# TOWARDS CONTEXTS WITH MATHEMATICAL MODELLING TASKS

**Hanti Kotze**
**University of Johannesburg**
hantik@uj.ac.za

**ABSTRACT**– This paper aims to determine the extent to which Biomedical Technology students contextualise real world problems. First year students in the Health Sciences are challenged to transfer their mathematical knowledge and skills to other contexts. Conversely, in many mathematics classrooms, students have limited exposure to the vast variety of contexts and settings on offer from daily life. This research seeks to address this gap. A mathematical modelling approach offers students opportunities to understand the real world contexts of tasks, to work mathematically and interpret solutions in real world terms. A mathematical modelling task was designed that required students to contextualise the universal significance of an abstract model. The task contexts summoned practical considerations that helped students to understand the real world utility of the mathematical model. Some students remained somewhat detached from reality but could perform inner-mathematical work with prudence. Biomedical Technology students need more opportunities to model real life problems since contexts provide scope for deeper meaning to abstract concepts. In turn, contexts can elicit a subtle balance between knowledge and skill, a triad that should be inseparable. The custom- designed task will supplement the body of contextually-rich exemplars to advance Biomedical Technology students' modelling skills.

**Keywords:** Biomedical Technology; mathematical modelling; real world contexts; error analysis.

## 1. INTRODUCTION AND BACKGROUND

Mathematics is embedded in daily life phenomena but often remains isolated and unconnected with the real world. Contextually-rich problems offer learning opportunities that allow students to connect the world of mathematics with real life. Contexts not only suggest strategies to problem-solving but can be an avenue to develop diversified skills. For Freudenthal (1991, p. 75), contexts are "domains of reality disclosed to the learner in order to be mathematised". Schoenfeld (2001) warns that when skills are isolated from contexts, they are destined to be without meaning and purpose.

Biomedical Technology students register for a semester module in mathematics in their first year of study at the University of Johannesburg (UJ). The module aims to endorse mathematical knowledge and skills that can be applied to biomedical contexts. Although the teaching and learning style remains largely traditional, Kotze, Jacobs and Spangenberg (2015) report that students in a pilot study at UJ are enthusiastic to learn in a modelling environment that offers greater articulation of skills in biomedical contexts. The current study aims to examine the extent to which students relate to real world contexts when exposed to a mathematical modelling approach for the first time. The research question is: *To what extent can Biomedical Technology students contextualise real world problems?*

## 2. THEORETICAL PERSPECTIVES

Mathematical modelling is the process whereby a real life phenomena is translated into a mathematical problem with the aim to find a realistic solution for the real world situation (Blum & Leiß, 2007). Although these protagonists of a mathematical modelling approach advocate its implementation at all levels of mathematics education, it is still a novel undertaking for most students at UJ. Often, classroom activities focus more on abstract mathematical concepts and principles with undue regard for their applicability, usefulness and connections with real life. According to Blum and Leiß (2007:225), the modelling cycle (Figure 1) originates in the physical world and relates to a real world problem. A non-mathematical situation must be abstracted as a situation model; this helps to untangle the reality-based problem. Only relevant information is retained such that the situation model develops into a simplified and idealised real model. The real model is then translated into a mathematical model; Freudenthal (1991) labels this process mathematising. In the next phase of the modelling process, the mathematical model must be manipulated mathematically; activities encompass all relevant mathematical domains and applicable skills, algorithms, processes and calculations to produce a mathematical solution. Mathematical results are then interpreted and validated in relation to the situation model. It may be necessary to repeat the modelling cycle if mathematical results are in conflict with the real world situation. The modelling process concludes with the presentation of results. As an indispensable component of the modelling process, contexts allow for greater flexibility in the modelling cycle (Freudenthal, 1991).

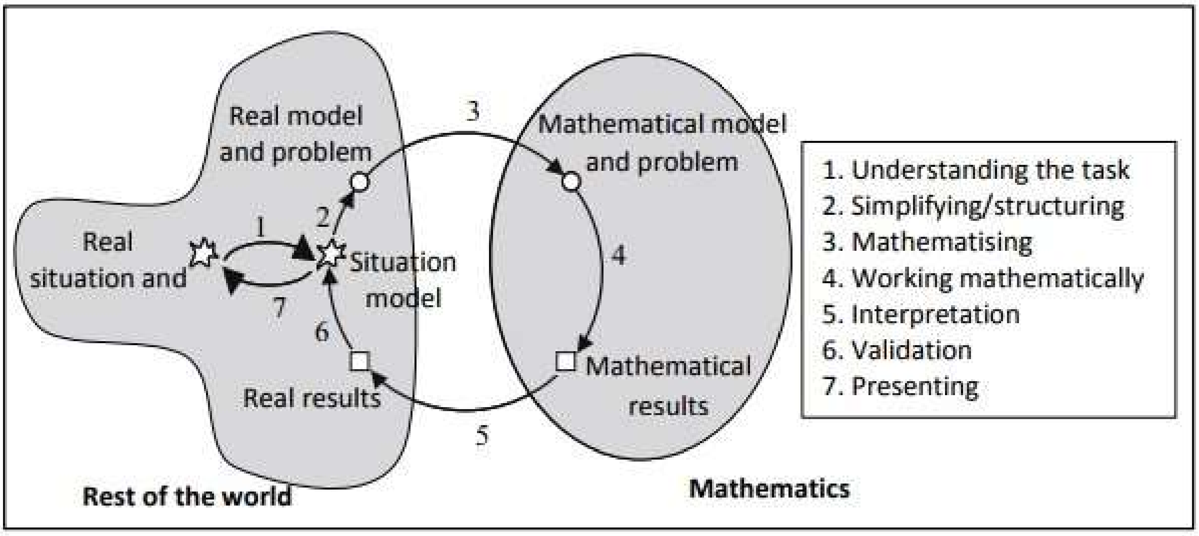


**Figure 1: The mathematical modelling cycle according to Blum and Leiß (2007:225).**

Mathematical tasks**,** situated in contexts that students can easily relate to**,** have the potential to enhance students' sense-making of problems (Wijaya, van den Heuvel-Panhuizen, Doorman & Robitzsch, 2014). Context-rich tasks can even be relatively simple problems, as long as contextual settings contribute to learning and deeper understanding of concepts. Students are likely to show more interest in tasks with contexts that draw on their own daily-life experiences and curiosities (Wijaya et al., 2014). Contexts can be created by way of stories, illustrations, pictures and maps. Contextually-rich settings can provide information, serve as a problem-solving stimulus, elicit model building and support the validation of processes and solutions.

Newman (1977) identifies five categories of errors to analyse students' difficulties when solving word problems; they are reading, comprehension, transformation, mathematical processes and encoding. Wijaya et al. (2014) point to remarkable similarities between the Newman error categories – associated with word problems – and errors made in the modelling processes as described by Blum and Leiß (2007). According to Wijaya et al. (2014), difficulties in context-rich modelling tasks involve four phases namely understanding, translating the real world situation into a mathematical problem, working mathematically and interpreting the mathematical solution in real world terms. Arguably, reading is part of understanding and therefore, Newman's reading and comprehension categories can be merged (Wijaya et al., 2014). The studies of Newman (1977), involving 11-13 year old Australian students and that of Wijaya et al. (2014), involving 14-18 year old Indonesian students, reveal similar results: low performing students experience difficulties with understanding and translation of word problems and modelling tasks. Consequently, students in these two studies struggle to progress to the phase where mathematical work must be performed. Taking heed of these outcomes, this study investigates errors made by Biomedical Technology students in a context-rich modelling task.

## 3. RESEARCH DESIGN AND METHODOLOGY

Supported by a contextualist world view (Schraw, 2013), this study values knowledge that is not only applicable to real world contexts, but is acquired from settings where new facts can collectively be debated. Within this paradigm, lecturers act as facilitators to promote learning experiences wherein students can negotiate a "consensual reality within a specific context" (Schraw, 2013, p. 3).

The participants were a 50-strong cohort of first year Biomedical Technology students at UJ. Students were divided into ten groups of four to six students per group. Using recent semester test scores, each group included low, middle and high performers. With this group structure, it was hoped that low performers would be more inspired and high performers would demonstrate their 'better' abilities. This was students' first exposure to a mathematical modelling approach.

Over the course of a semester, students were exposed to four mathematical modelling tasks; the third task (Appendix A) is reported on in this paper. The custom-designed task was completed within a scheduled 90-minute tutorial period and related to the area of the Bermuda Triangle (Figure 2). A narrative described the enigmatic disappearance of aircraft and ships over the triangle. Although the model for the area of the triangle was given, it still had to be interpreted in terms of the real world contexts. Students had to use technology to source the global positioning system (GPS) coordinates of the triangle. In effect, each different set of GPS coordinates would result in a different area.


**Figure 2: The Bermuda triangle map.**

All students had the opportunity to participate in the modelling task. To encourage students' attendance and participation, the task contributed to the semester mark. Each group submitted a completed worksheet which was assessed by the researcher and moderated by another subject expert. The inter-rater reliability of the task was measured with Cohen's kappa $(\kappa = 0.81)$. A pilot study was conducted the previous year which confirmed the content validity of the task.

**4. DATA ANALYSIS**

Group documents were analysed with content analysis. Following Wijaya et al. (2014), Table 1 aligns the Newman (1977) error categories in column one with the mathematical modelling phases as described by Blum and Leiß (2007) in column two. However, *mathematising* and *interpreting* were further sub-divided to allow for more nuanced analyses of these two important but difficult phases in the modelling cycle (Blum & Leiß, 2007). This brings the amount of modelling phases in column two to six. In scoring the task, all six modelling phases were considered equally important. In each phase, a zero score was awarded for an inappropriate response or a score of one for a relevant response; therefore the maximum score that could be awarded was six. Since there were ten groups, the total amount of possible errors was 60. Overall, 21 errors were recorded in the task. The task was analysed according to the modelling phases of Blum and Leiß (2007) where errors were not allowed to accumulate. For example, a wrong conversion of the GPS coordinates in phase one was not again penalised in follow-up phases. The most common errors appear in the third column of Table 1.

Table1. Newman's error categories, modelling phases and typical errors made in the task.

| Newman's error categories | Modelling phases (% errors) | Errors |
|---|---|---|
| Comprehension | Understanding (0%) | No errors were made as all groups could fully understand the task and source omitted data (GPS coordinates) |
| Transformation | Mathematising_1 (GPS coordinates) (23.8%) | Incorrect conversion of GPS coordinates from degrees minutes and seconds (DMS), to decimal degrees (DD)<br>Incorrect positioning of GPS coordinates on vertices of triangle<br>Unable to associate North and East with positive coordinates and South and West with negative |
| | Mathematising_2 (set up determinant) (0%) | No errors were made in representing relevant vertices of the triangle in order to set up the appropriate |
| Process skills | Working mathematically (9.5%) | Using wrong co-factor expansion in the evaluation of the determinant<br>Calculation mistakes in evaluating determinant |
| Encoding | Interpreting (23.8%) | Mistaking $\pm$ signs in given determinant model for $\pm$ signs associated with longitude or latitude<br>Confusing $\pm$ sign in given model with $\pm$ (as in taking the square root) |
| | Validating and presenting (42.9%) | Converting the area of the triangle from degrees to $km^2$ using 1 $= 111, 701km$<br>Presenting area as a negative value<br>2<br>Unrealistic answer given in $km$ instead of $km$<br>Unable to judge correctness of Bermuda triangle area<br>Presenting area in degrees (without converting to $km^2$) |

In the *understanding* phase, students had to understand *what* to do in terms of the contextual setting of the task. The given Bermuda triangle map (Figure 2) probed students to visualise the task and aimed to stimulate a sense of familiarity with the real world contexts of the task. The sub-tasks (Appendix A) were designed to stimulate understanding and help students to structure, simplify and delineate processes (Blum & Leiß, 2007). Since all groups could successfully source the omitted data (GPS coordinates) with technology and acquired a full understanding of the task, no errors were recorded in the *understanding* phase. In the first *mathematising* phase, students had to apply their prior knowledge of longitudes, latitudes and GPS coordinates to convert from degrees, minutes and seconds (DMS) to decimal degrees (DD). Analogous to Cartesian coordinates, a point on the longitude-latitude grid is positive if it lies North of the Equator (representing an East-West axis or $0$ latitude); alternatively, a point is negative if it lies South of the Equator. Similarly, a point is positive when it lies East of the Prime Meridian (representing a North-South axis or $0$ longitude) and negative if it lies West of the Prime Meridian. For example, the DMS coordinates $(32\ 18'28.08''N;64\ 45'1.8''W)$ of St George in Bermuda, had to be converted to DD coordinates $(32,3078\ ;-64,7505\ )$. Altogether, 23.8% of all errors were made in the first mathematising phase. On the contrary, no errors were recorded in the second *mathematising* phase. This means that all groups could accurately present their assumed GPS coordinates in terms of the given determinant – which represented the model for the area of the triangle. *Working mathematically* seemed to be the easiest part of the task. Procedural errors in this phase amounted to 9.5% of all errors made in the task; these included calculation mistakes and incorrect co-factor expansions to evaluate the determinant. Errors in *interpreting* the solution also amounted to 23.8% of all errors made, thus the second highest alongside the first *mathematising* phase. The plus-minus sign in the determinant was incorrectly interpreted by five groups. Group 8 spontaneously associated the plus-minus sign with the quadratic formula – this perhaps being the *only* instance where students have encountered this dual sign before – and trustingly computed the square root of the area. Three other groups confused this plus-minus sign with signs associated with North, East, South and West. Unexpectedly, the *validating* *and presenting* phase amassed most errors (42.9%). Since the Equator is divided into 360 degrees on the longitudinal axis, each decimal degree on the equator represents approximately 111.701 km. A conversion was therefore required to present the area of the Bermuda triangle in square kilometers. Most groups erred by multiplying the area by 111,701km instead of $(111,701km)^2$. In a former study unit, students were fluent with comparable conversions such as $m\,s^{-1}$ to $km\,h^{-1}$. If the *interpreting* phase is combined with the *validating and presenting* phase, this collective phase (labelled Encoding, according to Newman) will account for 66.7% of all errors made in the task. Figure 3 shows the correct responses in each modelling phase for each group. Only Group 2 obtained the maximum score (six) while five groups succeeded in only three of the modelling phases.

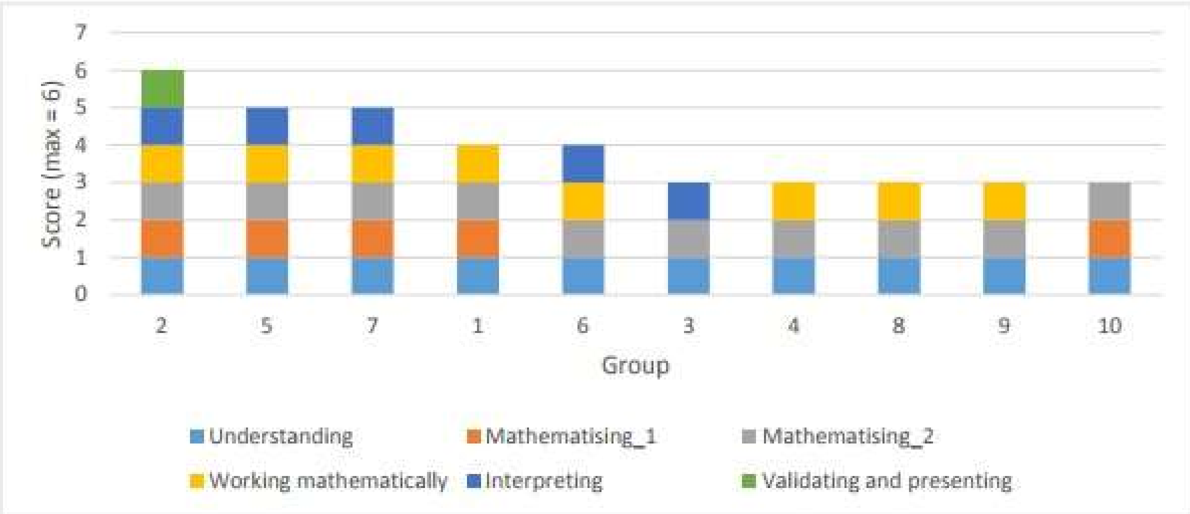


**Figure 3: Percentage correct responses of ten groups.**

**5. DISCUSSION**

According to Schoenfeld (2001), mathematics in contexts relate to the use of ordinary tools and skills in new contexts. The Bermuda triangle contexts encouraged students to source missing data, mathematise global coordinates of real world places and transfer data to an unfamiliar model (area of the triangle). Students had to exhibit ownership of prerequisite knowledge and technological skills to make sense of the task. The task commanded various mathematical concepts to be integrated into new contexts: areas, determinants, coordinates, conversions between DMS and DD and the dual meaning of the plus-minus sign in the given model. Results indicate that students could fully *understand* the intention of the task which eased the way for five groups to *mathematise* the real world data without errors. The given mathematical model for the area of a triangle was a novelty, yet all groups could correctly transfer their mathematised data to the model. Inner *mathematical work* was performed with prudence and errors were only observed in Group 3. Up to this point in the task, a mere 33% of the total amount of errors were noted. Most likely, this acclaimed success can be ascribed to the design of subtasks that had a scaffolding impact on students' modelling processes; in particular the modelling phases of *understanding*, *mathematising* and *working mathematically*. This outcome is contrary to the findings of Wijaya et al. (2014) who report that most errors were made in *understanding* and *mathematising*. However, the study of Wijaya et al. also indicates that fewer errors were made in *working mathematically* and *interpreting* the solution. A major concern was students' inability to take the contexts of the Bermuda triangle problem into account in the final phases of the task. Most students seemed to be reluctant – or perhaps unused – to reflect on contexts in order to 'find truth' in their answers. This reluctance may even suggest that *interpreting*, *validating* and *reporting* on the task, which accumulated 66.7% of errors, are not part of students' standard classroom repertoire. Classroom practices which rarely stimulate these modelling phases where errors predominated, may induce a lack of skills to reason about solutions; as Schoenfeld (2001, p. 53) pointedly says, all that is often required of students is "to draw a box around the correct answer".

When the Bermuda triangle task was analysed with the Newman (1977) error categories, errors were considered to be hierarchical and were allowed to accumulate from step to step (Wijaya et al., 2014). Even so, relatively few errors (39%) were made in the Newman error categories *comprehension, transformation* and *process skills.* Per illustration, students in Group 3 used incorrect signs when they converted the GPS coordinates from DMS to DD; consequently, they obtained a negative area for the Bermuda triangle. This error was however 'fixed' in their next subtask when they correctly interpreted the plus-minus sign as a means to eliminate the negative value obtained for the area. This is evidence that contexts were employed even though errors were made in previous steps. In this sense, subtasks served as a diagnostic tool to identify weaknesses in the modelling phases. The modelling phases of Blum and Leiß (2007) are considered to be more suitable to analyse a modelling task, which ultimately, is more concerned with a meaningful solution than with adequate procedural processes.

## 6. CONCLUSION

The contextual setting of the Bermuda triangle task demonstrated that when students understand the meaning of real world contexts, they are more likely to be successful to operate, transfer and model with real world data. Yet, the interpretation and validation of solutions remained largely unconnected to contexts. Evidently, the ability to solve a problem mathematically is no guarantee that solutions will be correctly interpreted. Although mathematics can be used to unlock understandings and interests in the public domain (Schoenfeld, 2001), students were unaccustomed to contextually-rich tasks and need more opportunities to connect their own world with the world of mathematics. How students will respond to a task with a more local context may be an area for further investigation.

## REFERENCES


Blum, W. & Leiß, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum & S. Kahn (Eds.), *Mathematical modelling: Education, Engineering and Economics – ICTMA 12* (pp. 222–231). Chichester: Horwood Publishing.

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht: Kluwer Academic Publishers.

Kotze, H., Jacobs, G. & Spangenberg, E. (2015). Attitudes of biomedical technology students towards mathematical modelling. *Proceedings of the ISTE International conference on Mathematics, Science and Technology Education*, 25 – 29 October, Mopani Camp in Kruger National Park, South Africa, (pp. 136-148).

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin*, 39, 31-43.

Schoenfeld, A. (2001). Reflections on an impoverished education. In L.A. Steen (Ed.), *Mathematics and democracy: the case for a quantitative literacy* (pp. 49-54). National Council on Education and the Disciplines

Schraw, G. (2013). Conceptual integration and measurement of epistemological and ontological beliefs in educational research, Review article, *Hindawi Publishing Corporation ISRN Education*, Volume 2013, Article ID 327680. Available from http://dx.doi.org/10.1155/2013/327680

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M. & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: an analysis of students' errors, *The Mathematics Enthusiast*, 11(3): 555-584.


**Appendix A: Task 2 – The Bermuda triangle**

a) Search for the GPS coordinates of St George in Bermuda, San Juan in Puerto Rico and Miami in Florida. b) Convert these GPS coordinates to decimal degrees.

c) The area of a triangle with vertices $(x\ ,y\ );(x\ ,y\ )$ and $(x\ ,y\ )$ is given as

$$A = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

Use this formula to calculate the area of the Bermuda triangle.

d) What do you think is the meaning of the plus/minus sign in the formula given in Question c?

e) Convert the area of the Bermuda triangle to square kilometers; assume 1 degree = 111,701km.