A CASE STUDY: INVESTIGATING A MODEL THAT INTEGRATES DICTIONARY AND POLYGON PIECES IN TEACHING AND LEARNING OF GEOMETRY TO GRADE 8 LEARNERS.

by

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ABSTRACT

Considering that geometry is taught according to certain principles that do not encourage creativity, I have decided to employ the mixed methods philosophical framework applying the concurrent transformative design in the form of an exploratory case study. The case study to (i) explore and design a model that influences learning using polygon pieces and mathematics dictionary in the teaching and learning of geometry to grade 8 learners; (ii) investigate if the measurement of angles and sides of polygons using polygon pieces assisted by mathematics dictionary promote learners' comprehension of geometry and (iii) investigate how mathematics teachers should use polygon pieces along with mathematics dictionary to teach properties of triangles in order to promote learners' conceptual understanding.

Drawing from my research findings a model has been developed from the use of polygon pieces and mathematics dictionary. The model use of mathematics dictionary in teaching and learning geometry is to develop learners' mathematics vocabulary and terminology proficiency. Polygon pieces are to enhance the comprehension of geometric concepts.

The quantitative data emerged from marked scripts of the diagnostic and post-intervention tests, the daily reflective tests and intervention activities were analysed as percentages and presented in line and bar graphs. Qualitative data obtained from observation notes and transcribed interviews were analysed in three forms: thematically, constant comparison and keywords in context.

These findings support other research regarding the importance of using physical manipulatives with mathematics dictionary in teaching and learning geometry. They align with other findings that stress that manipulatives are critical facilitating tools for the development of mathematics concepts. The investigations led into the designing of a teaching model for the topic under study for the benefit of the mathematics community in the teaching and learning of geometry, focusing on properties of triangles. The model developed during this study adds to the relatively sparse teaching models but growing theoretical foundation of the field of mathematics.

Key terms:

Polygon pieces; physical manipulatives; teaching and learning; reflective model; influence; learners; grade 8; geometry; language difficulties, properties of triangles.

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DEDICATION

This thesis is dedicated to my immediate family for their time and resources they sacrificed to support my studies.

DECLARATION OF ORIGINALITY

I declare that "A case study: investigating the influence of the use of polygon pieces in teaching and learning of geometry to Grade 8 learners" is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.

Shakespear M E K Chiphambo August 2017

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Signature: ______________________________

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ACRONYMS AND ABBREVIATIONS

1. CAPS	: Curriculum Assessment Policy Statements
2. CDASSG	: Cognition Development and Achievement in Secondary School Geometry
3. DBE	: Department of Basic Education
4. DT	: Diagnostic Test
4. F	: Female
6. FiMs	Foebel-inspired Manipulatives
7. L	: Learner
8. M	: Male
9. MiMs	: Montessori-inspired Manipulatives
10. NCTM	: National Council of Teachers of Mathematics
11. PT	: Post-Test
12. R	: Researcher
13. RME	: Realistic Mathematics Education
14. RT	: Reflective Test
15. RDP	: Rural Development Programme
16. SA	: South Africa
16. SOLO	: Structural of Observed Learning Outcomes
17. TIMSS	: Trends in International Mathematics and Science Study
18. UNISA	: University of South Africa
19. USA	: United States of America

CHAPTER ONE: INTRODUCTION AND OVERVIEW

1.1 Introduction of the chapter

This chapter presents the summarised overview outline of this thesis.

1.2 Background of this research study

One of the subjects that enhance critical thinking among the learners in schools is mathematics. It has to be acknowledged that mathematics has got several branches and one of them is geometry. Geometry is paramount to the learners for it helps them to fully understand other topics of mathematics, that is, if properly taught from the basic level of schooling. Despite its importance, research shows that this area of mathematics is often disregarded or given minimum attention in the early years of schooling (Clements & Sarama, 2011). Failing to lay a solid foundation in the early years of schooling has a negative impact on learners later in high school mathematics.

According to Bassarear (2005), most students only identify an equilateral triangle in its standard position as a true triangle when shown in a different position it is something else. The way the triangles are introduced to the learners brings all these varied alternative conceptions regarding properties of triangles. Matthews (2005) highlights the fact that in a world of clichés and simplifications, in the disarray of the classroom the triangle in all its glory, with its many diverse properties is being deserted.

Matthews (2005) further proposes that learners should be exposed to a variety of cognitively demanding interactive and educational activities to promote conceptual understanding in geometry. In view of Matthews' (2005) proposition, it is viable to incorporate physical manipulative assisted by mathematics dictionary into the teaching and learning of geometry. Research emphasizes that physical manipulative stimulates sureness and develops spontaneous understanding of spatial situations (Jones 2002).

Mathematicsvisual symbols play an imperative part as a means of communicating mathematical concepts (Panaoura, 2014). Learners' failure to conceptually understand geometric is a result of most teachers' activities which promote rote learning instead of critical thinking (Bobis, Mulligan, Lowrie & Taplin, 1999). Rote learning disconnects most learners from mathematics (Boaler, Cathy & Confer, 2015).

According to Reddy, Visser, Winnaar, Arends, Juan, Prinsloo and Isdale (2016) to assess mathematics achievement thirty-six countries participated in Trends in International Mathematics and Science Study (TIMSS 2015) at the Grade 8 level and three countries at the Grade 9 level (Norway, Botswana and South Africa). To assess the countries' achievement TIMSS 2015 established a set of international benchmarks to assess learners' achievement in mathematics. The categories of score are grouped as: scores between 400 and 475 (achievement at a low level), scores between 475 and 550 points (achievement at an intermediate level), scores from 550 to 625 points (achievement at a high level) and scores above 625 points (achievement at an advanced level).

Internationally, the top five classified countries with almost all learners scored above 400 points of TIMSS 2015, were from East Asia–Singapore (621), the Republic of Korea (606), Chinese Taipei (599), Hong Kong SAR (594) and Japan (587). The five lowest performing countries were Botswana (391), Jordan (386), Morocco (384), South Africa (372) and Saudi Arabia (368) (Reddy et al., 2016).

The countries which out-performed others were from Asia and the Northern Hemisphere. The countries from Middle-Eastern and African were at lower level of performance. One of those countries was South Africa.

In South African (SA) context, the schooling structure is comprised of 7% independent schools and 93% public schools (categorised as fee paying and no-fee schools). Reddy et al. (2016) show that of all the South African participants, 65% attended public no-fee schools, 31% public fee-paying schools and 4% independent schools. The TIMSS 2015 scores achieved by members from these various categories were as follows: public no-fee schools 341 points, public fee-paying schools 423 and independent schools 477 points. Reddy et al. (2016) further highlight that in SA less than 20% of learners attending no-fee schools achieved a score of over 400. It is suggested that the change in mathematics performance in all other South African school types from 2011 to 2015 indicates that the no-fee schools still need the most interventions to improve their performance (Reddy et al., 2016).

1.3 The research problem

Essential to effective teaching of geometry is to help learners develop the abilities of imagining, rational thinking, insight, perception, problem solving, inferring, empirical reasoning, rational argument and evidence (Jones, 2002). The use of polygon pieces in geometry instruction promotes comprehension of geometric concepts. Also as indicated in Van Hiele, vocabulary plays a significant role in developing geometrical understanding hence the use of dictionary is similarly important in this study. This research study was therefore, done from a theoretical perspective in the form of a case study aimed at investigating the influence of integrating polygon pieces and mathematics dictionary in the teaching and learning of geometry to grade 8 learners.

The exploration and investigation conducted using polygon pieces and a mathematics dictionary paved the way to the development of an important model. The model uses: (i) polygon pieces assisted by mathematics dictionary in teaching and learning for the comprehension of geometric concepts; (ii) the mathematics dictionary in teaching and learning geometry for mathematics vocabulary and terminology proficiency.

1.4 Research questions

The study was guided by two research questions: How will the use of polygons pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning of geometry influence learners' conceptual understanding of geometry concepts, specifically properties of polygons?

How can polygons pieces with mathematics dictionary be used as physical manipulatives to influence the teaching and learning of angle measurement in geometry for learners' conceptual understanding?

1.5 Underlying assumptions that influenced the intervention

The social constructivism paradigm put forward by Vygotsky influenced the intervention that made use of polygon pieces assisted by mathematics dictionary in the teaching and learning geometry. The social constructivist paradigm views the setting in which the learning happens as fundamental to the learning itself (Vygotsky, 1929; McMahon 1997). This hypothesis adds that formal learning that takes place when the learner interacts with the environment makes much meaning because the concepts that are learnt stay for so long in the brain, what the eye sees the brain never forgets. Despite having many interpretations, constructivist paradigm implies two main goals (Cobb, 1988): (i) Learners should cultivate mathematical structures that are more multifaceted, abstract and dominant than the ones previously existed in their minds so that they can be more capable of problem solving in a wide range of situations. (ii) It adds, learners should be independent and self-motivated when dealing with mathematical problems.

By employing social constructivism paradigm this research study was allowed to: (i) investigate the influence of using polygon pieces as physical manipulatives assisted by mathematics dictionary in the teaching and learning of geometry to Grade 8 learners, specifically properties of polygons; (ii) Explore if measurement of angles and sides of polygons using polygons pieces assisted by mathematics dictionary (cut pieces of 2-dimensionals) promote learners' geometric conceptual understanding; (iii) Examine how mathematics teachers should use polygon pieces as physical manipulatives assisted by mathematics dictionary to teach properties of polygons in order to promote learners' proficiency in geometry.

1.6 Rationale

In reference to the TIMSS (2015) report that states: no-fee schools in SA still need the most interventions to improve their performance in mathematics (Reddy et al., 2016). This research study was aimed at responding to the call by developing geometry teaching and learning model that integrates polygon pieces assisted by mathematics dictionary to help in enhancing learners' conceptual understanding of geometry. The model will also help mathematics teachers globally with new methods of teaching and learning geometry to promote learners' geometric proficiency.

1.7 Conceptual framework

This research study was framed by the van Hiele's (1999) levels of geometry thinking in an intervention programme that extensively used polygon pieces and mathematics dictionary in the instruction of geometry. I investigated how the use of polygon pieces assisted by mathematics dictionary influenced the learning and instruction of geometry. It has to be learnt that the hypothesised ideologies by the van Hiele model of geometric thinking for learners' learning of geometry are as follows:

Level 0 of geometry thinking – visualization: At this level, polygons are judged according their visual characteristics where by learners may for example judge a square as not being a parallelogram.

Level 1 of geometry thinking – analysis: At this level, through reflection and testing geometric shapes' characteristics gradually emerge and then used to describe the given shape.

Level 2 of geometry thinking – abstraction: At this level, figures are well ordered. They are construed one from another. Properties are arranged chronologically when describing a certain shape.

Level 3 of geometry thinking – formal deduction: At this level a learner's rational reasoning is considered to be at an advanced level of making meaning out of the given figures. For instance the learner can prove situations with valid reasons.

Level 4 of geometry thinking – rigor: At this level, learners can make a comparison between systems based on diverse axioms and can study geometric concepts without tangible mean (p. 311).

Clements and Battista (1991) extended the levels of van Hiele by adding the pre-cognition level (level 0) to give us five levels of geometry thinking.

Van Hiele (1999) counsels that in order to ensure that there is transition from one level of geometric thinking to the next teaching and learning must be in a sequence depicted in the five-phase structure, namely:

Phase 1: Inquiry phase: In this phase, resources lead learners to discover and realise definite features of geometric figures.

Phase 2: Direct orientation: In this phase, activities are presented in such a way that their features appear steadily to the learners, i.e. through brainteasers that disclose symmetrical sections.

Phase 3: Explication: The terms are introduced and learners are encouraged to use them in their discussion and written geometry exercises.

Phase 4: Free orientation: The teacher presents a variety of activities to be done using different approaches and this instils in learners capabilities to become more skilled in what they already know.

Phase 5: Integration: Learners are given opportunities to summarise what they have acquired during instruction, possibly by creating their personal activities.

Van Hiele's (1999) framework formed the basis of my analysis when examining the effect of the intervention (which made use of polygon pieces) on learners' geometric proficiency.

1.8 Research methodology

The philosophical framework that addressed the research questions of the study is the mixed method approach in order to provide the most informative, broad, composed, and expedient study outcomes (Johnson, Onwuegbuzie & Turner, 2007).

1.9 Research design

This research study applied the concurrent transformative design in the form of an exploratory case study which allowed the employment of both quantitative and qualitative research methods to rigorously examine a distinct unit (Yin, 1981; Yin 1994).

This study's sample was from one of the section 21 secondary schools in the Eastern Cape Province of South Africa in the Queenstown district. A cohort of 56 (40 females and 16 males) eighth graders volunteered to undertake the diagnostic test. Thereafter, purposeful sampling was done in order to obtain the prolific target group (9 eighth grade learners) irrespective of the sexual characteristics (Marshall, 1996).

To ensure the authenticity of the assertions arose from the pilot study, triangulation was employed to collect data. The quantitative data emerged from the diagnostic and post-intervention tests, the daily reflective tests and intervention activities scripts. The scores of quantitative data were analysed as percentages and presented in both linear and bar graphs. It has to be observed that the observation notes, transcribed interviews and qualitative data were obtained and analysed in three forms: thematically, constant comparison and keywords-in context. The results of this research were presented in five major themes identified during the data analysis processes.

Ethical issues were taken into consideration as follows: consent to do the research was obtained from the Department of Education and the school governing body through the principal, parents and learners. The anonymity was ensured to all the parties involved in this research study.

Threats to both internal and external validity were given a special attention and minimised. Threats to internal validity constitutes: history and maturation, selection bias, mortality, implementation, the attitudes of the subjects, data collector bias and data collector characteristics. Threats to external validity includes: history effects, setting effects and construct effects. The details of how each of the identified threats were minimised are found in chapter 3 section 3.3.6.

1.10 Significance of my research study

Although the present study was based on a relatively small sized sample of learners, the in-depth exploration makes the study transferable. This study offers an important teaching and learning model to help in minimising challenges that learners face in the learning of geometry.

The findings suggest that: (i) polygon pieces assisted by mathematics dictionary have a tremendous influence on the teaching and learning of geometry to high school learners also; (ii) how teachers incorporate mathematics dictionary and polygon pieces into teaching and learning of geometry has a greater influence on high school learners' learning of geometry.

1.11 Limitations

The model suggested by this study may have better influence if they were used during normal school hours, however, this study managed to achieve this after school hours, a time when learners were tired.

During school day lessons learners were learning about exponents, a topic that demands critical application of the mind. For this reason it is possible that some learners attended the research session mentally exhausted, the condition that could hinder their normal active participation in the research session.

1.12 An overview of the research methodology and its design

This research study applied the concurrent transformative design which allowed data to be collected within a short space of time; this was relevant to this research study, as it is a case study. In the case study, employed are both quantitative and qualitative research methods to examine a distinct unit rigorously (Yin, 1981).

The research study's sample comprised nine Grade 8 learners purposely selected from the cohort of 56, based on the results of the diagnostic test. The research process was designed in five phases, namely:

Phase 1: Pilot and diagnostic test. The test was first administered to a group of 28 learners who willingly volunteered to write the test.

Phase 2: The test was later administered to nine purposefully selected learners (low, middle and high achievers) who were engaged in the research project for the entire scheduled period.

Phase 3: Design of the intervention tasks. The diagnostic test results informed the final design of the intervention activities which made use of polygon pieces in the teaching and learning of geometry. Appropriate intervention approaches were designed to address alternative conceptions that learners demonstrated in the diagnostic test.

Phase 4: Administering of intervention tasks and observations. The intervention contained activities that focussed on informal ways of identifying properties of the triangles.

An observation schedule with criteria aligned to the levels of the van Hiele's (1999) model of geometric thinking was used to observe learners engaged in the intervention activities. The whole intervention programme covered 14.4 hours. The intervention programme comprised nine activities, seven daily reflective tests and daily reflective oral sessions to emphasize key mathematical concepts.

Table 1.1: An outline of the study

Chapter one	Chapter two	Chapter three	Chapter four	Chapter five
-Introduction and overview of the study -Background of my research study -Research problem - Conceptual framework -Research process -Significance of the study -Thesis overview	- Literature review relevant to my study.	- Detailed description of my research design - Methods - Sampling and sampling techniques - Description of instruments used in collecting data	Data analysis and discussion of the findings based on data emerging from: - pilot and diagnostic tasks - intervention and post intervention tasks - observations - semi-structured interviews	- The findings and critique of the research - Key findings - Unexpected outcomes - The support from the previous research, - The contradiction of my results in relation to the previous research - The detailed explanation of my research results -Advice to the researchers and educators in the interpretation of my research findings -Suggestions of the teaching and learning model -Presentation of the implications of my research -Recommendations for future research work

Table 1.1 gives a summarised outline of my research study. Under each of the chapters are the key elements that make a particular chapter.

This section briefly highlights the following: (i) an outline of my thesis; (ii) the background of my research study in the context of teaching and learning of geometry, particularly properties of triangles using polygons pieces, (iii) the research problem and the rationale for my study, (iv) the context of my research study and its theoretical underpinnings, (v) an overview of the research methodology and its design, (vi) some limitations of the study and an overview of the thesis.

1.13 Outline of my thesis

The organisation of this thesis takes on the system of five chapters including this opening chapter.

The second chapter examines and presents the literature review relevant to my study of the influence of polygon pieces assisted by mathematics dictionary in the teaching and learning of geometry to eighth-grade learners. The research focussed on how polygon pieces assisted by mathematics dictionary could be used as physical manipulatives to promote learners' conceptual understanding of geometry (Kilpatrick, Swafford, & Findell, 2001). Furthermore, the focus is on how mathematics teachers should use polygon pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning to promote learners' mathematical proficiency in geometry particularly properties of triangles.

The third chapter presents a detailed description of the research methodology and design illustrating the devised strategies employed when conducting this research study. In addition, it also presents justification of each of the selected methods employed in conducting my research study. The following sections have been epistemologically justified:

(a) the research methodology and (b) the research design, which comprises (i) the methods used to collect data, (ii) sample selection, (iii) sampling techniques, (iv) description and advantages of the instruments used in collecting data, (v) a detailed description of how the diagnostic and post-intervention tests were developed and validated to ensure that there were of an appropriate level and relevant standard for the target group, (vi) the analysis of data, (vii) the ethical issues and (viii) research validity.

The fourth chapter is concerned with the research findings focusing on the developed model. The model responds to the two questions: (i) how will the use of polygons pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning of geometry influence learners' conceptual understanding of geometry concepts, specifically properties of polygons? (ii) How can polygons pieces be used as physical manipulatives assisted by mathematics dictionary influence the teaching and learning of angle measurement in geometry for learners' conceptual understanding?

The last chapter discusses and combines the entire thesis, putting together the numerous academic and pragmatic components in order to present the link between the identified literature, the conceptual framework and the results of my research in view of the following subheadings:

- The findings and critique of research
- Key findings
- Unexpected outcomes
- The support from the previous research
- The contradiction of my results in relation to the previous research
- The detailed explanation of my research results
- Advice to the researchers and educators in the interpretation of my research findings
- Suggestions of the teaching and learning model
- Presentation of the implications of my research
- Recommendations for future research work

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

Geometry as a branch of mathematics plays a great role in the development of individuals as well as a country as a whole. It is one of the keys that makes one attain critical thinking and it acts as a catalyst among the learners in understanding other mathematical concepts without struggle. As such, this literature review wittingly examines how polygon pieces and mathematics dictionary can be used as physical manipulatives to promote learners' theoretical understanding of geometry, in particular properties of triangles (Kilpatrick et al., 2001). Furthermore, the focus is on how learners' geometric proficiency can be enhanced through the integration of polygon pieces as physical manipulatives assisted by mathematics dictionary into the teaching and learning. Several researchers acknowledge that the use of physical manipulatives positively influence learners by affording opportunities to classify, measure, order, count and learn fractions (Prawat, 1992; Kilpatrick et al. 2001; Van de Walle, 2004; Wolfgang, Stannard & Jones, 2007; Carbonneau, Marley & Selig, 2013)

2.2 The background of geometry

Some researchers stress out that geometry as an ancient branch of mathematics, it deals with points, linear segments, surfaces, solids and how they relate to each other (Kenneth, 2004). Recently, Clements and Sarama (2011) have defined geometry as a distinct kind of mathematical language used for the conversation of fundamentally spatial ideas which range from number lines to arrays. Even computable, numerical and mathematical concepts depend on a geometric base. Socially, geometry has been a contributing factor to the development of a number of mathematical theories. In addition, geometry stimulates mathematical reasoning, promotes communication skills and creativity in learners as they are engaged in a well-structured lesson (Bankov, 2013). This implies that mathematics teachers have a challenging task to promote the geometrical skills in learners so that they become productive and competent citizens nationally and globally.

In its broad nature, geometry is the compulsory and key area of study in science fields such as nuclear physics, space science, chemistry (for the study of atom and molecule arrangements) art mechanical drawing, natural science (for cell organisation) and geology (for crystalline structure) (Sherard, 1981). Recently, researchers have shown that geometrical skills acquired at primary and high school levels are also needed in architectural design; engineering and different areas of construction sector (Alex & Mammen, 2014; Van den Heuvel – Panhuizen, Elia & Robitzsch, 2015). According to Fujita and Keith (2003) the problems learners face in learning geometry emanate from how it is taught by most mathematics teachers. Its double – folded nature (theoretically and practically) still poses a challenge to most of the learners, which results in it acting as a chasm that is very difficult to bridge. This calls for mathematics teachers to be knowledgeable, creative enough in the subject matter.

In consideration of the above real-life fields of study; it has been proposed that geometry should be of the highest priority in school curricula right from primary level (Clements & Sarama, 2011). Hence, Current research outputs show constant attention in mathematics education in general and geometry education precisely (Alex & Mammen, 2014; Moss, Hawes, Naqvi and Caswell, 2015). Van den Heuvel-Panhuizen et al. (2015) add that geometry inculcates spatial reasoning skills, which in turn develop a sense of how to imagine situations which lead to real-life problem-solving. Although in some countries for the past years geometry seemed to be less considered in the school curriculum. For example in South Africa (SA) at high school level, geometry was examined in paper three which was an optional paper for the learners and geometry teaching was optional as well.

From 1994, when the new democratic government was inaugurated in South Africa, the South African Department of Education (DoE) several times, has been working on the revolution of educational policy. In 1995, the DoE provisionally implemented a new syllabus to replace the former of the apartheid regime. Later in 1998, after revisiting and transforming the initial syllabus, Curriculum 2005 (C2005) was introduced (King 2003). The curriculum review in 2000 resulted in the release of "Draft National Curriculum Statement" (NCS) in 2001. According to King (2003) the NCS was then replaced by, a Revised National Curriculum Statement (RNCS). The introduction of RNCS in the Further Education and Training Band (FET) in 2006, excluded Euclidean from the compulsory mathematics curriculum section (Alex & Mammen, 2014). Inadequate emphasis on geometry in the mathematics curriculum from primary grades has been a longstanding issue in the field of mathematics education (Moss et al., 2015)

Despite geometry being an important branch of mathematics most learners still do not get it right (Alex & Mammen, 2014). This evidence not only raises the questions about the learning of geometry, but also raises questions about the effectiveness of the teaching and learning strategies used by teachers when engaging learners in geometric activities (Goos, Brown & Markar, 2008). For example, the study done by Van Hiele (1999) reveals that school geometry is presented based on certain principles assuming that learners think at a formal logical level, yet most of the learners lack the basic conceptual understanding about geometry (Steele, 2013).

Most of the researchers are in agreement with the fact that in most cases teaching and learning of geometry are not done as it is supposed to be done. Most teachers do not help learners to establish connections between relationships of mathematical concepts and terminology (Usiskin 1982; Mayberry, 1983; Van Hiele-Geldof, 1984; Fuys, 1985; Senk, 1985; Burger & Shaungnessy, 1986; Van Hiele, 1986; Crowely, 1987; Fuys, Geddes & Tischler, 1988; National Council of Teachers of Mathematics (NCTM), 1989; Teppo, 1991; Clements & Battista, 1992; Baynes, 1998; Prescott, Mitchelmore &White, 2002; Thirumurthy, 2003; Ubuz & Ustün, 2003; Steele 2013). Failure to balance geometrical concepts with terminology poses a challenge to most of the learners; that is why most of them ended up operating at the lowest level which is not relevant to their grade as expected by the van Hiele levels of geometric thinking. Bhagat and Chang (2015) propose that teaching and learning should allow learners to explore different geometrical figures and their properties in different orientations if it has to be effective in helping learners with geometric conceptual understanding.

Geometry is significant to everyone even a person who does not want to become a mathematician needs it in order to be able to interpret the world and make sense out of it. Research has shown that anyone who has learnt geometry well has visualisation skills, improved reasoning capabilities and is able to appreciate the creation within the surrounding (Duatepe, 2004). The implication of this is that geometric-literate individuals gain all the mentioned skills and intuitively understand the world around them and have the ability to interpret it for conceptual understanding.

2.3 Proposed strategies for teaching and learning geometry

Starcic, Coctic and Zajc (2013) propose that teaching and learning geometry is not a simple and straight forward activity; there are so many alternative conceptions that need to be clarified in order for the learners to conceptually understand geometry. For example, the emphasis should not only be on giving the meaning and obviating analysis of the properties of shapes with no emphasis on the visualisation of the shapes (Blanco, 2001). Visualisation gives a vivid picture that lasts longer in the memory and is more influential than the spoken or written precepts.

Mosvold (2008) used video data from The trends in International Mathematics and Science Study (TIMSS) (1999) that reveal the concept that in order to promote learners' curiosity in mathematical concepts, real-world examples are used in Japanese classrooms. This implies that mathematics teachers must do away with the traditional teacher-centred approaches, procedure-based and rigid ways of teaching that do not instil creativity, visualisation and mathematical conceptual development in learners (Baynes, 1998; Keiser, 1997; Mayberry, 1983 & Duatepe, 2004). It is evident that traditional teaching practices deny learners creativity and cripple learners' problem- solving skills.

However, there is a realisation at a greater scale of improving geometry achievement in schools from lower grades (NCTM, 1989; NCTM, 2000). Hence, Jones (2002) argues that geometry's high demands to our visual, aesthetic and intuitive senses, compels teachers to structure lessons in a way that promote high quality mathematics learning. For example, learners have to be engaged in practical lessons which put all the senses to task so that interpretation of the world around them becomes real and vivid. Jones (2002) further highlights that by operating imageries, learners' confidence is stimulated and spontaneous skills of understanding spatial situations are developed.

At this point, it is worth noting that when learners are only taught the routine of the skills of a particular process they become unenthusiastic to attach meaning to the notion being taught (Van de Walle, 2004). These findings resonate with Steel's (2013) findings that state, improper implementation of geometry in the classroom lead to the learners' lack of conceptual understanding in geometry, which poses many challenges to mathematics teachers in the long run. The idea of teaching geometry for conceptual understanding applies to all levels of schooling including primary school level. Conceptual understanding does not come spontaneously; it requires an instructional process that matches figural and conceptual components using specific intervention strategies and well-integrated teaching and learning resources, in this case physical manipulatives (Luria, 1976; Bussi & Frank, 2015).

The argument is that failure to engage primary school learners in worthwhile geometrical activities significantly have a negative effect on their geometric learning practices at secondary school level (NCTM, 2006). Worthwhile activities are the ones that lead learners to cognitive learning and help them make logic of geometry. However, it seems that most teachers just focus on procedural teaching and learning of geometry ignoring conceptual teaching and learning (Browning, Edson, Kimani & Aslan-Tutak, 2014). Numerous studies suggest that learners need to be engaged in activities that allow the exploration of geometry in order to acquire conceptual reasoning to promote geometric conceptual understanding (Van Hiele, 1959; Battista, 2007; Leung, 2008; Browning et al., 2014).

Research reveals that teachers have a major role to play in helping learners learn geometry with conceptual understanding (Rice, 2003). This implies that teachers should recognize that the teaching and learning of geometry should be based on realistic practical approaches and not on a bunch of axioms and formulae to be kept in the memory every day (Bankov, 2003). Forcing learners to memorise axioms and procedures gradually rob them of their imagination, creativity and argumentation skills. Learners need a high level of engagement in geometrical activities in order to conceptually understand geometry.

Since research shows that high level of learner engagement and collaboration in geometry is enhanced by the use of hands-on activities (Morgan & Sack, 2011; Cited Research Center, 2010; Starcic et al., 2013), there is a need to integrate physical manipulatives in the instruction and acquiring of geometric concepts. Researchers further explain that when practical activities are used in the teaching and learning of geometry, learners' conceptual and procedural fluency are enhanced (Kilpatrick et al., 2001). It is also recommended that teachers should understand and take into consideration that the production of knowledge cannot be separated from the wide range of external representations of geometrical knowledge which surrounds the learning learner (Sutherland, Winter & Harries, 2001). From my own experience as a mathematics teacher mathematical concepts presented abstractly are easily lost to memory, but learning by doing helps to enhance retention of the taught ideas.

This implies that geometrical concepts should be presented using multiple representations, imagination and methodological skills for learners' deep conceptual understanding of geometry (Bankov, 2013). Teachers need to know the effect of integrating physical manipulatives into the teaching and learning of mathematics, for example cutting the given shape into pieces. By cutting out the angles and sides of the figure, learning opportunities are created for learners to conceptually understand the properties of the given figure before the use of protractors or even before the use of symbols that define a particular figure (Koyuncu, Akyuz,& Cakiroglu, 2015). Conceptual understanding refers to the ability to use various strategies in presenting mathematical ideas (Kilpatrick et al , 2001). For the empirical reasons stated, my study made use of polygon pieces physical in the teaching and learning of geometry, specifically properties of triangles.

2.4 Definition of physical manipulatives

According to Heddens (1986), Sowell (1989), Moyer (2001) and Van de Walle (2004), the term manipulative refers to concrete materials, real objects, images or drawings onto which a mathematical concept can be imposed in order to clarify the real concept.

Zuckerman, Arida and Resnick (2005) further define manipulatives as physical objects that are mainly designed to foster learning in a teaching and learning environment. Kilpatrick et al. (2001) used the following terms interchangeably: physical (concrete) materials or physical models or manipulatives. For the sake of consistency, in this study, the term physical manipulatives is to be used interchangeably with polygon pieces.

2.5 The history of physical manipulatives use

Research shows that the use of physical manipulatives started with the use of pebbles and abacuses which are still used today in many countries to teach place value (Gifford, Back & Griffins, 2015). Later in the nineteenth century, Froebel used the wooden rods to represent numbers up to 12, base ten, odd and even numbers this was the time when physical manipulatives were presented as structured materials in education. In the twentieth century, Montessori, Cuisenaire and Stern developed overlapping cards to teach the place value concept of numbers.

Over time the use of physical manipulatives waxed and diminished in Europe and North America due to the dominant mathematical theories that emerged at that time. For example, United States of America progressives, Dewey (1938) and Kilpatrick et al. (2001) considered Montessori's approach as too structured and ridged to be used in teaching and learning mathematics (Gutek, 2004). In the 1970s, the Netherlands, Realistic Mathematics Education (RME), placed emphasis on the use of diagrammatic models in teaching problem-solving in mathematics (Streefland, 1991).

Recently, the English government (in England) has decided to use Singapore mathematics textbooks to promote a Bruinerian concrete-pictorial-abstract approach in teaching and learning to improve learners' conceptual understanding in mathematics (Gifford, et al., 2015). Over the years, most of the researchers have reported many advantages of using concrete manipulatives in education when teaching and learning mathematics over their disadvantages (Ball, 1992; Moyer 2001; Van de Walle, 2004; Bankov, 2013; Paparistodemous, Potari & Pita – Pantazi, 2013; Carbonneau et al., 2013; Bhagat & Chang, 2014; Gifford et al., 2015).

2.6 Classification of physical manipulatives

Physical manipulatives come in different forms such as real-life objects, drawings and computer-operated objects. Zuckerman, et al. (2005) broaden the perspective by classifying manipulatives into (i) Foebel-inspired Manipulatives (FiMs) which were used to promote the modeling of real-life configurations. For instance, they used blocks made of wood to build a structure that was in the form of a castle, (ii) Montessori-inspired Manipulatives (MiMs) were used solely to instill the skill of modeling, which focused mainly on more mathematically intangible structures, for instance, Cuisenaire bars were arranged in diverse patterns that make mathematical quantities. MiMs appear in both forms: physical oriented or digital.

The MiMs that are in a digital form are the products of the physical ones. The computerised MiMs work in a form of simulations if they are to represent a certain concept (Zuckerman et al., 2005). Although the digital manipulatives are beneficial to the instruction of mathematics, they are not the area of attention for my research study. The focus is on physical manipulatives assisted by mathematics dictionary for the reason that they are cost effective and easily accessible even to rural schools that cannot access digital utilities.

Moyer, Bolyard and Spikell (2002) describe two main categories of manipulatives as concrete and virtual. It is argued that virtual manipulatives are either static or dynamic; static are visual representations of concrete manipulatives, for example, drawings and sketches. Dynamic visual representations are visual images on the computer that can be manipulated, they also represent concrete manipulatives.

According to Sowell (1989), from olden times, in different civilization physical manipulatives have been used to help them solve daily mathematical situations, for example, the Middle-East used counting boards; the Romans adapted the counting board to produce the world's first abacus, which was advanced by the Chinese and the Americans. The Mayans and the Aztecs both used corn kernels for counting. The Incas used knotted string called quipu. The uses of manipulative materials were also included in the activity curricular of the 1930s just to enhance the teaching and learning of mathematics.

In addition, Seefeldt and Wasik (2006:7) describe that physical manipulatives can be selected from household objects or purchased from the shops, for example "unifix cubes, counters, calculators, toothpicks, pattern blocks, bottle tops, skittles, base-ten blocks, coins, etc."

2.7 The use of physical manipulatives assisted by mathematics dictionary in the teaching of mathematics

Research regarding the use and integration of physical manipulatives in the instruction of mathematics gives us mixed outcomes. Fennema (1972) argues that physical manipulatives only benefit the learners at the entry level of school not those in high school. Suydam and Higgins (1997) report that physical manipulatives seem to benefit learners of all ages provided they are well incorporated into teaching and learning. This implies that there must be a decisive way of incorporating physical manipulatives assisted by mathematics dictionaryinto teaching and learning, not just making them available. In addition, mathematics teachers must play a major role in planning the activities that go together with the use of physical manipulatives assisted by mathematics dictionary to promote teaching and learning of mathematics.

The NCTM (2000) noted that the use of physical manipulatives is not only relevant to one specific mathematical topic and a particular level of learners. However, they can be used in teaching mathematical concepts to any grade in different topics. For example, topics such as categorising, ordering, distinguishing patterns, recognizing geometric shapes and understanding relationships among them, proportionality, place value, algebra, geometry, probability, exploring and relating spatial relationships, engaging in problem solving, learning about and investigating with transformations, the list goes on.

Many research studies show that learners in different grades who were taught geometry with physical manipulatives performed better in measures of retention and application than their counterparts who were taught without or with textbooks only (Prigge, 1978; Threadgill-Sowder & Juilfs, 1980; Olkun, 2003; Steen, Brooks & Lyon, 2006; Yuan, Lee & Wang, 2010; Carbonneau et al., 2013).

Another astounding performance has been reported in the use of physical manipulatives to teach fractions to primary school learners as compared to those used textbooks (Miller, 1964; Jordan, Miller& Mercer, 1999; Cramer, Post & delMas, 2002; Butler, Miller, Crehan, Babbit& Pierce, 2003; Witzel, Mercer & Miller 2003; Suh& Moyer, 2007; Gürbüz, 2010). Although most results of the research studies seem to be in favour of the use of physical manipulatives, it has been discovered that two groups taught fractions: one with the use of physical manipulatives and the other without them. The two groups performed the same on the measure of retention (Shoecraft, 1971; King, 1976; Robinson, 1978; Nishida, 2007).

There have been several research studies on the use of physical manipulatives to teach arithmetic to learners and they seemed to produce three different results. The first group of researchers reported that learners who were taught with physical manipulatives performed better on the measure of retention and transfer than without physical manipulatives (Aurich, 1963; Lucow, 1964; Carmody 1970; Wallace, 1974; Paolini, 1977). The second group of researchers, conducted individual research studies on the effect of using physical manipulatives in the teaching and learning arithmetic report those learners who used physical manipulatives achieved the same as those who used the textbook only (Nasca, 1966; Cook, 1967; Ekman, 1967; Weber, 1970; Nickel, 1971; Kuhfitting, 1974; Babb, 1975; Slaughter, 1980; Garcia, 2004; Battle, 2007). Lastly, according to Fennema (1972) and Egan (1990) learners who used physical manipulatives performed worse in a measure of retention and transfer than learners taught with the textbooks.

These results are consistent with recent research that revealed that physical manipulatives increase scores on retention and comprehension of geometrical concepts in learners (Gürbüz, 2010; Starcic et al., 2013). Feza and Webb (2005) note that teachers' ways of presenting geometrical concepts may be misunderstood by learners who are at the van Hiele low level of geometric thinking as compared to their expected grade level. In such cases physical manipulatives need to be the medium used to presenting geometrical concepts.

Another example from one of the studies regarding the use of physical manipulatives, TIMSS (2003), reveal that American grade 8 learners who were mostly taught procedural-based lessons scored lower in the mathematics test than 12 of the 23 schools that participated in the study. Recent 55 studies on the comparison of the influence of teaching and learning geometry using physical manipulatives to abstract teaching, the results favoured the use of physical manipulatives (Carbonneau et al., 2013).This study incorporates physical manipulatives assisted by mathematics dictionary in the teaching and learning of geometry.

Physical manipulatives assisted by mathematics dictionary can be used to complement and enhance teacher practice. Clements and Bright (2003) further describe that, with the use of physical manipulatives the instruction can change from the traditional that focuses on the end result to the one that focuses on the method of how to reach the expected solution. Instruction that uses physical manipulatives inevitably focuses on what it means to draw a triangle, without focusing much on the definition of the term ‘triangle’. Therefore, it enables students and teachers to represent abstract concepts as a reality in the mathematics class which links mathematical concepts to the previous knowledge of the same concepts (Cited Research Center, 2010).

This means that the teaching and learning of geometry cannot be done successfully without the use of physical manipulatives assisted by mathematics dictionary that promote effective learning of mathematics (Van de Walle, 2004; Sherman & Bisanz, 2009; Gürbüz, 2010). For example, according to the research findings by Alex and Mammen (2014),the twelfth-grade learners in some of the South African schools, geometrically, are still operating at concrete and visual levels of Van Hiele’ s theory, yet they are supposed to deal with level 3 van Hiele geometric thinking concepts. This suggests that physical manipulatives also need to be used at high school level, because empirically, research has shown that children who used physical manipulatives outperformed those who did not (Clements, 1999). The gap that has been identified is that, most studies do not specify the type of physical manipulatives to incorporate into the teaching and learning of geometry and how to incorporate them. Also they do not incorporate physical manipulatives assisted by mathematics dictionary in the teaching and learning of geometry.

These apparent contradictions regarding the use of physical manipulatives are due to methodical instructional factors like: (i) the extent to which learners were guided in the use of physical manipulatives, (ii) the nature of physical manipulatives used for instruction and imparting mathematical knowledge, (iii) the age of learners, (iv) the characteristics of teaching and learning environment (Carbonneau et al., 2013). Bryant, Bryant, Kethley, Kim and Pool (2008) dispute the third point; they argue that physical manipulatives are for every learner, regardless of the age.

As described above, Van Hiele (1999) draws a parallel with the notion that conceptual development is more inclined to teaching strategies than biological factors (i.e. age). Effective teaching and learning strategies that promote conceptual development from one level to the other should have a series of activities. Activities should range from exploration to gradually building concepts and appropriate mathematical language related to what learners already know about the topic. It is my premise that the incorporation of physical manipulatives assisted by mathematics dictionary is ideal for instruction and imparting of geometric knowledge.

In addition, polygon pieces assisted by mathematics dictionary allow self-exploratory learning and creativity that avoid telling method of teaching and learning. It is based on these premises that this research project will focus on investigating the influence of using cut pieces of polygons as physical manipulatives in the teaching and learning of geometry. Furthermore, the focus is on how the polygon pieces assisted by mathematics dictionary can be used as physical manipulatives to teach properties of the same polygon in order to promote learners' conceptual understanding in geometry. I call my teaching: the use of a triangle to teach properties of the same triangle.

Mathematics has a terminology that contains of words and symbols that permit people to have a shared base of understanding of mathematical concepts (Patterson & Young, 2013). This implies that the use of mathematics dictionary cannot be divorced from the teaching and learning of geometry. Patterson and Young, (2013) suggests that the use of mathematics dictionary help learners to develop understanding of mathematical terminology and their exact connotations. Research shows that the understanding of mathematics language and terminology pose unique challenges that are different from regular reading conditions (Lamb, 1980; Lamberg & Lamb, 1980). It is believed that the use of mathematics dictionary encourages active participation of the students in mathematics lesson (Vesel J & Robillard 2013).

2.8 Suggestions on useful ways of using physical manipulatives

Although physical manipulatives promote teaching and learning, it is important for teachers to note that they are not the only factors that help a learner understand the mathematical concepts they represent (Ball, 1992; Moyer, 2001; Van de Walle, 2004; Bankov, 2013).

It has been pointed out that teachers should know that physical manipulatives do not automatically provide mathematical meaning for the learners. Well-structured guidance to exploration and visualisation is needed for the learners to develop conceptual understanding of what the physical manipulatives represent (Moyer 2001; Bhagat & Chang, 2014).

The effective use of physical manipulatives assisted by mathematics dictionary is intricately linked with good teaching practice. Clements (1999) argues that good teaching practices entail teachers' guidance of learners in the use of physical manipulatives in the setting of instructive activities to actively involve them in worthwhile geometric learning to promote conceptual understanding. Clements (1999) further suggests that teachers need to know that the use of physical manipulatives in instruction and imparting geometric knowledge is to construct what is known as 'Integrated-concrete ideas' that support learners in the production of significant thoughts. For this reason physical manipulatives should be used before the introduction of geometric formal symbolic instruction such as teaching definitions and axioms.

Van de Walle (2004) argues that it is quite challenging for both educators and learners to guide and perceive mathematical notions in physical manipulatives. The challenges are to: (i) construe students' illustrations of their mathematical thinking, (ii) conceal and represent relationships among mathematical ideas and (iii) develop relevant concrete contexts for learning mathematical concepts. This implies that there are certain limits in the development of geometrical concepts if physical manipulatives are not well – integrated into the lesson (Paparistodemous et al., 2013).

Researchers' results on the need for physical manipulatives in geometry instruction and learning reveal that teachers have to be proficient in integrating physical manipulatives into their teaching. According to Kilpatrick et al. (2001), proficiency is related to effectiveness, to regularly help learners learn worthwhile mathematical content knowledge. Worthwhile mathematical content knowledge refers to activities that focus on directing learners' attention not only on specific skills, but also on empowering the learners with the abilities to process facts by giving evidence (Lester, 2003). Researchers argue that if there are no proper instructions written as guidelines on what to do over the use of physical manipulatives, learners may just have amusement instead of using them for the intended effective learning (McNeils, 2007; Ogg, 2010).

Suydam (1984) and Heddens (1986) suggest that to help learners in transition from one phase of learning to another, physical manipulatives from real-world settings are used to represent mathematical concepts in a way that can be more simplified than without them. This implies that the physical manipulatives help learners to be attached to the real-world conditions which ultimately provide opportunities of worthwhile learning of geometry (Chester, David & Reglin, 1991). Such teaching and learning approaches provide learners with logical settings for abstract advancement in geometry (Cai, 2003).

Prawat (1992) adds that learners' conceptual development is linked to the use of physical manipulatives, therefore, the incorporation of physical manipulatives into the teaching and learning of geometry is an essential condition for worthwhile learning. Van de Walle (2004) claims that the use of physical manipulatives is essential in the development of new concepts in learners. Physical manipulatives allow learners to think and reflect on new ideas that emerge during the justification of mathematical reasoning (Heddens, 1997). Gentner and Ratterman (1991) note that such extensive instruction and practice provide learners' with opportunities to observe and understand relationships between physical manipulatives and supplementary arrangements of geometric expressions.

The use of physical manipulatives in the instruction and practice of geometry, directs teachers to use open-ended activities that appeal to several of the learners' senses such as, touching, pictorial, auditory, etcetera. Such activities help the reduction of errors made by learners and the maximisation of opportunities to improve their scores for tests that focus on problem-solving and investigation (Carrol & Porter, 1997; Clements, 1999; Sebesta & Martin, 2004).

Researchers note that physical manipulatives stimulate a child-centred lesson where the former learning experiences are challenged, rather than the promotion and over-emphasising of rote learning of concepts. The conceptual understanding promoted by a learner-centred teaching and learning approach is developed from well-grounded ideas through exploration. The Cited Research Centre (2010) further argues that the challenging of former learning experiences fosters more detailed and richer conceptual understanding of geometry (Huang & Witz, 2010). The implication here is that physical manipulatives need to be used in order for the learners to have a well-developed conceptual understanding of geometry.

Physical manipulatives create conducive learning environment that allows learners to reflect on their past and present learning experiences (Cain-Caston, 1996; Heuser, 2000). When learners use physical manipulatives to reflect on their own learning have the opportunity to access geometric concepts that were inaccessible during the lesson (Uttal, Scudder & Deloache, 1997) For this reason, this research views physical manipulatives as objects that add support as mathematical lessons develop from known to unknown concepts (Papert, 1980).

According to Boggan, Harper and Whitemire (2007), if physical manipulatives are used for the reflection of previously learnt mathematics concepts, they have potential to improve learners' short-and long-term retention. Furthermore, other research adds that physical manipulatives are to be used as a means to improve learners' achievement of all levels. The learners' levels include a wide range of abilities in teaching and learning of geometry from slow learners to the gifted ones (Peterson, Mercer& O'Shea, 1998).

According to Uttal et al. (1997), when used effectively, physical manipulatives demystify the meaning of different mathematical symbols and concepts. For example, learners are given the opportunity to develop new geometric conceptions, create links between concepts and symbols and evaluate their conceptual understanding of the concepts being presented (Van de Walle, 2004).

On the other hand, researchers argue that physical manipulatives give the teacher an opportunity to present learners with resources, conditions and skills that allow them to discern learning. Unlike in the traditional method of teaching where no learning behind the scenes is promoted, except rote learning that leaves learners without conceptual understanding (Wearne& Hiebert, 1992).

Other researchers also propose that when learners use physical manipulatives, they acquire numerous skills and abilities that are easily retained and used in future as the need arise at the same time, learners also acquire skills like counting skills, computational skills, problem-solving skills and the ability to present one concept in two or many different ways (Carrol & Porter, 1997; Krach, 1998; Jordan et al., 1998; Clements, 1999; Chappel & Strutchens, 2001; Sebesta & Martin 2004). All these skills and abilities help learners to see the relationship that exists between mathematical concepts within the topic and between concepts in different topics.

Grouwns (1992), Cain-Caston (1996) and Heuser (2000) also comment that the use of physical manipulatives draws learners' curiosity in learning geometry and eliminates anxiety towards mathematics. Anxiety is eliminated when learners develop their own conceptual understanding of geometric concepts through the use of physical manipulatives (Vinson 2001).

Van de Walle (2004) claims that allowing learners to engage and participate in their own learning using physical manipulatives is an imperative motivational force in effective learning, which helps learners to perceive mathematics learning as worthwhile. Moyer (2001) suggests that learners should use physical manipulatives to reflect on their own actions in the process of learning mathematics.

Research done regarding the instruction and practice of mathematics using physical manipulatives reveals that learners who were engaged in the use of physical manipulatives to learn mathematics outperformed others who learnt without it (Driscoll, 1983; Greabell, 1978; Raphael & Wahlstrom, 1989; Sowell 1989; Uttal et al., 1997). This ratifies that physical manipulatives promote leaners' curiosity and positive attitude towards mathematics learning.

Suydam (1984) counsels that physical manipulatives must be well integrated into the lesson in order to help learners reason, solve problems, be imaginative of the idea behind the symbols and for easy communication with other learners as the lesson progresses. For example, to develop the conceptual understanding of the properties an isosceles triangle, learners should be engaged in an investigation in order to conceptualise the idea of properties of an isosceles triangle.

Lesson planning plays a major role in the effective use of physical manipulatives assisted by mathematics dictionary. According to Resnick and Omanson (1987),Wearne and Hiebert (1988),Fuson and Briars (1990) and Ball (1992), well-planned instruction and practice are required before employing a variety of physical manipulatives that cater for learners with diverse mathematical learning abilities. Van de Walle (2004) argues that teachers should not communicate with learners on how to use physical manipulatives, but let learners do self-exploration of the mathematical concepts being represented by physical manipulatives. The investigation way of teaching and learning can help learners link several ideas and being able to integrate their knowledge to gain a deeper conceptual understanding of the mathematics topic being presented (Suydam, 1984).

Steedly (2008) suggests that teachers should incorporate physical manipulatives into teaching and learning by using a special teaching method known as 'Concrete Representational Abstract (CRA)' which is a three-segmented instructional strategy. In the first step, the teacher must use concrete material to represent mathematical concepts to be learnt, the second step is to demonstrate the concept in representational verbal form, and, finally, in abstract or symbolic form. From my own knowledge, some teachers always rush learners into the use of physical manipulatives and do not allow them time to comprehend and have full appreciation of the mathematical concept imposed on physical manipulatives. As a result, learners' external actions would not always be in line with the activities intended by the teacher (Clements, 1999). It is clear that the acquisition of mathematical concepts has to be guided by a well – structured instruction that makes use of physical manipulatives.

To counsel on well-structured instruction, Resnick and Omanson (1987),Wearne and Hiebert (1988),Fuson and Briars (1990) and Ball (1992) suggest that wide-ranging instruction and practice are vital before physical manipulatives can be employed in mathematical teaching and learning. For example, the teacher must plan the lesson which accommodates learners with different learning abilities. This implies that polygon pieces should be used as physical manipulatives in teaching and learning of geometry.

2.9 Theoretical framework

My research study is framed by the van Hiele levels of geometric thinking in an intervention programme that makes extensive practice of physical manipulatives assisted by mathematics dictionary in the instruction and practice of geometry. I investigated how the use of polygon pieces assisted by mathematics dictionary influences on the learning and teaching of geometry by using van Hiele levels of thinking in the teaching and learning of geometry.

According to the van Hiele levels of geometric thinking, level 0-visualisation. At this level, learners are expected to describe figures by using their physical appearance. This implies defining a figure as a whole without breaking it down into various features, for example a square is not a parallelogram. To expound more on what each of the van Hiele levels of geometric thinking entails Crowley (1987) elaborates that at this level learners are engaged in hands-on-activities which require them to manipulate, shade, fold, design and measure geometric shapes. Learners use constructed geometric figures to identify related and different orientations in a given figure. In addition, they also make and orally define shapes using appropriate standard and non – standard language, for example, a rectangle looks like a door.

Level 1-analysis: Van Hiele (1999) proposes that at this level, learners are supposed to use distinct features to define figures, for example, a rhombus is a parallelogram. To explain this further, Crowley (1987) suggests that at this level, learners are provided with opportunities to use relevant properties to identify, classify, order and describe given shapes. Properties of shapes are explored by tiling in order to differentiate figures and ascertain more features that can be used to identify and categorise a certain figure. In addition, learners are supposed to formally use the language of mathematics, for example a square has four equal sides, two equal diagonals bisect each other and are perpendicular to each other, and it has four right angles.

Level 2-abstraction: Learners must logically classify figures using their properties which are construed one from another in an orderly way. The ordering of properties is done in a way that can be easily understood and remembered later when the need arises. Crowley (1987) adds that at this level learners begin to form systems of ideas that are related to each other regarding properties of shapes. Furthermore, learners must clearly make meaning out of definitions of geometric shapes in order to provide arguments based on well-supported steps. Learners are supposed to use more than one explanation to justify a certain situation, for example a rectangle has four sides, two pairs of sides parallel, each one of the angles is equal to 90^0 and diagonals bisect each other. In other words, at this level inclusion plays a vital role.

Level 3 – formal deduction: At this level, thinking is concerned with conceptually understanding the meaning of mathematical definitions and proving unfamiliar theorems.

According to Crowley (1987), at level 3 of the van Hiele theory, learners must have conceptual understanding of theorems, postulates and axioms. Furthermore, they are supposed to use a variety of skills of proofs to prove certain situations. In addition learners should be able to derive the ways of proving based on the given information. Learners have to "think about geometric thinking" (Crowley, 1987: 9) in order to be able to perform proofs and give meaning to a given mathematical situation.

Level 4-rigor: Learners are supposed to use a variety of axioms to compare geometric systems. Learners at this level are also expected to deal with abstract concepts in defining mathematical situations.

Clements and Battista (1991) extended the levels of Van Hiele by adding pre-cognition level (level 0), which is going to be included in this study to give us five levels of geometric thinking. At this level, the researchers claim that learners cannot distinguish a circle from quadrilateral or from a triangle without being given the images of reference (Clements & Battista, 1991; Clements, Swanimatha, Hannibal & Sarama, 1999).

Van Hiele (1999) counsels that in order to ensure that there is smooth movement from one level of geometric thinking to the subsequent level, teaching and learning must follow a five-phase structure of activities, namely: Phase 1: Inquiry phase – in this phase, learners use physical manipulatives to discover the characteristics of the geometric figures under investigation. Phase 2: Direct orientation – in this phase, learners are engaged in activities that have some guiding statements to the solution, for example, matching the given items with the appropriate definitions.

Phase 3: Explication – in this phase, the use of geometric terms becomes prominent, learners are encouraged to use geometric terms in both verbal and written form for geometric proficiency. Phase 4: – free orientation – in this phase, different activities are given to the learners in which they are required to respond to each of the activities in more than one way. This is done in order to promote learners' conceptual understanding in geometry. Phase 5: Integration – in this phase learners are given opportunities to reflect on the previous activities and are asked to design their own activities and provide solutions to the designed problems, this is for the consolidation of what has been learnt in the past.

This study is very important because the literature highlights that there is poor performance in geometry for the following reasons: (i) failure to present geometry to enhance learners' conceptual understanding, (ii) teachers' insufficient knowledge in teaching the concepts (Kelly, 2006; Bankov, 2013). For these reasons, I have to come up with teaching and learning models that are can help learners to learn geometry as well as to empower mathematics teachers in the teaching and learning of geometry.

2.10 Research into the van Hiele levels of geometric thinking

Khembo (2011) investigated the sixth-grade teachers' understanding of geometry based on the van Hielelevels of geometric thinking model. The outcomes reveal that most teachers operate at a lower level of the van Hiele levels of geometric thinking than expected. On the other hand, Usiskin (1982) argues that primary school learners are supposed to operate at the first two levels of geometric understanding in the van Hiele model and teachers should not be at those levels. This poses a challenge to the type of geometrical knowledge imparted to primary school learners. However, researchers suggest that teacher education should take into consideration the van Hiele geometric thinking model when developing and rectifying teachers' geometric alternative conceptions (Khembo, 2011). Researchers argue that if van

Hiele's theory is properly implemented, it helps teachers to make their pedagogical tasks as clear as possible to develop learners' understanding of geometrical concepts (Bankov, 2013).

For three years, Burger and Shaughnessy (1986) conducted research into the description of the van Hiele levels of thinking in geometry, particularly using clinical interview tasks that were based on triangles and quadrilaterals. The study involved 13 learners from the first grade to the twelfth grade and a university student majoring in mathematics. According to Burger and Shaughnessy (1986), the intention of the research study was to describe learners' thinking processes, categorise learners' behaviour and the use of manipulatives to identify main levels of reasoning in relation to van Hiele's model of geometric thinking.

Burger and Shaughnessy (1986) participants were given experimental tasks in an audiotape clinical interviews that were conducted in rooms that were only occupied by the interviewer for a session of 40 to 90 minutes. These interviews involved tasks on geometry, drawing, identifying, defining and sorting of shapes, comparing geometric shapes and describing the properties of parallelograms. The participants were also engaged in both informal and formal reasoning about geometric shapes. According to Burger and Shaughnessy (1986), diagram sketching, identifying and sorting were used to obtain data about van Hiele's levels 0 to 2. An inference game and questions based on axioms and poofs were used with the intention to obtain data about level 2(abstraction) and level 3(deduction) of the van Hiele levels of geometric thinking.

Burger and Shaughnessy's (1986) research study discovered that the van Hiele levels of geometric thinking are useful in describing learners' thinking on the activities based on polygons. They also discovered that most of the learners are not strongly grounded in basic concepts of Euclidean Geometry; seemingly, rote learning might be the cause of deficit in geometry conceptual understanding. The college student revealed some axiomatic thinking level in geometry, but none at high school level demonstrated this level.

In another case, Fuys et al. (1988) developed a monograph after a three-year research study of the van Hiele model of geometric thinking among adolescents. Their research work was focused on translating Van Hiele's work from Dutch into English with the aim of developing a working document to categorise the sixth and ninth graders and identifying the challenges they encounter during the lessons. They also analysed an American text series for K grade to the twelfth grade using van Hiele's levels of geometric thinking.

To achieve all these objectives, the analysis was done based on van Hiele's source material, particularly from Dina van Hiele-Geldof's (1957; 1984) doctoral thesis and Pierre van Hiele's (1959; 1984) article. Clinical interviews conducted in three different phases which involved 16 sixth – grade learners and 16 ninth-grade learners examining entry level of learners' geometry thinking. Furthermore, one-on-one interviews were conducted at some stage followed by the use of van Hiele's levels of geometric thinking to rate the standard of textbook content.

The research study of Fuys et al. (1988) supports the use of the van Hiele model in teaching and learning of geometry. The results further show that high school learners engaged in the research, progressed towards level 2 (informal theoretical) but with no sign of axiomatic thinking.

Another research conducted by Serow (2002) on learners' understanding of class inclusion in geometry considered the van Hiele theory as the theoretical framework for the study. The topic was researched in the context: relationships among triangle figures, relationships among triangles properties and the quadrilaterals. Serow's (2002) study involved learners from six different secondary schools of the age range between 12 and 18 years. The research findings indicate that the topic class inclusion in geometry is a topic difficult for learners to grasp. In addition, it has been discovered that in geometry class inclusion learners' behaviours that were described as at level 3 of van Hiele such as identifying class inclusion of polygons and its implications (Maryberry, 1981), have now been characterised by Serow's (2002) research study to be at level 4. Based on the above findings Serow (2002) admonishes the researchers to collaborate in the Structural of Observed Learning Outcomes (SOLO) model of the van Hiele level of geometric thinking in order to explore most of the difficulties that learners encounter in geometric conceptual understanding.

In a three-year Cognitive Development and Achievement in Secondary School Geometry (CDASSG) research project conducted in United States of America (USA), Usiskin (1982) aimed to find out the distribution of learners' performance and how their performance changed after one year of teaching and learning of geometry. A total of 2500 learners from a broader social-economical spectrum were engaged in the project. Mainly the project investigated the efficiency of the van Hiele theory in describing and predicting learners' performance in high school geometry.

According to Usiskin (1982), the seven questions were to test different attributes of the van Hiele levels as briefly described. Question 1, tested how the learners can be assigned to the van Hiele level of thinking with regard to conceptual understanding in geometry. Question 2, tested how static each level was in characteristics. Questions 3, 4 and 5, tested how students' achievement in geometry could be explained and predicted by these levels. Questions 6 and 7 focused on the comparison between the levels of van Hiele in terms of their properties and provided a somewhat less formal test of the validity of such properties

(Usiskin, 1982). In order to achieve the intended goal of the investigation, project learners were engaged in activities dominated by abstract concepts, for example on postulates, theorems definitions and proving.

Usiskin's (1982) results of the findings demonstrate that the van Hiele theory can guide to identify the reason why there is low achievement in geometry among learners. Furthermore, learners should be exposed to proving theorems as from lower grade so that they are equipped to achieve high marks in high school geometry. This implies that the reason why there is such high failure rate in geometry in high school is because the base (which is primary school) is not well established in geometric conceptual understanding.

Halat (2006) conducted a research study based on sex-related variances in the acquirement of the van Hiele levels and inspiration in learning geometry, which focused on the influence of gender on attaining the van Hiele geometry levels. Secondly, how boys or girls are motivated when doing an activity in the mathematics curriculum linked to van Hiele's levels of geometric thinking model.

The teaching and learning activities were designed based on the van Hiele theory and they were used by sixth-grade learners in a public middle school from low socio-economic income families in USA. The learners were engaged in twenty-five multiple-choice questions which were administered to them before and after the instructional period of thirty-five minutes each. The outcomes of Usiskin's (1982) research project shows that gender has no effect on students learning geometry. In addition, when teaching and learning of geometry make use of the application of the van Hiele levels of geometric thinking, equity may be achieved among learners. The teaching that uses the van Hiele theory is an intervention to remedy the problem of geometry; for this reason, Fennema and Hart (1994) propose that such mediations can achieve impartiality in learning mathematics.

Feza and Webb (2005) investigated the learners' level of geometric conceptual understanding after their primary school level, teaching geometry through investigation and how to develop learning activities based of learners' alternative conceptions. The research involved 30 learners from the previously disadvantaged primary schools. The results of the research study showed that none of the learners attained level 2 of the van Hiele levels of geometric thinking. One of the suggested reasons demonstrated during the research study is the language proficiency, which acts as barrier to learning and leads to learners' poor performance.

To date, there exist such significant amounts of research done using the van Hiele levels of geometry thinking; however, in my research, the van Hiele levels of geometric thinking are to be used, to critically look into how my intervention tasks influence can learners' learning when using polygon pieces as physical manipulatives in instruction and practice of geometry. In addition, the focus is also on developing a teaching model for teaching and learning of geometry that is to be relevant in a South African context and elsewhere. It is understandable that there are so many models of teaching and learning of geometry, but there is a possibility that they are not relevant to the South African's current situation of geometry teaching and learning. Nevertheless, in view of what has been discussed, learners become the focal point because most researchers have been focusing on the teachers and other areas, but very limited research have focussed on the involvement of the learners.

Van Hiele (1999) and Clements and Battista's (1992) levels of geometric thinking form the basis of my analysis, its emphasis on successive higher thought levels gives a way and are likely to improve the teaching and learning of geometry (Alex & Mammen, 2014). Research shows that the van Hiele levels of geometric thinking take learners' thinking ability into account at the same time that new geometric concepts are being introduced (Bankov, 2013; Alex & Mammen, 2014). This indicates that van Hiele (1999) and Clements and Battista's (1992) levels of geometric thinking give direction as the lesson progresses.

2.11 Physical manipulatives for visualisation

Visualisation is the basic level (level 0) of the van Hiele model of geometric thinking. Visualisation has also been identified as having a significant role to play in the mathematics curriculum (Rivera, Steinbring, & Arcarvi, 2014), for example a learner visualises and then gets confidence to articulate mathematical facts presented by the object under scrutiny. In other words visualisation enhances confidence to communicate and promotes the ability to think with certainty (Dean, 2010). Learners fail to develop the visualisation and exploration skills required for geometric conceptual understanding, problem-solving skills and geometric reasoning due to the way the concepts are presented by most of the teachers (Battista 1999; Idris, 2006, Bhagat & Chang, 2015).

In addition, Bhagat and Chang (2015) argue that physical manipulatives provide a guide to learners' exploration and visualisation of mathematics, such as geometrical concepts which seem to be too abstract in nature. Through the exploration and visualisation of mathematical representations in physical manipulatives learners acquire manipulative skill which is essential for psycho-motor coordination.

At visualisation level a learner is expected to recognize shapes and to draw the given shape. In the case where a learner is far below this level of geometric thinking, physical manipulatives play the major role of introducing and remedial mathematics skills so that the learner can operate at the expected level (Ogg, 2010).The past decade has seen rapid development of the usefulness of physical manipulatives in promoting the low-achievers, learners with learning disabilities and those hands-on learners to a basic level of geometrical thinking (Waycik, 2006).

However, Gluck (1991) points out that mathematics teaching and learning are associated with the structural building that needs a strong foundation to stand the test of times. This infers that in order for the learner to be at level 0 of geometric thinking, the cognitive development must be supported by the use of physical manipulatives to form schemas which are later followed by the use of mathematical symbols (Piaget, 1973).

Evidently, the development of geometry ideas progresses in a hierarchy of levels, for example learners have to recognize a shape first and then analyse that particular shape's properties (Teppo, 1991). The mentioned hierarchy can only be achieved with the use of physical manipulatives. Once physical manipulatives are properly incorporated into teaching and learning, learners are given opportunities to have vivid pictures and understanding of the world around them.

Prawat (1992) highlights that learners' engagement through the use of physical manipulatives is considered not only viable, but also an essential condition for worthwhile learning which leads to conceptual development. Chester et al. (1991) argue that using physical manipulatives help learners becoming connected to the real-world situations which eventually afford them the opportunities to acquire worthwhile learning.

Clement (1999) also claims that physical manipulatives help learners with skills needed to connect different mathematical representations in order to understand meaningful structures that lead to conceptual understanding of geometry. In addition, physical manipulatives promote the retention of mathematical concepts which learners have been engaged in. In other words, physical manipulatives allow learners to have full control of their own learning.

Clement (1999) concurs with Greeno and Riley (1987) regarding the idea that states that physical manipulatives help learners to create and develop a mental representation of the necessary mathematical information that “bridges the gap between informal and formal mathematics” (Boggan et al., 2007: 2-3). For example, physical manipulatives must give meaning to the concept they represent or the one that needs to be clarified (Uttal et al., 1997). This implies that teachers must make sure that learners visualise the mathematical concept being addressed through the use of physical manipulatives because they promote meaningful visualisation (Chiphambo, 2011).

Starcic et al. (2013) further highlight that the conception of geometry ideas is a prerequisite component in the procedure of geometric cognitive growth in learners and should be well-thought through as a compulsory stride at the concrete-experiential level in the progress of rational practices. Thus, for learners to conceptually understand geometry there is a need to be engaged in the manipulation of diverse didactic resources, like mosaics, geo-plates, tangrams, designs and figures of bodies (Cotic, Felda, Mesinovic & Simcic, 2011).

2.12 Physical manipulatives for the analysis of geometric concepts

The Longman dictionary of contemporary English defines analysis as a careful scrutiny of a phenomenon under study in order to make meaning out of which is better than before (2003). The implication of this definition for the use of physical manipulatives assisted by mathematics dictionary, is that they can be used in the teaching and learning of geometry critically to conceptually understand the distinct features of a given figure, In order to analyse the given figures Van de Walle (2004) proposes that physical manipulatives give learners something to use in the connection to real-world situations and mathematical symbols that are abstract.

As learners are engaged in the use of physical manipulatives they are trained to think rationally and analytically in areas where the application of problem-solving and decision-making skills is essential (Abudullah & Zakaria, 2013). Researchers argue that the process of learning mathematics without any mediating factor is difficult to comprehend, but with physical manipulatives, geometrical ideas are broken down into concepts that are easy to grasp (Ogg, 2010). The use of physical manipulatives in instruction and practice of geometry facilitates and afford a learner-centred environment in which learners are actively engaged in exploration and discovery of mathematical concepts in a collaborative way (Hohenwarter & Fuschs, 2004; Gawlick, 2005; Leung, 2008).

Other researchers argue that physical manipulatives allow low-level learners to have a deeper conceptual understanding of mathematical concepts. Hands-on learning promotes conceptual understanding (Peterson et al., 1998; Ogg, 2010). This implies that what has been acquired through experiential learning is not easily lost to memory. For example, to establish the properties of triangles, learners must make use of physical manipulatives where they are given opportunities to measure, and use geo-boards to make meaning. If the teaching and learning of geometry is done abstractly, the meaningful learning cannot be acquired as expected (Skemp, 1976; Herbert & Carpenter, 1992).

Physical manipulatives assisted by mathematics dictionary help learners revise and refine the acquired mathematical skills in order to think mathematically to learn. This allows learners to do self-evaluation of their new emerging mathematical ideas so that the ideas are well internalised and retained for the future use. In addition, the use of physical manipulatives assisted by mathematics dictionary provides learners opportunities to organise and classify shapes systematically and define their relationships in both verbal and symbolic languages (Paparistodemous et al., 2013). The relationship between geometry instruction and practice and the usage of physical manipulatives assisted by mathematics dictionary in instruction and practice of geometry cannot be separated from one another. It is said that by using physical manipulatives assisted by mathematics dictionary which are well integrated into the lesson, learners obtain a deeper geometric conceptual understanding as they investigate properties of shapes and relationships among these properties in order to derive conjectures and test hypothesis (Teppo, 1991).

The incorporation of physical manipulatives assisted by mathematics dictionary in instruction and practice of geometry facilitates a mathematical modelling process. Goos et al. (2008) argue that physical manipulatives allow learners to practice measurement in real life settings where they are able to narrate the given mathematical questions to the real-life setting. According to Cai (2003), when learners use physical manipulatives to model mathematical concepts, important aspects of the presented idea are learnt and conceptually understood.

Laridon, Barnes, Jawurek, Kitto, Myburgh and Pike (2006) clarify that, the approach for teaching and learning mathematical modeling should follow the process of translating the real – life encountered problem into a mathematical scenario in order to give learners a chance to make assumptions that lead to simplification of real-life ideas. This implies that by engaging learners in processes of interpreting real-life situations into mathematical models, they are equipped for problem-solving in real-life situations at any level.

2.13 Physical manipulatives for abstraction

According to Van Hiele (1999) claims at the abstraction level, a learner is able to describe informally the properties of the shape as it stands alone and describe the relationships between two or more shapes in terms of their properties. Other researchers conclude that physical manipulatives help to close the conceptual gap between formal mathematics and informal mathematics (Smith, 2009; Ogg, 2010). For example, physical manipulatives help learners to move from the concrete world to an abstract level as they construct concepts through investigation (Charlesworth, 1997). In other words, physical manipulatives assisted by mathematics dictionary help with learners' conceptual understanding of geometry and then to transit from concrete level to the abstract level of understanding geometry. Hence, it has been suggested that extra care should be taken when choosing physical manipulatives to help learners learn geometry with conceptual understanding (Crowley, 1987).

According to Hwang and Hu (2013) learners need to develop critical thinking in order to conceptually understand the abstract form of geometry. To ensure that learners acquire critical thinking skills which in turn promote geometric conceptual understanding there is a need to explore mathematics formulae and verbal explanations with physical manipulatives assisted by mathematics dictionary.

The NCTM (2000) adds that in order to support learners attain rational thinking abilities the instruction should be structured in such way that learners are afforded an opportunity to: (i) construct their own mathematical representations, categorise and communicate mathematical ideas effectively, (ii) select and apply appropriate mathematical representations which can be used to solve given problems, (iii) use available physical manipulatives to mode and interpret, physical social and mathematical phenomena (Hwang & Hu, 2013).

Moyer (2001) maintains that these tangible situations, conventional mathematical language and notation enable learners' abstract thinking to be closely coordinated with their concrete perceptions of the world. Thomas (1994) says that active manipulation of physical manipulatives offers learners opportunities to cultivate a range of imageries that can be used in the mental operation of intangible notions and enhance mathematical manipulation skills (Suydam, 1984). Integrating physical manipulatives assisted by mathematics dictionary into geometry teaching and learning may bridge the gap that most learners have between conceptual understanding and learning of geometry.

Dutton and Dutton (1991) argue that teaching for conceptual understanding should follow Bruner's theory of the stages of cognition, which stresses that learning starts with the use of semi-concrete or pictorial concepts and then symbolical problems. Kilpatrick et al. (2001) add that physical manipulatives provide models that pave the way to learners' conceptual understanding of mathematics being presented in order for them to think mathematically when learning. In the stages of cognition mentioned here physical manipulatives guide learners from the environment that is context embedded into the environment that is context reduced, which leads to abstract thinking (Alex & Mammen, 2014).

Incorporating physical manipulatives in the instruction and practice of mathematics promotes learners' abstract mathematical thinking, and cognitive mathematical relationships are developed through constructive abstraction in the problem-solving (Kamii, Lewis & Kirkland, 2001). In addition, physical manipulatives assisted by mathematics dictionary provide opportunities to the learners' ways of abstractions; it has been argued that learners do not automatically develop abstract thinking in the way they learn to speak a certain language. Abstract thinking takes time and that is why there is a need to introduce relevant physical manipulatives to represent abstract mathematical notions which are intended for learners to acquire (Tom, 1999).

Charles worth (1997) suggests that exploration of physical manipulatives draws learners' curiosity in learning mathematics, which, in the long run, allows the construction of mathematical concepts. Furthermore, physical manipulatives facilitate the development of abstract reasoning and give learners the opportunity to discover mathematical concepts through the exploration of learning (Bruner, 1961; Piaget, 1962; Bruner, 1964; Montessori, 1964; Piaget & Coltman, 1974; Papert 1980). This implies mathematical knowledge that learners acquire from the real-world settings are stimulated as they are engaged in the use of physical manipulatives assisted by mathematics dictionary.

The use of physical manipulatives enables learners to become mathematically proficient. Learners who are mathematically proficient are able to form mental images of the physical manipulatives which they use as a guide for the construction of their geometric thinking for problem-solving (Chao, Stigler& Woodward, 2000). Research argue that the use of physical manipulatives in the instruction and practice of geometry is spontaneously and manipulatively appealing (Thompson, 1994). The appealing situation promotes the development of spatial perceptions that help learners in acquiring diversity of mathematical expertise, for example, general cognitive thinking skills and problem solving capabilities (Sherrard, 1981). The development of spatial relationships help in the improvement of memory and story enacts and it leads learners from a context-embedded setting into the context-reduced setting (Biazak, Marley & Levin, 2010; Alex & Mammen, 2014).

2.14 Conclusion

To conclude; firstly, it has been discovered that, if properly followed, the van Hiele levels of geometric thinking promote learners' conceptual understanding in geometry. Furthermore, in teaching and learning of geometry teachers must integrate physical manipulatives because they help learners to progress from one level to the next. The incorporation of physical manipulatives in mathematics teaching and learning has a long account in education. For this reason most education departments globally are now promoting the integration of physical manipulatives in order for learners to acquire mathematical skills for conceptual understanding (Moyer, 2001; Clements & Bright, 2003). Conceptual understanding in this regard refers to both creating meaning and constructing systems of meaning so that what has been learnt is not lost to memory.

Research shows that not all manipulatives are appropriate for geometric teaching; some do not represent the mathematical concept behind the representation (Van de Walle, 2004). Clements (1999) describes that good physical manipulatives provide learners with opportunities to have control of the lesson. It also provides them with must-have features that reflect the real-life mathematical situations that help learners to in link geometric concepts with various types of knowledge for conceptual understanding.

I have also discovered that using physical manipulatives assisted by mathematics dictionary in teaching and learning is cost – effective because they can be made from locally available materials despite the geographical setup of the school. They are also user friendly.

In the next chapter, I present my research methodology and design.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1 Introduction

The chapter presents, a detailed description of the research methodology and the designs which were employed to illustrate the devised strategies that were employed when conducting this research study. I also justify why I have to employ each of the selected methods in conducting my research study. The following sections are to be epistemologically justified: (a) the research methodology and (b) the research design which comprised:(i) the methods used to collect data, (ii) sample selection, (iii) sampling techniques, (iv) description and advantages of the instruments used in collecting data, (v) a detailed description of how the diagnostic and post- intervention tests were developed and validated to ensure that they are at the appropriate level and relevant standard for the target group, (vi) the analysis of data, (vii) the ethical issues and (viii) research validity.

3.2 Research methodology

In this section, the research methodology, which is the philosophical framework that addresses the research questions in relation to the entire research processes is presented and described in detail (Creswell & Plano Clark, 2007). This research study is informed by the mixed methods paradigm which is defined as the unification of quantitative and qualitative data analysis in a distinct research study from which the simultaneously collected facts are given priority. The paradigm involves the amalgamation of the facts in one or more phases in the procedure of investigation to ensure that no part is left without being examined (Creswell, 2003). Researchers argue that the mixed methods paradigm provides the most instructive, comprehensive, composed, and convenient study outcomes (Johnson et al., 2007). The philosophical and epistemological foundation for employing mixed methods in association with my research study was to obtain different but complementary data on the same topic to best understand and get solutions to the difficulties learners face in learning geometry.

The advantages of using the mixed methods approach to this research were the following:

(i) To augment research outcomes to ensure that one form of data did not consent to obtain a deeper understanding of one or more of the constructs under study (Brewer & Hunter, 1989; Tashakkori & Teddlie, 1998).

(ii) To had an opportunity to simultaneously generalise results from a sample in order to gain a deeper understanding of how the use of polygon pieces assisted by mathematics dictionary influenced teaching and learning of geometry in an interesting way. The deeper understanding was gained by uniting numerical trends from quantitative data and specific details of the phenomenon under study from qualitative data (Hanson, Creswell, Plano Clark, Petska & Creswell, 2005).

(iii) To have a prospect to experiment hypothetical models and to adapt them based on my research participants' response which they gave after being engaged in the intervention programme (Hanson et al., 2005).

The listed advantages imply that the mixed methods paradigm gave more room for a thorough data analysis. All aspects identified by the different research instruments of my study were to be analysed from different angles of focus so that a true reflection of how the use of physical manipulatives assisted by mathematics dictionary influenced teaching and learning of geometry was eventually brought to light.

According to Denscombe (2008),the mixed methods paradigm offers quite a number of opportunities to the researcher, which are: (i) to advance the precision of the collected data (ii) to produce a more multi-faceted picture by merging information from a variety of kinds of sources that complement each other (iii) as a means of complementing for the specific strength or weakness which a particular method has (iv) as a way of developing the analysis and construct on initial findings using distinct kinds of data and (v) used as an utility for assessing the appropriateness for the research sample (Collins, Onwuegbuzie & Sutton, 2006). In my study, the following were reasons for using a mixed methods paradigm:

(i) To get an opportunity to scrutinise and understand the complexity of the phenomenon under study at a deeper level to ensure that there is strong correlation between the interpretation and usefulness of research findings (Collins et al., 2006). The understanding of how the use of physical manipulatives assisted by mathematics dictionary in teaching and learning geometry influenced learners' conceptual understanding allowed me to develop a model that can help to improve the situation of teaching and learning geometry (Creswell, 2003).

(ii) To had an opportunity to strategically position myself to explore, experiment and to have an in-depth understanding of how polygons pieces can be used in the teaching and learning of geometry.

(iii) To assess the appropriateness and relevance of the chosen instruments which were scheduled for data collection (Collins et al., 2006). Terre Blanche and Durrheim (1999) argue that the in-depth understanding of the meaning of human inventions of ideas, words and experiences can only be established in relation to the context in which they happen. Hence, in view of Terre Blanche and Durrheim's (1999) proposition and by employing the mixed methods paradigm, I was in a position to conduct an in-depth research in a context that was highly restricted and explicitly related to experiences of nine eighth-grade learners.

When learners were engaged in various tasks of an intervention programme that utilised pieces of polygons assisted by mathematics dictionary to facilitate conceptual understanding of geometry it was possible to observe them working. For this reason an opportunity was created to collect relevant and rich data which was required in this study (Collins et al., 2006). Lastly, the mixed methods paradigm also opened a window of exploring learners' mathematical proficiency as they were engaged in using polygon pieces as physical manipulatives assisted by mathematics dictionary for teaching and learning of geometry.

3.3. Research design

A detailed description of the contextual issues in relation to this study is provided. These contextual issues consist of the geographical background of: (i) the site where my research study was conducted, (ii) a clear description of the South African senior phase mathematical content in relation to van Hiele's (1999) stages of geometric intellectual and mathematical background of the chosen sample, (iii) the methods used to collect data, (iv) learners' sample selection, (v) learners' sampling techniques, (vi) description and advantages of the instruments used in collecting data, (vii) a detailed description of how the diagnostic and post-intervention tests were developed and validated to ensure that they were at an appropriate level and relevant standard for the target group, (viii) the analysis of data, (ix) the ethical issues and (x) research validity.

3.3.1 Geographical background

My research site was one of the section 21 secondary schools in the Eastern Cape Province of South Africa in the Queenstown district. Section 21 secondary schools are semi-urban secondary schools in a low-income group residential area.This was one of the schools within my reach, which means that I could easily obtain access and informed consent.

Secondly, selecting this school gave me an opportunity to observe and interact more interactively with participants for a more extended period. Lastly, this gave me a greater understanding of the context due to my own prior knowledge of section 21 schools.

The learners who formed my research sample were all in high school for the first time. All the learners who were the participants in my research study were admitted to the selected high school (which was the research site for this study) from semi-urban primary schools in a location which is a low-income group residential area; these primary schools are within the same location with my research site. My research site has an intake of at least 1300 learners yearly which includes not less than 300 eighth-grade learners. The sample chosen for this study comprised of learners whose background is summarised as follows: six learners were aged 13 (L1, L2, L3, L5, L6 & L7); L4, L9 and L8 were 14, 15 and 16 years old, respectively. These learners (my participants) came from four different primary schools as follows: L2, L3, L5, L6, L7 and L9 came from the same school while L1, L4 and L 8 attended primary education from three different primary schools which were situated within the area of my research site.

Regarding the family setup of my participants, of the nine learners, three learners (L1, L3 & L6) came from grandmother-headed families, four learners (L2, L7, L8 & L9) came from mother-headed families and one (L5) came from the child-headed family. Their economic status is categorised as: L1 and L8 came from a family where no family member is employed, L2, L3, L6, L7 and L9 came from families where mothers only are working, L4 has a mother and the aunt who are employed, and then L5 has a sister who is the only member that is employed.

These participants came from different accommodation structures, L2, L4, L7, L8 and L9 each one of them resided in a Rural Development Programme (RDP) house. RDP houses in South Africa are government-subsidised houses to enable people who do not earn enough money to qualify for a normal home loan, to have their own houses. L3 and L6 live in their own family houses. The families of the last two learners (L1 &L5) rent flats.

3.3.2 A case study

This research study applies the concurrent transformative design as it allows data to be collected within a short space of time; this was relevant to my research study which is a case study. A case study is a method of inquest that practically allows the researcher to investigate and study individuals' conceptual understanding of a particular concept within its real-life settings, especially when the precincts between the concept and setting are not evidently obvious (Yin, 2003).

In a case study, the researcher is free to employ both quantitative and qualitative research methods to rigorously examine a distinct unit (Yin, 1981; Yin 1994). According to Van Maanen (1985), quantitative and qualitative research methods are not mutually exclusive. This implies that quantitative and qualitative tools can both be employed rigorously together to capture an understanding of the complexities in teaching and learning (Feuer, Towne & Shavelson, 2002) hence, this study used the mixed methods approach. The quantitative approach should mirror events that are significant in a tabulated form in order to make sense of what the case study is all about (Yin, 1981). In this study the quantitative data were obtained from the diagnostic and post-intervention tests, the daily reflective tests and intervention activities scripts.

On the other hand, the qualitative method presents the description of the data in the form of words and pictures rather than numerical values (Bogdan & Bilken, 1998). In this case qualitative data were obtained from observation notes and transcribed interviews. The combination of the two (qualitative and quantitative) provided me the opportunity to do an in-depth study of how the use of physical manipulatives influences learners' conceptual understanding of geometry within a restricted time frame (Bell, 1993).

In addition, the simultaneous collection of data counterbalances the weakness that can be identified in one of the data collection methods in any phase of the research study (Terrell, 2011). A case study was chosen for the following reasons:

(i) To study an aspect of learners' identified alternative conception in geometry with in-depth scrutiny within the limited time frame (Bell, 1993; McMillan & Schumacher, 1993).

(ii) Participants were engaged in real-life actions that could allow for the situation to improve should the need arose (Cohen & Manion, 1985).

(iii) For the design of a teaching model for the teaching and learning of geometry.

(iv) To create an opportunity to think creatively and critically when dealing with the collected data (Patton, 1990).

This study took the form of an exploratory case study which is defined as a study that is used to investigate a scenario where the intervention is used as a strategy to study the participants. But the outcomes of such an intervention are not what appeared to be definite to the researcher (Yin, 2003). In order to design and implement rigorous case study research, the following components were taken into consideration:

(i) Propositions (Yin, 2003; Miles & Huberman, 1994): Included in my case, factors that influenced the use of physical manipulatives in teaching and learning of properties of triangles. According to Stake (1995) and Yin (2003), propositions have an influence in the development of the conceptual framework.

(ii) The application of a conceptual framework (Miles & Huberman, 1994). The conceptual framework directed and ensured the smooth flow of the research by determining who was to be included in the research and identifying the relationship that exists between logic, theory and experiences. Conceptual framework is based on literature or the personal experiences of the researcher (Baxter & Jack, 2008). In this research the conceptual framework is based on Van Hiele's (1999) model of geometric thinking and my personal experience as a mathematics teacher.

(iii) The design of the research question which was supposed to be in the form of 'how' or 'why'.

(iv) To have the criteria for the interpretation of the research findings (Baxter & Jack, 2008).

According to Hanson et al. (2005) there are six major forms of mixed methods design - three are successive and three are concurrent. Of the six, this research employs the concurrent transformative design of data collection (Terrell, 2011). Figure 3.1 below shows the concurrent transformative design which this study has employed.

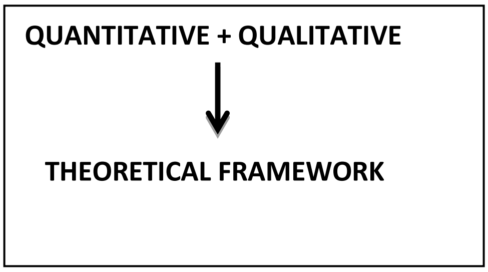


Figure 3.1: Shows the structural of concurrent transformative research design. Adapted from Terrell, S. (2011). Mixed-methods research methodologies. The Qualitative Report, 17(1), pp. 254-280.

In this study the concurrent transformative research design employs thematic analysis to analyse the collected data, the details of how each of the phases of thematic analysis is done is discussed in section 3.3.4 of this chapter. Boyatzis (1998) reports that thematic analysis functions as:

(i) A way of seeing how the research participants act towards a given situation in relation to the research question.

(ii) A way of making sense of apparently distinct behaviours demonstrated by the participants in a research study either verbally or in writing.

(iii) A way of analysing qualitative information into well-structure ideas for the audience's consumption.

(iv) A systematic way of observing how research participants interact in a given situation and make meaning out of their actions.

(v) A way of translating descriptive information (qualitative) into numerical data (quantitative) or the other way round.

To ensure that the five functions of thematic analysis recently mentioned are accomplished the four factors have been considered:

(i) Theoretical perspective –the van Hiele levels of geometric thinking model was explicitly applied in analysing the collected data.

(ii) In mixed method research, both quantitative and qualitative strategies of collecting data were equally prioritised so that one strategy covers the shortfalls of the other.

(iii) No predetermined sequence was followed in data collection instead it was done concurrently.

(iv) The integration: that is the combination of two forms of data (quantitative and qualitative) in this study was done in two phases - during analysis and interpretation of data to obtain a unified view of the data (Lenzerini, 2002).

Thematic analysis to be implemented in a concurrent, transformative design has to follow six phases. This study has taken them all into consideration (familiarising, coding, probing for themes, revising themes, describing and giving names to the themes, and generating a report). The steps are presented and discussed in detail in section 3.3.4 of this chapter.

The application of thematic analysis in this research study provided me with the opportunity to identify, analyse and report the emerging themes within the collected data (Braun & Clarke 2006). Boyatzis (1998) adds that the adoption of the thematic analysis into a research study allows the researcher to give the description of data in a detailed manner that is clear to the reader. In case of interviews development, they also provided an opportunity to identify themes and concepts and potentially revealed the reality of the research participants as they were engaged in a research project. Lastly, thematic analysis' flexibility provided me with an opportunity to determine themes in a variety of ways; since there was no restriction in theme identification.

In order for a researcher to analyse the data thematically, there must be themes that emerged from the data, hence the name 'thematic analysis. Braun and Clarke (2006) suggest that in principle, emerging themes capture the valuable information that was collected that is in relation to the research question. The valuable information that was captured from the data has been reported in the form of experiences, meanings and the reality of participants as the research unfolds. According to Boyatzis (1998), the use of themes that emerge from the collected data can be done at one of two levels - at a semantic (explicit) level or at a latent (interpretive) level. Boyatzis (1998) further describes that at latent levels of analysis themes are developed based on the interpretation of the participants' work in order to produce both descriptive and theorised analysis. In this research study, data were analysed at a latent level where by. In the process started to examine individual participants' geometrical ideas, suppositions, conceptualisations and beliefs (Boyatzis, 1998; Braun & Clarke, 2006) that surfaced as they were engaged in the research project from diagnostic test through intervention activities to the post-intervention test.

In this study, the sample was nine eight-grade learners engaged in using polygon pieces in teaching and learning about the properties of triangles. In order to ensure that enough data were collected from different perspectives, different research instruments were employed for both quantitative and qualitative data in my research project. Cohen, Manion and Morrison (2000) describe triangulation as the use of a range of methods of data collection in a single research study. The reasons for employing triangulation are: (i) to explore the research questions from different angles (Flick, Von Kardorff & Steinke, 2004) and (ii) to help in authenticating the assertions that might arise from an initial pilot study (Bogdan & Bilken, 1998).

3.3.3 Context of the study

3.3.3.1 Research sampling

Maxwell (2005) defines sampling as having a decision of whom to talk to or what data sources to focus on and where to conduct the research project? Sampling is done in many different ways, but in my research study, the research site sampling was based on the criterion sampling. According to Patton (1990), the systematic ways of conducting criterion sampling is to review and conduct an investigation at a site (in my research, it was a school) that meets some prearranged criterion of significance, for instance, proximity of the research site. My research site was within reach for the reasons mentioned below. The advantages of conducting a research study in a neighbouring school were that, due to its proximity, access and informed consent were easy to obtain. Secondly, it was possible to observe and interact more intensively with participants for a more extended period. Thirdly, the research site provides me with an opportunity to obtain a greater understanding of the context due to my prior knowledge of the section 21 schools. A section 21 school is a semi-urban secondary school in a location which is a low-income group residential area. Lastly, the cost of transport to the research site was reasonable per day which was not the case with other section 21 schools in the district which were very far.

Of the five grades at the research site I have chosen the eighth grade for two reasons. Firstly, properties of triangles form an important part of the curriculum for grade 8. Secondly because they are the lowest grade of the selected high schools and if the study proves that physical manipulatives assisted by mathematics dictionary play a positive role in teaching and learning, this can then form a springboard to help all the learners in the other grades of the school to clear their alternative conceptions in geometry, especially properties of the triangles.

From the eighth-grade, a cohort of 56 eighth graders volunteered (40 females and 16 males) to take part in writing the diagnostic test. After that, purposeful sampling was done to obtain the required number for a productive target group (nine grade 8 learners, low, middle and high achievers).The main research sample was a group of nine eighth-grade learnerspurposefully selected from a cohort of 56eighth graders. Purposeful selection was done according to individual learners' performance in the diagnostic test and gender representation. In this study, it included three learners with a high percentage, three with an average percentage and three with a below average percentage. Purposeful sampling was done because researchers argue that a small number of participants provides an opportunity for eliciting more in-depth data (Tashakkori & Teddlie, 2003) about the influence of physical manipulatives assisted by mathematics dictionary in the teaching and learning of geometry, which was the phenomenon under study (Teddlie & Yu, 2007).

Although researchers propose that another aim of doing purposeful sampling was to obtain the required number of the productive target group, regardless of gender (Marshall, 1996). My sample consisted of learners of different achievement (low, middle and high achievers). This was considered as a productive sample to be used in identifying learners' alternative conceptions in learning of geometry so that the relevant intervention is employed to help learners with geometric conceptual understanding.

Table 3.1: The cohort of 56 volunteers' performance in the diagnostic test

Learners' mark interval as a percentage	**Number of learners**	**Gender**	
		F	**M**
0 - 18,4	45	31	14
18,5 - 29,6	8	7	1
29,7 - 45	3	2	1
Total	**56**	**40**	**16**

Table 3.1 illustrates how the cohort of 56 volunteers performed in the diagnostic test. They performed as: 45 learners obtained marks between 0% and 18,4%, eight scored between 18,5% to 29,6% and three learners ranged between 29,7% and 45%.

The cohort of 56 learners included many females as compared to males giving the ratio of females to males as 5: 2. The selection of nine learners for the main research sample was exclusively based on the diagnostic test's results (low, middle and high achievers), regardless of gender (Marshall, 1996). Learners' achievement in the diagnostic test was the only factor influenced my sampling of nine learners for this study; hence it has many females as compared to males (seven females and two males). The purposefully selected nine learners had to be engaged in my research study in all the activities designed including the post-intervention test which has the same content as the diagnostic test. For details, refer to appendix 10.

3.3.3.2 Research intervention

From a theoretical perspective my research study had to investigate the influence of using of polygon pieces as physical manipulatives assisted by mathematics dictionary in the teaching and learning of geometry to grade 8 learners. Secondly, it had to explore whether the measurement of angles and sides of polygons using polygons pieces assisted by mathematics dictionary (cut pieces of two-dimensional) promote learners' conceptual understanding in geometry. In addition, it had to investigate how mathematics teachers should use polygon pieces assisted by mathematics dictionary to teach the properties of triangles in order to promote learners' conceptual understanding in geometry. Lastly, the investigations had to lead me into the designing of a teaching model for the topic under study so that the mathematics community can benefit from the model by using it when teaching and learning of geometry.

As a researcher my role during the research programme was to: (i) to provide teaching and learning resources, (ii) give clarity where need arose in the class, (iii) observe and jot down notes, (iv) mark the intervention and reflective scripts (iv) facilitate the revision of previous days' intervention activities and reflective tests.

3.3.3.3 Phase one: Pilot of the diagnostic test

In this study, the first phase was the piloting of a diagnostic test that was administered to 28 eighth-grade volunteers. For details of the content of the piloted test, refer to appendix

8. According to Van Teijlingen and Hundley (2001) one of the benefits of piloting the test was to obtain well-informed notice in advance about where the main research study could not be successful as well as where research etiquettes may not be properly shadowed. In view of this research study, the main aim was to discover whether anticipated approaches or instruments were inappropriate or too complicated for the research study (Van Teijlingen & Hundley, 2001). De Vaus (1993) counsels that failing to pilot the test is to take a great risk in your study. After the diagnostic test was piloted, no adjustments were made because the results showed that the standard of the questions was at the relevant level for the grade8 syllabus. Subsequently, the task was subsequently administered to the cohort of 56 eighth grade learners. Both the diagnostic and intervention tests contained five questions; which were set based on the van Hiele levels of geometric thinking. To minimise contamination the pilot group was obtained from one of the six-eighth grade classes while the cohort of 56 volunteers was from the other five eighth grade classes.

I designed the diagnostic test (with questions aligned to the van Hiele levels of geometric thinking),first was administered to the pilot group of 28 eighth grade learner sand was later administered to the cohort of 56 eighth grade learners from the same research site. The cohort of 56 volunteers wrote the test under the same standards and controlled conditions as the pilot group (after school, for 0.7 hours, to write as individuals and no physical manipulatives or mathematics dictionary could be used in the test) to ensure reliability of the instrument. The cohort of 56 eighth grade learners wrote the diagnostic test for the following three reasons:

(i) To identify alternative conceptions and misunderstandings that learners had regarding geometry.

(ii) To capture and explore the learners' conceptual understanding of geometry before employing the intervention.

(iii) To help in designing a suitable intervention strategy that aims to address some of the alternative conceptions that the selected grade 8 learners had in learning geometry.

Both the diagnostic and post-test questions 1.1 to 1.5 each with its three sub-questions were aligned to different levels of the van Hiele theory of geometric thinking as presented below.

Seven questions that were aligned to level-0, visualisation of the van Hiele theory were: 1.1(i); 1.2(i); 1.3(i) and (iii); 1.4(i) and (iii) and 1.5(i). For the detailed content of each of the questions refer to appendices 10 and 11. These questions were considered to be under the named level of the van Hiele theory since they provided learners with opportunities to use visual skills to determine the properties of triangles and also allowed them to recognize various triangles based on their unique properties.

Question 1.3(ii) was the only question aligned to level 1-analysis of the van Hiele theory of geometric thinking. The question focused on learners' ability to identify a geometric shape's properties given all the symbols to describe it.

Six questions, 1.1(ii) and (iii); 1.2(ii) and (iii); 1.4 (ii) and 1.5(ii) were aligned to level 2-abstraction of the van Hiele theory of geometric thinking, for details of the content refer toi appendices 10 and 11. Through these questions learners were given opportunities to solve problems where properties of figures and interrelationships were significant (Crowely, 1987).

Under level 3-formal deduction of the van Hiele theory was question 1.5(iii). The question required the learners to think logically in order to provide the properties of a given triangle. The question at this level was set to assess whether they could interact necessary and sufficient conditions of a triangle without the use of polygon pieces.

3.3.3.4 Phase two: The intervention programme

The intervention programme I have designed for this study was to address not only the alternative conceptions learners demonstrated in the diagnostic test but also to teach the concepts of the properties of triangles in an informal activity-based way so that learners would be able to identify, classify and name triangles based on their properties. In the intervention programme, physical manipulatives had to be used in order to engage learners in developing conceptual understanding of the properties of triangles. The intervention activities were also designed based on the needs that arose in the diagnostic activity.

Below is the generic diagram explaining how the intervention activities were used in order to help the learners develop the skills mentioned in this study.

In every intervention activity, learners were provided with an A4 paper. For instance, on the paper triangle ABC was drawn – along with the A4 paper were the two copies of triangles ABC provided to each one of the learners. Figure 3.2 clarifies how the process of using the original triangle and its copies was done.

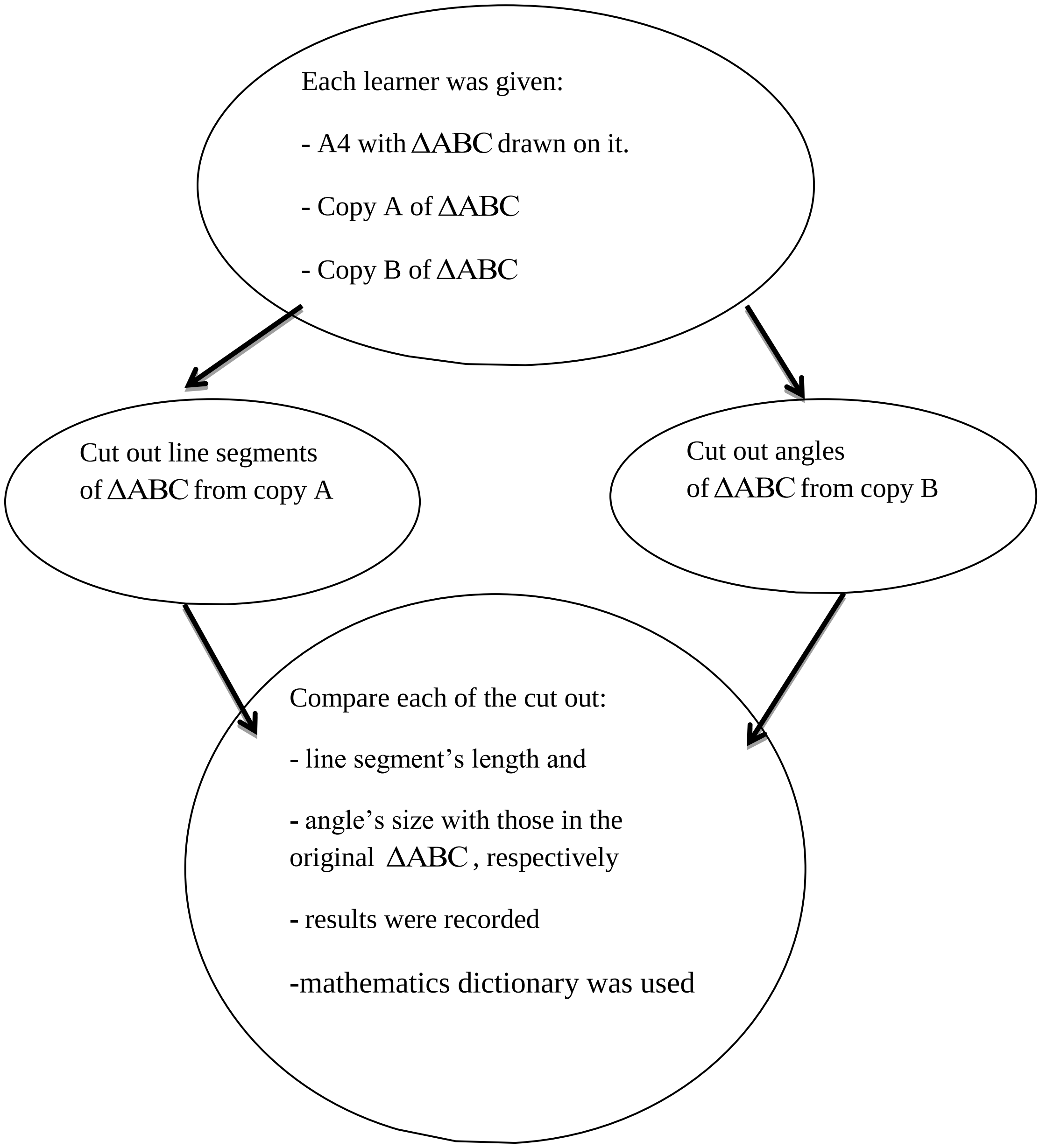


Figure 3.2: How the intervention process of cutting out polygon pieces

Figure 3.2 shows how the process of cutting out line segments and angles from the given triangle was done. The comparison was done by placing each of the cut out line segments or angles one at a time on top of the other line segment or angle in the original triangle and for every measure taken the results were recorded down. The findings of how the line segments and angles were related in the given triangle were used to describe the properties of that particular triangle. In this activity no rulers and protractors were used, only cut out line segments and angles were used. Even in describing how line segments were related, the informal mathematical language was used, i.e. longer than, shorter than or equal to. For angles, learners would use greater than, smaller than and equal to. When the properties were identified and described the name of that particular triangle was to be written down, the mathematics dictionary was made available to help in enhancing mathematics vocabulary and terminology.

In each of the planned intervention activities, learners were supposed to answer each and every question after measuring and comparing angles and sides of the given triangles using polygon pieces. Each intervention activity was scheduled for one hour. The total time spent to complete the nine intervention activities was nine hours. The use of physical manipulatives assisted by mathematics dictionary was applied in all the intervention activities. As shown in Figure 3.2 activities were done by cutting out the line segments and angles from the copies provided in order to explore the properties of specific provided triangles. The cut pieces were for the conceptual development of learners in geometry (Hwang & Hu, 2006).

Intervention activity 1 consisted of eight questions. For details of the content, refer to appendix 12. All eight questions were aligned to different levels of geometric thinking as follows: question 1.1 was aligned to level 0-visualisation, questions 1.2; 1.4 and 1.5 were aligned to level 3-formal deduction, questions 1.3 and 1.6 were aligned to level 2-abstraction, and question 1.7(i) – (ii) were aligned to level 1-analysis.

Intervention activity 2 had only two main questions that required learners to classify triangles based on their properties and to match the given properties of triangles with the relevant triangles. For details refer to appendix 13. Both questions were at level 1-analysis of van Hiele theory of geometric thinking.

Intervention activity 3 required learners to identify triangles by name and apply the use of symbols. This intervention had three main questions of which question 3.1 was related to levels 0-visualisation and 1-analysis of the van Hiele theory geometry thinking, while questions 3.2 and 3.3 were aligned to level 1-analysis of van Hiele theory. For details see appendix 14.

Intervention activity 4 had one question with sub-sections 4.1 to 4.6 which required learners to match given triangles with the list of properties given. The activity was at level 1-analysis of the van Hiele theory of geometric thinking. For details, refer to appendix 15.

Intervention activity 5 consisted of two questions 5.1(i) – (iii) and 5.2(i) – (v) which required learners to identify and explore the properties of a right-angled triangle. All questions in this activity were aligned to level 1-analysis, except question 5.2 (vi) which was aligned to level 2-abstraction of geometric thinking, for details of the question refer to appendix 16.

Intervention activity 6 requires learners to explore the properties of obtuse-angled triangles. There are only two questions which are divided into sections as shown in appendix 17. The contents of both questions 6.1(iii) and 6.2(i) – (iii) were at level 1-analysis of the van Hiele levels of geometric thinking while 6.2(iv) – (vi) were at level 2-abstraction of the van Hiele levels of geometric thinking. According to the structure of the intervention activity 6(i) and 6.1(ii) were instructions which learner were supposed to follow in order to do question 6.1(iii), refer to appendix 17 for details

Intervention activity 7 consisted of questions 7.1 and 7.2 in which learners were asked to explore the relationship of angles and line segments by using the physical manipulatives. For the details of intervention activity 7's content, refer to appendix 18. In this intervention activity questions 7.1(iii) and 7.2 (i) – (iii) were aligned to level 1-analysisof the van Hiele levels of geometric thinking while question 7.2(iv) – (vi) was at level 2-abstraction of the van Hiele levels of geometric thinking.

Intervention activity 8 contained questions 8.1 to 8.4 which required learners to explore and discover the properties of an equilateral triangle. For the detailed content of intervention activity 8 refer to appendix 19. Questions 8.1(i) – (iii) ; were aligned to level 0-visualsation of the van Hiele levels of geometric thinking while question 8.1(iv) and 8.2(i) - (iii) were aligned at level 1 of the van Hiele levels of geometric thinking. Question 8.3(i) – (ii) was at level 3-formal deduction of the van Hiele theory while question 8.4 was at level 2-abstraction of the van Hiele theory.

Intervention activity 9 consisted of questions 9.1 to 9.4 which focused on investigating properties of an isosceles triangle using polygon pieces. For more information, refer to appendix 20. In this activity, questions 9.1(i) – (iii) were at level 0-visualisation of the van Hiele theory. Questions 9.1(iv), 9.2(i) – (iii) were at level 1 of the van Hiele levels of geometric thinking. Question 9.3(i) – (ii) were at level 3-deduction while question 9.4 was at level 2-abstractionof the van Hiele levels of geometric thinking.

According to Feza and Webb (2005), it is evident in both the assessment standards of the South African curriculum and Van Hiele's descriptors that by the time South African learners exit the seventh grade, they should have been at the van Hiele level 2-abstraction of van Hiele theory. However, in the case of most of the learners, by the time they exit the seventh grade, they would still operating below level 2-abstraction of the van Hiele theory. Table 3.3 below shows the link between SA senior phase curriculum and the van Hiele levels of geometric thinking.

One of the components of the intervention programme designed for this research study was a set of seven tests. Each reflective test session was scheduled for 0.5 hours. The total time spent for the seven tests was 3.5 hours. The focus for each of the tests is briefly described below and the appendices have detailed information of the content:

Test 1 is consisted of three questions – questions 1.1 1.2 and 1.3. Of the three questions, question 1.1 was aligned to the van Hiele's level 0-visualisation. In addition, the other two questions, questions 1.2 and 1.3, were all aligned to level 1-analysis of van Hiele's level of thinking. At level 1-visualisation, learners are required to contrast different classes of figures based on their characteristics (Crowley, 1987).For detailed information regarding the first test, refer to appendix 21.

Test 2 consisted of only two questions, questions 2.1and 2.2. These questions had 6 and 5 sub-questions, respectively. Question 2.1 was at van Hiele's level 1of geometric thinking (analysis).This question required learners to identify certain shapes from visual clues (Crowley, 1987). Question 2.2 required learners to describe a figure using the set of properties. This was at level 2of the van Hiele theory (abstraction). For the details of the second test, refer to appendix 22.

The third test was consisted of two questions; each with four sub-questions as shown in appendix 23. Question 3.1 and its sub-questions focused on the van Hiele level 1-analysis of geometric thinking. Learners were asked to make use of symbols to illustrate the types of triangles drawn, i.e. an isosceles, an equilateral. Question 3.2 required learners to "identify what is given and what is to be proved in a problem" (Crowley, 1987:12), therefore, it was at level 3 of Van Hiele's levels of geometric thinking.

The fourth test is consisted of questions 4.1 and 4.2, both of which focused on the van Hiele level 0-visualisation. The two questions provided learners with an opportunity to identify shapes based on the given descriptions. For details of the two questions, refer to appendix 25.

The fifth test consisted of four questions. The first to the third questions required learners to write down the properties of right-angled scalene, obtuse-angled and acute-angled triangles. The three questions were at level 2-abstraction of the van Hiele's theory. The fourth question which had three sub-sections required learners to categorise the given triangles into the groups which have been mentioned in the first three questions. Question 4 was at level 0-visualisation of the van Hiele theory. For details of the test, refer to appendix 26.

The sixth and seventh tests contained two questions each. For both tests, the first questions were at level 1 and the second questions were at level 2-abstraction of the van Hiele theory. For content details of the questions, refer to appendices 27 and 28, respectively.

The designed teaching and learning model that is intended to help learners with conceptual understanding of the properties of various triangles is illustrated below. In all intervention activities, each learner was provided with a pair of scissors, a pencil, an eraser and a pen. There was also one mathematics dictionary which was for the enhancement of mathematics vocabulary and terminology proficiency.

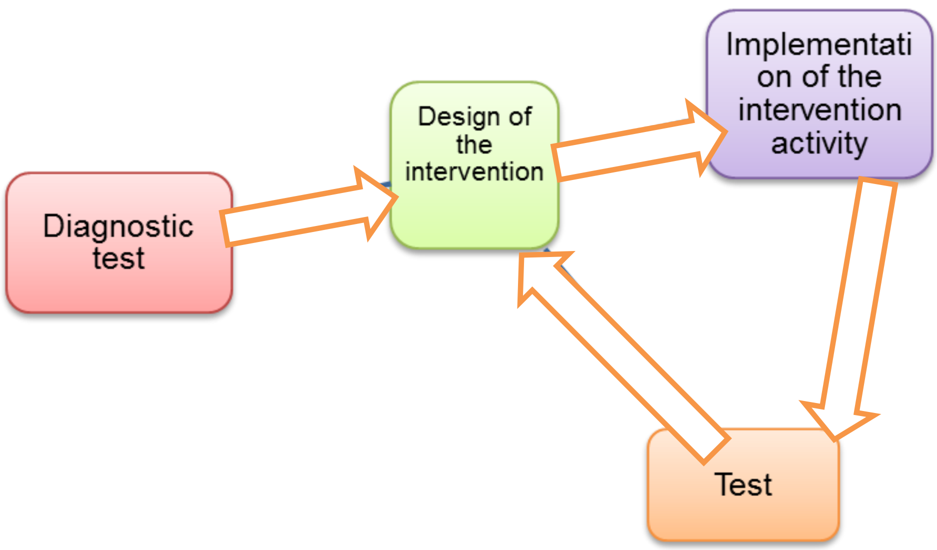


Figure 3.3: The planned intervention model

Figure 3.3 was the planned intervention model. The arrows in the model indicate the sequence of how the model was to be used to address alternative conceptions learners had in learning geometry. The proposed designed model of teaching and learning geometry shown in Figure 3.3 had four steps. The first step was to administer the diagnostic test to the cohort of 56 grade8 learners. The diagnostic was administered once, in the first day only and then the subsequent steps were to be implemented daily. The subsequent steps are: designing of the intervention activity, implementation of the intervention and a test to assess learners' conceptual understanding of the previous day's activity. During the intervention activities in each and every session, I was supposed to give individual assistance to learners who demonstrate alternative conceptions in learning geometry.

The proposed intervention model for this study consisted of nine intervention activities that focused on informal ways of identifying properties of the triangles and seven reflective tests. Each daily test impacted the design of the next intervention activity. The scripts were used during the intervention programme and collected daily for marking in order to identify learners' alternative conceptions in geometry.

Table 3.2: Senior phase mathematics general content focus as aligned to the van Hiele's level 2-abstraction of geometric thinking

	Content focus in general	**Specific content focus for senior phase**	**Description of Van Hiele's level 2 of geometric thinking**
Space and shape (Geometry)	The study of Space and shape improves understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions and transformations of two-dimensional shapes and three-dimensional objects.	• Drawing and constructing a wide range of geometric figures and solids using appropriate geometric instruments • Developing an appreciation for the use of constructions to investigate the properties of geometric figures and solids • Developing clear and more precise descriptions and classification categories of geometric figures and solids • Solving a variety of geometric problems drawing on known properties of geometric figures and solids.	Descriptive or analytical level. • A shape is recognized and defined by its properties • Properties of shapes are established experimentally, i.e. by measuring, drawing and making models. • Learners discover that some properties of shapes combined define a figure and some do not

Table 3.2 shows a summary of how South Africa’s geometry curriculum is linked to the van Hiele levels of geometric thinking. This gives a picture of how important it is for the teachers to consider the van Hiele framework of geometry as a guide when structuring teaching and learning activities.

According to Klausmeier (1992), the instruction should incorporate both expository and discovery methods for it to be more effective. Discovery learning is mediated by the use of physical manipulatives and they help with the construction of good representations of geometry concepts (Clements & Battista, 1992). This implies that if physical manipulatives are well incorporated into the teaching and learning can help learners in developing the higher levels of geometric thinking. It is in this view of what the researchers say in this paragraph that I have decided to design the intervention activities the way they are presented so that the identified learners operating at a lower levels of Van Hiele’s levels of geometric thinking than expected can be helped to move up and those at the appropriate expected level can be helped to reinforce their conceptual understanding of geometry.

3.3.3.5 Phase three: Observation technique

An observation schedule which I designed with its criteria aligned to the van Hiele’s (1999) levels of geometric thinking guided by the focus of my research question was used to collect data and evaluate learners’ performance. For the details of the contents in the observation schedule, refer to appendix 28.

This study implemented an observation technique of collecting data in order to have an opportunity to collect existing data from lived situations (Cohen et al., 2000), which gives more meaning about the issue being studied. The observations of learners working with physical manipulatives in the intervention programme were done daily in order to make rational decision about learners' learning of geometry. Since "qualitative research emphasises on process rather than outcomes" (Clarke& Ritchie, 2001:309), the emphasis of the collected data is in a form of a "thick description" (Clarke & Ritchie, 2001:277).

Clarke and Ritchie (2001) define thick description as an extensive narrative that captures the sense of behaviour as it occurs in a real-life scenario. The observation data for this research were generated when the participants engaged in the intervention activities. During observations, I had to engage myself in taking notes on: daily processes of activities, the dialogue between a learner and a learner and as well as a learner and me as a researcher. The notes were also taken form how each learner behaved and interacted in all the episodes. I also had to capture voice recordings during interviews to ensure that the details of events were not lost to memory during the time of interpretation of data (Mulhall, 2003).

3.3.3.6 Phase four: Post-test

After the intervention activities, learners were to write a post-intervention test that provided an opportunity for me to analyse the influence of the use of cut polygons in the teaching and learning of geometry. The post-intervention test was similar to the diagnostic test in content. Refer to appendices 10 and 11 for their content details. In the post-intervention test, learners worked under the same conditions as those of the diagnostic tests, for example, finding solutions to the questions without the use of neither physical manipulatives nor mathematics dictionary, working individually and working exactly 0.7 hours.

3.3.3.7 Semi-structured interviews

Soon after the post-intervention test, all nine learners were engaged in semi-structured interviews individually (each session took 0.2 hours) for the following reasons:

(i) For the exploration of their experiences in the use of polygons pieces as physical manipulatives in the teaching and learning of geometry.

(ii) To make the best use of the complexity and abundance of the data to address the research question (DiCicco-Bloom & Crabtree, 2006).

(iii) To capture what a learner was thinking at a particular moment, this made it likely for me to understand what learners conceptually understood, enjoyed or distasted and thought (Tuckman, 1972) regarding the intervention programme.

(iv) To help in exploration and understanding of learners' feelings in learning geometry using polygons pieces. The leading questions of the semi-structured interview were designed based on the research questions' focus. Details of the questions contained in the semi-structured interviews are shown in appendix 29.

3.3.4 Analysis of data

Feza (2015) defines data analysis as a way in which a researcher uses the collected data to search for meaning from the observed situation. My data analysis was done manually in order for me to acquire a clear understanding of the collected data. Since my research adopted the mixed methods approach, the collected data have been analysed quantitatively, i.e. intervention activities scripts, diagnostic and post-intervention tests scores have been analysed as percentages for ease of interpretation. In addition, the data have been presented in bar and line graphs.

On the other hand, the qualitative data have been analysed in three different ways:

(i) Using thematic analysis (Feraday & Muir-Cochrane, 2006). Braun and Clarke (2006) define thematic analysis as the method aimed at systematically identifying, analysing and giving a report of the themes identified in the collected data.

(ii) Constant comparison analysis (Tesch, 1990). In this method, the researcher reads through the collected data or critically watches video clips or observes photos in order to extract prominent themes from such a data (Leech & Onwuegbuzie, 2007).

(iii) Keywords in context analysis. This analysis exposes how the words have been used in context (Fielding & Lee, 1998) by matching words "that appear before and after keywords" (Leech & Onwuegbuzie, 2007:566).

According to Lacey and Luff (2009), my data analysis has to follow the six phases of thematic analysis:

(i) Familiarisation

In order to familiarise myself with the collected data, I read it repeatedly, listened attentively to the audio tapes that were used to collect data. In addition I also read the notes critically in order to make memos and summaries of the data. I also did a thorough analysis of the numerical data in order to seek trends and interactions in the data by considering a wide range of measures of dispersion.

Transcription of data – the data obtained from the recorded semi-structured interviews, and handwritten observation notes have been transcribed into a thick description. A comprehensive analysis of the transcribed semi-structured interviews was done using the 'keywords-in-context analysis', that is an analysis that exposed how the words have been used in context (Fielding & Lee, 1998) by matching words "that appear before and after keywords" (Leech & Onwuegbuzie, 2007:566).

(ii) Generating initial codes

Coding is part of data analysis that is done (Miles & Huberman, 1994) as the collected data is organised into meaningful categories (Tuckett, 2005).

The data from the diagnostic, the post-intervention and daily tests was analysed quantitatively through the use of graphs and tables comparing how learners performed initially in the diagnostic test and in the post-intervention test. In addition, I had to illustrate how individual learners performed during each episode of the intervention activities. After the comparison, the statistical information was qualitatively analysed in the form of descriptive scripts giving details on learners' performance in relation to the van Hiele levels of geometric thinking.

Data from the diagnostic, post-intervention tests and semi-structured interviews have to be organised by putting them into retrievable sections, i.e. by giving each interviewee a code for easy analysis. Secondly, observations notes were to be categorised into sections as per recorded date. The narrative data were number coded for easy tracing of the originality of the context when needed at some later stage during the research processes.

After number coding names and other identifiable materials were removed from the transcript to ensure anonymity. However, in order to identify the source of data later, each set of data was attached to the anonymised identifier. Furthermore, to avoid loss of data, the hard copies were made into several copies and stored independently until the end of the analysis of data. These copies should then be destroyed at a later stage.

The unstructured observations were in a form of notes recorded directly and jotted down in phrases from key events and dialogue; these were written up in a more detailed form in a private space for confidentiality (Mulhall, 2003). In order to do an analysis of my observations as useful as possible, the description of unstructured observations was in the form of thick descriptions to provide a more detailed and nuanced account for the identified keywords in context (Fielding & Lee, 1998). On the other hand, the data from the structured observations where the observation schedule with the criteria was aligned to the van Hiele levels the analysis was done in a form of thick description of how learners engaged in van Hiele's levels of geometric thinking in the intervention programme that made use of the polygon pieces to learn geometry.

At this stage, I coded different sets of information from the transcript and semi-structured interviews using highlighters of different colours. I also coded using numbers, titles and descriptions of my choice in order to look for predominant and repeated themes in the research study.

(iii) Searching for themes

According to Feza (2015), thematic analysis allows a researcher to identify themes that are prominent in the data. Themes are searched when the collected data have been initially coded and collated (Braun & Clarke, 2006). The identified codes are systematically sorted into possible themes and ordering all the significant coded data excerpts were ordered within the documented themes. The sorting of codes into themes was done in the form of a table, mind map or theme piles.

In order to uncover themes that lie in the data, I conducted thematic analysis of the collected data. Each sentence of learners' responses was number coded, for example as 1:2, which is interpreted as: 1 represents L1 and 2 is for the sentence number. This was to ensure that the first step in thematic analysis was done as described by Glesne (2006) that coding is the first step to do. After that, each sentence was annotated with specific annotations;

annotations are low-inference phrases that summarise each row (Ely, Anzul, Friedman, Garner & Steinmetz, 1991) for conducting descriptive analysis.

The identified annotations were colour coded according to their similarities and differences to allow moving on to the next stage of grouping the annotations together. By rigorously engaging with different sets of annotations the themes surfaced and then similar annotations were grouped under each theme as shown in table 26 in chapter four.

(iv) Reviewing of themes

In this section, I used the list of coded information to re-code and identify well-defined ideas that fell into the same category and were grouped together under a certain theme. For example, in semi-structured interviews, I have to extract similarities and differences in learners' responses to the interview questions. Themes were also well refined (Braun & Clarke, 2006), and those without enough supporting data were identified and discarded. This was to help in eliciting how the polygon pieces and use of mathematics dictionary influenced the teaching and learning of geometry for learners' conceptual understanding.

(v) Defining and naming themes

Themes that were used to represent and analyse the collected data were refined and defined. The refined themes highlighted the aspect of collected data that each one of them captured and also their own importance within the data (Braun & Clarke, 2006). Themes were considered with respect to how they related to each other within the data. In addition, the identification of sub-themes was done in this phase. Sub-theme is a theme within a theme; the process of identifying sub-themes helped to present the hierarchy of significance within the collected data (Braun & Wilkinson, 2003).

(vi) Producing the report

The information in the report is presented in a concise, coherent and logically interesting way with no repetition of statements (Braun & Clarke, 2006). The presentation is based on the main themes captured during the data analysis. Different extracts that captured the essence of the point were highlighted or put across to capture the attention and give meaning to the reader.

As much as this study has taken into consideration the dos of thematic analysis, it avoided the don’ts as Braun and Clarke (2006) propose that there are five pitfalls to avoid. These pitfalls are briefly described below:

(i) The researcher should avoid failure to analyse the collected data. This implies that what the data means must actually be brought to light with all supporting evidence based on literature.

(ii) Research questions should not be categorised as themes. The implication is that the researchers’ themes must emerge from the data and their patterning must make sense to the readers.

(iii) Unconvincing analysis due to the overlapping of almost all the themes. This means there should be adequate evidence from the collected data in order to present the analysis that is clear and catches the eye.

(iv) In compatibility between the data and analytic claims, vivid examples must be used to support the theme.

(iv) Mismatch between the research questions and analysis used to interpret the data. To do an analysis of each of the emergent themes, I followed Bazeley's (2009) simple three-step formula: describe-compare-relate.

Some photos and video clips that were taken during the research process will be systematically observed, interpreted and analysed using constant comparison analysis (Tesch, 1990). The constant comparison analysis also gives an opportunity to categorise the collected data into similar or different sentences or words that are easy to interpret. This method also promotes member-checking just to confirm with the participants if the interpretation of the data is not diverted from the actual description of the participants (Merriam, 1998; Leech & Onwuegbuzie, 2007).

Table 3.3 shows how my research considered the tools that were needed in the collection of data, the purpose of each research instrument and the type of data that was obtained as I engaged through the research processes.

Table 3.3: The summary of my research process

PHASE	**TOOLS**	**PURPOSE**	**DATA**
1	Administering diagnostic test	To identify alternative conceptions and misunderstandings that learners have regarding geometry, with regard to properties of the triangles.	Quantitative and qualitative results. The data will be used to design the intervention activities.
2	Engaging learners in intervention activities and tests: (a) use of dictionaries (b) use of physical manipulatives	(a) To establish the meanings of concepts such as an angle, line segment (b) To help the learners to identify properties of the triangles (c) To establish the trend of learning from one episode to another	Qualitative results Naturalistic Quantitative results from the tests
3	Observation	To explore how learners work with physical manipulatives to identify properties of triangles	'Thick description' of how learners worked and achieved when working with physical manipulatives. Descriptive data obtained from observation schedule and videos.
4	Post-intervention test	To analyse the influence of the use of physical manipulatives in learning of properties of the triangles.	To get quantitative and qualitative data Descriptive data
5	Semi-structured interviews	To explore how learners feel about the use of physical manipulatives in learning properties of the triangles. To explore how physical manipulatives have helped learners to learn geometry.	Qualitative results in the form of interview transcripts.
Time taken for each data gathering research session Pilot = 0.7hours Diagnostic test = 0.7hours Reflective tests (7 x0.5hrs) = 3.5 hours Intervention activities (1 hr x 9) = 9 hours Post-test = 0.7 hours Interviews (0.2 hrs x 9) = 1.8hours Total time taken = 16.4hours			

3.3.5 Ethical issues

Research suggests that the value of the best research is likely not to cause injury to a person engaged in it, the researcher's manner of doing the research should be good and ethically strict (Stake, 2000; Terrell, 2011). Hence, to conduct this study ethically, the steps explained below were followed.

Since this study involved eighth-grade learners, letters were written to the school governing body through the school principal, Queenstown Department of Education and parents of the participants asking for the consent to do the research at the selected school. In order to avoid disturbance of the school programme, the research was conducted after school hours. Appendices 1, 3 and 5 attest to this.

To ensure the issue of bias in selecting the nine learners from the cohort of 56 volunteers the purposeful selection was based on their performance in the diagnostic test (low, middle and high achievers).

For each learner involved in the study special codes were used instead of their names for anonymity purposes and this was communicated to their parents/guardians in writing. Refer to appendix 5. Furthermore, the school's name was considered anonymous. To avoid the abuse of power by researcher over the participants during the research, both participants and parents were informed that the members of the sample had the freedom to withdraw from the project at any stage. Due to the fact that learners were the participants in this research study, they filled in a consent form with the conditions mentioned in this paragraph, for details refer to appendix 7.

In order to ensure that the diagnostic and post-intervention tests were ethically free, ethical clearance was obtained from the University of South African (UNISA).For the details contained in the ethical clearance certificate refer to appendix 8. In addition, I have made it a point that the writing of my research report is free of bias towards any of these aspects: age, ethnicity, sexual orientation, race, gender, etc. Lastly, the report covers every aspect in detail so as to give interested readers the opportunity to critic its originality and ethical quality if they want to.

3.3.6 Validity

According to Fraenkel and Wallen (2006) validity is seen when the research measures that which it is supposed to measure. In simple terms it is the accurateness of the research processes and outcomes. Wiersma (1991) adds that validity involves two concepts concurrently:

(i) The extent to which the results can be accurately interpreted.

(ii) The extent to which the results can be generalised to populations and conditions. The former concept is called internal validity, and the latter is external validity. (p. 4).

The two outlined concepts of validity have threats as the research study progresses. The solutions to these threats are described in sections 3.3.6.1 and 3.3.6.2 below.

3.3.6.1 Internal validity

According to Le Compte and Goetz (1982), threats to internal validity are: history and maturation, selection bias, mortality, implementation, the attitudes of the subjects, data collector bias and data collector characteristics. These threats are defined below and the solution to each one is given in detail.

History refers to incidences that may bring about a change in the overall research study setup (Fraenkel & Wallen, (2006). To avoid or minimise such incidences in my research study all the participants were interviewed during the same day. Maturation refers to changes in relationships of individuals due to the passage of time or progressive development of the individuals (Fraenkel & Wallen, 2006). To reduce the effects of maturation on this study, I decided to follow a case study design which provides the opportunity of studying a particular phenomenon within a restricted time frame (Bells, 1993).

Selection bias refers to the situation where the participants in the research are different from each other in terms of age, gender, ability, etc. (Fraenkel & Wallen, 2006). The effect of this threat was dealt with by the use of diagnostic scores for purposeful sampling, regardless of gender (Marshall, 1996).

Fraenkel and Wallen (2006) define mortality as the changes in sample size as a result of dropout participants. In this study, purposeful selection helped in the selection of those who seem to be more willing to participate and committed.

Data collector characteristics like age and gender affect the results of the research study. To deal with this threat throughout the research study, data collection from both genders was done by the research (Fraenkel & Walle, 2006).

Data collector bias is when there is unintentional distortion of collected data which poses a threat to the research study (Fraenkel & Walle, 2006). To avoid this threat to occur Fraenkel and Wallen (2006)'s double blind technique was applied. According to WordNet 3.0 (2003 – 2012) double blind technique is defined as:

> An experimental procedure in which neither the subjects of the experiment nor the persons administering the experiment know the critical aspects of the experiment. A double blind procedure is used to guard against both experimenter bias and placebo effects.

Implementation is also another threat to the internal validity. To avoid this threat, it was ensured that the conditions were standardised throughout the research study. For example, the diagnostic test was piloted with 28 learners to ensure that the standard was consistent with the grade 8 mathematics syllabus and that all the questions were unambiguous.

If there is an ambiguity or lack of clarity with respect to participants' written work of their own processes, individual participants were informally interviewed in order to get illuminated oral explanation. Member checking was used as a system of external validation (Lewis & Ritchie, 2003).

3.3.6.2 External validity

According to Le Compte and Goetz (1982), there are three factors to be considered as threats to the external validity: history effects, setting effects and construct effects. In order to minimise the mentioned threats to the transferability of my research results to the populations and conditions, below are the strategies and procedures to be considered are set out below.

History effects, according to Serow (2002) it simply means the background of the participants must be known and acknowledged. My research sample was purposefully selected from a cohort of 56 eighth-grade learners based on the diagnostic test results, i.e. three were from the group that ranges from 0% to 18.4%, three from the group that have a scored range of 18.5% to 29,6% and the last three were from the group that have scores ranging from 29.7% to 45% range. In addition, the demographic characteristics of the participants in this study were reported, which is one of the prerequisites to avoid the threat. For demographic details refer to section 3.3.1.

Setting effects, Serow's (2002) strategy was adopted although it was done in a different country. But I ensured that all the participants in this study were enrolled in a South African high school system and were taught the same mathematics content.

Construct effects. This is the degree to which abstract expressions, overviews, or connotations are shared across times, sceneries, and populations (Le Compte & Goetz, 1982). All the participants were familiar with the mathematics frameworks chosen as they are described as geometry content within the syllabus of the Department of Basic Education [South Africa. DBE] (2009). The diagnostic and post-intervention tests were piloted using learners from the same research site. The observation schedule, semi-structured interview, diagnostic test and post-intervention tests were assessed and ethically cleared by the University of South Africa's ethics review committee. Intervention activities were structured in relation to the outline of van Hiele's geometric thinking levels activities as proposed by Crowley (1987).

3.3.7 Reliability

According to Bloor and Wood (2006) reliability is the degree to which a research findings remain the same when collected data are analysed several times by different researchers. Reliability is categorised into two: internal reliability and external reliability. These are described in detail below.

3.3.7.1 Internal reliability

To ensure that the same results would be found if other researchers are given a chance to replicate the research processes, I adopted two methods of reliability by Serow (2002) – low inference descriptors and mechanically reported data. According to Serow (2002:105):

> Low inference descriptors refer to the precise and descriptive accounts of findings, which allow for the accurate presentation of evidence. This presentation should provide the reader with means to reject or accept the findings based upon the richness of the material presented.

Based on the above quotation, this research study employed a range of data collecting instruments which includes: diagnostic and post-intervention test scripts, transcribed response to semi-structured interviews. Refer to appendix 30 for the details of the semi-structured interviews questions. For daily intervention activities questions that individual learners correctly answered, refer to appendices 31. For daily reflective tests questions that individual learners could not correctly answer, refer to appendix 32. When inferences were put together into groups based on similarities and differences, it was easy to identify codes (Feza, 2015). The advantage of low-inferences analysis gave the researcher the opportunity to do member checking in order to confirm whether the interpreted data was still relevant to what was collected or whether the information was altered as compared to the original data (Merriam, 1998; Leech & Onwuegbuzie, 2007).

Mechanical recording equipment was to be utilised in voice recording and video recording during interviews and in video recording during the interventions activities sessions. This is aimed at helping when coding and for future use by any researcher who might require the information in detail.

3.3.7.2 External reliability

To ensure the external reliability of my research findings, suggestions of Lacey and Luff (2007) and Wiersma (1991) were considered, therefore, the methods and processes for data analysis were well documented so that other researchers can follow the process in the form of an audit trail at any time after my research has been completed. I also justified the appropriateness of my analysis within the context of my study.

3.4 Conclusion

In this chapter, I presented a detailed account of how my research study which was informed by the mixed methods paradigm. The aim mixed methods gave me the opportunity to scrutinise and understand the complexity of the phenomenon under study at a deeper level to ensure that there is strong correlation between the interpretation of research findings and the usefulness of research findings (Collins et al., 2006). In addition, included is a description of the research design which is comprised of (i) the methods used to collect data, (ii) sample selection, (iii) sampling techniques, (iv) description and advantages of the instruments used in collecting datato answer my research question, (v) a detailed description of how the diagnostic and post-intervention tests were developed and validated to ensure that they are at an appropriate level and relevant standard for the target group, (vi) the analysis of data, (vii) the ethical issues, how they can be dealt with so that no one is injured in any form and (viii) content validity (internal and external) and reliability (internal and external).

In the next chapter, I present a detailed descriptive analysis of the collected data.

CHAPTER FOUR: RESULTS

4.1 Introduction

According to my research findings this chapter represents a model with three components:

(i) Ways on how to improve learners' mathematics vocabulary and terminology proficiency in mathematics using the mathematics dictionary. This responds to the first question of this study: "How will the use of polygons pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning of geometry influence learners' conceptual understanding of geometry concepts, specifically properties of polygons?"

(ii) Insights on how do polygon pieces assisted by mathematics dictionary develop geometric knowledge and understanding of learners. In responding to the second question: "How can polygon pieces be used as physical manipulatives assisted by mathematics dictionary to influence the teaching and learning of angle measurement in geometry for learners' conceptual understanding?"

(iii) Suggestions to teachers and researchers on how to use these polygon pieces assisted by mathematics dictionary to promote the learning of geometry.

This chapter gives the comprehensive outcomes of my research and themes that emerged during the analysis process.

4.2 Results

In this section, I present the results of how individual learners performed in the diagnostic test as compared to the post test and how they developed their mathematical concepts with the help of the intervention programme I designed. From the results of my research five themes were identified themes based on the similarities and differences in the collected data. The themes were singled out during the data analysis process. The identified themes seem to be of much influence to the designed model of teaching and learning geometry.

Below are five identified themes that emerged from the intervention activities, observations and transcribed interviews:

Theme 1: Mathematics dictionary, a tool for making meaning

Theme 2: Polygon pieces assisted by mathematics dictionary mediating conceptual understanding

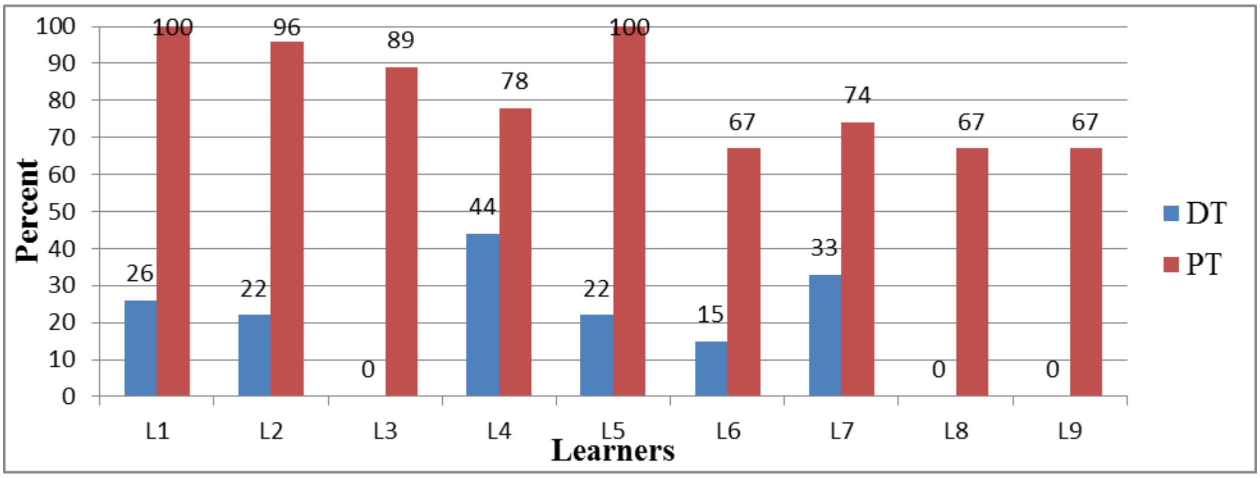
Theme 3: Language incompetence influencing meaningful learning

Theme 4: Polygon pieces assisted by mathematics dictionary unpack meaning and stimulate interest

Theme 5: Polygon pieces assisted by mathematics dictionary encourage active learning and long-term gains

Figure 4.1 below presents a comparative summary of learners' results in both the diagnostic test and the post-test.

4.2.1 Overall results of the diagnostic and post- tests



DT = Diagnostic test; **PT**= Post-Test **L**= Learner

Figure 4.1: Diagnostic test and the post-tests results.

The comparison of the diagnostic test and post-test results is illustrated in Figure 4.1 above. The results show that each learner's post-test results improved after being engaged in the intervention programme that made use of the polygon pieces in teaching and learning geometry. Two learners (L1 & L5) in the diagnostic test obtained 26% and 22%, respectively, but in the post-test they both obtained 100%.

L2 scored 22% in the diagnostic test, but in the post-test moved up to 96%. In another group of three learners (L3, L8 & L9), each learner obtained 0% in the diagnostic test, but in the post-test, they obtained 89%, 67% and 67%, respectively. The last three learners (L4, L6 and L7) initially obtained 44%, 15% and 33%, respectively, but their post-test marks were: 78%, 67% and 74%, respectively.

The route to such an improvement for each learner has been an up-and-down trend throughout the intervention programme they were engaged in (the intervention programme was comprised of nine intervention activities and seven reflective tests). The nine graphs below illustrate how each of the learners developed in the teaching and learning episodes throughout the intervention programme to the post-test.

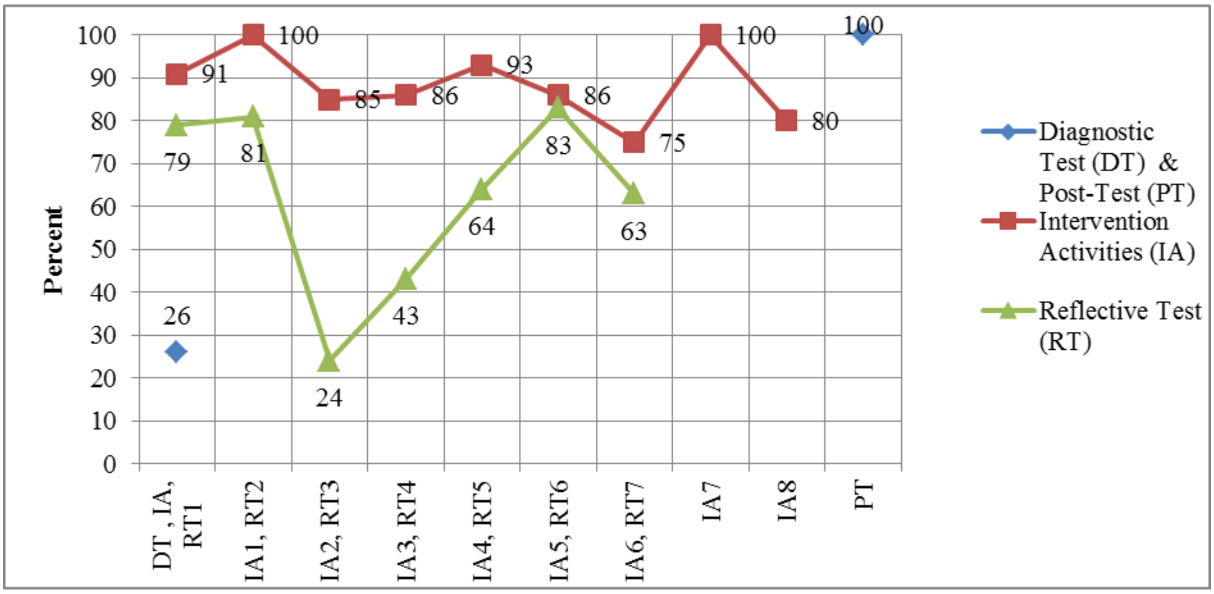


Figure 4.2 L1's developmental pattern throughout the intervention programme.

Figure 4.2 shows L1's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme. L1 achieved 26% in the diagnostic test because of failure to perform as per the levels suggested by van Hiele's (1999) model of thinking. L1 did not do well in the question described under each of the three levels of van Hiele's geometric thinking: Level 0 (visualisation), the questions under this level are: 1.2(i), 1.3(i), 1.4(iii) and 1.5(i). These four questions required learners to use visual skills to determine the properties of the given triangles, but L1 failed; therefore this learner performed at pre-recognition level as suggested by Clements and Battista (1991).

Level 2 (abstraction), the questions that fall under this level are: 1.1(ii) and (iii), 1.2(iii), 1.4(ii) and 1.5(ii). In each of the listed questions, learners had given triangles based on their properties. Level 3 (formal deduction): the question under formal deduction that L1 could not perform well is 1.5(iii). This question required the learners to think logically to provide the properties of triangles. Since this could not give a correct response to any one of the listed

question above, L1 performed at pre-recognition level hypothesised by Clements and Battista (1991) in these questions.

After a series of intervention activities which included the use of the mathematics dictionary and polygon pieces, L1 managed to respond to the very same question in the post-test, which could not be answered correctly in the diagnostic test. The 26% mark in the diagnostic test to 100% in the post- test, shows that L1 was able to operate at all four levels of geometric thinking hypothesised in the van Hiele theory. The levels addressed in each of the questions in the intervention activities and reflective tests were: level 0-visualisation, level 1-analysis, level 2-abstraction and level 3-formal deduction.

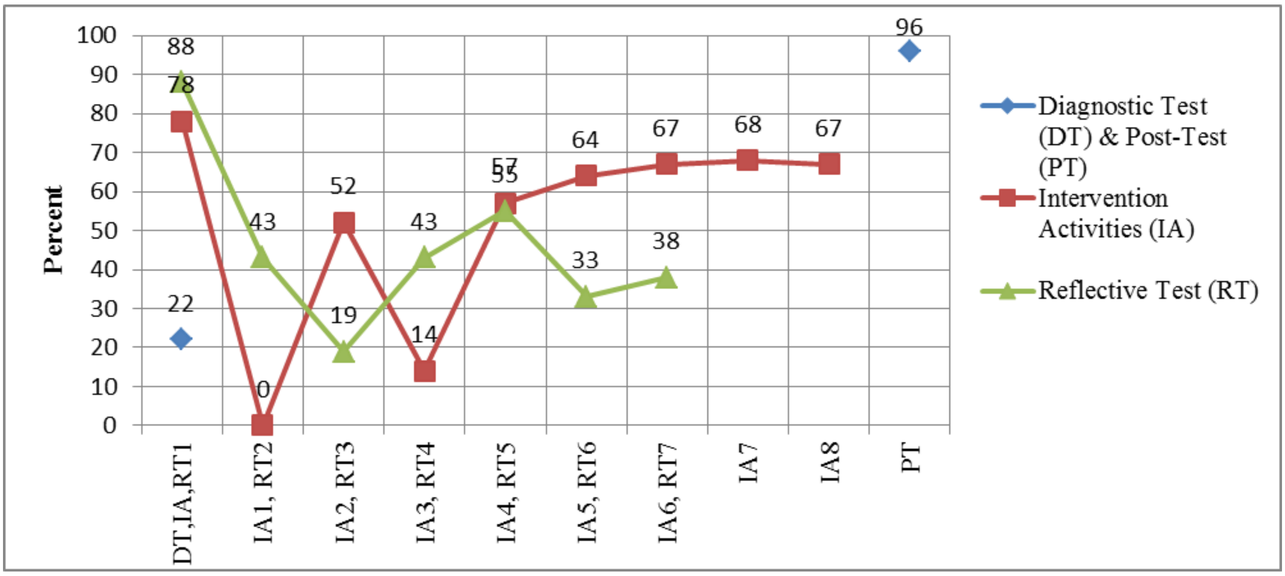


Figure 4.3 L2's developmental pattern throughout the intervention programme.

Figure 4.3 is L2's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme. In the diagnostic test, L2 scored 0% as shown in Figure 4.3 the reason being that the learner could not perform in questions that were at different levels of thinking in geometry according to van Hiele's (1999) model. Therefore, this learner was operating at the pre-recognition level as described by Clements and Batista (1991). The questions at visualisation level (level 0) are: 1.3(i), 1.4(iii) and 1.5(i). The question at level 1-analysis is: 1.3(ii).Questions at levels 2-abstraction are: 1.1(iii), 1.2(ii), 1.4(ii) and (iii), 1.5(i) and (ii).Questions at level 3-formal deduction are: 1.3(i), 1.4(ii) and 1.5(iii). In the post-test, L2 got all these questions correct because the use of polygon pieces allowed the learners to be able to use visual skills, analyse, work on abstract questions and be able to deduce mathematical ideas from a given scenario.

L2 was now able to perform at distinct levels of the van Hiele theory described in this paragraph for a given set of questions. The improvement in performance by L2 from22% in the diagnostic test to 96% in the post test as shown in figure 4.3shows how polygon pieces in the intervention activities and reflective tests helped L2.These helped L2 to migrate in geometric conceptual understanding from the pre-recognition level depicted by Clements and Battista (1991) to various prescribed levels of the van Hiele theory in different questions.

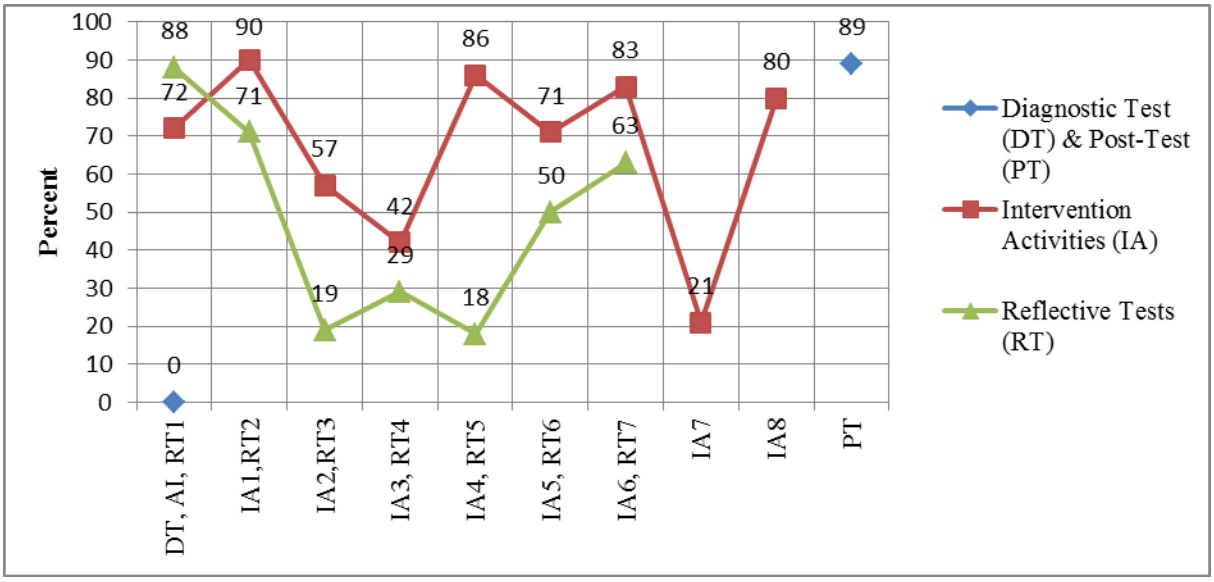


Figure 4.4 L3's developmental pattern throughout the intervention programme.

Figure 4.4 shows L3's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme. The questions below are the ones that L3 could not answer correctly, that led to the score of 0% in the diagnostic test. The questions belonged to each of the first four levels (visualisation, analysis, abstraction and formal deduction) of the van Hiele's theory. In the whole diagnostic test, L3 demonstrated the thinking that was at level 0-pre-cognition as theorised by Clements and Battista (1991). Questions at level 0-visualisation: 1.1(i), 1.2(i), 1.3(iii), 1.4(i), 1.4(iii), 1.5(i) and 1.5(iii).The questions at level1-analysis is: 1.3(ii); questions at level 2-abstraction: 1.1(i), 1.1(iii), 1.2(ii), 1.2(iii), 1.4(ii) and 1.5(ii); and questions at level 3-formal deduction, are: 1.4(ii) and 1.5(iii).

After the intervention episodes that made use of polygon pieces L3 scored 89% in the post-test as compared to 0% in the diagnostic test. Later in the post test, L3 improved by the help of the polygon pieces and the use of mathematics dictionary for vocabulary proficiency. Questions under the mentioned geometry levels of thinking suggested by the van Hiele model that could not be answered earlier on were now responded to confidently. This shows that in each one of the mentioned questions above, L3 migrated from the pre-recognition level of Clements and Battista (1991) to the expected van Hiele levels of geometric thinking such as level 0-visualisation, level 1-analysis, level 2-abstraction, and level 3-formal deduction.

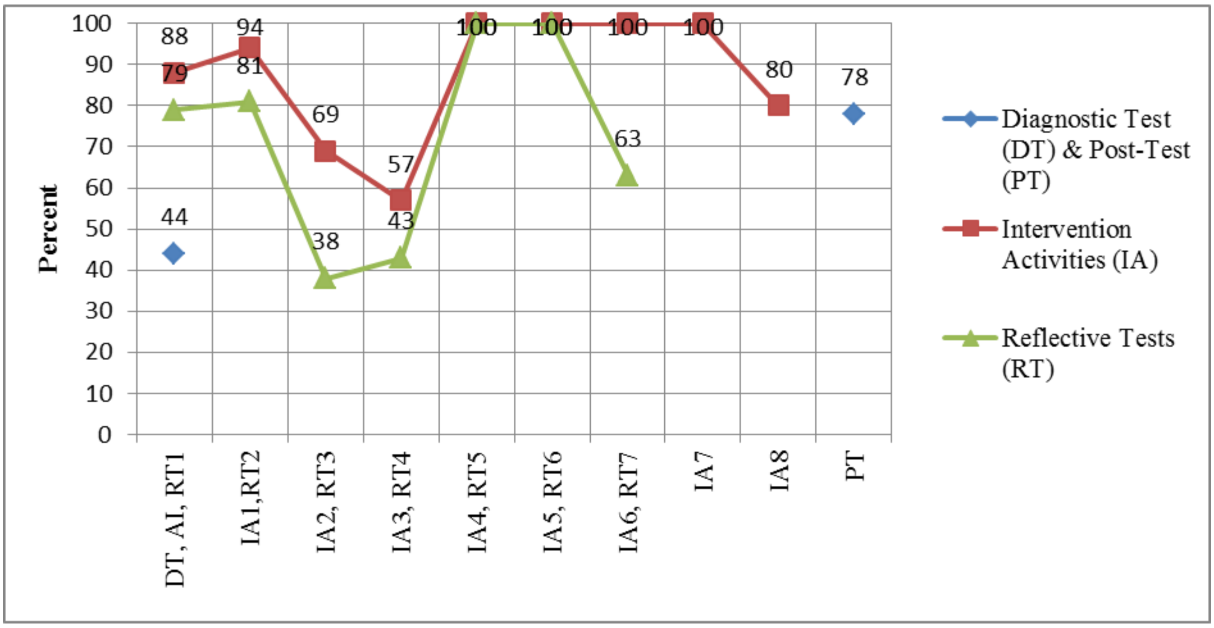


Figure 4.5 L4's developmental pattern throughout the intervention programme.

L4's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme are shown in Figure 4.5.A percentage of 44% is the mark that L4 scored in the diagnostic test in. The reason for such a low mark is that some of the questions that were at level 0 (visualisation) and level 2 (abstraction) of the van Hiele model of geometric philosophy were not answered correctly. This indicates that in those specific questions L4 was operating at level 0 as posited by Clements and Battista (1991). The following are the categories of the questions that the learner could not answer correctly. Questions at level 0-visualisation: 1.1(i), 1.2(i) and 1.5(i); questions at level 2-abstraction: 1.1(iii) and 1.2(ii).

After the series of intervention episodes that made use of polygon pieces, L4 obtained 78% in the post-test; this shows that learner was now able to use visual skills to determine the properties of triangles and to recognize given triangles based on their various properties. This showed a stride up to levels 0-visualisation and level 2-abstraction as stated by the van Hiele model of geometric thinking. The use of polygon pieces and mathematics dictionary contributed to such an improvement.

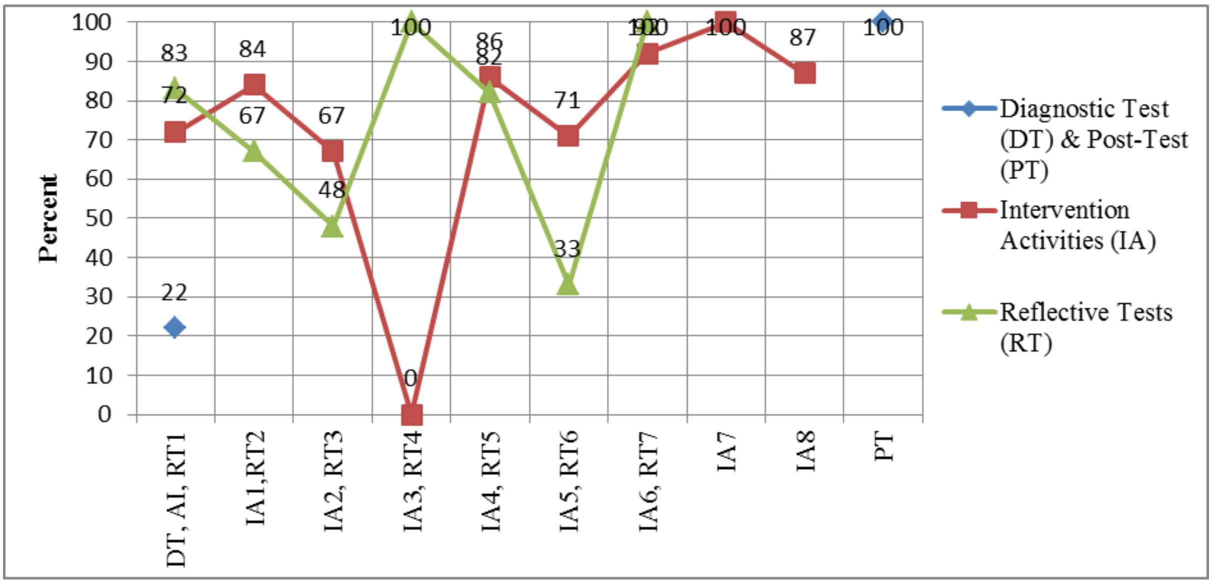


Figure 4.6 L5's developmental pattern throughout the intervention programme.

Figure 4.6 illustrates L5's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme. In the diagnostic test, L5 obtained 22% as shown in Figure 4.6. Questions that contributed to this low mark were those of level 0-visualisation, level 2-abstract and level 3-formal deduction of the van Hiele theory. This means that, conceptually L5 was operating at level 0-pre-recognition as put forward by Clements and Battista (1991). The details are as explained below: Questions at level 0-visualisation: 1.1(i), 1.2(i), 1.3(i), 1.3(iii), 1.4(i) and 1.5(i). Questions at level 2-abstraction: 1.1(iii), 1.2(ii), 1.2(iii), 1.4(ii) and 1.5(ii); questions at level 3-formal deduction: 1.3(i), 1.4(ii) and 1.5(iii). After the intervention activities that made use the polygon pieces and a mathematics dictionary L5scored 100% in the post-test. This is an indication of how the learner was now able to operate at the level 0-visualisation, level 2-abstraction and level 3-formal deduction after the polygon pieces helped to clarify some conceptually misunderstood ideas.

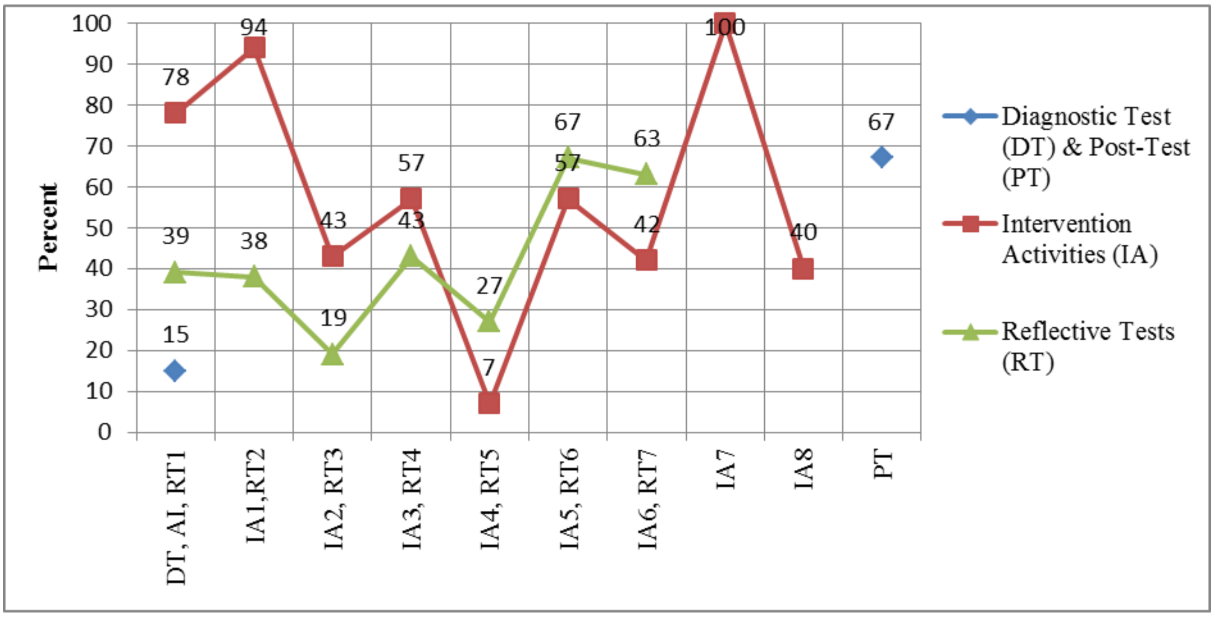


Figure 4.7 L6's developmental pattern throughout the intervention programme.

L6's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme are presented in Figure 4.7.As shown in Figure 4.7, L6 obtained 15% in the diagnostic test. This learner could not operate at three of the levels of van Hiele's (1999) geometric thinking model, namely level 0-visualisation, level 2-abstraction and level 3-formal deduction. The questions that L6 did not do well are categorised as follows: At level 0-visualisation, questions: 1.1(i), 1.2(i), 1.3(iii), 1.4(i) and 1.5(iii); questions at level 2-abstraction: 1.1(ii) and (iii), 1.2(iii) and (iii), 1.4(ii) and 1.5(ii); and question at level 3-deduction: 1.4(ii) and 1.5(iii). Intervention teaching and learning episodes that made use of polygon pieces and mathematics dictionary helped to address the challenges L6 had. As a result of this, this learner obtained 67% in the post-test, the reason being that the questions highlighted earlier on in this paragraph were also answered correctly. This gives us an idea that, initially L6 operated at pre-recognition level as suggested by Clements and Battista (1991), but the 67% mark illustrated that per each question mentioned above the learner was now able to operate at the respective levels of the van Hiele philosophy of geometric thinking, which were level 0-visualisation, level 2-abstraction and level3- formal deduction.

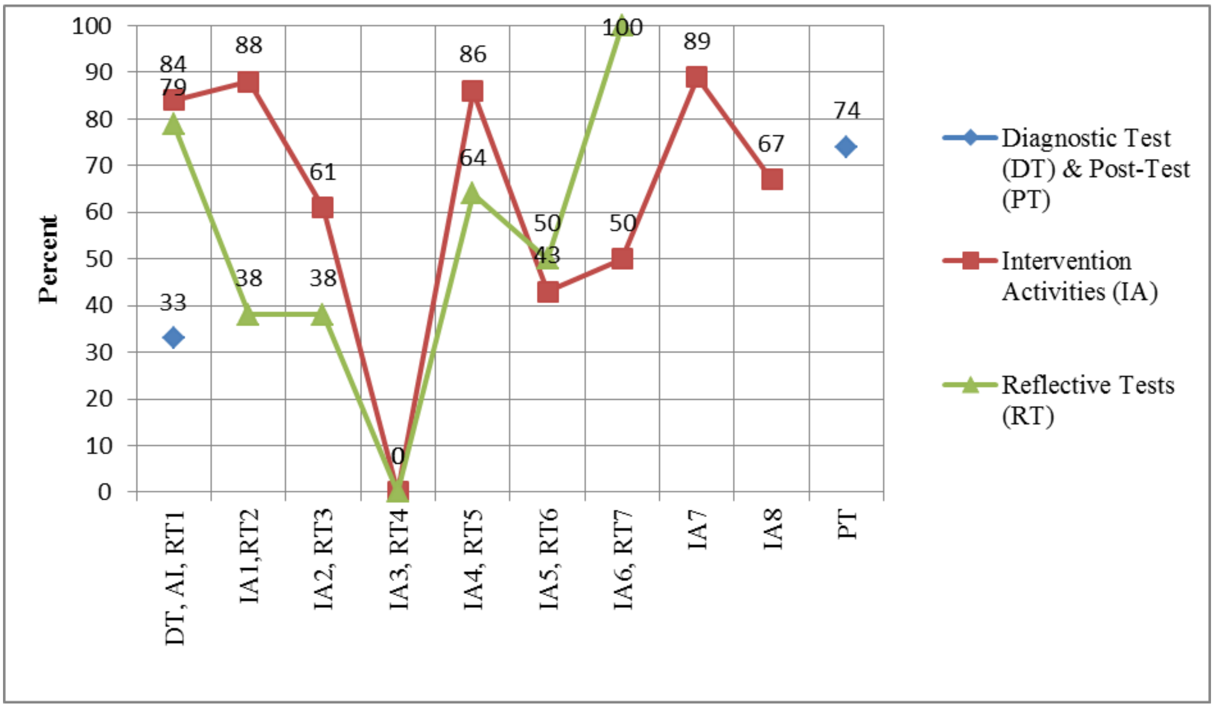


Figure 4.8 L7's developmental pattern throughout the intervention programme.

Figure 4.8 shows L7's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme. As shown in Figure 4.8, L7 obtained 33% in the diagnostic test because the learner could not respond correctly to some of the questions at different levels of the van Hiele theory of geometric thinking, level 0-visualisation, level 1-analysis, level 2-abstraction and level 3-formal deduction. Such results are an indication that in such questions, L7 was recognised as operating at level 0-pre-recognition as assumed by Clements and Battista (1991). In the following questions L7 could not perform: Questions at level 0-visualisation: 1.2(i) and 1.4(i); question at level 1-analysis: 1.3(ii); questions at level 2-abtsraction: 1.1(ii) and (iii), 1.2 (ii) and (iii) and a question at level 3-formal deduction: 1.5(iii).

After being engaged in the interventions episodes that made use of polygon pieces L7 managed to answer all the questions mentioned in this paragraph correctly resulting there was an increase in the score of the post-test as compared to the diagnostic test. The post-test mark was raised to 74% to show that in the above questions the learner was now operating at the respective mentioned levels of the van Hiele model of geometric thinking.

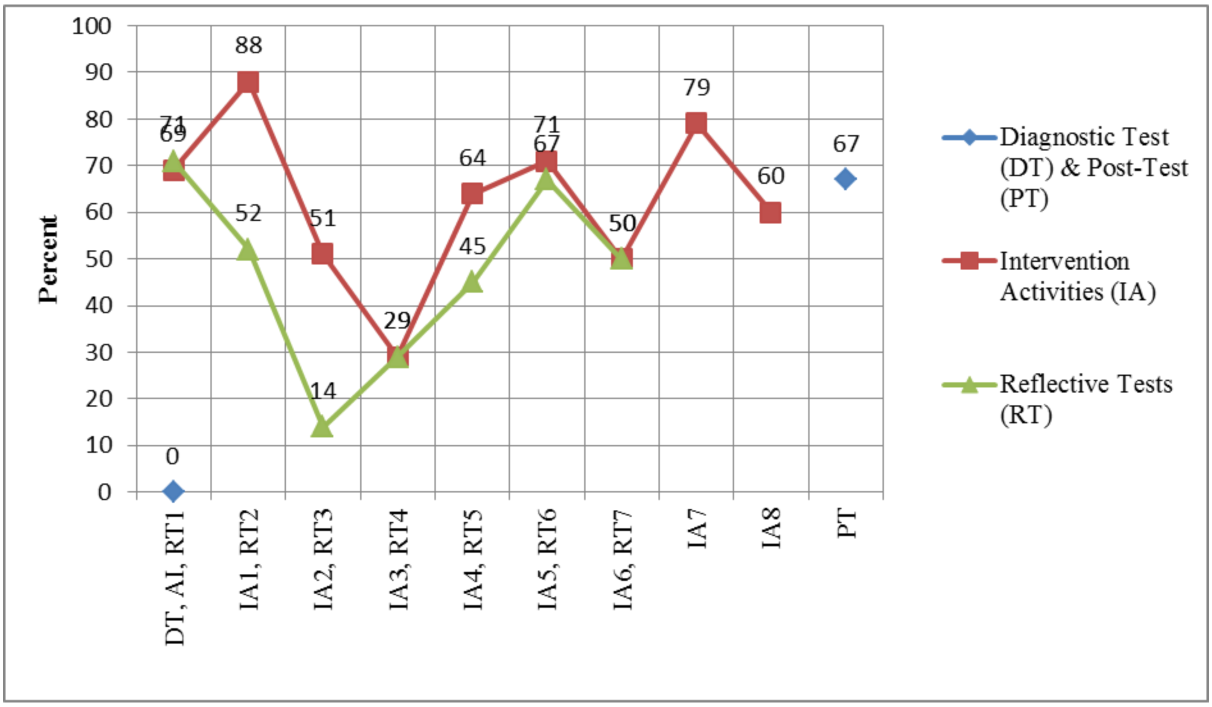


Figure 4.9 L8's developmental pattern throughout the intervention programme.

Figure 4.9 illustrates L8's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme.L8 obtained 0% in the diagnostic test. This learner struggled with questions that were identified as at level 0-visualisation, level 1-analysis, level 2-abstraction and level 3-formal deduction according to the van Hiele model of geometric thinking. These results have shown that L8's level of geometric thinking was still at level 0-pre-recognition as theorised by Clements and Battista (1991).The detailed account of the questions that L8 found difficult to answer is given below. Questions at level 0-visualisation: 1.1(i), 1.2(i), 1.3(i), 1.4(i) and (iii), these questions required learners to use their visual skills to describe triangles' properties; and a question at level 1-analysis: 1.3(ii).This question L8 had to describe the given triangles using all the properties.

Questions at level 2-abstraction: 1.1(iii), 1.2(ii), 1.4(ii) and 1.5(ii); and questions at level 3-formal deduction: 1.3(i), 1.4(ii) and 1.5(iii), required the learner to recognise triangles based on their properties. This learner was engaged to work with polygon pieces and the use of mathematics dictionary in order to be helped with the challenges identified in the questions listed in this paragraph. After the intervention activities this learner scored 67% in the post-test. The questions that posed a problem in the diagnostic test were now conceptually understandable. In those questions L8 was now operating at level 0-visualisation, level 1-analysis, level 2-abstraction and level 3-formal deduction according to the van Hiele model of geometric thinking. The understanding of questions at different levels of the van Hiele theory helped to improve L8's performance.

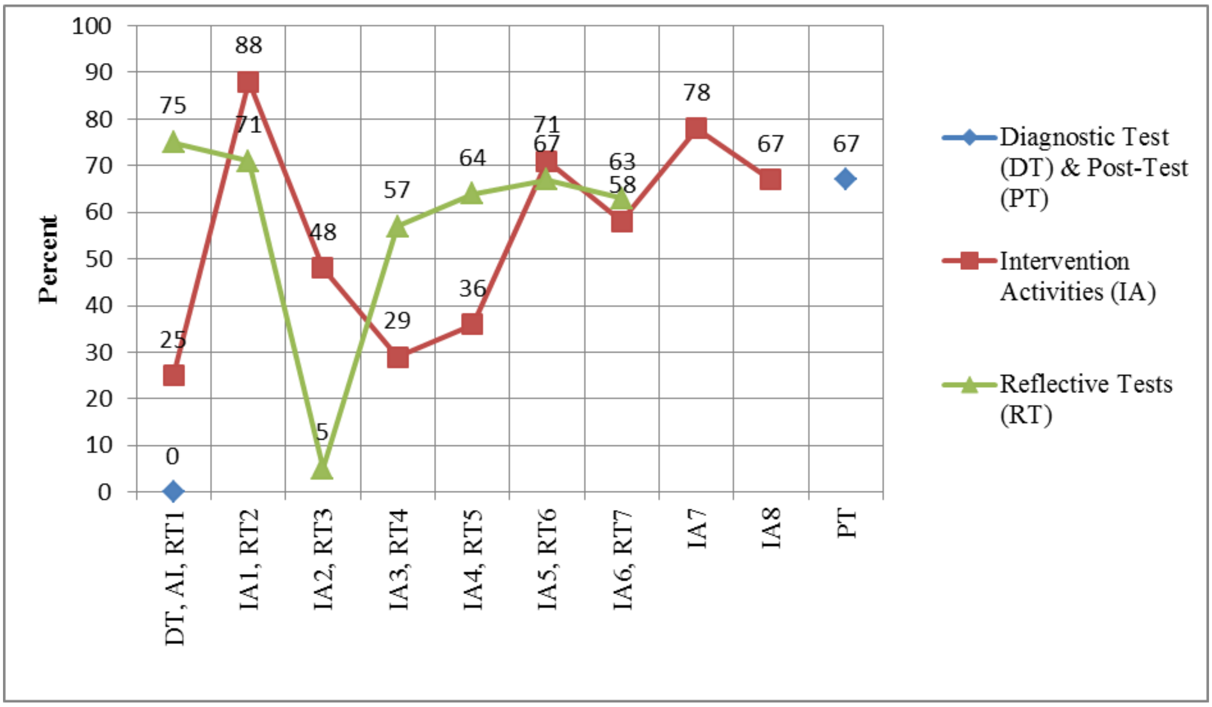


Figure 4.10 L9's developmental pattern throughout the intervention programme.

Figure 4.10 shows L9's developmental patterns of geometric conceptual understanding throughout the teaching and learning episodes of the intervention programme. According to Figure 4.10, L9 obtained 0% in the diagnostic test, but in the post-test the same learner scored 67%. In the diagnostic test L9 failed to respond correctly to questions that were at level 0-visualisation, level 1-analysis, level 2-abstraction and level 3-formal deduction according to the van Hiele theory of geometric thinking. L9's thinking before the intervention activities were implemented was operating at level 0-pre-recognition as hypothesised by Clements and Battista (1991). Presented below are the details of where this learner could not score: Questions at level 0-visualisation: 1.3(i) and (iii), 1.4 (i) and (iii); a question at level 1-analysis: 1.3(ii), questions at level 2-abstraction: 1.1(ii), 1.2(iii), 1.4(ii) and 1.5(ii); questions at level 3-formal deduction: 1.3(i) and 1.4(ii).

When L9 was engaged in the use of polygon pieces and the use of mathematics dictionary eventually managed to recognise triangles based on their properties, using visual skills and managed to demonstrate the application of logical thinking in order to provide the properties of the given triangles. This illustrates how L9 ended up operating at level 0-visualisation, level 1-analysis, level 2-abstraction and level 3-formal deduction according to the van Hiele's model of geometric thinking in the post-test.

4.2.2 Results of intervention activity 1 and reflective test 1

4.2.2.1 Results of intervention activity 1

All the learners improved their marks in intervention activity 1, as compared to how they performed in the diagnostic test. Eight learners (L1 to L8) scored marks above 68%, (refer to Figures 4.2 to 4.9), while L9 improved to 25% (refer to Figure 4.10). During intervention activity 1, mathematics dictionary was provided as a resource to help learners with mathematical concepts like definitions of different triangles and other terminologies. With the conceptual understanding of the definition of a triangle and the individual help provided to L1, L6 and L7, they were able to identify all triangles from the set of two-dimensional shapes.

The results of the learners who were not be able to identify all the triangles from the set of two-dimensional shapes in question 1.1 of the intervention activity 1, are presented in Table 4.1 below.

Table 4.1: Responses of learners to the intervention activity 1

LEARNERS CODE	SHAPES REGARDED AS TRIANGLES	TRIANGLES THAT WERE LEFT OUT IN LEARNERS' RESPONSES
L2		q
L3	c and n	h, i, p, q
L4		a, b, g
L5		b, h, p, q, r
L7	c and n	
L8		h, l, q
L9		b, h, l, p, q,

Table 4.1 shows learners who had difficulties in responding to question 1.1 of the intervention activity 1. Shapes 'c' and 'n' were included in the list of responses by L3 and L7 as triangles which they were not; such responses also demonstrate that, conceptually, the two learners had information regarding properties of triangle that was not well established. As shown in Table 4.1, the triangles that were supposed to be part of the list in learners' responses were left out. Their thinking level was at pre-recognition level as posited by Clements and Battista (1991).

L8's response to question 1.2 of intervention activity 1 was quite unique. The learner said that *"a triangle has 3 vertices and faces,"* in this case the two-dimensional could not be differentiated from the three-dimensional objects. This learner was operating at pre-recognition level of Clements and Battista (1991).

Another learner who responded differently to question 1.2 is L6. This learner responded as follows: *"Because they are use to be the or triangles shape is to be identified."* In this question, L6 was operating at pre-recognition level of Clements and Battista (1991).

In question 1.2 of intervention activity 1 which was aligned to level 0-visualisation as suggested by the van Hiele theory, four learners' responses were correctly done (L1, L3, L4 & L7). The use of mathematics dictionary helped these learners to respond correctly to the question. These learners were able to read from the dictionary with conceptual understanding of the definition of a triangle. The other group of learners (L2, L5 & L9) could not get the question right, according to their understanding, 'a triangle has three equal angles and sides'. Those with no three equal sides and angles were non-triangles to them. This indicates that this group of learners needed more work to develop their conceptual understanding of a triangle. It is clear that the three learners were operating at level 0- pre-recognition as theorised by Clements and Battista (1991).

In question 1.2 of intervention activity 1, the identification of all four scalene triangles was done successfully by L8, while L4 identified only three, which includes the triangle labelled 'b' from the set. The other seven learners (L1, L2, L3, L5, L6, L7 & L9) only identified two shapes, 'h' and 'q' as triangles with all sides not equal (scalene triangles), yet triangles labelled 'b' and 'p' were not considered as part of this group. In some instances, learners would just ignore to follow the instructions and operate as they wanted. Those learners who identified a few numbers of scalene triangles were not fully confident to be at level 1-analysis according to the van Hiele model of geometric thinking. It is possible that some were at level 0-visualisation or even far below that, at pre-recognition level as posited by Clements and Battista (1991).

In question 1.3 of intervention activity 1, all the learners were provided with two copies of intervention activity 1, each learner had to cut out the angles and line segments from each of the triangles in the first and second copy, respectively. After those angles' sizes and line segments' lengths were compared by placing each of the cut out angles and line segments on top of the angles and line segments in the original document, respectively. All the learners managed to identify triangles based on specific features, i.e. triangles with: (i) two equal sides, (ii) all sides equal and (iii) a right angle. By responding correctly to question 1.3 correctly learners demonstrated that there were operating at level 2-abstraction, as suggested by the van Hiele model of geometric thinking.

In response to questions 1.4 of intervention activity 1, seven learners managed to get it right, as it was linked to what they did in question 1.3 where the triangles were grouped based on their properties. The responses of L3, L4 and L5 demonstrated that triangles could be group based on either the lengths of sides or the sizes of angles or both properties. The use of polygon pieces (cut out angles and line segments) in order to identify the category in which a certain triangle belongs had a positive influence on learners' conceptual understanding in question 1.4. That is why the responses were correct. During the intervention, as I was observing learners' work, all the learners were busy measuring. This question was at level 3-formal deduction of the van Hiele model of geometric thinking that is where the three learners were exactly operating.

In intervention activity 1, two learners (L6 & L8) responded to questions 1.4 and 1.5 incorrectly. In question 1.4, L6 said that *'a' and 'i' are or have two equal sides"* while L8 said that *"we can use lines to identified the group of the triangles."* In question 1.5, L6 said that *"yes, because they don't have equal sides like 'b' and 'h',* while L8 said that *"yes, it is because we" [incomplete response].* This might be due to failure to link the information about the properties of triangles dealt with in question 1.2. Another reason seems to be the mathematics vocabulary that led to inadequate conceptual understanding of the question. The two learners' responses revealed that their level of thinking was at pre-recognition as suggested by Clements and Battista (1991).

In question 1.5 of intervention activity 1, four learners (L1, L2, L7 & L9) responded that there is no other property that could be used to categorise the given triangles. Three of the four (L1, L2 & L7) believed that triangles are grouped according to side property while L9 believed that only angle property is used. It seems the way in which these learners responded to question 1.3 was exactly applied to question 1.5 also. All four the learners have demonstrated an alternative conception regarding properties of triangles. They thought that the only property of a triangle was regarding the relationship of its sides nothing else, yet in reality there is an angle property also. These learners were operating at the van Hiele model level 1-analysis instead of level 3-formal deduction as per question's level.

In question 1.6 of intervention activity 1, eight learners mentioned the right-angled triangle, except L9 who left the space blank. The reason why this was well answered is that the comparison of angles and lengths of the triangles that was done in question 1.3 using polygon pieces enhanced most of the learners' conceptual understanding. The eight were operating at level 2-analysis of the van Hiele model of geometric thinking.

In question 1.6 of intervention activity 1, L2 and L3 said *"right angle triangle"* instead of right-angled triangle. This revealed mathematics language barrier. Furthermore, the distorted name might be due to negligence since the name is well spelt correctly in question 1.3, but these two learners decided to write it the other way. With L4, the name was written correctly, but the problem was the indefinite article that was used before the word right-angled triangle this learner used 'an'……instead of 'a'.

The only problem encountered by L2, L5 and L7 in question 1.6, of intervention activity 1, was that of spelling the word 'isosceles', for example it was spelt by each of the learners as follows*: 'isoslece, isosceles and isocelice',* respectively. Despite the fact that the dictionary was available to be used at any time, the three did not see the need to confirm the correct spellings.

Question 1.6 of intervention activity 1 is rated at level 2-abstraction of the van Hiele model. Six learners (L1, L2, L4, L5, L7 & L8) identified the triangle with two opposite sides as equal to an isosceles. Although L2, L5 and L7 belong in this group, they also fall under theme 4. Because of their failure to spell the word ‘isosceles’, it seems that knowing their problems in spelling well, they did not see the need to use the provided dictionary.

Question 1.6 of intervention activity 1, the triangle with all sides equal was identified as an equilateral by seven learners (L1, L2, L4, L5, L6, L7 & L8). The use of the mathematics dictionary helped learners L1, L4, L5, L7 and L8 to respond to the question correctly, including the spelling of the word ‘equilateral’, while L2 and L6 could not spell the word ‘equilateral’, correctly.

Question 1.6 of intervention activity 1, the fourth group of triangles was correctly mentioned by L1, L4 and L5 as scalene triangles. Even though the mathematics dictionary was provided to help learners respond to some of the questions in activity 1, L2, L3, L6, L7, L8 and L9 were not able to identify the fourth group of triangles as scalene. These learners were operating at the pre-recognition level of Clements and Battista (1991).

In question 1.6 of intervention activity 1, L1, L4 and L8 used the dictionary and managed to write the word ‘isosceles’ correctly. As a result, three learners were place at level 2-abstraction as suggested by the van Hiele model of geometric thinking.

In question 1.7 of intervention activity 1, learners were asked to draw and name triangles according to their classes based on the size of angles and the length of sides. L1, L3, L4, L6 and L7 managed to draw and correctly named of the four different triangles as isosceles, equilateral, scalene and right-angled triangles.

The learners to be regarded as being at level 1-analysis according to the van Hiele model of geometric thinking needed to answer question1.7 correctly, which the five relevant learners managed to. I concluded that those who failed were still operating at pre-recognition level as suggested by Clements and Battista (1991).

In question 1.7 of intervention activity 1, L5 managed to draw four triangles and named three of the triangles correctly. The fourth was named an equilateral, but without symbols. L8 managed to draw three different triangles and named them isosceles, equilateral and scalene triangles, but mathematical symbols were not used to show that the two were equilateral and isosceles triangles. Failure to insert mathematical symbols showed that the two were not developed to fully operate at level 1-analysis of the van Hiele theory, resulting in them being migrating to level 0-pre-recognition of Clements and Battista (1991).

In question 1.7, of intervention activity 1, L2 and L9 have drawn acute angled - triangles and named them as right-angled triangles. On the other hand L8 and L9 did not attempt to draw and name a right-angled triangle and a scalene triangle, respectively. Both of these groups were at level 0-pre-recognition of Clements and Battista (1991).

In question 1.7 of intervention activity 1, the only concept that seems to be ignored by most of the learners was the angle property. In the three types of triangles (isosceles, equilateral and scalene), all the learners focused on the side property, except in, a right-angled triangles where the angle property was applied because there was no option for the side property to be used.

For those learners who could not attempt to draw scalene triangles, I can conclude that it was due to failure to link and apply the knowledge acquired in the previous questions into the new situation.

4.2.2.2 Results of reflective test 1

The summary of how learners performed in the diagnostic test and reflective test 1 is presented in Table 4.2 below. For the content of the test refer to appendix 21.

Table 4.2: Learners' overall performance in the diagnostic test and reflective test 1

Item	Min	Mean	SD	Median	Maximum
Diagnostic test	0	18	14.82	22	44
Reflective test 1	42	76	13.11	79	88

Note: Values of minimum, mean and standard deviation and maximum for learners' (N=9) marks obtained in two activities, diagnostic test and reflective test 1 are displayed.

Table 4.2 shows that the reflective test's minimum, mean, median and maximum values were greater than those of the diagnostic test, with the exception of the standard deviations that is the other way round. These results show that the use of polygon pieces and mathematics dictionary influenced the learning of geometry.

In question 1.1 of reflective test 1, six learners (L1, L2, L4, L5, L6 & L7) managed to identify triangles from the pool of different two-dimensional shapes. Of the six learners who responded incorrectly to question 1.1of reflective test 1, L2 included a triangle labelled 'x' as one of the responses, yet there was no triangle labelled 'x' in the question. L3 and L8 did not include the triangle labelled 'q' in their responses. The reason for such an error might be due to an unchecked solution to verify whether all the answers were correct. The learners mentioned in this paragraph were at level 0-pre-recognition of Clements and Battista (1991).

In question 1.2 of reflective test 1, learners were to categorise the identified triangles in question 1.1 into: scalene, isosceles, equilateral and right-angled triangles. The main aim for question 1.2 was to check whether learners could be able to identify different triangles and match each one of them with its right name from the given list. Table 4.3 below shows how learners responded to question 1.2.

Aligned to level 0-visualisation of the van Hiele model of geometric thinking was question 1.2 of reflective test 1 that required learners to identify right-angled triangles from the given set of triangles. The solutions expected were (‘h’, ‘m’ & ‘o’). L1, L2, L3, L5 and L8 managed to identify the required three; the use of polygon pieces in the previous intervention seemed to have a positive influence on some of the learners in reflective test 1. Those who failed were at pre-recognition level of Clements and Battista (1991).

Table 4.3: Learners’ responses to question 1.2 in the reflective test 1

LEARNER CODE	SCALENE TRIANGLES	ISOSCELES TRIANGLES	EQUILATERAL TRIANGLES	RIGHT-ANGLED TRIANGLES
L1	a, i, q	b, e	c, p	h, k, l, m, o
L2	a, h, g	b, e	p, c	h, m, o
L3	a, i, q	b, e	c, p	h, m, o
L4	o	b	p	h
L5	a, i, p	b, e	c, p	h, m, o
L6	b, c, e, p	a, i, o, q	h, m	b, c, e, p, q,
L7	a, i, q, o	b, e	c, p	m, o
L8	b, c, e, p	m, a	i, p	h, m, o
L9	a, q	b	c, p	h, m

Table 4.3: shows how individual learners responded to question 1.2 in the reflective test that was based on the content of intervention activity 1.

As shown in Table 4.3 above, learners responded differently to question 1.2. The expected choices were triangles labelled ‘a’, ‘i’, ‘m’, ‘q’ and ‘o’. Out of the five scalene triangles, L7 identified four only; L1 and L3 identified three; L9 identified only two, triangles ‘a’ and ‘a’ and ‘q’. L4 identified only one. L6 and L8 did not manage to identify the correct triangles as scalene triangles instead triangles labelled ‘b’, ‘c’, ‘e’, ‘p’ were identified. In addition to one correct triangle identified by L2, triangles labelled ‘g’ and ‘h’ were also included as part of the solution. L5 correctly identified triangles ‘a’ and ‘i’, but also included triangle ‘p’ which was not part of the suggested solutions.

The identification of two isosceles triangles out of the required three (‘b’, ‘e’& ‘h’) in question 1.2 of reflective test 1, was correctly done by L1, L2, L3, L5 and L7. This group was at level 0-visualisation instead of level 1-analysis of the van Hiele model of geometric thinking. Another group of learners (L4 & L9) each identified only one isosceles triangle, ‘b’. L6 and L8 could not identify isosceles triangles from the set of triangles. For details of how these learners responded to the question, refer to Table 4.3 above. Failure to identify all three the triangles shows that the learners were operating at level 0-pre-recognition of Clements and Battista (1991) in some of the concepts.

In question 1.2 of reflective test 1, six learners (L1, L2, L3, L5, L7 & L9) correctly identified the two equilateral triangles (‘c’ & ‘p’) from the given set. L4 identified only one of the triangles (‘p’) while L8 had two choices ‘i’ and ‘p’ of which ‘i’ is incorrect. Finally, L6 identified triangles labelled ‘h and m’ as equilateral, yet both were not equilateral. L6 and L8’s responses show that their level of thinking was at pre-recognition as described by Clements and Battista (1991).

Responding to the last part of question 1.2, four learners (L2, L3, L5 & L8) correctly identified all three required triangles, ‘h’, ‘m’ and ‘o’. L1 included triangles labelled ‘k’ and ‘l’ which are not right-angled triangles, as part of the responses. L7 and L9 managed to identify two of the three required triangles; refer to the Table 4.14 above. L4 identified the triangle labelled 'h' only while L6 identified 'b', 'c', 'e', 'p' and 'q' as right-angled triangles, yet they were not. These learners seemed to be operating at level 0-pre-recognition of Clements and Battista (1991) because there were not sure of the type of triangles to select from the given set.

In question 1.3 of reflective test 1, L2 said that *"all sides are not equil."* This learner could not spell and write down the word equal correctly while L1 said *"shapes that are all not equal."* The problem with L1 is sentence construction. L6 could not answer the question correctly, the way in which L6 responded to the question is illustrated in the learner's own handwriting in Figure 4.11 below:

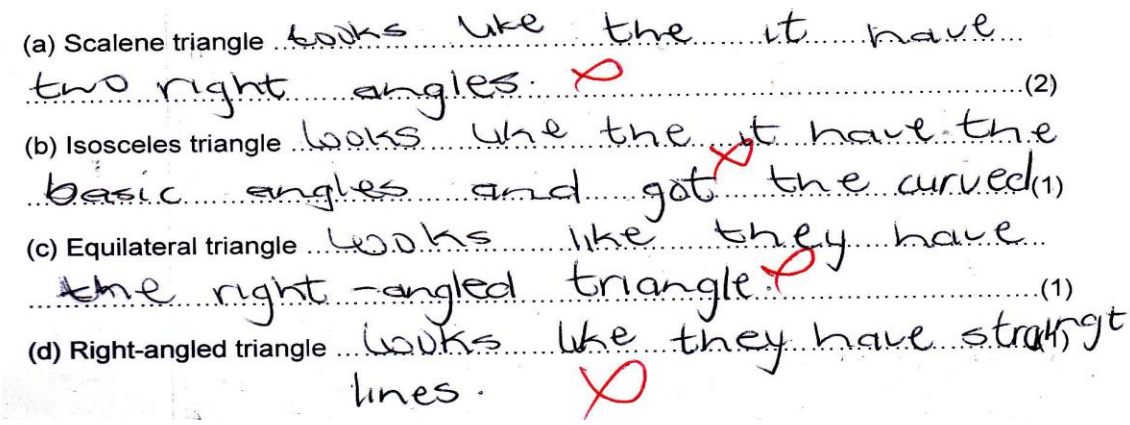

(a) Scalene triangle looks like the it have two right angles. (2)
(b) Isosceles triangle looks like the it have the basic angles and got the curved (1)
(c) Equilateral triangle looks like they have the right-angled triangle (1)
(d) Right-angled triangle looks like they have straingt lines.

Figure 4.11:L6 responded to question 1.3 of reflective test 1

L6 was not able to categorise triangles into their respective groups namely: scalene, isosceles, equilateral and right-angled triangles.

In question 1.3 of reflective test 1, learners were required to give a description in their own words of what each of the triangles looks like, i.e. a scalene, an isosceles, an equilateral or a right-angled triangle. This question is at level 1-analysis as suggested by the van Hiele model of geometric thinking. Five learners (L3, L4, L5, L8, & L9) were able to give a clear description of what a scalene triangle is. The five were operating at level 1-analysis of the van Hiele geometric thinking model.

Learners, who managed to describe what a scalene triangle looks like, used their visualisation skills. As I was observing learners writing the test, for example, L1 could not use the visualisation skills instead pieces of papers were used to measure the sides of the triangles in order to be sure whether the sides of the given triangle were all equal. Those who could not make it in this question it shows that they were operating at level 0-pre-recognition as suggested by Clements and Battista (1991).

4.2.3 Results of intervention activity 2 and reflective test 2

4.2.3.1 Results of intervention activity 2

In question 2.1.1 of intervention activity 2 learners were asked to identify and categorise the 10 triangles into five main groups based on their angle properties: the set of the triangles is shown in appendix 13. In order to do this activity each learner was provided with a copy of the question paper for activity 2. From the copy each learner had to cut out all three angles of each of the triangles labelled ‘a’ to ‘i’ one at a time. After that, in each of the original triangles (‘a’ to ‘i’), one angle’s magnitude was compared to the other two angles in the same triangle; for example, the angles cut out from the copy of triangle ‘a’ were placed on top of each of the other two angles in the original triangle ‘a’ one at a time and the results were recorded for each measure taken.

The same procedure done to the triangle labelled ‘a’ was followed for all other triangles one at a time. Through such an activity, each learner was given an opportunity to investigate the angle property of each of the triangles in intervention activity 2 without being told the properties for any triangle.

In question 2.1.1 of intervention activity 2, all the learners managed to identify triangles labelled ‘e’ and ‘i’ as having all equal angles. The exception was L7 who included the triangle labelled ‘a’ as one of the triangles with all angles equal, yet it did not belong there.

In question 2.1.1 of intervention activity 2, the second category of triangles (with two equal angles), eight learners were able to clearly identify triangles labelled ‘a’, ‘c’, ‘d’ and ‘h’, except L7, who included the triangle labelled ‘g’ as part of the solution, instead of the triangle labelled ‘d’. The error showed that L7 did not make use of the cut angles to confirm the solution.

In question 2.1.1of intervention activity 2, the third category of triangles (with all angles less than 90^0), L1 managed to identify all triangles that belong to this category, namely: triangles labelled: ‘a’, ‘e’, ‘i’ and ‘j’. Five learners, L2, L3, L4, L8 and L9, correctly identified triangles labelled ‘a’, ‘e’ and ‘j’, but left out the triangle labelled ‘i’.

In question 2.1.1 of intervention activity 2, another group of two learners (L5 & L6) identified the triangles labelled ‘a’, ‘e’ and ‘j’, but both left out triangles labelled ‘i’ and included the triangles labelled ‘f’ as part of the solution. Such errors showed that the angle-cutting activities that learners were engaged in were not taken seriously, resulting in them guessing in order to identify some of the mentioned triangles.

In question 2.1.1 of intervention activity 2, the last learner (L7) managed to identify the triangles labelled ‘a’ and ‘j’, but left out ‘e’ and ‘i’. This learner identified the triangles labelled ‘b’ and ‘g’ as part of the solution. Correctly responding to question 2.1.1 showed that the learners were at level 1-analysis of the van Hiele model of geometric thinking, but both groups those who mixed up response or left out some required responses were operating at pre-recognition level of Clements and Battista (1991).

Question 2.1.2 of intervention activity 2 was at level 1-analysis of the van Hiele model of geometric thinking. Learners were supposed to identify triangles and categorise them using their real names, like isosceles, equilateral, an acute-angled triangle, etc. In question 2.1.2.1, seven learners (L1, L2 L3, L4, L6, L7 & L9) were able to identify all the isosceles triangles correctly while one learner (L8) identified only three isosceles triangles correctly, included in the list was triangle ‘b’, which is not an isosceles. From my observation L8, has chosen triangle labelled ‘b’ considering the fact that it had one obtuse angle and the other two were acute angles which were regarded as equal in size. L8 did not use polygon pieces to confirm if the two acute angles in the triangle labelled ‘b’ were equal or not. L5 identified four triangles as isosceles, but also included triangle ‘e’ in the response, which does not belong to the category of isosceles triangles.

The identification of equilateral triangles in question 2.1.2.2 of intervention activity 2, was correctly done by all the learners. The polygon pieces used made it possible for the learners to measure accurately both the sides and angles of each of the triangles labelled ‘e’ and ‘i’ accurately. Those learners with errors in their choices proved that they were not well developed to be at level 1-analysis of the van Hiele model. Such responses showed that they were migrating to and from, level 0-pre-recognition of Clements and Battista (1991) to level 1-analysis of the van Hiele model of geometric thinking.

In question 2.1.2.3 of intervention activity 2, the identification of acute-angled triangles was done differently. Out of nine learners, only L1 and L8 managed to correctly identify triangles labelled: ‘a’, ‘e’, ‘i’ and ‘j’. L1 and L8 used the dictionary and understood the definition of an acute angled-triangle, which was why their responses were correct.

In question 2.1.2.3 of intervention activity 2, two learners (L4 & L6) identified only three acute-angled triangles, ‘a’, ‘e’ and ‘j’. Another group of learners (L2, L3 & L9) identified only ‘a’ as an acute-angled triangle. Lastly, L7 identified ‘j’ only as an acute-angled triangle. The three groups mentioned in this section managed to identify acute-angled triangles, the dictionary helped the learners to have an idea of what an acute angled-triangle looks like.

Although the learners were able to identify acute-angled triangles in question 2.1.2.3 of intervention activity 2, in their responses they also included other triangles which were not acute-angled; for example L1 and L2 identified ‘c’, ‘d’ and ‘f’. L4, L5, L7 and L9 identified ‘b’ and ‘g’. L8 identified ‘b’. L5 also identified ‘d’. This group could not identify all the acute-angled triangles due to the incorrect interpretation of the definition of an acute-angled triangle. These learners thought that a triangle that has two acute angles is also an acute-angled triangle. Learners in this paragraph seemed not to be well developed to be at level 1-analysis of the van Hiele model of geometric thinking. The responses showed that they were migrating to pre-recognition level of Clements and Battista (1991).

Learners’ responses to question 2.1.2.4 of intervention activity 2 were in different categories based on their similarities. L1, L2, L3, L5, L6 and L9 correctly identified triangles labelled ‘b’ and ‘g’ as obtuse-angled triangles. The use of both mathematics dictionary and pieces of angles in comparison of the three angles in the origin triangle helped learners to conceptually understanding the meaning of obtuse-angled triangles and what it actually it looks like. Since the question was at level 1-analysis of the van Hiele model of geometric thinking, this means that the six learners were able to operate at that level.

In the second category, L4 identified ‘b’ and ‘g’, also included ‘j’ as an obtuse-angled triangle. L8 identified ‘b’ and also included ‘f’ in the response. L7 identified ‘g’, and also included ‘a’, ‘c’ and ‘h’ as obtuse-angled triangles.

In question 2.1.2.5 of intervention activity 2, all the learners managed to identify triangles labelled ‘c’, ‘d’, ‘f’ and ‘h’ as right-angled triangles. The discussion of the definition from the mathematics dictionary, the measurement of various angles and the discussion held in the previous sessions helped the learners to conceptually understand that a right- angled triangle has one right angle.

Since the entire question 2 of intervention activity 2 addressed level 1-analysis of the van Hiele model of geometric thinking, this simply means those learners who could not get any of its sub-questions correct can be categorised to be at pre-recognition level 0, as hypothesised by Clements and Battista (1991).

4.2.3.2 Results of reflective test 2

Reflective test two was written on the third day of the data collection, the main aim was to recap previous lessons’ work and as a measure to determine whether learners conceptually understood and mastered the concepts that were covered during the intervention activity two. Table 4.4 below illustrates how learners performed in test two as compared to the diagnostic test.

Table 4.4: Learners’ overall performance in the diagnostic test and reflective test 2

Item	Min	Mean	SD	Median	Maximum
Diagnostic test	0	18	14.82	22	44
Reflective test 2	38	60.22	16.65	67	81

Note: Values of minimum, mean and standard deviation and maximum for learners’ (N=9) marks obtained in two activities, diagnostic test and reflective test two. In both a diagnostic and reflective test 2, the mean < median the data negatively skewed.

As in the comparison shown in Table 4.4, generally, learners’ performance improved in reflective test two as compared to the results obtained in the diagnostic test. Although the data sets are skewed to the same side, but the minimum, mean, median and the maximum measures for the reflective test two are far greater than those in the diagnostic test. The results of test two showed that learners' overall results improved as compared to those of the diagnostic test, which showed the positive influence on the use of polygon pieces to learners' conceptual understanding of geometry.

Most of learners' marks for test two showed a stride up. For example, L1, L3, L4, L5, L7, L8 and L9 improved, which was an indication that the use of cut polygon pieces helped them with conceptual understanding. Even though they did not get everything correct, but the move in the positive direction showed how polygon pieces influenced their understanding of geometry concepts. The percentage of less than 50% obtained by L2, L6 and L7 was an indication that with some learners the use of polygon pieces to teach mathematical concepts needs more time in order to positively influence their conceptual understanding. Some alternative conceptions cannot be unlearnt within a short space of time, more time and strategies need to be invested in order for such learners to conceptually understand what has been taught.

For both groups of learners, the overall performance in test two was mostly affected by how each of the learners responded to the questions. For example, in question 2.1.1 which was at level 1-analysis as posited by the van Hiele model, they were not able to identify all the scalene triangles from the given set of triangles. L1 was the only learner who identified all four scalene triangles; L7 identified three and L2, L4, L5, L6 and L8 identified only one. Those who identified three, two or one triangle were at level 0 - visualisation of the van Hiele model of geometric thinking. L3 could not manage to identify any. This learner was operating at level 0-pre-recognition as suggested by Clements and Battista (1991).

Questions 2.1 and 2.2 of the reflective test 2 were at level 1-visualisation and level 2-abstraction. In question 2.1.2 of reflective test 2, learners were supposed to identify four isosceles triangles, but not all the learners managed to do that, for example, L7 and L9 identified three, and L6 identified only two. The other group of six learners (L1, L2, L3, L4, L5 and L8) managed to identify all the required triangles.

In question 2.1.3 of reflective test 2, learners were asked to identify equilateral triangles from the given set of triangles. Instead of identifying all two equilateral triangles, L6 identified only one and L8 could not identify any. All other learners managed to identify the required triangles with an exceptional cases of L2 and L7, who, amongst their correct choices included triangles labelled ‘g’ and ‘i’, respectively.

In question 2.1.4 of reflective test 2, learners were supposed to identify right-angled isosceles triangles, from the given set of different triangles. Only three right-angled isosceles triangles were to be identified. Five out of nine learners (L1, L3, L4, L5 & L9) managed to identify all the required triangles. In that group of five there were cases likeL5 and L9also included one other triangle that was not right-angled isosceles, i.e. triangle labelled ‘j’ and ‘c’, respectively. L6 and L8 each identified only one right-angled isosceles triangle, but in their responses they also included other triangles which do not belong to that particular group, i.e. L6 included triangles labelled ‘c’ and ‘g’ while L8 included ‘j’.

In questions 2.1.5 of reflective test 2, learners were supposed to identify obtuse-angled triangles from the list. Only two were the required responses. Six learners (L1, L3, L4, L6, L8 & L9) managed to identify the two triangles, with two exceptional cases from the group being L6 and L8 who included triangle labelled ‘j’ as part of the solution. The other two learners (L2 & L7) could not identify any of the required triangles, their responses included the following triangles: ‘f’, ‘h’, ‘i’, ‘j’ and ‘a’, ‘c’, ‘f’, respectively. L5 identified one triangle and included triangles labelled ‘j’ as part of the solution.

Question 2.1.6 of reflective test 2 required learners to identify a right-angled scalene triangle. Of the nine, only one learner (L4) managed to identify triangle labelled ‘j’ without including any other triangles. The other group of five learners (L2, L3, L5, L7 & L9) identified triangle labelled ‘j’ as well, but also included other triangles, for example, L2: ‘h’ and ‘c’; L3: ‘b’ and ‘e’; L5: ‘a’; L7: ‘b’ and ‘e’; L9: ‘a’, ‘b’, ‘d’, ‘e’, ‘f’.

The learners who failed to identify all the required triangles in all the questions of reflective test 2 showed that they were still operating at Clement and Battista’s (1991) pre-recognition level.

In question 2.2.1 of reflective test 2, only three learners (L1, L3 & L9) managed to describe the angle properties of a scalene triangle, for example their responses are shown in figures 4.12, 4.14 and 4.20.

Figures 4.13 to 4.15, 4.19 and 4.20 show that in question 2.2.2 of reflective test 2, (L2, L3, L4, L8 & L9) correctly described the angle properties of an isosceles triangle. The polygon pieces that were used in the previous intervention activity made it possible for the learners to understand the concepts being asked in the questions.

Question 2.2.3 of reflective test 2 was correctly answered by three learners (L4, L6 & L8). To see each of the three learners’ responses to the question, refer to Figures 4.15, 4.17 and 4.19. The use of polygon pieces in which learners were engaged in intervention activity 1 positively influenced the three learners’ results in question 2.2.3 of reflective test 2.

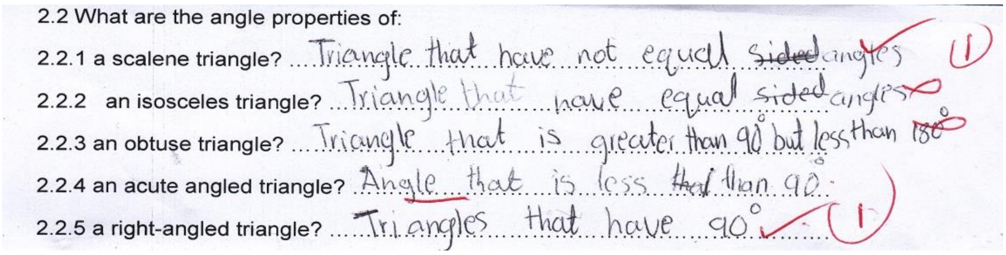
In question 2.2.4 of reflective test 2, learners were supposed to describe what an acute-angled triangle looks like based on the angle property. Only two learners (L4 & L5) were able to give the correct responses. For the details of their responses refer to Figures 4.15 and 4.16.

Responses to question 2.2.5 of reflective test 2 show that six learners (L1, L3, L4, L5, L6 & L9) have a clear conceptual understanding of the properties of a right-angled triangle, details of the six learners responses to question 2.2.5 is shown in Figures 4.13, 4.14 to 4.17 and 4.20.

Since reflective test 2 questions 2.2.1 to 2.2.5 described above were rated at level 2 – abstraction as suggested by the van Hiele model of geometric thinking. The group of learners who correctly answered the individual questions (2.2.1 to 2.2.5) correct are therefore identified to be at the van Hiele thinking level 2.

In reflective test 2, three learners (L1, L6 & L8) were not able to identify the required triangle, they identified, ‘b’ and ‘e’; ‘g’ and ‘h’; ‘g’, ‘h’ and ‘i’, respectively. Learners’ performance in this question shows that most of them were not really sure of the properties of a scalene triangle, thus why there are so many triangles listed as scalene triangles in their responses.The results tell us that L1, L6 and L8 all belong to the pre-recognition level of Clements and Battista (1991).

Question 2.2.1 to 2.2.5 of reflective test 2, required learners to describe each of the triangles based on the angle property. Figures 4.12 to 4.20 show how each of the learners responded to questions 2.2.1 to 2.2.5.



2.2 What are the angle properties of:

2.2.1 a scalene triangle? Triangle that have not equal ~~sided~~ angles (1)

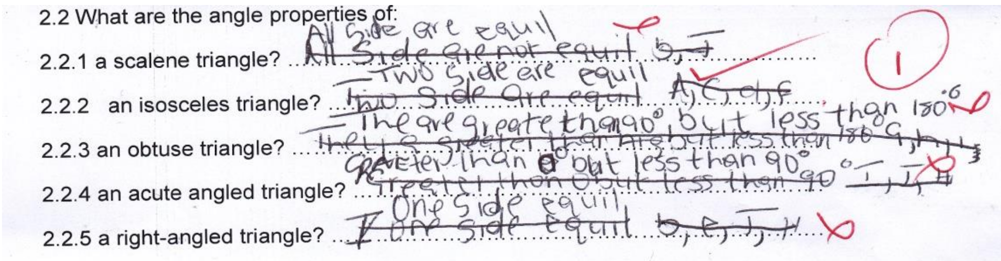
2.2.2 an isosceles triangle? Triangle that have equal ~~sided~~ angles

2.2.3 an obtuse triangle? Triangle that is greater than 90° but less than 180°

2.2.4 an acute angled triangle? Angle that is less ~~that~~ than 90°

2.2.5 a right-angled triangle? Triangles that have 90° (1)

Figure 4.12: Reflective test 2: L1's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? All Side are equil ~~All Side are not equil b, f~~

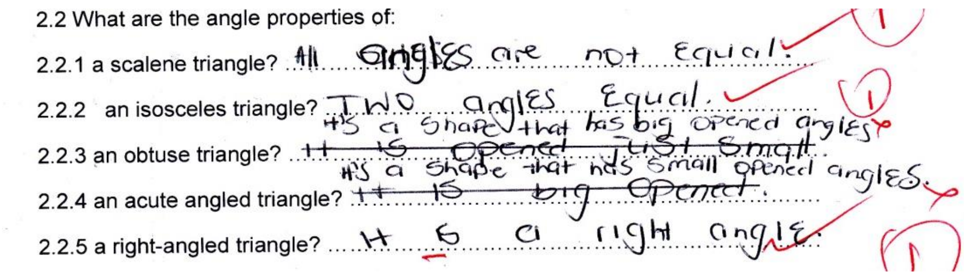
2.2.2 an isosceles triangle? Two Side are equil ~~Two Side are equil A, C, e, f~~ (1)

2.2.3 an obtuse triangle? The are greate than 90°, but less than 180°

2.2.4 an acute angled triangle? Greater than 0 but less than 90°

2.2.5 a right-angled triangle? One Side equil ~~One side equil b, e, j, H~~

Figure 4.13; Reflective test 2: L2's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? All angles are not Equal. (1)

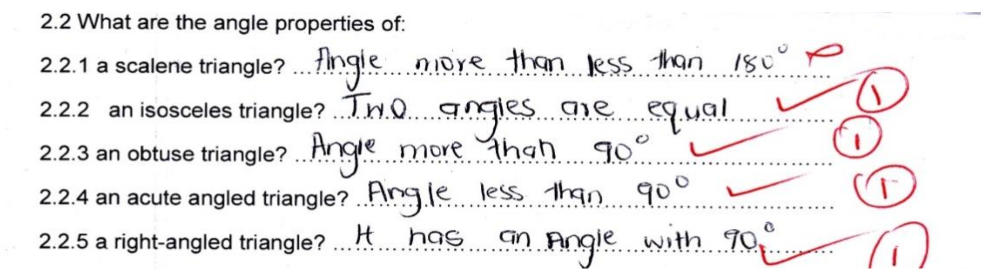
2.2.2 an isosceles triangle? Two angles Equal. (1)

2.2.3 an obtuse triangle? It's a shape that has big opened angles ~~It is opened just small~~

2.2.4 an acute angled triangle? It's a shape that has small opened angles. ~~It is big opened.~~

2.2.5 a right-angled triangle? It is a right angle. (1)

Figure 4.14: Reflective test 2: L3's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? Angle more than less than 180°

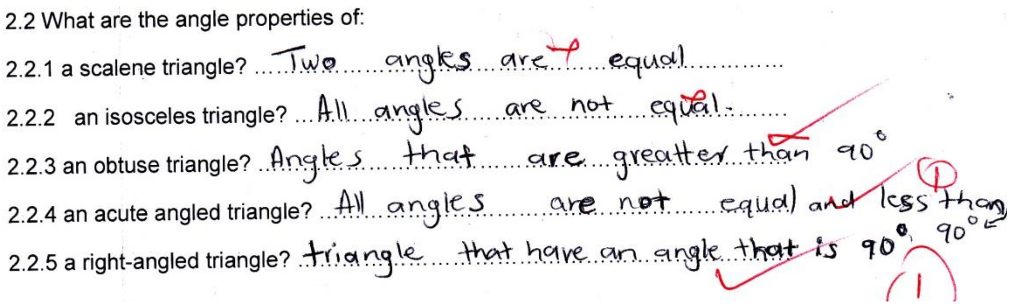
2.2.2 an isosceles triangle? Two angles are equal (1)

2.2.3 an obtuse triangle? Angle more than 90° (1)

2.2.4 an acute angled triangle? Angle less than 90° (1)

2.2.5 a right-angled triangle? It has an Angle with 90° (1)

Figure 4.15: Reflective test 2: L4's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? Two angles are equal

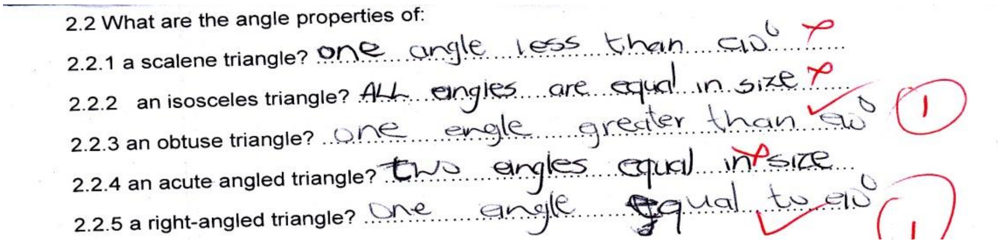
2.2.2 an isosceles triangle? All angles are not equal.

2.2.3 an obtuse triangle? Angles that are greatter than 90°

2.2.4 an acute angled triangle? All angles are not equal) and less than 90°

2.2.5 a right-angled triangle? triangle that have an angle that is 90°,

Figure 4.16: Reflective test 2: L5's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? one angle less than 90°

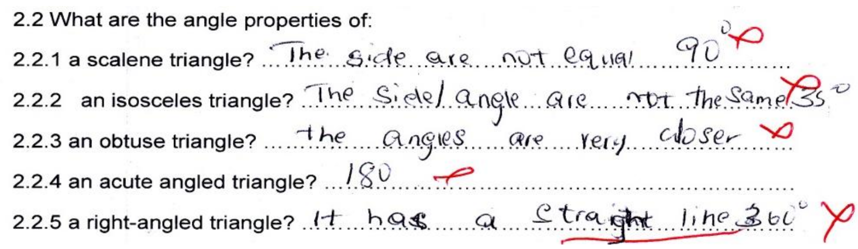
2.2.2 an isosceles triangle? ALL angles are equal in size

2.2.3 an obtuse triangle? one angle greater than 90°

2.2.4 an acute angled triangle? two angles equal in size

2.2.5 a right-angled triangle? One angle equal to 90°

Figure 4.17: Reflective test 2: L6's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? The side are not equal 90°

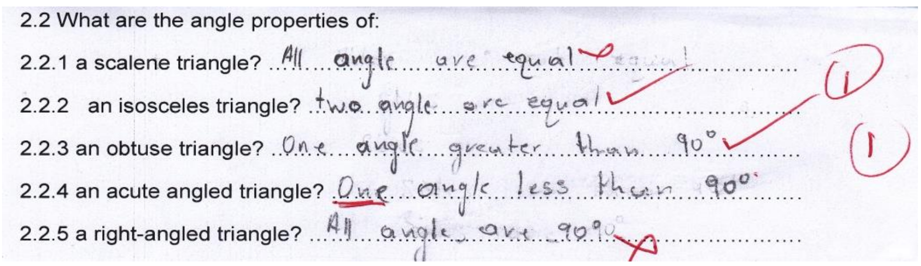
2.2.2 an isosceles triangle? The Side/angle are not the Same 35°

2.2.3 an obtuse triangle? the angles are very closer

2.2.4 an acute angled triangle? 180

2.2.5 a right-angled triangle? It has a straight line 360°

Figure 4.18: Reflective test 2: L7's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? All angle are equal

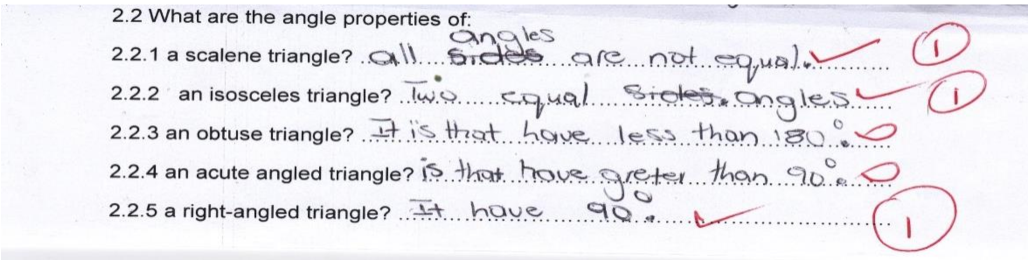
2.2.2 an isosceles triangle? two angle are equal

2.2.3 an obtuse triangle? One angle greater than 90°

2.2.4 an acute angled triangle? One angle less than 90°

2.2.5 a right-angled triangle? All angle are 90°

Figure 4.19: Reflective test 2: L8's detailed responses to question 2.2



2.2 What are the angle properties of:

2.2.1 a scalene triangle? all ~~sides~~ angles are not equal. ✓ (1)

2.2.2 an isosceles triangle? Two equal ~~sides~~ angles ✓ (1)

2.2.3 an obtuse triangle? It is that have less than 180°.

2.2.4 an acute angled triangle? is that have greter than 90°.

2.2.5 a right-angled triangle? It have 90°. ✓ (1)

Figure 4.20: Reflective test 2: L9's detailed responses to question 2.2

In question 2.2.1 of reflective test 2 the other six learners (L2, L4, L5, L6, L7 & L8) could not respond to the question correctly; refer to figures 4.13, 4.15 to 4.19. All these learners were operating at level 0-pre-recognition of Clements and Battista (1991) instead of meeting the question's level 2 -abstraction according to the van Hiele model of geometric thinking.

In question 2.2.2 of reflective test 2, the other four (L1, L5, L6 & L7) could not correctly respond to the question. Figures 4.12, 4.16 to 4.18 illustrate how each of the four learners presented their alternative conceptions regarding the properties of an isosceles triangle. From four learners' responses it was apparent that the two (L1 & L6) regarded an isosceles triangle as having the property of an equilateral, while (L5 & L7)'s responses revealed that conceptually the difference between isosceles triangle and a scalene was not well established. These learners operated at the pre-recognition level of Clements and Battista (1991).

In question 2.2.3 of reflective test 2, the other six L1, L2, L3, L5, L7 and L9 responded incorrectly. For details of responses; refer to Figures 4.12 to 4.14, 4.16, 4.18 and 4.20.

In question 2.2.4 of reflective test 2, seven learners (L1, L2, L3, L6, L7, L8 & L9) incorrectly described the properties of a scalene triangle. How each of the learners responded to question 2.2.4, refer to Figures 4.12 to 4.14, 4.15 to 4.20.

In question 2.2.5 of reflective test 2, the other three learners could not answer the question right. To see how each of these learners responded, refer to Figures 4.13, 4.17 and 4.19.

In question 2.2 of reflective test 2, learners had to operate at level two of the van Hiele's geometric thinking model; this is the analysis level where learners are supposed to identify shapes based on their properties. According to the learners' responses no one stayed at level one as from question 2.2.1 to 2.2.5, they were migrating from level 0-visualisation to level 1-analysis and back to level 0 from one question to another. From what I have observed from the way each learner responded to individual questions there are several reasons that caused this migration in geometric thinking levels; for example, L3 demonstrated a lack of competence in the use of proper mathematical language. The responses to questions 2.2.2 and 2.2.3 in Figure 4.14 attested to this.

Secondly, the sentence construction has also been another challenge and this emanates from the language barrier which played a major role in ensuring that learners gave alternative responses to questions 2.2. For example, response as indicated in Figure 4.13, L2's response shows that the learner had an idea, but could not find the correct way to write it down. Some learners lacked conceptual understanding regarding the properties of triangles, for example, L6's responses to question 2.2.4 in Figure 4.17 alluded to this.

4.2.4 Results of intervention activity 3 and reflective test 3

4.2.4.1 Results of intervention activity 3

Question 3.1 of intervention activity 3, required learners to identify types of triangles by estimating the lengths of sides and magnitude of angles and then in question 3.2, learners had to use polygon pieces to verify if their responses in question 3.1 were correct regarding triangles labelled ‘a’ to ‘o’. In question 3.3 learners had to use symbols, for example, showing that a particular triangle is an isosceles or an equilateral or otherwise.

In question 3.1.1 of intervention activity 3, four learners (L1, L3, L4 & L8) identified triangles labelled: ‘a’ ‘e’, ‘g’, ‘h’ and ‘n’ as isosceles. L2 identified triangles labelled ‘a’, ‘e’ and‘d’ as isosceles. L7 identified ‘a’, ‘e’, ‘g’ and ‘h’ as isosceles triangles. L6 has chosen triangles ‘a’, ‘e’, ‘h’ and ‘n’ as isosceles triangles. Triangles labelled ‘a’ and ‘e’ were identified by L9 as isosceles. L5 identified ‘d’ and ‘e’ as isosceles triangle.

In question 3.1.1 of intervention activity 3, although the learners were able to identify isosceles triangles from the given set of triangles, each could not identify all six the isosceles triangles, but they also included other triangles that were not isosceles. For instance, L1 included ‘c’, L2 included ‘i’, ‘k’ and ‘l’, L3 included ‘c’, ‘i’ and ‘o’; L4 had ‘l’ and ‘p’, L5 included ‘b’, ‘f’, ‘i’, ‘j’, ‘p’ and ‘o’. L6 considered ‘c’ and ‘i’ as part of the solution. L7 decided to include ‘f’, ‘i’, ‘j’, ‘o’ and ‘p’ in the solution. L9 included ‘b’, ‘c’, ‘i’, ‘l’, ‘m’ and ‘p’ as part of the isosceles triangles. The reason why these learners regarded figures which are not isosceles triangles as isosceles triangles was a lack of basic visualisation skills; they were not able to use visual properties to differentiate triangles.

In question 3.1.2 of intervention activity 3, the identification of equilateral triangles was not perfectly done, some shapes included in the solutions were not equilateral triangles.

Five learners (L1, L3, L5 & L7) identified ‘k’, ‘l’ and ‘m’ as equilateral triangles. L2 was able to identify triangles ‘l’ and ‘m’. L4 identified ‘k’ and ‘m’ while L9 identified ‘k’ only as an equilateral triangle. The reason why these learners had such varied responses was the fact that they could not make an informed judgement based on sight. All triangles were not labelled with symbols as a result they used physical appearance for their identification.

The activity was solely on obtaining responses using visualisation skills to estimate the lengths of a given triangle.

In the same question 3.1.2 there were extreme cases, for example learners who could not identify even one triangle as an equilateral. L6 identified choices ‘b’, ‘f’, ‘j’, ‘o’ and ‘p’ while L8 identified triangle, ‘b’ only. According to Sarwadi and Shahrill (2014) such responses were due to learners’ inability to do estimation, a skill which would have been developed in early grades. L8’s responses showed that the learner was at pre-recognition level of Clements and Battista (1991).

Question 3.1.3 of intervention activity 3 required learners to identify obtuse-angled triangles by estimation. The expected solutions are triangles labelled ‘b’, ‘f’ and ‘j’. L1, L2 and L9 managed to identify triangles labelled ‘b’, ‘f’ and ‘j’. These three learners were at level 1-analysis of the van Hiele model of geometric thinking. The other three learners (L1, L2 & L9) identified other triangles as obtuse-angled triangles, which they were not, for example; L1 included ‘i’. L2 included ‘c’, ‘i’ and ‘p’ while L9 included ‘i’ and ‘p’. L5 identified triangle labelled ‘b’ and also included ‘d’, ‘o’ and ‘p’. L4 identified triangle ‘j’ along with ‘o’. This simply showed that learners were at pre-recognition level 0 of Clements and Battista (1991).

Other triangles which three learners (L3, L6 & L7) considered as obtuse-angled triangles in question 3.1.3 included triangles labelled ‘a’, ‘c’ and ‘h’; ‘d’, ‘g’, ‘k’, ‘l’ and ‘m’; ‘a’, ‘c’, ‘h’ and ‘n’, respectively. L8 did not give any response as required.

In question 3.1.4 of intervention activity 3, learners were required to identify right-angled isosceles triangles which should include triangles labelled ‘a’, ‘e’, ‘h’ and ‘n’. Learners came up with different choices, for example, L5 managed to give all four correct responses without including any other triangle. L5 was able to make connections with what was done in the previous intervention activities where they made use of the polygon pieces and a mathematics dictionary.

In question 3.1.4 of intervention activity 3, L1 and L9 managed to identify the four triangles as expected, but they also included other triangles labelled ‘c’ and ‘b’; ‘c’, ‘I’, ‘l’, ‘m’ and ‘p’, respectively

In question 3.1.4 of intervention activity 3, each respondent in another group of five learners (L2, L3, L4, L6 & L8) responded with different responses as follows: L2 identified ‘h’, but also included triangles labelled ‘f’, ‘o’ and ‘p’. L3’s three correct responses (‘a’, ‘e’ and ‘h’, triangle ‘c’) were also included in the solution. L4 correctly identified triangles labelled ‘a’, ‘h’ and ‘n’ and also included are triangles labelled ‘c’ and ‘k’ which were not the required responses. Only one correct response, triangle labelled ‘a’ was identified by L6 along with triangles ‘c’, ‘k’ and ‘p’. L8 identified triangles labelled ‘a’ and ‘n’ only.

In question 3.1.4 of intervention activity 3, the extreme case was L7, who identified all incorrect triangles, viz: triangles labelled ‘a’, ‘b’, ‘j’ and ‘o’. This learner could not link the knowledge acquired in the previous intervention activities with what has been asked in question 3.1.4. These results showed that learner was not committed and focused in the previous intervention activities. Those who failed to answer question 3.1.4 correct were operating at level 0-pre-recognition of Clements and Battista (1991).

Question 3.1.5 of intervention activity 3’s suggested responses were: triangles labelled ‘c’, ‘i’ and ‘o’. In this question too, learners responded differently. Table 4.5 below gives summarised results of how each of the learners responded to question 3.1.5.

Table 4.5: How learners responded to question 3.1.5

Learner's code	Right-angled scalene triangles identified	Incorrect choices included in the responses
L1	i and o	
L2	o	h, j, p
L3	o	b, n
L4	c, i, o	
L5	c	d, k, l, m
L6	o	b, f, g, q
L7	c	a, b, d, e, g, k, n
L8		
L9	c, o	a, e, f, h, i, j, n

Table 4.5 indicates how each of the learners made choices by using visualisation skills to identify the right-angled triangles from a set of different types of triangles. Only L4 managed to identify the three triangles as required. L1and L9 managed to identify two of the three triangles as shown in Table 4.5 above. The other group of six learners identified only one correctly, but their responses included other triangles that were not supposed to be part of the solutions. L8 did not have any choice.

In question 3.1.5 of intervention activity 3, L4 managed to identify all three correct solutions without including any other triangle. L1 identified triangles labelled 'i' and 'o' only.

L2, L3 and L6 identified the triangle labelled 'o' and also included others which were not right-angled triangles. L5 and L7 identified the triangle labelled 'c', along with other triangles. L9 managed to identify triangles 'c' and 'o' and included other triangles which were not right-angled. Table 4.5 shows which triangles were also regarded as right-angled by the learners mentioned in this paragraph. L8 could not give any response at all.

Learners who responded correctly to both questions 3.1.4 and 3.1.5 demonstrated that their conceptual understanding of the two questions was at both level 0-visualisation and level1-analysis of the van Hiele model of geometric thinking. Those who mingled the correct triangles with the wrong ones demonstrated that they were not well developed to be at level 0-visualisation and level 1-analysis. In some concepts they were still at pre-recognition level as suggested by Clements and Battista (1991), together with those who failed dismally like L8 in question 3.1.5.

In questions 3.2.1 to 3.2.5 of intervention activity 3 learners cut out the line segments from the copies of each of the triangles labelled 'a' to 'p'. After cutting out three line segments of the triangle, they compared the lengths of the line segments by placing each of the cut out line segments on top of the ones in each of the corresponding original triangles labelled 'a' to 'p'. This activity was aimed at guiding learners in establishing the side properties of the given triangles without being told by the teacher. These questions were aligned to level 1-analysis of the van Hiele model of geometric thinking.

In question 3.2.1 of intervention activity 3, L1 and L4 managed to identify all six solutions without including any other triangle that was not an isosceles. In question 3.2.2, L1, L4, L5, and L7 managed to get the expected responses correctly without including any other triangles.

In question 3.2.3 of intervention activity 3, the expected responses were: 'b', 'c', 'f', 'i', 'j', 'o' and 'p'. Of the nine learners, four (L1, L4, L7 & L8.) managed to give the correct responses without including other shapes that were not scalene triangles.

The results of the learners who answered the questions correct as described in the three paragraphs above showed that L1 was well developed and stable at level 1-analysis of the van Hiele model of geometric thinking. L1 answered all five questions correctly. Other learners were not consistent in achieving at the very same level (level 1-analysis); for example, L4 and L5 answered three out of the five questions correctly, L7 managed to answer two questions correctly and L8 answered one correctly.

In question 3.2.4 of intervention activity 3, learners were asked to use polygon pieces to identify right-angled isosceles triangles. The expected responses were: triangles labelled ‘a’, ‘e’, ‘h’ and ‘n’. Of the nine learners engaged in the activities, only two L1 and L5 managed to identify all the required triangles correctly. The two learners were at level 1-analysis of the van Hiele model of geometric thinking in this question because that is where the question belongs.

In question 3.2.5of intervention activity 3, learners were supposed to identify right-angled scalene triangles labelled ‘c’, ‘i’ and ‘o’ as correct responses to the question. L1 identified the three triangles correctly with no any other triangle included in the list.

Reponses to questions 3.2.1 to 3.2.5 showed that the four (L4, L5, L7 & L8) were not stable at level 1- analysis. Sometimes an individual was migrating to pre-recognition level as hypothesised by Clements and Battista (1991) in one question or the other.

In question 3.2.1 of intervention activity 3, L7 identified the required six, but also included the triangle labelled ‘b’ in the list as one of the responses. Triangles labelled ‘a’, ‘e’, ‘h’ and ‘n’ were identified by L2 and L9 as isosceles triangles. The only difference between the two was that L9 did not include other triangles in the responses while L2 included triangle ‘f’.

In question 3.2.1 of intervention activity 3, L3 and L5 identified triangles labelled ‘a’, ‘d’, ‘e’, ‘h’ and ‘n’ as isosceles triangles, the difference between the two is that L3 included other triangles which were not isosceles, for example, triangles labelled ‘i’, ‘o’ and ‘p’. L6 managed to correctly identify triangles ‘d’, ‘e’, ‘g’, ‘h’ and ‘n’. L8 identified triangles labelled ‘a’, ‘e’, ‘g’, ‘h’ and ‘n’, as well as triangle ‘c’ which has been included in the list of responses.

In question 3.2.2 of intervention activity 3, L2, L3, L6, L8 and L9 also included triangles ‘k’, ‘l’ and ‘m’, but their responses included other triangles that were not equilateral. One thing that all the incorrect responses had in common was the triangle labeled ‘g’. On the other hand L2, L6 and L9 included triangles ‘c’, ‘i’ and ‘d’, respectively.

In question 3.2.3 of intervention activity 3, L5 included other triangles, like the triangle labelled ‘g’. L2 and L9 identified triangles labelled ‘b’, ‘c’, ‘f’, ‘i’ and ‘j’ only. L3 was able to identify only the triangles labelled ‘b’, ‘c’, ‘f, ‘j’ and ‘o’ as scalene triangles. L6 identified shapes labelled ‘i’, ‘j’, ‘o’ and ‘p’ as scalene triangles, in the list non scalene triangles ‘a’ and ‘n’ were also included.

In question 3.2.3 of intervention activity 3, of the seven triangles required as expected responses; the minimum of four and maximum of seven scalene triangles were identified by the learners because of the polygon pieces which they used to compare each of the triangles’ line segments one against the other. By the comparison of line segments learners’ confidence was stimulated and spontaneous skills of understanding of spatial situations developed (Jones 2002).The reason for incorrect responses was that some learners ignored the use of polygon pieces and mathematics dictionary; they decided to use visual observances without actually taking the measurement, which resulted in them failing. These learners were at pre-recognition level 0 of Clements and Battista (1991)

In question 3.2.4 of intervention activity 3, L8 and L9 both identified triangles ‘a’, ‘e’ and ‘n’. The only difference in their responses was that L8 only focused on the above mentioned three triangles while L9 included triangles labelled ‘c’ and ‘g’ in the list of responses which two shapes were not right-angled isosceles triangles. L2 only identified triangles ‘a’, ‘e’ and ‘h’. Triangles labelled ‘h’ and ‘n’ were identified by L4. L3 identified triangles ‘a’ and ‘h’, and included ‘c’ which was not a right-angled isosceles triangle. L7 identified triangles ‘a’, ‘h’ and ‘n’, and also included other triangles labelled ‘b’, ‘d’ and ‘g’. Lastly, L6 identified only one of the expected triangles, ‘a’, and the other included were triangles labelled ‘d’, ‘l’ and ‘o’. Failing to identify all the expected triangles it means that those learners were at level 0-pre-recognition, as described by Clements and Battista (1991).

Other learners could not identify all the responses, they did not follow the instruction of the intervention thoroughly and, as a result, they opted to use the sight to identify right-angled isosceles triangles.

In question 3.2.5 of intervention activity 3, L7, L8 and L9 managed to identify the required triangles, but their responses were inclusive of other triangles that were not supposed to be part of the responses. For example, L7 included triangles ‘b’, ‘f’, ‘j’ and ‘p’. L8 also included the triangle labelled ‘f’ and L9 included triangles ‘a’, ‘b’, ‘e’, ‘f’, ‘j’ and ‘n’ in the list of responses.

In question 3.2.5 of intervention activity 3, L2 and L3 identified the triangle labelled ‘o’, but they both included other triangles that were not part of the correct responses, such as ‘d’, ‘p’ and ‘n’, ‘b’. L6 only identified the triangle labelled ‘c’ along with other triangles which were not right-angled scalene triangles, for example; ‘b’, ‘f’ and ‘p’. Lastly, L5 identified only two triangles labelled ‘c’ and ‘o’. The two learners were at level 0 – pre-recognition of Clements and Battista (1991).

In question 3.3 of intervention activity 3, 16 different triangles were drawn. Learners cut out line segments and angles from the copies of the given triangles. The polygon pieces were used to help learners to indicate whether a certain triangle was a right-angled triangle or a right-angled isosceles triangle, or a scalene, an equilateral or an isosceles triangle by inserting relevant mathematical symbols to each of the given triangles.

In question 3.3 of intervention activity 3, seven learners (L2 to L8) showed only two properties of triangle labelled 'a'; one-90^0 angle and two equal sides. The only difference was L9 showed that triangle labelled 'a' has two equal sides only.

In question 3.3, of intervention activity 3 in triangle labelled 'e', L4 showed that the triangle labelled 'e' had only two equal sides. L3 indicated that the hypotenuse of triangle 'e' was equal to one other side and L7 has marked one of the angles and no sides were marked by any symbol. In triangle labelled 'h', L8 and L9 did not use any symbol to show its properties.

In question 3.3 of intervention activity 3, L1 indicated that triangle labelled 'n' was a right angle and all three sides are equal. L7 also inserted the right angle symbol and used double slashes on one of the sides of a triangle while a single slash was inserted on the other side. This learner wanted to show that the two sides were equal in length. L2, L8 and L9 did not use any symbol to illustrate the properties of triangle labelled 'n'.

In question 3.3 of intervention activity 3, L3 and L6 showed that in the triangle labelled 'n', all angles were equal, the hypotenuse and one other side of the triangle were equal in length. Such responses indicated that polygon pieces and mathematics dictionary were not used as instructed. L4 showed that the triangle labelled 'n' had two equal sides only while L5 used the right angled symbol and the equality symbols to show the equality of two opposite sides.

In question 3.3 of intervention activity 3, in all the four triangles (a, e, h & n) many learners did not consider the property of the equal angles as part of the solution. Such results indicated that learners used their own preconceived ideas regarding the right-angled triangles instead of making use of the polygon pieces and mathematics dictionary which were provided.

In question 3.3 of intervention activity 3, the second category of triangles comprised the triangles labelled ‘k’, ‘l’ and ‘m’, which were equilateral triangles. In triangle ‘k’, seven learners (L1 to L7) used the symbol to show that all sides were equal in length. The angle properties were not considered at all. L7 managed to illustrate that triangle ‘k’ was an equilateral triangle with both symbols for equal sides and equal angles, while L8 did not use any symbols in the triangle labelled ‘k’.

In question 3.3 of intervention activity 3, in triangle labelled ‘l’, L1, L3, L4, L6 and L8 only used the symbols, while L7 showed that triangle ‘l’ was an equilateral triangle using both symbols for equal sides and equal angles. The same triangle was identified as an isosceles by L2 and L5 who indicated that two sides were equal in length, while L9 did not use symbol to illustrate that triangle labelled ‘l’ was an equilateral.

In question 3.3 of intervention activity 3, L4 and L6 inserted symbols to show that all sides of triangle labelled ‘m’ were equal, while L3 and L7 showed that the triangle labeled ‘m’ had equal sides and equal angles. In their responses L5 and L8 have shown that triangle ‘m’ was an isosceles. L2 illustrated that the triangle labelled ‘m’ had a right angle while L9 indicated that it was a right-angled isosceles triangle.

In question 3.3 of intervention activity 3, the third category contained right-angled scalene triangles labelled ‘c’, ‘i’ and ‘o’. L2, L3 and L5’s responses showed that triangle ‘c’ was a right-angled isosceles, L7 and L8 did not use any symbol to show that the triangle ‘c’ was a right-angled triangle.

In the triangle labelled ‘i’, L2, L4, L5, L7 and L8 did not insert any symbol while L3 and L6 used symbols to show that triangle ‘i’ is an isosceles. L9 has inserted some marking in all the three angles of triangle ‘i’ as if it is an equilateral.

In triangle ‘o’ five learners (L2, L6, L7, L8 & L9) did not insert any symbol. L4 shaded all the three angles, which was an indication that all the angles of triangle ‘o’ were equal.

The correct responses given by most learners serve as evidence that mathematics dictionary and the use of polygon pieces enhanced learners’ conceptual understanding of what a right-angled scalene triangle looks like. The learners who could not do well, for example those who used an incorrect symbol for a right angle and those who did not use any symbols at all, the problem might be the mathematics vocabulary barrier, which was the major problem with most learners. Some could not use the dictionary as others did, even to conceptually understand that the meaning of the sentence was a challenge.

Another category of triangles in question 3.3of intervention activity 3, was a set of triangles labelled ‘d’ and ‘g’, which were isosceles triangles, L4, L5, L6 and L9 only inserted one symbol (two opposite sides are equal in length) to show that triangle labelled ‘d’ was an isosceles. L3 and L7 managed to inset the symbol for the equality of the sides in an isosceles triangle, but the angle property was not correctly done, they indicated that all angles were equal. L2 inserted a symbol to show that two opposite sides were equal, but also included the right angle symbol in the diagram. L8 used no symbols at all and just left the triangle as it was.

In question 3.3 of intervention activity 3, regarding the triangle labelled 'g', three learners (L1, L4 and L8) indicated that the given triangle was an isosceles by using the symbols of the equality of the sides only. L7 identified it as having two equal sides and marked one angle. L2 and L9 did not insert any symbol in the triangle. Failure to insert all the symbols as it was supposed to be done is evidence that their measurement skills were still not well developed. In addition it is clear that the learners had difficulties with the conceptual understanding of the real meaning of each of the geometric symbols.

The responses of L3, L5 and L6's responses above demonstrated the lack of basic conceptual understanding of types of triangles and their properties. The use of hands-on activities also seemed to be a new thing to most of the learners. The shift from the learners' former ways of learning geometry seemed to require an extended time to gradually allow their conceptual and procedural fluency be enhanced (Kilpatrick et al., 2001).

In question 3.3 of intervention activity 3 regarding triangles labelled 'b', 'f', 'j' and 'p', L3 and L6 measured incorrectly the length of triangle 'p', evidence in both answer scripts indicated that two opposite sides were equal, but the other three triangles (b, f & j) were identified as scalene triangles.

In question 3.3 of intervention activity 3, L8's response showing that 'f' is an isosceles was incorrect; it showed that the learner did not bother to use the polygon pieces to compare the length of the sides of triangle labelled 'f', the visual interpretation misled the learner because triangle labelled 'f' had two sides that seem to be equal visually, but in fact, they were not equal at all. Lastly, L4 inserted symbols to show that triangle labelled ‘p’ was an equilateral, yet it was not.

In question 3.3 of intervention activity 3, L1 showed that triangle ‘a’ had a 90^0 angle, two equal sides and two equal angles. In triangle labelled ‘e’, L1, L2, L5, L6, L8 and L9 inserted a right-angle symbol and also symbolically indicated that two sides were equal. Triangle labelled, ‘h’ was indicated by seven learners (L1 to L7) as having a right angle and two equal sides.

In question 3.3 of intervention activity 3, the third category was of right-angled scalene triangles labelled ‘c’, ‘i’ and ‘o’. In the triangle labelled ‘c’, L1, L4, L6 and L9 inserted only a right-angle symbol correctly into the triangle. In triangle labelled ‘i’ only L1 managed to insert the correct symbol to indicate that the triangle was a right-angled scalene. Triangle ‘o’ was correctly presented as a right-angled triangle by only L1, L3 and L5.

In question 3.3 of intervention activity 3, another category of triangles was a set of triangles labelled ‘d’ and ‘g’, were isosceles triangles. L1 managed to show both properties of an isosceles triangle, two opposite sides were equal and angles opposite equal sides were equal.

In question 3.3 of intervention activity 3, the last category of triangles labelled ‘b’, ‘f’, ‘j’ and ‘p’ were clearly identified by L1, L2, L5, L7 and L9 as scalene. That was after the measuring exercise which was done using polygon pieces and the use of mathematics dictionary helped the five learners to conceptually understand that a scalene triangle has sides of different lengths and three angles of different sizes.L8 managed to identify shapes labelled ‘b’, ‘j’ and ‘p’ as scalene triangles.

4.2.4.2 Results of reflective test 3

Reflective test 3 was written during the first hour of the fourth day of my data collection. The main aim was to assess learners' conceptual understanding of the content covered in intervention activity two. Table 4.6 below shows how learners performed in test three as compared to the diagnostic test.

Table 4.6: Learners' overall performance in the diagnostic test and reflective test 3

Item	Min	Mean	SD	Median	Maximum
Diagnostic test	0	18	14.82	22	44
Reflective test 3	5	24.89	12.90	19	48

Note: Values of minimum, mean and standard deviation and maximum for learners' (N=9) marks obtained in two activities, diagnostic test and reflective test three.

The statistics in Table 4.6 above show that the intervention activity which learners were engaged in had an influence in their conceptual understanding of the mathematical symbols used in different triangles. For example, when a given triangle was a right-angled triangle most learners were able to insert the right angle symbols in the triangle. As shown in Figure 4.6 the measures of central tendency, the: minimum, mean median and the maximum for the reflective test 3 are greater than those of the diagnostic test. These results show that in some cases, some learners were able to identify a particular triangle by its name and describe why that particular triangle is called by that name. The detailed description of how learners responded to each of the questions in the reflective test is given below.

In question 3.1.1 of reflective test 3, learners were supposed to show that the triangle labelled 'a' was an isosceles using all symbols for an isosceles triangle. Out of the nine learners, seven (L2, L4, L5, L6, L7, L8 & L9) inserted the symbol correctly. In this question, the seven learners performed at level 1-analysis of the van Hiele model of geometric thinking.

In question 3.1.1 of reflective test 3, L1 did not use any symbol, but just left the triangle unmarked and L3 just put a mark in one of the angles. L1, L2, L3 and L9 could not use symbols to show that triangle labelled 'b' in question 3.1.2 is an equilateral. Instead of showing both properties for an equilateral triangle, five learners (L4, L5, L6, L7 & L8) all used the side property only.

In question 3.1.3 of reflective test 3, triangle labelled 'c' was left with no symbol to indicate that it was a right-angled triangle by L1 and L8. A group of three learners (L2, L6 & L9) came up with different responses. For example, L2 and L9 said that *"all angles are equal",* while L6 inserted a symbol to show that two opposite sides are equal.

In question 3.1.3 of reflective test 3, four learners (L3, L4, L5 & L7) were able to indicate that the triangle labelled 'c' was a right-angled triangle by using a right angle symbol. This means that the four learners in this question were operating at an appropriate level 1-analysis posited by the van Hiele theory.

In question 3.1.4 of reflective test 3, learners were supposed to show that the triangle labelled 'd' was a right-angled isosceles. The responses to this question were categorised into six categories. L4 and L5 managed to insert two symbols, a 90^0 symbol and slashes to show that the two lines were equal in length. Question 3.1.4 was aligned to level 1-analysis of the van Hiele model of geometric thinking. The correct responses given by the two learners confirm that they performed at level 1-analysis.

In question 3.1.4 of reflective test 3 learners were supposed to show that the triangle labelled 'd' was a right-angled isosceles. L2, L3 and L7 only inserted a 90^0 symbol while L6 inserted slashes to show that two opposite sides were equal. L1 did not insert any symbol.

L9's response showed that all sides were equal and L8 showed that the height of triangle labelled 'd' was equal to its hypotenuse.

In question 3.2.3 of reflective test 3 seven learners (L2, L3, L4, L6, L7, L8 & L9) came up with different descriptions of an obtuse-angled triangle. Below are the responses from each of the seven learners:

L2: *"all sides are equal."*

L3: *"they have straight line."*

L4: *"Angled isosceles triangles."*

L6: *"They are not equal sides"*

L7: *"All the sides are not equal."*

L8: *"it because the angles are less than* 90^0*."*

L9: *"greater than* 90^0*"*

In question 3.2 of reflective test 3, learners were supposed to identify right-angled scalene, acute-angled, obtuse-angled and scalene triangles from the four triangles. For each choice reasons were to be given. In questions 3.2.1 to 3.2.4, no learners could identify the correct triangles as required in each of the questions. For details of question 3.2, refer to appendix 23. All the learners used the diagrams in question 3.1 to respond to question 3.2. This showed that learners did not follow the instruction and conceptually understand what the question required them to do. Evidently this shows that learners had the mathematical language barrier, which made them not to comprehend the questions.

In question 3.2.4 of reflective test 3; L3, L4 and L8 described a scalene triangle based on the sizes of its sides only. For example, they said that *"All sides are not equal"*. L1 focused only on the angle property and said that *"the angles are all not equal."*

In question 3.2.4 of reflective test 3, two learners (L6 & L9) could not give the correct answer to the question. Each one had a different response to the other, for example, L6 said that *"Because they are not equal and they are not have* 60^0 / 90^0*."* L9 said that *"Two angles are equal."*

Despite the fact that learners have chosen incorrect triangles, some managed to give the reasons that were relevant to questions 3.2.1 to 3.2.4. In question 3.2.1of reflective test 3, four learners (L1, L4, L5 & L7) gave two descriptions of what a right-angled scalene triangle looks like while L3 and L6 only gave one description. Three learners (L2, L8 & L9) could not give correct descriptions. For example, L8 said that "*it because two sides are not equal, they are greater than* 90^0*."* Such a response showed that some learners still had some difficulties that hindered them to conceptually understanding of the properties of triangles.

The mathematical vocabulary challenges demonstrated by learners were also noted in the results of the reflective test 3, specifically in question 3.2; for example, when question 3.2.3 required the learners to describe what an obtuse-angled triangle looks like, L4 said *"angled isosceles triangle."* This shows that besides having problems with conceptual understanding of geometry, L4 could not also construct a simple sentence, yet sentence construction is an essential element in geometry teaching and learning. A learner needs to know the language of teaching and learning well in order to do well in geometry, for example giving reasons to support a point needs good sentence construction skills.

In question 3.2.3 of reflective test 3 learners were supposed to give one description of an obtuse-angled triangle. Out of the nine only two (L1 & L5) managed to give the correct description. Their responses were: *"One angle is greater than* 90^0 *but less than* 180^0*"*.

Questions 3.2.4 of reflective test 3 required learners to describe a scalene based on its two properties. Three learners (L2, L5 & L7) managed to give the correct descriptions of a scalene triangle, for example, they said that: *"All angles are not equal in sizes and all sides are not equal in length."*

Questions 3.2 and 3.3 of intervention 3 were aligned to level 1-analysis of the van Hiele model of geometric thinking, therefore if a learner who failed to perform well in responding to a particular section of either one of questions 3.2 or 3.3, it means that, in that section the learner was operating at pre-recognition level hypothesised by Clements and Battista (1991).

Comparing the diagnostic test with reflective test 3's results showed that the use of polygon pieces in learning geometry helped the learners with conceptual understanding of the properties of various triangles despite the fact that there was a slight difference in performance. The uses of polygon pieces and mathematics dictionary have influenced learners' learning of geometry. These findings are in agreement with Duatepe's (2004) findings that have shown that anyone who has learnt geometry well has visualisation skills, improved reasoning capabilities and is able to appreciate the creation within the surrounding.

The skills learnt from the intervention activities where they used polygon pieces to explore the properties of triangles made it possible for them to get correct responses to question 3.2. As I was observing learners writing reflective test 3 some learners such as L5 were using small pieces of paper to confirm the lengths of a certain triangles.

In question 3.2 of reflective test 3, most learners could not operate at level 1-analysis of the van Hiele geometric thinking model. The results described above show that some learners could not stay at level 1-analysis of the van Hiele model of geometric thinking. At some stage they would move back to level0-visualisation of the van Hiele geometric thinking model, for example, in question 3.2.1, L5 could describe what a right-angled scalene triangle looks like, yet in question 3.2.2, L5 could not describe how an acute angled triangle looks like and in question 3.2.4, L5 managed to describe fully what a scalene triangle looks like. Getting one property of a triangle correct and fail another one was an indication that the conceptual understanding of the properties of triangles at that time needed more exposure to the geometric concepts for advancement. Most of the learners were at the pre-recognition level posited by Clements and Battista (1991).

4.2.5 Results of intervention activity 4 and reflective test 4

4.2.5.1 Results of intervention activity 4

In intervention activity 4 learners were required to match the six triangles drawn with their properties described in questions 4.1.1 to 4.1.6. For details of the content refer to appendix15. Learners were to use two copies provided for each of these shapes: $\Delta LMN, \Delta ABC, \Delta JKL, \Delta OPQ, \Delta DFH, \Delta RST$. From each of the first six copies of $\Delta LMN, \Delta ABC, \Delta JKL, \Delta OPQ, \Delta DFH, \Delta RST$ line segments were cut out. From each of the second copies of $\Delta LMN, \Delta ABC, \Delta JKL, \Delta OPQ, \Delta DFH, \Delta RST$ angles were cut out.

After the cutting of line segments and angles learners compared the lengths of the three line segments in each of the triangles by placing the cut line segment on top of the other two in the original triangle one at a time, for example in ΔLMN they compared the length of $\overline{LM}$ against $\overline{MN}$ and $\overline{LN}$. Thereafter, they also compared the sizes of the three angles in each of the triangles; for example in ΔLMN, they cut out the three angles. $\hat{L}$'s size was compared in relation to the other two, $\hat{M}$ and $\hat{N}$, by placing $\hat{\mathrm{L}}$ on top of the other two angles, one at a time. The results were recorded down and then used to respond to questions 4.1.1 to 4.1.6.

In question 4.1.1 of intervention activity 4, the solution had to be written as follows: $\Delta \mathrm{ABC}$, but learners responded to this question differently L1, L3, L4, L6 and L8 managed to match the triangle with the correct statement as required, meaning that they performed at level 1-analysis as suggested by the van Hiele model of geometric thinking.

Those who gave incorrect responses to question 4.1.1of intervention activity 4, were: L2 said that it is MNL, L5 said LMN, ABC, OPQ. L7 said that ABC while L9's response was written as '*A,b,c*'.

The last category of responses is made up of one learner – L8, who gave the response in question 4.1.1 as ABC instead of $\Delta \mathrm{ABC}$.

At the analysis level of the van Hiele model is question 4.1.2of intervention activity 4. The expected correct response to this question was triangle DFH, but the learners came up with different responses. Those who responded correctly were L1, L2, L3 L4, L6 and L8. This tells us that the six learners were performing at level 1-analysis theorised by van Hiele (1999).

In question 4.1.2of intervention activity 4, were three learners had problems in the use of symbols in their responses, for example, L5 just said DFH. L7 said $\mathrm{D\hat{H}F}$. L9's response was, D, F,H.

The expected answer to question 4.1.3of intervention activity 4 was triangle OPG. Most of the learners came up with other options, for example, L2 and L3 just said RST. L5's response was given as ABC. L8 chose triangle ABC. L6 opted for two triangles as the answer to question 4.1.3, triangles RST and LMN. L7 and L9 chose the correct response, but the problem lied in a way the triangle has been written down, each wrote the response as $O\hat{P}Q$ and O,P,Q, respectively, which were wrong representations mathematically.

In question 4.1.3 of intervention activity 4, L1 and L4 gave the correct response. These results show that in this question the two learners were operating at level 1-analysis as suggested by the van Hiele model of geometric thinking.

Another category was of learners who did not name the triangles correctly, for example, in question 4.1.3 instead of giving the answer as ΔOPQ L9 said O, P, Q. There were other such responses by some of the learners in other questions. The fourth category was the group that mentioned a triangle as if it was an angle, for question 4.1.3of intervention activity 4, L7 has written $O\hat{P}Q$ instead of ΔOPQ.

In question 4.1.4 of intervention activity 4, the expected correct responses were triangles LMN and ABC. L1, L3, L4 and L8 gave only one correct response – triangle LMN.

In question 4.1.4of intervention activity 4, L9's response was written as: triangle *lmn*. L2 chose one of the correct responses, but did not use the symbol for a triangle, the response was just written as ABC. L5 also did not indicate that LMN and ABC were triangles. L6's responses have been written as Δlmn and ΔIJK. L7 wrote the response as $L\hat{M}N$.

The correct answer to question 4.1.5of intervention activity 4, was supposed to be triangle RST, only L1 managed to identify the response without including other triangles which were not part of the required response.

In question 4.1.6 of intervention activity 4, the correct response was supposed to be triangle JKL, but learners responded to it differently based on their conceptual understanding of the question. Only L1 has only chosen triangle JKL as the one that has the properties defined in question 4.1.6.

In question 4.1.5 of intervention activity 4, L5 chose three responses that were written as RST, OPQ and IJK. L2 and L9 wrote their response as JKL while L3 and L4 wrote their responses as ΔJKL. L6's answer was ΔOPQ. L8's response was given as triangle ABC. L7's response was presented as $R\hat{S}T$.

In question 4.1.6 of intervention activity 4, L6 gave two choices for the same question triangles JKL and RST. L2, L3 and L5 had the same choices of responses, but presented it differently, L3 presented the answer as ΔOPQ while L2 and L5 just said OPQ. L4, L7 and L9 had the same choice of response to question 4.1.6, but differed in the way in which they presented their answers, they said ΔRST, $R\hat{S}T$ and RST, respectively. L8 chose ΔDHP as the response.

In intervention four, identified six different categories of responses. The first group of learners managed to match the given triangle with the defined properties, for example L1. The clarity given to the learners enabled them to give correct responses to questions that they could not conceptually understand at first. In addition, their measuring skills were well applied when using pieces of line segments and angles.

The second category was of those learners who could not match given triangles with their specific properties, for example L6 in question 4.1.5of intervention activity 4.

In the third category was a group of learners who used lower case in naming certain triangles, for example, ΔLMN was written as Δlmn by L9 and others. According to White (2005) such problems are associated with coding problems where the choice of the solution was correct, but the learner failed to present the solution in an acceptable written mathematical form. Learners with such problems needed extended time to be engaged in the activities so that they can conceptually understand different mathematical concepts.

In summary, all the learners who managed to respond to questions 4.1.4 to 4.1.6 correctly demonstrated each one of them was operating at level 1-analysis of the van Hiele geometric thinking model.

4.2.5.2 Results of reflective test 4

In the fifth day of data collection, the first activity administered to the learners was reflective test 4. The test was written in order to check learners' conceptual understanding of concepts learnt in intervention activity 3 that was performed the previous day. The reflective test was set at level 0-visualisation as hypothesised by the van Hiele model of geometric thinking. It focused on the identification of acute- angled and scalene triangles using the angle properties. For content details of reflective test 4, refer to appendix 24. The statistics of the comparison of learners' overall performance in the diagnostic test and reflective test 4 are presented in Table 4.7 below:

Table 4.7: Learners' overall performance in the diagnostic test and reflective test 4

Item	Min	Mean	SD	Median	Maximum
Diagnostic test	0	18	14.82	22	44
Reflective test 4	0	43	25.14	43	100

Note: Presented in this table are values of minimum, mean and standard deviation and maximum for learners' (N=9) marks obtained in two activities, diagnostic test and reflective test 4.

Table 4.7 above shows the comparison between learners' overall performance in the DT and RT4. Both data sets are skewed to the left with the same minimum scores. The data differs in the mean, standard deviation, median and maximum values. As shown in Table 4.7 aboveRT4's measures of central tendency are greater than those of the diagnostic test which is an indication of how learners improved in conceptual understanding of the properties of acute-angled and scalene triangles. The improvement shown in test 4's results demonstrated how polygon pieces positively influenced learners' conceptual understanding of the properties of acute-angled and scalene triangles.

Responding to question 4.1 of reflective test 4, four learners (L2, L4, L5 & L9) managed to identify three acute-angled triangles ($\Delta DEF, \Delta GKL \,\&\, \Delta LMN$) correctly without including any that was not an acute-angled triangle.

In question 4.1 of reflective test 4, L1 identified the three required triangles and also included ΔOPQ, which was not an acute-angled triangle. L6 and L8 identified one triangle each, ΔDEF and ΔGKL, respectively. In the list of acute angled-triangles identified, L6 included ΔOPQ and ΔDBC while L8 identified and presented the second and third triangle as $\hat{D}\hat{E}\hat{F}$ and ΔSTR , respectively. L7 responded to the same question as follows: *"DEF, GKL, MNL and RST."* Of the three triangles required, L3 managed to identify ΔLMN only.

In question 4.2 of reflective test 4, L5 managed to identify all four triangles that were scalene ($\Delta XYZ, \Delta UWV, \Delta OPQ \,\&\, \Delta RST$). The two groups of learners who managed to respond correctly to questions 4.1 and 4.2 of the reflective test 4 demonstrated that they were able to operate at level 0-visualisation according to the van Hiele model of geometric thinking.

Questions 4.2 of reflective test 4 required learners to identify scalene triangles from the set of six. For the detailed content of the test, refer to appendix 24. L6 identified three triangles; of the three, two are scalene triangles namely: ΔXYZ and ΔUWV, the third is ΔGKL which was also identified by L7, yet it was not a scalene triangle. L3, L8 and L9 identified ΔXYZ as a scalene, but the three learners differed in the sense that L3 chose only one triangle while L8 also identified ΔGKL and, instead of saying ΔSTR, this learner used incorrect symbols as follows: $\hat{S}\hat{T}\hat{R}$. In addition to the choice of ΔXYZ, L9 made three other choices but presented them mathematically incorrect as shown: *"UVW, OPQ, RST and Kgl."* L2 also had the same conceptual understanding as L9 in terms of writing the triangles without inserting the triangle symbols, for example all the chosen responses are written as follows: *"UVW, OPQ and RST"*

In questions 4.2 of reflective test 4, two other learners (L1 & L4) were not clear of the characteristics of a scalene triangle, both learners listed down the entire set of the given triangles.

Some learners were clear in identifying the required triangles; for example, in question 4.2, L2 made correct choices, but did not insert any symbol. This was an example of a learner operating at level 0-visualization of the van Hiele geometric thinking, to be at level 1-analysis mathematical vocabulary and symbols were supposed to be understood and used accordingly.

Some learners regarded an isosceles triangle as a scalene, for example L8 identified ΔGKL, yet the angles and length of edge as indicated on ΔGKL clearly showed that the shape was an isosceles. From the responses given it could be deduced that these learners did not make use of properties within each of the figures in order to make informed decisions which are categorised as at level 2-abstraction of the van Hiele geometric thinking. Failure to use the relationships of angles and sides in a triangle shows that a learner is operating at level 0-visualisation of the van Hiele geometric thinking.

4.2.6 Results of intervention activity 5 and reflective test 5

4.2.6.1 Results of intervention activity 5

In intervention activity 5, learners were given the diagram of ΔABC and its two copies. From the first copy they had to cut out line segment AC, AB and BC. After cutting, each learner compared the length of each of the line segments in relation to the other two in the original triangle by placing the cut out piece on top of each of the lines segments, i.e. they compared $\overline{AB}$ with $\overline{AC}$; $\overline{AC}$ with $\overline{BC}$ and $\overline{BC}$ with $\overline{AB}$ .For each measurement taken the results were recorded using comparative adjectives: longer than, shorter than or equal to. This activity was to help learners to conceptually understand the properties of a right-angled scalene triangle without being told by the teacher, but through self-exploration using polygon pieces.

In question 5.1 (iii) of intervention activity 5, out of nine learners, seven (L1, L2, L3, L4, L5, L7, & L9) managed to measure as per instruction and correctly used the terms: *'longer than, shorter than, equal to'* in their responses. Since this question was at level 1-analysis of the van Hiele geometric thinking model, the learners who got it right performed at the very same level 1-analysis.

Two learners came up with different responses to question 5.1(iii)of intervention activity 5, L6's response was: *'AC is longer than B and BC are shorter than A and AB are shorter than the C'*, while L8 presented the responses as follows:

$\Delta \hat{A}B$ *shorter than* $\Delta \hat{A}\hat{C}$

$\Delta \hat{B}C$ *shorter than* $\Delta \hat{A}\hat{B}$

$\Delta \hat{A}\hat{C}$ *buger than* $\Delta \hat{B}\hat{C}$

Seven learners who managed to answer question 5.1 (iii) of intervention activity 5 correctly for the reason that they were able to follow instructions. In instances where the instructions were not clear they used to call for individual help. On the other hand L6 did not ask for any clarity in this question, which resulted in presenting the answers as presented in the recent paragraph.

In question 5.2 of intervention activity 5, learners used the second copy of triangle ABC and carefully cut out the three angles. They were left with the shaded apex. After that each learner compared an angle's size with the sizes of the other two angles by placing one angle on top of each of the angles in the original triangle ABC. Placing an angle on top of the other two was done in order to determine how the three angles are related to each other in terms of sizes.

After being engaged in such an activity, in response to a question which was at level 1-analysis of the van Hiele model that was question 5.2 (i), of intervention activity 5, seven learners (L1, L3, L4, L5, L7, L8 & L9) concluded that "$\hat{A}$ *is smaller than* $\hat{C}$." Their responses were correct according to what was required. This simply demonstrated that the seven were comfortably operating at the stated level of the van Hiele's theory.

Responding to question 5.2 (i) of intervention activity 5, two learners (L2 & L9) came up with different incorrect responses, as follows: L2 said that "*A is maller than* $\hat{C}$. $\hat{C}$ *is bigger than A" w*hile L6's said: "*A is bigger than C and C is smaller than A."* In question 5.2(ii) of intervention activity 5, L2's response was "$\hat{B}$ *is bigger than* $\hat{C}$ *, but C is bigger than B."*L6 said that "*C is bigger than B and B is smaller than C.*"L9 responded by saying: "*angle* $\hat{B}$ *is shorter than* $\hat{A}$ *."*

In questions 5.2(i) to 5.2(iii) of intervention activity 5, not all learners managed to give the correct responses in each of the questions. Some learners did give the correct responses in one, two or all the questions, for example L1 and others they were able to strictly follow the instructions in the activity. The present finding, for example, the way L8 responded to question 5.2(iii) was due to what is known as an encoding error. The learner had correctly identified the solution to a problem, but could not express this solution in an acceptable written form. This learner used the comparative form 'longer than' instead of 'smaller than or greater than or bigger than'.

In all four questions, 5.2(ii) to 5.2(v), two learners L1 and L4 were operating comfortably at level 1-analysis hypothesised by the van Hiele model. The geometric thinking of the four learners (L3, L5, L7 & L8) was identified to be at level 1-analysis in only three questions of the mentioned four. In question 5.2(v) L3 and L5 were at level 0-pre-recognition as theorised by Clements and Battista (1991). L7 and L8 were at pre-recognition level in question 5.2(iii) according to Clements and Battista (1991).

In question 5.2(iii) of intervention activity 5, four different responses were given by each of the learners L6, L7, L8 and L9. According L6, "*B is smaller than A and A is bigger* *than B."* L7 said that *"B is bigger than A and B is maller than A."* L8 responded that "$\hat{B}$ *is longer than* $\hat{A}$." L9 responded a bit different from L8, *"angle* $\hat{B}$ *is longer than* $\hat{A}$ *'.*

In question 5.2(iv) of intervention activity 5, L2 and L6 obtained 33%. The way each of the two learners responded to the question made them to obtain a mark of 33%, for example L2 said *"* $\overline{AB}$ *is longer than* $\overline{BC}$ *and AC is longer than BC and AC."* L6 said "$\overline{AB}$ *is longer than* $\overline{AC}$ *and* $\overline{BC}$ *is shorter than* $\overline{AC}$*."*L9 said that *"Because* Δ *are angles; because* $\overline{AB}$ *are the lines segment."*

In question 5.2(v) of intervention activity 5, L3 and L5 obtained 67% in the same question due to the mixed responses which the learners presented, for example L3 correctly did the comparison between $\hat{A}$ and $\hat{B}$ and between $\hat{B}$ and $\hat{C}$ correctly, but there was nothing mentioned about the relationship between $\hat{A}$ and $\hat{C}$.

In question 5.2(v) of intervention activity 5, L5 responded with three statements showing how the three angles were related. One of the statements was incorrectly presented, for example: "$\hat{C}$ *is smaller than* $\hat{A}$ *and is bigger than* $\hat{B}$*."*

The extreme cases in question 5.2 (v) of intervention activity 5, were responses given by two learners, (L6 & L9) who both obtained 0% in this question alone; such responses directly affected their overall results in intervention activity 5. For more information on how L6 and L9 performed in intervention activity 5, refer to Figure 4.7 and 4.10, respectively. L6 said that *"* $\hat{A}$ *is smaller than B and C"* while L9 responded that: *"triangles are angles* $\hat{A}$ $\hat{B}$ *are less than and C."* L6's problem in the response was the distortion of the meaning of angles by using letters like B and C, referring to them as angles. L6 omitted the symbolic information required to illustrate that B and C are angles. L9 also fell into the same category as L6, but also had a problem with the sentence construction, which is a result of mathematics language difficulties.

Questions 5.2(ii) to 5.2(v) presented below were all aligned to level 1-analysis of the van Hiele levels of geometric thinking. In question 5.2 (ii) of intervention activity 5, six learners (L1, L3, L4, L5, L7 & L8) gave the correct response to the question by saying that:"$\hat{B}$ *is bigger than* $\hat{C}$."Responses to question 5.2 (iii) were also presented differently as shown: "$\hat{B}$ *is bigger than* $\hat{A}$*,"*. This was correctly done by L1, L2, L3, L4 and L5.

In question 5.2 (iv) of intervention activity 5, learners were supposed to give the properties of ΔABC in terms of: $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$. In this question learners were supposed to use their findings in question 5.1(iii) to give the properties of ΔABC. Six learners (L1, L3, L4, L5, L7 & L8) obtained 100%, they managed to describe correctly the properties of the given triangle based on the length of all its sides. The terms 'longer than, shorter than, all' were correctly used in comparative form in the description of how each of the line segment's length is in relation to the other two.

In question 5.2(iv) of intervention activity 5, L2 and L6 obtained 33% in question 5.2 (iv). Each obtained this mark because of the way they have responded, for example L2 said *"*$\overline{AB}$ *is longer than* $\overline{BC}$ *and AC is longer than BC and AC."* L6 said "$\overline{AB}$ *is longer than* $\overline{AC}$ *and* $\overline{BC}$ *is shorter than* $\overline{AC}$*."*L9 said that *"Because* Δ *are angles; because* $\overline{AB}$ *are the lines segment."*

In question 5.2 (v) of intervention activity 5, one of the learners (L6) obtained a mark of 0%. The response was presented as: *"(iv) it is longer than and smaller than (v) longer than and shorter than"*.

Question 5.2 (v) of intervention activity 5 required the learners to determine the properties of ΔABC in terms of, $\hat{A}$, $\hat{B}$, and $\hat{C}$. Different responses were given according to how each of the learners conceptually understood the question. After using the polygon pieces, L1, L4, L7 and L8 managed to get 100% in this question.

In question 5.2 (vi) of intervention activity 5, learners had to mention a specific name given to a triangle with properties mentioned in 5.2 (iv) - (v). The response was supposed to be: 'right-angled triangle'. L2 and L4 managed to respond to this question correctly, they both obtained 100%, although L2's response was, *'right angle triangle'*. These two had the answer to the question correct because they were able to link the findings in question 5.2 (i)-(v) to the required specific name of the triangle with the explored properties. They also used the mathematics dictionary where the definitions of different triangles were explained clearly. Questions 5.2 (iv) - 5(v) and 5(vi) were aligned to level 1-analysis and level 2-abstraction, respectively, of van Hiele's (1999) model of geometric thinking. Learners who could not get questions 5.2 (iv) - 5(v) and 5(vi) there were operating at pre-cognition level suggested by Clements and Battista (1991).

In question 5.2(vi) of intervention activity 5, other learners could not spell the names as they were supposed to be spelt; for example, L3 spelt it as "*scelen,*" L7 said *"scalen"* and L8's solution was written as: *"it is a right scalen triangle."*

In question 5.2(vi)of intervention activity 5, a group of six learners (L1, L3, L5, L7, L8 & L9) each one scored 50% in question 5.2 (v). Most of these six, managed to mention the word ‘scalene’.

4.2.6.2 Results of reflective test 5

The sixth day of data collection, before learners were engaged in the intervention activity five they were supposed to write reflective test 5, which required them to mention all the properties of each of the scalene triangles: right-angled, obtuse-angled and acute angled-scalene triangles. Also they were supposed to match the given names of the triangles with the correct triangles. For the detailed content of reflective test 5, refer to appendix 25.

Table 4.8: Learners’ overall performance in the diagnostic test and reflective test 5

Item	Min	Mean	SD	Median	Maximum
Diagnostic test	0	18	14.82	22	44
Reflective test 5	18	57.67	24	64	100

Note: Values of minimum, mean and standard deviation and maximum for learners’ (N=9) marks obtained in two activities, diagnostic test and reflective test five.

Table 4.8 shows the comparison of mark distribution between the diagnostic and reflective test 5, all the measures presented in Table 4.8 above are greater for the reflective test than those of the diagnostic test, from these statistics I can deduce that the intervention activity in which learners were engaged positively influenced their conceptual understanding of the properties of triangles. Each learner’s responses to reflective test 5 are presented below.

The first three questions of reflective test 5 belonged at Van Hiele’s level 2-abstraction of geometric thinking.

In response to question 1 of reflective test 5, L4, L5, L7 and L8 managed to give all three properties of a right-angled scalene triangle. L1, L6 and L9 mentioned two of the three properties of a right-angled scalene triangle, i.e. the relationship of angles and sides of a given triangle.

Question 2of reflective test 5 required learners to give three properties of an obtuse-angled scalene triangle, upon which different responses were given. I categorised learners' responses into four. The first category consisted of three learners L2, L4 and L5 who managed to give all the required properties correctly. Their conceptual understanding of the properties of obtuse-angled scalene triangles was made possible by the use of polygon pieces that they used to explore the properties of different triangles.

In question 2 of reflective test 5, the second category was comprised of L6, L7, L8 and L9, who said that all the angles are not equal in sizes and the lengths of the sides of the triangle are different. However, the third property was not described. Each of the three learners from the group above mentioned the third different reason which was based on their own conceptual understanding of the properties of triangles. For example, L6 said *"It is because they do not have equal angled."* L7 said that, *"all angles less than 90^0."* L8 said, *"They have greater than 90^0."* L9 said that, *"obtuse angles are greater than 90^0 are 180^0."*

L1 was in the third category based on how this learner responded to question 2 of reflective test 5; for example, instead of giving three properties of an obtuse-angled scalene triangle, L1 gave only one. Such responses revealed that the learner lacked conceptual understanding of the properties of an obtuse-angled scalene triangle.

The last category has only one learner, L3 who responded as follows: *"it is less than 90^0, it is right angle."* In the former statement it seems that the learner wanted to say it has 90^0, but due to difficulties in language comprehension the statement could not be put together correctly. In addition, L3 seems not to know that 90^0 is the same as saying a right angle, the two concepts are regarded as different.

In question 3 of reflective test 5, L1, L2, and L5 each managed to give one property of an acute-angled triangle, side property. The use of polygon pieces helped the learners in conceptually establishing how the sides of an acute-angled triangle are related to each other. The second response for each of the learners above was as follows: L1 said that *"The size of an acute angle are less than* 90^0*."* L2 said *"it is less than* 90^0 *but greater than."* L5 said that *"It have an angle that is less than* 90^0*."*

In response to question 3 of reflective test 5, L3, L6 and L8 could not clearly describe the properties of an acute angled triangle. L3 said that, *"it is more than* 90^0*. Two sides and angles are equal."* L6 said that, *"it is the smaller or bigger than other angles."* L8's said that, *"It is because they are not the same sizes."* Learners' responses showed that the conceptual understanding of the properties of cute-angled triangle was not yet clear by the time the reflective test was written. Up to the sixth day of intervention, L3 seemed not to be able to differentiate an acute-angled triangle from an obtuse-angled triangle. In the same test, L6 could not respond clearly, i.e. two comparative adjectives were used in the same sentence.

Question 3 of reflective test 5 required learners to mention two properties of an acute-angled-scalene triangle. According learners' responses three categories were identified. L4, L7 and L9 were in the first category; they were able to give both the all the required properties of the mentioned triangle.

The results of the learners who managed to answer questions 1, 2 and 3 of reflective test 5 well, giving all the required properties of the triangles, showed that they were operating at level 2-abstraction of the van Hiele model of geometric thinking. The other learners who managed to mention only one property seemed to be not well developed at level 2-

abstraction, but at some stage they were operating at level 0-visualisation posited by the van Hiele model.

In question 4 of reflective test 5, learners were supposed to match each of the triangles' names given in questions 1 to 3 with the correct triangles drawn in question 4. In question 4.1, three learners (L1, L4 & L5) correctly identified ΔABC as an acute-angled triangle. In question 4.2of reflective test 5, three learners (L1, L4 & L9) correctly mentioned that ΔDEF was an obtuse angled triangle. In question 4.3of reflective test 5, four learners (L1, L3, L4 & L5) managed to respond to the question with the correct response, i.e. ΔLMN *is a right-angled triangle*'. All the learners who managed to respond to questions 4.1 to 4.3 proved that they were operating at level 0 -visualisation as suggested by the van Hiele model of geometric thinking.

Responding to question 1 of reflective test 5, L2 described the triangle based on the following: the sides and a right angle. In addition, this learner tried to give the relationship of other angles within a right-angled scalene triangle, but could not do so due to spelling errors; for example, the learner said that *"all egle are not equil."*

Although L3 mentioned the right angle property in response to question 1, the learner could not construct the sentence well, for example, it was written as: *"it is a right angle."* L3 used 'is' which is the third person singular of the present tense of 'be' instead of using 'has' which is the third person singular of the present tense of 'have'. In addition, L3 could not come up with the other property clearly, for example, it was said that *"it has equal angle and sides."*

In question 4.1 of reflective test 5, L6 said that "ΔABC *is bigger than* ΔDGF *and their sides are not equal". This was also a typical example of a learner who has mathematics language barriers; the response simply showed that the question was not understood.

In question 4.1of reflective test 5, L2 and L9 said that "ΔABC *is a revolution*." L3 said that *"*ΔABC *is an obtuse triangle"* while L7 and L8 said that *"it is a scalene triangle."*

In question 4.2 of reflective test 5, L5 chose the correct answer, but the word obtuse was not spelt correctly; for example, it has been spelt *"obtuce."*L6 said that *"their angles are not equal."*

On the other hand, in responding to question 4.2, L2, L3 and L7 said that *"*ΔDEF *is an acute-angled triangle."* L8 said that *"*ΔDEF *is a scalene triangle."* L6 said *"their sides are not equal."*

The correct responses given by some learners in question 4.2 of the reflective test 5 were an indication that the use of polygon pieces and mathematics dictionary played a major role in learners' conceptual understanding of the properties of an obtuse-angled triangle. Learners, who could not answer the question correctly, indicated that the language barrier was the biggest problem. For example, L5, misspelt the word obtuse. Also L6 thought that the question required them to compare the differences between the three triangles, therefore, such a response was given.

In question 4.3 of reflective test 5, the other six learners responded differently. L2 and L7 said that *"*ΔLMN *is an obtuse angled scalene triangle."* L8 said that *"*ΔLMN *is an isosceles triangle."* L9 responded by saying *"*ΔLMN *is an acute angles."*

From the number of learners who answered question 4.3 correctly, I could deduce that the concept of properties of a right-angled triangle was conceptually understood. The intervention programme that made use of polygon pieces and mathematics dictionary in helped learners to learn geometric concepts with ease. When other learners were clear about the geometric concepts of properties of triangles, L6 did not conceptually understand exactly what the question require.

4.2.7 Results of intervention activity 6 and reflective test 6

4.2.7.1 Results of intervention activity 6

In intervention activity 6, learners were given the papers with ΔGHI drawn and its two copies to use for the cutting activity. From the first copy of ΔGHI, each learner had to cut out line segments GH, HI and GI, and then compare the length of each of the line segments with the other two in the original ΔGHI by placing each of the cut out line segment on top of the other two line segments one at a time, i.e. compare $\overline{GH}$ with $\overline{HI}$; $\overline{GI}$ with $\overline{HI}$ and $\overline{GI}$ with $\overline{GH}$.

The aim of this activity was to give learners an opportunity to explore and conceptually understand the properties of an obtuse-angled triangle by using polygon pieces without being told what the properties were. For this reason, one of the two copies of ΔGHI was for the exploration of the side property and the other one was for the angle property. Questions 6.1(i) and 6.1(ii) were part of instruction that learners had to follow in order to do question 6.1(iii).

In question 6.1(iii) of intervention activity 6 that was at level 1-analysis of the van Hiele model of geometric thinking, learners compared the lengths of the line segments using the polygon pieces and for each measurement taken they recorded down their findings. They used these comparative adjectives to answer questions: *'longer than, shorter than, equal to.'*

Three learners (L4, L5, & L6) responded to this question correctly and obtained 100%. The three managed to compare the lengths of $\overline{GH}$, $\overline{HI}$ and $\overline{GI}$ using the pieces of the line segments of ΔGHI. This simply tells us that L4, L5 and L6 were operating at level 1-analysis according to the van Hiele model of geometric thinking.

Another group of learners (L1, L2, L3, L7, L8 & L9) scored 67% in question 6.1 (iii) of intervention activity 6 only. These learners were categorised into three categories based on how they responded to the question. The categories are as follows: (a) L1, L3 and L9 used correct comparative term to illustrate the difference between $\overline{GI}$ and $\overline{HI}$, for example, L9 said *" $\overline{HI}$ is longer than $\overline{GI}$."* (b) L1, L3, L7, L8 and L9 correctly used polygon pieces to compare the difference between the lengths of $\overline{GH}$ and $\overline{HI}$, for example, L7 said that *" $\overline{HI}$ is shorter than $\overline{GH}$."* (c) The difference in lengths between $\overline{GH}$ and $\overline{GI}$ was given correctly by L7 and L8 with the aid of polygon pieces, for example L8's response showed that " $\overline{GI}$ *is shorter than* $\overline{GH}$ *".*

The three groups identified in the recent paragraph could not obtain 100% in question 6.1(iii) for the reason that responses given revealed that their level of thinking was that of level 0-visualisation of the van Hiele model of geometric thinking in some geometric concepts.

In question 6.2 of intervention activity 6, learners had to cut out angles from the second copy of ΔGHI and compared each of the angles' size with the other two angles by placing the cut out angle on top of each of the angles in the original ΔGHI. They discovered the relationship between the three angles as shown below. Question 6.2(i) – (iii) is aligned to level 1-analysis of the van Hiele model of geometric thinking while questions 6.2(iv) – (vi) is of level 2-abstraction according to the van Hiele model.

In question 6.2 (i) of intervention activity 6, L1, L2, L3, L4, L5, L6, L8 and L9 managed to write the correct comparison between $\hat{G}$ and $\hat{H}$. In question 6.2(ii) learners responded differently and they have been categorised based on how they responded to the question. The first category was comprised of Ll, L3, L4, L5 and L6 who responded to the question correctly. In question 6.2(iii)of intervention activity 6, the correct comparisons of the two angles were given by L1, L2, L3, L4, L5, L6, and L8.

Responses to questions 6.2(i) – (iii) showed that five learners (L1, L3, L4, L5 & L6) were comfortably operating at level 1-analysis of the van Hiele model of geometric thinking in those three questions. In questions 6.2(ii), three learners (L2, L8 & L9) were identified to be at pre-recognition level of Clements and Battista (1991). L9 in question 6.2(iii) was also at pre-recognition level of Clements and Battista (1991).

Responding to question 6.2(i) of intervention activity 6, only L7 responded differently to the question, said that: *"G is longer than H; H is shorter than G."*

The learners who managed to get questions 6.2(i) to 6.2(iii) of intervention activity 6, correctly, they were able to use polygon pieces in comparing the angles one against another with a focused mind. There are learners who could not differentiate a point from an angle, for example, instead of saying $\hat{H}$, a learner just said H.

In question 6.2(ii), L8, who was in the third category said, *"$\hat{G}$ is bigger than $\hat{H}$."* In another category was L9 who said that *"$\hat{G}$ is shorter than $\hat{I}$."* In question 6.2 (iii) of intervention activity 6, the only learner who made an error in the group was L2 whose comparative adjective has been written as: *'smalle'* instead of *'smaller'*. A different response was given by L9 who said that *"*$\hat{H}$ *is shorter than* $\hat{I}$*."*

In the second category in question 6.2(ii) of intervention activity 6, L2 said that *"*$\hat{G}$ *is maller than* $\hat{I}$*."*The last category comprised of one learner, L7 who responded as follows: *"G is shoter than I; I is linger than G."* The responses given by L7 to question 6.2 (iii) are as follow: *"it is longer than I; I is shother than H."*

Question 6.2 (iv) of intervention activity 6, required the learners to mention the properties of ΔGHI based on the responses they got in questions 6.2(i) to 6.2 (iii) after using the cut out line segments. The question 6.2(iv) was correctly responded to by L1, L4, L5, L7, L8 and L9.

In question 6.2(iv) of intervention activity 6, L1's response said *'all sides have different length.'* The minor error identified in this case was the word 'length' which was supposed to be written in plural form, but the letter 's' was left out.

In question 6.2(iv) of intervention activity 6, L3 obtained 67% in the question. In the responses given this learner did not give the properties of ΔGHI in a summary form like "all sides are different in length".

L2 and L6 obtained a mark of 33% in question 6.2(iv) of intervention activity 6. In their responses L2 said *"GH is the longest to all"* while L6 gave two statements, one of which was correct; for example *"It is* $\overline{GH}$ *longer than* $\overline{GI}$ " and the other one was incorrect for example, " $\overline{HI}$ *is shorter than* $\overline{GI}$ *is longer than* $\overline{GH}$ *"*.

In question 6.2 (v) of intervention activity 6, learners were required to give the angle property of ΔGHI. The responses to this question were categorised into four different groups which are under different themes: five learners (L1, L2, L4, L8 & L9) managed to respond correctly with the exception of L8 whose sentence construction was as follows, *"All angles are different sizes."* This learner left out the preposition 'of' between 'are' and 'different'. These learners were operating at level 2-abstraction according to van Hiele model of geometric thinking.

In question 6.2 (v) of intervention activity 6, L2's responses were as shown: "*GH is the longest to all side; GI is the shortest to all side."* The errors in L2's responses were grammatical errors. The learner used the preposition 'to' after the adjectives, longest and shortest, in their superlative forms instead of using 'of'. In addition, the article 'the' has been left out and the noun 'side' is in singular instead of plural form, 'sides'. Another problem identified was that there was no any description of how $\overline{HI}$ was related to the other two line segments, yet the question required the learners to compare the lengths of all three line segments of ΔGHI.

The errors which L7, L8 and L9 made when responding to question 6.2(v)of intervention activity 6, were of using comparative adjectives wrongly; for example, L7 said *"* $\overline{GI}$ *is longer than* $\overline{HI}$ *,"* instead of using 'shorter than'. L8 used 'shorter than' instead of 'longer than' in comparing the lengths of $\overline{HI}$ and $\overline{GI}$. L9 said that *"GH is 'shorter than' IG"* instead of using the comparative 'longer than'.

In question 6.2 (v) of intervention activity 6, L8's sentence construction was as follows, *"All angles are different sizes."* This learner left out the preposition 'of' between 'are' and 'different'. Such minor errors might be as a result of unverified solutions or else poor sentence construction which emanates from the deficit in conceptual language understanding (Movshovitz-Hadar, Inbar & Zaslavsky, 1987; Sarwadi & Shahrill, 2014). The other four learners managed to make connections between what they practically did in questions 6.2 (i) to (iii), to what they were required to do in question 6.2(v).

L5's response to question 6.2(v) of intervention activity 6, "*All angles different size,"* demonstrates the inability to construct the sentences in order to present the solution clearly. These findings resonate with the proposition of Starcic, et al. (2013) which states that teaching and learning geometry is not a simple and straightforward activity, and there are so many alternative conceptions that need to be made clear in order for the learners to conceptually understand geometry. This implies that learners need to be taught how to respond to the question so that the answers they give in geometry are grammatically correct.

According to L3's response to question 6.2 (v) of intervention activity 6, only the comparison of two angles was used to give the properties of ΔGHI; the third angle was ignored, yet for all the properties of ΔGHI to be completed, all three angles were to be taken into consideration. According to Movshovitz-Hadar et al. (1987), such errors were due to misuse of the provide data, the learner neglected the given information which could lead to 100% correct solution.

L6 responded to question 6.2 (v) of intervention activity 6, as follows*: "*$\hat{G}$ *is bigger than* $\hat{H}$*,* $\hat{H}$ *is bigger than* $\hat{I}$*,* $\hat{I}$ *is smaller than* $\hat{G}$*."* The first statement was done correctly, but the second and third statements did not really give the true impression of how the said angles relate to each other in sizes. There was a mismatch of the comparative adjectives used in comparing angles H and I, as well as in angles I and G, which shows that L6's language proficiency is questionable because this learner did not actually know when to use the words: “bigger than” and “smaller than”.

L7 responded to question 6.2 (v) of intervention activity 6 as follows: *“angles are equal.”* This response was not the correct one to describe the angle property of ΔGHI. Such conceptual misunderstandings are due to lack of geometric conceptual understanding. Another reason: L6 conceptually did not understand the exact meaning of the word “equal”, which showed the mathematical language barrier. This tells us that to help learners in dealing with such alternative conceptions from the extended contact time with several activities was required.

In question 6(vi) of intervention activity 6, learners were supposed to mention a specific name given to a triangle with properties mentioned in 6.2 (iv) – (v). The correct response was supposed to be obtuse scalene triangle. L4 obtained 100%, but the only error was that of misspelling of the word scalene, which has been written as, *“scalen”*.

In question 6(vi) of intervention activity 6, each learner in another group (L1, L2, L3, L6, L7 & L9) scored 50% of the question. Four of these learners (L1, L2, L7 & L9) said that, *“scalene triangle”* while the other two (L3 & L6) responded as follows: *“it a scelen triangles”* and *“It is an scalene triangle,*” respectively.

Two learners (L5 & L8) who scored 0% in the same question described in the previous paragraphs. L5 named the triangle “*an isosceles”* while L8 said *“all sides are not the same size.”* L5’s response showed inadequate conceptual understanding of different types of triangles resulting in the learner failing to make connections with the already known ideas. L8’s response approved that the learner was able to read the question clearly, but did not conceptually grasp the overall meaning of it and, therefore, was unable to proceed further to produce the correct solution to the question (White, 2005).

4.2.7.2 Results of reflective test 6

Before learners were engaged in intervention activity 6 they wrote reflective test 6, which was comprised of only two questions. Question 6.1 required learners to use a ruler, a protractor and a pencil to draw a right-angled isosceles triangle and insert the necessary symbols. In question 6.2 learners were to mention three properties of a right-angled isosceles triangle. For the content details of reflective test 6, refer to appendix 26. The comparative results of both reflective test 6 and the diagnostic test are shown in Table 4.9 below. Questions 6.1 and 6.2 were at level 1-analysis and level 2-abstraction, respectively, of the van Hiele model of geometric thinking.

Table 4.9: Learners' overall performance in the diagnostic test and reflective test 6

Item	Min	Mean	SD	Median	Maximum
Diagnostic test	0	18	14.82	22	44
Reflective test 6	33	61.11	20.88	67	100

Note: Values of minimum, mean and standard deviation and maximum for learners' (N=9) marks obtained in two activities, diagnostic test and reflective test 6.

As shown in Table 4.9 the measures of central tendency for the reflective test 6 are greater than those of the diagnostic test which is an indication that the intervention activities which made use of the polygons pieces influenced learners' conceptual understanding of geometry.

In response to question 6.1 of reflective test 6, L1, L4 and L6 managed to draw a right-angled triangle and inserted correct symbols to show that it was a right-angled isosceles triangle. The exploration of different triangles using polygon pieces helped the three learners to conceptually understand what a right-angled isosceles triangle looks like. The three learners were categorised to be at level 1-analysis of the van Hiele model of geometric thinking.

In the same question 6.1 of reflective test 6, L3, L5, L7, L8 and L9 managed to draw the triangle and inserted the 90^0 symbol only. This revealed that the level of conceptual understanding of the properties of a right-angled isosceles triangle was still at its infancy. Many intervention activities were needed in order for the five learners to grasp the concepts fully. This concealed that in geometric concepts regarding right-angled isosceles triangles the five learners were at level 0-visualisation as theorised by the van Hiele model of geometric thinking

Responding to a question at level 2-abstraction of the van Hiele model of geometric thinking – question 6.2of reflective test 6, L4, L8 and L9 listed all three properties of a right-angled isosceles triangle as required in the question. This showed that the three learners were able to operate at level 2-abstraction in terms of geometric thinking.

In question 6.1 of reflective test 6, L2 was the only learner who could not draw the required triangle. This learner had an alternative conception regarding how a right-angled isosceles triangle looks like, the illustration in Figure 4.21 below shows how the triangle was drawn.

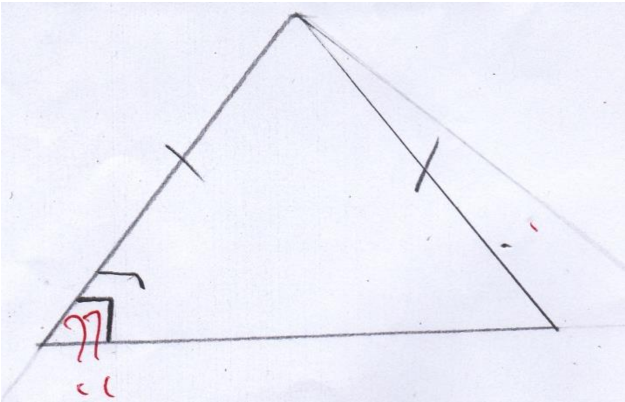


Figure 4.21: L2's response to question 6.1 in the reflective test 6

The response in Figure 4.21 above shows how L2 misinterpreted the mathematical language given in the question.

In question 6.2 of reflective test 6, L1, L2 and L7 each came up with two correct responses, but the third response was not correct; for example, L1 said that *"two lines are not equal in length"*. L2 responded as follows: "one angle is not equal." L2 wanted to *say "one angle is different in size from the other two angles*". L7 said that *"all sides are not equal*."

L1's response contradicts how the very same learner responded to question 6.1 of reflective test 6, where symbols for isosceles triangle were correctly inserted, but in this question, only two properties were mentioned, *"two angles are equal in size and one right angle"* while the third reason said *"two line are not equal in length"*. L7's first response to question 6.2 of reflective test 6, was *"All sides are not equal"* while the third point said *"Two sides are equal"*. The former, which is incorrect, contradicted the latter, which was one of the correct responses. In addition, L7 in responding to question 6.2 said that *"it have right angle"*.

In the recent sentence the L7 used ‘have’ instead of ‘has’ which is the third person singular of the present tense of ‘have’ followed by an article ‘a’. Both ‘has’ and ‘a’ were left out.

Two other learners only managed to mention that the triangle drawn in question 6.1 of reflective test 6, had a right angle, the other two reasons were incorrectly written, for example both learners (L3 & L5) said that *“All sides and angles are not equal”*. L3 further said that *“It is equal to 90^0”*.In the latter, L3 omitted a word or two, i.e. the learner would have said “*it has an angle equal to 90^0”*. Such errors emanated from the mathematical language difficulties which hindered learners to do proper sentence construction.

In question 6.2 of reflective test 6, L6 gave three responses presented as follows: (a) “two angles are equal.” (b) “One angle is over 900.” (c) “Two angles are less than 900.” From these responses, response (b) shows that L6 did not know what the word ‘over’ implies, which is a clear demonstration that the learner has language difficulties. The learner might have the correct response in mind, but the language has played a negative role. The third response was too general, yet the properties of the triangles needed were supposed to be specific; for example, the sides opposite two equal angles are equal or two angles are equal, each one is 45^0.

4.2.8 Results of intervention activity 7 and reflective test 7

4.2.8.1 Results of intervention activity 7

In intervention activity 7, learners used the first copy of triangle DEF and carefully cut out line segments DE, EF and DF. After that they used the pieces of line segments, one at a time, compared its length with the lengths of other two sides of the original triangle GHI by placing the cut out pieces on top of each of the line segments, they compared $\overline{DE}$ with $\overline{EF}$, $\overline{DE}$ with $\overline{DF}$ and $\overline{DF}$ with $\overline{EF}$. The purpose for this activity was to afford learners opportunities to explore and find out by themselves that some of the right-angled triangles were isosceles triangles. The activity required learners to have two copies of triangle DEF, copy a and b. The first copy was for the line segments cut outs and the second was for the angles cut outs.

Questions 7.1(iii), 7.2(i) – (iii) were at level 1-analysis of the van Hiele model of geometric thinking, while questions 7.2(iv)-(vi) are at level 2-abstraction of the van Hiele levels of geometric thinking.

In question 7.1 (iii) of intervention activity 7, learners compared the lengths of $\overline{DE}$ with $\overline{EF}$, $\overline{DE}$ with $\overline{DF}$ and $\overline{DF}$ with $\overline{EF}$ using polygon pieces and for each measurement taken the findings were recorded and they responded to the question using the comparative adjectives: *'longer than, shorter than, equal to.* 'Learners came up with different responses that were categorised as shown below.

The first group of learners, L3, L4, L5 and L6 managed to compare the lengths of the three line segments for ΔDEF correctly. The three learners applied their minds and skills to the comparison of the line segments using the polygon pieces. These learners' conceptual understanding of question 7.1(ii) was of the van Hiele's level 1-analysis.

In question 7.2 (i) to (iii) of intervention activity 7, learners used the second copy of ΔDEF, they cut out the three angles from the given copy and did the following: they took each of the cut out angles one at a time and compared its size with the other two angles by placing it on top of each of the angles in the original ΔDEF in order to establish the relationship between the three angles of the given triangle. When the activity was done a variety of answers came up as described below.

In question 7.2(i) of intervention activity 7, L1, L2, L3, L4, L5, L8 and L9 managed to give the correct responses. In question 7.2 (ii), L1, L2, L3, L4, L5 and L8 gave the correct responses. L3, L4, L5, L8 and L9 responded correctly to question 7.2(iii).

The results for questions 7.2(i)-(iii) demonstrated that L3, L4, L5 and L8 were conceptually at level 1-analysis of the van Hiele model of geometric thinking for the reason that they had correct responses in all the three questions. On the other hand, L1 and L2 were at level 0-pre-recognition of Clements and Battista (1991) in question 7.2(iii), while in question 7.2(i)-(ii), they were at level 1-analysis of the van Hiele model of geometric thinking. L9 operated at level 1-analysis of the van Hiele model in questions 7.2(i) and 7.2(iii) while in question 7.2(ii) L9 was regarded to be at level 0-pre-recognition of Clements and Battista (1991).

Question 7.2 (iv) of intervention activity 7, required learners to describe the properties of ΔDEF based on $\overline{DE}$, $\overline{EF}$ and $\overline{DF}$. This question was designed to help learners conceptually understand the properties of ΔDEF based on its line segments. Four learners (L2, L3, L4 & L5) gave the correct answer to this question after using the pieces of line segments as instructed.

Question 7.2 (v) required the learners to use the knowledge gained from the questions 7.2(i) to (iii) in order to give the angle property of ΔDEF. L3, L4, and L5 managed to obtain 100%. Responses to question 7.2 (vi) of intervention activity 7 were categorised into different groups based on how learners responded to the question. The first group was made up of learner who managed to give correct responses to the question, L1, L4 and L6.

According to learners' responses to questions 7.2(iv) – 7.2(vi) only L4 could operate at level 2-abstraction of the van Hiele model of geometric thinking in all three questions. L3 and L5 were at level 2-abstraction in questions 7.2(iv) – (v) only, but in question 7.2(vi) they were at pre-recognition level of Clements and Battista (1991). L1 and L6 were at level 2-abstraction in question 7.2(vi), but at pre-recognition level of Clements and Battista in questions 7.2(iv) – (v). L2 has been identified to be operating at level 2-abstraction of the van Hiele model of geometric thinking only in question 7.2(iv), but at level 0-pre-recognition level of Clements and Battista (1991) in questions 7.2(v) - (vi).

The learner whose response to question 7.1 (iii) of intervention activity 7, was different from any other learners was L8. This learner presented the solution as follows: *"* ΔDE *is shorter than* ΔDF, ΔDF *is longer than* ΔEF, ΔDE *is equal to* ΔEF*."*The comparative adjectives used in this case were appropriate, but the only problem L8 had was the use of the symbol Δ (delta) mathematically it is a symbol used for triangle not line segment.

In question 7.2(i), two learners (L6 & L7) could not give the correct answers. L6 wrote *"D is equal to F"* while L7 said that "$\hat{D}$ *is longer than* $\hat{F}$."

In question 7.2(ii) of intervention activity 7, L6, L7 and L9 could not respond as required by the question. In their responses to the same question L6 said "*D is smaller than E and E is bigger than D*" while L7 said that "$\hat{D}$ *is shoter than* $\hat{E}$*."*L9 said that "$\hat{D}$ *shorter than* $\hat{E}$*."*

In question 7.2(iii) of intervention activity 7, L1, L2, L6, and L7 gave their own responses, which were incorrect. L1 said that *"* $\hat{E}$ *is longer than* $\hat{F}$*."* L2 said that *"E is bigger* *than F."* L6's response says *"E is smaller than F and F is bigger than E."* The statement: $\hat{E}$ *is equal to F"* was a response given by L7.

Even though in question 7.1(iii) L6 was identified to be in a group under theme 2, in one of the statements the learner said that *"DE and EF is shorter than DF."* The sentences that had been written to compare different line segments was in a singular form, instead of using the verb 'are' L6 used 'is'. Another alternative conception identified was how L4 spelt the word "longest"; it was spelt as "*longestes".*

Four other learners (L1, L2, L7 and L9) each obtained 67% in question 7.1(iii)of intervention activity 7, but they differed in errors committed when they responded to the question; for example, L1 repeated that " $\overline{DE}$ *is equal to* $\overline{EF}$ *".*L2 wrote: *"DF is the longest but DE//EF is equal."* In L2's response, first part of the sentence was sensible, but the latter part of it had the sentence construction problem and the symbols used were not relevant to what was required in the question.

The other four learners (L1, L7, L8 & L9) in question 7.2(iv)of intervention activity 7, did not specify the exact line segments that were equal; for example, L1 said "*two lines are equal*". L7 said that *"two sides are equal. Two angles are equal and have right angle".* L8 said that *"two lines are equal."* L9 said that *"two sides and angles are equal".* The common response of these four learners said *"two lines are equal*". There is no specification of which lines are equal, yet the question requires specific answers. Another problem identified was that some learners; for example, L7 and L9, responded pertaining to the angles that were not asked. This was an indication that the learners were not quite sure of what was required actually.

One of the learners (L6) could not give the correct response, the response given was as follows *“it is longer than and shorter than and equal to”*.

In question 7.2(v) of intervention activity 7, the other three learners (L1, L7, & L8) obtained 50% for the question; their responses were as follow: “*two angles are equal*”. L9 said that *“one angle got right angle”*. The response was not specific as to which angles are equal. Two learners (L2 & L6) could not respond to the question correctly, they both obtained 0%. The responses given were as follows: for L2, “$\hat{D}$ *is equal to F and E is the biggest angled”*.L6 said that *“it is bigger than and smaller than and equal to”*.

L6 responded to question 7.2(v) of intervention activity 7, not as the question required, the response given was: *“it is bigger than and smaller than and equal to.”* From my experience as a mathematics teacher such a response implies two things: (a) the learner has no idea of what has been asked or (b) the learner does not conceptually understand the question because of the language difficulties.

In question 7.2(vi) of intervention activity 7, the second category of L2 and L7 had problems with the spelling of some words that were required in their responses. For example L2 wrote “*right angled* <u>*iscosles*</u>*”* while L5 said: *“right angle isoscelise triangle”*. The two learners could not spell the word “isosceles” correct, which shows that they had mathematical language difficulties. The same problem of failing to spell the word “isosceles” correct was also demonstrated by L9. Knowing very well that they could not spell the word correct, they did not bother themselves using mathematics dictionary that was available for them.

Two learners (L5& L9) obtained 50% in question 7.2(vi) of intervention activity 7, because they could not give the complete response of saying ‘right-angled isosceles triangle’ as required. In their responses as individuals they said that “*it is an isosceles”* and "*isosiase* Δ", respectively. L3 and L8 could not answer the entire question right. L3 said *"scalene"* while L8 said that *"two lines and two angles are equal"*.

4.2.8.2 Results of reflective test 7

Reflective test 7 consisted of two questions; the first one required learners to draw an equilateral triangle and then insert all the symbols that describe it. In the second question, learners were asked to mention the properties of an equilateral triangle. For the details of reflective test 7, refer to appendix 27. The performance of learners in this test in comparison to the diagnostic test is shown in Table 4.10 below.

Questions 7.1 and 7.2 are at the van Hiele model of geometric thinking, level1-analysis and level 2-abstraction, respectively.

Table 4.10: Learners' overall performance in the diagnostic test and reflective test 7

Item	Min	Mean	SD	Median	Maximum
Diagnostic test	0	18	14.82	22	44
Reflective test 7	38	67	19.40	63	100

Presented in table 4.10 are values of minimum, mean and standard deviation and maximum for learners' (N=9) marks obtained in two activities, diagnostic test and reflective test 7.

When comparing the diagnostic test with the reflective test 7, the measures of central tendency for the reflective test 7 were found to be higher than those of the diagnostic test. This was an indication that learners' conceptual understanding of geometry improved after having been engaged in intervention activities that made use of the polygon pieces and mathematics dictionary.

As required in question 7.1 of reflective test 7, L5 and L7 managed to draw the triangle and inserted all the symbols that described an equilateral triangle. The two learners managed to achieve this because during the activity of cutting and comparing the line segment against other line segments and an angle against other angles in an equilateral triangle they followed instructions and did exactly what was required of them.

In question 7.1of reflective test 7, six learners (L1, L2, L3, L4, L6 & L9) managed to draw an equilateral triangle and the symbols to show that all sides are equal were correctly inserted, but no symbols of the equality of the angles were inserted. These results revealed that the six learners were not fully developed in this question's concepts. I can conclude that their understanding at some stage of conceptual understanding of symbols for an equilateral triangle belonged to pre-recognition level of Clements and Battista (1991).

In the same question 7.1 of reflective test 7, L8 managed to draw the triangle and showed that the two sides are equal using similar signs (one slash for each side). However, on the third side, two slashes were used as if the third side is not equal to the other two sides. How L8's response to question 7.1 is shown in figure 4.22.

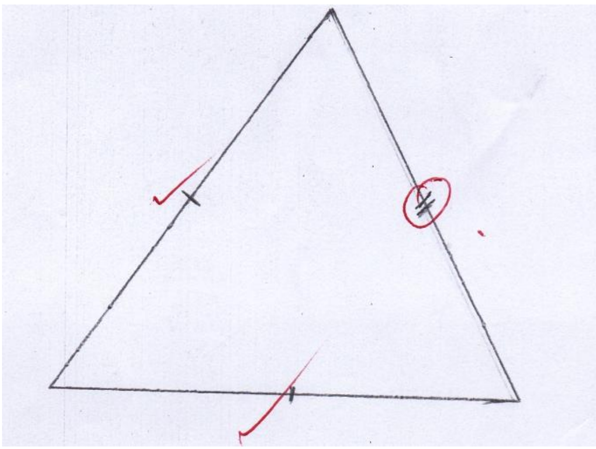


Figure 4.22: Shows how L8 responded to question 7.1 of the reflective test 7

In addition, no symbol was used to show that all the angles were equal. Such responses indicated that even though learners were engaged in the intervention activity, not all the concepts were conceptually understood and could not be remembered when needed to respond to reflective test 7. In this question it was evident that seven learners' understanding of the concepts was not yet developed to be operating at level 1-visualisation of the van Hiele geometric theory, at this level the learner were supposed to apply what was already learnt, into a new situation.

In question 7.2 of reflective test 7, eight learners (L1, L3. L4….to… L8) managed to mention the properties of an equilateral triangle. In the diagnostic test all the learners could not describe the properties of an equilateral triangle, except L4. However, in intervention activity7, eight learners responded to the question correctly which is an indication that the use of polygon pieces helped the learners to conceptually understand the properties of an equilateral triangle to a certain extent. The eight learners had moved from the pre-recognition level of Clements and Battista (1991) to level 2-abstraction as hypothesised by the van Hiele model of geometric thinking.

In question 7.2 of reflective test 7, L2 said that *"Two sides are equal. Two angles are equal"*. The description of an equilateral triangle was not done as expected.

4.2.9 Results of intervention activity 8

4.2.9.1 Results of intervention activity 8

In this section the results of intervention activity 8, were presented based on how learners responded to the questions pertaining to the properties of ΔXYZ. The activity engaged learners in the use of polygon pieces to explore and learn about the properties of an equilateral triangle without being told by the teacher or friends.

In question 8.1 (i) and (ii) of intervention activity 8, learners compared by estimation the sizes of $\hat{X}$, $\hat{Y}$ and $\hat{Z}$ as well as the lengths of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$. These two questions were designed to enhance visualisation skills. According to Starcic et al. (2013) visualisation of geometric concepts is a prerequisite element for enhancing geometric reasoning and should be considered as a compulsory stride at the concrete-experiential level in the progress of cognitive processes.

Questions 8.1(i)-(iii) are aligned to level 0-visualisation of the van Hiele model of geometric thinking.

In questions 8.1(i) of intervention activity 8, learners did estimate and came up with different responses which were categorised into two categories: the first group of eight learners (L1, L2, L4, L5, L6, L7, L8 & L9) managed to respond to the question correctly. They were able to make connections with the cutting and comparing of angles in the previous intervention activities. Their responses showed that the previous activities instilled the visual skills and conceptual understanding of the properties of an equilateral triangle; for example, learners conceptually understood that if all the sides in a triangle were equal, all the angles were equal too.

In questions 8.1(i) of intervention activity 8, L3 was the only learner obtained a mark of 33% in the activity. From the responses given, it was clear that this particular learner had an alternative conception of the meaning of mathematical symbols. For example, the symbols used to show that all sides in an equilateral triangle are equal were not used by this learner.

In question 8.1(ii) of intervention activity 8,L9 obtained 67% and L3 got 0%. L9 had a problem of lack of conceptual understanding in mathematics, for example, this learner said that: “ $\overline{xy}$ *is equal* $\overline{yz}$, $\overline{xZ}$ *is equal* $\overline{yz}$ ”. In addition, L9 did not consider that capital letters were used when presenting line segments. The learner who got 0% did not use the correct symbols for the line segments, for example, L3 said that *"ΔXY is longer than ΔYZ; ΔYZ is shorter than ΔXY; ΔXZ is equal to ΔXY."*

Although L9 managed to get the question right in question 8.1 (iii)of intervention activity 8, but the problem identified was that the learner had used lower case letters, instead of capital letters to name the line segments. L3 said that *"ΔXY is longer than ΔYZ; ΔYZ is shorter than ΔXY; ΔXZ is equal to ΔXY"*. L3 committed the same type of error in both questions 8.1 (ii) and 8(iii) of intervention activity 8, where the symbol for the triangle was used to describe the line segment.

In question 8.1 (ii) of intervention activity 8, learners compared by estimation the lengths of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ and wrote down the responses using these terms: 'shorter than, longer than, equal to, the longest of all.' Of the nine learners only seven (L1, L2, L4, L5, L6, L7 & L8) managed to give responses that are 100% correct. The seven learners made use of the symbols that illustrate that all line segments of ΔXYZ are equal.

In question 8.1 (iii) of intervention activity 8, learners used the first copy of ΔXYZ to cut out the line segments: XY, YZ and XZ. After that they compared each line segment's length with the lengths of the other two line segments of the original ΔXYZ by placing the cut out piece on top of each of the line segments, for instance comparing $\overline{XY}$ with $\overline{YZ}$, $\overline{YZ}$ with $\overline{XZ}$ and $\overline{XZ}$ with $\overline{XY}$ they recorded their findings. The line segments cutting activity helped the following learners: L1, L2, L4, L5, L6, L7, L8 and L9 to answer question 8.1(iii) correctly.

The results for question 8.1(i)-(iii) showed that seven learners (L1, L2, L4, L5, L6, L7 & L8) were at level 0-visualisation of the van Hiele model of geometric thinking in all the three questions. L9 operated at level 0-pre-recognition level hypothesised by Clements and Battista (1991) because could not answer question 8.1(ii) correctly, but managed to answer questions 8.1(i) and 8.1(iii) correctly. Therefore, in those two questions L9 was at level 0-visualisation of the van Hiele model.

In question 8.2 learners used the second copy of ΔXYZ to cut out the three angles and then compared the size of one of the angles with the sizes of the other two angles by placing one on top of the other angle in the original ΔXYZ. This question was at level 1-analysis of the van Hiele model of geometric thinking. After the activity, learners determined the relationships between $\hat{X}$ and $\hat{Y}$, $\hat{Y}$ and $\hat{Z}$, $\hat{X}$ and $\hat{Z}$, their responses are shown below.

In question 8.2 (i) of intervention activity 8, all the learners managed to use the pieces of angles correctly and every learner got the answer right.

In question 8.2 (ii) of intervention activity 8, eight learners were able to compare the sizes of the three angles of ΔXYZ with the help of the polygon pieces, except L7.In question 8.2 (iii) seven learners were able to use polygon pieces to compare the sizes of the angles resulting in them giving the correct responses, except L3 and L7.

In question 8.2(ii) of intervention activity 8,the only learner who could not answer the question as expected L7 said that *"Y is equal to Z"*.

In question 8.2(iii) of intervention activity 8, when other learners used polygon pieces to compare the sizes of the angles they gave the correct responses L7 said that *"Z is equal to Y"* and L3 said: " $\hat{Z}$ *is bigger than* $\hat{x}$ ".

The results revealed that seven learners (L1, L2, L4, L5, L6, L8 & L9) were at level 1-analysis of the van Hiele model of geometric thinking in all the three question 8.2(i) - (iii. L3 was at level 1-visualisation in questions 8.2(i) and 8.2(ii), but operated at level 0-pre-cognition level of Clements and Battista (1991) in question 8.2(iii). On the other hand L7 was at level 1- visualisation of the van Hiele model of geometric thinking in question 8.1(i), but was at level 0- pre-recognition level according to Clements and Battista (1991) in questions 8.2(ii) – (iii).

Question 8.3(i)-(ii) was at level 3-formal deduction according to the van Hiele model of geometric thinking. In question 8.3 (i) of intervention activity 8, learners were supposed to give the properties of ΔXYZ in terms of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$. The knowledge and skills obtained when doing questions 8.1(ii) and (iii) were to be applied in giving the properties of the mentioned triangle. Of the nine learners, only six (L1, L2, L4, L5, L6 & L7) managed to mention the properties of ΔXYZ. This means that in question 8.3(i) the six learners were at level 3-formal deduction of the van Hiele model of geometric thinking.

There are three learners who could not answer question 8.3(i) of intervention activity 8 correctly. The responses of L3, L8 and L9were in the same category – they all used comparative adjective longer than, shorter than, to describe how the sides of ΔXYZ are related to each other. For example L3 said " $\overline{XY}$ *is shorter than* $\overline{XZ}$, yet the given triangle is an equilateral. L8 said that *"All size are equal."*

In question 8.3(ii) of intervention activity 8, L2 and L3 obtained 0% and 33%, respectively. From my observation during the activity, it was quite clear that most of the learners who had this question right were actually referring to their previous responses in question 8.1(i) and 8.2(i) to (iii). But it was not the case with L2 and L3. By referring to the previous questions' responses most learners were able to link what was previously done to the required information in question 8.3(ii).

In question 8.3(ii) of intervention activity 8, L3 could not get the properties of ΔXYZ since the responses to question 8.2(iii) were incorrect; the learner needed the three responses in question 8.2(ii) as the point of reference when responding to question 8.3(ii). L2 failed completely to describe the properties of ΔXYZ, yet the very same learner gave the correct answer to question 8.1(i) and 8.2(i) to (iii).

Question 8.3 (ii) of intervention activity 8 required learners to give the properties of ΔXYZ based on the angle relationships investigated in question 8.1(i) and 8.2(i)-(iii). Out of nine learners, seven (L1, L4, L5, L6, L7, L8 & L9) managed to give 100% correct responses. This showed that they were operating at level 3-formal deduction according to the van Hiele theory.

My observation during the activity was that most of the learners who answered this question 8.3(ii) correct were actually referring to their responses in question 8.1(i) and 8.2(i) - (iii). EventuallyL1, L4, L5, L6, L7, L8 and L9 were able to link what was previously done in order to respond to question 8.3 (ii) and they then had it correct. This means the seven learners were operating at level 3-formal deduction of van Hiele model of geometric thinking.

In question 8.4 of intervention activity 8, learners were to give the name of the ΔXYZ. Out of the nine learners, seven learners (L1, L2, L4, L5, L6, L7 & L9) were able to say equilateral triangle. This revealed that their level of thinking was at level 2-abstraction of the van Hiele model of geometric thinking since the question was set at that particular level.

In question 8.4 of intervention activity 8, L3 identified ΔXYZ as an isosceles triangle, yet the properties show that it is not.

In question 8.4of intervention activity 8, of the seven learners who answered this question correctly, L2 and L9 could not spell the triangle's name correctly, they said *"equalateral and equlateral,"* respectively. In question 8.4, L8 said *"all size are equal and angles are equal."* This finding supports previous research into this brain area which links mathematics vocabulary barriers and alternative conceptions in mathematics. Failure to spell a word and giving responses did not address the question. For example L2, L8 and L9's responses showed that the learners had language barriers. In the case of spellings, the provision was made for the learners use the mathematics dictionary that was made available; however, it seemed that L2 and L9 did not see the need to make use of it to support them in spellings. The three learners seemed to be at level 0-pre-recognition level as posited by Clements and Battista (1991).

4.2.10 Results of intervention activity 9

This section gives a brief overview of the intervention activity nine: its purpose and then I give highlights of the research findings on how the learners performed. A description on what made the learners pass or fail some of the questions will be presented.

Intervention activity nine required learners to investigate the properties of an acute - angled isosceles ΔPQR using the polygon pieces, this was to avoid the abstract method of teaching, but to allow learners to learn by the method of discovery.

Questions 9.1(i)-(iii) were all at level 0-visualisation of the van Hiele model of geometric thinking.

In question 9.1(i) of intervention activity 9, learners estimated and compared the sizes of $\hat{P}$ and $\hat{Q}$, using the terms: 'equal to, greater than and smaller than'. In response to question

9.1 (i), L1, L4, L5, L8 and L9 managed to apply visualisation skills to ΔPQR in order to obtained the correct responses and also it seems that learners linked what they have learnt in intervention activity 7 to the current activity. The learners who had question 9.1(i) correct were at level 0-visualisation of the van Hiele model of geometric thinking.

In question 9.1 (i) of intervention activity 9, four learners (L2, L3, L6 & L7) could not respond correctly because of alternative conceptions they held as illustrated in their responses: L2 said that *"*$\hat{P}$ *is equal to Q."* – this learner ignored the use of angle symbols in Q. L3 said that *"*$\hat{P}$ *is greater than* $\hat{Q}$. $\hat{Q}$ *is smaller than* $\hat{P}$." L7 said that *"Q is greater than P. P is smaller than Q".* Responding to the same question 9.1 (i), L6 said that *"Two sides are equal."* The four learners were operating at level 0-pre-recognition level as hypothesised by Clements and Battista (1991).

In question 9.1(iii) of intervention activity 9, a mark of 0% was obtained by L6 because of the way the question was answered. The responses given are as follow: *"*$\overline{QR}$ *is smaller than* $\overline{PR}$ *,* $\overline{QR}$ *is maller than* $\overline{QP}$ *, and QP is bigger than* $\overline{PR}$ *."* The responses showed that the learner did not bother using the pieces of ΔPQR. It seems that all that has been written down is form of guess work. What I have also observed is that this learner was not conceptually clear on the symbols used to show that the two sides of an isosceles are equal. Such alternative conceptions are due to failure to make connection with the previous concepts. L6 was at level 0-pre-recognition as described by Clements and Battista (1991).

In questions 9.2(ii) and (iii) of intervention activity 9, three learners each one obtained 0% because of the way they responded to the question. For example, L2 said that *"Q is greater than R"* and *"P is greater than R".* The comparative adjective used is correct for both answers, but the symbols to show that 'Q' and 'R' are angles were not inserted; this puts L2 to level 0-pre-recognition of Clements and Battista (1991).

L7 said that *"*$\hat{Q}$ *is longer than* $\hat{R}$*."* and *"*$\hat{P}$ *is shoter than R."* The problems identified in both responses were the comparative adjectives that the learner has used to show the difference between the mentioned angles. The use of the words 'longer' and 'shorter' when comparing the angles is an indication of language difficulties and also shows that the mathematics dictionary that was provided for the learners to use was never consulted in order to support the conceptual understanding and the correct spelling for the comparative adjectives used for the comparison of the angles. Failure to spell places a learner at level 0-visualisation of the van Hiele model of geometric thinking.

In question 9.1 (ii) of intervention activity 9, L1, L8 and L9 scored 67%. The reason why the three learners did not get the entire question correct is that they could not fully describe the relationship between $\overline{RP}$ and $\overline{PQ}$; $\overline{QR}$ and $\overline{PQ}$. A learner would describe the relationship between one of the three pairs of line segments and leave the other two pairs, for example L9 just said that *"*$\overline{PQ}$ *is shorter than* $\overline{RP}$*."* The learner was operating at level 0 – pre-recognition level as posited by Clements and Battista (1991).

L2 and L4 obtained 33% in question 9.1 (ii) of intervention activity 9, because they could not describe the relationship of the three line segments clearly, for example, L4 said that *"*$\overline{PQ}$ *is shorter than* $\overline{RP}$ *and QR is the longest of all."* The latter part of the response shows that L4 was not well conversant with the symbols used in an isosceles triangle. L2 said that *"PQ is smaller than RP and QR is equal to Rq."*In this question, L4 was at level 0 pre-recognition according to Clements and Battista (1991).

L6 obtained 0% in question 9.1(ii) of intervention activity 9. The response was as follows: *"PQ is equal to RP is shorter than QR is shorter than all."* From this response I deduced the following: (i) the sentence construction shows that this learner had language difficulties and (ii) also did not understand the meaning of geometric symbols, like the ones that show that the opposite sides of an isosceles triangle are equal. Such responses showed that the learner was at level 0-pre-recognition according to Clements and Battista (1991).

L9's responses to questions 9.2(ii) and (iii) of intervention activity 9 were: *"*$\hat{Q}$ *is equal to* $\hat{R}$ *"* and *"*$\hat{P}$ *is equal to* $\hat{R}$ *",* respectively. These responses showed that the learner either did not use the cut pieces of angles as instructed to respond to question 9.2 (ii) and (iii). L9 did not also understand that the slashes at the two sides of ΔPQR as shown in appendix 20 are symbols to illustrate that the two opposite sides are equal in length; therefore the angles opposite the two sides are also equal in size.

In question 9.1 (ii) of intervention activity 9, learners did estimations and compared the lengths of $\overline{PQ}$, $\overline{RP}$ and $\overline{QR}$. They used the following terms: 'longer than, equal to and the longest of all'. The aim of this question was to evaluate how established learners' geometric visual skills were. Out of nine learners, three (L3, L5 & L7) managed to answer this question 100% correctly. Therefore, these three belonged to level 0-visualisation according to the van Hiele model of geometric thinking. Clear conceptual understanding of geometric symbols helped these three learners to make an informed decision regarding the lengths of $\overline{PQ}$, $\overline{RP}$ and $\overline{QR}$.

In question 9.1(ii) of intervention activity 9, L1, L8 and L9 scored 67%. They all correctly mentioned the relationship in length between $\overline{RP}$ and $\overline{QR}$, which showed that they conceptually understood the meaning of the symbols that were used in an isosceles triangle, but the only problem is that they did not consider the other side as well. This revealed that their level of thinking was not well developed at level 0-visualisation of the van Hiele model, but at some concepts there were migrating to pre-recognition level 0 of Clements and Battista (1991).

In question 9.1(iii) of intervention activity 9,learners were instructed to take the first copy of ΔPQR and carefully cut out line segments QP, QR and PR and then take each of the pieces of the line segments, one at a time, and compare its length with lengths of two other sides by placing the cut out pieces on top of each of the lines segments of the original ΔPQR, i.e. compare $\overline{QR}$ with $\overline{PR}$, $\overline{QR}$ with $\overline{QP}$ and $\overline{QP}$ with $\overline{PR}$ and record their findings.

In question 9.1(iii) of intervention activity 9, four learners (L1, L5, L7 & L9) managed to use the pieces of ΔPQR as instructed and obtained 100% marks in this question. They responded as follows: *"*$\overline{QR}$ *is equal to* $\overline{PR}$, $\overline{QR}$ *is longer than* $\overline{QP}$ *, and* $\overline{QP}$ *is shorter than* $\overline{PR}$*."* These learners got the question right for they were able to make connections of what was required to what they already knew from the previous activity in question 9.1(ii); for example the proper selection of the comparative adjectives. This implies that the way in which the learning activity has been designed played a role in instilling learners' conceptual understanding. This led me to the conclusion that in this question the learners were at level 0 –visualisation of the van Hiele model of geometric thinking.

In question 9.1(iii) of intervention activity 9, a group of three learners (L2, L3 & L4) each scored 67% in question 9.1 (ii) since they could not use the pieces of polygon as instructed, highlighted were the incorrect responses, for example L2 said *"QR is longer than*

PR." L3 said that *"$\overline{QP}$ is shorter than $\overline{PR}$."* This was a repetition of the second response that has been written as: *"$\overline{PR}$ is longer than $\overline{QP}$."*

On the other hand, L4 said that *"$\overline{QP}$ is equal to $\overline{QP}$."* This demonstrates the consequences of unverified solutions where the learner just rushed to the other question without confirming of the response that was written. Analysing this from another angle, considering the comparative adjective used $\overline{QP}$ is not equal to any of the sides of ΔPQR. This therefore, implies that L4 did not use the pieces of polygon to the optimum.

One of the learners (L8) obtained 33% in question 9.1(iii)of intervention activity 9. The learners said that: *"$\overline{QR}$ is equal to $\overline{PR}$, $\overline{QR}$ is bigger than $\overline{QP}$, and $\overline{QP}$ is smaller than $\overline{PR}$."* The comparative adjectives used in the two last responses were not relevant to the comparison of the line segments. The learner misinterpreted the mathematical language by translating words from the literal home language into English which was an indication of the mathematical language difficulties that are mostly prevalent in learners who study mathematics in a classroom in their second language.

In question 9.2(i) of intervention activity 9, all nine learners obtained 100%. Each one of them managed to cut out angles of ΔPQR and accurately compared the relationship between $\hat{Q}$, $\hat{P}$, and $\hat{R}$, which showed that they were all operating at level 1-analysis according to the van Hiele model of geometric thinking.

Responding to question 9.2(ii) and (iii) of intervention activity 9, six learners (L1, L3, L4, L5, L6 & L8), compared the relationships in sizes between $\hat{Q}$ and $\hat{R}$; and $\hat{P}$ and $\hat{R}$ using pieces of polygon and in both questions, these learners all obtained a mark of 100% .As I was observing them working individually, each piece of an angle was carefully fit into each one of the three angles in the original ΔPQR.In these two questions the six learners were at level 1-analysis of the van Hiele model of geometric thinking.

Question 9.3(i) and (ii) of intervention activity 9, required learners to give the properties of ΔPQR based on the edges and angles, respectively. Five learners (L2, L3, L4, L7 & L9) gave correct answer to question 9.3(i), thus giving the properties of ΔPQR based on its line segments. In question 9.3(ii) regarding the angle property of ΔPQR, only three learners (L3, L4 & L6) mentioned that angles were equal.

Questions 9.3(i)-(ii) were rated at level 3-formal deduction of the van Hiele model of geometric thinking, L3 and L4's thinking was also at the same level because they managed to answer both questions correctly. In question 9.3(i) L2 and L7 were at level 3-formal deduction of the van Hiele model, but in question 9.3(ii) they were at level 0-pre-recognition of Clements and Battista (1991). On the other hand L6 was at level 3 according to the van Hiele model, but in question 9.3(i) was at pre-recognition level suggested by Clements and Battista (1991).

Question 9.3(i) and (ii) of intervention activity 9, required learners to give the properties of ΔPQR based on the edges and angles, respectively. Five learners (L2, L3, L4, L7 & L9) responded to question 9.3 (i) correctly, thus giving the properties of ΔPQR based on its line segments. The three were at level 3-formal deduction as suggested by the van Hiele model of geometric thinking.

L1 and L5 just said "*two sides are equal",* without specifying the sides that were equal resulting in them obtaining 50% of the question. L6 and L8 could not get the whole question 9.3 (i) right. Their responses were: L6 said that *"$\overline{QR}$ is shorter than $\overline{PR}$ and $\overline{PR}$ is shorter than QP."* On the other hand L8 said *"two sizes are equal."*

Even though L7 was recognised as one of those learners who obtained 100%, in question 9.1(ii) of the intervention activity 9, the only error identified was a spelling error made by L7 who spelt the word 'shorter' as 'shoter'. This means when some concepts were not clear, the learner operated at level 0-pre-recognition level of Clements and Battista (1991).

Regarding the angle property of ΔPQR in question 9.3(iii) of intervention activity 9, five learners (L1, L2, L5, L7 & L8) said that "*two angles are equal."* L9 said that *"two sides are equal."*

In question 9.4 of intervention activity 9, learners were asked to give the specific name of ΔPQR and eight out of nine learners (L1, L2, L4, L5, L6, L7, L8 & L9) were able to identify the triangle as an isosceles. This means the eight learners were able to operate at level 2-abstraction of the van Hiele model of geometric thinking because question 9.4 has been set to be at that same level. In question 9.4 of intervention activity 9, L3 was the only learner who could not get the correct answer and said that "ΔPQR *is an equilateral triangle."* This learner was at level 0- pre-recognition according to Clements and Battista (1991). Such a response after being engaged in the use of polygon pieces an indication that much time is needed for such learners to undo the previously learnt alternative conceptions regarding the names of triangles.

4.2.11 Presentation of learners' transcribed interviews

Table 4.11: L1's transcribed interview, words before keywords and words after keywords

Keywords	Words before	Words after
help	Programme can	A person, me
like	I	it
learn	Me to, used to	Geometry, geometry construction
learning	Me in	Mathematics
measure	To, pieces	Angles and sides
measuring	Giving me	skills
learnt	We	All in

Table 4.11 indicates that L1 liked the use of polygon pieces when teaching and learning of geometry because it is easy to learn geometric concepts.

L1 was optimistic that the use of polygon pieces in teaching and learning of geometry in is tils essential skills in a learner's mind; for example, construction and measuring skills. This learner preferred to be taught geometry and other mathematics topics using polygon pieces in order to enhance conceptual understanding.

Table 4.12:L2's transcribed interview, words before keywords and words after keywords

Keywords	Words before	Words after
understand	Do, made me to	Them, the relationship
angles	Relationship of, measure	And, of given
sides	And, relationship of	Of different, of triangles
triangles	Of, classify the	Made me, well

Table 4.12 indicates how L2 feels about the use of polygon pieces in the teaching and learning of geometry.

L2 suggested that the best way to help learners understand the relationship of angles and sides in a triangle was to engage learners in the activities that made use of polygon pieces since they allowed learners to measure angles and lengths of sides of triangles. Also the classification of triangles was made clear once polygon pieces are incorporated into teaching and learning.

Table 4.13:L3's transcribed interview, words before keywords and words after keywords

Keywords	Words before	Words after
understand	Now I, I did not, helped me to	Things, before, mathematics concepts, at all, types of.
triangles	Types of, learning properties of, given, by using	And angles, by cutting out
Angles and sides	Cutting out, pieces of, against and	And compare them, of a triangle, against a

Table 4.13 shows that before the use of polygon pieces L3 did not clearly conceptually understand the properties of triangles, but after being engaged in the intervention activities that made use of pieces of polygons, most concepts were conceptually clear. For the fact that the investigation of properties of triangles was based on the physical comparison of the sides and angles of triangles, L3 felt that the concepts were well presented and motivating.

L3 suggested that the teaching and learning of mathematics, for example, geometry using polygon pieces was interesting and was of meaning that learner. To teach conceptual understanding an angle must be placed on top of other angles and a side of a triangle must be placed against another side in order to establish their relationships.

Table 4.14:L4's transcribed interview, words before keywords and words after keywords

Keywords	**Words before**	**Words after**
Triangles	About what a, types of, sides of the same	Is and, and their, using cut pieces, sides have the same, found out
Properties	And their, learning about, to learn, understand the	Of triangles, has helped me
Measure	I can now, helped me with	Angles, skills, its size
Construction	Lesson of, asked to	Of angles, an angle

Table 4.14 indicates that the use of polygon pieces that were incorporated into the teaching and learning of geometry was helpful to L4 in conceptual understanding of the properties of different triangles. From L4's responses, it can be deduced that the way in which the lessons were arranged and activities performed during the intervention programme, it helped in the building of conceptual understanding of geometric concepts regarding properties of triangles. L4 further said that the intervention programme enhanced the measuring skills for the reason that polygon pieces were used to determine the relationship between angles and sides in a given triangle. This learner suggested that such programmes can also be used in the teaching and learning of the construction of triangles and other geometric shapes.

Table 4.15:L5's transcribed interview, words before keywords and words after keywords

Keywords	**Words before**	**Words after**
Knowledge	Gained mathematical, have gained	Of how to
Triangles	To identify, lines and the types of, understand that a, construction of	And also how, if you want, has three sides, and angles
Measure	How to, by, have acquired	Lines and triangles, as we were
Angles	Three, triangles and, two bisected	Are the same

Table 4.15 shows that by using polygon pieces in teaching and learning geometry, L5 has gained important mathematical knowledge for the reason that the programme focused on how to measure and not on what it means to measure. It also shows that the polygon pieces used in the intervention programme helped this learner to conceptually understand and be able to classify triangles using their properties. This learner suggested that by being engaged in the measuring of angles and sides of triangles, mathematical skills were developed and enhanced at the same time.

This learner suggested that the use of polygon pieces could also be used to investigate the relationship between two bisected angles. This means that the use of polygons pieces is not limited to one topic only.

Table 4.16:L6's transcribed interview, words before keywords and words after keywords

Keywords	**Words before**	**Words after**
Angles	How, the sides and, other two, to measure	In a triangle, of triangles, and the sides of, are equal
Triangles	In a, of sides of a, in a, equal then	Are related, using cut pieces, with other two, sides, two angles, is an isosceles
Measure	By	The sides, without using a protractor

Table 4.16 indicates that the use of physical manipulatives helped L6 in determining the properties of triangles. L6 strongly believed that the use of polygons pieces gave them an opportunity to explore the properties of triangles. Also through such activities it was easy to determine the names of given triangles, therefore, L6 suggested that before learners are introduced to the measuring angles of shapes using the protractor, the use of cut angles and line segments must be introduced first in order to arouse curiosity and give meaningful explanations.

Table 4.17:L7's transcribed interview, words before keywords and words after keywords

Keywords	**Words before**	**Words after**
Understand	I did not, I now	Geometry, clearly now
Angles	Are two, all the, cut the, compared an	Equal, are equal, and sides, with other
Triangles	Sides of, a scalene	Out, all sides

Table 4.17 shows that the use of polygon pieces in the teaching and learning of geometry has helped L7 with the conceptual understanding of geometry through the exploration of the properties of different triangles. This tells us that teaching and learning of geometry should not only be done in terms of giving the meaning and obviating analysis of the properties of shapes with no emphasis on the visualisation of the shapes (Blanco, 2001).

Table 4.18:L8's transcribed interview, words before keywords and words after keywords

Keywords	**Words before**	**Words after**
Understand	Now I, I did not	And know what to do, the properties of
Angles	Cut the, construction of	And line for, and triangles
Properties	Did not learn the, the	Of triangles, of an obtuse

Table 4.18 indicates that before the use of polygon pieces, L8 did not conceptually understand the properties of different triangles, but after the use of polygon pieces the learner conceptually understood the properties of different triangles. This learner suggested that polygon pieces can also be used for the construction of the geometric shapes.

Table 4.19:L9's transcribed interview, words before keywords and words after keywords

Keywords	**Words before**	**Words after**
Triangles	Properties of, a given, that some, of a given, sides	Using pieces, is an isosceles, and also, all three sides, that are
Learn	Able to, have, we can	About the properties, the properties of
Learnt	I, I have	Mathematics, that we can
Pieces	Using cut, used those, cu out	Of shapes, to compare, before we actually

Table 4.19 indicates that the use of polygon pieces in the teaching and learning of geometry can help learners to conceptually understand the properties of different shapes since the work is not based on abstract concepts that are difficult for learners to grasp. L9 liked the use of cut polygon pieces for the reason that learning took place in an exploratory way without being told how an equilateral triangle looked like. This learner wanted to be taught mathematics using polygon pieces for the fact that the learner has realised that these polygon pieces simplify and instil mathematical skills, such as observation, calculation and communication skills, which are necessary for in real-life settings.

4.2.12 Data from the observations

Day 1: Observation results

Some learners could not conceptually understand the instructions in activity one of the intervention. The learners also demonstrated some difficulties in conceptually understanding the questions, for example, after being given materials to do intervention activity 1, L4 and

L8 spent almost 12 minutes without writing anything on the paper. L8 kept on asking for individual help where an explanation was needed in most of the questions in the activity. All learners were not able to name the identified group of triangles as an isosceles, they were not to be familiar with the mathematical symbols used to illustrate that line segments opposite two equal angles in an isosceles triangle are equal. L3 asked me the question, *"what is the meaning of the word properties?"*. Responding to this question the researcher referred the learners to the mathematics dictionary.

Day 2: **Observation results**

A number of learners did not follow the instructions that explained how to do intervention activity 2. For example, when given the paper, L4 immediately started to write, but when I checked some of the responses were incorrect for the reason that instructions were not adhered to. All other learners managed to cut out the angles and line segments from the provided copies of a particular triangle.

L6 at first did not cut out the angles and line segments in order to use them to compare how the three angles and line segments of a given triangle were related. I had to tell the learner to read the instructions clearly, L6 then read the instructions, but still did not know what to do. I discovered that the learner could not conceptually understand the question due to language difficulties. I intervened and clarified what was expected of them to do.

Day 3: Observation results

In other questions of the intervention activity 3 most of the learners managed to cut out the angles and line segments from the given copies of triangles.

In intervention activity 3, learners had to estimate the sizes of angles and lengths of triangle's sides, L1 decided to use a ruler to measure the sides of a given triangle. L8 could not use the polygon pieces in measuring and struggled until I gave this learner individual assistance to move forward.

Day 4: Observation results

On this day, some learners could not do the cutting activity well; for example, L4 got stuck on how to cut out the three line segments of a triangle. The learner asked for help, which was given.

Others learners, for example L1, managed to cut line segments by allowing the pair of scissors to cut through the apex of the triangle from its centre, so that no line segment was reduced in its original length.

Day 5: **Observation results**

All learners could not start the activity for the reason that they could not conceptually understand the meaning of $\overline{AB}$. Some asked "what does this mean?" Despite being engaged in the programme for four days most learners could not cut out the angles and line segments as required by the question; for example, L5 ended up cutting one line segment correctly, but left the other two cut into halves and they were of no use. I had to give this learner another copy of the triangle.

On the other hand, L4 could not understand how to do intervention activity 5 and said to me *"Sir, I don't understand what I am supposed to do in this question."* The learner asked me for clarity. A number of learners were not able to conceptually understand what was asked in the question. The learners needed individual help, which I gave, in simple terms. I ended up explaining to the whole class.

L1 marked all the line segments before they were cut out. When I asked why the line segments were labelled, the response was, *"I want to be able to identify them when I am measuring."*

Day 6: Observation results

All learners were able to cut out the line segments correctly which showed that they understood how to do the task. The pieces of the line segments were compared against each other to establish their relationships in terms of the lengths. In intervention activity 6.2(v), L7 said *that "angles are equal",* instead of saying all 'angles are different in sizes'. Therefore, it was clear that L7 could not differentiate the angles of triangle GHI both visually and by using polygon pieces which were used to compare the sizes of the three angles.

Day 7: **Observation results**

Most learners labelled all the angles in a particular triangle before they were cutting them out. I asked the learners, why they were labelling the angles? L8 said that *"for easy identification when I am measuring."* In another instance, L8 did not respond correctly to the question that required them to compare the line segments of a triangle; it was said that "………*two sizes are equal in length."* On the other hand, L1 and other learners recorded every measure that was taken as per the question's instruction and as a result, they were correct in their responses.

Day 8: Observation results

Although learners were doing the intervention activity individually, they also had an opportunity to explain some ideas to each other. For example, L3 explained to L7 on how to do intervention activity 8. It took most of the learners less time to do activity 8. Seemingly, they conceptually understood the question. L9 was not clear with the questions in intervention activity 9; this learner asked for clarity more than any other learner during that day.

Day 9: Observation results

In intervention activity 9 question 9.1(ii) learners were given choices of adjectives to use in their responses, but L6 decide to use different comparative adjectives 'smaller than.' Generally, L6 struggled to conceptually understand the idea behind the questions. On the other hand, other learners managed to cut out the angles and line segments. For example, L1 labelled the cut out angles and line segments, when I asked why the angles and line segments were labelled, the learner said that *"for easy identification when I have to use them in measuring.*

Table 4.20: Keywords from the field observation notes, words before keywords and words after keywords

Keywords	**Words before**	**Words after**
Conceptually understand	Could not, difficulties in, they, struggling	The instructions, the question, the meaning. The idea behind
Could not	But L8, all learners, some learners, L4, the learner	Use the pieces, be able to, angles and line segments, move on, conceptually understand
Individual help	Ask for, needed	When an explanation, which I did give
Cut out	At first did not, managed to, did not, managed to do, were able, there were	The angles and line segment, the line segments, correctly, used for
Mathematical symbols	Differentiate between the	Used for
Easy identification	for	When I am measuring, when I have

Table 4.20 indicates that during the intervention activities not all the learners were able to conceptually understand what some of the question required them to do. Although having polygon pieces in their hands, they did not know what to do. Such learners asked for help; for example, L4 said that *"Sir, I don't understand what I am supposed to do in this question."* Another learner (L3) also asked *"what is the meaning of the word properties?"* This was an indication of how mathematical language difficulties had a negative impact on learners' conceptual understanding of some of the questions. Not only that mathematical language was a problem, but also some mathematical symbols were not known by most of the learners; for example, the meaning of $\overline{AB}$ was not conceptually understood by all the up until it was clarified during revision.

When cutting out the angles and line segments from the given copies of triangles, some learners were creative enough; for example, L1 and others labelled all the angles and line segments before cutting. When I asked the question why the labelling was done, L1 responded as shown in the section of observations of day 9.

Over time, most of the learners were now able to cut the angles and line segments without any problems, such results showed how the intervention activities helped learners in skills development and enhancement.

Despite the use of polygon pieces and the mathematics dictionary each learner had a unique path of development. The way in which an individual learner's mathematical development took place from the diagnostics test results through the intervention activities to the post-test results is shown in Figure 4.2 to 4.10.

Table 4.21: How learners responded to diagnostic test and post-test.

Question Number in both DT and PT	van Hiele's levels of geometric thinking for each of the questions	Learners' codes	Did learners achieve questions in the DT at given Van Hiele's level of geometric thinking?	Did learners achieve questions in the PT at given Van Hiele's level of geometric thinking?
1.1 (i)	Level 0	L: 3, 4, 5, 6 and 8	No	Yes
(ii)	Level 2	L: 1, 3, 6, 7 and 9	No	Yes
(iii)	Level 2	L: 1,2,3,4,5,6,7 and 8	No	Yes
1.2 (i)	Level 0	L: 1, 3, 4, 5, 6, 7 and 8	No	Yes
(ii)	Level 2	L: 2, 3, 4, 5, 6, 7 and 8	No	Yes
(iii)	Level 2	L: 1, 3, 5, 6, 7 and 9	No	Yes
1.3(i)	Level 0 and 3	L: 2, 5, 8 and 9	No	Yes
(ii)	Level 1	L: 2, 3, 7, 8 and 9	No	Yes
(iii)	Level 0	L: 1, 3, 5, 6 and 9	No	Yes
1.4(i)	Level 0	L: 3, 5, 6, 7, 8 and 9	No	Yes
(ii)	Level 2 and 3	L: 1, 2, 3, 5, 6, 8 and 9	No	Yes
(iii)	Level 0	L: 1, 2, 3, 6, 8 and 9	No	Yes
1.5(i)	Level 0	L: 1, 2, 3,4 and 5	No	Yes
(ii)	Level 2	L: 1, 2, 3, 5, 8 and 9	No	Yes
(iii)	Level 0 and 3	L: 1, 2, 3, 5, 6, 7and 8	No	Yes

Table 4.21 illustrates the results of learners who could not respond correctly to certain questions in the diagnostic test, but managed to respond correctly to the very same questions in the post-test. The reason why the learners listed in Table 4.21 were not able to respond to questions which were considered to be at level 0-visualisation of the van Hiele model of geometric thinking, was possibly due to lack of well-established visualisation skills which helped in making a judgement regarding the given mathematical situation.

As shown in Table 4.21, the results further show that some learners who could not respond to questions that were at level 1-visualisation, level 2-analysis and 3-deduction of the van Hiele geometric thinking in the diagnostic test, but they managed to respond correctly to the very same corresponding questions in the post-test. Diagnostic test results showed that most of the learners were at level 0-pre-recognition level as described by Clements and Battista (1991). After learners were engaged in the observation and experimentation activities using the cut out line segments, angles and the use of mathematics dictionary most of the learners managed to migrate from pre-recognition level as described by Clements and Battista (1991) to the van Hiele levels of geometric thinking. The learners migrated to: level 0-visualisation, level1-analysis, level 2-abstraction and level 3- deduction in all the questions that belonged to the mentioned specific level.

The route to such an improvement was of ups and downs for all the learners, refer to Figures 4.2. to 4…10. The activities I designed were to engage learners in hands-on and minds- on learning. In order to be established and conceptually understand a particular triangle's properties learners had to cut out the line segments and angles in order to compare each one of the polygon pieces with the line segments and angles in the original triangle. Figure 4.22 shows how the process was done. In all nine activities and seven reflective tests each one of the learners kept on moving up and down in achievement, but they all eventually obtained the higher results in the post-test as compared to how they achieved in the diagnostic test.

The up and down results of the intervention activities were as a result of the following:

(i) Some learners could not spell the words correctly for the reason that they did not see the need of using the provided mathematics dictionary.

(ii) Some learners were not be able to visualise and describe given figures based on their properties

(iii) Most of the learners it was their first time to learn geometry using polygon pieces.

Table 4.22: Learners who could not answer certain questions correctly in both the diagnostic test and post-test

Question Number in both DT and PT	**van Hiele's levels of geometric thinking for each of the questions**	**Learners' codes**	**Did learners achieve questions in the DT at given Van Hiele's level of geometric thinking?**	**Did learners achieve questions in the PT at given Van Hiele's level of geometric thinking?**
1.1 (i)	Level 0	L7 and L9	No	No
(ii)	Level 2	L8	No	No
(iii)	Level 2	L9	No	No
1.2 (i)	Level 0	L9	No	No
(iii)	Level 2	L2, L4 and L8	No	No
1.3(i)	Level 0 and 3	L3, L4 and L6	No	No
(iii)	Level 1	L2 and L8	No	No
1.5(i)	Level 0	L6, L8 and L9	No	No
(ii)	Level 0 and 3	L4, L6and L7 and L9	No	No

Table 4.22 shows learners who could not improve their results in certain questions in both the diagnostic and the post-test. These learners were stuck for the following reasons:

(a) Mathematical language barriers which includes:

(i) Failure to spell the names of triangles correctly; for example, responding to question 1.5(i) in a diagnostic test, L9 said that *'it is PQR'* while responding to the same question in a post-test, L9 said that *'isosietive triangle'* instead of saying isosceles triangle.

(ii) Failure to make sense from what has been asked; for example in post-test, L6 said that *"it is 90^0, $\hat{D}$ and $\hat{F}$ "*, yet the question required the sizes of each of the angles not their sum.

(iii) Failure to use appropriate comparative adjectives when distinguishing the sizes of angles in a triangle, for example in question 1.2(i) L9 said that *"$\hat{G}$ is longer than $\hat{H}$ "* instead of using comparative 'greater than' or 'smaller than', the angles were considered as the line segments.

(b) Lack of conceptual understanding of mathematical symbols. For example, in question 1.1(ii) where the question required the learners to give the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$, L9 said that "*$\hat{A}\hat{B}$ is bigger than $\hat{B}\hat{C}$, $\hat{A}\hat{C}$ is longer than $\hat{B}\hat{C}$ and $\hat{B}\hat{C}$ is shorter than $\hat{A}\hat{B}$."* This learner could not conceptually understand what the different symbols represent, explaining why the angle symbols have been used in place of line segment symbols.

Also in question 1.3(i) of the post-test, L3 said that *"$\hat{D}$ is bigger than $\hat{F}$, $\hat{F}$ is smaller than $\hat{D}$"*, yet symbols were shown on the triangle that triangle EFD was an isosceles, therefore, the expected response was supposed to say $\hat{D} = \hat{F}$. The way L3 responded shows that the meanings of different geometric symbols are not yet clear.

The table 4.23 below gives a summary of how the learners performed initially in the diagnostic test as compared to how they performed in the post-intervention test.

4.3 Distribution of diagnostic and post-intervention tests marks

In this section, I present learners' distribution based on the marks obtained in the two tests, the diagnostic test and the post-intervention test.

Table 4.23: Comparison of diagnostic test results and post-test results

Percentage obtained →	0 - 20	21 - 40	41 – 60	61 - 80	81 - 100
Diagnostic test No. of boys	1	1			
No. of girls	3	3	1		
Post-intervention test No. of boys				1	1
No. of girls				4	3

Table 4.23 gives a summary of how the learners developed through the intervention programme. The number of learners whose scores are between 0% and 20% is four, one boy and three girls; between 21% and 40% there were four, one boy and three girls; and one girl obtained marks between 41% and 60%. The results of the post-intervention test showed a different picture where there was one boy in each of the following categories: between 61% and 80% and between 81% and 100%, while in the categories between 61% and 80%, between 81% and 100%, there were four and three girls, respectively.

Learners' improvement in the post-test was exclusively attributed to the intervention programme since the diagnostic test results showed clearly that despite that the topic of geometry was learnt in the previous grade learners could not get it right still. The intervention activities which made use of polygon pieces and mathematics dictionary helped the learners to acquire skills and comprehend relevant geometric terminologies ascribed to different triangles. The acquisition and comprehension of geometric terms like, equilateral, isosceles, line segment, etc. led to an improvement in post results.

4.4. Themes emerged from the research data

Five themes emerged from the intervention activities, observations and transcribed interviews. The five major themes emerged from the interview scripts, under each of the identified themes are annotations from which the major themes emerged. The annotations gave a reflection of how the participants felt about the intervention programme which was used in order to address the alternative conceptions that learners had in geometry as demonstrated in their responses in the diagnostic test. The annotations are presented by the two numbers, i.e. 1:2, this is interpreted as 1 is for learner 1 and 2 is the line 2 in the transcribed interview

Below is the detailed description of where each of the themes emerged from.

Theme 1: Mathematics dictionary, a tool for making meaning

During the intervention activities learners demonstrated that the use of mathematics dictionary enabled them to make sense of geometric terms.

Table 4.24: The themes emerged from the transcribed interview data

Theme number	**Themes**	**Annotations**
1	Mathematics dictionary, a tool for making meaning	For conceptual understanding of mathematical symbols (7:11 & 8:3) Learn the meaning of geometrical symbols (4:13)

Table 4.24 shows the theme emerged from the transcribed interviews data. The theme emerged from the annotations identified from three learners: L4, L7 and L8. The three learners stressed that the use of mathematics dictionary helped them to make meaning of most of mathematical symbols.

Theme 2: Polygon pieces assisted by mathematics dictionary mediating conceptual understanding

Table 4.25: The themes emerged from the transcribed interview data

Theme number	**Themes**	**Annotations**
2	*Polygon pieces mediating conceptual understanding*	For conceptual understanding of geometry (1:5, 1:6, 3:3, 7:5, 7:7 & 8:7) Help to clarify geometric concepts (2:5, 2:6, 2:17, 3:7, 4:10, 4:20, 5:9, 6:5, 7:8, 8:1 & 8:5) For conceptual understanding of mathematical symbols (7:11 & 8:3)

Table 4.25 shows the three major themes which emerged from the interview script, under each of the identified themes are annotations that give a reflection of how the participants felt about the intervention programme which was used in order to address the alternative conceptions that learners had in geometry as demonstrated in their responses in the diagnostic test.

During the semi-structured interviews eight learners (L1… to… L8) stressed how the use of polygon pieces helped them to learn geometry. As shown in the diagnostic test results in Figure 4.1, L4 knew some geometric basics before engaged in the intervention programme, but still the learner felt that the use of polygon pieces in teaching and learning geometry promotes conceptual understanding. One of the responses given by L4 attests to this, *"Yes, sir, I got a clear picture because now I clearly understand the concepts of triangles and their properties."* On the other hand even L3 and L8 who scored 0% in the diagnostic test as shown in Figure 4.1, also recognised that the use of polygon pieces in teaching and learning of geometry promote conceptual understanding. A quote from L8 expressed how the use of polygon pieces influenced this learner's learning, *"In grade 7 I did not learn the properties of triangles, but with what we have done, now I understand and know what to do."*

To be specific in how the polygon pieces influenced conceptual understanding, L3 and L8 said that the meaning of mathematical symbols was made clear to them. For example, L8 said: *"I did not understand the properties of an obtuse triangle. Even the slashes that are used to show that two opposite sides of an isosceles triangle are equal, I did not know the meaning of such slashes, but now after your programme it is clear to me".* This implies that, at first, before learners were engaged in the intervention programme that made use of the polygon pieces mathematical symbols were of little or no meaning at all to some of them.

Theme 3: Language incompetence influencing meaningful learning

Some learners could not conceptually understand the instructions in activity one of the intervention. The learners also demonstrated some difficulties in conceptually understanding the questions. For example, L4 and L8 after being given materials to do intervention activity one, the two spent almost 12 minutes without writing anything on the paper. L8 kept on asking for individual help almost in all the questions. Some learners were not familiar with the mathematical symbols used to illustrate that line segments opposite two equal angles in an isosceles triangle are equal. For instance, L3 asked me a question, *"what is the meaning of the word properties?*

Theme 4: Polygon pieces assisted by mathematics dictionary unpack meaning and stimulate interest

Table 4.26: The theme that emerged from the transcribed interview data

Theme number	Themes	Annotations
4	*Polygon pieces unpack meaning and stimulate interest*	Promote measuring skills (1:2, 2:12, 4:8, 5:15, 6:10 & 8:10) Taught how to measure (1:3, 1:12,1:15, 6:10, 6:11, 6:12, , 6:16 & 8:16), Learning mathematics (1:6, 2:8, 3:14, 4:12, 4:19& 9:18) Arouse learners' interest in learning mathematics (1:4, 2:3 & 2:7) Liked the use of polygon pieces (2:4, 3:13, 4:7, 5:3 &6:4)

Table 4.26 shows the fourth theme that emerged from the interview script. Under the identified theme are annotations that gave a reflection of how the participants felt about the intervention programme which was used in order to address the alternative conceptions which learners had in geometry as demonstrated in their responses in the diagnostic test.

During the semi-structured interviews the following learners stressed that the use of polygon pieces in teaching and learning of geometry instilled and promoted mathematical measuring skills which could be applied in other learning areas. For example L4, who initially could not score above 50% in the diagnostic test, voiced in favour of using polygon pieces by saying *"It has helped me with measuring skills. I can now measure angles and sides of triangles using the pieces of the same triangle"*. In addition, L1, L6 and L8 felt that the use of polygon pieces in the teaching and learning of mathematics have taught them how to measure. For example, L6 said that "*it was exciting to use triangle pieces to learn how angles in a triangle are related to each other, also the sides"*. The association within the theme is that there were two annotations that belonged to it as described in the direct quotations from L4 and L6's statements.

Five learners (L1, L2, L3, L4 & L9) said that the use of polygon pieces during the series of teaching episodes helped them to learn mathematics in a simple way. For example, they mentioned that the use of polygon pieces that was of help and made them enjoy the learning of mathematics, L3 said that their joy came through *"The learning of properties of triangles by using the pieces of angles and sides of triangles*". From another point of view regarding how the polygon pieces have been of a benefit to the learning of mathematics L4 said that *"And also that when the letter is written like this,* $\hat{Z}$ *it means angle Z"*. From L4's response, I conclude that the use of polygon pieces in teaching and learning of geometry helped in clarifying the meaning of geometrical symbols, which were not clearly explained to the learners in the previous lessons.

Under theme 4; six learners (L1… to… L6) said that they liked the programme that made use of polygon pieces to teach and learn geometry. When asked how the learners felt about the use of pieces of polygon in teaching and learning of geometry, L2 said that *"I feel excited, sir"*. This is an indication that the use of polygon pieces in teaching and learning of geometry arouse and promoted learners' interest in learning geometry. The excitement came when their curiosity was drawn to the teaching and learning of geometry.

Theme 5: Polygon pieces assisted by mathematics dictionary encourage active learning and long-term gains.

Table 4.27: The themes that emerged from the transcribed interview data

Theme number	**Themes**	**Annotations**
5	*Polygon pieces encourage active learning and long-term gains*	Promote hands-on-teaching and learning (2:10, 4:17, 7:11, 9:17) Explore properties of triangles (3:12, 3:9, 5:11, 9:5) Classification of triangles (2:13, 4:4, 4:11, 5:6, 6:12, 6:13, 6:14, 8:10 & 9:6) First time to learn geometry using polygon pieces (1:9, 2:16, 3:15, 4:6, 8:6 & 9:8) Limited time used for geometry (1:10 & 1:12)

Table 4.27 shows the fifth theme that emerged from the interview script; under the identified theme were annotations. This gave a reflection of how the participants felt about the intervention programme that was used to address the alternative conceptions that learners had in geometry as demonstrated in their responses in the diagnostic test.

The semi-structured interview results showed that nine learners (L1, L2, L3, L4, L5, L6, l7, L8 & 9) talked about this theme; they claimed that hands-on learning was the benefit of using polygon pieces in the teaching and learning of geometry. This theme is characterised by five different annotations that were identified from different learners' responses, namely: (i) first time to learn geometry using pieces of polygons, (ii) limited time used to learn geometry, (iii) promote hands-on-teaching and learning, (iv) explore properties of triangles and (v) classification of triangles.

Their responses were clustered in three different categories: (i) the polygon pieces helped to measure the sizes of angles and length of sides in a given triangle from: L2, L4, L7 and L9. Two learners, L3 and L9 said that the use of polygon pieces provided them with opportunities to explore the properties of triangles and compared their properties against each other. The polygon pieces assisted by mathematics dictionary allowed learners to go into an investigation process of the properties of different triangles (L3 & L9). According to L2, L4, L5, L6 and L9, the use of polygon pieces made it easy for them to classify triangles; for example, L2 said that "….*and also as I am speaking, I now know well the names of triangles."*

The responses from the learners listed under this theme clearly showed how the polygon pieces assisted by mathematics dictionary were used as physical manipulatives to influence the teaching and learning of geometry, specifically comparing the angles and length of the sides of a particular triangle in order for them to develop grounded conceptual understanding of the properties of triangles.

Six learners (L1, L2, L3, L4, L8 & L9) under theme 5 said that in the previous grade they did learn geometry, but without using any tangible items, for example L2 said that *"No sir, this is the first time I have been using small pieces of paper to learn geometry".* From the group of five, L1 further said that *"Yes sir, we learnt all geometry in those five days only".* The statement implies that they did learn geometry, but for a shorter period of time than expected.

L8 claimed that *"In grade 7, I did not learn the properties of triangles, but with what we have done now I understand and know what to do."* This response was an evidence of the 0% mark obtained in the diagnostic. For information on how the learner achieved, refer to Figure 4.9. This theme is directly linked to theme number two in the sense that if learners are not taught in the previous grade, the impact is reflected in their conceptual understanding of a particular phenomenon.

4.5 Why did the model influence mathematical development?

In this section, I briefly describe reasons that made my reflective teaching and learning model work in influencing learners' geometric conceptual understanding. The following are a few pointers that made the model work:

(i) The model was driven by learners. During the lessons no one had to tell the learners, for example, how an equilateral triangle looked like. But the learners were engaged in activities of cutting out line segments and angles and used them to explore, observe and experiment by comparison in order to establish the properties of the given triangle. The use of polygon pieces assisted by mathematics dictionary for teaching and learning geometry drew learners' curiosity to learn and as a result, they were very much focused and curious to do the assigned task.

(ii) The daily design of intervention activities which was informed by learners' previous activity's results. The previous activity's results were actually a guide for me in areas in which learners needed the most help.

(iii) The integration and use of the mathematics dictionary and polygon pieces into the teaching and learning of geometry were also a crucial part to be taken into consideration. Proper integration required the following: each and every learner was given all the required resources, like pair of scissors, three A4 papers, one with the original triangle drawn and two copies of the original triangle. Instructions were ready and emphasised by the facilitator. During the lesson, regular supervision was done to ensure that all the requirements in doing each activity were adhered to by all the learners.

(iv) The reflective tests and reflective sessions that were conducted daily before the beginning of a new activity also played a major role in ensuring that learners' retention was enhanced. Reflective tests were tests which learners wrote on a daily basis. The content of each of the tests was based on the previous day's intervention activity's content. After the reflection test, a reflection session was held where the previous day's alternative conceptions were rectified by the facilitator. After this session, the learners were engaged in a new intervention activity for that particular day.

4.6 Lessons learnt from these results

Although the current study is based on a small sample of participants, the findings suggest the following:

(i) Polygon pieces assisted by mathematics dictionary have influenced learners in the teaching and learning of geometry,.

(ii) When using polygon pieces assisted by mathematics dictionary, the teacher should not take a back seat, but must always move around the class observing and giving individual help where needed. It is important for the teacher to keep on moving around for the reason that most learners come to a mathematics class with preconceived ideas regarding geometry. When advised to use polygon pieces assisted by mathematics dictionary which were aimed at clarifying concepts, some learners did not want to take instructions. They just want to respond to a question based on their own previous knowledge which is sometimes correct, but mostly incorrect. The continuous observation and guidance enhance learners' performance and ensure the development of conceptual understanding.

(iii) Learners needed many activities for the extended exposure and enrichment in the use of polygon pieces and the mathematics dictionary in order to develop and enhance conceptual understanding of the properties of triangles.

(iv) It was important to have a reflective test and a reflective session of the previous lesson's concepts before engaging learners in the new lesson for the enhancement of their retention.

4.7 The actual model of teaching and learning geometry emerged during my research.

In this section, I present the teaching and learning model that helped the learners who participated in my research project to improve their post-intervention results with a wide margin as compared to the marks obtained in the diagnostic test. My presentation includes: what made the learners develop conceptually or become stuck in the process, tools and strategies at times that made the learners move up in post-test results.

This research project presents the model that can be used in the teaching and learning of geometry, specifically properties of triangles. The model is entitled: **Chiphambo's reflective model for teaching and learning geometry.** The model is the combination of different approaches to teaching and learning of geometry. The aspects includes: (i) the use of polygon pieces assisted by mathematics dictionary in teaching and learning of geometry; (ii) the use of mathematics dictionary for mathematics vocabulary enhancement and terminologies proficiency and (iii) the teacher's responsibilities during the teaching and learning.

Chiphambo's reflective model for teaching and learning geometry

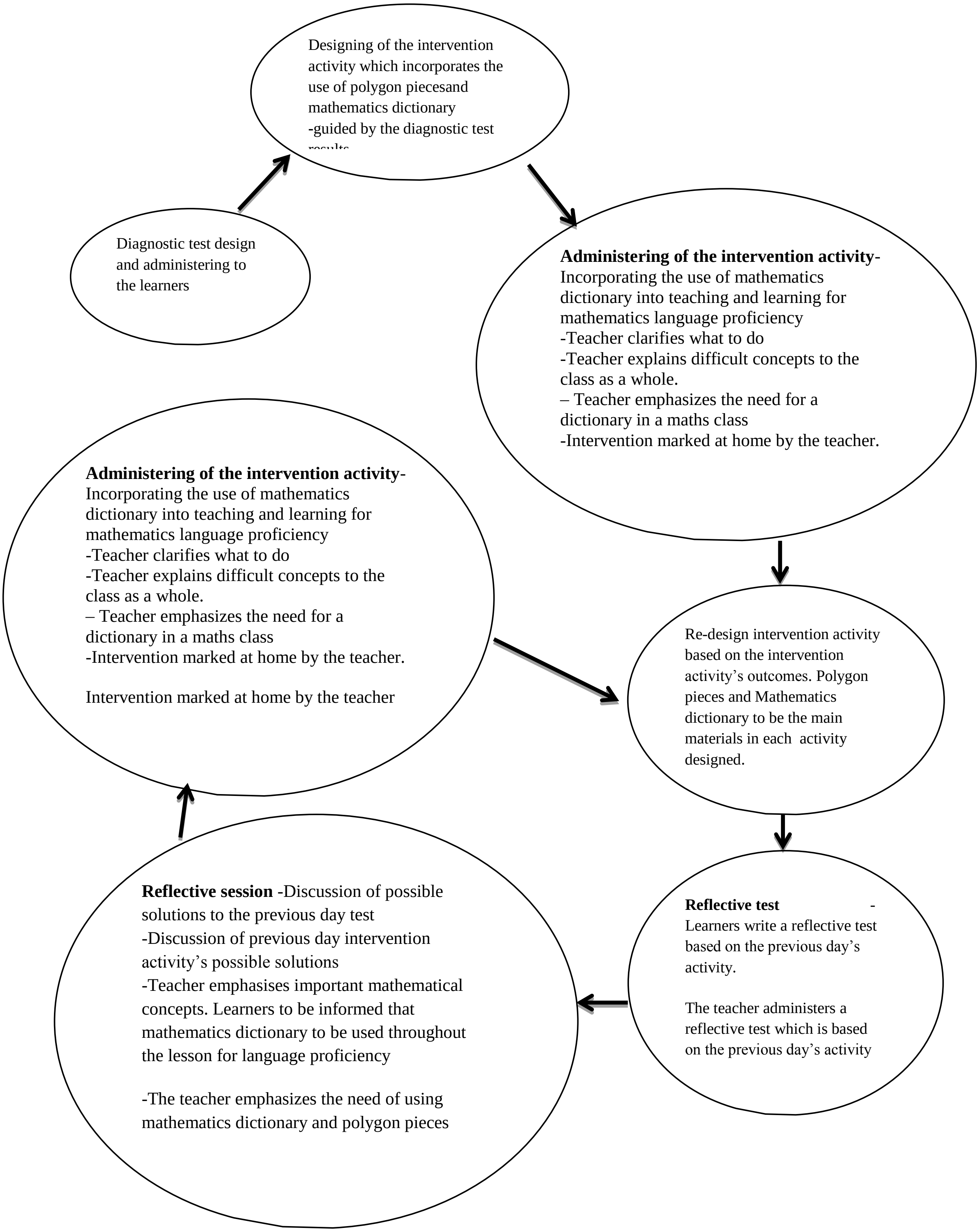


Figure 4.23: Chiphambo's reflective model for teaching and learning geometry.

Figure 4.23 shows Chiphambo's reflective model for teaching and learning geometry incorporating mathematics dictionary for mathematics vocabulary enhancement and terminologies proficiency; and the use of polygon pieces for geometric conceptual understanding.The arrows indicate the sequence of how the model was used to address alternative conceptions learners had in the learning of geometry. At each of the stages in the model are the accounts of what was expected of both the teacher and the learner as the teaching and learning progresses. The use of polygon pieces and the mathematics dictionary was of paramount importance for the development of learners' mathematical vocabulary and geometry terminology proficiency. Also it was for the development of conceptual understanding of mathematics.

The strengths and other aspects of importance in my reflective teaching and learning model are described below.

4. 8 Chiphambo's reflective model for teaching and learning geometry contributions

In this section, I present the findings of my research regarding the contributions made by the model which made use of polygon pieces assisted by mathematics dictionary for learners' development of mathematical conceptual understanding.

The results of my research show that the following are the contributions of the developed model to the learners' mathematical development: (i) geometric language development, (ii) mathematics discourse, (iii) development of geometric concepts, for example comparison of the angles and measurement of the line segments, (iv) developing the knowledge of the properties of triangles, (v) development of visualization skills, (vi) development and enhancement of psycho-motor-manipulative skills and (vii) development of the conceptual understanding of geometric symbols.

4.9 Conclusion

In this chapter, I presented the following results of my research findings: (i) how the use of polygons pieces as physical manipulatives assisted by mathematics dictionaryin the teaching and learning of geometry improve learner performance, (ii) how measurement of angles and sides of polygons using polygons pieces assisted by mathematics dictionary (cut pieces of 2-dimensionals) promote learners' geometric conceptual understanding; (iii) how mathematics teachers should use polygon pieces as physical manipulatives assisted by mathematics dictionary to teach properties of polygons in order to promote learners' proficiency in geometry.

In the next chapter, I present a discussion of how the conceptual framework and the associate literature are linked to my research findings.

CHAPTER FIVE: DISCUSSION

5.1 Introduction to the chapter

As it was stipulated in the first chapter, the study was set out to investigate a model that integrates dictionary and polygon pieces in teaching and learning of geometry to grade 8 learners. The investigation focused on how polygon pieces can be used as physical manipulatives assisted by mathematics dictionary to promote learners' conceptual understanding of geometry (Kilpatrick, et al., 2001). Furthermore, the study also wanted to investigate how mathematics teachers should use polygon pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning to promote learners' mathematical proficiency in geometry.

In this chapter, I present the link between the identified literature, the conceptual framework and the results of my research in view of the following subheadings:

- The findings and critique of research
- Key findings
- Unexpected outcomes
- The support from the previous research
- The contradiction of my results in relation to the previous research
- The detailed explanation of my research results
- Advice to the researchers and educators in the interpretation of my research findings
- Suggestions of the teaching and learning model
- Presentation of the implications of my research
- Recommendations for future research work

5.2 Findings and critique of the research

The previous studies that have noted the significance of physical manipulatives assisted by mathematics dictionary in geometry teaching and learning strongly support their use for the reason that it is the only economical way to help learners with the conceptual understanding of geometry concepts regardless of their location. In addition, strong relationships between geometry teaching and learning and the use of physical manipulatives assisted by mathematics dictionary have been reported to have a positive influence in the literature. For example, geometry is considered to be double-folded (theoretically and practically), which makes it difficult for most of the learners to achieve, and physical manipulatives break this barrier (Fujita & Keith, 2003). It is suggested that physical manipulatives once used effectively in the lesson spatial skills are inculcated for learners to be problem-solvers in real-life situations (Van den Heuvel-Panhuizen et al., 2015).

As mentioned in the literature review, the way geometry is taught made it to be regarded by the learners as the most difficult branch of mathematics. Van Hiele (1999) noted that most geometry is presented based on certain principles to the learners who have no basic conceptual understanding about it (Steele, 2013). In the view of van Hiele (1999) and Steele (2013), it is apparent that mathematics teachers must revisit how they teach geometry, regardless of the level of learners they are teaching.

Research is quite clear that teaching and learning of geometry that does not afford learners opportunities to manipulate objectives when learning, deny them opportunities to establish solid connections that link geometry concepts and terminology (NCTM, 1989; Teppo, 1991; Clements & Battista, 1992; Baynes, 1998; Prescott et al., 2002; Thirumurthy, 2003; Ubuz & Ustün, 2003; Steele 2013). My experience as a mathematics teacher is that allowing learners to learn by exploring and using hand-on activities draw their curiosity, resulting in them becoming very inquisitive in whatever stage they are going through during the lesson. For example, TIMSS' (1999) video data as reported by Mosvold (2008), states that to serve the same purpose in classrooms in Japan real-life examples are used when teaching mathematics. The use of such examples helps learners to attach meaning to their own learning, unlike the rote learning that leaves them with numerous unanswered questions regarding their alternative conceptions.

5.3 Key findings

In this section, I present the three key findings of my research study:

- The use of polygon pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning of geometry influenced learners' conceptual understanding of geometric concepts.
- Polygon pieces used as physical manipulatives assisted by mathematics dictionary influenced the teaching and learning of angle measurement in geometry for learners' conceptual understanding.
- Engaging learners in hands-on-learning using polygon pieces as physical manipulatives assisted by mathematics dictionary to teach properties of polygons promote learners' proficiency in geometry.

5.3.1 The use of polygon pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning of geometry influenced learners' conceptual understanding of geometric concepts.

Table 5.1 below presents the first key findings of my research findings. The first key finding showed that the teaching model was able to promote learners' geometric thinking levels from lower levels to higher levels of geometric thinking according to the van Hiele model.

Table 5.1: Learners' van Hiele levels during diagnostic test and after intervention, post-test

Learners	**Levels during diagnostic test**	**Levels after intervention**
1	Pre-recognition	Visualisation, analysis, abstraction and formal deduction
2	Pre-recognition	Visualisation, analysis, abstraction and formal deduction
3	Pre-recognition	Visualisation, analysis, abstraction and formal deduction
4	Pre-recognition	Visualisation, abstraction
5	Pre-recognition	Visualisation, abstraction and formal deduction
6	Pre-recognition	Visualisation, abstraction and formal deduction
7	Pre-recognition	Visualisation, analysis, abstraction and formal deduction
8	Pre-recognition	Visualisation, analysis, abstraction and formal deduction
9	Pre-recognition	Visualisation, analysis, abstraction and formal deduction

Table 5.1 shows that out of nine learners, six have moved through the levels step by step from pre-recognition level 0 suggested by Clements and Battista (1991) to the formal deduction level of geometric thinking hypothesised by the van Hiele model. The other two learners (L5 & L6) moved to the same higher level 3-formal deduction as the first six, but did not perform in questions at level 1-analysis of the van Hiele model of geometric thinking. L4 performed from pre-recognition of Clements and Battista (1991) to level 0-visualisation and level 2-abstraction of the van Hiele model of geometric thinking, but both could not answer questions at level1-analysis and level 3-formal deduction.

The group of six learners (L1, L2, L3, L7, L 8 & L9) were able to move step by step through the levels as described below. The percentage shown below represents the questions that each learner managed to answer correctly.

During the intervention six learners' performance showed an up and down movement until the post-test. Of all the questions at level 0-visualisation of the van Hiele model in both intervention activities and reflective test individual learners responded correctly as follows: L1 - 73%; L2 and L8 - 33%; L3 - 20%; L7 - 33% and L9 - 40%. For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

Questions that were at level 1-visualisation of the van Hiele model of geometric thinking in both intervention activities and reflective test were answered correctly as follows: L1-77%; L2- 41%; L3 - 75%; L7 - 33%; L8 - 30% and L9 - 39%.For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

The van Hiele model level 2-analysis questions in both intervention activities and reflective test were answered correctly by individual learners as follows: L1-13%; L2 and L7-4%; L3 and L8 - 17%, and L9 - 21%.For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

Intervention activities and reflective test questions at level 3-formal deduction of the van Hiele model of geometric thinking were attempted by individual learners as follows: L1 - 43%; L2 - 29%; L3 and L9 - 21%; L7 - 50% and L8 - 7%. For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

As shown in Table 5.1 above there were three learners who had unique movements along the levels, L4, L5 and L6. L4 moved from pre-recognition level of Clements and Battista (1991) to level 2-abstraction of the van Hiele model without performing in questions at level 1-analysis of the van Hiele model and could not reach level 3-formaldeduction. L5 and L6's movements were similar. The summary of how L4, L5 and L6 performed per question during the intervention activity is presented below: 73% of questions at the van Hiele level 0-visualisation were answered correctly by L4. The questions at level 1-analysis, 68% of them were correctly responded to by L4. Out of all the Level 2-abstraction questions, L4 answered 26% correct. Forty-three per cent of the questions at level 3-formal deduction were correctly answered by L4.For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

L5 and L6's post-test results showed that both learners moved from pre-recognition level of Clements and Battista (1991) to level 3-formal deduction of the van Hiele model, but both could not perform in questions at level 1 of the van Hiele model. How each of the two learners performed in the post-test is contrary to the performance in the series of intervention activities.

Seventy-three percent of the questions at level 0-visualisationof the van Hiele model were answered correctly by L5, while at the same level, L6 responded to 33% of the questions. For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

Questions at level 1-analysis of the van Hiele model were answered correctly by L5 while L6 managed to respond to 41% of the questions. For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

Both learners responded to 13% of the questions at level 2-abstraction of the van Hiele model. Lastly, questions at level 3-formal deduction as suggested by the van Hiele model were answered correctly as follows: L5 - 36% while L6 - 29% of the questions. For the details of how individual learners correctly responded to intervention activities and reflective test questions, refer to appendix 31 and 32.

L5 and L6's convincing performance in both intervention activities and reflective test questions at level 1-analysis of the van Hiele model indicated that the use of physical manipulatives assisted by mathematics dictionary had positive effects in teaching and learning of geometry.

In summary, improvement in learners' results in the post test revealed the positive effect of the use of polygon pieces to the learners and understanding of geometry concepts.

5.3.2 Polygon pieces used as physical manipulatives assisted by mathematics dictionary influenced the teaching and learning of angle measurement in geometry for learners' conceptual understanding.

In this section, I present the way in which how the measurement of angles and sides of polygons using pieces of the same polygons assisted by mathematics dictionary promoted learners' geometric conceptual understanding influenced the learning of geometry in the individual learners.

What caused most of these learners to be at pre-recognition level of operation as suggested by Clements and Battista (1991) was revealed during the intervention activities. Some of the identified challenges included: mathematics language barriers, mythical thinking, and unjustified jump in a logical inference and lack of proficiency in geometry. The use of polygon pieces assisted by mathematics dictionary while learning was regarded as unnecessary exercise for the reason that they never used them in mathematics classes before.

In question 1.2 of intervention activity 1 described in appendix 12, L6 responded as follows: *"Because they are use to be the or triangles shape is to be identified."* This shows that L6 had a mathematical language barrier which resulted in the learner assigning a given piece of information a meaning that was different from the asked question (Movshovitz-Hadar et al., 1987). These findings echo the same sentiments as recent research findings that state: failure to comprehend the meaning of some sentences most learners cannot make meaning of the mathematical concepts and terminology being presented (Usiskin 1982; Mayberry, 1983; Van Hiele-Geldof, 1984; Fuys, 1985; Senk, 1985; Burger & Shaungnessy, 1986; van Hiele, 1986; Crowely, 1987; Fuys et al., 1988; NCTM, 1989; Teppo, 1991; Clements & Battista, 1992; Baynes, 1998; Prescott et al., 2002; Thirumurthy, 2003; Ubuz & Ustün, 2003; Steele 2013).

Even though L4 managed to write the name of the identified triangle in question1.6 that is shown in appendix 12, L4the problem was the indefinite article that has been used before the words right-angled triangle. This learner used 'an' instead of 'a'. According to Christophersen and Sandved (1996), the indefinite article is an adjective used only before singular countable nouns. Such a mistake committed by L4 is categorised by Radatz (1980) as a language error due to mathematical language barrier.

In question 1.6 of reflective test 1 shown in appendix 21, L6 was not able to categorise triangles into their respective groups because this learner was not able to make sense of the properties of triangles. This was a typical example of a learner operating at level 0-pre-recognition according to Clements and Battista (1991). Such a learner was characterised as having mathematical language difficulties in most cases (Serow, 2002; Feza

& Webb, 2005). L6's challenges described above indicated that L6 struggled to comprehend the question.

Responding to intervention activity 5, question 5.2(iv) that required the learners to mention the properties of ΔABC in terms of: $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$, L9 said that *"Because Δ are angles; because $\overline{AB}$ are the lines segment* "This response revealed that the L9 did not comprehend what was really asked in the question. Failure to make sense of the question is classified by Sarwadi and Shahrill, (2014) as errors that occur due to mathematics language difficulties.

In Question 5.2(v) both L6 and L9 obtained 0%, such scores directly affected their overall results of the whole intervention activity 5. For more information on how L6 and L9 performed in IA 5 refer to Figure 4.7 and 4.10, respectively. L6 said that *"$\hat{A}$ is smaller than B and C"* while L9 responded as: *"triangles are angles $\hat{A}$ $\hat{B}$ are less than and C."* L6's problem in the response is the distortion of the meaning of angles by using letters like B and C, referring to them as angles. L6 omitted the symbolic information required to illustrate that B and C are angles. L9 falls within the same category as L6, but also had a problem with the sentence construction, which is a result of both mathematics and English language difficulties.

Van Hiele (1999) argues that in order for geometry teaching and learning activities to be effective they need to be placed in a context that is an indication of the importance of English language in the development and assessment of geometric understandings. It is further argued that instruction can foster or impede development in teaching and learning of geometry (Feza & Webb, 2005). This implies that English language proficiency has a role to play in the instruction of geometry. On the other hand, Van Hiele (1999) suggests that mathematical language is of prominent importance for describing geometric shapes. This also implied that at each level of the van Hiele model of geometric thinking, mathematical terms need to be introduced gradually to be mastered by the learners.

One of the learners (L6) responding to question 5.2(v) in appendix 16 obtained 0%. The response was presented as: *"(iv) it is longer than and smaller than (v) longer than and shorter than."* According to Sarwadi and Shahrill, (2014) such an error is due to insufficient quality of understanding of the whole question which emanates from both mathematical and English language difficulties. This was demonstrated by failure to comply with sentence construction in order to describe the mathematical situation.

In reflective test question 3 shown in appendix 23, L1 and L5 describe the characteristic of an acute angle and not the property of an acute-angled triangle. L2 seems not know how to describe the characteristic an acute-angled triangle. Furthermore, sentence construction seemed to be a challenge to these learners; such problems were derived from mathematical and English language barriers. Due to mathematical and English language difficulties, it was possible that what the learner wanted to say was completely different from what has been written down on the paper.

L3 was not able to differentiate an acute-angled triangle from an obtuse-angled triangle. L6 was still not sure of what to say, two comparative adjectives have been used in the same sentence. It was the same case with L8.According to Ashlock (2002), such results were a typical sign of mathematical language barriers that most learners demonstrate in a mathematics classroom. L8 had problems with mathematical language proficient; the learner could not make sense of what the question really required. From this learner's response the properties of a triangle were only based on angle sizes.

Question 6(iii) required learners to compare the lengths of $\overline{GH}$, $\overline{HI}$ and $\overline{GI}$ and the sides of triangle GHI L9 said *"GH is 'shorter than' IG"* instead of using the comparative 'longer than'. These findings seem to be in agreement with other research which discovered that failure to use adjectives correctly in their comparative form for the two line segments by the learners is due the language difficulties (Sarwadi & Shahrill, 2014).

Some learners failed to use the given comparative adjectives in their responses, for example, L7 whose response was *"I is longer than G."* According to the research findings such alternative conceptions might be due to a number of reasons. Firstly, incorrect interpretation of geometric symbols (Movshovitz-Hadar et al., 1987), secondly mathematical and English language barrier as a contributing factor. This was evident in a situation where a learner did not know what comparative adjectival form to use when comparing two angles (Sarwadi and Shahrill, 2014).

In question 6.2(iv) shown in appendix 17, the minor error identified in L1's answer showed that the solution was never verified (Movshovitz-Hadar et al., 1987).For example, *'all sides have different length.'* In this case the word 'length' was supposed to be written in plural form, but the letter's' was left out.

L6's responses to question 6.2(iv) shown in appendix 17, had two statements, one of which was correct. One was *"It is* $\overline{GH}$ *longer than* $\overline{GI}$ " and the other one was incorrect. " $\overline{HI}$ *is shorter than* $\overline{GI}$ *is longer than* $\overline{GH}$*."*In the latter response, this learner could not use the comparative adjectives correctly, which shows alternative conception in the meaning of the two comparative adjective, 'shorter than and longer than'. These findings further support the idea of mathematical and English language difficulties as one of the contributing factors to errors committed in mathematics by the learners (Sarwadi and Shahrill, 2014).

In question 6(vi) shown in appendix 17,L3 and L6 responded as follows: *"it a scelen triangles"* and *"It is an scalene triangle,"* respectively. The common errors demonstrated by these learners were poor sentence construction and lack of spelling skills. Research reveals that such errors emanate from mathematical language difficulties, which was a barrier that contributes to misunderstanding of what the question was about (Sarwadi & Shahrill, 2014).

The response in Figure 4.13 shows how L2 misinterpreted the mathematical language given in the question. This finding corroborates with the ideas of Movshovitz-Hadar et al. (1987), who suggest that such an error occurs when the learner translates an expression from the mathematical statement into a diagram form. Such results also demonstrate that there are some challenges in proficiency in the language of teaching and learning (LoTL), in this case mathematical terminologies. No matter how effective the intervention was, but if the language proficiency did not exist, the results remained affected negatively. A learner with such challenges did not qualify to be even at level 0-visualisation of the van Hiele geometric thinking, but was operating at pre-recognition level as suggested by Clements and Battista (1991).

In question 6.2 shown in appendix 17, L2 responded as follows: *"one angle is not equal."* L2 wanted to say one angle is different in size from the other two angles. L7 said that *"all sides are not equal."* The sentence construction was not correct; this was an indication of language difficulties, which have been confirmed by research in several instances as one of the impediment to the learning of geometry concepts (Feza &Webb, 2005).

In question 7.1(iii) shown in appendix 18 another alternative conceptions identified was how L4 spelt the word 'longest'. It has been spelt as "*longestes".* Such errors are identified as the products of language difficulties (Sarwadi & Shahrill, 2014).

As shown in appendix 18, one of the learners (L6) could not give the correct response in question 7.2(iv).The response given reads as follows: *"it is longer than and shorter than and equal to."* This learner took the listed options of responses for question 7.1(iii), which was a sign of mathematics language barrier that ensured that the learner did not conceptually understand what was required in the question (Sarwadi & Shahrill, 2014).

Responding to question 9.1(i) in appendix 20, L6 said *"Two sides are equal."* This learner did not answer the question; such a response further supports the idea of Radatz (1980) which proposes that when the learner has inadequate conceptual understanding of the text, the answer that was given, was in contradictory to what was asked. In addition my observation revealed that such responses demonstrated that the learner had a language barrier.

Even though L7 obtained a mark of 100% in question 9.1(ii) described in appendix 20, the error identified was a spelling error. The learner spelt the word 'shorter' as 'shoter', which was an indication of technical error that occurred during the process of extracting information from the list of given options (Movshovitz-Hadar et al., 1987).

Responding to question 9.2(i) described in appendix 20, L2 said *"PQ is smaller than RP and QR is equal to Rq."* The comparative adjective used in the former part of the response was not given as an option and was also not suitable for the comparison of the length of the line segments. This finding is in agreement with Feza and Webb's (2005) findings which showed that language incompetence acts as barrier to learning and leads to learners' poor performance in most cases. The latter part of L2's response to question 9.1(ii) showed that the learner did not abide to the rule which says that upper case must be used when presenting line segments. This might be due to learner's negligence of mathematical rules or a lack of conceptual understanding.

Each one of a group of three learners (L2, L3 & L4) scored 67% in question 9.1(ii) because they could not use the pieces of polygon as instructed, for example L2 said *"QR is longer than PR."* From L2's response I concluded that this learner was not conceptually clear about the symbols used for an isosceles triangle. It is also possible that L2 used the pieces of polygon as instructed, but could not conceptually understand the meaning of the word 'longer' and, therefore, was unable to give the correct comparative adjective, as in the words of White (2005), such problems are characterised as comprehension error.

L3 said *"*$\overline{\text{QP}}$ *is shorter than* $\overline{\text{PR}}$*."* This was a repetition of the second response that has been written as: *"*$\overline{\text{PR}}$ *is longer than* $\overline{\text{QP}}$*."* This was a typical example of a learner who had a mathematical language barrier. The two responses were regarded as different, yet they both had the same meaning. This demonstrated a lack of mathematical and English language proficiency as highlighted earlier by Feza and Webb (2005).

L7 said that *"*$\hat{\text{Q}}$ *is longer than* $\hat{\text{R}}$ *"*and *"*$\hat{\text{P}}$ *is shoter than R"*. The problems identified in both responses were the comparative adjectives which the learner used to show the difference between the mentioned angles. The use of the words 'longer' and 'shorter' when comparing the angles was an indication of both English and mathematics language difficulties. It also showed that the mathematics dictionary that was provided for the learners to use was never consulted in order to support the conceptual understanding and the correct spelling for the comparative adjectives used for the comparison of the angles. According to Crowely (1987),a person functioning at level 0-visualisation of the van Hiele model of geometric thinking can learn geometric vocabulary, but it was not the case with L7 who I rate to be operating at the level below zero of the van Hiele geometric thinking.

According to White's (2005) research findings, such responses as L9's response in question 7.1(iii) the learner failed to express the solution in an acceptable written form are categorised as encoding errors. Such errors are the products of English and mathematics language barriers where the learner cannot do a simple sentence construction to describe how two line segments are related to each other.

Responding to the same question 9.1 (i) described in appendix 20, L6 said that *"Two sides are equal."* This learner did not answer the question; such a response further supports the idea of Radatz (1980) which proposes that when the learner has inadequate comprehension of the text, the learner gives an answer that is contradictory to what has been asked. In addition my observation revealed that such responses were a result of mathematics language barrier.

L8's response to question 1.2 given in appendix 12 was quite unique. The learner said that *"a triangle has 3 vertices and faces."* The concept of three faces is applicable to the three-dimensional objects. The alternative conception has shown that this learner misused the information provided in the dictionary by imposing the information that disagrees with what the triangle exactly looked like (Movshovitz-Hadar et al., 1987).

In question 1.6 described in appendix 12, L2 and L6 could not spell the word equilateral correctly. This might be because of a mismatch between the learners' knowledge and instruction; the two learners were at different thinking levels as compare to the level of instruction where they had to use dictionary in a mathematics lesson (Crowely, 1987).

Even though the dictionary was provided to help learners respond to some of the question in activity 1, L2, L3, L6, L7, L8 and L9 were not able to identify the fourth group of triangles as scalene. This was due to what Steele (2013) calls a lack of the basic conceptual understanding about geometry, specifically properties of a triangle, which included the sides and angles.

In this study, just like in other research findings, (Feza & Webb, 2005), English language barriers have been found to be one of the factors causing learners' poor performance in geometry. For example, responding to question 1.2 of the intervention activities L6 said *"Because they are use to be the or triangles shape is to be identified."* For the content of question 1.2, refer to appendix 12. The response showed that this learner could not read and analyse the question due to both mathematics and English language difficulties, as a result no meaning was attached to the question asked that made the learner to give the response with some discrepancies in relation to the asked question (Movshovitz-Hadar et al., 1987).

L8's response to question 1.2 described in appendix 12 is quite unique. The learner said that *"a triangle has 3 vertices and faces".* The concept of three faces has been applicable to the three-dimensional objects. The alternative conception shows that this learner has misused the information provided in the dictionary by imposing the information that disagrees with what the triangle exactly looks like (Movshovitz-Hadar et al., 1987).

In question 1.6 described in appendix 12, L2 and L6 could not spell the word 'equilateral' correct. This might be because of a mismatch between the learners' knowledge and instruction; the two learners were at different levels with the level of instruction where they had to use dictionary in a mathematics lesson (Crowely, 1987).

The two learners (L2 & L9) who had drawn acute-angled triangles and inserted 90^0 symbols and called them right-angled triangles had alternative conceptions of the interpretation of the word 'right-angled'. This distortion of the definition is described as an imprecise citation of a recognizable definition of a right-angled triangle (Movshovitz-Hadar et al., 1987) which is due to language barriers.

Even though the dictionary was provided to help learners respond to some of the questions in activity 1, L2, L3, L6, L7, L8 and L9 were not able to identify the fourth group of triangles as scalene. This might be due to what Steele (2013) calls a lack of the basic comprehension of geometry concepts, specifically properties of a triangle which included the sides and angles as well.

In question 1.2 described in appendix 12, the three learners' responses (L2, L5 & L9) were based on what is known as mythical thinking where a logical quantifier like 'all' has been used in a wrong place (Movshovitz-Hadar et al., 1987). These learners were operating at the level lower than visualisation level (level 0) of the van Hiele geometric thinking model for the reason that visually, the learners could not identify different triangles from a set of the other two-dimensional shapes. To be precise, they were operating at the pre-recognition level as suggested by Clements and Battista (1991).

5.3.3 Engaging learners in hands-on-learning using polygon pieces as physical manipulatives assisted by mathematics dictionary to teach properties of polygons also promote high school learners' proficiency in geometry.

I present how the use of physical manipulatives assisted by mathematics dictionary played a vital role in promoting learners' proficiency in geometry. The eighth graders' diagnostic test results were in agreement with those of Alex and Mammen's (2014) research findings which revealed that the twelfth-grade learners in some of the South African schools were operating at concrete and visual levels of Van Hiele's theory in geometry. Instead of dealing with abstract mathematical concepts which are at level 3 (formal deduction) of the van Hiele geometric thinking.

Surprisingly, my research findings showed that in the post-test no learner obtained marks that were equal to or less than what was achieved in the diagnostic test. However, the findings of the current study do not support the previous research results by Fennema (1972) who claimed that physical manipulatives only benefit learners at entry level of school not those in high school.

What was surprising was that the three learners (L3, L8 & L9) who could not get any question correct in the diagnostic test after being engaged in the use of polygon pieces assisted by mathematics dictionary to learn about properties of triangles improved their results in the post-test by a very wide margin. This finding was unexpected and suggested that the way in which physical manipulatives assisted by mathematics dictionary were integrated into the teaching and learning had a vital role in influencing learners' performance in geometry. Even though these outcomes contrast from some already published studies (Fennema, 1972 & Egan, 1990),they were consistent with those of Prigge (1978); Threadgill-Sowder and Juilfs (1980); Suydam and Huggins (1997); Van Hiele (1999); NCTM (2000); Olkun (2003); Steen, Brooks and Lyon (2006); Yuan et al. (2010); Gürbüz (2010); Starcic et al. (2013) and Carbonneau et al. (2013) who reported that physical manipulatives benefit learners of all ages in geometry retention and application as long as they are well incorporated into teaching and learning.

In question 1.3as described in appendix 12, the reason why most of the learners were not able to identify triangles labelled 'b', 'h', 'p' and 'q' as scalene triangles from the given set might be that they only used their eyes to make a judgement regarding the magnitude of each of the angles and the lengths of line segments in each of the given triangles, instead of using cut out angles and line segments to confirm their decisions. These results are consistent with those of Steele (2013) which suggests that improper implementation of geometry activities in the lower grades is one of the factors that lead to the learners' lack of conceptual understanding and leaves them unable to develop proficiency in geometry. In the long run, this poses many challenges to mathematics teachers.

The reason why most of the learners managed to identify more than three out of six isosceles triangles in question 3.2.1 was that the polygon pieces used in the lesson helped to mediate learning. Such responses approved the notion argued by Thomas (1994) which states that active manipulation of physical manipulatives offers learners opportunities to develop a range of images that can be used in the mental manipulation of abstract concepts and enhance mathematical manipulation skills. Such an integration of physical manipulatives into geometry teaching and learning has shown how to bridge the gap that most learners had between conceptual understanding and learning of geometry. The reason for choosing incorrect might be inaccurate measuring of the line segments or else some did not measure at all they just applied their own ideas of over generalisation of the properties of triangles rules (Ashlock, 2002).

The two learners (L2 & L9) who drew acute-angled triangles and inserted 90^0 symbols and called them right-angled triangles had alternative conceptions in the interpretation of the word 'right-angled'. This distortion of the definition is described as an imprecise citation of a recognizable definition of a right-angled triangle (Movshovitz-Hadar et al., 1987).

The outcomes of this study specified that the use of polygon pieces in teaching and learning of geometry influenced learners' geometry proficiency. The positive influence of polygon pieces assisted by mathematics dictionary was evident in learners' post-test results; all learners obtained marks above 60%.For the detailed statistics, refer to Figure 4.1.

It was interesting to note that in all the nine semi-structured interview cases for this study, no learner talked bad about using polygon pieces when learning geometry as suggested in one of the research studies that the use of physical manipulatives is for primary school learners not high school learners. They all felt that during the intervention activities session, polygon pieces gave them something to manipulate and reflect on when learning geometry. In other words the polygon pieces were for the mediating of teaching and learning of geometry. In addition, the post-test results, which were high as compared to the diagnostic test results as shown in Figure 4.1, indicate that the use of polygon pieces assisted by mathematics dictionary addressed some of learners' alternative conceptions regarding types of triangles and their properties.

5.4 Unexpected outcomes

This study has shown a variety of outcomes and presented in this section are the unexpected outcomes. One unanticipated finding in question 4.1 of reflective test 5, L2 and L9 said that "ΔABC *is a revolution*". This term, was not even mentioned during my intervention activities, but it was given as the answer. This showed that some learner's had geometric terms lingering around in their minds which were inadequately understood in terms of what they mean, how they were supposed to be used and when were they applied to a mathematical situation (Radatz, 1980). The two learners' (L2 & L9) responses also indicated the mathematics language difficulties. For instance, when they were asked a certain concept that required them to mention the category in which triangle ABC showed in appendix 25 belonged. The two learners (L2 & L9) responded contrary to the question. Yet, in the true sense of the matter in the process of teaching learning geometry the van Hiele model expects learners to describe geometric shapes and concepts verbally using applicable standard and nonstandard language.

Contrary to anticipations, this study found a substantial variance between the planned model (Figure 3.2) in chapter 3 and the real model in chapter 4 (Figures 4.25). The planned model was developed to have four different stages, namely: diagnostic test, design of the intervention, implementation of the intervention activity and back to the design of the intervention. Learners' performance during intervention activities brought in a different setup of the model, the reflective learning model with seven stages. The model dealt with the use of polygon pieces in teaching and learning for conceptual understanding and the use of the mathematics dictionary in instruction of geometry for mathematical language proficiency. The model was learner driven. The mathematics dictionary was brought in during intervention activities when most of the learners showed some difficulties in understanding mathematics vocabulary spelling and could not make meaning out of some geometric terms.

5.5 Reference to previous research

This study also highlighted that failure to engage primary school learners in worthwhile geometrical activities significantly affect their future geometric learning experiences in the higher grades (NCTM, 2006). The diagnostic test results attested to this claim for the reason that there was no learner who managed to obtain a mark above 50% , yet the work in the diagnostic test consisted of the some contents that were from primary school mathematics syllabus. For details of how each learner achieved in the diagnostic test, refer to Figure 4.1.

In addition, the inconsistency of the results during the intervention activities as depicted in Figures 4.2 to 4.10 was due to the fact that most of the learners' geometric conceptual understandings were not well developed in the early grades. Another possible explanation for this was that those who were taught primary geometry, it was by rote learning; no physical manipulatives were used. The use of physical manipulatives, createdlearning opportunities for the learners to conceptually understand the properties of the given polygon even before the use of protractors or symbols that defined a particular figure (Koyuncu et al., 2015).

In question 3.1.1 of intervention activity 3 described in appendix 14, the learners managed to identify isosceles triangles from the given set of triangles. Two reason led to such an outcome: (i) in the previous intervention activities, learners were given opportunities to manipulate geometric figures in different orientations and (ii) learners were given opportunity to describe geometric shapes verbally using appropriate standard and nonstandard language (Crowely, 1987).

5.6 The detailed explanation of my research results

The observed improvement in learners' results in the post-test could be attributed to methodical instructional factors like: (i) the extent to which learners were guided in the use of physical manipulatives; (ii) the type of physical manipulatives used for teaching and learning geometry; (iii) the characteristics of the teaching and learning environment which entails reflective tests and reflective sessions (Carbonneau et al., 2013). The relevance of physical manipulatives was very important. The physical manipulatives used were relevant to the content that learners were engaged in. For example, in my research I used a method which I call *'use of a triangle to teach properties of the same triangle.'* This was where learning opportunities were developed and enhanced as learners worked with polygon pieces assisted by mathematics dictionary to establish the properties of an equilateral triangle without being told how such a triangle looked like.

The reason why most of the learners managed to identify more than three out of six isosceles triangles in question 3.2.1 described in appendix 14 was that the polygon pieces helped to mediate learning. The achievement made in question 3.2.1 further support Thomas' (1994) idea of active manipulation of physical manipulatives offers learners opportunities to develop a range of images that can be used in the mental manipulation of abstract concepts and enhance mathematical manipulation skills. Integrating physical manipulatives assisted by mathematics dictionary into geometry teaching and learning has shown how to bridge the gap that most learners have between conceptual understanding and learning of geometry.

The way learners responded to question 3.2.2 as described in appendix 14 was in agreement with Ogg's (2010) propositions which state that learning mathematics without any mediating factor is a difficult process to comprehend, but with physical manipulatives, geometrical ideas are broken down into concepts easy to grasp. By using polygon pieces assisted by mathematics dictionary, learners were able to identify equilateral triangles. In cases where other triangles have been chosen as equilateral, I can conclude that such learners did not do the actual measurement as required; the concept of over generalisation was applied to determine the answer.

The two learners, who responded correctly to question 3.2.4 described in appendix 14, did not take it for granted that once a triangle has a right angle, it is part of the solution. These two actually took their time to do the activity of cutting and measuring each right-angled triangle. As in the words of Luria (1976) and Bussi and Frank (2015) who affirm that conceptual understanding does not come spontaneously; it requires an instructional process that matches figural and conceptual components using specific intervention strategies and well-integrated teaching and learning resources. In this case the polygon pieces assisted by mathematics dictionary were used to ensure that what has been said by the research in this paragraph was fulfilled.

By getting 100% in question 5.2(v) L1, L4, L7 and L8 proved the claims made by Van de Walle (2004) that learners' conceptual understanding improved after using physical manipulatives since they were afforded with opportunities to create links between concepts and symbols and evaluate their conceptual understanding of the concepts being presented.

The findings of reflective test 7 were in agreement with the findings of Paparistodemous et al. (2013) which showed that if well incorporated into teaching and learning physical manipulatives provide learners with opportunities to organise and classify shapes systematically and define their relationships in both verbal and symbolic languages. For the content of reflective test 7 refer to appendix 27.

Most of the learners' outstanding performance in questions 8.1(i) to (iii) after using polygon pieces assisted by mathematics dictionary proved the idea that learners' engagement in the use of polygon pieces should considered not only viable, but also an essential condition for worthwhile learning which leads to conceptual development (Prawat, 1992). For the content details of questions 8.1(i) to (iii) refer to appendix 19

Question 9.2(i) described in appendix 20, all nine learners obtained 100%.The individual assistance which some learners needed also helped them balance geometrical concepts with terminology. This served as an alleviation of the challenges that most of the learners face in geometry lessons; that resulted in them operating at the level relevant to their grade as expected by the van Hiele levels of geometric thinking which is level 2 - abstraction.

In question 9.1(ii) the three (L3, L5 & L7) obtained 100% for the reason that they had a clear conceptual understanding of geometric symbols helped them to make informed decisions regarding the lengths of line segments.

Six learners' responses to question 9.2(ii) and (iii) described in appendix 20, differ from the claims made by Van de Walle (2004) that teachers should not communicate with learners on how to use physical manipulatives, but rather let learners do self-exploration of the mathematical concepts being represented in the physical manipulatives. The idea is broadly consistent with Wearne and Hiebert's (1988) earlier research findings which suggest that extensive instruction and practice is required before physical manipulatives are employed in mathematical teaching and learning.

In this study Wearne and Hiebert's (1988) statement implied that clear guidance was needed when learners were using physical manipulatives, otherwise besides the fact that physical manipulatives assisted by mathematics dictionary support learning of mathematics, teachers should know that they do not automatically provide mathematical meaning to the learners (Thompson, 1994). In this research according to Gentner and Ratterman (1991) the necessity for extensive instruction and practice gave learners opportunities to perceive and conceptually understand relationships between physical manipulatives and other forms of mathematical expressions.

The way learners answered question 9.4 given in appendix 20 proved that mathematical proficiency does not come spontaneously; it requires an instructional process that matches figural and conceptual components using specific intervention strategies and well integrated teaching and learning resources, in this case polygon pieces (Luria, 1976; Frank, 2015).

As shown in Table 4.11, L1's comment supports research findings that state that if the teaching and learning of geometry is done in an abstract way, the worthwhile learning cannot be acquired as expected (Skemp, 1976; Herbert & Carpenter, 1992).

As suggested by L2 in Table 4.12, Teppo (1991) echoes the same sentiments that when physical manipulatives are well incorporated into teaching and learning learners get a deeper geometric proficiency as they investigate properties of shapes and relationships among these properties in order to derive conjectures and test hypothesis.

L3's ideas in Table 4.13 resonate with what Bhagat and Chang (2015) propose that teaching and learning should allow learners to explore different geometrical figures and their properties in different orientations if it has to be effective in helping learners with geometric proficiency.

According to Blanco's (2001) proposition that teaching and learning of geometry should not only be on giving the meaning and obviating analysis of the properties of shapes with no emphasis on the visualisation of the shapes. L4's interviews responses shown in Table 4.11 alluded to the same notion. Therefore, this study concludes that the use of polygon pieces in the teaching of geometry enhances teaching and learning.

In Table 4.19, L5's suggestions are in agreement with the research that says by cutting out the angles and sides of the figure creates learning opportunities for learners to comprehend the properties of the given figure before even the use of protractors or even the use of symbols that define a particular figure (Koyuncu et al., 2015).

L6's ideas approved research findings which state that in order for the learners to conceptually understand geometry there is a need to be engaged in the manipulation of a variety of educational resources, such as mosaics, geo-plates, tangrams, designs and figures (Cotic, Felda, Mesinovic, & Simcic, 2011).

L8's suggestion regarding the use of physical manipulatives was supported by the research findings which state that hands-on-learning promotes geometric proficiency (Peterson et al., 1998; Ogg, 2010).

Research has shown that for the same reasons given by L9 in Table 4.22, the education departments globally are now promoting the use of physical manipulatives in order for the learners to acquire mathematical skills for conceptual understanding (Moyer, 2001; Clements & Bright, 2003).

The reason why the learners listed in Table 4.22 were not able to respond to questions which were considered to be at level 0-visualisation of the van Hiele levels of geometric thinking was probably due to lack of well-developed visualisation skills which would help them make a judgement regarding the given mathematical situation. Research has shown that visualisation skills are always not developed and enhanced if learners are denied opportunities to manipulate, create, describe and manage given shapes verbally using standard and nonstandard mathematical language (Crowely, 1987).

5.7 Advice to the researchers and educators in interpretation of my research findings

These outcomes need to be deduced with attentiveness. Conversely, with a relatively small sample size, caution must be applied. The results might not be convenient for an overcrowded classroom. During the period of mediating of teaching and learning constant monitoring and individual assistance was required to ensure that all the processes of how to use physical manipulatives assisted by mathematics dictionarywere adhered to.

The results produced by the nine learners in the post test made me to propose the replication of this study's intervention to a larger class as long as the processes followed in this study are carefully done.

These research findings may help us to understand conceptually how to incorporate polygon pieces into the teaching and learning of geometry so that learners can be assisted in establishing and enhancing the conceptual understanding of geometry. The amalgamation of the findings affords some support for the theoretical principle that polygon pieces assisted by mathematics dictionary have an influence on teaching and learning of geometry to the learners and can be used as physical manipulatives to promote learners' comprehension of geometry concepts. In addition, teacher should incorporate polygon pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning in a way that was done during my research to promote learners' mathematical proficiency in geometry.

5.8 Suggestions from Chiphanbo's reflective model for teaching and learning geometry

These findings suggest that the use of polygon pieces assisted by mathematics dictionary in teaching and learning geometry has an influence in the teaching and learning of geometry, specifically the properties of triangles. In general, therefore, the use of polygon pieces assisted by mathematics dictionary in teaching and learning geometry needs a proper way of incorporating polygon pieces into the lesson in order for them to be of influence to the teaching and learning of geometry.

Another important finding was that in order for the polygon pieces assisted by mathematics dictionary to be of influence in the teaching and learning of geometry, must be incorporated into the lesson tactfully. If they are not well incorporated into the teaching and learning of geometry might not serve the purpose at all, they end up being white elephants in the classroom. The following are some tips to ensure that polygons pieces serve their purpose (i) make sure that each and every learners has the necessary material to cut out the angles and line segments; (ii) consistent monitoring of learners is necessary to ensure that each learner uses the resources as instructed – I have discovered that some learners tend to ignore instructions.

5.9 Presentation of implications of the research

These research findings have important implications for developing the activities for teaching and learning of geometry for mathematical proficiency. The issues that emerged from these findings regard learners' and teachers' roles when teaching and learning geometry, as well as how mathematics dictionaries and polygon pieces can be incorporated into teaching and learning for mathematics proficiency. These outcomes provide further provision for the premise that states that polygon pieces have great influence on learners' learning of geometry. Mathematics dictionary helped the learners to learn mathematics terminologies and spellings of some mathematics terms like isosceles, equilateral, scalene, etc.

5.10 Commenting on findings

The current outcomes are substantial in at least three main aspects, namely:

(i) The incorporation of the mathematics dictionary into teaching and learning enhances learners' English and mathematics language proficiency for example Table 5.2 below illustrates how learners' comprehension of English and mathematics languages improved after the intervention activities.

Table 5.2: How some learners improved their mathematical terminologies and spellings in the post-test

Learner code	**Question number**	**Question content refer to**	**Diagnostic test learners' responses**	**Post-test Learners' responses**
L9	1.1(iii)	Appendix 10	It is equal	Right-angled scalene △
L1	1.1(iii)	Appendix 10	……it is a triangle	Right-angled scalene triangle
L3	1.2(iii)	Appendix 10	The name of a triangle is a spentil triangle	It is called scalene triangle
L7	1.2(iii)	Appendix 10	It is GHI are the properties of the triangle	It is a scalene triangle
L9	1.2(iii)	Appendix 10	It is GHI	Scalene △
L3	1.3(iii)	Appendix 10	A given a triangle longer than equal	It is an isosceles triangle
L9	1.3(iii)	Appendix 10	It is length	Isosceles △
L3	1.4(i)	Appendix 10	The types of a triangle is a length and breadth and sides	It is an equilateral triangle
L3	1.4(ii)	Appendix 10	X is when you analysis to improve you're a x. Y is a shorter than when you are wait Z. A is A partience	$\hat{X}$ is equal to $\hat{Y}$ $\hat{Y}$ is equal to $\hat{Z}$ $\hat{Z}$ is equal to $\hat{X}$
L8	1.4(ii)	Appendix 10	$\hat{X}$ and $\hat{Y}$ is equal than $\hat{Z}$ and $\hat{Y}$ and $\hat{Z}$ and $\hat{Y}$ is equal than $\hat{Z}$ and $\hat{X}$	$\hat{X}$ is equal to $\hat{Y}$ $\hat{Y}$ is equal to $\hat{Z}$ $\hat{Z}$ is equal to $\hat{Y}$
L6	1.4(ii)	Appendix 10	X is longer than Y and Y is equal to Z	$\hat{X}$ is equal to $\hat{Y}$ $\hat{Y}$ is equal to $\hat{Z}$ $\hat{Z}$ is equal to $\hat{X}$
L6	1.4(iii)	Appendix 10	xy is shorter than YZ and YZ is equal to XZ	XY is equal to YZ YZ is equal to XZ XZ is equal to XY
L3	1.5(i)	Appendix 10	A PQR sides that you can all in this triangle	It is an isosceles triangle

Table 5.2 shows how the use of the mathematics dictionary influenced learners' English and mathematics language proficiency. The influence is reflected in the column that has post-test responses where most of the learners managed to respond correctly to the questions correctly.

Remarkable improvement was also demonstrated by two learners (L4 & L7) who improved in spelling.

(ii) The use of polygon pieces has a greater influence in learners' geometric proficiency, specifically properties of triangles. Table 5.3 below shows how learners' comprehension of geometric concepts improved after the use of polygon pieces during their learning.

Table 5.3: Polygon pieces developed learners' comprehension of geometric concepts

Learner code	**Question number**	**Question content refer to:**	**Diagnostic test learners' responses**	**Post-test Learners' responses**
L1	1.1(ii)	Appendix 10	AB-parallel; AC- horizontal BC-horizontal	The lines are not equal
L1	14(i)	Appendix 10	Equal triangle	Equilateral triangle
L2	1.3(iii)	Appendix 10	The name of a triangle is DEF	Isosceles triangle
L2	1.4(ii)	Appendix 10	$\hat{X}$ is 60^0 Y is 110^0 and Z is 120^0	$\hat{X}$ is equal to $\hat{Y}$ and $\hat{Z}$
L2	1.4(iii)	Appendix 10	XY is bigger than YZ and XZ is bigger than XY	XY is equal to YZ and XZ
L3	1.1(ii)	Appendix 10	It is small side and A bigger side AC small bigger from A to another A deduce about size	$\hat{A}$ is smaller than $\hat{C}$
L4	1.1(iii)	Appendix 10	A and B are equal and C is less than A and B	It is a scalene triangle
L4	1.2(iii)	Appendix 10	Triangular prysom	It is a scalene triangle
L4	1.5(i)	Appendix 10	It is an equilateral triangle because all sides are equal	Isosceles triangle
L5	1.1(i)	Appendix 10	$\hat{A}$ is longer than $\hat{C}$ A is bigger than C	$\hat{A}$ is smaller than $\hat{C}$
L5	1.2(iii)	Appendix 10	2.Dimentional shapes	Scalene triangle
L5	1.3(iii)	Appendix 10	2.Dimentional shapes	It is a scalene triangle
L5	1.4(i)	Appendix 10	2. D shape	Equilateral triangle
L5	1.5(i)	Appendix 10	2. D shape	Isosceles triangle
L5	1.5(ii)	Appendix 10	They were not equal and they are used make a shape	They are equal in size
L6	1.1(iii)	Appendix 10	AB are associated and AC are the convection and BC are the associated	It is the scalene triangle
L6	1.4(i)	Appendix 10	It is an triangular	It is the equilateral triangle
L7	14(i)	Appendix 10	It is a equal triangle	Equilateral triangle
L8	1.4(i)	Appendix 10	Triangular	Equilateral triangle
L9	1.3(iii)	Appendix 10	D is 4cm and F is 3cm	Two angles are equal
L9	1.4(i)	Appendix 10	Rectangle	Equilateral $\triangle$

Table 5.3 shows how individual learners improved in their comprehension of geometric concepts after being engaged in the intervention activities that made use the polygon pieces. This is shown in their responses in the post-test. Table 5.3 above also highlights how each learner's alternative conception regarding geometry concepts; this is reflected in their responses in the diagnostic test.

(iii) How teachers incorporate polygon pieces into teaching and learning of geometry has greater influence on high school learners' learning of geometry.

5.11 Limitations of my research study

The model suggested in this study may had had a better influence if it was done during school hours, however, this study managed to achieve this after school day hours – a time when learners were a little bit tired.

School day learning might had a negative impact to the learner in terms of mental fatigue. In classes learners were learning about exponents, a topic that demands the critical application of the mind. For this reason it was possible that some learners attended the research session tired mentally, that possibly hindered active participation in the research session.

5.12 Recommendation for future research work

The uses of a mathematics dictionary and polygon pieces is a way of influencing learners' mathematical vocabulary proficiency when learning about geometry. Further research would be able to explore how properties of other shapes can be introduced and taught to learners using mathematics dictionary and polygon pieces. This study further suggests that in order for the use of polygon pieces assisted by mathematics dictionary to be effectively implemented in the teaching and learning geometry to a large class, mathematics teachers must ensure that the resources are adequate. No learners should be left as spectators; hands on learning must be the order in each and every class.

There is enough room for further progress in the use of other physical manipulative assisted by mathematics dictionary in determining the properties of polygons. Research could also focus on the properties of two-dimensional concepts that are formed after the cutting out of three angles of a triangle'' angles worth researching.

Finally, there is also a need for the research to critically focus on how best teachers can select and integrate polygon pieces into teaching and learning of three-dimensional objects.

5.12 Conclusion

In conclusion, the research was successful as it was able to investigate a model that integrates dictionary and polygon pieces in teaching and learning of geometry to eighth grade learners. The investigation focussed on how polygon pieces can be used as physical manipulatives assisted by mathematics dictionary to promote learners' comprehension of geometry concepts (Kilpatrick, et al., 2001). It also investigated how mathematics teachers should use polygon pieces as physical manipulatives assisted by mathematics dictionary in teaching and learning to promote learners' mathematical proficiency in geometry.

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Appendix 1: Letter of consent to the department of education

S. M. Chiphambo
36 Prince Alfred Street
Queenstown
5320.

28th October 2015

To:
The District Director
Queenstown Department of Education
Private Bag X7053
Queenstown
5320

Dear Sir/Madam

RE: REQUESTING FOR THE CONSENT TO CONDUCT PhD RESEARCH AT NKWANCA SENIOR SECONDARY SCHOOL FROM MARCH TO APRIL 2016

I would like to request for the consent to do my PhD (Mathematics Education) research project at the above mentioned school from March 2016 to April 2016. My research topic is "A case study: investigating the influence of the use of polygon pieces as physical manipulatives in teaching and learning of geometry in Grade 8". This will involve next year's Grade 8 learners (80). The sampling will be voluntary, if the number exceeds 80, then purposeful sampling will be done.

This project is aimed at investigating ways of improving mathematics teaching, specifically in "geometry". I want to do it at this school (where I am teaching) because I want the school and learners to benefit from this project.

In addition, I will observe the following ethical issues: the school's name will be anonymous and codes will be used instead of learners' names during my data analysis, in order to prevent social stigmatisation and/or secondary victimisation of respondents.

If the consent is given for this project to take place, the participants (the learners) will be engaged as follows:

Duration: 1.5 hours

Days : Mondays to Thursdays for a period of 3 to 4 consecutive weeks.

It will be done in the afternoon to avoid interruption of the normal teaching periods since the participants will be involved in diagnostics tasks, intervention programme, post- intervention programme and interviews.

Lastly but not the least I will be glad if my request reaches your favourable consideration and promptly attended to, so that I plan ahead before the next session begins.

Yours faithfully

SHAKESPEAR M. CHIPHAMBO (Student number 55717012).

Appendix 2: Response from the department of education

Provinceofthe
EASTERNCAPE
EDUCATION

IDS&GSECfiON:QUEENSTOWN
HOMESTEADSITE,2LIMPOPODRNELAURIEDASHWOODQUEENSTOWN5320.Private BagX7053QUEENSTOWN,5320
REPUBLICOFSOUTHAFRICA,Website:www.ecdoe.gov.za

Ref: Chiphambo SM
Enquiries: Tel.:0458085712 CELL: 0842510032

JONKER W.O. Fax:0458588906

TO	:MR S.M. CHIPHAMBO Cc Principal : Nkwanca SSS
FROM	:DISTRICT DIRECTOR
SUBJECT	: REQUEST TO CONDUCT RESEARCH
DATE	: 2 November 2015

This serves to approveyourrequesttoconductresearchforyourPHDatNkwancaSeniorSecondary school.Youaretoarrangewiththeprincipaloftheschoolfordetailarrangements,withtheprovisothat normalteachingandlearningwillnotbeaffectedattheschool.

Your'ssincerely

....
H.N. GODLO
DISTRICT DIRECTOR

DEPARTMENT OF EDUCATION
P/BAG 7053
QUEENSTOWN 5320
2/11/15
DATE
2 NOV 2015
CES MANAGEMENT & GOVERNANCE
QUEENSTOWN DISTRICT

Appendix 3: Letter of consent to the research site

To:

  (1) The Principal, P. O. Box 468, Queenstown, 5320.
  (2) The School Governing Board, P. O. Box 468, Queenstown, 5320.

From: S. M. Chiphambo, 36 Prince Alfred street, Queenstown, 5320.

Dear Sir/Madam

RE: REQUESTING FOR THE CONSENT TO CONDUCT PhD RESEARCH AT YOUR SENIOR SECONDARY SCHOOL FROM MARCH TO APRIL 2016

I would like to request for the consent to do my PhD (Mathematics Education) research project at the above mentioned school from March 2016 to April 2016. My research topic is "A case study: investigating the influence of the use of polygon pieces as physical manipulatives in teaching and learning of geometry in Grade 8". This will involve next year's Grade 8 learners (80). The sampling will be voluntarily, if the number exceeds, then purposeful sampling will be done.

This project is aimed at investigating ways of improving mathematics teaching, specifically in "geometry". I want to do it at this school (where I am teaching) because I want the school and learners to benefit from this project.

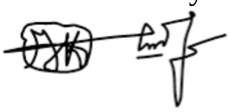
In addition, I will observe the following ethical issues: the school's name will be anonymous and codes will be used instead of learners' names during my data analysis, in order to prevent social stigmatization and/or secondary victimization of respondents.

If the consent is given for this project to take place, the participants (the learners) will be engaged as follows:
Duration: 1.5 hours
Days : Mondays to Thursdays for a period of 3 to 4 consecutive weeks.

It will be done in the afternoon to avoid interruption of the normal teaching periods since the participants will be involved in diagnostics tasks, intervention programme, post- intervention programme and interviews.

Lastly but not the least I will be glad if my request reaches your favourable consideration and promptly attended to, so that I plan ahead before the next session begins.

Yours faithfully



SHAKESPEAR M. CHIPHAMBO (Student number 55717012).

Appendix 4: Response from the research site

TO : MR SM CHIPHAMBO

FROM : N. LUTSEKE

SUBJECT : A LETTER OF NO OBJECTION TO DO RESEARCH

DATE : 15/11/2015

This is to serves to confirm that our High School has no the request that Mr S.M Chiphambo conduct research at our School

However the following conditions will apply:

1. Mr Chiphambo must get research consent also from the learners and parents of the participants

2.Your research must not interfere with teaching and teaching and learning in our institution

Appendix 5: A sample of a letter of consent to the parents/ guardians

DATE........................

From:

S.M. Chiphambo

To : Parents

Dear Parents

I am going to conduct the research project in mathematics education. I hereby request for the consent to engage your child.. as one of the participants, s/he has already volunteered to do so. The project will be conducted as follows:

Days : Mondays to Thursdays

Time : 1 hour 30 minutes after school

Duration : 3 to 4 weeks

The project is aimed to investigate the influence of the use of polygon pieces as physical manipulatives in teaching and learning of geometry in Grade 8.This project will benefit both the researcher as a teacher and the learner since mathematics is one of the crucial subjects.

For the confirmation of allowing your child to be engaged in this research projects would you please complete the attached consent form and return it to me.

Thank for your cooperation in this regard.

Yours faithfully

SHAKESPEAR M. CHIPHAMBO (Student number 55717012)

(Grade 8-12 Mathematics teacher)

Appendix 6: Consent form to the parents/ guardians

CONSENT FORM

I, ..., accept that my child,...(name of the child) be one of the participants in the research project conducted by Mr S.M. Chiphambo at..school as follows

Duration : 1 hour 30minutes

Days : Mondays to Thursdays

Time : After school for 3 to 4 weeks

The participant is free to quit the project at any given time.

By signing this form, I solemnly accept the conditions of the project and I also declare that the information given above is true

Parent(s) signature :.......................................

Contact number :...

For details of the research contact:

S.M. Chiphambo: 0760279032

Email: schiphambo@yahoo.com

Appendix 7: A sample of consent for learners

<u>CONSENT FORM</u>

I, ..., accept voluntarily to be one of the participants in the research project conducted by Mr S.M. Chiphambo (from KwaKomani Comprehensive School) at Nkwanca Senior Secondary School as follows

Duration : 1 hour 30minutes

Days : Mondays to Thursdays

Time : After school for 3 to 4 weeks

I am fully aware that I am free to quit the project at any given time and no one will penalize for withdrawing from the research.

By signing this form, I solemnly accept the conditions of the project and I also declare that the information given above is true

Learner's signature :..

Contact number :..

For details of the research contact:

SHAKESPEAR M. Chiphambo: 0760279032

Email: schiphambo@yahoo.com

Appendix 8: UNISA ethical clearance certificate

UNISA university of south africa

ISTE-SUBRESEARCHETHICSREVIEWCOMMITTEE

Date: 19/11/2015

Ref#:2015_CGS/ISTE_020
Nameofapplicant (student/researcher):**Mr. ShakespearMEK Chiphambo**
Student#:55717012
Staff#:

Dear**Mr.ShakespearMEKChiphambo**
Decision:EthicsApproval

Name: Shakespear M.E.K Chiphambo schiphambo@yahoo.com

Proposal: A case study: investigating the influence of the use of polygon pieces in teaching and learning of geometry to Grade 8learners.

Qualification: Postgraduate degree (PhD)research

Thank you for the application for research ethics clearance by the *ISTE SUB* Research Ethics Review Committee for the above mentioned research. Final approval is granted for the duration of the study

The application documents were reviewed incompliance with the Unisa Policy on Research Ethics by the Committee/Chairperson of ISTE SUB RERC on 19November, 2015. The decision will be tabled at the next RERC meeting for ratification.

The proposed research may now commence with the proviso that:

1) *The researcher will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics, which can be found at the following website: http://www.unisa.ac.za/cmsys/staff/contents/departments/res_policiesjdocs/Policy_ Research%20Ethics_rev%20app%20Counci/_22.06.2012.pdf. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study, as well as changes in the methodology, should be communicated in writing to the ISTE Sub Ethics Review Committee. An amended application could be requested if there are substantial changes from the existing proposal, especially if those*

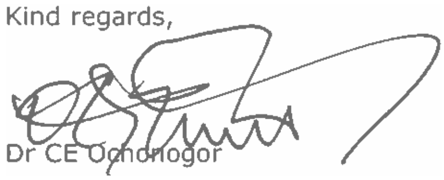
University of South Africa
Preller Street. Muckleneuk Ridge City of Tswane

P O Box 392 UNISA 0003 South Africa Telephone+27124293111Facsmile 127124294150www.unisa.ac.za

Changes affect any of the study-related risks for the research participants.

2)The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study.

Note:
The reference number [top right comer of this communique} should be clearly indicated on all forms of communication [e.g. Webmail, E-mail messages, letters] with the intended Research participants,as well as with the JST Sub RERC.



Signature

Title & Name of the chairperson

Title & Name of the Executive dean

Institute for Science and Technology Education (ISTE)
College of Graduate Studies
RobertSobukweBuilding,Office: 4th
418 Nana Sita Street (Old Skinner Street), Pretoria
Tel: 0123376189 Fax: 0865968489
Email: ochonec@unisa.ac.za
Floor, Room 4

Appendix 9: Piloted diagnostic test

Grade 8 Geometry March 2016

Instructions:

(i) Answer all the questions

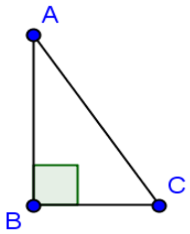
(ii) Write neatly

(iii)Provide your answers on the spaces provided under each question.

Question 1

Study the 2 Dimensional figures below and then answer the questions that follow:

1.1.



(i) What can you deduce about the sizes of $\hat{A}$, and $\hat{C}$?

………………………………………………………………………………………

(ii) What are the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$?

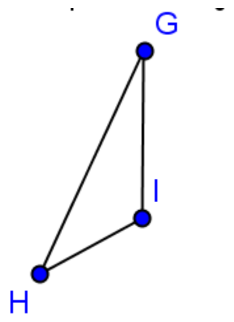
....

....

(iii)According to answers in 1.1. (ii) and (iii), what specific name is given to a shape with the properties mentioned above?

……………………………………………………………………………………..

1.2.



(i) What are the properties of the triangle GHI in terms of $\hat{G}$, $\hat{H}$ and $\hat{I}$?

……………………………………………………………………………………

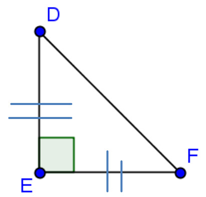
(ii) Determine the properties of triangle GHI in terms of $\overline{GH}$, $\overline{HI}$ and $\overline{GI}$.

……………………………………………………………………………………

(iii)What name is given to a triangle with such properties?

……………………………………………………………………………………

1.3.



(i) Write down the size of each of the following angles $\hat{D}$ and $\hat{F}$.

……………………………………………………………………………………

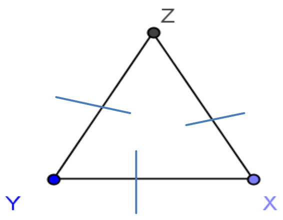
(ii) Determine the length of $\overline{EF}$, $\overline{DE}$ and $\overline{DF}$ use terms: shorter, longer than, equal, the longest of all.·

……………………………………………………………………………………

(iii)What name is given to triangle DEF?

…………………………………………………………………………………..

1.4.



(i) What type of a triangle drawn above?

…………………………………………………………………………………………

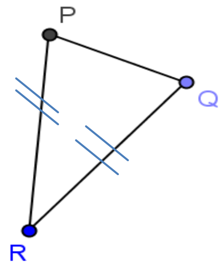
(ii) Determine the size of: $\hat{X}$, $\hat{Y}$ and $\hat{Z}$.

…………………………………………………………………………………………

(iii)Write down the length of: $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ use terms: shorter, longer than, equal, the longest of all.·

…………………………………………………………………………………………..

1.5.



(i) What name is given to triangle PQR?

…………………………………………………………………………….................

(ii)What is the relationship between $\hat{Q}$ and $\hat{P}$?

…………………………………………………………………………………………

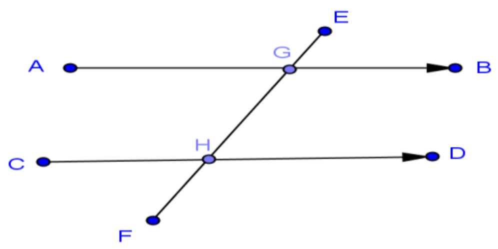
(iii) What can you conclude about the properties of triangles PQR?

…………………………………………………………………………………………

……………………………………………………………………………………

Question 2

2.1. In the diagram, AGB is parallel to CHD and both line are cut by EGHF (a transversal)



(i) Which three other angles are equal to $E\hat{G}B$? Give a reason for each statement?

…………………………………………………………………………………

…………………………………………………………………………………

(ii) Identify and list down all the pairs of supplementary adjacent angles in the diagram drawn above.

…………………………………………………………………………………

…………………………………………………………………………………

(iii)Name pairs of corresponding angles in the diagram drawn above.

…………………………………………………………………………………

…………………………………………………………………………………

(iv)Which angles add up to 360^0 in the diagram above?

…………………………………………………………………………………

(v) Identify and list all the pairs of alternate angles in the diagram above.

…………………………………………………………………………………

…………………………………………………………………………………

(vi)Name the pairs of vertically opposite angles in the diagram above.

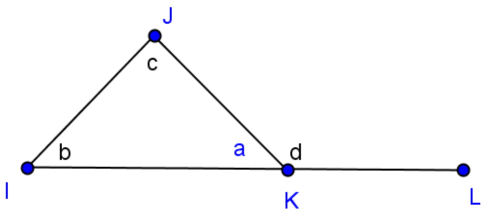
…………………………………………………………………………………...

(vii) Which pairs of angles are known as co-interior angles in the diagram above? List them all.

…………………………………………………………………………………

Question 3

In the diagram JIKL, $\hat{d} = 140^0$, $\hat{c} = 70^0$



3.1. Determine the size of each of the following angles: $\hat{b}$ and $\hat{a}$.

…………………………………………………………………………………………..

3.2. Is there any relationship between $\hat{b}$, $\hat{a}$ and $\hat{c}$?

……………………………………………………………………………………….

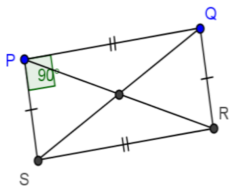
3.3. If your answer in 3.2 is 'Yes', show in two ways how these angles are related to each other.

…………………………………………………………………………………………

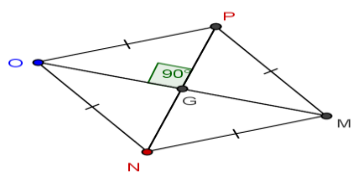
Question 4

Study the figures below (a - e) and then complete the table by naming them and putting a tick if the quadrilateral has the stated properties.

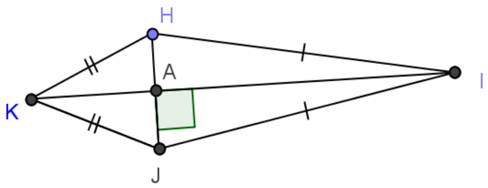
a.



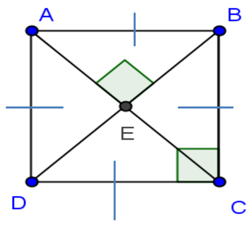
b.



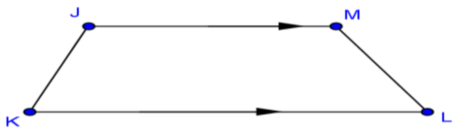
c.



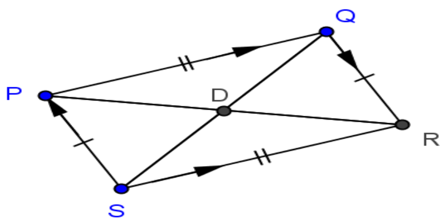
d.



e.



f.



Shape number	Shape name	Opposite sides parallel	Opposite sides equal	Adjacent sides equal	Opposite angles equal	Adjacent sides equal	Diagonals equal	Diagonals bisect each other	Diagonal intersect	Diagonals bisect angles
a	Rectangle									
b										
c										
d										
e										
f										

Appendix 10: Diagnostic test

Grade 8 Geometry diagnostic   March 2016

Instructions:

(i) Answer all the questions

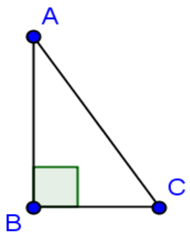
(ii) Write neatly

(iii)Provide your answers on the spaces provided under each question.

Question 1

Study the 2 Dimensional figures below and then answer the questions that follow:

1.1.



(iv)What can you deduce about the sizes of $\hat{A}$, and $\hat{C}$?

…………………………………………………………………………………

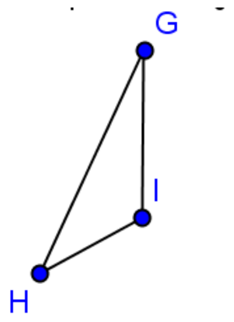
(v) What are the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$?

....

(vi)According to answers in 1.1. (ii) and (iii), what specific name is given to a shape with the properties mentioned above?

………………………………………………………………………………..

1.2.



(iv) What are the properties of the triangle GHI in terms of $\hat{G}$, $\hat{H}$ and $\hat{I}$?

……………………………………………………………………………………

(v) Determine the properties of triangle GHI in terms of $\overline{GH}$, $\overline{HI}$ and $\overline{GI}$.

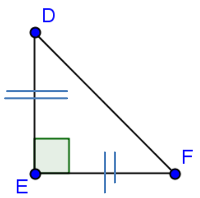
……………………………………………………………………………………

……………………………………………………………………………………

(vi) What name is given to a triangle with such properties?

……………………………………………………………………………………

1.3.



(iv) Write down the size of each of the following angles $\hat{D}$ and $\hat{F}$.

……………………………………………………………………………………

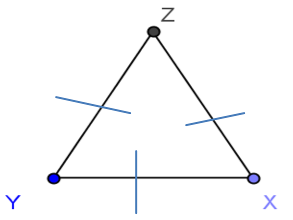
(v) Determine the length of $\overline{EF}$, $\overline{DE}$ and $\overline{DF}$ use terms: shorter, longer than, equal, the longest of all.

....

(vi) What name is given to triangle DEF?

…………………………………………………………………………………..

1.4.



(iv) What type of a triangle drawn above?

………………………………………………………………………………………

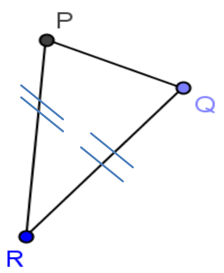
(v) Determine the size of: $\hat{X}$, $\hat{Y}$ and $\hat{Z}$.

………………………………………………………………………………………

(vi) Write down the length of: $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ use terms: shorter, longer than, equal, the longest of all.·

………………………………………………………………………………………

1.5.



(iv) What name is given to triangle PQR?

………………………………………………………………………………………

(v) What is the relationship between $\hat{Q}$ and $\hat{P}$?

………………………………………………………………………………………

(vi) What can you conclude about the properties of triangles PQR?

………………………………………………………………………………………
………………………………………………………………………………………

Appendix 11: Post-test

GRADE 8 Geometry March 2016

Instructions:

(i) Answer all the questions

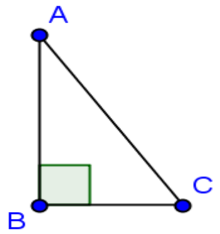
(ii) Write neatly

(iii)Provide your answers on the spaces provided under each question.

Question 1

Study the 2 Dimensional figures below and then answer the questions that follow:

1.1.



(vii) What can you deduce about the sizes of $\hat{A}$, and $\hat{C}$?

…………………………………………………………………………………………

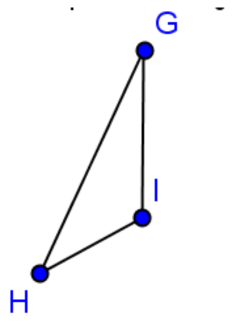
What are the properties of triangle ABC in terms of: $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$?

……………………………………………………………………………………………………

(viii) According to answers in 1.1. (ii) and (iii), what specific name is given to a shape with the properties mentioned above?

………………………………………………………………………………………..

1.2.



(vii) What are the properties of the triangle GHI in terms of $\hat{G}$, $\hat{H}$ and $\hat{I}$?

………………………………………………………………………………………

Determine the properties of triangle GHI in terms of $\overline{GH}$, $\overline{HI}$ and $\overline{GI}$.

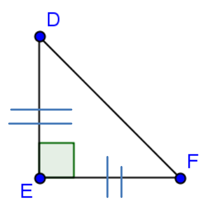
………………………………………………………………………………………

What name is given to a triangle with such properties?

………………………………………………………………………………………

………………………………………………………………………………………

1.3.



(vii) Write down the size of each of the following angles $\hat{D}$ and $\hat{F}$.

………………………………………………………………………………………

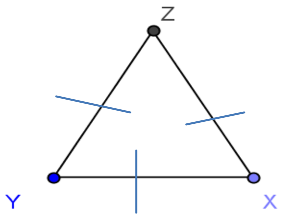
(viii) Determine the length of $\overline{EF}$, $\overline{DE}$ and $\overline{DF}$ use terms: shorter, longer than, equal, the longest of all.·

....

(ix)What name is given to triangle DEF?

……………………………………………………………………………………..

1.4.



(vii) What type of a triangle drawn above?

……………………………………………………………………………………

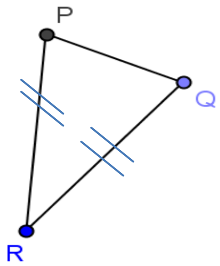
(viii) Determine the size of: $\hat{X}$, $\hat{Y}$ and $\hat{Z}$.

……………………………………………………………………………………

(ix)Write down the length of: $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ use terms: shorter, longer than, equal, the longest of all.·

……………………………………………………………………………………

1.5.



(vii) What name is given to triangle PQR?

……………………………………………………………………………….................

(viii) What is the relationship between $\hat{Q}$ and $\hat{P}$?

……………………………………………………………………………………

(ix) What can you conclude about the properties of triangles PQR?

……………………………………………………………………………………………

Appendix 12: Intervention activity 1

1.1. Drawn below are different 2-dimensional shapes, study them carefully and identify the triangles (show your answer by writing down the letter that represents the particular shape identified)

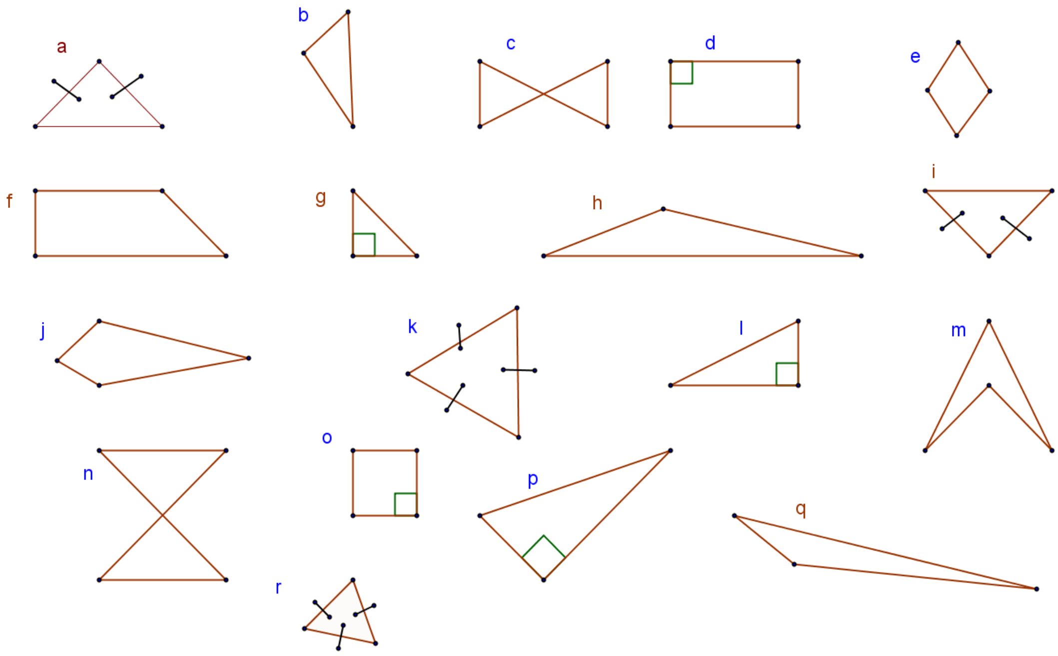


Figure 1.1: Two dimensional polygons

1.2. Explain why you are saying the identified shapes are triangles.

1.3. Now sort the triangles into groups according to their properties.

…………………………………………………………………………………………………

1.4. What property/properties have you used to group the identified triangles?

……………………………………………………………………………………………

1.5 Is there any other property that can be used to group these triangles? If yes, please explain.

……………………………………………………………………………………………

1.6 What specific name is given to each of the identified groups of triangles?

....

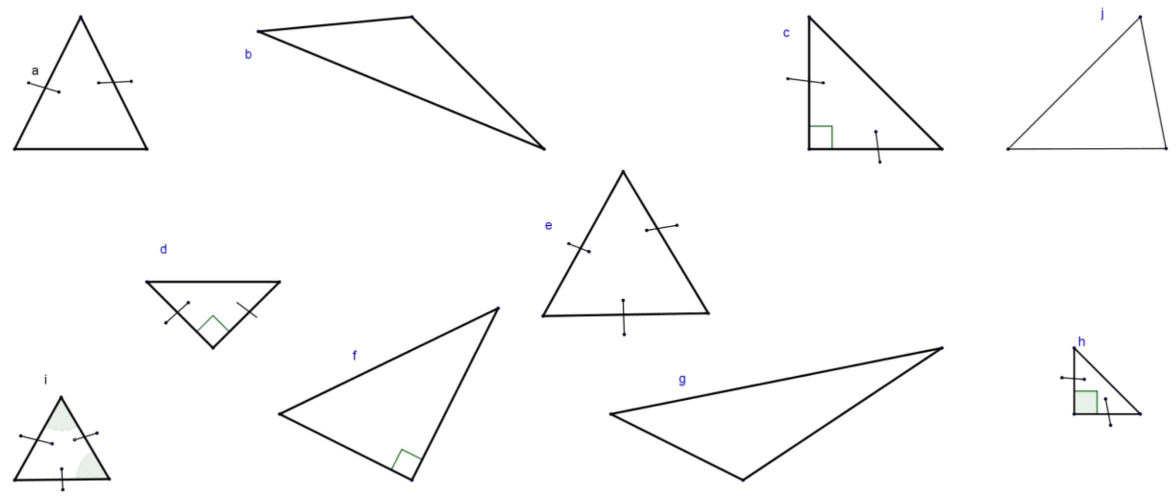
1.7 Draw different triangles according to their class based on

1.7.1 the size of angles

1.7.2. length of sides, and then name them accordingly.

Appendix 13: Intervention activity 2

2.1 Study the triangles below carefully and then answer the questions that follow.



2.1.1 Measure the angles in each and every shape above and then write down your findings for each triangle in the table below. Write the letter that represents that particular triangle under each of the headings

All angles equal in size	Two angles equal in size	All angles less than 90^0	One angle greater than 90^0	One angle equal to 90^0

2.1.2 Which of the triangle(s) is/are:

2.1.2.1 an isosceles ?...

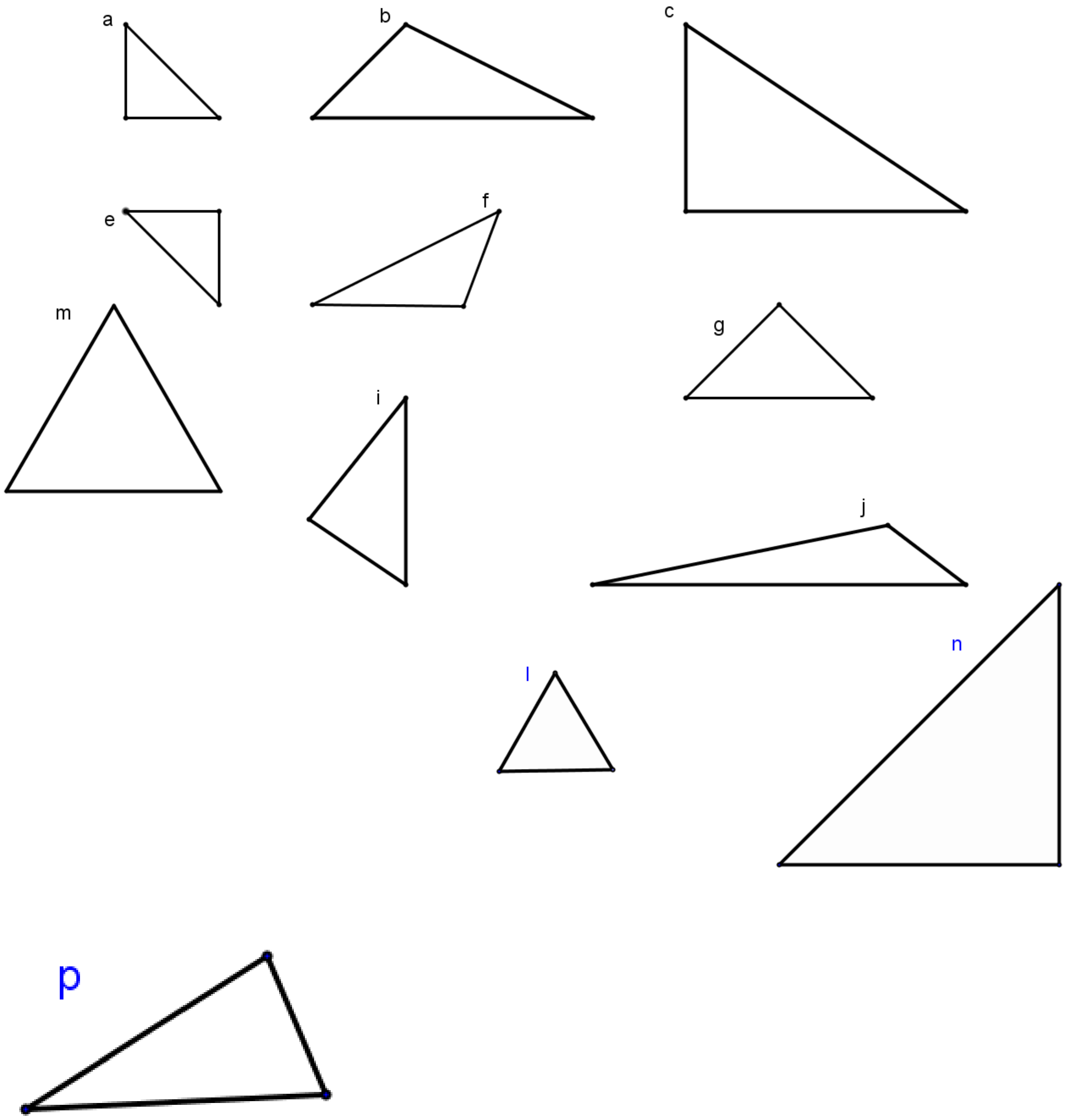
2.1.2.2 an equilateral? ……………………………………………………

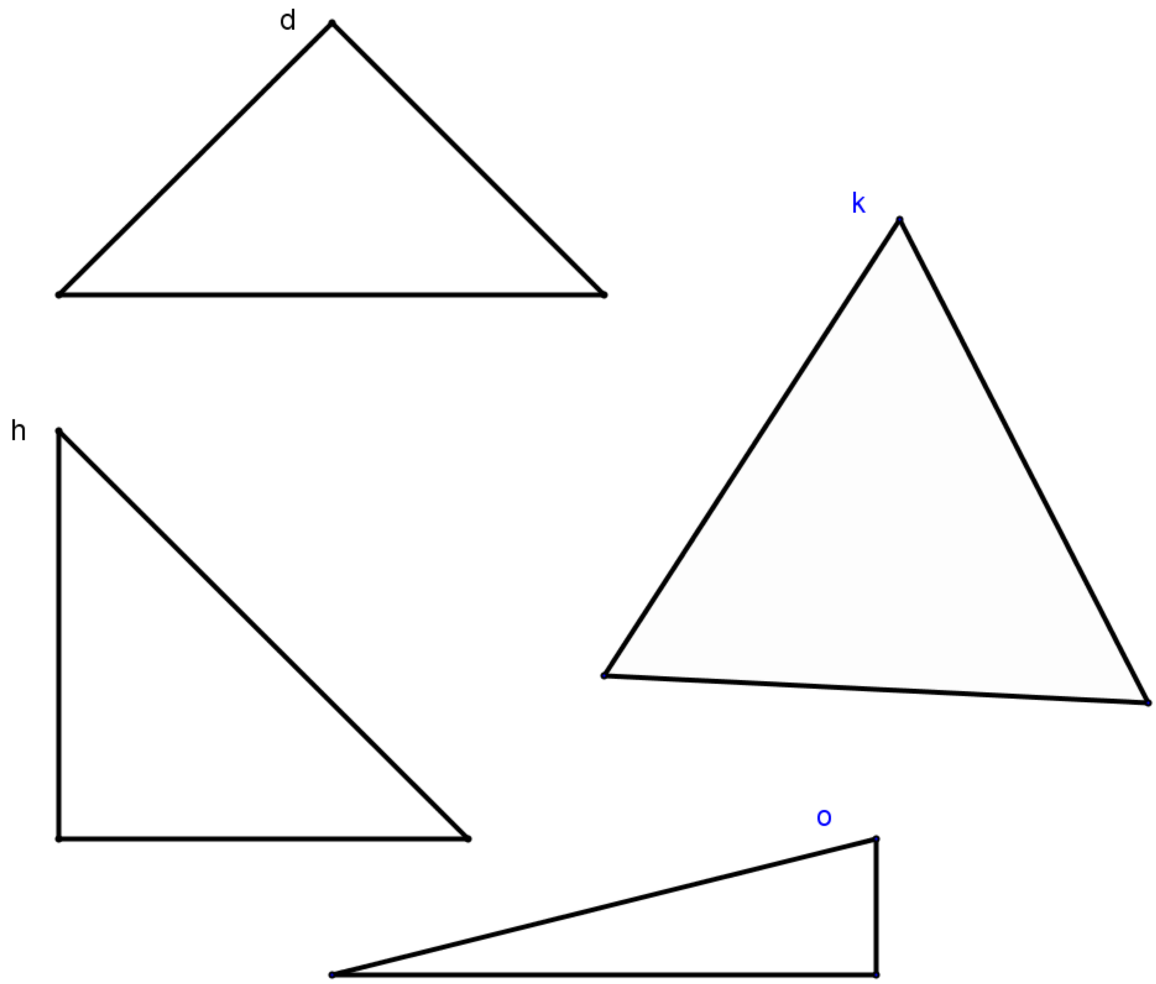
2.1.2.3 an acute angled? …………………………………………………

2.1.2.4 an obtuse? ……………………………………………………………

2.1.2.5 a right angled? ………………………………………………………….

Appendix 14: Intervention activity 3





3.1 Identify triangles with the names written below from the shapes drawn above by estimation of the length of sides and size of each of the angles and write down which ones are:

3.1.1 Isosceles triangles? …………………………………………………………………

3.1.2 Equilateral triangles? ………………………………………………………….. …….

3.1.3 Obtuse triangles? …………………………………………………………………

3.1.4 Right angled isosceles triangles? …………………………………………………

3.1.5 Right angled scalene triangles? …………………………………………………..

3.2. Now measure the sides and angles of each of the triangles and write down each triangle in the correct category below. (Write down only the letter that represents that particular triangle)

3.2.1. Isosceles triangles: ……………………………………………………

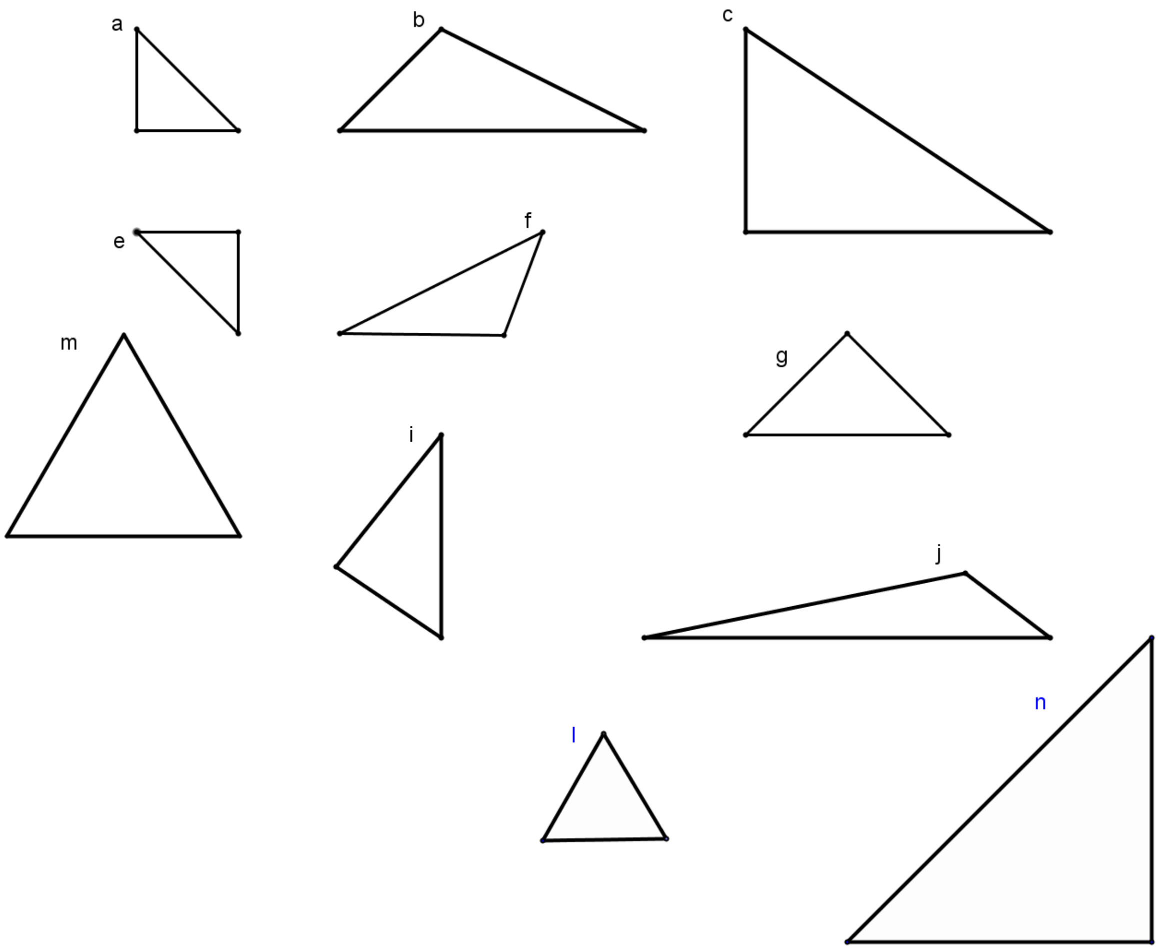
3.2.2 Equilateral triangles: …………………………………………………

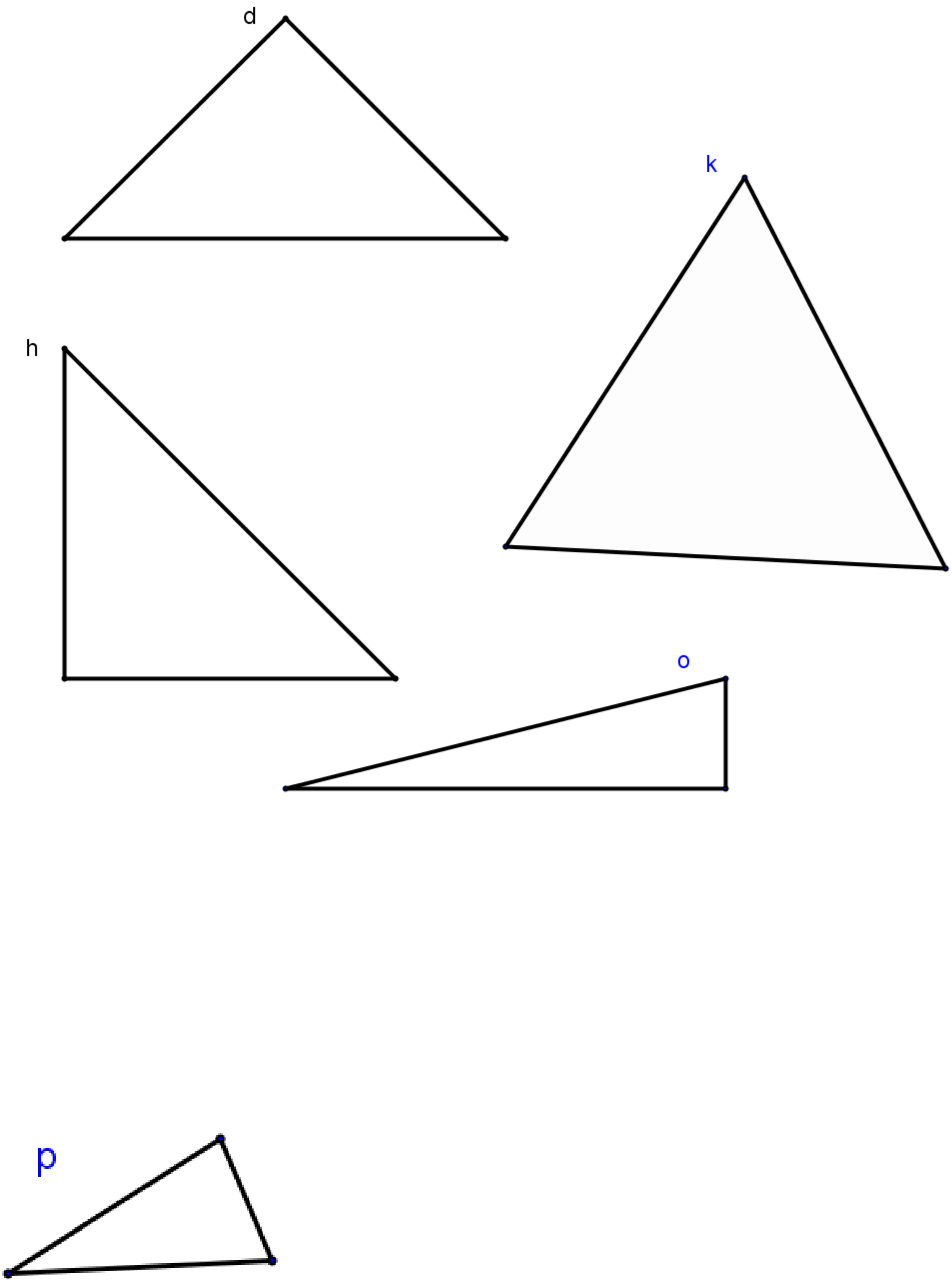
3.2.3 Scalene triangles……………………………………………………………

3.2.4 Right-angled isosceles triangles…………………………………………

3.2.5 Right-angled scalene triangles………………………………………………

3.3. In each of the triangles, use necessary symbols to indicate whether each of the triangles is a **right-angled** or **an isosceles** or **an equilateral triangle.**




d
k
h
o
p

Appendix 15: Intervention activity 4: Matching a triangle with its properties

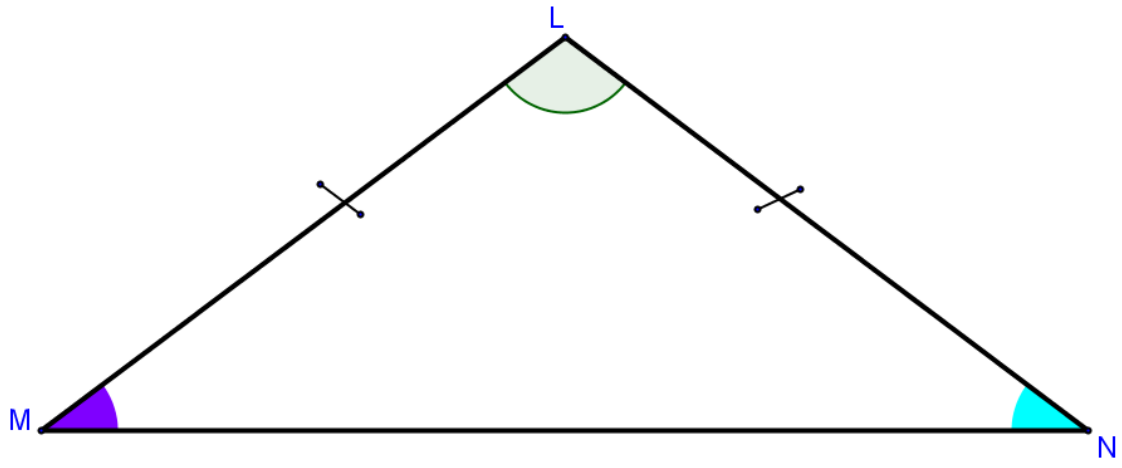


Figure 4.1

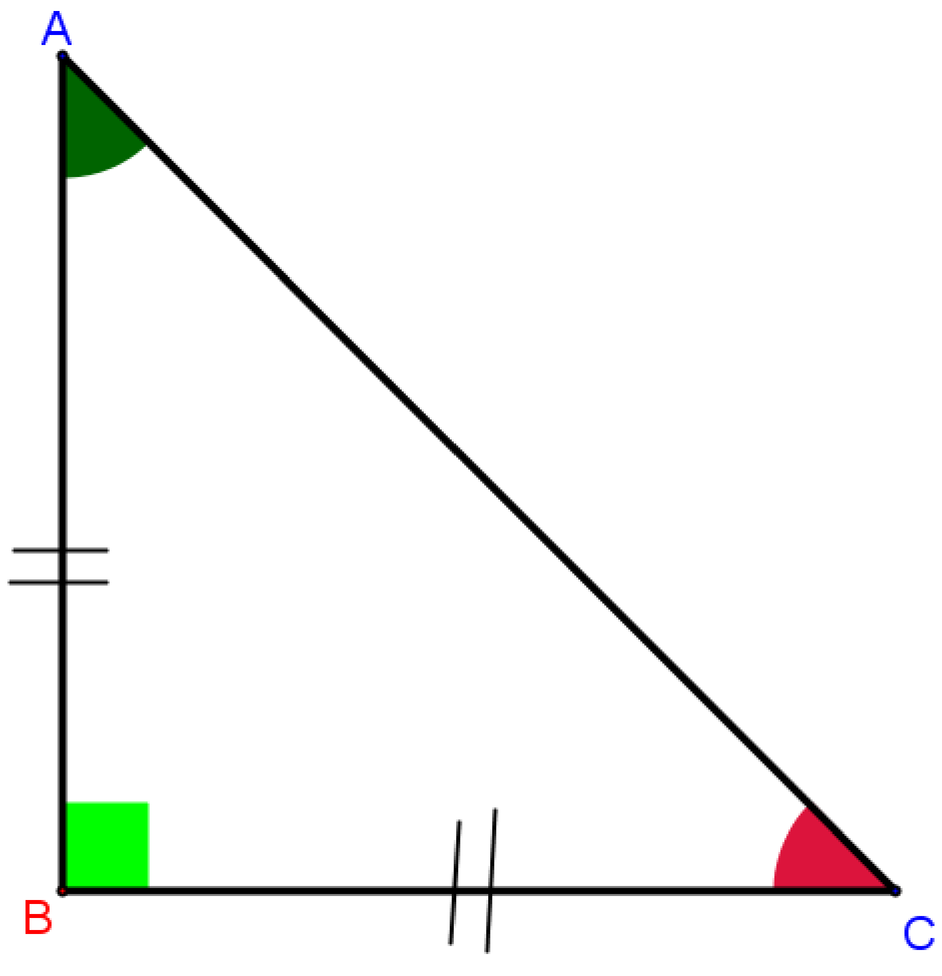


Figure 4.2

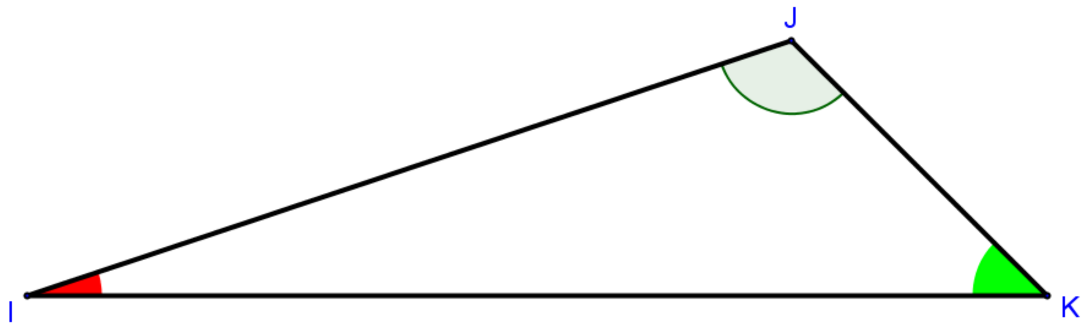


Figure 4.3

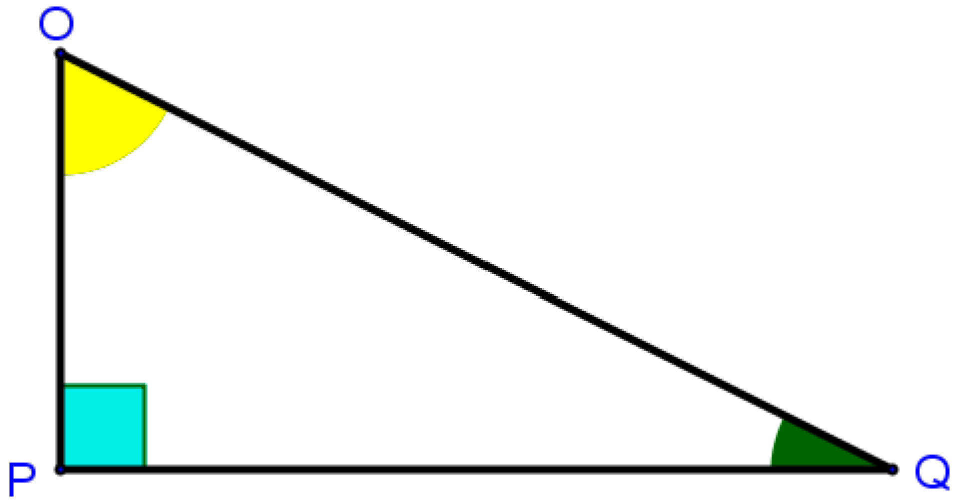


Figure 4.4

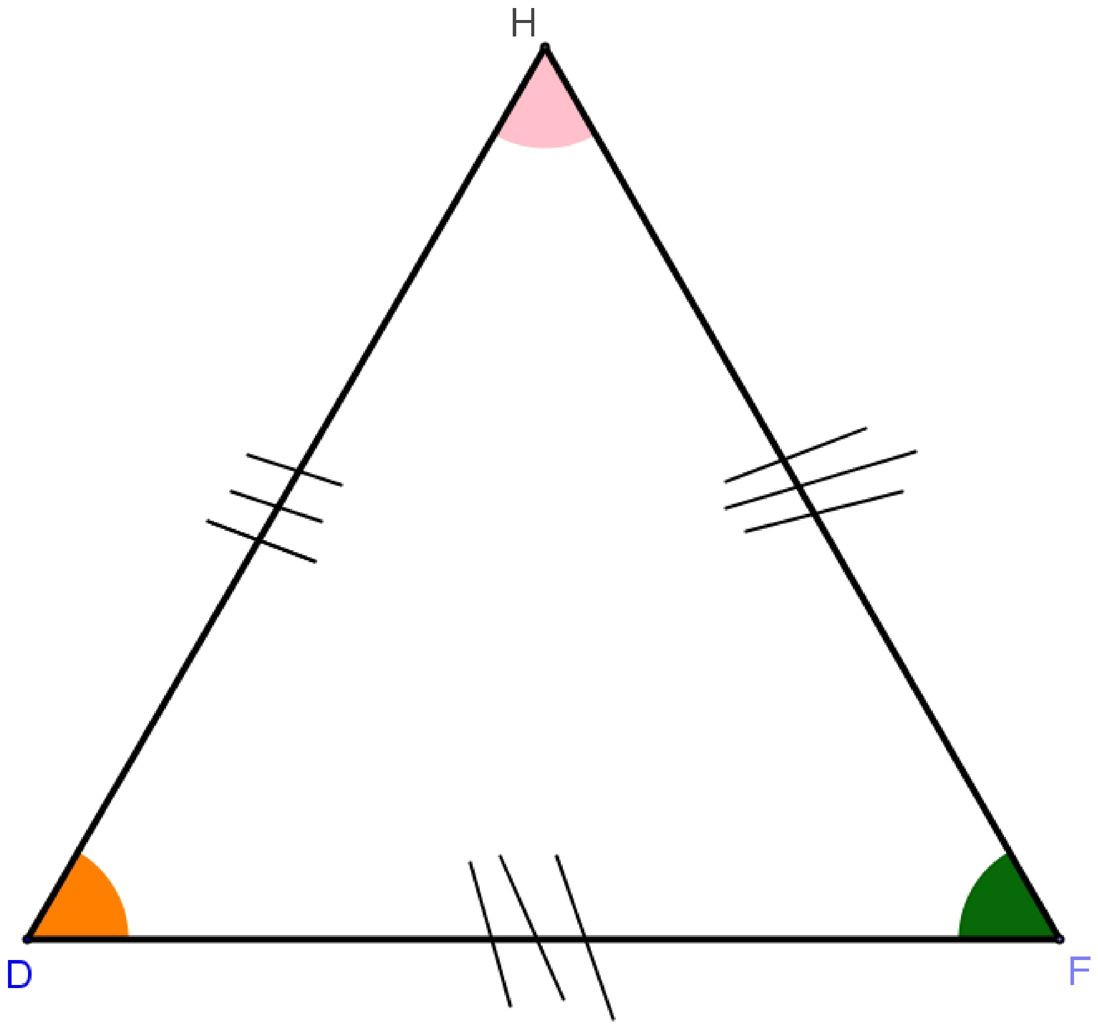


Figure 4.5

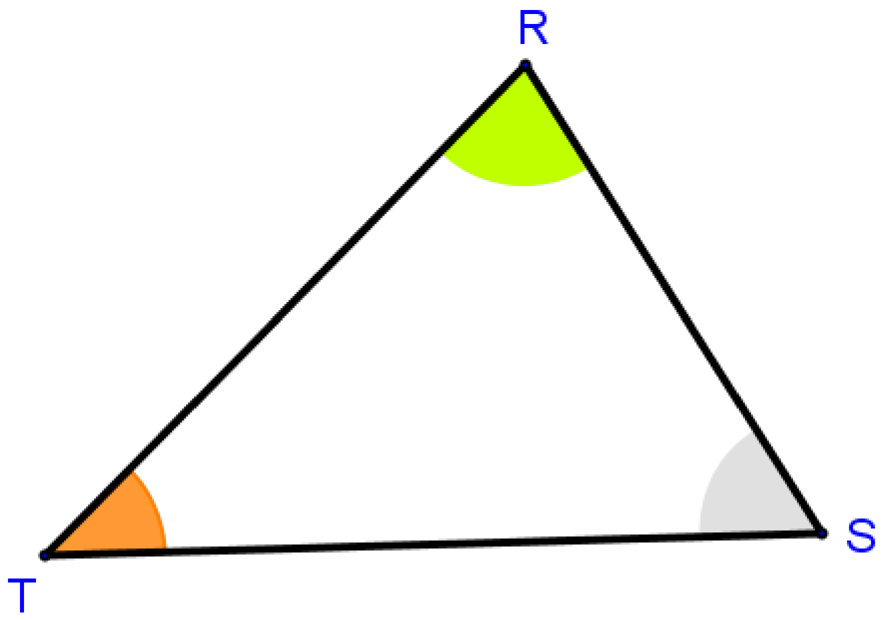


Figure 4.6

Take copies 4.1(a) – 4.6 (a) and 4.1 (b) - 4.6(b) of ΔLMN, ΔABC, ΔJKL, ΔOPQ, ΔDFH, ΔRST and then cut out the line segments and angles, respectively. Now put the cut out line segments and angles on top of the original diagram one at a time, compare the lengths of the three line segments and sizes of the angles. Respond to the question below:

4.1. Which of the triangles drawn above has the following set of properties?

4.1.1. - has 1 right angle

- angles opposites two equal sides are equal

……………………………………………………………………………………

4.1.2. - all angles are equal to each other

- all sides are equal in length

…………………………………………………………………………………..

4.1.3. - has 1 right angle

- all angles have different magnitude (sizes)

- all three sides have different lengths

…………………………………………………………………………..

4.1.4. – two opposite sides are equal

- angles opposite two equal sides are equal in size

……………………………………………………………………………

4.1.5. – all angles have different magnitude (sizes) and are acute

- all sides have different lengths (dimensions)

…………………………………………………………………………………

4.1.6. – all angles have different magnitude (sizes);

- all sides have different lengths (dimensions);

- has an obtuse angle

…………………………………………………………………………………

Appendix 16: Intervention activity 5

5.1 Given ΔABC and its twocopies on a hard paper provided:

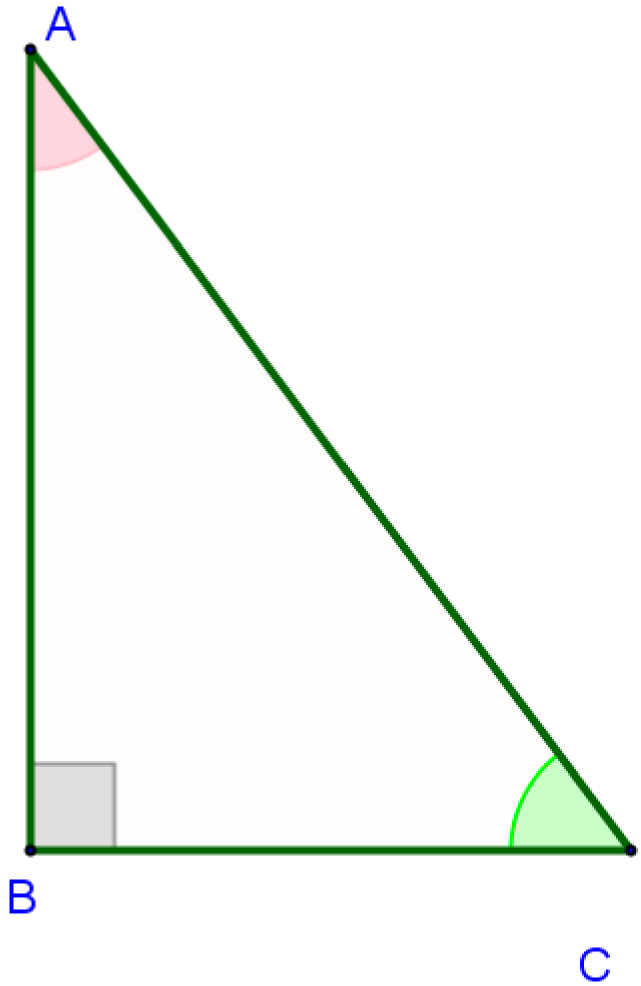


Figure 5.1: Triangle ABC

(i) From the first copy cut out line segment AC, AB and BC

(ii) Now take each of the cut out pieces one at a time, compare its length with those of the sides of the original triangle by placing the cut out piece on top of each of the lines, i.e. compare $\overline{AB}$ with $\overline{AC}$; $\overline{AC}$ with $\overline{BC}$ and $\overline{BC}$ with $\overline{AB}$

(iii) For each measurement taken record down your findings, use terms: longer than, shorter than, equal to.

…………………………………………………………………………………………………

5.2. Now, take the second copy of triangle ABC and cut out the angles through the broken lines, make sure you are left with the shaded apex and then do the following:

Take each of the cut out angles one at a time and compare its size with the other 2 angles by placing on top of each of the angles in the original triangle ABC. What is the relationship between:

(i) $\hat{A}$ *and* $\hat{C}$?

……………………………………………………………………………………………

(ii) $\hat{B}$ *and* $\hat{C}$?

……………………………………………………………………………………………

(iii) $\hat{B}$ *and* $\hat{A}$?

……………………………………………………………………………………………

(iv) What are the properties of ΔABC in terms of $\overline{\text{AB}}$; $\overline{\text{AC}}$ and $\overline{\text{BC}}$?

…………………………………………………………………………………….

(v) What are the properties of ΔABC in terms of $\hat{A}$, $\hat{B}$ and $\hat{C}$?

……………………………………………………………………………………..

(vi) What specific name is given to a triangle with properties mentioned in 5.2 (iv) - (v)?

………………………………………………………………………………………………………

Appendix 17: Intervention activity 6

6.1 Drawn below is ΔGHI , examine it carefully in order to do the activities below:

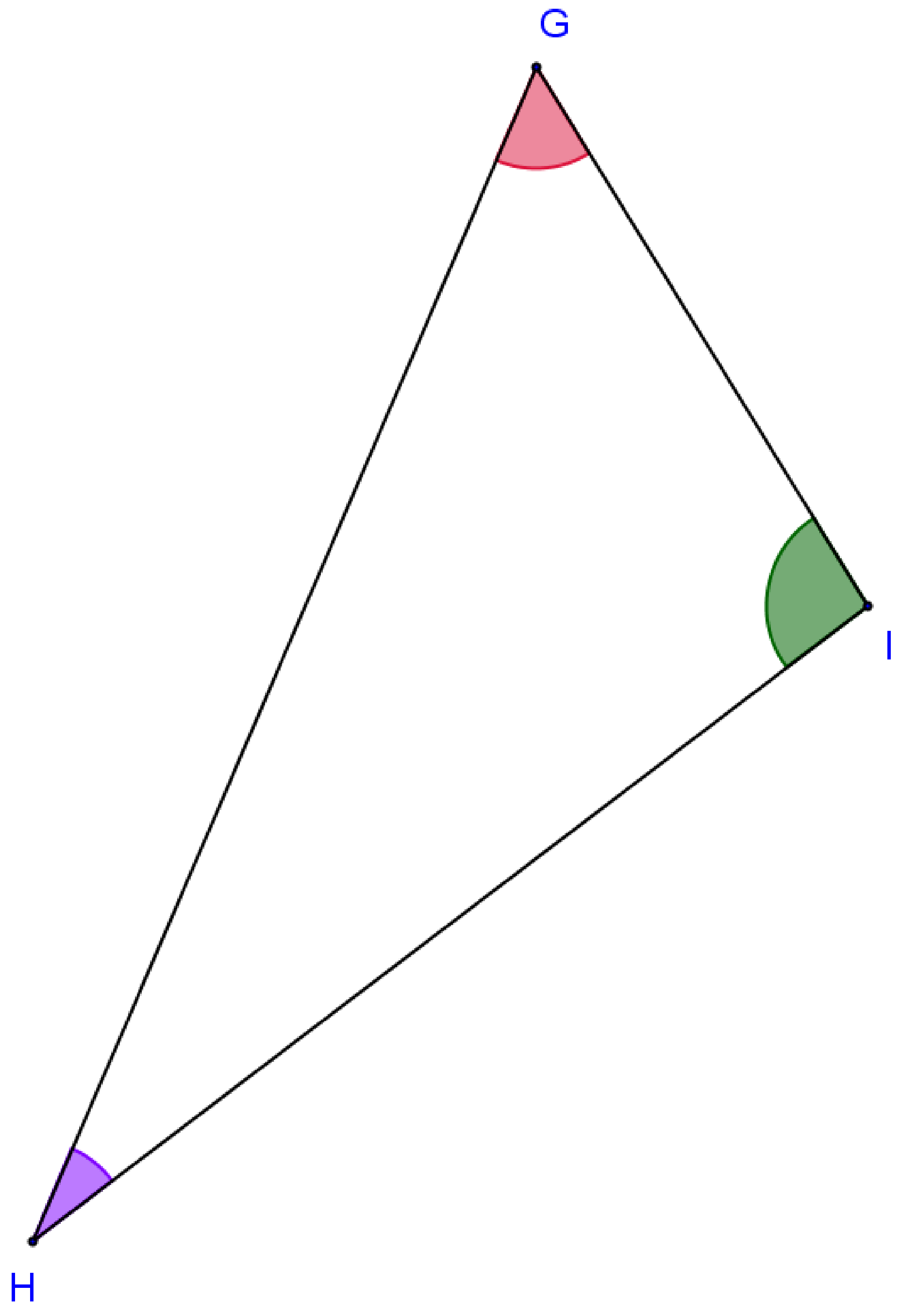


Figure 6.1: triangle GHI

(i) From the first copy CAREFULLY cut out line segment GH, HI and GI

(ii) Now take each of the cut out pieces of the line segments, one at a time, compare its length with those of the sides of the original ΔGHI by placing the cut out piece on top of each of the lines, i.e. compare $\overline{GH}$ with $\overline{HI}$; $\overline{GI}$ with $\overline{HI}$ and $\overline{HI}$ with $\overline{GH}$.

(iii) For each measurement taken record down your findings, use terms: longer than, shorter than, equal to.

………………………………………………………………………………………………………

6.2 Now, take the second copy of triangle GHI and cut out the shaded angles, make sure you are left with the shaded apex and then do the following:

Take each of the cut out angles one at a time and compare its size with the other 2 angles by placing it on top of each of the angles of the original triangle GHI. What is the relationship between:

(i) $\hat{G}$ and $\hat{H}$?

………………………………………………………………………………………………………

(ii) $\hat{G}$ *and* $\hat{I}$?

………………………………………………………………………………………………………(

iii) $\hat{H}$ *and* $\hat{I}$?

………………………………………………………………………………………………………

(iv) What are the properties of ΔGHI in terms of $\overline{GH}$; $\overline{HI}$ and $\overline{GI}$?

……………………………………………………………………………………

(v) What are the properties of ΔGHI in terms of $\hat{G}$, $\hat{H}$ and $\hat{I}$?

……………………………………………………………………………………..

(vi) What specific name is given to a triangle with properties mentioned in 6.2 (iv) - (v)?

……………………………………………………………………………………

Appendix 18: Intervention activity 7

7.1 Drawn below is triangle DEF, use it to do the activity as instructed below:

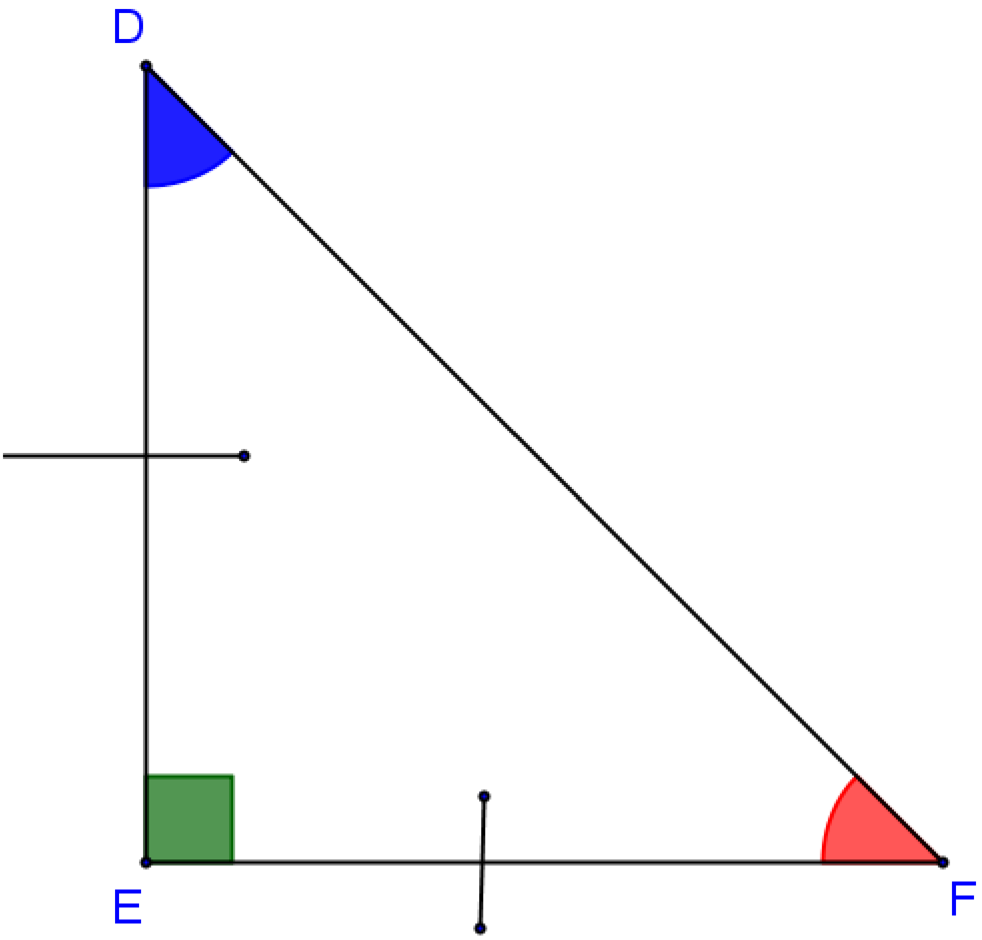


Figure 7.1: Triangle DEF

(i) From the first copy CAREFULLY cut out line segment DE, EF and DF

(ii) Now take each of the cut out pieces of the line segments, one at a time, compare its length with the length of other two sides of the original triangle GHI by placing the cut out piece on top of each of the lines, i.e. compare $\overline{DE}$ with $\overline{EF}$; $\overline{DE}$ with $\overline{DF}$ and $\overline{DF}$ with $\overline{EF}$.

(iii) For each measurement taken record down your findings, use terms like: longer than, shorter than, equal to.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

7.2 Now, take the second copy of triangle DEF and cut out the shaded angles, make sure you are left with shaded apex and then do the following:

Take each of the cut out angles one at a time and compare its size with the other 2 angles by placing it on top of each of the angles of the original triangle DEF. What is the relationship between:

(i) $\hat{D}$ and $\hat{F}$

……………………………………………………………………………………………………(

ii) $\hat{D}$ and $\hat{E}$

……………………………………………………………………………………………………(

iii) $\hat{E}$ and $\hat{F}$

……………………………………………………………………………………………………

(iv) What are the properties of ΔDEF in terms of $\overline{DE}$; $\overline{EF}$ and $\overline{DF}$?

………………………………………………………………………………….

(v) What are the properties of ΔDEF in terms of $\hat{D}$, $\hat{E}$ and $\hat{F}$?

…………………………………………………………………………………..

(vi) What specific name is given to a triangle with properties mentioned in 7.2 (iv) - (v)?

………………………………………………………………………………

Appendix 19: Intervention activity 8

8.1 Study ΔXYZ below carefully and then answer the questions below:

NB. Different colours have been used for easy identification when doing the activity, they are not necessarily determining the size of the angle.

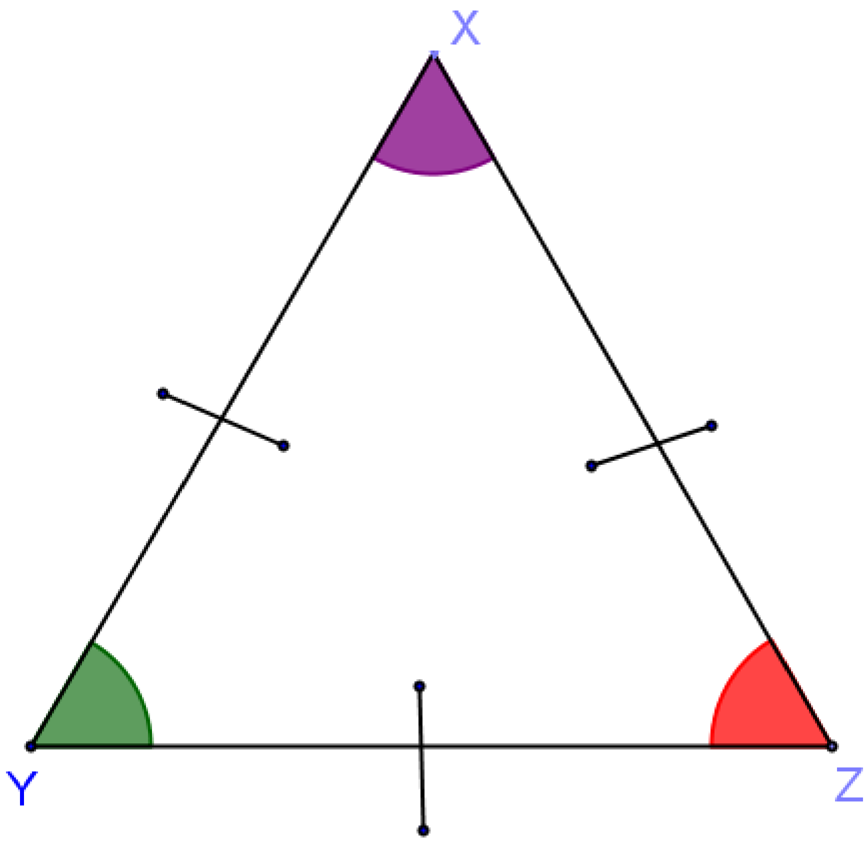


Figure 8.1: Triangle XYZ

(i) Estimate the sizes of $\hat{X}$, $\hat{Y}$ and $\hat{Z}$?

……………………………………………………………………………………………

(ii) By estimation compare and write down the length of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$. Use terms: **shorter than, longer than, equal to, the longest of all.**

…………………………………………………………………………………………………

From the first copy CAREFULLY cut out line segments: XY, YZ and XZ

(iii) Now take each of the cut out pieces of the line segments, one at a time, compare its length with those of the sides of the original triangle XYZ by placing the cut out piece on top of each of the lines, i.e. compare $\overline{XY}$ with $\overline{YZ}$, $\overline{YZ}$ with $\overline{XZ}$ and $\overline{XZ}$ with $\overline{XY}$. Record down your findings:

…………………………………………………………………………………………………
…………………………………………………………………………………………………

8.2 Now, take the second copy of triangle XYZ and cut out the shaded angles, make sure you are left with the shaded apex and then do the following:

Take each of the cut out angles one at a time and compare its size with the other 2 angles by placing it on top of each of the angles in the original triangle XYZ. What is the relationship between:

(i) $\hat{X}$ and $\hat{Y}$?

……………………………………………………………………………………

(ii) $\hat{Y}$ and $\hat{Z}$?

…………………………………………………………………………………..

(iii) $\hat{Z}$ and $\hat{X}$?

………………………………………………………………………………..

8.3 What are the properties of ΔXYZ in terms of:

(i) $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$?

………………………………………………………………………………………….

(ii) $\hat{X}$, $\hat{Y}$ and $\hat{Z}$?

……………………………………………………………………………………………..

8.4. What name is given to ΔXYZ ?

…………………………………………………………………………………………….

Appendix 20: Intervention activity 9

9.1 Drawn below is ΔPQR, use it to do the activities below:

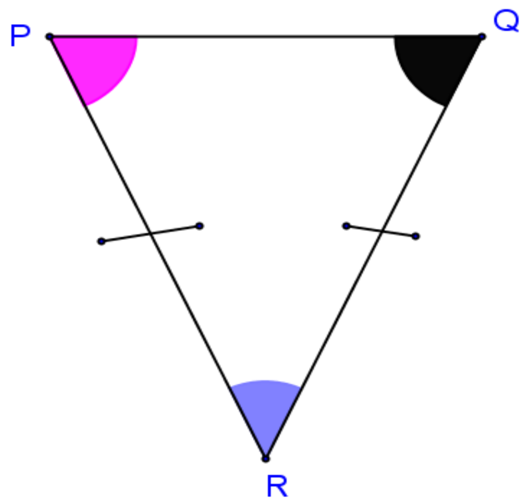


Figure 9.1: Triangle PQR

(i) Estimate the sizes of $\hat{P}$ and $\hat{Q}$? Use terms: equal to, greater than, smaller than

……………………………………………………………………………………

……………………………………………………………………………………

(ii) Estimation and write down the length of $\overline{PQ}$, $\overline{RP}$ and $\overline{QR}$. Use terms: shorter than, longer than, equal to, the longest of all.

……………………………………………………………………………………

……………………………………………………………………………………

(iii) From the first copy CAREFULLY cut out line segments: QP, QR and PR, now take each of the cut out pieces of the line segments, one at a time, compare its length with lengths of two other sides of the original triangle PQR by placing the cut out piece on top of each of the lines, i.e. compare $\overline{QR}$ with $\overline{PR}$, $\overline{QR}$ with $\overline{QP}$ and $\overline{QP}$ with $\overline{PR}$. Record your findings:

……………………………………………………………………………………

……………………………………………………………………………………

9.2. Now, take the second copy of triangle QPR and cut out the shaded angles, make sure you are left with the shaded apex and then do the following:

Take each of the cut out angles one at a time and compare its size with the other 2 angles by placing it on top of each of the angles in the original triangle QPR. What is the relationship between:

(i) $\hat{Q}$ and $\hat{P}$?

……………………………………………………………………………………

(ii) $\hat{Q}$ and $\hat{R}$?

…………………………………………………………………………………..

(iii) $\hat{P}$ and $\hat{R}$?

………………………………………………………………………………..

9.3 What are the properties of ΔQPR in terms of:

(i) $\overline{QR}$, $\overline{PR}$ and $\overline{QP}$?

……………………………………………………………………………………………………
……………………………………………………………………………………

(ii) $\hat{Q}$, $\hat{P}$ and $\hat{R}$?

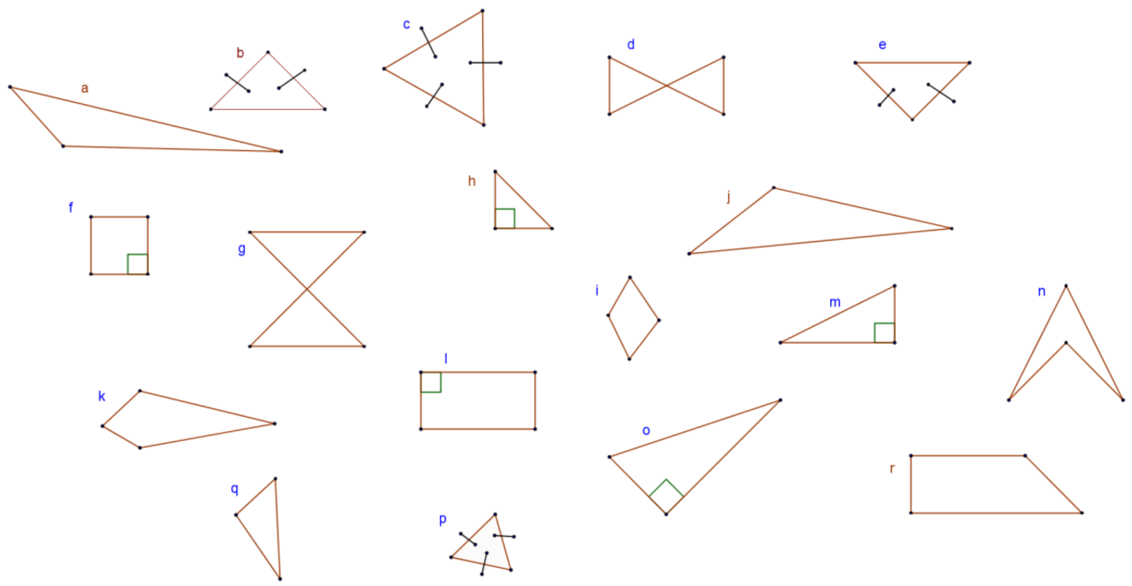
…………………………………………………………………………………………..

9.4. What specific name is given to ΔQPR?

……………………………………………………………………………

Appendix 21: Reflective test 1

Study the shapes below carefully and then answer the questions that follow:



1.1 Which of the shapes drawn above are triangles? (9)

………………………………………………………………………………………………

………………………………………………………………………………………

1.2 Match each of the triangles identified in QUESTION 1.1 with the correct category. Just write down the letter that represents the identified triangles. (9)

Scalene triangles	Isosceles triangles	Equilateral triangles	Right angled triangles

1.3. From what you can see write down how each of the triangles looks (its properties):

(a) Scalene triangle ……………………………………………………………………

……………………………………………………………………………………………...(2)

(b) Isosceles triangle ……………………………………………………………………

……………………………………………………………………………………………..(1)

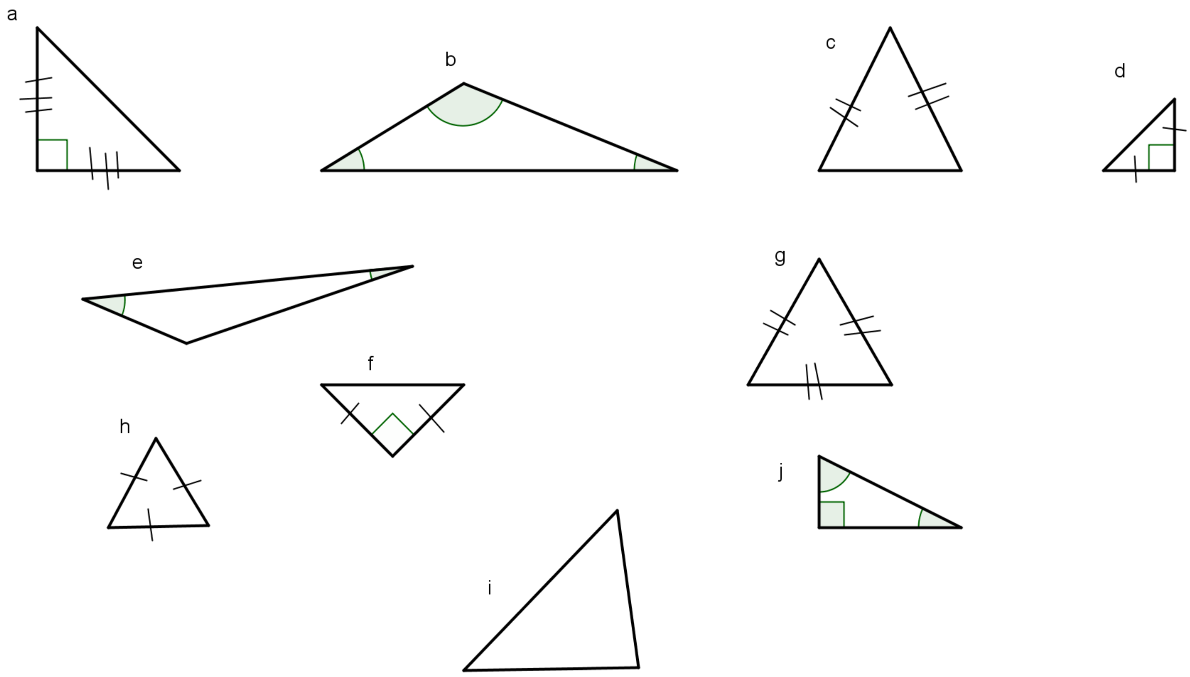
(c) Equilateral triangle …………………………………………………………………..

…………………………………………………………………………………………… (1)

(d) Right-angled triangle ………………………………………………………………....(1)

Appendix 22: Reflective test 2

Drawn below, are different types of triangles, use them to answer the questions that follo



2.1 Which of the shapes drawn above is/are: (write down the letter that represents that particular triangle(s).

2.1.1 an acute triangle(s)? ....…………………………………

2.1.2 an isosceles triangle(s)? ……………………………………………………….

2.1.3 an equilateral triangle(s)? ……………………………………………………….

2.1.4 a right angled isosceles triangle(s)? ………………………………………..

2.1.5 an obtuse triangle(s)?………………………………………………………….

2.1.6 a right angled scalene triangle? ……………………………………………..

2.2 What are the angle properties of:

2.2.1 a scalene triangle? ……………………………………………………………

2.2.2 an isosceles triangle? ……………………………………………………….

2.2.3 an obtuse triangle? ……………………………………………………………

2.2.4 an acute angled triangle? ……………………………………………………….

2.2.5 a right-angled triangle? ……………………………………………………….

Appendix 23: Reflective test 3

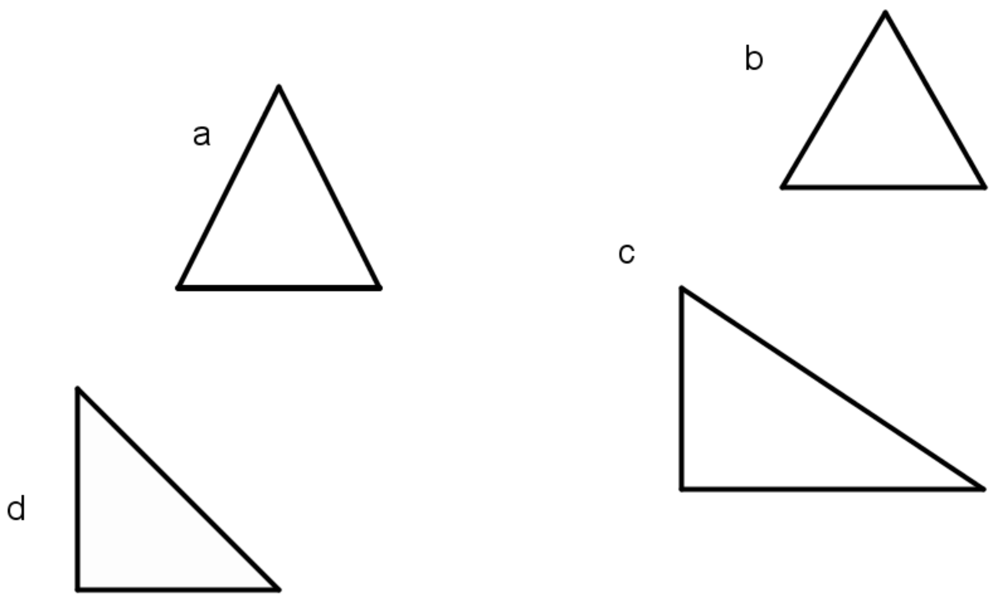
3. In each of the triangles drawn below show that:

3.1.1 ‘a’ is an isosceles triangles.

3.1.2 ‘b’ is an equilateral triangle.

3.1.3 ‘c’ is a right angled triangle

3.1.4 ‘d’ is a right –angled isosceles triangle.



3.2 Which of the diagrams drawn below is/are:

3.2.1 a right-angled scalene triangle? ………………………………………………………

Give two reasons: …………………………………………………………………

3.2.2 an/ acute angled triangle(s)?: ……………………………………………………..

Give a reason: ……………………………………………………………………

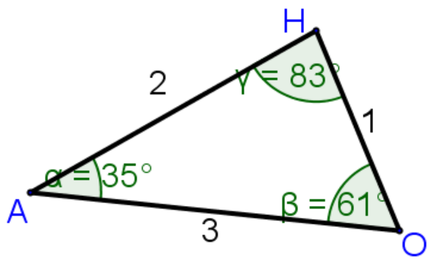
3.2.3 an obtuse triangle?...

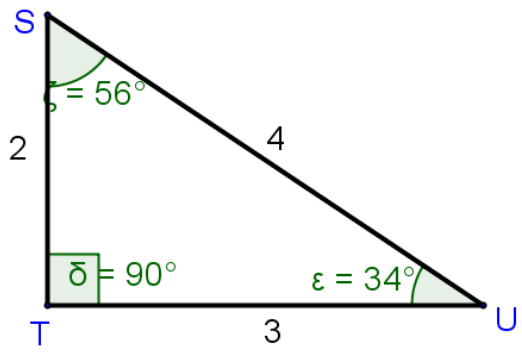
Give a reason……………………………………………………………………

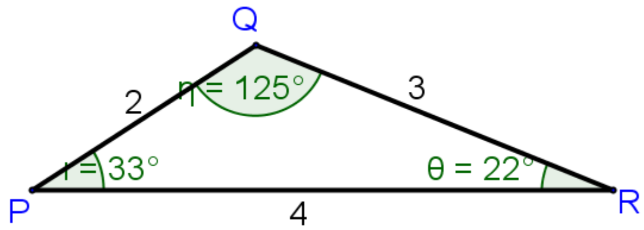
3.2.4 a/ scalene triangle(s)?……………………………………………………………….

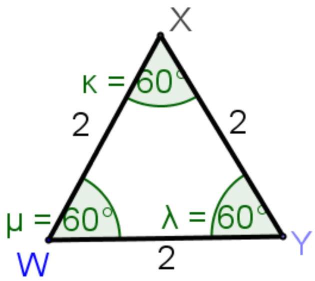
Give two reasons: ……………………………………………………………….

…………………………………………………………………………………..


H
A
O
2
1
3
γ = 83°
α = 35°
β = 61°


S
T
U
2
4
3
ζ = 56°
δ = 90°
ε = 34°


Q
P
R
2
3
4
η = 125°
ι = 33°
θ = 22°

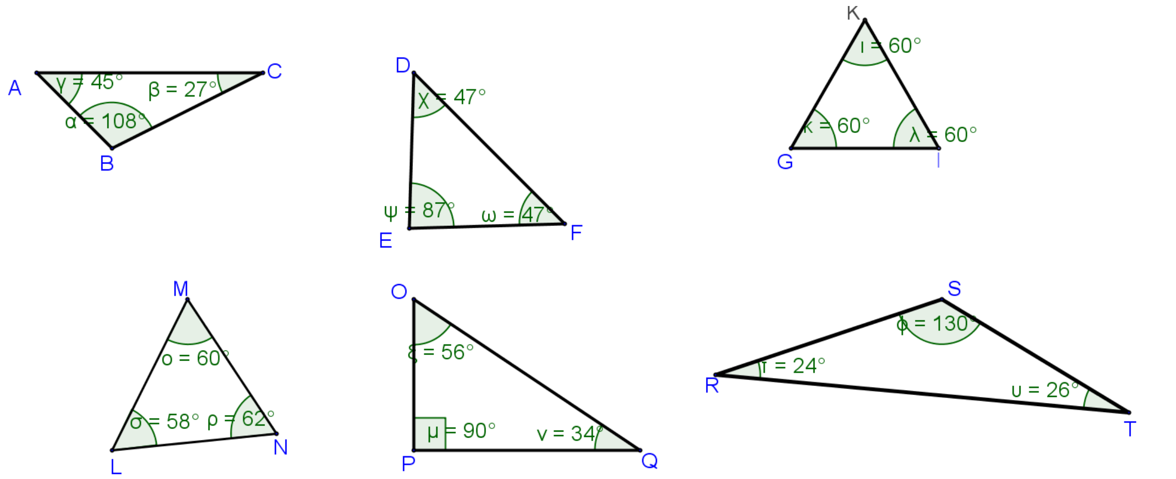

X
W
Y
2
2
2
κ = 60°
μ = 60°
λ = 60°

Appendix 24: Reflective test 4

4.1. An acute angled triangle has ALL angles less than 90^0.

Which of the shapes drawn below are acute angled triangles?

……………………………………………………………………………………………

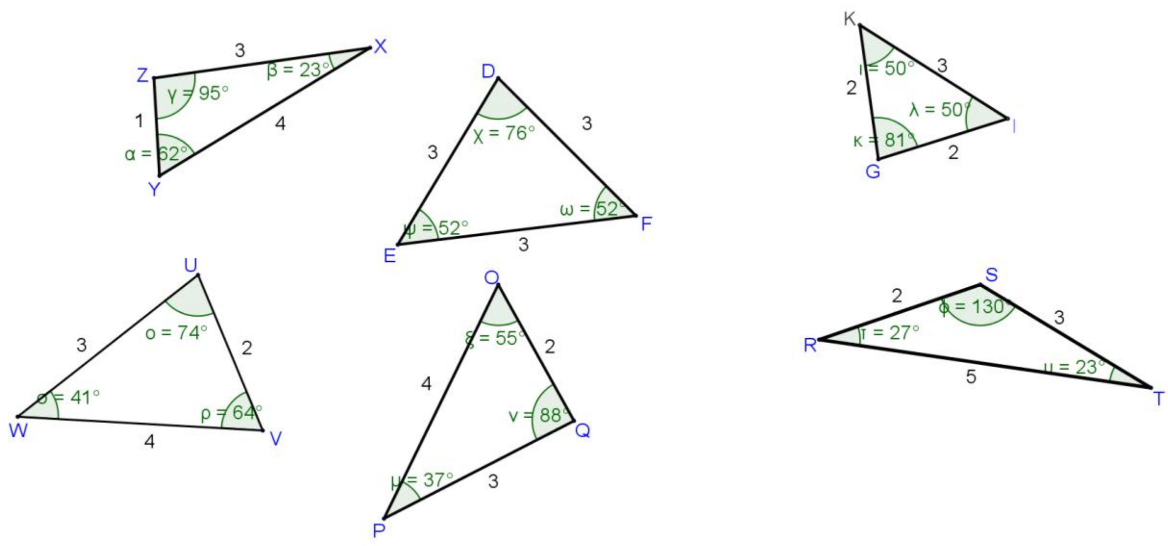


4.2 A scalene triangle has: (i) three angles of different sizes.

(ii) three sides of different lengths.

Which of the diagrams below are scalene triangles?

……………………………………………………………………………………………..



Appendix 25: Reflective test 5

1. Describe THREE characteristics of a **right angled scalene** triangle.

…………………………………………………………………………………………………

…………………………………………………………………………………

2. Mention THREE properties of an **obtuse angled scalene** triangle

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

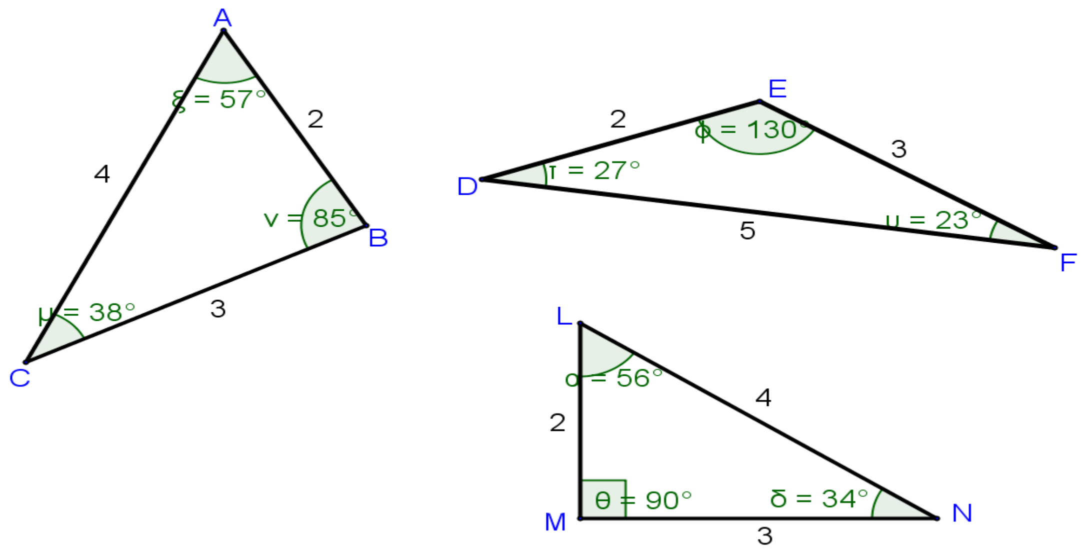
3. Write down TWO properties of an **acute angled scalene** triangle.

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

4. Under which of the three groups mentioned above does each of the triangles drawn below belong?



4.1. ΔABC is a ………………………………………………………………

4.2. ΔDEF is a ………………………………………………………………

4.3. ΔLMN is a ……………………………………………………………

Appendix 26: Reflective test 6

6.1. Use a ruler, a protractor and a pencil to draw a **right-angled isosceles triangle**. Indicate with all necessary features that it a right-angled isosceles triangle.

6.2 What are the **THREE** properties (characteristics) of a right-angled isosceles triangle?

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Appendix 27: Reflective test 7

7.1. Use a ruler, a protractor and a pencil to draw an **equilateral triangle**. Indicate with all the necessary features to show that your triangle is an equilateral.

7.2 What are the **TWO** properties (characteristics) of an equilateral triangle?

…………………………………………………………………………………………

…………………………………………………………………………………………

……………………………………………………………………………………

Appendix 28: An observation schedule

No	**Description**	**Alignment to van Hiele's levels 0-3**	**Ratings**		
			3	**2**	**1**
1.	The learner were able to measure the sides of the given polygons using physical manipulatives (cut out pieces of 2D shapes)	Analysis		√	
2.	The learners were able to get correct solutions to the question guided by physical manipulatives.	Formal deduction		√	
3.	The learners were able to move from concrete stage through pictorial to abstract stage of identifying and giving the relationship of angles and polygons based on sides also.	Analysis		√	
4.	The learners actively participated in learning and used physical manipulatives for conversation on how to get the solutions to various problems	Formal deduction		√	
5.	Physical manipuatives provided learners with an opportunity to reflect on their own mathematical experiences in order to define the terms i.e. scalene, line segments, angles, etc.	Abstraction	√		

6.	After using physical manipulatives learners were able to make connection between concepts and symbols	Formal deduction	√		
7.	In their small groups each and every member was able to differentiate shapes and angles of polygons by the help of physical manipulatives.	Visualisation	√		
8.	The learners' discussions of the given questions were guided by the physical manipulatives.	Analysis		√	
9.	The use of the programme allowed the learners to gain the skills i.e. communication skills, calculation skills.	Abstraction		√	
10.	The learners really used physical manipulatives in order to determine relationship of angles in a triangle	Visualisation		√	
11.	The learners were actively engaged in doing the task at hand using physical manipulatives.	Analysis	√		
12	The learners were motivated to do the task at hand (each and every learner was involved in doing the task).	Visualisation	√		
13.	There is an ability to understand the question that is presented diagrammatically (shown by solving the questions accurately)	Abstraction		√	

14.	The learners were able to cut out traced polygons into pieces and used them to identify the properties of that particular polygon.	Visualisation and Analysis		√	
15.	In their small groups learners were able to discuss and then determine the properties of each of the polygons using physical manipulatives: scalene triangles, isosceles triangle, etc.	Formal deduction	√		
16.	The learners used physical manipulatives in order to determine the types of angles formed when two opposite lines in a triangle are equal.	Abstraction		√	

KEY: 3.To a great extent. 2. Moderate. 1. No attempt has been made

Appendix 29: Semi-structured interview questions

1. After participating in this research project, what is your comment on the use of polygon pieces as physical manipulatives in teaching and learning of geometry. Specifically with regard to the properties of the triangles?
2. Why do you like the use of the program [physical manipulatives] in learning about the properties of the triangles?
3. Did the program that you have used help you to get the clear picture and explanation of how to identify the properties of the triangles?
4. Now tell me, how did you investigate the properties of the triangles?
5. Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
6. How can that topic(s) be taught? Please, explain your answer in details.

Appendix 30: Transcribed interview for learner 1 to learner 9

Key:

R: Researcher

L1: Learner 1

R	[I called for the first learner for the interviews. The learner came in, I welcomed the learner]. Welcome to this short interview session I am going to ask you few questions regarding the cut pieces of polygons in learning geometry, feel free to express yourself.
L1	1.1 Ok, sir
R	[I asked question 1] After participating in this research project, what is your comment on the use of cut polygon pieces as physical manipulatives in teaching and learning of geometry? In particular with regard to the properties of the triangles.
L1	1.2 The programme can help a person on how to measure
R	[Follow up question] To measure what?
L1	1.3 To measure angles and sides of triangles, sir.
R	[Follow up question] Do you like the programme?
L1	1.4 I like it, sir
R	[Follow up question] Why do you like the programme?
L1	1.5 I like it because it can help me in learning mathematics.
R	[Follow up question] In which topics of mathematics can the programme help you?
L1	1.6 In geometry, it can help me learn geometry.
R	[I had to ask another follow up question because geometry is what they were doing. I did not want to discourage L5 in answering other questions by saying you are repeating what you have been doing]. In what ways can the programme you have recently used help you in learning geometry?

L1	[The learner kept quiet for a while and then responded] 1.7 By giving me measuring skills
R	[After that response question 2 from the list was asked] What do you like about the programme you have recently used in learning the properties of triangles?
L1	[L1 just gave me s short response] 1.8 It is good
R	[Follow up question] Have you ever used such a programme before in learning geometry?
L1	1.9 No sir, we did not use cut pieces of polygons to measure angles like what we have just done.
R	[Follow up question] Did you learn geometry at primary school?
L1	1.10 Yes sir, we did learn geometry at primary, but for a very small time.
R	[Follow up question] What do you mean by saying for a very small time?
L1	1.11 For five days only sir.
R	[Follow up question] Did you learn properties of triangles?
L1	1.12 Yes sir, we learnt all geometry in those five days only.
R	[I then asked question 4 according to the list] How did you investigate the properties of the triangles?
L1	1.13 By estimating the sizes of angles and lengths of sides of triangles and then we cut out the angles and 1.14 the sides of different copies of triangles in order to measure the angles and sides of original triangles.
R	Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
L1	1.15 This can be used to learn geometry construction of angles and triangles
R	[Follow up question] How can you use the programme?
L1	1.16 To measure lines and angles
R	Do you have anything to say regarding the programme that has been used in teaching and learning of geometry?

L1	1.17 No sir
R	Thank you for your time, you may go now. [L5 left and I called the next learner, L2]

Key:

R: Researcher

L2: Learner 2

R	[As soon as L1 left, L2 was called in for the interview. The learner came in and was welcomed]. Welcome to this short interview session feel free to express yourself.
L2	2.1 Thank you sir
R	[Without wasting time I went into the questioning session]. After participating in this research project, what is your comment on the use of cut polygon pieces as physical manipulatives in teaching and learning of geometry? Specifically with regard to the properties of the triangles.
L2	[Paused for a while, seemingly she was thinking of what to say, then she responded] 2.2 No comment sir
R	[Such a response made me to think that the question was not clear, then I paraphrased the question] What is your feeling about the programme you have just used to learn geometry?
L2	2.3 I feel excited sir.
R	[Follow up question] Was if good or bad to be engaged in such a programme?
L2	2.4 It was good sir
R	[Follow up question] In what ways was it good?
L2	2.5 The things that I did not understand now I do understand them
R	[Follow up question] Things like what?
L2	2.6 The relationship of angles and sides of different triangles

R	[After that I asked question 2] What do you like about the use of physical manipulatives in teaching and learning about the properties of the triangles?
L2	2.7 It was learning of mathematics using pieces of papers, that was interesting to me
R	[Follow up question] Did you enjoy the use of the programme?
L2	2.8 Yes sir.
R	[I then asked question number 3] Did the programme you have used help you to get the clear picture and explanation of how to identify the properties of the triangles in regard to sides and angles?
L2	2.9 Yes sir
R	[Follow up question] How did the programme help you?
L2	2.10 It helped me to measure angles and sides of triangles and 2.11 made me to understand the relationship of sides and angles of given triangles.
R	[Then question 4 was asked] How did you investigate the properties of the triangles?
L2	[L3 kept quiet, smiled and then responded to the question] 2.12 By measuring the angles and sides of triangles. 2.13 And also as I am speaking I now know well the names of different triangles.
R	[Follow up question] Do you mean that you were not quite clear about the classification of triangles?
L2	2.14 We were taught in primary school, 2.15 but I could not classify the triangles well.
R	[I asked another follow up question] As you said that you were taught the properties of triangle at primary school, did you use any programme to learn that?
L2	2.16 No sir, this is the first time I have been using small pieces of paper to learn geometry.
R	[Question 4 was asked]

	How did you investigate the properties of the triangles?
L2	2.17 It helped me to learn geometry
R	[Follow up question] What else?
L2	2.18 [Kept quiet for a moment, seems to be puzzled or not quite knowing the other topics in mathematics]
R	[I had to ask the follow up question] Do you know that what you have been doing is geometry?
L2	2.19 No sir.
R	[Follow up question] Which one is geometry to you?
L2	2.20 Like shapes
R	[I asked the follow up question] Which shapes?
L2	[L3 kept quiet for a long time with no response] 2.21 No answer sir
R	Ok, if you have no answer, please answer this question. Do you have any comment regarding the programme you have been using to learn geometry?
L2	2.22 No sir
R	[L2 was then released after the last question]
L2	2.23 Left the interview room

Key:

R: Researcher

L3: Learner 3

R	[The third learner was called for interviews welcomed and I told the learner what was expected her during the interviews]. I welcome you to these interviews, feel free. I am going to ask you questions regarding what you have been doing in my research study for the past three weeks.
L3	[Nodded her head and smiled, then talked]. 3.1 Ok sir
R	[I asked her the first question from the list of semi-structured questions I prepared in advance]. After participating in this research project, what is your comment on the use of cut polygon pieces as physical manipulatives in teaching and learning of geometry, specifically with regard to the properties of triangles?
L3	[She responded with a smile]. 3.2 I feel happy to be part of this research programme because 3.3 now I understand things that I did not understand before
R	[Follow up question]. Ok, what other comments do you have?
L3	(Grinned and smiled seemed to be puzzled, and then answered]. 3.4 Nothing else to say sir.
R	[I moved on with questioning, I asked the second question from the list]. What do you like about the use of physical manipulatives in teaching and learning about the properties of the triangles?
L3	[She looked aside, seemingly she was thinking what to say, she took a deep breath and then responded]. 3.5 I like the programme because it helped me to understand mathematics 3.6 concepts that I did not understand at all.
R	[I asked the follow-up question, because she just said to understand concepts with no specification of what exactly]. To understand mathematics concepts like what?
L3	[She looked at me with a worrisome look, and then she answered].

	3.7 It helped me to understand types of triangles and angles in those triangles.
R	[By giving the recent response, she responded to my question 3 according to my questionnaire, so I move d onto ask question 4]. Now tell me, how did you investigate the properties of the triangles?
L3	3.8 [Smiled for a moment, then looked at me with no response given]
R	[I looked at her and asked]. Do you understand what the question says?
L3	[She responded] 3.9 Yes, sir I do. I investigated the properties of triangles by cutting out angles and 3.10 sides and compare them.
R	[I asked a question for clarity]. You said, you compared angles and sides of triangles, can you make this clear please.
L3	3.11 Ok, sir, I compared cut out angles and side from a given triangle, an angle against and 3.12 angle and a side against a side in order to come up with the properties of a given triangles.
R	[I moved on to the next question]. Did you enjoy the programme of cutting and comparing angles and sides of triangles?
L3	[She looked excited and seemed to be ready to answer]. 3.13 Yes, sir I did enjoy the programme.
R	[I asked a follow up question]. What made you enjoy the programme?
L3	3.14 The learning of properties of triangles by using the pieces of angles and sides of triangles.
R	[I asked another follow-up question in order to probe more responses]. Have you ever used such a programme before in learning mathematics?
L3	3.15 No sir, this is the first time.
R	[I then moved on to question 5]. Apart from learning the properties of the triangles, what other

	mathematics topic(s) can you learn using these physical manipulatives?
L3	[She kept quiet for a while and then responded with a low voice]. 3.16 I do not know any sir.
R	[No further questions were asked, I then allowed her to leave]. Thank you for your time, this is the end of the interview, you may leave.
L3	3.17 [L3 left the interview room]

Key:

R: Researcher

L4: Learner 4

R	[As soon as L3 left, I called for the 4th learner for the interviews. The learner came in, was greeted and welcomed]. Welcome to this short interview session feel free to express yourself.
L4	4.1 Ok sir
R	[I immediately asked question 1]. After participating in this research project, what is your comment on the use of cut polygon pieces as physical manipulatives in teaching and learning of geometry? In particular with regard to the properties of the triangles.
L4	4.2 I have learnt a lot sir
R	[Follow up question] You have learnt a lot like what? Please elaborate on this learnt a lot
L4	4.3 I have learnt about what a triangle is and 4.4 also different type of triangles and their properties.
R	[Follow up question] Which type of triangles did learn using physical manipulatives?
L4	4.5 Isosceles, equilateral, right-angled triangle and scalene
R	[Follow up question] Have you ever used the programme like you have been using to learn geometry?
L4	4.6 No sir
R	[I immediately, asked the second question according the list I had] Do you like the programme that you have just used to learn properties of

	triangles?
L4	4.7 Yes sir, I like the programme
R	[I asked the follow up question] Why do you like the programme?
L4	4.8 It has helped me with measuring skills. 4.9 I can now measure angles and sides of triangles using the cut pieces of the same triangle.
R	[I then asked question number 3] Did the programme you have used help you to get the clear picture and explanation of how to identify the properties of the triangles in regard to sides and angles?
L4	4.10 Yes, sir, I got a clear picture because now I clearly understand the concepts of triangles and their properties.
R	[Question 4] How did you investigate the properties of the triangles?
L4	4.11 We compared the sides of triangles using cut pieces of the same triangle and 4.12 found out that when all the sides of a triangle's sides have same slashes, it simply means that all the sides are equal. 4.13 And also that when the letter is written like this $\hat{Z}$, means angles Z.
R	[I then asked question number 5] Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
L4	4.14 [Kept quiet for a long time, this question was not answered]
R	[I paraphrased question 5, to make it clear to L4] Can the programme you have used recently be used in learning of geometry also be used for teaching and learning other mathematics topic?
L4	4.15 Yes, sir, it can be used in lesson of construction of angles and triangles
R	[Follow up question] How can that be done?
L4	4.16 Like……..[kept quiet for some time and then continued]. When you have been asked to construct an angle and 4.17 to know the size of that particular angle you need this programme to measure its size.

R	Do you have anything to say regarding the programme that has been used in teaching and learning of geometry?
L4	4.18 Yes, sir [She smiled and then said something]
R	What is it?
L4	4.19 I just want to say the programme we have used to learn about triangles and 4.20 their properties has helped me to understand the properties of triangles better than before.
R	Ok, if you have nothing else to say, you may go. Thank you for the information you have just given to me throughout this interview.
L4	4.21 [Left the interview room]

Key:

R: Researcher

L5: Learner 5

R	[When the L5 came in, I welcomed the learner] Welcome to this interview session, feel free to answer the questions I am going to ask you during the process.
L5	5.1 Thank you sir.
R	[Question 1 was asked] What are your comments regarding the programme you have recently used to learn geometry?
L5	5.2 No comment sir
R	[Follow up question] Do you like the programme?
L5	5.3 Yes sir
R	[Follow up question] Why do you like the programme?
L5	5.4 By being engaged in the programme, I have gained mathematical knowledge
R	[Follow up question] What sort of knowledge have you gained?
L5	5.5 I have gained knowledge of how to identify triangles and 5.6 also how to

	measure lines and triangles if you want to know the type of triangles.
R	[The follow up question seemed to have covered question 2 from the list, I then moved on to question 3] Did the programme that you have recently used help you to get a clear picture and explanation of how to identify the properties of the triangles in regard to their sides and angles?
L5	5.7 Yes sir
R	[Follow up question] How did it help you?
L5	[Smiled, took a deep breath, and then responded to the question] 5.8 It helped me to understand that a triangle has three sides and three angles.
R	[I immediately asked question 4] How did you investigate the properties of the triangles?
L5	5.9 By measuring as we were instructed in the activities.
R	[Question 5] Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
L5	5.10 In construction of triangles and angles
R	[Question 6] How can that topic be taught? Please, explain your answer in details.
L5	5.11 The cut pieces can be used to compare if two bisected angles are the same or different in sizes.
R	Do you have anything to say regarding the programme you were engaged in?
L5	5.12 No sir, but thank you for the skills you have taught us.
R	[Follow up question] Which skills, have you acquired from this programme?
L5	5.13 I have acquired measuring skills from the use of cut pieces of polygons
R	[After the recent response, I then thanked and allowed L7 to exit] Thank you for time, you may leave now.
L5	5.14 [L7, Left the interview room]

Key:

R: Researcher

L6: Learner 6

R	[When the L6 came in, I greeted the learner.] How are you doing?
L6	6.1 I am fine thanks, and you sir?
R	[I responded and then explained the aim for the interviews] I am also fine. Welcome to this short interview, I just want to hear from you how felt about the teaching and learning programme that you have recently used to learn geometry.
L6	6.2 Ok, sir
R	What are your comments regarding the programme you have recently used to learn geometry?
L6	6.3 I have no comment sir.
R	[Follow up question] Do you like the programme?
L6	6.4 Yes sir I like the programme.
R	[Follow up question] Why do you like the programme?
L6	[L6 looked down for some time and then responded] 6.5 I like it because last time when I was in grade 7, the teacher taught us properties of triangles, but I did not understand. 6.6 With what we have been doing I do understand now.
R	[Follow up question] What do you understand now?
L6	6.7 How angles in a triangle are related to each other, the same applies to the sides of the same triangles.
R	[Follow up question] For how long did you learn the topic of geometry in grade 7?
L6	6.8 I cannot remember sir.
R	[I then asked question 3 from the list] Did the programme that you have recently used help you to get a clear picture and explanation of how to identify the properties of the triangles in regard to

	their sides and angles?
L6	6.9 Yes sir
R	[Follow up question] How was that so?
L6	6.10 By measuring the sides and angles of triangles using triangle pieces. 6.11 And that was exciting to use cut pieces to learn how angles in a triangles are related to each other, also the sides.
R	[I asked question 4 from the list] How did you investigate the properties of the triangles?
L6	6.12 By measuring without using a protractor, we just compared one angle with other two angles and the side of a triangle with other two sides. 6.13 I discovered that if the angles are not the same in size and sides as well then the triangle is a scalene.6.14 If in a triangle, two angles are equal and two sides are equal then the triangle is an isosceles.
R	[Question 5 from the list was asked] Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
L6	6.15 Geometry sir
R	Geometry is broad, which part of geometry?
L6	6.16 To measure angles
R	Do you have comment?
L6	6.17 No sir
R	Thank you for your time, you may go. [L6 left the interview room]

Key:

R: Researcher

L7: Learner 7

R	[**L7** was called in for interviews and I greeted the learner] Next please, How are you doing?
L7	7.1 I am ok sir and you?
R	Please, feel free to respond to any question I will ask you. This is an interview regarding the mathematics programme that you have been doing for the past two weeks.
L7	7.2 Ok sir
R	After participating in this research project, what is your comment on the use of cut polygon pieces as physical manipulatives in teaching and learning of geometry, specifically with regard to the properties of triangles?
L7	7.3 No comment sir.
R	Do you like the programme that you have recently used to learn geometry?
L7	7.4 Yes, I do like it.
R	[Follow up question] Why do you like the programme?
L7	7.5 I have learnt many things, I now understand geometry.
R	[I then asked question 3] Did the programme that you have recently used help you to get a clear picture and explanation of how to identify the properties of the triangles in regard to their sides and angles?
L7	7.6 Yes sir
R	[Follow up question] How did the programme help you?
L7	7.7 The things that I did not understand clearly now I do.
R	[Follow up question] Things like what?
L7	7.8 In a scalene triangle all side are not equal. 7.9 In an isosceles two opposite side are equal and there are two angles equal 7.10 In an equilateral, all the angles are equal in size the same as the sides, they are equal in length.

R	[I then asked question 4 from the list of semi structured questions] How did you investigate the properties of the triangles using the programme?
L7	[Kept quiet for a while and then responded] 7.11 By using symbols which were shown on some of the shapes and also we cut the angles and sides of triangles out, we compared an angle with other angles and a side with other sides.
R	[Question 5] Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
L7	7.12 [kept quiet for a long time]
R	Do you understand the question?
L7	7.13 No quite clear sir
R	[I paraphrased the question] The way you have been learning geometry for the past two week, in which other mathematics topic can you use that way of learning?
L7	7.14 In algebra, sir
R	[Question 6] How can that topic be taught?
L7	[kept quiet or a long time and then answered] 7.15 No idea sir
R	[No explanation could be given to the response to question 6, I then allowed the learner to leave the room]. If you have no answer this is the end of the interview. You may leave now, thank you for time.
L7	7.16 Ok sir **L7** left the interview room].

Key:

R: Researcher

L8: Learner 8

R	[I called the eighth learner for the interview]. You are welcome; please feel free to answer all the questions I will ask you with no hesitation.
L8	8.1 [Looked at me and then said] ok sir
R	[Immediately I asked the first question]. After participating in this research project, what is your comment on the use of cut polygon pieces as physical manipulatives in teaching and learning of geometry? With regard to the properties of the triangles.
L8	8.2 I did not understand the properties of an obtuse triangle. Even the slashes that are used to show that two opposite sides of a isosceles triangle are equal, 8.3 I did not know the meaning of such slashes, but now after your programme it is clear to me.
R	[Follow up question]. Is there any other comment you would like to make regarding this?
L8	8.4 No, sir
R	[I then asked question 2] What do you like about the use of physical manipulatives in teaching and learning about the properties of the triangles?
L8	8.5 I like them because they have helped me to understand the properties of triangles
R	[Follow up question] Please elaborate, how did it help you?
L8	[He looked at me for a while then responded] 8.6 In grade 7 I did not learn the properties of triangles, but with what we have done 8.7 now I understand and know what to do.
R	[Follow up question] Are you saying that the teacher did not teach you the properties of triangles in grade 7 completely?
L8	8.8 No, sir nothing was done.

R	[Question 3] Did the programme you have used help you to get the clear picture and explanation of how to identify the properties of the triangles in regard to sides and angles?
L8	8.9 Yes, sir
R	[I then asked question number 4] How did you investigate the properties of the triangles?
L8	[L2 looked up the ceiling seemed puzzled with the question and then answered] 8.10 We cut the angles and lines for triangles and measure them to find out if they were the same or different in sizes and length.
R	[Follow up question] Do you have any other response besides the one you have just given?
L8	8.11 No, sir.
R	[I then asked question 5] Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
L8	[Kept quiet for a while and said] 8.12 Please, repeat the question.
R	[I paraphrased the question] In which other mathematics topic can you use what you have just used to learn mathematics?
L8	[L2, could not give the answer immediately, kept on repeating the word contra…….for a long time up until the he said it all] 8.13 Contra, contra, contra,………… Contraction.
R	[Follow up question for clarity on the word contraction] What do you mean by contraction?
L8	8.14 [Explained by demonstrating using a pencil on the desk]
R	Is it construction?
L8	8.15 Yes, sir, construction of angles and triangles can fit into the programme you

	have used
R	[I asked the question] How can that topic be taught?
L8	8.16 It can be used to measure the angles whether they are equal or not equal after construction of shapes.
R	What else can be taught using this programme?
L8	8.17 Lines and angles sir [seemed to be repeating the same thing. I had to move on to the next question]
R	Is there anything you want to say regarding the use of cut pieces of polygon that have been used to learn geometry?
L8	8.18 No sir.
R	Thank you for coming to the interview.
L8	8.19 Immediately, L8 left the room.

Key:

R: Researcher

L9: Learner 9

R	[I called for another learner, L8 came in and I greeted the learner] How are you doing?
L9	9.1 I am fine thanks and you sir?
R	I am also fine. Please relax; I am going to engage you in a sort interview regarding the programme you have recently used to learn mathematics.
L9	9.2 Ok sir
R	[I asked the first question from the list] What are your comments regarding the programme you have recently used to learn geometry?
L9	9.3 I do not have any comment sir.
R	[Question 2] What do you like about the programme you have recently used in learning the properties of triangles?

L9	9.4 Yes sir, I like the programme, it is a very good programme of teaching and learning.
R	[Follow up question] Why do you like the programme?
L9	[Smiled and scratched the nose, kept quiet for a while and then responded] 9.5 We were able to learn about the properties of a triangle using pieces of the same triangle.
R	[Follow up question] What else did you learn?
L9	9.6 It has helped me to know how to identify whether a given triangle is an isosceles, an equilateral or a scalene.
R	[Follow up question] Did you learn about types of triangles and their properties in grade 7?
L9	9.7 No sir
R	[Follow up question] Have you ever used such a programme to learn mathematics?
L9	9.8 No sir, this is the first time I have learnt mathematics using cut pieces of shapes.
R	[I then asked question 3 from the list] Did the programme that you have recently used help you to get a clear picture and explanation of how to identify the properties of the triangles in regard to their sides and angles?
L9	9.9 Yes sir.
R	[Follow up question] How did it help you?
L9	9.10 It tells us that some triangles have three sides and two opposite sides are parallel. 9.11 And other triangles all three sides are not equal.
R	[Follow up question] Do you know how parallel lines look like?
L9	9.12 Yes sir
R	Give me an example from any shapes you know.
L9	9.13 Two sides in triangles that are facing each other.
R	[Follow up question]

	Any other example?
L9	9.14 No sir
R	[I decided to move on to question number 4 from the list] How did you investigate the properties of the triangles using the programme?
L9	[L8 responded, but not to the asked question] 9.15 Right angle has 90 degrees
R	[Follow up question] Do you understand the question that I have asked you?
L9	9.16 Yes sir I do understand it.
R	[I decided to paraphrase the question] What were you actually doing during the lesson?
L9	9.17 Ok sir, we were cutting out angles and sides of triangles, we used those pieces to compare the sides of a given triangle and also the three angles of a triangle.
R	[Follow up question] From such activities what did you learn?
L9	9.18 I have learnt that we can learn the properties of a triangle just by measuring its angles and 9.19 sides with cut out pieces of angle and sides before we actually use a protractor.
R	[Question 5 was then asked from the list] Apart from learning the properties of the triangles, what other mathematics topic(s) can you learn using these physical manipulatives?
L9	9.20 There is no other topic I have in my mind sir.
R	[I thanked the learner for accepting to be interviewed] Thanks fr your time and for accepting to be interviewed.
L9	9.21 Ok sir. [L9 left the interview room immediately].

Appendix 31: Intervention activities questions that individual learners correctly answered

Question number	**Van Hiele's levels**	**Learners**	**Activity done**
1.1	0	L1, L6 and L7	were able to identify all triangles from the set of 2-dimensional shapes.
1.2	3	L1, L3, L4 and L7	To describe how a triangle looks like
1.3	2	L8	The identification of all four scalene triangles was successfully done
1.4	3	L1, L2, L6, L7, L8 and L9	Mention the property used to group triangles
1.5	3	none	All learners could not identify another property of a triangles
1.6	2	L1, L4 and L8	Identified the triangle with two opposite sides equal as an isosceles.
1.7	1	L1, L3, L4, L6 and L7	Managed to draw and correctly mentioned the names of the four different triangles namely: isosceles, equilateral, scalene and right-angled triangles.
2.1 2.1.1	1	L1, L2, L3, L4, L5, L6, L8 and L9	learners were asked to identify and categorise the 10 triangles into five main groups based on their angle properties
2.1.2.1	1	L1, L2, L3, L4, L6, L7 and L9,	were able to identify all the isosceles triangles
2.1.2.2	1	L1, L2, L3, L4, L5, L6, L7, L8 and L9	identification of equilateral triangles
2.1.2.3	1	L1 and L8	the identification of 'a', 'e', 'i' and 'j' as acute-angled triangles
2.1.2.4	1	L1, L2, L3, L5, L6 and L9	Identified obtuse angled triangles.
2.1.2.5	1	L1, L2, L3, L4, L5, L6, L7, L8 and L9	To identify triangles labelled 'c', 'd', 'f' and 'h' as right-angled triangles.
3.1.1	0 and 1	L1, L3, L4 and L8	Identified triangles labelled: 'a' 'e', 'g', 'h' and 'n' as isosceles.
3.1.2	1	L1, L3, L5 and L7	identification of equilateral triangles
3.1.3	1	L1, L2 and L9	To identify obtuse-angled triangles by estimation.
3.1.4	1	L5	to identify right-angled isosceles triangles
3.1.5	0	L4	identify the right-angled triangles from a set of different types of

			triangles
4.1.1	1	L1, L3, L4, L6 and L8	managed to match the triangle with the correct statement as required
4.1.2	1	L1, L2, L3, L4, L6 and L8	managed to match the triangle with the correct statement as required
4.1.3	1	L1 and L4	managed to match the triangle with the correct statement as required
4.1.4	1	L1, L3, L4 and L8	managed to match the triangle with the correct statement as required
4.1.5	1	L1	managed to match the triangle with the correct statement as required
4.1.6	1	L1	managed to match the triangle with the correct statement as required
5.1(i)	1	L1, L2, L3, L4, L5, L7 and L9	Learners compared $\overline{AB}$ with $\overline{AC}$; $\overline{AC}$ with $\overline{BC}$ and $\overline{BC}$ with $\overline{AB}$
5.2(i)	1	L1, L3, L4, L5, L6, L7, L8 and L9	compared an angle's size with the sizes of the other two angles
5.2(ii)	1	L1, L3, L4, L5, L7 and L8	$\hat{B}$ *is bigger than* $\hat{C}$
5.2(iii)	1	L1, L2, L3, L4 and L5	$\hat{B}$ *is bigger than* $\hat{A}$
5.2(iv)	1	L1, L3, L4, L5, L7 and L8	To give the properties of ΔABC in terms of: $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$.
5.2(v)	2	L1, L4, L7 and L8	required the learners to determine the properties of ΔABC in terms of, $\hat{A}$, $\hat{B}$, and $\hat{C}$
6.1(iii)	1	L4, L5 and L6	learners compared the lengths of the line segments using the pieces of polygon
6.2(i)	1	L1, L2, L3, L4, L5, L6, L8 and L9	managed to write the correct comparison between $\hat{G}$ and $\hat{H}$
6.2(ii)	1	L1, L3, L4, L5 and L6	managed to write the correct comparison between $\hat{G}$ and $\hat{I}$
6.2(iii)	1	L4, L5 and L6	managed to write the correct comparison between $\hat{H}$ and $\hat{I}$
6.2(iv)	2	L1, L4, L5, L7, L8, and L9	required the learners to mention the line segment property of ΔGHI
6.2(v)	2	L1, L2, L4 and L9	learners were required to give the angle property of ΔGHI
6.2(vi)	2	L4	Learners were supposed to mention a specific name given to a triangle with properties mentioned in 6.2 (iv) - (v).
7.1(iii)	1	L3, L4, L5 and L6	learners compared the lengths of $\overline{DE}$ with $\overline{EF}$, $\overline{DE}$ with $\overline{DF}$ and $\overline{DF}$ with $\overline{EF}$ using polygon pieces
7.2(i)	1	L1, L2, L3, L4,	cut out angles one at a time and compared its size with the other

		L5, L8 and L9	two angles $\hat{D}$ *is equal to* $\hat{F}$.”
7.2(ii)	1	L1, L2, L3, L4, L5 and L8	$\hat{D}$ *smaller than* $\hat{E}$
7.2(iii)	1	L3, L4, L5, L8, and L 9	$\hat{E}$ *is greater than* $\hat{F}$
7.2(iv)	2	L2, L3, L4 and L5	To describe the properties of ΔDEF based on $\overline{DE}$, $\overline{EF}$ and $\overline{DF}$.
7.2(v)	2	L3, L4 and L5	to use the knowledge gained from the questions 7.2(i) to (iii) in order to give the angle property of ΔDEF.
7.2(vi)	2	L1, L4 and L6.	a specific name given to ΔDEF based on properties identified in question 7.2(iv) and 7.2(v)
8.1(i)	0	L1, L2, L4, L5, L6, L7, L8 and L9	compared by estimation the sizes of $\hat{X}$, $\hat{Y}$ and $\hat{Z}$
8.1(ii)	0	L1, L2, L4, L5, L6, L7, and L8	compared by estimation the lengths of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$
8.1(iii)	0	L1, L2, L4, L5, L6, L7, L8, and L9	In ΔXYZ compared $\overline{XY}$ with $\overline{YZ}$, $\overline{YZ}$ with $\overline{XZ}$ and $\overline{XZ}$ with $\overline{XY}$
8.2(i)	1	L1, L2, L3, L4, L5, L6, L7, L8 and L9	In ΔXYZ determined the relationships between $\hat{X}$ and $\hat{Y}$
8.2(ii)	1	L1, L2, L3, L4, L5, L6, L8 and L9	In ΔXYZ determined the relationships between $\hat{Y}$ and $\hat{Z}$
8.2(iii)	1	L1, L2, L3, L4, L5, L6, L8 and L9	In ΔXYZ determined the relationships between $\hat{X}$ and $\hat{Z}$,
8.3(i)	3	L1, L2, L4, L5, L6 and L7	Learners were supposed to give the properties of ΔXYZ in terms of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$.
8.3(ii)	3	L1, L4, L5, L6, L7, L8 and L9	Required learners to give the properties of ΔXYZ based on the angle relationships investigated in question 8.1 (i) and 8.2 (i) to (iii).
8.4	2	L1, L2, L4, L5, L6, L7 and L9	Learners were to give the name of the ΔXYZ.
9.1(i)	0	L1, L4, L5, L8 and L9	In ΔPQR learners estimated and compared: (i) the sizes of $\hat{P}$ and $\hat{Q}$, using the terms: ‘equal to, greater than and smaller than’
9.1(ii)	0	L,5 and L7	In ΔPQR learners did estimations and compared the lengths of $\overline{PQ}$, $\overline{RP}$ and $\overline{QR}$, in comparison of the line segments they used these terms: ‘longer than, equal to and the longest of all’.
9.1(iii)	0	L1, L5, L7 and L9	In ΔPQR learners compared $\overline{QR}$ with $\overline{PR}$, $\overline{QR}$ with

			$\overline{QP}$ and $\overline{QP}$ with $\overline{PR}$
9.2(i)	1	L1, L2, L3, L4, L5, L6, L7, L8 and L9	In ΔPQR learners compared the size of $\hat{Q}$ with $\hat{P}$
9.2(ii)	1	L1, L3, L4, L5, L6 and L8	In ΔPQR learners compared the size of $\hat{Q}$ with $\hat{R}$
9.2(iii)	1	L1, L3, L4, L5, L6 and L8	In ΔPQR learners compared the size of $\hat{P}$ with $\hat{R}$
9.3(i)	3	L2, L3, L4, L7 and L9	required learners to give the properties of ΔPQR based on the edges: $\overline{QR}$, $\overline{PR}$ and $\overline{QP}$
9.3(ii)	3	L3, L4 and L6	required learners to give the properties of ΔPQR based on the angles: $\hat{Q}$, $\hat{P}$, and $\hat{R}$,
9.4	2	L1, L2, L4, L5, L6, L7, L8, and L9	Learners managed to give a specific name of ΔPQR based on properties mentioned in question 9.3(i) and 9.3(ii)

Appendix 32: Reflective test questions that individual learners correctly answered

Question number	Van Hiele's levels	Learners	Activity done
RT 1			
1.1	0	L1, L2, L4, L5, L6 and L7	Identified triangles from the pool of different 2-dimensional shapes.
1.2	1	L1; L3 and L7	Categorised the identified triangles in question 1.1 into: scalene
	1	L1, L2, L3, L5and L7	Categorised the identified triangles in question 1.1 isosceles,
	1	L1, L2, L3, L5, L7 and L9	Categorised the identified triangles in question 1.1 equilateral
	1	L2, L3, L5 and L8	Categorised the identified triangles in question 1.1 right-angled triangles
1.3	1	L1, L3, and L7	Described how each of the triangles looks like i.e. a scalene in their own words.
	1	L1, L2, L3, L5 and L7	Described how each of the triangles looks like i.e. an isosceles in their own words.
	1	L1, L2, L3, L5, L7 and L9	Described how each of the triangles looks like i.e.an equilateral in their own words.
	1	L2, L3, L5 and L5	Described how each of the triangles looks like i.e. a right-angled, in their own words.
RT 2			
2.1.1	1	L1	Identified all four scalene triangles
2.1.2	1	L1, L2, L3, L4, L5 and L8	Identified four isosceles triangles
2.1.3	1	L1, L3, L4, L5 and L9	Identified equilateral triangles from the given set of triangles.
2.1.4	1	L1, L3, L4, L5 and L9	Identified right-angled isosceles triangles, from the given set of different triangles,
2.1.5	1	L1, L3 and L9	Identified obtuse-angled triangles
2.1.6	1	L4	Identified a right-angled scalene triangle
2.2.1	2	L1, L3 and L9	Described the angles property of a scalene triangle,
2.2.2	2	L2, L3, L4, L8 and L9	Described the angle property of an isosceles triangle
2.2.3	2	L4, L6 and L8	Described the angle property of an obtuse-angled triangle
2.2.4	2	L4 and L5	Described how an acute angled triangle looks like based on the angle property
2.2.5	2	L1,L3,L4,L5,L6 and L9	Described the properties of a right-angled triangle,
RT 3			
3.1.1	1	L2, L4, L5, L6, L7, L8 and L9	Showed that triangle labelled 'a' is an isosceles using all symbols for an isosceles triangle.
3.1.2	1	L4, L5, L6, L7 and L8	Showed that triangle labelled 'b' in question 3.1.2 is an equilateral.
3.1.3	1	L3, L4, L5 and L7	Indicated that triangle labelled 'c' is a right-angled

			triangle by using a right angle symbol
3.1.4	1	L4 and L5	to show that triangle labelled 'd' is a right-angled isosceles.
3.2.1	3	L1, L4, L5 and L7	Managed to give two descriptions of how a right-angled scalene triangle looks like
3.2.2	3		identification
3.2.3	3	L1 and L5	One description of an obtuse angled triangle
3.2.4	3	L2, L5 and L7	Described a scalene based on its two properties.
TEST 4			
4.1	0	L2, L4, L5 and L9	Managed to identify three acute-angled triangles
4.2	0	L5	Identified scalene triangles from the set of six,
TEST 5			
1	2	L4, L5, L7 and L8	Indicated all the three properties of a right-angled scalene triangle.
2	2	L2, L4 and L5	Have given three properties of an obtuse-angled scalene triangle,
3	2	L4, L7 and L9	Have mentioned mention two properties of an acute-angled-scalene triangle.
4.1	0	L1, L4 and L5	Identified ΔABC as an acute-angled triangle.
4.2	0	L1, L4 and L9	Mentioned that ΔDEF is an obtuse angled triangle.
4.3	0	L1, L3, L4 and L5	to respond to the question with the correct response, ΔLMN is a right-angled triangle
TEST 6			
6.1	1	L1, L4 and L6	Required learners to use a ruler, a protractor and a pencil to draw a right-angled isosceles triangle and insert necessary symbols
6.2	2	L4, L8 and L9	Have mentioned three properties of a right-angled isosceles triangle
TEST 7			
7.1	1	L5 and L7	Have drawn an equilateral triangle and then insert all the symbols that describe it
7.2	2	L1, L3, L4, L5, L6, L7, L8 and L9	Managed to mention the properties of an equilateral triangle