CHAPTER 1

ORIENTATION

1.1 INTRODUCTION/BACKGROUND

The junior secondary school curriculum is in many ways a bridge between primary education with its concreteness and the more formal high school curriculum. The mathematics syllabus covers such areas as numbers, shapes, measurement, transformations, sets, ratio, algebra, probability and statistics.

According to MOET (2002: 2), the aims of teaching mathematics are to:

- provide students with knowledge and skills by enhancing their abilities to think logically and analytically;
- promote a positive attitude towards the subject as mathematics provides investigative environment that stimulates curiosity to investigate and solve problems.

Main themes that are stressed in the teaching of mathematics are thus:

- Knowledge and skills;
- Application and problem-solving;
- Appreciation of the environment.

(MOET 2002:2).

The aims of teaching Mathematics are to prepare students for life. They must provide knowledge, skills, power to think logically and analytically. Mathematics teaching should enhance curiosity, develop pupils' ability to solve problems in many spheres. Mathematics teaching should promote willingness to investigate, enjoyment and a feeling of confidence, in the pupils.

Knowledge and skills: This refers to the content knowledge that pupils should acquire in the learning of mathematics. Pupils should develop:

a) an understanding of numbers and the ability to use them confidently.

b) an understanding of spatial(things relating to size, area or position relationships between shapes, spaces or areas) concepts.

c) an understanding of measurement and the ability to estimate approximate and measure.

d) an ability to collect, represent and interpret data.

e) Manipulative skills in Mathematics.

Application and problem-solving: The Cockroft Report (1982:73) indicates that ‘‘ the ability to solve problems is at the heart of mathematics.’’ It further points out that ‘‘the idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very many fields.’’ It is imperative therefore, that pupils should be helped to develop abilities to apply mathematical skills and techniques of investigation to solve problems in mathematics, other school subjects and in various areas of everyday life.

Appreciation of the environment: This refers to teaching mathematics such that pupils are provided the opportunity to develop positive attitudes towards the subject. Pupils should develop awareness and appreciation of mathematical patterns embedded in their socio-cultural environment. They should develop the ability to apply mathematical skills to address issues in the environment they live in.

Knowledge of mathematics is essential in order to live meaningfully in this technologically, rapidly growing society. Mathematics is used in our homes, in industry, in business, in sports, everywhere. Psychologists consider mathematics to be a “ … particularly concentrated and powerful example of the functioning human intelligence" (Harre & Lamb 1986:154). It is a product of many minds in response to man's needs and curiosity. In learning mathematics, the concepts have to be constructed anew by an individual in his own mind. Once acquired, mathematics can greatly increase man's ability to understand and control his environment. It is necessary therefore that all children are helped to acquire some form of mathematics to their potential levels.

According to a document for Curriculum 2005 of South Africa, mathematics is described as "the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem -solving, logical thinking, etc., in an attempt to understand the world. This understanding is expressed, developed and contested through language, symbols and social interaction" (National Department of Education 1997:108).

The UK document, Mathematics for ages 5 to16, cited by Pimm (1995:10) offers the following explanation about Mathematics:

> In the broadest sense, mathematics provides a means for organizing, communicating and manipulating information. The language of mathematics consists of diagrams and symbols, with associated conventions and theorems.

According to Kieran (1992: 391), algebra is conceived as a branch of mathematics that deals with symbolizing and generalizing numerical relationships and mathematics structures, and with operating within those structures. Van de Walle (2004: 417) notes, "algebraic reasoning involves representing, generalizing, and formalizing patterns and regularity in all aspects of mathematics." Doing mathematics includes among many other processes, representing mathematical situations in multiple ways, investigating, formalizing patterns and regularities, making generalizations and solving mathematical problems. It can be observed from these that understanding of algebra is central to pupils' ability to do mathematics.

Traditionally pupils only begin learning algebra when they enter secondary school, approximately at the age of 15. At junior secondary level, typical topics treated under algebra include algebraic representation and formulae, graphs, relations and function notation, solution of linear equations and inequalities, indices, and sets. Reports from research projects conducted by mathematics educators such as Kuchemann (1981: 104), Kieran (1989: 39), Linchevski and Herscovicks (1996), indicate that learners at this level experience serious problems in understanding pre-algebraic concepts. The late start in algebra learning, only taking place in the junior secondary school, could be another source of pupils' difficulties in dealing with algebraic concepts even beyond high school education. According to MacGregor (1990:1), algebra learning should not await high school freshmen or the precocious eight graders as teaching of algebra through the use of concrete models is well within intellectual grasp of primary school pupils.

As established from the studies such as those conducted by Phillip and Schappelle (1999:312), Usiskin (2003:137), and observed from the researcher's experience as a mathematics teacher and marker in the J.C. (Junior Certificate) examinations, solution of linear equations poses a lot of problems to pupils at this level. The main problem here seems to be that most pupils tend to use algorithms associated with transformations of equations to simpler equivalent forms incorrectly. The results of the study that Miles (1999) conducted indicate that " there is a possibility that some students who have learning problems in Mathematics have difficulty in Algebra not because their difficulty is mathematics-based, but because it is language–based, thus delaying their assimilation and comprehension of mathematics instruction" (Miles1999: 46).

Successful learning of mathematical concepts and skills is a function of the approaches and strategies that teachers use in their teaching. The manner in which mathematics is taught, to a large extent, is influenced by the perceptions that teachers have about the subject and of what they believe good teaching to be.

Traditionally, Mathematics was viewed as a "well-defined field of inquiry that is timeless and unchanging, its bedrock being a reasoning process considered infallible" (Cooney & Shealy1997: 89). The goal of mathematics teaching, on the other hand, "has been to teach students to become fluent in applying a set of discrete procedures in order to solve problems. Within this orientation, Mathematics is considered to be a set of memorized rules and procedures for correctly solving certain qualitative problems” (Goldsmith & Schifter 1997:32). In recent years there is a shift in views on the nature of Mathematics and Mathematics learning and thus teaching.

Mathematics is now seen by many as a human activity, a way of thinking about the external world; a category of constructing meaning. It is viewed as a function of our experience and far more tentative and uncertain than it was previously thought. According to socio-constructivism, “knowledge is the dynamic product of the work of individuals operating in the communities, not a solid body of immutable facts and procedures independent of mathematicians. In this view learning is considered more as a matter of meaning-making and of constructing one’s own knowledge than of memorizing mathematical results and absorbing facts from the teacher’s mind or the textbook; teaching is the facilitation of knowledge construction and not delivery of information” (Jones 1997: 145).

The problem-centered approach selects constructivism as its approach to learning. In a problem-centered learning environment, it is accepted that pupils construct their own knowledge and therefore attempt to establish individual and social procedures to monitor and improve the nature and quality of those constructions. According to Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier and Wearne (1996: 12) teaching should be geared towards “making the subject problematic”. Teaching should allow “students to wonder why things are, to inquire, to search for solution and to resolve incongruities. It means that both curriculum and teaching should begin with problems dilemmas and questions for students” (1996:12). The constructivist theory of learning, indicates that effective learning can only take place when pupils are given chance to grapple with problems, they reflect on their solution procedures and then check the reasonableness of their results.

1.2 STATEMENT OF THE PROBLEM

Algebra is a powerful problem-solving tool (Nickson, 2000:109), therefore understanding of algebra is central to pupils’ ability to do mathematics. It follows from this that, in order to improve pupils performance in mathematics in general, the teacher should enhance a profound understanding and acquisition of algebraic concepts and thinking skills. According to Kieran (1989:39), difficulties that pupils have in algebra are centered around:

- The meaning of letters;
- The shift to a set of conventions different from those used in arithmetic; and
- The recognition and use of algebraic structure.

From past experiences, one has realized that pupils, even today, experience great difficulties when solving problems that involve linear equations. Nickson (2000:112-113) also points out that pupils encounter difficulties when solving problems that involve manipulation of algebraic expressions and equations. It is imperative, therefore, that an investigation is carried out to identify and critically analyze the problems that pupils and teachers encounter at the early stages of algebra learning, and in understanding linear equations.

In an attempt to find the answer to the above problem, the study sought to provide answers to the following sub-problems:

- What background knowledge, concepts and sub-concepts are necessary for understanding of linear equations?
- Do Grade 9 pupils have the background knowledge necessary for understanding of the linear equations?
- What obstacles do the pupils encounter in understanding the concepts and sub-concepts underlying solution of linear equations?
- What opinions do Grade 9 algebra teachers have regarding pupils' difficulties in understanding linear equations?
- Do Grade 9 algebra teachers have clear understanding of the concepts and sub-concepts in linear equations?
- Is there any correlation between teachers' content knowledge and pupils' understanding of linear equations?
- What teaching methods are preferred in the teaching of mathematics?

1.3 AIMS AND OBJECTIVES OF THE STUDY

The purpose of the research was to investigate the problems that teachers and pupils of Grade 9 algebra encounter on understanding linear equations. The aim of the study was therefore

- To identify the problems that teachers and pupils of Grade 9 algebra encounter in teaching and learning linear equations respectively.

This aim was achieved through the following objectives:

- To identify the relevant background knowledge, concepts and sub-concepts required for the understanding of linear equations.
- To assess pupils' understanding of the above identified concepts;
- To assess pupils' problems in understanding linear equations
- To assess teachers' perceptions and opinions regarding pupils' difficulties in understanding linear equations;
- To assess teachers' content knowledge related to linear equations.
- To investigate if there is any correlation between teachers' content knowledge and pupils' understanding of linear equations?
- To assess the methods of teaching and learning mathematics preferred by teachers and pupils respectively.

1.4 RATIONALE/ MOTIVATION

Examination results in mathematics across the different levels of secondary education indicate that pupils encounter a lot of problems in acquiring mathematical knowledge and skills. From the researcher's experience as a teacher and marker in the junior and senior secondary school mathematics examinations, it was realized that candidates at both levels displayed inability to handle problems that involved formulation and/or manipulation of algebraic expressions and equations. Solving algebraic word problems has been identified as the greatest problem for most pupils.

One possible source of these problems could be pupils' lack of adequate mathematical vocabulary to develop, express and contest their mathematical knowledge and understanding. Mathematical language consists of symbols, terminology, notations, conventions, models and expressions that can only be interpreted by the mathematically literate. The branch of mathematics where this language is mostly dominant is algebra.

Since mathematics is hierarchical in nature, understanding of higher order mathematical concepts is dependent on proper understanding of related lower order concepts. The research therefore, is intended to identify the problems that learners encounter in the early stages of learning algebra. The investigation into Grade 9 pupils' ability/inability to understand linear equations, both from the pupils' and teachers' perspective, was intended to bring out and expose these problems. At the end of the study some recommendations were made regarding the teaching of pre-algebraic concepts and sub-concepts underlying linear equations. Appropriate interventions by the teacher, targeted to address the identified problems would facilitate better acquisition of mathematical concepts by pupils even in the later years of study in the subject and/or related fields.

1.5 RESEARCH METHODOLOGY

1.5.1 Research Design and Methodology

This study is a quantitative survey research. A survey research may be used to learn about aspects such as people's behavior, attitudes, believes, values, habits, ideas, and opinions. This type of research is popular in education due to efficiency and generalizability. Accurate information for large population can be obtained with a small sample at relatively low costs (Macmillan & Schumacher 2001:304-305). In this study, the problems that Grade 9 pupils encounter in understanding linear equations was investigated through observations of the manner in which pupils solve linear equations. The solution procedures that pupils use when solving linear equations that require them to transform algebraic expressions by expanding brackets or factorizing, collecting and combining like terms, were analyzed quite critically. The way pupils tackle word problems that give rise to linear equations was also looked into carefully. The study also involved gathering information about teachers' and pupils' views on practices related to the teaching and learning of linear equations.

This research may also be considered evaluative. An evaluation research is meant to determine the merit and worth of a particular practice. It determines whether the practice works and is worth adopting, and may stimulate further research relating to the practice under study

(Macmillan & Schumacher, 2001:22). In this particular research, teaching methods preferred by teachers and pupils for teaching and learning of mathematics were assessed and some recommendations were made, based on the findings from the teachers and pupils participating in the study.

1.5.1.1 Sampling

According to de Vos (2002:145), in a survey research, the researcher must first identify the research population, after which data collection methods may then be used to gather information. In this study, the population is all Grade 9 pupils in schools. As this is definitely a very large population to handle, it was therefore necessary to work with a sample of the population. A sample is a group of subjects or persons selected from the target population. This is a group of individuals with the same characteristics as the target population and is trusted to provide the relevant data as it would be obtained from the whole population.

Four schools were chosen for the study. Random sampling was used to draw a sample of twenty (20) pupils from each school with a total of eighty (80) pupils in the study. A table of random numbers was used in this regard. The sample used represents approximately ten percent (10%) of the population size in each school, which is the sample size supported by de Vos (2002:202). As the study was on investigating problems that pupils and teachers of Grade 9 algebra encounter on understanding linear equations, it was necessary that information from teachers of mathematics at this level be obtained. Between three to four teachers were involved in each school, ending up with a total of fifteen (15) teachers participating in the study.

1.5.1.2 Literature Review

The researcher conducted a critical literature research. According to Ary, Jacobs and Razavieh (1990), review of related literature serves several important functions:

1. Knowledge of related research enables the investigators to define the frontiers of their field.
2. A thorough review of related theory and research enables researchers to place their questions in perspective.
3. Reviewing related literature helps researchers to limit their question and to clarify and define the concepts of the study.
4. A critical review of related literature often leads to insights into the reasons for contradictory results.
5. Through studying related research, investigators learn which methodologies have proved useful and which seem less promising.
6. A thorough search through related research avoids unintentional replication of previous studies.
7. The study of related literature places researchers in a better position to interpret the significance of their own results.

1.5.1.3 Instrumentation

Data was collected by means of tasks and questionnaires.

Tasks. Pupils were given a test containing some multiple-choice items and items that required them to show their working towards the solutions to the problems. The multiple-choice items were designed such that their solution required procedural and structural operations. The context problems on the other hand, were not only designed to focus on to pupils' ability to perform the mentioned operations, but also to assess their ability to interpret the questions correctly and formulate correct mathematical representation, using symbolic notation.

Questionnaires. There were two different questionnaires; one for the pupils and one for the teachers. These were intended to get both teachers' and pupils' perceptions and attitudes towards mathematics teaching/learning with special emphasis on pre-algebraic concepts and solving linear equations. Some items in the questionnaires also focused on assessing teachers' and pupils' content knowledge related to linear equations. Responses were on a 5-point Lickert scale. There was an open-ended question at the end of each questionnaire to give the respondents opportunity and space to discuss their experiences regarding solution of algebraic word problems.

1.5.1.4 Pilot Testing

A pilot study is a " … small study conducted prior to a larger piece of research to determine whether the methodology, sampling, the instruments and analysis are adequate and appropriate" (Bless & Higson-Smith 2000: 155). This mini-research is intended to expose deficiencies of the measuring instruments or the procedure to be followed in the actual project.

In this study, the piloting was achieved by consulting experts in the fields and preliminary exploratory study. The researcher in this particular study, referred to colleagues in the field and the supervisor for advise particularly on the wording of the statements in the questionnaires and items in the pupils' tasks.

The purpose of the preliminary exploratory study is to get a clearer picture of the demands of the investigation with regard to time, finance and transport factors. The amount of time within which the pupils' tasks may be completed was observed during piloting. Necessary corrections, e.g. wording of certain items were made due to feedback from the pilot study. This exercise is very helpful as it makes the researcher aware of any possible unforeseen problems that may emerge during the main investigation.

1.5.1.5 Strategy for Data Presentation and Analysis

After the data was collected, it was then analyzed. The analysis of pupils' responses to the tasks focused on the nature of the responses exhibited per item in the test. These were expressed in terms of their frequencies (percentages). Responses to the context problems were analyzed paying particular attention to pupils':

1. ability to interpret the question/text correctly;
2. ability to identify the variable;
3. ability to formulate a correct algebraic equation;
4. ability to choose a suitable strategy leading to the correct solution, i.e. correct structural and procedural operations; and
5. ability to interpret the solution in terms of the original question.

A comparison of responses from teachers and pupils on statements in the questionnaires that bear the same opinion was also made. Correlation between the responses on these items was then investigated. The Spearman Rank method was used in this regard.

1.6 DEMARCATION OF STUDY

The target population was Grade 9 pupils as the topic under investigation is taught at this level. In most cases, linear equations are usually dealt with towards the end of the school year and sometimes the topic is deferred to the beginning of the next school year, when the learners will then be in Grade 10. It is for this reason that grade 10 learners were used in the study.

The research was conducted in four (4) high schools in the Maseru area. The schools are very easy to reach and were chosen because it is where the researcher has good contacts. Two (2) of the schools are in the Maseru city area, while the other two are in the outskirts, but alongside the main road hence easily accessible. It needs mentioning that even though the two sets of schools are located at different places, there is no significant difference in the physical as well as human resources available in the schools. The numbers of mathematics teachers in the selected schools are almost the same. The language of instruction in all the four schools is English and this is the case with the rest of the high schools in the country.

The researcher had taught in one of the two schools outside town, and it was during that time that she became familiar with the mathematics teachers in the neighbouring schools. Teachers in this region used to work together, planning their teaching, developing instructional materials together during their weekly meetings. It was in those meetings that teachers shared their teaching experiences and even arranged class visits, through which they observed each other's lessons and gave advises as necessary. The school visits by the LCE (see section 1.7) staff supervising student teachers on teaching practice, also provided the researcher the opportunity to become more familiar with teachers in the other two schools in town.

1.7 LIST OF ACRONYMS USED

The following abbreviations will be used in the script:

MOET ---	Ministry of Education and Training
INSET ---	In-service Education and Training
NCTM ---	National Council of Teachers of Mathematics
LCE ---	Lesotho College of Education
ECOL ---	Examinations Council of Lesotho

1.8 CLARIFICATION OF TERMS

Pre-algebra: This refers to the basic algebraic concepts taught to learners both at the elementary and secondary school level preparing them for the more rigorous algebra that they may have to do at tertiary level. This includes concepts such as variable, terms (like and unlike) and expressions. The skills that pupils must acquire in relation to these concepts are simplifying, expanding, factorizing and collecting like terms.

A first-degree polynomial equation: A polynomial equation of the first degree in x is an equation of the form

$$a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0 = 0$$

where

$a_n, a_{n-1}, \dots, a_1, a_0$ are coefficients and the highest power of x with a non-zero coefficient is 1, that is, $n = 1$.

Grade 9: the ninth class in the formal school system, after kindergarten. This is equivalent to Form B according to the system in Lesotho.

Junior secondary education: the formal education that a learner receives after the primary education, before higher secondary education. This normally takes 3 years.

Learning problem: an obstacle in the learning process. It is a cognitive hiccup or hurdle that delays or prevents the learner from understanding a learnt concept.

Learners: the term 'learners' is used in the title of the dissertation and that reflects the preference in South Africa. However, the term 'pupils' will be used in the text because that is the common term for learners up to Grade 10 in Lesotho.

1.9 OVERVIEW OF THE STUDY

In chapter 1, the background to the study is provided. Orientation regarding the position of Mathematics in the school curriculum and the role of algebra in the learning of Mathematics is given. The problem is formulated and outlined. The aim of the study is also provided and broken into accompanying objectives to be achieved.

In chapter 2, information from reviewed literature in the area of study is given.

Chapter 3 provides a description of the research design and methodology. This outlines the procedure followed in conducting the research, paying particular attention to the design, sampling procedures and methods used in the collection of data.

In chapter 4 data analysis and discussion of research findings are provided.

In chapter 5 the summary of the study and the conclusion and recommendations are found.

1.10 CONCLUSION

This chapter has provided the background and orientation to the study, leading to the statement of the problem. The problem was then broken down into sub-questions. The research aims and objectives were formulated to address the research problem and its sub-questions. A description of the research design and the methods that were used in collecting data were provided. Finally an overview of the investigation was outlined.

The next chapter provides an in-depth review of the literature relating to the topic under investigation.

CHAPTER 2

LITERATURE REVIEW ON THE LEARNING AND TEACHING OF LINEAR EQUATIONS

2.1 INTRODUCTION

In Chapter 1, understanding linear equations was identified as a problem area for most pupils. The process of solving linear equations requires pupils to be competent in several other preliminary skills involved in algebraic manipulations. This chapter examines what learning algebra at the junior secondary level entails. The content knowledge and skills that pupils need to acquire and develop in Grade 9 algebra are identified. A brief discussion of how mathematical concepts are formed in the mind of an individual learner is also made and what implication this has on the teaching of mathematics is looked into. The traditional method(s) of teaching solution of linear equations are reviewed and contrasted with a problem-centered approach to the teaching of the topic based on Constructivism.

The information related to the above-mentioned issues is sought from the literature study and some is based on the author's experiences as a mathematics teacher and examiner in the Grade 10 national examinations.

2.2 WHAT IS ALGEBRA?

2.2.1 Definition

The word algebra, as explained by Mason (1996:73) is "derived from the problems of *al-jabr* (literally, adding or multiplying both sides of an equation by the same thing in order to eliminate negative/fractional terms), which were paralleled by problems of *al-muqabala* (subtracting the same thing from or dividing the same thing into both sides)". From this explanation, it can be realized that the definition of algebra reflected a limited view of the subject, restricted only to the process of solving equations. It is acknowledged that the meaning of algebra has developed and broadened from algebra as a process to object (an algebra). In an attempt to define algebra, Wheeler (1996:319) describes algebra as a symbolic system (its presence is recognized by symbols), a calculus (its use in computing numerical solutions to problems), and also as a representational system (it plays a major role in the mathematization of situations and experiences).

While to some people algebra is merely a collection of symbols, rules and procedures, to mathematicians it is much more than that. According to Kieran (1992:391), algebra is conceived as a branch of mathematics that deals with symbolizing and generalizing numerical relationships and mathematical structures, and with operating within those structures. Algebra is conceived, by some, as a study of a language and its syntax; a study of procedures for solving certain classes of problems. In the latter, algebra is not only seen as a tool for problem-solving but also as a tool for expressing generalizations. It is also viewed as the study of regularities governing numerical relations; a conception that centers on generalization and that can be widened by including the components of proof and validation (Bednarz, Kieran & Lee 1996:4).

Algebra is about identifying patterns and generalizing those patterns. Generalizing involves seeing a pattern, expressing it clearly in verbal terms, and then using the symbols to express the pattern in general terms. Sfard, quoted by Bednarz et al. (1996:103), affirms that most authors unanimously agree to the early origins of algebra because they " … spot algebraic thinking wherever an attempt is made to treat computational processes in a somehow general way." One of the most salient features that distinguish algebra from arithmetic is its generality. According to Mason (1996:74), " … generalization is the life-blood, the heart of Mathematics."

2.2.2 WHY ALGEBRA IS TAUGHT IN SCHOOLS

Several authors have advanced different, though not contradicting views, regarding the goals of algebra instruction. These are cited in Thorpe (1989), and some indicate that algebra is taught so as to develop students' skills in the solution of equations, that is, finding numbers that meet specific conditions (Fey1984:14). Schoenfeld sees the purpose of algebra teaching as " … to teach students to use symbols to help solve real problems, such as mixture problems, rate problems and so forth"(Schoenfeld 1986, remarks at the Mathematical Science Education Board Conference). Algebra is taught in schools to equip learners with knowledge and skills that would enable them to become sufficiently at ease when working with algebraic formulas and that they can read scientific literature more intelligently (Mathematics Proficiency Committee discussion1975). According to Flanders (1987) the goal of algebra teaching is " … to prepare students to follow derivatives in other subjects, for example, in physics and engineering". In addition, French (2002: 3), on the other hand holds the view that algebra ''provides a valuable training in thinking skills and a respect for rigorous argument'', and also gives insight into the explanations of a wide range of phenomena in the world.

As indicated earlier, mathematics is a human activity that deals with qualitative and quantitative relationships of space and time, algebraic concepts, principles and methods provide powerful intellectual tools for representing the quantitative information and reasoning about this information. The main topics covered in algebra include variables, relations, function, equations and inequations (inequalities), and graphs. It is imperative, therefore, that pupils should be able to handle and manipulate the symbolic mathematical language—algebra. Competency in algebra will also enable pupils to cope with learning more advanced mathematics.

According to the MOET curriculum document, typical topics in algebra at the junior secondary school level are:

1. properties of real numbers;
2. algebraic representation and formulae;
3. solution of linear equations and inequalities;
4. indices;

5. algebra of matrices;
6. coordinates, graphs, relations and function notation;
7. sets

(MOET 2002: 3-8)

Since this study was concerned with problems that are encountered in understanding linear equation, only those aspects that relate to linear equations were considered. The specific objectives for teaching linear equations are; that pupils should be able to correctly represent a realistic mathematical problem using the symbolic mathematical notation, perform the necessary operations on the formulated equation, interpret the results and provide a meaningful solution to the original problem. The next section will therefore discuss linear equations, paying particular attention to the sub-concepts that make up linear equations and the skills that are involved in learning linear equations.

2.3 UNDERSTANDING LINEAR EQUATIONS

According to James & James Mathematics Dictionary (1976), equations are mathematical statements that indicate equality between two expressions. Equations may express identities or conditional relationships between numbers and/or variables. An identity is a statement that is true for all values of the variables, except for those values of the variable for which each member of the statement of equality does not have meaning (James & James1976: 191). A conditional equation is one that is true for certain values of the variables involved. For example, $(x + 1)(x - 1) = x^2 - 1$ is an identity while equations such as $x + 7 = 12$, $2x - y = 17$ are conditional equations. A linear equation is one that is of the first degree in its variable. Graphs of linear equations form straight lines.

Equations are such an important part of mathematics, particularly algebra, because of the many uses they have. Equations may define functions; express one variable in terms of the others; or provide information about when a particular quantity is maximized or minimized, a property used in determining extreme points for functions (Usiskin, Perissini, Marchisoto, & Stanley 2003:135).

At grade 9, learning of linear equations comprises formulating the equations from context problems, solving the equations, and finally giving the solutions to the original problems. Pupils at this level also learn graphs of linear equations. Even though there is such a close connection between linear equations and graphs, the latter were not part of the study.

2.3.1 Solving Linear Equations

Kaput (1989:180) states "since the early days of algebra as we know it (since Viete), to *solve an equation* has meant to use the available syntactic methods to transform the expression in an equation until the resulting representation makes the roots of the equation cognitively accessible." Solving a linear equation is a process of finding a numerical value for the unknown (usually represented by a letter) or making the unknown the subject in the given or formulated equation. In some cases, this process is preceded by formulation of an algebraic equation for the given situation, which will then be followed by the solution process. The unknown can be on one side of the equation or may also appear on both sides of the equation. The success of the process of solving linear equations depends on the solver's conceptual, procedural and conditional knowledge, and thus his/her understanding of linear equations. The next section will discuss what understanding of mathematical concepts entails.

2.3.2 Mathematical Knowledge and Understanding

2.3.2.1 Conceptual knowledge

Davis quoted by Wessels (1990: 382) explains that a mathematical concept is a collection of meanings that one associates with a word used in mathematics. According to Cangelosi (1996:80) "a concept is a category people mentally construct by creating a class of specifics possessing a common set of characteristics". The process by which certain qualities of actual objects or events are internalized as concepts while other qualities are ignored, is called abstraction. Thus a concept can be described as an abstracted meaning, a mental picture or idea that an individual has about something.

Concept formation involves processes of construction and reconstruction of knowledge of the concept as a result of experiences with examples and non-examples of the concept. As Carroll quoted by Wessels (1990:176) puts it “One necessary condition for the formation of a concept is that the individual must have a series of experiences that are in one or more respects similar; the constellation of ‘respects’ in which they are similar constitutes the ‘concept’ that underlies them. … A further necessary condition for the formation of a concept is that the series of experiences embodying the concept must be preceded, interspersed or followed by other experiences that constitute negative instances of the concept”.

In learning, a pupil is engaged in building up a conceptual structure. This is a mental representation of that which is learnt. When a mathematical concept that is being leant is assimilated, and accommodated and fitted into the mental structure of knowledge, then the concept is said to be understood. As Hiebert and Carpenter (1992:67) put it, “a mathematical idea … more specifically mathematics, is understood if it’s mental representation is part of a network of representations.” These networks of knowledge may be vertically and hierarchically structured or may be web-like structures.

As learning occurs, connections already existing within knowledge structures may be strengthened, and /or the new information may be connected to the existing networks and new relationships constructed. As the networks become larger and more organized, and the connections become strengthened with reinforcing experiences, understanding increases. The number and strength of the connections within the network determine the degree of understanding. According to Hiebert & Carpenter (1992:70), past experiences of the pupil create mental networks and these existing networks influence the relationships that are constructed, thereby helping to reshape the new networks that are formed.

Conceptual knowledge consists of connected relationships between ideas and concepts internalized by an individual. As Hiebert and Carpenter (1992:78) put it, “A unit of conceptual knowledge is not stored as an isolated piece of information; it is part of a network”. It is the quality of this type of knowledge that determines the degree of understanding of mathematical concepts; the stronger the connections in the networks, the greater the understanding.

Pupils exhibit conceptual understanding when they are able to compare and contrast objects and form interrelationships between concepts. For example, realization that subtraction is the inverse of addition, that division is the inverse of multiplication, indicates that pupils have clear understanding of the concepts.

It is clear from the above discussions that the main purpose of instruction in mathematics should be to help pupils build up and enlarge their mental mathematical knowledge structures, such that they are able to transfer this knowledge to new situations. Teaching should take into consideration the quality of knowledge that already exists in the mind of the learner as the basis upon which new knowledge can be constructed. Meaningful learning is influenced by the pupil's past experiences related to the new information/concept to be learnt. Based on the cognitive theory on concept formation, it is important that teachers provide opportunities or environments that optimize the pupils' learning experience.

2.3.2.2 Procedural Knowledge

Procedural knowledge is commonly referred to as 'knowing how' (English & Halford 1995:23). The procedural knowledge of mathematics is therefore the knowledge about the rules and procedures that can be used in solving routine mathematical problems, and of the symbolism that is used to represent mathematics (Van de Walle 1998:25). This type of knowledge has a significant role in learning and doing mathematics. For example, algorithmic procedures help learners to do computations much faster with less concentration and thus free their minds to concentrate on more challenging tasks. It is worth noting however, that mastery of these rules and procedures does not in any way suggest that the pupil understands the involved concepts. In most cases pupils apply algorithms wrongly because during their learning, no meaning is attached and pupils are not aware of the logical reasoning underlying the algorithms. This means, when the algorithm is forgotten, it is difficult for the pupils to re-discover the actions. Orton (1992: 29) also affirms that "a worrying feature about algorithms in mathematics is that many which we expect our pupils to remember and use with confidence, are meaningless to the pupils, in terms of worthwhile knowledge, and are sometimes completely irrelevant." Letting pupils use a certain procedure repeatedly also does not help them develop understanding of the related concepts.

Pupils display **conditional knowledge** when they are able to identify and apply appropriate procedures for solving certain classes of problems. Conditional knowledge may be defined as knowledge of when to use conceptual or procedural knowledge. For example, knowing when to use a particular algorithm; choosing appropriate representations such as graphs, tables, equations, etc. to solve a problem. It needs mentioning that these types of mathematical knowledge are both important for mathematical expertise. As already mentioned, procedures allow mathematical tasks to be completed quickly with little mental effort. If a pupil can make a connection between a procedure and some of the conceptual knowledge upon which the procedure is based, the procedure then becomes part of the network, closely related to the conceptual knowledge. It can thus be realized that the conceptual knowledge contributes to mathematical expertise through its relationships with procedural knowledge. Hiebert and Carpenter (1992: 78) affirm, "procedures connected to networks gain access to all information in the network."

2.3.2.3 Pupils' Errors and Misconceptions

According to Behaviorism, all knowledge originates in experience. The theory assumes that pupils learn what they are taught, or at least some subset of what they are taught. It is assumed that knowledge can be transferred from one person to another in its intact form. This view of how learning occurs implies that the pupils' current knowledge is not necessary to learning. From the behaviorist perspective, errors and misconceptions are not important as Gagne, quoted by Olivier (1989), puts it "the effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules … This means that teachers would best ignore the incorrect performances and set about as directly as possible, teaching the rules for correct ones".

From the constructivist perspective, however, "misconceptions are crucially important to learning and teaching, because misconceptions form part of a pupils' conceptual structure that will interact with new concepts, and influence new learning, mostly in a negative way, because misconceptions generate errors" (Olivier, 1989). Olivier further notes that errors and misconceptions should be seen as the natural result of children's effort to construct their own knowledge, and that these misconceptions are intelligent constructions based on correct or incomplete (but not wrong) previous knowledge. Misconceptions can form serious barriers that may impede pupils' learning and understanding of new concepts.

From the latter perspective, it becomes apparent that the role of the teacher is to identify the type of thinking in the pupils that results in the errors displayed, so that the underlying misconceptions may be overcome. Errors are signposts that direct teachers to pupils' problem areas. According to Heiman, Narode, Slomianko and Lochhead (1987:18) "misconceptions can even be used as instructional aids; when a teacher presents a misconception that is troublesome for pupils, s/he forces them to test their conceptions against paradox and against their common sense." Through listening to what pupils say in reasoning and justifying their solutions, and watching them carefully as they execute problem solving, the teacher can discover the misconceptions that pupils may have. Sharing the misconceptions with the pupils and other teachers can improve pupils' understanding of concepts as well as instructional activities designed by the teachers.

2.3.3 Sub-Concepts in Understanding Linear Equations

Pre-requisite knowledge required for learner's understanding linear equations comprises:

1. number sense and operations;
2. properties of operations;
3. concept of a variable;
4. algebraic terms and expressions-(manipulation of algebraic expressions);
5. identifying and expressing relationships.
6. proper interpretation of concept of equation/equality;
7. ability to read and interpret the symbolic form of an equation; and
8. ability to identify appropriate strategies for solving the equations.
9. ability to formulate equations from context problems

2.3.3.1 Number sense and Operations

According to the Curriculum and Evaluation Standards of the NCTM (1989: 41), understanding of the basic operations, addition, subtraction, multiplication and division is central to knowing mathematics. Understanding operations here refers to ability to recognize conditions in real life where the operations would be useful, awareness of the properties of the operations (including the order in which operations should be performed), the ability to identify relationships among operations and having an insight into the effects of the operations on numbers. When pupils have good operation sense, they are able to apply operations meaningfully and flexibly, and can make thoughtful decisions about reasonableness of their results.

In order for them to be able to solve linear equations, pupils need to have developed number sense for whole number, fractions, decimals, integers, and rational numbers. French (2002: 24) also affirms that emphasizing mental calculation at earlier stages of mathematics learning can enhance these prerequisite skills. Pupils with good number sense have well understood number meanings and have developed multiple relationships among numbers. They are able to recognize the relative magnitudes of numbers; they know the relative effect of operating on numbers and are able to develop referents for measures of common objects and situations in their environments (NCTM 1989: 38). Ability to use multiple representations of numbers is a very important skill in solving mathematical problems. Good operation sense and number sense together would enable pupils to judge reasonableness of their solutions.

2.3.3.2 Properties of operations

Knowledge of the basic laws of algebra is essential in solving linear equations. These laws are very useful especially in transforming expressions to get their equivalent reduced forms. The following is a list of such laws, governing operations in algebra.

1. Commutative property. In addition and multiplication the order of terms makes no difference, e.g. $a + b = b + a$
2. The associative property. In addition and multiplication the grouping of terms and factors respectively makes no difference. For example,

$(a + b) + c = a + (b + c)$

$(a\,b)\,c = a(b\,c)$

3. The distributive property. This property can be used to change an expression involving brackets to its equivalent without the brackets. For example,

   $a\,(b + c) = ab + ac$; and $a(b - c) = ab - ac$

4. Properties of Zero.
   i. Any number times zero is equal to zero, i.e. $a \times 0 = 0$
   ii. Division by zero is impossible
   iii. Adding zero to any number gives that number (identity property)
5. Properties of one.
   i. multiplying any number by one (1) gives that number (identity property)
   ii. any number (except zero) divided by itself gives one (1). For example,

      $\frac{5}{5} = 1$, $\frac{b}{b} = 1$

   iii. Any number divided by one (1) gives that number.

      For example, $\frac{8}{1} = 8$, $\frac{p}{1} = p$

6. Inverse Operations.
   i. Addition. Example, $a + -a = 0$
   ii. Multiplication. Example, $\frac{a}{b} \times \frac{b}{a} = 1$

The teaching of arithmetic is usually done in isolation from algebra. This is particularly the case at the elementary level. This situation deprives pupils of the powerful ways of thinking about mathematics in the early grades and makes learning of algebra more difficult in the later grades, i.e. junior secondary level and higher.

Teaching arithmetic should develop mathematical thinking in the pupils. Pupils should be able to apply fundamental properties of numbers in computation. They should recognize that to group and regroup numbers when performing, for example, addition and subtraction and/or a combination of both operations simplifies the task. Understanding the relationship between multiplication and division should enable pupils to deduce other equivalent statements from those that involve those operations.

When pupils begin learning algebra, they should be able to recognize that they can apply the same properties they used on numbers to simplify algebraic expressions and in solving equations. For example, Carpenter, Franke and Levi (2003:2) support this by illustrating that in working 50 plus 30, this can be represented as :

$$\begin{aligned} 50 + 30 &= 5 \times 10 + 3 \times 10 \\ &= (5 + 3) \times 10 \\ &= 8 \times 10 \\ &= 80. \end{aligned}$$

Thus in algebra, they would immediately recognize that $5x + 3x = 8x$. In this way children would be learning arithmetic in a manner that paves way to effective learning of algebra.

Earlier studies, as indicated by Phillip and Schappelle (1999:312), also reveal that pupils' understanding of algebra as generalized arithmetic is quite fragile. For example, when asked to say whether a statement such as

$$\frac{2x+1}{2x+1+7} = \frac{1}{8}$$

was true, many students "manipulated these equations to see whether they could make the two sides of the equations look the same," but only a few attempted to check their results with numbers.

Mastery of skills and knowledge required for correct manipulation of algebraic expressions plays a very significant role in pupils' ability to solve linear equations correctly. Nickson (2002:27) indicates that lack of successful development and understanding of fractions at the earliest stages of children's learning will lead to further difficulties in their mathematics learning, through secondary education and beyond. Rules that govern manipulation of fractions are also important in solving many algebraic problems and equations. French (2002: 47) notes, "a proper understanding of algebraic processes is inevitably very dependent on a corresponding understanding and facility with arithmetical operations”. A smooth transition has to be facilitated for pupils to move from arithmetic into algebra.

2.3.3.3 The concept of a variable.

In introducing algebra, first pupils encounter the concept of a variable, then algebraic terms and expressions, after which equations would follow. This sequence is based on the fact that equations involve expressions, while expressions in turn involve variables. Understanding of the concept of a variable is fundamental to the study of algebra. According to Van de Walle (1998:474), “a variable is a symbol that can stand for any one of a set of numbers or other objects.” In some cases the referent set may have only one value, while it may have an infinite number of values in others, and the variable represents each one of them. Pupils need to develop a clear concept of a variable, that is, an understanding of how the values of an unknown change. A variable provides an algebraic tool for expressing generalizations. Unlike constants which can be defined in numerous ways, variables cannot be defined in terms of cardinal or ordinal values but can only be defined by number system reference (e.g. $a = b + c$) or by expression reference (e.g. $v = \frac{s}{t}$).

Research studies indicate that pupils experience a lot of difficulties in dealing with letters, in algebra. From the study conducted by Kuchemann (1981:104), six stages through which pupils progress in acquiring a mental model of a variable are identified. These are as follows:

1. *Letter evaluated*. In this case pupils avoid operating on a specific unknown and as such simply assign a numerical value to the unknown from the outset. The pupil may recall any number or recall the number fact about the expressed relationship.
2. *Letter not used*. Here a pupil may just ignore the existence of the letter, or at best acknowledge it, but does not give it meaning. For example, If $a + b = 5$, $a + b + 2 =$? and the pupil gives 7 as an answer.
3. *Letter used as an object*. In this case the child regards the letter as shorthand for an object in its own right. For example, ‘$3b$’ as ‘3 balls’. At this level pupils are able to regard expressions like $5 + 2a$, $p + 1$ as meaningful.

4. *Letter used as a specific unknown*. The learner here regards the letter as a specific but unknown number, and can operate on it directly.

5. *Letter as a generalized number*. The letter is here regarded as representing, or at least as being able to take several values rather than just one value.

6. *Letter used as a variable*. This is the final stage where the pupil sees the letter as representing a range of unspecified values and understands that a relationship exists between two such sets of values.

It was found in that study that a greater number of the 13-15 year old pupils treated the letters as specific unknowns than as generalized numbers; despite the classroom experiences they had in representing number patterns as generalized statements. The majority either treated the letters as objects or ignored them.

It is important that instruction is geared towards helping pupils construct a clear concept of a variable. Pupils need to have experiences with the different meanings and uses of variables so that they can become comfortable in dealing with them in different contexts. Clement (1982:22) further confirms that "understanding an equation in two variables requires an understanding of the concept of variable at a deeper level than that required for one variable equations." Activities that offer pupils opportunity to explore and investigate patterns, and require them to make verbal formulations of rules that describe the observed patterns and finally generalize situations can play a significant role in the development of the concept of a variable. While English and Warren (1998: 168-169) support this idea, they have however realized that "students find it easier to verbalize a generalization than to express it symbolically".

It is clear therefore that a clear understanding of variable is essential as variables provide the algebraic tool for expressing generalizations.

2.3.3.4 Algebraic terms and expressions

Mathematical knowledge is communicated through the symbolic mathematical language. This language uses numbers, letters and other conventional symbols. Austin and Howson (1979) assert " mathematical symbolism in its now internationally accepted form, is shorthand, the bulk of which has been devised by speakers of a few related languages. The use of this symbolism can accordingly cause considerable difficulties to those whose mother tongue has different structures" (Austin and Howson 1979: 176). The use of this formal mathematical language requires the pupil to have a clear understanding of the relevant mathematical concept in order that she/he can translate into the correct symbolic notation, manipulate the symbols and then be able to translate back into meaningful concepts (Brodie 1989:49). Use of brackets in mathematics is also one of the complications in interpreting mathematical expressions or statements, as this structure is not present in ordinary language.

Earlier research provides evidence that simplification of algebraic expressions creates serious difficulties for many pupils (Linchevski & Herscovics 1996). Pupils experience serious problems in grouping or combining like terms. Whereas in arithmetic, operations yield other numbers, in algebra operations may yield algebraic terms and/or expressions.

.For example,

<u>Arithmetic</u>	<u>Algebra</u>
2 x 4 = 8 (8 is a new number)	2 x a = 2a (a term)
1 + 2 =3	$a + b$ (expression)
$\frac{6}{3} = 2$	$\frac{6}{b}$ (a term)
2 x 3 + 4 can be simplified to a numerical term.	While 2($x + y$) can only be reduced to 2$x + y$.

When given a problem whose final answer is, say, 2x + 3, some pupils would go further to give 5x as their answer. This results from the fact that to these pupils 2x +3 is not acceptable as the solution; to them a solution should always be a single term. It is clear from this that pupils need to be helped to appreciate the dual nature of expressions. They should be able to see expressions as a process and as a product. Expressions encapsulate a process as instructions to calculate a numerical value, but they are also a product as objects which can be manipulated in their own right (French 2002:24). As French puts it "failure to appreciate this dual nature of expressions is a major barrier to success in algebra" (French 2002: 24). Tall and Thomas cited in French (2002:

15) and Nickson (2000:142) identified four obstacles frequently met by pupils in making sense of algebraic expressions. These are:

- the parsing obstacle;
- the expected answer obstacle;
- the lack of closure obstacle; and
- the process-product obstacle.

To some pupils, the + sign signals that they have to do some calculation; they expect to produce an answer. This is what is referred to as the expected answer obstacle. The way we read from left to right is also noted to influence pupils to interpret for example $3 + 2x$ as saying ‘add 3 and 2 and then multiply by x.’ This obstacle is what is termed the parsing obstacle. This obstacle also leads pupils into reading “*ab* as *a* and *b*” and thereby end up thinking that it is the same as $a + b$. When pupils show discomfort when they have to accept, say, $2x + 3$ as a final answer after some algebraic manipulations is said to be due to the ‘lack of closure’ obstacle. To the pupils this is an incomplete answer. The process-product obstacle refers to pupils’ failure to appreciate the dual nature of algebraic expressions i.e expressions can indicate an instruction and at the same time they can represent the result of the operations (French 2002:15-16).

From the study that Bishop and Stump conducted, many pre-service teachers lacked full understanding of what algebra was and could not explain the kind of activities which may characterize a classroom that promotes algebraic reasoning. It is worth noting that when teachers’ knowledge about the role of letters in mathematics is rich, it is very likely that the necessary understandings will be passed over to their pupils. As Bishop and Stump puts it ‘‘a teacher with a rich understanding of connections between mathematical ideas is more likely to reveal and represent them, at the same time, a teacher who lacks them is unlikely to promote deep insight in his or her students’’(Bishop & Stump, 2000:108).

2.3.4.5 Manipulation of algebraic expressions and equations

The above-mentioned problems with algebraic terms and expressions lead to further problems that are usually seen when pupils solve equations. Pupils who have insufficient conceptual knowledge about terms and expressions experience serious problems when they have to read and interpret the symbolic form of equations. Pupils are usually not able to make sense of the algebraic equations, as they do not really understand the structure of the relations in the equation (Kieran, 1992: 397).

Kieran identified some complexities in the use of the word structure in the context of algebra. *Surface structure,* according to Kieran (1992:397), refers simply to the arrangement of different terms and operations that go to make up an algebraic (or arithmetic) expression. As Nickson (2000:112) puts it, ‘‘*Systemic structure* refers to the properties of operations within an algebraic expression and the relationships between the terms of the expression that come from within the mathematical system’’. For example, rewriting 2 + 5(x + 2) as 5(x + 2) + 2 using the commutative law or as 5x + 12 using the distributive law and addition. There is also the notion of the *structure of an equation*, which incorporates the systemic structure and the relationship of equality (Nickson, 2000:112).

Solution of linear equations involves both procedural and structural operations. Procedural operations refer to the arithmetic operations carried out on numbers to yield numbers, while structural operations refer to a set of operations carried out on algebraic expressions. For example, substituting p and q to obtain 13 is a procedural operation. Simplifying an expression such as 5p + q – 2p to yield an equivalent expression 3p + q is a structural operation.(Kieran 1992:392).

Algebraic manipulations include processes such as simplifying, expanding brackets, collecting like terms, factorizing, etc. In solving linear equations these processes are performed when transforming the original equation to its simpler equivalent forms. Earlier research studies e.g. Kieran (1992), and the experience of the researcher as a mathematics teacher, reveal that most pupils do actually encounter difficulties when they are confronted with solving equations involving negative numbers. This is usually evident where brackets are involved, particularly when the brackets follow the minus sign.

Other cases include where the like terms are collected and there is a gap in between them, with a minus sign before the other terms. Nickson (2000:120) indicates that these result from a static view of the use of brackets and jumping off with the posterior operation by pupils. According to Nickson, pupils at this level could not realize that "926 – 167 - 167 was the same as 926 - (167 + 167) only two of the 27 pupils thought this was the case. "The jumping off with the posterior operation" refers to cases where pupils, in an attempt to collect like terms where the distance between the terms is involved, they tend to focus on the operation sign that follows the term. For example x + 7 – 2x – 3 may be simplified to 3x – 4. Mastery of skills and knowledge required for correct manipulation of algebraic expressions plays a very significant role in pupils' ability to solve linear equations correctly. French (2002: 24) affirms that understanding and fluency in performing operations with negative numbers is an essential pre-requisite for learning algebra successfully. He notes, "a proper understanding of algebraic processes is inevitably very dependent on a corresponding understanding and facility with arithmetical operations"(French (2002: 47).

2.3.3.6 The Concept of Equation / Equality

Equations are mathematical statements that indicate equality between two expressions. Many pupils at the elementary level and junior secondary level fail to interpret the equal sign as a symbol denoting the relation between two equal quantities. To them the sign is interpreted as a command to carry out a calculation. Experience in classroom teaching has it that in solving for the unknown in an equation of the form

$$7 + 3 = x + 9$$

Pupils would respond as follows:

$$7 + 3 = 10 + 9 = 19.$$

Usiskin et al (2003: 137) further illustrate this situation by indicating that when pupils are asked to find out what number would make the statement

$$7 + \_\_ = 10 + 5 \text{ true,}$$

many would give the answer as 3, seeing 10 as the result after addition, ignoring the 5 on the right. This kind of response also indicates that to pupils the equality has a direction from left to right. In their workings, particularly those that involve extended computations; learners would calculate, for example, 13 + 45 + 7 as

$$13 + 45 = 58 + 7 = 65.$$

This clearly indicates that pupils interpret the equal sign as the command to carry out the calculation; it does not represent the relation between the left hand side and the right hand side of the equation.

The above-indicated problem suggests that teachers should emphasize the meaning of the equality and the role of the equal sign in an equation to the pupils. It is very important that learners are helped to develop proper interpretation of the equal sign as this understanding is essential in algebraic manipulations.

2.3.3.7 Formulating equations from context problems

Mathematics is taught in schools to develop in the pupils, knowledge and skills that they require in solving problems they may encounter in their daily life. It has however been realized that when confronted with realistic mathematical problems, pupils often find it very hard to formulate the given problem situation into the symbolic mathematical language. This is mainly due to lack of correct interpretation of the question, which involves identification of variables and relationships that exist between those variables.

As mentioned earlier, mathematics is sometimes considered a language, due to its strong lingual base, often spoken in symbolic notations. Mathematical terms are well defined and symbols are used to express the mathematical relationships in shorthand. From previous studies, it has been realized that, when solving algebraic word problems pupils experience serious difficulties when interpreting a problem and translating it into the symbolic mathematical language.

MacGregor (1991: 25) indicates that several researchers have confirmed that the sequential left to right translation from ordinary language to mathematical symbolism is a common procedure taught to pupils. The common error associated with this approach is the ‘reversal error’. The *student-professor* problem is the well-known example that illustrates this. The problem requires students to write an equation for this statement: “At this university there are six times as many students as professors” using *S* for the number of students and *P* for the number of professors (MacGregor 1991:19).

From research reports, some responses to the *student-professor* problem were $6S = P$ which is actually wrong. Although this question was asked to university students, their wrong response is a result of their inherent misconception from early years in algebra learning. Kieran (1992:403) reckons “some semantic knowledge is often required to formulate these equations; but solvers only typically use nothing more than syntactic rules.” As English & Halford (1995) puts it “students rely on direct syntactic approach to solving these problems, that is, they use a phrase-by-phrase translation of the problem into variables and equations” (English & Halford 1995: 241). Berger and Wilde (1987:23) also affirm “algebra word problems have been a source of consternation to generations of students.” Even for those pupils who are most able in solving linear equations, the moment these equations are cloaked in a verbal cover story (Berge & Wilder1987:123), they also struggle to solve such problems.

From experiences that one has gathered as a mathematics teacher, it has become very clear that in order for pupils to become successful problem solvers, they need to be taught problem solving as a skill for them to acquire. According to the MOET (see section 1.7) curriculum document (2002: 2), one of the goals of mathematics teaching should be to develop the learners’ ability to solve problems. It has been noted that this skill is best acquired when pupils are given relevant practice and are motivated into practicing the skill.

According to Constructivism effective learning occurs when pupils are actively involved in the process, therefore exposing learners to solving many real life mathematical problems is very advantageous in developing their problem solving skills. According to Polya (1973), there are four phases through which a problem solver must proceed in order to solve the confronting problem successfully. These are sequentially;

1. understanding the problem.
2. devising a plan or decide on an approach for tackling a problem.
3. carrying out the plan.
4. looking back at the problem, the answer and what you have done to get there.

Solving algebraic word problems requires careful thought, analysis and a general synthesis of previously acquired mathematical content knowledge and skills. The problem solver needs to understand the problem clearly, distinguishing between pertinent and redundant information. He/she needs to be able to identify the target variable and establish relationships that exist the different variables in the problem. The problem solver has to be able to select appropriate strategies that will help him/her finally work to a logical solution. Such strategies include reducing and simplifying the problem; identifying and using patterns; organizing data in tables, picture graphs, diagrams; guessing and testing and working backwards. Logical thinking and reasoning are the backbone to a successful problem solving process (Krulik & Rudnick 1982:2). It is imperative therefore to suggest that pupils need to be exposed to situations that require them to use these strategies in order to build their confidence and competence in problem solving. Problems that pupils meet in real life are not the same as textbook problems, therefore if mathematics learning is to benefit pupils; it must be related to pupils' lives. As Nickson puts it "for the notion of an equation to be meaningful to children, they need experience of generating their own equations from situations with which they are familiar (2000:145)."

Meaningful learning is a process through which new knowledge is assimilated and accommodated in the network of already existing structures of knowledge in the mind of the pupil through active participation. Discussions provide the best opportunity for this process as it is through classroom discourse that pupils engage in making conjectures, analyzing problems at hand, critical thinking and making connections between the known and the new, wondering why, and finally drawing conclusions. The aim of mathematics teaching is that pupils should become mathematical problem solvers, learn to communicate mathematically and learn to reason mathematically (NCTM 1989:5), surely discussions provide opportunity for this to happen.

2.3.3.7.1 Approaches to solving algebraic word problems

Ho Kheong and Tian Hoo (1995:34) illustrate two approaches that can be used in solving algebraic word problems: The 'model' approach and the algebraic method

2.3.3.7.1.1 The model approach

In this approach concrete representation of objects that are involved in the question are used. For example, in solving the problem:

"A ruler and two pencils cost \$1.40. A ruler costs 20c more than a pencil. Find the cost of a ruler." The solution would be as follows:

Reading from the diagram,

$$\begin{aligned} 3 \text{ parts} &= 140 - 20 \\ &= 120 \\ 1 \text{ part} &= \frac{120}{3} = 40 \end{aligned}$$

thus the cost of a ruler

$$\begin{aligned} &= 1 \text{ part} + 20 \\ &= 40 + 20 \\ &= 60 \text{ cents} \end{aligned}$$ (Ho Kheong & Tian Hoo 1995:34-35)

2.3.3.7.1.2 The algebraic method

The same solution could be obtained as follows:

$$\begin{aligned} r + 2p &= 140 \\ r - p &= 20 \\ 3p &= 120 \\ p &= 40 \\ r - p &= 20 \\ r - 40 &= 20 \\ r &= 60 \text{ cents (cost of ruler)} \end{aligned}$$

When compared to the model approach, this is a more abstract method. It has been realized however, that in Singapore where the model approach is highly emphasized at the primary level, pupils at the secondary level usually fall back to their original approach (the model approach) when similar problems are given (Ho Kheong & Tian Hoo 1995:35).

The implication of this situation is that secondary school teachers need to be aware of the solution methods that are familiar to the pupils so that they may use the integrated approach, which would step by step offer pupils a smooth transition to the more abstract algebraic method. Allowing pupils to use their own methods in solving mathematical problems gives them courage and boosts their morale and ego and thus sustains their motivation to learn, since they realize that they are trusted to make a contribution to the learning process. Earlier research reports indicate "part-whole relationship seems to have positive results with children solving word problems" (Nickson 2000:129). The main feature of this approach is helping children identify the parts of the context that are needed to answer the question and the relationships that exist between and among the parts and the whole text. According to the Standards (NCTM 1989:104), formal methods for solving equations can be developed and supported by informal methods.

In order for pupils to solve algebraic word problems successfully, they should be able to read written mathematics and understand what the text is all about. According to Chaiklin (1989:97), this comprehension process involves:

a) reading the problem,
b) forming a mental representation that interprets the information in the problem into objects with associated properties
c) organizing the relations among those objects and
d) representing the relations as equations.

This means pupils should be able to interpret the text in mathematical terms and use representations either in the form of a graph, diagram or picture, or equation to express the ideas in the given situation. This is very similar to three phases in Polya's problem-solving approach.

The NCTM Standards (1989:146) express the view that instructional programs should enable students to create and use representations to organize mathematical ideas. Pupils should be able to select, apply and translate among different forms of representations to solve problems. They should be able to use representations to model and interpret physical, social and mathematical phenomenon. Dossey et al. (2002:83) also supports this by saying that “students’ ability to develop and interpret various representations increases their ability to do and understand mathematics” (2002:83).

Dossey et al. (2002:165) suggest that, in helping pupils develop the ability to identify and describe relationships symbolically or graphically, pattern analysis activities can be of vital importance. These activities would offer pupils opportunity to develop expressions and equations. In establishing relationships, pupils may think through a given situation in terms of the actions involved. In the process they could identify the target variable and the numerical operations required. As Dossey et al. (2002:170) puts it “developing an operation sense and ability to target the variable of interest, puts pupils well on their way to being able to represent problems in terms of expressions and equations.”

2.3.4 Strategies for solving linear equations

Once the problem is interpreted in mathematical terms, the pupils’ task is to select the appropriate strategy to use in solving the problem. Chaiklin (1989:109) observes that in order for pupils to solve equations successfully, they must use both their conditional and procedural knowledge. Conditional knowledge enables the problem solver to locate the target variable, examine the structure of the expression in the equation and decide on which strategy to use in solving the problem.

2.3.4.1 The Trial and Error Method

In solving an equation such as $2x + 5 = 9$, some pupils would prefer to construct a table of values of x and their corresponding outputs for $2x + 5$ until they get the output 9 and thus read the desired value of x. Dossey et al.(2002:177). This method has some limitations, particularly for larger values, as these would require long calculations, therefore is not economical.

2.3.4.2 The Balance Approach

As indicated earlier on, an equation is a statement of equality involving constants, variables and/or expressions. The two sides of an equation represent equal quantities. The balance method is based on this notion.

The analogy used here is that of pan balances. In order to keep the pans balancing, whatever is added or removed to/from one pan must also be added or removed to/from the other pan. In solving equations the policy is therefore similar; whatever is done on the left side of the equal sign must also be done on the right side. The process generates a series of equivalent equations, each much more simpler than the one just generated before. This approach seems to bring meaning to the solution process. Pupils transit to manipulating algebraic transformation rules first through working with a concrete reference.

Experience indicates that teachers just show pupils that in order to solve an equation such as $y + 10 = 3y - 2$, they need to transfer 10 to the right side of the equal sign and change the sign, and then move $3y$ to the left side of the equal sign and also change the sign. The expected resulting equation would be $y - 3y = - 10 - 2$. Even though pupils may manage to get the equivalent equation correct, they would do so without really understanding what they are doing and why the procedure works. Pupils taught in this way even fail to realize that the new equation is equivalent to the original one. To them, algebra learning is seen as manipulation of signs, letters and symbols without any meaning attached. It is therefore clear from this that algebra needs to be taught in a more meaningful way, in order for pupils to appreciate its power in mathematical reasoning.

2.3.4.3 The Unfolding Process

In this method, students reason through to the solution of the equation. This reasoning includes identifying the operations that are involved connecting the terms and/or variables in an equation; and then reversing the operations until the solution is reached. In solving the equation $2x + 3 = 9$, the solver would identify the target variable first. He would realize that he needs to find the number, which is such that when multiplied by 2, adding 3 would give 9. Now starting with 9, he would realize that to get to the required number needs to reverse the operations. As the last operation was adding 3, then subtracting 3 should be done before dividing by 2. This requires the problem solver’s ability to apprehend the structure of the relation in the problem (Kieran 1992: 397).

2.3.4.4 The Graphing Method.

Another strategy that can be used in solving an equation such as $2x + 3 = 9$, is by creating two equations from the given one, i.e $y = 2x + 3$ and $y = 9$. Pupils may then draw the graphs of these two equations and the point of intersection will provide the value of x for which $2x + 3 = 9$. This method is not popular for solving linear equations, as it is usually introduced when teaching how to solve simultaneous linear equations.

2.4 CONCLUSION

In this chapter a literature survey was carried out by the researcher to gain further insight into the study. Professional journals, scholarly books and other publications were used to gather information related to the study, providing in-depth knowledge about earlier research activities that have been carried out in the area. Reports of work done earlier by researchers such as Kieran, Lins, Kuchemann and others do reveal that solution of linear equations poses a major problem to most pupils. As the study is mainly focused on problems that Grade 9 pupils encounter in understanding linear equations, only information related to this topic was sought.

From the literature study and the researcher's experience as a mathematics teacher at Grade 9, it was realized that the main difficulties that pupils encounter in understanding linear equations arise from insufficient background knowledge. As Skemp (1971) indicated, mathematics is hierarchical in nature, so learning of higher order concepts is successful only when related lower order concepts are fully grasped by the pupils.

Construction of knowledge and understanding of a new idea requires that the pupils be actively thinking about it. The pupils needs to make connections between old and new knowledge; he/she needs to engage in reflective thinking, sifting through existing ideas to find those that seem most useful in giving meaning to new concepts being learnt (Van de Walle 2004:22). It is therefore essential that before pupils are exposed to the learning of linear equations, they are first equipped with the necessary background knowledge upon which new knowledge can be build. This pre-requisite knowledge comprises:

- Number sense and operations
- Algebraic laws

- Concept of variable
- Algebraic terms and expressions (the symbolic language of mathematics)
- Manipulation of algebraic expressions

Lack of knowledge on any of the above mentioned concepts may lead to serious hiccups in the pupil's ability to understand linear equations. Proficiency in dealing with operations with decimals, fractions, directed numbers lessens pupils' problems in solving linear equations. When pupils are knowledgeable about laws governing algebraic manipulation, including order of operations, they are less likely to encounter any problems in transforming equations to obtain to their simpler equivalent forms. In order for them to be able to represent wordy mathematics information into the symbolic form, pupils need to have a sound knowledge of various uses of letters in mathematics. They need to be fluent in the symbolic mathematical language; they need to be able to interpret algebraic terms and expressions. With these abilities they would then be able to represent relationships expressed among variables in the problem statement in symbolic form.

It is clear from this that teachers need to help pupils acquire knowledge and skills on the above cited areas through problems that engage them in:

a) generalization activities
b) performing operations involving bracketing and order of operations
c) solving problems that require them to detach parts of an expression and perform manipulations before re-insertion (Linchevski 1995:116-117).

Problems of this nature will help pupils make a smooth transition from arithmetic to algebra.

Pupils are said to be proficient with linear equations when they are able to:

- represent a realistic mathematical problem using the symbolic mathematical notation in the form of an equation;
- perform the necessary operations on the formulated equation to get the value of an unknown;
- Interpret the result and provide a meaningful solution to the original problem.

In order to achieve these, pupils should be helped to acquire both conceptual and procedural knowledge as well as conditional knowledge needed for successful solution of linear equations.

Conceptual knowledge is attained through logical connections constructed internally in the mind of the pupils existing as part of a network of ideas. Procedural knowledge is one about rules and procedures that pupils use in carrying out computations. Knowledge of algorithms that can be used when performing certain computations is part of procedural knowledge (Van de Walle 2004:27). Conditional knowledge refers to knowledge about identifiable methods that pupils may use in solving certain problems.

CHAPTER 3.

RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

In Chapter 2, a literature study relating to the problem of investigation was undertaken. Publications of earlier researchers such as Kuchemann, Kieran, Lins, and Linchevski were read, broadening the researcher's background knowledge on the subject. This chapter is devoted to outlining the methodology and design of the research.

A research design is a "plan, recipe or blueprint for the investigation"(de Vos 2002:165) and thus offers a clear description of how the research is going to be conducted. This gives an indication of the procedures that will be followed in the sampling and data collection in order to reach the research aim and objectives. The ultimate goal for a good research design is to provide a credible answer to the research question (Macmillan & Schumacher 2001:166).

This study is a quantitative survey research in which random selection of participants was used. According to de Vos (2002:145) in this type of design, the researcher must first identify the research population, after which data collection methods may then be used to gather information. A survey research is used to investigate people's beliefs, values, habits ideas, opinions etc. this type of research is popular in education due to efficiency and generalisability. Accurate information for large population can be obtained with a small sample at relatively low costs (Macmillan & Schumacher 2001:304-305).

3.2 DEMARCATION OF STUDY

The target population is Grade 9 pupils as the topic under investigation is taught at this level. In most cases, linear equations are usually dealt with towards the end of the school year and sometimes the topic is deferred to the beginning of the next school year when the pupils will be in Grade 10. It is for this reason that grade 10 pupils were used in the study.

The research was conducted in four (4) high schools in the Maseru area. The schools are very easy to reach and were chosen because it is where the researcher has good contacts. Two (2) of the schools are in the Maseru town while the other two are outside town, but alongside the main road, hence accessible. It needs mentioning that even though the two sets of schools are located at different places, there is no significant difference in the physical as well as human resources available in the schools. The number of mathematics teachers in the schools is almost the same. The language of instruction in all the four schools is English and this is the case with the rest of the high schools in the country.

Random sampling was used to draw a sample of twenty (20) pupils from each school with a total of eighty (80) pupils in the study. A table of random numbers was used in this regard. The sample used represents approximately ten percent (10%) of the population size in each school, which is the sample size supported by de Vos (2002:202). As the study is investigating problems that Grade 9 pupils encounter in understanding linear equations, it is necessary that information from teachers of mathematics at this level be obtained. Between three to four teachers were involved in each school, ending up with a total of fifteen teachers participating in the study.

3.3 DATA COLLECTION INSTRUMENTS

Data for this study were collected by means of questionnaires and tasks. There were two types of questionnaires; one for the pupils and the other for the teachers. The tasks were specifically designed for the pupils. The instruments were personally taken to the schools by the researcher, and were administered by the researcher with the help of the mathematics teachers for the classes involved in the study.

3.3.1 Pupils' Tasks

The tasks were in four categories based on the areas that were identified in chapter 2 as relevant to pupils' understanding of linear equations. The tasks were designed to focus on the following areas:

SECTION A: Pupils' ability to form algebraic expressions;

SECTION B: Pupils' mastery of the mathematical language often used in real life problems;

SECTION C: Pupils' ability to manipulate algebraic expressions correctly;

SECTION D: Pupils' ability to formulate linear equations from word problems based on real life contexts, and their ability to solve the given linear equations.

The tasks in Sections A, B and C comprised five multiple-choice questions in each section, each with four alternatives. Section D consisted of two types of questions. Two questions were algebraic word problems that required respondents to formulate linear equations from real life contexts and then work out the solutions. The other four questions were readily formatted linear equations which respondents had to solve. All the tasks in Section D required the respondents to show their working towards their final solutions. Thus respondents had a total of 21 tasks to work on.

3.3.1.1 The Rubric for Scoring the Tasks

The scoring for tasks in Sections A, B and C was one mark for each correct response, while for Section D the scoring was on a 6-point scale based on the rubric designed by Randall Charles as presented by Ottis and Offerman (1988:49) in Table 3.1 below.

Number of points	Observed characteristics of the student's solution
0	• Blank paper • Numbers from problem recopied- no understanding of problem evidenced • Incorrect answer and no work shown
1	• Inappropriate strategy started-problem not finished • Approach unsuccessful-different approach not tried • Attempt failed to reach a sub-goal
2	• Inappropriate strategy- but showed some understanding of the problem • Appropriate strategy used-did not find the solution or reached a sub-goal but did not finish the problem • Correct answer and no work shown
3	• Appropriate strategy but __ ignored a condition in the problem __ incorrect answer for no apparent reason __ thinking process unclear
4	• Appropriate strategy or strategies • Work reflects understanding of problem • Incorrect answer due to a copying or computational error
5	• Appropriate strategy or strategies • Work reflects understanding of the problem • Correct answer

Table 3.1: The Rubric for Scoring Problem-Solving performance

Although there are more recent schemes that are recommended for assessing problem solving performance such as one suggested by Van de Walle (2004: 66), the researcher found one by Ottis and Offerman closer to what she uses in evaluating mathematical performance of her students at LCE and also in the marking of National Examinations under ECOL.

Problem-solving is a complex process that involves other sub-processes. According to Charles, Lester and O'Daffer (1997:7-9), in solving a mathematical problem, a pupil must show understanding of a question in the problem, understand the conditions and variables in the problem. She/he should be able to identify relevant and useful data needed to solve the problem. She/he must be able to choose and apply correctly appropriate strategy or strategies that would help him/her to reach the solution. At the end of it all the problem-solver should check the reasonableness of his/her solution. It is on the basis of these that evaluation of problem-solving performance should not only focus on the correct answer but should also consider the intermediate steps that the learners go through before reaching the final solution.

3.3.2 Pupils' Questionnaires

The questionnaire consisted of two sections. Section A sought general and personal information about the respondent, while Section B consisted of 17 statements that were to be rated and one (1) open-ended question.

The statements in this questionnaire were designed to investigate;

a. pupils' experiences in learning mathematics;

b. the perceptions that pupils have about the nature of mathematics;

c. knowledge that pupils have about the structure of an equation, expressions and terms;

d. what knowledge pupils consider important and necessary for successful solution of linear equations; and

e. pupils’ problems in solving linear equations from their own perspective.

The ratings of the responses are on a 5- point Lickert scale in which:

1 = strongly disagree   2 = agree

3 = not sure   4 = agree and

5= strongly agree.

The open-ended question provided the respondents opportunity and space to write what they find problematic when solving algebraic word problems

3.3.3 Teachers' Questionnaires.

The questionnaire consisted of two sections. Section A sought general and personal information about the respondent, while Section B consisted of 22 statements that were to be rated and one (1) open-ended question.

The 22 items in Section B that were responded to using a five-point Lickert scale in which:

1– Strongly disagree   2 – disagree

3 – not sure   4 – agree

5– strongly agree

Just as with pupils' questionnaire, the statements in this section were designed to investigate:

i. Teachers' experiences with mathematics and mathematics teaching.
ii. Teachers' perceptions with regard to the nature of mathematics and mathematics teaching.
iii. Knowledge that teachers have concerning the concept of variable and algebra as a whole.
iv. What teachers consider as important background knowledge to understanding linear equations.
v. Pupils' problems in learning linear equations from teachers' perspective.

The last item on the questionnaire is an open-ended one. Here teachers are provided with opportunity and space to say what seems problematic to pupils in solving linear equations and how they think these problems may be addressed.

3.3 EXECUTION OF THE INVESTIGATION

3.4.1 The Pilot Study

A pilot study is a “small study conducted prior to a larger piece of research to determine whether the methodology, sampling, the instruments and analysis are adequate and appropriate” (Bless & Higson-Smith 2000: 155). This mini-research is an investigation into feasibility of the planned research. It is intended to expose deficiencies of the measuring instruments or the procedure to be followed in the actual project.

Cilliers (de Vos 2002:211) holds the view that a pilot study does not only involve a small-scale trial run on the target group but also includes the literature study, seeking advice from experts in the field, preliminary exploratory studies, and the intensive study of the strategic units. In this study, the piloting was achieved by means of literature study, experience of experts in the fields and preliminary exploratory study.

3.4.1.1 The Study of Literature

The information related to the topic of investigation was gathered from journals, books, and dissertations. This was used to get a clear formulation of the research problem. Earlier studies conducted in the field also provided the researcher with a broad background knowledge about learning pre-algebra in general and more specifically linear equations. According to de Vos (2002:212), a prospective researcher “must trace all available literature that is broadly and specifically relevant to his subject”. From studying earlier researchers’ experiences in the field, the researcher will be able to decide on the suitability of his/her procedure and be aware of possible obstacles on executing the study.

3.4.1.2 Experience of Experts

Through personal interviews with people with expertise in the field, the researcher was able to refine the problem of investigation further (de Vos 2002:212). The purpose of the interviews was to help the researcher get broader perspectives about the subject. The researcher in this particular study referred to colleagues in the field and the supervisor for advice particularly on the wording of the items in the questionnaires and those in the pupils' tasks.

3.4.1.3 Preliminary Exploratory Studies

This involves the execution of the investigation to subjects with similar characteristics as the sample identified for the study. The purpose here is to get a clearer picture of the demands of the investigation with regard to time, finance and transport factors. In this case, the researcher carried out the exploration of the research areas i.e finding out appropriate times when the research may be carried out at the different sites. The amount of time within which the pupils' tasks may be completed was observed during piloting. Necessary corrections, e.g. wording of certain items were made due to feedback from the pilot study. This exercise is very helpful as it makes the researcher aware of possible unforeseen problems that may emerge during the main investigation.

3.4.2 Validity and Reliability

In order for the results of an investigation to be considered true and trust-worthy, the instruments used for collecting the information from which inferences would be made should be tested for validity and reliability**.**

3.4.2.1 Validity: Cangelosi (1996:311) notes that "a measurement is valid to the same degree that it is both **relevant** and **reliable**".

3.4.2.2 Relevance: A measurement is considered relevant if it is aligned to the mathematical content as well as the learning level reflected by the instructional objectives. This implies that much as items must match the objectives, several objectives that address attainment of the goal must be represented in the test (measurement tool) according to their importance. In this study this was ensured by identifying the respective areas that have to be tested and by checking whether the designed tasks actually address the purpose for which they were designed. The pilot testing became useful in this regard.

3.4.2.3 Reliability: A measurement is said to be reliable if it has both internal consistency and scorer consistency. Internal consistency refers to the degree to which the results from various items comprising the measurement are in harmony. Cangelosi (1996:316) notes that “a measurement has scorer consistency to the same degree that:

a) a teacher (or whoever scores the test) faithfully follows the items’ keys so that the measurement results are not influenced by when the measurement is scored, and

b) different teachers (or scorers) who are familiar with the measurement’s content agree on the same score warranted by each item response so that results are not influenced by who scores the measurement.”

To address the reliability concerns in a measurement, a scoring guide(rubric) was used. This identified the particular skills, knowledge, and understandings that were being assessed, and described different levels of quality for each.

Reliability and validity of the questionnaires was ensured by means of the pilot study conducted.

3.5 STRATEGY FOR DATA ANALYSIS

The type of responses that participants provided to the tasks were analyzed with regard to information gathered from the literature study. The statements in both the teachers’ and pupils’ questionnaires were categorized into themes and responses were presented on bar charts in which the frequencies for the related items were reflected. As with the responses to the tasks, the nature of the results observed were analyzed with reference to the literature study. The last part of the analysis was concerned with the comparison of the responses from the participating teachers and pupils on similar or related statements concerning pupils’ problems in understanding linear equations. This was done using the scatter diagrams for the mean scores for the responses. The correlation coefficient was also calculated to investigate the type of relationship between the responses using the mean scores. The Spearman Rank method was used.

3.5.1 The Spearman Rank Order Coefficient

This is used to investigate if correlation exits between two sets of data. The corresponding values that are compared are ranked according to the relative position in the group. The difference between the ranks are calculated and then squared. According to this method the correlation coefficient indicated by r or the Greek letter *ñ* (rho) and is calculated using the formula:

r = 1 - $\frac{6\sum d^2}{n(n^2-1)}$, where d = difference between ranks

and n = number of pairs of values compared.

The correlation coefficient must be a value between –1 and 1 (inclusive):

-1 r 1 or -1 *ñ* 1

If r = + 1, this indicates that a positive and strong correlation between the sets of data compared.

If r = -1, this also indicates a strong correlation between the compared sets of data though with a negative slope

If r = 0 there is no linear relationship between the values. (Steffens 2002: 167).

3.6 CONCLUSION

In this chapter, the procedure followed in conducting the research was discussed. This included the description of the target population sampling methods. The instruments used in collecting data and a description of how their validity and reliability were ensured was also provided. A method used in determining whether there is any correlation between responses from participating teachers and pupils to the statements, in the questionnaires, that bear similar meaning was also discussed. The next chapter will be on the presentation, analysis and interpretation of the results from the study.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

In the preceding chapter, the procedure that was followed in conducting the study was discussed**.** Two sets of questionnaires (one for the pupils and another for the teachers) were used. A set of tasks was also given to pupils to work on. In this chapter the results of the investigation will be presented, analyzed and interpreted.

4.2 STUDENTS' RESPONSES TO TASKS

The tasks were in four categories based on the areas that each group addressed. The categories were based on concepts underlying understanding of linear equations. Tasks in Section A were designed to assess pupils' ability to formulate algebraic expressions

4.2.1 SECTION A: Formulating Algebraic Expressions

Table 4.1 shows the distribution of responses on tasks in Section A .

ITEM\ OPTION	A	B	C	D	BLANK
1	1(1.3 %)	3(3.8%)	**67(83.8%)**	9(11.3%)	0 (0%)
2	2 (2.5 %)	**71 (88.8%)**	6 (7.5%)	1 (1.3%)	0 (0%)
3	**40(50%)**	21(26.3%)	11(13.8%)	7 (8.8%)	1(1.3%)

4	4 (5%)	5 (6.3%)	8 (10.0 %)	**61(76.3%)**	2 (2.5%)
5	29 (36.3%)	5 (6.3%)	13 (16.3%)	**31(38.8%)**	2 (2.5%)

Table 4.1

The highlighted (bolded) numbers indicate the correct responses.

In all the five (5) tasks that were meant to test pupils' ability to formulate algebraic expressions for the worded mathematical expressions, the majority of the respondents were able to formulate the correct expressions. For items 1, 2 and 4, above 75% of the respondents were able to form the correct algebraic expressions while only 50% and 38.8% of the respondents correctly got the expressions for items 3 and 5 respectively. In the latter two cases, the tasks involve the concept of ratio and there seems to be much confusion in the responses, the subjects were not sure of when to multiply and when to divide. This is particularly evident in item where there is a very slight gap between alternatives A and D, which both involved ratio. It is surprising though that the next popular response to item 3 is B (26.3%) which has nothing to do with ratio. As was realized from the literature study, in deed, pupils experience serious problems when they have to deal with letters in expressions (Kuchemann1981: 104).

4.2.2 SECTION B: Mathematical Language for expressions

Table 4.2 below shows the distribution of responses to tasks in Section B. The tasks were intended to assess pupils' mastery of the mathematical language often used in real life problems that involve solution of linear equations.

ITEM\ OPTION	A	B	C	D	BLANK
6	21 (26.3%)	3(3.8%)	**52 (65.0%)**	3 (3.8%)	1(1.3%)
7	**76 (95%)**	0 (0%)	2(2.5%)	0(0%)	2 (2.5%)
8	10 (12.5%)	38 (47.5%)	**26 (32.5%)**	2 (2.5)	4 (5.0%)
9	6 (7.5%)	16 (20%)	26 (32.5%)	**31 (36.3%)**	1(1.3%)
10	8 (10.0%)	35 (43.8%)	**36 (45.0%)**	1 (1.3%)	0 (0%)

Table 4.2

The highlighted (bolded) numbers indicate the correct responses.

This seems to have been a more challenging section compared to Section A. Although 65% and 95% of the respondents were able to score correctly in items 6 and 7 respectively, the correct scores for items 8, 9 and 10 were obtained by less than 50% of the respondents. Very small gaps exist between the percentages for the correct and the next popular alternatives.

It appears that for item 6, alternative A was the next popular response with 26.3% frequency after the correct alternative C (65.0%). It seems that this choice results from what is termed the 'parsing obstacle' (see section 2.3.3.4). This obstacle leads learners into reading '*a* and *b*' as '*ab*' and end up thinking that it is the same as '*a* + *b*'. Alternative B (47.5%) was even more popular than the correct alternative C (32.5%) for item 8. This shows that most respondents confused 'quotient' for 'product'. A further indication of this is reflected by the choice of alternative A (12.5%) against alternative D (2.5%). Item 8 seems to have been more difficult to the respondents; it has the highest number (4.8%) of subjects who left an item unanswered. Responses to item10 further indicate that for some pupils indeed '*a* and *b*' means the same as '*ab*' since almost the same number of respondents chose alternatives A and B, where the latter is the correct alternative.

4.2.3 SECTION C: Manipulation of Algebraic Expressions

Tasks in this section were designed to test pupils' ability to manipulate algebraic expressions correctly.

Table 4.3 shows the distribution of subjects' responses to the tasks.

ITEM\OPTION	A	B	C	D	BLANK
11	31(38.8%)	2(2.5%)	12(15%)	**32 (40.0%)**	3(3.8%)
12	**38 (47.5%)**	13 (16.3%)	26(32.5%)	3 (3.8%)	0(0%)
13	31 (38.8%)	7 (8.8%)	**39(48.8%)**	2 (2.5%)	1 (1.3%)
14	7 (8.8%)	**37 (46.3%)**	25 (31.3%)	10 (12.5%)	1 (1.3%)
15	21 (26.3%)	**27 (33.8%)**	17 (21.3%)	14 (17.5%)	1 (1.3%)

Table 4.3

The highlighted (bolded) numbers correspond to the correct solutions.

Algebraic manipulations seem to have presented much challenge to the respondents. Even though larger percentages of the respondents were able to get the correct alternatives, these are with lower percentages than what was observed in Section A. Only 40% of the respondents got item 11 correct. A very narrow gap can be observed between the correct answer D (40%) and A (36.3%), which is the next popular but wrong alternative. On simplifying $\frac{x+2}{2x+4}$ one of the respondents showed the working as follows:

$$\frac{x+2}{2x+4} = \frac{1}{2} + \frac{2}{4} = \frac{3}{6} = \frac{1}{2}$$

Even though this eventually resulted in the correct option, the working reveals some misconceptions regarding manipulation of fractions. It is not surprising that the same respondent chose $\frac{4m}{7}$ as a simplified expression for $\frac{3m}{4} + \frac{m}{3}$ in fact 38.8% of the respondents chose $\frac{4m}{7}$ as the correct answer. The option A i.e. ¼ for the same task arises from 'canceling' the *x*'s and the 2's. It was noted earlier that understanding and facility with arithmetical operations enables pupils to acquire proper understanding of algebraic processes. (See section 2.3.3.2).

For item 12, 47.5% of the respondents were able to get the correct answer A. The next popular response was alternative C (32.5%). From one respondent's working:

$5 (2 – t)- 3 (2t + 1)$

$=10 –5t- 6t – 3$

$=10 – 3 - 5t – 6t$

$= 7 – t$

It can be realized that even when the expansion of brackets is correctly done (which is hard to achieve for many pupils), combining the like terms is also another obstacle for the respondents. This is a clear indication that this is one of the problems that pupils encounter in learning linear equations.

Even though many respondents, 46.3% and 33.8% got the correct answers for items 14 and 15 respectively compared to other alternatives, this is still with low percentages. The next popular alternatives do indicate that indeed manipulation of fractions poses a great challenge to pupils at this level. It seems that for item 14 although respondents could recall that where division of fractions is involved that could be changed to multiplication. How this is done was not clear to them, hence $\frac{48}{15}$ was chosen , arising from 8 x 6 (for the numerator) and 3 x 5 for the denominator. In item 15 however, respondents suggest that $2\frac{1}{2}k \div 1\frac{1}{2}k$ can be simplified to 2 as the *k*'s and the ½'s divide or 'cancel' each other leaving 2 ÷ 1 = 2. This is a further indication of misconceptions that pupils have in fractions. Rules that govern manipulation of fractions are also very important in solving many algebraic problems and equations. (See section 2.3.3.2).

4.2.4 SECTION D: Solving Algebraic Equations

The items in this section were focused on pupils' ability to formulate algebraic equations from word problems that are based on real life contexts; and their ability to solve readily existing linear equations. The scores for each respondent were worked out according to the rubric outlined in section 3.4.

Table 4.4 shows the distribution of scores of respondents for tasks in this section.

ITEM\SCORE	0	1	2	3	4	5
16	42 (52.5%)	0 (0%)	1 (1.3%)	1 (1.3%)	4 (5.0%)	32 (40.0%)
17	93 (93.8%)	0 (0%)	4 (5.0%)	0 (0%)	1 (1.3%)	0 (0%)
18	6 (7.5%)	30 (37.5%)	8 (10.0%)	8 (10.0%)	8 (10.0%)	20 (25.0%)
19	18 (22.5%)	11 (13.8%)	5 (6.3%)	7 (8.8%)	6 (7.5%)	33 (41.3%)
20	15 (18.8%)	25 (31.3%)	4 (5.0%)	0 (0%)	2 (2.5%)	34 (42.5%)
21	7(8.8%)	2 (2.5%)	6 (7.5%)	0 (0%)	2 (2.5%)	63(78.8%)

Table 4.4

Items 16 and 17 were real life problems and 52.5% and 93.8% of the respondents scored zero (0) marks in both questions respectively. Only 40% of the respondents scored the total score (5 marks) for item 16, while none of them got the total score (5 marks) for item 17. The 5% who scored the 2 marks for item 17 only managed to get them as 'benefit of doubt'(BoD) marks, they were not the clear 2-mark cases suggested in the rubric. Indeed item 17 was the most poorly done, perhaps it was more involved than item 16. From the literature study, it was indicated that pupils indeed have problems when they have to translate from words to mathematical symbolism.(See section 2.3.3.7.) In their attempt to solve item 16, the most common wrong equation formulated by the respondents was $3x + 15 = 2x$.

Problems that people encounter in real life are not the ready equation-formatted problems, they need to be interpreted and translated into the symbolic mathematical terms (equations) before they can be successfully solved. From the responses to these two items, it becomes very clear that this is not a very easy task. This is further highlighted by what can be observed in the next items 18, 19, 20,and 21 in which subjects had to solve the already existing linear equations. Here the number of respondents who scored zero dropped drastically. It was observed that even respondents who scored zeros for items 16 and 17 were able to score 5 marks on some of the next items (18, 19, 20 and 21). Generally respondents were able to score higher marks in the latter items, greater percentages of respondents scored the total 5-mark for each question.

Although items 18 and 21 are similar in that they both have a variable on one side of the equal sign, item 18 further requires knowledge about expanding the brackets and knowledge about operations on directed numbers. It is not surprising therefore that respondents performed poorly on item 18 than on item 21. Only 25% of the respondents scored the total 5-mark for item 18 against 78.8% in item 21.

Very interesting revelations were observed in responses for item 21. As was discovered from the literature study, the use of the '=' sign as a symbol for equality, and that an equation is a statement of equality of two quantities on the two sides of the equation, really do not seem to be obvious to the pupils as one would have thought. To some the equal sign is a directive to carry an operation, hence responses such as :

$$
\begin{aligned}
7 + 9 &= x + 10 \\
&= 16 + 10 \\
&= 26
\end{aligned}
$$

were also found. (See section 2.3.3.6.)

It was also realized that some respondents did not begin by adding 7 and 9 first, but immediately used the inverse operations throughout, i.e. they did their working as follows:

$$
\begin{array}{rcl}
7 + 9 &=& x + 10 \\
\underline{-\ 9} && \underline{-\ 9} \\
7 &=& x + 1 \\
\underline{-\ 1} && \underline{-\ 1} \\
6 &=& x, \qquad x = 6.
\end{array}
$$

This type of working reveals that respondents applied the learnt algorithm/procedure and worked mechanically to the right solution without making sense out of the equation.

Responses such as: $7 + 9 = x + 10$

$$
\begin{array}{rcl}
\underline{-7} && \underline{-\ 7} \\
9 &=& x + 3 \\
\underline{-\ 9} && \underline{-\ 9} \\
x &=& -\ 6
\end{array}
$$

also indicate that the respondent did not really care to understand what the statement tells, he/she just applied the procedure carelessly and does not even bother to see if the solution makes the original equation true. This could also be an indication that pupils are not able to make sense of algebraic statements or equations. They do not really understand the structure of the relations in the problem. (See section 2.3.3.4.)

In both items 19 and 20, a variable appears on the two sides of the equal sign. Of the two items, it was observed that item 19 has the highest percentage of respondents who scored zero i.e. 22.5% against the 18.8% in item 20. It is surprising though that item 19 which 'looks' less difficult than item 19 is the one that gave the respondents more difficulty. For this item 50% of the respondents were only able to score up to 1-mark, while for item 19 there was only 36.5%, which indicates that 63.5% of the respondents scored more than 1-mark. The most common misconception here was in the algorithm involving 'transferring terms to the other side of the equal sign'. This gave rise to workings such as :

$$
\begin{array}{rcl}
11 - 3x &=& 18 - 4x \\
29 &=& 7x
\end{array}
$$

This clearly indicates that the algorithm is applied without any meaning or understanding attached. Again after a series of wrong operations, one respondent worked to the step $7 = x + 1$, and in an attempt to transfer the 'x' to the other side the new equation was $7x = 1$, but in the next step the final answer was $x = \frac{1}{7}$ and not $x = 71$ or 17 which would perhaps be consistent with his/her interpretation of the 'transferring process'.

4.2.5 Synthesis

What is revealed by responses to pupils' tasks can be outlined as follows:

1. Formulating equations from word problems is the most difficult part in pupils' attempt to solve mathematical problems, as reflected by respondents' performance on items 16 and 17 compared to responses to items 18, 19, 20, and 21. (See section 2.3.3.6.)

2. Pupils experience difficulties particularly when confronted with forming algebraic expressions in cases where either or both multiplication and division are involved. (See section 2.3.3.1.)

3. Mathematical language poses a lot of difficulty for most pupils e.g '*a* and *b*' is interpreted as '*ab*', and quotient' is confused with 'product'. (See section 2.3.3.4.)

4. Algebraic manipulations pose a greater challenge to pupils. (See section 2.3.3.4)

5. Pupils rely on applying algorithms that they do not even seem to understand and ultimately get wrong solutions. (See section 2.3.2.2)

6. Operations that involve directed numbers; especially negative numbers are another source of pupils' frustration in solving linear equations.

7. Pupils have difficulty in working on algebraic expressions involving fractions

- the ‘cancelling’ as opposed to versa vis ‘dividing’ poses a lot of problems
- the ‘change sign and turn upside down’ algorithm in which the dividing fraction is turned up-side down and the division sign is changed to multiplication, is not well-understood

4.3 STUDENTS’ QUESTIONAIRE ANALYSIS

4.3.1 SECTION A: Personal And General Information

4.3.1.1 *Gender of respondents*

N = 80

Gender	Number	%
Male	42	52.0
Female	38	48.0

Table 4.5

From Table 4.5 it can be noticed that 52% of the respondents were male, while 48% were female pupils.

4.3.1.2 *Age of respondents*

N= 80

Age	Number respondents	%
14 Years	10	12.5
15 Years	31	38.8
Over 15 Years	39	48.7

Table 4.6

It can be observed from Table 4.6 that 12.5% of the respondents were 14 years old, 38.8% were15 years and 48.7% were above 15 years of age.

4.3.1.3 *Grade obtained in PSLE(primary School Leaving Examinations)*

N = 80

Grade in PSLE	Number of respondents	%
1	43	53.8
2	18	22.5
3	17	21.3
4	2	0.25

Table 4.7

It can be observed from the table that 52.8% of the respondents had obtained a very good pass (Grade 1) in mathematics in their PSLE, while 22.5% obtained Grade 2 which is a fair pass and 21% obtained Grade 3 which is the lowest level for a pass. About 3% of the respondents had completely failed mathematics in the said examinations.

4.3.2 Section B

The statements in this section were designed to investigate:

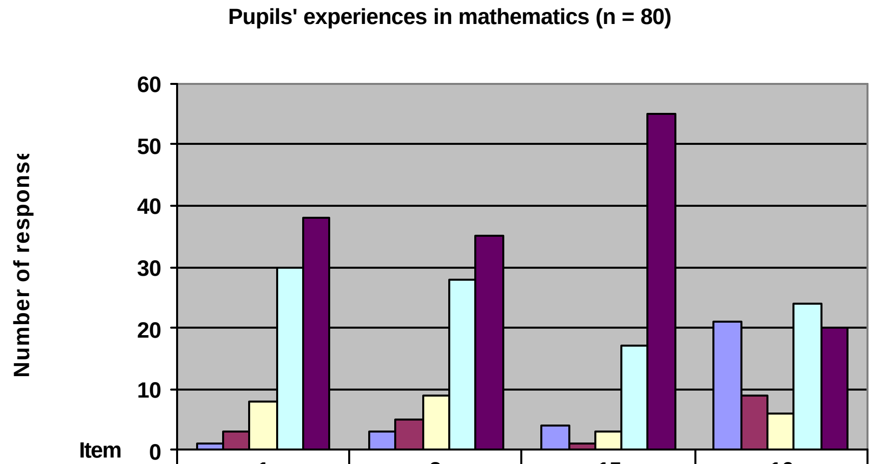
A. Pupils' experiences in learning mathema tics
B. The perceptions that pupils have about the nature of mathematics
C Knowledge that pupils have about the structure of an equation, expressions and terms
D What knowledge pupils consider important and necessary for successful solution of linear equations
E Pupils' problems in solving linear equations from their own perspective.

The ratings of the responses are on a 5- point lickert scale in which:

1 = strongly disagree 2 = agree
3 = not sure 4 = agree and
5 = strongly agree.

4.3.2.1 Pupils' Experiences in Learning Mathematics

Fig. 4.1 shows the distribution of subjects' responses to statements addressing theme 1 i.e. Pupils' experiences in learning mathematics



Item	1	3	15	16
Strongly Disagree	1	3	4	21
Disgagree	3	5	1	9
Not Sure	8	9	3	6
Agree	30	28	17	24
Strongly Agree	38	35	55	20

Fig.4.1

Statements/items addressed were:

1. I enjoy learning Mathematics

3. Solving linear equations is interesting

15. I learn better through discussions.

16. My teacher allows me to use my own methods

These responses indicate that above 50% of the respondents has a positive attitude towards Mathematics. Of the respondents, 85% enjoying learning Mathematics, and 78.8% enjoy solving linear equations; 90.1% feel they learn Mathematics better through class or peer discussion. Discussions provide the best opportunity for meaningful learning. There is freedom of expression and pupils learn in a more relaxed atmosphere and are therefore most likely to enjoy the experience. (See section 2.3.3.7.)

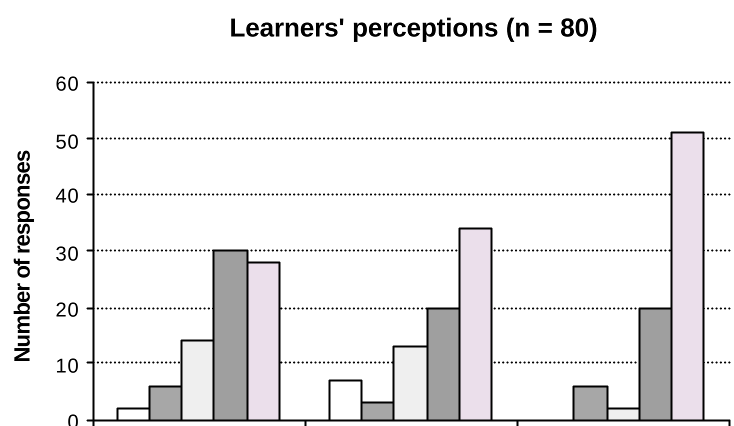
While 55% of the respondents indicated that their mathematics teachers allowed them to use their own methods in solving problems, 37.5% indicated that they were not allowed to do so and 7.5% were not sure whether they were allowed or not. It was indicated earlier that it is important that teachers allow their pupils to use their own methods in solving problems. This would help teachers to know the solution procedures that pupils are familiar with, and so help them (pupils) develop more formal (algebraic) methods of solution. (See section 2.3.3.7.)

4.3.2.2 Learners' Perceptions About Mathematics and its Teaching and Learning

Statements that were intended to assess pupils' perceptions on the nature of mathematics and what learning mathematics involves are:

2. All life activities involve mathematics
5. Mathematics is remembering formulas, rules, and procedures
7. I enjoy doing mathematics if the teacher first demonstrates how to do it.

Figure 4.2 is a bar chart indicating the distribution of responses to the statements.



item	2	5	7
Strongly disagree	2	7	0
Disagree	6	3	6
Not sure	14	13	2
Agree	30	20	20
Strongly agree	28	34	51

Fig. 4.2

Although 72.5% of the responders do appreciate and acknowledge that all activities that we carry out in our everyday life involve application of mathematics, 67.5% believe mathematics to be about remembering formulas, rules, algorithms and applying them to mathematical problems. Many respondents (63.8%) indicate that they enjoy doing mathematics, i.e solving problems, if the teacher first demonstrates the method of solving a given problem.

These attitudes and feelings are perhaps a result of the respondents' experiences and exposure to the manner in which mathematics has been taught to them. Experience confirms that traditionally mathematics' lessons are characterized by a teacher presenting a problem to the pupils and then going on to show/demonstrate how that problem can be solved. What then remains for the pupils, is to practice the demonstrated skill on problems that are similar to the one used as an example. This is why respondents enjoy the mathematics they learn. To them doing mathematics is just a matter of applying the rules and procedures correctly. There is definitely need to change from this kind of approach to mathematics teaching to one that promotes learning with understanding. Allowing pupils to discover the intended mathematical content, acquiring appropriate mathematical skills is an effective way of teaching of mathematics.

4.3.2.3 Knowledge About Structure of Equations, Expressions and Terms

The third theme of the investigation is on the knowledge that pupils have about the structure of an equation, expression and terms. The following items addressed this:

4. Letters can be used to represent specific numbers;

6. A letter can be used to represent any number

11. 3x + 4 is an example of algebraic equations;

12. There are 3 terms in 3x + 4.

Figure 4.3 below is a bar chart indicating the distribution of pupils' responses to these items.

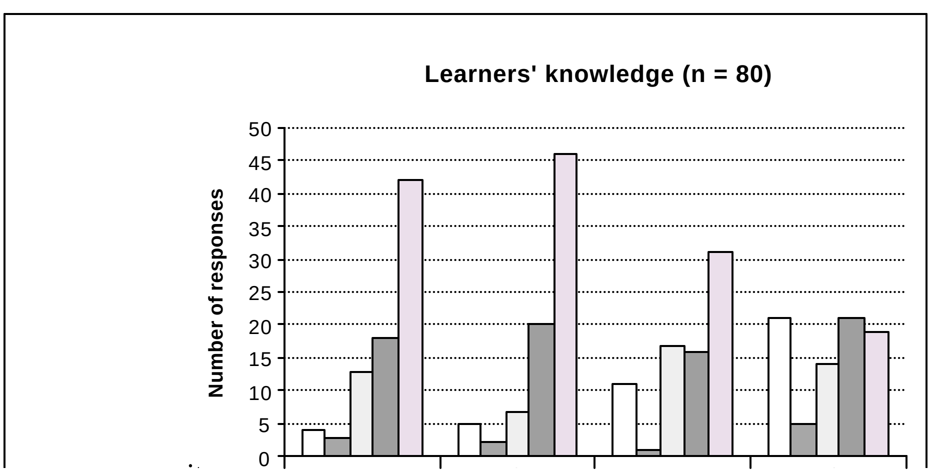


Fig.4.3

For items 4 and 6, 75% and 82.5% of the respondents agree that letters can be used to represent specific numbers and any number, respectively. Although it is encouraging to note that a greater number of respondents do have knowledge about use of letters in mathematics, it was surprising to find out that 58.8% of the respondents believe that 3x + 4 is an equation and only 15.1% do not agree to this; 21.3% of the respondents are not sure. For item 12, 50.1% of the respondents think there are 3 terms in 3x + 4, 32.6% do not agree with this and 17.5% are not sure.

These results are in line with earlier studies carried out. Responses indicate that indeed, pupils have some confusion regarding the concept of a variable (see section 2.3.3.3) and about the structure of an equation (see section 2.3.3.4).

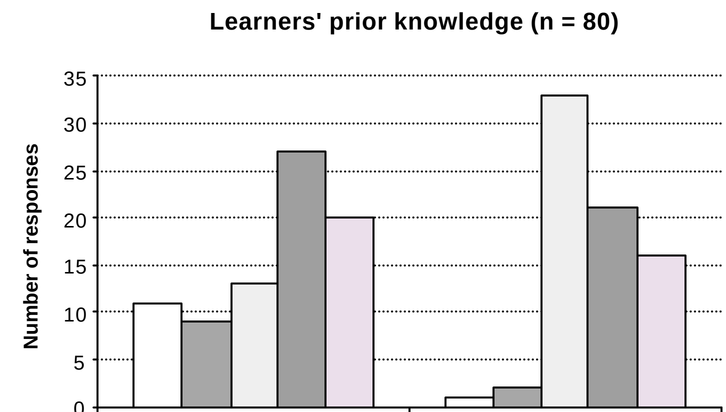
The observation that 50.1% of the respondents believe that there are three (3) terms in the expression 3x + 4 illustrates that they do not have proper interpretation for expression and term. Their third term is most probably from seeing 3x as two terms. This problem is due to lack of understanding of this symbolic notation.(See section 2.3.3.4.).

4.3.2.4 Background Knowledge to Understanding Linear Equations

The items were designed to assess whether pupils were aware of the background knowledge that would help them solve linear equations. These items were:

8. I have no problem solving linear equations involving directed numbers
9. Knowledge of arithmetic rules can help one to do algebraic manipulations correctly.

Figure 4.4 below shows the distribution of participants' responses to the items.



Item	8	9
Strongly disagree	11	1
Disagree	9	2
Not sure	13	33
Agree	27	21
Strongly agree	20	16

Fig.4.4

Responses to item 8 indicate that 58.8% of respondents do not have difficulties with handling equations with directed numbers, while 16.3% were not sure and 25% indicate in clear terms that they do encounter problems whenever operation with the negative sign is involved. This is in line with the observations made from pupils' responses to the tasks, where it was realized that respondents do show difficulties where negative numbers were involved (example on 20). Pupils' inability to solve linear equations which involved directed numbers is usually seen where brackets are involved, following a minus sign and where the problem solver needs to collect like terms.(See section 2.3.3.5.)

Responses to item 9 indicate that only 46% of the subjects agree that knowledge of arithmetic rules is useful in performing algebraic manipulations, 41.3% is not sure and 3.8% does not believe that such knowledge is useful. One may suspect here that this 41.3% of respondents may have not quite understood the statement in order for them to make the correct decision. But since the number that agrees is more than those who do not agree with the statement, it may still be concluded that the majority of the respondents realize the need for knowledge of arithmetic rules in order for them to be able to manipulate algebraic expressions correctly.

Indeed an observation was made earlier that it is necessary that pupils are helped to appreciate the relationship between algebra and arithmetic, as most of the rules in arithmetic can be applied in algebra to simplify algebraic expressions and in solving equations.(See section 2.3.3.2.)

4.3.2.5 Problems That Pupils Encounter in Solving Linear Equations (pupils' perspective).

The items in this category were designed to investigate pupils' problems with regard to solving linear equations. These items were;

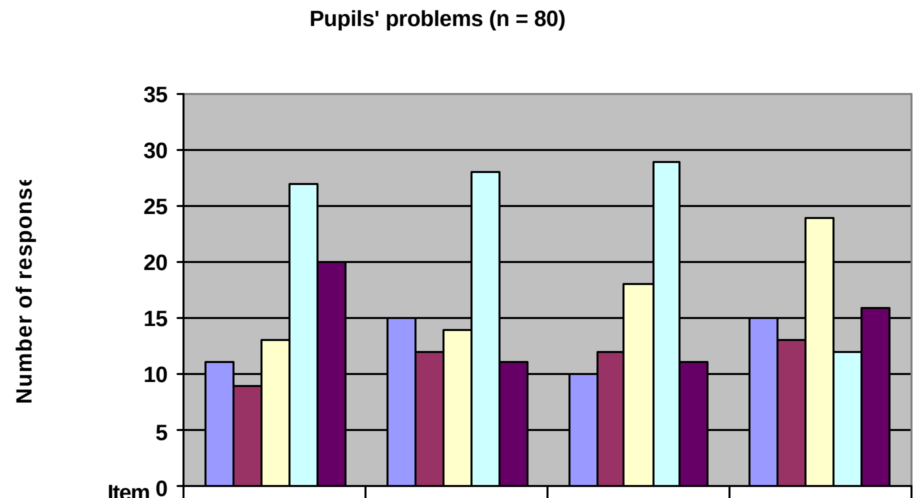
8. I have no problem solving equations that involve negative numbers;

10. I enjoy forming equations from word problems;

13. I enjoy solving algebraic word problems; and

17. I have no problem solving linear equations that have the unknown on both sides.

Figure 4.5 below shows the distribution of pupils' responses to the above statements. Here the majority of the respondents disagreed with all the statements. The respondents indicated that they do not enjoy formulating equations from word problems. They do not enjoy solving those linear equations that involve negative numbers as well. This indeed, is in line with what they have expressed earlier, and was also confirmed by the literature study.(See section 2.3.3.6 and section 2.3.3.7.)



Item	8	10	13	17
Strongly Disagree	11	15	10	15
Disagree	9	12	12	13
Not sure	13	14	18	24
Agree	27	28	29	12
Strongly agree	20	11	11	16

Fig 4.5

4.3.2.6 Pupils' Responses To The Open-Ended Question

The last part of the pupils' questionnaire was an open-ended question that provided the respondents opportunity and space to write what they find problematic when solving algebraic word problems. Here a lot of ideas/opinions were brought up, although most mainly emphasized the opinions expressed in the earlier section of the questionnaire.

The majority of the respondents indicated that they experience great difficulties in translating the algebraic word problems into symbolic equations. They say sometimes they do not understand what the question really wants them to do. The language is too difficult to understand. One respondent wrote, " The difficulty is that some word equations have no operation sign." This further illustrates that translating form word to symbols is not an easy task. This is also confirmed by one respondent who wrote "when I am given a problem I always experience difficulties in translating it into an equation. But if I were able to translate it correctly I would have absolutely no problem in solving such equations." These responses only confirm what was gathered from the literature study. (See section 2.3.3.7.)

Another problem area mentioned by respondents though not new, is that concerning their interpretation of terms and expressions. One respondent indicates “I find it difficult where I multiply letters with numbers and add numbers with letters and [I] get stressed.” This shows that for some pupils at this level, terms and expressions do not convey any meaning to them. Another respondent revealed that he/she has a problem when he has to “give answers mixed with letters and numbers”, which of course confirms what has been discussed earlier, that for pupils at this level giving an answer in a form of expressions is not acceptable. (See section 2.3.3.3.)

As indicated earlier, most findings in this part of the questionnaire merely emphasize what has been expressed before. Directed numbers, particularly negative numbers are a stumbling block to successful solution of linear equation. Collecting like terms is another problematic area. One respondent reckons that moving terms to another side of the equation creates problems; he/she thinks, “it goes with its sign.” The same respondent mentioned that the many different methods or strategies used in solving equations make the process even more difficult to understand. What these suggest probably is that to some pupils the whole process of solving linear equation is mere manipulation of numbers and letters, applying rules and procedures that give no meaning to them. This perception usually results when pupils are provided with algorithms, rules and procedures without any meaning attached.

The manner in which a mathematics teacher handles his/her lessons greatly influences the performances of pupils in the subject. While some respondents indicated the enthusiasm of their mathematics teachers toward the success of their students, some responses reveal that pupils have even developed a negative attitude toward the subject because of the way they are treated particularly when they do not understand something. It was interesting to find out that some pupils were aware of the problem solving strategies as suggested by Polya (see section 2.3.3.7). One respondent, in explaining how he/she tackles problems involving linear equations, wrote, “I read the question to understand, then solve the problem. When I have found the answer I check my solution. If the equation does not balance, I start working again.”

4.3.2.7 Synthesis

Findings from pupils’ questionnaires can be outlined as follows;

1. Formulating algebraic equations from algebraic word problems is the most difficult part in solving linear equations. (See section 2.3.3.7.)
2. When confronted with an equation ready for them to solve, some pupils indicate that:
    - They find working with negative numbers quite problematic.
    - Collecting like terms is also not an easy task, especially when it involves transferring from the other side of the equal sign.
    - The many procedures, methods used when transforming equations to their simpler equivalent forms are difficult to follow and create confusion. They find if difficult to add or multiply letters with numbers, in other words they cannot comprehend what these collection of number and letters mean.
    - They do not know what they are supposed to do; the only method available is guess and check and is tiresome. (See section 2.3.3.5.)

3. Most pupils enjoy learning mathematics through discussions (class and or peer). (See section 2.3.3.5.)
4. Most pupils enjoy solving linear equations if the teacher first demonstrates how they should work out the problem and then gives them exercises to practice the skill.
5. Pupils like to be given chance to use their own methods in solving equations.
6. Some pupils believe that doing mathematics is applying rules, procedures and algorithms and mastery of these will help to solve linear equations successfully.

4.4 TEACHERS' QUESTIONNAIRES ANALYSIS

The questionnaire consists of two sections, Section A and Section B. Section A is focused basically on personal and general information about the respondents. Section B 22 items designed to seek information related to the teaching and learning of solving linear equations. The questionnaires were distributed to 15 teachers and were all collected back.

4.4.1 SECTION A PERSONAL AND GENERAL INFORMATION

4.4.1.1 Respondents' Gender

N = 15

Gender	Number	%
MALE	9	60
FEMALE	6	40

Table 4.8

It can be observed that 60% and 40% male and female teachers, respectively, took part in the study.

4.4.1.2. Age of Respondents

N = 15

Age in Years	17-20	21-24	25-28	29-32	Over 32
Number	0	0	1	4	10
%	0	0	6.7	26.7	66.7

Table 4.9

Table 4.10 shows that all the respondents were above 25 years of age. While 6.7% of the respondents were within 25-28 years, 26.7% were within 29-32 years and 66.7% were above 32 years of age.

4.4.1.3 Marital Status of Respondents

N = 15

Marital status	Single	Married	Divorce	Widowed
Number	3	11	0	1
%	20	73.3	0	6.7

Table 4.10

Table 4.11 indicates that 20% of the respondents were single, 73.3% were married and 6.7% were widowed. None were divorced.

4.4.1.4 Qualifications of Respondents

N = 15

Certificate	C.O.S.C	STC	Dip.Sc.Ed	Bsc.Ed	Hons/Masters
Number	0	2	1	10	2
%	0	13.3	6.7	66.7	13.3

Table 4.11

Key: C.O.S.C. : Cambridge Overseas Schools' Certificate
S.T.C. : Secondary Teachers Certificate
Dip. Sc. Ed.: Diploma in Science Education
Bsc.Ed. : Bachelor of Science with Education

4.4.1.5. Teaching Experience of Respondents

N = 15

Teaching Experience in Years	0-3	4-7	8-11	12-15	Above15
Number	2	3	2	5	2
%	13.3	20	13.3	33.3	13.3

Table 4.12

4.3.2 SECTION B

This section was made up of 22 items that were responded to using a five-point Lickert scale in which:

1 = Strongly disagree 2 = disagree

3 = not sure 4 = agree

5 = strongly agree

Just as with the pupils' questionnaire, the statements in this section were designed to investigate:

A. Teachers' experiences with mathematics and mathematics teaching.

B. Teachers' perceptions with regard to the nature of mathematics and mathematics teaching.

C. Knowledge that teachers have concerning the concept of variable and algebra as a whole.

D. What teachers consider as important background knowledge to solving linear equations.

E. Pupils' problems in solving linear equations from teachers' perspective.

The last item on the questionnaire is an open-ended one. Here participants were provided with opportunity and space to say what seems problematic to pupils in solving linear equations and how they thought these problems could be addressed.

4.4.2.1. Teachers' experiences with Mathematics and Mathematics Teaching

Fig. 4.6 shows the distribution of responses addressing this aspect as reflected by items 1 and 2.

1. I enjoy teaching mathematics. 2. Algebra is the most interesting part of mathematics

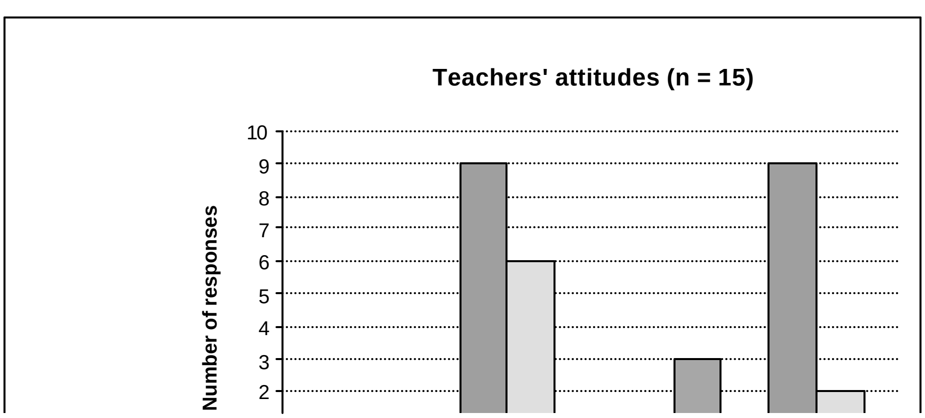


Fig 4.6

Fig. 4.6

Responses here indicate that although 100% of the respondents enjoy teaching mathematics, 73.3% of them find algebra teaching interesting, 6.7% are not sure and 20% do not find algebra teaching interesting. Perhaps this is due to the fact that this area is problematic to many pupils and even to teachers. The latter is reflected in the findings from pupils' tasks in Section 4.2 and responses to the pupils' questionnaires in section 4.3, and also supported by literature. The fact that 100% of the respondents have acquired the relevant certificates for teaching mathematics at this level is one contributing factor to their positive attitude to the teaching of the subject. When teachers have a liking for their subject, they are also more likely to inject this love to their pupils, no wonder why 85% of the responses from pupils indicate love for mathematics.

4.4.2.2 Teacher -perceptions about Mathematics, and Mathematics teaching and learning.

Quite a number of items focused on this aspect. These are items 3,4,6,7,14,15,18,19,21 and 22.

The purpose of these statements is to seek information on what teachers regard as good teaching and practices that they engage in helping their learners develop understanding of taught concepts. The items address specifically the fifth objective of the study: to assess and evaluate the teaching of pre-algebraic concepts.

Fig. 4.7 below is a bar chart showing the distribution of the responses to these items.

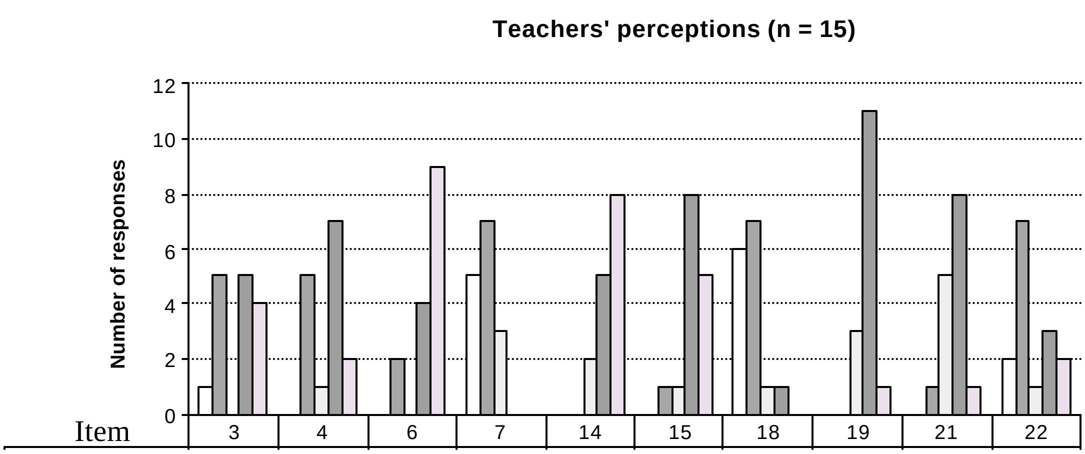


Fig. 4.7

From the above responses it can be noticed that 60% of respondents believe that learning mathematics is remembering formulas, rules and procedures and applying them to problems. In fact, 85% of respondents say that in solving linear equations it is important that pupils learn correct procedures. Of the respondents, 60% believe that the best way to teach mathematics is to show pupils how to workout solutions and then let them practice the skill, while only 33.3% do not agree with this and 6.7% are not sure.

These opinions tally very well with what the majority of these pupils expressed, they enjoy mathematics lessons when the teacher first demonstrates how to work out the solutions and then they practice the skills through further exercises. This, unfortunately, is typical of what happens in traditional mathematics classrooms. Pupils are not given opportunity to explore and discover the mathematics. Surely this practice must stop. According to the NCTM Standards (1991:89) instructions should not only focus on memorizing definitions or executing computational algorithms. Instead they should incorporate a wide range of objectives such as making conjectures, constructing arguments and validating solutions.

It is encouraging though that 86.7% of the respondents do agree that pupils should be allowed to use their own methods and procedures in solving problems, and only 13.3% disagree with this. This is in line with what was gathered from the literature study. Allowing pupils to use their own informal methods of solving word problems will pave way to teachers to lead them into more abstract ways of solving linear equations. (See section 2.3.3.7.)

With regard to real life problems, 86.6% of the participants acknowledge that real life problems are useful in introducing new topics. The majority (86.7%) of the respondents also do not believe that the word problems should just be given to those gifted pupils to solve. Most of the respondents (80%) also indicate that they spend some time in their lessons teaching problem-solving strategies to pupils. This is confirmed by responses to pupils' questionnaire where there is indication that participants were aware of the problem-solving process.

Many respondents (80%), indicated that they use discussions in their teaching, these provide more opportunity for pupils to verbalize their thoughts. During problem solving, it remains the responsibility of the teacher to make discussions successful, where all participate freely without fear of rebuke. It is worth noting though that not everybody can manage to do the same kind of mathematics. All pupils must, however, be given opportunity to experience mathematics to their potential, leaving them with confidence that they can do mathematics.

Most of the respondents, (60%), agree that they benefit from the workshops organized by the Ministry of Education and Training. They are equipped with skills that enable them to generate interesting real life problems for use in their classrooms. One unfortunate revelation is that only 33.3% of the respondents agree that they often discuss problems they encounter in their teaching, in their departmental meetings. As part of professional development, teachers of mathematics need to open doors for sharing ideas and experiences about mathematics teaching. At school level, the mathematics departmental meetings should provide opportunities for such discussions, whether informally or in the form of organized seminars of workshops. Collegial links with teachers in other schools can play a vital role in teacher development; these can give teachers chance to explore instructional strategies and perform better in their teaching. There is absolutely no reason why teachers would not attend workshops, especially when they are fully sponsored and are beneficial to them.

4.4 2.3. Content Knowledge that Teachers Have About Variable and Algebra as a Whole.

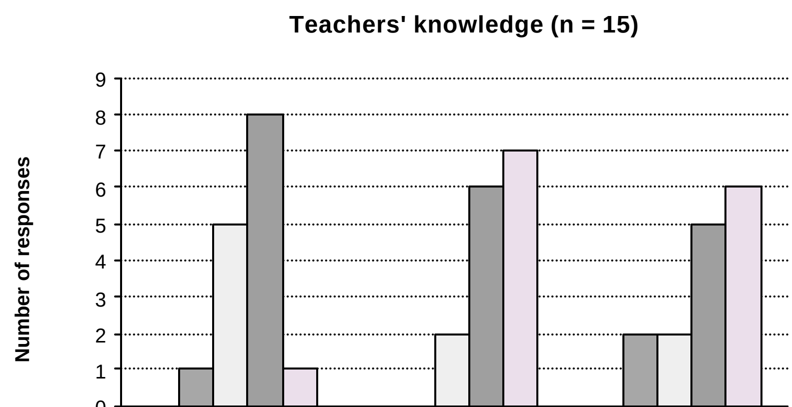
Items that addressed this aspect are:

8. Algebra is generalized arithmetic

10. Letters are used in Mathematical expressions as variables

11. Letters are used in equations to represent specific unknowns

Figure 4.8 shows the distribution of responses to these items.



Item	8	10	11
Strongly disagree	0	0	0
Disagree	1	0	2
Not sure	5	2	2
Agree	8	6	5
Strongly agree	1	7	6

Fig.4.8

Only 60% of the respondents agree that algebra is generalized arithmetic while 33.3% are not sure and 6.7% do not agree to the statement. While 86.7% of the participants believe that letters are used in mathematical expressions as variables, 73.3% also agree that letters are used in equations to represent specific unknowns. While 13.3% of the respondents are not sure about the role of letters in equations, another 13.3% do not agree that in an equation, letters stand for specific unknowns.

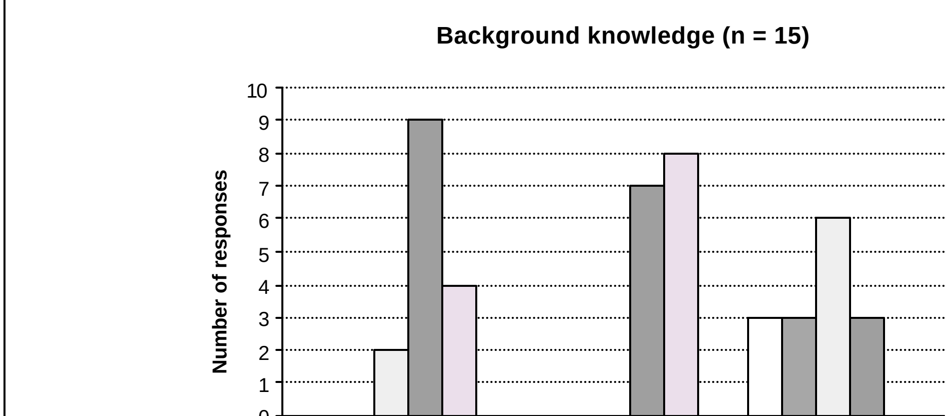
As it has been mentioned earlier knowledge about the role of letters in Mathematics is fundamental to understanding algebra. It is therefore of vital importance that teachers have clear understanding of the role of variables so that they will pass this knowledge over to their pupils. (See section 2,3.3.3.)

4.4.2.4 Background Knowledge to Solving Linear Equations (teachers' perspective).

Items that address this aspect are:

9. Understanding of arithmetic is essential for algebra learning
12. In teaching algebra, it is important that pupils understand the role of variables
13. Even when they do not understand directed numbers, pupils can still perform algebraic manipulation correctly

Fig 4.9 below shows the distribution of responses to these items



Item	9	12	13
Strongly disagree	0	0	3
Disagree	0	0	3
Not sure	2	0	6
Agree	9	7	3
Strongly agree	4	8	0

Fig.4.9

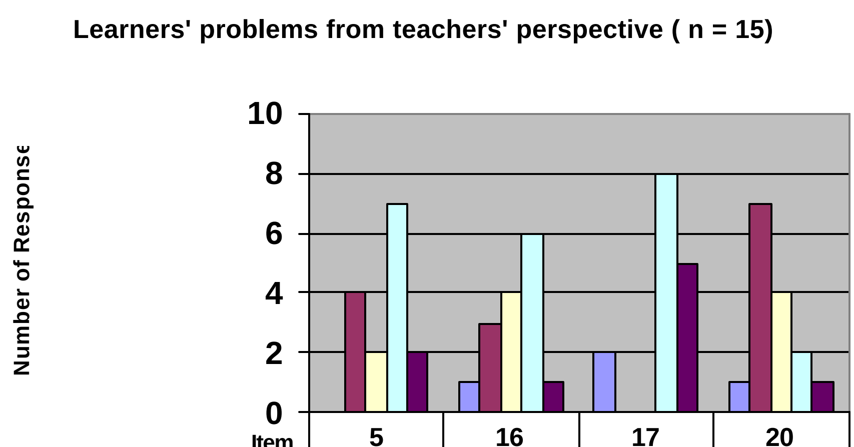
Responses to item 9 indicate that 86.7% of the participants appreciate the importance of arithmetic to algebra learning; only 13.3% of the respondents are not sure whether understanding of arithmetic is essential for algebra learning. All participants (100%) believe that understanding the role of variables is important for pupils in the learning of algebra. There is difference of opinion when considering the effect of knowledge of directed numbers on pupils' ability to perform algebraic manipulations. While 40% of the respondents do not believe that pupils can do algebraic manipulations successfully without proper understanding of directed numbers, another 40% is not sure about this and 20% of the respondents believe that it is possible for pupils to do algebraic manipulations successfully even if they lack knowledge of operations on directed numbers.

As mentioned earlier, it important that pupils develop proper understanding of the properties of operations in arithmetic as this understanding is essential for algebra learning (see section 2.3.3.2). In section 2.3.3.3, the importance of understanding the role of letters in algebra was also established. Knowledge of directed numbers is necessary for correct manipulation of algebraic expressions and equations (see section 2.3.3.5). When teachers are aware of the pre-requisite knowledge required for understanding of linear equations, it is more likely that they would prepare their pupils sufficiently for learning linear equations. When this is not the case, it is also unlikely that pupils would be helped to develop the necessary background knowledge and skills. (See section 2.3.3.4 and section 2.3.3.5.)

4.4.2.5 Pupils Problems in Solving Linear Equations (Teachers' Perspective)

The items that addressed this aspect are:

5. Pupils experience difficulties in solving linear equations with a variable on both sides of the equal sign
16. When solving linear equations, pupils fail to interpret the equal sign correctly
17. Pupils experience problems handling equations that involve fractions
20. Pupils normally like solving algebraic word problems



Item	5	16	17	20
strongly disagree	0	1	2	1
Disagree	4	3	0	7
Not Sure	2	4	0	4
Agree	7	6	8	2
Strongly Agree	2	1	5	1

Fig.4.10 shows the distribution of

participants’ responses to these items.

Fig 4.10

The responses indicate that 60% of the participants agree that pupils do encounter problems when solving linear equations with a variable on both sides of the equal sign; 13.3% is not sure and 26.7% of the respondents do not agree that having a variable on both sides of the equal sign creates any difficulties for the pupils. In solving equations of this nature, pupils usually apply algorithms such as ‘change side-change sign’. The fact that some teachers indicate that pupils have no problems solving equations of this nature could be due to their observation that pupils are able to apply the algorithms correctly; although pupils themselves have indicated that sometimes these rules confuse them. The usefulness of algorithmic procedures in doing routine tasks easily and thus freeing the mind to concentrate on more challenging tasks cannot be disputed. It is important though that pupils are taught the logical reasoning underlying algorithms as well.(See section 2.3.2.2.)

Fractions are also viewed as another source of difficulties for learners in understanding linear equations by 86% of the respondents. Only 13.4% of the respondents do not agree to this. It has been established that fractions have always presented a considerable challenge to pupils in the learning of mathematics from the early grades through to higher levels of secondary education.. It follows that when these have not been fully grasped by pupils, they (pupils) will have difficulties in learning and understanding linear equations with fractions. (See section 2.3.3.2.)

From what was gathered from pupils' questionnaires, respondents indicated that they actually hate solving algebraic word problems due to the difficulties that they experience during the exercise. Responses from the teachers' questionnaire also confirm this. While 53.4% of the respondents indicate that pupils do not like solving algebraic word problems, 26% remain undecided and only 20% of the participants are of the opinion that pupils enjoy solving algebraic word problems. As seen in the literature study, earlier research studies indicate that learners have always found it very difficult to generate equations that represent relationships expressed in algebraic word problems (see section 2.3.3.7). With regard to use and interpretation of the equal sign, respondents to the teachers' questionnaire also confirmed that sometimes pupils show wrong interpretation of the equal sign. Although only 46.7% of the respondents have observed this, pupils' responses to item 20 of the Pupils' Tasks do reveal that to some the '=' sign is not a symbol for equivalence, but they see it as a command to carry out an operation. (See section 2.3.3.6.)

4.4.2.6 TEACHERS' RESPONSES TO THE OPEN-ENDED QUESTION.

In this part of the questionnaire, participants were required to indicate whether or not their pupils usually solve algebraic word problems that give rise to linear equations without any difficulties. Following from their first response they were also asked to discuss the sources of the difficulties and/or suggest ways that pupils could be helped to improve their problem solving abilities.

The majority (86.7%) of the respondents indicated that pupils have great difficulties in handling algebraic word problems. Only 13.3% of the view said that their pupils were able to solve such problems. From the respondents' perspective, pupils' main problem concerns formulation of the equations from the word problem. According to them, pupils sometimes show lack of comprehension of the problem statement. They fail to identify the relationships between the variables in the problem. Respondents, however, feel that:

1) If pupils are given word problems to solve in every lesson, this might help improve their problem solving skills.

2) Problem solving skills should be taught to the pupils, as this would help them to interpret and analyze the problem before engaging in the solution process. Pupils should be encouraged to check if their solutions actually answer the questions.

3) If word problems are introduced to the pupils from the early stages of their mathematics learning then they (pupils) will get used to solving such problems.

4) The problems that are used in the class should be related to pupil's world, everyday life experience. When this is the case then, pupils would understand the stories told in the problem and find solving such problems interesting and worthwhile.

5) If pupils are properly taught how to deal with negative numbers and how to perform operations on brackets, then they would have less problems in understanding linear equations.

6) Pupils should be given opportunity to work individually and sometimes in groups. Working in groups would help them gain more knowledge as they interact and exchange ideas with each other. This would also give them chance to use logical reasoning in justifying their solution procedures to their peers.

4.4.2.7 Synthesis

Nothing new came from the teachers' questionnaire; the responses only emphasized what was gathered earlier from pupils' tasks as well as pupils' questionnaires. Findings from the teachers' questionnaires may be outlined as follows:

- Despite the many problems that pupils have in understanding mathematical concepts, generally participants indicated a positive attitude towards teaching the subject. Some respondents have indeed pointed out that teaching algebra is not a very enjoyable experience at all. This may be due to the fact that pupils find learning algebra difficult and this therefore places heavy demands on the part of the teachers.

- While most respondents were of the view that teachers should teach pupils procedures of working out linear equations, some still believe that pupils should be allowed to use their own methods in solving mathematical problems and that discussions would help pupils realize shortcomings or strengths of their solution procedures.

- Without sufficient background knowledge of skills and concepts, understanding of linear equations will always be a problem to pupils. This basic knowledge includes:

-- knowledge of basic operations on directed numbers and fractions. This includes mastery of the rules that govern the order in which operations are carried out on mathematical expressions and equations.

-- the role of brackets, i.e the use and meaning of brackets in mathematics

-- understanding of the role of letters in mathematics

-- understanding of the meaning of expressions, the dual nature of expressions

-- correct interpretation of the equal sign.

- Lack of understanding of the symbolic mathematical notation and the technical language as used in mathematics could also be another source of pupils' difficulty in understanding linear equations. This comprises interpretation of word and/or phrases like 'twice as much as', '*a* and *b*', 'product of', '*ab*', etc.

- Effective teaching and learning of linear equations can be realized through practice of learnt skills in a problem based learning environment where pupils can conjecture, critique, analyze generalize situations and use logical reasoning to justify and judge reasonableness of their results. Word problems from real life context that are within pupils' environment and involve their daily experiences can play a significant role here.

- Teachers' content knowledge with regard to concepts and sub-concepts in linear equations needs to be strengthened. The type of understandings that teachers have on the concepts they teach affect the quality of understanding that pupils would develop.

- Participation in professional workshops is very important, as it is in such gatherings that mathematics teachers and experts in the field exchange views and share knowledge and skills as well as experiences on the teaching of the subject.

4.5 RELATIONSHIP BETWEEN GROUP RESULTS

Both the Teachers' and the Pupils' Questionnaires contained statements that sought information on similar or related aspects on the problems that pupils encounter in understanding linear equations. Table 4.13 below shows the related or similar statements from both questionnaires.

4.5.1 Similar Statements from the Teachers' and Pupils' Questionnaires

Group	Statements					
Teachers	1	3	4	10	9	11	20
Pupils	1	5	7	6	9	4	13

Table 4.13

Responses from the teachers as well as from pupils to such statements were then compared. Table 4.14 below shows the distribution of pupils' responses to the statements.

4.5.2 Pupils' Responses to the Statements

Statement	Score	Mean Score

1	1	3	8	30	38	4.3
5	7	3	9	20	34	3.6
7	0	6	2	20	51	4.4
6	1	2	33	21	16	3.4
9	5	2	7	20	46	4.3
4	4	3	13	18	42	4.1
13	10	12	18	29	11	3.2

Table 4.14

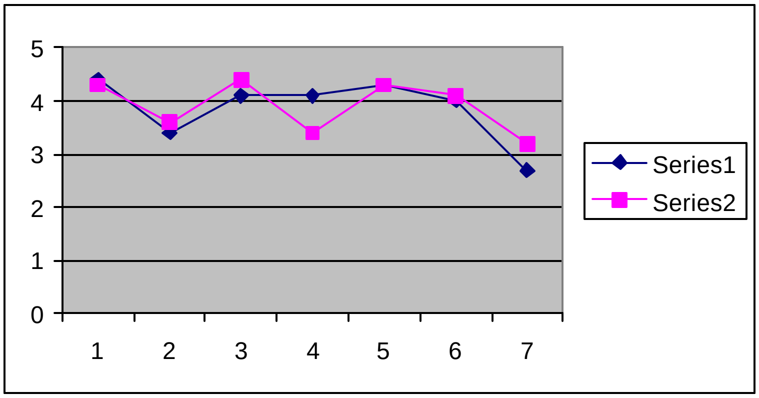
4.5.3 Teachers' Responses to the Statements

Statement	Score					Mean Score
1	0	0	0	9	6	4.4
3	1	5	0	5	4	3.4
4	0	5	1	7	2	4.1
9	0	0	2	9	4	4.1
10	0	0	2	6	7	4.3
11	0	2	2	5	6	4
20	1	7	4	2	1	2.7

Table 4.15…

4.5.4 Comparison of the Responses from Teachers and Pupils

The comparison of the responses from the participating teachers and pupils was done using the mean scores of the responses**.** Fig 4.11 shows the graph of these mean scores.



key:

series 1: teachers'mean scores

series 2: pupils' mean scores

Fig 4.11

From the figure, it can be realized that the two graphs closely overlap on each other, indicating similar views from both the teachers and the pupils on related or similar aspects. A scatter diagram of teacher responses against pupils' responses was also obtained. Fig .12 shows the results.

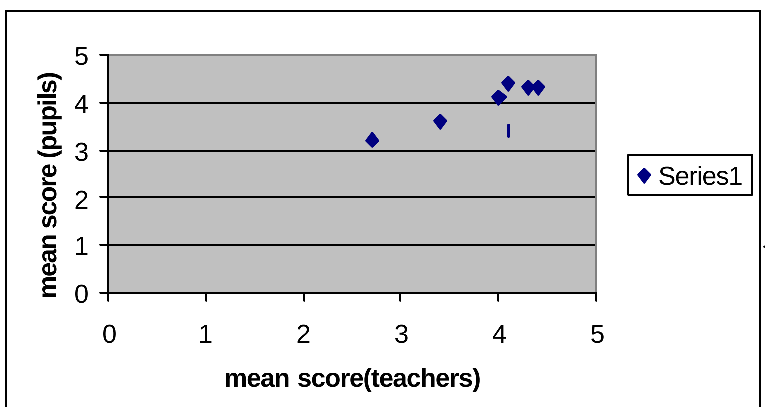


Fig. 4.12

From the scatter plot, it can be generally realized that there is a positive correlation between pupils' and teachers' responses. A correlation coefficient for this set of data may be calculated using the Spearman Rank method.

Table 4.16 below shows the ranks assigned to the mean scores in each set of data.

pupils		teachers		Difference between the ranks	d^2
Mean score	rank	Mean score	rank		
4.3	2	4.4	1	1	1
3.6	5	3.4	6	1	1
4.4	1	4.1	3	2	4
3.4	6	4.1	3	3	9
4.3	2	4.3	2	0	0
4.1	4	4	5	1	1
3.2	7	2.7	7	0	0

Table 4.16

According to the Spearman Rank method, the correlation coefficient, r or ñ, is given by:

$$r = 1 - \frac{6\sum d^2}{n^3 - n}$$

where d = difference between corresponding ranks, and n = number of values compared

$$r = 1 - \frac{6(16)}{7^3 - 7}$$

$$= 1 - \frac{96}{366}$$

$$= 0.738$$

This value is fairly close to +1, so it can be concluded that there is a strong relationship between teachers' and pupils' opinions on statements that had similar meanings.

4.6 CONCLUSION

In this chapter an attempt has been made to present, analyze and interpret the findings of the investigation. The responses to pupils' tasks, pupils' questionnaire and the teachers' questionnaire have brought out similar information with regard to Grade 9 pupils' problems in understanding linear equations. From the investigation it can observed that:

1. Pupils encounter great difficulties in understanding linear equations due to lack of pre-requisite background knowledge of:

    - Basic operations on directed numbers and fractions
    - Order of operations and use of brackets.
    - Meaning and role of letters in algebra.
    - Meaning of equation and use of equal sign.
    - Proper interpretation of algebraic expressions.

2. The manner in which mathematics is taught also has great influence on the pupils' level of understanding of the taught concepts. Views that teachers have about the nature of mathematics and what they perceive as good teaching, to a large extend dictates their ways of teaching. Some teachers still believe that mathematics is about rules, formulas and procedures and as a result their teaching is mainly focused on drilling pupils on the practice of the skills and procedures that they demonstrate to them. In order to understand linear equations, pupils still need to have procedural, conceptual and strategic knowledge. However, pupils appear to have mechanistic knowledge of solving linear equations such that the algorithms that they attempt to use are wrongly applied and they (pupils) do not even seem to be able to recognize their mistakes. This needs to be addressed, as it denies pupils opportunity to learn mathematics with understanding.

3. There is a general observation amongst teachers that solving word problems is a major difficulty that pupils encounter in their learning of linear equations. Despite all this they have a strong feeling that this experience must be brought to pupils much earlier and be a must for every mathematics lesson. This is actually in line with what the NCTM standards (1989:5) advocate for: that mathematics teaching should be aimed at developing problems solving skills and abilities in pupils, using problems that are related to pupils' life's.

4. Pupils have indicated that they hate solving word problems because they sometimes do not understand what the problem is all about. This is due to lack of comprehension of the problem statement, in ability to identify target variables and establish relationships between the variables in the problem. Teachers and pupils believe that sustained exposure to solving problems that require the said skills would definitely help them improve their abilities in this regard.

It appears also that most mathematics departments within the school offer very little support to the teaching and learning of mathematics. Teachers do not discuss their classroom experiences among themselves; they do not share any difficulties or achievements. This must be corrected.

CHAPTER 5

FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In this chapter conclusions are drawn and recommendations are made concerning the problems that pupils and teachers of Grade 9 algebra encounter on understanding linear equations. These are based on findings from the literature study in chapter 2, the investigation carried out in chapter 3 and the findings of the research as presented and interpreted in chapter 4.

5.2 FINDINGS

The investigation on the problems that learners and teachers of Grade 9 algebra encounter on understanding linear equations was carried out as described in Chapter 3. Through the literature study, the researcher was able to get deeper understanding of the topic under investigation. Works of earlier researchers such as Kieran, Linchevski, MacGregor, Kuchemann etc. relating to the problems that learners and teachers of Grade 9 encounter in understanding linear equations were visited. Readings from publications by widely-known mathematics educators associations such as the NCTM, professional journals and other scholarly books provided the researcher with very useful information. Tasks were designed to help investigate the problems identified in Chapter 2. These were given to the sample group to work on and the responses noted. Two types of questionnaires were also designed to seek information from Grade 9 pupils and their teachers concerning the topic under investigation.

It is from the literature study and the empirical investigation that the following were identified as some of the obstacles/problems that pupils and teachers encounter on learning/teaching linear equations:

a) Lack of relevant background knowledge;

b) Lack of conceptual, procedural and strategic knowledge required for solving linear equations.

c) Inappropriate approaches and methods used in the teaching of algebra and more specifically, linear equations.

a) Background knowledge required for understanding linear equations comprises:

1. The concept of a variable. The different roles of variables in algebra as seen or interpreted by pupils hinder their (pupils) ability to express and / or generalize situations;
2. Correct interpretation of terms and expressions. Pupils have difficulty in interpreting the relations that exist amongst the terms in algebraic expressions eg. '*ab*' is often read as 'a and b' which later gives rise to a misconception that it is the same as '$a + b$';
3. Basic laws of algebra and order of operations. Operations that involve directed numbers, fractions and use of brackets also pose a greater challenge to pupils;
4. Understanding the structure of an equation i.e the structure of the relation(s) in an equation.
5. The concept of equality. The interpretation of the equal sign as a command to carry out an operation as opposed to a symbol relating equal quantities on the two sides of an equation also creates problems for pupils;

b) Lack of knowledge of procedures that can be used in solving linear equations. This includes:

1. the logical reasoning behind the procedures. For example, solving linear equations that have the unknown on both sides of the equal sign is also problematic;
2. strategies for solving linear equations such as making graphical, tabular or pictorial representations. Solving algebraic word problems was identified as a serious problem to pupils. The major problem concerns translations from words to symbols.

c) Approaches used in the teaching of algebra and in particular, linear equations.

1 Some teachers still hold a formalistic view of mathematics; as such believe that teaching mathematics is drilling pupils on procedures (and rules) on working out solutions to linear equations. For example, algorithms that are taught to pupils are usually given without any meaning attached, there is usually no presentation of logical reasoning behind these, justifying why they work;

2. Some teachers think that discussions consume a lot of time and allowing discussions may even cause confusion to slow learners. This is not in line with what most pupils prefer. Pupils prefer learning mathematics through discussions, they get encouraged to learn

more mathematics when their ideas are responded to in a positive manner and are allowed to use their own methods in solving problems

3. Some teachers are not aware of the importance of linking algebra learning to pupils previous experiences with arithmetic;

If the above-mentioned aspects would be given the necessary attention, that would promote pupils' success in understanding linear equation. Pupils are said to be proficient about linear equations when they are able to:

- represent a realistic mathematical problem using the symbolic mathematical notation in the form of an equation;
- perform the necessary operations on the formulated equation to get the value of the unknown; and
- interpret the result and provide a meaningful solution to the original problem.

5.3 CONCLUSION

The main aim of this study was to investigate and make a critical analysis of the problems that Grade 9 pupils and teachers encounter on understanding linear equations. The study revealed that the problems encountered relate to pupils' conceptual and procedural knowledge and to the teaching approaches used in the teaching of the topic. Teachers' content knowledge on the topic also play a significant role on promoting high levels of understanding of the subject matter on the part of the pupils.

5.3.1 Pupils' conceptual and procedural knowledge.

This refers to the background knowledge and skills that are required to help pupils gain better understanding of linear equations; and the content knowledge and skills that are acquired in the process of learning linear equations. These comprise:

- basic operations on directed numbers and fractions, including the rules governing the order of operations and the use of brackets;
- the different uses of letters, i.e. concept of variable;
- the meaning of equation and use of equal sign;

- proper interpretation of algebraic expressions; and
- strategic knowledge or problem-solving skills and strategies.

5.3.2 Teaching methods used by the teachers.

From the study, it was realized that pupils experience more difficulties when solving algebraic word problems involving linear equation than when solving ready formulated linear equations. The main difficulty is translating from words to symbols. The reasons behind this are that pupils are usually drilled on using procedures and algorithms for solving ready formulated equations and only get very little exposure to solving the 'hated' word problems. This means that the approach used in the teaching of linear equations is 'drill and practice' and not the problem-based discovery learning that is so much advocated for in the current curriculum reforms in mathematics education. Even though some discussions are allowed in the mathematics classrooms it is only to a limited extent.

One of the reasons why teachers may resort to teaching by 'drill and practice' could be their limited knowledge of the subject matter.

5.4 RECOMMENDATIONS

The following recommendations are made, based on the findings of the study; these will be in three categories:

5.4.1 Conceptual and Procedural knowledge.

1. Transition from arithmetic to algebra needs to be made as smooth as possible so that pupils can build up the arithmetic-to-algebra connection and vice versa. pupils should be helped to develop good number sense and operation sense as these would help them to apply operations meaningfully and flexibly. They would also be able to recognize the relative magnitudes of numbers and use multiple representations of numbers. Pupils have to be used to working back and forth between algebra and arithmetic so as to be able to identify the advantages of each as dictated by the nature of tasks with which they have to deal with.

2. The equal sign. Two important principles of equality i.e. symmetry and transitivity need to be stressed very explicitly in the early stages of the teaching and learning of algebra. It is important that pupils recognize that the equal sign does not imply that a numerical answer is expected but is a symbol relating equal quantities on two sides of an equation.
3. The concept of variable. More attention has to be paid on the different uses of letters in mathematics. Pupils need to realize that sometimes a letter may have a fixed value or may be a variable.
4. Pupils have to be helped to develop a better understanding of the dual nature of algebraic expressions as entities or objects in their own right.
5. Structure of an equation. In solving linear equations, pupils need to understand why certain transformations have to be done. This will be the case if they already understand the surface and the systemic structure of an equation. When these are well-recognized, then even if they do not remember the algorithm(s) to apply in solving an equation they can still work their way through to the solution.
6. Pupils encounter great difficulties when they have to translate word problems into the symbolic mathematical form. When they have not generated the problems themselves, it is advisable that they be allowed to use their own words to translate the problem into a form that is more accessible to them.
7. Teachers have a limited content knowledge on concepts and sub-concepts underlying solution of linear equations. Participation in in-service workshops may be of great help; as it is in such gatherings that teachers may get more insight knowledge on the subject they teach.

5.4.2 Teaching Methods

Problem-based learning environments promote better/effective acquisition of mathematical knowledge and skills. Pupils need to be exposed to formulating equations from situations with which they are familiar. Discussions play a vital role here.

Success of discussion in a mathematics classroom is dependent on the teacher’s ability to create an environment that is conducive to discussions. In a classroom where contributions from individual pupils are considered valuable (not necessarily correct) and are dealt with in more positive ways, discussions are more likely to sustain and produce positive results. Every individual needs to be respected. Through classroom discussions pupils are encouraged to inquire and seek clarification and even feel more comfortable and confident in justifying their responses or solutions to mathematical problems. It is noted that “cultivating a tone of interest when asking a student to explain or elaborate on an idea helps to establish norms of civility and respect rather than criticism and doubt” (NCTM1991:35).

According to the NCTM Standards (1991:35), the role of a teacher in classroom discourse should be that of a facilitator. The mathematics teacher should provoke pupils’ thinking through tasks that are interesting, yet challenging. It is the responsibility of the teacher to choose and/or create tasks that are more likely to promote better understanding of mathematical concepts and procedures while at the same time building up pupils’ ability and confidence in problem-solving and mathematical reasoning.

As Linchevski (1995:117) has indicated, problems that are designed to promote effective learning of solution of linear equations should be focused on:

a) Providing pupils opportunity to experience substitution of numbers for letters (numerical verification);
b) Dealing with equivalent equations through substitution;
c) Building cognitive schemes through reflective activity that allows students to use their own procedures and
d) Give pupilsopportunity to practice forming equations from context problems.

Problem-solving skills and strategies should be taught to pupils. Appropriate problems that require use of different strategies should be provided to offer pupils opportunity to practice using the strategies. Expertise in Mathematics is characterized by the ability to do mathematics, ability to solve mathematical problems, ability to communicate mathematically, and ability to reason mathematically. A person with these traits will have developed the mathematical habits of mind and possesses mathematical power. Mathematical power “denotes an individual’s ability to explore, conjecture, and reason logically, as well as ability to use a variety of mathematical methods effectively to solve non-routine problems”(NCTM 1989:5).

5.4.3 Professional Development of Mathematics Teachers

Professional growth in a mathematics teacher stems from proficiency in the content knowledge, pedagogy and psychology of learning mathematics. This knowledge enables the teacher to interpret the curriculum, design learning programs and appropriate teaching and learning materials. Once a teacher is competent in the mathematics he or she has to teach, her confidence in teaching the subject also increases. She / he would be broad-minded and able to understand different approaches that pupils use in solving mathematical tasks. She/he would be able to lead students into re-inventing the mathematics through multiple representations and modeling strategies.

It is important therefore that teachers get regular refresher courses to keep them up-dated with new developments in the teaching and learning of mathematics. The purpose of this IN-Service Education and Training (INSET), is to help teachers improve the quality of their teaching. This means that teachers are re-trained into new ways of doing the practice of teaching. With the new perspectives in Mathematics and Mathematics Education, they are inducted and re-orientated into these new directions. As a result of their experiences in an INSET program, teachers will change from doing things in their old traditional way and adopt the new approaches that are advocated for.

At school level, teachers of mathematics may organize meetings, with the help of the Head of Department, in which they can discuss their successes and frustrations with regard to mathematics teaching. They could also organize meetings with other mathematics teachers in the neighboring schools to share ideas and experiences about their practice of teaching. Participation in national and international workshops, seminars and conferences offer teachers opportunity to become part of a larger family of professionals in mathematics education. As teachers engage in the given practices, they will see and hear things that have impact on their beliefs"(Franke, Fennema & Carpenter1997:256).

5.4 **RESEARCH CONSTRAINTS**

Depth of the investigation

Even though there is such a close relationship between graphs and linear equations, no attention was given to the aspect of graphs. There was no intention on the part of the researcher to go beyond solution of linear equations as treated in the investigation.

Execution of the study

It was a bit difficult getting pupils to work on the tasks as this took part of their school time. However, the researcher was given greater support particularly from the mathematics teachers for the classes concerned. Although all the teacher questionnaires were returned to the researcher, the open-ended question was not well-responded to. The participants complained that they did not have time to give it the attention it deserves. Interviews with the teachers could have, perhaps, given more information for this question. The number of mathematics teachers that were anticipated to take part in the study was not met. This is due to lack of mathematics teachers in the schools; one teacher in most cases was teaching more than one class in the target group.

5.7 FURTHER RESEARCH

It will be important that an investigation is carried out to investigate on the extent to which mathematics teachers use problem-based teaching in their mathematics classrooms and perhaps through observations see how much pupils are being prepared to cope with the tasks. Even if this is not focused on algebra teaching, it will provide a picture of the type of teaching practices that are prevailing in the schools.

Another area that needs investigation is that of language in mathematics. It was found in the study that pupils have a difficulty in interpreting this symbolic mathematical language, it would be appropriate also to carry out further investigations into this problem and perhaps design a support program to support algebra learning, especially, the second-language learners.

A further investigation may also be carried out on the teachers' content knowledge necessary for successful teaching of algebra in the secondary schools.

5.6 FINAL SUMMARY

The study has revealed the nature of obstacles that pupils and teachers of Grade 9 algebra encounter on understanding linear equations. Due to the hierarchical nature of mathematics, it is important that pupils fully understand lower order concepts before they are taught related concepts of a higher order. Much of the problems that pupils have in understanding linear equations are due to lack of the necessary background knowledge. Teachers lack adequate content knowledge necessary for effective teaching of solution of linear equations. Another source of their problems is the manner in which they are taught this topic.

Children learn mathematics effectively when they are given opportunity to grapple with problems related to their everyday experiences. Such problems should not only be of application type, but should provide pupils with interesting challenges that require them to extend their thinking and thus gaining more knowledge. Since most textbooks used in the schools do not provide any support to this approach, it is important that teachers are offered the necessary professional support in this regard.

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APPENDIX A

PUPILS' TASKS

INSTRUCTIONS

1. Answer all questions.
2. All answers will be treated with the strictest confidence.

SECTION A

Indicate your answer by marking with an **X** the appropriate box that shows the intended response, on the answer sheet provided.

Question 1

What is the perimeter of a square of side **r** cm?

a) (2 + r) cm. b) $\frac{r}{4}$ cm. c) 4r cm. d) $2r^2$ cm.

Question 2

A girl is **q** meters shorter than her mother who is **p** meters tall. What is the girl's height?

a) pq meters. b) (p-q) meters. c) (q-p) meters. d) (p+q) meters.

Question 3

A family buys a sack of rice weighing **e** kg. If **f** kg are used each day, how long will the sack of rice last?

a) $\frac{e}{f}$ days. b) (e-f) days. c) ef days. d) $\frac{f}{e}$ days.

Question 4

A man has **x** Maloti. He gives his son **y** Maloti, and divides the remainder equally between his two daughters. How much money does each daughter get?

a) $\frac{x+y}{2}$ Maloti. b) 2xy Maloti. c) $\frac{y-x}{2}$ Maloti. d) $\frac{x-y}{2}$ Maloti.

Question 5

A lorry travels at an average speed of **s** km/hr. What is the distance covered if it travels for **t** hours at that speed?

a) $\frac{s}{t}$ km. b) $(s - t)$ km. c) $(s + t)$ km. d) st km.

SECTION B

Find the correct algebraic expression for each statement given and mark **X** to the correct option in the given answer sheet.

6. Multiply the sum of ***a*** and ***b*** by **3**.

   a) $3ab$. b) $6ab$. c) $3(a + b)$. d) $3(a-b)$.

7. Add ***p*** and ***q*** and subtract ***s*** from the result.

   a) $(p+q) - s$. b) $(p-q)-s$. c) $s-(p+q)$. d) $s-(p-q)$.

8. Subtract **2** from the quotient of ***c*** and ***d***.

   a) $2 - cd$. b) $cd - 2$. c) $\frac{c}{d} - 2$. d) d/c – 2.

9. Take away **7** from the product of ***m*** and ***n*****+1.**

   a) $7 - mn + 1$. b) $7 - m(n+1)$. c) $mn + 1 - 7$. d) $m(n+1) - 7$.

10. Multiply the sum of ***e*** and ***f*** by **½.**

    a) e x f x ½. b) e + f x ½. c) (e + f) x ½. d) (e – f) x ½.

SECTION C

Find the correct solution for each problem and mark with an **X** the box that shows the desired option on the answer sheet provided.

11. Simplify the following expression: $\frac{x + 2}{2x + 4}$

    a) ¼ b) $\frac{2}{5}$ c) $\frac{0}{4}$ d) ½

12. Collect the like terms in; $5(2-t) - 3(2t + 1)$.

a) $7 - 11t$. b) $13 - 11t$. c) $7 - t$. d) $11 - 7t$.

13. Express as a single fraction; $\frac{3m}{4} + \frac{m}{3}$

a) $\frac{4m}{7}$ b) $\frac{4m}{12}$ c) $\frac{13m}{12}$ d) $\frac{3m^2}{12}$

14. Simplify the expression; $\frac{8a}{3b} \div \frac{6a}{5b}$

a) $\frac{16}{5}$ b) $\frac{20}{9}$ c) $\frac{48}{15}$ d) $\frac{4}{5}$

15. Express as a single fraction; $2\frac{1}{2}k \div 1\frac{1}{2}k$

a) 2. b) $\frac{5}{3}$. c) $\frac{10\,k^2}{6}$. d) $\frac{4\,k^2}{15}$

SECTION D

Solve the following problems, showing all the working:

16. Mpho is 6cm taller than Paballo. If the total height of the two girls is 294 cm, what is the height of each girl?

17. A man is 3 times as old as his son. In 15 years time, he will be twice as old as his son at that time. How old is the son?

Solve for **x** in the following:

18. $5(2 - x) - 3(2x + 1) = 40$

19. $\frac{x + 5}{3} = \frac{x + 2}{6}$

20. $11 - 3x = 18 - 4x$.

21. $7 + 9 = x + 10$

THANK YOU

APPENDIX B

PUPILS' QUESTIONAIRES

INSTRUCTIONS.

1. Answer all questions.
2. All answers will be treated with strictest confidence.
3. When presented with a choice circle the number next to the alternative chosen.

QUESTIONS

SECTION A

PERSONAL AND GENERAL INFORMATION

1. Gender

Male	1
Female	2

2. Age in years

12	1
13	2
14	3
15	4
Over 15	5

3. Grade obtained in Mathematics in the PSLE

Grade	No. of pupils
1	1
2	3
3	4
F	4

SECTION B

INSTRUCTIONS

1. Please answer all questions as honestly and carefully as you can.
2. Circle 1,2,3,4,or 5 to indicate how strongly you disagree or agree with each statement.

Ratings are;

1 – strongly disagree;2 – disagree; 3 – not sure; 4 – agree; 5 – strongly agree.

STATEMENTS	RATING				
1. I enjoy learning mathematics	1	2	3	4	5
2. All life activities involve application of some mathematics.	1	2	3	4	5
3. Solving linear equations is an interesting topic to learn.	1	2	3	4	5
4. Letters can be used to represent specific numbers.	1	2	3	4	5
5 Mathematics is remembering the formulas and rules, and applying them to appropriate situations.	1	2	3	4	5
6 A letter can be used to represent any number.	1	2	3	4	5
7 I enjoy solving mathematics problems if the teacher first shows me how to work out the solutions.	1	2	3	4	5
8 I have no problem solving linear equations that involve negative numbers	1	2	3	4	5
9 Knowledge of arithmetic rules can help one to do algebraic manipulations with understanding	1	2	3	4	5
10 I enjoy forming equations from word problems	1	2	3	4	5

11	3x + 4 is an example of an algebraic equation	1	2	3	4	5
12	There are three (3) terms in 3x + 4	1	2	3	4	5
13	I enjoy solving algebraic word problems	1	2	3	4	5
14	Solving real life problems makes learning mathematics interesting	1	2	3	4	5
15	I learn mathematics better when I discuss problems with my classmates	1	2	3	4	5
16	My mathematics teacher allows me to use my own methods in solving problems	1	2	3	4	5
17	I have no problem solving linear equations that have the unknown on both sides of the equal sign	1	2	3	4	5

When solving algebraic word problems it is important that the problem is translated into mathematical symbols. This is then followed by working on the equation formed to obtain equivalent other equations, solving for the unknown. Are you able to do these translations successfully, and finally get the correct solution? (YES / NO)

If your answer is yes, explain how you manage

If your answer is no, explain where you experience the difficulties.

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I THANK YOU.

APPENDIX C

TEACHERS' QUESTIONAIRE

INSTRUCTIONS.

1. Answer all questions.
2. All answers will be treated with strictest confidence.
3. When presented with a choice circle the number next to the alternative chosen.

QUESTIONS

SECTION A

PERSONAL AND GENERAL INFORMATION

1. Gender

Male	1
Female	2

2. Age in years

17 –20	1
21 –24	2
25 –28	3
29 –32	4
Over 32	5

3. Marital Status

Single	1
Married	2
Divorced	3
Widowed	4

4. Qualifications

COSC	1
STC	2
DIP Sc Ed	3
Bsc Ed	4
Honors/Masters	5

5. Teaching Experience in years

0 –3	1
4 - 7	2
8 – 11	3
12 – 15	4
Above 15	5

SECTION B

INSTRUCTIONS

1. Please answer all questions as honestly and carefully as you can.

2. Circle 1,2,3,4,or 5 to indicate how strongly you disagree or agree with each statement. **Ratings are: 1 – strongly disagree; 2 – disagree; 3 – not sure; 4 – agree; 5 – strongly agree.**

STATEMENTS	RATING				
1. I enjoy teaching mathematics	1	2	3	4	5
2. Algebra is the most interesting part of mathematics	1	2	3	4	5
3. Learning mathematics is remembering the formulas and rules, and applying them to problems.	1	2	3	4	5
4. The best way to teach mathematic is to show learners how to work out solutions and then let them practice the skills.	1	2	3	4	5
5. Pupils experience difficulties in solving linear equations with a variable on both sides of the equal sign	1	2	3	4	5
6. Allowing learners to use their own procedures in solving problems is appropriate in the learning of mathematics.	1	2	3	4	5
7. Classroom discussions usually lead to confusion for slow learners, therefore should be avoided.	1	2	3	4	5
8. Algebra is generalised arithmetic.	1	2	3	4	5
9. Understanding of arithmetic is essential for algebra learning.	1	2	3	4	5
10. Letters are used in mathematical expressions as variables.	1	2	3	4	5
11. Letters are used in equations to represent specific unknowns.	1	2	3	4	5
12. In teaching algebra, it is important that pupils understand the role of variables.	1	2	3	4	5
13. Even when they do not have a clear understanding of directed numbers, pupils can still perform algebraic manipulations correctly.	1	2	3	4	5
14. When solving linear equations, it is important that pupils learn the correct procedures.	1	2	3	4	5

15. Real life problems are useful in introducing new topics.	1	2	3	4	5
16. When solving linear equations, pupils fail to interpret the equal sign correctly.	1	2	3	4	5
17. Pupils experience problems when solving equations that involve fractions.	1	2	3	4	5
18. Word problems should only be given to those gifted pupils to solve.	1	2	3	4	5
19. I usually spend some time in my lessons teaching students problem solving strategies.	1	2	3	4	5
20. Students normally like solving algebraic word problems.	1	2	3	4	5
21. Mathematics workshops organized by the Ministry of Education equip teachers with skills for generating interesting real life problems.	1	2	3	4	5
22. In our departmental meetings we often discuss problems that we encounter when teaching mathematics.	1	2	3	4	5

When solving algebraic word problems, pupils should be able to identify the target variable, establish relationships that are expressed in the problem statement and then formulate the correct equation using mathematical symbols. The solution processes would then follow these.

From your experience, are pupils able to do these successfully? (YES / NO)

If no, how do you suggest these skills can be developed in the pupils?

If yes, what can other teachers do to help their pupils acquire the above-mentioned skills?