AN INVESTIGATION GRADE 11 LEARNERS ERRORS WHEN SOLVING ALGEBRAIC WORD PROBLEMS IN GAUTENG, SOUTH AFRICA.

by

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A thesis submitted in fulfilment of the requirements for the degree of

Masters in Mathematic Science Education

From the

Institute of Science and Technology Education

at the
University of South Africa

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September 2017

ABSTRACT

South African learners struggle to achieve in both international and national Mathematics assessments. This has inevitably become a serious concern to many South Africans and people in the education arena. An algebraic word problem holds high preference among the topics and determines success in Mathematics, yet it remains a challenge to learners. Previous studies show there is a connection between learners' low performance in Mathematics and errors they commit. In addition, others relate this low performance to English language inproficiency. This has encouraged the researcher to investigate the errors Grade 11 learners make when they solve algebraic word problems. The researcher used a sequential explanatory mixed approach to investigate Grade 11 learners from Gauteng, South Africa when they solve algebraic word problems. Accordingly, a convenient sampling helped to select three schools, and purposive sampling to choose the learners. In this study, the researcher employed a quantitative analysis by conducting a test named MSWPT with 150 learners. In addition, the researcher used qualitative analyses by conducting the Newman (1977) interview format with 8 learners to find out areas where errors are made and what kind of errors they are. Findings discovered that 90 learners demonstrated unfitness due to poor linguistic proficiency, while the remaining 60 learners fall into three main categories, namely those who benefitted from researcher unpacking of meaning; those who lack transition skills from arithmetic to algebra; and those who lack comprehension and calculation knowledge. Conclusively, the researcher found linguistic, comprehension, semantic and calculation errors. The reasons learners make these errors are due to (i) a lack of sufficient proficiency in English and algebraic terminology (ii) the gap between arithmetic and algebra.

Keywords: Algebra, Algebraic word, Problem, Error, MSWPT Mathematics Strategic Word Problem

DECLARATION

I declare that, to the best of my knowledge, this thesis is my work and that I have not been assisted by anyone. I submit this work for Masters in Science and Mathematics Education at the University of South Africa and declare that this work had not been submitted to any other university or institution. In addition, I declare that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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ACKNOWLEDGEMENTS

I sincerely appreciate the contribution of the following people towards the completion of my thesis:

- My supervisor, Prof. N.N. Feza for her constructive criticism, knowledge, continuous guidance and support which helped pave the way for the completion of this work of research;

- Ms. Eva Makwakwa, the research assistant for her encouragement and support towards my thesis;

- Mr. Aaron Tshikotshi, the ISTE Mathematics researchers' librarian for always making available articles when requested;

- All the educators, the HODs of Mathematics and the principals of schools investigated for their cooperation and support, not forgetting the love and support from my colleagues at Tshwane North College TVET;

- Prof. C.E. Ochonogor, the ISTE's Master & Doctorate Supervisor for his support and advice, which gave me the inspiration needed.

- My late husband, Marvin Salihu, for all financial support he had made available, the self-determination and confidence he had built in me.

- My most loving children, Samuel, Joseph and Emmanuel for their continuous prayers and encouragement received during the difficult times;

- My late parents Benjamin and Christiana Okundaye for their moral support and numerous ways in which they nurtured my ability in life, especially to the completion of this thesis;

- My wonderful siblings, Esther, Comfort, Patience, Ephraim, Carol and late sister Deborah for their advice, guidance and support; and

- Most of all, my Lord and Saviour, El Shaddai, who made all people around me to be a blessing, which led to the success of this dissertation.

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CHAPTER 1: The background of the study

1.1 INTRODUCTION

This chapter presents the background of the study, the importance of algebra, the problem statement, the purpose and significance of the study, limitations, ethical considerations for the study, the definition of terms and outline of chapters for the study.

1.1.1 Contextual background of the study

Contrary to lay belief, Mathematics is not all about computation. Kaput (2008) makes us understand that for those specialized in this scientific discipline, Mathematics computation is all about understanding structures, associations, designs of mathematical concepts and constructing answers to story problems. Ilany and Margolin (2010) opine Mathematics is a difficult and abstract subject in terms of its structures. According to them, these structures include operating symbols and equations such as the linear and quadratic as well as algebraic expressions. Mathematics structures also use a particular language, grammar, and three-dimensional positioning of numbers that portray the given situation in a problem (Ilany & Margolin, 2010). The rapid advancement in today's world has equally added to Mathematics becoming crucial and gaining a greater following. Nowadays, the new imperative is for people to acquire, investigate and apply mathematical knowledge successfully to grow into effective citizens. In particular, learners need to be prepared properly to acquire essential mathematical knowledge. This is necessary because Mathematics is an important subject, and a prerequisite for studying subjects like Computer Science, Commerce, Engineering and Music, to mention but a few examples.

Mathematics is a useful tool to access the Science, Technology, Engineering and Mathematics (STEM) discipline. Tella (2008) discusses the substantial role Mathematics plays in shaping, inter alia, the reasoning, calculating or guiding principles in subjects like Commerce,

Economics, Education and Humanities. Hsieh, Lin, & Wang (2012) identify two major roles of mathematics competence (MC): mathematical knowledge and thinking processes. Mathematical knowledge signifies those areas specified in the Mathematics curriculum topics, for example, the understanding of numbers (Niss, 2003). The utilization of Mathematics to solve problems in real life situations is one example of thinking processes in mathematical competence (Common Core State Standard for Mathematics and National Governors Association Centre, 2010). It also provides a wide range of job opportunities (Norris, 2012). Learners' success or failure in Mathematics at school level determines their access into higher education and selecting a career (OECD, 2010). According to Njagi (2015), Mathematics is a worldwide tool useful to develop technological careers and improve coherent ability. The importance of learning Mathematics is not only focusing on content alone, but also on the context such as empowering learners to solve everyday life problems.

Given this background, many countries are striving to provide quality education that emphasizes the importance of Mathematics. However, there have been mixed results in terms of the outcomes obtained from the assessment of learners in Mathematics. Some countries have been able to teach their learners to perform better in Mathematics, while in other countries learners' performance continues to be a matter of great concern.

The results of the Trends in International Mathematics and Science Study (TIMSS, 2011, 2015) show that, among the countries that participated, South Korea, Singapore and Taiwan had significantly higher performance than all other countries in both Grade 4 and Grade 8 Mathematics assessments (Reddy et al, 2012; 2016). The three afore-mentioned countries registered average scale scores of 613, 611 and 609 respectively in the TIMSS (2011) Mathematics achievement, and are higher than the TIMSS centre point of 500 (Mullis et al, 2012). TIMSS (2011; 2015) reveal

that South Africa is amongst the poorly performing countries with an average mean of 352, far from the international set mean of 500. The South African Grade 9s participated in the Grade 8 Mathematics assessment for the above two years' assessment (Reddy et al, 2016).

In addition, other international studies have also shown that South Africa is performing poorly in Mathematics when compared to other countries (Makgato & Mji, 2006). According to a Department of Basic Education report (2011), only 23.5% of South African learners achieved more than 50% in the 2010 National Senior Certificate (NSC) Mathematics examination. The Annual National Assessment (ANA), a local South African government initiative through the Department of Basic Education (DBE) that aims at improving the quality of education, records that its Mathematics record analysis for Grade 6 and 9 was a sub-standard performance of 30% and less (ANA, 2012). The Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ) is a monitoring project to determine educational quality for numeracy and literacy skills of Grade 6 learners in these countries (Van der Berg et al. 2011). Out of the 14 countries which participated in SACMEQ II (2000), South Africa ended in the ninth position in Mathematics and performed below less developed countries, such as Swaziland (Van der Berg, 2007). Among the 15 participating countries in SACMEQ III (2007), South Africa had the eighth lowest position for Mathematics and performed worse than countries like Kenya, Tanzania and Swaziland. This shows that even though Mathematics is an essential subject, it remains a major challenge to learners in South Africa. If learners are struggling with Mathematics, it could be that an understanding of algebra, which forms the basis of most concepts in this subject, is not firmly established in them and due to their shaky background in arithmetic (Booth, 1988; Skemp, 1987).

1.1.2   Importance of algebra

Proficiency in algebra is the fundamental mathematics required for learners to proceed into higher-level studies in STEM courses (Adelman, 2006). The importance of algebra can be seen as it appears fourth in the hierarchy among ten Mathematics topics specified in the Curriculum and Assessment Policy Statement (CAPS, 2011, p.9) - the new policy requirement for Grade 11 Mathematics learners. Algebra constitutes 40% of the TIMSS Mathematics assessment. Most importantly, algebra serves as an access towards learning other areas of Mathematics (Adams & Ely, 2012). The Department of Basic Education emphasizes learner problem solving should focus on application problems; this is the ability to apply mathematical learning to all facets of real life situations (CAPS, 2011, p. 8). Algebraic word problem solving is a most useful tool, because it simulates the knowledge and skills that incorporate the application of Mathematics in real life situations. The more algebra proves its importance, the more algebraic word problems typically show they deal with using mathematical ideas in tangible world conditions (Haghverdi, Semnani & Seifi, 2012).

Drijvers, Boon, & Van Reeuwijk (2010) note the central problem of solving algebraic equations is with representing known and unknown values. This topic encourages thinking among learners, because it does not only present intellectual images, but also provides complex associations between quantities. Learners develop many misconceptions during transitioning from arithmetic to algebraic thinking. (Ladele, 2013). Poor understanding of the equals sign is one of the challenges of solving arithmetic problems. (Knuth, Stephens, McNeil, & Alibali, 2006; Chesney & McNeil, 2014).   The inability to manipulate negative signs (Vlassis, 2004), and

learners' poor understanding that variables can signify more than one value, are explained (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014b).

Given the preceding discussions, it is reasonable to deduce that learners in South Africa are performing poorly in Mathematics (TIMSS, 2011). Studies reveal that a relationship exists between learners' poor performance in Mathematics and errors they make when solving algebraic word problems (Norton, 2009; Luneta & Makonye, 2010). Fuchs, Fuchs, & Compton (2012) are of the notion that learners' poor performance could be due to errors with mathematical concepts. In particular, these errors include when learners have to solve algebraic word problems. The aspect of generating equations to represent the relationship between the quantities also remains difficult for most learners (Kieren, 2007). Drijvers, Goddijn, & Kindt, (2011) argue that learners find it difficult to transit from arithmetic to algebra and this could be the reason for the errors they make in solving algebra. Maat, Ibrahim and Zakaria (2010) also reveal that errors in solving equations are due to a lack of understanding. One such error, French (2002) explains, is that some learners perceive the equation $(a + b)^2 = a^2 + b^2$ to be correct.

Scholars reveal that the reasons for learners' challenges and the errors they make in Mathematics are linked more to their poor language proficiency than to their lack of computation ability (Fuchs, Fuchs and Compton, 2012; Le Febvre et.al, 2010; Turner, 2011; Riordain & O'Donoghue, 2009). This means learners' computation skills challenges are due to their inability to identify and understand key words in the problem statement. More of these knowledge gaps prompted the researcher to investigate errors that Grade 11 learners commit when answering algebraic word problems. What are these errors learners encounter? Errors are the symptoms of misconceptions, also the deviation from conceptions, while misconceptions are like a fully-fledged disease (Luneta and Makonye, 2010). Scholars have documented their views on learners'

errors (Olivier, 1989; Ryan & Williams, 2007; Oliver & Caglayan, 2008; Luneta & Makonye, 2010). Oliver (1996) and Makonye and Luneta (2010) view errors as systematic or non-systematic respectively. Luneta (2008) explains systematic errors as insistent and continually wrong solutions, while, according to Khazanov, non-systematic errors are non-natural, unusual and unintended wrong answers (Khazanov, 2008). Highlight from the study of Shalem, Sapire and Sorto (2014) is that South African learners are involved in both national and international assessments. So it is imperative that error analyses are done to enable effective learning. Logically the serious research on error analyses can help to correct challenges South African learners have in Mathematics. Therefore, the researcher adopted the analyses of errors by Haghverdi and his colleagues (2012) with the following error categories: linguistic knowledge, comprehension knowledge, communicational knowledge, and calculation knowledge.

The researcher's opinion is that errors learners make, start in earlier grades and as the learners progress through the grades, the errors are compounded. From this researcher's experience in the teaching environment, it has become clear that most learners have problems dealing with questions with more than one operation, equations with two variables, one at each side of the equation, factorization and particularly translating word problems into algebraic form. All these stem from inadequate conceptual knowledge. Therefore, to bridge this knowledge gap, this study will investigate errors Grade 11 learners commit when solving algebraic word problems; particularly in these three algebraic areas of the curriculum, namely linear equations, quadratic equations and equations of two unknowns with one linear and other quadratic equation.

1.2   Problem statement

The 2011 and 2012 annual matric Mathematics examination examiners' reports of the Gauteng Department of Education (GDE) have persistently highlighted errors that learners commit

when solving some of the algebraic word problems in examination tasks (GDE, 2011; 2012). Among other factors, the reports have linked learners' errors committed during examination to poor linguistic comprehension and interpretation of certain aspects of the problem task (GDE, 2011; 2012). This aforementioned report includes learners in the Tshwane South District where the researcher is a Mathematics lecturer and where the study is conducted.

As a Mathematics educator in this district, the researcher observed that some learners resort to solving problems with methods which could be time-consuming. These methods can also be termed as non-algebraic in approach. This can lead to learners committing errors when solving algebraic word problems. The researcher also observed that this problem is recurring among learners, and that it cuts across all grade levels in high school. In addition, learners misinterpret the use of operations in word problem processes, by multiplication instead of addition. For example, consider this question from Duru and Koklu (2009): write down the symbolic representation of the statement "4 more than 3 times of a number is 16". In the study 128 out of 185 of the learners correctly translated it as $3x + 4 = 16$, but some miscomprehended the operation as $3(x+4)=16$. Evidently, it shows learners have challenges with manipulating negative numbers and difficulties calculating equations with two variables. This prompted the researcher to conduct this study.

1.3   Purpose of study

The researcher is deeply concerned about errors learners make, a situation, which effected the desire to alleviate these problems and minimize the possibilities of these errors continuing into the higher grades. These prior reasons have encouraged the researcher to investigate the nature of these errors, and determine learners' understanding and perceptions of the errors they commit when solving word problems. Solving algebraic word problems involves the translation of questions in

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word form into algebraic terminology, such as equations. Algebraic inadequacy or English deficiency, amongst others, is one of the challenges learners face. These are discussed in subsequent chapters.

In attending to the purpose of this study, the researcher administered a sequential explanatory mixed method (Creswell & Plano-Clark, 2011; Van Wyk & Taole, 2015). This method explains the quantitative preceding the qualitative method. The quantitative part will address the second and the third questions below by assessing and checking learners' knowledge displayed, using their Mathematics test scores to analyze and identify algebraic knowledge types. The quantitative approach focuses on frequencies of methods used by learners, adapting the work of Mamba (2012) and Haghverdi et al (2012) on analysing algebraic errors. The qualitative approach responds to the study's first question below by conducting interviews with selected learners. The research design will respond to these research questions:

- What type of errors do Grade 11 learners make when solving algebraic word problems?

- What are the possible causes of errors made by Grade 11 learners when they solve algebraic word problems?

- What are the possible strategies Grade 11 learners can use to minimise errors when they solve algebraic word problems?

This study's aim is to gain insight into errors Grade 11 learners make in their algebraic solutions. The researcher has chosen to conduct this research within the context of error analysis discourse. In this area of knowledge, the researcher will study the nature of Grade 11 learners' errors, and their understanding and conceptions of the errors they commit when they solve algebraic word problems. In order to achieve the aims of this study, the following are the objectives proposed:

- To determine the types of errors Grade 11 learners made when translating algebraic word problems;

- To determine causes of errors Grade 11 learners made when solving algebraic word problems; and

- To determine strategies, Grade 11 learners can use to minimise errors when they solve algebraic word problems.

1.4   Significance of the study

Previous results from international and national Mathematics examinations show that South African learners experience challenges when solving algebraic word problems. The researcher observes that learners' poor performance in Mathematics is linked to errors they make. For example, learners commit errors because they are attempting to solve problems arithmetically instead of algebraically. In addition, the researcher finds that learners have difficulty in translating language in the word problems into mathematical symbols. Thus, while manipulating operations, learners commit errors.

This research in errors will be significantly helpful to improve the teaching of Mathematics in Grade 11 and should lead to subsequent better performances in Grade 12. The results from this study will help teachers to design effective instructional methods and strategies to improve learners' understanding of algebraic word problems in Grade 11 and in high school at large. It will enrich teachers in the teaching process with intervention initiatives to eliminate or reduce errors.

This research on mathematical errors will encourage learners involved with this study to be cautious of errors and check their solutions properly before submitting them for marking. It will also make learners diligent, particularly when solving algebraic word problems; that is, they will avoid confusing the use of certain operations when interpreting word problems.

The recommendations that will come up from this research could also help Mathematics curriculum planners, assessors (examiners) and textbook authors to the extent of improving their presentation of algebraic word problems. Research in algebraic word errors will specifically assist in the learning and teaching environment, and particularly enhance learners' capability to solve algebraic problems, translating from word problems to algebraic symbols and eliminating errors.

1.5   Limitations of the study

The researcher collected data from three Gauteng high schools in South Africa. This data, giving a picture of a certain group, should not be generalized but can be transferred. The second limitation is the accessibility of learners to the researcher. This forced the researcher to work with only three schools. The researcher conducted the study with 150 learners for the written test and 8 learners for the interviews, which gave enough information to make the inferences this work is contributing.

1.6   Definition of terms

This study defines the following operational terms:

Problem

A problem refers to a situation, quantitative or otherwise, that confronts a learner or group of learners. The problem requires a solution or calculation for which the learner sees no apparent or obvious means or path to obtain a solution.

Algebra

Algebra is a part of Mathematics and deals with the study of variables, terms, expressions and relationships between these quantities. It involves solving equations, using operations.

Algebraic word problems

Algebraic word problems refer to textual questions requiring translation from English into mathematical language (algebra) that eventually leads into equation questions that are either linear or quadratic equations.

Errors

Errors refer to the wrong solution because of miscomprehended expressions and equations. Algebraic word problems mostly contain abstract or ambiguous language frequently miscomprehended by learners rather than when the problem is written in simple language.

1.7   Chapter outline for the thesis

Chapter 1 Introduction

 It will include background of the study, problem statement, research aims and objectives, research questions, significance of the study and the operational definitions of some key terms used throughout the study.

Chapter 2 Literature review and theoretical framework

This will include the theoretical framework, reviews of the nature of algebra, algebraic word problems, and analyses of errors in algebra from literature, errors in algebraic word problems, linear equations and quadratic equations.

Chapter 3 Research methodology

This chapter will include elucidating the research method design and approach of the study, the population, samples and sampling procedure, instrumentation, reliability and validity issues, as

well as methods of data collection and analysis. This chapter discusses the ethical consideration issues.

Chapter 4 Data presentation and analysis

This chapter contains data presentation analysis and findings.

Chapter 5 Conclusion

It involves discussion of findings, conclusion and recommendations.

1.8   Conclusion

This chapter presented the introduction to this study followed by the problem statement.. Thereafter, the aims, including the objectives, are explained with four ways of attaining these objectives. The research study's aims and objectives are focused on the achievement of research questions. The chapter includes the significance of the study, the limitation of the study, ethical considerations for the study, the definition of terms and the chapter's outline.

The next chapter will be the review of related literature.

Chapter 2: Literature Review

This section discusses the literature review that influences this study under three headings, namely:

- the introduction of the study as situated in literature;

- the theoretical framework of the study;

- and literature on errors learners make relating to these three areas of the curriculum: linear, quadratic, solving an equation with one linear and quadratic by factorization.

2.1   Introduction

Scholars have different perceptions in defining the subject Mathematics. However, they share common ideas. Researchers like Kaput (2007), Usiskin (1997) and Kieran **(1997)** similarly describe that Mathematics is all about generalization, symbolism and structure. A European-based researcher views Mathematics as a concept that has an internal structure of controlled connections that cause it to have essential properties as a magnitude of its context (Tall, 2011**)**. Mathematics is viewed through the following three perspectives: conceptual-embodied, perceptual-symbolic and axiomatic-formal (Tall Lima & Healy 2014). The conceptual-embodied is perception and reflection on properties of an object which before are viewed and felt in the actual world, but later imagined in the mind (Tall, 2008). In addition, the term perceptual-symbolic refers to all procedures of developing symbols such as counting numbers (Tall, 2008). Tall et al (2014) affirm axiomatic-formal as the act of constructing formal theory, definition and proof.

The West African group of researchers indicate that Mathematics is central to human activity in general (Andam, Atteh, Obeng-Denteh & Okpoti, 2015). According to Andam and his colleagues, this human activity is a course of action displayed right from childhood. However, from the perspective of South African researchers Barwell, Barton & Setati (2007), this subject

incorporates language as well. Mathematical language is a language of signs, known as operations and such that language refers to conception, meanings and deductions (Boulet, 2007). Mathematics depends a lot on conceptual understanding of how ideas relate and coherently build on one another. Possession of previous knowledge plays an important role in this conceptual understanding of Mathematics (Bush & Karp, 2013).

Davis (2010) argues that Mathematics is either routine or creative activity. Routine means to solve arithmetic problems, using mathematical operators $(+, -, \times, \ and \ \div)$. Sfard (1991; 2007; 2008) perceives Mathematics to be operational or structural. The concept 'operational' refers to it being useful for all calculations, including the use of operators such as addition, subtraction, multiplication and division. 'Structural' is all aspects of Mathematics that link disjointed ideas to arrive at a solution.

Mathematics entails computation. Literature shows that understanding constructions, associations and designs of mathematical ideas is useful to produce solutions for complex real life problems (Hiebert & Carpenter, 1992). The development on mathematical facts has gained more attention and importance with the rapid advancements in today's world (Arseven, 2015). In his journal "Mathematics modelling approach in Mathematics Education", he states that Mathematics has become an essential activity for individuals regardless of age, to achieve, examine, and use mathematical understanding effectively and efficiently to be achieving people in our information age. It is imperative that learners are well equipped with mathematical knowledge to function in other areas of education related to it and also to function adequately in the real world (Arseven, 2015). In addition, CAPS (2012) emphasizes the development of the following skills:

(a)   to recognise representation of different and same concepts in algebra

(b)   to represent and describe situations in algebraic language, formulae and expressions

(c)   to analyse and interpret equations and to communicate effectively mathematical ideas in visually, symbolically and linguistically.

The DBE focuses more on this area of the curriculum, which entails algebraic thinking with specific concepts such as computation, symbols formulae, relationships and patterns to mention but a few. Hence, this study becomes essential for enhancing mathematics learning and teaching. The preceding records show South African learners are performing poorly in Mathematics. This means it is important to address the challenge right from the onset.

2.2   Learners' performance

Research reveals that learners encounter difficulty in Mathematics (Adu, 2012). The challenge of poor learner performance in Mathematics is not peculiar to South Africa only. This has proved to be a challenge in Nigeria as well, where in the West Africa Secondary Council Examinations (WASCE) 30% of Nigerian learners scored 50% and above in Mathematics (Uwadiae, 2011). In Singapore, Grade 4 learners recorded underachievement in Mathematics TIMSS 2007 (Kuar, Koay, Fooneg & Sudarshan, 2012). South Africa is not an exception, as many schools are also struggling to produce school leavers with the standard of Mathematics required in the workplace (Adler, 2015).

Comparative SACMEQ (2000, 2007) studies of participating Southern and Eastern African countries show South Africa as being among the lowest performing countries in Mathematics (Moloi & Strauss, 2005; Moloi & Chetty, 2011). South Africa participated in SACMEQ for the years 2000, 2007 and 2013 (Taylor and Spaull, 2015). The results of SACMEQ II and III show

that while some countries like Botswana and Tanzania made progress in Mathematics performance at Grade 6 level, South Africa did not improve (Spaull, 2013). The report from SACMEQ shows that in SACMEQ 11, with an average mean of 500, South Africa scored an average of 486; while in SACMEQ lll, with an average mean of 507, South Africa had 495 (Spaull, 2013). However, the SACMEQ IV result showed a slight improvement as South Africa ranked a mean score of 587 in Mathematics, which is higher than the SACMEQ average mean of 584 (Taylor & Spaull, 2015). How is South Africa performing in the Mathematics international assessment? Is there improvement or is it still a challenge as is the case in SACMEQ, an African Mathematics assessment? The following TIMSS discussion will unveil this answer.

The TIMSS, (1995, 1999, 2003, 2011 & 2015) records that South Africa participated in the Mathematics assessment (Reddy et al., 2016). Subsequent TIMSS results (2003 - 2015) showed no improvement in these years, because the average South African Grade 9 learners displayed 2 grades less performance in the Grade 8 Mathematics assessment among 11 other middle-income countries (Spaull & Taylor, 2015)**. The Grade 8 and 9 learners wrote and performed poorly in the 2003 Grade 8 TIMSS, which resulted in the South African Grade 9 learners only writing the assessment meant for grade 8 learners in the subsequent years of 2011 and 2015 (Reddy et al., 2016). The result of TIMSS (2011), having an international mean of 500, showed South Africa scored 352 with their Grade 9s assessed for Grade 8 Mathematics (Reddy et al, 2012). In the 2015 TIMSS South Africa also performed poorly with scores of 376 and 372 respectively in the Grade 4 and Grade 8 Mathematics assessment (Mullis et al, 2016). Evidently, this indicates that South African learners are struggling with poor Mathematics performance.

Reports have also showed internationally that South Africa is performing poorly in Mathematics as compared to other countries (Spaull & Simkins, 2013). Similarly, the Annual

National Assessment (ANA) result documented that South African learners are struggling with a low pass rate in Mathematics (Spaull, 2013). In addition, the ANA, a local South African government initiative through the Department of Basic Education (DBE), aims at improving the quality of education. According to Spaull (2013), the Mathematics record analysis for Grade 9 in the ANA (2012) showed low performance: less than 5% of learners were able to achieve 40% and above in Mathematics. The table below shows the South African Grade 9 learners' performance in the ANA Mathematics Assessment.

Table 1: 2012 Grade 9 Mathematics scores by percentage range

Range	Percentage of score %
Less than 30%	91.9
30 – 39%	3.8
40 -49%	2.1
50 59%	1.1
60 -69%	0.6
70 – 79%	0.3
80 and over	0.2
Total	100

Simkins (2013), Report for CDE, Performance in the South African Educational system - What do we know? (p. 6)

The above table shows that most South African learners struggle with poor performance in Mathematics because the pass mark is 30% and most of the learners, that is 91.9%, scored less than 30%. In addition, percentages of learners who meet the requirements for university entry were 20.1% in 2008 and 30.6% in 2013 (Spaull, 2013). This means the numbers of learners with average and above scores in Mathematics are few. It can be that the difficult domain of Mathematics

referred to above is word problems. As long as Mathematics is an intimidating subject for many learners so particularly is the area of algebraic word problems (Appoh Andam et al., 2015). Solving algebraic word problems has been proven to be more difficult compared to solving mathematical problems (Forsten, 2004; Fuch, Powell, Seethaler, Cirino, Fletcher & Fuch, 2009; Gersten, Jordan, & Flojo, 2005; Hart, Petrill, Plomin, & Thompson, 2009 & VanSciver, 2009). Clement (1982) acknowledges that the reason for learners' difficulties is their inability to construct meaningful symbolic relationships with algebraic equations. On the other hand, researchers view that a language problem is the reason for learners' difficulties in comprehending how to solve word problems (Koedinger and Nathan, 2004; Kotospoulos, 2007). Therefore, when learners encounter these difficulties, they make errors.

2.3   Algebraic Thinking

Arithmetic teaching in the elementary classes is the basis of algebra learning in school. This later develops and becomes Algebra in the later years of learning. Research shows that learners find it difficult to transit from arithmetic to algebra (Ladele 2013). Therefore, Kieran (2004) suggests that elementary learners in higher grades should have more experience in algebra. In this same vein, the United States has introduced algebraic thinking through number patterns during elementary education and this allows for the smooth transition towards advanced algebra (Kaput, 2000; Kaput & Blanton, 2003). Learners find it challenging to shift from solving concrete problems to abstract problems. It has proved to be more difficult to improve from the step of solving word problems arithmetically than to solving them algebraically (Reimer & Moyer, 2005). Learners find solving arithmetic word problems difficult. For example, representing algebraic word problems into mathematical equations and then calculating is more difficult (Tolar et al, 2012; Hecht & Vagi, 2010).

Arithmetic word problems are generally solved using mind calculation, while algebraic word problems require translation from the word problems into mathematical formulae and equations (Daroczy, Meurers, Nuerk, & Wolska, 2015; Gasco & Villarroel, 2014).

English (2010) and Kaput (2008) have made a massive contribution to the understanding of algebraic thinking. English (2010) denotes that algebraic thinking is essentially the procedure of a learner possessing vital mathematical knowledge, skills, and a way of thinking required to achieve in Mathematics. Algebraic reasoning involves generalization and expressing such generalization of symbols in a straight sequential and conventional manner, such as identifying the meanings of these symbols hidden in the words (Kaput, 2008). Algebraic thinking is viewed not only as means to manipulate and simplify symbols, but also as a way of thinking which involves using patterns, functions, structures or modelling situations (Radford 2006a; 2009a; 2010). Algebraic thinking refers to algebraic forms, which deal with indeterminate quantities, such as unknowns or variables; these make difference from arithmetic known with determinate quantities (Radford, 2006). According to Lin, Rajano, Bell and Sutherland (2001), unless the learners interact with the algebraic question in the expression form, they will not be able to get the meaning of the problem. Up to this point they cannot ascertain the level of difficulty of the problem. Scholars recognize the presence of algebra in STEM, but the presentation manner in the classroom does not depict the relationship of algebra with other STEM subjects (Drijver Boon & Van Reeuwijk, 2010). Bakker et al (2008) suggest a strategy to connect algebra in class to algebra in STEM. This relationship between algebra and STEM subjects makes Drijver Boon & Van Reeuwijk (2010) recognize that technology has a positive connection with algebra. This implies that algebra should have an advantage in its importance in technology. Kieran and Drijver (2006) suggest the use of technology to teach algebra, which can enhance the teaching and learning of

algebra. Dekker and Dolk (2011) opine that the combination of concrete to abstract will assist the shifting from arithmetic to algebra. Dekker and his colleague view arithmetic reasoning differently from algebraic reasoning. They do not regard algebraic thinking as advanced arithmetic. The researchers refer in their article "Arithmetic to Algebra" to learners' processes of shifting from concrete into abstract as being subtle and time-consuming. This means that one thought is separate from the other in the sense that learners at a point in time should have practiced and mastered the basic arithmetic, before proceeding to be in control of adequate algebraic knowledge (Dekker & Dolk, 2011). Drijver, Goddijn and Kindt (2011) explain that Algebraic thinking involves deeper processes than the simple movement of arithmetic to algebraic. In addition, Kaput and Blanton, (2011) mention Algebraic thinking means constructing a structure that represents the problem by simplifying, using quantifiable variables to have equations, formula and expression.

The point that algebra occurred historically after arithmetic in the curriculum and that arithmetic is focus on operations of numbers, while algebra is comprehensively about numbers, functions and variables, makes them separate topics (Carraher, Schliemann, Brizuela & Earnest, 2006). There are steps to unite two topics that appear separate in the curriculum, although it could be difficult (Carraher et al, 2006). Star and his colleagues (2015) concur that an adequate understanding of arithmetic operation is the essential to become proficient in algebra.

Their article, "Teaching Strategies for Improving Algebra Knowledge" implies algebra is a new skill for learners. It constitutes a primary part of Mathematics that requires far-reaching and abstract thinking development from them. Mathematicians emphasise that algebraic understanding and skills are essential for successful tertiary education, particularly in the field of science and technology (Star et. al., 2015).

The two types of knowledge identified by Schneider, Johnson & Star (2011) for a proficient algebraic solver are conceptual and procedural knowledge and these two have a positive relationship with each other. Baroody, Fail & Johnson (2007) describe conceptual knowledge as applying generalization and giving values to mathematical principles. In addition, Rittle-Johnson and Schneider (2015) state that conceptual knowledge is about building, choosing the right solution for problem solving methods, while Canobi (2009) views procedural knowledge as various actions or methods, which are towards the fulfilment of a target. Hallet, Nunes and Bryant (2010) denote there is a connection between conceptual knowledge and procedural knowledge in topics like fractions and decimals. Furthermore, Patol & Canobi (2010) identify the above-mentioned knowledge type's relationship also exists in addition and subtraction operations. Studies noted that flexible procedural knowledge (which has similar meanings as procedural knowledge) is the skill to select among various methods the most suitable to solve problems correctly (Star & Rittle-Johnson, 2007; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009). Lefevre et al (2006) mentioned that there is a correlation between conceptual and procedural knowledge in the numeric part of Mathematics. According to Durkin, Rittle-Johnson & Star, (2011) noticed the link of these two kinds of knowledge when solving equations.

These two kinds of knowledge are termed as having a bi-directional and iterative connection with one another. This means that the development of one type of knowledge can bring about the improvement of the other type of knowledge and vice versa. Hecht & Vagi (2010) noticed that conceptual knowledge has power to determine the existence of procedural knowledge and vice versa. However, in their article, conceptual knowledge has a stronger influence on procedural knowledge than vice versa (Hecht & Vagi, 2010). Conceptual knowledge is independent of procedural knowledge and they have a positive correlation with one another.

Schneider and colleagues declare that a proficient algebraic solver must have conceptual and procedural knowledge. In addition, algebraic thinking propels scientific interpretations and understanding such as the manner humans relate with the world. Kirshner & Awtry, (2004) describe Algebraic thinking as finding patterns and simplifying them into known and unknown situations. There are many instances of algebraic thinking situations in activities such as comparing job offers, or trying to settle for a better agreement, or choosing the better one in of terms the time and distance when travelling with a car or a bus. According to EduGains (2013), Algebraic thinking or reasoning is involved when:

- Architects and construction experts design buildings and determine materials needed to build structures;

- Software developers create codes;

- Bankers figure out mortgage and interest rates; and

- Scientists in almost every field go about their daily work

Algebraic reasoning is the process of simplification and calculation that builds on past knowledge with numbers and uses concepts of signs and operation such as investigation with notion of shapes and function (Van de Walle, Karp, & Bay-Williams, 2011). According to Drijver et al (2011), learners most times answer arithmetic problems using simple procedure while algebraic problems solving requires abstract thinking. Solving algebraic problems involves learners putting together various pieces of compound information concurrently (Star et al, 2015). They opined that the learner have difficulty to build up algebraic thinking because of pieces they need to put together. Next to be addressed are the challenges learners encounter when solving algebraic problems.

2.4   Algebraic problem solving

Star et al (2015) view algebra as a domain of Mathematics, which requires learners to be proficient in many skills. They assert that one of such skills is linguistic skills, in conjunction with others, which must work simultaneously in order for the learner to arrive at the correct solution. Star et al (2015) note that the procedure of putting together numerous pieces of compound information simultaneously, most often restricts solvers of an algebraic problem from developing new knowledge. According to these researchers, most learners find it challenging to deal with numerous pieces of information concurrently. This then forms a barrier in their ability to develop this new knowledge.

Algebra remains a powerful problem-solving tool. It is an important domain in Mathematics and is fundamental for mathematical proficiency. The understanding of algebra is the central phase in learners' ability to excel in Mathematics. Consequently, learners' profound understanding and acquisition of algebraic concepts and thinking skills are necessary for good performance in Mathematics.

The fact that arithmetic is termed the simplification of algebra, pinpoints that there exists a structural similarity between arithmetic and algebraic problems (Banerjee and Subramaniam, 2012). Scholars in their research "Evaluation of a teaching approach for beginning algebra" also signify their differences in the method of solving arithmetic and algebraic expression problems.

In addition, a Singapore study discusses that solving word problems accelerates in learners the effective use of operators $(-,+,\div,\times)$ which is the basis of lower primary school algebra (known as arithmetic) and a prerequisite for higher levels of primary, secondary and tertiary schooling algebra (Chan, 2014).

It is important to note learners do not have difficulties with the one-step word problem but with multi-steps word problems (Chang, Ghani & Koay, 2012). Similarly, Anderson, Edward & Maloy, 2009, affirmed learners frequently have problems understanding and solving multiple operation word problems. At this point, it is important to clarify that arithmetic word problems demand one-step solutions while algebraic word problems require more than one step to solve. Take, for example, the algebraic problem solving a quadratic equation which requires using the substitution and then factorization procedure. These mentioned and explained learners' difficulties with solving algebraic problems create a situation for them to make errors.

Murray (2012) explains that a learner difficulty is not with numeric problems (which is arithmetic) but with algebraic word problems, because of words used. Similarly, Cummins et al. (1988; 2006) and Griffin & Jitendra (2009) find that linguistic knowledge together with mathematical computation is needed by learners to enable them to solve algebraic word problems successfully. The effective solution of algebraic word problems is not dependent on learners' ability to perform requiring mathematical operation alone, but also on the accurate understanding of text in the word problem (Jitendra & Star, 2012; Van der Schoot et al, 2009). When solving algebraic word problems the proper understanding of text in the word problem precedes the correct mathematical computation. Similarly, Kieran (2011) underpins that algebra is about cognitive processes, involving not only literal symbols. Studies also explain the major challenge for learners solving algebraic word problems is the failure to understand the problem statement (Boonen et al., 2013; Lee et al., 2009; Thevenot, 2010). The more Algebra serves as a bridge to tertiary learning, the more difficult it is for many learners (Subramanian & Banerjee, 2011). In order to solve word problems meaningfully and successfully, Verschaffel, Greer, & De Corte (2000) identify these four steps necessary for learners: the need to comprehend the text, model, execute and evaluate the

answer for them to execute the word problems. The highlight from points listed in this study is that effective solving of algebraic word problems goes together with meaningful reading and understanding. Reading difficulties are the reasons for learners' challenges of solving algebraic problem and this leads to learners committing errors.

The main goal of CAPS (2011) - the schools' curriculum - is to focus on the application of mathematics to everyday problems and the solving of algebraic word problems. The act of learners solving algebraic word problems helps to achieve this goal of applying mathematics because it inculcates into learners the ability to solve real life problems. Algebraic word problems constitute those mathematical ideas and abilities that require learners to decode and encode and thereafter develop in them the ability to apply mathematics to daily activities or real life situations (Verschaffel, Greer & De Corte, 2000, 2012; Boaler, 1993; Herbert & Carpenter, 1992 and Hiebert et al, 1997)**.** In addition, algebraic word problems are a domain of mathematical problems where numbers and relationships are used to represent given problems in text form (Nortvedt, 2011). In summary, algebraic word problems are a major component of Mathematics that creates a problem solving ability for learners (NCTM, 2007, 2010; OECD, 2010; 2013).

Studies have shown solving algebraic word problems are a difficult task for most learners (Star, 2005; Moreno et al, 2011; Boonen et al, 2013; Schleppegrell, 2007). Therefore, it is inevitable for learners to make errors when they encounter these difficulties.

2.5   Importance of algebra

Algebra is an important component of Mathematics; that is the reason it appears third in the hierarchy of topics listed in CAPS, and constitutes 40% of TIMSS Mathematics assessment. Algebra consists of the concepts of variables, expressions and equations (Usiskin, 1988). Kieran (1992), on the other hand, refers to algebra as a component of Mathematics, which includes symbol

simplification, representation, number connections, Mathematics configuration and working on this configuration. Cathcart, Pothier, Vance & Bezuk (2006) explain that algebra forms the basis of reasonable associations in the domain of Mathematics and it consists of arrangement. Algebra is the advance form of mathematical expression made in words (Fearnley-Sander, 2000). It answers to categories of knowledge in terms of sum and operation (Usiskin, 1999). Algebra is a Mathematical expression in the form of texts, letters and variables. The term 'variable' in algebra has the capability to characterize whatever value is given to it, for example, 3*n* (Adams & Ely, 2012). In addition, algebraic concepts are useful and are components in most topics of Mathematics (Adams & Ely, 2012).

Liston and O'Donoghoe (2010) assert that algebra is an aspect of Mathematics and it provides the foundation for mathematical thinking. Proficiency in algebra manipulation is essential for learners who are interested in science, technology, engineering and a mathematic STEM career. Algebraic knowledge and skills enhance the productivity of a skilled workforce for scientific and technical careers.

Algebraic word questions appear (interwoven) in most Mathematics topics like geometry, calculus and trigonometry to mention a few (Department of Education, 2011, p.13). In addition, Kieran (1992; 2004) insists that the algebra lessons covered in the early secondary school are generalized arithmetic. In addition, Sonnerhed (2009) states that algebraic word problems are the means of translating text presented in real life cases into algebraic structures by the use of algebraic symbols. In support of the previous statement, the Department of Education prioritises solving algebraic word problems because it encourages the learning of application-based subjects. Haghverdi et al (2012) affirm that algebraic questions in the form of words are grouped as application questions because they handle mainly practical mathematical ideas in day-to-day situations. Algebraic text

questions consist of values, which can be familiar or at times unfamiliar, such as having explicitly or implicitly stated relationships between the values (Wright, 2014). In order to solve word problems algebraically, there is the need to make use of equations.

2.6   Learner algebraic challenges

The researcher noticed that, although South African learners experience challenges with word problems, they experience most difficulty with algebraic word problems. It proves to be a complex task for many learners to solve these, because it entails integrated skills. Also, the researcher discovered from her experience of teaching that learners exhibit difficulty in areas of factorization, quadratic formula solving for *x*, and others.

Many studies at primary, secondary and tertiary levels have documented that poor proficiency in algebra results in low performance in Mathematics. Ladele (2013) discusses this at the primary level. Egodawatte (2011) explains this at the high school level, while Moru (2006) asserts this among first year university students. Furthermore, Luneta & Makonye (2010) discovered that the fact that learners are challenged in calculus, is because of poor factorization, indices, equations and numbers. Murray (2012) observes that learners often have more difficulty in algebraic word problems than in straight numeric computations. This low performance in algebraic based word problems has been a concern to many educational practitioners. As observed by policy makers, the errors learners commit emanates mostly from algebra (NAEP, 2008). Algebra is pivotal to most errors committed by learners in Mathematics because it forms the basis for this subject. However, it can be said that, if learners are underperforming in Mathematics, then they could be committing errors in areas of arithmetic, functions, quadratics, statistics and algebra, to mention a few. The researcher observes that learners are committing errors in the areas of algebraic word problems. Norton (2009) denotes that the poor performance of learners while

answering algebraic text questions, is as a result of errors learners make. Similarly, Luneta & Makonye (2010), and Oliver & Caglayan (2008) reveal that a relationship exists between learners' poor performance in Mathematics and errors they commit when solving algebraic word problems.

The GDE (2011; 2012) result analysis of the National Curriculum Statement (NCS) Grade 12 Mathematics exams (which includes Tshwane South District) highlighted that learners were committing mostly language and interpretation errors when solving algebraic word problems. Many studies have also shown that poor performance could be because of errors (Nesher, 1987; Makonye, 2012). There were 80 studies on errors reviewed from the period of 1970s to the beginning of the 21st century (Jiang, 2013). Errors are answers to miscomprehended problems. The errors are symptoms of misconceptions and a deviation from conception. Errors analyses according to Newman (1977) and Casey (1978) focused on algebraic text questions which have five groups, namely reading, comprehension, transformation, processing and encoding. In this line, Vaiyavutjamai and Clements (2006) concur with Skemp (1987) that errors learners commit when solving algebraic word problems, are categorized as relational and instrumental errors, particularly with the concept of solving quadratic equations. Lewis and Mayer (1987) opine that errors arise from these two parts: the failure to understand the syntax and semantics of the word problem. In addition, studies recommend that for learners to interpret these words adequately, they must have the microstructure and macrostructure understanding of the word problem (Cummins et al, 1988, 2006). Learners' errors stem from their inability to comprehend the words in the problem and to recognize the relationship between the words (Macgregor & Stacey, 2000; Nortvedt, 2010). Research has documented the necessity for basic operational understanding to supplement linguistic skills to enhance learners' ability to generate the needed equations (Ahmad, Tarmizi, & Nawawi, 2010). In other times, errors are linked a to lack of procedural and conceptual

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understanding (Egodawatte & Stoilescu, 2015). Researchers emphasize that it is essential learners are deep-rooted in the conceptual understanding (Hiebert & Grouws, 2007); this will assist learners in formulating equations from given word problems. It is discovered that linguistic skills correlate with conceptual understanding. Although there are many studies about algebraic errors, there are limited studies recorded on algebraic word problem errors, particularly in the areas of solving linear and quadratic equations by factorization. This has prompted the researcher to investigate errors Grade 11 learners commit when solving algebraic word problems in the areas of solving linear equations, linear and quadratic equations, that is, by factorization.

2.7   Theoretical Framework

This study employs the work of Haghverdi et al. (2012) and Mambo (2012) as an analysis tool in getting insight into learners' errors. Haghverdi and his team contribute to error analysis by outlining the following means or tools for analysing errors:

- Linguistic knowledge;

- Comprehension knowledge (a) semantic (b) structural and (c) intuitional;

- Communicational knowledge;

- Calculation knowledge; and

- **No answer**

2.8   **Linguistic knowledge**:

Studies have established that learners have difficulty symbolizing questions in the form of text expression into the relevant equation (**Kieran, 2007;** Howie et al. 2012; Ilany & Margolin, 2010**). Similarly, studies conducted in Turkey where Grimm (2008) and Duru & Koklu (2011) revealed that learners' reading comprehension is linked to a more conceptual understanding of Mathematics. The language of Mathematics refers to terms as words, symbols and numbers used

in solving Mathematics. Learners have difficulty understanding some English words, which have a different meaning in Mathematics or vice versa.   Kotsopoulos (2007) presents the following examples:

> 1. Words, which exist only in mathematical situation: Examples: quotient, parabola, and hypotenuse;
> 2. The meanings in Mathematics terms are different from everyday language, such as *table, factor, limit* and *round*. Another example is the word *circle*: it is a round object, but can also mean to round numbers to a decimal place.
> 3. Some mathematical words sound similar to everyday English words. Example: 'sum' could mean 'some'; 'pi' can refer to 'pie'.

.   Therefore, learners' understanding of certain English words will determine their success in solving the algebraic text question. Research conducted in the US reveals that there is a relationship between mathematical expression and English expression (Ilany & Margolin, 2010). This means the ability to solve problems using mathematical terms is associated with the understanding of the English expression used. The PIRLS (2006) and TIMSS (2011) assessment, recorded that South African learners who had low literacy levels, had equally poor performance in Mathematics (Howie et al. 2012). These previous studies on the importance of language in solving algebraic text problems pinpoint the role of language for effective understanding, conceptualization and solving algebraic text questions. In order for learners to extract the right meaning from mathematical text questions, they need to demonstrate effective reading abilities (Fuch, Fuch, Hamlett et al, 2008**;** Williams, 2008). In addition, Goodman & Goodman (2009) indicate that the procedure of reading successfully involves comprehension. Klapwijk (2015) explains comprehension of the text correlates with proper reading of the text when dealing with questions in text form. His article, "Comprehension: Reading Strategy Instruction Framework for all

Teachers", pinpoints that reading strategies include speeding, reading, skimming and scanning. The steps of speeding and reading bring about comprehension, while skimming and scanning result in understanding information topics specified in the subject contents (Klapwijk, 2015). Block & Duffy (2008) defined comprehension as a planned way in which the learner uses hints from the text question together with prior knowledge to create and check prediction that will help to build meaning from the text as well. Edwards & Turner (2009) distinguish between the meaning of the reading process and reading comprehension. In their book, "Family Literacy and Reading Comprehension", they explain that the reading process is the relationship between the learner, the word and the socio-cultural situation, while other? researchers describe reading comprehension as the connection between the learner, the method used, the text being read and the situation in which reading is occurring (Edwards & Turner, 2009). Since algebraic word questions consist of text, it means understanding of the language in the question can determine the performance in solving algebra word questions considerably.

Barwell (2005) indicated that most difficulties learners encounter in Mathematics are more from linguistic deficiency and less from quantitative deficiency. Similar research has shown learners are not challenged with questions in numeric and symbolic form, but with questions, which are expression oriented (Morgan, 2007). South Africa is a typical example where multilingual classroom environments have resulted in learners encountering difficulties in teaching and learning because of language barriers (Barwell, 2005). Learners who hail from different language backgrounds experience problems of fluency and proficiency in the language of instruction and study (Barwell, 2005). The fact that they come from different language settings has weakened their learning of Mathematics. This is because they struggle to understand the spoken and the printed words and to convey words verbally and in writing. The result of this is that it

subsequently affects their conceptual understanding of mathematical concepts and procedures (Barwell, 2005; 2006). Language has a role in facilitating mathematical knowledge and is a crucial component in the teaching and learning of algebra (Barbu, 2010; O'Donoghue, J. & Riordain, M., 2009 & Turner, 2011).

Ilany & Margolin (2010) opine that learners encounter difficulties in both natural and mathematical language when solving algebraic word problems. They suggested that learner difficulties originate from missing knowledge between words used and the numerical construction. According to their studies learners' errors mostly occurred from the steps of converting the question in text form into numeric form. Their article "Language and Mathematics" states these errors are because of learners' limited language skills, which affect their ability to formulate solutions for the given question. Ilany & Margolin (2010) argued that most learners' errors are because of inadequate language skill. Consequently they are unable to create a significant understanding from the information in the question, including data and a solution method. Therefore, linguistic knowledge is the bridge that links reading skills to constructing meaning from the algebra text questions (Fuch, Fuch, Hamlett et al 2008; Williams, 2008).

2.9   Comprehension Knowledge

The significance of the comprehension process, while algebra text questions are being solved, shows by it connecting and activating the learner's prior knowledge. The learner, who comprehends the word problem, is the one who can create a link between this word understanding and previous mathematical knowledge. Duru & Koklu (2011) explain comprehension as cognitive skills, which include knowing the constituent and circumstance in the Mathematics problem in such a way that the solver knows what is required and can solve the problem in the right way. Mathematics comprehension refers to when learners possess the understanding of the language for

instruction (Bernado, 2002) together with the understanding of mathematical terms, which consist of text, operations and numbers (Duru & Koklu, 2011). In addition, Lumpkin & McCoy (2007) explain that a learner's comprehension of a text question means an adequate understanding of the problem statement of the mathematical text question, such as to facilitate the construction of an appropriate solution.

Yerushalmy (2006) notes that increased proficiency to answer application problems is the learner's ability to transfer conveniently algebraic questions in expression form into numbers and symbols. The effective solution of algebraic word problems necessitates that learners should have comprehension knowledge to convey proficiency in language (natural language) to mathematical concepts (Mathematics language) and vice versa. Aunola, Nurma & Vilenius-Tuohimaa (2007), in a study of the investigation between mathematical word problem solving skills and comprehensive reading, provided explicit understanding of this relationship. Aunola and his fellow researchers suggest in the results of their studies that the two skills are in fact connected, and they found a strong relationship between all Mathematics word problems and reading comprehension skills.

The main struggle learners have when solving word problems constitutes the proficiency to understand how to assemble the mathematical problem that is entrenched in the problem text. The difficulty with understanding the problem structure often leads to faults in the choice of the solution tactic (Wright, 2014). In addition, Yerushalmy (2006) suggests that the increasing competence of solving algebraic questions is centred on the understanding of real-life problems; meaning learning to move freely between words, expressions, numbers, and symbols. Ilany and Margolin (2010) state:

The difficulty with the solution of mathematical word problems is the need to translate the event described in the natural language to arithmetic operations expressed in mathematical language. The translation from natural language includes syntactic, semantic, and pragmatic understanding of the discourse (p. 139).

It is important that learners possess comprehension skills that enhance the translation of words and expressions from instructional language into mathematical language, that is numbers and operations.

Voyer (2011) suggests that solving word problems is dependent on cognitive understanding. Most importantly, the conceptual understanding is similar to cognitive understanding, which is used for generating problem representation that is loaded in ideas and leads to the suitable choice of action schemata and the solution. Baroody, Feil & Johnson, (2009) opine that to translate successfully words and expressions from the question problem statement into the appropriate equation, using right principles, depends upon the learner's conceptual understanding. This conceptual understanding of the learner comprises the understanding of words used in the question and the understanding of mathematical formulas, which best relate to the situation and relationships expressed within the words. Comprehension also refers to conceptual understanding. (See NCTM, 2001; Conceptual understanding: comprehension of mathematical concepts, operations, and relations) Therefore, comprehension knowledge is essential for the learner to connect mathematical formulas to the best situation and relationships expressed within the algebraic word problem.

In order to solve algebraic word problems, the learners must create an appropriate equation that correctly matches the relationships expressed in the problem. Therefore, identifying the relationships precedes the creation of the equation(s). Nathan & Koedinger (2000a) found that

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learners encounter more challenges with algebraic word problems than they do with equation problems. Nathan, Long, and Alibali (2002)'s analyses of textbooks support preceding researchers' findings that learners encounter greater difficulty with word problems. Highlight from the studies of Cummins et al. (1988), Lewis & Mayer (1987) and Mayer (1982) states that the solving of algebraic text questions has two parts, namely the comprehension part and the solution part. In the comprehension part, the learner practices the expression of the question to build subsequent internal representations of the numeric and position-centered interaction of words in the question (Nathan, Kintsch, & Young, 1992). In the solution phase, the problem solver uses or transforms the quantitative relationships that are represented both internally and externally to arrive at a solution. The learner in the solution stage converts the numeric association to get the answer, using internal and external forms. Multiple readings of the algebraic text problems are essential for learners to have the comprehension of the problem statement and context (Cook, 2006). An equation is the manipulation that has equal signs, joining two algebraic expressions, which consist of one or two variables. Learners are to create the appropriate mathematical terms of either an equation or inequality, based on an understanding of the relationships stated or implied within the text problem.

Furthermore, Polya (1973) indicates that understanding the problem is the first step a learner must take in the solution of a problem. Understanding the problem requires identification and comprehension of explicit and/or implicit relationships within the word problem. In addition, Ilany and Margolin (2010) explain that the learner gaining understanding of the text problem and circumstances around the ability to construct the form of mathematical formula, relating to the problem. Therefore, the translation is a difficult but crucial task, which includes the learner separating the relevant from the irrelevant information. They mentioned that this includes being

able to bring about the correct identification of the mathematical operations in the text of the algebraic word problems. This is essential to the subsequent creation of the equation(s) to besolved in the complete algebraic word problems process (Ilany & Margolin, 2010). Therefore, errors most times emerge when learners do not adequately comprehend the text problem and cannot identify the connections between texts (Nortvedt, 2011).

2.10   Communication Knowledge

Scholars highlight the role of communication together with numbers and symbols that constitute mathematical concepts, which are necessary for learning to take place (Adams, Campbell & Davis, 2007; Lim & Presmeg, 2011; Chitera, Essien & Setati, 2009). In addition, Khalid and Tengah (2007) explain the role communication plays for teaching and learning to take place: that between teacher and learner, and its effect on the learner in order to have meaning of what is learned. Schleppegrell (2007) reveals that during the process of teaching the learner Mathematics, the language ambiguity creates difficulty in communicating the steps of solving the mathematical word problems. Poor performance in Mathematics among English language learners is an indicator of the difficulties encountered in the teaching and learning situation. Schleppegrell (2007) alludes to the fact that language complexity is a major difficulty experienced by learners while learning mathematical terms, and one of the reasons for low performance in Mathematics is that learners are not English speaking. Barwell (2005, 2006) explains that the lexical ambiguity of algebraic problems results in the learners not being able to make sense of the problems. Leh, Jitendra, Caskio & Griffin (2009) indicate that for learners to solve algebraic word problems successfully, effective communication is required. In order to obtain the needed understanding and effective communication to enable learners to connect different meanings, interpretations and relationships, the text needs to be free of ambiguity.   According to Haghverdi et al (2012),

communication knowledge is required for expressing and connecting the text question to the mathematical symbols, concepts and their organisation. Haghverdi and his fellow researchers disclose that this communication knowledge will assist learners with the appropriate selection of schema from mathematical concepts that will bring the relevant solution. Researchers assert that proper communication during the teaching process enhances the learners' understanding of the problem; they discover ways of bringing out the solution between the condition stated in the text problem and appropriate Mathematics ideas and constructions (Haghverdi et al, 2011).

In addition, researchers refer to communication as being different, but also having similar meaning. Kevin (2002) asserts that undergraduate mathematicians mostly display errors because of inproficiency in communication. While Eibrink (2008) opines that learner errors originate from problems of understanding, transforming and process skills. Scholars refer to communication as a means at which understanding can take place. Also, transformation and process skills are dependent on the ability to encode communication. Error analyses developed by scholars (Newman 1977; Clement & Ellerton, 2011) (mentioned earlier in 2.6 under Learners challenges on p.28) were summarised by Prakitipong & Nakamura (2006) to have these two errors:

1. Errors originating from linguistic fluency and conceptual understanding; errors corresponding with simple reading and understanding the meaning of problems and

2. Errors emerging from Mathematics processing that consists of transformation, process skills and encoding answers.

2.11   Calculation Knowledge

Learners require multiple skills and approaches when solving Mathematics text questions (Moses & Cobb, 2001). One of such skills includes calculation knowledge that forms the basis of

mathematic knowledge ((Haghverdi et al, 2012). They mentioned procedures taken while doing problem solving are also known as process skills Calculation knowledge involves activities of solving mathematics using mathematical operations, practical skills and numerical actions, which constitute a mathematical idea (Haghverdi et al, 2012). Similarly, calculation knowledge is also known as instrumental understanding (Star, 2005). Calculation knowledge is the knowledge of formula, managing the formula with steps and solving the mathematical problem, without understanding how the methods are derived (Star, 2005; Schneider & Stern, 2010). Skemp (1987) and Yang Sherman, & Murdick (2011) point out that systematic errors come from the use of algorithms that lead to incorrect answers or the use of procedures that have not been fully understood. In the process of solving the problem, learners have to select which knowledge to use and which not to use. Many steps are involved when a learner tries to solve a mathematical word problem. The researcher refers to this type of knowledge displayed by learners as method orientated and algorithm problem solving skills, which involve the use of operations.

Operations and steps are vital to solving algebraic word problems. Studies have contributed to the use of addition and subtraction operators to calculate problems in Mathematics (Fuchs, Fuchs, Seethaler, et al., 2009; Griffin & Jitendra, 2009; Jitendra, Griffin, Haria, et al., 2007; Jitendra et al., 2007; Leh & Jitendra, 2013; Owen & Fuchs, 2002; Seo & Bryant, 2012). It is important that learners first master basic skills of addition and subtraction after which they can proceed to the more difficult skills of multiplication and division skills of solving text questions (Griffin and Jitendra, 2009). It is necessary for learners to be equipped with techniques of generalization and the solving steps of addition and subtraction to enable them to transfer to steps of multiplication and division.

Studies have revealed that transformation errors occurred mostly during the computation process, especially when the multiplication operator is being used (Kotsopoulos, 2007; Ibrahim, Maat & Zakaria, 2011). According to Noraishiyah (2002), poorly performing learners mostly display process errors, particularly when solving equations using the quadratic formula. Noraishiyah (2002) found that the problems of these particular learners do not only come from process errors but more so from comprehension errors. Learners commit process skills errors in the operation of addition, subtraction, multiplication and division while solving algebraic problems. Zankara et al (2011) indicate that learners commit process skill errors (also known as calculation errors) when they use positive instead of negative numbers and vice visa. However, in their article "Analysis of Student's Error..." the mentioned errors displayed when factorizing was mostly from transformation and process skill errors. In addition, Zankara and colleagues also show process skills outnumbered other error types by 55 in completing squares and 71 errors in quadratic problems. In support of the studies of Zankara et al (2011), Norasiah (2002) and Intanku (2003) also find that process skills errors occurred the most frequently. Process skills errors are more frequently made than transformation errors, when learners are solving quadratics equations using quadratic formulae.

2.12   No Answer

The learner's lack of strategy or clear method of solving word problems makes him face a challenging situation, which can be described as a point of cross roads or a dilemma situation. The challenging situation is such that the learner does not understand the problem, is not able to select the correct operation, does not have experiences with similar problems; the problem is not related to him- or herself (Krulik & Rudnick 1987; Raduan, 2010). When this happens, learner ventures no answer (no response). It may be a lack of knowledge, meaning there is a cross between

understanding and the answer to be given. Could errors be the reason for learners having no answer for some questions? Could this be one of the procedures in problem solving?

This study is about errors learners are committing. The researcher will discuss how these errors come up in the content of the study. The researcher will expatiate on errors which learners made when solving algebraic word problems that require solving by quadratic equation, linear equation and both linear and quadratic equations. The next area of focus is the discussion on errors found in this study.

2.13   What are Errors?

The researcher gathered from literature that errors originate from misconceptions. Nesher (1987) and Oliver (1989) identify these mistakes as slips, errors or misconceptions. Oliver (1989) defines errors according to the causes. Some are slips, which are wrong answers due to carelessness. Luneta and Makonye (2010) and Riccomini (2008) define errors as inaccuracy and deviations from accuracy. "Errors are seen as part of learning" (Ainley, Patts & Hansen, 2006, p.16). As Oliver (1989) argues, errors are important ways learners form conceptual structure of knowledge. The researcher sees errors as faulty answers to problems. The errors could be as a result of carelessness or poor conception. Oliver (1989) pinpoints errors at other times are symptoms of underlying conceptual structure. Nesher (1987) refers to errors as incorrect premises or misconceptions.

Researchers classify errors in different ways, depending on scholars' understanding of it. Errors could either be systematic or non-systematic errors (Oliver, 1989). Systematic errors are repeatedly wrong solutions to calculations, and occur consistently (Makonye & Luneta, 2010), whereas non-systematic errors are those wrong solutions that can be easily corrected, are non-persistent and happened by chance (Khazanov, 2008). Both expert and novice are committing non-systematic

errors. Systematic errors are wrong answers that come from poor planning. Brodie & Berger (2010) explain systematic errors as wrong answers that learners give recurrently while non-systematic errors are wrong solutions that learners can easily detect and amend themselves while evaluating the solution. Errors are term relational and instrumental, and they are also known as procedural and conceptual errors. Constructivists' studies opine that errors are a result of underlying conceptual structures (Ryan & Williams, 2007; Peng & Luo, 2009). Ryan and Williams (2007) highlighted that systematic errors are a product of learners' conceptual limitations. These aforementioned studies bring about the awareness of those errors made by learners and form the natural means at which knowledge is constructed. This implies that the learners' process of acquiring new knowledge is related to their prior knowledge forming the basis and this will consequently elicit errors within the process. The misconceptions learners have about learning concepts cause errors and the fact that these misconceptions were not revised. They therefore interfere with new concepts of learning in a negative way (Olivier, 1989). An error is the product of learners' poor conceptual understanding. The misconceptions that learners have are systematic errors and linked to both conceptual and cognitive principles.

The learners' possession of transmitted knowledge is a useful tool to solve algebraic word problems because it can rebuild, deduce, improve and relate to the learners' previous understanding.

Errors that learners make when solving algebraic word problems are not only linked to challenges of signifying and converting problem statements. Kieran (1992) identifies the main difficulty learners have when solving algebraic word problems, is associated with the ability of formulating equations. The process of formulating equations in algebra is challenging. In addition, Clement (1982) and Bernado & Okagaki (1994) researched on reverse errors learners make when

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solving algebraic word problems. They explain the reverse error is of two types, namely syntactic and semantic. Clement (1982) illustrates this problem of reverse error as such that "There are six times as many students as professors at this university" (p. 17). In addition, Clement explains with this question, "At the last company cocktail party, for every six people who drank hard liquor, there were eleven people who drank beer" (p. 17). Syntactic errors occur when learners overemphasize the literal translation of words, and semantic errors when learners lack the understanding of how to convert one variable into another (Clement, 1982). Next, the researcher will discuss relevant literature regarding errors, which occur from using equations to solve algebraic word problems.

2.14 Errors emanating from Linear Equations

What is a linear equation? This is the use of the equals sign as an assertion of equality between two expressions (Sherman & Bisanz, 2009). Take the value example: $4a + 3 = 20$. This means to solve any equation correctly, there is a need to understand that the equality sign indicates the expression on the right side is equal to the expression on the left side. Kieran (1992) opines that solving any equation requires the understanding that algebraic equations are objects. It includes understanding the right meaning of the equality sign and the instruction on the use of substitution for verification. Firstly, Filloy & Rojano (1989) denote the expression on the left hand side of the equal sign is the same as the one on the right hand side. Secondly, Knuth, et al (2006) describe the relational knowledge of the equal sign enhances in the learner the skill to create, interpret and use operations to solve equations. It also means any operation performed on equation should be on both sides of the equation. In order to solve the equation correctly, learners have to be equipped with strategies for solving different types of equations, for example, linear, quadratic and cubic equations. In order to effectively solve the equations, there is need to apply the

appropriate algorithms to solve different types of equations. In order to use an appropriate algorithm, a learner must first analyse the structure of the equation. It also means having a better understanding of linear equations and the explanation about the concept expression and all that entails to simplify the linear equation. French (2002) explains, "A proper understanding of algebra is inevitable and very dependent on a corresponding understanding with arithmetic operations" (p. 47). Learners encounter difficulties to progress from concrete to abstract; such that these result in errors. The error of dual nature is common among learners moving from arithmetic to algebraic steps. For example, given the problem 2x +3, the learner gives an answer of 5x, which is a single term. The learner fails to recognize that solving an expression means to apply both a procedure and the operation signs. French (2002) opines that a learner who displays dual error definitely lacks the understanding of the dual character of an expression. Expression comprises of variables, letters, constants, numbers used in various operations $(+, -, +, \div, \times, =$ and others). Derya (2004) emphasizes the importance of linear equation and its application has a wide advantage in the field of physics, biology, economics, finance, engineering and mostly applied in Google and the Global Positioning System (GPS). Although Derya's (2004) study focuses on undergraduates, it states that linear equation poses the most challenges in solving algebraic problems.

Algebra, that comprises arithmetic as its foundational part, is a significant part of Mathematics; it requires establishing the right concepts from the very early grades. Learners with insufficient arithmetic knowledge solve algebraic word problems using arithmetic reasoning. This means that learners, instead of representing algebraic word problems abstractly, prefer to solve them using the concrete arithmetic means. Kieran (1992; 2004) argue that most learners do not always perceive the manipulation of algebraic equations, because they do not properly understand the construction of equations and the connection among the terms of the equations. The challenges

in transforming algebraic word problems into equations make it difficult for learners to solve problems algebraically. In the following, the researcher will elaborate on errors learners commit when solving quadratic equations.

2.15 Errors when solving Quadratic Equations

There are three forms of quadratic relations in the curriculums (NCTM, 2000; CAPS, 2011). These are:

1. Factored form, $y = a(x - r)(x - s)$

2. Standard form, $y = ax2 + bx + c$

3. Vertex form, $y = a(x - h)2 + k$.

Scholars (Healy, Lima & Tall, 2014; Kotsopoulos 2007; Lima, 2008 and Zakaria et al, 2010) have made considerable contributions in understanding the errors learners make in their solving of quadratic equations in algebra. Zakaria et al (2010) indicate that errors, which come from solving quadratic equations by factorization, relate to comprehension, transformation and process skill errors. The researcher observes that learners, when solving quadratic formulae, find it difficult to perform the calculation of the square root sign in quadratic equations. Kotsopoulos (2007) asserts that learners experience an inability to recall multiplication facts when solving quadratic equations in algebra. Lima (2008) and Tall et al (2014) maintain that learners' problems with quadratic equations originate from poor understanding of linear equation concepts. With reference to point 2.14 Error from Linear equations, it points to the fact that linear errors occur as a result of poor understanding of arithmetic knowledge. While arithmetic forms the basis of algebra, scholars share the same view that algebraic errors originate mostly from poor arithmetic understanding. In addition, the study of Haghverdi et al, (2012) reveals that most errors learners commit come from the gap between arithmetic and algebra. Clements & Vaiyavutjamai (2006) state that relational

and instrumental knowledge is required to solve quadratic equations. In their journal "Effects of Classroom Instruction on Students' Understanding", they state that learner errors when solving quadratic equations come from a lack of these two types of knowledge. Haghverdi et al, (2012) allude to the fact that the learner with

the correct approach to solving any equation will analyse the structure of the problem and afterwards apply a suitable formula, which is the right equation. The learner analysis of the structure of the problem is his ability to determine if a linear, quadratic, or cubic equation is the required equation. This means the learner is capable to appropriate the equation with the right formula. Their article, "The Relationship between different Kinds of Students' Errors and Knowledge", further explains that learners find it a challenge to identify the basic surface structure of a quadratic equation as in these equations below:

$2sin^2 - 5sin\theta + 2 = 0$ Is same as $2k^2 - 5k + 2 = 0$

Another error comes from the failure to interpret the structure of the equation properly. Consequently, learners develop errors through the over generalization of concepts from one domain to another. For example, this error occurs when quadratic equations like $x^2 - 5x + 6 = 12$ are solved without a proper understanding of the zero property (Oliver, 1989). This is illustrated in the examples below.

$x^2 - 5x + 6 = 12$

$(x - 3)(x - 2) = 12$

$x - 3 = 12$ Or $x - 2 = 12$

$x = 15$ Or $x = 14$ (p. 6)

This error is difficult to eradicate permanently (Oliver, 1989).

It is supposed to be : $$x^2 - 5x + 6 = 12$$

$$x^2 - 5x + 6 - 12 = 12 - 12$$

$$x^2 - 5x - 6 = 0$$

$(x - 6)(x + 1) = 0$

$x = 6$ Or $x = -1$ (p.6)

In addition, Kotsopoulos (2007) reveals that learners find it difficult to factorize quadratic equations not posed in the standard way, such as $x^2 - 3x - 1 = x + 4$. Next to be examined are errors learners make when solving a system with two equations, that is, one linear and one quadratic equation.

2.16 Errors when solving one quadratic and one linear equation

Simultaneously simplifying a system consisting of a quadratic and a linear equation requires factorizing first the quadratic, which is most often challenging to learners. Kotsopoulos (2007) indicates that simplifying a quadratic and a linear equation goes simultaneously with factorizing the quadratic equation. In his study "It's like hearing a Foreign Language" the factorizing of quadratic equations involves application of basic multiplication tables, which generally is difficult for learners to do. Kotsopoulos (2007) asserts that learners require procedural knowledge and the exclusion of conceptual knowledge to recall basic multiplication tables.

2.17 Knowledge gap

The skill to translate the expressions in the algebraic problem into algebraic terms is crucial for learners solving algebraic word problems. Proficiency in algebraic skills requires learners to be competent in number sense and symbol understanding. Algebra is a tool used to relate with real world situations. Understanding it helps to develop algebraic thinking which also depends on

proficient use of symbol sense. Herbert et al. (1997) mention that the abstract nature of algebra makes it difficult for many learners to accomplish this. It is difficult, as it results in learners using the route method to learn it, such as the trial and error method. Route method means a short course that makes use of formulae (not standardised or applied procedure (steps)) without having a conceptual understanding of it. Therefore, Foster (2007) encourages learners to develop a deeper understanding, which, according to Skemp, is instrumental understanding of the use of symbols. A highlight of his study shows that learners' errors are a result of the poor transition from arithmetic to algebra (Cooper & Williams, 2001). Errors come from areas like equal signs, operational laws, and operation on variables. For example, the area of challenge is the learner solves an expression question by equating it to zero, as if it is an equation. Learners are of the notion that algebraic problems entail much formula and as such this is difficult for learners, because they have a poor arithmetic background (particularly topics like ratio). An example of a ratio problem where learners show poor arithmetic skills is the student and professor problem. The researcher gathered from studies that learners are not able to translate to algebra because they have not understood the elementary part of algebra, that is, arithmetic. This creates a knowledge gap in learning. Since this study is not dependent alone on algebraic problems but also explores algebraic problems in words form, it is therefore necessary to review studies that deal with the influence of language on the knowledge gap (towards the translation from arithmetic to algebra knowledge).
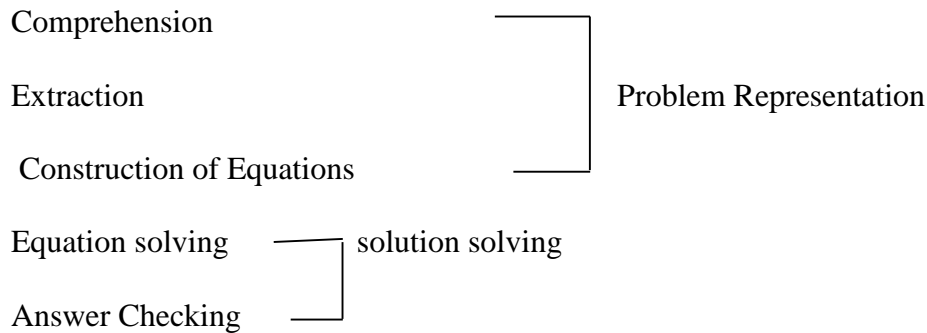
Deatline-Buchman, et al., (2007) pinpoint the need to instruct learners about how to read, understand, plan, solve and check in order to help them successfully solve word problems.

The algebraic word problem translation processes comprise of three steps, namely interpreting, performing and assessing.

The Mayer (1982; 1987) types of knowledge needed for successful word problem solving processes include:

- Linguistic and factual knowledge (about encoding the text);

- Schema knowledge (about relations among problem types);

- Algorithmic knowledge (about procedures); and

- Strategic knowledge (about approach).

Below are the steps Ahmed, Salim & Zainuddin (2008) present to successfully solve word problems.

Comprehension

Extraction   Problem Representation

 Construction of Equations

Equation solving   solution solving

Answer Checking

From Mayer's categories cited above, language is first among the steps of successfully solving algebraic word problems. Ahmed et al (2008) cite two major categories, namely problem representation and solution solving needed for the effective solving of algebraic word problems. Comprehension among the list for problem representation is dependent on reading; this is synonymous with language mentioned in Mayer's categories. Newman (1983) also identified errors: these are categorized as reading, comprehension, strategy know- how, transformation, process and skill. The researcher identifies with the first two, namely reading and comprehension analysis from Newman because these two explain the reason for the knowledge gap between arithmetic and algebraic understanding.

Lester et al (2011) argue that learners' errors are not only a function of variables, but they are also a function of their characteristics. The researcher postulates that learners' characteristics as explained from these aforementioned studies (which refer to learners' language abilities in the studies of Mayer and Newman), are reading and comprehension inclusively. Cercone, Naruedomkul & Supap (2010) promote the real life situation in learners by motivating in them the ability of reading and understanding, such as being able to create the numeric illustration of circumstances described in the word problem. Mathematics questions referring to real life problems are multipart, because such questions are combinations of words, numbers, letters, symbols and, at times, graphics. The language abilities of learners will determine, if they have a knowledge gap or not.

Scholars (Pimm, 1991; Feza, 2011) have pointed out the role word language (text) plays in errors learners commit when solving algebraic word problems. Ladele (2013) denotes that in the analyses of errors Newman (1983) and Polya (1973) are both emphasizing the role of language in similar ways: the first step that Polya states is 'understanding of the problem', while Newman's first step is of two categories, namely: reading and comprehension. Chinen (2008) indicates that Newman's two categories refer to understanding word problems that centre around the importance of language. The successful solving of such problems requires the understanding of Mathematics, which is the language of both words and of Mathematics (Kersaint, Thompson, & Petkova, 2009; Morgan, 2005).

Learners' display of weakness, (also known as incompleteness when handling arithmetic, particularly in areas of multiplication sums) is an indication of their having poor conceptual understanding (Chan, Leu & Chen, 2007). The study of Yershalmy (1997) reveals that learners have difficulties when solving and symbolizing story problems that involve multiple operations.

Learners with these mentioned challenges have gaps between arithmetic and algebraic understanding. The following are examples of common words and phrases that indicate the basic mathematical operations of addition, subtraction, multiplication and division (Bittenger, Charles, Dossey, Keedy, & Smith, 1990). According to the explanation of Bittenger et al, (1990), these mathematical operations are used to represent word phrases like "more than, less than, greater than, added to, subtracted from, the sum of, twice, three times, multiplied by, the product of, divided among, double, half of, fewer than, increased by, difference of, ratio of, quotient of" (p. 30). The simple stated phases in word problems constitute the basic arithmetic operation, which most learners misinterpret when solving word problems.

Chan et al., (2007) denotes that learners make errors when interpreting multiplication operations and experience more difficulty when interpreting multiplication signs together with another operation to form a two-operation scenario. When learners are challenged with this, it indicates they experience a knowledge gap between arithmetic understanding and algebraic understanding. Scholars identify learners who depict poor understanding of concepts like fractions, decimals, negative numbers, equivalence, ratios, percentages and rates, as having a knowledge gap (Chick & Stacey, 2004; Sangwng, 2007). In addition, learners who display problems of over-generalization or under-generalization, reflect deficient understandings and have challenges with the transition from arithmetic to algebraic understandings.

The superficial understanding of arithmetic can also be explained when learners lack in number sense and find it difficult to manipulate operations ($+, -, \times, \div$). French (2002) asserts the role number sense has is inexplicable in areas of fractions decimals, integers and rational number. There is a need to have adequate number sense and good operation skills in order to be a proficient mathematical problem solver. He identifies algebraic reasoning as dependent on proper

development of arithmetic reasoning, while Clement (1987) opines that learners require firstly the proper understanding of one variable equation before proceeding to the concept of two variable equations.

In summary, the importance of the smooth transition of arithmetic to algebra is emphasized in this section because an inability to achieve this transition is termed as a knowledge gap that leads to learners having difficulty and low performance in mathematical problems. Panah (2010) opines the knowledge gap between learners' prior knowledge and new knowledge is the reason for their weakness, difficulties and errors in algebraic word problems.

2.18   Language proficiency

The following are topics of studies that identify the necessity of language comprehension when solving algebraic problems: the distance rate problem, the student-professor problem and speed problem. Barbu and Beal (2010) find from their analyses of learners' difficulties that most difficulties arise from text comprehension.

Studies affirm that the translation of algebraic word questions into algebraic representation requires a combination of difficult concepts (Adu-Gyamfi & Bossé, 2014; Adu-Gyamfi, Bossé & Chandler, 2015). In order to obtain this required representation there is the need to understand the structures and as they interact within the representation (Adu-Gyamfi & Bossé, 2014; Adu-Gyamfi, Bossé & Chandler, 2015). Studies reveal that learners therefore frequently make errors in the process of translating linguistics to algebra, which involves representation (Adu-Gyamfi & Bossé, 2014; Adu-Gyamfi, Bossé & Chandler, 2015). This process requires the use of language. Scholars assert that learners are struggling with algebraic word problems because it uses language to bring out real life situations (Gerofsky, 2010; Sepeng, 2011a, 2011b, Sepeng & Webb, 2012).

Researchers indicate that reading mathematical word problems does not depend alone on the understanding of words in the problem but also involves special reading skills which may not

apply in other learning areas (Barton, Heidema, and Jordan, 2002). According to Barton, et al (2002) the expectation is that Mathematics learners should have the ability to translate and understand the technical and mathematical symbols, operations and graphics, such as the ability to read Mathematics words and interpret data presented in odd ways. Mathematics is about both natural thought and language processes (Barbu, 2010; Schleppegrell, 2004). Roe & Taube (2006) describe Mathematics knowledge using eight competencies, among which is the competency of communication. Roe and Taube (2006) refer to communication proficiency as the ability to read, interpret and understand mathematical words. Barbu (2010) and Fuch, Fuch, Hamlett et al, (2008) suggest that the learning of Mathematics is dependent on reading and comprehension, and it is reasonable to have reading and comprehension clearly integrated into Mathematics teaching, learning and examinations.

Literature documented that reading well is not only an advantage to learning of Mathematics but also for other learning areas (Callahan, & Clark, 1988; Corcoran, & Mamalakis, 2009). Educational research has been undertaken to determine if there is a relationship between the two (that is, reading and learning Mathematics) to establish if learners' reading proficiency can enhance Mathematics performance (Corcoran, & Mamalakis, 2009; Balaas, 2014). Effective reading is a factor that influences achievement in Mathematics (Balaas, 2014). Vilenius Tuchimaa, Anda & Nurmia, (2008) established the existence of the positive correlation between understanding Mathematics and reading. Walker, Zhang & Surber, (2008) find that reading skills improve learners' Mathematics performance, such as making it easier for them to assess higher-level cognitive skills. Abedi & Lord (2001) affirm the relationship between language and achievement in mathematical word problems. In addition, Grimm (2008) revealed the relationship between early reading skills and improvement in learning Mathematics. Early developmental

theory concurs with the relationship between language skills and Mathematics (Carey & Bootstrapping, 2004; Gelman & Butterworth, 2005). Researchers opine the positive relationship helps learners have good language skills and the advantage to develop number concepts (Carey & Bootstrapping, 2004). As a language, it allows an individual to use letters of the alphabet, known also as variables or literal symbols, in general numeric forms across other mathematical domains (Boulet, 2007**).**

Adams & Ely, (2012) reveal that one of the algebraic language expressions is variables, which is letter from alphabet to represent literal symbols, in numeric forms, which can take any value given to it. The value of a variable, which is a major constitute of algebra makes algebra an integral in all topics of Mathematics (Adams & Ely, 2012). Algebra representation is a useful tool in problem solving in advanced Mathematics such as applied Mathematics, engineering and applied sciences (Bezuk, Cathcart, Pothier & Vance, 2006).

Cimmiyotti's (2013) study on the correlation of reading with academic performance in Mathematics informs that although reading skills are needed in lower grades, more of them are needed in the higher grade level of 3, 4 and 5. Further studies suggest that learners are not performing well in their Mathematics tests because they struggle to read and comprehend the test given in words (Crane, Huang, Derby, Makkonen & Goel, 2008). According to Crane and colleagues secondary school learners are expected to have developed the necessary language skills, which primary learners are still struggling to build. The role of language is becoming obvious: that is, to aid comprehension not only in Mathematics as a subject but also in Science. For example, an international study involving seven countries and the US found a correlation between language and science with a correlation coefficient of 0.819 (Cornley, 2009). Precisely other scholars have deeper insights into the correlation between language and ability to solve algebraic word problems

(Vilenius et al, 2008; Grimm, 2008). The former group used 225 Grade 4 learners in their findings and discovered that learners with better reading comprehension skills perform well in word problems. It follows that if such learners are to succeed in algebraic word problems then they have to add technical reading skills to their reading skills. This implies that learners who struggle to decode text will equally perform poorly in problems as algebraic word problems because they require logical reasoning strategies. In addition, Grimm (2008) has established that there is a link between reading skills and other components to form conceptual knowledge. He explains that with Grade 3 learners having higher levels of reading comprehension, they tend to learn faster problem solving and data interpretation of words. The emergence of this study has been encouraged by the effect language has on the comprehension of algebraic problems; particularly discussing South African learners with respect to these discrepancies. Now, the researcher explains that multi-lingual countries' problems (when not incorporated in their educational system) result in inequality of education.

The Education for All (EFA) goals are to ensure access to quality education for all learners in the language they understand through which they can develop the essential foundation in Mathematics, Science and argumentative reading and writing (Heugh, 1999; Heugh & Skutnabb, 2012). However, having equal access and an equally provisioned educational system has been the global concern in sub-Saharan Africa countries and mostly less developed countries. South Africa and the United States indicate typical examples of this problem. The 2005 Setati analysis of DOE documents from the periods 1996, 1997a, 1997b, 2002a, 2002b, and 2005a to 2005b opine that South Africa had several failed attempts to equalize its educational system.

In order for learners to achieve, and understand Mathematical problems in general and algebraic word problems in particular, language proficiency is required. Studies reveal that

language is one of the major factors that determine English learners' understanding and performance in Mathematics in the second or third language (Barbu, 2010; 2004; Barwell, 2005; 2006; Kleemans et al, 2011). Subsequent literature mentioned earlier has shown that Mathematics is not void of language as such that it contains a particular vocabulary, syntax and discourse, which makes it difficult for learners learning English as their second language at all levels of education (Barbu, 2010). Fakeye & Ogunsiji (2009) emphasize the correlation between English language proficiency and achievement in science learning areas, which includes Mathematics. Mathematics learners are frequently faced with the problem of misreading and simplifying: learners perceive that the word problems do not align with the actual meaning of the language of the Mathematics problem (Fakeye & Ogunsiji, 2009).

South Africa is a country with eleven official languages but uses only English as its medium of instruction in its schools. Similar to this education inequality situation in South Africa, the US have EL (those learners whose mother tongue is not English, but who learn English as English learners) and EP (those learners whose mother tongue is primarily English). Studies have established that the EP learners are more proficient in English than the EL learners and the result of this is that they perform better in Mathematics (Beal, Adam & Cohen 2010; Gugliemi, 2008). Similar studies indicate the EL learners demonstrate poorer achievement in Mathematics than the EP learners (August & Shanahen, 2006; Kieffer, Leseiux, Rivera & Francis, 2009). In addition, Beal & Cirett, (2010) highlight the role of language in determining learners' performance in Mathematics. However, a few other countries like Ethiopia practised system-wide education and experience language influence in a different way (Heugh, 2011). Studies reveal that Ethiopia has administered its assessment through several different languages system-wide (Heugh, 2009; Heugh, 2011). In 1994, the Ethiopian Ministry of Education decentralized to 11 administration

regions with 32 Ethiopian languages used for teaching and learning in the primary school. Five of these languages are used annually for regional assessment in Grade 8 and federally every four years (Heugh et al., 2011). These, are some of the influences language has on algebraic understanding.

2.19   Conclusion: In this chapter, the researcher extensively reviewed relevant literature on the study, starting with the most crucial part of the discussion on learners' errors taken from the first diagram of this study. The researcher saw it fit to start with learners' performance, followed by algebraic thinking, then algebraic problem solving and then the importance of algebra and learners' algebraic challenges as they relate to errors learners commit. The researcher discussed relevant literature according to the categories of error analysis by Haghverdi et al (2012) which are lack of the following: linguistic knowledge, comprehension knowledge, communication knowledge and calculation knowledge. The researcher further reviewed more literature, that is additional to errors different from those previously mentioned. The researcher addressed literature relating to the content of written tests which focus on the CAPS curriculum statement for grade 11 topics such as when solving linear equations, quadratic equations and both quadratic and linear equations, also errors that emanate from a knowledge gap and language proficiency.

In the following chapter, the researcher will discuss the suitable method and design for this study.

CHAPTER 3: Research Methodology

3.1   Introduction

This chapter discusses the study's methodology, research design, sampling, procedure, instrument validity and instrument reliability, data analysis and ethical consideration for this study.

3.2   Methodology

The study method unfolds with nine sections. These sections consist of research design, sampling, procedure, instrument, validity, reliability, data analysis, ethical consideration and limitation of study. This study used a mixed method design. Creswell and Plano Clark (2010) define mixed method research as methodology of a single type, which involves the integration of data collection and the analysis of the quantitative (such as experiment, survey) and qualitative (such as structured interview) method. Creswell, 2013; Denzin & Lincoln, (2013) explain the mixed method not as any one of the approaches, but rather as the combination of quantitative and qualitative design to have a more holistic comprehension of research problem.

The strength in using mixed methods in this study stems from Azarin and Cameron (2010), as well as Niglas (2004). Together they denote firstly that this method gives the opportunity of having a deeper understanding of problems; secondly, it helps to validate the findings of one method by using the other method (the qualitative method has to validate the quantitative method); thirdly, it provides a more comprehensive understanding. Fourthly, using both methodologies does not only guarantee the use of diversified method. It also maximizes the micro and macro aspect of the methods in terms of benefits derived from methods' objectivity and subjectivity. Lastly, using mixed method is most suitable in addressing this study's research questions in terms of recognising the types of errors and possible causes of errors. The knowledge of these errors will assist teachers and learners to eliminate errors learners make when solving algebraic word problems. Although

Driscoll et al (2007) affirm that mixed method design can be complex and time consuming to plan and implement, the researcher, however, finds it most suitable for this study.

3.2.1 Mixed method design

Mixed method design is a new field in research that brings together different perspectives in responding to problems (Van Wyk & Taole, 2015). The scholars in this field have suggested four kinds of mixed method design, namely: convergent parallel mixed method design, explanatory sequential mixed method design, exploratory mixed method design and embedded mixed method design (Creswell & Plano-Clark, 2011; Van Wyk & Taole, 2015).

3.3 Research Design

This study uses sequential explanatory mixed method with the quantitative preceding the qualitative research design. This method focuses on collecting data at different stages (Van Wyk & Taole, 2015). Initially the quantitative data assists the researcher in identifying and observing types of errors committed by the learners before gaining insight into these errors qualitatively. This strengthens this study as a form of triangulating the quantitative findings with the qualitative explanation. The qualitative design responded to the second and third research question of the study through interviews conducted with selected learners. Conversely, the quantitative design addressed the first question by assessing and exploring learners' knowledge and using scores to analyse such knowledge in algebra. The quantitative design focused on frequencies of learners' scores and adapted the work of Mamba (2012) and Haghverdi et al., (2012) (which analyses algebraic errors) to analyse the learners' scores for errors. The researcher's background information will also assist in looking at frequencies of skills that influence learning. Then this data will help to inform recommendations for learners' strategies to eliminate the errors they commit. Therefore, the research design will respond to these research questions:

1. What type of errors do Grade 11 learners commit when solving algebraic word problems?

2. What are the possible causes of errors committed by Grade11 learners when they solve algebraic word problems?

   2.1 What are the possible methods learners can use to eliminate errors made solving algebraic word problems?

3.3.1 Qualitative method

Creswell (2008) defined "qualitative research as an inquiry process of understanding a social or human problem, based on building a complex, holistic picture, formed from words reported detailed views of informant and conducted in a natural setting" (p.1). Qualitative research underpins the explanatory and understanding processes in depth rather than focusing on outcomes. The qualitative stage aims to uncover prevalent trends in learners' thinking, while they commit errors when solving algebraic word problems. Buetow, Adair, Coster, Hight Gribben and Mitchell (2002) view the qualitative method as the more valid approach in saving time. It gives a richer study situation. The researcher chose the qualitative approach because this approach provides a better understanding of learners' thinking as they commit errors when solving algebraic word problems. This approach is suitable for the study as it unveils patterns and relationships about ways learners commit errors when solving algebraic word problems. Qualitative data in this study entails the results of the structural interviews of eight selected learners.

3.3.2 Quantitative method

The quantitative method provides numeric data from test scores; that is the Mathematics Strategic Word Problem Test (MSWPT) administered to learners. The researcher was able to link the study statistically with the aid of the quantitative method. It also makes it very possible to have

a clearer picture of how poorly or excellently the learners are performing. The method's structural formal and specific procedure of instrument development and research data collection make it most applicable for this study.

3.3.3   Sampling

Sampling refers to taking a portion number of units from the population as representative of the population. It is a representative of the population, because it has the particular characteristics of that population (Denscombe, 2008). The sample is the representation, while the portion taken is the generalized part. Graziano and Raulin (2000) explain the importance of understanding the concept of representativeness and its relationship to generalisability.

Neuman (2003) affirms that drawing the sample size is dependent on the fact that the larger the population, the smaller the percentage of that sample needed and vice versa. Therefore, this study sample consists of 150 questionnaires answered by Grade 11 learners, and unveils the algebraic word problem errors they commit.

The basic quantitative paradigm focuses on randomization, generalisability, representativeness and both probability and non-probability sampling techniques, while the qualitative paradigm focuses on non-probability sampling techniques (Alston & Bowles, 2003). Qualitative paradigm sampling involves mostly interpretive or constructive paradigm. Creswell (2008) defines qualitative research "as an inquiry process of understanding a social or human problem based on building a complex, holistic picture, and formal from words reporting detailed view or informants and conducted in a natural setting" (p.1).

The researcher adopts both the quantitative and qualitative paradigm sampling. Specifically, in terms of the quantitative paradigm, this study adheres to the purposive sampling technique, because it is the most suitable. Rubin and Babbie (2005) describe purposive sampling

as judgmental sampling. The sampling method is, according to the researcher's judgement in terms of the sample size, the components, characteristics, that which best suits the purpose of the study.

3.3.3.1 Participants

The participants of this study consist of 150 Grade 11 learners in semi-urban schools from three public secondary schools located in Tshwane district of Gauteng Province, South Africa. The participating learners are 60 boys and 90 girls between 15–18 years of age and predominantly Xhosa and IsiZulu speaking, precisely 70% and 30% respectively. The learners here hail from semi-urban schools located in Tshwane South, District of Gauteng. It is important to state that these schools are located in areas with informal settlements. Hence, students hail from a poor economic background.

3.4   Pilot Study

3.4.1   Reliability

Reliability is a means of checking if a test is a good test. A test is reliable if one gets consistent test results even though other researchers administer it. Adebule & Ayodele (2007) quote Payne (1997) as saying, "Reliability is the extent or degree to which the test is likely to produce a consistent score." There are basically, three types of reliability, namely: stability, equivalence and internal reliability.

This study verifies its test using stable (otherwise known as test re-test) reliability. Stability reliability gives the same measure of test to the same group of learners at two different points in time.

Seven Mathematics high school teachers, including the district subject adviser and the researcher's supervisor, scrutinized the test. Other modifications were made to have test items written within the Grade 11 curriculum for Mathematics and distributed according to Bloom's taxonomy level.

The researcher selected 30 learners to write the test. The researcher ensured that test items appropriately reflected relevant topics in the school curriculum statement (CS: educational goals) (see test in appendix).

3.4.2 Validity of the instrument

Validity is an important measure of a good test. It refers to how well the instrument claims to measure what it truly measures (Cohen & Manion, 1994). Rubio (2005) asserts that the validity of any test is determined by the extent it accurately assesses what its purpose is to measure. Marguerite et al. (2006) affirm that validity encompasses three types of validation, namely: construct, content and criterion. This study chose to verify content validity, because this requires an in-depth understanding of errors which only can be revealed from tests or assessments given to learners.

To ensure the validity of the study, the researcher conducted a pilot test. This helped the researcher to have the main study appropriate, unambiguous and effective for the purpose of the study. To accomplish this, seven teachers with 5 to 13 years' teaching experience (Mathematics in high school), including a subject adviser, helped the researcher to ascertain the content validity of the test. The researcher's supervisor also put in an expert assessment on the study's test question to establish test content validity. The test thus reflected to a considerable degree the objectives stipulated by the South African DBE policy for Grade 11 learners stated in the CAPS document (CAPS, 2011).

Research Procedure: It involves these two phases, namely; the pilot and the main study.

Bless, Higson-Smith and Kagee (2006) comprehensively define pilot study "as a small study conducted prior to a larger piece of research which is used to determine whether the methodology, sampling, instrument and analysis are adequate and appropriate" (p.184). Barker (2003) highlights

the benefit of a pilot study by his definition that pilot is a procedure for testing and validating an instrument by administering it to a small group of participants from the intended test population. In this case, participants in the pilot study should not be on the list for the main study (Rubin & Babbie, 2005).

This means the research tool or pilot test contains 12 algebraic word problems, which consist of equation questions that are linear, quadratic, one linear and one quadratic question and real life problems. Questions 1, 3 and 7 require linear solutions, while Questions 2, 5, 8 and 9 involve quadratic algorithms, and Question 4 constitutes one linear and one quadratic equation. In addition, Questions 10, 11 and 12 are real life problems. There were 15 questions.

The researcher selected 30 learners to write the pilot test and ensured that test items appropriately reflected relevant topics in the school curriculum statement (CS: educational goals). In addition, the pilot study helped the researcher to embark on processes of re-correction, which result in a better main study (Rubin & Babbie, 2005).

		POST-TEST	PRE-TEST
POST-TEST	Pearson Correlation	1	.827[**]
	Sig. (2-tailed)		.000
	N	30	30
PRE-TEST	Pearson Correlation	.827[**]	1
	Sig. (2-tailed)	.000	
	N	30	30

**. Correlation is significant at the 0.01 level (2-tailed).

Table 2: Correlations

A pilot test was giving to ensure the validity of the instrument (MSWPT). The researcher correlated the first and second pilot tests to have a coefficient of 0.8, which has strong test and re-test reliability. The researcher used the Pearson correlation method to analyse the test and re-test to have correlation significance at 0.01 levels as indicated in the table results.

The table above shows a Pearson correlation of r=0.827; p=0.000<α=0.01. Therefore, there is a significant relationship between pre-test and post-test. This implies that the test-instrument is reliable.

3.5   Main Study

The researcher obtained approval from the Tshwane District and ethical clearance approval from the University of South Africa. Initially, the researcher intended to carry out the study's research in four schools, but later conducted research in three schools. Testing the last school at the given time would have resulted in unfair treatment of the other learners who were tested previously in terms of teaching received. The researcher refers to unfair treatment in terms of the time between the third school test administration and that of the last (4th) school would have had a wide gap of many months. This would have resulted in the fourth school having had more of an advantage than the preceding three schools, because they would then have covered and mastered most of the topics for the grade curriculum. This is the reason the researcher reframed the study to use three schools for the investigation. The schools chosen were through convenient sampling: consideration was given to the schools' accessibility to the researcher, while learners selected were through purposive sampling because they are predominately 70% Xhosa and 30% IsiZulu speakers. Most learners hail from informal settlements and are consequently from a poor economic background. Referring to content, the test instrument initially had 15 items, but later it was reduced to 12. Questions 2 and 3 were similar questions testing the same thing in the content. Therefore, the researcher chose Question 2 and eliminated Question 3. In addition, the researcher removed Questions 8 and 13, because they did not fit into Bloom's taxonomy categories and were above the standard required for Grade 11. The time initially allocated for the test was two hours, but was

changed to one hour. These were the adjustments made on the instrument, after the pilot study revealed the shortcomings.

3.5.1 Instrument

This study selects 3 schools taken from Tshwane South District of Gauteng, which is a semi-urban area. The researcher noticed that learners in these schools have challenge with poor performance in Mathematics. In addition, Servaas van der Berg (2012) indicates that socio-economic status of African learners in South Africa influences their Mathematics performance. Spaull (2012) confirms that as much as teacher content knowledge plays a role in learners' performance, economic status plays a more significant role. Therefore, to ascertain the role played by teacher's content knowledge or the socio-economic status of learners in South Africa in their Mathematics performance, a test had to be given to the learners.

The test used is referred to as MSWPT. The researcher developed it in 2013. It consists of 12 word problems, designed according to the DBE CAPS specifications for Grade 11 in South Africa and distributed in five levels of Blooms taxonomy: 1) knowledge 2) comprehension 3) application 4) analysis and 5) synthesis. The test has Question 1 and Question 7 in the knowledge level and Question 9 and Question 11 in the comprehension level. The analysis level has

Question 2 and the synthesis level has Question 12. Most questions (Questions 3-6, Question 8 and 11) are application level.

Knowledge	Comprehension	Analysis	Synthesis	Application
Q1	Q9	Q2	Q12	**Q3**
Q7	Q10			Q4

				Q5
				Q6
				Q8
				Q11

Table 3 : Research Question distribution according to Bloom's taxonomy

Why are, the researcher test questions developed mostly from the application level? In this study, the analysis level and synthesis level became unimportant due to the level of the learners that will be intensively in the findings. According to CAPS, the major highlight goal of Mathematics education is for learners to solve problems in a creative manner. The DBE(2012) emphasized that not only should algebraic word problems aim at helping learners to acquire skills in solving problems, but learners should also be able to apply such skills to solve real life problems. Considering the highlights of CAPS, it is imperative to have most questions in the application level. The test questions are such that require learners to depict higher order thinking which involves more than one algorithm. For example, Question 5 (Appendix A) requires learners to substitute the linear with a quadratic equation, which is simplifying and then factorizing the quadratic. These procedures expose learners to making errors because it entails two or more algorithms. In this study, the findings demonstrate intensively that the analysis level and synthesis level became unimportant due to the competency? level of the learners. Knowledge level questions only require the learner to recall formulae. For example, Question 1 requires the learner to recall the meaning and the symbol for the phrase "consecutive odd" and solve for the unknown. The comprehension level questions require from the learner to have the correct understanding and to

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convert the word problem into numeric form. In Question 10, the age of the father and the son is one quarter of the father. There is a serious need for word understanding ability and to translate it into algebraic numeric form. The analysis level questions require of the learner the ability to break down the question into components or parts. Question 2 is an example of an analysis question involving factorization. Synthesis level questions require learner ability to build up parts of information into one new whole such as in Question 12.

While measuring the outputs that are learners' responses, this instrument allows the researcher to identify the nature of inputs learners receive. Such inputs will be teacher instructional strategies and knowledge provided.

3.5.2   Data collection

The researcher categorised the learners into three groups: those who scored between 0 to 29% in the low group, those between 30 to 39% in the average group and 40% and above in the high group (see figure 1 in page 76). The researcher selected eight learners from the average and high groups for the interviews.

The researcher collected two types of data: (i) test (ii) interview. Firstly, the researcher administered the MSWPT to 150 learners, that is, 50 learners from each of the three schools in 2013. The test constitutes the content of CAPS for Grade 11 Mathematics (see Appendix, topic 2, points 2 & 4). **Flanagan, Mascolo and Hardy-Braz (2009)**, assert that tasks assist researchers to detect the strengths and weaknesses of learners; without it, it will be difficult to find errors learners make. The researcher marked the scripts and tests were analysed for errors. Secondly, the researcher tried to understand learners' thinking for errors make by conducting interviews with 8 learners selected according to the errors they made. The researcher selected eight learners in ratio 2:3:3 from the low, average and high scores respectively of learners' test marks.

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The researcher arrived at the learner categories from the learners' scores. Out of a total raw score of 50 marks, those learners who had a 18-25 raw score were classified as high score, those with a 10-17 raw score as average score and those with a 0-9 raw score as low score (see Table 4).

3.5.3 Interview

The researcher used Questions 1, 3 and 11(see App. A and J) and interviewed the learners following the Newman (1977) interview format below. (See also Appendix E) The interview used consists of three questions from the test learners have written. Question 1 addresses errors in the domain of knowledge, Question 3 helps to determine errors in the area of comprehension and Question 11 reveals errors made in the application area. The Newman interview format instructions given to the learners were as follows:

- Please read these questions to me. (Can the learners read the problems);

- Tell me: what is the question asking you to do? (Can the learners comprehend or understand the question, by writing an equation);

- Solve the question. (Can the learners translate the word problem into mathematical formula or can the learners transform the question into procedure and solve it?)

- Apply the steps of process chosen to solve the question. (Can the learners use the method or formula or process skills well?)

- Encode your answer to this question.

- Check your solution in order to be sure your answer is correct (verification).

  The interviews lasted 30-45 minutes per learner and were administered to the learners in a classroom set aside after school hours. Learners were enthusiastic about the interview.

3.6   Data analysis

This study makes use of both quantitative and qualitative analyses of data. This analysis

aims to answer research questions stated above (research design) by explaining the data.

3.6.1   Quantitative analysis

The quantitative analysis adopted the error analysis of Haghverdi et al. (2011) which is as follows:

- Linguistic knowledge;

- Comprehension knowledge (a) semantic (b) structural and (c) intuitional;

- Communicational knowledge; and

- Calculation knowledge.

Linguistic knowledge: the problem solvers use this knowledge to read the text in the word

problem. The lack of linguistic knowledge at the beginning of the problem solving process

disenable the learners' progress to solve the problem. The learners' mathematical learning and

linguistic learning are two sides of the same coin (Barwell, 2005). This determines whether the

learners have the basis, which Bloom taxonomy explains as knowledge level. This implies that

any learners who have the basic ability to read, will demonstrate the knowledge level (to recall

formula).

Comprehension knowledge means the knowledge learners acquire when they read to understand

algebraic word problems.   Comprehension is the process of reading and understanding a word

problem. Cercone, Naruedomal & Supap, (2011) explain comprehension as identifying the

problem situation, the characteristics of the problem and the problem type. Comprehension

knowledge enhances the ability to construct the equation for the word problem. This

comprehension knowledge consists of three knowledge types, namely semantic knowledge, structural knowledge and intuition knowledge.

Semantic knowledge. This knowledge enables problem solvers to understand word problems, such as getting the meaning of what the text question requires. The presence of semantic knowledge allows for data and mathematical expressions not to be seen literally anymore. Semantic knowledge helps learners to form meanings of the word problem. Supap et al (2011) describe semantic knowledge as the ability to have the meaning of the word and the meaning of the word order in a sentence; such as to be able to interpret these words as mathematical concepts. Semantic knowledge enhances learners' understanding of what the problem requires and the ability to interpret the problem correctly. For example, learner A in Question 11 could not interpret the problems correctly. Question 11 stated: A rectangular parking area has a dimension of $50\ m$ by 120m. If the parking area is doubled….' Lack of semantic knowledge let the learner misinterpret the wording and have an incoherent problem representation of the word problem (Cummins, 1988; 2006). For example, learner A here doubled the length and doubled the width; he had a misconception of the question. The learner had no understanding of how to apply the method to solve the problem. Therefore, he lacked semantic knowledge of the question.

 Intuition knowledge refers to the knowledge such as the learner's formal and informal education, past knowledge, objective experiences, and environment, as well as the learner's capabilities. This knowledge also deals with the significance of problem-related data and information (Burton & Jarrette, 1999). The learner, after reading and solving the word problem, uses intuition knowledge (also called common sense) to examine the answer's correctness or incorrectness. Polya (1973) refers to this knowledge as looking back. Take the example of Question 10. The question informs that the age of a learner is half times less than that of the elder brother. Intuition knowledge helps

the learner to know he is wrong when calculating the answer and finding it is different from the expected or realistic answer given the information.

Structural knowledge refers to schemas of mathematical concepts existing as internal representations stored in the memory. Schemas are data structures for representing the generic concepts stored in the memory (Rumelhart & Norman, 1985). Similarly, Fischbein (1999) opines that schemes provide the strategy for solving problems, such as schemes for solving quadratic equations. A case in point is the scheme used for completing the square is different from the one required for solving using a quadratic formula. The schemata are knowledge structures, which help learners to classify problems and enable them to find the appropriate solution. Therefore, schemata (also the structural knowledge of mathematical concepts) are given to learners, or constructed by learners themselves. Schemata or structural knowledge helps learners to select a proper method or pattern for their solutions when solving word problems. Nesher and Herskovits (2003) find in their research that schemata are significantly more ordered and more available. In addition, the meaning of structures is more easily accessible by expert solvers than by slow solvers when solving word problems. The majority of the learners had difficulties in Question 1 to figure out the algebraic equations to be constructed, such as to identify the variable, operation and equations to use for the phase "consecutive of odd numbers". Learners were found to lack structural knowledge.

Communicational knowledge is a kind of knowledge, which links the words in the problem representation to mathematical concepts and structures. Learners encounter challenges to link expressions in the word problem with the mathematical symbols and concepts. In Question 11, which requires translating the word into an algebraic equation, learner C, for example, could only write the double of the area is 6000, but could not continue to write the required algebraic equation

which then would have to be factorised to get the value of the increase in the length and breadth of the area. Learner C lacks communication knowledge to understand this question.

 Calculation knowledge is mathematical knowledge, which relates to computation, operation and solving algebraic word problems. Hiebert & Carpenter (1992) explain calculation knowledge as "any individual task that can be performed correctly without understanding" (p.89). Many of the learners are doing or solving the mathematics questions given in the research test correctly, but do not really understand what they are doing. Take learner G, for example. In Question 1 he arrives at answers 25 and 27, the consecutive numbers, without actually knowing the meaning of 'consecutive odd numbers' and how to construct these words into algebraic terms or equations.

However, Majid and his team assert that learners' lack of linguistic knowledge results in poor semantics knowledge. Similarly, Greeno (1985) emphasizes that lack of learners' linguistic knowledge leads to poor performance in word problems. They also highlighted errors learners make when solving word problems as a result of a lack of linguistic and comprehension knowledge, in particular the comprehension, comprising semantic and structural knowledge.

In this study, the researcher used 150 learners. Among them are 90 learners who scored zero; the remaining 60 learners are analysed for errors. If 150 learners are 100%, then 60 learners will be 40% who write the MSWPT set according to the CAPS requirements for Grade 11(Appendix I, Topic 2., Point 2 & 4: quadratic equations by factorization and quadratic formula; equations in two unknowns, one linear and other quadratic).   The tests written are analysed for error types. Following the analyses of algebraic errors by Mamba (2012) and Haghverdi et al., (2011), the error types found are a lack of the following knowledge: linguistic and comprehension (consisting of the following knowledge types: semantic and structural, intuitional, communicational and calculation).

3.6.2   Qualitative analysis: In the following section, eight learners' interviews were analysed, adopting the Newman six errors categories namely: reading skills, comprehension, transformation, process skills, encoding and verification.

Setati and Adler (2001) supported that poor language is one of the main reasons for errors when learners solve word problems. In order to understand the reasons for the errors committed by learners and thereafter understand the things which hinder learners from making progress when solving algebraic word problems, the qualitative analysis helps to answer the research question 2 and 3 qualitatively.

Having followed Newman's questioning, the researcher was able to identify the domain errors (primary domains) and find their relationship to the research question, which enabled the answering of the research question. Moreover, the researcher listed into columns the errors or disability of knowledge discovered while interviewing selected learners.

3.6.3 Ethical considerations for the study

The researcher ensured that this study followed these steps to achieve the ethical principles: voluntary participation, informed and understood consent, and ensuring confidentiality in dealing with the research participants.

Participants had the choice either to take part or not to take part in this research. The informed consent ensures that learners who voluntarily participate are adequately informed about the aims, methods and purpose of the research, and whether it has any effect on participants (whether physical or psychological).

- This study aims to preserve anonymity in that under no condition will learners' names or school names be disclosed for record in the research analysis.

- The researcher obtained ethical clearance from the UNISA committee clearance board.

- The researcher also obtained a letter of consent from Tshwane South District, where the three schools studied are situated.

The researcher received consent and permission from school principals, the schools' Heads of Department of Mathematics and teachers.

.3.7   Conclusion

This chapter described the study methodology, the research design, sampling and procedure: (i) pilot study (ii) main study. It described the instrument having (i) validity, (ii) reliability. The data analysis consists of (i) quantitative analysis (ii) qualitative analysis and the ethical consideration of the study. The instrument of study was developed aiming at the curriculum statement specified for grade 11(see appendix I, Topic 2 & 4) of solving algebraic word problems. In addition, the quantitative analysis answers the research question 1, while the qualitative analysis answers research questions 2 & 3. The next chapter is the study's discussion on findings.

CHAPTER 4 FINDINGS

This chapter presents the results and analysis responding to the research study's questions in chapter 1 (see chapter1.3). The aim of the study is to investigate errors made by grade 11 learners and determine learners' understanding of the errors they made when solving these three areas of algebraic word problems in the Grade 11 CAPS curriculum (see Appendix I, Topic 2, point 2 & 4). The researcher brings results from the MSWPT and the interviews with learners. The findings report on errors that came out of the analysis of the remaining 40% learners' test scores are shown in Figure 1 below. The researcher's presentation of findings is referenced from (i) learners' scores in figure 1 (ii) Bloom's taxonomy (1981) (iii) Haghverdi et al (2012) and Newman (1977). The researcher presented the result and analysis in two sections: firstly, the results from the marked 40% learners' scripts and secondly, the results from eight learners interviewed by the researcher using the Newman (1977) error analysis model.

The first section contains the results of learners' scores in frequencies as presented in figure (i). In the following section, the researcher presents the detailed results of the learners' scores sorted into various schools in Table 4. This gives a better picture of learners' performance. The researcher arrived at the results in Table 5 by marking and analysing the 40% learners' scripts for errors according to the error analysis of Haghverdi et al (2012). The researcher used Bloom's concepts of standardising tests (particularly algebraic tests) in levels of (a) knowledge (b) comprehension and (c) application and analysed the test questions in Bloom's knowledge levels for errors. In Table 6 the researcher had these results in the learners' scripts sorted into the various schools they belong to. The researcher analysed these learners' results using the error analysis of Haghverdi et al (2012). In Table 7, the researcher analysed these learners' results with reference to Bloom's levels of standardising questions for errors, and expressed them in percentages.

Secondly, the researcher, wanting to understand the reasons for errors made by learners, analysed the eight selected learners' interviewed results. Looking thoroughly at their results, the researcher arrived at the themes found.

4.1   Learners' scores

This presentation is supposed to focus on errors. However, the thorough analyses of learners' scores depict more than just errors, especially when one looks at the number of learners with the lowest scores in the test (see figure 1 below). Therefore, these results present errors and the different Mathematics levels the participating learners operate in. The high frequency of learners with low test percentages depict there were many errors made by learners.
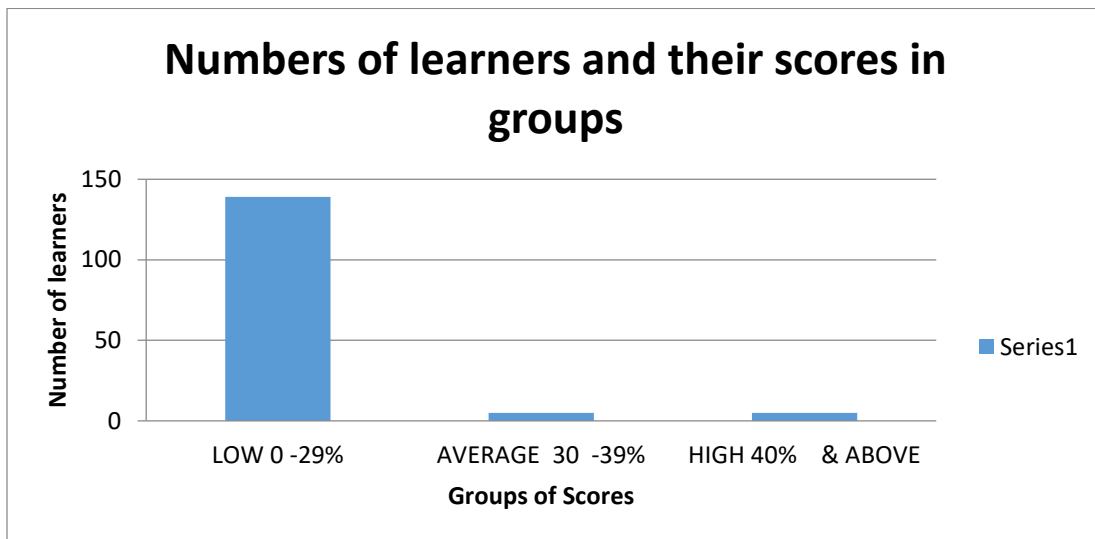
Figure 1: Learners' Scores in Percentages and their groupings (Low, Average & High)

Figure 1 shows the number of low performing learners is 30 times more than that of the average and high performing learners. This implies that the prevalent scores of learners are within the 0 to 29% range, but few learners achieve the class average of 30 to 39% or the high group of 40% and above. The frequency for the low performing group is 140, while for both the average performing group and the high performing group the frequency is 5. However, this is the highest score and these are the most marks achieved. Those five learners who are in the high group scored 40%,

40%, 44%, 46% and 50%. This range shows clearly that this high is average. In fact, it is an indication that these learners could not perform on higher levels in the test.

The researcher presented the findings following the following categories: learners' scores in percentages; errors made by learners; and percentages of errors found in learners in correlation with the three domains of questions according to Bloom's taxonomy (1981) which focuses on knowledge, comprehension and application questions.

4.2 Detailed findings

Low Score0 - 29%	RAW SCORE	SCH A	SCH B	SCH C	Total
	0 - 14	48	42	50	140
Average 30- 39%	15 – 19	1	4	NONE	5
HIGH 40% &>	20 AND ABOVE	NONE	5	NONE	5

Table 4: School Group and their Score type

Table 4 presents a deeper understanding of how learners' frequency scores are distributed. The researcher presents them according to each of the three schools investigated. The results in the above table clearly depict that the number of learners who achieved average and high group results are quite few, as compared to those in the low group. School C learners only fall in one category, which is the low scoring one, whereas school A has only one learner in the average group and none in the high scoring one. School B is almost even on the average and high scoring learners. Table 4 shows school B has learners represented in the average scores and high scores group, but the other schools have all learners represented only in the lower scores group. Therefore, school B

performed better than the other two schools. School B indicates some potential for improvement, while the other two schools are more challenging.

The CAPS (DBE, CAPS, 2011) stipulates that 40% is specified as the pass mark. In this study, only 5 out of 150 of the learners were able to attain this pass mark.

Table 2 presents the results following being grouped in accordance with Bloom's taxonomy levels of questions for algebra tests. This study concentrates on three aspects of distribution, namely: knowledge, comprehension and application levels. This study referred to Bloom's levels of standard questions for algebraic tests, because it has similarities with the study's adopted analyses of errors by Haghverdi et al(2011); both are pinpointing knowledge levels among learners. In addition, among the application questions the researcher focused on questions 3 and 11, while knowledge questions centred on questions 1 and 7 and the comprehension questions on questions 9 and 10. Therefore, in Table 5, the researcher presents the number of errors in these three groups of questions when analysing the 40% learners who attempted and had scores for the MSWPT.

Types of questions	Linguistic	Comprehension	Semantic	Calculation	No Error
Knowledge	79	1	1	None	None
Comprehension	26	6	4	3	None
Application	35	6	14	2	1

Table 5: Types of Questions and Errors found

Table 5 depicts learners' linguistic deficiency in all the questions types. The highest influence of the language deficiency is observable in the knowledge domain. This implies learners who experience a language deficiency will be quick to display huge knowledge inefficiency. Therefore, such learners cannot make any progress or begin with problem solving. Hence, there is a negligible number of comprehension, semantic and other error types. This means a learner who can neither read nor understand what the problem entails, will not be able to derive an equation. This means,

if a learner displays such poor language competency in knowledge questions, then that learner cannot achieve in either comprehension or semantic questions. The presence of a considerable number of linguistic errors in the knowledge basis will be the same for the comprehension stage that follows and then for the application question; they follow in the order of understanding. Therefore, comparing the record 26 linguistic errors in the comprehension questions to the 79 linguistic errors in the knowledge questions, it is evident that there are few occurrences of comprehension, semantic and calculation errors in the comprehension questions. Consequently, the rate of language error occurrences has a major influence on other errors. The above findings also concur with the statement in the previous chapter that language efficiency is the backbone determinant to all other knowledge types. Table 6 below gives a better understanding and identification of types of errors reflected by learners in the three schools researched:

Type of Error	Linguistic Error	Semantic Error	Comprehension Error	Calculation Error	No answer
School A	82	13	2	15	6
School B	126	19	30	9	14
School C	100	16	18	-	11
Total	308	48	50	24	31
Percentage (%)	67	10.4	10.8	5	6.7

Table 6: Percentages of *the* Learners' Errors .

The researcher followed the error analysis of the Haghverdi et al (2012) approach to focus on this kind of knowledge (linguistic, semantic, comprehension and calculation). Another error that emerges is the no error type. Table 6 shows that the lack of linguistic knowledge has the highest average of 67% followed by semantic knowledge with an average of 10.4%, while comprehensive knowledge has an average of 10.8%. No answer has 6.7% and lastly, calculation has a 5% average. The researcher discovered that linguistic knowledge has six times as many errors as the others. The researcher also noticed that both lack of comprehension and semantic knowledge have almost the same average percentage: 10.4% and 10.8% respectively. The reason could be that learners who have the ability to read and understand what the problem needs, have progressed a step in the knowledge level. Therefore, the learners lacked comprehension or semantic knowledge while solving word problems.

While learners are getting word problems solved at this level of knowledge, the challenge learners have could be comprehension or semantic knowledge, since these knowledge levels are closely related. The inability to understand the word problem in order to derive an equation, or to represent the word problem in algebraic terms, is close to also choosing the wrong equation. Also from the table, it is explicit that the number of learners lacking linguistic proficiency is 10 times the number of learners with no answer. Linguistic errors make up 67% of the total, where no answer is 6.7%. Therefore, according to these results, the learners with no answer is an indication of an intersection of insufficient knowledge and language deficiency.

Next, the researcher took the study's test results (that is 40% learners), sorted them into the participating schools and then analysed the errors found into percentages according to Bloom's questions knowledge levels. See Table 7 below.

Schools	Knowledge	Comprehension	Application

A	60%	56%	28%
B	100%	100%	0%
C	83%	100%	25%

Table7: Learners' errors categorized according to Bloom's taxonomy

The above table gives a better understanding of the effect of language on other error occurrences. School B had the most language errors as they had the highest percentage of knowledge errors and a maximum number of errors in the comprehension questions. This school had no errors in the application questions, because there were no errors here. School A which had 60% of errors in knowledge had almost equal the number of errors in the comprehension question, that is, 56% fewer errors in the application question, namely 28%.

Although this study has its data analysed with Bloom's guidelines, it, however, adopted the above knowledge categories of knowledge, with comprehension subdividing to semantic and structural knowledge and then application. Many learners demonstrated unfitness because most scored 0 and as such it shows they lacked linguistic ability (it was a reading problem) and could not demonstrate the first level of knowledge, neither could they access comprehension or application knowledge. The 90 learners with a 0 score showed they lack linguistic knowledge. The researcher analysed the remaining 60 learners (40%) into these categories of errors such as comprehension: semantic or structural and no error. The learners' lack of linguistic proficiency influences their ability to proceed or continue with the problem. Their poor reading skills are the reason for their weak comprehension; this deters them from performing well in algebraic word problems. The English linguistic proficiency determines if learners either increased or decreased in their performance when solving algebraic word problems (Neville-Barton & Barton, 2005, Feza, 2011**,** Henry et al.,

2014, Anthony and Setati, 2007). Generally, this means the learners' ability to read problems influences their performance in the basic algebraic knowledge needed. Out of the three schools investigated in this study, only five learners demonstrated English language proficiency such as to have the average marks they had attained.

These results record 140 learners in the low score group It is a very high number and relates to an English language deficiency among the learners. The five learners who were able to achieve average marks do not have linguistic English deficiency because they can read to identify specific terms and algebraic symbols in the problem. The fact is that linguistic error influences other errors. Therefore, the researcher, desiring to understand the reasons for learners' errors, thoroughly looked at the eight learners' interview comments and calculations and had learners display these categories of themes. Some of the learners could not progress towards the solution of a word problem because of language deficiency. For some other learners it was their poor arithmetic bases which made them unable to translate into the algebraic understanding of the word problem. The other learners were able to break through after the researcher unpacked some information in the test questions. Below is the thematic report that discusses the exploratory section of the study. Four themes that emerged from the analysis are as follows:
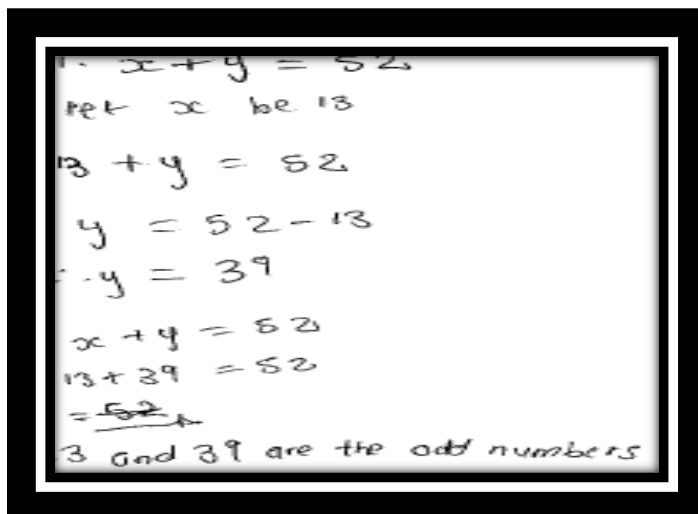
- Language deficiency with subthemes: English language deficiency and algebraic language inadequacy;

- the unpacking of meaning;

- errors; and

- the lack of transition from arithmetic knowledge to algebraic knowledge.

1(a)   Language deficiency

The results below are slightly complex, because some learners belong to one sub-theme and some belong to both. Therefore, our presented episodes will show this complexity. Learners A and B in questions 1 and 3, and also learners D and H in question 3 demonstrate language deficiency, which has to do with the difficulty of reading and understanding words in English. Conversely, learners C, E and H showed algebraic language inadequacy. Below are learner episodes presented firstly by introducing the question.

Question1: The sum of two consecutive odd numbers is 52. What are the two odd numbers?

Learner A written response [Appendix, 2013]

Learner A interview response

Researcher: Please read question 1.

Learner: The sum of two consecutive odd numbers is 52. What are the two odd numbers (he struggled to read the word 'consecutive'

Researcher: Do you mean 'consecutive'?

Learner: Yes, consecutive.

Researcher: What is the question asking you to do?

Learner: The question wants me to get two numbers that sum up to 52. (Then she wrote

   13 + y=52)

Researcher: How do you think that you can represent the two odd numbers in symbols?
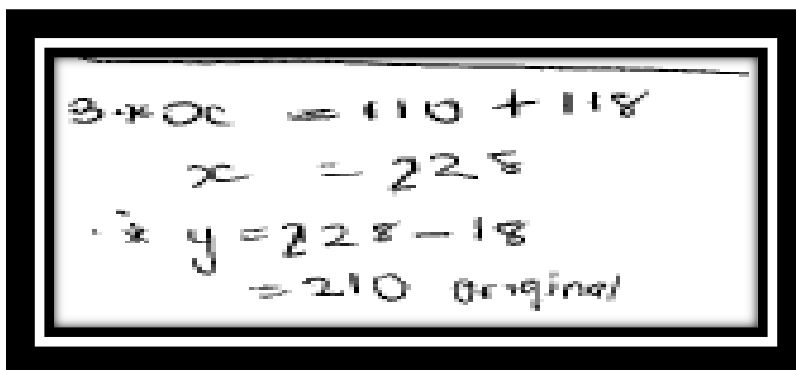
   Take for example number 1 as the first odd number and the next and next will Odd

   number and for example take it to be 3. How do you represent these with symbols?

Learner: If I add 13 +29 =52 that is why I add 13 to *y* and will get 52. I guess.

Learner A could neither read nor understand the word 'consecutive.' He tried to solve the question

by using any two odd numbers adding up to 52. The learner thought 13 plus 39 would be 52,

without considering the meaning of the word 'consecutive odd numbers.' Therefore, the learner

was struggling with the word 'consecutive,' which became a barrier to him. Since the learner did

not understand the meaning of the word 'consecutive', he then ignored it and presumed to solve

the question in his own way. More challenges are shown by this learner in question 3 below.

Question 3: The sum of two digit numbers and the number formed by interchanging the digit is

110. The new number 18 is more than the original number. Find these numbers.

Learner A written response [Appendix, 2013]

Learner A interview response

Researcher: Read question 3.

Learner: The sum of a two-digit number and the number formed by interchanging the digits, is 110. The new number is 18 more than the original number. Find these numbers.

Researcher: What does the question ask you to do?

Learner: It wants me to add numbers 110 and 18. So I wrote line 1.

Researcher: Two-digit numbers refer to TU (tens and units), for example, 24 and the interchange 42.To represent these algebraically will be *xy* and then the interchange is *yx*.

Learner: Writes line 2 and the next and last line.

Here learner A shows he did not understand specific terms stated in the word problem to enable him to derive the required equation. He lacked comprehension knowledge as clearly shown in line 1. Although the learner derives equations, they are meaningless. Where did the learner get 13 +x = 110 +118? He also wrote the original number as 210, that is calculated from 228-18. He became confused. He could not perform the algebraic calculation because of this language barrier. Evidently, the learner suffered from a huge gap between arithmetic and algebra. He is operating at a very low level of arithmetic. This shows that the learner's arithmetic ability is very low, whereas he is expected to comprehend algebraic problems. Here, the learner has a challenge with both his English language deficiency and algebraic language inadequacy.

Learner D written response to question 1 [Appendix, 2013]
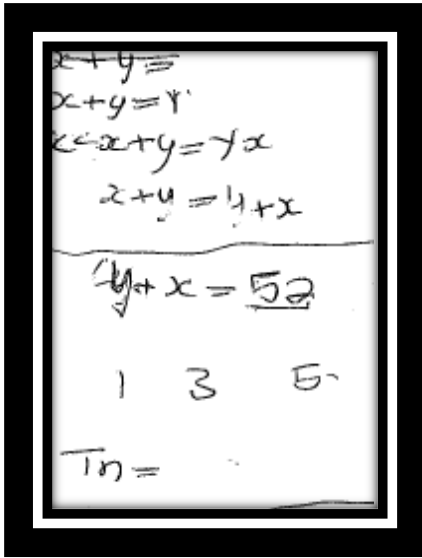
$x + y =$

$x + y = Y$

$x < x + y = y x$

$x + y = y + x$

$y + x = \underline{52}$

1 3 5

$Tn =$

Learner D interview response

Researcher: Please read question 1.

Learner: The sum of two consecutive odd numbers is 52. What are the two odd numbers?

Researcher: What is the question asking you to do?

Learner: I write line 1 and cancel it, then I write lines 2, 3 and then 4, rule off.

Researcher: How do you represent the two odd numbers that add up to 52?

Learner: Write $y + x = 52$; that is, the first odd number can be x and the second one can be y.

Researcher: How then can you solve the sum algebraically to get the two odd numbers, if x is the

first? Take the first odd number, then you jump one number and the next is the second odd number.

Learner: I wrote the last line: 1, then 3 and 5. I had no clue, so I stopped.

Learner D displayed a serious lack of language competence as he uses x and y with no meaning.

He wrote meaningless equations. As lines 2 and 3 do not correlate, how can x and y interchange

and be equal to y and x? Nevertheless, it is not clear if this problem is language or algebraic.

1(b)   Algebraic inadequacy

The episodes below present Learner B in question 1, while learner F in question 3 demonstrates algebraic inadequacy.

Learner B written response to question 1[Appendix, 2013]
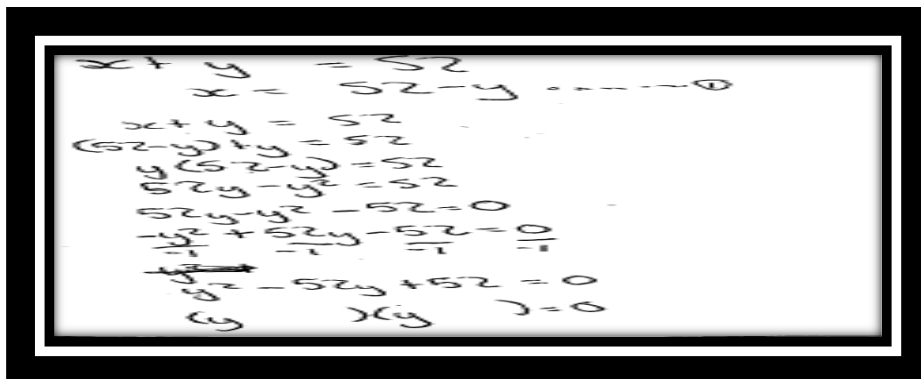
Learner B Interview response

Researcher: Please read question 1.

Learner: The sum of two consecutive odd numbers is 52. What are the two odd numbers?

Researcher: Show how you can solve the question.

Learner: It means writing line 1 and then line 2 solving for x.

Researcher: How then can you arrive at the two odd numbers using an equation?

Learner: I substituted for x in line 4 and then arrived at the quadratic equation in line 6.

The learner tried to factorize, got confused, and could not continue with the solution.

Here the learner could read the word 'consecutive', but could not bring meaning to it. The learner used rote approach to solve the problem. Although the initial equation shows, the learner understands 'sum' in the question, but lacks the conceptual understanding of the word 'consecutive' as line 1 shows. Hence, this word poses a big challenge; it is an algebraic term, which learner B cannot access. Therefore, looking at line 4, it shows the learner tries to substitute for x

into the equation and he messes all up by multiplying instead of adding. Also, line 5 shows he arrives at a quadratic equation and tries to factorize it. However, he could not manage this and was stuck. Therefore, the learner could not work with algebraic terminology.

Learner F written response [Appendix, 2013]
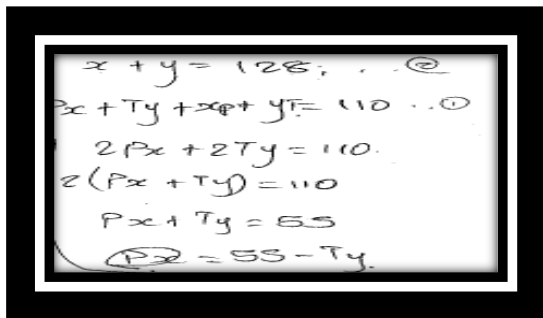
Learner F Interview response

Researcher: Please read question 3.

Learner: The sum of a two-digit number and the number formed by interchanging the digits, is 110. The new number is 18 more than the original number. Find these original numbers.

Researcher: How do you solve the question in order to get these required two numbers?

Learner: The question wants me to add number 110 and 18 to get the number.

Researcher: Do you know that you are required to solve for two numbers and not one number? As said, "the number" that is one number, is the original and the other is the interchange.

Learner: Write line 2 with two of each these digit variables, that is Px and Ty equal to 110 with two of the digits the same.

Researcher: Take the original two-digit number to be *xy* and the other interchange to be *yx*. How do your variables *Px* and *Ty* represent the number and the interchange?

Here Learner F struggled with understanding the meaning of the question. This struggle of the learner stems from poor language proficiency, which hides the meaning of algebraic language; that is, the variable used and the equation needed to solve the question become difficult. He had a

problem identifying the sum of the two-digit number and its interchange number giving 110. Evidently, the poor language proficiency renders the learner unable to comprehend the mathematical terms specified in the word problem.

The big word 'interchange' is the one learner F has problems understanding. In addition, the learner could not solve the problem using the right variable to represent two digit numbers. However, step 1 shows the learner has a clear conception of the word 'sum' between the *x* and *y*. However, the learner lacks the algebraic terminology to solve the problem. This is also algebraic inadequacy. The following theme explains the unpacking of meaning given by the researcher.

2. Unpacking of meaning

Learners C, F and G were able to achieve the correct solution to question 1 after benefitting from the unpacking of meaning by the researcher. The unpacking of the meaning given by the researcher is different for each learner, because of his or her diverse challenges. The learner benefits firstly from the researcher's use of a clue, secondly through the researcher's scaffolding or use of examples, and thirdly by the researcher establishing a relationship between *x* and *y* which inspires the learner to reason algebraically. The differences in learners' facial expressions and actions result in the researcher varying the approaches used.

Learner C written response [Appendix, 2013]

Learner C Interview response

Researcher: Please read question 1.

Learner**:** The sum of two consecutive odd numbers is 52; what are the two odd numbers?

Researcher: How do you solve the question?

Learner: If I add the two odd numbers, I will get 52.

Researcher: How do you solve for the first odd number and then the second odd number, if the second odd number is two more than the first odd number?

Learner: I write from line 2 to line 5, solving the first odd number a is 25, and then in the next column solve for the second odd number b is 27.

Learner C benefitted from the defining of the term 'consecutive' where the researcher used simple language. The researcher using further explanations to question 1, gave the learner a clue as to how to solve the question. This helped the learner to understand the question and he was then able to solve the question. Moreover, the learner was able now to see that the difference between the two consecutive numbers is 2. This shows that this learner had sound number sense. It became obvious to the learner that the difference between the first odd number and next odd number is two. He solved the problem without any error. The learner understood the meaning of the word 'consecutive' with researcher's guidance.

Next is learner F benefitting from the unpacking guidance by the researcher.

Learner F written response [Appendix, 2013]

Learner F Interview response

Researcher: Please read question 1.

Learner**:** The sum of two consecutive odd numbers is 52. What are the two odd numbers?

Researcher: How do you solve this question if the first odd number is, for example, 1 and then the next odd number is 3, that is the second odd number.

Learner: I write line 1, 2, 3 and got the first odd number y is 25, which later is substituted into the original equation to have the second odd number x is 27.

Learner F was able to benefit from the researcher providing clearer meaning to the word "consecutive" using examples of odd numbers. The researcher provided clues with the example, that is, if the first odd number is 1, and the next odd number is 3, that means the second odd number is 2 more than the first odd number. Hence, the learner wrote the equation as y as the first odd number. When added to the second odd number it is represented as $y + 2 = 52$. Here the learner was able to solve the problem without any error.

Next is Learner G benefitting.

Learner G written response [Appendix, 2013]

$$x + (x + 2) = 52$$

$$2x + 2 = 52$$

$$\frac{2x}{2} = \frac{60}{2}$$

$$x = 25$$

$$y = 27$$

∴ The two odd numbers 25 and 27

Learner G Interview response

Researcher: Please read question 1.

Learner**:** The sum of two consecutive odd numbers is 52. What are the two odd numbers?

Researcher: If the first odd number is x and the next odd number is two numbers ahead, how do you solve this algebraically? (Using my fingers to denote the first and then second odd number to explain.)

Learner: I write line 1, 2, 3 and 4 solving for the first odd number to be 25 and the next odd number is 27.

Similarly, Learner G benefitted from a concrete explanation of the word 'consecutive' by the researcher using fingers as learning tools. The researcher stirred abstract and algebraic reasoning in the learner by unpacking meaning to the question, using the relationship between variable x and y. With this researcher's assistance, the learner was able to solve correctly for the required two odd numbers. This means that the learner had a better understanding of the question by listening to the researcher's explanation of 'if the first odd number is x and the next odd is two numbers ahead.'

In addition, the researcher, using her fingers as teaching aids helps to revive the learner's reasoning to represent the first odd with x and the second odd in terms of x as seen in lines 1 and 2. After solving for x, the learner should understand that y could be solved by substituting x in the previous equation. He was able to solve it without any error.

Learners who benefitted from Unpacking of meaning in question 3.

Learner D written response [Appendix, 2013]
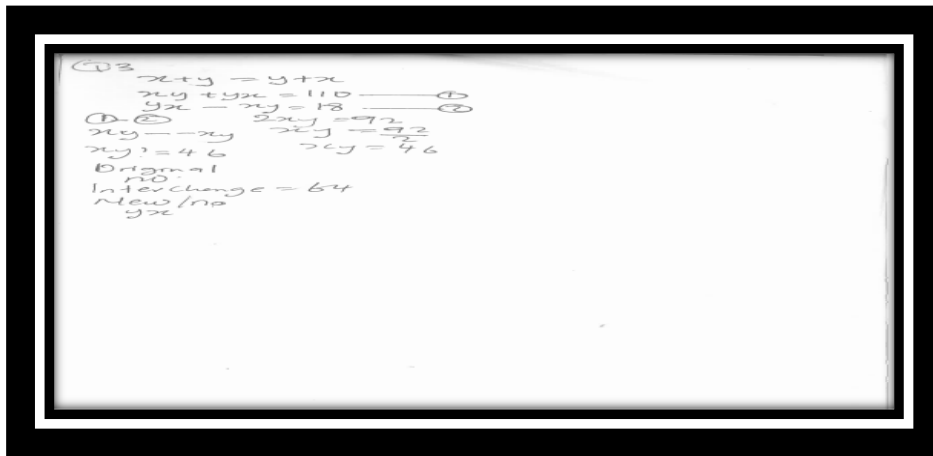
Learner D Interview response

Question 3: The sum of a two-digit number and the number formed by interchanging the digits is 110. The new number is 18 more than the original number. Find the original numbers.

Researcher: Please read question 3

Learner: The sum of a two-digit number and the number formed by interchanging the digits is110. The new number is 18 more than the original number. Find the original numbers.

Researcher: How do you solve for these required two-digit numbers?

Learner: Write $x + y = y + x$

Researcher: Do you mean two numbers are represented as x? What about TU (TENS and UNITS? Example 24). How then can you write the sum of the original two digits and the interchange is 110?

Learner: I write $xy + yx = 110$

Researcher: What about the interchange digit (new number) being 18 more than the original digit number?

Learner: I got two equations and solved them simultaneously to get the original digit number xy as 46

$Xy + yx = 110$

$Yx - xy = 18$

By checking I got the original digit number as 64.

Learner D was able to benefit from the researcher unpacking meaning of the word "interchanging" in problem Question 3. It is obvious that learner D was struggling with this sum. He found it difficult in the initial attempt during the test. However, the learner was able to get the clear and desired meaning of the word problem that resulted in the correct solution of the problem after the researcher administered the unpacking of meaning during the interview. In the first equation, the learner wrote $x + y = y + x$ showing lack of comprehension on part of the learner. However, after the researcher shed some light on question 3, the learner was able to derive the correct equation in line 2 and subsequent ones which eventually allowed the learner to arrive at the right solution. He had no error here.

Below is learner E benefitting from unpacking in question 3.

Learner E written response [Appendix, 2013]
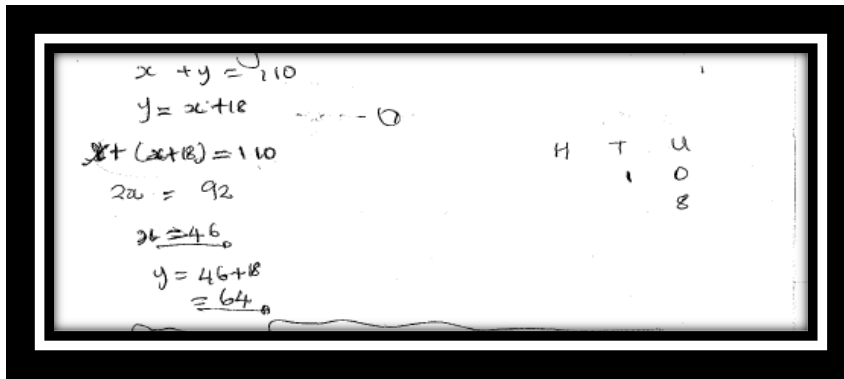
Learner E Interview response

Researcher: Please read question 3.

Learner: The sum of a two-digit number and the number formed by interchanging the digits is 110.

The new number is 18 more than the original number. Find the original numbers.

Researcher: How do you solve this question to arrive at the original required digit numbers?

Learner: I write line one, then try to solve for the new digit number with y that is 18 more than the

original number. Thereafter I have line 1 and 2 solving to get the new digit.

Researcher: You mean the original digit is x while the new is y. (I write this in the right hand

corner of the page)

What about your HTU that is H- Hundreds, T- tens and U- units? How do you write that?

This is mathematically, if the new two-digit number is 18 more than the original two-digit number.

Learner: Then I think the new number which is 18 more than the original number must be written

as the original 46, plus number 18 line 6 and proceed to get 64.

The researcher unpacked the meaning of the word problem when she used question 3 to bring

clarity to the learner. Furthermore, the researcher explained how to represent the phase "two-digit

number" in the word problem with HTU as shown in lines 1and 2 of the second column. Thereafter,

the learner was able to write the correct equation in line 3: the first digit is x and the second digit

is represented in terms of x such that the new number is 18 more than the original number. Hence, the learner benefitted from the unpacking of meaning, because he solved the problem without any error. He was able to solve algebraically in the last 2 lines: that is, solve for y in terms of x, when x is known. Although the learner got the sum right, his calculation shows that he is poor with place value representation. He wrote th two- digit number as x and the interchange as y.

Learner G written response [Appendix, 2013]
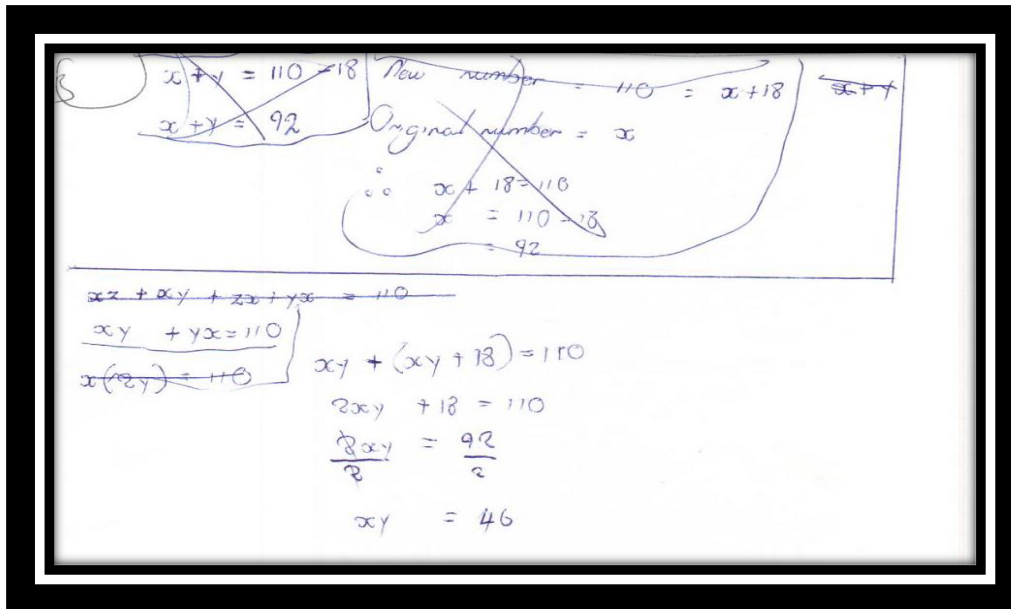
Learner G Interview response

Researcher: Please read question 3.

Learner: The sum of a two-digit number and the number formed by interchanging the digits is 110. The new number is 18 more than the original number. Find the original numbers.

Researcher: How do you solve this question?

Learner: I wrote many equations and then cancelled.

Researcher: Are you not supposed to add the original two digits which you correctly wrote xy and its interchange two-digit number to get 110? Also, the interchange two- digit number is 18 more than the original two-digit number. How do you represent this?

Learner: I write line 1, 2 3 and 4 solving for the original two-digit number to get 46.

Learner G initially made a comprehension error as is seen in the first few steps with many cancellations. However, with the help of the researcher emphasizing the sum of the original and the new two-digit number to equal 110, and the new number is 18 more than the original, unpacked the meaning of question 3. Therefore, the unpacking of meaning of certain specific terms that are unclear in the word problem enables the learner to effect the correct solution of the question to arrive at the answer 46 as seen above. Although the learner was able to solve for the original number correctly, he got confused and could not arrive at the reverse of the number (which is 64) as is required in the question. His language deficiency of the question is partial, because he shows some extent of understanding of what is needed through adequate reading. However, he displayed a certain amount of algebraic language inadequacy. This means that the learner displayed partial comprehension knowledge of the problem.

The following presents learners E and G who had no errors because they benefitted from the unpacking of meaning by the researcher.

Learner E written response [Appendix, 2013]

Learner E Interview response

Researcher: Read question 11.

Learner: A rectangular parking area's dimensions are 50m by 120m. If the parking area is doubled by increasing both dimensions by x meters, determine the dimension of the new parking area.

Researcher: How do you solve this question for the new dimension?

Learner: I write line 1 to 3 to calculate the parking area.

Researcher: Doubling the new parking area, how do you solve for the increased dimension?

Learner: I multiply the parking area by two to get the new parking area. I then have the new area solved with a quadratic equation as shown in line five. Then I factorize to get new dimension in the second column.

Here learner E benefitted from the unpacking of meaning by the researcher: that is, emphasizing 'doubled the area' to get the new area. The learner, with the help of unpacking, was able to solve the question without any error, as shown above.

Learner G written response [Appendix, 2013]

Learner G Interview response

Researcher: Please read question 11.

Learner**:** A rectangular parking area's dimensions are 50m by 120m. If the parking area is doubled by increasing both dimensions by *x* meters, determine the dimensions of the new parking area.

Researcher: How do you solve this question?

Learner: I know that when I multiply the length by breadth will get the area.

Researcher: This is the correct procedure to get the answer for the parking area. Therefore, to solve for the new packing area, you double the original parking area and solve it by factorization to get the new dimensions of length (l) and breadth (b).

Learner: I write line 5, 6 and other subsequent lines to solve for new l and b.

From the explanation given by the researcher, the learner has an idea what the question requires. In addition, the learner stated that the area should be doubled and the equation from that has to be factorized. This enabled the learner to solve the question without any error. Therefore, the learner has sufficient language proficiency and algebraic language efficiency, which helped him to solve the question. In the following theme, the researcher presents learners A, B, C, D, F and H with various errors.

3. **Errors displayed by learners**

The following learners, A, B, C, D, F and H, exhibited process skills errors rather than calculation errors in question 11 as referred to in this study. Learner B also displayed comprehension and process skills errors in Question 3.

Learner B written response [Appendix, 2013]

Learner B Interview response

Researcher: Read question 3.

Learner: The sum of a two-digit number and the number formed by interchanging the digits is 110. The new number is 18 more than the original number. Calculate the original numbers.

Researcher: How do you solve this question?

Learner: I write x to represent the original number and y as the interchange digit number and their sum will be 110. I initially make x the subject and then later y, as in lines 2 and 3.

Researcher: But two-digit refers to TU (tens and units, that is xy). How do you represent the sum of the original two-digit number and its interchange two-digit number?

Learner: I write lines 1 and 2 and then rule off. I subtract the interchange yx from the original xy, adding 18 to it. I continue solving by factorizing which equals to 110, then arrive at yx is equal to 50. In the next column, I have the original xy and interchange digit yx are equal to 110, I write line 2 that the original is equal to the interchange minus 18. After this I try to factorize, then, using the quadratic method, I arrive at 64 which is the original number.

Learner B displays a lack of comprehension knowledge in all steps of solving the question. The learner's problem stems from a lack of comprehension as he tries to make x the subject in line 2 and later resolves to make y the subject in line 3 and writes the initial equation and then rules off. Looking at column 1, the learner is factorizing a linear equation, a serious mathematical error. In line 3 after the rule off, then learner finally gets $yx=50$ in the last line. This shows a huge comprehension error. Also in column 2, the learner is trying to solve for the interchange number using a quadratic formula. Doing this, he makes many process errors to arrive at $xy=64$ Although the learner uses algebraic steps in solving this question, he displays comprehension and process skills errors. The next section presents learners' display of errors in question 11.

Question 11: A rectangular parking area's dimensions are 50m by 120m. If the parking area is doubled by increasing both dimensions by x meters, determine the dimension of the new parking area.

Learner A written response [Appendix, 2013]

Learner A response to question 11 (Semantic error)
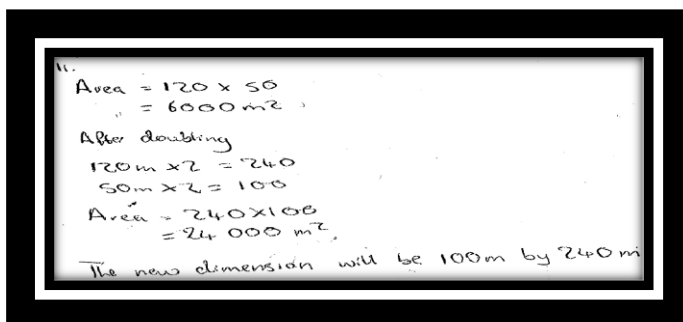

Researcher: Read question 11.

Learner: A rectangular parking area's dimensions are 50m by 120m. If they double the parking area by increasing both dimensions by x meters, determine the dimension of the new parking area.

Researcher: How do you solve this question?

Learner: I suppose the area of the rectangle is l x b, so I write line 1, to solve for the area of the

parking.   Also the new parking area will be two times the dimension, so I solve for them, to

have lines 4, 5 and 6.

The learner's first three lines are correct; he was able to exhibit a knowledge level that recalls the

formula of the area of a rectangle. However, the above shows he displayed a semantic error in line

3 and following lines, as he tries to solve the doubled area by doubling the variable l and b to have

100 and 240. The learner said, I think the new parking area will be 100 multiplied by 240 equalling

24 000. Although he wrote an equation to represent this specific term in the word problem, it was

the wrong equation or method, consequently getting the new area dimension wrong. A similar

thing happened to learner B below. The learner showed some comprehension of the question, but

other steps indicate he chose the wrong method to solve the problem. This was a semantics error.

Learner B written response [Appendix, 2013]

Learner B Interview response (Semantic error)

Researcher: Read question 11.

Learner: A rectangular parking area's dimensions are 50m by 120m. If the parking area is

doubled by increasing both dimensions by x meters, determine the dimension of the new parking

area.

Researcher: How do you solve this?

Learner: I write line 1 and 2 solving for the area of the parking space. Then I thought of multiplying each dimension by two and then using the area formula l x b will get the new parking area.

Researcher: Are you not supposed to double result of the previous area or is the dimension supposed to be doubled?

Learner: I write lines 4, 5, 6 and 7 to solve for the new parking dimension.

Learner B has the first equation for the area right. However, he has the second equation for doubling the area wrong. He could not arrive at the quadratic equation as shown in the three preceding lines. Therefore, he has the new dimension of length and breadth but it is incorrect. He exhibits a semantic error in lines 3-5 by employing the wrong method. However, the first and second steps show the learner has comprehension knowledge.

Learner C written response [Appendix, 2013]

Learner C Interview response (Calculation error)

Researcher: Please read question 11.

Learner: A rectangular parking area's dimensions are 50m by 120m. If the parking area is doubled by increasing both dimensions by *x* meters, determine the dimension of the new parking area.

Researcher: Can you explain how you solved the question?

Learner: I calculated the area of the parking, using length multiplied by breadth I multiplied the area of parking by two as I wrote in lines 3 and 4. Then I got the quadratic equation in lines 6 and line 7.

Researcher: How do you factorize the quadratic equation to get the new dimensions of the parking area?

Learner: I tried to factorize and then could not and got stuck.

Learner C got the first equation for the area right as well as the second equation of doubling the area solved to have the quadratic equation in line 6. However, from line 8 continued to column 2, it shows he could not factorize to arrive at the new dimension for length (l) and breadth (b). Therefore, he displayed process skills errors according to Newman but in this study, it is known as a calculation error.

Learner D written response [Appendix, 2013]

Learner D Interview response (Comprehension and Calculation error)

Researcher: Read question 11.

Learner: A rectangular parking area's dimensions are 50m by 120m. If the parking area is doubled by increasing both dimensions by x meters, determine the dimension of the new parking area.

Researcher: How do you solve the question?

Learner: I multiplied the dimensions to get the area of the parking, after which I multiplied the area by two in lines 6 and 7 to get the new parking area. Then I tried to factorize as I did in lines 9, 10 11 and others.

For the first equation learner D correctly solved the first area of the rectangle. He also got right the area of the second rectangle equation, that is the doubled area. However, comparing learner D with learner C above, it is clear that D got the second equation right (doubling the area) but could not represent these into quadratic equations as seen in line 8 and following. Therefore, he displayed both calculation and comprehension errors. This shows a deficit in comprehension knowledge, which disenabled him from paraphrasing the word problem into the required equation in line 8. His poor calculation skills caused him to default in solving the factorization correctly, as is evident in the solving of the quadratic equation in lines 8 and 9.

Learner F written response [Appendix, 2013]

Learner F Interview response (Calculation error)

Researcher: Read question 11.

Learner: A rectangular parking area's dimensions are 50m by 120m. If the parking area is doubled by increasing both dimensions by x meters, determine the dimension of the new parking area.

Researcher: How do you solve this question?

Learner: I calculate the area of the original parking space, then the new parking space, try to factorize, using the quadratic formula and cannot continue.

The above response shows learner F arrives at the first and second equations correctly. The last line equation shows he could not factorize the quadratic equation to arrive at the new length and breadth dimension. This was a calculation error. He has adequate language proficiency but lacks a little in algebraic language efficiency.

Learner H written response [Appendix, 2013]
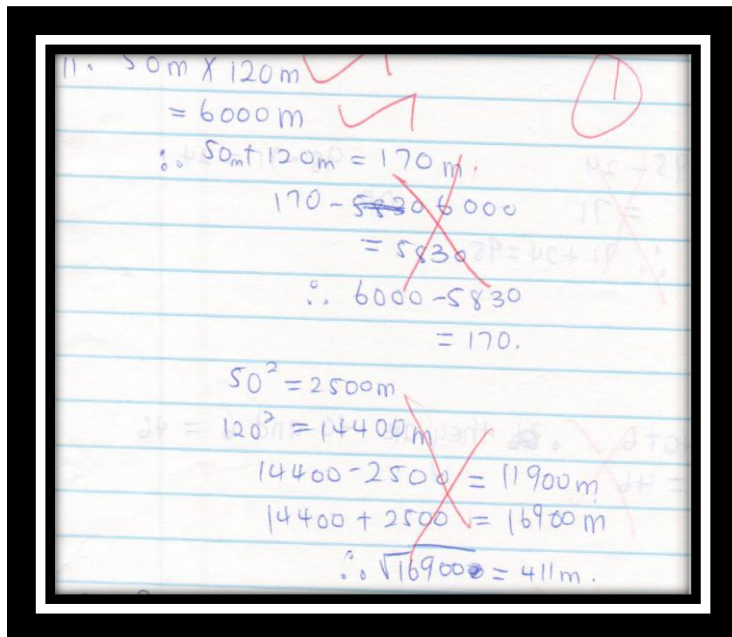
Learner H Interview response (Comprehension error)

Researcher: Read question 11.

Learner: A rectangular parking area's dimensions are 50m by 120m. If the parking area is doubled by increasing both dimensions by x meters, determine the dimension of the new parking area.

Researcher: How do you solve this question?

Learner: I solve for parking area in line 1 and 2. After which I do not know what to do.

Researcher: If the new parking area is two times the former area, how do you solve for the new dimensions?

Learner: I calculate other lines and could not arrive at any solution.

Here learner H got the equation for the area correct as shown in lines 1 to 2. However, the subsequent equation is not really an equation as can be seen in line 3 and the following lines. This is clearly a comprehension error; the learner is unable to present the question in the required algebraic equation. The learner exhibited language fluency but has some algebraic language gaps.

4. **Lack of transition from arithmetic to algebra**:

Learner H displays this in questions 1 and 3 and learner E in question1 as shown below.

Learner H written response to question 1 [Appendix, 2013]
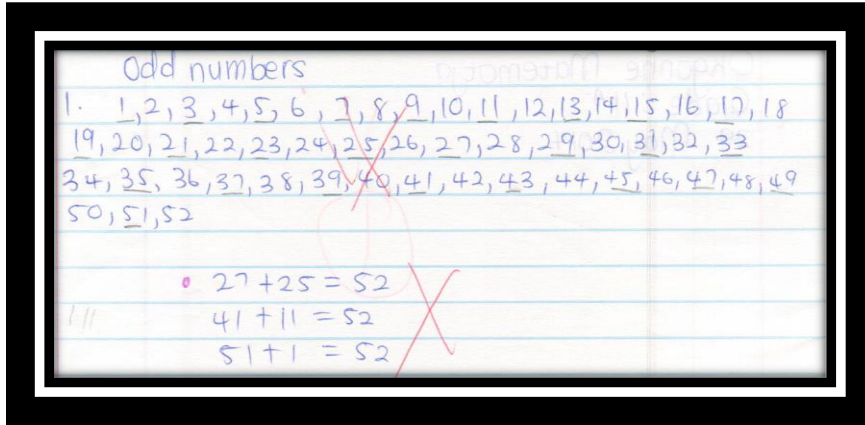
Learner H Interview response

Researcher: Read question 1.

Learner: The sum of two consecutive odd numbers is 52. What are the two odd numbers?

Researcher: How do you answer this question?

Learner: I think by listing the numbers from 1 to 52 and underlining the odd numbers. Then I guess having two different numbers adding up to give 52 is the solution.

The learner uses a trial and error method because he lacks algebraic understanding. This learner understood the question clearly, that is why he could list all numbers between 1 and 52 and underline the odd numbers.

27+25=52   point 1 is correct and consecutive

41+11=52   point 2 is also correct but not consecutive

51+1=52   point 3 is correct but not consecutive

Primary teachers have done well with this learner. However, it is noticed that learner H's barrier starts from middle school work, that is from Grade 7 upwards. The problem identified here is that learner H has a knowledge gap of transiting from arithmetic to algebra.

Learner H written response to question 3[Appendix, 2013]
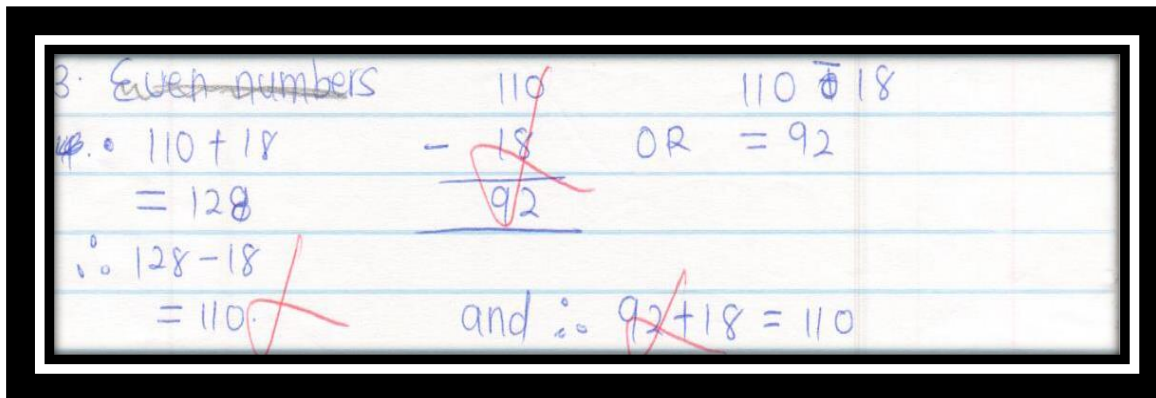
Learner H interview response

Researcher: Please read to me question 3.

Learner: The sum of a two-digit number and the number formed by interchanging the digits is 110. The new number is 18 more than the original number. Find the original numbers.

Researcher: How do you solve this question?

Learner: I think of adding these numbers and then I write these.

Learner H's calculation showed much confusion. He shows that he understands that the question needs to be solved with an equation. He tries to solve the problem using equations, but he cannot, because what he is writing, are not equations; they all are meaningless. Learner H shows he can neither represent word "interchange" nor "sum" into algebraic equations as required. Here the learner reasoning ability is at primary school level.

Learner E written response [Appendix, 2013]
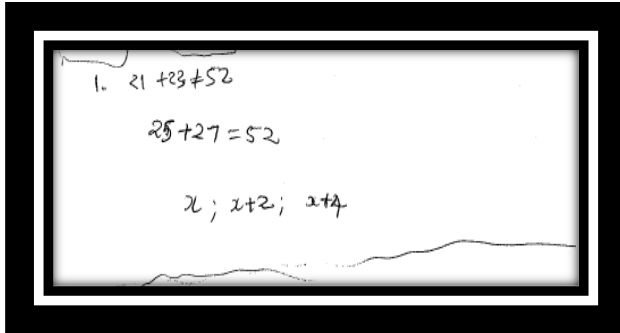
Learner E Interview response

Researcher: Please read question 1.

Learner: The sum of two consecutive odd numbers is 52. What are the two odd numbers?

Researcher: What are you required to do in the question?

Learner: I write line 1, then 2, trying to introspect, that is, guess what numbers can add up to 52.

Researcher: If x is the first odd number, jump the next number, then the next will be the second.

 Learner: I write line 3 (last line showing more confusion).

There is some understanding shown by the learner in line 2: the two numbers used are consecutive odd numbers that add up to 52. Although the learner is not working on an algebraic level but at a concrete level (arithmetic), he shows full understanding of the problem. Here the learner is operating at a very low level: he did not follow solving by the equation method using x and y. In summary, the analysis shows the learner has a knowledge gap between arithmetic and algebra. As the learner uses a trial and error method, it indicates that there is a knowledge gap, which impedes the learner from making progress towards algebraic thinking in problem solving. This calls for the teacher's attention.

4.3   CONCLUSION

The result from the analysis shows there is a correlation between English language deficiency and algebraic language inadequacy. These language problems co-exist; the

inability to comprehend the algebraic terms specified in the word problem is caused by poor language skills. For example, learner E was able to solve the problem at the concrete level and not at the algebraic level, because he tried to add together the next odd number that forms 52 and could not make use of an equation as expected. Another example is question 1, where learners solve arithmetically by listing all numbers between 1 and 52. Such learners lack algebraic terminology to solve the question. In question 1 another learner who was supposed to sum the next two odd consecutive numbers, could not figure this out into its correct equations. Therefore, learners guessed to arrive at any two odd numbers that sum up to be 52.

The researcher provided unpacking of meaning to certain questions by encouraging learners to reason algebraically. This unpacking of meaning by the researcher varies depending on learner differences. Some learners required the researcher to give just a clue, while others needed scaffolding and at other times creating a relationship between variables (x & y) helped. Learners C, F, and G benefitted from unpacking of meaning by the researcher in question 1, while learners D and E benefitted in this way in question 3.

Learners A, B, C, D, F and H exhibited errors such as comprehension, semantic and calculation errors. In question 11 some of the learners made calculation errors by using the wrong procedure to factorize the quadratic equation.

 Learner E and H in question 1 displayed challenges with translating arithmetic to algebra knowledge. This means that learners had algebraic language inadequacy problems. The learners' written work makes it clear they have achieved the arithmetic level but have problems expressing word problems on the algebraic level. It is obvious that learner E understands the meaning of the word "consecutive" but can only solve at the concrete level and not at the algebraic level because

he tries to add together the next odd number that forms 52 and cannot make use of an equation as expected.

The study supports the findings of Haghverdi et al (2012) that error types found are linguistic, semantic, comprehension and calculation. This is the response to research question 1. While responding to research question 2 (the reasons for errors learners made), the researcher found in the following themes that errors made, were due to learner inadequacy in the English or algebraic language or both; errors; unpacking of meaning and the inability to translate arithmetic to algebraic knowledge.

CHAPTER 5: Discussion

5.1   Introduction

This chapter entails the discussion of the results by responding to research questions of this study. Firstly, the overall picture in figure 1 indicates that the majority of these Grade 11 learners shows very little algebraic understanding. This result challenged this study, because the researcher eliminated many learners who could not perform any algebraic calculations. The small number of learners who managed to indicate some knowledge and understanding showed up a variety of mixed errors and knowledge levels that speak to the education system.

This chapter will focus on four key findings:

1. Language deficiency

2. The unpacking of meaning

3. Errors and

4. The lack of transition from arithmetic to algebraic thinking.

The findings consist of English and algebraic language categories. The main aim of this study was to tease out errors. However, more in-depth investigation brought to the fore major challenges in the teaching and learning of Mathematics in South African classrooms. This discussion will highlight the findings and look back at the curriculum to find the origins of these challenges, and in the literature review to see if these findings can be supported or refuted. In addition, the literature will determine the extent to which this study contributes to new knowledge. The study will make recommendations for teaching and learning purposes and future research.

5.2   Learners' Scores

Figure 1 presents the learners' scores in frequencies: the low, average and high group. The low group have scores within 0-29 %, while the average group has scores of 30-39 % and the high group is 40% and above. As seen in Figure 1, the frequency for the low group is the largest with 140 learners, while the average group frequency is 5 and the high group's frequency is 5. This figure generally shows that the majority of South African learners experience challenges with Mathematics and are struggling with it. The findings concur with other assessments like SAMEQ 11 (2000) and 111 (2007), TIMSS (2011) and the ANA results as recorded in Chapter 2. Highlights from SACMEQ, SACMEQ 11 (2000) and 111(2007) show this poor performance. Van der Berg (2007), reports that among 14 participating countries in SACMEQ II (2000), South Africa came ninth in Mathematics, even lower in performance than Botswana, Swaziland and Kenya. Similarly, highlights from SACMEQ III (2007) show South Africa, among 15 countries assessed, came tenth in reading and eighth in Mathematics, achieving a poorer score than countries such as Kenya, Tanzania and Swaziland (Van der Berg, 2007). TIMSS (2003- 2011) results show South African learners are struggling in Mathematics assessment, because both Grade 8 and Grade 9 pupils wrote the Grade 8 test in 2003, but in 2011 only Grade 9 pupils wrote the Grade 8 test (Reddy, 2012). In summary, TIMSS (2003 - 2011) record that the average South African grade 9 learners 2 grades scored two grades lower in the grade 8 Mathematics assessment among 21 other middle-income countries (Spaull, 2013).

This indicates that South African learners are struggling with poor performance. The ANA (a local South Africa government initiative through the DBE) analysis in Table 1, Chapter 2, shows that grade 9 recorded a low performance, as less than 5% of learners were able to achieve 40% and above in Mathematics (ANA, 2012). The results from the matric Mathematics exam have also

shown these difficulties South Africa learners encounter. The results of the South African National

Certificate examination of 2011 indicate that the pass rate for Mathematics in the matriculation

examination was 46.3% in 2011, a decline from 47.4% in 2010 (DBE, 2012)**.** This study supports

and could see the reason the TIMSS assessment had the grade 9 learners not able to write the

assessment for their grade level, but instead doing the Grade 8 one which is a year lower for them.

This study's finding agrees not only with TIMSS, but also with that of ANA and SACMEQ.

5.3.1   English language deficiency

English is the language of instruction in Mathematics and English language deficiency

refers to the learners' inability to use the English language effectively for learning. The researcher

reviews the English language and its role in Mathematics learning, before focusing on its

inadequacy. Mathematics and language have something in common, as the former is conceptual,

abstract and has a specific register (Pimm, 1987; 1991). Studies (Pimm, 1987; 1991) and

curriculum developers (NCTM, 1991; 2000; 2006) recognize that Mathematics has to be

communicated using language. Language is significant for thinking and learning to take place.

Mathematics teaching and learning is indisputably an interactive one, required worldwide. South

Africa is a multi-lingual nation that has a common language of instruction. This language for

instruction happens to be English. The current Language in Education Policy recognizes 11 official

languages. Amongst languages specified in the South African educational policy, English remains

dominant among the 11 official languages for teaching and learning (Adler & Setati, 2001; Adler,

Bapoo & Reed and Setati, 2002). English takes the lead as language of power and educational and

socio-economic advancement in South Africa (Bourdieu, 1991). English dominance has authority

not only from the pedagogic and cognitive part, but also from the political aspect (Gee 1999; Setati

2003). Gee (1999) argues that people who gain political racemisation do that by using language as

a tool to project their views in speaking and writing. The findings report supports the argument of Njagi (2015) and Howie (2003) that learners' English inadequacy influences their Mathematics achievement. Howie (2003) and Njagi (2015) note that the poor performance of South African learners in TIMSS signifies the need to develop English language proficiency, because it is the only recognized and reasonable language for instructing Mathematics. Howie's opinion led to the revision of the education policy to launch CAPS, which aims to correct this poor performance in Mathematics.

Hence, these research findings are consistent with other research that language proficiency greatly determines learners' success in Mathematics and in particular, algebraic word problems. The researcher emphasizes the fact that most learners use English as a second language to learn Mathematics. It therefore becomes more difficult for such learners to access Mathematics knowledge which it becomes obscures as the language is not their own.

Citing from a written learner response to Question 1 in the findings, it is clear the lack of English language capability is the reason the learner found it difficult to extract the meaning of 'consecutive' and to arrive at an algebraic understanding; he wrote:

$$13 + y = 52$$

$$y = 52 - 13$$

$$y = 39$$

This second key finding refers to the low score group of 140 learners. The following discussion centres on the fact that the high frequency of low-scoring learners is due to the algebraic inadequacy of the group.

5.3.2 Algebraic language inadequacy

Algebra is a language used for expressing mathematical relationships (Turner, 2011; Barwell, 2012). Algebra uses symbols for generalizing arithmetic (van de Walle et al, 2011). Arithmetic teaching proceeds algebraic teaching in the school curriculum therefore it means arithmetic is the basis for learning algebra (EduGrains, 2013). Reviewing the test for this study makes it clear the test questions consist of algebraic terms, such as manipulating with variables. The scholar states that arithmetic deals with operations involving particular numbers, whereas algebra deals with generalized numbers, variables, and functions (Carraher et al, 2006). In addition, Ilany and Margolin (2010) reveal that most difficulties learners have with algebraic words, show they have poor algebraic knowledge. They have poor algebraic knowledge, because they found it difficult to identify mathematical structures hidden within textual information. In the findings, it has learners' B and F display algebraic inadequacy that supports the previous study discovery on problems with algebraic language (Ilany and Margolin.2010).

The researcher noticed learners' problems with algebraic language inadequacy are mostly from algebraic word problems and not from numeric algebraic problems. Therefore, this study agrees with Clement (1982) suggested that much time is required to achieve the proper process of transiting from arithmetic reasoning to algebraic manipulation. This study concurs with the EduGrains (2013) explanation that it is important for learners to understand the multiple uses of variables, which can enable them to distinguish and manoeuvre with variables. EduGrains (2013) emphasizes learners should differentiate between a variable that gives one value ($x + 4 = 5$) and those variables with a range of values ($x + y = 5$). In addition, the particular variable does not make any difference as long as within a given problem the same variable has the same meaning. The value associated with a particular variable in one problem can be different in another problem.

Usiskin (1988) describes variable as one of the concepts of algebra, which could be complex and can have different meanings depending on the situation. He refers to variables in 5 major aspects namely: (1) unknowns (2) formulae (3) generalized patterns (4) place values and (5) relationships between quantities. This study test assessed learners similar to Usiskin's points (1988): (1) solving for unknown(s) is required in most questions. In point (2) deriving the formula of area and generalizing, such as in Question 11 and using point (3) formula to solve quadratic formula was needed in Question 4 and point (5) relationship between patterns was applied in Question 11. While the researcher reviewed the learners' solutions, it was clear that poor understanding of arithmetic concepts was the reason for misinterpreting variables. In the following, the discussion is about the effect of English language deficiency and algebraic language inadequacy on learners' solution of algebraic word problems.

5.4 English language deficiency and algebraic language inadequacy

The English language deficiency and the algebraic language inadequacy contribute to the learners' language problems, when they solve algebraic word problems. It follows that a learner with English language problems will display algebraic language problems. Hansson (2012) suggests that learners' English language (for instruction) and algebraic language (which is the language of Mathematics) should improve simultaneously. This means that a learner, who is proficient in English language, will definitely have adequate algebraic knowledge. This is the reason: in this study the learners with English language deficiency also struggled when solving algebra word problems. Learners struggle because English language as a medium for instruction is not their mother tongue, but they learn algebra as an English learner. The researcher discovered that in less demanding questions like Question 1 and Question 3 learners displayed language problems because of algebraic language inadequacy and not necessarily English language deficiency. This

means when learners are solving these less demanding questions, they exhibit algebraic inadequacy due to their low level of reasoning.

For example, Learner A in Question 3 could not understand specific terms in the word problem, such that the big problem he faced, was words such as "interchange" and also "18 more than the original number". Therefore, he displayed a lack of algebraic understanding to enable him to derive the required equation. He has this written response:

$$13 + x = 110 + 118$$

Learners had challenges with both English language and algebraic language inadequacy.

This study agrees with Kersaint et al (2009) that learning Mathematics demands mathematical language together with an understanding of the English language. Hence, the researcher suggests learners need to have adequate understanding of both English and mathematical languages in order to make meaning of any given algebraic word problem. Newman (1983a) indicates that algebraic word problems always "require constantly translating words to gain a correct meaning for the mathematical context"(p.6). Learners' most challenges when solving algebraic word problems are due to the deficiency in linguistic processes as opposed to deficits in the quantitative processes (Le Febvre et.al; 2010). Although learners find problems requiring English language and algebraic language challenging, they encounter more problems particularly with algebraic word problems where two languages intermingle. Reviewing these research test questions (Question 1 and Questions 3 – 6), it is clear that they consist of words such as scholars' explanations of words used in both Mathematics and everyday English. These words have similar meanings, but the meaning in Mathematics is more specific (Rubenstein & Thompson, 2002). The study gives examples of words in English such as 'sum', 'difference' and 'more than' meaning addition, subtraction and increase in Mathematics (Rubenstein & Thompson, 2002). Scholars explain other examples of

words. For example, words like 'round' and 'square' have more than one meaning in the field of Mathematics: 'round' means 'the shape of a circle' or 'the task of rounding a number to the nearest tenth' and the word 'square' can refer to a shape or to a number times itself (Rubenstein & Thompson, 2002).

Reviewing this study test question 12, it agrees with what scholars Chamot and O'Malley (1994) and Heinze (2005) refer to as multiple meanings for the same word in Mathematics, such that learners should learn words together with their similar words for proper understanding of Mathematics words. Chamot and O'Malley (1994) and Heinze (2005) also give other examples, such as the word 'sum' and similar words like plus, combine or increase. The scholars' explanation and study findings concur with the content of research Question 12, such that a similar word for length and breadth is 'dimension'.

Considering Questions 11 and 12 from this research test question, one finds they obviously agree with the point these scholars have made with regard to the fact that English learning learners find Mathematics word problems difficult, because they lack the background knowledge about specific problems (Barwell, 2001; Short & Spanos, 1989). Reflecting on Question 11, for example, it could be difficult for these learners (who hail from informal settlements) to understand questions using words or terms centred on a car parking area, which these learners are not familiar with. Consequently, they commit errors for these reasons. The Department of Education document affirms the reason learners are not familiar with the information given in the word problem. This is why they struggle with understanding the text and solving these problems. In addition to this, a study indicates that Minnesota's Sample Fourth Mathematics word problems contain culturally specific information, in which learners make errors and get poor marks in the MCA test (Minnesota Department of Education, 2009). This explains why the English second language learners tend to

score lower than their native English-speaking peers do. English second language learners have a poor understanding of mathematical word problems due to their limited English proficiency. As a result, this affects their mathematical abilities. (Martiniello, 2008).

Scholars have recorded syntax and semantic errors in Mathematics. Semantics refers to meaning attached to certain words or phrases in Mathematics. For example, the phrase 'divided by' gives a completely different equation than the phrase 'divided into' (Heinze, 2005). For example, 6 divided by 12 is 0.5, whereas 6 divided into 12 is 2 (Irujo, 2007).

5.5 The unpacking of meaning

In the findings, 30% of the learners received unpacking of meaning from the researcher. These were precisely learners C, F and G in Question 1, while learners D, E, and G in Question 3 and learners' E and G for Question 11. Only learner G did not benefit fully from this intervention given by the researcher. Reviewing the previous literature, the researcher found that teachers unpacked the curriculum content briefly and not in the manner the researcher unpacked the test question. This makes it difficult for learners. The teacher unpacking content - like solving two simultaneous equations with two variables - is by assisting learners to know how to eliminate one of the variables to get the solution for the first variable and then the second. Learners should be assisted by teachers on how to get the meaning of solving by substitution, a system consisting of one linear and one quadratic equation, such as solving for one variable first by substituting one equation for the other one to arrive at the third equation and then solving for the second variable. Teachers' help to learners will enable them to unpack the meaning of how to solve either algebraic or graphical systems consisting of a linear and a quadratic equation. In this line, Caputo (2015) engaged in unpacking.

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Caputo (2015) contributed by explaining the effects of lengthy and uninteresting word problems, compared to those that are precise and interesting to learners solving these algebraic problems. His study explained it with this scenario:

A set of numbers is said to be closed under a certain operation, if, when you perform the operation on any two numbers in the set, the result is also a number in the set. Is the set of irrational numbers closed under addition? Explain.' (p.45)

Caputo (2015) opined that the concept of closure was new for many Algebra I learners, and that is why it was difficult for them to understand, especially when they were first introduced to it during a test. His study also denotes that the concept of classifying numbers required by a question is a routine exercise, which learners are familiar with. However, the lengthy wording of the question makes it confusing and intimidating to learners (Brunner, 2013). The scholar affirms that for this reason, learners make errors, have low scores and a decreased interest in the subject. These all result in a high number of dropouts (Brunner, 2013).

In addressing these learners' problems, the researcher explained to learners with examples, using integers with addition, and then integers with division, in order that learners can access how to differentiate sets with closure and those without closure (Caputo, 2015). The subsequent test shows learners' low performance and the need to increase their conceptual understanding of the question (Caputo, 2015). It is clear learners lacked conceptual understanding of the question. Caputo (2015) uses diverse means to aid learners' conceptual understanding of the question, firstly by extracting a paragraph from the Old Testament to form the test questions. Learners' results in this reframed test were not encouraging, as those with grades above 80% increased from 15% to 25% of the class population and the average ELL showed a 5% improvement, compared to the previous test (Caputo, 2015). He tried to improve learners' understanding of the test questions by

having test wordings aligned according to an old newspaper to attract their interest and better their marks (Caputo, 2015). This idea was successful, as he could gain learners' interest and obtain a grade above 80% with an increase of up to 40% of the class.

Caputo (2015) unpacks the meaning given to the test by substituting the lengthy words initially used in the test with interesting, precise, familiar words and applicable to the area of content. This unpacking narrated by Caputo is similar to the way the researcher unpacked meaning in this study of learners C, F and G in Question 1 and learners D and E in Question 3, by using the relationship between variables and scaffolding.

5.6 Errors

Ilany and Margolin (2010) affirm that lack of language proficiency is the reason for most errors learners make when changing from instruction language into algebraic terminology. According to Ilany and Margolin (2010), learners find it difficult to build a significant body of knowledge from the information in a question, including numbers and a solution format. Results from this study show that a lack of proficiency in the English language affects learners' algebraic knowledge; so much so that out of 150 learners who participated in the quantitative test, 60 learners were analysed into groups of errors, because most learners (90 of them) scored zero. Studies also indicated that comprehension is greatly influenced by language (Beringer et al., 2010; Fuchs, Fuch, Hosp & Jenkin 2001; Wise et al. 2010).

Reviewing the investigation in Chapter 3, it becomes clear that errors from the Newman analysis emanate from a lack of linguistic knowledge, comprehension knowledge, communication knowledge (which includes semantic, structural and intuitional) and processing skills. In the literature review in Chapter 2 it is highlighted that the errors learners make in the algebraic content areas, stem mainly from systematic and non-systematic errors. Learners A, B, C, D, F and H

exhibited errors such as comprehension errors, calculation errors, at times semantic errors, but also no errors. In Question 11, the learners made calculation errors by using wrong procedures to factorize the quadratic equation. In addition, the researcher also found that in the more demanding questions like Question 11, learners showed errors of comprehension and calculation, and then there were those responses without errors. In the investigation in Chapter 3, this study's findings also support the analysis of Majid et al. (2011) with its four types of errors. In support of this, the study findings have those errors known as knowledge gaps in comprehension, calculation and no error.

5.6.1 Comprehension Error

In the findings, 26% of the learners interviewed displayed comprehension errors such as learners A, B, G and H in Question 3; also learners A, B and H in Question 11. Some of the learners showed both comprehension errors together with calculation errors, while some showed only comprehension errors. The explanation below shows learner A in Question 11 displayed partial comprehension: semantic errors.

He was able to exhibit a knowledge level recalling the formula of the area of a rectangle. However, the figure above shows that he displayed a **semantic error** in line 3 and in following lines as he tries to solve the doubled area and multiplies the variables l and b, 100 and 240. The learner said: "I think the new parking area will be 100 multiplied by 240 to get 24 000." Although he wrote an

equation to represent this specific term in the word problem, it was a wrong equation or method, thereby getting the new area dimension wrong. Learner D's written work for Question 11 below showed he showed both comprehension and calculation errors.

Learner D solved the first equation for the first area of the rectangle correctly. He also got right the area of the second rectangle equation, that is the doubled area. D got the second equation right, that is doubling the area, but he could not represent these in a quadratic equation as seen in line 8 and following. Therefore, he displayed both a calculation error and a comprehension error. His deficiency in comprehension knowledge disabled him from paraphrasing the word problem into the required equation in line 8. His poor calculation skills caused him to default in solving the factorization correctly, as seen in the solution of the quadratic equation in lines 8 and 9.

5.6.2   Calculation Error

About 19% of learners showed a calculation knowledge gap in Question 11, such as learners A, B, C, F and H. As mentioned earlier, 90 learners scored zero, because the questions were unattempted. A typical example is learner F in Question 11.

The figure above shows learner F correctly arriving at the first and second equation. The last line equation shows he could not factorize the quadratic equation to arrive at the new length and breadth dimension. Therefore, he displayed a calculation error.

5.7 Lack of transition from arithmetic to algebraic understanding

Arithmetic lessons precede algebraic lessons, as it appears in the school curriculum statement (Carraher et al, 2006). The problem is that many learners exit the primary school and enter into middle secondary school with a shaky foundation knowledge of arithmetic upon which algebraic understanding is supposed to be? built (Ketterline-Geller Chord & Fien, 2008).

In order to solve any equation correctly, it involves the comprehensive manipulation of features like equals signs (=), operation signs (+, -, ×, ÷), variables (x, y, z) and other numbers like (4, 10,1/2, 0.75), coefficients (4x) and exponents ($x^3, 2^5$). Knuth, Stephens, McNeil, & Alibali (2006) indicate that understanding equals signs and negative numbers is essential for solving algebraic problems. Research has emphasized that getting the meaning of equality is the major challenge learners encounter when transiting from arithmetic to algebraic thinking (Knuth et al., 2006). The

research findings concur with these studies' indication that conceptual understanding (also referred to as relational understanding instead of rote learning) is required for learners to successfully transit from arithmetic to algebra (Law & Shahrill, 2013; Pungut & Shahrill, 2014; Sarwadi & Shahrill, 2014; Vaiyavutjamai, 2004; Vaiyavutjamai, Ellerton & Clements, 2005; Vaiyavutjamai & Clements, 2006). The following examples explain how learners' lack of conceptual understanding is the reason for their displaying rote or memorized learning when solving algebraic word problems. Learner H in Question 1 and Question 3 displayed the challenge of shifting from arithmetic to algebraic knowledge. This learner understood the arithmetic aspect of the question; that is why he listed all numbers between 1 and 52 and underlined the odd numbers, after he provided the sum of any two consecutive numbers that equal 52. Also in Question 3, it is clear the learner was successful in the arithmetic aspect of the question (the sum and difference of the problem), but could not interpret the phrase "interchange" and "two-digit numbers" in algebraic terms. In the same line is learner E, who used a guess method to provide the sum of the two numbers adding up to 52, and tried to show consecutive numbers, but in a concrete way, not using algebraic terminology (see chapter 4, p. **106 – 108**).

This research supports previous literature that problems in simplifying and factorizing equations, particularly quadratic equations, are an indication of learners' challenges with transmitting from arithmetic to algebra (Linchevski & Sfard, 2005; Kotospoulos, 2006, 2007; Oliver, 1995). Question 11 among learners interviewed shows that 3 out of 8 learners (that is 37%) displayed problems of poor transition from arithmetic to algebra. Therefore, the research supports Cummins et al. (1988) that learners' main difficulties are poor conceptual understanding of word problems and not the cognitive aspect. Didis and Erbas (2015) explain that a lack of conceptual ability creates difficulties in comprehending and interpreting the problem, such as in representing the

relationships symbolically. This is consistent with what learners experience in this study. Kotsopoulos (2007) indicates that conceptual knowledge is the deeper understanding which reflects in learners' ability to assess whether a solution to a problem makes sense or not. The researcher also explains procedural knowledge is mainly to perform mathematical computation; that is, doing calculations and algorithms effectively and efficiently. Hiebert and Lefevre (1987) have made huge contributions towards the roles of procedural and conceptual knowledge in Mathematics education. Haapasalo and Kadijevich, (1987; 2000) suggest that conceptual knowledge builds procedural fluency. Berger, (2004) opines that neither of them can be separated, but support each other. Kotospoulos (2006) asserts that learners need to rely on procedural knowledge (for example multiplication facts) and conceptual understanding (the relationship between a, b and c). For example, procedural knowledge is required for quick factorization of a quadratic equation. Generally speaking, it is evident from this study of learners' written responses, that both conceptual and procedural knowledge are necessary for them to successfully transit from arithmetic to algebra. In some questions, learners are challenged by poor conceptual understanding, while at other questions they showed procedural problems. Learner A, B and H displayed a comprehension gap in Question 3, while in Question 11 learners A, B, C, F & H had calculation problems. Previous literature explains the reason learners in this study do not adequately understand the quadratic formula. Learners in this case know and can use quadratic formula. However, they lack conceptual understanding of the quadratic concept that will help them use procedure or formula correctly. Findings in this research are consistent with studies which show that learners' difficulties to transit from arithmetic to algebra are due to a lack of understanding of how to solve equations, compelling them resort to guess work (Filloy & Rojano, 1989; Lima & Tall, 2008). This study also agrees with Panah's assertion (2010) that learners'

difficulties when solving algebraic word problems are due to a gap between their prior and new knowledge, namely the lack of transition from arithmetic to algebra.

5.8   RECOMMENDATIONS

The following are recommendations based on the findings of this study. The findings fall in the following categories: language inadequacy, errors and lack of transition from arithmetic to algebra.

Language inadequacy- It is important to seriously address English language deficiency because it is the source of the poor Mathematics performance in schools. Research studies done in this area, have led to the language policy in South Africa. The recommendation is that the teachers' implementation process in the classroom should be aligned with that of the teachers' workshop activities. It should also ensure that reports are produced regularly and efficiently in order to inform of any necessary changes during project implementation. In addition, the role of monitoring and evaluation should not be underestimated during intervention. All problems encountered during the process of implementation should be directed to implementers and the project directors.

Algebraic inadequacy- Government focuses on grades 3, 6 and 9 (that is the end of three curriculum years), preparing them, but not all grades, for ANA.   Due to the low performance of grade 9s in ANA, the government has opted to use an intervention programme, focusing on improving grade 8 and 9 teachers. However, the biggest problem this study is showing, is with Grade 7, the beginning of secondary phase. The researcher suggests grade 7 be integrated into the secondary school intervention plan for teacher training. The problem has to be addressed from the foundation, because building on weak foundations is very dangerous. In addition, the researcher

advocates that government should address the problem of assigning teachers who are not specialized to teach the subject. For example, an engineer or information technologist is assigned to teach Mathematics. Adler's findings (2014) should be taken seriously and amended. In many schools, too many teachers teaching Mathematics at lower secondary level have little, if any, training as Mathematics teachers. This researcher likens it to "putting a round peg in a square hole".

Errors

The results of the findings show that only 5 learners achieved 40% and above with the highest score less than 50%. This means that these few learners were the ones who elicited errors in this study. This study agrees with the ANA results, as it means the higher the grade assessed, the lower the learner performance. The poor performances of South African grade 9 learners in the ANA for Mathematics in 2012 (Department of Basic Education, 2012) highlights, among other factors, the need for quality professional teacher development or learning programmes. The Guskey (2000) initiatives about expert teacher improvement are procedures and actions planned to enhance expert understanding, skills and behaviour so that teachers may also improve that of the learners.

Reviewing the levels of the errors, the researcher suggests that these errors can be corrected using professional teacher development like that used by Adler (2015). This study agrees with Adler's research work, which used the Transition Mathematics 1 (TM 1) and Transition Mathematics 2 (TM2) courses as backbone to the professionally development of teachers. The intervention courses ultimately translate into increased learner achievement, although they were principally directed at increasing teacher knowledge, rather than learner attainment. His design to improve learners' attainment, should not strictly be for developing countries like South Africa, but can also

be applicable in developed countries. McMeeking et al. (2012), equally using this intervention of professional development for their teachers in the US, report similar results of learner attainment. This programme of intervention and such of those noted by the researcher as using scaffolding, could equally be a solution to South African learners' challenges.

Generally, it is evident that some of the learners have good arithmetic knowledge. It is clear there is no transition in the learning from arithmetic to algebra. This study is quite different from other studies, because it calls attention to the knowledge gap, and not necessarily errors as expected. Learners have not attained the level of algebra, but lack algebraic terminology.

Lack of transition from arithmetic to algebra

Booth (1988) asserted that learners' poor arithmetic skills are the reason for difficulties they have in algebra. For example, learner H's written response to Question 1, shows he does not have the algebraic terminology for the question; he resorted to listing all numbers between 2 and 52 and then circled the even numbers. Similarly, in Question 3 learner H shows confusion and he could not transit the arithmetical word problem into algebraic terms. This study's findings concur with previous research that learners' lack of semantic understanding of the problem makes learners solve example equations by memorizing the rules; they are not embedded in understanding. Therefore, they give inexplicit responses, which are not flexible.

Learners mostly display conjoin problems like writing $5x + 3$ as $8x$, trying to 'finish up' the algebraic expression. Others show an incorrect generalization of operation like $3 + \frac{1}{4}$ as $\frac{31}{4}$ (Matz, 1982). Learners struggle with simplifying algebraic expressions with parenthesis. For example, 2(x + 5) becomes 10x. Mathematics lesson should eradicate these basic problems soonest (Linchevski & Herscovics, 1994). Studies document that low ability learners most often display

problems of conjoin expression, just as better performing learners have limited procedural understanding of algebraic problems.

To overcome this problem and equip learners with better comprehensive schema, they need to be more familiar with everyday language vocabulary. In addition, they need to use a variety of mathematical language and symbols in order to explore and express mathematical meanings. The classroom should encourage the various ways expression can be written. These could help to explore and improve mathematical understanding.

The fact that arithmetic thinking is more of numerical determinacy and algebra is of numerical indeterminacy, makes them distinct from each other (Radford, 2006). There are valid reasons for having arithmetic placed before algebra in the Mathematics curriculum. In addition, these are compelling reasons for incorporating algebraic thinking into arithmetic in early Mathematics. There is a need to avoid separating arithmetic from algebra learning and the challenge of transition from arithmetic to algebra. This poses great challenges to learners. Therefore the researcher has deemed it fit to support the recommendations of the National Council of Teachers of Mathematics suggesting the integration of algebraic thinking in all elementary grades (NCTM, 2006). The integration of the before-mentioned algebraic thinking should then bring a solution to the discrepancies between the two. The advantage advocated are to interweave the abstract and the concrete instead of having them each separately (Carraher & Schliemann, 2002). If learners are involved too much in the concrete nature of arithmetic, it could result in them experiencing feelings of discouragement and difficulty with Mathematics. Conversely, if these are properly and reasonably integrated, it provides the opportunity to bring together concepts, which are supposed to be separate. This should solve the challenge of transition from arithmetic to algebra rather than the separation of the two by time and content. Foundation Phase to secondary school educators are

advised to incorporate algebraic thinking not only in letter, but also in deed in the classroom. Teachers should be encouraged to gradually incorporate algebraic thinking into all levels of arithmetic learning.

Conclusion: the researcher found that errors learners made in this study are similar to those found by Haghverdi et al (2013): liguististic, comprehension, semantic and calculation errors. The errors learners made were due to a non-proficiency in English and at times an inadequacy with algebraic language and at other times the gap between arithmetic and algebra.

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**APPENDIX A – MATHEMATICAL STRATEGIC WORD PROBLEM (MSWP) TEST**

TIME -1HR   **GRADE** 11

ASSESSMENT STANDARD: 11.2.4, 11.2.5 & 11.2.6   **TOTAL: 50 MARKS**

ANSWER ALL QUESTIONS AND SHOW ALL YOUR WORK ON ANSWER SHEET

1 The sum of two consecutive odd numbers is 52 what are the two odd numbers?

2 The product of 2 consecutive odd numbers is 483 Find the product of the next 2 consecutive positive even number after the largest odd number.

3. The sum of a two-digit number and the number formed by interchanging the digits is 110.The new number is 18 more than the original number. Find the original number.

4. The sum of 2 numbers is8. The difference of their squares is144. Find the numbers.

5. The sum of two numbers is 7 and the square root of their sum of the squares of the two numbers is 5.Find the two numbers.

6. The product of two integers is 95. Find the integers if their sum is 24.

7. One number is 6 more than another and the sum of the two numbers is 46. Find these numbers.

8. When 2 cm is subtracted from each side of a certain square its area is decreased by $100cm^2$ what was the original area of square?

9**.** There are two integer numbers, the first is 3 more than the second and the product is 54.find the numbers.

10**.** Tom is now a half times the age of his elder brother. 5 years ago the elder brother was 3 times the age of Tom. Find the present age of each.

11. A rectangular parking area has dimension of 50 m by 120 m If the parking area is doubled by increasing both by x meters, determine the dimension of the new parking area.

120m 50m	
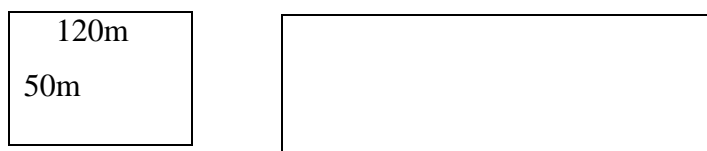
12. A bricklayer and his apprentice build a wall in 24 days when each person works separately; the apprentice takes 20 days longer than the bricklayer to complete the job. Calculate the number of days each person takes to complete the job on his own.

APPENDIX B (MEMORANDUM)

1 Let first odd be n and next consecutive odd be n+2

$$n + n + 2 = 52$$
$$2n + 2 = 52$$
$$2n = 50$$

$n = 25\sqrt{\frac{1}{2}}$[1]

$$n + 2 = 25 + 2 = 27 \sqrt{\frac{1}{2}}$$

2. Let 1st odd number will be n and 2nd odd number will be n+2

$$n(n + 2)\sqrt{\frac{1}{2}} = 483\sqrt{\frac{1}{2}}$$

$$n^2 + 2n - 483 = 0\sqrt{\frac{1}{2}}$$

$$(n + 23)(n - 21) = 0$$

$$n = -23, n = 21\sqrt{\frac{1}{2}}$$

Hence the two consecutive odd numbers, from the positive odd number: $n = 21 \& n = 23$

Therefore the two consecutive even numbers:

1st even num. will be $(n + 1)$ and 2nd will be $(n + 2)$ **[5]**

$(21 + 1) = 22\sqrt{\frac{1}{2}}$ and $(22 + 2) = 24\sqrt{\frac{1}{2}}$

$$22 \times 24 = 528 \checkmark$$

3. Let Ist digit be x and 2nd digit be y

The two digit =xy and interchange =yx

Sum of two digit and num. formed by interchanging is110

$$xy + yx = 110\sqrt{\frac{1}{2}}$$

The new number is greater than original number by 18

$$yx - xy = 18\sqrt{\frac{1}{2}}$$

$xy + yx = 110$ ....(1)

$yx - xy = 18$.......................(2)

(1)   −(2) $2xy = 110 - 18\sqrt{\frac{1}{2}}$

$$2xy = 92\sqrt{\frac{1}{2}}$$

$$xy = 46\sqrt{\frac{1}{2}}$$

Solve for yx in (2) when $xy = 46$**[4]**

$$yx - xy = 18\sqrt{}$$

$$yx - 46 = 18\sqrt{\frac{1}{2}}$$

$$yx = 18 + 46$$

$$yx = 64\sqrt{\frac{1}{2}}$$

CHECKING $xy + yx = 110$

$$46 + 64 = 110$$

4. Let 1st number be x and 2nd be number be y

Sum of two numbers:

$x + y = 8\sqrt{\frac{1}{2}}$ ....(1)

 Difference of these two numbers squares

$x^2 - y^2 = 144\sqrt{\frac{1}{2}}$ ....(2)

From (1) $x = 8 - y\sqrt{}$ ....(3)

Substitute into (2) $x^2 - y^2 = 144$ ....(2)

$$(8 - y)^2 - y^2 = 144$$

$$(8 - y)(8 - y) - y^2 = 144$$

$$64 - 8y - 8y + y^2 - y^2 = 144\sqrt{}$$

$$64 - 16y = 144\sqrt{}$$

$$-16y = 80 \qquad\qquad\qquad \textbf{[8]}$$

$$y = -5\sqrt{}$$

Solve for x: $x = 8 + 5$

$$x = 13\sqrt{}$$

CHECKING: Sum of two is 8

$$x = 13 - 5$$

$$8 \ \sqrt{}$$

5. Let first number $=x$

  Let second number $=y$

$$x + y = 7\sqrt{}$$

$$x^2 + y^2 = 25\sqrt{}$$

From (1) $x = 7 - y\sqrt{}$

Substitute (3) into (2)

$$(7 - y)^2 + y^2 = 25$$

$$(7 - y)(7 - y) + y^2 = 25\sqrt{}$$

$$49 - 7y - 7y + y^2 + y^2 = 25$$

$$49 - 14y + 2y^2 = 25$$

$$2y^2 - 14y + 24 = 0$$

$$y^2 - 7y + 12 = 0\sqrt{}$$

$$(y - 4)(y - 3) = 0$$

$y - 4 = 0$ / $y - 3 = 0$

$y = 4\sqrt{\frac{1}{2}}$ / $y = 3\sqrt{\frac{1}{2}}$

6. Let the first integer $=x$

  Let the second integer $=y$

$xy = 95\sqrt{\frac{1}{2}}$eqn (1)

$x + y = 24\sqrt{\frac{1}{2}}$eqn (2)

From (2) $x = 24 - y\sqrt{}$eqn(3)

Substitute (3) into (1)   **[4]**

$$(24 - y)y = 95$$

$$24y - y^2 = 95$$

$$y^2 - 24y + 95 = 0\sqrt{\frac{1}{2}}$$

$$(y - 19)(y - 5) = 0\sqrt{\frac{1}{2}}$$

$y - 19 = 0 \text{ or } y - 5 = 0$

$y = 19\sqrt{} \text{ or } y = 5$

Then solving for x; when $y = 19x + 19 = 24x = 5$

Checking: $xy = 955(19) = 95$

7. Let the 1st number be x

  Let the 2nd number be y

  One number is 6 more than another

$x = y + 6\sqrt{} \text{eqn}(1)$

  The sum of the two numbers is 46   **[3]**

$x + y = 46\sqrt{} \text{eqn (2)}$

  [Substitute x] $x = y + 6$ into eqn (2)

$$y + 6 + y = 46$$
$$2y = 40$$
$$\text{Implies} \quad y = 20$$
$$x = 20 + 6 = 26\sqrt{}$$

8. Let x equal the length of the side of the square

  Then $(x - 2)(x - 2)\sqrt{} = x^2 - 100\sqrt{}$

$$4x = 104$$
$$x = 26\sqrt{}$$
$$\text{Therefore the area} = X \ x \ X \qquad \textbf{[5]}$$
$$\text{Area} = 26 \times 26\sqrt{}$$
$$\text{Area} = 676\sqrt{}^{\text{cm2}}$$

9. Let first integer be x

  Let second integer be $x + 3$

$$x(x + 3) = 54\sqrt{}\frac{1}{2}$$
$$x^2 + 3x - 54 = 0\sqrt{}\frac{1}{2}$$
$$(x + 9)(x - 6) = 0\sqrt{}\frac{1}{2}$$

$x + 9 = 0 \text{ or } x - 6 = 0$

$x = -9 \text{ or } x = 6\sqrt{\frac{1}{2}}$

10.   Let x be Tom and y be the brother

$px = \frac{1}{2}y\sqrt{} \qquad$ Implies that $\quad y = 2x\sqrt{}$

$$y - 5 = 3(x - 5)\sqrt{}$$
$$y - 5 = 3x - 15\sqrt{}$$
$$2x - 5 = 3x - 15$$
$$x = 10\sqrt{}$$

Substitute $\quad x = 10 \quad$ into $\quad y = 2x$

$$y = 2(10)$$

$y = 20\sqrt{} \qquad$ Tom is 10 years and brother is 20years $\hfill$ **[6]**

11. Area $= l \times b$

Area $= 50 \times 120\sqrt{\frac{1}{2}}$

$$= 6000m^2\sqrt{\frac{1}{2}}$$

If the Area is doubled $= 2(6000)m^2$

$$= 12000m^2\sqrt{}$$

New Area $= l \times b = 12000m^2$

$$= 80 \times 150\sqrt{\frac{1}{2}} = 12000$$

$$= (50 + x)(120 + x) = 12000$$

$= (50 + 30)(120 + 30)\sqrt{\frac{1}{2}} = 12000 \hfill$ **[3]**

It isincreased by $30m$

12. Let the bricklayer take x days

The apprentice take $= (x + 20)$ days

The bricklayer and apprentice build $\frac{1}{x}$ and $\frac{1}{x+20}$ of wall each day

Together they build the wall in 24 days so in one day they will build

$\frac{1}{24}$of the wall.

$\frac{1}{24} = \frac{1}{x} + \frac{1}{x+20}$

$$\frac{24(x+20)+24x}{24(x+20)} = \frac{x(x+20)}{24x(x+20)}$$

$$` 24x + 480 + 24x = x^2 + 20x$$

$$48x + 480 = x^2 + 20x$$

$$x^2 - 28x - 480 = 0\sqrt{}$$

$(x-40)(x+12) = 0\sqrt{}$ **[4]**

x−40 = 0   or $x + 12 = 0$

$x = 40\sqrt{}$or$x = -12$ (ignore negative value it is not real)

  The bricklayer takes 40 days on his own and

   Apprentice takes $60\sqrt{}$ days on his own.

APPENDIX C

58 KRUGER STREET

EIKEHOF 16

BRONKHORSTSPRUIT

Dear Name of Principal

Re: Request for Permission to conduct research in your school

I am a M.Sc. (Mathematics Education) student at the University of South Africa. As a requirement for my studies, I am expected to carry out some research and compile a report thereof. I intend to conduct a study on *Errors grade 11 learners commit when solving algebraic word problems*. This will entail administering a test I would have developed according to the Curriculum Assessment Policy Statement and interviewing a selected sample of the learners. I therefore request permission to conduct my study in your school. Please be rest assured that the identities of the school, teachers and learners will be highly protected in order to comply with Ethical requirements laid down by UNISA.

Thanks for your kind consideration.

Yours Sincerely

Salihu F.O

PS: For any enquires, you may contact me with number- 0817505823 or Email 44632789@mylife.unisa.ac.za.

APPENDIX D

LETTER OF CONSENT FROM PARENT/GUARDIAN OF LEARNERS

Dear Parent/Guardian

I am a second year Master of Science (Mathematics Education) student at the University of South Africa under the supervision of Prof. N. N. Feza I hope to conduct a research study in the final year thesis, which examines Grade 11 students' errors in solving Algebraic word problem.

The purpose of this study is to identify errors, difficulties and possible causes of these errors learners commit in solving Algebraic word problems .This will also help suggest to schools teaching framework that could minimize such errors. I wish to administer a test to 40 learners in Grade 11 Mathematics class. The test is approximately 1 hour; it contains 12 standardized word questions. Based on the result of the test, your child may be requested to participate in an interview. This is aimed to have depth understanding of learners' thinking process and reasons for their errors. The interview will take about 30 minutes.

I would like to request the participation of your child in this study. Participation of your child in this is study, is voluntary issue and any information collected will be kept anonymous.

Please indicate on the attached form whether you permit your child to take part in this study, your cooperation will be very much appreciated. For further enquires please contact cell phone 0817505823 or email at 44632789@mylife.unisa.ac.za or supervisor at telephone 0123376049

Thanks for your cooperation

Yours Sincerely,

Salihu F.O

I………………………………………………………………………………………………..

Agree/disagree   to   allow   my   child

……………………………………………………………………………………..   to

participate in

1.   Test   YES   NO   (TICK ONE)

2.   INTERVIEW

PARENT/GUARGIAN SIGNATURE……………………………………………………

DATE   …………………………………………………………………..

APPENDIX E

STUDENT INTERVIEW FORMAT

PROCESS
INTERVIEW QUESTION

1. READING   PLEASE READ QUESTION

2. COMPREHENSION/INTERPRETATION   WHAT DOES THE QUESTION
MEAN?

3. STRATEGY SELECTION/SKILL SELECTION   HOW WILL YOU DO THE
QUESTION?

4. PROCESS   WORK OUT THE QUESTION. TELL ME WHAT
YOU DO   AS YOU PROCEED.

5. ENCODING   WRITE DOWN THE ANSWER.

6. VERIFICATION   IS THERE ANY WAY YOU CAN CHECK TO
MAKE SURE YOUR ANSWER IS RIGHT

GAUTENG PROVINCE

Department: Education

REPUBLIC OF SOUTH AFRICA

For administrative use:
Reference no: D2014/251

GDE RESEARCH APPROVAL LETTER

Date:	18 September 2014
Validity of Research Approval:	10 February to 3 October 2014
Name of Researcher:	Salifu F.O.
Address of Researcher:	58 Kruger Street
	16 Eikehof
	Bronkhorstspruit
	1020
Telephone Number:	076 799 9705
Email address:	samuel.salihu@gmail.com / 44632789@mylife.unisa.ac.za
Research Topic:	Investigating Grade 11 learners' errors in solving algebraic word problems in Mamelodi East schools, Tshwane South District; Gauteng; South Africa
Number and type of schools:	FOUR Secondary Schools
District/s/HO	Tshwane South

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

2013/10/0 2

The following conditions apply to GDE research. The researcher may proceed with the

1

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

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above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

Appendix G

1. The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.
4. A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
5. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
6. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
7. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
8. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
9. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
10. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
11. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
12. On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
13. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
14. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

[signature]
....
Dr David Makhado
Director: Education Research and Knowledge Management

DATE: 2013/10/02
....

2

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

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Appendix H

Dear Mrs Salihu, F. O. (44632789)

UNISA

college of
science, engineering
and technology

Date: 2014-09-10

Application number:

2014_CGS/ISTE_020

REQUEST FOR ETHICAL CLEARANCE: (An investigation of grade 11 learners' errors when solving algebraic word problems in Gauteng Province, South Africa.)

The College of Science, Engineering and Technology's (CSET) Research and Ethics Committee has considered the relevant parts of the studies relating to the abovementioned research project and research methodology and is pleased to inform you that ethical clearance is granted for your research study as set out in your proposal and application for ethical clearance.

Therefore, involved parties may also consider ethics approval as granted. However, the permission granted must not be misconstrued as constituting an instruction from the CSET Executive or the CSET CRIC that sampled interviewees (if applicable) are compelled to take part in the research project. All interviewees retain their individual right to decide whether to participate or not.

We trust that the research will be undertaken in a manner that is respectful of the rights and integrity of those who volunteer to participate, as stipulated in the UNISA Research Ethics policy. The policy can be found at the following URL:
http://cm.unisa.ac.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf

Please note that the ethical clearance is granted for the duration of this project and if you subsequently do a follow-up study that requires the use of a different research instrument, you will have to submit an addendum to this application, explaining the purpose of the follow-up study and attach the new instrument along with a comprehensive information document and consent form.

Yours sincerely

Prof Ernest Mnkandla
Chair: College of Science, Engineering and Technology Ethics Sub-Committee

Prof IOG Moche
Executive Dean: College of Science, Engineering and Technology

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2015 -02- 1 8

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nce, Enginee
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Appendix I

ANNEXURE 3: EXAMPLE OF A GRADE 11 WORK SCHEDULE FOR MATHEMATICS

Grade 11 Mathematics Work Schedule

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
1		11.2.4 Manipulate algebraic expressions: (a) by completing the square, (b) simplifying algebraic fractions with binomial denominators. 11.2.5 Solve (a) quadratic equations (by factorisation, by completing the square, and by using the quadratic formula) and quadratic inequalities in one variable and interpret the solution graphically;		
2		(b) equations in two unknowns, one of which is linear and one of which is quadratic, algebraically or graphically.		

3	**Mathematical Modelling with Quadratic Equations**
	(including: manipulating algebraic expressions; solving quadratic equations and equations in two unknowns; non-real numbers)
		11.2.6 Use mathematical models to investigate problems that arise in real-life contexts:
		   (a) making conjectures, demonstrating and explaining their validity,
		   (b) expressing and justifying mathematical generalisations of situations,
		   (c) using various representations to interpolate and extrapolate,
		   (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and pertinent features.		

4		(Examples should include issues related to health, social, economic, cultural, political and environmental matters.) 11.1.1 Understand that not all numbers are real. (This requires the recognition but not the study of non-real numbers.) 11.1.6 Solve non-routine, unseen problems.		
5	**Exponents and Surds** (consolidation of laws etc. as needed in other topics)	11.1.2 (a) Simplify expressions using the laws of exponents for rational exponents. (b) Add, subtract, multiply and divide simple surds (e.g. see Subject Statement) (c) Demonstrate an understanding of error margins.		
6	**Number Patterns** (including simple and compound decay)	11.1.3 Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence (a) make conjectures and generalisations		

Appendix J

QUESTIONS ANALYSIS

NO	BLOOM TAXONOMY LEVEL	CURRICULUM SECTION(CAPS)	KNOWLEDGE REQUIREMENT	POSSIBLE ERRORS
1	KNOWLEDGE	11.2.4	1. SIMPLIFICATION OF LINEAR EQUATION. 2. MANIPULATIONALGEBRA EQUATION	1. DIFFICULTY O IDENTIFYING(2) CONSECUTIVE NUMBERS WITH (N+2) AFTER N=
2	1.KNOWLEDGE 2.COMPREHENSION 3.APPLICATION 4. ANALYSIS	11.2.5(a)	1. MEANING OF ALGEBRA EQUATION. 2. SOLVING QUADRATIC EQUATION BY FACTORIZATION.	1. PROBLEM OF IDENTIFYING NEXT TWO POSITIVE EVEN NUMBERS.
3	1.KNOWLEDGE 2.COMPREHENSION 3.APPLICATION 5 SYSTHESIS	11.2.4	1. MANIPULATION OF ALGEBRA EQUATION. 2. ABILITY TO THE SUBTITUTE. 3. ABILTY TO FIND THE INTERCHANGE.	1.PROBLEM OF UNDERSTANDIN THE INTERCHANGE OFTWO DIGIT X YX 2. PROBLEM OF UNDERSTANDIN THE DIFFERENC BETWEEN

				ORIGINAL AND NEW DIGIT.
4	2.COMPREHENSION 3.APPLICATION 4. ANALYSIS	11.2.4 11.2.5(a) 11.2.5(b)	1. MANIPULATE ALGEBRAIC EQUATION. 2. SOLVING TWO UNKNOWN ONE LINEAR & ONE QUADRATIC EQUATION.	1. PROBLEM OF REPRESENTING THE SUM OF TH (2) NUMBERS. 2. PROBLEM OFABLE TO SUBSTITUTE.
5	2.COMPREHENSION 3. ANALYSIS 4. APPLICATION	11.2.4 11.2.5(a) 11.2.5(b)	1. MANIPULATION OF ALGEBRAIC EQUATION. 2. SUBSTITUTING ONE LINEAR TO A QUADRATIC EQUATION. 3.FACTORIZATIONTO GET THE NUMBERS X,Y	1. PROBLEM OF ARRIVING AT THE DIFFERENC BETWEEN TWO NUMBERS. 2. PROBLEM OF SUBSTITUTING LINEAR INTO QUADRATIC. 3. PROBLEM OF FACTORIZING THE QUADRATI TO GET ONE NUMBER(+) & ANOTHER NUMBER(-)
6	1.KNOWLEDGE 2. COMPREHENSION	11.2.5(b)	1. MANIPULATION OF ALGEBRAIC EQUATION. 2. ABILITY TO INTERPRETE THE PRODUCT AS XY.	1. PROBLEM OF INTERPRETING THE WORD "PRODUCT' ANI ALSO THE WORI 'TOTAL' TO

	3. ANALYSIS		3. ABILITY TO COMBINE THE TWO EQUATIONS WITH TWO UNKNOWN AND ARRIVE AT A QUADRATIC EQUATION. 4. ABILITY TO FACTORIZE THE QUADRATIC TO GET THE INTEGERS.	ARRIVE AT ALGEBRAIC EQUATIONS. PROBLEM OF FACTORIZATION OF QUADRATIC EQUATIONTO GET THE INTEGERS.
7	1.KNOWLEDGE 2. COMPREHENSION	11.2.4	SIMPLIFYING ALGEBRAIC EQUATION.	1.INABILITY TO INTERPRETE '6 MORE' AS ADDING 6 TO VARIABLETO MAKE THE OTHER VARIABLE.
8	1.KNOWLEDGE 2. COMPREHENSION 4.ANALYSIS 5. SYNTHESIS	11.2.6(b)	1 .EXPRESSING AND JUSTIFYING MATHEMATICALGENERALIZATIONSOF SITUATIONS. 2. RECOGNISE THAT (20 SHOULD BE SUBTRACTED FROM THELENGTH. 3. MANIPULATING THE QUADRATIC EQUATION TO GET VALUE OF LENGTH	1. INABILITY TO RELATE IT TO AREA OF SQUARE. 2. PROBLEM OF NOT BEING ABL TO SUBTRACT (FROM EACH LENGTH L.
9	1.COMPREHENSION 3. APPLICATION	11.2.5(a)	SOLVING QUADRATIC EQUATION BY FACTORIZATION.	1. PROBLEM OF ARRIVING AT THE VARIABLE.

	4. ANALYSIS			2. PROBLEM TO RECOGNISE THA[ ] MULTIPLYING THE (2) VARIABLE WILL[ ] GIVE A QUADRATIC EQUATION. 3. PROBLEM OF KNOWING TO USE THE (+) VALUE OF FACTORIZATIO[ ]
10	1.KNOWLEDGE 2.COMPREHENSION 4. SYNTHESIS	11.2.6(a)	1. KNOWING HOW TO REPRESENT A QUARTER OF SOMETHING. 2. KNOWING ALGEBRAIC EQUATION TO REPRESENT 10 YRS AGI OF BOTH TOM& BROTHER.	1. PROBLEM OF REPRESENTING TOM AS THE QARTER OF BROTHER. PROBLEM OF 2.MANIPULATIN[ ] THE ALGEBRAI[ ] EQAUTION BY SUBSTITUTING OF ONE VARIABLE INT[ ] ANOTHER TO ONE
11	1.KNOWLEDGE 2. COMPREHENSION	11.2.6(c)	1. RECOGNISE THE AREA OF RECTANGLE. 2. KNOWING THAT MULTIPLICATION OF L & B WILL GIVE AREA.	PROBLEM OF REASONING ACCORDINGLY.

	3. APPLICATION		3. ALSO KNOWING THAT MULTIPLYING BOTH L & B BY A CERTAIN VALUE WILL GIVE DOUBLE THE AREA.	
12	2. COMPREHENSION 3. ANALYSIS 5. SYNTHESIS	11.2.4(b) 11.2.6(d) 11.2.5(a)	1. RECOGNISE THE BRICKLAYER & APPENTICE IN PARTS. 2. ABLE TO SIMPLIFY ALGEBRAIC FRACTIONS. 3. ABLE TO FACTORISE QUADRATIC THEN REGET THE (-) VALUE.	PROBLEM TO RECOGNISE THE PARTS OF THE INDIVIDUAL WORK OR WHICH FORM FRACTION PROBLEM OF BEING ABLE TO SIMPLIFY QUADRATIC EQUATION.