South African
Computer
Journal
Number 18
December 1996

Suid-Afrikaanse
Rekenaar-
tydskrif
Nommer 18
Desember 1996

Computer Science and Information Systems

Special Edition: Computer Security

Rekenaarwetenskap en Inligtingstelsels

The South African Computer Journal
An official publication of the Computer Society of South Africa and the South African Institute of Computer Scientists

Die Suid-Afrikaanse Rekenaartydskrif
'n Amptelike publikasie van die Rekenaarverenging van Suid-Afrika en die Suid-Afrikaanse Instituut vir Rekenaarwetenskaplikes

Editor
Professor Derrick G Kourie
Department of Computer Science
University of Pretoria
Hatfield 0083
dkourie@dos-lan.cs.up.ac.za

Subeditor: Information Systems
Prof Lucas Introna
Department of Informatics
University of Pretoria
Hatfield 0083
lintrona@econ.up.ac.za

Production Editor
Dr Riël Smit
Mosaic Software (Pty) Ltd
P.O.Box 23906
Claremont 7735
gds@mosaic.co.za

World-Wide Web: http://www.mosaic.co.za/sacj/

Editorial Board

Subscriptions

	Annual	Single copy
Southern Africa:	R50,00	R25,00
Elsewhere:	$30,00	$15,00

An additional $15 per year is charged for airmail outside Southern Africa

to be sent to:
Computer Society of South Africa
Box 1714 Halfway House 1685

Phone: +27 (11) 315-1319 Fax: +27 (11) 315-2276

Drukkers Printers

Guest Contribution

A Pragmatic Approach to Development Information to Provide Service on a Wide Scale

Stephen S Mncube, PhD
Divisional Manager: Development Information Services, Centre for Policy and Information, Development Bank of Southern Africa

1 Introduction

The rapid technological growth in our time has produced an explosion of information. This, in turn, has spawned information systems based on the use of computers and automated systems. These mechanised devices with their seemingly infinite capacity to store and retrieve knowledge on command have myriad applications. But the use of computer and automated information devices pose serious problems to individuals, groups and societies on an international scale in disseminating the available information. This is even more true in information flow between regions with high information capabilities than in those that have little or none. The information flow between these regions has been varied and frequently haphazard whenever it has existed.

In this context the philosophy for development information speaks to interdependence and humanitarian concern in information flow to various regions of the earth. Information must be perceived as a universal entity. The initial failure of one region and the ability of another to acquire information should not dictate a permanent global demarcation into 'developed and less developed' categories.

2 Objectives

The objective of development information is for it to function as an interlinking mechanism between a complex industry of information resources and the users of information. In addition, development information should be a catalytic agent that aims at providing objective clarification in information needs between regions with high information capabilities and those with little.

3 Mission

The mission is to strive for a move in all regional, national and international agencies and organisations concerned with information to give full co-operation and assistance in setting criteria and standards, formulating policy, and assessing the information needs of a given region of the world. Development information is capable of realistically assessing information related to socio-economic development in the light of the unique requirements of world regions in need of information and information technology.

4 The Dynamics of Information Processing

The proliferation of information technology production has resulted in the growth and development of an ever expanding information packaging industry. This area has become so large in size and scope that it is necessary to discuss some of the important developments that are taking place in the area.

The newest phrases now being used in information packaging range from electronic archives, compact discs (CDs), computer tapes, microfiche, teletexts, video discs, magnetic tapes and interactive imaging systems (optical systems) to word processing and the use of laser technology. Developing countries will not escape this new wave of information packaging. It soon will be bombarded with vendors of these products, to a point where some adaptation will be inevitable.

Assuring the quality of technical processes and the accuracy of packaging information is becoming an increasingly difficult task. Rapid increases in the volume of information, the sophistication of information uses, and the complexity of material flows and processes are characteristic of most modern technical environments. As complexity increases, the risk of introducing significant errors into material processes increases. The very complexity of such systems makes the detection of error itself a complex task. With increasing frequency, public and private organisations are seeking help from corporations with experience in quality control and information validation to ensure that technical process and information packaging meet performance and accuracy standards. The problems association with quality control and validation can be minimised by following these guidelines:

- Establishing ways of aiding in planning, organisation and control of software purchasing and development

through
  - creating a directory of software suppliers
  - evaluating the quality of software supplied
  - keeping abreast of the state-of-the-art in software production
- Providing leadership in the innovative use of software materials and the utilisation of extensive market research on software before making a major purchase
- Establishing cost-efficient ways of packaging and designing your own software by learning how to design and evaluate software for your own use

Preparation for these new technologies for packaging information ought to be made in institutions of higher learning. Perhaps it would be timely to introduce some of these concepts in technical institutes in order that future demands imposed by the new information technology may be met.

The successful growth of developing countries information technology will ultimately depend upon the commitment of substantial resources, especially financial resources. The successful application of this information technology will require more than the mere receiving and storing of it. In addition to the tasks of acquiring and organising informational materials, channels must be established to analyse incoming information. Too much of the information technology that does get transferred out of the industrialised nations is never utilised because it is unsuited for the consumption of users in developing countries. A great deal more effort must be made to analyse, package and disseminate materials on existing and forthcoming information in all vital areas of work and study so that these technologies will be accessible to the developing countries' information-user communities.

5 The Need to Establish a Consortium

Information technology specialists need to establish a consortium of regional, national and international information networks and associations. The consortium could be an open structure inviting any institutions, organisations and agencies existing for the purpose of forming a network or documentation clearinghouse and of providing information technology not as an end-product but as a means for human change.

The element common to all membership is an interest in and dedication to providing useful and accurate information that can bring about humanistic change. Equally important is a commitment to the development of relevant information resources to meet the needs of regions with low information capacity.

The philosophical outlook of the consortium would therefore be to crystalise and emphasise broad knowledge, deep understanding, and imaginative efforts, including a dedication to great ideas in providing accurate solutions to the information needs of various regions on an international scale.

6 The Organisation of Services for Members

The ideas constituting a conceptual framework for a service-oriented consortium are as multitudinous as the Kalahari sands but in this instance the consortium could function to:

- support creative change within its membership
- facilitate and support new educational enterprises and programs addressed to meet the needs of previously disenfranchised persons
- develop and implement co-operative programs and projects among it members
- provide a meeting ground for a diversity of persons, institutions, and agencies with common values and purposes
- provide a forum for the exchange of ideas among its member associates
- encourage methods of solving social problems
- influence public policy to be consistent with its mission and purpose.

7 An Appraisal of Internet

While casual observers have the leisure to observe unobtrusively the growth and development in Internet to be a world-wide phenomenon in information sharing, they do so at no cost. On the other hand, information specialists have to judge and weigh the work of an ongoing Internet program and estimate its usefulness as a network or networks to their daily operations. Information specialists are, therefore, still more sceptical about the scope and magnitude of the Internet. They alone are facing challenges of adding another performance task of being evaluation researchers of Internet in order to provide objective clarification of incorporating Internet as an integral component of their information system. This can be a tedious undertaking because it entails not only knowing how to navigate the Internet network but also cognisance of the following key factors:

- how appropriate the Internet is to your information environments
- to what extent the databanks provided through Internet are relevant to the mission and objective of your environment
- what the relationship is between costs and benefits of having Internet at your disposal

All these factors need to be addressed to determine the effectiveness of Internet in any given information environment, be it in a government setting or in other work environments.

8 Training in Information Networks

A seminar for the network should be designed to launch the co-operative exchange of knowledge and experience with information accessibility and utility of the participant's respective information holdings. To succeed in this effort, the organisers will marshal appropriate interdisciplinary experts and technical resources from within the regions involved. The content of the seminar will consist of information related to formalising and establishing a plan for information networking. It is important that information ministries achieve a high level of knowledge and sensitivity to the information needs of their individual country, region, and ultimately the world, in order to assess, prescribe, design, manage and evaluate the most appropriate uses of information technology for enhancing the advancement of their world countries.

Although the organisers will determine who will be invited to participate in this seminar, special effort will be made to ensure that representatives come from a diversity of backgrounds, and have some knowledge or experience relating to information systems. Additionally, consideration will be given to the level of information technology which is currently utilised by the representative's country.

The content of the seminar will be tailored to meet the specific needs and issues designated by the participants through a pre-seminar survey and needs assessment, which will be administered by the organisers.

9 Summary

The above aims at presenting some possible scenario and does not pretend to be exhaustive. The issues, however vital to development information, are given cursory treatment here. It remains important for the luminaries in this area to expand on some of the thoughts contained above.

Bibliography

1. A D Burnett. *Technology for information in development: proceedings of the sixth conference of the International and Comparative Librarianship Group of the Library Association.* International and Comparative Librarianship Group of the Library Association, Brighton Polytechnic, Falmer, Sussex, August 21–23 1987.
2. M Cook. Guidelines on curriculum development in information technology for librarians, documentalists, and archivists. Paris Unesco, 1986.
3. O Fairfax, J Durham, and W W. Audio-visual materials: development of a national cataloguing and information service: Report of a joint feasability study for the british library and the council for educational technology. Distributed by Councils and Education Press, 1976.
4. M Fransman. *1948 – The market and beyond: Co-operation and competition in information technology development in the Japanese system.* Cambridge University Press, New York, 1990.
5. N K Hanna. The information technology revolution and economic development. World Bank Discussion Papers, World Bank Washington DC., 1991.
6. N Heaton and L W MacDonald. *Human factors in information technology product design and development.* Ellis Horwood, 1991.
7. T Koizumi and G E Lasker, eds. *Advances in education and human development: new trends in learning and teaching evolution of human systems and institutions, individual freedom and sociopolitical controls in the high technology environment of the advanced information society.* International Institute for Advanced Studies, Baden-Baden, West Germany, 1990.
8. V Sethi. The development of measures to assess the extent to which an information technology application provides competitive advantage. Ann Arbor: MI, University Microfilm International, University of Pittsburgh, 1988.

SACJ is produced with kind support from
Mosaic Software (Pty) Ltd.

The Complexity of Petri Net Transformations*

Stephen R. Donaldson† Falko Bause‡ Pieter S. Kritzinger§

†*Oxford University Computing Laboratory. sd@comlab.ox.ac.uk*

‡*Lehrstuhl Informatik IV, Universität Dortmund. bause@ls4.informatik.uni-dortmund.de*

§*Department of Computer Science, University of Cape Town. psk@cs.uct.ac.za*

Abstract

We consider the complexity of several property preserving Petri net transformations found in the literature. These transformations were candidates for inclusion in a software tool to perform liveness and boundedness analysis on Petri nets. These results show that it is infeasible for certain transformations in general to assist in liveness and boundedness analysis. We also consider synthesis transformations and show, for each transformation, their complexities. In each case, we consider the complexity of the decision problem with regard to the applicability of the transformation.

Keywords: *Petri Nets, Analysis of Algorithms, Problem Complexity, Program Verification, Model Validation and Analysis, Model Development, Reduction and Synthesis Transformations, Liveness, Boundedness*

Computing Review Categories: *D.2.2, D.2.4, F.2.0, I.6.4, I.6.5*

1 Introduction

The authors considered implementing certain Petri net transformations in a Petri net analysis tool in order to assist with the liveness and boundedness analysis of large Petri nets. Petri net transformations have been proposed to be used in two different ways: as *synthesis* or *reduction* transformations. In each case, only property preserving transformations are investigated. Synthesis transformations are applied to a Petri net in which the desired properties are known to hold (either by construction or because state space analysis is possible), producing a larger Petri net in which the desired properties are now guaranteed to hold. Reduction transformations are intended to reduce the state space in such a way that a state space analysis may be performed to determine if the desired properties are held by the original Petri net. The distinction between reduction and synthesis transformations is usually determined by their use, as each can often be reversed and used as the other type of transformation.

While compiling our list of transformations from the literature, it became evident that the decision problems regarding their applicability fell into one of three classes. One could classify two of these classes as structural, where only the incidence matrices and the initial marking need be inspected in order to determine whether the transformation is applicable. We consider a property dependent on the initial marking, that can be determined without state space exploration, to be a structural property, rather than a behavioural property. The third class depends on the structure of the state space and may require the (partial) generation of the state space in order to determine whether the transformation applies. Furthermore, we do not directly assume or show that state space exploration, partial or full, is required for any of the problems that we consider in this paper. Since we use problem reductions to prove our results, this fact is hidden in the decision problem from which the reduction emanates. The structural classes mentioned above are the following: the first includes decision problems that can be solved simply by scanning the incidence matrices and the initial marking of the Petri net; the other class involves deciding whether some relation exists between the entries of the incidence matrices, and possibly the entries of the initial marking. We have found that all transformation decision problems that can be decided by scanning the incidence matrices and initial marking can be decided in polynomial-time. Those problems that determine whether there exists a dependency among the entries of the incidence matrices and the initial marking, range in complexity from polynomial-time problems to $\mathcal{NP}$-complete ($\mathcal{NPC}$). Those problems that depend on the behaviour of the resultant net are **EXPSPACE**-hard [5, 22, 12].

For the **EXPSPACE**-hard results we use Liptons results cited in Jones, Landweber and Lien [13], and use problem reduction and the **EXPSPACE**-hardness of the liveness and reachability problems. For polynomial-time results, Donaldson [6] shows code fragments and asserts that these are a valid interpretation of the applicability conditions of the particular transformation.

In the remainder of the paper, we give the notation used, followed by definitions of transformation rules. We then consider the complexity of various reduction transformations followed by the complexity of various synthesis transformations.

2 Definitions and Notation

We include here the "standard definitions" related to Petri nets simply to establish a notation that we can use through-

*This work was carried out at the Data Network Architectures Laboratory, Department of Computer Science, University of Cape Town, Private Bag RONDEBOSCH, 7700, South Africa. URL: http://www.cs.uct.ac.za

out the paper.

An *unmarked Petri net* is a 4-tuple $N = (P, T, B, F)$ where: $P = \{p_1, \ldots, p_m\}$ is a finite and non-empty set of *places*. $T = \{t_1, \ldots, t_n\}$ is a finite and non-empty set of *transitions*. $B \in \mathbb{N}^{m \times n}$ is the *backward incidence matrix* and describes the arcs and their respective weights from the places to the transitions. $F \in \mathbb{N}^{m \times n}$ is the *forward incidence matrix* and describes the arcs and their weights from the transitions to the places. Further, $P \cap T = \emptyset$. The *incidence matrix* C of the Petri net N is defined by $C = F - B$.

A *marking* M of a *marked Petri net* $N = (P, T, B, F, M_0)$ is a vector $M \in \mathbb{N}^m$ giving a distribution of tokens among the places in P. M_0 is the *initial marking* and is the marking of the Petri net in its initial state. A marking M' is said to *cover* a marking M'' if and only if $M'(p) \geq M''(p)$, $\forall p \in P$,written $M' \geq M''$. $M(p)$ is the number of tokens on the place $p \in P$ at marking M.

This matrix notation of Petri nets will be used throughout this paper but it is equivalent to the functional notation often used in the literature. $B(p, t)$, for example, describes the arc from the place $p \in P$ to the transition $t \in T$ and its arc weight.

Given Petri net $N = (P, T, B, F)$,a transition $t_i \in T$ is said to be *enabled* at a marking M' of N if and only if $M' \geq Be_i$ where e_i is the unit vector with a 1 as the i-th component. A transition $t_i \in T$ enabled at a marking M of N may *fire* yielding a new marking M'' of N defined by $M'' = M' + (F - B)e_i = M' + Ce_i$.

The firing of a transition usually changes the marking of a Petri net. If M_1 was the marking before the firing of a transition $t \in T$ and if M_2 the marking after firing t then M_2 is said to be *directly reachable* from M_1 by firing t and is written $M_1 \xrightarrow{t} M_2$. The *reachability set* is the transitive and reflexive closure of direct reachability and is written $R(M_0)$. $R(M)$ is the set of all possible markings reachable from the marking M.

Let $N = (P, T, B, F, M_0)$ be a Petri net with an initial marking, a sequence

$$\sigma = M_0 t_{i_1} M_1 t_{i_2} M_2 \ldots t_{i_k} M_k$$

of transitions $t_{i_1}, t_{i_2}, \ldots, t_{i_k} \in T$ and markings $M_0, M_1, M_2, \ldots, M_k \in R(M_0)$ is called a *firing sequence* or *occurrence sequence* if and only if $M_0 \xrightarrow{\sigma} M_k$. The firing sequence is often shown without any markings and in this case it is called a *transition sequence* and is written $\sigma = t_{i_1} t_{i_2} \ldots t_{i_k}$.

If $\sigma = t_1 t_2 \ldots t_k$ is a firing sequence then $\#(\sigma, t_i)$ is a count of the number of times the transition t_i appears in the sequence σ.

For an element $u \in P \cup T$ the set $\bullet u$ is called the *pre-set* of u and $u \bullet$ is called the *post-set* of u and are defined as follows: $\bullet t = \{p \in P : B(p, t) \neq 0\}$ and is the set of all input places of the transition $t \in T$. $t \bullet = \{p \in P : F(p, t) \neq 0\}$ and is the set of output places of the transition $t \in T$. $\bullet p = \{t \in T : F(p, t) \neq 0\}$ and is the set of all input transitions of the place $p \in P$. $p \bullet = \{t \in T : B(p, t) \neq 0\}$ and is the set of all output transitions of the place $p \in P$.

Let $N = (P, T, B, F)$ be a Petri net. A net $N' = (P', T', B', F')$ is called a *subnet* (Genrich and Stankiewicz-Wiechno [11]) of N if and only if $P' \subseteq P$,$T' \subseteq T$ and

$$F'(p,t) = \begin{cases} F(p,t) & \text{if } p \in P' \text{ and } t \in T', \text{ and} \\ 0 & \text{otherwise, and} \end{cases}$$

$$B'(p,t) = \begin{cases} B(p,t) & \text{if } p \in P' \text{ and } t \in T', \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

A Petri net $N = (P, T, B, F, M_0)$ is said to be *k-bounded*,$k \in \mathbb{N}$, or simply *bounded* if and only if $\forall M \in R(M_0)$,$M(p) \leq k \; \forall p \in P$.

In a Petri net $N = (P, T, B, F, M_0)$,a transition $t \in T$ is said to be *live* (Reisig [17]) if and only if $\forall M \in R(M_0)$, $\exists M'$ reachable from M such that M' enables t. The Petri net N is said to be *live* if and only if $\forall t \in T$,t is live.

We have concentrated on the *well behaved* properties, that is, the properties of liveness and boundedness, and on well behaved Petri nets, that is, those that are both live and bounded.

An unmarked Petri net $N = (P, T, B, F)$ is said to be *structurally bounded* (Esparza and Silva [9]) if and only if for all possible initial markings M'_0 the Petri net $N' = (P, T, B, F, M'_0)$ is bounded.

An unmarked Petri net $N = (P, T, B, F)$ is said to be *structurally live* (Esparza and Silva [9]) if and only if there exists an initial marking M'_0 such that the Petri net $N' = (P, T, B, F, M'_0)$ is live.

Some of the results and transformations mentioned in this paper pertain to subclasses of Petri nets. The following classes of Petri net feature quite prominently in the literature on transformations of Petri nets.

A Petri net $N = (P, T, B, F)$ is called an *ordinary* Petri net if and only if

$$\begin{aligned} B(p,t) &\in \{0,1\} \forall p \in P \text{ and } \forall t \in T \text{ and} \\ F(p,t) &\in \{0,1\} \forall p \in P \text{ and } \forall t \in T. \end{aligned}$$

A *marked graph* is an ordinary Petri net $N = (P, T, B, F)$ such that $|\bullet p| = 1$ and $|p \bullet| = 1$ for all $p \in P$.

A *free choice net* is an ordinary Petri net $N = (P, T, B, F)$ such that $|p \bullet| \leq 1$ or $\bullet(p \bullet) = \{p\}$ for all $p \in P$.

3 Transformation Rule

Consider a Petri net $N = (P, T, B, F, M_0)$ and let S_N be the set of all Petri nets, a *transformation rule* $\mathcal{R}$ is a rewrite rule together with a predicate $\mathbf{P} : S_N \longrightarrow \{$*true*,*false*$\}$ that produces a new Petri net $N' = (P', T', B', F', M'_0)$, written $N \xrightarrow{\mathcal{R}} N'$,provided $\mathbf{P}(N)$ holds. The transformation rule describes how the Petri N' is derived from N. The predicate $\mathbf{P}$ is called the *application condition* for the transformation rule $\mathcal{R}$. The Petri net N is called the *source* and N' the *target*.

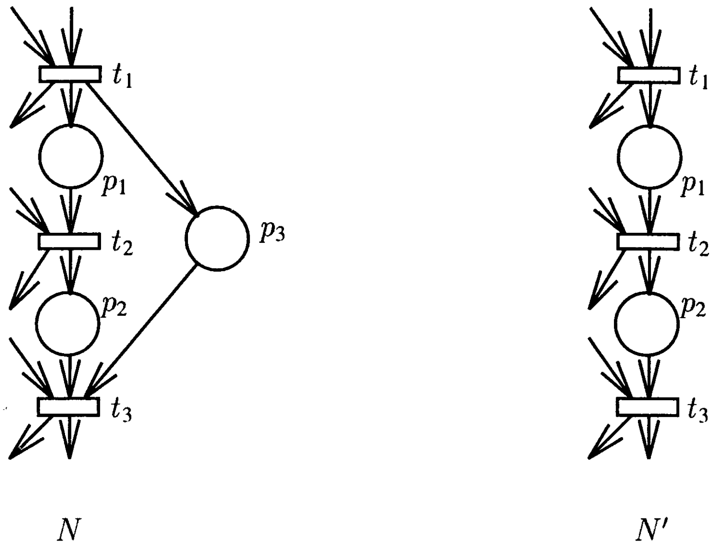

Figure 1. Simplification of redundant places

Let $\mathcal{R}$ be a transformation rule, $N = (P, T, B, F, M_0)$, $N' = (P', T', B', F', M_0')$ be Petri nets such that $N \xrightarrow{\mathcal{R}} N'$ and let $\mathcal{C}$ be a class of Petri nets characterised by a set of properties $\mathcal{C}_P$. The transformation rule $\mathcal{R}$ is said to be *property preserving with respect to* $\mathcal{C}_P$ if $N \in \mathcal{C} \iff N' \in \mathcal{C}$. Such a transformation rule is said to be *strongly sound* with respect to $\mathcal{C}_P$. If $N \in \mathcal{C} \implies N' \in \mathcal{C}$ then the transformation is said to be *sound* with respect to $\mathcal{C}_P$ (according to Esparza [8]).

Synthesis transformations often start with a particular Petri net in which the properties of liveness and boundedness are known to hold. The smallest such Petri nets have only one place and one transition: A Petri net $N = (\{p_1\}, \{t_1\}, B, F)$ where $B, F \in \mathbb{N}^{1\times 1}$ is called an *atomic net*, whether or not an initial marking is supplied. The atomic net is an example of a Petri net in which liveness and boundedness analysis is straightforward.

4 Complexity of Reduction Transformations

In this section we review each of the reduction transformations surveyed and consider the complexity of the decision problems regarding their applicability. These transformations are due to Berthelot [1, 2] and Berthelot, Roucairol and Valk [3]. The transformations make localised changes to the structure of the Petri net and sometimes also to the initial marking. Other transformations in Berthelot [1], which Berthelot calls *addition of nets* are synthesis transformations, these will be dealt with in Section 5.

Simplification of Redundant Places

If a place is such that its marking can never inhibit the firing of any transition connected to it then it is a redundant place. This transformation removes redundancies with regard to a place by removing arcs from transitions surrounding the place. If all arcs are removed then the place can be removed from the Petri net. Such places are said to be *redundant* which Berthelot [1] defines in terms of the application conditions of this transformation:

Definition 1 Redundant Place *A place $p' \in P$ is said to be* redundant *if and only if there exists an m-vector $v \in \mathbb{Z}^m$ such that the following four conditions apply:*

$$\begin{aligned} v(p) & \begin{cases} > & 0 \quad \text{if } p = p' \text{ and} \\ \leq & 0 \quad \text{if } p \neq p', \end{cases} && (1) \\ \exists b & \in \mathbb{N} : v^T M_0 = b, && (2) \\ v^T B & \leq b e^T \text{ and} && (3) \\ v^T (B - F) & \leq O^T. && (4) \end{aligned}$$

(where e is the n-vector with a one for each component.)

These conditions are a reformulation in linear algebra of the conditions given in Berthelot [1].

After having identified the redundant place the transformation can be performed by removing all edges in $\bullet p'$ and $p' \bullet$, then for every transition, $t_i \in T$, an edge with weight

$$w_i = -v^T C e_i \qquad (5)$$

is added from t to p'. If all the weights w_i are zero then the place p' is isolated and can be removed.

Figure 1 from Berthelot [1] shows a part of a Petri net that has a redundant place, p_3 in N. N' is the same part of the Petri net after the transformation of simplifying redundant places. In this example all the weights are zero and the place p_3 has been removed.

Re-arranging, Eqs (2), (3) and (4) gives an augmented matrix

$$D = [(B - M_0 e^T) | (B - F) | (-M_0)]$$

and the matrix inequality

$$v^T D \leq O^T.$$

Define an m-vector u such that

$$u(p) = \begin{cases} v(p) & \text{if } p = p' \text{ and} \\ -v(p) & \text{if } p \neq p'. \end{cases}$$

and a matrix $A = (a_{ij})$ with the same dimensions as $D = (d_{ij})$, that is $|P| \times (2|T| + 1)$, such that

$$a_{ij} = \begin{cases} d_{ij} & \text{if row } i \text{ corresponds to place } p' \text{ and} \\ -d_{ij} & \text{otherwise.} \end{cases}$$

The problem of deciding whether the place p' is redundant reduces to whether there exists a nonnegative solution to the matrix inequality

$$u^T A \leq O^T \text{ where } u(p) \begin{cases} \geq 1 & \text{if } p = p' \text{ and} \\ \geq 0 & \text{if } p \neq p'. \end{cases} \quad (6)$$

We need a few definitions and results in order to show the complexity of deciding place redundancy:

Definition 2 Linear Programming Problem *Given a matrix $A \in \mathbb{Q}^{m\times n}$ and a column vector $b \in \mathbb{Q}^m$ does there exist an $x \in \mathbb{Q}^n$ such that $Ax \leq b$?*

Definition 3 Integer Linear Programming Problem *Given a matrix $A \in \mathbb{Q}^{m\times n}$ and a column vector $b \in \mathbb{Q}^m$ does there exist an $x \in \mathbb{N}^n$ such that $Ax \leq b$? The $\{0, 1\}$* integer linear programming problem, *or ZERO-ONE ILP, is defined as for the ILP problem except that the solution vector x is required to belong to $\{0, 1\}^n$.*

Definition 4 Homogeneous Integer Linear Programming Problem *Given a matrix $A \in \mathbb{Q}^{m\times n}$ does there exist an $x \in \mathbb{N}^n$ such that $Ax \leq O$ and $x \neq O$? The $\{0, 1\}$* homogeneous integer linear programming problem, *or ZERO-ONE HILP, is defined as in the HILP case except that the solution vector x is required to belong to $\{0, 1\}^n$.*

The sequel assumes that the entries of A and b are integers as a transformation from rational entries (pairs of integers) to integer entries will at most square the size of the problem without changing the existence of a solution (van Emde Boas [21]).

Theorem 1 [15] *Deciding whether there exists a solution to a given LP problem is in $\mathcal{P}$.*

Theorem 2 *Deciding whether a solution exists to a given HILP problem is in $\mathcal{P}$. If any component of a solution were constrained by a positive lower bound then the problem is still in $\mathcal{P}$.*

Proof Given an instance of HILP, $Ax \leq O$, the problem can be formulated as an LP problem by adding the constraints

$$\begin{aligned} x_1 + x_2 + \cdots + x_n &\geq 1 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \\ &\vdots \\ x_n &\geq 0. \end{aligned}$$

A solution to the HILP problem exists if and only if a solution to the constructed LP problem exists, because the homogeneity of the problem allows any rational solution to be scaled to an integer solution. Since the scaling involves multiplying by positive integers, any component satisfying a positive lower bound will satisfy the constraint after scaling. □

Theorem 3 *Consider a given Petri net $N = (P, T, B, F, M_0)$. The problem of deciding whether a given place $p' \in P$ is redundant is in $\mathcal{P}$.*

Proof By Theorem 2 deciding whether there exists a vector u satisfying Inequality (6) is in $\mathcal{P}$. □

It is possible that there might be bounds imposed on the arc-weights of the reduced Petri net, for example if the Petri net were required to be an ordinary Petri net, or if it is required that redundant places be isolated and removed from the Petri net (arc-weights of zero). Another reason that one would be interested in *total* redundancy is that *partial* redundancy does not preserve strong connectedness of Petri nets and live and bounded Petri nets are strongly connected (Best [4]). In the following we show that deciding whether a place is redundant such that this arc-weight bound is satisfied is an intractable problem.

The following definition and result are cited in Garey and Johnson [10]:

Definition 5 The Knapsack Problem *Given a vector $a \in \mathbb{N}^n$ and $B, K \in \mathbb{N}, B \geq K$, does there exist a vector $x \in \{0, 1\}^n$ such that $a \cdot x \leq B$ and $a \cdot x \geq K$?*

Theorem 4 *The knapsack problem in Definition 5 is $\mathcal{NPC}$.*

Definition 6 A Special Bounded Homogeneous Integer Linear Programming Problem (SBHILP) *Given a matrix $A \in \mathbb{Z}^{m\times n}, b \in \mathbb{N}$ and a component index k, does there exist an $x \in \mathbb{N}^n$ such that $Ax \geq O, Ax \leq be$ (e is the vector with each component equal to one) and $x_k \geq 1$?*

All problems of systems of linear equations and inequalities over $\mathbb{Z}$, $\mathbb{N}$ or $\{0, 1\}$ are in $\mathcal{NP}$:

Theorem 5 [23] *Given $A' \in \mathbb{Z}^{m\times n}, b' \in \mathbb{Z}^m, C' \in \mathbb{Z}^{p\times n}$ and $d' \in \mathbb{Z}^p$, then the problem of deciding whether there exists an $x \in \mathbb{Z}^n$ (or $\{0, 1\}^n$) such that $A'x = b'$ and $C'x \geq d'$ is in $\mathcal{NP}$.*

Theorem 6 *The SBHILP problem is $\mathcal{NPC}$.*

Proof The SBHILP problem is in $\mathcal{NP}$ by Theorem 5. The proof that the problem is $\mathcal{NPH}$ follows from a reduction from the knapsack problem: Consider an instance of the knapsack problem in Definition 5; $a \in \mathbb{N}^n$ and $B, K \in \mathbb{N}$. The system

$$\begin{aligned} a \cdot x &\leq B \\ a \cdot x &\geq K \end{aligned}$$

where $x \in \{0, 1\}^n$ can be rewritten

$$\begin{aligned} a \cdot x - Ky &\leq B - K \\ a \cdot x - Ky &\geq 0 \end{aligned}$$

where $y = 1$ and $x \in \{0, 1\}^n$. This system can be augmented with additional constraints to get an equivalent system with solutions $x \in \mathbb{N}^n$:

$$\begin{aligned}
a \cdot x - Ky &\leq B - K \\
(B-K)x_i &\leq B - K \text{ for } i = 1, 2, \ldots, n \\
(B-K)y &\leq B - K \\
a \cdot x - Ky &\geq 0 \\
(B-K)x_i &\geq 0 \text{ for } i = 1, 2, \ldots, n \\
(B-K)y &\geq 0
\end{aligned}$$

where $y = 1$ and $x \in \{0,1\}^n$. Since $y \in \{0,1\}$, the requirement that $y = 1$ can be written $y \geq 1$. Thus an instance of the SBHILP problem has been created which has a solution if and only if the given knapsack problem has a solution. Thus SBHILP is $\mathcal{NPH}$ and hence $\mathcal{NPC}$. □

Theorem 7 *Given a Petri net $N = (P, T, B, F, M_0)$, a place $p' \in P$ and a bound $b \in \mathbb{N}$, deciding whether there exists a vector v satisfying Eq. (5) and Condition (4) such that for all $t_i \in T$: $w_i \leq b$ is $\mathcal{NPC}$.*

Proof By Theorem 5 the problem is in $\mathcal{NP}$. In order to show that the problem is $\mathcal{NP}$-hard, we use a reduction from the SBHILP problem: Given an instance of SBHILP, a matrix $A = [a_{ij}] \in \mathbb{Z}^{m \times n}, b \in \mathbb{N}$ (we can assume $k = 1$), we want to know if there exists an $x \in \mathbb{N}^n$ such that $Ax \geq O, Ax \leq be$ and $x_1 \geq 1$? Construct a Petri net with the incidence matrix $C = [c_{ij}]$ where

$$c_{ij} = \begin{cases} a_{ij} & \text{if } i = 1 \text{ and} \\ -a_{ij} & \text{otherwise.} \end{cases}$$

Initially, for the backward incidence matrix B take the negative entries of C as the positive entries of B with zeroes elsewhere, and take the positive entries of C as the positive entries of F with zeroes elsewhere. We want to know if there exists a solution to $Cx \geq O$ and $Cx \leq be$ where $x \in \mathbb{Z}^n$ is such that $x_1 \geq 1$ and $x_i \leq 0, i \neq 1$. Whether the place corresponding to x_1 is redundant is dependent on the existence of a solution to

$$x^T[(B - M_0 e^T)|(B - F)|(-M_0)] \leq O^T$$

as before. Splitting this system up again we see that we have the following systems of linear equations that need to be satisfied by the same vector x: $[B - M_0 e^T]x \leq O$, $[B - F]x \leq O$ and $-M_0 x \leq O$. If we choose $M_0 = O$ then this last system is always satisfied and the first system reduces to $Bx \leq O$. The second system is the one that we are interested in as it is simply the system $Cx \geq O$. We would like to show that the additional constraints can also be made redundant by choosing suitable values for the entries of B (the entries of B can be increased by any amount as long as the corresponding entries of F are increased by the same amount thus not affecting the values of C). Von zur Gathen and Sieveking [23] showed that if there existed a solution to a system of linear equations and inequalities then there existed a solution which was polynomially bounded. Unfortunately this upper bound, based on the maximum of the absolute values of sub-determinants of a certain size depending on the rank of the coefficient matrices, implies an exponential number of computations. The computed bound, however, need not be as fine as their computation delivers, and we can use the following crude upper bound:

$$Q = \prod_{1 \leq i \leq n} \left(1 + \sum_{1 \leq j \leq m} a_{ij}^2 \right)$$

which we modified from Hadamard's inequality (see Knuth [14]). This bound can be computed in polynomial-time. Now replace the system $Bx \leq O$ by $B'x \leq O$ where $B' = [b'_{ij}]$ is such that

$$b'_{ij} = \begin{cases} b_{ij} & \text{if } i = 1 \text{ and} \\ \max(b_{ij}, Qb_{1j}) & \text{otherwise.} \end{cases}$$

Since this last adjustment only increases the values of B, they can be offset by corresponding changes to F so as not to effect the matrix C. We want to show the redundancy of this new system of equations. Assume first that there are only solutions to $Cx \geq O$ and $Cx \leq be$ with $x_2 = 0, x_3 = 0, \ldots, x_n = 0$, that is $x = (t, 0, \ldots, 0)$ where $t \in \mathbb{N} \setminus \{0\}$. The equations $Ax \geq 0$ then reduce to $a_{11}t \geq 0, a_{12}t \geq 0, \ldots, a_{1n}t \geq 0$ and we have $a_{11} \geq 0, a_{12} \geq 0, \ldots, a_{1n} \geq 0$, thus $x = (1, 0, \ldots, 0)$ is a solution to $Ax \geq O$. Furthermore, $a_{11}t \leq b, a_{12}t \leq b, \ldots, a_{1n}t \leq b$ implies $a_{11} \leq b, a_{12} \leq b, \ldots, a_{1n} \leq b$ and so $x = (1, 0, \ldots, 0)$ is also a solution to $Ax \leq be$. The existence of a solution of the form $x = (t, 0, \ldots, 0)$ can be checked in polynomial time simply by checking whether $x = (1, 0, \ldots, 0)$ is a solution. Assume now that a solution of the form $x = (t, 0, \ldots, 0)$ does not exist. Then there must be at least one other non-zero component to x, assume $x_2 \neq 0$ (it is sufficient to show the redundancy of the additional constraints when only one other component is non-zero and we can take this value to be -1 as this is the worst possible value in terms of satisfying the additional constraints). In this case the system of equations $B'x \leq O$ reduces to $b'_{1j}t_1 - b_{2j} \leq 0$ where $1 \leq t_1 \leq Q$. To show that the system is redundant, we substitute for t_1 its worst possible value: If $b'_{2j} = b_{1j}Q$ then we have $b_{1j}Q - b_{1j}Q = 0$, thus satisfying the constraints. If $b'_{2j} = b_{2j}$ then $b'_{2j} \geq b_{1j}Q$ and so $b_{1j}Q - b_{2j} \leq b_{1j}Q - b_{1j}Q = 0$ which also satisfies the constraints. A polynomial-time algorithm for the problem in the statement of Theorem 7 implies a polynomial-time algorithm for SBHILP. The problem is thus $\mathcal{NP}$-hard and this completes the proof that the problem is $\mathcal{NPC}$. □

Fusion of Doubled Places

When one combines two places into a single place and is able to distinguish to which of the original two places tokens in the combined place belonged (by markings on sets of other places) then the two places are doubled. Berthelot [1] defines doubled places in terms of the application conditions of this transformation:

Definition 7 Doubled Places [1] *Two places $p, p' \in P$, $p \neq p'$ are* doubled *if and only if the following application*

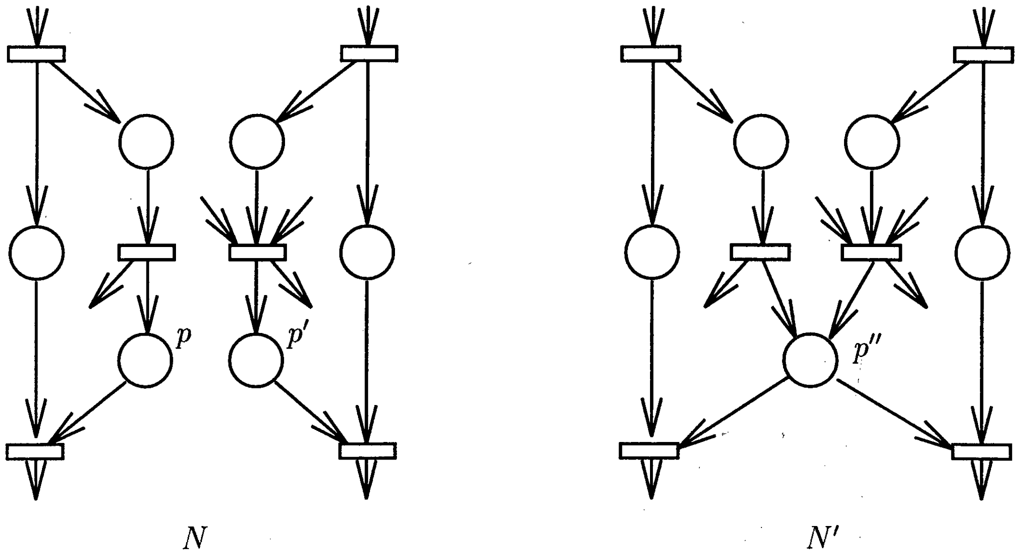

Figure 2. Fusion of doubled places

conditions hold:

$$\begin{array}{l} \exists I_1, I_2, \subseteq P : p \in I_1 \textit{ and } \forall t \in T : \\ p \in \bullet t \Longrightarrow \exists q \in I_2 : q \in \bullet t \\ \textit{and an m-vector } v \in \mathbb{Z}^m \textit{ such that} \\ \qquad v(p) \left\{ \begin{array}{lll} > & 0 & \textit{if } p \in I_1, \\ < & 0 & \textit{if } p \in I_2 \textit{ and} \\ = & 0 & \textit{otherwise,} \end{array} \right. \\ \qquad v^T M_0 \leq 0, \\ \qquad v^T B \geq O^T \textit{ and} \\ \qquad v^T(F-B) \leq O^T. \end{array} \tag{7}$$

$$\begin{array}{l} \exists I'_1, I'_2, \subseteq P : p' \in I'_1 \textit{ and } \forall t \in T : \\ p' \in \bullet t \Longrightarrow \exists q \in I'_2 : q \in \bullet t \\ \textit{and an m-vector } v' \in \mathbb{Z}^m \textit{ such that} \\ \qquad v'(p) \left\{ \begin{array}{lll} > & 0 & \textit{if } p \in I'_1, \\ < & 0 & \textit{if } p \in I'_2 \textit{ and} \\ = & 0 & \textit{otherwise,} \end{array} \right. \\ \qquad v'^T M_0 \leq 0, \\ \qquad v'^T B \geq O^T \textit{ and} \\ \qquad v'^T(F-B) \leq O^T. \end{array} \tag{8}$$

$$I_2 \cap I'_2 = \emptyset. \tag{9}$$

$$\begin{array}{l} \forall M \in R(PN, M_0) : \\ \quad \sum_{q \in I_2} M(q) > 0 \Longrightarrow \sum_{q \in I'_2} M(q) = 0 \textit{ and} \\ \quad \sum_{q \in I'_2} M(q) > 0 \Longrightarrow \sum_{q \in I_2} M(q) = 0. \end{array} \tag{10}$$

$$\begin{array}{l} \forall t \in T : \\ \quad B(p,t) > 0 \Longrightarrow B(p',t) = 0 \textit{ and} \\ \quad B(p',t) > 0 \Longrightarrow B(p,t) = 0. \end{array} \tag{11}$$

After having identified two places as being doubled places the transformation can be performed. This is done by fusing the two doubled places, that is, places p and p' are removed and a single place, p'', is inserted into the Petri net such that $\bullet p'' = \bullet p \cup \bullet p'$ and $p'' \bullet = p \bullet \cup p' \bullet$.

Figure 2 from Berthelot [1] shows a part of a Petri net N that contains doubled places p and p' and the same part of the Petri net N' after the transformation of fusing the doubled places.

The following discussion centres around the subproblems concerning the application conditions of doubled places. Whether the whole problem is intractable depends on whether the subproblems are restricted by their interdependence.

The problem of deciding whether all conditions apply, that is, whether two places are doubled, can also be shown to be intractable. The following result is used in the proof.

Theorem 8 *Consider a Petri net $N = (P, T, B, F, M_0)$ with doubled places $p, p' \in P$. Then for all $M \in R(M_0)$, $M(p) > 0 \Longrightarrow M(p') = 0$ and $M(p') > 0 \Longrightarrow M(p) = 0$.*

Proof By considering the firing rule, $M' = M + Ce_i$ if $M \xrightarrow{t_i} M'$ and, by Condition (7), $v^T M_0 \leq 0$ and $v^T(F - B) \leq O^T$, it follows that $v^T M \leq 0$ for all $M \in R(M_0)$. This can be rewritten

$$\sum_{q \in I_1} v(q)M(q) + \sum_{q \in I_2} v(q)M(q) \leq 0.$$

Assume that $M(p) > 0$ for some marking $M \in R(M_0)$, then $\sum_{q \in I_1} v(q)M(q) > 0$. Hence, $\sum_{q \in I_2} v(q)M(q) < 0$ or $\sum_{q \in I_2} M(q) > 0$. Hence, $\sum_{q \in I_2} v(q)M(q) < 0$ or $\sum_{q \in I_2} M(q) > 0$. Thus, by Condition (10), $\sum_{q \in I'_2} M(q) = 0$. However, it follows from Condition (8), that for all $M \in R(M_0)$,

$$\sum_{q \in I'_1} v'(q)M(q) + \sum_{q \in I'_2} v'(q)M(q) \leq 0.$$

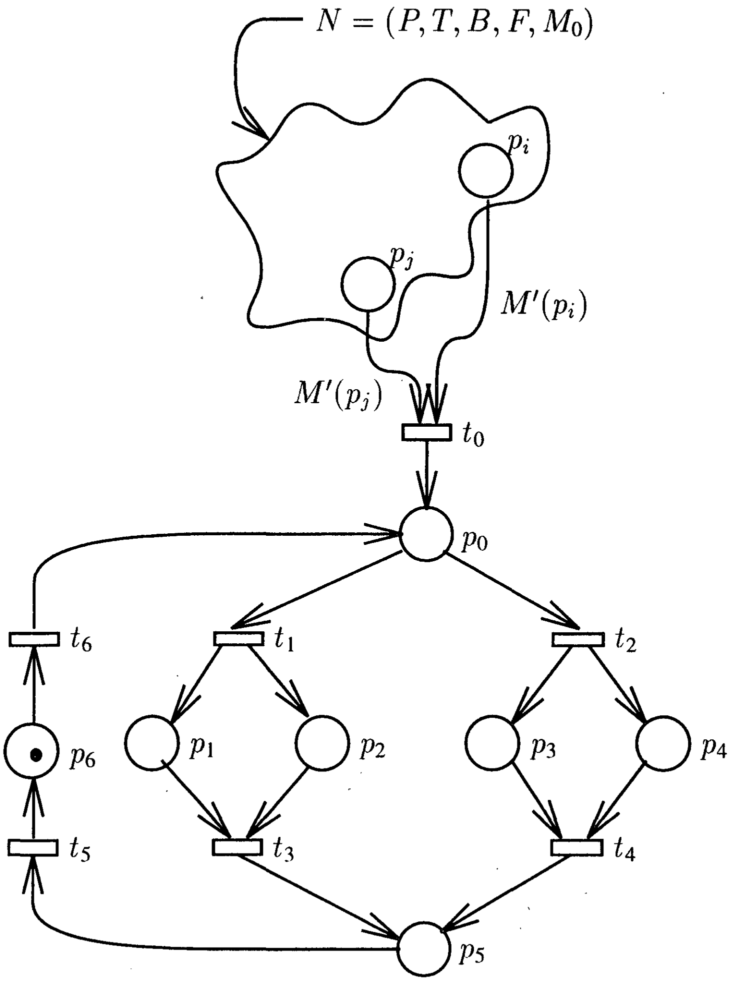

Figure 3. Reduction of the coverability problem to doubled places

Hence $\sum_{q \in I'_1} v'(q)M(q) \leq 0$. However, $v'(q) > 0$ for all $q \in I'_1$, and hence $\sum_{q \in I'_1} M(q) = 0$. Since $p' \in I'_1$, $M(p') = 0$. Similarly,if $M(p') > 0$, then $M(p) = 0$. □

We need the following definition and result, due to Lipton and cited in Jones, Landweber and Lien [13], in order to prove our next result:

Definition 8 The Coverability Problem [16] *Given a Petri net $N = (P, T, B, F, M_0)$ and a marking M'. Does there exist a marking $M \in R(M_0)$ such that $M \geq M'$?*

Theorem 9 *The coverability problem is* **EXPSPACE**-*hard.*

Theorem 10 *Consider a given Petri net $N = (P, T, B, F, M_0)$. Deciding whether two places $p, p' \in P$ are doubled is* **EXPSPACE**-*hard.*

Figure 3 illustrates the construction used in the proof showing a reduction from the coverability problem.

Proof Consider a given instance of the coverability problem: A Petri net $N = (P, T, B, F, M_0)$ and a marking M'. Construct a transition t_0 with input arcs from each place $p_i \in P$ with arc-weight $M'(p_i)$, and no other inputs. For the rest of the construction, all the constructed arcs have an arc-weight of one. Construct a place p_0 which is the only output place of transition t_0. Construct transitions t_1 and t_2 which are the only outputs of p_0. Transitions t_1 and t_2 have no other inputs. Construct places p_1 and p_2 which are the only outputs of t_1. Places p_1 and p_2 have no other inputs. Construct transition t_3 which only has inputs from p_1 and p_2. Transition t_3 is the only output of places p_1 and p_2. Construct places p_3 and p_4 which are the only outputs of t_2. Places p_3 and p_4 have no other inputs. Construct transition t_4 which only has inputs from p_3 and p_4. Places p_3 and p_4 have no other outputs. Construct place p_5 which only has as inputs t_3 and t_4. Neither transition t_3 nor transition t_4 have any other outputs. Construct transition t_5 which has as only input p_5. Place p_5 has no other outputs. Construct place p_6 which only has as input t_5. Transition t_5 has no other outputs. Construct transition t_6 which only has as input p_6 and output p_0. Place p_0 has no other inputs and place p_6 has no other outputs. At the initial marking, p_6 is marked with exactly one token, and no other constructed places are marked. With respect to Definition 7 let $I_1 = \{p_2\}$, $I_2 = \{p_1\}$, $I'_1 = \{p_3\}$, $I'_2 = \{p_4\}$,$p = p_2$,$p' = p_3$,

$$v(s) = \begin{cases} 1 & \text{if } s = p_2, \\ -1 & \text{if } s = p_1 \text{ and} \\ 0 & \text{otherwise, and} \end{cases}$$

$$v'(s) = \begin{cases} 1 & \text{if } s = p_3, \\ -1 & \text{if } s = p_4 \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

By considering the initial marking as well as the backward and forward incidence matrices of the Petri net N with the additional places and transitions, it is easy to verify that v satisfies Condition (7), v' satisfies Condition (8), $p = p_2$ and $p' = p_3$ satisfy Condition (11). Since $I_2 \cap I_2' = \emptyset$,$p_2$ and p_3 are doubled if Condition (10) is satisfied. If t_0 does not ever fire then Condition (10) is satisfied. However, if t_0 can fire then there exists a marking in the reachability set of the Petri net N augmented with the construction such that places p_2 and p_3 are simultaneously marked, and hence by Theorem 8 places p_2 and p_3 cannot be doubled. Transition t_0 will fire if and only if there exists a marking M,covering M', in the reachability set of the original Petri net N. Thus, places p_2 and p_3 are doubled if and only if the given instance of the coverability problem has no solution. The result follows from Theorem 9 (and the fact that deterministic space complexity classes are closed under compliment). □

Other Reduction Transformations

In Donaldson [6] it is shown that the *Fusion of Equivalent Places*, *Post-fusion of Transition*, *Pre-fusion of Transitions*, and *Lateral Fusion of Transitions*, all proposed by Berthelot [1], are in $\mathcal{P}$.

5 Complexity of Synthesis Transformations

We next consider the complexity of the applicability of each of the synthesis transformations surveyed from the literature. These transformations are due to Berthelot [1], Valette [20], Suzuki and Murata [19] Esparza [7], Esparza and Silva [9] and Savi and Xie [18].

Addition of a Derivable Subnet

This transformation duplicates a subnet of a Petri net. This refines the process by adding an alternate path along which tokens can flow. The tokens which take this alternate route are removed from the original Petri net, flow through the added subnet and then are returned to the original Petri net. This transformation does not change the dynamic behaviour of the Petri net as each firing sequence in the transformed Petri net has a corresponding firing sequence in the original Petri net (replacing any transitions in the added subnet with the corresponding transitions of the subnet in the original Petri net) and each firing sequence in the original Petri net is still a firing sequence in the transformed Petri net (Berthelot [1]).

Berthelot [1] defines a derivable subnet in terms of the application conditions that must be satisfied before this transformation can be applied:

Definition 9 Derivable Subnet [1] *Let* $N = (P, T, B, F, M_0)$ *be a marked Petri net. A* subnet $N_s = (P_s, T_s, B_s, F_s)$ *of* N *is* derivable *if and only if the following conditions hold:*

1. N_s *is an* open subnet *of* N*,that is,*P_s *contains all the places of* P *which are connected to transitions of* T_s.
2. N_s *is a marked graph.*
3. N_s *is connected and contains no circuit.*
4. $\exists t_1 \in T_s$ *such that there is a path from* t_1 *to every other transition of* N_s.
5. N_s *is not* re-entrant*; for all markings* $M \in R(M_0)$ *reached by* N*,every path included in* N_s *(a marked graph) contains at most one token.*
6. P_s *can be partitioned into:*
   - $P_E = \{p \in P_s : p \notin T_s\bullet\}$.
   - $P_S = \{p \in P_s : p \notin \bullet T_s\}$.
   - $P_I = \bullet T_s \cap T_s\bullet$.
7. N_s *can be emptied by firing transitions in* T_s *and non-conflicting transitions of* T.
8. $(T \setminus T_s) \bullet \cap P_I = \emptyset$.

After having identified a derivable subnet the addition of the subnet transformation is completed in two steps (let N_d be the duplicate of the derivable subnet):

1. The inputs of N_d are set to the inputs of N_s and the outputs of N_d are set to the outputs of N_s, i.e. $\bullet N_d = \bullet N_s$ and $N_d\bullet = N_s\bullet$.
2. N_d is added to N by fusing inputs and outputs of N_s and N_d leaving the internal places distinct.

Figure 4 shows a part of a Petri net N with a derivable subnet and the same part of the Petri net N' after duplicating the subnet.

Let $N = (P, T, B, F, M_0)$ be a Petri net to which a subnet $N_s = (P_s, T_s, B_s, F_s, M_{0s})$ is to be added as a derivable subnet (Definition 9):

Theorem 11 *Deciding whether a given subnet of a given Petri net is derivable is* **EXPSPACE**-*hard.*

The proof is by reduction from the coverability problem. Figure 5 illustrates the construction used in the proof:

Proof Given an instance of the coverability problem, $N = (P, T, B, F, M_0)$ and a marking M', construct a transition t_0 with inputs from all places $p \in P$ with arc-weights $M'(p)$. Construct places p_1,p_2 and p_3,and transitions t_1 and t_2 such that p_1 has as only input the transition t_0 with arc-weight 2 and as only output the transition t_1 with arc-weight one. Place p_2 has as only input transition t_1 and as only output transition t_2, both with arc-weights of one. Place p_3 has as only input transition t_2 and as only output transition t_3, both with arc-weights of one. Initially the places p_1,p_2 and p_3 are not marked. Denote by $N_s = (P_s, T_s, B_s, F_s, M_{0s})$ the subnet where $P_s = \{p_1, p_2, p_3\}$,$T_s = \{t_1, t_2\}$,

$$B_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \; F_s = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and ${M_{0s}}^T = [0, 0, 0]$. It is obvious that N_s satisfies parts 1, 2, 3, 4, 6, 7 and 8 of Definition 9. Thus N_s is a derivable subnet if and only if part 5 of Definition 9 holds, that is, if and only if N_s is not re-entrant. Transition t_0 can only fire if the marking M' is coverable in the original Petri net, and if it does, it will deposit two tokens in N_s in which case N_s is re-entrant. If t_0 cannot fire then N_s will never have any tokens placed in it. The result follows since the coverability problem is **EXPSPACE**-hard (Theorem 9 on page 51). □

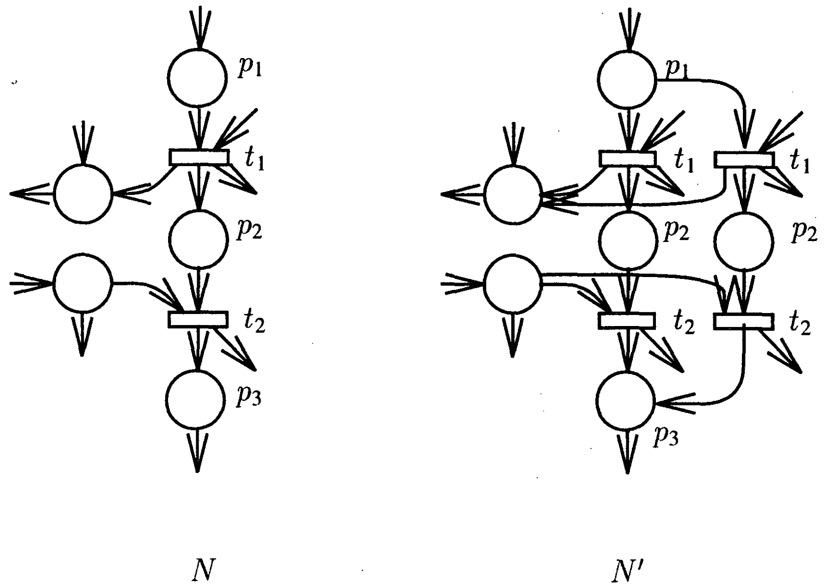

Figure 4. Addition of a derivable subnet

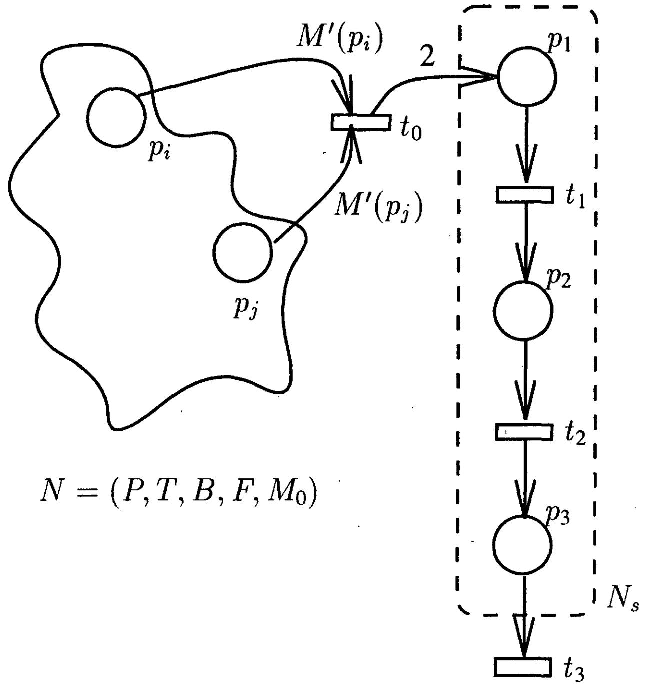

Figure 5. Reduction of coverability to the derivable subnet problem

The complexity of each of the parts of Definition 9 may be important if the hard questions have been answered by, for example, the modeller building a Petri net. Donaldson [6] discusses these problems individually.

Stepwise Refinement of Transitions

This refinement transformation is performed by replacing a transition of a Petri net by another Petri net. In order that the properties of liveness and boundedness are preserved there has to be imposed on the transition and the replacing Petri net certain conditions regarding their dynamic behaviour. For example, if the subnet could generate tokens regardless of the tokens supplied to it, then its inclusion could not possibly preserve boundedness. Similarly, if the subnet consumed tokens and never returned them to the original Petri net then it may be possible that that the refinement did not preserve liveness. Conversely, the subnet may behave in a "proper" way provided that not too many tokens are delivered to it by the original Petri net. Thus a restriction has to be placed on the transition being refined. We describe these conditions below. This transformation has its origins in Valette [20] and was generalised by Suzuki and Murata [19]:

Definition 10 **k-enabled Transitions [19]** *A Petri net $N = (P, T, B, F, M_0)$ has a transition $t \in T$ which is said to be k-*enabled *for $k \in \mathbb{N}^+$ in N if and only if there exists a* *marking* $M \in R(M_0)$ *such that*

$$kB(p,t) \leq M(p), \forall p \in P.$$

Definition 11 Block and Associated Petri Net [19] *A Petri net* $N = (P, T, B, F, M_0)$ *with two distinct transitions* t_{in}, $t_{out} \in T$ *is called a* block. *The Petri net* $B(N, t_{in}, t_{out}, k) = (P \cup \{p_0\}, T, B_B, F_B, M_{0B})$ *is called the* associated Petri net *of the block N, and is defined as follows:*

$$B_B(p,t) = \begin{cases} 1 & t = t_{in}, p = p_0, \\ 0 & t \neq t_{in}, p = p_0 \text{ and} \\ B(p,t) & p \in P. \end{cases}$$

$$F_B(p,t) = \begin{cases} 1 & t = t_{out}, p = p_0, \\ 0 & t \neq t_{out}, p = p_0 \text{ and} \\ F(p,t) & p \in P. \end{cases}$$

$$M_{0B}(p) = \begin{cases} k & p = p_0 \text{ and} \\ M_0(p) & p \in P \end{cases}$$

where $k \in \mathbb{N}^+$. *The place* $p_0 \notin P$ *is called the* idle place.

Definition 12 k**-well-behaved [19]** *A Petri net* $N = (P, T, B, F, M_0)$ *is said to be* k-well-behaved *with respect to two distinct transitions* $t_{in}, t_{out} \in T$ *if and only if the following conditions hold:*

1. t_{in} *is live in* $B(N, t_{in}, t_{out}, k)$.
2. *For each* $\sigma_1 \in L(B(N, t_{in}, t_{out}, k))$ *such that* $\#(\sigma_1, t_{in}) > \#(\sigma_1, t_{out})$, *there exists* $\sigma_2 \in (T \setminus \{t_{in}\})^+$ *such that* $\sigma_1\sigma_2 \in L(B(N, t_{in}, t_{out}, k))$ *and* $\#(\sigma_1, t_{in}) = \#(\sigma_1\sigma_2, t_{out})$.
3. $\#(\sigma, t_{in}) \geq \#(\sigma, t_{out})$ *for any* $\sigma \in L(B(N, t_{in}, t_{out}, k))$.

Suzuki and Murata [19] shows that for $n \in \mathbb{N}^+$, if a Petri net N is $(n+1)$-well-behaved with respect to t_{in} and t_{out}, then N is n-well-behaved.

Let $N = (P, T, B, F, M_0)$ and $N' = (P', T', B', F', M'_0)$ be Petri nets such that $P \cap P' = \emptyset$ and $T \cap T' = \emptyset$. Assume that for some $k \in \mathbb{N}^+$, a transition $t_0 \in T$ is *not* (k+1)-enabled in N and that N' is k-well-behaved with respect to two distinct transitions $t_{\text{in}}, t_{\text{out}} \in T'$. The transformation of refining transition t_0 yields

$$N'' = \text{TR}(N, N', t_0, t_{\text{in}}, t_{\text{out}}) = (P'', T'', B'', F'', M''_0)$$

defined as follows:

$$P'' = P \cup P'.$$

$$T'' = (T \cup T'') \setminus \{t_0\}.$$

$$B''(p,t) = \begin{cases} 0 & t \in T \setminus \{t_0\}, p \in P', \\ 0 & t \in T' \setminus \{t_{\text{in}}\}, p \in P, \\ B(p,t) & t \in T \setminus \{t_0\}, p \in P, \\ B'(p,t) & t \in T', p \in P' \text{ and} \\ B(p,t_0) & t = t_{\text{in}}, p \in P. \end{cases}$$

$$F''(p,t) = \begin{cases} 0 & t \in T \setminus \{t_0\}, p \in P', \\ 0 & t \in T' \setminus \{t_{\text{out}}\}, p \in P, \\ F(p,t) & t \in T \setminus \{t_0\}, p \in P, \\ F'(p,t) & t \in T', p \in P' \text{ and} \\ F(p,t_0) & t = t_{\text{out}}, p \in P. \end{cases}$$

$$M''_0(p) = \begin{cases} M_0(p) & p \in P \text{ and} \\ M'_0(p) & p \in P'. \end{cases}$$

Figure 6 from Suzuki and Murata [19] shows a Petri net N with a transition t_0 that is 3-enabled and *not* 4-enabled. Figure 7 shows the Petri net N' which is a block with respect to transitions t_{in} and t_{out} and is 3-well-behaved and *not* 4-well-behaved. Figure 8 shows the Petri net $B(N', t_{\text{in}}, t_{\text{out}}, 3)$ associated with the block and Figure 9 shows the Petri net N'' which is the result of refining transition t_0 in N using N'.

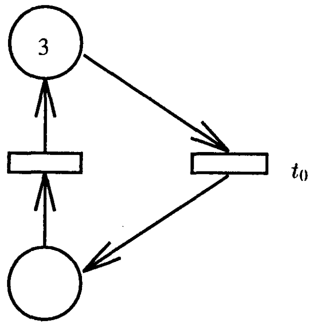

Figure 6. Petri net N in which t_0 *is 3-enabled*

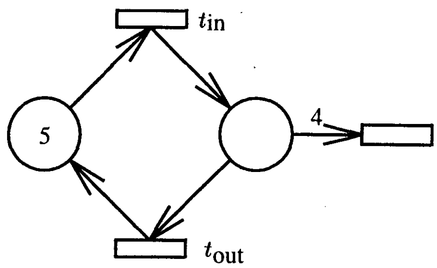

Figure 7. Block N' *which is 3-well-behaved and not 4-well-behaved*

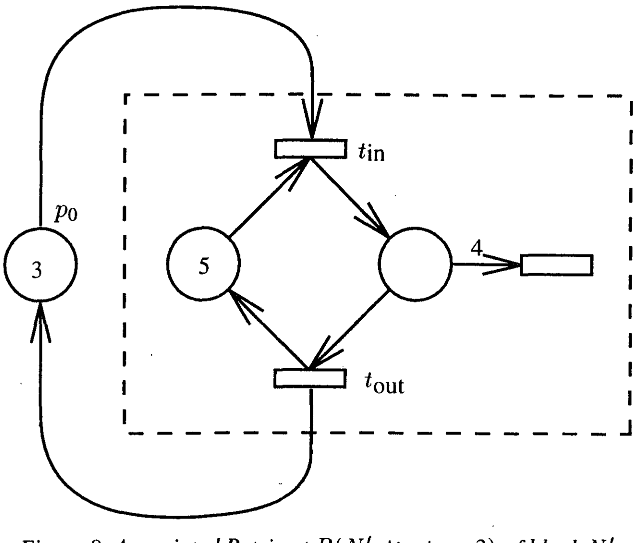

Figure 8. Associated Petri net $B(N', t_{in}, t_{out}, 3)$ *of block* N'

Deciding whether a given transition in a given Petri net is k-enabled (Definition 10) is an intractable problem:

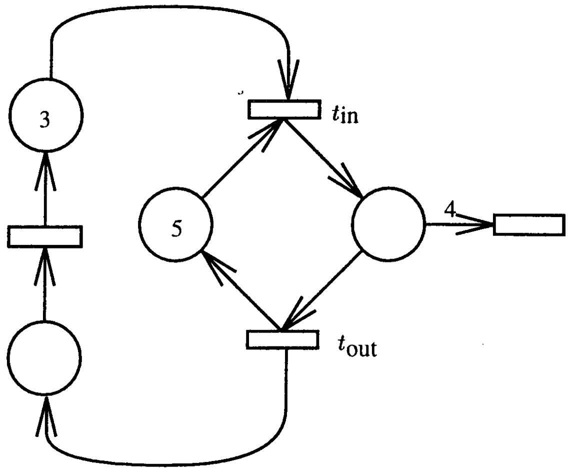

Figure 9. Petri net N'' produced by refining t_0 in N using N'

Theorem 12 *Consider a given Petri net $N = (P, T, B, F, M_0)$, a transition $t_0 \in T$ and a nonnegative integer k. Deciding whether t_0 is k-enabled and not $(k+1)$-enabled is* **EXPSPACE**-*hard.*

The construction described in the following proof is illustrated in Figure 10.

Proof The proof is by a polynomial time reduction from the coverability problem. Let $N = (P, T, B, F, M_0)$, and a marking M' be an instance of the coverability problem. Construct a transition t' which has as inputs each of the places $p \in P$ with arc-weight $M'(p)$. Construct a place p_0, initially not marked, which has as input only the transition t' with arc-weight one. Transition t' has no other inputs or outputs. Construct another transition t_0 with an input from p_0 with arc-weight one and no outputs. Place p_0 has no other outputs. Finally, construct a place p_1, initially marked with one token, with no inputs and which is the only other input to transition t_0 with arc-weight one. Place p_1 has no other outputs. The transition t' will fire if and only if there exists a marking $M \in R(M_0)$ such that $M \geq M'$; this will ensure that at least one token could be placed on p_0. In this case t_0 could fire, removing the token from p_1. Thus there exists a reachable marking of the constructed Petri net such that t_0 is 1-enabled and not 2-enabled if and only if the given marking is coverable in the given Petri net. The hypothesis follows from Theorem 9 (the coverability problem is **EXPSPACE**-hard). □

Leaving out the construction of the place p_1 and the arc to transition t_0 gives a reduction which shows that deciding whether a transition is k-enabled is **EXPSPACE**-hard.

If it is known that the transition t_0 is k-enabled but not (k+1)-enabled in some Petri net in which it is to be refined, then deciding whether a given Petri net is k-well-behaved (Definition 12) is also intractable (Donaldson [6]).

Other Synthesis Transformations

The complexity of further synthesis transformations are considered in Donaldson [6]. For the transformations *Regulation of Laterally Fusible Transitions* and *Regulation of Identical Transitions*, the application conditions of these transformations (and hence the complexity of the decision problem regarding their complexity) are the same as the corresponding reduction transformations (Berthelot [1]) and can be performed in polynomial-time. The transformation *Stepwise Refinement of Places* (Suzuki and Murata [19]) is always applicable to any place in a Petri net.

Esparza [7] considers generating classes of nets from kits of rules starting from the atomic net. Of these kits, it is only the *SL&SB kit* which preserves liveness and boundedness. The SL&SB kit produces only *structurally live* and *structurally bounded* Petri nets. In this stepwise refinement technique the atomic net is not marked and nothing is said of the actual markings which ensure that the system is live and bounded, only that such a marking exists. This transformation is also described by Esparza and Silva [9]. Donaldson [6] also examines the *Free Choice Kit* of Esparza [7] and show that for both sets of kits, it is possible to decide in polynomial-time each of the refinement rules.

Finally, Donaldson [6] considered the *Event Graph Module Decomposition* of Savi and Xie [18] and provides an argument showing that the process can be performed in polynomial-time.

6 Conclusion

We have shown that the applicability of certain Petri net transformations range in complexity from polynomial-time (*simplification of redundant places* — unbounded arc-weights) and $\mathcal{NPC}$ (*simplification of redundant places* — bounded arc-weights) to **EXPSPACE**-hard (*doubled places, addition of a derivable subnet*, and *stepwise refinement of transitions*) decision problems, and thus discovered that they are not candidates for inclusion in a software tool for analysis of the general Petri net.

We have also considered the other reduction transformations proposed by Berthelot [1]: *Fusion of Equivalent Places, Post-fusion of Transitions, Pre-fusion of Transitions* and *Lateral Fusion of Transitions* (Donaldson [6]). Deciding the applicability of these transformations requires only the inspection of the backward and forward incidence matrices and, in the cases of pre-fusion of transitions and lateral fusion of transitions, the initial marking as well. The applicability of these transformations can be decided in polynomial-time.

Similarly, we have also considered the synthesis transformations of Berthelot [1] *Regulation of Laterally Fusible Transitions* and *Regulation of Identical Transitions*, the *SL&SB* and *Free Choice* kits of Esparza [7], and the *Event Graph Module Decomposition* of Savi and Xie [18], all of which can be decided in polynomial-time.

References

1. G Berthelot. 'Checking properties of nets using transformations'. In G Rozenberg, ed., *Advances in Petri Nets 1985, Lecture Notes in Computer Science, volume 222*, pp. 19–40. Springer-Verlag, (1986).

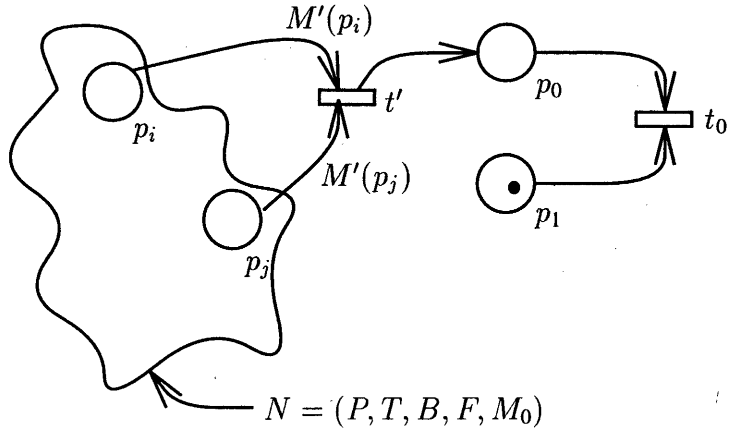

Figure 10. Reduction of the coverability problem to the k-enabled problem

2. G Berthelot. 'Transformations and decompositions of nets'. In W Brauer, W Reisig, and G Rozenberg, eds., *Petri Nets: Central Models and Their Properties, Lecture Notes in Computer Science, volume 254*, pp. 359–376. Springer-Verlag, (1986).
3. G Berthelot, G Roucairol, and R Valk. 'Reductions of nets and parallel programs'. In W Brauer, ed., *Net Theory and Applications, Lecture Notes in Computer Science, volume 84*, pp. 213–223. Springer-Verlag, (1980).
4. E Best. 'Structure theory of Petri nets: The free choice hiatus'. In W Brauer, W Reisig, and G Rozenberg, eds., *Petri Nets: Central Models and their Properties, Advances in Petri Nets 1986 Part I*, pp. 213–223. Springer-Verlag, (1986).
5. T H Corman, C E Leiserson, and R L Rivest. *Introduction to Algorithms*. MIT/McGraw-Hill, 1990.
6. S R Donaldson. The complexity of Petri net transformations. Master's thesis, University of Cape Town, Private Bag RONDEBOSCH, 7700, South Africa, September 1993.
7. J Esparza. 'Synthesis rules for Petri nets, and how they lead to new results'. In J C M Baeton and J W Klop, eds., *CONCUR '90 Theories of Concurrency: Unification and Extension, Lecture Notes in Computer Science, volume 458*, pp. 182–198. Springer-Verlag, (1990).
8. J Esparza. 'Reduction and synthesis of live and bounded free choice Petri nets'. *Information and Computation*, **114**(1):50–87, (1994).
9. J Esparza and M Silva. 'Top-down synthesis of live and bounded free choice nets'. In *Application and Theory of Petri Nets: 11th International Conference*, pp. 63–83, (June 1990).
10. M R Garey and D S Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman and Company, 1979.
11. H J Genrich and E Stankiewicz-Wiechno. 'A dictionary of some basic notations of net theory'. In W Brauer, ed., *Net Theory and Applications, Lecture Notes in Computer Science, volume 84*, pp. 519–535. Springer-Verlag, (1980).
12. D S Johnson. 'A catalog of complexity classes'. In J van Leeuwen, ed., *Handbook of Theoretical Computer Sceince: Volume A - Algorithms and Complexity*, chapter 2, pp. 67–161. Elsevier/The MIT Press, (1990).
13. N D Jones, L H Landweber, and Y E Lien. 'Complexity of some Petri net problems'. *Theoretical Computer Science*, **4**:277–299, (1977).
14. D E Knuth. *Seminumerical Algorithms*, volume 2 of *The Art of Computer Programming*. Addison-Wesley, 1969.
15. G L Nemhauser and L A Wolsey. *Integer and Combinatorical Optimization*. Wiley, 1988.
16. J L Peterson. *Petri Net Theory and the Modeling of Systems*. Prentice-Hall, 1981.
17. W Reisig. *Petri nets: An Introduction*, volume 4 of *EATCS Monographs on Theoretical Computer Science*. Springer-Verlag, 1985.
18. V M Savi and X Xie. 'Liveness and boundedness analysis for Petri nets with event graph modules'. In K Jensen, ed., *Application and Theory of Petri Nets 1992, Lecture Notes in Computer Science, volume 616*, pp. 328–347. Springer-Verlag, (1992).
19. I Suzuki and T Murata. 'A method for stepwise refinement and abstraction of Petri nets'. *Journal of Computer and System Sciences*, **27**(1):51–76, (1983).
20. R Valette. 'Analysis of Petri nets by stepwise refinements'. *Journal of Computer and System Sciences*, **18**(1):35–46, (1979).
21. P van Emde Boas. 'Complexity of linear problems'. In *Conference on Fundamentals of Computation Theory*, pp. 117–120, (1979).
22. P van Emde Boas. 'Machine models and simulations'. In J van Leeuwen, ed., *Handbook of Theoretical Computer Sceince: Volume A - Algorithms and Complexity*, chapter 1, pp. 1–66. Elsevier/The MIT Press, (1990).
23. J von zur Gathen and M Sieveking. 'A bound on solutions of linear integer equalities and inequalities'. *Proceedings of the American Mathematical Society*, **72**(1):155–158, (October 1978).

Received: 2/96, Accepted: 4/96, Final copy: 8/96.

Notes for Contributors

The prime purpose of the journal is to publish original research papers in the fields of Computer Science and Information Systems, as well as shorter technical research notes. However, non-refereed review and exploratory articles of interest to the journal's readers will be considered for publication under sections marked as Communications or Viewpoints. While English is the preferred language of the journal, papers in Afrikaans will also be accepted. Typed manuscripts for review should be submitted in triplicate to the editor.

Form of Manuscript

Manuscripts for *review* should be prepared according to the following guidelines.

- Use wide margins and $1\frac{1}{2}$ or double spacing.
- The first page should include:
  - title (as brief as possible);
  - author's initials and surname;
  - author's affiliation and address;
  - an abstract of less than 200 words;
  - an appropriate keyword list;
  - a list of relevant Computing Review Categories.
- Tables and figures should be numbered and titled.
- References should be listed at the end of the text in alphabetic order of the (first) author's surname, and should be cited in the text in square brackets [1–3]. References should take the form shown at the end of these notes.

Manuscripts accepted for publication should comply with the above guidelines (except for the spacing requirements), and may be provided in one of the following formats (listed in order of preference):

1. As (a) LaTeX file(s), either on a diskette, or via e-mail/ftp – a LaTeX style file is available from the production editor;
2. As an ASCII file accompanied by a hard-copy showing formatting intentions:
   - Tables and figures should be original line drawings/printouts, (not photocopies) on separate sheets of paper, clearly numbered on the back and ready for cutting and pasting. Figure titles should appear in the text where the figures are to be placed.
   - Mathematical and other symbols may be either handwritten or typed. Greek letters and unusual symbols should be identified in the margin, if they are not clear in the text.

   Contact the production editor for markup instructions.
3. In exceptional cases camera-ready format may be accepted – a detailed page specification is available from the production editor;

Authors of accepted papers will be required to sign a copyright transfer form.

Charges

Charges per final page will be levied on papers accepted for publication. They will be scaled to reflect typesetting, reproduction and other costs. Currently, the minimum rate is R30-00 per final page for LaTeX or camera-ready contributions that require no further attention. The maximum is R120-00 per page (charges include VAT).

These charges may be waived upon request of the author and at the discretion of the editor.

Proofs

Proofs of accepted papers in category 2 above may be sent to the author to ensure that typesetting is correct, and not for addition of new material or major amendments to the text. Corrected proofs should be returned to the production editor within three days.

Note that, in the case of camera-ready submissions, it is the author's responsibility to ensure that such submissions are error-free. Camera-ready submissions will only be accepted if they are in strict accordance with the detailed guidelines.

Letters and Communications

Letters to the editor are welcomed. They should be signed, and should be limited to less than about 500 words.

Announcements and communications of interest to the readership will be considered for publication in a separate section of the journal. Communications may also reflect minor research contributions. However, such communications will not be refereed and will not be deemed as fully-fledged publications for state subsidy purposes.

Book reviews

Contributions in this regard will be welcomed. Views and opinions expressed in such reviews should, however, be regarded as those of the reviewer alone.

Advertisement

Placement of advertisements at R1000-00 per full page per issue and R500-00 per half page per issue will be considered. These charges exclude specialized production costs which will be borne by the advertiser. Enquiries should be directed to the editor.

References

1. E Ashcroft and Z Manna. 'The translation of 'goto' programs to 'while' programs'. In *Proceedings of IFIP Congress 71*, pp. 250–255, Amsterdam, (1972). North-Holland.
2. C Bohm and G Jacopini. 'Flow diagrams, turing machines and languages with only two formation rules'. *Communications of the ACM*, **9**:366–371, (1966).
3. S Ginsburg. *Mathematical theory of context free languages*. McGraw Hill, New York, 1966.

South African Computer Journal

Number 18, December 1996
ISSN 1015-7999

Suid-Afrikaanse Rekenaar-tydskrif

Nommer 18, Desember 1996
ISSN 1015-7999

Contents