Exploring Misconceptions of Grade 9 learners in the concept of fractions in a Soweto (township) school.

by

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DECLARATION

I declare that **EXPLORING MISCONCEPTIONS OF GRADE 9 LEARNERS IN THE CONCEPT OF FRACTIONS IN A SOWETO (TOWNSHIP) SCHOOL** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.

SEPTEMBER 2020

Moyo M. Date

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LIST OF ABBREVIATIONS

NCS	National Curriculum Statement
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
ANA	Annual National Assessment
GET	General Education and Training
FET	Further Education and Training

ABSTRACT

The study aimed to explore misconceptions that Grade 9 learners at a school in Soweto had concerning the topic of fractions. The study was based on the ideas of constructivism in a bid to understand how learners build on existing knowledge as they venture deeper into the development of advanced constructions in the concept of fractions. A case study approach (qualitative) was employed to explore how Grade 9 learners describe the concept of fractions. The approach offered a platform to investigate how Grade 9 learners solve problems involving fractions, thereby enabling the researcher to discover the misconceptions that learners have/display when dealing with fractions. The research allowed the researcher to explore the root causes of the misconceptions held by learners concerning the concept of fractions. Forty Grade 9 participants from a township school were subjected to a written test from which eight were purposefully selected for an interview. The selection was based on learners' responses to the written test. The researcher was looking for a learner script that showed application of similar but incorrect procedures under specific sections of operations of fractions, for example, multiplication of fractions. Both performance extremes were also considered, the good and the worst performers overall.

The written test and the interviews were the primary sources of data in this study. The study revealed that learners have misconceptions about fractions. The learners' definitions of what a fraction is were neither complete nor precise. For example, the equality of parts was not emphasised in their definitions. The gaps brought about by the learner conception of fractions were evident in the way problems on fractions were manipulated.

The learners did not treat a fraction as signifying a specific point on the number system. Due to this, learners could not place fractions correctly on the number line. Components of the fraction were separated and manipulated as stand-alone whole numbers. Consequently, whole number knowledge was applied to work with fractions. A lack of conceptual understanding of equivalent fractions was evident as the common denominator principle was not applied.

In the multiplication of fractions, procedural manipulations were evident. In mixed number operations, whole numbers were multiplied separately from the fractional parts

of the mixed number. Fractional parts were also multiplied separately, and the two answers combined to yield the final solution.

In the division of fractions, the learners displayed a lack of conceptual knowledge of division of fractions. Operations were made across the division sign numerators separate from the denominators. This reveals that a fraction was not taken as an outright number on its own by learners but viewed as one number put on top of the other which can be separated. Dividing across, learners rendered division commutative. A procedural attempt to apply the invert and multiply procedure was also evident in this study. Learners made procedural errors as they showed a lack of conceptual understanding of the keep-change-flip division algorithm. The study revealed that misconceptions in the concept of fraction were due to prior knowledge, over-generalisation and presentation of fractions during instruction.

Constructivism values prior knowledge as the basis for the development of new knowledge. In this study, learners revealed that informal knowledge they possess may impact negatively on the development of the concept of fractions. For example, division by one-half was interpreted as dividing in half by learners. The prior elaboration on the part of a whole sub-construct also proved a barrier to finding solutions to problems that sought knowledge of fractions as other sub-constructs, namely, quotient, measure, ratio and fraction as an operator.

Over generalisation by learners in this study led to misconceptions in which a procedure valid in a particular concept is used in another concept where it does not apply. Knowledge on whole numbers was used in manipulating fractions. For example, for whole numbers generally, multiplication makes bigger and division makes smaller.

The presentation of fractions during instruction played a role in some misconceptions revealed by this study. Bias towards the part of a whole sub-construct might have limited conceptualisation in other sub-constructs. Preference for the procedural approach above the conceptual one by educators may limit the proper development of the fraction concept as it promotes the use of algorithms without understanding.

The researcher recommends the use of manipulatives to promote the understanding of the fraction concept before inductively guiding learners to come up with the algorithm. Imposing the algorithm promotes the procedural approach, thereby depriving learners of an opportunity for conceptual understanding.

Not all correct answers result from the correct line of thinking. Educators, therefore, should have a closer look at learners' work, including those with correct solutions, as there may be concealed misconceptions.

Educators should not take for granted what was covered before learners conceptualised fractions as it might be a source of misconceptions. It is therefore recommended to check prior knowledge before proceeding with new instruction.

KEY TERMS

Errors, Fractions, Relational understanding, Instrumental understanding, Conceptual understanding; Procedural understanding; Procedural errors; Misconceptions; Part of a whole.

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1 CHAPTER ONE: INTRODUCTION TO THE STUDY

1.1 BACKGROUND TO THE STUDY

Learners reveal mathematical misconceptions as they attempt to solve mathematical problems. Although there may be many misconceptions in the mathematical world, this research intends focusing on misconceptions to the concept of fractions at the Grade 9 level. Fractions are a part of daily mathematics as evident in daily activities like sharing, estimating and prices. As learners make constructs in their everyday learning, misconceptions are inevitable. Vamvakoussi and Vosniadou (2010) believe that it is essential to identify misconceptions as early as possible to guide students in changing those misconceptions and hence allow for future understanding of more complex connected concepts. In this study, learners were allowed to give their own account of the topic of fractions.

Charalambous and Pitta-Pantazi (2007) state that learning fractions is a serious challenge and stumbling block to mathematical maturation of children. This emphasises that some learners face difficulties when it comes to fractions, and this affects their progress in mathematics. Fraction knowledge is not only localised to numerical operations as Brown & Quinn (2007) lament that the number of algebraic generalisations that depend on fractional constructs increases as learners move to each subsequent mathematics level. There is a need for skills in fraction work to maximise competence levels as learners progress from level to level.

Unlike whole number work, which starts development from kindergarten, fractions get introduced at later stages. Poems, songs and games exist from the pre-primary school level all aimed at developing the whole number concept. Comparatively, efforts to cement the concept of the fraction at tender years is nearly non-existent. This impacts on familiarity as learners will be more familiar with whole numbers, which are promoted through their everyday activities. Informally the whole number concept is developed, whereas the fraction concepts await formal development at a later stage.

Experience with whole numbers makes the fraction concept somewhat abstract as Bruce, Bennett and Flynn (2014) are of the opinion that fractions are multiple digits (numerator and denominator) that represent one quantity. Furthermore, with multiple digits as it is, a fraction can be interpreted differently as Bruce et al. (2014) lament that a fraction possesses several meanings depending on how it is used or the context. This complicates fractions, considering they are not practised as often as whole numbers. To fully understand, all situations where fractions portray different meanings may be challenging to learners who might be having limited exposure to the concept of fractions. Bruce et al. (2014) add by stating that part-whole, part-part, operator, quotient and measure are the five sub-constructs of fractions which, unfortunately, are often not made explicit to learners. This may leave learners prone to misconceptions as they create their own understandings of fractions.

Technological advancements come along with curriculum changes, which embrace and make use of the latest developments. With fractions, however, the use of calculators has reduced efforts by learners to conceptualise operations with fractions. The effects are evident at higher grades where knowledge of fractions become a pre-requisite to handling algebra. Learners get stuck as the calculator become of little use with algebraic fractions. Lack of conceptual understanding becomes evident. This study focuses on Grade 9s as they are at the summit of the GET phase of South African education system. It is the ideal focus group for the study as basic work on algebraic fractions is covered, hence prone to misconceptions on fractions. Conceptual understanding of fractions at this level is crucial as the next FET phase deals with more demanding algebraic fractions, which cannot be done with total dependency on a calculator. Competence in fractional operations is key at the FET phase.

1.2 RATIONALE FOR THE STUDY

The rationale of doing this research is to determine the misconceptions that learners have when dealing with problems that relate to fractions. Nesher (1987) states that misconceptions detected can explain a cluster of errors. According to Haser and Ubuz (2003), fractions form a basis for various concepts in elementary school mathematics such as decimals, rational numbers, ratio, proportion and percentage. A panel of researchers, such as Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto and Wray (2010) argue that understanding fractions is essential for algebra and other more advanced areas of mathematics. Jigyel (2007) points out that many studies found middle primary and junior secondary school learners to have difficulties in understanding and working with fractions. It is worth an effort on the researcher's part to undertake this study focusing on junior secondary learners' misconceptions in fractions. Junior secondary school learners are not yet exposed to algebraic fractions so will be ideal subjects to provide data for the study, misconceptions in the concept of fractions. Higher secondary school leaners may focus more on the application of the skills acquired before the specific lesson than the actual conceptualisation of fractions.

From my primary schooling days, the researcher always wondered why $\frac{1}{10}$ was said to be bigger than $\frac{1}{100}$. This misconception has been confirmed to me now as a teacher, surprisingly, exhibited by secondary school learners. My learners appear to hold more misconceptions pertaining to fractions. For instance, on giving them their test papers with marks given as a percentage, that is, 25%, they further ask, "25% out of what Sir?" Wait until you give them problems on fractions for another shocker, $\frac{3}{5}+\frac{2}{7}=\frac{5}{12}$. Through mental mathematics which my teacher liked back then, the researcher ended up getting confused as to when the researcher thought he had found the trick, “small denominator is the big fraction”, some did not work out. The researcher, however, relied on this as it coincidentally helped me survive punishment. Manipulating fractions had a set of rules, which had to be followed without necessarily worrying about the derivations or reasons that supported the rules. For example, when dealing with $\frac{1}{2} \div \frac{3}{4}$; maintain the first fraction $\frac{1}{2}$ .change $\div\, to\ \times$ and invert the second fraction $\frac{3}{4}$ to $\frac{4}{3}$, so that $\frac{1}{2} \times \frac{4}{3}$ is the new problem statement. Siegler et al. (2010) term this the invert-and-multiply rule. Skemp (1976) describes this kind of understanding as an instrumental understanding and contrasts this with a relational understanding. An instrumental understanding is a process in which learners possess rules and formulae and have the ability to use them without reason, not knowing the origin of those rules and formulae. The invert-and multiply procedure highlights this: $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$.

Siegler et al. (2010) suggest that teachers can begin by noting that multiplying any number by its reciprocal produces a product of 1 and dividing by 1 leaves the number unchanged. This implies that the first lessons on the division of fractions have to be through the multi-step procedure to help deduce the invert and multiply procedure. The multi-step procedure: $\frac{1}{2} \div \frac{3}{4} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} \times \frac{\frac{4}{3}}{\frac{4}{3}}$ making the denominator product1 (multiplying by the reciprocal) at the same time not changing the value of the fraction as you have multiplied by $1(\frac{\frac{4}{3}}{\frac{4}{3}})$.

At the end of an activity or two, learners can be guided onto discovering that the solution is the product of the dividend and the reciprocal of the divisor, hence the invert and multiply procedure.

Skemp (1976) further states that learners should develop a relational understanding even in dealing with fractions. In other words, learners should know both what to do and why when dealing with the problems that involve fractions. This also implies that learners should be able to relate the concept of fractions with their everyday life experiences. For example, $\frac{3}{4} + \frac{3}{4}$ may be related to bread which learners are well versed with. Contextualising to bread will make learners realise that since a loaf of bread has four quarters, then the solution is one loaf of bread plus two quarters.

Work in assessment also reflects a growing sensitivity to the importance of student misconceptions according to Smith III, diSessa & Roschelle, (1994). The importance of misconceptions in education is evident at schools where diagnostic reports are compiled at the end of each major exam marking session. Misconceptions by learners on fractions will help educators approach lesson delivery with pointers to help minimise potential misconceptions. The researcher is interested in the learners he works with and their understanding, which is why the researcher conducted this study to investigate how learners describe and work with the concept of fractions.

Siegler et al. (2010) are of the opinion that a greater understanding of fractions, knowledge of students' conceptions and misconceptions about fractions, and effective practices for teaching fractions are critically important for improving classroom instruction. Nesher (1987) affirms that the road to a state of learner expertise is paved with errors and misconceptions. Hence this study is looking at learners' misconceptions in the concept of fraction.

Grade 9, in South Africa, is a transition stage from the General Education and Training band (GET) to the Further Education and Training band (FET). At the FET band, learners are expected to specialise in either Mathematics, Mathematical Literacy or Technical Mathematics. Each of these requires knowledge of fractions. The researcher intends to identify misconceptions on fractions at the GET band so that reacting on these will minimise challenges with fractions at the FET band. This study is a step ahead of categorising learners' responses as right or wrong. An understanding of how learners solve problems involving fractions may reveal misconceptions and through interaction, inform the researcher on the root causes of these misconceptions. Brodie (2014) believes that errors are rarely taught directly by teachers, and yet all learners, even gifted learners, develop them at some point. This gives the researcher reason to undergo this study and discover the misconceptions in fractions that lead to errors. Smith III et al. (1994) describe this as a way of uncovering structure and meaning in learners' responses.

This study will benefit me as a researcher and other educators in terms of our teaching strategies because the researcher will be able to understand how learners describe the concept of fractions and how they reason and solve problems related to fractions. The findings, implications and recommendations of the study will provide clues to curriculum developers as they develop the curriculum pertaining to fractions. Students construct their mathematical knowledge and build on previous knowledge they learn; this means that any misconceptions they develop as they learn mathematics might affect their future learning of similar related mathematical concepts (Vamvakoussi & Vosniadou, 2010). This study will help us to know what misconceptions are evident and what might be the cause of these misconceptions, so that we may change and improve the way we teach the concept of fractions. This study aimed to explore misconceptions that Grade 9 learners in a school in Soweto have with the concept of fractions.

1.3 STATEMENT OF THE PROBLEM

One of the reasons why this study focuses at Grade 9 learners is because Grade 9 learners were subjected to the Annual National Assessments (ANA). These revealed major concerns as the mathematics pass rate was 10,8% nationally in 2014 as shown by the Table 1.1, extracted from the report on the 2014 Annual National Assessments from the Department of Basic Education (DBE) (DBE, 2014, p. 63).

Table 1.1: Average percentage mark in Grade 9 mathematics by province in 2012, 2013 and 2014.

PROVINCE	**AVERAGE MARK (%)**		
	2012	**2013**	**2014**
Eastern Cape	14.6	15.8	13.3
Free State	14.0	15.3	12.9
Gauteng	14.7	15.9	12.4
KwaZulu Natal	12.0	14.4	10.7
Limpopo	8.5	9.0	5.9
Mpumalanga	11.9	13.7	11.3
Northern Cape	13.2	12.6	9.7
North West	11.2	13.3	10.6
Western cape	16.7	17.0	13.0
National	**12.7**	**13.9**	**10.8**

Over the three years (2012-2014), the National average percentage mark does not surpass 14%. Despite improving from 12.7% to 13.9% from 2012 to 2013, the national average percentage mark dropped to 10.8% in 2014. Amongst the 10.8% who achieved in 2014, only 3% managed to get 50% or better. This implies that 97% of the 2014 Grade 9 learners had more incorrect solutions than the correct ones in their examinations.

Table 1.2 shows percentage of learners obtaining at least 50% of the Mathematics marks according to the 2014 Annual National Assessments report (DBE, 2014, p.12)

Table 1.2: Percentage of learners achieving 50% or more

GRADE	PERCENTAGE OF LEARNERS ACHIEVING 50% OR MORE		
	2012	2013	2014
3	36	59	65
6	11	27	35
9	2	2	3

The worst achievement trend over three years drew the researcher’s interest to the Grade 9 class. Grade 9 is the summit of the Senior Phase (phase in the South African education system that comprises of Grades

7, 8 and 9) and a step towards Further Education and Training (FET) hence the researcher found it most suitable for this research. This coincides with the Gauteng Department of Education's (GDE) (2017) Mathematics Strategy whose purpose is to ensure that learners do not reach Grade 10 with foundational knowledge gaps required for success in mathematics at the FET level. According to the mathematics Curriculum Assessment Policy (CAPS) document, the Department of Education (DBE) (2011), Algebraic Expressions and Exponents, Number Patterns, Sequences and Series, Equations and Inequalities, Trigonometry, Functions, Euclidean Geometry, Analytical Geometry, Finance and Growth and Statistics make up the topics in the Grade 10 South African mathematics curriculum. This implies that Grade 9 is the highest level where fractions are dealt with independently as a topic. In Grade 10, fractions are taken as assumed knowledge of yesteryears, and as such, it is found embedded in other topics such as ratio, proportion and rate. It is therefore of interest to the researcher to investigate how Grade 9s wrap up work on the topic fractions, in readiness for application at Grade 10 level.

1.4 THE AIMS AND OBJECTIVES OF THE STUDY

Research aims:

- To explore how Grade 9 learners describe the concept of fractions.
- To investigate how Grade 9 learners solve problems involving fractions.
- To discover misconceptions that learners have/display when dealing with fractions.
- To discover the root cause of misconceptions held by learners concerning the concept of fractions.

The study was done to answer the research questions set out in the following section.

1.5 THE RESEARCH QUESTIONS

Work on fractions by Grade 9 learners was explored through the following research questions;

1. How do Grade 9 learners describe the concept of fractions?
2. How do Grade 9 learners solve problems involving fractions, especially common fractions?
3. What misconceptions are evident when learners are solving problems involving fractions?
4. What are the root causes of learners' misconceptions in the learning of fractions?

1.6 HOW MY INTEREST IN THIS RESEARCH TOPIC DEVELOPED

Over the years in my teaching career as a mathematics educator for Grades 10-12, the researcher realised that learners made mistakes in mathematical calculations. Mistakes made with whole numbers were mostly on arithmetic errors and not as prevalent as those made when rational numbers were involved. On the contrary, the researcher noticed that errors pertaining to fractions were not just the surface arithmetic errors, but something more deeply entrenched in leaners. Comparatively, learner feedback from fractional activities showed some resemblance though answers were incorrect. This raised curiosity in me, wondering how the concept of fractions was perceived by learners that led to similar conceptual misunderstandings. The urge to find answers concerning the concept fractions, learner perceptions and the resulting misconceptions inspired the researcher undertake this study in case his quest for answers might be quenched as the researcher uncovers the reasons behind the challenges in work on fractions.

It is disheartening to witness multitudes of the young generation fall prey to a trap that may disarray their future mathematical orientation. Efforts to be proactive are evidenced by the researcher's undertaking to explore misconceptions that learners have in the topic of fractions.

1.7 DEFINITION OF KEY TERMS

- Fraction—relational representations that can be perceived as continuous or discreet quantities and are an integral part of human life from birth (Bruce, Bennett & Flynn, 2014).
- Relational understanding—a way of understanding that reflects knowing what to do supported by the reasons why it has to be done that way. Knowing what to do and why (Skemp, 1976).
- Instrumental understanding—use of rules or algorithms without the knowledge of why they work. "Rules without reasons" (Skemp, 1976).
- Errors—mistakes made by learners due to carelessness, misinterpretation of symbols and texts, lack of relevant experience or knowledge related to a mathematical topic, learning objective or concept, lack of awareness, or inability to check solution relevancy/meaningfulness (Hansen, 2006).
- Part of a whole—a sub-construct of fractions that interprets $\frac{x}{y}$ as representing the partitioning of the whole entity into y equal shares and then taking x parts out of y shares (Charalambous & Pitta-Pantazi, 2007).
- Conceptual understanding—ability to connect ideas of a particular concept. This is explicit or implicit of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain (Rittle-Johnson & Alibali, 1999)
- Procedural understanding—competence in applying steps to solution finding, a feat that Rittle-Johnson and Alibali (1999) term "action sequences for solving problems".
- Procedural errors—errors that occur due to incorrect application of the step by step process towards solution finding.
- Misconceptions—synthetic concepts that do not match the accepted view and that form as learners attempt to integrate existing knowledge with new information before a deeper conceptual change occurs (Vamvakoussi & Vosniadou, 2010). These misunderstandings of a particular concept result in misinterpretations based on incorrect meanings.
- Sub-constructs—the composition of a concept. In fractions, the Kieren Model proposes five related but distinct sub-constructs, which are part of a whole, a measure, quotient, operator and ratio.

1.8 ORGANISATION OF THE DISSERTATION

This study was aimed at gathering much information about misconceptions surrounding the concept of fractions as portrayed by learner perceptions and engagement in fraction activities.

The literature review in Chapter 2 will therefore dwell much on the causes of misconceptions supported by examples of some activities that illustrate these misconceptions. The historical review showcases the nature of the study in retrospect.

Chapter 3 contains the design and the methodology followed during the fieldwork. It explains the overall design, the sampling techniques employed and the criteria used in the choice of sample size. Furthermore, full details of the data collection techniques (how data was collected), and ways to gain access to the subjects will be discussed. The discussion on the data collection process in Chapter 3 sums up with justifications on the suitability of some questions as part of the written test instrument.

Chapter 4 contains the observations and analysis of the study. Firstly, tables summarising performances are presented. It is important to note that percentages in tables were rounded off to the nearest percentage. Secondly, against test items, the occurrence of misconceptions relative to the test item is presented. Interview extracts evidencing learners' line of thinking are then introduced before several analyses are presented.

Chapter 5 blends the results from previous chapters, co-ordinating with new analyses to clarify the types of misconceptions. In this chapter, a discussion of findings, recommendations and the conclusion of the study will be drawn. The research questions of the study will serve as guidelines to sub-headings which will be used in the analysis of data. The findings will be stated explicitly, and lastly the implications and reflections on the findings will be discussed.

1.9 CHAPTER SUMMARY

This chapter portrayed the learning of fractions as a serious challenge that not only inhibits mathematical progress but also comes along with misconceptions. Fractions were discussed as essential and an important part of the learning of algebra at higher educational levels. The rationale of doing this research was discussed as the determination of misconceptions that learners have when dealing with problems that relate to fractions. The aims and objectives of the study were brought forward to explore how Grade 9 learners describe the concept of fractions, to investigate how Grade 9 learners solve problems involving fractions, to discover misconceptions that learners have/display when dealing with fractions and to discover the root cause of misconceptions held by learners in relation to the concept of fractions. The researcher pointed out the prevalence of similar errors learners had in the topic of fractions as the push factor that aroused curiosity and eventually led to the undertaking to study on misconceptions in the topic of fractions. This chapter also presented an outline of the dissertation, which concludes with implications and reflections after reviewing the historical aspect of the study.
Chapter 2 provides the literature review and follows.

2 CHAPTER TWO: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 INTRODUCTION

Learners have prior conceptions that explain some of the mathematical phenomena even before they are formally taught expert concepts. This means new instruction finds learners with some knowledge, thereby defying the tabula rasa view of learners' knowledge state before instruction. New instruction may be said to find the learner with some baseline knowledge providing a platform for take-off. Gardee and Brodie (2015) assert that the theory that proposes active knowledge construction through the use of prior knowledge as a foundation to build new knowledge is constructivism. In this chapter, the researcher first discusses the theory of constructivism as the underpinning theory of this study. The researcher will then review and analyse the literature through the lens of the theory of constructivism.

This section will, therefore, focus on the description of the concept of fractions, including the differences between misconceptions and errors, which may result in incorrect solutions when learners are dealing with fractions. This section highlights some of the misconceptions in fractions, particularly, concerning an understanding of the concept of fractions, in addition and subtraction of fractions and multiplication and division. Causes of misconceptions, such as prior-knowledge, overgeneralisation and teaching, will also be highlighted.

2.2 THEORETICAL FRAMEWORK

The theory of constructivism informs this study. The general principles of constructivism are based largely on the work of Piaget. According to Piaget (1964), when a person interacts with an experience/situation/idea, one of two things may happen. Either the new experience is integrated into the person's existing schema (a process called assimilation), or the existing schema is adapted to accommodate the new idea/experience (a process called accommodation or adaptation). Assimilation refers to the use of an existing schema to give meaning to new experiences. Assimilation is based on learners' ability to notice similarities among objects and match new ideas to those they already possess. Accommodation is the process of altering existing ways of seeing things or ideas that do not fit into existing schemata. Accommodation is facilitated by reflective thought and it results in the changing or modification of existing schemata.

The main principle of constructivism is that learners engage in constructive learning on their own, working quietly through set tasks, allowing their minds to sift through the materials they are working with, and consolidating new ideas with existing ideas. Constructivism rejects the notion that children are "blank slates" with no ideas, concepts and mental structures. Noureen, Arshad and Bashir (2020) point that constructivism acknowledges that learners are not blank slates or empty vessels waiting to be filled with knowledge. Noureen et al (2020) further state that from a rich array of previous experiences, knowledge, and beliefs, learners construct new knowledge. Gupta and Gupta (2017) agree as they believe that just like in cell theory where all cells arise from pre-existing cells, knowledge already resides in human body and what is needed are ways to explore it. This confirms that learners have knowledge inherent in them as they come to instruction. They do not absorb the ideas as teachers present them, but rather, they are creators of their own knowledge (Hatano, 1996). This study focuses on the exploration of misconceptions in Grade 9 learners in the concept of fractions. Thus in this study, the theory of constructivism is pertinent in a sense that when learners construct knowledge by themselves, misconceptions do arise. This means that misconceptions are part of learning.

Cobb (1994) concurs that constructivism is the generally accepted view that learners construct their mathematical ways of knowing as they strive to be effective by restoring coherence to the world of their personal experiences. Similarly, Ultanir (2012) agrees that real understanding is constructed based on experience and background knowledge. This implies that experience gathered by learners as they progress through the early childhood development phase, foundation phase, intermediate phase and all the way up to the senior phase is vital because it will give meaning to the new knowledge. At these previous stages, fractions were introduced and developed to a certain level.

The ideas of Skemp on what he terms relational understanding gives a viewpoint of what constructivism entails. Skemp (1976) distinguishes between two types of understanding, relational understanding and instrumental understanding. Skemp (1976) states that instrumental understanding can be described as “rules without reasons” whereas relational understanding is knowing what to do and why. The rules without reasons approach takes doing mathematics as a step by step procedure with rules that guide the path towards solution finding.

Savery and Duffy’s (2001) primary proposition of constructivism states that knowledge evolves through social negotiation and through the evaluation of the viability of individual understandings. This study will provide such an evaluative platform on learners’ work on fractions. Dubinsky and Schwingendorf (1997), are of the opinion that constructivism directs people to matters of what others know and how they might have come to know it. As part of data collection techniques, some participants in this study were interviewed to understand their line of thinking on dealing with problems on fractions. During the interviews, posing the question “Why” invoked justifications by interviewees and fulfilled Dubinsky and Schwingendorf’s (1997) perception of constructivism as directing people to matters of what others know and how they came to know it. Dubinsky and Schwingendorf (1997) further state that constructivism forces people to speak in the active voice. This implies that constructivism is empirical in the sense that what is spoken is supported by underlying action. This study followed the same format in that misconceptions on the topic of fractions were noted as they unfolded and summaries compiled based on empirical evidence.

Lebow (1993) summarises the constructivist framework as having values of collaboration, personal autonomy, generality, reflectivity, active engagement, personal relevance and pluralism. These descriptions characterised this study as participants freely engaged in work involving fractions, linking with previous experiences and generalising. This study also incorporated participants' interpretational skills on the work on fractions. This adheres with Cobb's (1994) idea that constructivists are typically concerned with the quality of the individual interpretive activity, with the development of ways of knowing at a more micro level. Thompson (2000) adds that constructivists believe that people have explicit control over what knowledge they end up constructing. This implies that concerning fractions; learners also have control over fraction constructions though these constructions may be founded on one or a series of misconceptions. The effects of this developmental stage on knowing fractions played a crucial role in this study.

According to Naylor and Keogh (1999), the central principles of constructivism are that learners can only make sense of new situations in terms of their existing understanding. Similarly, Ultanir (2012) contends with the idea that constructivism entails individuals constructing or creating an individual understanding or knowledge through the interaction of ideas, events, activities with which they come into contact as well as what they already believe. This means learners must have a foundation on which to build on for proper accommodation and assimilation. The accommodation aspect of constructivism proved crucial to this study as misconceptions, or the application thereof may have resulted. Naylor and Keogh (1999) consolidate by stating that learning involves an active process in which learners construct meaning by linking new ideas with their existing knowledge.

Dewey (1998:22) thus emphasises this as he asserts; "Finding material for learning within experience is only the first step. The next step is the progressive development of what is already experienced into a fuller and richer and also more organised form, a form that gradually approximates that in which subject-matter is presented to the skilled, mature person". The progression process, a transition towards solution finding on working with problems on fractions, will provide data for this study as misconceptions result.

Ciot (2009) asserts that in constructivism, the teacher is a guide, facilitator and co-explorer who encourages learners to question, challenge and formulate his or her own ideas, opinions and conclusions. Formulation of these ideas by learners was an important aspect of this study as misconceptions surfaced.

As much as the theory of constructivism emphasises the fact that the construction of knowledge depends on what the learner already knows, it also highlights the importance of social interactions. Hatano (1996), Smith, Disessa and Roschelle (1989), all argue that knowledge is constructed when learners interact with each other. When ideas are shared, the construction of meanings ensue. In the process of construction, misconceptions can occur, and this occurrence formed an important part of this study. The researcher explored misconceptions that Grade 9s held about the concept of fractions. Thompson (2000) maintains that the rate of misconceptions is high enough to support a popular understanding of constructivism as being about discovery, learning, cooperation, and a ban on lecturing. In the context of fractions, this study focused on the misconceptions that learners hold, those they applied in their discovery of learning undertakings; also those misconceptions that are part of the product of their discovery endeavours in the topic of fractions. Dubinsky and Schwingendorf (1997) asserted:

As a background theory, constructivism orients us to formulating descriptions, problems, explanations, and theories in specific ways (Dubinsky & Schwingendorf, 1997). Dubinsky and Schwingendorf, (1997) believe that adherents of constructivism must be willing to look introspectively at their certainties and be in a position to question their knowledge. From the study's point of view, this also applies to the Grade 9 learners. The interview sessions conducted gave learners the platform to reflect and question what they initially regarded as obvious knowledge.

Constructivism is appropriate for this study as Brooks and Brooks (1999) view it as not a theory about teaching, but one about knowledge and learning. Thompson (2000) believes constructivism provides a basis for reconceptualising mathematics teaching though it does not offer a particular model of mathematics teaching. Reconceptualising mathematics may change learners' existing schemas through the application of knowledge learners already have. This may render them prone to misconceptions.

This study is about the misconceptions learners have in the topic of fractions at the Grade 9 level at a township school. The misconceptions were uncovered from the knowledge the learners possess. Brooks and Brooks (1999) further emphasise the point that constructivism defines knowledge as temporary, developmental, socially and culturally mediated and subsequently non-objective. The non-permanence of constructivist knowledge may be attributed to continuous knowledge replenishing as an individual is subjected to new instruction.

Hatano (1996) states that the reconstruction of knowledge results from a combination of assimilation and accommodation. This implies that the reconstructed knowledge will undergo assimilation and accommodation when the individual is confronted with new instruction. On and on, the incremental knowledge process renders knowledge non-static but developmental. Formally or informally, knowledge sources may be a result of interactions. Zain, Rasid, and Abidin (2012) state that in a classroom situation, learners are encouraged to think beyond what is presented, thereby exploring concepts further in collaboration with the teacher. During such conceptual explorations, misconceptions may result. Zain et al. (2012) further put across the focus of constructivism as learner empowerment to negotiate and come up with solutions through sharing and exchanging of ideas. This is in line with Atherton's (2011) belief that constructivism presents the existence of a learner-teacher collaboration with the learner much more actively involved in the creation of new meanings. Concerning this study, the product of the learner-teacher collaborations was evidenced in learner responses, which made use of the meanings that were created regarding fractions during instruction. Follow-ups on written evidence during the interviews summoned clarifications from selected participants, which then paved the way to the identification of probable causes of misconceptions.

Ultanir (2012) exemplifies the constructivist approach as the different individual perceptions, interpretations and explanations of the same object. This implies that constructivism allows for subjectivity, autonomy, different points of view and ultimately does not always lead to the same conceptualisations.

This study is underpinned by interpretive research paradigm. According to Du Plooy, Davis and Bezuidenhout (2014) The main idea of interpretivism paradigm rests on the fact that people are fundamentally different from objects. Consequently, researchers cannot study human beings in the same way that we study objects in the natural sciences, because, unlike objects, human beings change all the time and the environment in which they find themselves constantly influences them. Interpretivists, for example, argue that it does not make sense to study people in laboratory settings, as people do not live in laboratories, and as they are always influenced by the things that are happening in their environment.

Interpretivism believe that if we want to understand human behaviour, you need to grasp what people view as common sense. For interpretivists, this is an essential source of information for understanding people. Hence this study is more on understudying how learners define the concepts of fraction, what misconceptions are associated with fraction when learners are solving fractions related problems and what might be the cause of this misconceptions.

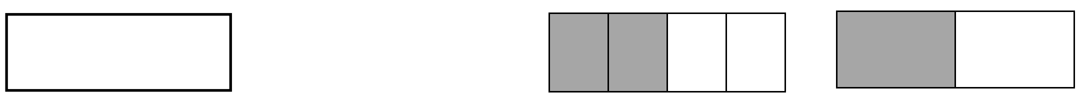
2.3 LITERATURE REVIEW

2.3.1 What is a fraction?

According to Bassarear and Moss (2016), the word "fraction" is derived from the Latin word *fractio*, which comes from the word *frangere*, meaning to break. They further explain that the fraction $\frac{1}{8}$ could be understood to mean the quantity formed by one part when a whole is partitioned into eight equal parts. The key idea in understanding fractions is to be able to identify the whole and the equal fractional parts. Most importantly, fractions cannot be regarded as separate, independent entities. This means that fractions have meaning only in relation to the whole in which they apply. Van de Walle (2016) indicates that the concept of a fraction tells us only about the relationship between the part and the whole. For example, an example of a fraction as part of the whole is given in Figure 2.3: This whole is a square. The whole is divided into four equal parts. Each part is one-fourth (a quarter) of the whole. Four quarters, therefore, make one whole.

Kerslake (1991) explains fractions as numbers. Kerslake (1986) argues that to explain fractions as part of geometric shapes is not sufficient as it eludes understanding of some aspects of fractions. In the “part of a whole” geometric perspective, part shading clearly outlines a fraction but does not bring forth the idea of a fraction as a number. Kieren (1980) cited in Kerslake (1986) believes that the “part of a whole” type limits the development of the opinion that a fraction can be greater than one. This implies that the “part of a whole” perspective narrows thinking to partition one whole. Kerslake consolidates this aspect by the idea that adding $\frac{2}{3}$ to $\frac{3}{4}$ is not helped by the ‘part to whole’ model as it portrays $\frac{2}{3}$ of one circle being added to $\frac{3}{4}$ of another circle. Siegler et al. (2010) agree with this as they recommend ensuring that learners know that fractions are numbers that expand the number system beyond whole numbers and regard the use of number lines as a key representation tool to convey fraction concepts.

However, Orton and Frobisher (1996, cited in Haser & Ubuz, 2003, p. 64) believe that the part-of the whole model has continuous representations and discrete representations. Van de Walle, Karp, and Bay-Williams (2016) refer to the two part-whole representations as continuous wholes (area model) and the discontinuous wholes (set model). The continuous whole or area model of equivalent fractions is illustrated as follows:



Whole

Shading $\frac{2}{4}$

Shading $\frac{1}{2}$

Figure 2-1: Continuous/area model

The following is an illustration of the discontinuous whole or set model of equivalent fractions. In the set model, area models may be taken to resemble the subsets as shown in Figure 2-2:

Alvin has 30 rabbits and wants to sell $\frac{2}{5}$ of them. He puts them in 5 cages and chooses the rabbits in 2 cages.

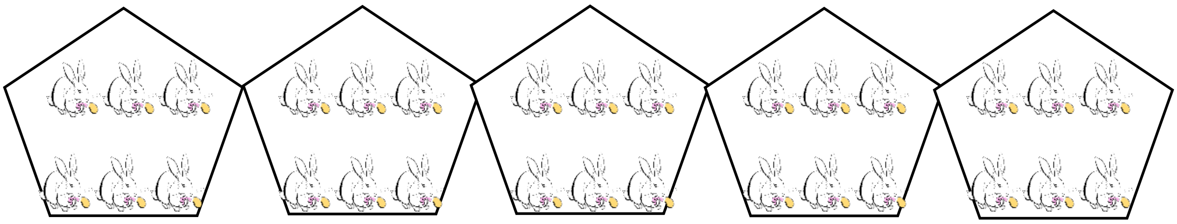


Figure 2-2: Discontinuous/set model

The part of a whole from the set model may be used during instruction on equivalent fractions. Clearly, two cages of the five contain 12 rabbits of the 30 implying that $\frac{2}{5}$ is equivalent to $\frac{12}{30}$.

Thus the definition of a fraction which this study adopts refers to a fraction as defined as a part of a whole, but a whole is being divided by "equal parts" or equal shares or an equal–sized portion of a whole (Van De Walle 2016). The limitation and the lack of this understanding of a whole being divided into equal parts resulted in learners not understanding the concept of a fraction. The researcher understands that the definition of a fraction as "part of a whole" has been captured in many South African textbooks (Laridon et al. 2005).

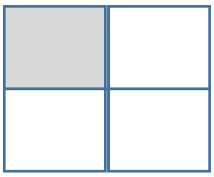


Figure 2-3: Example of a fraction as part of a whole

Van De Walle (2016) indicated that learners should able to say, "*My whole is a square. It is divided into four equal parts. Each part is a fourth of the whole. My whole is a circle. To find* <u>*one eighth*</u> *of the whole, I divide it into* <u>*eight*</u> *parts of equal size, and shade one part. The shaded part is* <u>*one eighth of the whole."*</u> As you can see in Figure 2.4, their whole is "cut up" or partitioned into several equal sized pieces.

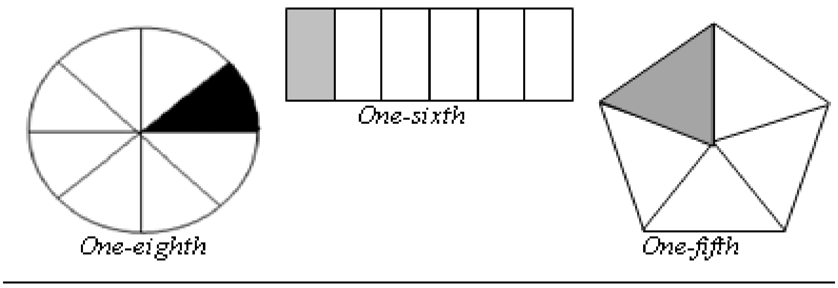


Figure 2-4: Whole partitioned into equal sizes

She emphasised the language pattern of a fraction is very important, and it could help to eliminate some of the misconceptions that learners possess. Concerning the three circles in Figure 2.5, learners would tend to indicate that they are all represented by $\frac{1}{3}$, not taking into consideration that in some of them their whole are not divided into equal parts. This, in my view, is a misconception.

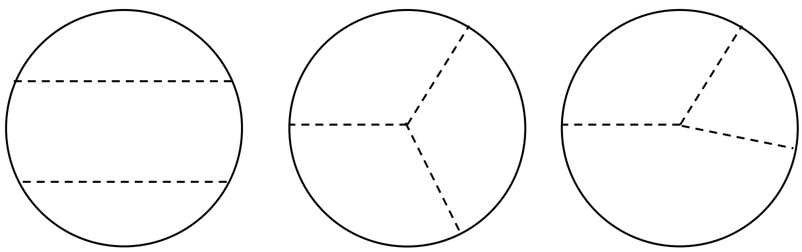


Figure 2-5: Whole partitioned unequal sizes

Weinberg (2001) believes that learners tend to see a fraction as a calculation on its own (division) or as a complex set of two numbers written on top of each other. Jigyel (2007) uplifts developing and consolidating the notion of a fraction as representing a relationship between two counts; a count of shaded parts and that of total parts, as some learners perceive the two as unrelated. This brings about interpreting $\frac{2}{5}$ as shading 2 parts out of 5 leading to $\frac{10}{2}$ being interpreted as shading 10 parts out of 2. On the contrary, taking $\frac{10}{2}$ to be $10 \div 2$ leads to $\frac{2}{5}$ to be $2 \div 5$ and this way $\frac{2}{5}$ may not be perceived as one single quotient value but a number divided by another number.

Hart (1993) clarifies the components of a fraction, the meaning of $\frac{2}{5}$. He indicated that most learners do not have an idea that the denominator (the bottom number that tells how many equal parts the whole is divided into) and the numerator (the top number that tells how many parts one has of the available equal parts) represent the part-whole representation of a fraction. In other words, the numerator ***counts*** and the denominator ***tells*** **what is being counted.** When asked the meaning of $\frac{2}{5}$, they would often say 2 is the numerator and 5 is the denominator without the understanding of these parts of fractions.

According to Van de Walle et al. (2010), understanding fractions means understanding all the possible concepts that fractions could represent, part-to-whole, ratio, measurement, and to indicate operations. However, Bruce et al. (2014) split the meanings of fractions into what they term five sub-constructs; part-whole, part-part, operator, quotient and measure. Leinhardt and Smith (1984, cited in Haser & Ubuz 2003, p. 64) attribute these sub-contracts to the different meanings that can be attached to fractions, depending on the different contexts making their relationship somewhat complex. Doyle, Dias, Kennis, Czarnocha and Baker (2015) contend that the five fraction sub-constructs are related though distinct. The following is the representation of the sub-constructs by Kieren with an extension now including fraction operations through efforts of Behr, Lash, Post and Silver (1983). The Kieren Model and Behr et al. (1993) extension of the Kieren model.

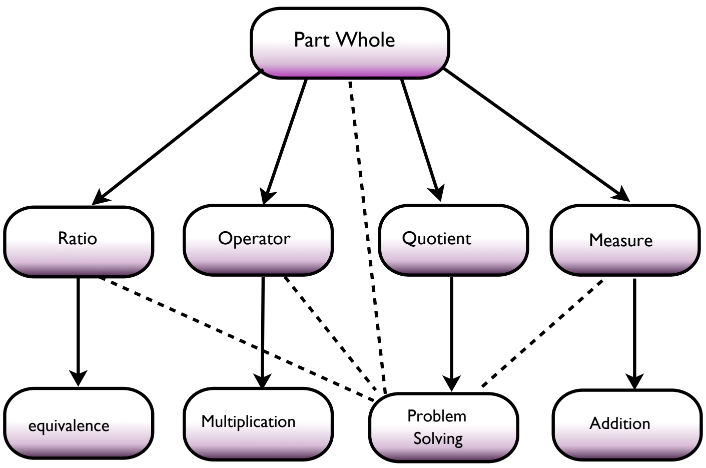


Figure 2-6: Fraction model of Behr et al. (1983, p. 100).

Doyle et al. (2015) consider the part-whole as the primary sub-construct leading to the other four sub-constructs which are regarded as secondary and concerned with the conceptual knowledge of fractions. The bottom row, the extension by Behr et al (1993), addresses the problem-solving aspect of fractions as well as procedural knowledge of fractions. The primary and secondary sub-constructs (Kieren Model) that bear conceptual knowledge can be summarised as follows: Part-whole sub-construct is a sub-construct of fractions that interprets $\frac{x}{y}$ as representing the partitioning of the whole entity into y equal shares and then taking x parts out of the y shares (Charalambous & Pitta-Pantazi, 2007). According to Doyle et al., the part-whole sub-construct is used for introductory lessons in mathematics curricula as it has limitations when it comes to improper fractions. The measure sub-construct is synonymous with the linear presentation of rational numbers. Doyle et al. (2015) state that the measure is for the placement of fractions on the number line and the placement of $\frac{x}{y}$ on an interval with some unit incorporating improper fractions as well.

The quotient sub-construct of fractions provides the solution to the division problem. Doyle et al. (2015) puts it as the provision of $\frac{x}{y}$, the amount obtained when x quantities are divided into y equal shares. The ratio sub-construct caters for comparisons as Doyle et al. (2015) are of the opinion that $\frac{x}{y}$ extends the part-whole to include the part through a comparison between two quantities x and y. The operator sub-construct is a calculation multiplying by the numerator and dividing by the denominator. Doyle et al. (2015) believe the operator sub-construct implies taking a fraction of some quantity, that is, $\frac{x}{y}$ of some quantity ending up as multiplying the quantity by x and dividing by y.This research focuses on misconceptions relating to efforts to showcase an understanding of fractions by learners holistically. All the fraction sub-constructs are taken into consideration in this study.

2.3.2 Misconceptions vs Errors

Smith et al. (1994) have the impression that misconceptions are flawed ideas that are strongly held by a student, that interfere with learning. Olivier (1989) states that misconceptions are errors that are made repeatedly each time learners are confronted with similar situations. In other words, the constant incorrect application of a mathematical concept is perceived as routine by the learner. Ojose (2015) describes misconceptions as misunderstandings and misinterpretations based on incorrect meanings.

Sarwadi and Sharhill (2014) however, view systematic errors as usually a consequence of student misconceptions. The “consequence perspective” is supported by Clement, in Confrey et al. (1990), as he asserts; “A misconception is a ‘conceptual stumbling’ block, inconsistent semi-autonomous schemes, and cognitive process responsible for errors in problem-solving” (Confrey, 1990:18). All the above definitions illustrate differing views about misconceptions. This research project’s focus is on misconceptions, as outlined by Confrey. Moreover, Vamvakoussi and Vosniadou (2010) are of the opinion that misconceptions are “synthetic concepts” as they result when learners broaden their number system knowledge further than natural numbers. Misconceptions are, therefore, the by-product of broadening the number system knowledge. These definitions imply that misconceptions are an un-objected reality. They go hand in hand with knowledge development; they will always be there but can be appropriately dealt with on surfacing. As learners interact with new material, they interlink with ideas they already have, as stated by Smith et al. (1994) who say students do not come to instruction as blank slates. When learners construct knowledge, the activity of reconstructing and reorganising what they already know and synthesising new knowledge may cause misconception synthesis to occur as well. Given an opportunity, the learners can defend their inductive conclusions by applying concepts from elsewhere incorrectly. Several learners may give similar incorrect feedback to some task. This may point to a viewpoint or understanding that they do possess misconceptions

Confrey (1990) states that numerous studies provide empirical support, that there are significant qualitative differences in the understandings that students develop in instructional situations, and that these understandings are frequently very different from those that the teacher intends. This implies that as learners engage in mathematical calculations, they make numerous errors at times that may lead to incorrect solutions. Not all incorrect solutions are a result of conceptual misunderstandings. Brodie, Shalem, Sapire, & Manson (2009) direct attention to errors and mistakes, which are not necessarily misconceptions but also lead to incorrect solutions. In their quest to distinguish between mistakes and errors, Brodie et al. (2009) put across mistakes as easily rectified and obtained from mathematical slips. This implies that $2 \times 3 = 5$ may exemplify a mistake as there is probably only a slim chance that a learner can give the same answer on another trial. Gardee and Brodie (2015) regard mistakes as sporadic and as a result of carelessness and hence view them as easier to mend when identified.

As opposed to mistakes, which are minor mathematical slips, Brodie et al. (2009) describe errors as those flaws that are more difficult to address. Brodie et al. (2009) elaborate, stating that errors are often repeated in different contexts owing to a learner having an underlying misconception. Gardee and Brodie (2015) concur that errors are systematic, pervasive, persistent and occur regularly over a wide variety of contexts. This draws a line between errors and mistakes.

What then, is the relationship between errors and misconceptions? Gardee and Brodie (2015) believe that one is a consequence of the other; that is, errors are a consequence of misconceptions. Nesher (1987) also points out that clusters of errors, which are not sporadic, are a product of misconceptions. In other words, misconceptions may be said to promote the generation of errors. Having clarified the three words, mistake, error and misconception, it then follows that a misconception is neither a mistake nor an error but may lead to an error.

Hodes and Nolting (1998) proposed four types of errors; careless errors, application errors, procedural errors and conceptual errors. Hodes and Nolting (1998) hold that that mistakes made when the learner does not understand the properties or principles covered in the textbook or lesson are conceptual errors. This implies that a misconception is a type of error that relates a particular concept.

Ojose (2015) believes certain misconceived methods and errors in calculations could somehow lead to a correct solution. Credit provided by the correctness of the solution reinforces the steps taken by the learner, and they are bound to be repeated. The following scenario illustrates how a situation making sense to the learner, unfortunately, leads to a wrong mathematical interpretation. You are buying eggs from a shop on two consecutive days. On day one, out of the five eggs you buy, two are rotten. On day two, out of five eggs you buy, one is rotten. Of all the eggs you have purchased, what fraction is rotten? Out of the total of ten eggs in two days, three are rotten. For a learner who approaches the problem by recollecting that on day one two out of the five eggs were rotten ($\frac{2}{5}$), and on day two, one out of the five eggs were rotten ($\frac{1}{5}$), and then considering that $combined\ day\ 1\ \&\ day\ 2;\ \ 3\ out\ of\ 10\ were\ rotten\ \left(\frac{3}{10}\right),$ it may be difficult to convince that $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}\ not\ \frac{3}{10}$.

Gardee and Brodie (2015) believe that a constructivist framework suggests that errors illuminate important aspects of learner reasoning, and they are both sensible and reasonable to learners. Smith et al. (1994) contend that learners develop durable conceptions with explanatory power, but these conceptions are inconsistent with the accepted mathematical and scientific concepts presented in instruction. To the learners, their approach may seem logical and bound to be done over and over again. Idris and Narayanan (2011) declare that once students have been introduced to a variety of strategies to assist with adding and subtracting, many overgeneralise, confuse or misapply the strategies. Correcting such may require knowledge of the root cause as it may call for thorough convincing to re-direct learners' mind-set from what they already believe to be the undisputed truth or to be simply obvious. An intervention to help reorganise their thinking is necessary. For example, consider faulty thinking leading to the impression that $\frac{1}{2}+\frac{1}{2}=\frac{2}{4}$. This study is, however, not focusing on careless errors but conceptual ones, that is, misconceptions.

Brodie et al. (2009) have it that flaws that exist in incorrect conceptual understanding, but that seemingly makes sense to learners, must be recognised as such before the knowledge is restructured. Restructuring through the application of incorrect concepts may cement relationships of concepts incorrectly, further perpetuating the misconceptions. From another angle, a misconception in one concept may affect the conceptualisation of the other. Lindquest, in Smith et al. (1994, p. 121), attributes the challenges learners are confronted with to learning notions of probability that lead to interference of misconceptions about probabilistic events. This implies misconceptions in one concept have been carried over to affect learning negatively in the other concept.

Smoothing correct conceptualisation might require special attention to misconceptions. Smith et al. (1994) lobby for the development of expert ideas which must substitute misconceptions. Sarwadi and Sharhill (2014) are of the view that misconceptions by learners must not be taken as dead ends but regarded as an opportunity for reflecting and learning.

As misconceptions may impact negatively on learning, identifying them may be a step towards minimising or even eradicating them. Smith et al. (1994) maintain that taking up from Piaget's cognitive development work, research on misconceptions legitimised attempts to bring to surface structure and meaning in learners' responses. It was one of the objectives of this study to specifically identify misconceptions that Grade 9 learners had on the topic of fractions at a township school. In a list of characteristics of misconceptions, Graeber and Johnson (1991) pointed out that misconceptions are self-evident, coercive, and prevalent in both naïve and academically gifted learners; they are persistent, robust and frequently supported by informal language and symbol use. Acknowledging that indeed students have misconceptions, this work focuses on learners' misconceptions on the concept of fractions. Origins, effects and ways to limit misconception manifestations will be suggested when discussing the analysis of findings.

2.3.3 Misconceptions in fractions

Durkin and Ritle-Johnson (2014) believe that as learners transition from their conception of number as natural numbers to a conception of number as including natural, rational and real numbers, they generate misconceptions. Resnick, Nesher, Leonard, Magone, Omanson, and Peled (1989) classified misconceptions in fractions under three categories, namely; the whole number misconception, the role of the zero misconception and the fraction misconception. The whole number misconception is what Ni and Zhou (2005) referred to as the whole number bias, limiting one's scope of numbers to whole numbers. Bruce et al. (2014) hold that this bias is a result of emphasis on whole number counting at an early age, which tends to reinforce a strong concept of numbers as whole numbers.

Niemi (1996) has it that having learners talk and write about how they create or recognise equivalent fractions can strengthen their understanding and provide valuable information to educators. Jigyel (2007), in a study on equivalent fractions, concluded that some learners perceived the components of fractions, the numerator and denominator, as two unrelated whole numbers, which led to misconceptions. Jigyel (2007) declares that geometrically, learners were able to compare and accept knowledge on equivalent fractions. This implies that visualising figures representing fractions being compared served as empirical proof that the two fractions are indeed equivalent. However, Jigyel commented on the evolution of misconceptions when learners were dealing with numerical comparisons of equivalent fractions. According to Jigyel (2007), learners viewed one fraction as a multiple of the other just like the relationship between the object and the image under enlargement in the topic transformations. This can be illustrated as follows; e. g. comparing

$$\frac{2}{5} \text{ and } \frac{4}{10}.$$

$\frac{4}{10} = 2 \times \frac{2}{5}$. This is interpreted as twice as big or double $\frac{2}{5}$. From this perspective, the misconception is clear from the fact that $\frac{2}{5}$ cannot be equivalent to its double value, $\frac{4}{10}$.

Test or assessment items that teachers subject learners to may be a source of misconceptions. According to Guarino, Sykes and Santagata (2013) in an assessment task on simplifying fractions, learners were asked to simplify fractions similar to $\frac{4}{10}$; $\frac{6}{8}$ and $\frac{12}{50}$. Correct solutions resulted but asking learners "why" revealed a misconception. Guarino et al. (2013) stated that some learners regarded simplifying to mean divide both numerator and denominator by 2, and difficulties in simplifying fractions like $\frac{6}{15}$ confirmed this. The teacher's bias towards fractions, which coincidentally yielded the same result as dividing both numerator and denominator by 2, may be said to be the source of the simplifying misconception.

In their study on manipulating fractions, Smith et al. (1994) maintain that two strategies are sufficient for reasoning to solve most (if not all) problems; conversion to common denominator and conversion to decimal. According to Vamvakousi and Vosniadou, (2010) research indicates that children have difficulty integrating fractions into their already well-established understanding of whole numbers. Jigyel (2007) believes there is a need to devote attention to developing and consolidating the notion that takes a fraction as representing how two counts relate.In this study, it was interesting to see students' limitations/improvisations.

2.3.3.1 Misconceptions in addition and subtraction of fractions

Lestiana, Rejeki and Setyawan (2016) point out that most learners believe that fractions' numerators and denominators can be treated as separate whole numbers. Most learners often add or subtract the numerators and denominators of two fractions. For example: $\frac{1}{3} + \frac{2}{3} = \frac{3}{6}$; $\frac{3}{6} - \frac{1}{2} = \frac{2}{4}$. Most learners also fail to recognise the relationship between the denominators—that is, the denominator is the number of equal parts into which one whole is divided, and that the numerator signifies the number of those parts. The fact that numerators and denominators are essentially treated as whole numbers in multiplication only adds to the confusion.

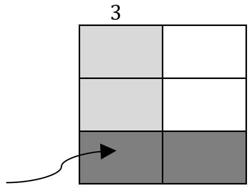
To overcome this misconception, you will be presented with a real-world problem. For example: "If you have $\frac{3}{4}$ of an orange and give $\frac{1}{3}$ of it to a friend, what fraction of the original orange do you have left?"

Subtracting the numerators and denominators separately would result in an answer of $\frac{2}{1}$ or 2. You will immediately recognise that it is impossible to start with $\frac{3}{4}$ of an orange, give some of it away, and end up with two oranges. Such examples help students see why treating numerators and denominators as separate whole numbers is inappropriate, and will make them more receptive to appropriate procedures. The other way you can address the misconception about the addition and subtraction of fractions is by using model representations.

2.3.3.2 Misconceptions in multiplication and division

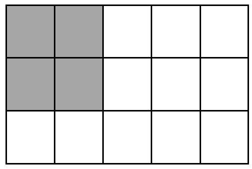
According to Bruce et al. (2014), multiplication and division of fractions proved challenging to either teach (educator perspective) or learn (learner point of view) probably due to the multi-faceted nature of fractions. The complexity of fractions due to multiple constructs may be the result of the obstacles to teaching or learning concepts on fractions. Struggling to conceptualise may result in learners applying formulas they do not understand. As Bruce et al. (2014) point out, learner misunderstandings on the meaning behind algorithms applied in fractions negatively impact the understanding of other topics like algebra. Empson, Junk, Dominguez and Turner (2006) suggest three ways of thinking about the multiplication of fractions as outlined by the table below;

Table 2.1: Strategies in Multiplication of fractions

Multiplication Strategy	**Description**	**Example**
In measurement	Groupings with equal contents. Number of groups and the size of contents are used to come up with the total quantity.	A recipe requires $\frac{1}{4}$ cup of baking powder. How much baking powder would be required for $\frac{1}{2}$ the recipe? A quarter of one-half, $\frac{1}{4} \times \frac{1}{2}$.
Partial groups	Multiplying one fraction quantity with another fraction quantity	Each bag has $\frac{1}{4}$ kg of candy. If there are $5\frac{1}{2}$ bags of candy, how much candy is available altogether?
Cartesian Product	Multiplication taken as the shared space of two numbers	$\frac{1}{3}$  The intersection cell, cell on both $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{1}{6}th$.

Division of fractions can also be approached from different perspectives as indicated by Sinicrope, Mick and Kolb (2002) in Table 2.2 as follows.

Table 2.2: Strategies in the division of fractions

Division Strategy	**Description**	**Example**
In measurement	Model for determining the number of times the denominator goes into the numerator	
Fair share	A model for sharing equal amongst a number of individuals	If two people share $\frac{1}{2}$ a loaf of bread, how much of the loaf will each get?
As the inverse of a Cartesian product	Involves determining the side of a rectangle	Given the area of a rectangle is $\frac{4}{15}$ square units. If one side measures $\frac{2}{3}$ units, what is the size of the other side? 
Unit rate determination	Emphasis is on the size of one group	A dozen eggs cost R10. What is the cost of one egg?
Inverse of multiplication	Inverting a fraction and multiplying	60 learners doing Art in Grade 11 are one and a half times more than Grade 12 learners doing Art. How many Grade 12 learners do Art?

However, the outlined strategies to multiplication and division of fractions come along with misconceptions. Bruce et al. (2014) instigate challenges to multiplication and division of fractions during instruction to emphasise procedures rather than conceptual understanding, have a bias towards the part-whole meaning of fractions, make limited use of representations of multiplication and division with fractions and have insufficient instructional time. Preferences for a procedural approach rather than a conceptual one results from the comparatively less demanding use of algorithms which, according to Bruce et al. (2014), are relatively easy to memorise and apply. Learners may, therefore, exhibit competence in the procedure but conceptually be found wanting. Bruce et al. (2014) further state that the appropriate application of algorithms does not signify the understanding of the concept. Wu (2001) points out that the procedural approach becomes insufficient when dealing with more complex fraction problems.

Petit, Laird, & Marsden (2010) holds the view that educators tend to focus more on instruction relating to procedures and neglect the underlying concepts. This implies that getting the correct solutions is paramount for educators at the expense of proper conceptualisation by learners. Vale and Davies (2007) attribute choosing procedural over conceptual approach by educators to be due to the challenging nature of teaching underlying concepts as they require deep content understanding, unlike teaching only rules. Lo and Lou (2012) contend that despite the solid content and conceptual knowledge educators possess, prospective educators struggle to represent fractions with pictures, diagrams or word problems. This implies limitations to models or ways of presenting fractions during instruction by educators.

Hackenburg and Tillema (2009) note that the difficulty with the multiplication of fractions depends on the type of fractions being multiplied and the order in which the fractions are multiplied. Learners are familiar with part of a whole type of problems, for example, $\frac{1}{2} \times 5$. Applying knowledge from whole numbers, they view multiplication as repeated addition The same problem can be expressed as $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ giving $\frac{5}{2}$. It becomes challenging to conceptualise and deal with part of a part type of problems. Empson and Levi (2011) refer to such as partial groups and exemplify by the following example: I have $\frac{3}{4}$ of a bag of candy. Given that a full bag weighs $\frac{1}{2}$ pound. How many candy pounds do I have? The problem translates to $\frac{3}{4}$ of $\frac{1}{2}$, part of a part; $\frac{3}{4} \times \frac{1}{2}$. Bruce et al. (2015) state that the "×" operator is generally interpreted as meaning repeated addition, but when dealing with fractions it means taking an amount of another amount, that is, a part of another fraction. The dual interpretation of the " $\times$ " may be the source of conceptualisation hitches as thinking of a $\frac{3}{4}$ being repeatedly added $\frac{1}{2}$ times might render fraction operations as abstract. The repeated addition approach is no longer convenient in such part of a part type of problems.

The literature provides the misconception that multiplication makes bigger and division makes smaller, and this includes operations on fractions. Graeber and Tirosh (1990) identified the learner notion that multiplication always results in a bigger number and division in a smaller number and considered the belief problematic as it impacts negatively on the understanding of operations with fractions.

According to Fitri and Prahmana (2019), misconceptions in the multiplication of fractions include multiplying mixed fractions without converting to common fractions. This leads to whole numbers multiplied to give the whole number answer and fractional parts multiplied to give the fractional component of the final solution. For example, $2\frac{3}{5} \times 7\frac{1}{8}$ manipulated as $2 \times 7 = 14$ for whole numbers and $\frac{3}{5} \times \frac{1}{8} = \frac{3}{40}$ for the fractional part leading to the combined solution of $14\frac{3}{40}$. The misconception is also reflected in the division of fractions. As Fitri and Prahmana (2019) contend, learners dividing mixed fractions without changing to common fractions exhibit the lack of conceptual understanding of division of fractions.

Fitri and Prahmana (2019) point out that equalising denominators in multiplication is a misconception which shows a lack of conceptual understanding in the multiplication of fractions. It is an unnecessary step which does not exist to those who have conceptualised multiplication of fractions. Misconceptions in multiplication and division of fractions may affect conceptualisation in other topics. As Brown and Quinn (2007) point out, the use of algorithms or short-cuts applied without reasoning about the actions can lead to learners having difficulty with more complex algebra later on. Experience with the multiplication of natural numbers may be used as an aid in conceptualising the multiplication of fractions. According to Tsankove and Pjanic (2009) the area model of multiplication can be used as an effective way of linking multiplication with fractions to whole number multiplication.

2.3.3.3 Misconceptions on comparing of fractions (A bigger denominator implicates bigger fraction misconception)

Ozkan (2011) suggests that a poor understanding of a concept promotes the development of an own understanding of the concept by learners. According to Ozkan (2011), the misconceptions on comparing fractions include the tendency to compare numerator with numerator and denominator with denominator. Jigyel (2007) believes misconceptions on comparing fractions emanate from the learner's perception that the numerator and denominator are two separate and not related whole numbers. Their conclusions are therefore influenced by prior whole number knowledge. This implies the extension of whole number knowledge to fractions. Siegler, Thompson and Schneider (2011) contend that the conceptual barrier to the learning of fractions is the knowledge of natural numbers.

Jigyel (2007) states that learners compare fractions better when they are presented geometrically as area models than when presented numerically as $\frac{a}{b}$. Visualising shaded portions of circles helps learners decide with ease on ranking the fractions being compared. However, Jigyel (2007) laments that learners do not develop any in-depth understanding of pictorial and numerical representations other than visual-spatial features (matching areas). The shallow learner understanding may therefore be sufficient for geometrical comparisons and lead to misconceptions on comparisons of numerical fractions.

Comparing fractions includes determining whether two fractions are equivalent or not. Misconceptions that relate to a comparison of equivalent fractions include, according to Jigyel (2007), learners' view that one fraction is a multiple of the other just like the relationship between the object and the image under enlargement in the topic transformations. The misconception further exacerbates the view discussed earlier on about comparing numerator with numerator and denominator with denominator. For instance, when comparing $\frac{3}{4}$ and $\frac{6}{8}$, since 6 is bigger than 3 and 8 is bigger than 4, $\frac{6}{8}$ is taken as greater than $\frac{3}{4}$, which strengthens the misconception that $\frac{6}{8}$ is double $\frac{3}{4}$. Bruce et al. (2015) believe challenges in comparing fractions stem from the nature of the fractions themselves as they point out that the multiple digits (numerator and denominator) representing one quantity are considered separately when comparing fractions.

2.3.4 Causes of misconceptions

Prediger (2006) is of the view that obstacles faced by learners can be didactical obstacles or epistemological obstacles. Clarifying further, Prediger (2006) brings forward didactical obstacles as those stumbling blocks that are evoked by the way of teaching and the epistemological obstacles as those that stem from the structure of mathematical content. That which misleads is the source of the misconceptions held by learners, whether didactical or epistemological it is of importance to this study. Van de Walle, Folk and Bay-Williams (2010) believe that learners build on their prior knowledge of whole numbers which supports and inhibits their work with fractions. Ashlock (2002) attributes manifestations of misconceptions to over generalisation and over specialisation of rules to make sense of new information. This implies that as much as previous knowledge is essential, it contains elements that can prejudice conceptualisation in fractions. The informal knowledge learners possess from everyday experiences can be for or against conceptualisation in fractions. Machaba (2016) asserts that when learners construct knowledge, reconstructing and re-organising prior-knowledge with the new knowledge, misconceptions are likely to arise. This means the inappropriate transition affecting the reconstruction and re-organisation of prior knowledge may lead to misconceptions. In other words, an improper link and association of experience with material at hand can jeopardise conceptual development.

Sarwadi and Shahrill (2014) point out that since mathematics is cumulative, learner failures to link new knowledge with previous knowledge (assimilation and accommodation) creates a gap in conceptualising and leads to misconceptions. This means that a learner experience or previous knowledge must be meaningful relative to new instruction for conceptualisation to occur otherwise misconceptions may result instead.

Ojose (2015) believes “naive theories” impede rational reasoning of learners and cause misconceptions. Naïve theories may be referring to self-imposed unproven theories that learners hastily deduce from their everyday mathematics. New instruction may require adjustments in the existing knowledge, and Ojose (2015) argues that the adjustment requires unlearning of what has previously been learned. This implies that cognitive conflicts during the learning-unlearning process lead to misconceptions. Ojose (2015) also links the manifestation of misconceptions with the nature of mathematics. Ojose (2015) contends that learners view rules as changing from one concept to the other. The supposed inconsistency in rules probably provides for misconceptions. Learners may be confronted with a selection crisis: Which rule to use and under what circumstances? Widjaja and Stacey (2009) condemned over-reliance on rules and memorised facts without understanding.

Woodward, Baxter and Howard (1994) pointed out that a continued surface understanding of mathematics lets learners apply improper algorithms or repair strategies resulting in firmly established and deep-rooted misconceptions. This implies that the shallowness of mathematical understanding of particular concepts may lead to misconceptions as learners resort to algorithms which they use inappropriately. Surface understanding may limit learner alternatives to solving problems at hand. This can be carried over through schooling years.

Sarwadi and Sharhill (2014) are of the opinion that mathematical misconceptions may originate in the primary stage of schooling and develop severely at the secondary school level as a result of inattention. This inattentiveness is levelled against teachers who generally assume that learners well understood the content covered in previous grades. Teachers presumably care less about a progress check on yesteryears' concepts but proceed with new instruction on unstable conceptual backgrounds in learners. Sarwadi and Sharhill (2014) state that misconceptions may be caused by faulty prerequisite knowledge or faulty existing schema. From the didactic perspective, it is not only inattention by teachers that may lead to the continued manifestations of misconceptions. Sarwadi and Shahrill (2014) attribute some misconceptions to learner incompatibility with teachers' instructions or techniques. This means the way teachers present their lessons may lead to misconceptions on the part of learners. The choice of words to use, examples used on giving instruction, reference to previous concepts and how illustrations or demonstrations are put across may be unfortunate sources of misconceptions in lesson delivery.

Sequencing of topics as teachers implement curriculum delivery can also lead to misconceptions. According to Widjaja and Stacey (2009), misconceptions resulting from overgeneralisation of fraction knowledge to decimals was more prevalent in U.S.A and Israel where the teaching of fractions comes before that of decimals than in France where decimals are taught first. This implies that the way the curriculum is delivered in terms of ordering of related concepts may in itself be a source of misconceptions. The reconstruction process is also prone to errors as it mixes knowledge from yesteryears with new instruction.

2.3.4.1 Prior-knowledge

According to Smith et al. (1994), learners do not come to instruction as blank slates. This implies the content they possess when subjected to new instruction might influence reception, accommodation and assimilation of new material. Content inappropriately used may therefore be the beginning of a series of misconceptions at different stages due to restructured knowledge.

2.3.4.2 Overgeneralisation

Olivier (1989) and Machaba (2016) argue that the source of misconceptions is mostly an overgeneralisation of previous knowledge, to an extended domain. For example, knowledge of whole numbers incorrectly used when dealing with fractions. An example of a likely misconception is $\frac{7}{10} - \frac{3}{5} = \frac{4}{5}$, where learners work out the numerators separately $(7 - 3 = 4)$ and denominators separately too $(10 - 5 = 5)$, giving the incorrect solution $\frac{4}{5}$. Ojose (2015) attributes this misconception to the learner application of the wrong algorithm in solving the problem. Smith et al. (1994) puts this across as the application of knowledge that is correct in a domain, in this instance a whole number, to a completely different domain where this knowledge no longer works like in the fraction domain.

Prediger (2008) states that the intuitive notion that multiplication makes bigger, which is true for natural numbers, has been shown by former empirical studies to be connected to the repeated addition rule, puts across another example of the impact of over-generalisation. If multiplication is indeed repeated addition, then the general expectation is an increase being the result. Vom Hofe et al. (2005) contemplate in their study about the multiplication of fractions that only those learners who got rid of their misconception of repeated addition could change their conception about multiplication's order attributes.

Prediger (2008) adds that aside from the generalisation of natural numbers, limited individual models of fractions (mostly only parts of a whole) proved an obstacle for the formation of multiplication models. Failure in the formulation of models for multiplication may be the reason why learners resort to the notion that multiplication is repeated addition. This alternative may compel them to use models for the addition of fractions in multiplication and end up in a mix up which more often leads to undesirable outcomes.

According to Lestiana et al. (2016), most learners in their study used a cross-multiplication strategy for adding fractions. For example, $\frac{1}{5} + \frac{2}{5} = \frac{5\times2}{1\times5} = \frac{10}{5}$. This implies that they over-generalised the cross-multiplication method of comparing fractions and applied it to the addition operator. Lestiana et al. (2016) further state that some learners did what they termed crossed-addition, that is, $\frac{1}{5} + \frac{2}{5} = \frac{5+2}{1+5} = \frac{7}{6}$. The misconception is that as we can cross-multiply, cross addition is also possible. Bruce et al. (2014) advocate this learner tendency to the complexity of fractions as a "multi-faceted construct". This means fractions can be applied in a variety of ways making use of shortcuts. Bruce et al. (2014) further state that student misunderstandings of the meaning behind algorithmic "shortcuts" with fractions can lead to problems later in other areas of mathematics, such as algebra.

2.3.4.3 Teaching

Brodie et al. (2009) hold the view that an understanding of where misconceptions are coming from and where they are being made is a valuable tool which may be used to improve teaching and to improve learner understanding of concepts. This means informed instruction will be in such a way as to address and minimise chances of misconceptions.

“If algebra is for everyone, then all students must first become familiar and fluent with fractions”, (Brown & Quinn, 2007, p.12). De Turk (2008) laments that without a foundation in fractions, learners who come to study of rational expressions in algebra will be severely handicapped. This brings in the role of the educator in the conceptualisation of fractions. Mathematical learning is a systematic process that involves building on prior knowledge and mixing different skills and basic concepts to achieve mastery of mathematical calculations and procedures (Sarwadi & Shahrill, 2014).

Siegler et al. (2010) are of the opinion that learners sometimes learn computational procedures (fractions algorithms) through rote learning; hence they often quickly forget or become confused by routines. Rote learning deprives learners of in-depth understanding, as Siegler et al. (2010) assert, “forgetting and confusing algorithms occur less often when learners understand how and why computational procedures yield correct answers”. Without a proper approach (on instruction), inductive in some way, learners will make their deductions/shortcuts, which may not be universally correct. Siegler et al. (2010) recommend that educators should explain why some non-standard procedures that learners generate yield correct answers despite not looking like conventional algorithms.

Expertise on the teacher part comes in handy as Bruce et al. (2014) state that researchers have found that even when teachers possess solid content knowledge and conceptual understanding of fractions themselves, prospective teachers struggled to represent fractions conceptually (with pictures, diagrams or in word problems). This implies that on teaching, their learners would be subjected to procedural instruction. Van Steenbrugge, Lesage, Valcke and Desoete (2014) contend that misconceptions and understandings of teachers mirror those of elementary school students; hence their call to address teachers’ preparation to increase expectations from learners in terms of performance.

Li and Li (2008) believe that the mathematics curriculum deficiencies or teaching strategies that deny learners opportunities to assimilate new concepts can result in misconceptions. Curriculum planners and the curriculum implementers, therefore, play a part in promoting or avoiding misconceptions.

According to Sleeter (2005), the race, gender and socio-economic status of children need to be considered when teachers try to convey fractions to students using word problems or pictorial representations. This implies learner background impacts on understanding. Examples and exercises given must be within learner contexts and drawing from their experiences. Siegler et al. (2010) state that teachers are aware of learners’ difficulty in learning about fractions and are often frustrated by it. They further recommend professional development programmes for teachers which will empower them.

Nesher (1987) suggests that at least two kinds of expertise are a requirement for one to teach the given subject matter. Nesher (1987) puts the subject matter expert, who has deep insight into the conceptual structure and can, therefore, knowledgeably handle the discipline to be learnt, upfront. This implies that educators handling fractions must be well versed with the content. Nesher (1987) further explains that as a teacher, being a knowledge expert may not be adequate, the role of an expert teacher calls for this knowledge via engagements with learners in various pedagogical techniques to bring the learners to know the given subject matter successfully.

Prediger (2008) believes that learners who are only conversant with the part of a whole model for fractions suffer from a didactic obstacle that can be avoided by use of a variety of interpretations of fractions. The didactic aspect calls for educators to be empowered well in terms of fraction knowledge, its dissemination and alternative instruction delivery ways. The application of numerous teaching approaches that considers differences in learners’ learning styles might do away with the barriers caused by exposing learners to only part of a whole model of fractions.

During instruction, educators interact with learner errors. Gardee and Brodie (2015) suggest that educators may avoid, correct, probe or embrace errors. Probing and embracing may bring misconceptions held by learners in the topic of fractions to surface.

Employing alternative ways to get to the gist of where misunderstandings emanate from may require the subjects’ perspective.

Probing is one such way in which learners can be prompted to respond to explain themselves. According to Brodie and Shalem (2011), an effort by teachers to get first-hand information on how errors make sense to learners involves asking the learners themselves probing questions or pressing questions to gain access to learner thinking. Brodie (2014) supports the idea of probing as she believes it improves the reasoning capabilities of learners as learners explain their thinking, thereby justifying their ideas. Guarino et al. (2013) suggest cycles of planning, teaching and reflecting deepen teacher understandings of how learners reason about key mathematical ideas and common misconceptions. They further argue that teachers get insight into fruitful instructional strategies, visual representations and mathematical tasks from such exposures. Gersten, Schumacher and Jordan (2017) define fractions as part of a whole, stating that the key to teaching fractions is to keep it concrete like using fraction circles and other visual manipulatives.

Olivier (1989) argues that misconceptions are part of learning. This means that in the process of learning, misconceptions cannot be ruled out. Teachers should be empowered enough to courageously grab the opportunity to redirect learners and utilise misconceptions on enhancing learning. Vosnaidou (1999) declares that the treatment of misconceptions as opportunities for reflection is important and supported by conceptual change research that outlines the meta-conceptual awareness as an invaluable condition for a successful process of change. Duit (1999) describes this as a conceptual change approach in which the discrepancies between the intended mathematical conceptions and the individual conceptions are not seen as individual deficits but as typical stages of transition in the process of reconstructing knowledge.

Prediger (2008) is of the opinion that opportunities for developing mathematical literacy are enhanced through reflecting on misconceptions as they can clearly outline the patterns and aims of processes of concept formation in mathematics. Teachers should, therefore, not limit their role to classwork and corrections. The misconceptions they come across on learner feedback should be reflected upon, so that future lessons are not only informed by the current findings but are also better in terms of efforts to avoid misconceptions.

Gardee and Brodie (2015) believe that if educators work on ways to understand why learners may have made errors, they may come to appreciate learners' thinking and discover methods to assist learners in engaging their current knowledge to develop new knowledge. This implies that the solutions learners provide are a result of thinking though it might not be in line with expectations. An understanding of learners' thinking may provide educators points of departure in their quest to facilitate the creation of new knowledge at the expense of learners' existing knowledge. This renders this study relevant as it aims at coming up with misconceptions that learners have in the concept of fractions at the Grade 9 level. This further strengthens the idea that misconceptions are a passage to knowledge acquisition.

Brodie (2013) advocates teachers' embracing of errors as this involves teachers utilising errors constructively to generate new knowledge for the learner who made the error and others. Gardee and Brodie (2015) refer to this as using errors and misconceptions as utilities to enhance epistemological access. For this to happen with success, it follows that educators must desist from taking wrong solutions as signifying a total mathematical breakdown by the learner. The teacher's attitude towards a learner's incorrect feedback impacts accordingly on the learner's mathematical way forward.

Hansen (2011) argues that sensitivity on the part of teachers when it comes to errors and misconceptions may render errors productive as errors can be motivational tools that assist learners in developing their conceptual knowledge from their errors. Nesher (1987) agrees with this as he states that unexpected feedback on the part of learners, if not condemned for it, intrigues and motivates the learner to pursue an inquiry, thereby learning. Nesher (1987) elaborates on the idea of embracing misconceptions by stating that each misconception and the resulting error have the potential to become a significant milestone in learning; hence they must be welcome. Hansen (2006) adds by stating that the most effective teaching strategy is to cultivate an ethos where learners do not mind making mistakes. Ojose (2015) believes that teachers should acknowledge that misconceptions held by learners can be overcome through planning, employing effective teaching strategies or generally deliberately offering learning opportunities for learners. Sarwadi and Sharhill (2014) recommend that teachers should be diagnostically oriented as they prescribe appropriate instructional strategies.

Yearly and Bruce (2014) assert that the way teachers read fractions might also contribute to misconceptions. They are of the opinion that reading $\frac{4}{10}$ as “four over 10” leads to the following representation of the fraction by learners.

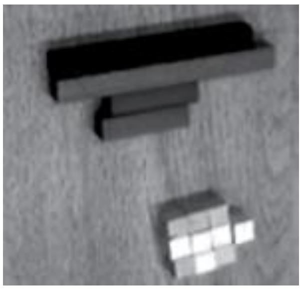


Figure 2-7: Learner representation of 4 over 10

Four items are literally seen over ten items. Brodie (2014) states teachers at times promote chances of errors through the taken-for-granted use of language and concepts. This means educators must choose words carefully avoiding vague language on instruction as this may breed misconceptions in learners. Jigyel (2007) terms this the pedagogical influence on learners’ developing fraction language since learners tend to reproduce the sanctioned classroom language.

In a bid to check the didactic influence, Jigyel (2007) conducted a test on learners to choose the most appropriate way of reading $\frac{2}{5}$. Options provided were; two-fifths, two over five, two upon five and none of the above. Jigyel (2007) states that two-fifths was most popular though a significant number of learners chose two over five. According to Jigyel (2007), a learner interviewed (selected from the two over five batch) explained that there is a two on a line above five, so it is two over five. Already, there is a separation of the two from the five which may influence the perception that $\frac{2}{5}$ is not a number which can be positioned on the number line. Two out of five is not amongst Jigyel’s (2007) options but may also come along with its own misleading ways in the concept of the fraction.

Niemi (1996) challenges teachers to involve learners in activities where learners get an opportunity to express (verbally or in writing) how they create or recognise equivalent fractions as this may strengthen understanding and provide valuable information to teachers. Learners' perceptions may reveal inherent misconceptions. This may offer a platform for the exposure of an incorrect line of thinking that may coincidentally yield the anticipated final solutions. Guarino et al. (2013) believe that posing the question "why" during instruction uncovers learners' ways of thinking as they provide critical insight into their understandings or misunderstandings. This might inform the teacher on possible sources of misconceptions. Concerning incorrect learner thinking that results in expected answers, Nesher (1987) encourages teachers to consider the existence of non-discriminating items so that they guard against using such on assessments. Non-discriminating items disguise misconceptions as follows;

$$\frac{4}{15} \div \frac{2}{5} = \frac{2}{3}$$

Getting the correct solution to this problem may not necessarily mean learners well conceptualised the division of fractions. The solution does not discriminate between learners who manipulated correctly from those who got the correct answer due to mere coincidence. However, the following procedure discriminates, it separates those who conceptualised from the rest. Not everyone who got the above correctly could also do the following manipulations correctly, some may face challenges.

$$\frac{3}{5} \div \frac{2}{25}$$

Kilpatrick, Swaffold and Findell (2001) state that teachers should be prepared and present contrasting cases, items that cover a wide range of aspects of the concept at hand so that learners use strategies that work across a class of cases. Ojose (2015) holds that teachers who plan knowing the nature of a misconception and its source usually fathom appropriate ways to deliver instruction that is beneficial to learners. In addition to analysis of procedures and their pre-requisites, Nesher (1987) adds that teachers must know how new knowledge is embedded in learners' existing larger meaning system as they prepare instruction for a new piece of knowledge. Nesher (1987) acknowledges how highly improbable it is for a teacher to fully predict the effects of learners' previous knowledge systems on the current instruction and therefore suggests the following approach as depicted in Figure 2.8.

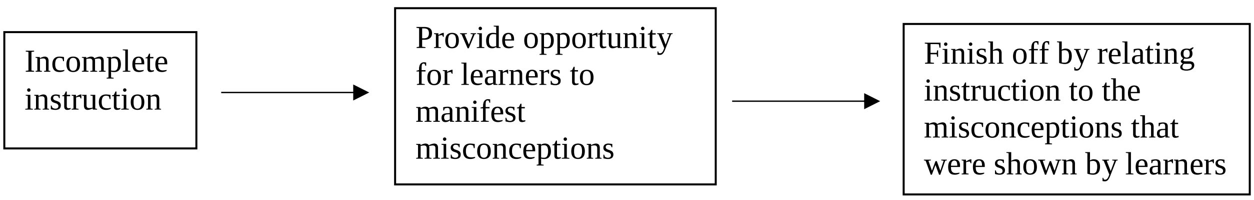


Figure 2-8: Classroom instruction delivery

Informed instruction after learners have revealed misconceptions is what Guarino et al. (2013) term facilitated learner-directed instruction. It may lead to in-depth understanding as instruction directly addresses the mishaps learners presented. This implies that a proper interlink of new and existing knowledge may promote conceptualisation, minimising misconceptions. According to Nesher (1987), chances of the occurrence of multiplication and division word problems in textbooks is very low and worrying as misconceptions related to such may not be detected. As teachers, more often than not, rely on textbook material as guidelines towards fulfilling curriculum implementation, some misconceptions may remain locked up in this part of fraction work which is hardly addressed. Ojose (2015) suggests that teachers should be sensitive and look out for misconceptions and misunderstandings that learners may bring to class so that they work purposefully towards correcting them. Guarino et al. (2013) believe it is important for teachers to anticipate learner misconceptions at the planning phase of the instruction to be delivered.

2.4 CHAPTER SUMMARY

In this chapter, the focus was on literature related to the study. The researcher also looked at the study in retrospect, discussing findings by previous researchers in matters pertaining to the study.

The theoretical framework, constructivism, which is the backbone of this study was discussed. The essence of constructivism is that learners partake in constructive learning on their own, quietly attempting tasks, sifting through the materials they are working with, and consolidating new ideas with existing ideas. In relation to this study, the theory of constructivism is pertinent in the sense that when learners construct their own knowledge, misconceptions are bound to arise.

The definition of a fraction which this study adopted was discussed in this chapter. Van De Walle et al.'s (2016) perception of a fraction as a part of a whole is the guiding definition of a fraction used in this study. The whole referred to, is the whole subdivided into equal parts or equal shares or equal-sized portions.

Misconceptions were explained in relation to errors before zooming into misconceptions in the concept of fractions and the possible causes from literature. Misconceptions were differentiated from errors following Gardee and Brodie (2015)'s guidance that one is a consequence of the other; that is, errors are a consequence of misconceptions. The types of misconceptions on fractions discussed in this chapter are misconceptions in the addition and subtraction of fractions, misconceptions in multiplication and division of fractions as well as misconceptions on comparing fractions. Prior knowledge, over generalisations and teaching, were discussed as causes of misconceptions in this chapter. As this study was about exploring misconceptions in the topic *fractions* at Grade 9 level in a particular school in Soweto, the next chapter discusses the design and methods of collecting data.

3 CHAPTER THREE: RESEARCH METHODOLOGY AND RESEARCH DESIGN

3.1 INTRODUCTION

This chapter documents the design and the methodology followed during the fieldwork. The researcher shall explain the overall design, the sampling techniques employed and the criteria used in the choice of the sample size. The researcher shall also give full details of the data collection process (how the data was collected) and how the researcher gained access to the subjects. The design and methodology were enacted to achieve the objectives of the study, which were:

- To explore how Grade 9 learners describe the concept of fractions.
- To investigate how Grade 9 learners solve problems involving fractions.
- To discover misconceptions that learners have/display when dealing with fractions.
- To discover the root cause of misconceptions held by learners concerning the concept of fractions.

3.2 RATIONALE OF THE RESEARCH APPROACH

This part of the report gives a description of the route undertaken and the methods employed in the study.

3.2.1 Qualitative approach

The researcher focused on how learners deal with fractions at Grade 9 level, the misconceptions they hold as well as the root causes of these misconceptions. The learners seek an understanding of the world they live in, in relation to fractions. In their pursuit of knowledge, the learners develop varied and multiple subjective meanings of their experiences. These subjective meanings are negotiated socially and historically-formed through interaction with others. Inductively, learners develop a theory or pattern of meaning. All of the above augurs well with the assumptions of the socialist constructivist worldview, according to Creswell (2009:10). Thus the nature of the research methodology that was used in this study is qualitative.

3.2.2 Research design

In the case study approach, one or more events, environments, programmes, social groups or interrelated systems will be analysed in detail (McMillan & Schumacher, 2014). Case studies are carried out to define and see the details comprising an event, develop potential explanations for an event, and to evaluate an event (Gall, Borg, & Gall, 1996). It is the qualitative research methods which allow the researcher to make an in-depth analysis and interpretation, and through the findings arrive at analytical generalisations rather than realistic estimates for similar situations (Cohen, Manion, & Morrison, 2000). The researcher intended to make an inquest on learners' work on fractions and identify misconceptions they held. The quality of learners' work was the primary source of data; therefore, this qualitative study adopted the case study design.

3.3 RESEARCH SETTING

This study was carried out in one of the Department of Basic Education's secondary schools in Soweto, a school in Circuit 4 of Johannesburg North district in Gauteng. The organisational structure of the Department of Basic Education (DBE) in the Gauteng province can be illustrated as follows:

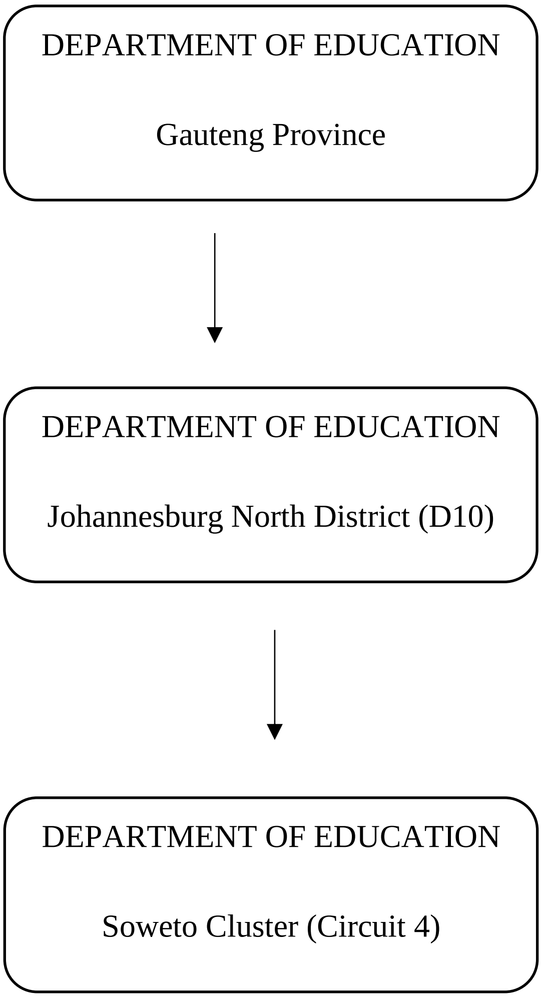


Figure 3-1: Gauteng Department of Education organisational chart

The data was collected during the first term, a term allocated for the number systems by the South African CAPS curriculum. Participants were constituents of a class in a Soweto school in Circuit 4 under the Johannesburg North region with seven Grade 9 classes.

3.4 RESEARCH SAMPLE AND DATA SOURCES

3.4.1 Population and sampling procedure

The research site was a Secondary school in Soweto Township. The participants were Grade 9 learners that were not known to the researcher. Convenience sampling was used in light of accessibility to the subjects at the school, while one class in the school was chosen randomly. According to Creswell (2009), random sampling is the random selection of a small sample. From a population of 320 learners making up seven classes, one class was chosen randomly for the study. An interview was conducted with four boys and four girls; selection based on their responses in the test. The researcher was looking for a learner script that showed application of similar but incorrect procedures under specific sections of operations of fractions, for example, questions on multiplication of fractions. Both performance extremes were also considered, the good and the worst performers overall.

3.4.2 Ethical consideration

The researcher observed the importance of ethical considerations and went through the requisite process, as evidenced by the issuance of the ethical certificate (Appendix B). Since the study was conducted at a government school, the researcher had to follow protocol and get clearances at all structures that govern the school up to district level. The researcher completed a Gauteng Department of Education (GDE) form seeking permission to conduct research in one of its institutions. The researcher wrote a letter to the principal of the school at which the researcher was conducting the research. The letter requested permission to conduct the study with Grade 9 learners and described the purpose and the rationale of the study. The school governing body (SGB) was informed of the research in a meeting with the principal. After permission was granted, the Grade 9 learners were informed of the study in time for them to decide whether to participate in the study or not without being forced.

The learners who agreed to participate in the study were guaranteed anonymity and confidentiality. They, however, had to fill a participant information sheet for record purposes (Appendix G). Letters describing the study were sent to parents/guardians of potential participants with an optional tear-off return slip for providing consent. The letters had full details of the researcher and encouraged parents/guardians to make an appointment to meet the researcher for clarification purposes. The parents/guardians of the participants signed letters of consent approving the participation of their children in the study. Pseudonyms were used in the study, and responses were kept confidential. The researcher made it clear to participants that the study had nothing to do with their course work hence did not contribute towards promotion marks. However, the researcher told the participants that they could see their scripts and marks if they wanted to. All collected data was put under lock and key.

3.5 DATA COLLECTION METHODS

In this qualitative research, the instrumental case study research method was employed. A test and interviews were the main sources of data. To guard against questions that may not bring forth feedback relevant to the study, a pilot was conducted. The pilot also helped adjust interview timing and duplications.

A Grade 9 class of 40 learners was then subjected to a test on the topic of fractions. The researcher chose Grade 9 with the assumption that Grade 9 learners would have done fractions in their previous learning. Eight learners were chosen for an interview based on how they had answered the test items. After the written test, the researcher marked the questions in the test in terms of correct, incorrect and partially correct categories per learner. In other words, the recordings of the learners' responses were recorded per learner, and per question, as indicated in Table 1(Appendix I). Codes were assigned to each learner to serve as references whenever necessary in this study, for instance, L1, for Learner 1. No specific criteria were used for coding. This helped the researcher to have an overview of all the learners' performance. The interview was tape-recorded so that the researcher could transcribe the information correctly.

3.5.1 Written test

According to McMillan & Schumacher (2014), student proficiency which is the major concern of much educational research is usually assessed using tests. This study is no different as a test was one of the sources of data. McMillan & Schumacher (2014) further state that a test is a rich resource for doing research. Downing (2009) agrees as he states that in the testing of learning, abilities and achievement or in the measurement of cognitive knowledge, written tests are useful. In particular, the test 'strength to expose what Downing (2009) terms the "knows" and "knows how" of the base level of Miller's Pyramid. To gain an understanding of learners' thoughts about the concept of fractions, a fifteen–item test was constructed and administered to learners. Its content was based on the conceptual understanding of fractions, insights and misconceptions on fractions. The items varied from decontextualised to contextualised problems. Learners were asked to show all their solutions or answers on the answer scripts provided. All the test items were corresponding to the purpose statement and research questions of the research study.

A test on fractions was administered following Bloom's Taxonomy of question levels. It featured all the levels of questioning, from the *recall type* up to the *high order reasoning* type. Its content was based on the conceptual understanding of fractions, insights and misconceptions on fractions. A variety of questions on fractions were included, some drawn from learners' everyday contexts. Learners were asked to show all their working or solutions on their answer scripts. McMillan & Schumacher (2014) believe tests differ in use and it is what is done with the results that create distinctions such as achievement and aptitude. Results of the administered test in this study were used to select learners for interviews.

Table 3.1: Written Test

TEST

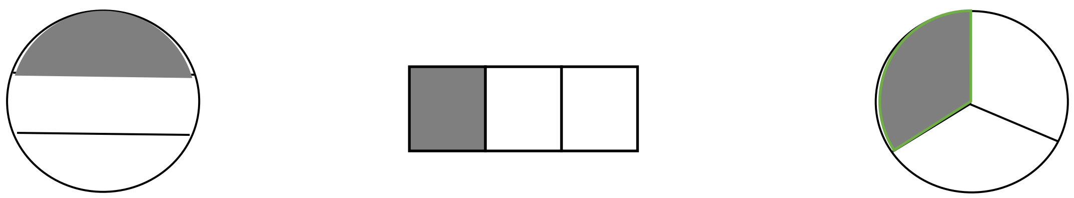
Answer all the questions

Show all your calculations where possible

Question 1

1.1 What is a fraction?

1.2 Write the fractions represented by the following diagrams



A B C

A ---------------------------------

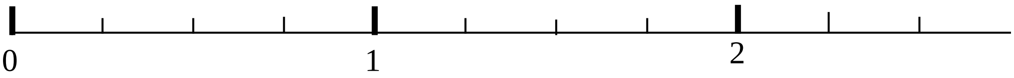
B ---------------------------------

C---------------------------------

1.3 Which is bigger, $\frac{3}{4}$ or $\frac{5}{8}$ of a given cake? Support your choice by showing how you compared the two fractions.

1.4 Show the following on the number line below:

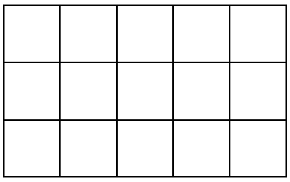
$\frac{1}{4}$; $\frac{3}{4}$; $\frac{7}{4}$; $\frac{8}{4}$; $\frac{10}{4}$



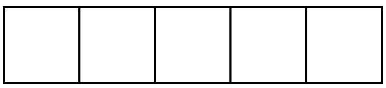
1.5 Arrange the following fractions from biggest to smallest

$\frac{5}{8}$; $\frac{4}{4}$; $\frac{2}{6}$; $\frac{3}{7}$; $\frac{1}{2}$

1.6a). Shade $\frac{3}{5}$ of the rectangular mat below.



b) Write down in words what you will do to shade $\frac{3}{5}$ of the rectangular mat.

1.7  Shade $\frac{3}{5}$ of the whole

In the fraction $\frac{3}{5}$, what does the 5 mean? ___________________

What does the 3 mean? _________________________________

<u>Question 2</u>

Use any strategies to find answers of the following

2.1 $\frac{1}{3} + \frac{1}{6}$

2.2 $\frac{1}{4} + \frac{3}{4}$

2.3 $\frac{7}{10} - \frac{2}{5}$

2.4 $7\,\frac{1}{2} + 10\frac{1}{2} + \frac{3}{4}$

Question 3

Use any strategies to find answers of the following:

3.1 $\frac{1}{2} \times \frac{1}{3}$

3.2 $2\frac{1}{2} \times 3\frac{1}{5}$

Question 4

Use any strategies to find answers of the following

4.1 $8 \div \frac{1}{2}$

4.2 $\frac{1}{2} \div 10$

3.5.2 *RATIONALE FOR THE CHOICE OF QUESTIONS*

Questions 1.1 – 1.7 focus on the conceptual understanding of the meaning of the concept, fraction

QUESTION 1.1

This question tests learners' conceptual understanding of the concept of fractions. As learners define fractions, they showcase what they know and understand about the concept, fractions. This question deals with the formal dimension aspect of the concept of fractions. It compels the respondents to portray individual understandings of the concept of fractions by means of their definitions.

QUESTION 1.2

Question 1.2 checks for learner awareness on visual representations of fractions. It tests if learners could name a fraction represented diagrammatically. Specifically, the question seeks to check whether learners are aware of the importance of equal parts as sub-divisions of a whole. Again, this question assesses the conceptual understanding of the concept, fraction, such as “My whole is a circle. It is divided into three equal parts. Each part is a third of the whole”.

QUESTION 1.3

Question 1.3 tests whether learners can compare fractions and decide on which fraction is bigger or smaller than the other. Use of knowledge on equivalent fractions may be a requirement; hence the question checks for learner competence in this regard. It also seeks clarity on how the comparison is done as there are alternatives to equivalence or compare the following two fractions, using blocked paper.

QUESTION 1.4

This question checks whether learners can represent fractions of the same denominator on the number line. This question test learners understanding of fractions as numbers between 0 to 1 (This is one unit). The way in which we demarcate (iterate) the number line, will tell us into what fraction parts the unit is divided. Viewing $\frac{8}{4}$ as a quotient will help learners realise that it is the same as two and eventually allow them to place it correctly on the number line.

QUESTION 1.5

This question tests learner’s ability and knowledge to sort more than two fractions in such a way that it becomes easier to arrange them chronologically. It tests learners’ understanding of the ordering of fractions. The fractions in question have different denominators so the question checks for the fraction comparison skill, whether through equivalence or other means. This question also seeks an explanation of the method/s undertaken by the learner in arranging fractions.

QUESTION 1.6

This question tests the learners’ ability to represent fractions diagrammatically. It further tests the understanding of the meaning of fraction. The first area to be shaded has the number of sub-divisions equal to the denominator of the fraction to be represented. The second area to be shaded checks for knowledge of equivalent fractions or application of a fraction as an operator as the number of partitions in the area to be shaded is not equal to the denominator of the fraction under consideration. This question further seeks clarity on why the portions were shaded the way they were; a link between the parts of a whole concept and the concept of equivalent fractions.

QUESTION 1.7

Question 1.7 requires learners to represent a fraction diagrammatically by shading parts of a given whole. Learners are further expected to explain what a numerator represents and also explain what a denominator represents in a fraction. The question seeks the meaning of each component of a common fraction.

Questions 2 to 4 focus on operations of fractions.

QUESTION 2

This question tests learners' understanding of the addition and subtraction of operations of fractions. This question checks for the underlying steps, procedure and conceptual understanding of adding and subtracting fractions.

This question requires learners to showcase how they add common fractions with different denominators, add common fractions with the same denominators, subtract common fractions with unlike denominators as well as add three fractions which include mixed fractions.

QUESTION 3.

This question focuses on multiplication of fractions. The question is broken down into multiplication of common fractions and multiplication of mixed fractions

QUESTION 4.

This question tests learners' understanding of the division operation of fractions. It also seeks to uncover how learners divide fractions and the application of standard procedures (use of the invert and multiply rule). The question focuses on division of a whole number by a fraction and also the division of a common fraction by a whole number

The researcher marked the scripts, completing the question by question score sheet. The eight chosen learners were interviewed face-to-face in relation to their test responses.

3.5.3 Interviews

Scrutiny of just the written feedback by learners may not have been adequate in addressing the objectives of this study. Written feedback might have limited the study to the algorithmic dimension of knowledge learners displayed. The algorithmic dimension involves learners' capability to use successive steps of standard procedures, according to Prediger (2008). Leaners may blindly follow the steps without necessarily fully understanding the concepts. In addition to written feedback, interviews were conducted to augment the written feedback. Interviews are comparatively better positioned than written feedback when it comes to making learners reveal how much they know and collaborate on building structures of the knowledge they have. In-order to elicit the learners' understanding, misunderstanding or both, the interviews were conducted in a clinical fashion. Prediger (2008) asserts that the intuitive dimension of knowledge entails representing concepts and operations using ideas and beliefs about mathematical entities and mental models. The flexibility of interviewing in terms of follow-up questioning puts the researcher at a vantage point allowing prying deeper into the learner insights.

According to Vamvakoussi and Vosniadou (2010), interviewing students allows researchers to understand the process of thinking involved in solving mathematical problems better and, consequently, better identifying what misconceptions students hold. An interview helps check for what Skemp (1976) terms "the Devil's Advocate", a situation where educators teach in such a way that learners get correct solutions through instrumental means (knowing what to do but not why it works).

Creswell (2009) explains that in the interview guide approach, topics are selected in advance, but the researcher decides the sequence and wording of the questions during the interview. Creswell (2009) further states that the interview guide is conversational and situational, probes can increase comprehension. The interview, utilising a semi-structured format, adhered to these guidelines, as each selected learner was subjected to the same questions in any order beginning with general questions. The probing related to specific feedback from interviewees. The learners' written responses guided their selection, and the selected learners were called one by one to be interviewed about what they had written. They were then asked to justify their responses. Participants were probed to gather as much information as possible regarding their responses. They were allowed to give their interpretations, experiences, insights and their reasoning concerning the concepts of fractions.

The researcher utilised an interview guide approach on participants. Topics were selected in advance, but the researcher had to decide on sequencing and wording during the interview. McMillan and Schumacher (2014) believe that in-depth interviews must assume a conversational tone rather than an interrogative one. The focus was therefore on covering pre-selected topics in any order in a smooth flowing conversational way. These included learner fears on the topic fractions, favourite sections in the topic as well as interpretation of some content on fractions. Content stretched to include all four operations with fractions: addition, subtraction, multiplication and division.

The interview helped gather information about learners' experiences, expectations and the scope of fractions as they approached a test on the concept fractions. The interview was tape-recorded so that the researcher could transcribe the information correctly.

3.6 DATA ANALYSIS METHODS

The written test and interviews were the major sources of data in this study. Analyses of data from both sources was usually done simultaneously. The researcher realised that the findings from these sources supported or emphasised one another. Furthermore, on interviews, the researcher ended up reporting on data from four learners instead of the intended eight as they were resemblances in some of the responses. The data provided by four interviewees was representative and provided adequate data to address the objectives of the study. Considering what Skemp (1976) termed the devil's advocate, the interviewed learners were expected to explain the thinking involved even on test items they got correct. The data analysis was an ongoing process from the stage of collecting data onwards. This was a qualitative study. McMillan & Schumacher (2014) believe that the data collection and data analysis of such studies are interwoven and influence one another. The interview recordings were transcribed and the resulting word documents kept as files in a computer.

The scripts from the written test were marked, and a question by question analysis table completed referring to the individual participant solutions. Three categories were used to classify learner solutions, "C" for a correct solution, "PC" for a partially correct solution and "IC" for an incorrect solution. The data provided by this table was used to enact another table revealing a prevalence of correct or incorrect solutions per question.

A diagnostic analysis report was completed. The purpose was to zoom into the similarities in the steps or procedures that learners took in solving test items, especially those that led to incorrect solutions. Most importantly for this study, the diagnostic analysis highlighted the most common misconceptions per question.

3.7 DISCUSSION OF THE LEARNERS' RESPONSES FROM THE PILOT

To maximise the chances of getting data relevant to the study that addresses the questions that guide the study, the researcher carried out a pilot. The pilot helped in the timing of interviews, the rephrasing of some questions and even in removing some questions from the initial test items. Some questions were found to be too vague, so the pilot assisted and the vague ones were adjusted accordingly. Some questions/test items duplicated the outcomes; they brought nothing new or different from what other questions/test items had provided. Such questions/test items were then also removed.

QUESTION 1.1

This is a question checking on the conceptual understanding of fractions through definitions. The question was not changed. The learners displayed a vague idea of what a fraction is. On explaining their cases, the learners preferred writing an example of a fraction instead of verbally stating what a fraction is. Those learners who tried had their definitions of a fraction relating to dividing a whole into pieces though it did not clarify the equality in size of the pieces. The learners struggled to come up with a simple definition of a fraction. They however resorted to utilising real life examples; like cutting of a cake. On being asked how cutting of a cake makes a fraction, some learners explained that the number of pieces taken out of the number of pieces that were cut makes a fraction. Though it may sound proper, learners took it for granted that pieces cut are equal in size as this was never mentioned. None of the explanations went beyond a whole. This implies that the fraction concept is confined to one whole drawing from learners' definitions. According to Van de Walle et al. (2016), the learners' scope is biased towards the continuous whole or area models of fractions. This follows Abosalem's (2013) concern about fractions having different meanings or not being clearly defined, which may result in misconceptions. Abosalem (2013) clarifies by adding that at some point $\frac{1}{4}$ of a chocolate bar indicates part of a whole and the next three out of four students $\frac{3}{4}$ represents part of a set. Definitions by learners did not cater for the set model or the discontinuous whole.

QUESTION 1.3

This is a question checking on the conceptual understanding of fractions by means of comparing fractions. The question was not changed. Basically, learners used denominators to compare the two fractions. Some chose the bigger denominator implying the bigger fraction. Those who chose the smaller denominator justified their choice by means of the size of cake slice when divided into eight parts being smaller than when divided into four parts. In both cases, learners disregarded the numerator. The focus was directed to the denominators and the numerators played no significant role in comparing fractions. One learner, L13, however, compared both numerators and denominators. The learner said, "As numerator, 5 is bigger than numerator 3 and denominator 8 is bigger than denominator 4, then it follows that $\frac{5}{8}$ "IS .This showcases a lack of conceptual knowledge of fractions by the learner.

QUESTION 1.5

A question checking on the conceptual understanding of fractions by means of ordering fractions. The question was not changed. The part of a whole area model influenced the learner responses to this question. Most learners held the view that the bigger the denominator, the bigger the fraction. The denominators guided the learners and the numerators played no part in the ranking. Numerators were insignificant and played no part in arranging the fractions. L13's advice, however, was 'arrange according to numerators, from small to big". Responding to the question "Why?" the learner said, "numbers increase according to the number line, one piece remains smaller than five pieces taken from a whole". L13 clarified that pieces taken from a whole help determine the small fraction and the numerators denote these. Denominators, in this case, were not considered. This may mean L13 may not take the numerator and denominator as components of one number, which is a fraction; hence, this renders the denominator not important.

L1 explained the part of a whole concept well referring to each fraction. The learner, however, had no idea of using equivalent fractions for comparisons. Lack of alternatives led to the learner recommending drawing pictorial representations of each fraction and matching. L1 explained that circles should be drawn and divided into the denominator partitions of the fraction to be represented. The numerators provided the number of parts to be shaded. Shaded circles can then be compared. The method proved complicated and can be tedious considering the number of fractions that may be compared and the inaccuracy of size of partitions. This shows that the learner did not develop the fraction concept across grades but was still operating at the introductory level the learner was exposed to at primary school.

QUESTION 1.6

This was a question checking on the conceptual understanding of fractions by means of representing fractions diagrammatically. The question was not changed. Many learners correctly shaded the first area where the denominator is equal to the area partitions. Knowledge inadequacies were evident in the second area where the area partitions were not equal to the denominator of the fraction to be represented. The learners seemed not to understand what was expected of them. L25 resorted to considering only five items which may have been due to the interpretation of the part of a whole concept from the "out of" point of view. The restricted conceptualisation limited to the continuous whole (area model) might have compelled L4 and L25 to focus on five items only. This total breakdown showcased a lack of knowledge on equivalent fractions or on the use of the fraction $\frac{3}{5}$ as an operator on 15 to get nine as the number of blocks to be shaded. L35's representation of the fraction $\frac{3}{5}$ has eight shaded blocks. For the rationale, learner 35 clarified that, "three blocks shaded separate from the rest are for the numerator and the other five represent the denominator". This displays a lack of basic knowledge of fractions. L35 might be striving to display $\frac{3}{5}$ literally so that at a glance three shaded blocks would be seen over five shaded others.

QUESTION 1.7

This was a question checking on the conceptual understanding of fractions by means of explaining what components of fractions mean. The question was not changed. The idea of parts of a whole was used by some learners to explain what $\frac{3}{5}$ meant. Most learners, however, wrote 3 is a numerator, and 5 is the denominator instead of the explaining meanings of each in the fraction $\frac{3}{5}$. It was interesting how they adjusted their explanatory approach when it came to the fraction $\frac{10}{3}$. Shading parts of a whole perspective might have complicated the fraction and rendered it outside learners' scope. This might have led to some learners viewing $\frac{10}{3}$ from the division point of view as some had no idea what it really meant. L2 interpreted the two fractions $\frac{3}{5}$ and $\frac{10}{3}$ differently. When asked why interpret $\frac{3}{5}$ as three out of five and $\frac{10}{3}$ as ten divided by three, the learner said, "you cannot shade ten parts out of the available three so as such it is a division problem". The two views might have put the learner in a difficult position to define both the numerator and the denominator. This also cements the idea of the complexity of fractions as they can be perceived differently probably depending on the context.

QUESTION 2.1

This was a question checking learners' understanding of the addition and subtraction operations of fractions. The question was not changed. The most common solution was as a result of adding across, numerator adding to numerator, likewise, denominator adding to denominator. Confidently, a learner justified this approach by stating, "if I get $\frac{2}{3}$ in a maths exercise today and get $\frac{1}{4}$ in tomorrow's exercise, together I have correctly answered three out of the expected seven marks". This is the exact result of adding numerators and adding denominators. Such kind of interpretation destabilises the fraction concept as learners isolate and treat numerators and denominators separately. Learners avoided using, for example, thirds, fifths, sevenths, tenths, and the like on reading fractions. The learners resorted to such terms as "out of" and "over" on reading fractions. This might promote the separation of the numerator from the denominator and eventually leading to not treating a fraction as one number which can be located on the number line. Some read off fractions using the operator "divide" which might render a fraction as a mathematical division operation in progress.

QUESTION 4.1

This was a question checking learners' understanding of the division operation of fractions. The question was not changed. All interviewees confidently gave four as the answer. The learners said they interpreted the question to mean the same as $\frac{1}{2}\ of\ 8$. The misconception may be an extension of the widely used phrase, "share half-half", inevitably leading to the divide by half misconception. L13 interpreted divide by half as divide into two halves. Most probably, the language had an effect leading to the learner's approach. L9 had some understanding about the invert and multiply rule of the division of fractions, however, the first fraction was inverted instead of the second fraction. Due to inadequate conceptualisation, L9 may not have been empowered to reflect on the solution, or better still, why the invert and multiply rule works. Reducing the complexity of the problem came out as the major reason why the division of fractions had to be dealt with via the invert and multiply rule. L27 also preferred multiplication to division, though the division sign was switched to multiplication without inverting. Again, the reason was that multiplying is better than dividing. This indicates that changing to multiplication may be taken as a way to make a division problem simpler and not as a result of a series of mathematically correct manipulations.

Another question on division was removed from the test items as it brought similar findings to those evidenced in the remaining questions. Data from the remaining questions was inclusive the data from the remaining questions fully represents the outcomes from the removed questions. It would have been a duplication and waste of resources to include the removed question as it does not affect the nature of the findings. The questions on word problems were also removed as the data provided by the remaining questions was adequate in answering the research questions.

Piloting the study provided an opportunity to see how long the interview would take as well as to see how the combinations from different studies worked. Data collection, analysis and the interpretation of the data will be discussed in the following chapter.

3.8 ISSUES OF TRUSTWORTHINESS

According to Shenton (2004), trustworthiness in qualitative research has four criteria to be considered, credibility, transferability, dependability and confirmability. Participants were a randomly selected Grade 9 class of 40 learners in a school in Soweto township. A test and interviews were the sources of data. The participants' work or responses in the case of interviews were identified using codes throughout this study. The researcher adhered to confidentiality stipulations as per pre-study arrangements. Sources were acknowledged in accordance with the college regulations and plagiarism avoided. Deviation occurred on interview reporting. Initially, the researcher intended to report on all eight interviewees. Similarities in findings compelled the researcher to only use data from four participants. This data was representative of findings from all eight interviewees, so no essential data relevant to the study was ignored. Instead, duplications of findings in terms of reporting was avoided. The data was adequate to address the objectives of the study.

3.9 LIMITATIONS AND DELIMITATIONS OF THE STUDY

The researcher is a full-time FET educator at the institution where the study was carried out. The study was carried out during the course of the term when the researcher had to attend to the core duties of teaching mathematics to Grade 10-12 learners. Time constraints were inevitable and resulted in the study focusing only on one class in a school with seven Grade 9 classes. Purposeful sampling was used to narrow the target participants to those with potentially rich data valuable for the study.

3.10 CHAPTER SUMMARY

This chapter discussed the nature of the study and the approaches to data collection. Being qualitative, a written test and interviews were the methods of data collection in the study and they have been discussed in the study. For the written test, the rationale justifying the inclusion of some questions was incorporated. As for the interview, its semi-structured nature was put forward as a way of getting as much data as possible from participants in a scientific way, directed by specific learner responses. The chapter also provided the research setting before a section on population sampling and ethical considerations. The limitations to the study and the issues around trustworthiness were also discussed. The data analysis methods were discussed as well as the outcome of the pilot which was conducted.

4 CHAPTER FOUR: DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

This chapter documents the findings and data analysis of the study. To reiterate, the purpose of the study was to explore misconceptions of Grade 9 learners in the concept of fractions in a Soweto (township) school. A written test was administered to 40 learners in a classroom, and an interview was undertaken with eight selected learners. However, in this study, the researcher only used the result of four (4) learners from the interview because with these four learners only the researcher had saturated data and it was more or less similar to the other four learners. The findings are presented thematically, drawing from analysis of collected data. Despite the norm that summarising data using tables, percentages and frequencies is associated with quantitative research, the need to quantify the prevalence of types of misconceptions compelled the researcher to use tables in this study. This section presents the data analysis process followed by tables summarising data obtained from written scripts. The last section, in this chapter offers interview extracts of the four interviewees and the corresponding analysis.

The results of the interview and the written test will be viewed together for this report.

The data analysis and observation will be directed towards answering the research questions of the study, which are:

1. How do Grade 9 learners describe the concept of fractions?
2. How do Grade 9 learners solve problems involving fractions, especially relating to common fractions?
3. What misconceptions are evident when learners are solving these problems?
4. What might be the root causes of these misconceptions?

4.2 DATA ANALYSIS PROCESS

Table 4.1 below presents each learner's nature of response to test questions using the codes C, PC and IC where C stands for correct, PC partially correct and IC stands for incorrect.

Table 4.1: Categories of learners' responses to each test question.

	1.1	1.2	1.3	1.4	1.5	1.6a	1.6b	1.7	2.1	2.2	2.3	2.4	3.1	3.2	4.1	4.2
L1	PC	IC	C	IC	PC	IC	IC	IC	IC	C	IC	IC	PC	IC	IC	IC
L2	PC	IC	IC	IC	IC	IC	IC	IC	IC	PC	IC	IC	C	IC	IC	IC
L3	IC	PC	IC	PC	IC	IC	IC	IC	IC	PC	IC	PC	PC	IC	IC	IC
L4	PC	IC	C	IC	IC	PC	PC	IC	IC	C	C	PC	C	IC	IC	IC
L5	IC	IC	IC	C	IC	IC	IC	PC	IC	C	IC	PC	IC	IC	IC	IC
L6	PC	IC	C	IC	IC	C	C	C	IC	PC	IC	IC	C	IC	C	IC
L7	PC	IC	IC	IC	IC	IC	IC	IC	IC	C	C	PC	IC	IC	IC	IC
L8	IC	IC	IC	IC	IC	IC	IC	IC	IC	PC	IC	PC	IC	IC	IC	IC
L9	IC	IC	IC	IC	IC	IC	IC	IC	IC	PC	IC	IC	C	IC	IC	IC
L10	IC	IC	PC	IC	IC	IC	IC	C	IC	C	IC	C	PC	IC	IC	IC
L11	IC	PC	IC	PC	IC	IC	IC	C	C	C	C	PC	IC	IC	C	IC
L13	IC	IC	IC	PC	IC	IC	PC	C	IC	PC	IC	PC	C	IC	IC	IC
L14	PC	PC	IC	C	IC	IC	IC	IC	IC	PC	IC	PC	C	IC	IC	IC
L15	PC	C	C	C	C	C	C	C	C	PC	IC	PC	PC	IC	IC	IC
L16	IC	IC	IC	IC	IC	IC	IC	IC	IC	PC	IC	PC	IC	IC	IC	IC
L17	PC	IC	C	IC	IC	IC	IC	IC	IC	C	IC	PC	C	IC	IC	IC
L18	IC	IC	IC	IC	IC	IC	IC	IC	IC	IC	IC	PC	C	IC	IC	IC
L19	IC	IC	IC	IC	IC	IC	IC	IC	IC	PC	IC	PC	IC	IC	IC	IC
L20	IC	IC	IC	IC	IC	IC	IC	IC	IC	PC	IC	IC	IC	IC	IC	IC
L21	IC	IC	IC	IC	IC	IC	IC	PC	IC	PC	IC	PC	C	IC	IC	IC
L22	IC	IC	IC	IC	IC	IC	IC	C	IC	C	IC	PC	C	IC	IC	IC
L23	IC	IC	C	PC	PC	C	C	IC	C	PC	IC	IC	IC	IC	IC	IC
L24	IC	PC	IC	PC	IC	IC	IC	IC	IC	C	C	PC	C	IC	IC	IC
L25	PC	PC	C	IC	IC	IC	IC	PC	IC	C	IC	PC	C	IC	IC	IC
L26	PC	PC	C	C	PC	C	C	C	C	C	C	C	C	IC	C	C
L27	IC	IC	IC	IC	IC	C	IC	C	IC	PC	IC	PC	PC	IC	IC	IC
L28	IC	IC	IC	PC	IC	IC	IC	IC	IC	C	IC	IC	IC	IC	C	IC
L29	PC	PC	C	PC	IC	IC	C	IC	IC	C	IC	PC	C	IC	IC	C
L30	PC	PC	C	IC	IC	IC	IC	C	C	C	C	C	C	IC	C	IC
L31	IC	IC	IC	PC	IC	C	C	PC	IC	PC	IC	PC	C	IC	IC	IC
L32	PC	PC	C	IC	PC	IC	IC	IC	C	C	C	IC	C	IC	IC	IC

L33	PC	IC	C	C	PC	IC	IC	C	IC	C	IC	C	PC	PC	IC	IC
L34	PC	PC	PC	IC	IC	IC	C	PC	C	C	IC	C	IC	IC	IC	IC
L35	PC	IC	IC	PC	IC	IC	IC	IC	IC	C	IC	PC	PC	IC	IC	IC
L36	IC	PC	IC	IC	IC	IC	IC	C	IC	C	IC	IC	PC	IC	IC	IC
L37	PC	IC	IC	IC	IC	IC	IC	IC	IC	C	IC	PC	PC	IC	IC	IC
L38	PC	IC	C	IC	PC	IC	IC	IC	IC	IC	IC	PC	C	IC	IC	IC
L39	PC	IC	C	IC	IC	C	IC	IC	IC	C	IC	PC	PC	IC	IC	IC
L40	PC	IC	IC	IC	IC	IC	IC	IC	C	C	C	IC	PC	IC	IC	IC

C= correct, PC= partially correct, IC= incorrect.

Tables 4.1 and 4.2 show the performance and the percentages of correct, partially correct, and incorrect solutions of all 40 learners who wrote the test.

Table 4.1 lays out each learner's responses to each test question under categories C, PC and IC, where C stands for correct, PC partially correct and IC stands for Incorrect. Table 4.2 shows the percentages of correct/incorrect learner solutions per question. In-order to get a clear-cut success rate per question, PC was incorporated with the IC to give the total number of incorrect solutions per question in Table 4.2. The PC category showed that learners had an idea about the fraction problem though they deviated from the ideal solution due to potential misconceptions or arithmetic challenges.

Table 4.2: Percentages of correct/Incorrect learner solutions per question.

Question	Number of correct solutions	Correct solutions in terms of %	Number of incorrect solutions	Incorrect solutions in terms of %
1.1	0	0%	40	100%
1.2	1	3%	39	97%
1.3	15	37%	25	63%
1.4	5	13%	35	87%
1.5	1	3%	39	97%
1.6a	7	18%	33	82%
1.6b	7	18%	33	82%
1.7	11	27%	29	73%
2.1	8	20%	32	80%
2.2	22	55%	18	45%
2.3	8	20%	32	80%
2.4	6	15%	34	85%

3.1	17	43%	23	57%
3.2	0	0%	40	100%
4.1	5	13%	35	87%
4.2	2	6%	38	94%

Looking at the table, the solutions from learners are so depressing. The table reflects poor performance in almost all questions, except Question 2.2, which shows 55% of learners answering the question correctly. It is also evident that it is likely that most of the learners had no understanding of the concept of fraction. The definition of the concept of fraction as indicated in many textbooks, defining fraction as "part of a whole", seems to be the cause of confusion and misconceptions. It is evident in the responses of many learners that the phrase "equal parts" in the concept of fraction was missing in most learners' understanding of fractions. Hence, only one learner (3%) got question 1.2, showing awareness of the existence of a diagram drawn with parts which are not equal. Figure 4-1 shows L15's response..

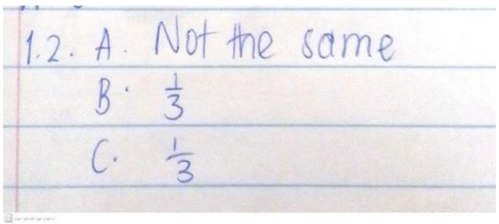
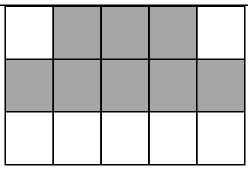
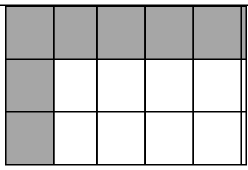


Figure 4-1: L15's response to Question 1.2

As it is evident from Table 4.1, no learner defined a fraction as "part of a whole which is divided into equal parts". The understanding of "a whole divided into equal parts" is very important in fractions. The researchers' assumption was that if learners lack an understanding of the concept of fraction, which was tested in Question 1.1 – 1.7, they would be unable to solve problems which included fractions, as indicated in Questions 2-4. It appears that the researcher's assumption is correct because most learners did not do well in these questions.

Table 4.3: Most common incorrect learner solutions (misconceptions) and the corresponding frequencies.

Question	**Misconception**	**Frequency**	**%**	**Misconceptions**	**Frequency**	**%**
1.1.	Part of a whole	20	50			
	Explanation: 50% of learners did not point out the equality of parts in the sub-divided whole in their definitions					
1.2	$A = \frac{1}{3}$; B =$\frac{1}{3}$; C=$\frac{1}{3}$	30	75	$\frac{Number \neq 1}{3}$	4	10
	Explanation:75% of learners did not notice that a geometrical figure should be divided into equal parts, while 4 learners had solutions with denominator 3 (because of three					

Question	Misconception	Frequency	%	Misconceptions	Frequency	%
	partitions) although numerators differed					
1.3	$\frac{5}{8}$	25	63			
	Explanation: 63 % of leaners perceive that the bigger the numerator and denominator the bigger the fraction					
1.4	$\frac{8}{4}$ located elsewhere other than at 2	17	43			
	Explanation: 43% of learners did not consider $\frac{8}{4}$ as equivalent to 2 on linear presentations.					
1.5	$\frac{5}{8}; \frac{3}{7}; \frac{2}{6}; \frac{4}{4}; \frac{1}{2}$	6	15	$\frac{5}{8}; \frac{4}{4}; \frac{3}{7}; \frac{2}{6}; \frac{1}{2}$	6	15
	$\frac{1}{2}; \frac{2}{6}; \frac{3}{7}; \frac{4}{4}; \frac{5}{8}$	8	20	$\frac{1}{2}; \frac{4}{4}; \frac{2}{6}; \frac{3}{7}; \frac{5}{8}$	7	18
	Explanation:68% of learners focussed on either numerators only or denominators only on ordering as follows; 15% - big numerator implies big fraction 20% - ascending order of numerators 15% - big denominator implies big fraction 18% - small denominator implies big fraction.					
1.6 (a)		15	38		14	35
	Explanation: 38% of the learners believed that you have to literally shade 3 blocks on top of 5 blocks and 35% perceive shadings should be on the leading row and the leading column					
1.6(b)	Shade 3 blocks over 5 blocks	15	38			
	Explanation: 38% of learners aimed at visualising 3 blocks on top of five blocks.					
1.7	3 numerator 5 denominator	29	73			
	Explanation: 73% of the learners named the components of the fraction instead of explaining the meaning of each component					
2.1	$\frac{1}{3}+\frac{1}{6}=\frac{2}{9}$	20	50	$\frac{1}{3}+\frac{1}{6}=\frac{1}{9}$	4	10
	Explanation: 50 % of learners added the numerator with numerator and denominator with denominator when adding fractions					
2.2	$\frac{1}{4}+\frac{3}{4}=\frac{4}{8}$	16	40			
	Explanation: 40% of the learners added numerator with numerator and denominator with denominator					
2.3	$\frac{7}{10}-\frac{2}{5}=\frac{5}{5}$	20	50	$\frac{7}{10}-\frac{2}{5}=\frac{5}{10}$	5	13
	Explanation: 50% of learners subtracted numerators to get the numerator answer and					

Question	Misconception	Frequency	%	Misconceptions	Frequency	%
	subtracted denominators to get the denominator answer. 5 learners found the common denominator and just subtracted the numerators					
2.4	$7\frac{1}{2}+10\frac{1}{2}+\frac{3}{4} = 17\frac{5}{8}$	21	53			
	Explanation: 53% of learners added whole numbers separately from the fractional parts. However, on adding fractional parts, numerators were added to give a numerator answer and denominators were added to give the denominator answer.					
3.1	$\frac{1}{2}\times\frac{1}{3}$ $\frac{1\times 3}{2\times 3}+\frac{1\times 2}{3\times 2}=\frac{5}{6}$	12	30			
	Explanation:30% of the learners takes finding the common denominator as the first step (regardless of the operator) when dealing with fractions with different denominators.					
3.2	$2\frac{1}{2}\times 3\frac{1}{5}=6\frac{1}{10}$	13	33	$2\frac{1}{2}\times 3\frac{1}{5}=6\frac{2}{10}$	6	15
	Explanation: 48% of learners believed that whole numbers are multiplied separately from fractional parts before combining the two answers					
4.1	$8\div\frac{1}{2}=4$	15	38	$8\div\frac{1}{2}=\frac{1}{4}$	6	15
	Explanation: 38% of learners perceived dividing by a half as getting half of the quantity. 6 learners however divided guided by the whole number division that two goes into eight, four times. Position of the two in the question(denominator) influenced the positioning of four in the solution $\frac{1}{4}$					
4.2	$\frac{1}{2}\div 10=5$	9	23	$\frac{1}{2}\div 10=\frac{1}{5}$	5	13
	Explanation: 23% of learners calculated half of ten whereas 5 learners divided ten by two to get 5 which they reciprocated to maintain the denominator status 2 initially had.					

Table 4.1 and 4.2; however, do not show how the learners solved the problems. Although Table 4.3 presents the most common incorrect learner solutions, it does not cater for learner explanations (which were part of the interview process) including how they obtained correct solutions. The discussion, which follows, is an analysis of how four selected learners responded to each of the questions in the test and during the interview. The researcher interviewed eight learners, and the data obtained revealed similarities. The researcher have therefore decided to use data of four learners because the data of eight learners would have reflected too many repetitions.

According to Piaget (1964) Constructivism rejects the notion that children are "blank slates" with no ideas, concepts and mental structures. They do not absorb ideas as teachers present them, but rather, they are creators of their own knowledge. When learners interact with new knowledge, their existing knowledge give meaning to that new knowledge. In the process of constructing, restructuring of knowledge misconceptions could arise. Thus, prior knowledge could be attributed as a cause of misconceptions.

As indicated above, data obtained from four learners will be discussed. The researcher will refer to them as L1, L2, L3 and L4 without implying through the labelling the order in which they were interviewed. The interviewer will be represented as the "interviewer (I)'' in the transcript.

Question 1

Q 1.1 *What is a fraction?*

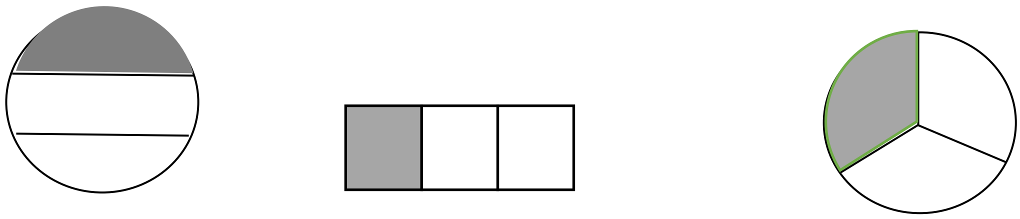
The researcher intended to get a description of the concept of fractions and what it entails from the learners' point of view. This question aimed to test learners' understanding of the concept of *fraction.* It is evident from Table 4.1 that no learner responded to this question correctly. Table 4.3 displays "part of a whole" as the most popular response accounting for 50% of the incorrect solutions. In defining the concept of a fraction, most learners invoke everyday context. However, the concept of "equality" appears to be missing in their definitions and understanding of fractions. Most learners defined fractions as "part of a whole", "a piece of something which was complete", "a section cut from something". When probed more during the interview, L6, L11, L15 and L17 said the following:

I	*You defined a fraction as a section cut from something. Please elaborate*
L6	*Ok, when you eat a slice of Pizza it is a fraction*
I	*If you cut out a section, will the remaining part be a fraction also?*
L6	*No! The part of a whole cut out or removed is the one which is a fraction.*
I	*Why not the remaining part?*
L6	*the removed part is like the shaded part of a whole which is used for showing fractions.*
I	*You talk about whole numbers and cake cutting in your definition of a fraction, how are they linked?*
L11	*When you buy a cake and cut it into two halves; that is a fraction and to write the fraction you put a whole number on top of another whole number.*

I	*So a fraction is formed by cutting cakes*
L11	*Kkkk , giggling, it's an example Sir. When you cut things, you form fractions.*
I	*You wrote that a fraction is a part of a whole, what do you mean by part of a whole?*
L15	*a piece of something which was complete.*
I	*Say I cut a piece of a cake, it is a fraction according to you. What about the remaining cake?*
L15	*it is another fraction because it is also part of something that was complete.*
L17	*a number formed by two numbers*
I	*What happens with two numbers to form this other number?*
L17	*the two numbers, one is the numerator and the other is the denominator*
I	*Please elaborate*
L17	*Yes, the numerator shows the number of parts shaded in a shape divided into parts shown by the denominator*

It is evident from the extract above that some learners, such as L17, define a fraction as a number that "contains two numbers, numerator and denominator. When asked to clarify in the interview L17 says, *Yes, the numerator shows the number of parts shaded in a shape divided into parts shown by the denominator. Learner 38: Yes, it is an improper fraction if parts are not equal.* It appears that learners' responses revealed a vague idea of what a fraction is. On clarifying, learners resorted to examples of sharing a cake/pizza or colouring parts of the circle. The parts of a whole perspective supported by cake or pizza slicing dominated as evidenced by narrations above by L6, L15 and L11. Learners, however, did not mention the equality of sizes cut on a cake/pizza or on shading parts of circle. The lack of "equal parts" in the understanding of the concept of fraction was also evident in Question 1.2

Q 1.2 Write the fractions represented by the following diagrams



A **B** **C**

A ---------------------------------

B --------------------------------

C----------------------------------

Figure 4-2: Diagrams Question 1.2

As in question 1.1, this question also aimed at testing the understanding of the concept of fraction. The researcher wanted to find out if learners understand that fraction is about a whole divided into equal parts. Table 4.3 shows that 75% of the learners had $\frac{1}{3}$ as the answer. This accounted for 30 out of the 39 incorrect solutions. These learners did not notice that Diagram A was not divided into three equal parts. The researcher has taken only three learners' examples (L6, L11 and L38) that indicated that the fraction that represented Diagram A, B and C was $\frac{1}{3}$. When asked in the interview to justify their responses, they said:

L6	*All show wholes divided into three parts so they represent the same fraction*
I	*And the fraction represented is* $\frac{1}{3}$*?*
L6	*: Yes, 1 whole divide by 3 hence* $\frac{1}{3}$
L11	*All are divided into three parts so they are thirds.*

I	*Ok, so what matters is the number of parts the shapes are divided into?*
L11	*yes, that is where the denominator and name come from*

L38	*Dividing a whole into parts tells you what fraction you are making*
I	*How does the fraction and parts relate?*
L38	*If there are five parts for example, fifths, the fraction is $\frac{1}{5}$ but in this case there are three parts, thirds, so the fraction shown is $\frac{1}{3}$*

As the shapes are divided into three parts, L11 justified the $\frac{1}{3}$ answer by stating that the shapes are divided into thirds. This might imply that the perception is that the size of the parts does not matter as we express parts as fractions of a whole. The denominator is more often carelessly taken to represent the number of parts a whole is divided into without emphasising the equality of these parts. The misconception held by L11 might be a result of teachers taking definitions for granted and assuming all learners got the concept right in previous years. L38 explained the solution from the naming of fractions. L38 reiterated that three parts mean thirds and thirds are mathematically represented as a fraction $\frac{1}{3}$. L6 echoed similar sentiments, revealing the misconception that wholes divided into equal parts represent the same fraction regardless of the inequality in size of parts.

Q 1.3 Which is bigger, $\frac{3}{4}$ *or* $\frac{5}{8}$ of a given cake?, support your choice by showing how you compared the two fractions.

This question tests whether learners can compare fractions and decide on which fraction is bigger or smaller than the other. It also seeks clarity on how that comparison is made. In addition, it also tests the understanding of the concept of fraction, because for one to compare the two fractions, one should understand the meaning of those fractions. As indicated in Table 4.2, 63% of learners got this question incorrectly. Most learners indicated that $\frac{5}{8}$ is bigger than $\frac{3}{4.}$. Table 4.3 shows that all incorrect solutions (63%) were for $\frac{5}{8}$ being bigger than $\frac{3}{4.}$. When asked to justify their responses, L6, L15, L11 and L 34 said:

L6 *Eighths are many than quarters so $\frac{5}{8}$ is bigger than $\frac{3}{4}$.*

I *What do you mean by many?*

L6 *Eighths mean 8 parts and quarters is four parts.*

I *So you look at denominator values.*

L6 *Yes, denominators show which pieces are many.*

L11 *Both numbers making up the fraction $\frac{5}{8}$ are bigger than corresponding numerator and denominator values of the fraction $\frac{3}{4}$*

I *Why did you do it that way?*

L11 *It makes sense Sir, a fraction composed of big numbers is bigger than the one with small numbers.*

This is the correct answer but an incorrect explanation. L15, for example, got the answer correct but by using incorrect reasoning.

I *Please explain why you are saying $\frac{3}{4}$ is bigger than $\frac{5}{8}$*

L15 *For $\frac{3}{4}$ the fraction is divided into four parts and for $\frac{5}{8}$ the fraction is divided into eight parts. Eight parts will be smaller in size than four parts so $\frac{3}{4}$ is bigger.*

I *You referred to a fraction being divided into a number of parts, which fraction?*

L15 *oh! Askies, I wanted to say the shape is divided…*

I *Ok, not an issue, so you looked at the size of the parts to help you decide on the bigger fraction.*

L15 *Yes Sir.*

L34 *$\frac{5}{8}$ and $\frac{3}{4}$. Comparing numerators, 5 is bigger than 3. Comparing denominators, 8 is bigger than 4 so $\frac{5}{8}$ is bigger than $\frac{3}{4}$*

I *Are you looking at individual numbers making up a fraction or the fraction itself?*

L34 *A fraction is a fraction because of those numbers so if I look at them I am looking at the fraction.*

L15's response showed that not all correct answers were arrived at through proper mathematisation. Indeed $\frac{3}{4}$ is bigger than $\frac{5}{8}$ although the method employed encompassed a misconception. L15 views the size of parts of a shape as the determining factor. Pictorial visualisations become incomplete if only the size of a portion is considered without considering the number of portions as indicated by the numerator. As the size of parts of a shape depends on the denominator value, L15 may be said to follow the idea that the smaller the denominator, the bigger the fraction. The learner's approach is based wholly on denominators ignoring numerator values. Although L6 compared fractions using denominators, unlike L15 who considered size of parts, L6 focused on the number of parts. Eight-eighths meant more than four quarters. This shows knowledge inadequacies in quantifying fractions (viewing fractions as measures).

Disintegrating the fractions through using the phrase "out of" compelled the learners to revert to whole number knowledge. The focus was then directed to either numerator comparisons only or denominator comparisons only, never the complete fraction. This was exhibited by L11 and 34 who extended knowledge from whole numbers to the fraction concept. Learners 11 and 34 separated components of the fraction, isolated numerators and treated them as stand-alone whole numbers. Denominators also were treated similarly. The misconception held by the learners may fall under those misconceptions which at times co-incidentally give correct solutions. For instance, consider $\frac{1}{2}$ and $\frac{7}{8}$. $\frac{7}{8}$ is bigger that $\frac{1}{2}$ not because 7 is greater than 1 and 8 is greater than 2. Getting the problem correct may exacerbate the misconception as learners may think they manipulated correctly.

Q 1.4 Show the following on the number line below:

$\mathbf{\frac{1}{4}}$; $\mathbf{\frac{3}{4}}$; $\mathbf{\frac{7}{4}}$; $\mathbf{\frac{8}{4}}$; $\mathbf{\frac{10}{4}}$

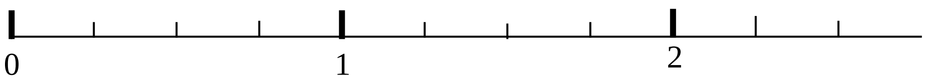


Figure 4-3: Diagram Question 1.4

The question aimed to test ability and knowledge to sort more than two fractions in such a way that it becomes easier to arrange them and make a linear representation chronologically. This question also sought an explanation of the method/s undertaken by the learner in arranging fractions. This question also tests the knowledge of understanding the concept of fraction, that is, understanding dividing a fraction into equal parts and the concept of numerator and denominator. Eighty-seven per cent of the learners had an incorrect solution. Forty-three per cent of the learners had $\frac{8}{4}$, located in a different location than at 2 on the number line, accounting for the most common incorrect arrangement, as shown in Table 4.3. Learner L6 gave a representation of the fractions on the number line as depicted in Figure 4.4.

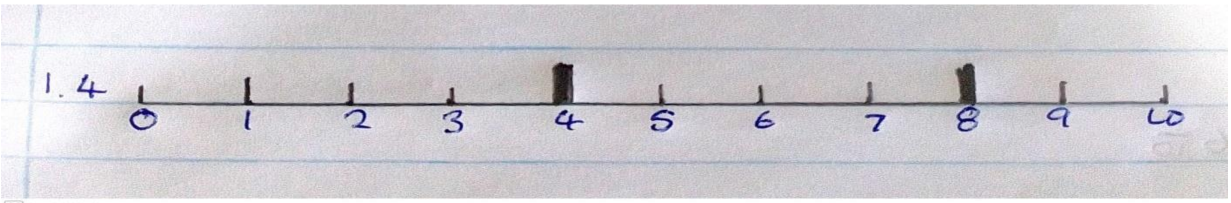


Figure 4-4: L6's response to Question 1.4

Learner L6 had the following to say about the representation;

L6 *I looked at the top numbers and the biggest is 10 so I had to correct the number line so that it caters for all top numbers in the list.*

I *If those numbers on the number line represent numerators, what about the denominators?*

L6 *I tried Sir because I know shading shapes for fractions and use a number line for adding or subtracting numbers not showing fractions.*

Learner L11 showed the fractions in the number line as depicted in Figure 4.5

L11:

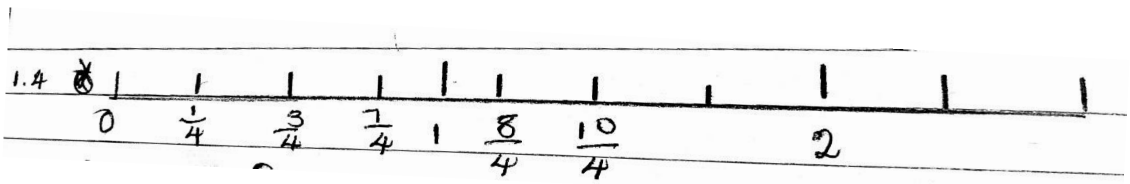


Figure 4-5:L11's response to Question 1.4

When asked to explain his thinking and reasoning behind the answer L11 said:

L11 *Since the fractions have the same denominator, I arranged them according to size of numerators and filled in available spaces on the number line from left to right. As there are five fractions so the first five spaces were filled.*

I *Did you recognise the presence of the numbers 0, 1 and 2 on the number line?*

L11 *Yes, since there are three gaps in between 0 and 1 so it means the first three fractions will fit and the remaining fractions will fill* *gaps after 1.*

Learner 34 represented her representation as depicted in Figure 4.6.

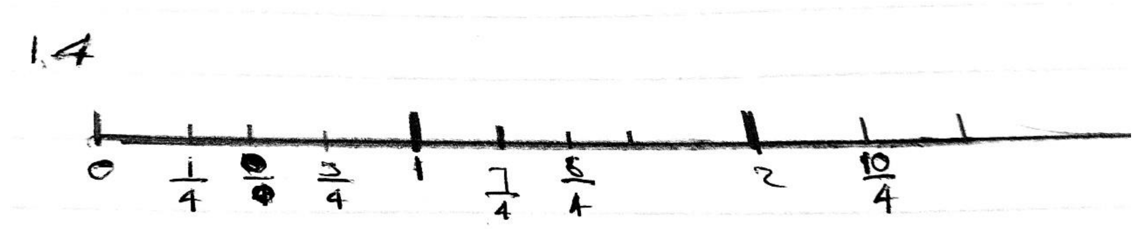


Figure 4-6: L34's response to Question 1.4

When asked to justify his thinking behind the response, L34 said:

L34: *$\frac{1}{4}$ and $\frac{3}{4}$ are smaller than 1whole so I placed them before 1; $\frac{7}{4}$ and $\frac{8}{4}$ are more than 1 so I placed them between 1 and 2; $\frac{10}{4}$ is more than 2 so it is to the right of 2.*

I: *Enlighten me about the location of $\frac{8}{4}$.*

L34: *It follows $\frac{7}{4}$.*

I: *According to your number line, $\frac{8}{4}$ is bigger than $\frac{7}{4}$ but less than 2.*

L34 *Yes, $\frac{10}{4}$ is the only fraction bigger than 2.*

Learner 38 represented her answer as depicted in Figure 4.7.

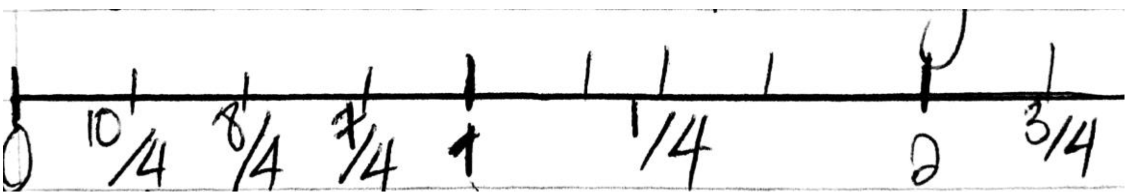


Figure 4-7: L38's response to Question 1.4.

When asked to explain her representation, she said:

I *Please explain why you placed the fractions the way you did*

L38 *If a whole is already divided into 4 parts and you must come up with 10* *parts out of those 4, ($\frac{10}{4}$), the resulting pieces will be smaller than pieces of $\frac{8}{4}$, $\frac{7}{4}$, and so on.*

I *What about the position of $\frac{3}{4}$?*

L38: *It was supposed to come before $\frac{1}{4}$, I made a mistake.*

Learner L6 displayed a conceptual breakdown in relating fractions to number line representations. L6 changed the numbering on the number line showcasing a whole number bias. The interview revealed that the learner's experience with number lines relates to whole numbers only and previous knowledge on representing fractions focused on shading shapes.

L11 could not relate fractions to number lines. L11's arrangement showed that the learner did not give numerical values to fractions in relation to the provided linear scale. The scale of the number line played no part in his thinking as the fractions were just placed following each other in terms of size but not according to scale. Learners generally said they preferred colouring segments on circle partitions. This implies that more work on fractions was done with shapes than number lines. L38 argued that $\frac{8}{4}$ was not the same as 2 when it came to fractions. This approach probably clouded the learner's judgement when it came to $\frac{8}{4}$ and two. L38 argued that eight pieces from the original four pieces of a whole will definitely be smaller than two pieces from the original one piece (one whole) as 2 is $\frac{2}{1}$. Reading off fractions using "out of" and seeking a pictorial arrangement might have overshadowed taking $\frac{8}{4}$ as $8 \div 4$. L38's misconception is diagrammatically illustrated in Figure 4.7 below.

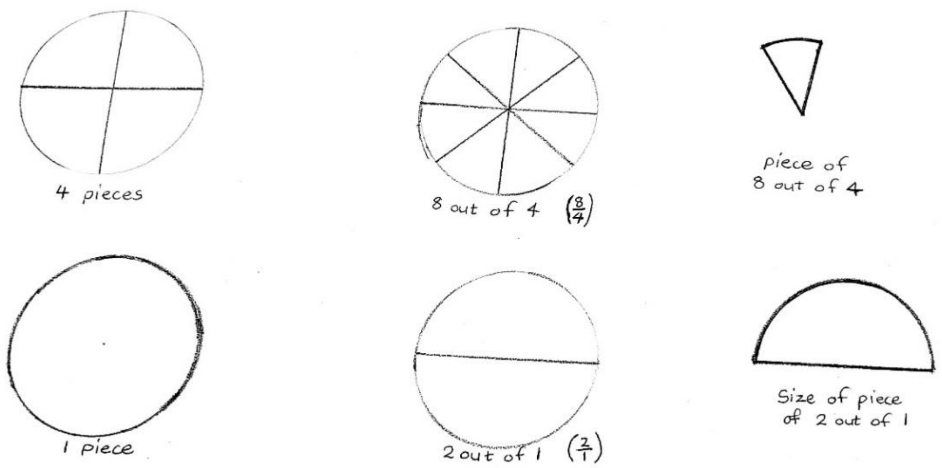


Figure 4-8: L38's misconception regarding size of partitions of wholes.

Most learners who managed to place the fractions according to the numerator values (since denominators are the same) were unable to recognise that $\frac{8}{4} = 2$ because of the misconception on partitioning and the interpretation of the term "out of" concerning fractions.

Q 1.5 Arrange the following fractions from biggest to smallest;

$$\mathbf{\frac{5}{8}} \quad ; \quad \mathbf{\frac{4}{4}} \quad ; \quad \mathbf{\frac{2}{6}} \quad ; \quad \mathbf{\frac{3}{7}} \quad ; \quad \mathbf{\frac{1}{2}}$$

Table 4.2 shows that 97% of the learners ordered the fractions incorrectly. Table 4.3 further clarifies that the most popular incorrect arrangements were influenced by singling out components of a fraction when comparing the fractions. Table 4.3 shows that 68% of the arrangements were due to learners focusing on denominators only or numerators only. Thirty-three per cent (33%) of the learners ordered the fractions according to the denominators and 35% considered the numerators only.

L6 arranged the fractions as follows:

$$\frac{5}{8} \quad ; \quad \frac{4}{4} \quad ; \quad \frac{3}{7} \quad ; \quad \frac{2}{6} \quad ; \quad \frac{1}{2}$$

I: *Please explain how you came up with this arrangement*

L6: *I looked at the number of pieces shaded in each whole.*

I: *How?*

L6: *The numerator shows us how many pieces are shaded, for example, in* $\frac{5}{8}$ *, five pieces are shaded,* $\frac{4}{4}$ *, four pieces are shaded, in* $\frac{3}{7}$ *three pieces are shaded, in* $\frac{2}{6}$ *two pieces are shaded and lastly one piece shaded in* $\frac{1}{2}$.

I: *In other words you arranged in decreasing order of numerators.*

L6: *Yes from highest shaded parts to lowest.*

L11 organised the fractions as follows: $\frac{5}{8} \quad ; \quad \frac{3}{7} \quad ; \quad \frac{4}{4} \quad ; \quad \frac{2}{6} \quad ; \quad \frac{1}{2}$

I: *Please explain how you came up with this arrangement.*

L11: *I counted the parts formed when wholes shown by the numerator are divided by the denominator. The one that forms more parts is bigger so they follow each other that way.*

I: *Please use an example to show how the number of parts you counted are formed.*

L11: *In $\frac{2}{6}$, 2 wholes divided into 6 parts simply means each whole is cut into 6 parts. The resulting 12 parts are many so combined are greater than one whole divided into 2which will give 2 parts, hence $\frac{2}{6}$ is bigger than$\frac{1}{2}$.*

I: *Just roughly illustrate how the 12 parts you are talking about are formed.*

L11: *[illustrates diagrammatically]. Two wholes divide by six yielding 12 parts.*

Figure 4-9: L11's explanation of two wholes divided by six.

L17 arranged the fraction as follows:

$$\frac{1}{2} \quad ; \quad \frac{4}{4} \quad ; \quad \frac{2}{6} \quad ; \quad \frac{3}{7} \quad ; \quad \frac{5}{8}$$

When asked to explain his thinking behind the answer L17 said:

L17: *The bigger the denominator, the smaller the fraction.*

I: *Why is it so?*

L17: *The denominator tells us how many parts we cut our whole into. If the denominator is big, then it means we are cutting the whole into many small pieces. If the denominator is small, then the pieces will be big.*

I: *So the size of the piece informs you on whether the fraction is big or small?*

L17: *Yes, that is the easier way out.*

I: *Please explain how you came up with this arrangement.*

L34: *$\frac{4}{4}$ is total, everything, so it is biggest and comes first. For those which are not total we use the denominator to tell us which is big. 8 is big* *followed by 7, 6 and 2 is the smallest so fractions will follow each other that way.*

I: *You mean, generally, the big fraction is the one that has a big denominator?*

L34: *Yes, as long it is not total.*

I: *What about the numerators, what is their purpose?*

L34: *They show us how many parts are shaded in a shape*

I: *On arranging you did not consider them*

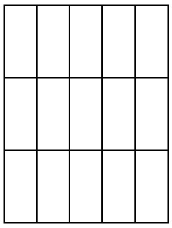
L34: *Yes, they only tell us about shaded parts in each shape.*

The learner responses varied and their explanations showcased several misconceptions on comparing fractions. L17 justified their ordering of fractions through what they termed the ‘big is small’ phenomenon when referring to denominators of fractions being compared. According to L17, from biggest to smallest means the smallest denominator has to come first as it is for the biggest fraction. The misconception revealed was that numerators played no part in the ordering. This might be a result of using unit fractions to compare fractions during instruction. For unit fractions, $\frac{1}{2};\frac{1}{3};\frac{1}{4}$, the phenomenon, big denominator small fraction applies. On the other hand, L34 generally ordered fractions according to denominators, ranking them by size. The ordering seemed guided by the impression that bigger the denominator was, the bigger the fraction. The bigger the numerator, the larger the fraction with the exception of $\frac{4}{4,}$ which L34 rightfully regarded as a complete whole.

L6 used numerators as the basis of ordering. The idea of using shaded parts (of fractions with different denominators) as a basis for comparison reveals that L6 has little knowledge about equivalent fractions and levelling values before comparing.

L11 perceived the numerator as depicting the number of wholes which are each being divided into parts shown by the denominator. From the whole number perspective, L11 considered $\frac{2}{6}$ as 2÷ 6 before interpreting it as 2 wholes each divided by 6. The misconception led to each whole cut into six parts. The resulting 12 parts are relatively many so combined are greater than one whole divided into two; hence $\frac{2}{6}$ is bigger than $\frac{1}{2}$. According to L11, the more the number of parts formed through the division of the numerator wholes, the bigger the fraction. L11 displays a misconception on division. Instead of focussing on the value of each piece upon dividing, L11 considers the number of pieces formed. Dividing one whole into 6 yields 6 pieces but the solution to 1 ÷ 6 is certainly not 6.

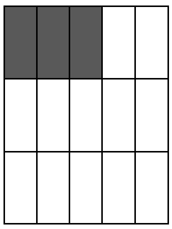
Q 1.6. **a) Shade $\frac{3}{5}$ of the rectangular mat below.**



b) Write down in words what you will do to shade $\frac{3}{5}$ of the rectangular mat.

This question is also testing learners' understanding of the concept of fractions. Learners have to know what $\frac{3}{5}$ mean. They have to know that their whole is divided into five equal parts, and three of those parts should be shaded. Eighty-two per cent of the learners had incorrect solutions. Table 4.3 shows that the most popular solutions were those where shadings representing the numerator were shown separately from those representing the denominator on the diagram. Thirty-eight per cent of the learners believed that you have to literally shade three blocks on top of five blocks and 35% perceived the shadings should be on the leading row and the leading column showing three shaded for the numerator and five shaded for the denominator.

L6 demonstrated her answer as follows:



L6 said, *"I shaded 3 blocks"*. The interview extract shows that L6 did not understand the concept of fractions at all. When probed she said:

I: *Why did you shade three blocks?*

L6: *The numerator shows the number of parts that must be shaded. As the numerator is three, I shaded three blocks. Got confused though, I* *expected only five blocks*

I: *If I may ask, what fraction does your diagram represent as it is now?*

L6: $\frac{3}{15}$*, that's why I said there must be five blocks only*

I: *Why only five blocks?*

L6: *The denominator shows the number of parts the shape is divided into. Since the denominator is five, there has to be five blocks in one whole*

I: *What is the relationship between* $\frac{3}{5}$ *and* $\frac{6}{10}$*?*

L6: $\frac{6}{10}$ *is* $\frac{3}{5}$ *times two.*

I: *How so?*

L6: $\frac{3}{5}+\frac{3}{5}=\frac{6}{10}$

L17 had a similar display only that the learner drew an extract by the side probably to emphasise the point that only five blocks are considered.

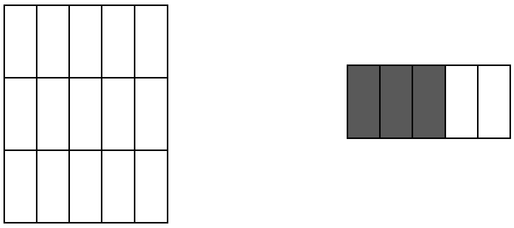


Figure 4-2: L17's response to Question 1.6

L17 commented on her solution as below.

I: *Why did you draw a separate diagram?*

L17: *I only need 5 blocks to represent* $\frac{3}{5}$

L: *But there were 15 blocks.*

L17: *Ya, I chose the 5 I need.*

I: *What is the relationship between* $\frac{3}{5}$*and*$\frac{6}{10}$*?*

L17: $\frac{6}{10}$ *divide by two is* $\frac{3}{5}$.

I: *How so?*

L17: $6 \div 2 = 3$ *and* $10 \div 2 = 5$.

L34 also seemed not to understand the meaning of $\frac{3}{5}$. L34 represented the answer as follows:

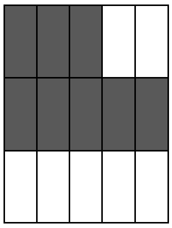


Figure 4-11: L34's response to Question 1.6

The interview below shows that L34 does not understand the concept of fractions.

I: *Make me understand the way you shaded.*

L34: *I shaded 3 blocks on top of 5 blocks, 3 blocks over 5 blocks.*

I: *Why?*

L34: *The fraction is 3 over 5, so 3 shaded parts must be over or on top of 5 shaded parts so that it is easily seen indicating* $\frac{3}{5}$.

L38 represented the answer as follows.

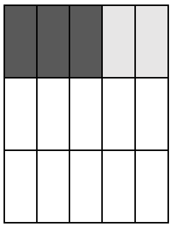


Figure 4-12: L38's response to Question 1.6.

The interview below shows that L38 does not understand the meaning of $\frac{3}{5}$ in particular and the concept of fraction in general.

I: *Make me understand the way you shaded.*

L38: *I shaded three out of five blocks and had to use different colours to show which five blocks have I chosen to use.*

I: *So you used five blocks only?*

L38: *Yes, isn't the fraction is* $\frac{3}{5}$*?*

I: *Off course,* $\frac{3}{5}$ *, meaning…*

L38: *Meaning shade 3 out of 5. I had to choose my own 5 and leave the rest.*

I: *What happens with the rest of the blocks?*

L38: *They are not part of my answer, I only used 5blocks.*

I: *What is the relationship between* $\frac{3}{5}$ *and* $\frac{6}{10}$*?*

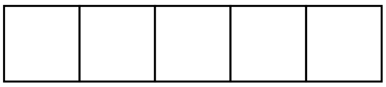
L38: *Two times* $\frac{3}{5}$.

I: *Two multiply by* $\frac{3}{5}$

L38: *Yes.*

L6, L17 and L38 held the view that since the denominator was five, they expected the whole to have exactly five partitions. The learners explained that they, therefore, had to deal with only five blocks and disregard the rest. L38 clarified that different colours helped show the five blocks which were chosen from the rest. This misconception is probably due to limited knowledge on equivalent fractions. L34 believed that by looking at the colouring of blocks, one must visualise $\frac{3}{5}$ at a glance. Three blocks were, therefore, shaded over the five shaded blocks. The part of a whole concept might have blinkered learners onto emphasising the three and the five in their diagrams. The misconception displayed by L34 may be due to interpreting the word "over" on the pictorial display literally.

Q 1.7 Shade $\frac{3}{5}$ of the whole



In the fraction $\frac{3}{5}$, what does the 5 mean? _________________

What does the 3 mean? _______________________________

This question focused on the interpretation of a fraction and its components. The question tested if a learner would shade $\frac{3}{5}$ and be able to interpret the meaning of $\frac{3}{5}$ as "my whole has been divided into five equal parts, and three of them are identified or shaded." Or, five indicates the name of the fraction, *Fifth* and three is counting, that is, *three-fifths.* Twenty-seven per cent (27%) of learners gave the correct answer. Seventy-three per cent (73%) of the learners were incorrect. Table 4.3 shows that all of the incorrect answers gave "three means numerator" and "five means denominator" as their answers. Misunderstanding what the question required might have led learners to name the components instead of addressing the meaning of the question. L17, L6, L15 and L30 had the following to say regarding the three denominator and five numerator answer:

L6 had the following comments:

L6: *L6: 3 is the numerator because it is the top number in a fraction and 5 is the denominator because it is the bottom number in a fraction.*

I: *Do you know what the top number tells us about the fraction and also what the bottom number tells us about the fraction?*

L6: *Denominator tells how many parts the shape is cut into and numerator is for the shaded parts.*

L15 commented:

L15: *Denominator is the number that gives the fraction a name.*

I: *A name how? Please elaborate.*

L15: *For example if the denominator is 5, fifths, 6, sixths, and so on.*

I: *What about the numerator?*

L15: *Numerator is the number divided by the denominator.*

L17: *Three is the numerator and five is the denominator.*

I: *Yes, I agree with you, what do they mean?*

L17: *It means three out of five.*

I: *Still, I need clarity what does three mean and five mean in three out of 5?*

L17: *Three parts out of five parts.*

L38 added:

L38: *Three is the numerator and five is the denominator.*

I: *Yes I agree with you, but what do three and five mean?*

L38: *It means three divide by five.*

I: *Ok, let me put it this way. From the three divide by five, what does three as a numerator mean and what does five as a denominator mean?*

L38: *It means how many times five goes into three. Five is dividing into three.*

It is clear from the above extract that L38 does not understand the meaning of a numerator and denominator. Consequently, the learner does not understand the meaning of a fraction. Surprisingly, this learner could shade $\frac{3}{5}$, correctly but did not understand what she shaded. Similarly, L17 had the same understanding as L38. The learner responses revealed a lack of understanding of what the components of a fraction really mean as they preferred naming the components instead. The use of common fractions differently in a variety of contexts where fractions are named differently may be the confusing part. L6 seemed to have a vague idea of what a denominator was but equality of parts is not mentioned. L6's solution to Question 1.1 and 1.2 attest to it that sizes of parts are not of importance. L15 showcased the complexity of the concept of fraction as the efforts to define the numerator are from the quotient sub-construct, whereas the denominator seems to be part of the whole perspective.

<u>Question 2</u>

Use any strategies to find answers of the following

2.1 $\frac{1}{3}+\frac{1}{6}$

This question focuses on the addition of common fractions with different denominators. Eighty per cent (80%) of the learners did not get the right answer. Table 4.3 shows that 50% of the learners got $\frac{2}{9}$ as the answer. Evidently, the learners just added across, numerator 1 plus numerator 1 to get the numerator the answers 2. The denominators were treated in the same way, denominator 3 plus denominator 6 to provide the denominator answer 9. Probably the misconception emanates from the prior-knowledge of the addition of whole numbers. For example, L6 indicated that the answer of the question would be $\frac{2}{9}$. The evidence below explains L6's solution.

I: *How did you get your answer?*

L6: *1+1=2 and 3+6=9 Sir, so* $\frac{2}{9}$.

I: *Why did you separate numbers from their fractions?*

L6: *I combined them at the end.*

I: *What makes you add that way?*

L6: *Adding is bringing things together so numerator adds numerator and denominator adds denominator,*

I: *May you please explain why you add numerator to numerator and denominator to denominator.*

L6: *If one shape has 1 part shaded and the other shape also has 1 shaded part, together there are 2 shaded parts. If the shape has 3 parts and* *the other shape has 6 parts so all in all there are 9 parts. From the two shapes we have 2 shaded parts from 9 parts.*

L15 had the following solution:

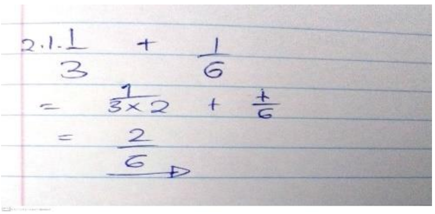


Figure 4-3: L15’s solution for Question 2.1.

L15 justified the solution in the interview set out below.

I: *Why did you multiply denominator 3 by 2?*

L15: *To have a common denominator because denominators are not the same.*

I: *Ok, what happens after multiplying by 2?*

L15: *The denominators are now the same so I can simply add numerators.*

The same thinking as evidenced in L6, was also reflected by L17, who provided the solution:

$$\frac{1}{3} + \frac{1}{6} = \frac{2}{6}.$$

I: *Take me through your working, how did you get your answer?*

L17: *1+1=2 and we use the common denominator as the denominators are not the same. The common denominator is 6. The answer is* $\frac{2}{6}$

I *What is the purpose of the common denominator?*

L17: *It represents the denominators in the question. The numbers must not leave a remainder when they divide into the common denominator.*

However L38 reads “+” as multiplication, as evidenced below:

L38: $\frac{1}{3} + \frac{1}{6} = 6 \times 1 = 6; 3 \times 1 = 3 = \frac{6}{3} = 2$

I: *Make me understand how this multiplication comes in in an addition problem.*

L38: *it's fractions so cross multiplication works*

I: *Why does cross multiplication work for fractions?*

L38: *It's a short cut, the denominator multiplies the numerator of the other fraction.*

I: *Can you do the same problem using the longer way, without taking a short-cut.*

L38: *I am now used to the short cut and have forgotten the steps of the longer method*

I: *Ok, you chose the answer to* 3×1 *to be the numerator in the answer, why not the answer to* 6×1?

L38: *The answer of the numerator first fraction gives us the numerator answer and the answer of the denominator of the first fraction the denominator answer.*

I: *you are guided by the first fraction.*

L38: *Yes.*

I: *Is it a rule?*

L38: *It's like that Sir.*

I: *Does this mean that* $\frac{1}{6} + \frac{1}{3}$ *would give* $\frac{1\times 3}{6\times 1} = \frac{3}{6}$.

L38 *Yes Sir.*

I: *Do you know any other way of doing the same problem?*

L38: *I always do it this way, it's easy to understand.*

I: *And you get correct answers.*

L38: *Sometimes.*

Learner responses reveal a misappropriation of rules. Due to knowledge on whole numbers, L6 treated the fractional components distinctly, which led to the numerators being manipulated on their own and the denominators on their own as well. L6 treated fractions as whole numbers and displayed no conceptual understanding of fractions. L6's way of thinking is attributed to an application of the wrong algorithm. L17 and L 15 have an idea of the involvement of common denominators when dealing with the addition of fractions with different denominators. However, the common denominator is placed at the end without including it in calculations. This displays partial conceptualisation by the learners as knowledge from equivalent fractions becomes the missing link. L38 executed a series of steps showing a lack of understanding of fractions and failed to justify the calculations mathematically. The learner might be failing to apply their own generalisations that may have been deduced during instruction.

Q 2.2 Use any strategies to find answers of the following

$\frac{1}{4} + \frac{3}{4}$

This question focused on the addition of common fractions having the same denominator. Fifty-five per cent (55%) of the learners got it right, 40% were partially correct, and 5% had incorrect solutions. Sixteen of the 18 incorrect solutions were $\frac{4}{8}$. This represents 40% of the learners and was the most common incorrect answer, as shown in Table 4.3. The effect of knowledge from whole number addition is evidently applied as the numerators were added separately to give the numerator answer and the denominators were summed up to give the denominator answer. L6's solution of $\frac{4}{8}$ attests to this:

I: *Please explain how you got your answer?*

L6: *It's like in 2.1 Sir; we are adding like terms.*

I: *What do you mean by that?*

L6: *Numerator to numerator and denominator to denominator. You can even say one block shaded out of four blocks combined with three blocks shaded out of four blocks will undoubtedly give you four shaded blocks out of a total of eight blocks.*

L15 had the following to say about the $\frac{4}{8}$ solution:

L15 *I added shaded parts and got 4. Denominators show number of parts so I added the denominators to get a total of 8. That's why my answer is* $\frac{4}{8}$.

I *Does this mean you are adding parts of different wholes?*

L15 *Two fractions are added so it is parts in one whole plus parts in the other whole.*

L17 wrote $\frac{4}{4}$ as an answer and seem to understand from the solution point of view and had the following to say:

I: *How did you get* $\frac{4}{4}$ *in this case?*

L17: *This question has fractions having the same denominator so it means we are adding parts of one circle so the answer will have the same* *denominator.*

I: *Hhmm, please clarify the other case where you add parts of different circles.*

L17: *When denominators are not the same in fractions to be added, we are adding parts of two different circles so the denominator in the answer will change .*

I: *Ok, explain how you ended up with* $\frac{4}{4}$ *.*

L17: *Denominators are the same so all is happening in one circle. One part of the 4 parts is shaded then we have to further shade 3 more parts out of the 4. This leads to everything shaded, all the 4 parts out of 4, hence* $\frac{4}{4}$*.*

L38 produced $\frac{1}{4} + \frac{3}{4} = \frac{4\times1}{4\times3} = \frac{4}{12} = 3$.The evidence below shows how L38 refers to "+" as multiplication.

I: *How do you get the answer three?*

L38: *I used the short-cut and cross-multiplied to get* $\frac{4}{12}$*. 4 then gets into 12 three times hence the answer three.*

I: *What happened to the numerator four?*

L38: *It cancelled as it divided into 12 to get three.*

L6 added numerators and added denominators. The misconception might be the effects of the perception of adding shaded regions from different wholes. The misconception leads to having four shaded parts from a combined total of eight, hence $\frac{4}{8}$. L38 cross multiplied, products forming numerator and denominator of the solution fraction. Although the method was regarded as a short cut by the learner, L38 did not put forward a convincing rationale. Some manipulations by teachers as they take it for granted that learners follow or understand may cause such misunderstandings. Considering $\frac{2}{7} + \frac{3}{5}$; with common denominator 35, there is a stage $\frac{(5\times2)+(7\times3)}{35}$ of getting to equivalent fractions. If the teacher deliberately skips explanations on how this stage is reached or the teacher is expertly seen as multiplying with ease diagonally, then a misconception can result. L38 might have generalised from such that the method is cross multiplication, though unfortunately the common denominator was completely left out.

Q 2.3 **Use any strategies to find answers of the following**

$\frac{7}{10} - \frac{2}{5}$.

This question focused on subtraction of common fractions with different denominators. Twenty per cent (20%) of the learners got it correct, 30% partially correct, and 50% were incorrect. Table 4.3 shows that $\frac{5}{5}$ was the most common incorrect answer and accounted for 50% of the learner solutions. As in the addition of common fractions exhibited by Questions 2.1 and 2.2, experience with whole numbers undoubtedly influenced operations of numerator subtracting from numerator and denominator subtracting from the denominator. L6 had the following to say regarding the $\frac{5}{5}$ solution:

I: *Brief me on how you got your answer?*

L6: *Aah obvious Sir, 7-2 = 5 and 10-5 =5*

I: *So numerators have their own answer and denominators have their own, then they form the fraction answer*

L6: *Exactly, top numbers give the top answer and bottom numbers give bottom answer.*

L15 justified $\frac{5}{5}$ being the final answer as evidenced by the following:

L15: *I removed 2 shaded parts from a whole with 7 shaded parts and remained with 5 shaded parts.*

I: *That's for the numerator I suppose, what about the denominator?*

L15: *Yes, for the denominator I removed 5parts from 10 parts so it means the answer will be having 5 parts.*

I: *Numerator is 5 and the denominator is also 5.*

L15: *Yes, it means all the parts in the answer are shaded parts.*

However, L17 had the following to say about the solution $\frac{5}{5}$:

I: *Brief me on how you got your answer.*

L17: *7-2=5 for top numbers. The answer must be over the common denominator as the numbers are not the same. The common denominator is 10, I made a mistake. The answer must be* $\frac{5}{10}$ *not* $\frac{5}{5}$ *sir.*

I: *So you were correcting the denominator otherwise the numerator stays*

L17: *yes it must be over the common denominator 10*

L38 related subtraction to division as can be seen from his justification below.

L38: $\frac{7}{10} - \frac{2}{5} = \frac{7 \div 5}{10 \div 2} = \frac{1rem2}{5} = \frac{2}{5}$

I: *Please get me through your working.*

L38: *When it is addition we cross-multiply, when it is subtraction, we must cross divide.*

I: *Go on, elaborate on answers after cross-dividing.*

L38: *7* $\div 5 = 1remainder\ 2$ *and 10*$\div\ 2 = 5$. *1 is the whole so the fraction part is* $\frac{2}{5}$.

I: *So the 1 whole is not part of the answer*

L38: *yes, the final answer is a fraction*$\frac{2}{5}$

L6, L15 and L17 subtracted across, numerator minus numerator and denominator minus denominator to get their answers. L17, however, remembered the common denominator, although the learner did not understand its purpose. The learner displayed a lack of knowledge of levelling the ground, which is achieved through the use of equivalent fractions. Forgetting some steps in calculations might be the consequence of original instrumental understanding.

L38 explained that when it is addition, you cross multiply so when it is subtraction, you must cross divide. L38 might be mixing concepts, multiplication implying addition and division implying subtraction might be algorithms from laws of exponents. Application of these algorithms by L38 in fractions shows a conceptual breakdown.

Q 2.4 **Use any strategies to find answers of the following:**

$$\mathbf{7\ \frac{1}{2} + \ 10\frac{1}{2} + \ \frac{3}{4}}$$

This question focused on the addition of three fractions, two of which are in mixed number format. Fifteen per cent (15%) of the learners got it right, 60% were partially correct, and 25% were incorrect. Eighty-five per cent (85% of the learners did not get it correct. Table 4.3 shows that 53% of the learners had $17\frac{5}{8}$ as their answer. This might be a result of adding whole numbers separately before adding the fractional parts of each fraction involved. Isolating components of the fractions and adding them separately showcases the extension of whole number knowledge to the concept of fractions. Corresponding values in the sense of position on the fraction are added the same way whole numbers are added. L6 had the following to say about getting the solution $18\frac{5}{8}$.

I: *How did you get your answer?*

L6: *I added whole numbers separately to get an answer for whole numbers before adding fractions to get an answer for fractions.*

I: *Now explain how you got 18 and* $\frac{5}{8}$.

L6: *Adding whole numbers 17+10+1=18 then 1+1+3=5 and 2+2+4=8 so the answer for the fraction is* $\frac{5}{8}$.

I: *For whole numbers, you said 17+10+1, where is the 1 coming from?*

L6: *The teacher said when it is* x, *it means* $1x$. *The 1 is optional, you can write or leave it. Invisible.*

I: *Ok, but there is no x.*

L6: *Fractions are like x as whole numbers are behind fractions.*

I: *Ok*

L11 interpreted addition to mean multiplication and justified the calculations through the following dialogue:

L11:

$$17\frac{1}{2}\times\frac{1}{2}\times\frac{3}{4}=17\frac{3}{16}$$

I: *Please get me through your work on this problem*

L11: *I added whole numbers to get 17 before multiplying fractions to get the answer.*

I: *Why multiply?*

L11: *When adding, the short-cut is to multiply.*

I: *So whole numbers can be added but the fraction parts are multiplied.*

L11: *Yes, the short cut is applicable to fractions*

The line of thinking of L17 was similar to that of L6 as evidenced by the following conversation reflected below:

I: *How did you get your answer?*

L17: *I used the method of adding whole numbers before combining the answer to that of adding fractions.*

I: *Whole numbers7and 10 give a sum of 17, where is* $\frac{5}{8}$ *coming from?*

L17: *Adding fraction numerators 1+1+3 gives 5 and adding denominators2+2+4, I got 8 so the answer* $\frac{5}{8}$.

L38 obtained:

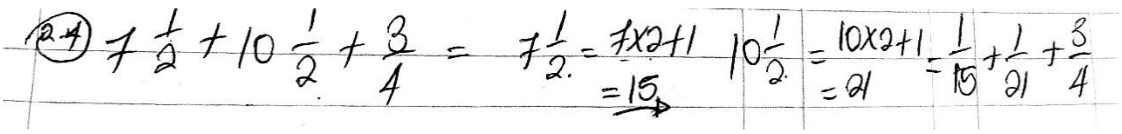


Figure 4-4: L38's answer for Question 2.4

The following is what L38 said in relation to steps which were taken in dealing with the problem:

I: *What is your final answer here?*

L38: $\frac{5}{15+21+4}$ *which is* $\frac{5}{40}$.

I: *Ok, please get me through the steps.*

L38: *First I changed mixed numbers* $7\frac{1}{2}$ *and* $10\frac{1}{2}$ *.7* $\frac{1}{2}$, $7 \times 2 + 1$ *gave me 15 and10*$\frac{1}{2}$, $10 \times 2 + 1$ *gave me 21. As these are fractions, it is* $\frac{1}{15}$ *and* $\frac{1}{21}$*. I then added* $\frac{1}{15} + \frac{1}{21} + \frac{3}{4}$ *to get answer* $\frac{5}{40}$.

L6 explained that whole numbers were added separately. Since $x = 1x$, the learner explained that as such $\frac{3}{4}$ has a whole number 1 leading to the sum of whole numbers $7 + 10 + 1$ being 18. The fractional parts were added across giving $\frac{1+1+3}{2+2+4} = \frac{5}{8}$. Bringing the whole number answer and the fraction answer together leads to the final answer $18\frac{5}{8}$. The misconception on $\frac{3}{4}$ supposedly having a whole number 1 may be a result of work on algebra. The explanation that a variable represents a number might have played a part in L6's misconception. Drawing from this, the interpretation of $7\frac{1}{2}$ by learner L6 might be seven halves. This brings the whole problem to be 7 halves + 10 halves + 1 three-quarters. Adding the whole numbers includes the 1 giving 18 before the misconception on the addition of fractions results in

$\frac{5}{8}$. L11 added whole numbers correctly to get 17. Fractional parts were surprisingly multiplied by L11. The learner explained that when we add, we put down the whole number answer and multiply. The learner might be confusing fractions with work on the laws of exponents. L38 had a conceptual misunderstanding on changing mixed numbers to improper fractions. The denominator is dropped, resulting in a whole number answer. This implies that L38 changes mixed numbers to improper fractions through following what he was told to do (instrumental instruction) without actually understanding the justifications of the steps taken. Changing a mixed number and ending up with a whole number shows a lack of understanding of the fractions. The reciprocal of the answer is then used for further calculations.

Q 3.1 Use any strategies to find answers of the following:

$\frac{1}{2} \times \frac{1}{3}$

This question focused on the multiplication of common fractions. Forty-two point five per cent (42,5%) of the learners got it right, 27, 5% were partially correct and 30% of the learners were incorrect. Table 4.3 shows that the most common anomaly was finding the common denominator when multiplying fractions. Thirty per cent (30%) of the multiplication of fractions. Use of cross manipulations with fractions might be leading the learners to just applying the manipulations without proper conceptual reasons underlying their actions.

L11 arrived at the following solution:

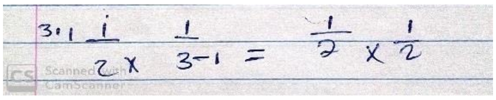


Figure 4-5: L11’s answer to Question 3.1

L11 also had his efforts directed to working with a common denominator and had the following to say:

L11: *Denominators are different so my teacher encouraged us to use same denominators*

I: *How did you make the denominators to be the same?*

L11: *I subtracted 1 from 3 so that both fractions have denominator 2*

I: *Now you have a different fraction problem*

L11: *Yes that was the reason why I subtracted 1, to make it easy.*

L15 arrived at the following solution:

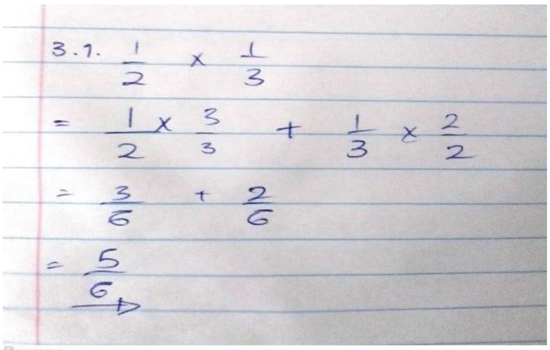


Figure 4-6: L15's answer to Question 3.1

L15 used a similar approach to that used by L38 although with a variation in that addition and multiplication were both used. L15, when asked about the procedure used, had the following to say:

L15: *I first found the common denominator and made the fractions to have the same denominator*

I: *Why did you find the common denominator?*

L15: *So that I can add easily the two fractions.*

I: *It's multiplication, how come you talk of addition now?*

L15: *With fractions, addition and multiplication work together.*

Working with common denominators was evident in the solution of L17, who arrived at the following solution:

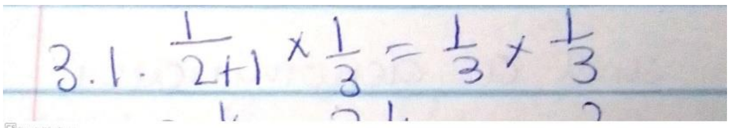


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Figure 4-7: L17's answer for Question 3.1

L17 explained the steps leading to the solution.

L17: *First I made the denominators the same.*

I: *How did you do that?*

L17: *I added one to denominator two to make it three.*

I: *Why?*

L17: *Denominators must be the same if you want to work easily with fractions.*

L38 seemed to use cross-addition in a problem of multiplication and had the following to say:

L38: $\frac{1+3}{2+1} = \frac{4}{3}$

I: *It's a problem on multiplication, how does it change to addition?*

L38: *The easier way is to add?*

I: *How then do you add?*

L38: *add across, the top with the bottom in the second fraction gives answer to the top. The bottom and the top give an answer to the bottom.*

I: *Why add across in particular?*

L38: *it is very fast and easy.*

I: *Taking you back a bit, for addition you were cross multiplying. Now it's multiplication, you use cross addition. Why switch operations?*

L38: *They are short cuts to solving fractions*

L38 added values diagonally supporting the step by stating that when it is multiplication, the easier and faster way is to add. This misconception can be attributed to utilising an algorithm for a particular topic in a concept where it is irrelevant. Multiplication may be taken as repeated addition, but in this case, L38 explained from the context of laws of exponents. L11, L17 and L15 over-elaborated the common denominator aspect of fractions as they decided to work with common denominators even on multiplication of fractions. However, inadequate conceptualisation of fractions led to mathematically incorrect manipulations. Determining the common denominator proved a challenge for learners, thereby displaying weak conceptions of equivalent fractions knowledge.

Q 3.2 Use any strategies to find answers of the following:

$2\frac{1}{2} \times 3\frac{1}{5}$.

This question focused on the multiplication of mixed fractions. This resulted in no correct and 100% incorrect solutions by the learners. The learners matched and multiplied corresponding components of the fractions as if they were multiplying natural numbers. was The most common incorrect solution, as shown in Table 4.3, was $6\frac{1}{10}$. Forty-eight per cent of learners believed that whole numbers are multiplied separately from fractional parts before combining the two answers. Learner L6 had the following to say when asked about the $6\frac{1}{10}$ solution:

L6: $6\frac{1}{10}$.

I: *Please explain how you arrived at your answer?*

L6: *I multiplied whole numbers alone* $2 \times 3 = 6$ *and fractions separately* $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$ *hence combined answer of* $6\frac{1}{10}$.

I: *Why do you separate wholes from fractional parts as you carry out your calculations?*

L6: *Whole numbers can be treated separately in fractions and the fractions as well.*

I: *Do you separate wholes in some operations or whether it's addition, subtraction, multiplication or division the same approach works?*

L6: *It works all round, you can add wholes/fractions separately, you can subtract wholes/fractions separately, multiply wholes/fractions like in this case and you can even divide wholes/fractions separately.*

I: *Please give me an example of a real life situation where multiplication of fractions is used.*

L6: *Aaah, I don't know.*

A combination of multiplication and addition was applied by Ll5 who had the following to say in justifying the steps taken.

L15: $2 \times 3 = 6 \;\; and \;\; \frac{1 \times 5}{2 \times 5} + \frac{1 \times 2}{5 \times 2} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

$\frac{7}{10} + 6 = 6\frac{7}{10}$.

I: *How did you work out this one?*

L15: *After multiplying whole numbers to get 6, I then added the fractions.*

I: *It is a multiplication problem, why did you add specifically the fraction parts, not whole numbers as well?*

L15: *Some rules work on fractions and not on whole numbers.*

I: *State the rule you applied here*

L15: *When it is multiplication, you can choose to add as multiplication means adding again and again.*

I: *Please give me an example of a real life situation where multiplication of fractions is used*

L15: *Hmm, I can't think of any.*

An approach used by L34 involved converting to improper fractions before multiplication. The following is what L34 had to say about the steps taken to find the solution:.

L34: $\frac{4}{2} \times \frac{15}{5} = \frac{60}{10}$.

I: *Please shed light on your calculations*

L34: *I converted mixed fractions then multiplied*

I: *How did you convert?*

L34: *$2\frac{1}{2}$ can be written as $\frac{2\times2\times1}{2} = \frac{4}{2}$ and $3\frac{1}{5}$ as $\frac{5\times3\times1}{5} = \frac{15}{5}$*
It is therefore $\frac{4}{2} \times \frac{15}{5} = \frac{4\times15}{2\times5} = \frac{60}{10}$.

I: *Are you in a position to explain what actually happens when converting a mixed fraction to an improper number, say from the shapes point of view?*

L34: *I know that we multiply and the answer must be over the denominator we used.*

I: *Please give me an example of a real life situation where multiplication of fractions is used.*

L34: *I don't know.*

Like L34, L38 also converted the fractions and had the following to put across:

L38: $2\frac{1}{2} = 2 \times 2 + 1 = 5 \qquad 3\frac{1}{5} = 3 \times 5 + 1 = 16 \qquad = \frac{5}{16}$

I: $\frac{5}{16}$ *— how do you get that?*

L38: *I changed mixed fractions so that I get answers easily.*

I: *get me through changing mixed fractions*

L38: $\frac{1}{2} = 2 \times 2 + 1 = 5 \qquad 3\frac{1}{5} = 3 \times 5 + 1 = 16$

I: *What happened to the denominators?*

L38: *All the numbers in the mixed number have been used to get the answers. To get 5, the denominator 2 was multiplied with whole number 2 and then added to numerator 1.*

I: *Are you in a position to explain what actually happens when converting a mixed fraction to an improper number, say from the shapes point of view?*

L38: *No.*

I:	*Ok. From fractions, you end up with whole numbers.*
L38:	*The final answer is a fraction as these whole numbers will form the final fraction..*
I:	*How do you know which one would be the numerator?*
L38:	*The first fraction forms the numerator and the second fraction will provide the denominator.*
I:	*Please give me an example of a real life situation where multiplication of fractions is used.*
L38:	*I don't know.*

L6 believed that whole number parts of the mixed numbers are multiplied separately and the fractional parts separately also before combining the respective answers to form the product. This misconception may emanate from a similar process which yields correct solutions in the addition of fractions.

L34 and L38 displayed misconceptions relating to changing a mixed number into an improper fraction. L38 manipulated correctly but dropped the denominators. When the researcher asked for clarity on $2\frac{1}{2} = 5$, the learner replied that it is how they were taught. Learners could not relate the mixed fraction-improper fraction conversion to circles or the part of a whole concept. This shows that these manipulations were not approached inductively by the educator/s concerned. Learners were probably told how manipulations were done without attending to the rationale behind such manipulations.

Q 4 Use any strategies to find answers of the following

4.1 $\mathbf{8 \div \frac{1}{2}}$

This question focused on the division of a whole number by a common fraction. Thirteen per cent (13%) of the learners got it right. Eighty=seven per cent (87%) of learners did not get it right. Table 4.3 shows that four was the most popular incorrect answer. Thirty-eight per cent (38%) of learner solutions had four as the answer. Clearly, learners interpreted the question as to mean halving an eight. The misconception might be a result of inappropriate use of mathematical language informally in daily experiences. L6 had the following to say in justifying a solution of four:

I: *Please explain your calculation.*

L6: $Simple, \frac{1}{2} of\ 8\ is\ 4.$

I: *But the question is* $8 \div \frac{1}{2}$.

L6: *Off course,* $8 \div \frac{1}{2}$ *means half of 8.*

I: *Please explain your calculation.*

L6: *When you have 8 blocks and you divide making smaller you cut at the centre you get four-four.*

I: *Why cut at the centre?*

L6: *Yes, divide by half means have two parts, half-half*

I: *What about divide by two?*

L6: *You can say it that way if you want, answers are the same*

L11 provided the same answer though through an attempt of applying the invert and multiply algorithm. L11 justified the steps taken as follows:

L11: $\frac{8}{1} \times \frac{1}{2} = \frac{8}{2} = 4$

I: *Please explain your calculation*

L11: *When dividing fractions, you change the division sign to a multiplication before multiplying normally.*

I: *Does this mean* $8 \div \frac{1}{2}$ *is the same as* $8 \times \frac{1}{2}$*?*

L11: *For some questions you are lucky they give you as times (×) you do not need to change but others you need to change because you would not get the answer right if you do not change* $\div$ *to* $\times$.

Splitting the division of the whole number by the numerator and the same whole number by the denominator led to the following clarifying dialogue with L17:

L17: $8 \div 2 = 4$ $\quad 8 \div 1 = 8 \quad$ $4 + 8 = 12$

I: Please let me understand how you worked out this one.

L17: I divided 8 by the numerator and by the denominator. I then added the answers together.

I: Elaborate on dividing 8 by numerator and denominator.

L17: Ok, $8 \div \frac{1}{2}$ *. 8 is divided by 1 and the same 8 is divided by 2.*

I: What about adding?

L17: It is one problem so the answers must be added together to have the final answer.

L38 had the following to say when asked about the thinking involved in the steps taken to solve the problem:

L38: $8 \div \frac{1}{2} = 8\frac{1}{2} \quad = \quad 8\times 2 + 1 = 17 \div 2 = 8rem1 \quad (8 \div 1 = 8) \quad 8\div 2 = 4 \quad = \frac{4}{8}$.

I:l *How did you work out this problem?*

L38: *I first got 8* $\frac{1}{2}$ *then changed it by saying* $8 \times 2 + 1$*which is 17.* $\frac{17}{2}$ *is 8 remainder 1.*

I: *How did you get 8* $\frac{1}{2}$ *in the first place?*

L38: *I was not sure of my answer that is why I did another one by the side.*

I: *Please explain this other one.*

L38: *I divided eight by one and got eight. I then divided eight by two and got four. The final answer is* $\frac{4}{8}$.

The learners explained that $8 \div \frac{1}{2}$ means the same thing as $\frac{1}{2} of$ 8. L6 and L11 clarified in such a way that they ended up concluding that dividing by two is the same as dividing by $\frac{1}{2}$. This misconception may be a result of interpretation of divide by half to mean divide into two equal parts, which in essence implies divide by two. Learners use the two interchangeably in everyday life. L6 pointed out that, informally, they use divide by half more often than divide by two when instructing two individuals to share equally.

L11 had a vague idea of changing division to multiplication. The learner declared that the $\div$ has to be changed to $\times$ because at times you get remainders when you divide across. If you multiply, however, the learner continued, you will never get a remainder so we must always change to multiplication. Maybe due to rote learning, the learner just changed the operator $(\div)$ to $(\times)$ without inverting the second fraction.

L17 and L38 divided the whole number eight by separate components of the fraction $\frac{1}{2}$. Eight was divided by the numerator treating these as whole numbers. Eight was similarly divided by the denominator in a whole number fashion. L17 added the resulting whole numbers whilst L38 formed a solution fraction using the two whole numbers. Resorting to inappropriately using whole number knowledge might mean learners do not understand what fractions are, let alone the underlying concepts.

_Q 4.2_ Use any strategies to find answers of the following

$$\mathbf{\frac{1}{2} \div 10}$$

This question focused on the division of a common fraction by a whole number. Six per cent (6%) of the learners got it correct. Of the 94% incorrect answers, Table 4.3 shows that five was the most prevalent answer. Twenty-three per cent (23%) of the learners had “five” as the solution. The learners might have expected an answer smaller than ten, generalising from the natural number perspective that division makes smaller. Learner L34 had the following to say as a justification for getting the answer of five.

An attempt to the invert and multiply is evidenced in the following conversation with L6:

L6: $\frac{1}{2} \div 10 = 20.$

I: *Please guide me onto how you got your answer*

L6: *I converted* $\frac{1}{2}$ *to* $\frac{2}{1}$ *so that the sign changes to* $\times . \frac{2}{1} \times 10 = 20.$

I: *Do you know why inverting and changing to multiplication works?*

L6: *I know that it makes working with division of fractions simple.*

I: *In what way? Please elaborate.*

L6: *My teacher said it is not always easy to divide some numbers so co…converting a fraction makes it multiplication and you can multiply easy numbers.*

I: *Do you think you can use this question to clarify what you mean?*

L6: *Yes Sir, for* $\frac{1}{2} \div 10$ *I cannot easily get the answer to* $1 \div 10$ *but when I put upside down and multiply, it becomes very simple.* $\frac{2}{1} \times 10 = 20.$

From a different approach, L15 also got five and clarified the steps taken in the following manner:

L15: $\frac{1}{2} \div \frac{10}{1} = \frac{10}{2} = 5.$

I: *Please get me through your working..*

L15: *I changed 10 to* $\frac{10}{1}$ *so that it does not confuse me.*

I: *Explain further to illustrate how this helped you avoid confusion.*

L15: *It is clear now as numerators will divide each other and denominators divide each other also.*

I: *Alright I get you, now the problem looks*

$\frac{1}{2} \div \frac{10}{1}$, *continue then.*

L15: $\frac{1 \div 10 = 10}{2 \div 1 = 2}$ *so it leads to* $\frac{10}{2}$ *and 10 divide by 2 is 5.*

The fraction was split to divide it into the whole number separately by L17 who had the following to say;

L17: $1 \div 10 = 10$ $\quad 2 \div 10 = 5$ $\quad 5 + 10 = 15$

I: *Please guide me through your solution to this question.*

L17: *Ok, divided all parts of the fraction into ten and got answers for numerator and denominator:*

I: *Please elaborate on your division.*

L17: $1 \div 10 = 10$ *and* $2 \div 10 = 5$ *so the answer is* $10 + 5 = 15$.

I: *Why divide both 1 and 2 by 10?.*

L17: *Ten is a common number to numerator one and denominator two.*

I: *I get you, but* $1 \div 10 = 10$ *and* $2 \div 10 = 5$ *how is it so*?

L17: *Yes, one goes into 10 ten times and two goes into 10 five times.*

L30 also got an answer of five and supported the calculations as follows:

I: *Please explain how you arrived at your answer.*

L30: *It means ten divide by half.*

I: *And what is the answer to that?*

L30: *Ten divide by half is five.*

I: *How is it so?*

L30: *You end up with two halves with 5-5.*

Responses to Question 4.1 and 4.2 were similar. On justifying their solutions, the learners believed that $\frac{1}{2} \div 10$ is the same as $10 \div \frac{1}{2}$ and that both translated to $\frac{1}{2}$ of 10. Learner 15 insisted on changing $\div$ to $\times$ without inverting the second fraction. This cemented the misconception by the learner that $\frac{1}{2} \div 10$ is indeed equal to $10 \div \frac{1}{2}$.

In reality $\frac{1}{2} \times 10$ and $10 \times \frac{1}{2}$ are the same because of the commutative property, which however does not apply to division.

4.1 CHAPTER SUMMARY

In this chapter, the observations and the data analysis of the study was presented. Tables were used to summarise the findings from the written test. Codes were used to signify the state of answers by each participant relative to each test item. C in the table (Table 4.1) meant correct solution, PC, partially correct and IC was used for an incorrect solution. The prevalence of correct against incorrect solutions for each test item was presented as percentages in Table 4.2. The chapter had a section on analysis based on what the tables portrayed.

Extracts from the learner written test scripts were analysed in conjunction with verbal responses from the interviews. The chapter presented interview reports of four interviewees and the corresponding analysis. Analysis was done simultaneously for data from both sources, the written test and interviews. The findings revealed that indeed learners have misconceptions in the topic fractions. Knowing the procedure of dealing with fractions as opposed to conceptual understanding was indicated by the findings as the participants' work method when engaging in fraction work. A conclusion and recommendations based on the study findings are found in the next chapter.

5 CHAPTER FIVE: CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In this chapter, a discussion of findings, recommendations and conclusion of the study will be drawn. The researcher shall focus again on research questions and answer them based on the analysis of my data. The researcher shall state explicitly the research findings. And lastly, the implications and reflections of the findings will be discussed.

In concluding this research, the researcher reiterates the critical questions. The critical motivations that guided this study were:

- To explore how Grade 9 learners describe the concept of fractions.
- To investigate how Grade 9 learners solve problems involving common fractions.
- To discover misconceptions that learners have/display when dealing with fractions.
- To discover the root cause of misconceptions held by learners in relation to the concept of fractions.

In this chapter, conclusions will be drawn referring to each aim, the relevant data collected, and the analysis thereof. The findings will be explicitly put forward before discussing the implications.

5.2 CONCEPT OF FRACTION FROM THE LEARNERS' POINT OF VIEW.

One of the objectives of this study was to get an understanding of how Grade 9 learners perceive the concept of fractions. A test (Appendix H) was administered to provide data in this regard, depending on learner responses. Interviewing selected learners added the additional oral input to the collected written feedback, and the following section lays out how Grade 9 learners view fractions as a concept.

5.2.1 Lack of conceptual understanding of Fraction

Questions 1.1 to 1.7 were designed to test learners' conceptual understanding of fractions. While most learners perceive fractions as a part of a whole, those who could not define fractions in words managed to illustrate their understanding through the use of examples. The illustrations were on pizza slices, dividing a circle and colouring some parts or sub-dividing any other shape which they considered a whole into many parts. Some learners used the word "sharing" to stress their idea of a fraction being a share of the whole unit. The findings reveal that learners' scope of fractions is limited to one whole. However, examples, explanations and illustrations by learners all referred to "cutting, dividing and sharing a whole". Their explanation did not refer to a whole as being divided by "equal parts" or equal shares or an equal–sized portion of a whole (Van De Walle 2016). The limitation and the lack of this understanding of a whole being divided into equal parts resulted in learners not understanding the concept of fraction. The researcher understands that the definition of a fraction as "part of a whole" has been captured in many South African textbooks (Laridon et al., 2005). It is evident from learners' responses that this definition has caused misconceptions on fractions. For example, this misconception was evident when 97% of learners indicated that the diagram in Figure 5.1 below represents $\frac{1}{3}$. It appears that these learners do not have an understanding of a fraction representing a whole, which has to be divided into "equal parts."

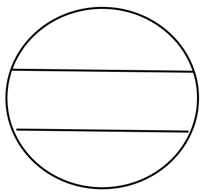

Figure 5-1: Division of whole into three unequal parts.

Similarly, the learners responded incorrectly to part of question 1.7 which required them to provide the meaning of the "3" and "5" in the fraction $\frac{3}{5}$. Almost all the learners responded by saying 3 is the numerator, and 5 is the denominator. Surprisingly, most learners could shade $\frac{3}{5}$ as required in the question, but were unable to give the meanings of a numerator and denominator. Furthermore, the fact that most learners responded that $\frac{5}{8}$ is bigger than $\frac{3}{4}$ indicates that learners do not have a conceptual understanding of a fraction. The inability to represent a fraction on a number line indicates a lack of a conceptual understanding of a fraction. Most learners represented fractions on the number line based on the degree or size of a numerator or denominator, but not on the meaning of the concept of fraction. It appears that the faulty representation of fraction on the number line was caused by faulty use of the prior-knowledge of whole numbers. It appears that the prior- knowledge of whole numbers gave the wrong meaning to the concept of fraction (Machaba, 2016).

Most learners responded to Question 2, the addition and subtraction of fractions, procedurally without a conceptual understanding of the concept of fraction. For example, in Question 2.2 they indicated that the answer to $\frac{1}{4}+\frac{3}{4}$ would be $\frac{4}{8}$, which is $\frac{1}{2}$. Ordinarily, this does not make sense. How can a one-quarter and three-quarters make one half? It shows that most learners did not apply their minds when responding to question such as this one. If the learners had drawn from their everyday knowledge in answering this kind of question, they would have realised that a quarter of bread plus three-quarters of bread cannot make one-half of bread. This does not make sense. This is an indication that learners solved this problem procedurally without an understanding of fractions. Even those learners who got the answer correctly did not get it procedurally without an understanding of the concept of fractions. For example, when L17 was asked to explain how she got $\frac{4}{4}$, she said, "*this question has fractions having the same denominator so it means we are adding parts of one circle so the answer will have the same denominator"*. It appears that this learner has the rule, "when adding fractions of the same denominator then add numerator". This learner has developed an instrumental understanding rather than a relational understanding (Skemp, 1976). This is in line with what Skemp (1976) said, namely, that instrumental understanding can be described as "rules without reasons" whereas relational understanding is knowing what to do and why.

Clearly, she does not seem to understand the origin and context of this rule. This suggests that sometimes a learner can solve the problem correctly but without a correct understanding of the concept. Again, a lack of an understanding of the concept of fractions was also evident in Question 2.4: $7\frac{1}{2} + 10\frac{1}{2} + \frac{3}{4}$. Most learners indicated that the answer is $17\frac{5}{8}$. This question just required learners to use a common sense of mixed fractions (the combination of fractions and whole numbers). It is clear that most learners had knowledge of whole numbers, because they could add 7 + 10 to be 17, but they were unable to work with fractions. If they had a conceptual understanding of a fraction, they would have used the everyday context of bread as indicated in this discussion. The perception of a fraction being part of a whole might have restricted the learners' scope of fractions to one whole. The study revealed that learners could not comprehend questions involving improper fractions. On arranging fractions on a number line, $\frac{8}{4}$ was regarded as different from 2.

5.3 HOW DO GRADE 9 LEARNERS SOLVE PROBLEMS INVOLVING COMMON FRACTIONS?

Questions 2.1 to 4.2 were designed to test learners' ability to manipulate and solve problems on fractions. The learners treated fractions as two numbers written one above the other. Despite displaying the knowledge that $\frac{1}{3}$ means colouring one part out of three, the learners did not treat $\frac{1}{3}$ as a number on its own. As such, calculations were made for the numerator independently of the denominator. Operations on fractions were therefore generally an extension of the whole number operations except that the numbers were written one above the other. The following equations illustrate how most learners dealt with operations on fractions.

$$\frac{1}{2}+\frac{2}{5}=\frac{1+2}{2+5}=\frac{3}{7} \qquad \frac{5}{8}-\frac{2}{3}=\frac{5-2}{8-3}=\frac{3}{5} \qquad \frac{2}{3}\times\frac{5}{7}=\frac{2\times5}{3\times7}=\frac{10}{21} \qquad \frac{15}{6}\div\frac{5}{3}=\frac{15\div5}{6\div3}=\frac{3}{2}$$

It is evident that learners take fractions as a form of style of presenting numbers differently. Numerators independently operate on numerators and denominators likewise operate on denominators. Numerators add to or subtract from other numerators and denominators add to or subtract from other denominators for addition/subtraction of fractions. For multiplication and division, numerators are multiplied/divided by other numerators and the denominators are multiplied/divided by other denominators. This is in line with what Jigyel (2007) argued in his study on equivalent fractions, namely, that some learners perceived components of fractions, the numerator and denominator, as two unrelated whole numbers, which led to misconceptions.

Again, misconceptions arose when learners were solving addition and subtraction of fractions. One of the most common misconceptions in this regard is when learners add/subtract numerator with other numerators and add/subtract denominators with denominators. This strategy seems to have been drawn from working with whole numbers in the early grades. This is in line with Olivier (1989) and Machaba (2016) who argued that the source of misconceptions is mostly an overgeneralisation of previous knowledge to an extended domain. For example, knowledge of whole numbers is incorrectly used when dealing with fractions.

It came as no surprise when most learners, for example, L38, used multiplication on the problem that required the addition of fractions. For example, L38 wrote $\frac{1}{4}+\frac{3}{4}=\frac{4\times1}{4\times3}=\frac{4}{12}=3$. When asked to explain why she did that, she said, "*I used the short-cut and cross-multiplied to get* $\frac{4}{12}$*. 4 then gets into 12 three times hence the answer 3".* In their study Lestiana, Rejeki and Setyawan (2016) found that most learners used cross multiplication strategy in adding fractions

5.4 MISCONCEPTIONS THAT LEARNERS HAVE/DISPLAY WHEN DEALING WITH FRACTIONS.

Most of the misconceptions that learners showed when dealing with fractions have already been discussed in Section 5.1 and 5.2. The researcher will, therefore, not reiterate what has been discussed but just summarise for the reader. Misconceptions have been found in the following areas:

- Faulty or incomplete definition of the concept fraction;
- Bigger denominators implied bigger misconceptions;
- The addition and subtraction of fractions;
- Misconceptions relating to the division of fractions; and
- Misconceptions relating to the multiplication of fractions.

5.4.1 Definition of the concept fraction.

As discussed in Section 5.2, the learners defined a fraction as a part of a whole. L15 clarified the part of a whole perception by saying, "*a piece of something which was complete".* Table 4.3 shows that 50% of the learners defined the fraction as a part of a whole. They, however, did not point out that the partitions of the whole should be equal in size. This misconception played itself out negatively when learners were comparing and solving fractions. While most learners define fraction in the context of using words such as "sharing, dividing and cutting", the researcher concurs with Van Der Walle (2016) that fractional parts are equal shares or equal portions of a whole unit. I reiterate that the word, "equal", is very significant because the analysis of written feedback by learners in this study showed that learners referred to partitions into three parts as "thirds", despite those parts not all being of the same size.

The definitions used by the learners limited the scope of fractions to less than one. L6, for example, portrayed a fraction as part of something complete. This completeness refers to a unit item or a single item, as L6 went on to say, "*when you eat a slice of Pizza it is a fraction".* Efforts by learners to relate the fraction concept to real-life always referred to solitary items. This learner tendency might have set a boundary within which learners' mindset relative to fractions operated. The researcher agrees with Kieren (1980, cited in Kerslake, 1986) who believed that the "part of a whole" definition limits the development of the idea that a fraction can be greater than one. Learners' definitions also excluded the consideration of fractions that can be formed by part of a collection of discrete items. This is in line with Jigyel's (2007) findings that learners in her study linked fractions to pictures of shaded parts of a model like circles or rectangles and less frequently to part of a group.

5.4.2 Bigger denominator implicates bigger fraction misconception.

The interviews brought to the surface the misconception that the bigger the denominator, the bigger the fraction. Some learners compared fractions guided by this belief and came up with incorrect conclusions. Fifteen per cent (15%) of the learners ordered fractions in descending order of denominators implying the bigger the denominator, the bigger the fraction. Instead of comparing the shaded parts of these fractions, learners' attention was drawn to denominator values which they arranged in descending order, as the question indicated from biggest to smallest. On explaining the solution to Question 1.5, L34 said "*For those fractions which are not total we use the denominator to tell us which is big. Eight is big followed by 7, then 6 and 2 is the smallest so fractions will follow each other that way."* L34's intention was to compare and order fractions but L34 instead ordered denominators. Learners perceive the comparison question as if it refers to denominators or the number of parts into which a shape representing each fraction is divided. This misconception emanates from looking at individual values of components of fractions being compared. This resembles Siegler et al.'s (2010) contention that believing

$\frac{3}{8} > \frac{3}{5}$ results from focusing on numerators and denominators as separate numbers and as such ending up comparing the two denominators and ignoring the essential relation between each fraction's numerator and denominator. Put differently, learners seemed to answer the question, "Which fraction represents a whole which is divided into many parts" intending to answer to the question, "Which is the bigger fraction". L6 on explaining a solution to question 1.3 said *"Eighths mean 8 parts and quarters is four parts".* Further probing on whether the focus should be directed on denominators led L6 to clarify and say. "*Yes, denominators show which pieces are many.*" This replicates a similar finding by Vosniadou and Stafylidou (2004) as their study found the belief that the value of the fraction increases as the value of the denominator increases.

5.4.3 Misconception relating to addition and subtraction of fractions

The most popular way learners (50%) added fractions in this study was to add numerators to give a numerator answer and denominators to get the denominator answer. This is an indication of a lack of basic knowledge of fractions. In question 2.1, L6 said, "*if one shape has one part shaded and the other shape also has one shaded part, together there are two shaded parts. If the shape has 3 parts and the other shape has 6 parts so all in all there are 9 parts. From the two shapes, we have 2 shaded parts from 9 pars".* As much as addition means bringing things together, the learners applied it incorrectly, and it is evident that they added as if they were dealing with whole numbers. Basic knowledge of fractions would have deterred learners from adding parts which are not of the same size as indicated by different denominators. Siegler et al. 2010) are of the opinion that a push factor leading to the misconception might be the fact that this approach works for multiplication. Learners generalise the multiplication approach and extend it to addition in this case.

From another angle, merging the two sets draws the thinking to focus on a new whole which is a sum combination of the previous independent two wholes. This affinity towards one whole is what Haser and Ubuz (2003) take as being responsible for interpreting fractions incorrectly.

Crossed-multiplication is another strategy which was used by some learners on Question 2.1. L38 said, *"I used the short-cut and cross-multiplied to get* $\frac{4}{12}$*. 4 then gets into 12 three times hence the answer 3".* This method is used on comparing two fractions to decide on which fraction is bigger. The learners inaccurately applied the procedure to the addition of fractions. The researcher agrees with Lestiana et al.'s (2016) belief that some learners in her study generalised the cross multiplication strategy to apply to the addition of fractions. Also, they may not have any knowledge of how to add fractions.

5.4.4 Misconceptions relating to the division of fractions.

Question 4 focused on the division of fractions. Learners were tested on the understanding of the processes that underlie the division of fractions. Most learners did not follow the correct procedure to divide fractions. Thirty-eight per cent (38%) of the learners divided the quantity in half instead of dividing by half. The interpretation might have been a factor as divide by half seems to be taken to mean divide in half. Interviewed on why four was the solution to $8 \div \frac{1}{2}$ in Question 4.1, L6 said, "*when you have 8 blocks and you cut at the centre you get four-four"*. L6 divided in half. This translates to eight blocks considered as a unit (whole) divided at the centre, not eight separate blocks (eight wholes) or discrete objects, each block divided at the centre. This misconception also draws from centralising work on fractions to one whole. Decontextualised fractional problems may not cause learners to apply their minds. Reverting to the bread example, learners would have responded comparatively better if they thought of eight loaves divided into halves with the focus being on the resulting number of pieces.

On trying to apply the invert and multiply approach learners made numerous mistakes. Some learners inverted the first fraction and others just changed the division sign to multiplication. This displayed a lack of conceptual understanding of the standard division procedure. The learners were applying a rule they did not understand. In Question 4.1, L11 said, *"when dividing fractions, you change the division sign to a multiplication before multiplying normally"*. L11 did not realise the implication of the statement, which is, division is the same as multiplication. However, in Question 4.2, L6 said, *"I converted* $\frac{1}{2}$ *to* $\frac{2}{1}$ *so that the sign changes to* $\times$. $\frac{2}{1} \times 10 = 20$". With such findings in this study, the researcher agrees with Siegler et al. (2010) who lament that learners misapply the invert and multiply procedure for dividing due to a lack of conceptual understanding of the procedure.

Learners also approached division in a manner as if the whole number was divided by components of a fraction separately. In Question 4.1, L17 said, *"ok,* $8 \div \frac{1}{2}$ *. 8 is divided by 1 and the same 8 is divided by 2"*. This shows that the learner does not regard $\frac{1}{2}$ as a meaningful number on its own as whole number operations were done separately using parts of the fraction. This also shows a misunderstanding of the procedures that govern the division of fractions. In Question 1.6, L17 said, $\frac{6}{10}$ divide by two is $\frac{3}{5}$. evidently, the learner divided both six and ten separately as whole numbers to get $\frac{3}{5}$. This is due to the conception that fractions are just two whole numbers that can be treated separately (Bruce, Bennett & Flynn 2014).

The learner written feedback revealed that learners extended the commutative property of addition and multiplication to the division operator as alluded to in Section 5.3.9 above.

5.4.5 Misconceptions relating to the multiplication of fractions.

Some learners in this study were doing cross multiplication, multiplying the numerator of the first fraction by the denominator of the second, then the denominator of the first fraction by the numerator of the second. Thirty per cent (30%) of the learners had to find the lowest common multiple or common denominator first before attempting to multiply. L15 said, "*I first found the common denominator and made the fractions to have the same denominator"*. L15 did not realise that finding a common denominator on multiplying fractions is an unnecessary extra burden. This illustrates use of a procedure applicable in comparing fractions and the addition/subtraction of fractions in multiplication. Partial conceptions by learners compel them just to apply fractional procedures they remember even on situations where they do not apply.

Multiplication of mixed numbers was a challenge to learners. Forty-eight per cent (48%) of the learners had the misconception that whole numbers were multiplied separately from the fractional parts. Fractional parts were also multiplied separately, and the answers brought together to give a solution comprising of the whole number answer and the fractional part answer. The learners again generalised a procedure adequate in the addition of fractions to the multiplication of fractions. In Question 3.2, answering to whether isolating whole numbers across fractional operations is feasible, L6 said, "*It works all round, you can add wholes separately, you can subtract wholes separately, multiply wholes like in this case and you can even divide wholes separately"*. This consolidates the generalisations learners made in different computations with fractions.

The learners revealed another misconception on multiplying a fraction by a whole number. In Question 1.6, L38 put two times $\frac{3}{5}$ forward as the relationship between $\frac{3}{5}$ and $\frac{6}{10}$. This is a misunderstanding of the concept of fractions as equivalent fractions were considered different in size or magnitude, one double the other in this case. The general perception that multiplication always results in a bigger number further exacerbates this (Graeber & Tirosh 1990). The researcher concurs with Jigyel's (2007) view that learners view one fraction as a multiple of the other, just like the relationship between the object and the image under enlargement in the topic of transformations.

5.5 ROOT CAUSE OF MISCONCEPTIONS HELD BY LEARNERS IN RELATION TO THE CONCEPT OF FRACTIONS

5.5.1 Prior knowledge

This study has the theory of constructivism as its backbone. According to Cobb (1994), constructivism is the generally accepted view that learners construct their mathematical ways of knowing as they strive to be effective by restoring coherence to the world of their personal experiences. Ultanir (2012) concurs with the idea that based on experience and background knowledge; the real understanding is constructed.

This implies that if prior knowledge is infested with misconceptions, constructions are likely to enforce the misconceptions.

The findings in this study reveal that incorrect application of prior knowledge is a source of misconceptions in the concept of fractions. In question 4.1, as reflected in Table 4.3, thirty-eight per cent (38%) of the learners interpreted $8 \div \frac{1}{2}$ as dividing the quantity eight into two halves taking it from prior informal knowledge. L6 said, *"Yes, divide by half means have two parts, half-half."* The incorrect informal use of mathematical language clearly had an impact in formal settings.

The analysis revealed that divide by two and divide by $\frac{1}{2}$ are used interchangeably by learners. In other words, learners regard them as one way of saying the other. This may be a result of the informal use of divide by half in their everyday lives instead of divide in half. This follows Siegler et al.'s (2010) view that learners may solve a problem framed in everyday context but be unable to translate, solve and present it using formal notation. This led to division questions with a $\frac{1}{2}$ to be interpreted as seeking half the quantity. In Question 4.1, L6 said $8 \div \frac{1}{2}$ *"means half of 8",* and in Question 4.2, L34 said, "*It is another way of saying the same thing, it means half of ten",* referring to $\frac{1}{2} \div 10$. The researcher agrees with Ashlock's (1994) suggestion that misconceptions in mathematical concepts are dependent on individuals' experience and application of these concepts in everyday life.

Poor transitioning along the continuum of learning fractions in terms of concept development may also be a source of misconceptions. Some learners believed the bigger the denominator, the smaller the fraction. In this study, in Question 1.5, on explaining why the bigger denominator denotes the smaller fraction, L15 said, "*the denominator tells us how many parts we cut our whole into. If the denominator is big, then it means we are cutting the whole into many small pieces. If the denominator is small, then the pieces will be big".* This is a characteristic of visual comparisons using divisions of a whole.

L15, however, stuck to the generalisation even when the compared fractions had numerators other than one. In this context, numerators may be regarded as insignificant to the learner and therefore play no role in comparing fractions. L15 did not develop his conception of fractions, conceptually leading to being stuck with definitions and approaches which were used in introductory lessons in primary school. As the content increases in depth along with the grades, the definitions and approaches have to be adjusted accordingly, accommodating new material. L15 seems to have developed a misconception from prior knowledge. What was generalised at a primary level might have been suitable for assessment standards at that level but might not be suitable to be carried over and might not suffice at higher levels. Prior knowledge can therefore be a source of misconceptions in this sense.

The learners focused on denominators to compare fractions. This led to a misconception on equivalent fractions. At a glance, with the notion "big denominator, big fraction" in mind, the learners did not take $\frac{3}{5}$ to be the same as $\frac{9}{15}$. Some learners, however, viewed $\frac{3}{5}$ as not equal to $\frac{9}{15}$ because they viewed $\frac{9}{15}$ as a multiple of $\frac{3}{5}$, being three times as large. The misconception roots from singling out numerators and concluding that transitioning from 3 to 9 is a result of multiplying by 3. Considering the denominators the same way leads to the same conclusion that 15 is arrived at through multiplying by 3. Lack of conceptual awareness distracts the learners from noticing that they actually multiplied by $\frac{3}{3}$. This approach then leads to the misconception that $\frac{9}{15}$ is an enlargement of $\frac{3}{5}$; hence they cannot be equal.

In this study, L38 said, "$two\ times \frac{3}{5} = \frac{6}{10}$". This is similar to Jigyel's (2007) assumptions on how learners might be relating equivalent fractions. Jigyel (2007) believes learners view the relationship between $\frac{2}{3}$ and $\frac{4}{6}$ either as $2\left(\frac{2}{3}\right)$ or $\frac{2}{2}\left(\frac{2}{3}\right)$. Either way, learners acknowledge this as the doubling effect having a 1:2 relationship. This may consolidate the argument that the two fractions are not the same and affirm the misconception. Abosalem (2013) is of the opinion that when the numerator and denominator are increased, learners tend to believe that the fraction always increases in value. This implies that $\frac{9}{15}$ would have most likely been regarded as greater than $\frac{3}{5}$ by learners. The learners failed to represent $\frac{3}{5}$ correctly when provided with 15 blocks. The role of the denominator from the part of a whole point of view suppressed the learner thinking to five being the limit. Perceiving fractions as part of a whole caused learners to consider only five blocks, the whole in this case. Prior knowledge on the denominator being the determining value as to how many parts the whole is divided into, might have limited expectations to five parts not the presented 15 parts. The need to working within one whole was evident. In this study, referring to Question 1.6, L38 said, *"meaning shade 3 out of 5. I had to choose my own 5 and leave the rest"*.

5.5.2 Over-generalisations

The study findings show a generalisation of knowledge from whole numbers to work on fractions. On comparing fractions in Question 1.3, L11 said, *"both numbers making up the fraction* $\frac{5}{8}$ *are bigger than the corresponding numerator and denominator values of the fraction* $\frac{3}{4}$".This led to L11 concluding that $\frac{5}{8}$ is bigger than $\frac{3}{4}$. Sixty-three per cent (63%) of the learners had such a conclusion, as shown in Table 4.3. The misconception is caused by comparing components of fractions as if comparing whole numbers. Whole number comparisons eventually influence overall judgement.

Numerator-numerator and denominator-denominator operations showed that learners isolated components of fractions so that they can apply their whole number knowledge on the separated components. L6 on, explaining steps to calculating $\mathbf{\frac{1}{3}+\frac{1}{6}}$ said, *"1+1=2 and 3+6=9 Sir, so* $\frac{2}{9}$". Evidently, when it comes to computations with fractions, L6 treated fractions as numbers placed in some style retaining their whole number status. Table 4.3 shows that 50% of the learners added the corresponding components of fractions. The researcher supports Bruce, Bennett and Flynn (2014) who put forward that this is a result of regarding a fraction as just two whole numbers that can be treated separately.

Some learners revealed a misconception in the multiplication of fractions which can be attributed to not considering a fraction as a number on its own. Viewing a fraction as two numbers that can be treated separately may be the cause of the misconception shown by learners on multiplying a whole number by a fraction. This interpretation led to the separation of the fraction and eventually separate manipulations. In Question 1.6, L38 said, *"2 times* $\frac{3}{5}=\frac{6}{10}$*."* The misconception was supported by other misconceptions identified in the study. L6 said, "$\frac{3}{5}+\frac{3}{5}=\frac{6}{10}$", and L17 said, "$\frac{6}{10}$ *divide by two is*$\frac{3}{5}$*"*.

A multiplication procedure was used in division. L15, in Question 4.2 said *"It is clear now as numerators will divide each other and denominators divide each other also"*. Whole number knowledge had an influence in the division of fractions in this study. The learners isolated components of the fractions and treated the resulting numbers as stand-alone whole numbers. Fractions were treated as two whole numbers that can be manipulated separately. Dividing across ensued, numerator divided by numerator and denominator divided by denominator.

Learners in this study generalised and used properties true to some operations in other operations where they were not valid. The learners took it for granted that properties true in additive operations and true in multiplicative operations are generally also true for division. L17 wrote, "$1 \div 10 = 10$ *and* $2 \div 10 = 5$" and said, *"Yes, 1 goes into 10 ten times and 2 goes into 10 five times"*. This misconception is caused by the assumption that since the reverse operations are true for addition and multiplication, they have to be true for division as well. Gardie and Brodie (2015) subject such to the commutative property, true for addition and multiplication, being applied in division.

The learners assumed that procedures possible in multiplication are also applicable in division. This led to the misconception that division is also commutative. Referring to Question 4.2, L30 said, *"It means 10 divide by half*', when asked about the meaning of $\frac{1}{2} \div 10$. This conforms to findings by Gardie and Brodie (2015) who found the commutative property true for addition and multiplication being applied in division.

The learners in this study also generalised and used procedures relevant to some operations in other operations where they do not fit. The study results show that the addition problem question, Question.4, was generally manipulated with viewing the whole number parts alone and the fractional parts being added separately before combining the separate answers for the final solution. The learners, however, generalised this procedure by extending it to multiplication involving mixed numbers. Table 4.3 shows that 48% of the learners generalised the addition of the mixed numbers procedure to the multiplication in Question 3.2. The fact that it is a correct procedure giving correct answers in addition might be the source of the misconception. In Question 3.2, a problem on multiplication of mixed fractions, L6 said, *"It works all round, you can add wholes/fractions separately, you can subtract wholes/fractions separately, multiply wholes/fractions like in this case and you can even divide wholes/fractions separately"*. L6 generalised by treating whole numbers separately across all operators, addition, subtraction, multiplication and division. This shows the influence of whole number knowledge on the concept fraction.

Some learners in this study used addition procedures in multiplication. For example, L15 said, *"I first found the common denominator and made the fractions to have the same denominator"*. Probably emphasis on making sure the fractions have a common denominator during instruction contributed to finding the common denominator when multiplying. This concurs with Haser and Ubuz's (2003) thinking that equalising denominators of fractions when multiplying is because the multiplication concept is not well developed or methods of addition are generalised to multiplication.

Generalising also played a part in the misconception that multiplication leads to an increase in value, and division leads to a decrease. Learners interpreted divide by one-half in Question 4.1 as divide in half, as the expectation was probably a solution smaller than eight from the whole number perspective that division makes smaller. This concurs with Bruce et al.'s (2014) idea that learners think dividing by one-half is the same as dividing in half. Table 4.3 shows that 53% of the learners had an answer less than eight confirming the learner expectation that the solution has to be smaller due to division.

For whole numbers, division reduces the value, so learners in this study had the same expectation. It was therefore not surprising that the most popular answer to Question 4.1 was four. L6 said, *"When you have 8 blocks and you divide making smaller you cut at the centre you get four-four"*. This is supported by Graeber and Tirosh (1990), who stated that beliefs that impede an accurate understanding of fraction operations include believing multiplication always results in a bigger number and division in a smaller number.

5.5.3 Presentation of fractions to learners.

Analysis revealed that learners mostly defined fractions from the perspective of part of a whole, and Table 4.3 indicates that 50% had incorrect definitions aligned to the part of a whole sub-construct. In Question 1.1, L15 said, *"a piece of something which was complete"*. This shows emphasis and bias towards the part of a whole sub-construct by educators during instruction. This might be the source of the misconception that narrowed learner scope to visualise a whole as only one item or shape being divided into a number of parts. Practical examples on pizza slices or cake pieces might have cemented the perception that fractions are only part of one unit (or whole). This is in line with Gabriel (2016), who suggests that learners always consider fractions as entities smaller than one; hence they have difficulties placing fractions on the number line.

The part of a whole sub-construct addressed the area model type of situations and learners were not exposed to the continuous model as evidenced by their failure to comprehend equivalence questions. In this study, L38 managed to represent $\frac{3}{5}$ on five blocks but had the following to say when presented with 15 blocks to represent the same fraction, *"I shaded 3 out of 5 blocks and had to use different colours to show which 5 blocks have I chosen to use"*. Seventy-three per cent (73%) of the learners showed a lack of conceptual understanding on equivalent fractions as shown by representations in Table 4.3.This shows rigidity towards a whole with denominator partitions and the belief that $\frac{3}{5}$ can only be represented on a whole with five parts. The misconception roots from the conceptual misunderstandings of equivalent fractions as learners did not relate fractions in terms of equivalence. Fractions were viewed independently as parts of one whole.

Results of the study show that the fractions were viewed as parts of a whole through the area model, with less exposure to the continuous model, and the linear model of fractions led to misconceptions. The way fractions were represented on the number line by learners displayed a lack of conceptual knowledge on the linear model. In Question 1.4, Table 4.3 shows that 43% of the learners regarded the fraction $\frac{8}{4}$ as different from 2. Conceptual challenges were also exhibited by L11 who said, *"Since the fractions have the same denominator, I arranged them according to size of numerators and filled in available spaces on the number line from left to right. As there are five fractions, so the first five spaces were filled"*. L11 did not take into cognisance the value of the fractions relative to the number line scale. The number line stretches further than one; this did not fit well with the part of a whole conception and led to misconceptions. The learners failed to link the part of a whole approach with the number line. Linking shaded parts of a whole with a linear scale proceeding further than one proved a challenge for learners. Apart from approaching fractions from the part of a whole perspective, the researcher agrees with Brown and Kim (2016) who challenges teachers to address other sub-constructs of fractions during instruction;

fraction as a quotient, fraction as a ratio, fraction as an operator and a fraction as the size of an object when a bigger object is divided into equal smaller objects. Instruction should make use of a variety of approaches or methodologies to make an appeal to a number of learning styles that different learners may possess.

The multi-faceted nature of the concept fraction on its own can be a source of misconceptions. The five sub-constructs of fractions seem many for learners to conceptualise and deal with appropriately. L6 evidently had no idea about the linear representation of fractions. In Question 1.4, L6 said, *"I looked at the top numbers, and the biggest is 10, so I had to correct the number line so that it caters for all top numbers in the list".* Adjusting the number line to make it fit the learner's conception of fraction relative to the number line suggests that the learner lacks exposure to a linear representation of fractions. This supports Moseley and Okamoto (2008) who suggest that the different meanings of fractions are not made explicit to learners, more often instruction focuses on part of a whole construct. This showcases Abosalem's (2013) concern on fractions having different meanings or not being clearly defined, which may result in misconceptions. Abosalem (2013) clarifies by adding that at some point $\frac{1}{4}$ of a chocolate bar indicates part of a whole and the next three out of four students $\frac{3}{4}$ represents part of a set.

Fraction problems which seek only computations may be a source of misconceptions because of the multi-faceted nature of fractions. The researcher agrees with Bruce et al. (2014), who state that fractions involve different procedures than those learners are familiar with in whole number multiplication or division. Contexts bring sense to the problem and eliminate some sub-constructs towards focusing on the most applicable one. In this study, learners were not able to represent $\frac{3}{5}$ when provided with 15 blocks. Contextualising and challenging learners to calculate $\frac{3}{5}$ of 15 items would have eliminated other sub-constructs of viewing $\frac{3}{5}$. The researcher agrees with Sharp and Adams (2002) as he suggests posing word problems as a means to help learners gain a better understanding of fractions.

This study also revealed that imposing rules during instruction can be a source of misconceptions. Ojose (2015) laments that when learners are armed with rules, they have no ways of self-checking sense in their solutions. This suggests that algorithms and other short cuts should be presented to learners only after a thorough coverage of the concept using seemingly longer but inductive methods. The learners attempted to use rules without success. In this study, for example, L11 just changed division to multiplication in Question 4.1, attempting the invert and multiply algorithm. L11 said, *"when dividing fractions, you change the division sign to a multiplication before multiplying normally".* L11 might have approached the problem based on procedure rather than an in-depth understanding of the fraction concept. Obviously, the learner has forgotten some important aspects of the rule due to rote learning. The researcher concurs with Alghazo and Alghazo (2017) as they asserted that learners find it hard to memorise the division procedure so tend to forget it with time. Teachers need to teach for understanding before making procedures available, which are short and give correct solutions at the expense of understanding.

The learner who successfully applied the invert and multiply principle, L30 said, *"I only know how to use it"*. This is in line with Hanson (2001), who attributes misconceptions related to imposed formulae or rules to memorising methods instead of comprehending the underlying concepts behind fractions. Although L30 got Question 4.1 correctly by applying the rule, the interview brought to surface what Skemp (1976) terms "the Devil's Advocate", a situation where educators teach in such a way that learners get correct solutions through instrumental means (knowing what to do but not why it works). The fact that learners in this study used the "memorise and recall" strategy is in line with Alghazo and Alghazo's (2007) suggestion that because fractions are complicated, learners spend more time trying to memorise the rules for calculations instead of understanding them.

Presenting fractions as complexes that call for abstract manipulations governed by rules might have led to learner misconceptions. Learners did not use the estimation skill to make sense of their solutions. For example, in Question 2.3, L15 wrote $\frac{7}{10}-\frac{2}{5}=\frac{5}{5}$. It is amazing how reducing a fraction less than a whole can result in one whole. This shows a lack of estimation skills and agrees with Siegler et al. (2010) who suggest that learners should use benchmark numbers like $0, \frac{1}{2}$, and 1 to compare fractions as learners' understanding of computational procedures can be developed.

The multifaceted nature of fractions leads to a number of ways in which they are read, at times depending on the context. Learners were using "out of", "over", "into", "divide by" and sometimes the extension –*ths* on the denominator value, for example "fifths" for denominator 5. Misconceptions resulted from some ways of reading fractions.

In this study, the way some learners read fractions became sources of misconceptions. L34 said:

"The fraction is 3 over 5, so 3 shaded parts must be over or on top of 5 shaded parts so that it is easily seen indicating $\frac{3}{5}$". The learner literally interpreted the word "over" which comes from the reading of the fraction. This falls under the same category as what Yearly and Bruce (2014) term the use of imprecise fraction language.

Another misconception resulted from using the phrase "out of" when reading fractions. L38 said: *"I shaded 3 out of 5 blocks and had to use different colours to show which 5 blocks have I chosen to use"*. Interpretation of "out of" directed the learner to consider only five blocks in Question 1.6 as the phrase stipulated the whole must have five parts. Jigyel (2007) believes "out of"" is another viable way which descriptively emphasises the part-whole relationship.

The use of the word "into" on doing fractional computations also caused misconceptions in this study. Using the word "into" made learners conclude that division holds the commutative property. L17 wrote, $1 \div 10 = 10$ and $2 \div 10 = 5$ and supported it by saying, *"Yes, 1 goes into 10 ten times and 2 goes into 10 five times"*. Reading fractions using "into" was prevalent in operations of fractions. In such calculations, simplifying fractions did not restrict division to numerator dividing denominator or denominator dividing numerator. It was done either way and seemed to give the same answer. For example; $\frac{10}{2}$ and $\frac{2}{10}$ on simplifying two into ten either way gives five. This approach, unfortunately, leads to a misconception that $10 \div 2 = 2 \div 10$ hence it portrays division as having the commutative property.

5.6 CONCLUSION

The analysis of the findings of the study indicates that learners have misconceptions when dealing with the topic of fractions. Conceptual misunderstandings contributed to several misconceptions identified in this study. The way learners perceived fractions as shown by their definitions painted a picture that fractions were not considered as part of the number line system but a way of showing part of a whole of geometric figures or similar everyday objects.

On engaging in activities involving fractions, the work was done procedurally as interviewee justifications on work they presented indicated that they had no conceptual underpinnings but utilised a step by step approach. Misconceptions were identified and the simultaneous analysis of data from the written test and the interviews led to the identification of possible causes of the misconceptions in this study.

Identification of possible causes of misconceptions informed the researcher to suggest recommendations pertaining to work on fractions.

5.7 IMPLICATIONS AND RECOMMENDATIONS

This study highlights some precautions to be taken by educators as they deliver content on the topic fractions. Borasi (1994) points out that knowledge of misconceptions that learners have on the concept of fractions helps in empowering educators to prepare and deliver lessons being informed. Precautionary measures can then be taken on delivering content on fractions.

The study also points out the importance of presenting fractions from different angles by authors to avoid indoctrinating learners onto a particular viewpoint. Over-reliance on the "part of a whole" aspect may mislead learners or narrow learner fraction spectrum, as this downplays the conception of other sub-constructs. The utilising of a variety of strategies during instruction is encouraged as this appeals to numerous learning styles held by different learners. The researcher supports Jigyel's (2007) advice that multiple contexts and fraction representations should be used to promote flexible interpretations and understanding of fractions to challenge and broaden learner understanding.

Learner feedback revealed little coherence between the part of a whole aspect and number line usage. Stephens and Pearn (2003) recommend the use of different procedures which in turn have to be inter-related. Educators have to vary activities to include use of the number line to boost the linear representation of fractions. The number line will help eradicate the misconceptions inherent in the part of a whole approach, especially the limitation to one whole. This will help learners acknowledge and accommodate fractions in their number system.

Learners treated fractions as the result of combining two numbers and evidence from the study show that the learners perceived the two numbers as still maintaining their whole number properties. They were independent in terms of operations, though written one above the other. Educators have to be cautious when naming fractions as some misconceptions may result from careless referencing to fractions.

Though the part of a whole approach to fractions is helpful, the study showed that it comes along with misconceptions if not properly supported by other approaches. For a better understanding of the fraction concept, several approaches are a necessity. Stephens and Pearn (2003) lobby for the teaching of checking strategies and asking learners to make number line representations of fractions as they contend this can assist some learners in identifying and correcting their misconceptions. Checking strategies are reflective ways introspecting into thought processes applied to deal with a particular problem. The part of a whole approach must not be limited to working with one whole only.

Educators must avoid rote learning, as learners might memorise and forget in the long run. The invert and multiply approach has to be derived inductively. Learners must use the longer way for some activities before they are guided to discover that inverting the second fraction and multiplying yields the same answer. Siegler et al. (2010) agree with this as they suggest that teachers should help learners understand the multi-step calculation, which is the basis for the invert and multiply procedure.

Freedom for calculator use also impacts on arithmetic competence. Interviewed learners blamed the non-use of calculators in the test for their below-par performance. Reliance on calculators should be restricted in the topic fractions at Grade 9 level. Authors should prefix some questions by "without use of a calculator" so that practices which started at primary school regarding fractions can be developed and eventually sustained. The "calculator dependency syndrome" impacts negatively on operations with fractions. It lowers the cognitive effort for success by learners. Alghazo and Alghazo (2017) conclude that allowing learners to use calculators has made them less eager to learn how to do calculations on fractions and gradually made learners dependent on the calculator.

Interviews showed that some learners got correct solutions by coincidence, not applying the concept correctly. This implies that not all correct answers come from the correct line of thinking. Educators must, therefore, probe learners even on getting correct solutions, checking for proper conceptualisation. Stephens and Pearn (2003) advocate the importance of uncovering whole number thinking amongst learners through interviews as such forms of thinking can give correct answers. Educators must give learners opportunities to showcase their understanding of the concept through activities like peer teaching and posing the question “Why?”.

The study revealed that misconceptions exist amongst learners in the concept of fractions. It is therefore important for educators to work towards eradicating them, directing learners to the proper conceptualisation of fractions. Stephens and Pearn (2003) suggest that incorrect fractional thinking must be dealt with explicitly. The study showed that learners bring with them experiences from everyday life which may facilitate or inhibit conceptualisation to the mathematics class. The National Research Council (2001) has it that educators should anticipate that learners will bring a form of informal knowledge acquired from their outside lives into the school. Whether for or against the formation of misconceptions, learners’ informal knowledge should be used by educators as a stepping stone towards proper conceptualisation.

A closer interest by educators into learner responses will help reveal learner misconceptions on fractions. Brodie (2014) contemplates that misconceptions should be welcome in the learning environment as they provide clues for the educator on how to direct learners onto proper conceptualisation. There may be a hierarchical development of fractional knowledge across the stages of education in the South African education system. Activities at each stage are of paramount importance and a requirement in the next educational phase. Analysis of the study revealed gaps in essential aspects of fraction knowledge which had to be dealt with at earlier stages of the education system. For instance, subjecting learners to algorithms without checking on the understanding of basics in fractions defeats the constructivist approach of accommodation (developing from what learners already know). Rote learning inevitably leads to poor relational understanding.

Some information from primary grades did not get refined along with the grades and impacted negatively to work on fractions at higher grades. Educators are therefore encouraged to come up with diagnostic means to check whether learners are in a proper state (content-wise) to engage in new material. Interventions are recommended for cases where gaps are identified to minimise misconceptions.

Textbooks should also include many word problems on fractions so that learners may contextualise and appreciate the application of fractions in everyday life. Their comprehension skills may then also improve. Concentrating more on directly given fraction number problems compels work on fractions in the abstract world. The researcher challenges future studies on fractions to work on ways to promote proper conceptualisation rather than procedural mastery when learners deal with fractions. Such an approach may also contribute greatly to the reduction of misconceptions.

5.8 CHAPTER SUMMARY

The chapter discussed the findings before recommendations were made, and a conclusion of the study was drawn. The focus was on the research questions, and respective answers were arrived at based on the analysis of the collected data. Each question served as a sub-heading of the sections stating the findings. The deliberations under these sub-headings brought forth answers around learner perceptions of the fraction concept, how work on fractions is dealt with, the misconceptions evident and the likely root causes of misconceptions about fractions. The findings informed the deliberations of the study. Most findings exhibited that the learners extended knowledge from work on whole numbers onto work on fractions even where it was not appropriate. The findings were stated before their implications and reflections of the findings were discussed.

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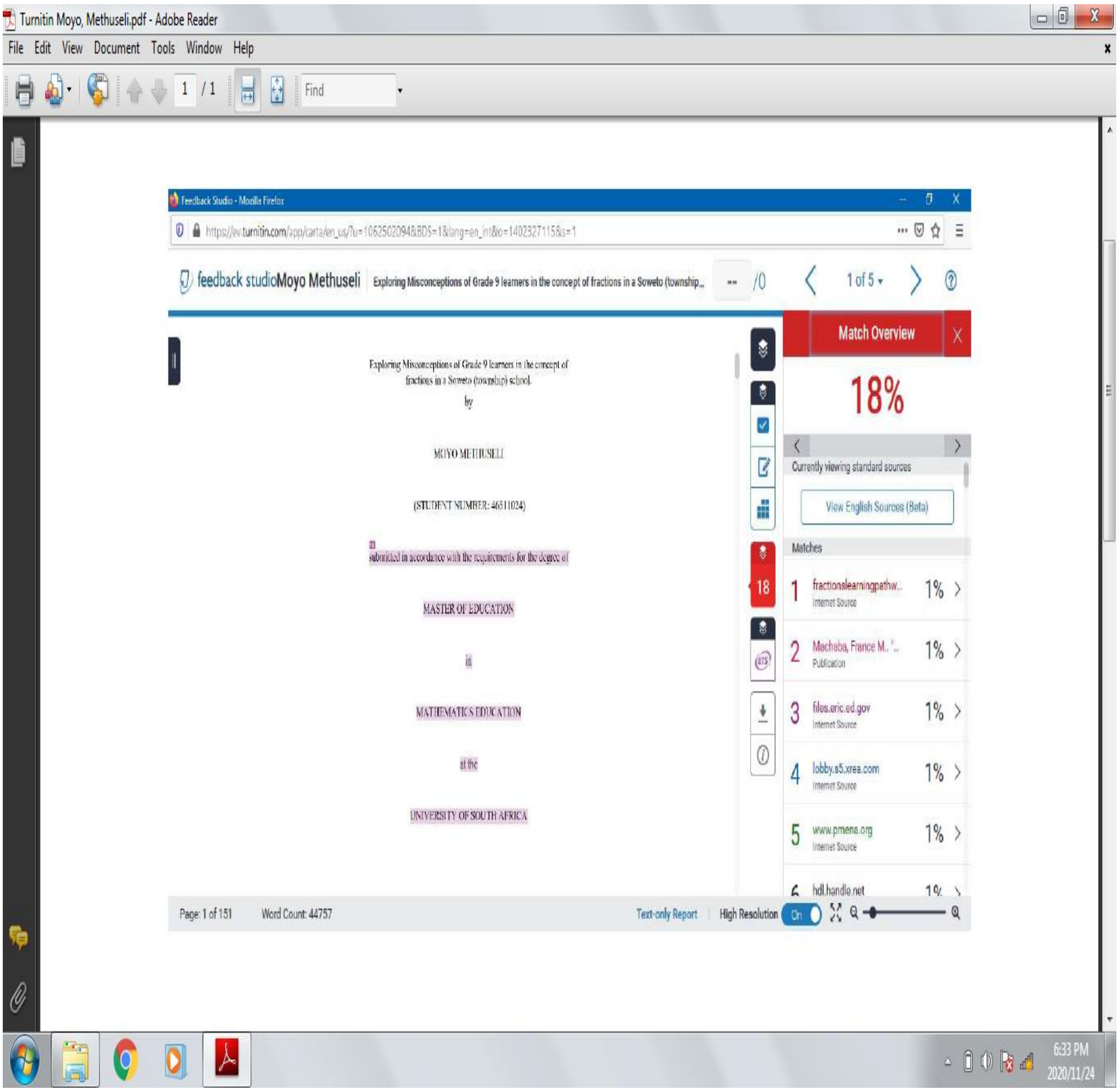
7 LIST OF APPENDICES

7.1 APPENDIX A: TURNITIN REPORT



7.2 APPENDIX B: ETHICAL CLEARANCE LETTER

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2018/11/14

Ref: **2018/11/14/46511024/43/MC**
Name: Mr M Moyo
Student: 46511024

Dear Mr Moyo

Decision: Ethics Approval from 2018/11/14 to 2021/11/14

Researcher(s): Name: Mr M Moyo
E-mail address: methuselim@yahoo.com
Telephone: +27 73 249 3160

Supervisor(s): Name: Prof MF Machaba
E-mail address: emachamf@unisa.ac.za
Telephone: +27 12 429 8582

Title of research:

Exploring the misconceptions of Grade 9 learners in the concept of fractions in Soweto township schools

Qualification: M. Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2018/11/14 to 2021/11/14.

The ***medium risk*** *application was reviewed by the Ethics Review Committee on 2018/11/14 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date **2021/11/14**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2018/11/14/46511024/43/MC** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

fp:

Prof V McKay
EXECUTIVE DEAN
Mckayvi@unisa.ac.za

7.3 APPENDIX C: LANGUAGE EDITOR'S CERTIFICATE

Member South African Translators' Institute www.language-services.online

PO Box 3172

Lyttelton South

0176

10 September 2020

TO WHOM IT MAY CONCERN

The thesis titled " Exploring Misconceptions of Grade 9 learners in the concept of fractions in a Soweto (township) school." by Moyo Methuseli has been proofread and edited for language by me.

I verify that it is ready for publication or public viewing in respect of language and style and it has been formatted as per the prescribed style of the relevant institution.

Please note that no view is expressed in respect of the subject-specific technical contents of the document or changes made after the date of this letter.

Kind regards

AMdeWet

Anna M de Wet

BA (Afrikaans, English, Classical Languages) (Cum Laude), University of Pretoria.
BA Hons ((Latin) (Cum Laude), University of Pretoria.
BA Hons (Psychology), University of Pretoria.

7.4 APPENDIX D: PERMISSION LETTER TO THE PRINCIPAL

Date ____________________

The Principal

____

____

Dear Sir/Madam,

REF: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

I, Methuseli Moyo, am doing research under supervision of Prof M. F. Machaba, an Associate Professor in the Department of Mathematics Education towards an M Ed at the University of South Africa. We are inviting grade 9 mathematics learners at your school to participate in a study entitled, _Exploring Misconceptions of Grade 9 learners in the concept of fractions in township schools.

The aim of the study is to gain understanding of how learners perceive the concept fractions in mathematics and explore misconceptions they hold. This study will employ qualitative research method. It will follow a case study design to explore the main research question: Exploring Misconceptions of Grade 9 learners in the concept of fractions in township schools.

We will be studying a grade 9 mathematics class through test and face-to-face interviewing.

This study will bring to surface misconceptions held by Grade 9s on the concept fractions. It will also bring forth the learners' general perception of the concept of fraction. The study poses no potential risks and there will be neither incentives nor reimbursements attached to the research.

If you would like to be informed of the final research findings, contact Mr M Moyo on 073 249 3160 or email methuselim@yahoo.com. Should you have any concerns about the way in which the research has been conducted, contact Prof. M F Machaba on 012 429 8582 or email emachamf@unisa.ac.za.

Yours faithfully,

Moyo M. (Mr)

7.5 E: PERMISSION FROM THE PARENT/GUARDIAN FOR THE LEARNER TO TAKE PART (IF THEY CONSENT)

Date ___________________

Dear ____________________,

I, Methuseli Moyo, am doing research under supervision of Prof M. F. Machaba, an Associate Professor in the Department of Mathematics Education towards an M Ed at the University of South Africa. Your child is kindly invited to participate in a research study entitled, ‗Exploring Misconceptions of Grade 9 learners in the concept of fractions in township schools'.

It is important for you to fully understand what is entailed in the research to enable you to make an informed decision whether to let your child participate or not. If you have any queries regarding the research study, after reading this form please do not hesitate to consult me or my supervisor on the contact details below.

The aim of the study is to gain understanding of how learners perceive the concept fractions in mathematics and explore misconceptions they hold. We will be studying a grade 9 mathematics class through test and face-to-face interviewing.

This study will bring to surface misconceptions held by Grade 9s on the concept fractions. It will also bring forth the learners' general perception of the concept of fraction. Your child will be asked to write a test (60 minutes) and if selected may be further asked for a 10-minute interview session on the topic fractions. Please note that the exercise is voluntary. The study poses no potential risks and there will be neither incentives nor reimbursements attached to the research.

All information your child provides is completely confidential. Your child's name will not appear in any publication resulting from this study and any identifying information will be omitted from the report. However, with your permission, anonymous quotations may be used. Data collected during this study will be retained on a password protected computer for 5 years in my locked office.

If you would like to be informed of the final research findings, please contact Mr M Moyo on 073 249 3160 or email methuselim@yahoo.com. Should you have concerns about the way in which the research has been conducted, you may contact Prof. M F Machaba on 012 429 8582 or email emachamf@unisa.ac.za.
Thank you for taking time to read this information sheet and for letting your chid participate in this study.

Thank you.

Moyo M. (Mr)

REPLY SLIP ✂---

I, ______________________ (participant full name), hereby agrees to participate in the in the research study entitled, ‗Exploring Misconceptions of Grade 9 learners in the concept of fractions in township schools'.

I have read and understood my role what it entails in the study. I am willing to be part of the research study.

Participant full name …………………………………………….. Signature ……………………………………… Date..……..…………………

Researcher's signature………………………………………….. Date………………………………………

7.6 F: ASSSENT LETTER TO THE LEARNER PARTICIPANT

Date ___________________

Dear ____________________,

I, Methuseli Moyo, am doing research under supervision of Prof M. F. Machaba, an Associate Professor in the Department of Mathematics Education towards an M Ed at the University of South Africa. You are kindly invited to participate in a research study entitled, ‗Exploring Misconceptions of Grade 9 learners in the concept of fractions in township schools'.

It is important for you to fully understand what is entailed in the research to enable you to make an informed decision whether to participate or not. If you have any queries regarding the research study, after reading this form please do not hesitate to consult me or my supervisor on the contact details given below.

The aim of the study is to gain understanding of how learners perceive the concept fractions in mathematics and explore misconceptions they hold. We will be studying a grade 9 mathematics class through test and face-to-face interviewing.

This study will bring to surface misconceptions held by Grade 9s on the concept fractions. It will also bring forth the learners' general perception of the concept of fraction. Your participation through subjecting yourself to test writing and an interview on the topic will be appreciated. Please note that the exercise is voluntary. The study poses no potential risks and there will be neither incentives nor reimbursements attached to the research.

All information you provide is completely confidential. Your name will not appear in any publication resulting from this study and any identifying information will be omitted from the report. However, with your permission, anonymous quotations may be used. Data collected during this study will be retained on a password protected computer for 5 years in my locked office.

If you would like to be informed of the final research findings, please contact Mr M Moyo on 073 249 3160 or email methuselim@yahoo.com. Should you have concerns about the way in which the research has been conducted, you may contact Prof. M F Machaba on 012 429 8582 or email emachamf@unisa.ac.za.
Thank you for taking time to read this information sheet and for letting your chid participate in this study.

Thank you.

Moyo M. (Mr)
I, _______________________ (participant full name), hereby agrees to participate in the in the research study entitled, ‗Exploring Misconceptions of Grade 9 learners in the concept of fractions in township schools'.

I have read and understood my role what it entails in the study. I am willing to be part of the research study.

Participant full name …………………………………………….. Signature ……………………………………… Date..……..…………………

Researcher‘s signature………………………………………….. Date………………………………………

7.7 G: PARTICIPANT INFORMATION SHEET

PARTICIPANT INFORMATION SHEET

Surname				
Full Name(s)				
Date of Birth				
Number of years in Grade 9 (tick)	1	2	Other (Specify)	
Gender *(tick)*	M ale		Female	
Race *(tick)*	African	White	Coloured	Indian
Home Language *(tick)*	SeSotho	SeTswana	IsiZulu	Other (Specify)
Are you a South African? *(tick)*			Yes	No
If not insert Nationality				
Cellular number				
Physical Address				
Mode of transport used from school to home *(tick)*	Walker	Organised transport	public transport	Other (Specify)
Parent/Guardian Information:				
Name				
Surname				
Relationship to Participant				
Cellular number				
Work number				

7.8 APPENDIX H: SELECTION TEST

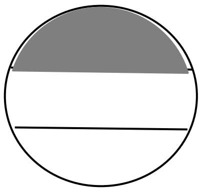
Answer all the questions

Show all your calculations where possible

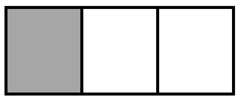
Question 1

1.1 What is a fraction?

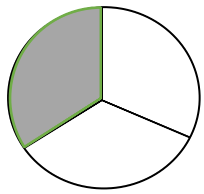
1.2 Write the fractions represented by the following diagrams



A



B



C

A ---------------------------------

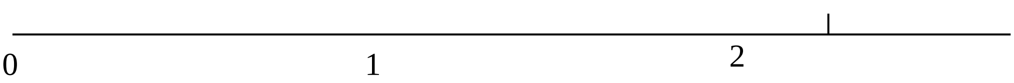
B --------------------------------

C----------------------------------

1.3 Which is bigger, $\frac{3}{4}\, or\, \frac{5}{8}\, of\, a\ given\ cake$?, support your choice by showing how you compared the two fractions.

1.4 Show the following on the number line below:

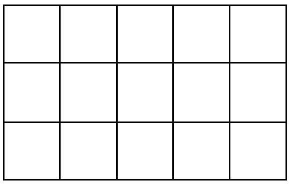
$\frac{1}{4}$; $\frac{3}{4}$; $\frac{7}{4}$; $\frac{8}{4}$; $\frac{10}{4}$



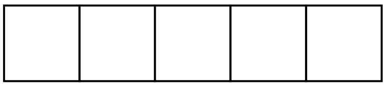
1.5 Arrange the following fractions from biggest to smallest;

$$\frac{5}{8} \quad ; \quad \frac{4}{4} \quad ; \quad \frac{2}{6} \quad ; \quad \frac{3}{7} \quad ; \quad \frac{1}{2}$$

1.6a) Shade $\frac{3}{5}$ of the rectangular mat below.



b) Write down in words what you will do to shade $\frac{3}{5}$ of the rectangular mat.

1.7  Shade $\frac{3}{5}$ of the whole

In the fraction $\frac{3}{5}$, what does the 5 mean? ___________________

What does the 3 mean? ___________________________________

Question 2

Use any strategies to find answers of the following

2.1 $\frac{1}{3} + \frac{1}{6}$

2.2 $\frac{1}{4} + \frac{3}{4}$

2.3 $\frac{7}{10} - \frac{2}{5}$

2.4 $7\frac{1}{2} + 10\frac{1}{2} + \frac{3}{4}$

Question 3

Use any strategies to find answers of the following

3.1 $\frac{1}{2} \times \frac{1}{3}$

3.2 $2\frac{1}{2} \times 3\frac{1}{5}$

Question 4

Use any strategies to find answers of the following

4.1 $8 \div \frac{1}{2}$

4.2 $\frac{1}{2} \div 10$

7.9 APPENDIX I: TEST SCRIPTS ANALYSIS TABLE OF RESULTS

Questions→ Participant code ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
L1															
L2															
L3															
L 4															
L5															
L6															
L7															
L8															
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L36															
L37															
L38															
L39															
L40															

7.10 APPENDIX J: SUMMARY OF RESULTS FROM APPENDIX C

QUESTION	Nunber of correct solutions (C)	%	Nunber of incorrect solutions (IC)	%	Nunber of partially correct solutions (PC)	%
1.1						
1.2						
1.3						
1.4						
1.5						
1.6a						
1.6b						
1.7						
2.1						
2.2						
2.3						
2.4						
3.1						
3.2						
4.1						
4.2						

7.11 APPENDIX K: TEST SCRIPTS DIAGNOSTIC ANALYSIS

education
Department: Education
GAUTENG PROVINCE

TEACHER		**TASK**		**GRADE**	

DIAGNOSTIC ANALYSIS PER QUESTION

Report on weaknesses, common errors, misconceptions, conceptual misgivings, etc.

Question 1

Question 2

Question 3
Question 4

Mention topics which were answered well.

Question
Topic
Question
Topic

Question
Topic
Question
Topic

Remedial measures/ suggestions for improvement/ interventions (what will be done to remedy weaknesses, common errors, misconceptions, etc.)

____________________________ ____________________________

Subject teacher Signature

Date

____________________________ ____________________________

HOD Signature

Date

7.11 APPENDIX L: LEARNERS' TEST SCRIPTS

L6

Maths

Question 1

1.1 A section cut from something

1.2

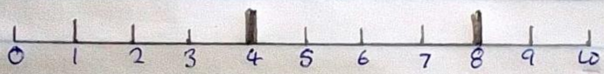
A - $\frac{1}{3}$

B - $\frac{1}{3}$

C - $\frac{1}{3}$

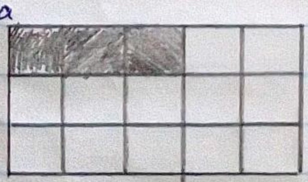
1.3 $\frac{5}{8}$

1.4



1.5 $\frac{5}{8}, \frac{4}{4}, \frac{3}{7}, \frac{2}{6}, \frac{1}{2}$

1.6 a



b) $\frac{3}{5}$ 3 is the numerator

5 is the denominator

b) You have to Shade 3 blocks in 5 blocks

1.7

Question 2

2.1 $\frac{1}{3}+\frac{1}{6}=\frac{2}{9}$

2.2 $\frac{1}{4}+\frac{3}{4}=\frac{4}{8}$

2.3 $\frac{7}{10}-\frac{2}{5}=\frac{5}{5}$

2.4 $7\frac{1}{2}+10\frac{1}{2}+\frac{3}{4}=18\frac{5}{8}$

Question 3

3.1 $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

3.2 $2\frac{1}{2}\times 3\frac{1}{5}=6\frac{1}{10}$

Question 4

4.1 $8\div\frac{1}{2}=4$

4.2 $\frac{1}{2}\div 10=20$

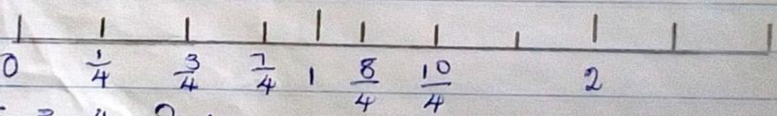
L11

Question1

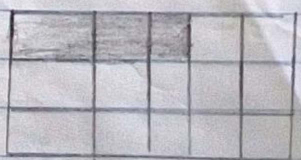
1.1 What is a fraction?
A fraction is a whole numbers that shows us How to use number for example when you buy a cake and cut it in two half that is a fraction.

1.2 (a) $\frac{1}{3}$
(b) $\frac{1}{3}$
(c) $\frac{1}{3}$

1.3 The bigger is $\frac{5}{8}$ because it is bigger one and $\frac{3}{4}$ is the Small one.

1.4 ~~a~~ 

1.5 $\frac{5}{8}, \frac{3}{7}, \frac{4}{4}, \frac{2}{6}, \frac{1}{2}$.

1.6


(b) I will Shade with and I will get halfs and give them halfs too.

1.7 

In the fraction $\frac{3}{5}$ what does the 5 mean Columns.
What does the 3 mean.
you have to colour

Question2

Question 2.

2.1 $2\frac{\times 1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$

2.2 $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$

2.3 $\frac{7}{10} - \frac{2 \times 2}{5 \times 2} = \frac{7}{10} - \frac{4}{10}$

2.4 $7\frac{1}{2} + 10\ \frac{1}{2} + \frac{3}{4} = 17\ \frac{1}{2} \times \frac{1}{2}$

Question 3

3.1 $\frac{1}{2} \times \frac{1}{3-1} = \frac{1}{2} \times \frac{1}{2}$

3.2 $2\ \frac{1}{2} \times 3\ \frac{1}{5} = 7\ \frac{1}{2} \times \frac{1}{5}$

Question 4

4.1 $\frac{8}{1} \times \frac{1}{2} = \frac{8}{2} = 4$

4.2 $\frac{1}{2} \div 10 = 20$

L15

Question 1

1.1 A fraction is a piece ~~of~~ or part of something that is whole

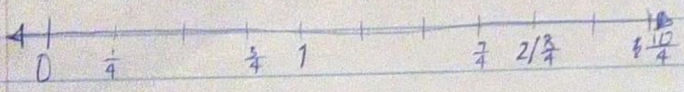
1.2. A Not the same

B. $\frac{1}{3}$

C. $\frac{1}{3}$

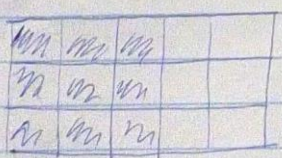
1.3. $\frac{3}{4}$ because the fraction was divided into four parts not eight making fraction $\frac{3}{4}$ bigger

1.4.



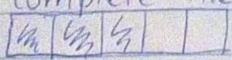
1.5. $\frac{4}{4}$; $\frac{5}{8}$; $\frac{1}{2}$; $\frac{3}{7}$; $\frac{2}{6}$.

1.6. a)



b) I will shade or colour in 9 squares to make or complete the fraction that they have given.

1.7.



1.7. a) It means that the thing that should be coloured in was divided into 5 parts. (Denominator)

b) It means that it is the number that is being shaded or devided into.

Question 2

2.1. $\frac{1}{3} + \frac{1}{6}$

$= \frac{1}{3\times 2} + \frac{1}{6}$

$= \frac{2}{6}$

2.2. $\frac{1}{4} + \frac{3}{4} = \frac{4}{8}$

2.3. $\frac{7}{10} - \frac{2}{5} = \frac{5}{5}$

2.4. $7\frac{1}{2} + 10\frac{1}{2} + \frac{3}{4}$

$= 7 + 10 = 17$

$\frac{1}{2} + \frac{1}{2} + \frac{3}{4} = \frac{5}{8}$

$= 17 + \frac{5}{8}$

$= 17\frac{5}{8}$

Question 3

3.1. $\frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{2} \times \frac{3}{3} + \frac{1}{3} \times \frac{2}{2}$

$= \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

3.2. $2 \times 3 = 6$ and $\frac{1\times 5}{2\times 5} + \frac{1\times 2}{5\times 2} = \frac{5}{10} + \frac{2}{10}$ $\therefore \frac{7}{10} + 6$

$= \frac{7}{10}$

$= 6\frac{7}{10}$

Question 4

4.1. $8 \div \frac{1}{2}$

$= \frac{8}{1} \times \frac{1}{2}$

$= \frac{8}{2}$

$= \underline{4}$

4.2. $\frac{1}{2} \div 10$

$= \frac{1}{2} \div \frac{10}{1}$

$= \frac{10}{2}$

$= \underline{5}$

L17

Question 1

1.1. A fraction is a numerator and a denominator

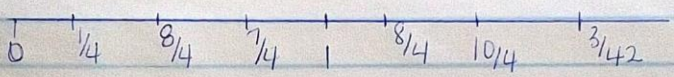
1.2. A Denominator

B. Numerator

C. Denominator

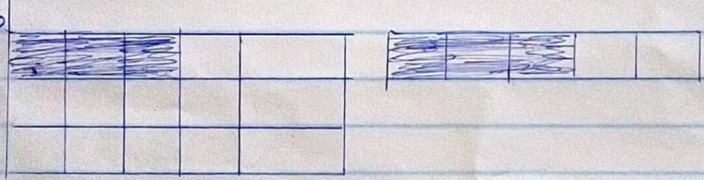
1.3. $3/4$ Because

1.4.



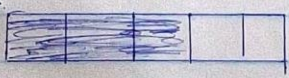
1.5. $1/2$, $4/4$, $2/6$, $3/7$, $5/8$

1.6



6. I will draw 5 mats and then I colour 3 mats and leave two that is how I shade $3/5$

1.7.



a. denominator

b. Numerator

Question 2

2.1. $1/3 + 1/6 = 2/6$

2.2. $1/4 + 3/4 = 4/4$

2.3. $7/10 - 2/5 = 5/5$

2.4. $7\frac{1}{2} + 10\frac{1}{2} + 3/4 = 17\ 5/8$

Question 3

3.1. $\frac{1}{2+1} \times \frac{1}{3} = \frac{1}{3} \times \frac{1}{3}$

3.2. $2\frac{1}{2} \times 3\frac{1}{5} = 6\frac{2}{10}$

Question 4

4.1 $8 \div \frac{1}{2}$

$= 8 \div 2 = 4$

$8 \div 1 = 8$

$4 + 8 = 12$

4.2. $2 \div 10 = 5$

$1 \div 10 = 10$

$5 + 10 = 15$

L30

Question 1.

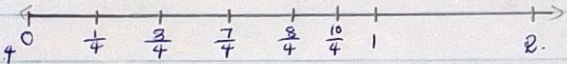
1.1. fraction is a number with a numerator and a denomenator.

1.2. a) $\frac{3}{3}$

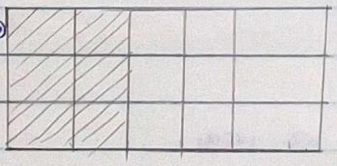
b) $\frac{1}{3}$

c) $\frac{3}{4}$

1.3. $\frac{3}{4} > \frac{5}{8}$

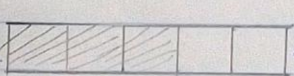
1.4 

1.5 $\frac{1}{2}$; $\frac{4}{4}$; $\frac{5}{8}$; $\frac{3}{7}$; $\frac{2}{6}$.

1.6(a)  $\frac{3}{5}$.

1.6(b) 5 mean the denomenator

3 mean the numerator.

1.7.  $\frac{3}{5}$.

Question 2.

2.1. $\frac{1}{3} + \frac{1}{6}$ $\frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$.

2.2. $\frac{1}{4} + \frac{3}{4}$ $\frac{1+3}{4} = \frac{4}{4} = 1$

2.3 $\frac{7}{10} - \frac{2}{5}$ $\frac{7-4}{10} = \frac{3}{10}$.

2.4 $7\frac{1}{2} + 10\frac{1}{2} + \frac{3}{4}$

$\frac{15}{2} + \frac{21}{2} + \frac{3}{4}$

$\frac{30 + 42 + 3}{4} = \frac{75}{4}$

Question 3.

3.1. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

3.2. $2\frac{1}{2} \times 3\frac{1}{5} = \frac{5}{2} \times \frac{16}{5}$

$\frac{55}{10} = 5,5.$

Question 4.

4.1 $\frac{1}{2} \div 8 = \frac{1}{2} \div 8 = 4$

4.2. $\frac{1}{2} \times 10 = 5.$

L38

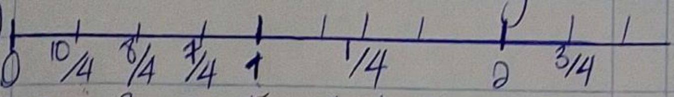
Question 1

(1.1) fraction is a certain shape that has equal sizes.

(1.2) (a) Equal faction.
(b) Proper fraction
(c) Inproper fraction

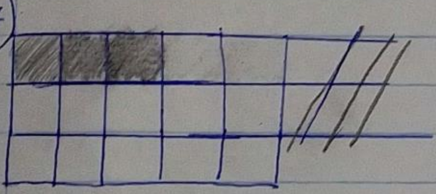
(1.3) 3/4
5/8 when we convert it in gives us unequal numbers.

(1.4)



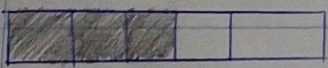
(1.5) $\cancel{\frac{5}{8}, \frac{3}{7}, \frac{2}{6}, \frac{1}{2}}$, $\frac{4}{4}, \frac{1}{2}, \frac{5}{8}, \frac{3}{7}, \frac{5}{8}$.

(1.7)



$\frac{3}{5}$

(1.8)



(a) 5 means numerator

(b) 3 means. dinominator.

Question 2.

(2.1) $\frac{1}{3} + \frac{1}{6} = \frac{6 \times 1 = 6}{3 \times 1 = 3} = \frac{3}{6} = 2 \rightarrow$

(2.2) $\frac{1}{4} + \frac{3}{4} = \frac{4 \times 1 = 4}{4 \times 3 = 12} = 3 \rightarrow$

(2.3) $\frac{7}{10} + \frac{2}{5} = \frac{7 \div 5 = 1 \text{ rem } 2.}{10 \div 2 = 5} = \frac{2}{5}.$

(2.4) $7\frac{1}{2} + 10\frac{1}{2} + \frac{3}{4} = 7\frac{1}{2} = \frac{7 \times 2 + 1}{= 15} \rightarrow \quad 10\frac{1}{2} = \frac{10 \times 2 + 1}{= 21} = \frac{1}{15} + \frac{1}{21} + \frac{3}{4}$

Question 3.

3.1 $\frac{1}{2} \times \frac{1}{3} = \frac{1+3=4}{1+2=3} = \frac{4}{3} = 12 = \frac{1}{6}$

&

3.2 $2\frac{1}{2} \times 3\frac{1}{5} = 2\frac{1}{2} = 2\times2+1 = 5$ $= 3\frac{1}{5} = 3\times5+1 = 16$ $= \frac{5}{16}$

Question 4

4.1 $8 \div \frac{1}{2} = 8\frac{1}{2} = 8\times2+1 = 17 \div 2 = 8$ rem 1. (8 ÷ 17 8 ÷ 2 = 4 $\frac{4}{8}$

4.2 [illegible] $\div \frac{10}{10} \div \frac{1}{2} = \frac{10}{5}$