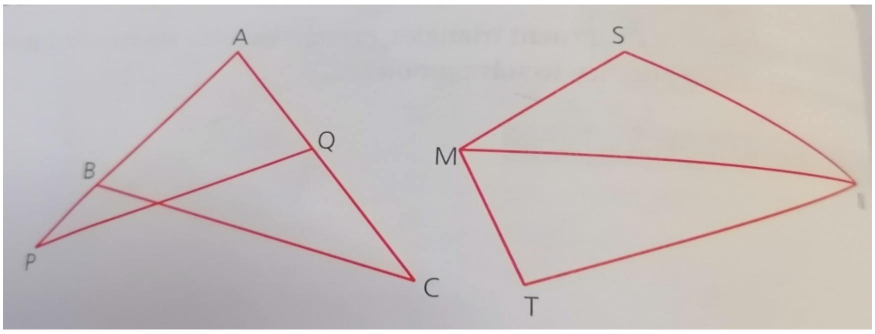


Figure 4. 19 Photography of Example showing common sides and common angles (Groenewald et al, 2012:138)

The authors of this textbook showed the concept of common angles and common sides in diagram above (Figure 4.20). The first diagram demonstrates that ∠A is common to ΔAPQ and ΔACB and that line MN is common to ΔSMN and ΔTMN. It appears that the main focus in the given diagram was to show common sides and common angles. It leaves learners to assume that the other corresponding sides and corresponding angles not indicated on the diagrams to be obviously equal. The examples of common sides and common angles are followed by an exercise where learners are expected to prove with reasons why pairs of triangles are congruent or not. The questions in the exercise also required the learners to perform some mathematical operations before they prove congruence in the triangles. Knowledge of angles on different lines is essential in proving congruent triangles. Learners are directed to specific shapes to check congruency. The textbook gives pictures where they identify congruent triangles.

Groenewald et al. (2013) pointed out that congruent triangles are similar triangles but similar triangles are not necessarily congruent triangles. The concepts of similar triangles and congruent triangles have been treated together.

4.2.5.4 Platinum Mathematics Grade 9 Learners' Book

The book Platinum Mathematics Grade 9 Learners' book (Cambell, Heany, Mant, Rossouw and Williers, 2013) was also used by the educator as he was preparing for the congruent triangles lessons. Campbell et al. (2012:132) assert, "Equiangular triangles are not congruent triangles and congruence cannot be proved without at least one pair of equal sides". The points noted here appear to be key facts about congruent triangles. The information tends to address the crucial difference between similar triangles and congruent triangle.

The writers of this book present each congruence case with an explanation. For example, SAS is explained as if two pairs of sides and one pair of angles are equal then the triangles are not necessarily congruent (Campbell et al. 012). This statement is followed by a diagram showing an ambiguous case where the angle is not between the given sides. The diagram is followed by another statement which says that triangles with pairs of equal sides and a pair of equal included angles are congruent (Campbell et al. 012). I have noted that the writers have explained each of the five cases of congruence in triangles in the same way. I think what the writers have done explains why SSA or ASS cannot be conditions for congruent triangles. I think when data has been presented in this way the learners will tend to know beyond the five conditions of proving congruent triangles.

According to Pythagoras, the third side can always be determined. For example in Figure below learners are required to calculate the third angle before they prove congruence in the triangles.

Campbell et al. (2012) say that congruence cannot be proved using only angles. Therefore the AAA condition is explained and it leaves learners with no doubt of taking the condition for congruence. There are examples on naming pairs of congruent triangles. Learners have to apply knowledge of properties of 2-D shapes in order to match congruent triangles. There is need for careful reading and noting of all the details in the questions on proving congruent triangles.

The examples are followed by an exercise. The first question required learners to be able to name congruent triangles and state the case of congruence for each. The symbols used here are very important as they are a language which adds to the vocabulary of the learners. Knowledge of angles on intersecting line, parallel lines and perpendicular lines is required for the learners to answer the questions. The knowledge about parallel lines is also required.

There are questions which were asking learners to state the four cases of congruence in abbreviated form. Other questions were asking learners to prove congruence, application of the congruence cases and proof of congruency in triangles with common sides or common angles.

The properties of a circle are required in the proving of congruence on triangles. There were some questions which required learners to apply different properties of 2-D shapes to prove congruent triangles.

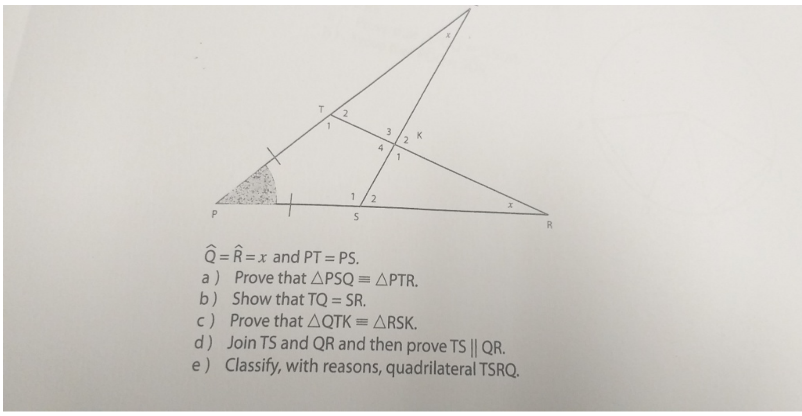


Figure 4. 20 Photography of a question on proving congruent triangles (Campbell, Heany, Mant, Rossouw and Williers, 2013: 134)

The above diagram shows an example of questions requiring proof of congruent triangles (see figure 4.21). This is the kind of information I indicated that leads to learners saying that it is 'obvious' where the learners need to provide reasons. Notice that the learners are expected to assume that all the lines are straight lines including PQ, PR PS and RT. This information is not provided in the diagram and without it learners may not be able to come to expect conclusions.

Although the angles and sides are clearly indicated for example, Q=R = x and PT= PS, the learners are expected to move from known to unknown. The learners are expected to study the diagrams before they attempt to answer any question. The diagram contains a lot of information that must be interpreted before proving congruence. The learners are expected to concentrate on all the given information and instruction in order to answer the questions accurately. Congruent triangles cannot be treated as an isolated concept in the curriculum. Figure 4.21 shows that only the learners who have mastered properties of 2-D shapes may attempt to answer questions. The question in figure 4.21 is one of the questions which learners tend to term 'difficult' because it involves a lot of concentration. On the other side, these are some of the questions which may help learners to develop their reasoning skills.

I noticed that the text book, Platinum Mathematics Grade 9 does not address the problem of cases which do not prove congruence in triangles like the ASS and AAA. Learners may be tempted to pick SSA as a reason for congruent triangles. If this happens at the foundation proving congruence, it may be difficult for learners to quickly grasp the concept correctly in the upper grades.

4.2.5.5 Mathematics in English Book 1 Grade 9 Terms 1 and 2

In addition to the two main textbooks discussed above, the teacher also used the Mathematics in English workbook (Department of Education (DBE), 2019). Most of the worksheets used by the teacher during classroom observations were extracted from this workbook. Like the textbooks, the workbook started with definition of congruent triangles (see Figure 4.22 below). The book states the cases of congruency in triangles as shown below.

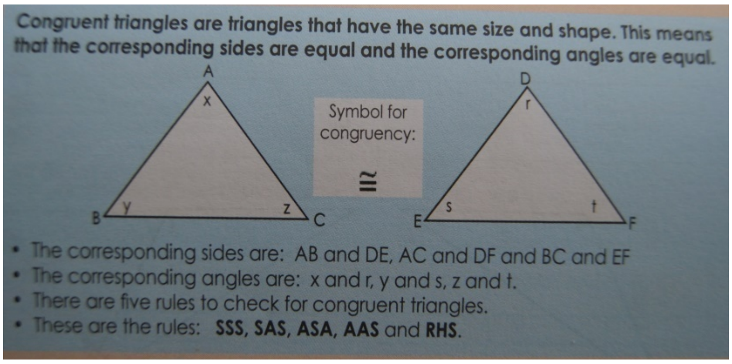


Figure 4. 21 Photography of summary of Rules for Congruent Triangles (DBE, 2019:138)

One of the weaknesses of the textbooks analysed in this research was that the diagrams lacked details which learners were supposed to grasp. There are no symbols or measurements used to show equal corresponding sides. This follows to say that the learners must read both the diagram and the information provided so that they will be able to make conclusions. Learners may be tempted to take diagram information for granted.

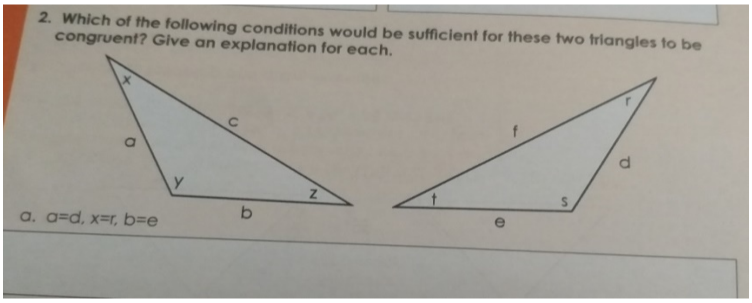


Figure 4. 22 Photography of the conditions for Congruent Triangles (DBE, 2011:141)

The conditions for two triangles to be congruent are outlined in the CAPS curriculum. Figure 4.23 shows an example of questions requiring learners to determine with reasons sufficient conditions for triangles to be congruent. The questions tend to be suitable for revision exercise to those learners who have grasped the conditions for congruence in triangles. I have noticed that the questions allow learners to have different correct answers.

I noticed that the Mathematics CAPS curriculum is to a great extent aligned to the levels of Van Hiele’s theory. The Senior Phase CAPS curriculum Grades 7 to 9 is organised in the following way:

- In Grade 7, learners are expected to recognise and describe congruent figures by comparing shape and size which is the same as Visualisation (Level 1) of Van Hiele's levels of geometric thinking.
- In Grade 8, learners are expected to identify and describe the properties of congruent shapes which is the same as Analysis (Level 2) of Van Hiele's levels of geometric thinking.
- In Grade 9, the learners are expected to establish the minimum conditions for congruent triangles which is the same as Informal Deduction (Level 3) of Van Hiele's levels of geometric thinking, (DBE, 2011).

The CAPS curriculum suggests that constructions are useful context for establishing the minimum conditions for 2 triangles to be congruent (DBE, 2011). Therefore the CAPS curriculum tends to suggest that Grade 9 learners are expected to operate at Level 3 of Van Hiele's theory. From the above analysis of Grade 7 to 9 Mathematics Curriculum, the content layout is hierarchical as proposed by VanHiele's theory. The findings concur with other research that textbooks have a lot of information and exercises prepared for learners to grasp the concept of congruence in triangles. Fujita and Jones (2016) found that the aspect of proof is over-emphasised in the Japanese textbooks thereby shadowing them from seeing the rationale for grasping arguments based on empirical evidence while learning to write proofs on geometry. The workbook the teacher used tends to be suitable for revision as it is more summative than a teaching tool.

4.3 Comparing Classroom Observation and Document

4.3.1 Analysis Results

The lessons observed were informed by the documents the educator used to prepare. There is a direct link between Mathematics lessons observed and documents analysed. The educator used his experience and knowledge of interpreting the CAPS and some textbooks.

Each lesson taught began with checking learners' knowledge of shapes especially the names of triangles and their properties (see lesson1-5). The CAPS document and the textbooks used refer to the prior knowledge of the triangles. It was evident from the delivered lessons that the educator introduced each lesson with the inclusion of prior knowledge. The prior knowledge is stated in the textbooks as well as in the CAPS documents that the educator was using. At the beginning of each lesson the educator checked whether the learners were aware of the properties of triangles and quadrilaterals, theorem of Pythagoras and types of angles on different lines. The prior knowledge was checked either through questioning or through a class discussion.

The educator defined what congruent triangles are in each of the delivered lessons. This is in line with what the textbooks say when they define what congruent triangles are at the beginning of the topic on congruent triangles. The books do not state how the definition of congruent triangles should be taught. The teacher explained to the learners orally how triangles are congruent. There are examples from the textbooks which the educator used to explain the congruence rules.

One thing the educator did not do was to allow the learners to use their textbooks in any one of the five double lessons observed. The textbooks used are different approaches and one of them would have helped him explain one of the concepts. Learners understand that books have authority and rarely would we have learners arguing with what the book says but rather than they seek to understand what is in the books. For example, Mathematics Today says that congruent triangles are not proved by corresponding angles only. There should be at least one pair of corresponding sides. This could have helped some of the learners to see for themselves that AAA cannot be used to prove congruent triangles. Giving learners a scenario or case in the textbook for them to discuss or solve in groups helps them to acknowledge the facts in the book.

The textbooks as well as the CAPS document outline the minimum requirements for triangles to be proved congruent. The methods advocated by the curriculum were not followed by the educator. The CAPS document states clearly that checking of minimum facts required in proving of congruence in triangles should be through investigation. This is one point were learners were supposed to display their reasoning skills. However, the educator resorted to the use of the lecture method which is less time consuming. One of the methods would be by construction that the learners were able to determine the minimum number of facts required to establish congruence in triangles. The curriculum requires the learners to rediscover congruence in triangles by engaging themselves in the construction of the triangles.

The textbooks as well as the curriculum treated congruence and similarity in the same topic. Mathematics Today Grade 9 learner's book has taken Congruent and Similar triangles as a topic to be taught together while in Platinum the topics are separated but follow each other starting with congruent triangles. The learner's workbook started with similar triangles then congruent triangles. The textbooks especially Platinum have categorically stated the difference between similar triangles and congruent triangles. The educator pointed out in his teaching that similar and congruent triangles are different. The educator said that congruent triangles are similar and similar triangles are not necessarily congruent. Platinum Mathematics Grade 9 explains that equiangular triangles are not congruent and that there is at least one pair of corresponding sides in the triangles that are congruent.

4.4 Comparing Classroom Observations and Focus Group Discussion

The focus group interviews were done one week after the lesson observations. Focus group participants were selected from those who were observed during classroom observation to participate in the study. The interviews were an extension of the lesson observations or a clarification of some problematic question in the learners' minds. This part helped to establish what learners know about proving congruent triangles.

Properties of triangles and quadrilaterals were revised with some of learners failing to understand some of the important facts like a square having the same properties as a rectangle thereby rendering it to be a rectangle. During classroom observations there was no deep discussion on the properties of quadrilaterals as compared to what happened during the lesson observation. Learners were eager to know more about the differences between shapes.

The learners revealed their views and interests in the topic under discussion, congruent triangles. They acknowledged that congruence of triangles is an important topic that leads them to develop reasoning skills. Three learners of six learners in the focus group interviews expressed their feelings about the subject, Mathematics. They were saying that given a choice about Mathematics, they, rather choose other subjects because of topics like Congruent Triangles which is difficult for them. Two of them said that they understand the importance of the subject although it is difficult they will continue doing it. One of the learners said that he had passion and interest in Mathematics and likes topics like Congruent Triangles.

The traits of learners' proving of congruent triangles in the classroom observations were revealed in the focus group interviews. Generally the learners were quiet during classroom observation and were reluctant to answer the worksheet problems. This was answered by their expression of discomfort in proving congruent triangles. Some learners categorically expressed their dislike of both the topic and Mathematics during our focus group interviews. One learner said, "I would rather do another subject than doing Mathematics it is stressing me every day". It is an attitude issue which takes a long time to build as compared to failure to grasp the concept or a result of poor teaching methods. The learner has told herself that Mathematics is not palatable and has fixed herself to failure than take pains to do the subject patiently.

One of the participants of the focus group interview mentioned that congruent triangles can be proved by the rule of only corresponding angles of triangles being equal. Most of the learners supported her. Amongst the participants who supported her was the learner with notes showing that AAA can be used to prove congruence in triangles. The educator did not control the learners' notes and error went unnoticed. The same notes were taken from the learner's workbook. However it is the learner who wrote this point that AAA of corresponding angles proves congruence in triangles and it is not in the book. This gives a picture that an educator's failure to notice mistakes or errors in learners' written work can cost the learner dearly. The educator had uttered a statement about corresponding angles resulting in similar triangles and not congruent triangles. Telling learners facts about what they should know results in more harm than good. In the present Mathematics curriculum, educators are expected to be facilitating learning rather than imposing knowledge. This could be one of the reasons why learners tend to struggle with concepts of the previous years. The focus group debated this and they remained divided over what is true. Although, the participants were later corrected in another discussion after the designated discussion, a point was proved that learners did not know the truth about AAA. Primasatya and Jatmiko (2018) claim that in order to improve the ability to think critically for learners, it is necessary to develop a learning that allows learners to explore in order to find a particular concept.

In the discussion, the question of whether SSA or ASS may be used as a rule for proving congruent triangles was raised. Most of the learners were of the view that *it is one of the possible rules for proving congruent triangles.* The learners had this problem not cleared during the classroom observation as there was not any one practical activity used to prove that they do not work or any provision of a counterexample. One learner argued during the focus group discussion that the rules are not a way of proving congruent triangles. However, she could not convince her colleagues as this proof was not tangible. Her point of view could have come from her personal studies. All the three textbooks have mentioned that SSA or ASS is not a way of proving congruence in triangles. This shows that the educator ignored or did not consult these books when he was preparing his lessons. It was important for the educator to take this fact as a teaching point.

The participants indicated that they saw no reason why they should justify their obvious reasons of congruent triangles. One of the participants said that the triangles are of the same size and shape by merely looking at them. DBE, (2018) suggested that learners must not make assumptions as the diagrams are not drawn to scale. Learners must be taught that giving reasons for every step taken in proving congruence is the correct way to solve problems in geometry than make assumptions. Some accepted that they lack proper terms to use to prove congruent triangles. The participants generally agreed that when proving congruent triangles in class or in groups it appears simple but the problem comes when they are doing the work individually. They are somehow inclined to the use one word answers. This is evidenced by the exercise they were asked to prove with reasons why pairs of triangles are congruent or not. The majority of the participants wrote the answers only without reasons.

Summary of Research Findings6

Observation	Focus Group Interviews	Document Analysis
Learner could identify 2-D shapes	Learners were more precise on the types of 2-D shapes	The three textbooks plus the CAPS document explained what 2-D shapes are with given examples
Less than half the learners could use the properties of 2-D shapes	Learners lacked objects to explain 2-D shapes Vocabulary was a challenge to two thirds of the learners.	Each shape or class of shape was explained in simple terms for learners to grasp.

Most of the learners failed to define terms involving 2-D shapes	Two out of six in each group could define terms accurately.	Most of the terms are defined in each textbook.
Learners struggled to differentiate between 2-D and 3-D shapes	More than half of the learners could not differentiate 2-D from 3 –D shapes.	The books separated 2-D from 3-D shapes.
Learners failed to interpret complex questions involving proving of congruent triangles	Two out twelve learners were able to explain their answers with reasons	One book gave a lot of thought provoking questions on proving congruent triangles.
Learners could identify axioms but failed to attach reasons.	Most of the learners understood the axioms of congruence of triangles through discussions.	All the textbooks explained the axioms with reasons to each step.

Table 4. 3 Summary of Research findings

4.5 Discussion of the Findings

4.5.1 Introduction

The Van Hiele's theory indicates that effective learning takes place when learners activity experience the objects of study in appropriate contexts, and when they engage in discussion and reflections (Mason 2002). In this section of the study, the findings will be discussed in relation to how Van Hiele has suggested for the development of geometric reasoning of learners to occur. The purpose of this study was to investigate the reasoning skills of the ninth graders in the proving of congruent triangles. Three instruments were used to investigate the learners' geometric reasoning according to Van Hiele's theory. The methods used were classroom observation, focus group discussion and document analysis. Each of the research findings will be discussed in light of Van Hiele's levels of geometric thinking. The Van Hiele levels included in this study are visualization, analysis and informal deduction. The last level of Van Hiele's theory was purposefully left out because it is beyond the scope the high school learners' geometric development level. The Van Hiele levels will be discussed under the following subtopics:

Visualisation

Analysis

Informal Deduction and

Deduction

4.5.2. Findings and the Van Hiele Theory

Van Hiele puts forward a hierarchy of levels of thinking ranging from about the age of five years through to academic adults. The theory states that the learners' geometrical thinking is sequential. One of the theories that help greatly and effectively in teaching geometry is Van Hiele which attracted scholars and educationists' attention throughout the world over because it helps effectively in teaching geometry to the students through the school stages (Al-ebous 2016). The findings of this study are measured against the Van Hiele levels of geometric thinking.

I observed five lessons taught by an experienced teacher. The teacher used the lecture method where he dominated throughout the five lessons. One or two axioms were presented in each lesson. There was a minimal involvement of the learners as depicted in transcript 1(Appendix K). In lesson 2, 3 and 4, learners participated in about the last 15 minutes of each lesson. The discussion time constituted about 25% of the lesson time. The teacher's practice contradicted what Van Hiele advocated in the teaching and learning of geometry concepts. The learners were expected to be more of hands-on as opposed to being passive recipients. The teaching method was contrary to what Mason (2016) says about teaching geometry concepts, when he says that the Van Hiele theory indicates that effective learning takes place when learners actively experience the objects of study in appropriate contexts, and when they engage in discussion and reflection. The lessons were more of factually oriented as opposed to what Van Hiele suggested. Van Hiele advocates for teachers to concentrate on the development of insight in learners to help them move from one level of thinking to another higher level (Brodie 2000).

The teacher used an already made lesson plan which barely had an objective and appropriate learner activities (see Appendix M). A lot of our lesson plans are missing an understanding of the Van Hiele levels and how it plays into understanding geometry concepts (Geometry, 2017). The findings concur with what the literature has said about lack of proper planning and knowledge about the Van Hiele's theory. There was a great difference between classroom observations and focus group interviews. The majority of the learners in focus group interviews were more involved as compared to classroom observation lessons (see Appendices K and L).. The focus group interviews included the identification of shapes, description of the shapes according to their properties and the making of informal deductions about the shapes. The findings showed that two learners in both focus groups were still on level 1, seven out of 12 learners were on level 2 and three out of 12 learners were on level 3.

The educator tended to focus mainly on the most important aspects of proving of congruent triangles. I observed that learners were not given opportunity to discover the minimum conditions for congruence in triangles for themselves. Cirillo et al. (2015) state that it is fairly typical for teachers to tell their learners which triangle congruence criteria are valid and then have learners use those postulates in proofs. One textbook was clear on how to prove that both AAA and SSA are not rules for congruent triangles (Groenewald et al. 2012). The educator appeared to be ignorant of the Mathematics CAPS curriculum suggestion on how to investigate the minimum facts needed to prove congruence in triangles. The learners were limited to the information provided by the educator. The learners were not referred to the textbooks as an authority on proving congruent triangles.

Learners struggled to find the correct terms to describe shapes. Language is undoubtedly an essential tool in communication (Jojo, 2015) and Geometry tends to stress more on the use of language than any other part of Mathematics. When learner 4 in focus group interviews 1 failed to find the correct term for an acute-angled triangle, he resorted to drawing the triangle on the chalkboard. He said, *I want to draw the triangle, I know* (learner drew an acute-angled triangle on the chalkboard).

The learners found it difficult to identify the properties of a circle that would enhance the proving of congruent triangles on the circle. Only Learner 2 in focus group 2 managed to identify radii in the circle with triangles to be proved congruent. All the learners in group 1 agreed that a diagonal BD on quadrilateral ABCD is a common side to ΔABC and ΔACD. Focus group 2 struggled to figure out the, if… then… concept questions about properties of triangles. Learner 2 from group 2 asked why BC would appear in two triangles when it is one side. The learners in both groups view relationships of lines and shapes differently. For example, learners took time to see that there are vertically opposite angles on intersecting. Most of the learners in group 1 failed to see that congruent triangles are also similar triangles. In group 2 half of the learners (three out of six) could explain the difference between similar triangles and congruent triangles by way of drawing diagrams. Learner 4 in group 1 explained *(pointing at the drawn diagrams*) saying, *"Congruent triangles are equal in all respect meaning that all their corresponding sides are equal and all their corresponding angles are also equal, while similar triangles have corresponding angles equal. Sides are different (see Appendix G).*

The focus group interviews showed that most of the learners (an average of 76% of the learners in the focus groups) believed that proving of congruent triangles was difficult for them (Appendices K and L). For example, learner 6 in focus group 1said, *honestly speaking I do not like Mathematics especially proving of congruent triangles. It is very difficult for me*. Again learner 4 in focus group 2 said, *Iii eish it's mixed up I know Mathematics is very important but it is very difficult. Congruence appear easy to me but when I am doing it alone I become confused. I think I started to fail Mathematics in Grade 8. I used to like Mathematics in primary school. There are times when I know it and at other times I am off, off.* The learners were affirming Miyazaki, Fujita and Jones' (2017) study which found that students at the secondary school level and beyond experienced difficulties in understanding proof in mathematics in general, and in geometry in particular. The fourteen year old learners are expected to be starting to learn to construct proofs in geometry.

I found that the Mathematics curriculum for the Senior Phase Grade 7-9 was designed in a spiral manner to cater for learners' geometrical development. It tends to be aligned to Van Hiele's theory. Van Hiele suggested that learners develop from one level of geometric reasoning to the next through structured instruction. Congruency starts in Grade 7 where learners are expected to recognize and describe similar and congruent figures by comparing shape and size. There is a suggestion that learners have to be given careful instructions about how to do the constructions of the various shapes. In Grade 8 learners are required to identify and describe the properties of congruent shapes. The CAPS document showed that the same learners were required to recognise that two or more figures are congruent if they are equal in all respects that is, corresponding angles and sides are equal (DBE, 2011). In Grade 9, learners are required to investigate through construction the minimum number of facts to prove congruency. The Mathematics CAPS curriculum tends to promote learners' hands-on experience where they are required to identify, to construct and to investigate 2-D shapes (DBE, 2011). To a great extent, the Mathematics CAPS curriculum shows that the document was designed taking into consideration the importance of theories like the Van Hiele's theory of geometric thinking. I other words, aspects of Van Hiele's theory are enshrined in the current Mathematics CAPS curriculum.

The analysed textbooks tend to be biased towards content as compared to geometric reasoning development. Most of the problems in almost all the textbooks appeared to be knowledge based questions (Groenewald et al. 2012, Campbell, 2013 and DBE, 2019). The problems have very little evidence of discussion aspects which develops reasoning skills. The textbooks tended to promote the memorization of conjunctures, rules and algorithms. However, there are a few questions which provoke learners' thinking. An example of such questions is outlined below in figure 4.22.

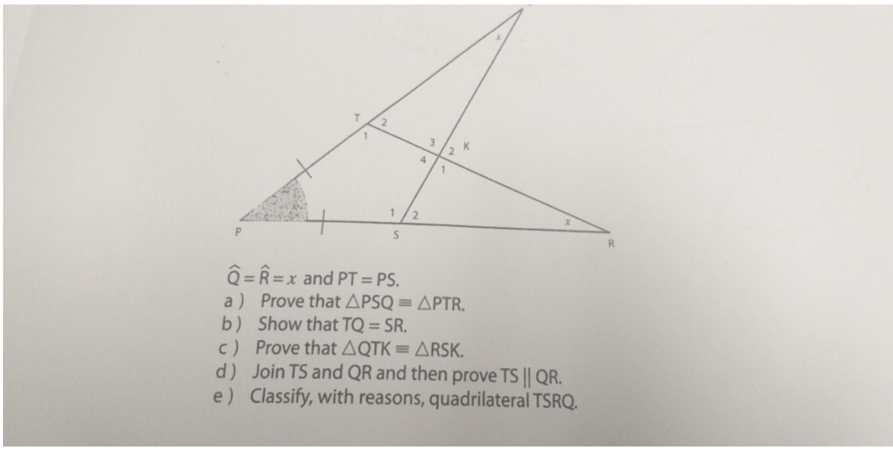


Figure 4. 23 Photography of Question 6 in the book Platinum Mathematics Grade 9 Learner's book (Campbell et al, 2013:134)

This question allows learners to think before attempting to answer them. Learners should be able to see properties of different triangles and be able to identify congruent triangles with reasons. When learners manage to prove that ΔPSQ ≡ΔPTR in (a) on the diagram, they might be able to see other relationships between the triangles. Question 4 is one of the problems that enhance thinking on the part of the learners. If question 4 (d) and 4(e) are done properly, they are likely to prepare learners for the Midpoint theorem found in Grade 10 curriculum.

4.5.3 Classroom Observation

In the five lessons observed, the teacher did not use any concrete shape for the learners to identify as suggested in level 1 of Van Hiele's theory of geometric thinking (Van Hiele 1986) and the recognition of figures by appearance alone (Chimuka, 2017). This could be that the teacher took for granted that learners have passed level 1of visualisation and that he assumed that learners can now comprehend abstract concepts. Failure to use physical aids in the teaching of geometry leads to weakness in the development of geometric thinking at Van Hiele's level 1 (Armah and Kissi, 2019). Research showed that even if the learners are familiar with visual aids, they still need them in their lessons. If pre-service educators were found to be in need of visual aids to understand (Armah and Kissi, 2019), then the Grade 9 learners would need more of visual aids.

In lesson 1 to 4 almost all learners were able to name shapes like triangle, squares rectangle and rhombus. The learners could recognize and represent the figures (triangles) as given by De Villiers and Njisane (1987) who stated that learners at level 1 recognise and represent figure types. In lesson 5 there were compound diagrams which confused the majority of the learners. Learners could see the triangles but were unable to pick individual shapes and describe them. The majority of the learners could see the shape as a whole as outlined in Van Hiele's theory. The learners could identify 2-D shapes easily. This gives a picture that almost all the learners were able to answer Van Hiele's Level 1 concepts.

In the lessons observed, a few of the learners could argue in support of their suggested answers. The learners failed to see the interrelationship between different triangles as advocated by Van Hiele in his theory of geometric thinking. In lesson 2, 3 and 4, learners were able to make one step deduction. The majority of the learners saw no reason why they should justify their answers (Figure 4.9). They have taken the proving of congruent of congruent triangles as an obvious thing.

In all the five lessons observed, the educator emphasised the idea of recognising the properties of the pairs of shapes (triangles) to be proved congruent. It was in lesson 1 and 2 where learners paid particular attention to the properties of the triangles. The majority of the learners were able to define different types of triangles like Learner 2 in Lesson 2 who said *an equilateral triangle is a* *triangle where all the sides are equal and the angles are each 60°.* The majority of learners were able to compare figures according their properties.

In lesson 3, the educator asked learners to explain what it meant to say that triangles are congruent using the SSS axiom. About 40% of the learners understood that SSS meant that the sides of two or more triangles are equal. The Van Hiele expects a learner at level 2 to identify and tests relationships between parts of figures, congruence of sides (Fuys et al. 988). In lesson 3 the learners were required to state with reasons why figures were congruent or not using the given properties. Learners answered the questions differently. Learner 1 used insightful approach to answer the question. She first of all converted the millimeters to centimeters as she could convert different units to the same unit and she also compared the corresponding sides and angles of the triangles before she wrote her suggested answers. Learner 3 guessed his answer since he failed to justify his answer.

Learners struggled to define most of the simple terms in the proving of congruent triangles. Only three out of 32 learners (about 10%) appeared to be grasping the concept of congruent triangles abstractly. Learner 4 of lesson 4 indicated that *AAA is not an axiom of proving congruent triangles. S*he explained the concept of proportionality in the triangles under discussion. She explained that *triangles are not always congruent when considering their corresponding angles only. Their corresponding sides are not necessarily congruent but proportional rendering them to be similar triangles (see Appendix K).* Three of the 32 learners tried to solve Question 5 in lesson 5. These learners were able to use interrelationships of properties of triangles to prove congruent triangles.

Both the Mathematics CAPS curriculum and Van Hiele's theory expect the Grade 9 learners to operate at level 3 (DBE 2011 and Crowley 1987). Wang et al. (2018:92) say, "From the perspective of development of geometry content, congruent triangles reasoning and proof is the beginning of formal mathematical reasoning and proof". This may be interpreted to mean proving of congruent triangles is the first step towards formal reasoning for the Grade 9 learners. This tends to suggest that learners are not expected to be operating at level 4.

4.5.4 Focus Group Interviews

The focus group interviews were more interactive than most of the observed lessons. The two focus groups were able to name different shapes (see Appendix L). Two of the learners in group 2 acknowledged that they did not know a trapezium. Learner 4 in group 1 wanted to draw the shape on the chalkboard. Learner 3 in Group 2 could not differentiate 2-D shapes from 3-D shapes. The learner was not sure whether a cube was a 2-D shape or a 3-D shape. These examples were an indication of the need to have concrete shapes in the learning congruent triangles. More than 90% of the learners in the interviews were able to identify 2-d shapes. In other words, both focus groups showed that there was need for visual aids in the proving of congruent triangles.

The learners could describe figures according to their properties for example, equilaterals as triangles with congruent sides and congruent angles but they did not understand the interrelationship between these different types of figures (Armah and Kissi, 2018 and Siew et al. 2013). The learners failed to compare classes of figures according to their properties as advocated by Howse and Howse (2015) who indicated that the learners are expected to interpret verbal and symbolic statements of rules so that they can apply them. The learners also failed to use insightful approaches to solve geometric problems.

Both focus group interviews compared quadrilaterals using properties. The learners agreed that diagonals divide a quadrilateral into triangles that may or may not be congruent. For example, a square gave them two triangles which were congruent. The learners recognised that the formed triangles had equal corresponding side and equal corresponding angles. One of the learners in focus group 2 mentioned that *the diagonal is a common side of the formed two triangles*. Most of the learners in both focus groups were not familiar with quadrilaterals like a trapezium and kite (Appendix K). When they were explaining the properties of the quadrilaterals, none of the learners could describe these shapes. Learner 3 from group 2 said that *a trapezium is like a scalene triangle which has all sides different.* Another learner in the same group added saying that he *thinks that the angles of a trapezium are different and are more than 360°.* A parallelogram was drawn on the chalkboard for discussion. The two groups agreed that the name parallelogram comes from the parallel sides of the shape. They also concurred that *a parallelogram has equal opposite angles and equal opposite sides.* They could all identify the corresponding sides and angles of the triangles formed by drawing a diagonal across the parallelogram. Not all the learners in this study were able to describe shapes using properties.

In separate interview sessions both groups agreed that *when two sides of a triangle are equal then the opposite angles to those sides are equal* (Appendix L). Learners demonstrated that they can calculate the measures of the remaining angles of an isosceles triangle when given any one angle of the triangle. Learners grasped the concept of supplementary angles accurately.

The learners found it difficult to identify the properties of a circle that would enhance the proving of congruent triangles on the circle. Only one learner, learner 2 in focus group 2 managed to identify radii in the circle with triangles to be proved congruent. All the learners in group 1 agreed that a diagonal BD on quadrilateral ABCD is a common side to ΔABC and ΔACD. Focus group 2 struggled to figure out the, if… then… concept questions about properties of triangles. Learner 2 from group 2 asked why BD would appear in two triangles when it is one side. The learners in both groups view relationships of lines and shapes differently. For example, learners took time to see that there are vertically opposite angles on intersecting. Most of the learners in group 1 failed to see that congruent triangles are similar triangles. In group 2 half of the learners (three out of six) could explain the difference between similar triangles and congruent triangles by way of drawing diagrams. Learner 4 in group 1 explained *(pointing at the drawn diagrams*) saying, *"Congruent triangles are equal in all respect meaning that all their corresponding sides are equal and all their corresponding angles are also equal, while similar triangles have corresponding angles equal. Sides are different (*see Appendix L).

It was difficult for learners in both groups to comprehend the relationships between triangles in a compound diagram. Only three learners in group 1 could separate triangles in a diagram. Two of the three learners were the same learners who also dominated in the classroom observation. The idea of triangles sharing a common side or a common angle was new to the learners. The focus group discussions also affirm that the learners were not engaged in any activity where they were expected to reason formally (Appendix L).

4.5.5 Document Analysis

The topic Geometry of 2-D shapes begins with a revision of the shapes like parallelogram, rectangle, square, rhombus, trapezium and kite (DBE, 2011). The shapes are then described according to their properties. The learners are expected to identify different types of triangles like the equilateral, isosceles and scalene and right-angled triangles. The textbooks consulted by the teacher for his lessons were also checked to see whether they included the identification of shapes. In the book, Mathematics Today Grade 9 the authors refer the learners to the identification of congruent triangles that was given in Grade 7 and 8 as important aspects to note (Goenewald et al. 2012). In the textbook Platinum Mathematics Grade 9 the definitions and identification of triangles and quadrilaterals are outlined as important aspects (Campbell et al. 2013). The textbooks lacked some detailed information which helps clear misconceptions in learners. For example, in Figure 4.21 there are no symbols for corresponding sides and angles being equal. This is likely to lead learners to take for granted that triangles are equal without an indication of symbols. .

The Mathematics CAPS curriculum specifies that the learners should be able to identify and describe the properties of congruent shapes including congruent triangles (DBE, 2011). The relationship between the properties of shapes is vital. The properties of similar triangles and congruent triangles are clearly spelt out in all the textbooks the educator used (DBE, 2019, Groenewald et al. 2012 and Campbell et al. 2013). In the book Mathematics in English Grade 9 Term 2 Book 1 and 2 presents a topic on polygons where learners are expected to give differences and similarities between polygons (DBE, 2019). This is where lines of symmetry and or diagonals are drawn to determine the shapes formed. The concept of congruent triangles can be easily picked from diagonals drawn on some shapes like on a rhombus or on a square. Learners are expected to compare the properties of triangles formed by drawn diagonals.

The CAPS document suggests that conditions of congruency in triangles can be done through investigation. DBE (2011:136) says, "Constructions are useful contexts for establishing the minimum conditions for two triangles to be congruent". The documents used by the teacher tend to agree with Van Hiele's theory which states that at the informal deduction level, learners identify different sets of properties that characterise a class of figures and tests that these properties are sufficient. In both the curriculum and textbooks used by the teacher to prepare his lessons, the topic on construction comes before the congruence topic. Therefore, the skills gained in construction of shapes are used in determining the minimum conditions for congruence in triangles. The curriculum tended to be aligned to Van Hiele's theory which says that at the ordering level the learners follow a deductive argument of their own. The learners also discover new properties of deduction.

In the book Platinum Grade 9, the last exercise on the topic congruent triangles contains difficult questions which challenge learners to use deductive skills. There are complicated diagrams which require the learners to go beyond the knowledge of identifying properties of shapes. An example of such questions is found in the observed lessons where learners were given to answer a question on Figure 4.8. It is interesting to note that both the diagrams and the written information are equally important in the answering of the questions. Learners need to be able to read the instructions properly in order to answer accurately.

Learners should also be able to solve geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties of triangles and quadrilaterals, as well as properties of congruent triangles. DBE (2019) points the learners to the fact that when two angles of one triangle are equal to two angles of another triangle then the third angles are also equal. This fact leads learners to the "if … then" concept in proof. Again the learners are expected to deduce that if only the corresponding angles of triangles are proved to be congruent then the triangles are similar and not congruent. Campbell et al. (2013) state that equiangular triangles are not congruent and that congruency cannot be proved without at least one pair of equal sides.

I noted that learners did not refer to any of the textbooks during the lessons. The learners struggled to explain a point that is explained in the textbooks. For example, the class debated the issue of ASS and AAA as rules for proving congruence when Groenewald et al. (2012) in their book Mathematics Today explained the concept (Appendix K).

4.5.6 Overall Results

The research findings showed that learners were operating at different levels of Van Hiele's geometrical thinking levels. About 10% of the learners (three out of 32) showed that they were operating at level 3 of Van Hiele's levels of geometric reasoning. About 23 out of 32 learners (72%) showed that they were operating at level 2. About eight out of thirty-two learners were still operating at level 1 of Van Hiele's levels of geometric reasoning. These findings concur with a number of researchers who found that Senior Phase learners operate below expected level of Van Hiele's geometric levels (Onda et al. 2017 and Wang et al. 2017). If the learners were encouraged to use the textbooks in doing their classwork or homework, we could have had a better response. The deficits could also include poor teaching methods, lack of learner participation and lack of attractive teaching aids.

According to Van Hiele's theory at the deduction level, learners can construct proofs, develop a proof in more than one way is seen, the interpretation of necessary and sufficient conditions is understood and the distinction between a statement and its converse is made (Stefanowicz, 2014). Crowley (1987: 16) says, "No method of instruction allows a learner to skip a level; some methods enhance progress, whereas others retard or even prevent movement between levels". There was no learner who attained level 4 of Van Hiele's geometric thinking levels. The findings of this study agree with what is in the literature review.

4.6 Conclusion

The purpose of this research section was to present, analyse and discuss the research findings. The instruments used were the participant observation, focus group discussion and document analysis. The findings from the three methods used, show similarities and differences about the reasoning skills of ninth graders. The findings of this study were discussed in light of the Van Hele's levels of geometric thinking. The findings show that most of the learners operated at level 2 of Van Hiele's geometrical thinking. Each of the research findings were presented separately. The findings of the three research methods were discussed and analysed under the same section. In the discussion of each research method included four of Van Hiele's levels of geometric thinking, that is, visualization, analysis, abstraction and deduction. I decided not to include the fifth level which is above the scope of secondary school learners. The last and final chapter of this study will make conclusions, implications and suggestions for future researchers.

CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

5.1 Introduction

This chapter serves to summarise the entire study on the reasoning skills of the ninth graders in the proving of congruent triangles. Each chapter is highlighted, conclusions are made, implications are pointed out and the limitations are outlined. Comments are also made in light of the literature review and the theoretical framework used. The research sub-questions will be discussed with regards to results obtained from the classroom observation, the document analysis and the focus group discussions. This chapter concludes with suggestions for future research on the reasoning skills of ninth graders in the proving of congruent triangles.

5.2. Summary of the study

In Chapter 1, a brief background of the study was articulated and its purpose was explained. The purpose of this study was to explore the reasoning skills of the ninth graders in the proving of congruent triangles. Learners' challenges in the proving of congruent triangles were also highlighted. A research question was formulated. To expand the main question, three sub-questions were formulated. The rationale of the study was also explained in detail.

Chapter 2 discussed the literature related to the proving of congruent triangles and proof in general. The literature review showed the importance of deductive reasoning in the proving of congruence. Proving of congruent triangles forms the basis for Euclidean Geometry proof in the Mathematics CAPS curriculum. Based on the analysis of the literature, the Van Hiele theory was identified as the most suitable theoretical framework for this study. The conceptual framework was then discussed in depth where levels of geometrical reasoning were explained. The framework helped to determine the levels of learners' geometric reasoning skills.

In chapter 3, the research methodology was discussed with intrepretivism as the underlying philosophy. The paradigm allows the researchers to view the world through the perception and experiences of the participants (Thanh and Thanh, 2015). A case study was adopted where learners were studied in their natural learning environment whilst they were learning the proving of congruent triangles. A grade 9 class of 32 learners together with their Mathematics educator was conveniently chosen to participate in the research. Furthermore, two groups of six participants in each group were purposively selected to participate in separate focus group discussions. Classroom observation, focus group interview and document analysis were used as instruments for collecting data. Issues of trustworthiness and ethical considerations were also discussed in this section of the study.

In chapter 4, the research findings were presented and discussed in depth. Each instrument findings were treated separately. The audio and video taped data were transcribed into transcript 1 and 2 consecutively. The participant observation was the main instrument used in seeking to find the learners' reasoning skills in the proving of congruent triangles. Document analysis worked as a complementary instrument in establishing learners' reasoning skills. The focus group discussions were used to consolidate what has been revealed by the other two instruments. Lastly, the results from the different instruments were compared and contrasted.

5.3 Discussion of research questions

The aim of the study was to investigate the reasoning skills of ninth graders in the proving of congruent triangles. It also investigated the challenges faced by the learners in communicating their reasoning skills while proving congruent triangles. The main research question was formulated as: What are the challenges faced by ninth graders in communicating reasoning skills in the proving of congruent triangles? This question was further broken down into manageable sub-questions as follows:

How do ninth graders use properties of 2-dimensional shapes in proving congruency of triangles?

How do the ninth graders use congruence axioms to make deductions?

How do the ninth graders communicate their reasoning skills in the learning of congruent triangles?

5.3.1 Research sub-question 1

To answer the question, how ninth graders use properties of 2-dimensional shapes in proving congruency of triangles, the learners were observed proving congruent triangles in their natural learning environment. They were also engaged in focus group discussions where they expressed their understanding of the concept of proving congruent triangles. The analysed textbooks revealed what was expected of them in the learning of congruent triangles. The data from observed lessons and focus group discussions were transcribed into transcripts 1 and 2 respectively.

The findings based on this study indicated that the learners were able to identify the properties of 2 dimensional shapes. There appeared to be no connection between knowing the properties of the 2 dimensional shapes and using the properties to prove congruence in triangles. The same learners who were able to identify properties of 2 dimensional shapes could not see the properties while answering congruent triangles questions. The educator's teaching method, the lecture method could have influenced how the learners reasoned when answering questions. The lessons were treated as separate entities which led learners to see different concepts not related to each other as they proceeded with the topic. Most of the talking was done by the educator leaving learners to be more of passive recipients. Throughout the five lessons, learners were bound to be listeners than active participants.

In most cases, teachers tend to dominate in a lesson so that they can be able to complete the curriculum. The National Senior Certificate Examination diagnostic report suggested that more time needs to be spent on the teaching of Euclidean geometry in all grades (DBE, 2018). In the lessons I observed, the teacher appeared to have wanted to impress that he knows the content at the expense of the learners' display their reasoning skills. Again, lack of prepared learning aids could have contributed to the teacher's choice of the lecture method.

Three out of 32 learners in the observed lessons were able to use the properties of triangles to prove congruence (see Appendix F, lesson 5). The learners were able to pick the triangles from a compound diagram to prove congruence. These learners were able to identify the corresponding sides and corresponding angles that were equal. The learners were aware that the triangles discussed were congruent but would not be able to state reasons for their congruence. The learners generally, got confused to work with compound shapes with a lot of detail about the diagram. Learners did not pay particular attention to the instructions and symbols in the diagram. The skill of reading instructions and identifying meaning of symbols tend to be lacking in the majority of the learners. Most of the learners acknowledged that they did not pay particular attention to the instructions and the information on each diagram presented to them. It was after a long discussion when the majority of the learners would be able to grasp the relationship between the properties of the 2 dimension shapes and the proving of congruent triangles. The results show that learners were not able to see the connection between properties of 2 dimension shapes and the proving of congruent triangles.

The discussions done during focus group interviews indicated that learners were quick to identify the properties of 2-D shapes (see Appendix L) while the majority of them failed to pick radii on the circle, vertically opposite angles on intersecting lines and a diagonal being a common side on new triangles created when proving congruence in triangles. From the focus group interviews done, only two out of six learners were able to use the properties of triangles to prove congruence. The two learners dominated the interviews throughout the 3 sessions in group 2. These were the same learners who constantly corrected other learners' misconceptions (see Appendix L, Transcript 2). About 3 learners out of 32 were able to use knowledge on 2-D shapes to prove congruent triangles. According to van Hiele's theory, the majority of these learners were operating at level 2. This shows that the learners were operating below the expected level which is level 3 according to other researchers quoted in the literature review of chapter 2.

Observations show that the learners rarely used the textbooks to refer to any question or problem before them. The textbooks treated the concept of congruent triangles as one of the concepts which belong together with theorem of Pythagoras, similarity, angles and measurement of length (see chapter 4). It is true that an educator should not be a slave of the textbook, but in this study the educator decided not even to refer to anything in any of the textbooks. When learners were struggling to prove congruence in triangles, I expected the learners to refer to any of their textbooks. Mironychev et al. (2018) aver that textbooks are considered the key tool when studying a subject. I strongly believe that the textbook would have made a difference if the learners were encouraged to use during lessons and also for their homework. The text book was not used by the learners during the observation and group discussions as portrayed by their responses to questions about the previous work.

5.3.2 Research sub-question 2

To determine whether learners were able to use the congruence axioms, they were observed proving congruent triangles in five Mathematics lessons and were also given an opportunity to discuss the proving of congruent triangles in three sessions of focused group discussions.

The learners were given exercises to do after each lesson delivered in class. The learners seemed to know most of the congruence axioms. Unfortunately some of the learners worked backwards, to get the answers. Instead of giving reasons which led them to a particular axiom, they started by stating the axiom without reasons. DBE (2018: 182) says, "Attention should be paid to reasons. Teachers should not condone the use of incorrect reasons in classwork and class based assessment tasks". Although, learners were aware of the process of proving congruent triangles they decided not to give reasons but to state the rule only. Some of the learners argue that congruent triangles are clear and obvious such that there is no need to write down the reasons (see Appendix K). Learners must refrain from making assumptions (DBE, 2017). Some of the learners were stuck on the questions they were supposed to show their understanding of congruent axioms. They however showed signs that they cannot connect the knowledge on 2-D shapes and the axioms for proving congruence in triangles. The same learners who could identify 2-D shapes were failing to use the facts about these shapes to explain the axioms.

In this study, it was found out that almost all the learners were aware of the congruent axioms. The learners were quick to identify the axioms but their challenge came when they were supposed to justify answers. Most of the learners did not see the need to write down the reasons why two triangles are congruent. This finding concurred with what the literature review says about learners who often see no need to go beyond their observations to prove congruence of two more triangles. For example, the NSC examination diagnostic report declares that learners must refrain from making assumption (DBE, 2018). Some of the learners believed that diagrams were clear enough to convince them that the triangles were congruent. They held the idea that diagrams were drawn to scale. I observed that it was a misconception that learners build over time and it was now difficult for the learners that it was not always the case. They believed that some of the issues should just be taken for granted. For example, if two triangles look alike they should be automatically taken to be congruent triangles. Writing down reasons for congruence was insignificant for some the learners.

Observations show that the educator taught all the five observed lessons using the lecture method. This had an impact on the grasping of the concept when five lessons are taught in the same way. The learners' interests were taken for granted. Learners had challenges in the use of the axioms they know to prove congruence in triangles. There were situations where the learners were supposed to prove for themselves that certain rules do apply to proving of congruent triangles. For example, when a question on ASS was raised as a rule for congruence, the learners were supposed to investigate on their own giving counterexamples to prove the invalid criterion for congruence in triangles. The learners argued at length about the ASS as a criterion for congruence in triangles. The educator did exactly what Cirillo et al. (2015) discovered that some teachers tell their learners which triangle congruence criteria are valid and they have learners use them as postulates in proofs. In lesson 2 of the observed lessons, one learner raised a question as to why AAA does not hold for congruent triangles. Her argument was based on the SSS which results in corresponding angles of the triangles being equal. She believed that the same principle should work for the AAA. However, the learner was corrected by the class discussion and the teacher's explanation of similar triangles. Other learners wrote AAA in their personal notes as a rule for congruence in triangles (see Appendix A). Learners possess knowledge and should be given an opportunity to display what they know. Some of the learners had misconceptions of the axioms of congruent triangles.

About a third of the learners had problems with the order in which triangles are named. They failed to identify congruent triangles, because of the order of naming the corresponding sides and or corresponding angles. The same learners argued with the rest of the class about an example that was done on the chalkboard (see Appendix K, lesson 2).

In the focus group interview, the learners were quick to identify the RHS rule as a rule used for congruence of right-angled triangles. They were not sure of what to say when the right angle was an included angle. Most of the learners were still of the view that the rule remains RHS. They tended to be rigid on the Right angle (R) in the triangle. Most of the learners could not see that the 'hypotenus' is missing in the reason. This argument is related to the educator's idea of checking the 'equal corresponding sides' and or 'equal corresponding angles' of the congruent triangles. In some cases the learners would give the rule without stating the minimum number of reasons to establish congruence in the triangles.

It was noted that both verbal and written proof was a challenge to most of the learners. Some learners could verbalise their reasoning skills but failed to put that in writing. There was need for training learners to verbalise their thoughts and learn also to put their thoughts in writing. The literature review proved to this study that learners improve their deductive reasoning in all aspects of proving after being trained to think logically.

5.3.3 Research sub-question 3

To check whether the Grade 9 learners were able to communicate their reasoning skills in the learning of congruent triangles, they were exposed to focus group interviews as well as classroom observations. I observed that learners were struggling to express themselves mathematically. One term meant different things to different learners. In lesson 2 of the observed lessons, most of the learners failed to explain what SSS meant (Appendix A, lesson 2). This meant a term can mean different things to different learners in the same grade. Learners used inappropriate terms to express their thoughts. For example, where the learners wanted to talk about ‘corresponding sides of two triangles are equal’ most of the learners would just say the ‘sides are equal’.

The majority of the learners wanted to work out problems in the same way the educator did it in the example. They showed different reasoning skills when they were discussing questions. About four of the learners in the class held the view that order of the naming of the triangles does not matter as long as the triangles have the same properties, that is, same size angles and same size sides. This was an indication that the cncept of congruent triangles was not yet grasped. They were supposed to have noticed that for triangles to be congruent, the corresponding sides and corresponding angles should fit in perfectly well when one triangle is placed on top of the other.

The focus group interviews showed that learners were limited in the choice of formal mathematical vocabulary. In both focus group interviews, learners failed to find suitable words to distinguish similar triangles from congruent triangles. The learners were aware of the difference but could not express the difference in the formal mathematics terms (see Appendix L, Transcript 2). For example, Learners 1, 3 and 4 in focus group interview 1 explained congruent triangles as, *look* *alike*, *same size and equal sides* respectively. However not all learners failed to express themselves, two out of six learners from focus group discussion 2 were able to express their views clearly. Learner 5 in focus group interview 2 says that as long as all the corresponding sides of a triangle are equal then all the other angles of that triangle are automatically equal. She added saying that triangles are not equal if we have only corresponding angles equal. In focus group interview 1, Learner 3 described similar triangles as having corresponding sides in proportion and equal corresponding angles. Learner 4 in the same focus group added that AAA is used only when proving similar triangles.

Generally, learners had a poor formal mathematics language. They tended to lack the proper terms to express what they know in mathematics. Learners acknowledged that tests are more difficult than the classwork and homework. Learner 2 in focus group interview 2 says that he understands concepts better when he discusses the problems with someone. He went on to say that if it were possible to have 'group test', he would get all the questions correct. The reason could be that he fails to comprehend the meaning of the question on his own. In both focus groups, learners were more comfortable in drawing the diagrams to represent congruent triangles than to explain (see Appendix L). Opting to draw the diagram instead of describing the shape could be a result of poor mathematical vocabulary.

5.4 Concluding remarks concerning the study

The results of this investigation show that learners can display their reasoning skills in limited ways. However, the study found that there are a number of factors that hinder learners' communication of their reasoning skills in the proving of congruent triangles. From the literature review it emerged that knowing the learners' levels of geometrical thinking assists the educator to plan meaningful lessons (Evbuomwan, 2013). Knowledge of the learners' levels of geometric thinking is of paramount importance in the learning of proof. Educators need constant refresher courses where they are reminded about the place of teaching and learning theories which enhance their teaching of proof. For example, Van Hiele's theory is one of the important theories to apply when proving congruence in triangles (Jaime and Gutierrez, 1995).

The use of visual aids is of vital importance in the teaching of 2-D shapes especially in the teaching of congruent triangles. Learners need a hands-on approach to learning of proof of congruent triangles. Knowing the learners' reasoning skills levels helps the educator to be able to know the starting point (Van Hiele, 1984). There is need to have more time allocated to concepts where learners are involved in exercising their reasoning skills like in proving congruent triangles. In order for learners to be able to think deductively, learners need to be trained not only to receive information but also to criticise various things related to that information (Primastya and Jatmiko 2018).

Learning Mathematics must be about understanding the concepts rather than just covering the scope. The learners need to enjoy doing mathematics when innovative methods are employed. Mironychev (2018) suggests that teachers should substitute proofs of theorems with demonstrations of computer animation or hands-on activities with numerical examples. Most of the challenges learners face in the proving of congruent triangles is cleared by employing practical activities where they discover for themselves. For example, if learners construct congruent triangles using ruler and compass, they were likely to discover the minimum facts required to establish congruence in triangles.

The lecture method deprived the learners' full participation in the proving of congruence in triangles (see Appendix K). This study was driven by the interpretivism philosophy where learners create their own knowledge (see chapter 3). The use of visual aids cannot be over-emphasised as it very crucial in establishing learners' understanding of 2-D shapes. Learners should be led to rediscover the knowledge themselves. I concur with the literature which revealed that it was the quality and nature of the experience in the teaching and learning program that influences a genuine advancement from a lower to a higher level of geometric thinking. The educators are encouraged to act as a facilitator while learners take an active part. Textbooks are an important tool in the teaching and learning of congruent triangles. Learners should be encouraged to use the textbook as one of the sources of knowledge in their Mathematics lessons. Mironychev (2018) states that learners are required to use books during the lessons and read books at home when doing home assignments.

5.5 Limitations of this study

This is a case study research where it focused mainly on a particular group of learners who had a particular background. The outcome of this study was influenced by the particular factors around this group of participants. The participants were a class of thirty-two learners and their mathematics teacher in an urban setting of KwaZulu-Natal Province. The school and the class were conveniently selected to participate in the research. Therefore, this study brings about unique findings which are difficulty to generalise. The results can only provide an insight into the reasoning skills of ninth graders in proving of congruent triangles. The purpose of this research was to develop a deep understanding of the reasoning skills of the ninth graders as they prepare for the Euclidean Geometry in Grade 10.

The research involved a class of 32 mathematics learners and their educator proving congruent triangles in their natural learning environment. The lessons were according to the educator's understanding of congruent triangles. There was no prescription on the method of teaching the lessons. It was left open to the educator to use his experience and understanding of congruent triangles. Furthermore the two groups of six learners each, which were purposively selected to participate in focus group discussion, answered open-ended questions about the proving of congruent triangles. The scenario warranted for different results that depends on the respondents' experience and knowledge. The results could be slightly different if participants were drawn from a private school or from a rural setting.

5.6 Recommendations for future research

The proving of congruent triangles is an essential activity that should be taken seriously by the high school educators as they prepare learners for Euclidean geometry in Grade 10. It is of utmost importance to prioritise the preparation for the teaching of congruent triangles so that learners are ready for the much feared concept of Euclidean Geometry. This study was done at a public school as a case study investigating the reasoning skills of ninth graders. Another research that could be done is to compare the reasoning skills of ninth learners in public schools with the reasoning skills of learners in the private schools. Further research could examine more closely on the learners' communication skills improving of congruent triangles with a fewer participants for an in-depth study.

If the proving of congruence in triangles is to be successful, the teachers should be equipped with the knowledge of learners' geometrical level of reasoning.

5.7 Final Reflections

When I was teaching Euclidean Geometry, I was worried about the FET band learners who were struggling with the concept of proof. I tried different methods to teach the same concept but there were no satisfactory results. I then decided to check whether the ninth graders were prepared to tackle the concept of proof at Grade 10. I chose the proving of congruent triangles as it is the beginning of proof in the curriculum. It was this challenge in the topic that kept me focused on my studies.

I have discovered in the literature review that there was a lot of knowledge that was not put into practice. I became more aware that there are theories which educators need to consider when preparing to teach proof. These theories inform educators about how learners reason at different levels depending on their experience. The literature also gave insight on the ideal methods of handling proving of congruence in triangles.

The observations were challenging for me as I had to remain focused on what I had to observe and at the same time I had to guard against distraction. Collecting data was not an easy thing for me as a lot of data emerged which could have distracted me from focus. Being a participant observer made it even worse because I was supposed to participate and at the same time capture data.

I want to appreciate and thank the Grade 9 class and their mathematics teacher for their support and cooperation during the data collection period. The class behaved in a natural way as we observed them learning proof of congruence in triangles. The class proved to be a well organised and self-directed group of participants. They were consistent throughout the data collection period. They were not influenced by our presence as they displayed what they are capable of doing and were not ashamed of displaying their lack of knowledge. The flowing of the lessons showed how organised and prepared the teacher was during the observation. The two focus groups displayed maturity and their determination in the discussions. Almost all the learners were free to participate in the discussions.

I would like to acknowledge that this research has changed the way I viewed the teaching and learning of congruent triangles. I now have a clear picture of how the learners progress from lower level to higher level of Van Hiele's geometric thinking levels. I have grown in my work as a mathematics educator as well as becoming more astute academically.

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Appendix A

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT HUNT ROAD SECONDARY SCHOOL

Title: Exploring ninth graders' reasoning skills in proving congruent triangles in Ethusini, KwaZulu-Natal Province.

Date: 25 June 2019

Dear Sir/Madam

KwaZulu-Natal Department of Education

Ethusini Circuit Office

Durban

4001

Dear Circuit Manager

I, Mapedzamombe Norman am doing research under the supervision of M. M Phoshoko, a professor in the Department of Mathematics Education towards a Masters of Education at the University of South Africa. We do not have funding from any organization. We are inviting you to participate in a study entitled Exploring ninth graders' reasoning skills in proving congruent triangles in Ethusini, KwaZulu-Natal Province

The aim of the study is to develop the reasoning skills of ninth graders in proving congruent triangles.

Your circuit has been selected because of its convenience to the researcher. I work and stay in the area.

The study will entail observing learners proving congruent triangles, interviewing focus groups and analyzing related documents like learners' written work, lessons plans and main textbook used by Grade 9 learners in the school.

The benefits of this study are that learners are exposed to different ways of proving congruent triangles and teachers will realize the

There are no potential risks involved in this study as the students are studied in their natural learning environment.

There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will entail

Yours sincerely

____

Norman Mapedzamombe

Unisa Student

Appendix B

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT HUNT ROAD SECONDARY SCHOOL

Title: Exploring ninth graders' reasoning skills in proving congruent triangles in Ethusini, KwaZulu Natal Province.

Date: 25 June 2019

Dear Sir/Madam

Hunt Road Secondary School

Durban

4001

Dear Sir/Madam

I, Mapedzamombe Norman am doing research under the supervision of M. M Phoshoko, a professor in the Department of Mathematics Education towards a Masters of Education at the University of South Africa. We do not have funding from any organization. We are inviting you to participate in a study entitled Exploring ninth graders' reasoning skills in proving congruent triangles in Ethusini, KwaZulu-Natal Province

The aim of the study is to develop the reasoning skills of ninth graders in proving congruent triangles.

Your school has been selected because of its convenience to the researcher. Again your school has more than one class of the Grade 9 learners from which I can choose a class to include in the research.

The study will entail observing learners proving congruent triangles, interviewing focus groups and analyzing related documents like learners' written work, lessons plans and main textbook used by Grade 9 learners in the school.

The benefits of this study are that learners are exposed to different ways of proving congruent triangles and teachers will realize the:

There are no potential risks involved in this study as the students are studied in their natural learning environment.

There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will entail

Yours sincerely

Norman Mapedzamombe

Unisa Student

Appendix C

PARTICIPANT INFORMATION SHEET

Date: 25 June 2019

Title: EXPLORING THE NINTH GRADERS' REASONING SKILLS IN PROVING CONGRUENT TRIANGLES IN ETHUSINI, KWA ZULU NATAL.

DEAR PROSPECTIVE PARTICIPANT

My name is Norman Mapedzamombe and I am doing research under the supervision of Professor M.M. Phoshoko, a professor in the Department of Mathematics Education towards a Master of Education at the University of South Africa. We do not have funding from any organization. We are inviting you to participate in a study entitled, "Exploring the ninth graders' reasoning skills in proving congruent triangles in Ethusini, KwaZulu-Natal."

This study is expected to collect important information that could help improve the reasoning skills of the ninth graders in the proving of congruent triangles. Grade 9 Mathematics educators will be able to learn how to handle learners with different reasoning skills in the proving of congruent triangles.

You have been included in this research by the virtue that you are teaching a Mathematics class which was selected for observation in this research.

I obtained your contact details from your school Principal, Mr. Mkhize. I will observe Grade 9 class proving congruent triangles for two weeks.

Your role in this study is to teach congruent triangles to the Grade 9 learners as it appears on the timetable. I will focus on how the learners display their reasoning skills in the proving of congruent triangles. I am kindly asking you to allow me to audiotape and videotape your class during the observation period. I also request to access your teaching documents. The study will take about two weeks for observation. Document analysis will run concurrently with the lesson observation period.

Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. It is difficult to withdraw from this research once you have started participating as this will affect the whole project outcomes.

Participating in this research will help you as a Mathematics educator to improve your approach to proving of congruent triangles.

There are no foreseen risks to your participation in this research. The possible benefits in participating in this research include the establishment of learners' reasoning skills in the proving of congruent triangles and the improving of teaching and learning of congruent triangles.

You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research. Your answers will be given a code number or a pseudonym and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. Your identity is only exposed to the transcriber and cameraman who will sign a confidentiality agreement.

Your answers may be reviewed by people responsible for making sure that research is done properly, including the transcriber, external coder, and members of the Research Ethics Review Committee. Otherwise, records that identify you will be available only to people working on the study, unless you give permission for other people to see the records.

Your anonymous data may be used for research report, journal article and or conference proceedings. Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard and all electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable.

This study has received written approval from the Research Ethics Review Committee of the College of Education, Unisa. A copy of the approval letter can be obtained from the researcher if you so wish.

If you would like to be informed of the final research findings, please contact Norman Mapedzamombe on 0721125345 oremail46323716@mylife.unisa.ac.za.

Should you have concerns about the way in which the research has been conducted, you may contact Professor M. M. Phoshoko on mmphoshoko@unisa.ac.za or on 012 429 6993.

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

____

Norman Mapedzamombe
(Unisa Student)

Appendix D

A LETTER REQUESTING PARENTAL CONSENT FOR MINORS TO PARTICIPATE IN A RESEARCH PROJECT

25 June 2019

Dear Parent

Your ____________________________ (son/daughter/child) is invited to participate in a study entitled Exploring ninth graders' reasoning skills in proving congruent triangles in Ethusini, KwaZulu Natal Province.

I am undertaking this study as part of my master's research at the University of South Africa. The purpose of the study is to improve the teaching and learning of congruent triangles at grade 9 level and the possible benefits of the study are the improvement of proving of congruent triangles and the increase of learners' reasoning skills. I am asking permission to include your child in this study because his/her class has been sampled for this study in the school. I expect to have him/ her and other children participating in the study.

If you allow your child to participate, I shall request him/her to:

- Take part in a participant observation. The learners are observed while proving congruent triangles for a period of two weeks. They are observed during their Mathematics lessons as they come on the timetable.
- Take part in a group interview where they will discuss how they prove congruent triangles. Three 30 minute sessions will be conducted over a period of two weeks.

I am kindly asking you to allow me to audiotape and or videotape your child during the lesson observations and group interviews.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are improvement in the learning and teaching of Mathematics. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available. Your child will be asked to do the same lessons in the other class that is not involved in the research.

In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

The benefits of this study are exposure to different ways to prove congruent triangles, innovative methods of teaching congruent triangles and develop attitude of defending their answers.

There are no anticipated risks to participating in this study.

There will be no reimbursement or any incentives for participation in the research.

If you have questions about this study please ask me or my study supervisor, Prof M MPhoshoko Department of Mathematics Education, College of Education, University of South Africa. My contact number is 072 112 5345 and my e-mail is 46323716@mylife.unisa.ac.za. The e-mail of my supervisor is mmphoshoko@unisa.ac.za. Permission for the study has already been given by the Principal of your child's school and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:

Sincerely

_______________________________ _____________________________

Parent/guardian's name (print) Parent/guardian's signature: Date:

NORMAN MAPEDZAMOMBE ________________25/06/2019

Researcher's name (print) Researcher's signature Date:

Appendix E

CONSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, ____ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the lessons I am going to teach.

I have received a signed copy of the informed consent agreement.

Participant Name and Surname (please print) ___________________________

Participant Signature__________________ Date ___________________

Researcher's Name & Surname: Norman Mapedzamombe

____________________________ 25/06/2019

Researcher's signature Date

Appendix F

ASSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, ________________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording (both videotape and audiotape) of the observation lessons and focused group interviews.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) _____________________

_____________________ _________________________

Participant Signature Date

Researcher's Name & Surname: Norman Mapedzamombe

_______________________ 25/06/2019

Researcher's signature Date

Appendix G

FOCUS GROUP ASSENT AND CONFIDENTIALITY AGREEMENT

I____ grant assent that the information I share during the focus group may be used by Norman Mapedzamombe for research purposes. I am aware that the group discussions will be digitally recorded and grant assent for these recordings, provided that my privacy will be protected. I undertake not to divulge any information that is shared in the group discussions to any person outside the group in order to maintain confidentiality.

Participant‘s Name (Please print): ___________________________________

Participant Signature: ____

Researcher’s Name: (Please print): NORMAN MAPEDZAMOMBE

Researcher’s Signature: ______________________________

Date: 25 July 2019

Appendix H

Classroom Observation Protocol

Pre-observation Data

Teacher…………………………… Date………………………………..

School…………………………..…. Grade/Level……………………....

Class period ………………………

Topic

……………………………………………………………………………………………....
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Lesson within the topic of study

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……..…………………………………………………………………………………………
…………..…..

Objectives

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………..............………………………………………………………....……………………
…………….………….…………

Intended outcome ……………………………….……………………….……………...

Teaching/Learning Aids

………………………………………………………………………………………………
…….…………………………………………………………………………………………
……………...

Assessment Tool ……………………………………………………………………

Classroom Activities

<u>Introduction to lesson:</u>

………………………………………………………………………………………………………
…..……………………………………………………………

Time Taken……………………………….…….……………………………………..

First Activity

………………………………………………………..……………………………….……
…..………………………………………………………………………………………………
………………………………………………………………………………………………………
………………...................

Duration……………………………….

Second Activity

………………………………………………………………………………………………………
………………………………………………………………………………………………………

Duration…………………………………

Third Activity

………………………………………………………………………………………………………
.…………………………………………….…………….........

Duration……………………………….

Summary ……………………………………………………………………………………

The Sequence of activities

Description of the class

…………………………………………………………………………………………..
…………………………………………………………………………………………..
………………………………………………

Teaching aids …………………………………………………

Assessment strategies used

………………………………………………………………………………………….
……………………………………………………………………………………………
………………………………………………

Time not devoted to teaching

……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………

Student data

Number of students………………………………..........……………………..

Number of Girls ……………………………......…………………………..

Number of Boys…………………………………........……………………….

The content of a student's notebook

……………………………………………………......………..……………………………………
…………………………………………………………………..……………………………………
…...

Student behaviour

……………………………………………………………………………………………….…
………………………………………………………………..……………………………….........
....

Interaction

……………………………………………………………………………......................................
....…………
…………………………………………………..
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Involvement

………………………………………………………………………...
....
....

<u>Reflections and Interpretations</u>

……………………………………………………………………………..…...………………...
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What did not happen?

……………………………………………………………………………………………….....
..................…………………………………………………………………..……………….......
....................…………………...

Non-verbal behaviour

……...

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Classroom Observation Protocol https://ed.fnal.gov.instruct. Classroom

Appendix I

FOCUS GROUP INTERVIEW QUESTIONS

Introduction

Hello my name is Norman Mapedzamombe. I am glad that you have accepted to talk to me about proving of congruent triangles in Mathematics. I am interested in understanding your reasoning skills in Geometry concepts. I will be asking you questions where you will display your views and feelings about the topic. There are no absolute correct answers. You are free to express your thoughts and opinions on the subject. I will be recording our discussion so that I will not be able to me miss anything that you say. I will not include your name in the outcome of this study. Your answers will be treated with confidentiality.

Questions

1.	In your own words, can you explain what congruent triangles are? What do you already know about congruent triangles?
2.	What is your general feeling about the topic congruent triangles? Why do you think you feel this way?
3.	What words or phrases come to your mind when you think of the topic congruent triangles? I want to make sure I understand, can you explain more?”
4	

	What conditions make triangles congruent? Can you explain your answer?
5	How do you conclude that triangles are congruent? Can you give an example?
6	What are the congruent triangles axioms that you know? Is there any axiom which you do not understand? Is there anything you can add?
7	How do you conclude that triangles are congruent?
8	There are situations where facts do not add up to make congruence in triangles. Can you identify such situations?
9.	What other challenges are involved in the proving of congruent triangles?
10.	Is there anything else you would like to say about why you like or dislike the proving of congruent triangles?

APPENDICES

Appendix J

Document Analysis Tool

All the teaching documents used to teach Congruence in triangles were analysed using this tool.

Teaching Documents

Document	**Yes**	**No**	**Comments**
a. Lesson Plans			
1. The educator uses lesson plans to teach his/her lessons.			
2. The lesson plans are derived from the curriculum.			
3. The educator plans new lessons every year.			
4. There is innovation and creativity in the structure of the lesson plans.			
5. The educator includes a variety of resources in his/her lessons plans.			

6. The lesson plans show problems which enhance development of reasoning skills.			
7. The lesson objectives incorporate development of reasoning skills.			
8. The educator prepares quality lesson plans.			
9. The educator evaluates his/her lesson plans.			
10. The lesson plans carters for learners' different learning styles.			
11. The educator uses teaching notes to present his/her notes.			
12. The structure of the work in the lesson plans allows learners to share knowledge.			
(b). Textbook(s)			
1. The introduction of proof of congruent triangles has enough background in the book.			

2. There is a link between Grade 9 work and Grade 7-8 work.			
3. The books use symbols and signs consistently.			
4. There are colourful pictures in the book. Do you think the pictures are appealing?			
5. The examples in the book are adequate per each concept.			
6. The language used in the book(s) is clear and relevant for the grade level.			
7. The books provide learners with challenging problems.			

Document Analysis	Yes	No	Comments
(c). Mathematics Exercise Books			
1. Learners write legibly.			
2. Learners write neatly.			

3. Learners write orderly in their books.			
4. Learners do correction after every feedback.			
5. The feedback is of assistance to the learners If yes in what ways are they helped?			
6. Learners use exercise books to do their work.			
7. Learners often write class work and tests.			
8. Do learners often write homework?			
9. Learners often write class tests.			
10. Learners share their work. Comment why you say that.			
11. Learners draw diagrams in the Exercise books. Comment on the quality and quantity of the diagram			
12. There are enough questions for learners to practice. Comment on the variety of the questioning.			

13. The teacher accepts only one way of answering a particular problem.			
14. Learners copy the problems in their exercise books before they solve them.			
15. The learners can express their thoughts clearly on paper.			
16. The learners are given with challenging questions.			
17. The structure of the work in the book allows learners to share knowledge.			
18. The introduction of prove of congruent triangles has adequate information.			
19. There is a link between Grade 9 work and Grade 7-8 work.			
20. Learners are consistent on the use of symbols and signs in the book.			

21. Learners have different ways of answering the same question.			

Appendix K

Transcript 1

Classroom Observation

Direct quotations are written italics.

LESSON 1 **Date: 16/09/2019** **Time: 09h00 – 10h00**
The educator introduced us as colleagues who had come to observe them learning about congruent triangles. *Let us cooperate with them. They will audiotape/videotape our lessons*. The educator drew the learners' attention to the 2D shapes they know. *Let us name the 2 dimensional shapes we know.* Reponses from learners: *square, parallelogram, rectangle, rhombus, kite, rectangular prism* Educator: *Is a rectangular prism a 2 D shape?* Learner1: *No, no that is very wrong, a rectangular prism.* Teacher: *Sorry, we do not say it like that let us help her. What kind of shape is a rectangular prism?* Learner 2: *It is a 3 dimensional shape.* Educator: *Why do we say that?*

Learner 3: *A rectangular prism is a solid shape and has 3 directions* (class laughed at this answer).

Educator: *She is correct*. *I see you have mentioned a square, rhombus, kite and rectangles, Is there anything common about these shapes? The shapes are called quadrilaterals because they have each four sides.*

Let us focus on some kind of triangles. Do we know different kinds of triangles?

That is very easy, responded one learner (raising up his hand).

The learners mentioned *Right-angled triangle, isosceles triangle, equilateral triangle and scalene*.

Teacher: An*yone to give us triangles that you know*

Learners 4: [shouted] *Scalene, equilateral, isosceles, right-angled triangle*

Teacher: *Triangles are named according to their characteristics/ properties. For example here, an equilateral triangle is named so because all its sides are equal. How about its angles. Also the angles are equal. What is the size of each angle? Each angle of an equilateral triangle is 60° because the sum of a triangle is 180°.*

Teacher: *Today we want to look at congruent triangles.*

Do we all know congruent triangles and if so, can you describe them. Congruent triangles are triangles that are the same in all respect.

Educator: *Can I have someone to come and draw congruent triangles?*

Learner 2(drew two triangles on the chalkboard) *These two triangles are congruent*,

Teacher*: There are conditions which make 2 or more shapes to be congruent and this week we are going to look at these conditions as we look at the topic Congruent triangles.*

The educator wrote the topic *Congruent Triangles* on the chalkboard and drew two labelled triangles. Discussion

The class discussed the conditions for having triangles congruent.

It was done clearly and all the learners were glued to the chalkboard.0
LESSON 2 **Date: 17/09/2019** **Time: 09h00 – 10h00**
The educator started the lesson by referring the learners to the previous day lesson. The learners explained in their own understanding what they know about the rule SSS in proving congruent triangles. Learner1: *SSS refers to triangles with equal sides.* Another learner called out from the class saying that *the triangles have 3 sides*. Learner 2: *The 2 triangles have corresponding sides*. Learner 3: *SSS means that the 2 or more triangles have their corresponding sides equal and also their corresponding angles equal too*. The learners agreed with this answer. The educator proceeded to explain congruence in another set of triangles. The educator emphasized the importance of an included angle. The *axiom SAS means the angle is an included angle*. The class noted that the angle is always an included angle not just any angle on the triangles. The class discussed the importance of the order of naming corresponding sides and corresponding angles in the congruent triangles. Teacher: *The order is not an alphabetical order of the angles of the triangle but the order of the sides of the congruent triangles.*

While discussing the rule SAS, learner 1 raised a question as to whether it was wrong to have SSA as a rule for congruence in triangles. The question drew the attention of many learners. Learner 4: *Is there anything wrong in having the reason SSA as for* $\Delta ABC \equiv \Delta DEF$. There was an argument when learners 2 and 3 agreed with her while a few disagreed. The teacher explained (figure 4.2) saying that the learners should check whether the corresponding sides and corresponding angles of the triangles were equal before making a conclusion. The majority of the learners seemed not aware of the correct thing to do. The teacher explained that *the SSA or ASS does not hold as two different triangles can be drawn from such a situation*. This discussion made the educator to rub the written example so that SAS will hold.
LESSON 3 **Date: 18/09/2019** **Time: 08h00 – 09h00**
The educator asked learners to summarise the previous lesson on congruent triangles. Learner 1: *SSS and SAS are rules for proving congruent triangles.* Learner 2: *The "A" in SAS is an included angle.* The educator emphasized the importance of corresponding angles and corresponding sides of the congruent triangles to be equal.

The educator went on to introduce the lesson of the day.

Educator: *Today we are going to look at the triangles with two given angles and a side. The side may be an included one or not as you shall see in the following diagrams*. The educator drew the diagrams as indicated on Figure 4.3 and Figure 4.4.

The educator explained that *AAS and ASA are both rules for proving congruent triangles.* He went to explain that *their difference is that ASA rule has an included side while AAS do not have an include side.*

A learner raised a question in comparison to the previous rule (SAS).

Learner 4 : My question is, Why is that we denied ASS as a binding rule for proving congruent triangles but now we are accepting AAS as a binding rule? These 2 rules are similar.

Teacher: *Yeah, you see when we construct the image triangles of triangle given AAS or ASS we get different triangles. The triangle with AAS will give us one and only triangle. The triangle with ASS will give us more than one triangle with the same measurements but different sizes.*

Teacher: It is unfortunate that we do not have time we could have done the activity for you to see for yourselves.

Learners were given worksheet to answer questions on proving congruent triangles. The learners were asked to determine with reasons whether pairs of triangles were congruent.

The learners had different answers to the given 4 questions and some of the answers are:

I became interested in the different answers written by the learners so I asked them for the reason of their particular answers.

Observer: *Can you tell me the reasons for your answers?*

Learner 1:I forgot to write the reasons for my answers

Learner 2: *I did not get the instructions correctly.*

Learner 3: *It is one and the same thing what is important is to indicate that the two triangles are congruent or not*. Learner 4*: Sir, it is obvious that the two triangles are equal and there is no reason for writing that down*. About a quarter of the class (8 learners) did not write anything on their worksheets. They said that they do not understand everything on congruent triangles. Some of the learners were waiting for their colleagues to explain to them. Learner 6: *I desperately need help!* Exclaimed one girl. The educator went to the girl. The lesson ended when some learners were still writing their work. The teacher concluded the lesson by pointing the learners that *proving of congruent triangles is about the relationship between figures. Every step you take should be supported by a reason. We need three facts to prove that triangles are congruent. So far, the rules we have learned include, SSS; ASA;AAS and SAS*. The educator thanked the learners and dismissed them for lunch.
LESSON 4 **Date: 19/09/2019** **Time: 09h00 - -10h00**
The educator greeted the learners with a happy smile. He introduced the topic by calling upon learners to state the properties of right-angled triangles. The learners were able to identify the following about right-angled triangles: *the presence of a right angle, the hypotenuse, the sum of the interior angles of a triangle is 180°, and the sum of the other 2 non-right angles are complementally* and that a *right-angled triangle reminds them of the theorem of Pythagoras*.

The class discussed the theorem of Pythagoras. Learner 2 pointed out that *the theorem of Pythagoras is used to solve the length of the unknown side in a right-angles triangle.*

The teacher drew Figure 4.5 on the chalk board. He explained the RHS rule in the proving of congruent triangles.

While we were discussing the calculation of the unknown third angle on a right-angled triangle, learner 1 had a question on the AAA. *Is AAA a rule is a reason for proving congruence in triangles?*

There was quick answer from the educator saying, *No we need at least one pair of corresponding sides equal.*

Learner 2: *If two triangles have their corresponding angles equal, they are also congruent*.

The same learner went on to explain that *the sum of the interior angles of a triangle is supplementary.*

Learner 3 argued that *if three pairs of corresponding sides result in corresponding angles equal then when three pairs of corresponding angles are congruent, this also should result in corresponding pairs of sides congruent.*

He went on to say that *if we start with three pairs of corresponding angles equal then the corresponding sides would also be equal.*

These questions sparked an interesting environment where most of the learners were keen to hear the educator's response. Learner 4 explained why AAA is not a rule for congruence in triangles. He said that *AAA reason is for similar triangles since we can draw 2 shapes of the same shape but of different sizes.*

Learner 4 said that *we cannot have congruence proved without at least one given side.*

The educator did not answer the learner's argument about SSS leading to all corresponding angles equal and AAA failing to lead to sides being equal.

I refer you to information on equiangular triangles and proving of congruence cannot be possible without one pair of equal sides.

The lesson ended with a given homework for learners to write down notes on the proving of congruent triangles.
LESSON 5 **Date: 20/09/2019** **Time:10h30 – 11h30**
The teacher started a new lesson without referring to the given homework. It is like he forgot to check what they wrote as notes about congruent triangles. I checked what the learners wrote as their notes and I found one like what is on Figure 4.7. There was need to revise the notes so that points like (a) can be corrected. I found that about 5 learners wrote the same notes. Learner 1*: I got the notes from my friend.* Learner 2 claimed that she wrote the notes herself. I noticed that some of the notes like (b)-(d) were taken from the textbook. The teacher started the lesson by revising the congruence axioms. They were able to point out the axioms as *SSS, SAS, AAS, ASA and RHS*(chorus answers) The teacher emphasized that *congruent triangles are similar triangles but similar triangles are not necessarily congruent triangle.* The learners were given one long question to answer. The question needed the learners to be able to interpret the symbols used and read instructions carefully. Figure 4.10 represents the question to be answered. Most of the learners had a challenge to answer the question. The learners discussed in pairs and in threes.

There was little success in the majority of the learners who attempted to answer the question. A few learners (about 3 out of 32) managed to have a breakthrough.

One of the learners whose answers are on Figure 4. 11 and Figure 4. 12 say, *we have decided to breakdown the shapes into triangles. We compared the triangles using the given sides and angles.*

The class discussed the concept of a common angles or common sides between triangles.

Most of the learners were excited to learn that sides or angles can be common between shapes.

Much time was spent practicing question 5 (a).

The class was asked to take question 5 as homework.

The teacher said, *We do not have enough time to finish this question together today. When we come next week we will first look at this question before we start a new topic.*

The teacher gave us time to talk to the learners. I thanked the educator and his class for hosting us during the time they were learning how to prove congruent triangles.

Appendix L

Transcript 2

Focus Group Interviews

<u>Group 1 Responses</u>

In this exercise the learners were assigned names as Learner 1-6 for identification purposes in this report.

DAY 1

Date: 30/09/2019

Time: 14h00 – 15h00

Facilitator: Good morning boys and girls? You are welcome to our focus group interview. We are going to discuss proving of congruent triangles. We are going to have 3 sessions at 3 different times. We are going to video/audio tape these discussions so that we capture the correct information from you. You are free to contribute by way of asking questions or answering questions. I invite you to make your contribution freely as there is no right or wrong answer in these discussions. Our first question says, Name any 2-D shapes you know. Classify these shapes according to their properties.

Learner*s:* *Pentagon; hexagon; square; rectangle; triangle; octagon; rectangular prism; rhombus; cylinder; not cylinder sir.*

Facilitator*: May I know from you learners are all these shapes 2 dimensional? Let us separate 2-D shapes from 3-D shapes.*

Learner 1*: Ok that is simple sir a cylinder is not a 2 dimensional shape.*

Learner 2 Sir these shapes are confusing 2D or 3D shapes?

Facilitator: *May I know from you learners how many of you have forgotten the Grade 8 work you did last year?*

Learners*: (Four out of six indicated that they had forgotten Grade 8 work)*

Facilitator: *Thank you boys and girls. Let us try to classify the shapes according to their properties?*

Learner 3: *There are shapes we call quadrilaterals like a square and a rectangle.*

Facilitator: *What is a quadrilateral?*

Learner 2: *A quadrilateral is a shape with four sides.*

Learner 1*: Example is rectangle, rhombus and square.*

Learner 3: *There is also a parallelogram with parallel sides. Yeah the sides are equal.*

Learner 6: *Another class of 2-d shapes is the triangles. Triangles are many like the right-angled triangle, isosceles triangle and ……. Ok.*

Learner 5: *Equilateral triangles*

Learner 1: *Isosceles triangles are triangles with 2 equal sides***.**

Facilitator: *Are there other kind of triangles you know*

Learner 3: *A scalene is also a triangle. I know of acute angles also.*

Learner 5: *An acute-angled triangle all angles are less than 90°.*

Learner 4*: I want to draw the triangle I know [draws an acute- angled triangle]. The triangle is called scalene angles are less than 90°.*

Learner 1: *Sir what about a hexagon? Is it a quadrilateral or is 6 sides shape? [the group laughed].*

Facilitator: *We said a quadrilateral is a four-sided figure and how many sides is a hexagon?*

Learner 1: *Okay, sir I see it is a polygon not a quadrilateral.*

Facilitator*: Thank you. Let us now have the next question. In your own words, can you explain what congruent triangles are? What do you know about congruent triangles?*

Learner 3: *I think congruent triangles are triangles which are similar.*

Learner 4: *They also have equal sides.*

Facilitator*: Yes any other responses*.

Learner3: *They are of the same size.*

Learner1*: They look alike*.

Learner 3: *I don't think so. They are the same sides and same angles*.

Facilitator: *What do mean when you say they look alike?*

Learner 5: *They are the same shape*.

Facilitator: *Our next question says, what is your general feeling about the topic congruent triangles? Why do you think you feel this way?*

Learner 4: *The concept of congruent triangles is an interesting topic which gives us an opportunity to think. I also want to say it is difficult as compared to other topic.*

Learner 6*:* *Honestly speaking I do not like Mathematics especially proving of congruent triangles. It is very difficult for me.*

Learner 3*:* *I am comfortable doing Mathematics especially challenging work in proving congruent triangles. I know Mathematics is an important subject.*

Learner 1: *I do not have a choice, I have to do Mathematics including proving of congruent triangles. I will try my level best to do well. I feel this topic helps us to think independently.*

Learner 2: *Like my colleague (name mentioned) learner 1 I like the topic because it is important in life. I will try hard.*

Learner 5*:* *Sir, this Mathematics of yours is very difficult (the whole group burst into laughter). I do not know why my teacher sets difficult questions in a test but when in class the questions appear simple.*

Facilitator*: Let us look at the next question which says, what words or phrases come to your mind when you think of the topic congruent triangles? I want to make sure I understand, can you explain more?"*

Learner 1*: I think of corresponding sides. I also think of identical twins which look alike. [Everyone laughs].*

Learner 4*: [Shouted]. You got the concept correct 'mfuwethu' (meaning my brother). Sir I think of a hypotenuse and a side on right-angled triangles.*

Learner 3*: I see the rules of congruent triangles like the SSS and SAS.*

Learner 1: *I think of Pythagoras theory used to solve for the unknown side.*

Learner 4: *Sir, Is that congruent triangles? No, congruent triangles topic is different from Pythagoras.*

Facilitator: *It is ok, there is no wrong answer here we are all learning. Let us have someone to expand on that one.*

Learner 3: *We use Pythagoras to find the third side as said by Learner 1 so that we can be able to determine corresponding angles which are congruent.*

Facilitator: *Thank you boys and girls our time is up. Let us meet again tomorrow at the same time. Thank you once again for your cooperation.*

DAY 2

Date: 02/10/2019

Time: 14h00-15h00

Facilitator*: Good morning everyone! Welcome to our second session where we are discussing how the ninth graders communicate their reasoning skills. Our first question today says that what conditions make triangles congruent? Can you explain your answer?*

Learner 6*:* *I know SSS as a condition for proving congruent triangles. The sides of the triangles are equal.*

Facilitator*: What do you mean by the sides of the triangles are equal?*

Learner 2: I think the SSS refer to the corresponding sides of the triangles that are equal.

Learner 3*: The lengths of the corresponding sides are congruent, meaning they are the same size.*

Learner 5: *I think of Side-Angle-Side where 'Angle' is the included angle. And we cannot have SSA as a rule of congruence in triangles.*

Facilitator: *Who can tell us the reason why SSA fails to be a recognised rule for congruent triangle?*

Learner 3*:* *I think, yah, the reason is that A is not an included angle.*

Facilitator: *Is there anyone who wants to try to explain more on why SSA does not form part of the congruent triangles rules.*

Learner 6*: We can have a small triangle and big triangle which we can draw.*

Facilitator: *What do you mean by saying we will have a small triangles and a big triangle?*

Learner 6*: I don't know how to explain it but there will be 2 two triangles from the same triangle.*

Facilitator*:* *Alright boys and girls. How about if it is ASS? Does it satisfy the condition of congruence in triangles?*

Learner 3*:* *SSA and ASS are the same they do not satisfy the conditions of congruent triangles.*

Learner 2: *AAA satisfies the conditions of congruent triangles. She went to the chalkboard to draw two triangles with corresponding angles equal (see Figure4. 13).*

Facilitator*: Are congruent triangles and similar triangles the same?*

Learner 1*:* *No they are not. Similar triangles are the same yeah and congruent triangles are like the same size and shape.*

Facilitator: *How do you conclude that triangles are congruent?*

Can you give an example?

Learner 1: *You can check a minimum of 3 facts about the triangles.*

Facilitator: *What kind of information do you check so that congruence is determined?*

Learner 5*:* *In the facts that we check to consider congruent triangles, there must be at least one pair of corresponding sides equal.*

Learner 2*:* *We check for equal sides.*

Facilitator: *Yes, any other ideas on this question?*

Learner 4*: If all the sides are equal then we can conclude that the two triangles are congruent.*

Learner 3*: Triangles are congruent if their corresponding angles are 90° each, hypotenuses equal and any corresponding sides equal. This results in having the RHS rule.*

Facilitator: *What reason do we give if there is a right angle and 2 corresponding sides equal and no hypotenuse included?*

Learner 3*:* *We can still have the same reason as RHS?*

Facilitator*:* *How many people agree with that? If you do not agree can you give your views on this point?*

[Three learners agreed that the reason remain as RHS. Two of the learners disagreed with learner 3 while one remained undecided].

Learner 5: *The reason can be SAS since the right angle is an included angle.*

Facilitator: *Thank you for your idea. You have linked your to the previous rules done yesterday.*

Facilitator: *What are the congruent triangles axioms that you know? Is there any axiom which you do not understand? Is there anything you can add?*

Learner 1*:* *SSS all the corresponding sides are equal.*

Learner 2*:* *SAS, the angle is included.*

Learner 3*:* *ASA, the side is included.*

Learner 4*:* *RHS, there is right angle.*

Learner 5*: SSA, angle is not included*

Learner 6: *AAS, angle is not included.*

Facilitator: *How many rules or axioms do have, and are all our answers correct?*

Learner 4*:* *SSA is not true. We said it in class that it does not work as a rule.*

Facilitator*:* *Any other contribution or question about what we were discussing today. [There was silence].*

Thank you every one for your fruitful discussion. We again here on Friday at the same time. When we happen to have changes with regard to venue and time I will let know

DAY 3

Date: 04/10/2019

Time: 14h00 – 15h00

Facilitator*:* *Good morning all members of the group. Welcome to our last session of the topic which says, ninth graders' reasons skills in the proving of congruent triangles. I am reminding you that we will be video/audio taped in this session. Are we happy with that? I am encouraging everyone of you to take part in today's discussion.*

Our first question says that are there situations where facts do not add up to make congruence in triangles?

Learner 1: *The sum of the interior angles of any triangle is 180°. So we can have triangles that are different whose interior angles are supplementary.*

Learner 3*: If triangles have their corresponding sides in proportion, then they are said to be similar triangles.*

Learner 4: *I think when Angle-Angle-Angle is the only reason for considering triangles to be congruent. We need one side to prove that 2 or more triangles are congruent.*

Facilitator*: What other challenges are involved in the proving of congruent triangles? Is there anything else you would like to say about why you like or dislike the proving of congruent triangles?*

Learner 1*: There are many difficult questions in the tests which are difficult. Test questions are different in the tests than in class work. This is the reason why I hate Mathematics.*

Learner 6: *Myself, I like Mathematics especially congruent triangles because I know how to solve the problems*

Learner 3: *I like Mathematics very much. I like the challenging questions in the topic. When I am doing Mathematics I feel very happy. There is nothing difficult.*

Learner 2: *Proving congruent triangles is difficult for me. I do not like it. Hey, it's tough for me. There are too many new words involved.*

Facilitator: (Drew three diagrams on the chalkboard. The diagrams were on triangles on a circle, angles on intersecting line and a diagonal on a quadrilateral). Let us discuss the relations of the properties of each diagram.

Learner 3: (pointed to the chalkboard indicating that she wanted to write down). $\angle ABC$ is equal to this angle (pointing at $\angle DBC$)

Focus Group Discussion

<u>Group 2 Responses</u>

Day 1

Date: 01/10/2019

14h00 -15h00

In this exercise (like in the first group) the learners were assigned names as Learner 1-6 for identification purposes in this report.

Facilitator: *You are all welcome to our focus group discussion this morning. We have general question we want answer through this discussion. The main question we want to answer is: How do ninth graders communicate their reasoning skill in the proving of congruent triangles? We are going to have 3 sessions in 3 different days. Name any 2-D shapes you know. Classify these shapes according to their properties.*

The learners named the following shapes randomly: square; circle; rectangle; kite; rhombus, triangle; pyramid; cylinder; pentagon; cube; rhombus; parallelogram

Learner 1: *Some of the shapes can be quadrilaterals. These are shapes like square and rectangle. Some shapes are called polygons*

Facilitator: *How many of you have forgotten what we did in Grade 8 about 2-D shapes?*

Learners: *(Five learners indicated by way of raising their hands)*

Facilitator: *Can you give us examples of polygons?*

Learner 3*: Pentagon, hexagon, octagon and all the '….gon' shapes.*

Learner 4*: Sir how about the cube? Is it not has many sides?*

Learner 1:*A polygon is a shape with many sides so a cube is a polygon.*

Learner 3*: I think a cube is not a polygon because it is a solid shape.*

Facilitator: *What is the difference between 2-D shapes and 3-D shapes? Can you give an example of each?*

Learner 4*: A 2-D shape has 2 like a side or directions on the shape like a square.*

Learner 1*: 2-D shape is plain shape while a 3-D shape has volume and area.*

Learner 4:*3-D shapes are solid shapes like a cylinder or a cube.*

Learner 2: *Triangles is another group. We have the isosceles triangle, scalene triangle and the right-angled triangle.*

Facilitator: *What is your general feeling about the topic congruent triangles?*
Why do you think you feel this way?

Learner 5*: I feel excited when proving congruent triangles. It helps me to open my mind*
Learner 1: *The topic is difficult but very important one. Many people get reasoning from proving congruency.*

Learner 3*: I do not like the topic because there are too many things.*

Learner 4: *Eish it's mixed up I know that Maths is very important but it is very difficult. Congruence appear easy to me but when I am doing alone I become confused. I think I started to fail Maths in Grade 8. I used to like Maths in primary school. There are times when I know it and at other times I am off, off [Kkkkkk]*

Learner 6: *No problem with congruence. It is easy for me. I can answer any question on congruent triangles.*

Learner 2: *It has been said by learner 1.*

Facilitator: *Let us look at the following question***.** *What words or phrases come to your mind when you think of the topic congruent triangles? I want to make sure I understand, can you explain more?*

Learner 1*: Corresponding sides. The corresponding sides should be equal in order for triangles to be congruent*

Learner 2*: Corresponding angles. The angles should be equal.*

Learner 3*: Included angle on the triangles. Yeah that's it.*

Learner 4*: I think of right angles on the triangles.*

Learner 5*: Included side equal.*

Learner 6: *Equal sides and equal angles*

DAY 2

Date: 03/10/2019

Time: 14h00 -15h00

Facilitator*: Good morning everyone! Welcome to our second session where we are discussing how the ninth graders communicate their reasoning skills. Our first question today says that what conditions make triangles congruent? Can you explain your answer?*

Learner 4: *Triangles are congruent when the sides are equal, like SSS.*

Learner 1: *When the angles are also equal in all respect.*

Learner 2: *When we have RHS*

Learner 4: *We have SAS included angle, Sir.*

Facilitator*: How do you conclude that triangles are congruent? Can you give an example?*

Learner2: *I think you check the number of facts that make triangles congruent. The facts should be only 3.*

Leaner 6: *I think all the sides and angle should be equal.*

Learner 1: *We need corresponding angles and corresponding sides to be equal for triangles to be congruent.*

Learner 4: *There must be one pair of corresponding sides equal.*

Learner 2*: The triangles should be of the same size and same shape.*

Facilitator: *How do you know that the triangles are of the same and same size.*

Learner 2: *The size of the triangle you check the corresponding sides. The angles I think you can measure. I'm not sure.*

Learner 5: *I think on the angles you can see what you are given and at times you calculate the angles.*

Facilitator: *What are the congruent triangles axioms that you know? Is there any axiom which you do not understand?*

Is there anything you can add?

Learner1: *Right angle-Hypotenuse -Side*

Learner 2: *Angle-Side-Angle*

Learner 3: *Side-Side-Side*

Learner 4*: Angle-Side-Angles*

Learner 5: *Angle-Angle-Side*

Facilitator: *Is this all we can say about the axioms of congruent triangles.*

Learner 6*: There is SAS, the angle in an included angle.*

Learner 1: *I do not understand AAS and why does it work when ASS does not work?*

Facilitator*: Anyone to help learner 1, please?*

Learner 4*: I think we do not need argue with the law. It is law there is no reason.*

Facilitator*: Although it is a law there is reason why it works for AAS and no for ASS. We need a practical lesson were we will draw diagrams to prove a point.*

I think we need to construct diagrams to explain what they all mean mathematically.

Facilitato*r:* *How do you conclude that triangles are congruent?*

Learner ***4:*** *Triangles are congruent when you check their sides.*

Learner 2: *I check the angles of the two triangles to be equal.*

Learner 3: *I check all the angles and all the sides they should be equal or corresponding equal shape.*

Learner 6: *I think I check only 3 facts like SSS*

Learner 1***:*** *I also check any 3 reasons like RHS if the triangles are right angle.*

DAY 3

Date: 07/10/2019

Time: 14h00 -15h00

Facilitator*:* *There are situations where facts do not add up to make congruence in triangles. Can you identify such situations?*

Learner 2: *Triangles are not congruent if some of the sides are not equal.*

Learner 3: *I have got a question***.** *Do we have situation when triangles have corresponding sides but one pair of corresponding angles not equal?*

Learner 5*:* *As long as all the corresponding sides of a triangle are equal then all the other angles are also equal. I wanted to say that triangles are not equal if we have only corresponding angles equal.*

Learner 6: *When the triangles are ASS.*

Facilitator: *Why do you say that? Can someone explain what ASS is?*

Learner 5*:* *ASS or SSA is not a reason for congruent triangles because the triangles are not congruent. We have done this together in class.*

Learner 2: *I think when you draw the triangles they are not the same.*

Facilitator: *What other challenges are involved in the proving of congruent triangles?*

Learner 5*:* *I think there is nothing*

Facilitator: *Is there anything else you would like to say about why you like or dislike the proving of congruent triangles?*

Learner 6*:* *Proving of congruent triangles is very interesting but wants someone who reads every day. I think I can improve my Maths through studying.*

Learner 2: *I understand when I am discussing with someone than when I am doing alone. If we can have a group test we will all pass.*

Facilitator: I drew on the chalkboard three diagrams seeking knowledge to related properties of 2-D shapes to the proving of congruent triangles. The diagrams were on triangles on a circle, angles on intersecting line and common sides on triangles. Let us discuss the relations of the properties of each diagram.

Learner 3: *I can see AB = BD and AC=DC and the reason is that they are given.*

Learner 4: *$\angle A = \angle D$ and the reason is give*

Appendix M

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2019/09/11

Ref: **2019/09/11/46323716/10/MC**
Name: Mr N Mapedzamombe
Student No.: 46323716

Dear Mr Mapedzamombe

Decision: Ethics Approval from 2019/09/11 to 2022/09/11

Researcher(s): Name: Mr N Mapedzamombe
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Supervisor(s): Name: Prof MM Phoshoko
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Title of research:

Exploring the ninth graders' reasoning skills in proving congruent triangles in Ethuseni, Kwa Zulu Natal

Qualification: M. Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2019/09/11 to 2022/09/11.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2019/09/11 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.

University of South Africa

APPENDIX N

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15 JULY 2020

TO WHOM IT MAY CONCERN

LANGUAGE CLEARANCE CERTIFICATE

This serves to inform that I have read the final version of the dissertation titled:

Exploring Ninth Graders' Reasoning Skills in Proving Congruent Triangles in Ethusini Circuit, KwaZulu-Natal Province,
by Norman Mapedzamombe.

To the best of my knowledge, all the proposed amendments have been effected and the work is free of spelling and grammatical errors. I am of the view that the quality of language used meets generally accepted academic standards.

Yours faithfully

S. Govender

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B Paed. (Arts), B.A. (Hons), B Ed.
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Dissertation 1

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