AN EXPLORATION OF MATHEMATICAL KNOWLEDGE FOR TEACHING FOR GRADE 6 TEACHERS IN THE TEACHING OF FRACTIONS: A CASE STUDY OF THREE SCHOOLS IN CAPRICORN SOUTH DISTRICT

by

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DECLARATION

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AN EXPLORATION OF MATHEMATICAL KNOWLEDGE FOR TEACHING FOR GRADE 6 TEACHERS IN THE TEACHING OF FRACTIONS

I declare that the above dissertation is my own work and that all the sources that I have used or quoted been indicated and acknowledged by means of complete references.

I further declare that I submitted the dissertation to originality checking software and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at Unisa for another qualification or at any other higher education institution.

_________________________ _________________________

Signature Date

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DEDICATION

This dissertation is dedicated to my children, Javis, Jasley Rose and Mahlatse Eddy who always gave me the courage to continue with the journey and told me never to give up.

I also dedicate this dissertation to my late husband, Ramakola Elias and my first son, Mangapa Jason. Let their souls rest in eternal peace.

LIST OF ABBREVIATIONS AND ACRONYMS

CAPS	Curriculum and assessment policy Statement
CCK	Common content Knowledge
DSG	Development Support Group
HCK	Horizon content Knowledge
KCC	Knowledge of content and curricular
KCS	Knowledge of content and students
KCT	Knowledge of content and teaching
LCM	Least common multiple
LS	Learners
L	Learner
MKT	Mathematical knowledge for teaching
PCK	Pedagogical Content Knowledge
SA	School A
SB	School B
SC	School C
SCK	Specialised content Knowledge

ABSTRACT

The study aimed to explore teachers' mathematical knowledge in respect of teaching the concept of fractions to Grade 6 learners. To that end a qualitative study was done, using a case study design. Data were collected through the observation of, and interviews with, three teachers at three schools in the Capricorn South district. Rooted in the theory of constructivism, the study was supplemented by the conceptual framework of mathematical knowledge for teaching (MKT) (Ball et al., 2008) and Shulman's (1986) notion of pedagogical knowledge for teaching (PCK). The key finding of this investigation revealed that, of the three teachers, two did not develop the concept of fractions for their learners, but merely followed the traditional method of teaching the concept by encouraging their learners to memorise rules without understanding. Only one teacher emphasised an understanding of mathematical concepts. The main observation which the researcher made, was that teachers require a great deal of knowledge and expertise, in carrying out the work of teaching subject matter related to fractions.

Keywords

Conceptual understanding, constructivism, fraction concepts, fraction names, fractional notations, fractions, misconceptions, models, pedagogical content knowledge (PCK), mathematical knowledge for teaching (MKT).

TSHOBOKANYO

Maikaelelo a thutopatlisiso e ne e le go tlhotlhomisa kitso ya dipalo ya barutabana malebana le go ruta barutwana ba Mophato wa 6 mogopolo wa dikarolwana. Go fitlhelela seo, go dirilwe thutopatlisiso e e lebelelang mabaka, go dirisiwa thadiso ya thutopatlisiso ya dikgetsi. Go kokoantswe *data* ka go ela tlhoko le go nna le dipotsolotso le barutabana ba le bararo kwa

dikolong tsa kgaolo ya Capricorn Borwa. Thutopatlisiso eno e e theilweng mo tioring ya kago ya kitso e ne e tshegeditswe ke letlhomeso la sediriswa sa tokololo sa kitso ya dipalo ya go ruta (MKT) (Ball et al. 2008) le mogopolo wa ga Shulman (1986) wa kitso e e kgethegileng ya go ruta (PCK). Phitlhelelo ya botlhokwa ya patlisiso eno e senotse gore mo barutabaneng ba le bararo, ba le babedi ga ba a tlhamela barutwana ba bona mogopolo wa dikarolwana, mme ba latetse fela mokgwa wa tlwaelo wa go ruta mogopolo ka go rotloetsa barutwana go tshwarelela melawana kwa ntle ga go tlhaloganya. Ke morutabana a le mongwe fela yo o gateletseng go tlhaloganngwa ga megopolo ya dipalo. Temogo e kgolo e e dirilweng ke mmatlisisi ke gore barutabana ba tlhoka kitso le boitseanape jo bogolo go tsweletsa tiro ya go ruta dithuto tse di amanang le dikarolwana.

Mafoko a botlhokwa

Go tlhaloganya megopolo, tiori ya kago ya kitso, megopolo ya dikarolwana, maina a dikarolwana, matshwao a dikarolwana, dikarolwana, megopolo e e fosagetseng, dikao, kitso e e kgethegileng ya diteng tsa go ruta (PCK), kitso ya dipalo ya go ruta (MKT).

SETSOPOLWA

Dinyakišišo di ikemišeditše go utolla tsebo ya dipalo ya baithuti mabapi le go ruta kgopolo ya dipalophatlo go baithuti ba Kreiti ya 6. Ka lebaka la se go dirilwe dinyakišišo tša boleng, go šomišwa tlhamo ya dinyakišišo tša seemo. Tshedimošo e kgobokeditšwe ka go lekodišiša, le go dira dipoledišano le, barutiši ba bararo ka dikolong tše tharo ka seleteng sa Borwa bja Capricorn. Ka ge di theilwe go teori ya gore baithuti ba itlhamela tsebo, dinyakišišo di tlaleleditšwe ke tlhako ya boikgopolelo ya tsebo ya dipalo go ruteng (MKT) (Ball le ba bangwe, 2008) le kgopolo ya Shulman (1986) ya tsebo ya diteng tša thuto (PCK). Kutollo ye bohlokwa

ya dinyakišišo tše e utollotše gore, go barutiši ba bararo, ba babedi ga se ba ba le kgopolo ya dipalophatlo go baithuti ba bona, eupša fela ba no latela mokgwa wa setlwaedi wa go ruta kgopolo ye ya dipalophatlo ka go hlohleletša baithuti ba bona go tsenya melawana ye ka hlogong ka ntle le go e kwešiša. Ke fela morutiši o tee yo a gateletšego gore go swanetše go ba le kwešišo ya dikgopolo tša dipalo. Temogo e tee yeo monyakišiši a bilego le yona, ebile gore barutiši ba hloka tsebo ye kgolo le botsebi, go phethagatša mošomo wa go ruta diteng tša thuto tšeo di amanago le dipalophatlo.

Mantšu a bohlokwa

Kwešišo ya dikgopolo, go itlhamela tsebo, dikgopolo tša dipalophatlo, maina a dipalophatlo, dinotheišene tša dipalophatlo, dipalophatlo, go se kwešiše gabotse, mekgwa, tsebo ya diteng tša thuto (PCK), tsebo ya go ruta dipalo (MKT).

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CHAPTER 1: ORIENTATION OF THE STUDY

1.1. INTRODUCTION

This chapter intends to give an overview and the orientation to the study. The purpose of this research was to explore teachers' mathematical knowledge for teaching the concept of fractions in Grade 6. Thus, this chapter provides the background of the study, followed by a statement of the problem, followed by the rationale of the study, the aims and objectives of the study and the research questions.

1.2. BACKGROUND OF THE STUDY

The content area of fractions has proved itself to be very complicated and troublesome for learners to master. Van de Walle, Karp, and Bay-Williams (2010. p.313) identified many possible factors contributing to poor understanding of fractions. They identified the reasons for learners' difficulties in fractions as follows:

- fractions include many meanings such as part-whole, measurement, division operator.
- the written format of fractions is unusual for learners.
- the conceptual understanding of fractions is ignored in instructions; and
- whole-number knowledge is overgeneralized by learners.

Pienaar (2014) alluded that one of the reasons teachers experience difficulties when teaching fractions may be how mathematics as a subject is viewed in the South African curriculum. In support of the reasons above, the researcher believes that because the concept of fractions is one of the topics in the mathematics curriculum, which is challenging for learners, it is therefore important for teachers to teach learners the concept of fractions meaningfully and effectively. Teachers' mathematical knowledge for teaching fractions plays a significant role in this case, especially in primary schools or at the elementary level. Ball et al.(2008) framework of mathematical knowledge for teaching serves as a point of reference in this regard.

The teaching of fractions is vital since it connects other topics such as decimals, percentages, ratios, and proportions. Sowder and Wearner (2006) pointed out that learners consistently perform weakly, and as such, they have a weak understanding of fractions. Teachers' poor content knowledge of teaching fractions is one reason for South African learners' poor performance. Any incorrect teaching of fractions can affect learners' understanding of the topic and become a lifetime problem, influencing their schooling, tertiary education, and even at their performance at their workplace. Shulman (1986) stated that teachers must have a knowledge base specific to the subject matter. Pienaar (2014) concurs, saying that fractions play an important role in our ever-advancing technological society. Many careers today rely heavily on the ability to compute accurately, proficiently, and insightfully with fractions.

1.3. RATIONALE OF THE STUDY

The researcher is a primary school teacher offering mathematics in the intermediate and senior phase, Grade 6 and Grade 7. In her teaching experience, the researcher observed that year by year when the Grade 5s are promoted to Grade 6, they bring along with them shallow and insufficient mathematical knowledge on fractions, referring to the mathematical knowledge on fractions expected to have been mastered in lower grades. Most learners seem to have insufficient knowledge of the concept of fractions, which is surprising to experience at this level of study. I observed that teachers seemed to experience problems when teaching fractions. Many will agree with the idea that fractions are challenging concepts that most learners find it difficult to understand. Furthermore, Ma 1999; asserted that the understanding of fractions continues to be a challenging topic both for learning and for teaching. He also pointed out that teachers and researchers have typically defined the teaching of fractions as a thought-provoking area of the Mathematics curriculum Moreover, it is true that fractions cannot be divorced from our daily life usage, and this is the reason enough for teachers to develop the fraction concept effectively to the learners. Steffe & Olive, (2010)alluded that this is especially tricky in light of the fact that learners have many everyday life experiences with fractions before they are introduced to formal teaching and learning about them.

The Curriculum Assessment and Policy Statement (CAPS), for Foundation Phase Mathematics, Grade R-3 (Department of Basic Education [DBE], 2011) outlines Grade 2 fraction sub-topics such as use and name unitary fractions including halves, thirds, and fifths; recognise fractions in diagrammatic form and write fractions as 1half ($\frac{1}{2}$) as stipulated in the CAPS document. This implies that the teaching of fractions in primary schools starts in Grade 2 and progresses to higher grades. As the fractional concepts progresses, the Grade 5s are at this level expected to have mastered fractional concepts like comparing and ordering fractions to at least twelfths, adding and subtracting fractions with same denominators ,mixed numbers, recognise and use the equivalency of fractions as outlined in the CAPS: Intermediate Phase Mathematics, Grade 4-6 (DBE, 2011).

The biggest concern is why these learners bring with them shallow and inadequate fractional mathematical knowledge, because when introduced to the concept of fractions, using baseline assessment and also the diagnostic assessment, trying to find their prerequisite skills, their responses indicate that they have insufficient knowledge about the topic. The shallow and insufficient mathematical knowledge about fractions concept that learners bring to Grade 6 could be attributed to mathematical knowledge for teaching. Moreover, teachers from other schools also shared similar sentiments about the poor performance of learners regarding fractional concepts.

Mathematics performance in the indicated grade is therefore not satisfactory Clarke ,Rocke& Michell(2007) pointed out that learners have a weak conceptual understanding of fractions. Pantziara & Philippou, 2012) also ,highlighted that learners have a weak conceptual understanding of fractions.

This situation aroused my interest to attempt to identify the problems and the challenges underlying this shallow and insufficient mathematical knowledge at our schools. When the promoted grade 5s learners were given class exercises and assessments, they performed poorly in fractions and that is reason that prompted me to find out the challenges behind this.
The researcher believes that teachers who offer mathematics at this level should have sufficient mathematical content knowledge and pedagogical content knowledge. Shulman (1987) alerted teachers not to separate content knowledge and pedagogy because both are needed to enable the teachers to carry out their work effectively. The problem is that, instead of continuing with other concepts or following the tracker and planner, the gap created must be addressed. That causes a delay that leads to incomplete work coverage of the pacesetter/schedule, which may disrupt the plan.

1.4. STATEMENT OF THE PROBLEM

From my experience as a teacher and Head of Department at my school, I have noticed that teachers struggle or neglect the teaching of fractions. They experience challenges when teaching fractions. Kong (2008) alluded that the topic of fractions is important in the primary Mathematics curriculum. Pienaar (2014) supports that the teaching of fractions is difficult, and Ma (1999) stated that teachers have insufficient knowledge of fractions necessary for classroom teaching.

It seems that South African teachers struggle with content for mathematics that they teach (Bansilal, Brijlall & Mkhwanazi, 2014). They further alluded that teachers' poor content knowledge of teaching fractions and the incorrect way of teaching fractions could be one reason for South African learners' poor performance in national assessments in mathematics. Pienaar (2014) indicated that the teaching of fractions is difficult.

Taylor and Vinjevold (1999), Carnoy, Chisholm and Chilisa (2012) alluded that over the past years, the ongoing low learner performance in mathematics has led to increasing interest in understanding how teacher pedagogical practices and content knowledge may contribute to patterns of poor academic performance. Research and evaluation of mathematics intervention point to the lack of foundational mathematical knowledge as one of the key factors for poor performance.

In addition, Fleisch (2008) maintained that poor performance crises start early in the foundation phase where learners acquire basic skills that they need as they further their studies. This is where primary school teachers should equip learners with the relevant mathematical knowledge, skills, and attitudes. Primary school mathematics teachers should have extensive mastery of the fractional mathematical knowledge for teaching.

According to my observation, it is surprising to find learners who cannot tell what a fraction is. The researcher's concern is that if learners were taught or mastered fractional concepts from Grade 2, it could not be difficult for them to recall what they learned in lower grades. Fleisch (2008) argues that the foundation phase is the level where learners should acquire the basic and foundational mathematics skills. If they fail to acquire these fundamental mathematical skills, they will continue performing poorly as they progress to higher grades.

Learners, who are inadequately prepared in lower levels, pose lots of challenges to the intermediate phase teacher; this causes the intermediate teacher to deviate from the original pacesetter and struggle to close the gap caused by foundation phase teachers. This may result in the incomplete coverage of the syllabus at the end of the term or year.

This situation calls for the intermediate phase mathematics teacher to develop intervention strategies in trying to address the identified content gaps. If the syllabus content is not covered, learners leaving the phase with some information not dealt with properly will result in many errors and misconceptions.

All these concerns will create a significant problem for the entire economy which may lead to high failure rate and high unemployment rates of young people as they would have performed poorly with no attainment of a complete qualification.

In the light of the reasons given above, one is likely to think and believe that South Africa will continue to perform poorly as long as the teachers' mathematical knowledge is lacking. Fleisch et al. (2008) indicated that mathematics learning problems appear at a very early stage in children, but mostly in elementary school, and then that problem continues up to high school. Any incorrect teaching of fractions can affect learners' understanding of the topic and become a lifetime problem. Ultimately this will influence their schooling, tertiary education and working situations.

1.5. AIMS AND OBJECTIVES OF THE STUDY

Based on the rationale and statement of problem mentioned above, the study aims to explore the following aspects:

- To explore the mathematical knowledge for teaching the concept of fractions in Grade 6.
- To explore how teachers, unpack the mathematical knowledge for teaching fractions.
- To explore how teachers, work with learners' mathematical ideas when teaching fractions.
- To explore how teachers, respond to learners' mathematical questions when teaching fractions.
- To explore how teachers, pose mathematical questions to Grade 6 learners.
- To explore how do teachers restructure mathematical tasks to be understood by learners.

1.6. THE RESEARCH QUESTIONS

The research study was based on the following research questions**:**

1.6.1 Research question

How do mathematics teachers teach fractions in Grade 6?

1.6.2 Research sub-question

1. How do teachers unpack/introduce fractions to Grade 6 learners?
2. How do teachers work with learners' mathematical ideas when teaching fractions?
3. How do Grade 6 teachers respond to learners' mathematical questions when teaching fractions?
4. How do teachers pose mathematical questions to Grade 6 learners?

5. How do teachers restructure mathematical tasks to be understood by learners?

1.7. ORGANISATION OF THE RESEARCH STUDY

The research study aimed at exploring the mathematical knowledge for teaching the concept of fractions to Grade 6 learners. Chapter 1, serves as the background of the study that provides a general overview of the study, orientating the reader to the study. Chapter 1 is followed by Chapter 2 on the theoretical framework and literature review. The theoretical framework that serves as the lens of the study is constructivism theory and the conceptual framework of Ball and Bass (2008) for mathematical knowledge for teaching, and the PCK theory of Shulman (1986).

The literature review serves as the bloodstream of the research study as it is reflected in nearly every part of the study. The literature review dwells on the fraction concept, fraction names, fraction notation, fraction models, why fractions are difficult and confusing, and the teaching of fractions. The teaching of fractions includes topics such as comparing and ordering of fractions, changing common fractions to decimal fractions, proper and improper fractions, converting mixed fractions to improper fractions, equivalent fractions.as well as adding and subtracting of fractions. The literature review is followed by Chapter 3 on the methodology that outlines the type of methods used in data collection and the pilot study. The data collected from Chapter 3 was under scrutiny and is analysed and discussed in Chapter 4. Chapter 5 elaborates on the findings, recommendations, conclusion as well as reflection of the study.

1.8. CHAPTER SUMMARY

This chapter provided the overview and the orientation to the study. This research aimed to explore teachers' mathematical knowledge for teaching the concept of fractions in Grade 6. Therefore, this chapter provided the background of the study, followed by a statement problem. The reason for this study was the continual inadequate and insufficient knowledge that the learners bring as they progress to higher grades each year. The researcher then defined the research questions to determine the challenges underlying this shallow and insufficient mathematical knowledge of fractions at our schools.

The main aim and objectives of the study were to explore the mathematical knowledge for teaching the concept of fractions in Grade 6; to explore how teachers unpack fractions; how they define, explain, and represent the concept of fractions; how they work with learners' mathematical ideas and how they restructure the mathematical task. This study serves to answer the research questions outlined in Chapter 1.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1. INTRODUCTION

This study focused on mathematics for teaching, the kind of mathematical teaching teachers do when they go about their work of teaching. Several theoretical bases that enlighten the discussion in mathematics education have been used in this study. This chapter outlines the key theories used in the teaching and learning of mathematics. It discusses behaviourism and constructivism, two conflicting theories in the teaching and learning of mathematics. Short descriptions of the two theories are given, but more emphasis is placed on constructivism because the research study is pinned on it. Constructivism rejects the idea that learners are "blank slates", meaning that they do not just absorb ideas as teachers present to them, they construct their own knowledge. To construct is to build something in the physical world. Building something requires paraphernalia, resources, and effort. The paraphernalia learners use to build understanding is their **existing knowledge** they already have. The resources for building their understanding are things they see, hear or touch in our physical surrounding. The effort refers to the ability to construct their own knowledge and understanding.

According to constructivism, learners are perceived as originators of their knowledge; they give meanings to things they think about, that they see, hear or touch. Learners pose knowledge even before they are formally taught. They have a web of interconnected ideas in them. Constructivists discard the notion that learners are empty vessels or blank slates, which one can fill with information; instead, learners make sense of their knowledge.

This study focuses on the exploration of mathematical knowledge for teachers in the teaching of the concept of fractions. Therefore, the theory of constructivism is appropriate in the sense that when learners construct knowledge by themselves, they will discover the concept of fractions on their own and these will make learners master and understand the concept of fractions in various ways.

2.2. THEORETICAL FRAMEWORK

There are many theories in the teaching and learning of mathematics developed by researchers. Among the theories, there are two teaching approaches to mathematics namely behaviourism and constructivism. Schunk (2000) views behaviourism as a doctrine that explains learning as a system of behavioural responses to physical stimuli. Behaviourism theory concerns itself with the effects of reinforcement, external motivation, and behaviours. Blaise (2011) refer behaviourism as a theory of learning that focuses on observable behaviour. Behaviourists view learning as a permanent change in the form of behaviour.

Blaise (2011) further indicated that behaviourism is the theory that believes that learning occurs through teachers' rewards and punishment that lead to a change in behaviour. Behaviourism theory regards learners as inactive participants who require positive reinforcement and motivation to learn. In behaviourism, teachers use a teacher-centred approach with direct instruction. In this theory, there is a high possibility that learners may learn procedural content rather than the concept. In this approach, teachers use rewards and punishment to control learners' behaviours; for example, if a learner is praised or celebrated for a particular behaviour, they are more likely to repeat that behaviour, and if a learner is punished for a specific behaviour, they are less likely to repeat the behaviour. Behaviourism is centred on the idea that learners learn through responses to their behaviour by observing others' behaviour. Schunk (2000) argues that behaviourism focuses mainly on changing behaviour but offers no indication of learners' structural reasoning for change.

Unlike behaviourism, which centres around transmission of knowledge from the teacher with learners' being passive participants in the learning process, the constructivism approach, which this study employed, suggests teaching and learning mathematics happens quite differently from the behaviourism perspectives. Schunk (2000) indicates that constructivism focuses on the learner's intellectual development and higher levels of thinking and understanding. The constructivists' view of learning regards a learner as an active agent in the process of knowledge attainment. It is a theory based on how people construct their own understanding and knowledge of the physical world. From a constructivist's perspective, teachers do not teach in the traditional sense of delivering learning to a group of learners as behaviourists argue, instead, they use materials with which learners become actively engaged through social interaction. Learners are taught to be self-regulated and take an active role in their learning.

The key ideas in constructivism include schemata, which implies cognitive structures with pre-conceived ideas of the world. In a constructivist learning environment, teachers and learners share the knowledge and authority. The teacher's role is a facilitator, and a learning group comprise a small number of heterogeneous learners by preference. Learner-centredness and collaboration are learning goals in a constructivist learning environment. Learners actively engaged in the lesson by asking questions based on prior knowledge to construct new knowledge and understanding as per Piaget's (1964) theory as well as Ball et al.'s (2008) and Shulman's (1986) frameworks are explained in this chapter.

Piaget's theory

Piagetian constructivism focusses on inner mental processes and does not consider the societal aspects of the learner. The Piagetian equilibration theory focuses on two distinct learning processes of assimilation and accommodation. According to Piaget (1964), learners are repeatedly engaged in the adaptation of information in their schemata. This means that information is either assimilated or accommodated in the learners' schemata. Schemata are mental structures to help us understand how things operate and how we organise knowledge. As we take in new information, we connect it to other things we know or things we have experienced. Piaget (1964) reasons that if new information does not get assimilated or accommodated, a cognitive conflict arises. Therefore, the continuous organisation and re-organisation of information by assimilation and accommodation processes, lead to effective knowledge (cognitive development). This process of equilibration is like an engine that propels the construction of knowledge and so creates opportunities for mathematics knowledge to take place.

There is a need to consider social aspects, as Ernest (1996) argues that learners are not inactive receivers of knowledge but are actively constructing their own knowledge. Piaget (1964) argued that learners do not just inactively learn, but they actively try to make sense of their world. He argued that as these learners learn and mature, they develop schemas-patterns of knowledge in long term memory that helps them organise and respond to information. Furthermore, learners experience new things they attempt to reconcile with the new knowledge with existing schemas. Piaget believed that learners use two distinct methods of assimilation and accommodation in knowledge acquisition .

The term assimilation stemmed from the work of Jean Piaget and his work on the cognitive development of learners. As I have already explained above, assimilation is the cognitive process of fitting new information or ideas into an existing cognitive schema, perception and understanding. This means that when teaching learners about fraction concepts, they will automatically connect or fit the new information or make sense of the information by referring to what they already know about fractions. According to Piaget (1964), learners construct new ideas using what they know.

Accommodation, on the other hand, is a cognitive process of revisiting existing cognitive schemata, perception and understanding to integrate the new information. This means to make sense of some new information one had to adjust the information you already have to make room for the new information. **Assimilation and accommodation processes,** as explained, make Piaget`s theory appropriate to my research study. It is factual that if learners cannot accomplish both processes then no learning may take place.

Constructivism theory is relevant to my study because for learners to understand the fraction concept meaningfully, teachers are required to engage learners actively in the learning process, using prior knowledge and understanding as outlined by Piaget (1964).Teachers need to unpack, define, represent and explain the fraction concept to learners. As they unpack the fraction concept to learners, they are expected to apply their mathematical knowledge for teaching in developing the fraction concept meaningfully to learners in an understandable way. They are required to unpack, introduce, deal with learners' mathematical ideas, and restructure the mathematical tasks.

2.3. MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) FRAMEWORK

There will not be any effective teaching and learning if teachers do not know the subject they are supposed to teach. Ball et al (2008) indicates teachers must know the subject they teach because if they do not know it well, they are unlikely to have the information they need to help learners learn. This researcher further argues that just knowing the subject well is not good enough for teaching as teachers need to know mathematics in ways useful for making mathematical sense of learners work and choosing powerful ways of representing the subject so that it is understandable to learners.

Mathematical knowledge for teaching refers to the knowledge that is explicit to the teaching profession as opposed to the kind of knowledge used by other professions like engineering and accounting. Teachers need to have adequate, in-depth mathematical knowledge for teaching considering the reasoning in the paragraph above. Ball et al. (2005) came up with a question of what teachers need to know, and be able to do, to effectively convey the work of teaching mathematics.

Ball, Hill and Bass (2005) proposed that teachers need to know the topics and procedures that they teach. These researchers focused explicitly on *how* teachers need to know the content. They further argued about what else teachers need to know about mathematics and *how* and *wher*e teachers *use* mathematical knowledge for teaching in practice. In their scrutiny, they noticed the mathematical demands of teaching and concluded that the mathematical demands of teaching require mathematical knowledge and skill.

Ball, Thames and Phelps (2008) further referred to mathematical knowledge for teaching as the knowledge required in everyday tasks, such as explaining, defining, representing concepts to learners, listening to learners talk, working with learners' thinking or ideas, commenting on learners' work and controlling their work. This suggests that everyday tasks should be carried out effectively. The teaching of fractions also demands the teacher's mathematical knowledge and skill, which means mathematical knowledge for teaching the concept of fractions is essential for teaching the Grade 6 learners. Mathematical knowledge for teaching requires a teacher to have fractional mathematical reasoning that most adults do not require regularly.

Most of the subtopics like comparing and ordering common fractions, including tenths and hundredths, adding and subtracting fractions in which one denominator is a multiple of another, identifying which fraction is proper, improper or mixed, converting fractions to percentages and decimals and equivalent fractions start in Grade 6 and require a teacher who has a deeper understanding of fractions.

Teachers should know how to introduce, unpack and represent the concept of fraction using models or concrete objects to encourage more abstract thinking. These fractional concepts should be taught or conveyed to learners in a way that learners can grasp or understand. The researcher believes that before teachers can teach algorithms or procedural way of solving fractions, they should consider the conceptual understanding of fractions first.

Ball et al. (2008) outlined the domains of mathematical knowledge for teaching (MKT) that teachers need to carry out their work as teachers. They indicated that teachers require a great deal of knowledge and expertise in teaching the subject matter, as shown in Figure 2.1 below.

According to Ball et al. (2008), the teacher's knowledge, as indicated in Figure 2-1, is divided into two domains namely, *subject matter knowledge* and *pedagogical content knowledge*. Subject matter knowledge has three domains. The first domain is common content knowledge (CCK), referred to as the mathematical knowledge that anyone might know. Examples of common content knowledge include knowledge of algorithms and procedures, for example, adding fractions, comparing fractions, changing improper to proper fractions and recognising wrong answers.

Specialised content knowledge (SCK) is defined as the mathematical knowledge and skill uniquely needed by teachers in their work of teaching; it is also used in assessing learners' errors. The last domain is horizon content knowledge (HCK). Pedagogical content knowledge, according to Shulman (1986), also has sub-domains, namely knowledge of content and teaching (KCT), knowledge of content and students (KCS) and knowledge of content and curricula (KCC).

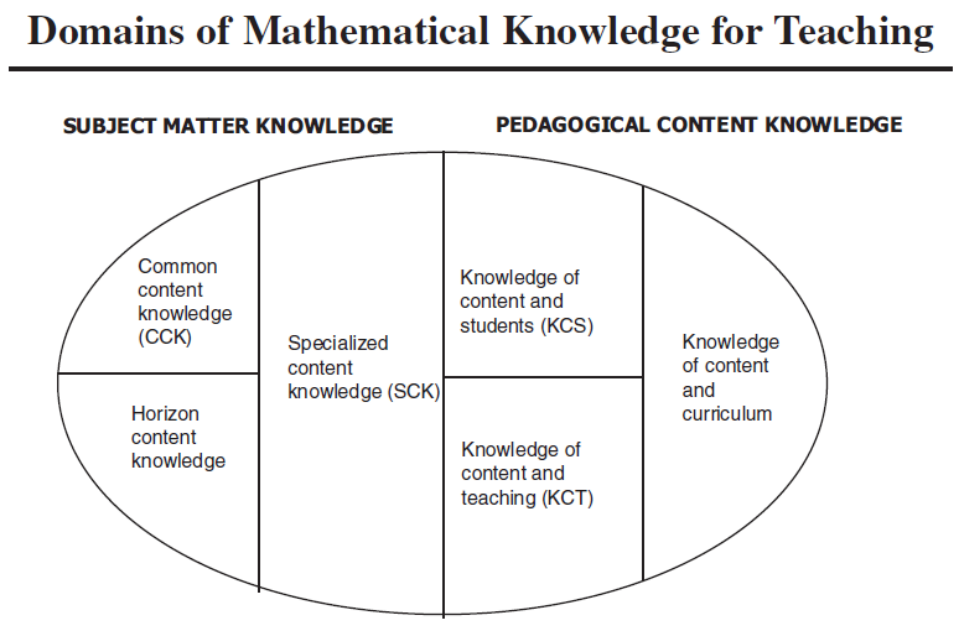


Figure 2.1: Domains of Mathematical knowledge for teaching

Among the sub-domains identified by Ball et al. (2008) the *specialised content knowledge*, needs the teacher to have a deeper understanding of fractions that allows the teacher to explain new ideas, work out fractional mathematics problems in various ways and analyse learners' explanations. The *specialised content knowledge* is unique and exceptional since it allows teachers to apply different ways of solving the mathematical problems. It is vital because it does not channel learners' thinking but allows them to explore.

It will be difficult for a teacher to teach learners about fractional mathematics concepts if they do not know the content. It is broadly accepted that what a teacher teaches, and how the teacher teaches, is a task of the teacher's own knowledge of the subject. Mathematics teachers should be knowledgeable about the mathematics they are teaching. Ball et al. (2005) stated that the specialised content knowledge includes the teacher's ability to use content knowledge to access different representations and knowledge of different methods for solving mathematics problems that may arise within the mathematics teaching.

There is no way one can separate Balls' notion from Shulman's pedagogical content knowledge (PCK) as both are intertwined or interwoven. In his presidential address, Shulman (1986) pointed out that teaching entails more than knowing the subject matter. He indicated that besides the content knowledge and curricular knowledge, teachers need a third type of knowledge. He recognised a special domain of teacher knowledge which is referred to as pedagogical content knowledge (PCK). He amalgamated the knowledge of teaching and the knowledge of the subject as equally important. Shulman (1986) alerted teachers not to separate content knowledge and pedagogy because both are needed to enable them to carry out their work effectively. He argues that teachers need to know and understand more of their subject than other users because he reasoned that teaching requires a transformation of knowledge into a form that learners can understand.

Shulman's (1986) notion of pedagogical content knowledge (PCK) is viewed as the knowledge of teachers to use their knowledge of mathematics to unpack, represent, formulate, explain, illustrate and make the concepts understandable to learners. This is in support of the idea that mathematics teachers should use their mathematical knowledge to unpack the fractional concepts and deliver them to learners in a way that learners fully comprehend.

Shulman (1986) also pointed out that teaching involves more than knowing the subject matter, but instead teaching entails transforming the knowledge for the learners in an understandable manner. He further argued that besides knowing the content well, the teacher needs the knowledge on how to deliver or convey the knowledge and to be understood. This means the teacher should know the fractional mathematics concepts well and be able to deliver them to learners in a way that is comprehensible. Teachers are, therefore, urged to apply Shulman's (1986) idea when teaching fractions.

Teacher content knowledge should represent a deep understanding of the concepts to be mastered by learners. Adler and Davis (2006) argue that teachers' mathematical knowledge is an important factor for learners' success. Teacher's mathematical knowledge has an impact in the classroom teaching. Adler and Davis (2006) indicated that the teacher requires a deeper and broader understanding of mathematics.

Ball et al. (2004) proposed eight categories of mathematical teaching that teachers frequently engage with. They argued that the eight categories are tasks of teaching, which appear more often in teacher's work. Kazima, Pillay and Adler (2008) decided to shorten the eight categories/aspects to six because they concluded that some of them were overlapping.

The six identified categories are as follows:

- *Defining* which implies that the teacher provides a definition of a concept to learners.
- *Explaining* which means that teachers explain problems to learners.
- *Representation* which means that teachers represent an idea in a variety of ways.
- *Working with learners' ideas* which means teachers engage with both learners expected and unexpected mathematical ideas.

- *Restructuring learners' tasks*–referring to simplifying a problem or making it more complex.
- *Questioning*–referring to posing and responding to questions for the lesson to go on.

These categories have been used as an indicator of the presence and the absence of teachers' mathematical knowledge in this study.

2.4. LITERATURE REVIEW

The researchers of mathematics education have over the past years grappled with the idea of trying to understand the problems teachers have in developing the fractional concepts effectively to learners and again trying to find out why fractions are difficult. Most of these researchers for example, Van de Walle et al. (2013) identified many possible reasons why fractions are difficult. They identified the reasons for learners' difficulties in fractions as including the following:

There are many meanings to fractions, fractions are sometimes written unusually, teaching does not always focus on a conceptual understanding of fractions and learners tend to overgeneralise their whole number knowledge.

Fractions are one of the mathematics concepts that teachers need to know and understand well. Cramer & Whitney, (2010) highlighted that learners' lack of understanding fractional concepts raises many problems in the topics such as fraction computation, decimals, percentages, and concepts areas like algebra that make use of fractions. Kong (2008) indicated that the topic of fractions was important in the primary mathematics curriculum. Pienaar (2014) indicated that the teaching of fractions is difficult.

2.4.1.The development and definition of the fraction concept

Bassarear and Moss (2016) highlighted that the word “fraction” was derived from the Latin word *fractus*, which comes from the word *frangere*, meaning to break. A fraction is breaking something that is a whole into smaller equal parts. When we work with learners, we talk about the concept of half. What is the meaning of it or what is a half actually? In most cases, we take for granted that learners know the meaning of the words and conclude that they understand. At some point in their lives, they encountered sharing things such as a pizza or a pie. Learners know how much half a pizza is. They know that when sharing the pizza or anything with someone or a classmate, the two pieces should be the same size (equal). After noticing that they understand the concept, then we can build on it. If they know that half is two parts that are the same, then thirds are three parts of the same size, fourths are four parts of the same size, and so on. This forms the basic construct for fractions. Van de Walle (2016) indicates that the concept of fraction tells us only about the relationship between the part and the whole. The following are examples of a fraction as part of the whole, as indicated by Van de Walle (2016).

Figure 2.2: Area model

This whole is a **rectangle.**

The whole is divided into two **equal parts.**

Each part is half of the whole.

Two parts, therefore, make one whole.

Figure 2.3: Circular model

This whole is a **circle.**

The whole is divided into five **equal parts.**

Each part is one-fifth of the whole.

Five fifths therefore make one whole.

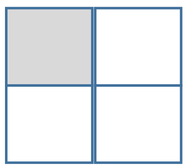


Figure 2.4: Square model

This whole is a **Square.**

The whole is divided into four **equal parts.**

Each part is one-fourth (a quarter) of the whole.

Four quarters, therefore, make one whole.

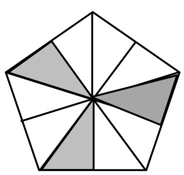


Figure 2.5: Pentagonal model

This whole is a **pentagon.**

The whole is divided into **ten equal parts.**

Each part is one-tenth of the whole.

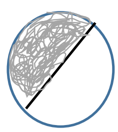
Ten tenths, therefore, make one whole.

As indicated above, the fraction names are very important when dealing with fractions. According to Van de Walle, 2016), using the examples above ,will be easy for learners to recognised that a fraction is a part of a whole.

2.4.2. Fraction names

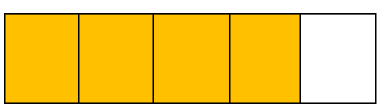
Naming a fraction is considered to refer to the whole to which the fraction applies. According to Van de Walle (2016), teachers **must** not just say *half* to learners, if teachers say *half* what do they mean? Half of what? Instead, they should say half of an apple, half an hour etc. Sharing an apple equally among four classmates, each will get a quarter of the apple. If six classmates share thirty chocolates, each will get five of the chocolates, and if they are two sharing the same chocolate, each will receive half of the chocolate. Representation of fractions at this stage of sharing, is a **verbal expression.** Lamon (2008) pointed out that the naming of fractions helps learners to use the correct language and to understand the concept of fractions well.

Examples are given below:



Half of a circle

Three thirds of a whole



Four fifths of a rectangle

Figure 2.6: Area models for Fractional names

2.4.3. Fractional notation

Once the teacher notices that learners have a solid understanding of or are conversant with the fraction names such as sixth of a pie, fourth of cookie eighth of an orange etc, and know the meaning of different wholes like half of an orange, the teacher now knows that they are ready to understand the symbol of fractions. Representation at this stage is **symbolic,** according to Van de Walle (2016).

What does the fraction $\frac{4}{5}$ mean?

Van de Walle (2016) alluded that the ***bottom*** part of the fraction tells us into how many parts the whole is divided. The bottom part is called the denominator. The word “denominator” names the fraction. It gives the fraction its name: fifth. The name tells us that the whole is divided into ***five equal parts.*** The word **denominator** comes from a Latin word meaning “namer” and the word numerator comes from a Latin word “numberer”. Van der Walle also referred to the **top** part of the fraction which tells us how many of the parts we shade or identify we are referring to. The top part is called the **numerator**. It tells us the number of fifths to consider. The explanation above can be represented diagrammatically as follows:

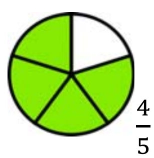


Figure 2.7: Fractional notation

A fraction can therefore be considered as a numeral that can be expressed in the form of ($\frac{a}{b}$),

where b$\neq 0$ (**b must not be zero**) (Van de Walle, 2016). In this case, the numeral at the bottom of the fraction (the denominator) has an entirely different function from the numeral at the top. A numeral is a **symbol** or **name** that stands for a number. For example, the fraction $\frac{2}{9}$ tells that the whole has been divided into nine equal shares (parts). The numerator tells us that two of those shares are under consideration. The denominator always gives the fraction its name, for example, “ninth” or two-ninths of something. The notation provides us with a fraction in terms of two numbers; one tells us its name, and the other tells us how many shares (parts)we have, or we consider (Van de Walle, 2016). We need to understand that one quarter can be written as $\frac{1}{4}$, seven fifths as $\frac{7}{5}$, etc. The diagram below shows $\frac{7}{10}$ (**seven tenths**) of the square are shaded and $\frac{3}{10}$ (**three tenths**) of the square are not shaded.

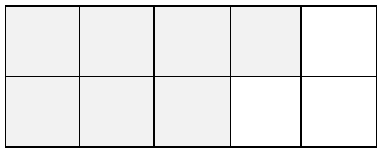


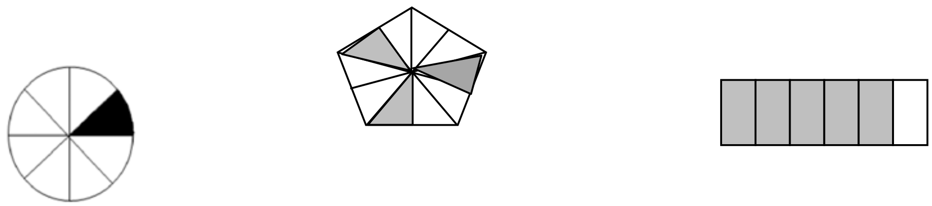
Figure 2.8: Area model

2.4.4. Models for fractions

There are two categories of concrete models for fractions, continuous (regions or lines) and discrete (sets of objects) (Van de Walle, Karp & Bay Williams, 2010)**.** The research is clear that concrete representations are key for learners' comprehension of fractions (Van de Walle et al., 2010; Cramer and Wyberg, 2009)

AREA MODELS (CONTINUOUS WHOLES)

Cramer, Wyberg and Leavit (2008) highlighted that teachers should use multiple representations such as area models, length/linear models and set models to deepen learners' understanding of fractions and encourage more abstract thinking. Siegler (2010), and Fazio and Siegler (2011), also pointed out that learners should be taught in fractions using number lines and visual representations to foster deep conceptual understanding. Humphreys and Parker (2015) agree with the above researchers that teachers should use multiple representations to build conceptual understanding embedded in multiple real-life contexts. There are three main types of fraction models, namely, **the area model**, **length model** and **the set model**. Teachers are encouraged to work with all three models. The three models play a significant role in developing a conceptual understanding of fractions. Van de Walle et al. (2016) alluded that the wholes for area models are continuous, which means they are not single pieces. In area models' diagrams and paper folding is usually used. The whole is "cut-up" or partitioned into several **equal-sized** pieces (Van de Walle et al., 2016)**.** The shapes below are examples of area models, and they show that fractional parts are not single pieces (not broken), but continuous.

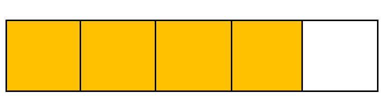


One -eighth three-tenths five -sixths

Figure 2.9: Area models

Area models set models and length models help learners visualize parts of a whole. In these models, tasks involve sharing things that are cut into **equal parts**. The fractions are based on parts of an area. Van de Walle (2004) points out that circular models are commonly used because they emphasise the amount that is remaining to make up the whole. In using a circular object, an apple, which will be an example of a concrete object, may be used to represent a whole, which will be cut into four equal parts. One (1) part of 4 will mean a quarter/fourth of a whole. For learners to understand how to develop fraction concepts they need to do or make their own area model. Cramer and Henry (2002) indicated that models could help learners clarify ideas that are often confused in the symbolic mode. The following are also examples of area models.

circular modelling



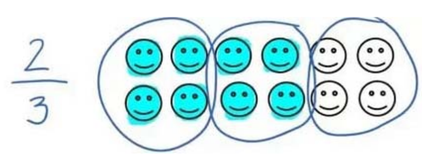
Area modelling

Figure 2.10: Circular and length models

Set models (discontinuous wholes)

The wholes for set models are discontinuous, which means that the whole consists of several **separate equal-sized pieces (**Van de Walle, 2016). Each portion makes up a part of the whole. Each portion is called a subset of the whole. In a set model, the whole is understood as a set of objects, and a subset of a whole makes up fractional parts. The task of the teacher is to assist learners in developing these concepts. A set model for fractional concepts is also important in developing learners' understanding of fractions (Petit, Laird, Marsden & Ebby, 2010). The following are examples of set models.

This is two groups of five



This is two groups of three.

Figure 2.11: Set Models.

Length models (number line presentation)

A length model for fractional concepts is also important in developing learners' understanding of fractions (Petit et al., 2010). In the following length model, the unit is divided into four equal parts, which means that each part is one fourth of the whole. The whole, in this case, is one. Therefore, this will be one-fourth or quarter of one ($\frac{1}{4}$ of 1) and two quarters or two fourths will be $\frac{2}{4}$ of 1 and the whole will be $\frac{4}{4}$ of 1. In placing the fractions on the number line, teachers should assist learners in making sure that the line is properly drawn. A length model is pictured below.

$\mathbf{\frac{1}{4}}$ $\quad$ $\mathbf{\frac{2}{4}}$ $\quad$ $\mathbf{\frac{3}{4}}$ $\mathbf{\frac{4}{4}}$

Figure 2.12: Number line model

Teachers are encouraged to provide learners with more activities in order to discover all these models and the sharing of things into equal parts on their own. Constructivism perceives learners as creators of their own knowledge and understanding.

2.5.WHY ARE FRACTIONS DIFFICULT AND CONFUSING

Van de Walle et al. (2013) identified many possible reasons why fractions are difficult. They identified the reasons for learners' difficulties in fractions as including the following: There are many meanings to fractions, fractions are sometimes written unusually, teaching does not always focus on a conceptual understanding of fractions and learners tend to overgeneralise their whole number knowledge. Cramer and Whitney (2010) alluded that learners think the numerator and the denominator are separate values and are confused seeing them as a single value.

Pienaar (2014), in support of this difficulty of fractions, alluded that one of the challenges that teachers experience when teaching fractions may be how mathematics as a subject is viewed in the South African curriculum. This can be one of the reasons for seeing fractions as difficult and confusing. Learners struggle with fractions because they use what they know about whole number to solve problems with fractions. Therefore, it is the teacher's role to help learners see how fractions are like and different from whole numbers. I am in a view that teachers should use various visuals or models that show parts of a whole for learners to understand, visualise and master the concept of fractions. Cramer and Whitney (2010) suggested that learners often misinterpret fractions, due to the fraction bar that appears between the numerator and denominator.

Cramer and Whitney (2010) pointed out that learners' first misconception results from not understanding the numerator and the denominator well. They stated that learners think that the numerator and the denominator are separate values and have trouble seeing them as one value or number. This means that learners find it hard to see $\frac{1}{2}$ as a single value. I am in a suggestion that in order to assist them, teachers, need to the find fraction value on the number line, illustrated in Figure 2.12 above to help learners develop this notion.

Siebert and Gaskin (2006), and Cramer and Whitney (2010), highlighted that teachers should avoid the phrase or the use of one out of two, two out of six, etc. to learners. Considering what Siebert and Gaskin (2006), and Cramer and Whitney (2010), are saying I am in a view that teachers should rather say two fourths, three eighths etc.

Learners may think that $\frac{2}{3}$ means two parts, not two equal sized parts. Learners may mistakenly think that a fraction like $\frac{1}{2}$ is smaller than $\frac{1}{10}$ because 2 is smaller than 10. In a situation like this, I am in a view that teachers should use various models that show parts of a whole for them to understand the fraction concept better, and learners can be asked to create their own representation of fractions using different models.

Learners incorrectly use the operation rules for the whole numbers to compute with fractions, for example, in teaching learners to add or subtract fractions a teacher may be aware that learners who often have difficulties in executing $\frac{2}{3}+\frac{1}{2}$ are likely to add the numerators and denominators separately, as in the task $\frac{2}{3}+\frac{1}{2}=\frac{3}{5}$. Cramer and Whitney (2010), and Siegler et al. (2010) indicated that learners who concluded that $\frac{2}{3}+\frac{1}{2}=\frac{3}{5}$ do not understand fractions and they will continue to make such errors and misconceptions until they are taught how to develop and understand the concept of fractions meaningfully. To overcome all these difficulties or confusion, I am in a suggestion that teachers should avoid teaching learners the fractional mathematical rules first because if they do so, the learners will continue to be confused and perceive fractions as a difficult notion to deal with. One main reason why learners find fractions difficult, is that they memorise rules and algorithms instead of understanding fractions.

Van de Walle (2009) suggested that teachers should not rush to teaching algorithm as it can hinder learners' understanding of the concept.

It is, therefore, the responsibility of the teachers to remedy this situation. If we as teachers can develop the fraction concept correctly, performance regarding fractions will improve radically in our country.

2.6. BEST PRACTICES OF TEACHING OF FRACTIONS

Ball et al. (2005) highlighted that for teachers to teach mathematics well, they need to unpack or decompress their mathematical ideas to be accessible to learners. This means that teachers need to know how to do mathematics and know how to use mathematics in practice (Adler, 2004). Van de Walle (2013) pointed out that fractions are complex but important concepts in mathematics. He further stated that they are heavily used in various measurements and calculations. The teaching of fractions requires teachers to shift emphasis from the learning of rules and develop a strong conceptual basis for fractions.

2.6.1. Teaching of fractions when dealing with the unpacking of the concept of fractions

The unpacking of fractions requires a teacher to have a deep understanding of the concept of fractions. In this case, the teacher should start with the learners' prior knowledge. Tall (1989) referred to learners' prior knowledge as cognitive roots which are essential for developing a concept in connecting and laying the foundation for learners 'conceptual thinking. On the other hand, Essien (2009) talked of the first approach , which emphasises the importance of starting with learners' prior knowledge and connecting it with the new knowledge. These researchers strongly argue that in any pedagogic practice, the first counter needs to be addressed with mathematical concepts. The unpacking of a concept also needs the teacher to design the first counters as well as cognitive roots. The teacher should explain to learners what a fraction is. The teacher may pose questions to learners to arouse their interest about the topic or just to find out learner's prior knowledge about fractions.

The teacher will then bring learners on board by defining, explaining, and representing what a fraction is. At this stage, a teacher should use models to help learners understand what a fraction is. For learners to understand fractional concepts, a variety of models may be used, for example area models, length models and set models to foster deep conceptual understanding. Cramer and Wyberg (2009) indicated that the effective use of models in fraction tasks plays a significant role. Learners seem to explore when a variety of models are used, and this builds learners' understanding of fractions (Cramer & Wyberg, 2009).

Different and appropriate representations of models broaden and deepen learners' understanding of fractions and helps learners to learn with ease. Van de Walle (2004) identified uses for models in the classroom, that is, to help learners develop new concepts, make connections between concepts and symbols, and assess learners' understanding.

When introducing fractions, the fraction symbol should be delayed until the fraction concept is stable. Van de Walle (2009) pointed out that the fraction symbol can prove to be a confusing notation for children, so learners should rather be encouraged to write the fraction names in words ,for example, 3 quarters or three quarters instead of $\frac{3}{4}$.

Stohlmann, Cramer, Moore and Maiorca (2013)pointed out that when unpacking the fraction concept, teachers are encouraged to refrain from teaching learners by using the old way of teaching. One of the elements that seem to be true is the way teachers are teaching fractions. Learners are encouraged to memorise rules without knowing where they came from. The learners will forget the memorised rules and become blind followers of the rules. Cramer & Whitney, (2010); Siegler et al., (2010) asserted that teachers are urged to use models instead of presenting the rules to overcome all this. Using models makes fractions more concrete to the learner and not just a number on top of another number without meaning. The learner will be able to estimate the answer before calculating, evaluating the reasonableness of the final answer. Learners should be motivated to discover the fraction concept on their own by drawing or folding papers into equal parts and be able to elaborate on them.

In support of what is said above Stohlmann, Cramer, Moore and Maiorca (2013) highlighted that if learners are taught the procedural way of working out with fractions first, the learners are **less likely** to master the fraction concept. They further pointed out that understanding the fraction concept first is **more powerful** and more generative than remembering mathematical procedures.

2.6.2. Teaching of equivalent fractions

Lamon (2002) alluded that equivalence with fractions refers to the fact that many different fractions can be used to name the same quantity, depending on how the quantity is subdivided, and Van de Walle (2016) added that equivalent fractions are ways of describing the same amount by using different sized fractional parts. He further argued that equivalence is about naming the same fractions in more than one way. Van de Walle (2016) stated that models may be used in developing conceptual understanding on equivalence, as illustrated in Figure 2.15 below. He indicated that two fractions are equivalent if they are representations for the same amount.

Van de Walle and Lovin (2006) expand upon the idea of equivalence when they state that to help learners create an understanding of equivalent fractions is to have them use models to find different names for a fraction. Van de Walle and Lovin (2006, p. 66) provide the following big idea about equivalent fractions: "Two equivalent fractions are two ways of describing the same amount by using different-sized fractional parts. For example, in the fraction $\frac{6}{8}$, if the eighths are taken in twos, then each pair of eighths is a fourth. The six-eighths then can be seen $\frac{2}{8} = \frac{1}{4}$

$\frac{6}{8} = \frac{3}{4}$ to be three fourths” Figure 2.13 illustrate the above statement.

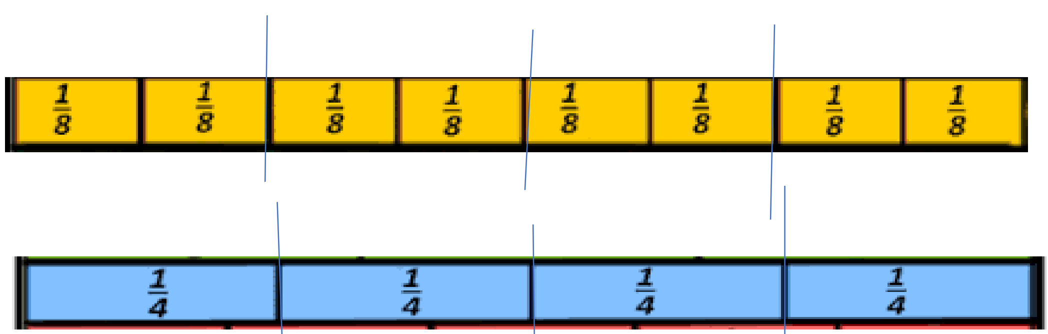


Figure 2.13: Equivalence Fractions

According to Gould (2005), set models may be used to develop the concept of equivalence. At The same time, Petit et al. (2010) stated that length models are very important in developing learners' understanding of fractions. He further alluded that length or measurement models can be paper folding strips; one piece is measured in terms of the smallest strip or rod. Each length is a different colour for ease of identification. Petit et al. (2010) state that strips of paper are also length models; it can be folded to produce fraction strips made by learners. The teacher is required to do the activity with learners to develop the concept of equivalent fractions and demonstrate or show equivalence in fractions. Working with learners will allow them to develop the concept on their own. Van de Walle et al. (2010), Cramer and Wyberg (2009), and Lamon (2008) highlighted that concrete representation is key for learners to comprehend fractions. Learners can be asked to have different colours on them as they will be colouring the strips. Learners can also be asked to take out a paper from their books and be instructed to cut the piece of paper into nine strips that are exactly equal in size and shape. The following stages can be followed for learners to engage actively in the task.

Stage 1: Cut an A4 piece of paper horizontally into nine congruent strips (the same in size and shape).

Stage 2: Fold one strip in half and colour it purple.

Stage 3: Fold one strip into quarters and colour it blue

Stage 4: Fold one strip into eighths and colour it dark yellow.

Stage 5: Fold one strip into thirds and colour it green.

Stage 6:Fold one strip into fifths and colour it in red

Stage 7:Fold one strip into tenths and colour it in light green

Stage 8: Fold one strip into sixths and colour it in yellow

Stage 9: Fold one strip into twelfths and colour it in cream white

Stage 10: Unfold each strip and label the fractional parts.

Stage 11: Leave one strip unfolded, which will serve as a whole.

Learners should reorganise the strips to resemble the fraction chart below.

This resulting fraction chart can be seen in Figure 2.14.

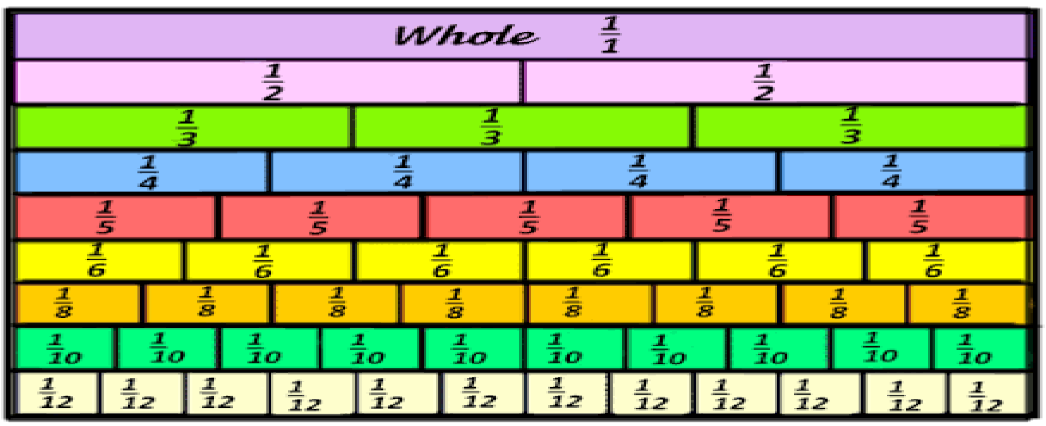


Figure 2.14: Fraction Chart

Siegler (2010) pointed out that paper strips are another method of creating fraction names. He further stated that paper strips or rods can be used as both a whole and a part. According to Siegler (2010), for learners to create fraction names like in the above activity, it will be easy for them to notice that the length $\frac{1}{2}$ of the strip is equal to the length $\frac{2}{4}$ of the strip, the length of $\frac{4}{8}$ of the strip, $\frac{5}{10}$ and also $\frac{6}{12}$ of the strip. We say that one half is equivalent to two quarters and equivalent to four-eighths, equivalent to five-tenths and also equivalent to six -twelves and that is represented mathematically as: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$. Learners will realize that the bottom number (denominator) is twice the top number (numerator). We can, therefore, conclude that equivalent fractions are fractions that have different numerators and denominators but are equal in size and value.

A number line can also be used to demonstrate the equivalence of fractions, as in the following example. It is clear on this number line that $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{2} = \frac{6}{6}$ .The number line is used for the action of counting. In the following number line, counting is in halves and sixths.

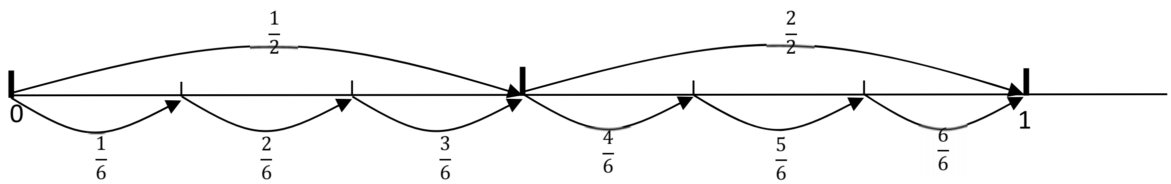
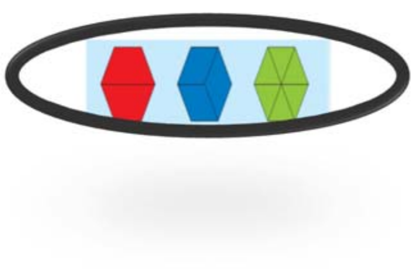


Figure 2.15: Number line

Diagrams like the one below can be used to find the equivalent fractions because Van de Walle et al. (2016) highlighted that teachers should use a variety of representations such as area models, length/linear models and set models to deepen learners' understanding of fractions and to encourage more abstract thinking.



$\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6}$

Figure 2.16: Area models

Learners can be given more activities to do to familiarise themselves with equivalent fractions using different representations.

2.6.3. Teaching of fractions about comparing and ordering of fractions

Petit et al. (2010) pointed out that when looking to see whether two or more fractions are equal, we are comparing them by identifying which is smaller or bigger than the other. Van de Walle (2016) added that comparing fractions is to check which part of the same whole is bigger or smaller than another part. He further alluded that when comparing fractions, the whole must always be the same. A fraction wall/chart can also be used in comparing fractions. According to the fraction wall, we can see that all have the same whole, and we see that $\frac{1}{2}$ is greater than $\frac{1}{3}$ and also greater than $\frac{1}{4}$; $\frac{1}{3}$ is greater than $\frac{1}{4}$ and so on ,as illustrated in Figure 2.17.

Petit et al.(2010) stated that comparing fractions using rules can be effective in getting the correct answer, but if learners are taught these rules, before they master the fraction concept about relative sizes, then the chance of making mistakes is too high. He further highlighted that using rules requires no thought about the size of a fraction. If learners are taught these rules before thinking about the relative size of different fractions, they will be less likely to develop a number sense about fraction size.

Petit et al. (2010)(alluded that the number line model is regarded as a good model for helping learners develop a better understanding of fractions' relative sizes. They stated that the number line must extend beyond 1 when comparing fractions like $4\frac{1}{2}$ etc. The main aim of using models is for learners to have a grounded understanding of the concept, not memorising an algorithm method. Learners should have a sound understanding of comparing and the ordering of fractions. Cramer and Whitney (2010) recommended that teachers should let learners know the meaning of the fractions, make sense of them and avoid rote procedures. Learners should also realise that fractions are numbers and use models. Fazio and Siegler (2011) argue that misconceptions with fractions stem from a lack of conceptual understanding.

When using the fraction wall illustrated in Figure 2.17, learners can also notice that the fraction with the bigger denominator is the smallest when comparing fractions with the same numerators.

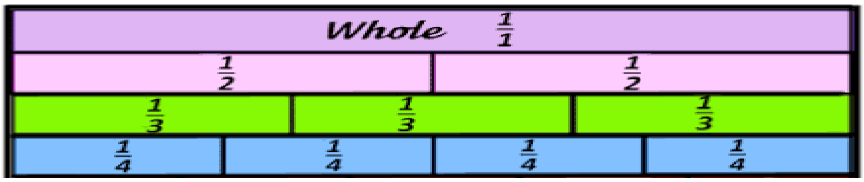


Figure 2.17: Fraction chart

Exercises like this may be given to learners using greater than signs **(>)** and less than signs **(<).** After learners mastered visual representations, they will be able to create their own visuals to reason about fractions.

2.6.4. Teaching addition of fractions

Teaching algorithms on adding and subtracting fractions requires a learner who has a deeper understanding of equivalence fractions, using the common denominator approach. It also requires a learner who has a strong conceptual foundation of equivalence. Henry (2012) indicated that learners with a good starting point with the fraction concept are more likely to add or subtract fractions appropriately. He further added that learners who lack conceptual understanding should be inspired and strengthened to use concrete models. Teachers are advised to stay away from a rule-oriented approach when dealing with the fraction concept. Learners' common error in addition of fractions is to add both numerator and denominator, which happens more often in our schools. Siegler et al. (2010) stated that learners sometimes ignore the denominator and add the numerators, for example $\frac{4}{5} + \frac{4}{10} = \frac{8}{10}$, and this is a clear sign that learners do not know that the different denominators indicate different sized pieces. The algorithm of saying when we add or subtract fraction with unlike denominators, we look for the least common multiple (LCM), should be avoided. It is not a good way of developing the concept. Teachers should refrain from using procedures but rather use models to develop a stronger understanding of the concept in learners. Henry (2012) pointed out that teachers should use multiple representations for high level of understanding of the fraction concept.

Harbour, Karp, and Lingo (2016) highlighted that misconceptions will never be avoided but teachers can intervene before the misconception becomes deeply rooted. Teachers should understand why their learners are making errors and experience misconceptions before they can address them and develop interventions to promote understanding. Teachers can start to discover the root of their learners' misconceptions and errors. Through the constructivist lens, we view errors as learners' attempt to construct their mathematical knowledge.

Van de Walle et al. (2009) indicated that learners should explore addition and subtraction of fractions using the three models: Area, set and linear models. Cramer et al. (2008) on the other hand, suggested that circular area models are the most effective for developing the concept of addition and subtraction of fractions.

Van de Walle (2016) highlighted tha**t** addition and subtraction of fractions occur in three **stages** of teaching. He alerted teachers to illustrate the activities by using models for conceptual development for learners to understand, because they seemed to struggle when adding or subtracting fractions. He explained the stages as follows:

Stage 1-Addition and subtraction of fractions with same denominators (like denominators).

Stage 2- Addition and subtraction of fractions where one denominator is a multiple of **the** other. (unlike denominators); and

Stage 3 Addition and subtraction of fractions where one denominator is not a multiple of the other (unlike denominators).

As the above researchers mentioned that models could be used, a linear model will be serving as an example in the following examples.

Stage 1

Addition of fractions with same denominators (like denominators)

Example :

1.$\frac{1}{5}$ + $\frac{3}{5}$ The sum is $\frac{1}{5} + \frac{3}{5}$

The whole of **five blocks** should be drawn and the parts of the whole involved in the calculation should be shaded (such as, one part and three parts) to find the answer, as depicted in Figure 2.18 below.

$\mathbf{\frac{1}{5}}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Added to $\frac{3}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Equals to $\frac{4}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ (Added numerators and denominator stays the same.

Figure 2.18:Length models

Stage 2

Addition of fractions where one denominator is a multiple of the other (unlike denominators).

Example :

$\frac{1}{5} + \frac{3}{10}$

In the above activity, learners should select a whole into which both five and ten can divide. According to this argument, the whole should be divided into ten equal parts.

$\frac{1}{5} + \frac{3}{10}$

$\frac{2}{10} + \frac{3}{10}$ (using equivalence, we must make the denominators the same)

$\frac{1}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

added to $\frac{3}{10}$

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$\frac{1}{5}$ (We notice that $\frac{1}{5}$ is twice $\frac{1}{10}$, the denominators are multiples of each other)

$\frac{1}{10}$	$\frac{1}{10}$								

added to

$\frac{1}{50}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Equals to $\frac{5}{10}$

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Figure 2.19: Length models

Stage 3

Addition of fractions where one denominator is not a multiple of the other (unlike denominators).

In Section 2.6.2, It was stated that according to Van de Walle (2016), equivalent fractions are ways of describing the same amount by using different sized fractional parts. He further argued that equivalence is about naming the same fractions in more than one way. Van de Walle (2016) stated that in developing conceptual understanding of equivalence, models could be used. He alluded that two fractions are equivalent if they are representations for the same amount. Using the paper strips in the activity described in Section 2.6.2, learners assimilated what was learned in the equivalent fraction concept and noticed that $\frac{1}{3}$ fits exactly into $\frac{2}{6}$ and $\frac{1}{2}$ fits exactly into $\frac{3}{6}$.

Ans example of the calculations follow below:

Example:

$\frac{1}{3} + \frac{1}{2}$ (using equivalence, we must make the denominators the same)

$\frac{2}{6} + \frac{3}{6}$

In the above activity, learners should select a whole into which both three and two can divide. According to this, the whole should be divided into six equal parts:

$\mathbf{\frac{1}{3}}$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Added to $\frac{1}{2}$

$\frac{1}{2}$	$\frac{1}{2}$

Equals to $\frac{2}{6}$

	$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$

added to $\frac{3}{6}$

		$\frac{1}{2}$			$\frac{1}{2}$

Equals to $\frac{5}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Figure 2.20: Length model

2.6.5. Teaching subtraction of fractions

Subtraction of fractions, like the addition of fractions, occurs in three **stages** of teaching and representation as set out in the example below (Van de Walle, 2016).

Stage 1.

Subtraction of fractions with same denominators (like denominators)

Here follows the illustration.

$\frac{3}{8}$ - $\frac{1}{8}$ The whole of eight equal blocks should be shown, shading three parts and one part of a whole

$\mathbf{\frac{3}{8}}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Subtract $\frac{1}{8}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Equals to $\mathbf{\frac{1}{8}}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Figure 2.21: Length model

Stage 2

Subtracting the fractions with one denominator a multiple of the other.

The learners should first find out into how many parts the whole should be divided. The learners should then select the whole into which can be divided by both two and four. The whole should be divided into four equal parts.

$\frac{1}{2}$ - $\frac{1}{4}$ using equivalence, we must make the denominators the same.

Illustration:

$\mathbf{\frac{1}{2}}$

$\frac{1}{2}$	$\frac{1}{2}$

Subtract $\frac{1}{4}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{\mathbf{1}}{\mathbf{2}}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Subtract $\frac{\mathbf{1}}{\mathbf{4}}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Equals to- $\frac{\mathbf{1}}{\mathbf{4}}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Figure 2.22: Length models

Stage 3

Subtraction of fractions with one denominator not a multiple of the other.

$\frac{1}{2}$ - $\frac{1}{3}$

Learners should find out into how many parts the whole should be divided. Learners should select a whole that can be divided by both two and three. The whole should thus be divided into six equal parts.

$\frac{1}{2}$ - $\frac{1}{3}$

$\frac{3}{6}$ - $\frac{2}{6}$ =$\frac{1}{6}$ (using equivalence, we must make the denominators the same).

$\frac{\mathbf{1}}{\mathbf{2}}$

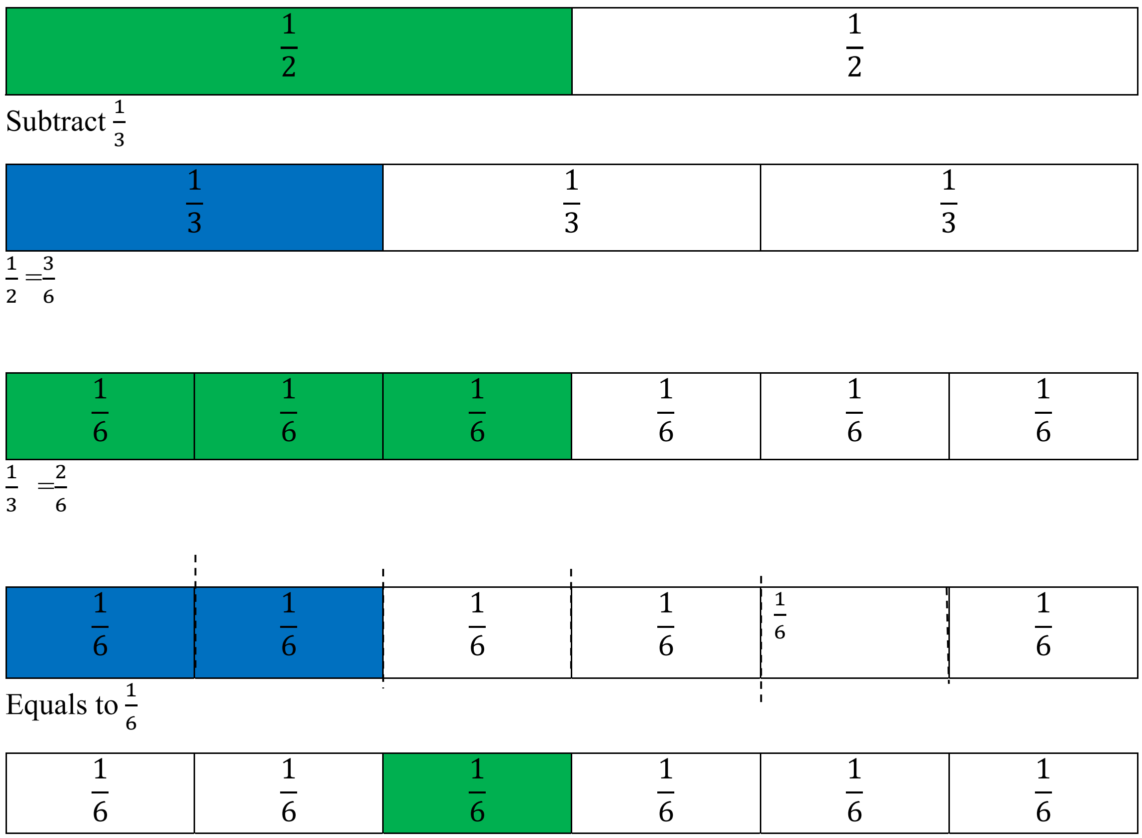


Figure 2.23: Length models

2.6.6.Teaching proper and improper fractions

It was indicated in Section 2.4.1.that a fraction is formed by breaking something that is a whole into smaller equal parts and in Section 2.4.2 it was stated that a fraction can be considered as a numeral that can be expressed in the form of

$(\frac{a}{b})$, where b$\neq$ 0 (**b must not be zero**)Van de Walle (2016).

In this case, the numeral at the bottom of the fraction (the denominator) has an entirely different function from the numeral at the top. The top numeral is the numerator. The above statements referred to the proper fraction. A fraction that is greater than 0 but less than 1 is called a proper fraction or a normal fraction. In a proper fraction, the numerator (top numeral) is less than the denominator (bottom numeral)

An example of a proper fraction

$\frac{4}{5}$ numerator

Denominator

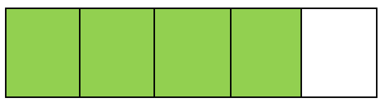


Figure 2.24: Proper fraction

An improper fraction has the numerator bigger than the denominator. This means an improper fraction is greater than 1. It is an improper fraction because it is not proper to leave it as the answer, it can be written as mixed numbers. All improper fractions are larger than 1. If illustrated by models, the number of shaded parts will be more than the total parts.

An example of improper fraction is reflected below:

$\frac{5}{2}$ Numerator

Denominator

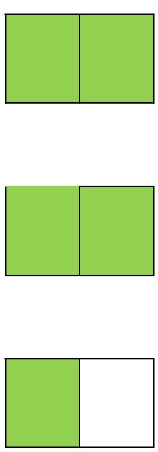


Figure 2.25: Improper Fraction

Teaching learners about proper and improper fractions requires a teacher to use different representations for learners to have a deeper understanding of the concepts. Any incorrect teaching of proper and improper fractions can lead learners to a misconception of the concept.

Part-whole representation is highly recommended in this regard. Van de Walle (2009) alerted teachers to pay close attention to the usage of models for learners to visualise the concept meaningfully.

2.6.7. Teaching about converting mixed fractions to improper fractions.

As indicated in Section 2.6.6, improper fractions are fractions with numerator larger than 1 and mixed numbers are formed by converting improper fractions, for example $\frac{3}{2} = 1\frac{1}{2}$. Teaching about converting mixed fractions to improper fractions require a teacher to first use models before using the algorithms .Van de Walle (2009) alerted teachers not to rush to algorithms because it can delay learners' understanding of the concept. Using an algorithm like "multiply the whole number part by the fraction's denominator, add the numerator then write the answer on top of the denominator, and the denominator remains the same", should be avoided unless learners have a strong and solid understanding of the concept. A misconception may arise if learners are taught about the procedure before understanding the concept well.

An example is reflected below:

$\frac{8}{5}$

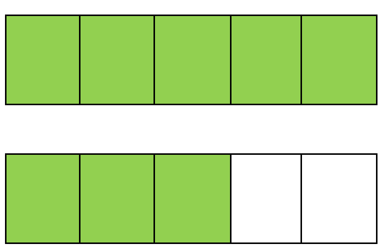


Figure 2.26: Improper fraction

2.6.8. Teaching about converting common fractions to decimal fractions

Common fractions is an umbrella name for proper fractions, mixed fractions and improper fractions. In Section 2.6.6 and Section 2.6.7, proper, improper and mixed fractions have already been explained. Decimal fractions usually have denominators of tenths, hundredths etc. Converting common fractions to decimal fractions can be done using a calculator or simply using the long division method, but this is the wrong way of teaching the concept because learners may have difficulties in applying the long division method and misconception might form in this way. Using the calculator is also not encouraged because a learner may end up not knowing how to master the concept. Teachers are therefore encouraged to use models, especially the rectangular model as illustrated in Figure 2.26.

The illustration shows the changing of common fractions to decimal fractions. The representation also shows percentages which are not part of the topic.

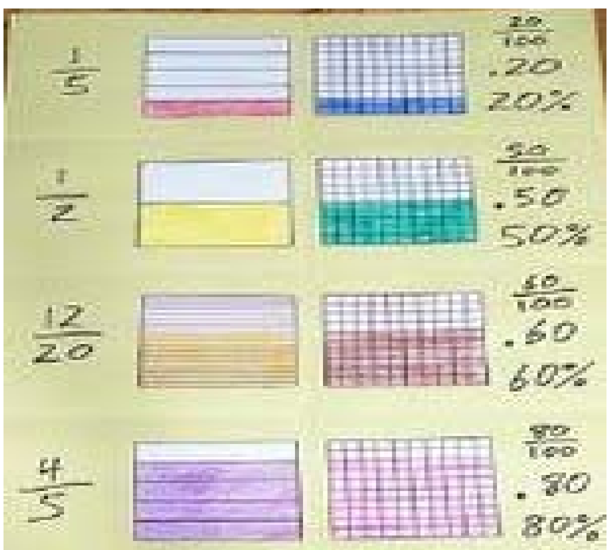


Figure 2.27: Changing common fractions to decimal fractions.

2.7. Teaching of fractions in dealing with learners' mathematical ideas

Dealing with learners' mathematical thinking/ideas when teaching or developing the concept of fractions requires teachers to shift or reform from the teacher-centred approach. Cazden (2001) mentioned that currently teacher-centred teaching continues to dominate. Cazden (2001) further alluded that teachers dominate classroom discourse. Chisholm et al. (2000) also argue that many classrooms remain teacher-centred, and teachers engage learners in shallow ways. Working with learners' thinking is key to reform many teachers across the globe. Many teachers who do not consider becoming "reform teachers'' and do not work with learners' thinking. Constructivists encourage learners to take part actively in their own learning. Learner-centeredness and collaboration is a learning goal in the constructivist learning environment.

Teachers are, therefore encouraged to make their lessons more learner-centred by encouraging learners to contribute to mathematical tasks. They are also encouraged to give learners tasks that are genuine and promote learner mathematical thinking. Brodie et al. (2009) indicated that teachers should select tasks of higher cognitive demand.

In learner-centred teaching, learner talk is regarded as a foremost factor of classroom teaching (Franke, Fennema, Carpenter, Levi and Empson, 1999). There is genuine engagement in learner talk, which implies that learners articulate their own mathematical ideas rather than produce what the teacher wants (Dawes, 2004). Dawes argues that genuine classroom talk includes reasoning about mathematical ideas. Learner talk is foremost crucial because of the following indicators, which were used in this study during data collection and analysis.

- Help teachers to monitor learners' mathematical ideas.
- Help learners to gauge one another's strategies and understanding.
- Increase learners' mathematical ideas.
- Allow learners to express and clarify their ideas.
- Enable learners to share ideas.
- Provide teachers with information about what learners know and don't know and how they think and try to make sense of their mathematical ideas.
- Require genuine involvement among members of a group.
- Help learners make sense of mathematics.
- Support learners to express and clarify their mathematical thinking.

Dealing with learner's mathematical ideas needs teachers to elicit learner questioning, posing mathematical problems that challenge and promoting their mathematical thinking. Posing problem-solving questions, require a teacher to listen to learners and watch the strategies that learners use. From a constructivist perspective, it is widely known that learners are not empty vessels in which the teacher fills in information, they have a web of interconnectedness ideas in them. They are **c**onstructor**s** of their own knowledge.

Teachers are encouraged to use mostly interpretive listening, allowing the teacher to listen carefully to learners making sense of their mathematics. In contrast, evaluative listening seeks the correct answer that the teacher already has in mind. Generative listening also is encouraged because it transforms one's mathematical understanding and can guide the direction of the lesson.

When a problem is posed to learners, it is important for the teacher to unpack the problem with learners. Unpacking the problem makes learners understand and engage in a problem, making sense of their ideas. Unpacking the problem implies that teachers need to explain what the question is all about. Franke et al. (2001) stated that learners infrequently ask questions. Teachers are therefore advised to encourage learners to ask questions frequently for clarity to remedy this situation. Teachers should create a situation conducive to learners for them to ask questions freely. Teachers should avoid asking questions with short answers, low-level questions that require learners to recall facts, rules, and procedures. Hiebert and Wearne (1993) elaborated that asking high-level questions requiring learners to conclude their findings and synthesise their ideas is highly recommended.

Asking open-ended questions like "Can you tell us how you solve the problem?" is a fruitful way of engaging learners in explaining the method they have used in solving the problem. Follow up questions also support learners to elaborate on their workings. Teachers should encourage learners to think aloud because some learners with learning disabilities are impulsive. When faced with a problem, they struggle to find the correct answer. Thinking aloud may help anchor skills and strategies, both behaviourally and mathematically.

What is required of teachers to understand or gauge their learners' mathematical thinking when dealing with a mathematical problem? If there was no time to ask or pose questions, teaching and learning would be meaningless. Teachers ought to pose questions to assess learners' mathematical thinking, which means posing or asking questions is one assessment technique that improves learners' understanding. The act of asking questions is cognitively challenging, it has been a challenging part of teaching and requires a teacher to know their learners well, as recommended in Shulman's domain of knowledge of content and students (KCS)

Questions asked can be used to challenge and stimulate learners thinking. Moyer and Milewicz (2002) stated that approximately 80% of teaching in schools is spent on posing or asking questions. They further pointed out that questions serve many purposes such as provoking learners and making them listen carefully, analysing their thoughts and thinking critically. The importance of questioning cannot be ignored or underestimated; hence, Boaler and Brodie (2004) concur that questioning is an important teaching strategy in establishing classroom atmosphere conducive to the development of learners' mathematical thinking.

There are different types of questions that may be used in the classroom situation, as set out below.

Probing questions

Probing questions are asked for clarification, explanation, or justification purposes. Teachers who encourage learners to elaborate, justify and explain their thinking use probing questions. Probing questions promote learning and push up learners to think more deeply. Moyer and Milewicz (2002) found that probing questions help the teacher better focus on learner thinking. Learners can be grouped to work as a team and solve a problem given. After solving the problem, they must explain or elaborate to other groups, even to the teacher on how they reached the answer.

High order questions

High order questioning is cognitively demanding, promoting critical thinking skills, making the learner apply, synthesise, and evaluate information. The purpose of these questions is to extend learners' knowledge, encourage learners' participation, and promote deeper thinking.

Like in probing questions, learners may be given a mathematical problem to solve, and because of its difficulty, learners will have to think deeply about how to solve a problem. They must justify, explain and elaborate on their findings.

Guiding questions

Boaler and Brodie (2004) stated that guiding questions guide learners to discuss problems and derive mathematical concepts and procedures. Guiding questions are like leading questions. The teacher leads or guides learners towards an answer and provides learners with a chance to respond. If the learners are not sure on how to solve a mathematics problem, the teacher may lead or guide with a question such as “which method do you need to use now”? or “which operation do you think you will use in solving your problem? “

For examples:

Mary drinks $\frac{1}{2}$ a glass of water, Sara drinks $\frac{1}{3}$ glass of water and John drinks $\frac{1}{4}$ glass of water. How many glasses of water did they drink altogether? A leading or guiding question here is, "Which operation do you think you will use in solving this problem?"

Factual questions

Factual questions allow teachers to check learners' recall of specific mathematical facts such as "What is a proper fraction?" Factual questions allow teachers to assess basic information before moving forward.

Specific questions

Specific questions need a specific response. They address something specific in a learner's explanation like $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$. Moyer and Milewicz (2002) indicated that posing questions to learners timely facilitates the cognitive growth of learners. I concur with the above statement. Posing questions to learners frequently or throughout the lesson makes the class more interactive and helps the teacher to measure and improves learning.

Maher and Martino (1994) argue that when learners are given a problem to solve individually or in small groups and have completed the activity given, they are eager to hear feedback from others and ready to discuss their ideas. After a lengthy discussion about which solution is correct the teacher then intervenes strategically because learners with conflicting ideas may be brought together to discuss their findings.

Maher and Martino (1994) indicated that learners alone do not question each other about their argument's detail. Therefore, the teacher's role in questioning learners becomes critical after learners worked individually or as a group. The teacher's timely asking of questions can estimate a learner's understanding of the mathematical ideas. Questions asked, in this case, should be connected to the learners' present thinking about the solutions.

The use of questioning enables the teacher to gain an understanding of their learners' current thinking. Teachers should encourage learners to ask questions of their own, and this improves their learning. Self-questioning is more effective and should be reinforced. The teacher's questioning should consider Bloom's Taxonomy's six levels: knowledge, comprehension, application, analysis, synthesis, and evaluation.

Teachers are encouraged to pose open-ended questions as they engage and encourage learners to engage critically in a discussion. Teachers are discouraged from posing leading questions. When posing a question, the teacher should give learners a chance to think and formulate responses, if they do not respond the teacher should rather reframe the question. An example of an activity that learners may engage in during a group discussion is as follows:
Six learners wanted to share a pizza among themselves. How many pieces of pizza will each get? If they are five, how many slices is each going to get?
Learners will be brainstorming about the activity given. In a group, the learners will work collaboratively as a team in a group, keeping a distance as per the COVID 19 national regulations. There will be a leader who ensures that proceedings stay in order, the scribbler who will note what they agreed upon, a reporter who will give answers to the class and other members who are part of the discussion group. At this stage, the teacher will intervene as little as possible where there are some misunderstandings and who will be playing a leading role. One of the characteristics of constructivism is that the teacher and learners share the authority of learning. The teacher is the facilitator, and the learners are active agents of their own learning.

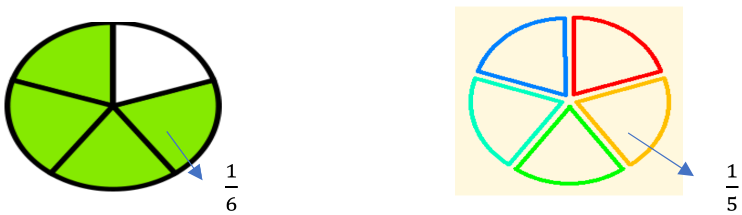


Figure 2.28: Group discussion of the activity

After brainstorming learners will conclude by saying that for six learners to share the pizza equally, each will get one sixth ($\frac{1}{6}$) piece of pizza and for five learners each will get one fifth ($\frac{1}{5}$) slice of pizza, as shown above.

Learners may further tell that in $\frac{1}{6}$ and $\frac{1}{5}$, both the top numbers are called numerators because they enumerate how many parts of a pizza each will receive or how many parts have been used, whereas the bottom numbers are denominators since they denominate into how many parts the pizzas were divided, they give the fraction a name. They can even further show their understanding that $\frac{1}{6}$ is called one-sixth and $\frac{1}{5}$ is called one-fifth, and they are both proper fractions.

In a classroom situation, it is proper for the teacher as well as learners to ask questions. If there are no questions asked, then learning and teaching will be worthless. Teachers and learners' questions are of the utmost importance for teaching and learning to take place smoothly or effectively. Questioning is one of the thinking process skills. The process of asking questions allows learners to articulate their existing understanding of a topic, make connections with other ideas, and become aware of what they do or do not know.

Teachers should always encourage learners to ask questions. Graesser and Olde (2003) found that learners ask few questions. This implies that learners rarely ask questions or do not ask at all if not encouraged to do so. In light of this, it shows that teachers seem not to encourage learners to ask questions. Graesser and Olde (2003) indicated that if they have the opportunity to ask questions, only few of them will ask high-quality thinking or cognitive questions, whereas most of them ask questions that are factual, procedural, or closed. Learners 'questions help the teacher to diagnose the learners' understanding and to tap into their thinking. It also helps to evaluate and interpret higher-order thinking.

Responses are answers or replies that the teacher is entitled to provide to learners if there is a need for clarification of mathematical tasks on certain aspects. Therefore, it is important for teachers always to create a favourable classroom environment for teaching and learning to enhance and promote learner participation and be free to ask questions for clarity on the topic of the day. Ball and Bass (2000) indicated that teaching creates an environment for learning mathematics conducive to learning.

There are ways in which a teacher can respond to questions from learners. According to Brodie (2002), listening to learners' question is important. If a learner asks a question, the teacher should be sure that the question is understood clearly and that other leaners have heard and understood the question. If a learner asks the teacher to do something, they can redirect the question to other learners to respond to it. Sometimes answering a question with a question may be attractive, but it cannot be very reassuring to a learner seeking information.

What learners need from the teacher is to resolve all their uncertainties, answer all their questions, or tell them what they do not remember. If learners do not understand, they are most likely to ask for a repetition ("Could you say that again please? or "Could you go through that again please?"). If you always agree to this request as a teacher, you condition learners to be dependent and preserve your role of authority. You should consider instead, to sometimes deflect to someone else to "say what they think you said" in order to stimulate learners to listen to and to learn from each other.

Tobin and Gallagher (1987) highlighted that teachers responded differently to learners' questions, depending on who asked the questions. Teachers seem to target learners who regularly contribute to classroom discourse by raising their hands, asking questions. Watts and Alsop (1997) suggested that when responding to learners' questions, teachers may sometimes re-direct the question to other learners because the teachers may sometimes find themselves in a situation where a learner asks or raises an unexpected question which they are unable to answer accurately. In case this situation happens, the teacher is advised not to ignore the question but rather ask other learners to suggest an answer or not hesitate but tell learners that they are going to refer the question.

Teachers are encouraged to listen to their learners' responses. Three types of listening may be used, namely evaluative, interpretive, and generative listening. Davis (1997) alluded that when teachers engage in evaluative listening, they seek correct answers that they already have in their minds. In this kind of listening, learners' responses are largely ignored and not taken into account. He further stated that in interpretive listening, teachers no longer assess the correctness of learners' responses, instead they are interested in making sense of the responses learners brought forward. In this kind of listening, teachers try to understand their learners' contributions and how they argue to produce solutions. Teachers at this stage are likely to change the lesson towards the learners' responses. In generative listening. teachers and learners seek next steps and question themselves about what should be done next. In this kind of listening, learners' contributions guide the direction of the lesson. In this study, teachers were evaluated as to whether they listen to their learners interpretively or evaluatively when asking questions in the teaching of fractions.

An essential part of mathematics teaching is giving feedback or responses. As Bard (2014) writes, "to teach is to provide feedback". The responding stage is the stage where the teacher provides verbal or non-verbal reactions. Teachers usually use reinforcement for learners giving a correct response; the correctness of learner's response is usually praised by saying words like "very good" (Brophy, 2013). The teachers should show interest in all answers. They should encourage them when they answer by nodding, looking at them and using a facial expression that indicates that the teacher is listening. Learners who respond should be thanked to show appreciation for their involvement.

Table 2.1 illustrates the kinds of questions that teachers should use when dealing with learner questioning. Brodie and Boaler (2004) developed nine categories of teacher questions. Their development of teacher questions was informed by Hiebert and Wearne (1993)

Table 2.1: Kinds of questions that teachers should use when dealing with learner questioning.

Question type	Description	Example
1.Gathering information, leading learners through method	Requires immediate answer. Rehearses known facts/procedures. Enables learners to state facts/procedures	What is the value of x in the equation? How would you plot that point?
2.Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called? How would we write this correctly?
3.Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationship and meaning. Makes links between mathematical ideas and representations.	Where is this x on the diagram? What does probability mean?
4.Probing, getting learners to explain their thinking	Asks learners to articulate, elaborate or clarify ideas	How did you get 10? Can you explain your idea?
5.Generating Discussion	Solicits contributions from members of the class	Is there another option about this? What did you say, Justin?
6.Linking and applying	Points to the relationships among mathematical ideas and mathematics and other areas of life	In what other situations could you apply this? Where else have you used this?
7.Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used.	Would this work with other members?
8.Orienting and focusing	Helps learners to focus on key elements or aspects of the situation to enable problem-solving	What is the problem asking you? What is important about this?
9.Establishing context	Talks about issues outside math in to enable links to be made with mathematics	What is a lottery? How old do you have to be to play the lottery

This study used these types of questions as indicators, measuring the presence or absence of questions during the teaching of fractions.

2.8. Teaching of fractions in dealing with the restructuring of the mathematical tasks (fraction concept)

The restructuring of mathematical tasks is an important aspect of mathematical problem solving for teachers to utilise. When restructuring the mathematical problem task, teachers must know how to develop the concept of fractions effectively. Restructuring is an important aspect of problem-solving that teachers can apply when they teach concepts like fractions.

Teachers need to realise that learners may not understand the method used in solving a certain problem but may understand another. According to Kazima (2008), restructuring mathematical tasks is important because it helps the teacher scale the mathematical task up or down. He highlighted that restructuring the mathematical task may include posing the right questions and working with learners' ideas.

Restructuring mathematical tasks refers to scaling the task down if it is too difficult or scaling the task up if it is not thought-provoking enough for learners. Scaling up and down enable learners to engage in a task effectively. Another kind of restructuring a mathematical task that teachers might need to enact is shifting the appropriate mathematical outcomes of the task. It is regarded as the additional component of restructuring of the task (Ball et al., 2004)

Teachers may restructure their task if the result of the first task does not function as required or as planned, which means that the result or the answer does not illustrate the concept as it was expected. For example, if the teacher unpacked the fractions procedurally, making learners memorise the fractional rules, they can restructure and use different models for learners to master the concept. The teacher may follow all the aspects of representing the fraction concept, sharing of a whole into equal parts, naming of fractions, pronouncing them using correct language and knowing the fractional notation and so on.

The using of a fraction wall used for finding equivalent fractions may also serve as an example. The teacher may restructure the same task by using a number line representation or shade diagrams to display the equivalency, using the same activity for learners to understand better.

2.9. CONCLUSION

This chapter outlined the main theory and related frameworks in the teaching and learning of fraction concept. The study is framed and guided by constructivism theory and the frameworks of Ball et al (2008) and Shulman (1986) PCK as the point of reference. The theory and frameworks explained above, have one good thing in common and that is – they have a room to encourage teachers to be facilitators of learning and transform from their old way of teaching and allow learners to construct their own learning and understanding. The kind of teaching and learning atmosphere allowed by this theory is that learning should be learner- centred where the teacher should be aware that learners construct their learning in an active way. All constructivists acknowledge that learning does not happen passively on the part of the learner.

Ball et al (2008) and Shulman (1986) PCK’s frameworks encourage teachers to know their subject matter and know how to deliver the content to learners in a way learner comprehend. This implies that teachers ‘mathematical knowledge should be beyond the subject matter.

CHAPTER 3: RESEARCH METHODOLOGY

3.1. INTRODUCTION

This chapter explains the methods of collecting data. It unfolds the research design, the sampling and data analysis. It further explains data collecting techniques which include observation and interview. It indicates issues regarding reliability and validity, ethical consideration, confidentiality, the pilot, and the limitations of the study.

3.2. RESEARCH METHODOLOGY

The study is mainly supported by a qualitative research approach. A qualitative research approach is suitable for this study since it researches the actual practice of the intermediate phase, teachers from the identified schools when teaching fractions. Through a qualitative research approach, the researcher can explore the mathematical knowledge for teaching on how to introduce, unpack, develop, and define fractions to Grade 6 learners of the intermediate teachers.

McMillan and Schumacher (2010) highlighted that the qualitative research approach focuses on exploring, understanding, and determining significance and describing a phenomenon through the participant's practices and viewpoints. The researcher observed the teachers in practice and conducted interviews with the participating teachers. This approach is subjective as the researcher cannot detach herself from the issues discussed. The qualitative research approach was used because it gave the researcher an opportunity to gain insight into the inner experience of the participants. Terre Blanche (2008) indicated that in any qualitative study the context is accepted in a naturalistic manner where the researcher will be physically present.

3.3. RESEARCH DESIGN

The researcher used a case study utilising two distinct methods, observation and interviews, while also observing teachers in practice. Creswell (2010) alluded that an interview is a two - way conservation in which the interviewer asks the participants questions to collect data. The researcher interviewed each teacher about what she had seen during the observation phase.

3.4. RESEARCH SITE

The research study took place in Limpopo Province in South Africa, at three primary schools in Capricorn South District. The primary schools where the research was conducted had foundation, intermediate and senior phases. Both sampled schools are public schools. They are in Lebowakgomo Township.

3.5. SAMPLING

The research study's sample population was three mathematics teachers who were teaching Grade 6 from neighbourhood schools who participated in the study. The population of the study was the three neighbouring Primary schools Mathematics teachers, in the Capricorn South District ,Limpopo. According to Cohen et al. (2011), convenience sampling involves choosing the nearest individuals to serve as participants. In contrast, Creswell and Clark (2011) state that convenience sampling involves identifying and selecting individuals or groups of individuals that are especially knowledgeable about or have experience with a phenomenon of interest. Furthermore, Leedy and Ormrod (2010) emphasise that sampling is convenient if it is dependent only on the accessibility and availability of participants.

According to Farrokhi and Mahmoudi-Hamidabad (2012), convenience sampling is a kind of non-probability or non-random sampling in which participants are selected for the purpose of study if they meet specific criteria. The researcher sampled Grade 6 mathematics teachers because of their accessibility, and their willingness to participate. The schools researched were sampled purposefully because of their accessibility, and their willingness to participate.

3.6. DATA COLLECTION TECHNIQUES/METHODS

The data collection was don through two distinct methods of observing and interviewing teachers who offered mathematics in the primary schools selected. The main data collection method was observation. The researcher observed the teachers in practice, in the role of a non-participant observer (complete observer) in the classroom.

Maree (2009) states that observation is an everyday activity whereby we use our senses (e.g., seeing, hearing, touching smelling and tasting), but also our intuition to gather bits of data. Maree adds that observation is an essential data-gathering technique that can provide the researcher with an insider perspective of group dynamics and behaviours in a specific setting.

Non-participatory observation

Maree (2011) indicated that a non-participatory observation is an unobtrusive qualitative data collection technique or strategy for gathering primary data about some aspect of the social world without interacting directly with the participants. The researcher visited the sampled participants and observed how they taught fractions to Grade 6 learners. The observation was done during normal teaching and learning hours with the participants' and their principals' permission. The observation lasted nine hours in total during normal (30 minute) or double (60 minute) mathematics periods. Only teachers teaching mathematics in Grade 6 classes were observed. The researcher observed three double lessons per participant, which gives a total of nine lessons.

Non-participatory observation is described by MacFarlan (2017) as observation where the researcher observes the participants without actively participating in their activities. She further adds that non-participatory observation is a technique used to understand the phenomenon studied by entering the community and the social systems involved. Maree (2011) concurs that during this observation period, the researcher only enters the situation to focus on their role as observer. The researcher might look for patterns of behaviour in a community to understand the assumptions, values, and beliefs of the participants and to make sense of the social dynamics, but the researcher remains uninvolved and does not influence the dynamics of the setting. This implies that the researcher was a complete observer.

During this study, data was collected through a non-participatory observation method where teachers teaching Grade 6 learners, had to teach with the researcher taking field notes of the patterns of behaviour of both the learners and the teacher. Kawulich (2012) indicated that field notes are quick notes about something about which the researcher wants to write more later. Field notes help the researchers recall a lot of material they do not have time to write down in detail during observation. Appendix B reflects the non-participatory observation schedule. During the observation period, the researcher used the running records to capture more detail which was continuous and could account for what was observed. Participants were allowed to provide or withhold consent before the observations could take place, and a copy of their consent has been attached.

An observation schedule was designed based on the categories condensed by Kazima (2008) from the eight categories/aspects developed by Ball et al. (2004). Ball et al. (2004) argued that the aspects of teaching appeared repeatedly in teacher's work. The conceptual framework used focused mainly on the following categories/aspects.

- How do teachers unpack/introduce, define, explain, and represent the concept of fractions to Grade 6 learners?
- How do teachers work with learners' mathematical ideas?
- How do teachers pose and respond to learner questioning?
- How do teachers restructure their mathematical tasks?

The researcher decided to merge two categories/aspects to the category of working with learners' mathematical ideas. The two categories merged were: *How* ***teachers pose*** *mathematical questions to learners* and *how do teachers respond to learners' mathematical questions* in *working with learners' mathematical ideas.* Therefore, the researcher will deal with three categories only namely,

- How do teachers unpack/introduce, define, explain, and represent the concept of fractions to Grade 6 learners?
- How do teachers work with learners' mathematical ideas?
- How do teachers restructure their mathematical tasks?

The necessary arrangements were made by the researcher with the principals of the identified schools to sit with affected teachers to get their consent to participate in the study. In these meetings, it was made clear when and for how long these observations would take place. The aim was to have a handful of these observations to see all the teachers' interactions and approaches with their mathematics learners.

During the time of sitting back and observing, the researcher took field notes about the proceedings which informed part of the interview being planned to take place some days after the observation phase.

The observer had a structured observation schedule which clearly indicated the above aspects/categories and how the teacher addressed each of the. Each category had indicators that the teacher was expected to meet when engaging with the mathematics work of teaching. The field notes taken provided an idea of what to ask during the interview session. The interview followed the observation process. The interviews with the teachers augmented the data collection strategy.

Unstructured interview was conducted some days after the lesson observations in the schools of the sampled teachers. The set of questions were follow-up questions from what the researcher observed during the lesson presentation. Creswell (2010) alluded that an interview is a two -way conservation in which the interviewer asks the participants questions to collect data. The researcher interviewed each teacher to confirm what she had seen during observation.

The interview considered the follow-up questions from notes made during the observation stage. This means the set of pre-determined questions were not rigidly defined beforehand but instead teachers were interviewed using follow -up questions generated from what was observed in the classroom. The interview was recorded and kept safe in the researcher's storeroom and will be kept safe for three years.

The information collected using these methods were enough to start analysing it. The collected data analysis was done in a meaningful, structured manner considering the key questions asked in the study.

3.7. DATA ANALYSIS

After collecting the data through observations and interviews, the researcher used the observation schedule designed to check if the categories followed agreed with the categories condensed by Kazima (2008) from the eight aspects developed by Ball et al. (2004).

Ball et al. (2004) proposed eight categories of mathematical work that teachers frequently engage in. They argued that the eight categories were teaching tasks which appeared most often in teachers' work. Kazima et al. (2008) decided to shorten the eight categories/aspects to six because they concluded that some of them were overlapping. The six categories selected were as follows:

- ***Defining,*** which means that the teacher provides a definition of a concept to learners.
- ***Explaining,*** which means teachers explains problems to learners.
- ***Representation,*** which means teachers represents an idea in various ways.
- ***Working with learners' ideas,*** which means teachers engage with both learners expected and unexpected mathematical ideas.
- ***Restructuring learners' 'tasks***, which refers to simplifying a problem or making it more complex.
- ***Questioning,*** referring to posing and responding to questions for the lesson to go on.

The researcher decided to merge two categories/aspects, namely *how teachers pose mathematical questions to learners* and *how do teachers respond to learner's mathematical questions,* in *working with learners' mathematical ideas.* Therefore, the researcher dealt with three categories only namely,

- How do teachers unpack/introduce, define, explain and represent the concept of fractions to Grade 6 learners?

- How do teachers work with learners' mathematical ideas?
- How do teachers restructure their mathematical tasks?

The researcher also checked if the observed teachers were acting in line with what Ball's (2008) framework was saying about the mathematical knowledge for teaching. The researcher also compared the teachers actions with Shulman's (1986) notion of PKC, and Adler and Davis' (2006) study indicating that a deeper and broader understanding of mathematics was required by the teacher. The unpacking of fractions involves *introducing, defining, explaining, and representing* of a concept; the teacher dealing with learners' mathematical ideas and the teacher restructuring their mathematical tasks.

3.8. RELIABILITY AND VALIDITY

Reliability and validity were considered in this study. Cohen, Manion and Morrison (2011) indicated that reliability is concerned with the precision and accuracy of data interpretation. On the other hand, Brown and Dowling (2001) further showed that reliability involved the consistency, dependability or stability of the results or a coding process. According to these authors, it may mean that if a test was repeated or used many times by different researchers, the same results should be achieved. Reliability is when the same instrument is used at different times but always gives similar results, implying that the instrument was repeatable and consistent.

An instrument is valid if it measures what it supposed to measure (Creswell, 2010). This means that validity concerns the accuracy of the questions asked, the data collected, and the explanations offered. It further means that the instruments used to collect data, in this case, the observation schedule, should be carefully designed by considering the scope of the research study. The questions designed and asked on the observation schedule enabled the researcher to get valid data from their responses.

The observations were done over a slightly longer period to come out with convincing, reliable, and valid data. The follow-up interview questions were well structured using simple words for meaning. This was done to avoid ambiguous interpretations. The probing questions or follow up questions which emanated from the observation process were double checked to maximise their quality for purposes of reliability and validity.

3.9. ETHICAL CONSIDERATION

The research study considered ethical issues. A formal request to do research at the identified schools was prepared and forwarded to the principals of those schools. Over and above this request, another formal request was directed to the identified teachers to get their consent for them to participate in the research study. All participants were well informed that the whole exercise was intended to gather information for the researcher to advance her studies at one of the country's recognised universities. The information was given willingly as the results of the study might be used to benefit the participants and ultimately improve performance in the teaching and learning of mathematics in their respective schools. Appendices C-G shows accent from learners and their parents as well as from the department

3.10. CONFIDENTIALITY

The participating schools were labelled by means of the letters of the alphabet from A to C to ensure confidentiality during the process of observation and interview. All the schools' real identities relating to the alphabet letters remain confidential, only known to the researcher. Information provided by the participants, particularly personal information, was protected and not made available to anyone other than the researcher to ensure confidentiality of the participants' personal information. . All participants were assured of confidentiality in writing. The participants were assigned pseudonyms to protect their identities and to ensure confidentiality, e. g. names like Eddy, Rose and William were be used. The researcher reassured the participants that their real names would be kept anonymous, and all data gathered would be kept confidential. The researcher introduced herself before the start of the research to gain the trust of the participants.

3.11. PILOT STUDY

A pilot study is regarded as a foretaste of the actual study. Polit (2001) alluded that a pilot study refers to the baseline study in preparation for the major study. Piloting in this study was not an easy task for the researcher. It exposed many unforeseen weaknesses in the design study. The piloting was very helpful because good research demands thorough planning and preparation for the actual research study.

A pilot study was conducted to prepare for a full-scale study by observing three teachers from neighbouring schools in practice, teaching one lesson. Through piloting, the researcher realised that observing one lesson per teacher was not enough to conclude whether the teachers' mathematical knowledge for teaching was enough or inadequate.

The researcher found that the instrument was not clear about the categories and indicators that teachers were expected to meet during their lesson presentation. I found that I was not clear about lesson transcripts or how to identify the episodes. The researcher did not use the analytical framework that underpins the main study for the pilot project. The piloting process contributed to the improvement of the researcher's analytical framework in terms of developing categories as well as indicators that teachers should meet when executing their tasks.

It was through piloting that I was aware of how to design an observation schedule as a tool to measure how mathematics teachers taught fractions in Grade 6. The researcher used six categories, as a conceptual framework, shortened by Kazima et al. (2008) from the original eight categories proposed by Ball et al. (2004), namely defining, explaining, questioning representing, working with learners' ideas, and restructuring tasks.

The piloting process helped me in ensuring that the observation schedule was useful in the sense that the final categories identified were certainly observable in every lesson, and there was no overlapping between the categories.

Piloting assisted the researcher significantly in reflecting on what she was observing in the class. The reflection helped me think about which questions to ask as follow-up questions that augmented the data collection strategy and helped design the interview guide.

Piloting provided important information on the efficiency and effectiveness of the data collection methods and analysis.

The researcher learned a great deal from the pilot study. It was an eye-opener in many ways.

3.12. LIMITATIONS OF THE STUDY

The research was done at a master's level, which is a higher level than the honours level. The expectation at this master's level is far bigger as a result. The researcher's inexperience in conducting research was a challenge in itself which could have brought some limitations on how to execute the task effectively and appropriately.

The study's intended sample was very small and may not be generalisable to all primary schools in the country. One of the factors was that the participants for this study were not from my workplace but from our neighbouring schools. There could issues of biasness in some processes of the study.

The other factor may be the time for collecting data because the content area of fractions is allocated five hours which is ten periods. According to the pacesetter, schools should be at a specific pace, which means the researcher had to observe two schools per day until the three lessons per teacher were completed. The reason was the time constraints.

The other factor is the era that we find ourselves in, the era of the Covid-19 pandemic. Learners are not supposed to be close to each other, they should abide and adhere to the national regulations of 1.5 m distancing. Group work, as well as learner-talk had to take place under the conditions as shown in Figure 2.28 above.

4. DATA COLLECTION AND ANALYSIS

4.1. INTRODUCTION

This chapter aims to present the data collected and the analysis of the data from the three neighbouring Primary Schools in Capricorn South District. The data was collected through observations and interviews to answer the research questions. There are now three research questions because the researcher **merged** the posing and responding of questions on working with learners' mathematical ideas, because of overlapping. The research questions are as follows:

- How do teachers unpack/introduce, define, explain, and represent the concept of fractions to Grade 6 learners?
- How do teachers work with learners' mathematical ideas?
- How do teachers restructure their mathematical tasks?

Further, the researcher designed the observation schedule (APPENDIX B) and the interview guide (APPENDIX C).

4.2. REPORT ABOUT THE SCHOOL AND THE TEACHERS

The researcher realised that to explore teachers' mathematical knowledge on teaching; it is important to acknowledge the teaching experience that each participant have, their qualifications, gender, and their years of teaching mathematics in Grade 6, as indicated in Table 4.1.

Table 4.2. identifies the topic taught and the number of learners in each school. There were 20 learners in class because the schools had to abide and adhere to the regulations of Covid-19 valid at the time of the study. The acronyms used for schools as well as the teachers' pseudonyms are as follows:

- Eddy in SA (SA stands for School A)

- Rose in SB (SB stands for School B)
- William in SC (SC stands for School C)
- LS stands for all learners.
- L stands for "a particular learner in class."

Table 4.1. shows the years of experience in teaching, the number of years of teaching mathematics in Grade 6, the qualifications, and the gender of each participant.

Table 3.1: Teaching Information

Participants	**Years of experience in teaching**	**No. of years teaching mathematics in Grade 6 Class**	**Qualifications**	**Gender**
Eddy	15years	10	Primary teachers Diploma/Degree	Male
Rose	25 years	16	Primary teachers Diploma/Degree	Female
William	20 years	12	Degree	Male

Eddy is attached to school A (SA). He has 15 years' experience in teaching, and he has taught mathematics in a Grade 6 class for ten years. Rose is attached to school B (SB). She has 25 years' experience in teaching and has taught Grade 6 Mathematics for 16 years. William has taught Grade 6 mathematics for 12 years with 20 years' experience in teaching. William was attached to school C (SC).

Table 4.2. indicates the topics that were taught during the observation process/phase.

Table 3.2: Mathematics topics observed and number of learners.

Participant	Topics observed	Number of learners in the class
Eddy	➢ Comparing and ordering of fractions ➢ Equivalent fractions ➢ Changing common fractions to decimal fractions	20
Rose	➢ Fractional Concept ➢ Fractional Notation ➢ Proper and improper fraction	20
William	➢ Converting mixed fractions to improper fractions ➢ Comparing fractions ➢ Addition of fractions with unlike denominators	20

The Covid-19 pandemic has disrupted the 2020 academic year and as a result, schools were advised to select the model that best suited them to obey the 1.5 m distancing. Each school chose a model based on the enrolment of the school. Among the models, there were daily rotational model, weekly rotational model, bi -weekly rotational model and so on.

School A selected the weekly rotational model. The grades were alternating weekly. The Grade 4 and 6 learners were paired in the same week and Grade 5 and 7 in the following week.

The researcher was fortunate to observe Eddy of school A during the Grade 4 and 6's rotational week. The first lesson was observed on Monday at 8:00-9:00, and the topic of the day was comparing and ordering of fractions.

The 2nd lesson was on Wednesday at 8:00-9:00, and the topic was equivalent fractions. The 3rd lesson was observed on Friday at 9:00-10:00, and the topic of the day was changing common fractions to decimal fractions. Each lesson ran for 30 minutes, and all the lesson durations were 1 hour, which means the mathematics period was a double lesson. School B of teacher Rose and school C of William chose the daily rotational model, which means their Grade 4 and 6 learners were paired on the same day and the Grade 5s and Grade 7s on the following day. The researcher observed the lessons during the Grade 4 and 6's rotational days. I was fortunate because the Grade 6 learners of school B and C came to school on different dates.

Rose was observed on Monday at 8:00-9:00 teaching about the fractional concept, and on Wednesday at 8:00-9:00, teaching fractional notation, and on Friday at 9:00-10:00 when she was teaching proper and improper fractions. This means that on Tuesday and Thursday, I was at school C, where William is attached. Monday of the following week was the last day of the observation of William. His 1st lesson of converting mixed fractions to improper fractions was observed at 8:00-9:00, the 2nd lesson of comparing fractions was observed from 8:00-9:00, and the 3rd lesson of addition of fractions of unlike denominators was observed from 9:00-10:00.

4.3. OBSERVATION

I was privileged during my observation because the schools researched or observed chose different scheduling models to accommodate the COVID-19 situation. School A, represented by teacher Eddy, chose the weekly rotational model of teaching School B represented by teacher Rose and school C represented by William chose the daily rotational model. I was fortunate because their grade 6 learners came to school on different dates.

The Covid-19 pandemic has disrupted the 2020 academic year and as a result, schools were advised to teach only the fundamental topics. I was also fortunate because common fractions are one of the basic topics to be taught in Grade 6 in Term 3 during the Covid-19 pandemic according to an amended school calendar.

The seating arrangement in the three schools observed was the same. Desks were aligned in rows with seating space for one learner per desk, due to the COVID-19 regulations requiring 1.5 m distancing. Each class was occupied by 20 learners.

Observations were carried out to explore how mathematics teachers teach fractions in Grade 6 and how they use their mathematical knowledge for teaching, in developing the fractional concept knowledge. The researcher observed three teachers, one teacher per school on different dates. Eddy was observed within the first week, and Rose and William were observed on the second subsequent week. The observation of Rose and William took almost two weeks, the last day of observation was Monday of the third week. Three double lessons per teacher were observed, captured and recorded.

During the observation, the researcher focused mainly on the following categories/aspects as an analysing conceptual framework, namely, the teacher unpacking fractions to Grade 6 learners. As they unpack the fraction concept, the researcher focused on how the three teachers applied their mathematical knowledge for teaching the fraction concept. When unpacking the fraction concept, the focus was also on how learners engaged in the lesson activities, how they used prior knowledge, how they assimilated information into their existing experiences and how they constructed new ideas. The researcher further focused on whether teachers defined, explained, represented, and introduced the fraction concept in a manner that learners understood. The above categories/aspects were observed as a package. Each category had indicators or expectations that required teachers to engage in meeting the requirements. The details of the categories are as follows:

4.3.1. Category 1: How do teachers unpack/introduce fractions to Grade 6 learners?

Indicators:

- Teachers introducing a concept.
- Teachers defining the concept.
- Teachers representing of the concept.
- Teachers explaining the concept.

As indicated in Chapter 2, in this category, the teacher is expected to unpack and introduce the concept of fractions. Unpacking the concept means to unwrap, decompress, unload whatever contents the concept might have. This implies that the teacher should introduce, define, explain, and represent the concept of fractions in a way that learners understand. When introducing a concept, the teacher is expected to link the previous knowledge with the existing knowledge, moving learners from the known to the unknown. *Defining, explaining,* and *representing* are aspects of the mathematics element that the teacher should engage in when teaching fractions. Defining means–a statement of the exact meaning of a word or the statement that explains a meaning of a word; explaining appears to be similar to defining. However, the teacher could explain something using practical examples to understand the concept; representation is to check if the teacher used different ways of representing the concept of fraction. The teacher could use numerical representation, diagrammatical representation, narrative representation or real-word context representations. The teacher should have a deeper understanding of fractions. Putting it differently, the concept should be shown to learners using fractional models. Learners seem to learn much better if models are used, representing a concept using diagrams such as area, length and other models such as a fraction wall.

4.3.2. Category 2: How do teachers work with learners' mathematical ideas when teaching fractions?

Indicators:

- Do teachers encourage learner talk/learner-centredness?
- Selecting tasks of high cognitive demand
- Genuine engagement with learners
- Teacher listening–Evaluative or interpretive or generative.
- Learners asking questions.
- Teacher listening to learners questioning.
- Leading /guiding questions
- Follow up questions.
- Probing questions
- Specific questions
- Open-ended questions

As indicated in Chapter 2, this category encourages, for example, working with learners' ideas through teamwork. Considering this, teachers are expected to encourage *learner talk* (engage learners in a discussion) and encourage genuine engagement or involvement in the mathematical task brought forward. They should provide learners with selected high cognitive thinking tasks, tasks that challenge learners' thoughts. Dealing with learners' mathematical ideas is one of the central points in mathematical problem-solving tasks. Allowing learners to brainstorm in a discussion, is therefore highly recommended. Teachers are also expected to reform teacher-centred approaches as it leads them to lesson domination. Learner talk increases learners' mathematical ideas, enables learners to share ideas with each other and help learners make sense of mathematics. Teachers are expected to encourage learners to ask questions for clarity, and teachers are therefore expected to listen to their learners' responses. Teachers are encouraged to use mostly interpretive listening since it enables the teacher to listen carefully to learners making sense of their mathematics unlike evaluative listening, because evaluative listening seek the correct answer that the teacher already has in mind. Generative listening is also encouraged because it transforms one's mathematical understanding and can guide the lesson's direction.

Dealing with learners' mathematical ideas is one of the central points in mathematical problem-solving tasks. Allowing learners to brainstorm in a discussion, is therefore highly recommended. The teacher is also expected to reform from teacher centred as it leads to lesson domination. Learner talk increases learners' mathematical ideas, enables them to share ideas, and helps them make sense of mathematics. The teacher is expected to encourage learners to ask questions for clarity, and the teacher is therefore expected to listen to their learners' responses.

In this category, the teacher is expected to create a situation where learners feel free, recognised, and welcomed. The teacher is expected to respond to learners' questions by using reinforcement to respond correctly. The teacher is also expected to listen to learners' questions using different types of listening, such as interpretive listening. The teacher should be certain that learners' questions are clear and understood by other learners. The responding stage is the stage where the teacher provides verbal or non-verbal reactions. The teacher may use reinforcement to learners giving a correct response; the correctness of learner's response can be praised by saying words like "very good".

Moreover, in this category, the teacher is expected to pose different types of questions such as probing questions, open-ended questions, and follow up questions to cater to learners' cognitive mathematical thinking and reasoning. Open-ended mathematical questions stimulate and strengthen learners' mathematical thinking. Probing questions promote learning and push up learners to think more deeply. Probing questions promote learning and push up learners to think more deeply about mathematics.

4.3.3. Category 3: How do teachers restructure mathematical tasks to be understood by learners?

Indicators:

- Scaling a mathematical problem task up or down
- Shifting of appropriate mathematical outcome

As highlighted in Chapter 2, teachers are expected to reduce the complex or complicated fractional tasks to their simplest form and increase the most straightforward fractional task to a complex form to cater to all the cognitive levels. In other words, if the task is too difficult for learners the teacher should scale it down or make it simpler, and if the task is not challenging enough, then the teacher should scale it up for learners to understand. The teacher may use the additional restructuring of the task called shifting of the appropriate mathematical outcome.

4.4. BACKGROUND TO THE LESSON OBSERVED

I have observed three teachers as they went about their work of teaching fractions to Grade 6 learners. I have attended three double lessons per teacher per school. In each school, I have observed three double lessons. One lesson takes 30 minutes, which means six lessons from each teacher were observed. This means eighteen lessons were observed in the three selected neighbouring schools. Lessons were transcribed and chunked into evaluative events/episodes.

4.4.1 Eddy's lesson observations

Category 1: How do teachers introduce, define, explain, and represent the concept of fractions to Grade 6 learners?

Comparing and Ordering of Fractions

Below is the extract that displays how the teacher (Eddy) defined, explained, and represented the concept of fraction in his teaching. In this extract Eddy is the teacher, LS represents "learners" and L represents "a learner in class", for example, L1 represents Learner 1, L2 represents Learner 2 etc.

00:08–00:09

1.	**Eddy:**	Good morning class
2.	**LS**:	Good morning sir
3.	**Eddy**:	Sit down
4.	**LS**:	(Sit down and listen to the teacher)
5.	**Eddy**:	Today we are going to learn about comparing and ordering of fractions [turns to the board and writes] ($\frac{5}{7}$; $\frac{2}{7}$; $\frac{6}{7}$; $\frac{4}{7}$; $\frac{1}{7}$), look at these fractions. What do you realise?

00:10

6. **L1**: They have the same denominators

7. **Eddy**: Yes, they have the same denominators. Comparing fractions like this once, is easy because if they have the same denominators, the fraction with the bigger numerator is the biggest. So, who can come and arrange them for us?

8. **L2**: [Stands and go to the chalkboard, writes] $\frac{6}{7}; \frac{5}{7}; \frac{4}{7}; \frac{2}{7}; \frac{1}{7}$

9. **Eddy**: Good, this is how we order and compare fractions. The same applies if they have the same numerators; the fraction with the bigger denominator is the smallest. [writes $\frac{1}{3}; \frac{1}{5}; \frac{1}{7}; \frac{1}{4}; \frac{1}{2}$ on the board]. Someone, come and arrange these fractions from the smallest to the biggest.

10. **L3**: [stands and go to the board and writes] $\frac{1}{7}; \frac{1}{5}; \frac{1}{4}; \frac{1}{3}; \frac{1}{2}$

00:20

11. **Eddy**: You are correct. Clap hands for him. Now because you understand, let us continue comparing fractions with different numerators and denominators. When comparing fractions with different numerators and denominators, we should make them to have the same denominators by looking for the LCM which means lowest common multiple for example [writes on the board] fractions that are multiple of the other $\frac{2}{3}$ and $\frac{1}{6}$ we should multiply $\frac{1}{6}$ by $\frac{2}{2}$ like $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ now because the denominator are the same, we can compare them and the answer is $\frac{2}{3} > \frac{1}{6}$. If they are not multiple of the other find the LCM. (Comparing and Ordering Fractions, Lesson 1)

It is evident in this lesson (line 7) that the teacher, Eddy, went on to explain a rule by saying, *yes, they have the same denominators. Comparing fractions like this once, is easy because if they have the same denominators, the fraction with the bigger numerator is the biggest. So, who can come and arrange them for us?* After the teacher had provided an explanation of comparing and ordering fraction of the same denominator, he then asked if there would be any learner who would come to provide an answer of his already given explanation of a “rule”. It appears that, while Eddy is explaining the rules of comparing fractions, Eddy does not show learners were this rule originates. It appears that the teacher is encouraging learners to master "rules, procedures" at the expense of developing the concept. This kind of teaching encourages memorisation rather than conceptual understanding.

In my understanding the teacher wanted learners to develop a concept of comparing and ordering fractions, the teacher should not have started by foregrounding a "rule", *if they have the same denominators, the fraction with the bigger numerator is the biggest.* The teacher could have used teaching strategies which would allow learners to discover a rule by themselves. For example, the teacher, Eddy, could have used number line representation, or diagram representation to develop the concept of comparing and ordering fractions. The teacher's explanation of a rule should have come as a reinforcement of what learners had already discovered through their investigation. It was no surprise that L2 (in line 8) provided the correct answer $\frac{6}{7}; \frac{5}{7}; \frac{4}{7}; \frac{2}{7}; \frac{1}{7}$.

Similarly, as evidenced in line 9 to 11, the teacher used a similar strategy of foregrounding procedures, routine and rules rather than developing the concept so that learners could discover these rules by themselves without being told when comparing a fraction of the same denominator to compare a fraction of the same numerator. Again, as in line 11, Eddy used procedures in his teaching to compare fractions of different denominators. He said:

> *Now because you understand, let us continue comparing fractions with different numerators and denominators". When comparing fractions with different numerators and denominators, we should make them to have the same denominators by looking for the LCM which means lowest common multiple for example (writes on the board)* $fractions\ that\ are\ multiple\ of\ the\ other$ $\frac{2}{3}$ *and* $\frac{1}{6}$ *we should multiply* $\frac{1}{6}$ *by* $\frac{2}{2}$ *like* $\frac{2}{3} \times \frac{2}{2}$ $=\frac{4}{6}$ *now because the denominators are the same, we can compare them and the answer is* $\frac{2}{3} > \frac{1}{6}$*.If they are not multiple of the other find the LCM. (Eddy)*

When Eddy said "*now because you understand"* this was Eddy's assumption that learners do understand based on the rules and procedures he provided to them and they mimic those rules, and some of them (L1 and L2) obtained the correct answer. Eddy assumed that all learners were at the same level of understanding as L1 and L2, without him checking on all learners understanding. In fact, we are not even certain that L1 and 2 understood the concept of comparing and ordering fractions based on algorithm and routine procedures. Based on his assumption, Eddy moved to compare fractions of different numerators and denominators. Even at this stage, he taught procedures, saying:

> When comparing fractions with different numerators and denominators, we should make them to have the same denominators by looking for the LCM which means lowest common multiple for example [writes on the board] fractions that are multiple of the other $\frac{2}{3}$ and $\frac{1}{6}$ we should multiply $\frac{1}{6}$ by $\frac{2}{2}$, at the expense of developing the concept. (Eddie)

As it appears in the lesson, the teacher used only one form of representation in his lesson, which is number representation. The teacher dominated the lesson by providing rules and procedures without letting learners discover them by themselves. The teacher could have approached this lesson differently if he wanted learners to develop the concept. The teacher could have used a number line, drawing, and fraction chart to compare and order fractions. In this lesson, there was no evidence of different representation to teach comparing and ordering of fractions. When asked to reflect on his lesson during the interview, Eddy seemed to suggest that teaching by emphasising procedures and rules without considering learners' understanding was the norm for his teaching. This was supported by an interview extract below:

Researcher: You mentioned that you have ten years' experience in teaching mathematics in Grade 6, and you have taught fractions many times. Do you always teach fractions the way you did this year in the lessons, which I have observed? Does your teaching develop learners to master fractional concepts?

Eddy: This is the way I normally teach these learners because I just assume that the lower grades teachers, already introduced the fractional concept, therefore I am teaching them rules on how to convert fractions and so on.

From the extract above, it appears that Eddy's teaching is always dominated by teaching rules without much more understanding of the concepts. He said, *this is the way I normally teach these learners.* Eddy's justification of teaching rules without understanding the concept is based on the assumptions that teachers at lower grades had already introduced the concepts with an understanding. Something to note again is that Eddy has been teaching in this way for the past ten years.

Equivalent Fraction

In his Lesson 2 of equivalent fraction, Eddy thought he used a chart as a representational model for learners to discover equivalent fractions, his lesson was also providing explanations without letting learners discover these by themselves. He said:

> We can also find the equivalent fractions by multiplying or dividing the numerator and the denominator of a fraction by the same whole number for example 1/(2) × 2/2 = 2/4 ; 1/(2) × 4/(4) = 4/8. (Eddy)

This is evidenced by the extract of Lesson 2 below (Line 9 and 10)

9. **Eddy**: Correct. [put the fraction chart on the board] This is a fraction chart; it will help us to find fractions that are equal in value. (learners recognised equivalent fractions using the fraction chart)

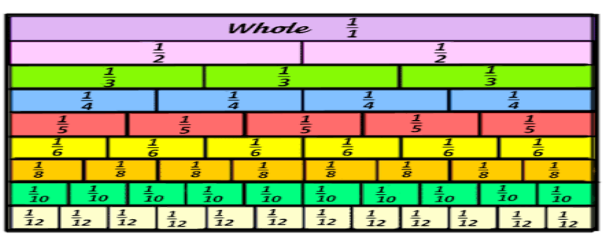


Figure 3.1: Fraction Chart

00:20

10. **Eddy**: we can also find the equivalent fractions by multiplying or dividing the numerator and the denominator of a fraction by the same whole number for example $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$; $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$ etc (Writes on the board) $\mathbf{\frac{2}{9}}$ **,** $\mathbf{\frac{1}{3}}$ **;** $\mathbf{\frac{2}{5}}$ **;** $\mathbf{\frac{4}{7}}$ find three equivalents fractions of each of these fractions. Discuss and I give you 10minutes to work them out. (Lesson 2, Equivalent Fraction)

As evidenced in line 9 to 10 of his second lesson on equivalent fraction, it appears that Eddy did not engage learners in using the fractional chart to discover equivalent fraction**.** He just put the fractional chart on the board and said this is a fraction chart; it will help us find fractions equal in value. He immediately moved to a rule of getting an equivalent fraction through "by multiplying or dividing the numerator and the denominator of a fraction".

Converting common fraction to decimal fraction

Similarly, in his Lesson 3, converting common fraction to decimal fraction, he asked learners the difference between a common fraction and decimal fractions. After realising that learners kept quiet (line 8), he then provided an answer as indicated in the extract below (Line 9 of lesson 3) It appears that Eddy knows that he can teach fraction in such a way that learners develop the conceptual understanding using representations such as models. However, he just chose not to do so based on the reasons he furnished in the interview extract below:

Researcher: From your knowledge and point of view, what do you think are the main things a teacher needs to know in order to develop conceptual understanding of fractions for learners to master?

Eddy: From my point of view, I think I should have concentrated on representation, but according to my knowledge from the lower grades, thus where the fraction concept is introduced using models. In grade 4 and 5 also the fractional concept should be emphasised. In my teaching, I just take it for granted that they have mastered the concept and therefore the how part of doing things should be taken care of.

00:18

6. **Eddy**: Our lesson today is about converting or changing common fractions to decimal fractions

7. **Eddy:** What is the difference between common fraction and decimal fractions?

8. **LS**: [kept quiet]

9. **Eddy**: (after a while because of learners' silence) A common fraction is a fraction where there is a top number called the numerator and the bottom number called the denominator whereas the decimal fractions have the decimal comma. Decimals are tenths, hundredths, thousandths etc. For example, 0,6 is 6 tenths and $\frac{6}{10}$ in its simplest form is $\frac{3}{5}$.do you understand?

10. **LS:** Yes sir.

11. **Eddy**: Converting a decimal fraction to a common fraction is easy because decimals are in tenths, hundredths or thousandths, for example 0,3 is 3tenths and is written as $\frac{3}{10}$; 0,08 is 8hundreths and is written as$\frac{8}{100} = \frac{2}{25}$ in its simplest form.

Let us try to convert the following to common fractions and simplify where possible. (Writes on the board)

e.g. o,4 =$\frac{4}{10}$ =$\frac{2}{5}$

a. 0,55
b. 0,02
c. 0,75
d. 0,5

(Lesson 3, converting common fractions to decimal fractions

After he had provided an explanation of differentiating a common fraction to a decimal fraction (line 9), he started to explain how to convert a common fraction to a decimal fraction without even involving learners in using any other strategies they know. From there, he moved on with the lesson by providing an activity for learners to do. This suggests that Eddy teaches in this way. This is also confirmed by an interview extract below.

Researcher: Your teaching was full of explaining fractional rules in all your lessons observed, is it your way of teaching? do these learners know where the rules come from? Can you briefly elaborate how these learners develop fractional concepts?

Eddy: As I have mentioned earlier, when these learners pass grade 5, they are expected to have mastered the fractional concept, in our case we are supposed to teach them the mathematical rules on how to find answers or to solve fractional problems.

Eddy gave a justification that learners should have mastered the fractional concepts in their lower grades. He responded by saying that he teaches the mathematical rules because, according to him, learners have already mastered the fractional concepts. He was just assuming that learners mastered fractional concepts. He should have checked if learners knew the concepts instead of assuming that they did. He further said that it was his way of teaching and this is an indication that he is not ready to transform from the traditional way of teaching. The way the teacher responded to the questions asked is not convincing enough that the teacher is ready to let his learners develop mathematical concepts. Instead of finding what learners know about the fractional concepts, he just assumed that they knew.

Category 2: How do teachers work with learners' mathematical ideas when teaching fractions?

The extract below shows how Eddy worked with learners' mathematical ideas when teaching fractions.

Comparing and Ordering of Fractions

00:09

5.	**Eddy:**	Today we are going to learn about comparing and ordering of fractions (turns to the board and writes ($\frac{5}{7}$; $\frac{2}{7}$; $\frac{6}{7}$; $\frac{4}{7}$; $\frac{1}{7}$), look at these fractions. What do you realise?
6.	**L1**	They have the same denominators
7.	**Eddy**:	Yes, they have the same denominators. Comparing fractions like this once, is easy because if they have the same denominators, the fraction with the bigger numerator is the biggest. So, who can come and arrange them for us?
8.	**L2:**	[Stands and go to the chalkboard) writes $\frac{6}{7}$; $\frac{5}{7}$; $\frac{4}{7}$; $\frac{2}{7}$; $\frac{1}{7}$]

00:27

9. **Eddy**: Good, this is how we order and compare fractions. The same applies if they have the same numerators; the fraction with the bigger denominator is the smallest. $[writes \frac{1}{3}\ ;\ \frac{1}{5};\ \frac{1}{7}\ ;\frac{1}{4}\ ;\frac{1}{2}\ on\ the\ board]$Someone, come and arrange these fractions from the smallest to the biggest.

10. **L3**: [stands and go to the board and writes $\frac{1}{7}$; $\frac{1}{5}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{1}{2}$]

11. **Eddy**: You are correct. Clap hands for him. Now because you understand, let us continue comparing fractions with different numerators and denominators. When comparing fractions with different numerators and denominators, we should make them to have the same denominators by looking for the LCM which means lowest common multiple for example [writes on the board fractions that are multiple of the other $\frac{2}{3}$ and $\frac{1}{6}$ we should multiply $\frac{1}{6}$ by $\frac{2}{2}$ like $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ now because the denominator are the same, we can compare them and the answer is $\frac{2}{3} > \frac{1}{6}$.If they are not multiple of the other find the LCM].

12. **Eddy**: [writes on the board] $\frac{1}{10}$; $\frac{3}{5}$; $\frac{2}{15}$; $\frac{2}{3}$; $\frac{4}{9}$; $\frac{1}{27}$ Discuss these activities and come up with answers. Arrange them from the smallest to the biggest, I give you five minutes to work them out.

13. **Eddy**: [after five minutes] Are you done, learners?

14. **LS**: Not yet sir, we are still discussing

15. **Eddy:** Get finished.

LS: [After a while] We are now done, sir

16. **Eddy**: Ok, let one of you give answers

17. **L4**: From our discussion here are the answers. [The LCM =30

$a)\ \frac{1}{10} \times \frac{3}{3} = \mathbf{\frac{3}{30}}\ ;\ \frac{3}{5} \times \frac{6}{6} = \mathbf{\frac{18}{30}}\ ;\ \frac{2}{15} \times \frac{2}{2} = \mathbf{\frac{4}{30}}$

Answer: $\mathbf{\frac{3}{30}\ ;\ \frac{4}{30}\ ;\ \frac{18}{30}}$

b) $\frac{2}{3} \times \frac{9}{9} = \mathbf{\frac{18}{27}}\ ;\ \frac{4}{9} \times \frac{3}{3} = \mathbf{\frac{12}{27}}\ ;\ \frac{1}{27} \times \frac{1}{1} = \mathbf{\frac{1}{27}}$

Answer: $\frac{1}{27}\ ;\ \frac{12}{27}\ ;\ \frac{18}{27}$]

18. **Eddy:** Go and practice other activities from your workbooks.
See you tomorrow.
(**Comparing and Ordering of fractions, lesson 1)**

The lesson was about comparing and ordering fractions from the smallest to the largest or from the largest to the smallest. In Turn 5, the teacher says, *today we are going to learn about comparing and ordering of fractions (turns to the board and writes* ($\frac{5}{7}$; $\frac{2}{7}$; $\frac{6}{7}$; $\frac{4}{7}$; $\frac{1}{7}$)*, look at these fractions. What do you realise?* It is good that the teacher had asked learners a question, *what do you realise*, in the given fraction. The teacher wanted to involve learners in this lesson when asking them this question. The teacher wished for learners to observe a pattern in the given fractions. In line 6, L1 responded by saying, *they have the same denominators*.

Though the teacher had asked the question, he did not probe more on L1 responses. The teacher could have probed more to say, if they have the same denominator, what it means in terms of their sizes. The lack of involvement of learners through questions was a dominating factor in this Eddy's lesson. Eddy said that sometimes if he involved learners, they took time to do what was expected and did not participate, or only a few did. If he allows them to discuss, he ends up not covering the pacesetter. This is proved by what he said in the interview.

Researcher: You were dominating your lessons with little interaction among learners is it how teaching should be? Elaborate on that.

Eddy: This is how I am teaching. Sometimes if I involve these learners, they take time to do what is expected and they do not take part, only few did. If I allow them to discuss, I end up not covering my pacesetter.

From the extract above, It appears that Eddy committed himself to finishing the syllabus at the expense of learners' understanding. If he allows them to discuss, he indicated that "[I am] ***up not covering my pacesetter".*** However, in his teaching, he provided learners with work to do in a group.

It appears to be common practice, as evidenced in line 12, in this lesson for Eddy to provide learners with work to do and discuss in their group. He wrote on the board, $\frac{1}{10}$; $\frac{3}{5}$; $\frac{2}{15}$; $\frac{2}{3}$; $\frac{4}{9}$; $\frac{1}{27}$ and said, *"discuss these activities and come up with answers. Arrange them from the smallest to the biggest, I give you five minutes to work them out"*. He asked learners after five minutes if they had finished with their work. The learners responded as in line 14 that they "*are still discussing*". It appears that these learners were given work and left alone to discuss without the teacher monitoring and checking what they were doing. Because if he was moving around checking on what they were doing, he would not have asked them if they were done, but he would have seen when he was moving around. Even the responses that one of the learners, L4, provided mimicked the rule he encouraged in his explanation in class. There was no other strategy that learners came up with different from the algorithmic one he gave them.

The fact that learners were given work to do appears to involve learners in the lesson, encouraging learner talk and learner centredness. However, after giving learners work to do, there was no indication that the teacher would move around, see what they were doing, ask them questions, listen to their questioning, and provide appropriate feedback. In the whole lesson observed, there was no evidence of where learners had asked questions. The teacher did not even bother to entertain responses that L4 had provided in line 17 or invite other learners to respond to what L4 wrote. He did not even ask L4 to explain his thinking behind his responses. Instead, he instructed his learners to "*go and practice other activities from your workbooks*". This is an indication that the teacher could not work with learners' ideas in his lesson. This kind of teaching approach was also evidenced in Lesson 2 and 3. In Lesson 2, for example, he asked learners how they knew if a fraction was a common fraction. One of the learners, L1, said, "*we know if there is a top number and a bottom number*", as indicated in the extract below.

Equivalent Fraction

5.	**Eddy:**	Learners, how do we know that a fraction is common fraction
6.	**L1:**	We know if there is a top number and a bottom number.
7.	**Eddy:**	Yes, you are correct, and we know the top number is the numerator and the bottom number is the denominator. Today we are to learn about equivalent fractions. Who can tell what equivalent fractions are?
00:50		
8.	**L2:**	Equivalent fractions are fractions with the same value, their portions are equal.

It appears that Eddy acknowledges the fact that leaving learners to work in groups without monitoring them is not a good thing. This is supported by an interview extract below.

Researcher:	You provided learners with the work to do and not monitor what they were doing ,is it the way it should be? Elaborate on that.
Eddy:	It is not the right thing to do or let me say it was not proper to leave them the way I did. I should have checked if they are doing the right thing. What I have done is not good for learners. (Lesson 2, Equivalent Fraction)

L1's response, "*we know if there is a top number and a bottom number", could not be followed up. Instead of engaging L1 and asking other learners to comment on L1's response, he moved on with his lesson and said: "*Yes, you are correct, and we know the top number is the numerator, and the bottom number is the denominator. Today we are to learn about equivalent fractions. Who can tell what equivalent fractions are*?"

Again, in Line 10 to 16 as indicated in the extract below, after Eddy had explained a rule of getting an equivalent fraction, he wrote the following fraction on the board $\frac{2}{9}$, $\frac{1}{3}$; $\frac{2}{5}$; $\frac{4}{7}$ and asked learners to find three equivalents fractions for each of these fractions.

10.	**Eddy**:	We can also find the equivalent fractions by multiplying or dividing the numerator and the denominator of a fraction by the same whole number for example $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$; $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$ etc (Writes on the board) $\frac{2}{9}$, $\frac{1}{3}$; $\frac{2}{5}$; $\frac{4}{7}$ find three equivalents fractions of each of these fractions. Discuss and I give you 10minutes to work them out.
11.	**L1:**	[One of the learners go to the board and asks his colleagues to give answers.] Which fraction is equivalent to $\frac{2}{9}$ **?**
12.	**LS**:	[raise their hands]. [The learner points out to the other learner and asks her to give the answer.]
13	**L2**	$\frac{2}{9} \times \frac{2}{2} = \frac{4}{18}$
14.	**L3**:	[the other learner] $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$
15.	**L4:**	$\frac{2}{5} \times \frac{2}{2} = \frac{4}{10}$
16.	**L5:**	$\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$

(Lesson 2, Equivalent Fraction)

Again, the teacher did not bother to engage or ask other learners to comment on L13, L14, L15 and L16's responses. Again, these learners' responses were a replica and an endorsement of the rule he preached to them in class.

Similarly, he did the same when asking learners to simplify as indicated in the extract below:

17. **Eddy** Well done learners, you have done so well. So, tell me, what is to simplify?

18. **L6**: To simplify is to reduce.

19. **.Eddy**: you are correct to simplify is to reduce something to its simplest form. When simplifying fractions, we get their equivalent forms. Look at the answers if we simplify them, we realise that we get the initial one by dividing both the numerator and the denominator by the whole number ,for example $\frac{12}{21} \div \frac{3}{3}$ $=\frac{4}{7}$

20. **Eddy:** [writes on the board]

Take out your books and work out the following:

21. **Eddy:** [Find the equivalent fractions of the following]:

a. .$\frac{1}{5}$

b. $\frac{4}{9}$

c. $\frac{4}{5}$

22. **Eddy:** [Simplify the following:]

a. $\frac{10}{20}$

b. $\frac{40}{50}$

c. $\frac{25}{75}$

(Lesson 2, Equivalent Fraction)

Eddy is asking questions in his lessons, as in lesson 2 line 5, 7 and, 17, his questioning style does not elicit learners' mathematical thinking nor encourage them to talk. He does not seem to probe further about learners' responses. There were no open-ended questions and follow-up questioning in his lessons. The teacher is expected to pose different types of questions to cater to learners' cognitive mathematical thinking and reasoning. Open-ended mathematical questions stimulate and strengthen learners' mathematical thinking. Probing questions promote learning and raise learners to think more deeply. Probing questions promote learning and raise up learners to think more deeply about mathematics. The lack of asking learners open-ended questions, follow up questions, and probing questions was also evidenced in his Lesson 3 on converting common fractions to decimal fractions as illustrated by the extract below.

Converting fractions to Decimal Fractions

00:13

7. **Eddy:** What is the difference between common fraction and decimal fractions?

8. **LS**: [kept quiet]

9. **Eddy:** [after a while because of learners' silence] A common fraction is a fraction where there is a top number called the numerator and the bottom number called the denominator whereas the decimal fractions have the decimal comma. Decimals are tenths, hundredths, thousandths etc. For example, 0,6 is 6 tenths and $\frac{6}{10}$ in its simplest form is $\frac{3}{5}$.do you understand?

Again, he gave learners work in the board without involving them.

00:49

11.: **Eddy:** Converting a decimal fraction to a common fraction is easy because decimals are in tenths, hundredths or thousandths, for example 0,3 is 3tenths and is written as $\frac{3}{10}$; 0,08 is 8 hundredths and is written as $\frac{8}{100} = \frac{2}{25}$ in its simplest form. Let us try to convert the following to common fractions and simplify where possible. [writes on the board]

e.g. $0,4 = \frac{4}{10} = \frac{2}{5}$

d. 0,55
e. 0,02
f. 0,75
g. 0,5

12. **LS:** [Write answers in their rough books]

13. **Eddy:** Who can give us answers?

14. **LS**: [raise their hands]

15. **Eddy:** [Called L1, L2 , L3 and L4 to come and write answers on the board]

16. **L1**: (stands and go to the board and writes answers)

$0,55 = \frac{55}{100} = \frac{11}{20}$

17. **L2**: (stands and go to the board and writes answers)

$0,02 = \frac{2}{100} = \frac{1}{50}$

18.: **L3**: (stands and go to the board and writes answers)

$0,75 = \frac{75}{100} = \frac{3}{4}$

19 **L4**: (stands and go to the board and writes answers)

$0,5 = \frac{5}{10} = \frac{1}{2}$

20 **Eddy:** Well done all of you.

Converting common fractions to decimals, make sure your denominator is either ten or hundred for example:

$\frac{1}{10} = 0,1$

$\frac{6}{25} \times \frac{4}{4} = \frac{24}{100} = 0,24$

21 **Eddy** Convert to decimals, L5 ,come and give us answers

22. **L5** [Gives answers]

a) $\frac{1}{10} = 0,1$

b) $\frac{20}{100} = 0,2$

c) $\frac{25}{100} = 0,25$

23. **Eddy:** Work out this once [writes on the board) and we will mark them tomorrow]

a) $\frac{1}{2} =$ ________

b) $\frac{9}{100} =$ ________

c) $\frac{3}{4} =$ ________

d) $\frac{4}{5} =$ ________

e) $\frac{3}{5} =$ ________

[Lesson ends]

(Lesson 3, Converting common fraction to decimal fraction)

Working with learners' ideas teamwork should be encouraged. Considering this, the teacher could not encourage learner talk (engage learners in a discussion) and encourage genuine engagement or involvement in the mathematical task brought forward. He did not provide learners with selected high cognitive thinking tasks, tasks that challenge their thoughts. Dealing with learners' mathematical ideas is one of the central points in mathematical problem-solving tasks. Allowing learners to brainstorm in a discussion is highly recommended but not reflected in Eddy's lesson. The teacher's lesson was teacher centred that is,. lesson domination. He could not encourage learners' mathematical ideas, enable them to share ideas, or help them make sense of mathematics. There was no evidence in the whole lesson where he encouraged learners to ask questions for clarity and wherein; he was expected to listen to his learners 'responses. There was no indication that the teacher listens interpretively to learners' responses. However, there was some evidence of him listening in an evaluative way because evaluative listening seeks the correct answer that the teacher already has in mind. In all three lessons, there were no instances where learners asked any questions of the teacher. This is supported by the following interview extract.

Researcher: In the three lessons observed your learners did not even ask a single question, is it a norm, is it the way they always do? What can be the cause?

Eddy: They did not ask questions even if you encourage them to do. I do not know the reason. I just think that maybe they are not confident enough to ask questions or they think their classmates will laugh at them if they ask questions.

Category 3: *How do teachers restructure mathematical tasks to be understood by* learners?

The teachers are expected to reduce the complex or complicated fractional tasks to their simplest form and increase the simplest fractional task to a complex form to cater to all the cognitive levels. In other words, if the task is too difficult for learners, the teacher should scale it down or make it simpler, and if the task is not challenging enough, then the teacher should scale it up for learners to understand. Or the teacher may use the additional restructuring of the task called shifting of the appropriate mathematical outcome.

In all three his lessons, there was no evidence of scaling up or down a mathematical problem task and shifting of appropriate mathematical outcome.

4.4.2 Rose's lesson observation

The second teacher observed, was Rose of school B, her observations were as follows:

Category 1: How do teachers define, explain and represent the concept of fractions to Grade 6 learners?

Unpacking the concept of fraction

Below is the extract that illustrates how the teacher (Rose) defined, explained, and represented the concept of fraction in her teaching.

00:23

5.	**Rose**:	[Give learners A4 paper sheet each] This is the A4 paper, I am going to tell you what to do and you should listen.
6.	**LS**:(	[Listening and paying attention to the teacher)**]**
7.	**Rose**:	Fold your A4 paper once and make sure the two parts are on top of each other. Are you following?
8.	**LS:**	Yes mam

00:26

9.	**Rose**:	Unfold your papers. What do you notice?
10.	**LS:**	**(**raise their hands)
11.	**Rose**:	L1 tell us, what can you about the A4 paper
12.	**L1**:	The paper has 2 parts now.
13.	**Rose**:	Do you agree with her learners?
14.	**LS:**	Yes mam
15.	**Rose**:	What else can you say?
16.	**L2**:	Two parts are equal. We had a whole of the A4 paper but now it is divided into 2.

17. **Rose**: What else can you say about the paper?

18. **L3**: We now have halves no longer a whole.

00:39

20. **Rose**: you are correct. Each part is half of the paper. Now let us fold it twice and unfold and see what happens. How many parts do you see now?

21. **L3:** I see four parts.

22. **Rose:** [Poses a question to L3 who has just answered] What can you say about those parts?

23. **L3:** They are equal.

24. **Rose:** [Orders learners] Fold the paper three times now.

25. **LS:** [do what was ordered]

00:45

26. **Rose:** Unfold your paper. (learners do what the teacher told them) How many parts do you see?

27. **LS4:** There are eight equal parts.

28. **Rose:** Do you agree with him?

29. **LS** [Answers by shouting] Yes mam.

00:49

30. **Rose:** : Learners, continue to fold the paper four times and unfold, then five times and unfold and see how many parts you see?

00:54

31. **Rose:** So, learners we were dealing with fractions all this while and a fraction is part of a whole, half is part of a whole, a quarter is a part of a whole, do you understand?

32. **LS:** Yes mam.

(Fractional Concept, lesson 1, time interval 00:20 -00:54)

The extract above illustrates how Rose executes her mathematical work of teaching. The aspects of defining, explaining, and representing were evident in Rose' lessons observed. In

Lesson 1, it was evident that the teacher wanted learners to develop a conceptual understanding and recognise the fractional concept on their own. Line 7, "*Fold your A4 paper once and make sure the two parts are on top of each other". Line 9, "Unfold your papers. What do you notice?" Line 20, "Now let us fold it twice and unfold and see what happens. How many parts do you see now*?" Line 30, "*Learners, continue to fold the paper four times and unfold, then five times and unfold and see how many parts you see?"*

There was evidence of verbal representation such as half of an A4 paper. In terms of different representation, the teacher could have used other representations such as circular, rectangular or square diagrams, i.e. a diagrammatical representation, on the board, and shade some parts of a whole for learners to have a clear picture of other representations. Using A4 paper may lead learners to think that it is the only object to use for the development of a fractional concept.

At this stage of developing the concept of fractions, the teacher has done well because at this stage, the teacher should only use verbal expression, which teacher Rose did. The teacher together with learners showed each other half, fourth, eighths, sixteenths. The only thing she could have done additionally was to show or let learners develop other fraction names like thirds, fifths, sixths, and the rest would follow.

It was also evident that the teacher explained and defined what a fraction is to learners Line 31 "So*, learners we were dealing with fractions all this while and a fraction is part of a whole, half is part of a whole, a quarter/fourth is a part of a whole etc ,do you understand ?"*

It appears that this is how Rose used to teach mathematical concepts when asked if she always taught fractions the way she did in these lessons, she said the following:

Rose: This is how I am teaching, when I introduce a lesson, I make sure that learners understand the concept before teaching them the rules. I want learners to master the concept first. I have realised that mastering the concept is important because after mastering the concepts they learn with ease when teaching them the how part of working out fractional rules.

It is evident from the above extract that Rose, when introducing a lesson, would make sure that learners understand the concept before teaching them the rules. Rose seemed to have found the secret of teaching learners a concept with an understanding and the benefit thereof. She said, "*they learn with ease*". When asked what the teacher would do to develop conceptual understanding in the teaching of fractions, she said in the absence of the teaching resources the teacher had to improvise, as indicated in the interview extract below.

Researcher: From your knowledge and point of view, what do you think are the main things a teacher needs to know in order to develop conceptual understanding of fractions for learners to master?

Rose: From my perspective, for learners to understand better ,teaching resources should be available and if they are not available ,as a teacher I must improvise.

Fractional Notation

1.	**Rose:**	On Friday we have seen that when we have an A4 paper and fold it, in that way, we were representing a fraction verbally
2.	**Rose:**	[Asks a question] Now tell me, what is a fraction?
3.	**L1:**	A fraction is a part of a whole;
00:17-00:25		The teacher went out and talked with the other teacher]
4.	**Rose:**	Yes, you are correct. Here with me is an apple, do you all see it?
5.	**LS:**	Yes mam.

6.	**Rose:**	Look at me all of you, I cut it like this [showing learners how she cut it] How many parts do you see?
7.	**L2:**	I see four parts
8.	**Rose:**	Look carefully at this part, are they equal or not equal?
9	**L3:**	They are equal.
10:	**Rose"**	Ok, if I give Mpho this part (referring to one part) how many parts did I give Mpho?
11.	**L4:**	One part
12.	**Rose:**	How many parts was this apple divided?
13	**LS:**	[some learners were not listening, were giggling when the lesson was on]
14. 00:40	**L5:**	It was divided into 4 equal parts.
15.	**Rose**:	Very good, and we write it like this (writes on the board) $\frac{1}{4}$. You see there is a top number and a bottom number. The top number tell us how many parts I have used and the bottom tells us how many parts our apple was divided into. Are you with me?
16. 00:52	**LS:**	Yes mam.
17.	**Rose:**	So, each time you see a number like this, know that is a representation of a fraction ,it is expressed in the form $(\frac{a}{b})$ where a represent a number and b also represent a number. The top number is called the numerator and the bottom number is the denominator. Are you following learners?
18.	**LS:**	Yes mam.
19.	**Rose:**	Numerator tells or indicates how many parts were used and the denominator tells or indicates how many parts the whole was divided. Our apple was divided into four parts and one part was used.

(Fractional notation, lesson 2, time interval ,00:15- 00:52)

In her Lesson 2 on fractional notation, it was evident that the teacher was convinced that her learners developed the fractional concept verbally in Lesson 1, **Line 1 "***On Friday we have* *seen that when we have an A4 paper and fold it ,in that way, we were representing a fraction verbally*". She seemed certain that her learners understood what they have done in the folding and unfolding process and when developing halves, fourths etc. In my view, that is why she continued to the other fractional notion, in this case, the fractional notation.

She took an apple and showed learners how she divided it into four equal parts and gave Mpho (one of the learners) one part and explained to learners what the numerator and denominator meant. **Line 15:** "*We write it like this(writes on the board)* $\frac{1}{4}$*. you see there is a top number and a bottom number. The top number tell us how many parts I have used and the bottom tells us how many parts our apple was divided into. Are you with me?"* Line **17: "** *So, each time you see a number like this, know that is a representation of a fraction, it is expressed in the form* $(\frac{a}{b})$ *where (a) represent a number and (b)also represent a number."* Line 17 (continue to say), "*the top number is called the numerator and the bottom number is the denominator."* Line **19:**" *numerator tells or indicates how many parts were used and the denominator tells or indicates how many parts the whole was divided. Our apple was divided into four parts and one part was used."*

In Lesson 2, the teacher used an apple, showing learners the numerator and the denominator and explaining what they meant. The teacher did well, but she could have used other models such as an area model, set models and length model for learners to explore and understand better.

Proper and Improper Fractions

1.	**Rose:**	Last time we discovered what the numerator and denominators are, do you still remember what they are?
2.	**LS:**	**Yes.**
3.	**Rose:**	Who can tell us?
4.	**L1:**	Numerator is the top number and denominator is the bottom number.
5.	**Rose**:	Remember the numerator shows how many parts were used, and denominator indicates how many parts the whole was divided
6.	**LS:**	Yes. [say it loud in a choral way]
00:17		
7.	**Rose:**	[Writes $\frac{2}{5}$;$\frac{7}{2}$;$\frac{1}{3}$;$\frac{9}{4}$;$\frac{6}{7}$;$\frac{3}{4}$; $\frac{11}{2}$ on the board.]
8.	**Rose:**	Here on the board are fractions and I want you to mention the once with numerator smaller than the denominator.

9		**L2:** $\frac{1}{3}$, **L3**: $\frac{3}{4}$; **L4**: $\frac{6}{7}$; **L5**: $\frac{2}{5}$
10:	**Rose:**	You are correct all of you, what can we say about them, look at them carefully
11.	**L6:**	All their numerators re smaller than the denominators
12.	**Rose:**	Good, they are called proper fractions and the rest on the board are called improper fractions because their numerators are bigger than the denominator.
00:24		
13.	**Rose:**	[writes more fractions on the board and ask learners to classify in a table in their rough books $\frac{21}{5}$; $\frac{2}{7}$; $\frac{1}{5}$; $\frac{12}{5}$; $\frac{2}{15}$; $\frac{25}{50}$; $\frac{4}{5}$; $\frac{2}{10}$]

Proper Fractions	Improper Fractions

14.	**LS.**	[Some get down and work individually but other learners giggle and are not participating]
00:48		
15.	**Rose:**	Are you done? Let us correct. [After correcting, most got them right except noise makers]
16.	**Rose:**	Now write proper and improper fractions of your choice, list five proper and five improper fractions
17.	**LS**	[Take time to list or write them in their mathematics books]
01:00		
	Rose:	It is time up; we will mark the work the next day. (**Proper and improper fractions, lesson3 time interval 00:13 -01:00)**

In the above extract, it was evident that the teacher reminded learners what they already knew before introducing the new concept. **Line 1:** "*Last time we discovered what the numerator and denominators are, do you still remember what they are?* Learner 1's answer was an indication of knowing what the numerator and denominator were. Line 4: "*Numerator is the top number and denominator is the bottom number.*" The teacher went on to define concepts, Line 5: "*Remember the numerator shows how many parts were used, and denominator indicates how many parts the whole was divided*".

It was evident that learners recognised and identified which fractions are proper and which are improper. Line 9**: L2:** $\frac{1}{3}$, **L3**: $\frac{3}{4}$; L4: $\frac{6}{7}$; $L5$ $\frac{2}{5}$ got answers correct. Line 10: "You are correct all of you." There was an indication of learners given an activity to do Line **13:** Writes more fractions on the board and ask learners to classify it in a table in their rough books.

$\frac{21}{5}$; $\frac{2}{7}$; $\frac{1}{5}$; $\frac{12}{5}$; $\frac{2}{15}$; $\frac{25}{50}$; $\frac{4}{5}$; $\frac{2}{10}$

Proper fractions	**Improper fractions**

The following extract illustrates how teacher Rose was confident in her way of teaching. She indicated that when she teaches, she makes it a point that conceptual development is fully understood before teaching the mathematical rules**.** The following extract illustrates her self-assurance in her teaching:

Researcher: You mentioned that you have 16 years' experience in teaching mathematics in Grade 6, and you have taught fractions many times. Do you always teach fractions the way you did this year in the lessons which I have observed? Does your teaching develop learners to master fractional concepts?

Rose: This is how I am teaching, when I introduce a lesson, I make sure that learners understand the concept before teaching them the rules. I want learners to master the concept first. I have realised that mastering the concept is important because after mastering the concepts they learn with ease when teaching them the how part of working out fractional rules.

Category 2: How do teachers work with learners 'mathematical ideas when teaching fractions?

Unpacking the concept of fraction

The extract below illustrates how teacher Rose worked with learners' mathematical ideas.

00:23

5. **Rose**: [Give learners A4 paper sheet each]
This is the A4 paper, I am going to tell you what to do and you should listen.
6. **LS**:([Listening and paying attention to the teacher)**]**

7. **Rose**: Fold your A4 paper once and make sure the two parts are on top of each other. Are you following?
8. **LS:** Yes mam

00:26

9. **Rose**: Unfold your papers. What do you notice?

10. **LS:** **(**raise their hands)

11. **Rose**: L1 tell us, what can you about the A4 paper

12. **L1**: The paper has 2 parts now.

13. **Rose**: Do you agree with her learners?

14. **LS:** Yes mam

15. **Rose**: What else can you say?

16. **L2**: Two parts are equal. We had a whole of the A4 paper but now it is divided into 2.
17. **Rose**: What else can you say about the paper?

18. **L3**: We now have halves no longer a whole.

00:39

20. **Rose**: you are correct. Each part is half of the paper. Now let us fold it twice and unfold and see what happens. How many parts do you see now?
21. **L3:** I see four parts.

22. **Rose:** [Poses a question to L3 who has just answered] What can you say about those parts?
23. **L3:** They are equal.

24. **Rose:** [Orders learners] Fold the paper three times now.

25. **LS:** [do what was ordered]

00:45

26. **Rose:** Unfold your paper. (learners do what the teacher told them) How many parts do you see?

27. **LS4:** There are eight equal parts.

28. **Rose:** Do you agree with him?

29. **LS** [Answers by shouting] Yes mam.

00:49

30. **Rose:** : Learners, continue to fold the paper four times and unfold, then five times and unfold and see how many parts you see?

00:54

31. **Rose:** So, learners we were dealing with fractions all this while and a fraction is part of a whole, half is part of a whole, a quarter is a part of a whole etc, do you understand?

32. **LS:** Yes mam.

(Fractional Concept, lesson 1, time interval 00:20 -00:54)

In Lesson 1, it was clear that the teacher was involving learners in folding and unfolding A4 paper for learners to understand the concept of fractions. Each learner was doing what the teacher instructed them to do. Learners were folding and unfolding the A4 paper. In the lesson, the teacher involved learners to elicit their thinking. For example, in line 9, she asked learners what they noticed when folding the paper in halves. It was clear that the teachers wanted learners to discover the knowledge of fraction by themselves. In line 9 and 11, she said what they noticed. In addition, she pointed at L1 to tell what he says about the A4 paper. Indeed, L1 recognised that the paper had two parts.

Interestingly, after L1 had responded, Rose did not move on in the lesson, but she involved other learners in Line 13, asking if they did agree with L1. It appears that asking other learners what they think about L1 response, was trying to solicit other learners' contributions. Another example of this is in Line 28: "*do you agree with him?*"

Rose lesson was dominated by probing questions such as what else can you say in line 15 as also observed in line 9 and 11. "*What do you notice and what can you say about A4?*" The fact that she invited other learners to comment on L1's responses showed that she listened carefully to L1 response. In line 17, 20 and 22 she probed learners again, saying, "*what else can you say*" (line 15), "*what else can you say about the paper?*" (line 17), "*how many parts do you see now* (line 20)?" and "*what can you say about those parts* (Line 22)?". It is clear that, in this style of questioning, Rose wanted her learners to articulate, elaborate and clarify their ideas.

However, in Line 6 (Listening and paying attention to the teacher), the teacher could have allowed learners to brainstorm at this stage, to come up with other fractions verbally. As she involved them in a task, they recognised that the A4 paper was a whole and if folded once it shows halves, if twice it shows fourths, three times eighths, and four times it gives sixteenths.

The following illustrates how learners were engaged in the activity.

Fractional Notation

12. Leaner1: *The paper has 2 parts now*.

16. Learner2*: Two parts are equal. We had a whole of the A4 paper but now it is divided into 2.*

18. Learner 3*: We now have halves, no longer a whole.*

21. Learner 3: *I see four parts.*

23. Learner 3: *They are equal.*

27. Learner 4: *There are eight equal parts.*

Although there was no indication of learners asking questions, the teacher tried to move the lesson by asking questions rather than using direct teaching.

6. Rose: Look at me all of you, I cut it like this (showing learners how she cut it) How many parts do you see?

7. **L2**: I see four parts.

8. Rose: Look carefully at this part, are they equal or not equal?

9. L3: They are equal.

10. Rose: Ok, if I give Mpho this part (referring to one part) how many parts did I give Mpho?

11. L4: One part

12. Rose: How many parts was this apple divided?

13. LS: (Some learners were not listening, were giggling when the lesson was on)

14. **L5**: It was divided into 4 equal parts.

00:40

15. Rose: Very good, and we write it like this (writes on the board) $\frac{1}{4}$. You see there is a top number and a bottom number. The top number tell us how many parts I have used and the bottom tells us how many parts our apple was divided into. Are you with me?

16. LS:Yes Mam

00:52

17. Rose: So, each time you see a number like this, know that is a representation of a fraction ,it is expressed in the form $(\frac{a}{b})$ where a represent a number and b also represent a number.

The top number is called the numerator and the bottom number is the denominator. Are you following learners?

18. LS: Yes mam

19. **Rose** : Numerator tells or indicates how many parts were used and the denominator tells or indicates how many parts the whole was divided. Our apple was divided into four parts and one part was taken out.

(**Fractional notation, lesson 2, time interval, 00:15- 00:52)**

In the above extract, Rose made it a point that her learners should look at her while cutting the apple in front of them. She wanted her learners to recognise or realise that the four parts that the apple was cut into equal parts. **Line 9, L3** confirmed that the parts were equal. Rose wanted to be sure that learners were following, by asking them into how many parts the apple was divided, then L5 responded by saying it was divided into four **equal** parts.

After Rose realised that her learners were aware of the equal parts, she continued by writing a fraction name ($\frac{1}{4}$.) on the board and letting learners know the top and the bottom numbers. She made it a point that her learners knew what the numerator and the denominator mean. It was interesting because learners saw all the parts to which their teacher was referring.

There was clear evidence that teacher Rose wanted her learners to master the fractional concepts. Her response correlates with what she did in her lessons, working with learners' mathematical thinking. She pointed out that learners master the concept of fraction using verbal representation first, and she only later let learners know the fractional notation and the meanings of the top number and the bottom number as the concept developed. The only disappointing element in her teaching, was the way her learners behaved during her lesson presentation. According to her response, she did not want to act because punishment is not allowed. She could have used other disciplinary measures rather than to leave them to do as they pleased.

In Lesson 2, there was no indication of learner-talk. The teacher dominated the lesson. Learners were sitting and listening to the teacher. It was clear that learners were not given a task to do. They were just passive participants in the classroom.

Proper and Improper Fractions

13. Rose: (writes more fractions on the board and ask learners to classify in a table in their rough books

$\frac{21}{5};\frac{2}{7};\frac{1}{5};\frac{12}{5};\frac{2}{15};\frac{25}{50};\frac{4}{5};\frac{2}{10}$

Proper fractions	Improper fractions

14. LS: Get down and work individually but other learners giggle and not participating.

There was no indication of learners doing an activity in Lesson 2, they just sit and listen to what the teacher was saying. The teacher dominated the lesson. Some learners were not listening, they were giggling, and the teacher ignored them, which seemed not to be good because it could negatively impact the performance of learners. The teacher coming to class some minutes late and leaving the class to talk with other teachers is highly discouraged as it consumes teaching time. (*00:17-00:25 The teacher went out and talk with the other teacher.)* She went out, and when she returned, she simply continued with her business of the day. Her teaching was good, but this kind of behaviour might jeopardise her good work.

00:48

15. Rose: Are you done? Let us correct.

(After correcting, most got them right except noise makers)

16. Rose: Now write proper and improper fractions of your choice, list five proper and five improper fractions.

17. LS: (Take time to list or write them in their mathematics books)

(Proper and improper fractions, lesson 3-time interval 00:13 -01:00)

The above extract shows how learners were involved in a task. There was no indication of monitoring learners' work. The learners were ordered to write individually instead of allowing discussions, and in the process, some learners were not participating, and the teacher just kept quiet and did nothing about them. **14.** *Get down and work individually but other learners giggle and not participating.* The teacher did not pose different types of questions, such as probing questions etc., to learners. It was clear that the learners did not even ask questions for clarity. Evaluative kind of listening was evident because the answers given were the answers that the teacher already knew.

Although Rose tried to involve learners in her lesson, she did not do this optimally. She seemed to be acknowledging the fact that involving learners in discussions was very important, but it was time-consuming. This is confirmed by the interview below.

Researcher:	You were dominating your lessons with little interaction among learners is it how teaching should be? Elaborate on that.
Rose:	I do not think is good, but this is how I teach because I want them to listen and understand what I am teaching, allowing them to discuss is good but is time consuming.

During the lesson, some of the learners were giggling and not participating, and the teacher appeared to have done nothing about it. When asked why she did not do something about it, she said it is difficult because learners just misbehave, and she cited the issue of lack of punishment. This is supported by the interview extract below.

Researcher:	Some of your learners were giggling and not participating and you ignored them, is it how your class always behave?
Rose:	This is difficult because these learners just misbehave, remember punishment is not allowed. I think they just did this on purpose**.**

Like in Eddy's lesson, none of the learners in Rose's class asked the teacher questions in the three observed lessons. When asked during the interview why learners were not asking question, she said, *they are shy to ask questions in front of their peers*. This is illustrated by the interview extract below.

Researcher:	In the three lessons observed, your learners did not even ask a single question, is it a norm, is it the way they always do? What can be the cause?
Rose:	I think these learners are shy to ask questions in front of their peers, even if you encourage them to ask questions, they will not do it, they do not have confidence at all.

As indicated in the extract above, apart from the fact that *learners are shy to ask questions in front of their peers*, Rose further indicated that learners seemed not *to be confident at all.*

Category 3: How do teachers restructure mathematical tasks to be understood by **learners?**

In all three of Rose's lessons, there was no evidence of scaling up or down of a mathematical task and shifting of appropriate mathematical outcome.

4.4.3 William's lesson observations

The third teacher observed, was William of school C, his observations are reflected below:

Category 1: How do teachers define, explain and represent the concept of fractions to Grade 6 learners?

Converting mixed fractions to improper fractions

Below is the extract that reveals how the teacher (William) defines, explains, and represents the concept of fractions in his teaching.

1.	**William:**	[Poses a question to learners] What is a proper fraction?
2.	**L1:**	A proper fraction is a fraction with the numerator smaller than the denominator
3.	**William:**	Correct. Can you give an example of a proper fraction?
4.	**LS:**	[raising up their hands]
5.	**William:**	Yes, let us hear from you.
6.	**L2:**	$\frac{3}{7}$
7.	**William:**	Now because you know what a proper fraction is, what can you say about a mixed fraction or mixed number?
8.	**L3:**	A mixed fraction is a fraction with the whole number and a proper fraction.
9.	**William:**	Give an example of a mixed fraction.
10.	**L4:**	I think $5\frac{1}{2}$ is a mixed number.
11.	**William:**	you are very correct. But what is a mixed fraction?

00:23

12.	**William:**	Ok, let us continue, yes, a mixed fraction has the whole number and a proper fraction like the answer given by this learner(pointing at her) $5\frac{1}{2}$ is a mixed fraction ,so we can change it to improper fraction by saying2 which is the denominator $\times$ 5 the whole number + 1 the numerator and the denominator remain the same.
13.	**William:**	[Writes on the board] The answer is $\frac{11}{2}$. Do you all understand?
14.	**LS:**	[Without confidence] Yes.
15.	**William:**	[repeats what he said for them to understand]

00:31

16. **William:** Ok change the following. [Writes on the board] $4\,\frac{2}{3}$,I give you few minutes to work it out. [learners work it out individually] [After a few minutes] Are you done?

17. **LS:** [some say yes, some say no and others just keep quiet.]

18. **William:** Let me give you another example,(Writes on the board) $2\,\frac{1}{3}$,you see this is a mixed number because it has the whole number and a proper fraction, so changing it to improper fractions the steps we have to follow are this: We say denominator multiplied by the whole number plus the numerator and the denominator remain the same, it does not change .Do you understand good people? It is like this, $\frac{3\times 2+1}{3}=\frac{7}{3}$

00:52

19. **William**: Now try $4\,\frac{2}{3}$,change it to improper fraction. Work it out in your rough books individually. I give few minutes to do it, start:

20. **L:** [after a while, a learner was pointed out and ordered to go to the board and show others how he got the answer.]

$$4\,\frac{2}{3}=\frac{3\times 4+2}{3}$$
$$=\frac{14}{3}$$

21. **William:** That is very good, well done. This is exactly how we work it out. Now do the following: (writes on the board):

a) $1\,\frac{2}{5}$
b) $3\,\frac{4}{7}$
c) $5\frac{1}{2}$

(Converting mixed fractions to improper fractions, Lesson 1, time interval 00:02-1:00)

The teacher introduces his lesson by posing a question, line **1,** (pose a question to learners) what a proper fraction is. This is an indication that the teacher started where the learners were, what they already knew to what they did not know. Learners already knew what proper and mixed fractions were, Line 2 “A proper fraction is a fraction with the numerator smaller than the denominator”, Line 8, “A mixed fraction is a fraction with the whole number and a proper fraction.”

The teacher's explanation of changing mixed fractions to improper fractions was procedural. His teaching encourages memorisation of rules, Line 12 "*Ok, let us continue, yes, a mixed fraction has the whole number and a proper fraction like the answer given by this learner(pointing at her)* $5\frac{1}{2}$ *is a mixed fraction ,so we can change it to improper fraction by saying 2, which is the denominator* $\times$ *5 the whole number + 1 the numerator and the denominator remain the same."* The teacher after realising that learners did not understand him, repeated the explanation for learners to understand, *Line 18:*

> Let me give you another example,(Writes on the board) $2\frac{1}{3}$,you see this is a mixed number because it has the whole number and a proper fraction, so changing it to improper fractions the steps we have to follow are this: We say denominator multiplied by the whole number plus the numerator and the denominator remain the same, it does not change. Do you understand good people? It is like this, " $\frac{3 \times 2+1}{3} = \frac{7}{3}$. (William)

The teacher could have allowed learners to investigate or find out how to work out these procedures on their own instead of spoon-feeding them with every explanation.

In the above extract, the learners were only asked about a proper fraction line 2, L1 answered what a proper fraction was and L3 answered what a mixed fraction was. The lesson was about converting mixed fractions to improper fractions. The learners were shown procedurally how the conversion is done but the teacher did not explain to learners that their answers were actually **improper fractions**. He did not explain to learners that $\frac{11}{2}$; $\frac{7}{3}$; $\frac{14}{3}$ were improper fractions. The teacher could at least have represented all these fractions (proper, mixed and improper fractions) diagrammatically, for learners to visualise all these concepts.

In Lesson 2 below, William introduces his lesson with a question, *"What is a mixed fraction?"* He was linking the previous knowledge with the existing knowledge. He made it clear that before they start with the new topic, they must remember what they have learned, (Line 3, 8 and 9). There was evidence that the teacher used models to represent which fractions were smaller and which were larger (Line 18 indicates the representation).

Comparing and ordering of fractions

1.	**William:**	We have learned about changing mixed fractions to improper fractions, let us recap. What is mixed fraction?
2.	**LS:**	[Raise their hands up]
3.	**L1:**	A mixed fraction is a fraction with the whole number and a proper fraction.
4.	**William:**	Good attempt, L1 can you give an example of it?
5.	**L1:**	$2\frac{1}{3}$
6.	**William:**	Correct, Is there anyone of you learners who disagree with L1?
7.	**LS:**	No sir.
8.	**William:**	What is an improper fraction?
9.	**L2:**	It is a fraction with the numerator bigger than denominator.
10.	**William:**	Good, can you supply us with an example L2?
11.	**L2:**	$\frac{31}{3}$
00:13		
12.	**William:**	Well done, now let us continue with our today s topic. [Writes $\frac{1}{3}$; $\frac{1}{2}$ on the board.] This are fraction names, one third and one half. I want us to compare them and see which is bigger than the other. Which one do you think is the biggest?
13.	**L3:**	I think is $\frac{1}{3}$
14.	William:	L3 can you tell us the reason why you say so?

15. L3: It is the biggest because if you look at them their numerators are the same, but the denominators are different so, $\frac{1}{3}$ has the denominator of 3, is 3 not bigger than 2?

16. William: What are you saying learners? Is there anyone who agrees with the speaker?

17. LS: [Silence]

18. William: Ok, let us represent them on a diagram and see,(draw models on the board)

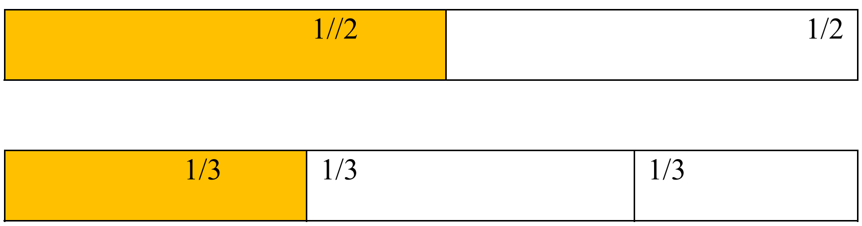


Figure 3.2: Length model

Which fraction is bigger, look at these diagrams?

19. LS: [hands up]

20. L5: ½ is bigger than 1/3

00:22

22. William: That is correct, so you can see that using diagrams help us to see which fraction is smaller or bigger. Try this one ¼ and 2/3, someone come and show us.

L6:

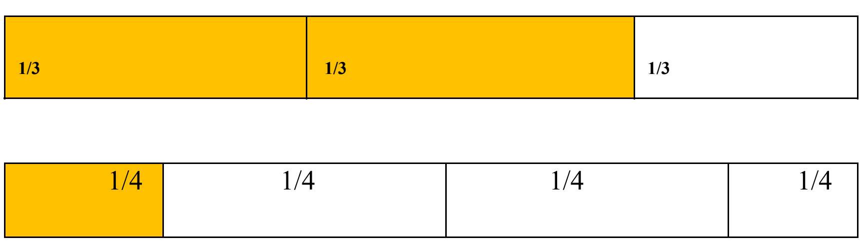


Figure 3.3: Length model

00:37

23. **William:** Good, this is how we compare fractions. Do the following:
Which fraction is bigger, which is smaller? Use this sign [writes on the board]
$<$; $>$; or = when we compare fractions, for example, from the one we did ½ $>$ 1/3

a) 1 /4 and 1/6

b) 1/3 and 1 /4

c) 2/4 and 1 /2

[after some minutes] who is done, if you are done what are the answers?

00:56

24. **L7:** ¼ $>$ 1/6

25. **L8:** 1/3 $>$ 1 /4

26. **L9:** 2/4 = 1 /2

27. **William:** Well done, I hope each one of you understood. Will see you tomorrow.

(Comparing and ordering fractions, lesson 2, time interval 00:00-01:00)

In Lesson 3, William explained an algorithm (procedural way of adding fractions with unlike denominators). He could have used area models to represent the two fractions by choosing the whole that both 2 and 6 can divide into, show the whole, that is, 6 blocks and shade $\frac{1}{2}$ + $\frac{2}{6}$ of the whole. He could have represented it this way instead of showing learners the rules; but he did not even show them where the rules originated.

He encouraged learners to memorise the mathematical rule for adding unlike fractions, saying we look for the LCM if the denominators are not the same.

Addition of fractions with unlike denominators

1.	**William:**	Ok, you do not understand heh: let me show you, these fractions have different denominators. So, we must look for the LCM, and LCM means lowest common multiple. We look for the multiples of 2 and 6 and select the smallest number that appears both in the multiples of 2 and 6,and in this case 6 is our LCM. LCM of 2 and 6 is 6. Therefore, we say $\frac{1}{2} + \frac{2}{6}$ $=\frac{1}{2} \text{ x } \frac{3}{3} = \frac{3}{6} + \frac{2}{6}$ $=\frac{5}{6}$ Try this one on your own $\frac{1}{5} + \frac{3}{10}$
2.	**LS:**	$\frac{4}{15}$
3.	**William:**	No, no, I have told you learners that we must look for the LCM, we cannot just add the way this girl did. By the way what is the LCM of 5 and 10? Let us find the multiples of 5 and 10, what is the LCM of 5 and 10?
4.	**L1:**	10.

00:56

5.	**William:**	Yes, so is [writes on the board] $\frac{1}{5} + \frac{3}{10}$ $\frac{1}{5} \text{ x } \frac{2}{2} \quad = \frac{2}{10} + \frac{3}{10} = \frac{5}{10}$ $= \frac{1}{2}$(simplification)

(Addition of fractions with unlike denominators, lesson 3)

Again, in the above extract, William could have corrected his learners about their misconceptions because they mistakenly added the numerators and the denominators (line 2, LS $\frac{4}{15}$). Instead of correcting them, he said: "*No, no, I have told you learners that we must look for the LCM" and* continued with his lesson. He could have rectified them and shown where their mistake was so that they did not repeat the mistake in the future.

The interview below illustrates how teacher William responded when asked about developing a fractional concept for learners to master. The teacher did not see the importance of using representation in all his lessons. He alluded that he sometimes used models, adding he did not see the need to use models, an unsatisfactory response. Learners should know how to add fractions, compare fractions, and convert fractions procedurally, but the importance in the matter is the mastery of the concept.

William is aware that model representation is useful but ignored using it on other topics. He could consider using representations in all the lessons to make learning easier for learners.

Researcher: You mentioned that you have 12 years' experience in teaching mathematics in Grade 6, and you have taught fractions many times. Do you always teach fractions the way you did this year in the lessons which I have observed? Does your teaching develop learners to master fractional concepts?

William: Yes this is my way of teaching, I sometimes use models where necessary like I did in lesson 2 of comparing fractions, but in other lessons like lesson 3, I don't see the need to use models when adding fractions, like converting mixed to improper fractions, learners should know the how part of changing them to improper .

The extract below indicates that the teacher teaches what is in the textbooks, not considering that before learners can solve a problem procedurally, they should have mastered the concept first. The researcher's questions appeared difficult for the teacher because his teaching was procedural. He did not even show any indication of changing his way of teaching.

Researcher: I have observed that in lesson 1 and 3, your teaching was about explaining mathematical rules, do your learners know where these rules come from?

William: Hei, Madam, this is a difficult question indeed, I don't know how to respond to it because I just teach them according to what is in the textbook.

Category 2: How do teachers work with learners' mathematical ideas when teaching fractions?

Converting mixed fractions to improper fractions

For ease of referencing, the following extract from Lesson 1 is repeated again:

In the extract above, William dominated the lesson as he explained how to convert mixed fractions to improper fractions. The learners were just sitting silently listening to their teacher's explanation of the algorithm. The teacher realised that his learners knew what proper fractions were and continued to ask learners what a mixed number is. Line 1"Now *because you know what a proper fraction is, what can you say about a mixed fraction or mixed number?"* A learner gave an example of a mixed number but when asked what a mixed number was, the learners did not answer until teacher William came up with an explanation, which is surprising, to give an example of something the learners do not know but know examples of it, it does not make sense. William's learners seemed not to understand because after feeding them with mathematical rules, when asked if they understood, their response was not satisfactory. Line **7** and 8 support the statement: "[*Writes on the board] The answer is* $\frac{11}{2}$ *. Do you all understand? 8.LS: [Without confidence) Yes.*" Instead of developing a concept meaningfully for learners, he continued explaining the rules. His teaching was full of rules for learners to memorise. Learners did not ask questions for clarity; they were just passive participants in the learning process.

Comparing fractions

For ease of referencing, the following extract from lesson 1 used in this discussion is reflected again.

1.William: We have learned about changing mixed fractions to improper fractions, let us recap. What is mixed fraction?
2.L 1: A mixed fraction is a fraction with the whole number and a proper fraction.
William: Good attempt, L1 can you give an example of it?
2.L1: $2\frac{1}{3}$
3.William: correct, is there anyone of you learners who disagree with L1?

4.LS: No Sir

5.William: What is an improper fraction?

6.L2: It is a fraction with the numerator bigger than denominator.

7.William: Good, can you supply us with an example L2?

8.L2: $\frac{31}{3}$

00:13

9.William: Well done, now let us continue with our today s topic, (writes $\frac{1}{3}$; $\frac{1}{2}$) on the board

This are fraction names, one third and one half. I want us to compare them and see which is bigger than the other. Which one do you think is the biggest?

10.. L3: I think is $\frac{1}{3}$

11. William: L3 can you tell us the reason why you say so?

12.L3: It is the biggest because if you look at them their numerators are the same, but the denominators are different so, $\frac{1}{3}$ has the denominator of 3,is 3 not bigger than 2?

13. William: what are you saying learners? Is there anyone who agrees with the speaker?

14.LS: (silence)

15. William: ok, let us represent them on a diagram and see,(draw models on the board)

1//2	1/2

1/3	1/3	1/3

Which fraction is bigger, look at these diagrams?

16.LS (hands up)

17..L5: ½ is bigger than 1/3

00:22

18.William: that is correct, so you can see that using diagrams help us to see which fraction is smaller or bigger. Try this once ¼ and 2/3, someone come and show us

19.L6:

1/3	**1/3**	**1/3**

¼	1/4	1/4	1/4

2/3 is bigger than 1/4

00:37

20.William: Good, this is how we compare fractions. Do the following:

Which fraction is bigger, which is smaller? Use this sign (writes on the board)

< ; > ; or = when we compare fractions, for example, from the one we did ½ > 1/3

a) 1 /4 and 1/6
b) 1/3 and 1 /4
c) 2/4 and 1 /2

In the above extract, William introduced his lesson by moving his learners from what they have learned the previous day to link it to the new topic of the day. He introduced a lesson through questioning. Line 1, *What is mixed fraction?* There was an indication that learners now know what a mixed fraction is and a learner could give an example of it. ***2**.L 1: A mixed fraction is a fraction with the whole number and a proper fraction.*

***William:** Good attempt, L1 can you give an example of it?*

2.L1: $2\frac{1}{3}$

There was an evidence of probing questions. Line L3 "can you tell us the reason why you say so?"

A misconception arose when William asked learners to compare fractions. line 9 to 14 below:
***9**.William: Well done, now let us continue with our today s topic, (writes* $\frac{1}{3}$ *;* $\frac{1}{2}$ *) on the board This are fraction names, one third and one half. I want us to compare them and see which is bigger than the other. Which one do you think is the biggest?*
***10**. L3: I think is* $\frac{1}{3}$
***11**.William: L3 can you tell us the reason why you say so?*
***12**.L3: It is the biggest because if you look at them their numerators are the same, but the denominators are different so,* $\frac{1}{3}$ *has the denominator of 3, is 3 not bigger than 2?*

***13.**William: what are you saying learners? Is there anyone who agrees with the speaker?*

14.LS: (silence)

Teacher William resolved the misconception by using a linear model for learners to compare fractions and this helped learners a lot. After using the model, learners could compare fractions with ease. There was no indication of learners asking questions. There was no learner-talk, the teacher was dominating.

The following extracts reveal that teacher William was dominating the lesson and did not encourage learners to ask questions. The teacher is complaining about time consumption if he allowed discussions by learners. I believe that learners should be monitored during class discussion. Learners should also be encouraged to pose questions for them to understand better and for clarification purposes. What is the use of finishing the work of teaching, while learners are left behind and do not understand what has transpired in the lesson presentation? The following extract justify the above statement:

Researche*r: You were dominating your lessons with little interaction among learners is it how teaching should be? Elaborate on that.*

William: If I allow these learners to discuss as a group or as a class, they consume time, which means when the period ends, you may find that the work that was supposed to be covered was far behind. Dominating the class help me to finish my work of teaching.

Researcher: You did not involve your learners in a discussion, there was no interaction among learners is it what you always do? elaborate

William: Allowing them in the discussion consumes time because one-hour period is not enough, if I allow them, we did not finish our lesson on time.

In the extract below, there is evidence that learners did not ask questions. Although the teacher indicated that he encouraged them to ask questions, they failed to do so.

Researche*r: In the three lessons observed your learners did not even ask a single question, is it a norm, is it the way they always do? What can be the cause?*

William: They are always like this; I really don't know the reason. I normally encourage them to ask me questions, unfortunately they do not ask questions.

Addition of fractions with unlike denominators

1.William: Ok, you do not understand heh: let me show you, these fractions have different denominators. So, we must look for the LCM, and LCM means lowest common multiple. We look for the multiples of 2 and 6 and select the smallest number that appears both in the multiples of 2 and 6,and in this case 6 is our LCM.

LCM of 2 and 6 is 6

Therefore, we say $\frac{1}{2} + \frac{2}{6}$

$$=(\frac{1}{2} \text{ x } \frac{3}{3}) + \frac{2}{6}$$

$$= \frac{3}{6} + \frac{2}{6}$$

$$= \frac{5}{6}$$

00:49

Try this one on your own $\frac{1}{5} + \frac{3}{10}$

2.LS: $\frac{4}{15}$

3.William: No, no, I have told you learners that we must look for the LCM, we cannot just add the way this girl did. By the way what is the LCM of 5 and 10? Let us find the multiples of 5 and 10, what is the LCM of 5 and 10?

4.L1: 10

00:56

5.William: yes, so is (writes on the board) $\frac{1}{5} + \frac{3}{10}$

$$\frac{1}{5} \text{ x } \frac{2}{2} \quad = \frac{2}{10} + \frac{3}{10} = \frac{5}{10}$$

$$= \frac{1}{2}\text{(simplification)}$$

The above extract revealed how William was teaching learners about how to add fractions with unlike denominators. His teaching was procedural. The traditional using of LCM. The concept was not developed for learners to understand. It appeared that William wanted his learners just to memorise rules. This may hinder learners ‘understanding of fractions. Rules are not even told where they were derived from.

Category 3: ***How do teachers restructure mathematical tasks to be understood by*** **learners?**

In all William’s three lessons observed, there was no any evidence of scaling up or down of a mathematical problem task and shifting of appropriate mathematical outcome.

4.5. ANALYTICAL FRAMEWORK

This study's analytical framework has its roots in Chapter 2 that provided many details on the aspects observed. The analytical framework was pinned on six aspects condensed by Kazima (2008), namely defining, explaining, representing, working with learners ‘mathematical ideas, and restructuring tasks. The participants were assessed based on the categories/aspects mentioned in Chapter 4.3.1–4.3.3, and the categories were observed as a package. Each category has indicators.

Each participant was rated using a rubric tool to assess their mathematical work of teaching and how they used their mathematical knowledge for teaching fractions to Grade 6 learners. The following is the quantification used to structure an overview of the data analysis using a rubric tool as follows:

- If none of the indicators was met during lesson presentation, the teacher would be rated between 0 and 1.
- If some of the indicators were met during lesson presentation, the teacher would be rated between 2 and 3.

- If most of the indicators were met during lesson presentation, the teacher would be rated between 4 and 5.

As indicated in the rating guide in Table 4.3 and Table 4.4 below, the three lessons observed will each be scored out of five (5) points. The scoring will depend on whether the indicators were met or not. The three lessons give a total of 15 points.

Rubric analytical tool.

Table 3.3: Rating guide

None of the indicators were met	Some of the indicators were met	Most of the indicators were met
0-1	**2-3**	**4-5**

Table 4.3 above was the rating guide used to rate the three teachers as they go about their teaching. It was used to judge/assess their mathematical work of teaching, how they used their mathematical knowledge for teaching as they were teaching fractions to Grade 6 learners.

Table 4.4 illustrates how the three teachers were analysed or judged when teaching fractions in Grade 6. They were judged based on the three categories/aspects indicated in Table 4.4, checking if the indicators were met or not. The three categories/aspects are: How do teachers unpack/introduce fractions to Grade 6 learners, how do teachers work with learners' mathematical ideas when teaching fractions and how do teachers restructure mathematical tasks to be understood by learners. Each participant taught three lessons, and each lesson was scored out of five (5), the three lessons give a total of 15 marks.

Table 3.4: Analysis of data

		Lessons observed. Rating			
		1	**2**	**3**	**total**
Participant/s	**Categories**	**5**	**5**	**5**	**15**
Eddy	1.How do teachers unpack/introduce fractions to Grade 6 learners? (introducing, defining, explaining and representing) a concept	3	3	3	9

	2. How do teachers work with learners 'mathematical ideas when teaching fractions? (working with learners' ideas, posing and responding to questioning)	2	2	2	6
	3.How do teachers restructure mathematical tasks to be understood by learners?	0	0	0	0
Rose	1.How do teachers unpack/introduce fractions to Grade 6 learners? (introducing, defining, explaining and representing a concept)	4	4	4	12
	2. How do teachers work with learners' mathematical ideas when teaching fractions? (working with learners' ideas, posing and responding to questioning)	3	3	2	8
	3.How do teachers restructure mathematical tasks to be understood by learners?	0	0	0	0
Total					
William	1.How do teachers unpack /introduce fractions to Grade 6 learners? (introducing, defining, explaining and representing a concept)	3	3	3	9
	2. How do teachers work with learners' mathematical ideas when teaching fractions? (working with learners' ideas, posing and responding to questioning)	2	2	2	6
	3.How do teachers restructure mathematical tasks to be understood by learners?	0	0	0	0
Total					

Figure 4.4 reflects a bar graph that represents the data that was analysed in Table 4.4. It illustrates how each participant executed their mathematical work of teaching fractions in Grade 6.

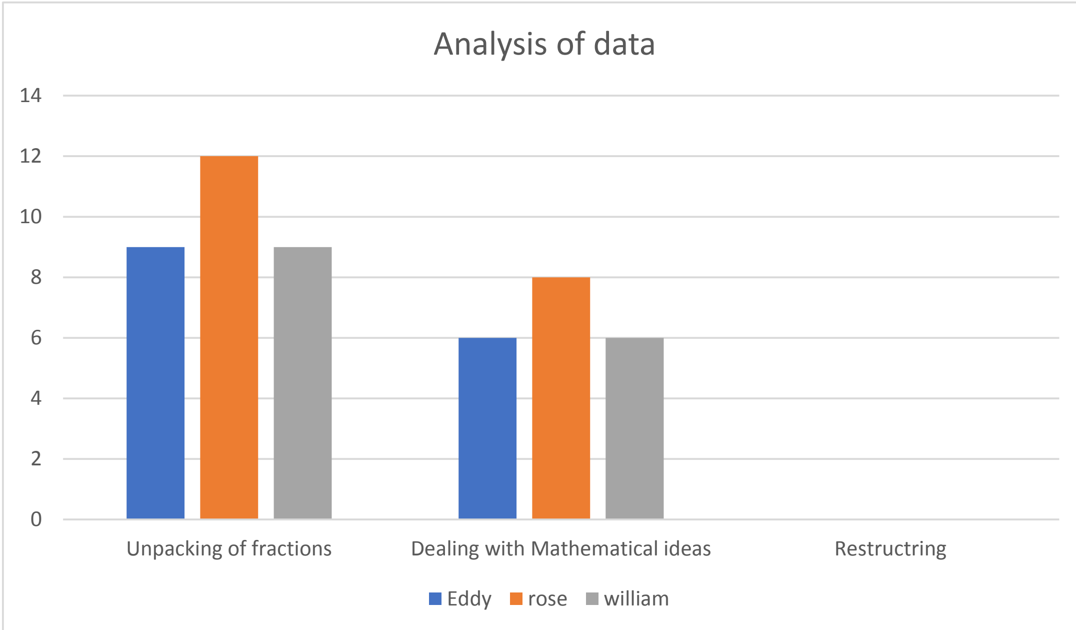


Figure 3.4: -Bar graph for data analysis

Table 4.5 below is a composite score of the three participants. It is an illustration of the total number of lessons observed, $\frac{total\ lessons\ score}{total\ score}$ x 100, which was used to find the percentages of each category/aspect.

Table 3.5: Composite data analysis of lessons observed.

Aspects	Occurrences=15	%Occurred
Mathematical work of teaching for William's lessons		
Unpacking of fractions (introducing, definition, explanation and representation)	9	60
Working with learners' mathematical ideas (questioning)	6	40
Restructuring the mathematical tasks	0	0
Mathematical work of teaching for Rose's lessons		
Unpacking of fraction (Definition, Explanation and Representation)	12	80
Working with learners 'mathematical ideas(questioning)	8	53
Restructuring the mathematical tasks	0	0

Mathematical work of teaching for William's lessons		
Unpacking of fraction (Definition, Explanation and Representation)	9	60
Working with learners 'mathematical ideas(questioning)	6	40
Restructuring the mathematical tasks	0	0

The data reflected in Table 4.5 indicates that Eddy scored 9 out of 15 in the aspect of unpacking of fractions (definition, explanation and representation) with a percentage of 60 per cent and scored 6 of 15 in the aspect of working with learners' mathematical ideas, achieving 40 per cent for the category. In all his lessons, the researcher recorded no occurrences in restructuring mathematical tasks.

The data reflected in Table 4.5 indicates that in all Rose's lessons, she scored 12 out of 15 which gives a percentage of 80% in working with learners' ideas, posing and responding to learner questioning (first category). She was then rated 8 out of 15, which gives a percentage of 53%. In all her lessons, the researcher recorded no occurrences in restructuring mathematical tasks.

The data reflected in Table 4.5 indicates that William scored 9 out of 15 in the aspect of unpacking of fractions (definition, explanation and representation) giving a percentage of 60%, and scored 6 out of 15 in the aspect of working with learners' mathematical ideas achieving a percentage of 40% In all his lessons the researcher recorded no occurrences in restructuring mathematical tasks.

Figure 4.2 is a bar graph that represents the composite data that was analysed in Table 4.5. .It illustrates the percentages that each participant achieved when executing their mathematical work of teaching fractions in Grade 6.

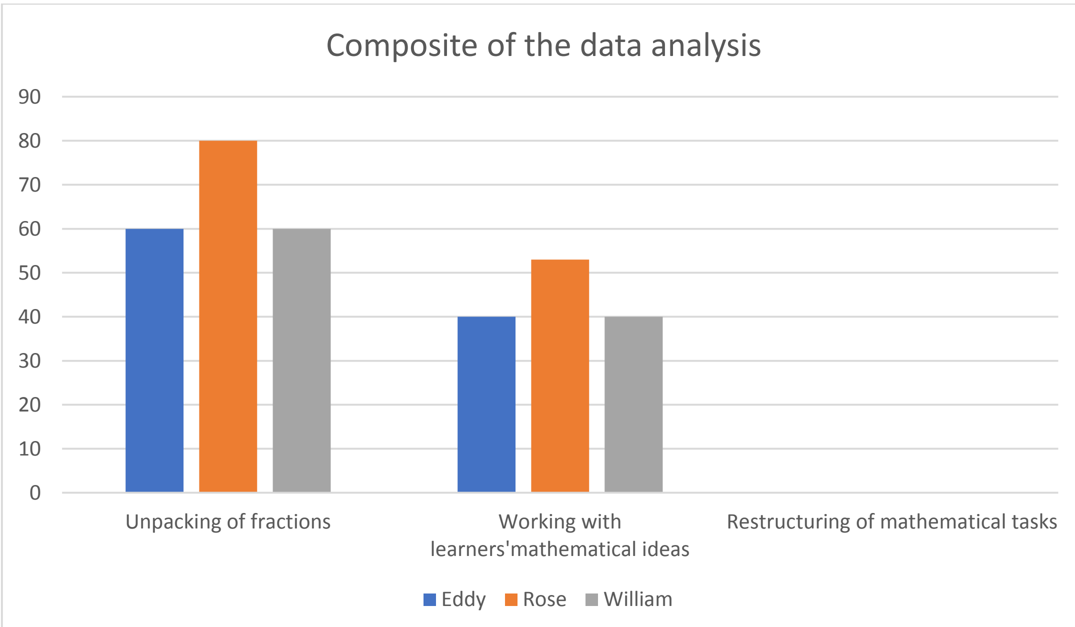


Figure 3.5: Bar graph for the composite of the data analysis

4.6. DISCUSSION OF OBSERVED LESSONS.

The researcher observed how the three teachers engaged in their mathematical work of teaching and demonstrated their mathematical knowledge for teaching. As they went through their teaching, the researcher observed how they introduced, defined, explained, and represented fractions (that is all about unpacking), how they dealt with learners' mathematical ideas and how they restructured their mathematical tasks to answer my research questions. The researcher decided to merge categories/aspects, namely *how teachers pose mathematical questions to learners* and *how do teachers respond to learner's mathematical questions* in *working with learners' mathematical ideas.* Therefore, the researcher's research questions were the following:

- How do teachers unpack/introduce, define, explain, and represent the concept of fractions to Grade 6 learners?

- How do teachers work with learners' mathematical ideas?
- How do teachers restructure their mathematical tasks?

The observation process revealed that the three teachers did not meet all six the aspects required for the mathematical work of teaching. In all lessons observed, working with learners' mathematical ideas was unsatisfactory. Teachers were mostly dominating the lessons. There was an indication of being unable to reform from the traditional way of teaching. It was clear that teachers were not restructuring their mathematical tasks.

From Table 4.5 above, we see the different emphasis in the mathematical work of teaching from the three teachers. We can see that while teacher Rose was developing the fractional concept verbally, William and Eddy were dealing with the procedural way of teaching fractions. Rose's teaching of fractions was far more pleasing than the two other teachers observed. Teachers Eddy and William's way of teaching was similar. Their pattern of teaching was the same. Their emphasis was on the memorisation of rules.

In all Rose's lessons, she introduced, defined, explained, and represented the concepts well. The researcher rated her 4 out of 5 in each lesson because it falls under the category ***most of the indicators were met*** in the rubric rating quide**.** In all lessons, she scored 12 out of 15 and achieved 80%. In working with learners' ideas, posing and responding to learner questioning (first category), she was rated 3 in the first two lessons and 2 in the last lesson, because even though she was working with learners' mathematical ideas, her learners did not even ask her any questions. There was no evidence of probing questions in her last lesson. She was then rated 8 out of 15 and achieved the percentage of 53%. In all her lessons the researcher recorded no occurrences in restructuring mathematical tasks.

As Eddy and William's teaching style was the same, their rating was the same. Both scored 9 in the first category achieving a percentage of 60%; and scored 6 out of 15 in all three their lessons under the aspect of unpacking of fractions (definition, explanation, and representation). They were rated 3 out of 5 on each lesson. They were rated under the rubric rating guide that states ***some of the indicators were met.***

Learners did not ask questions; no probing questions were asked. They were rated 2 out of 5 on the aspect of working with learners' mathematical ideas because there was no evidence of learners asking questions. They achieved 40 % each. In all their lessons the researcher recorded no occurrences in restructuring mathematical tasks.

4.7. CONCLUSION

The interviews were helpful as they broadened the knowledge of the three teachers who participated in the study. This means that some of the areas they overlooked all the time, but they have now realised that these aspects are important to apply as they go about their teaching.

The teachers were aware that for learners to understand better, model representations should always be used. The teachers seem to have a problem in shifting from the traditional way of teaching. Teachers are encouraged to use model representation to develop the fractional concept fully. The teachers should develop the concept of fractions using verbal representations, diagrammatical representation, narrative as well as numerical representation. Teachers should transform their way of teaching, involving learners actively in the lesson, allowing learner-talk, allowing them to brainstorm, use representations in developing the concepts before teaching the mathematical rules and encourage learners to take the lead because constructivism perceives learners as active participants of their learning. In my view, if we as teachers did not reform our old way of teaching, and continue to teach mathematical rules without understanding, learners will continually perform poorly in mathematics. In short, I can say all the work of teaching is in our hands. Teachers are encouraged not to be barriers to learning.

CHAPTER 4: **FINDINGS, RECOMMENDATIONS, CONCLUSION AND REFLECTION**

5.1. INTRODUCTION

In the previous chapter, the data generated was presented, analysed, and discussed. This chapter provides findings, recommendations, conclusions, as well as reflection of the study. This chapter provides the findings from the data collected from classroom observation and interview. The research study explored the mathematical knowledge for Grade 6 teachers in the teaching of fractions. The study aimed at exploring the following:

1. How do mathematics teachers teach fractions in Grade 6?
2. How do mathematics teachers unpack, define, explain, and represent fractions to Grade 6 learners?
3. How do mathematics teachers deal with learners' mathematical ideas?
4. How mathematics teachers restructure mathematical tasks for Grade 6 learners.

This study was motivated by my experience of seeing the continual inadequate and insufficient knowledge that the learners bring as they progress in higher grades each year. The researcher then came up with the research questions to find the challenges underlying this shallow and insufficient mathematical knowledge of fractions at our schools.

The research study also serves to answer the research questions, and they are as follows:

1. How do teachers unpack/introduce, define, explain, and represent the concept of fractions to Grade 6 learners?
2. How do teachers work with learners' mathematical ideas?
3. How do teachers restructure their mathematical tasks?

This chapter was structured in terms of the research questions. The theoretical lens that informed this study was drawn from Ball et al.'s (2008) framework, Shulman's (1986) pedagogical content knowledge (PCK) and the constructivism theory to elaborate on the notion of teaching fractions to the Grade 6 learners. The literature was used to answer the three research questions mentioned earlier. The following lays out how the three teachers taught fractions in Grade 6.

5.2. RESPONDS TO THE RESEARCH QUESTIONS

This section responds to the research questions:

5.2.1 How do teachers unpack/introduce, define, explain, and represent the concept of fractions to Grade 6 learners?

This study revealed that Eddy merely engaged in **explaining** procedures to learners when teaching fractions. He was spoon-feeding learners with mathematical rules; he wanted learners to memorise the rules without understanding where the rules came from. The memorised rules will automatically be forgotten in the long run. Stohlmann et al. (2013) advised teachers not to teach the procedural way of working out with fractions first because if they do so, the learners are **less likely** to master the fraction concept. Stohlmann et al. (2013) further highlighted that teachers should refrain at all cost from encouraging memorisation of rules. Ball and Bass (2005) also added that for teachers to teach mathematics well, they need to unpack or decompress their mathematical ideas to be accessible to learners, which implies that for teachers to teach the concept of fractions, they should know what is expected of them. Skemp (1976) pointed out that instrumental understanding refers merely to be able to apply a sequence of steps without knowing why they are being applied in that way, or what they mean, that is, rules without reasons. In contrast, relational understanding is knowing what to do and why, which means that learners should be told where and how the rules originated. Hiebert (1996) also added that mathematical tasks that encourage learners to use procedures that are not actively linked to meaning or that consists of memorisation are viewed as lower-level cognitive demands in the learning of mathematics fractional concepts.

Eddy's learners were passive participants in the class. Constructivism (Piaget, 1964) perceive learners as creators of their own learning and as active participants in the learning process, which means Eddy's learners were supposed to discover rules on their own, making sense of mathematics.

The study revealed that Eddy failed in providing learners with the opportunity to discover the mathematical rules on their own. They were just sitting and listening to what their teachers were saying. His lessons were teacher-centred with learner's passive participants. Few learners were active in answering the questions posed to them, but the majority wase passive. *Eddie asked: "Learners, how do we know that a fraction is [a] common fraction?"* L1 answered by saying: "*We know if there is a top number and a bottom number*." In contrast, Blaise (2011) stated that teachers use the teacher-centred approach with direct instruction in behaviourism.

In his first lesson, comparing and ordering of fractions, it was evidenced that he used procedures, routines and rules requiring fractions to have the same denominator. For example, he said, "*when comparing fractions with different numerators and denominators, we should make them to have the same denominators by looking for the LCM"*. For him to develop the concept of comparing and ordering fractions, he should have used different models such as linear, circular models to represent the concept meaningfully. His old-fashioned way of teaching may lead learners to make mistakes which can lead them to misconceptions. Sarwadi and Shahrill (2014) embrace the Piagetian view that when learners fail to assimilate or accommodate, a gap is formed in the learning of the concept and this leads to the birth of misconceptions coming into being.

Although Eddy used a fraction chart in his second lesson as one of the models to represent equivalent fractions the use of the fraction chart was not that very useful because the teacher put it on the board and instead of using it he simply told his learners that it was a fraction chart that would help them find equivalent fractions. Instead of developing the concept using the chart, he continued to teach them the procedure of how to find equivalent fractions. Eddy stated, "*we can also find the equivalent fractions by multiplying or dividing the numerator and the denominator of a fraction by the same whole number for example* $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$; $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$." He immediately moved to a rule of getting an equivalent fraction by saying: "*By multiplying or dividing the numerator and the denominator of a fraction*". As in the first lesson, his teaching was procedural, and teacher orientated. His way of teaching leads learners to be passive participants of learning. They were not allowed to discover equivalent fractions on their own as perceived by constructivists. Constructivism view learners as creators of their own knowledge and understanding, they make sense of their learning, so Eddy served as a barrier of his learners' understanding of the concept.

Likewise, in his third lesson of changing common fractions to decimal fractions, he used rules and procedure to teach. He spent a lot of time explaining what common fraction and decimals fractions are. He further taught learners the mathematical rules about how to change a common fraction to decimal fractions. Eddy stated:

This was what he did in comparing and finding equivalent fractions. Eddy merely engaged in explaining mathematical rules to learners. Eddy's teaching contrasts with what Ball et al. (2008) and Shulman (1986) pointed out, namely that teaching involves more than knowing the subject matter, but instead entails transforming the knowledge to the learners in an understandable manner. They further argued that besides knowing the content well, the teacher needs to know how to deliver the content and be understood. They further alluded that knowing the subject matter is not enough, knowledge should be beyond the subject matter.

Furthermore, Eddy's teaching did not resonate with the theory of constructivism (Piaget 1964), which suggests the teacher's role is that of the facilitator and motivator. In his teaching, Eddy was not facilitating learning (Machaba, 2018), where learners were supposed to take control of their learning. His teaching was more teacher-centred than learner-centred (Machaba, 2018). He failed to allow learners to construct their knowledge and understanding using their existing experiences. In all Eddy's lessons, the fraction concepts were not fully developed; only the mathematical rules were emphasised, which is regarded as poor delivery of the content. Van de Walle (2016) highlighted that rushing to procedures will lead learners to make errors and misconceptions, and that will hamper their conceptual understanding. Teacher Eddy should have developed the fraction concepts using different models for learners to have a solid and deeper understanding of fractional concepts. The researcher is of the view that Eddy's way of teaching indicated that his mathematical knowledge of teaching appeared to be insufficient because in all his lessons observed, he failed to develop the concept of fractions to learners meaningfully.

As indicated in Chapter 4, Eddy was rated 60% in the aspects of defining and explaining the concept. This suggests that although Eddy could not unpack the concept in such a way that learners develop conceptual understanding, he could explain and define fractional concepts in such a way that learners would understand. In Ball et al.'s (2008) terms, we could say Eddy posed the *content knowledge*, but not a specialised content knowledge that needs the teacher to have a deeper understanding of fractions and allows the teacher to explain new ideas, work out fractional mathematics problems in a variety of ways and analyse learners 'explanations.

Olivier (1989) indicated that errors are indicators of the existence of misconceptions and happen as a result of many factors, for example, the way teachers are teaching fractions. From the constructivists' point of view, errors are intelligent constructs of knowledge by learners.

Ball et al. (2008) further stated that a teacher should have mathematical knowledge of the subject they are teaching. The teacher should know the subject matter. Shulman's (1986) notion of pedagogical content knowledge (PCK) is viewed as the knowledge of teaching the subject matter, the knowledge of formulating and presenting the subject matter to be comprehensible to learners. The frameworks of Ball and Shulman further stated that teachers should know their learners and understand their common difficulties, errors, and misconceptions, which means that they should have a specialised content knowledge (SCK) and knowledge of the curriculum and students (KCS). Teachers should use their knowledge of mathematics to unpack, represent, formulate, explain, illustrate, and make the concepts understandable to learners. Ball et al. (2008) alluded that mathematics teachers should use their mathematical knowledge to unpack the fractional concepts and deliver to learners in a way they fully comprehend.

Analysis revealed that Rose wanted her learners to develop a conceptual understanding and recognise the fraction concept independently. From constructivism's perspective, learners construct knowledge and understanding on their own connecting their web of ideas. In Rose's first lesson of fraction concepts, she managed to develop the concept well by involving learners in an activity where each learner was folding and unfolding the A4 paper. In this lesson, learners were active participants of their learning, and this resonates with constructivism as it specifies that learners are active agents of their learning process. Rose knows that learners should master the fraction concept first before being introduced to algorithms. This is supported by Van de Walle (2009) when he alluded that teachers should not rush to algorithms as it can delay learners' understanding of the concept.

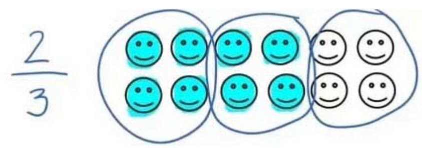
Rose used verbal expression and learners were able to tell that they were having half of the A4 paper, or a fourth, sixth etc. of the A4 paper. Learners discovered the concept of fractions verbally on their own. Analysis revealed that there was clear indication that learners were developing the concept of fractions with their teacher. Learners were actively engaged in this activity, and they could discover the fraction concepts on their own. Analysis revealed that Rose used linear modelling when her learners were doing the folding and unfolding of the A-4 paper in developing and naming of fractions and it supported by Petit, Laird, Marsden and Ebby (2010) stating that the length model for fractional concepts is important in developing learners' understanding of fractions, naming fractions such as one-sixth of the A4-paper etc. This is supported by Lamon (2008) pointing out that naming of fractions helps learners use the correct language to understand the concept of fractions.

In her second lesson of fractional notations, she used the circular area model using an apple to demonstrate part, whole and equal sized parts. This resonates with Van de Walle (2007) who states that teachers should emphasise fractional parts as equal shares or equal sized portions of a whole or unit.

After she realised that her learners understood the concept, she moved to fractional notation where she used an apple as a model. This resonates with Cramer et al. (2008), supporting the idea of using a circular area model because they alluded that circular area models are effective in developing the fractional concept. Rose cut an apple in front of all the learners and ordered them to look at what she was doing. She said: "*Look at me all of you, I cut it like this (showing learners how she cut it).*" The apple was cut or divided into four parts and one part of the apple was given to a learner in the class, with Rose stating: "*I give Mpho this part (referring to one part).*" . She then asks them probing questions about the parts of the apple such as "*How many parts was this apple divided?*" In her teaching, her learners discovered that when we talk about fractions, we are actually referring to **equal sized parts**. One of her learners responded that the apple was cut into four **equal parts.** L5 commented: *"It was divided into four **equal parts.**"* She then gave the notation of the concept of fractions symbolically, for example $\frac{1}{4}$. This resonates with Van de Walle (2016), highlighting that representation at this stage is **symbolic.** This is where she should have told learners that $\frac{1}{4}$ is named one-fourth of an apple not one over four. This resonates with Siebert and Gaskin (2006) and Cramer and Whitney (2010), highlighting that teachers should avoid the phrase or the use the format of one out of two, two out of six to learners.

The learners understood that the top number represents parts that were used, considered, or taken out whereas the bottom number indicates the number of equal parts into which the apple was cut. The bottom number also gives the fraction a name, for example fourth. Rose unpacked the fraction concept meaningfully, as is supported by Ball and Bass (2005), highlighting that for teachers to teach mathematics well they need to unpack or decompress their mathematical ideas so that they can be accessible to learners. Rose unpacked the fraction concept in a way learner could understand. Shulman's (1986) notion of pedagogical content knowledge is viewed as the knowledge of teaching the subject matter, formulating, and presenting the subject matter to be understandable to learners. Ball et al. (2008) alluded that mathematics teachers should use their mathematical knowledge to unpack the fractional concepts and deliver it to leaners in a way that learners fully comprehend, which Rose managed to do.

In her third lesson, proper and improper fractions, analysis revealed that she wanted her learners to know what proper and improper fractions are by asking questions to find out if they know about this concept. She merely engaged in explaining the concept for example: *"they are called proper fractions and the rest on the board are called improper fractions because their numerators are bigger than the denominator"* In this lesson, Rose could have used model representation showing learners proper and improper fractions. This is supported by Van de Walle, Karp and Bay-Williams (2010); with Cramer and Wyberg (2009) highlighting that concrete representations are key for learners to comprehension of fractions. Example of proper and improper fractions are as follows.



proper fraction

Figure 4.1: Proper Fraction

An example $\frac{8}{5}$

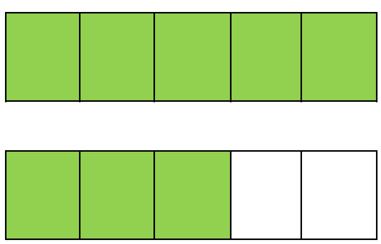


Figure 4.2: Improper Fraction

The study revealed that teacher Rose, who developed the conceptual understanding of fractions for learners the most effectively, appeared to be the most successful teacher of the three participating teachers. The way she unpacked, represented, defined, and explained the concept of fractions to Grade 6 learners is appreciated.

As specified in Chapter 4, Rose was rated 80% in defining, explaining, and representing the concept. This suggests Rose unpacked, explained, defined, and represented the concept in such a way that learners developed a conceptual understanding of the concept of fractions. This resonates with Ball et al.'s (2008) notion highlighting that knowing the subject well is not good enough for teaching as teachers need to know mathematics in ways useful for making mathematical sense of learners work and choosing powerful ways of representing the subject so that it is understandable to learners.

Analysis revealed that William introduced his first lesson by posing a question to learners: *"What is a proper fraction? Can you give an example of a proper fraction"?*

Posing a question was a good starting point. Asking questions arouse learners' prior knowledge and link what they have already learned to what they are yet to learn. This is justified by Tall (1989) when referring to learners' prior knowledge as cognitive roots to develop a concept in connecting and laying the foundation for learners' conceptual thinking. Essien (2009) emphasises the importance of starting with learners' prior knowledge and connecting it with new knowledge.

The study is pinned on constructivism, where a teacher is perceived as the facilitator of learning. William used a behaviourist approach as he was dominating the lesson. Blaise (2011) stated that in behaviourism, teachers use a teacher-centred approach with direct instruction. In a behaviourist approach, learning and teaching is teacher orientated, with learners being passive agents of their learning. This agrees with William's approach to his teaching. His teaching was more teacher-centred than learner-centred (Machaba, 2018). His teaching was procedural; he was dominating the lesson explaining rules, "[He wrote on the board $2\frac{1}{3}$], *you see this is a mixed number because it has the whole number and a proper fraction, so changing it to improper fractions the steps we have to follow are this: We say denominator multiplied by the whole number plus the numerator and the denominator remain the same, it does not change. . Do you understand good people? It is like this,* $\frac{3 \times 2+1}{3} = \frac{7}{3}$."

His teaching encouraged memorisation of rules which may be forgotten at any time. The literature recommends teachers should use various models to represent a concept to develop a deep and strong conceptual understanding rather than just explaining the procedures or provide step-by-step procedures. Cramer and Wyberg (2009) indicated that the effective use of models plays a significant role in fraction tasks. Van de Walle (2004) also highlighted the use of models in the classroom to help learners develop new concepts, make connections between concepts and symbols to assess learners' understanding.

Ball et al. (2008) further referred to mathematical knowledge for teaching as the knowledge required in everyday tasks such as explaining, defining, and representing concepts to learners. They indicated that teachers require a great deal of knowledge and expertise in carrying out the work of teaching the subject matter. Shulman (1986) alerted teachers not to separate content knowledge and pedagogy because both are needed to enable the teachers to carry out their work effectively. He argues that teachers need to know and understand more of their subject than other users because he reasoned that teaching requires a transformation of knowledge into a form that learners can understand.

Shulman's (1986) notion of PCK), is viewed as the knowledge of mathematics a teacher uses to unpack, represent, formulate, explain, illustrate, and make the concepts understandable to learners. Shulman's notion of PCK supports the idea that mathematics teachers should use their mathematical knowledge to unpack the fractional concepts and deliver to them to learners in a way that they fully comprehend.

William considered using the area model when teaching about comparing fractions, but in the two lessons observed, he did not develop the fraction concept for learners to master. His two observed lessons were procedural. He merely engaged in **defining and explaining** the mathematical rules. He encouraged learners to master rules at the expense of developing the concept. His teaching encouraged memorisation rather than conceptual understanding. He only used representation when comparing fractions. Fractions can only be compared if they are parts of the same whole. According to Van de Walle (2016), when comparing fractions, we want to check which part of the same whole is bigger or smaller than another part. William did it well because his learners were able to compare fractions.

Analysis revealed how William taught his learners about the addition of fractions, where the denominators are not multiples of the other. He provided learners with mathematical rules, not even explaining to learners how the rules came to be. He encouraged memorization although he could have used representations for learners to understand better. Van de Walle (2009) states that if learners have developed a deep understanding of the concept of fraction equivalence, they should use the knowledge gained in engaging with addition and subtraction of fractions. This implies that if learners understood how to find the equivalent fractions independently, then the information assimilated would be used when they were adding or subtracting fractions. The traditional way of teaching about LCM should at all cost be avoided. The literature discourages the use of LCM because it was too traditional and restrictive.

He could have used any model to represent the two fractions by choosing the whole that both 2 and 6 could divide into, then show the whole e. g. six blocks, and shade $\frac{1}{2} + \frac{2}{6}$ of the whole. Cramer et al. (2008) highlighted that teachers should use a multiple of representations such as area models, length/linear models and set models to deepen learners' understanding of fractions and to encourage more abstract thinking. Among the models, the length model may be used as follows:

$\frac{1}{2}$

$\frac{1}{2}$	$\frac{1}{2}$

$\frac{2}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{2}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Added to $\frac{2}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Equals $\frac{5}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Figure 4.3: Length model

In William's lesson of changing mixed fractions to improper fractions, the researcher is suggesting that he could have represented them on any model of representation, for example: Illustration

$3\frac{2}{5} = \frac{17}{5}$

$\frac{5}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{5}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{5}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{2}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Figure 4.4: Length Model

Askew (2000) and Van de Walle (2009) highlighted that a global understanding of the concept of fractions and a deep sense of their purposes are more important than learning a set of rules. They further indicate that the introduction of rules should be delayed until learners have arrived at a complete understanding of the concept. This means that before learners are introduced to rules, they should first have a strong and solid understanding of the fraction concept. Developing fractional concepts require a teacher to use different representations, using models such as circular, rectangular, square, area, length and set models for learners to visualise the concept of fractions (Van de Walle, 2016). The concept can be represented diagrammatically, symbolically, verbally or narratively. Learners who mastered the concept of fraction are likely to perform well in the computation of fractions. Stohlmann et al. (2013) highlighted that if learners are taught the procedural way of working out with fractions first, learners are **less likely** to master the fraction concept. They further pointed out that understanding the fraction concept first is **more powerful** and more generative than remembering mathematical procedures.

5.2.2. How do teachers work with learners' mathematical ideas?

In this research question, it was stated in Chapter 2 that working with learners 'mathematical ideas require a teacher to engage learners actively in a class discussion. It requires a teacher to provide learners with challenging mathematical tasks that stimulate their thinking. It requires learners to pose questions for clarity and respond to questions posed by the teacher. Moreover, in this research question, the teacher is expected to pose different types of questions such as probing questions, open-ended questions, and follow up questions to cater to learners' cognitive mathematical thinking and reasoning. Open-ended mathematical questions stimulate and strengthen learners' mathematical thinking. Probing questions promote learning and push up learners to think more deeply.

In Eddy's first lesson of comparing and ordering of fractions, learners were able to compare fractions using the routine procedure taught by their teacher. ***L4*** *"From our discussion here are the answers. The LCM =30*

$a) \frac{1}{10} \times \frac{3}{3} = \mathbf{\frac{3}{30}}\ ; \frac{3}{5} \times \frac{6}{6} = \mathbf{\frac{18}{30}}\ ; \frac{2}{15} \times \frac{2}{2} = \mathbf{\frac{4}{30}}$

Answer: $\mathbf{\frac{3}{30}\ ; \frac{4}{30}\ ; \frac{18}{30}}$

$b) \frac{2}{3} \times \frac{9}{9} = \mathbf{\frac{18}{27}}\ ; \frac{4}{9} \times \frac{3}{3} = \mathbf{\frac{12}{27}}\ ; \frac{1}{27} \times \frac{1}{1} = \mathbf{\frac{1}{27}}$

Answer: $\frac{1}{27}\ ; \frac{12}{27}\ ; \frac{18}{27}$"

The literature discouraged this kind of teaching because it encourages memorization of rules. Van de Walle (2016) encourages teachers to use various models at their disposal because the LCM rule has no conceptual basis for learners to understand fractions. The use of the LCM rule encourages learners to be blind followers of rules. I reiterate that Skemp (1976) pointed out that instrumental understanding refers merely to being able to apply a series of steps without knowing why they are being applied in that way, or what they mean. This implies using "rules without reasons", whereas relational understanding is knowing what to do and why, which means that learners should be told where and how the rules are derived.

Though learners answered the questions as above, they did not pose questions for clarity. No probing questions were asked. Moyer and Milewicz (2002) indicated that posing questions to learners timely, facilitate the cognitive growth of learners. Responding to learner questioning was not there because learners were not given that chance to ask questions.

In Eddy's second lesson, equivalent fractions, learners did not pose questions to their teacher for clarity, but there was an indication of a learner asking his classmates questions. ***L1:*** *"One of the learners go to the board and ask his colleagues to give answers. Which fraction is equivalent to* $\frac{2}{9}$ ***?"***

There was evidence of asking probing questions from the teacher such as, *"how do we know that a fraction is common fraction?"* There was an indication of learners answering questions posed by the teacher "*find three equivalents fractions of each of these fractions.*" Working with learners' mathematical ideas was not satisfactorily applied.

In his third lesson of changing common fractions to decimal fractions, there was an indication of probing questions such as "*What is the difference between common fraction and decimal fraction?" T*here was an indication of learners answering questions through writing ***Eddy*** *called L1, L2 , L3 and L4 to come and write answers on the board.* No learner questioning was evident. Working with learner's mathematical ideas was not fully applied. As specified in Chapter 4, Eddy got 40% in this research question because working with learners' mathematical ideas was not satisfactorily applied. One of the factors that might contribute to Eddy not working with learners' mathematical ideas meaningfully, might be how the learners were sitting due to the Covid-19 regulations, because desks were aligned in rows with seating space for one learner per desk, keeping 1.5 m distancing.

In Rose's first lesson of fractional concepts, learners were actively involved in folding and unfolding the A4 paper, discovering equal parts using verbal expressions. This is supported by constructivism views that learners are active participants of their learning. They were making sense of the mathematical ideas, such as realising that when they fold and unfold the A4-paper, equal sized parts were recognised.

The teacher posed probing questions to learners such as "*What do you notice?*" and *"tell us, what can you say about the A4 paper?"* In this lesson Rose was facilitating learning and learners were constructing their own ideas/knowledge and understanding as supported by the constructivist theory of learning. The teacher performed the activity with learners in a way that learners comprehended. This resonate with what Ball et al. (2005) alluded that the teacher should unpack, define ,explain and represent the fractional concept in a way that is accessible to learners, and this goes hand in hand with Shulman's (1986) notion of pedagogical content knowledge, stating that the teacher needs the knowledge on how to convey subject knowledge and to be understood. Rose displayed all these frameworks and use of the constructivism theory as well.

In this research question, Rose was scored 53% because there was an absence of learners asking questions as a result the research questions was not fully applied. One of the elements that appeared to contribute to this research question was the situation of Covid-19 regulations. Teacher Rose did not group learners to discuss or brainstorm because of how they were sitting due to COVID -19 regulations. Desks were aligned in rows with seating space for one learner per desk, due to COVID -19 regulations of 1.5 m distancing.

The analysis revealed that in her second lesson of fractional notation, learner involvement was not the same as in her first lesson. She was dominating the lesson because she was the one cutting an apple as a circular model, to introduce the concept of numerator and denominator. The apple was cut into four equal parts, and one part was used.

She was using the behaviourist approach because her teaching was centred around her transmitting the information to learners instead of constructivism approach. Her learners were inactive participants of learning. Though she was using an apple as a circular model, her learners were not that much involved except for answering questions posed to them such as: "*How many parts was this apple divided?"*

In her third lesson, Rose as in the second lesson, was dominating the class, explaining the concepts such as: "The *numerator shows how many parts were used, and denominator indicates how many parts the whole was divided"*. Learners were inactive participants of their own learning. In this lesson probing questioning was not evident. Questions asked was of low order level that did not demand deep thinking "*Last time we discovered what the numerator and denominators are ,do you still remember what they are?"*

Boaler and Brodie (2004) pointed out that questioning is an important teaching strategy in establishing a classroom atmosphere conducive to developing learners' mathematical thinking. This was not evident in Rose's third lesson because the learners were not encouraged to ask questions. Working with learners' mathematical ideas was not much in evidence because of learners' passiveness during their learning. The teacher was only engaged in defining, explaining and questioning aspects of teaching. As specified in Chapter 4, Rose was awarded 53% in this research question because working with learners' mathematical ideas was not satisfactorily applied.

In William's first lesson of converting mixed fractions to improper fractions, learners were involved through questioning. Learners were also answering the activity given using routine procedures taught by their teacher. *L5: (after a while) (a learner was pointed out and ordered to go to the board and show others how he got the answer.)* " $4\frac{2}{3} = \frac{3 \times 4+2}{3}$

$$= \frac{14}{3}$$"

Teachers should avoid encouraging learners to memorise rules. This is supported by Van de Walle (2009), suggesting that teachers should not rush to teach algorithms as it can delay learners' understanding of the concept.

There was no indication of learners asking questions. Working with learners' mathematical ideas was not satisfactory.

In William 's second lesson of comparing fractions, there was an indication that learners were asked questions: "*What is an improper fraction? What is mixed fraction?*" There was no learner questioning.

Learners were able to compare fractions using a linear model used. Working with learners' ideas was not satisfactorily done.

In William's third lesson of the topic: Addition of fractions with unlike denominators, working with learners' mathematical ideas were not evident at all. The teacher was just explaining the procedure of how to add fractions. He merely engaged an explanation of rules. There was no learner questioning. As specified in Chapter 4, William received 40% in this research question because working with learners' mathematical ideas was not satisfactorily applied. One of the factors that might have contributed to William not working with learners' mathematical ideas fruitfully might be how learners were sitting due to obeying Covid-19 regulations because desks were aligned in rows with seating space for one learner per desk, keeping 1.5 m distancing.

Working with learner mathematical ideas was not satisfactorily applied by any of the three teachers observed. One of the factors that might have contributed to these teachers not grouping learners might be how learners were seated during lesson presentation. Seating arrangement in the three schools observed were the same. Desks were aligned in rows with seating space for one learner per desk, due to COVID -19 regulations of 1.5 m distancing. Each class was only occupied by 20 learners. They were sitting individually because of the Covid-19 situation and as a result, it was not possible for learners working close to one another as a group or in pairs. The researcher is in a suggestion that learners could discuss fractions as a class as depicted in Figure **5-5** (Discussion group) as can seen in Figure 5.5 where learners were keeping their distance, obeying the regulations, discussing while wearing their face masks.

Figure 4.5: Discussion group

In conclusion, the study revealed that the aspect of working with learners' mathematical ideas was not satisfactorily evident.

5.2.3. How do teachers restructure their mathematical tasks?

It was discovered that the three teachers observed did not restructure the mathematical tasks for the Grade 6 learners. Restructuring mathematical tasks was not evident. The three observed teachers were rated 0% because none of them engaged with this research question.

The study revealed that of the three teachers observed, none of them met the six aspects of teaching. Working with learners' mathematical ideas was not convincing. They only engaged with defining, explaining and representations of the fraction concept.

The situation that seemed to be true is that the teachers seemed to lack mathematical knowledge in developing the concept of fractions. Any incorrect teaching of fractions can affect learners' understanding of the topic and become a lifetime problem. Ultimately this will influence their schooling, tertiary education and working situations. Shulman (1986) stated that teachers must have a knowledge base specific to the subject matter.

Let us take note that the three teachers observed have been teaching mathematics for many years, which implies that they have been teaching like they did this year. Let this be a concern to the Department of Basic Education to look into this matter intensively. This is supported by Ma (1999) when he indicated that teachers have insufficient knowledge of fractions necessary for classroom teaching. In support of him, it appeared that Eddy and William have been teaching like they were observed all this while. Let us think about numbers and the numbers of learners taught by them. This is supported by the following statement**:** I reiterate the statement: Any incorrect teaching of fractions can affect learners' understanding of the topic and can become a lifetime problem. Ultimately this will influence their schooling, tertiary education and working situations. Rushing to teach algorithms can impede learners' understanding of the concept development (Van de Walle, 2009). Shulman (1986) stated that teachers must have a knowledge base specific to the subject matter. This highlights that teachers should themselves know how to develop the conceptual understanding of learners.

On a little positive note, one of the findings that gives hope is that the observed teachers have defined, explained concepts to learners even though some of the explanation were procedural and not conceptual, and the teachers managed to use a fraction wall, A4 paper, an area model, and an apple when presenting the lesson. This is a good representation because learners seem to learn far much better when models are used.

The study revealed that the three teachers did not meet all the aspects/categories, or the indicators as expected. These shortcomings appeared to be an indication of the teachers' lack of mathematical knowledge for teaching. Knowing how to add fractions, comparing fractions, for example, is not enough. Knowledge should be beyond the subject matter. Knowing to define and explain fractions correctly is not enough for teaching because the mathematical work of teaching requires a teacher to engage with six aspects condensed by Kazima (2008) from the eight aspects identified by Ball et al. (2004).

5.3. RECOMMENDATIONS

From the findings, the researcher came up with the following recommendations.

Regarding the use of models, it is recommended that teachers should use different representations such as area models, circular, rectangular, set models and length models for developing the concept of fractions fruitfully. Cramer and Wyberg (2009) indicated that the effective use of models in fraction tasks plays a significant role. Learners seem to explore when a variety of models are used, which builds learners' understanding of fractions (Cramer & Wyberg, 2009).

It is therefore recommended that the procurement committee of the schools purchase different fractional charts for the teaching of fractions to be effective. It is also recommended that the school should have internet facilities to download information related to fractions.

Teachers were using the whole-class teaching approach when teaching fractions, which is a teacher-centred approach. This implies that teachers find it difficult to shift from the traditional way of teaching. Teachers are using the chalk and talk way of teaching. It is therefore recommended that the DBE should define the role of teachers in the classroom as outlined by the new approach of teaching. It is also recommended that the DBE should emphasise learner-talk in-class teaching to encourage learners working as a team. Teamwork promotes collaboration and interpersonal relationships among learners.

To address the issue of unpacking and introducing the concept of fractions, it is important that we as teachers fully consider learners' prior knowledge of the concept of fractions that learners should have acquired in earlier grades that will provide a foundation for the topic of the day. We should teach from the known to the unknown. We should teach learners to master the concept of fractions before providing mathematical rules. Teachers should at all-time develop the conceptual understanding of learners. It is recommended that learners discover the concepts or rules on their own with the teacher's assistance. Therefore, it is recommended that the mathematics subject committees in consultation of the managers of the school organise workshops for mathematics teachers to assist one another. Stohlmann et al. (2013) highlighted that if learners are taught the procedural way of working with fractions first, learners are less likely to master the fraction concept. They further pointed out that understanding the fraction concept first is more powerful and more generative than remembering mathematical procedures.

It is important to pose high order questions or probing questions to the learners to challenge their thinking or their ideas. Teachers should allow learners to learn to listen, communicate, think, reason logically and apply the mathematical knowledge gained. Teachers are therefore advised to use Karin and Boaler's (2004)'kind of questions that they can use when dealing with learner questioning.

On the issue of learners as passive participants in the class, it is recommended that teachers consider learner=talk and provide challenging fractional mathematical tasks that promote genuine engagement among learners.

It is recommended that the development support group (DSG) consisting of the immediate senior of the teacher and his/her peer teaching the same subject, organise a meeting and deal with issues that may impede learners from mastering the fractional concepts.

On the issue of teachers not meeting all the six categories of the mathematical work of teaching, it is recommended that teachers consider all six aspects of teaching because, during lesson presentation, teachers should define, explain, represent concepts to learners. They should also work with learners' mathematical ideas and restructure the mathematical tasks by scaling up and scaling down the mathematical tasks to cater to all the cognitive levels of learners.

It is recommended that teachers unpack the fractional concepts well for learners to have a mastery of fractions before the procedural way of working them out.

5.4. CONCLUSION

The research study was rooted on constructivism theory, Ball et al.'s (2008) framework of mathematical knowledge for teaching and Shulman's (1986) idea of pedagogical content knowledge. Ball et al. (2008) emphasised that teachers require a great deal of knowledge and expertise in teaching the subject matter, which is fractions. Mathematical knowledge for teaching means the mathematical knowledge common to everyone working in diverse careers and the subject matter knowledge that supports teaching, meaning the fractional mathematical knowledge that supports the specific teaching.

The research study highlights the fact that teachers should develop the concept of fractions using different model representations in order for learners to have a strong ,solid and deeper understanding of the concept of fractions. It also highlighted the idea of developing the fraction concept first before applying the procedure. It also makes teachers aware that making learners memorise the rules is not recommended according to the researchers mentioned in this study. **I reiterate** one of them is Stohlmann et al. (2013) who asserted that if learners are taught the procedural way of working with fractions first, learners are less likely to master the fraction concept. They further pointed out that understanding the fraction concept first is more powerful and more generative than remembering mathematical procedures.

The researcher is in a view that teachers should encourage learner participation in lessons through open discussion and should avoid teacher-centred teaching approaches. This implies that teachers should refrain from being the only sole source of information but allow learners to take a lead of their own learning and this is supported by constructivists theory.

The researcher suggest that teachers should listen to learners in order to understand their shortcomings for rectification, they should also reinforce learners' responses in a more human manner. The usage of models' representation is encouraged to assist learners in understanding the fractional concepts meaningfully. This study encourages teachers to be facilitators of learning, fosters critical thinking ,promote motivated learners to be independent and be creators of their own learning.

5.5. REFLECTION

This chapter concludes all processes elaborated in the research study from Chapter 1 to 5. Chapter 1, served as the background of the study that provides a general overview of the study, orientating the reader to the study. Chapter 1 was followed by Chapter 2 on the theoretical framework of constructivism, Ball et al.'s (2008) framework of mathematical knowledge for teaching (MKT) and Shulman's (1986) pedagogical content knowledge (PCK) as well as the literature review. The literature review serves as the bloodstream of the research study as it is found in almost every part of the study and serves to answer the research questions. It was followed by Chapter 3 on the methodology and outlines the type of methods used in data collection. The pilot study is also discussed in Chapter 3. The data was analysed and discussed in Chapter4. Chapter 5 elaborates on the findings, recommendations, conclusions as well as a reflection of the study.

This study has been an eye-opener in many ways, the importance of developing the fraction concept before teaching algorithms and the significance of using a variety of models for representing the concept. The recommendations given in this research study should be a "must implement" if the teaching of fractions is to be improved.

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APPENDICES

APPENDIX A: OFFICIAL LETTER OF REGISTRATION CONFIRMATION

2220

[illegible]
P O BOX 2145
CHUENESPOORT
0745

STUDENT NUMBER : [illegible]

ENQUIRIES TEL : [illegible]
FAX : [illegible]
eMAIL : [illegible]

[illegible]

Dear Student

I hereby confirm that you have been registered for the current academic year as follows:

Proposed Qualification: MED MATHS EDUC ([illegible])

CODE	PAPER	S NAME OF STUDY UNIT	NQF crdts	LANG.	PROVISIONAL EXAMINATION EXAM.DATE	CENTRE(PLACE)
Study units registered without formal exams:						
[illegible]		MEd - Mathematics Education (Dissertation)	**	E		

You are referred to the "myRegistration" brochure regarding fees that are forfeited on cancellation of any study units.

To avoid cancellation of your registration or examination entry and forfeiting your minimum initial payment, you must submit the following to the Registrar (Academic) by return of mail:
214 A copy of your identity document, passport or birth certificate reflecting your full names and date of birth.

\# Your attention is drawn to University rules and regulations (www.unisa.ac.za/register).
Please note the new requirements for reregistration and the number of credits per year which state that students registered for the first time from 2013, must complete 36 NQF credits in the first year of study, and thereafter must complete 48 NQF credits per year.
Students registered for the MBA, MBL and DBL degrees must visit the SBL's ESOnline for study material and other important information.
Readmission rules for Honours: Note that in terms of the Unisa Admission Policy academic activity must be demonstrated to the satisfaction of the University during each year of study. If you fail to meet this requirement in the first year of study, you will be admitted to another year of study. After a second year of not demonstrating academic activity to the satisfaction of the University, you will not be re-admitted, except with the express approval of the Executive Dean of the College in which you are registered. Note too, that this study programme must be completed within three years. Non-compliance will result in your academic exclusion, and you will therefore not be allowed to re-register for a qualification at the same level on the National Qualifications Framework in the same College for a period of five years after such exclusion, after which you will have to re-apply for admission to any such qualification.
Readmission rules for M&D: Note that in terms of the Unisa Admission Policy, a candidate must complete a Master's qualification within three years. Under exceptional circumstances and on recommendation of the Executive Dean, a candidate may be allowed an extra (fourth) year to complete the qualification. For a Doctoral degree, a candidate must complete the study programme within six years. Under exceptional circumstances, and on recommendation by the Executive Dean, a candidate may be allowed an extra (seventh) year to complete the qualification.

CREDIT BALANCE IN STUDY ACCOUNT: 21.00-

Yours faithfully,

Dr F Goolam
Registrar

[illegible]

UNIVERSITEIT VAN SUID-AFRIKA
PRETORIA
UNIVERSITY OF SOUTH AFRICA

University of South Africa
[illegible]
[illegible]
Telephone: [illegible] Facsimile: [illegible]
[illegible]

APPENDIX B: OBSERVATION SCHEDULE / GUIDE

<table>
<tr><td>ACTIVITY OBSERVED</td><td></td><td>GRADE 6</td></tr>
<tr><td colspan="3">Purpose of observation:
To explore the mathematical knowledge for teaching the concept of fractions to Grade 6 Learners</td></tr>
<tr><td>Observation Category</td><td>Expectations/ Strength of the category/Indicators</td><td>Action observed</td></tr>
<tr><td>1. How Grade 6 teachers unpack the concept of fractions to their teaching?</td><td>Teacher introducing a concept.
Teachers defining the concept.
Teacher explaining the concept.
Teachers representation of the concept</td><td></td></tr>
<tr><td>2. How do teachers work with learners ‘mathematical ideas when teaching fractions?</td><td>Do teachers encourage learner talk/learner -centeredness?
Selecting tasks of high cognitive demand
Genuine engagement with learners
Teacher listening-Evaluative, generative, or interpretive.
Learners asking questions.
Teacher listening to learners questioning.
Leading /guiding questions
Follow up questions.
Probing questions
Specific questions
Open ended questions
Teacher listening-Evaluative, generative, or interpretive.
Do teachers involve other learners</td><td></td></tr>
<tr><td>5. How do teachers restructure mathematical tasks to be understood by learner?</td><td>Scaling up or down a mathematical problem task
Shifting of appropriate mathematical outcome</td><td></td></tr>
</table>

APPENDIX C: REQUESTING PERMISSION FROM CAPRICORN SOUTH DISTRICT DIRECTOR

38 Gordon Verster Street
The Willows
Pretoria
0184

The District Director
Department of Education
LEBOWAKGOMO
0737

REF: PERMISSION TO CONDUCT RESEARCH IN CAPRICORN SOUTH DISTRICT

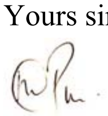
Dear Sir/Madam

I, Phuti Margaret Moloto, am doing a research study under the supervision of Prof FM Machaba in the Department of Mathematics Education at the University of South Africa(UNISA) We (my supervisor and I) are cordially inviting the Grade 6 Mathematics teachers in your District to participate in a study entitled, ***"Exploration of Mathematics knowledge for teaching fractions in Grade 6"***

The purpose of this study is to explore the mathematical knowledge in teaching fractions in Grade 6, for the exploration to contribute meaningfully to the improvement of teaching and learning. The study will discover the mathematical knowledge that teachers possess in carrying out their work effectively. There are no potential risks involved. Participants will be briefed on the nature and purpose of the study. The researcher will also seek consent from the teachers.

If you would like to be informed of the final findings, kindly contact Phuti Margaret Moloto on 072 465 7108 or email at margaretphutij@gmail.com. Should you have any concerns about the way in which the research has been conducted, you can contact Prof. M.F. Machaba on (012) 429 8582 or email emachamf@unisa.ac.za.

Yours sincerely



Phuti Margaret Moloto

APPENDIX D: REQUESTING PERMISSION FROM THE PRINCIPAL/SGB

Date ________________

THE PRINCIPAL

____

____

____

Dear Sir/Madam

I, Phuti Margaret Moloto, am doing a research study under the supervision of Prof FM Machaba in the Department of Mathematics Education at the University of South Africa(UNISA) We (my supervisor and I) are cordially inviting the Grade 6 Mathematics teachers in your District to participate in a study entitled, ***"Exploration of Mathematics knowledge for teaching fractions in Grade 6"***

The purpose of this study is to explore the mathematical knowledge in teaching fractions in Grade 6, for the exploration to contribute meaningfully to the improvement of teaching and learning. The study will discover the mathematical knowledge that teachers possess in carrying out their work effectively. The research study will take place in the classroom and the researcher will be observing the teacher teaching the concept of fractions to Grade 6 learners

There are no potential risks involved. Participants will be briefed on the nature and purpose of the study. The researcher will also seek consent from the teachers.

If you would like to be informed of the final findings, kindly contact Phuti Margaret Moloto on 072 465 7108 or email at margaretphutij@gmail.com. Should you have any concerns about the way in which the research has been conducted, you can contact Prof. M.F. Machaba on (012) 429 8582 or email emachamf@unisa.ac.za.

Yours sincerely

Phuti Margaret Moloto

APPENDIX E: ASSENT TO PARTICIPATE IN THIS STUDY (RETURN SLIP

Date ___________________

I, ____ (full name of participant), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read and understood the study as explained in the information sheet.

I have had enough opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to taking part in the observation process and in the interviews phase. I agree that all the data collected can be used in the research study, whilst keeping my results private and confidential.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) : ________________________________

Participant Signature :___________________ Date ____________________________

Researcher's Name & Surname (please print) : **Phuti Margaret Moloto**

Researcher's Signature Date___________________________

APPENDX F: LETTER TO REQUEST CONSENT FROM PARENTS(RETURN SLIP)

Dear Parents

Your son/daughter/child is invited to participate in the study entitled ***"An Exploration of Mathematical knowledge for teaching for Grade 6teachers in the teaching of fractions"***. I am undertaking this study of my master's research at the university of South Africa (Unisa).The purpose of the study The purpose of this study is to explore the mathematical knowledge in teaching fractions in Grade 6, for the exploration to contribute meaningfully to the improvement of teaching and learning. The study will discover the mathematical knowledge that teachers possess in carrying out their work effectively.

The research study will take place in the classroom and the researcher will be observing the teacher teaching the concept of fractions to Grade 6 learners. Learners will be listening to their teacher and participate by answering and posing questions from their teacher.

I am asking permission to include your child in this study because he/she is doing Garde 6.I am asking for permission to capture the information pertaining to your child 's understanding. Any information that is obtained about your child will remain confidential and will only disclosed with your permission. Her name and the name of the school will remain anonymous.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct advantage from participating in the study, neither you nor your child will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be part of the study now and change your mind later without any penalty.

In addition to your permission, your child must agree to participate in the study and you, and your child will also be asked to sign the consent form which accompanies this letter. If your child does not wish to participate in the study, he /she will not be included and there will be no penalty.

If you would like to be informed of the final findings, kindly contact Phuti Margaret Moloto on 072 465 7108 or email at margaretphutij@gmail.com. Should you have any concerns about the way in which the research has been conducted, you can contact Prof. M.F. Machaba on (012) 429 8582 or email emachamf@unisa.ac.za.

Permission for the study has already been given by the Capricorn South Department of Education District Director, the Principal and SGB of the school and the ethics committee of the College of Education, UNISA.

You are deciding about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child

Sincerely

____________________	____________________	____________________
Parent/Guardian's name(print)	Parent/Guardian's signature	Date
	**Moloto Phuti Margaret**	________________
Researcher's Signature	Researcher's name(print)	Date

APPENDIX G : REQUESTING PERMISSION FROM THE TEACHERS

TITLE OF THE RESEARCH: "***An Exploration of Mathematics knowledge for teaching for Grade 6 teachers in the teaching of fractions"***

Date ___________________

Dear Participant

I, Phuti Margaret Moloto, doing a research study under the supervision of Prof F.M Machaba, in the Department of Mathematics Education at the University of South Africa(UNISA) We (my supervisor and I) are cordially inviting a Grade 6 Mathematics teacher to participate in a study entitled, ***"An Exploration of Mathematics knowledge for teaching for Grade 6 teachers in the teaching of fractions"***

The purpose of this study is to explore the mathematical knowledge in teaching fractions in Grade 6, for the exploration to contribute meaningfully to the improvement of teaching and learning. The study will discover the mathematical knowledge that teachers possess in carrying out their work effectively.

The research study will take place in the classroom and the researcher will be observing the teacher teaching the concept of fractions to Grade 6 learners.

There are no potential risks involved. Participants will be briefed on the nature and purpose of the study. The researcher will also seek consent from the teachers.

If you would like to be informed of the final findings, kindly contact Phuti Margaret Moloto on 072 465 7108 or email at margaretphutij@gmail.com. Should you have any concerns about the way in which the research has been conducted, you can contact Prof. M.F. Machaba on (012) 429 8582 or email emachamf@unisa.ac.za.

Yours sincerely

Phuti Margaret Moloto

APPENDIX H: ETHICS CLEARANCE

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/06/10

Ref: **2020/06/10/44528418/16/AM**
Name: Mrs PM Moloto
Student No.: 44528418

Dear Mrs PM Moloto

Decision: Approved

Researcher(s): Name: Mrs PM Moloto
E-mail address: margaretphuti@gmail.com
Telephone: 0714657108

Supervisor(s): Name: Prof. M.F. Machaba
E-mail address: emachamf@unisa.ac.za
Telephone: (012)429 8582

Title of research:

An exploration of mathematical knowledge for teaching for Grade 6 teachers in the teaching of fractions

Qualification: MEd Mathematics Education

____

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above-mentioned research.

Research Ethics Committee Recommendations:

None

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

APPENDIX I:LANGUAGE EDITOR'S CERTIFICATE

Member South African Translators' Institute
www.language-services.online

PO Box 3172
Lyttelton South
0176
21 January 2021

TO WHOM IT MAY CONCERN

The thesis titled "An exploration of mathematical knowledge for teaching for Grade 6 teachers in the teaching of fractions: A case study of three schools in Capricorn South District" by Phuti Margaeret Moloto has been proofread and edited for language by me.

I verify that it is ready for publication or public viewing in respect of language and style and it has been formatted as per the prescribed style of the institution.

Please note that no view is expressed in respect of the subject-specific technical contents of the document or changes made after the date of this letter.

Kind regards

AMdeWet

Anna M de Wet

BA (Afrikaans, English, Classical Languages) (Cum Laude), University of Pretoria.
BA Hons ((Latin) (Cum Laude), University of Pretoria.
BA Hons (Psychology), University of Pretoria.

APPENDIX J: TURNITIN REPORT

MOLOTO PM_44528418_DFMED96

ORIGINALITY REPORT

19%	17%	7%	11%
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