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On Syntax and Semantics Related to Incomplete Information Databases

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Abstract

We propose a mathematical model of a database with incomplete information, which we call the N-system. Basically speaking, such a system stores information concerning properties of some objects. The information may be incomplete in that it may be known that the object has a property with probability from the interval $<\alpha,\beta> \subset <0,1>$. In the particular case where $<\alpha,\beta>=<0,1>$, means that it may be not known whether an object has a property, the N-system is equivalent to Lipski's system [3]. In another particular case, when $<\alpha,\beta>=<1,1>$ or $<\alpha,\beta>=<0,0>$ the N-system presented corresponds to Codd's relational model of databases [1].

The first level, the level of terms of a query language to communicate with the N-system, is described and its semantics and syntax are defined. A system of axioms, which serves as a basis for equivalent transformations of queries is presented (only for terms). Syntactical and semantic equivalences for terms are defined and completeness properties are shown.

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Introduction

The problem of information incompleteness seems to be very complex, and, at the same time, inherent in most human activities and particularly in the domain Data Bases.

In [11], the author has presented a common approach in some informational systems, namely: an ISR system [5], [6], [13], [16], an L-system [2], [3], [4], a stochastic system [14], [15], and has shown that it is possible to express all the abovementioned systems as a quadruple $S=<X,A,R_I,V>$ where X is a set of objects, A is a nonempty set of descriptors, R_I is an equivalence relation on A and V is a map defined on the product $A \times X$ with the values in the set of all subcontinua of the interval <0,1>.

In [11] a formal generalisation of the abovementioned systems is also presented. The generalised system is called an N-system.

In this paper the first level – the level of terms of a query language to N-systems is presented and its syntax and semantics are defined. In the first level of the language a query can define a property of objects and the expected response is then the set of objects satisfying this property.

A system of axioms is presented and discussed. Syntactical and semantic equivalences for terms are defined and completeness properties are shown.

For the sake of self-containment of this paper let us recall the basic definition of the N-system.

Basic notions

A mathematical model of an N-system is given below. Let C(<0,1>) be the set of all subcontinua (subsets which are closed and connected) of the interval <0,1>.

Let $V:Z \rightarrow C(<0,1>)$ be a function, such that for each $z \in Z$, V(z) is an interval $<a,b> \subset <0,1>$. We will call the numbers α and β the lower and upper values of the function V, denoted by the symbols $\alpha=V_*(z)$, $\beta=V^*(z)$.

Definition 1 (N-system)

By N-system we mean a quadruple

$S=<X,A,R_I,V>$

where

- X — finite, nonempty set of objects
- A — finite set of descriptors
- R — an equivalence relation on A; the equivalence classes of R_I are intended to be attributes and are denoted by $\{A_i\}_{i \in I}$
- $V:D_v \rightarrow C(<0,1>)$, where D_v is a subset of the product $A \times X$ and the following conditions are satisfied:

1) $\sum_{a \in A_i} V_*(a,x) \leq 1 \qquad \text{for } i \in I$

2) if for some $x \in X$ and $i \in I$;
$\{a \in A_i; (a,x) \in D_v\} = A_i$ then $\sum_{a \in A_i} V^*(a,x) \geq 1$.

Fact: So the notion of the N-system which has been defined is a common generalisation of the notions of the ISR system, the L-system and the stochastic informational system. See [8], [11].

In the case where the set of attributes contains only one set, i.e. the relation R_I has only one equivalent class, instead of $S=<X,A,R_I,V>$ we will write $S=<X,A,V>$ as an abbreviated form.

The first level of language to the N-system

The main task of an information system is to answer queries submitted by the user(s). To communicate with the system more specifically to formulate queries – the user has a query language at his disposal. In the special case when the information is complete, semantics is intuitively evident, and is "the only natural one". This is no longer true when the information is incomplete. The need for a precise formal semantics is essential here. It is also clear that a query can be interpreted in many different ways, of which none is distinguished as "the only natural one". This fact raises the problem of making sure that the user's intention, as expressed in a query, and the system's understanding of that query coincide.

We will describe the semantics of a query language to N-systems-$\mathcal{L}_N$ below. We will concentrate on the first level of the language $\mathcal{L}_N$ – the level of terms. The second level – the level of formulas will be presented in a separate paper.

Let $S=<X,A,V>$ be a single attribute N-system where $A=\{a_1,a_2,\ldots,a_n\}$.

Roughly speaking, the fundamental queries to an N-system are of the form:

> "List all objects $x \in X$ such that the probability that x is described by the property $a \in A$ belongs to the interval $<\alpha,\beta> \subset <0,1>$".

It is easy to see that in the case when the N-system is a maximal extension of some N-system [9], then an arbitrary object $x \in X$ is either described by $a \in A$ with probability, from the interval $<\alpha,\beta> \subset <0,1>$ or is not described.

Using only the information contained in the N-system we may state that:

(1) The object x has the property a, with the probability from the interval $<\alpha,\beta>$

$[\, <V_*(a,x),V^*(a,x)> \subset <\alpha,\beta> \,]$

(2) The object x may have the property a with the probability from the interval $<\alpha,\beta>$

$[\, <V_*(a,x),V^*(a,x)> \cap <\alpha,\beta> \neq \varnothing \,]$

(3) The object x has no property a with probability from the interval $<\alpha,\beta>$

$[\, <V_*(a,x),V^*(a,x)> \cap <\alpha,\beta> = \varnothing \,]$

We postulate the following form of a term of the language $\mathcal{L}_N$: A term is the pair (a,P), such that $a \in A$ and $P \subset \nabla$ where ∇ is the characterisation of the set $C(<0,1>)$. (For details of the geometric construction and some algebraic properties of the set ∇, see [10]).

Definition 2 (Alphabet of the language $\mathcal{L}_N$)

The alphabet of $\mathcal{L}_N$ consists of:

(i) All pairs $(b_1,\ldots,b_m,P)$, where $b_i \in \tilde{A}$, $P \subset \nabla^m$, $i=1,2,\ldots,m$, taken as constants of the language $\mathcal{L}_N$, where

$\tilde{A}=\{b=a_{i_1} \vee \ldots \vee a_{i_k} \,;\, a_{i_j} \in A,\ 1 \leq i_1 \leq \ldots \leq i_k \leq n\}$

is the set of algebraic expressions on A. ($\tilde{A}$ is an atomic Boolean algebra, with atoms $a_1,\ldots,a_n$ and operations $(\wedge,\vee,-,\mathbb{0},\mathbb{1})$)

(ii) Constants $\mathbb{0},\mathbb{1}$

(iii) Symbols of Boolean operations $+,\bullet,\sim,\rightarrow$

Note that our language does not contain variables.

Definition 3 (Terms of the language $\mathcal{L}_N$)

The set $\mathcal{T}'$ of terms $\mathcal{L}_N$ is the least set $\mathcal{T}$ satisfying the following conditions:

(i) $\mathbb{0},\mathbb{1}$ Œ T

(ii) $(b_1,\ldots,b_m,P) \in \mathcal{T}$ where $P \subset \nabla^m$, $b_i \in \tilde{A}$ $i=1,\ldots,m$.

(iii) if $t,s \in \mathcal{T}$ then $t+s \in \mathcal{T}$, $t \bullet s \in \mathcal{T}$, $\sim t \in \mathcal{T}$, $t \rightarrow s \in \mathcal{T}$

It is obvious that each term of the form (a,P) is a particular case of the general form of the term $(b_1,\ldots,b_m,P)$ in Definition 3.

Parentheses are used, if necessary, in the obvious way.

As will become apparent later on, the order of a sum or product is immaterial, so we shall abbreviate finite sums and products as

$\sum_{i \in J} t_i$ and $\prod_{i \in J} t_i$, respectively.

Intuitively, terms are the names of certain features of objects more "complex" than those expressed by descriptors and intervals.

Semantics – interpretation of terms

We shall now define the semantics of our query language $\mathcal{L}_N$. For any query Q we shall define its value denoted by $\| Q \|$, which intentionally is the response of the N-system to the query Q. It is intuitively clear what the value of query is if we think of $\mathbb{0},\mathbb{1},-,+,\bullet,\rightarrow$ as corresponding to $\varnothing$,X, and the set theoretical operations of complementation, union, intersection and the operation "$(X \setminus A) \cup B$" respectively.

More formally we have the following definition.

Definition 4 (Value of term)

The value of the term $t \in \mathcal{T}$ in an N-system $S=<X,A,V>$ denoted by $\|t\|_S$ is defined inductively as the following subset of X;

1) $\| \mathbb{0} \|_S=\varnothing$, $\| \mathbb{1} \|_S=X$,

2) $\| (b_1,\ldots,b_m,P) \|_S=\{x\in X;\ (\tilde{V}(b_1,x),\ldots,\tilde{V}(b_m,x)) \in P\}$ where $\tilde{V}$ is an extension of the function V on the set of algebraic expressions $\tilde{A}$; $\tilde{V}:\tilde{A}\times X\rightarrow C(<0,1>)$ defined as follows.

If $b=a_{i_1}\vee\ldots\vee a_{i_k}$ then $\tilde{V}(b,x)=<\tilde{V}_*(b,x),\ \tilde{V}^*(b,x)>$ where

$$\tilde{V}_*(b,x) = \max\left(\sum_{j=i}^{k} V_*(a_{i_j},x),\ 1-\sum_{i\notin\{i_1\ldots i_k\}} V^*(a_i,x)\right)$$

$$\tilde{V}^*(b,x) = \min\left(\sum_{j=i}^{k} V^*(a_{i_j},x),\ 1-\sum_{i\notin\{i_1\ldots i_k\}} V_*(a_i,x)\right)$$

3) $\| t+s \|_S = \| t \|_S \cup \| s \|_S$

4) $\| t\bullet s \|_S = \| t \|_S \cap \| s \|_S$

5) $\| t\rightarrow s \|_S = (X\setminus \| t \|_S) \cup \| s \|_S$

6) $\| \sim t \|_S = X \setminus \| t \|_S$

In practice it may also be useful to consider atomic formulas of the type $t \le s$. However, since $t \le s$ can always be expressed by $t\bullet\sim s=0$, we will not explicitly introduce $\le$ into the language.

We shall prove the following theorem.

Theorem 5

For each N-system $S=<X,A,V>$ we have:

(1) $\| (b,\varnothing \|_S = \varnothing$

(2) $\| (b,\nabla) \|_S =X$,

(3) $\| (a_1\vee a_2\vee\ldots\vee a_m,\{1\}) \|_S = X$

(4) $\| (b,P_1) + (b,P_2) \|_S = \| (b,P_1 \cup P_2) \|_S$

(5) $\| (b,P_1)\bullet(b,P_2) \|_S = \| (b,P_1 \cap P_2) \|_S$

(6) $\| (b_1,P_1)\bullet(b_2,P_2) \|_S = \| (b_1,b_2,P_1\times P_2) \|_S$

(7) $\| (\sim b,P) \|_S = \| (b, \nabla \setminus P \|_S$

where $b=(a_{i_1}\vee\ldots\vee a_{i_m}) \in \tilde{A}$, $P\subset\nabla^m$

Proof:

(1) and (2) are obvious from the definition of the value of the term,

(3) follows from the simple fact that for each $x\in X$ we have $\tilde{V}(a_1\vee a_2\vee\ldots\vee a_m,x)=\{1\}$

(4) $\| (b,P_1) + (b,P_2) \|_S$

$= \| (b,P_1) \|_S \cup \| (b,P_2) \|_S$

$= \{x\in X;\ \tilde{V}(b,x)\in P_1\} \cup \{x\in X;\ \tilde{V}(b,x)\in P_2\}$

$= \{x\in X;\ \tilde{V}(b,x)\in P_1 \vee \tilde{V}(b,x)\in P_2\}$

$= \{x\in X;\ \tilde{V}(b,x) \in P_1 \cup P_2\} = \| (b,P_1 \cup P_2) \|_S$

The proof of (5), (6) and (7) is similar to the proof of (4).

■

Definition 6

Two terms t and s are semantically equivalent, denoted $t\approx s$, if for every N-system S

$\| t \|_S = \| s \|_S$

A query entering an N-system can be replaced by any other semantically equivalent query. In practice, it is reasonable to transform the query into some semantically equivalent query which minimises the cost of retrieval. We want the transformation to be carried out by purely syntactical means without accessing the file which stores the information about the objects. To this end we can use the set of axioms (rules of transformation) during the transformation process.

Prior to a presentation of axioms for terms in $\mathcal{L}_N$, we introduce and consider some special auxiliary sets.

Let $p=(\alpha,\beta)\in \nabla$ and let its corresponding interval in the characterisation of the set $C(<0,1>)$ (see [10]) be $\bar{p} = <\alpha,\beta> \subset <0,1>$.

Let $f_{i_1,\ldots,i_k}$: $R^n \rightarrow R$ be a function defined as follows:

$$f_{i_1,\ldots,i_k}(z)= \sum_{j=1}^{k} z_{ij}$$

where $z= (z_1,z_2,\ldots,z_n)$.

With $p \in \nabla$ we associate a set

$P_{i_1,\ldots,i_k} = \{(\alpha,\beta)\in H' ;\ f_{i_1,\ldots,i_k}(<\alpha,\beta> \cap H) = \bar{p}\ \}$

where $H = \{(z_1,z_2,\ldots,z_n) \in I^n ;\ \sum_{i=1}^{n} z_i = 1\}$,

$H'=\{(\alpha,\beta)\in H'' ;\ \forall\ i=1,\ldots,n\ \alpha_i + \sum_{\substack{j=1\\ j\neq i}}^{n}\beta_j \ge 1$ and $\beta_i + \sum_{\substack{j=1\\ j\neq i}}^{n}\alpha_j \le 1\}$

$H''=\{(\alpha,\beta)\in R^{2n};\ \alpha,\beta\in I^n,\ \alpha \le\beta\ \sum_{i=1}^{n}\alpha_i \le 1\le \sum_{i=1}^{n}\beta_i\ \}$,

$I = <0,1>$.

For a more intuitive interpretation of the sets H, H', H", see [12].

The set $p_{i_1,\ldots,i_k}$ can be understood as a set of all possible proper representations (see [12]) of the object $x\in X$ such that

$\tilde{V}(b,x)=\bar{p}$ where $b=a_{i_1}\vee\ldots\vee a_{i_k}$

Let $P \subset \nabla^m$, where m is a natural number, then every point of the set P is an ordered sequence $(p^1,p^2,\dots,p^m)$ where $p^i \in \nabla$, $i=1,2,\dots,m$.

Now we shall define the following three auxiliary sets:

(6) $P^{1}_{i_1,\dots,i_k} = \{(h',p^2,\dots,p^m) \in H' \times \nabla^{m-1};$
$h' \in (p^1)_{i_1,\dots,i_k},\ (p^1,p^2,\dots,p^m) \in P\}$

(7) $P^{1,1} = \{(p^1,p^2,\dots,p^{m-1}) \in \nabla^{m-1};$
$(p^1,p^1,p^2,\dots,p^{m-1}) \in P\}$

(8) $P^{j,k} = \{(p^1,p^2,\dots,p^m) \in \nabla^m;$
$(p^1,\dots,p^{k-1},p^j,p^{k+1},\dots,p^{j-1},p^k,p^{j+1},\dots,p^m) \in P\}$

In the figure below we demonstrate a simple example of two intervals $<\alpha,\beta>$ and $<\alpha',\beta'>$ such that $(\alpha,\beta), (\alpha',\beta') \in P_{1,3}$.

The intersections of these two intervals with H are shown. The sums of the first and the third coordinates of points belonging to these intersections constitute the whole interval $\overline{p}$.

Axioms for queries to N-system

Definition 7 (Axioms for terms)

The set **B** of axioms consists of:

B1: Substitutions of terms into the axioms of Boolean algebra.

B2: $(b_1,\dots,b_m,P_1) + (b_1,\dots,b_m,P_2) = (b_1,\dots,b_m,P_1 \cup P_2)$

B3: $(b_1,\dots,b_m,P_1) \bullet (b_1,\dots,b_m,P_2) = (b_1,\dots,b_m,P_1 \cap P_2)$

B4: $\sim(b_1,\dots,b_m,P) = (b_1,\dots,b_m,\nabla^m \backslash P)$

B5: $(b_1,\dots,b_m,\varnothing) = \mathbf{0}$, $(a_1 \vee a_2 \vee \dots \vee a_m,\{1\}) = \mathbb{1}$

B6: $(b_1,b_2,\dots,b_m,P) = (a_1,\dots,a_n,b_2,\dots,b_m,P^{1}_{i_1,\dots,i_k})$

where $b_1 = a_{i_1} \vee a_{i_2} \vee \dots \vee a_{i_k}$ and $P^{1}_{i_1,\dots,i_k}$ is defined by (6)

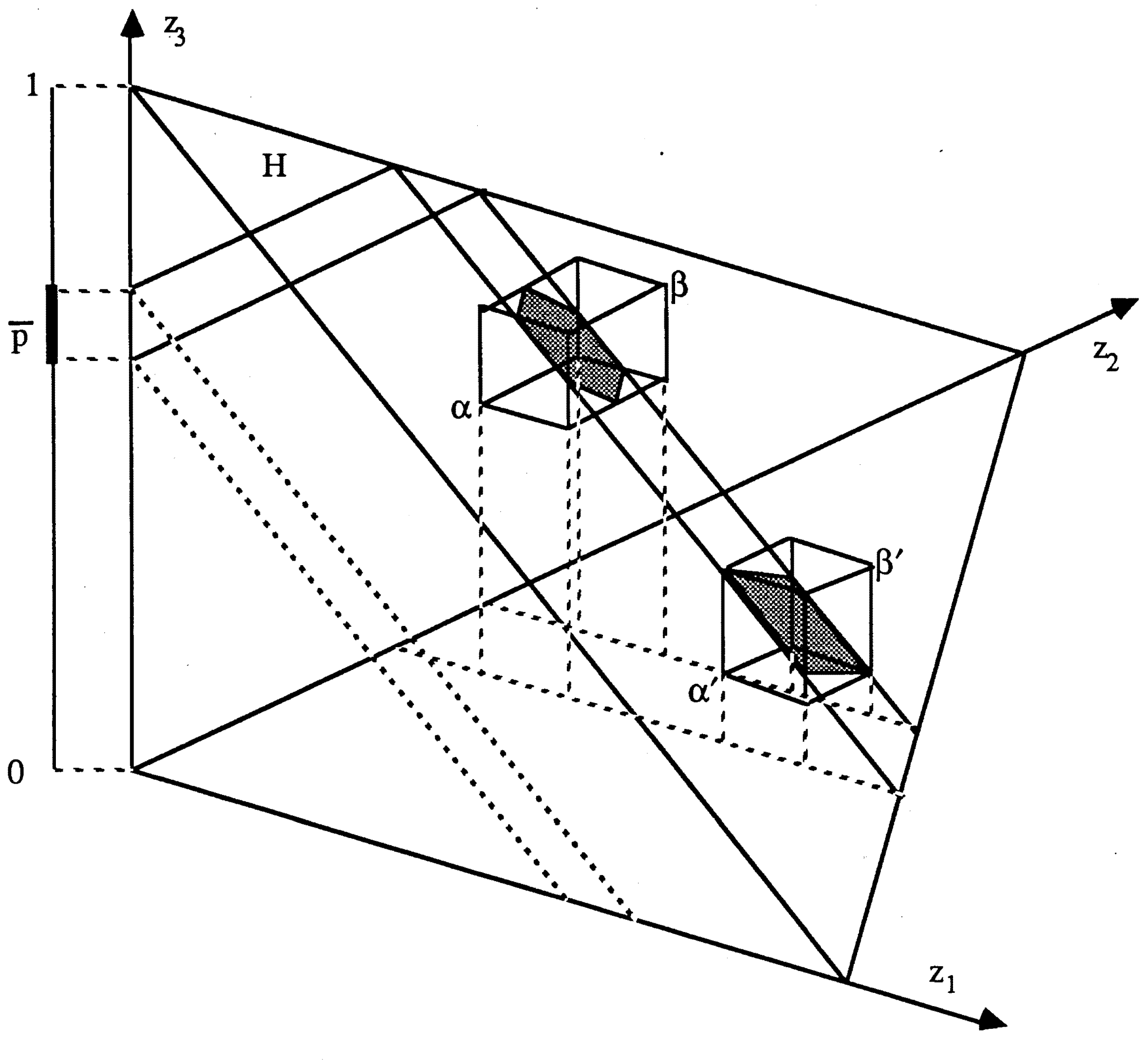

Figure 1

B7: $(b_1,b_1,b_2,\dots,b_{m-1},P) = (b_1,\dots,b_{m-1},P^{1,1})$
where $P^{1,1}$ is defined by (7)
B8: $(b_1,\dots,b_m,P)$
$=(b_1,\dots,b_{j-1},b_k,b_{j+1},\dots,b_{k-1},b_j,b_{k+1},\dots,b_m,P^{k,j})$
where $P^{k,j}$ is defined by (8), $1 \le j \le k \le m$.

Definition 8 (Syntactical equivalence of terms)

Two terms t and s are syntactically equivalent, denoted t=s, if and only if there exists a term $u \in \mathcal{T}$ such that the terms t and s can be transformed into the term u using the set of axioms $\mathbb{B}$ and the inference rule *Modus Ponens.*

Theorem 9

Syntactical equivalence of two terms $t,s \in \mathcal{T}$ implies their semantic equivalence;

$(t=s) \Rightarrow \| t \|_S = \| s \|_S$ for each N-system S.

Proof: We should show that the values of terms, from two sides of equalities in the set of axioms $\mathbb{B}$, are equal in any fixed N-system S.

B1–B5 are a direct consequence of Theorem 5.

B6 let $x \in X$

$x \in \| (b_1,\dots,b_m,P) \|_S \Leftrightarrow \forall (p^1,\dots,p^m \in p)$

$(\tilde{V}(b_1,x),\dots,\tilde{V}(b_m,x))=(p^1,\dots,p^m) \underset{*}{\Leftrightarrow}$

$V(a_1,x),\dots,V(a_n,x)) \in P^{1}_{i_1,\dots,i_k}$,

$\tilde{V}(b_2,x)=p^2,\dots,\tilde{V}(b_m,x)=p^m \Leftrightarrow V(a_1,x),\dots,V(a_n,x)$,
$\tilde{V}(b_2,x),\dots,\tilde{V}(b_m,x) \in P^{1}_{i_1,\dots,i_k}$

$\Leftrightarrow x \in \| (a_1,\dots,a_n,b_2,\dots,b_m,P^{1}_{i_1,\dots,i_k} \|_S$.

Let us explain equivalence $\underset{*}{\Leftrightarrow}$. Let $r_S(x)=<\alpha,\beta>$ where

$r_S(x)=\{(z_1,z_2,\dots,z_n) \in R^n ; V_*(a_i,x) \le z_i \le V^*(a_i,x)$
$i=1,2,\dots,n\}$.

Then

$\tilde{V}(b_1,x)=p^1 \Leftrightarrow f_{i_1,\dots,i_k}(<\alpha,\beta> \cap H)=\bar{p}^1$

$\Leftrightarrow (\alpha,\beta) \in P^{1}_{i_1,\dots,i_k} \Leftrightarrow V(a_1,x),\dots,V(a_n,x)$

$= (\alpha,\beta) \in P^{1}_{i_1,\dots,i_k}$.

B7 $X \in \| (b_1,b_1,b_2,\dots,b_m,P) \|_S$

$\Leftrightarrow (\tilde{V}(b_1,x),\tilde{V}(b_1,x)),\tilde{V}(b_2,x),\dots,\tilde{V}(b_m,x)) \in P$

$\Leftrightarrow \tilde{V}(b_1,x),\tilde{V}(b_2,x),\dots,\tilde{V}(b_m,x) \in P^{1,1}$

$\Leftrightarrow x \in \| (b_1,b_2,\dots,b_m,P^{1,1} \|_S$

B8 this is a direct consequence of the definition of the set $P^{k,j}$.

■

Normal form theorems play an important role in the theory of N-systems. On the one hand they are the main tool for proving completeness results and, on the other hand, they have far-reaching consequences on implementation.

In this paper we shall present only one normal form, but a far more intensive discussion of the normalisation of N-systems will be presented in a separate paper.

Definition 10 (Canonical form)

A term $t \in \mathcal{T}$ is in canonical form if $t=(a_1,a_2,\dots,a_n,P)$ where $P \subset H'$.

Theorem 11

For each term t there is a term s in canonical form such that $\vdash t=s$.

Proof: It is rather obvious that the terms $\mathbb{0}$ and $\mathbb{1}$ can be represented in the form:

$\mathbb{1} = (a_1,\dots,a_n,H')$

$\mathbb{0} = (a_1,\dots,a_n,\varnothing)$

Let $t=(b_1,\dots,b_m,P)$ be an arbitrary term. Using the axiom B6 in definition 9 we have

$t=(b_1,\dots,b_m,P)=(a_1,\dots,a_n,b_1,\dots,b_m,P^{1}_{i_1,\dots,i_k})$.

Now applying n-times the axiom B8 we may write

$t=(b_2,a_1,\dots,a_n,b_3,\dots,b_m,$
$((((P^{1}_{i_1,\dots,i_k})^{n+1,n})^{n,n-1})\dots)^{2,1})$

Repeated application of B6 and B8 gives us the following form

$t=(a_1,\dots,a_n,a_1,\dots,a_n,\dots,a_1,\dots,a_n,Q')$
where $Q' \subset (H')^m$

Now, using B7 and B8 repeatedly, we have:
$t=(a_1,\dots,a_n,Q)$, $Q \subset H'$
but this is the canonical form of the term $(b_1,\dots,b_m,P)$.

Therefore our proof is complete.

■

To complete our consideration of the completeness property, we have to prove that semantic equivalence for terms implies their syntactical equivalence.

Theorem 12

Let t and s be two terms. If for every N-system S
$\| t \|_S = \| s \|_S$ then $t = s$.

Proof: Firstly we shall show that for each term $t \in \mathcal{T}$, $t \ne 0$, there exists an N-system S such that
$\| t \|_S \ne \varnothing$

Using Theorem 11 we may transform t into canonical form
$(a_1,a_2,\dots,a_n,P)$
where $P \subset H'$ and $P \ne \varnothing$. There is then some $<\alpha,\beta>$ such that $<\alpha,\beta> \in P$.

where $P \subset H'$ and $P \neq \emptyset$. There is then some $<\alpha,\beta>$ such that $<\alpha,\beta> \in P$.

Let x be an object in an N-system $S=<\{x\},A,V>$ such that

$V(a_i,x)=<\alpha_i,\beta_i>$ i=1,...,n,

so far in this N-system S

$x \in \| t \|_S$ then $\| t \|_S \neq \emptyset$.

Let us suppose that $t \neq s$, $t,s \in \mathcal{T}$.

We have

$(t\cdot\sim s) + (\sim t\cdot s) \neq 0$

From the first part of this proof it follows that for some N-system S the value of the term

$\| (t\cdot\sim s) + (\sim t\cdot s) \|_S \neq \emptyset$

so

$\| t \|_S \cap (X \setminus \| s \|_S) \cup (X \setminus \| t \|_S) \cap \| s \|_S \neq \emptyset$

but this means that

$\| t \|_S \neq \| s \|_S$

and this is the end of the proof.

■

The conjunction of Theorems 11 and 12 is known as the Completeness Theorem.

The completeness property for terms for a multi-attribute N-system $S=<X,A,R_I,V>$ can easily be obtained by applying an algebraic operation called the direct product (see [9]), to single-attribute N-systems S_i, i=1,2,...,I. This procedure is rather technical and is not presented in this paper.

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