RELATIONSHIP BETWEEN LEARNERS' MATHEMATICS-RELATED BELIEF SYSTEMS AND THEIR APPROACH TO NON-ROUTINE MATHEMATICAL PROBLEM SOLVING: A CASE STUDY OF THREE HIGH SCHOOLS IN TSHWANE NORTH DISTRICT (D3), SOUTH AFRICA

Munyaradzi Chirove
Institute of Science and Technology Education
University of South Africa
South Africa
Email: munyaradzi.chirove25@gmail.com

David Mogari
Institute of Science and Technology Education
University of South Africa
South Africa
Email: Morgrld@unisa.ac.za

Abstract–This study aims to determine if there is a relationship between High School learners' mathematics-related belief systems and their approach to mathematical non-routine problem-solving. Data was collected, firstly, from 425 high school learners using survey questionnaires and mathematics non-routine problem test and, lastly, from six cases selected from the 425 learners using interview schedules, open-ended and retrospective questionnaires. The basic methods used to analyze the data were factor analysis, thematic analysis and methodological triangulation. Learners' mathematics-related beliefs were classified into three categories, according to Daskalogianni and Simpson's (2001) macro-belief systems: utilitarian, systematic and exploratory. A number of learners' problem solving strategies that reflected their general behaviour in non-routine mathematical problem solving were identified, that include unsystematic guess, check and revise; systematic guess, check and revise; trial-and-error; systematic listing; looking for a pattern; making a model; and considering a simple case. A weak positive linear relationship between learners' mathematics-related belief systems and their approaches to non-routine problem solving was discovered.

Keywords: mathematics-related belief system, problem solving, non-routine problem solving, approach to mathematical non-routine problem solving

1 INTRODUCTION

Studies conducted over the past twenty years on South African learners' problem solving abilities discovered that the learners lack adequate problem solving strategies and skills (e.g., Maree, Aldous, Swanepoel, & Vander Linde, 2006; and Wessels, 2012). Similar findings were also discovered in some other countries, for example, Netherlands (Kolovou, Heuvel-Panhuizen, Bakker, & Elia, 2011), Chile, Phillipines and Indonesia (HSRC, 2006), and Malaysia (Zanzali & Nam, 1997). There seems to be a debate among mathematics education researchers on to whom learners' deficiencies or incompetence on problem solving should be attributed to. Some researchers blame the high school classroom mathematics as not preparing learners adequately to solve non-routine problems (e.g., Mabilangan, Limjap, & Belecina, 2011). Some researchers (e.g., Maree et al., 2006; and Wessels, 2012), attach the blame to mathematics educators who fail to prepare learners for non-routine problem solving because of various reasons, such as, teacher adopting a traditional teaching approach and teacher's inadequate mastery of mathematics content.

Some researchers, e.g., Lester (2013) and Kolovou et al. (2011), argue that teachers are not to blame but the whole mathematics education community should bear the blame. Kolovou et al. (2011) posit that research findings have not yet revealed much on how learners approach non-routine mathematical problems and how teachers can facilitate or support learners to solve the problems. Lester (2013) states that, as of to date, among all the research studies conducted aimed at enhancing learners' meta-cognitive abilities in problem-solving, none has identified the proficiencies teachers need to do this.

In search of a solution to learner performance in mathematics, over the past twenty years, there seems to be an interest among some researchers on the effects of beliefs to performance in mathematics (e.g., Callejo & Vila, 2009; and Daskalogianni & Simpson, 2001). Though the researchers attempted to unravel the relationship between beliefs and other constructs such as learners' performance, problem solving behaviour, mathematics learning, motivation, meta-cognition, just to mention a few, there is still a lack of clarity on how learners' beliefs are related to their mathematical problem solving. Rather, contradictory conclusions were reached. For example, Daskalogianni and Simpson (2001) assume a causal relation; Spangler (1992) views the relationship as cyclic; and Callejo and Vila (2009) and Goldin, Rosken, & Torner (2009) assume no causality, but, rather, view the relationship between them as complex.

The present study determined if there is a relationship between high school learners' mathematics-related belief systems and their approaches to non-routine mathematics problem solving. As the classroom instruction do not prepare learners to solve non-routine mathematical problems effectively (Kolovou et al., 2011), the researcher conjectures that learners' mathematics-related belief systems do guide their mathematics problem solving behaviour.

2. CONCEPTUAL FRAMEWORK

2.1. Mathematics-related belief systems

Several scholars expressed their views on the definition of belief systems (e.g. Benbow, 2004; Callejo & Vila, 2009; Op`t Eynde, De Corte, & Verschaffel, 2006; Schoenfeld, 1985). Some researchers define a belief system in terms of its components (e.g, Op`t Eynde et al., 2006; Schoenfeld, 1985); while some researchers define it as a way one's beliefs are organized (e.g, Benbow, 2004; Callejo & Villa, 2009). In this study, the researcher viewed mathematics related belief systems as perceived by both Schoenfeld (1985) and Op`t Eynde et al. (2006). As such, he viewed mathematics-related belief systems as the conscious and unconscious subjective conceptions learners hold to be true about mathematics education, about themselves as learners of mathematics and about the mathematics class context that determine their approach to mathematics and mathematical tasks.

Daskalogianni and Simpson (2001), Jin, Feng, Liu, and Dai (2010), Lazim, Abu Osman, and Wan Salihin (2003), Op`t Eynde et al. (2006), and Schoenfeld (1992) are some of the researchers who gave different categories of mathematics-related belief systems. The categories proposed by these researchers reveal a lack of consensus among them on the precise categories of beliefs. Though there are quite a number of belief systems, in this study, the researcher classified learners' beliefs according to Daskalogianni and Simpson's (2001) belief systems (systematic, exploratory and utilitarian) (see table 1). Daskalogianni and Simpson's (2001) macro-belief systems appealed to the researcher because they provided examples of beliefs that fall into each category which guided him to classify learners' beliefs more appropriately into the respective belief systems.

Table 1: Macro- and micro-belief systems.

		MACRO-BELIEFS		
		Systematic	Exploratory	Utilitarian
MICRO-BELIEFS	Nature of mathematics	Methodical, logical	Problem-solving, linking things	Tool for other subjects, applied in life
	Focus of exercises	Follow series of steps	Understand different ways of thinking	Obtain correct exam answer
	Working in mathematics	Exact answer, similar exercises	Explore things, enjoy challenge	Known algorithms, study techniques
	Didactical contract	Dependence on notes and teacher	Dependence on own abilities	Dependence on teacher

Adopted from Daskalogianni and Simpson (2001, p. 15).

Learners whose macro belief systems can be classified as 'systematic' believe that mathematics is a static subject that is made up of a rigid body of knowledge. They are at easy with exercises where they have to apply previously practiced methods or strategies. As such, they also view mathematics as a logical and methodical subject, and view mathematical exercises as tasks that have exact answers and whose solution process involves a series of steps. When working on problems, they constantly refer back to notes and depend much on the teacher.

Learners whose macro belief systems can be classified as 'exploratory' view mathematics as dynamic. They believe that new mathematical truths, concepts and approaches to problem solving are under discovery. They view mathematics as involving problem solving, and having more than one correct answer. They also enjoy the challenge of new exercises and they do look for connections or links between the concepts learnt. They depend much on their own abilities on learning and solving mathematical problems.

Learners, whose macro belief systems can be classified as 'utilitarian', view mathematics as a tool for other subjects and as a subject that can be applied to solving real life problems. They focus much on study techniques and expect to obtain correct answers in exercises or exams. They approach mathematical problem solving by applying well known algorithms and numerical techniques. Utilitarian believers tend to depend much on the teacher in learning mathematics and problem solving.

2.2. Mathematical problem solving heuristics, approaches or strategies

The researcher defines problem solving as a process of applying previously acquired knowledge to new and unfamiliar situations. Unlike a routine mathematical question which a learner knows how to resolve immediately, a non-routine mathematical question requires a learner to apply creative thinking and resourcefulness in search of an appropriate problem solving approach (Bunday, 2013; Branca, 1980). In this study, what the researcher considered as important in problem solving were the methods, procedures, strategies, and heuristics that learners used in solving the problems.

Schoenfeld (1985) defines heuristic as rules of thumb that can be used when solving problems. Elia, Van den Heuvel-Panhuizen, and Kolovou (2009) classify heuristic strategies into cognitive (e.g., working backwards, finding a pattern; using analogies; considering extreme cases; visual representation; intelligent guessing and testing; systematically accounting for all possibilities; and deductive reasoning) and meta-cognitive strategies (e.g., decomposing the problem, monitoring the solution process, evaluating, and verifying results). However we should note that although a heuristic acts as a set of guidelines that a person applies to various situations, it does not guarantee success (as an algorithm) in problem solving. In this study, learners' cognitive and meta-cognitive approach strategies to problem solving were analyzed in connection to their mathematics-related belief systems.

In this context, this study attempted to determine if there is an observable relationship between, the 10th, 11th and 12th graders' mathematics-related belief systems and their approaches to non-routine mathematical problem-solving. The study addressed the following research questions:

1. What are the grade 10, 11, and 12 learners' mathematics-related belief systems?
2. Are the grade 10, 11, and 12 learners' approaches to mathematical non-routine problem solving?
3. Is there any relationship between learners' mathematics-related belief systems and their approach to mathematical non-routine problem-solving?

3. METHODOLOGY

This study employed a mixed methods study to determine and explain the nature of relationship between high school learners' mathematics-related belief systems and their approach to non-routine mathematical problem solving. A mixed methods design was adopted in consideration of the complexity of unravelling the relationship between the two constructs (Callejo & Vila, 2009). An explanatory sequential method design was used, and it is a design in which the qualitative data collected will be used to explain, in depth, the quantitative data previously collected (Cresswell, Klassen, Plano Clark, & Smith, 2011). In addition, the researcher adopted a positivist-interpretive paradigm in conducting the research study.

The primary purpose of this study was to use a beliefs questionnaire (close-ended) and mathematics non-routine problem solving test to determine learners' belief systems and approaches to problem solving, respectively. A secondary purpose was to gather qualitative data, by use of interviews and open-ended questionnaires, which explained the learners' beliefs and approaches to problem solving. The two data sets were, then, analysed to reveal the possible relationship between belief systems and approach to problem solving.

3.1 POPULATION AND CASE SELECTION

A convenience sample of 425 grade 10, 11 and 12 learners was selected from three high schools of Tshwane North District, Gauteng province of South Africa for participation in the study. All the selected learners, firstly, completed the mathematics beliefs questionnaire and then answered the mathematics problem solving test. The researcher used SPSS to cluster learners into two groups using the Hierarchical (complete linkage) method. He, then, purposefully selected six learners, three from each cluster as a case study on the basis of their degree of representative of learners belonging to each cluster. The selected learners participated in interviews and answered an open-ended questionnaire and a retrospective questionnaire.

3.2. DATA COLLECTION INSTRUMENTS

Mathematics beliefs questionnaire (BQ)

The researcher developed a 63-items mathematics-related beliefs questionnaire which he modelled on different beliefs questionnaires developed by Op't Eynde et al. (2006) and Physick (2010). The researcher conducted principal component factor analysis to assess 'construct validity' of the beliefs questionnaire. With the aid of a scree plot, five factors were identified that represented the 63 items loaded for factor analysis (See figure 1).

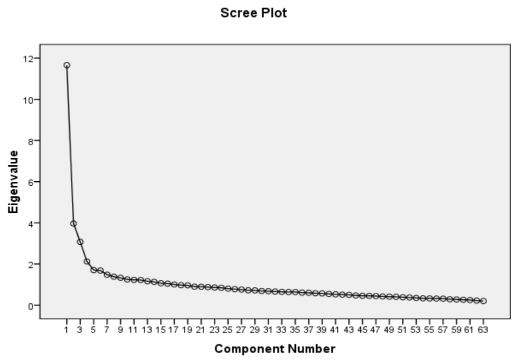

Figure 1: Scree plot showing number of components and eigenvalues of the correlation matrix

To facilitate analysis of BQ, the items were encoded in such a way that positive beliefs (that is, desirable or healthy beliefs about mathematics learning and problem solving) always gave a high value, while negative beliefs (that is, undesirable or unhealthy beliefs about mathematics learning and problem solving) gave a low value. For example, the belief: 'It is not important to understand why a mathematical procedure works as long as it gives the correct answer' was coded from 1 = "strongly agree" to 5 = "strongly disagree", and "For me the most important thing in learning maths is to understand" was coded from 5 = "strongly agree" to 1 = "strongly disagree".

To establish learners' beliefs about each scale used in this study, the researcher calculated the mean value of their responses to each scale. He considered a learner having an average score greater than 3 as holding that belief; and a mean score of 3 as holding a neutral belief. As such, a higher average belief score meant that the belief was held stronger than the other beliefs by the learner (Jin et al., 2010). The researcher used Daskalogianni and Simpson's (2001) key macro-belief systems (systematic, exploratory and utilitarian) to classify all the beliefs questionnaire items used in this study. Then, using the average score of each category in conjunction with the thematic analysis, he identified learners' predominant beliefs that largely influenced their behaviour in problem-solving.

To measure the reliability of the mathematics beliefs questionnaire, the researcher calculated the Cronbach's alpha on each factor. Factor 1 (I can do well in maths) had a very high reliability coefficient, **α** = 0,900. Similarly, factors 2 (I make sense of what I learn); 3 (Group work facilitates learning maths); and 4 (Maths is numbers, rules and techniques) had high reliability coefficients of 0,766; 0,816 and 0,696, respectively. Factor 5 (Maths is continuously evolving) had a slightly lower alpha of 0,557 as compared to the other four factors. After combining all the five factors, he concluded that the mathematics beliefs questionnaire was reliable.

To assess both convergent and discriminant validity of the beliefs questionnaire instrument, the researcher computed the correlation coefficients between the factors and represented them in form of a matrix (see table 2.1). The matrix of correlations revealed that the convergent coefficients were higher than the discriminant ones. Thus, the correlation matrix provided some evidence for both convergent and discriminant validity. By virtue of having evidence for both convergent and discriminant validity, the researcher concluded that the mathematics beliefs questionnaire was valid (Trochim, 2006).

Table 2.1: Extract of matrix of correlations between factors.

		FACTOR 1				FACTOR 2				FACTOR 3			
		Q16	Q14	Q12	Q43	Q34	Q33	Q36	Q37	Q21	Q27	Q30	Q38
FACTOR 1	Q16	**1**	**.629**	**.636**	**.437**	.333	.255	.227	.254	.173	.172	.119	.069
	Q14	**.629**	**1**	**.649**	**.447**	.312	.256	.310	.273	.066	.135	.094	.056
	Q12	**.636**	**.649**	**1**	**.482**	.339	.260	.271	.270	.101	.122	.111	.003
	Q43	**.437**	**.447**	**.482**	**1**	.250	.248	.189	.170	.156	.129	.095	.056
FACTOR 2	Q34	.333	.312	.339	.250	**1**	**.446**	**.389**	**.355**	.137	.138	.227	.024
	Q33	.255	.256	.260	.248	**.446**	**1**	**.340**	**.376**	.163	.158	.208	.049
	Q36	.227	.310	.271	.189	**.389**	**.340**	**1**	**.376**	.118	.109	.197	.044
	Q37	.254	.273	.270	.170	**.355**	**.376**	**.376**	**1**	.241	.229	.246	.196
FACTOR 3	Q21	.173	.066	.101	.156	.137	.163	.118	.241	**1**	**.370**	**.397**	**.355**
	Q27	.172	.135	.122	.129	.138	.158	.109	.229	**.370**	**1**	**.292**	**.311**
	Q30	.119	.094	.111	.095	.227	.208	.197	.246	**.397**	**.292**	**1**	**.404**
	Q38	.069	.056	.003	.056	.024	.049	.044	.196	**.355**	**.311**	**.404**	**1**

Mathematics problem solving test (PT)

The PT was made up of six multi-step, real life problems. The problems were adopted from Burton (1984); Greenes, Immerzel, Ockenga, Schulman, and Spungin (1986); Muis (2004) and Callejo and Vila (2009). The variables considered in selection of the problems were: (1) the problem should be comprehensible by the learners, (2) the problem requires no specialist mathematical knowledge, (3) facts and concepts to be applied, (4) strategies or approaches to be applied in solving the problem, and (5) the level of difficulty of the problem- demands of the questions should match the academic level of learners.

Problem 1 (P1) was an arithmetic sequence word problem. It required learners to look for pattern, list possible values or use an algorithm either derived from pattern analysis or from recognition. Problem 2 (P2) could be solved by use of logic (reasoning), or simple proportion, Problem 3 (P3) could be solved by use of an algebraic approach (forming an inequality). It expected learners to be able to manipulate inequalities, and look back to check if the answer makes sense.

Problem 4 (P4) could be solved by either logic (reasoning) or systematic trial and error. Problem 5 (P5) could be solved by an algebraic method (forming an equation) or by systematic trial and error. Problem 6 (P6) had no clear mathematics referents (i.e. had no numbers in its formulation). It could be solved by reasoning and algebraic method. Learners were also expected to be able to manipulate inequalities.

To identify the approaches applied by learners in solving problems, the researcher analyzed their written responses to the PT. Examples of approaches he sought from their written work were: random trial and error, systematic trial and error, seeking patterns, piece-wise, holistic, and non-attempts (blank answer sheet). He used the learners' approaches to problem-solving to infer their belief systems that explained their problem solving behaviour. To facilitate analysis of the learners' responses to the PT, the researcher formulated a coding scheme for each problem (Elia et al., 2009; Mabilangan et al., 2011) (see table 2.2).

Table 2.2: Extract of Coding scheme for the non-routine mathematics problem solving strategies

Category	Variable Name	Explanation
Systematic listing	SL	Making an organized list which is composed of at least three values. The steps are of the same size and trials 'move' in one direction.
Modeling	MD	Use of algebra (linear equations, simultaneous equations, linear inequalities), drawing diagrams, or sketches
Look for patterns	LP	Identifying some common characteristics that can be generalized and used to solve the problem.

Adapted from Elia, et al, (2009) and Mabilangan, et al (2011).

As suggested by Mabilangan et al. (2011), the researcher classified the problem solving strategies applied by learners into three main categories: (1) Thorough or insightful use of strategies, (2) Partial use of strategies, and (3) Limited strategies (see table 2.3). To facilitate analysis of the approaches or strategies employed by learners to problem solving, a point system was used: 5 points for insightful use of strategies, 3 points for partial use of strategies, and 1 point for limited use of strategies. 2 points were assigned to an approach in between limited and partial use of strategies. Similarly, 4 points were assigned to an approach in between partial and insightful use of strategies (Mabilangan et al., 2011). A score of 0 was assigned to a blank answer sheet. As such, the researcher used the table that classifies problem solving strategies as a marking rubric in this study.

Table 2.3: Classification of problem solving strategies

Thorough/ Insightful use of strategies	Partial use of strategies	Limited strategies
The strategies show some evidence of insightful thinking to explore the problem.	The strategies have some focus, but clarity is limited.	The strategies lack a central focus and the details are sketchy or not present.
The learner's work is clear and focused.	The learner applies a strategy which is only partially useful.	The procedures are not recorded (i.e., only the solution is present).
The strategies are appropriate and demonstrate some insightful thinking.	The learner starts the problem appropriately, but changes to an incorrect focus.	Strategies are random. The learner does not fully explore the problem and look for concepts, patterns or relationships.
The learner gives possible extensions or generalizations to the solution or the problem.	The learner recognizes the pattern or relationship, but expands it incorrectly.	The learner fails to see alternative solutions that the problem requires.

Adopted from Mabilangan et al. (2011, p. 28).

The content of the mathematics problem solving test was validated by two public high school heads of mathematics department and three high school mathematics teachers. To measure the reliability of the test, the researcher computed Spearman-Brown coefficient measure of reliability. The researcher obtained a coefficient of 0,509 which he regarded as lower than expected (0.70 and higher) (Maree et al., 2006) probably because most learners faced difficulties on solving the non-routine problems.

Open-ended Questionnaire (OQ) and Retrospective questionnaire (RQ)
The OQ was made up of seven questions the researcher wanted to follow up from the case studies. The RQ was made up of both closed and open questions (see Lucangeli & Cabrele, 2006). The RQ gave the respondents an opportunity to reflect on how they solved each problem, explain the approaches they employed, state any obstacles or difficulties met in problem solving, and their perception on whether they had solved the problems correctly. Basically, it tested learners' meta-cognitive skills of predicting, monitoring and evaluating their problem solving process.

Interview Schedule
The interview was semi-structured and pre-sequenced. The researcher selected five basic common questions that were based on learners' responses to the beliefs questionnaire. He requested the respondents to state reasons why they chose the specific rating for each belief item he selected from the beliefs questionnaire. Any other interesting leads that emerged during the interview were also discussed. Data from semi-structured interviews and questionnaires were cross-validated through convergent validity. Converging as well as diverging responses were noted and interpreted, thereof. In this regard, Anderson (1990) argues that conclusions suggested by different data sources are far stronger than those suggested by one alone. Reliability was determined by the consistency of the responses got from the interviewees.

4 FINDINGS

Learners' mathematics-related belief systems

As mentioned above, the items of the beliefs questionnaire were classified into three sub-categories according to Daskalogianni and Simpson's (2001) three key macro-belief systems, namely systematic, exploratory and utilitarian. By calculating the mean score of each of the three belief systems (see table 3.1), the learners' strongly held belief systems were identified as follows: The researcher considered a learner having a mean score greater than 3 as holding a certain belief, and the greater the score the more strongly held the belief is (Jin et al., 2010).

Table 3.1: Selected learners' mathematics related belief systems mean scores

	Mean belief score		
Learner	Systematic	exploratory	utilitarian
A26	3.39	3.65	3.68
A31	4.11	3.87	4.21
B08	3.94	3.48	4.05
B57	4.50	4.65	4.63
C27	3.94	3.87	3.74
F43	3.83	3.87	3.95

The table shows that each learner holds all the three belief systems, since their scores are all more than 3 in all the belief systems. The researcher considered the belief systems with higher scores as the predominant set of beliefs that could possibly explain the learners' approaches to problem solving. Using table 3.1 and based on the predominant belief systems, four learners, A26, A31, B08 and F43, were classified as utilitarian believers; one learner, B57, was classified as an exploratory believer; and one learner, C27, was classified as a systematic believer.

Out of 425 learners, 32 learners were systematic believers, 255 learners were exploratory believers and 122 learners were utilitarian believers. 10 learners were classified as neutral believers because they had mean belief scores which were less than or equal to 3. One learner had two equivalent highest mean scores of systematic and exploratory belief systems. 5 learners had, each, two equivalent highest mean scores of exploratory and utilitarian belief systems. In order to understand the constituents of each belief system, the researcher analyzed learners' responses to beliefs questionnaire in conjunction with their responses to retrospective questionnaire and interviews. The researcher, also, inferred some beliefs from learners' solutions to non-routine problems.

Most of the learners (99.1%) scored points between three and five, inclusively, on beliefs questionnaire (see table 3.2). This percentage (99.1%) could be extremely high and questionable.

The mean belief score of 425 learners was 3.73 and a standard deviation of 0.504. A mean score of 3.73 in beliefs indicated, in this study, that the learners held the beliefs in question. However, when answering the beliefs questionnaire, it is possible for a learner to choose the socially acceptable beliefs and fail to reveal his/her actual beliefs in mathematics problem solving. A similar finding was discovered by HSRC (2006) who discovered that South African learners stated some socially acceptable attitudes to mathematics other than their real attitudes. A qualitative analysis of the relationship between learners' mathematics-related belief systems and their approach to non-routine mathematical problem solving took into consideration learners' beliefs that were derived and inferred from responses on interviews, open-ended questionnaire and non-routine mathematics problem solving test to counteract the weaknesses of the closed-form beliefs questionnaire whereby learners might choose only the socially acceptable beliefs and hide their actual beliefs.

Table 3.2: Beliefs mean score frequencies. N = 425.

Mean Score (Nearest whole no.)		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	1	.2	.2	.2
	2	3	.7	.7	.9
	3	112	26.4	26.4	27.3
	4	303	71.3	71.3	98.6
	5	6	1.4	1.4	100.0
	Total	425	100.0	100.0	

Learners' non-routine mathematical problem solving strategies

For in-depth study, the researcher focused on the solution strategies and answers of the six learners he selected since they represented each category of learner clusters as indicated above. The six selected learners solved the problems using the following strategies: Systematic Listing (SL); Modeling (MD); Trial-and-error (TE); Use a Formula (F); Systematic Guess, Check and Revise (GCR(sys)); Unsystematic Guess, Check and Revise (GCR(unsys)); Consider a simple case (SC); Logical reasoning (LG); No logical reasoning (NLG); and Look for patterns (LP). (See table 3.3). Figures 2, 3 and 4 present examples of learners' written solutions to the problems.

Table 3.3: Summary of strategies applied by learners in solving the six non-routine problems.

							Problem					
							P1	P2	P3	P4	P5	P6
Learner	A26						GCR(unsys)	SC; LG	NLG	NLG; GCR(unsys)	NLG	SC; MD
	A31						SL	LG	GCR(unsys)	NLG; GCR(unsys)	SC; GCR(unsys)	MD; SC; LG
	B08						GCR(unsys)	SC; LG	GCR(unsys)	MD; GCR(unsys)	GCR(unsys)	MD
	B57						SL;LP; MD	SC; LG	MD	MD; TE	NLG; MD	MD; SC;TE
	C27					GCR(unsys)	LG	GCR(unsys)	NLG; GCR(unsys)	MD; GCR(unsys)	MD	
	F43	GCR(unsys)	LG	SC	MD; LG	MD	MD; LG					

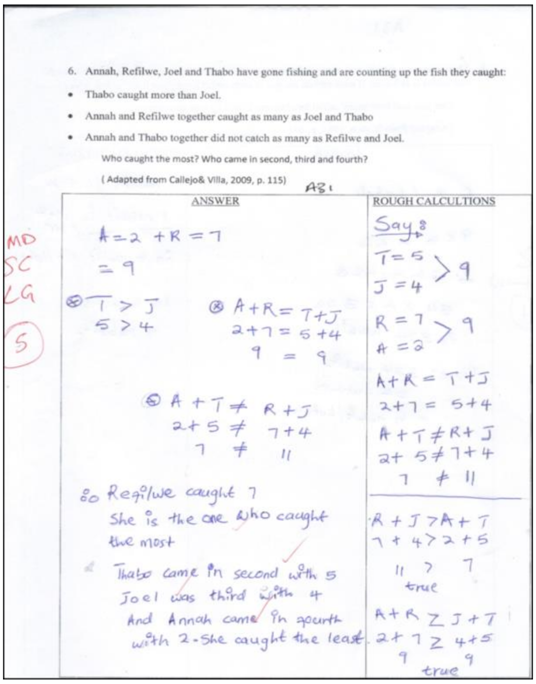

Figure 2: Learner A31's solution to P6.

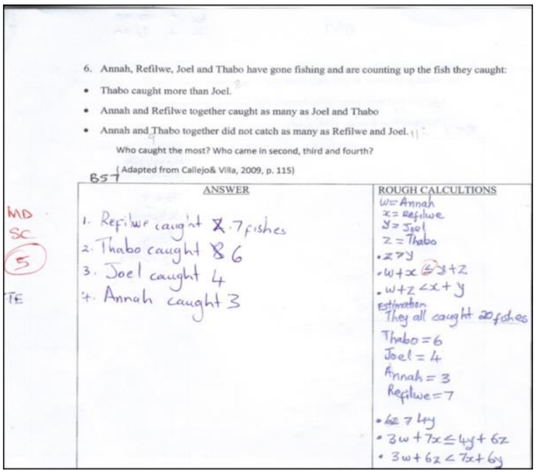

Figure 3: Learner B57's solution to P6.

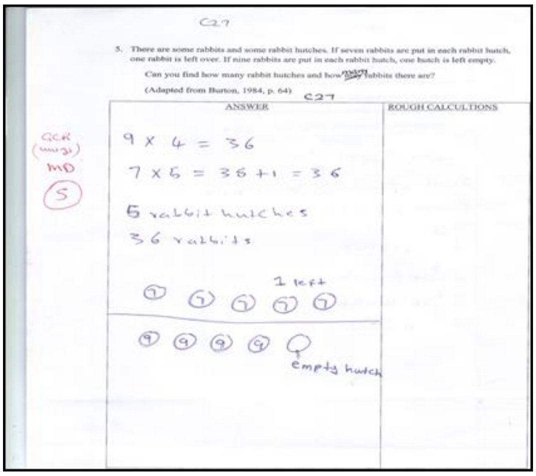

Figure 4: Learner C27's solution to P5

The majority of learners (85.6%) performed poorly and scored low points in mathematics non-routine problem solving test (see table 3.4). On average, the problem solving mean score of 425 learners was 1.62 (out of 5) and a standard deviation of 0.877. On average, the learners performed poorly in solving non-routine mathematical problems. Similarly, Maree et al. (2009) discovered that South African learners performed poorly in mathematics problem solving. The TIMSS (2003) discovered that, approximately, 30% of the South African grade 8 learners were able to solve non-routine mathematical problems. Similar results were discovered in other countries as well. For example, Mogari and Lupahla (2013) and Kolovou et al. (2011) discovered that Namibian and Dutch learners, respectively, performed poorly in mathematics non-routine problem solving even though they were classified as high performers in school mathematics.

Table 3.4: Problem solving mean score frequencies (N = 425)

Mean Score (Nearest whole no.)		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	36	8.5	8.5	8.5
	1	157	36.9	36.9	45.4
	2	171	40.2	40.2	85.6
	3	53	12.5	12.5	98.1
	4	8	1.9	1.9	100.0
	Total	425	100.0	100.0	

Relationship between learners' mathematics related belief systems and their approaches to mathematics problem solving

The researcher computed Pearson's correlation coefficient as a numerical measure of the relationship between belief systems and approaches to non-routine mathematics problem solving. Pearson's correlation coefficient obtained was 0.242 which means that there is a very weak statistical linear relationship between mathematics problem-solving approach and mathematics-related belief systems. The correlation coefficient of determination, $R^2 = 0.0586$ means that only 5.86% of the change in approach to problem solving is explained by the mathematics-related belief systems and vice versa. The researcher, also, analysed the relationship between approach to problem solving and each belief system.

Pearson's correlation coefficient of utilitarian belief system versus approach to problem solving was 0.241 which means that there is a very weak positive linear relationship between approach to problem solving and utilitarian belief system. The correlation coefficient of determination, $R^2 = 0.058$ means that only 5.8% of the change in approach to problem solving is explained by the utilitarian belief system and vice versa. Pearson's correlation coefficient of exploratory belief system versus approach to problem solving was 0.175 which means that there is a very weak positive linear relationship between approach to problem solving and exploratory belief system. The correlation coefficient of determination, $R^2 = 0.031$ means that only 3.1% of the change in approach to problem solving is explained by the exploratory belief system and vice versa. Pearson's correlation coefficient of systematic belief system versus approach to problem solving was 0.270 which means that there is a very weak positive linear relationship between approach to problem solving and systematic belief system. The correlation coefficient of determination, $R^2 = 0.073$ means that only 7.3% of approach to problem solving average score is explained by the systematic belief system and vice versa. A qualitative analysis of the relationship between the belief systems and approach to problem solving was, also, done to explain the nature of their relationship.

4. CONCLUSION

Though learners faced difficulties in solving non-routine mathematical problems, they employed several strategies to resolve the problems (see table 3.3). Grades 10, 11 and 12 learners hold mathematics related beliefs that could be classified into belief systems, namely, systematic, exploratory and utilitarian. Among the three belief systems, it was discovered that every learner held one of the belief systems stronger than the others. The predominant belief system influenced the behaviour of the learners in solving non-routine mathematical problems.

It was discovered that there is a weak positive linear relationship between high school learners' mathematics-related belief systems and their approaches to non-routine mathematical problem solving. The existence of a positive relationship might mean that a positive change of a learner's mathematics-related beliefs (i.e., development of more healthy beliefs) is likely to result in a positive improvement in use and application of problem solving strategies. A weak positive relationship might, also, mean that a relatively large change in development of positive, healthy mathematics-related belief systems among learners would result in some noticeable improvement in learner performance in mathematics (as measured by effective use and application of problem solving strategies). Though the correlation coefficient of determination (R^2) was relatively low, in general, the learners' predominant mathematics-related belief systems could explain their approach to mathematics non-routine problem solving (and vice versa). This means that one can possibly predict a learner's likely approach to a certain non-routine problem by taking stock of his/her predominant mathematics-related beliefs (and vice versa).

Similar finding was discovered by Spangler (1992), Schoenfeld (1992) and Mason (2003) who discovered that there is a positive relationship between beliefs and mathematics learning and problem solving. Schoenfeld (1992) concluded that beliefs determine the approach a learner applies to a given problem. Spangler discovered that mathematics-related beliefs and learning influence each other. Hence, Spangler (1992) described the relationship between them as cyclic. Mason (2003) concluded that learners' beliefs can be used to predict their achievement in mathematics. A contradictory finding was discovered by Goldin et al. (2009) and Callejo and Vila (2009). They discovered a complex relationship between beliefs and approach to problem solving. As a result, they could not identify if beliefs influence problem solving behaviour and vice versa.

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